Title: Quantum-spin-Hall-like phenomena and duality between order parameters in graphene

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Abstract: The quantum spin Hall effect relates seemingly unrelated degrees of freedom, i.e., charge and spin degrees of freedom. We will discuss such "duality" can be extended to much wider class of quantum numbers, and the corresponding order parameters. In particular, two valleys in graphene can be viewed as an SU(2) pseudo spin degree of freedom, which turns out to be "dual" to the charge degree of freedom, pretty much in the same way as spin in the quantum spin Hall effect is closely tied with charge. I.e., graphene can host "the quantum valley Hall effect" (QVHE). We will show that one of the best venues to observe the QVHE in graphene is actually superconductivity that can be induced in graphene by proximity effect, say, where passing supercurrent in one direction induces accumulation of pseudo spin ("valley spin") at the boundary of graphene sample. We will also discuss the "inverse QVHE" as a possible scenario to explain the highly resistive state found in N=0 Landau level in graphene in a high magnetic field.

Quantum-spin-Hall like phenomena, duality between order parameters, and graphene

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- Introduction: topological phases in condensed matter

    quantum Hall (QHE), quantum spin Hall effect (QSHE)

- topological defect in topological and semi-topological systems

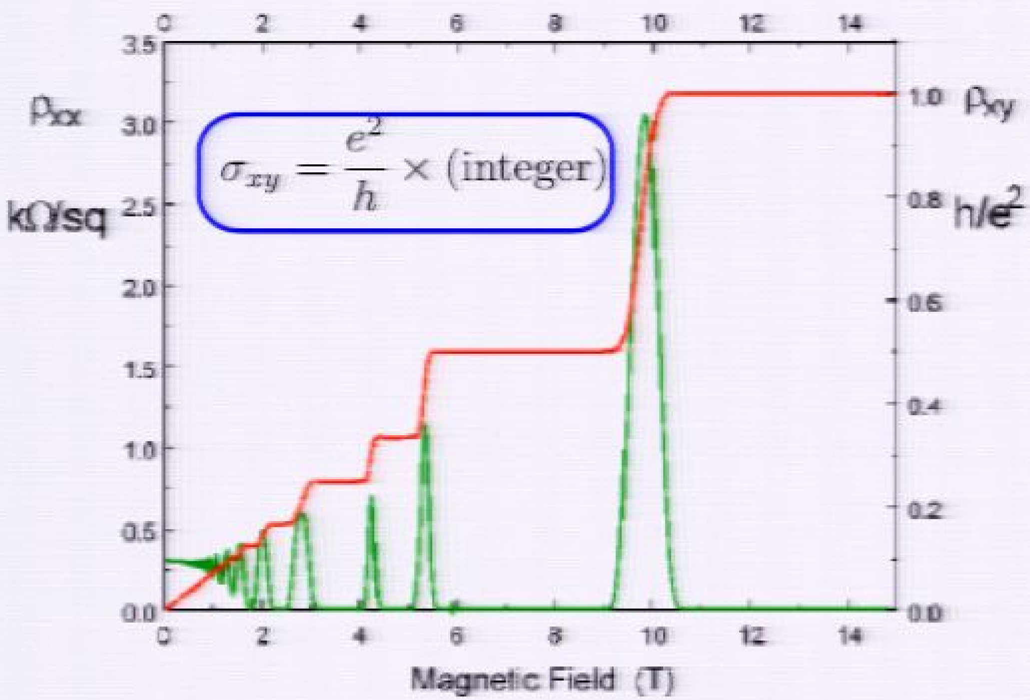
    -"dualities" between order parameters

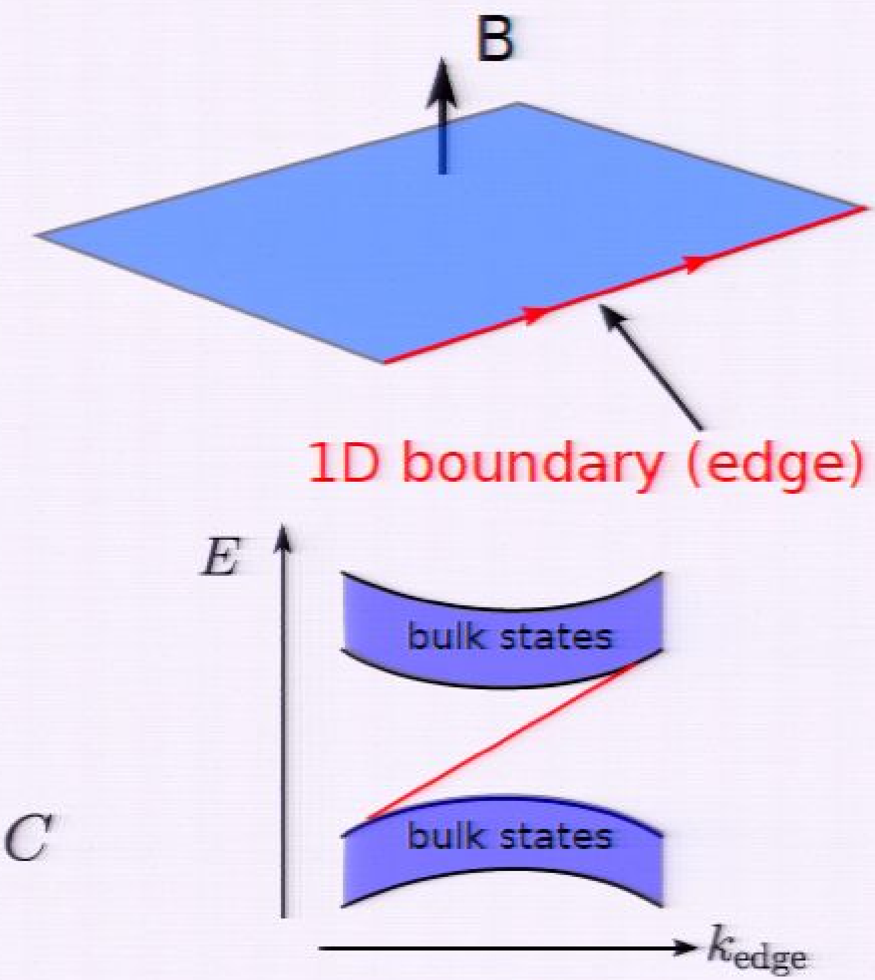
    - quantum valley Hall effect and superconducting graphene

- inverse quantum valley Hall effect in graphene in high magnetic field

integer quantum Hall effect (IQHE)

quantized transport by a gapless chiral edge mode





number of edge modes $= \frac{-\sigma_{xy}}{e^2/h} = C$

topological insulator labeled by an integer topological invariant

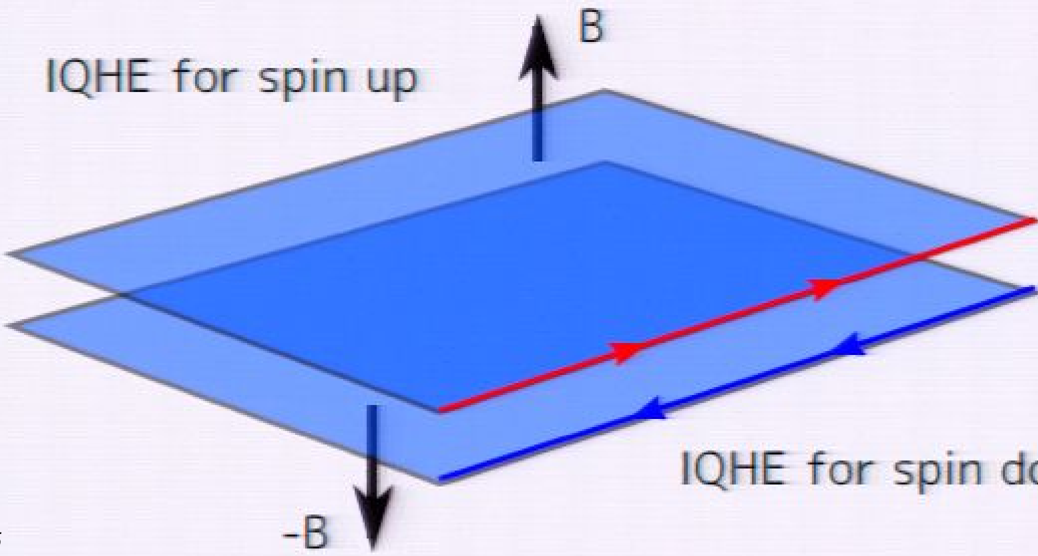
quantum spin Hall effect (QSHE)

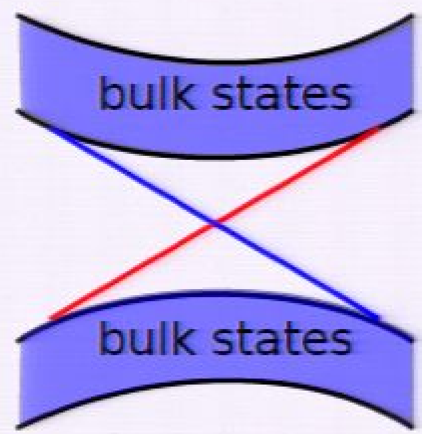
in d=2 spatial dimensions, with good T

- time-reversal invariant band insulator
- gapless Kramers pair of edge modes
- strong spin-orbit interaction

TRS

$$(i\sigma_y)\mathcal{H}^*(-i\sigma_y) = \mathcal{H}$$





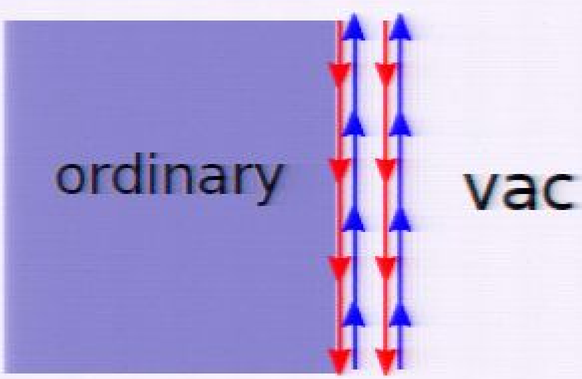
quantum spin Hall effect (QSHE)

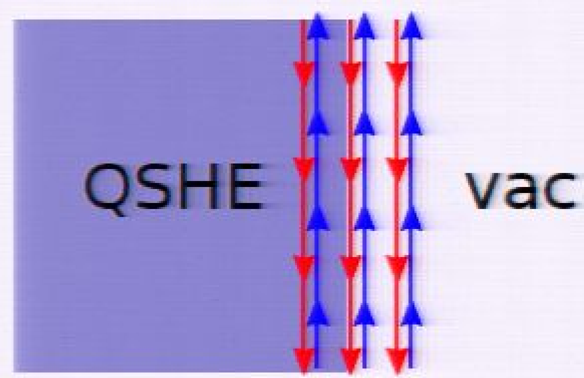
in d=2 spatial dimensions, with good T

quantum spin Hall insulator is charecterized by a binary ($\mathbb{Z}_2$) topological quantity.

- odd number of Kramers pairs at edge --> stable

  even number of Kramers pairs at edge --> unstable

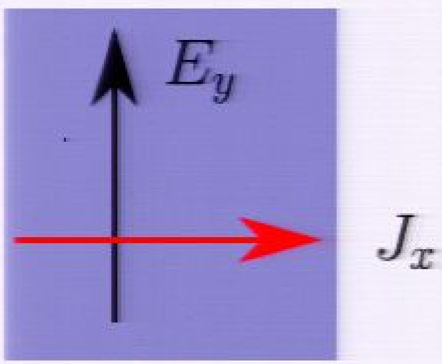




$1+1=0$ $\quad$ $3=1$

electromagnetic response in the QHE

- quantum Hall effect

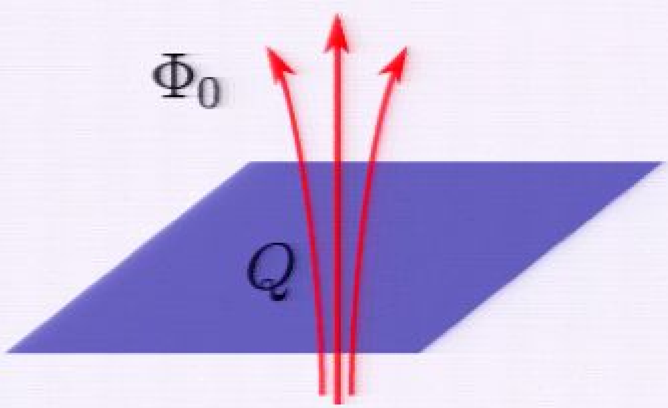


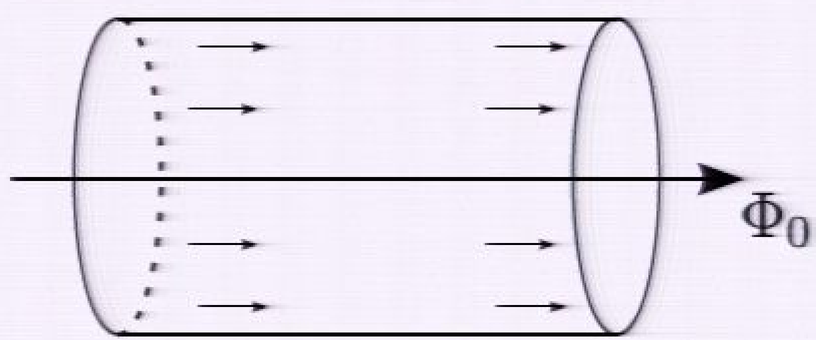
$$S_{\text{eff}} = \frac{n}{4\pi}\int d^3x\, \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

$$J_i = \frac{\delta S_{\text{eff}}}{\delta A_i} = \frac{n}{2\pi}\epsilon^{itj}\partial_t A_j$$

$$Q = \frac{\delta S_{\text{eff}}}{\delta A_t} = \frac{n}{2\pi}\epsilon^{tij}\partial_i A_j$$

- accumulation of unit charge

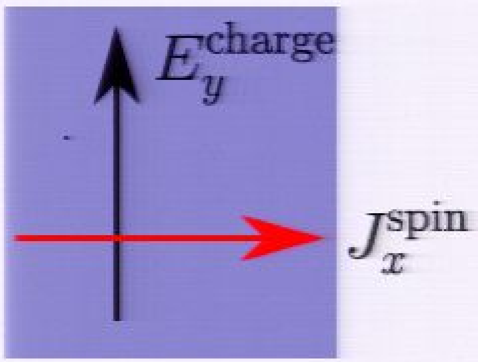




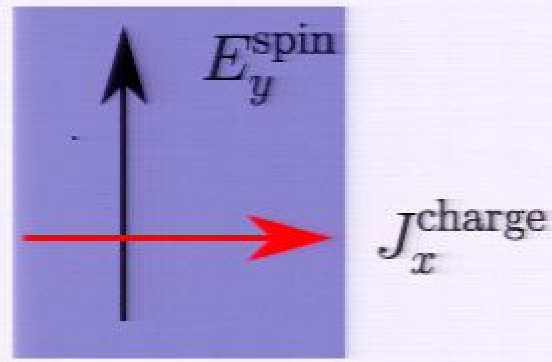
static analogue of Laughlin quasiparticle

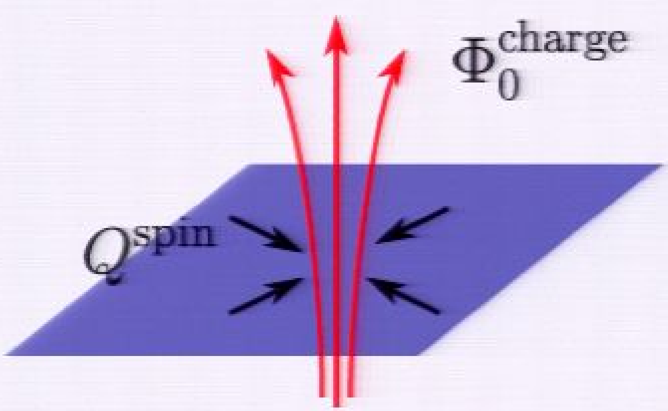
charge and spin in the quantum spin Hall effect

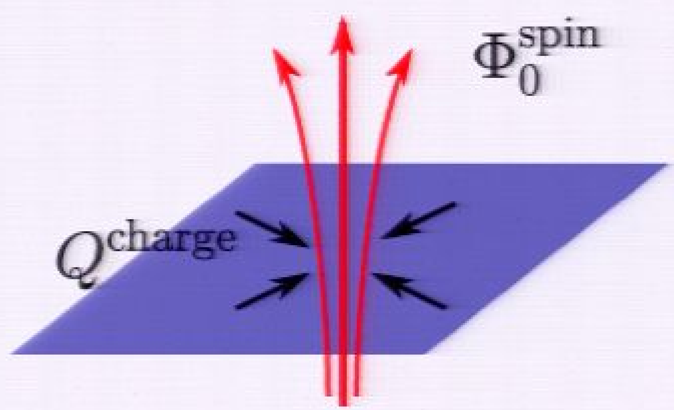
QSHE



"inverse" QSHE







N.B. spin needs not to be conserved

Qi-Zhang, Ran-Vishwanath-Lee

questions

- why charge and spin like each other ?

  valleys, orbitals, etc.

- how does the physics of deffects show up in topological and semi-topological systems ?

This talk:

- exhaustive study of "dualities" between order parameters on the honeycomb lattice
- link to experiments : superconducting graphene,
  N=0 Landau level in graphene

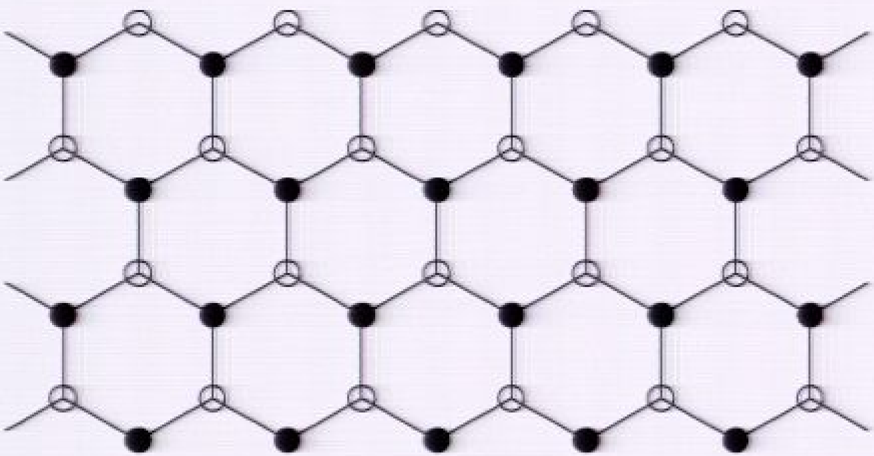
graphene kinetic term

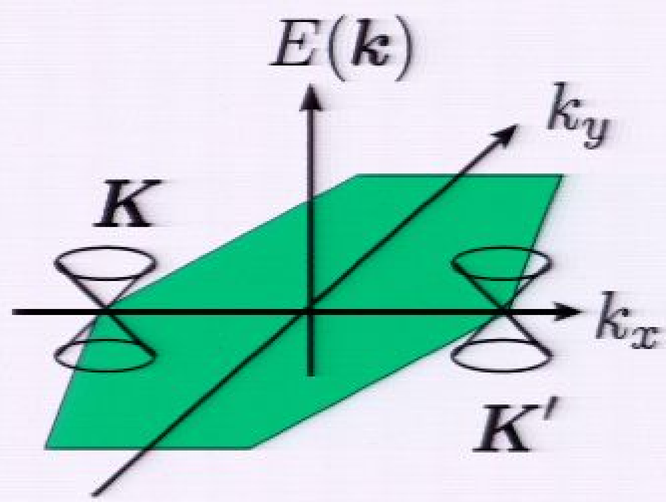
$$\mathcal{H}_0 = \sum_{s=\uparrow,\downarrow} \psi_s^\dagger \begin{pmatrix} \tau_x k_x + \tau_y k_y & 0 \\ 0 & -\tau_x k_x - \tau_y k_y \end{pmatrix} \psi_s$$

graphene kinetic term

$\boldsymbol{K}$ $\quad$ $\boldsymbol{K}'$

4x4x (spin)





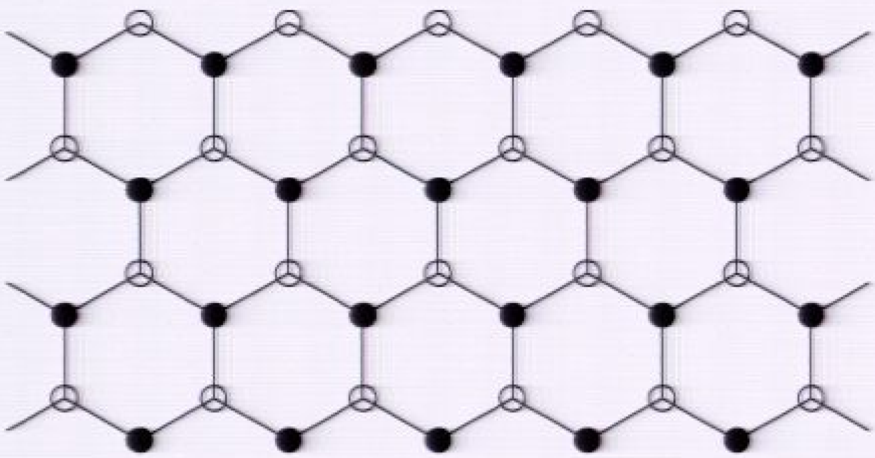
graphene kinetic term + mass

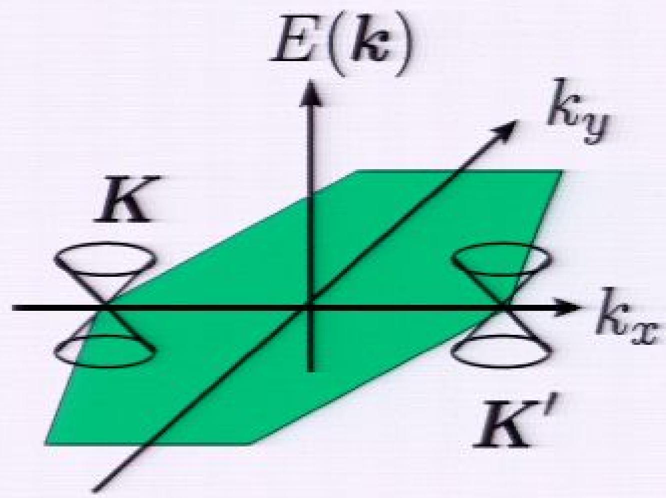
$$\mathcal{H}_0 = \sum_{s=\uparrow,\downarrow} \psi_s^\dagger \begin{pmatrix} \tau_x k_x + \tau_y k_y & 0 \\ 0 & -\tau_x k_x - \tau_y k_y \end{pmatrix} \psi_s$$

graphene kinetic term

$\boldsymbol{K}$ $\boldsymbol{K'}$

4x4x (spin)





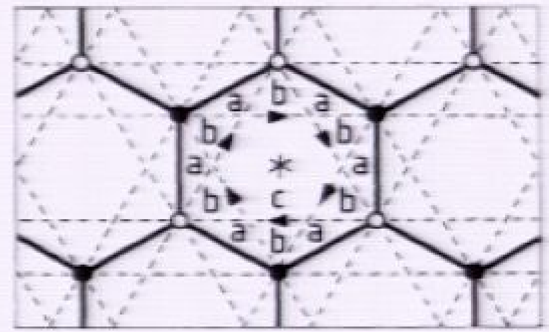
To generate a gap, perturb H_0 by "mass terms":

$$\mathcal{H}_0 \to \mathcal{H}_0 + \mathcal{H}_I \qquad \mathcal{H}_I \sim \Psi^\dagger M \Psi$$

"mass matrix"

example of masses

quantum Hall effect: QHE Haldane

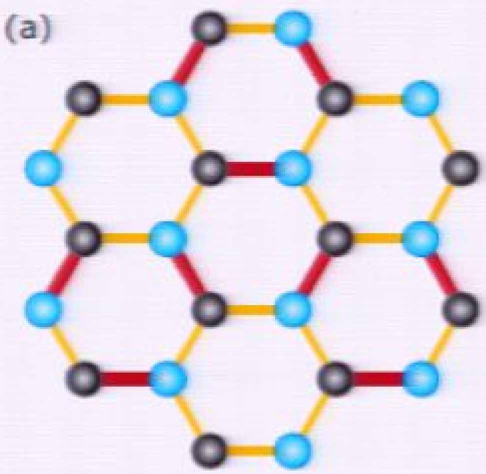


quantum spin Hall effect: $\mathrm{QSHE}_{x,y,z}$

Kane-Mele

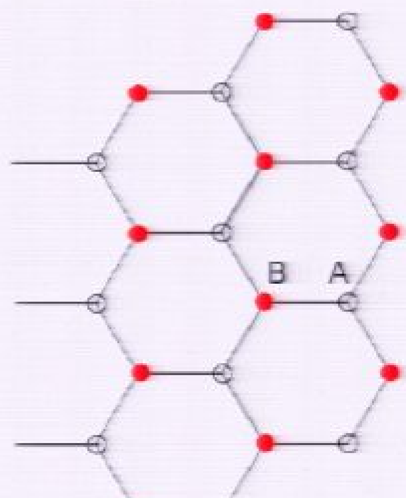
valence bond solid: $\mathrm{Re\,VBS}, \quad \mathrm{Im\,VBS}$

(a)



charge density wave: CDW

spin density wave: $\mathrm{Neel}_{x,y,z}$

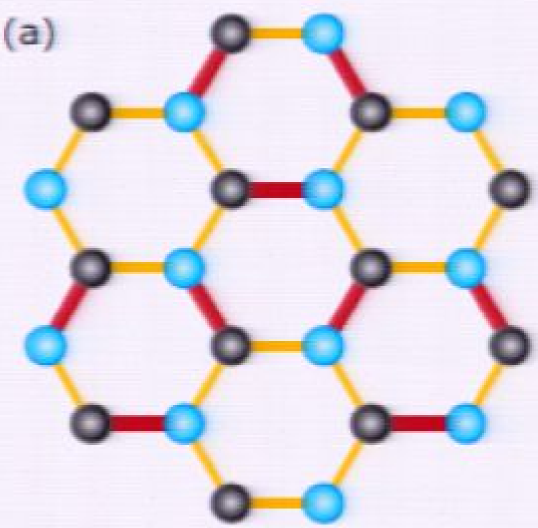


more masses (order parameters with full gap) ...

$\{\mathcal{H}_0(\boldsymbol{k}), M_*\} = 0$ --> full gap in energy spectrum   36 in total

quantum spin Hall effect: $\mathrm{QSHE}_{x,y,z}$ Kane-Mele

quantum Hall effect: QHE Haldane



singlet superconductor (SSC): $\mathrm{Re\,SSC}, \quad \mathrm{Im\,SSC}$

spin density wave: $\mathrm{Neel}_{x,y,z}$

valence bond solid: $\mathrm{Re\,VBS}, \quad \mathrm{Im\,VBS}$

charge density wave: CDW

n.n. spin-orbit coupling: $\mathrm{Re\,VBS}_{x,y,z} \quad \mathrm{Im\,VBS}_{x,y,z}$

triplet superconductors (TSC): $\mathrm{Re\,TSC}_{x,y,z}, \quad \mathrm{Im\,TSC}_{x,y,z}$

$\mathrm{Re\,TSC}_{02,x,y,z}, \quad \mathrm{Im\,TSC}_{02,x,y,z} \quad \mathrm{Re\,TSC}_{32,x,y,z}, \quad \mathrm{Im\,TSC}_{32,x,y,z}$

- Take any three masses $m_1 M_1 + m_2 M_2 + m_3 M_3$

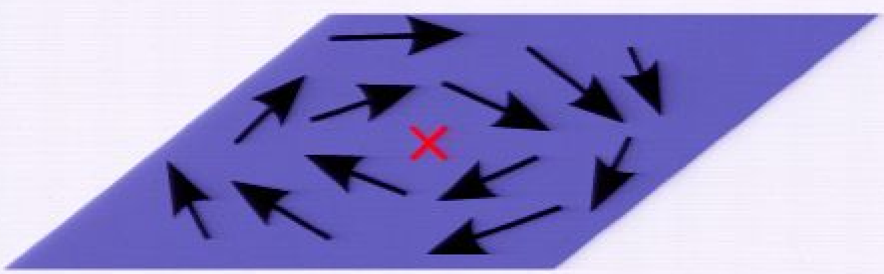
E.g. $\mathcal{H}_I = \Psi^\dagger \left(\lambda_x \hat{M}_{\mathrm{QSHE}_x} + \lambda_y \hat{M}_{\mathrm{QSHE}_y} + \lambda_z \hat{M}_{\mathrm{QSHE}_z} \right) \Psi$

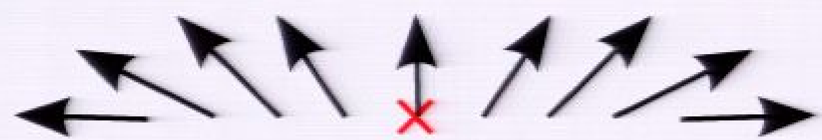
order parameter

"mass matrix"

order parameters
or "mass terms"

- Make a texture ("hedgehog defect") in masses: $\vec{m}(x) = \begin{pmatrix} m_1(x) \\ m_2(x) \\ m_3(x) \end{pmatrix}$





- When does the defect accumulate quantum number ("charge") of some sort ?

If it does, which type ?

Answer:

Find two more masses M_4, M_5

such that: $\{M_a, M_b\} = \delta_{ab}, \quad a, b = 1, \ldots, 5$

--> "charge" accumulation at the core of hedgehog

type of charge = U(1) rotating M_4 and M_5

Example:

hedgehog in $\mathrm{QSHE}_{x,y,z}$ binds charge of $\mathrm{Re}\,\Delta$ and $\mathrm{Im}\,\Delta$

= electric chage

Derivation:

non-linear sigma model field thoery with WZW term

along the line of Abanov-Wiegmann

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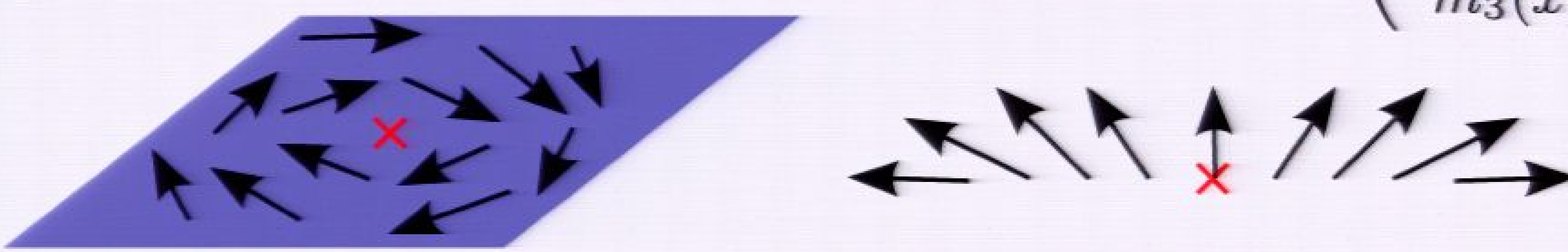

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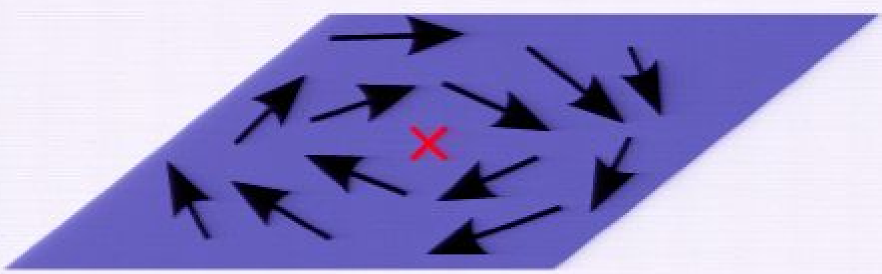

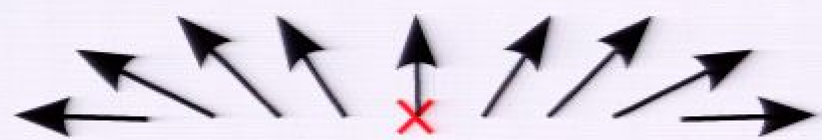

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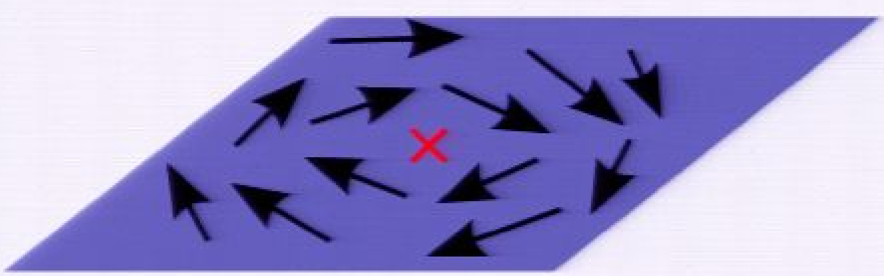

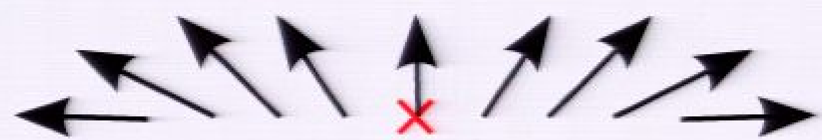

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non-linear sigma model field thoery with WZW term

along the line of Abanov-Wiegmann

systematic search for 5-tuplet

TABLE III. Enumeration of the 56 distinct five-tuplets of maximally pairwise anticommuting PHS $X_{\mu_1\mu_2\mu_3\mu_4}$. The 56 five-tuplets are broken into 28 pairs related by the operation of C conjugation (9.12).

Five tuplet	Partner five-tuplet by C conjugation
{Re VBS, Im VBS, Re SSC, Im SSC, CDW}	{Re VBS, Im VBS, Néel$_x$, Néel$_y$, Néel$_z$}
{Im VBS, CDW, Re VBS$_x$, Re VBS$_y$, Re VBS$_z$}	{Im VBS, Néel$_z$, Im TSC$_{32z}$, Re TSC$_{32z}$, Re VBS$_z$}
{Re VBS, CDW, Im VBS$_x$, Im VBS$_y$, Im VBS$_z$}	{Re VBS, Néel$_z$, Re TSC$_{02z}$, Im TSC$_{02z}$, Im VBS$_z$}
{Re SSC, Im SSC, QSHE$_x$, QSHE$_y$, QSHE$_z$}	{Néel$_x$, Néel$_y$, Im TSC$_z$, Re TSC$_z$, QSHE$_z$}
{Re VBS, Re SSC, Re TSC$_{02x}$, Im TSC$_{02y}$, Re TSC$_{02z}$}	{Re VBS, Néel$_x$, Re TSC$_{02x}$, Im TSC$_{02x}$, Im VBS$_x$}
{Re VBS, Im SSC, Im TSC$_{02x}$, Re TSC$_{02y}$, Im TSC$_{02z}$}	{Re VBS, Néel$_y$, Im TSC$_{02y}$, Re TSC$_{02y}$, Im VBS$_y$}
{Im VBS, Im SSC, Re TSC$_{32x}$, Im TSC$_{32y}$, Re TSC$_{32z}$}	{Im VBS, Néel$_y$, Re TSC$_{32y}$, Im TSC$_{32y}$, Re VBS$_y$}
{Im VBS, Re SSC, Im TSC$_{32x}$, Re TSC$_{32y}$, Im TSC$_{32z}$}	{Im VBS, Néel$_x$, Im TSC$_{32x}$, Re TSC$_{32x}$, Re VBS$_x$}
{CDW, Im SSC, Im TSC$_x$, Re TSC$_y$, Im TSC$_z$}	{Néel$_z$, Néel$_y$, Im TSC$_x$, Re TSC$_x$, QSHE$_x$}
{CDW, Re SSC, Re TSC$_x$, Im TSC$_y$, Re TSC$_z$}	{Néel$_z$, Néel$_x$, Re TSC$_y$, Im TSC$_y$, QSHE$_y$}
{Im VBS$_x$, QSHE$_y$, Im VBS$_z$, Re TSC$_{32y}$, Im TSC$_{32y}$}	{Re TSC$_{02z}$, Re TSC$_z$, Im VBS$_z$, Re TSC$_{32x}$, Im TSC$_{32y}$}
{Im VBS$_x$, QSHE$_y$, Re VBS$_x$, Néel$_x$, QSHE$_z$}	{Re TSC$_{02z}$, Re TSC$_z$, Im TSC$_{32z}$, Re SSC, QSHE$_z$}
{Im VBS$_x$, Re TSC$_{32y}$, Im TSC$_{32z}$, Im TSC$_{02x}$, Im TSC$_x$}	{Re TSC$_{02z}$, Re TSC$_{32x}$, Re VBS$_x$, Im TSC$_{02y}$, Im TSC$_x$}
{Im VBS$_x$, Re TSC$_{32z}$, Re TSC$_{02x}$, Re TSC$_x$, Im TSC$_{32y}$}	{Re TSC$_{02z}$, Re VBS$_y$, Re TSC$_{02x}$, Re TSC$_y$, Im TSC$_{32y}$}
{Im VBS$_x$, Re TSC$_{32z}$, Im TSC$_{32z}$, Im VBS$_y$, QSHE$_z$}	{Re TSC$_{02z}$, Re VBS$_y$, Re VBS$_x$, Im TSC$_{02z}$, QSHE$_z$}
{Im VBS$_x$, Re TSC$_x$, Im TSC$_x$, CDW, Re VBS$_x$}	{Re TSC$_{02z}$, Re TSC$_y$, Im TSC$_x$, Néel$_z$, Im TSC$_{32z}$}
{QSHE$_y$, Im VBS$_z$, QSHE$_x$, Re VBS$_z$, Néel$_z$}	{Re TSC$_z$, Im VBS$_z$, Im TSC$_z$, Re VBS$_z$, CDW}
{QSHE$_y$, Re TSC$_{02y}$, Re TSC$_y$, Im SSC, Im TSC$_{32y}$}	{Re TSC$_z$, Re TSC$_{02y}$, Re TSC$_x$, Néel$_y$, Im TSC[illegible]}
{QSHE$_y$, Re TSC$_{02y}$, Im TSC$_{02y}$, Re VBS$_x$, Re VBS$_z$}	{Re TSC$_z$, Re TSC$_{02y}$, Im TSC$_{02x}$, Im TSC$_{32z}$, Re VBS$_z$}

Out of $\binom{36}{5} = 376992$ possibilies, there are 56 5-tuplets

there are 560 triplets, 280 4-tuplets

there are no N-tuplets for N > 5

Examples:

$$\{\mathrm{Re\,VBS}, \mathrm{Im\,VBS}, \mathrm{CDW}, \mathrm{Re\,SSC}, \mathrm{Im\,SSC}\}$$

$$\{\mathrm{Re\,VBS}, \mathrm{Im\,VBS}, \mathrm{Neel}_x, \mathrm{Neel}_y, \mathrm{Neel}_z\}$$ Tanaka-Hu

$$\{\mathrm{Re\,SSC}, \mathrm{Im\,SSC}, \mathrm{QSHE}_x, \mathrm{QSHE}_y, \mathrm{QSHE}_z\}$$ Grover-Senthil

$$\{\mathrm{Neel}_x, \mathrm{Neel}_y, \mathrm{Im\,TSC}_z, \mathrm{Re\,TSC}_z, \mathrm{QSHE}_z\}$$

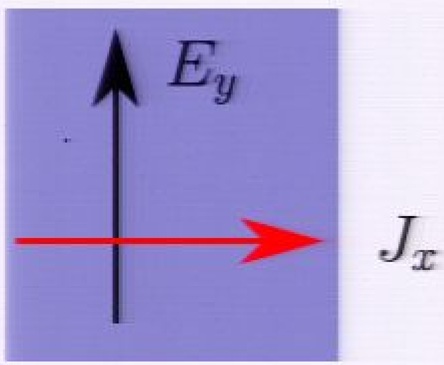
....

IQHE mass term commutes with all the other

QHE, Chern-Simons action

$$S_{\text{eff}} = \frac{n}{4\pi} \int d^3x \, \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

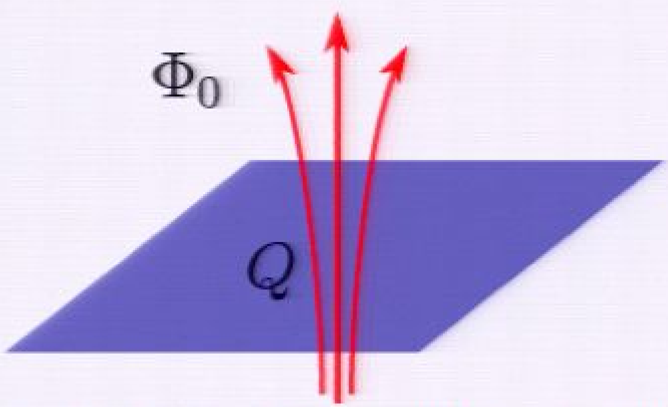
- quantum Hall effect

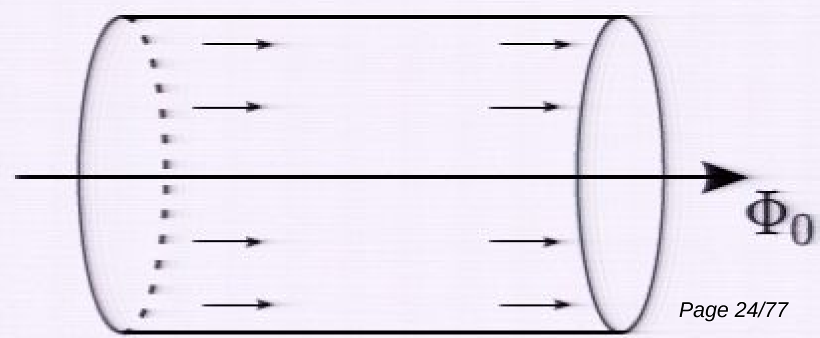


$$J_i = \frac{\delta S_{\text{eff}}}{\delta A_i} = \frac{n}{2\pi} \epsilon^{itj} \partial_t A_j$$

- accumulation of unit charge

$$Q = \frac{\delta S_{\text{eff}}}{\delta A_t} = \frac{n}{2\pi} \epsilon^{tij} \partial_i A_j$$





static analogue of Laughlin quasiparticle

classification of topological insulators and superconductors

- Schnyder, SR, Furusaki, Ludwig (08)

AZ\d	0	1	2	3	4	5	6	7	8	9
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	$\cdots$
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\cdots$

AZ\d	0	1	2	3	4	5	6	7	8	9
AI	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	$\cdots$
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\cdots$
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\cdots$
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\cdots$
AII	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	$\cdots$
CII	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\cdots$
C	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	$\cdots$
CI	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	$\cdots$

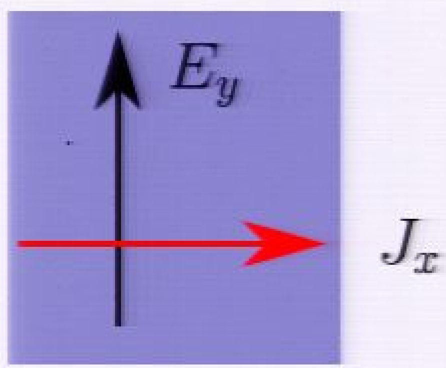
- Kitaev (all d and periodicity "Periodic Table", 2009)

- SR and Takayanagi (construction by D-branes, 2010)

QHE, Chern-Simons action

$$S_{\text{eff}} = \frac{n}{4\pi} \int d^3x \, \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

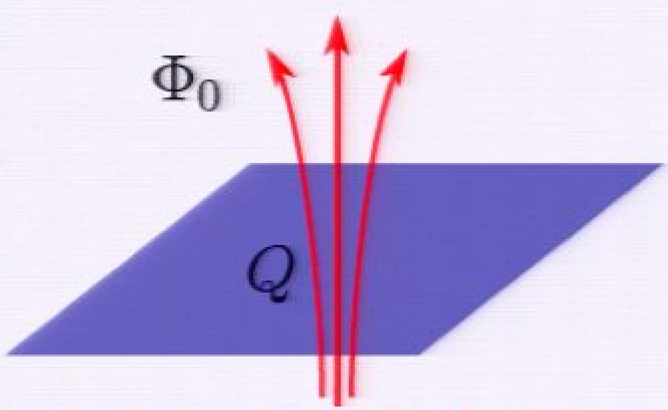
- quantum Hall effect

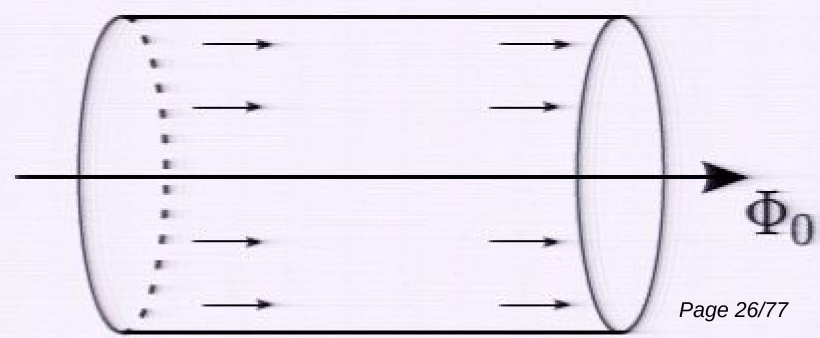


$$J_i = \frac{\delta S_{\text{eff}}}{\delta A_i} = \frac{n}{2\pi} \epsilon^{itj} \partial_t A_j$$

- accumulation of unit charge

$$Q = \frac{\delta S_{\text{eff}}}{\delta A_t} = \frac{n}{2\pi} \epsilon^{tij} \partial_i A_j$$





static analogue of Laughlin quasiparticle

double Chern-Simons effective action

$$\mathcal{L}^{\text{eff}}_{m,\eta=0} = \frac{m}{2\pi}\left(b'^{1\rho} b'^{1}_{\rho} + b'^{2\rho} b'^{2}_{\rho}\right) + \frac{\operatorname{sgn}\mu_{\rm s}}{2\pi}\,\epsilon^{\nu\rho\kappa} b'^{0}_{\nu}\partial_{\rho} b'^{3}_{\kappa} + \ldots .$$

$$n_1 := \frac{|\Delta|\cos\theta}{m}, \quad n_2 := -\frac{|\Delta|\sin\theta}{m}, \quad n_3 := \frac{\mu_{\rm s}}{m}$$

"charge" gauge field

$$b'^{0}_{\mu} = -a_{\mu},$$

$$b'^{1}_{\mu} = -\sin\alpha\cos\theta\left(a_{5\mu} - \frac{1}{2}\partial_{\mu}\theta\right),$$

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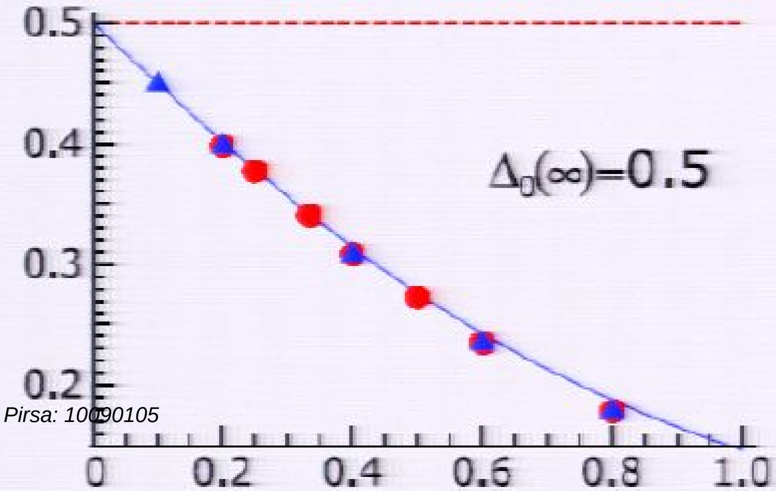
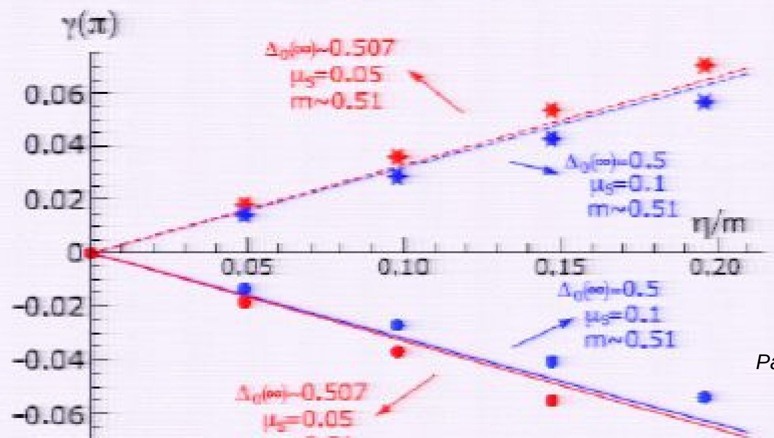
"spin" gauge field

fractional "charge"



fractional Berry phase



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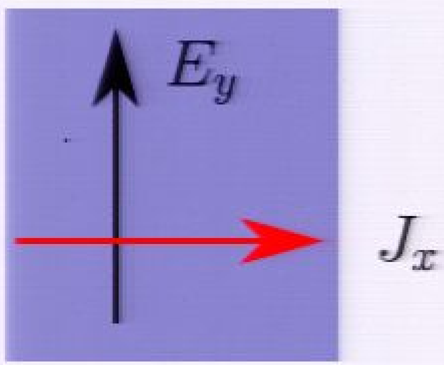

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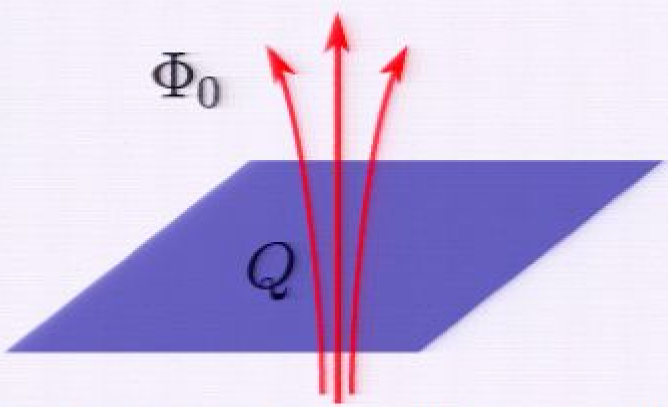

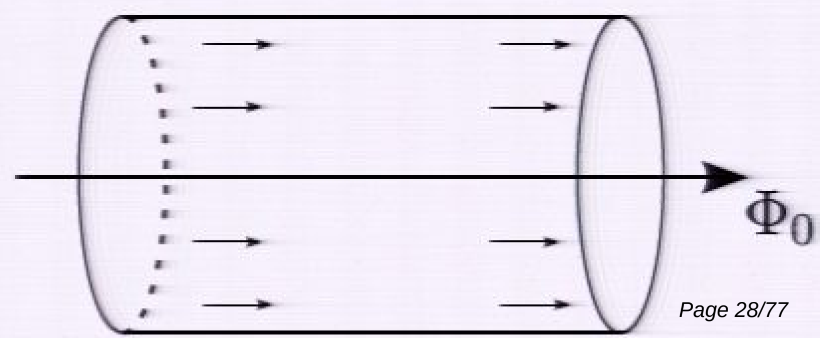

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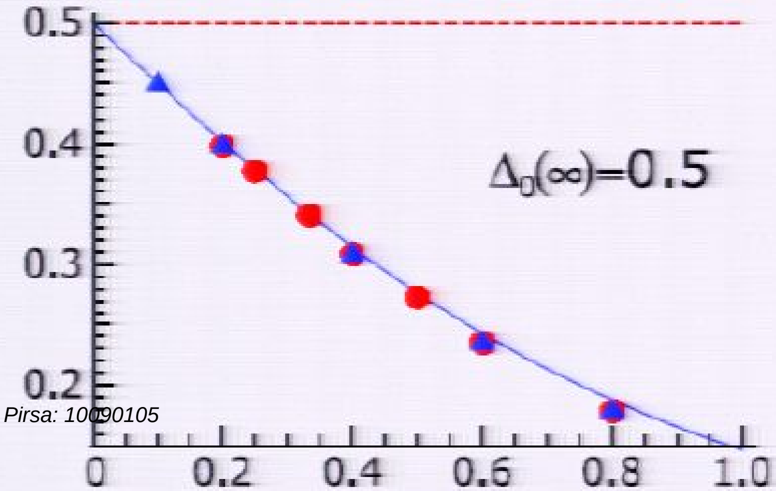
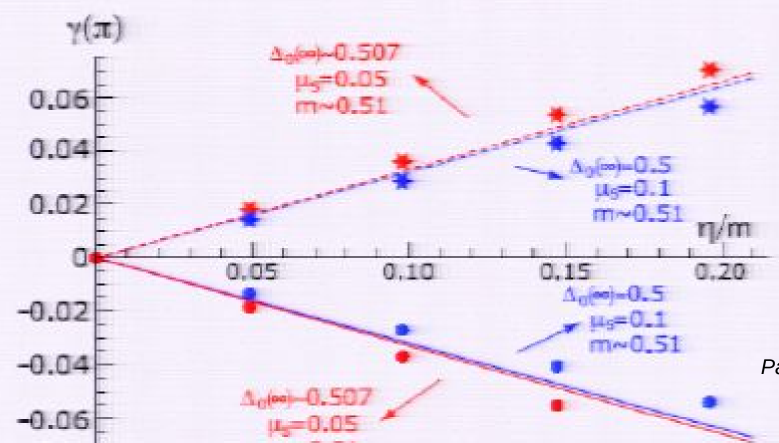
"spin" gauge field

fractional "charge"



fractional Berry phase



- Take any three masses $m_1 M_1 + m_2 M_2 + m_3 M_3$

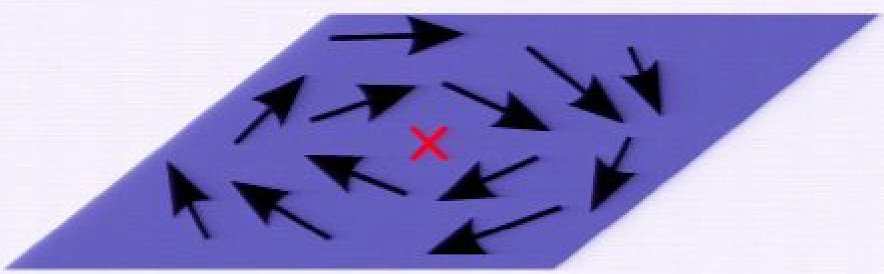
E.g. $\mathcal{H}_I = \Psi^\dagger \left(\lambda_x \hat{M}_{\mathrm{QSHE}_x} + \lambda_y \hat{M}_{\mathrm{QSHE}_y} + \lambda_z \hat{M}_{\mathrm{QSHE}_z} \right) \Psi$

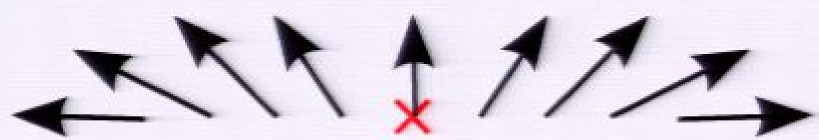
order parameter

"mass matrix"

order parameters
or "mass terms"

- Make a texture ("hedgehog defect") in masses: $\vec{m}(x) = \begin{pmatrix} m_1(x) \\ m_2(x) \\ m_3(x) \end{pmatrix}$





- When does the defect accumulate quantum number ("charge") of some sort ?

If it does, which type ?

Out of $\binom{36}{5}$ = 376992 possibilies, there are 56 5-tuplets

there are 560 triplets, 280 4-tuplets

there are no N-tuplets for N > 5

Examples:

$\{\mathrm{Re\,VBS}, \mathrm{Im\,VBS}, \mathrm{CDW}, \mathrm{Re\,SSC}, \mathrm{Im\,SSC}\}$

$\{\mathrm{Re\,VBS}, \mathrm{Im\,VBS}, \mathrm{Neel}_x, \mathrm{Neel}_y, \mathrm{Neel}_z\}$ Tanaka-Hu

$\{\mathrm{Re\,SSC}, \mathrm{Im\,SSC}, \mathrm{QSHE}_x, \mathrm{QSHE}_y, \mathrm{QSHE}_z\}$ Grover-Senthil

$\{\mathrm{Neel}_x, \mathrm{Neel}_y, \mathrm{Im\,TSC}_z, \mathrm{Re\,TSC}_z, \mathrm{QSHE}_z\}$

....

IQHE mass term commutes with all the other

double Chern-Simons effective action

$$\mathcal{L}^{\text{eff}}_{m,\eta=0} = \frac{m}{2\pi}\left(b'^{1\rho} b'^{1}_{\rho} + b'^{2\rho} b'^{2}_{\rho}\right) + \frac{\operatorname{sgn}\mu_s}{2\pi}\,\epsilon^{\nu\rho\kappa} b'^{0}_{\nu}\partial_\rho b'^{3}_{\kappa} + \ldots$$

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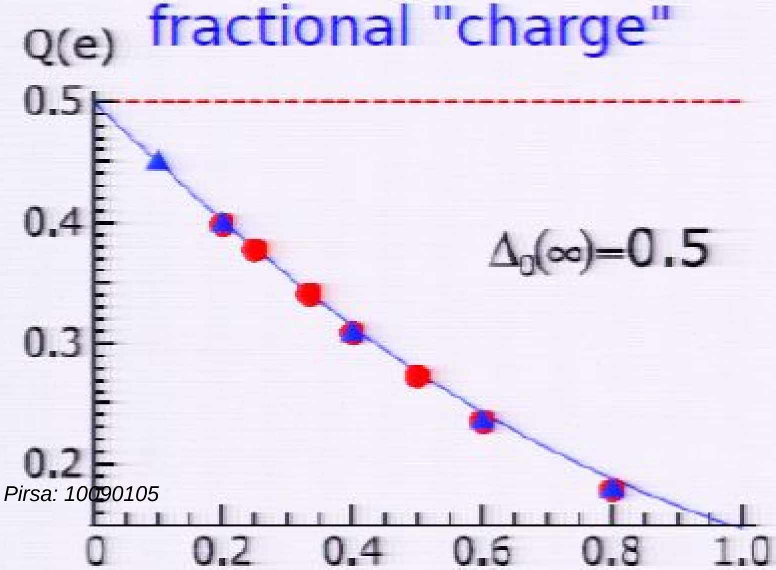
$$b'^{3}_{\mu} = +\left(\cos\alpha\, a_{5\mu} + \frac{1-\cos\alpha}{2}\partial_\mu\theta\right),$$

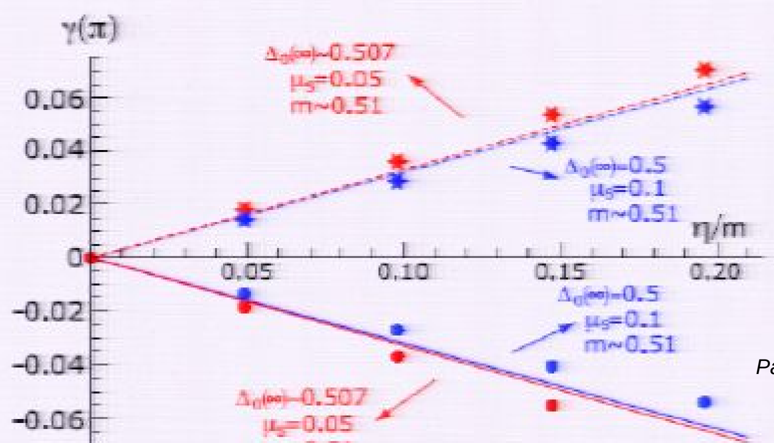
"charge" gauge field (b'^{0}_{μ})

"spin" gauge field (b'^{3}_{μ})

fractional "charge"



fractional Berry phase



classification of topological insulators and superconductors

- Schnyder, SR, Furusaki, Ludwig (08)

AZ\d	0	1	2	3	4	5	6	7	8	9
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	$\cdots$
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\cdots$

AZ\d	0	1	2	3	4	5	6	7	8	9
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BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\cdots$
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\cdots$
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\cdots$
AII	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	$\cdots$
CII	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\cdots$
C	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	$\cdots$
CI	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	$\cdots$

- Kitaev (all d and periodicity "Periodic Table", 2009)

- SR and Takayanagi (construction by D-branes, 2010)

classification of topological insulators and superconductors

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spatial dimensions →

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AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\cdots$

AZ\d	0	1	2	3	4	5	6	7	8	9
AI	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	$\cdots$
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\cdots$
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\cdots$
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\cdots$
AII	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	$\cdots$
CII	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\cdots$
C	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	$\cdots$
CI	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	$\cdots$

symmetry class of fermionic Hamiltonians

$\mathbb{Z}$ integer classification

$\mathbb{Z}_2$ binary classification

0 no top. ins./SC

- Kitaev (all d and periodicity "Periodic Table", 2009)

topological insulators/SCs in three-dimensions

AZ\d	1	2	3
A	0	$\mathbb{Z}$	0
AIII	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	0	0	0
BDI	$\mathbb{Z}$	0	0
D	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	$\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	$\mathbb{Z}$	0
CI	0	0	$\mathbb{Z}$

IQHE

p+ip wave SC

polyacetylene

3He B phase

TMTSF

Z2 topological insulator

QSHE

d+id wave SC

Dualities in three-dimensions

Pavan Hosur, SR, Ashvin Vishwanath

$\{\text{VBS}_x, \text{VBS}_y, \text{VBS}_z, \text{Neel}_x, \text{Neel}_y, \text{Neel}_z\}$

$\{\text{VBS}_x, \text{VBS}_y, \text{VBS}_z, \text{CDW}, \text{Re}\, \text{s-wave}\, \text{SC}, \text{Im}, \text{s-wave}\, \text{SC}\}$

$\{\text{CII}_x, \text{CII}_y, \text{CII}_z, \text{CDW}, \text{Re}\, \text{s-wave}\, \text{SC}, \text{Im}, \text{s-wave}\, \text{SC}\}$

Z2 topological insulator

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"Z" topological insulator

topological singlet supercondcutor

core of a hedgehog defect in magnetic order
looks like a topological insulator/superconductor !

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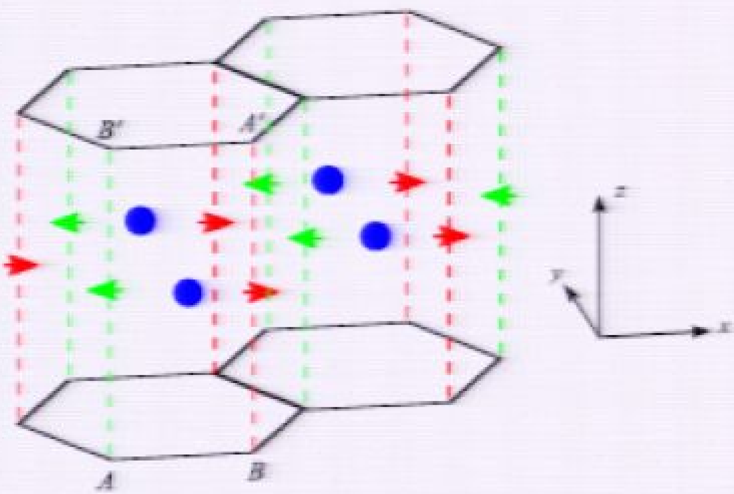
AI	0	0	0
BDI	$\mathbb{Z}$	0	0
D	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	$\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	$\mathbb{Z}$	0
CI	0	0	$\mathbb{Z}$

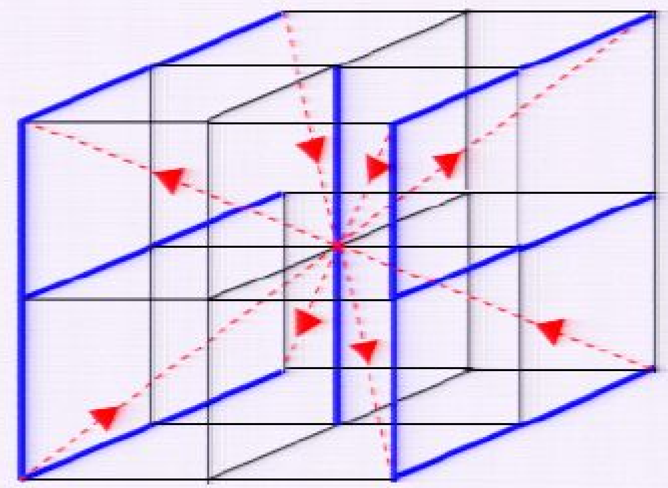
- IQHE (A, $d=2$)
- p+ip wave SC (D, $d=2$)
- polyacetylene (BDI, $d=1$)
- 3He B phase (DIII, $d=3$)
- TMTSF (DIII, $d=1$)
- Z2 topological insulator (AII, $d=3$)
- QSHE (AII, $d=2$)
- d+id wave SC (C, $d=2$)

Dirac models in three-dimensions

layered honeycomb model



cubic lattice with pi flux



3D Dirac kinetic term

$$\mathcal{H}_0 \;=\; \Psi^\dagger\Big(k_x\Gamma_1 + k_y\Gamma_2 + k_z\Gamma_3\Big)\Psi$$

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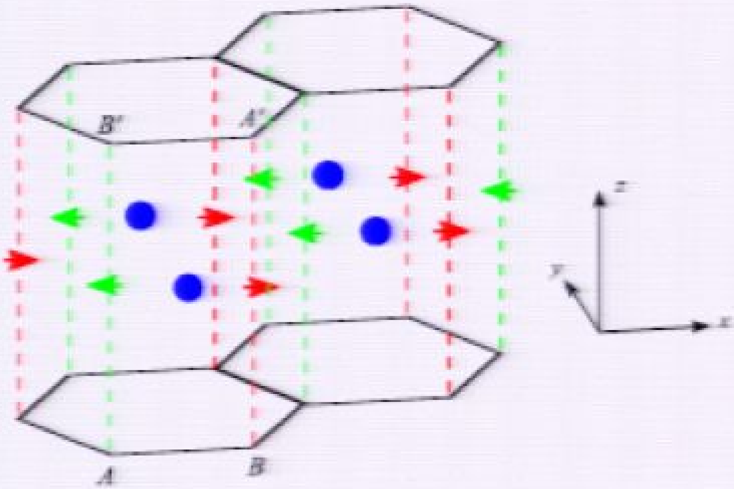

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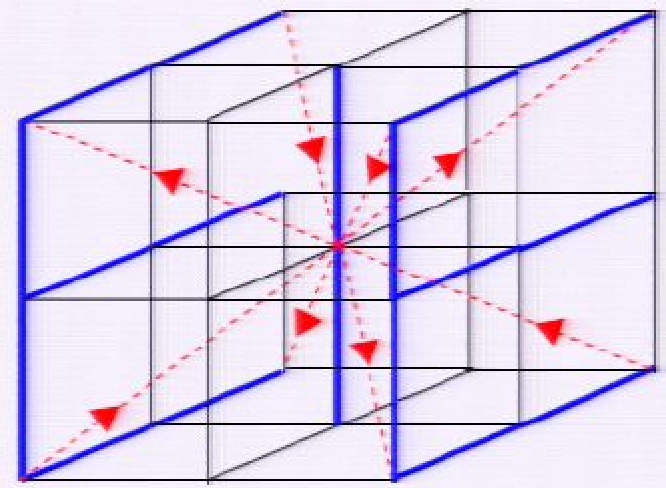

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p+ip wave SC
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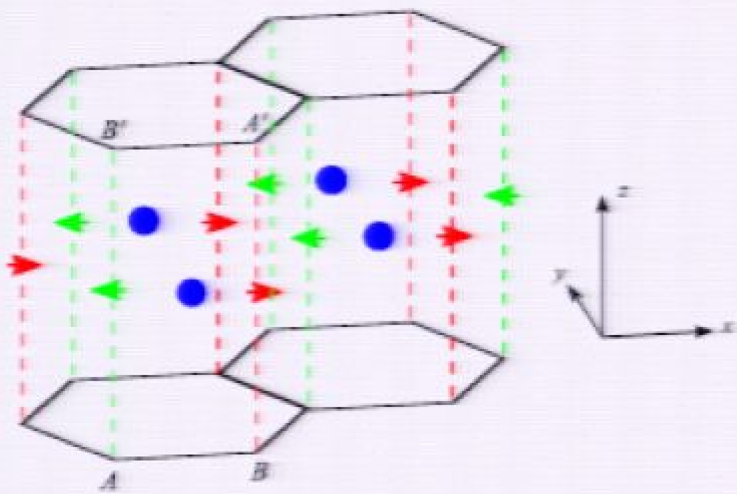

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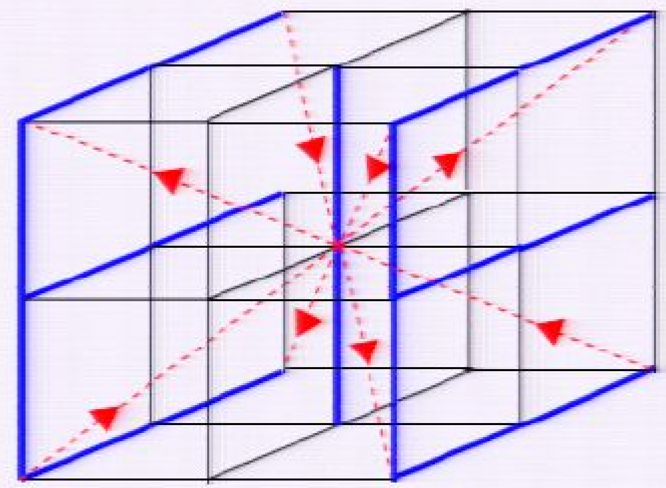

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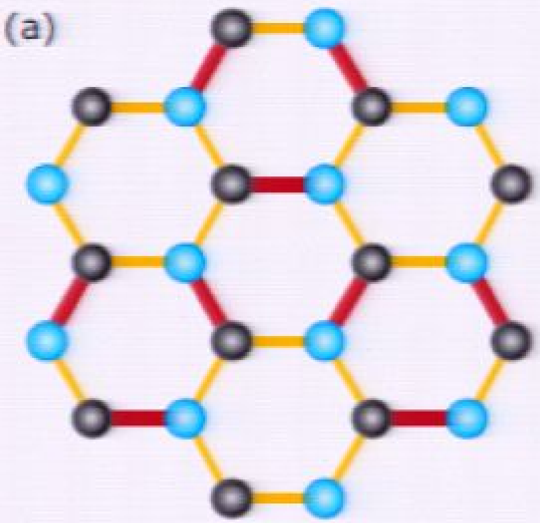
superconducting graphene

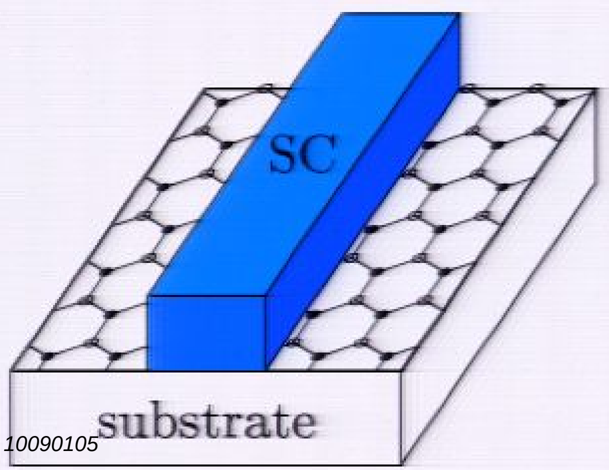
$$\{\mathrm{Re\,VBS}, \mathrm{Im\,VBS}, \mathrm{CDW}, \mathrm{Re\,SSC}, \mathrm{Im\,SSC}\}$$

valence bond solid

s-wave superconductivity

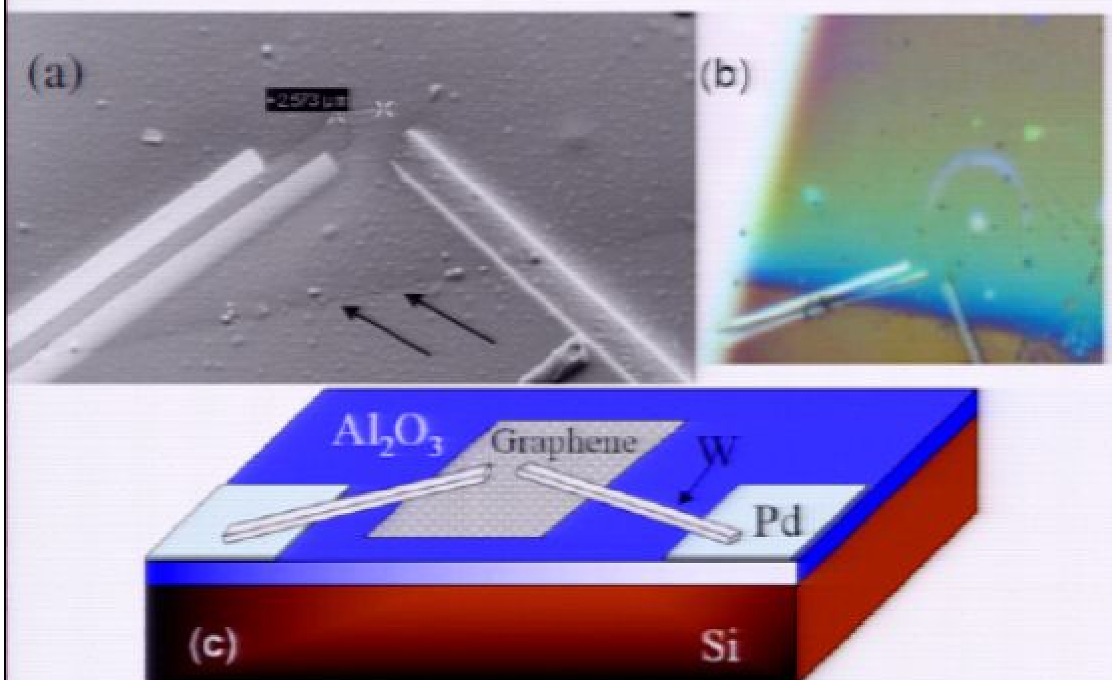
(a)

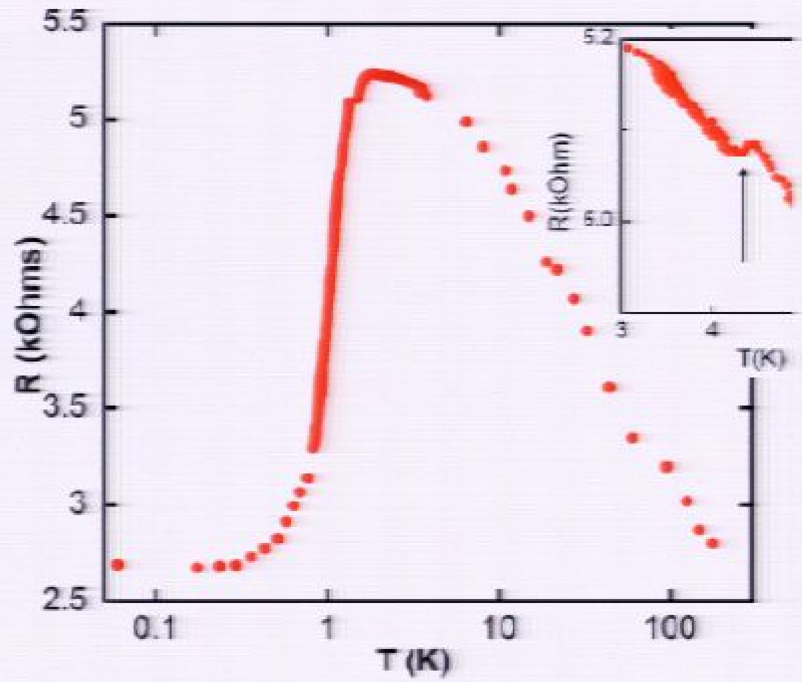




superconducting proximity in graphene:

- Heersche et al. Nature 446, 56 (2007)
- Shailos et al. Euro. Phys. Lett. 79, 57008 (2007)
- UC Berkeley, Zettel group


(a)
(b)
Al_2O_3
Graphene
W
Pd
(c)
Si

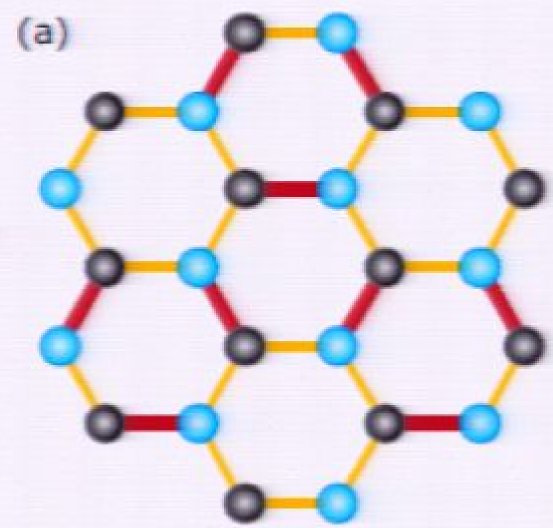

R (kOhms)
5.5
5
4.5
4
3.5
3
2.5
0.1
1
10
100
T (K)
R(kOhm)
5.2
5.0
3
4
T(K)

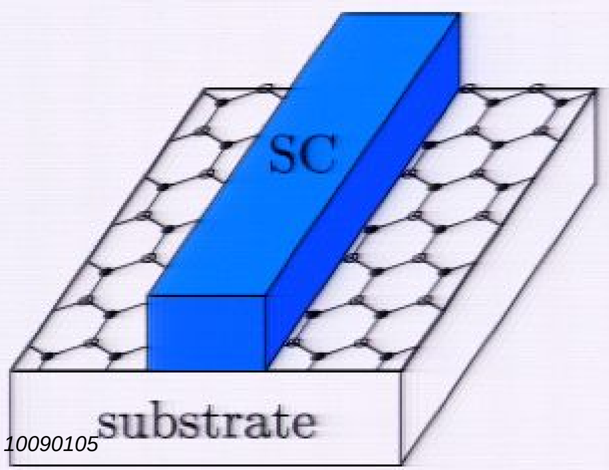
superconducting graphene

$$\{\mathrm{Re\,VBS}, \mathrm{Im\,VBS}, \mathrm{CDW}, \mathrm{Re\,SSC}, \mathrm{Im\,SSC}\}$$

valence bond solid

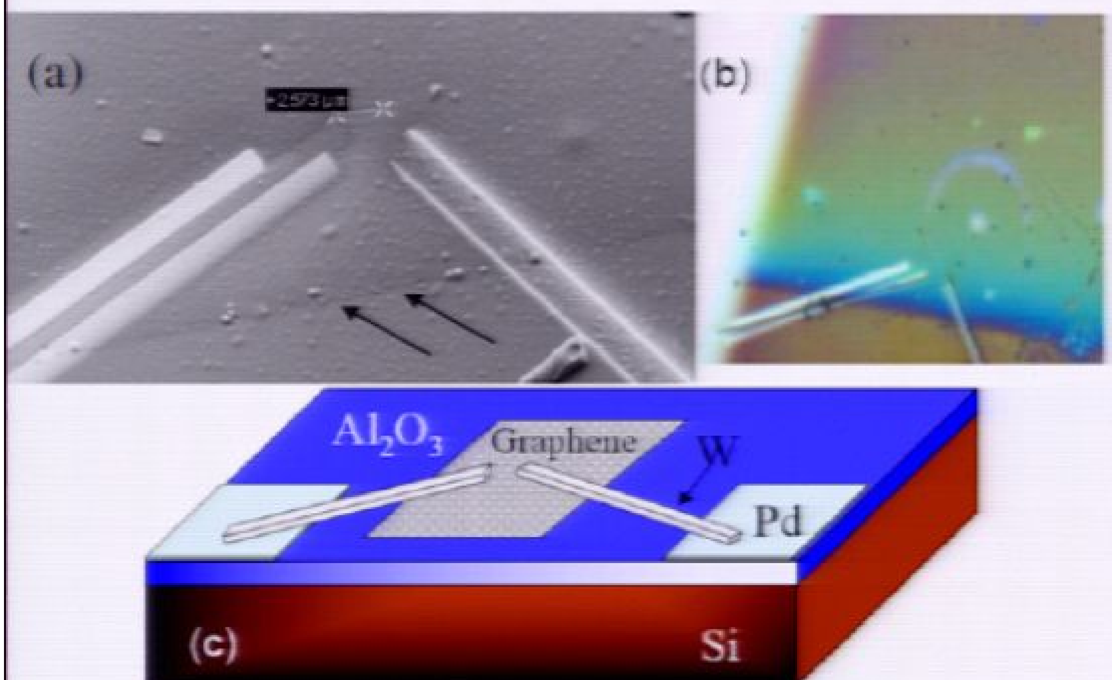
s-wave superconductivity



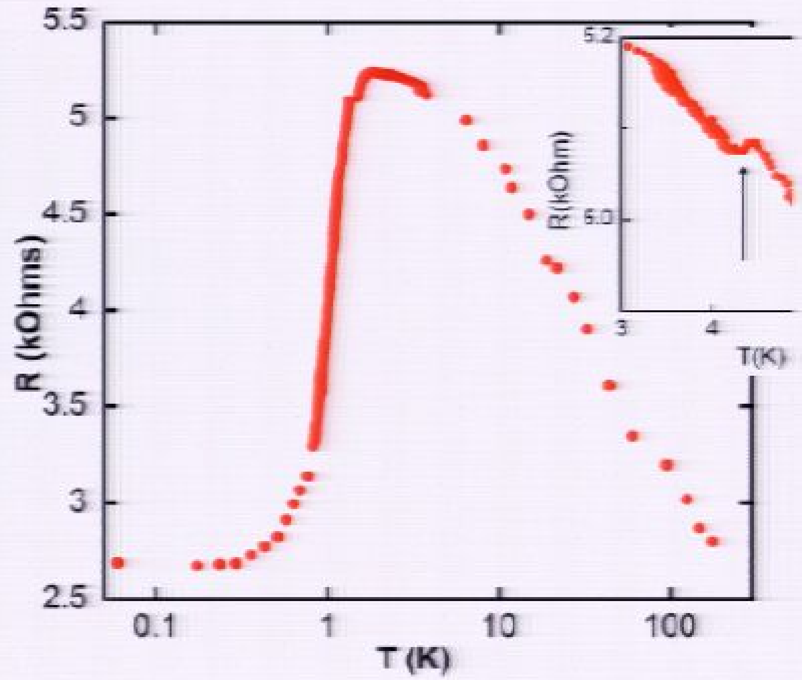


superconducting proximity in graphene:

- Heersche et al.
  Nature 446, 56 (2007)
- Shailos et al.
  Euro. Phys. Lett. 79, 57008 (2007)
- UC Berkeley, Zettel group

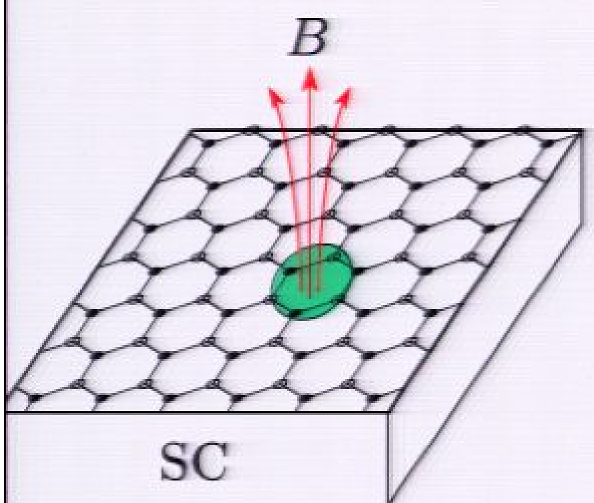


(a)
(b)
Al_2O_3
Graphene
W
Pd
(c)
Si



R (kOhms)
T (K)
5.5
5
4.5
4
3.5
3
2.5
0.1
1
10
100
R (kOhm)
T(K)
5.2
5.0
3
4

single vortex in superconducting graphene



$$\Delta(|r|)e^{i\theta}$$

$$\Delta(|r|)\begin{cases} \propto |r| & |r| \to 0 \\ \to \Delta & |r| \to \infty \end{cases}$$

$$\boldsymbol{A}(r) = A(|r|)\boldsymbol{e}_\theta$$

- two zero energy modes (4 Majorana modes)

Ghaemi-Wilczek (2007)

- invariant under SU(2) pseudo spin rotation

$$(Q_1, Q_2, Q_3) = (\tau_x\sigma_y\rho_0, -\tau_x\sigma_x\rho_z, \tau_0\sigma_z\rho_z)$$

$$[Q_i, Q_j] = 2i\epsilon_{ijk}Q_k$$

- filling one of two states -> pseudo spin doublet

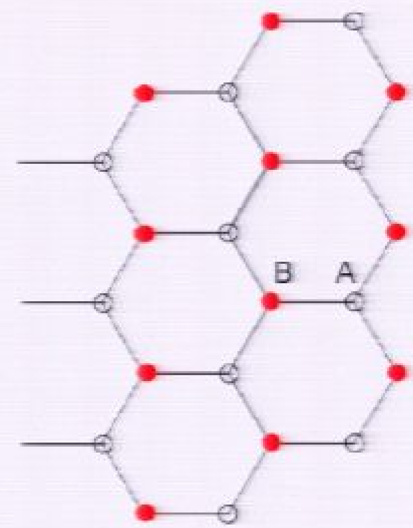
$$\left\langle \Psi^\dagger \vec{Q} \Psi \right\rangle \neq 0$$

vortex core hosts SU(2) pseudo spin

valley pseudo spin and order parameters

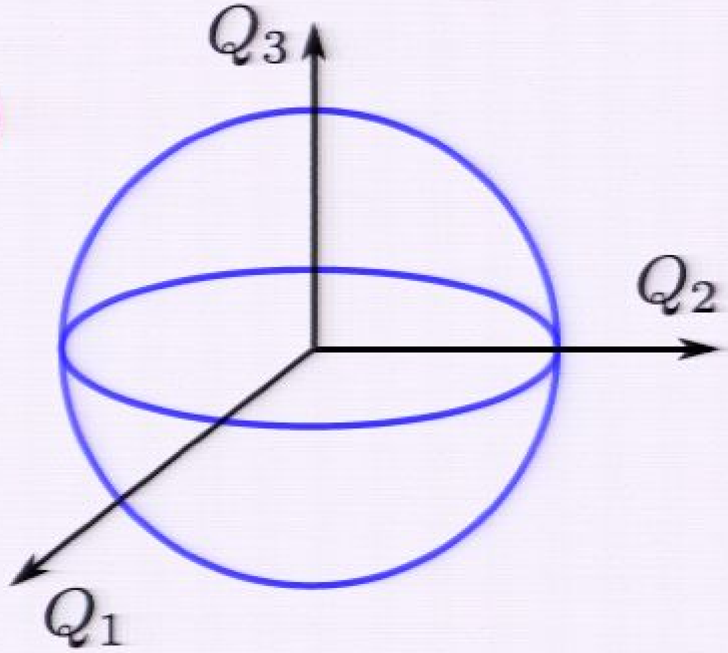
charge density wave (CDW)

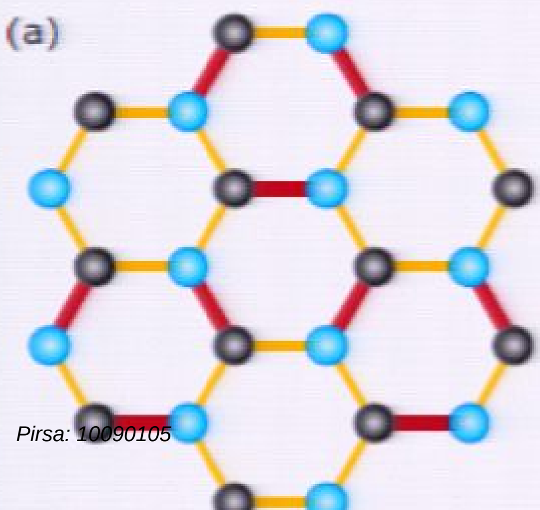
$$\langle c_A^\dagger c_B - c_B^\dagger c_A \rangle \neq 0$$



bond density wave or valence bond solid (VBS) "Kekule pattern"

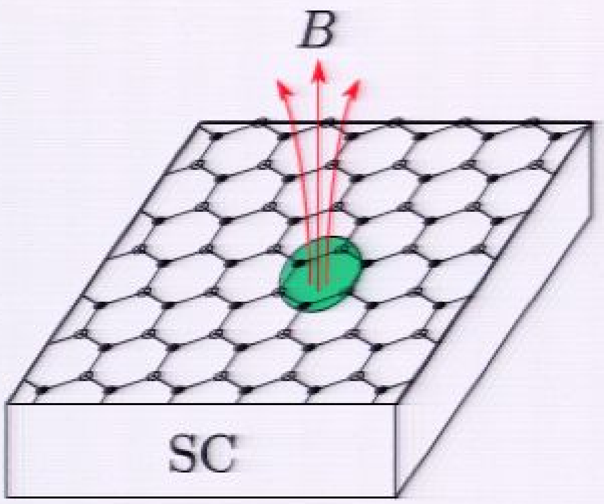
$$\langle c_A^\dagger c_B \rangle \propto e^{i\phi}$$

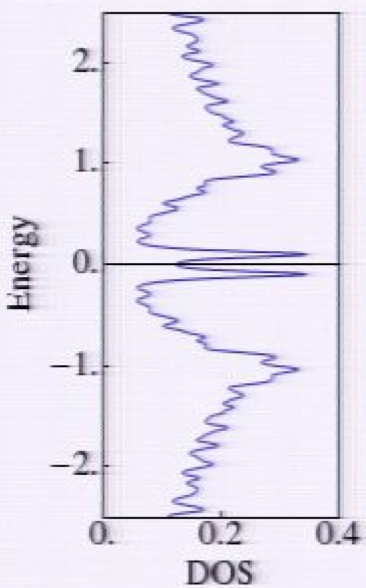


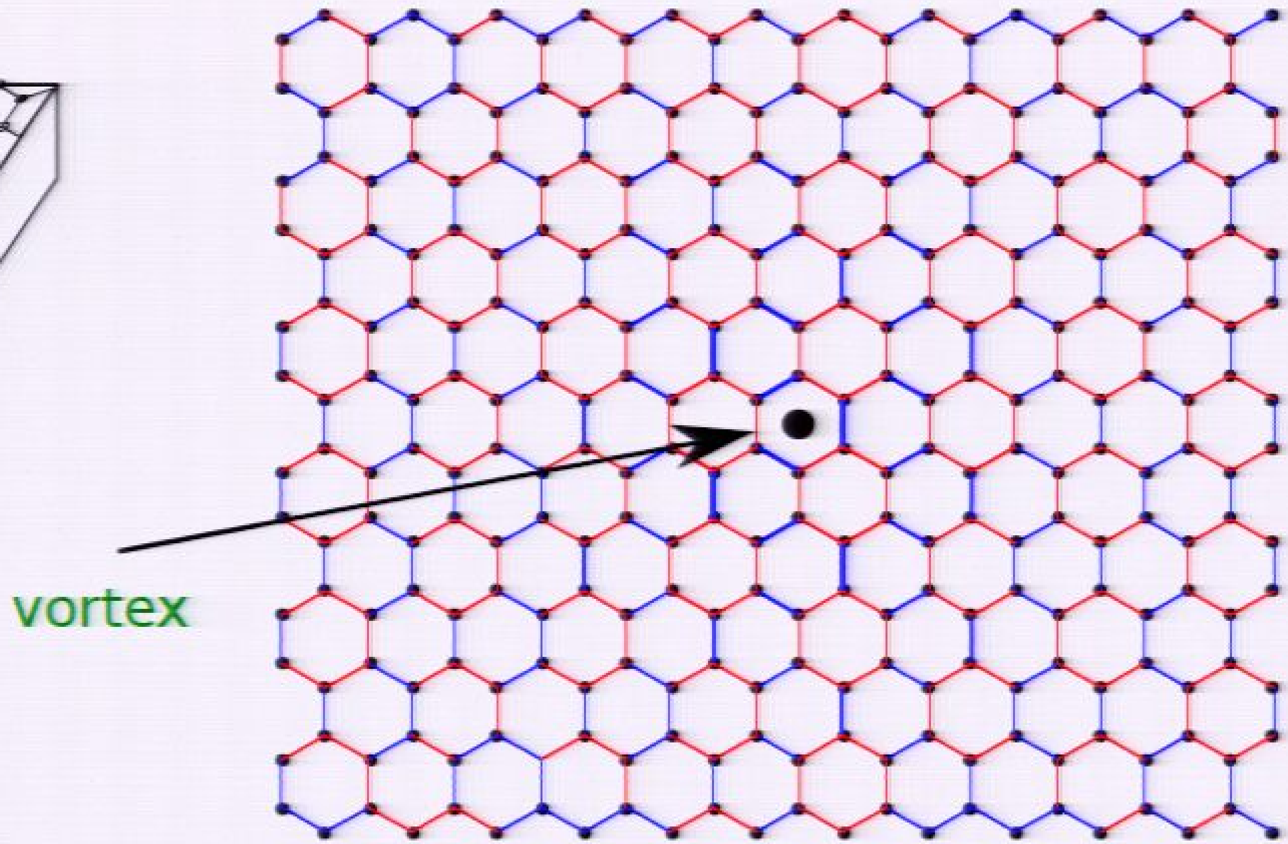


Mintmire-Dunlap-White
Veit-Ajiki-Ando
Hatsugai-Fukui-Aoki
Hou-Chamon-Mudry

vortex in superconducting graphene







$|\langle c_{i\sigma}^{\dagger} c_{j\sigma} \rangle|$ red: stronger bond, blue: weaker bond

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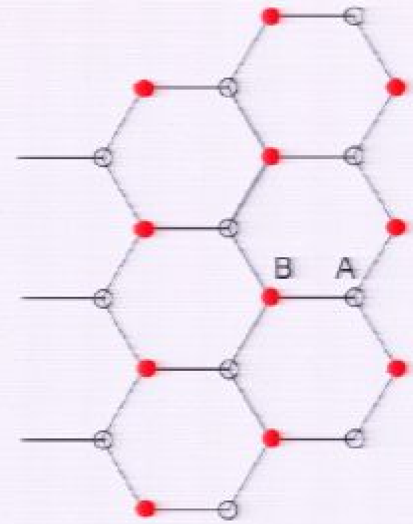

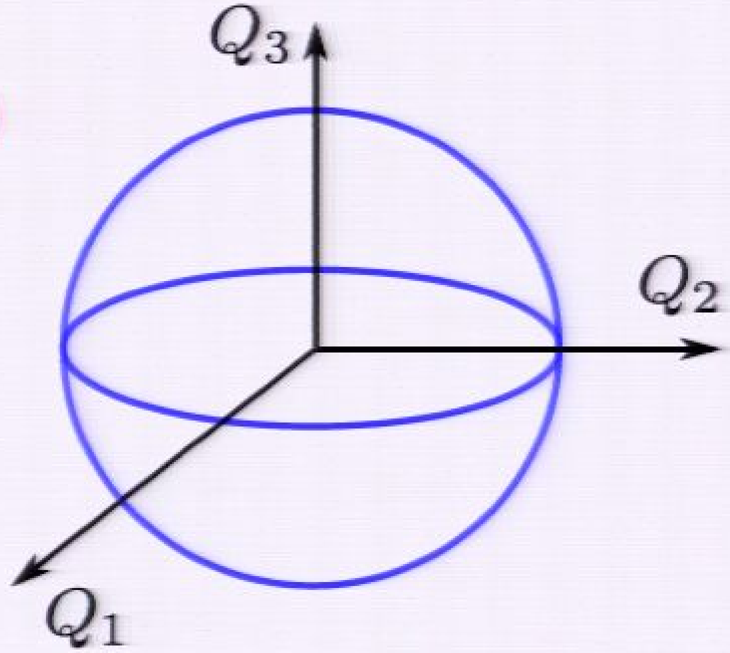

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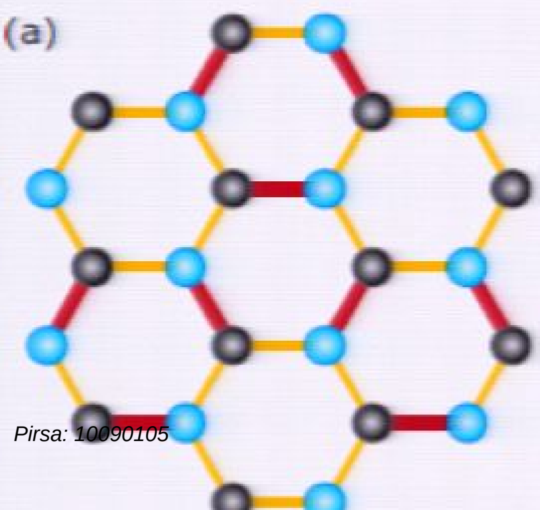
(a)



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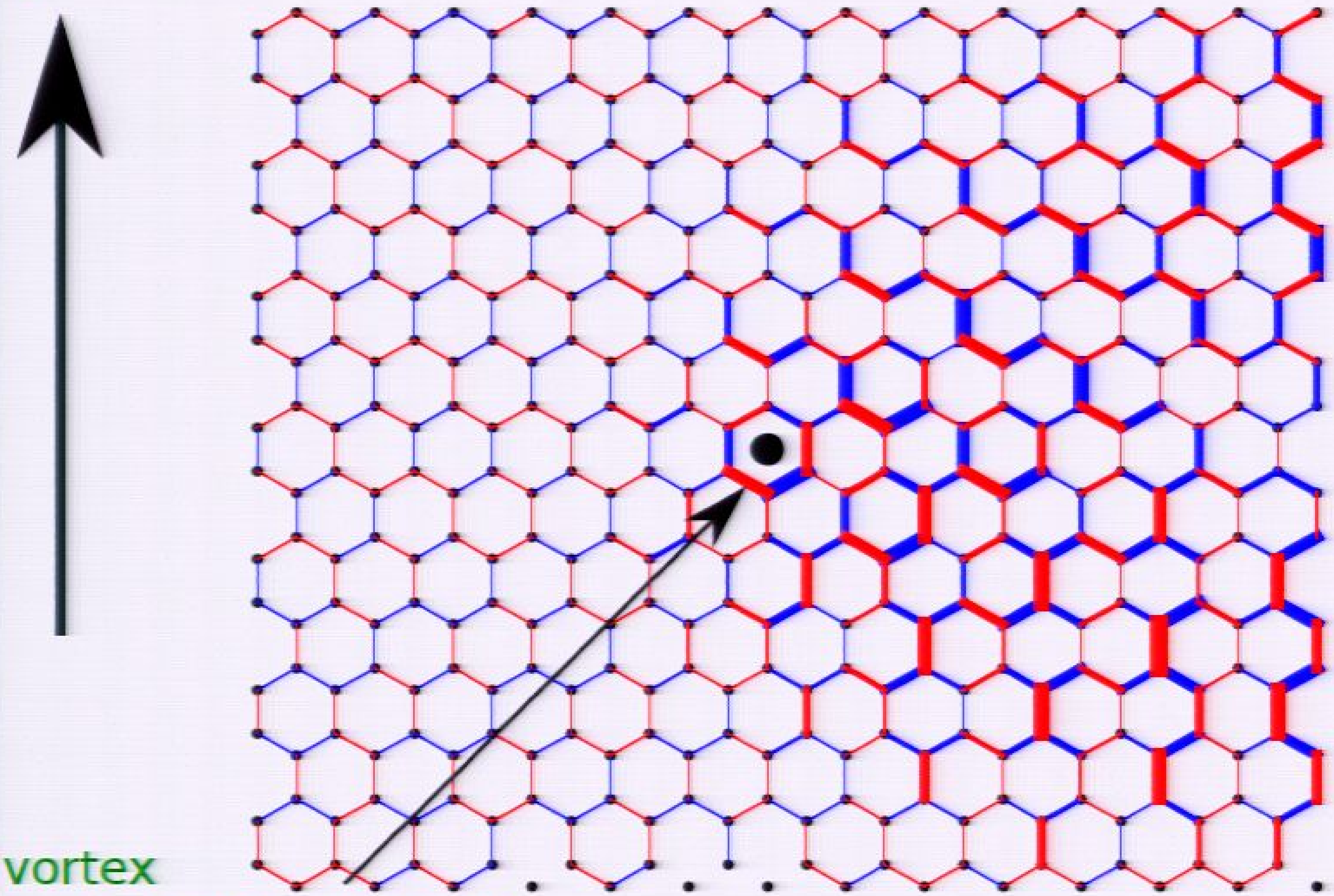
SU(2) symmetric quantum valley Hall effect

supercurrent



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vortex
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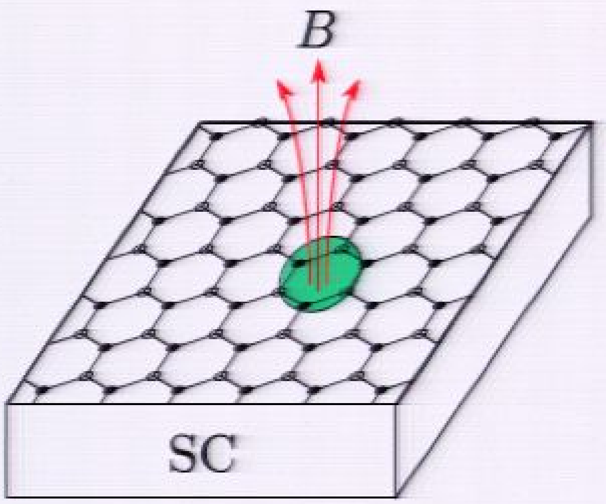

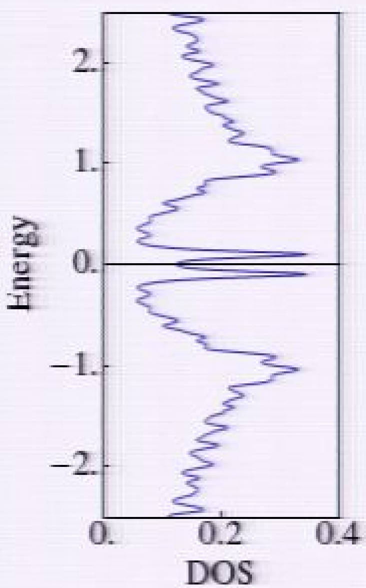

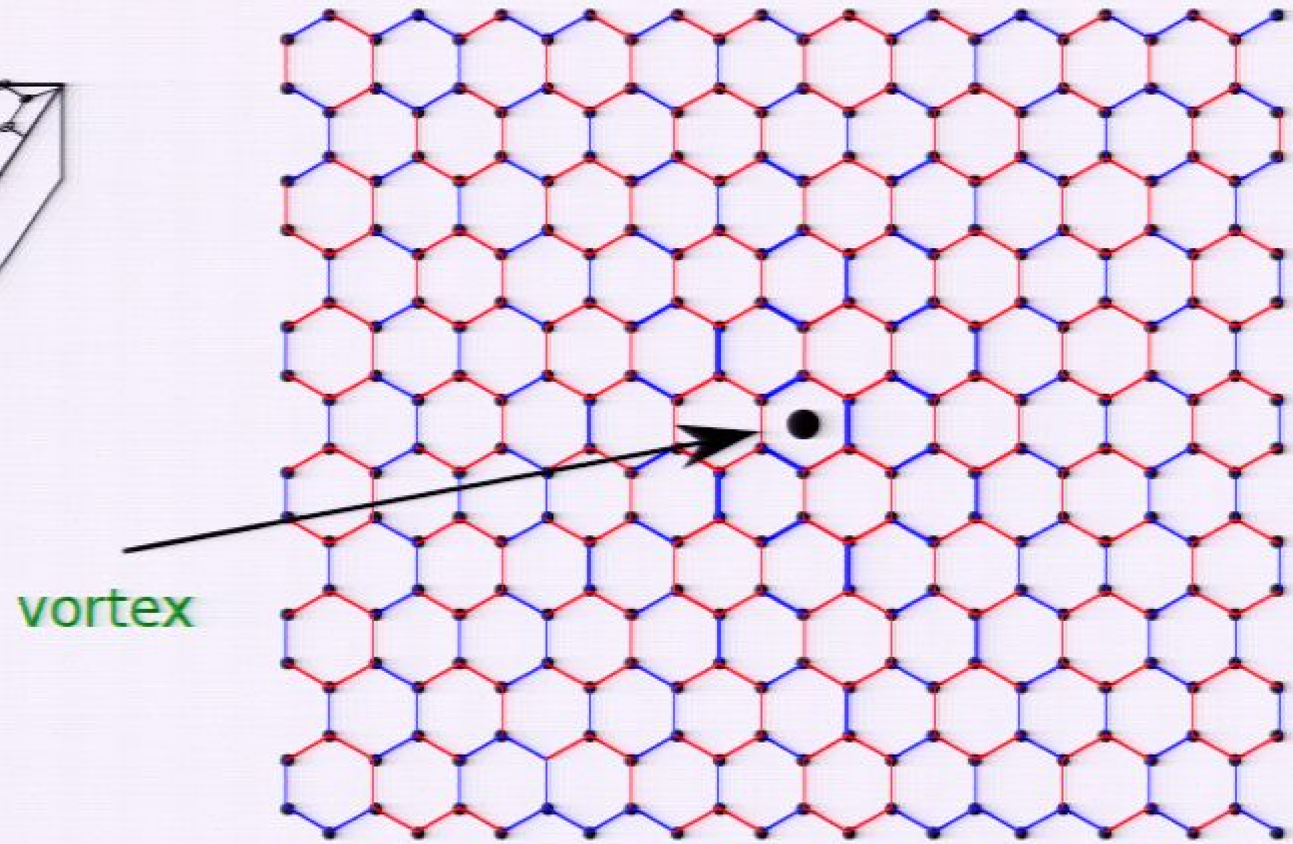

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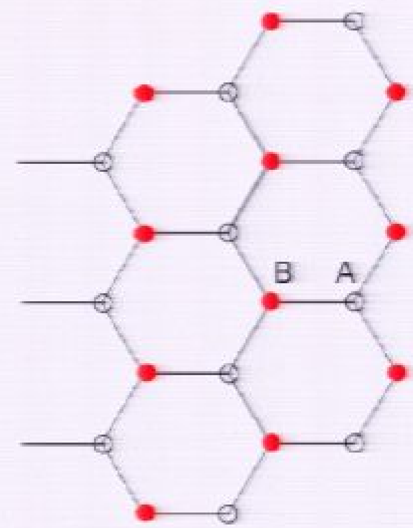

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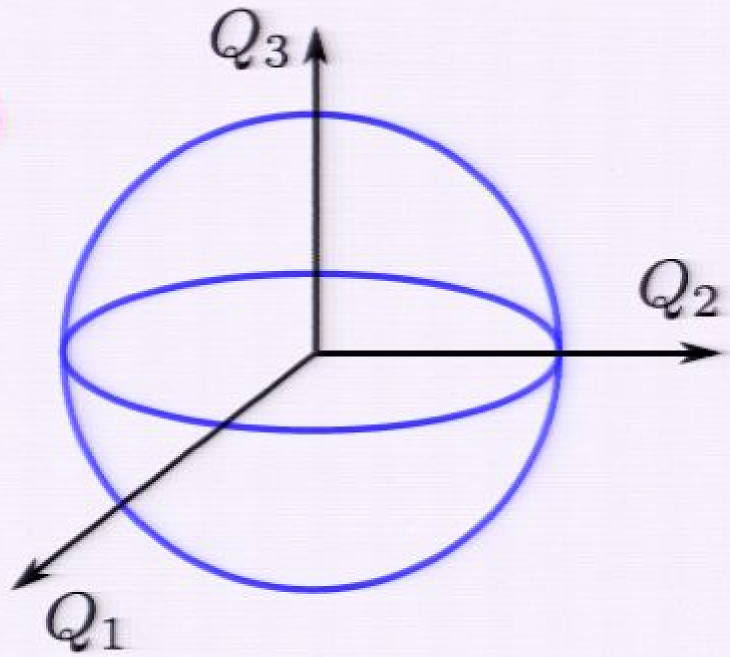

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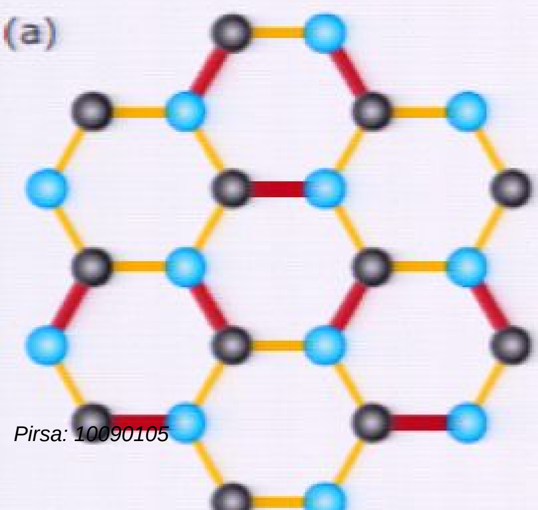

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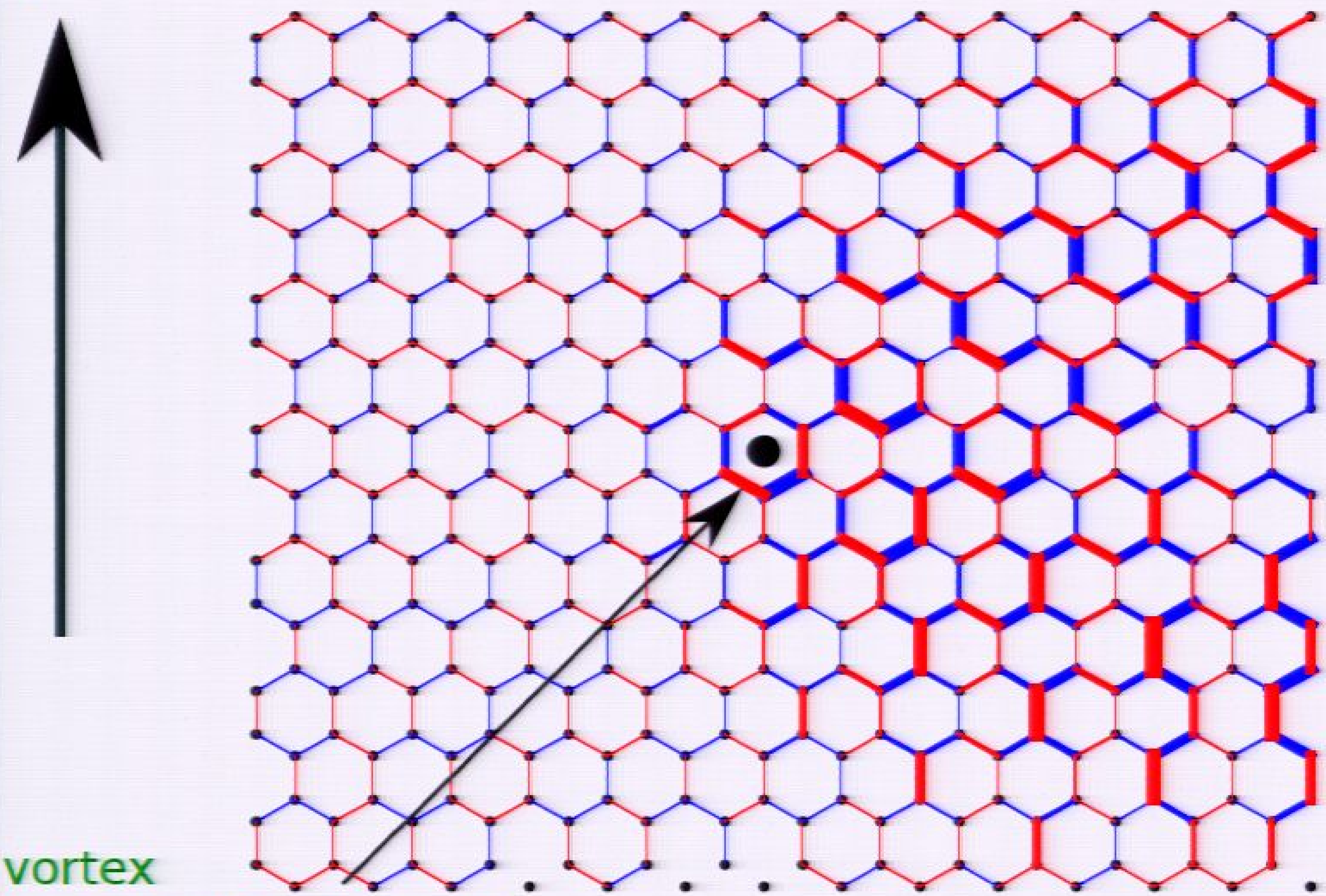

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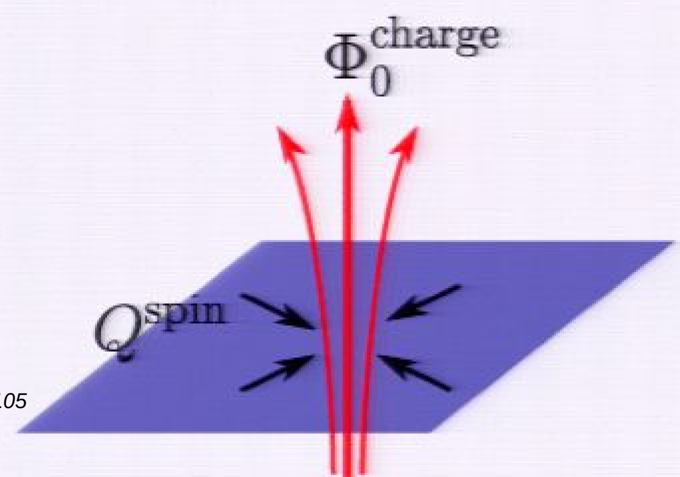
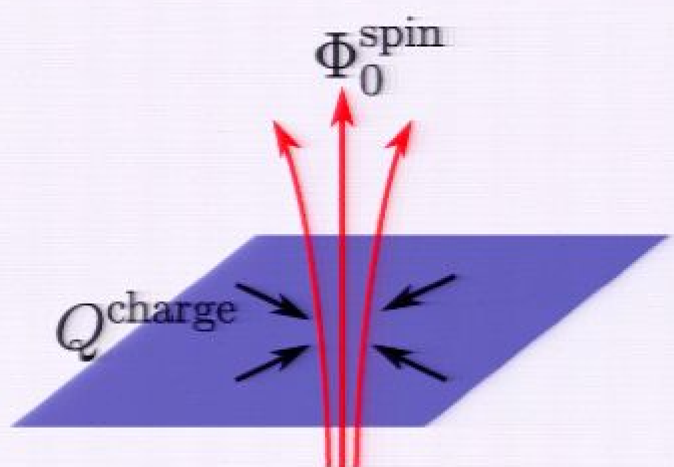
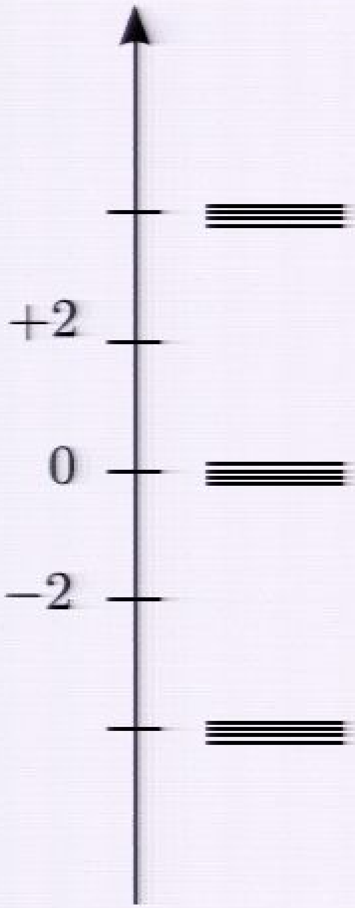




relativistic LLs in weak field

$\nu = 2\pi \ell_B^2 \rho$

$\sigma_{xy} = +6$



$\sigma_{xy} = +2$

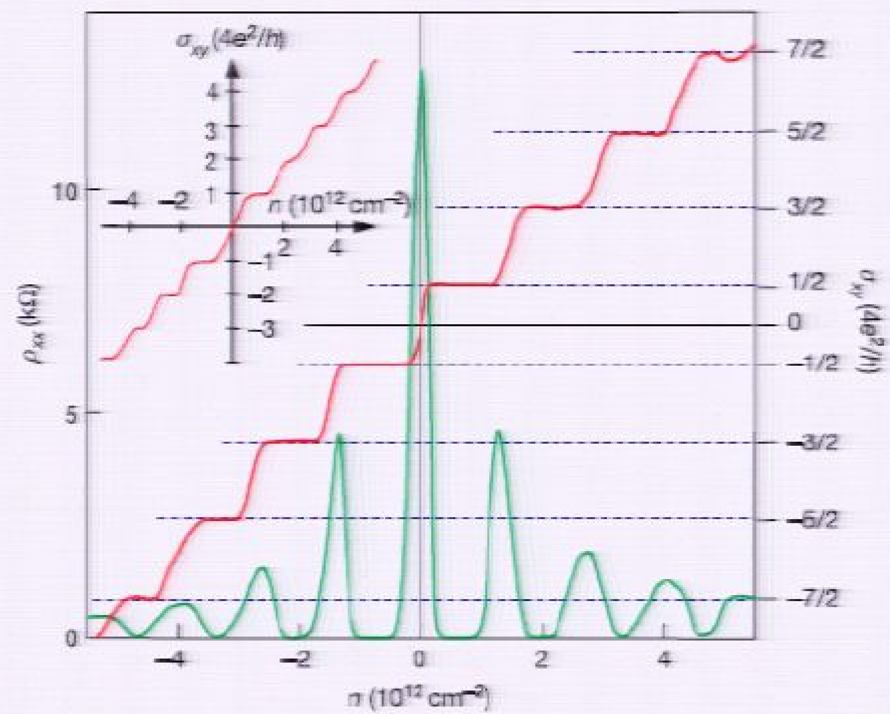
$\sigma_{xy} = -2$

$\sigma_{xy} = -6$

$E_{LL} = \pm\sqrt{2e\hbar B}\sqrt{N}$

$$\sigma_{xy} = 4 \times \left(n + \tfrac{1}{2}\right)$$

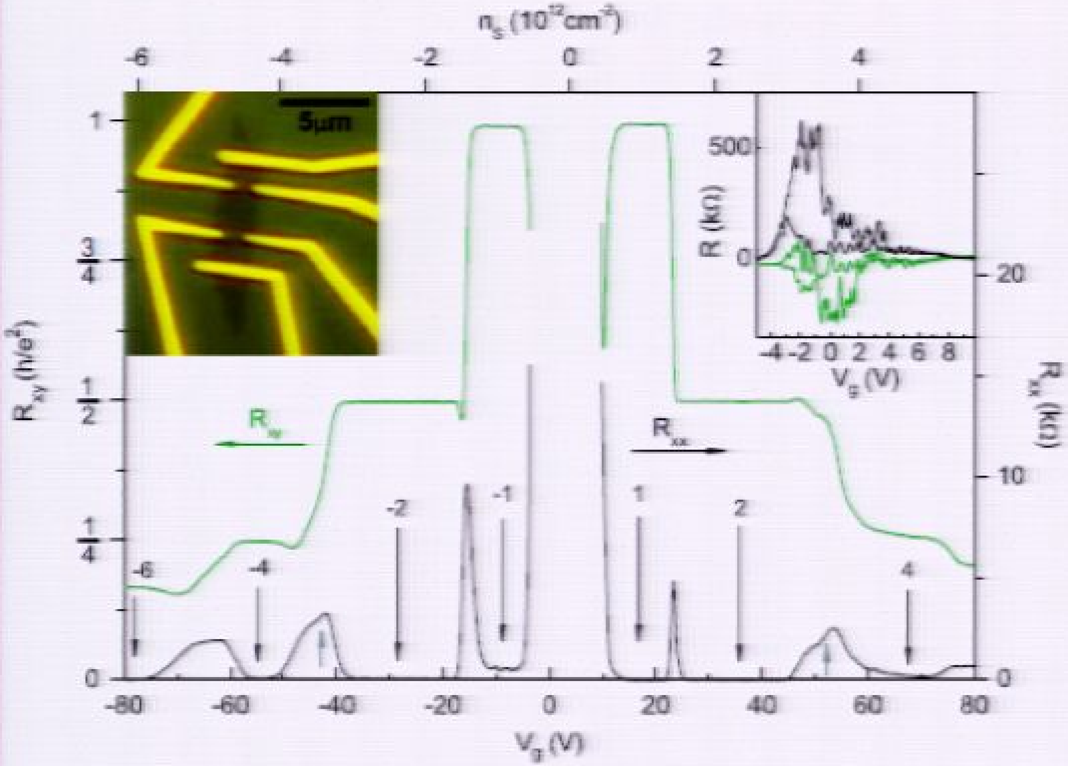
"1/2 shift"

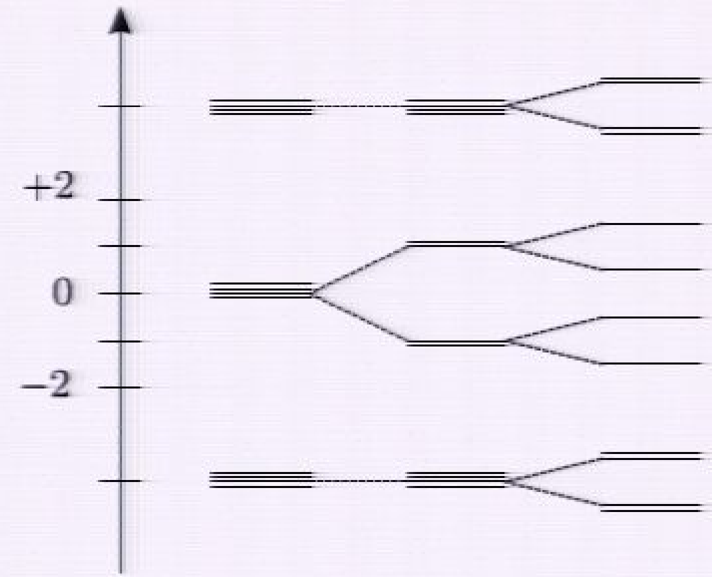


4-fold degeneracy = (spin) x (valley)

Morozov et al. (2005)

new QH states in high field





Y. Zhang et al. (PRL 2006)
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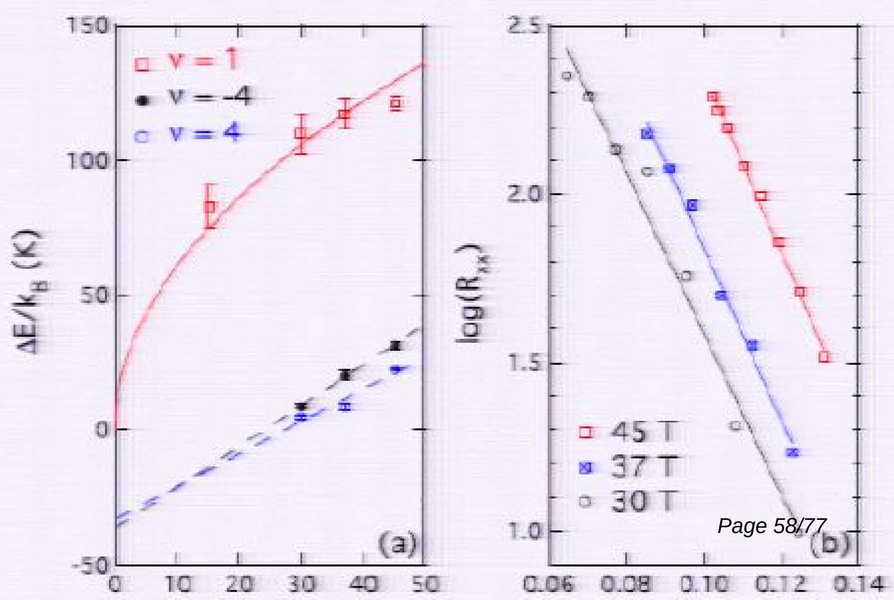

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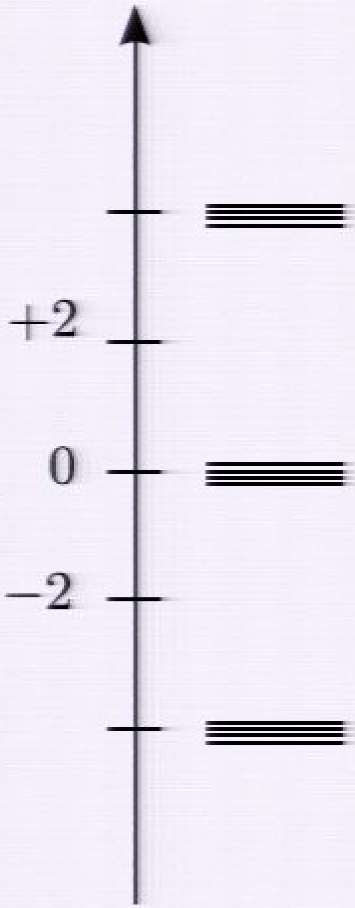

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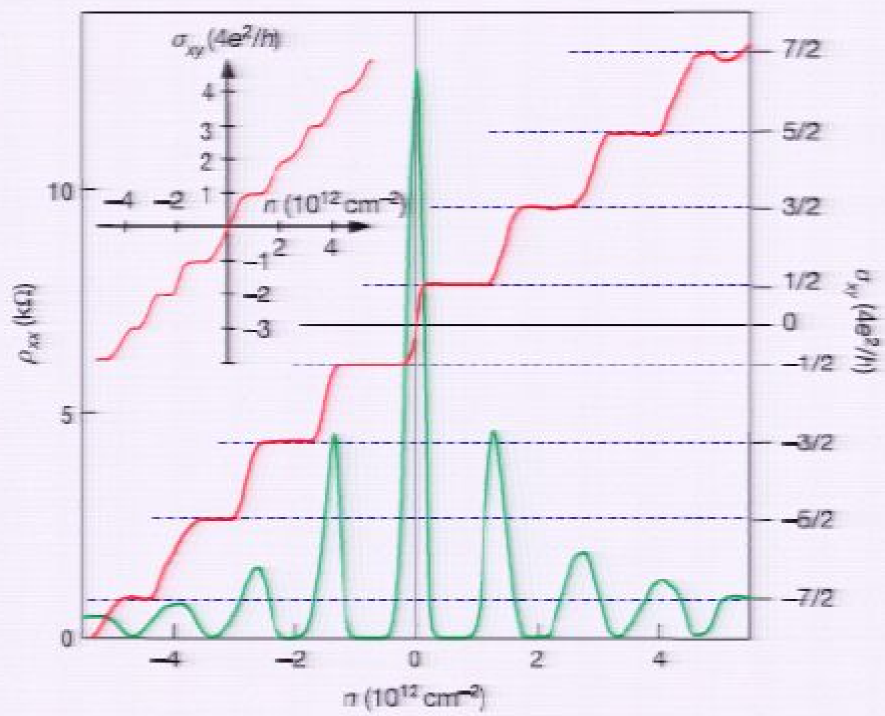

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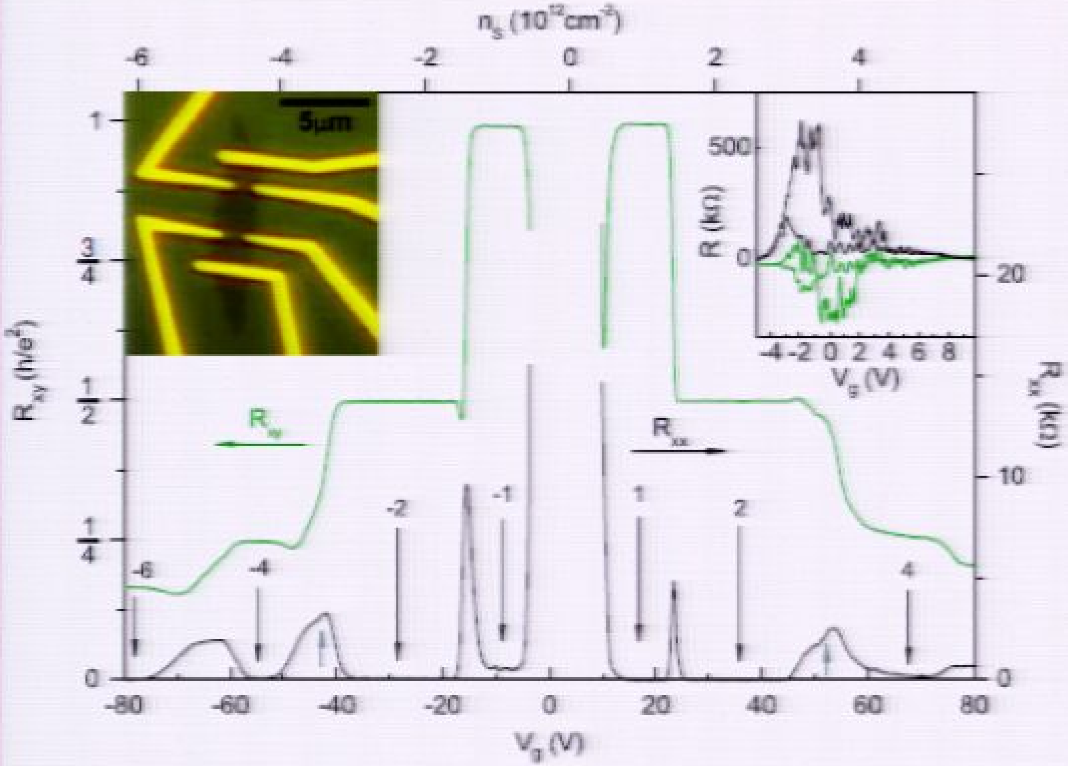

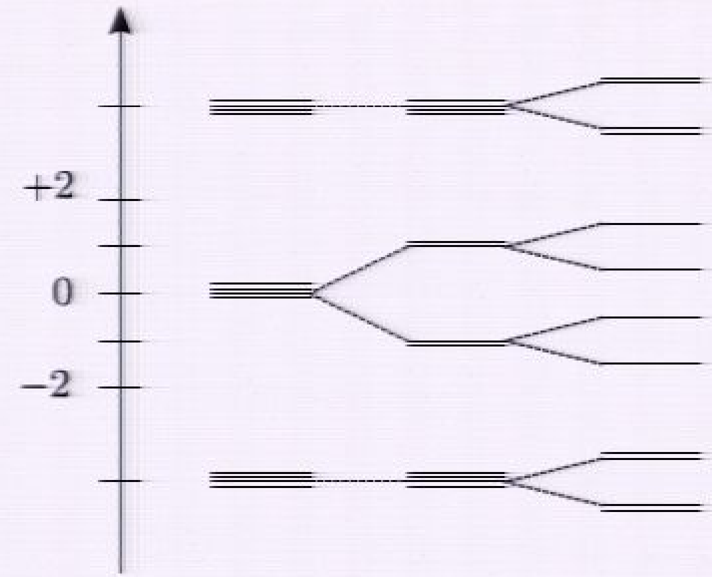

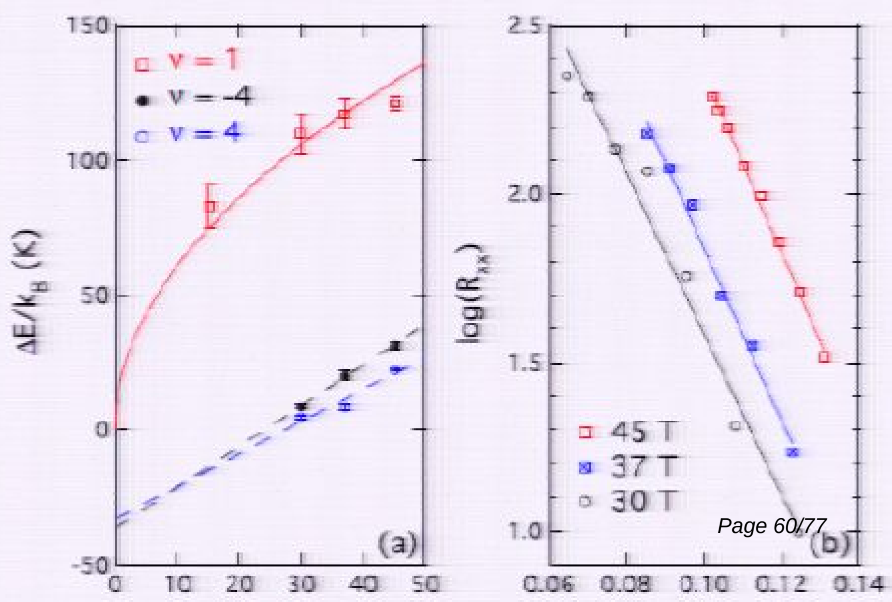

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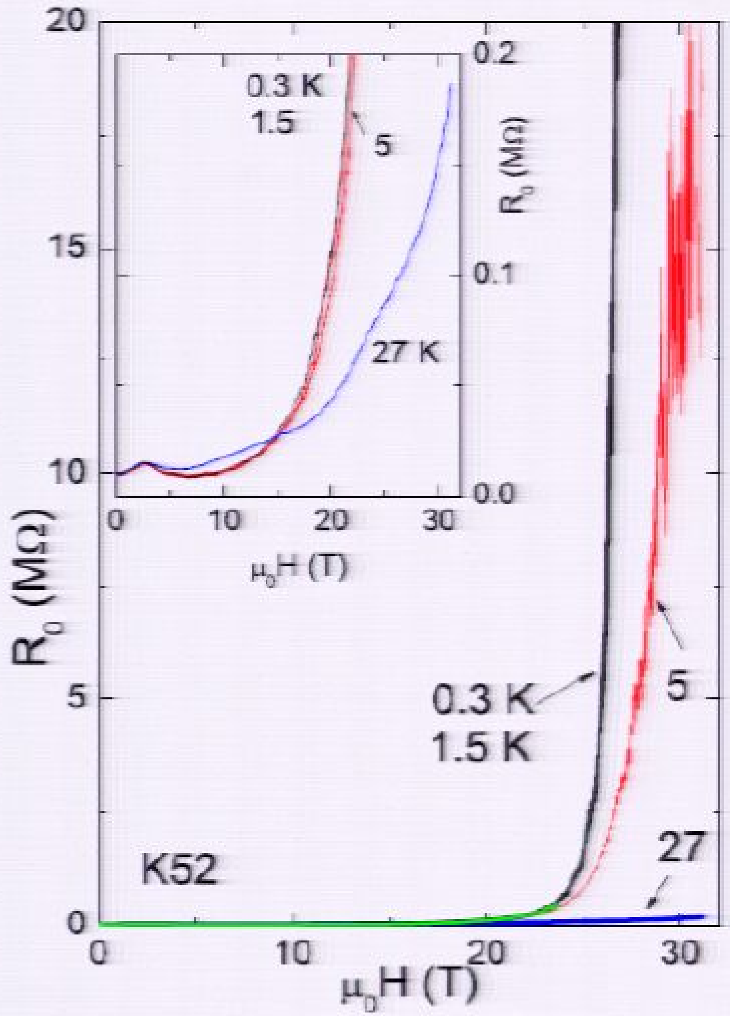
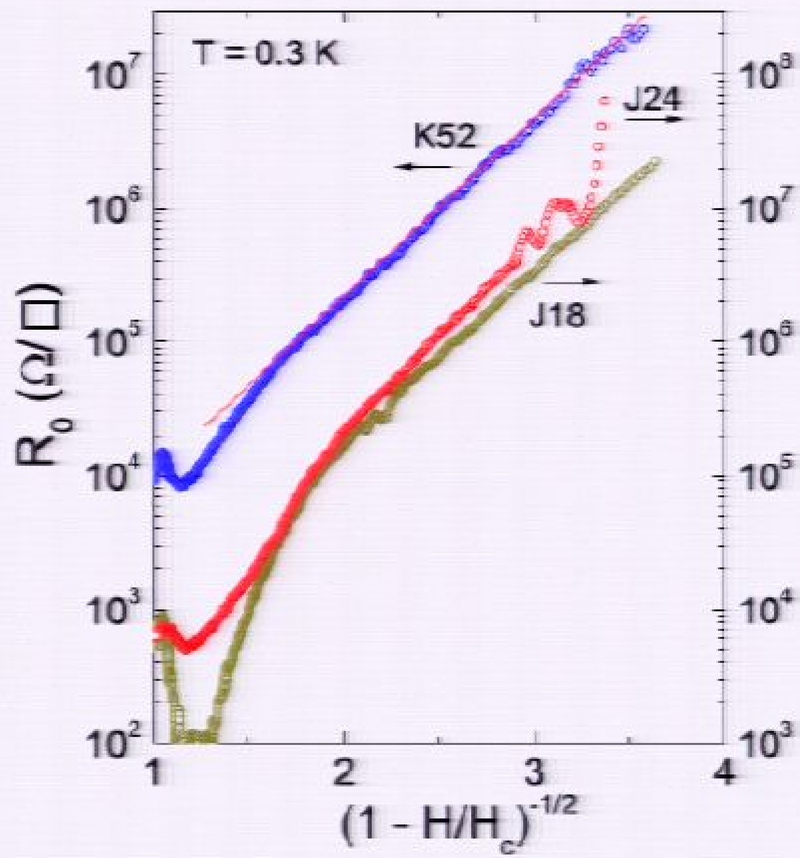
high resistance state and KT behavior at nu = 0

J. G. Checkelsky, Lu Li, and N. P. Ong, PRL (2007)





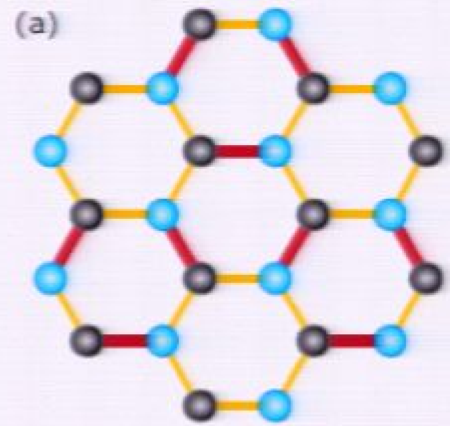
Kosterlitz-Thouless scaling:

$$R_0 = a \exp\left[b/\sqrt{1 - H/H_c}\right]$$

What is the nature of the insulating ground state at high field ?

insulating ground state: our proposal

ground state = broken pseudospin symmetry state
= bond ordering
("Kekule pattern")



XY pseudospin (quantum Hall) ferromagnet

$|\langle c^{\dagger}_{i\sigma} c_{j\sigma} \rangle|$

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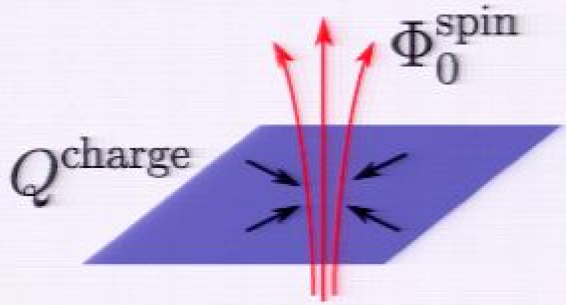

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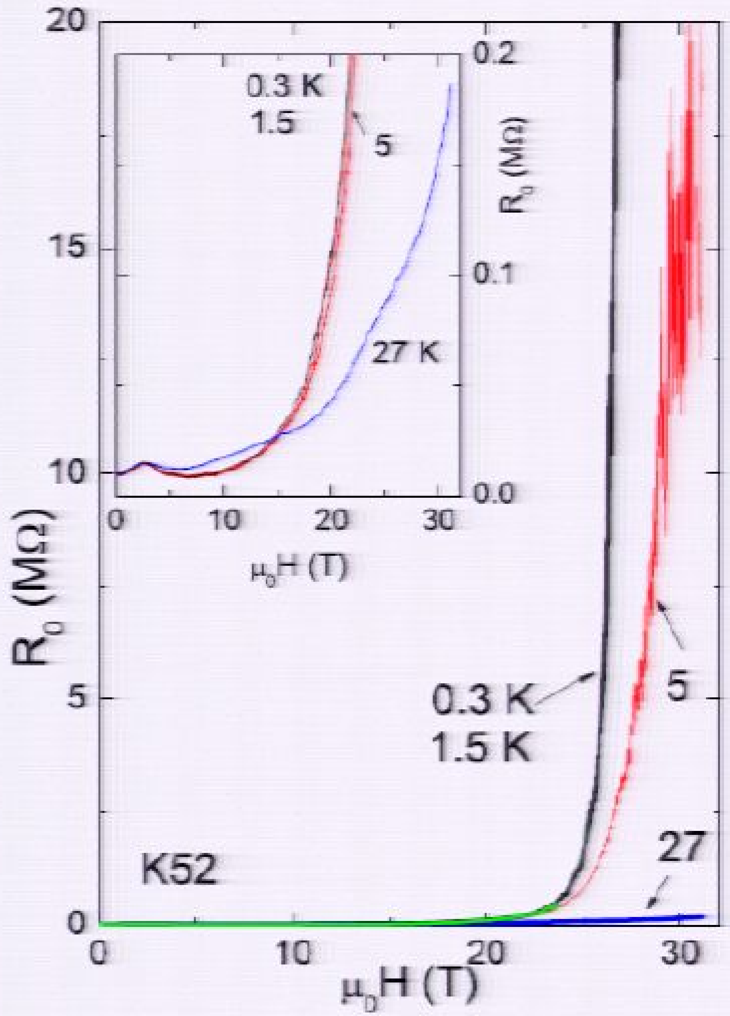

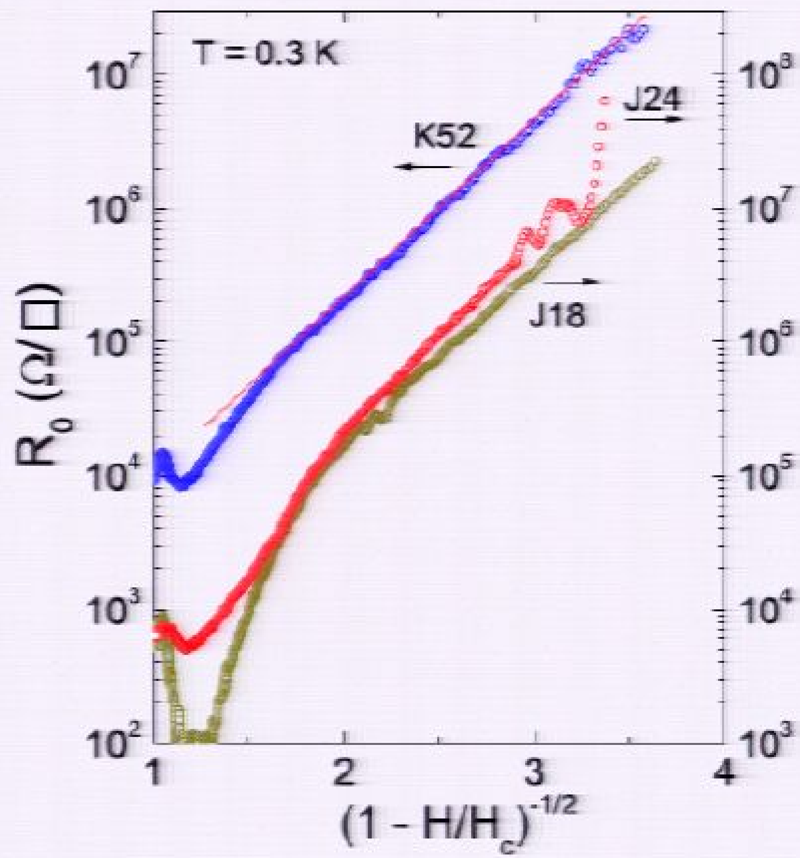

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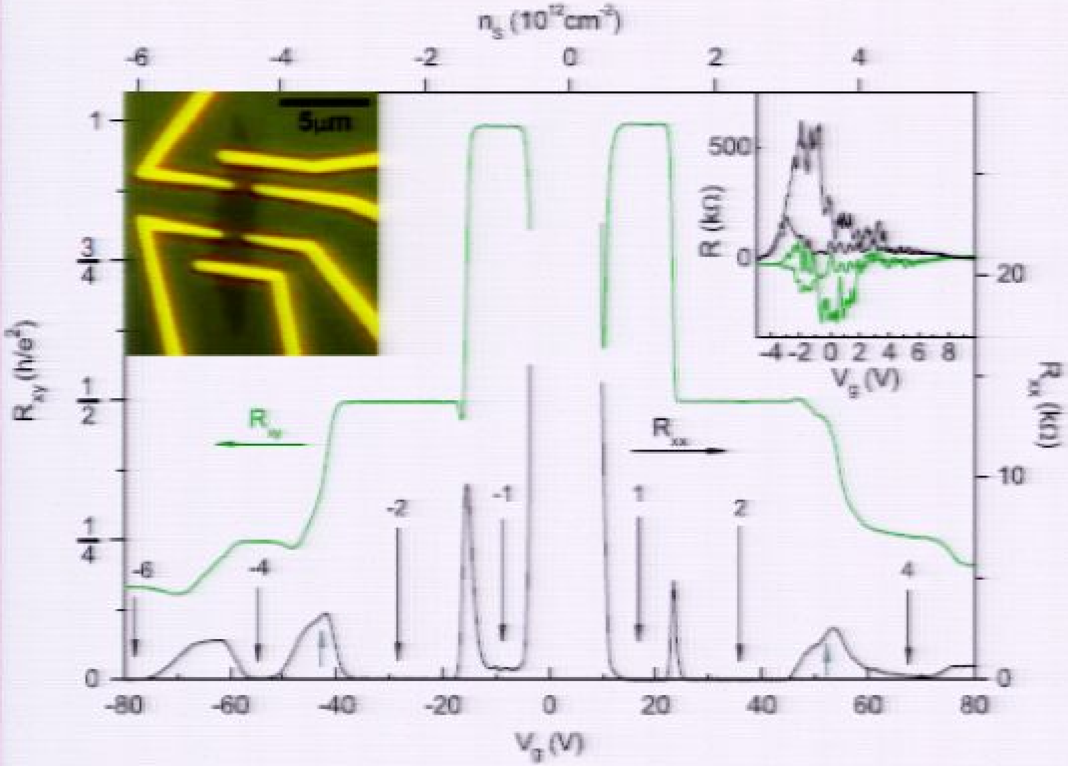

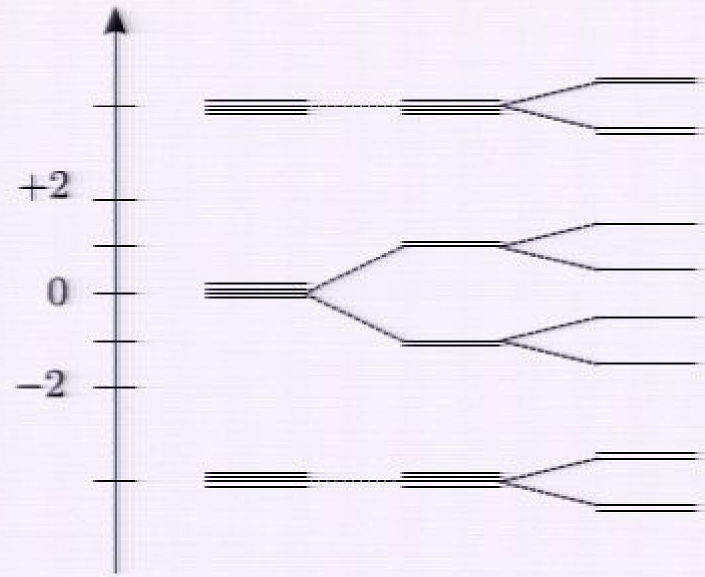

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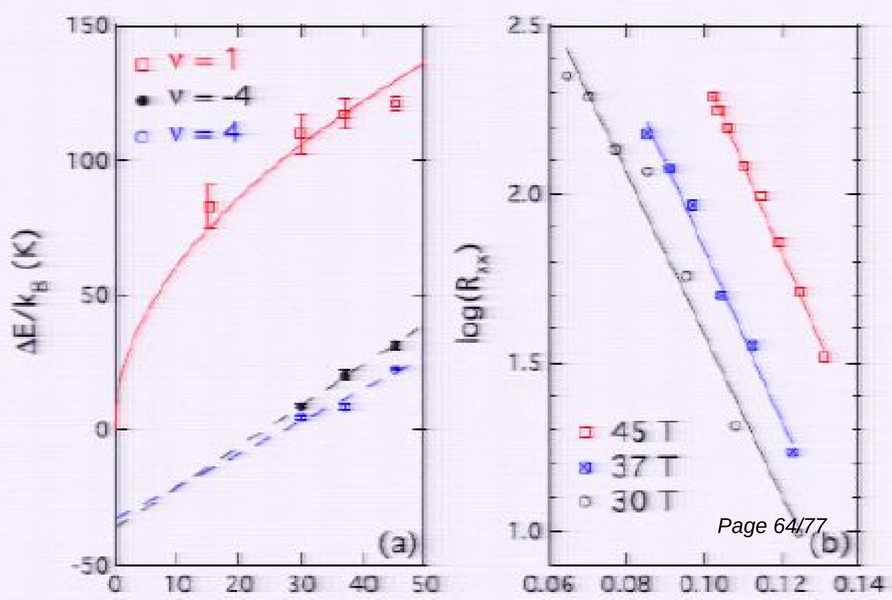

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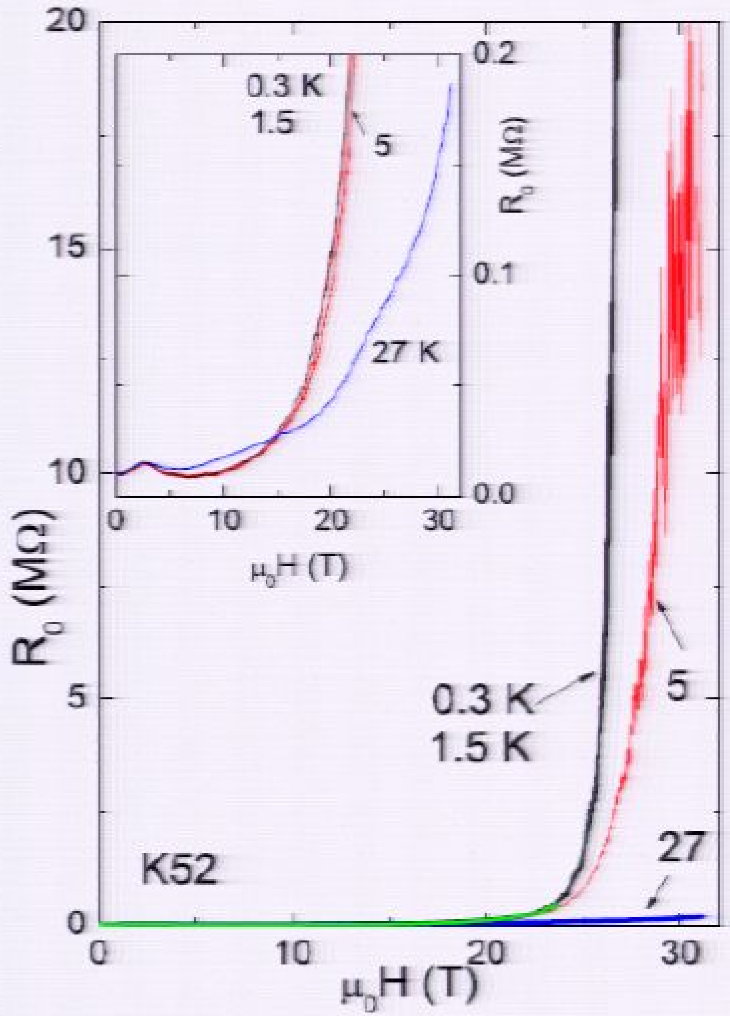

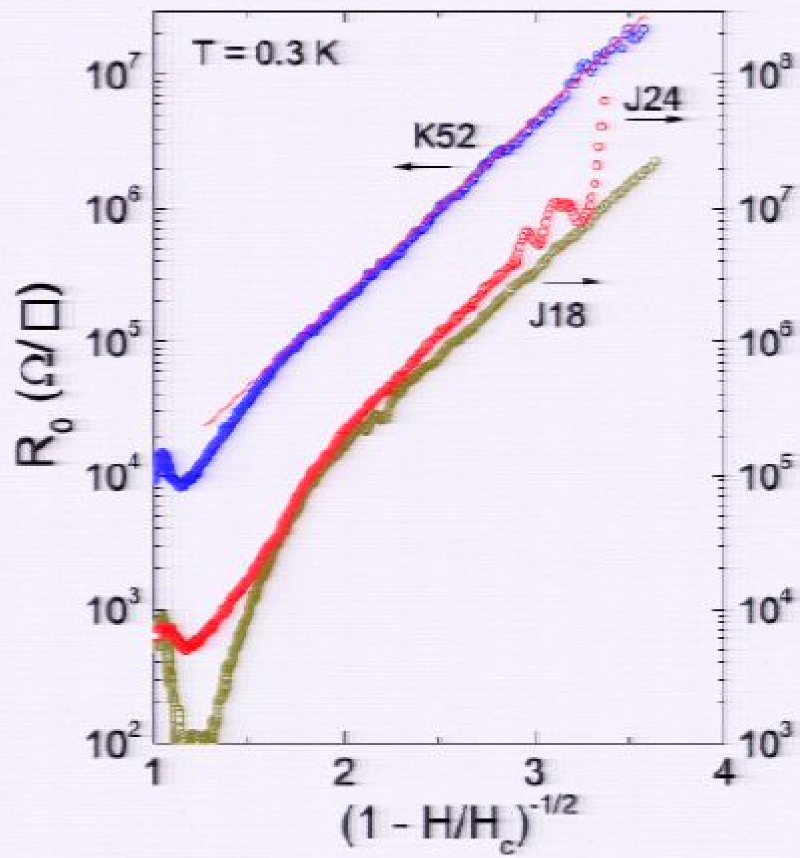

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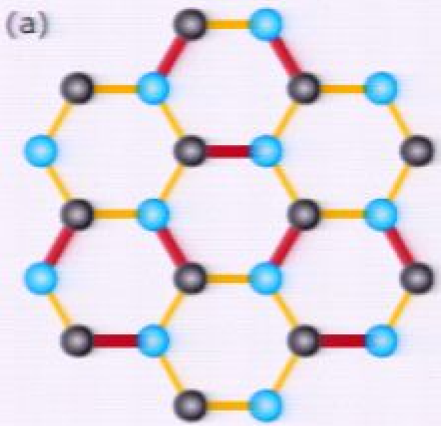

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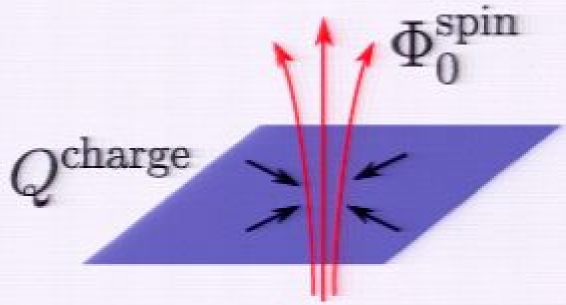

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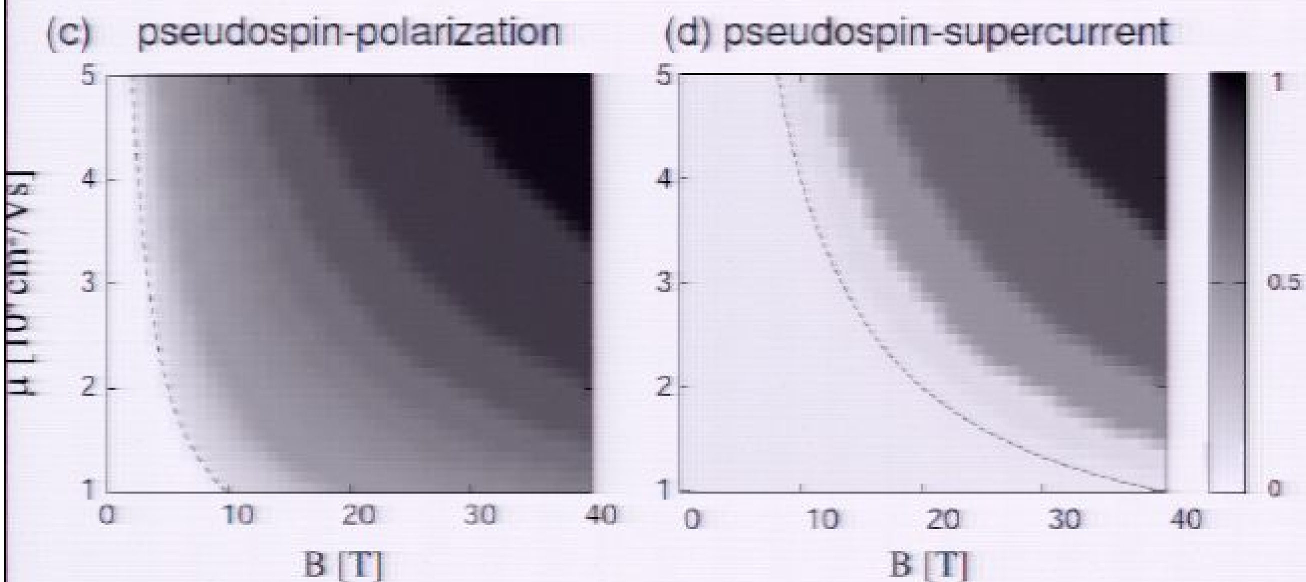
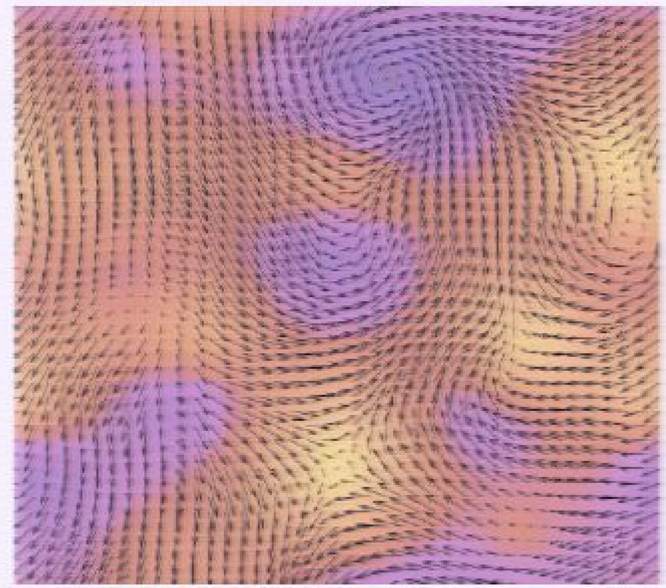
Hartree-Fock phase diagram and transport

Hartree-Fock phase diagram

typical pseudospin configuration





graphene in B field

valley-supercurrent: neutral

vortex current: charged

$$\rho_{xx} \sim \xi_{KT}^2$$

superconducting film

supercurrent: charged

vortex current: neutral

$$\sigma_{xx} \sim \xi_{KT}^2$$

$$\xi_{KT} \sim e^{a/\sqrt{B_c - B}}$$

summary of results and future issues

- dualities between order parameters

- fractional charge, and fractional statistics

- superconductivity in garaphene --> quantum valley Hall effect

  valley pseudo spin carried by a SC vortex

- N=0 LL in garaphene --> inverse quantum valley Hall effect

  an interesting arena to explore "topological Mott insulator".

  charge carried by a Kekule vortex ?

  effect of disorder: Hou, Chamon, Mudry (2010)

  new experiments in Corbino geometry, in suspended graphene

  KT in tilted-Dirac cone system α-$(\mathrm{BEDT\text{-}TTF})_2\mathrm{I}_3$

- effects of fluctuations

- dynamical questions: deconfining quantum crticality ?

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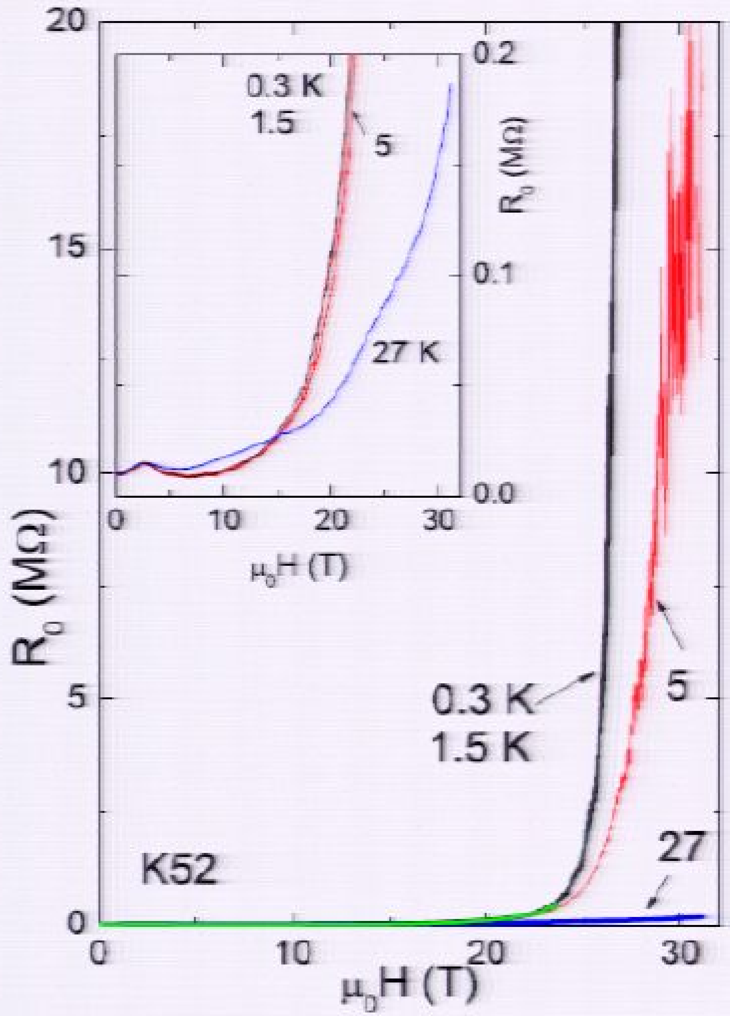

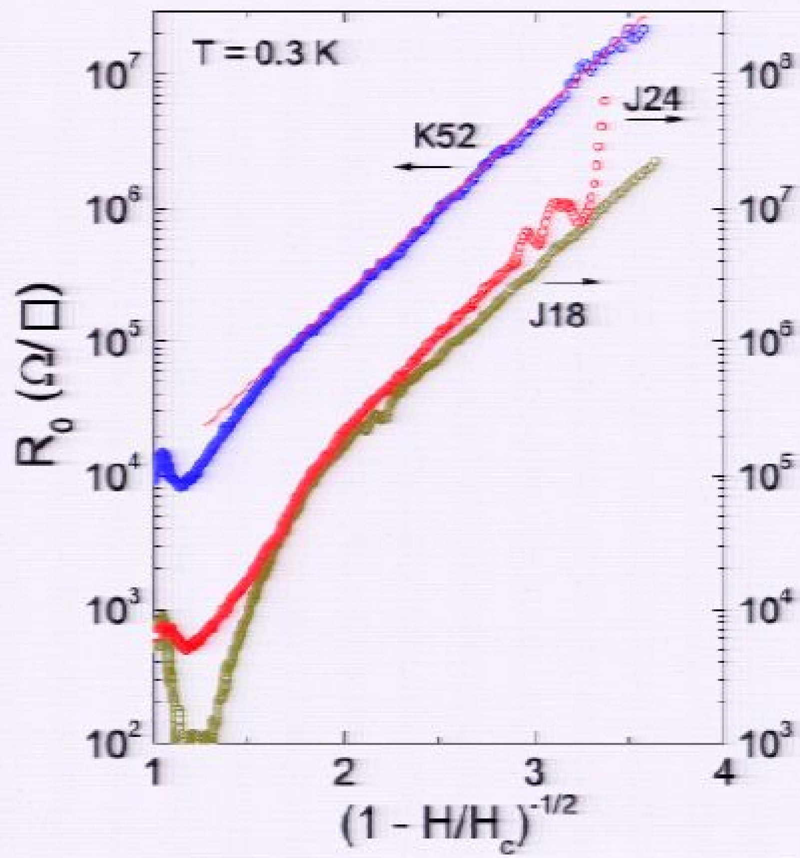

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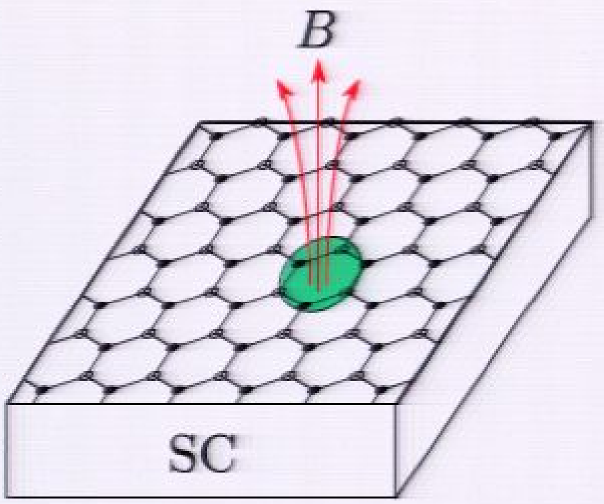
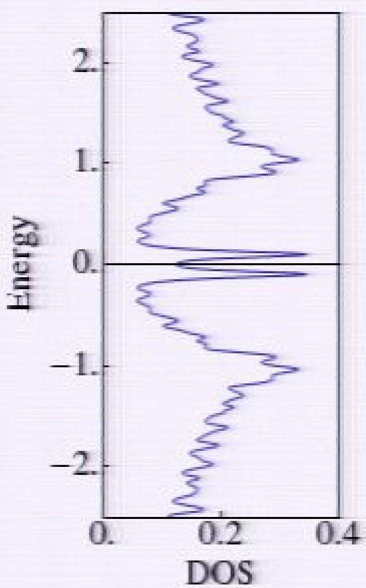
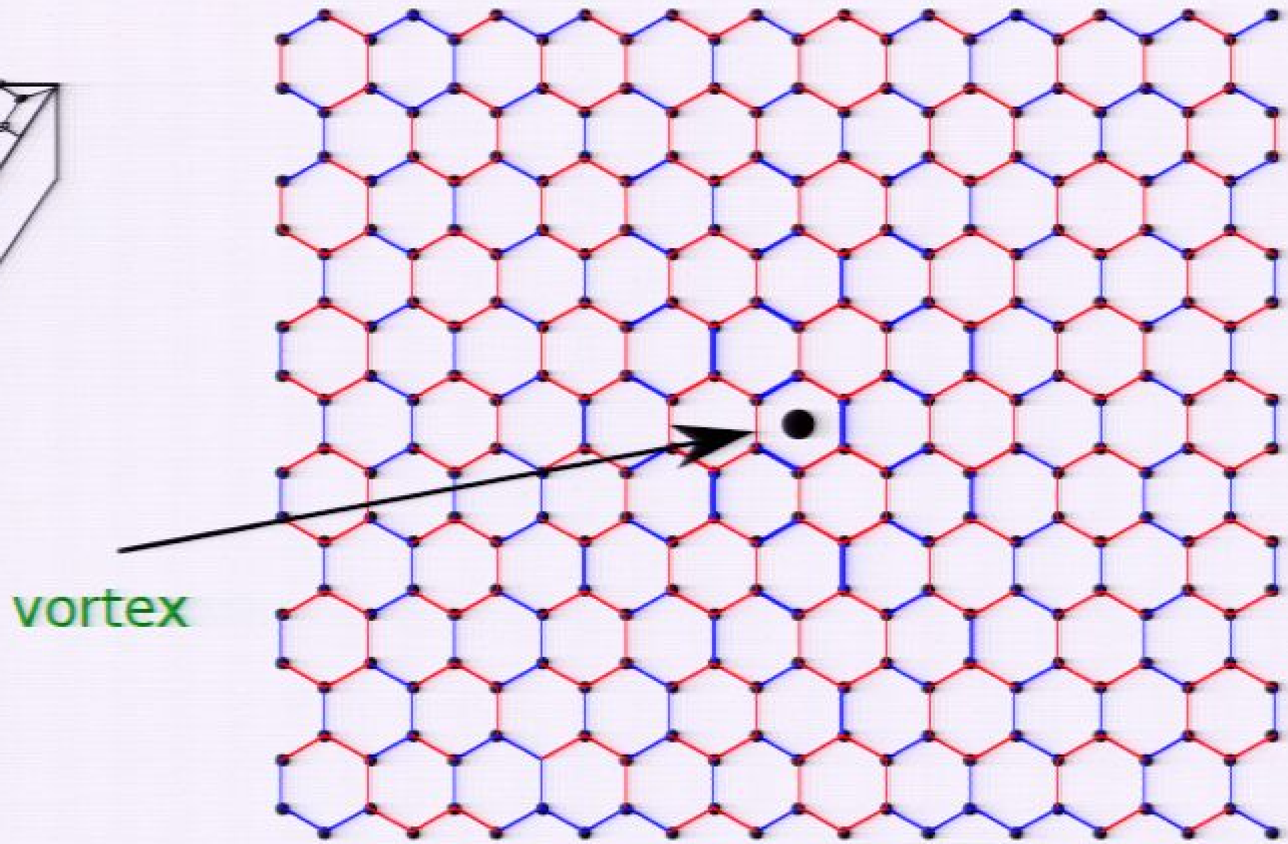
vortex in superconducting graphene







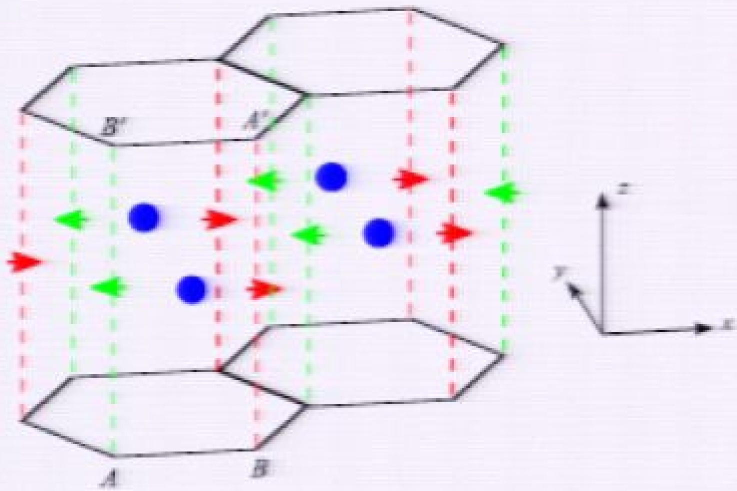
$|\langle c_{i\sigma}^{\dagger} c_{j\sigma} \rangle|$ red: stronger bond, blue: weaker bond

- degeneracy lift either by lattice effect or substrate

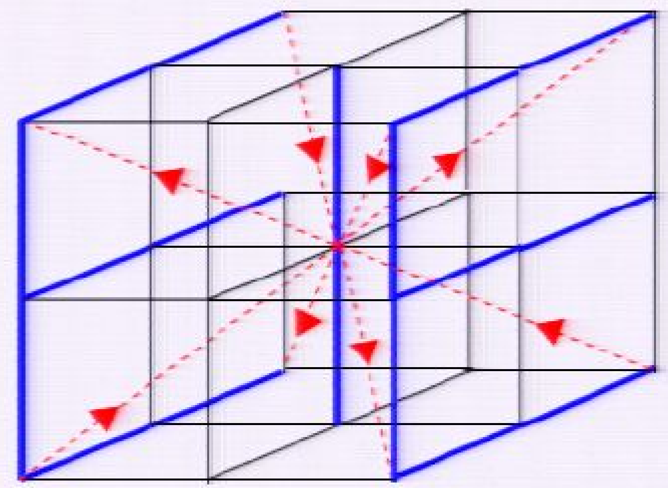
- accumulation of "valley pseudo spin" --> texture in VBS order paramet

Dirac models in three-dimensions

layered honeycomb model



cubic lattice with pi flux



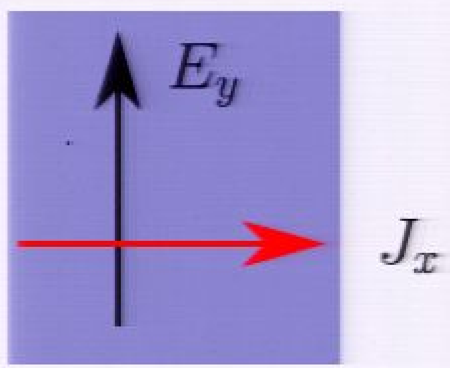
3D Dirac kinetic term

$$\mathcal{H}_0 \;=\; \Psi^\dagger \Big(k_x\Gamma_1 + k_y\Gamma_2 + k_z\Gamma_3 \Big)\Psi$$

QHE, Chern-Simons action

$$S_{\text{eff}} = \frac{n}{4\pi}\int d^3x\, \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

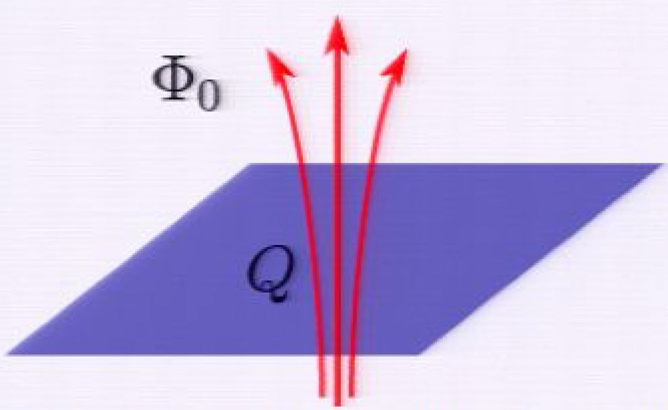
- quantum Hall effect

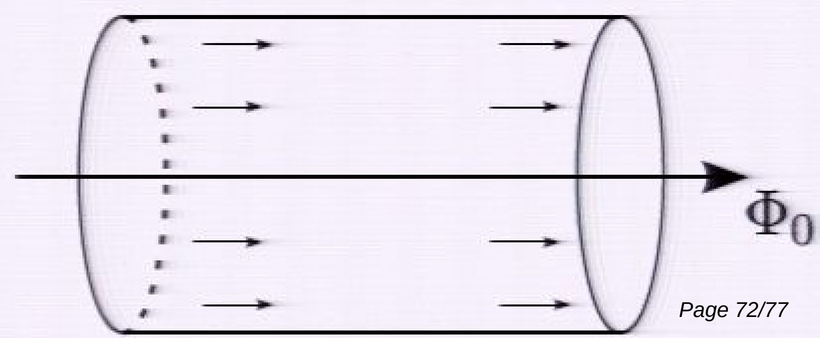


$$J_i = \frac{\delta S_{\text{eff}}}{\delta A_i} = \frac{n}{2\pi}\epsilon^{itj}\partial_t A_j$$

- accumulation of unit charge

$$Q = \frac{\delta S_{\text{eff}}}{\delta A_t} = \frac{n}{2\pi}\epsilon^{tij}\partial_i A_j$$





static analogue of Laughlin quasiparticle

Out of $\binom{36}{5} = 376992$ possibilies, there are 56 5-tuplets

there are 560 triplets, 280 4-tuplets

there are no N-tuplets for N > 5

Examples:

$\{\mathrm{Re\,VBS}, \mathrm{Im\,VBS}, \mathrm{CDW}, \mathrm{Re\,SSC}, \mathrm{Im\,SSC}\}$

$\{\mathrm{Re\,VBS}, \mathrm{Im\,VBS}, \mathrm{Neel}_x, \mathrm{Neel}_y, \mathrm{Neel}_z\}$ Tanaka-Hu

$\{\mathrm{Re\,SSC}, \mathrm{Im\,SSC}, \mathrm{QSHE}_x, \mathrm{QSHE}_y, \mathrm{QSHE}_z\}$ Grover-Senthil

$\{\mathrm{Neel}_x, \mathrm{Neel}_y, \mathrm{Im\,TSC}_z, \mathrm{Re\,TSC}_z, \mathrm{QSHE}_z\}$

....

IQHE mass term commutes with all the other

Answer:

Find two more masses M_4, M_5

such that: $\{M_a, M_b\} = \delta_{ab}, \quad a, b = 1, \ldots, 5$

--> "charge" accumulation at the core of hedgehog

type of charge = U(1) rotating M_4 and M_5

Example:

hedgehog in $\mathrm{QSHE}_{x,y,z}$ binds charge of $\mathrm{Re}\,\Delta$ and $\mathrm{Im}\,\Delta$

= electric chage

Derivation:

non-linear sigma model field thoery with WZW term

along the line of Abanov-Wiegmann

- Take any three masses $m_1 M_1 + m_2 M_2 + m_3 M_3$

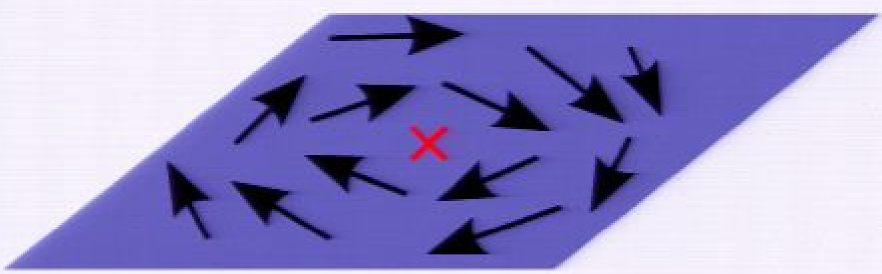
E.g. $\mathcal{H}_I = \Psi^\dagger \left(\lambda_x \hat{M}_{\mathrm{QSHE}_x} + \lambda_y \hat{M}_{\mathrm{QSHE}_y} + \lambda_z \hat{M}_{\mathrm{QSHE}_z} \right) \Psi$

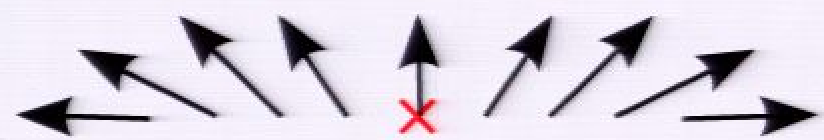
order parameter

"mass matrix"

order parameters
or "mass terms"

- Make a texture ("hedgehog defect") in masses: $\vec{m}(x) = \begin{pmatrix} m_1(x) \\ m_2(x) \\ m_3(x) \end{pmatrix}$





- When does the defect accumulate quantum number ("charge") of some sort ?

If it does, which type ?

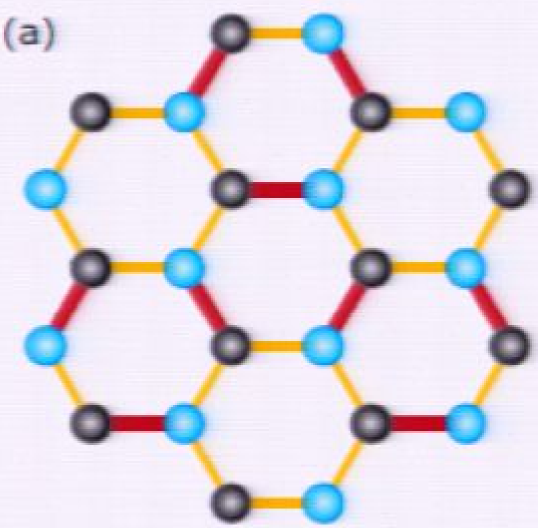
more masses (order parameters with full gap) ...

$\{\mathcal{H}_0(\boldsymbol{k}), M_*\} = 0$ --> full gap in energy spectrum

36 in total

quantum spin Hall effect: $\mathrm{QSHE}_{x,y,z}$ Kane-Mele

quantum Hall effect: QHE Haldane



singlet superconductor (SSC): $\mathrm{Re\,SSC}, \quad \mathrm{Im\,SSC}$

spin density wave: $\mathrm{Neel}_{x,y,z}$

valence bond solid: $\mathrm{Re\,VBS}, \quad \mathrm{Im\,VBS}$

charge density wave: CDW

n.n. spin-orbit coupling: $\mathrm{Re\,VBS}_{x,y,z} \quad \mathrm{Im\,VBS}_{x,y,z}$

triplet superconductors (TSC): $\mathrm{Re\,TSC}_{x,y,z}, \quad \mathrm{Im\,TSC}_{x,y,z}$

$\mathrm{Re\,TSC}_{02,x,y,z}, \quad \mathrm{Im\,TSC}_{02,x,y,z} \quad \mathrm{Re\,TSC}_{32,x,y,z}, \quad \mathrm{Im\,TSC}_{32,x,y,z}$

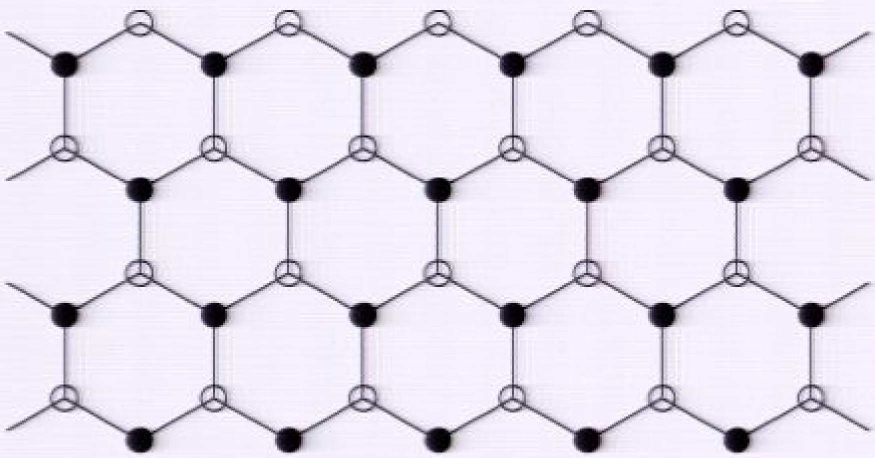
graphene kinetic term + mass

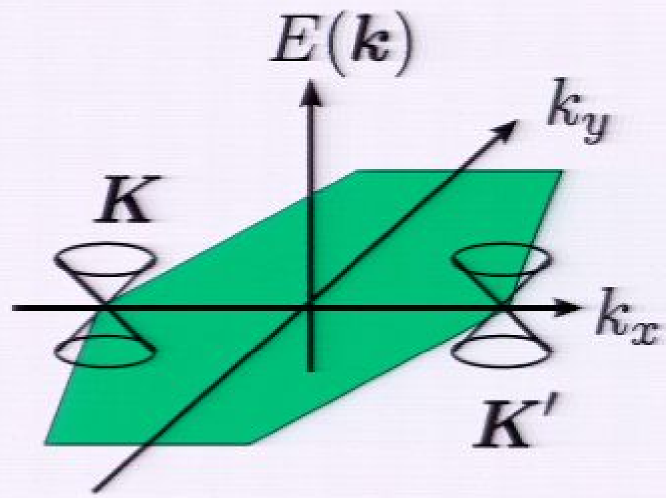
$$\mathcal{H}_0 = \sum_{s=\uparrow,\downarrow} \psi_s^\dagger \begin{pmatrix} \tau_x k_x + \tau_y k_y & 0 \\ 0 & -\tau_x k_x - \tau_y k_y \end{pmatrix} \psi_s$$

$\boldsymbol{K}$ $\quad$ $\boldsymbol{K'}$

graphene kinetic term

4x4x (spin)





To generate a gap, perturb H_0 by "mass terms":

$$\mathcal{H}_0 \to \mathcal{H}_0 + \mathcal{H}_I \qquad \mathcal{H}_I \sim \Psi^\dagger M \Psi$$

"mass matrix"