Title: Supersymmetric Configurations, Geometric Transitions and New Non-Kahler Manifolds

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Abstract: We give a detailed derivation of a supersymmetric configuration of wrapped D5-branes on a two-cycle of a warped resolved conifold. Our analysis reveals that the resolved conifold should support a non-Kahler metric with an SU(3) structure. We use this as a starting point of the geometric transition in type IIB theory. A mirror, and a subsequent flop transition using an intermediate M-theory configuration with a G2 structure, gives rise to the complete IR geometric transition in type IIA theory. A further mirror transformation gives the type IIB gravity dual of the IR gauge theory on the wrapped D5-branes. Expectedly non-Kahler deformations of the resolved and the deformed conifolds appear as the gravity duals of the confining gauge theories in type IIA and type IIB theories respectively, although in more generic cases these manifolds could also be non-geometric. In the local limit we reproduce precisely the scenarios presented in our earlier works. Our present work should therefore be viewed as providing a supergravity proof of geometric transitions in the full global scenarios in type II theories.

# Motivation for studying geometric transition

- QCD is asymptotically free, accessible to perturbation theory in UV, but not in IR, needs duality between UV and IR
- Gauge theory can be imbeded in string theory, and duality between UV and IR in gauge theory becomes geometric transition in string theory

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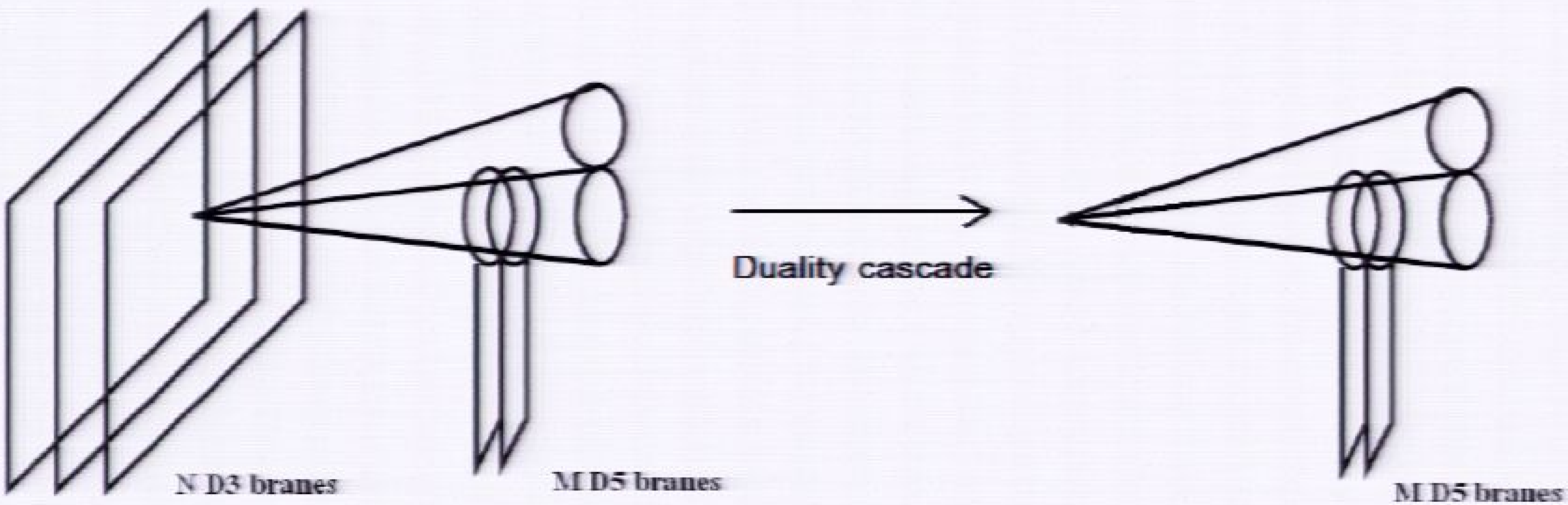
# Motivation

Gauge/Gravity dual:

| **Type IIB background** | **Gauge theory** |
|---|---|
| $AdS_5 \times S^5$ | N D3 branes<br>$\mathcal{N} = 4$ SU(N) SCFT |
| $AdS_5 \times M^5$ ($T^{1,1}$) | N D3 branes<br>$\mathcal{N} = 1$ (SU(N)$\times$ SU(N)) SCFT<br>**(Klebanov-Witten Model)** |
| $AdS_5 \times T^{1,1}$<br>NSNS and RR flux<br>near $r \to 0$ modified background<br>deformed conifold | N D3 branes and M D5 branes<br>$\mathcal{N} = 1$ SU(N)$\times$SU(N+M)<br>break CF invariance/running<br>far IR gaugino condensation<br>chiral symmetry breaking<br>**(Klebanov-Strassler Model)** |

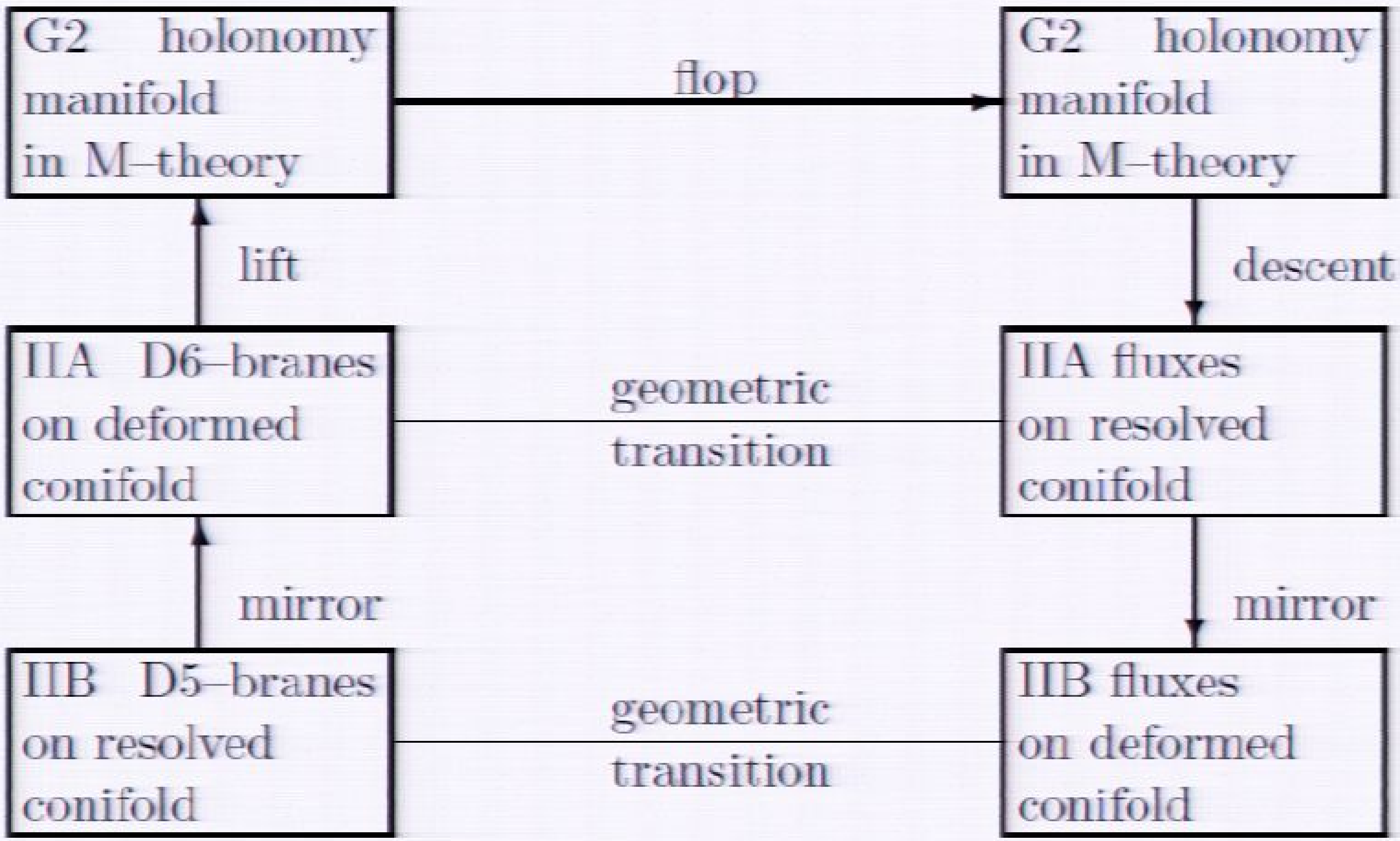
# Motivation

Duality Cascade: from UV to IR



# Motivation

Vafa's duality chain:



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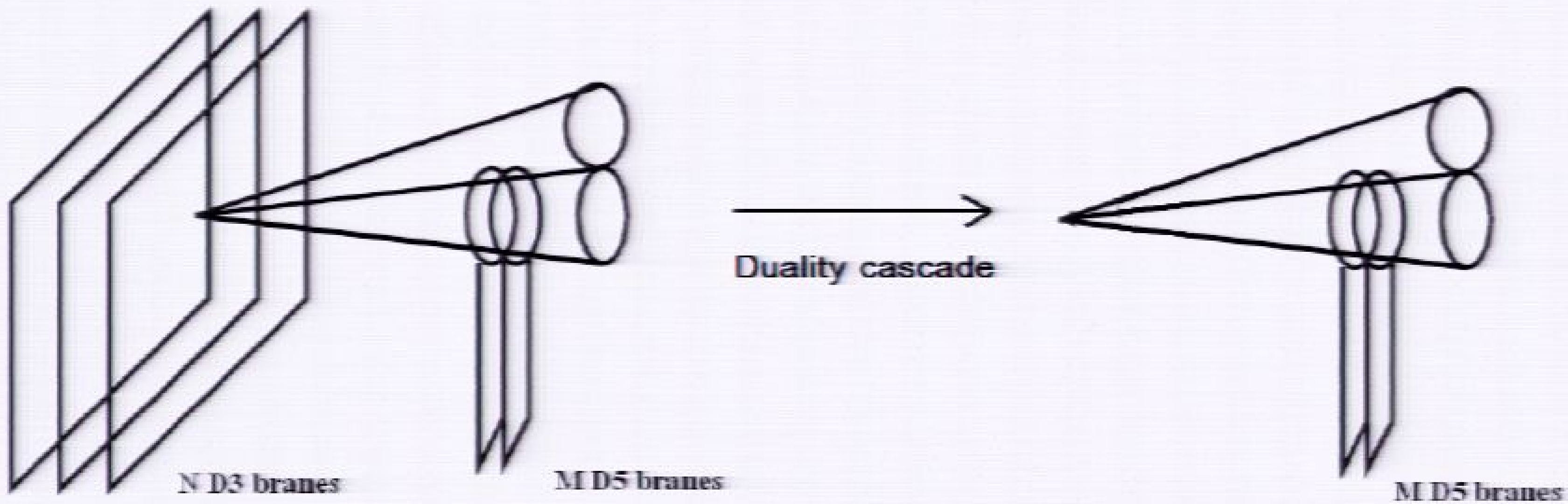

# Motivation

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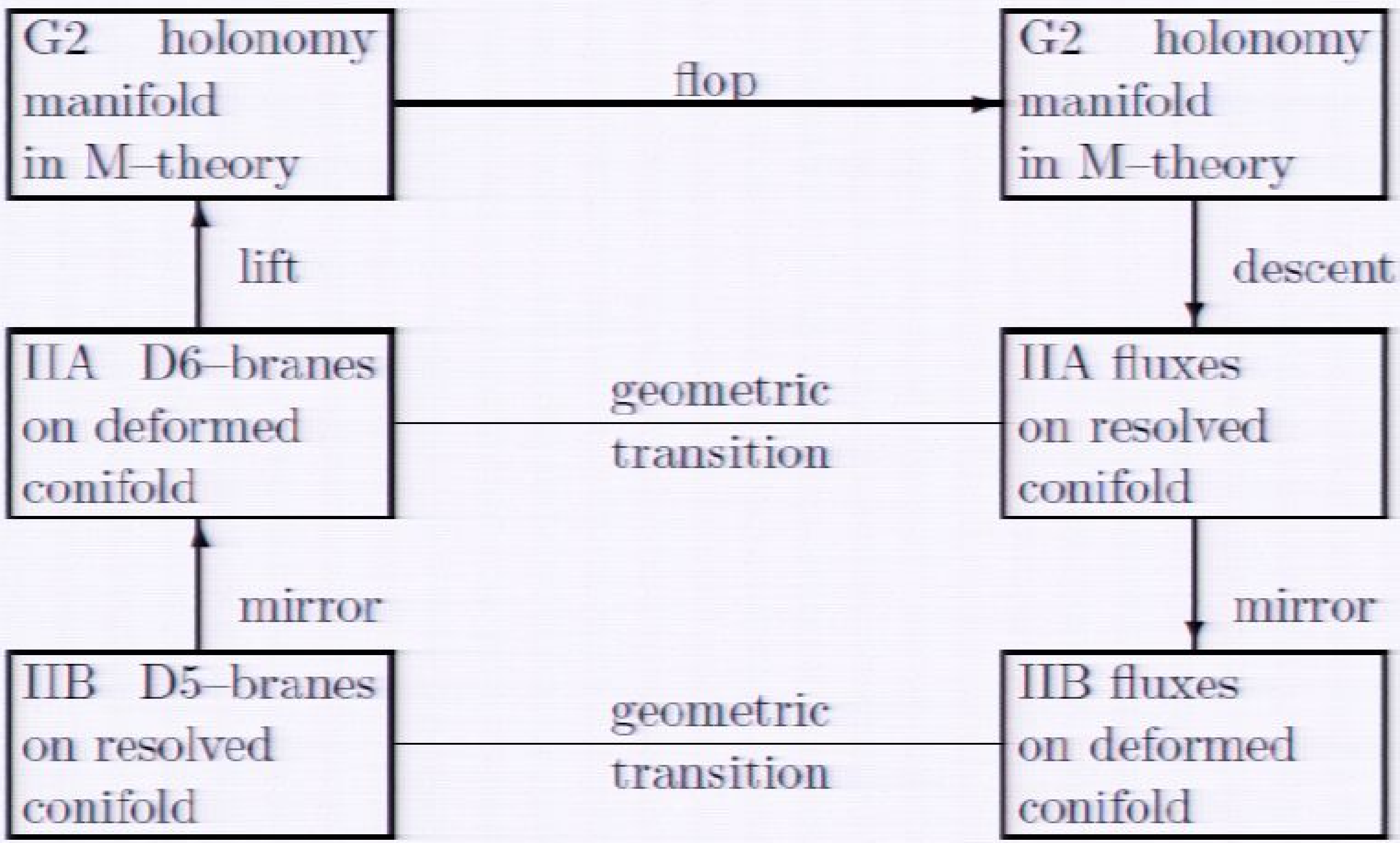


# Local picture (hep-th/0403288)

The metric is

$$ds^2 = dr^2 + \left(dz + \sqrt{\frac{\gamma'}{\gamma}} r_0 \cot\langle\theta_1\rangle dx + \sqrt{\frac{\gamma'}{(\gamma + 4a^2)}} r_0 \cot\langle\theta_2\rangle dy\right)^2 + \left[\frac{\gamma\sqrt{h}}{4} d\theta_1^2 + dx^2\right] + \left[\frac{(\gamma + a^2)\sqrt{h}}{4} d\theta_2^2 + dy^2\right] + \ldots (1)$$

The local $B_{NS}$ field was

$$B_{\rm NS} = b_{x\theta_i} dx \wedge d\theta_i + b_{y\theta_i} dy \wedge d\theta_i \qquad (2)$$

$F_3$ and $F_5$

# Local picture

The above background is invariant under the orbifold operation:

$$\mathcal{I}_{xy}: \; x \to -x, \quad y \to -y \tag{3}$$

and therefore can support D7/O7 states at the following orientifold points:

$$\frac{\mathbf{T}^2}{\mathcal{I}_{xy}\,\Omega\,(-1)^{F_L}} \tag{4}$$

Notice that $b_{xy}$ absent but $b_{xz}$ and $b_{yz}$ are allowed, so its mirror manifold can be non-geometric.

# Local picture

Away from the orientifold point, the local metric takes the following fibration form:

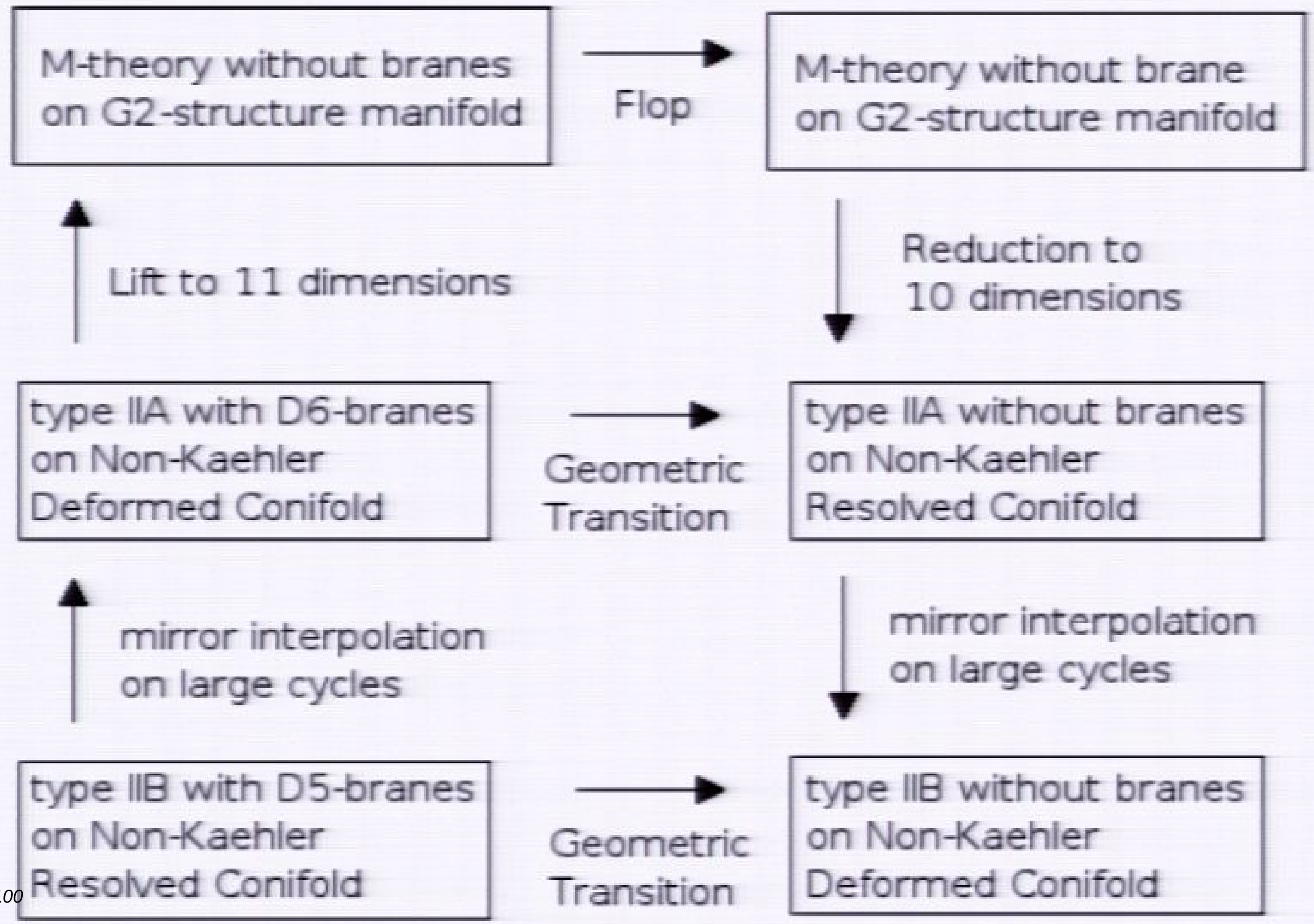
$$
\begin{aligned}
ds^2 &= dr^2 + (dz + \Delta_1 \cot\theta_1 dx + \Delta_2 \cot\theta_2 dy)^2 + \\
&\quad + \left(\frac{\gamma\sqrt{h}}{4} d\theta_1^2 + dx^2\right) + \left(\frac{(\gamma + 4a^2)\sqrt{h}}{4} d\theta_2^2 + dy^2\right) \\
H_3 &= d\mathcal{J}_1 \wedge d\theta_1 \wedge dx + d\mathcal{J}_2 \wedge d\theta_2 \wedge dy \\
F_5 &= K(r)\,(1 + *)\; dx \wedge dy \wedge dz \wedge d\theta_1 \wedge d\theta_2 \qquad (5) \\
F_3 &= c_1\,(dz \wedge d\theta_2 \wedge dy - dz \wedge d\theta_1 \wedge dx)
\end{aligned}
$$

# From local to global

- full global geometry is a six–dimensional Kahler manifold with F-theory seven-branes
- full global geometry is non–Kahler with or without F-theory seven-branes

# Global picture

Our duality chain, global and supersymmetric:



## Starting point

Start from

$$ds^2 = h^{1/2} e^{\phi} d\tilde{s}^2_{0123} + h^{-1/2} e^{\phi} ds_6^2 \quad (6)$$

where

$$h = \frac{e^{-2\phi} F_0^{-4}}{e^{-2\phi} h^{-2} F_0^{-4} \cosh^2\beta - \sinh^2\beta}, \qquad d\tilde{s}^2_{0123} = F_0 ds^2_{0123}$$

$$ds_6^2 = F_1\, dr^2 + F_2 (d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2)^2 + \sum_{i=1}^{2} F_{2+i} (d\theta_i^2 + \sin^2\theta_i d\phi_i^2)$$

To preserve supersymmetry, we have

$$H_{\rm NS} = e^{2\phi} * d\left(e^{-2\phi} J\right) \quad (7)$$

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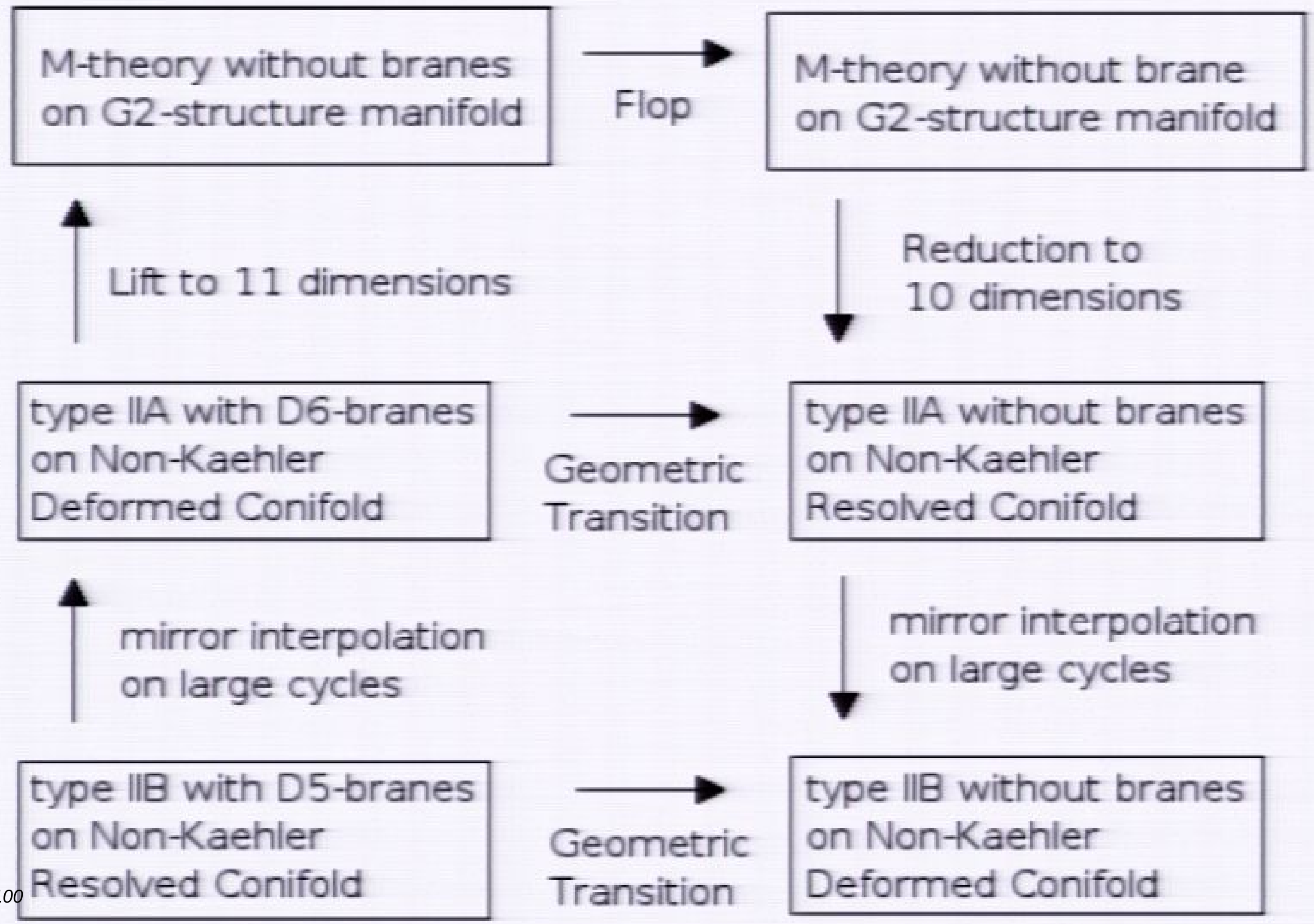

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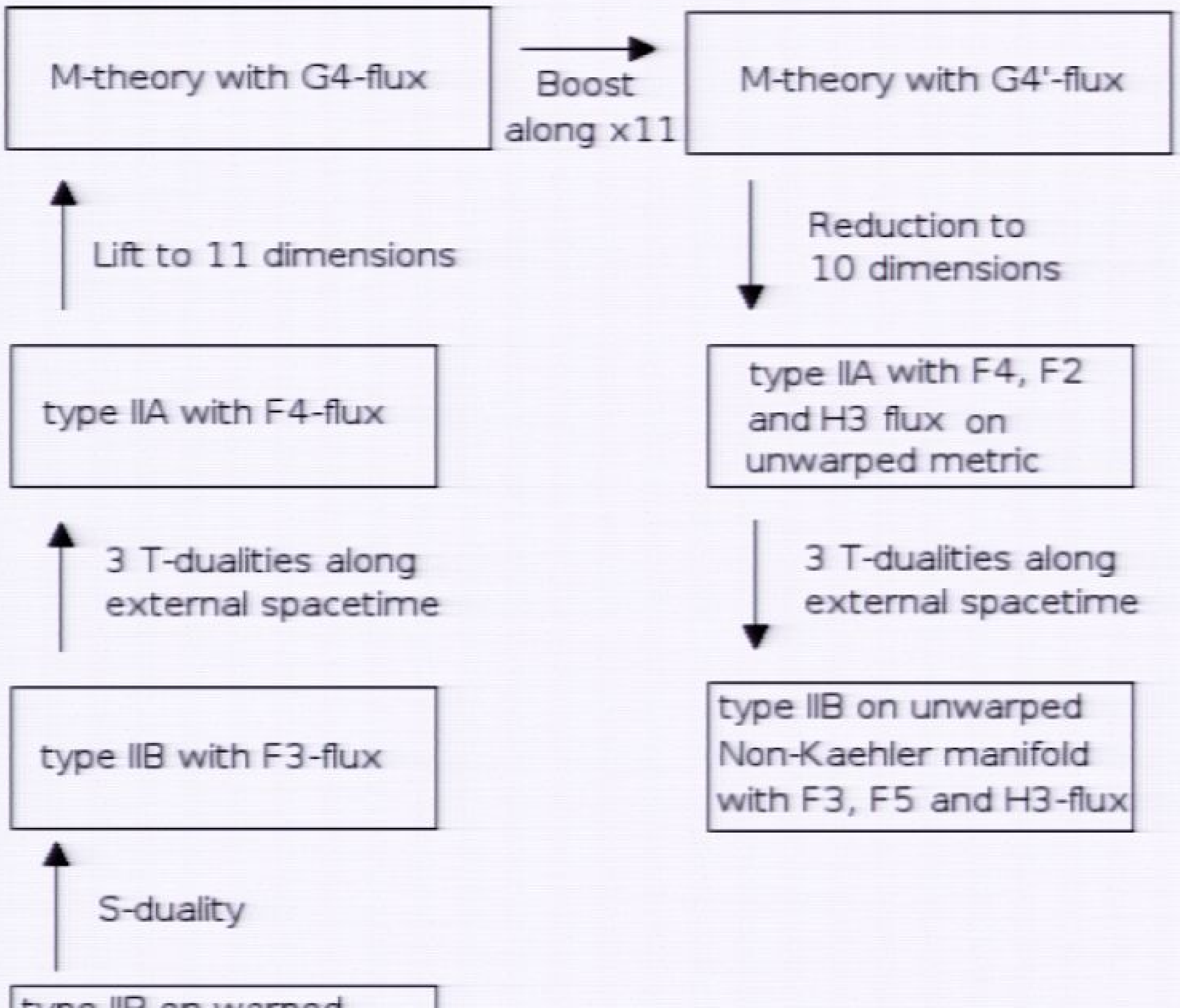
# Starting point(0906.0591)



# Starting point

We get Supersymmetric type IIB background:

$$F_3 = h \cosh\beta \; e^{2\phi} * d\left(e^{-2\phi} J\right),$$

$$H_3 = -hF_0^2 \sinh\beta \; e^{2\phi} d\left(e^{-2\phi} J\right)$$

$$F_5 = -\frac{1}{4}(1+*)dA_0 \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3, \qquad \phi_{\rm now} = -\phi$$

$$ds^2 = F_0 ds^2_{0123} + F_1\, dr^2 + F_2(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2)^2$$

$$+ \sum_{i=1}^{2} F_{2+i}(d\theta_i^2 + \sin^2\theta_i d\phi_i^2) \qquad (8)$$

# Mirror interpolation from IIB to IIA

Strominger Yau Zaslow(SYZ) Conjecture: Mirror symmetry is three T dualities.

- semi flat:the metric only depends on the base coordinates
- large base

# Mirror interpolation from IIB to IIA

SYZ works in the large base limit, to large the base:

$$d\psi \rightarrow d\psi + f_1 \cos\theta_1 \, d\theta_1 + f_2 \cos\theta_2 \, d\theta_2$$
$$d\phi_1 \rightarrow d\phi_1 - f_1 \, d\theta_1, \qquad d\phi_2 \rightarrow d\phi_2 - f_2 \, d\theta_2 \quad (9)$$

and we get

$$(1+f_i^2)d\theta_i^2 + 2f_i \sin^2\theta_i d\phi_i d\theta_i \qquad (10)$$

further more

$$g_{\psi\psi} \mapsto g_{\psi\psi}(1-\epsilon) \qquad (11)$$

where we take $f_i \mapsto \infty$ and $\epsilon \mapsto 0$ but $\frac{f_1 f_2}{\epsilon}$ fixed

# Mirror interpolation from IIB to IIA

Type IIA metric:

$$
\begin{aligned}
ds_6^2 &= F_1 dr^2 + \frac{\alpha F_2}{\Delta_1\Delta_2}\Big[d\psi - b_{\psi r}dr + \Delta_1 \cos\theta_1\Big(d\phi_1 \\
&- b_{\phi_1\theta_1}d\theta_1 - b_{\phi_1 r}dr\Big) + \Delta_2 \cos\,\theta_2\Big(d\phi_2 - b_{\phi_2\theta_2}d\theta_2 - b_{\phi_2 r}dr\Big)\Big]^2 \\
&+ \alpha j_{\phi_2\phi_2}\Big(d\phi_1 - b_{\phi_1\theta_1}d\theta_1 - b_{\phi_1 r}dr\Big)^2 + \Big(F_3 - F_2\beta_1^2\cos^2\theta_1\Big)d\theta_1^2 \\
&+ \alpha j_{\phi_1\phi_1}\Big(d\phi_2 - b_{\phi_2\theta_2}d\theta_2 - b_{\phi_2 r}dr\Big)^2 + \Big(F_4 - F_2\beta_2^2\cos^2\theta_2\Big)d\theta_2^2 \\
&- 2\alpha j_{\phi_1\phi_2}\Big(d\phi_1 - b_{\phi_1\theta_1}d\theta_1 - b_{\phi_1 r}dr\Big)\Big(d\phi_2 - b_{\phi_2\theta_2}d\theta_2 - b_{\phi_2 r}dr\Big) \\
&- 2j_{\phi_1\phi_2}\beta_1\beta_2\; d\theta_1 d\theta_2
\end{aligned}
\tag{12}
$$

where $\beta_i = f_i\sqrt{\epsilon}$

# Mirror interpolation from IIB to IIA

To get a "Non–Kahler deformed conifold", we do a rotation between $d\theta_1$ and $d\tilde{\phi}_1 = d\phi_1 - b_{\phi_1\theta_1}d\theta_1 - b_{\phi_1 r}dr$

$$
\begin{aligned}
ds_6^2 \;=\; & F_1 dr^2 + \frac{\alpha F_2}{\Delta_1\Delta_2}\Big[d\psi - b_{\psi r}dr - b_{\psi\theta_2}d\theta_2 \\
& + \Delta_1 \cos\theta_1 d\tilde{\phi}_1 + \Delta_2 \cos\theta_2 \cos\psi_0 d\tilde{\phi}_2\Big]^2 \\
& + \alpha j_{\phi_2\phi_2}\Big[d\theta_1^2 + d\tilde{\phi}_1^{\,2}\Big] + \alpha j_{\phi_1\phi_1}\Big[d\theta_2^2 + d\tilde{\phi}_2^{\,2}\Big] \\
& + 2\alpha j_{\phi_1\phi_2}\cos\psi_0\Big[d\theta_1 d\theta_2 - d\tilde{\phi}_1 d\tilde{\phi}_2\Big] \\
& + 2\alpha j_{\phi_1\phi_2}\sin\pi_0\Big[d\tilde{\phi}_1 d\theta_2 + d\tilde{\phi}_2 d\theta_1\Big] \qquad (13)
\end{aligned}
$$

# Mirror interpolation from IIB to IIA

- $g_{\psi\psi} \mapsto g_{\psi\psi}(1-\epsilon)$
- $\psi_0 \mapsto \psi$

SUSY?

# Mirror interpolation from IIB to IIA

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$$
\begin{aligned}
ds_6^2 \;\; = \;\; & F_1 dr^2 + \frac{\alpha F_2}{\Delta_1 \Delta_2}\Big[d\psi - b_{\psi r} dr - b_{\psi\theta_2} d\theta_2 \\
& + \Delta_1 \cos\theta_1 d\tilde{\phi}_1 + \Delta_2 \cos\theta_2 \cos\psi_0 d\tilde{\phi}_2\Big]^2 \\
& + \alpha j_{\phi_2\phi_2}\Big[d\theta_1^2 + d\tilde{\phi}_1^2\Big] + \alpha j_{\phi_1\phi_1}\Big[d\theta_2^2 + d\tilde{\phi}_2^2\Big] \\
& + 2\alpha j_{\phi_1\phi_2} \cos\psi_0 \Big[d\theta_1 d\theta_2 - d\tilde{\phi}_1 d\tilde{\phi}_2\Big] \\
& + 2\alpha j_{\phi_1\phi_2} \sin\pi_0 \Big[d\tilde{\phi}_1 d\theta_2 + d\tilde{\phi}_2 d\theta_1\Big] \qquad (13)
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$$
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ds_6^2 \;=\; & F_1 dr^2 + \frac{\alpha F_2}{\Delta_1\Delta_2}\Big[d\psi - b_{\psi r}dr - b_{\psi\theta_2}d\theta_2 \\
& + \Delta_1 \cos\theta_1 d\tilde{\phi}_1 + \Delta_2 \cos\theta_2 \cos\psi_0 d\tilde{\phi}_2\Big]^2 \\
& + \alpha j_{\phi_2\phi_2}\Big[d\theta_1^2 + d\tilde{\phi}_1^{\,2}\Big] + \alpha j_{\phi_1\phi_1}\Big[d\theta_2^2 + d\tilde{\phi}_2^{\,2}\Big] \\
& + 2\alpha j_{\phi_1\phi_2}\cos\psi_0\Big[d\theta_1 d\theta_2 - d\tilde{\phi}_1 d\tilde{\phi}_2\Big] \\
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\end{aligned}
$$

# Mirror interpolation from IIB to IIA

- $g_{\psi\psi} \mapsto g_{\psi\psi}(1-\epsilon)$
- $\psi_0 \mapsto \psi$

SUSY?

# M theory lift and flop

Lift to M theory:

$$
\begin{aligned}
ds_{11}^2 &= e^{-\frac{2\phi}{3}} ds_{10} + e^{\frac{4\phi}{3}} \Big[ dx_{11} + \widetilde{A}_{\phi_1} d\phi_1 + \widetilde{A}_{\phi_2} d\phi_2 \\
&\quad + \widetilde{A}_{\theta_1} + \widetilde{A}_{\theta_2} d\theta_2 + \widetilde{A}_r dr \Big]^2 \qquad (14)
\end{aligned}
$$

# M theory lift and flop

To do the flop we define the one forms:

$$
\begin{aligned}
\sigma_1 &= \sin\psi_1(d\phi_1 - \tan\lambda_4 dr) + \sec\lambda_1 \cos(\psi_1 + \lambda_1) d\theta_1, \\
\sigma_2 &= \cos\psi_1(d\phi_1 - \tan\lambda_4 dr) - \sec\lambda_1 \sin(\psi_1 + \lambda_1) d\theta_1, \\
\sigma_3 &= d\psi_1 - \frac{1}{2}\tan\lambda_3 dr + \Delta_1 \cos\theta_1(d\phi_1 - \tan\lambda_1 d\theta_1 - \tan\lambda_4 dr)
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_1 &= -\sin\psi_2(d\phi_2 - \tan\lambda_5 dr) + \sec\lambda_2 \cos(\psi_2 + \lambda_2) d\theta_2, \\
\Sigma_2 &= -\cos\psi_2(d\phi_2 - \tan\lambda_5 dr) - \sec\lambda_2 \sin(\psi_2 + \lambda_2) d\theta_2, \\
\Sigma_3 &= d\psi_2 + \frac{1}{2}\tan\lambda_3 dr - \Delta_2 \cos\theta_2(d\phi_2 - \tan\lambda_2 d\theta_2 - \tan\lambda_5 dr)
\end{aligned}
$$

# M theory lift and flop

Now the metric can be expressed in terms of these one forms:

$$
\begin{aligned}
ds_7^2 \quad &= g_r dr^2 + g_1(\sigma_3 + \Sigma_3)^2 + g_2(\sigma_3 - \Sigma_3)^2 \\
&+ g_3(\sin\psi_1\sigma_1 + \cos\psi_1\sigma_2)^2 + \widetilde{g}_3(\cos\psi_1\sigma_1 - \sin\psi_1\sigma_2)^2 \\
&+ g_4(\sin\psi_2\Sigma_1 + \cos\psi_2\Sigma_2)^2 + \widetilde{g}_4(\cos\psi_2\Sigma_1 - \sin\psi_2\Sigma_2)^2 \\
&+ g_5(\sin\psi_1\sigma_1 + \cos\psi_1\sigma_2)(\sin\psi_2\Sigma_1 + \cos\psi_2\Sigma_2) \\
&- \widetilde{g}_5(\cos\psi\sigma_1 - \sin\psi_1\sigma_2)(\cos\psi_2\Sigma_1 - \sin\psi_2\Sigma_2) \qquad (15)
\end{aligned}
$$

# M theory lift and flop

The flop:

$$\sigma_1 \mapsto a\sigma_1 + b\Sigma_1, \quad \Sigma_1 \mapsto e\sigma_1 + f\Sigma_1,$$
$$\sigma_2 \mapsto c\sigma_2 + d\Sigma_2, \quad \Sigma_2 \mapsto g\sigma_2 + h\Sigma_2,$$
$$\sigma_3 + \Sigma_3 \mapsto \sigma_3 - \Sigma_3, \quad \sigma_3 - \Sigma_3 \mapsto \sigma_3 + \Sigma_3 \qquad (16)$$

# M theory lift and flop

Metric after flop:

$$ds_{11}^2 = e^{-\frac{2\phi}{3}} F_0 ds_{0123}^2 + g_r dr^2 + g_1(\sigma_3 - \Sigma_3)^2 + g_2(\sigma_3 + \Sigma_3)^2 + a^2(k^2 G_2 + k G_3 + G_1)(\sigma_1^2 + \sigma_2^2) + b^2(\mu^2 G_2 + \mu G_3 + G_1)(\Sigma_1^2 + \Sigma_2^2) \quad (17)$$

# Reduction to Type IIA

Type IIA metric after reduction, which is "Non–Kahler resolved conifold":

$$
\begin{aligned}
ds_6^2 \;=\; & F_1 dr^2 + e^{2\phi}\Big[d\psi - b_{\psi\mu}dx^\mu \\
& + \widetilde{\Delta}_1 \cos\theta_1\Big(d\phi_1 - b_{\phi_1\theta_1}d\theta_1 - b_{\phi_1 r}dr\Big) \\
& + \widetilde{\Delta}_2 \cos\theta_2\Big(d\phi_2 - b_{\phi_2\theta_2}d\theta_2 - b_{\phi_2 r}dr\Big)\Big]^2 \\
& + e^{\frac{2\phi}{3}}a^2(k^2 G_2 + kG_3 + G_1)\Big[d\theta_1^2 + (d\phi_1^2 - b_{\phi_1\theta_1}d\theta_1 - b_{\phi_1 r}dr)^2\Big] \\
& + e^{\frac{2\phi}{3}}b^2(\mu^2 G_2 + \mu G_3 + G_1)\Big[d\theta_2^2 + (d\phi_2^2 - b_{\phi_2\theta_2}d\theta_2 - b_{\phi_2 r}dr)^2\Big]
\end{aligned}
$$

# Reduction to Type IIA

The fluxes generically have the following form

$$B_{\mathrm{now}} = \bar{b}_{ij} dx^i \wedge dx^j, \qquad F_{\mathrm{now}} = \bar{f}_{ijkl} dx^i \wedge dx^j \wedge dx^k \wedge dx^l \quad (18)$$

**No Branes!**

Type IIB mirror will be generically NON–GEOMETRIC!

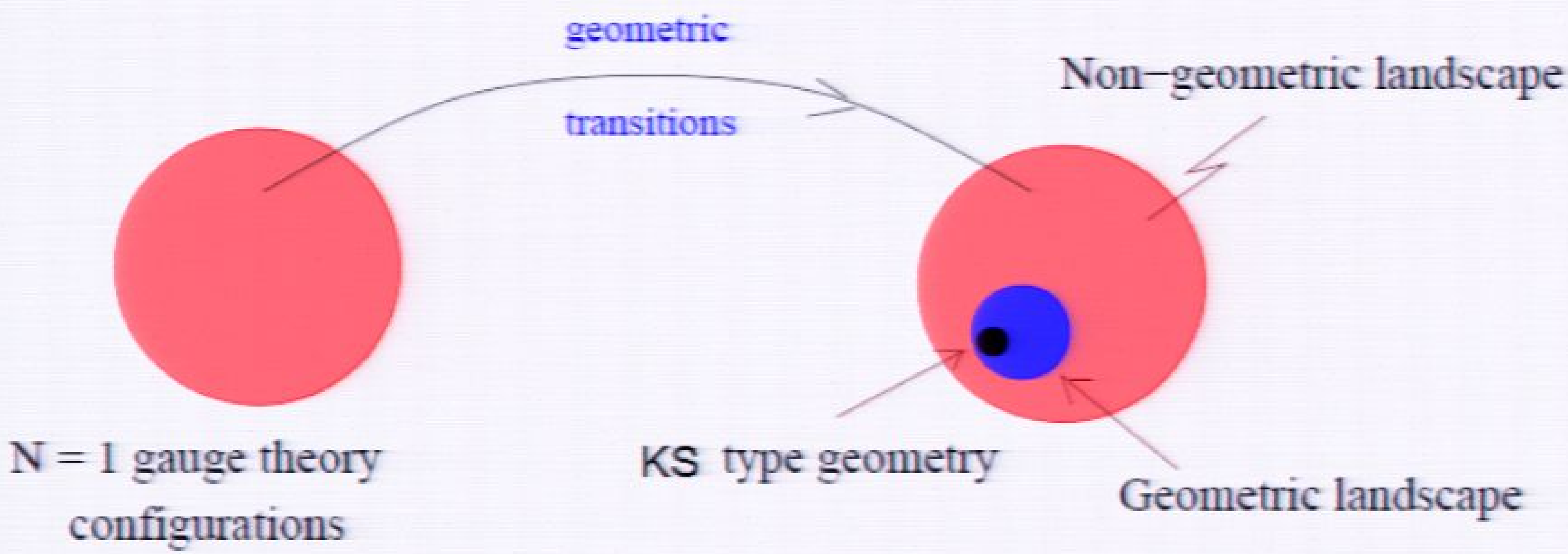
# Mirror interpolation from IIA to IIB

Taking $b_{r\phi_i} = b_{r\psi} = 0$, we find the Type IIB metric is "Non–Kahler deformed conifold":

$$
\begin{aligned}
ds^2 &= F_0^2 ds_{0,1,2,3}^2 + g_{rr} dr^2 + g_{\psi\psi}\left(\widetilde{\mathcal{D}}\psi + \widehat{\Delta}_1\, \widetilde{\mathcal{D}}\phi_1 + \widehat{\Delta}_2\, \widetilde{\mathcal{D}}\phi_2\right)^2 \\
&\quad + g_{\theta_1\theta_1}\left(d\theta_1^2 + \widetilde{\mathcal{D}}\phi_1^2\right) + g_{\theta_2\theta_2}\left(d\theta_2^2 + \widetilde{\mathcal{D}}\phi_2^2\right) \\
&\quad + g_{\theta_1\theta_2}\left(d\theta_1 d\theta_2 + \widehat{\Delta}_3\, \widetilde{\mathcal{D}}\phi_1 \widetilde{\mathcal{D}}\phi_2\right)
\end{aligned}
$$

**No Branes!**

# Type IIB Mirror configuration



# Discussions

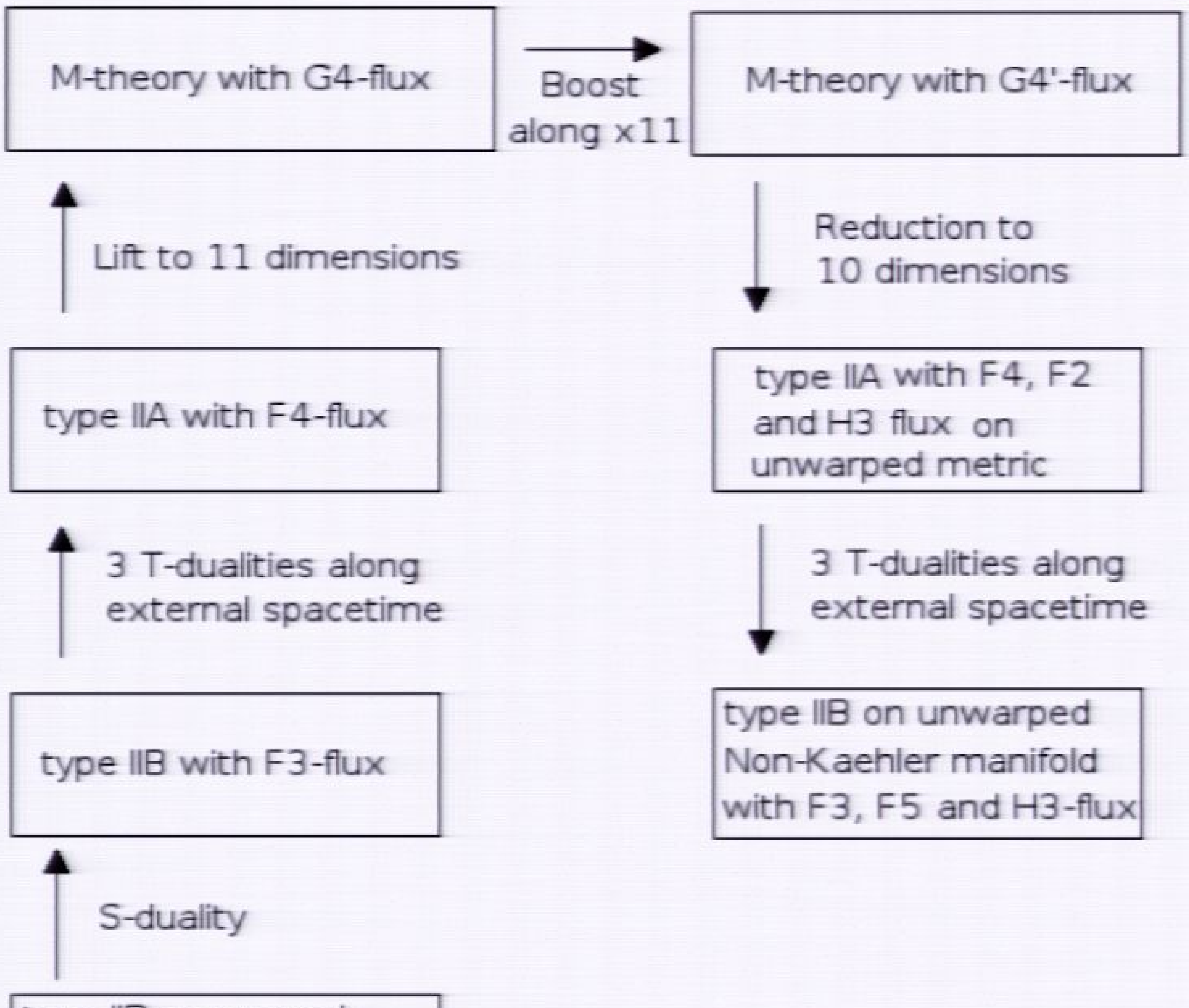
- Non–geometric aspects of the mirror configurations
- SUSY condition, in progress
- Heterotic GT, in progress
- UV completion

# Mirror interpolation from IIB to IIA

- $g_{\psi\psi} \mapsto g_{\psi\psi}(1-\epsilon)$
- $\psi_0 \mapsto \psi$

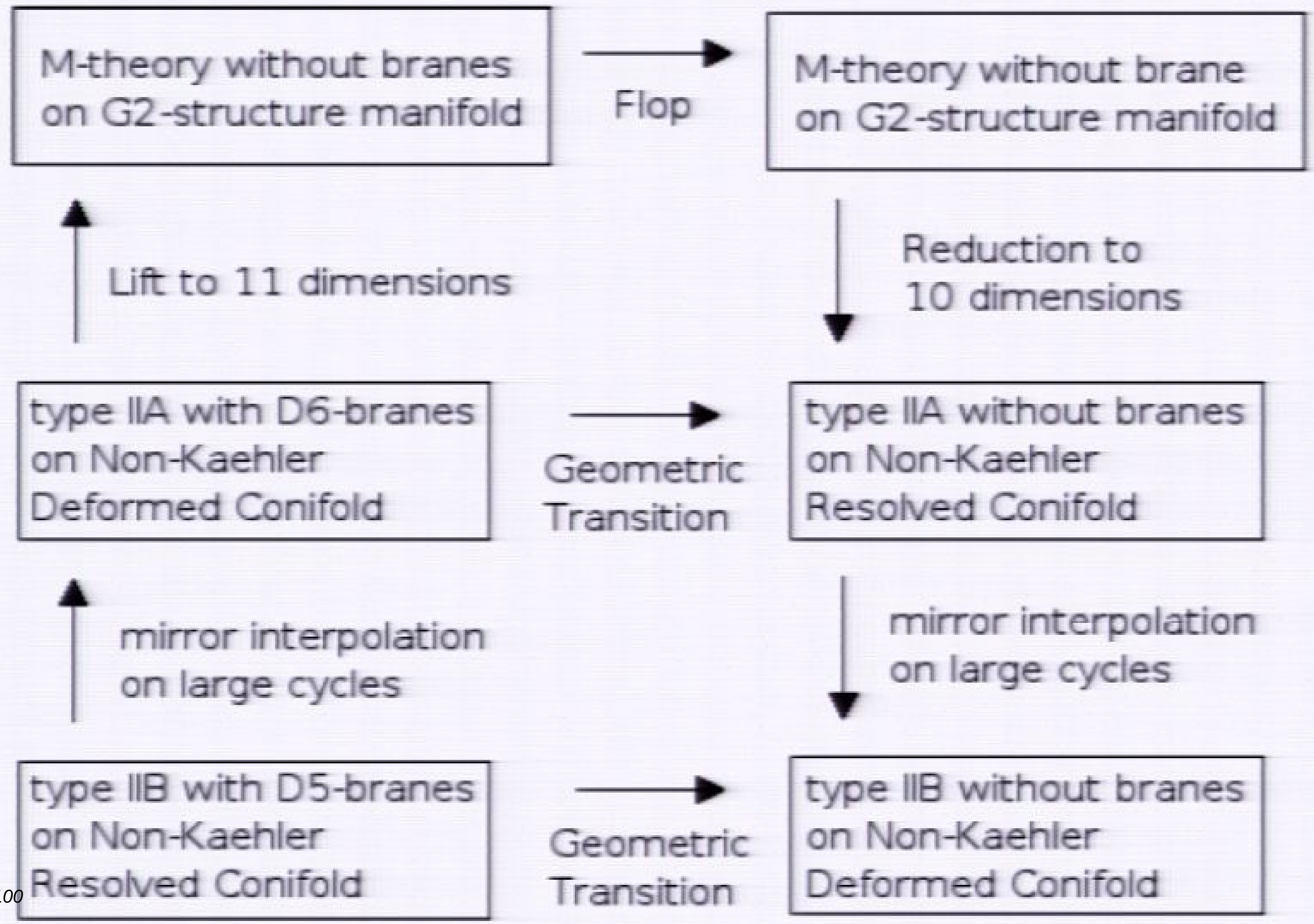
SUSY?

# Starting point(0906.0591)



# Global picture

Our duality chain, global and supersymmetric:

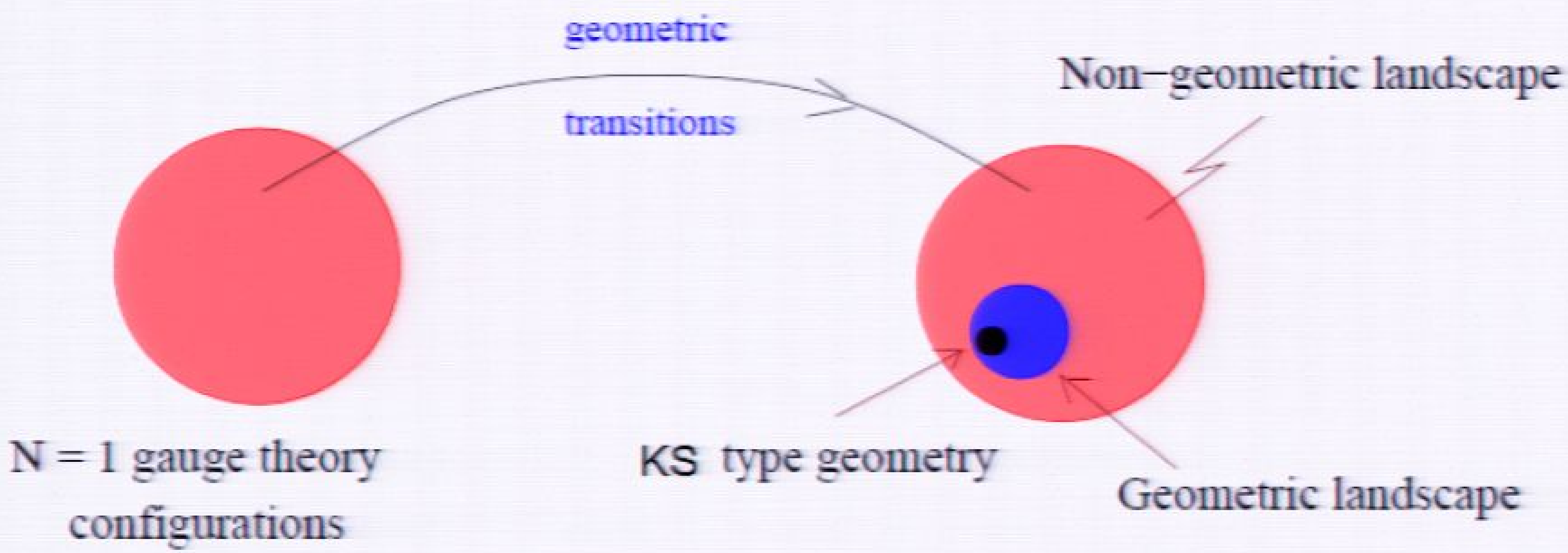


# M theory lift and flop

Lift to M theory:

$$ds_{11}^2 = e^{-\frac{2\phi}{3}} ds_{10} + e^{\frac{4\phi}{3}} \Big[ dx_{11} + \widetilde{A}_{\phi_1} d\phi_1 + \widetilde{A}_{\phi_2} d\phi_2 + \widetilde{A}_{\theta_1} + \widetilde{A}_{\theta_2} d\theta_2 + \widetilde{A}_r dr \Big]^2 \qquad (14)$$

# Type IIB Mirror configuration



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