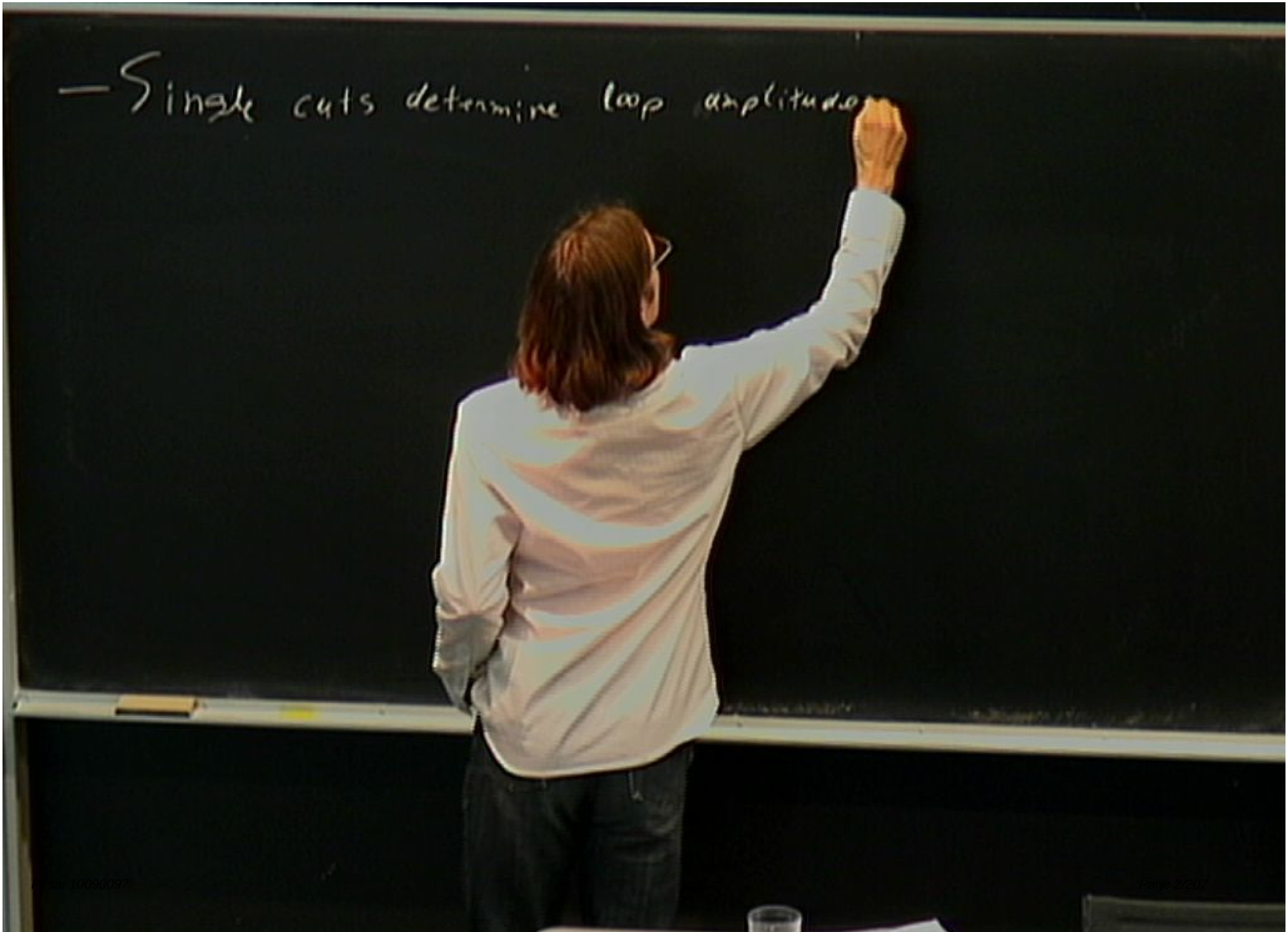
Title: Scattering amplitudes from single-cuts - Lecture 3

Date: Sep 17, 2010 02:00 PM

URL: http://pirsa.org/10090097

Abstract:



—Single cuts determine loop amplitudes

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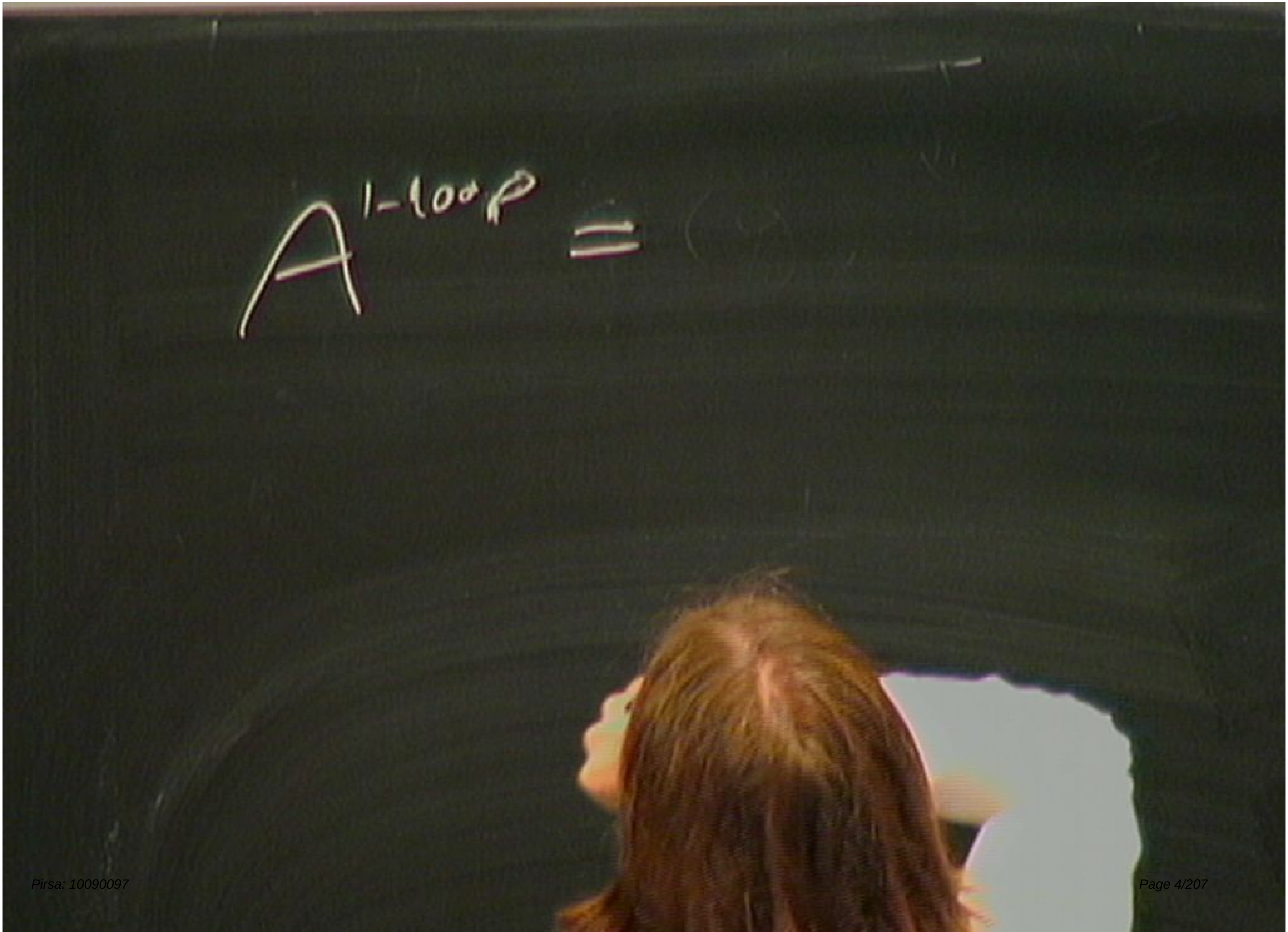
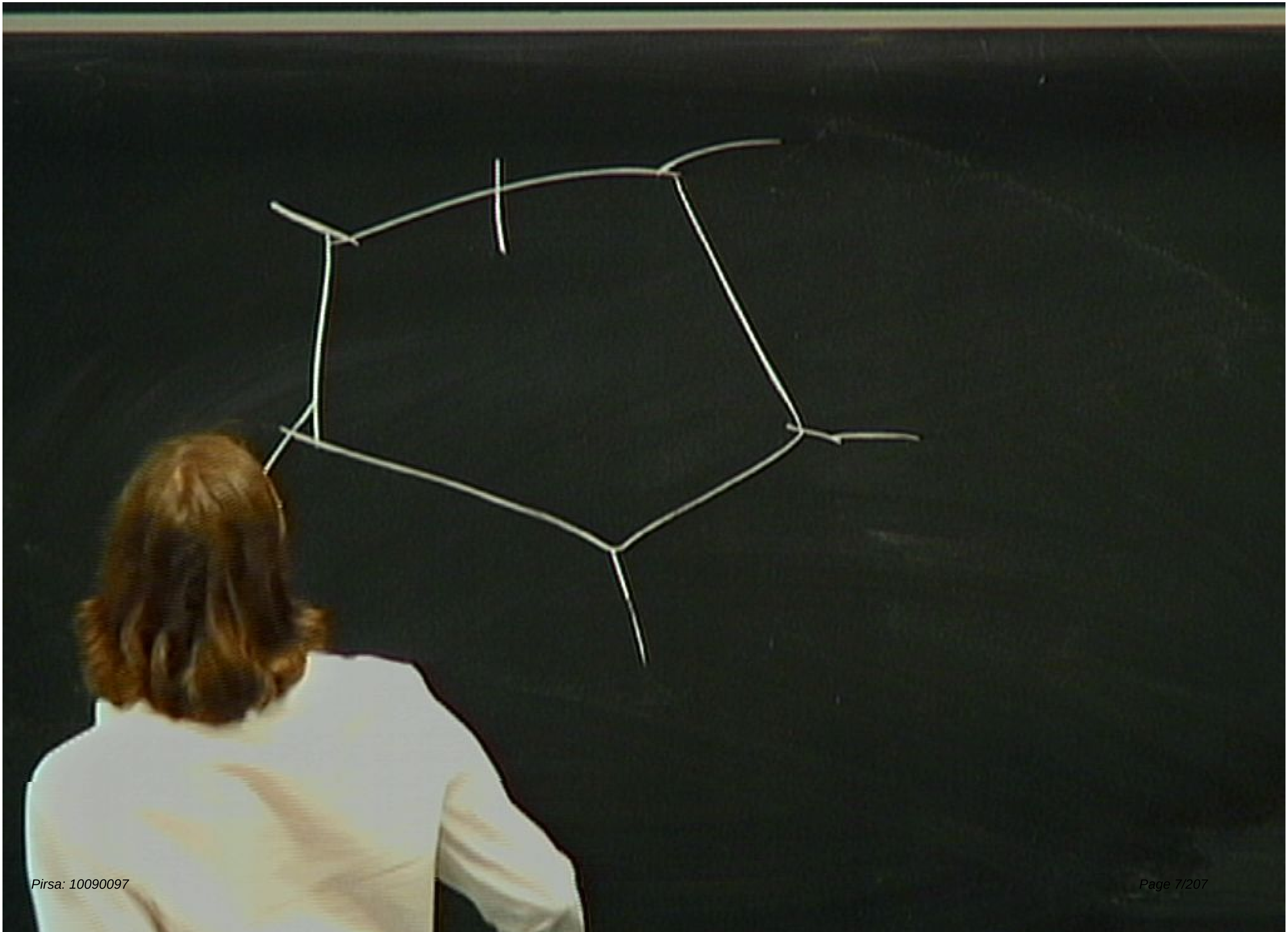
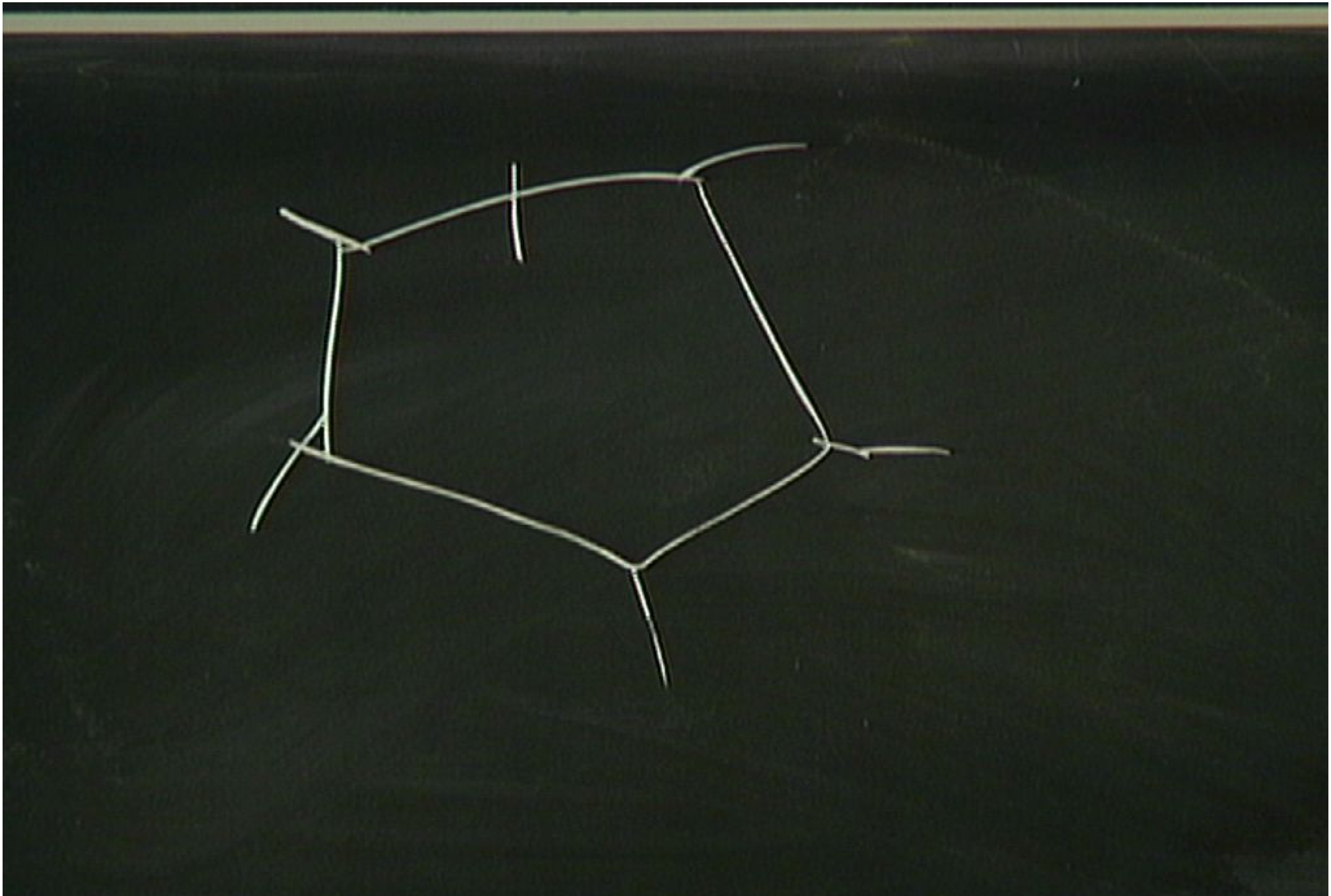
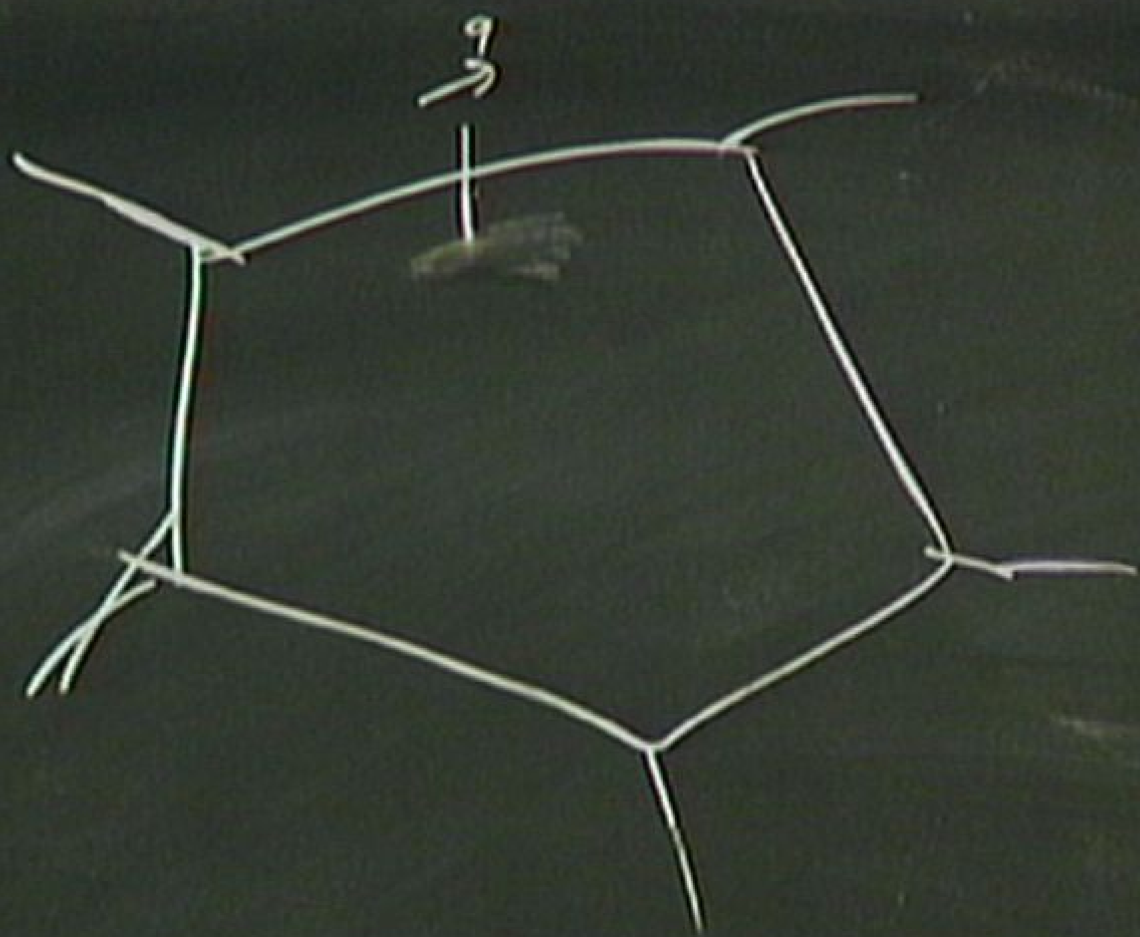

$A^{1\text{-loop}} =$

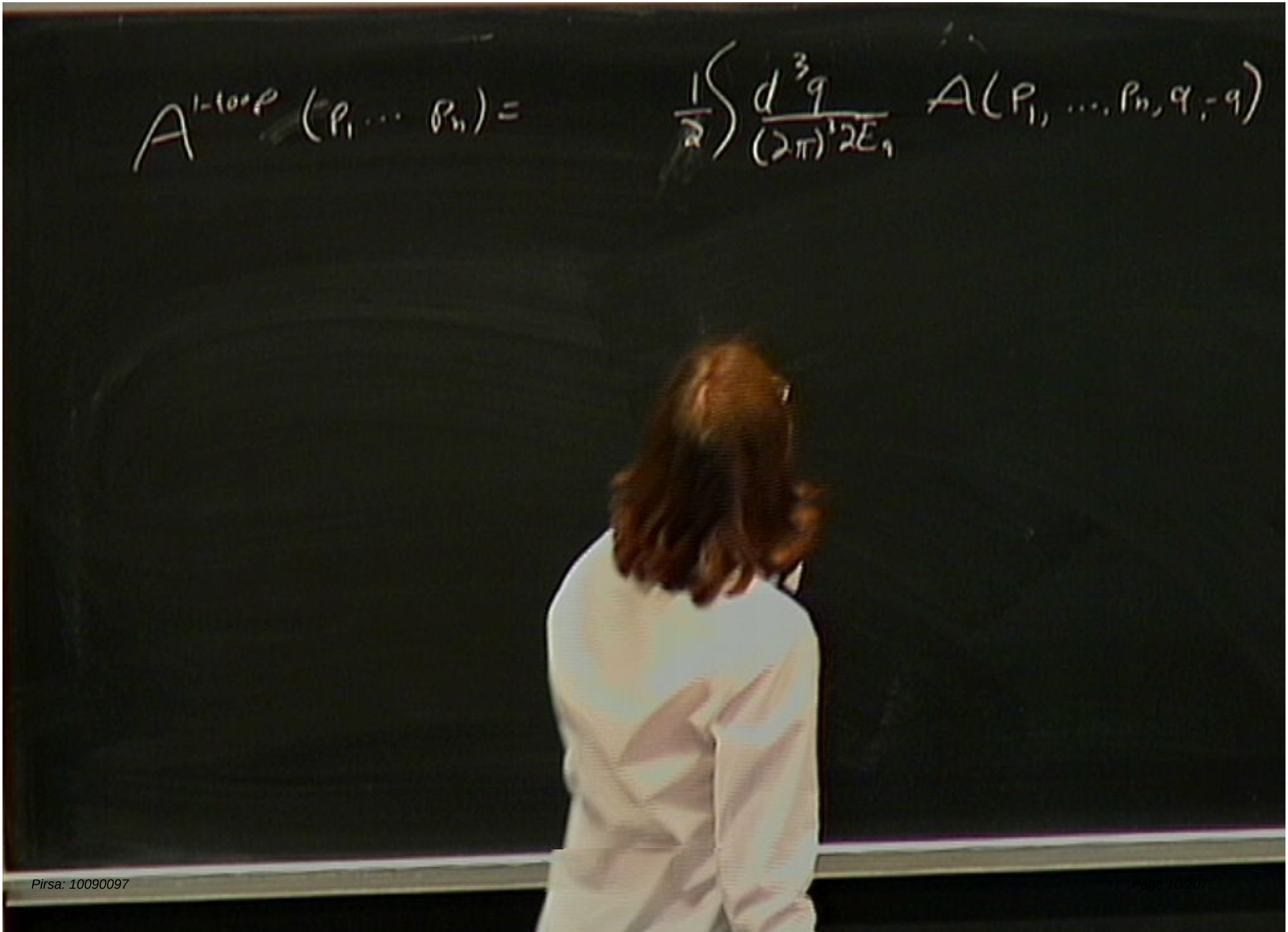
$$A^{1\text{-}loop}(p_1 \dots p_n) =$$

$$\frac{1}{g}\int \frac{d^3q}{(2\pi)^3\, 2E_q}\, A(p_1, \dots p_n, q; -q)$$



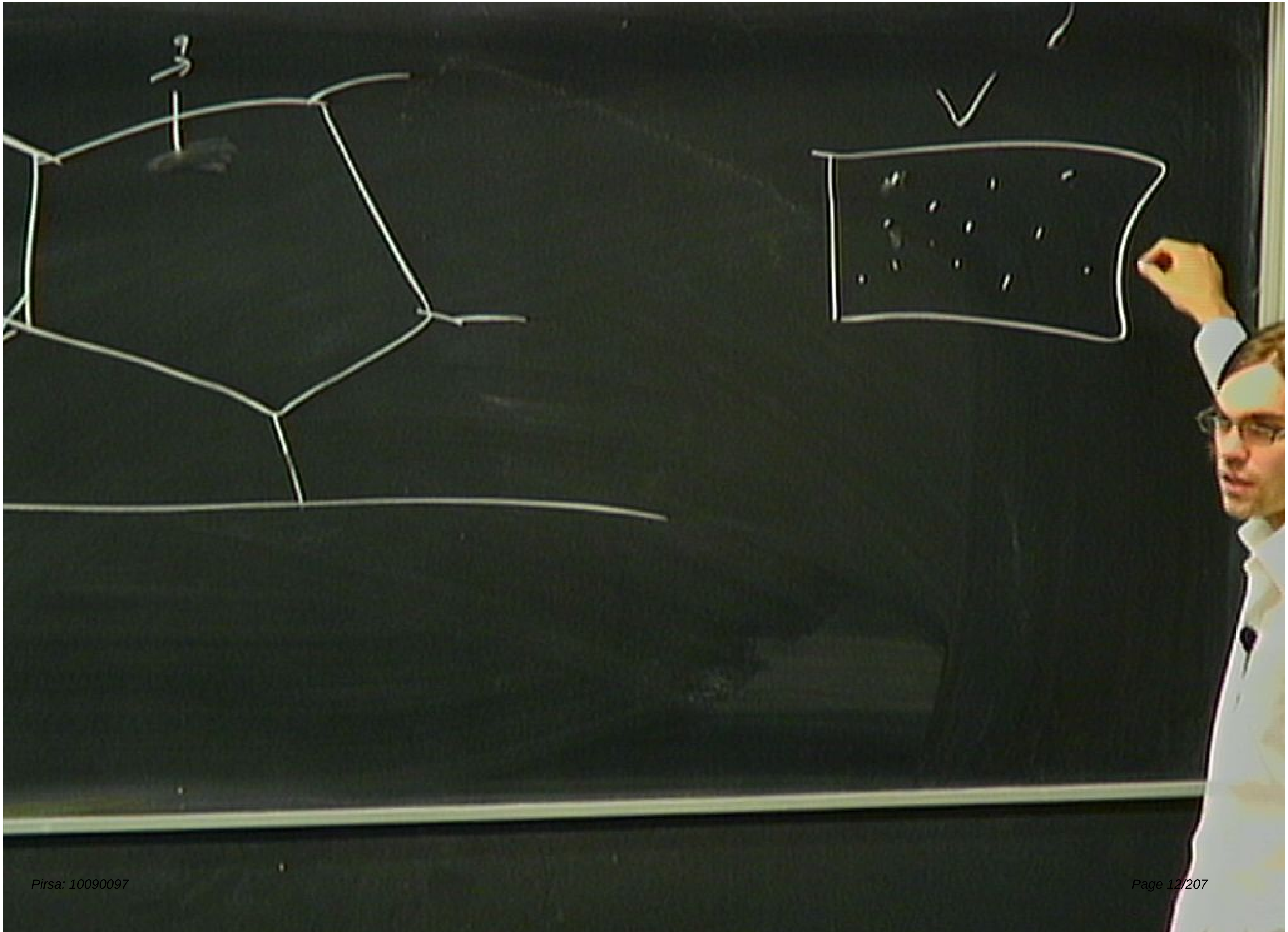


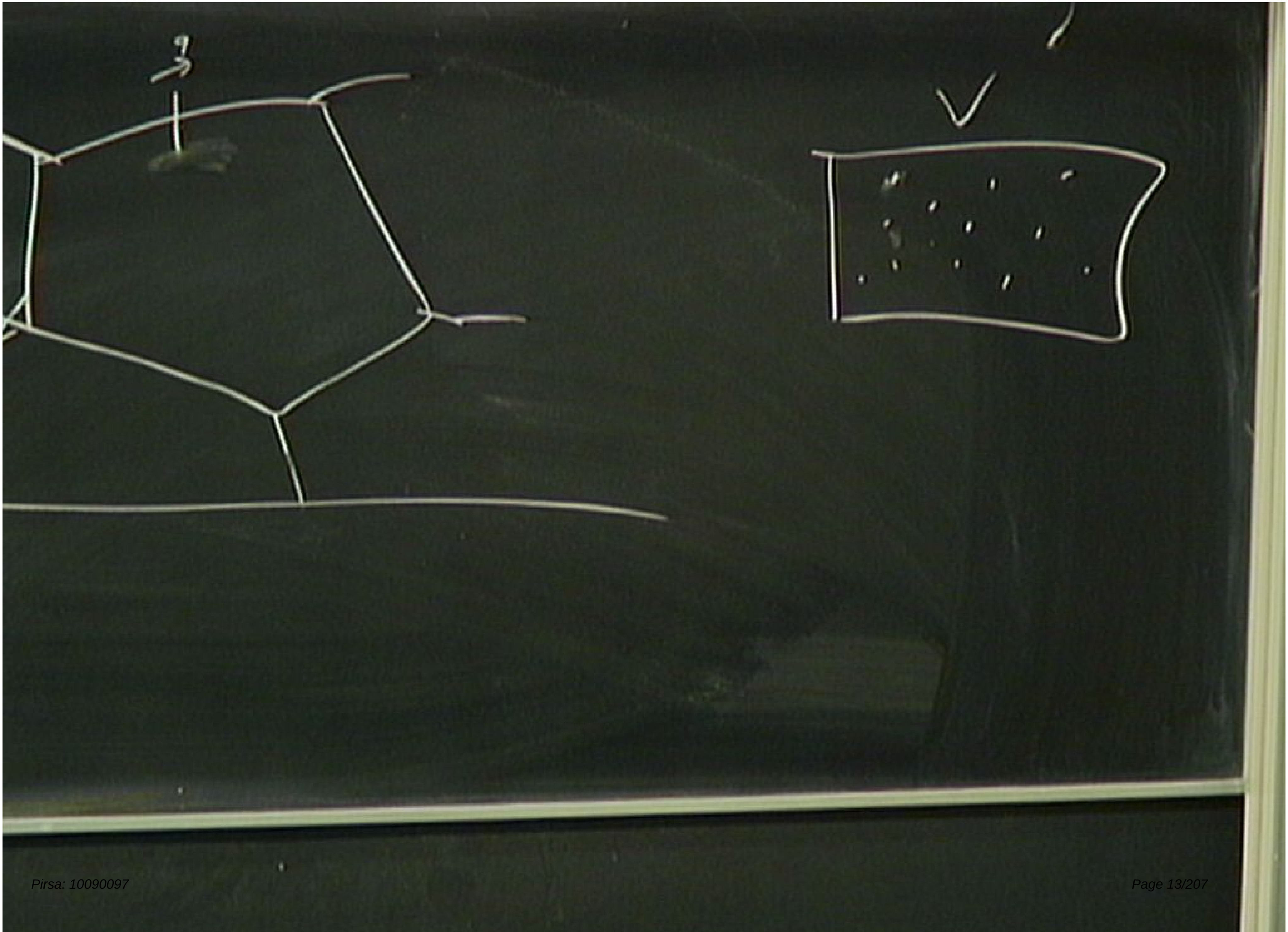

q

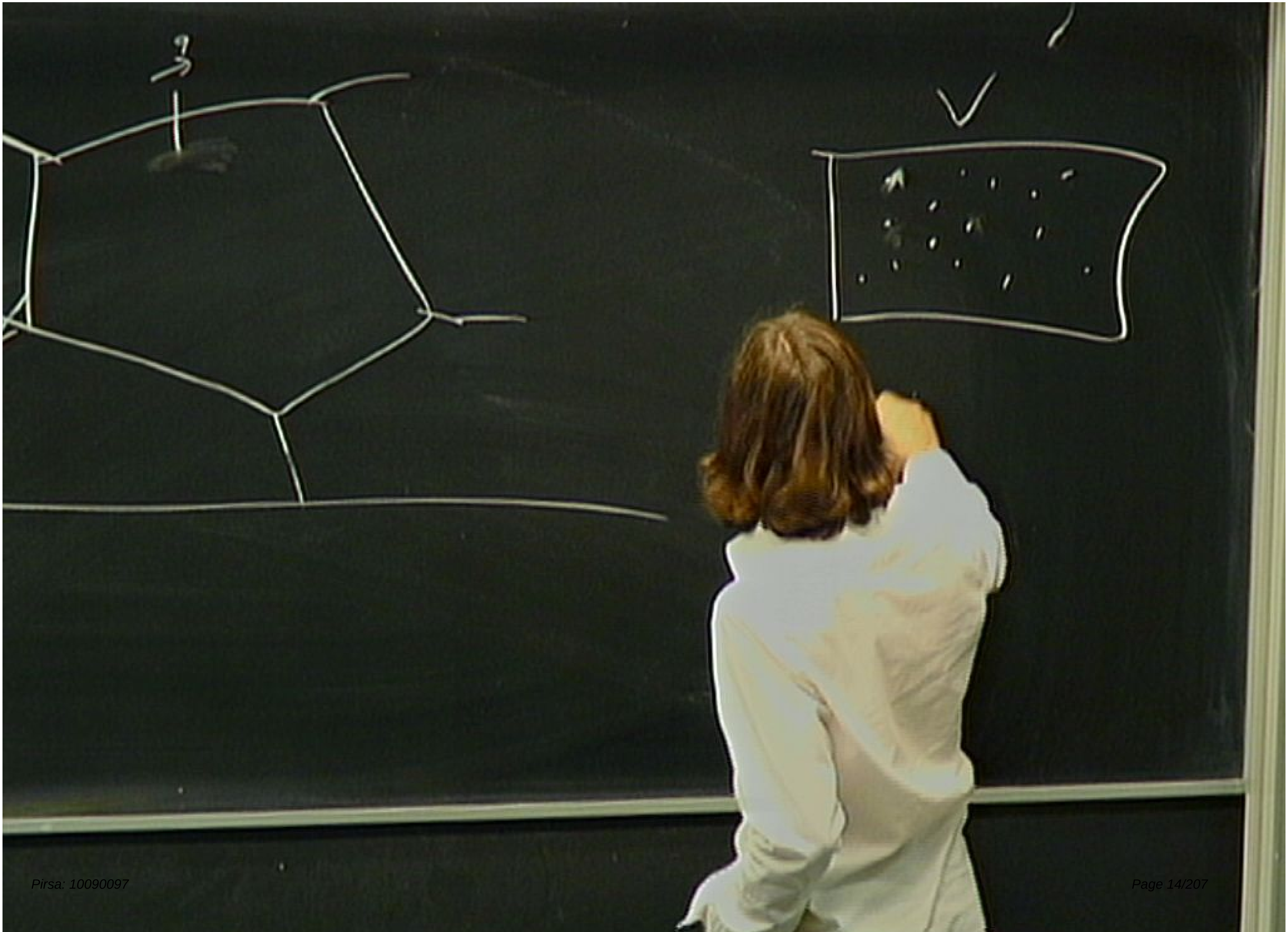

$A^{1\text{-loop}}(p_1 \dots p_n) = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3 2E_q} A(p_1, \dots, p_n, q, -q)$

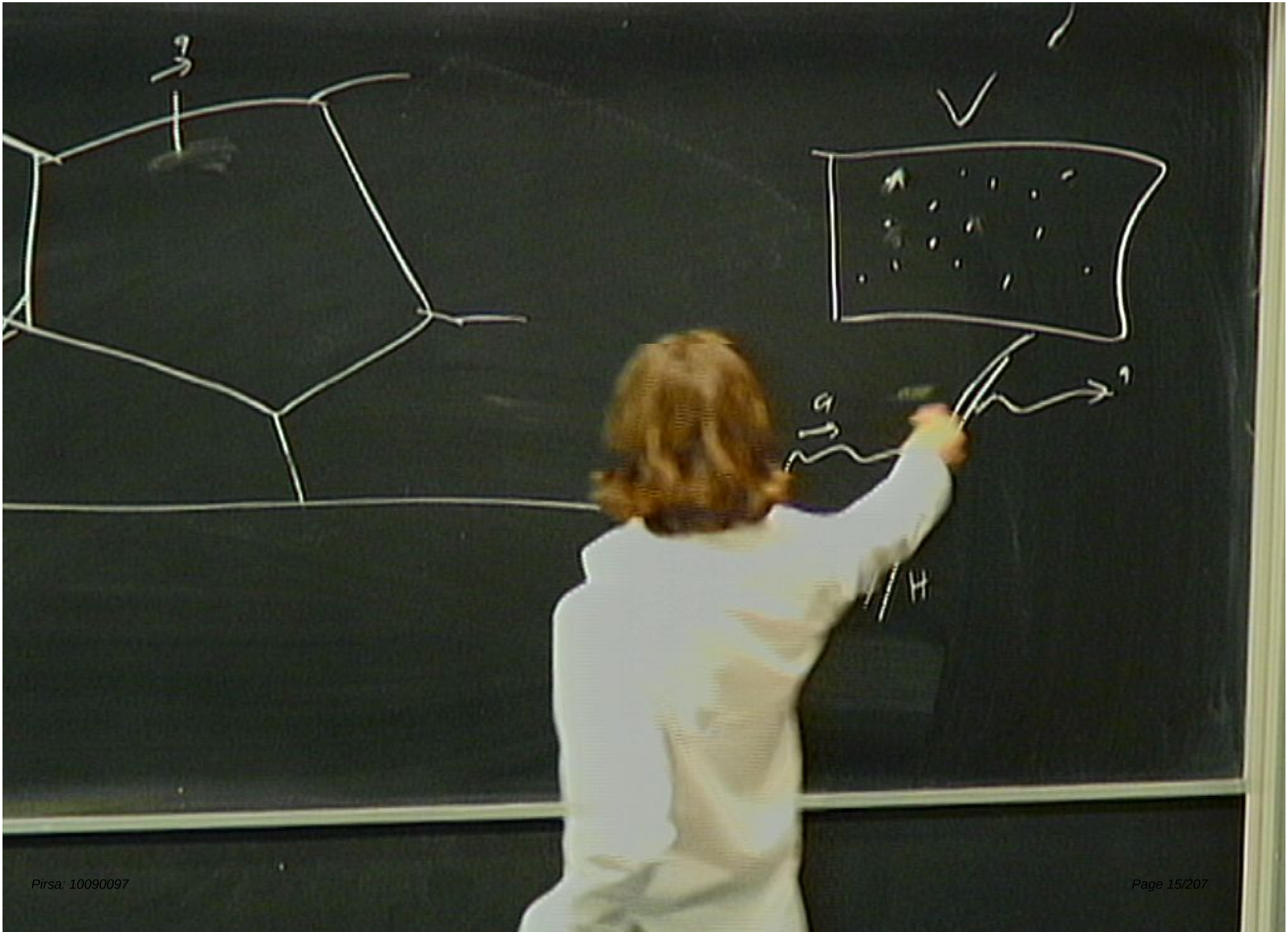
$$A^{1\text{-loop}}(p_1 \ldots p_n) = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3 2E_q} A(p_1, \ldots, p_n, q, -q)$$

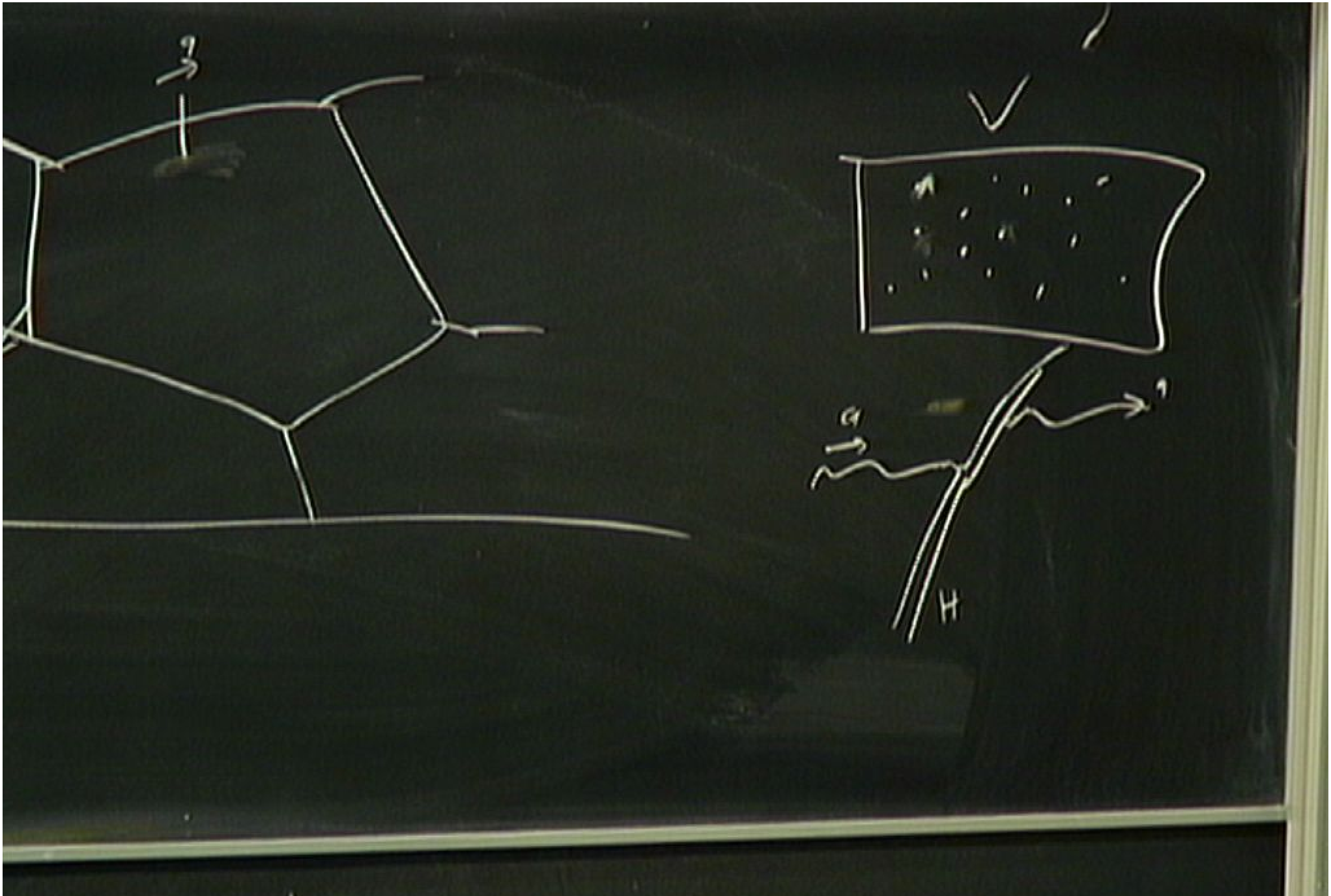
$$\hbar \frac{E_q}{2}$$







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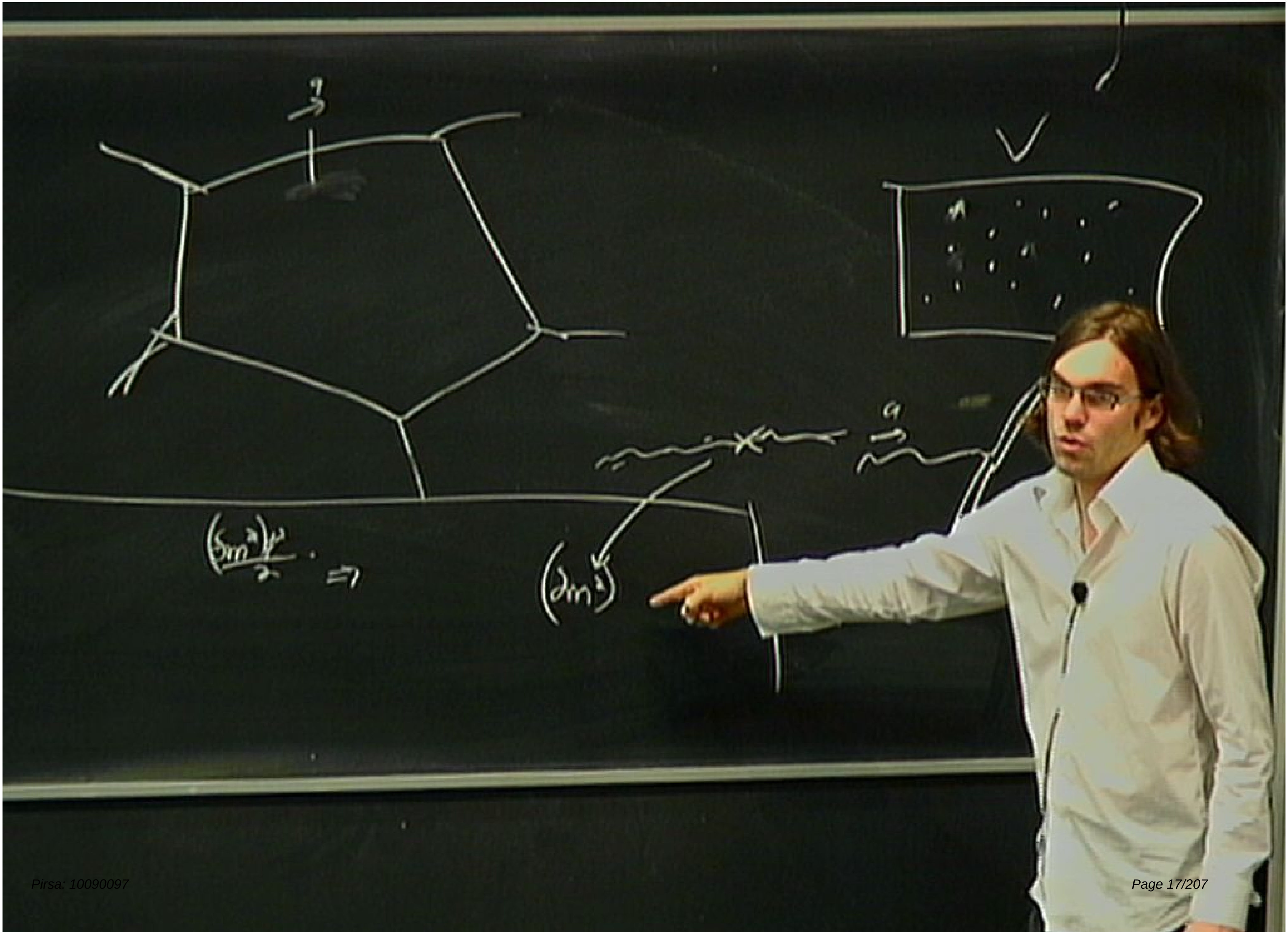

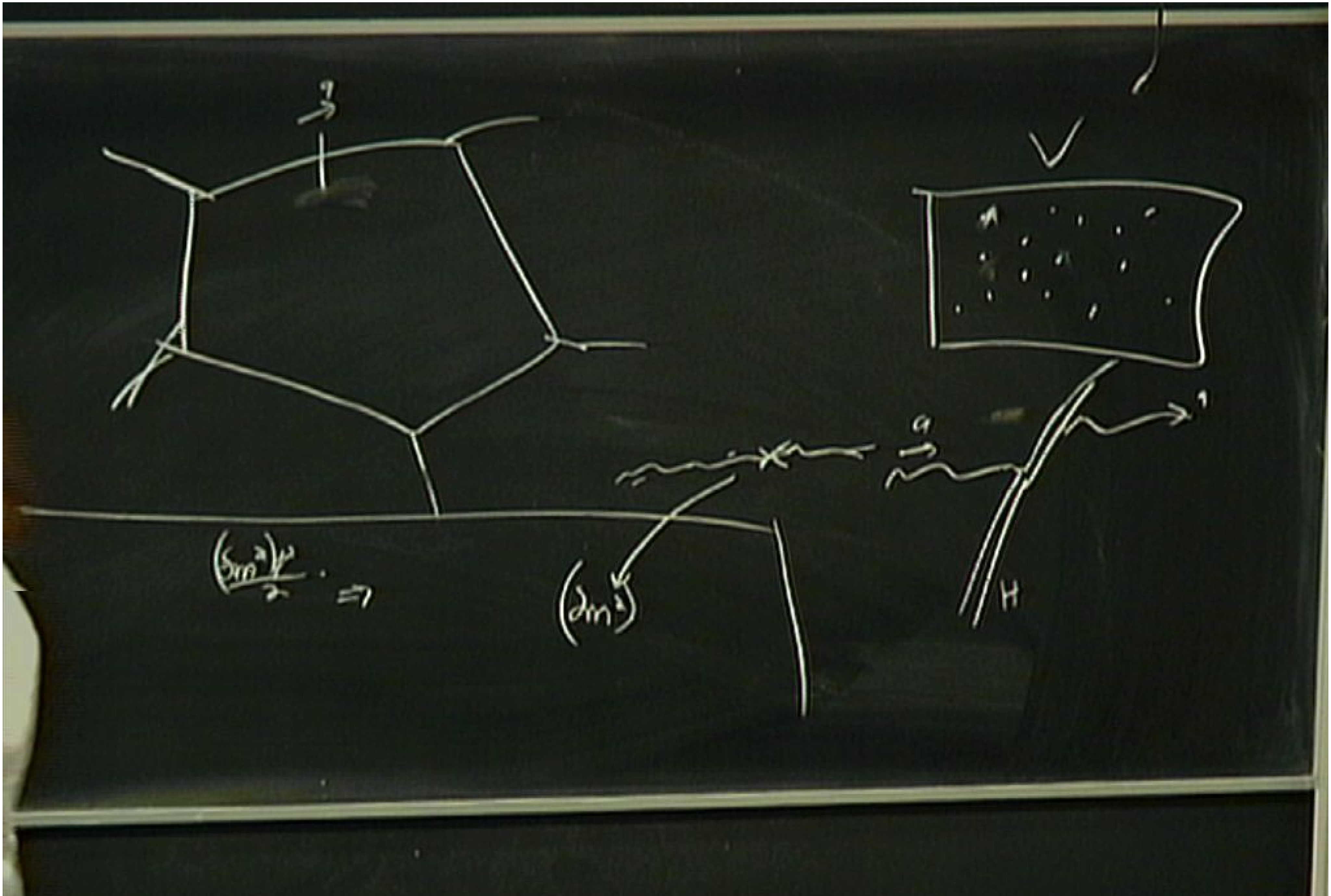
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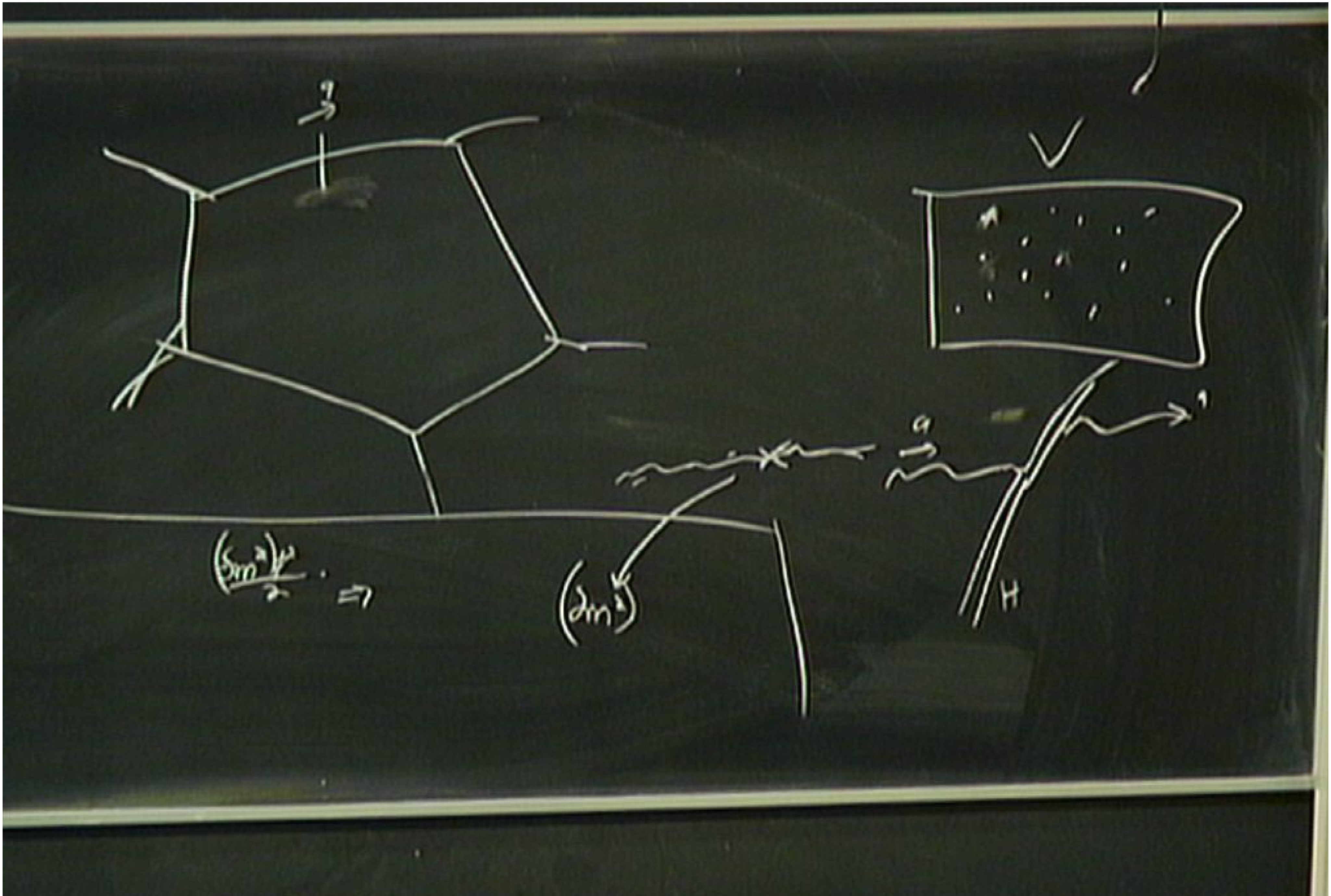
H

$$A^{1\text{-loop}}(p_1 \dots p_n) = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3 2E_q} A(p_1, \dots, p_n, q, -q)$$

$$E_{vac} = \int \frac{d^3q}{(2\pi)^3} \hbar \frac{1}{2} (E_q =$$

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$$A^{1\text{-loop}}(p_1 \cdots p_n) = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3 2E_q} A(p_1, \ldots, p_n, q, -q)$$

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$$\left(-\frac{E_q}{2}\right)$$

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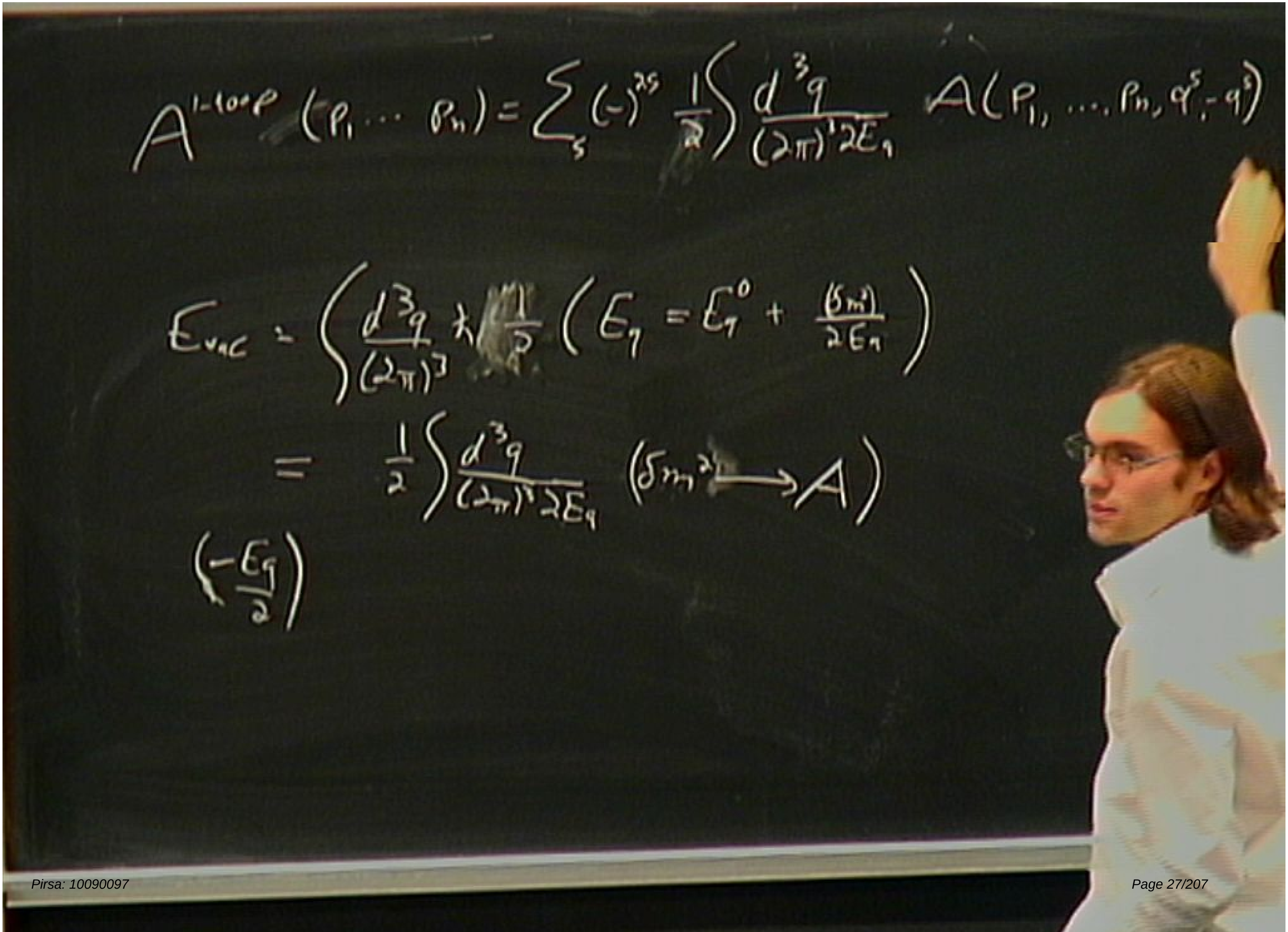
$A^{1\text{-loop}}(p_1 \cdots p_n) = \sum_s (-)^{2s} \frac{1}{2} \int \frac{d^3q}{(2\pi)^3 2E_q} A(p_1, \ldots, p_n, q^s, -q^s)$
$E_{vac} = \int \frac{d^3q}{(2\pi)^3} \frac{1}{2} \left(E_q = E_q^0 + \frac{\delta m^2}{2E_q} \right)$
$= \frac{1}{2} \int \frac{d^3q}{(2\pi)^3 2E_q} \left(\delta m^2 \to A \right)$
$\left(-\frac{E_q}{2} \right)$

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$$= \frac{1}{2} \int \frac{d^3q}{(2\pi)^3 2E_q} \left(\delta m^2 \longrightarrow A \right)$$

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— Single cuts determine loop amplitudes

— i) Match all single cuts to reconstruct $\int d^4 q$

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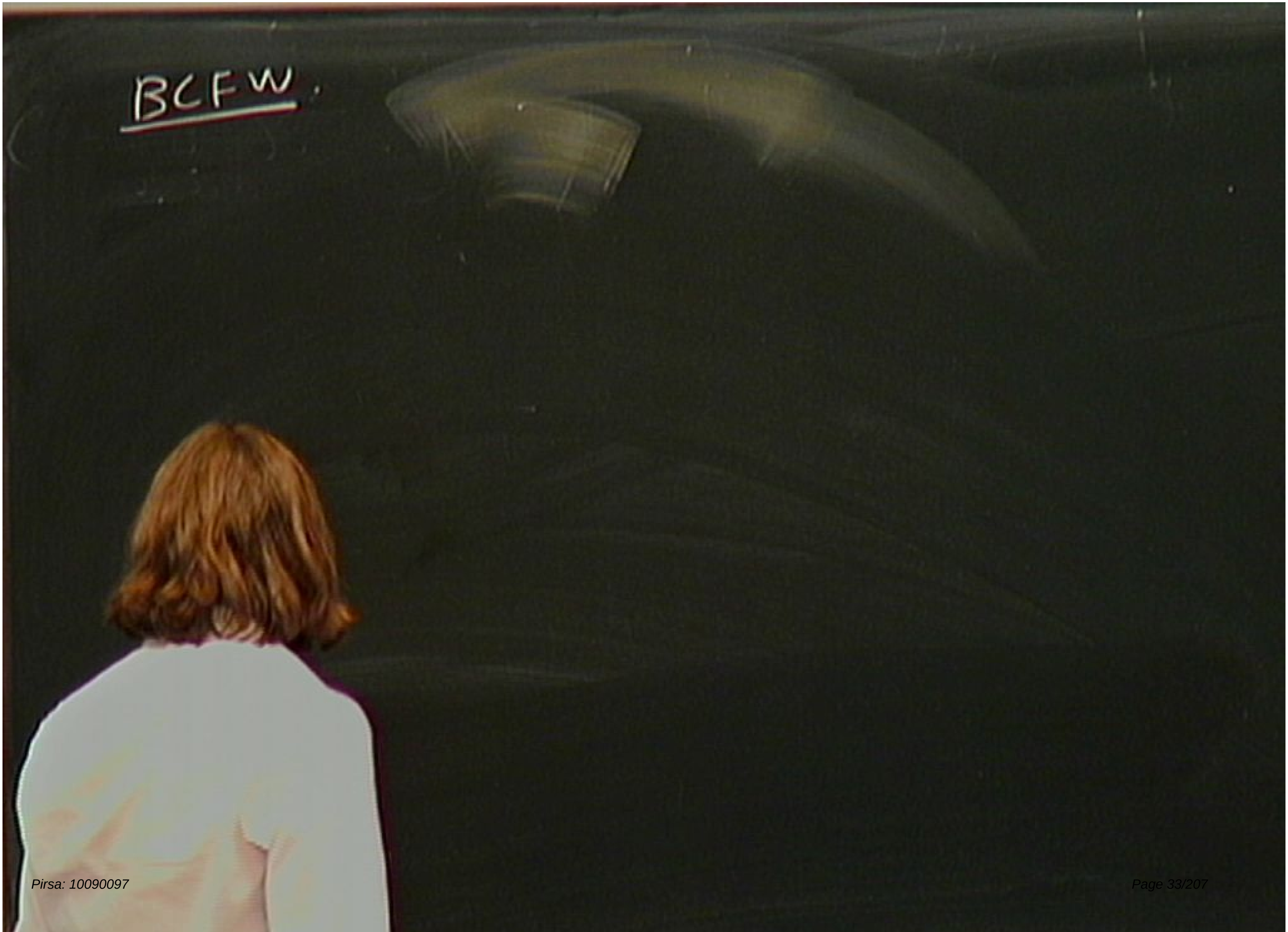
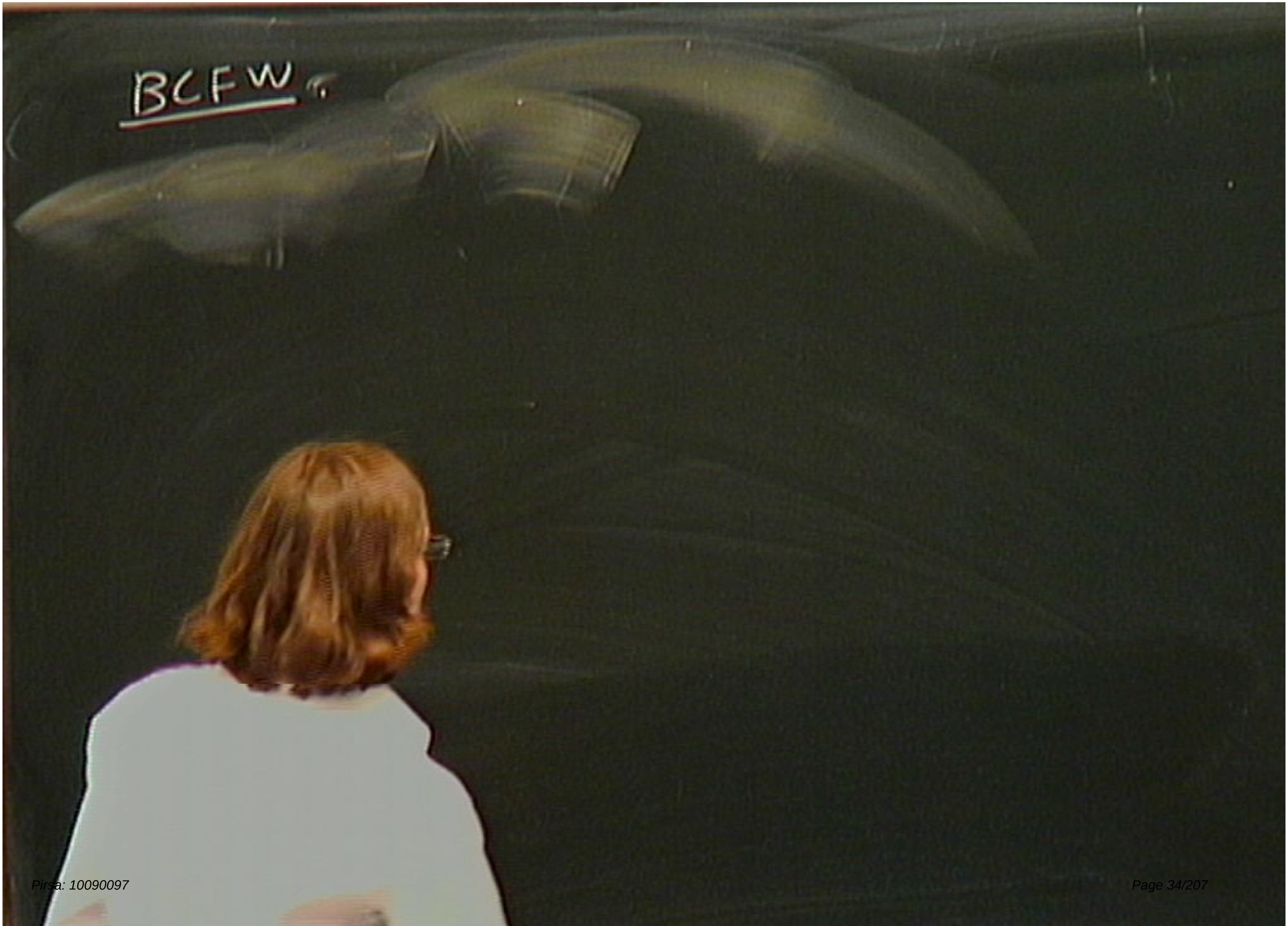
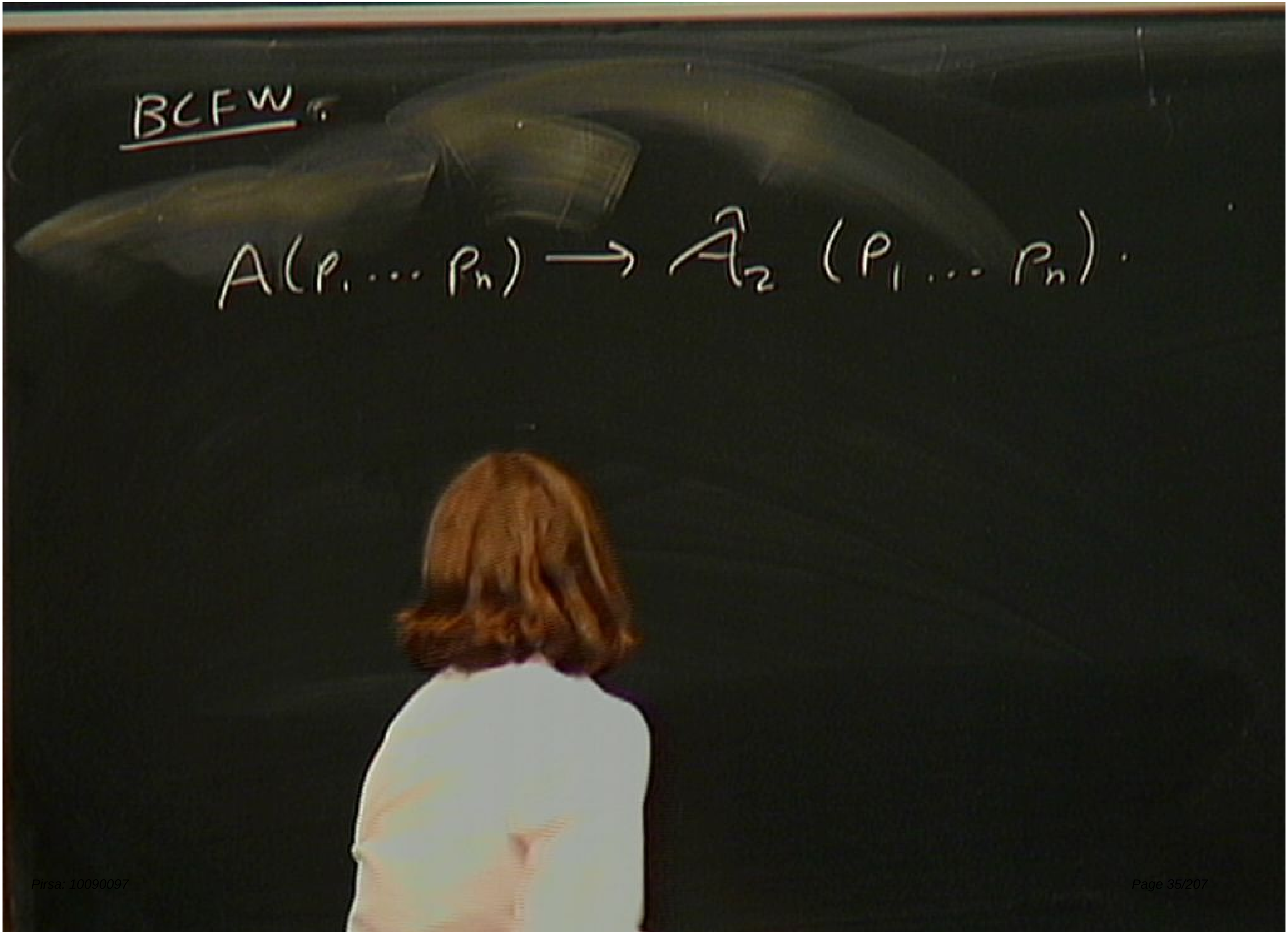
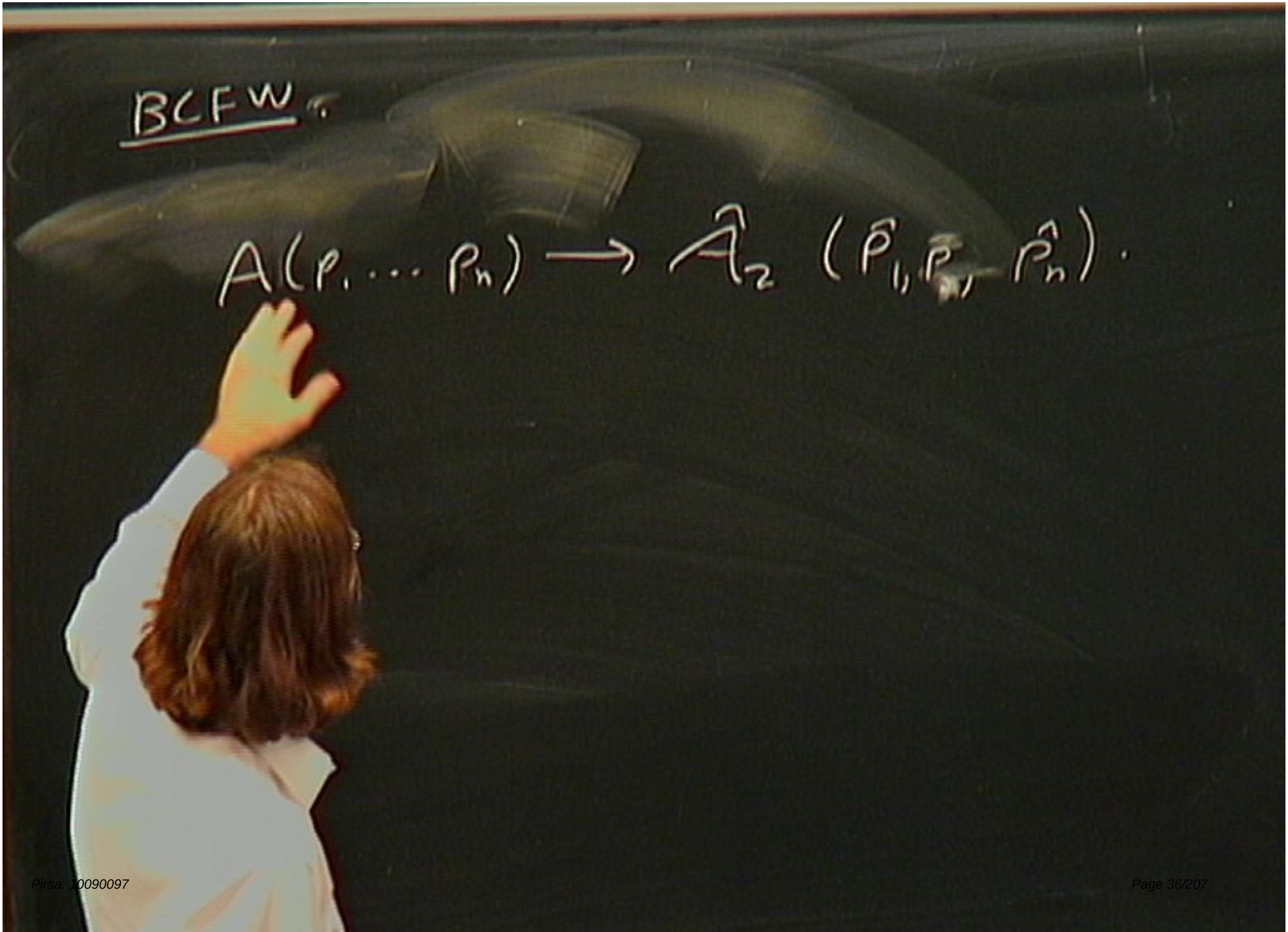
  ii) Recursion relations.


BCFW.


BCFW


BCFW
$A(p_1 \ldots p_n) \longrightarrow \hat{A}_z(p_1 \ldots p_n)$.


BCFW.
$A(p_1 \dots p_n) \longrightarrow \hat{A}_z(\hat{p}_1, \hat{p}, \hat{p}_n).$

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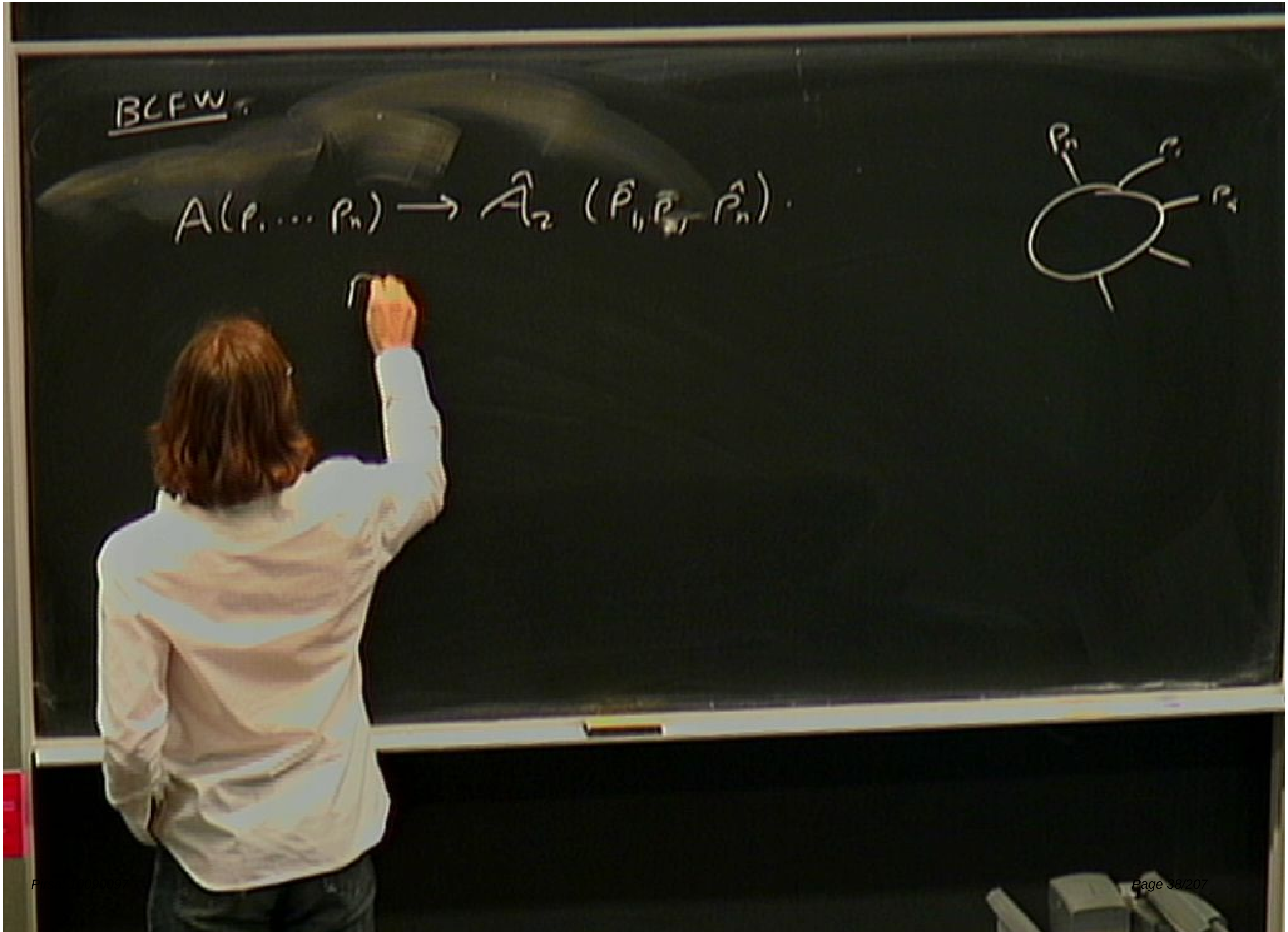
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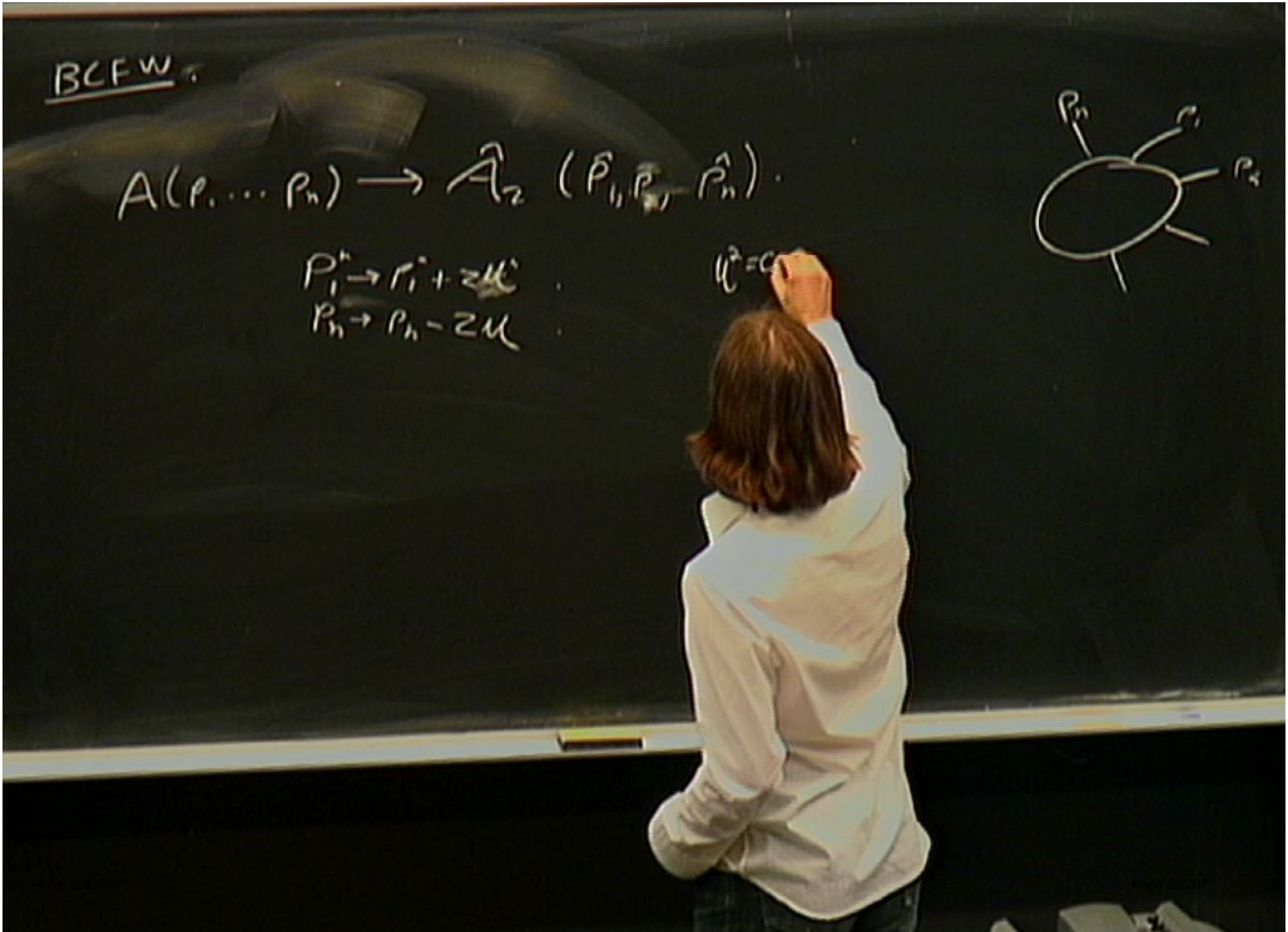
p_n
p_1
p_2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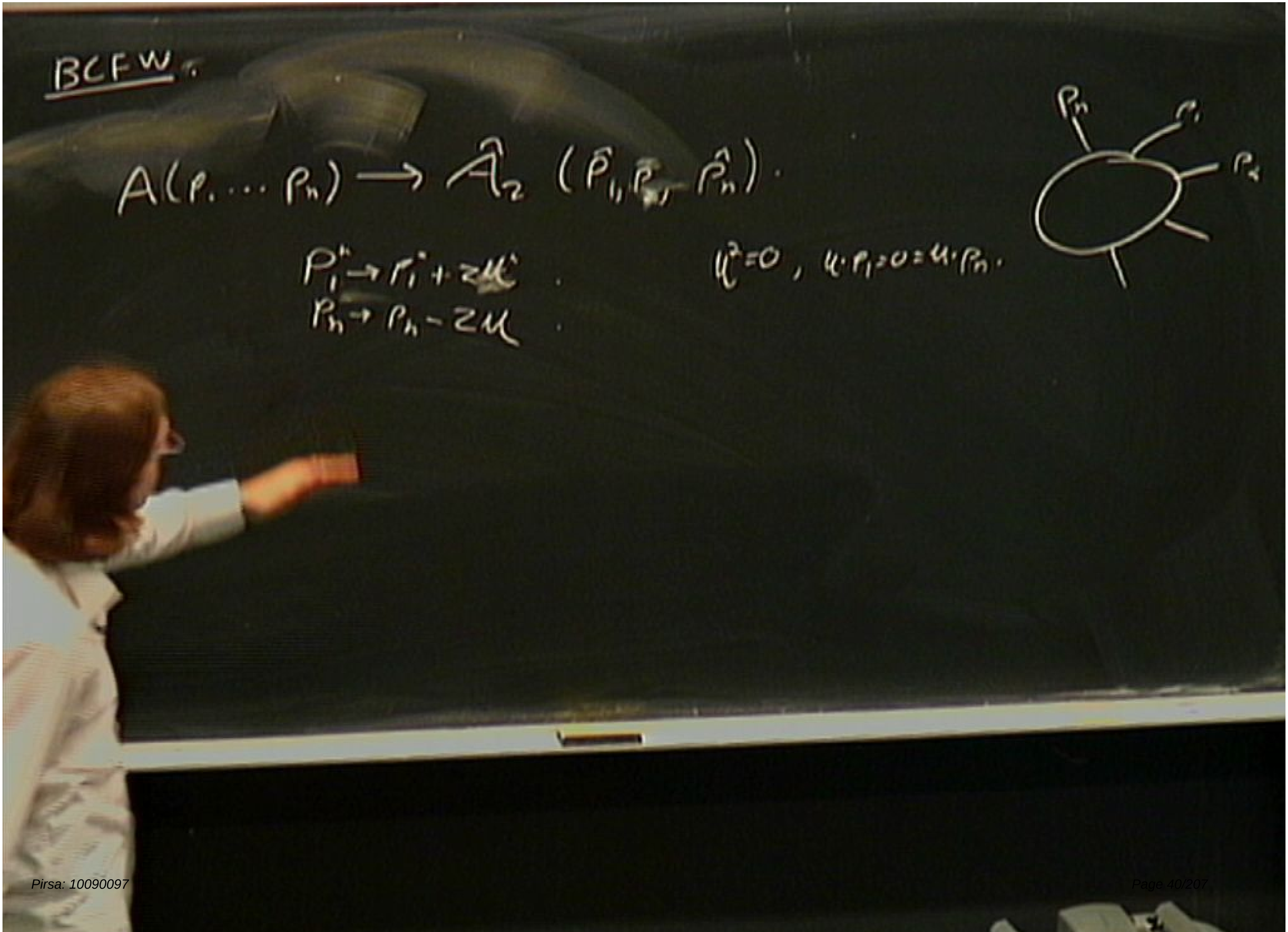
$A(p_1 \cdots p_n) \longrightarrow \hat{A}_z(\hat{p}_1, \hat{p}_2, \ldots \hat{p}_n)$.
$p_1^\mu \to p_1^\mu + z q^\mu$.
$p_n \to p_n - z q$.
$q^2 = 0$
p_n
p_1
p_2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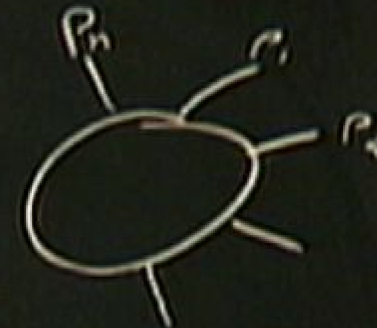
$A(p_1 \ldots p_n) \longrightarrow \hat{A}_z(\hat{p}_1, p_2, \ldots \hat{p}_n)$.
$p_1^\mu \to p_1^\mu + z q^\mu$.
$p_n \to p_n - z q$.
$q^2 = 0$, $q \cdot p_1 = 0 = q \cdot p_n$.
p_n
p_1
p_2

BCFW.

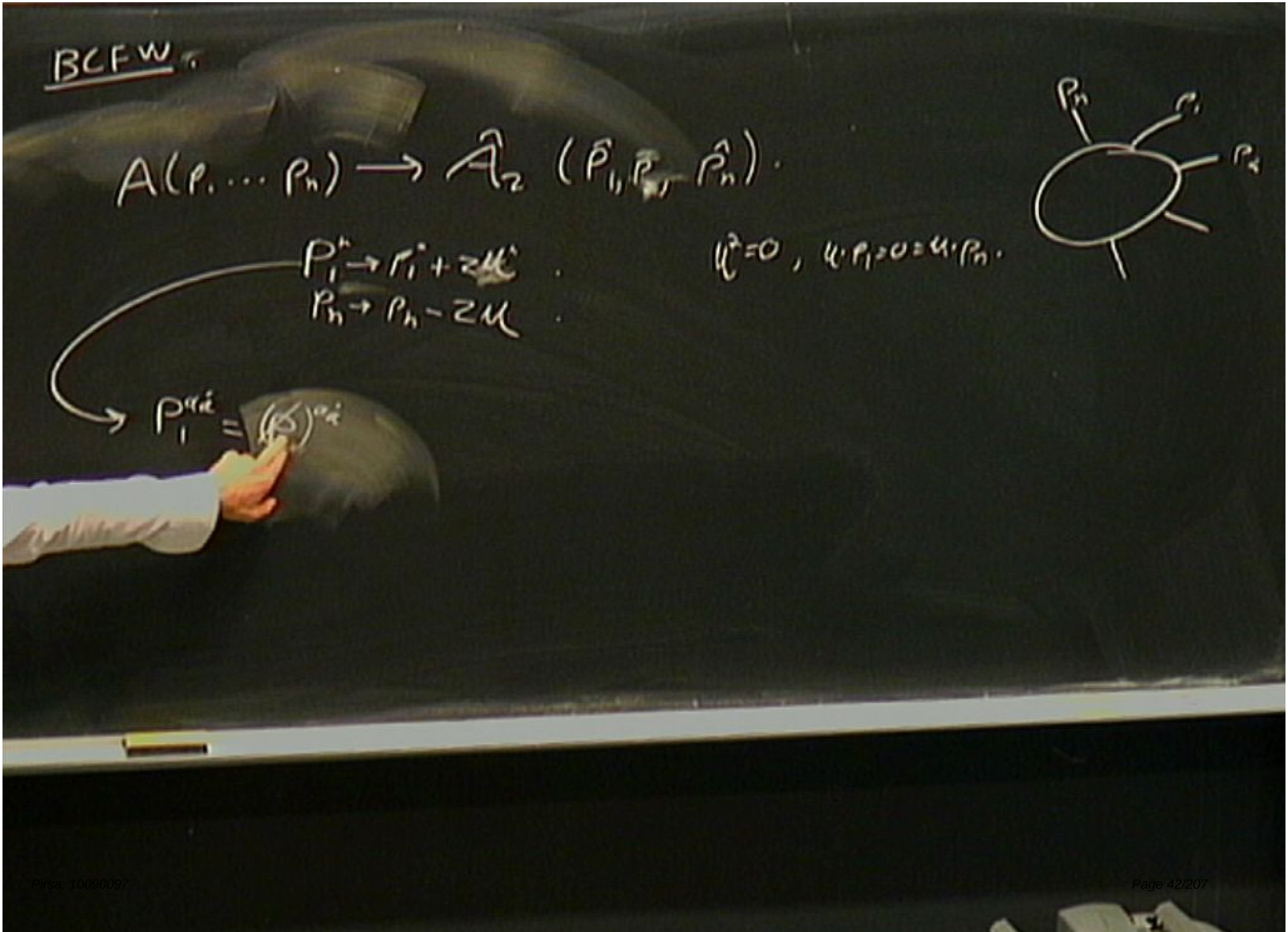
$$A(p_1 \dots p_n) \longrightarrow \hat{A}_z(\hat{p}_1, p_2, \dots \hat{p}_n).$$

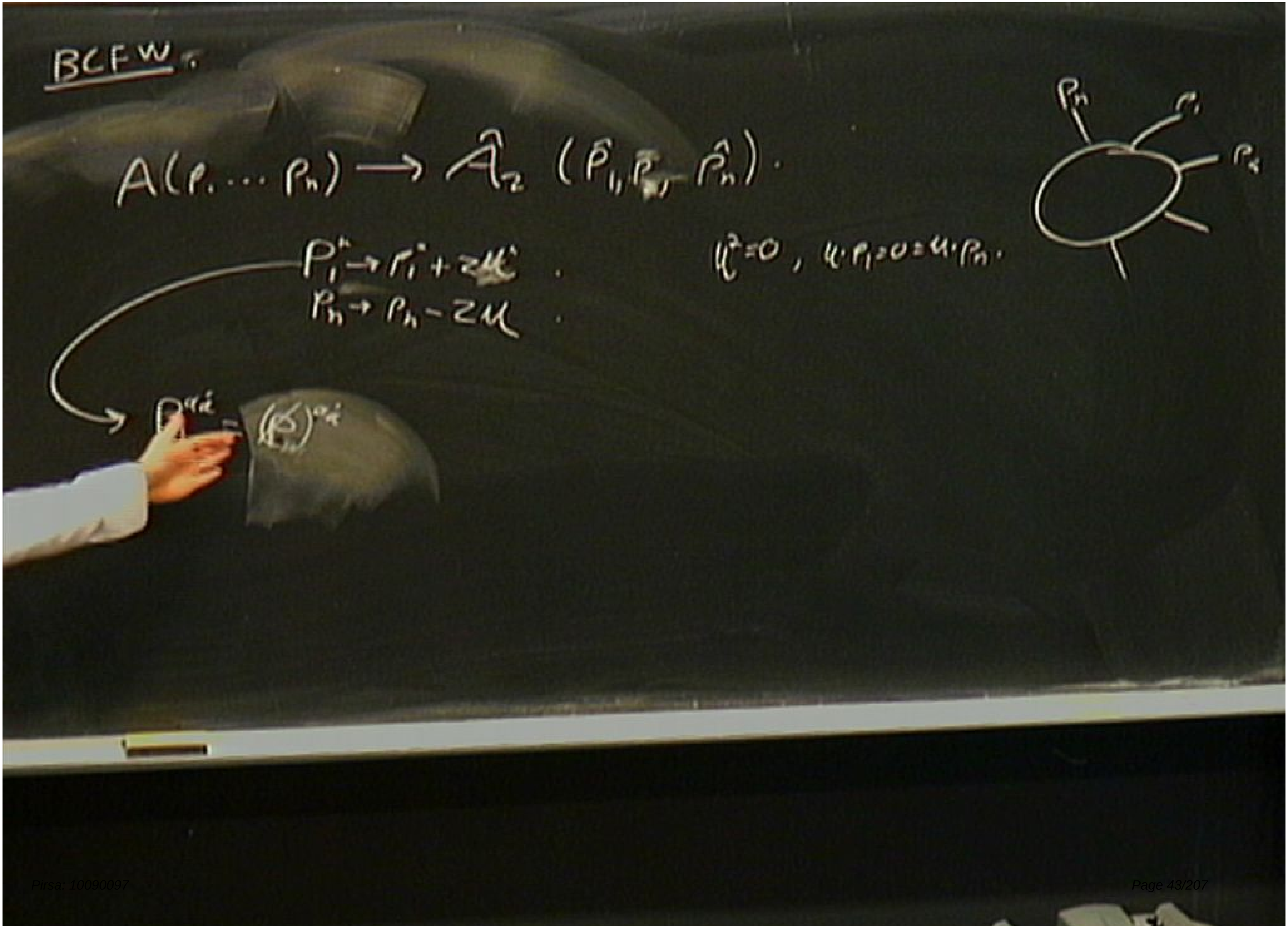


$$p_1^\mu \to p_1^\mu + z q^\mu .$$
$$p_n \to p_n - z q .$$

$$q^2 = 0 , \quad q \cdot p_1 = 0 = q \cdot p_n .$$

$$p_1^{\alpha\dot{\alpha}} = (\dots)^{\alpha\dot{\alpha}}$$


BCFW
A(p1 ... pn) → Âz (p̂1, p2, ..., p̂n).
p1^μ → p1^μ + z q^μ
pn → pn − z q
q^2 = 0, q·p1 = 0 = q·pn
p1^{αα̇} =
pn
p1
p2


BCFW
A(p1 ... pn) → Âz (p̂1, p2, ... p̂n)
p1 → p1 + zq
pn → pn − zq
q² = 0, q·p1 = 0 = q·pn
pn
p1
p2

BCFW

$$A(p_1 \ldots p_n) \longrightarrow \hat{A}_z\,(\hat{p}_1, p_2, \ldots \hat{p}_n)\,.$$

$$p_1^\mu \to p_1^\mu + z q^\mu \,. \qquad q^2 = 0\,,$$

$$p_n \to p_n - z q\,.$$

$$p_1^{\alpha\dot\alpha} = (p_1)^{\alpha\dot\alpha}$$

$$= \lambda^{\alpha}\tilde\lambda^{\dot\alpha}$$

BCFW

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$$= \lambda_1^\alpha \tilde{\lambda}_1^{\dot{\alpha}}$$

BCFW

$$A(p_1 \ldots p_n) \longrightarrow \hat{A}_z(\hat{p}_1, p_2, \ldots \hat{p}_n)$$

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$$p_n \to p_n - z q$$

$$p_1^{\alpha\dot\alpha} = (\not{p}_1)^{\alpha\dot\alpha} = \lambda_1^\alpha \tilde\lambda_1^{\dot\alpha} \qquad q^{\alpha\dot\alpha} = \lambda_1^\alpha \tilde\lambda_n^{\dot\alpha}$$

$$\lambda_1 \to \lambda_1 +$$

BCFW.

$$A(p_1 \cdots p_n) \longrightarrow \hat{A}_z(\hat{p}_1, p_2, \ldots \hat{p}_n).$$

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$$p_1^{\alpha\dot\alpha} = (p_1)^{\alpha\dot\alpha} = \lambda_1^\alpha \tilde\lambda_1^{\dot\alpha}$$

$$q^{\alpha\dot\alpha} = \lambda_1^\alpha \tilde\lambda_n^{\dot\alpha}$$

$$\lambda_n \to \lambda_n - z \lambda_1$$

$$\tilde\lambda_1 \to \tilde\lambda_1 + z \tilde\lambda_n .$$

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$$p_1^{\alpha\dot\alpha} = (p_1)^{\alpha\dot\alpha}$$
$$= \lambda_1^\alpha \tilde\lambda_1^{\dot\alpha}$$

$$q^{\alpha\dot\alpha} = \lambda_1^\alpha \tilde\lambda_n^{\dot\alpha}$$

$$\boxed{\begin{aligned}\lambda_n &\to \lambda_n \ldots \\ \tilde\lambda_1 &\to \ldots\end{aligned}}$$

BCFW

$$A(p_1 \ldots p_n) \longrightarrow \hat{A}_z(\hat{p}_1, p_2, \ldots \hat{p}_n).$$

$$p_1^\mu \to p_1^\mu + z q^\mu \qquad q^2 = 0, \quad q \cdot p_1 = 0 = q \cdot p_n.$$

$$p_n \to p_n - z q$$

$$p_1^{\alpha\dot\alpha} = (\sigma \cdot p_1)^{\alpha\dot\alpha} = \lambda_1^\alpha \tilde\lambda_1^{\dot\alpha} \qquad q^{\alpha\dot\alpha} = \lambda_1^\alpha \tilde\lambda_n^{\dot\alpha}$$

$$\lambda_n \to \lambda_n - z\lambda_1$$

$$\tilde\lambda_1 \to \tilde\lambda_1 + z\tilde\lambda_n.$$

BCFW.

$$A(p_1 \dots p_n) \longrightarrow \hat{A}_z(\hat{p}_1, p_2, \dots \hat{p}_n).$$

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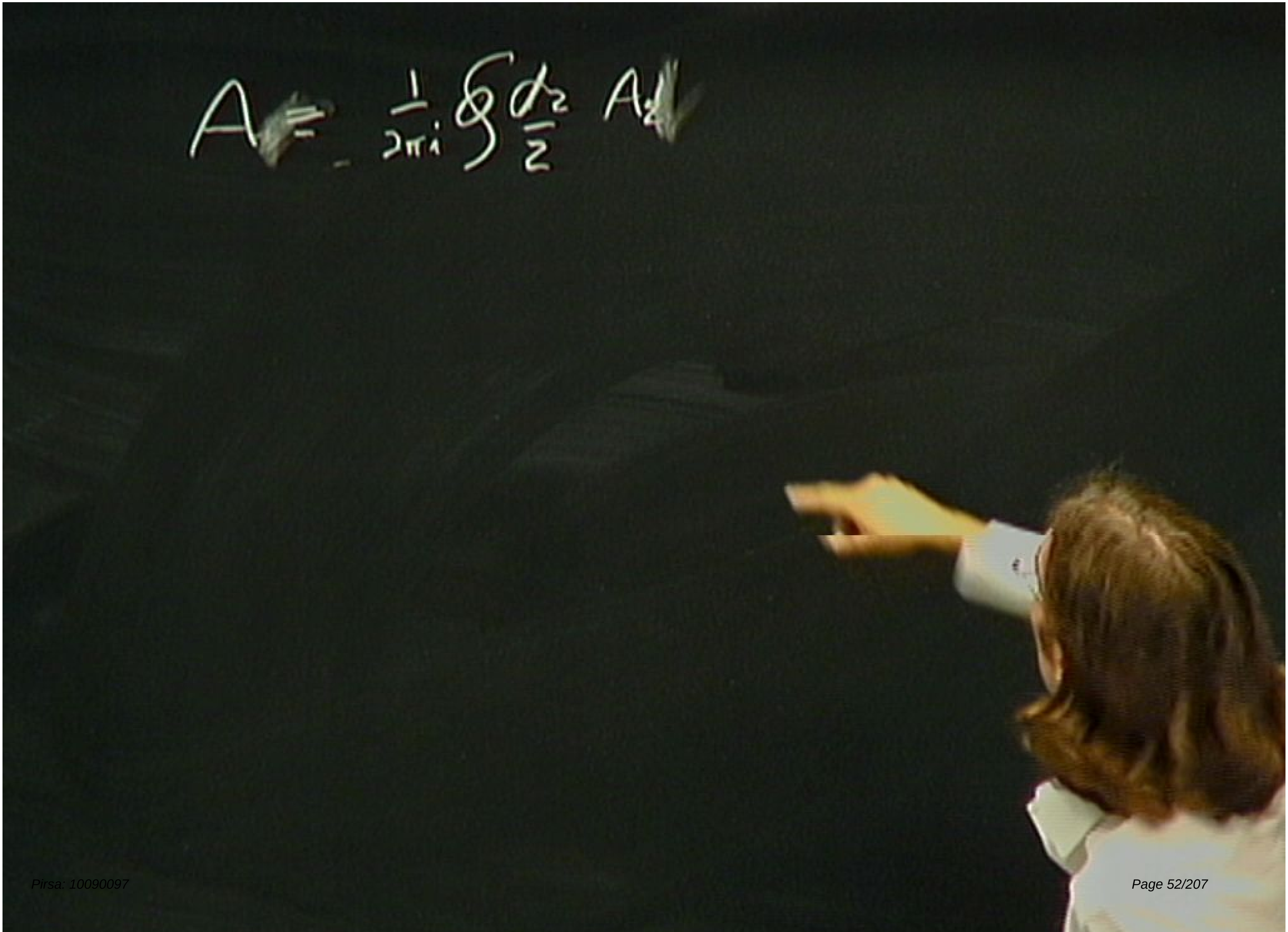
$$p_1^{a\dot{a}} = (\not{p}_1)^{a\dot{a}} = \lambda_1^a \tilde{\lambda}_1^{\dot{a}}$$

$q^{a\dot{a}} = \lambda_1^a \tilde{\lambda}_n^{\dot{a}}$

$$\lambda_n \to \lambda_n - z\lambda_1$$
$$\tilde{\lambda}_1 \to \tilde{\lambda}_1 + z\tilde{\lambda}_n.$$


$A = \frac{1}{2\pi i}\oint \frac{dz}{z} A$

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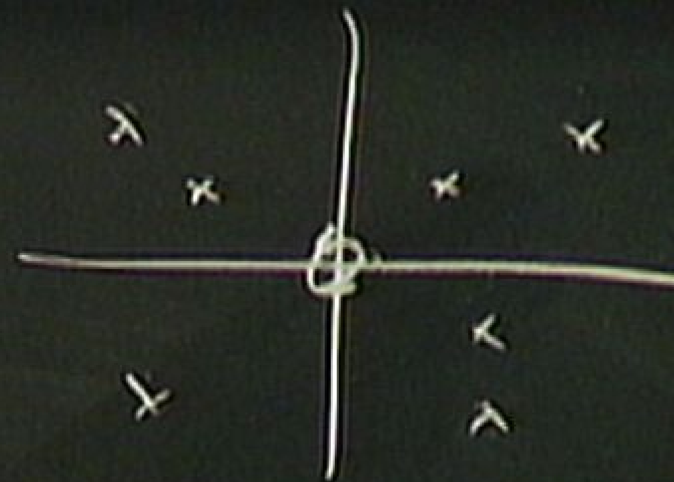
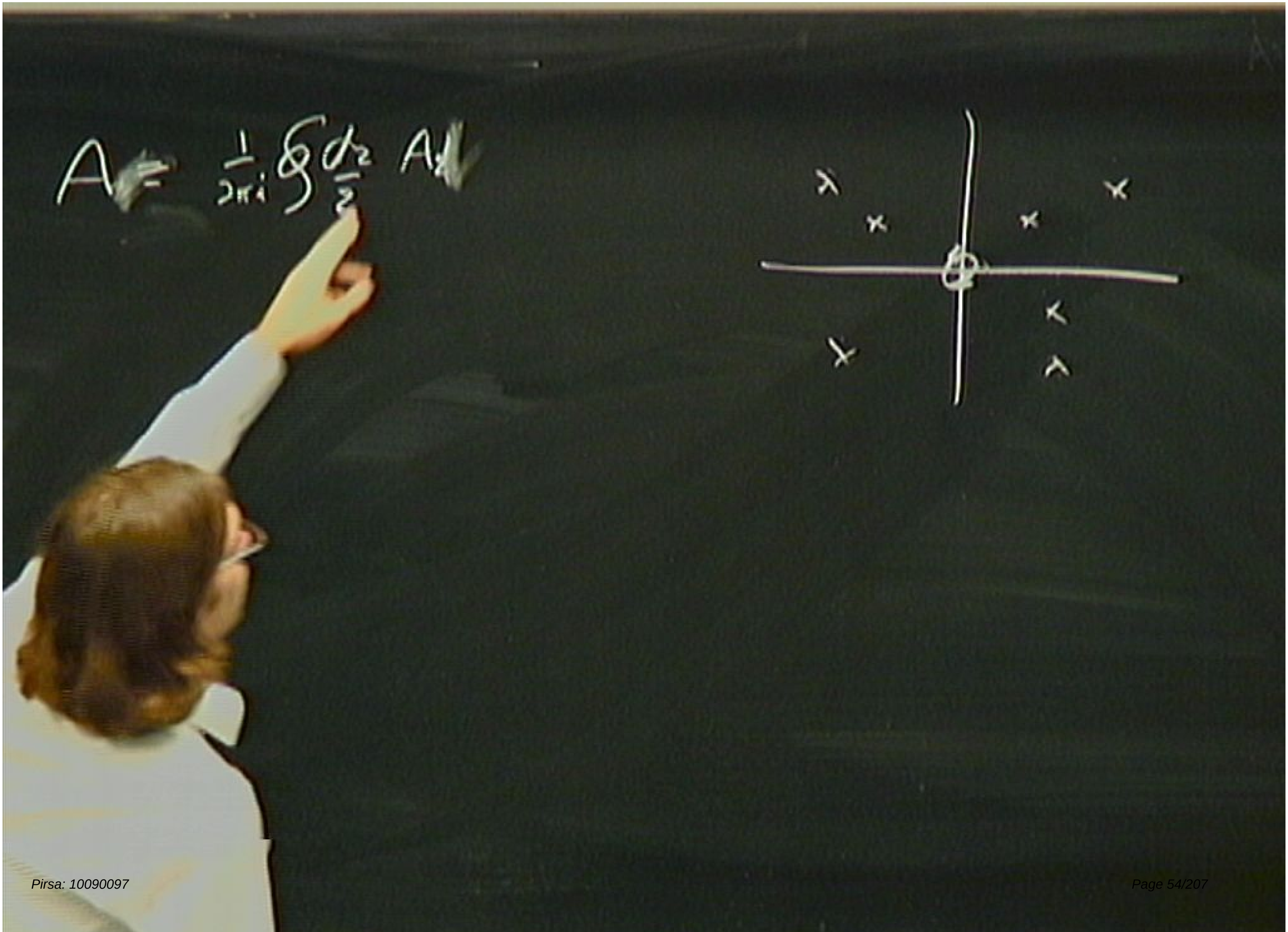
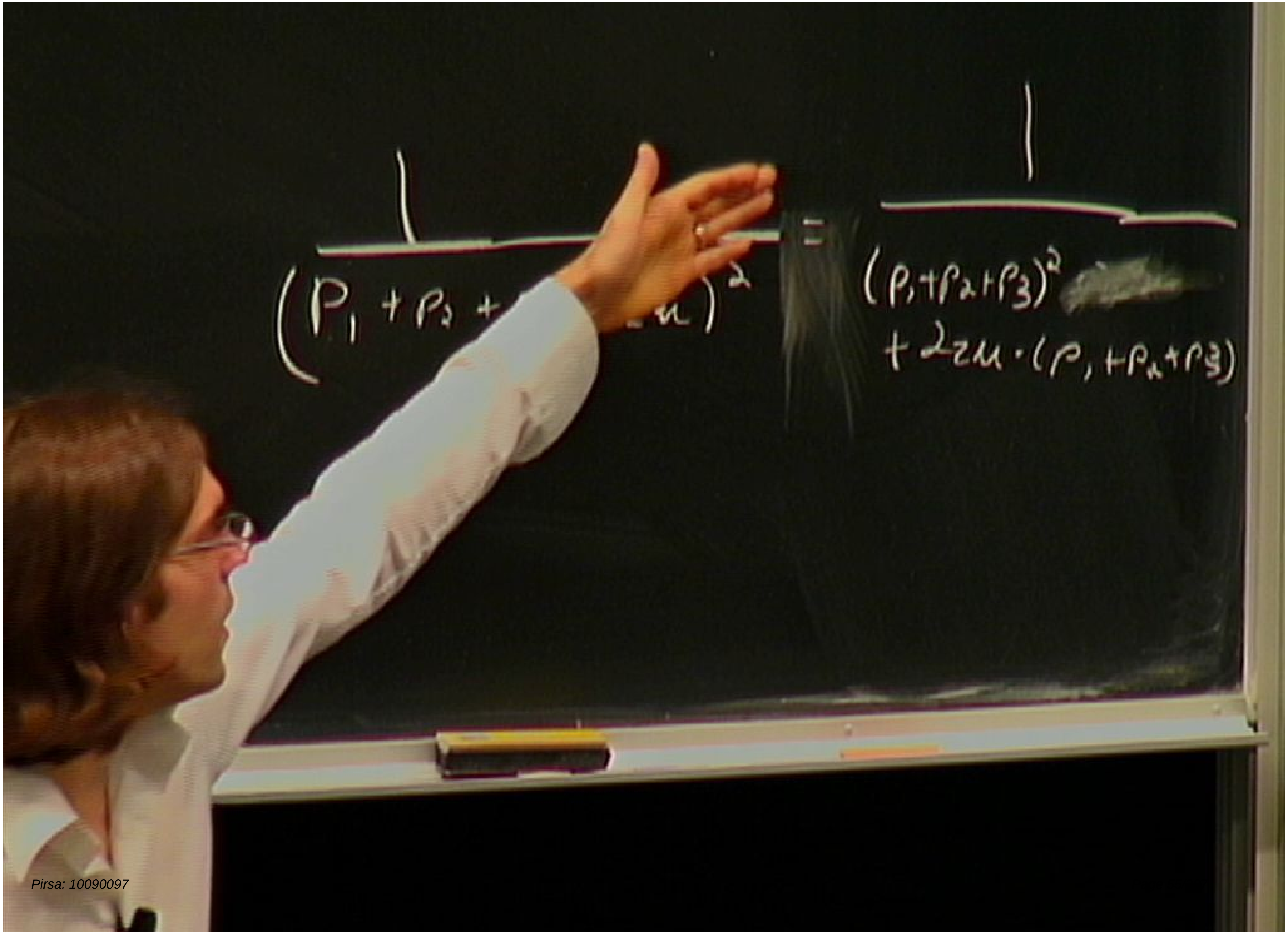
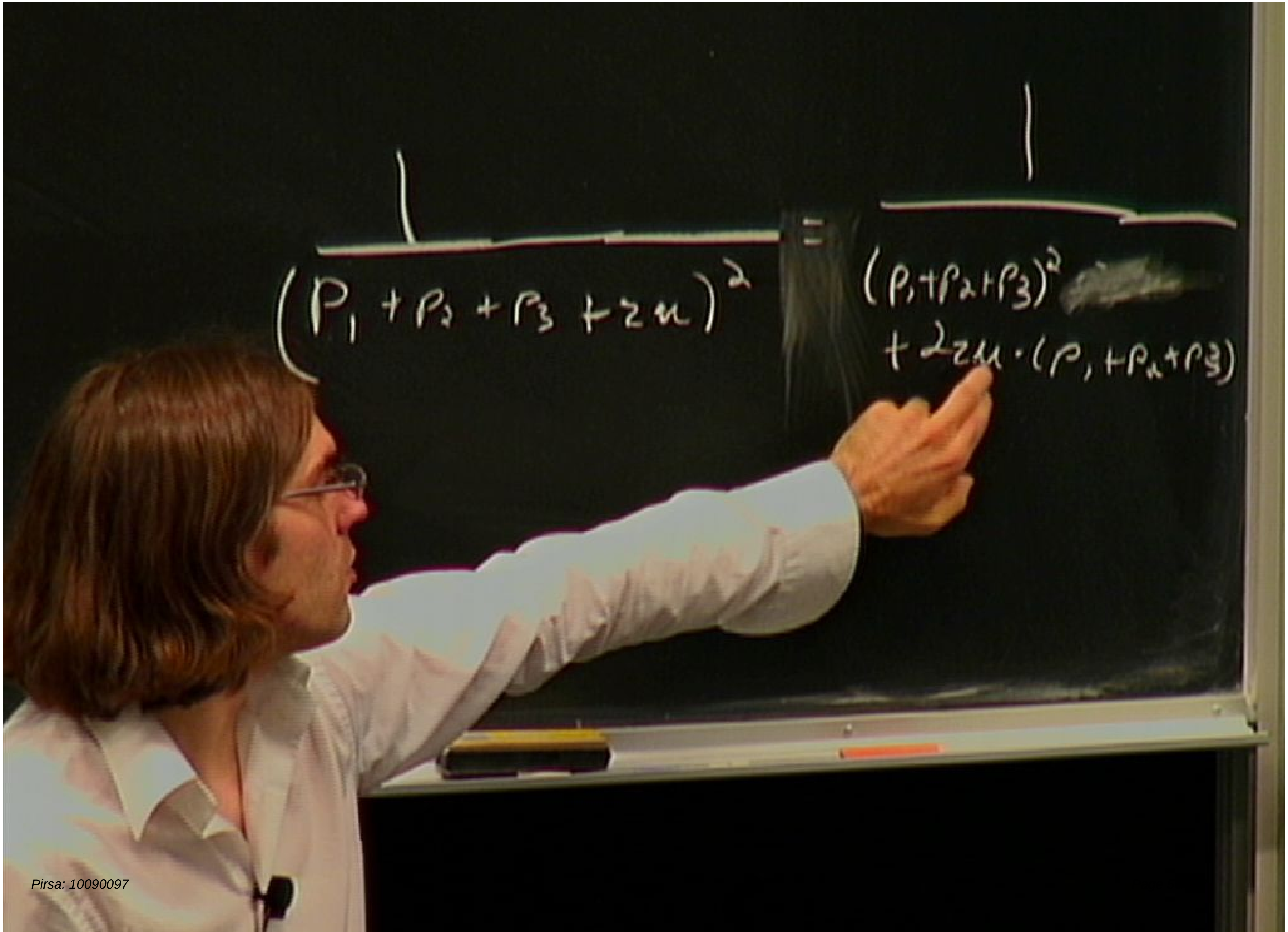



$A = \frac{1}{2\pi i}\oint \frac{dz}{z} A$


1
(p1+p2+p3)^2
+ 2zu·(p1+p2+p3)



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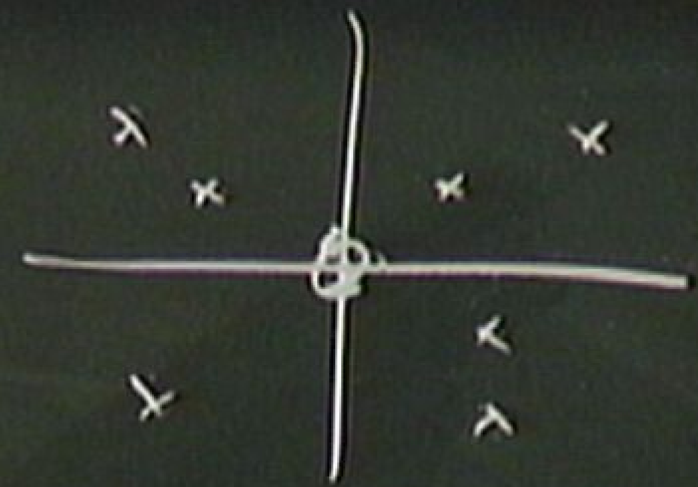
$$(p_1 + p_2 + p_3 + zu)^2 = (p_1 + p_2 + p_3)^2 + 2zu \cdot (p_1 + p_2 + p_3)$$

$$\frac{1}{\left(p_1 + p_2 + p_3 + 2m\right)^2} = \frac{1}{(p_1+p_2+p_3)^2 + 2m\cdot(p_1+p_2+p_3)}$$

$$A = \frac{1}{2\pi i}\oint \frac{dz}{z} A(z)$$



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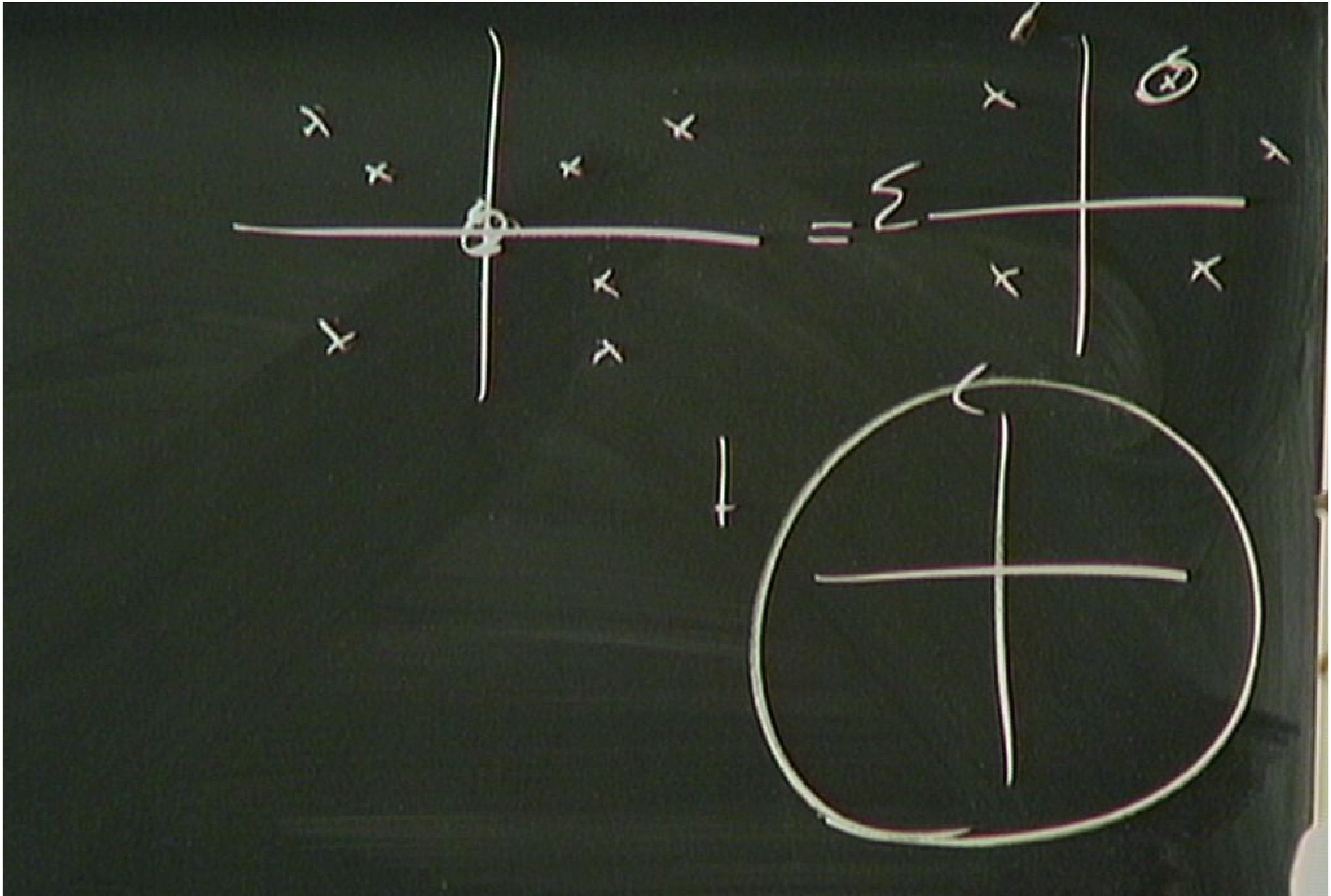
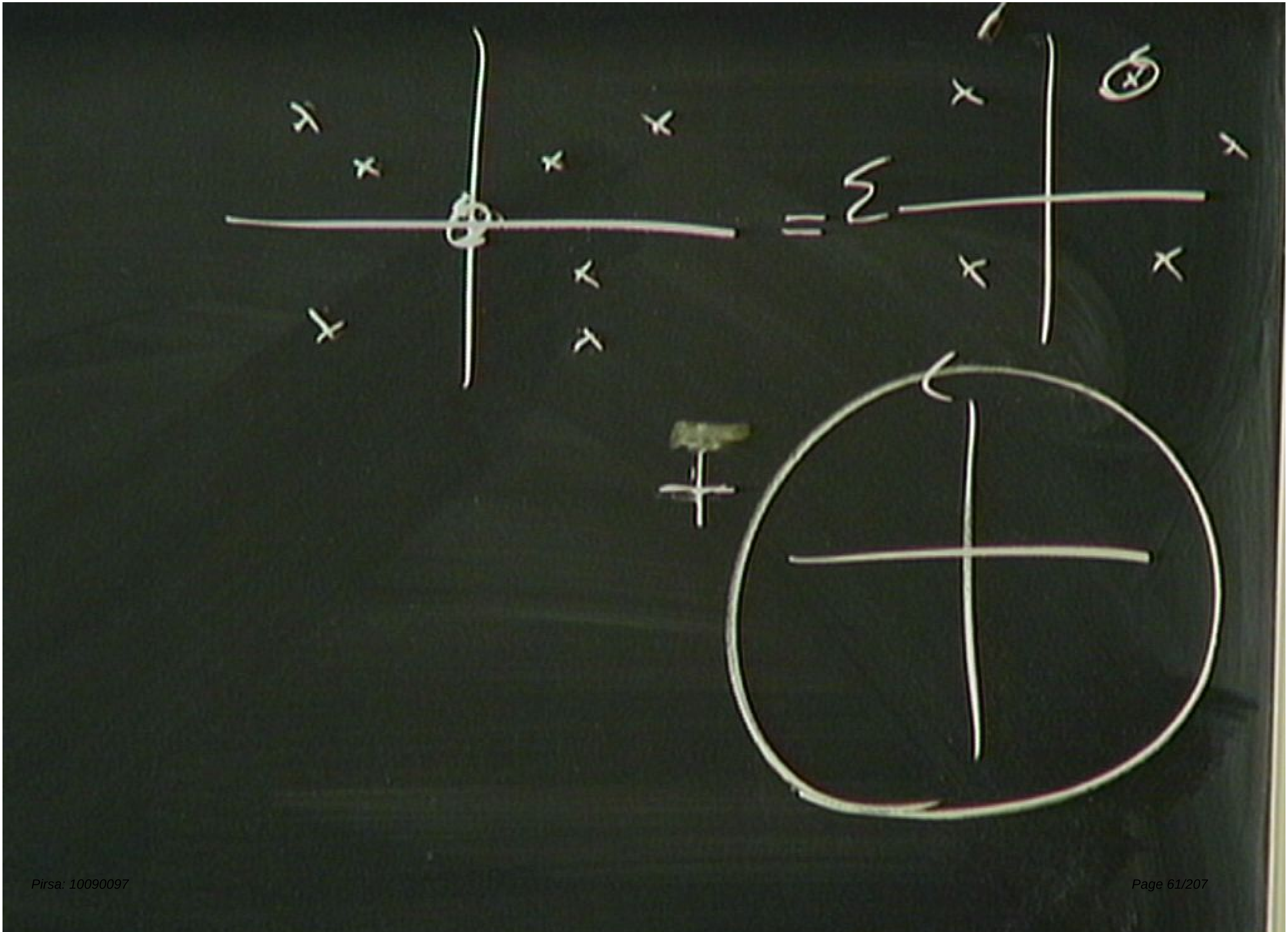
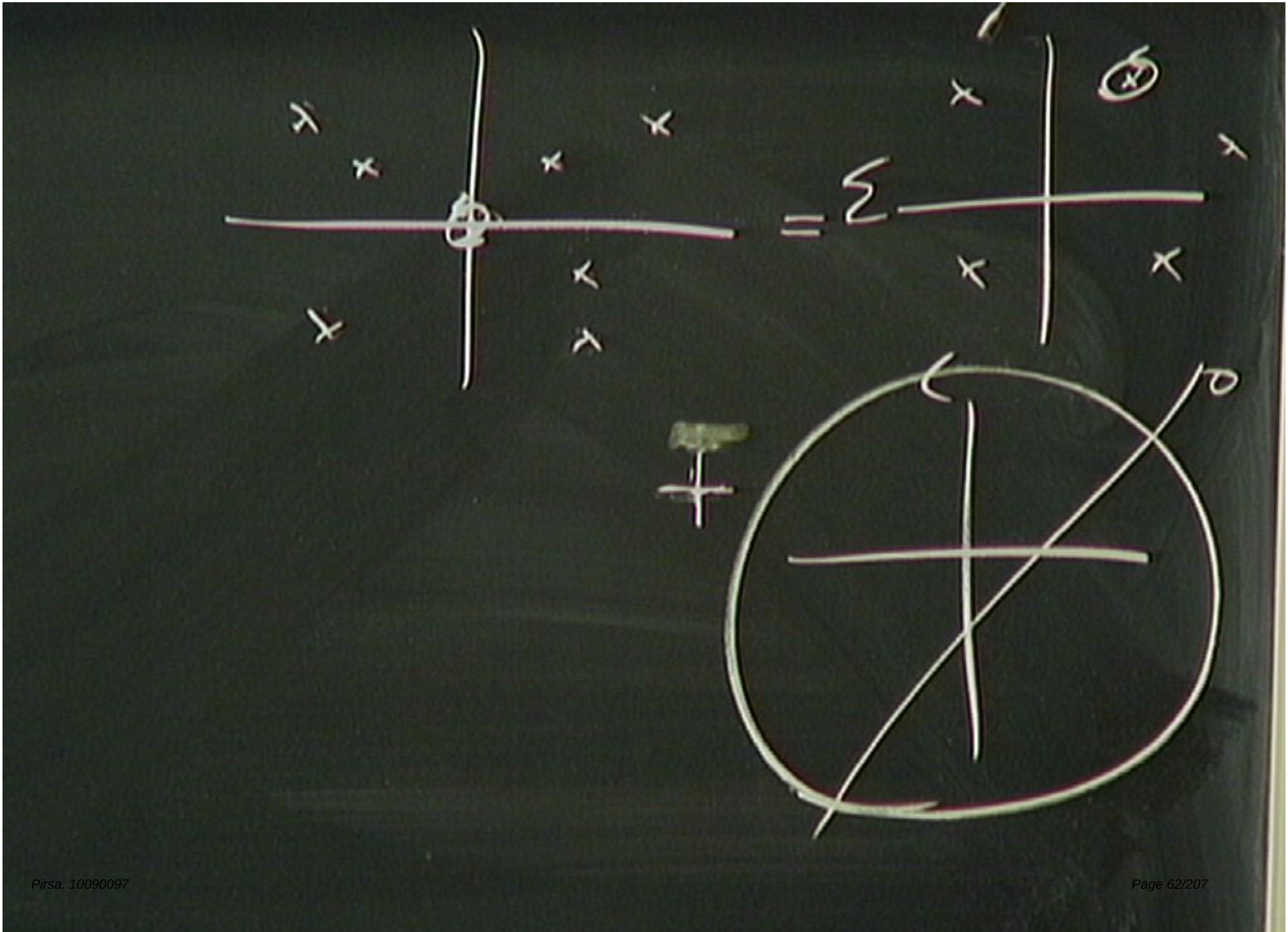
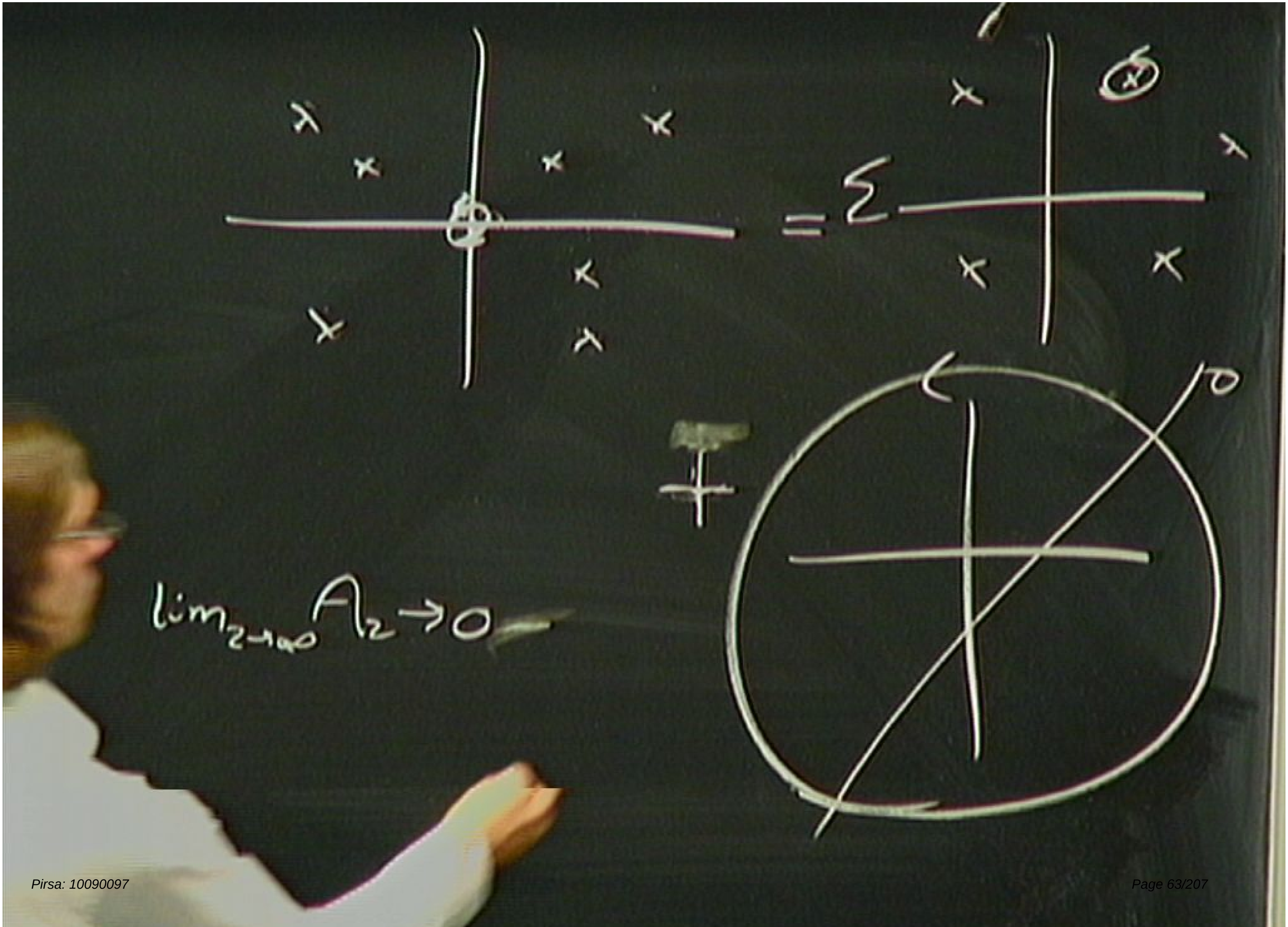
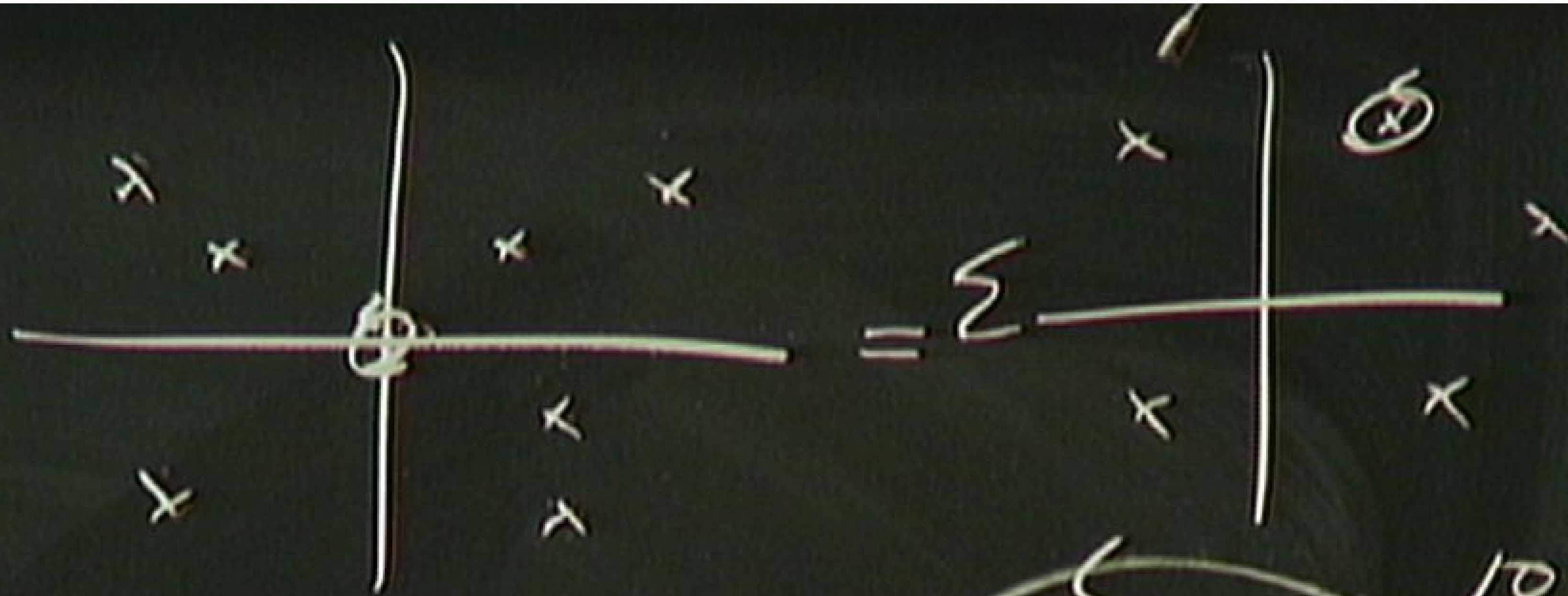
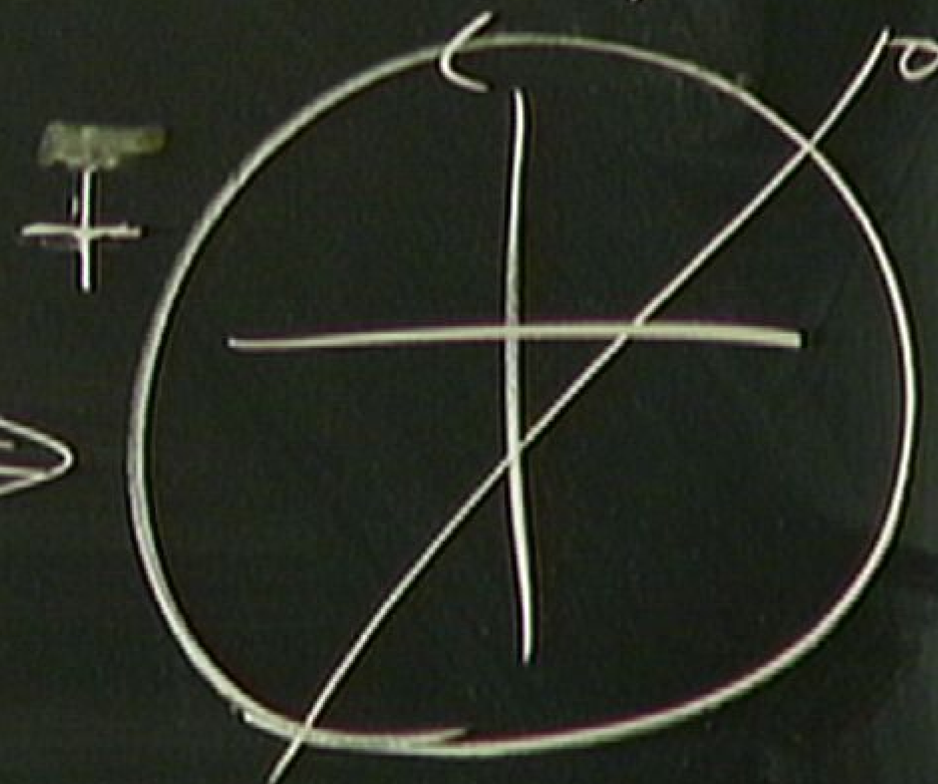



= Σ

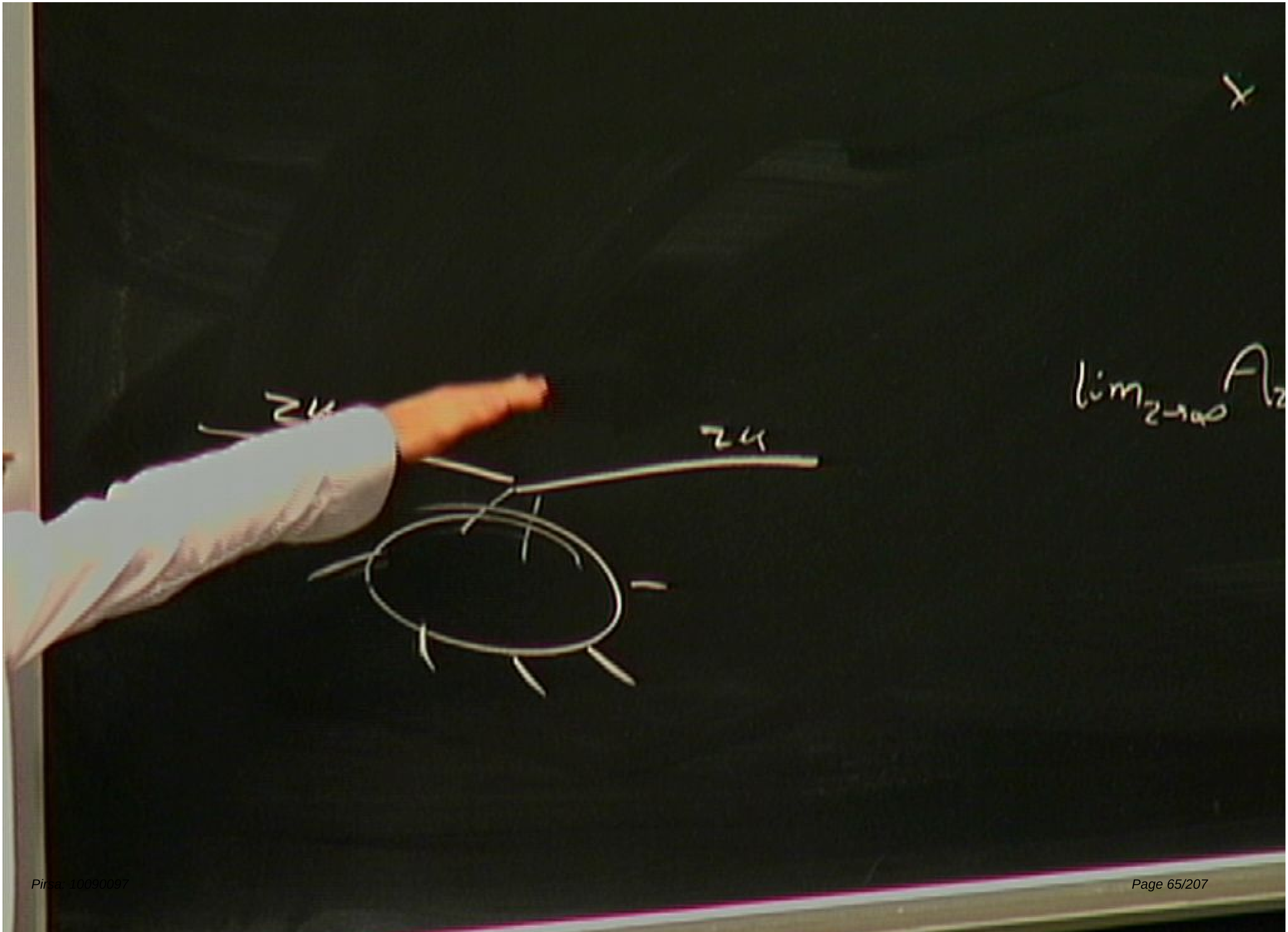


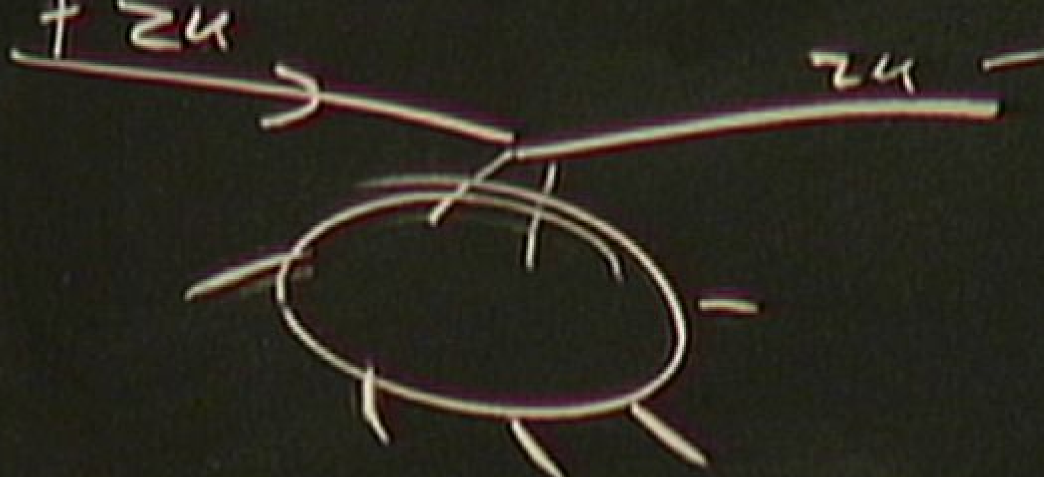

$= \Sigma$
$\lim_{z\to\infty} A_z \to 0$



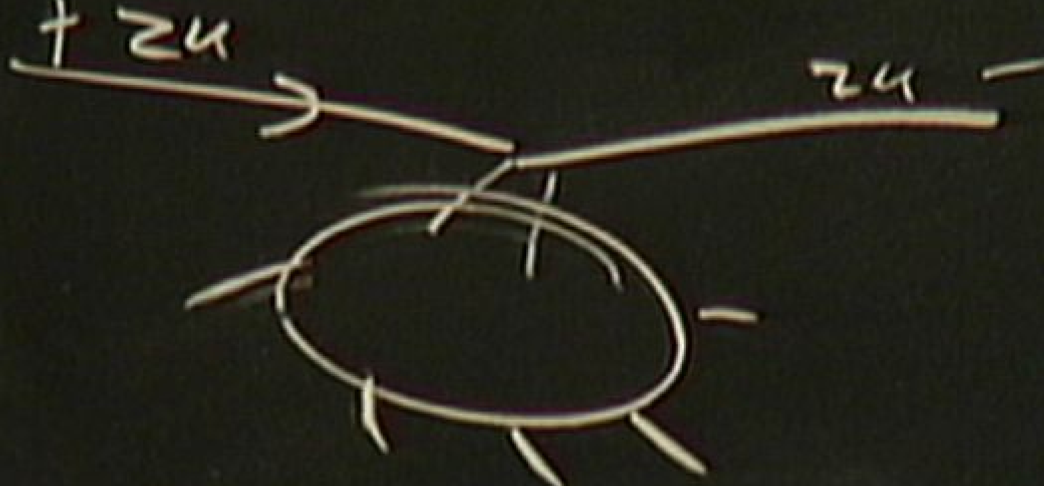


$$\lim_{z\to\infty} A_z \to 0 \iff$$

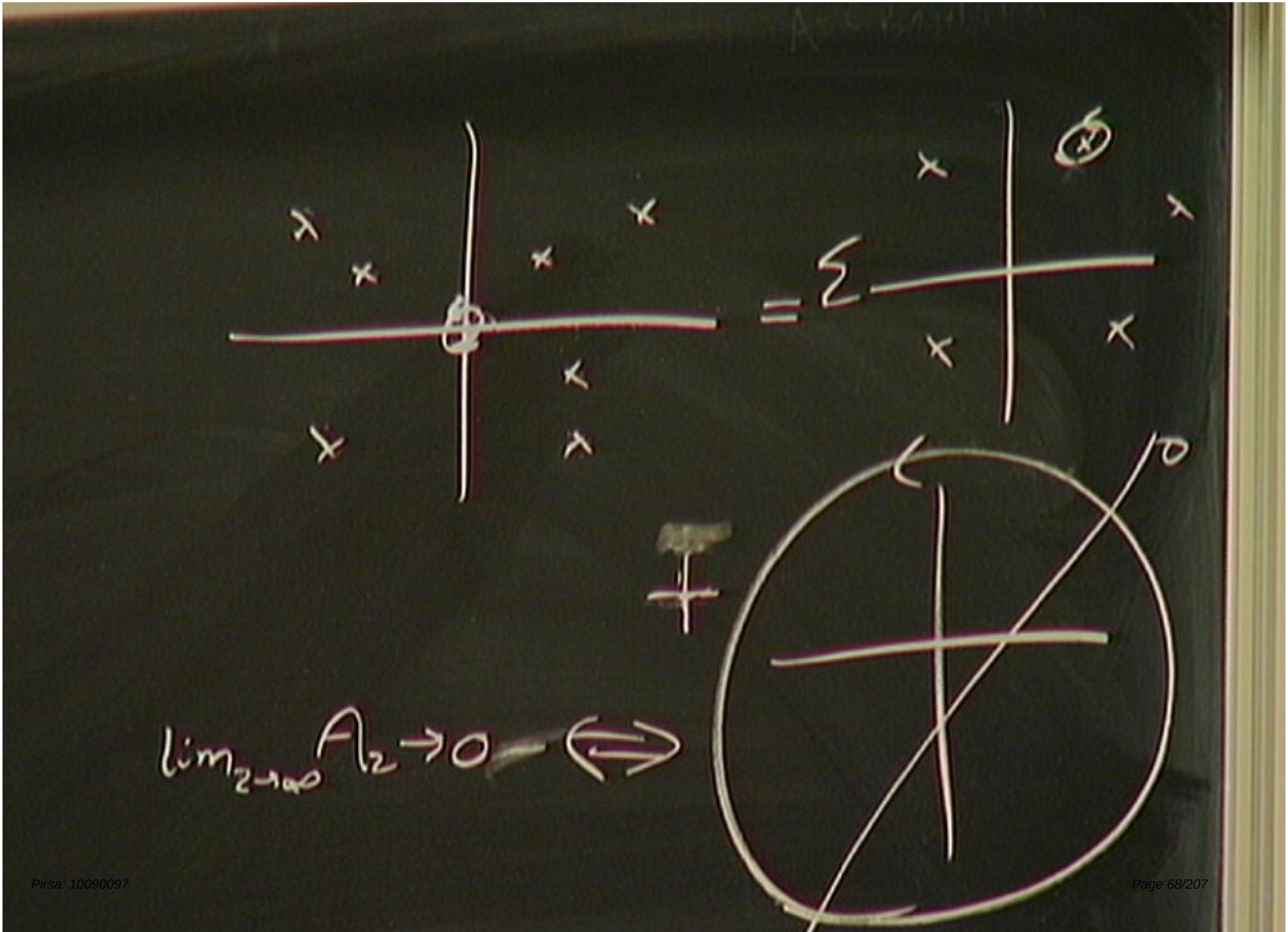

zu
zu
lim z→∞ A

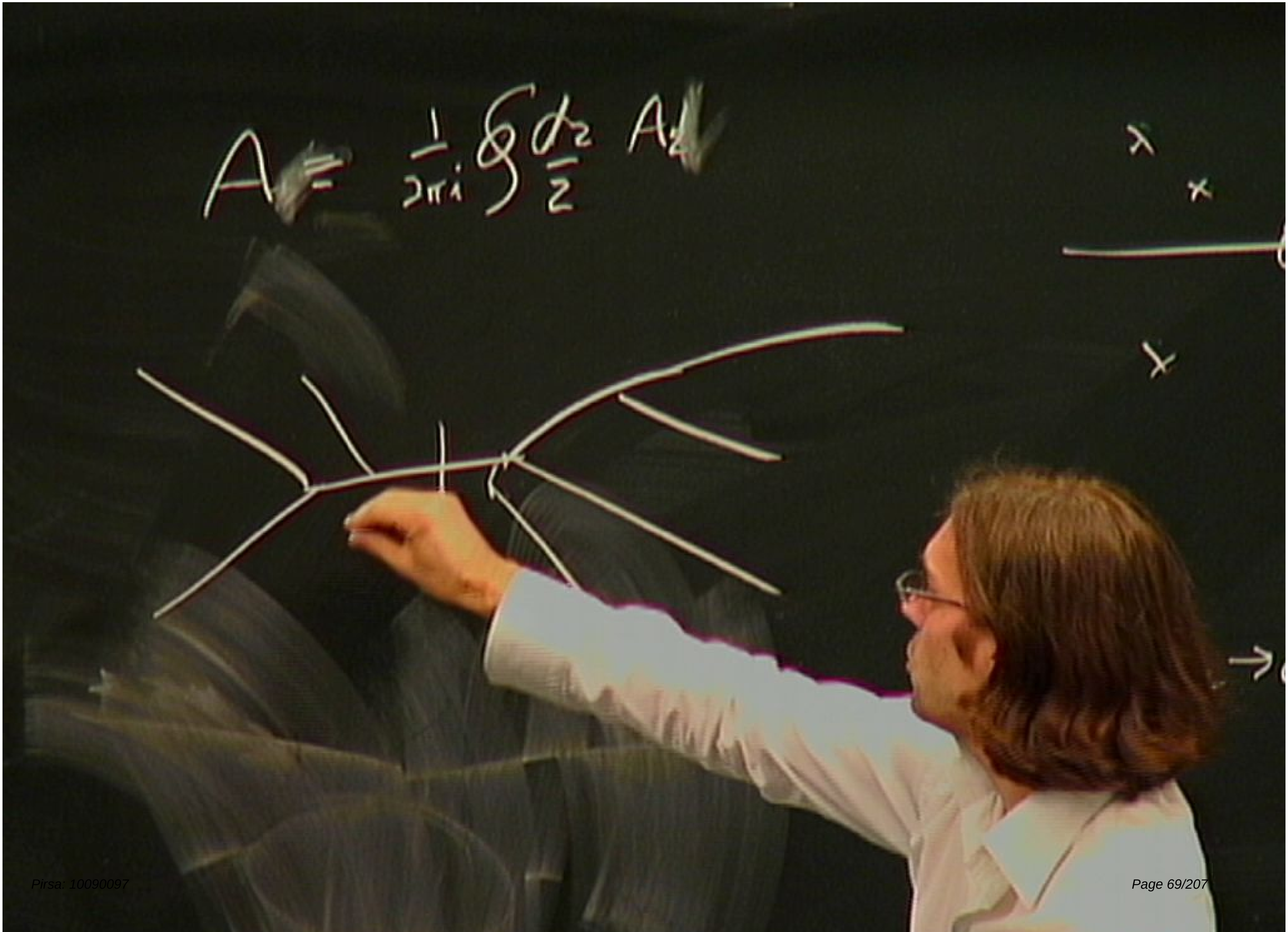


$\lim_{z\to\infty} A_z$

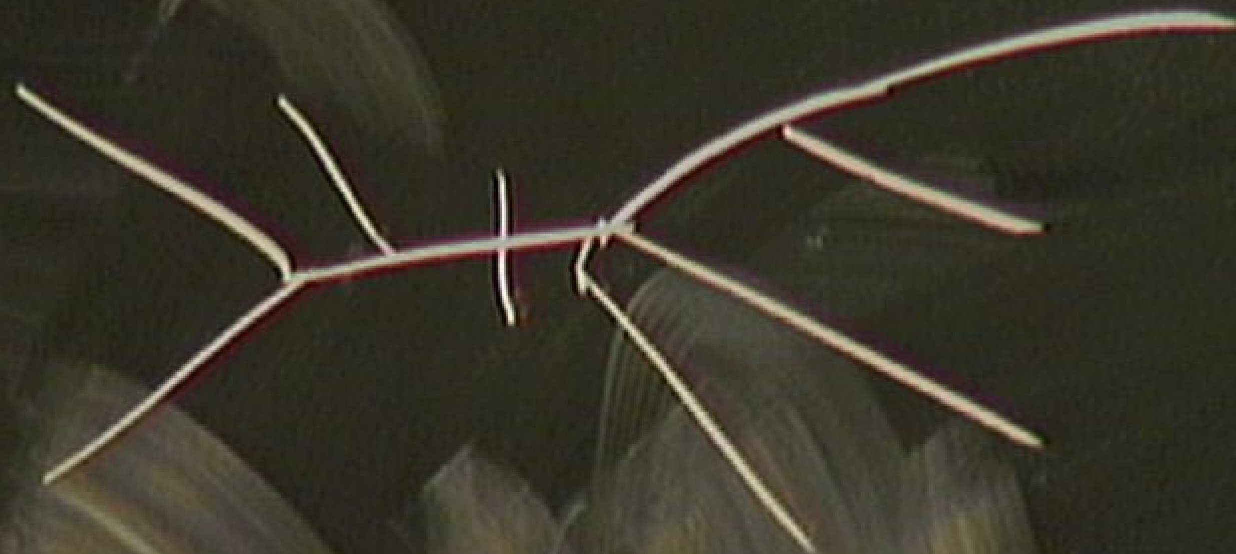


$\lim_{z\to\infty} A_z$


$= \Sigma$
$\lim_{z\to\infty} A_z \to 0 \iff$


$A = \frac{1}{2\pi i}\oint \frac{dz}{z} A_z$

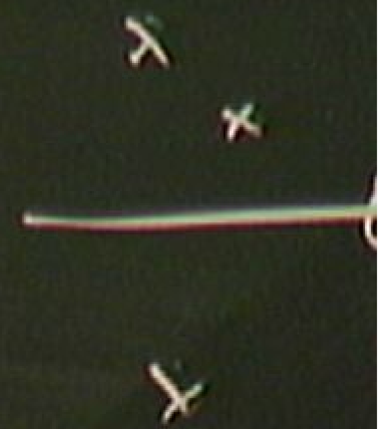
$$A = \frac{1}{2\pi i}\oint \frac{dz}{z} A_z$$



$$\lim_{z\to\infty} A_z \to$$

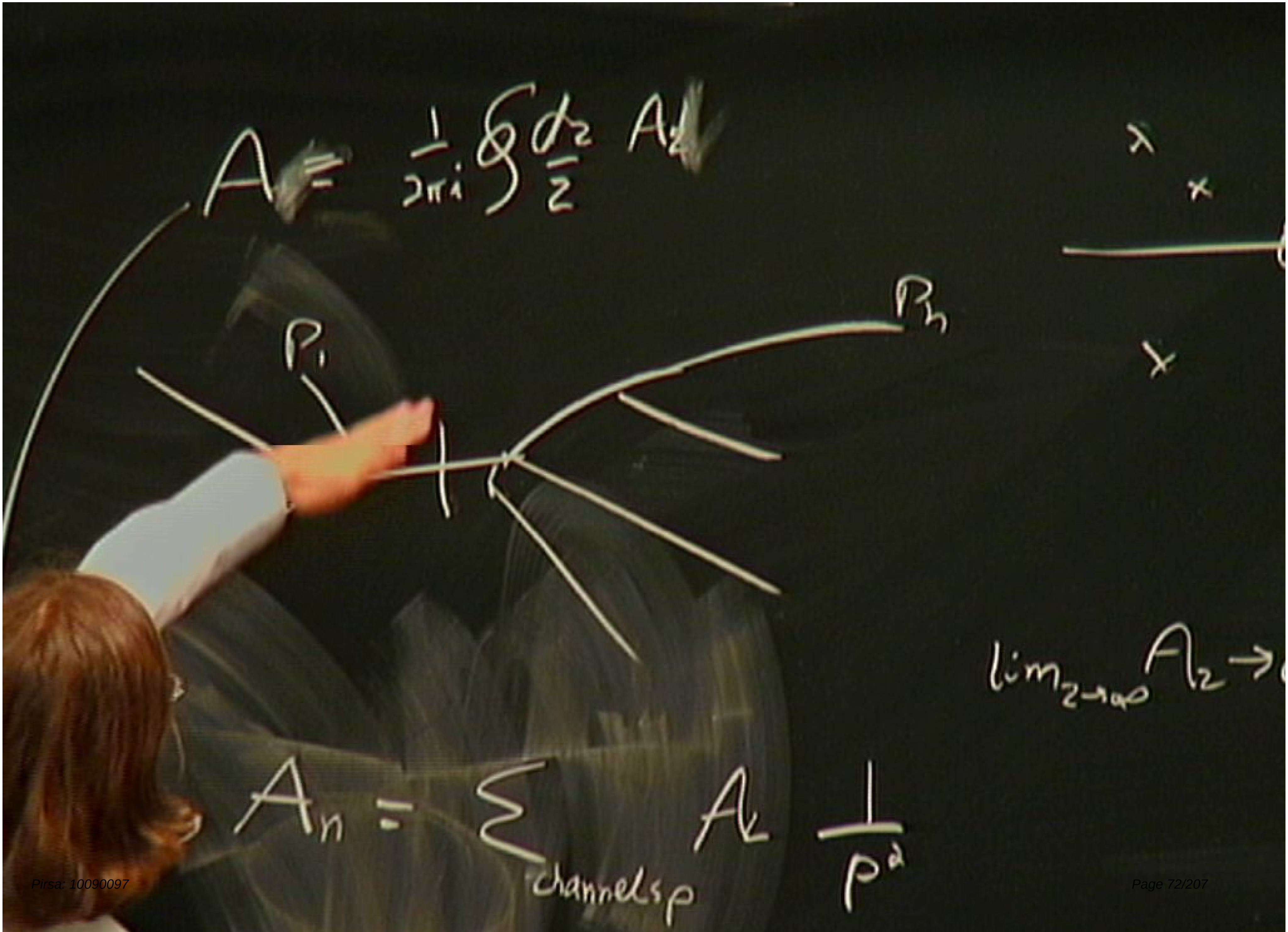
$$A_n = \sum_{\text{chan}}$$

$$A = \frac{1}{2\pi i}\oint \frac{dz}{z} A_z$$

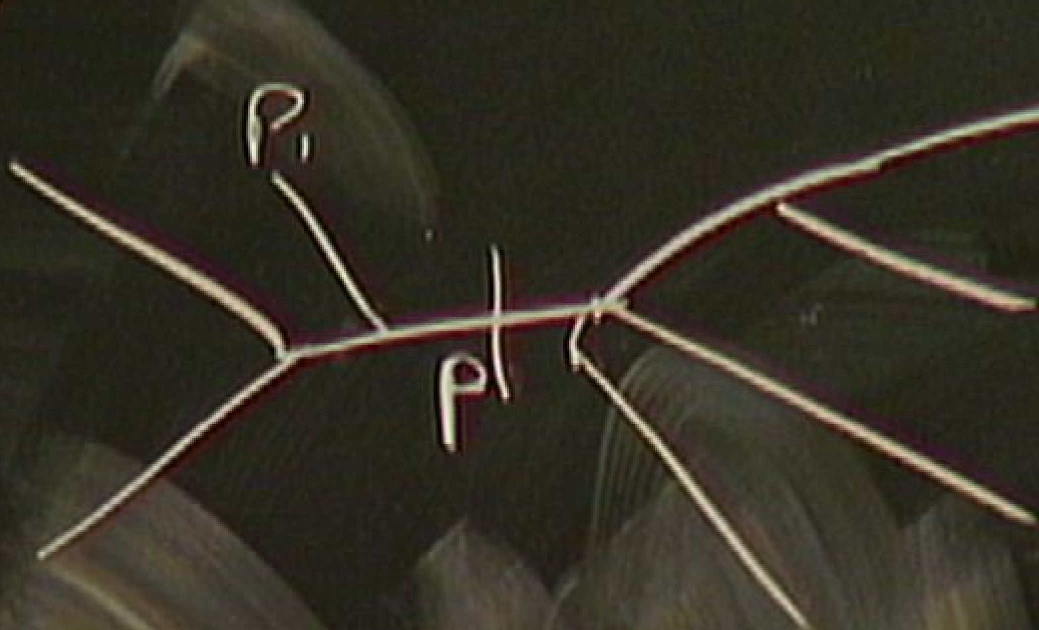


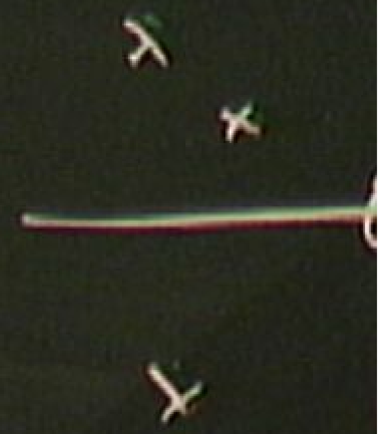
$$\lim_{z\to\infty} A_z \to$$

$$A_n = \sum_{\text{channels}}$$


A = 1/2πi ∮ dz/z A_z
p_1
p_n
lim_{z→∞} A_z →
A_n = Σ_{channels p} A_L 1/p^2

$$A = \frac{1}{2\pi i}\oint \frac{dz}{z} A(z)$$

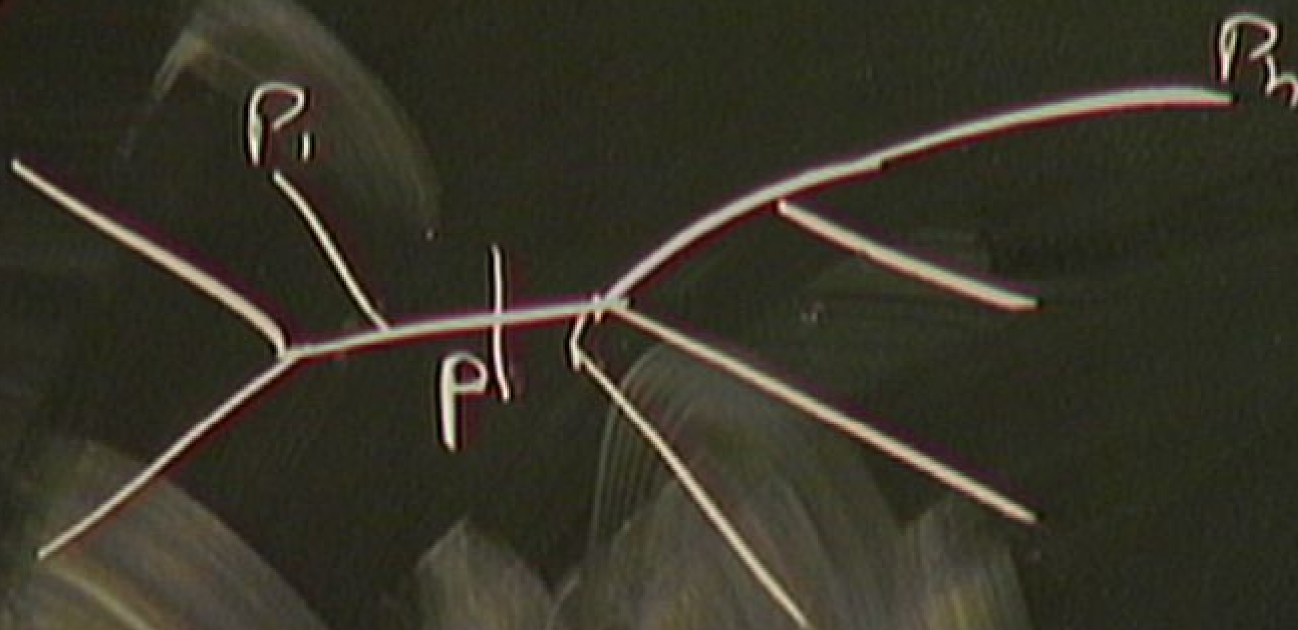




$\lim_{z \to \infty}$

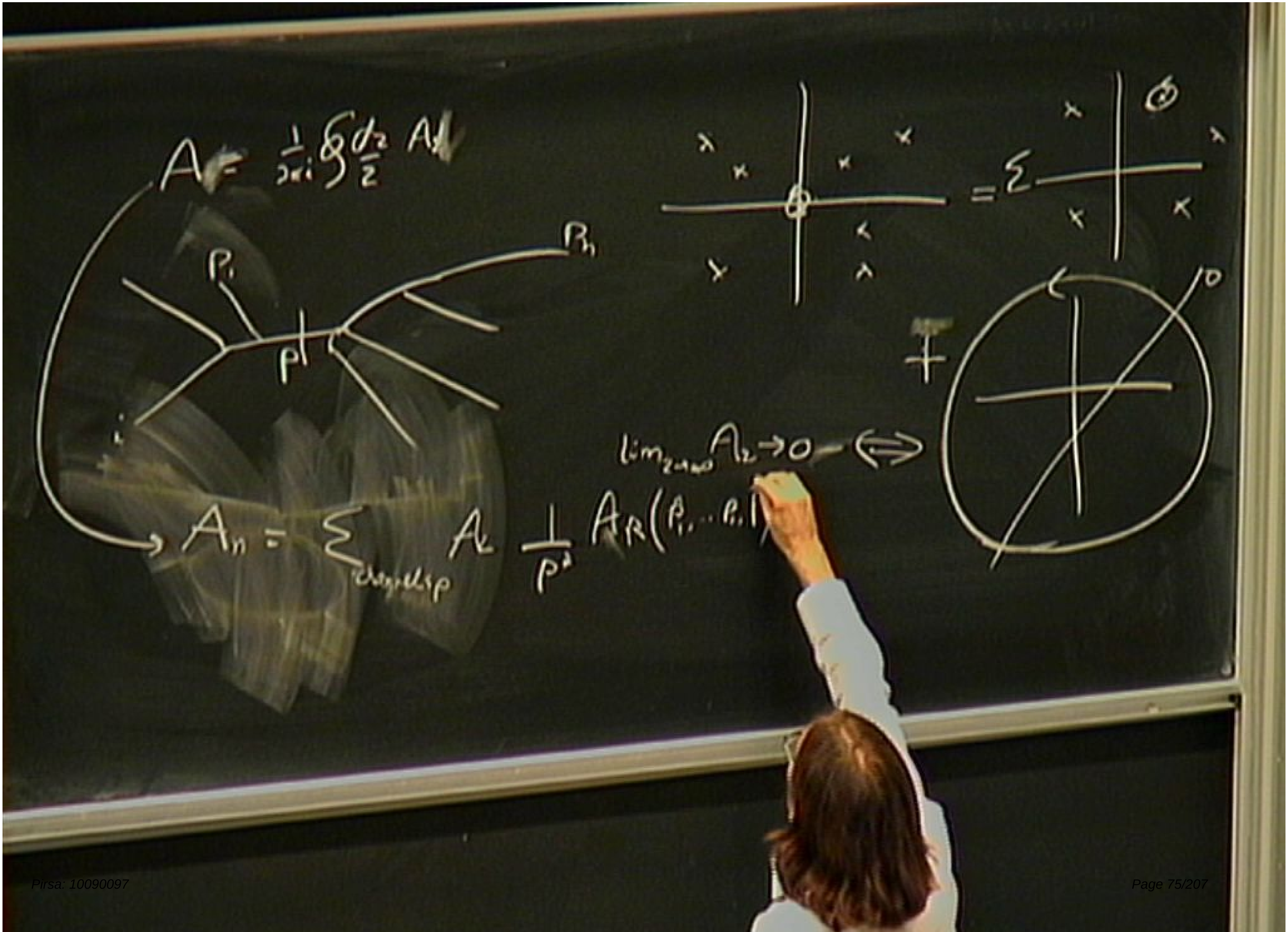
$$\rightarrow A_n = \sum_{\text{channels } p} A_L \frac{1}{p^2}$$

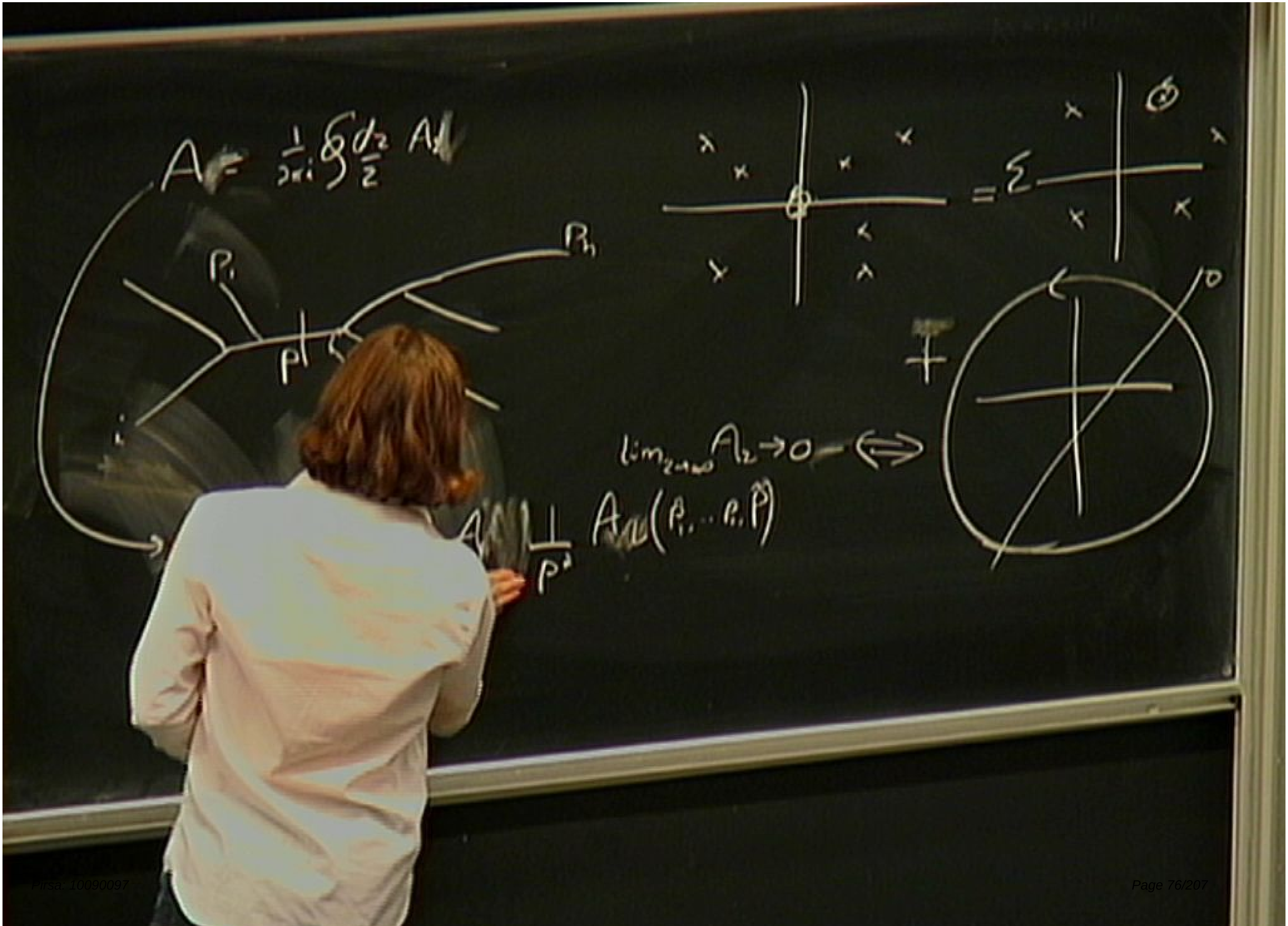
$$A = \frac{1}{2\pi i}\oint \frac{dz}{z} A_z$$

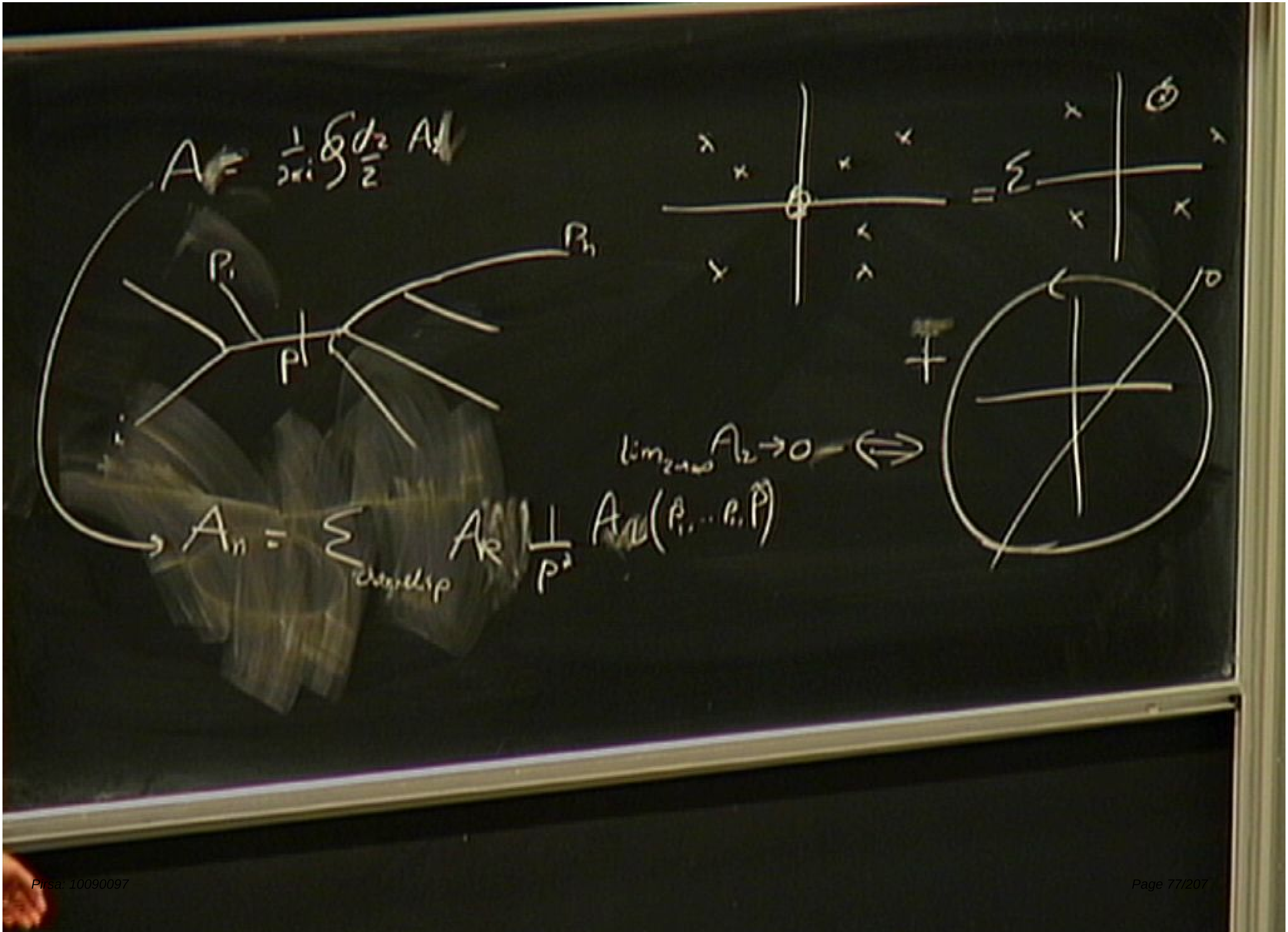


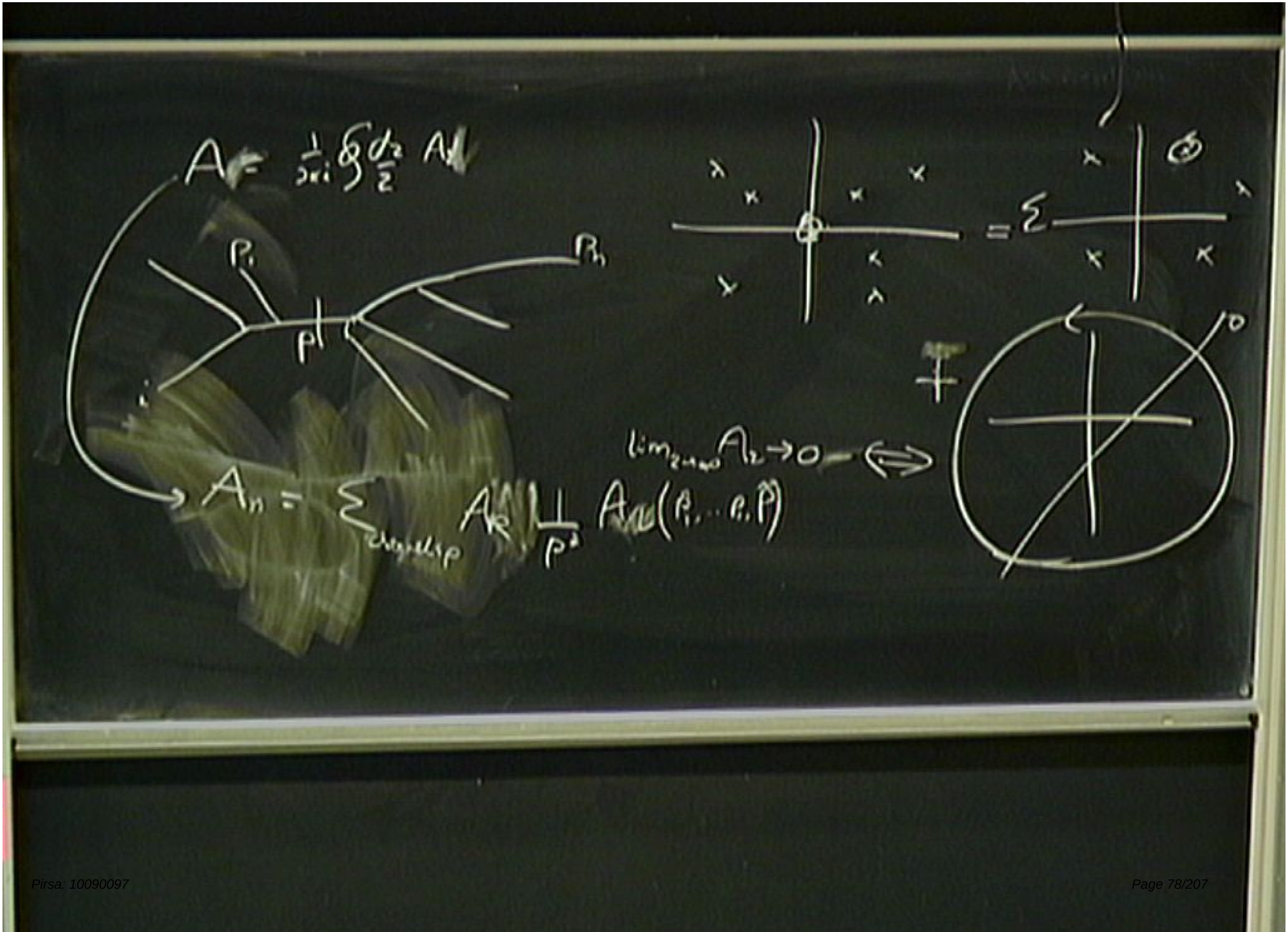
$$\lim_{z\to\infty} A_z \to$$

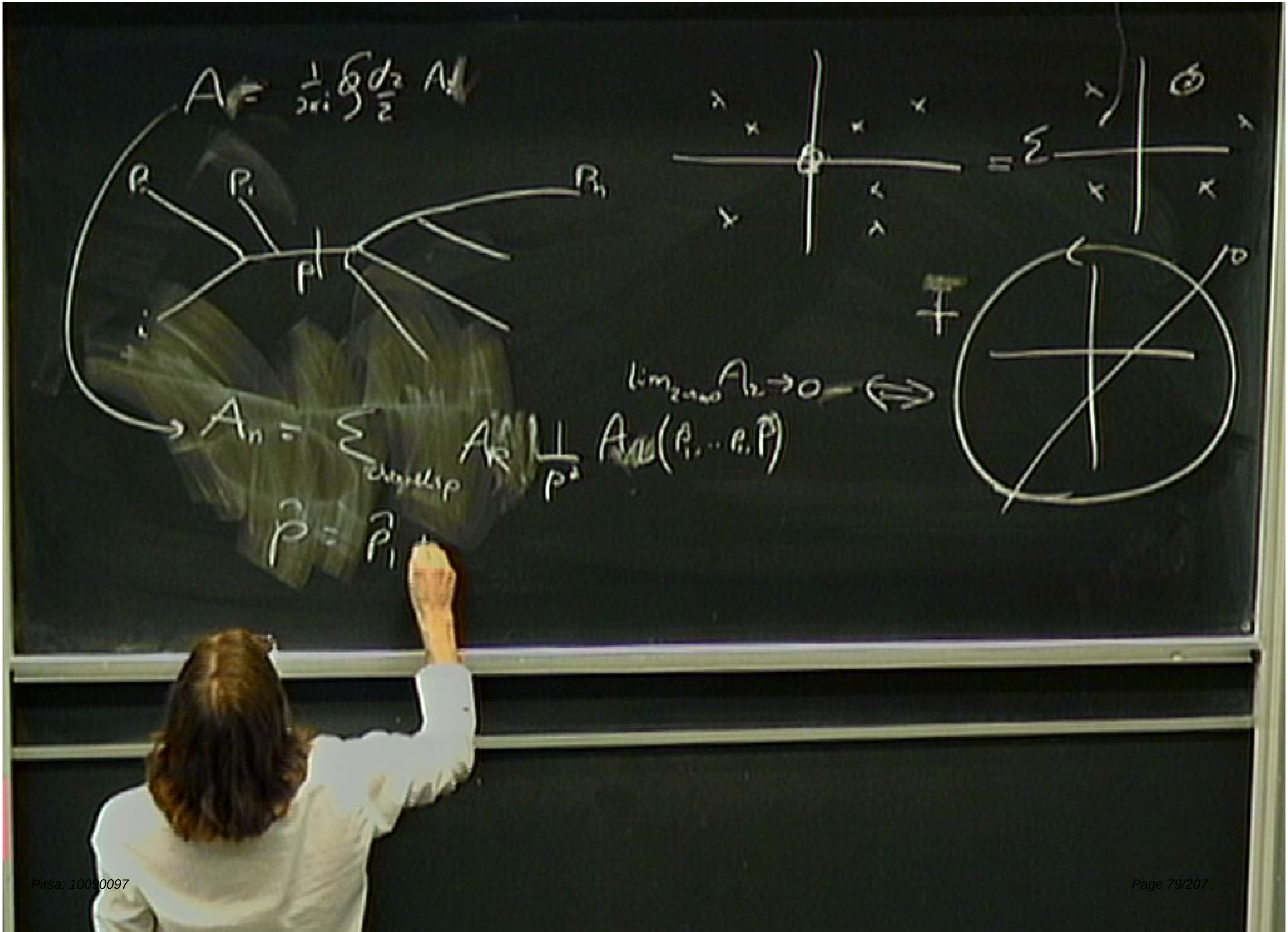
$$\to A_n = \sum_{\text{channels } p} A_L \frac{1}{p^2} A_R(\quad)$$

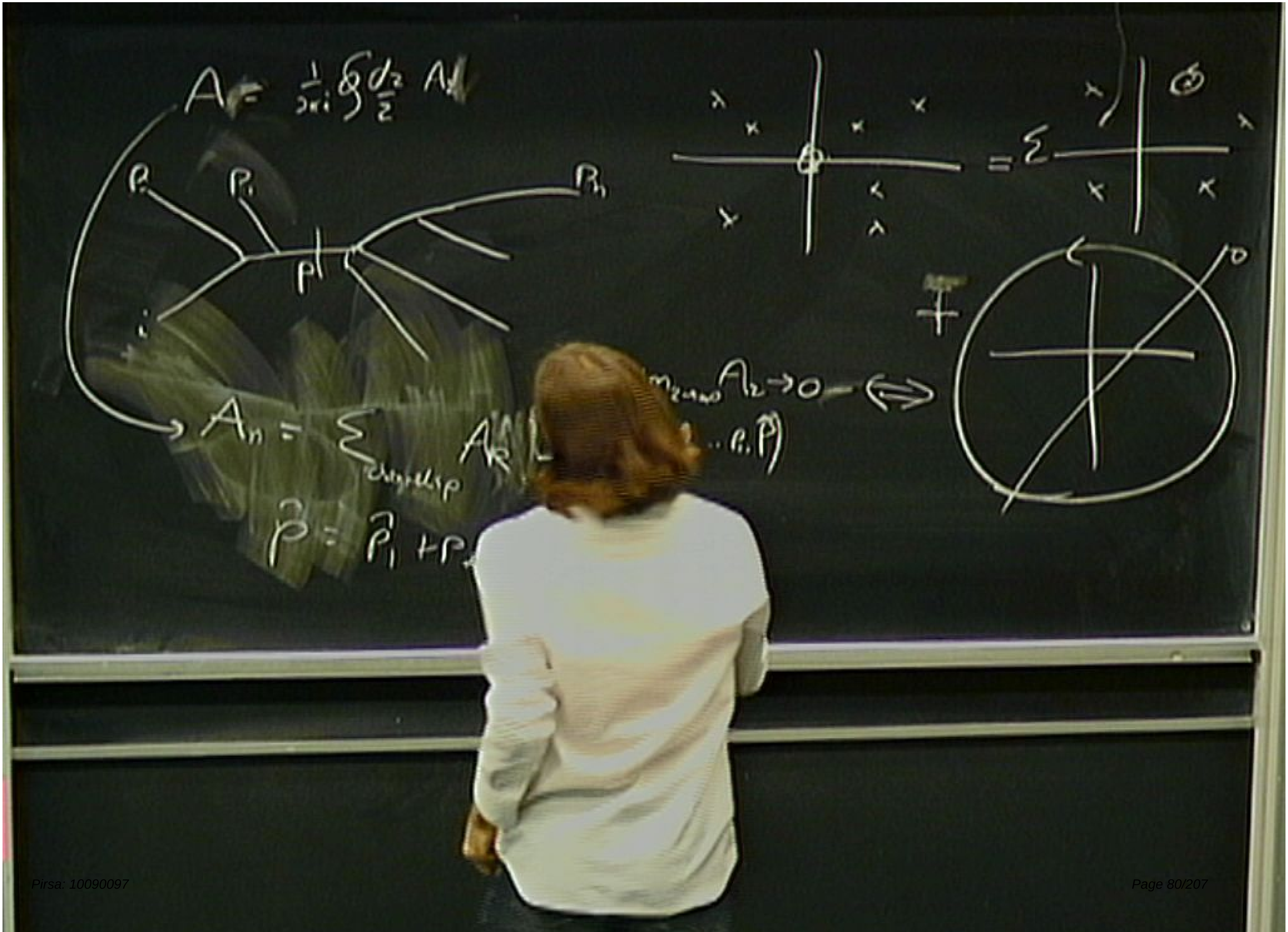

A = 1/2πi ∮ dz/z A
P1
Pn
P
= Σ
lim z→∞ Az → 0 ⟺
An = Σ AL 1/P² AR(P1 ... P

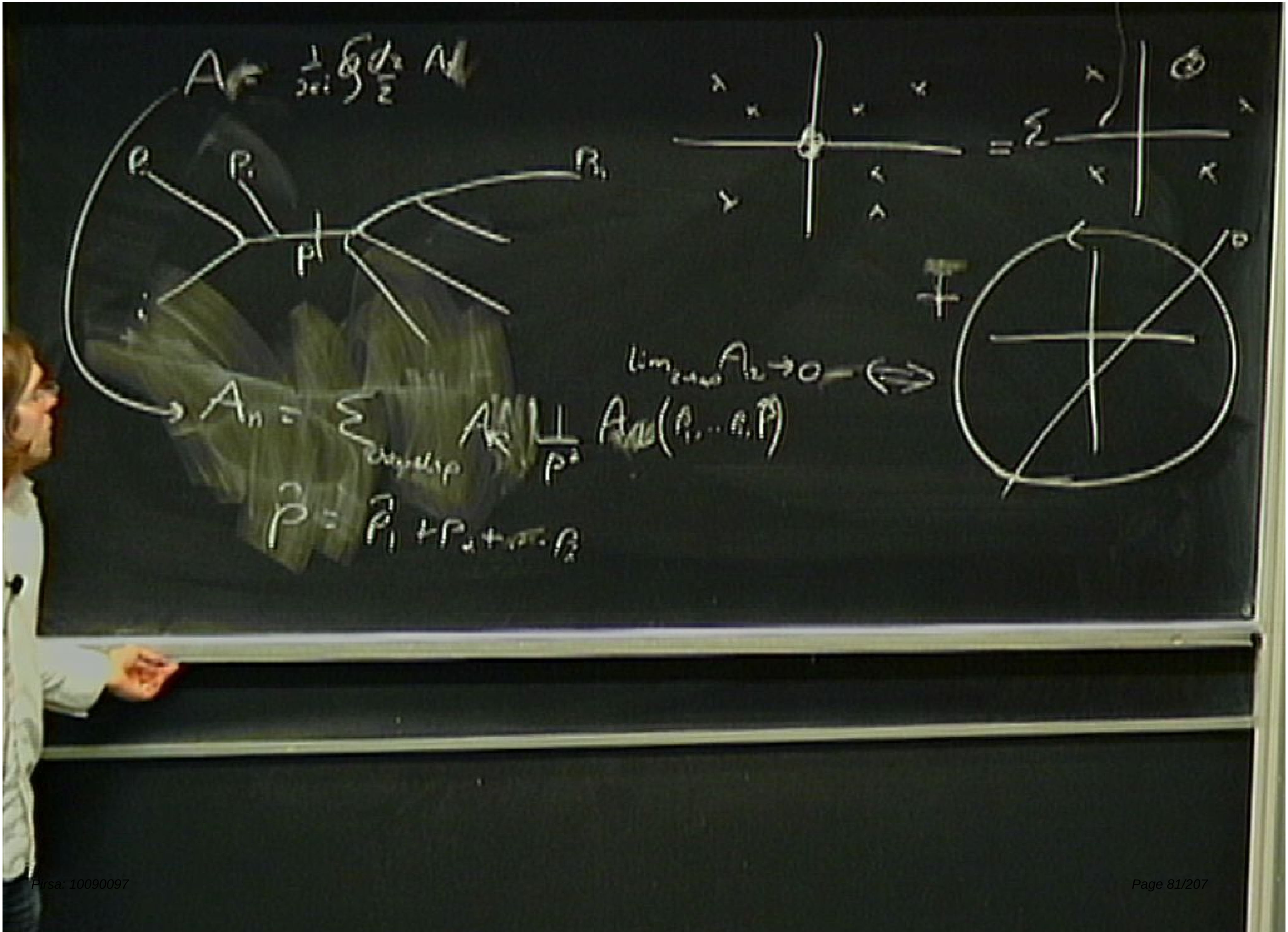

P1
Pn
lim A_z → 0 ⟺


A = 1/2πi ∮ dz/z A(z)
P1
Pn
P
i
= Σ
lim z→∞ A(z) → 0 ⟺
An = Σ diagrams,p AL 1/P² AR(P1 … Pi, P̂)

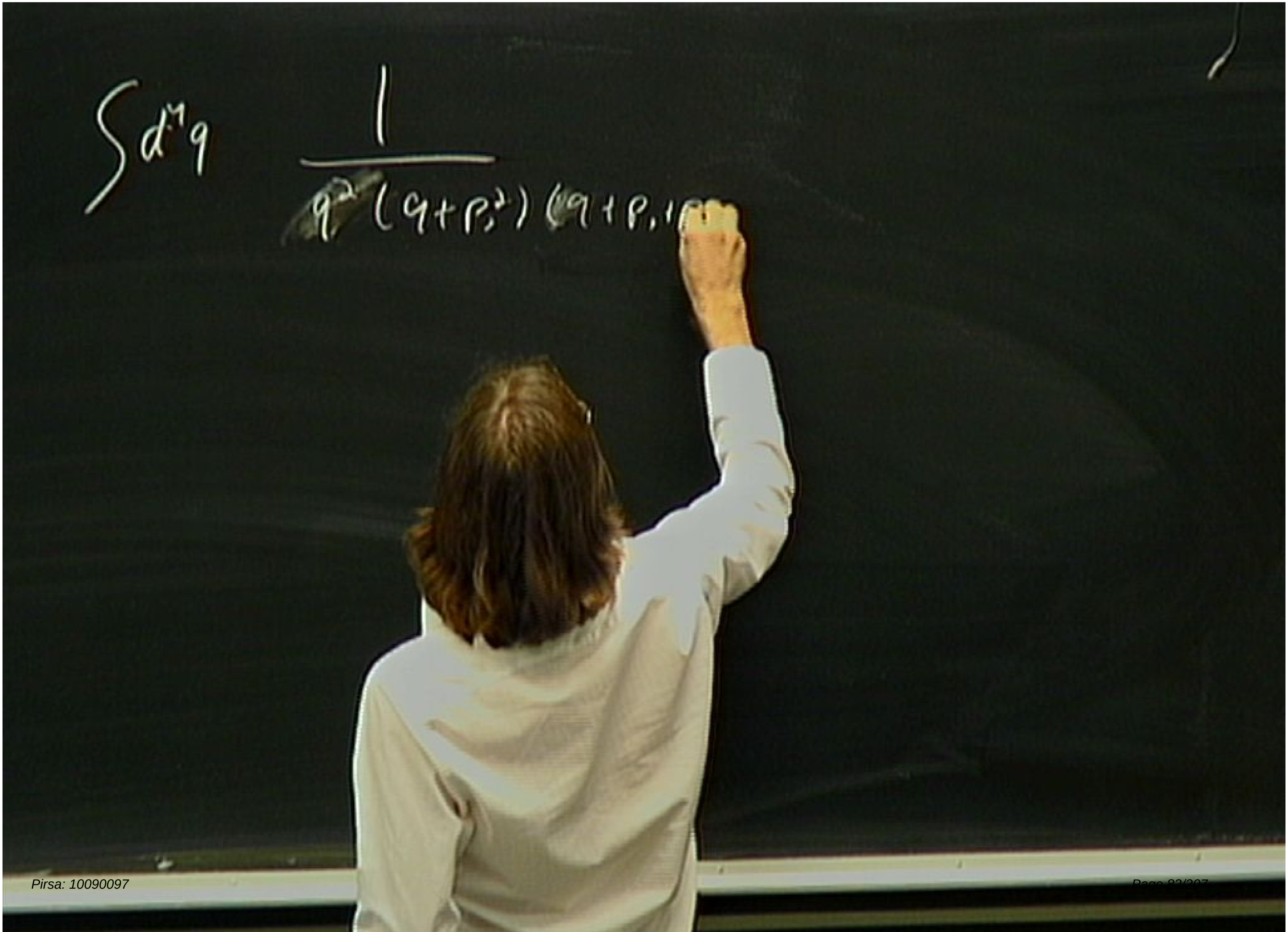

A = 1/2πi ∮ dz/z A(z)
P1
Pn
P
An = Σ A_L 1/P² A_R(P1 … Pi, P)
lim A_z → 0 ⟺
=Σ


A = 1/2πi ∮ dz/z A
P₁
P₁
Pₙ
P
i
lim A_z → 0 ⟺
Aₙ = Σ A_L 1/P² A_R(P₁ … Pᵢ P̂)
P̂ = P̂₁


A = 1/2πi ∮ dz/z A
Pᵢ
P₁
Pₙ
P
Aₙ = Σ A_L
P = P₁ + P
A_z → 0



$A = \frac{1}{2\pi i}\oint \frac{dz}{z} A(z)$
$\lim_{z\to\infty} A_z \to 0 \Leftrightarrow$
$A_n = \sum A_L \frac{1}{P^2} A_R(P_i \ldots P_j, P)$
$\vec{P} = \vec{P}_1 + P_2 + \cdots P_i$


∫d⁴q
1
q² (q+p₃²) (q+p₁

$$\int d^4q \, \frac{1}{(q+p_1)^2\,(q+p_1+p_2)^2}$$

$$\int d^4 q \left(\frac{1}{q^2 (q+p_3)^2 (q+p_1+p_2)^2} + \frac{1}{q^2 (q+p_3)^2 (q+p_3+p_4)^2} \right)$$

$$\int d^4q \left(\frac{1}{q^2 (q+p_1)^2 (q+p_1+p_2)^2} + \frac{1}{q^2 (q+p_3)^2 (q+p_3+p_4)^2} \right) \rightarrow \frac{1}{(q+p_1)^2} \quad (q+p_3+p_4) \ \ldots$$

$$\int d^4q \left(\frac{1}{q^2 (q+p_1)^2 (q+p_1+p_2)^2} + \frac{1}{q^2 (q+p_1)^2 (q+p_3+p_4)^2} \right) \rightarrow \frac{1}{(q+p_1)^2 \; (q+p_1+p_2) \ldots}$$

$$q^2 (q+p_3)^2 (q+p_3+p_4)^2$$

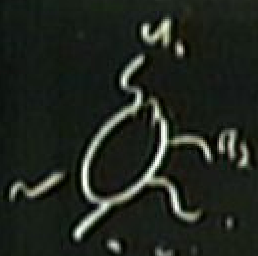
Planar gauge thy.

$$q^2(q+p_3)^2(q+p_3+p_4)^2$$

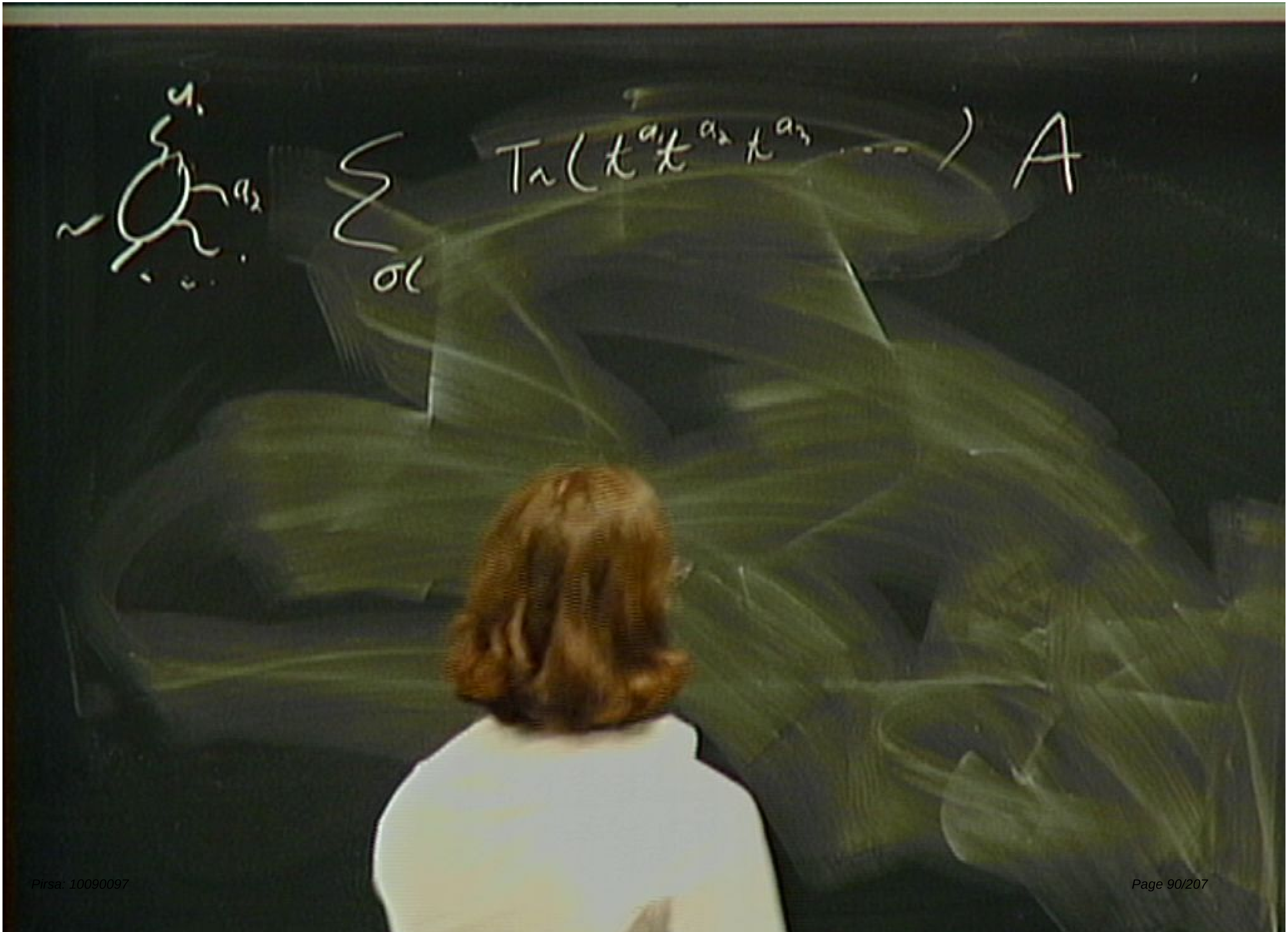
Planar gauge thy.

Σ

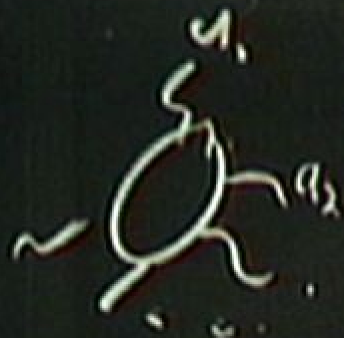
Planar gauge thy.



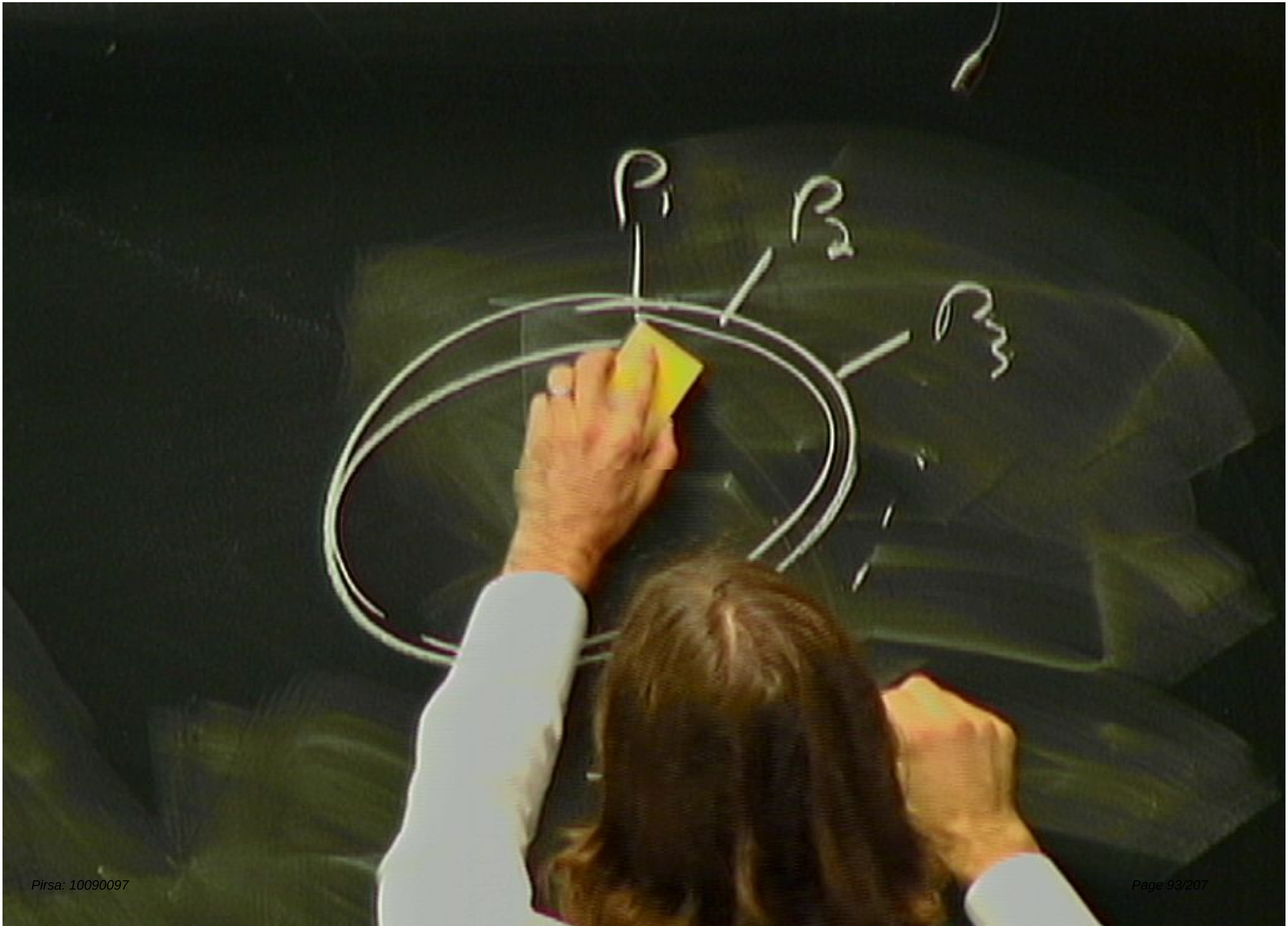
$$\sum_{\sigma} \mathrm{Tr}(t^{a_1} t^{a_2} t^{a_3} \cdots)$$

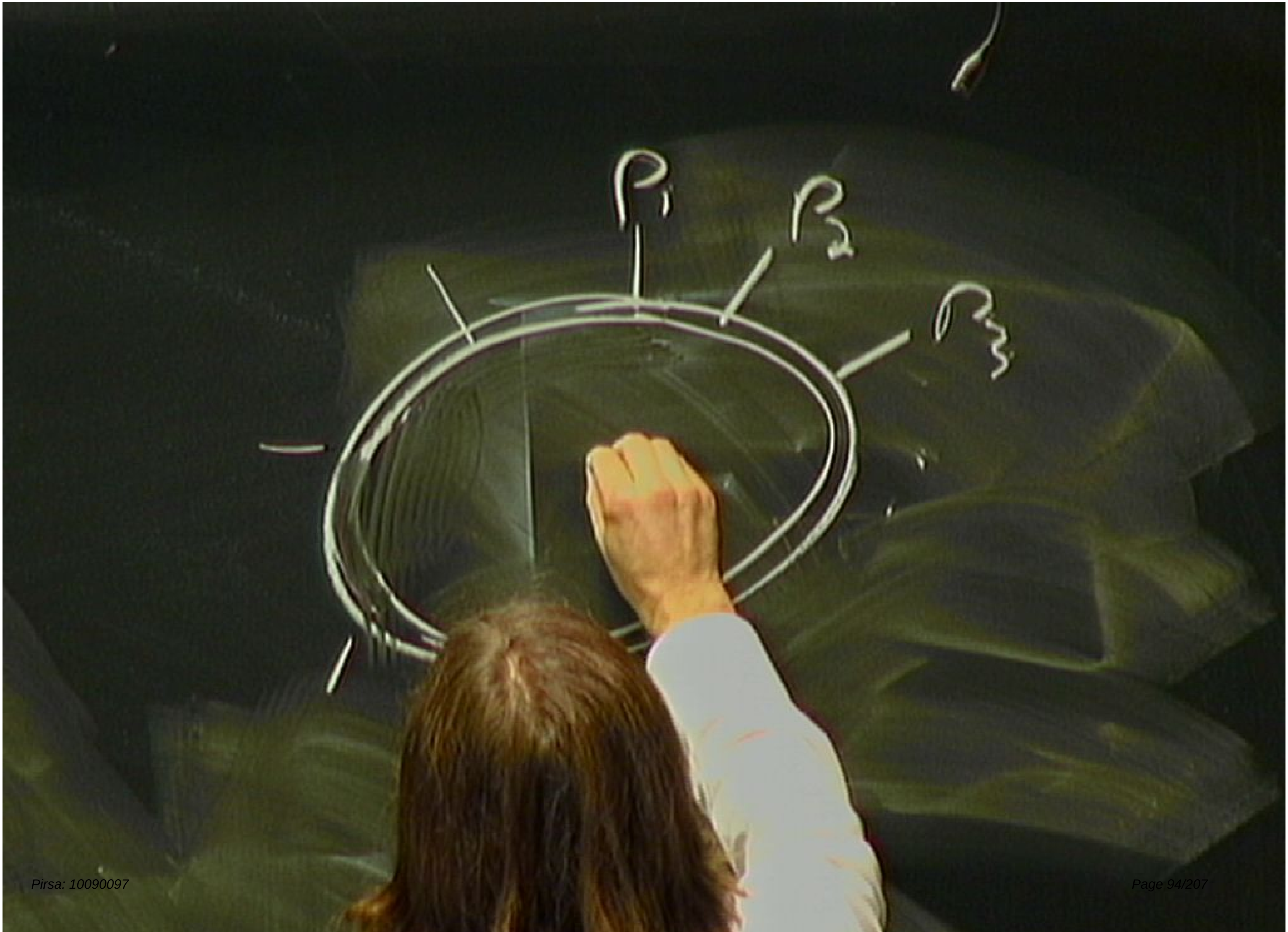

a_1
a_2
$\sum_{\sigma} \mathrm{Tr}(t^{a_1} t^{a_2} t^{a_3} \ldots) A$

$$\sum_{\sigma} \mathrm{Tr}\left(t^{\sigma(a_1)} t^{\sigma(a_2)} t^{\sigma(a_3)} \cdots \right) A$$

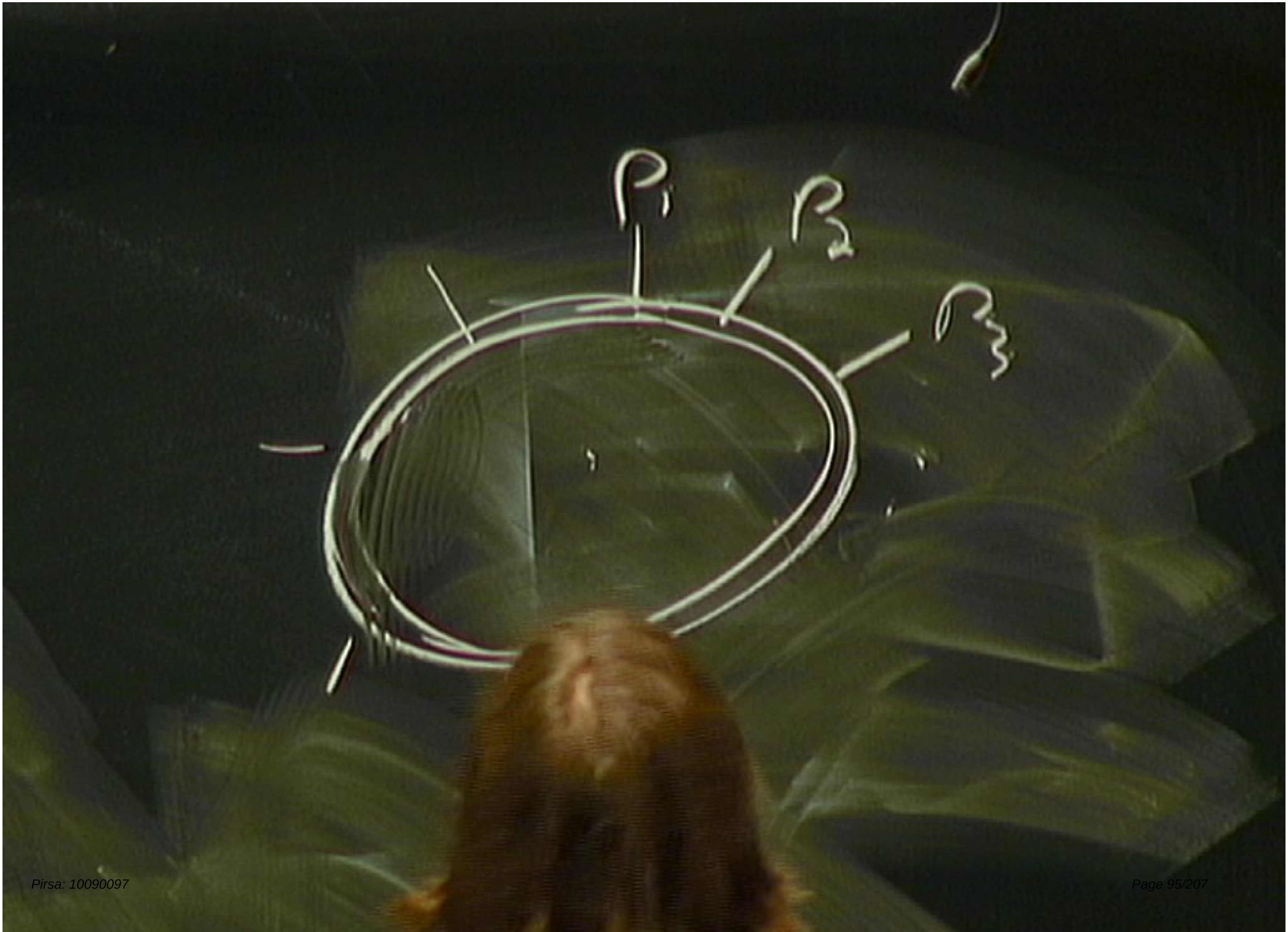


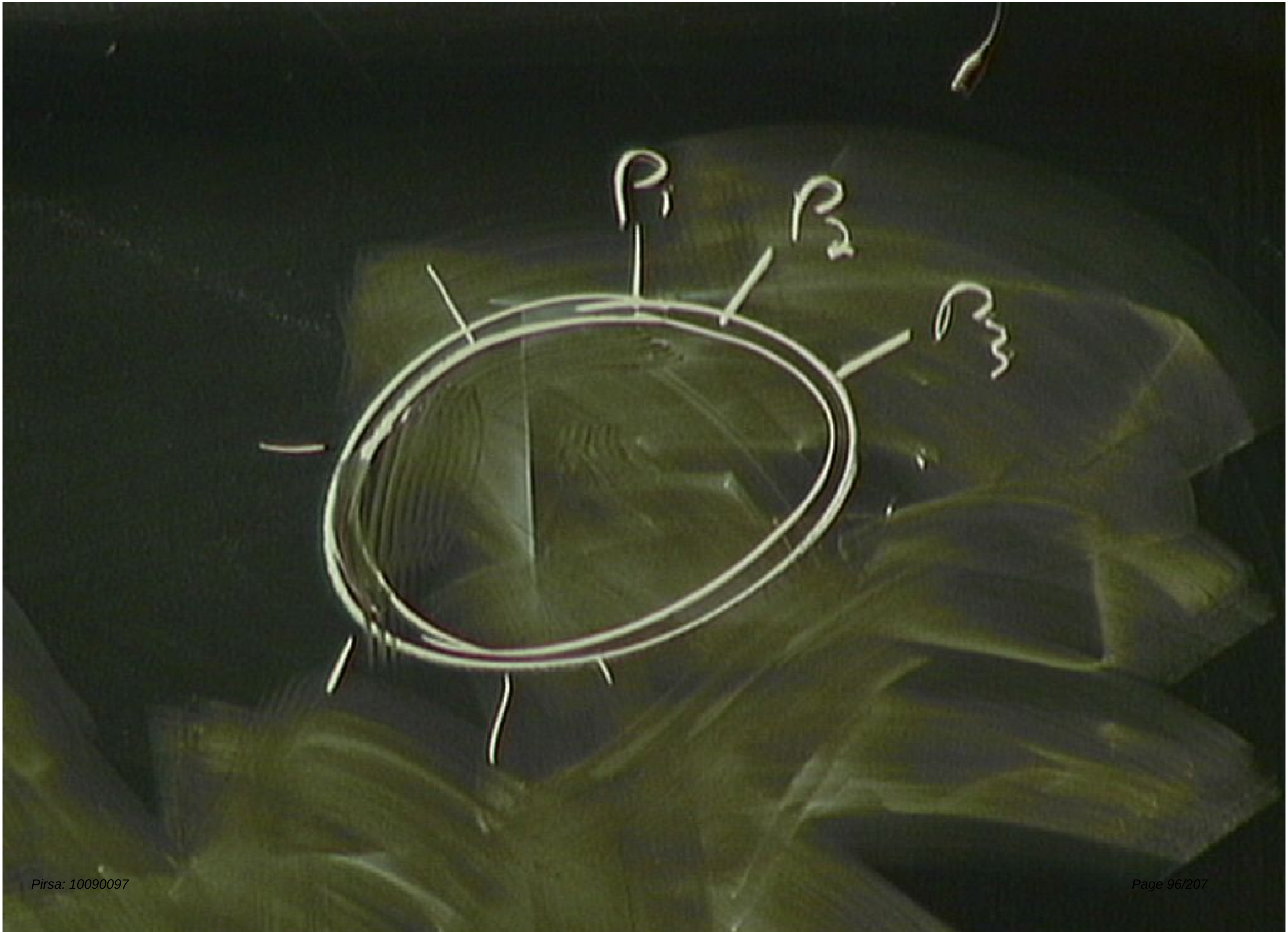
$$\sum_{\sigma} \mathrm{Tr}\left(t^{\sigma(a_1)} t^{\sigma(a_2)} t^{\sigma(a_3)} \cdots\right) A$$

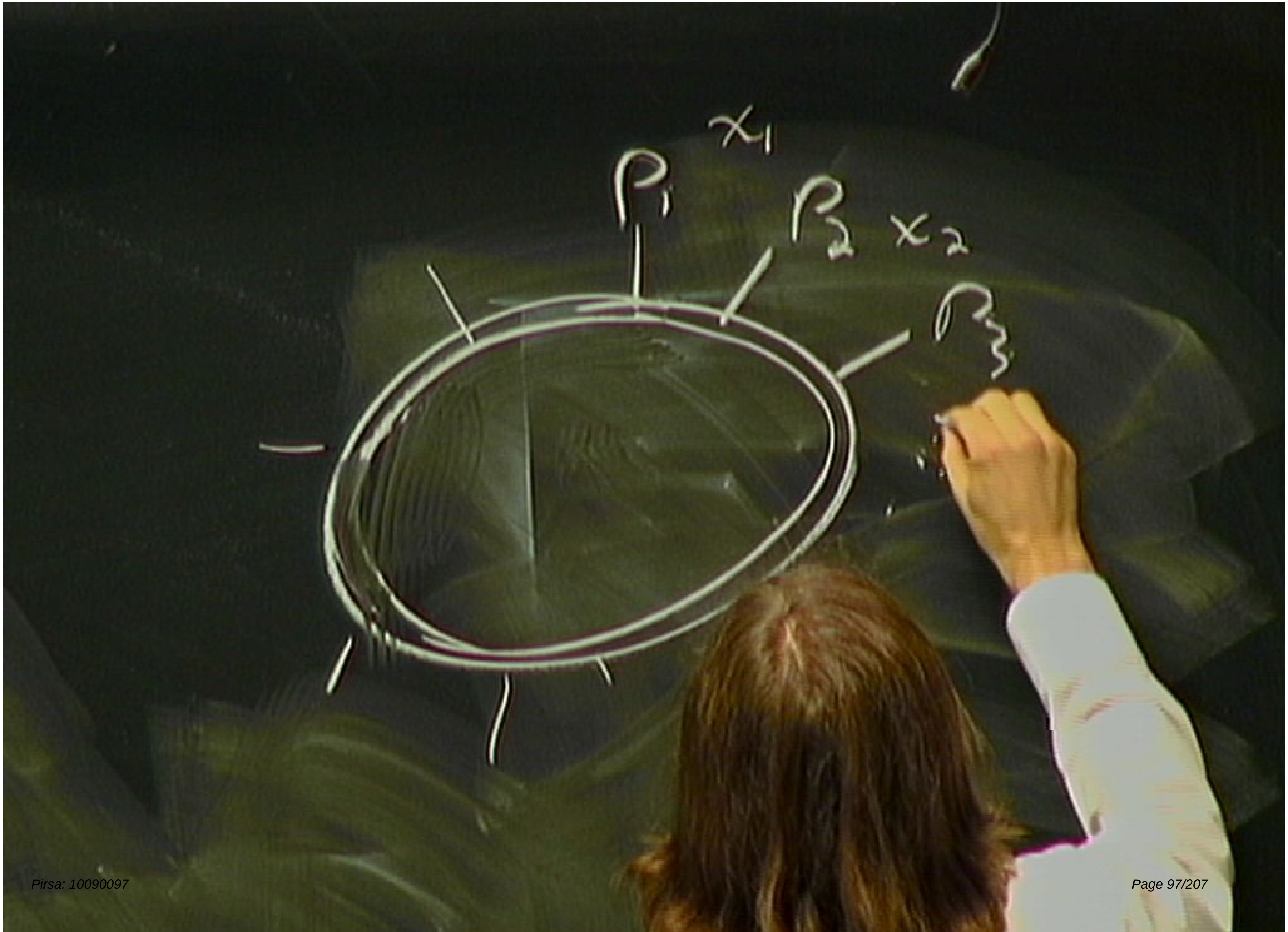

P1
P2
Pm



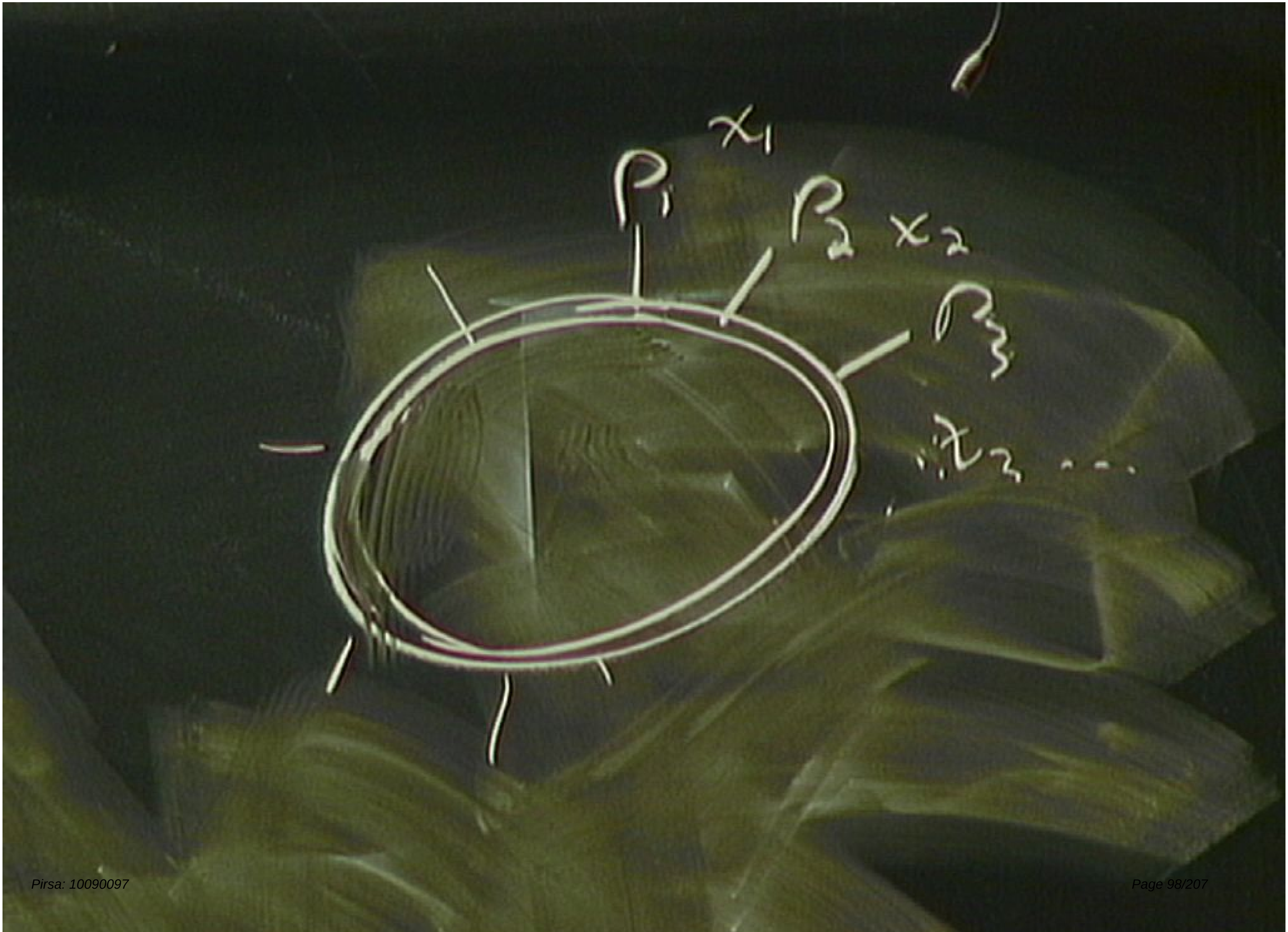
P_1
P_2
P_3

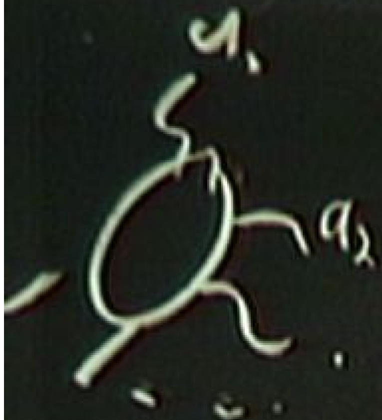

P_1
P_2
P_3


p_1
p_2
p_m



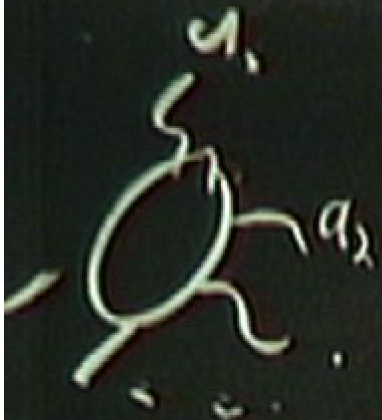
x_1
p_1
p_2 x_2
p_3


x1
P1
P2 x2
P3
x3 ...



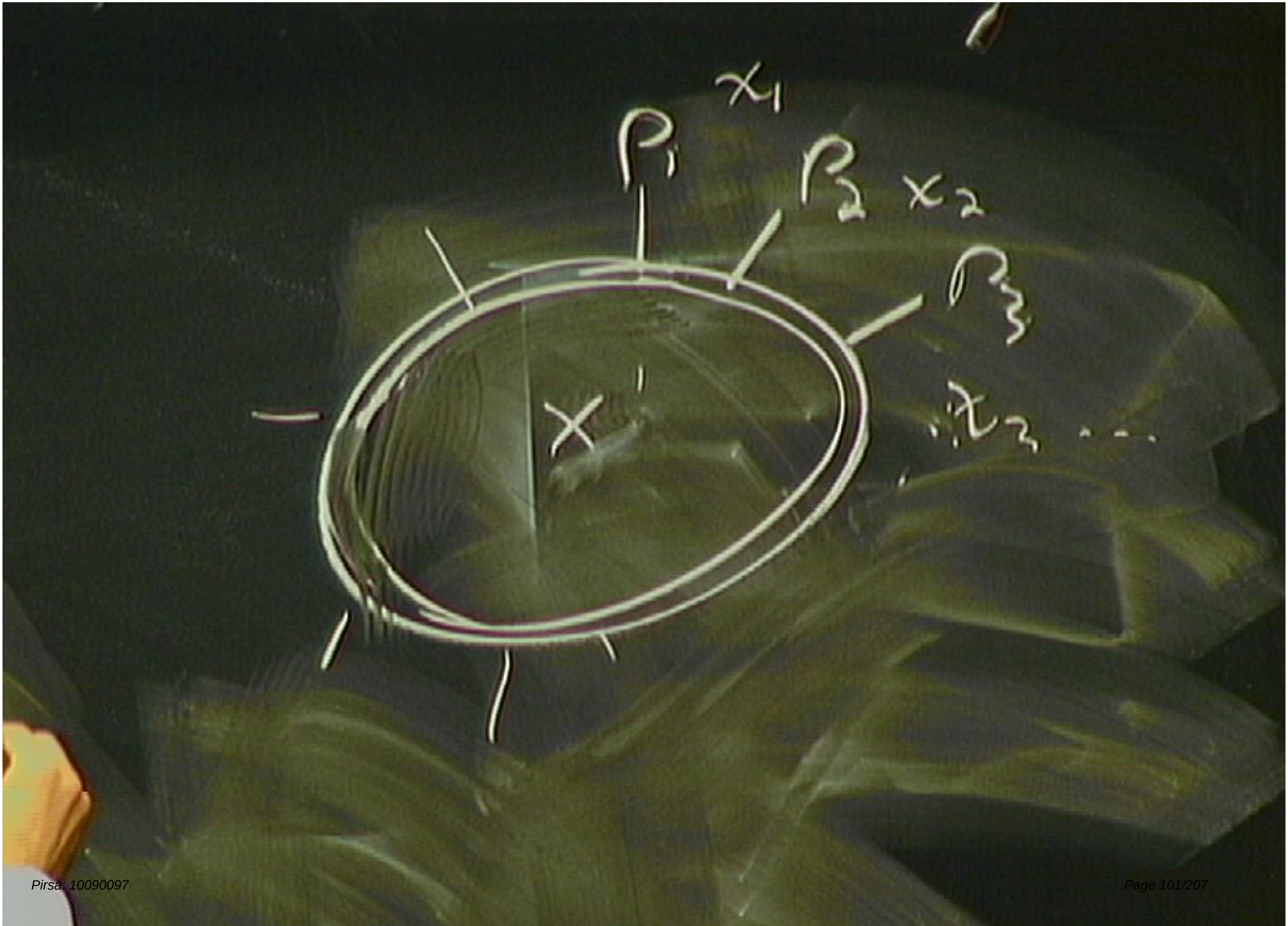
$$\sum_{\sigma} \mathrm{Tr}\left(t^{\sigma(a_1)} t^{\sigma(a_2)} t^{\sigma(a_3)} \cdots\right) A$$

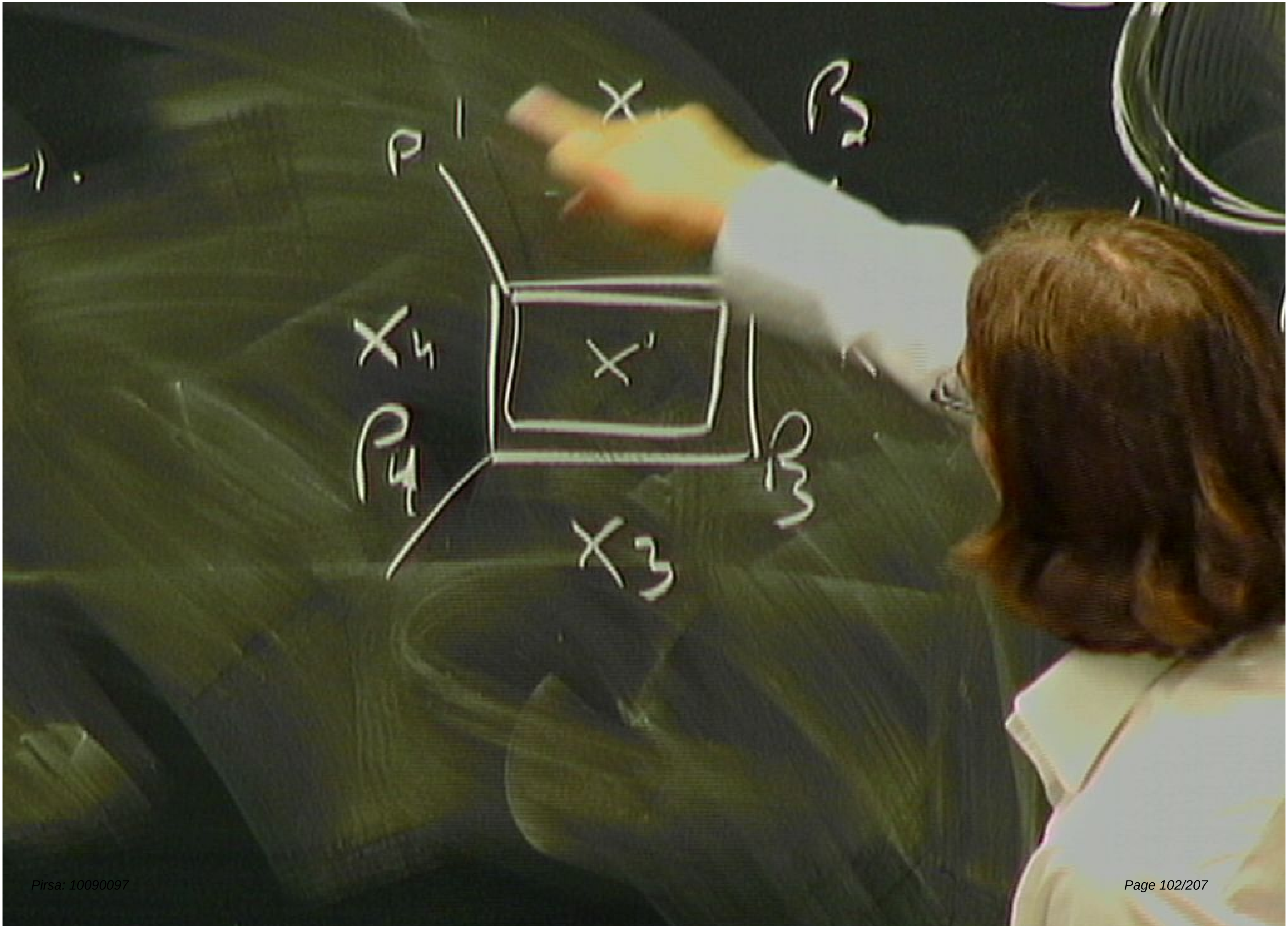
$$p_i = x_i - x_{i-1}.$$

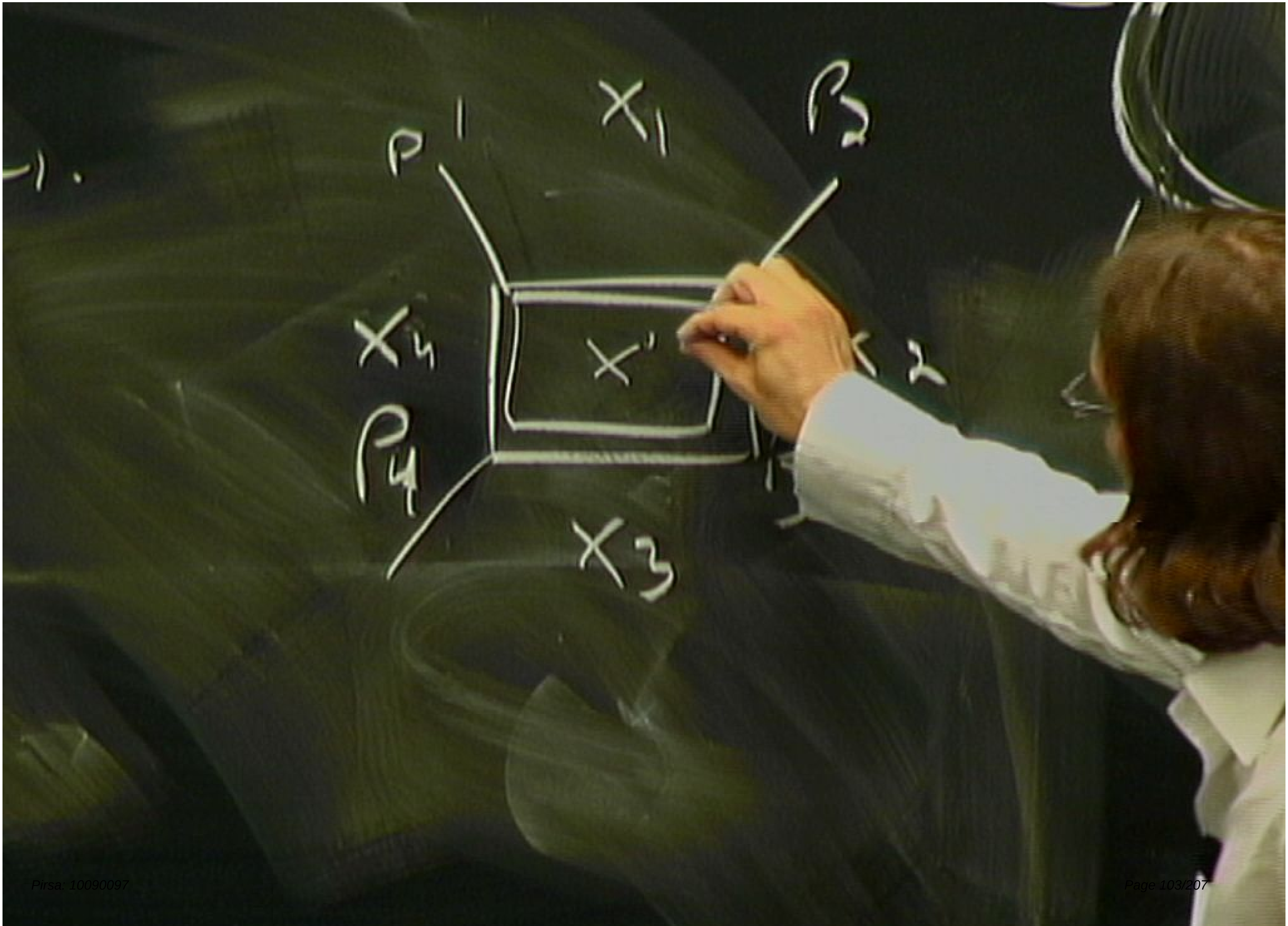


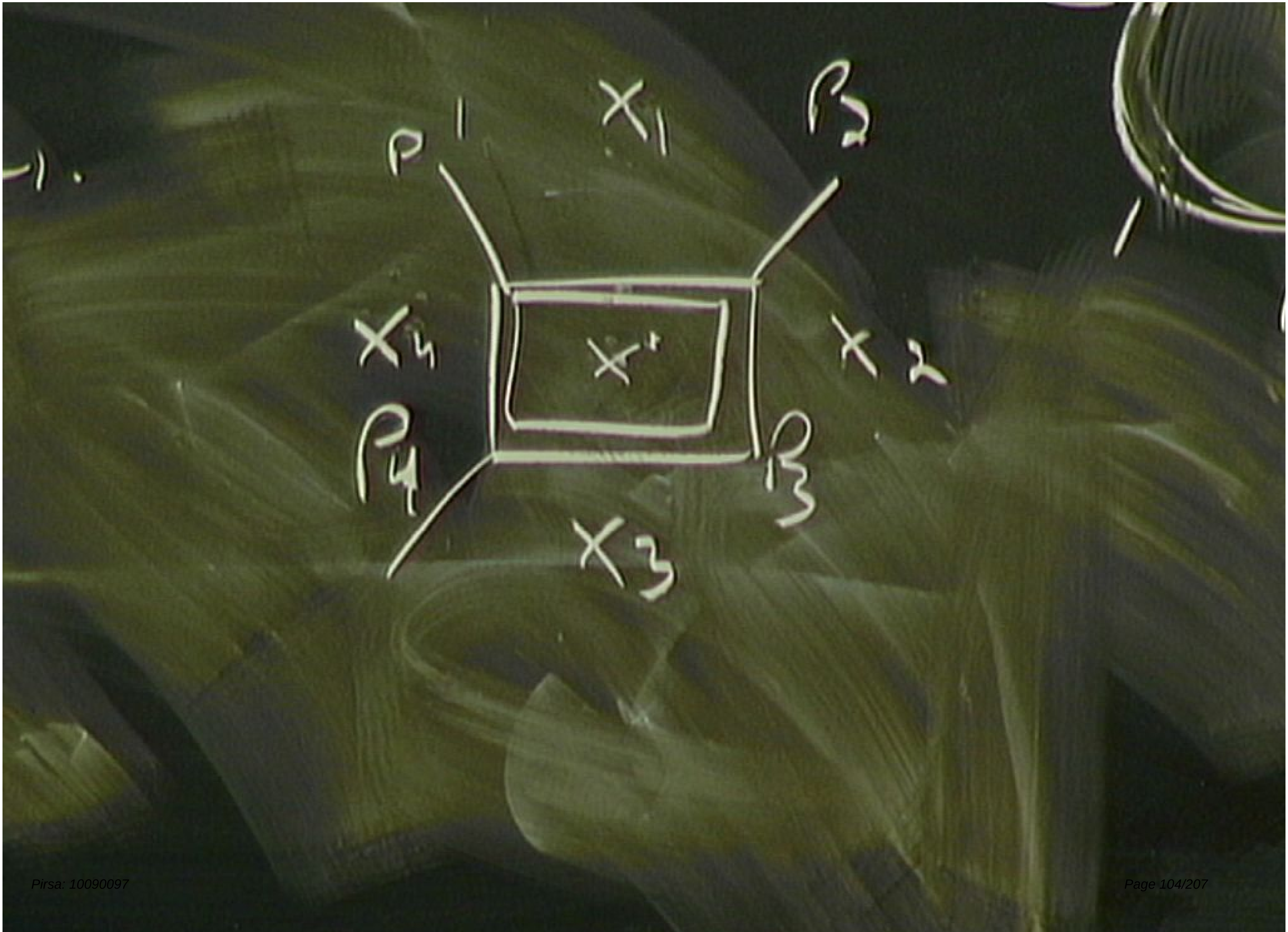
$$\sum_{\sigma} \mathrm{Tr}\left(t^{\sigma(a_1)} t^{\sigma(a_2)} t^{\sigma(a_3)} \cdots\right) A$$

$$p_i = x_i - x_{i-1} .$$

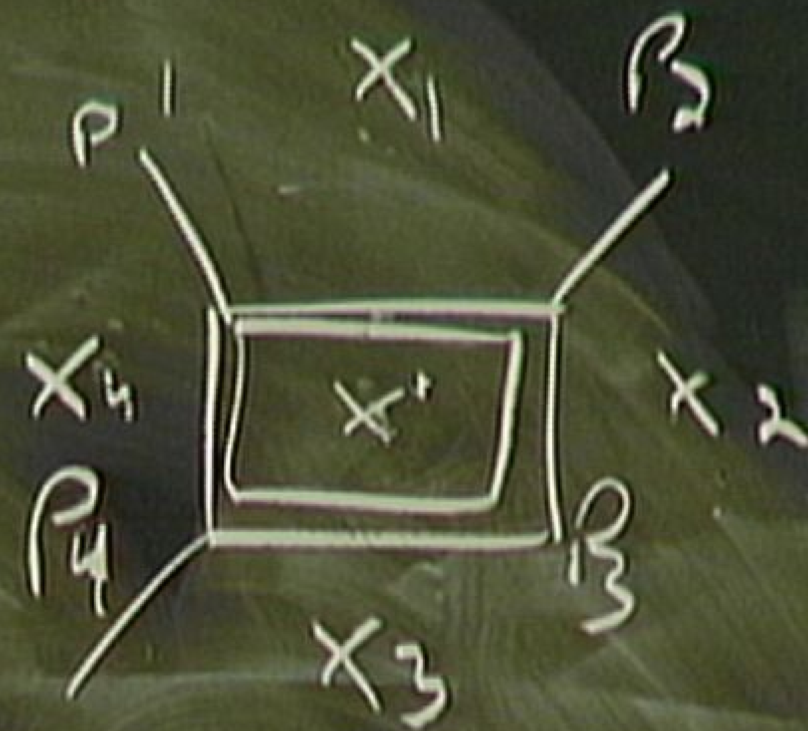

x_1
p_1
p_2
x_2
p_3
x_3 ...


p1
x
p2
x4
x'
p4
p3
x3


p1
x1
p2
x4
x'
x2
p4
x3

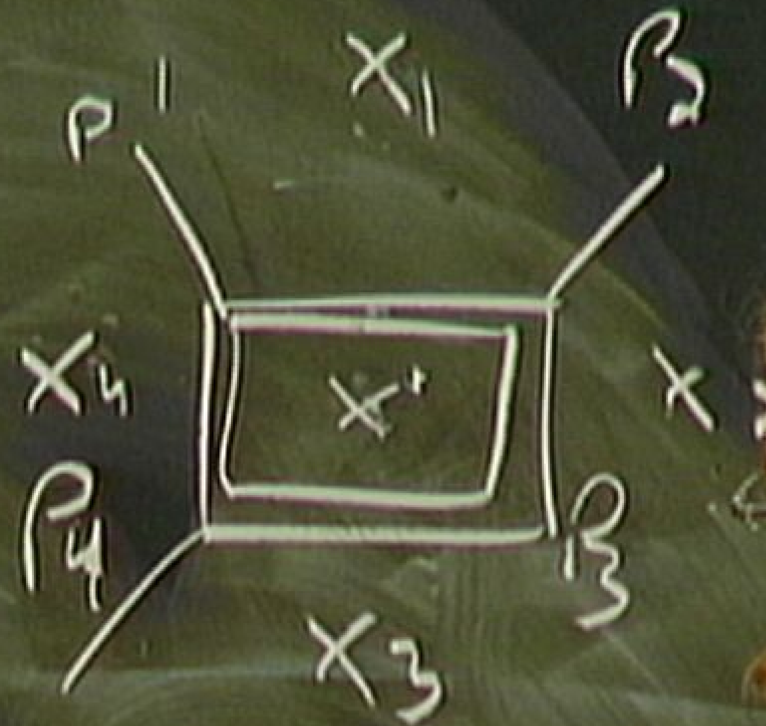

p1
x1
p2
x4
x'
x2
p4
p3
x3

$$p_i = x_i - x_{i-1}.$$



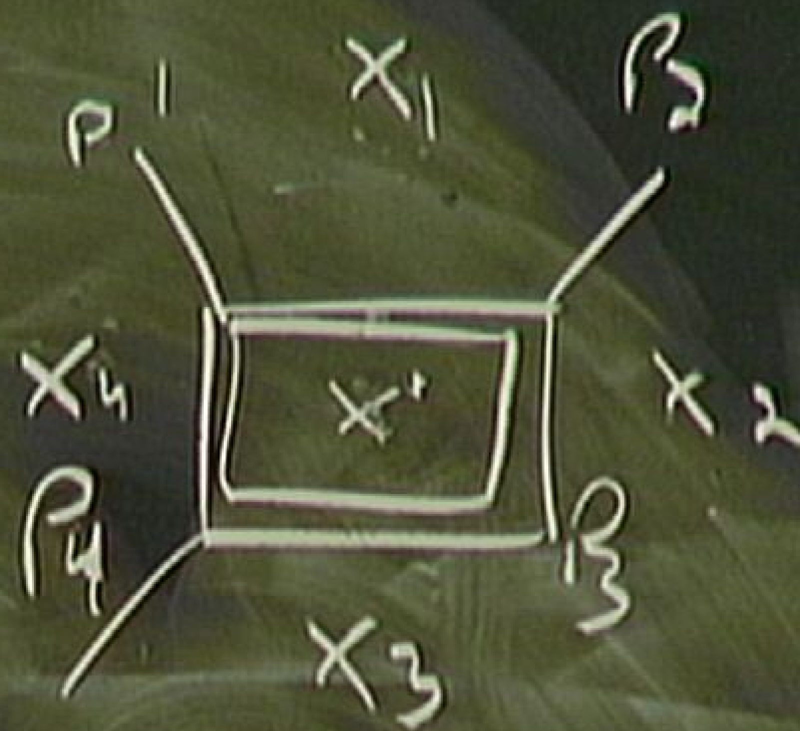
$$\int \frac{d^4x'}{(x'-x_1)^2(x'-x_2)^2\cdots(x'-x_4)^2}$$

$$p_i = x_i - x_{i-1}.$$

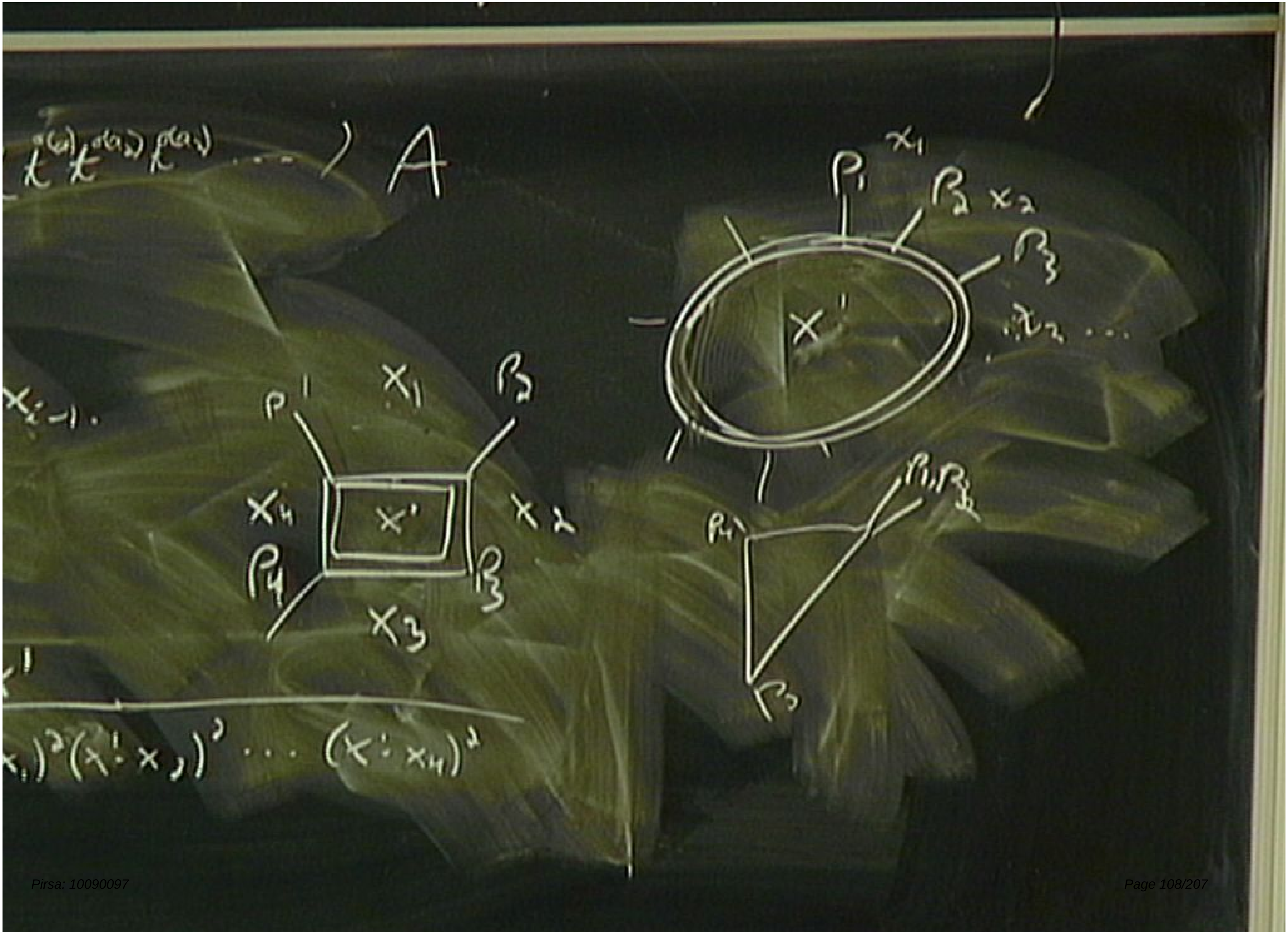


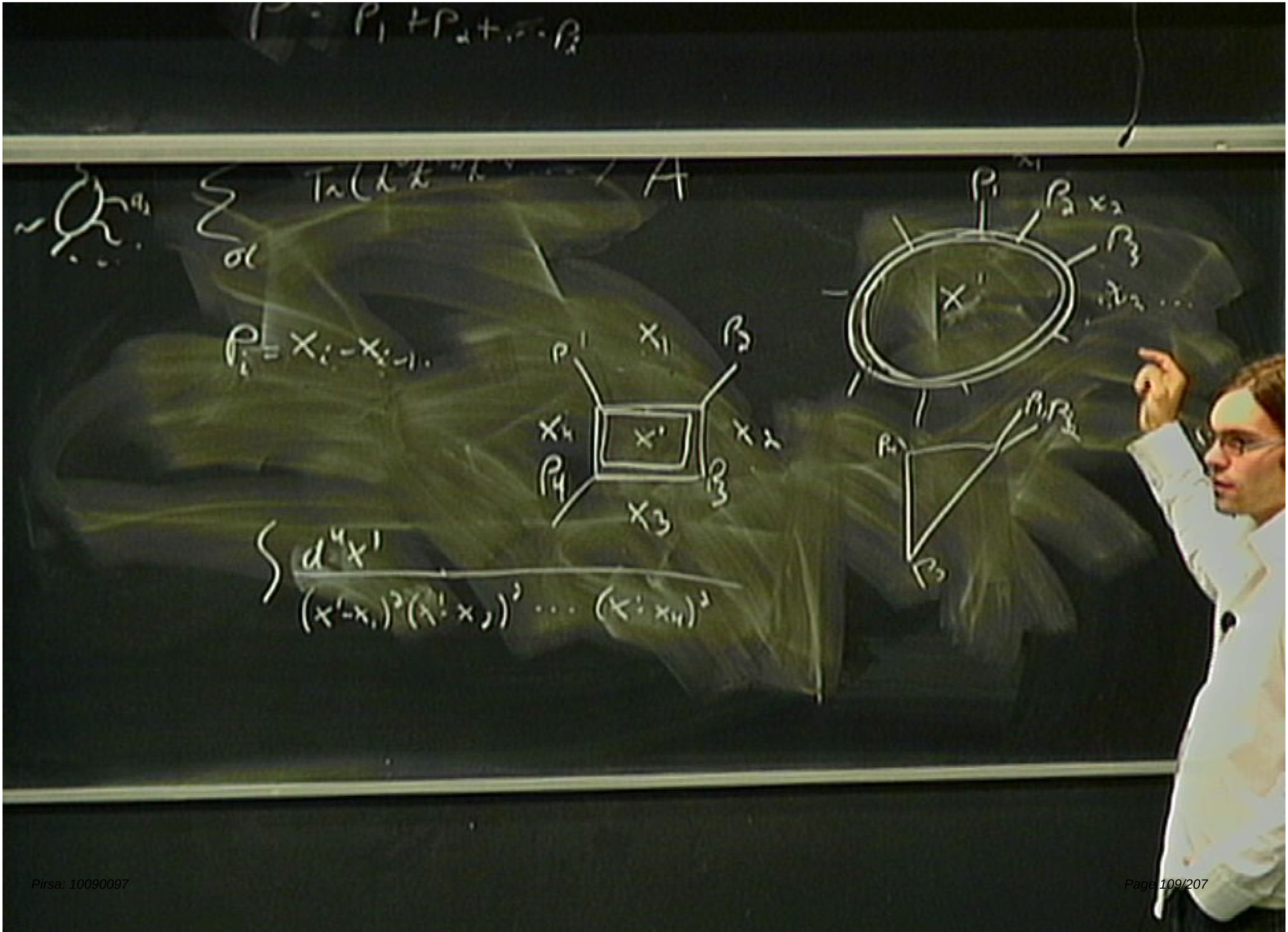
$$\int \frac{d^4x'}{(x'-x_1)^2(x'-x_2)^2\cdots(x'-x_4)^2}$$

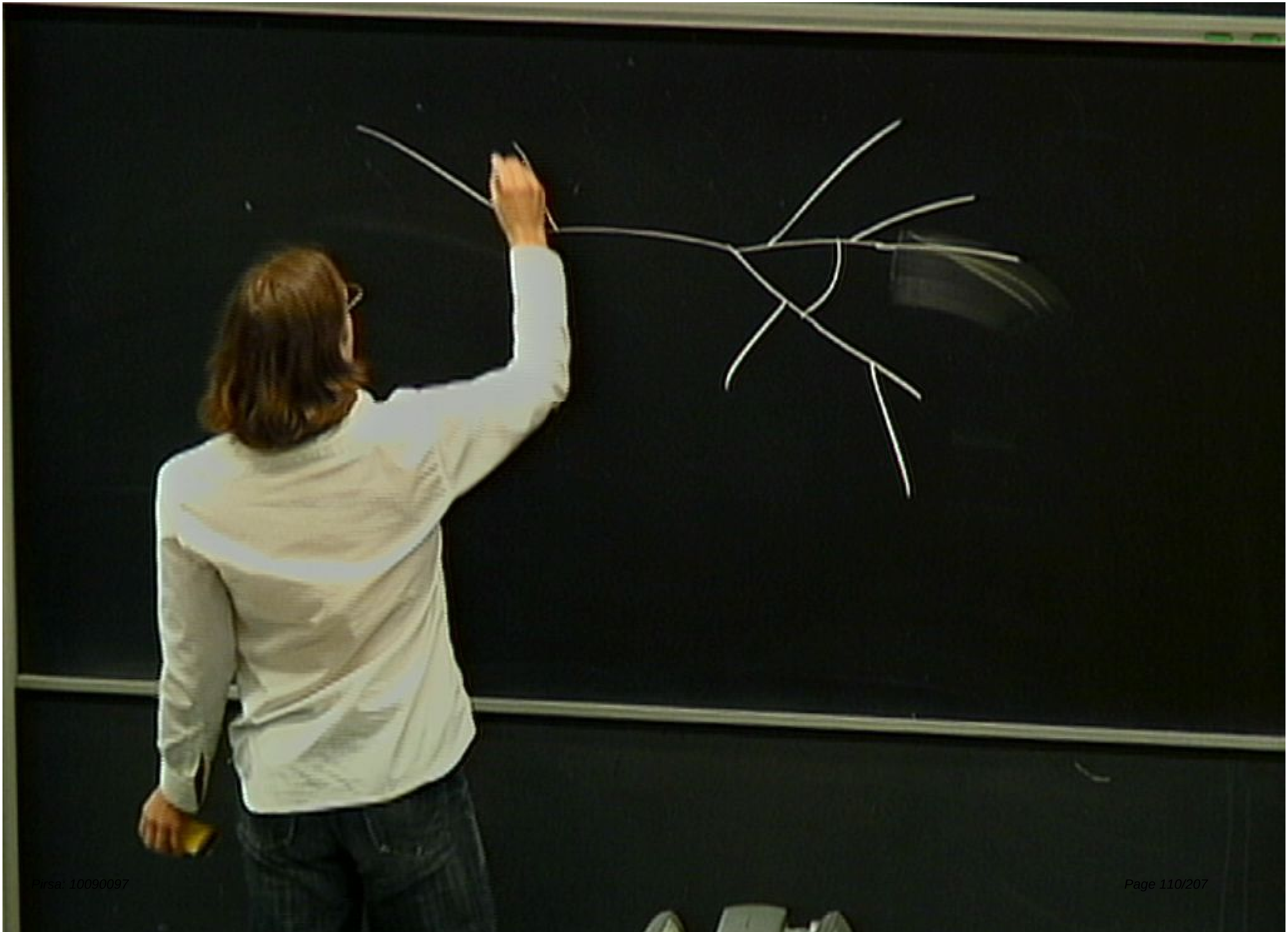
$$p_i = x_i - x_{i-1}.$$

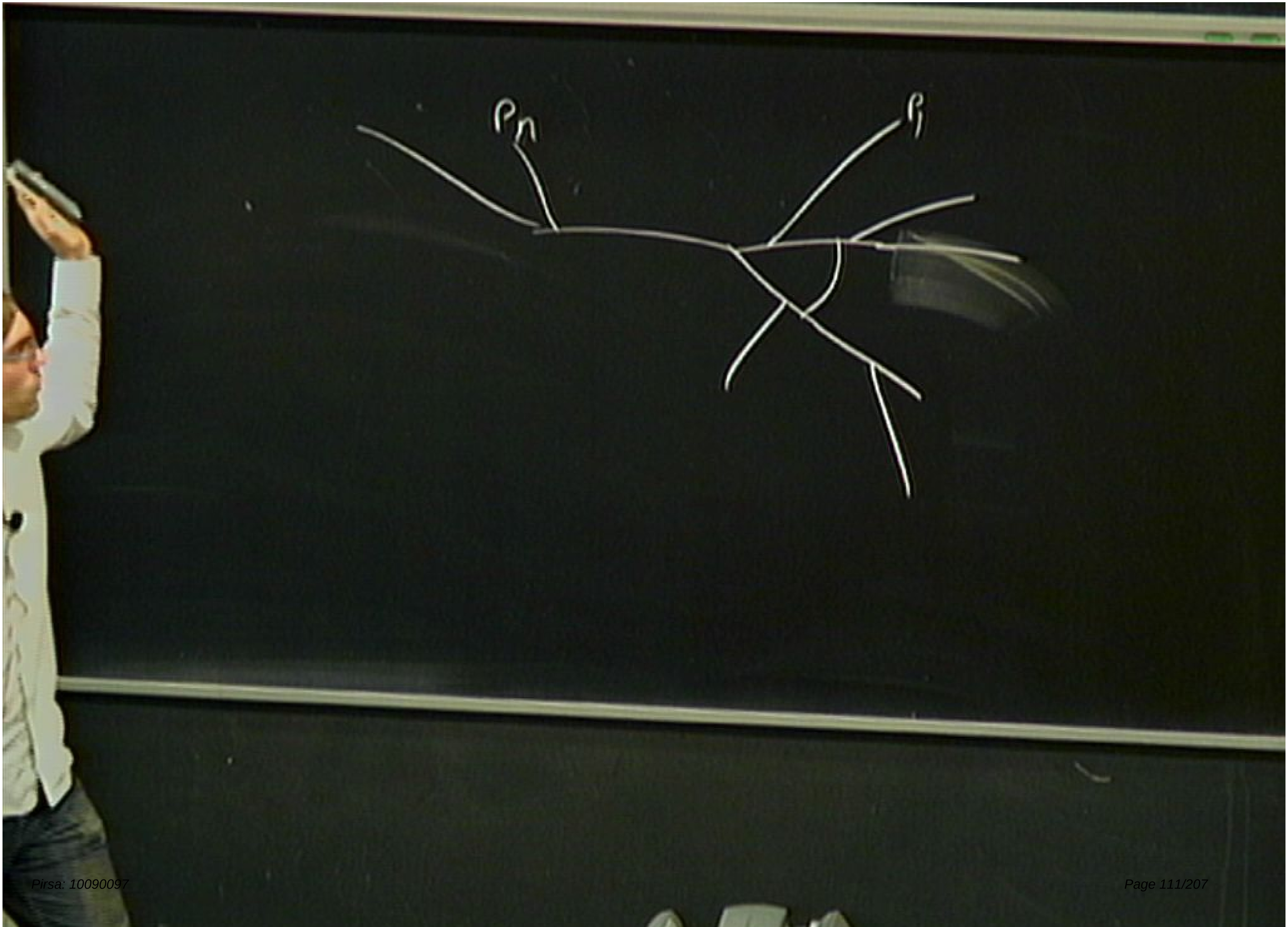


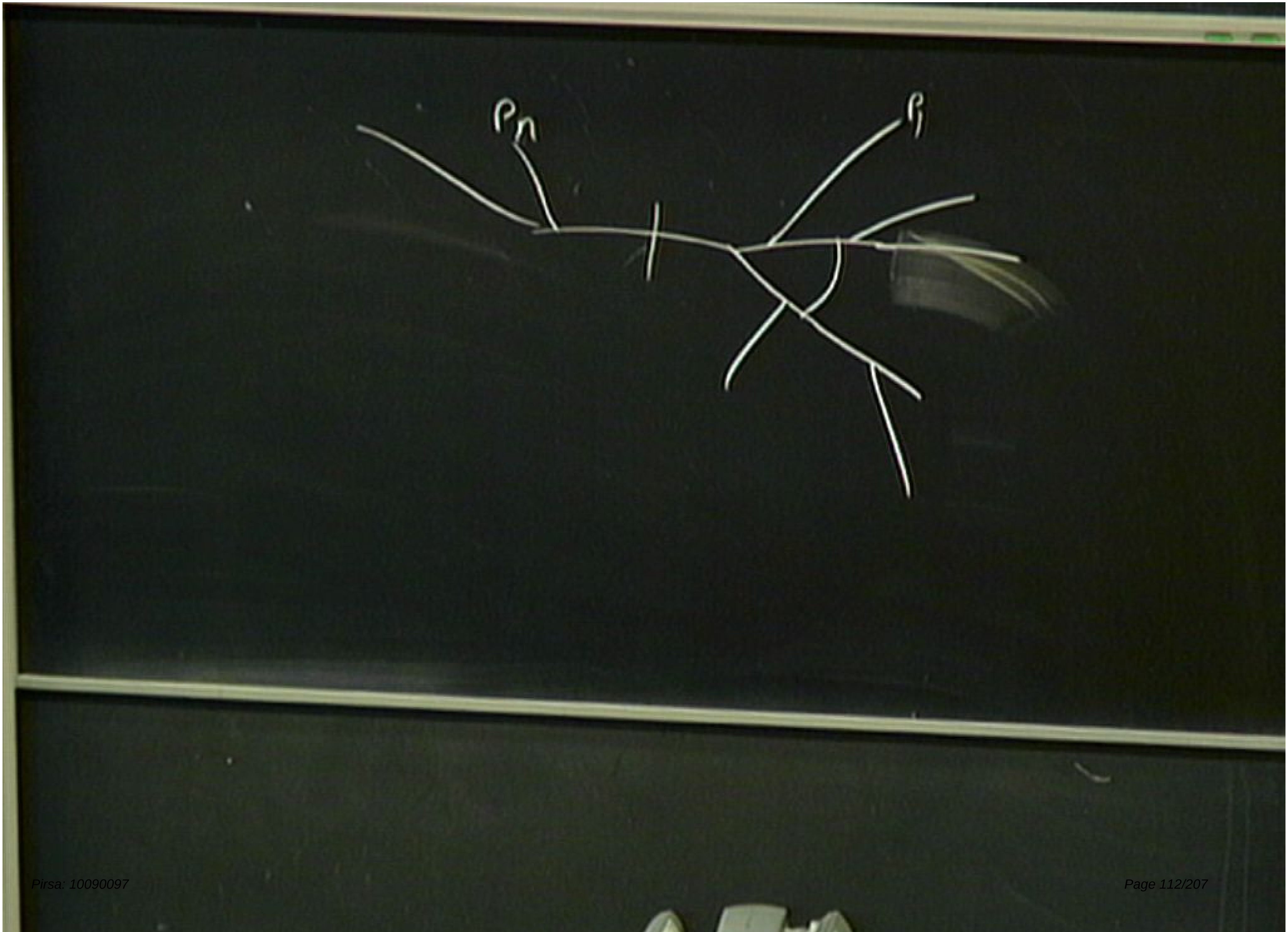
$$\int \frac{d^4x'}{(x'-x_1)^2(x'-x_2)^2 \cdots (x'-x_4)^2}$$


A
x1
P1
P2
x2
P3
x'
x4
x3
P4
P3

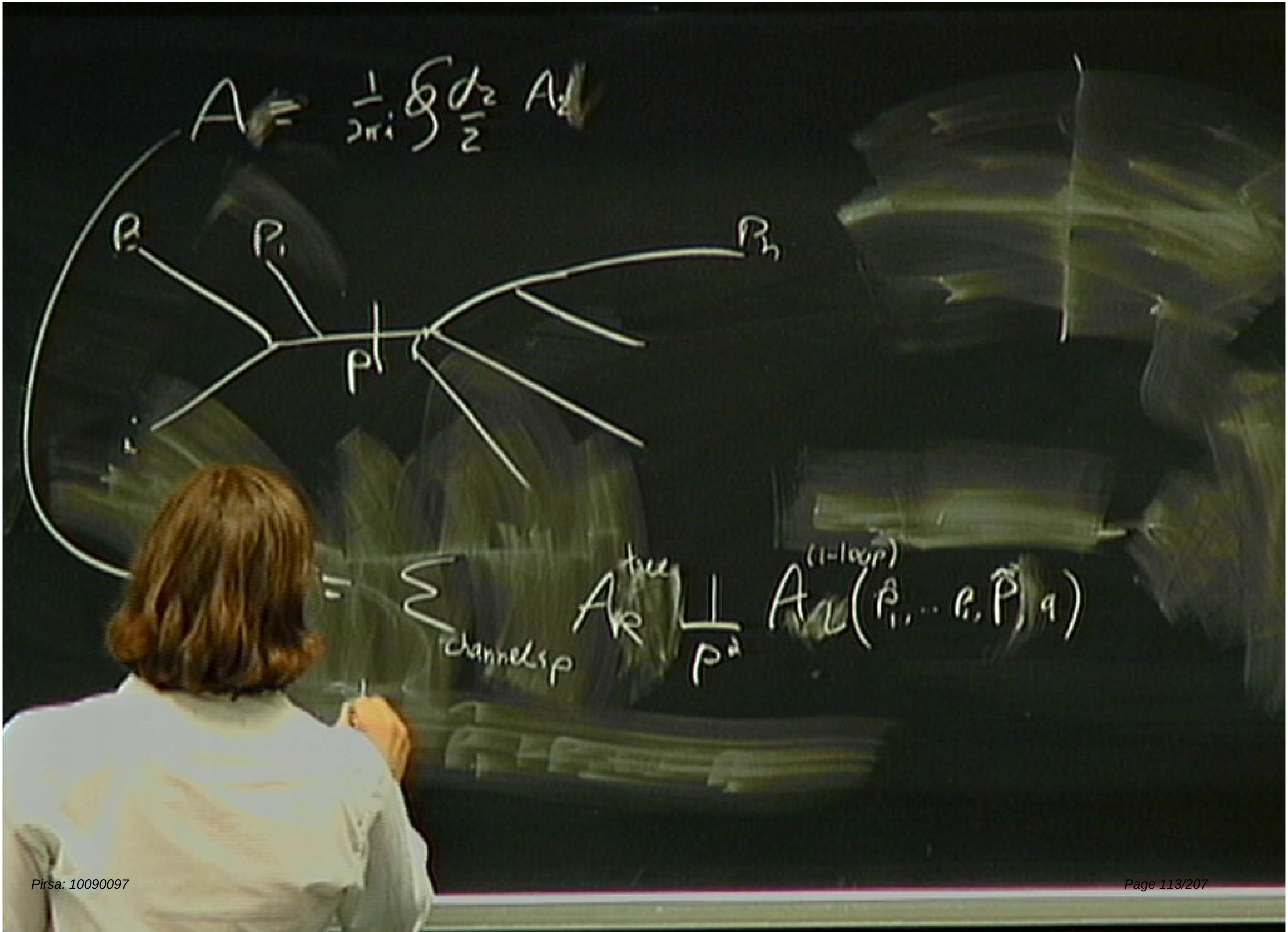

p_1 + p_2 + ... p_n
p_i = x_i - x_{i-1}
x_1
x_2
x_3
x_4
x'
∫ d^4x' / (x'-x_1)^2 (x'-x_2)^2 ... (x'-x_4)^2



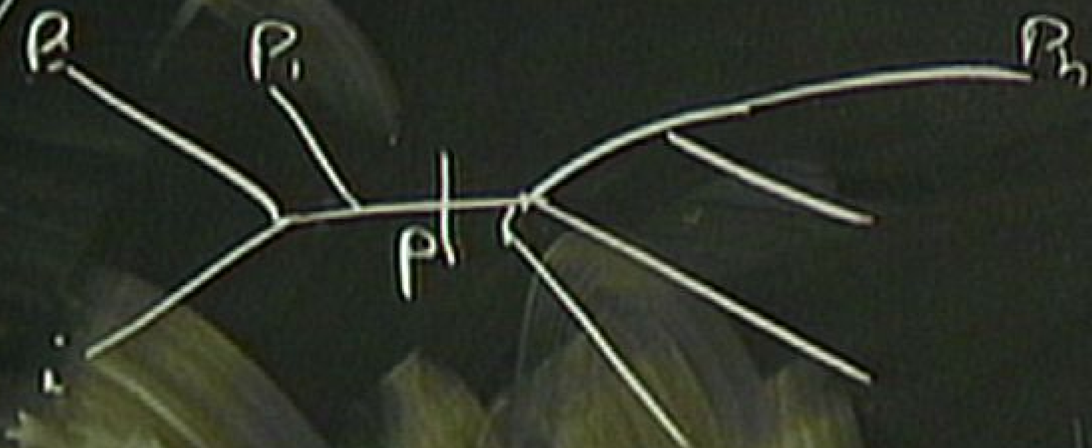

p_n
p_1



p_n
p_1


A = 1/2πi ∮ dz/z A(z)
P₁
Pₙ
p
= Σ A_R^tree 1/p² A_L^(1-loop)(P₁, ... P, P̂, q)
channels, p

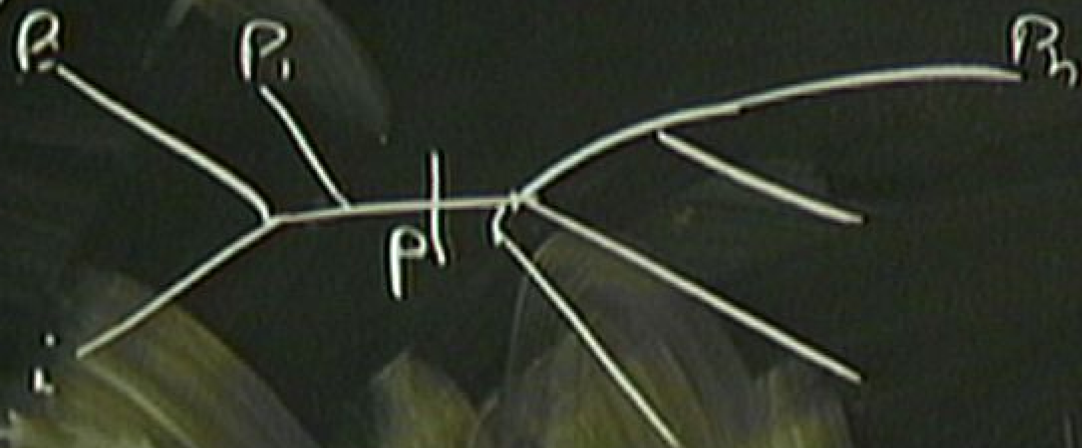
$$A = \frac{1}{2\pi i}\oint \frac{dz}{z} A$$



$$A_n = \sum_{\text{channels}, p} A_R^{\text{tree}} \frac{1}{p^2} A_L^{(1\text{-loop})}(\hat{P}_1, \ldots P_i, \hat{P}, q)$$

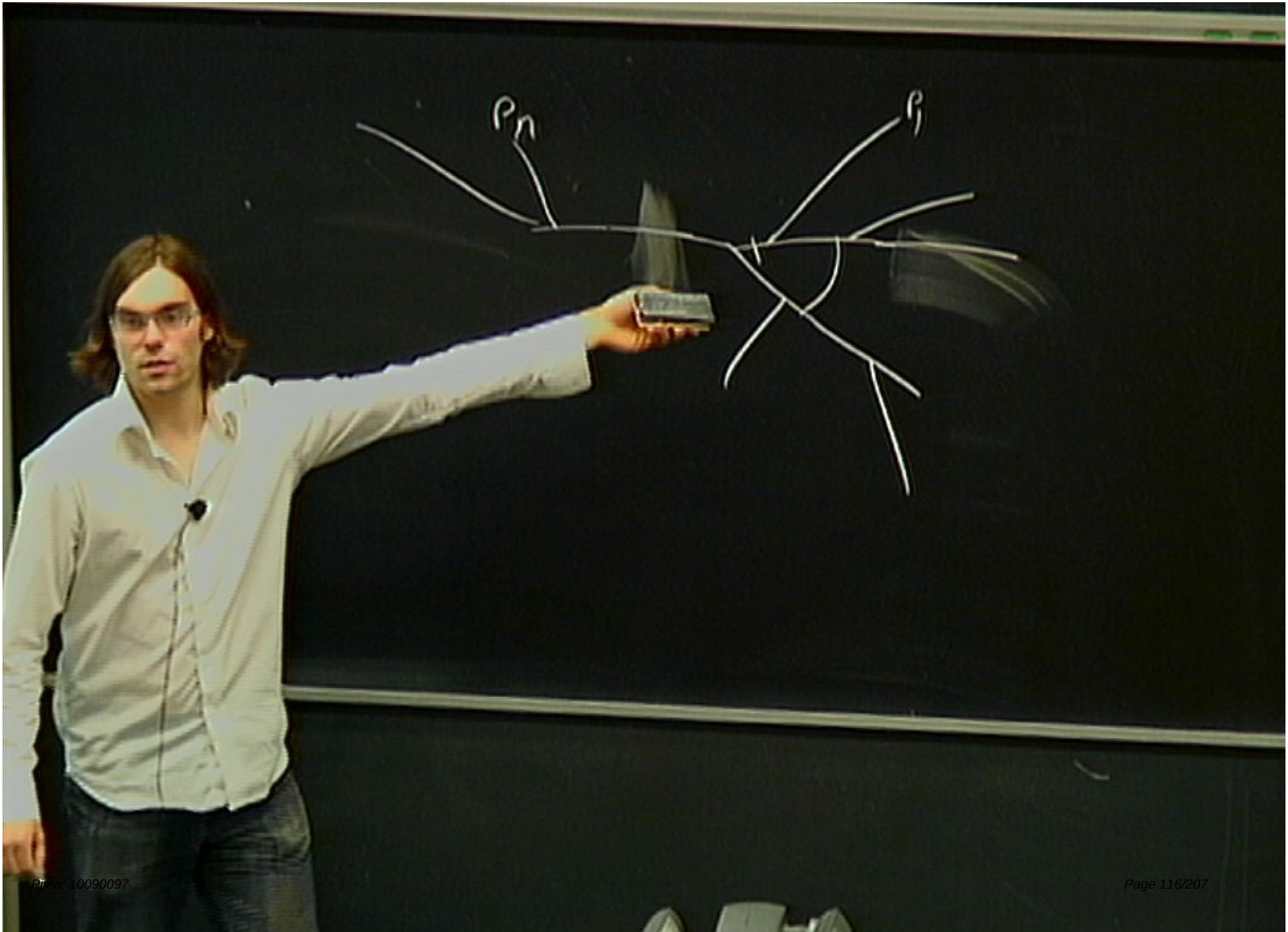
$$+ \sum_{\text{chans}} A_R^{1\text{-loop}} \frac{1}{p^2} A_L^{\text{tree}}(\qquad)$$

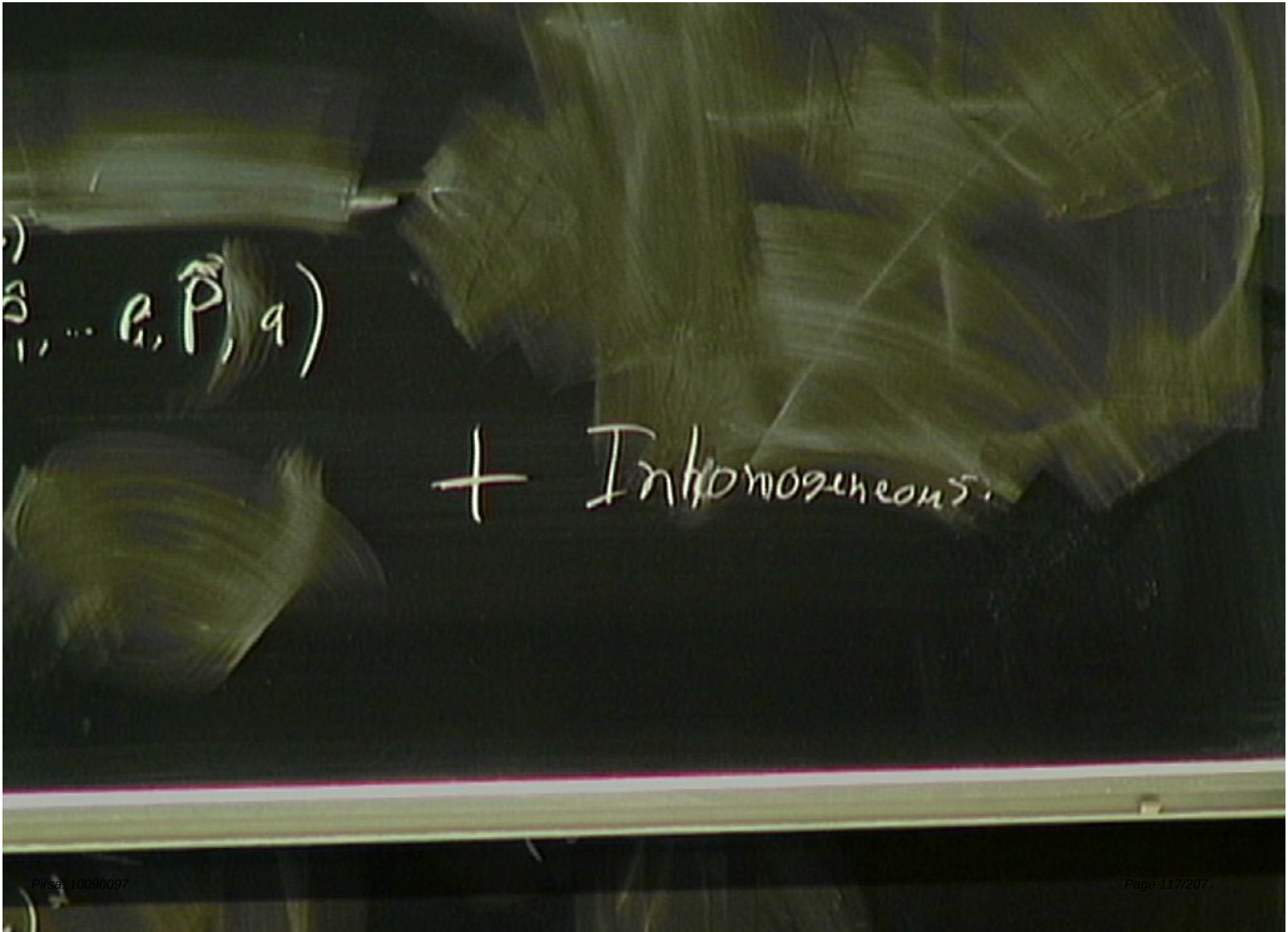
$$A = \frac{1}{2\pi i}\oint \frac{dz}{z} A(z)$$

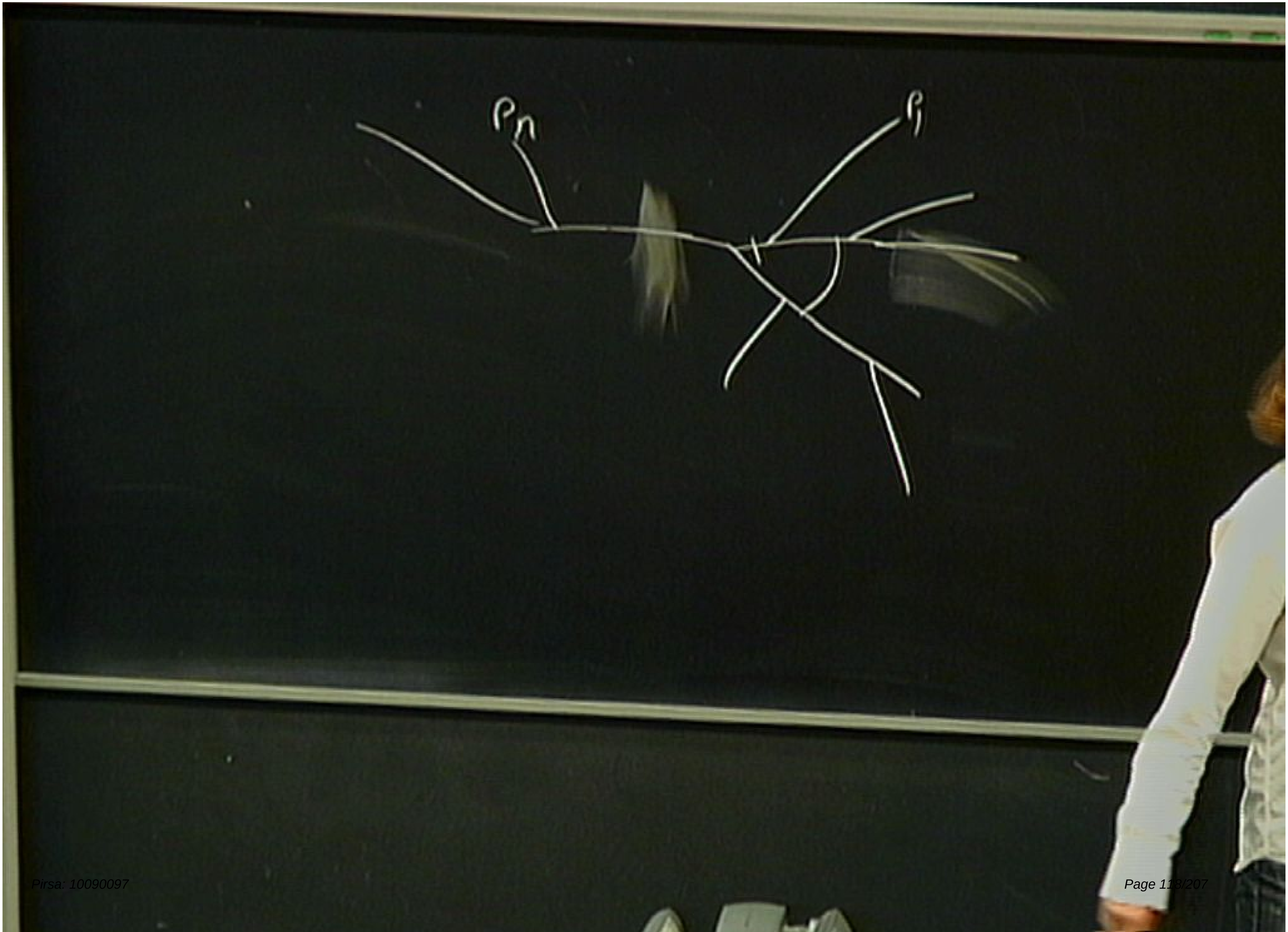


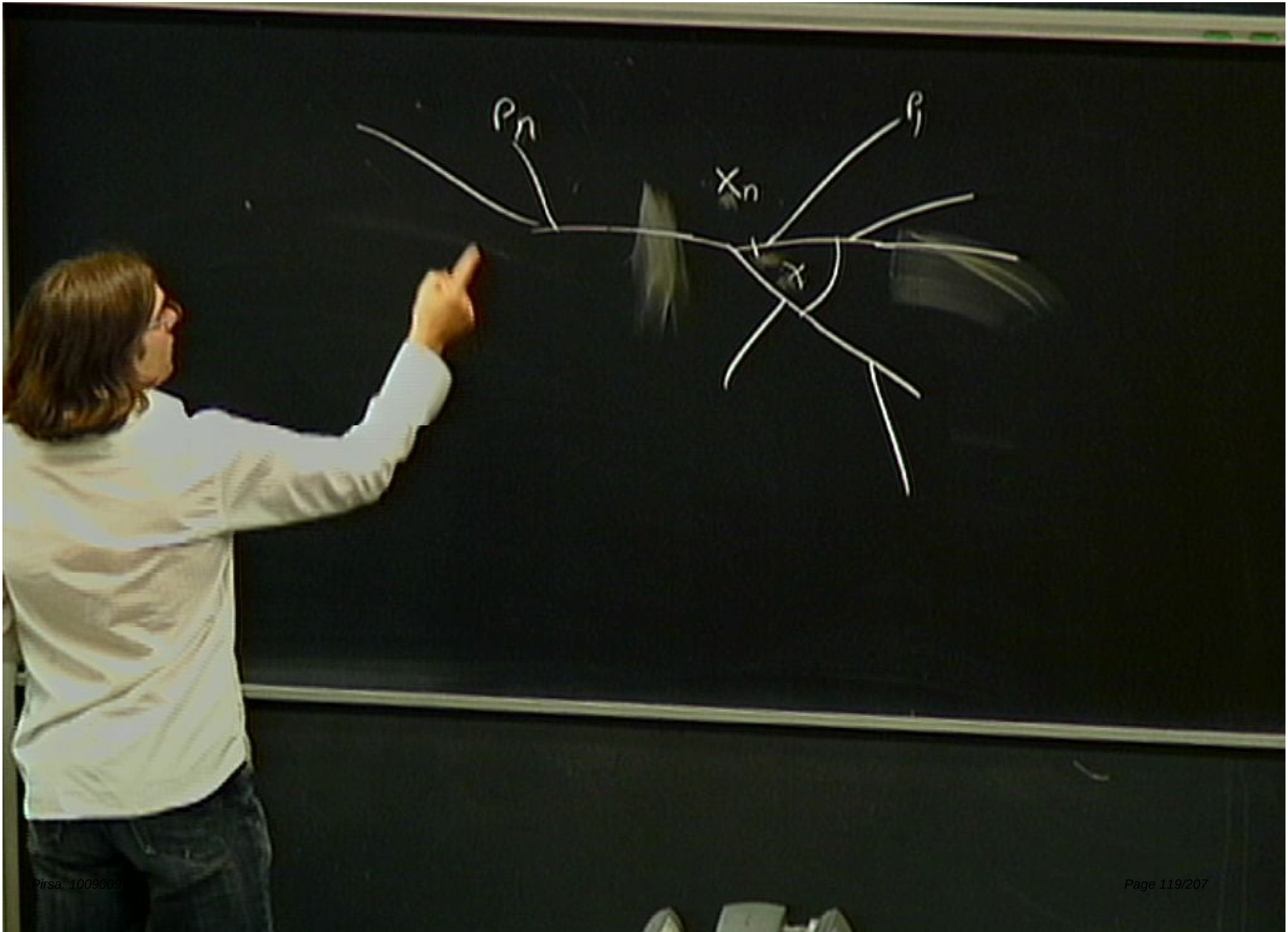
$$A_n = \sum_{\text{channels } p} A_R^{\text{tree}} \frac{1}{p^2} A_L^{(1\text{-loop})}(\hat{P}_1, \dots P, \hat{P}, q)$$

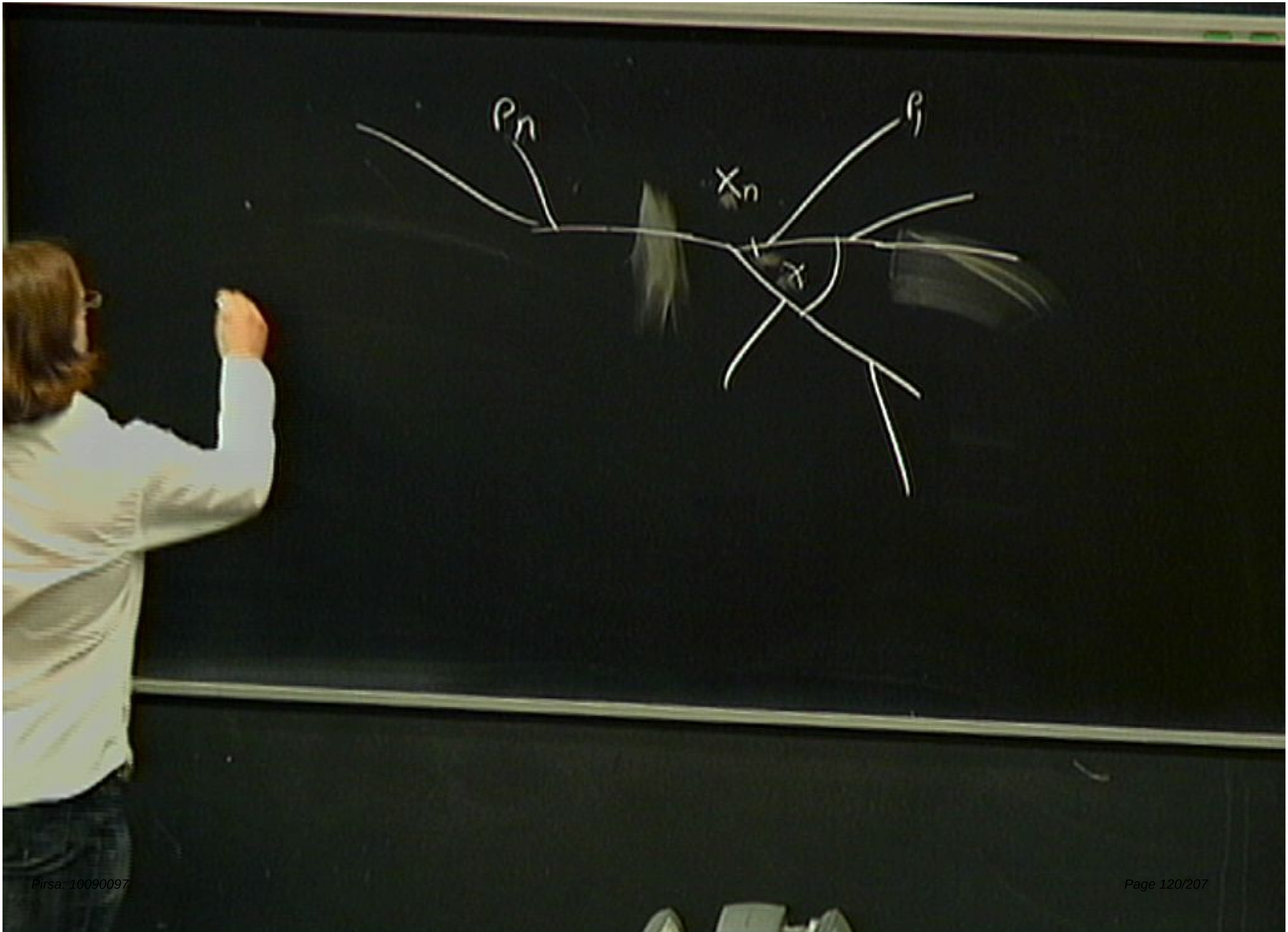
$$+ \sum_{\text{channels}} A_R^{1\text{-loop}}(\,, q) \frac{1}{p^2} A_L^{\text{tree}}$$




+ Inhomogeneous:

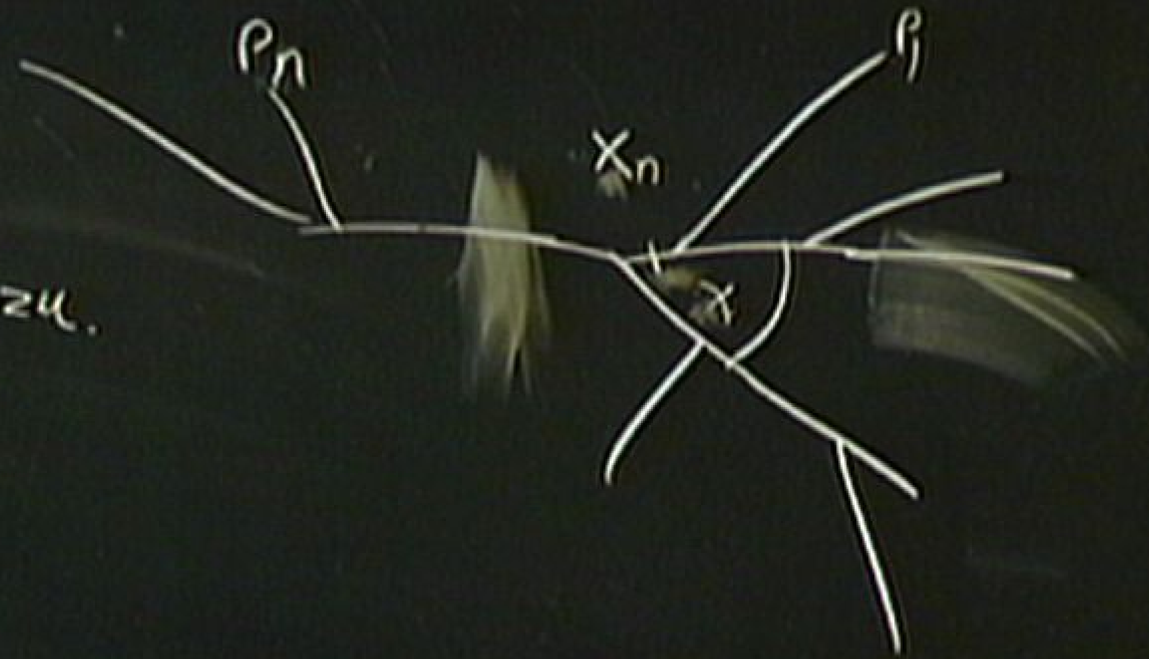

p_n
p_1

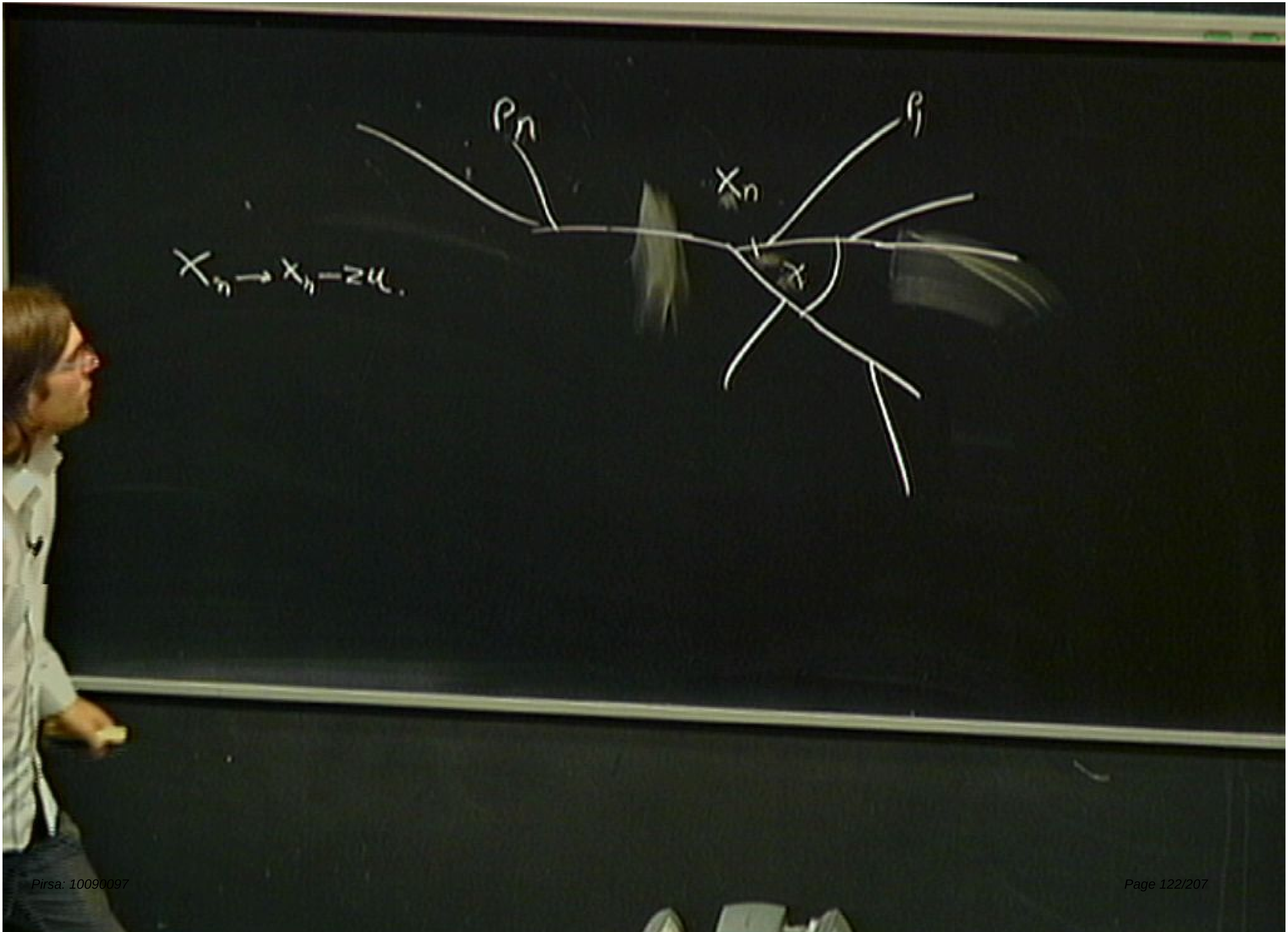

p_n
x_n
x



p_n
p_j
x_n
x

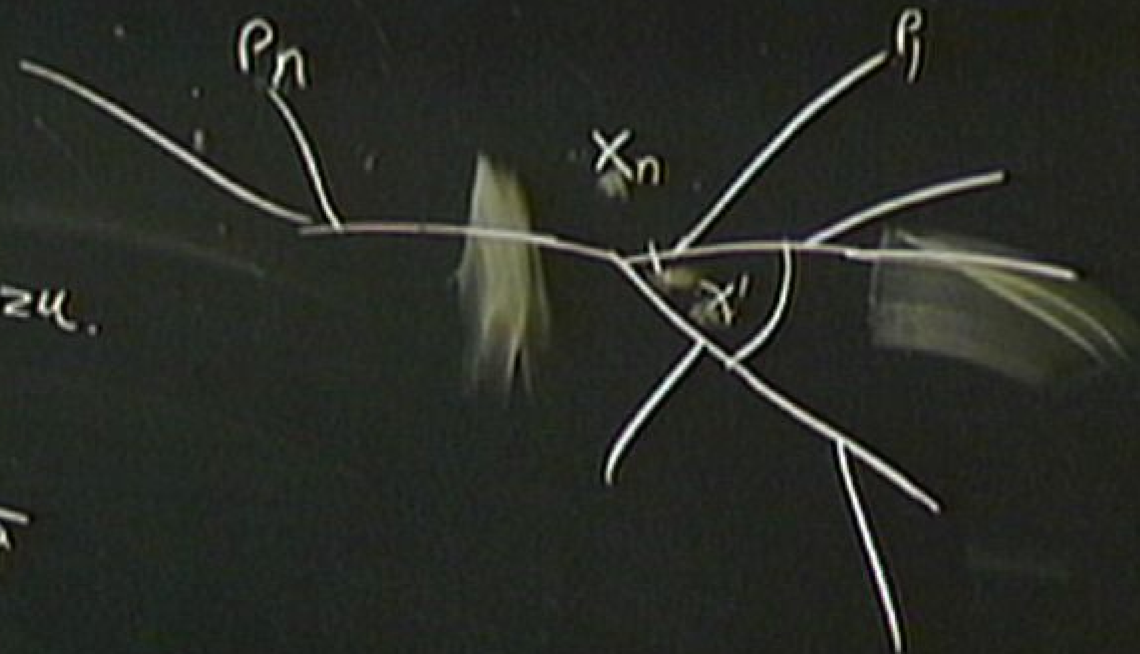
$$x_n \to x_n - 2u.$$




p_n
p_1
x_n
x
$x_n \to x_n - zu$.

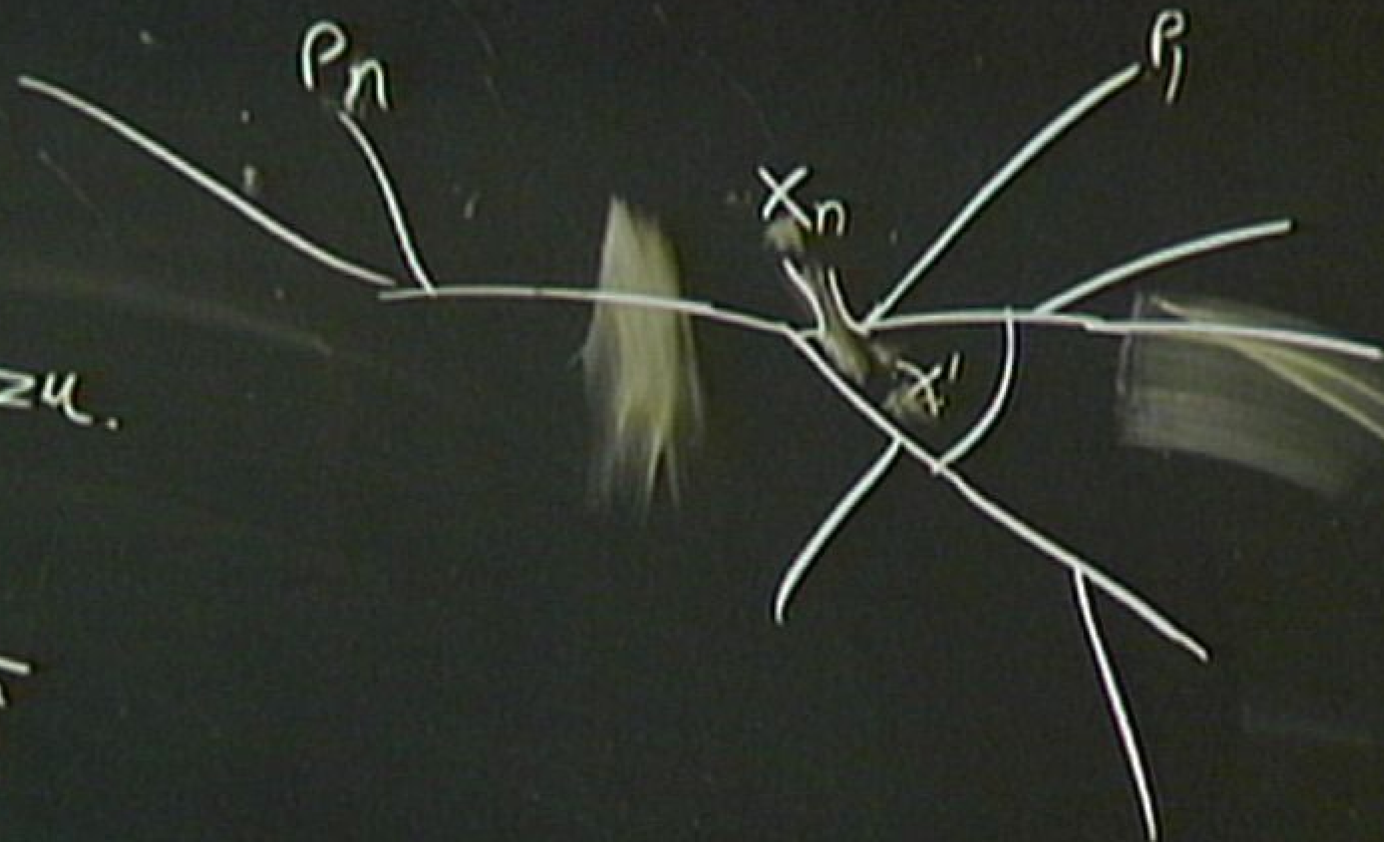
$$\hat{x}_n \to x_n - zu.$$

$$\frac{1}{(x' - \hat{x}_n)^2}$$



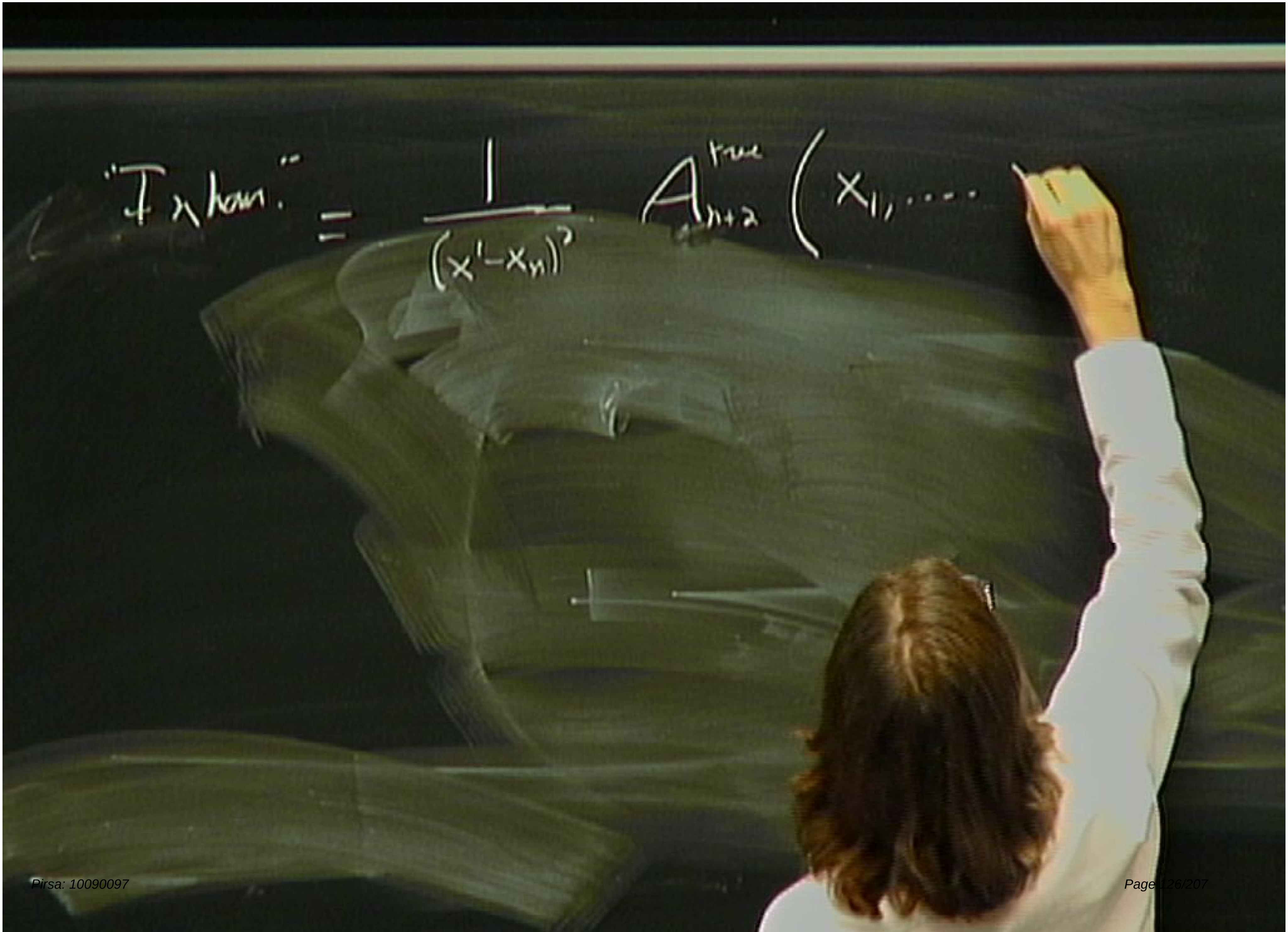
$$\hat{x}_n \to x_n - 2u.$$

$$\frac{1}{(x' - \hat{x}_n)^2}$$



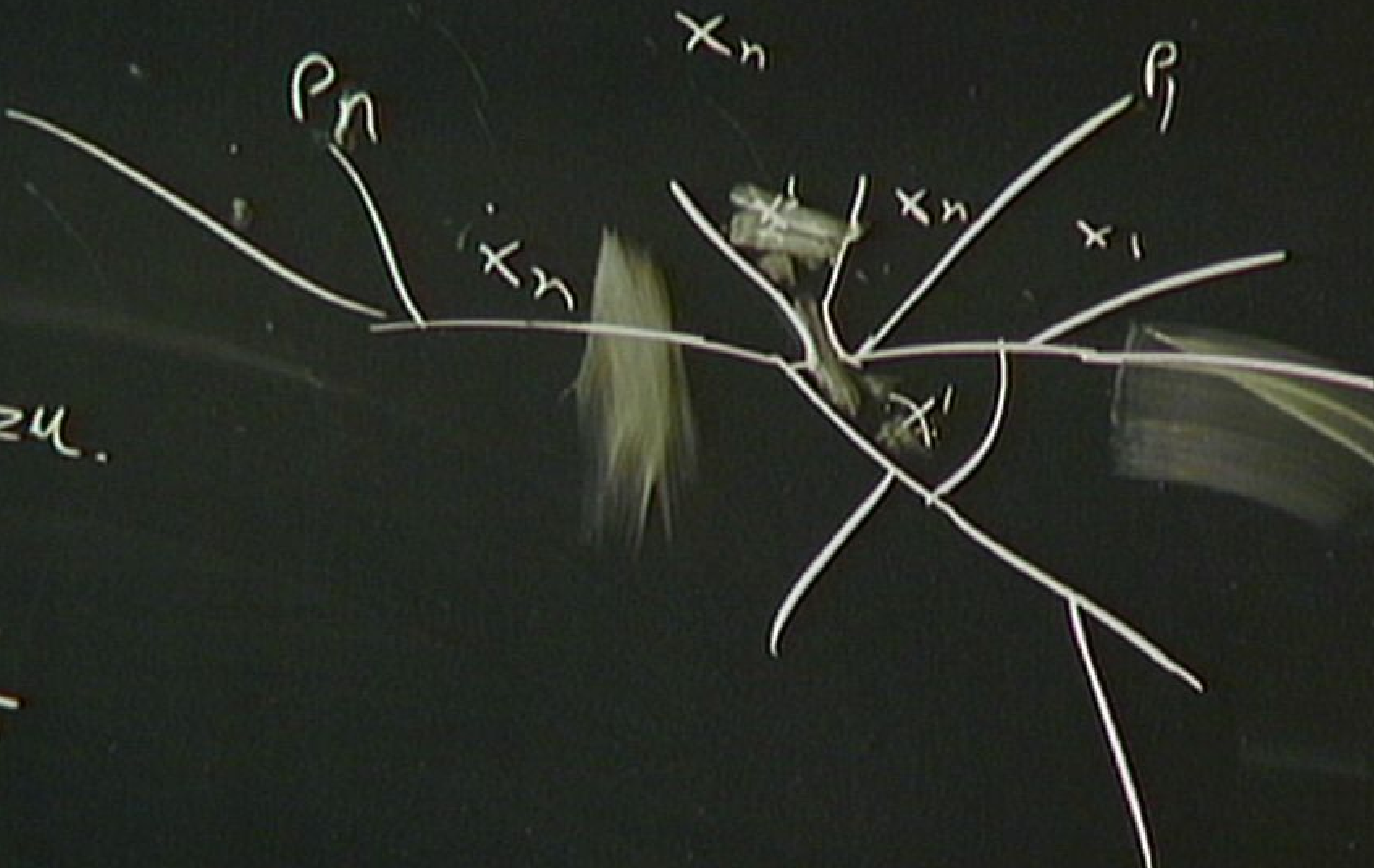
$$\hat{x}_n \to x_n - zu.$$

$$\frac{1}{(x'-\hat{x}_n)^2}$$


"Fxform" $= \frac{1}{(x'-x_n)^2} A^{tree}_{n+2}(x_1, \dots$

$$\text{"Inkhorn"} = \frac{1}{(x'-x_n)^2} A_{n+2}^{\text{true}}\left(x_1, \ldots \hat{x}_n\right.$$

$-zu.$



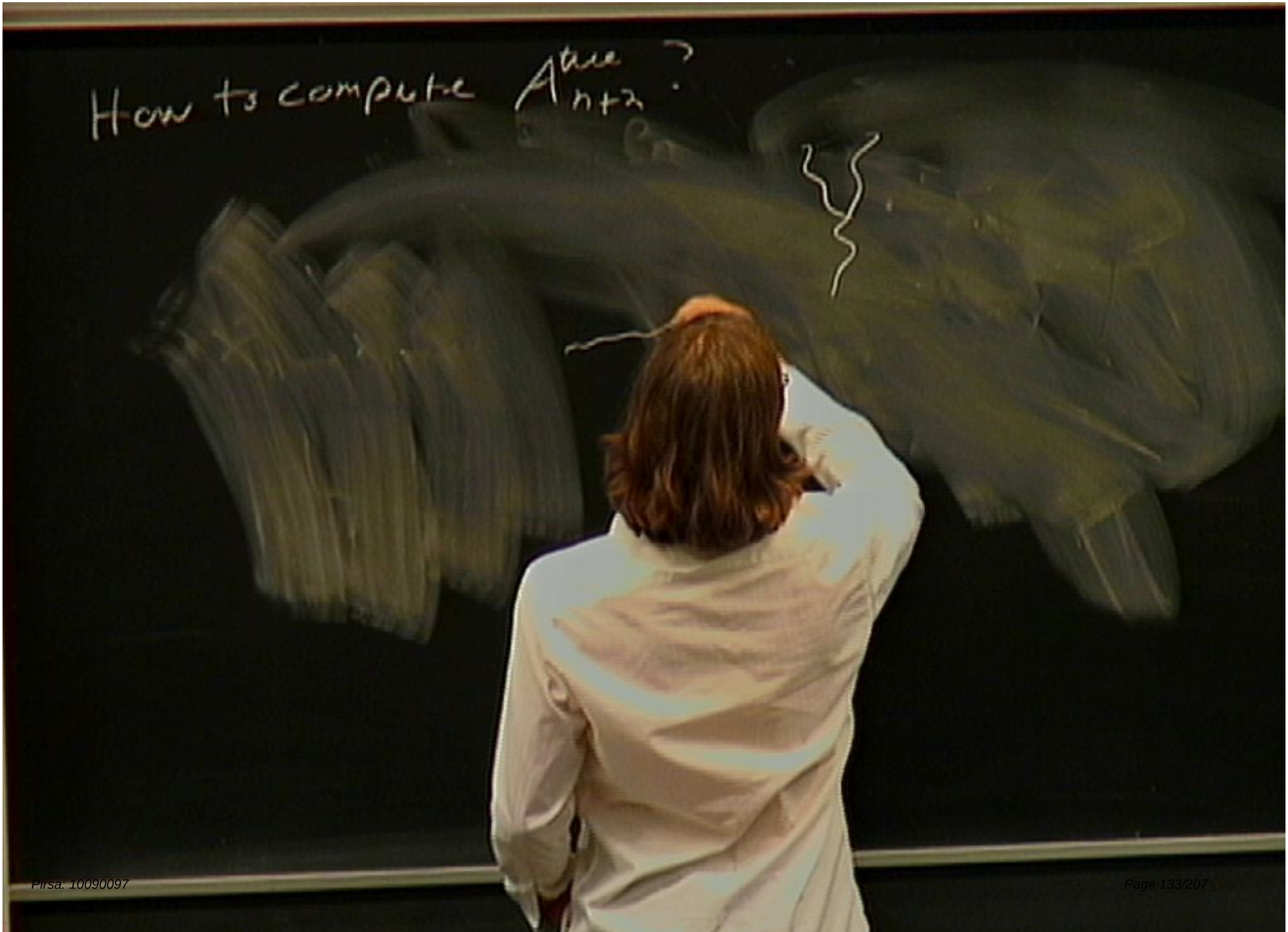
$)^2$

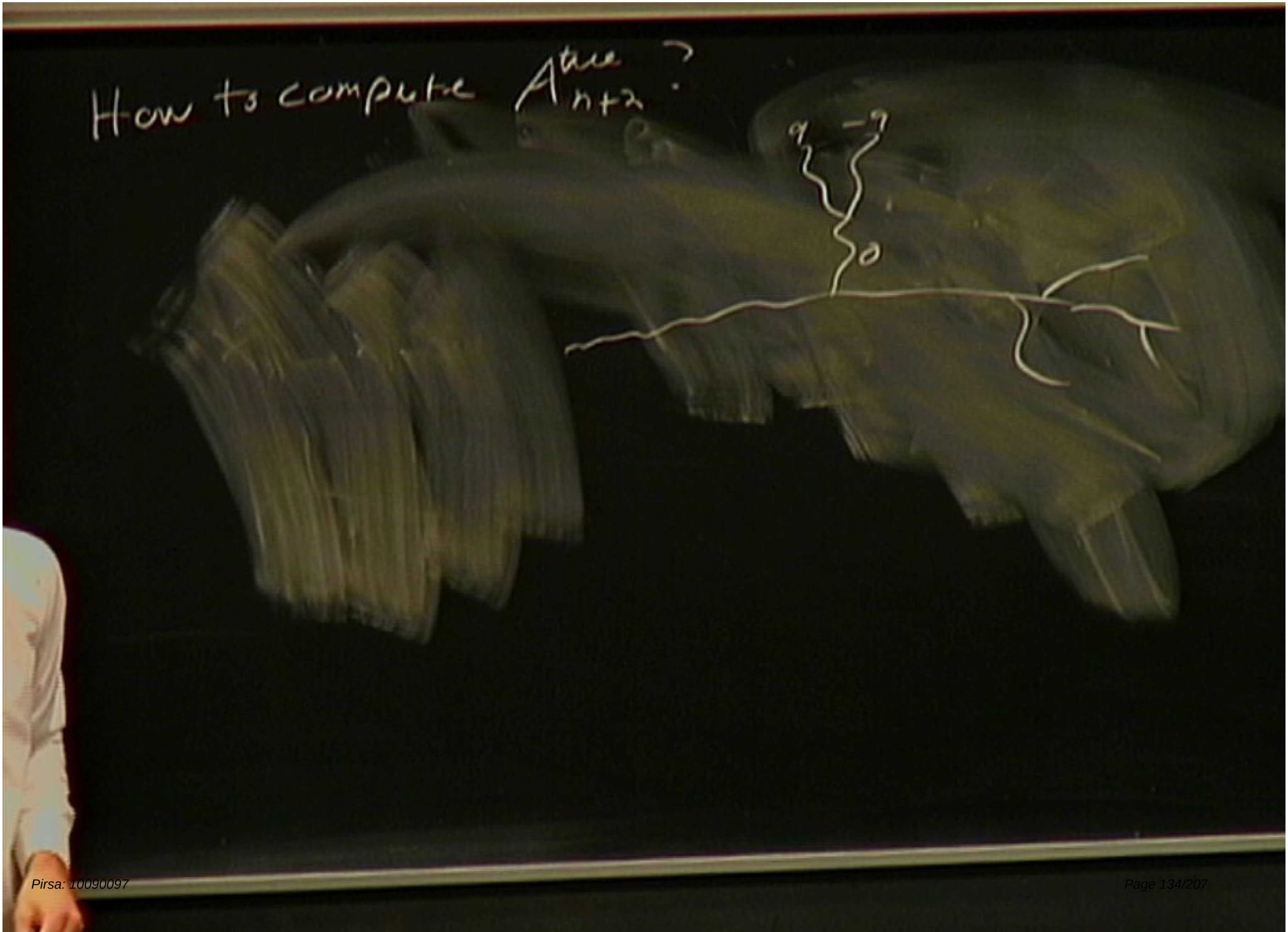
$$\text{"}I_n\,\text{kan.}\text{"} = \frac{1}{(x'-x_n)^2}\, A^{\text{tree}}_{n+2}\left(x_1, \ldots\, \hat{x}_n, x', \hat{x}_n\right), \quad \hat{x}_n = x_n + z_0 \ldots$$

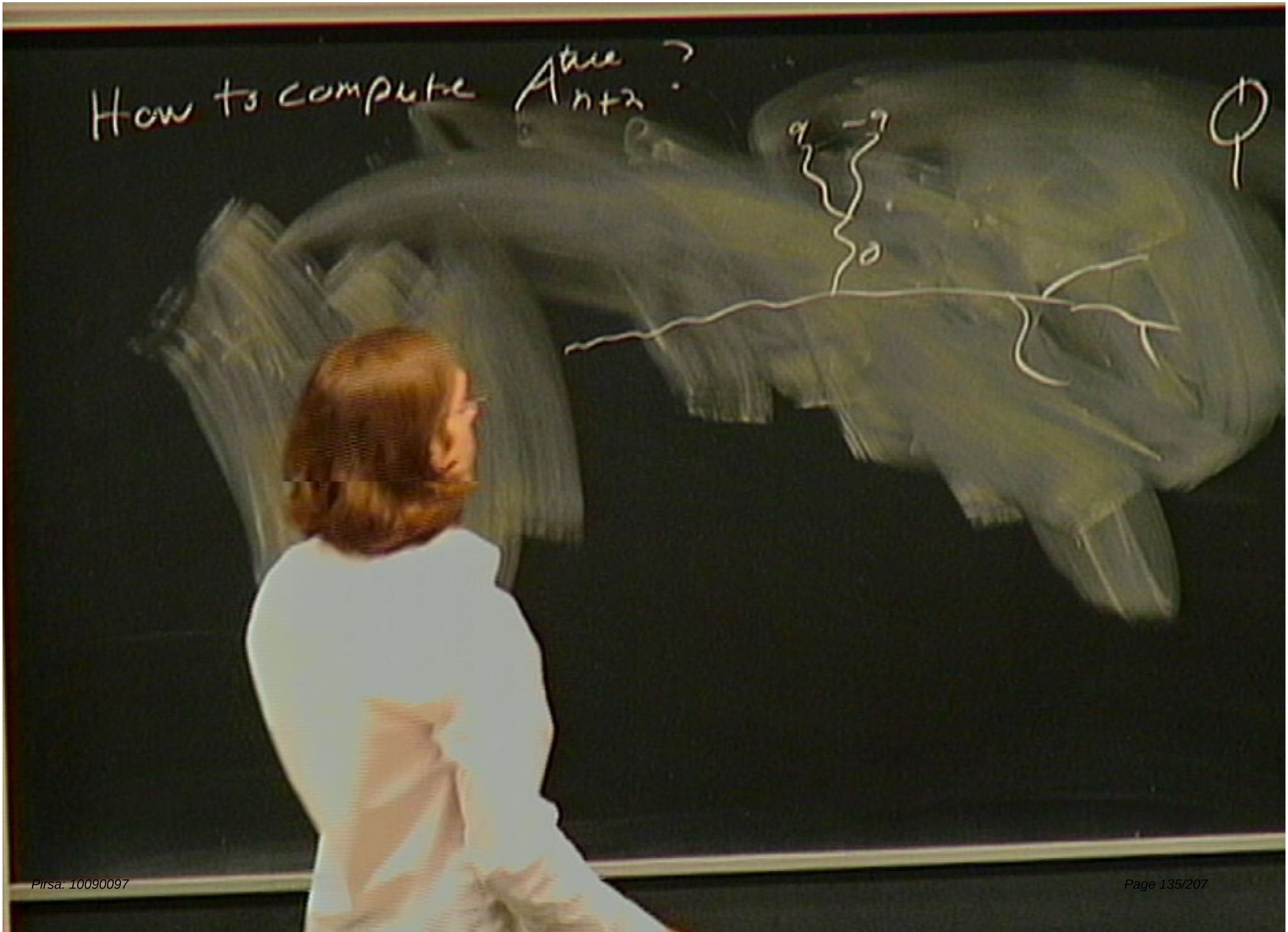
$$\text{"Inhom."} = \frac{1}{(x'-x_n)^2} A^{\text{tree}}_{n+2}(x_1, \ldots \hat{x}_n, x', \hat{x}_n) , \quad \hat{x}_n = x_n + z_* u \text{ s.t. } (\hat{x}_n - x')^2 = 0$$

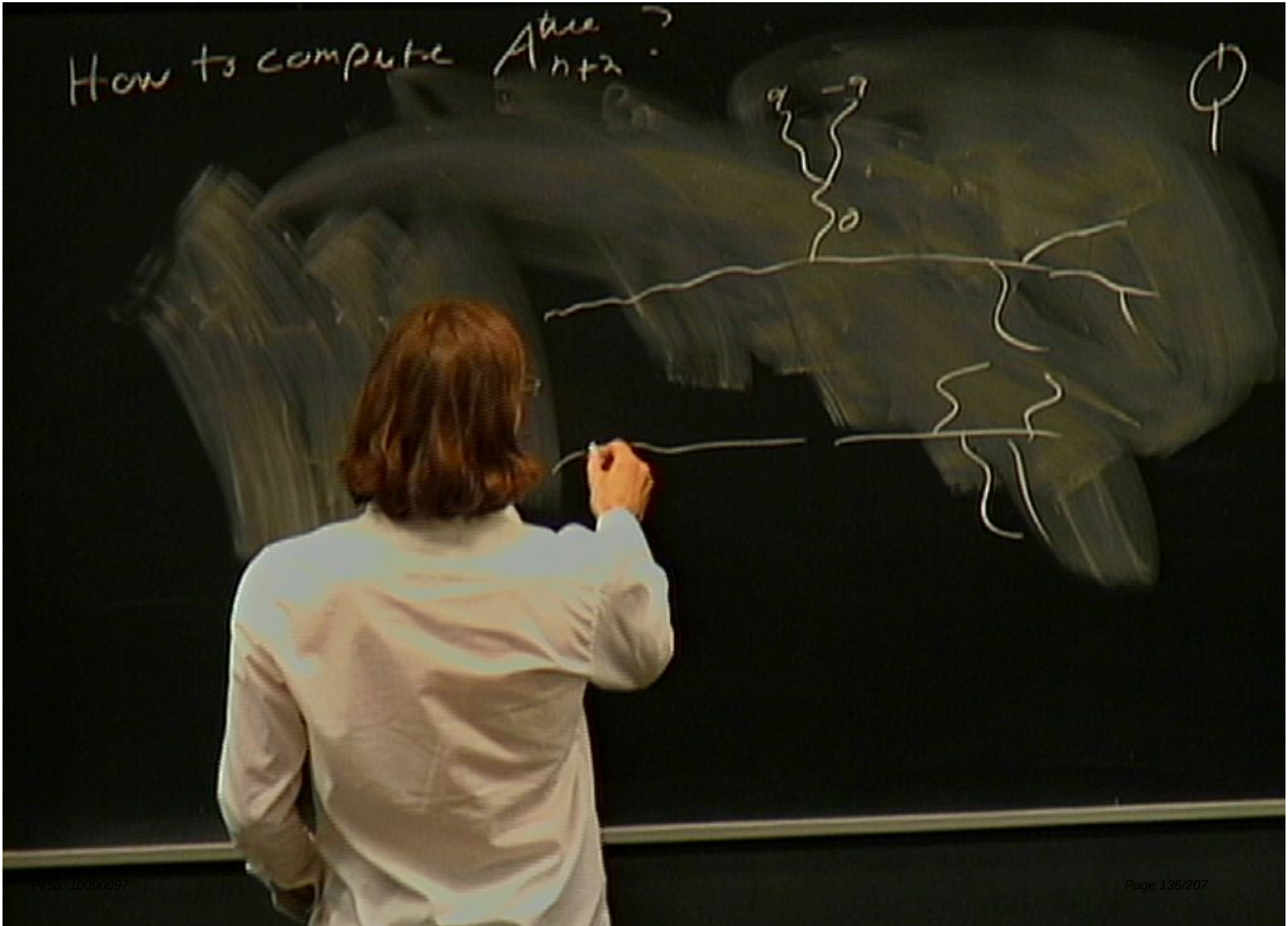
$$\text{"I}_n\text{kon."} = \frac{1}{(x'-x_n)^2}\, A^{\text{tree}}_{n+2}\left(x_1,\dots \hat{x}_n, x', \hat{x}_n\right), \quad \left(\hat{x}_n = x_n + z\omega \;\; \text{s.t. } (\hat{x}_n - x')^2 = 0\right)$$

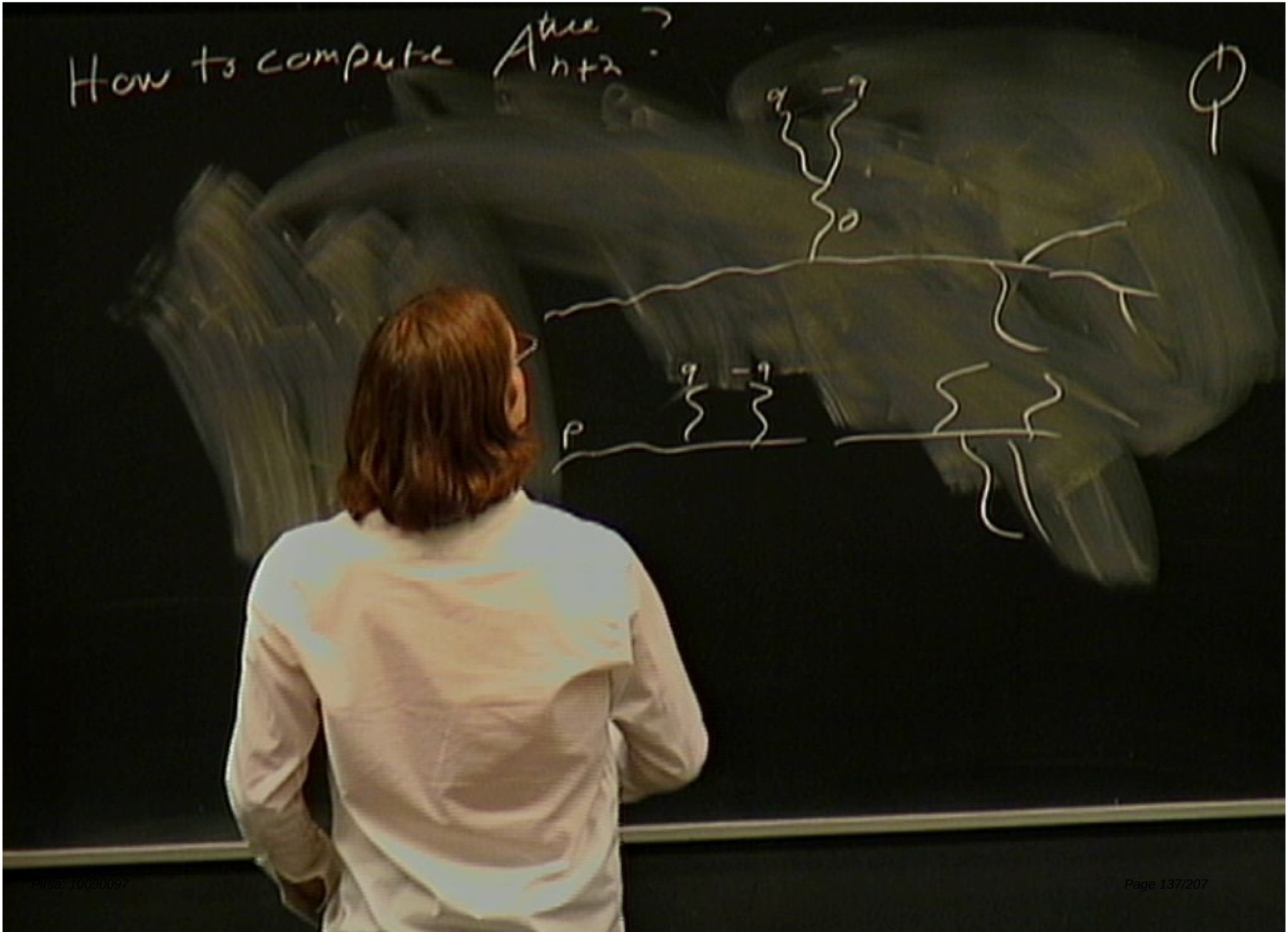
$$\text{"Fnlam"} = \frac{1}{(x'-x_n)^2}\, A^{\text{tree}}_{n+2}\left(x_1, \ldots \hat{x}_n, x', \hat{x}_n\right), \quad \begin{array}{l} \hat{x}_n = x_n + z\omega \\ \text{s.t. } (\hat{x}_n - x')^2 = 0 \end{array}$$


How to compute A^{tree}_{n+2}?

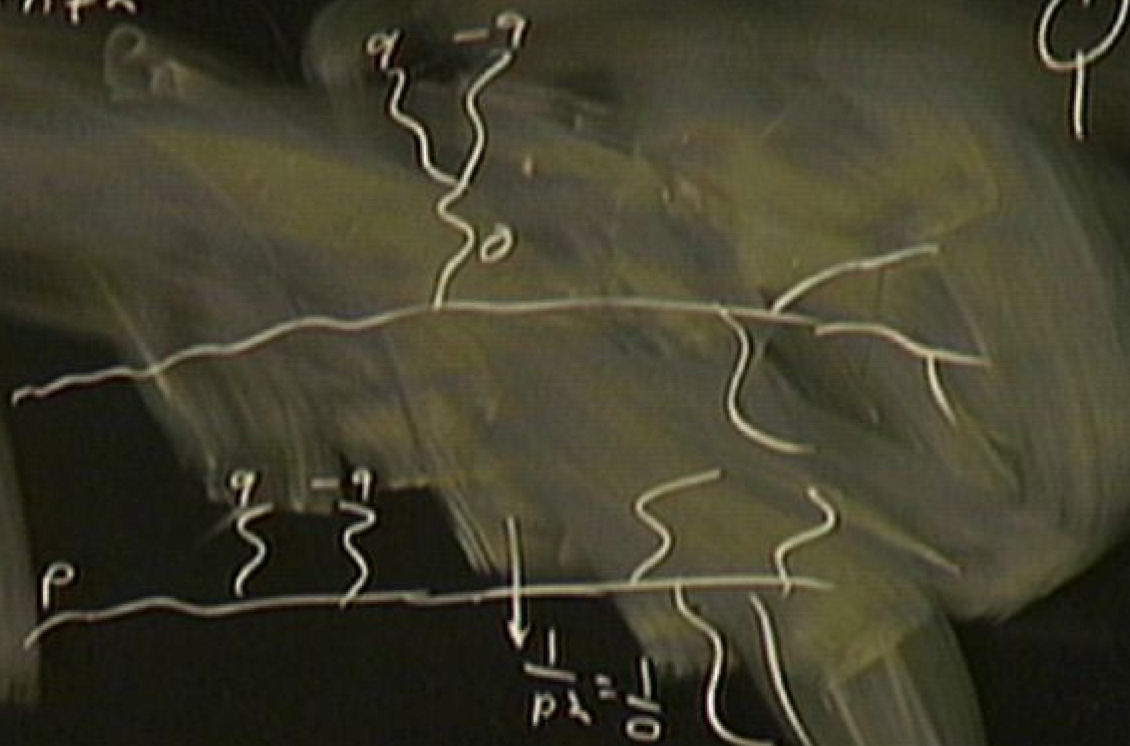

How to compute A^{tree}_{n+2} ?
q
-q
0

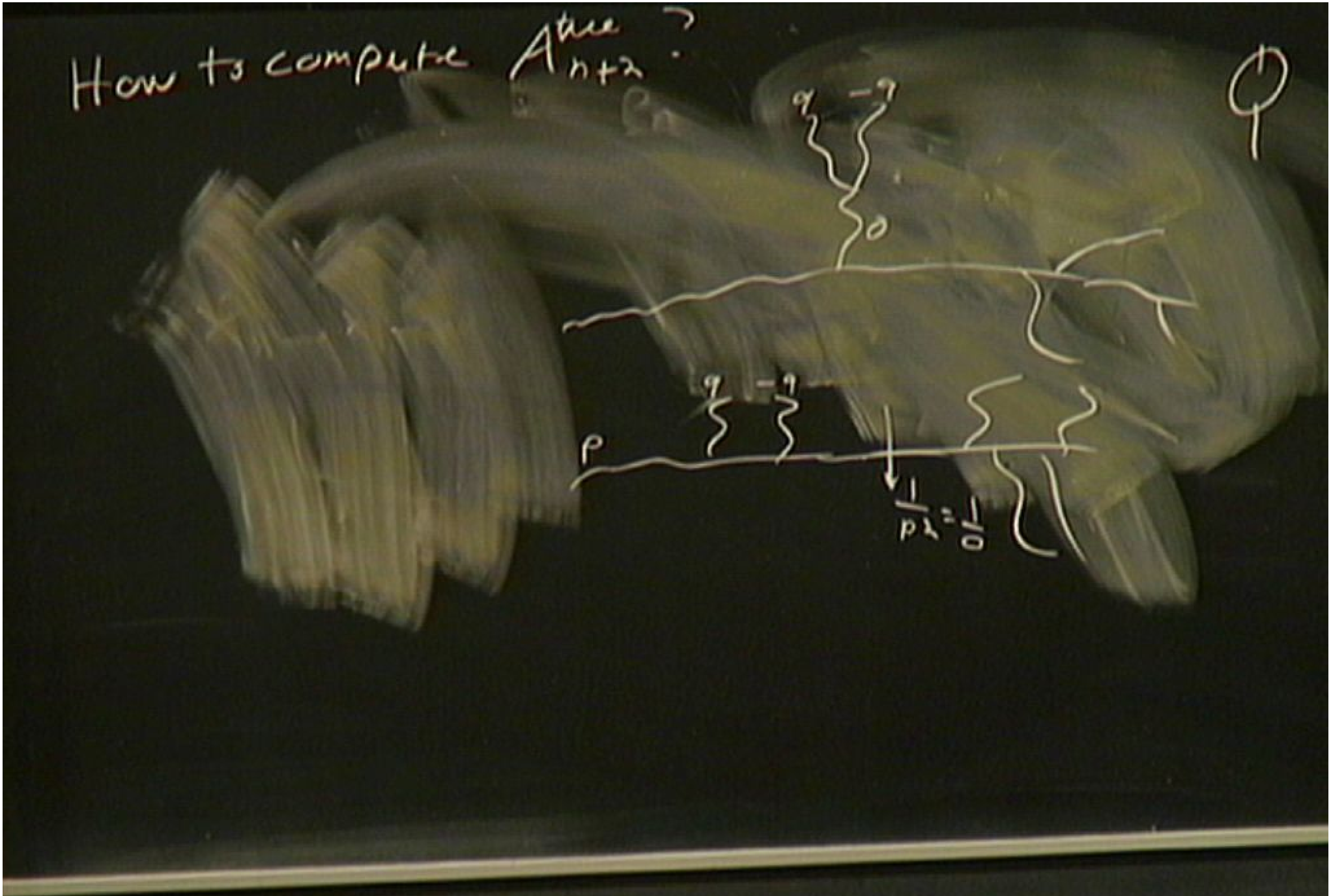

How to compute A^{tree}_{n+2} ?


How to compute A^{tree}_{n+2} ?
q
-q
0

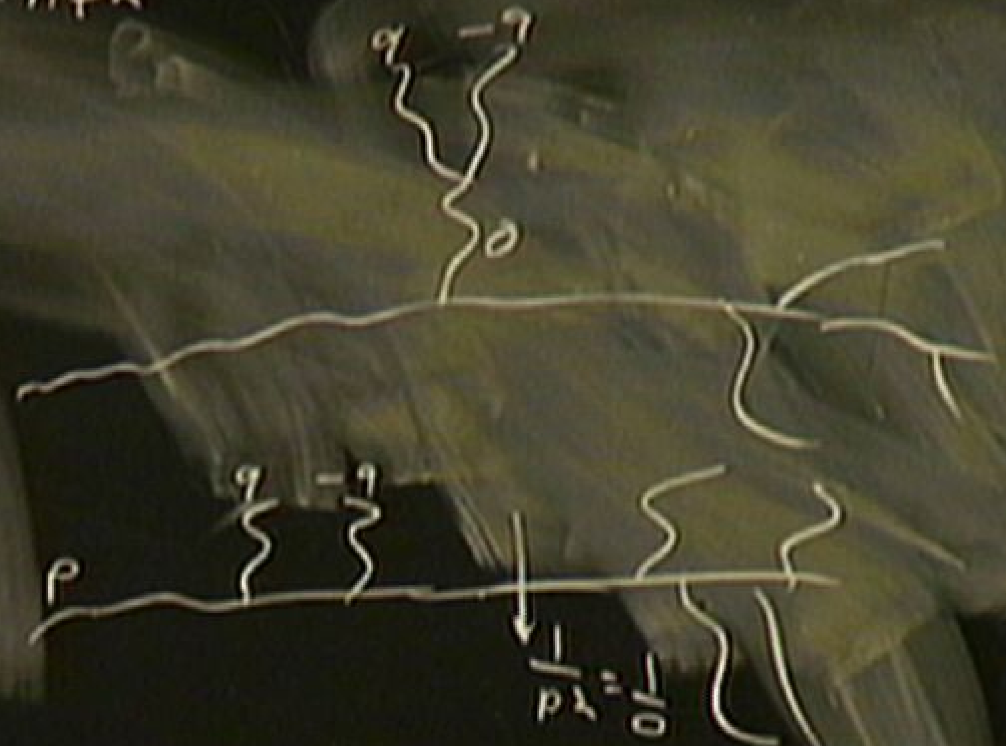

How to compute A^{tree}_{n+2} ?
q
-q
q
-q
p

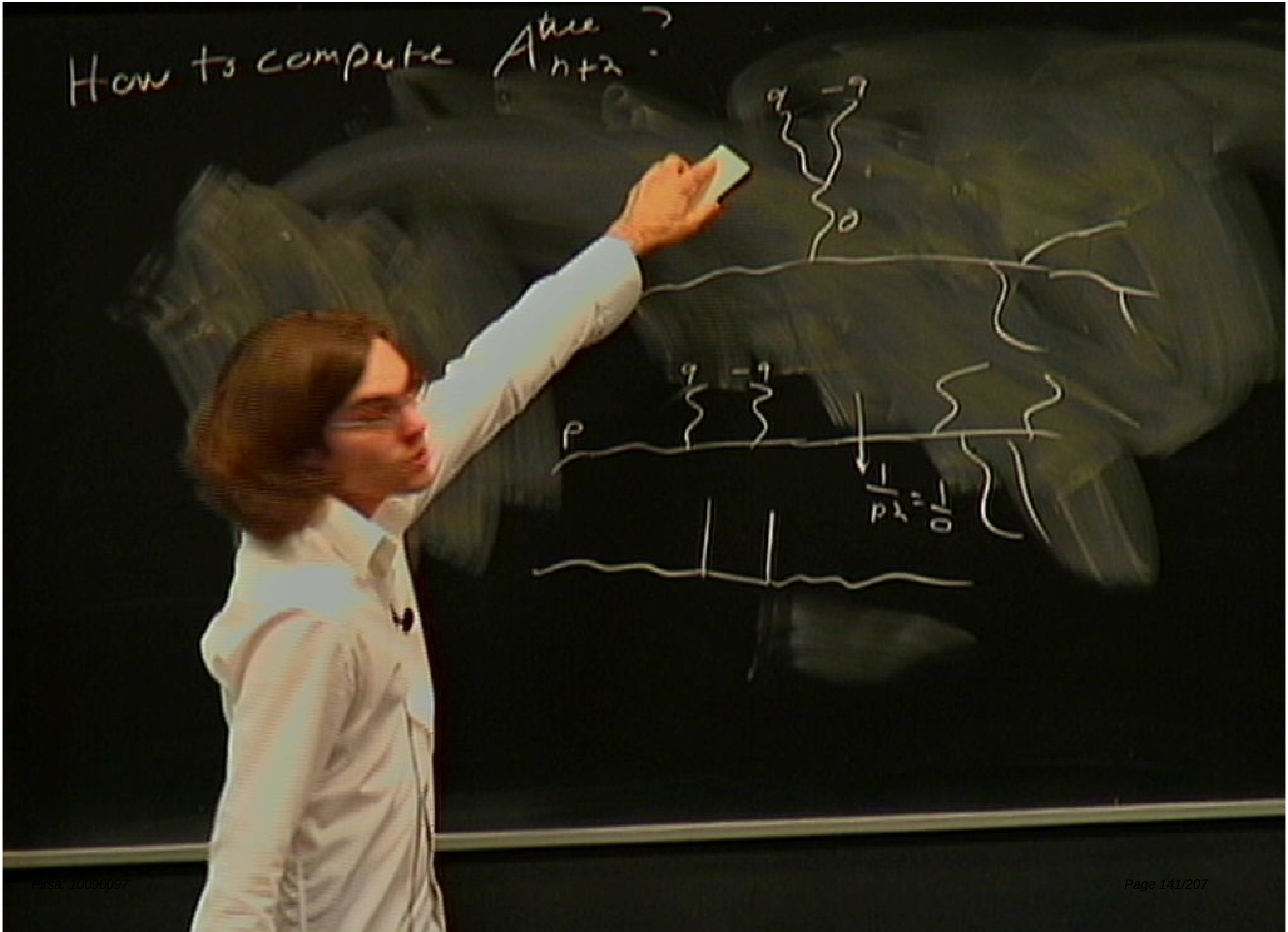
How to compute A^{tree}_{n+2} ?

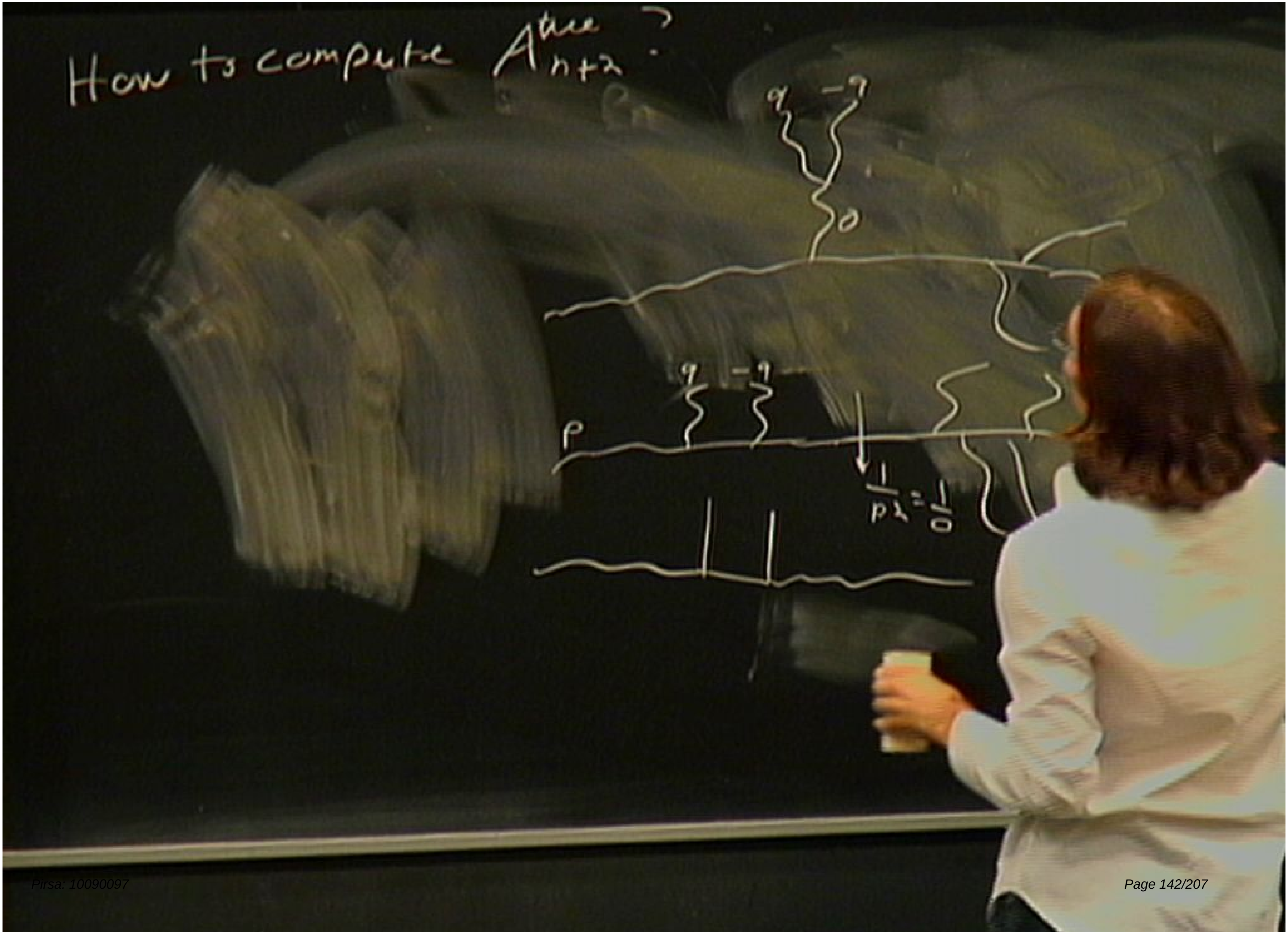


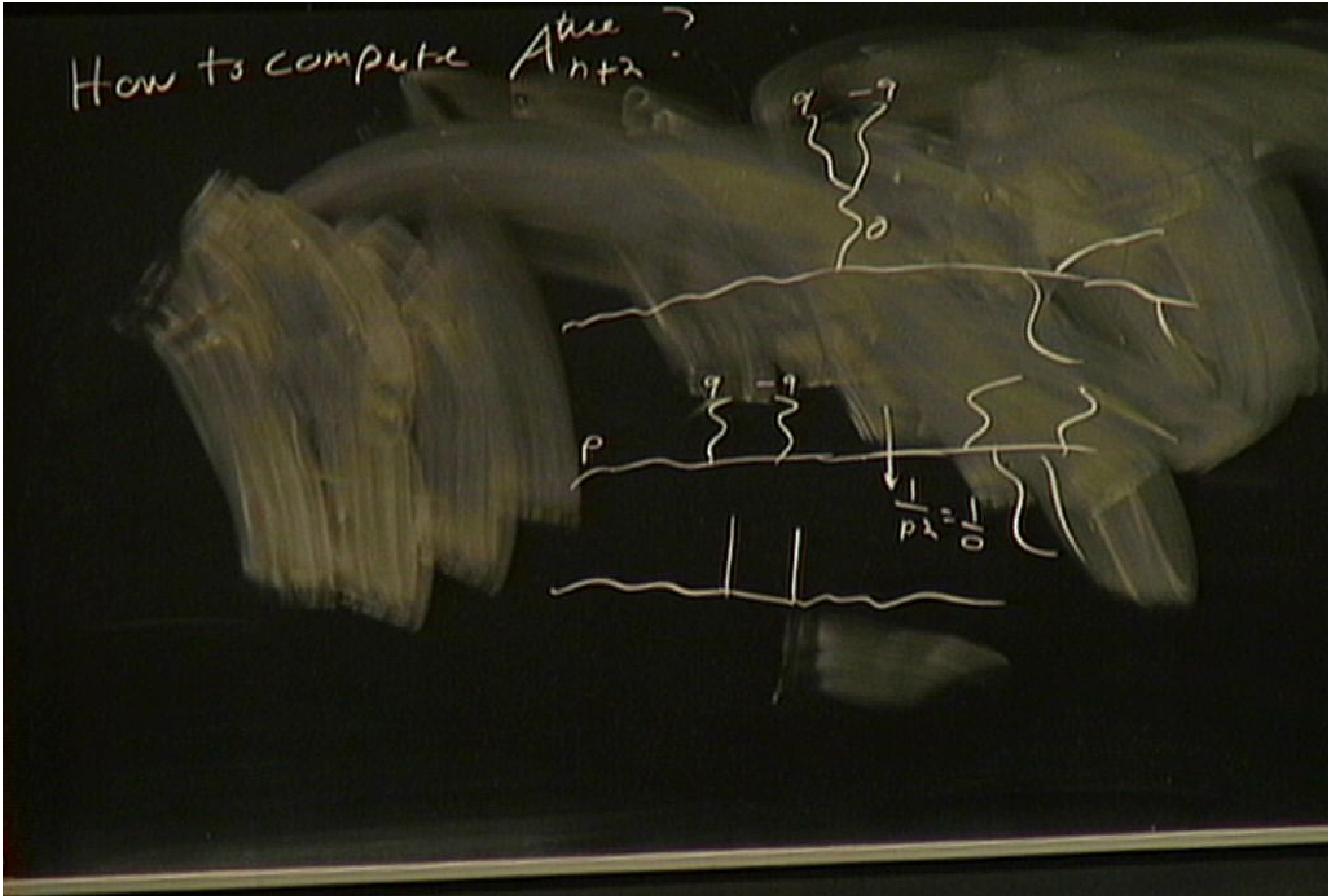

How to compute A^{tree}_{n+2} ?
q
$-q$
0
Φ
q
$-q$
p
$\frac{1}{p^2} = \frac{1}{0}$

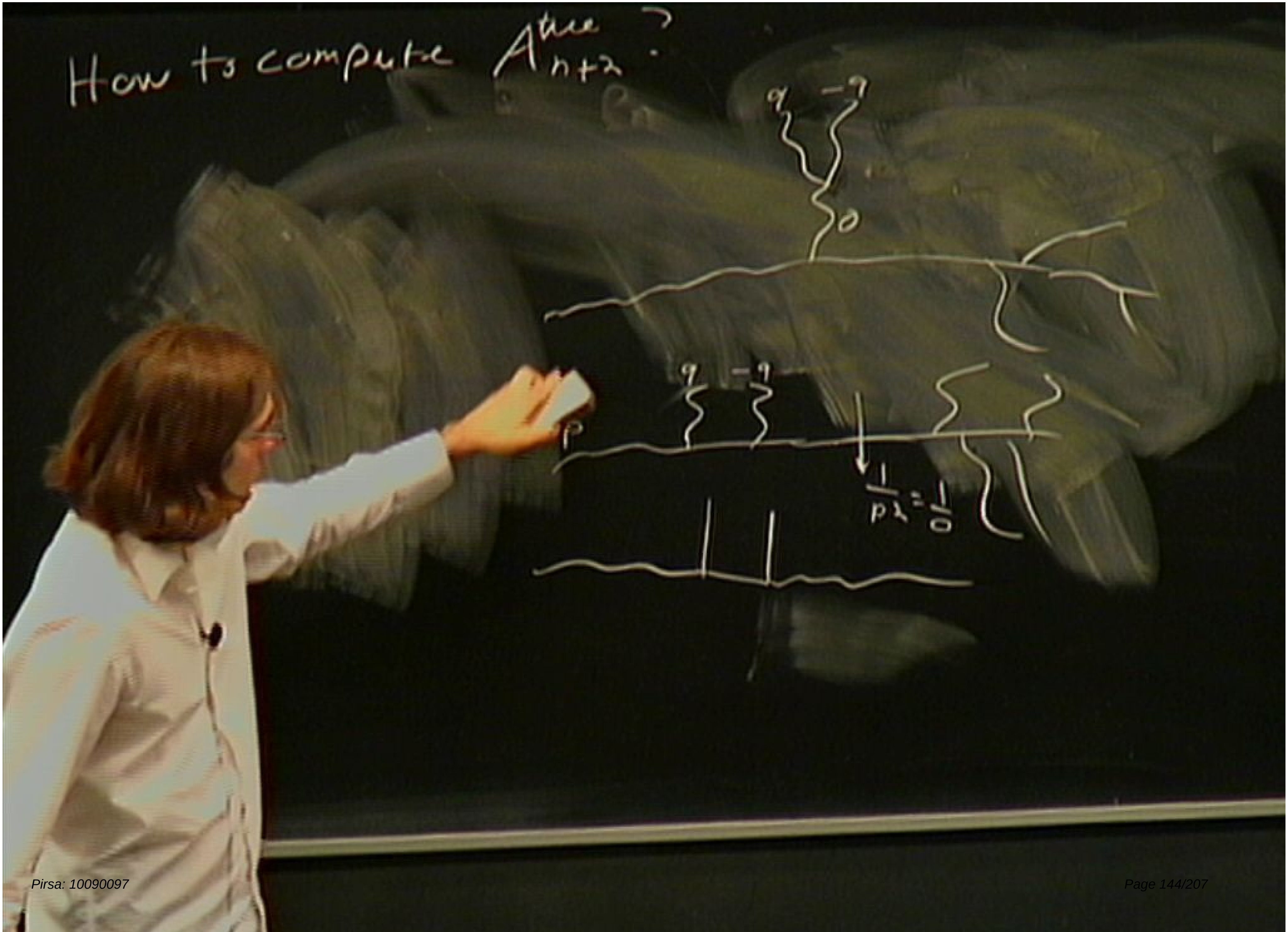
How to compute A^{tree}_{n+2}?

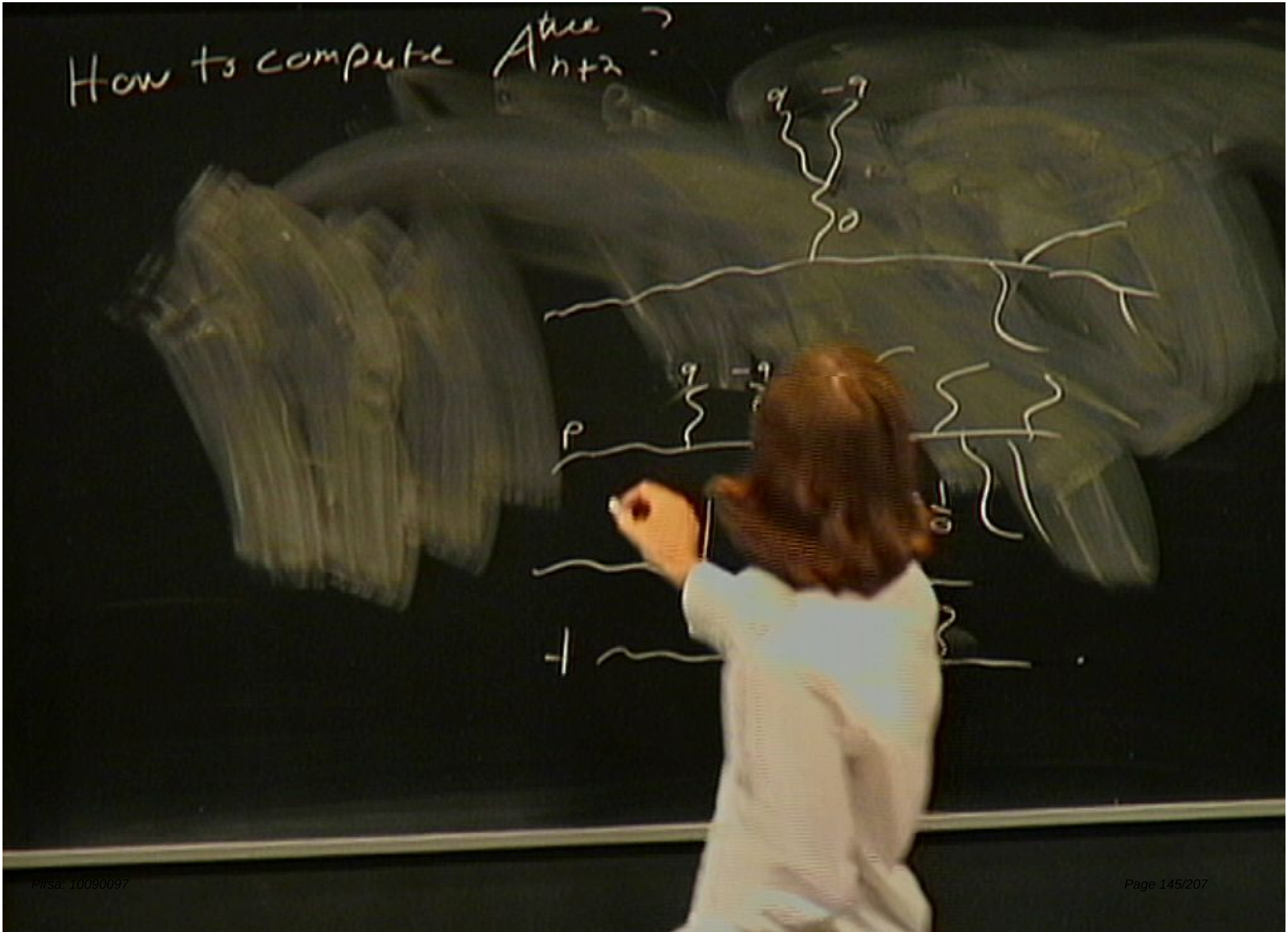


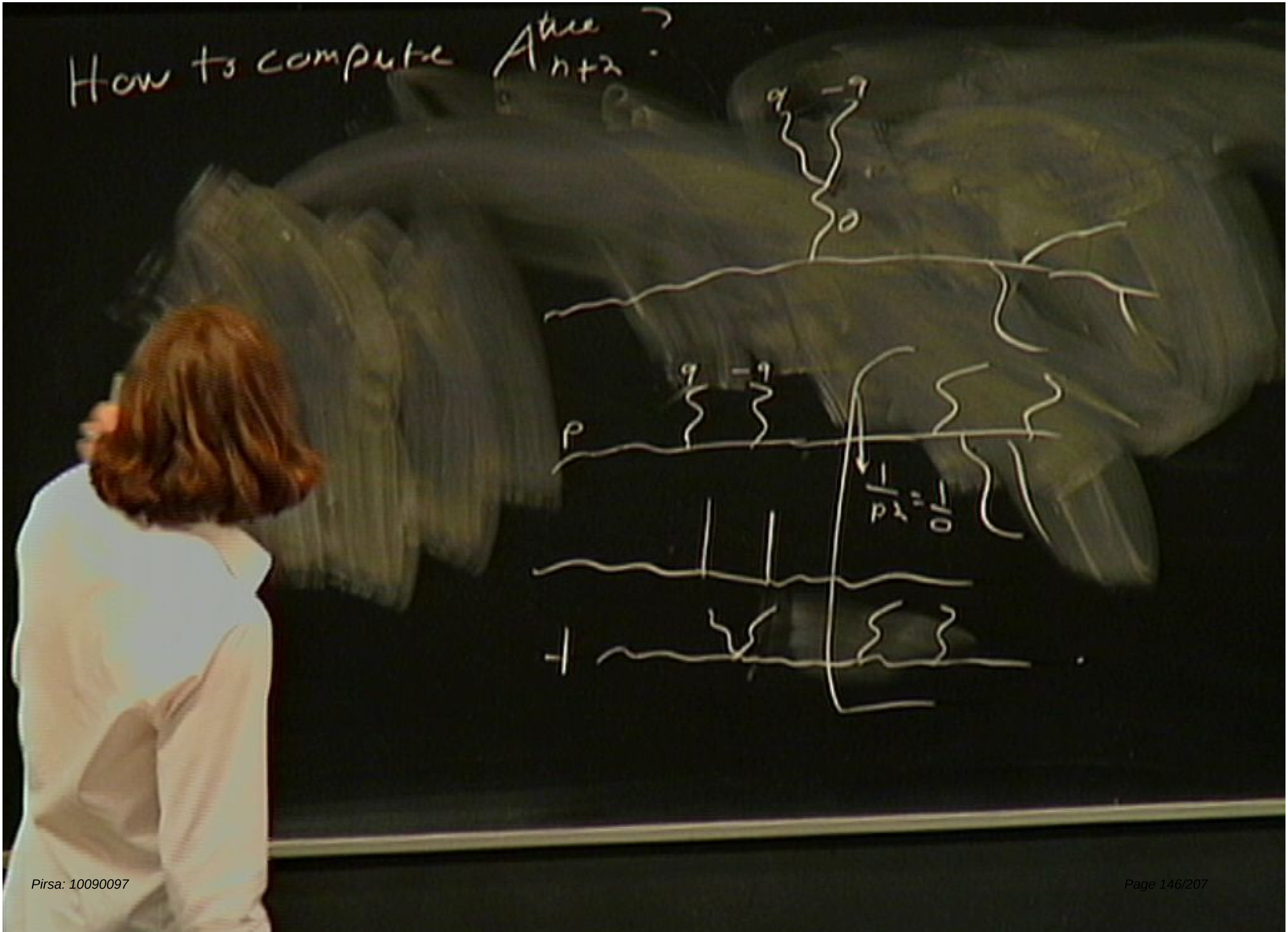

How to compute A^{tree}_{n+2} ?
q
-q
0
p
q
-q
$\frac{1}{p^2} = \frac{1}{0}$

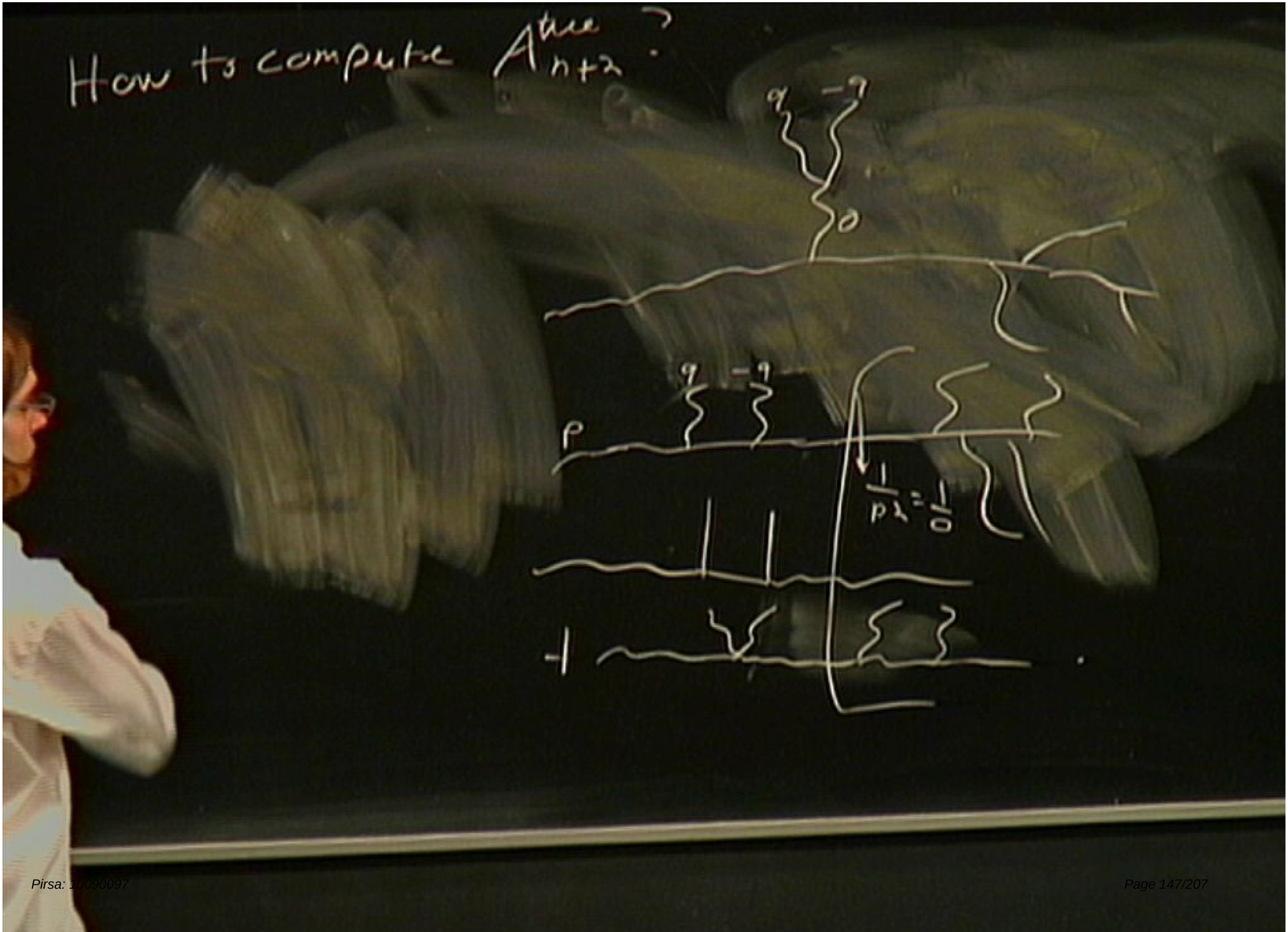

How to compute A^{tree}_{n+2}?
q
$-q$
p
$\frac{1}{p^2} = \frac{1}{0}$

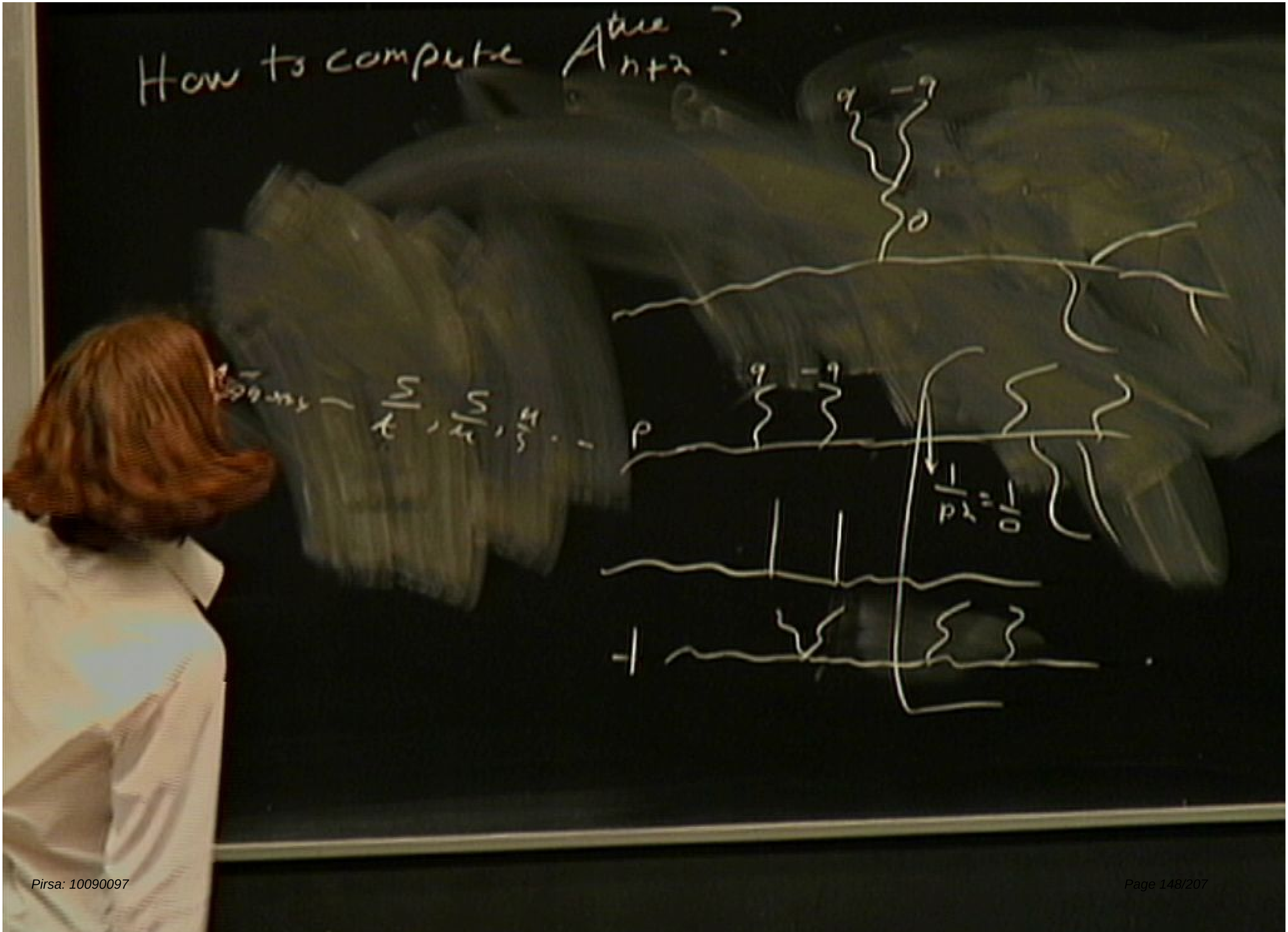

How to compute A^{tree}_{n+2} ?
q $-q$
0
q $-q$
p
$\frac{1}{p^2} = \frac{1}{0}$

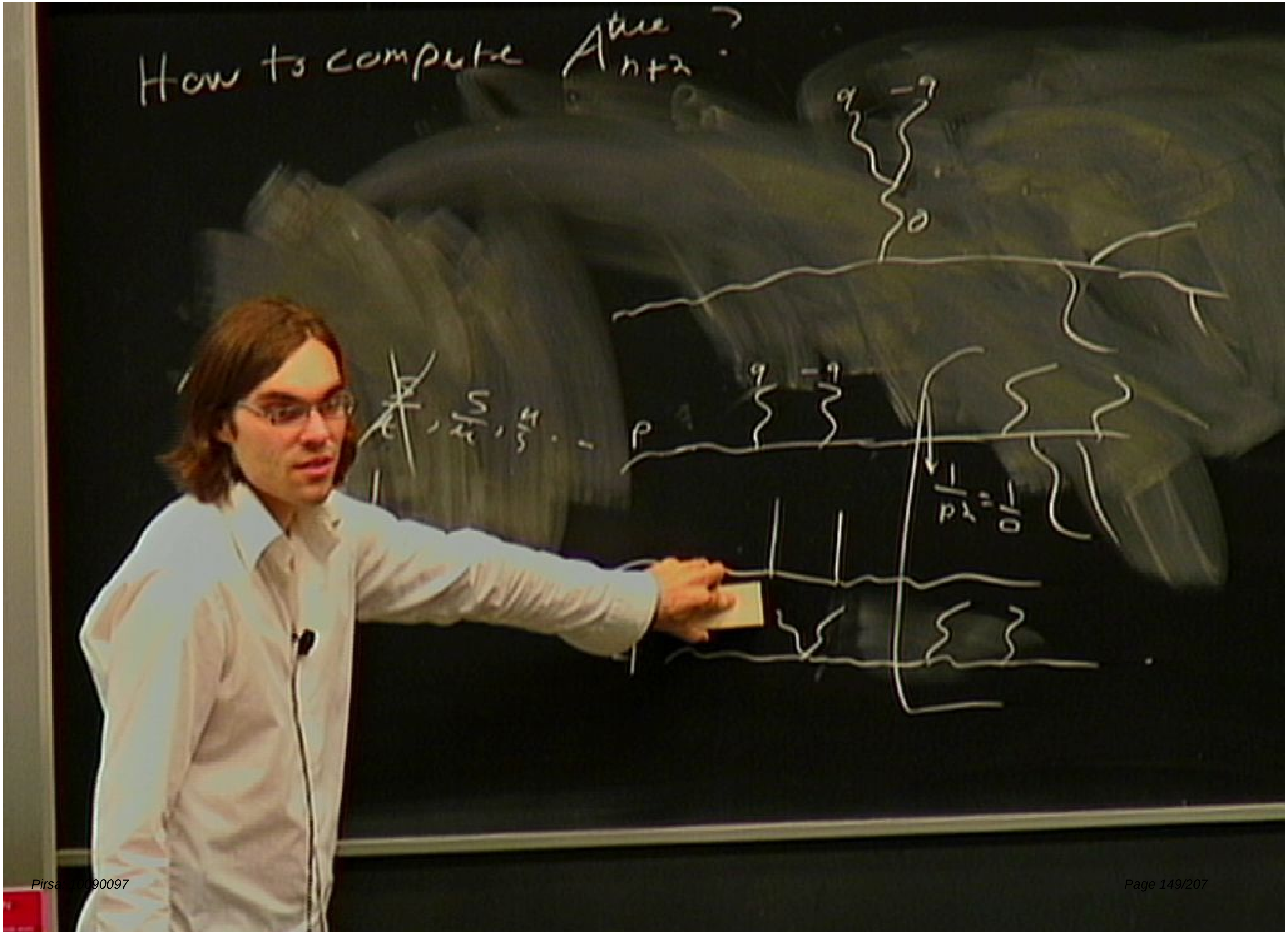

How to compute A^{tree}_{n+2}?
q
-q
0
p
q
-q
$\frac{1}{p^2} = \frac{1}{0}$


How to compute A^{tree}_{n+2} ?


How to compute A^{tree}_{n+2} ?
q
-q
0
p
q
-q
$\frac{1}{p^2} = \frac{1}{0}$


How to compute A^{tree}_{n+2}?
q -q
p
$\frac{1}{p^2} = \frac{1}{0}$
-1

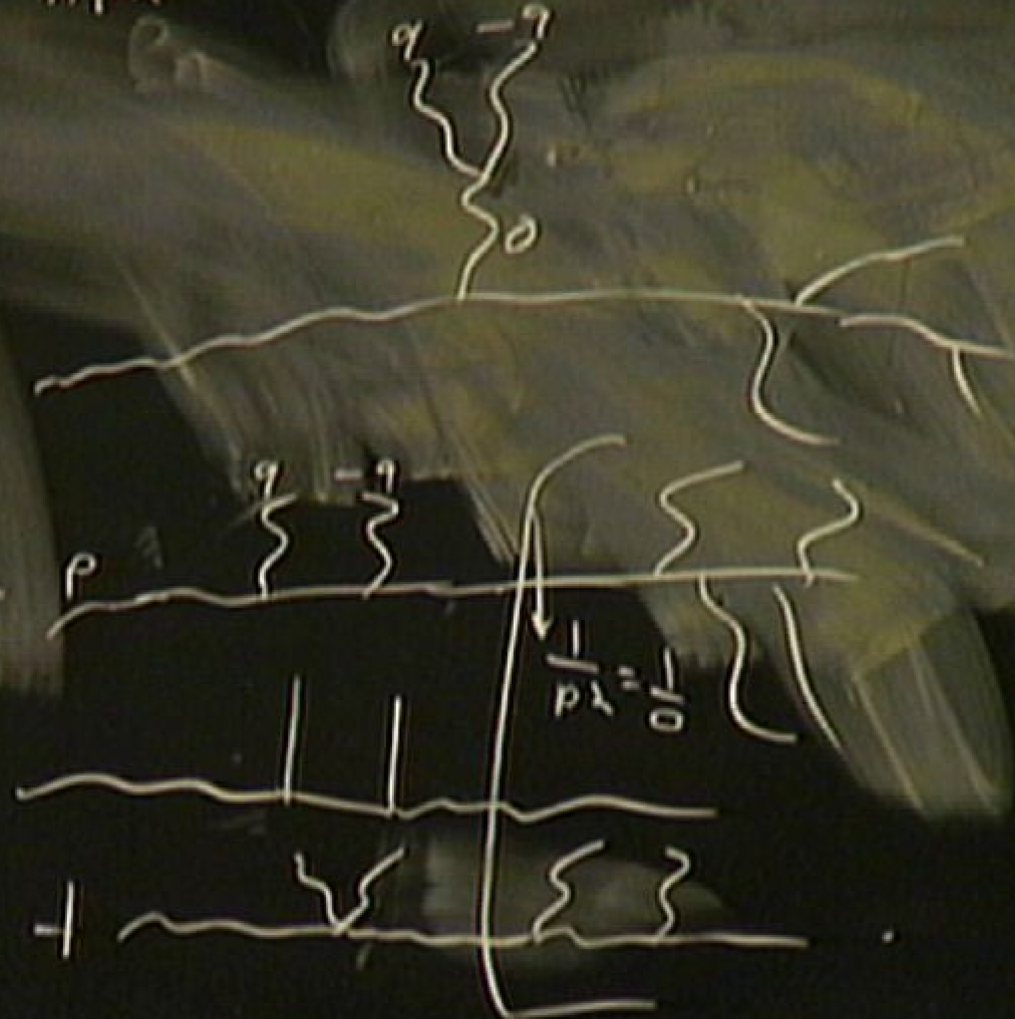

How to compute A^{tree}_{n+2}?
q
-q
0
p
-1
$\frac{1}{p^2} = \frac{1}{0}$

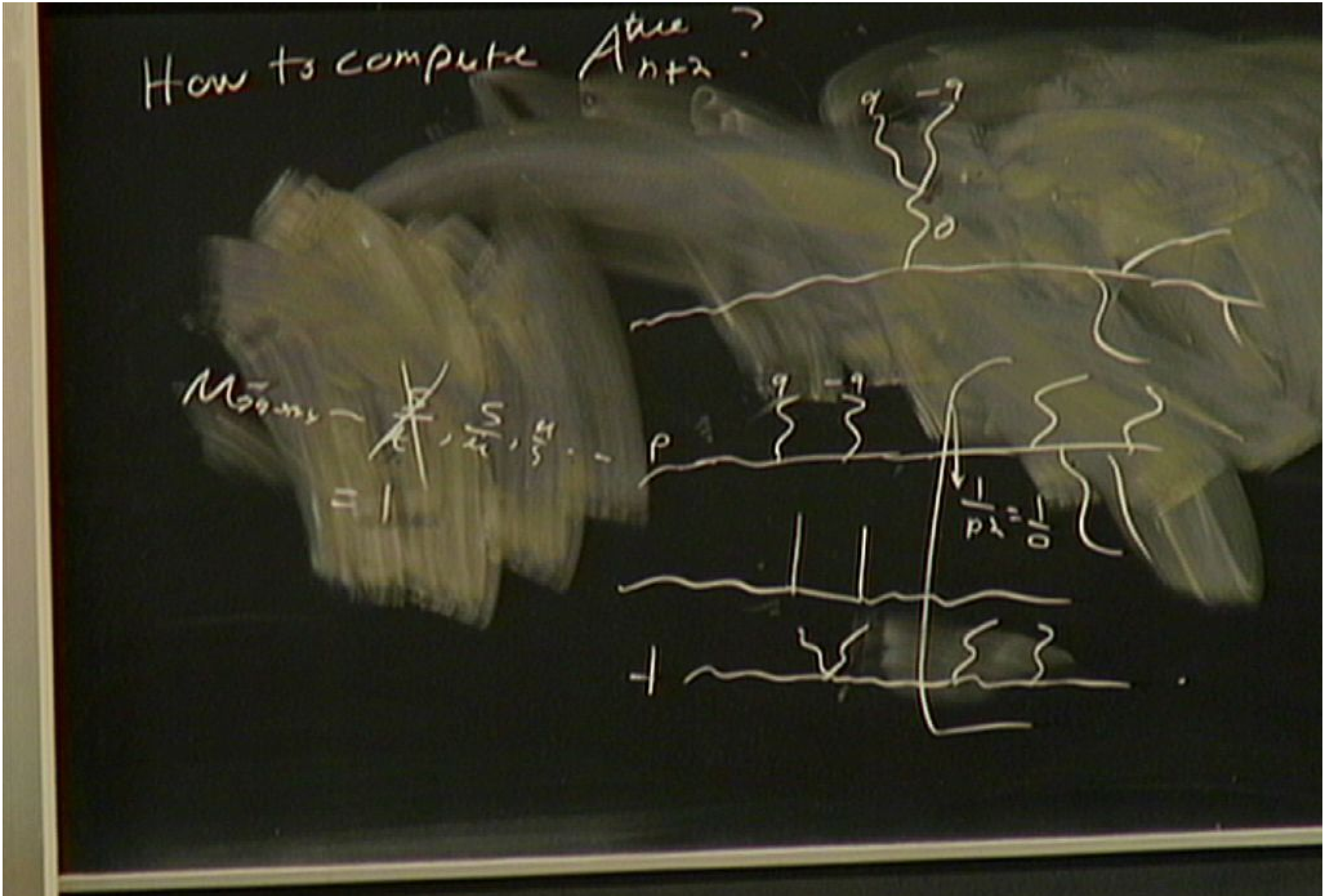

How to compute A^{tree}_{n+2}?

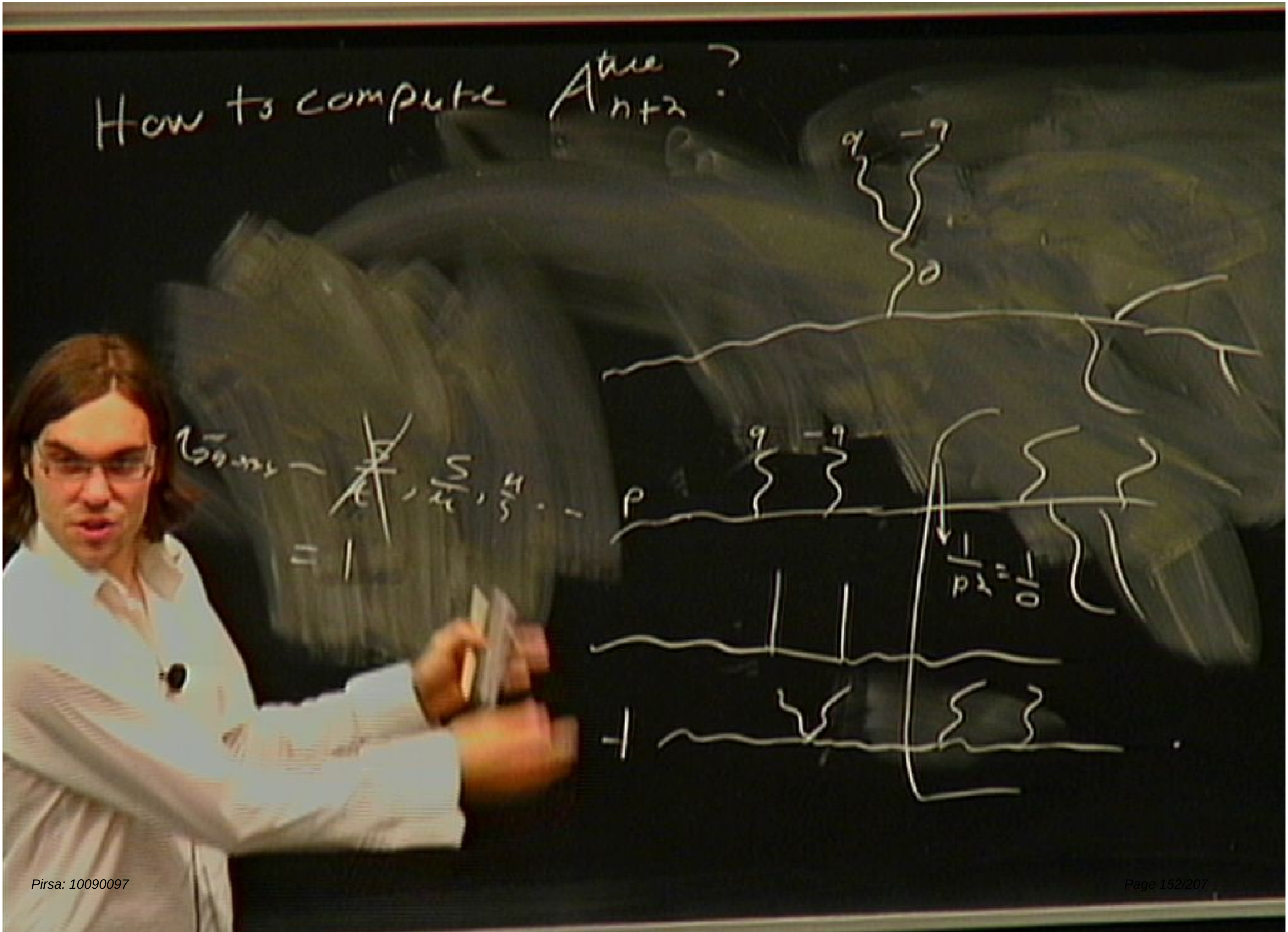
How to compute A^{tree}_{n+2}?

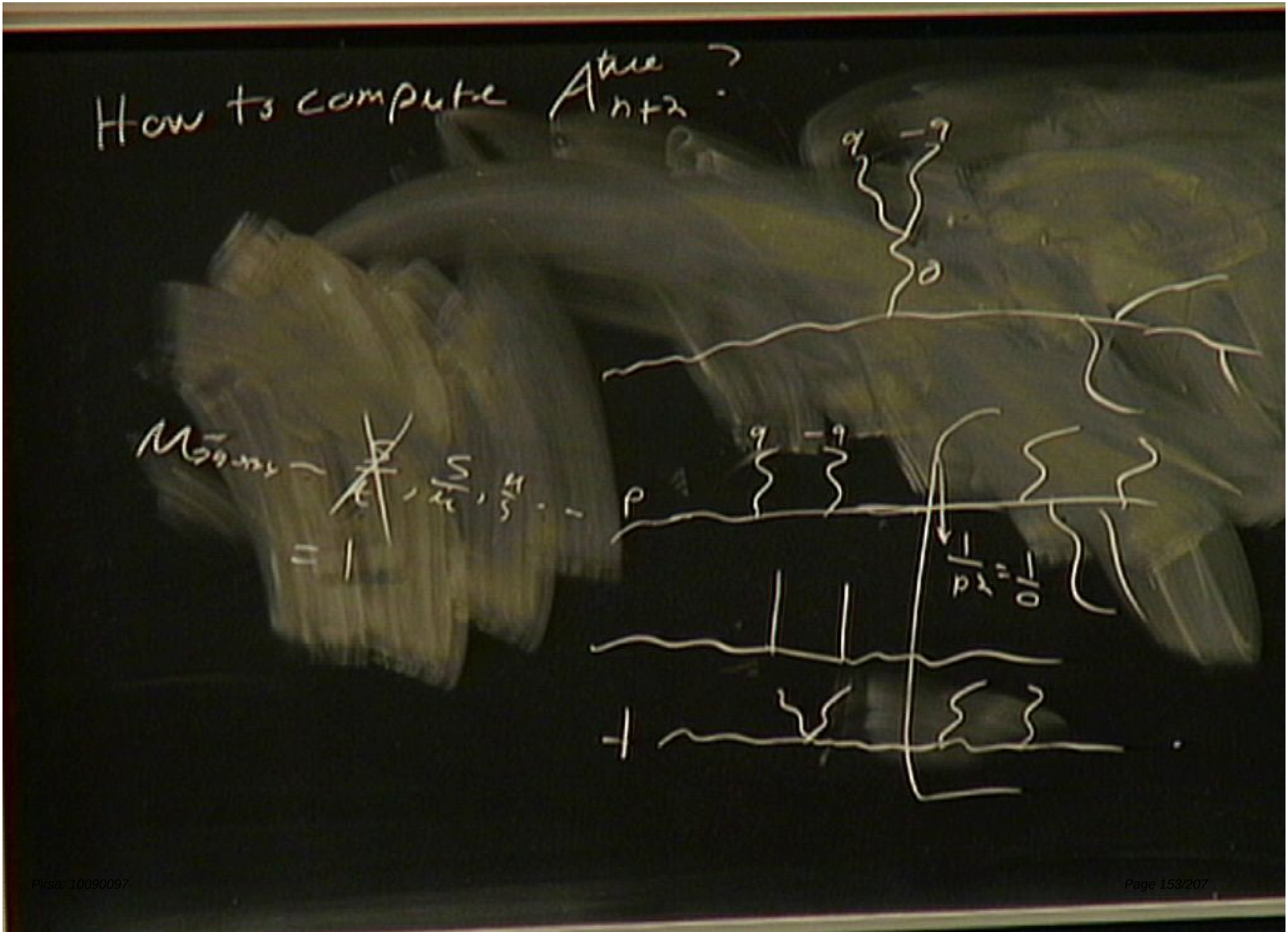
Moments — $\cancel{\frac{3}{6}}, \frac{5}{12}, \frac{4}{3} \ldots$

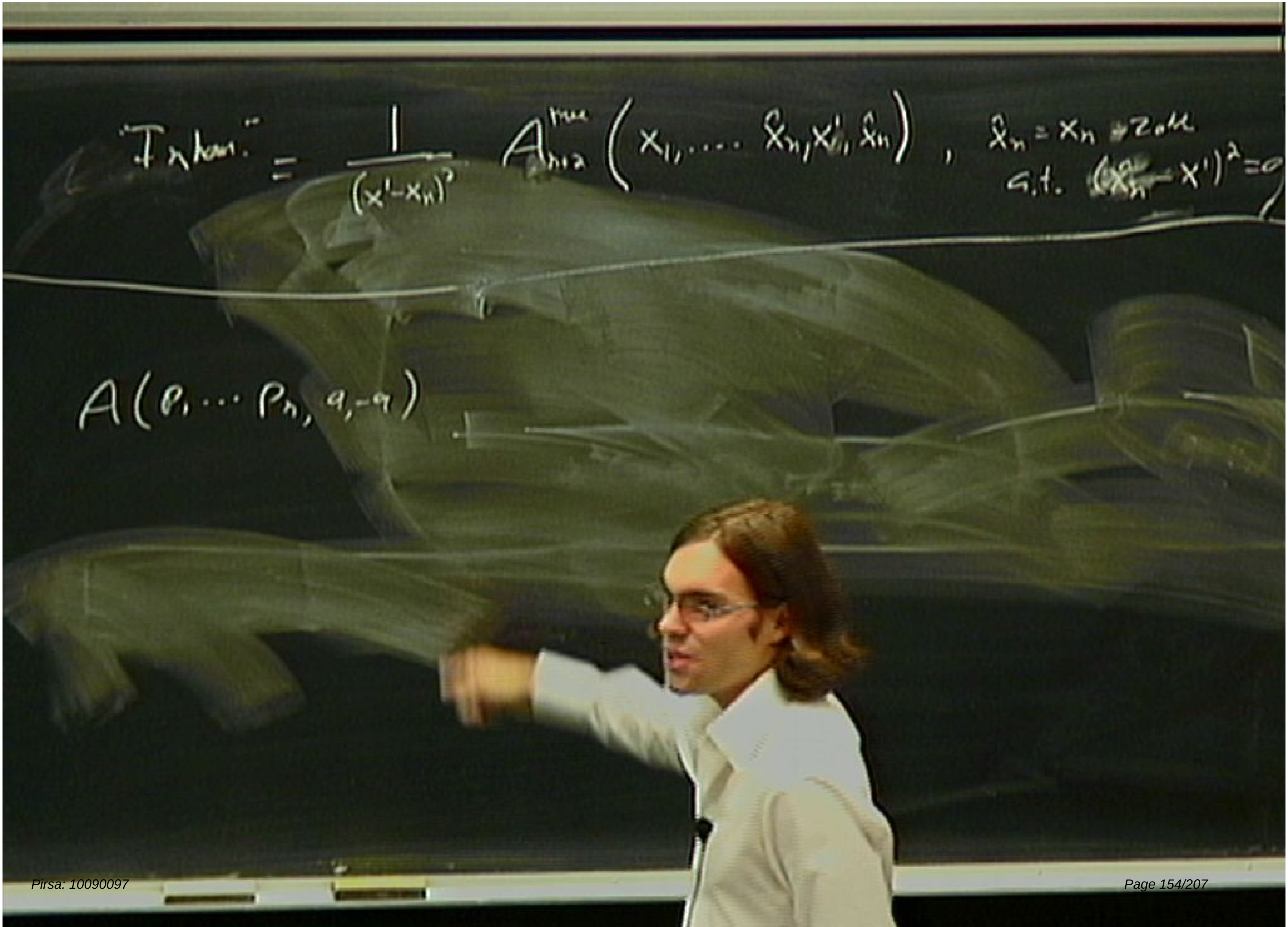
$= 1$

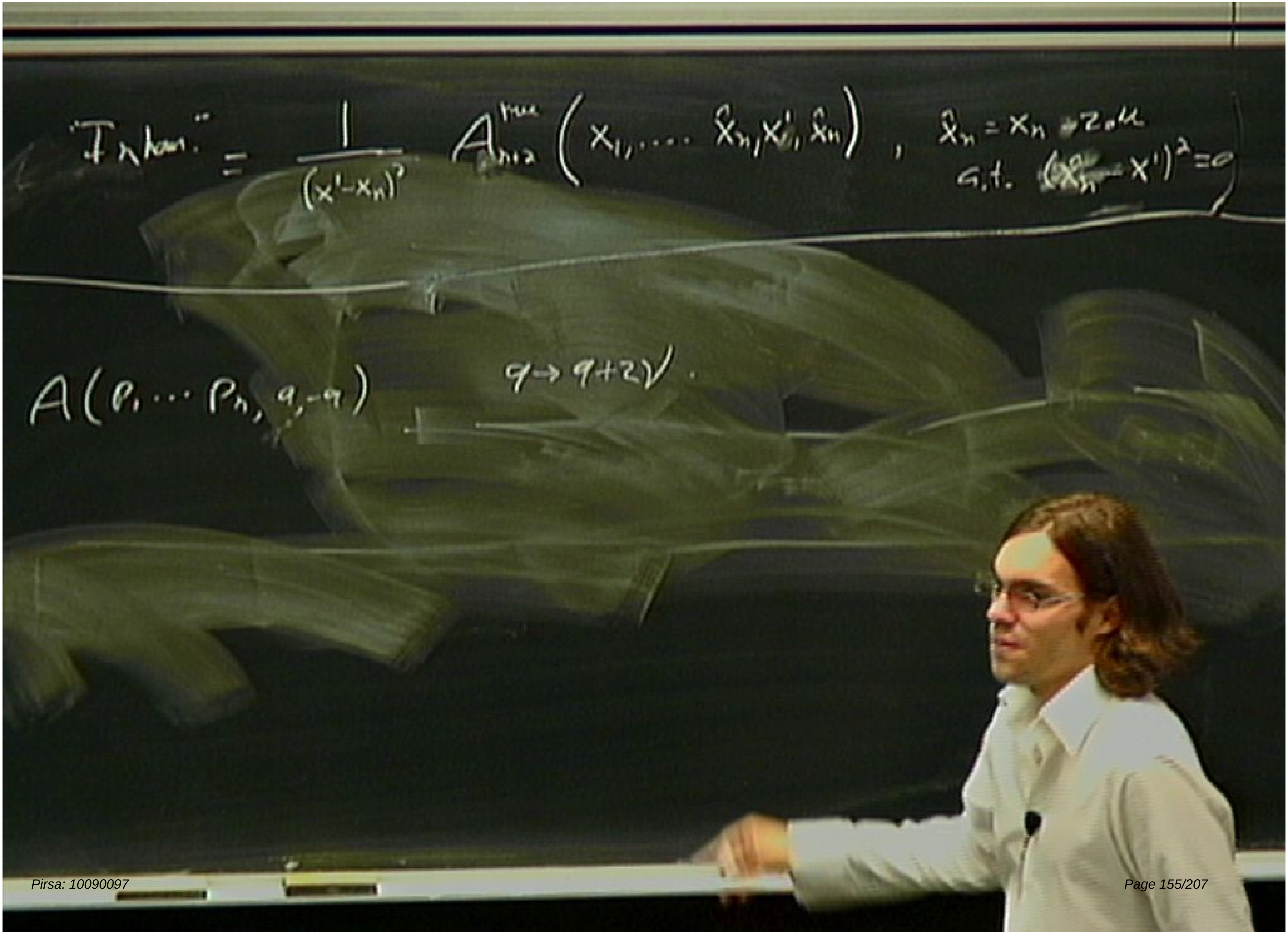



How to compute A^{tree}_{n+2}?
q
-q
0
p
= 1
$\frac{1}{p^2} = \frac{1}{0}$


How to compute A tree n+2 ?
= 1
1/p² = 1/0


How to compute A^{tree}_{n+2}?
$\frac{1}{p^2} = \frac{1}{0}$


"Fishbone" = 1/(x'-x_n)^2 A^tree_{n+2}(x_1, ... x̂_n, x', x̂_n)
A(p_1 ... p_n, q, -q)


"Fnhan." = 1/(x'-x_n)^2 A^tree_{n+2}(x_1, ... x̂_n, x', x̂_n), x̂_n = x_n + z_* u
s.t. (x̂_n - x')^2 = 0
A(p_1 ... p_n, q, -q)
q → q+zV.

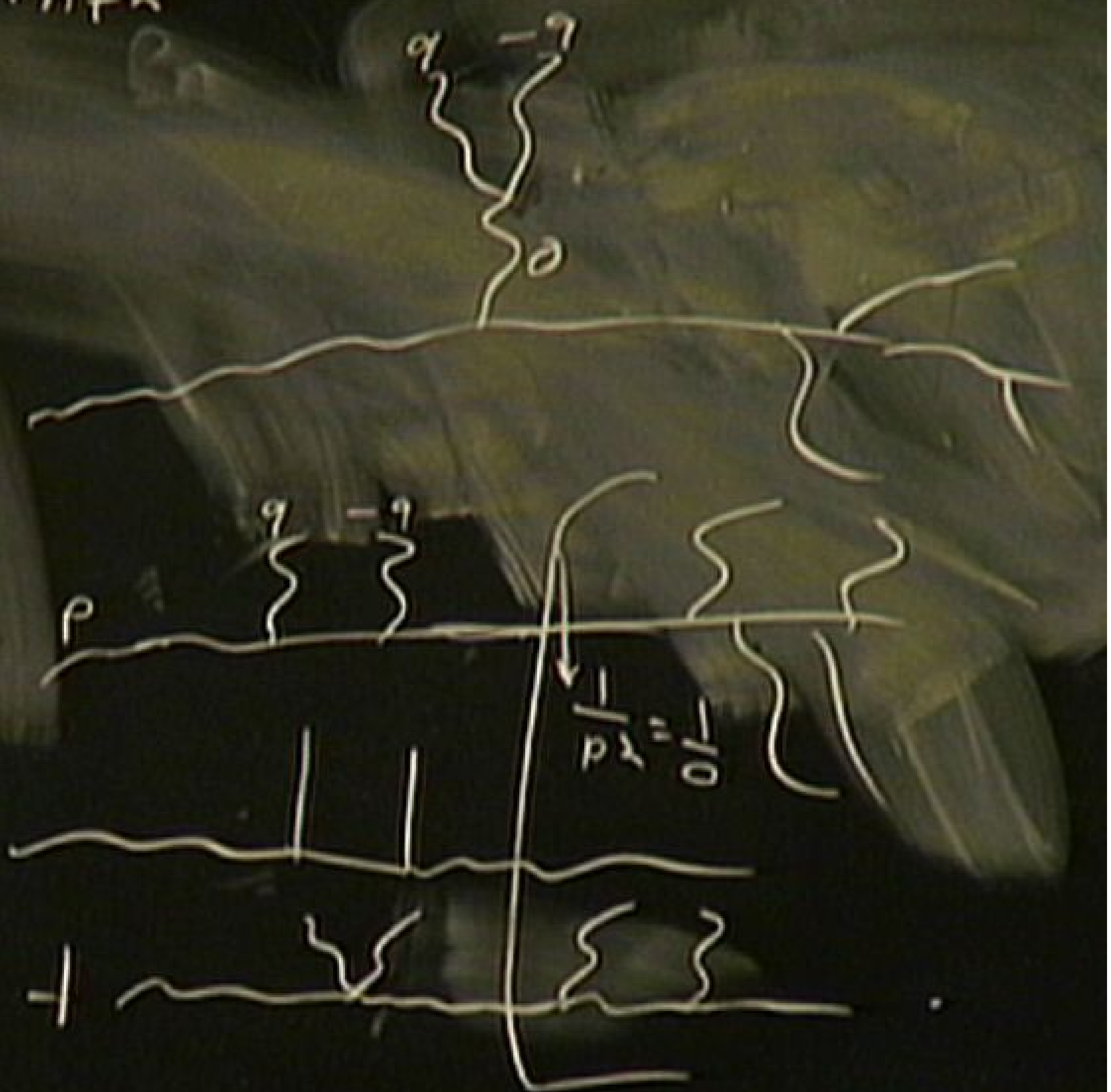
$$\text{"I}_{\lambda}\text{ham."} = \frac{1}{(x'-x_n)^2} A^{\text{tree}}_{n+2}\left(x_1, \ldots \hat{x}_n, x', \hat{x}_n\right), \quad \hat{x}_n = x_n + z\mu \quad \text{s.t. } (\hat{x}_n - x')^2 = 0$$

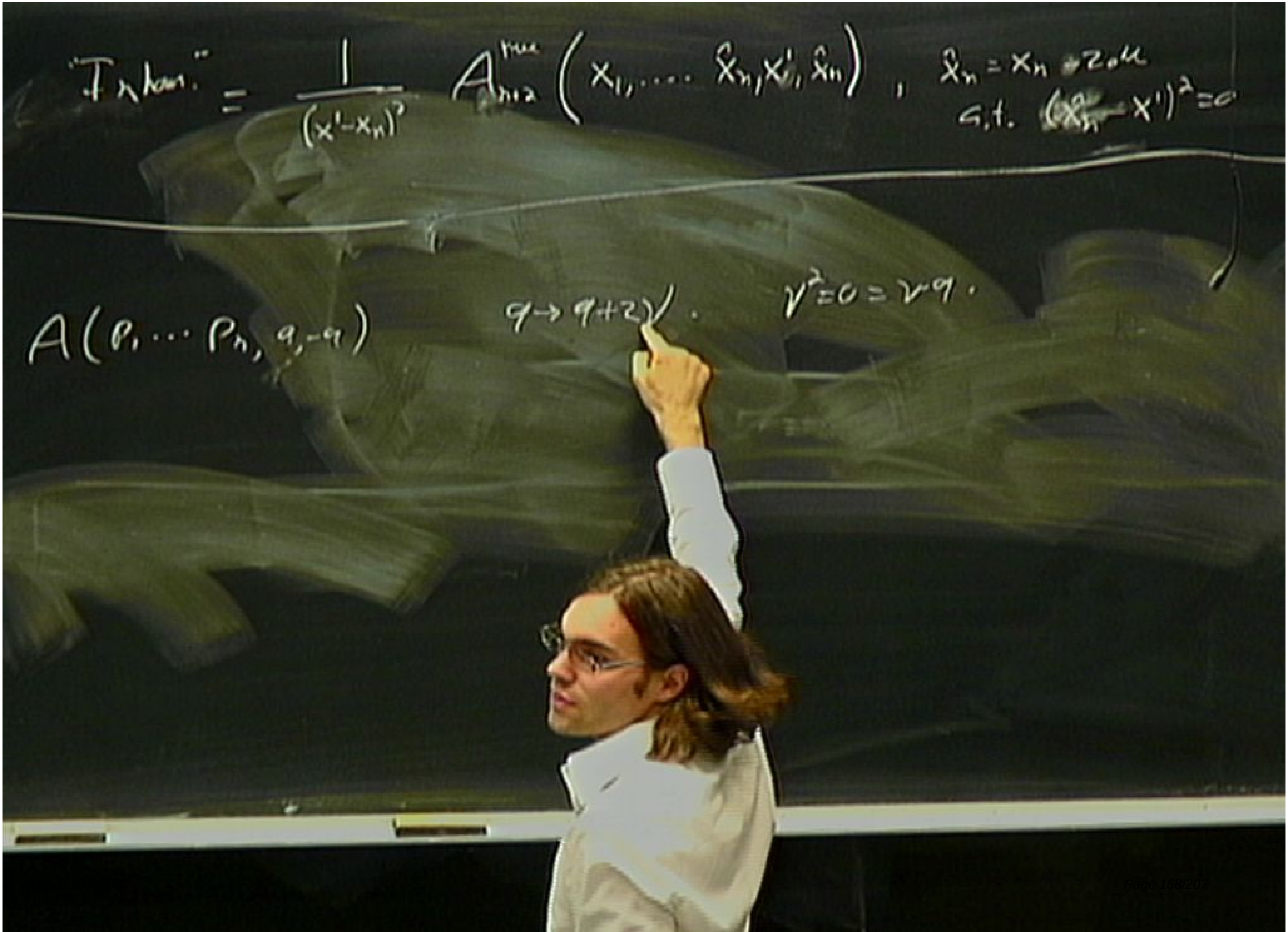
$$A(p_1 \cdots p_n, q, -q) \qquad q \to q + zv. \qquad v^2 = 0 = v \cdot q.$$

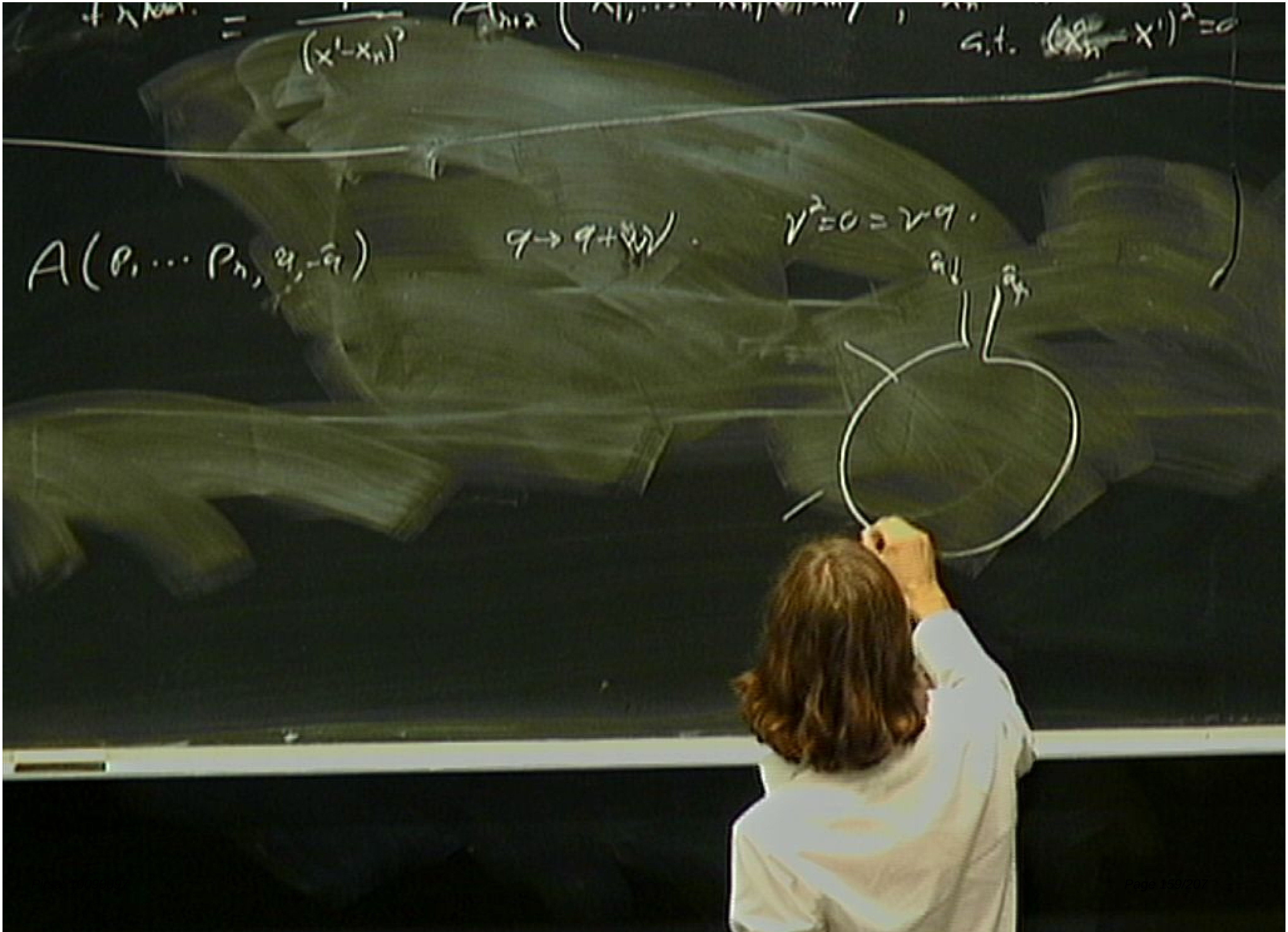
How to compute A^{tree}_{n+2}?

$M_{\bar{5}q\ldots m_2} \sim \cancel{\frac{2}{t}},\ \frac{S}{t},\ \frac{u}{s},\ \ldots$

$= 1$

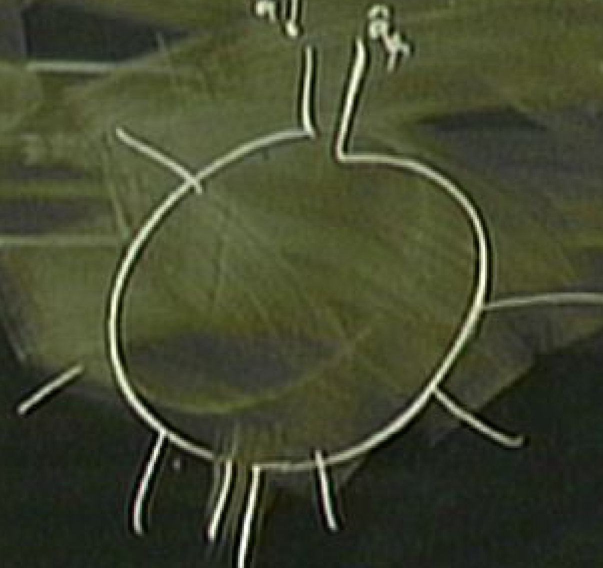


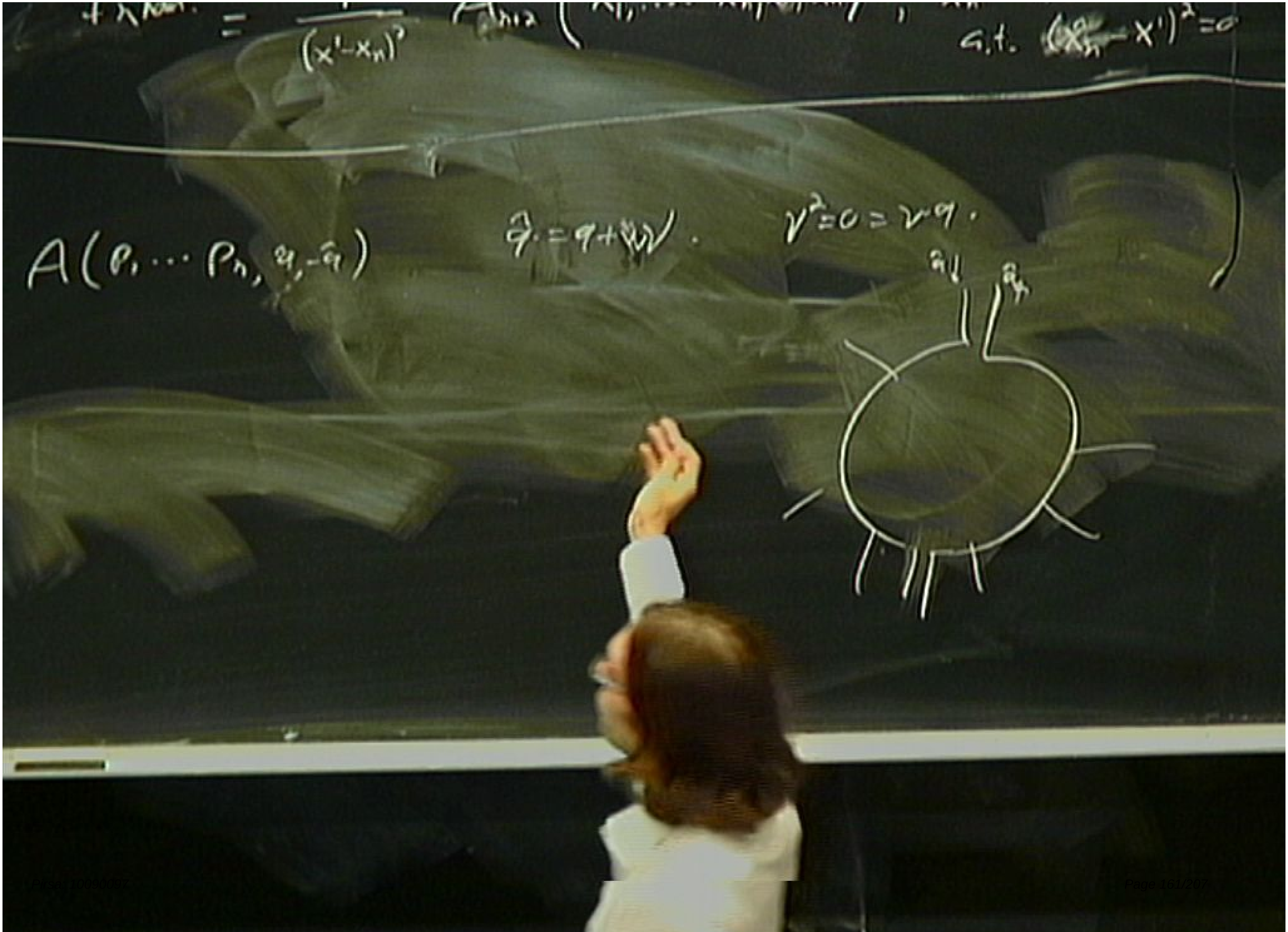

A(p1 … pn, q, -q)
q → q+2v.
v²=0=v·q.

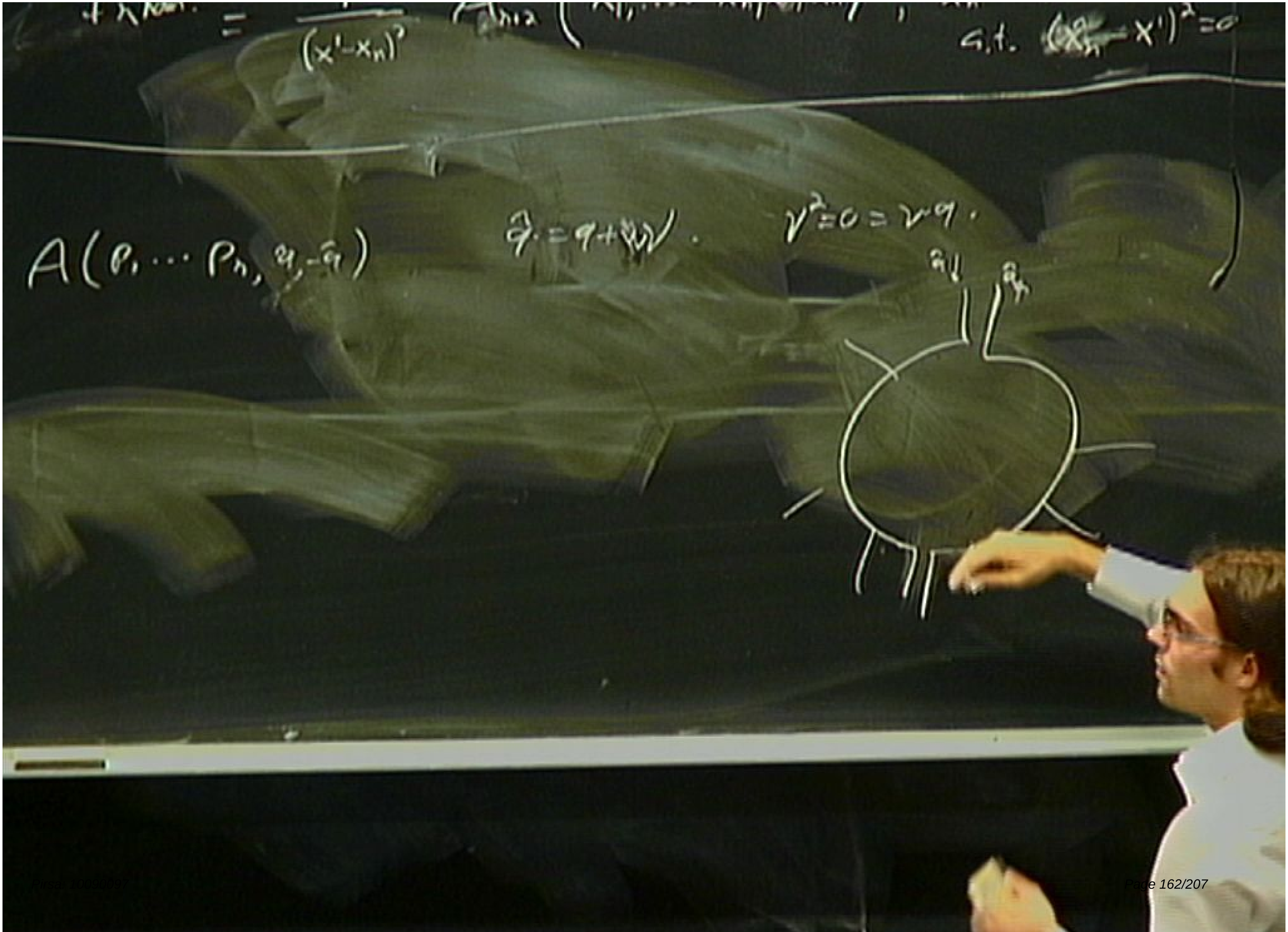

A(p₁ … pₙ, q, -q)
q → q + zv
v² = 0 = v·q

$= \frac{1}{(x'-x_n)^2} \, A_{n+2}\big(x_1, \ldots$ s.t. $(x_n - x')^2 = 0$

$A(p_1 \cdots p_n, q, -q)$ $\quad q \to q + z v$. $\quad v^2 = 0 = v \cdot q$.



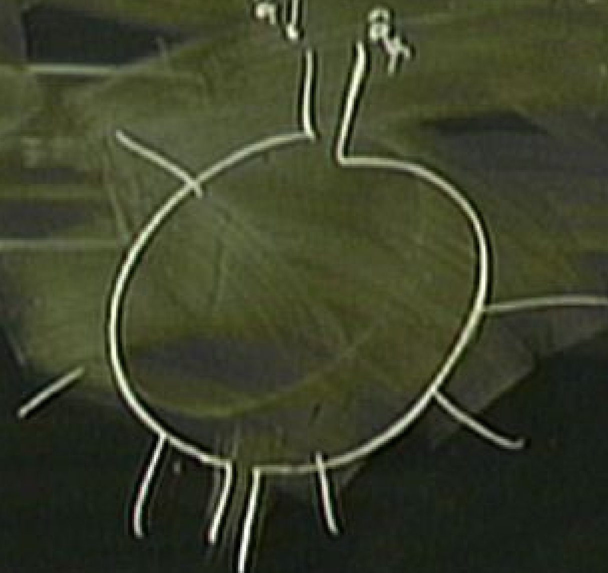

(x'-x_n)^2
s.t.
A(p_1 ⋯ p_n, q, -q)
ν^2 = 0 = ν·q.

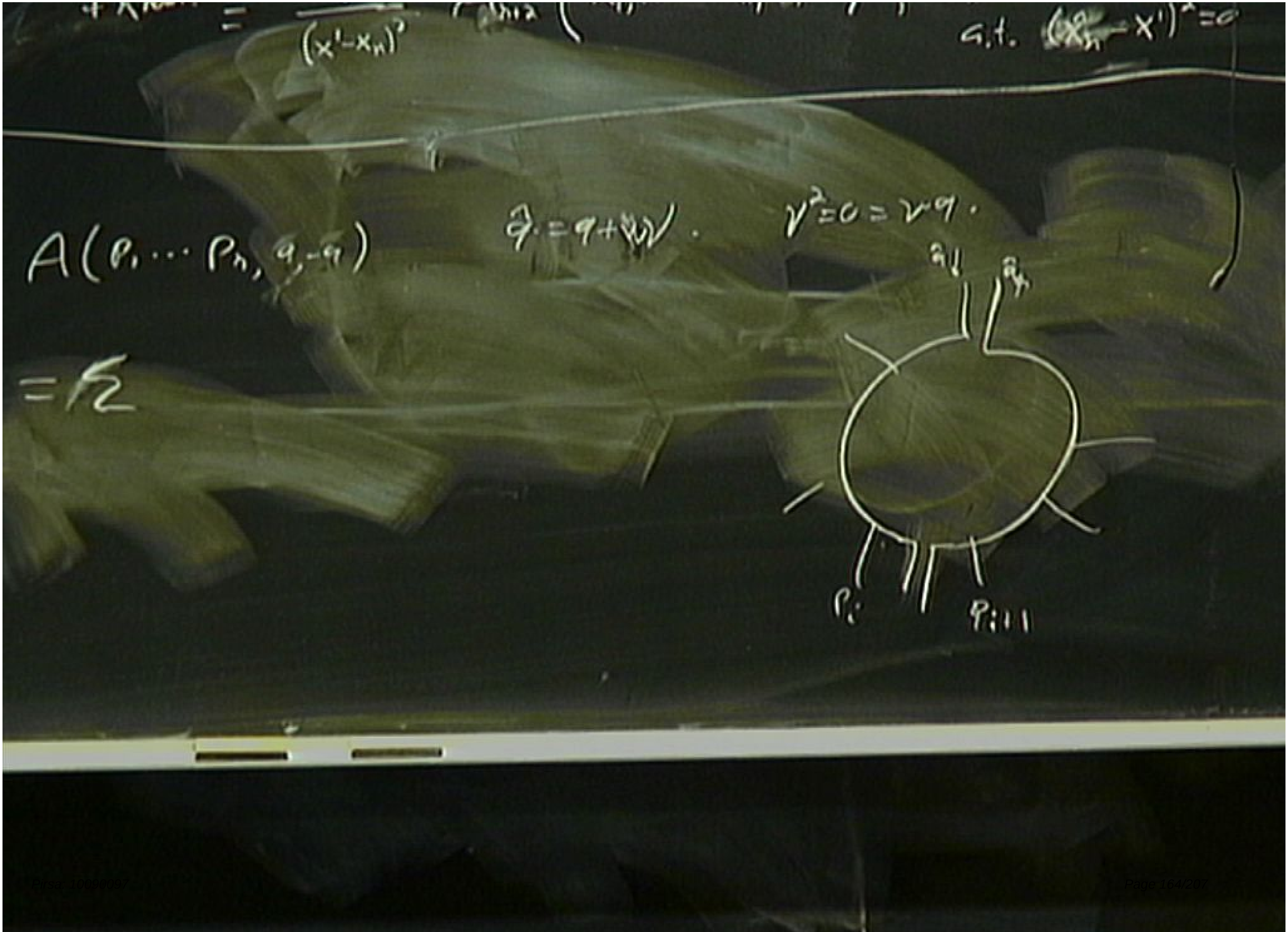


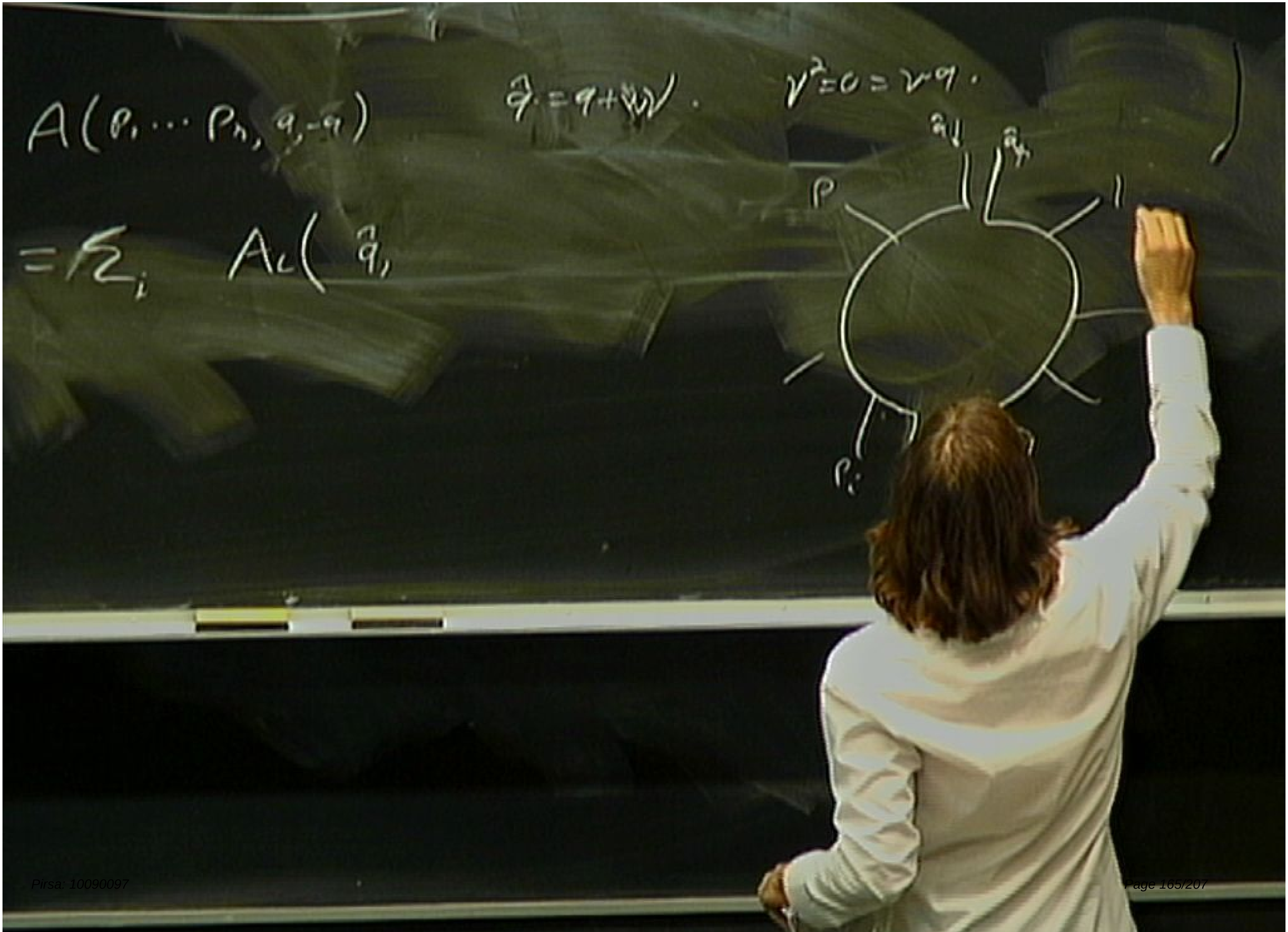
$(x'-x_n)^2$
s.t. $(x_n-x')^2=0$
$A(p_1 \cdots p_n, \hat{q}, -\hat{q})$
$\hat{q} = q + z\nu$
$\nu^2 = 0 = \nu \cdot q$

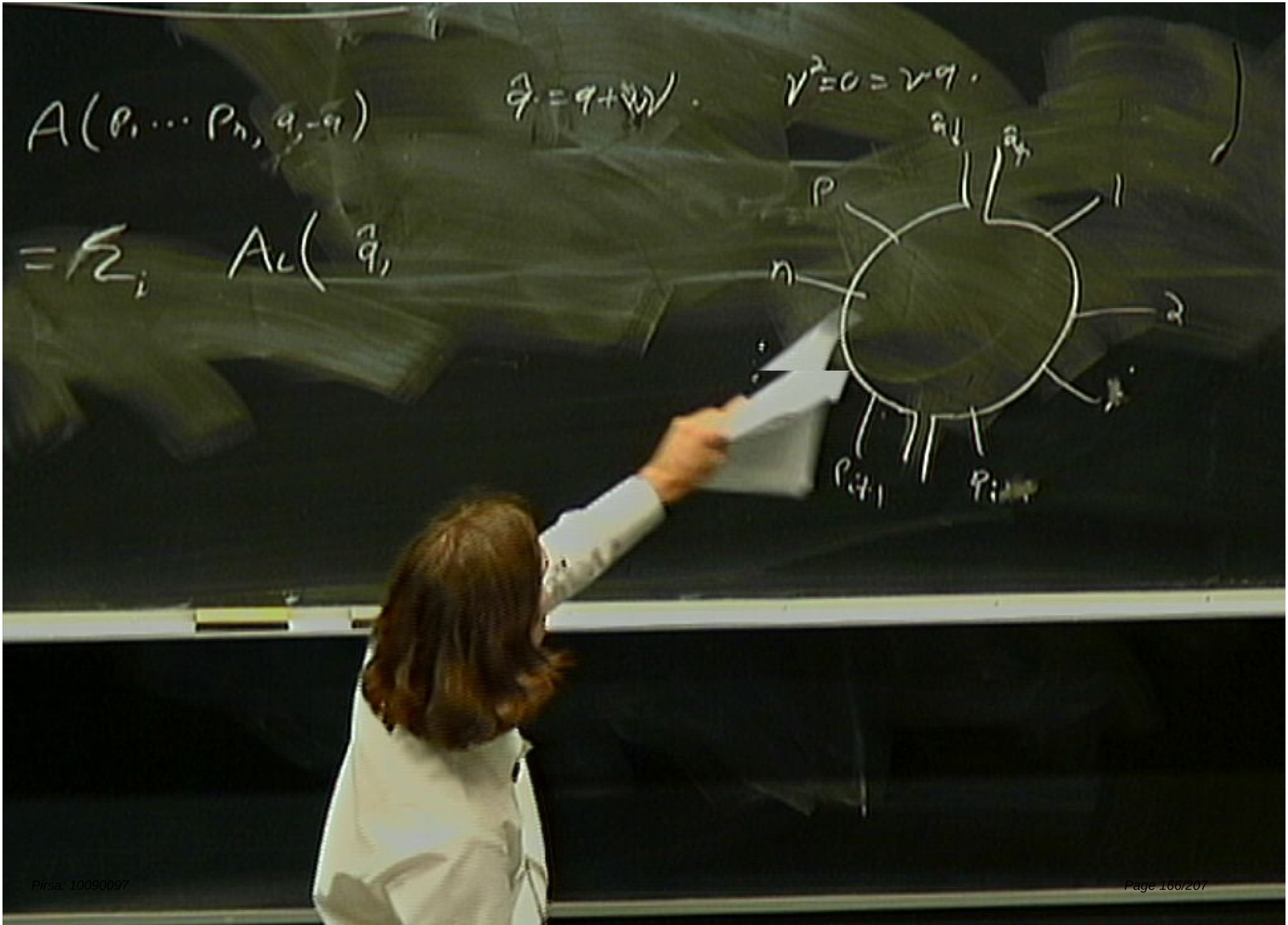
$$= \frac{1}{(x'-x_n)^2} \; A_{n+2}\big(\quad \text{s.t. } (x_n - x')^2 = 0$$

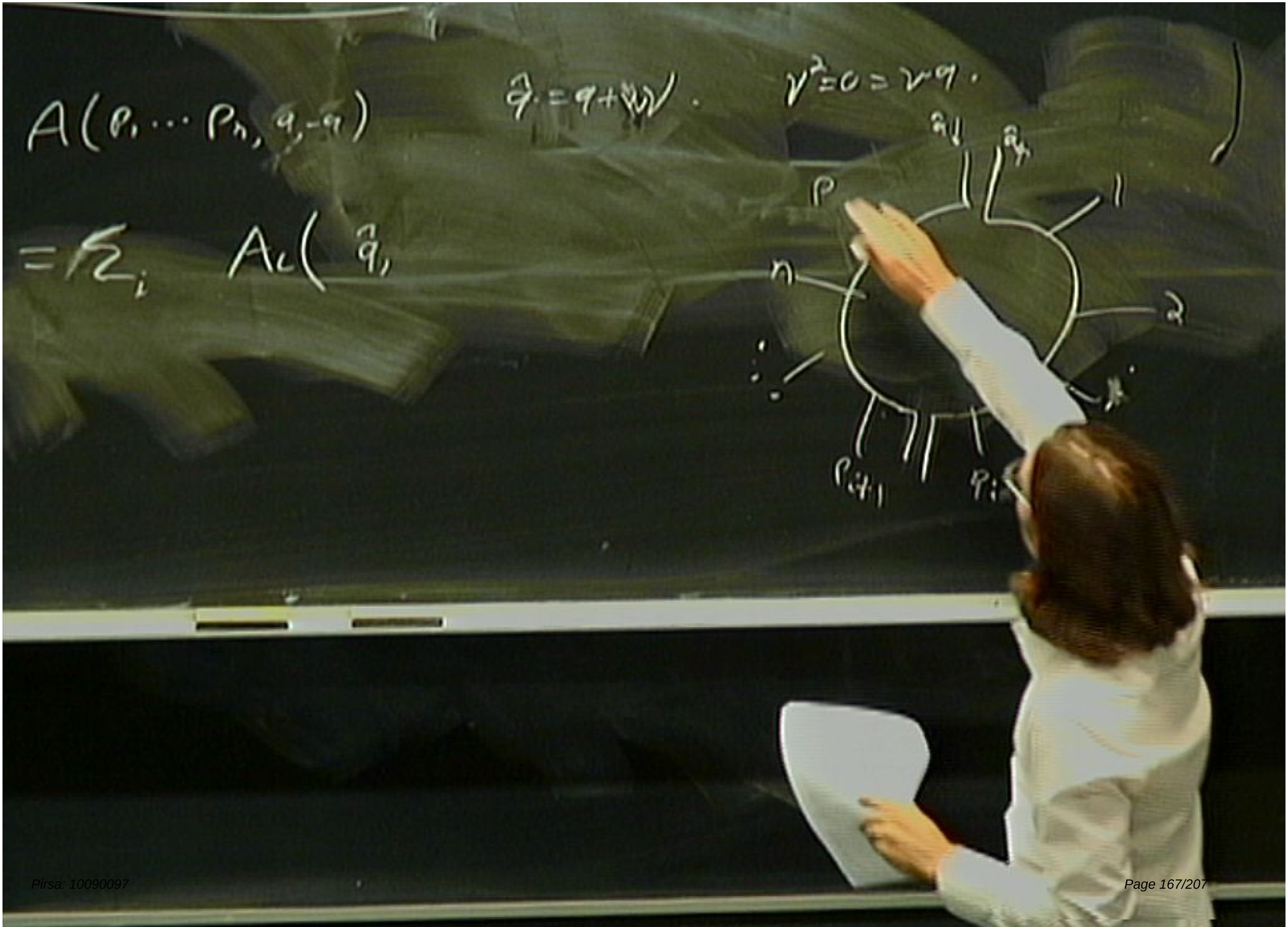
$$A(p_1 \cdots p_n, q, -\hat{q}) \qquad \hat{q} = q + z\nu . \qquad \nu^2 = 0 = \nu \cdot q .$$




$= \frac{}{(x'-x_n)^2}$
s.t. $(x_n - x')^2 = 0$
$A(p_1 \cdots p_n, q, -q)$
$\hat{q} = q + zv$.
$v^2 = 0 = v \cdot q$.
$= \int z$
p_i
p_{i+1}



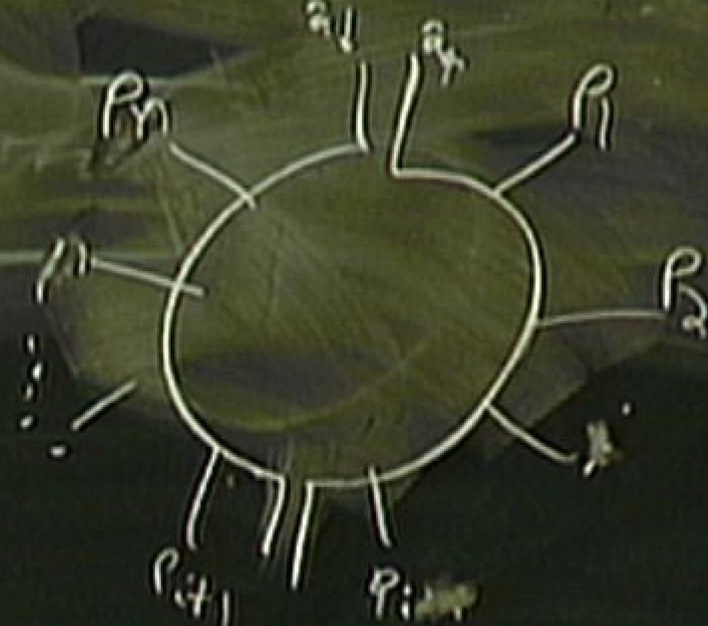


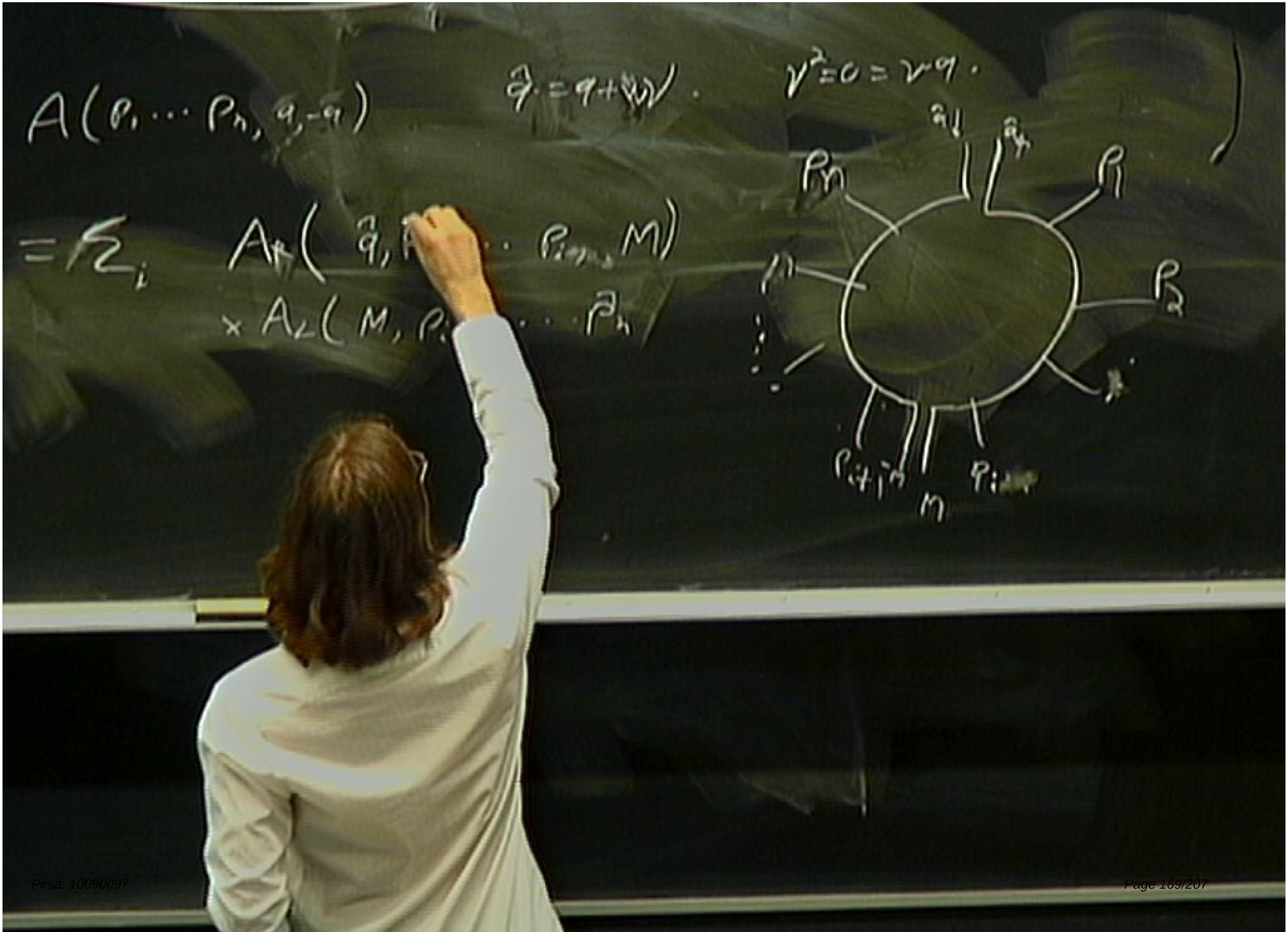

A(p1 ... pn, q, -q)
q̂1 = q + zv
v² = 0 = v·q
= Σi AL(q̂1
p
n
2

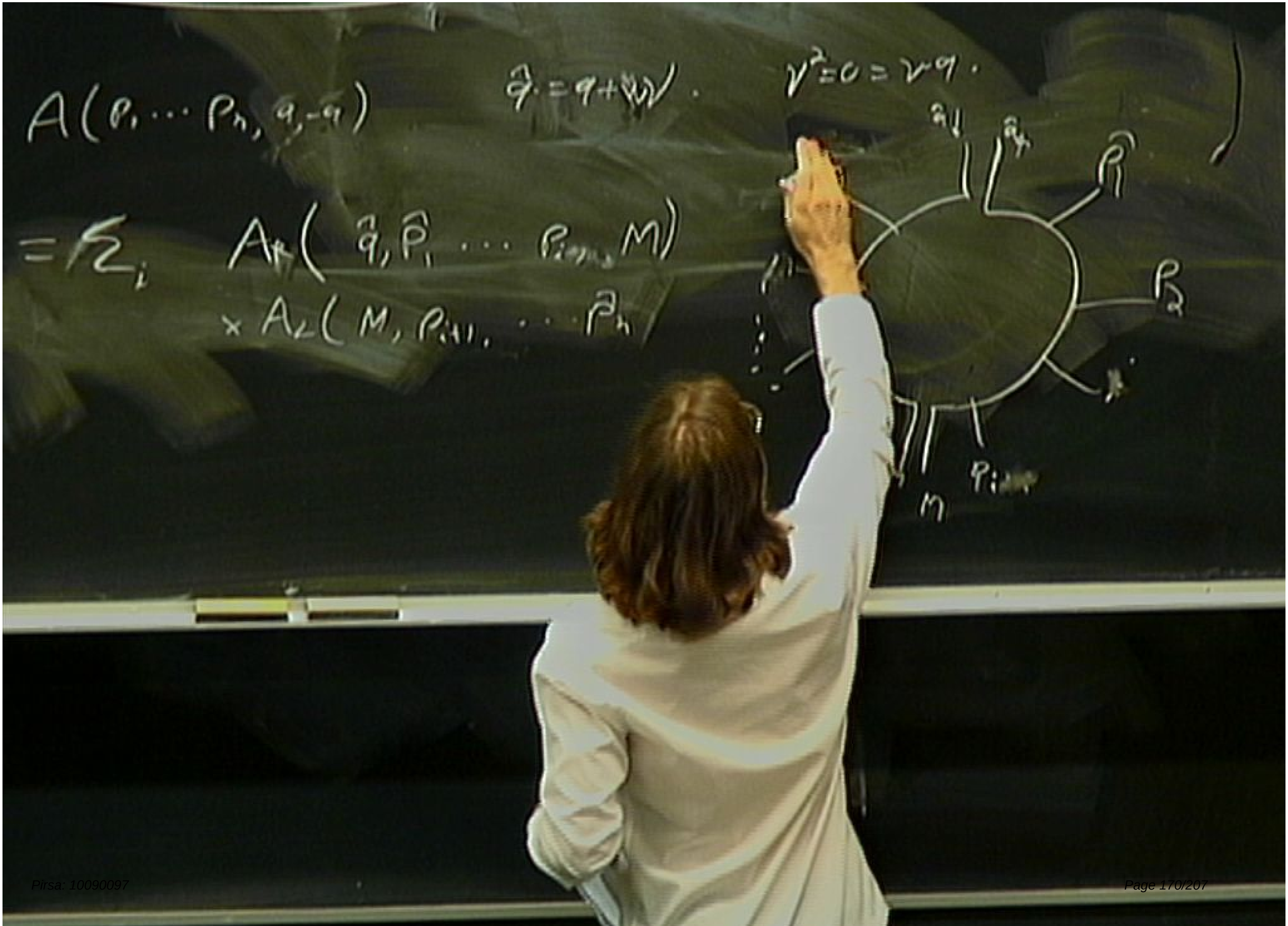
$$A(p_1 \cdots p_n, q, -q)$$

$$\hat{q}_i = q + z(i) v \,. \qquad v^2 = 0 = v \cdot q \,.$$

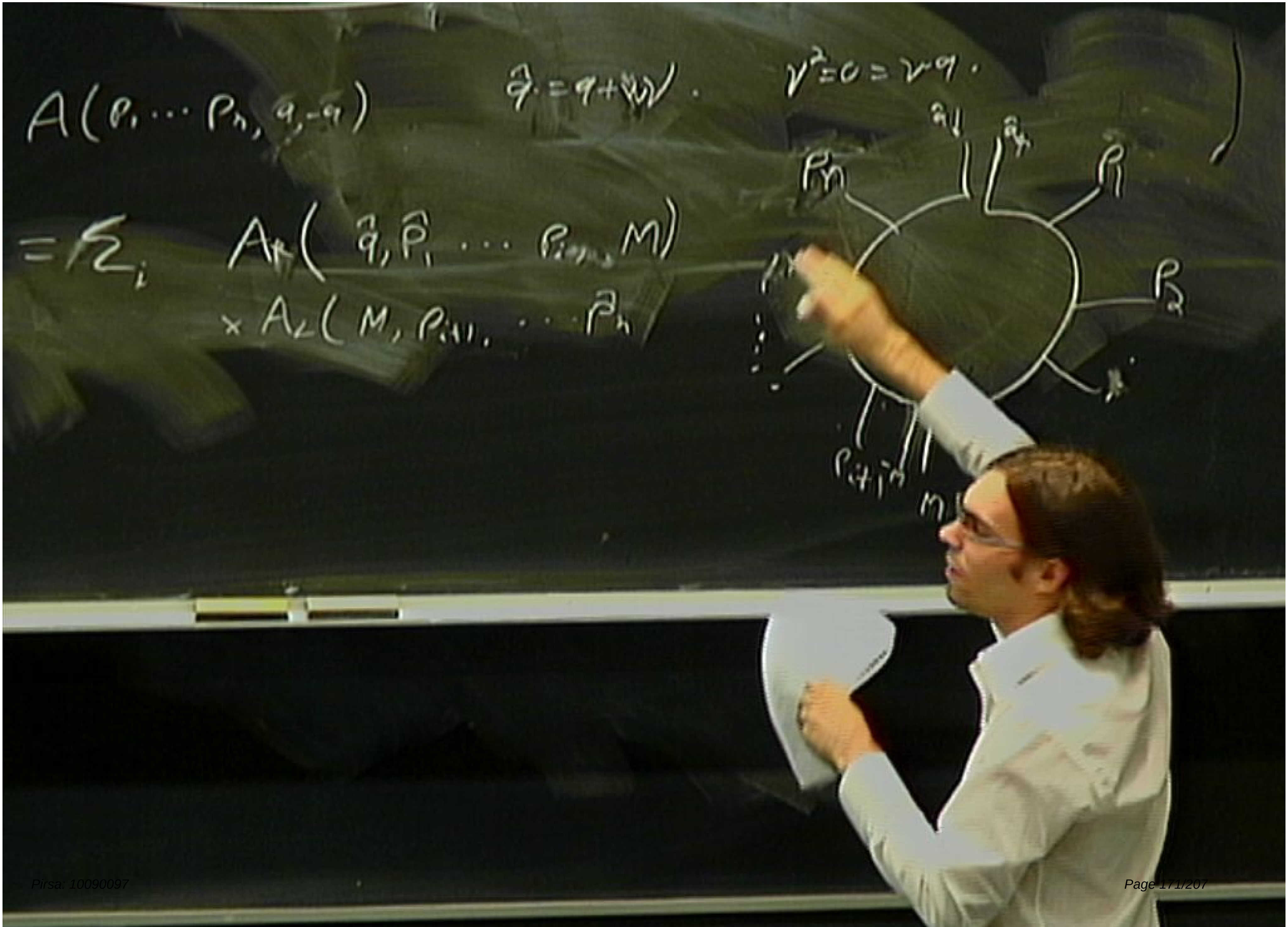
$$= \sum_i A_L(\hat{q}, \ldots$$





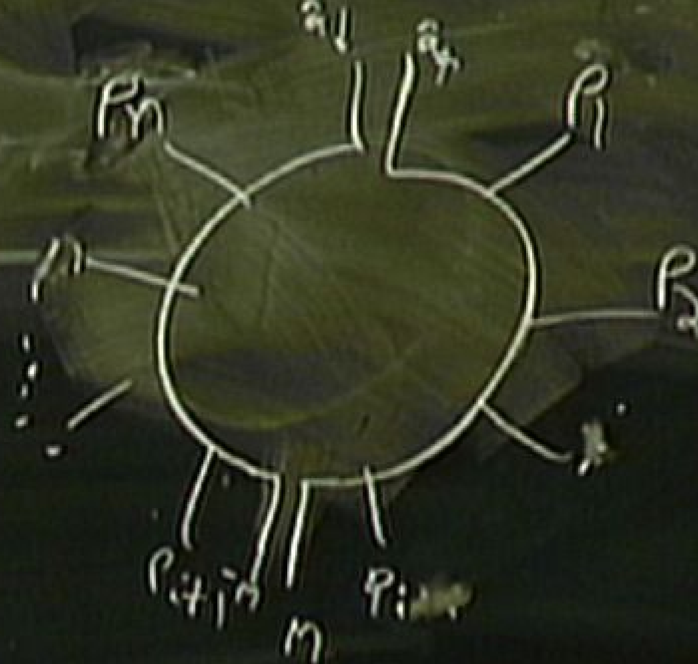


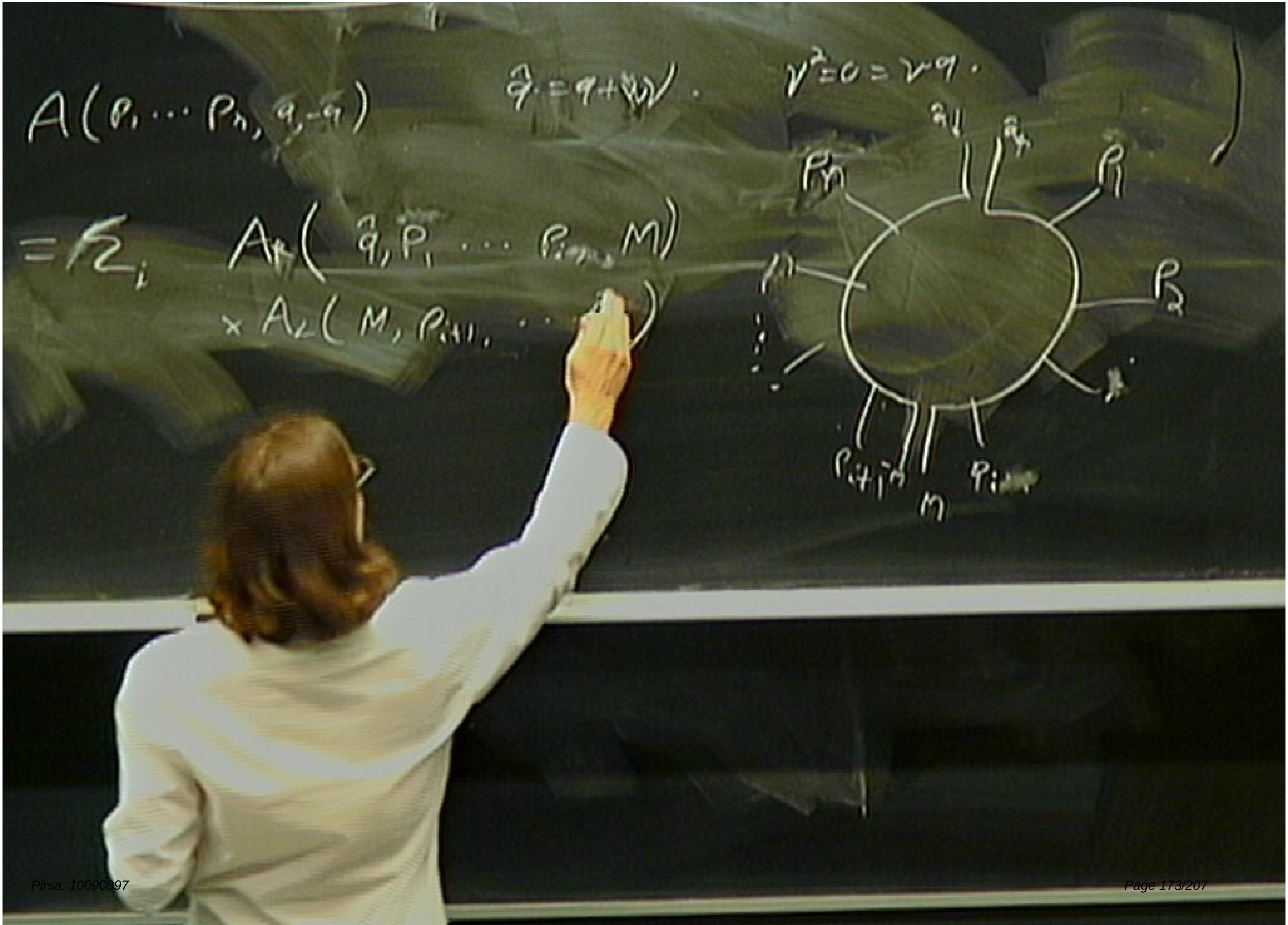
$A(p_1 \cdots p_n, q, -q)$
$\hat{q} = q + z\nu$
$\nu^2 = 0 = \nu \cdot q$
$= \sum_i A_R(\hat{q}, \hat{p}_1 \cdots p_{i+m}, M)$
$\times A_L(M, p_{i+1}, \cdots \hat{p}_n)$
$\hat{q}$
$\hat{p}_1$
p_2
m


A(p1 ... pn, q, -q)
= Σi AR(q̂, p̂1 ... pi,m, M)
× AL(M, pi+1, ... p̂n

$$A(p_1 \cdots p_n, q, -q) \qquad \hat{q} = q + z\nu \qquad \nu^2 = 0 = \nu \cdot q$$

$$= \sum_i A_R(\hat{q}, \hat{p}_1 \ldots p_{i-1}, M)$$
$$\times A_L(M, p_{i+1}, \ldots \hat{p}_n)$$

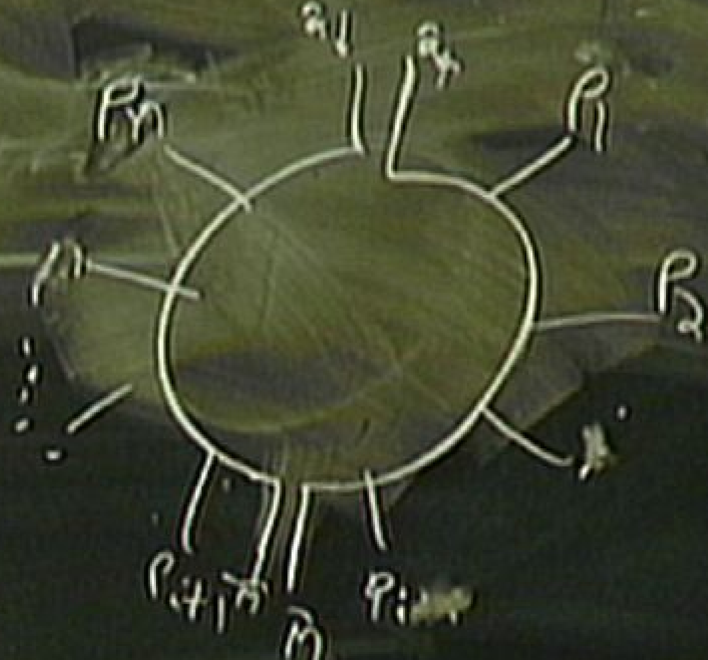


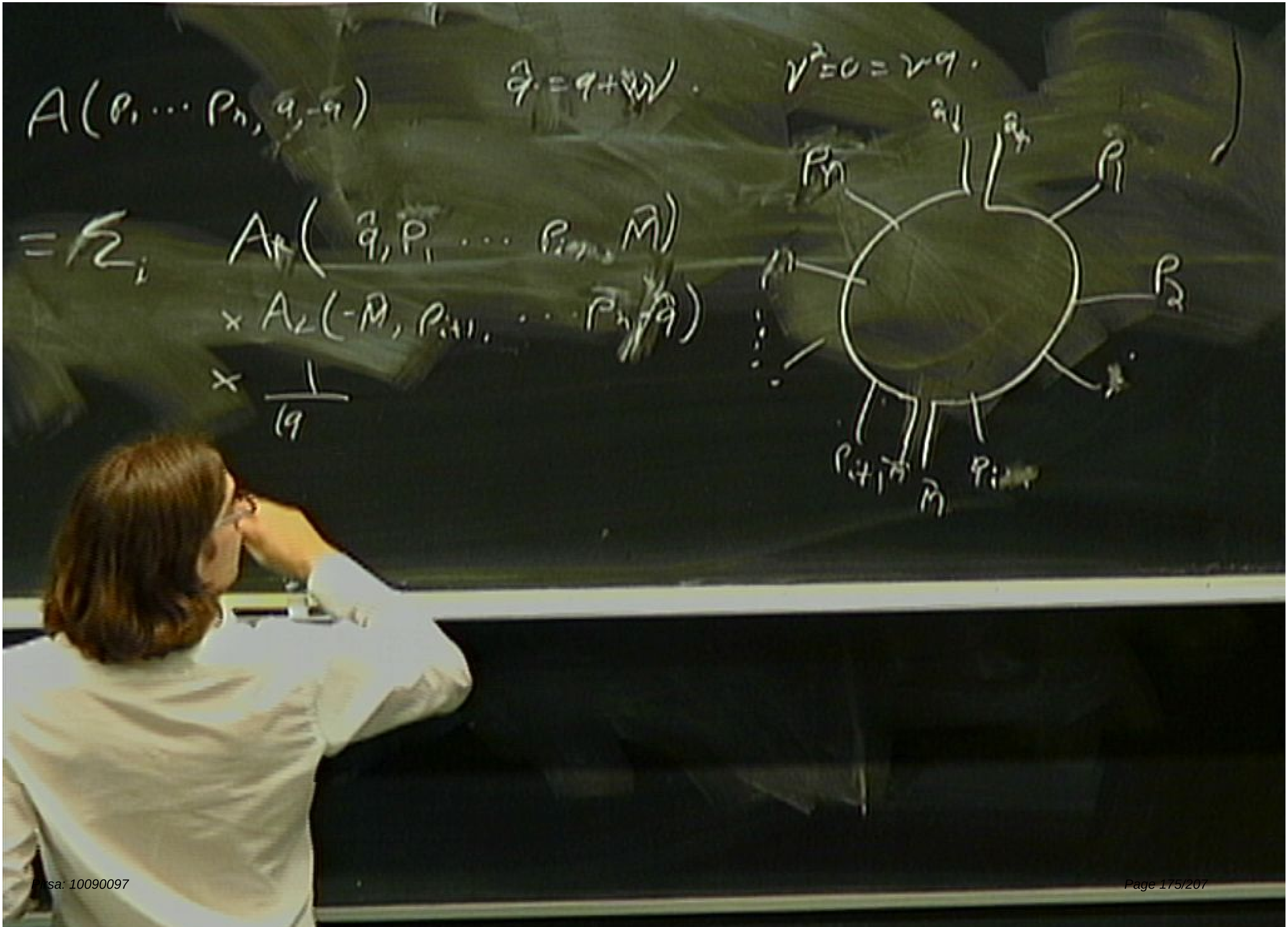

A(p1 ··· pn, q, -q)
× AL(M, pi+1, ...
M

$$A(p_1 \cdots p_n, q, -q)$$

$$\hat{q} = q + z v \qquad v^2 = 0 = v \cdot q$$

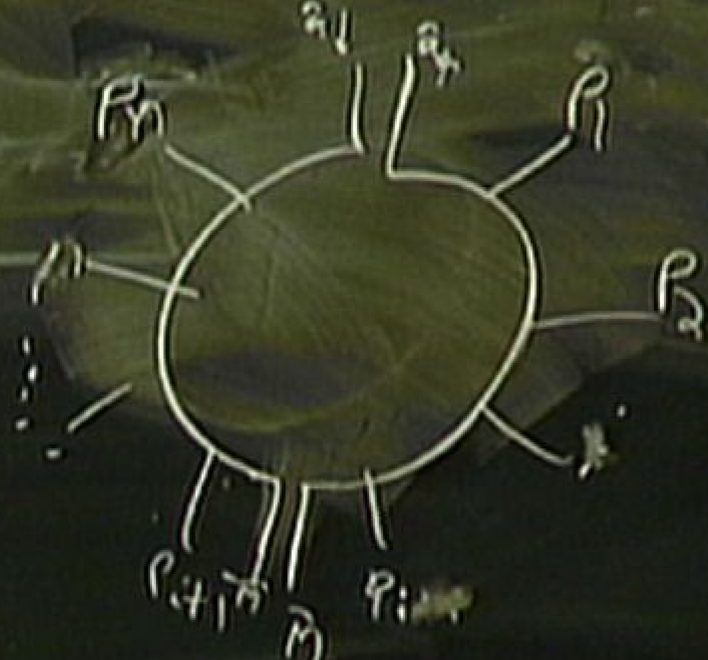
$$= \sum_i A_R(\hat{q}, \hat{p}_1 \cdots p_{i}, \hat{P}) \times A_L(-\hat{P}, p_{i+1}, \cdots \hat{p}_n, \hat{q})$$




A(p1 ... pn, q, -q)
= Σi AL(q̂, p1 ... pi, P̂)
× AR(-P̂, pi+1, ... pn, q̂)
× 1 / (q
v² = 0 = v·q

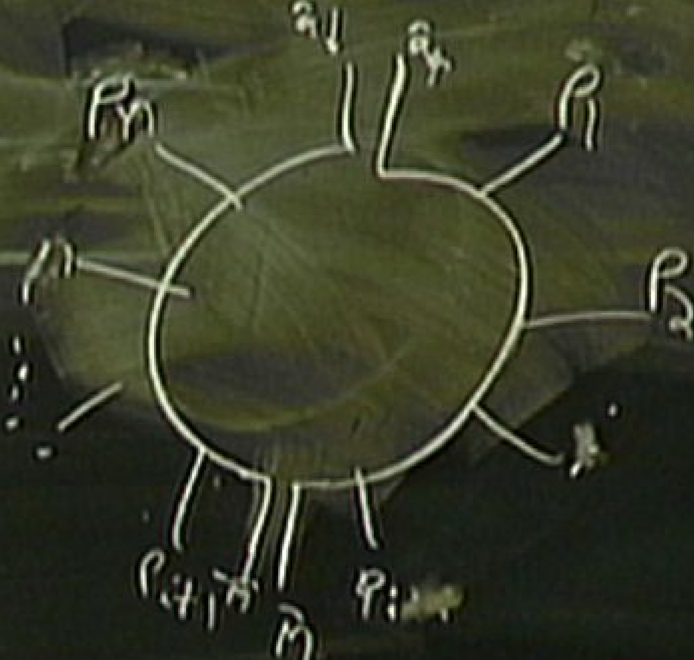
$$A(p_1 \cdots p_n, q, -q) \qquad \hat{q} = q + zv . \qquad v^2 = 0 = v \cdot q .$$

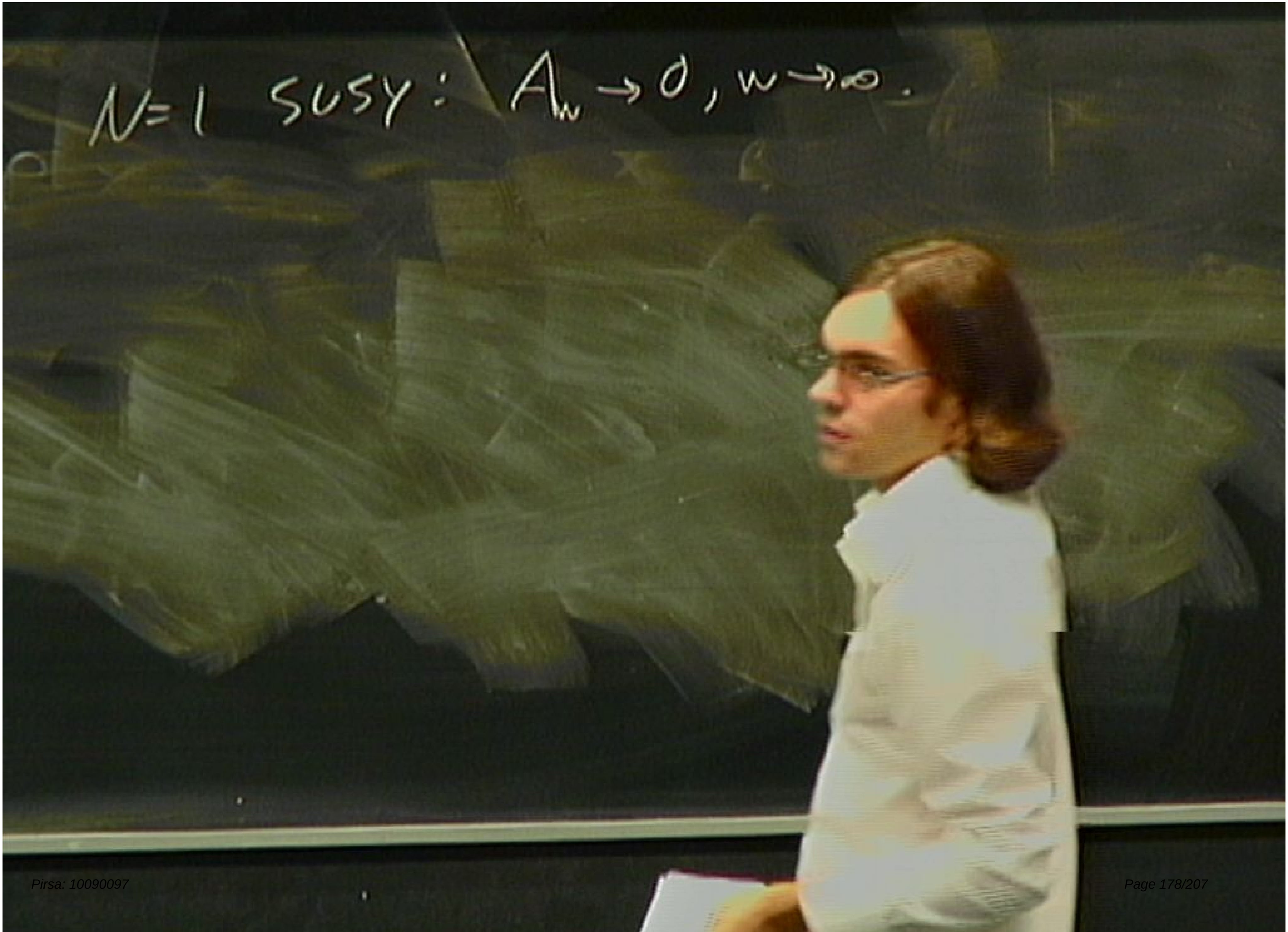
$$= \sum_i A_R(\hat{q}, p_1 \cdots p_i, P) \times A_L(-P, p_{i+1}, \cdots p_n, -\hat{q}) \times \frac{1}{(q + p_1 + \cdots + p_i)^2} .$$



$$A(p_1 \cdots p_n, q, -q) \qquad \hat{q} = q + z\nu. \qquad \nu^2 = 0 = \nu \cdot q.$$

$$= \sum_i (-) \; A_R(\hat{q}, p_1 \cdots p_i, \hat{M})$$
$$\times A_L(-\hat{M}, p_{i+1}, \cdots p_n, -\hat{q})$$
$$\times \frac{1}{(q + p_1 + \cdots + p_i)^2}.$$




N=1 SUSY: $A_w \to 0$, $w \to \infty$.

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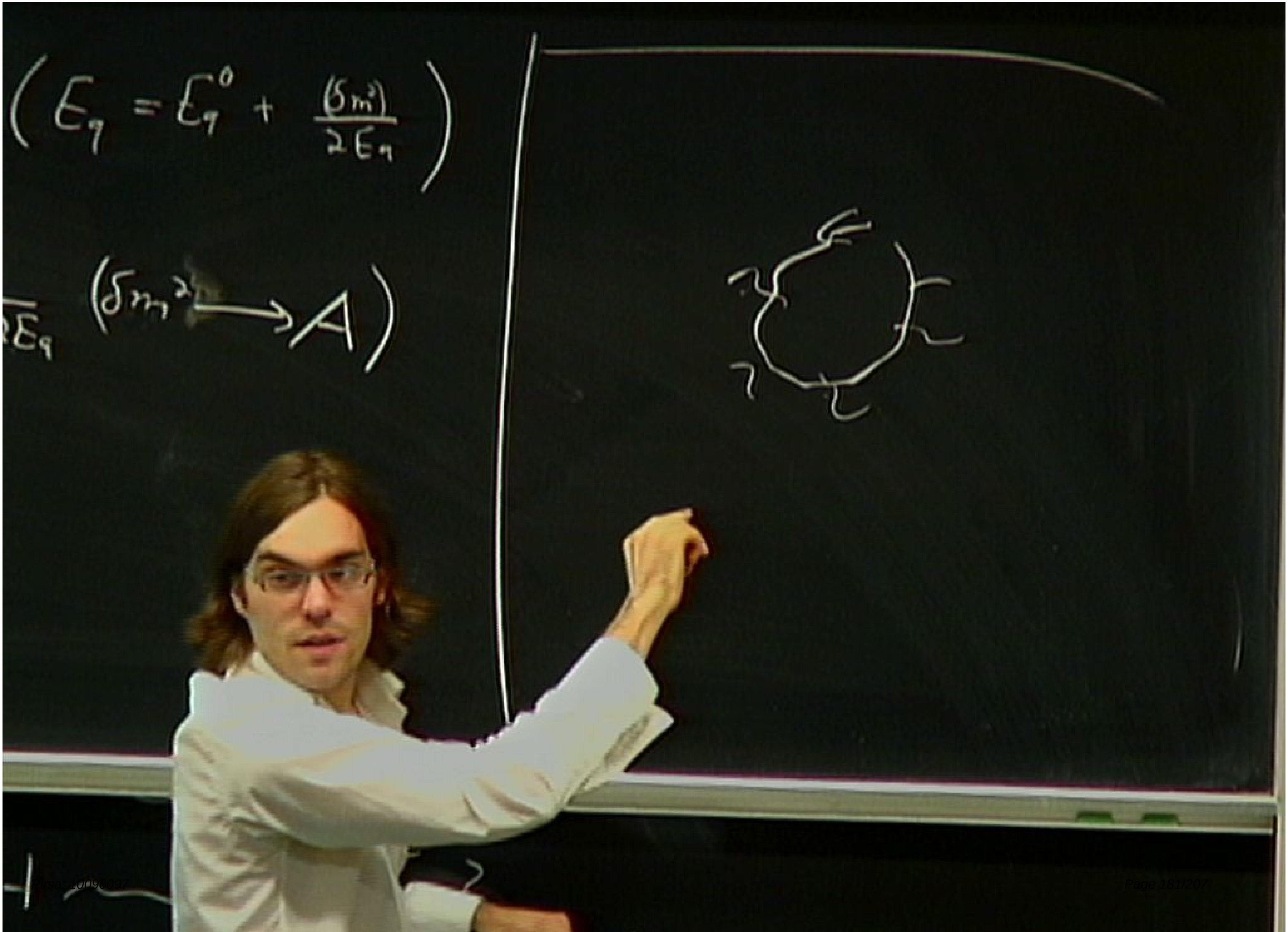
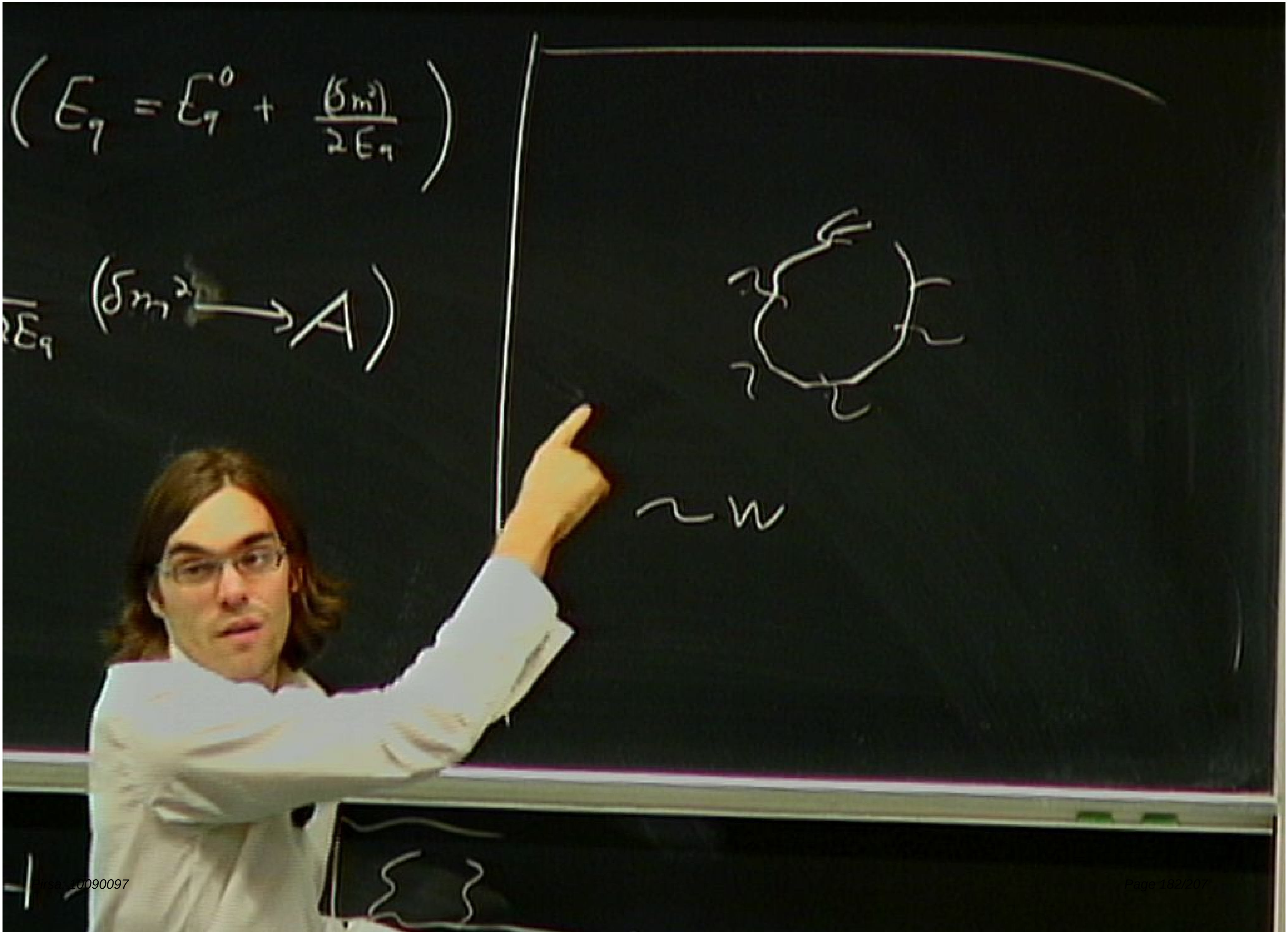
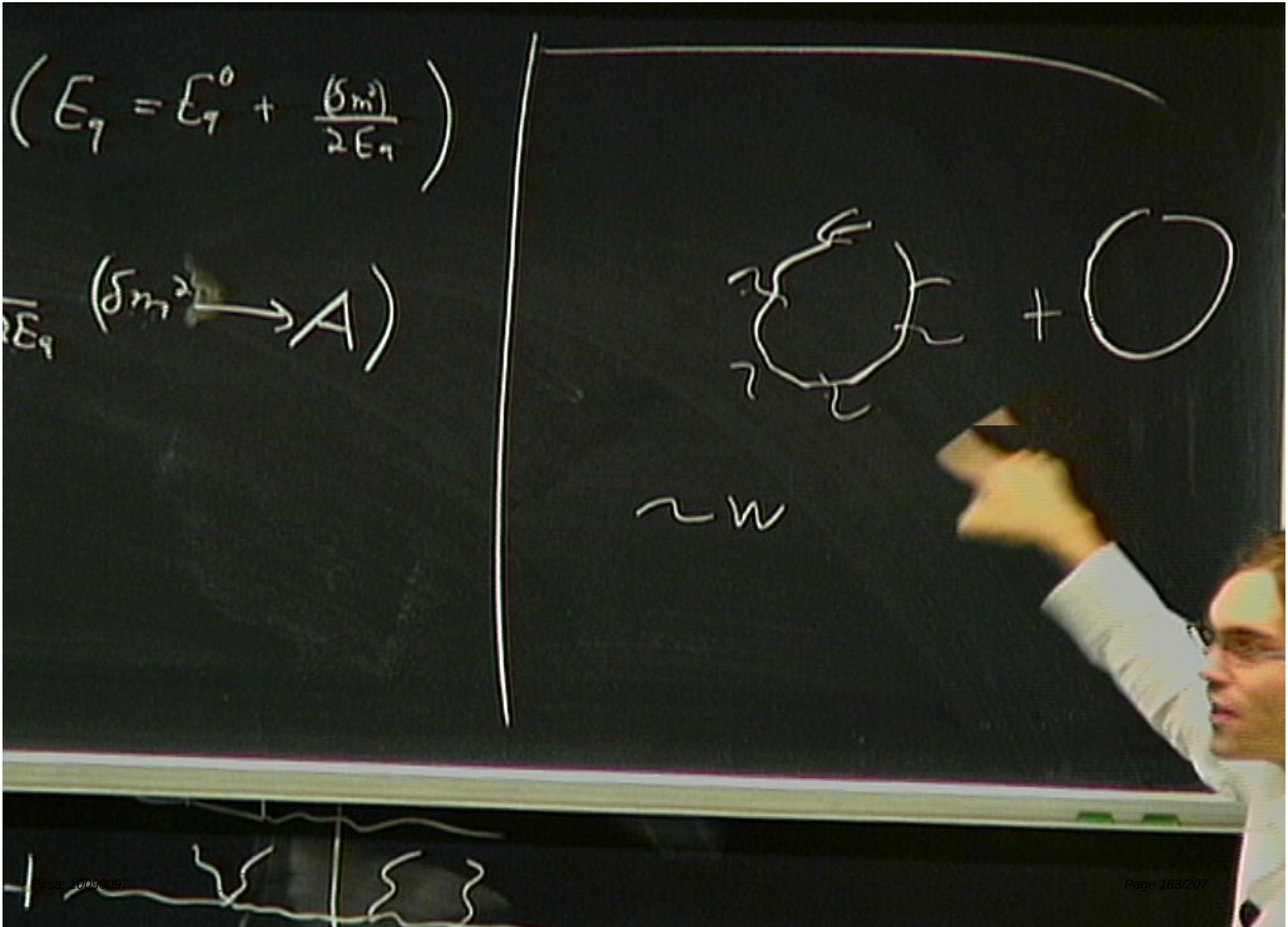
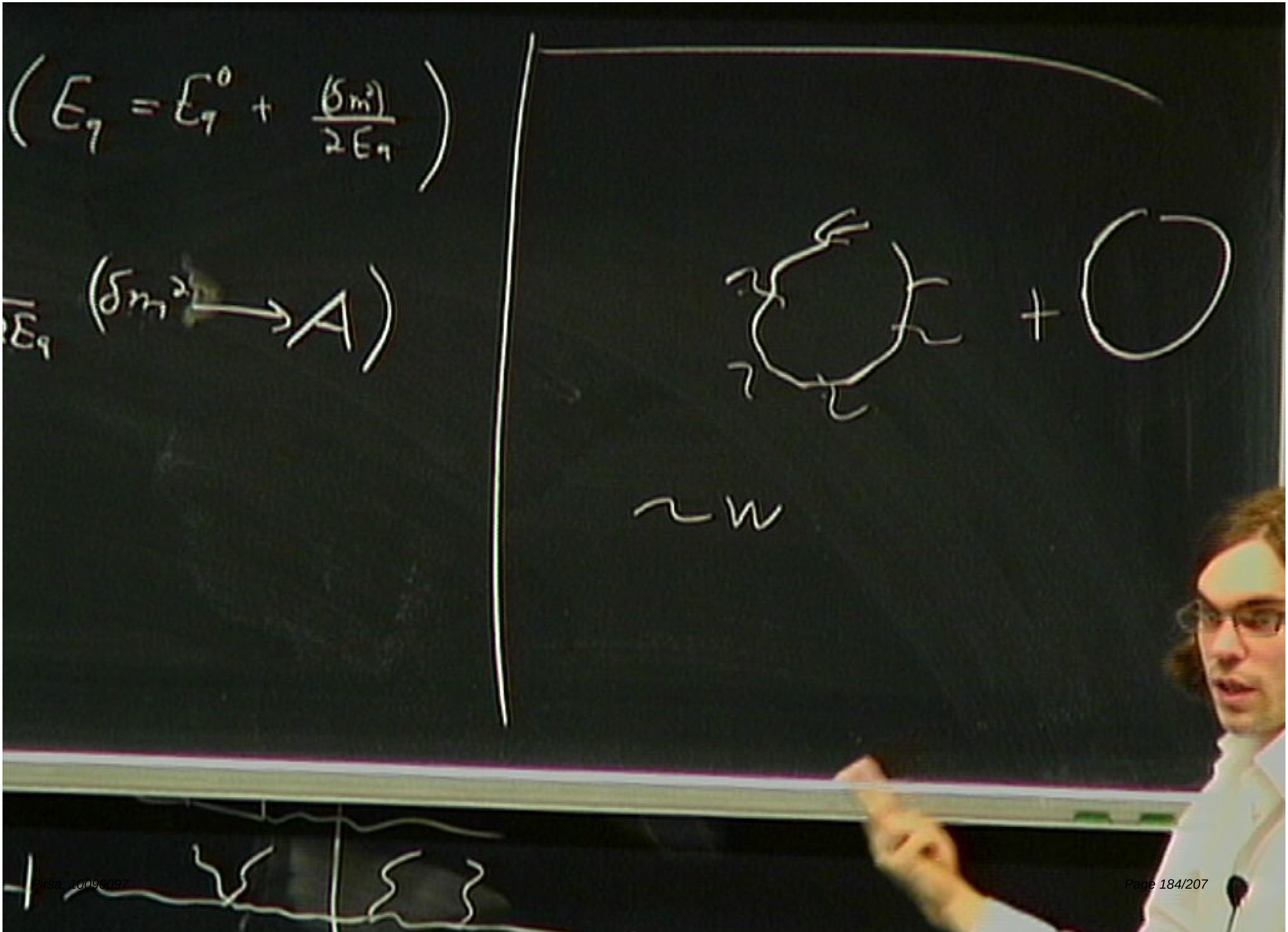
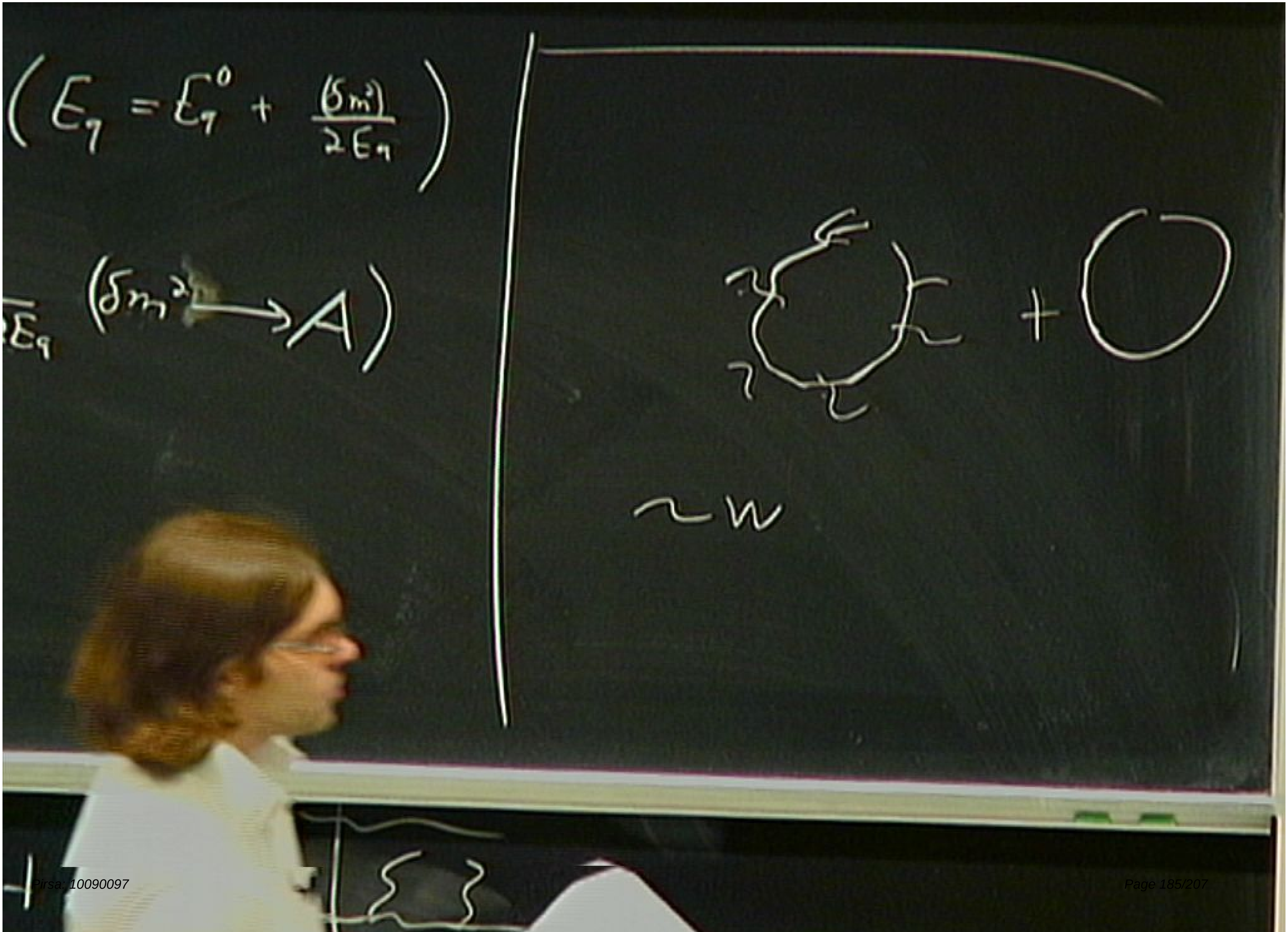
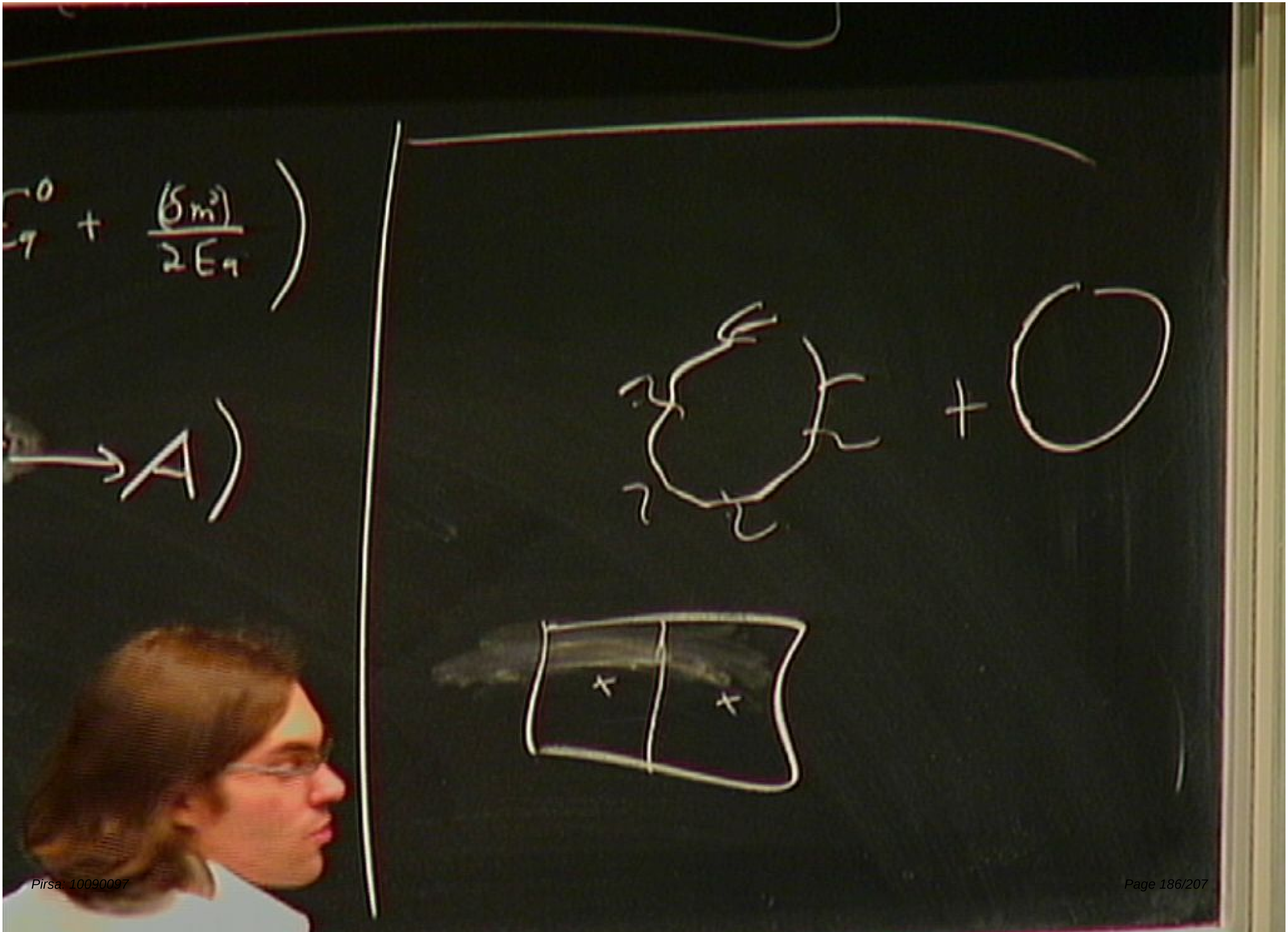

$\left(E_q = E_q^0 + \frac{(\delta m^2)}{2E_q} \right)$
$2E_q$ $\left(\delta m^2 \longrightarrow A \right)$


(E_q = E_q^0 + (δm²)/(2E_q))
(δm² → A)
~ W

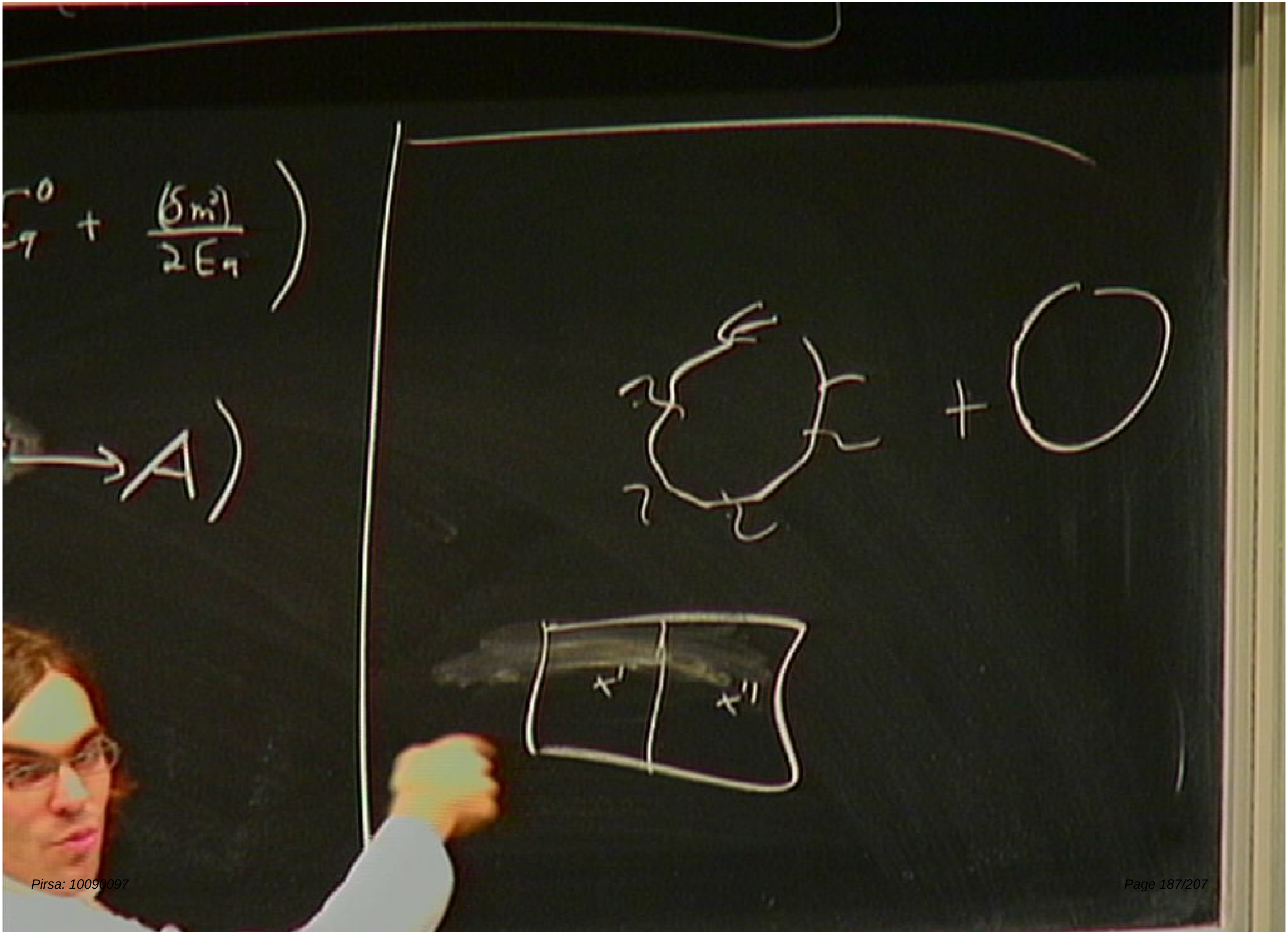

$\left(E_q = E_q^0 + \frac{(\delta m^2)}{2E_q} \right)$
$(\delta m^2 \rightarrow A)$
$\sim w$

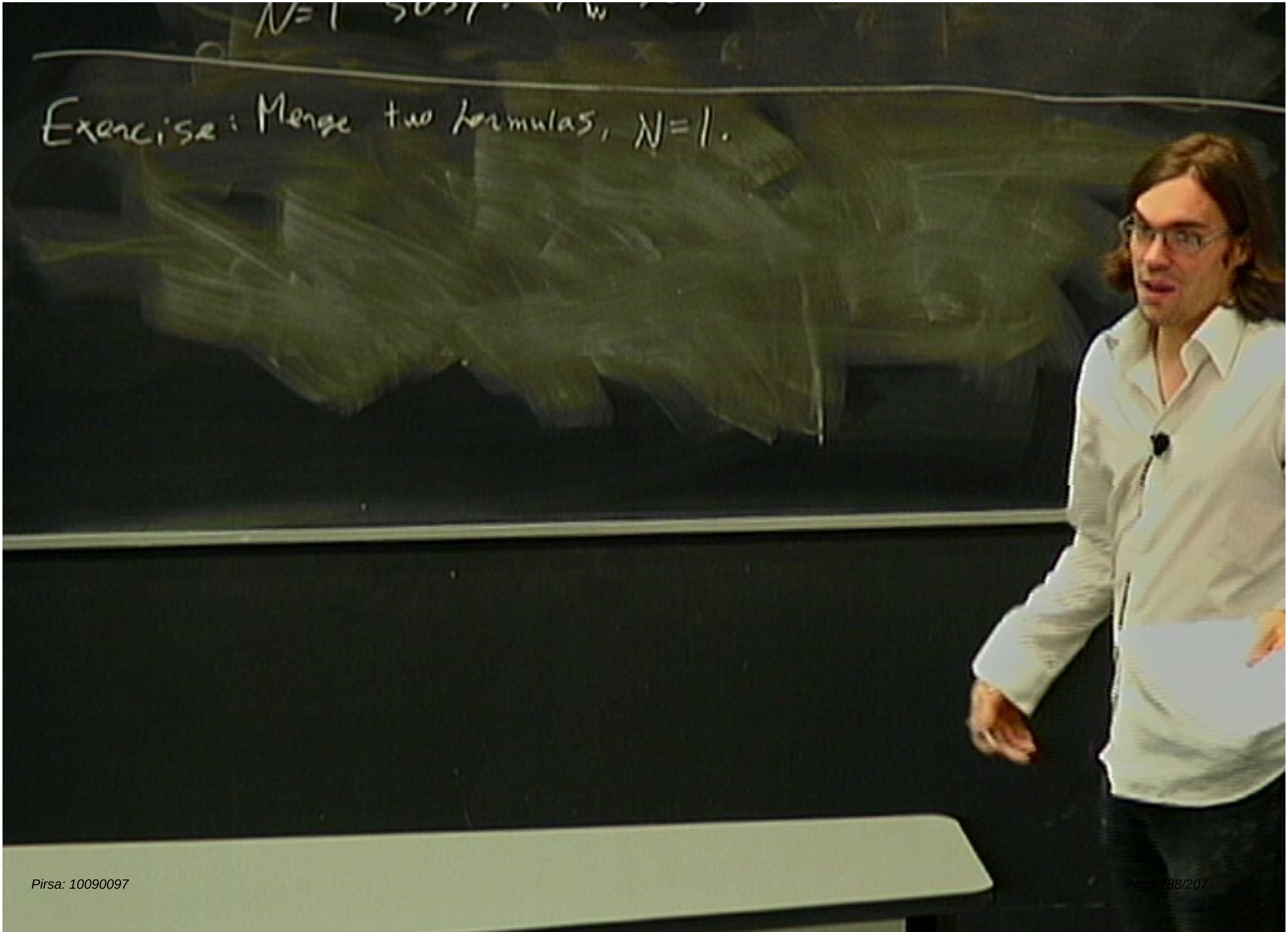

$(E_q = E_q^0 + \frac{(\delta m^2)}{2E_q})$
$(\delta m^2 \to A)$
$\sim w$

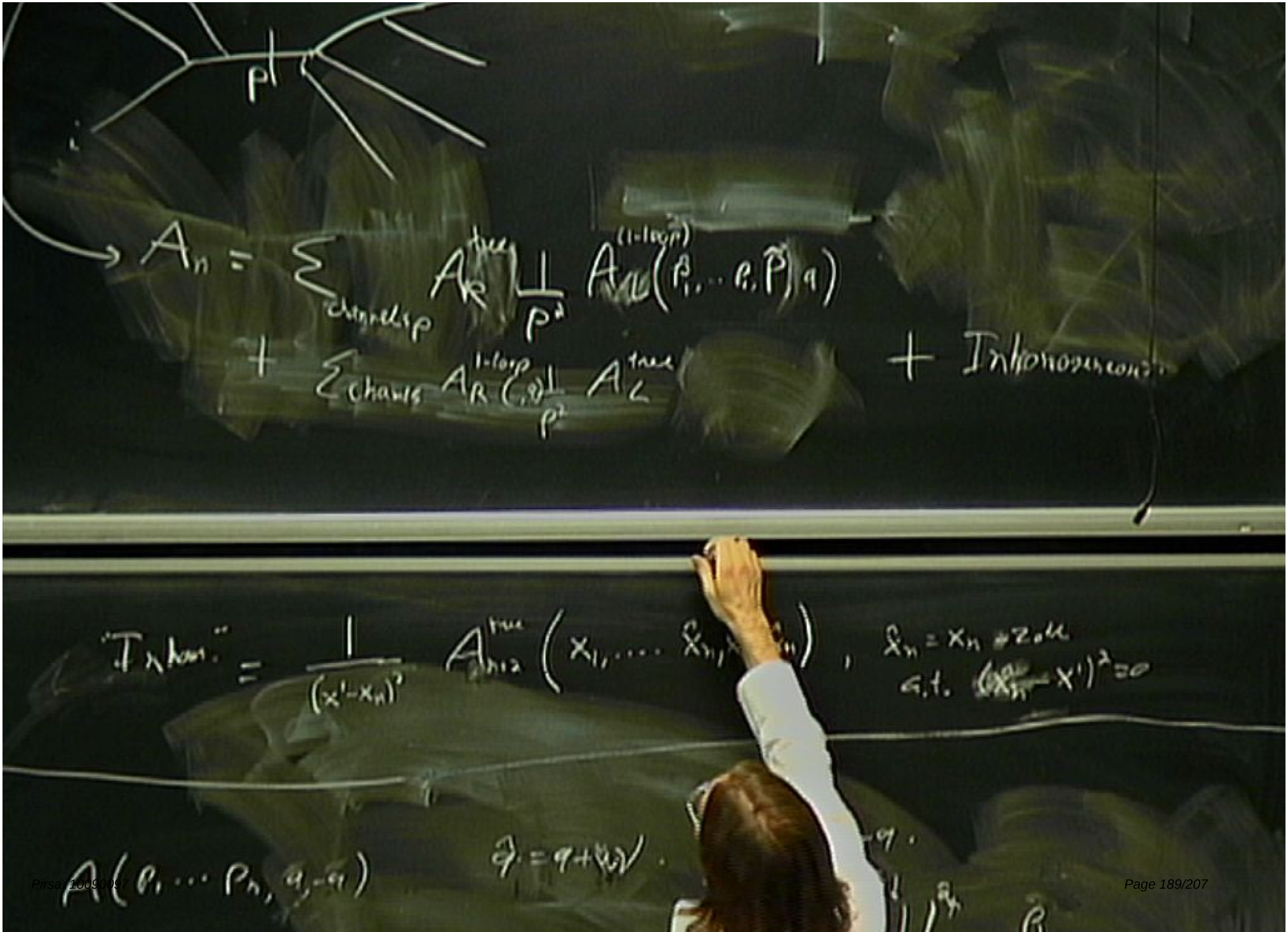

$\left(E_q = E_q^0 + \frac{(\delta m^2)}{2E_q} \right)$
$(\delta m^2 \longrightarrow A)$
$\sim W$

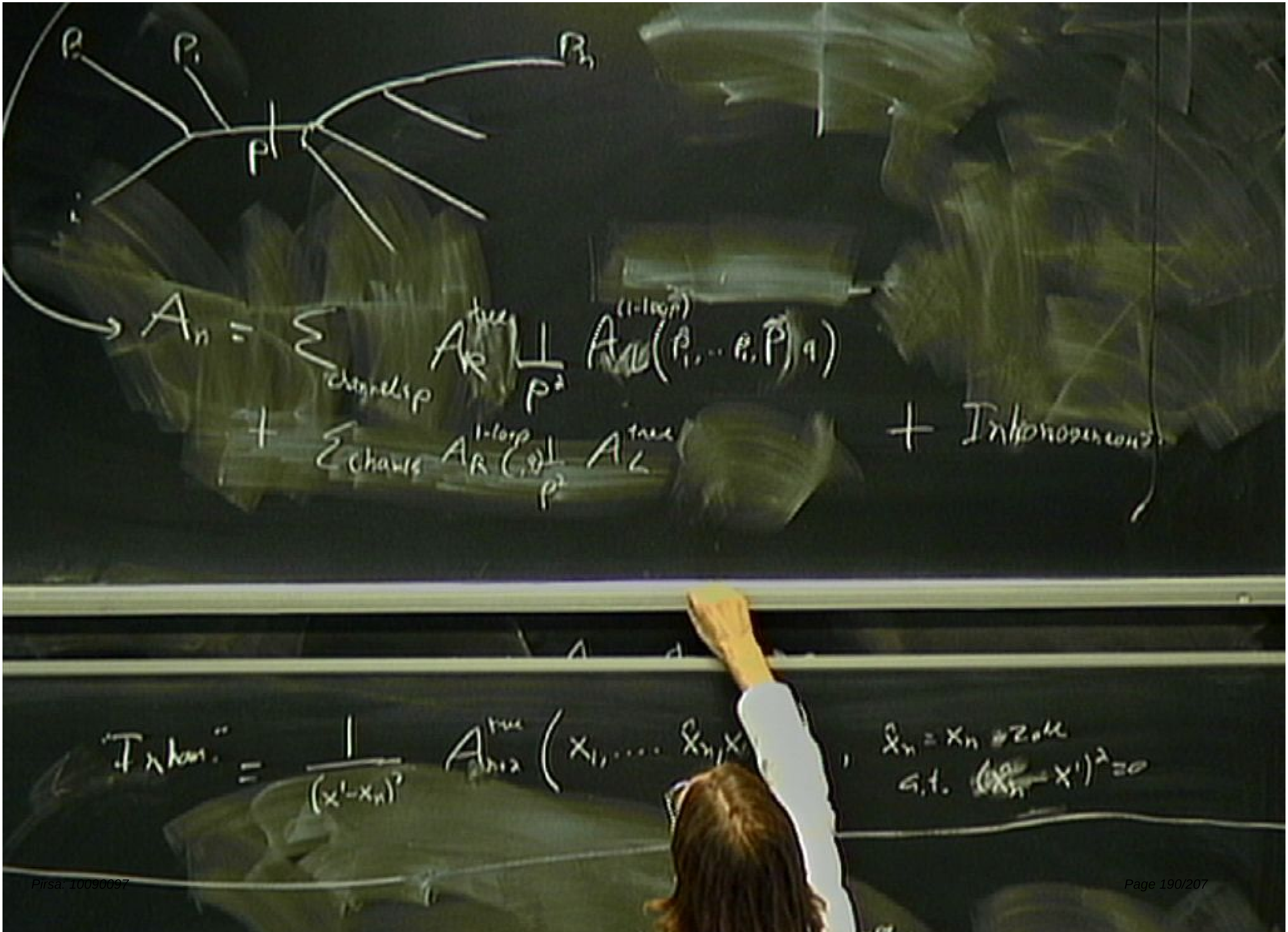


+ (δm²)/2E
→A)


$+ \frac{(\delta m^2)}{2E_q}$
$\to A)$
x'
x''


Exercise: Merge two formulas, N=1.


p
A_n = Σ_channels p A_R^tree 1/p² A_L^(1-loop)
+ Σ_channels A_R^1-loop 1/p² A_L^tree
"Inhom." = 1/(x′−x_n)² A_n+2



$A_n = \sum_{\text{channels}} A_R^{\text{tree}} \frac{1}{P^2} A_L^{(1\text{-loop})}(P_1, \ldots P, \hat{P}, q)$
$+ \sum_{\text{channels}} A_R^{1\text{-loop}} \frac{1}{P^2} A_L^{\text{tree}}$
$+$ Inhomogeneous
"Inhom." $= \frac{1}{(x' - x_n)^2} A_{n+2}^{\text{tree}}$

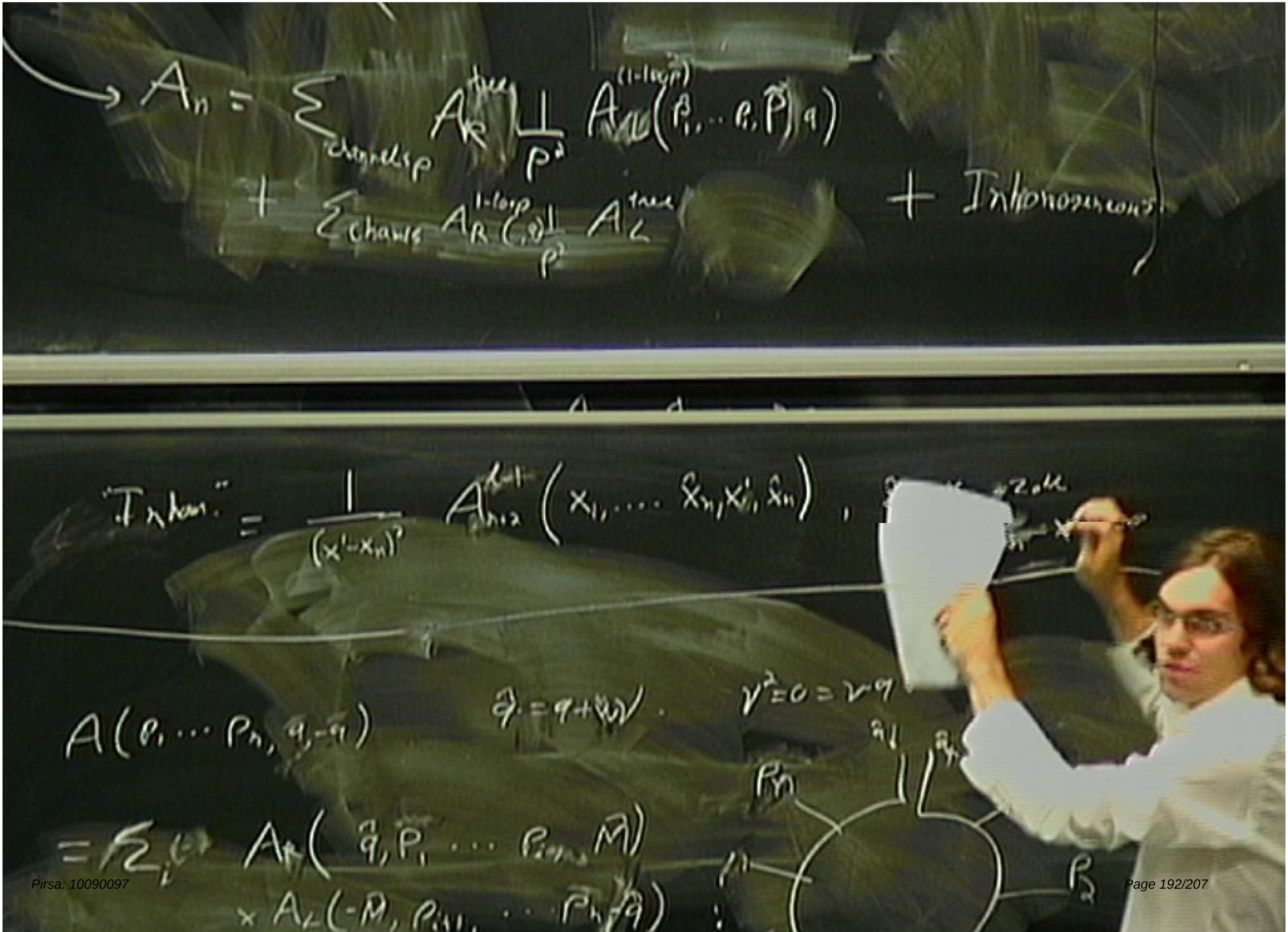
$$A_n = \sum_{\text{cyclic}, P} A_R^{\text{tree}} \frac{1}{P^2} A_L^{(1\text{-loop})}(P_1 \ldots P, P | q)$$

$$+ \sum_{\text{choices}} A_R^{1\text{-loop}}(\,, q) \frac{1}{P^2} A_L^{\text{tree}} \qquad + \text{Inhomogeneous}$$

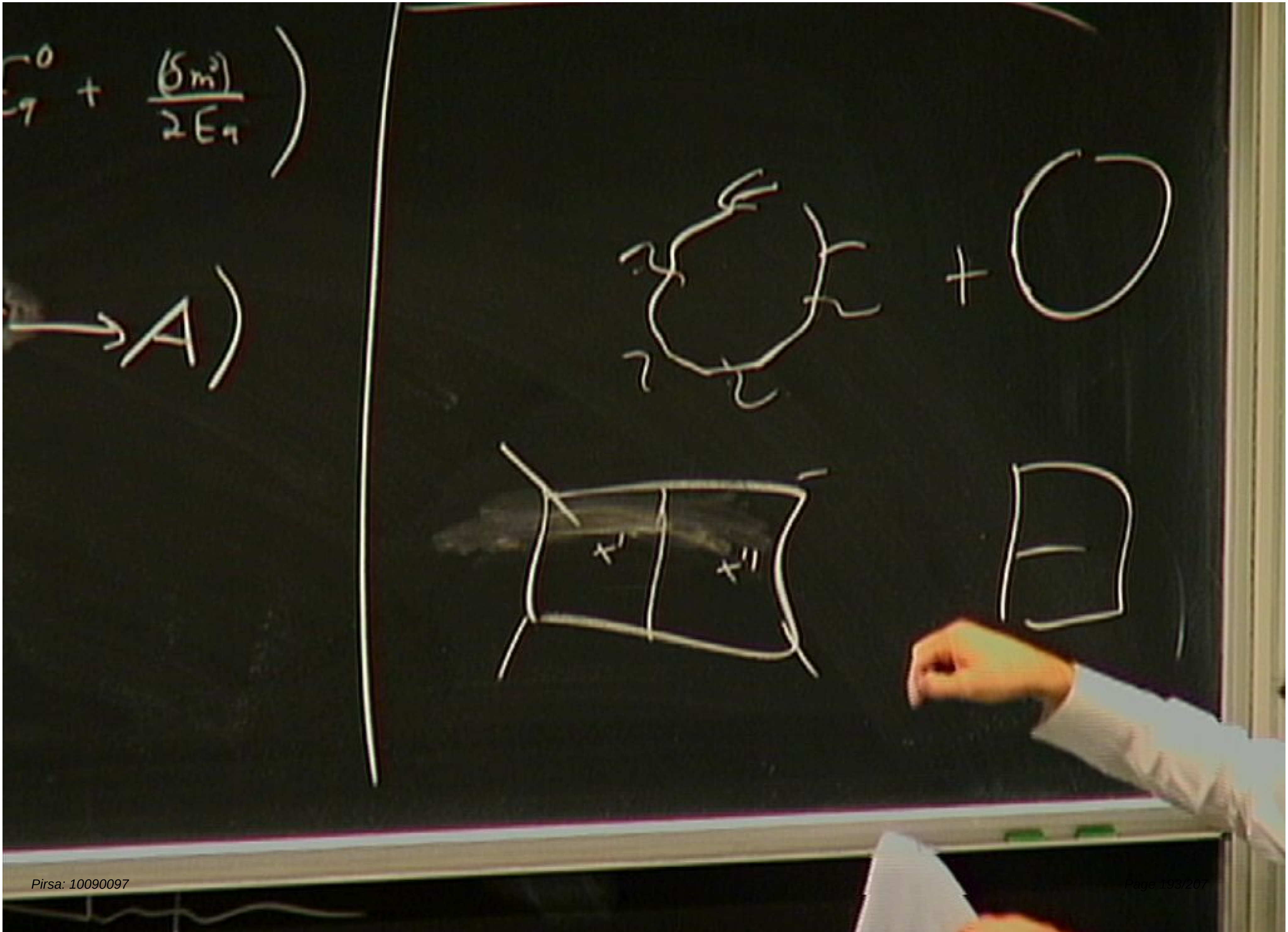
$$\text{Inhom.} = \frac{1}{(x' - x_n)^2} A_{n+2}^{L-1}\left(x_1, \ldots \hat{x}_n, x', \hat{x}_n\right), \quad \hat{x}_n = x_n + z\,\mu \quad \text{s.t. } (\hat{x}_n - x')^2 = 0$$

$$A(P_1 \cdots P_n, q, -q) \qquad \hat{q} = q + z\nu, \qquad \nu^2 = 0 = \nu \cdot q$$

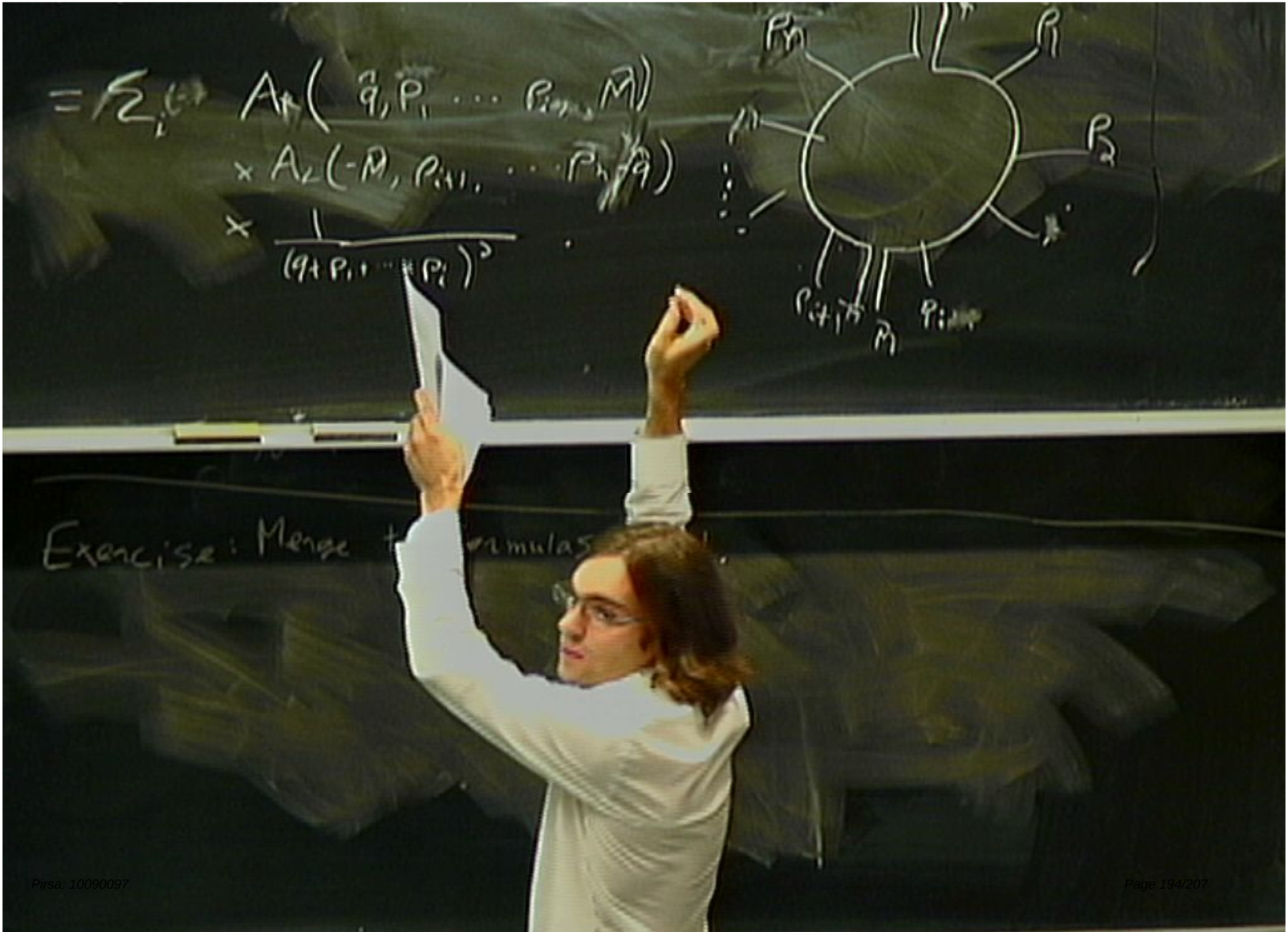
$$= \sum_i (\ldots) \, A_n(\hat{q}, P_i \ldots P_{i-1}, \hat{M})$$

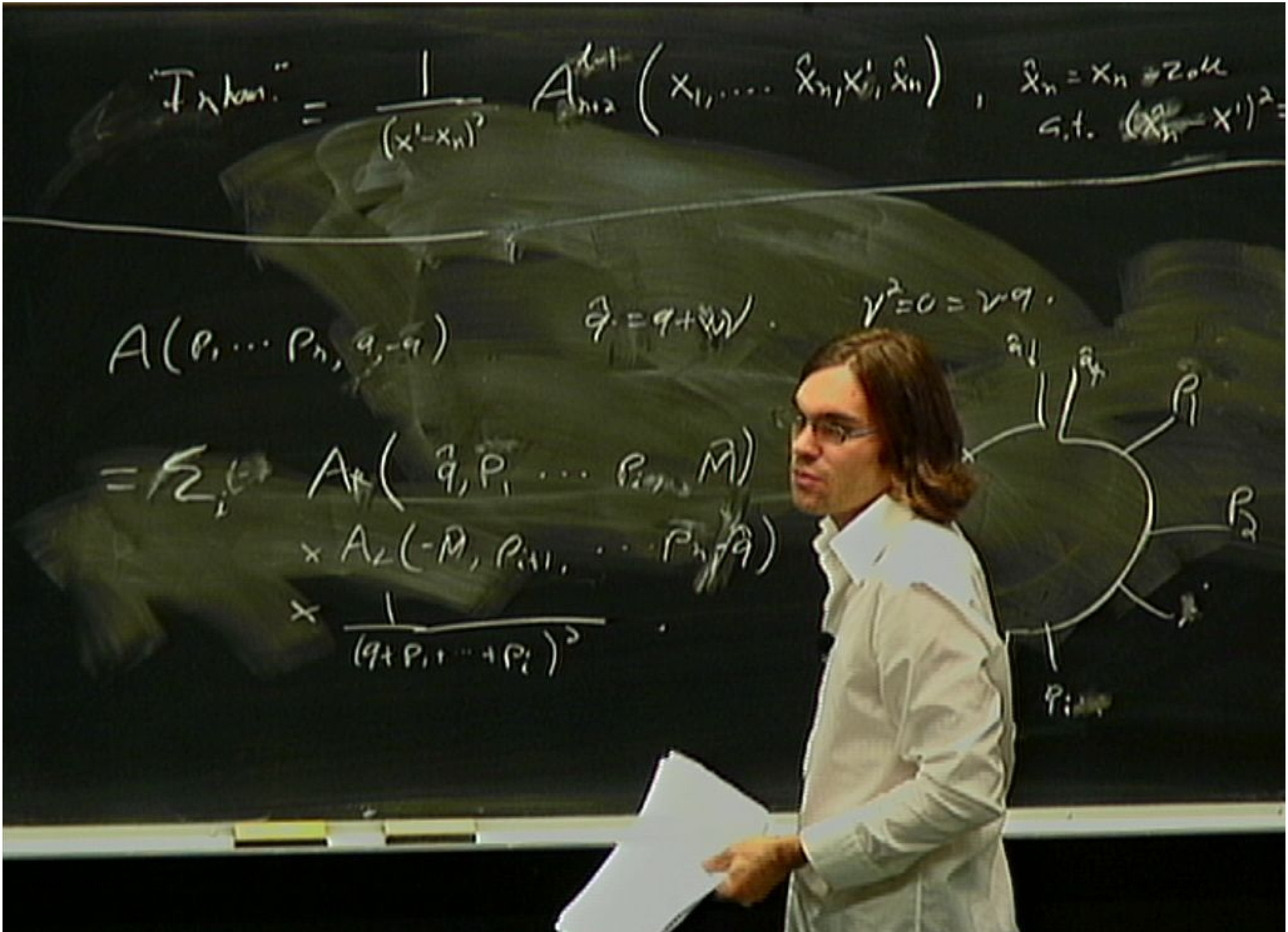


$A_n = \sum_{\text{channels } P} A_R^{\text{tree}} \frac{1}{P^2} A_L^{(1\text{-loop})}(P_1, \ldots P_i \hat{P} | q)$
$+ \sum_{\text{channels}} A_R^{1\text{-loop}} \frac{1}{P^2} A_L^{\text{tree}}$
$+$ Inhomogeneous
"Inhom." $= \frac{1}{(x' - x_n)^2} A_{n+2}^{l-1} (x_1, \ldots x_n, x_n', x_n)$
$A(p_1 \ldots p_n, q, -q)$
$\hat{q} = q + \hat{z} \nu$
$\nu^2 = 0 = \nu \cdot q$
$\times A_L(-\hat{P}, p_{i+1}, \ldots p_n, -\hat{q})$



$E_q^0 + \frac{(\delta m^2)}{2E_q}$
$\rightarrow A)$


Exercise: Merge



A(p1 ... pn, q, -q)
s.t.

$$\text{"Inhom."} = \frac{1}{(x'-x_n)^2} A^{d+1}_{n+2}\left(x_1, \ldots \hat{x}_n, x'_1, \hat{x}_n\right), \quad \hat{x}_n = x_n + z u$$

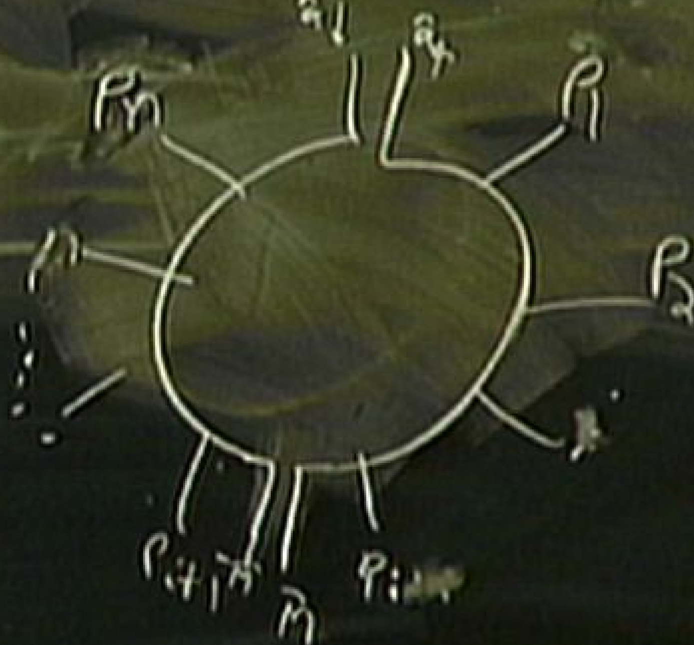
$$\text{s.t. } (\hat{x}_n - x')^2 =$$

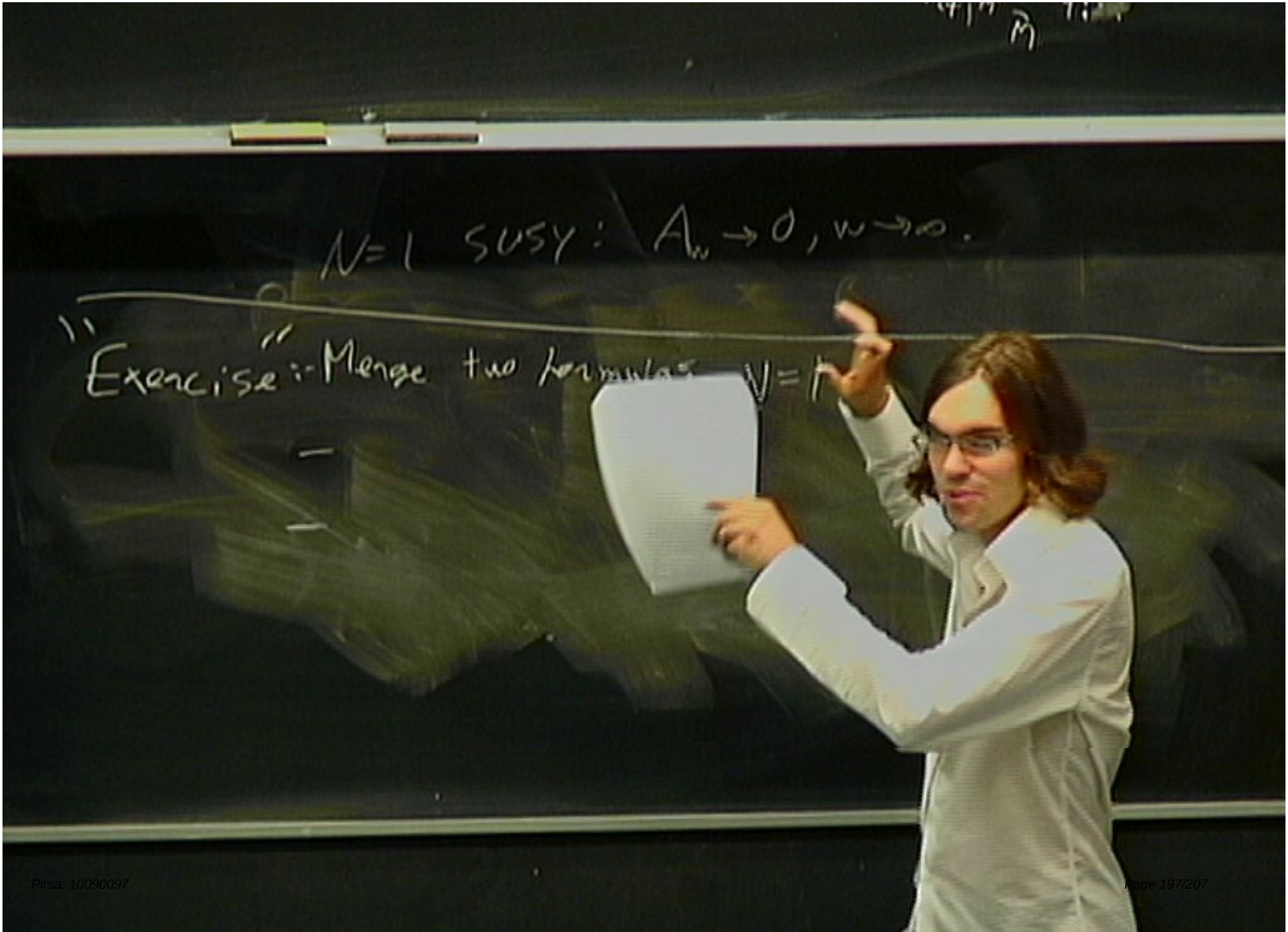
$$A(p_1 \cdots p_n, q, -q) \qquad \hat{q} = q + z\nu . \qquad \nu^2 = 0 = \nu \cdot q .$$

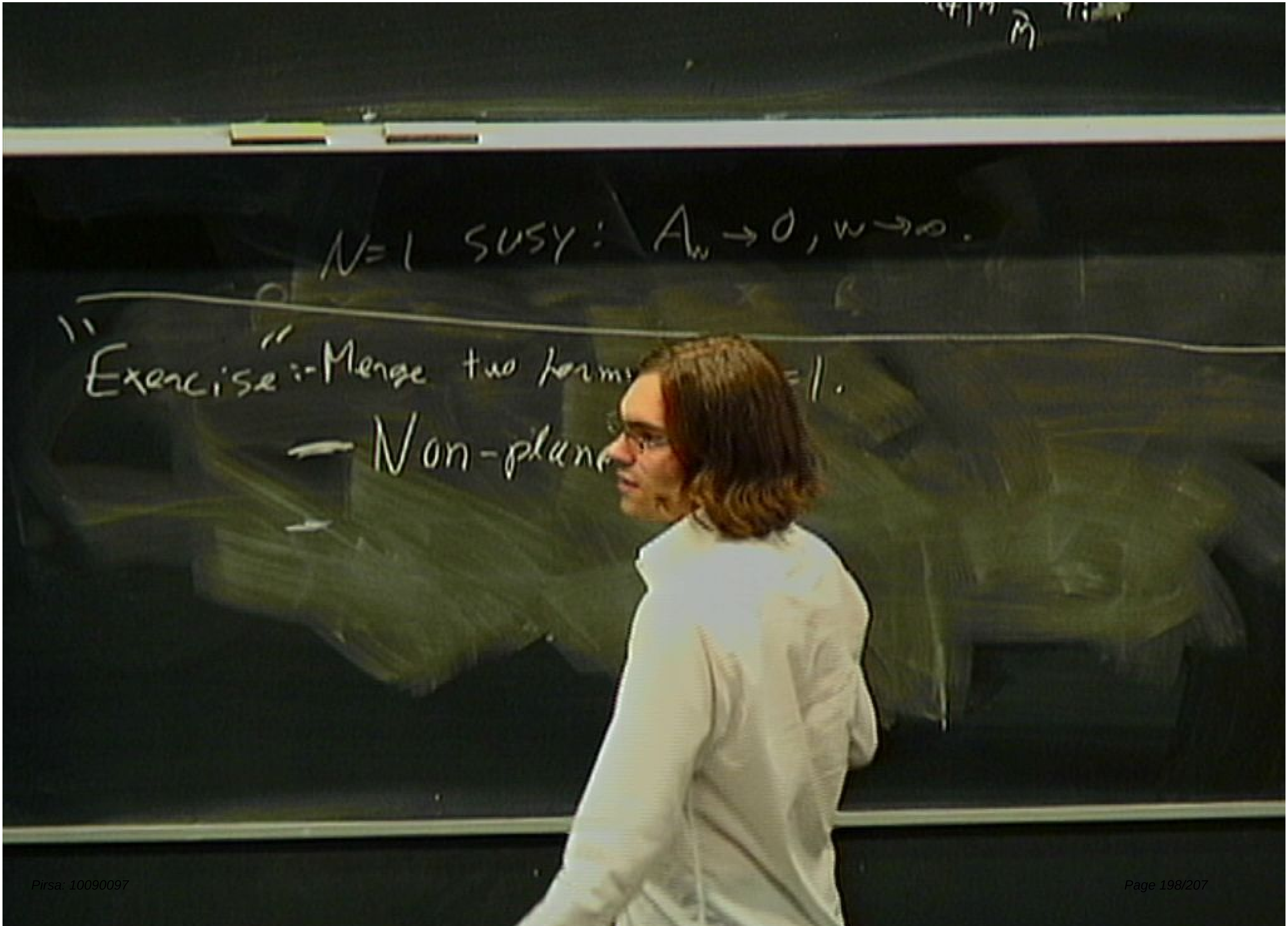
$$= \sum_i (-) A_L(\hat{q}, \hat{p}_1 \cdots p_i, \hat{P})$$

$$\times A_R(-\hat{P}, p_{i+1}, \cdots \hat{p}_n, -\hat{q})$$

$$\times \frac{1}{(q + p_1 + \cdots + p_i)^2} .$$




N=1 SUSY: A_w → 0, w → ∞.
"Exercise": Merge two formulas N=1


N=1 SUSY: A_w → 0, w → ∞.
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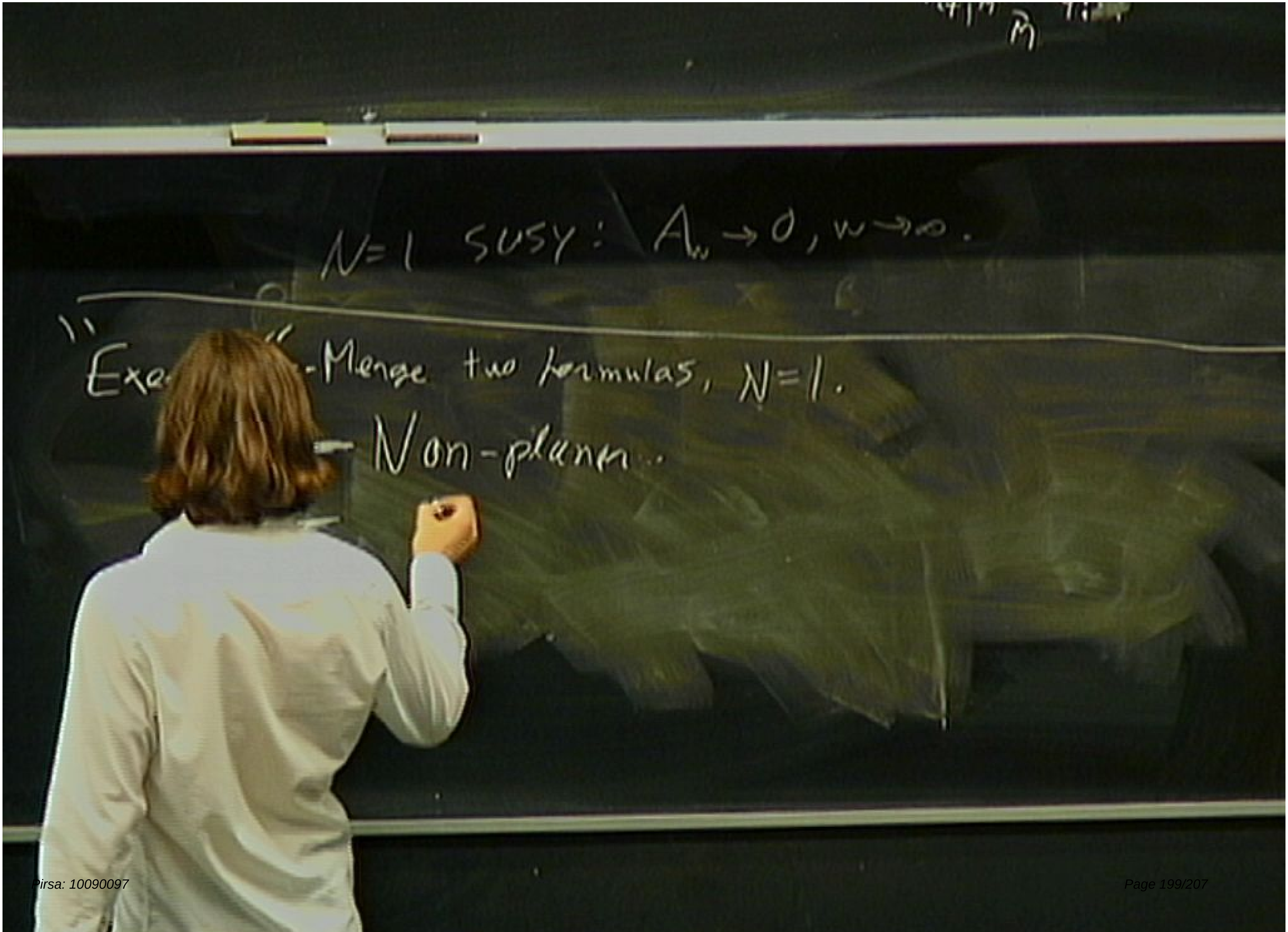
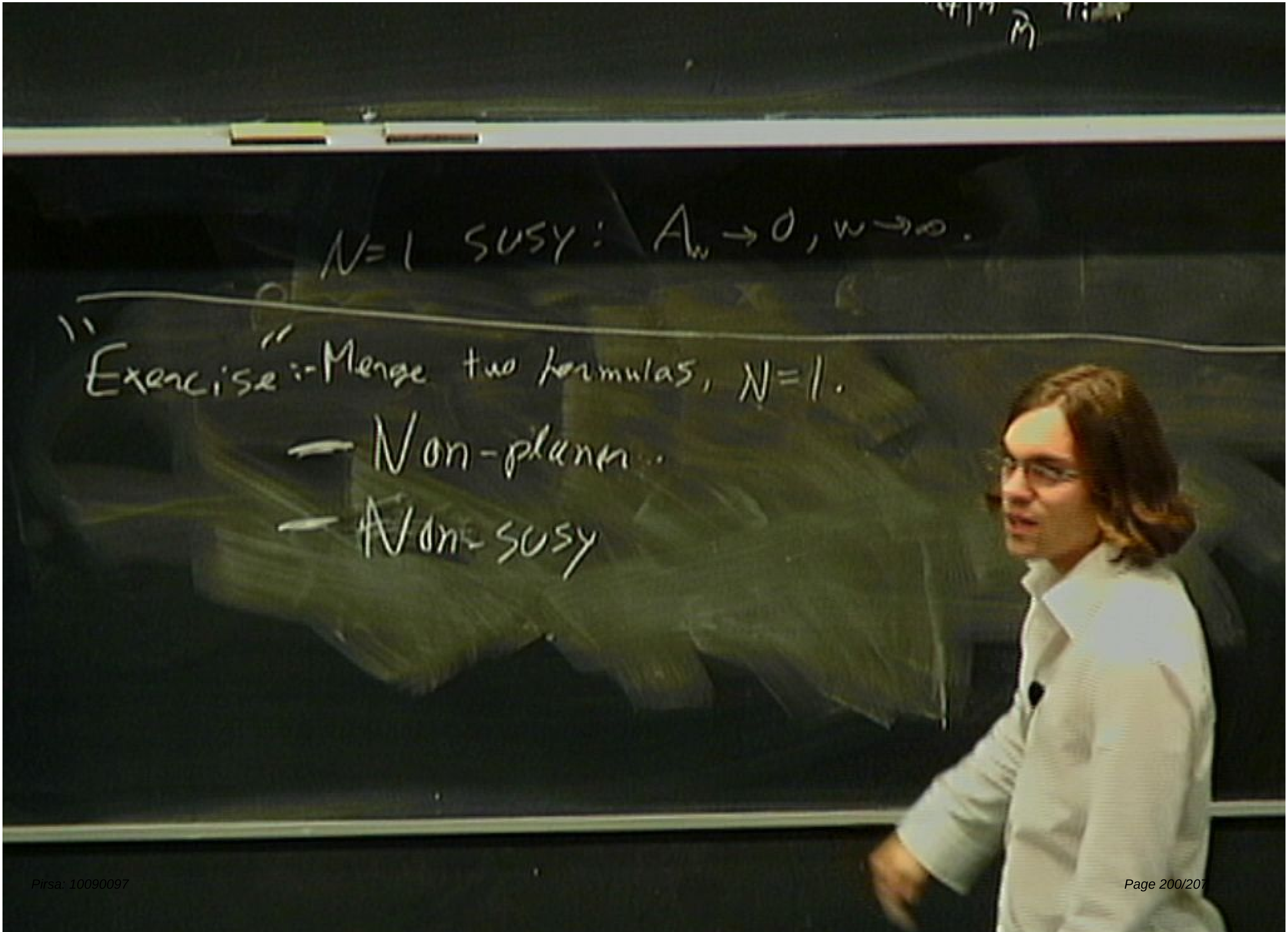
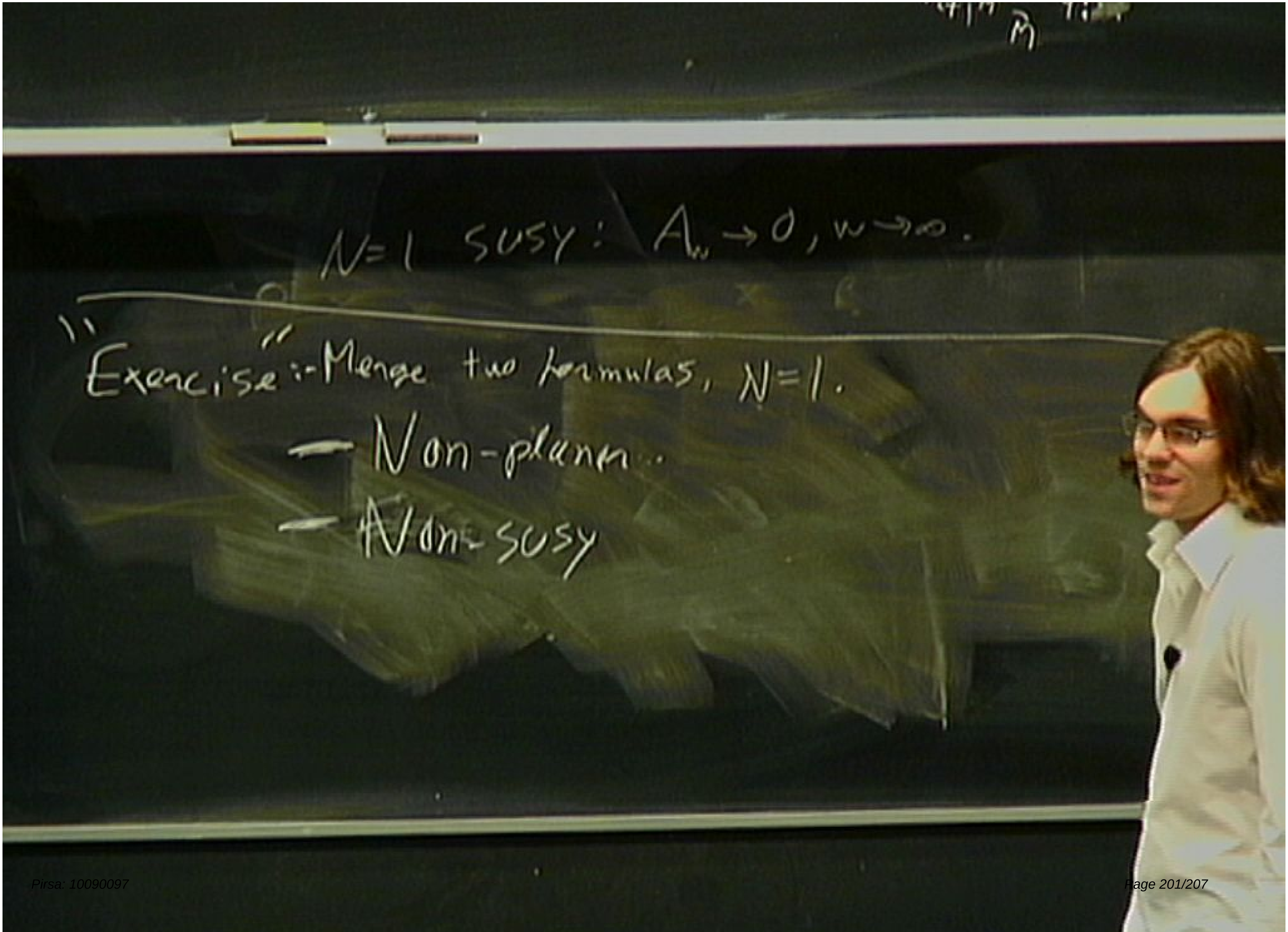
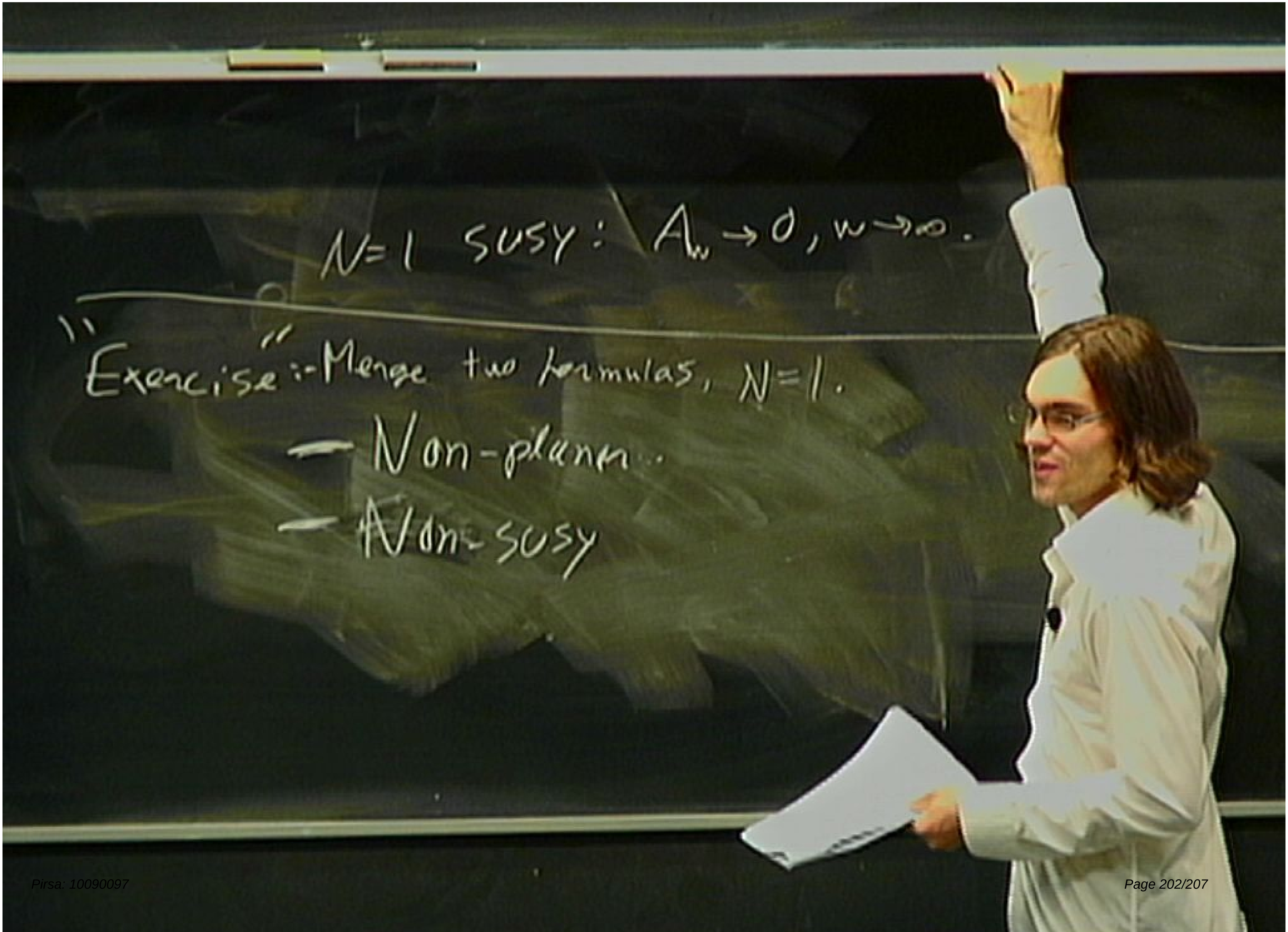

N=1 SUSY: $A_w \to 0, w \to \infty$.
-Menge two formulas, N=1.
Non-plan


N=1 susy: A_w → 0, w → ∞.
"Exercise":- Merge two formulas, N=1.
— Non-planar..
— Non-susy


N=1 susy: A_w → 0, w → ∞.
"Exercise": Merge two formulas, N=1.
– Non-planar..
– Non-susy

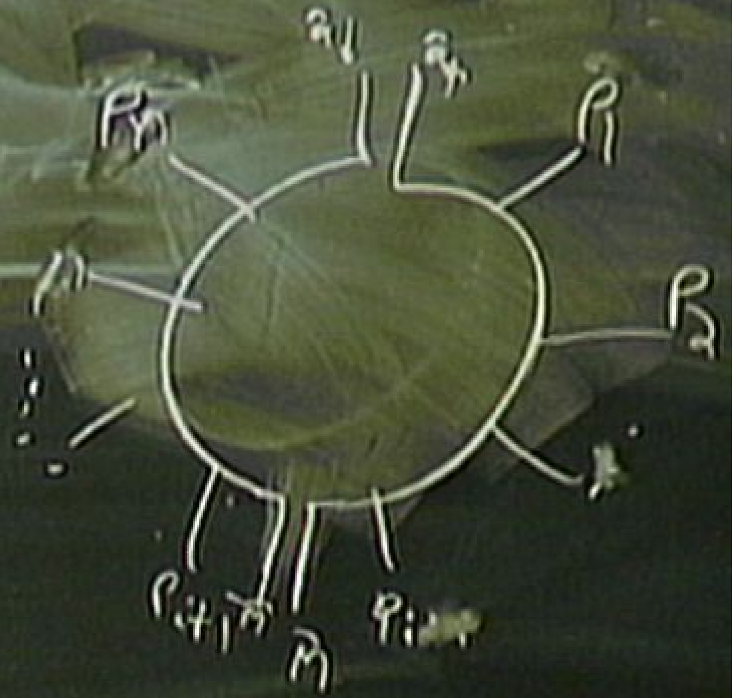

N=1 SUSY: $A_w \to 0$, $w \to \infty$.
"Exercise": Merge two formulas, N=1.
— Non-planar..
— Non-susy

$A(p_1 \cdots p_n, \hat{q}, -\hat{q})$ $\quad \hat{q} = q + wv. \quad v^2 = 0 = v \cdot q.$

$= \sum_i (-) \; A_R(\hat{q}, p_1 \cdots p_i, M)$

$\times A_L(-M, p_{i+1}, \cdots p_n, \hat{q})$

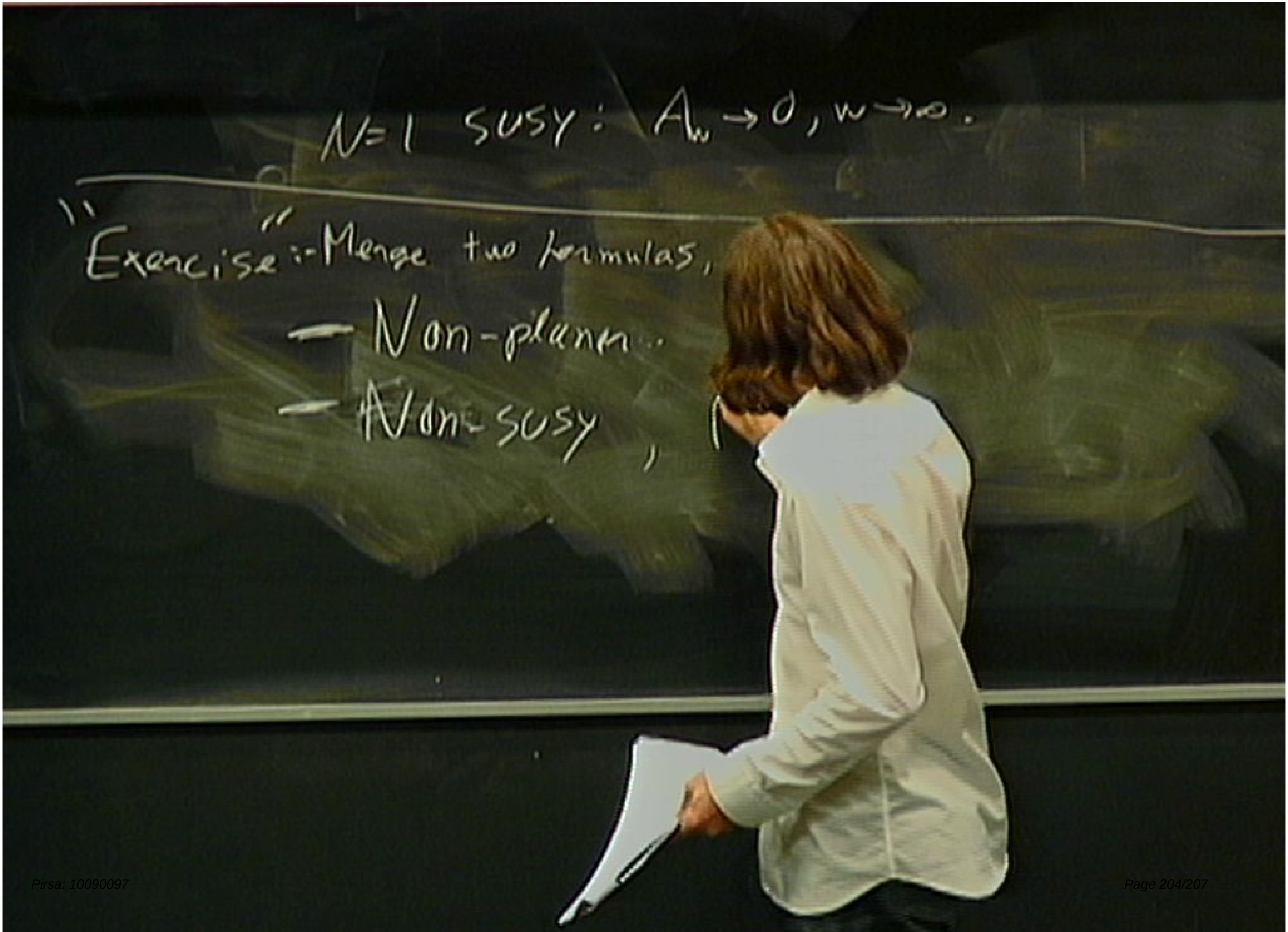
$\times \dfrac{1}{(q + p_1 + \cdots + p_i)^2}.$



$N=1$ SUSY: $A_w \to 0, \; w \to \infty.$

"Exercise": Merge two formulas, $N=1$.

— Non-plana...


N=1 susy: A_w → 0, w → ∞.
"Exercise": Merge two formulas,
— Non-planar..
— Non-susy

$N=1$ SUSY: $A_w \to 0,\ w \to \infty$.

"Exercise": Merge two formulas, $N=1$.

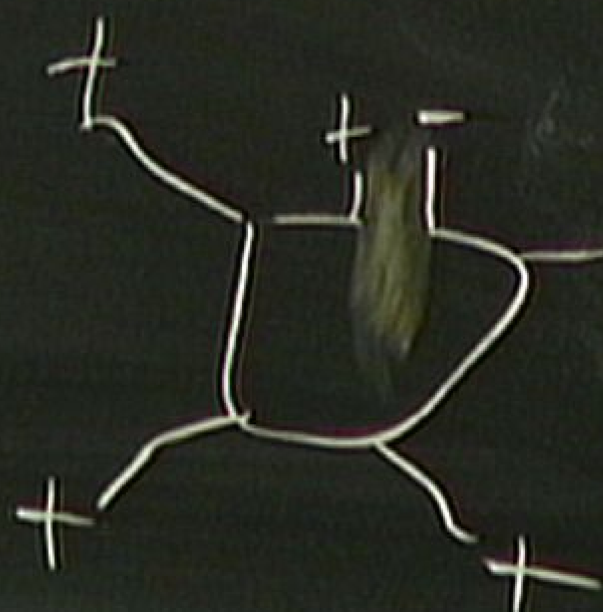
— Non-planar.

— Non-susy, Rational terms

$$E_{vac} = \int \frac{d^3q}{(2\pi)^3}\, \frac{1}{2} \left(E_q = E_q^0 + \frac{(\delta m^2)}{2E_q} \right)$$

$$= \frac{1}{2} \int \frac{d^3q}{(2\pi)^3\, 2E_q} \left(\delta m^2 \longrightarrow A \right)$$

$$\left(-\frac{E_q}{2} \right)$$

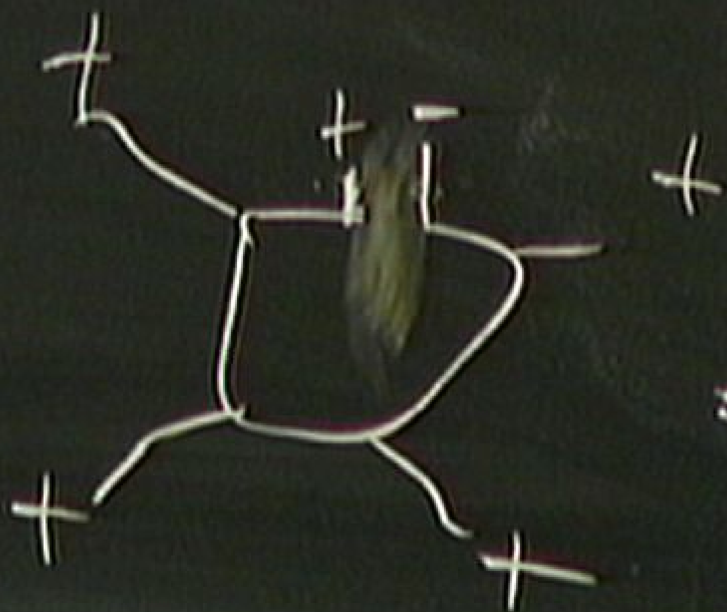


$$\neq 0$$

$$E_{vac} = \int \frac{d^3q}{(2\pi)^3} \hbar \frac{1}{2} \left(E_q = E_q^0 + \frac{(\delta m^2)}{2E_q} \right)$$

$$= \frac{1}{2} \int \frac{d^3q}{(2\pi)^3\, 2E_q} \left(\delta m^2 \longrightarrow A \right)$$

$$\left(-\frac{E_q}{2} \right)$$



$$\neq 0$$