

Title: Scattering Amplitudes from Single-Cuts: Lecture 2

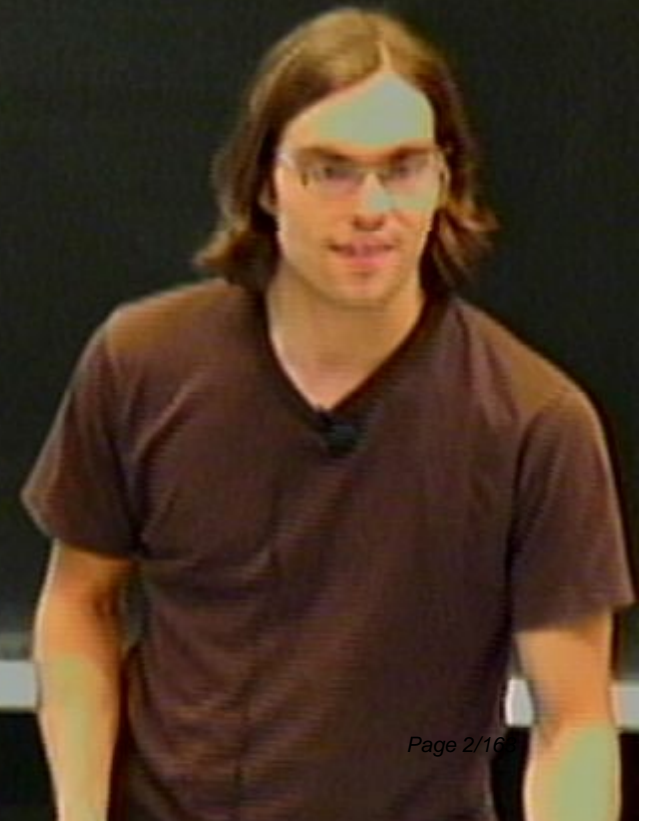
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URL: <http://pirsa.org/10090094>

Abstract: On-shell methods provide a powerful tool for the perturbative computation of scattering amplitudes in gauge theories, such as QCD. In these lectures I will focus on such methods which can be developed without invoking unitarity, but rather by setting one propagator by loop level on-shell ("single-cuts"). After summarizing the present status of applying these ideas to QCD, I will discuss their recent successful application to planar $N=4$ super Yang-Mills, leading to on-shell recursion relations for loop integrands.

Summary notes:

-



Summary solution:

$$- \text{Loops} = \int dLIPS \times [\text{Physical tree amps}]$$

-

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- Planar th₇ \Rightarrow Multi loop

- SU

Summary, so far:

- Loops = $\int dLIPS \times [\text{Physical tree amps}]$.

- Planar th₄ \Rightarrow Multiloop

- SUSY \Rightarrow Forward limits tree amplitudes.

Summary notes:

- Loops = $\int dLips \times [\text{Physical tree amps}]$.

- Planar th \Rightarrow Multiloop

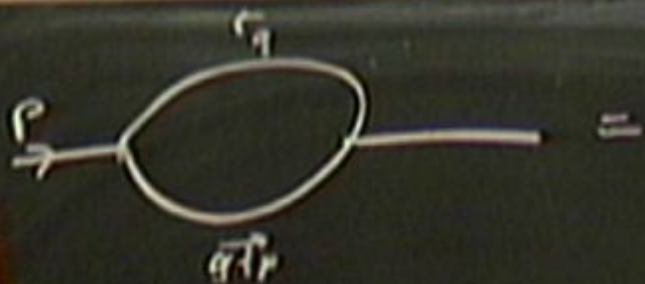
- SUSY \Rightarrow Forward limits tree amplitudes.

Summary notes:

- Loops = $\int dLIPS \times [\text{Physical tree amps}]_R$.

- Planar th \Rightarrow Multiloop

- SUSY: Forward limits two amplitudes.

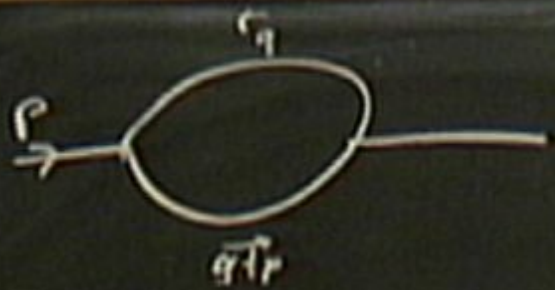


$$= \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+r)^2}$$

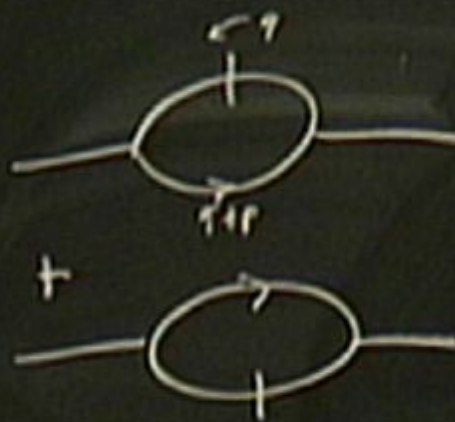


$$= \left\{ \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2 (q+r)^2} \right\} \pi.$$





$$= \left[\int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+r)^2} \right] \pi.$$

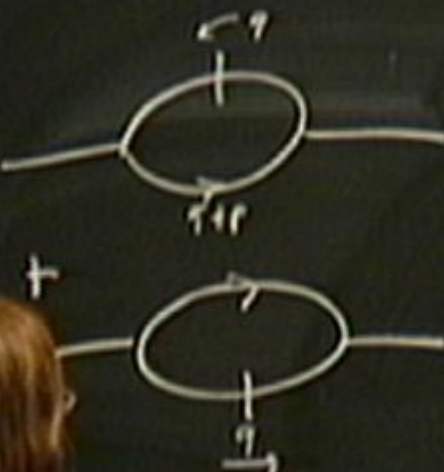


$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+r)^2 - i\epsilon} + \frac{1}{(q-r)^2 - i\epsilon} \right]$$



A Feynman diagram showing a bubble loop. An incoming line with momentum p enters from the left and splits into two lines that form a loop. The loop momentum is labeled q . The two lines of the loop meet at a vertex, and a single line exits to the right. The diagram is labeled with q at the top and $q+r$ at the bottom.

$$= \left\{ \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+r)^2} \right\} \pi.$$

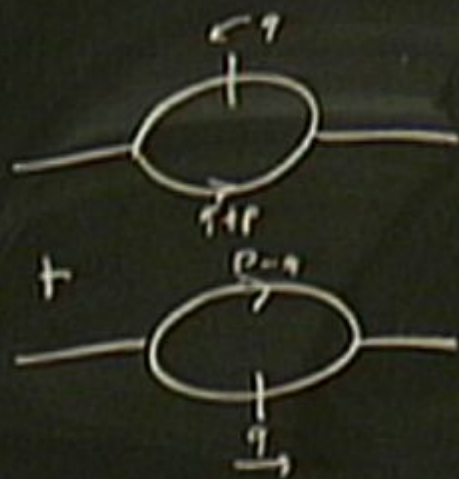


Two Feynman diagrams of bubble loops. The first diagram has an incoming line from the left and an outgoing line to the right. The loop momentum is q , and the external momentum is r . The second diagram is similar but with the external momentum r pointing in the opposite direction. The two diagrams are separated by a plus sign.

$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+r)^2 - i\epsilon(q^0 + |q|)} + \right.$$



$$= \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+p)^2} \pi.$$

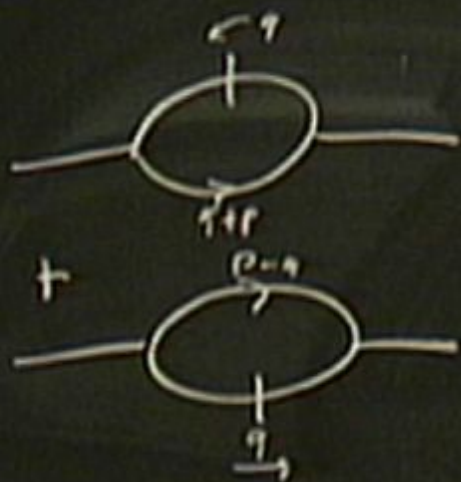


$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+p)^2 - i\epsilon(q^0+p^0)} + \frac{1}{(p-q)^2 - i\epsilon(p^0-q^0)} \right]$$



$$= \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+r)^2} \pi$$

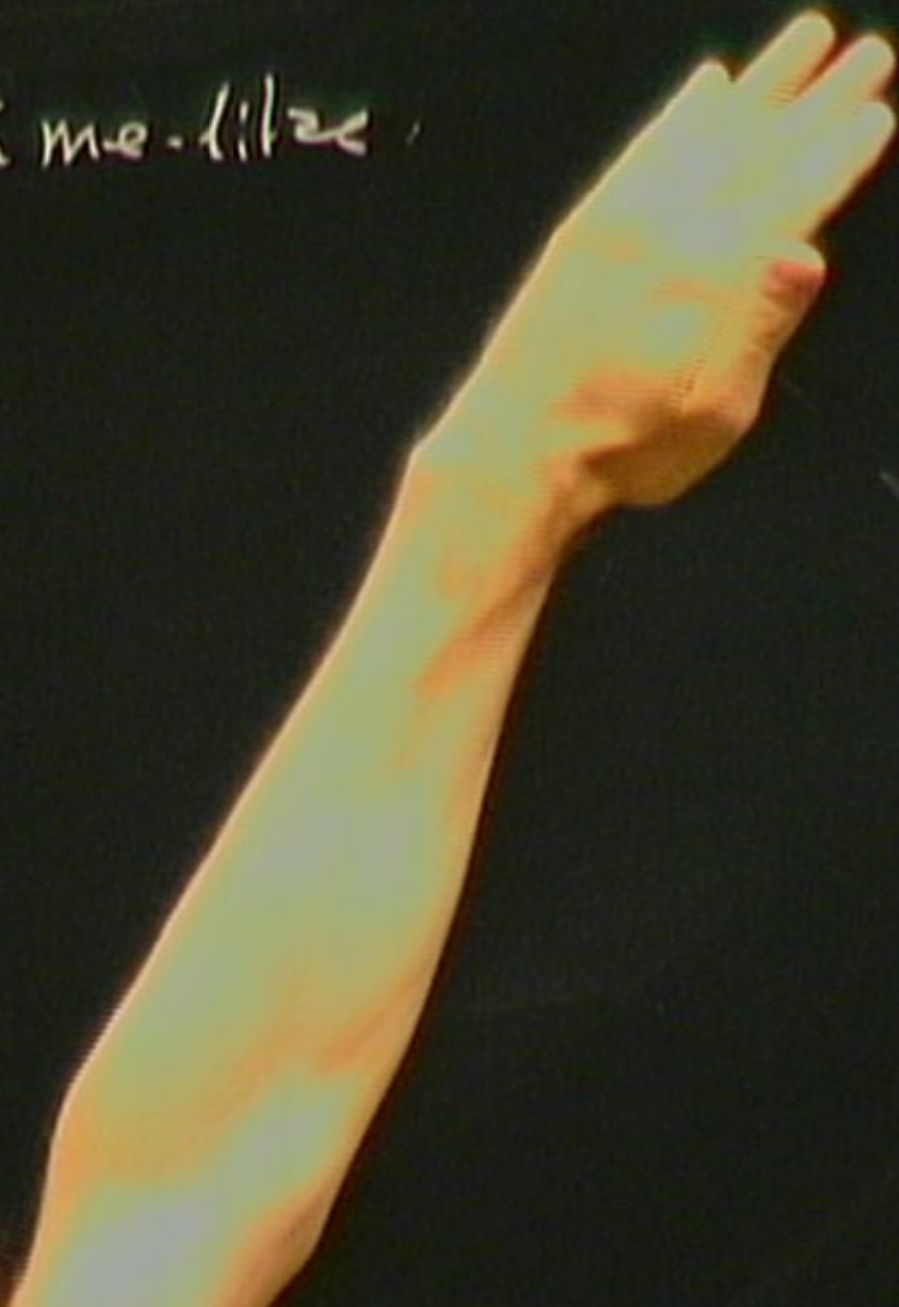
$$p^2 = -p_0^2 + \vec{p}^2$$



$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+r)^2 - i\epsilon(q^0+r^0)} + \frac{1}{(p-q)^2 - i\epsilon(p^0-q^0)} \right]$$

$p = \text{time}$

$\rho = \text{time-like}$



$\rho = \text{time.lit}$

$\rho = \text{time-like}$

$$\int \frac{d^{\rho} q}{(2\pi)^{\rho}} \pi \delta(q^0) \Rightarrow \int \frac{d^{\rho-1} q}{\phantom{(2\pi)^{\rho-1}}}$$

= time-like

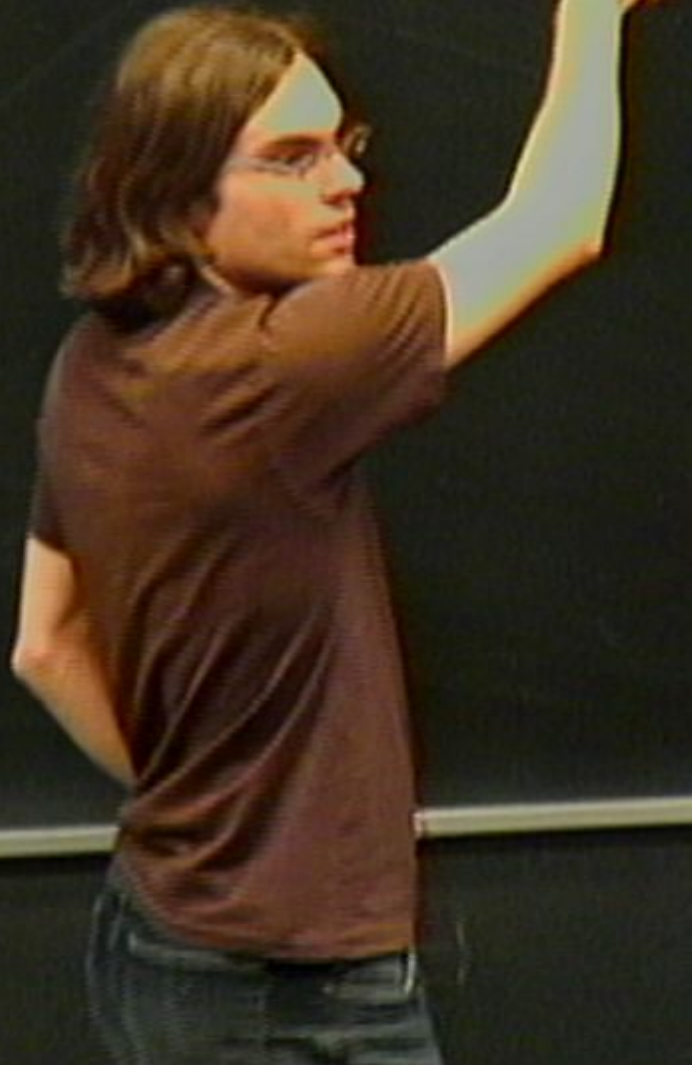
$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^0) \Rightarrow \int \frac{d^{D-1} \vec{q}}{2 (2\pi)^{D-1} 2|\vec{q}|}$$

= time-like

$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \Rightarrow \int \frac{d^{D-1} \vec{q}}{2 (2\pi)^{D-1} 2|\vec{q}|} =$$

= timeline

$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \Rightarrow \int \frac{d^{D-1} \vec{q}}{2 (2\pi)^{D-1}} = C \int_0^\infty dq^0 (q^0)^{D-3}$$



= timelike

$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^0) \Rightarrow \int \frac{d^{D-1} \vec{q}}{2 (2\pi)^{D-1} 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3}$$

$p = \text{time-like}$

$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \Rightarrow \int \frac{d^{D-1} \vec{q}}{2 (2\pi)^{D-1} 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + \{q^0 \rightarrow -q^0\}$$

$p = \text{time-like}$

$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \Rightarrow \int \frac{d^{D-1} \vec{q}}{2 (2\pi)^{D-1} 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + [\vec{q} \rightarrow -\vec{q}]$$

$$\int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{-q^0^2 - 2r^0 q^0 - i\epsilon (q^0 + r^0)} \right]$$

time-like

$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \Rightarrow \int \frac{d^{D-1} \vec{q}}{2 (2\pi)^{D-1} 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + \{q^0 \rightarrow -q^0\}$$

$$\int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{-p^2 - 2p^0 q^0 - i\epsilon (q^0 + r^0)} + \frac{1}{-p^2 + 2p^0 q^0 - i\epsilon (p^0 - q^0)} \right]$$

$p = \text{time-like}$

$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^0) \Rightarrow \int \frac{d^{D-1} \vec{q}}{2 (2\pi)^{D-1} 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + \{q^0 \rightarrow -q^0\}$$

$$2 \int_0^\infty dq^0 (q^0)^{D-3} \frac{1}{-2p^0 q^0 - i\epsilon(q^0 + p^0)} + \left. \frac{1}{-p^0 + 2p^0 q^0 - i\epsilon(p^0 - q^0)} \right\}$$

$p^0 > 0. \frac{2}{p^0} \int_0^\infty dq^0 (q^0)^{D-3}$

$p = \text{time-like}$

$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \Rightarrow \int \frac{d^{D-1} \vec{q}}{2 (2\pi)^{D-1} 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + \{q^0 \rightarrow -q^0\}$$

$$2 \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{-p^2 - 2p^0 q^0 - i\epsilon (q^0 + r^0)} + \frac{1}{-p^2 + 2p^0 q^0 - i\epsilon (p^0 - q^0)} \right]$$

$$p^0 > 0. \quad \frac{2}{p^0} \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{p^0 + 2q^0} \right]$$

$p = \text{time-like}$

$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \Rightarrow \int \frac{d^{D-1} \vec{q}}{2 (2\pi)^{D-1} 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + [\vec{q} \rightarrow -\vec{q}]$$

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$$p^0 > 0. \quad \frac{2}{p^0} \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{-1}{p^0 + 2q^0} + \frac{-1}{p^0 - 2q^0 + i\epsilon (p^0 - q^0)} \right]$$

$\rho = \text{time-like}$

$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \Rightarrow \int \frac{d^{D-1} \vec{q}}{2 (2\pi)^{D-1} 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + [\text{ghosts}]$$

$$2 \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{-p^2 - 2p^0 q^0 - i\epsilon(q^0 + r^0)} + \frac{1}{-p^2 + 2p^0 q^0 - i\epsilon(p^0 - q^0)} \right]$$

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$$p^0 > 0. \quad \frac{2}{p^0} \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{-1}{p^0 + 2q^0} + \frac{-1}{p^0 - 2q^0 + i\epsilon} \right]$$

$$q^0 \rightarrow p^0 - q^0 \quad 2(p^0)$$

$p = \text{time-like}$

$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \Rightarrow \int \frac{d^{D-1} \vec{q}}{2 (2\pi)^{D-1} 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + [\dots]$$

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$p^0 > 0$

$$\frac{2}{p^0} \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{-1}{p^2 + 2q^0} + \frac{-1}{p^2 - 2q^0 + i\epsilon} \right]$$

$q^0 \rightarrow p^0 - q^0$

$$2 (2\pi)^{D-4} \int_0^\infty \frac{du u^{D-3}}{u+1}$$

$p = \text{time-like}$

$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \Rightarrow \int \frac{d^{D-1} \vec{q}}{2 (2\pi)^{D-1} 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + \{q^0 \rightarrow -q^0\}$$

$$2 \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{-p^2 - 2p^0 q^0 - i\epsilon (q^0 + r^0)} + \frac{1}{-p^2 + 2p^0 q^0 - i\epsilon (p^0 - q^0)} \right]$$

$$\frac{2}{p^0} \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{-1}{p^2 + 2q^0} + \frac{-1}{p^2 - 2q^0 + i\epsilon} \right]$$

$$\frac{2}{p^0} \int_0^\infty \left(\frac{u}{2}\right)^{D-4} \frac{du^{D-3}}{u+1}$$

$\rho = \text{time-like}$

$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \Rightarrow \int \frac{d^{D-1} \vec{q}}{2 (2\pi)^{D-1} 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + [\dots]$$

$$2 \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{-p^2 - 2p^0 q^0 - i\epsilon (q^0 + p^0)} + \frac{1}{-p^2 + 2p^0 q^0 - i\epsilon (p^0 - q^0)} \right]$$

$$p^0 > 0. \quad \frac{2}{p^0} \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{-1}{p^2 + 2q^0} + \frac{-1}{p^2 - 2q^0 + i\epsilon} \right]$$

$$q^0 \rightarrow \frac{p^0 q^0}{2} \quad \frac{1}{2} \left(\frac{p^0}{2} \right)^{D-4} \int_0^\infty \frac{du u^{D-3}}{u+1}$$



$\rho = \text{time-like}$

$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \Rightarrow \int \frac{d^{D-1} \vec{q}}{2 (2\pi)^{D-1} 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + \{q^0 \rightarrow -q^0\}$$

$$2 \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{-\rho^2 - 2\rho q^0 - i\epsilon(q^0 + \rho)} + \frac{1}{-\rho^2 + 2\rho q^0 - i\epsilon(\rho - q^0)} \right]$$

$$\rho^0 > 0. \quad \frac{2}{\rho^0} \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{-1}{\rho^0 + 2q^0} + \frac{-1}{\rho^0 - 2q^0 + i\epsilon} \right]$$

$$q^0 \rightarrow \frac{\rho^0 q^0}{2} \quad \frac{2}{\rho^0} \left(\frac{\rho^0}{2} \right)^{D-4} \int_0^\infty \frac{du u^{D-3}}{u+1} \left[1 + e^{-i\epsilon(0-u)} \right]$$



$p = \text{time-like}$

$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \Rightarrow \int \frac{d^{D-1} \vec{q}}{2(2\pi)^{D-1} 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + [\dots]$$

$$\int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{-p^2 - 2p^0 q^0 - i\epsilon(q^0 + r^0)} + \frac{1}{-p^2 + 2p^0 q^0 - i\epsilon(p^0 - q^0)} \right]$$

$$\int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{-1}{p^2 + 2q^0} + \frac{-1}{p^2 - 2q^0 + i\epsilon} \right]$$

$$\int_0^\infty \frac{du u^{D-3}}{u+1} \left[1 + e^{-i\epsilon(D-1)} \right]$$

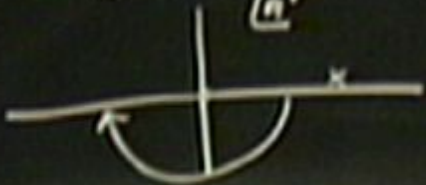
$\rho = \text{time-like}$


$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^0) \Rightarrow \int \frac{d^{D-1} \vec{q}}{2 (2\pi)^{D-1} 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + [\vec{q} \rightarrow -\vec{q}]$$

$$2 \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{-p^0 - 2p^0 q^0 - i\epsilon (q^0 + r^0)} + \frac{1}{-p^0 + 2p^0 q^0 - i\epsilon (p^0 - q^0)} \right]$$


$$p^0 > 0. \quad \frac{2}{p^0} \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{-1}{p^0 + 2q^0} + \frac{-1}{p^0 - 2q^0 + i\epsilon} \right]$$

$$r^0 \rightarrow \frac{p^0 q^0}{2} \quad \frac{2 \left(\frac{p^0}{2}\right)^{D-4}}{(p^0/2)^{D-4}} \int_0^\infty \frac{du u^{D-3}}{u+1} \left[1 + e^{-i\epsilon(0-q)} \right]$$





$$= \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+i)^2} \quad p^2 = -p^2 + p^2$$



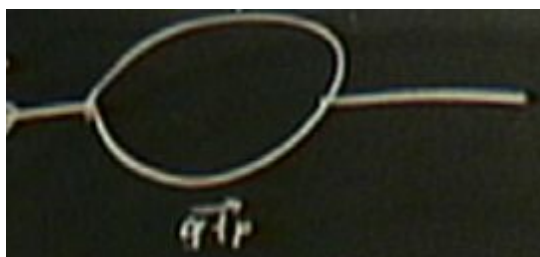
$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+i)^2 - i(q^2+i)} + \frac{1}{(p-i)^2 - i(q^2+i)} \right]$$



$$= \dots$$

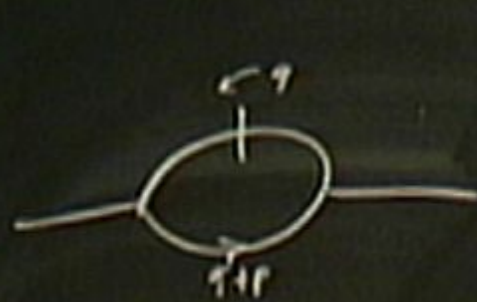


Handwritten notes and additional equations on the lower part of the chalkboard, including $p^2 = -p^2 + p^2$ and $p^2 = -2q^2 + i\epsilon$.

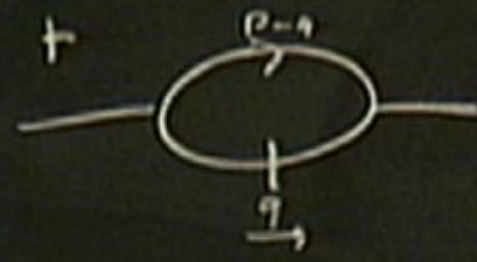


$$= \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+p)^2} \pi$$

$$p^2 = -p^2 + \vec{p}^2$$

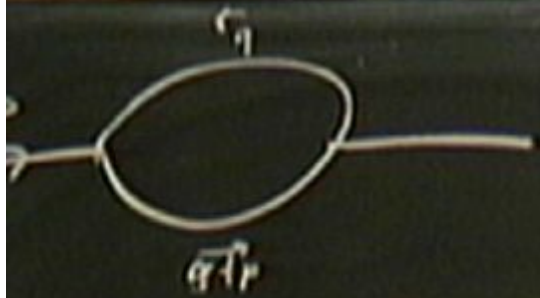


$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+p)^2 - i\epsilon(q^0+p^0)} + \frac{1}{(p-q)^2 - i\epsilon(p^0-q^0)} \right]$$



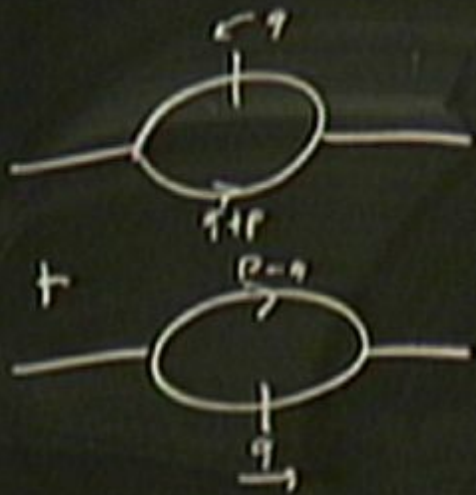
$$= C' (p^0)^{D-4} e$$

$p^0 > 0$
 $\int_0^{p^0} dq^0 (p^0 - q^0)^{D-3}$
 $\int_0^{p^0} \frac{du (p^0 - u)^{D-3}}{u+1}$
 $\frac{1}{(2\pi)^D} \int \frac{d^D q}{(2\pi)^D} \frac{1}{(q^0)^2}$



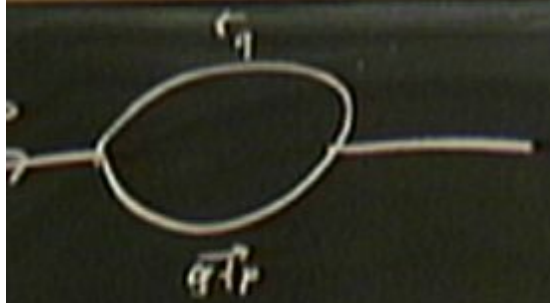
$$= \left\{ \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+r)^2} \right\} \pi.$$

$$p^2 = -p_0^2 + \vec{p}^2$$



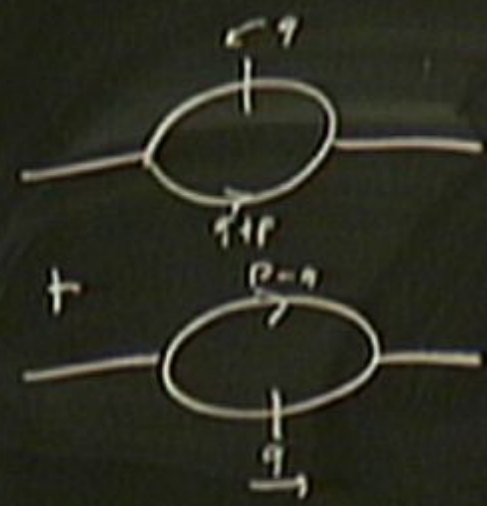
$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+r)^2 - i\epsilon(q_0+r_0)} + \frac{1}{(p-q)^2 - i\epsilon(p_0-q_0)} \right]$$

$$= C' (p_0)^{D-4} [1 + e^{-i\pi(D-4)}] \cdot e^{-i\pi(D-4)} \cos \dots$$



$$= \left\{ \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+p)^2} \right\} \pi.$$

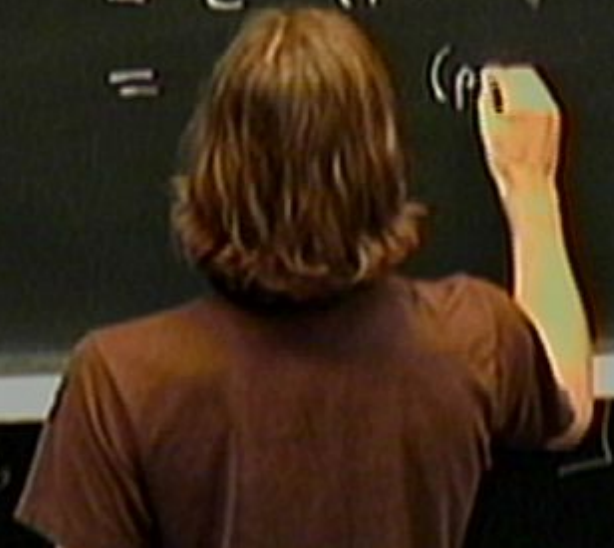
$$p^2 = -p_0^2 + \vec{p}^2.$$

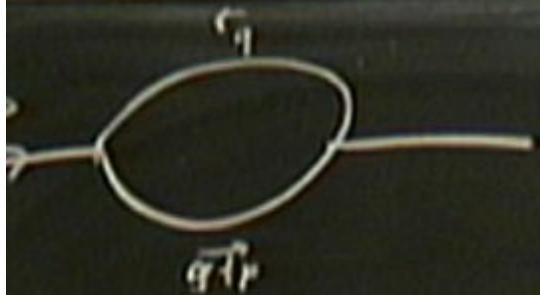


$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+p)^2 - i\epsilon(q_0^2 + p_0^2)} + \frac{1}{(p-q)^2 - i\epsilon(p_0^2 + q_0^2)} \right]$$

$$= C (p_0^2)^{D-4} \left[1 + e^{-i\pi(D-4)} \right] e^{-i\pi \frac{D-2}{2}} 2 \cos \pi \frac{D-1}{2}.$$

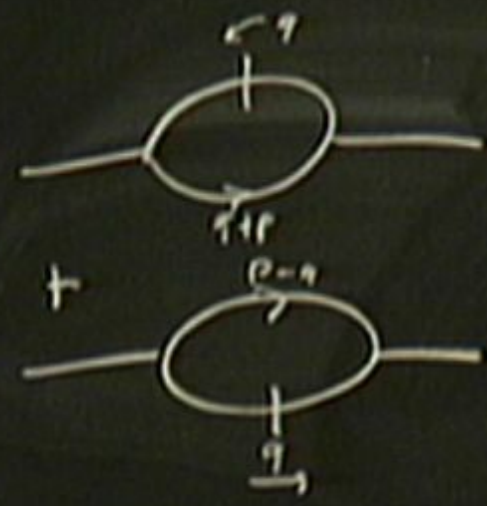
$$= (p_0^2)^{D-4}$$





$$= \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+r)^2} \pi$$

$$p^2 = -p_0^2 + \vec{p}^2$$

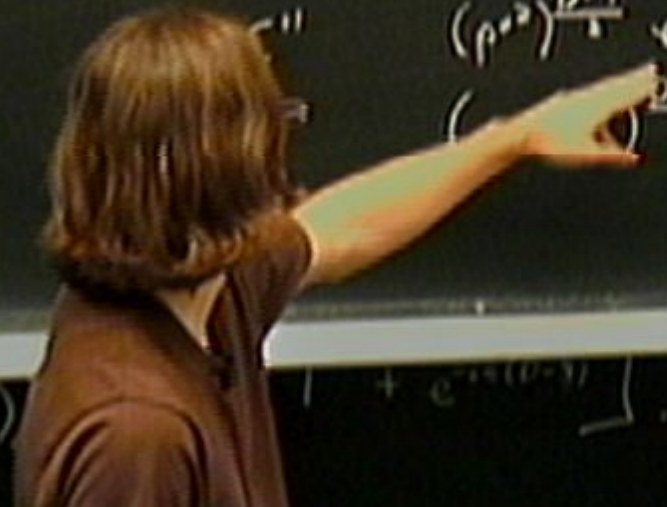


$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+r)^2 - i\epsilon(q^2+p^2)} + \frac{1}{(p-q)^2 - i\epsilon(p^2+q^2)} \right]$$

$$= C (p^2)^{D-4} [1 + e^{-i\pi(D-4)}] e^{-i\pi \frac{D-4}{2}} 2 \cos \pi \frac{D-4}{2}$$

$$(p^2)^{\frac{D-4}{2}} e^{-i\pi \frac{D-4}{2}}$$

$$(p^2)^{\frac{D-4}{2}}$$



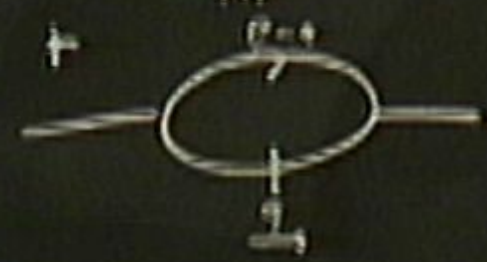


$$= \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+p)^2}$$

$$p^2 = p_1^2 + p_2^2$$



$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+p)^2 - i\epsilon(q^0)} + \frac{1}{(p-q)^2 - i\epsilon(p^0)} \right]$$



$$= C' (p^2)^{D-4} [1 + e^{-i\pi(0-4)}]$$

$$= C'' (p^2)^{\frac{D-4}{2}} e^{-i\pi \frac{D-4}{2}}$$

$$= (-p^2 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow (-p^2 - i\epsilon)^{D-4}, p^2 > 0$$



$$[1 + e^{-i\pi(0-4)}]$$



$$= \int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2 (q+p)^2}$$

$$p^2 = p_1^2 + p_2^2$$



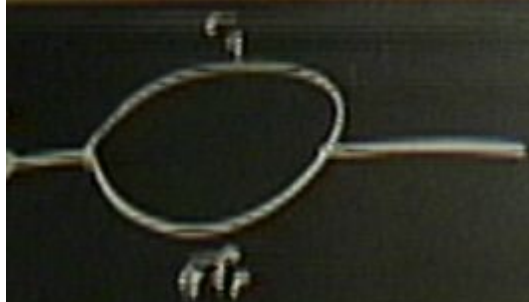
$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+p)^2 - i\epsilon(q_0 + |p|)} + \frac{1}{(p-q)^2 - i\epsilon(p_0 - |p|)} \right]$$



$$= C' (p^2)^{D-4} [1 + e^{-i\epsilon(p_0 - |p|)}]$$

$$= C'' (p^2)^{\frac{D-4}{2}} e^{-i\epsilon \frac{D-4}{2}}$$

$$= (-p^2 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow (-p^2 - i\epsilon)^{D-4}, p^2 > 0$$

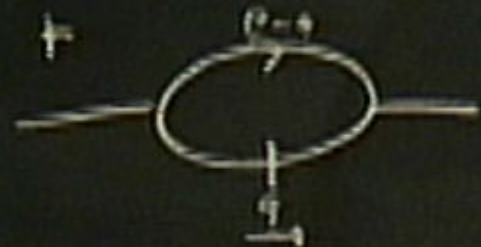


$$= \left[\int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+r)^2} \right] \pi$$

$$p^2 = -p^2 + p^2$$



$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+r)^2 - i\epsilon(q^2 + r^2)} + \frac{1}{(p-q)^2 - i\epsilon(p^2 + r^2)} \right]$$

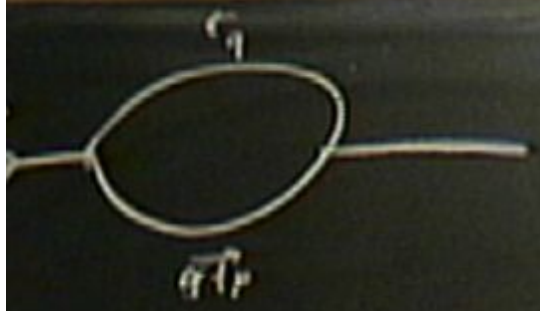


$$= C' (p^2)^{D-4} \left[1 + e^{-i\pi(D-4)} \right] \cdot e^{-i\pi D/2} \cos \frac{\pi D}{2}$$

$$= C'' (p^2)^{\frac{D-4}{2}} e^{-i\pi \frac{D-4}{2}}$$

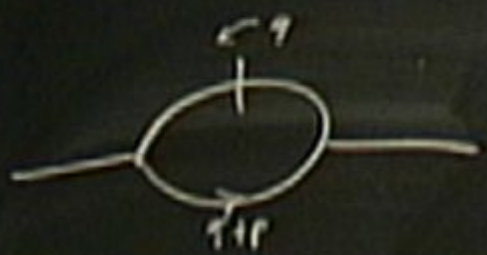
$$= (-p^2 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow (-p^2 - i\epsilon)^{D-4}, p^2 > 0$$

$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \Rightarrow \int \frac{d^D q}{(2\pi)^D} \frac{1}{|q|} = C \int_0^\infty dq (q^2)^{D-3} + [q^2 = -\epsilon^2]$$

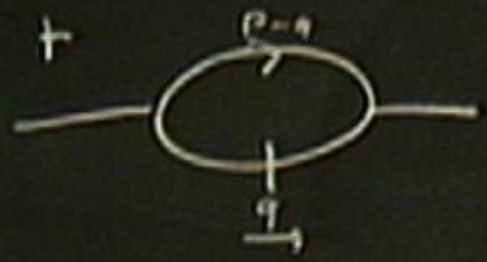


$$= \left\{ \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2 (q+p)^2} \right\} \pi.$$

$$p^2 = -p^2 + \bar{p}^2.$$



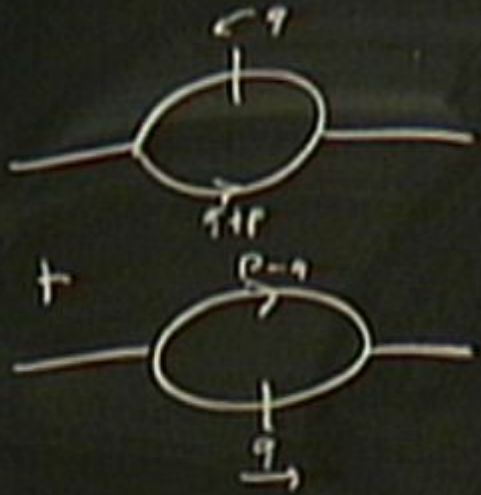
$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+p)^2 - i\epsilon(q^2+p^2)} + \frac{1}{(p-q)^2 - i\epsilon(p^2+q^2)} \right]$$



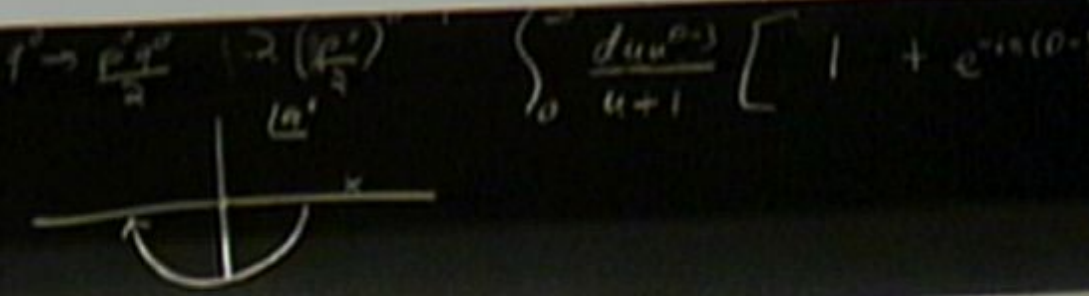
$$= C' (p^2)^{D-4} [1 + e^{-i\pi(D-4)}].$$

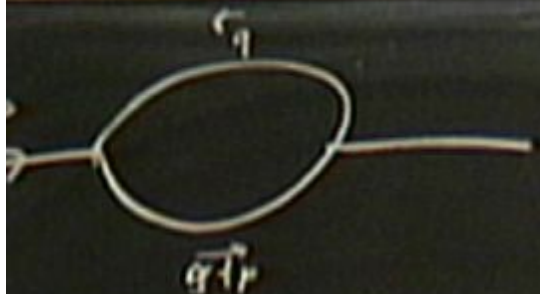
$$= C'' (p^2)^{\frac{D-4}{2}} e^{-i\pi \frac{D-4}{2}}.$$

$$= (-p^2 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow (-p^2 - i\epsilon)^{D-4}, p^2 > 0.$$



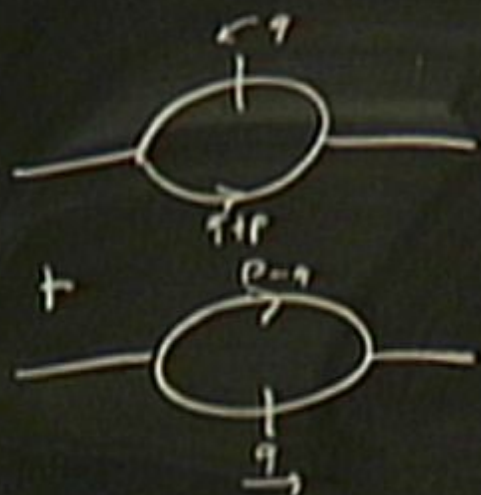
$$\begin{aligned}
 &= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+i\epsilon)^2 - i\epsilon(q^2 + \epsilon^2)} + \frac{1}{(p-q)^2 - i\epsilon(p^2 + \epsilon^2)} \right] \\
 &= C' (p^0)^{D-4} \left[1 + e^{-i\pi(D-4)} \right] \dots \text{in } \frac{D}{2} \text{ } 2 \cos \frac{\pi(D-4)}{2} \\
 &= C'' (p^0)^{\frac{D-4}{2}} e^{i\pi \frac{D-4}{2}} \\
 &= \dots \frac{(-p^2 - i\epsilon)^{\frac{D}{2}}}{(p^2)^{\frac{D-4}{2}}}, \quad p^2 > 0.
 \end{aligned}$$





$$= \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+r)^2} \pi$$

$$p^2 = -p_0^2 + \vec{p}^2$$

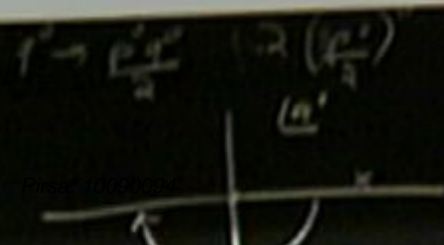


$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+r)^2 - i\epsilon(q_0+r_0)} + \frac{1}{(p-r)^2 - i\epsilon(p_0-r_0)} \right]$$

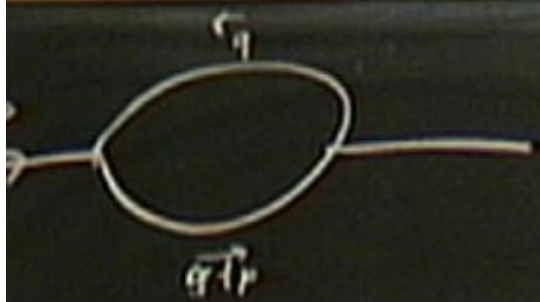
$$= C' (p^0)^{D-4} [1 + e^{-i\pi(D-4)}]$$

$$= C'' (p^0)^{\frac{D-4}{2}} e^{-i\pi \frac{D-4}{2}}$$

$$= (-p^2 - i\epsilon)^{\frac{D-4}{2}} \frac{1}{(p^2 - i\epsilon)^{\frac{D-4}{2}}}, p^0 > 0$$

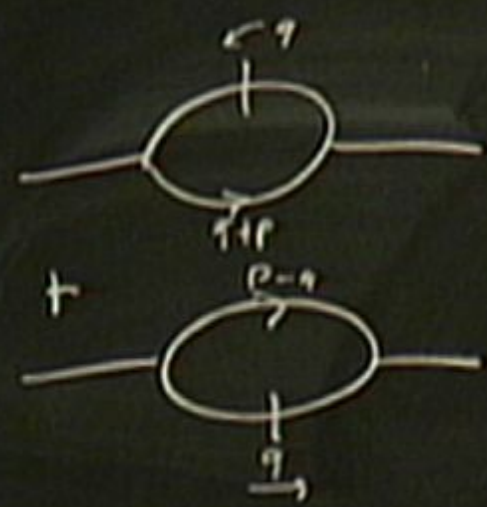


$$\int_0^\infty \frac{du u^{D-3}}{u+1} [1 + e^{-i\pi(D-4)}]$$



$$= \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+r)^2}$$

$$p^2 = -p_0^2 + \vec{p}^2$$

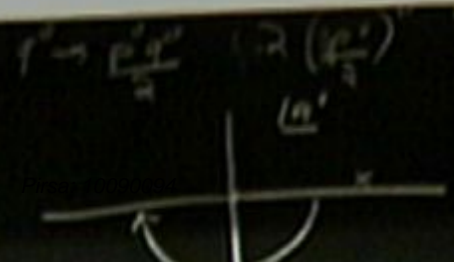
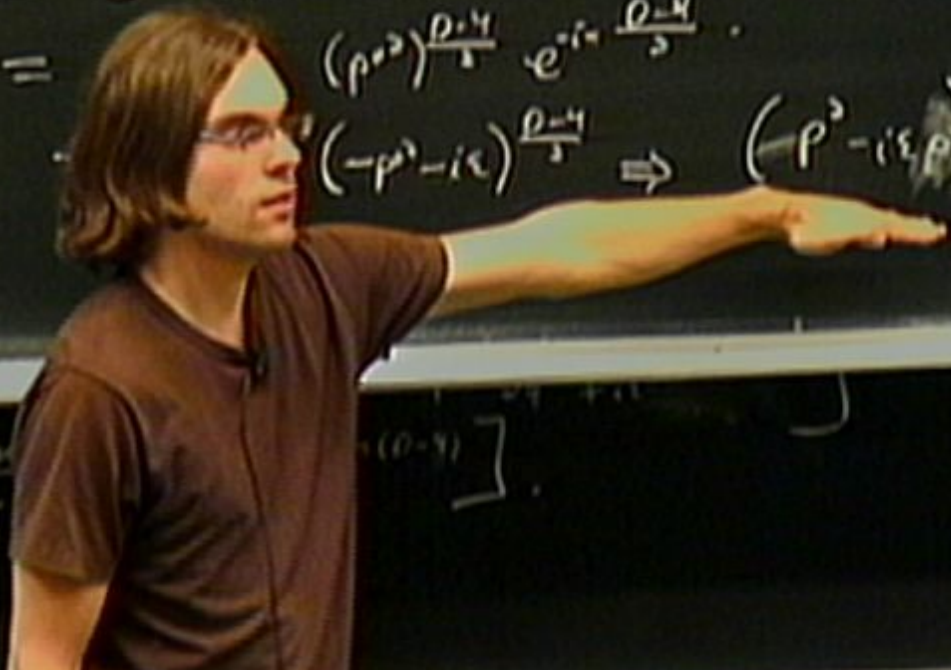


$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+r)^2 - i\epsilon(q^2+r^2)} + \frac{1}{(p-q)^2 - i\epsilon(p^2+q^2)} \right]$$

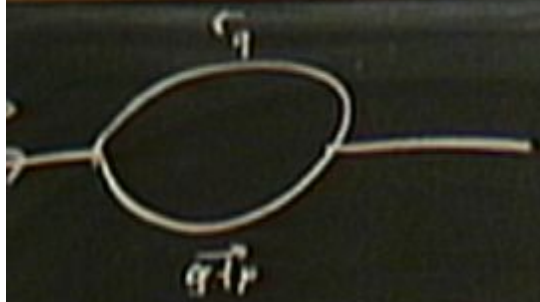
$$= C' (p^2)^{D-4} [1 + e^{-i\pi(D-4)}] e^{-i\pi \frac{D-1}{2}} 2 \cos \pi \frac{D-1}{2}$$

$$= (p^2)^{\frac{D-4}{2}} e^{-i\pi \frac{D-4}{2}}$$

$$(-p^2 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow (-p^2 - i\epsilon p^2)^{\frac{D-4}{2}}, p^2 > 0$$

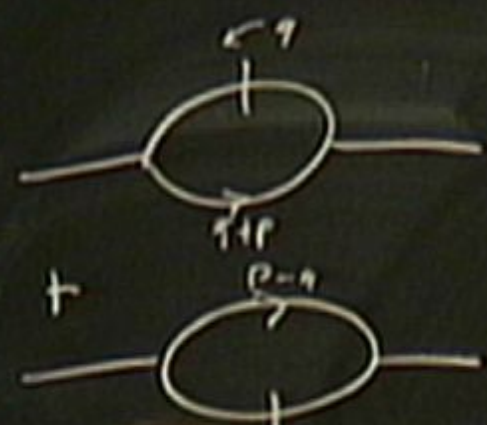


$$\int_0^{\infty} \frac{dx}{x+i\epsilon}$$



$$= \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+r)^2} \pi$$

$$p^2 = -p^2 + \vec{p}^2$$



$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+r)^2 - i\epsilon(q+r)^0} + \frac{1}{(p-q)^2 - i\epsilon(p-q)^0} \right]$$

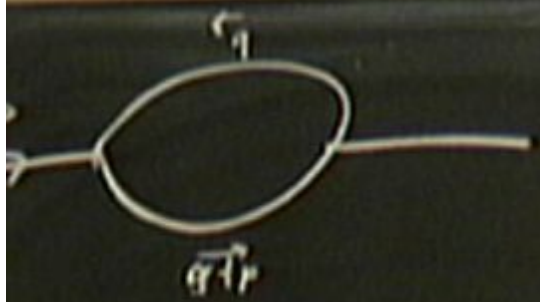
$$= C' (p^0)^{D-4} [1 + e^{-i\pi(D-4)}] e^{-i\pi \frac{D-1}{2}} 2 \cos \pi \frac{D-1}{2}$$

$$= C'' (p^2)^{\frac{D-4}{2}} e^{-i\pi \frac{D-4}{2}}$$

$$= (-p^2 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow (-p^2 - i\epsilon p^0)^{\frac{D-4}{2}}, p^0 > 0$$

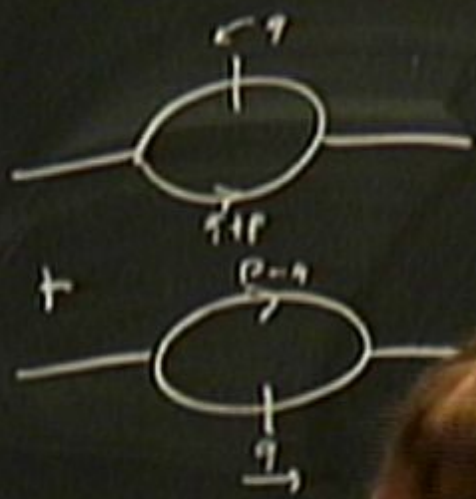
$$(p^2)^{\frac{D-4}{2}}, p \text{ spacelike}$$

$$\int_0^{\infty} \frac{du u^{D-3}}{u+1} [1 + e^{-i\pi(D-1)}]$$



$$= \left\{ \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2 (q+r)^2} \right\} \pi.$$

$$p^2 = -p^2 + \bar{p}^2$$



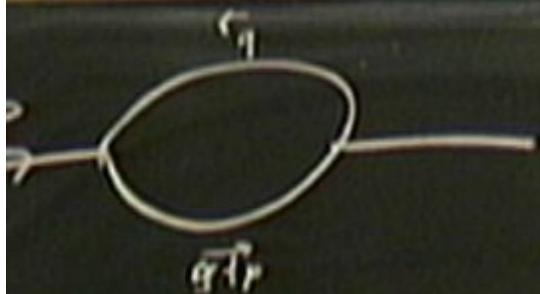
$$= \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+r)^2 - i\epsilon (q^2+r^2)} + \frac{1}{(p-q)^2 - i\epsilon (p^2+q^2)} \right]$$

$$= C' (p^2)^{D-4} [1 + e^{-i\pi(D-4)}].$$

$$= C'' (p^2)^{\frac{D-4}{2}} e^{i\pi \frac{D-4}{2}}$$

$$= (-p^2 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow \frac{(-p^2 - i\epsilon p^2)^{\frac{D-4}{2}}}{(p^2)^{\frac{D-4}{2}}}, p^2 > 0.$$

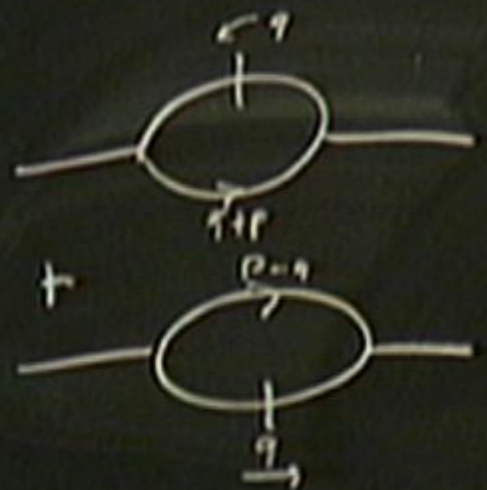
p^2 spacelike.



$$= \left\{ \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2 (q+p)^2} \right\} \pi$$

$$p = \begin{pmatrix} 0 \\ \vec{p} \\ p_0 \end{pmatrix}$$

$$p^2 = -\vec{p}^2 + p_0^2$$



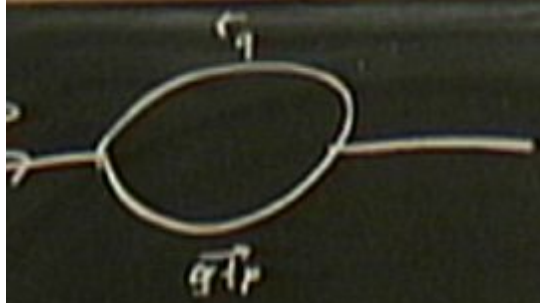
$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+p)^2 - i\epsilon (q^0+p^0)} + \frac{1}{(p-q)^2 - i\epsilon (p^0-q^0)} \right]$$

$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[1 + e^{-i\pi(0.4)} \right] \cdot e^{-i\pi \frac{q^0}{2}} 2 \cos \pi \frac{q^0}{2}$$

$$= (p^2)^{\frac{D-4}{2}} e^{-i\pi \frac{D-4}{2}}$$

$$= (-p^2 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow (-p^2 - i\epsilon)^{\frac{D-4}{2}}, p^2 > 0.$$

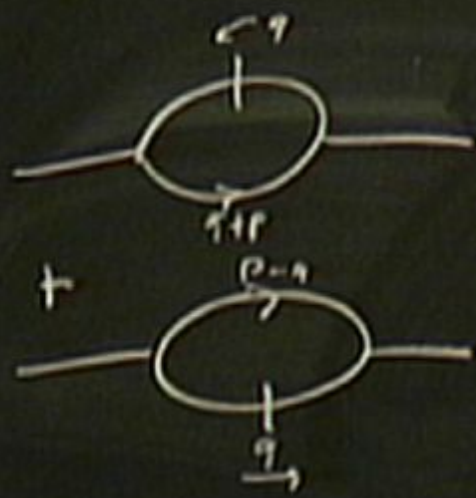
$$(p^2)^{\frac{D-4}{2}}, p^2 < 0 \text{ spacelike.}$$



$$= \left\{ \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+r)^2} \right\} \pi$$

$$p = \begin{pmatrix} 0 \\ 0 \\ p_3 \end{pmatrix}$$

$$p^2 = -p_3^2 + \bar{p}^2$$



$$= \left\{ \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+r)^2 - i\epsilon(q^2+r^2)} + \frac{1}{(p-q)^2 - i\epsilon(p^2+q^2)} \right] \right\}$$

$$= C' (p^2)^{D-4} \left[1 + e^{-i\pi(D-4)} \right] \cdot e^{-i\pi \frac{D-4}{2}} 2 \cos \pi \frac{D-4}{2}$$

$$= C'' (p^2)^{\frac{D-4}{2}} e^{-i\pi \frac{D-4}{2}}$$

$$= (-p^2 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow \begin{cases} (-p^2 - i\epsilon)^{\frac{D-4}{2}}, & p^2 > 0 \\ (p^2)^{\frac{D-4}{2}}, & p \text{ spacelike} \end{cases}$$

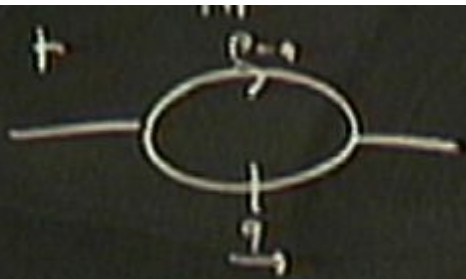
$$I_m \Rightarrow \int \pi d(q^2) \delta((q+p)^2) S$$

$$I_{\text{th}} \Rightarrow \int \pi d(r) \pi b((q+r)^2) \sin(q^2+r^2) .$$

$$I_{\text{th}} \Rightarrow \int \pi d(q^2) \delta((q+p)^2) \sin(q^0)$$

$$\left(\begin{matrix} q^0 \\ \vec{q} \end{matrix} \right), \left(\begin{matrix} -q^0 \\ \vec{q} \end{matrix} \right)$$

$$I_{\gamma} \Rightarrow \int_{\left(\begin{smallmatrix} \eta \\ \bar{\eta} \end{smallmatrix} \right), \left(\begin{smallmatrix} -\eta \\ \bar{\eta} \end{smallmatrix} \right)} \pi d(\eta) \sqrt{((\eta + \rho)^2)} \sin(\eta) \rightarrow 0.$$



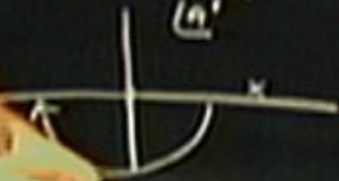
$$= C' (p^0)^{D-4} [1 + e^{-i\pi(D-4)}] \cdot e^{-i\pi \frac{D-4}{2}}$$

$$= C'' (p^0)^{\frac{D-4}{2}} e^{-i\pi \frac{D-4}{2}}$$

$$= (-p^0 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow \left(\frac{-p^0 - i\epsilon}{p^0} \right)^{\frac{D-4}{2}}, p^0 > 0$$

Spezialfall.

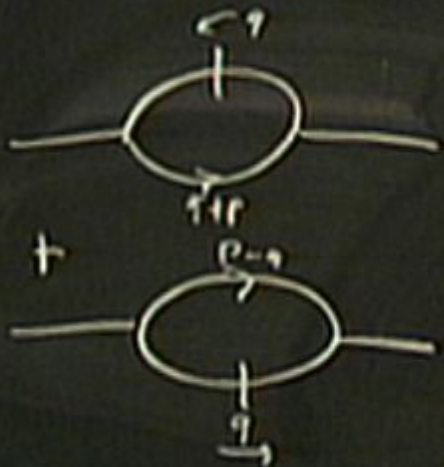
$$\int_0^\infty \frac{d^4 q (q^2)^{D-4}}{(q^2)^2} \left[\frac{1}{p^0 + 2q^0} + \frac{1}{p^0 - 2q^0 + i\epsilon} \right] [1 + e^{-i\pi(D-4)}]$$





$$= \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+r)^2} \Rightarrow C'''(p^2) \cdot p^2 + \bar{p}^2$$

$$p = \begin{pmatrix} p_0 \\ \mathbf{p} \end{pmatrix}$$



$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+r)^2} + \frac{1}{(p-r)^2 - i\epsilon(p^2+q^2)} \right]$$

$$= C' (p^2)^{D-4} \left[\dots \right]$$

$$= C'' (p^2)^D \left[\dots \right]$$

$$= C''' (p^2)^{D-4} \left[\dots \right]$$

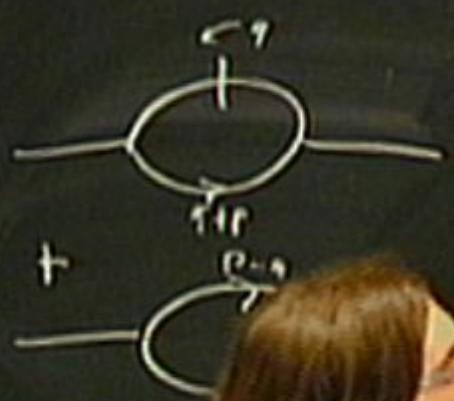
$e^{-in \frac{2\pi}{\epsilon}}$
 $2 \cos \frac{\epsilon}{2}$
 $(i\epsilon p^2)^{\frac{D-4}{2}}$, $p^2 > 0$
 $\frac{1}{\epsilon^2}$ spacelike



$$= \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+r)^2} \Rightarrow C''' (p^2 - i\epsilon)^{\frac{D-4}{2}}$$

$$p^2 = -p_0^2 + \vec{p}^2$$

$$p = \begin{pmatrix} p_0 \\ \vec{p} \end{pmatrix}$$



$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+r)^2 - i\epsilon(q+r)} + \frac{1}{(p-r)^2 - i\epsilon(p-r)} \right]$$

$$= C' (p^0)^{D-4} [1 + e^{-i\pi(D-4)}]$$

$$= C'' (p^0)^{\frac{D-4}{2}} e^{-i\pi \frac{D-4}{2}}$$

$$= (-p^2 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow \frac{(p^2 - i\epsilon p^0)^{\frac{D-4}{2}}}{(p^2)^{\frac{D-4}{2}}}, p^0 > 0$$

Spacelike

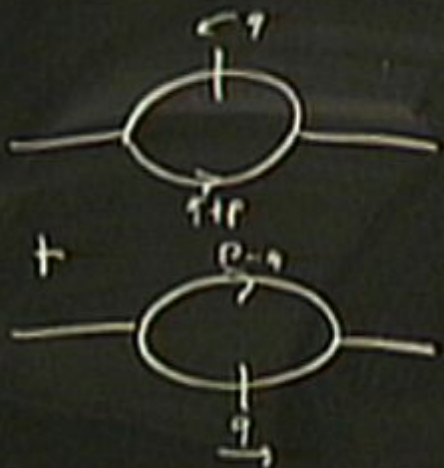


$$= \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+p)^2}$$

$$\Rightarrow C''' (p^2 - i\epsilon)^{\frac{D-4}{2}}$$

$$p^2 = -p_0^2 + \vec{p}^2$$

$$p = \begin{pmatrix} p_0 \\ \vec{p} \end{pmatrix}$$



$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+p)^2 - i\epsilon(q_0+p_0)} + \frac{1}{(p-q)^2 - i\epsilon(p_0-q_0)} \right]$$

$$= C' (p^0)^{D-4} [1 + te^{-i\pi(0-4)}]$$

$$e^{-i\pi \frac{D-1}{2}} 2 \cos \pi \frac{D-1}{2}$$

$$= C'' (p^0)^{\frac{D-4}{2}} e^{-i\pi \frac{D-4}{2}}$$

$$= (-p^2 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow \frac{(p^2 - i\epsilon p^0)^{\frac{D-4}{2}}}{(p^2)^{\frac{D-4}{2}}}, p^0 > 0$$

spacelike

$$I_m \Rightarrow \int \pi d(q^0) \delta((q+p)^2) \text{sgn}(q^0) \rightarrow 0.$$

$$\left(\begin{matrix} q^0 \\ \vec{q} \end{matrix} \right), \left(\begin{matrix} -q^0 \\ \vec{q} \end{matrix} \right)$$

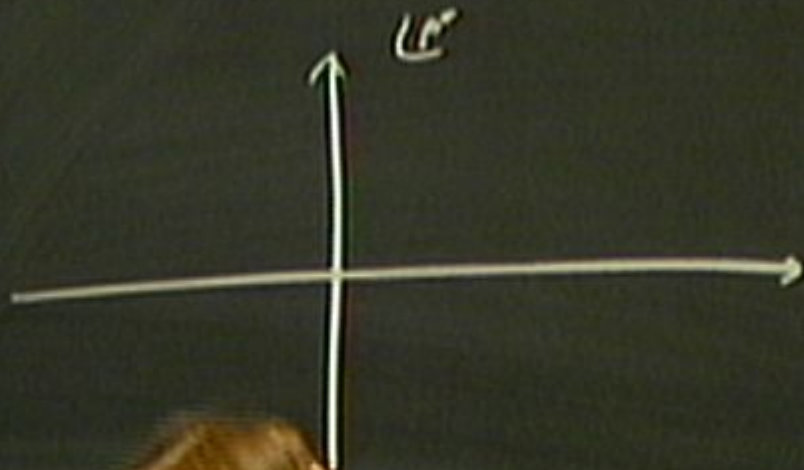
$$(p^2 - i\epsilon p^0)^{\frac{p-1}{2}} \Leftrightarrow (p^2 - i\epsilon)^{\frac{p-1}{2}}$$

$$I_m \Rightarrow \int \pi d(q^0) \pi \delta((q+p)^2) \text{sgn}(q^0) \rightarrow 0.$$

$$\left(\begin{matrix} q^0 \\ \vec{q} \end{matrix} \right), \left(\begin{matrix} -q^0 \\ \vec{q} \end{matrix} \right)$$

$$(p^2 - i\epsilon p^0)^{\frac{D-4}{2}} \Leftrightarrow (p^2 - i\epsilon)^{\frac{D-4}{2}}$$

$$(p^2)^{\frac{D-4}{2}}$$



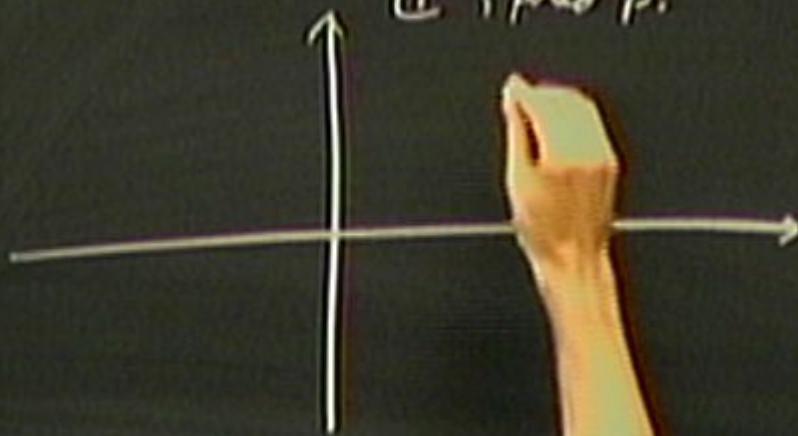
$$I_m \Rightarrow \int \pi d(q^0) \delta((q+p)^2) \text{sgn}(q^0) \rightarrow 0.$$

$$\left(\begin{matrix} q^0 \\ \vec{q} \end{matrix} \right), \left(\begin{matrix} -q^0 \\ \vec{q} \end{matrix} \right)$$

$$(p^2 - i\epsilon p^0)^{\frac{D-4}{2}} \Leftrightarrow (p^2 - i\epsilon)^{\frac{D-4}{2}}$$

$$(p^2)^{\frac{D-4}{2}}$$

ϵ^0 , fixed \vec{p} .



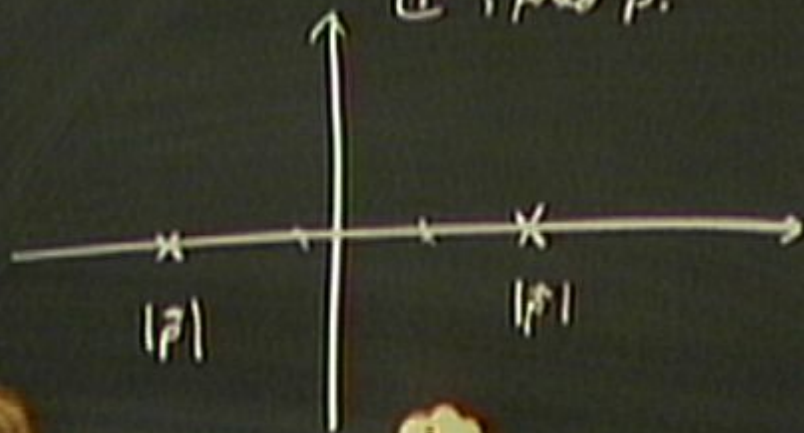
$$I_m \Rightarrow \int \pi d(q^0) \delta((q+p)^2) \text{sgn}(q^0) \rightarrow 0.$$

$$\left(\begin{matrix} q^0 \\ \vec{q} \end{matrix} \right), \left(\begin{matrix} -q^0 \\ \vec{q} \end{matrix} \right)$$

$$(p^2 - i\epsilon p^0)^{\frac{D-1}{2}} \Leftrightarrow (p^2 - i\epsilon)^{\frac{D-1}{2}}$$

\mathbb{R}^D , fixed \vec{p} .

$$(p^2)^{\frac{D-1}{2}}$$



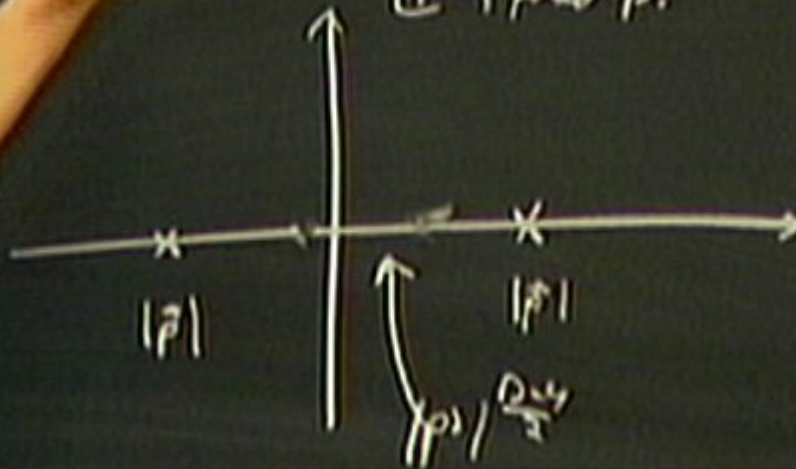
$$I_m \Rightarrow \int \pi d(q^0) \delta((q+p)^2) \text{sgn}(q^0) \rightarrow 0.$$

$$\begin{pmatrix} q^0 \\ \vec{q} \end{pmatrix}, \begin{pmatrix} -q^0 \\ \vec{q} \end{pmatrix}$$

$$(p^2 - i\epsilon p^0) \frac{p^0 - 1}{2} \Leftrightarrow (p^2 - i\epsilon) \frac{p^0 - 1}{2}$$

\mathbb{C}^0 , fixed \vec{p} .

$$(p^2) \frac{p^0 - 1}{2}$$



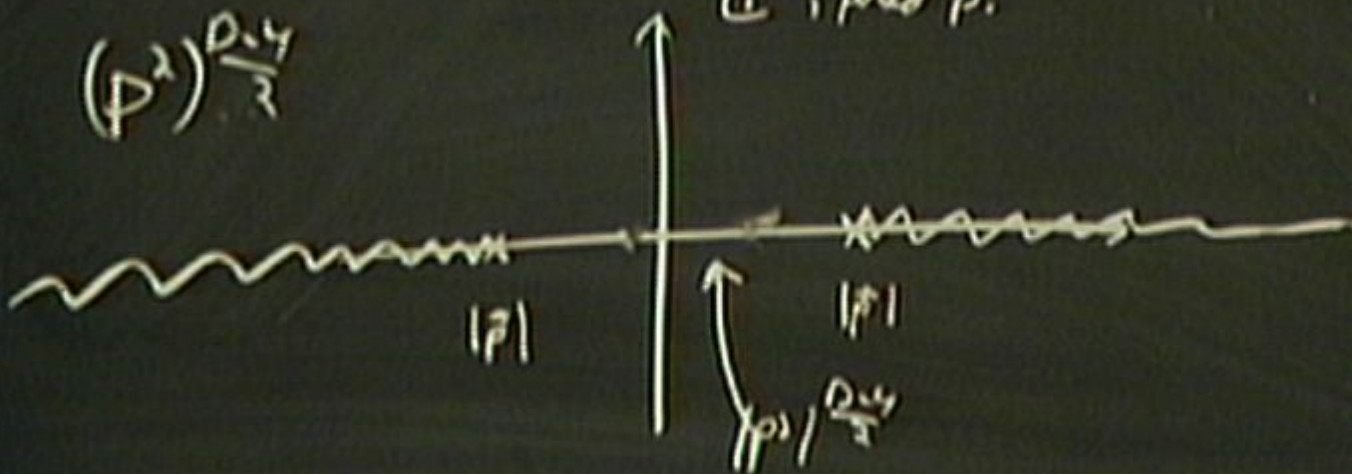
$$I_m \Rightarrow \int \pi d(q^0) \delta((q+p)^2) \text{sgn}(q^0) \rightarrow 0.$$

$$\left(\begin{matrix} q^0 \\ \vec{q} \end{matrix} \right), \left(\begin{matrix} -q^0 \\ \vec{q} \end{matrix} \right)$$

$$(p^2 - i\epsilon p^0)^{\frac{D-4}{2}} \Leftrightarrow (p^2 - i\epsilon)^{\frac{D-4}{2}}$$

\mathcal{L}^0 , fixed \vec{p} .

$$(p^2)^{\frac{D-4}{2}}$$



$$(p^0 + i\epsilon)^2 - \vec{p}^2$$

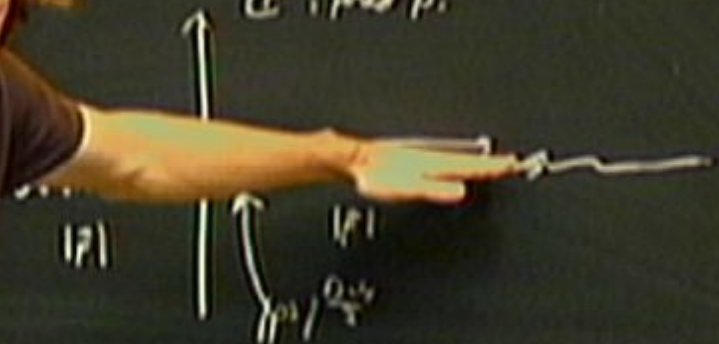
$$\text{Im} \Rightarrow \int_{-\infty}^{\infty} \frac{e^{i\omega t}}{\omega^2 + p^2} \sin(\omega t) d\omega \rightarrow 0.$$

$$\left(\begin{matrix} \omega \\ \omega \end{matrix} \right), \left(\begin{matrix} -\omega \\ \omega \end{matrix} \right)$$

$$(p^2 + i\epsilon)^2 - p^2$$

$$\frac{p^2 - i\epsilon}{2} \Leftrightarrow (p^2 - i\epsilon) \frac{p^2 - i\epsilon}{2}$$

LE, fixed p.



$$\text{Im} \Rightarrow \int_{-\infty}^{\infty} \tau d(\tau^2) \sqrt{((q+p)^2)} \sin(\tau^2) \rightarrow 0.$$

$$\left(\begin{matrix} q' \\ \tau \end{matrix} \right), \left(\begin{matrix} -q' \\ \tau \end{matrix} \right)$$

$$(p^2 - i\epsilon p^0)^{\frac{p-1}{2}} \quad (p^2 - i\epsilon)^{\frac{p-1}{2}}$$

$$(p^2)^{\frac{p-1}{2}}$$

$$(p^2 + i\epsilon)^2 - p^2$$



$$\text{Im} \Rightarrow \int_{-\infty}^{\infty} \frac{r d(q^2) \sqrt{s} ((q+p)^2)}{s^2 n(q^2)} \rightarrow 0.$$

$$\left(\begin{matrix} q^1 \\ q^0 \end{matrix} \right), \left(\begin{matrix} -q^0 \\ q^1 \end{matrix} \right)$$

$$\frac{(p^2 - i\epsilon p^0)^{\frac{D-4}{2}}}{(p^2 - i\epsilon)^{\frac{D-4}{2}}}$$

$$(p^0 + i\epsilon)^2 - p^2$$

$$(p^2)^{\frac{D-4}{2}}$$

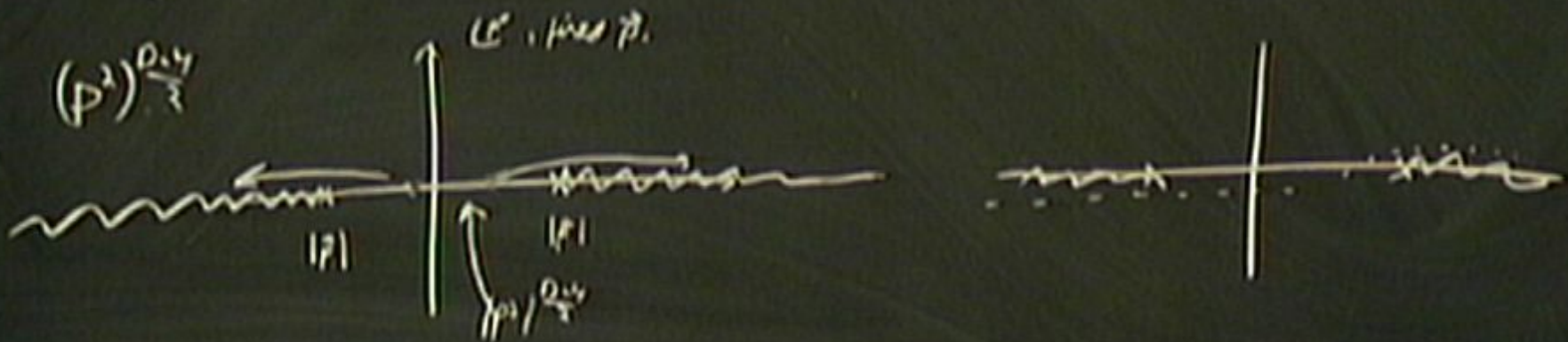


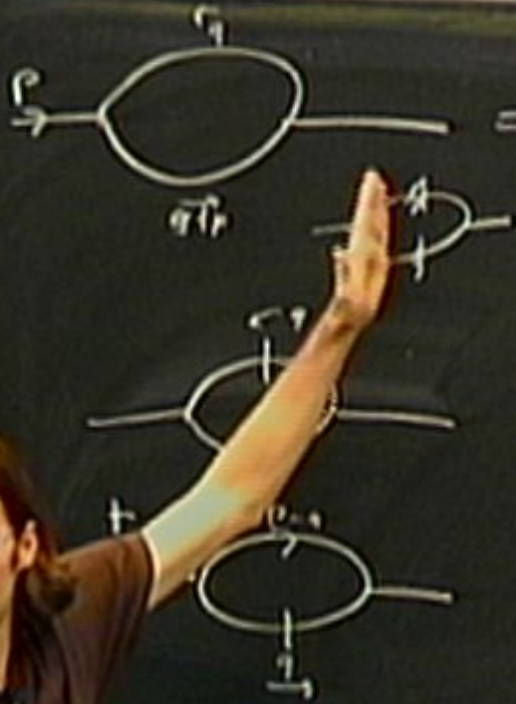
$$I_m \Rightarrow \int_{-\infty}^{\infty} \tau d(q^2) \delta((q+p)^2) \operatorname{sign}(q^2) \rightarrow 0.$$

$$\left(\frac{q^0}{\vec{q}} \right), \left(\frac{-q^0}{\vec{q}} \right)$$

$$(p^2 - i\epsilon p^0)^{\frac{D-1}{2}} \Leftrightarrow (p^2 - i\epsilon)^{\frac{D-1}{2}}$$

$$(p^0 + i\epsilon)^2 - \vec{p}^2$$





$$= \left\{ \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2(q+i)^2} \right\} \pi \rightarrow C' (p^2)^{\frac{D-4}{2}}$$

$$p = \begin{pmatrix} 0 \\ \vec{p} \end{pmatrix}$$

$$p^2 = -\vec{p}^2 + p^0^2$$

$$= \left\{ \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+i)^2 - i(q+i)} + \frac{1}{(p-i)^2 - i(p-i)} \right] \right\}$$

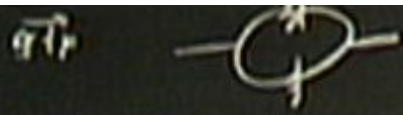
$$= C' (p^2)^{\frac{D-4}{2}} \left[1 + e^{-i\pi} \right] e^{i\pi \frac{D-4}{2}} 2 \cos \frac{\pi(D-4)}{2}$$

$$= C'' (p^2)^{\frac{D-4}{2}} e^{i\pi \frac{D-4}{2}}$$

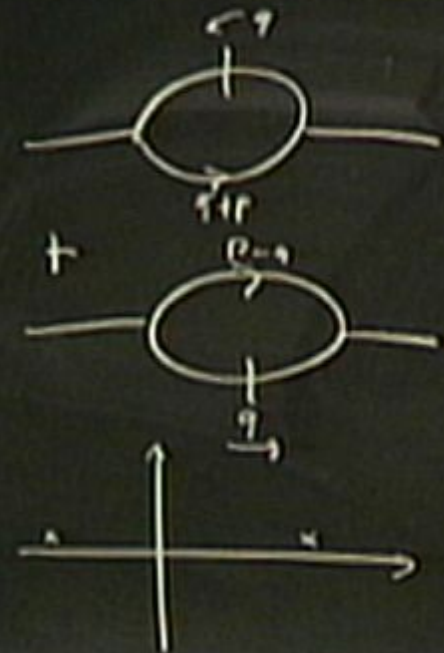
$$(-p^2 - i\epsilon)^{\frac{D-4}{2}} \rightarrow \left(-p^2 - i\epsilon \right)^{\frac{D-4}{2}}, p^2 > 0$$

$$\left(p^2 \right)^{\frac{D-4}{2}}$$

Specialize



$$P = \begin{pmatrix} p^0 \\ p^1 \\ p^2 \end{pmatrix}$$



$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q^2)^2 - i\epsilon(q^0 + r^0)} + \frac{1}{(p-q)^2 - i\epsilon(p^0 - q^0)} \right]$$

$$= C' (p^0)^{D-4} [1 + e^{-i\pi(D-4)}] e^{-i\pi \frac{D-1}{2}} \cos \frac{\pi(D-1)}{2}$$

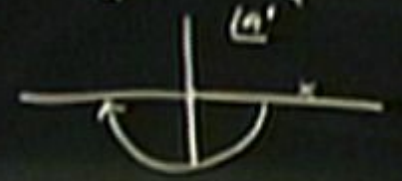
$$= C'' (p^0)^{\frac{D-4}{2}} e^{-i\pi \frac{D-4}{2}}$$

$$= (-p^2 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow \frac{(p^2 - i\epsilon p^0)^{\frac{D-4}{2}}}{(p^2)^{\frac{D-4}{2}}}, p^0 > 0.$$

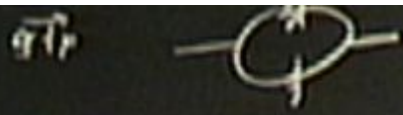
small like

$$p^0 > 0 \Rightarrow \int_0^\infty dq^0 (q^0)^{D-4}$$

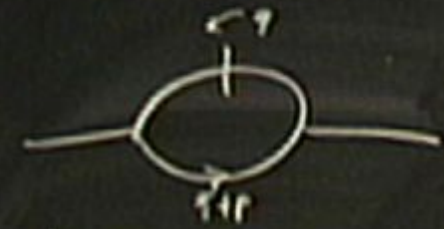
$$f \rightarrow \frac{p^0}{2} \Rightarrow \int_0^\infty \frac{du u^{D-4}}{u+1} [1 + e^{-i\pi(D-4)}]$$



$$\left[\frac{-1}{p^2 + 2q^0 - i\epsilon(q^0 + r^0)} + \frac{-1}{p^2 - 2q^0 + i\epsilon(p^0 - q^0)} \right]$$



$$P = \begin{pmatrix} p^0 \\ p^1 \\ p^2 \end{pmatrix}$$



$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+p)^2 - i\epsilon(q^0+p^0)} + \frac{1}{(p-q)^2 - i\epsilon(p^0-q^0)} \right]$$

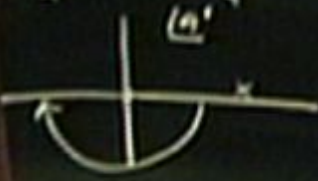
$$= C' (p^0)^{D-4} [1 + e^{-i\pi(D-4)}] e^{-i\pi \frac{D-1}{2}} \cos \frac{\pi(D-1)}{2}$$

$$= C'' (p^0)^{\frac{D-4}{2}} e^{-i\pi \frac{D-4}{2}}$$

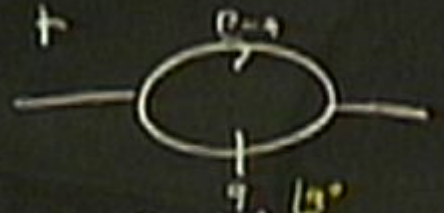
$$= (-p^0 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow \frac{(p^2 - i\epsilon p^0)^{\frac{D-4}{2}}}{(p^2)^{\frac{D-4}{2}}}, p^0 > 0.$$

space-like

$$\int_0^\infty \frac{du u^{n-1}}{u+1} \left[\frac{1}{p^2 + 2p^0 u - i\epsilon(p^0 + u)} + \frac{1}{p^2 - 2p^0 u + i\epsilon(p^0 - u)} \right]$$



$$p = \begin{pmatrix} p_0 \\ \vec{p} \end{pmatrix}$$



$$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+p)^2 - i\epsilon(q_0+p_0)} + \frac{1}{(p-q)^2 - i\epsilon(p_0-q_0)} \right]$$

$$= C' (p^0)^{D-4} [1 + e^{-i\pi(D-4)}] e^{-i\pi \frac{D-4}{2}}$$

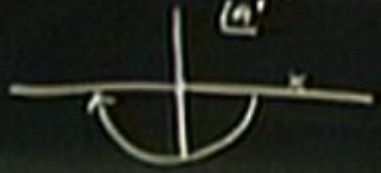
$$= C'' (p^0)^{\frac{D-4}{2}} e^{-i\pi \frac{D-4}{2}}$$

$$= \frac{(-p^2 - i\epsilon)^{\frac{D-4}{2}}}{(p^2)^{\frac{D-4}{2}}} \Rightarrow \frac{(p^2 - i\epsilon)^{\frac{D-4}{2}}}{(p^2)^{\frac{D-4}{2}}}, p^0 > 0.$$

speculato.

$$p^0 > 0, \frac{2}{p^0} \int_0^{p^0} dq^0 (q^0)^{D-3} \left[\frac{-1}{-p^2 - 2p^0 q^0 - i\epsilon(q^0+p^0)} + \frac{-1}{-p^2 + 2p^0 q^0 - i\epsilon(p^0-q^0)} \right]$$

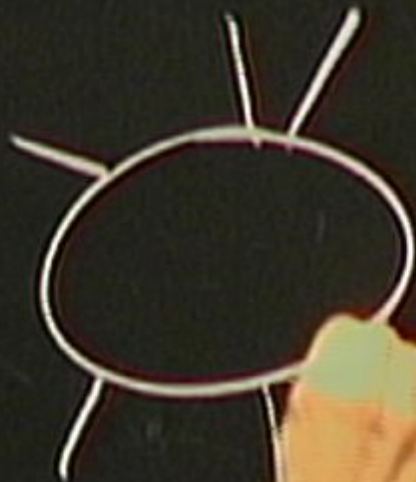
$$f \rightarrow \frac{p^0}{2} \int_0^1 \frac{du u^{D-3}}{u+1} [1 + e^{-i\pi(D-1)}] \left[\frac{-1}{p^2 + 2p^0 u} + \frac{-1}{p^2 - 2p^0 u + i\epsilon} \right]$$



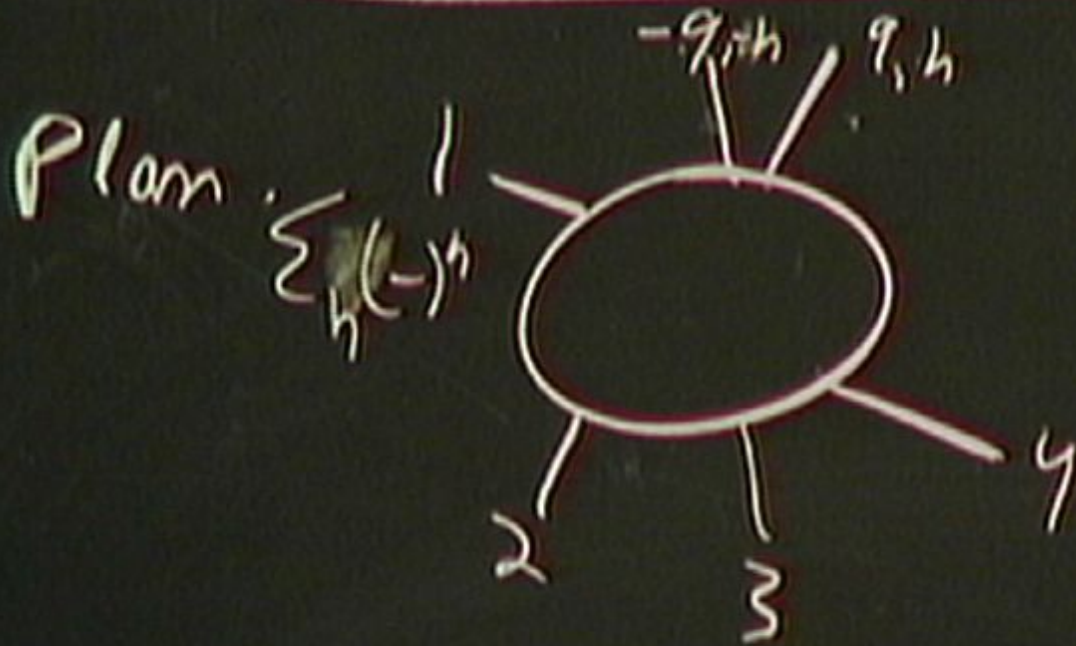
MHV 4-pt amp. 1-loop, $N=4$.

MITV 4-pt amp. 1-loop, $N=4$.

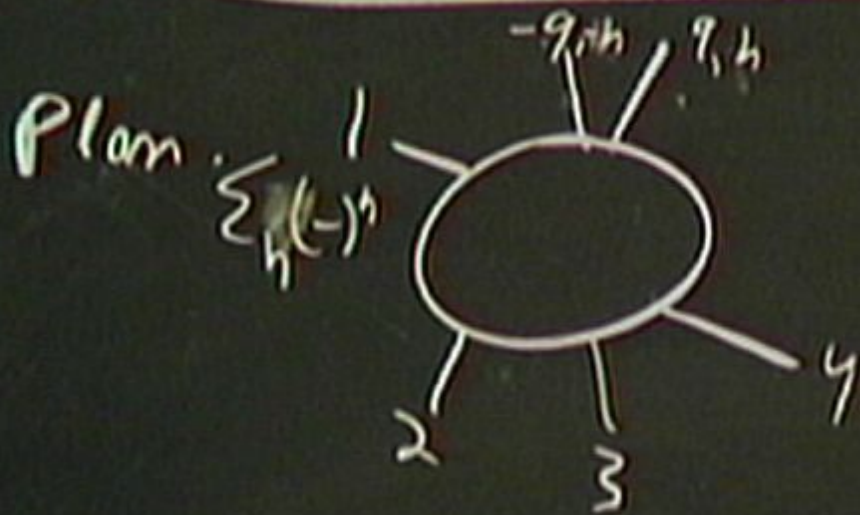
Plan.



MITV 4-pt amp. 1-loop, $N=4$.

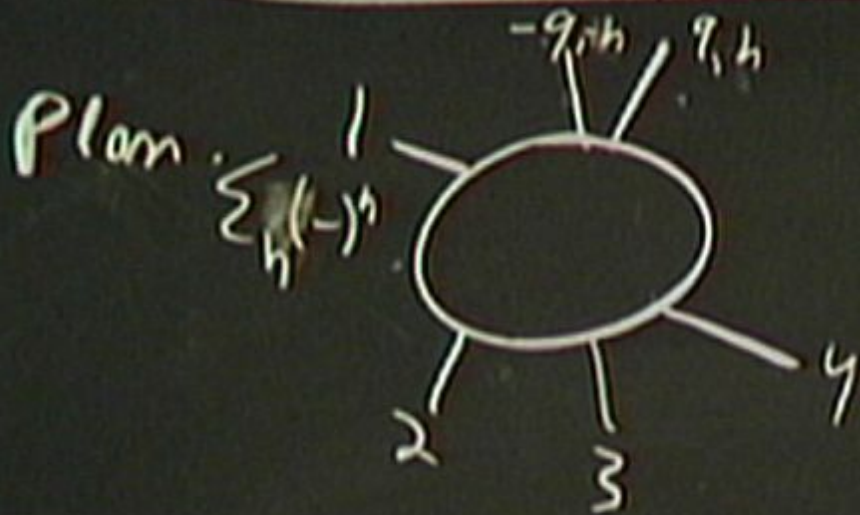


MHV 4-pt amp. 1-loop, $N=4$.



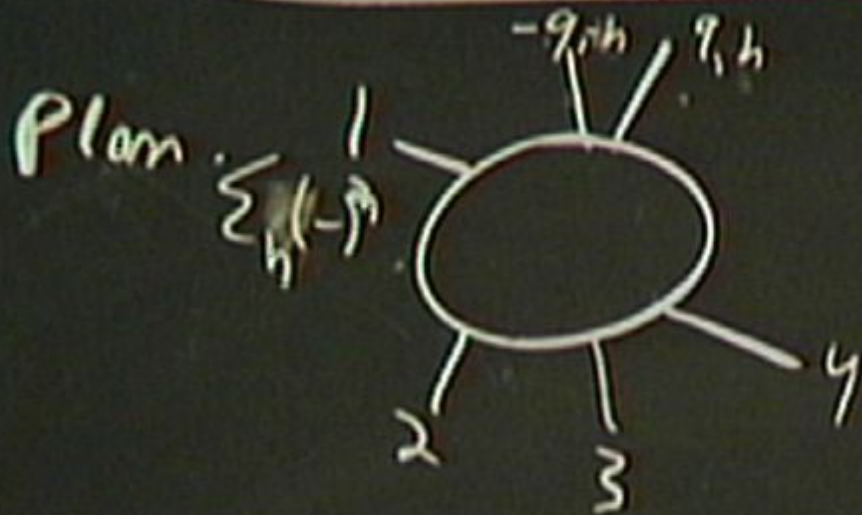
NMHV 6-pt amplitude

MHV 4-pt amp. 1-loop, $N=4$.



NMHV 6-pt amplitude

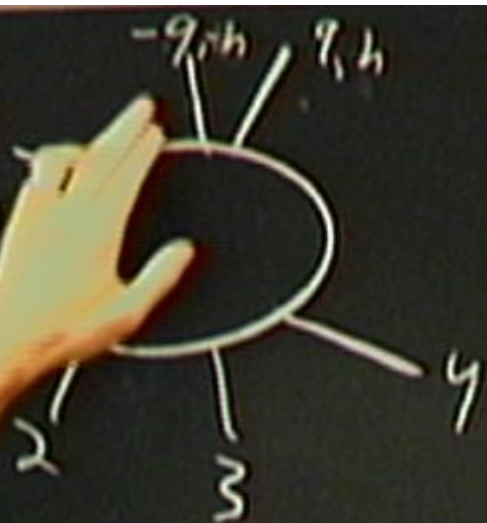
MHV 4-pt amp. 1-loop, $N=4$.



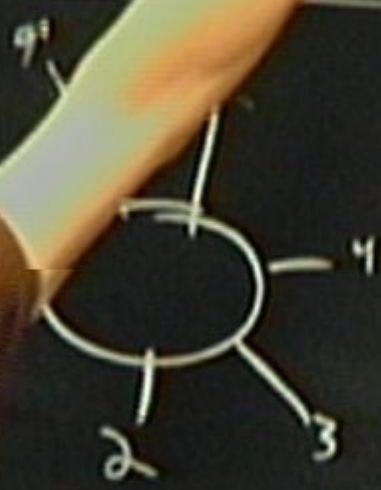
NMHV 6-pt amplitude



Plan $\sum_h (-1)^h$



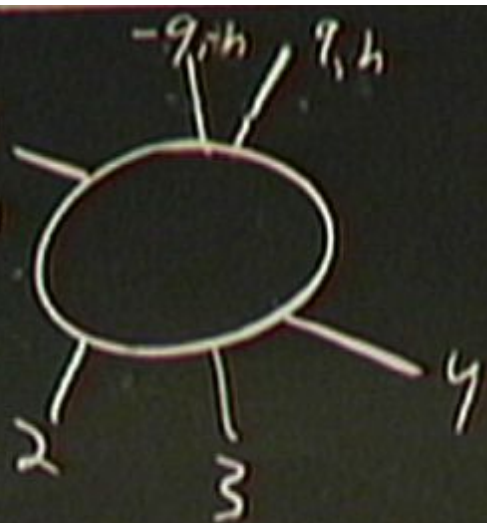
NMHV 6-pt amplitude



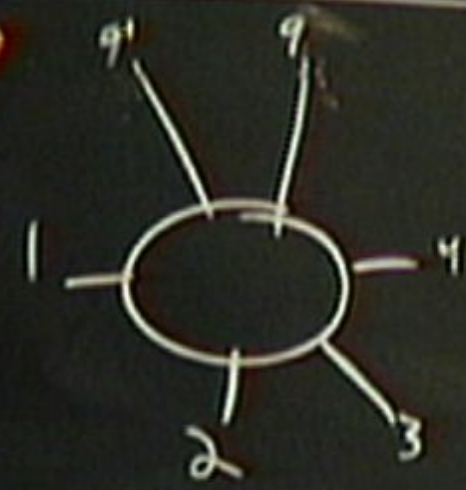
$$\lambda_{q'} = \lambda_{q'} + z \lambda_q$$

$$\bar{\lambda}_q = \bar{\lambda}_q - z \bar{\lambda}_{q'}$$

Plan $\sum_h (-1)^m$

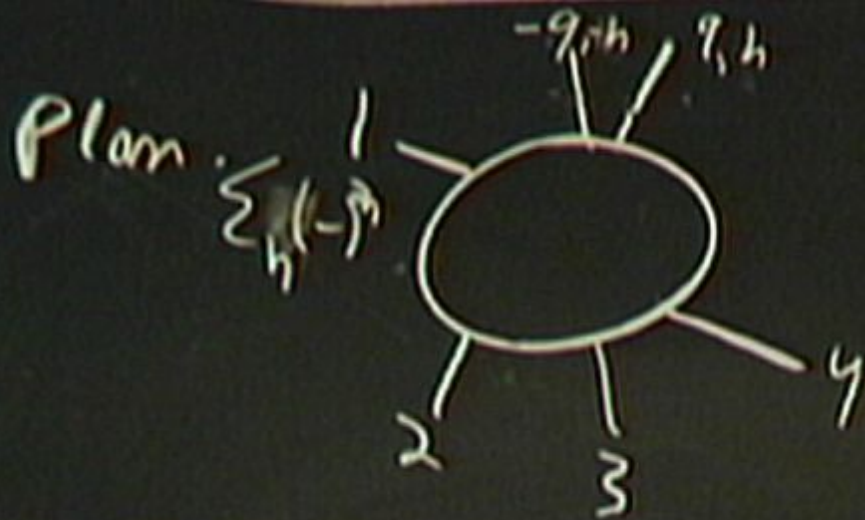


NMHV 6-pt amplitude

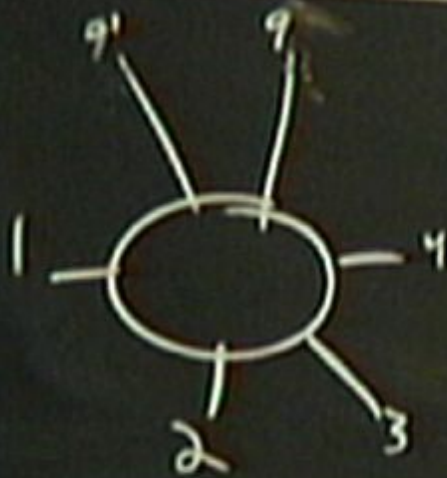


$$\lambda_{q'} = \lambda_{q'} + z \lambda_q$$

$$\bar{\lambda}_q = \bar{\lambda}_q - z \bar{\lambda}_{q'}$$



NMHV 6-pt amplitude

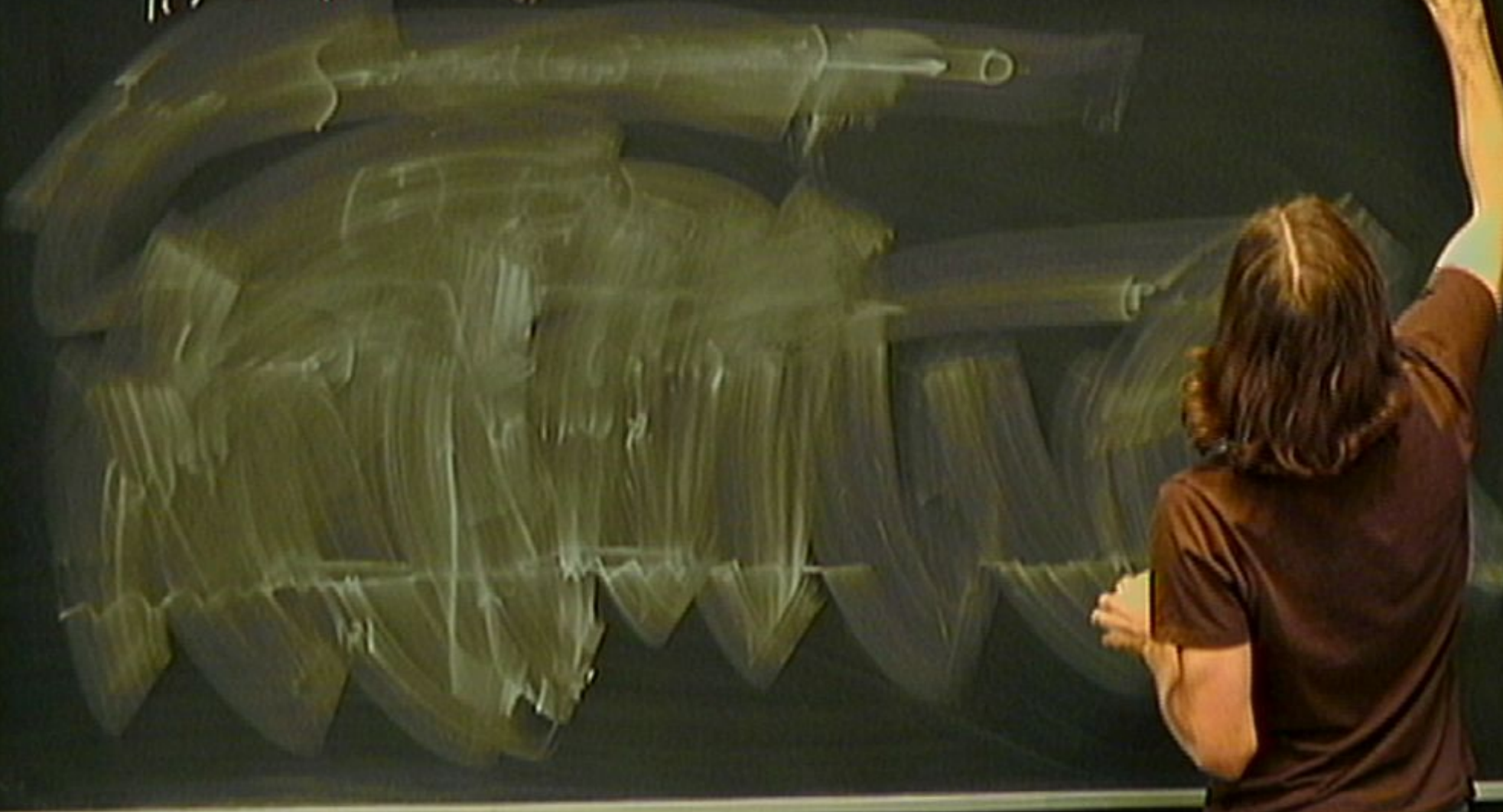


$$\begin{aligned} \lambda_{q'} &= \lambda_{q'} + z \lambda_q \\ \lambda_q &= \lambda_q - z \lambda_{q'} \\ \tilde{\eta}_q^A &= \tilde{\eta}_q^A - z \tilde{\eta}_{q'} \end{aligned}$$

$$|i\rangle = |H\rangle + \sum_{n=1}^4 c_n |\psi_n\rangle +$$

$$n=1 \dots 4$$

$$|i\rangle = |1\rangle + \eta^A |\psi_A\rangle + \frac{\eta^A \eta^B}{2} |\psi_{AB}\rangle + \frac{\eta^A \eta^B \eta^C}{3!} \epsilon_{ABC} |\psi_{ABC}\rangle$$



$$|i\rangle = |1\rangle + \eta^A |4\rangle + \frac{\eta^A \eta^0}{2} |9\rangle + \frac{\eta^A \eta^0 \epsilon_{\lambda\mu\nu} |\sqrt{0}\rangle}{3!} + \frac{(\eta^A)^4}{4!} |-\rangle$$

$$|i\rangle = |1\rangle + \eta^A |4\rangle + \frac{\eta^A \eta^B}{2} |9\rangle + \frac{\eta^A \eta^B \eta^C \epsilon_{ABCD} |\sqrt{0}\rangle + \frac{(\eta^A)^4}{4!} |-\rangle.$$

$$A_{ij}^{kl} = \frac{\delta^k(\epsilon \lambda n)}{\langle i | \lambda \rangle \dots \langle j | \lambda \rangle}$$



$$|i\rangle = |1\rangle + \eta^A |4\rangle + \frac{\eta^A \eta^B}{2} |6\rangle + \frac{\eta^A \eta^B \eta^C \epsilon_{ABCD}}{3!} |10\rangle + \frac{(\eta^A)^4}{4!} |15\rangle$$

$$P_i^{\alpha\dot{\alpha}} = P_i^\alpha \sigma_{\alpha\dot{\alpha}}^i = \lambda_i^\alpha \bar{\lambda}_{\dot{\alpha}i}$$

$$A_{\mu\nu}^{\alpha\dot{\alpha}} = \frac{\delta^{\mu\nu}(\epsilon \lambda \eta)}{\langle 12 \rangle \dots \langle 41 \rangle}$$

$$|i\rangle = |1\rangle + \eta^A |4\rangle + \frac{\eta^A \eta^B}{2} |9\rangle + \frac{\eta^A \eta^B \eta^C \epsilon_{ABCD}}{3!} |\sqrt{0}\rangle + \frac{(\eta^A)^4}{4!} |-\rangle$$

$$P_i^{\alpha\beta} \equiv P_i^\alpha \sigma_{\alpha\beta}^i = \lambda \sigma^i$$

$$A_{\mu\nu}^{\alpha\beta} = \frac{\delta^{\mu\nu}(\epsilon \lambda n)}{c_{12} c_{34}}$$

$$+ + - - : n_3^x n_4^y$$

δ^2

$$|i\rangle = |1\rangle + n_1^A |4\rangle + \frac{\lambda^1 n^0}{2} |4_{11}\rangle + \frac{\lambda^A \lambda^A n^C}{3!} \epsilon_{AA10} |\sqrt{0}\rangle + \frac{(\lambda^1)^4}{4!} |1-\rangle$$

$$P_i^{aa} \equiv P_i^a \sigma_{ii}^{aa} \Rightarrow \lambda_i^a \delta_i^a$$

$$A_4^{HHV} = \frac{\delta^R(\Sigma \lambda n)}{c_{12} \dots c_{41}}$$

$$+ + \dots : n_3^4 n_1^4$$

$$\Leftrightarrow \pi \delta^2(\lambda_3 n_3^4 + \lambda_1 n_1^4)$$

$$|i\rangle = |1\rangle + \eta^A |4\rangle + \frac{\eta^A \eta^B}{2} |6\rangle + \frac{\eta^A \eta^B \eta^C \epsilon_{ABCD}}{3!} |\sqrt{0}\rangle + \frac{(\eta^A)^4}{4!} |-\rangle$$

$$P_i^{aa} \equiv P_i^a \sigma_{ii}^a = \lambda \sigma_i^a$$

$$A_4^{abcd} = \frac{\delta^R(\Sigma \lambda \eta)}{\langle 12 \rangle \dots \langle 41 \rangle}$$

$$= \eta_3^3 \eta_4^4$$

$$\Leftrightarrow \pi \delta^3(\lambda_3 \eta_3^A + \lambda_4 \eta_4^A) = \epsilon_{342} \eta_3 \eta_4$$

$$|i\rangle = |1\rangle + \eta^A |4\rangle + \frac{\lambda^A \eta^0}{2} |F_{AC}\rangle + \frac{\eta^A \lambda^A \eta^C \epsilon_{AC10} |\sqrt{0}\rangle + \frac{(\eta^A)^4}{4!} |-\rangle.$$

$$P_i^{aa} \equiv P_i^a \sigma_{ii}^a \Rightarrow \lambda^a \delta^a_i.$$

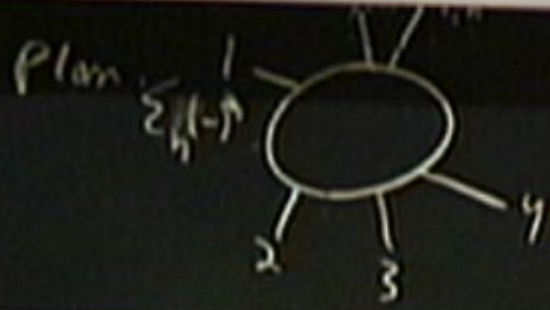
$$A_4^{ijkl} = \frac{\delta^k(\epsilon \lambda^i)}{\langle 12\rangle \dots \langle 41\rangle}$$

$$+ + - - : \eta_3^4 \eta_4^4$$

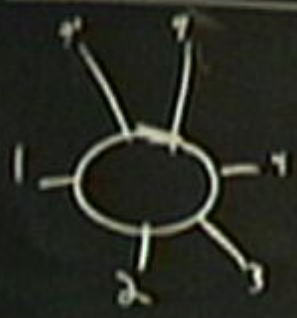
$$\frac{\langle 34\rangle^4}{\langle 12\rangle \dots \langle 41\rangle}$$

$$\hookrightarrow \pi \delta^2(\lambda_3 \eta_3^4 + \lambda_4 \eta_4^4) = \langle 34\rangle^4 \eta_3 \eta_4.$$

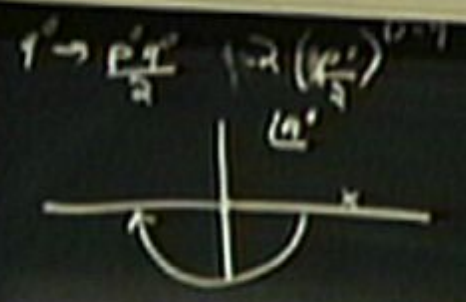
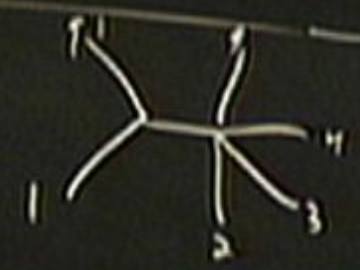
$(p^2)^{\frac{d-2}{2}}$ spacetime



NMHV 6-pt amplitude

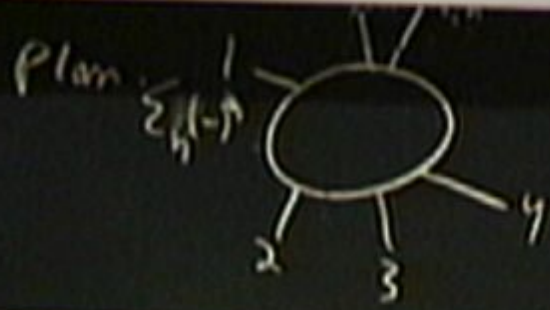


$$\begin{aligned} \tilde{\lambda}_{1'} &= \lambda_{1'} + z \lambda_4 \\ \tilde{\lambda}_4 &= \lambda_4 - z \lambda_{1'} \\ \tilde{n}_4^\wedge &= n_4^\wedge - z n_{1'} \end{aligned}$$

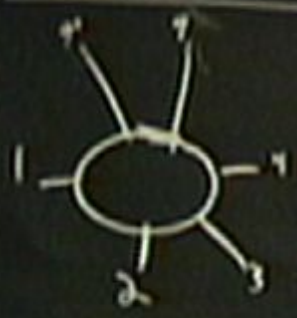


$$\left[\frac{-2q^2 + i\epsilon}{(0-y)} \right]$$

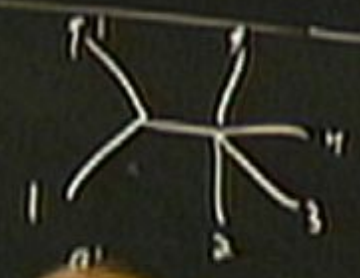
$(p^2)^{\frac{u+1}{2}}$ spacetime



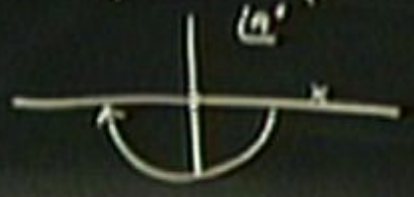
NMHV 6-pt amplitude



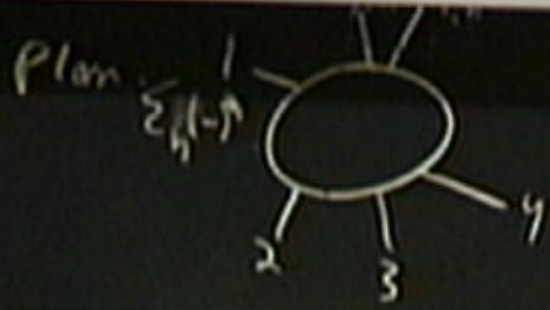
$$\begin{aligned} \tilde{\lambda}_{1'} &= \lambda_{1'} + z \lambda_4 \\ \tilde{\lambda}_4 &= \lambda_4 - z \tilde{\lambda}_{1'} \\ \tilde{n}_4^\wedge &= n_4^\wedge - z n_{4'} \end{aligned}$$



$i \rightarrow \frac{p_i}{2} \quad \int_0^1 \frac{du u^{u-3}}{u+1} \left[\dots \right]$



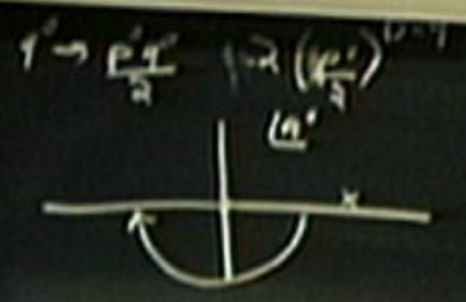
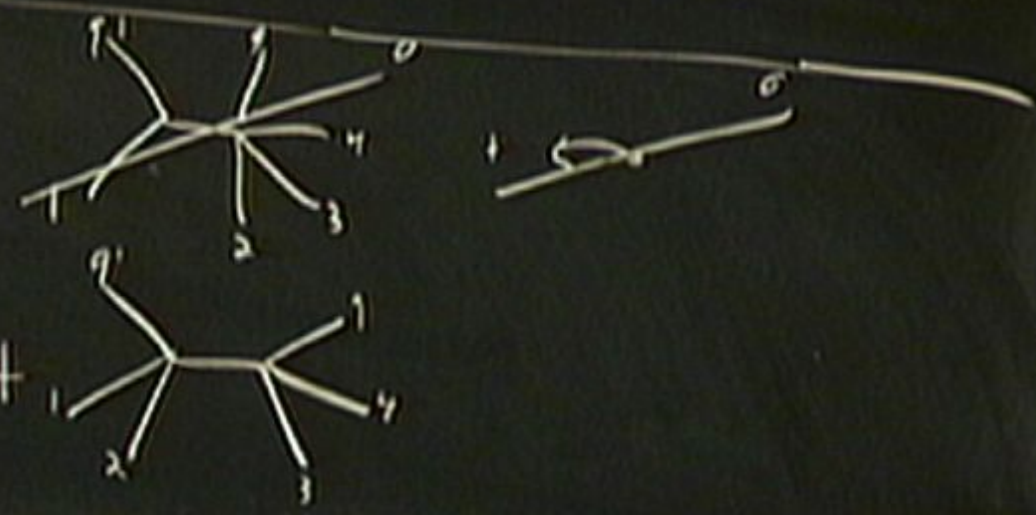
$(p^2)^{\frac{d-1}{2}}$ specialise.



NMHV 6-pt amplitude

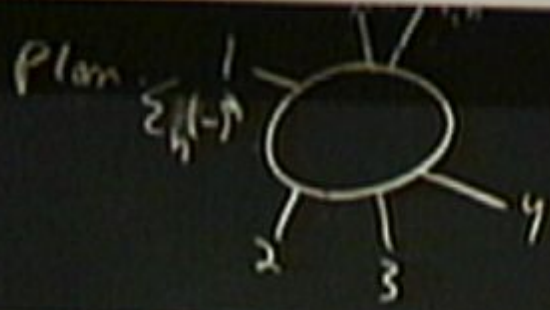


$$\begin{aligned} \tilde{\lambda}_{1'} &= \lambda_{1'} + z \lambda_4 \\ \tilde{\lambda}_4 &= \lambda_4 - z \lambda_{1'} \\ \tilde{n}_4^\wedge &= n_4^\wedge - z n_{1'} \end{aligned}$$

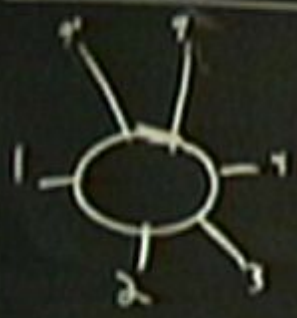


$$\int_0^\infty \frac{du u^{0-\gamma}}{u+1} [1 + e^{-i\pi(0-\gamma)}]$$

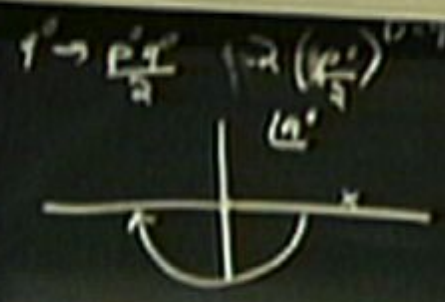
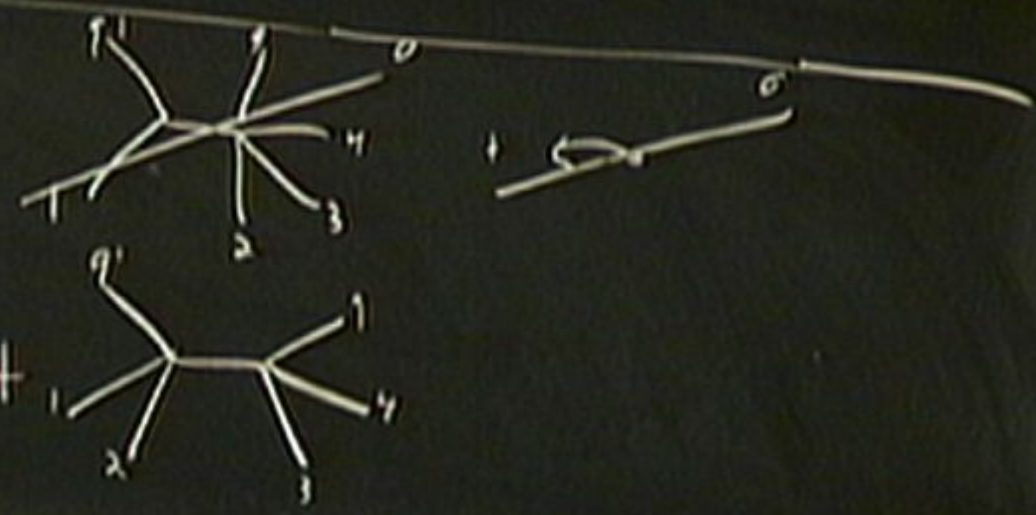
$(p^2)^{\frac{d-2}{2}}$ Speculator



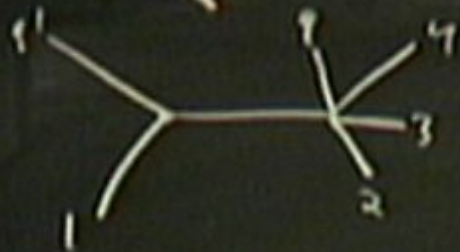
NMHV 6-pt amplitude

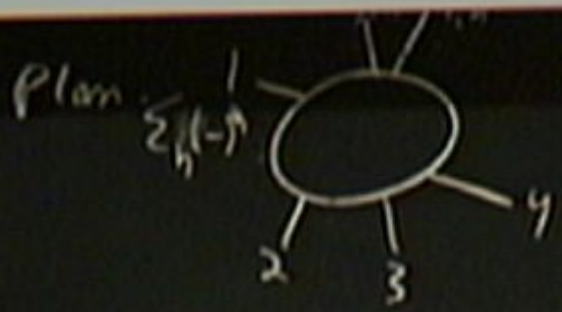


$$\begin{aligned} \lambda_{1'} &= \lambda_{1'} + z \lambda_{4'} \\ \lambda_{4'} &= \lambda_{4'} - z \lambda_{1'} \\ \tilde{n}_{1'}^\wedge &= \tilde{n}_{4'}^\wedge - z \tilde{n}_{4'} \end{aligned}$$



$$\int_0^\infty \frac{du u^{0-3}}{u+1} \left[1 + e^{-i\pi(0-\gamma)} \right]$$

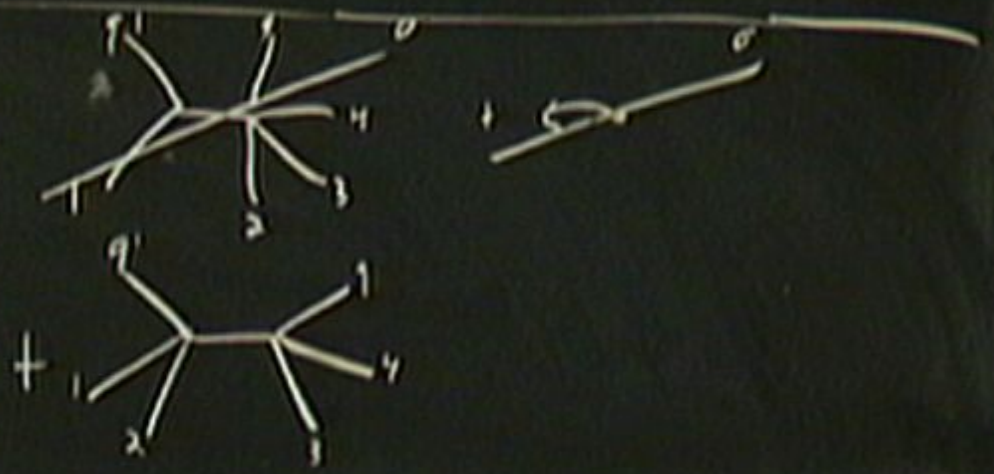




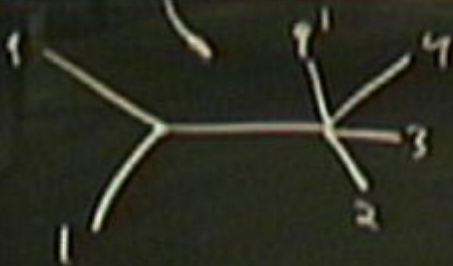
NMHV 6-pt amplitude



$$\begin{aligned} \lambda_{q_i} &= \lambda_{q_i} + z \lambda_{q_j} \\ \lambda_{q_j} &= \lambda_{q_j} - z \lambda_{q_i} \\ \tilde{\lambda}_{q_i} &= \tilde{\lambda}_{q_i} - z \tilde{\lambda}_{q_j} \end{aligned}$$



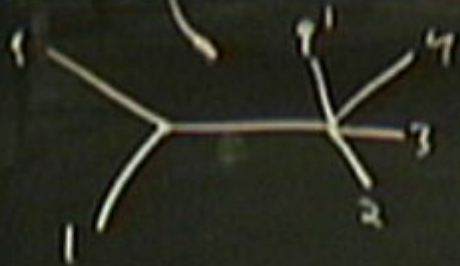
$$\int_0^1 \frac{du u^{u-1}}{u+1} \left[1 + e^{-i\pi(0-\gamma)} \right]$$



9



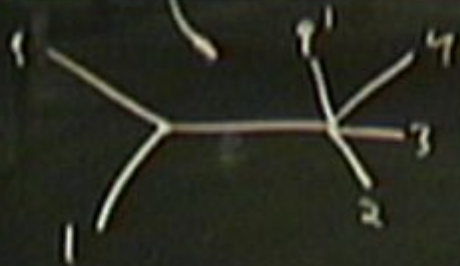
$$\{P_1 + () \}$$



$$[P_1 + \lambda_1(\bar{\lambda}_1 - z_0 \bar{\lambda}_1)]^2 = 0$$

$$[q_{11}] - z_0 [q_{11}] = 0$$

$$\frac{[q_{11}]}{[q_{11}]}$$



$$\left[P_1 + \lambda_1 (\bar{\lambda}_1 - z_0 \bar{\lambda}_1) \right]^2 = 0$$

$$[q_{11}] - z_0 [q'_{11}] = 0$$

$$z_0 = \frac{[q_{11}]}{[q'_{11}]}$$



$$\int d^4 n_M \int d^4 n_1 \delta^4(\lambda_1 n_1 + \lambda_M n_M + \lambda_1 (n_1 - z n_{1'}))$$

$$[P_1 + \lambda_1 (\bar{\lambda}_1 - z \bar{\lambda}_{1'})]^2 = 0$$

$$[q_1] - z_0 [q_{1'}] = 0$$

$$z_0 = \frac{[q_1]}{[q_{1'}]}$$



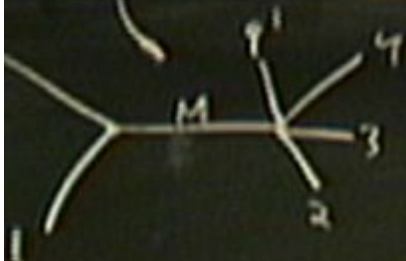
$$\int d^4 n_M \int d^4 n_1 \delta^4(\lambda_1 n_1 + \lambda_M n_M + \lambda_4 (n_4 - z n_4))$$

$$\times \delta^4(\lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4 + (\lambda_1 + z \lambda_1) n_4)$$

$$[P_1 + \lambda_1 (\bar{\lambda}_1 - z \bar{\lambda}_1)]^2 = 0$$

$$[q_1] - z_0 [q_1]$$

$$z_0 = \frac{[q_1]}{[q_1]}$$



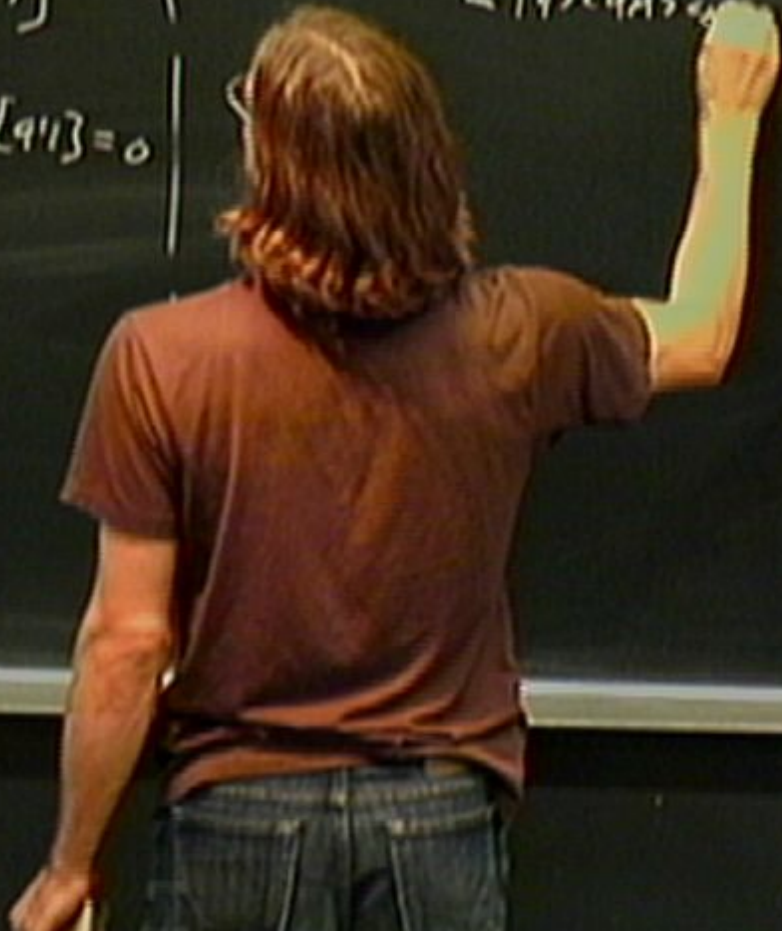
$$\int d^4 n_M \int d^4 n_T \delta^8(\lambda_1 n_1 + \lambda_M n_M + \lambda_T (n_T - z_0 n_{T'})) \times \delta^8(\lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4 + (\lambda_1 + z_0 \lambda_1) n_{T'})$$

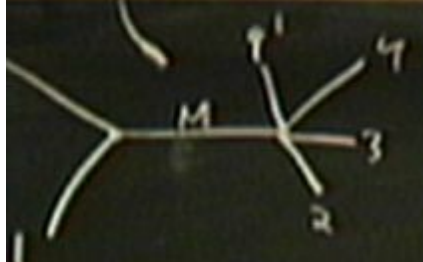
$$[P_1 + \lambda_1 (\lambda_1 - z_0 \lambda_1)]^2 = 0$$

$$[q_{13}] - z_0 [q'_{13}] = 0$$

$$z_0 = \frac{[q_{13}]}{[q'_{13}]}$$

$$\langle 1q \rangle \langle qM \rangle$$





$$\int d^n n_M \int d^n n_T \delta^D(\lambda_1 n_1 + \lambda_M n_M + \lambda_T (n_T - z_0 n_1)) \times \delta^D(\lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4 + (\lambda_1 + z_0 \lambda_T) n_1)$$

$$[P_1 + \lambda_1 (\lambda_1 - z_0 \lambda_1)]^2 = 0$$

$$[q_1] = 0$$

$$z_0 = \frac{[q_1]}{[q_2]}$$

$$\langle 1q \rangle \langle qM \rangle \langle M1 \rangle \langle q'1 \rangle \langle q'4 \rangle \langle 43 \rangle \langle 32 \rangle \langle 2M \rangle$$

CASE

$$\int d^4 n_M \int d^4 n_q \delta^8(\lambda_1 n_1 + \lambda_M n_M + \lambda_q (n_q - z n_{q'}))$$

$$\times \delta^8(\lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4 + (\lambda_{q'} + z_0 \lambda_q) n_{q'})$$

$$\times \frac{1}{\langle 1q \rangle \langle qM \rangle \langle M1 \rangle \langle q'M \rangle \langle q'4 \rangle \langle 43 \rangle \langle 32 \rangle \langle 2M \rangle}$$

$$\lambda_q \rightarrow \lambda_{q'}$$

$$\bar{\lambda}_q \rightarrow \bar{\lambda}_{q'}$$

$$n_q \rightarrow n_{q'}$$

$$z_0 \rightarrow$$

$$\int d^4 n_M \int d^4 n_q \delta^8(\lambda_1 n_1 + \lambda_M n_M + \lambda_q (n_q - z_0 n_{q'}))$$

$$\times \delta^8(\lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4 + (\lambda_{q'} + z_0 \lambda_q) n_{q'})$$

$$\times \frac{1}{\langle 1q \rangle \langle qM \rangle \langle M1 \rangle \langle q'M \rangle \langle q'4 \rangle \langle 43 \rangle \langle 32 \rangle \langle 2M \rangle}$$

$$\lambda_q \rightarrow \lambda_{q'}$$

$$\bar{\lambda}_q \rightarrow \bar{\lambda}_{q'}$$

$$n_q \rightarrow n_{q'}$$

$$N_c \rightarrow 1$$

$$\int d^4 n_M \int d^4 n_q \delta^8(\lambda_1 n_1 + \lambda_M n_M + \lambda_q (n_q - z n_{q'}))$$

$$\times \delta^8(\lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4 + (\lambda_{q'} + z_0 \lambda_q) n_{q'})$$

$$\times \langle 1q \rangle \langle qM \rangle \langle M1 \rangle \langle q'1 \rangle \langle 1q' \rangle \langle q'3 \rangle \langle 32 \rangle \langle 2M \rangle$$

$$\lambda_4 + z_0 \lambda_q \rightarrow 0$$

- $\lambda_q \rightarrow \lambda_{q'}$
- $\bar{\lambda}_q \rightarrow \bar{\lambda}_{q'}$
- $n_q \rightarrow n_{q'}$
- $N_0 \rightarrow 1$

Case 1

$$\int d^4 n_M \int d^4 n_q \delta^8(\lambda_1 n_1 + \lambda_M n_M + \lambda_q (n_q - z_0 n_{q'}))$$

$$\times \delta^8(\lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4 + (\lambda_{q'} + z_0 \lambda_q) n_{q'})$$

$\langle 1q \rangle \langle qM \rangle \langle M1 \rangle \langle q'M \rangle \langle q'4 \rangle \langle 43 \rangle \langle 32 \rangle \langle 2M \rangle$

$\lambda_4 + z_0 \lambda_q \rightarrow 0$

$$\int d^4 n_M \int d^4 n_q \delta^8(\lambda_1 n_1 + \lambda_M n_M + \lambda_q (n_q - z_0 n_{q'})) \times \delta^8(\lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4 + (\lambda_{q'} + z_0 \lambda_q) n_{q'})$$

$$|q\rangle \langle q_M\rangle \langle M1\rangle \langle q'M\rangle \langle q'4\rangle \langle 43\rangle \langle 32\rangle \langle 2M\rangle$$

$$\lambda_4 + z_0 \lambda_q \rightarrow 0$$

$$\sim \frac{\epsilon^4}{\epsilon^2} , \epsilon \rightarrow 0$$

$$\begin{aligned} & \lambda_{q'} \\ & \leftarrow \lambda_{q'} \\ & \leftarrow n_{q'} \\ & \leftarrow n_{q'} \\ & \leftarrow |0\rangle \end{aligned}$$

$$\int d^4 n_M \int d^4 n_q \delta^8 \left(\lambda_1 n_1 + \lambda_M n_M + \lambda_q (n_q - z n_{q'}) \right) \\ \times \delta^8 \left(\lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4 + (\lambda_{q'} + z_0 \lambda_q) n_{q'} \right)$$

$$\times \langle 1q \rangle \langle qM \rangle \langle M1 \rangle \langle q'M \rangle \langle q'4 \rangle \langle 43 \rangle \langle 32 \rangle \langle 2M \rangle$$

$$\begin{aligned} \lambda_q &\rightarrow \lambda_{q'} \\ \bar{\lambda}_q &\rightarrow \bar{\lambda}_{q'} \\ n_q &\rightarrow n_{q'} \\ |0\rangle_M &\rightarrow |0\rangle_{q'} \end{aligned}$$

$$\lambda_4 + z_0 \lambda_q \rightarrow 0$$

$$\sim \frac{\epsilon^4}{\epsilon^2} , \epsilon \rightarrow 0$$

$$\int d^4 n_M \int d^4 n_q \delta^8 \left(\lambda_1 n_1 + \lambda_M n_M + \lambda_q (n_q - z n_{q'}) \right) \\ \times \delta^8 \left(\lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4 + (\lambda_{q'} + z_0 \lambda_q) n_{q'} \right)$$

$$\times \langle 1q \rangle \langle qM \rangle \langle M1 \rangle \langle q'M \rangle \langle q'4 \rangle \langle 43 \rangle \langle 32 \rangle \langle 2M \rangle$$

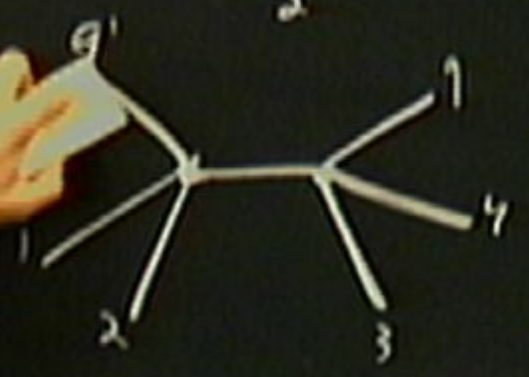
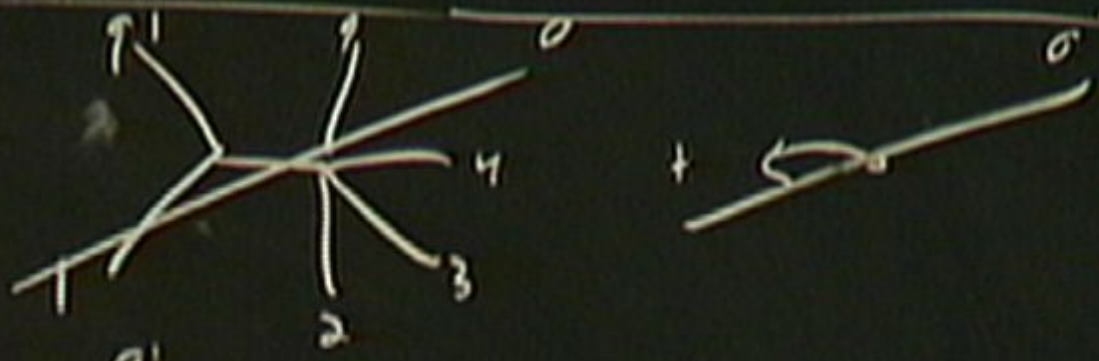
$$\begin{aligned} \lambda_q &\rightarrow \lambda_{q'} \\ \bar{\lambda}_q &\rightarrow \bar{\lambda}_{q'} \\ n_q &\rightarrow n_{q'} \\ |0_M\rangle &\rightarrow | \end{aligned}$$

$$\sim \frac{\epsilon^4}{\epsilon^2} \quad , \quad \epsilon \rightarrow 0$$

$$q_i + \sum \lambda_j$$

$$q_i - \sum \lambda_j$$

$$q_i - \sum n_j$$

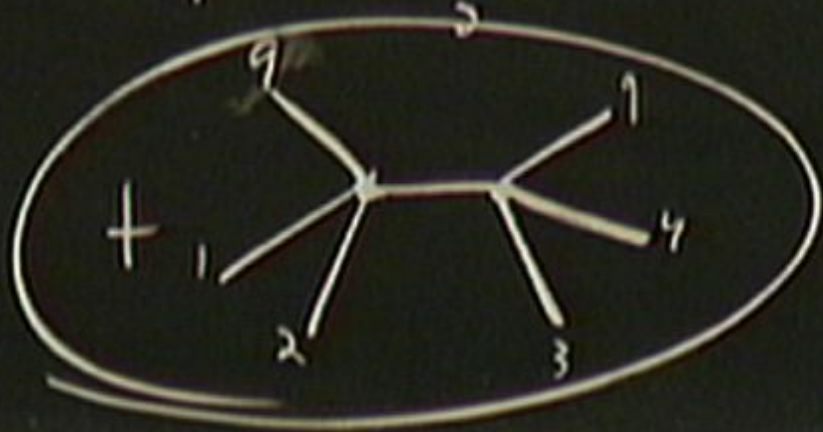
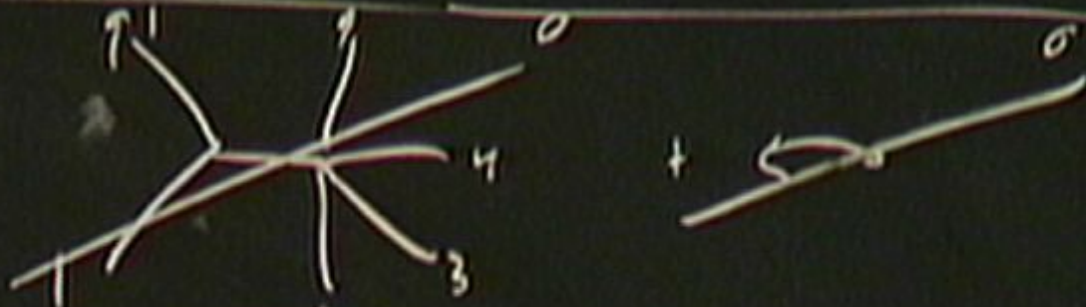


$$\frac{du u^{p-3}}{u+1} \left[1 + e^{-in(D-4)} \right]$$

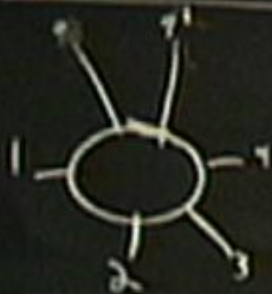
$$q' + z \lambda q$$

$$q - z \bar{\lambda} q'$$

$$q^A - z \eta q'$$



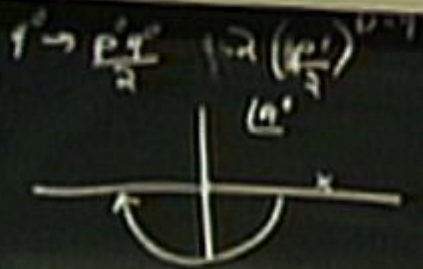
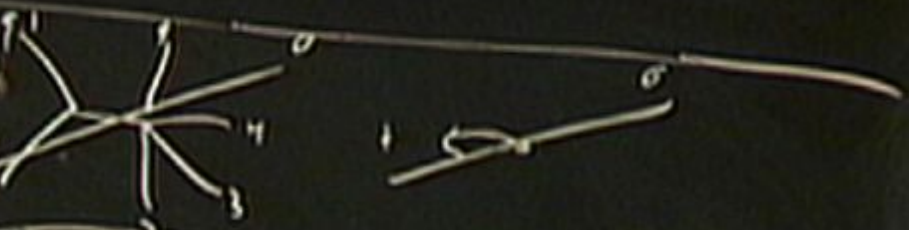
$$\frac{d\mu^{\rho-3}}{\mu+1} \left[1 + e^{-in(D-4)} \right]$$



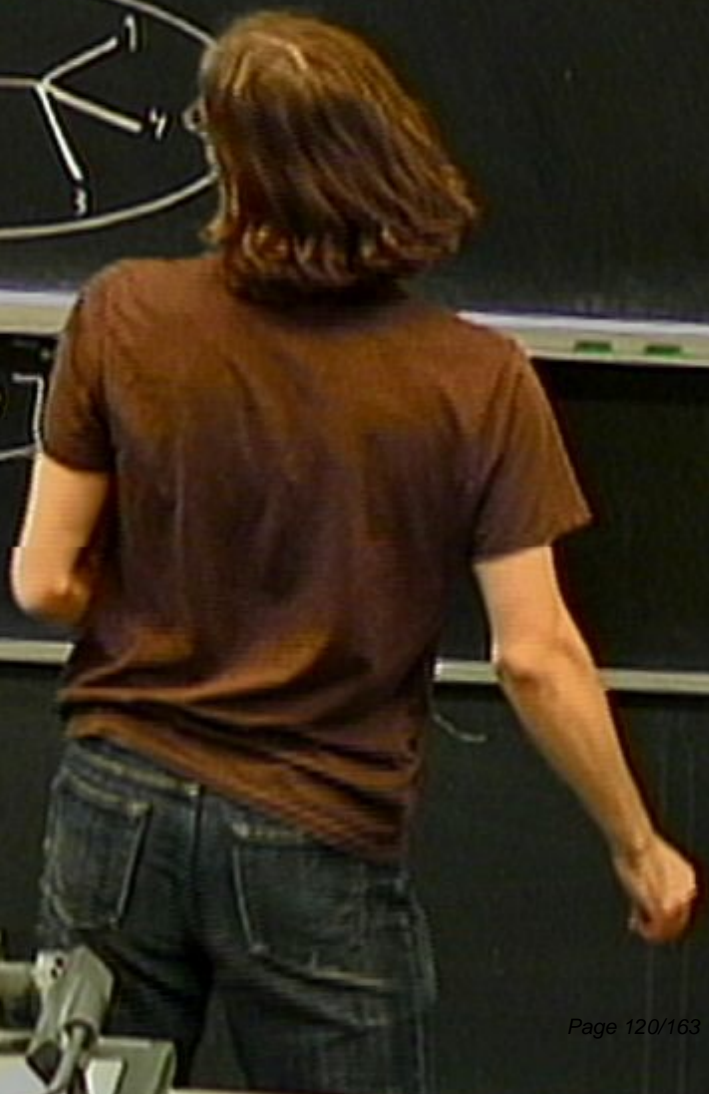
$$\lambda_1' = \lambda_1 + z \lambda_4 \rightarrow \lambda_1(1-z)$$

$$\lambda_2' = \lambda_2 - z \lambda_1 \rightarrow \lambda_2(1-z)$$

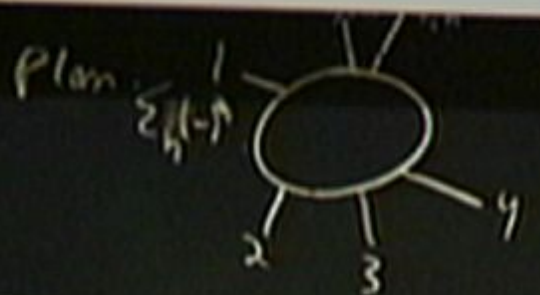
$$\lambda_3' = \lambda_3 - z \lambda_4 \rightarrow \lambda_3(1-z)$$



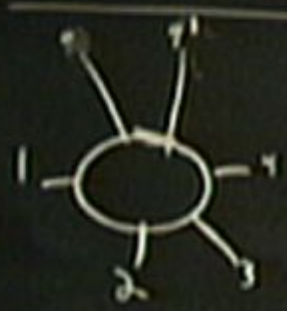
$$\int_0^{\infty} \frac{du u^{p-1}}{u+1} [1 + e^{-u(0-y)}]$$



$(p^0)^2$ spacelike



NMHV 6-pt amplitude

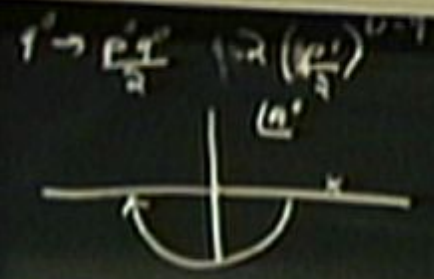
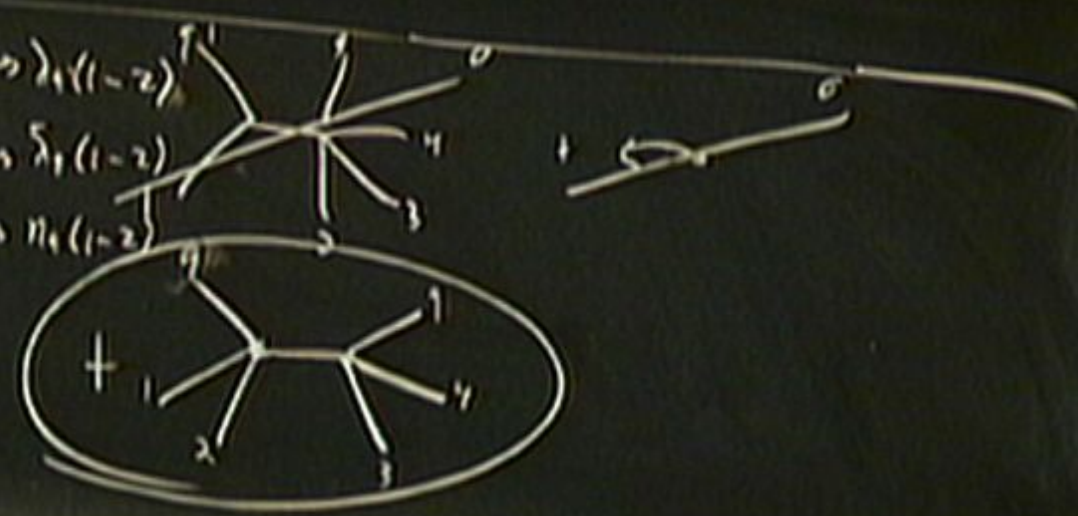


$$\lambda_{ij} = \lambda_{ij} + z \lambda_{ij} \rightarrow \lambda_{ij}(1-z)$$

$$\tilde{\lambda}_{ij} = \tilde{\lambda}_{ij} - z \tilde{\lambda}_{ij} \rightarrow \tilde{\lambda}_{ij}(1-z)$$

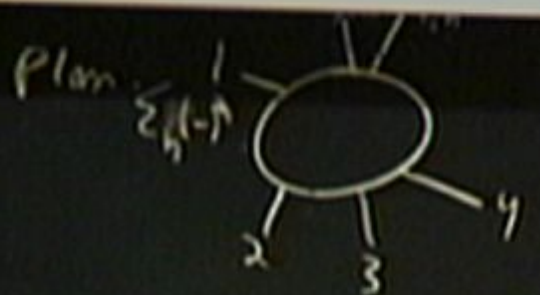
$$\tilde{\eta}_{ij} = \tilde{\eta}_{ij} - z \tilde{\eta}_{ij} \rightarrow \tilde{\eta}_{ij}(1-z)$$

$W = 1-z$

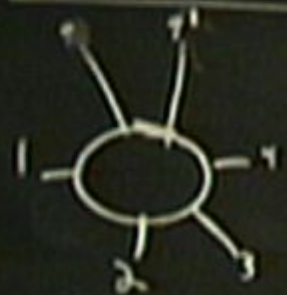


$$\int_0^{\infty} \frac{du u^{0-1}}{u+1} \left[1 + e^{-iu(0-y)} \right]$$

$(p^0)^2$ spacelike



NMHV 6-pt amplitude

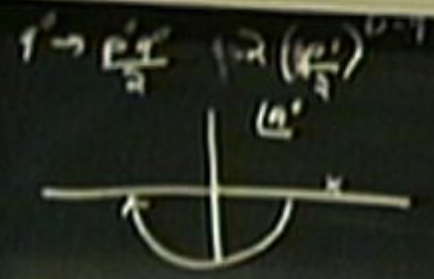
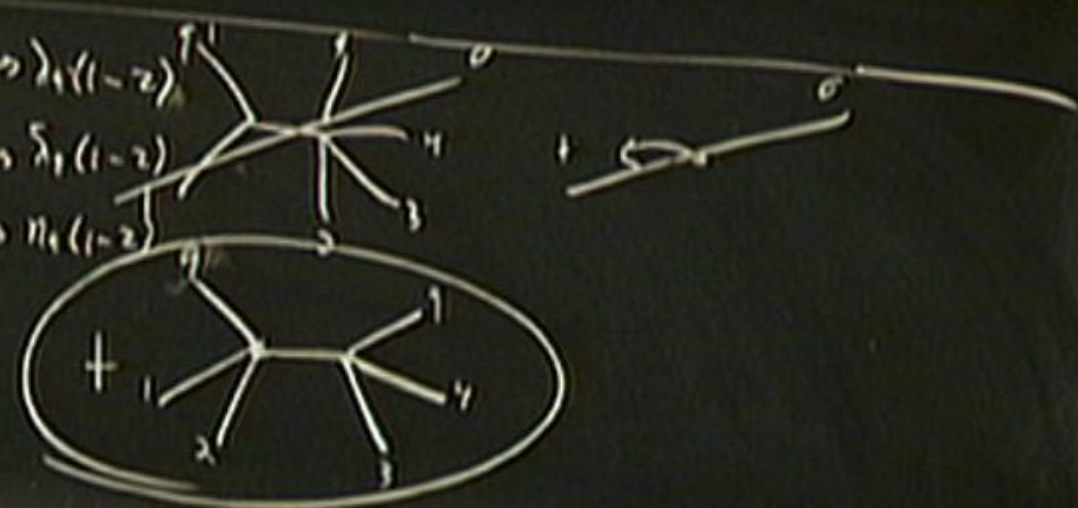


$$\lambda_{1'} = \lambda_1 + z \lambda_4 \rightarrow \lambda_1(1-z)$$

$$\lambda_2 = \lambda_2 - z \lambda_{1'} \rightarrow \lambda_2(1-z)$$

$$\eta_{1'}^A = \eta_1^A - z \eta_{1'}^A \rightarrow \eta_1^A(1-z)$$

$W = 1-z$



$$\int_0^{\infty} \frac{du u^{0-1}}{u+1} \left[1 + e^{-i\pi(0-1)} \right]$$

$$(P_1 + P_2 + w_0 q)^2 = 0 \Rightarrow w_0 = -\frac{P_1 \cdot P_2}{q \cdot (P_1 + P_2)}$$



$$(P_1 + P_2 + w_0 q)^2 = 0 \Rightarrow w_0 = -\frac{P_1 \cdot P_2}{q \cdot (P_1 + P_2)} \dots$$

$$(P_1 + P_2 + W_0 q)^2 = 0 \Rightarrow W_0 = \frac{-P_1 \cdot P_2}{q \cdot (P_1 + P_2)} \dots$$

$$\int d^4 \eta_a d^4 \eta_b$$

$$\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle$$

$$(P_1 + P_2 + W_0 q)^2 = 0 \Rightarrow W_0 = \frac{-P_1 \cdot P_2}{q \cdot (P_1 + P_2)} \dots$$

$$\int d^4 n_q d^4 n_m \delta^8(\lambda_1 n_1 + \lambda_2 n_2 + z_0 \lambda_3 n_3 + \lambda_4 n_4) \delta^8(\lambda_3 n_3 + \lambda_4 n_4 - z_0 \lambda_1 n_1 - \lambda_2 n_2)$$

$$\langle 12 \rangle \langle 2m \rangle \langle m q \rangle \langle q 1 \rangle W_0^2 \langle 34 \rangle \langle 4 q \rangle \langle q m \rangle \langle m 3 \rangle$$



$$(P_1 + P_2 + W_0 q)^2 = 0 \Rightarrow W_0 = \frac{-P_1 \cdot P_2}{q \cdot (P_1 + P_2)} \dots$$

$$\int d^4 n_q d^4 n_m \delta^6(\lambda_1 n_1 + \lambda_2 n_2 + z_0 \lambda_3 n_3 + \lambda_4 n_4) \delta^8(\lambda_3 n_3 + \lambda_4 n_4 - z_0 \lambda_1 n_1 - \lambda_2 n_2)$$

$$\langle 12 \rangle \langle 2M \rangle \langle M q \rangle \langle q 1 \rangle \omega_0^2 \langle 34 \rangle \langle 4 q \rangle \langle q M \rangle \langle M 3 \rangle$$



$$(P_1 + P_2 + W_0 q)^2 = 0 \Rightarrow W_0 = \frac{-P_1 \cdot P_2}{q \cdot (P_1 + P_2)} \dots$$

$$\int d^4 n_q d^4 n_m \delta^6(\lambda_1 n_1 + \lambda_2 n_2 + z_0 \lambda_3 n_3 + \lambda_4 n_4) \delta^8(\lambda_3 n_3 + \lambda_4 n_4 - z_0 \lambda_1 n_1 - \lambda_2 n_2)$$

$$\langle 12 \rangle \langle 2M \rangle \langle Mq \rangle \langle q1 \rangle W_0^2 \langle 34 \rangle \langle 4q \rangle \langle qM \rangle \langle M3 \rangle$$

$\overline{9 \cdot (A_1 + A_2)}$

$$\int d^4 n_q d^4 n_M \delta^8(\lambda_1 n_1 + \lambda_2 n_2 + z_0 \lambda_3 n_3 + \lambda_4 n_4) \delta^8(\lambda_3 n_3 + \lambda_4 n_4 - z_0 \lambda_1 n_1 - \lambda_2 n_2)$$

$\langle 12 \rangle \langle 2M \rangle \langle Mq \rangle \langle q1 \rangle \omega_1^2 \langle 34 \rangle \langle 4q \rangle \langle qM \rangle \langle M3 \rangle$

$$= \delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4)$$



$$(P_1 + P_2 + W_0 q)^2 = 0 \Rightarrow W_0 = \frac{-P_1 \cdot P_2}{q \cdot (P_1 + P_2)} \dots$$

$$\int d^4 n_q d^4 n_n \delta^8(\lambda_1 n_1 + \lambda_2 n_2 + z_0 \lambda_3 n_3 + \lambda_4 n_4) \delta^8(\lambda_3 n_3 + \lambda_4 n_4 - z_0 \lambda_1 n_1 - \lambda_2 n_2)$$

$$\langle 12 \rangle \langle 34 \rangle \langle 14 \rangle \langle 23 \rangle \langle 13 \rangle \langle 24 \rangle \langle 31 \rangle \langle 42 \rangle \langle 12 \rangle \langle 34 \rangle \langle 14 \rangle \langle 23 \rangle$$

$$= \frac{\delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4)}{\langle 12 \rangle \langle 34 \rangle \langle 41 \rangle}$$

$$(P_1 + P_2 + W_0 q)^2 = 0 \Rightarrow W_0 = \frac{-P_1 \cdot P_2}{q \cdot (P_1 + P_2)} \dots$$

$$\int d^4 n_q d^4 n_m \delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4) \delta^8(\lambda_3 n_3 + \lambda_4 n_4 - \lambda_1 n_1 - \lambda_2 n_2)$$

$$\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle \langle 12 \rangle \langle 34 \rangle \langle 41 \rangle \langle 12 \rangle \langle 34 \rangle \langle 41 \rangle$$

$$= \frac{\delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4)}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \times \frac{\langle 23 \rangle \langle 41 \rangle}{\langle 12 \rangle \langle 34 \rangle}$$



$$(P_1 + P_2 + W_0 q)^2 = 0 \Rightarrow W_0 = \frac{-P_1 \cdot P_2}{q \cdot (P_1 + P_2)} \dots$$

$$\int d^4 n_q d^4 n_n \delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4) \delta^8(\lambda_3 n_3 + \lambda_4 n_4 - \lambda_1 n_1 - \lambda_2 n_2)$$

$$\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle W_0^2 \langle 23 \rangle \langle 41 \rangle \langle 9M \rangle \langle M3 \rangle$$

$$= \frac{\delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4)}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \times \frac{W_0^2 \langle 23 \rangle \langle 41 \rangle \langle 9M \rangle^2}{\langle 91 \rangle \langle 94 \rangle \langle M2 \rangle \langle M3 \rangle}$$

$$(P_1 + P_2 + W_0 q)^2 = 0 \Rightarrow W_0 = \frac{-P_1 \cdot P_2}{q \cdot (P_1 + P_2)} \dots$$

$$\int d^4 n_q d^4 n_M \delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4) \delta^8(\lambda_3 n_3 + \lambda_4 n_4 - \lambda_1 n_1 - \lambda_2 n_2)$$

$$\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle W_0^2 \langle 23 \rangle \langle 41 \rangle \langle 9M \rangle \langle M3 \rangle$$

$$= \frac{\delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4)}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \times \frac{W_0^2 \langle 23 \rangle \langle 41 \rangle \langle 9M \rangle^2}{\langle 91 \rangle \langle 9M \rangle \langle M2 \rangle \langle M3 \rangle} \dots$$

$$(P_1 + P_2 + W_0 q) = 0 \Rightarrow W_0 = \frac{P_1 + P_2}{q}$$

$$\int d^4 n_q d^4 n_n \delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4) \delta^8(\lambda_3 n_3 + \lambda_4 n_4 - \lambda_1 n_1 - \lambda_2 n_2)$$

$$\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle W_0^2 \langle 23 \rangle \langle 41 \rangle \langle 9M \rangle \langle M3 \rangle$$

$$= \frac{\delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4)}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \left| \begin{array}{c} | \\ | \times \\ | \\ | \times \end{array} \right. \frac{W_0^2 \langle 23 \rangle \langle 41 \rangle \langle 9M \rangle^2}{\langle 91 \rangle \langle 92 \rangle \langle M2 \rangle \langle M3 \rangle} \times$$

$$\begin{aligned} \langle M \rangle &= \langle 9(\psi + 1) \rangle \\ &= \langle 9(P_1 + P_2) \rangle \end{aligned}$$

$$(P_1 + P_2 + w_0 q)^2 = 0 \Rightarrow w_0 = \frac{-P_1 \cdot P_2}{q(P_1 + P_2)} \dots$$

$$\int d^4 n_1 d^4 n_2 \delta^6(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4) \delta^8(\lambda_3 n_3 + \lambda_4 n_4 - \lambda_1 n_1 - \lambda_2 n_2)$$

$$= \frac{\delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4)}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \times \frac{w_0^2 \langle 23 \rangle \langle 41 \rangle \langle 9M \rangle^2}{\langle 91 \rangle \langle 92 \rangle \langle M2 \rangle \langle M3 \rangle} \times \frac{1}{(q + P_1 + P_2)^2}$$

$$\langle M \rangle = \langle 9(A+1) \rangle = \langle 9(P_1 + P_2) \rangle$$



$$(P_1 + P_2 + w_0 q) = 0 \Rightarrow w_0 = -\frac{P_1 + P_2}{q}$$

$$\int d^4 n_q d^4 n_m \delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4) \delta^8(\lambda_3 n_3 + \lambda_4 n_4 - \lambda_1 n_1 - \lambda_2 n_2)$$

$$\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle w_0^2 \langle 23 \rangle \langle 41 \rangle \langle 9M \rangle \langle M3 \rangle$$

$$= \frac{\delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4)}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \times \frac{w_0^2 \langle 23 \rangle \langle 41 \rangle \langle 9M \rangle^2}{\langle 91 \rangle \langle 94 \rangle \langle M2 \rangle \langle M3 \rangle} \times \frac{1}{(q + P_1 + P_2)^2}$$

$$\begin{aligned} \langle M \rangle &= \langle 9(\psi + 1) \rangle \\ &= \langle 9(P_1 + P_2) \rangle \end{aligned}$$

$$\times \frac{(P_1 + P_2)^2 \langle 23 \rangle \langle 41 \rangle}{\langle 91 \rangle \langle 94 \rangle}$$

$\langle M2 \rangle =$

$$(P_1 + P_2 + w_0 q)^2 = 0 \Rightarrow w_0 = \frac{-P_1 \cdot P_2}{q(P_1 + P_2)} \dots$$

$$\left\{ \frac{d^{\mu_1} n_1 d^{\mu_2} n_2 \delta^{\nu}(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4)}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \delta^{\sigma}(\lambda_3 n_3 + \lambda_4 n_4 - \lambda_1 n_1 - \lambda_2 n_2) \right\}$$

$$= \frac{\delta^{\nu}(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4)}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \times \frac{w_0^2 \langle 23 \rangle \langle 41 \rangle \langle 9M \rangle^2}{\langle 11 \rangle \langle 9P \rangle \langle M2 \rangle \langle M3 \rangle} \times \frac{1}{(q + P_1 + P_2)^2}$$

$$\langle M \rangle = \frac{1}{2} (q + P_1 + P_2)$$

$$\langle M2 \rangle = \dots$$

$$(P_1 + P_2 + \psi_0 q)^2 = 0 \Rightarrow \psi_0 = -\frac{P_1 \cdot P_2}{q \cdot (P_1 + P_2)} \dots$$

$$\int d^4 n_1 d^4 n_2 \delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4) \delta^8(\lambda_3 n_3 + \lambda_4 n_4 - \lambda_1 n_1 - \lambda_2 n_2)$$

$$\langle 12 | \langle 34 | \langle M | \psi_0^2 \langle 34 | \langle 49 | \times q M \rangle \langle M | \rangle$$

$$= \frac{\delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4)}{\langle 12 | \langle 34 | \langle 34 | \langle 41 | } \left| \begin{array}{l} \times \frac{\psi_0^2 \langle 23 | \langle 41 | \langle 9M \rangle^2}{\langle 91 | \langle 9 | \langle M2 | \langle M3 | } \\ \times \frac{(P_1 \cdot P_2)^2 \langle 23 | \langle 41 | }{\langle 91 | \langle 9 | } \end{array} \right. \times \frac{1}{(q + P_1 + P_2)^2}$$

$$\langle M | = \langle 9 | (q + P_1 + P_2)$$

$$\langle M2 | = \langle 9 | \langle 3 | \langle 12 |$$

$$\langle M3 | = -\langle 9 | (P_1 + P_2) | \rangle = +\langle 9 | \langle 4 | \langle 34 |$$

$$(P_1 + P_2 + w_0 q)^2 = 0 \Rightarrow w_0 = \frac{-P_1 \cdot P_2}{q \cdot (P_1 + P_2)} \dots$$

$$\int d^4 n_1 d^4 n_2 \delta^4(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4) \delta^4(\lambda_3 n_3 + \lambda_4 n_4 - \lambda_1 n_1 - \lambda_2 n_2)$$

$$\langle 12 \rangle \langle 34 \rangle \langle M \rangle \langle M \rangle \langle q \rangle w_0^2 \langle 23 \rangle \langle 41 \rangle \langle q M \rangle \langle M \rangle$$

$$= \frac{\delta^4(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4)}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \times \frac{w_0^2 \langle 23 \rangle \langle 41 \rangle \langle q M \rangle^2}{\langle q \rangle \langle q \rangle \langle M \rangle \langle M \rangle} \times \frac{1}{(q + P_1 + P_2)^2}$$

$$\times \frac{(P_1 \cdot P_2)^2 \langle 23 \rangle \langle 41 \rangle}{\langle q \rangle \langle q \rangle \langle 12 \rangle \langle 34 \rangle (q + P_1 + P_2)^2}$$

$$\langle M \rangle = \langle q \rangle \langle q + P_1 + P_2 \rangle$$

$$= \langle q \rangle \langle P_1 + P_2 \rangle$$

$$\langle M \rangle = \langle q \rangle \langle P_1 + P_2 \rangle$$

$$\langle M \rangle = \langle q \rangle \langle P_1 + P_2 \rangle = + \langle q \rangle \langle 34 \rangle$$

$$(P_1 + P_2 + \psi_0 q)^2 = 0 \Rightarrow \psi_0 = \frac{-P_1 \cdot P_2}{q \cdot (P_1 + P_2)} \dots$$

$$\int d^4 n_1 d^4 n_2 \delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4) \delta^8(\lambda_3 n_3 + \lambda_4 n_4 - \lambda_1 n_1 - \lambda_2 n_2)$$

$$\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle \psi_0^2 \langle 23 \rangle \langle 41 \rangle \langle 9M \rangle \langle M3 \rangle$$

$$= \frac{\delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 + \lambda_4 n_4)}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \times \frac{\psi_0^2 \langle 23 \rangle \langle 41 \rangle \langle 9M \rangle^2}{\langle 91 \rangle \langle 92 \rangle \langle M2 \rangle \langle M3 \rangle} \times \frac{1}{(q + P_1 + P_2)^2}$$

$$\langle M1 \rangle = \langle 9 \rangle \langle 4 \rangle \langle 1 \rangle \langle 2 \rangle$$

$$= \langle 9 \rangle \langle P_1 + P_2 \rangle$$

$$\times \frac{(P_1 \cdot P_2)^2 \langle 23 \rangle \langle 41 \rangle}{\langle 91 \rangle \langle 94 \rangle \langle 91 \rangle \langle 94 \rangle \langle 12 \rangle \langle 34 \rangle (q + P_1 + P_2)^2}$$

$$\langle M2 \rangle = \langle 9 \rangle \langle 3 \rangle \langle 1 \rangle \langle 2 \rangle$$

$$\langle M3 \rangle = -\langle 9 \rangle \langle P_1 + P_2 \rangle \langle 3 \rangle = +\langle 9 \rangle \langle 4 \rangle \langle 3 \rangle = \frac{st}{P_1 \cdot P_2 \cdot q \cdot P_4 (q + P_1 + P_2)^2}$$

$$(P_1 + P_2 + W_0 \gamma)^2 = 0 \Rightarrow W_0 = \frac{-P_1 \cdot P_2}{\gamma(P_1 + P_2)} \dots$$

$$\int d^4 n_1 d^4 n_2 \delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \cancel{\lambda_3 n_3} + \lambda_4 n_4) \delta^8(\lambda_3 n_3 + \lambda_4 n_4 - \cancel{\lambda_1 n_1} - \lambda_2 n_2)$$

$$\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle W_0^2 \langle 23 \rangle \langle 41 \rangle \langle 9M \rangle \langle M \rangle$$

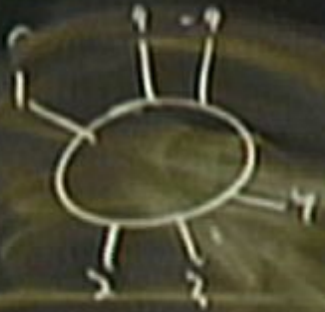
$$= \frac{\delta^8(\lambda_1 n_1 + \lambda_2 n_2 + \lambda_4 n_4)}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \left| \begin{array}{l} \times \frac{W_0^2 \langle 23 \rangle \langle 41 \rangle \langle 9M \rangle^2}{\langle 91 \rangle \langle 94 \rangle \langle M2 \rangle \langle M3 \rangle} \times \frac{1}{(\gamma + P_1 + P_2)^2} \\ \times \frac{(P_1 \cdot P_2)^2 \langle 23 \rangle \langle 41 \rangle}{\langle 91 \rangle \langle 94 \rangle \langle 91 \rangle \langle 94 \rangle \langle 12 \rangle \langle 34 \rangle (\gamma + P_1 + P_2)^2} \end{array} \right.$$

$$\langle M \rangle = \gamma(\gamma + 1) = \gamma(P_1 + P_2)$$

$$\langle M \rangle = \gamma \beta \gamma$$

$$\langle M \rangle = -\gamma(P_1 + P_2) = +\gamma \beta \gamma$$

$$= \frac{\gamma^4}{\gamma \cdot P_1 \cdot \gamma \cdot P_2 (\gamma + P_1 + P_2)^2} \quad |\epsilon| (1 - \cos \theta)$$



\Rightarrow

$$\int \pi d(q^2) \frac{st}{(s+m^2)^2 (s+p_1^2)^2 (s-p_4^2)^2}$$

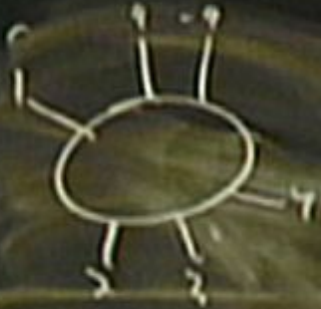


\Rightarrow

$$\int_{0+in} \pi \delta(q^2)$$

st

$$(q+p_1+p_2)^2 (q+p_1)^2 (q-p_4)^2$$



\Rightarrow

$$\int_{0+in}^{\infty} \pi \delta(q^2) \frac{st}{(q+p_1+p_2)^2 (q+p_1)^2 (q-p_4)^2}$$





$$\Rightarrow A^{\text{tree}} \times \left\{ \begin{array}{l} \pi \delta(q^2) \\ \text{cut} \end{array} \right\} \left\{ \frac{st}{(s+p_1+p_2)^2 (s+p_1)^2 (s-p_4)^2} + \text{cyclic} \right\}$$



$$\Rightarrow A^{111} \times \left\{ \sum_{0 \leq i \leq 11} \pi \delta(q^i) \right\} \left\{ \begin{array}{l} st \\ (q+P_1+P_2)^2 (q+P_1)^2 (q-P_4)^2 + \text{cyclic} \end{array} \right\}$$

$$-A^{111} \times \left\{ \frac{-i - st}{q (q+P_1+P_2)^2 (q+P_1)^2 (q-P_4)^2} \right\}$$



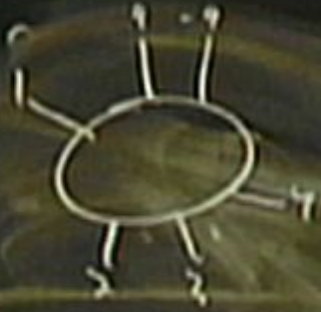
$$\Rightarrow A^{\text{tree}} \times \left\{ \sum_{0 \leq n} \pi \delta(q^2) \left[\frac{st}{(q+p_1+p_2)^2 (q+p_1)^2 (q-p_4)^2} + \text{cyclic} \right] \right\}$$

$$\Rightarrow A^{\text{tree}} \times \left\{ \frac{-i st}{q^2 (q+p_1+p_2)^2 (q+p_1)^2 (q-p_4)^2} \right\}$$



$$\Rightarrow A^{1111} \times \left\{ \sum_{0 \leq i \leq n} \pi \delta(q^2) \left[\frac{st}{(q+P_1+P_2)^2 (q+P_1)^2 (q-P_4)^2} + \text{cyclic} \right] \right\}$$

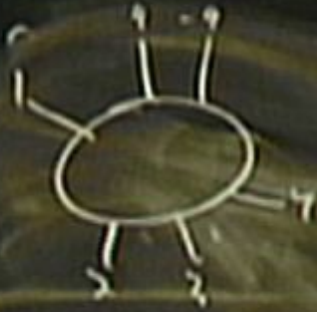
$$= A^{1111} \times \left\{ \frac{-i - st}{q (q+P_1+P_2)^2 (q+P_1)^2 (q-P_4)^2} \right\}$$



$$\Rightarrow A^{\text{tree}} \times \left\{ \begin{array}{l} \pi \delta(q^2) \\ 0 \text{ in} \end{array} \right\} \left\{ \frac{st}{(q+p_1+p_2)^2 (q+p_1)^2 (q-p_4)^2} + \text{cyclic} \right\}$$

$$= A^{\text{tree}} \times \left\{ \frac{-i st}{q (q+p_1+p_2)^2 (q+p_1)^2 (q-p_4)^2} \right\}$$

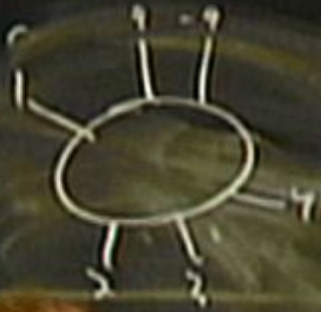




$$\Rightarrow A^{full} \times \left\{ \sum_{\text{orient}} \pi \delta(q^2) \left[\frac{st}{(q+p_1+p_2)^2 (q+p_1)^2 (q-p_1)^2} + \text{cyclic} \right] \right\}$$

$$= A^{full} \times \left\{ \frac{-i \, st}{q^2 (q+p_1+p_2)^2 (q+p_1)^2 (q-p_1)^2} \right\}$$

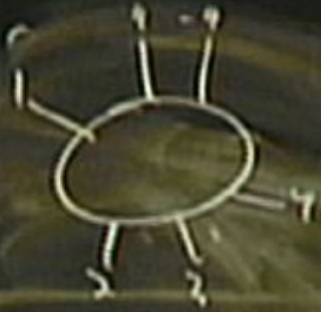




$$\Rightarrow A^{\text{tree}} \times \left\{ \frac{\pi \delta(q^2)}{0!} \left[\frac{st}{(q+p_1+p_2)^2 (q+p_1)^2 (q-p_4)^2} \right] \right\} \text{ cyclic}$$

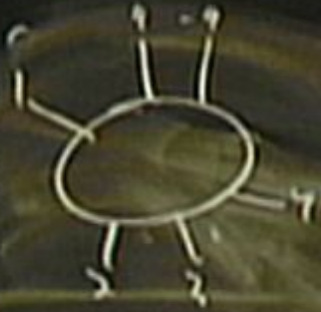
$$= A^{\text{tree}} \times \left\{ \frac{-i st}{q (q+p_1+p_2)^2 (q+p_1)^2 (q-p_4)^2} \right\}$$





$$\Rightarrow A^{\text{tree}} \times \left\{ \frac{\pi \delta(q^2)}{0! 1!} \left[\frac{st}{(q+p_1+p_2)^2 (q+p_1)^2 (q-p_4)^2} \right] \right\} \text{ + cyclic}$$

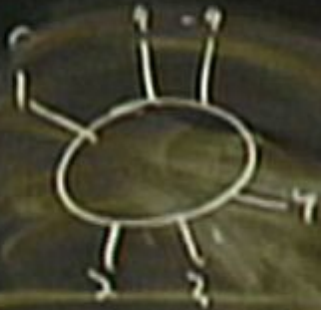
$$= A^{\text{tree}} \times \left\{ \frac{-i st}{q (q+p_1+p_2)^2 (q+p_1)^2 (q-p_4)^2} \right\}$$



$$\Rightarrow A^{\text{tree}} \times \left\{ \sum_{\text{0-in}} \pi \delta(q^2) \left[\frac{st}{(q+p_1+p_2)^2 (q+p_1)^2 (q-p_4)^2} \right] + \text{cyclic} \right\}$$

$$= A^{\text{tree}} \times \left\{ \frac{-i st}{q^2 (q+p_1+p_2)^2 (q+p_1)^2 (q-p_4)^2} \right\} + \left\{ \frac{q^2 F(q,p_i)}{q^2 () () ()} \right\}$$

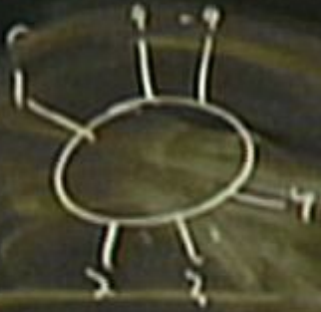




$$\Rightarrow A^{\text{tree}} \times \left\{ \pi \delta(q^2) \left[\frac{st}{(\pi + p_1 + p_2)^2 (q + p_1)^2 (q - p_4)^2} \right] + \text{cyclic} \right\}$$

$$= A^{\text{tree}} \times \left\{ \frac{-i st}{q^2 (\pi + p_1 + p_2)^2 (q + p_1)^2 (q - p_4)^2} \right\} + \left\{ \frac{q^2 F(q, p_i)}{q^2 (\dots) (\dots) (\dots)} \right\} = \frac{(q + p_1)^2 G^2(q, p_i)}{(\dots)}$$





$$\Rightarrow A^{\text{tree}} \times \left\{ \frac{\pi \delta(q^2)}{0!} \left[\frac{st}{(q+p_1+p_2)^2 (q+p_1)^2 (q-p_4)^2} \right] + \text{cyclic} \right\}$$

$$= A^{\text{tree}} \times \left\{ \frac{-i st}{q^2 (q+p_1+p_2)^2 (q+p_1)^2 (q-p_4)^2} \right\} + \left\{ \frac{q^2 F(q,p)}{q^2 () () ()} \right\} = \frac{(q+p_1)^2 G^2(q,p)}{() () ()}$$



$$|i\rangle = |H\rangle + \eta_1^A |4\rangle + \eta_2^A |8\rangle + \frac{\eta^A \eta^A \eta^A \epsilon_{\alpha\beta\gamma\delta}}{3!} |\psi_0\rangle + \frac{(\eta^A)^4}{4!} |-\rangle$$

$$P_i^{\alpha\beta} \equiv \langle H | \sigma_i^{\alpha\beta} | \psi \rangle$$

$$\frac{q^2(q+r)^2 H(q,r)}{r^2 - q^2}$$

$$|1\rangle = |H\rangle + \eta_1^A |\psi_1\rangle + \eta_2^A \frac{1}{2} |\psi_2\rangle + \frac{\eta_1^A \eta_2^A \epsilon_{\alpha\beta\gamma\delta} |\psi_3\rangle}{3!} + \frac{(\eta_1^A)^4}{4!} |1\rangle$$

$$P_i^{\alpha\beta} \equiv \langle H | \sigma_i^{\alpha\beta} | H \rangle$$

$$\frac{q^2(q+r)^2 H(q,r)}{...}$$



$$|h\rangle = |H\rangle + \eta_1^A |4\rangle + \frac{\eta_1^A \eta_1^B}{2} |8\rangle + \frac{\eta_1^A \eta_1^B \eta_1^C \eta_1^D}{3!} |16\rangle + \frac{(\eta_1^A)^4}{4!} |1\rangle$$

$$P_i^{aa} = (H^a \sigma_{ii}^{aa} \rightarrow) \eta_i^a$$

$$\frac{q^2(q+q^2)^2 H(q, P_i)}{1 - q^2 - q^4}$$



Summary, So far:

- Loops = $\int dLIPS \times [\text{Physical tree amps}]_R$.

- Planar th \Rightarrow Multiloop

- SUSY \exists Forward limits tree amplitudes.

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