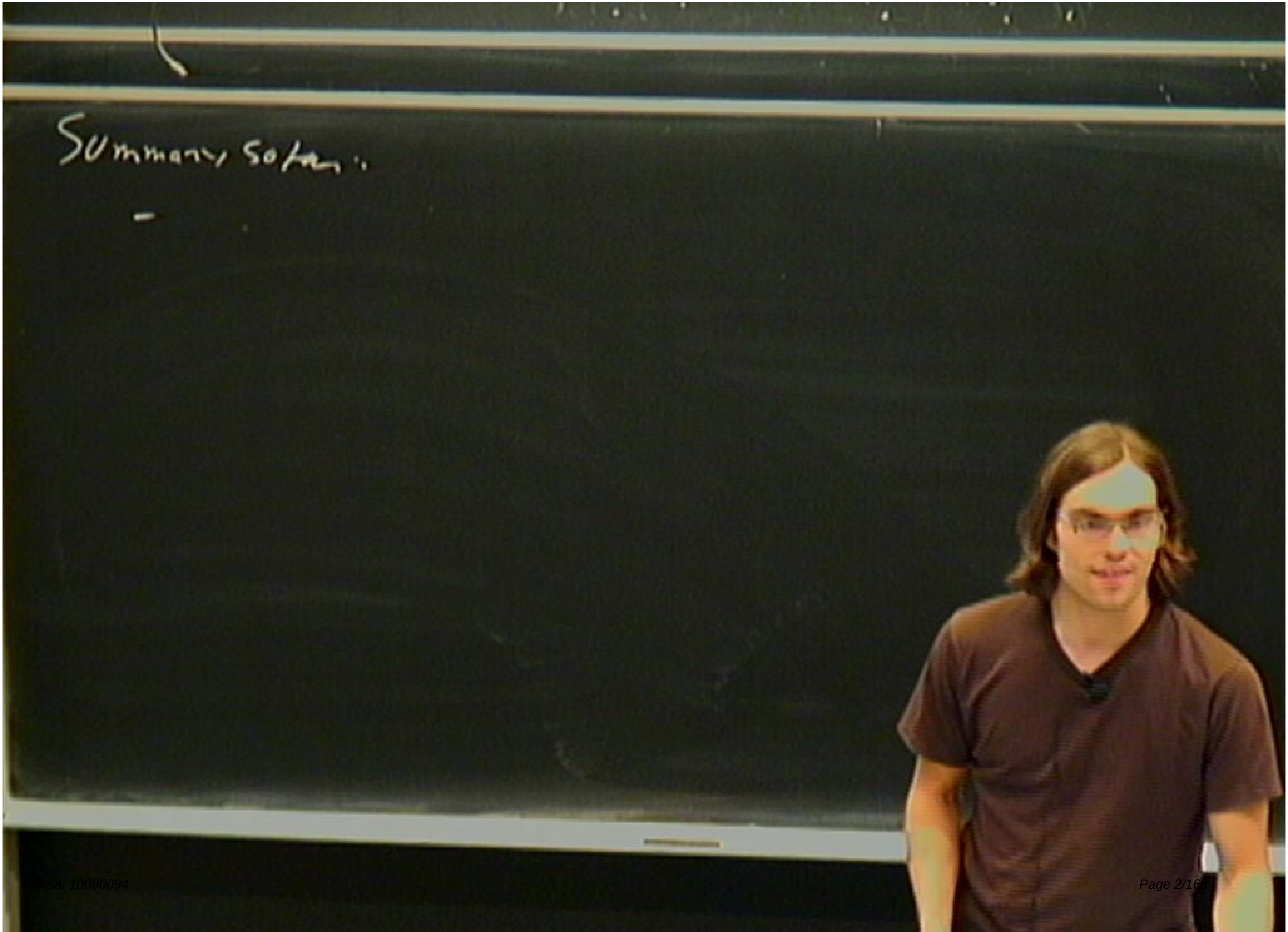
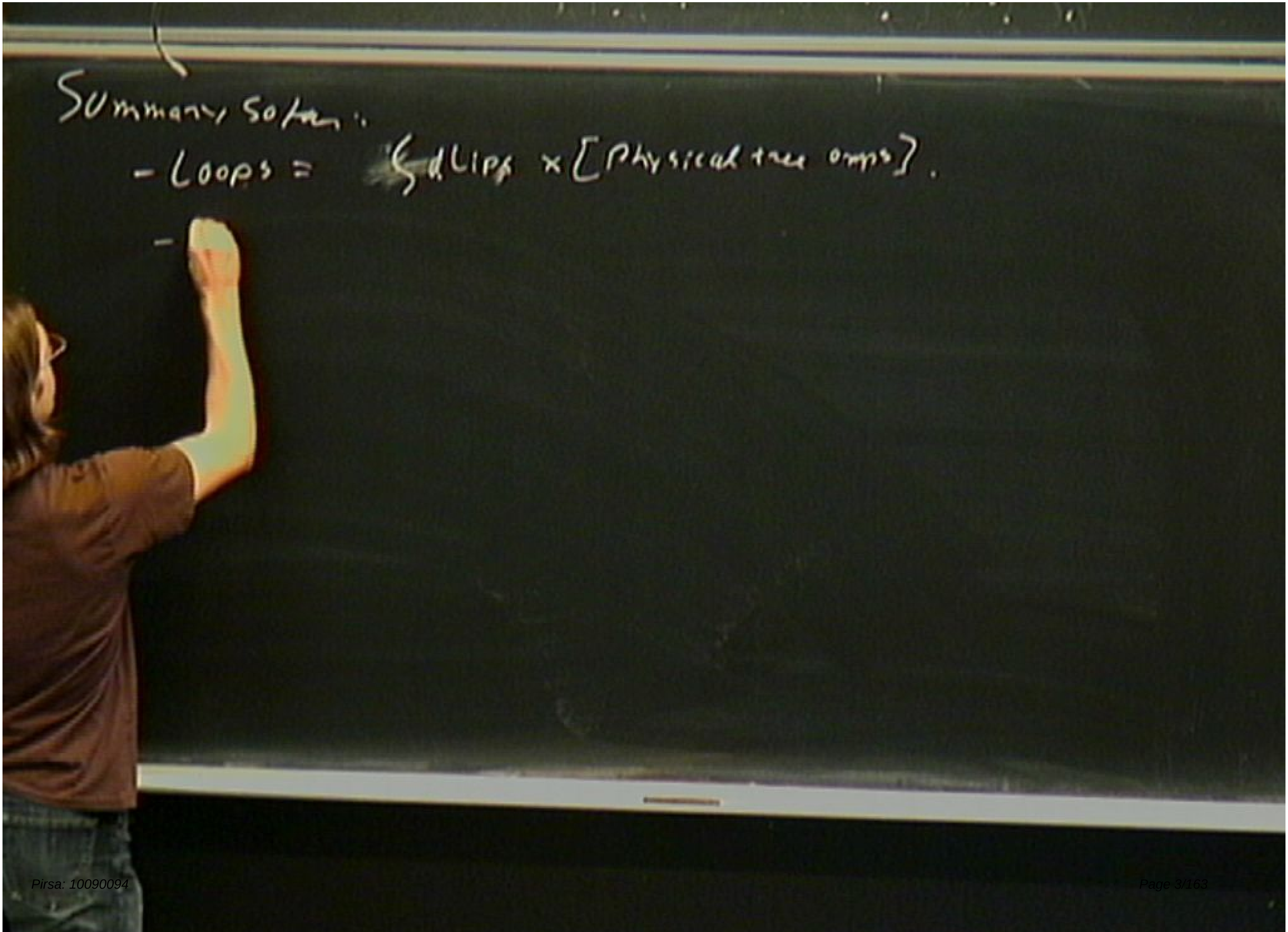
Title: Scattering Amplitudes from Single-Cuts: Lecture 2

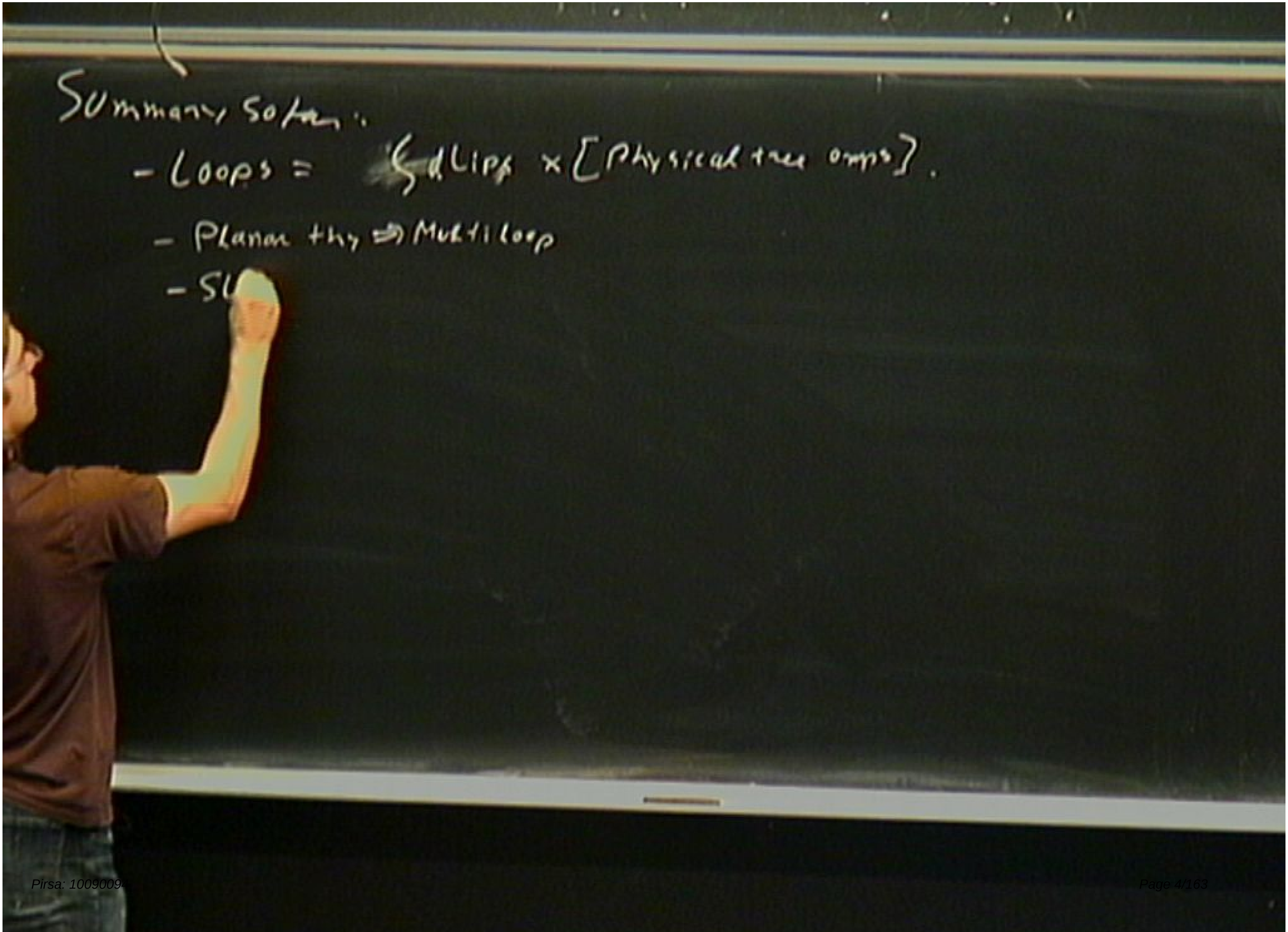
Date: Sep 14, 2010 02:00 PM

URL: http://pirsa.org/10090094

Abstract: On-shell methods provide a powerful tool for the perturbative computation of scattering amplitudes in gauge theories, such as QCD. In these lectures I will focus on such methods which can be developed without invoking unitarity, but rather by setting one propagator by loop level on-shell ("single-cuts"). After summarizing the present status of applying these ideas to QCD, I will discuss their recent successful application to planar N=4 super Yang-Mills, leading to on-shell recursion relations for loop integrands.


Summary so far:
-


Summary so far:
- Loops = ∫dLIPS × [Physical tree amps].
-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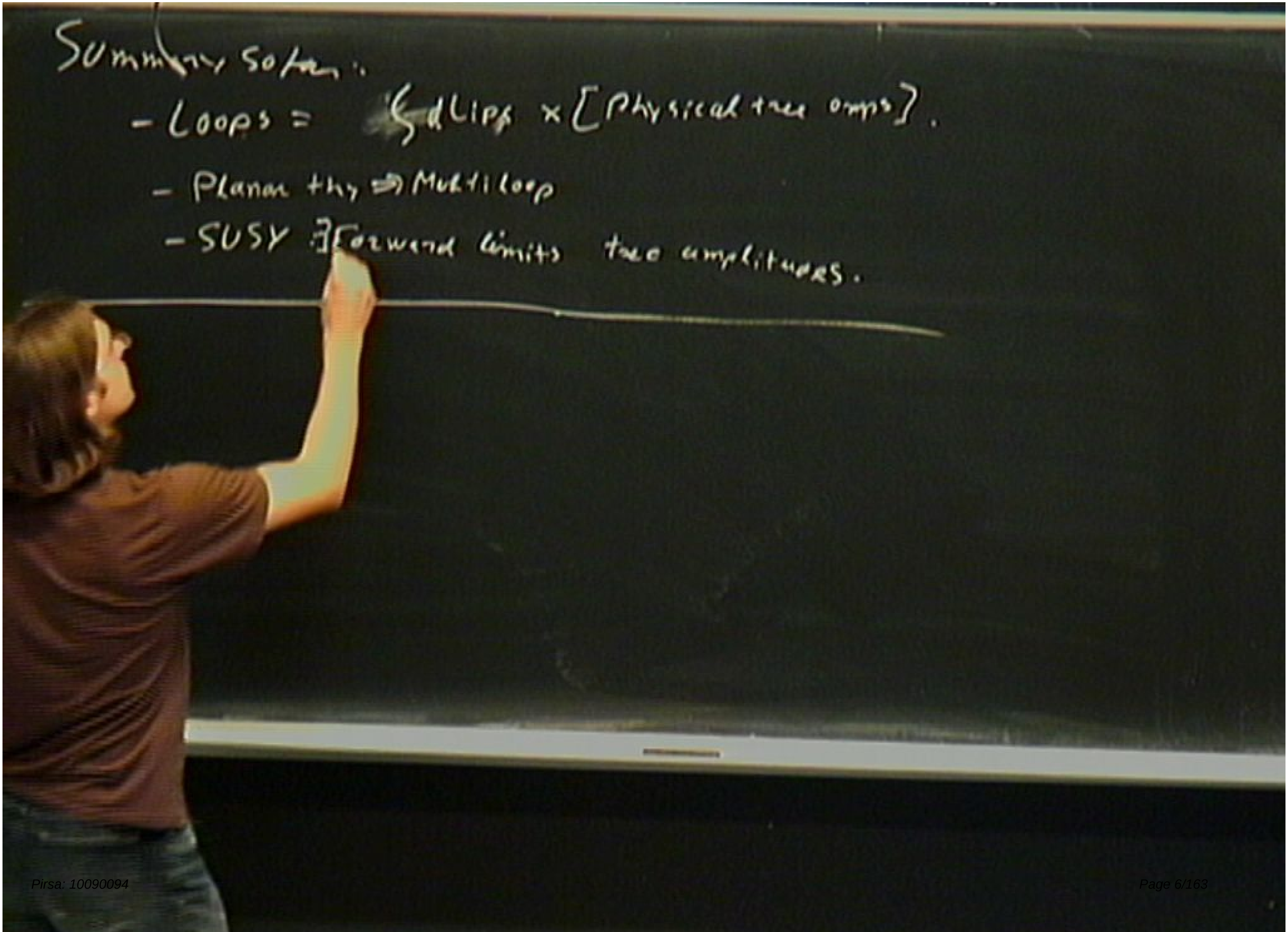
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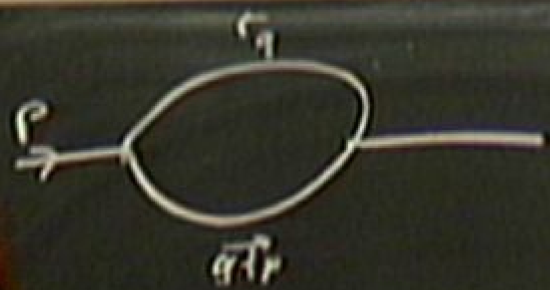
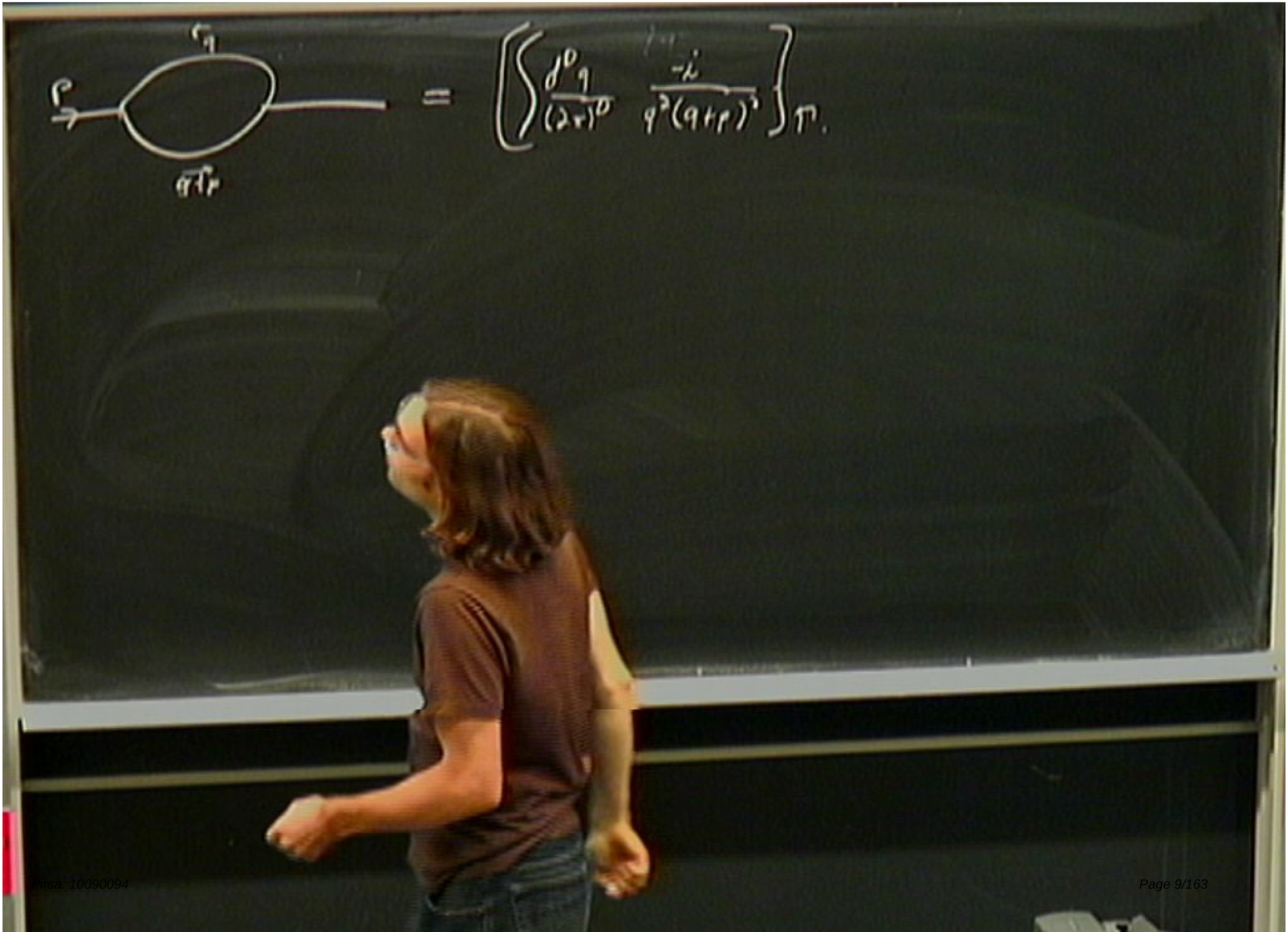
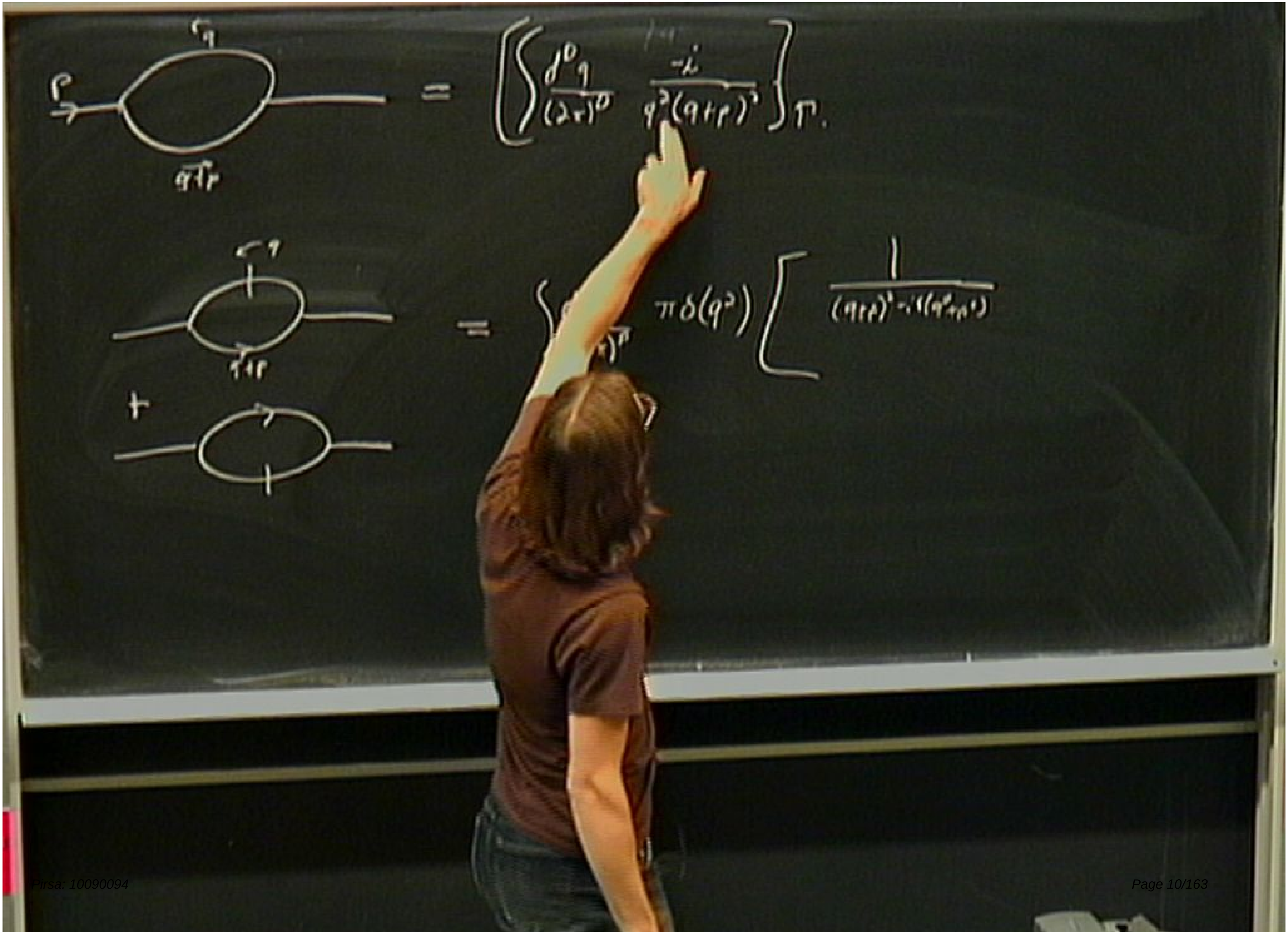
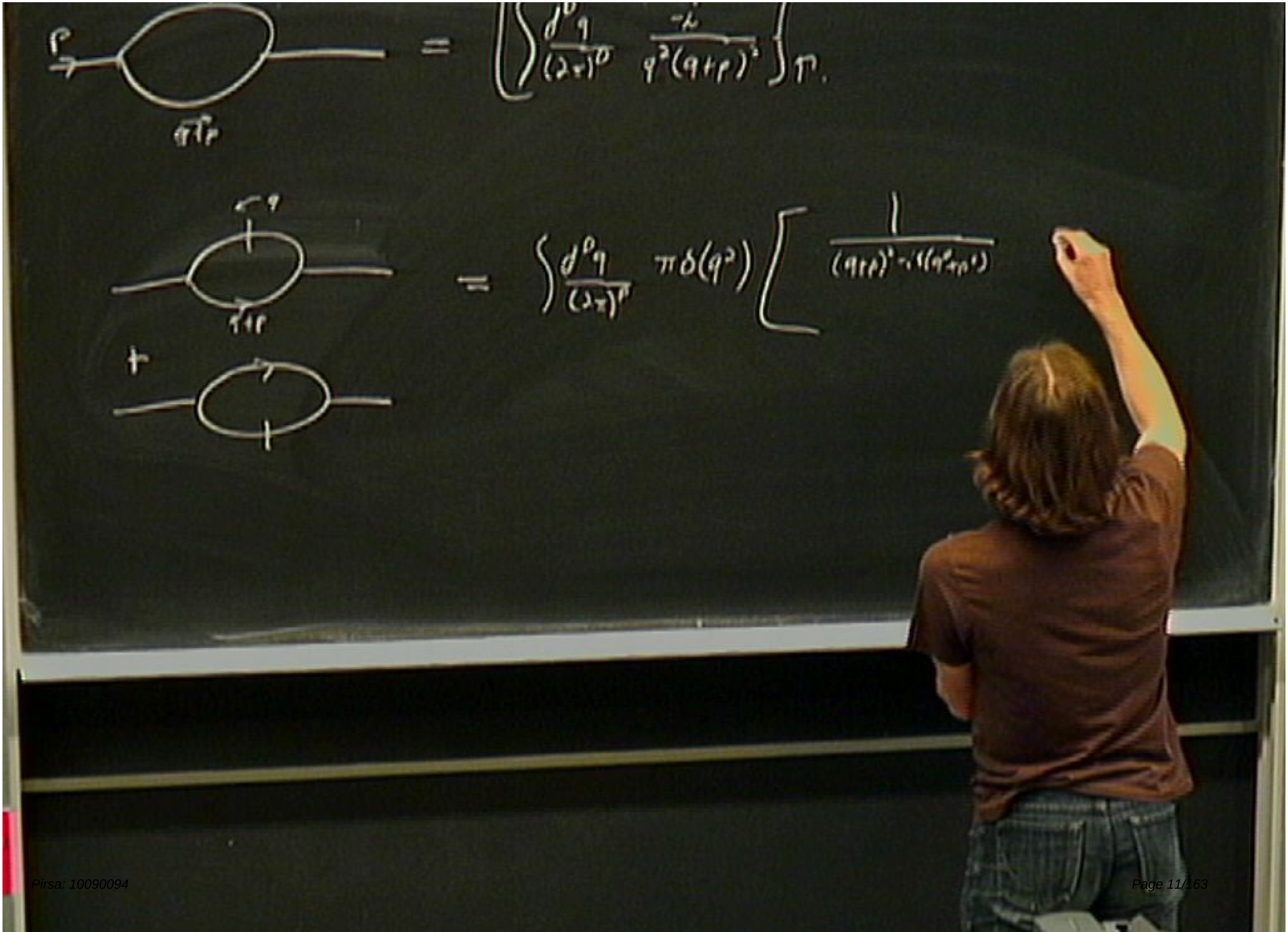


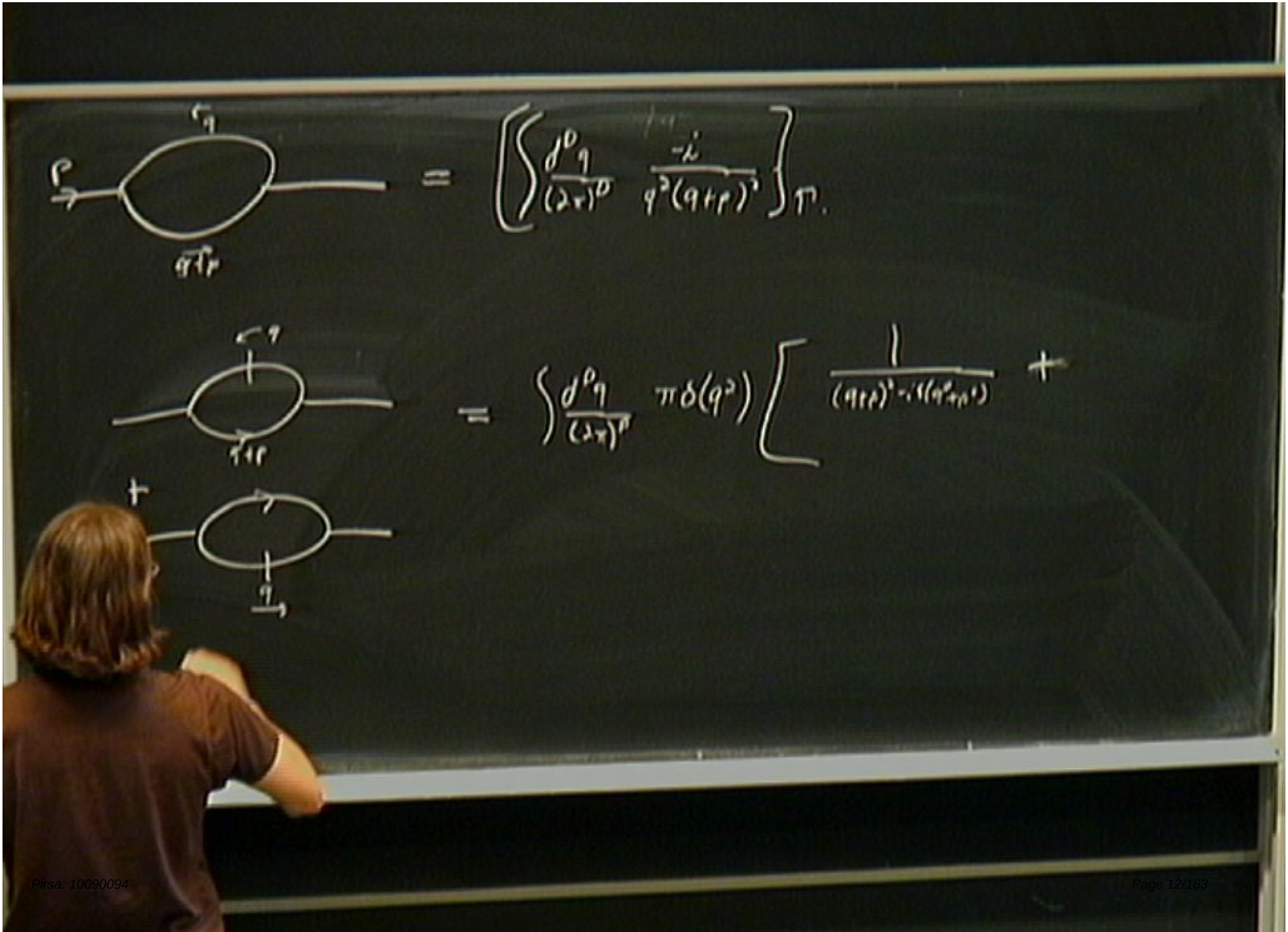
$$= \int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2 (q+p)^2}$$

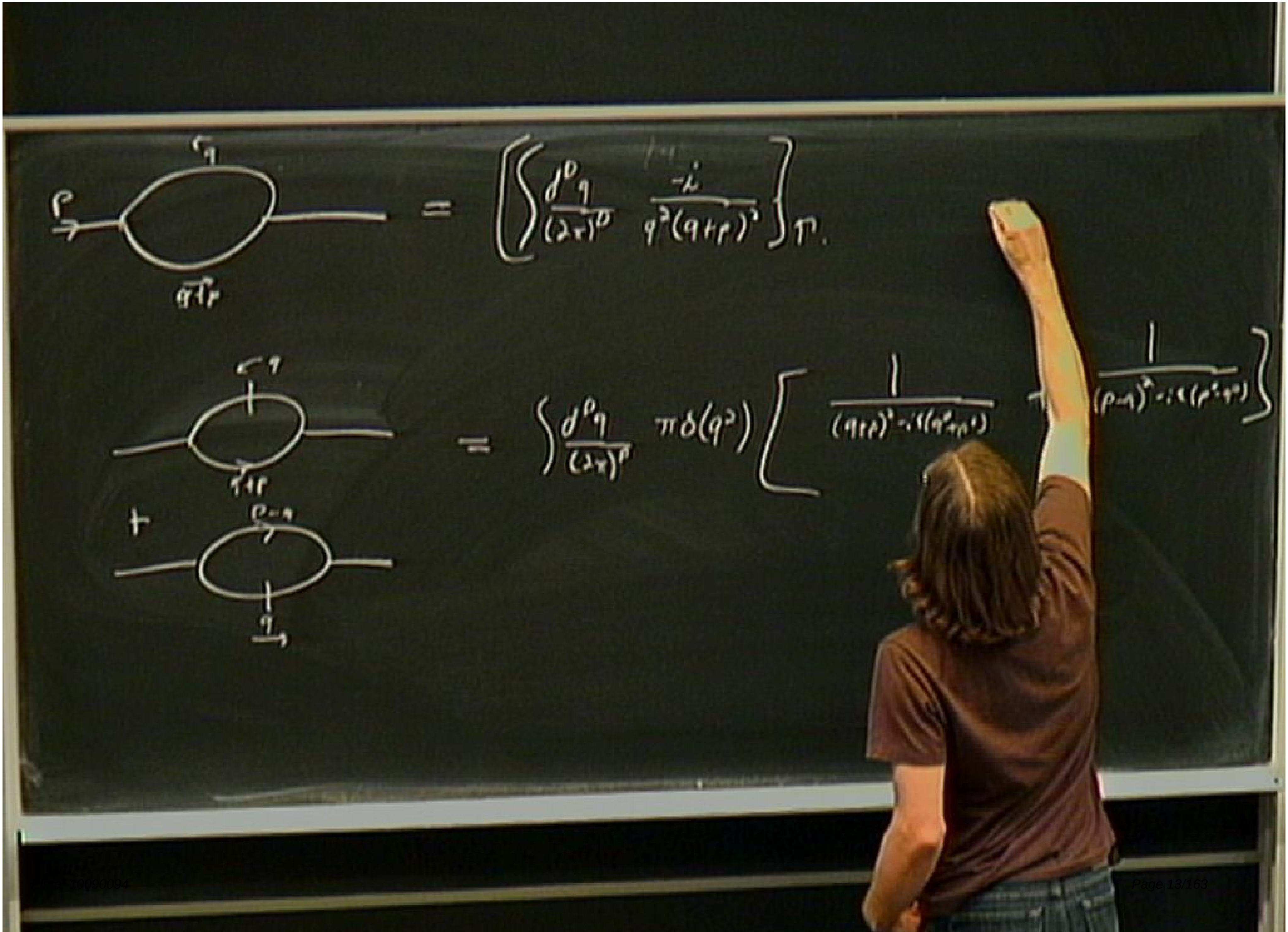

p
q
q+p
$$= \left[\int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2 (q+p)^2}\right]_{\text{fin.}}$$

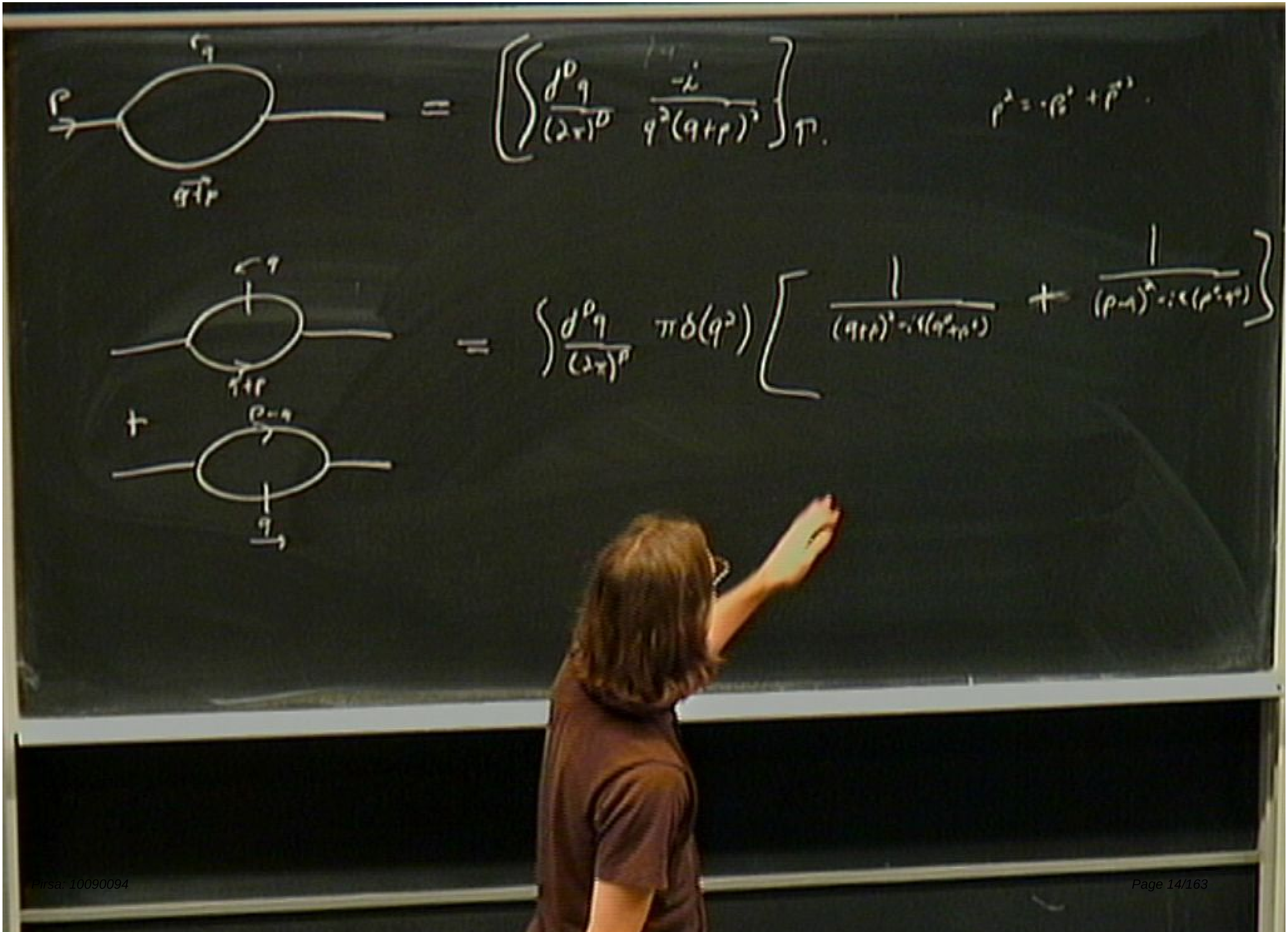

q
p
q+p
$= \left[\int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2 (q+p)^2} \right]_{\Gamma}$
q
q+p
$\pi\delta(q^2)$
+

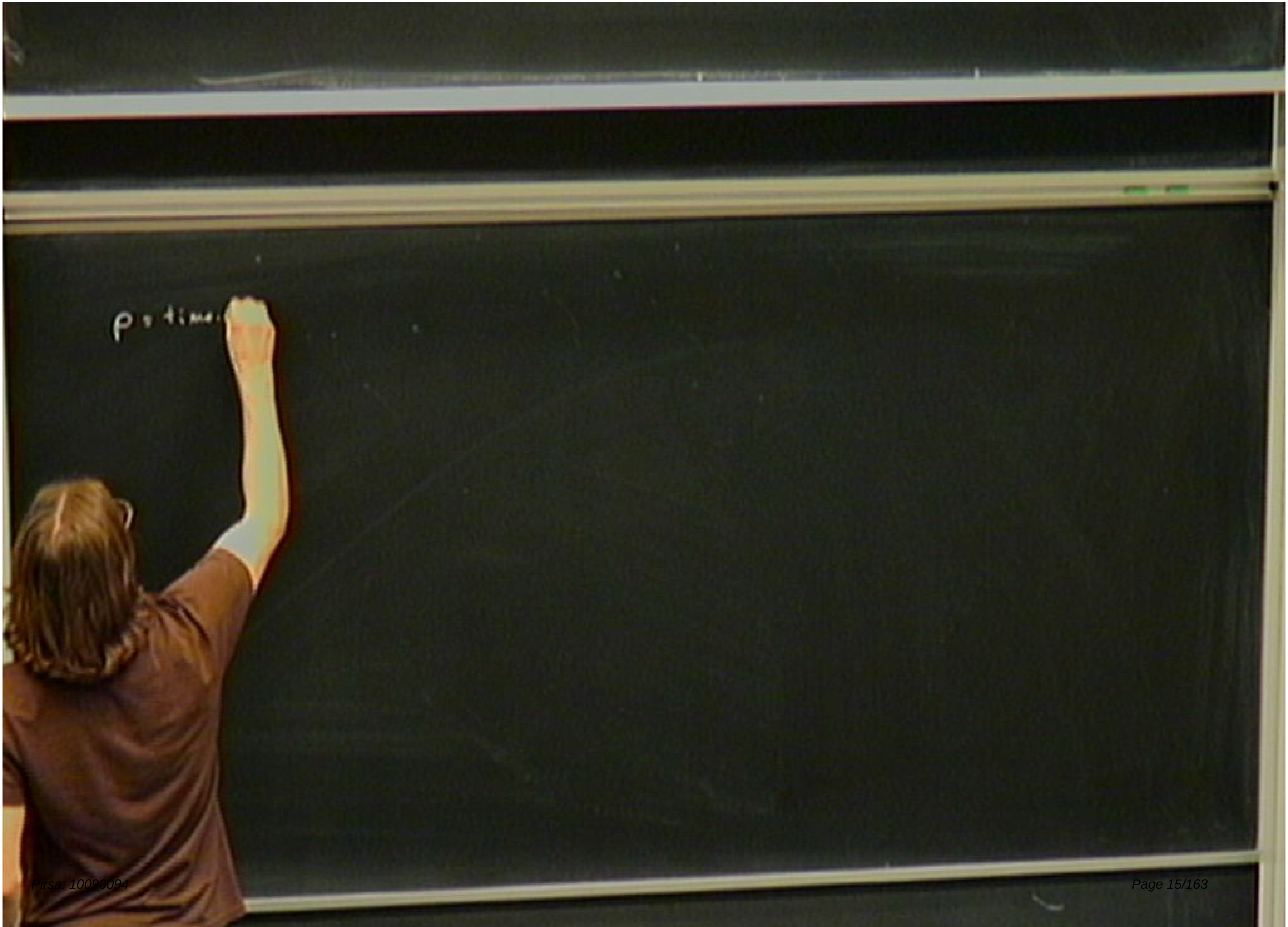


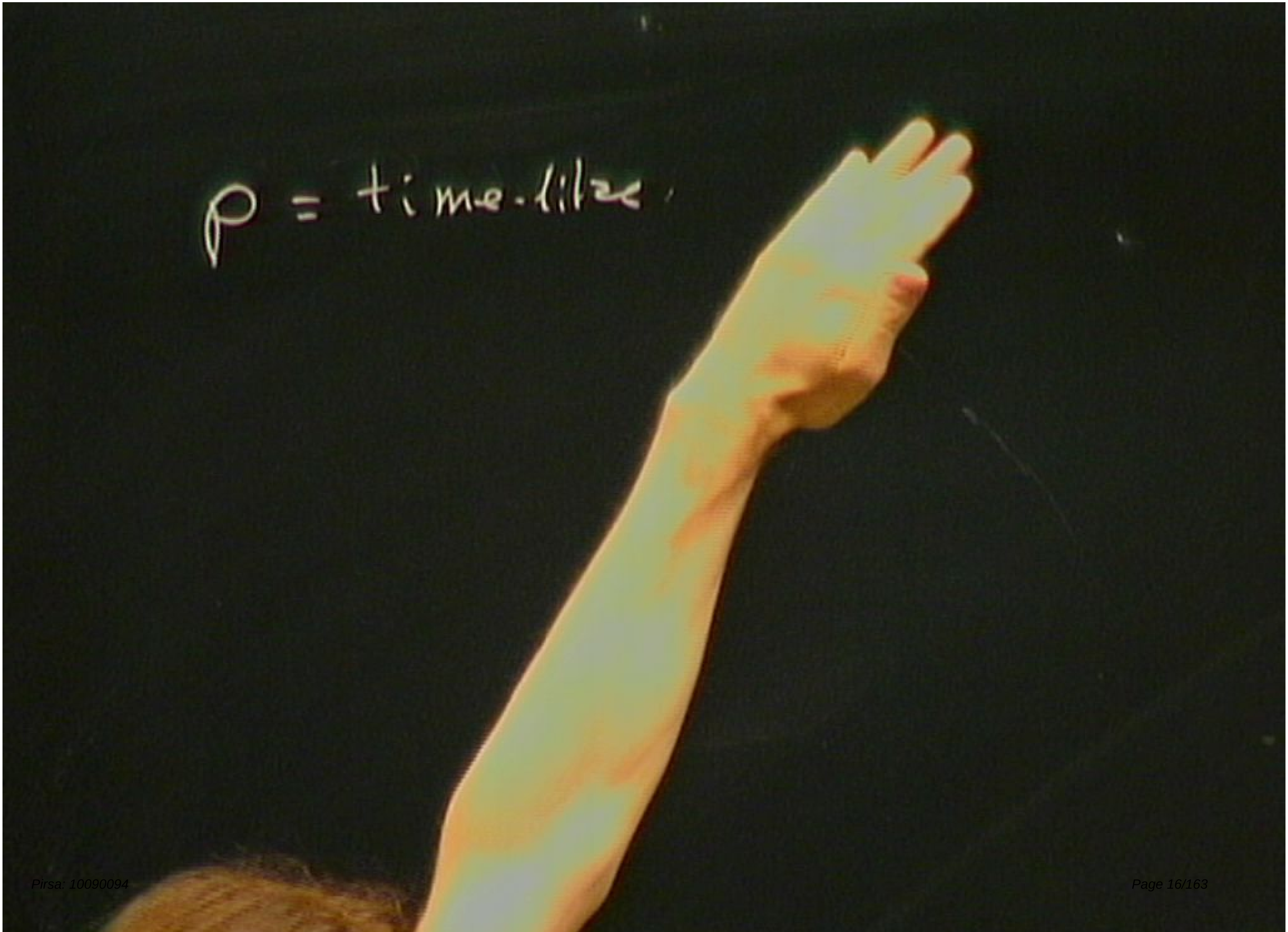
p
q+p
$= \int \frac{d^D q}{(2\pi)^D} \frac{-\lambda}{q^2(q+p)^2}$
q
$= \int \frac{d^D q}{(2\pi)^D} \pi\delta(q^2) \left[\frac{1}{(q+p)^2 - i(q^0+p^0)} \right.$
q+p
+


$= \left[\int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2 (q+p)^2} \right]_{\Gamma}$
$= \int \frac{d^D q}{(2\pi)^D} \, \pi \delta(q^2) \left[\frac{1}{(q+p)^2 - i(q^2+p^2)} + \right.$


p
q
q+p
= [∫ d^D q/(2π)^D · −i/(q²(q+p)²)]_π.
q
q+p
= ∫ d^D q/(2π)^D πδ(q²) [1/((q+p)² − iε(q⁰+p⁰)) + 1/((p−q)² − iε(p⁰−q⁰))]
+
p−q
q


q
p
q+p
$= \left[\int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2 (q+p)^2} \right]_{p^2}$.
$p^2 = -p_0^2 + \vec{p}^2$.
q
q+p
+
p-q
q
$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+p)^2 - i\epsilon(q^0+p^0)} + \frac{1}{(p-q)^2 + i\epsilon(p^0 - q^0)} \right]$




ρ = time-like

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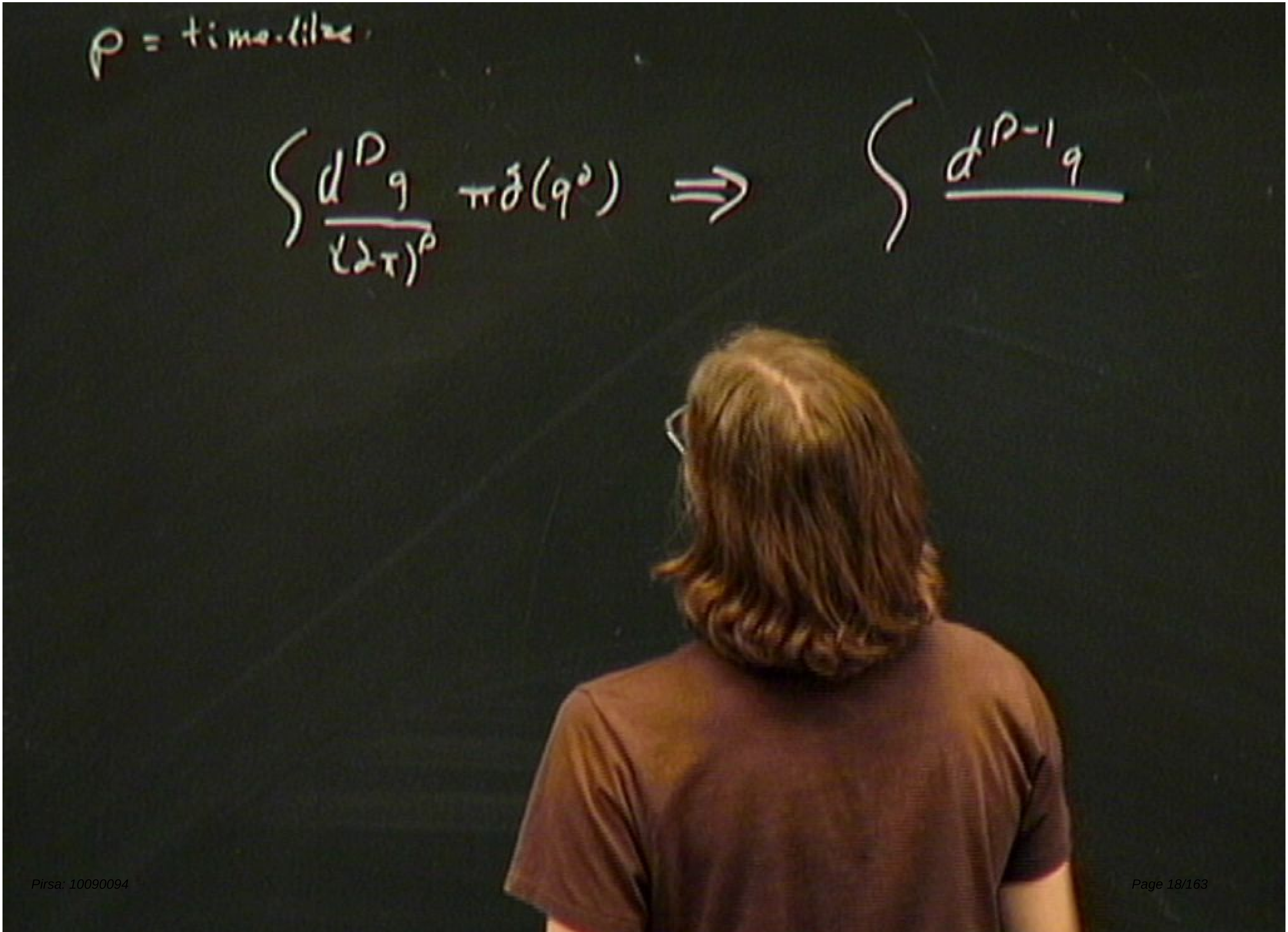
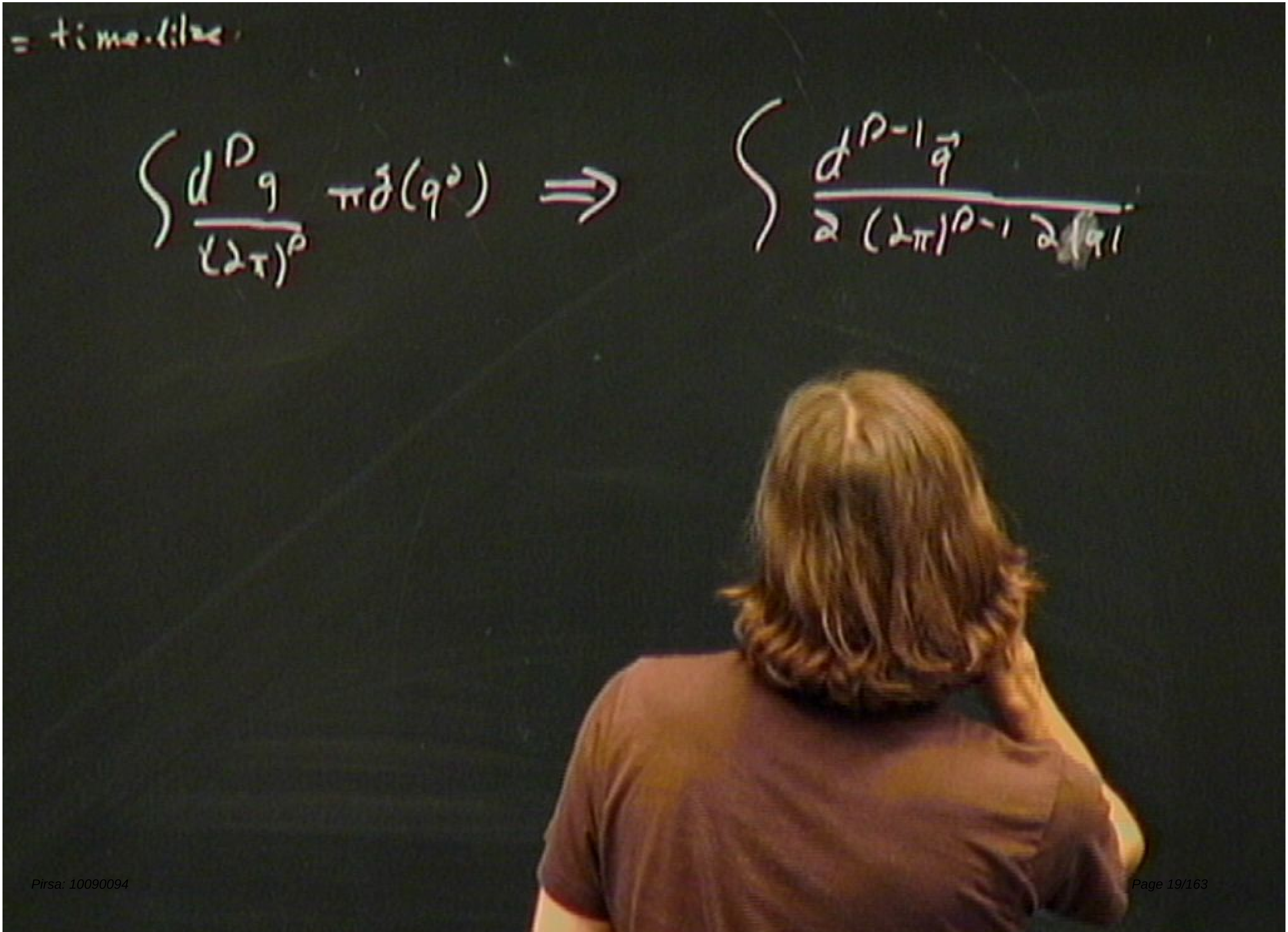
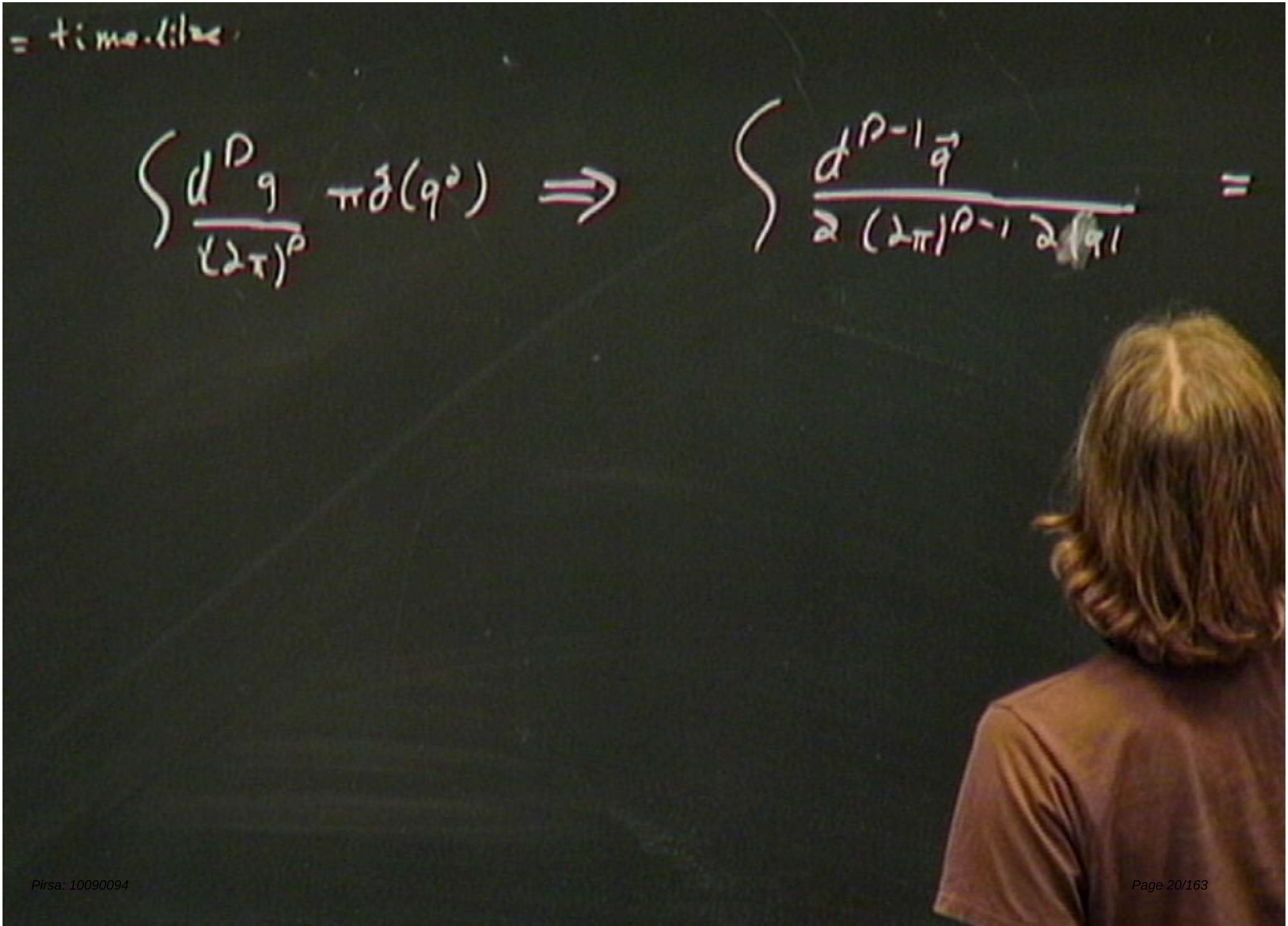
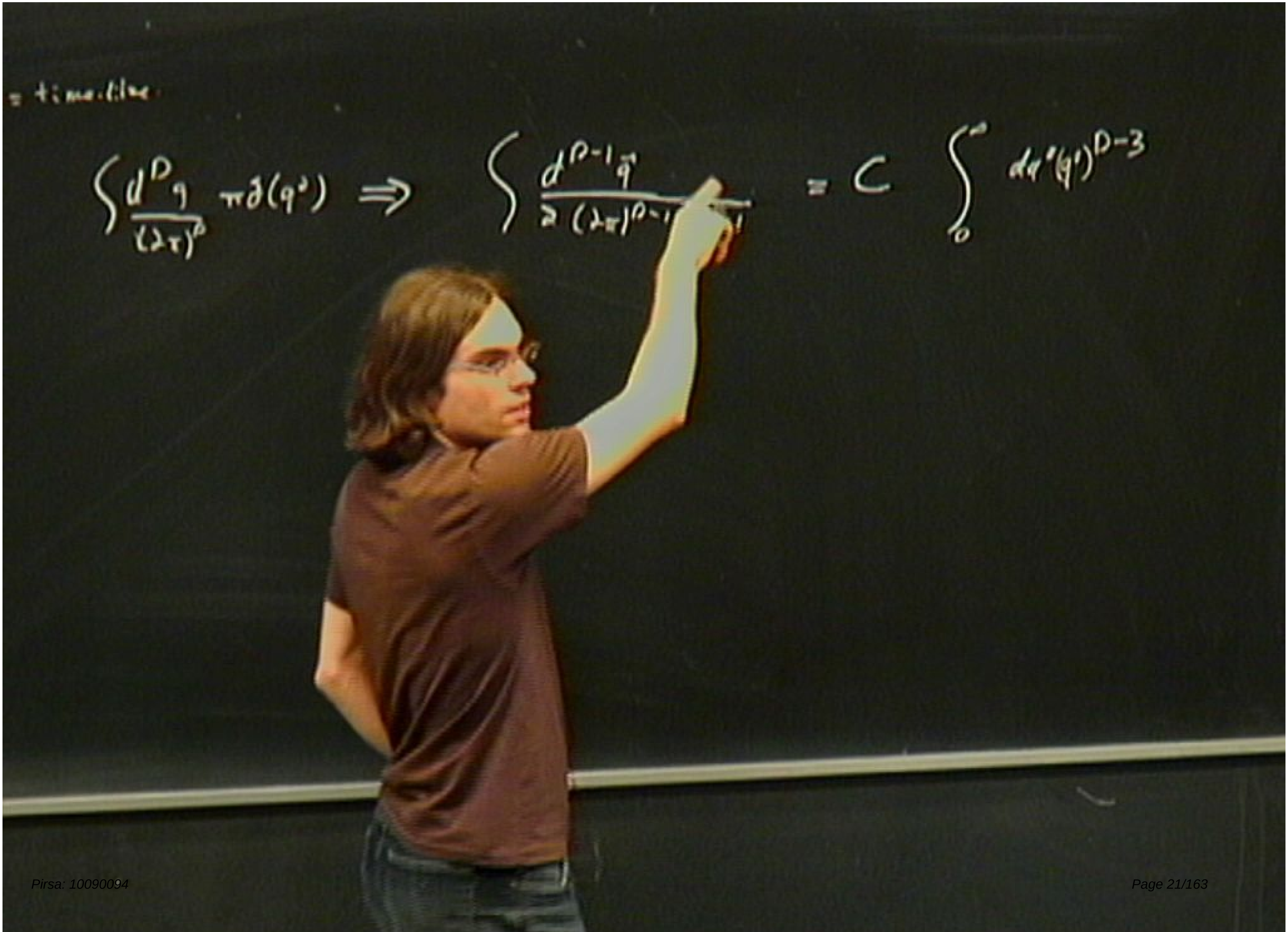
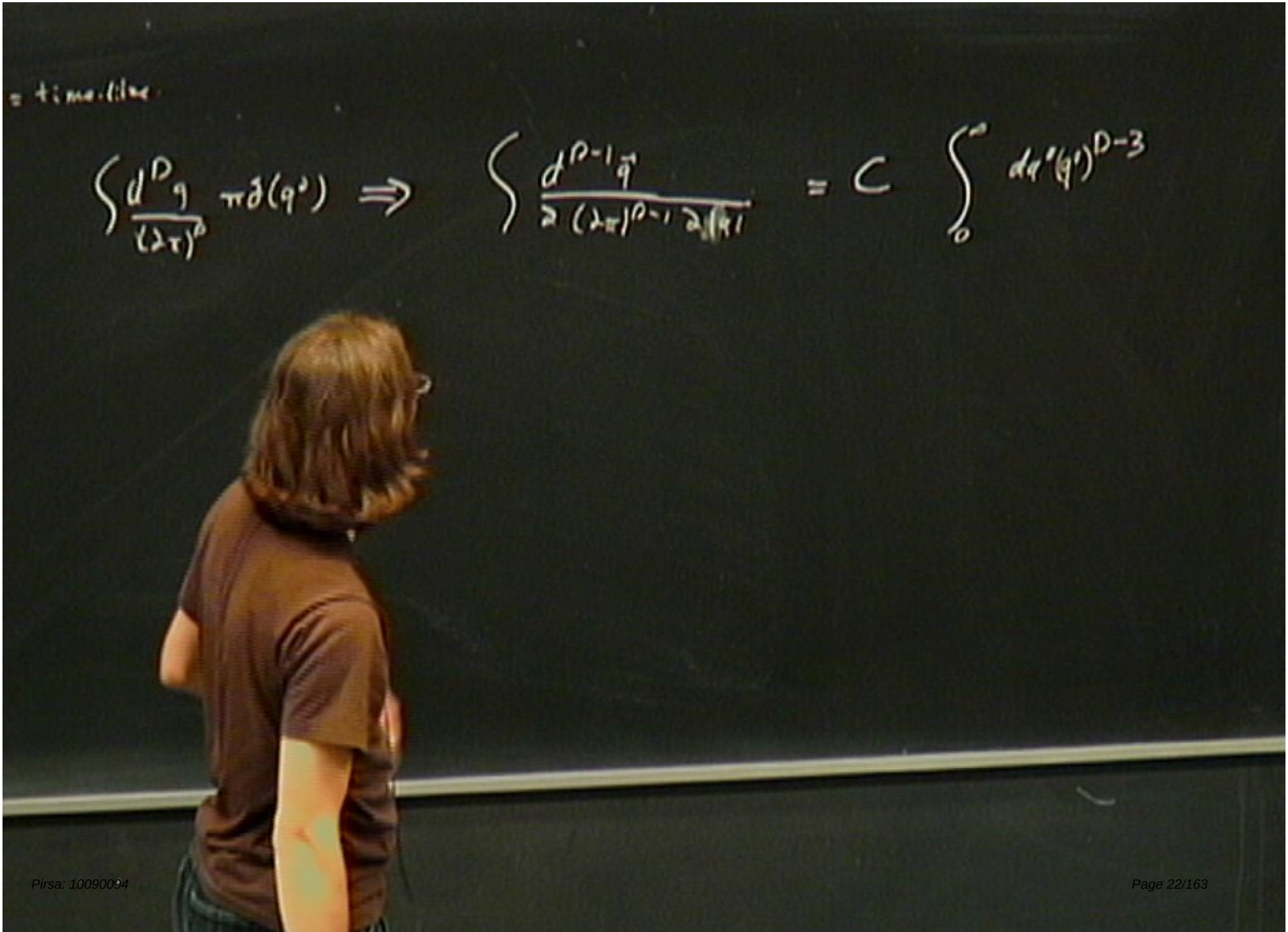
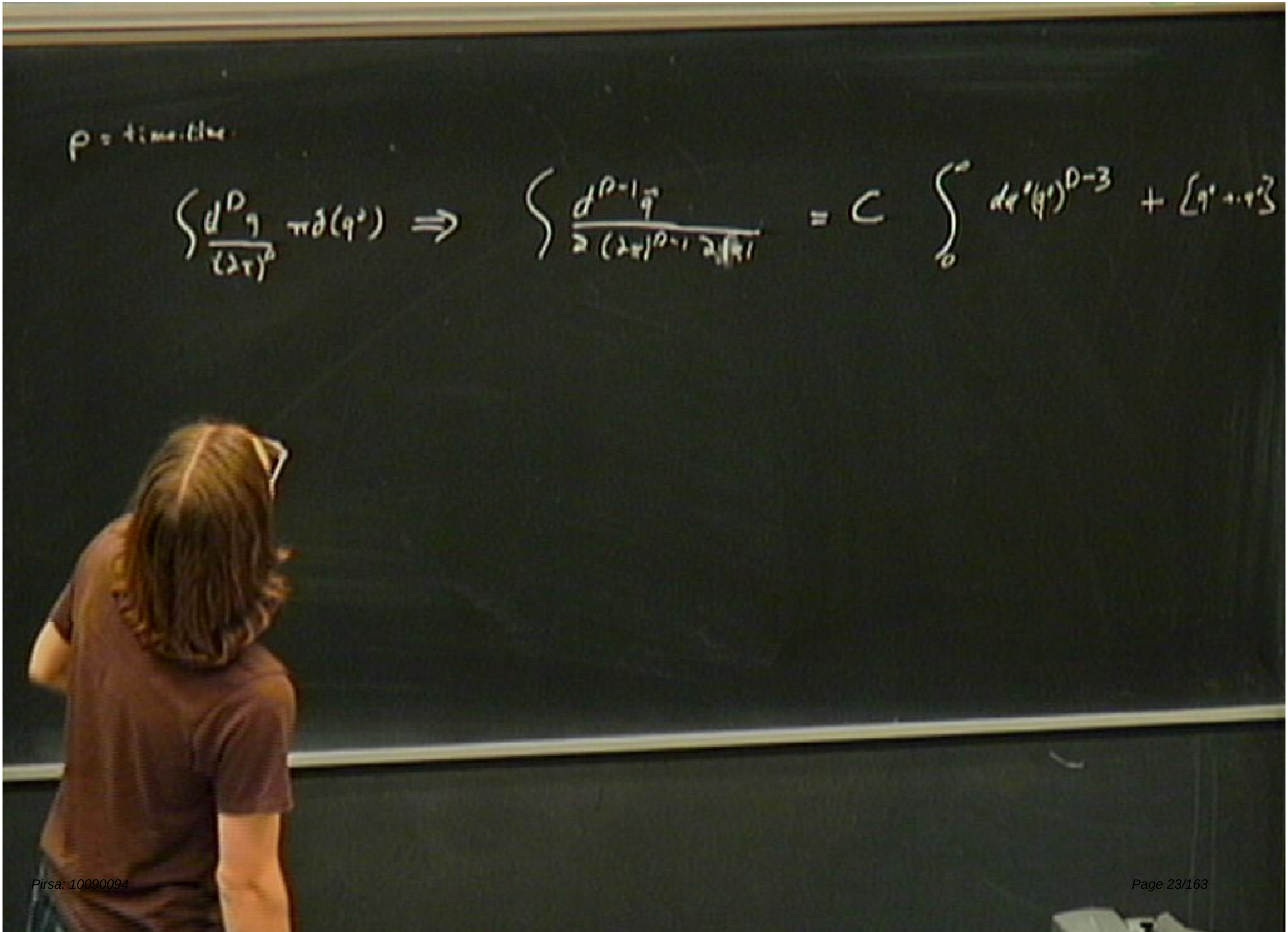

p = time-like
$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^0) \Rightarrow \int \frac{d^{D-1} q}{}$


= time-like
$\int \frac{d^D q}{(2\pi)^D} \pi\delta(q^2) \Rightarrow \int \frac{d^{D-1}\vec{q}}{2(2\pi)^{D-1}\, 2|q|}$


= time-like
$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \Rightarrow \int \frac{d^{D-1}\vec{q}}{2(2\pi)^{D-1} 2|q|} =$


= timelike


= timelike
$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \Rightarrow \int \frac{d^{D-1}\vec{q}}{2(2\pi)^{D-1} 2|\vec{q}|} = C \int_0^\infty dq' (q')^{D-3}$


p = timelike
$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \Rightarrow \int \frac{d^{D-1}\vec{q}}{2(2\pi)^{D-1}|\vec{q}|} = C \int_0^\infty dq^2 (q^2)^{D-3} + \{q^2 \to -q^2\}$

p = timelike

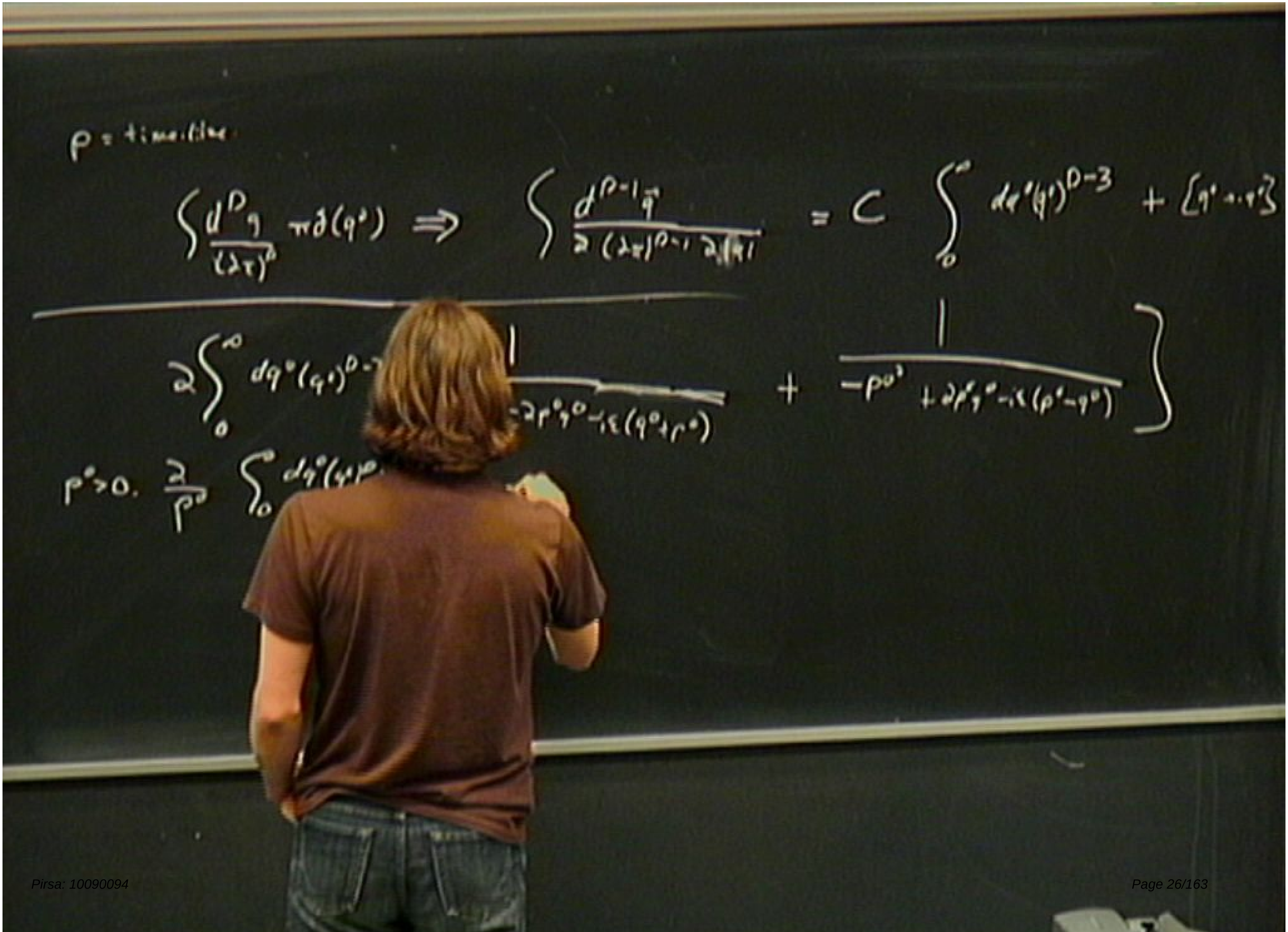
$$\int \frac{d^D q}{(2\pi)^D}\, \pi\delta(q^2) \Rightarrow \int \frac{d^{D-1}\vec{q}}{2\,(2\pi)^{D-1}\, 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + \{q^0 \to -q^0\}$$

$$\int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{-p^{0\,2} - 2p^0 q^0 - i\varepsilon(q^0 + p^0)} \right.$$

timelike

$$\int \frac{d^D q}{(2\pi)^D}\, \pi\delta(q^2) \Rightarrow \int \frac{d^{D-1}\vec{q}}{2(2\pi)^{D-1}\, 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + \{q^0 \to -q^0\}$$

$$\int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{-p^{0\,2} - 2p^0 q^0 - i\epsilon(q^0 + p^0)} + \frac{1}{-p^{0\,2} + 2p^0 q^0 - i\epsilon(p^0 - q^0)} \right]$$


p = timelike
∫ d^D q / (2π)^D π δ(q²) ⇒ ∫ d^(D-1) q / 2 (2π)^(D-1) 2|q| = C ∫_0^∞ dq⁰ (q⁰)^(D-3) + [q⁰ → -q⁰]
p⁰ > 0.

p = timelike

$$\int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \Rightarrow \int \frac{d^{D-1}\vec{q}}{2(2\pi)^{D-1} 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + \{q^0 \to -q^0\}$$

$$2\int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{-p^{0^2} - 2p^0 q^0 - i\epsilon(q^0+p^0)} + \frac{1}{-p^{0^2} + 2p^0 q^0 - i\epsilon(p^0-q^0)} \right]$$

$$p^0 > 0. \quad \frac{2}{p^0} \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{-1}{p^0 + 2q^0} \right.$$

p = timelike

$$\int \frac{d^D q}{(2\pi)^D}\, \pi\delta(q^2) \Rightarrow \int \frac{d^{D-1}\vec{q}}{2(2\pi)^{D-1}\, 2|\vec{k}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + \{q^0 \to -q^0\}$$

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$$p^0 > 0. \quad \frac{2}{p^0} \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{-1}{p^0 + 2q^0} + \frac{-1}{p^0 - 2q^0 + i\epsilon(p^0)} \right.$$

p = timelike

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$$2\int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{-p^{0\,2} - 2p^0 q^0 - i\epsilon(q^0+p^0)} + \frac{1}{-p^{0\,2} + 2p^0 q^0 - i\epsilon(p^0 - q^0)} \right]$$

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p = timelike

$$\int \frac{d^D q}{(2\pi)^D}\, \pi\delta(q^2) \Rightarrow \int \frac{d^{D-1}\vec{q}}{2(2\pi)^{D-1}\, 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + [q^0 \to -q^0]$$

$$2\int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{-p^{0\,2} - 2p^0 q^0 - i\varepsilon(q^0+p^0)} + \frac{1}{-p^{0\,2} + 2p^0 q^0 - i\varepsilon(p^0 - q^0)} \right]$$

$p^0 > 0$. $\frac{2}{p^0} \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{-1}{p^0 + 2q^0 \text{[illegible]}} + \frac{-1}{p^0 - 2q^0 + i\varepsilon \text{[illegible]}} \right]$

$q^0 \to p^0 q^0 \quad 2(p^0)^{\ldots}$

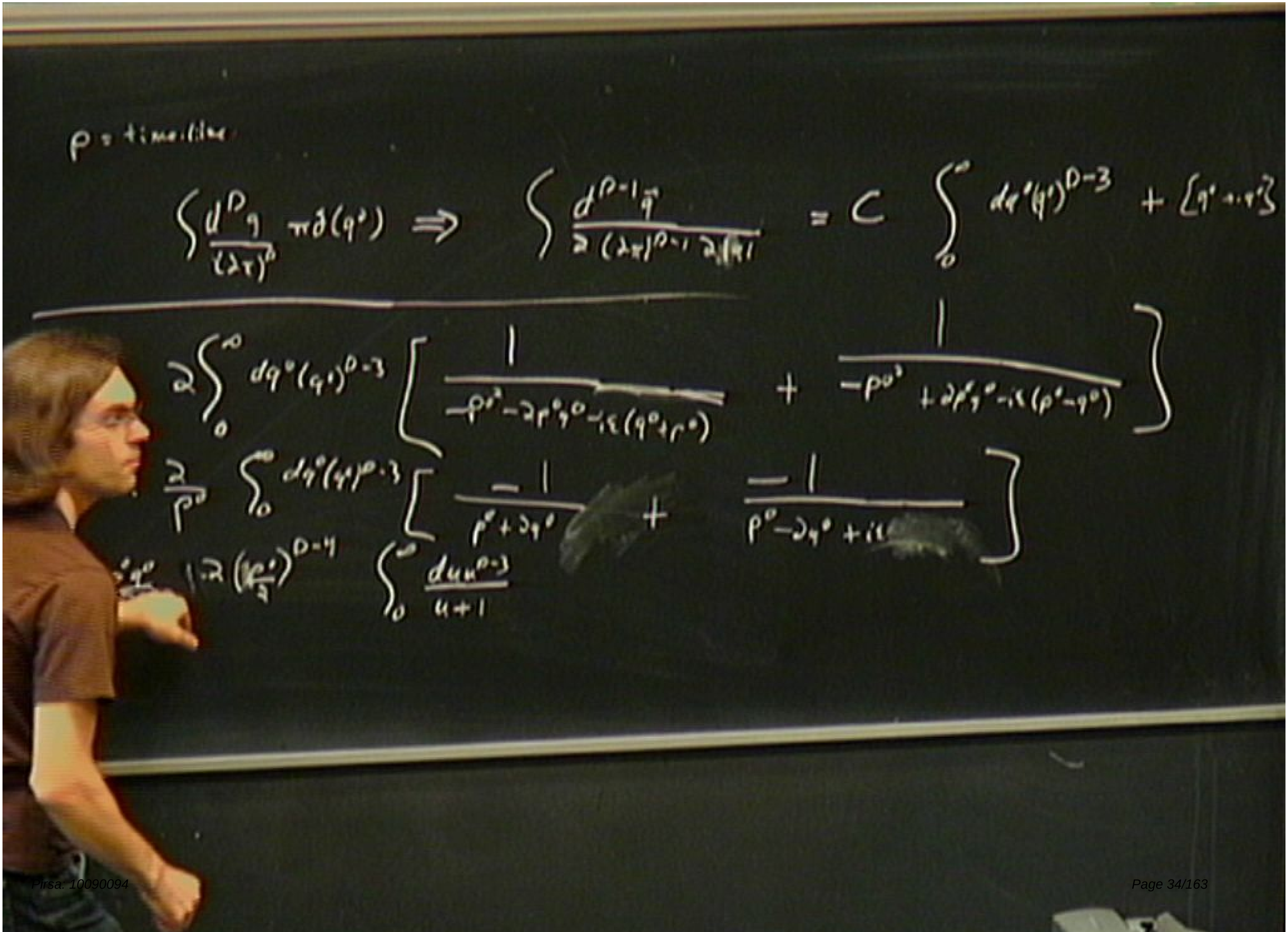
p = timelike

$$\int \frac{d^D q}{(2\pi)^D}\, \pi\delta(q^2) \Rightarrow \int \frac{d^{D-1}\vec{q}}{2\,(2\pi)^{D-1}\, 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + \{q^0 \to -q^0\}$$

$$2\int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{-p^{0^2} - 2p^0q^0 - i\epsilon(q^0+p^0)} + \frac{1}{-p^{0^2} + 2p^0q^0 - i\epsilon(p^0-q^0)} \right]$$

$$p^0>0. \quad \frac{2}{p^0} \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{-1}{p^0+2q^0} + \frac{-1}{p^0-2q^0+i\epsilon} \right]$$

$$q^0 \to p^0 q^0 \qquad 2(p^0)^{D-4} \int_0^\infty \frac{du\, u^{D-3}}{u+1}$$


p = timelike
∫ d^D q / (2π)^D π δ(q²) ⇒ ∫ d^(D-1) q / 2 (2π)^(D-1) 2|q| = C ∫_0^∞ dq⁰ (q⁰)^(D-3) + {q⁰ → -q⁰}
2 ∫_0^∞ dq⁰ (q⁰)^(D-3) [1 / (-p⁰² - 2p⁰q⁰ - iε(q⁰+p⁰)) + 1 / (-p⁰² + 2p⁰q⁰ - iε(p⁰-q⁰))]
2/p⁰ ∫_0^∞ dq⁰ (q⁰)^(D-3) [-1 / (p⁰ + 2q⁰) + -1 / (p⁰ - 2q⁰ + iε)]
∫_0^∞ du u^(D-3) / (u+1)

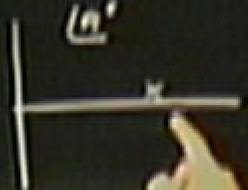
p = timelike

$$\int \frac{d^D q}{(2\pi)^D}\, \pi\delta(q^2) \Rightarrow \int \frac{d^{D-1}\vec{q}}{2(2\pi)^{D-1}\, 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + \{q^0 \to -q^0\}$$

$$2\int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{-p^{0\,2} - 2p^0 q^0 - i\epsilon(q^0+p^0)} + \frac{1}{-p^{0\,2} + 2p^0 q^0 - i\epsilon(p^0-q^0)} \right]$$

$$p^0>0.\quad \frac{2}{p^0}\int_0^\infty dq^0 (q^0)^{D-3}\left[\frac{-1}{p^0+2q^0} + \frac{-1}{p^0-2q^0+i\epsilon}\right]$$

$$q^0 \to \frac{p^0 q^0}{2} \quad \frac{-2}{p^0}\left(\frac{p^0}{2}\right)^{D-4} \int_0^\infty \frac{du\, u^{D-3}}{u+1}\Big[\,1$$



p = timelike

$$\int \frac{d^D q}{(2\pi)^D}\, \pi\delta(q^2) \Rightarrow \int \frac{d^{D-1}\vec{q}}{2(2\pi)^{D-1}\, 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + [q^0 \to -q^0]$$

$$2\int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{-p^{0 2} - 2p^0 q^0 - i\epsilon(q^0+p^0)} + \frac{1}{-p^{0 2} + 2p^0 q^0 - i\epsilon(p^0 - q^0)} \right]$$

$$p^0 > 0. \quad \frac{2}{p^0} \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{-1}{p^0 + 2q^0} + \frac{-1}{p^0 - 2q^0 + i\epsilon} \right]$$

$$q^0 \to \frac{p^0 q^0}{2} \quad 2\left(\frac{p^0}{2}\right)^{D-4} \int_0^\infty \frac{du\, u^{D-3}}{u+1} \left[1 + e^{-i\pi(D-3)} \right]$$

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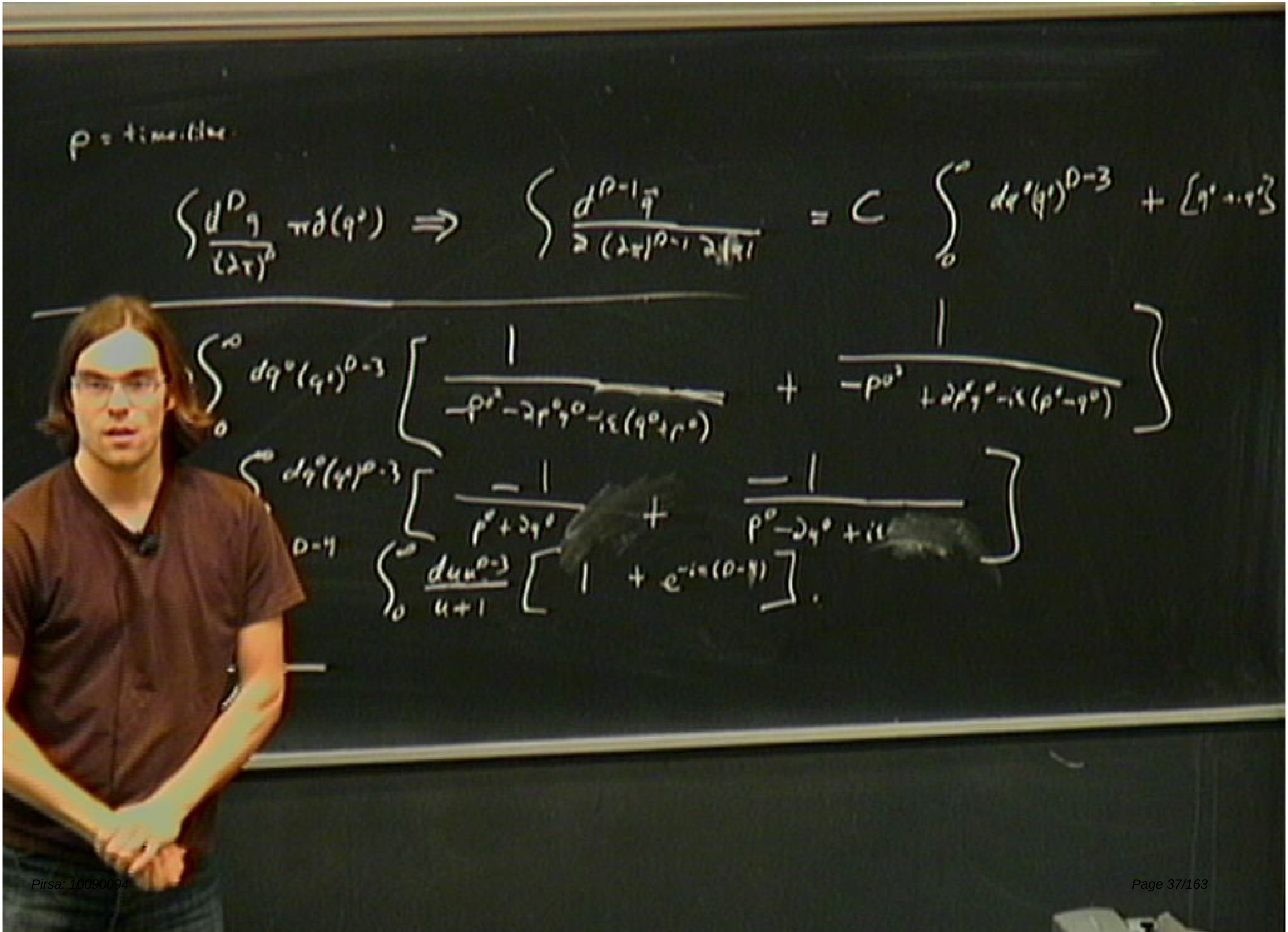
p = timelike

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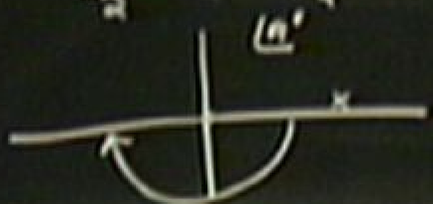
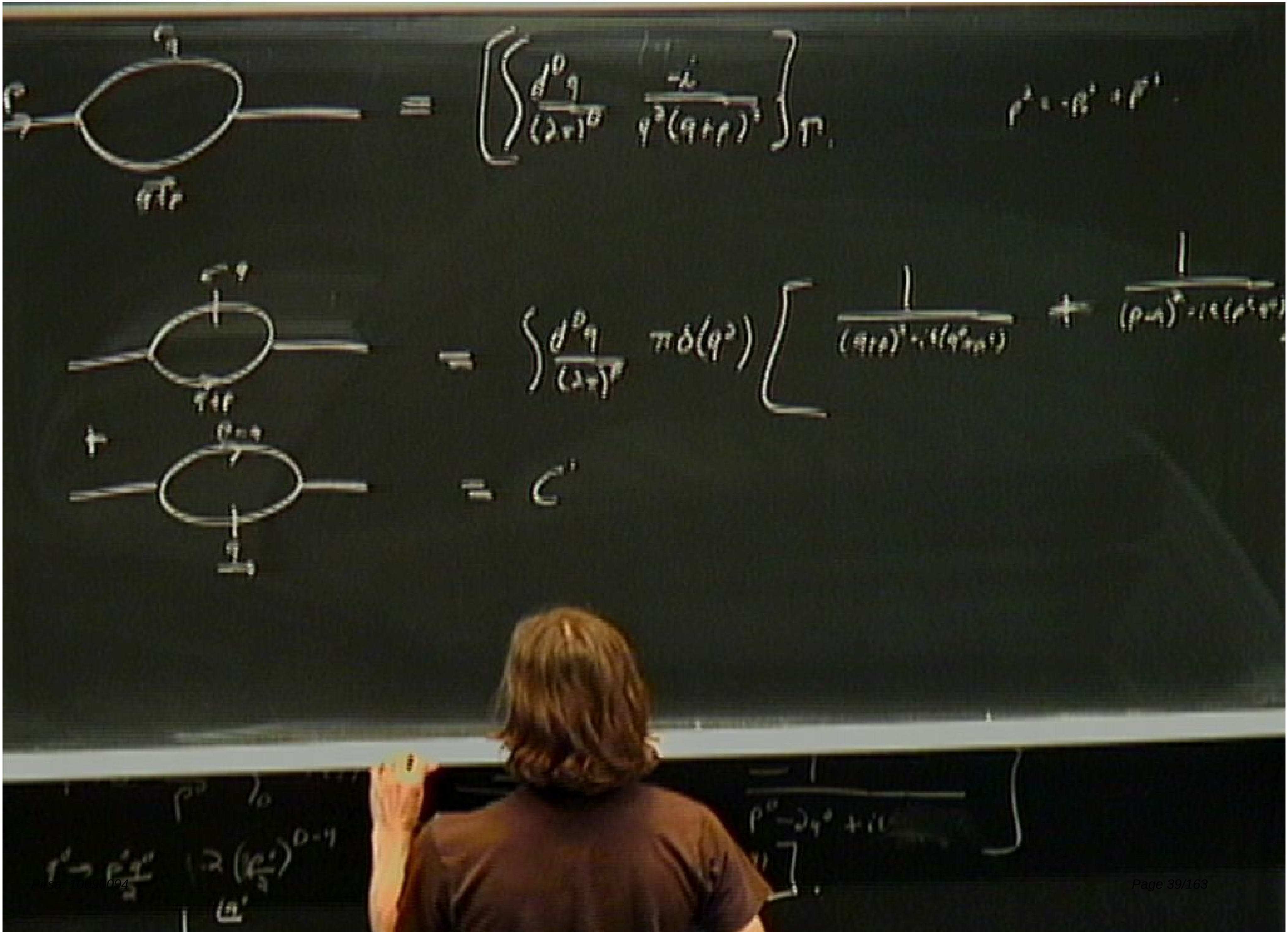
$$\int \frac{d^D q}{(2\pi)^D}\, \pi\delta(q^2) \Rightarrow \int \frac{d^{D-1}\vec{q}}{2\,(2\pi)^{D-1}\, 2|\vec{q}|} = C \int_0^\infty dq^0 (q^0)^{D-3} + \{q^0 \to -q^0\}$$

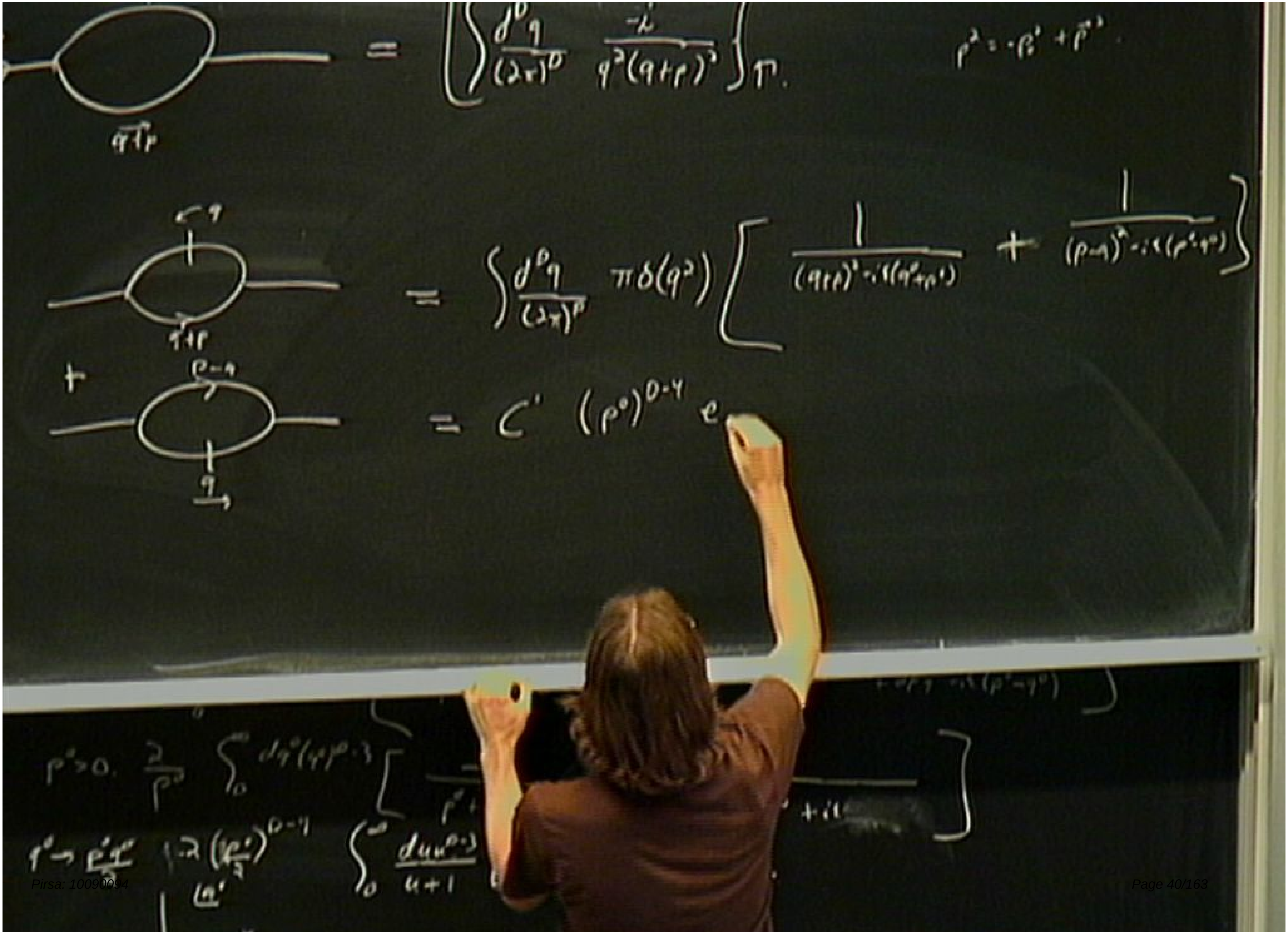
$$2\int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{1}{-p^{0^2} - 2p^0 q^0 - i\epsilon(q^0+p^0)} + \frac{1}{-p^{0^2} + 2p^0 q^0 - i\epsilon(p^0 - q^0)} \right]$$

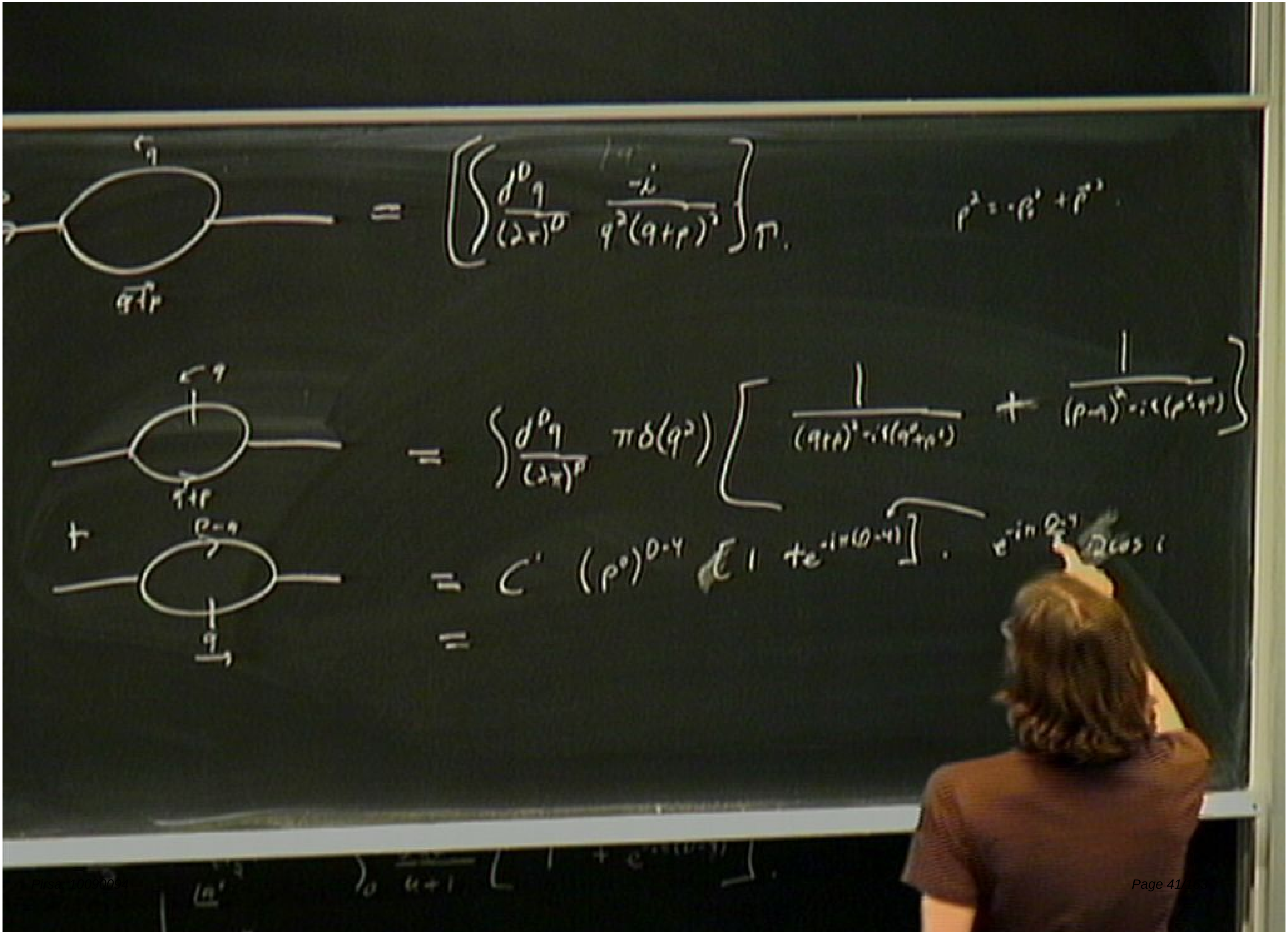
$$p^0 > 0. \quad \frac{2}{p^0} \int_0^\infty dq^0 (q^0)^{D-3} \left[\frac{-1}{p^0 + 2q^0} + \frac{-1}{p^0 - 2q^0 + i\epsilon} \right]$$

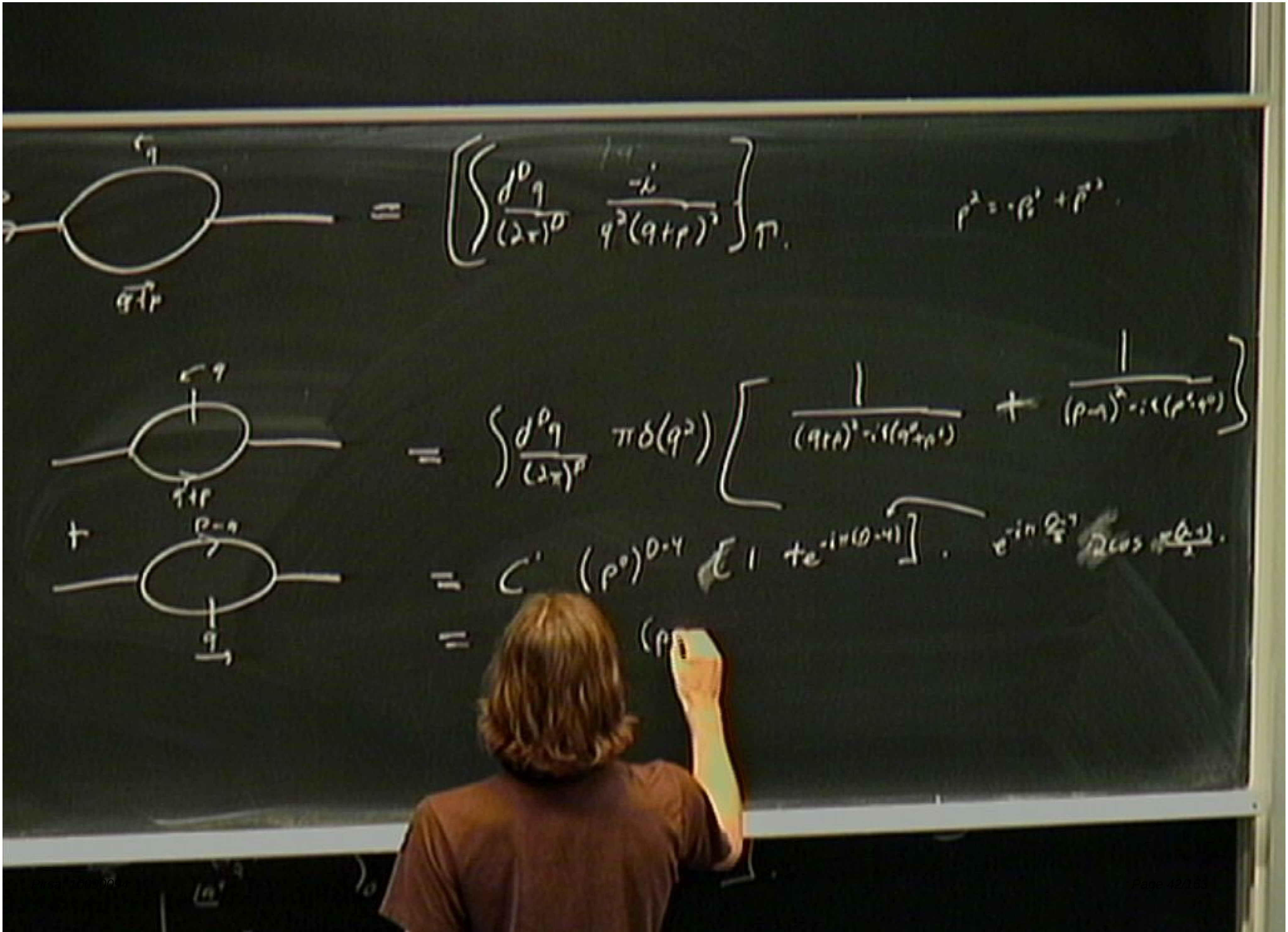
$$q^0 \to \frac{p^0 q^0}{2} \quad (-2)\left(\frac{p^0}{2}\right)^{D-4} \int_0^\infty \frac{du\, u^{D-3}}{u+1} \left[1 + e^{-i\pi(D-4)} \right].$$

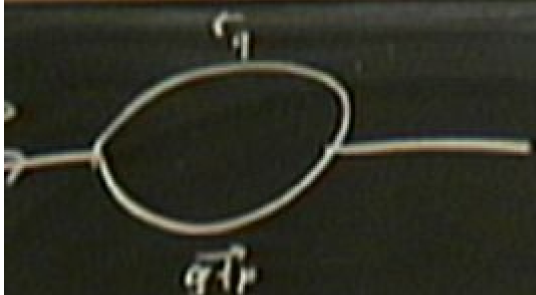






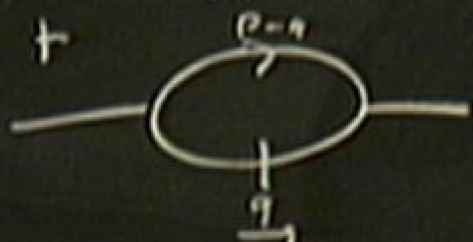






$$= \left[\int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2 (q+p)^2} \right]_{p^0}. \qquad p^2 = -p^{0\,2} + \vec{p}^{\,2}.$$

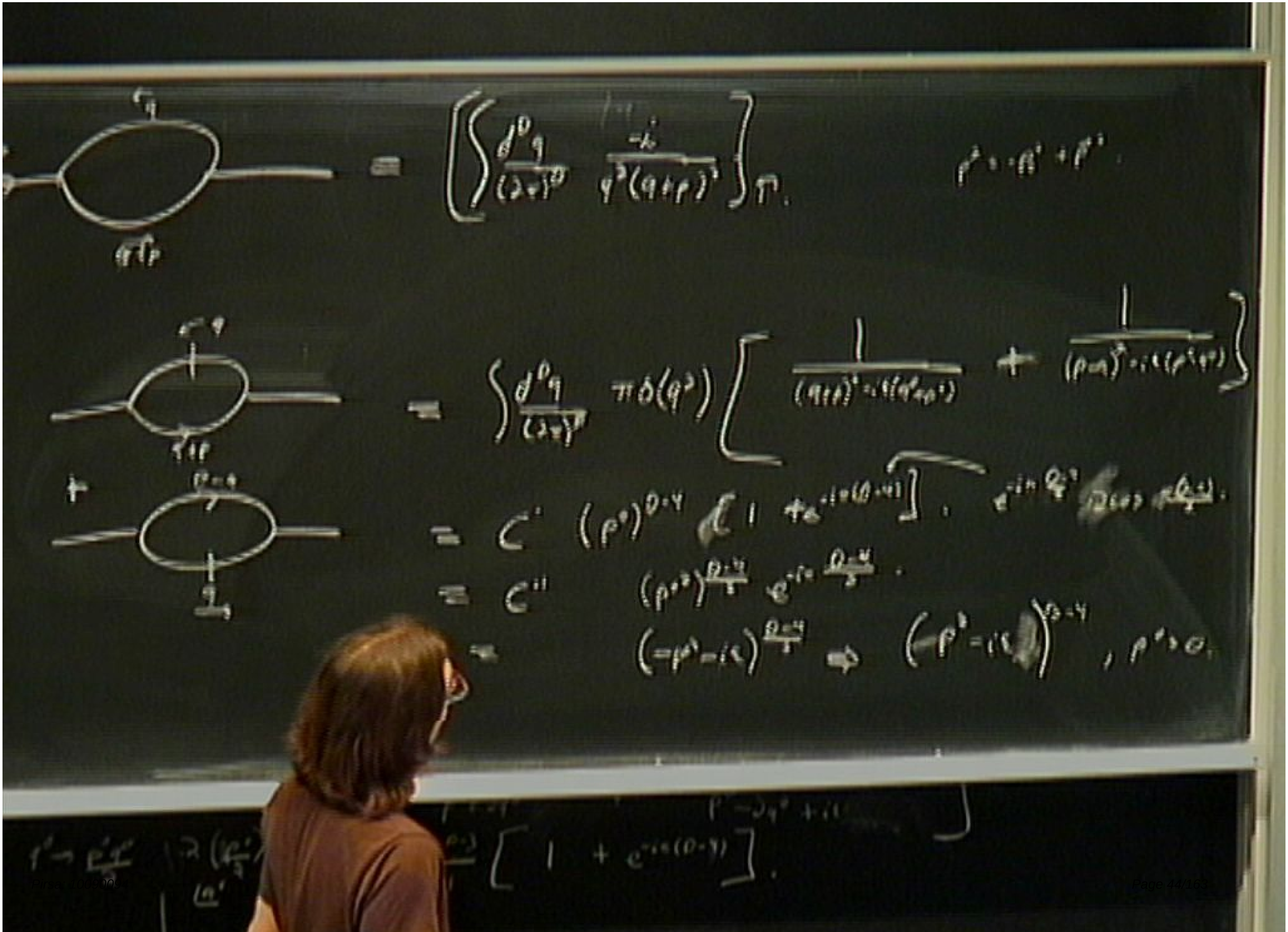
$$= \int \frac{d^D q}{(2\pi)^D} \, \pi \delta(q^2) \left[\frac{1}{(q+p)^2 - i\epsilon(q^0+p^0)} + \frac{1}{(p-q)^2 - i\epsilon(p^0-q^0)} \right]$$

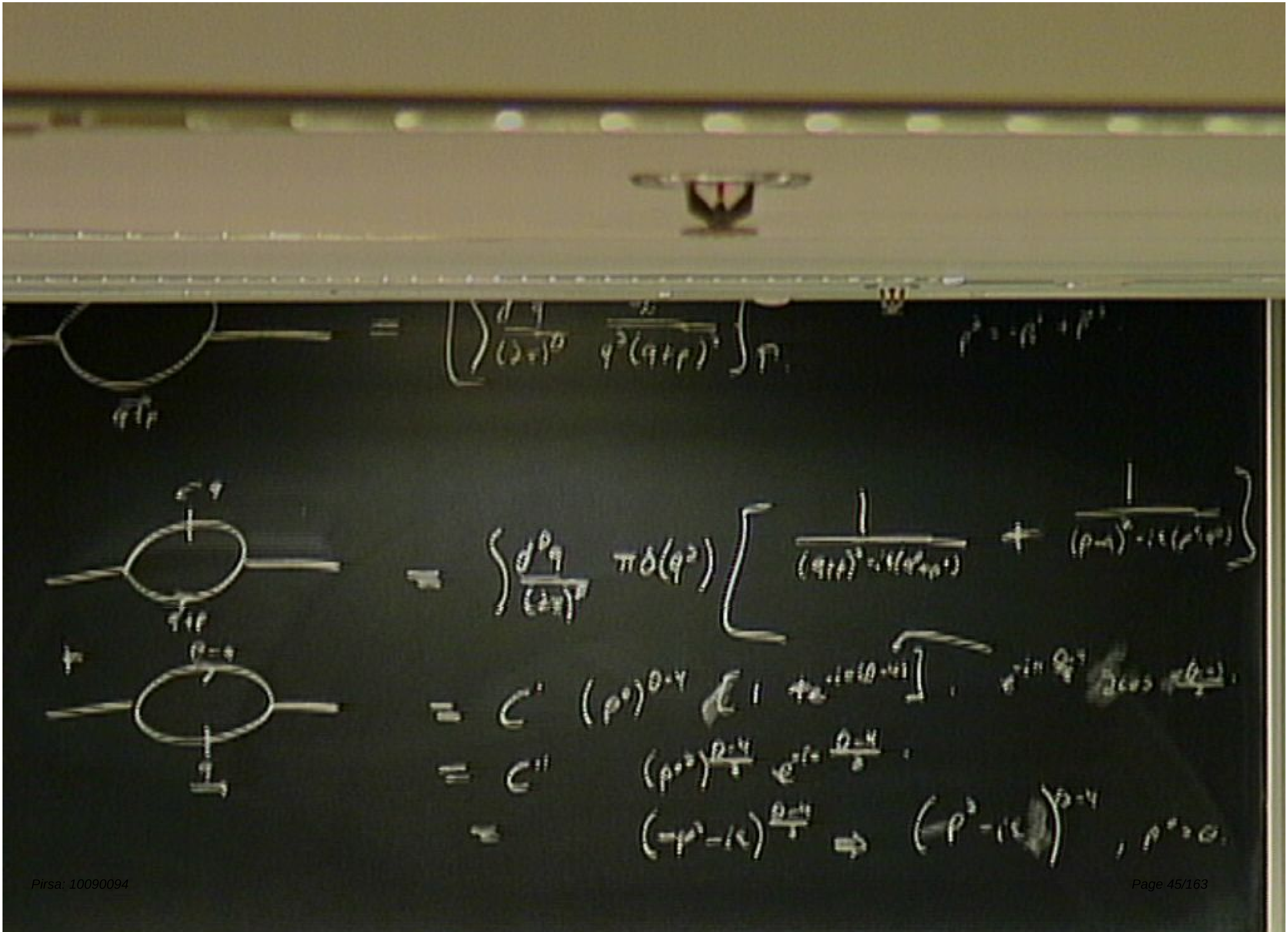
$+$ 

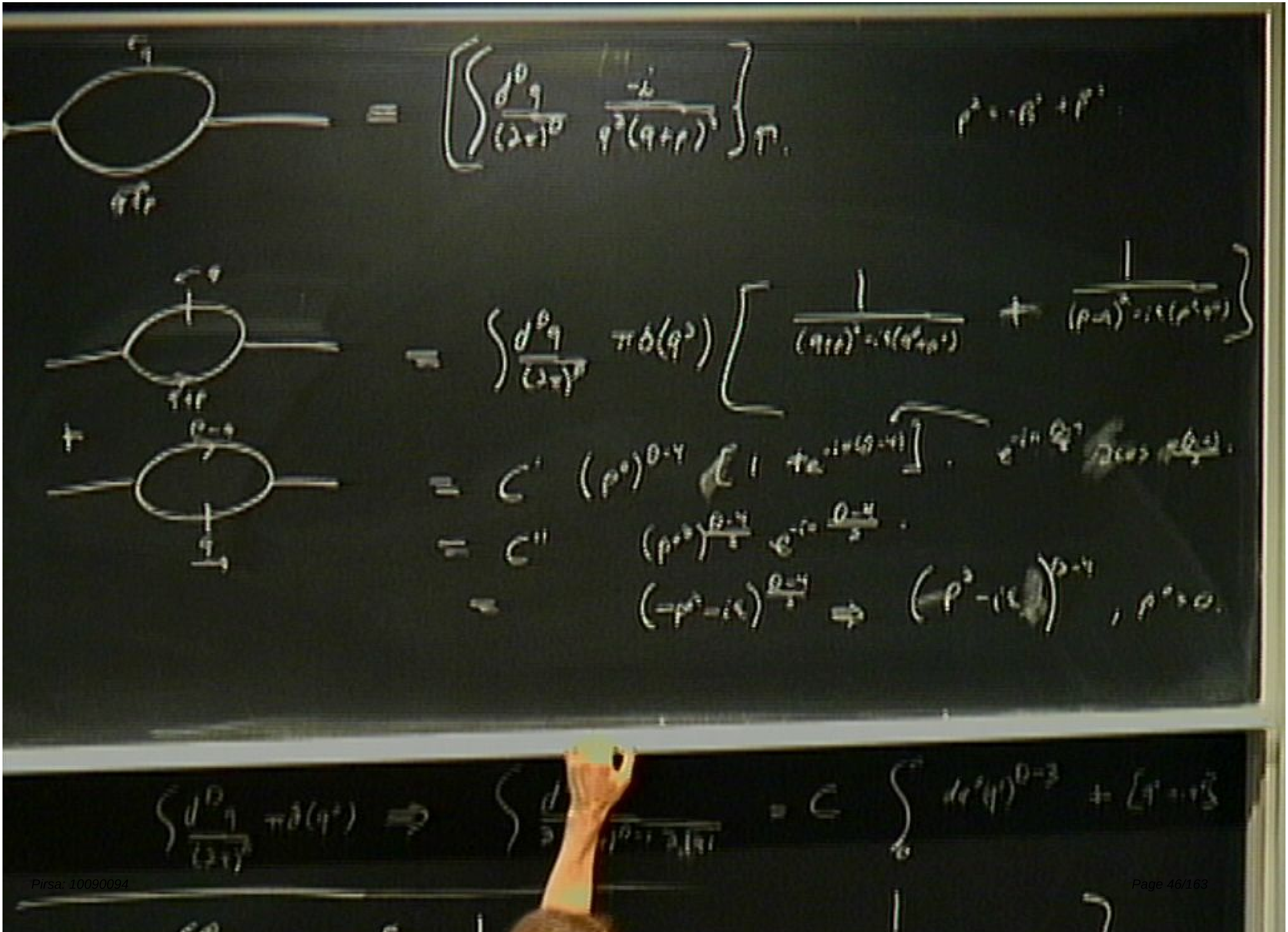
$$= C' \, (p^0)^{D-4} \left[1 + e^{-i\pi(D-4)} \right] . \quad e^{-i\pi\frac{D-4}{2}} \, 2\cos \pi\frac{D-4}{2}.$$

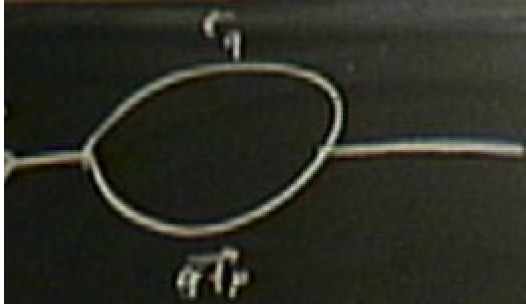
$$= C'' \, (p^2)^{\frac{D-4}{2}} \, e^{-i\pi\frac{D-4}{2}}.$$

$$(\ [illegible] \ )^{\frac{D-4}{2}}.$$


$= \left[\int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2 (q+p)^2}\right]_{\Gamma}$
$= \int \frac{d^D q}{(2\pi)^D} \, \pi \delta(q^2) \left[\frac{1}{(q+p)^2 + i\epsilon(q^0+p^0)} + \frac{1}{(p-q)^2 + i\epsilon(p^0-q^0)} \right]$
$= C' (p^2)^{D-4} \left[1 + e^{-i\pi(D-4)} \right]$
$= C'' (p^2)^{\frac{D-4}{2}} e^{-i\pi\frac{D-4}{2}}$
$= (-p^2 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow (-p^2 - i\epsilon)^{D-4}$
$\left[1 + e^{-i\pi(D-4)} \right]$

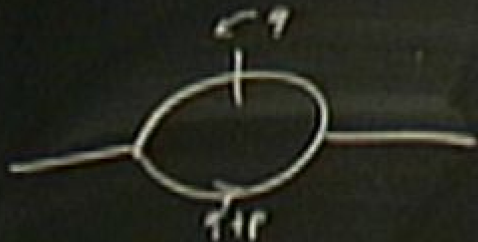






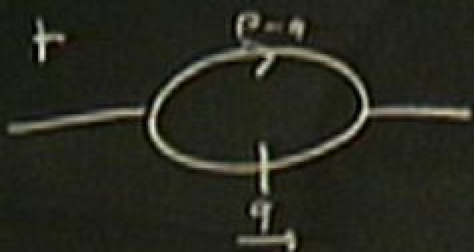
$$= \left[\int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2 (q+p)^2}\right]_{\Pi}.$$

$p^2 = -p^{0\,2} + \vec{p}^{\,2}.$



$$= \int \frac{d^D q}{(2\pi)^D}\, \pi\delta(q^2)\left[\frac{1}{(q+p)^2 - i\epsilon(q^0+p^0)} + \frac{1}{(p-q)^2 - i\epsilon(p^0-q^0)}\right]$$

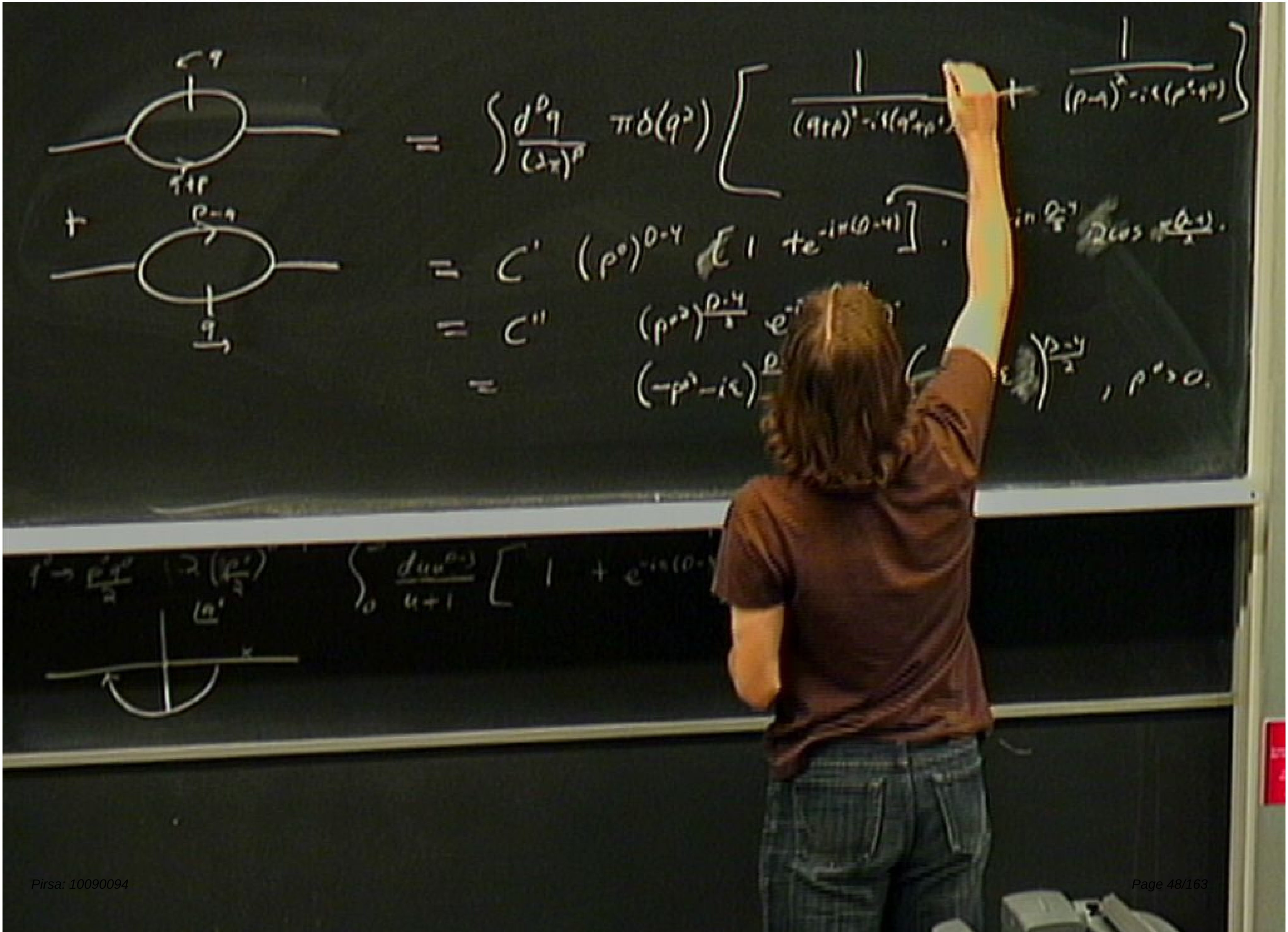
+

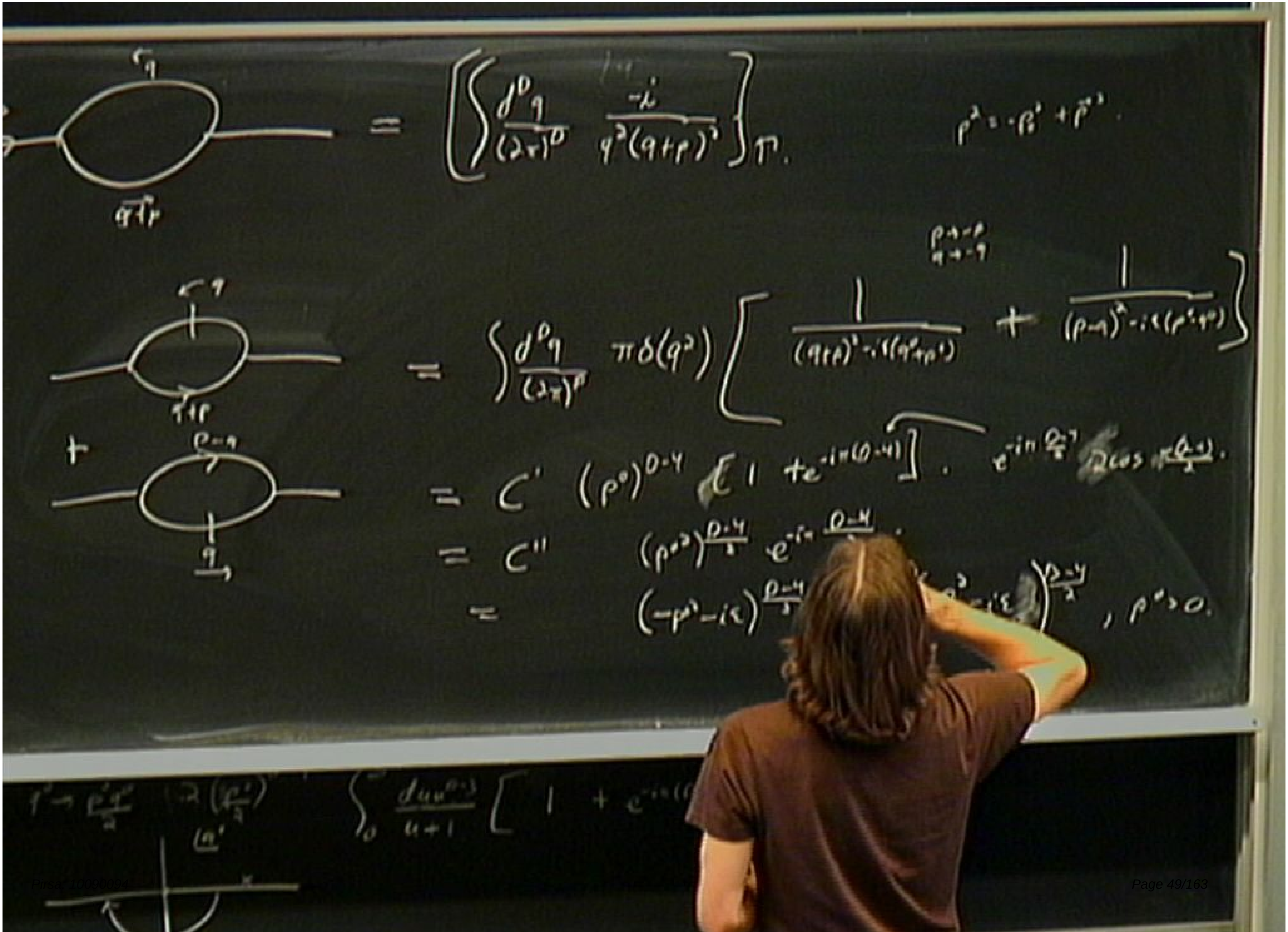


$$= C' (p^0)^{D-4}\left[1 + e^{-i\pi(D-4)}\right] \cdot \quad e^{-i\pi\frac{D-4}{2}}\, 2\cos\frac{\pi(D-4)}{2}.$$

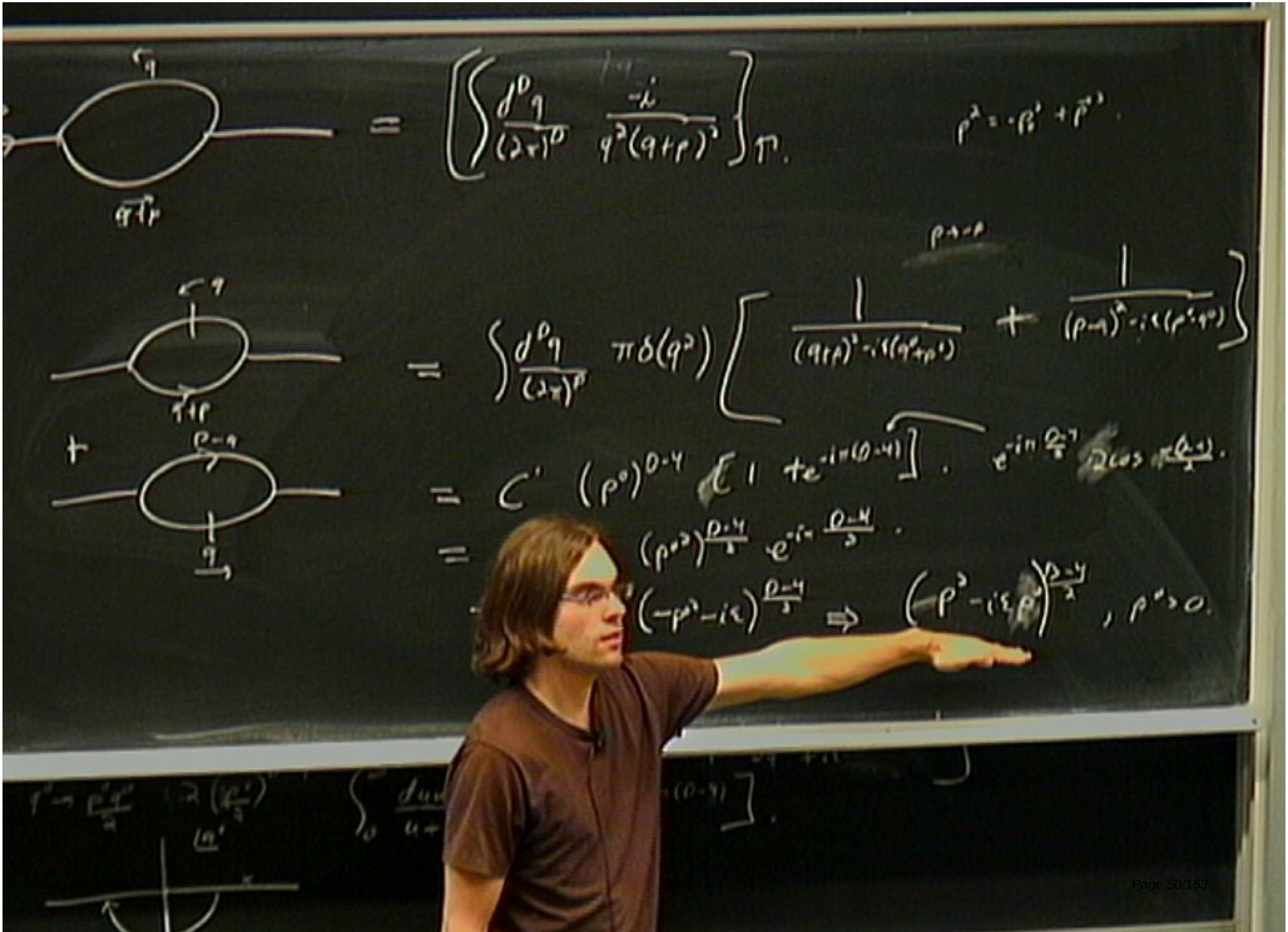
$$= C'' (p^{0\,2})^{\frac{D-4}{2}} e^{-i\pi\frac{D-4}{2}}.$$

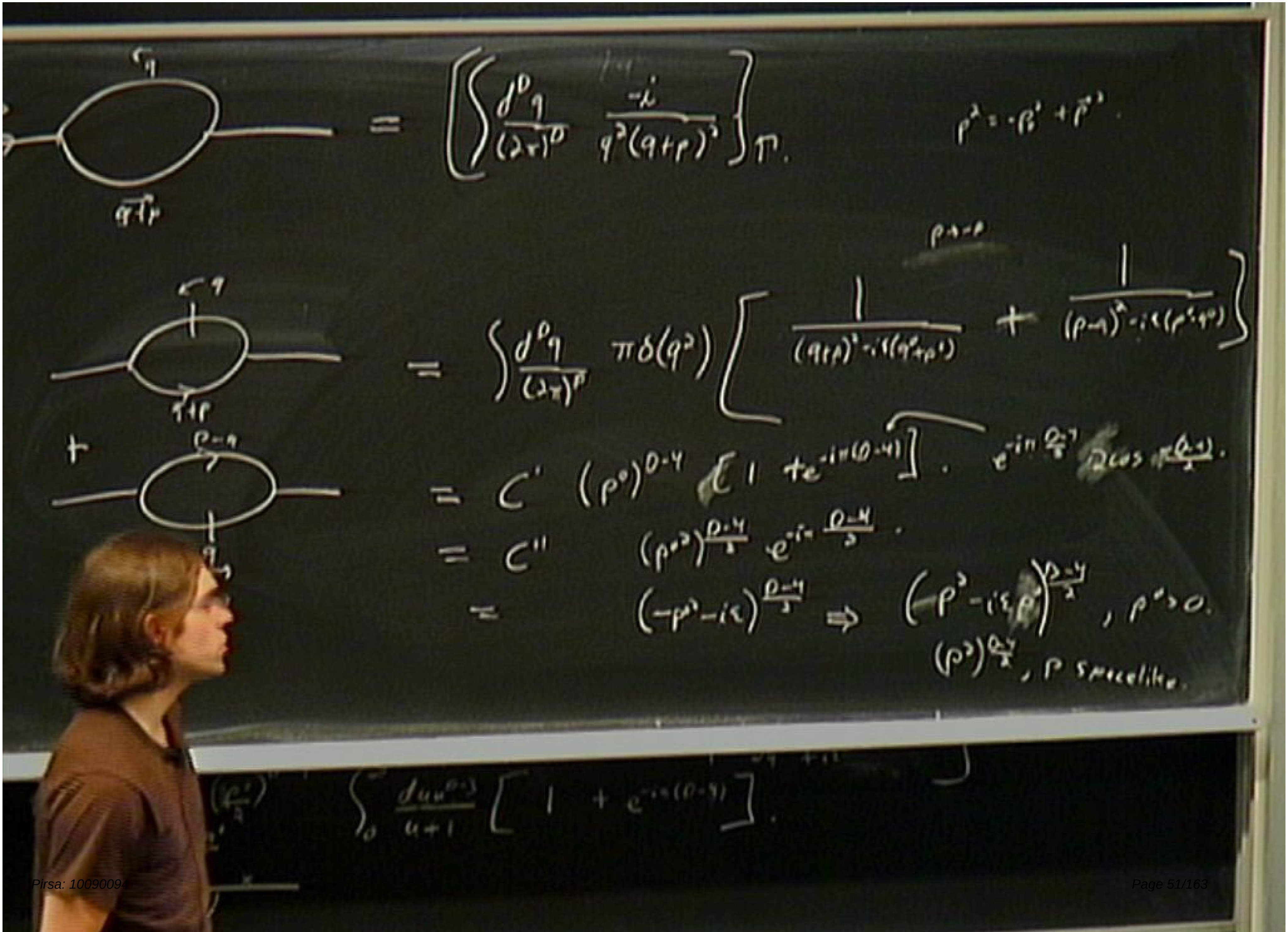
$$\approx (-p^2 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow (-p^2 - i\epsilon)^{D-4}, \quad p^0 > 0.$$



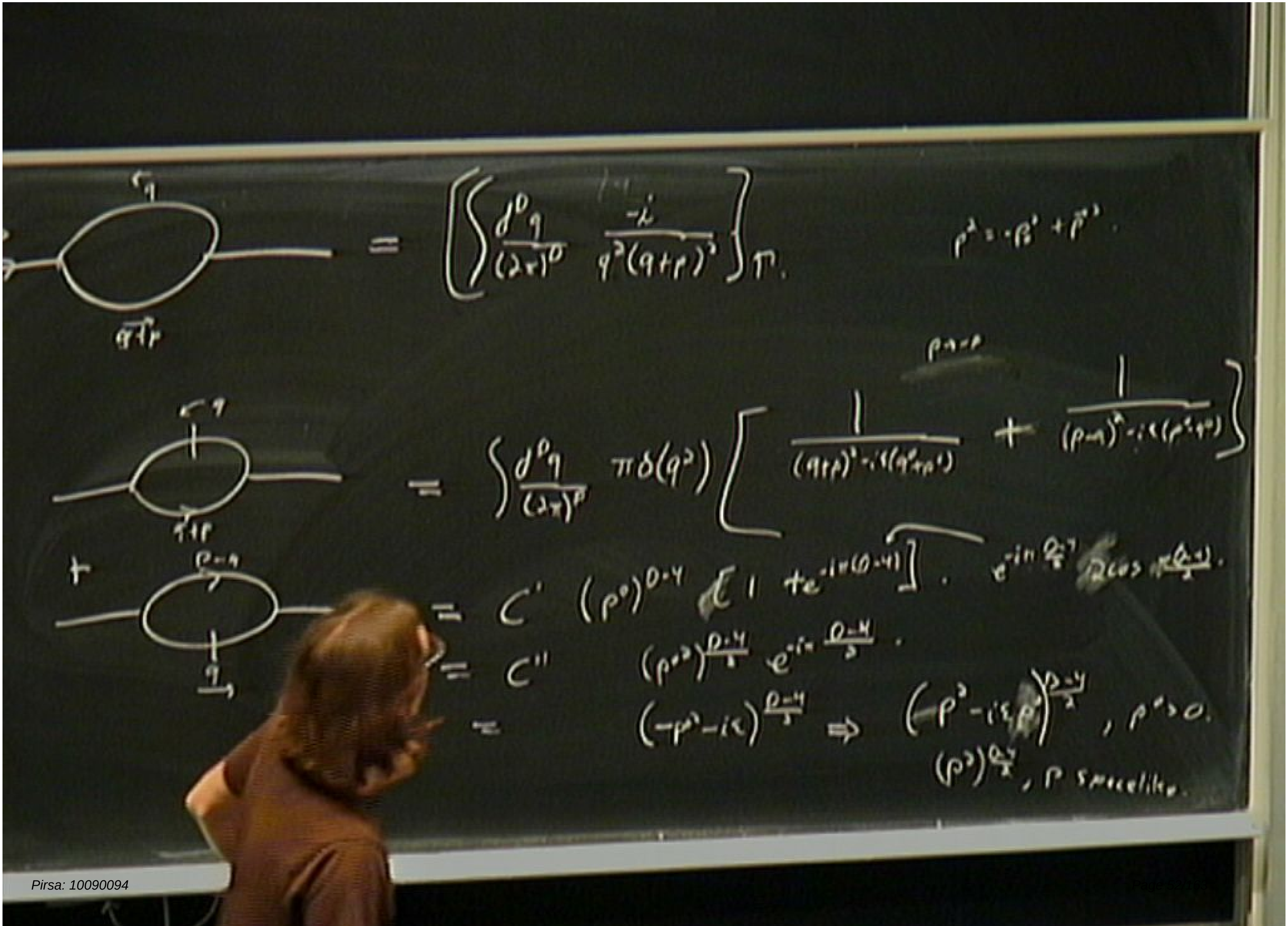


$= \left[\int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2 (q+p)^2}\right]_{\Gamma}$
$p^2 = -p^{0\,2} + \vec{p}^{\,2}$
$= \int \frac{d^D q}{(2\pi)^D} \pi\delta(q^2) \left[\frac{1}{(q+p)^2 - i\epsilon(q^0+p^0)} + \frac{1}{(p-q)^2 - i\epsilon(p^0 - q^0)}\right]$
$= C' (p^0)^{D-4} [1 + e^{-i\pi(D-4)}]$
$= C'' (p^{0\,2})^{\frac{D-4}{2}}$
$, p^0 > 0.$

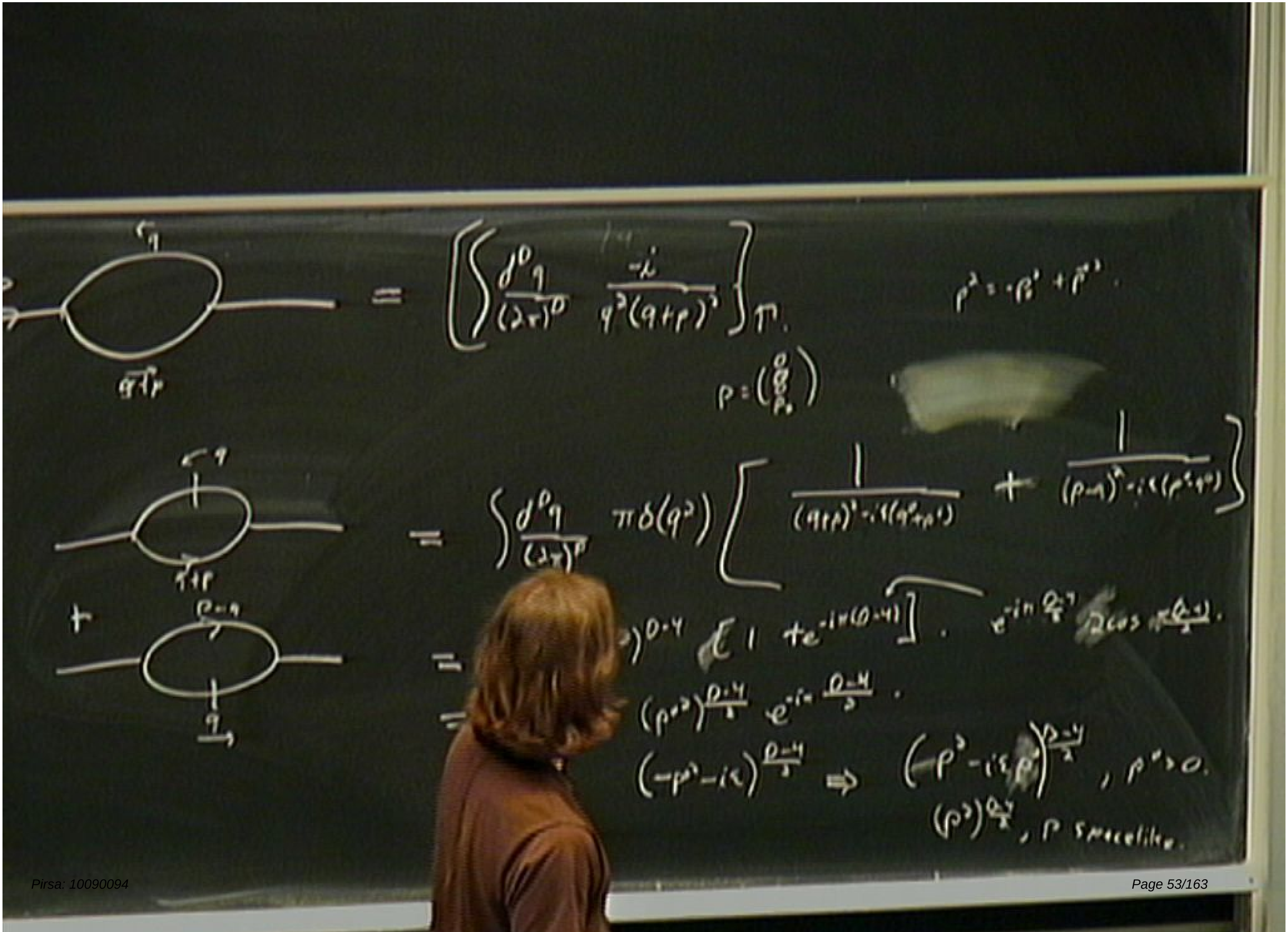


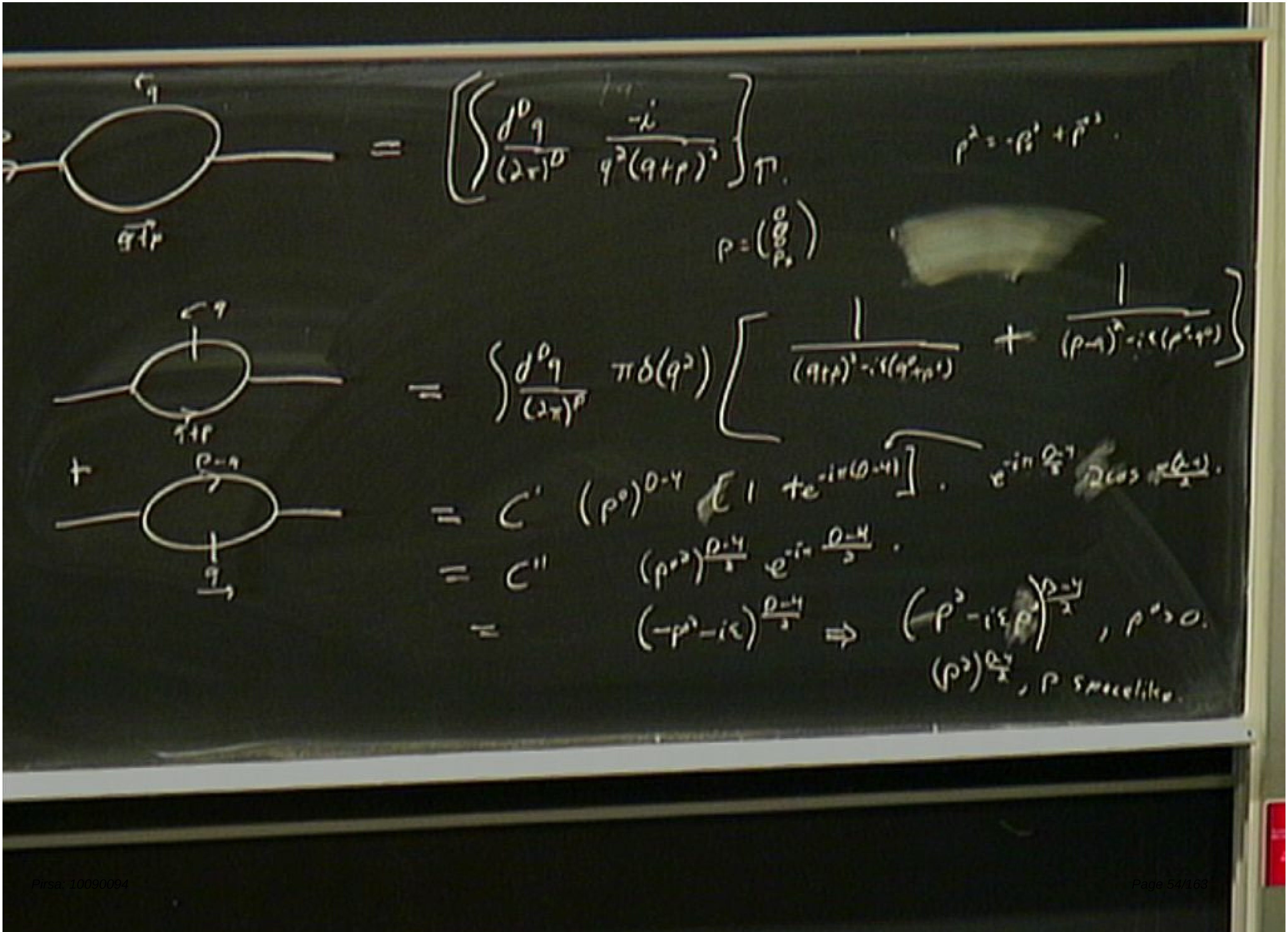


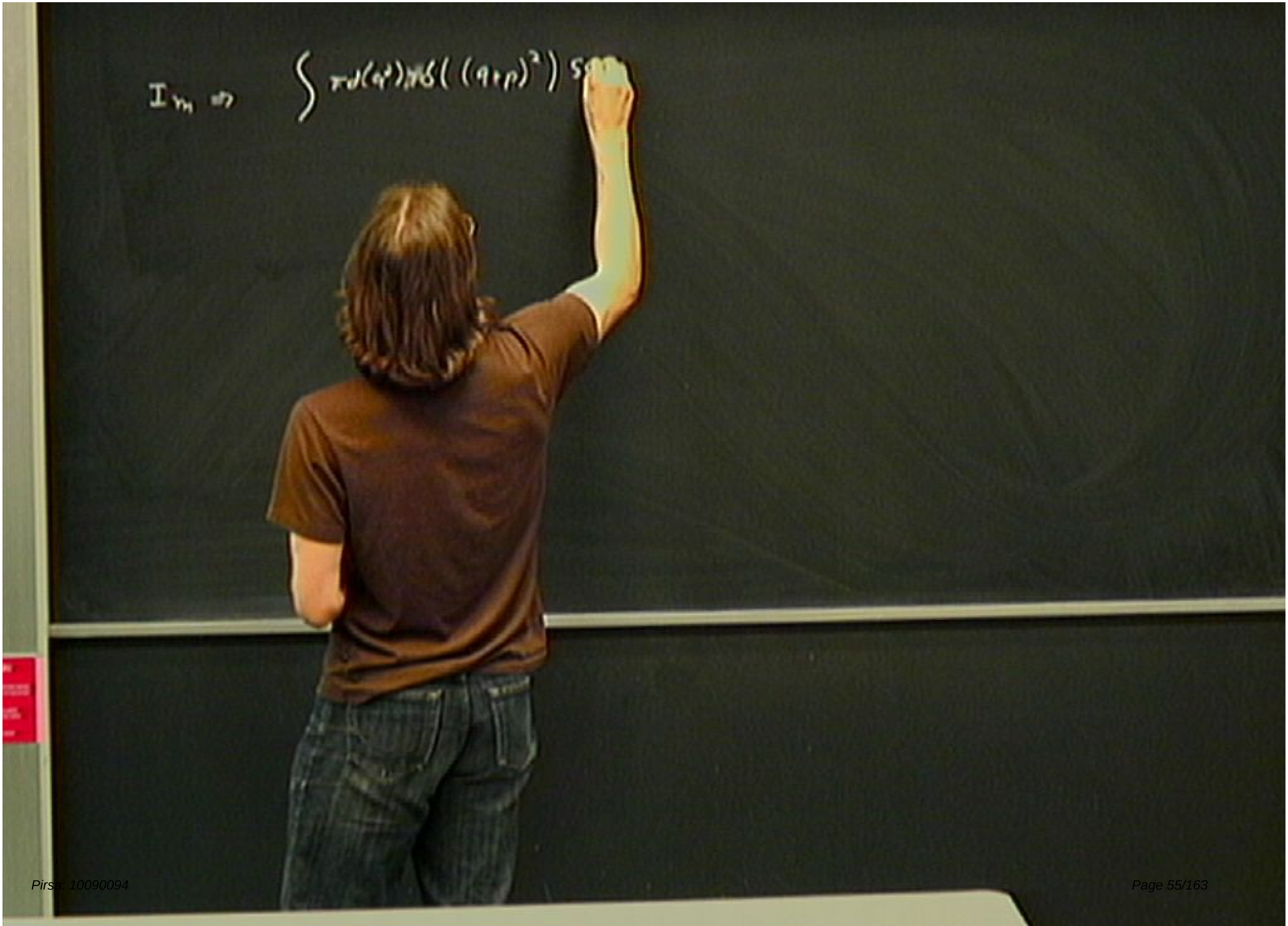
$= \left[\int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2 (q+p)^2} \right]_{\Gamma}$
$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+p)^2 - i\epsilon(q^0+p^0)} + \frac{1}{(p-q)^2 - i\epsilon(p^0-q^0)} \right]$
$= C' (p^0)^{D-4} [1 + e^{-i\pi(D-4)}]$
$= C'' (p^{02})^{\frac{D-4}{2}} e^{-i\pi \frac{D-4}{2}}$
$= (-p^2 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow (-p^2 - i\epsilon p^0)^{\frac{D-4}{2}}, \ p^0 > 0$
p spacelike



Pirsa: 10090094




$= \left[\int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2 (q+p)^2} \right]_{\Pi}$
$p^2 = -p_0^2 + \vec{p}^2$
$p = \begin{pmatrix} 0 \\ 0 \\ 0 \\ p_3 \end{pmatrix}$
$= \int \frac{d^D q}{(2\pi)^D} \pi \delta(q^2) \left[\frac{1}{(q+p)^2 - i\epsilon(q^0+p^0)} + \frac{1}{(p-q)^2 - i\epsilon(p^0 - q^0)} \right]$
$= C' (p^0)^{D-4} \left[1 + e^{-i\pi(D-4)} \right]$
$= C'' (p^{0\,2})^{\frac{D-4}{2}} e^{-i\pi\frac{D-4}{2}}$
$= (-p^2 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow (-p^2 - i\epsilon p^0)^{\frac{D-4}{2}}, \; p^0 > 0$
$(p^2)^{\frac{D-4}{2}}$, p spacelike



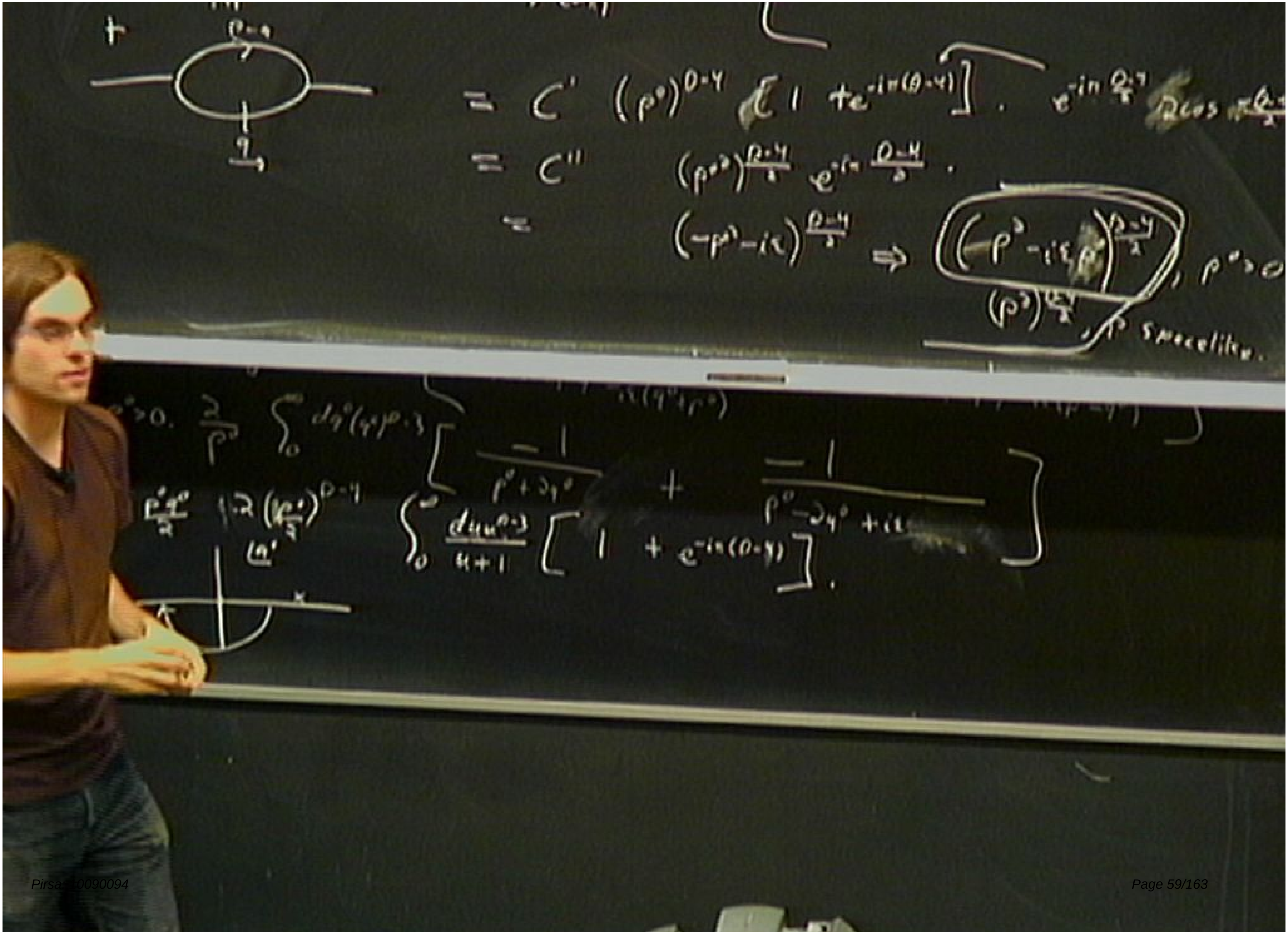
$$I_m \sim \int \pi\delta(q^2)\,\pi\delta\left((q+p)^2\right)\,\mathrm{sgn}(q^0+p^0) \;.$$

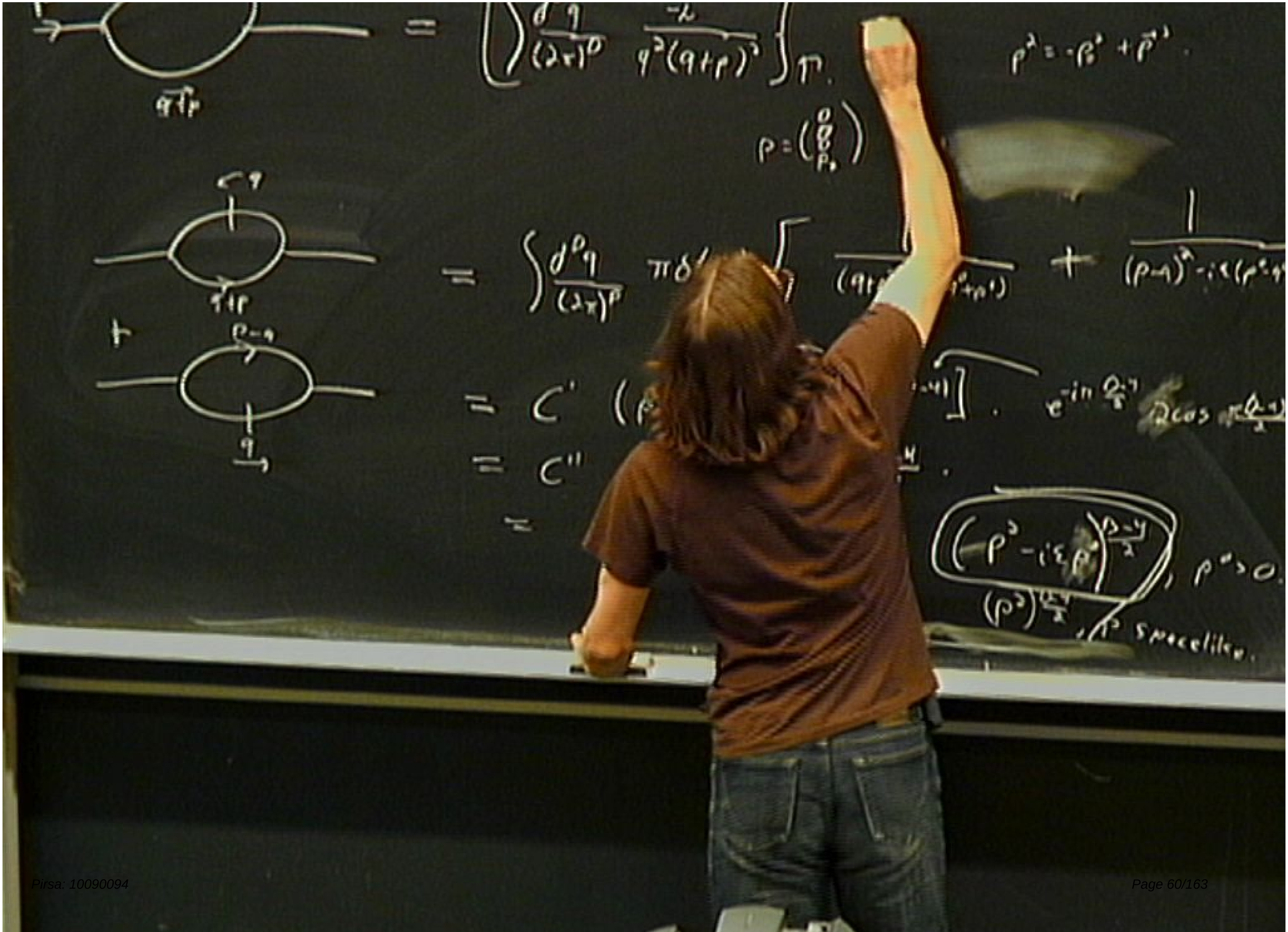
$$I_m \sim \int \pi\delta(q^2)\,\pi\delta\left((q+p)^2\right)\,\mathrm{sgn}(q^0)$$

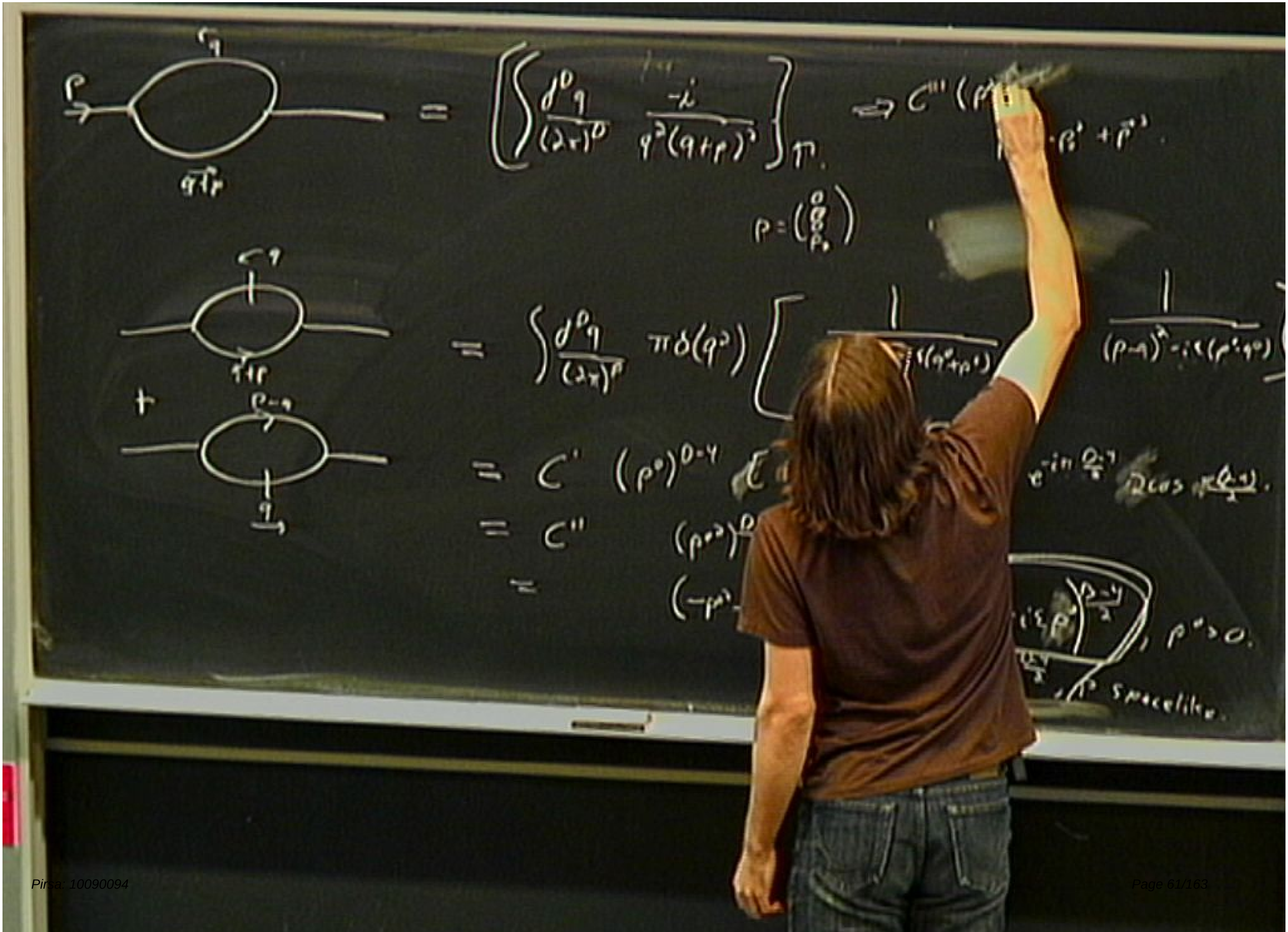
$$\begin{pmatrix} q^0 \\ \vec{q} \end{pmatrix}, \begin{pmatrix} -q^0 \\ \vec{q} \end{pmatrix}$$

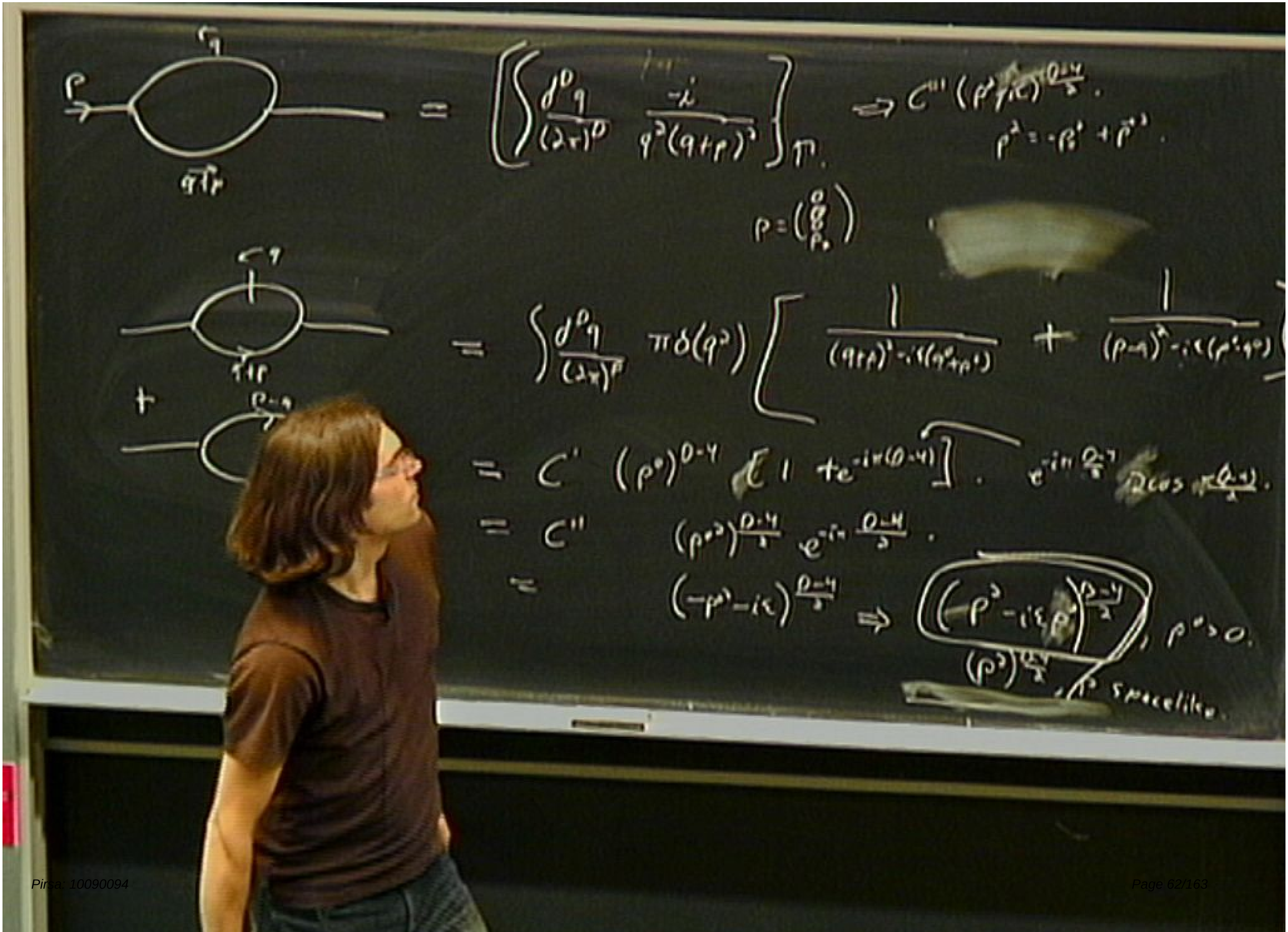
$$\mathrm{Im} \Rightarrow \int \pi\delta(q^2)\,\pi\delta\big((q+p)^2\big)\,\mathrm{sgn}(q^0) \longrightarrow 0.$$

$$\begin{pmatrix} q^0 \\ \vec{q} \end{pmatrix}, \begin{pmatrix} -q^0 \\ \vec{q} \end{pmatrix}$$





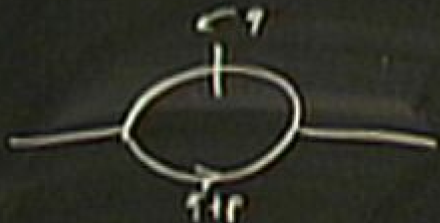

spacelike


spacelike

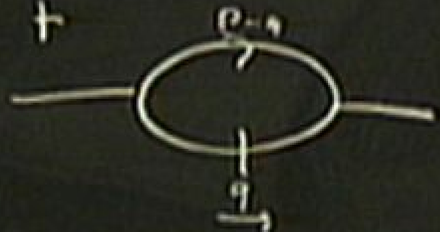
$$= \left[\int \frac{d^D q}{(2\pi)^D} \frac{-i}{q^2 (q+p)^2}\right]_{p^2} \Rightarrow C'''(p^2 - i\epsilon)^{\frac{D-4}{2}}.$$

$$p^2 = -p^{0\,2} + \vec{p}^{\,2}.$$

$$p = \begin{pmatrix} 0 \\ 0 \\ p_0 \end{pmatrix}$$



+



$$= \int \frac{d^D q}{(2\pi)^D} \pi\delta(q^2) \left[\frac{1}{(q+p)^2 - i\epsilon(q^0+p^0)} + \frac{1}{(p-q)^2 - i\epsilon(p^0-q^0)} \right]$$

$$= C' \, (p^0)^{D-4} \left[1 + e^{-i\pi(D-4)}\right] \cdot \; e^{-i\pi\frac{D-4}{2}} \, 2\cos\frac{\pi(D-4)}{2}.$$

$$= C'' \, (p^{0\,2})^{\frac{D-4}{2}} e^{-i\pi\frac{D-4}{2}}.$$

$$= (-p^2 - i\epsilon)^{\frac{D-4}{2}} \Rightarrow (p^2 - i\epsilon p^0)^{\frac{D-4}{2}}, \quad p^0 > 0.$$

$(p^2)^{\frac{D-4}{2}}$, p spacelike.

$$\mathrm{Im} \Rightarrow \int \pi\delta(q^2)\,\pi\delta\left((q+p)^2\right)\,\mathrm{sgn}(q^0) \longrightarrow 0.$$

$$\begin{pmatrix} q^0 \\ \vec{q} \end{pmatrix}, \begin{pmatrix} -q^0 \\ \vec{q} \end{pmatrix}$$

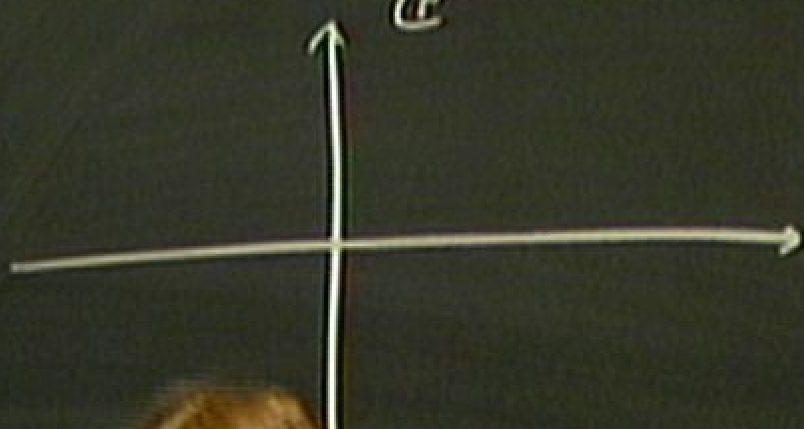
$$\left(p^2 - i\varepsilon p^0\right)^{\frac{D-4}{2}} \Leftrightarrow \left(p^2 - i\varepsilon\right)^{\frac{D-4}{2}}$$

$$\mathrm{Im} \Rightarrow \int \pi \delta(q^2)\, \pi \delta((q+p)^2)\, \mathrm{sgn}(q^0) \longrightarrow 0.$$

$$\begin{pmatrix} q^0 \\ \vec{q} \end{pmatrix}, \begin{pmatrix} -q^0 \\ \vec{q} \end{pmatrix}$$

$$(p^2 - i\epsilon p^0)^{\frac{D-4}{2}} \Longleftrightarrow (p^2 - i\epsilon)^{\frac{D-4}{2}}$$

$$(p^2)^{\frac{D-4}{2}}$$



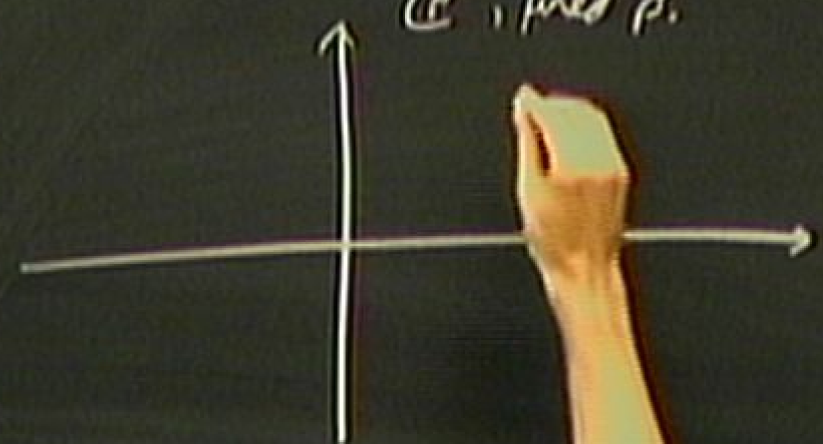
$$\mathrm{Im} \Rightarrow \int \pi\, d(q^2)\, \pi\, \delta\left((q+p)^2 \right) \mathrm{sgn}(q^0) \longrightarrow 0.$$

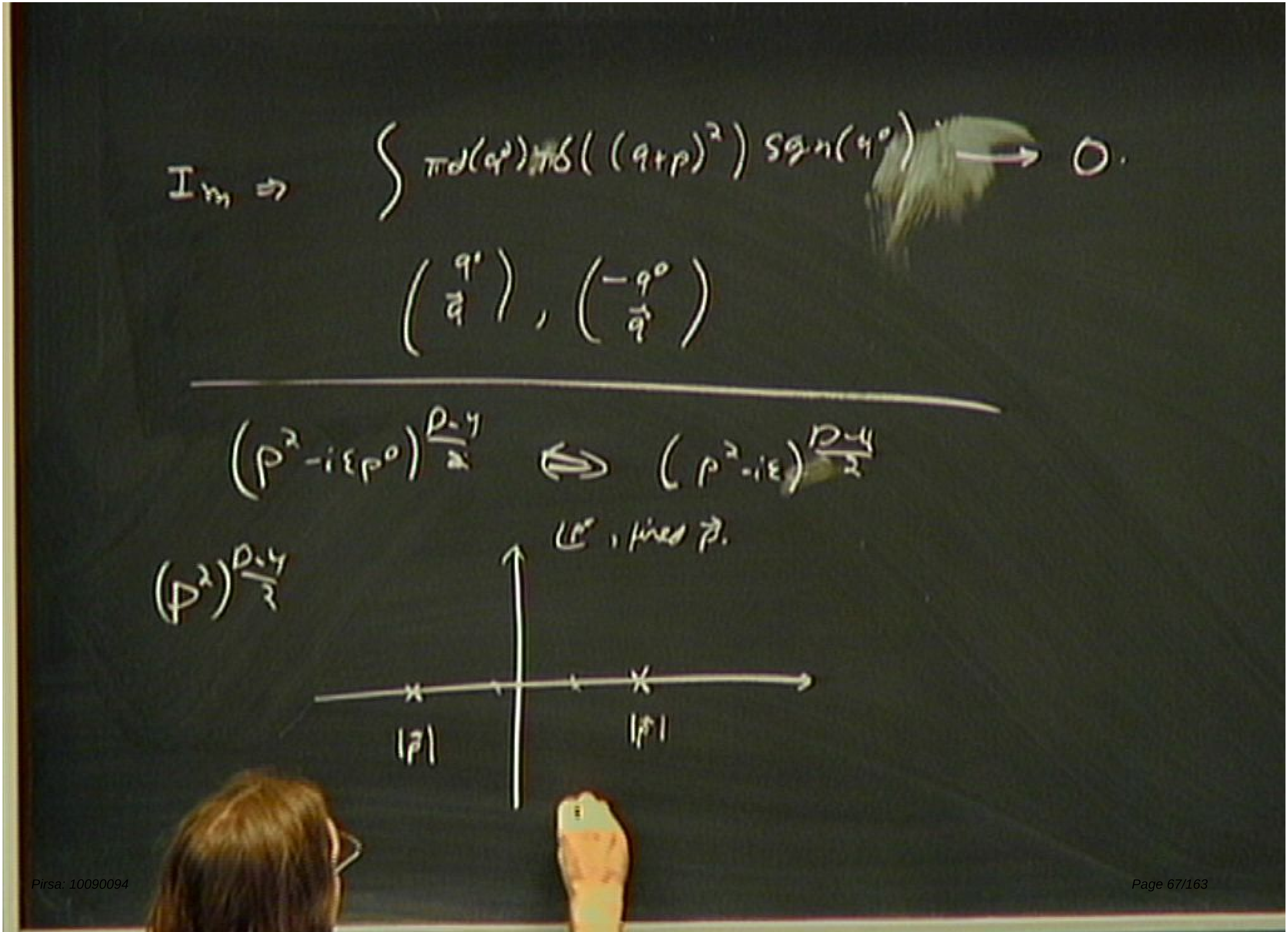
$$\begin{pmatrix} q^0 \\ \vec{q} \end{pmatrix}, \begin{pmatrix} -q^0 \\ \vec{q} \end{pmatrix}$$

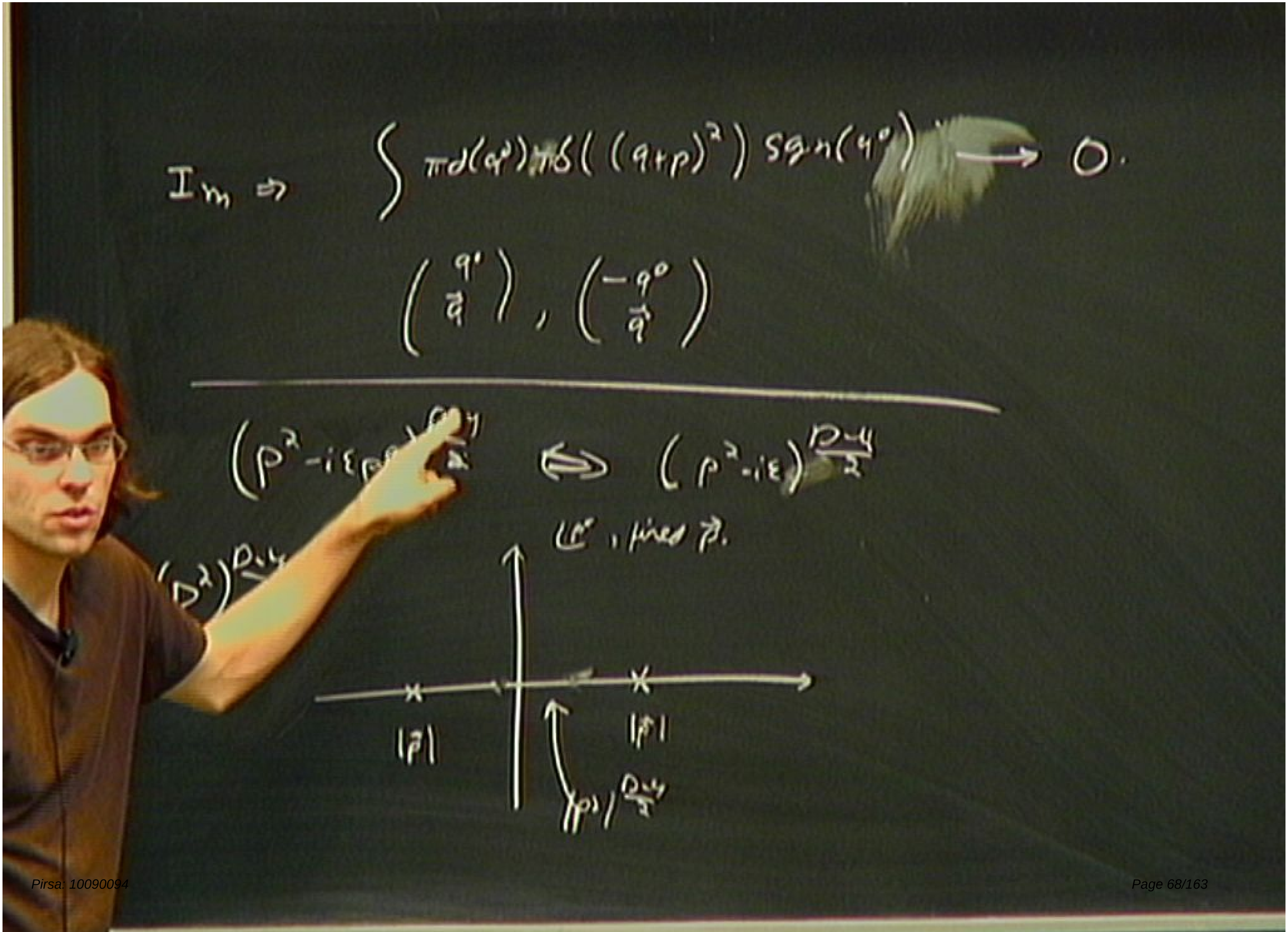
$$\left(p^2 - i\varepsilon p^0\right)^{\frac{D-4}{2}} \Longleftrightarrow \left(p^2 + i\varepsilon\right)^{\frac{D-4}{2}}$$

p^0, fixed $\vec{p}$.

$$\left(p^2\right)^{\frac{D-4}{2}}$$




Im ⇒ ∫ πδ(q²) πδ((q+p)²) sgn(q⁰) → 0.
(q⁰, q⃗), (−q⁰, q⃗)
(p² − iεp⁰)^((D−4)/2) ⇔ (p² − iε)^((D−4)/2)
(p²)^((D−4)/2)
p⁰, fixed p⃗.
|p⃗|
|p⃗|


Im ⇒ ∫ π d(q²) π δ((q+p)²) sgn(q⁰) → 0.
(q⁰, q⃗) , (−q⁰, q⃗)
⇔ (p² − iε)^((D−4)/2)
fixed p⃗.
|p⃗|
|p⃗|

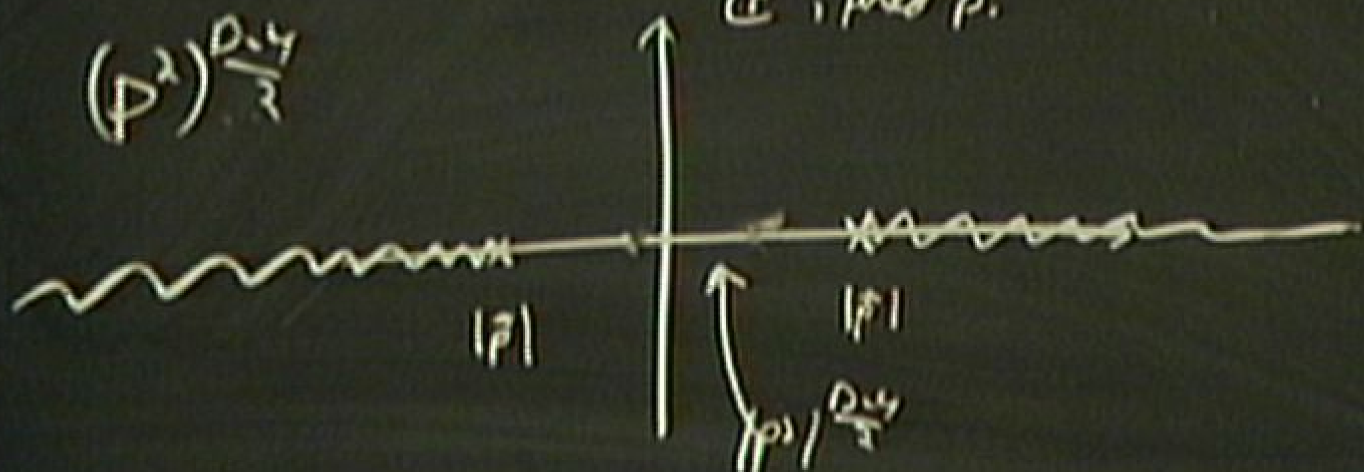
$$\mathrm{Im} \Rightarrow \int \pi\, \delta(q^2)\, \pi\, \delta\left((q+p)^2\right) \mathrm{sgn}(q^0) \longrightarrow 0.$$

$$\begin{pmatrix} q^0 \\ \vec{q} \end{pmatrix}, \begin{pmatrix} -q^0 \\ \vec{q} \end{pmatrix}$$

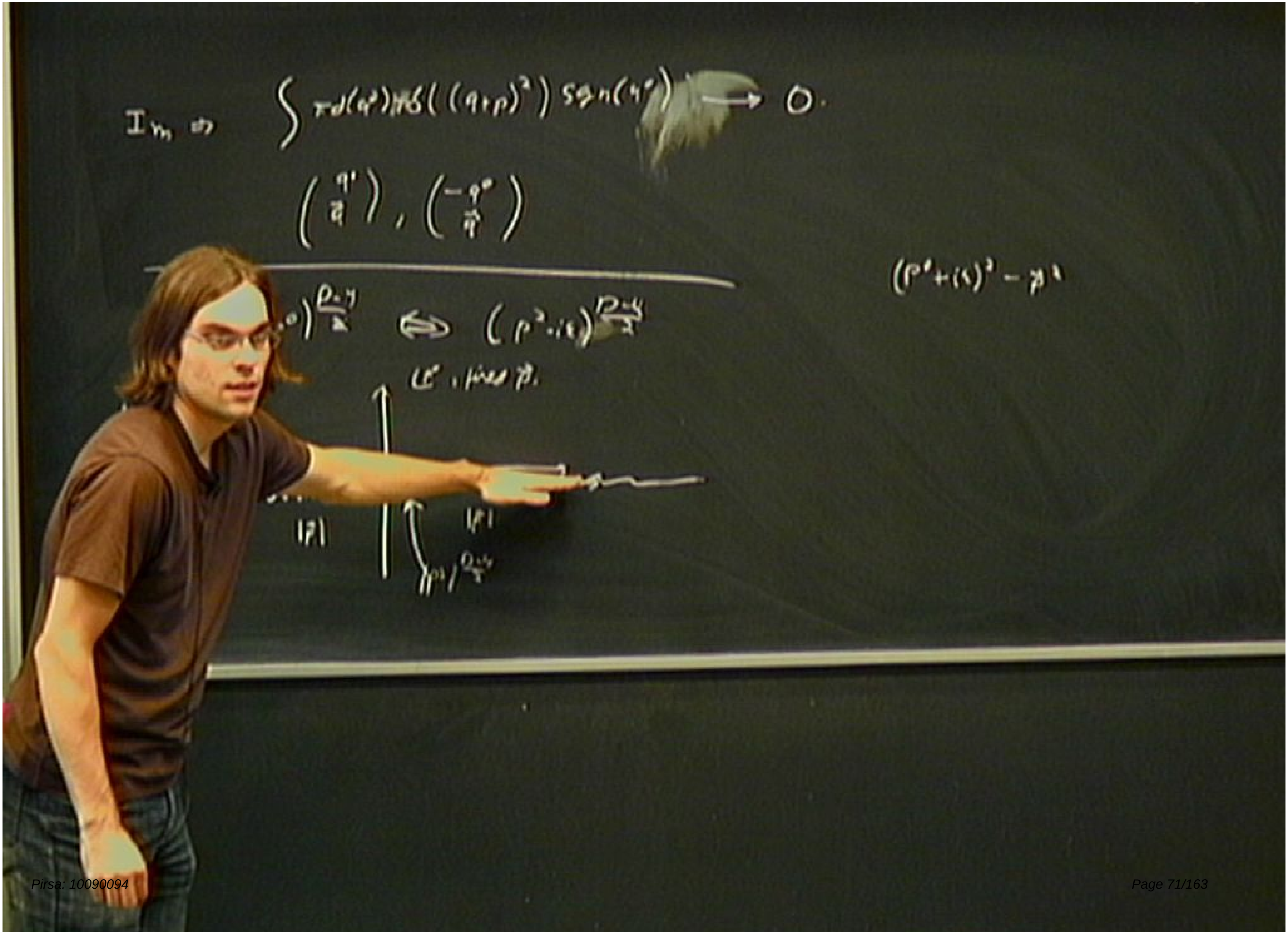
$$\left(p^2 - i\epsilon p^0\right)^{\frac{D-4}{2}} \Leftrightarrow \left(p^2 - i\epsilon\right)^{\frac{D-4}{2}}$$

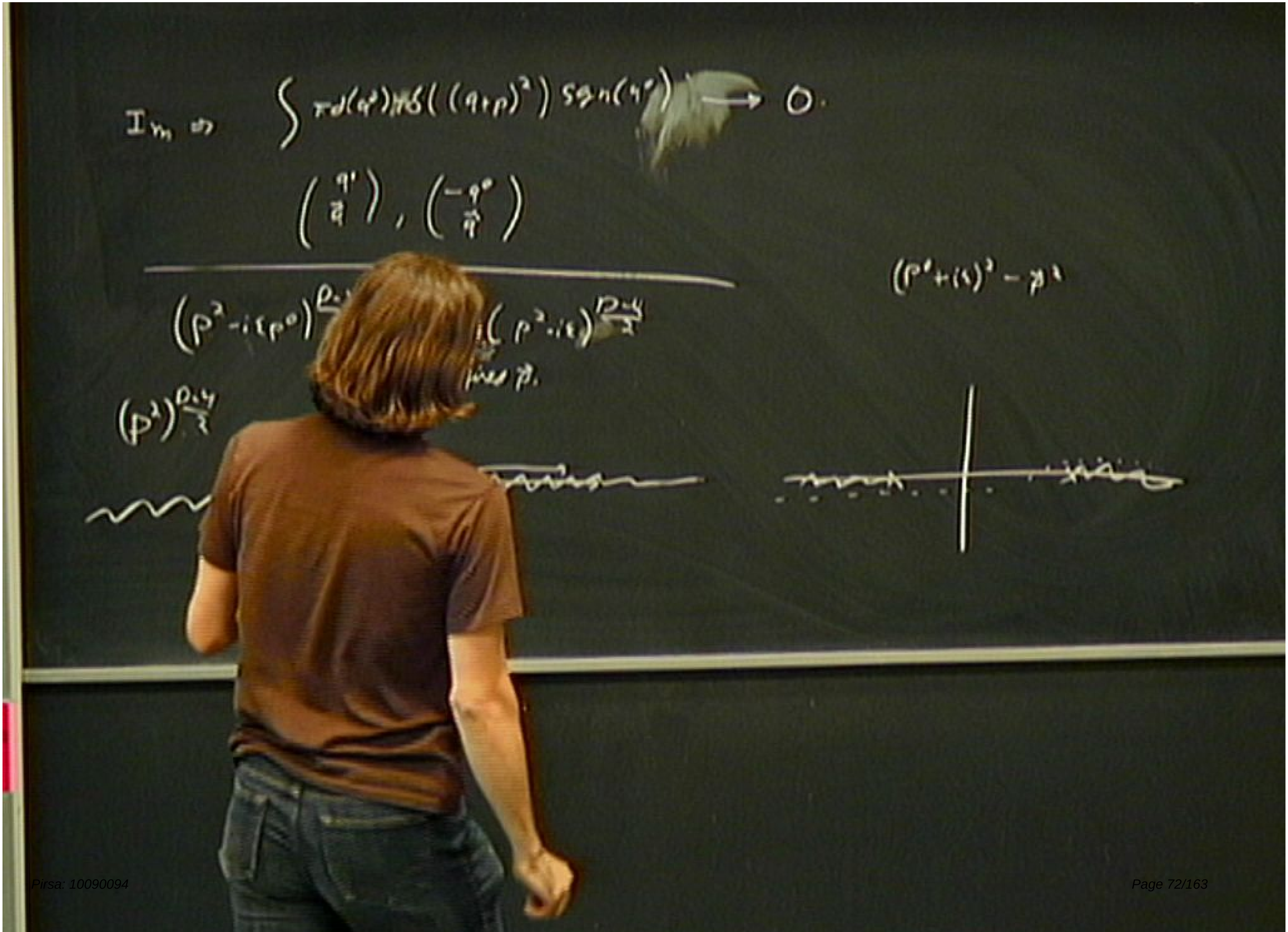
p^0, fixed $\vec{p}$.

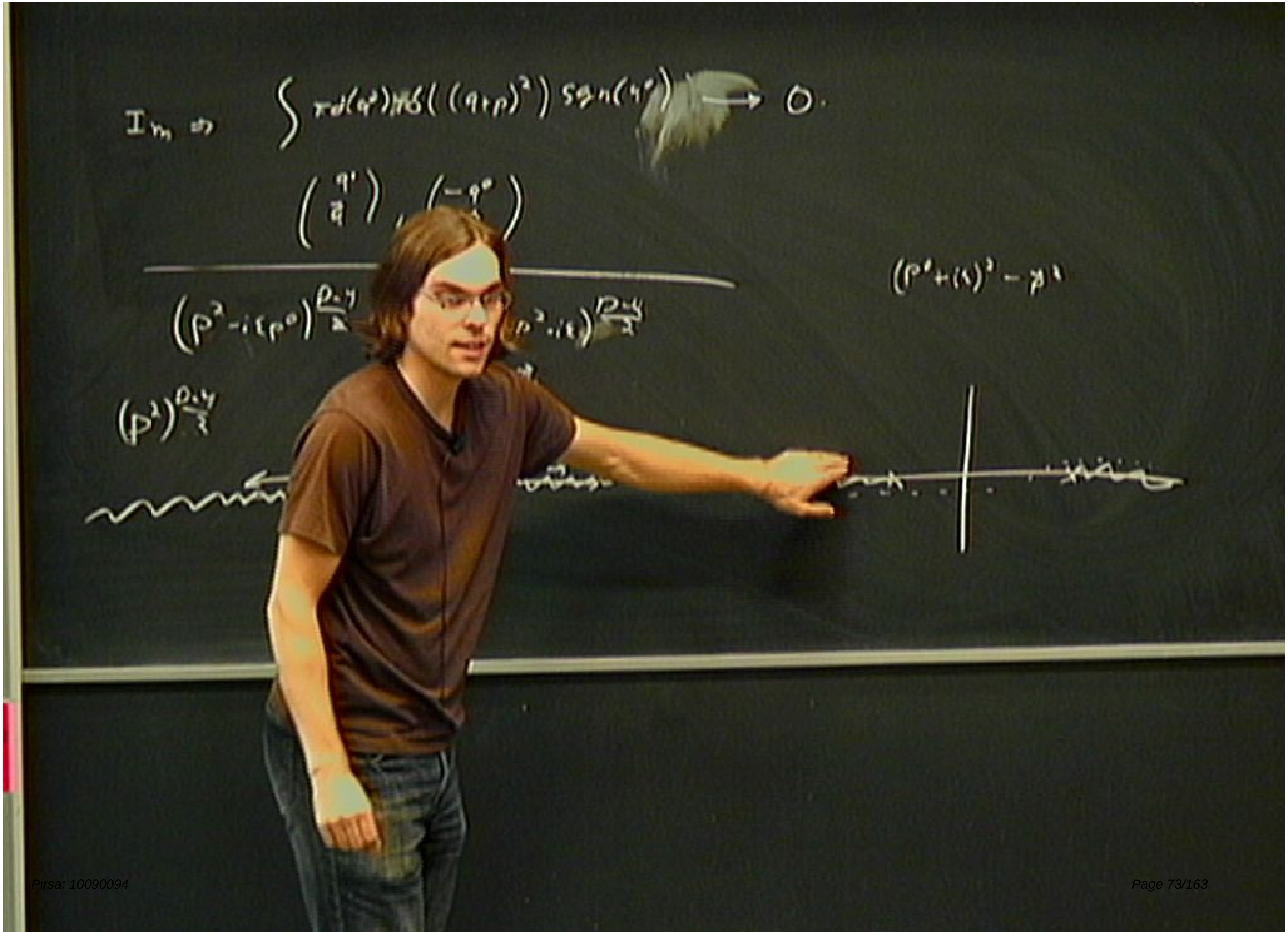
$$(p^2)^{\frac{D-4}{2}}$$



$$(p^0 + i\varepsilon)^2 - \vec{p}^{\,2}$$







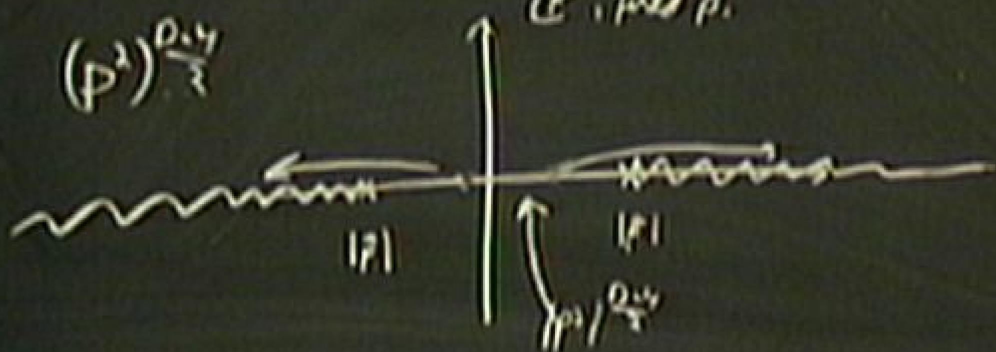
$$\mathrm{Im} \Rightarrow \int \pi\, d(q^2)\, \delta\left(((q+p)^2)\right) \mathrm{sgn}(q^0) \longrightarrow 0.$$

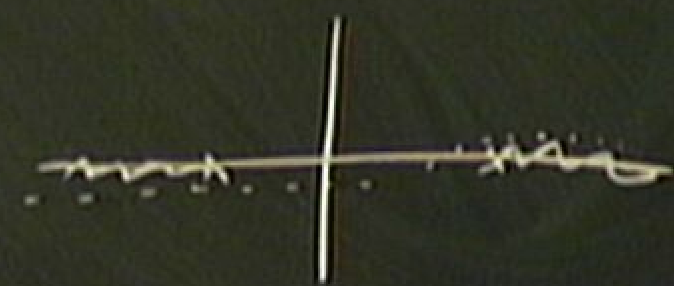
$$\begin{pmatrix} q^0 \\ \vec{q} \end{pmatrix}, \begin{pmatrix} -q^0 \\ \vec{q} \end{pmatrix}$$

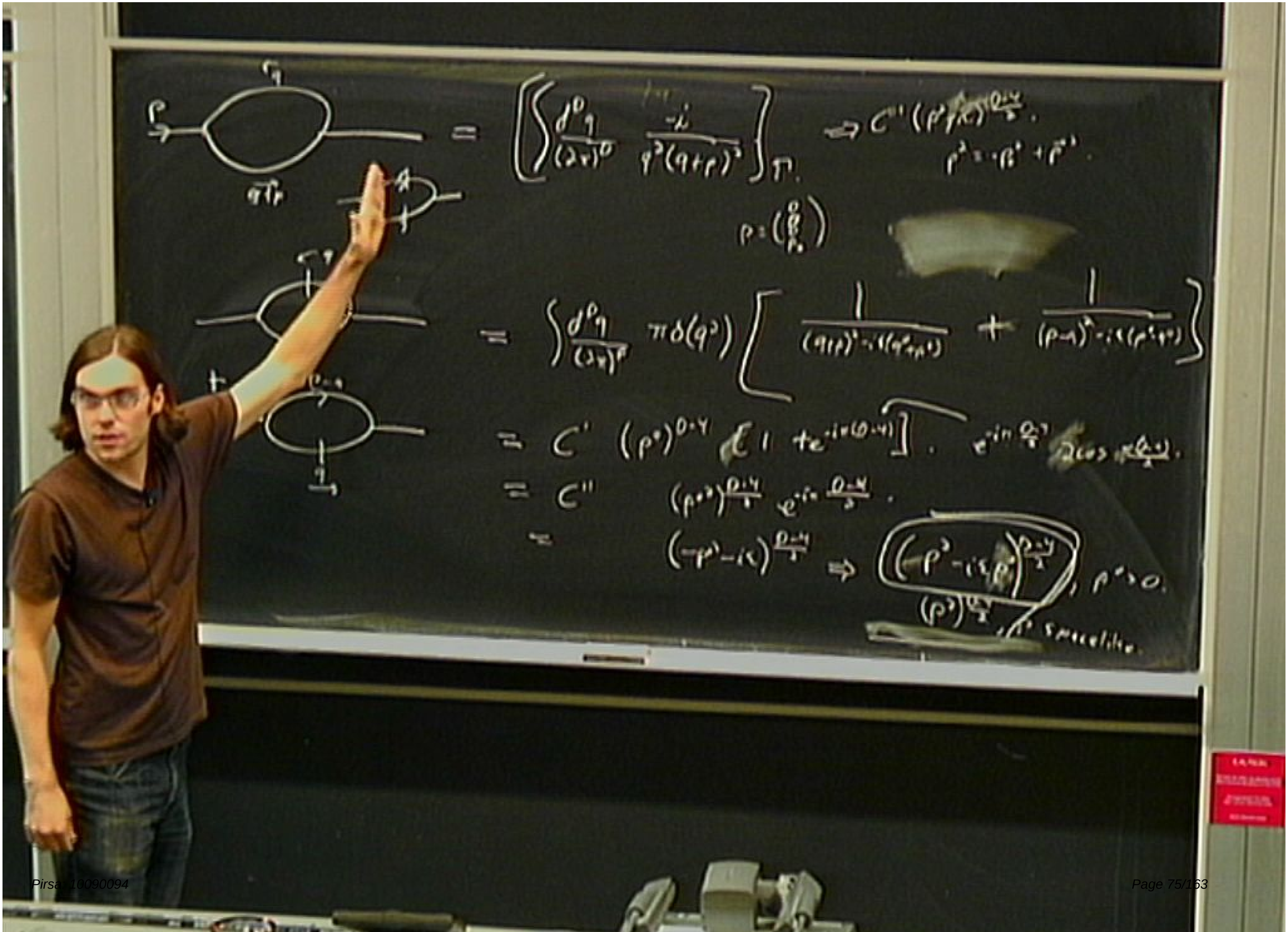
$$(p^2 - i\epsilon p^0)^{\frac{D-4}{2}} \Leftrightarrow (p^2 + i\epsilon)^{\frac{D-4}{2}}$$

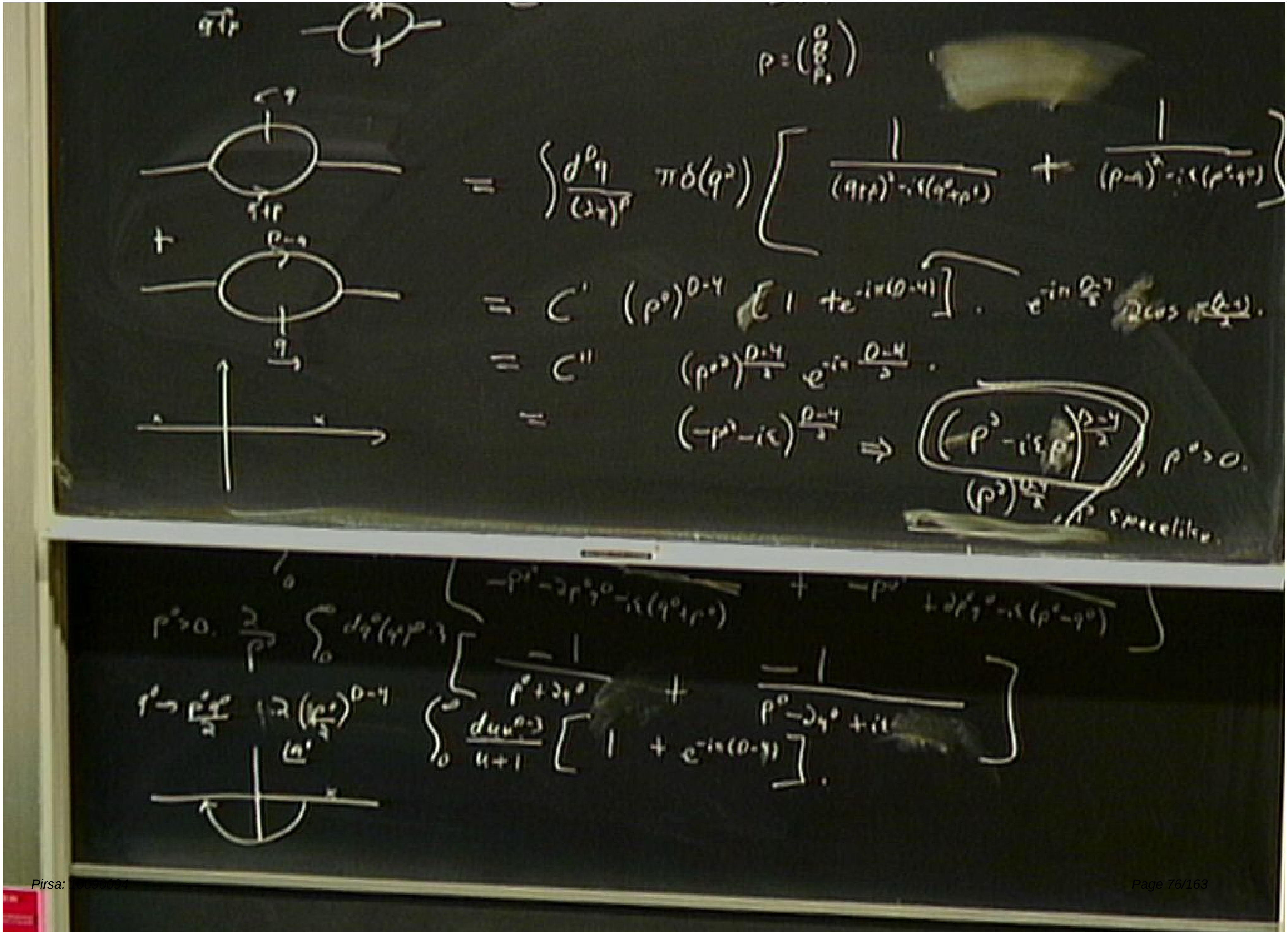
$$(p^0 + i\epsilon)^2 - \vec{p}^2$$

$$(p^2)^{\frac{D-4}{2}}$$

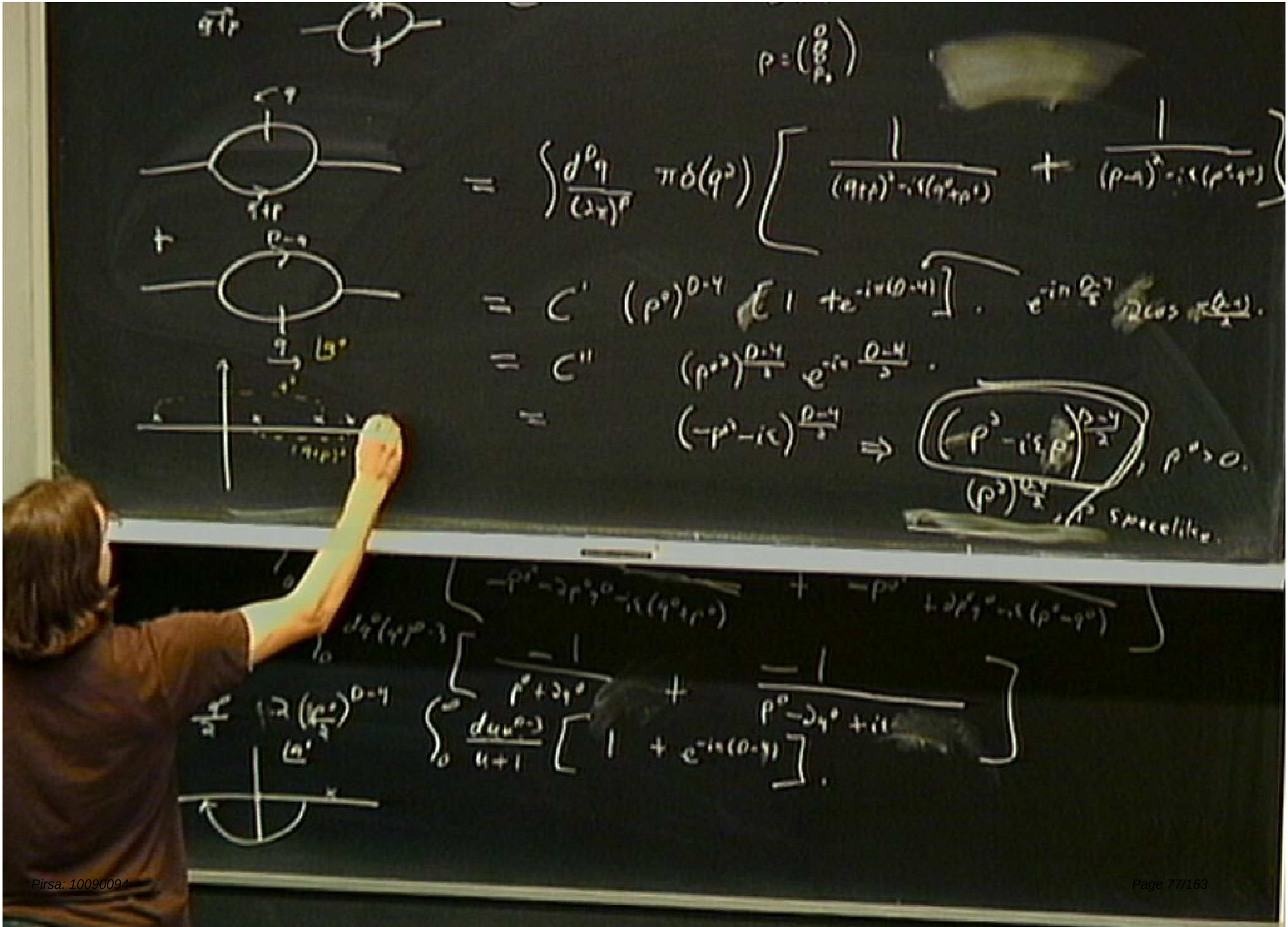


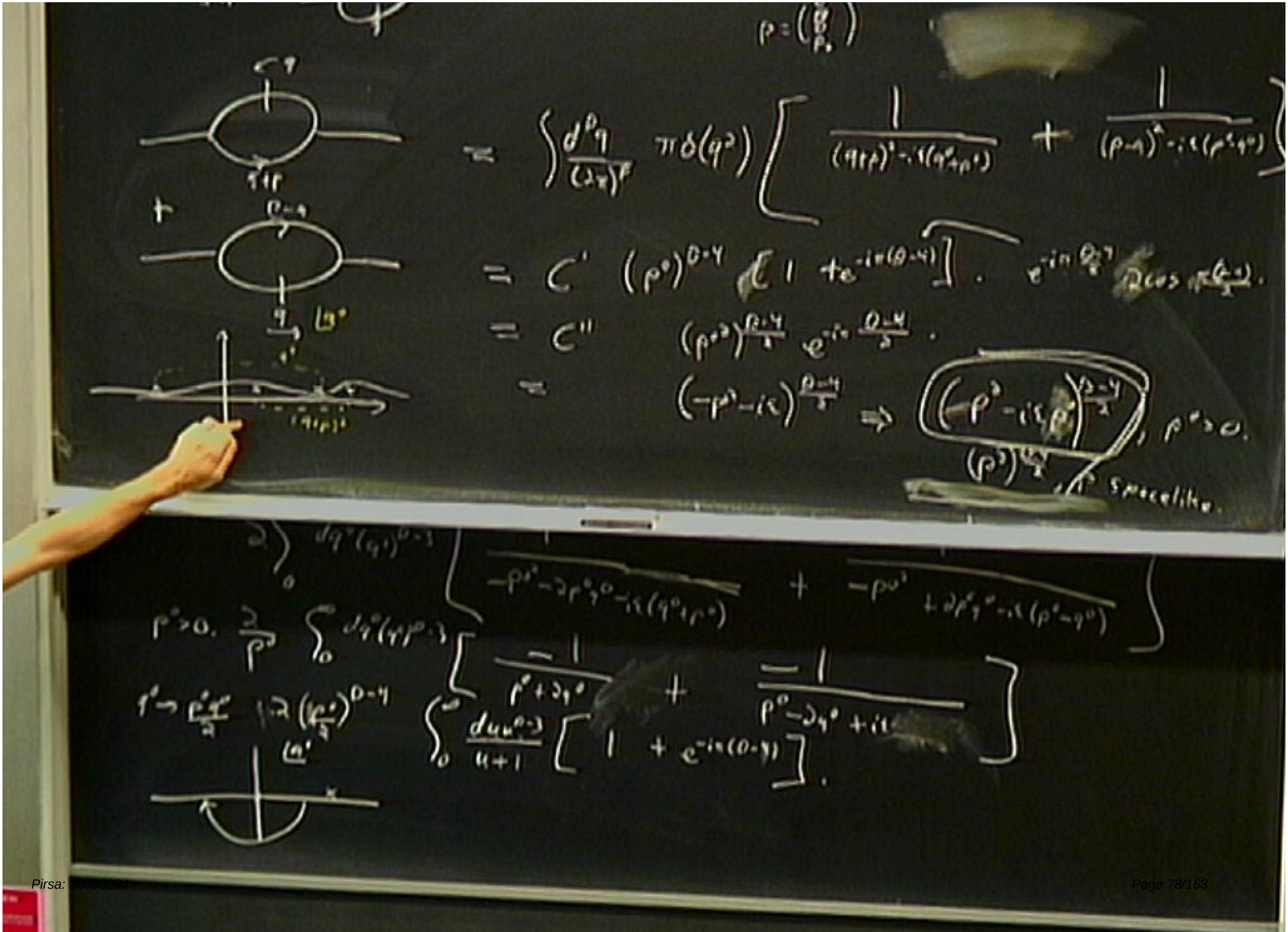


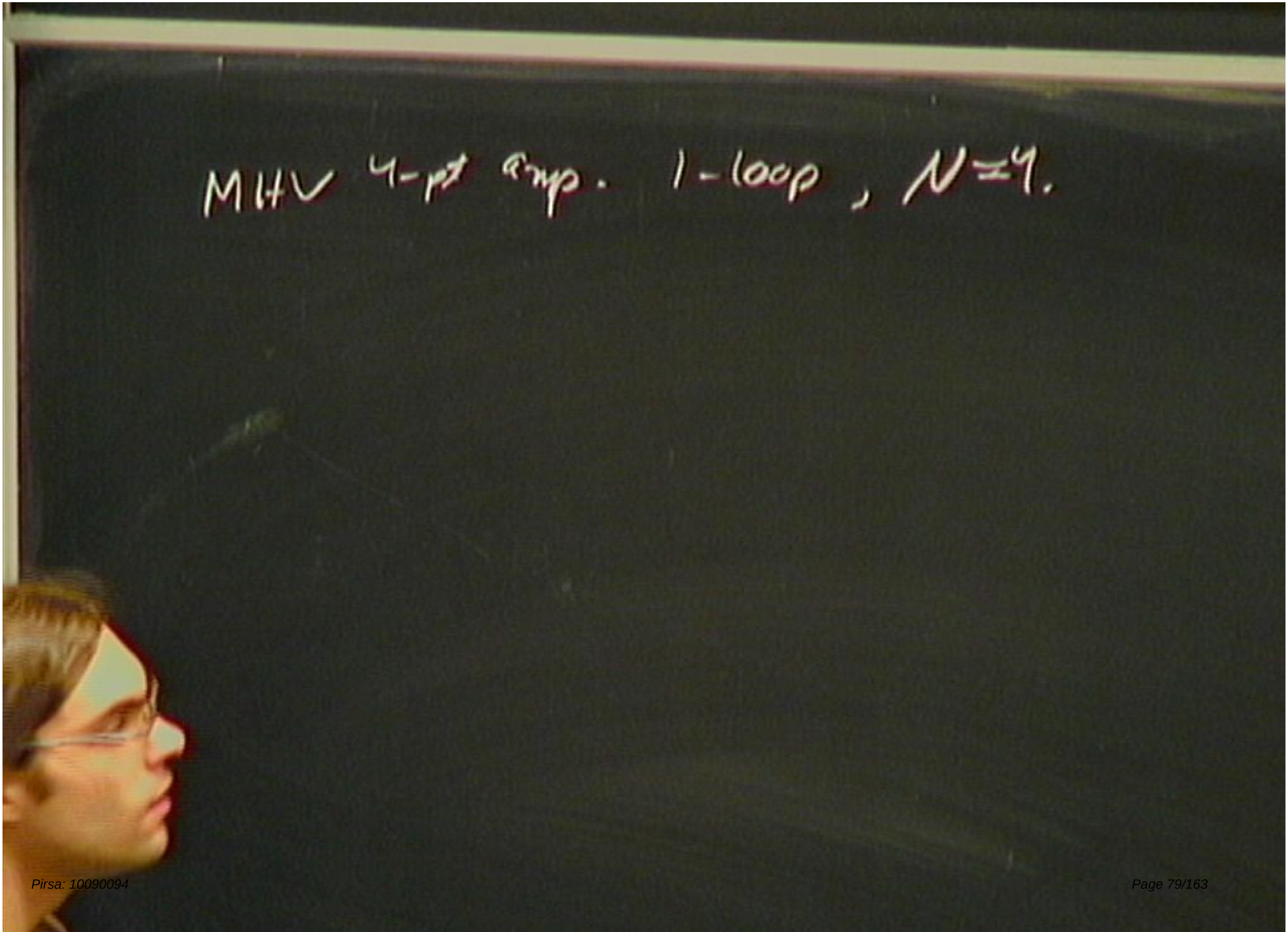

p^2 > 0
p^2 spacelike



p = (0 0 0 p_0)
= ∫ d^D q/(2π)^D πδ(q^2) [1/((q+p)^2 − iε(q^0+p^0)) + 1/((p−q)^2 − iε(p^0−q^0))]
= C' (p^0)^{D−4} [1 + e^{−iπ(D−4)}] · e^{iπ(D−4)/2} 2cos π(D−4)/2
= C'' (p^{02})^{(D−4)/2} e^{−iπ(D−4)/2}
= (−p^2 − iε)^{(D−4)/2} ⇒ (−p^2 − iεp^0)^{(D−4)/2}, p^0 > 0
(p^2)^{(D−4)/2} p spacelike
p^0 > 0




spacelike

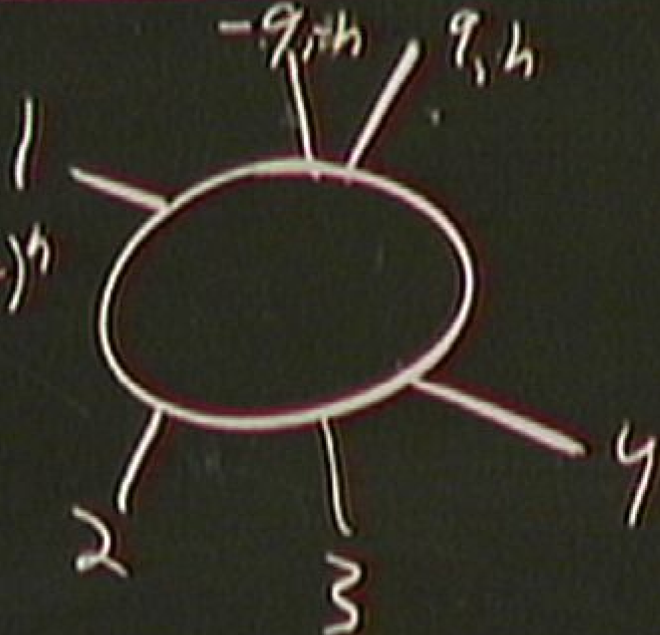

MHV 4-pt amp. 1-loop, N=4.

MHV 4-pt amp. 1-loop, $\mathcal{N}=4$.

Plan.

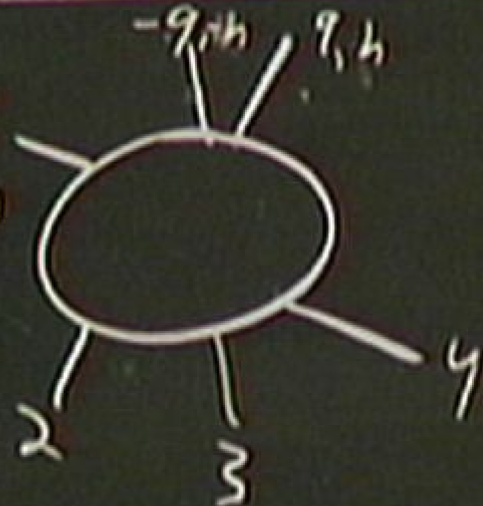
MHV 4-pt amp. 1-loop, $\mathcal{N}=4$.

Plan. $\sum_h (-)^h$



MHV 4-pt amp. 1-loop, $\mathcal{N}=4$.

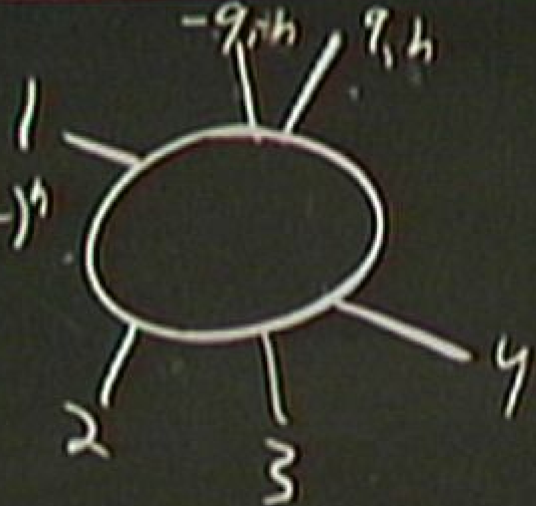
Plan. $\sum_h (-)^h$



NMHV 6-pt amplitude

MHV 4-pt amp. 1-loop, $\mathcal{N}=4$.

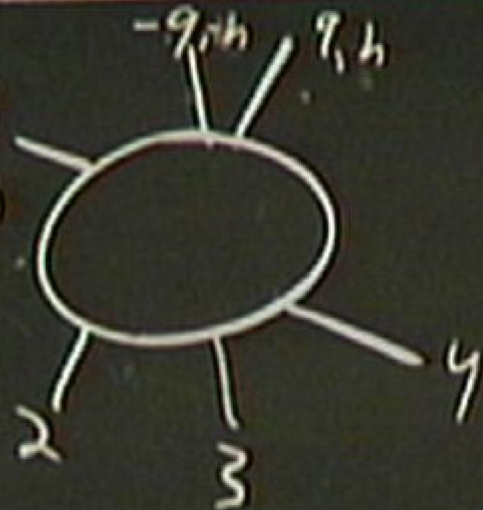
Plan. $\sum_h (-)^h$



NMHV 6-pt amplitude

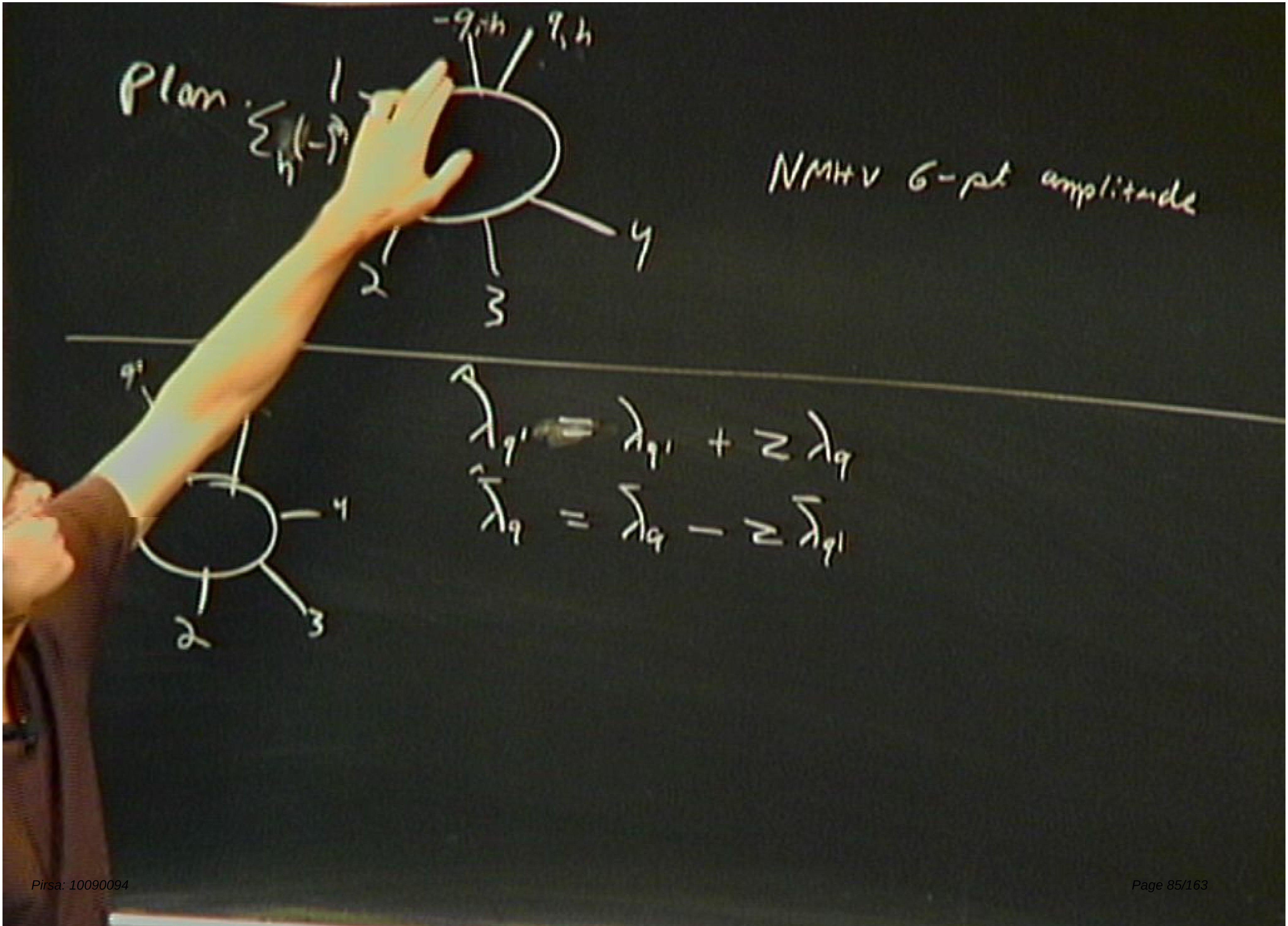
MHV 4-pt amp. 1-loop, $\mathcal{N}=4$.

Plan. $\sum_h (-)^h$

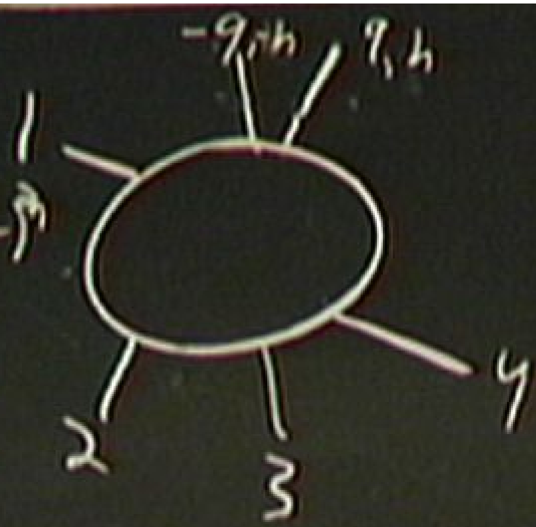


NMHV 6-pt amplitude

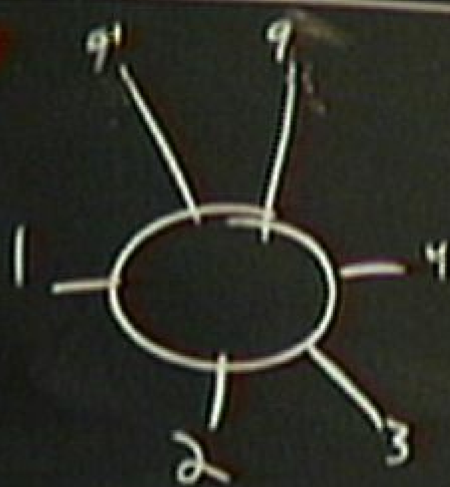
q' q


Plan
$-q, -h$
q, h
1
2
3
4
NMHV 6-pt amplitude
$\hat{\lambda}_{q'} = \lambda_{q'} + z\lambda_q$
$\hat{\tilde{\lambda}}_q = \tilde{\lambda}_q - z\tilde{\lambda}_{q'}$

Plan: $\sum_h (-1)^h$



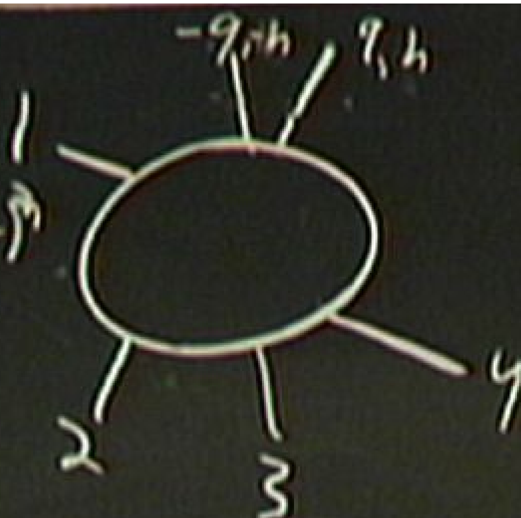
NMHV 6-pt amplitude



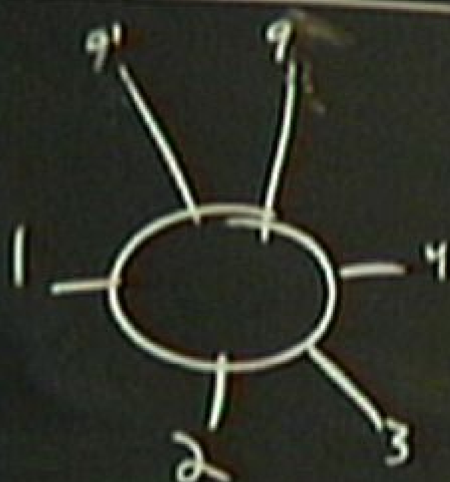
$$\hat{\lambda}_{q'} = \lambda_{q'} + z\lambda_q$$

$$\hat{\tilde{\lambda}}_q = \tilde{\lambda}_q - z\tilde{\lambda}_{q'}$$

Plan: $\sum_h (-1)^h$



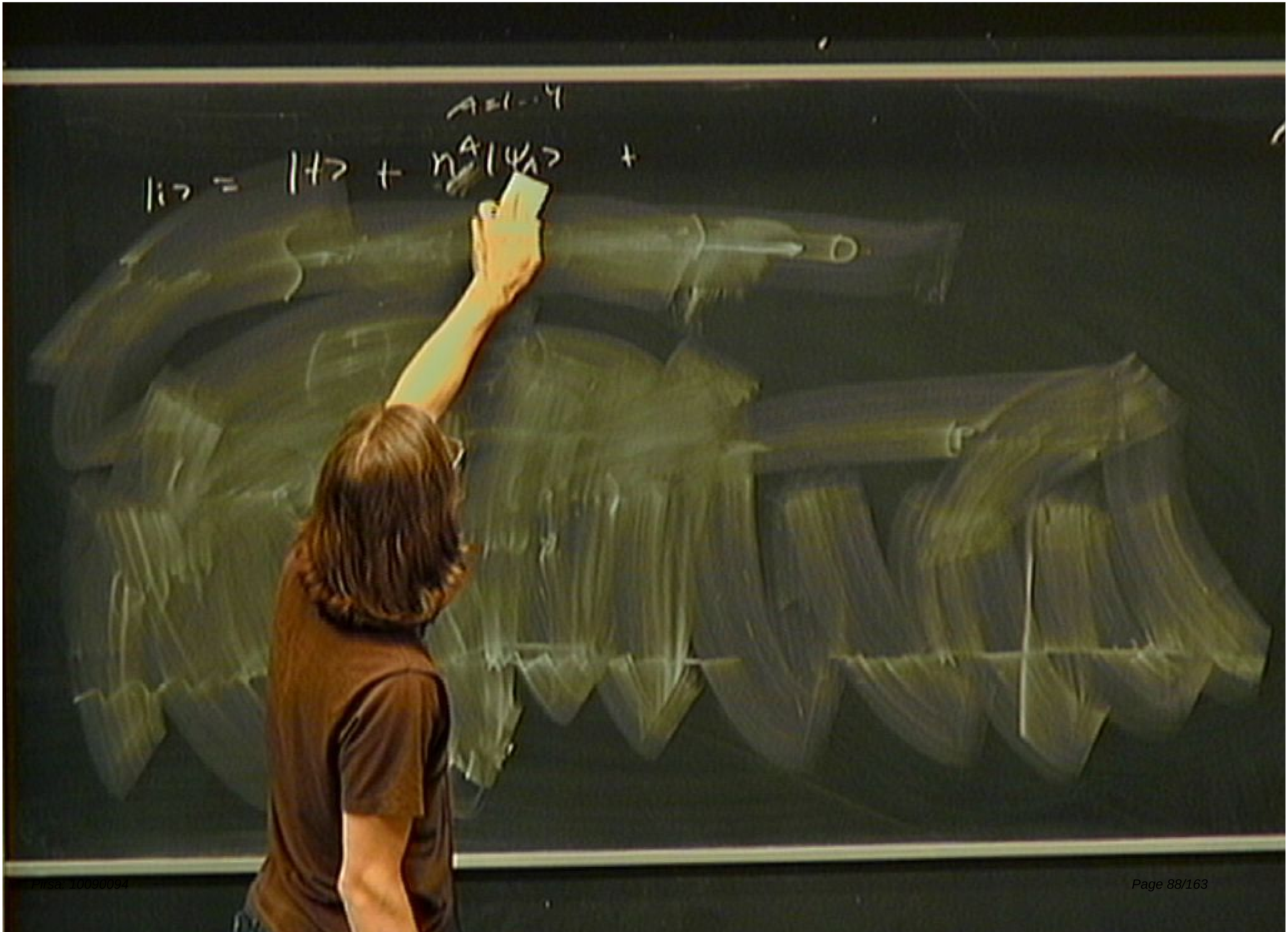
NMHV 6-pt amplitude

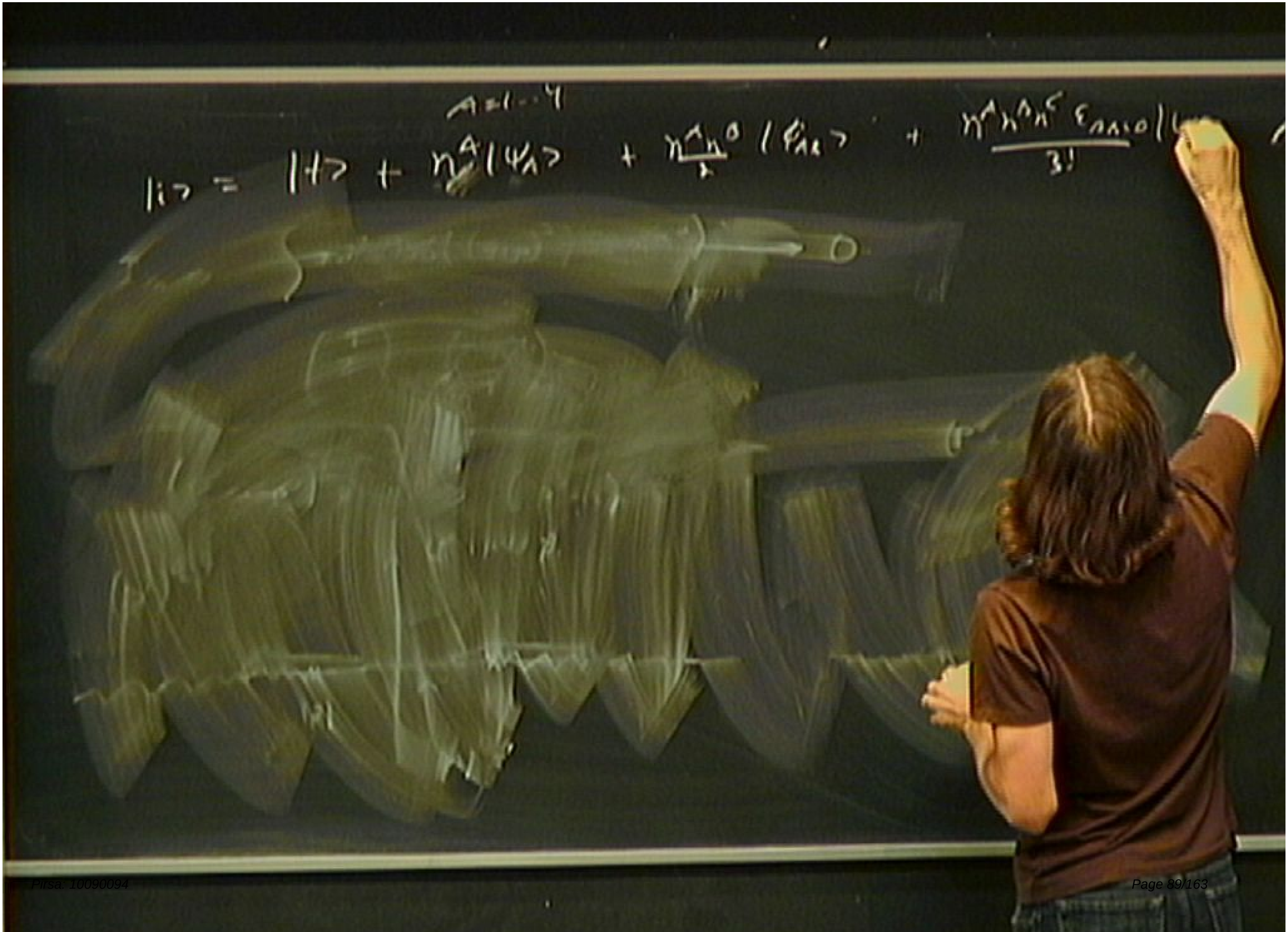


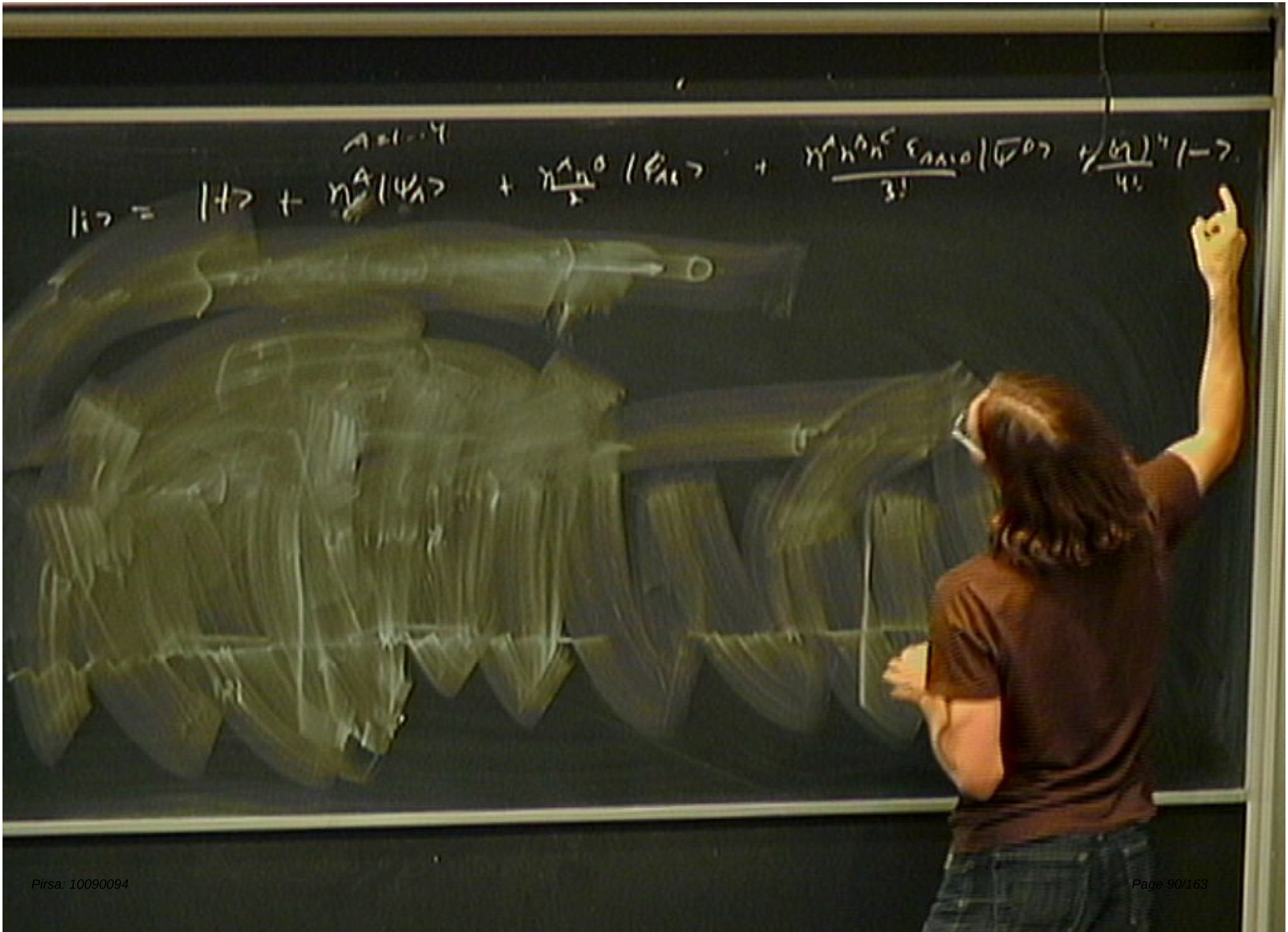
$$\hat{\lambda}_{q'} = \lambda_{q'} + z\lambda_q$$

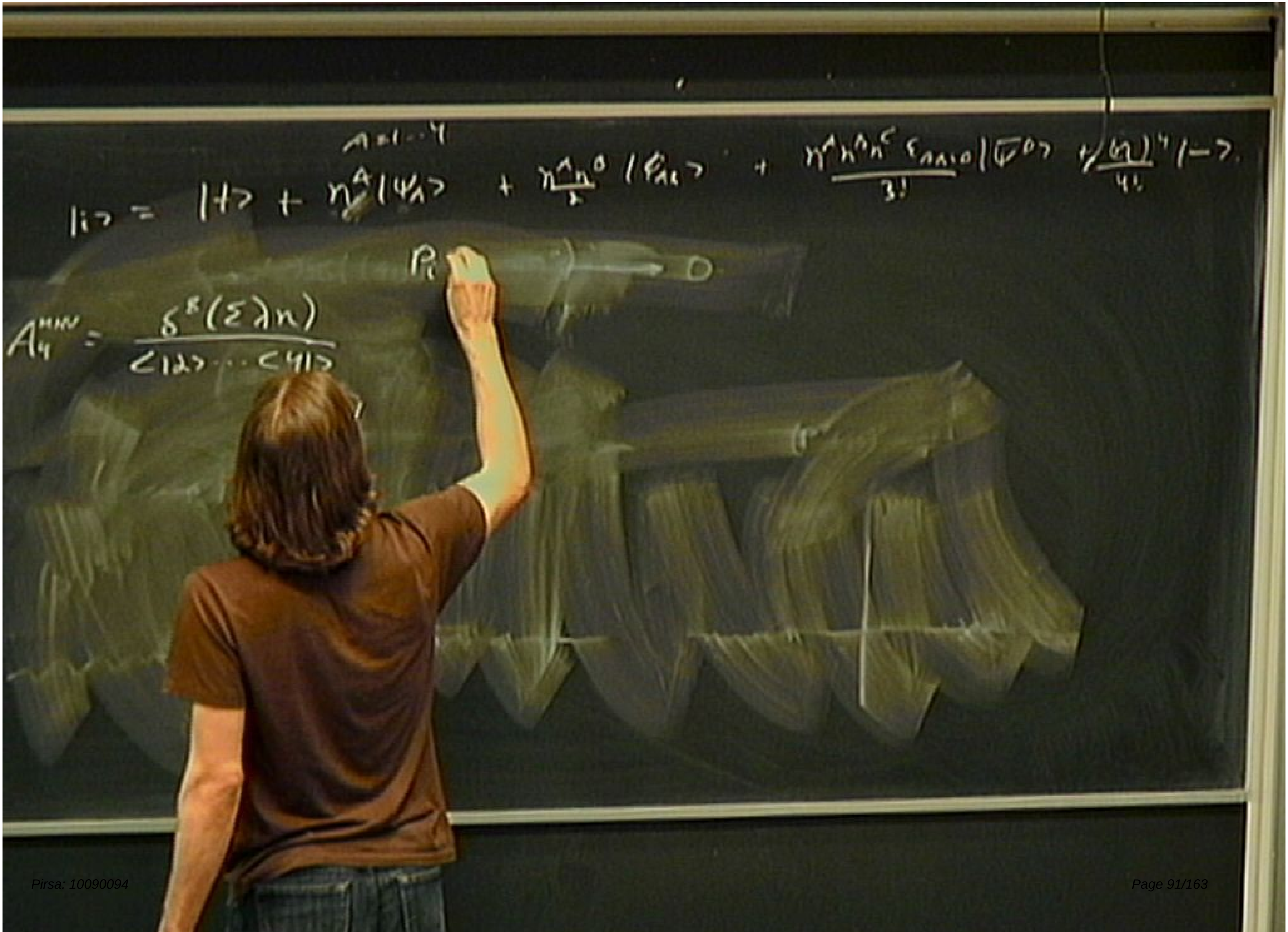
$$\hat{\tilde{\lambda}}_q = \tilde{\lambda}_q - z\tilde{\lambda}_{q'}$$

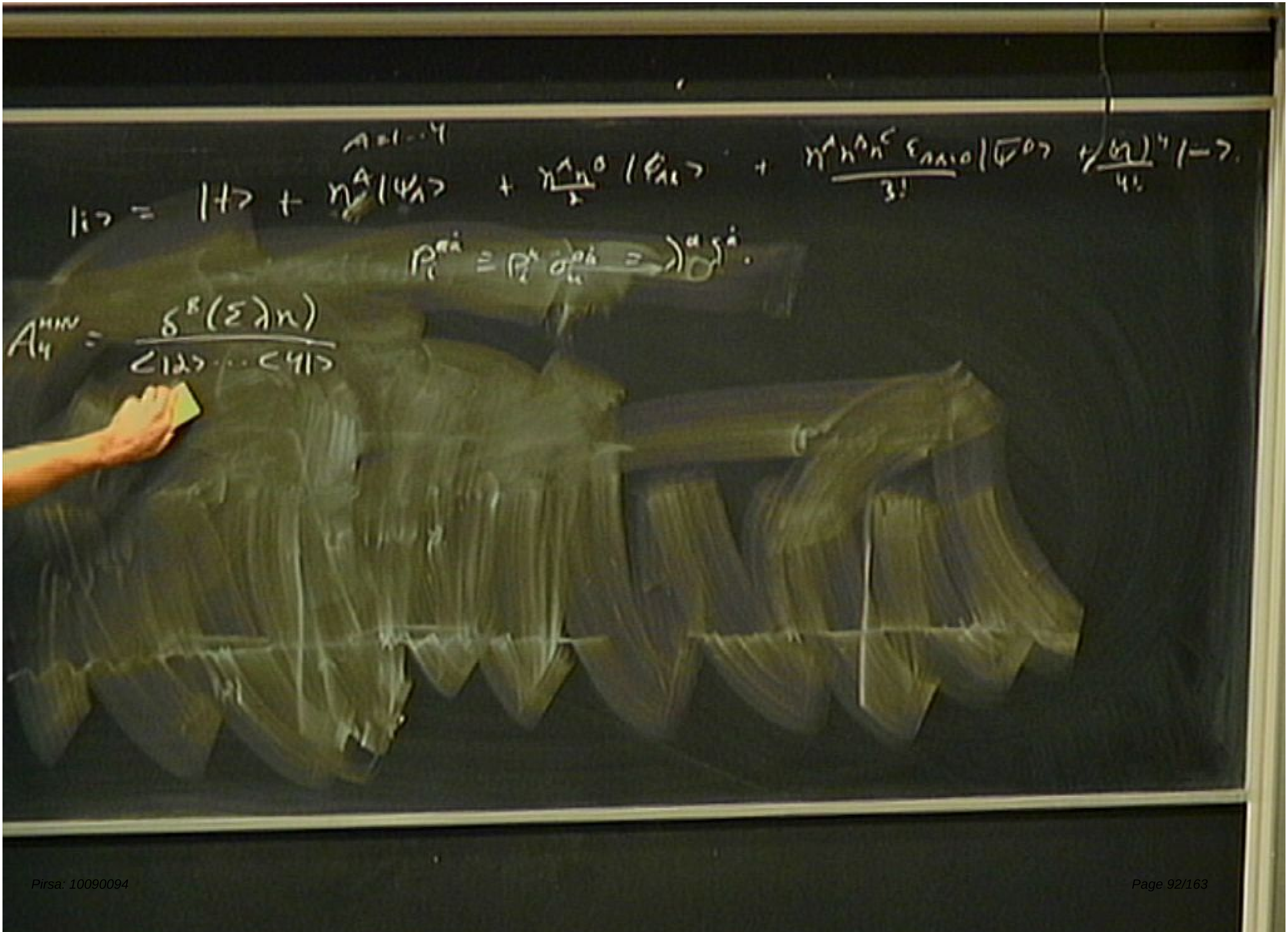
$$\hat{\eta}^A_q = \eta^A_q - z\eta_{q'}$$


A=1..4
|i> = |H> + η^A|ψ_A> +

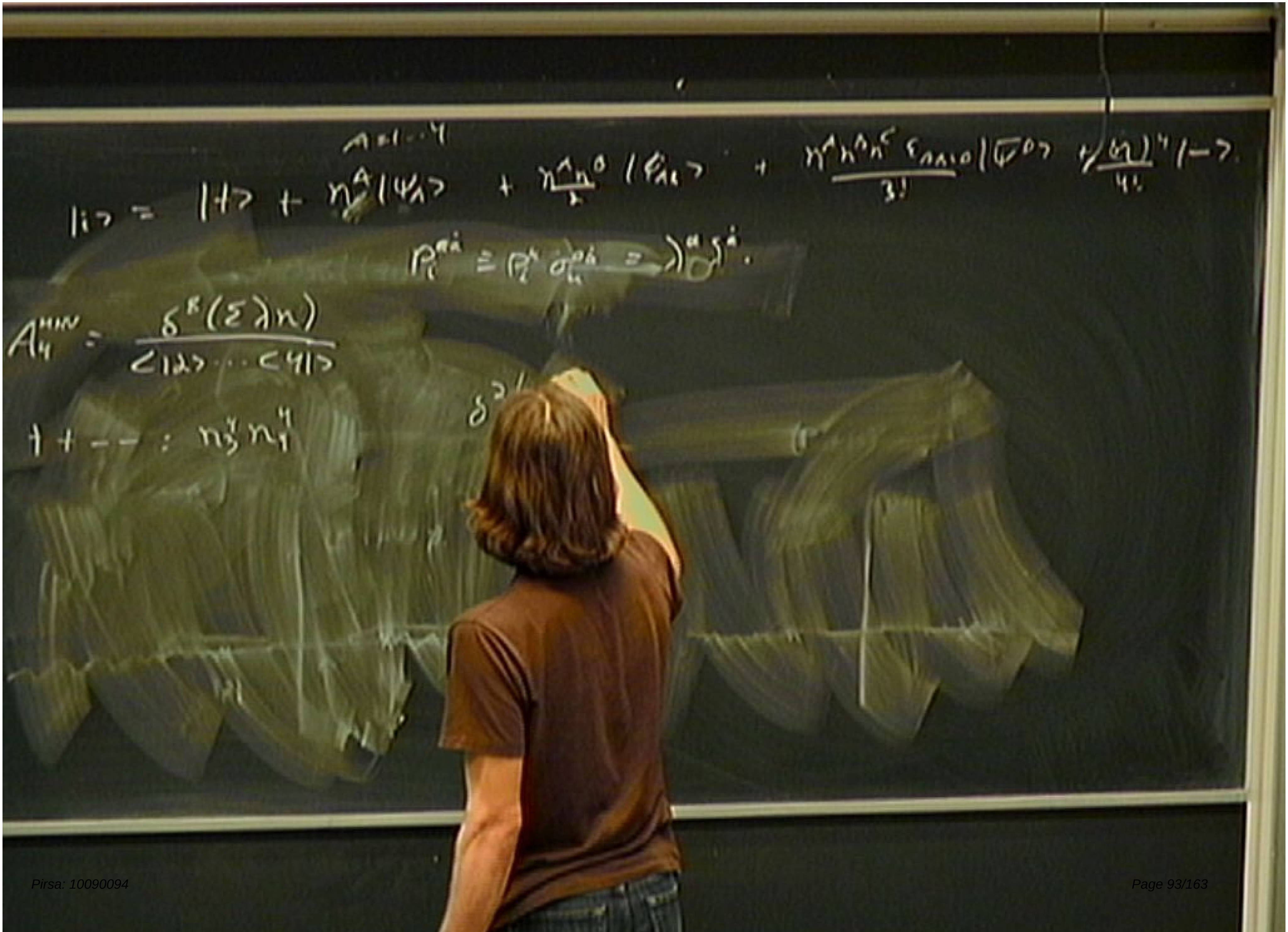


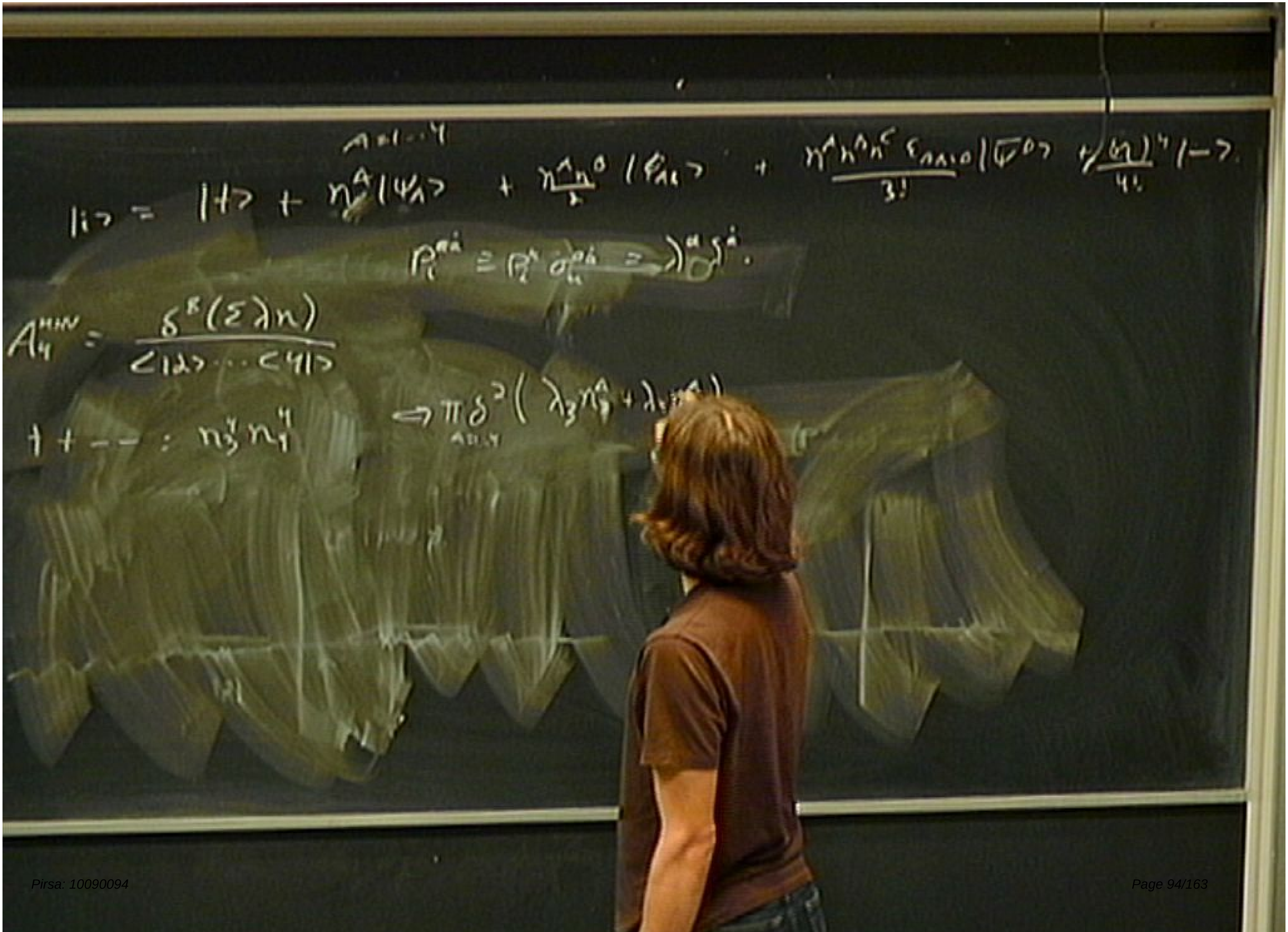


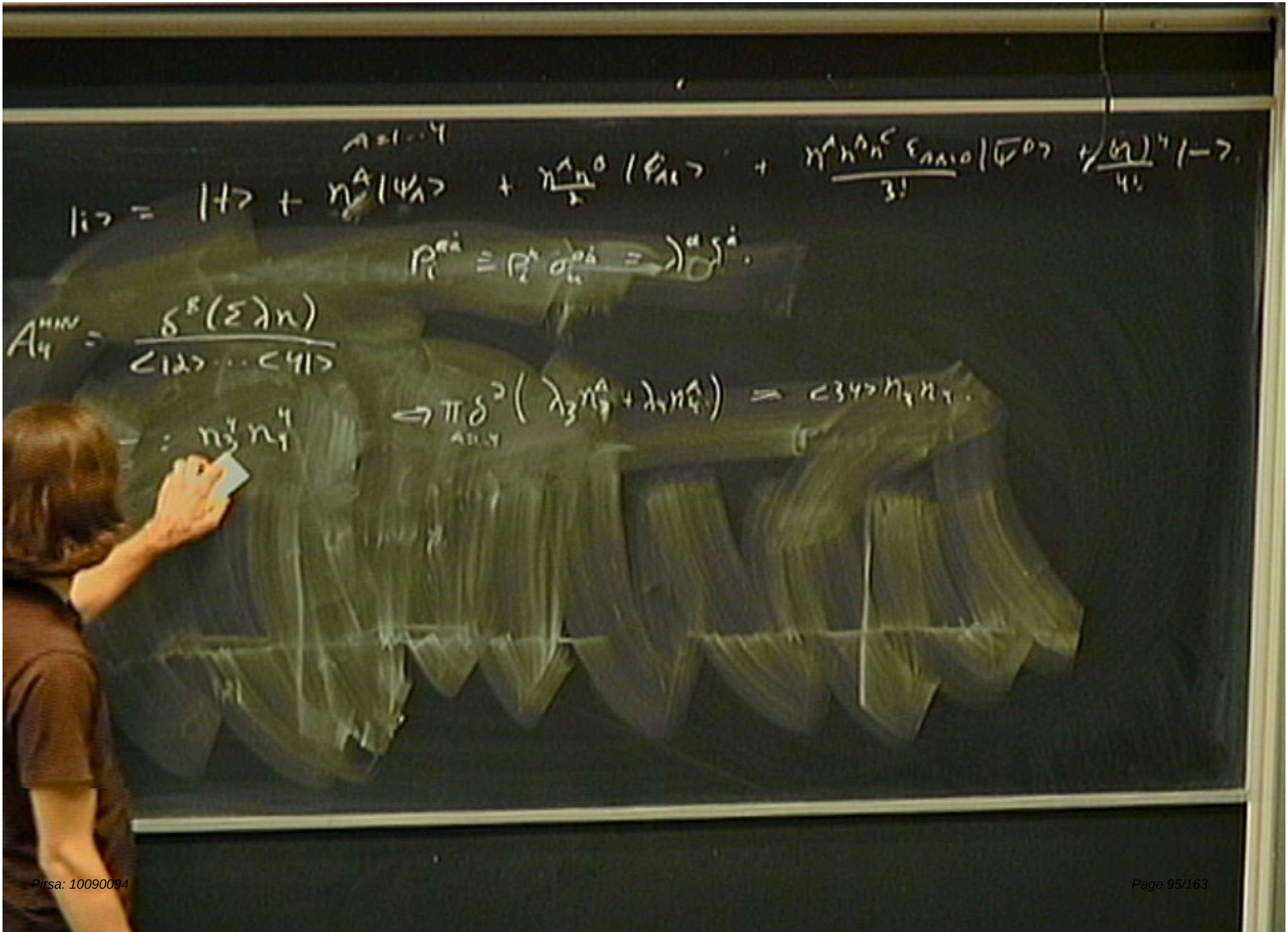

A = 1 ... 4
|i> = |+> + η^A |ψ_A> + η^A η^B / 2 |φ_AB> + η^A η^B η^C ε_ABCD / 3! |ψ̄^D> + (η)^4 / 4! |−>
P_i
A_4^MHV = δ^8(Σλη) / <12> ... <41>

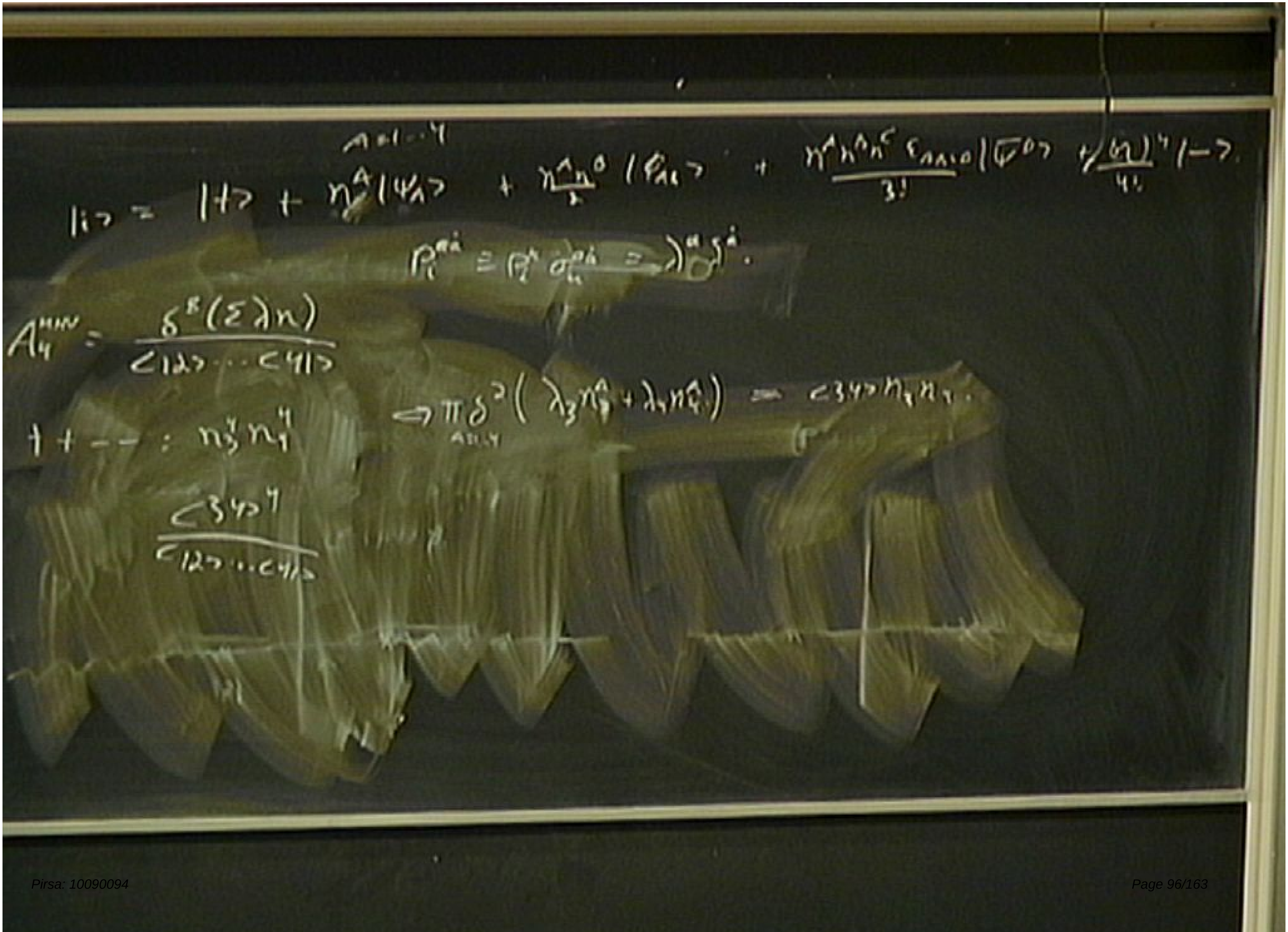


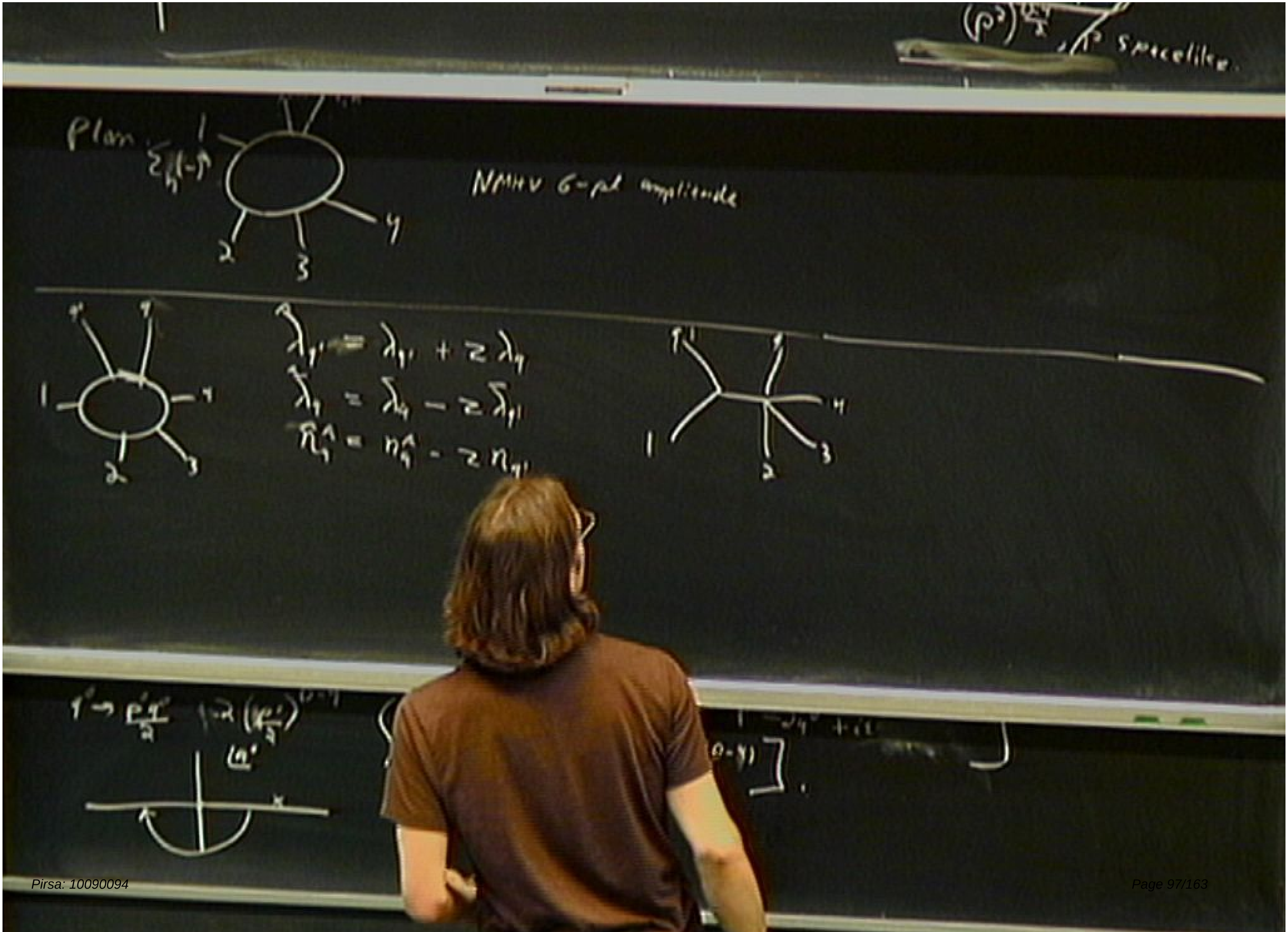
A=1…4
|i> = |+> + η^A|ψ_A> + η^A η^B/2 |φ_AB> + η^A η^B η^C ε_ABCD |ψ̄^D>/3! + (η)^4/4! |−>
p_i^{aȧ} = p_i^μ σ_μ^{aȧ} = λ^a λ̃^ȧ
A_4^{MHV} = δ^8(Σλη) / <12>…<41>

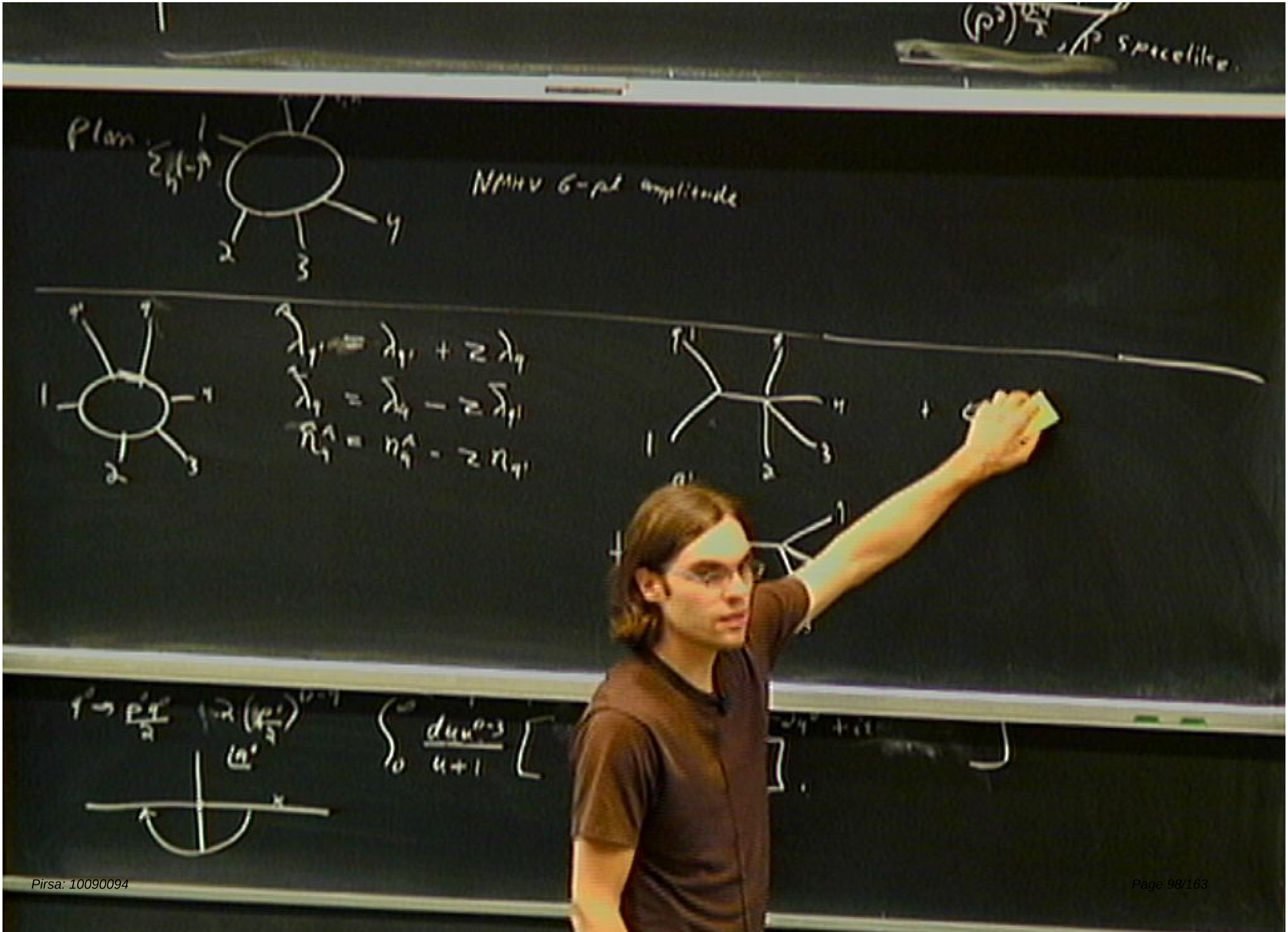

A = 1...4
|i> = |+> + η^A |ψ_A> + η^A η^B / 2 |φ_AB> + η^A η^B η^C ε_ABCD / 3! |ψ̄^D> + (η)^4 / 4! |−>
A_4^MHV = δ^8(Σ λη) / <12> ... <41>
+ + − − : η_3^4 η_4^4

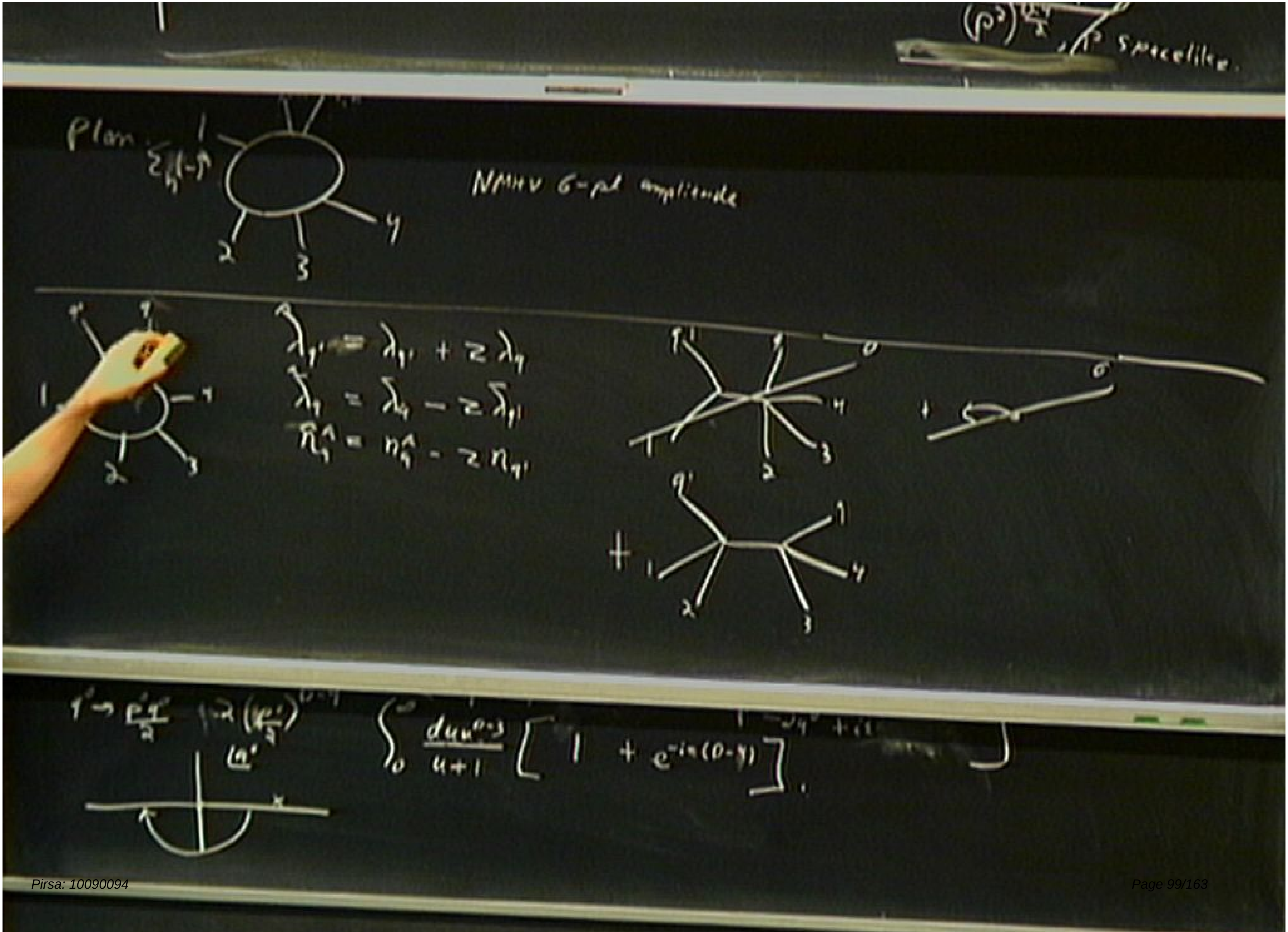

A = 1...4
|i> = |+> + η^A |ψ_A> + η^A η^B / 2 |φ_AB> + η^A η^B η^C ε_ABCD |ψ̄^D> / 3! + (η)^4 / 4! |−>
p_i^{aȧ} = p_i^μ σ_μ^{aȧ} = λ^a λ̃^ȧ
A_4^{MHV} = δ^8(Σλη) / <12>···<41>
++−− : η_3^4 η_4^4
⇒ Π_{A=1..4} δ^2(λ_3 η_3^A + λ_4 η_4^A)

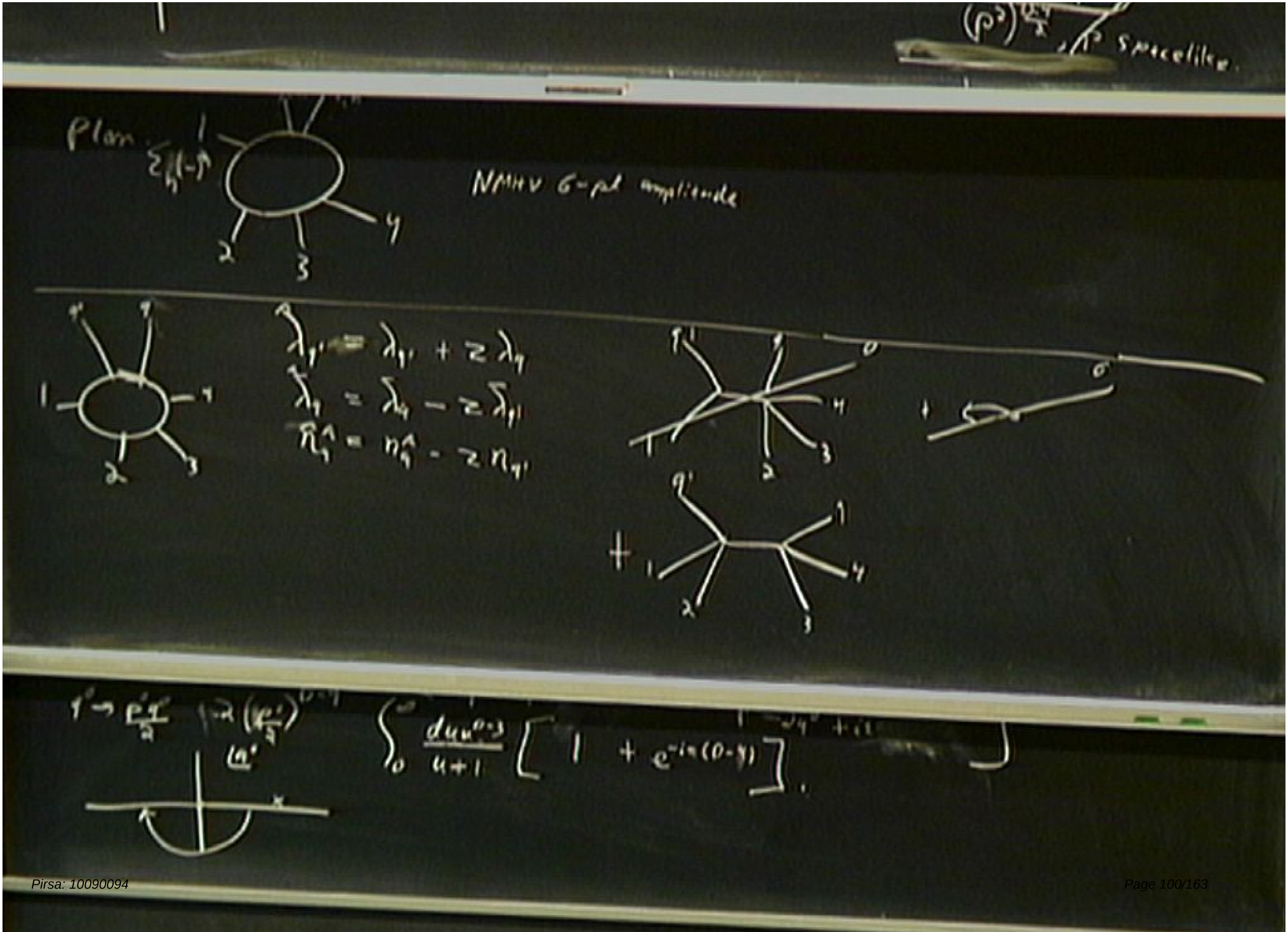

A = 1...4
|i> = |+> + η^A |ψ_A> + η^A η^B / 2 |φ_AB> + η^A η^B η^C ε_ABCD |ψ̄^D> / 3! + (η)^4 / 4! |−>
A_4^MHV = δ^8(Σλη) / <12>...<41>

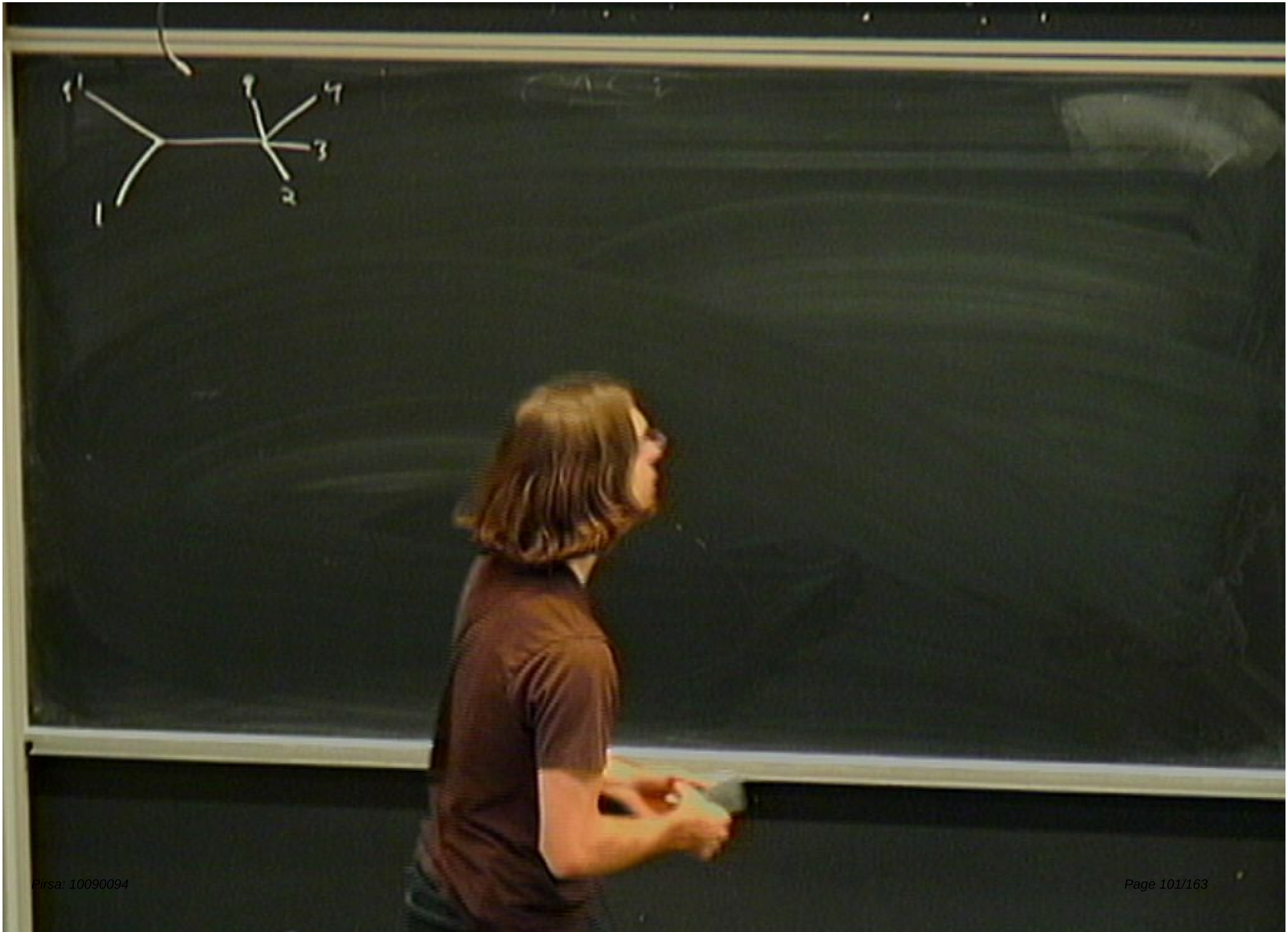

A=1...4
|i> = |+> + η^A|ψ_A> + η^A η^B/2 |φ_AB> + η^A η^B η^C ε_ABCD |ψ̄^D>/3! + (η)^4/4! |−>
p_i^{aȧ} = p_i^μ σ_μ^{aȧ} = λ^a λ̃^ȧ
A_4^MHV = δ^8(Σλη) / <12>...<41>
→ Π_{A=1..4} δ^2(λ_3 η_3^A + λ_4 η_4^A) → <34> η_3 η_4
++−− : η_3^4 η_4^4
<34>^4 / <12>...<41>

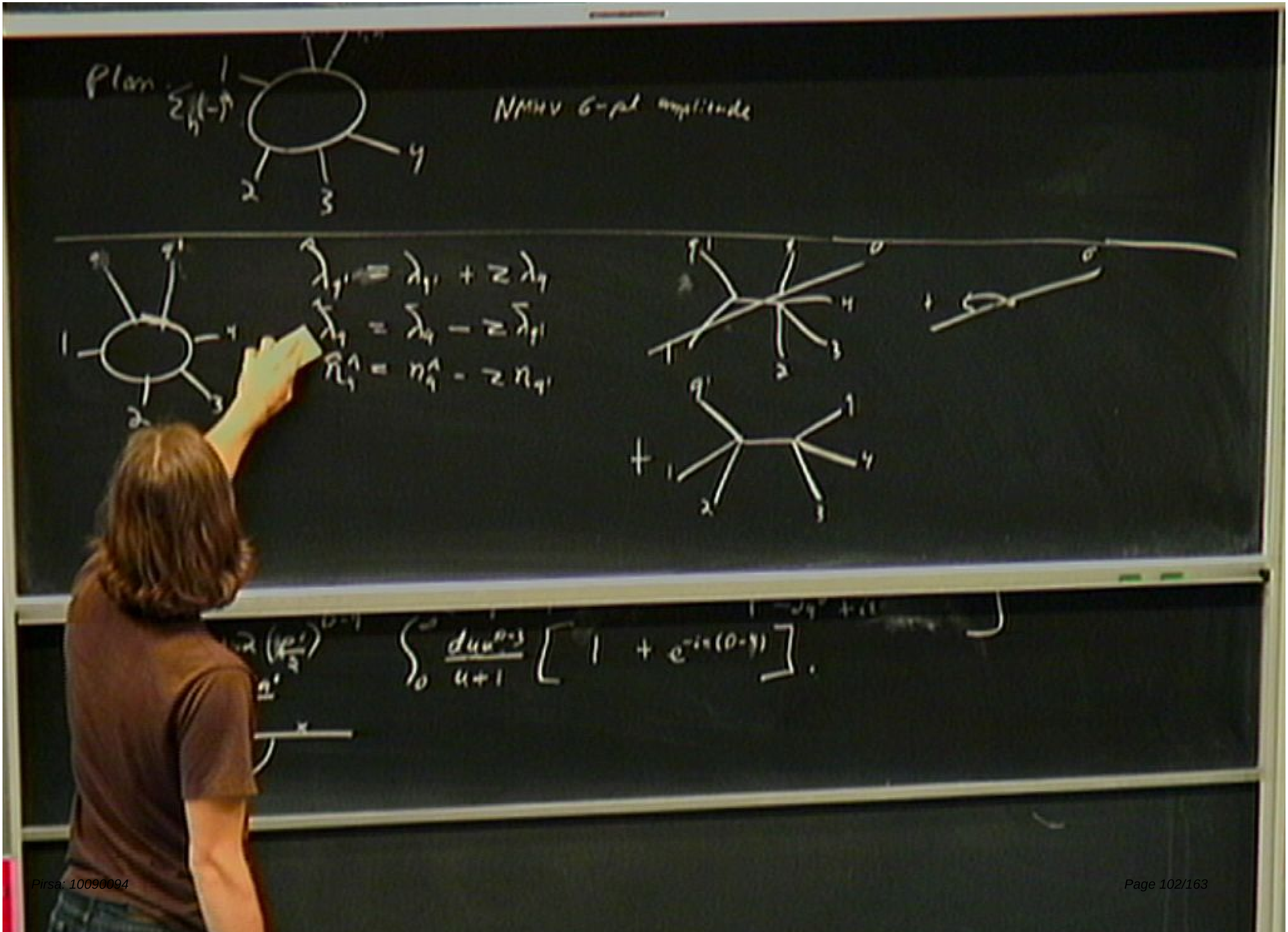

Plan
NMHV 6-pt amplitude
spacelike

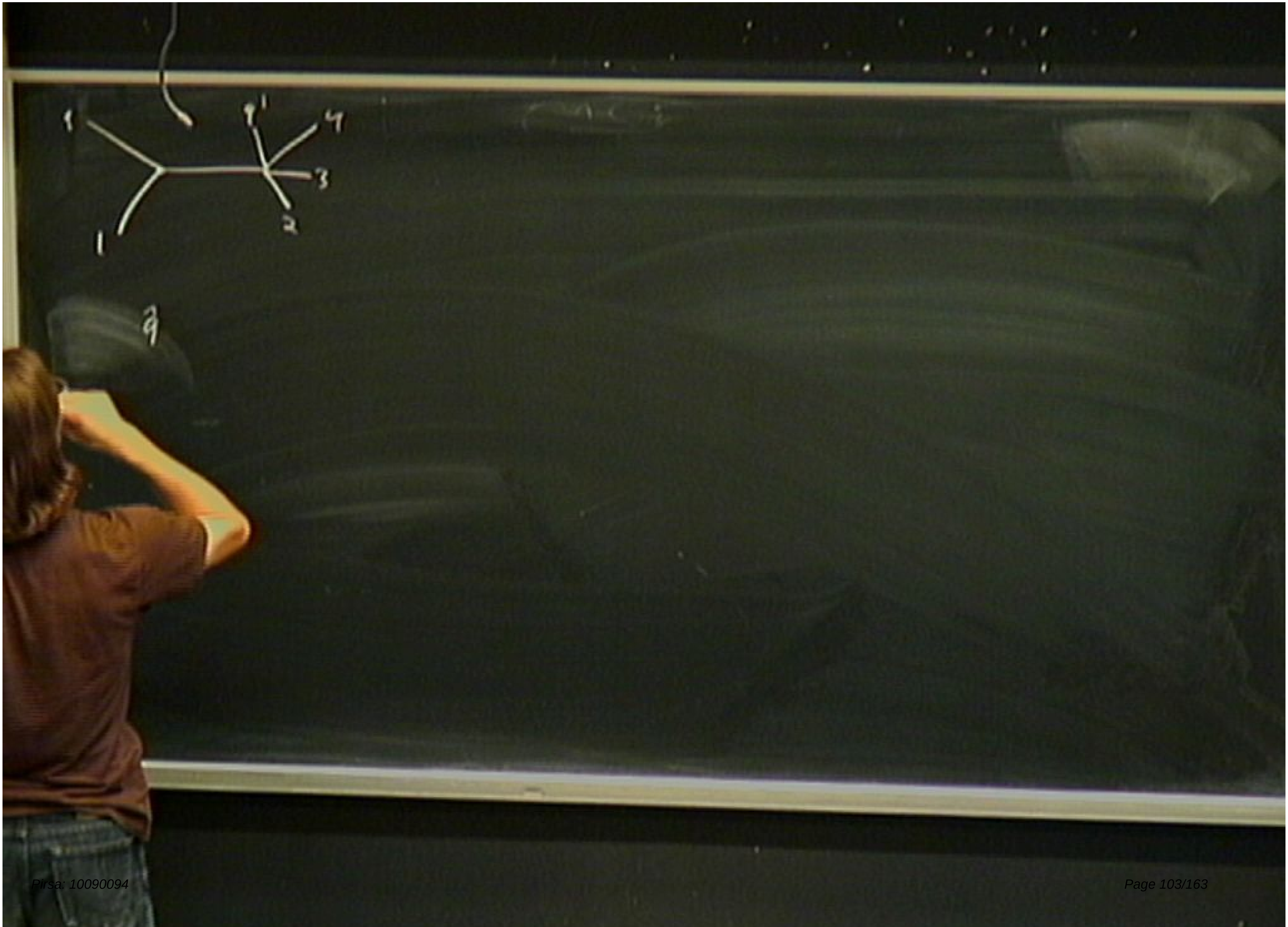

Plan
NMHV 6-pt amplitude
spacelike


Plan
NMHV 6-pt amplitude
$\lambda_{q'} = \lambda_{q'} + z\lambda_q$
$\tilde\lambda_q = \tilde\lambda_q - z\tilde\lambda_{q'}$
$\eta_q^A = \eta_q^A - z\eta_{q'}$
$\int_0^\infty \frac{du\,u^{D-3}}{u+1}\left[1 + e^{-i\pi(D-4)}\right]$
p^2 spacelike

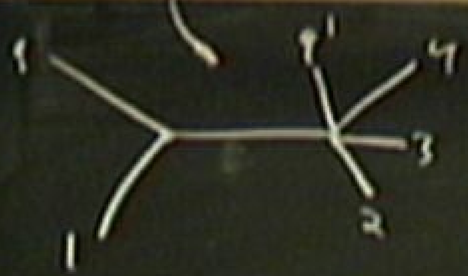

Plan
NMHV 6-pt amplitude
Spacelike


1'
4
3
2
1

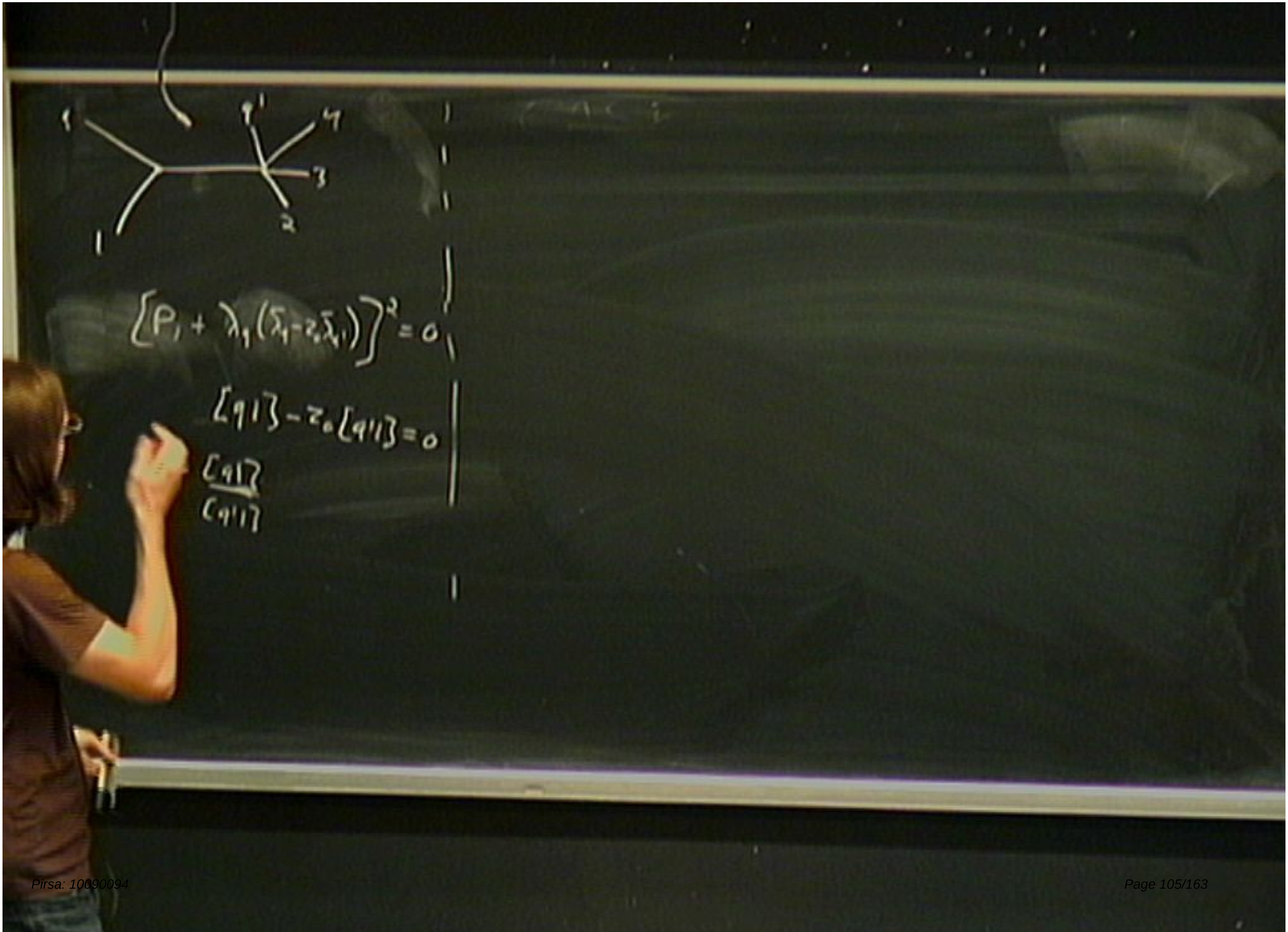

Plan
NMHV 6-pt amplitude

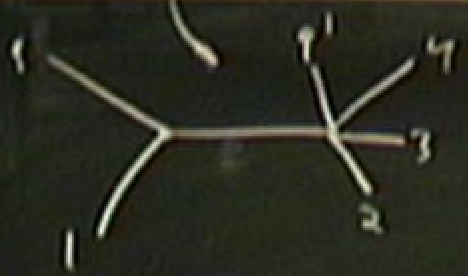


q
q'
4
3
2
1



$$\left[P_1 + (\quad) \quad \right]$$

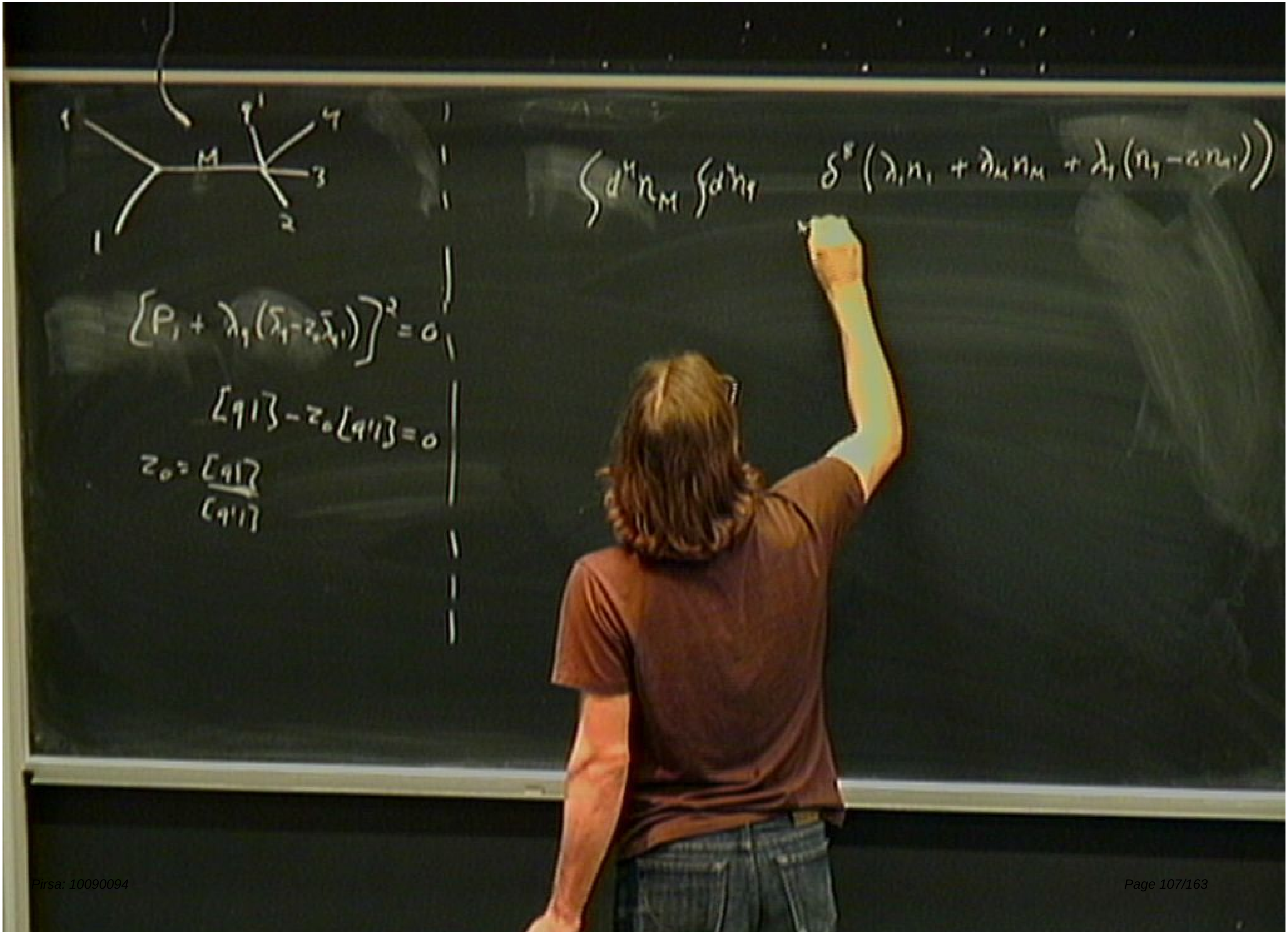

[P_1 + λ_4(λ̃_4 − z_0 λ̃_{4'})]^2 = 0
[q1] − z_0 [q'1] = 0
[q1] / [q'1]



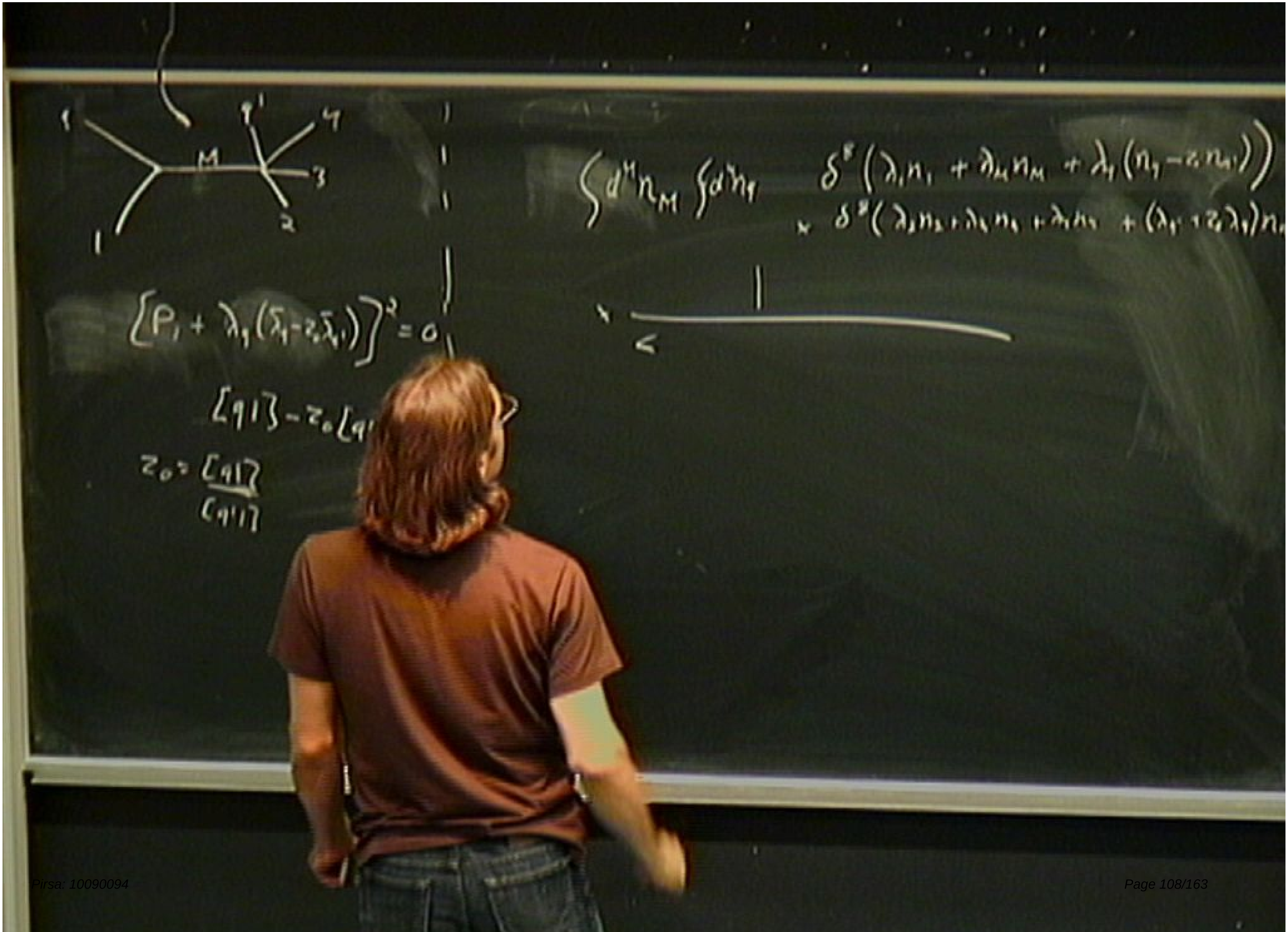
$$\left[P_1 + \lambda_q(\tilde{\lambda}_4 - z_0\tilde{\lambda}_1)\right]^2 = 0$$

$$[q1] - z_0[q'1] = 0$$

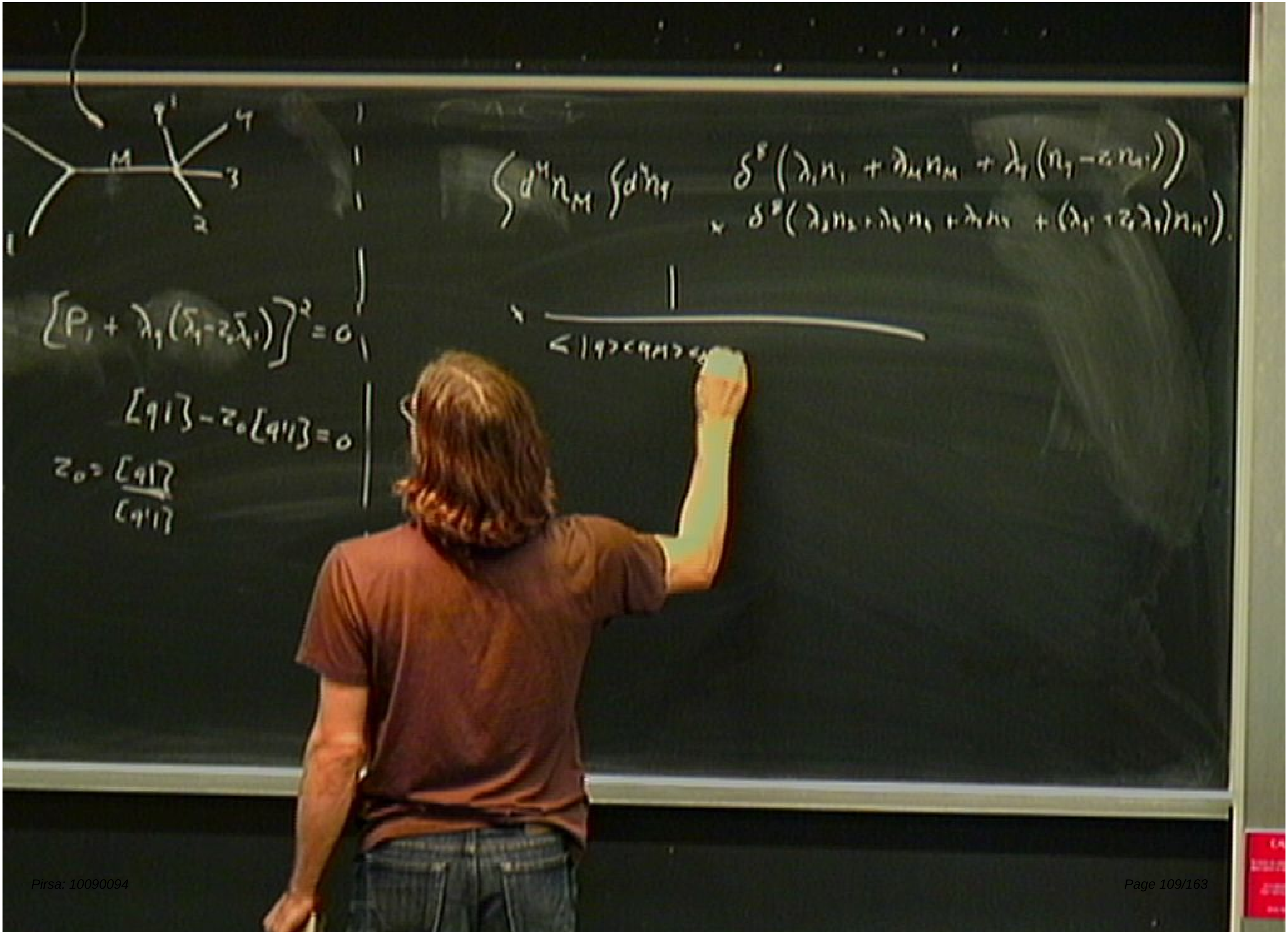
$$z_0 = \frac{[q1]}{[q'1]}$$

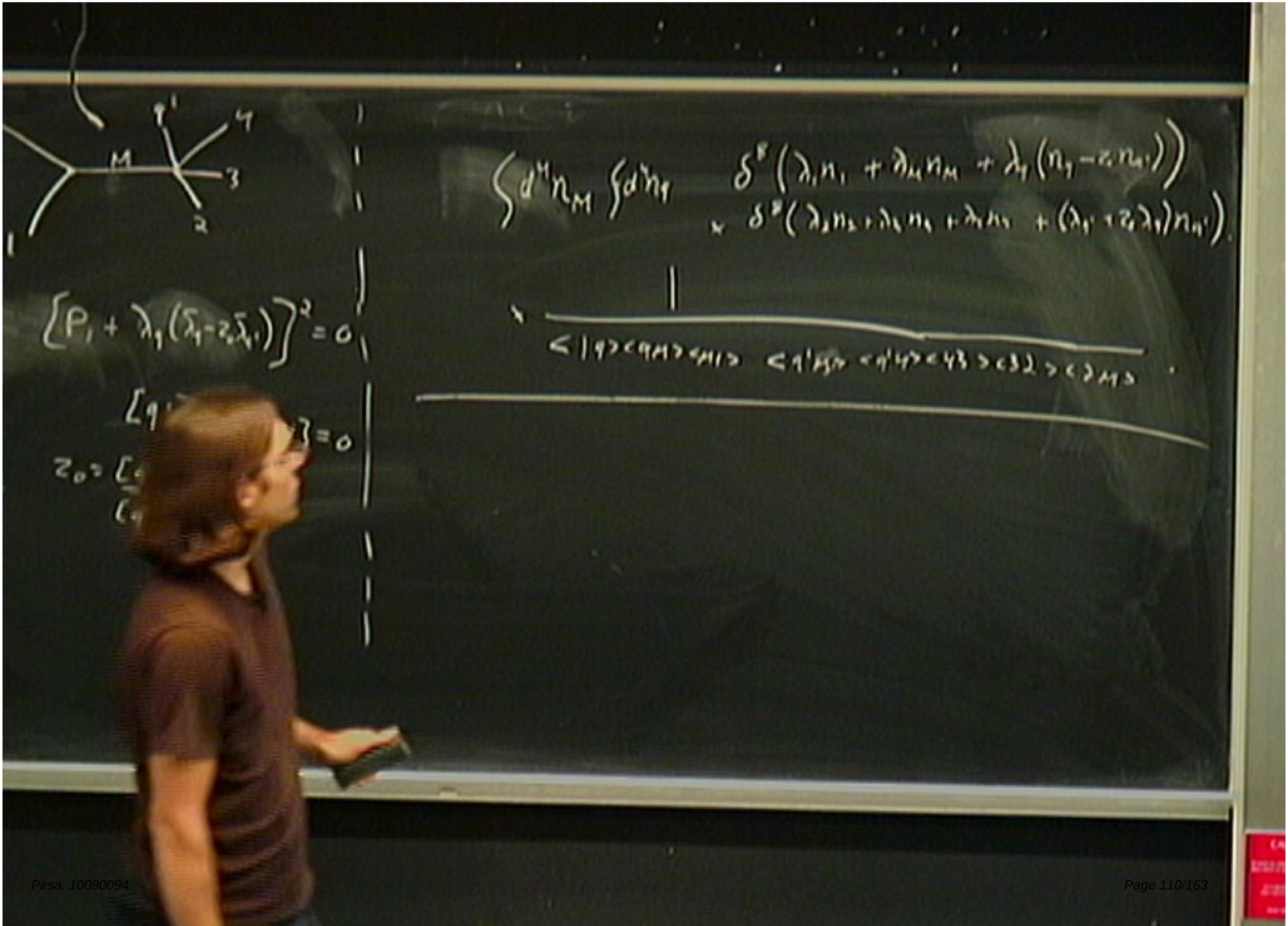


[P1 + λ4(λ̃4 - z0λ̃1)]^2 = 0
[41] - z0[4'1] = 0
z0 = [41] / [4'1]
∫d^4ηM ∫d^4η4   δ^8(λ1η1 + λMηM + λ4(η4 - z0η1))



[P_1 + λ_q(λ̃_q − z_0 λ̃_1)]^2 = 0
z_0 = [q1]/[q'1]


[q1] - z₀[q'1] = 0
z₀ = [q1] / [q'1]


M
1
2
3
4
4'
[P1 + λ4(λ̃4 − z0λ̃4')]² = 0
]= 0
z0 =
∫d⁴ηM ∫d⁴η4 δ⁸(λ1η1 + λMηM + λ4(η4 − z0η4'))
× δ⁸(λ2η2 + λ3η3 + λMηM + (λ4' + z0λ4)η4')
×
<14><4M><M1> <1'M> <4'4><43><32><2M>

$$\int d^4\eta_M \int d^4\eta_q \quad \delta^8\left(\lambda_1\eta_1 + \lambda_M\eta_M + \lambda_q\left(\eta_q - z_0\eta_{q'}\right)\right)$$

$$\times\ \delta^8\left(\lambda_2\eta_2 + \lambda_3\eta_3 + \lambda_4\eta_4 + (\lambda_{q'} + z_0\lambda_q)\eta_{q'}\right).$$

$$\times \frac{1}{\langle 1q\rangle\langle qM\rangle\langle M1\rangle \quad \langle q'M\rangle\langle q'4\rangle\langle 43\rangle\langle 32\rangle\langle 2M\rangle}.$$

$\lambda_q \to \lambda_{q'}$

$\bar\lambda_q \to \bar\lambda_{q'}$

$\eta_q \to \eta_{q'}$

$z_0 \to$

$$\int d^4\eta_M \int d^4\eta_q \quad \delta^8\left(\lambda_1\eta_1 + \lambda_M\eta_M + \lambda_q\left(\eta_q - z_0\eta_{q'}\right)\right)$$

$$\times\ \delta^8\left(\lambda_2\eta_2 + \lambda_3\eta_3 + \lambda_4\eta_4 + (\lambda_{q'} + z_0\lambda_q)\eta_{q'}\right).$$

$$\times \frac{1}{\langle 1q\rangle\langle qM\rangle\langle M1\rangle \quad \langle q'M\rangle\langle q'4\rangle\langle 43\rangle\langle 32\rangle\langle 2M\rangle}.$$

$\lambda_q \to \lambda_{q'}$

$\bar{\lambda}_q \to \bar{\lambda}_{q'}$

$\eta_q \to \eta_{q'}$

$z_0 \to 1$

$$\int d^4\eta_M \int d^4\eta_q \quad \delta^8\left(\lambda_1\eta_1 + \lambda_M\eta_M + \lambda_q\left(\eta_q - z_0\eta_{q'}\right)\right)$$

$$\times\ \delta^8\left(\lambda_2\eta_2 + \lambda_3\eta_3 + \lambda_4\eta_4 + (\lambda_{q'} + z_0\lambda_q)\eta_{q'}\right).$$

$$\times \frac{1}{\langle 1q\rangle\langle qM\rangle\langle M1\rangle \quad \langle q'M\rangle\langle q'4\rangle\langle 43\rangle\langle 32\rangle\langle 2M\rangle}.$$

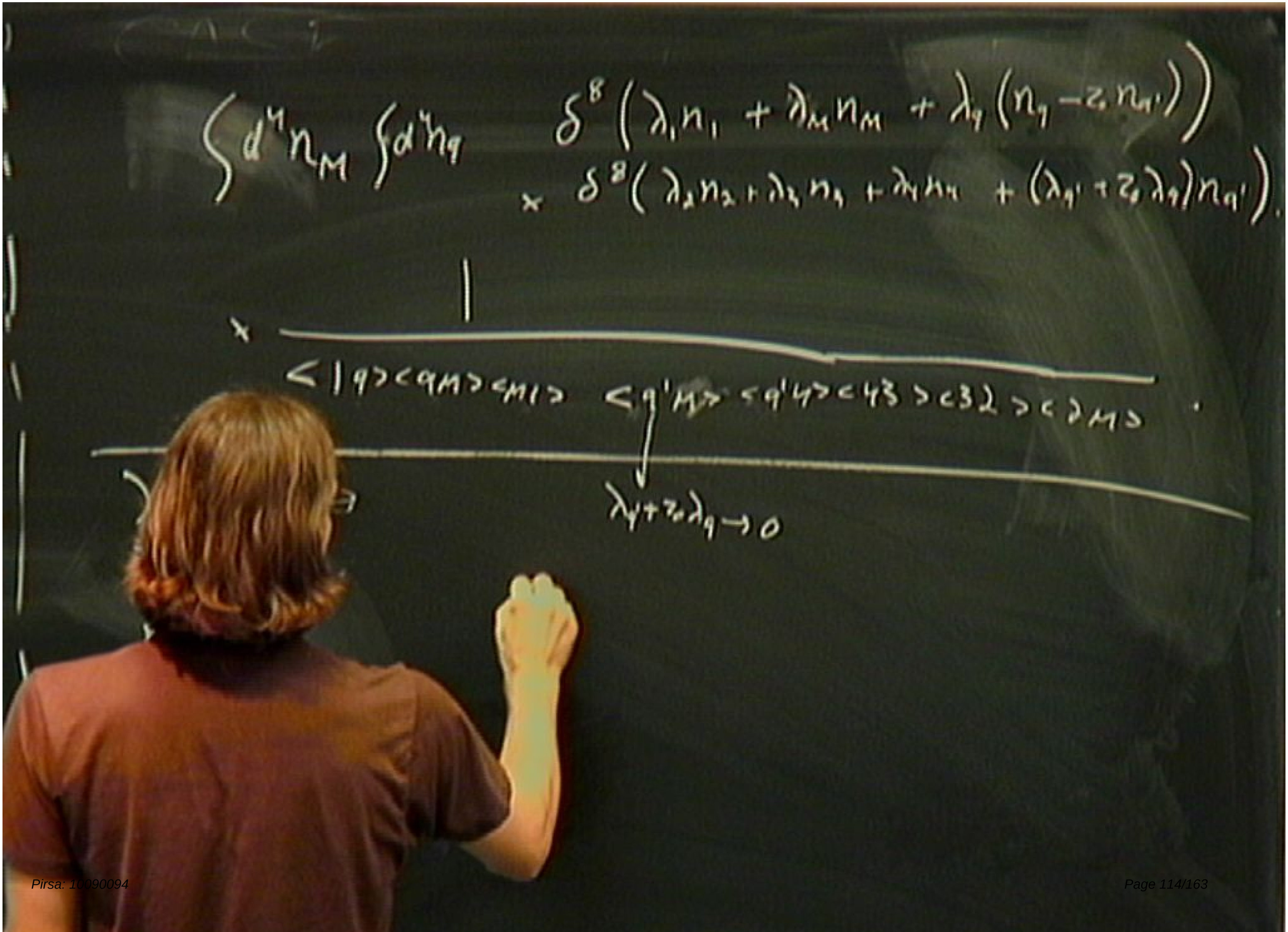
$\langle q'M\rangle$: $\lambda_{q'} + z_0\lambda_q \to 0$

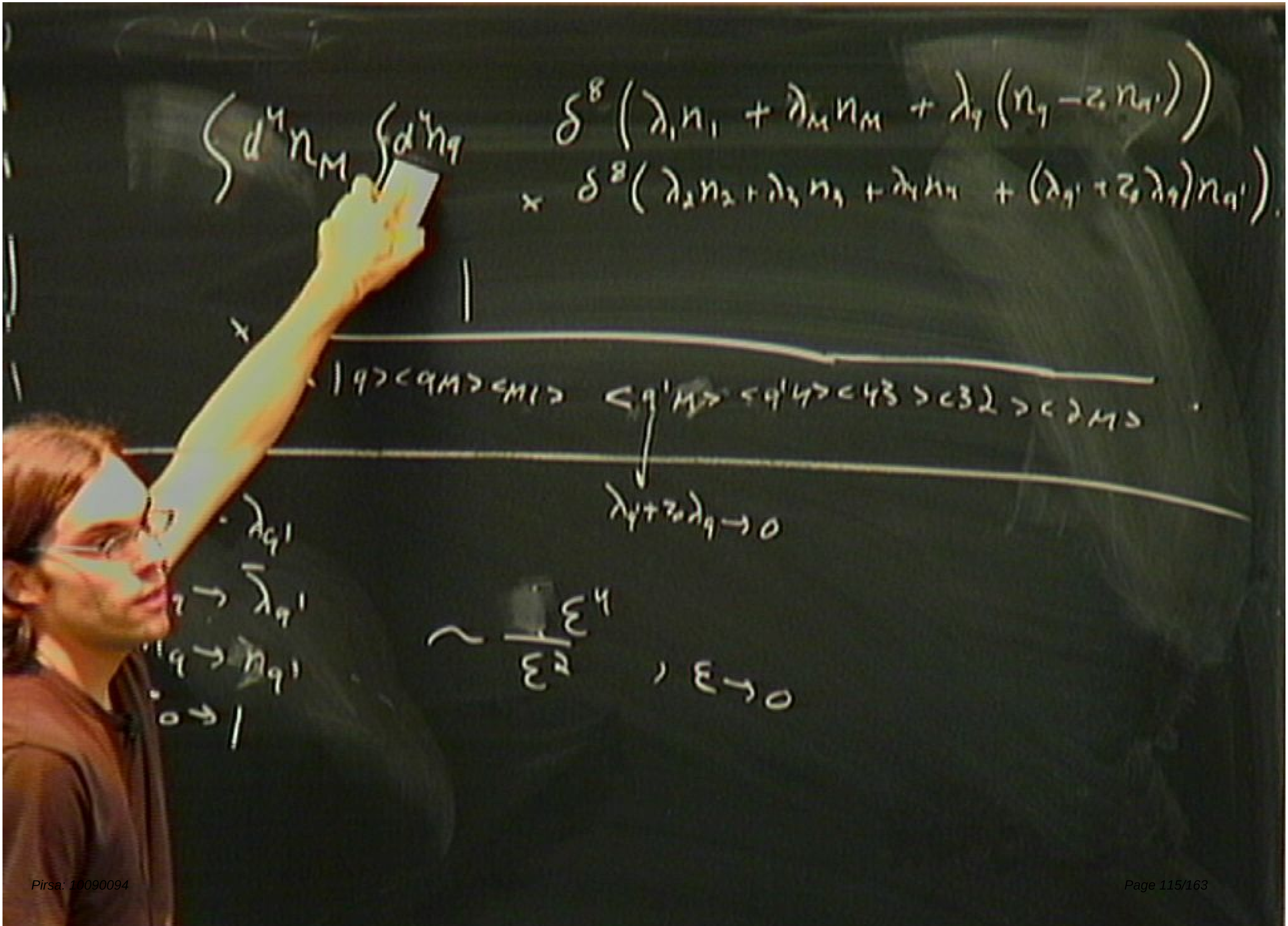
$\lambda_q \to \lambda_{q'}$

$\bar\lambda_q \to \bar\lambda_{q'}$

$\eta_q \to \eta_{q'}$

$z_0 \to 1$


∫d⁴η_M ∫d⁴η_q δ⁸(λ₁η₁ + λ_Mη_M + λ_q(η_q − z₀η_q'))
× δ⁸(λ₂η₂ + λ₃η₃ + λ₄η₄ + (λ_q' + z₀λ_q)η_q').
<1q><qM><M1> <q'M> <q'4><43><32><2M>
λ_q' + z₀λ_q → 0


∫d⁴η_M ∫d⁴η_q δ⁸(λ₁η₁ + λ_M η_M + λ_q(η_q − z₀η_q'))
× δ⁸(λ₂η₂ + λ₃η₃ + λ₄η₄ + (λ_q' + z₀λ_q)η_q')
<q'M> <q'4><43><32><2M>
λ_q' + z₀λ_q → 0
~ ε⁴/ε² , ε → 0

$$\int d^4\eta_M \int d^4\eta_q \quad \delta^8\big(\lambda_1\eta_1 + \lambda_M\eta_M + \lambda_q(\eta_q - z_0\eta_{q'})\big)$$

$$\times\ \delta^8\big(\lambda_2\eta_2 + \lambda_3\eta_3 + \lambda_4\eta_4 + (\lambda_{q'} + z_0\lambda_q)\eta_{q'}\big).$$

$$\times\ \frac{1}{\langle 1q\rangle\langle qM\rangle\langle M1\rangle\ \ \langle q'M\rangle\langle q'4\rangle\langle 43\rangle\langle 32\rangle\langle 2M\rangle}.$$

$\lambda_{q'} + z_0\lambda_q \to 0$

$\lambda_q \to -\lambda_{q'}$

$\bar\lambda_q \to \bar\lambda_{q'}$

$\eta_q \to \eta_{q'}$

$z_0 \to 1$

$$\sim \frac{1}{\varepsilon^2}\,\varepsilon^4, \quad \varepsilon \to 0$$

$$\int d^4\eta_M \int d^4\eta_q \quad \delta^8\left(\lambda_1\eta_1 + \lambda_M\eta_M + \lambda_q(\eta_q - z_0\eta_{q'})\right)$$
$$\times\ \delta^8\left(\lambda_2\eta_2 + \lambda_3\eta_3 + \lambda_4\eta_4 + (\lambda_{q'} + z_0\lambda_q)\eta_{q'}\right).$$

$$\times \frac{1}{\langle 1q\rangle\langle qM\rangle\langle M1\rangle \quad \langle q'M\rangle\langle q'4\rangle\langle 43\rangle\langle 32\rangle\langle 2M\rangle}.$$

$\lambda_{q'} + z_0\lambda_q \to 0$

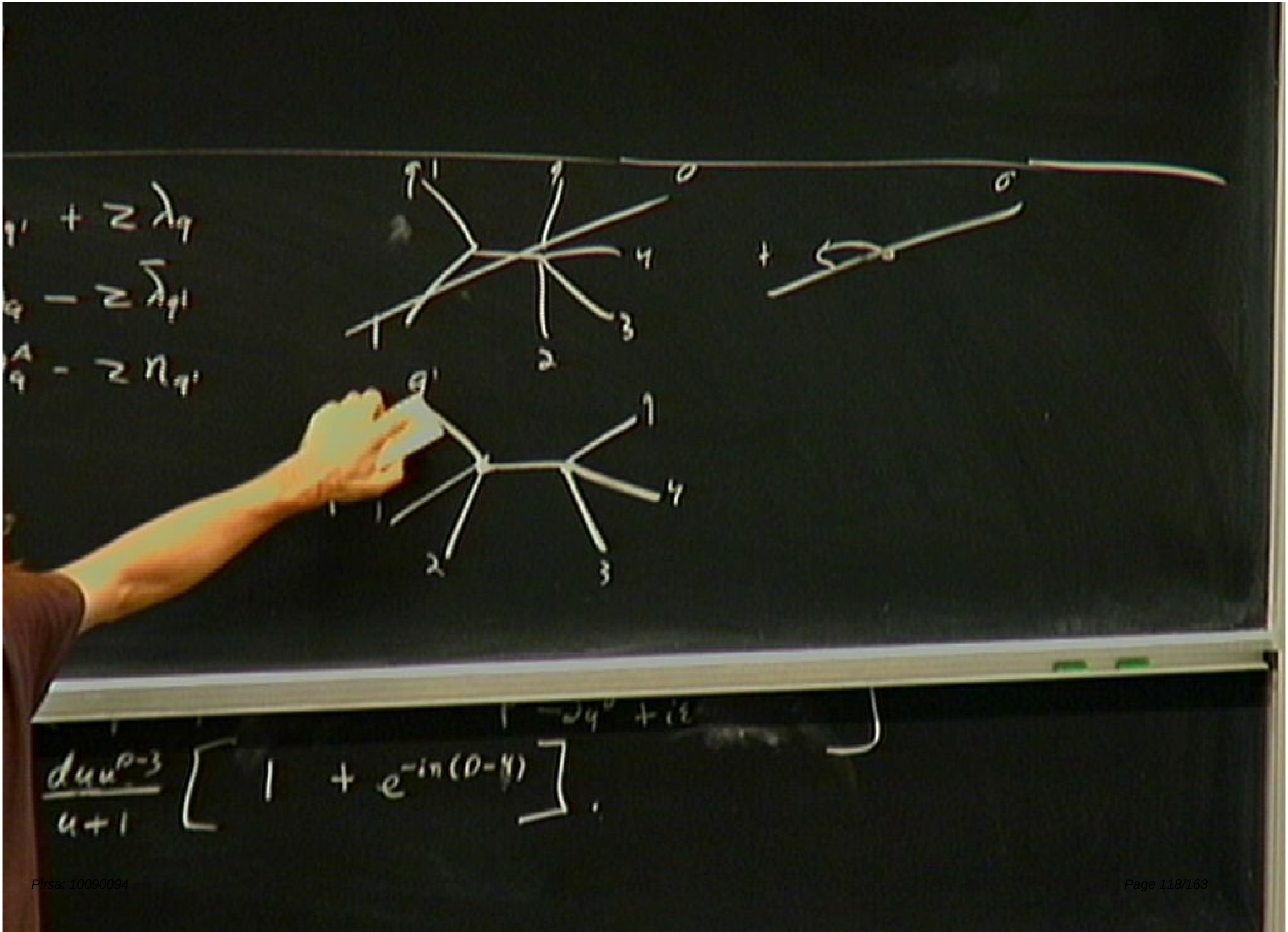
$\lambda_q \to -\lambda_{q'}$

$\bar\lambda_q \to \bar\lambda_{q'}$

$\eta_q \to \eta_{q'}$

$z_0 \to 1$

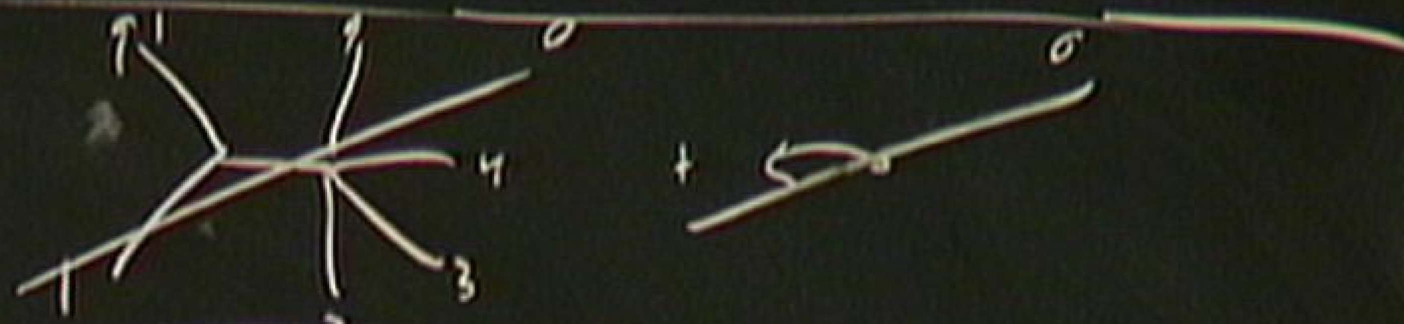
$$\sim \frac{\varepsilon^4}{\varepsilon^2}, \quad \varepsilon \to 0$$

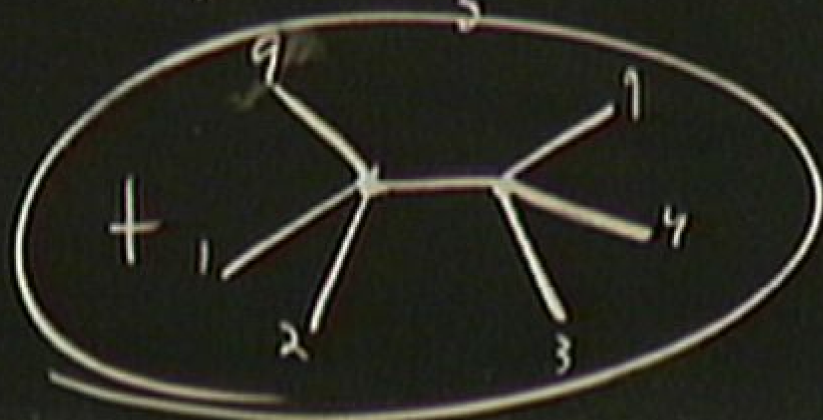

+ z λ_q
− z λ̄_q'
− z η_q'
q' q 0 4 1 2 3
+
q' 1 4 2 3
1 + e^{-in(D-4)}
u+1

$\ldots_{q'} + z\,\lambda_q$

$\ldots_q - z\,\bar{\lambda}_{q'}$

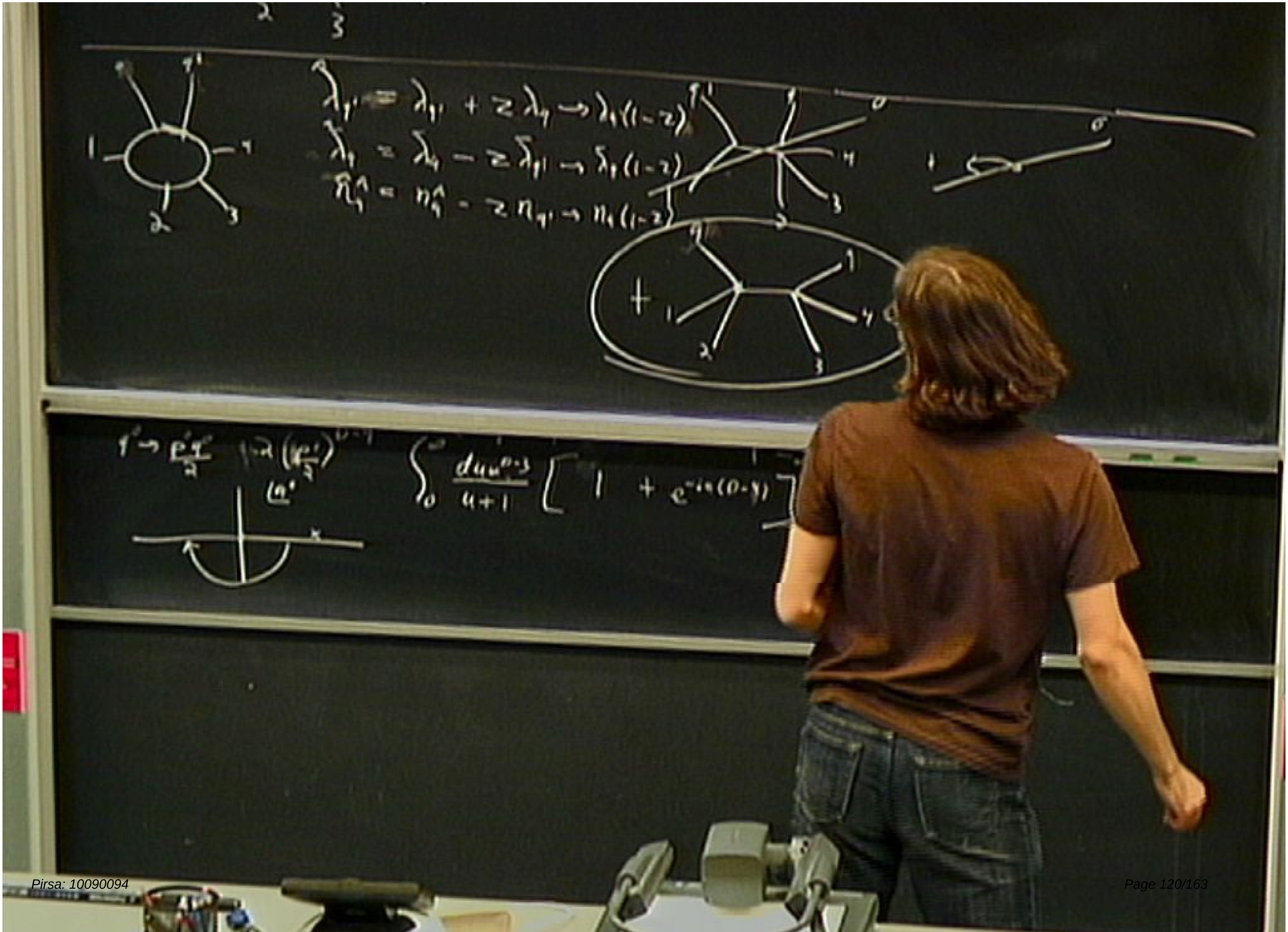
$\ldots^A_q - z\,\eta_{q'}$

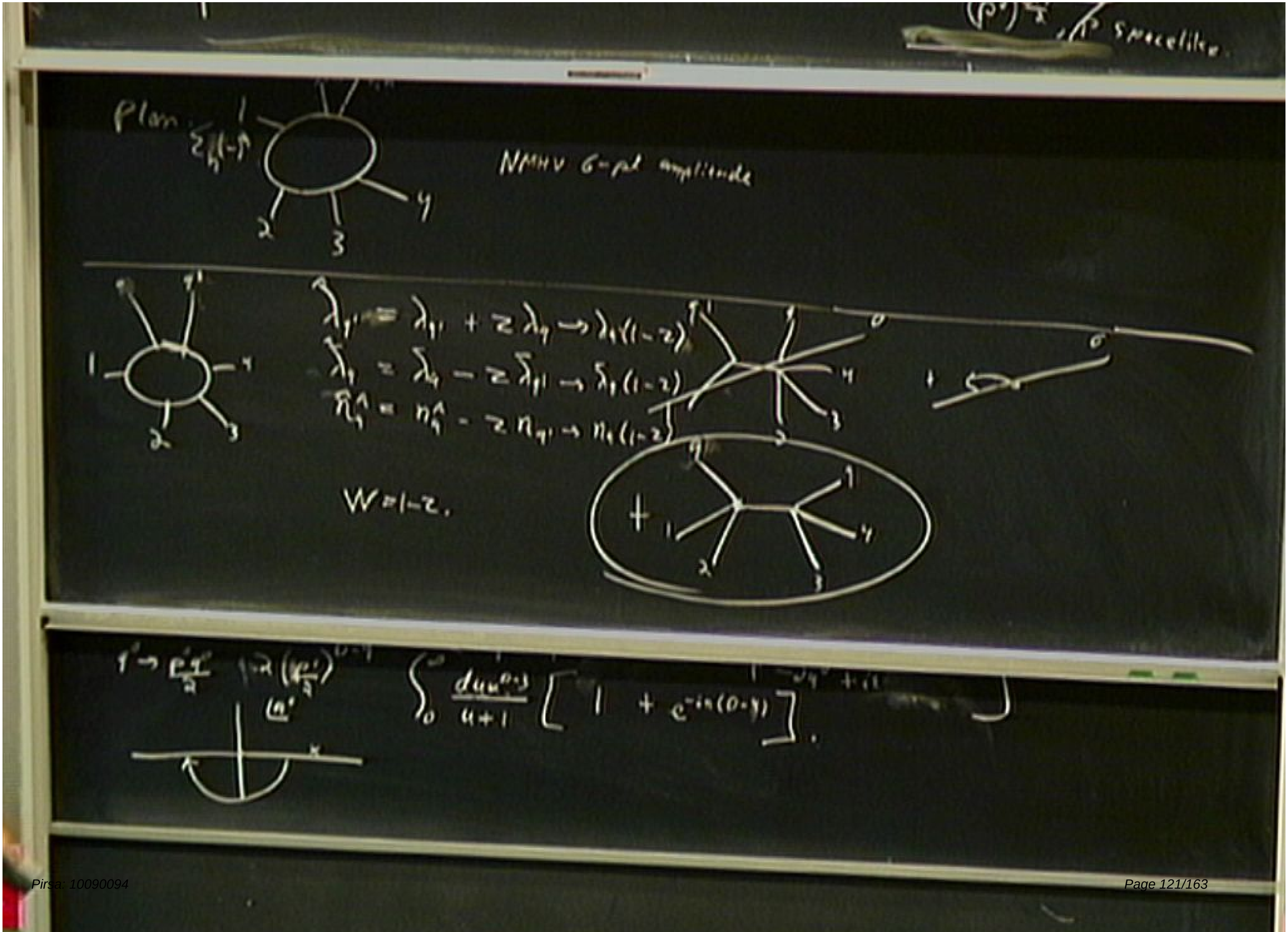


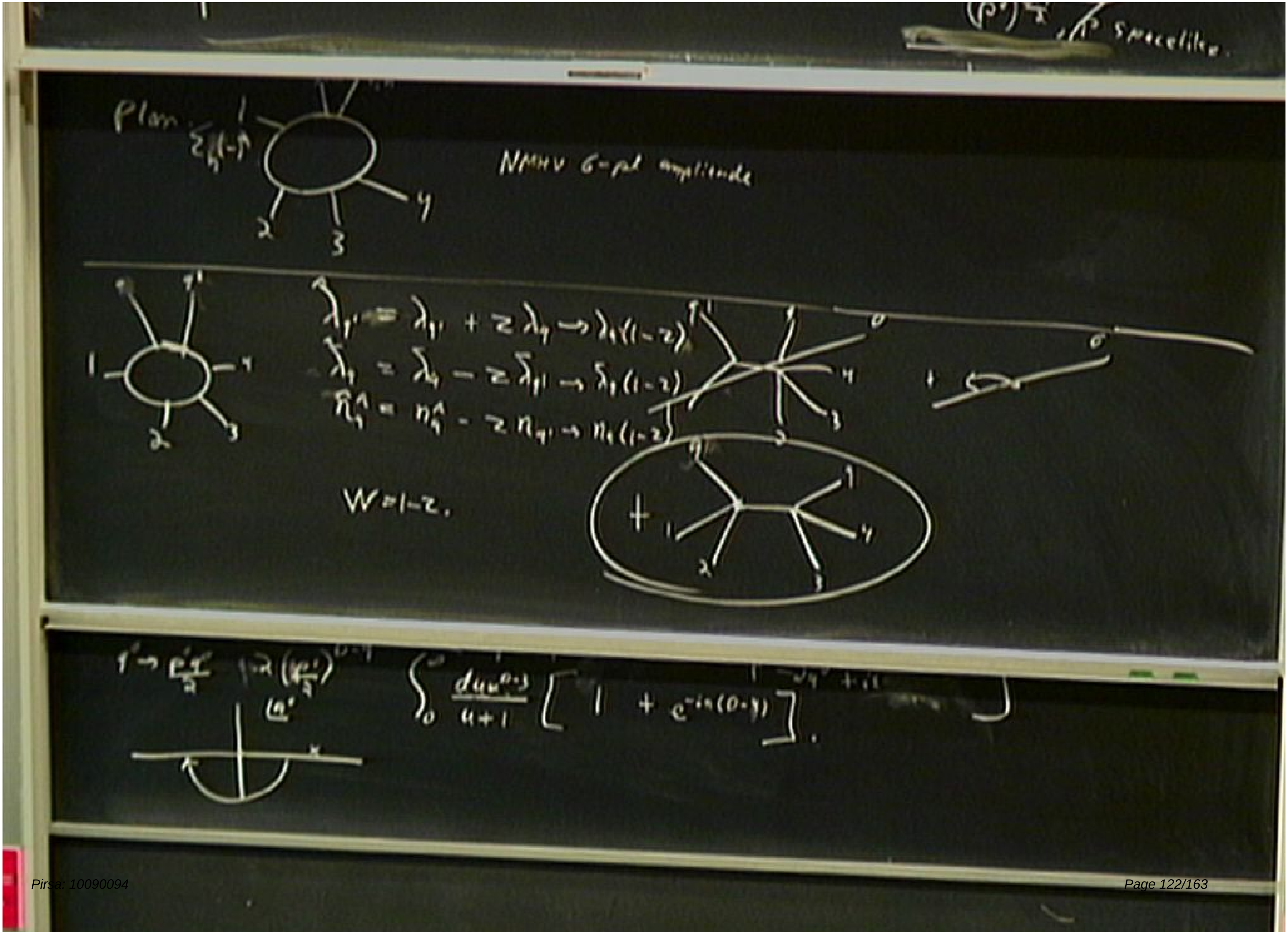


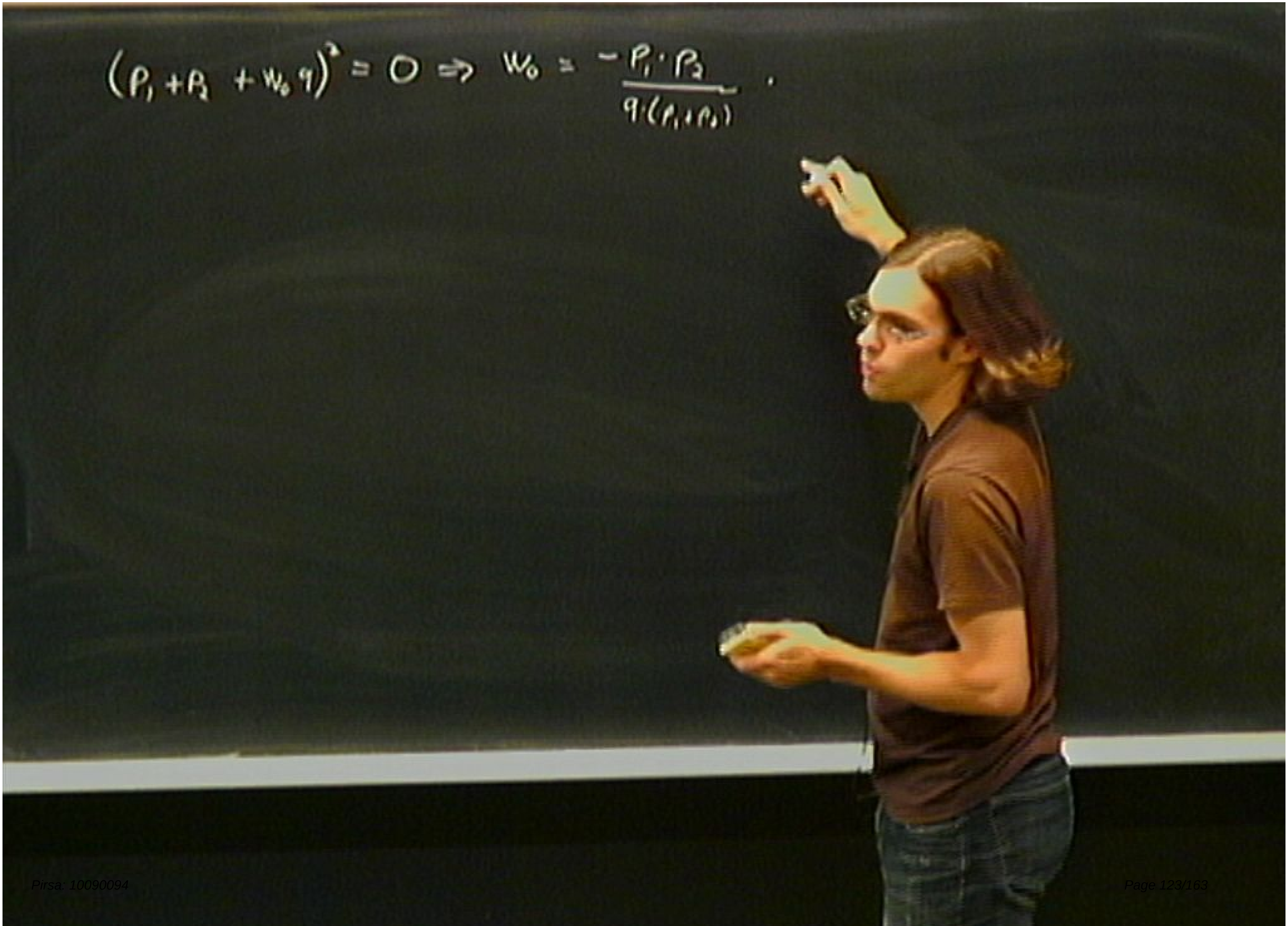
$\ldots - 2q^0 + i\varepsilon$

$$\int \frac{du\,u^{D-3}}{u+1}\left[\,1 + e^{-i\pi(D-4)}\right].$$

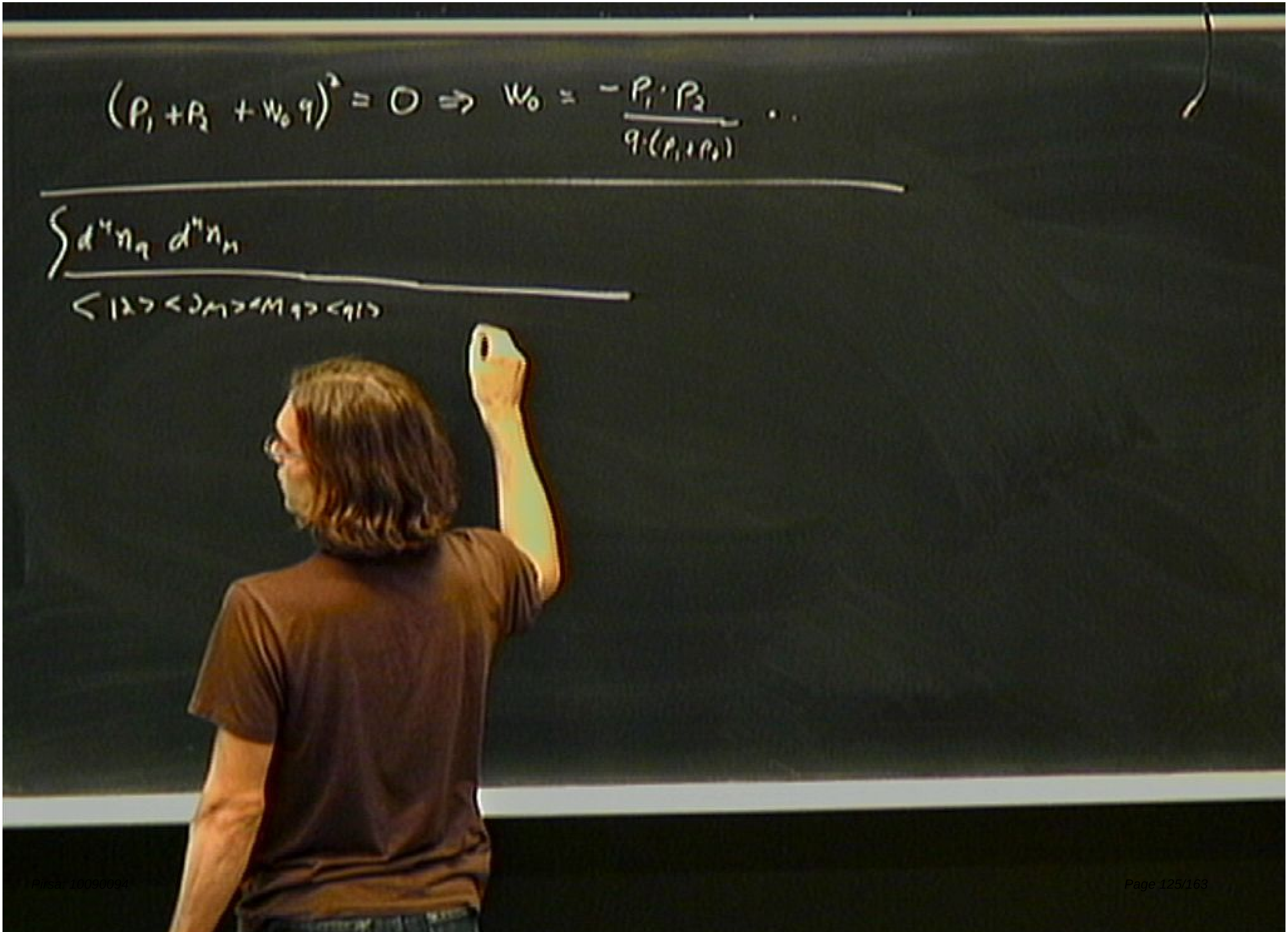


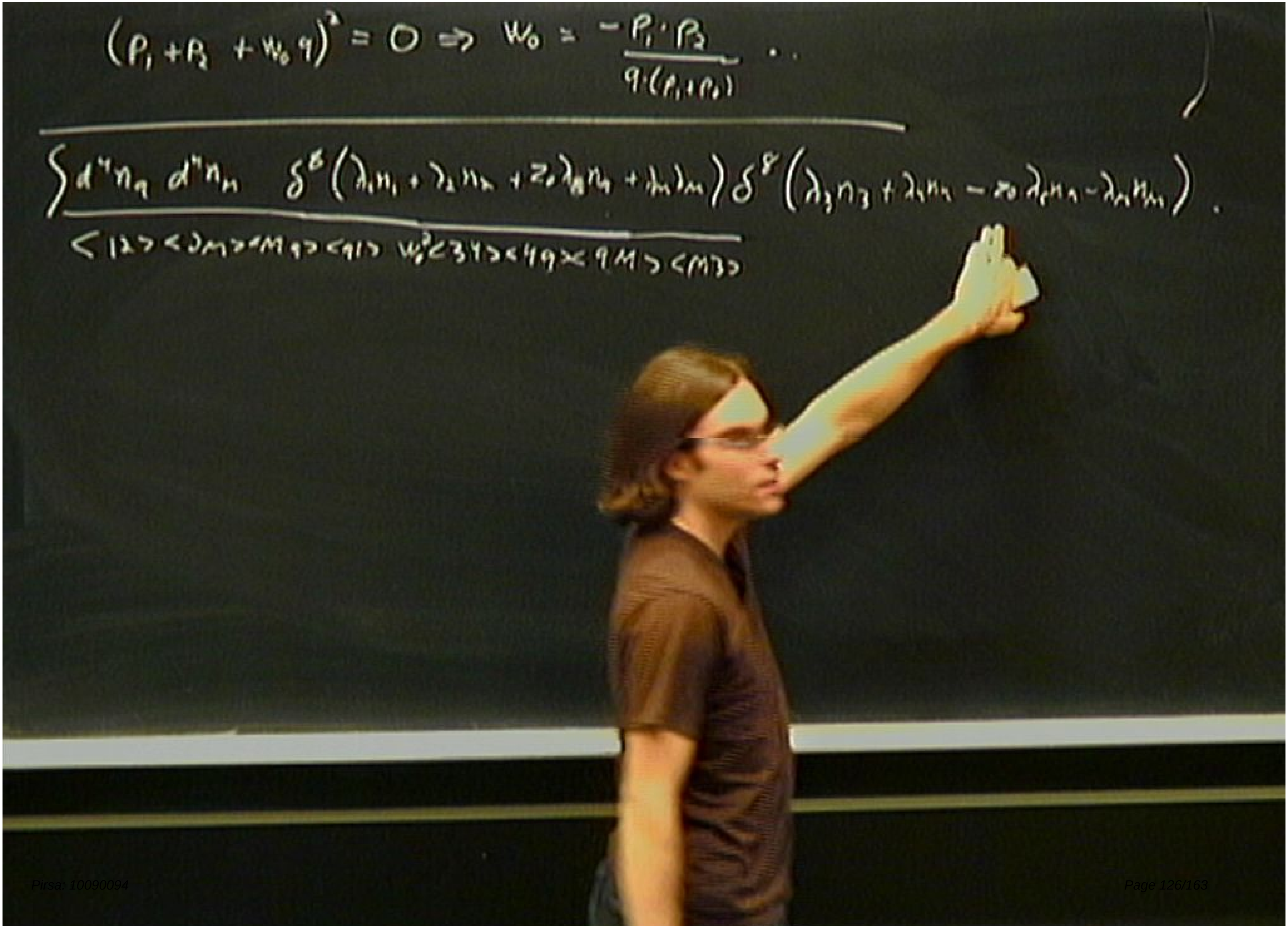

Plan
NMHV 6-pt amplitude
W = 1-z.
spacelike

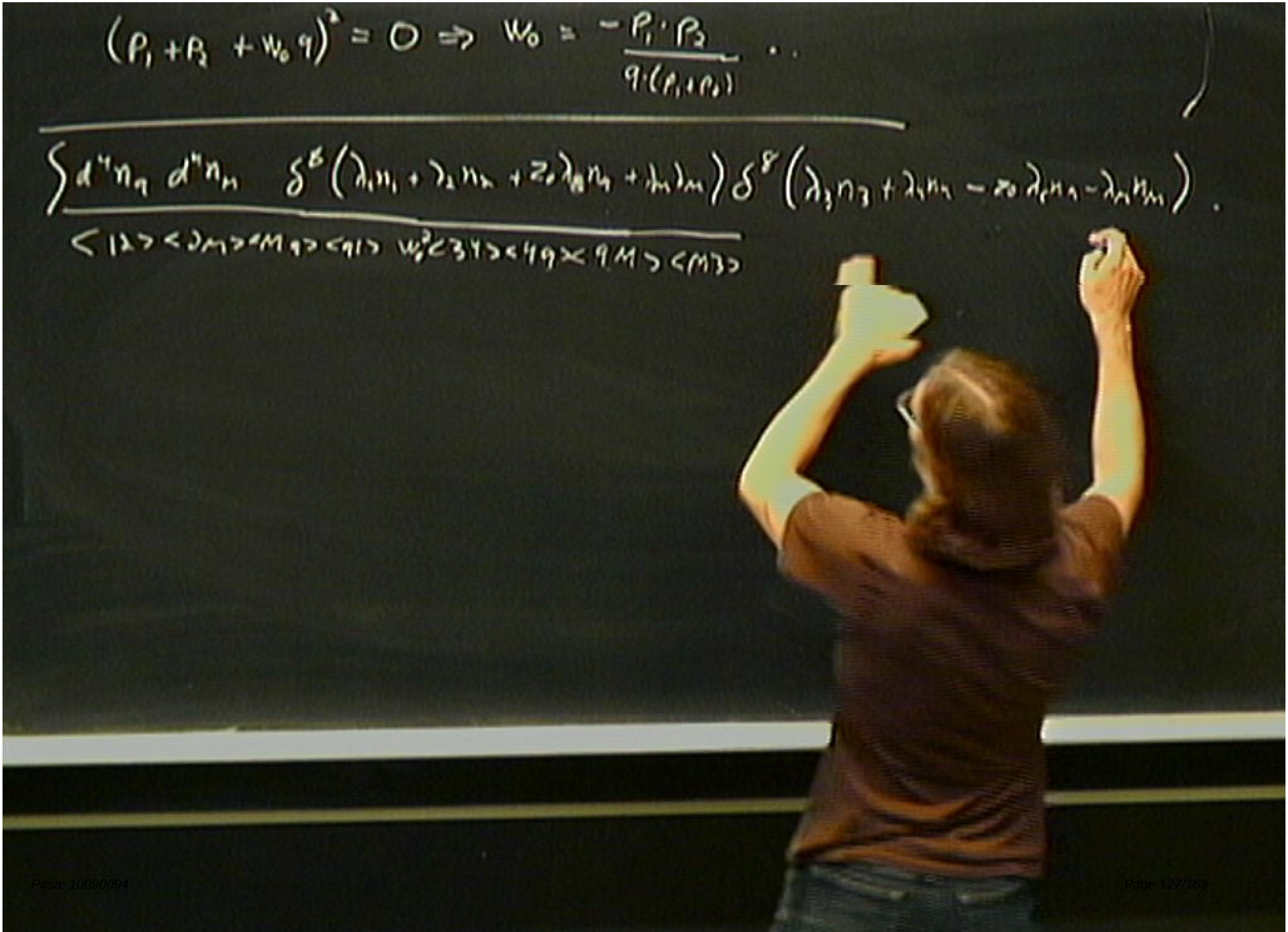

spacelike.
Plan.
NMHV 6-pt amplitude
W = 1-z.


$(p_1 + p_2 + w_0 q)^2 = 0 \Rightarrow w_0 = -\frac{p_1 \cdot p_2}{q \cdot (p_1 + p_2)}$.

$$(p_1 + p_2 + w_0 q)^2 = 0 \Rightarrow w_0 = -\frac{p_1 \cdot p_2}{q \cdot (p_1 + p_2)} \quad \cdots$$

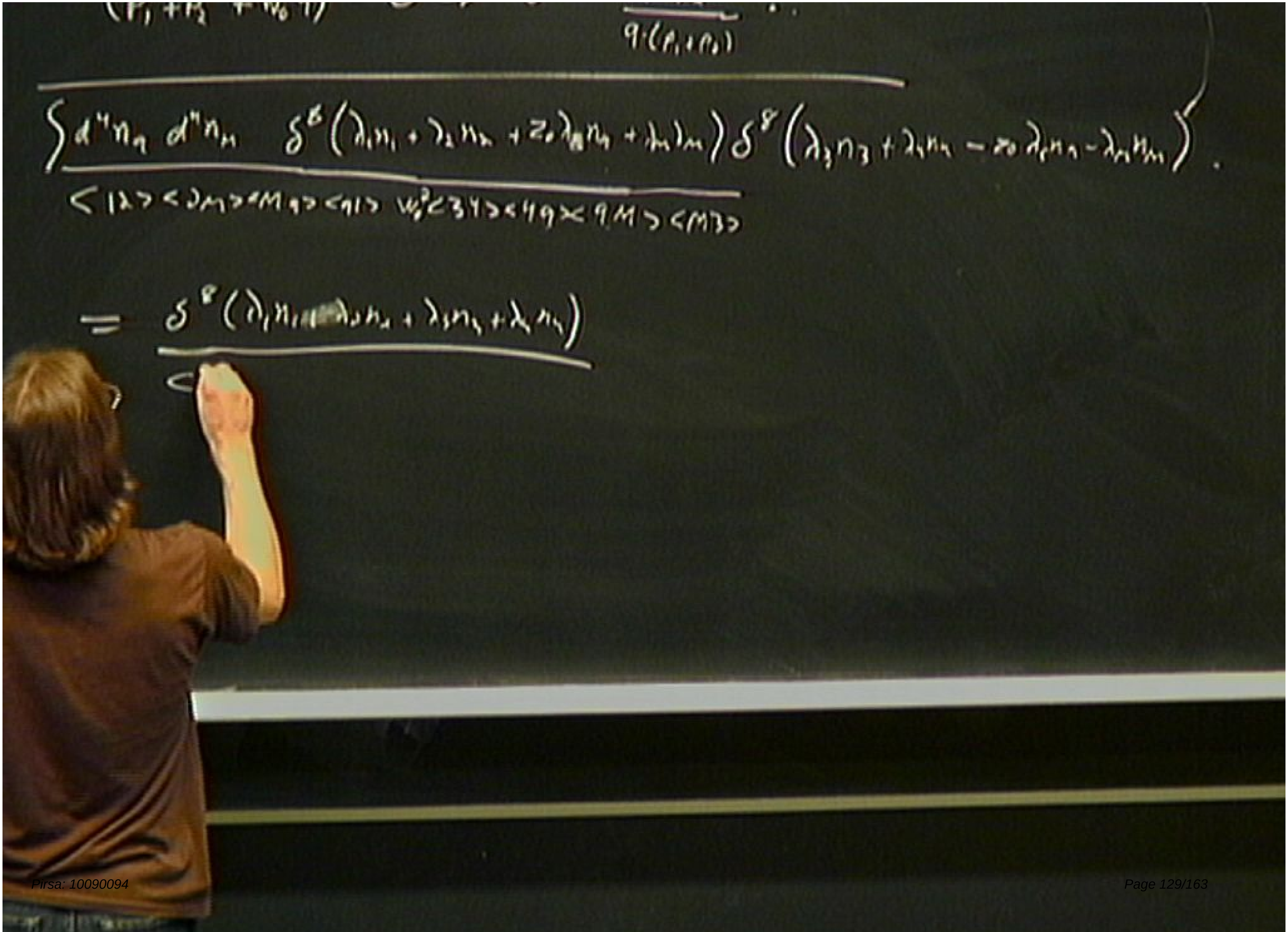

(P1 + P2 + w0 q)^2 = 0 ⇒ w0 = − P1 · P2 / q·(P1 + P2) . .
∫ d^4η1 d^4ηn / <12><2m><mq><q1>

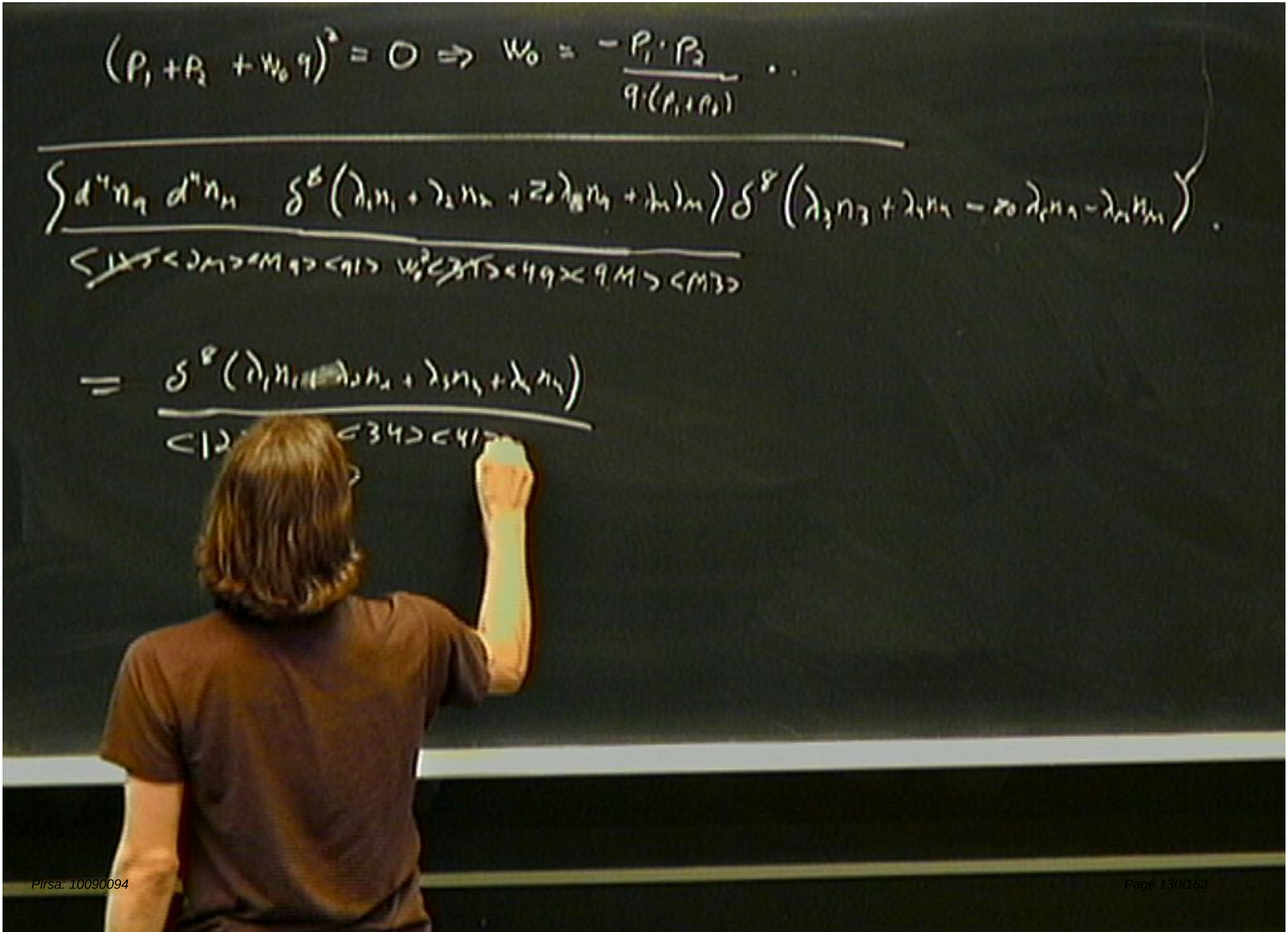

$(P_1 + P_2 + w_0 q)^2 = 0 \Rightarrow w_0 = -\frac{P_1 \cdot P_2}{q\cdot(P_1+P_2)} \ldots$
$\int \frac{d^4\eta_q \, d^4\eta_m \quad \delta^8(\lambda_1\eta_1 + \lambda_2\eta_2 + z_0\lambda_q\eta_q + \eta_m\lambda_m)\,\delta^8(\lambda_3\eta_3 + \lambda_4\eta_4 - z_0\lambda_q\eta_q - \lambda_m\eta_m)}{\langle 12\rangle\langle 2m\rangle\langle mq\rangle\langle q1\rangle \, w_0^2\langle 34\rangle\langle 4q\rangle\langle qm\rangle\langle m3\rangle}$.


$(p_1 + p_2 + w_0 q)^2 = 0 \Rightarrow w_0 = -\frac{p_1 \cdot p_2}{q \cdot (p_1 + p_2)}$ . .
$\int d^4\eta_q \, d^4\eta_m \; \delta^8(\lambda_1\eta_1 + \lambda_2\eta_2 + z_0\lambda_q\eta_q + \lambda_m\eta_m)\,\delta^8(\lambda_3\eta_3 + \lambda_4\eta_4 - z_0\lambda_q\eta_q - \lambda_m\eta_m)$.
$\langle 12\rangle\langle 2m\rangle\langle mq\rangle\langle q1\rangle \; w_0^2\langle 34\rangle\langle 4q\rangle\langle qm\rangle\langle m3\rangle$

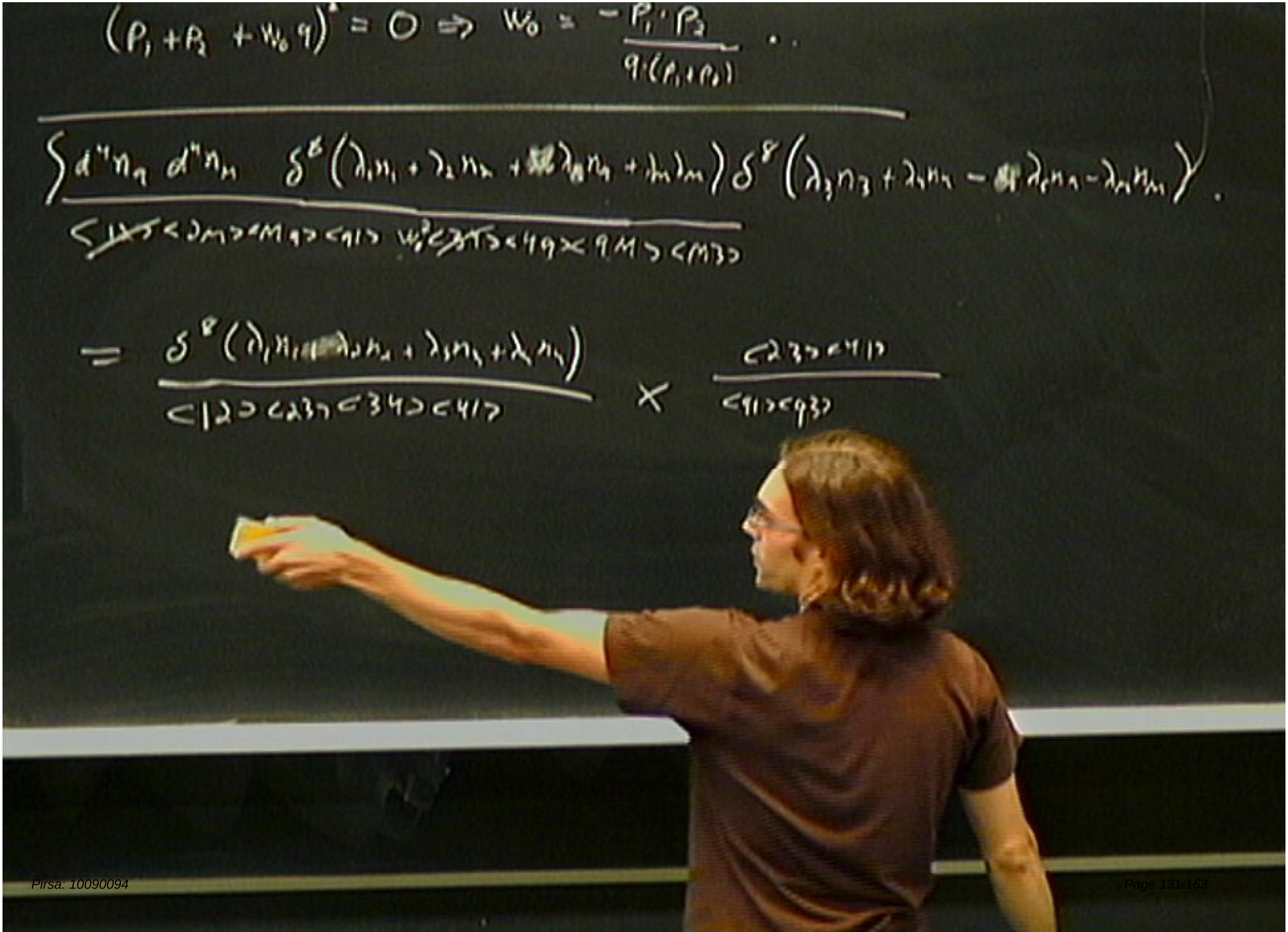
$$(P_1 + P_2 + w_0 q)^2 = 0 \Rightarrow w_0 = \frac{-P_1 \cdot P_2}{q \cdot (P_1 + P_2)} \ldots$$

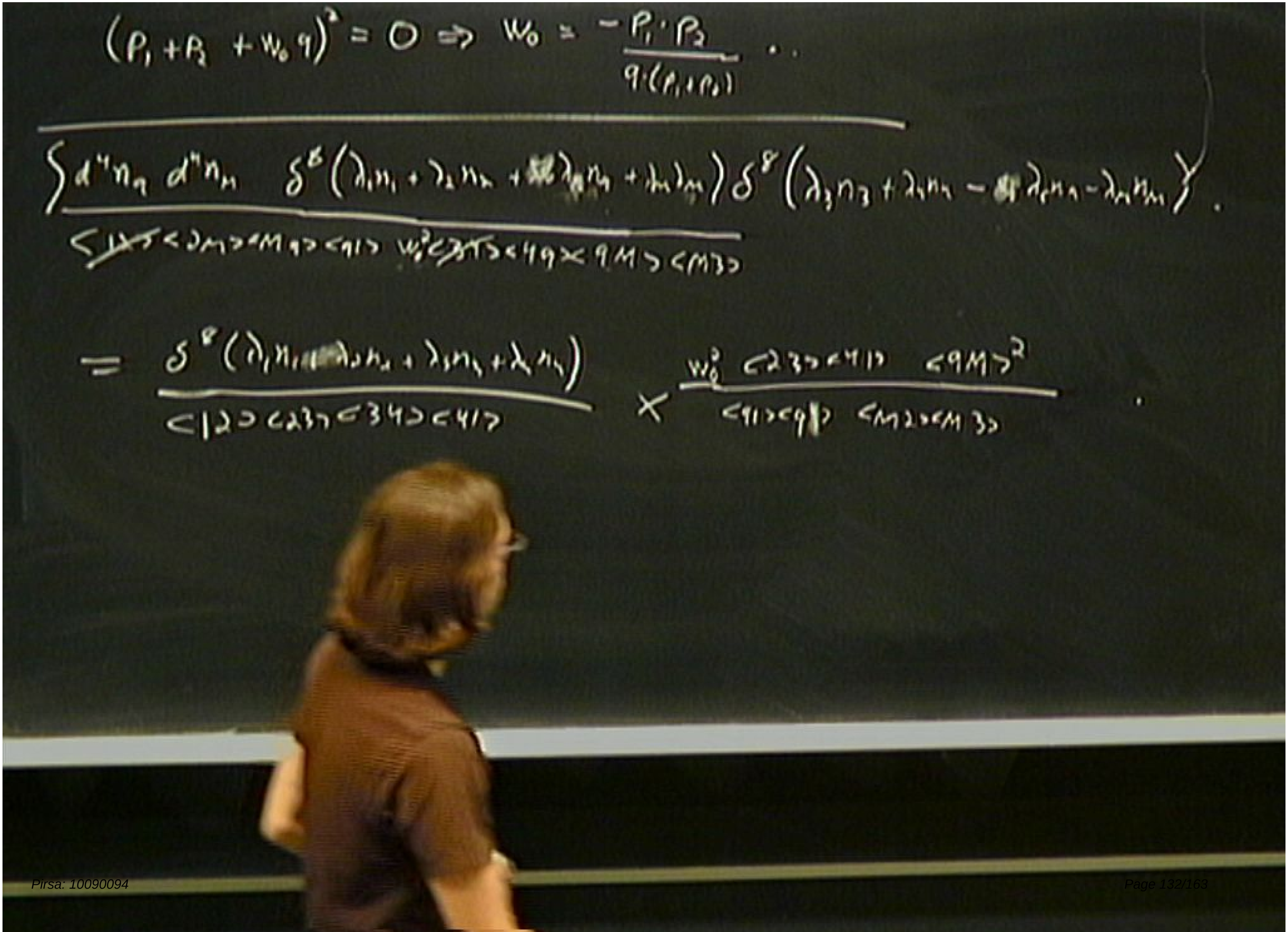
$$\frac{\int d^4\eta_q \, d^4\eta_M \; \delta^8(\lambda_1\eta_1 + \lambda_2\eta_2 + z_0\lambda_q\eta_q + \lambda_M\eta_M)\, \delta^8(\lambda_3\eta_3 + \lambda_4\eta_4 - z_0\lambda_q\eta_q - \lambda_M\eta_M)}{\langle 12\rangle\langle 2M\rangle\langle Mq\rangle\langle q1\rangle \; w_0^2\langle 34\rangle\langle 4q\rangle\langle qM\rangle\langle M3\rangle}$$


q·(p1+p0)
∫ d4ηq d4ηm δ8(λ1η1 + λ2η2 + z0λqηq + λmηm) δ8(λ3η3 + λ4η4 − z0λqηq − λmηm)
<12><2m><mq><q1> w0²<34><4q><qm><m3>
= δ8(λ1η1 + λ2η2 + λ3η3 + λ4η4)



$(P_1 + P_2 + w_0 q)^2 = 0 \Rightarrow w_0 = -\frac{P_1 \cdot P_2}{q \cdot (P_1 + P_2)}$ . .
$\int \frac{d^4\eta_q \, d^4\eta_M \; \delta^8(\lambda_1\eta_1 + \lambda_2\eta_2 + z_0\lambda_q\eta_q + \lambda_M\eta_M)\, \delta^8(\lambda_3\eta_3 + \lambda_4\eta_4 - z_0\lambda_q\eta_q - \lambda_M\eta_M)}{\langle 12\rangle\langle 2M\rangle\langle Mq\rangle\langle q1\rangle \; w_0^2\langle 34\rangle\langle 4q\rangle\langle qM\rangle\langle M3\rangle}$.
$= \frac{\delta^8(\lambda_1\eta_1 + \lambda_2\eta_2 + \lambda_3\eta_3 + \lambda_4\eta_4)}{\langle 12\rangle \; \langle 34\rangle\langle 41\rangle}$


(P1 + P2 + w0 q)^2 = 0 ⇒ w0 = − P1·P2 / q·(P1 + P2)
∫ d^4η_q d^4η_m δ^8(λ1η1 + λ2η2 + λqηq + λmηm) δ^8(λ3η3 + λ4η4 − λqηq − λmηm) / <12><2m><mq><q1> w0^2 <34><4q><qm><m3>
= δ^8(λ1η1 + λ2η2 + λ3η3 + λ4η4) / <12><23><34><41> × <23><41> / <q1><q3>


$(P_1 + P_2 + w_0 q)^2 = 0 \Rightarrow w_0 = -\frac{P_1 \cdot P_2}{q \cdot (P_1 + P_2)}$ . .
$= \frac{\delta^8(\lambda_1\eta_1 + \lambda_2\eta_2 + \lambda_3\eta_3 + \lambda_4\eta_4)}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \times \frac{w_0^2 \langle 23\rangle\langle 41\rangle \langle qm\rangle^2}{\langle q1\rangle\langle q2\rangle \langle m2\rangle\langle m3\rangle}$.

$$(p_1 + p_2 + w_0 q)^2 = 0 \Rightarrow w_0 = -\frac{p_1 \cdot p_2}{q \cdot (p_1 + p_2)} \cdots$$

$$\int d^4\eta_q \, d^4\eta_m \; \frac{\delta^8(\lambda_1\eta_1 + \lambda_2\eta_2 + \lambda_q\eta_q + \lambda_m\eta_m)\,\delta^8(\lambda_3\eta_3 + \lambda_4\eta_4 - \lambda_q\eta_q - \lambda_m\eta_m)}{\langle 12\rangle\langle 2m\rangle\langle mq\rangle\langle q1\rangle \; w_0^2\langle 34\rangle\langle 4q\rangle\langle qm\rangle\langle m3\rangle}$$

$$= \frac{\delta^8(\lambda_1\eta_1 + \lambda_2\eta_2 + \lambda_3\eta_3 + \lambda_4\eta_4)}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \times \frac{w_0^2 \langle 23\rangle\langle 41\rangle \; \langle qm\rangle^2}{\langle q1\rangle\langle q4\rangle \; \langle m2\rangle\langle m3\rangle} \cdots$$

$$(P_1+P_2+w_0 q) = 0 \Rightarrow w_0 = \frac{\ \cdots\ }{q\cdot(P_1+P_2)} \ \ldots$$

$$\int d^4\eta_q\, d^4\eta_m \; \frac{\delta^8(\lambda_1\eta_1+\lambda_2\eta_2+\lambda_q\eta_q+\lambda_m\eta_m)\,\delta^8(\lambda_3\eta_3+\lambda_4\eta_4-\lambda_q\eta_q-\lambda_m\eta_m)}{\langle 2m\rangle\langle mq\rangle\langle q1\rangle\; w_0^2\langle 34\rangle\langle 4q\rangle\langle qm\rangle\langle m3\rangle}.$$

$$= \frac{\delta^8(\lambda_1\eta_1+\lambda_2\eta_2+\lambda_3\eta_3+\lambda_4\eta_4)}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \times \frac{w_0^2\,\langle 23\rangle\langle 41\rangle\;\langle qm\rangle^2}{\langle q1\rangle\langle q2\rangle\;\langle m2\rangle\langle m3\rangle} \times .$$

$$\times$$

$$\langle m = \langle q(y + |1\rangle + |2\rangle)$$
$$= \langle q(P_1+P_2)$$

$$(P_1 + P_2 + w_0 q)^2 = 0 \Rightarrow w_0 = -\frac{P_1 \cdot P_2}{q \cdot (P_1 + P_2)} \cdots$$

$$\int \frac{d^4\eta_q \, d^4\eta_M \; \delta^8(\lambda_1\eta_1 + \lambda_2\eta_2 + \lambda_q\eta_q + \lambda_M\eta_M)\, \delta^8(\lambda_3\eta_3 + \lambda_4\eta_4 - \lambda_q\eta_q - \lambda_M\eta_M)}{\langle 2M\rangle\langle Mq\rangle\langle q1\rangle \; w_0^2 \langle 34\rangle\langle 4q\rangle\langle qM\rangle\langle M3\rangle}.$$

$$= \frac{\delta^8(\lambda_1\eta_1 + \lambda_2\eta_2 + \lambda_3\eta_3 + \lambda_4\eta_4)}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \times \frac{w_0^2 \langle 23\rangle\langle 41\rangle \; \langle qM\rangle^2}{\langle q1\rangle\langle q4\rangle \; \langle M2\rangle\langle M3\rangle} \times \frac{1}{(q + P_1 + P_2)^2}$$

$$\times$$

$$\langle M| = \langle q|(P_1 + P_2)$$
$$= \langle q|(P_1 + P_2)$$

$$(P_1+P_2+w_0 q)^2 = 0 \Rightarrow w_0 = -\frac{P_1\cdot P_2}{q\cdot(P_1+P_2)} \cdots$$

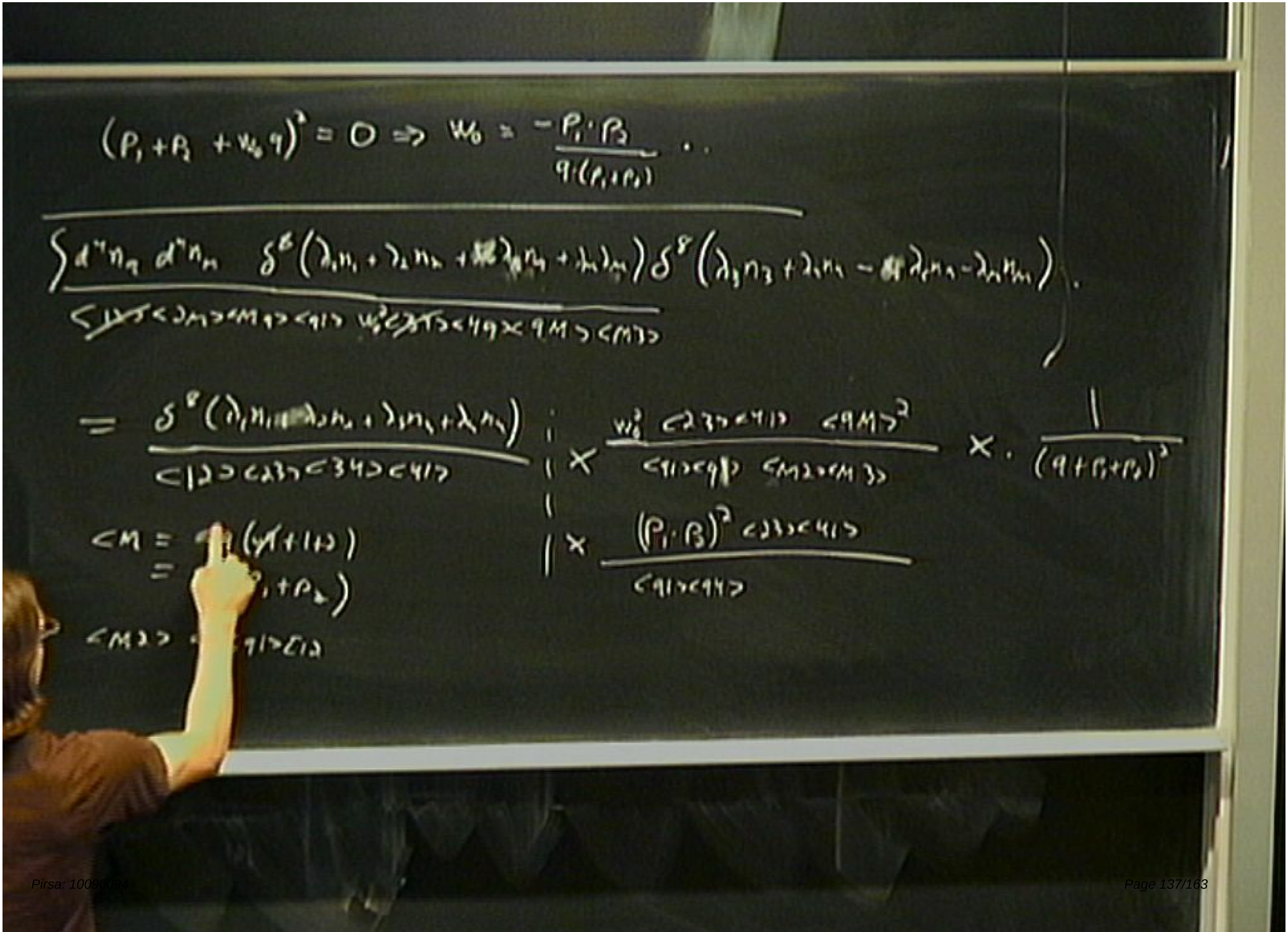
$$\int d^4\eta_q\, d^4\eta_m \frac{\delta^8(\lambda_1\eta_1+\lambda_2\eta_2+\lambda_q\eta_q+\lambda_m\eta_m)\,\delta^8(\lambda_3\eta_3+\lambda_4\eta_4-\lambda_q\eta_q-\lambda_m\eta_m)}{\langle 12\rangle\langle 2m\rangle\langle mq\rangle\langle q1\rangle\, w_0^2\langle 34\rangle\langle 4q\rangle\langle qm\rangle\langle m3\rangle}$$

$$= \frac{\delta^8(\lambda_1\eta_1+\lambda_2\eta_2+\lambda_3\eta_3+\lambda_4\eta_4)}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \times \frac{w_0^2\langle 23\rangle\langle 41\rangle\,\langle qm\rangle^2}{\langle q1\rangle\langle q2\rangle\,\langle m2\rangle\langle m3\rangle} \times \frac{1}{(q+P_1+P_2)^2}$$

$$\times \frac{(P_1\cdot P_2)^2\langle 23\rangle\langle 41\rangle}{\langle q1\rangle\langle q4\rangle}$$

$$\langle m = \langle q(q+1+2)$$
$$= \langle q(P_1+P_2)$$

$$\Rightarrow \langle m2\rangle = $$


(P1 + P2 + w0 q)^2 = 0 ⇒ w0 = − P1·P2 / q·(P1+P2) . .
∫ d^4η_q d^4η_m δ^8(λ1η1 + λ2η2 + λqηq + λmηm) δ^8(λ3η3 + λ4η4 − λqηq − λmηm)
⟨12⟩⟨2m⟩⟨mq⟩⟨q1⟩ w0²⟨34⟩⟨4q⟩⟨qm⟩⟨m3⟩
= δ^8(λ1η1 + λ2η2 + λ3η3 + λ4η4) / ⟨12⟩⟨23⟩⟨34⟩⟨41⟩
× w0² ⟨23⟩⟨41⟩ ⟨qm⟩² / ⟨q1⟩⟨q4⟩ ⟨m2⟩⟨m3⟩
× 1 / (q+P1+P2)²
× (P1·P2)² ⟨23⟩⟨41⟩ / ⟨q1⟩⟨q4⟩
⟨m = (+|2⟩)
= +P2)
⟨m2⟩ q1⟩[12

$$(p_1+p_2+w_0 q)^2 = 0 \Rightarrow w_0 = -\frac{p_1\cdot p_2}{q\cdot(p_1+p_2)} \;\ldots$$

$$\int \frac{d^4\eta_q\, d^4\eta_m\;\delta^8(\lambda_1\eta_1+\lambda_2\eta_2+\lambda_q\eta_q+\lambda_m\eta_m)\,\delta^8(\lambda_3\eta_3+\lambda_4\eta_4-\lambda_q\eta_q-\lambda_m\eta_m)}{\langle 12\rangle\langle 2m\rangle\langle mq\rangle\langle q1\rangle\; w_0^2\langle 34\rangle\langle 4q\rangle\langle qm\rangle\langle m3\rangle}\,.$$

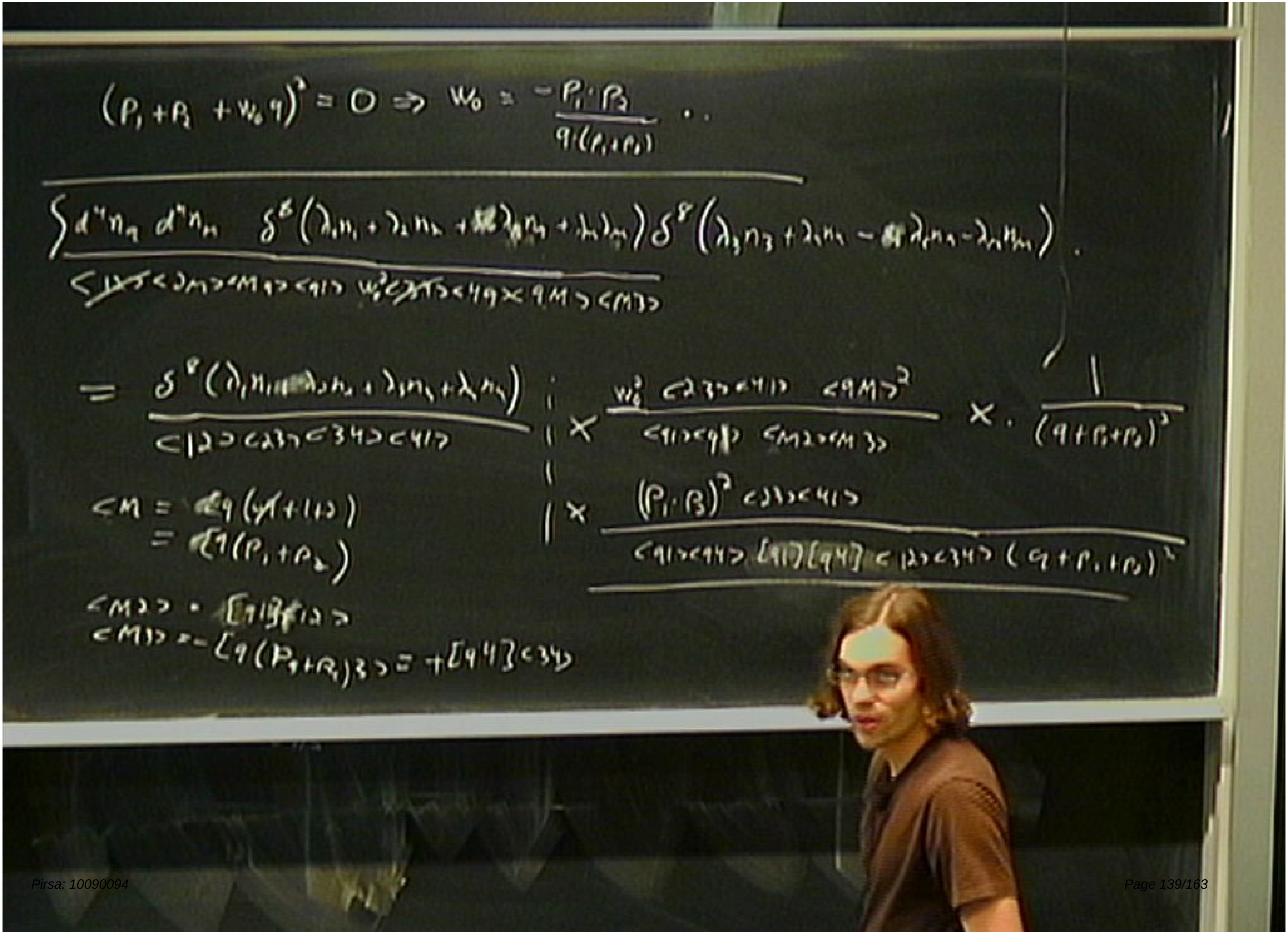
$$= \frac{\delta^8(\lambda_1\eta_1+\lambda_2\eta_2+\lambda_3\eta_3+\lambda_4\eta_4)}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \times \frac{w_0^2\langle 23\rangle\langle 41\rangle\;\langle qm\rangle^2}{\langle q1\rangle\langle q4\rangle\;\langle m2\rangle\langle m3\rangle} \times \frac{1}{(q+p_1+p_2)^2}$$

$$\times \frac{(p_1\cdot p_2)^2\langle 23\rangle\langle 41\rangle}{\langle q1\rangle\langle q4\rangle}$$

$$\langle m| = [q|(p_1+p_2) = [q|(p_1+p_2)$$

$$\langle m2\rangle = [q1]\langle 12\rangle$$

$$\langle m3\rangle = -[q(p_1+p_2)3\rangle = +[q4]\langle 34\rangle$$



$$(P_1+P_2+w_0 q)^2 = 0 \Rightarrow w_0 = -\frac{P_1\cdot P_2}{q\cdot(P_1+P_2)} \ldots$$

$$\int \frac{d^4\eta_q\, d^4\eta_m\ \delta^8(\lambda_1\eta_1+\lambda_2\eta_2+\lambda_q\eta_q+\lambda_m\eta_m)\,\delta^8(\lambda_3\eta_3+\lambda_4\eta_4-\lambda_q\eta_q-\lambda_m\eta_m)}{\langle 12\rangle\langle 2m\rangle\langle mq\rangle\langle q1\rangle\ w_0^2\langle 3m\rangle\langle 4q\rangle\langle qm\rangle\langle m3\rangle}.$$

$$= \frac{\delta^8(\lambda_1\eta_1+\lambda_2\eta_2+\lambda_3\eta_3+\lambda_4\eta_4)}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \times \frac{w_0^2\langle 23\rangle\langle 41\rangle\ \langle qM\rangle^2}{\langle q1\rangle\langle q4\rangle\ \langle M2\rangle\langle M3\rangle} \times \cdot \frac{1}{(q+P_1+P_2)^2}$$

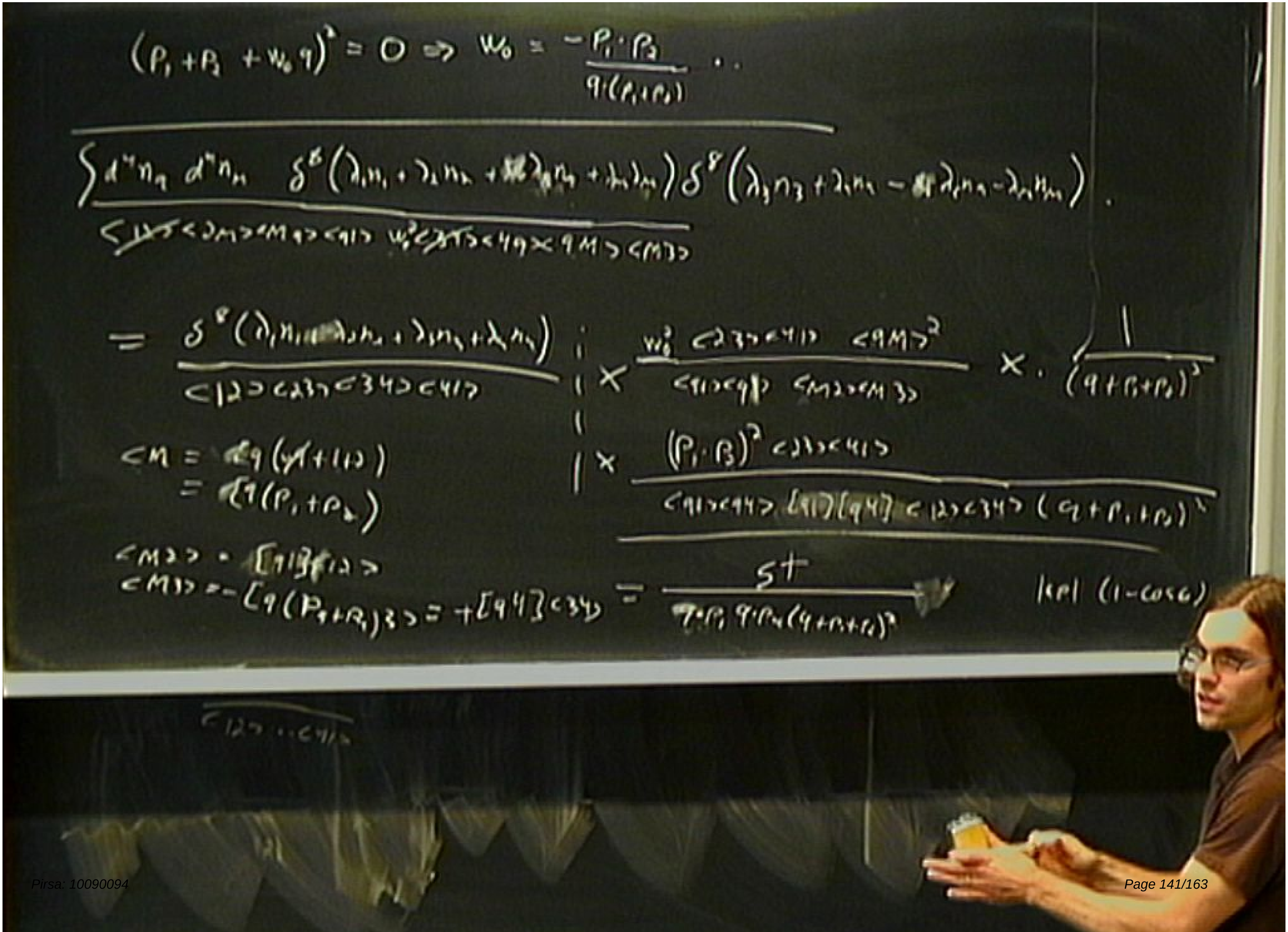
$$\times \frac{(P_1\cdot P_2)^2\langle 23\rangle\langle 41\rangle}{\langle q1\rangle\langle q4\rangle\,[q1][q4]\,\langle 12\rangle\langle 34\rangle\,(q+P_1+P_2)^2}$$

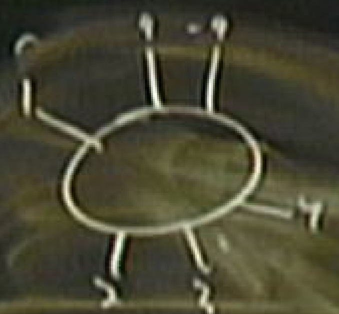
$$\langle M| = \langle q(q+1+2)) = [q(P_1+P_2))$$

$$\langle M2\rangle = [q1]\langle 12\rangle$$

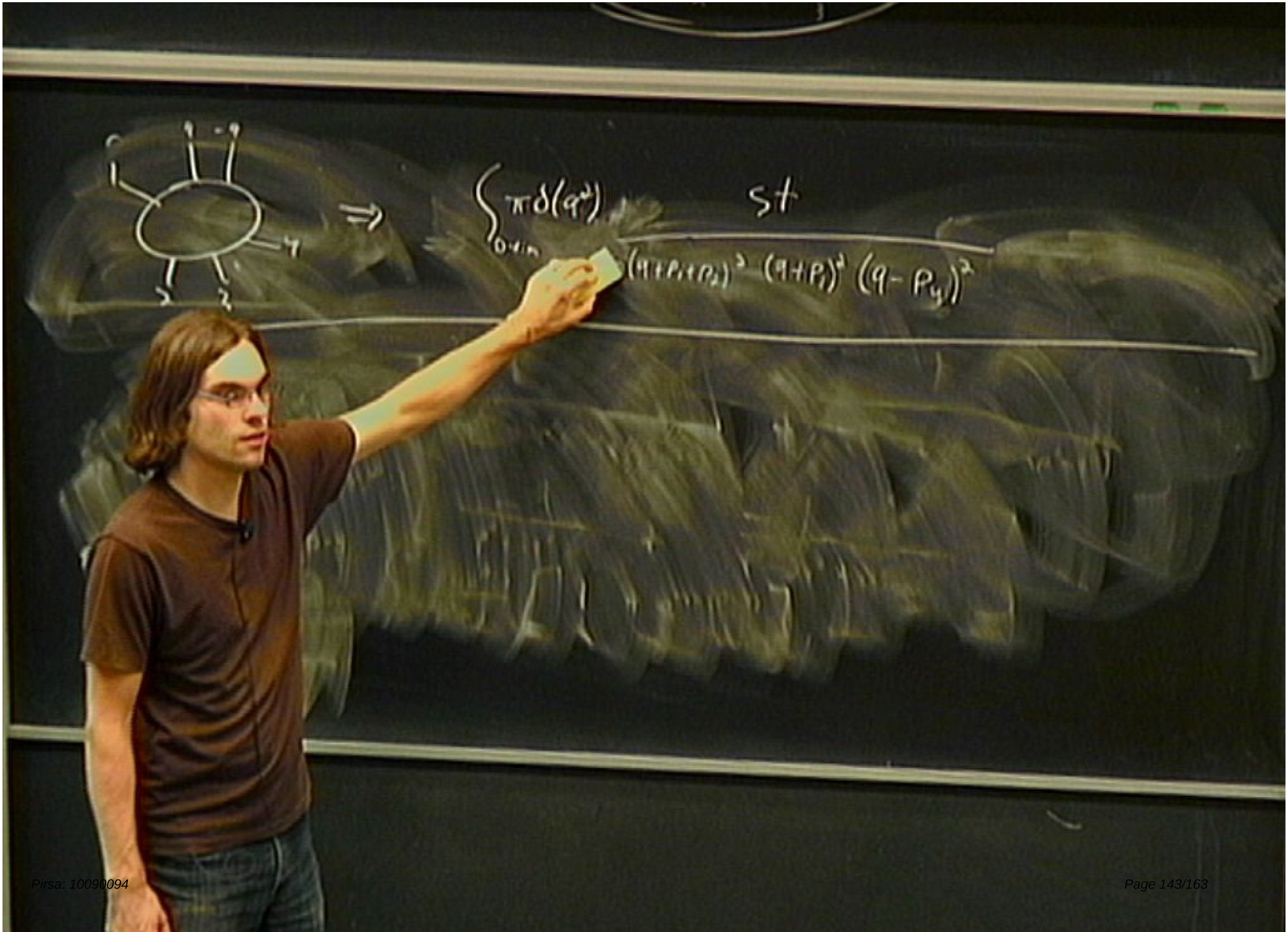
$$\langle M3\rangle = -[q(P_1+P_2)|3\rangle = +[q4]\langle 34\rangle = \frac{st}{q\cdot P_1\, q\cdot P_4\,(q+P_1+P_2)^2}$$

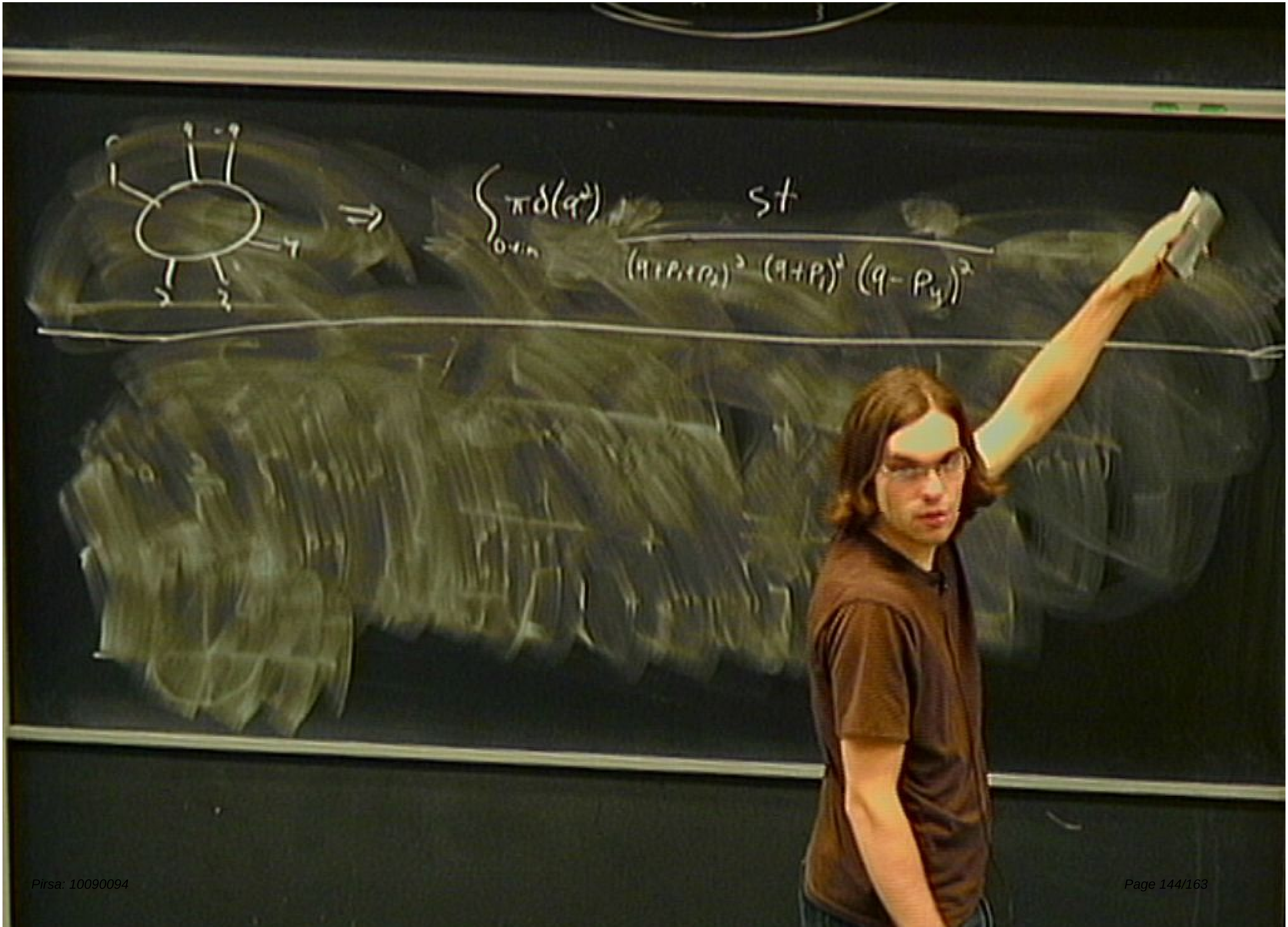
$\langle 12\rangle \ldots \langle 41\rangle$



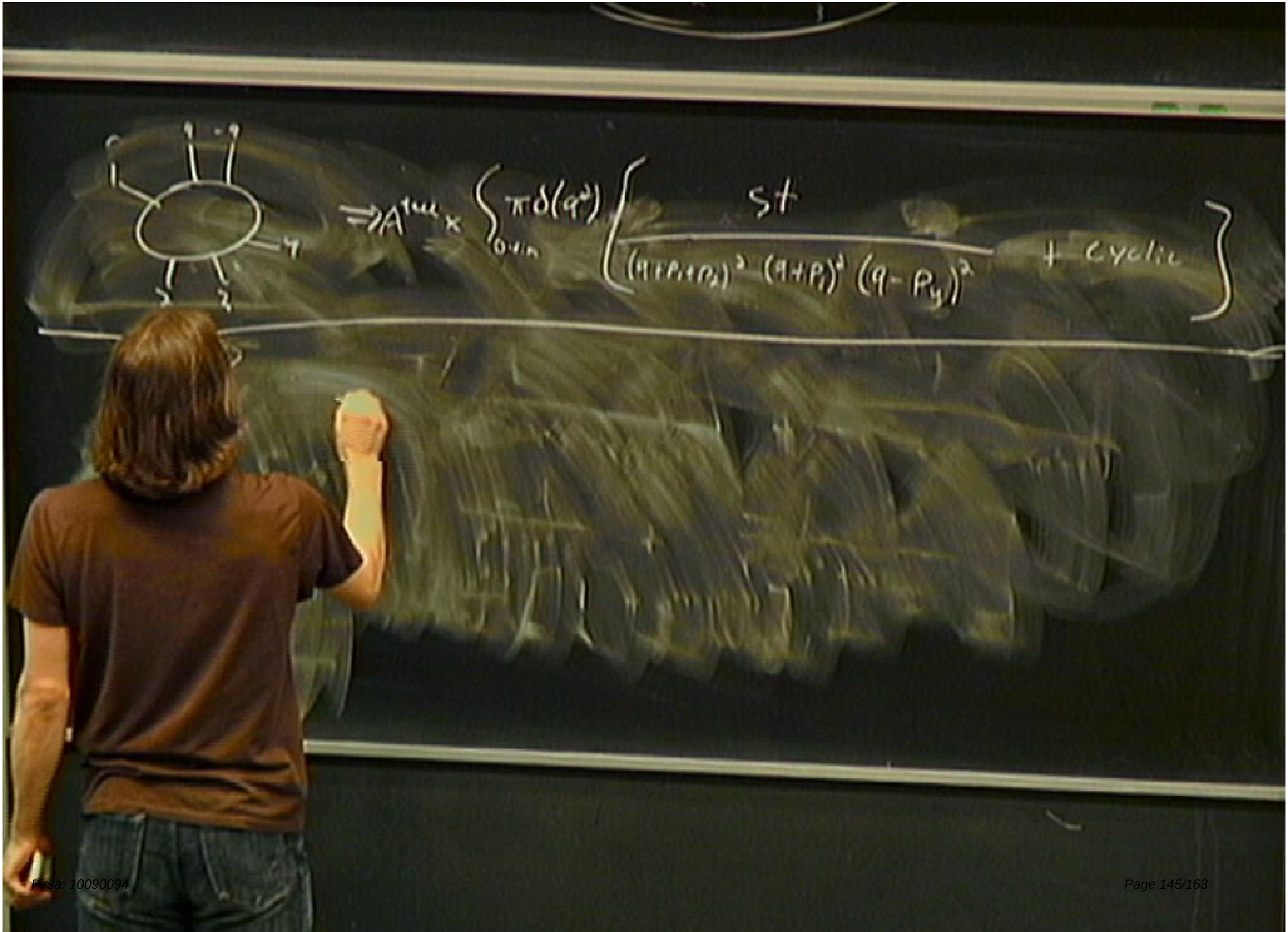


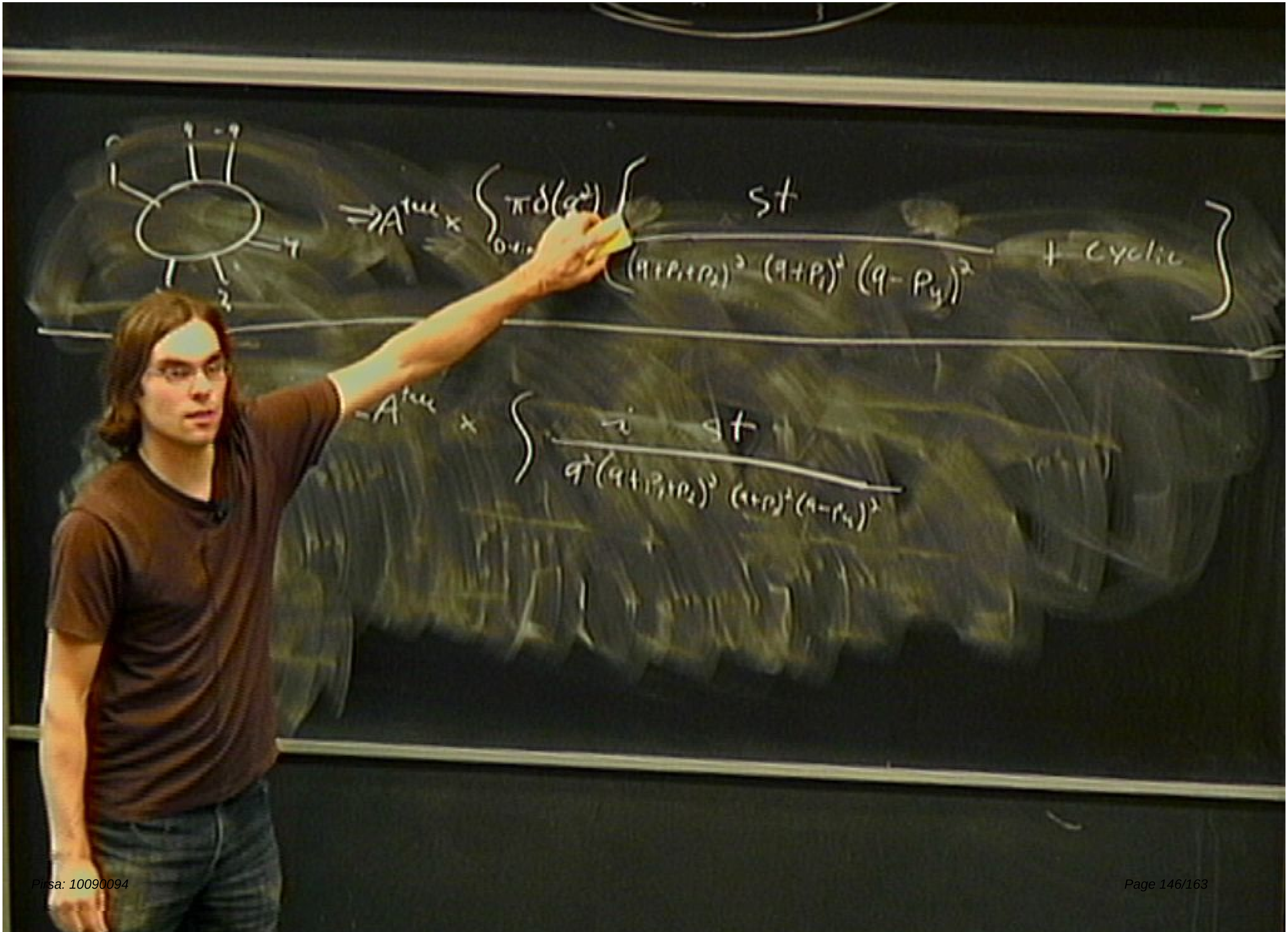
$$\Rightarrow \int \pi \delta(q^2) \frac{st}{(q+p_1+p_2)^2 (q+p_1)^2 (q-p_4)^2}$$

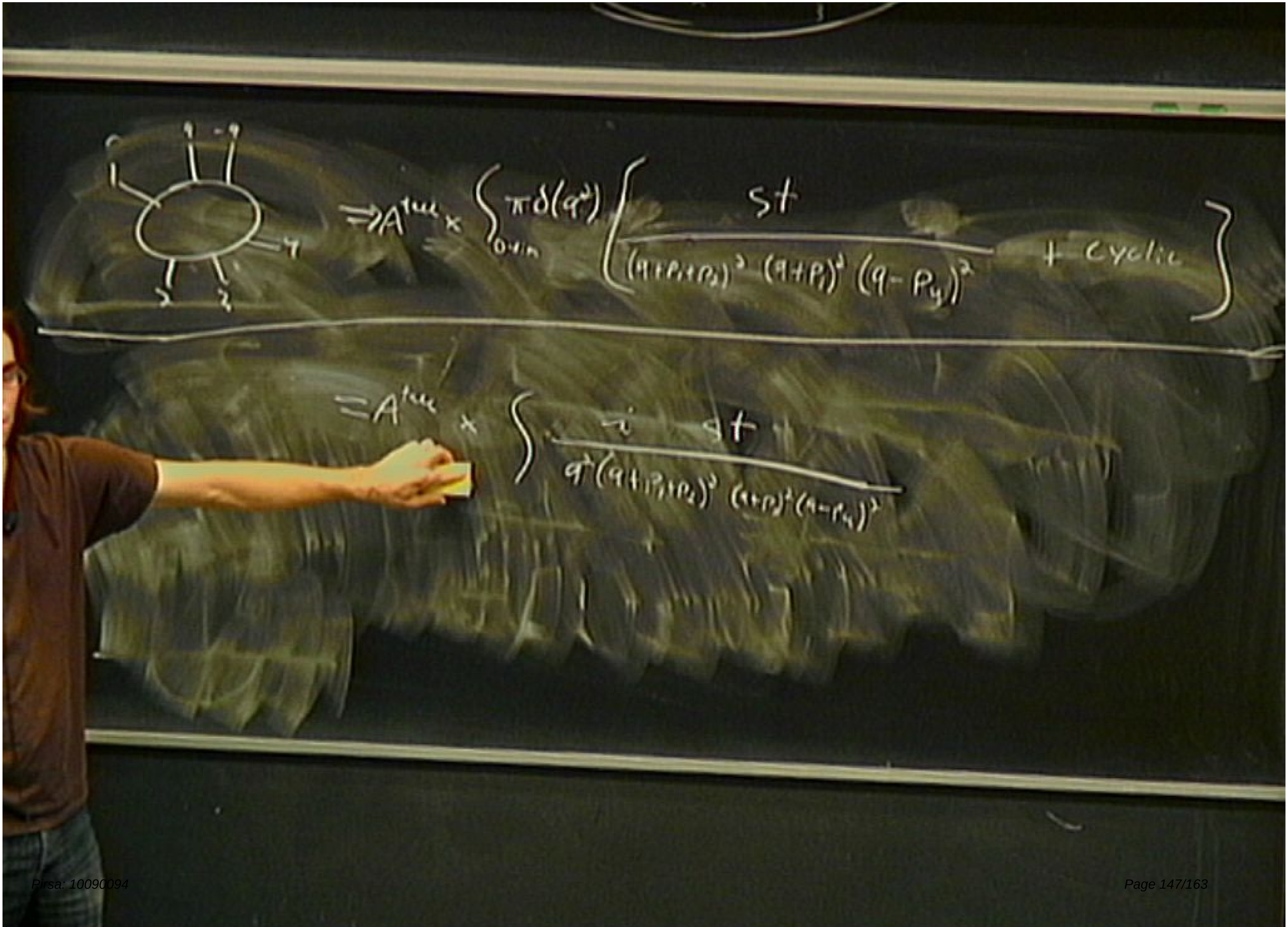


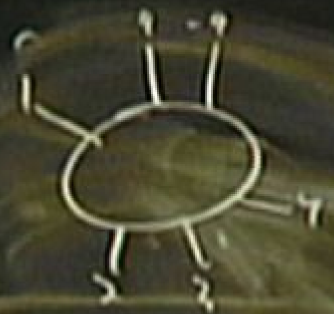


πδ(q²)
st
(q+P₁+P₂)² (q+P₁)² (q−P₄)²


+ cyclic


⇒ A^tree × { πδ(q²) ∫ st / (q+p1+p2)² (q+p1)² (q−p4)² + cyclic }
A^tree × ∫ i st / q²(q+p1+p2)² (q+p1)² (q−p4)²


⇒ A^tree × ∫ πδ(q²) { st / ((q+p1+p2)² (q+p1)² (q−p4)²) + cyclic }
= A^tree × ∫ i st / (q² (q+p1+p2)² (q+p1)² (q−p4)²)

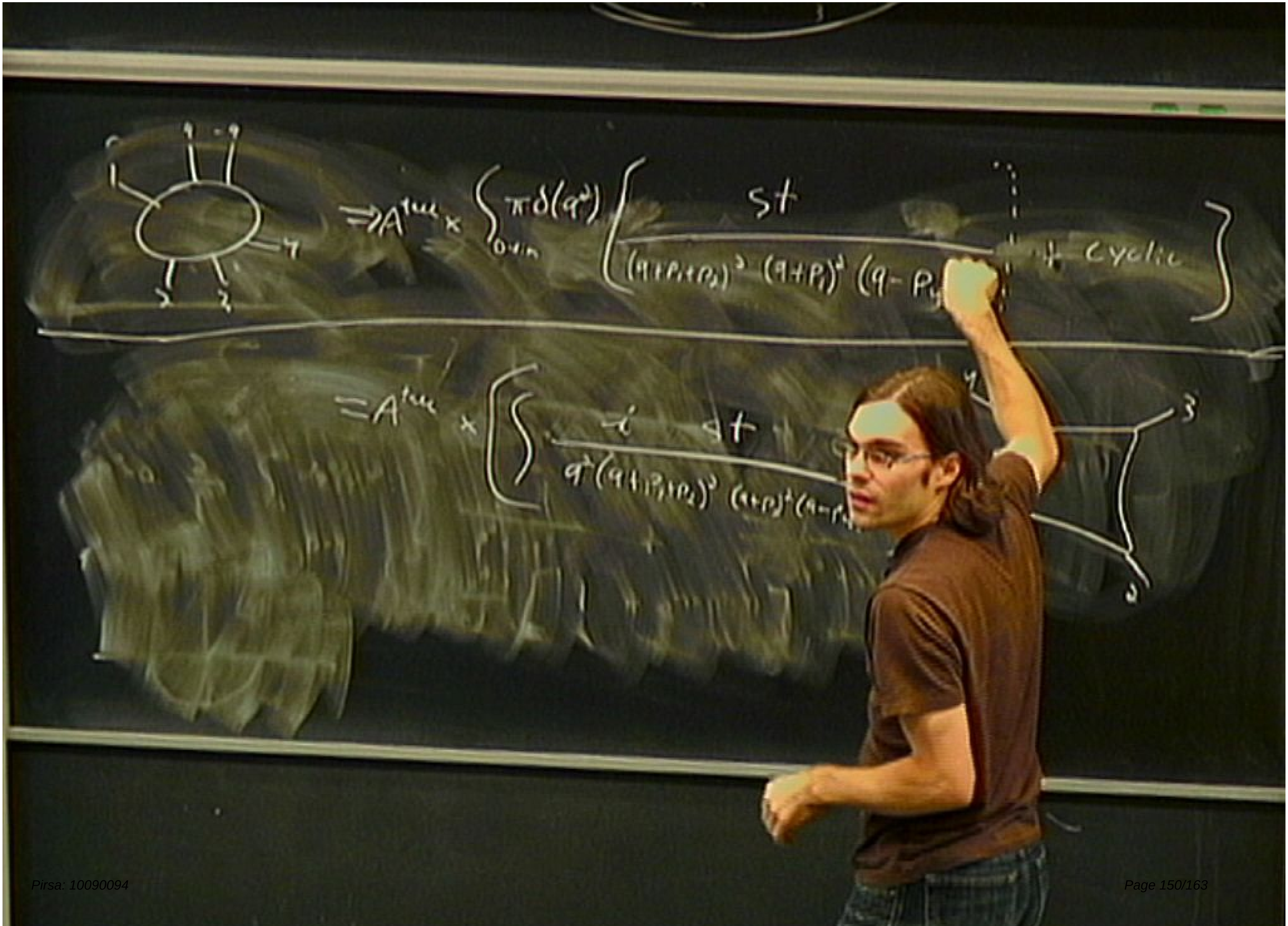


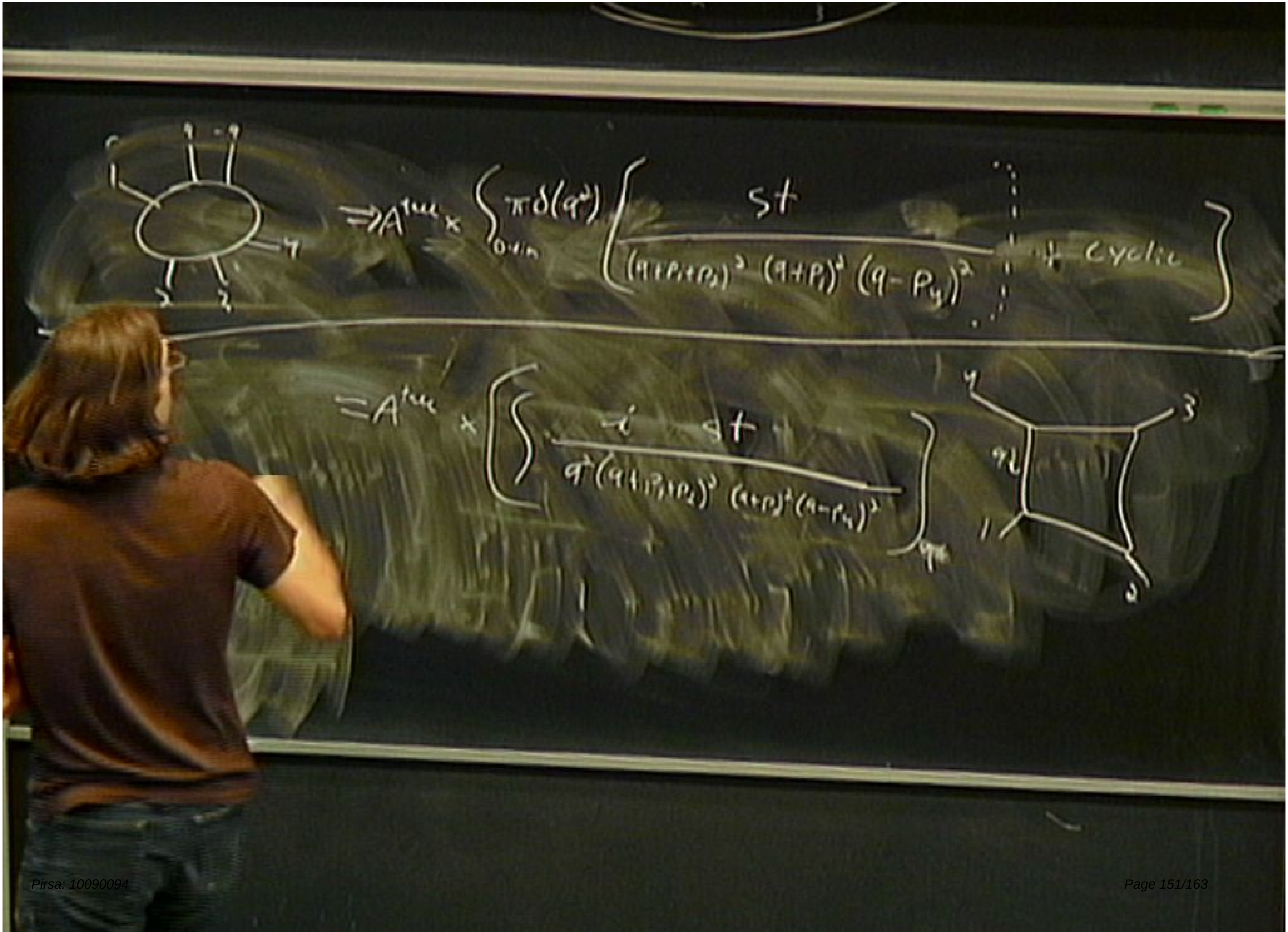
$$\Rightarrow A^{\text{tree}} \times \int_{D\text{-dim}} \pi\delta(q^2) \left[\frac{st}{(q+p_1+p_2)^2\,(q+p_1)^2\,(q-p_4)^2} + \text{cyclic} \right]$$

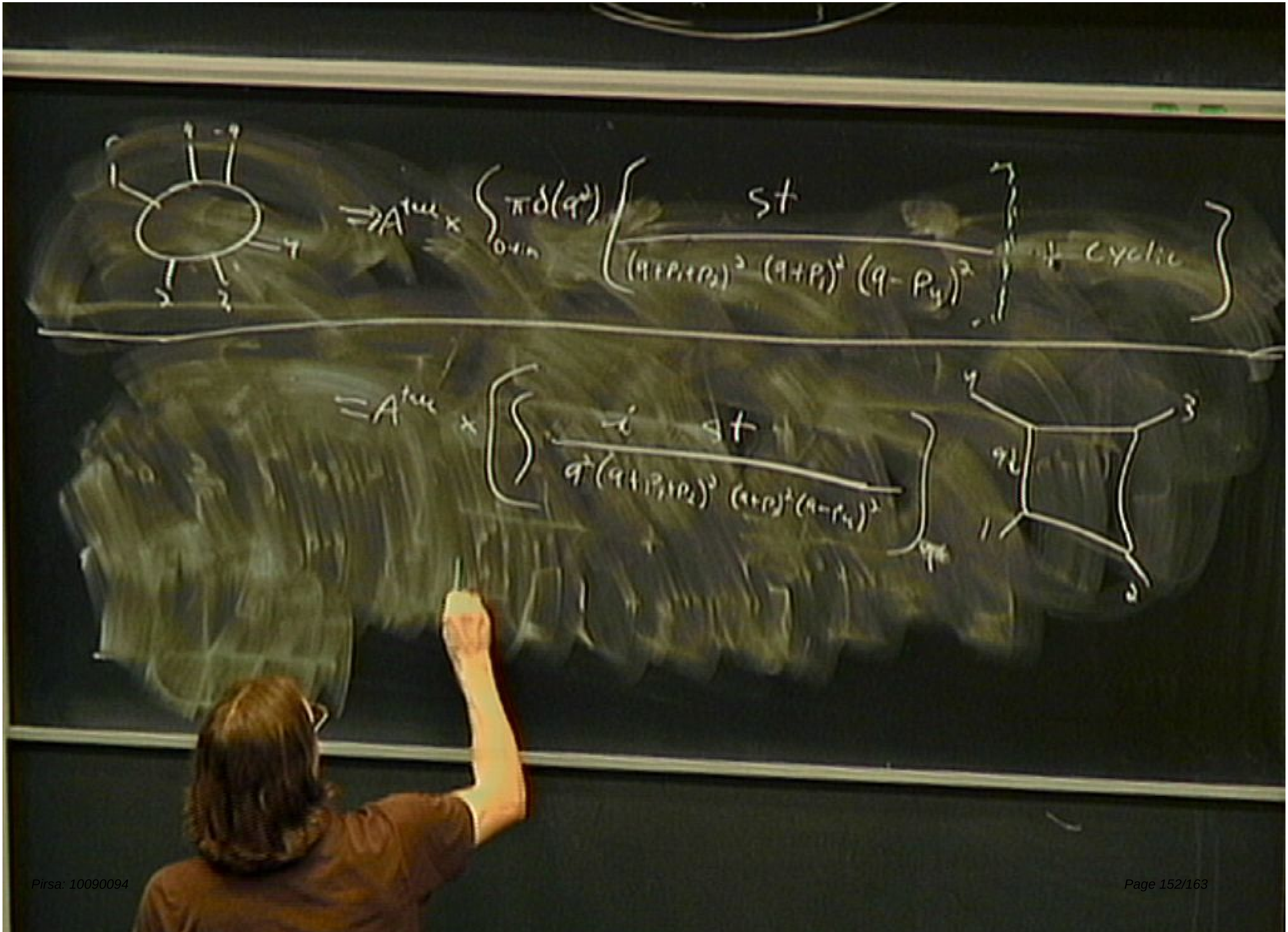
$$= A^{\text{tree}} \times \int \frac{i\,st}{q^2(q+p_1+p_2)^2\,(q+p_1)^2(q-p_4)^2}$$

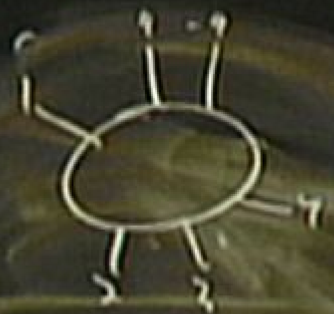
$$\Rightarrow A^{tree} \times \int_{D\text{-dim}} \pi\delta(q^2) \left[\frac{st}{(q+p_1+p_2)^2\,(q+p_1)^2\,(q-p_4)^2} + \text{cyclic} \right]$$

$$= A^{tree} \times \left[\int \frac{i\;st}{q^2(q+p_1+p_2)^2\,(q+p_1)^2(q-p_4)^2} \right]_{q^{2*}}$$


+ cyclic

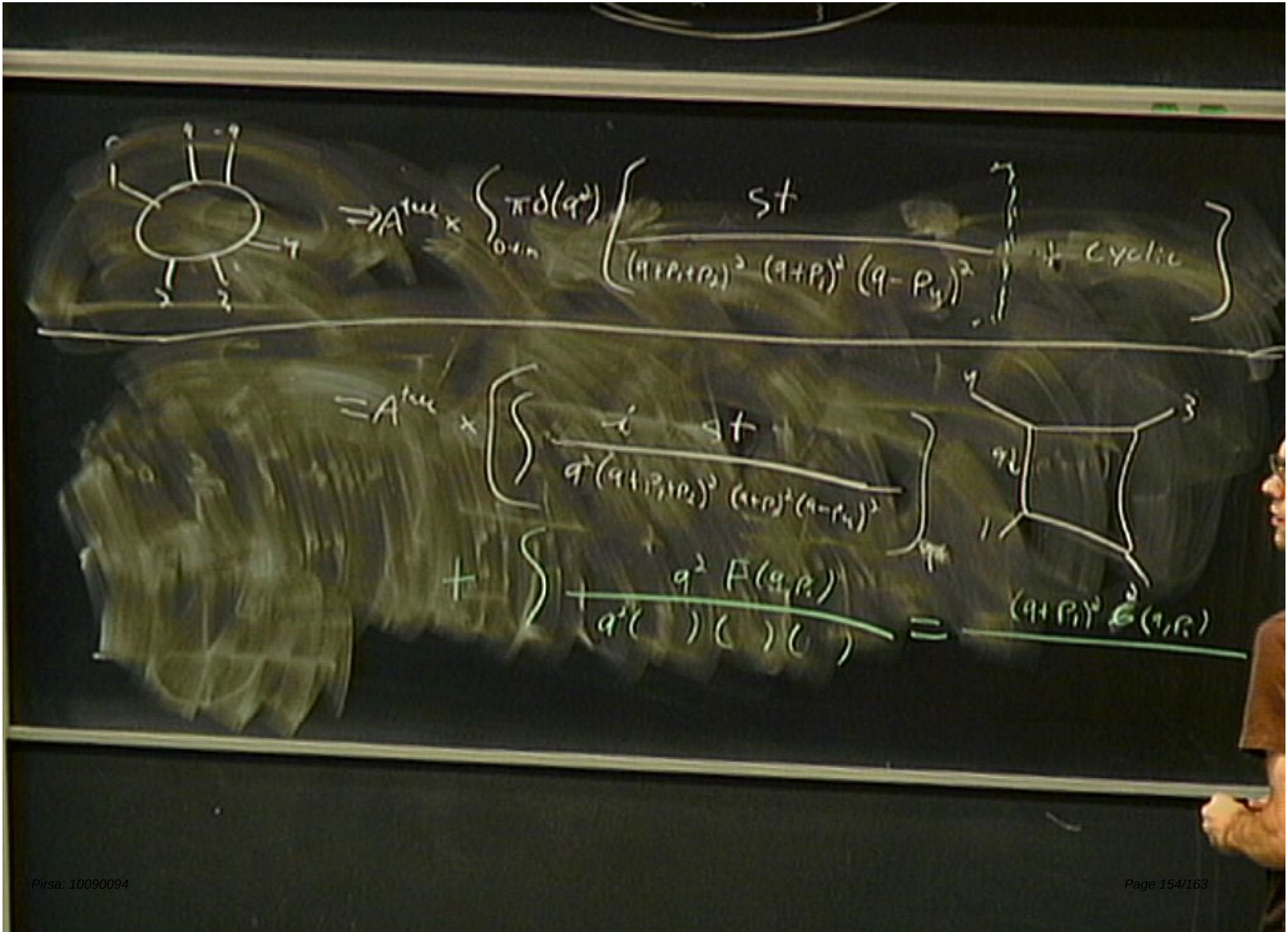

st
+ cyclic
st

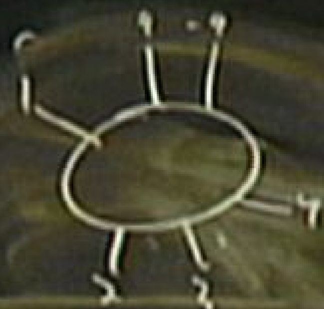

⇒ A^tree × ∫_{D=4} π δ(q²) [st / (q+p1+p2)² (q+p1)² (q−p4)²] + cyclic }
= A^tree × [∫ i st / q²(q+p1+p2)² (q+p1)²(q−p4)²]



$$\Rightarrow A^{\text{tree}} \times \left\{ \int_{D=4} \pi\delta(q^2) \left[\frac{st}{(q+P_1+P_2)^2\,(q+P_1)^2\,(q-P_4)^2} \right] + \text{cyclic} \right\}$$

$$= A^{\text{tree}} \times \left[\int \frac{i\; st}{q^2 (q+P_1+P_2)^2 (q+P_1)^2 (q-P_4)^2} \right]_{q^2} + \int \frac{q^2\, F(q,P_i)}{q^2(\quad)(\quad)(\quad)}$$

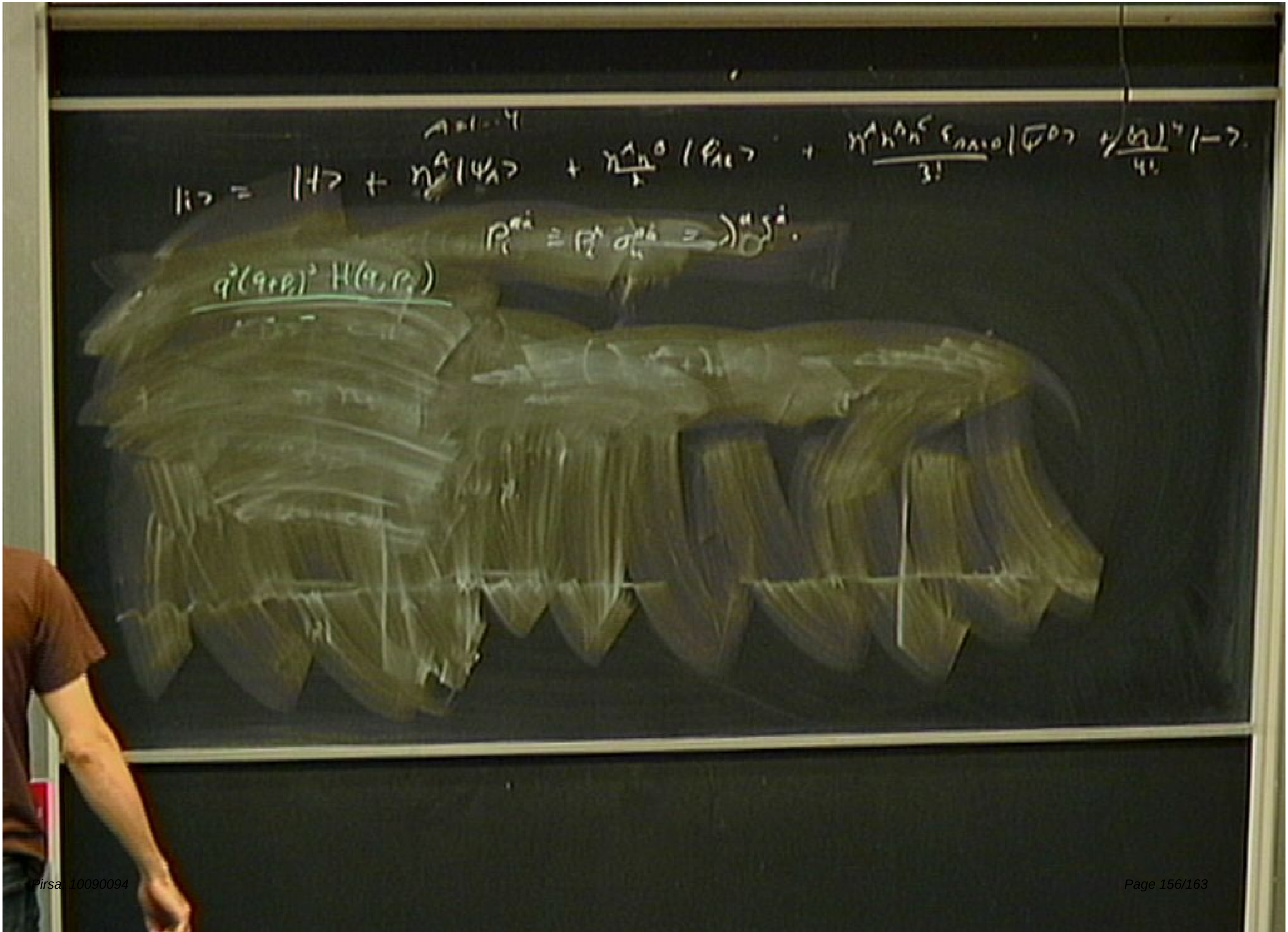

$\Rightarrow A^{tree} \times \int_{4\text{-}dim} \pi\delta(q^2)$
st
$(q+p_1+p_2)^2 \, (q+p_1)^2 \, (q-p_4)^2$
+ cyclic
$= A^{tree} \times$
st
$q^2 (q+p_1+p_2)^2 (q+p_1)^2 (q-p_4)^2$
$q^2 F(q,p_i)$
$(q+p_1)^2 G(q,p_i)$

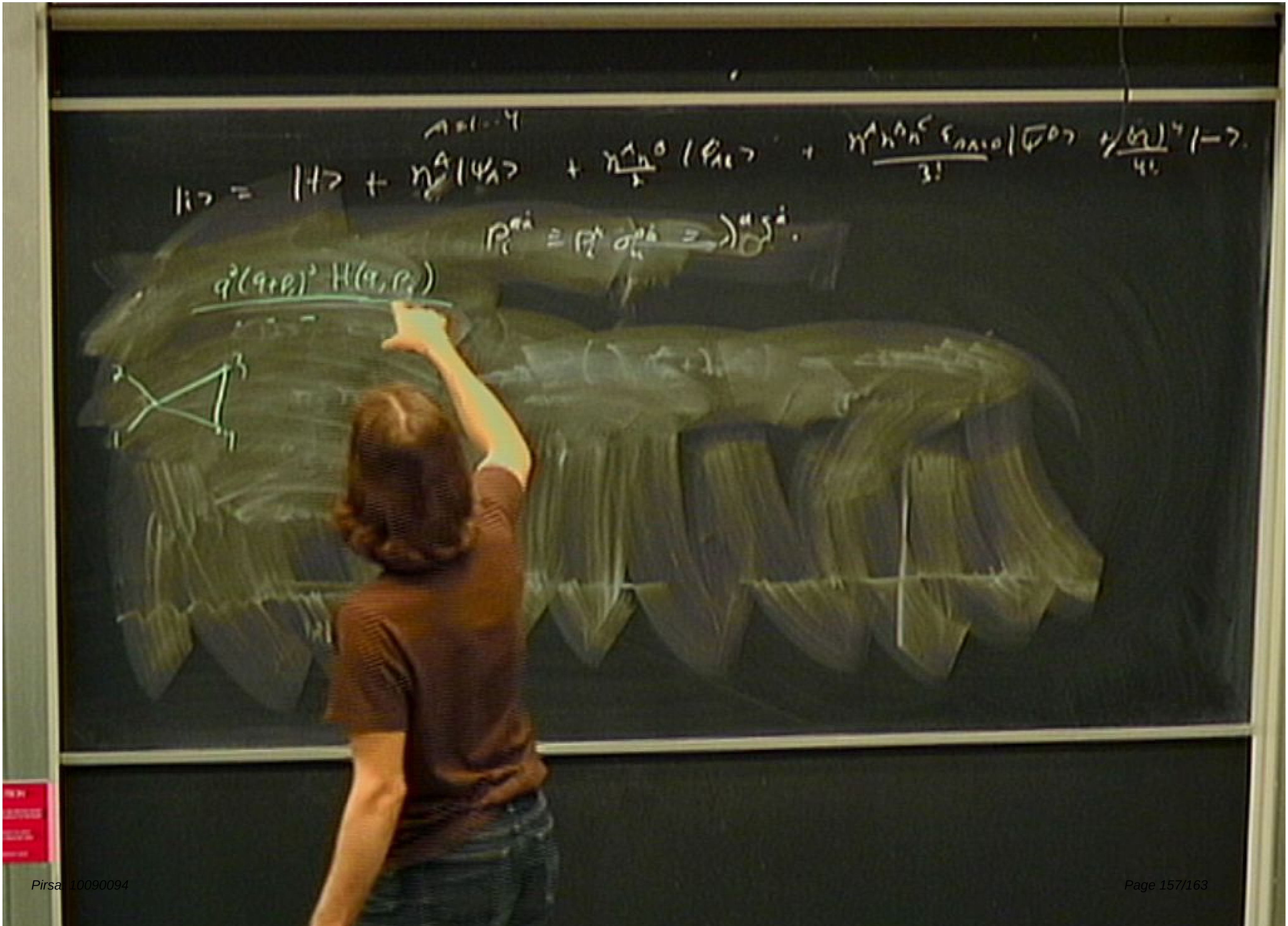


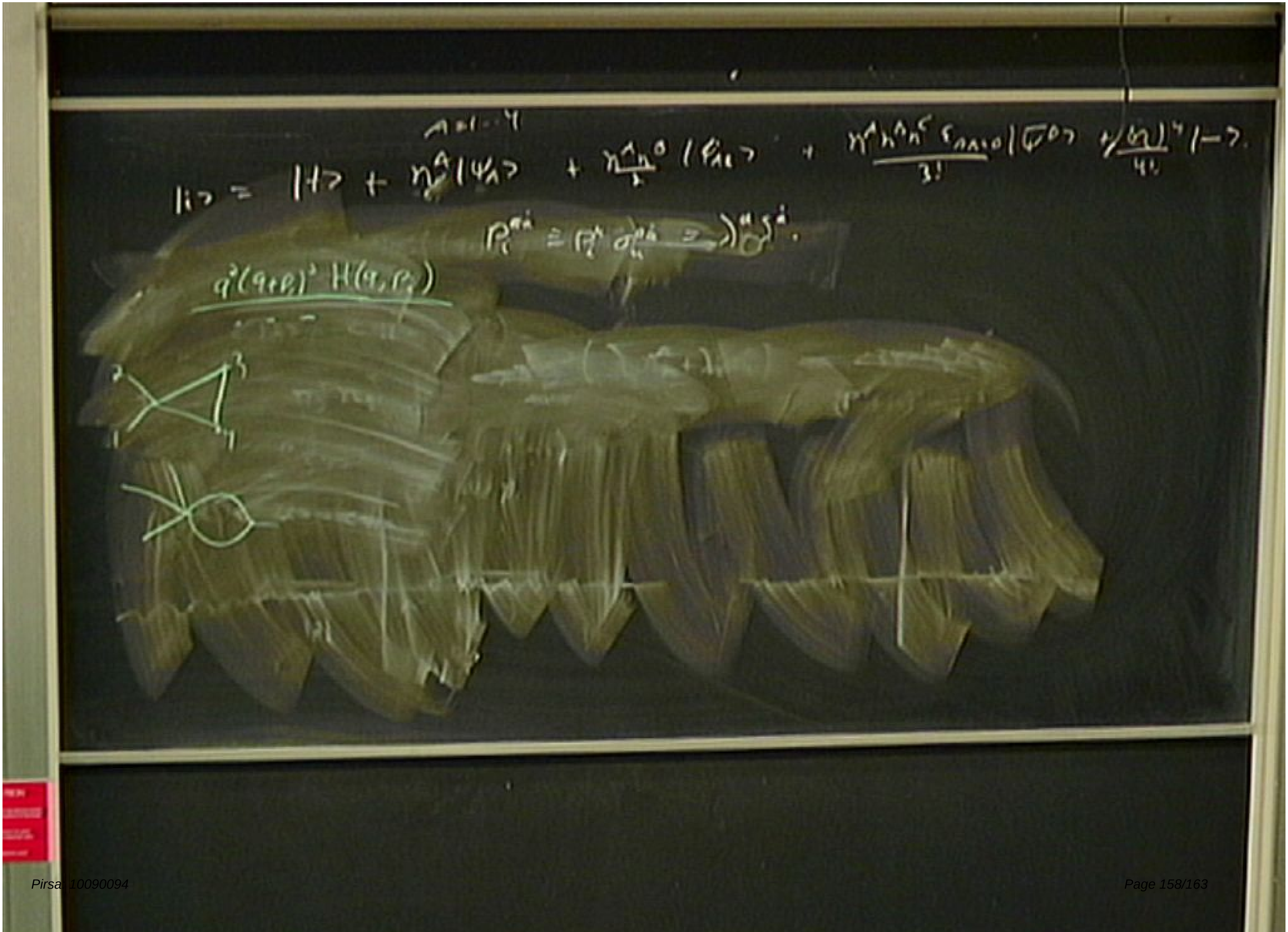
$$\Rightarrow A^{tree} \times \left\{ \int_{D=4} \pi\,\delta(q^2) \left[\frac{st}{(q+p_1+p_2)^2\,(q+p_1)^2\,(q-p_4)^2} \right] + \text{cyclic} \right\}$$

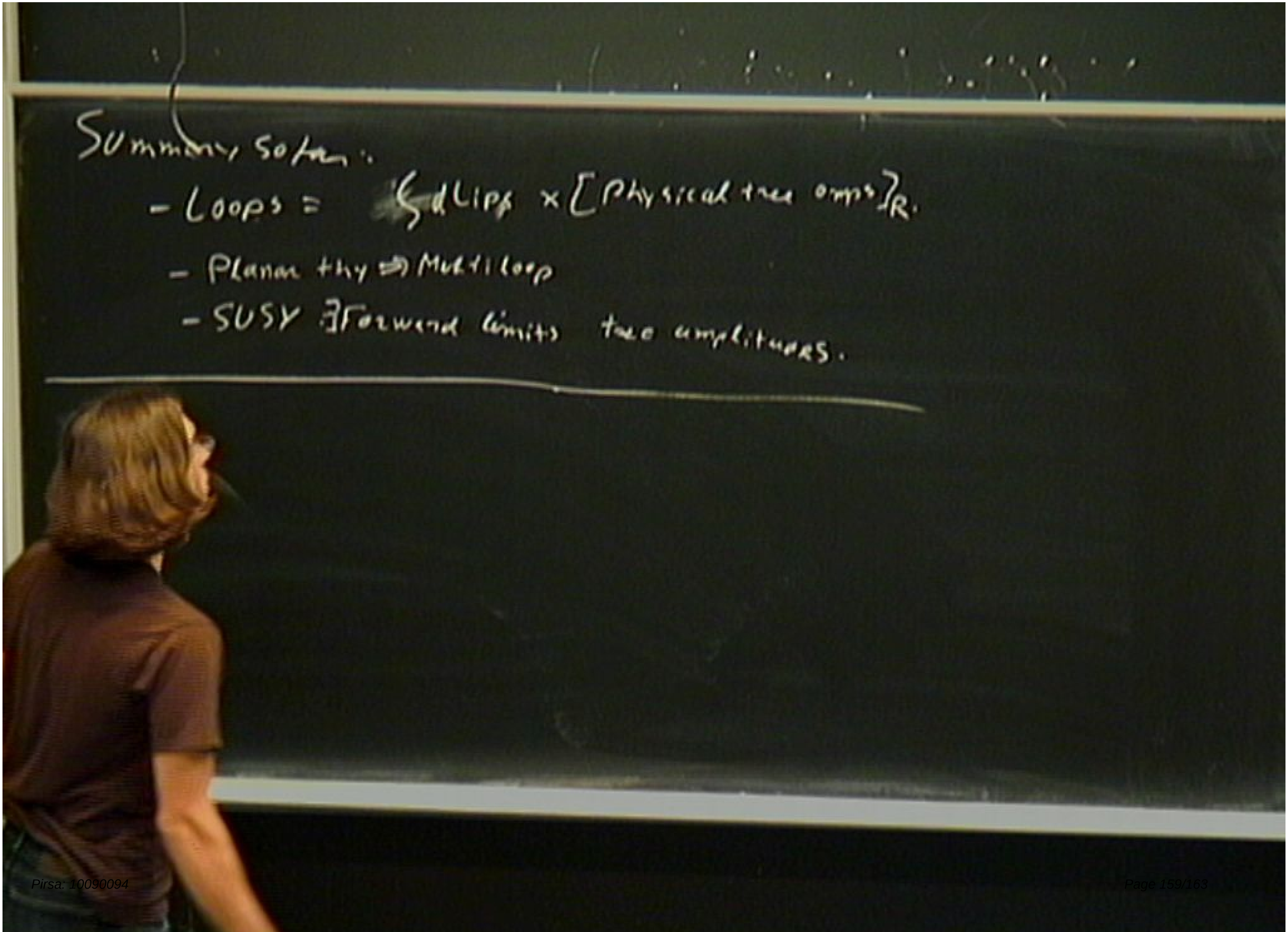
$$= A^{tree} \times \left\{ \int \left[\frac{i\ st}{q^2 (q+p_1+p_2)^2\,(q+p_1)^2 (q-p_4)^2} \right]_{q^4} \right.$$

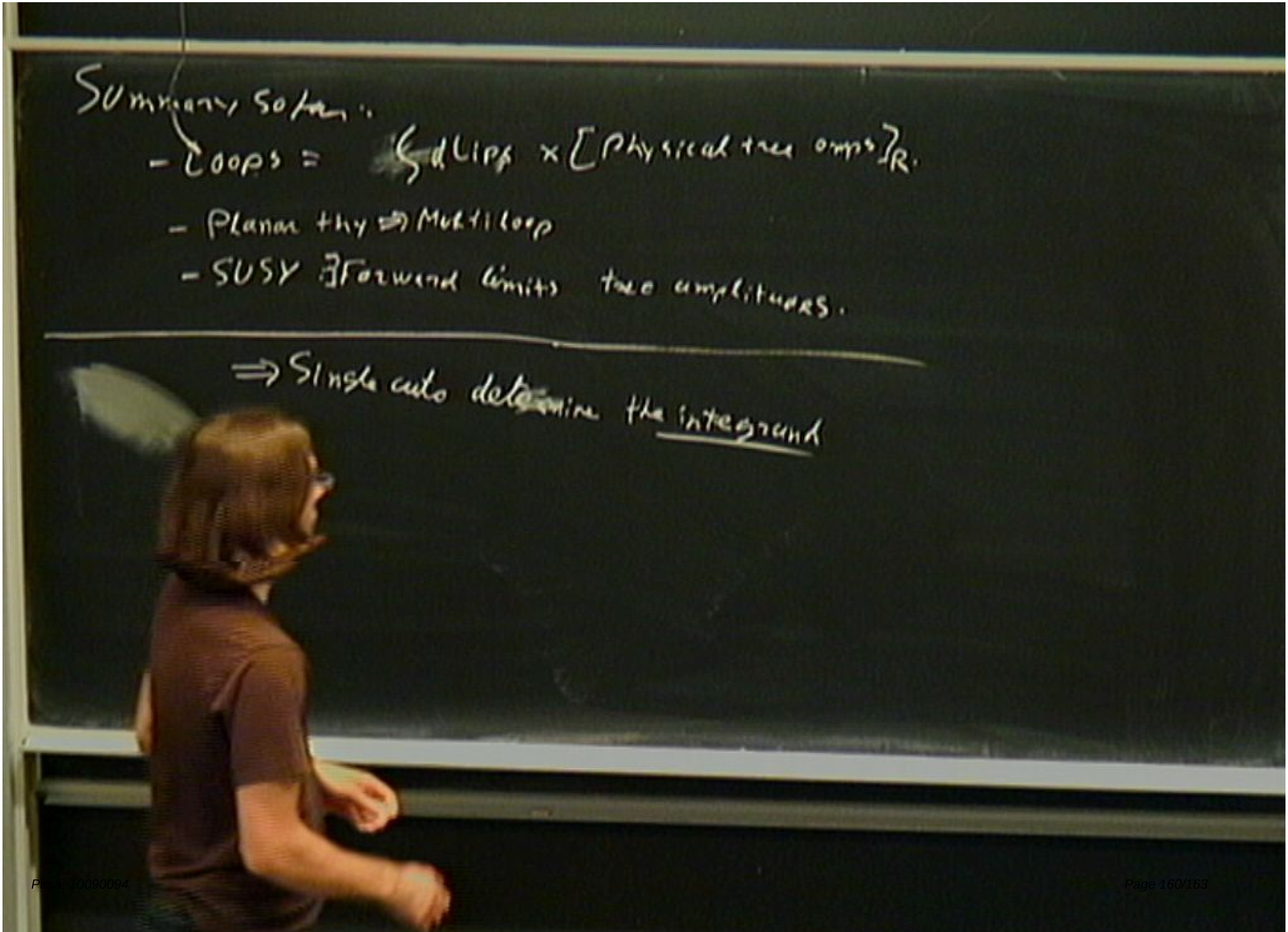
$$+ \int \frac{q^2\, F(q,p_i)}{q^2(\quad)(\quad)(\quad)} = \frac{(q+p_1)^2\, \mathcal{G}(q,p_i)}{}$$






|i> = |+> + η^A |ψ_A> + η^A η^B /2 |φ_AB> + η^A η^B η^C ε_ABCD |ψ̄^D> /3! + (η)^4 /4! |−>
q²(q+p₁)² H(q,p₁)


Summary so far:
- Loops = ∫dLIPS × [Physical tree amps]R.
- Planar thy ⇒ Multiloop
- SUSY ∃ Forward limits tree amplitudes.


Summary so far:
- Loops = ∫ dLIPS × [Physical tree amps]$_R$.
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⇒ Single cuts determine the integrand

Summary so far:

- Loops = $\oint d\text{Lips} \times [\text{Physical tree amps}]_R$.
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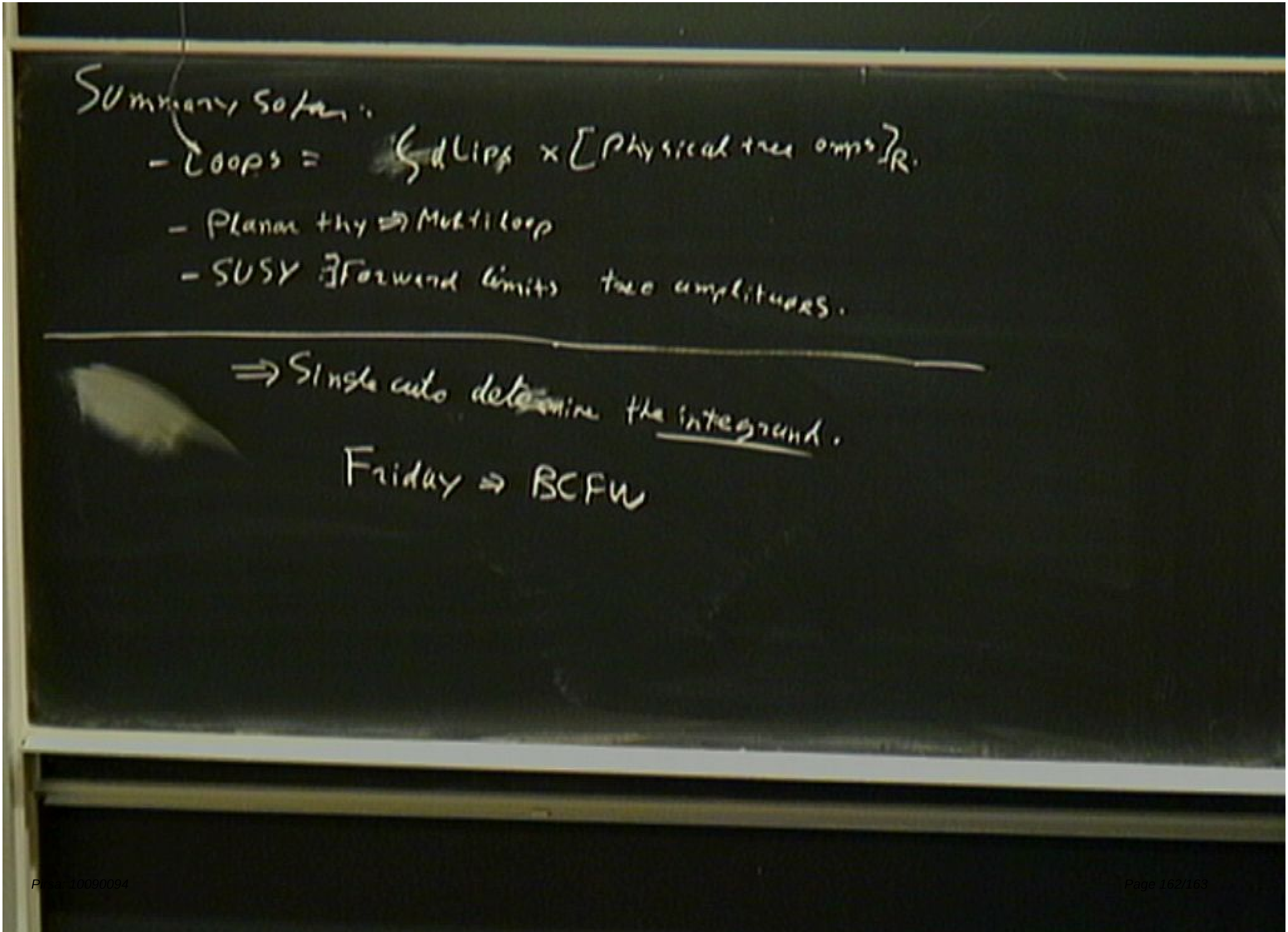
⇒ Single cuts determine the integrand.


Summary so far:
- Loops = ∫ dLIPS × [Physical tree amps]_R.
- Planar thy ⇒ Multiloop
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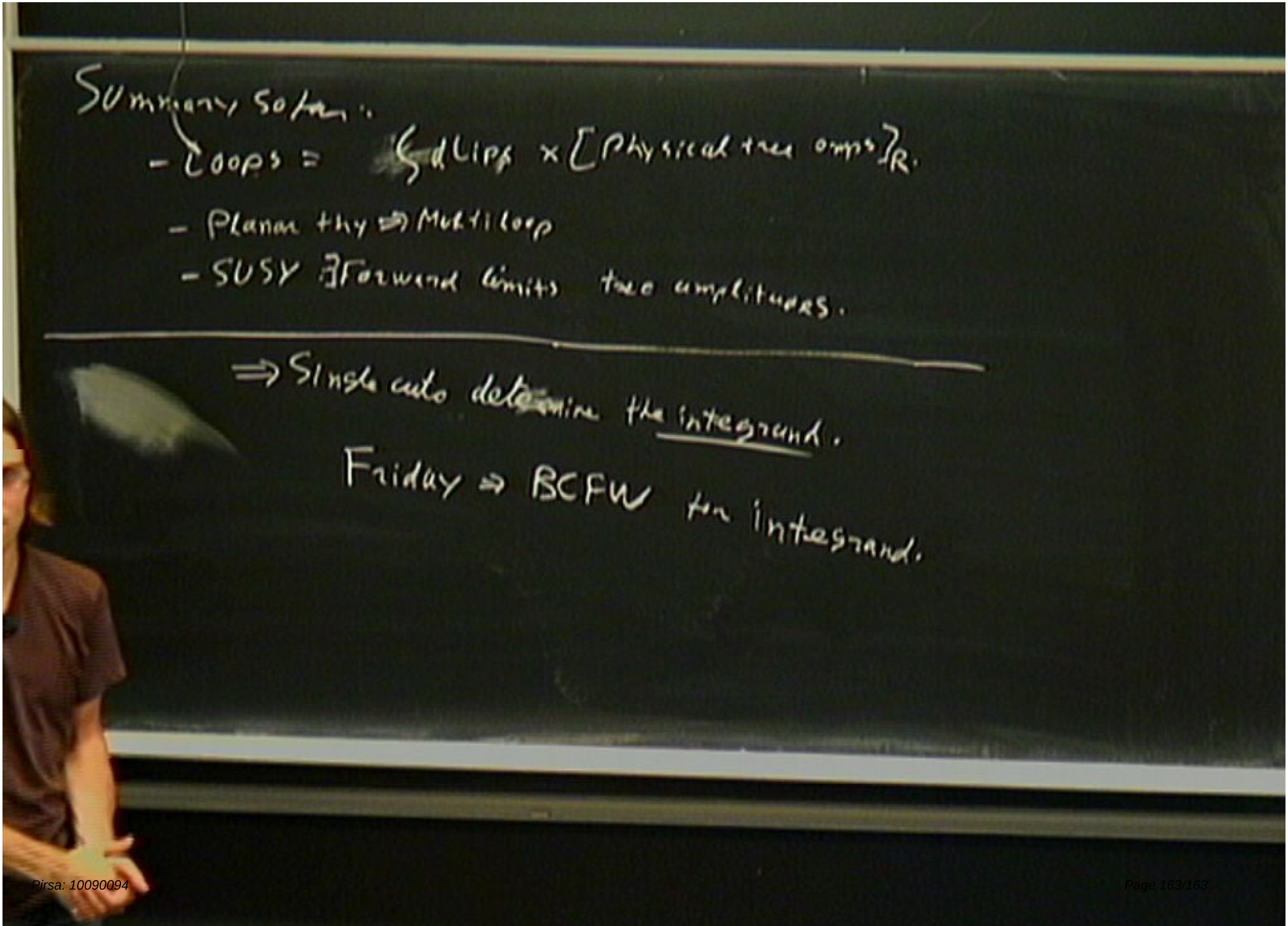
Friday ⇒ BCFW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