

Title: Scattering Amplitudes from Single-Cuts: Lecture 1

Date: Sep 13, 2010 02:00 PM

URL: <http://pirsa.org/10090093>

Abstract: On-shell methods provide a powerful tool for the perturbative computation of scattering amplitudes in gauge theories, such as QCD. In these lectures I will focus on such methods which can be developed without invoking unitarity, but rather by setting one propagator by loop level on-shell ("single-cuts"). After summarizing the present status of applying these ideas to QCD, I will discuss their recent successful application to planar $N=4$ super Yang-Mills, leading to on-shell recursion relations for loop integrands.

Brandhuber et al, hep-th/0510253

Catani et al 0804.3170, 1007.0194

E.W. Nigel Glover, C. Williams 0810.2964

SC H 1007.3224

MA-H et al, 1008.2958

R. Brits 1008.3101

"Scattering amplitudes
from single-cuts"

Brandhuber et al, hep-th/0510253

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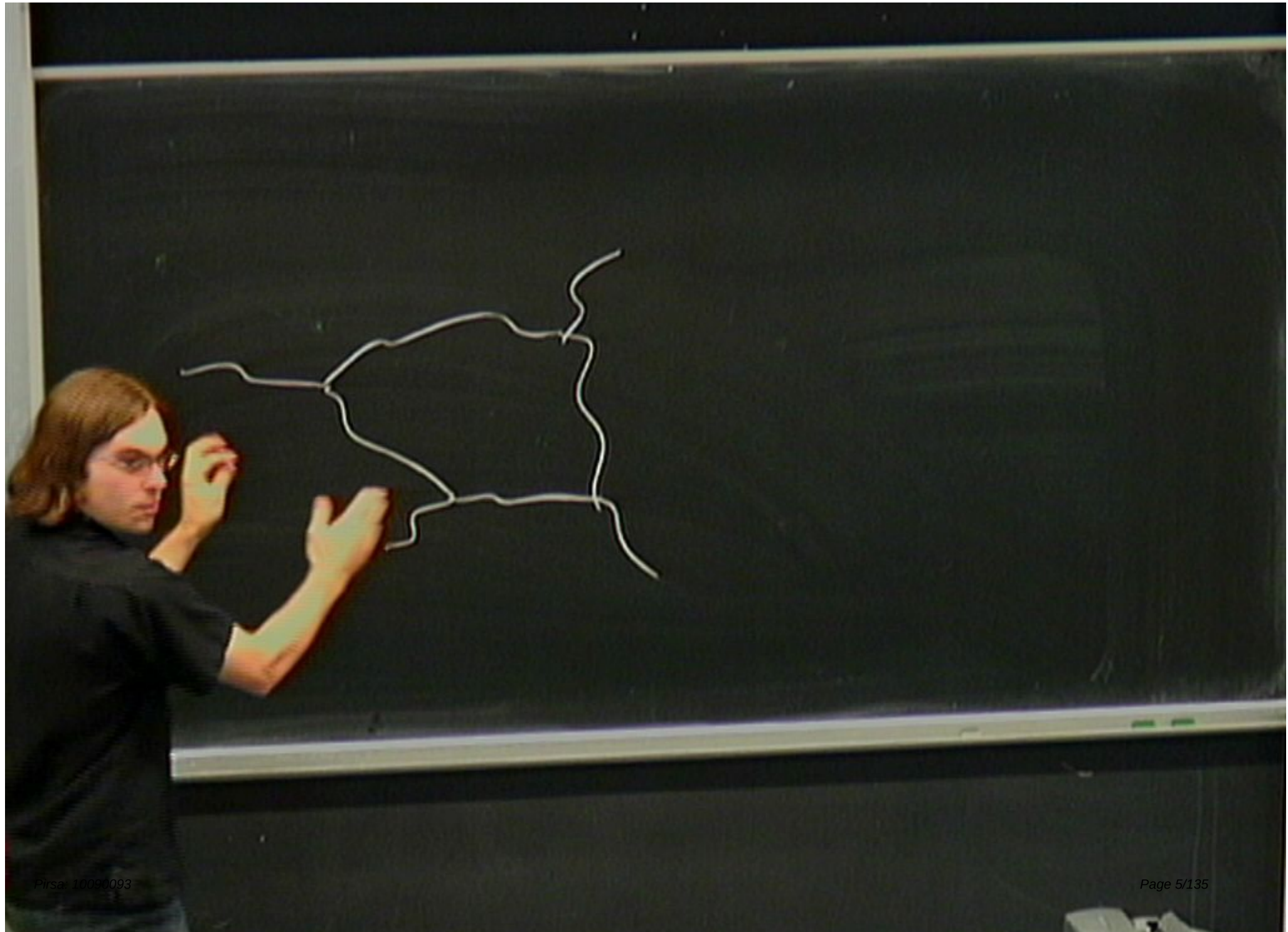
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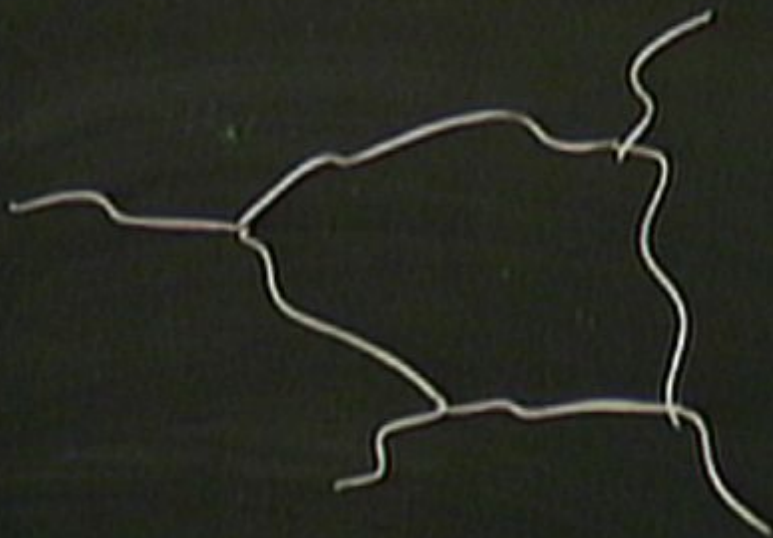
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"Scattering amplitude
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"Scattering amplitudes
from single-cuts"

- "A technique to do
num. algebra in GFT"

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"Scattering amplitudes
from single-cuts"

- "A technique to do
num. algebra in GFT"
- Build

Brandhuber et al, hep-th/0510253

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"Scattering amplitudes
from single-cuts"

- A technique to do
num. algebra in GET -

- Build on knowledge of tree
amps.

Brandhager et al, hep-th/0510253

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MA-Hell 1008.2958

R.P. 1008.3101

"Scattering amplitudes from single-cuts"

- "A technique to do num. algebra in QFT"

- Build on knowledge of tree amps.

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"Scattering amplitudes from single-cuts"

- "A technique to do num. algebra in GFT"

- Build on knowledge of tree amps.

Application to ^{planar} 4-5

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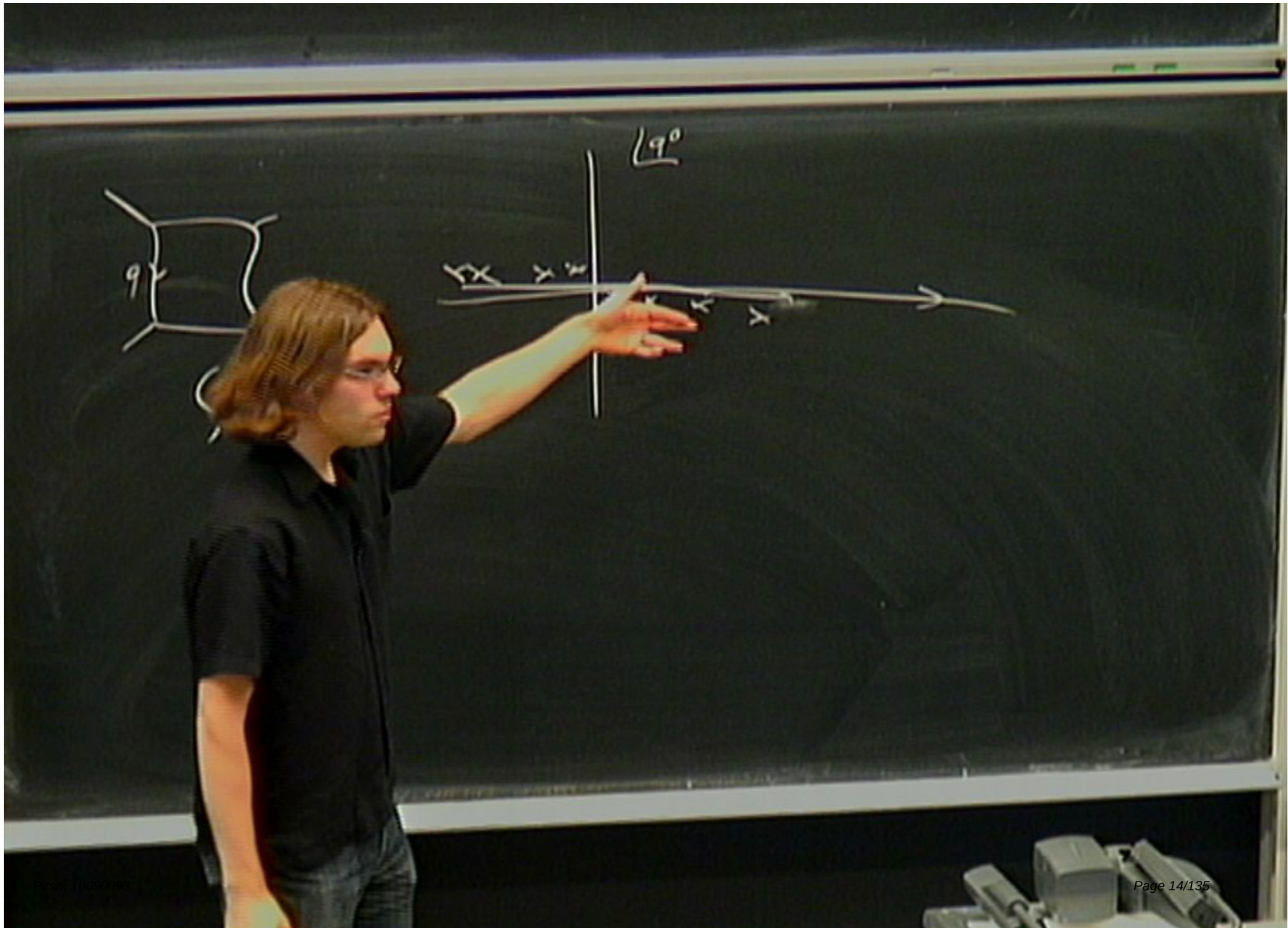
R. Boels 1008.3101

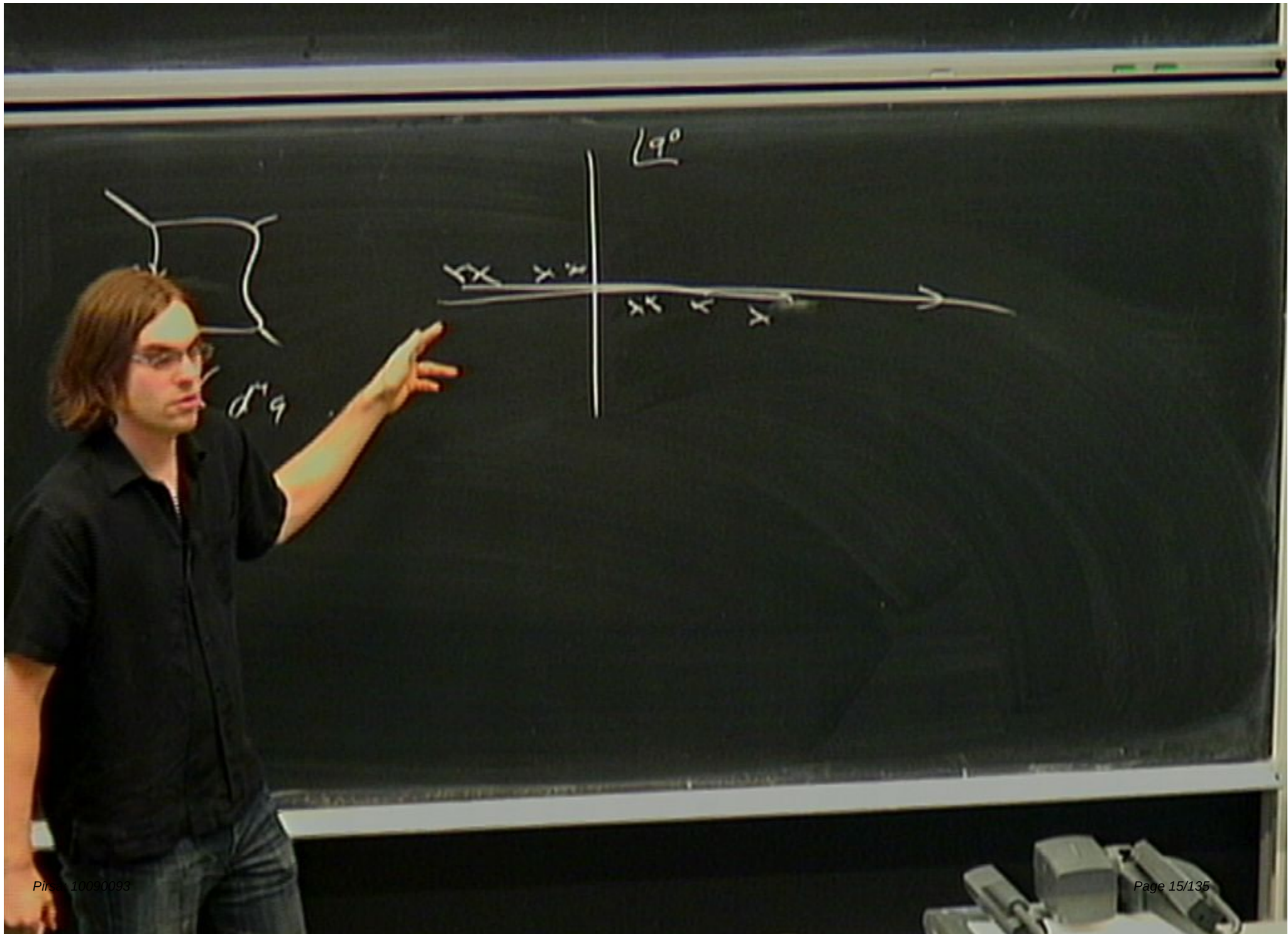
Application to ^{Planar} $\mathcal{N}=4$ SYM

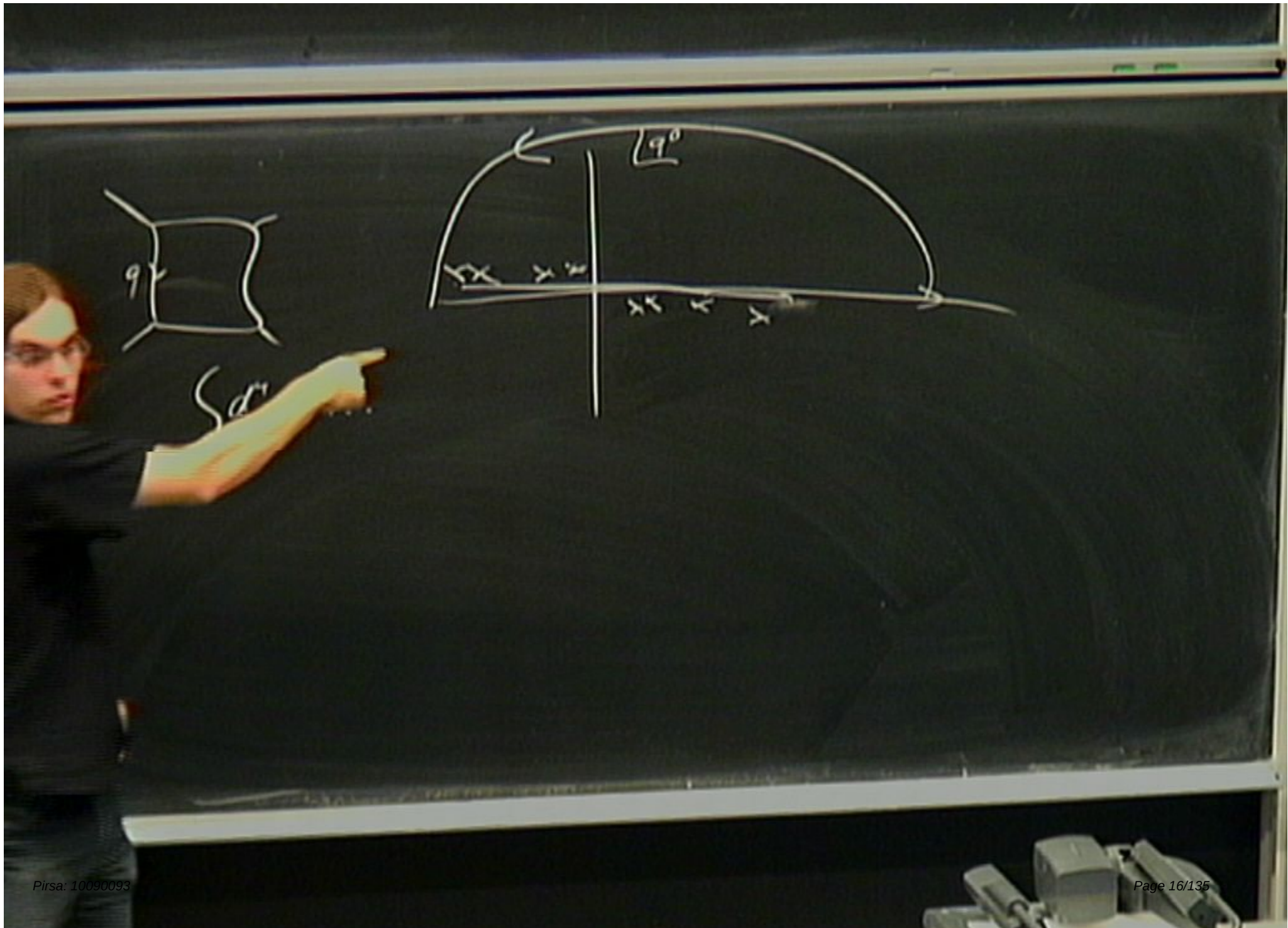
"Scattering amplitudes from single-cuts"

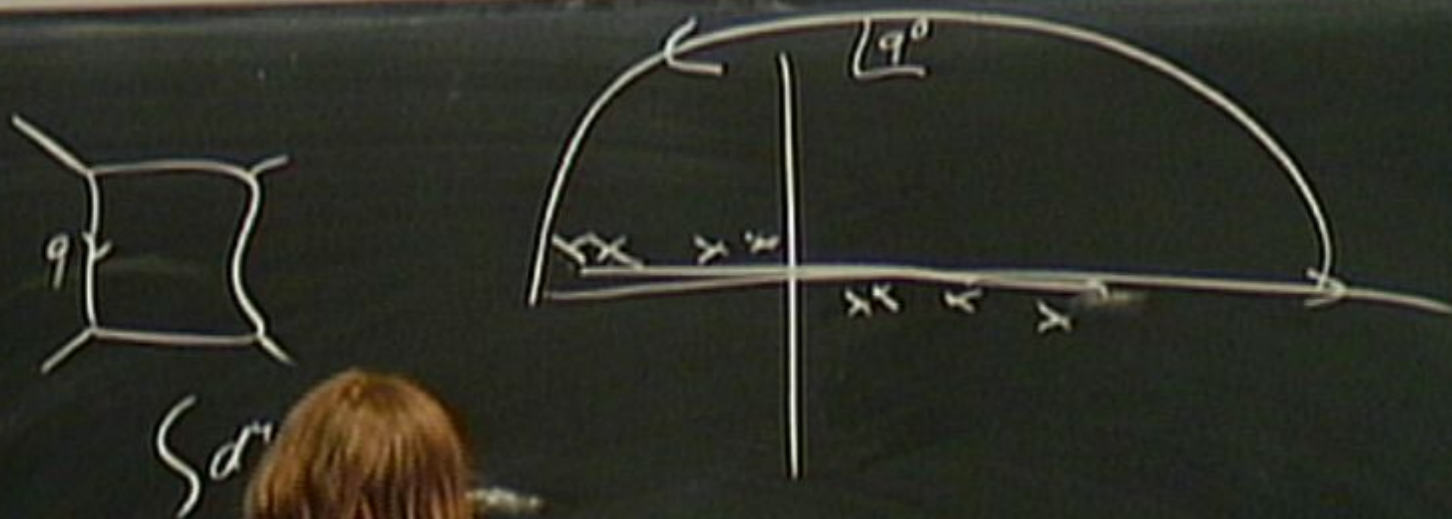
- "A technique to do num. algebra in GFT"

- Build on knowledge of tree amps.

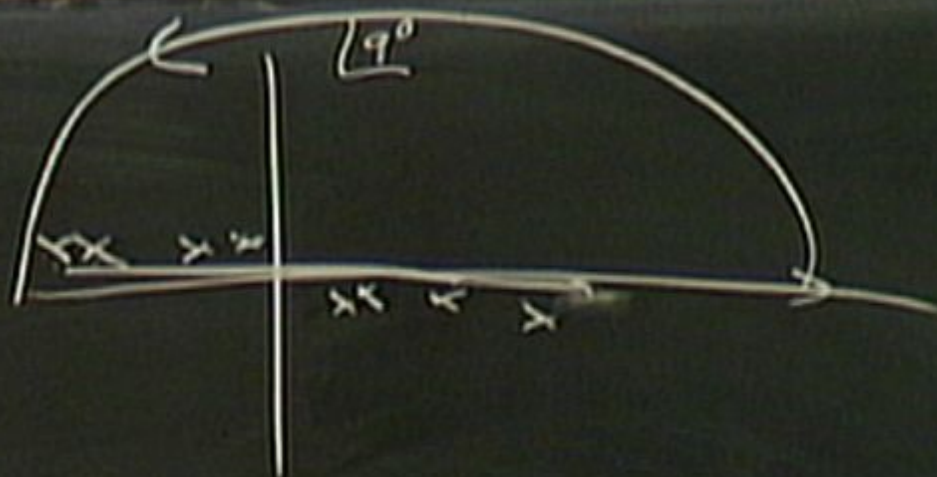






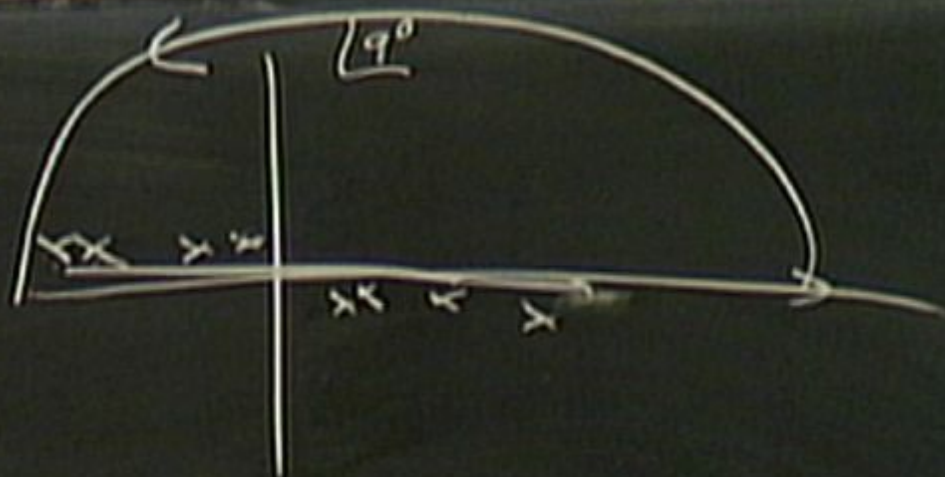


$$9 \approx 2\pi \delta /$$



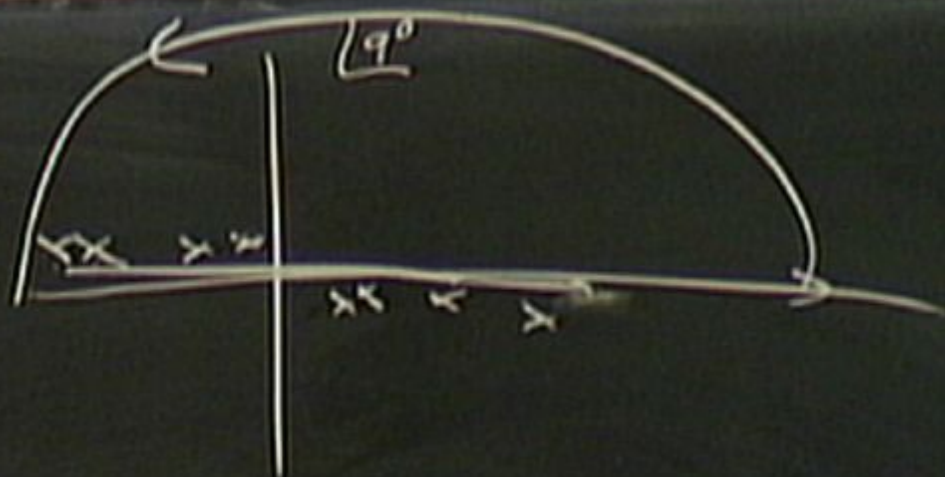
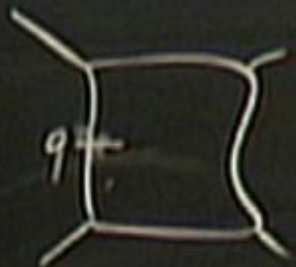
$$\int d^4 q \frac{1}{q^2}$$

$$\rightarrow \int d^4 q 2\pi \delta(q^2)$$



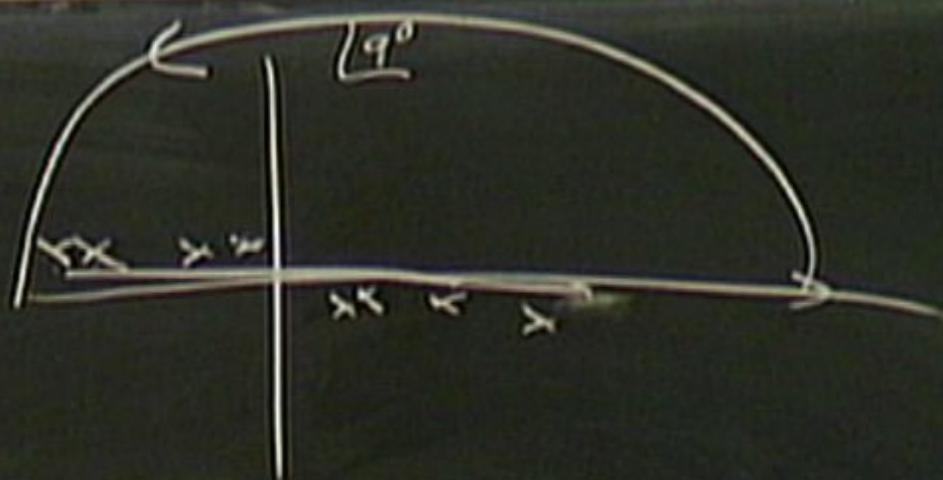
$$\int d^4 q \frac{1}{q^2}$$

$$\rightarrow \int d^4 q 2\pi \delta(q^2)$$



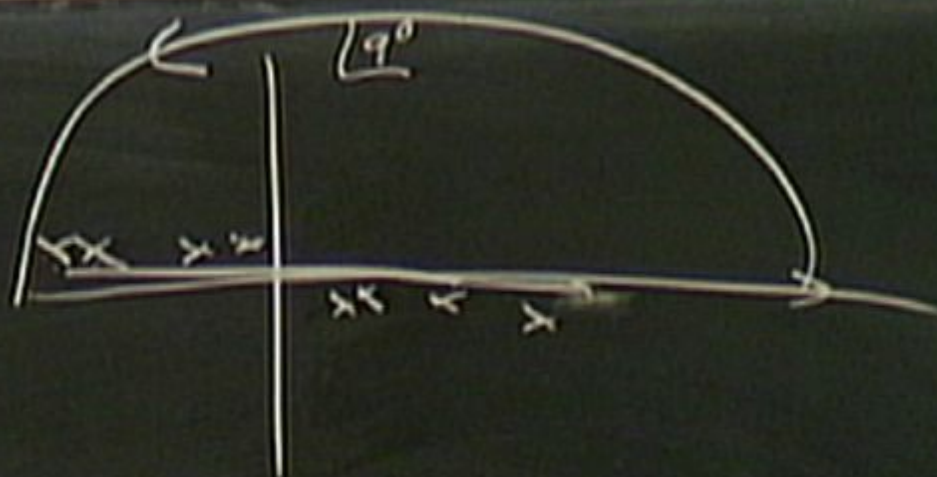
$$\int d^4 q \frac{1}{q^2}$$

$$\rightarrow \int d^4 q 2\pi \delta(q^2)$$



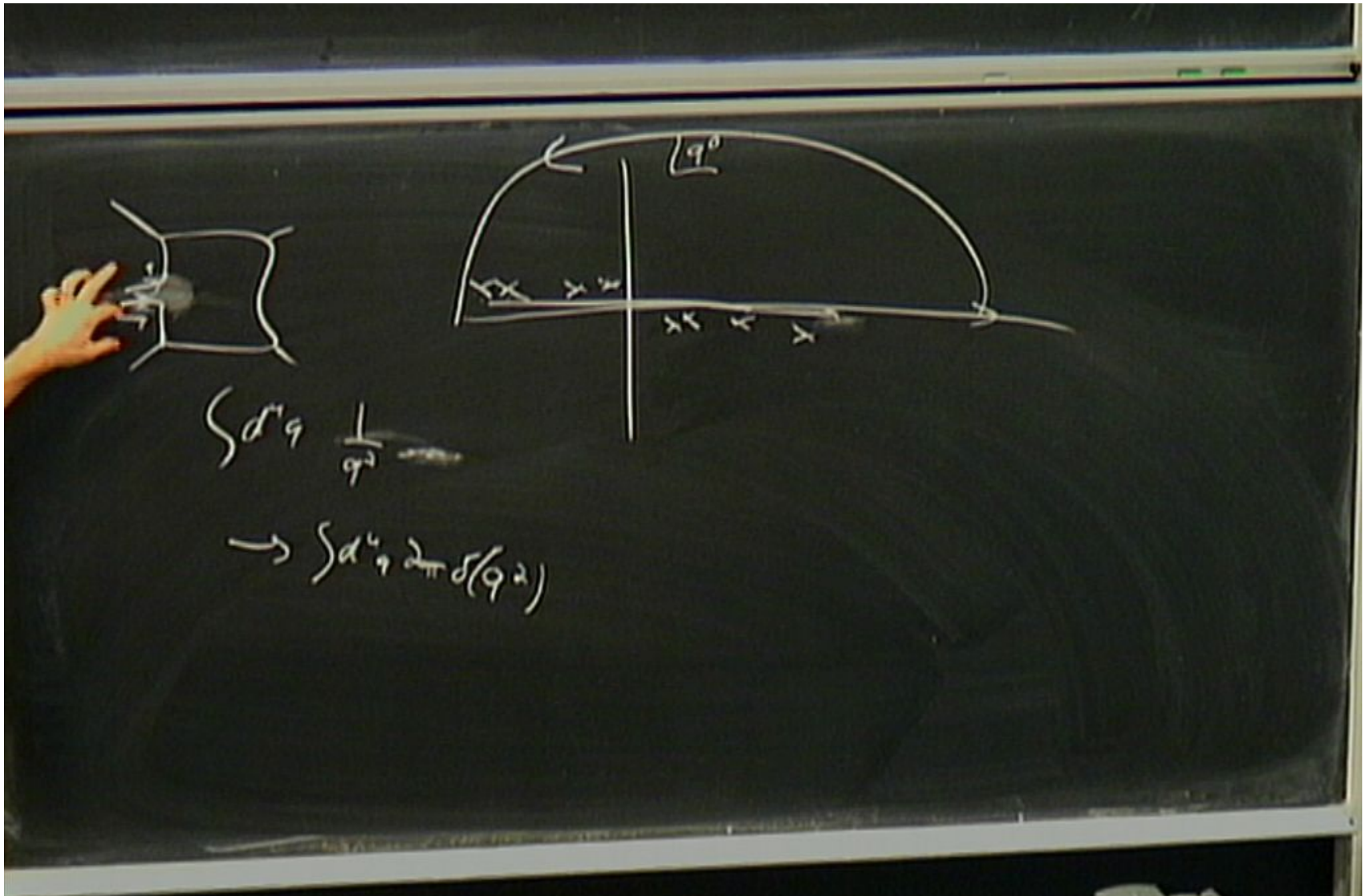
$$\int d^4 q \frac{1}{q^2}$$

$$\rightarrow \int d^4 q 2\pi \delta(q^2)$$



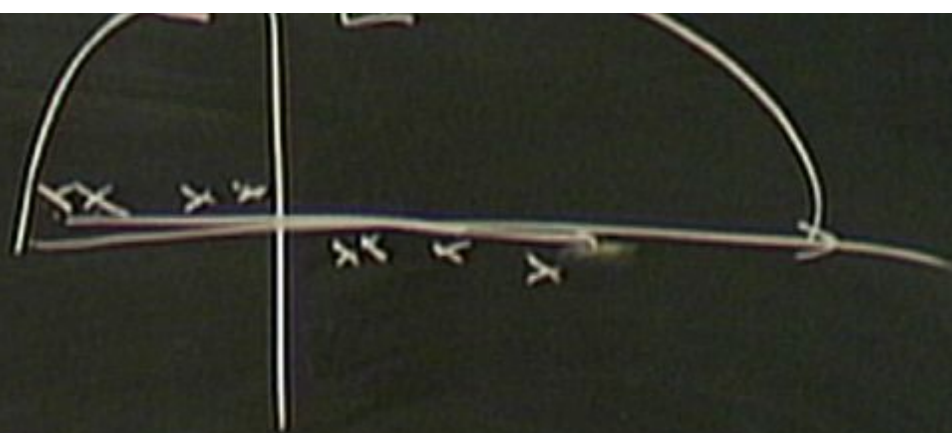
$$\int d^4 q \frac{1}{q^2}$$

$$\rightarrow \int d^4 q 2\pi \delta(q^2)$$



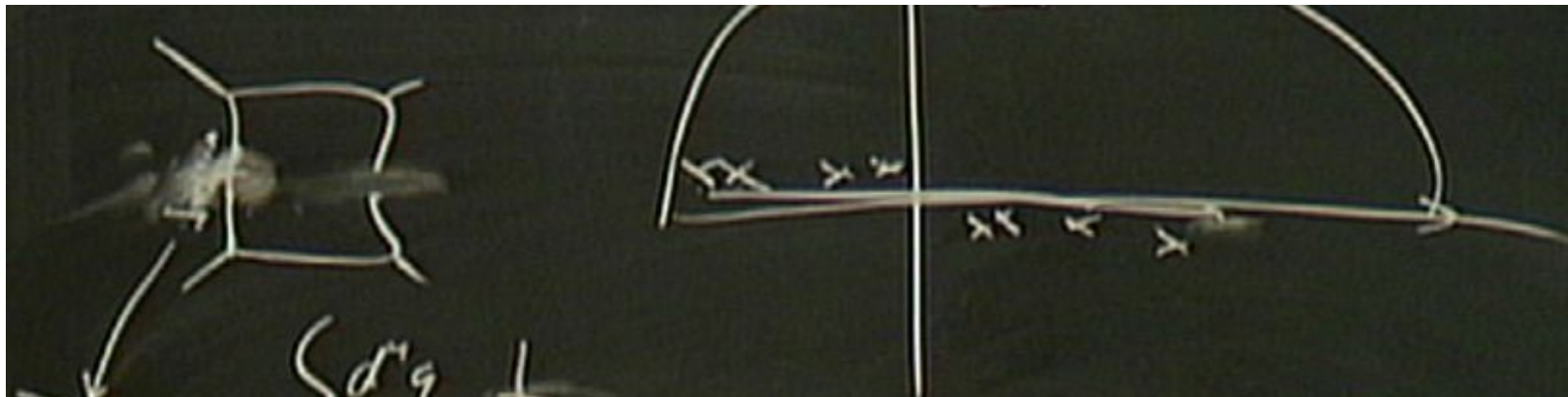
$$\int d^4 q \frac{1}{q^2}$$

$$\rightarrow \int d^4 q 2\pi \delta(q^2)$$



$$\int d^4 q \frac{1}{q^2}$$

$$\rightarrow \int d^4 q 2\pi \delta(q^2)$$



$\int d^4 q \frac{1}{q^2}$
 2-cuts.
 Zvi Bern
 Lance Dixon. $\rightarrow \int d^4 q 2\pi \delta(q^2)$

Brandhuber et al, hep-th/0510253

Catani et al 0804.3170, 1007.0194

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"Scattering amplitudes from single-cuts"

- "A technique to do num. algebra in GFT"

- Build on knowledge of tree amps.

Application to ^{Planar} 4-5 loop

today :- is

Brandhuber et al, hep-th/0510253

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Plan today :- ie.
- T

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Application to ^{planar} 2-loop

Planar 2-loop

2-loops (planar vs. non-planar)

SC H 1007.3224

N.A. Hetal, 1008.2958

R. Boels 1008.3101

0810.2764

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num. algebra in GFT

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amps

Application to ^{planar} ~~to~~ ⁴ ~~5~~ ⁶ ~~7~~ ⁸ ~~9~~ ¹⁰ ~~11~~ ¹² ~~13~~ ¹⁴ ~~15~~ ¹⁶ ~~17~~ ¹⁸ ~~19~~ ²⁰ ~~21~~ ²² ~~23~~ ²⁴ ~~25~~ ²⁶ ~~27~~ ²⁸ ~~29~~ ³⁰ ~~31~~ ³² ~~33~~ ³⁴ ~~35~~ ³⁶ ~~37~~ ³⁸ ~~39~~ ⁴⁰ ~~41~~ ⁴² ~~43~~ ⁴⁴ ~~45~~ ⁴⁶ ~~47~~ ⁴⁸ ~~49~~ ⁵⁰ ~~51~~ ⁵² ~~53~~ ⁵⁴ ~~55~~ ⁵⁶ ~~57~~ ⁵⁸ ~~59~~ ⁶⁰ ~~61~~ ⁶² ~~63~~ ⁶⁴ ~~65~~ ⁶⁶ ~~67~~ ⁶⁸ ~~69~~ ⁷⁰ ~~71~~ ⁷² ~~73~~ ⁷⁴ ~~75~~ ⁷⁶ ~~77~~ ⁷⁸ ~~79~~ ⁸⁰ ~~81~~ ⁸² ~~83~~ ⁸⁴ ~~85~~ ⁸⁶ ~~87~~ ⁸⁸ ~~89~~ ⁹⁰ ~~91~~ ⁹² ~~93~~ ⁹⁴ ~~95~~ ⁹⁶ ~~97~~ ⁹⁸ ~~99~~ ¹⁰⁰ ~~101~~ ¹⁰² ~~103~~ ¹⁰⁴ ~~105~~ ¹⁰⁶ ~~107~~ ¹⁰⁸ ~~109~~ ¹¹⁰ ~~111~~ ¹¹² ~~113~~ ¹¹⁴ ~~115~~ ¹¹⁶ ~~117~~ ¹¹⁸ ~~119~~ ¹²⁰ ~~121~~ ¹²² ~~123~~ ¹²⁴ ~~125~~ ¹²⁶ ~~127~~ ¹²⁸ ~~129~~ ¹³⁰ ~~131~~ ¹³² ~~133~~ ¹³⁴ ~~135~~ ¹³⁶ ~~137~~ ¹³⁸ ~~139~~ ¹⁴⁰ ~~141~~ ¹⁴² ~~143~~ ¹⁴⁴ ~~145~~ ¹⁴⁶ ~~147~~ ¹⁴⁸ ~~149~~ ¹⁵⁰ ~~151~~ ¹⁵² ~~153~~ ¹⁵⁴ ~~155~~ ¹⁵⁶ ~~157~~ ¹⁵⁸ ~~159~~ ¹⁶⁰ ~~161~~ ¹⁶² ~~163~~ ¹⁶⁴ ~~165~~ ¹⁶⁶ ~~167~~ ¹⁶⁸ ~~169~~ ¹⁷⁰ ~~171~~ ¹⁷² ~~173~~ ¹⁷⁴ ~~175~~ ¹⁷⁶ ~~177~~ ¹⁷⁸ ~~179~~ ¹⁸⁰ ~~181~~ 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planar
 $N=4$ SYM

knowledge of tree
amps

- BCFW for loop integrands
-

non)

planar
 $N=4$ SYM

knowledge of tree
amps

- BCFW for loop integrands
- $N=4$ application

non)

iq-o-677.

is o. by.

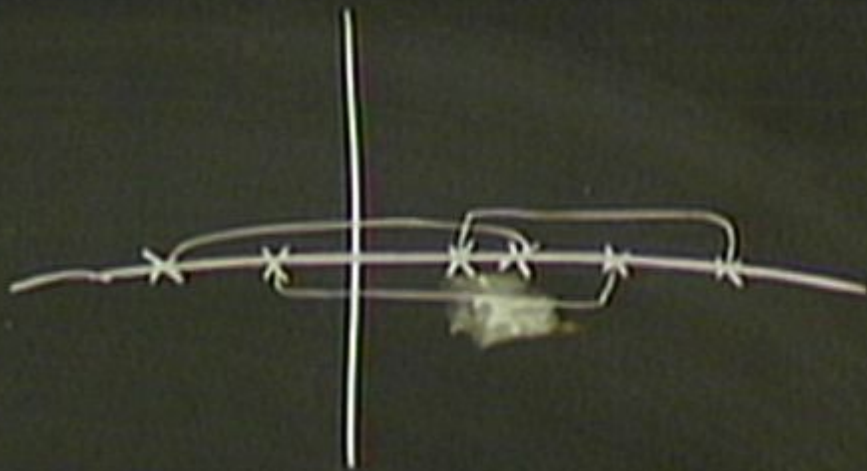
$$A_n^{\text{1-loop}}(p_1, \dots, p_n) =$$

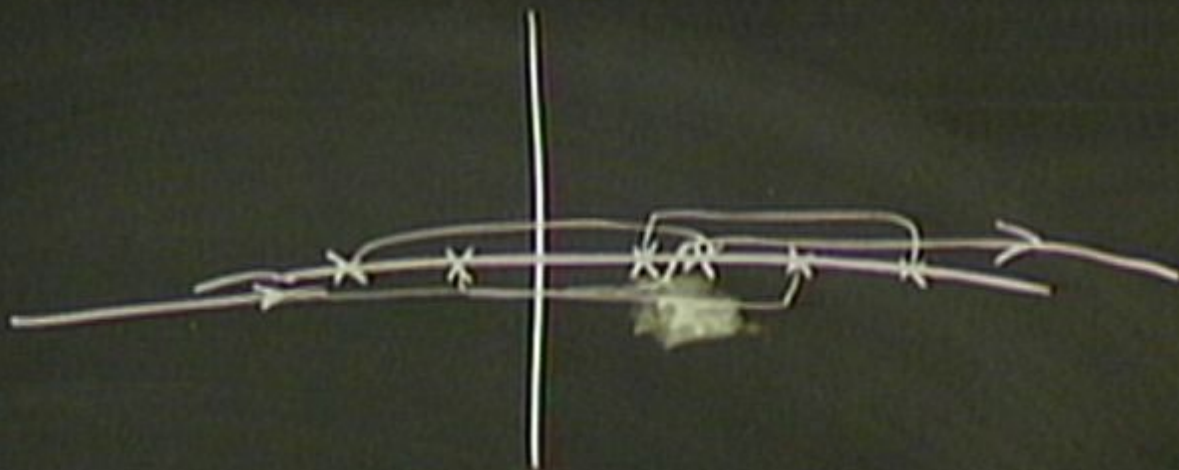
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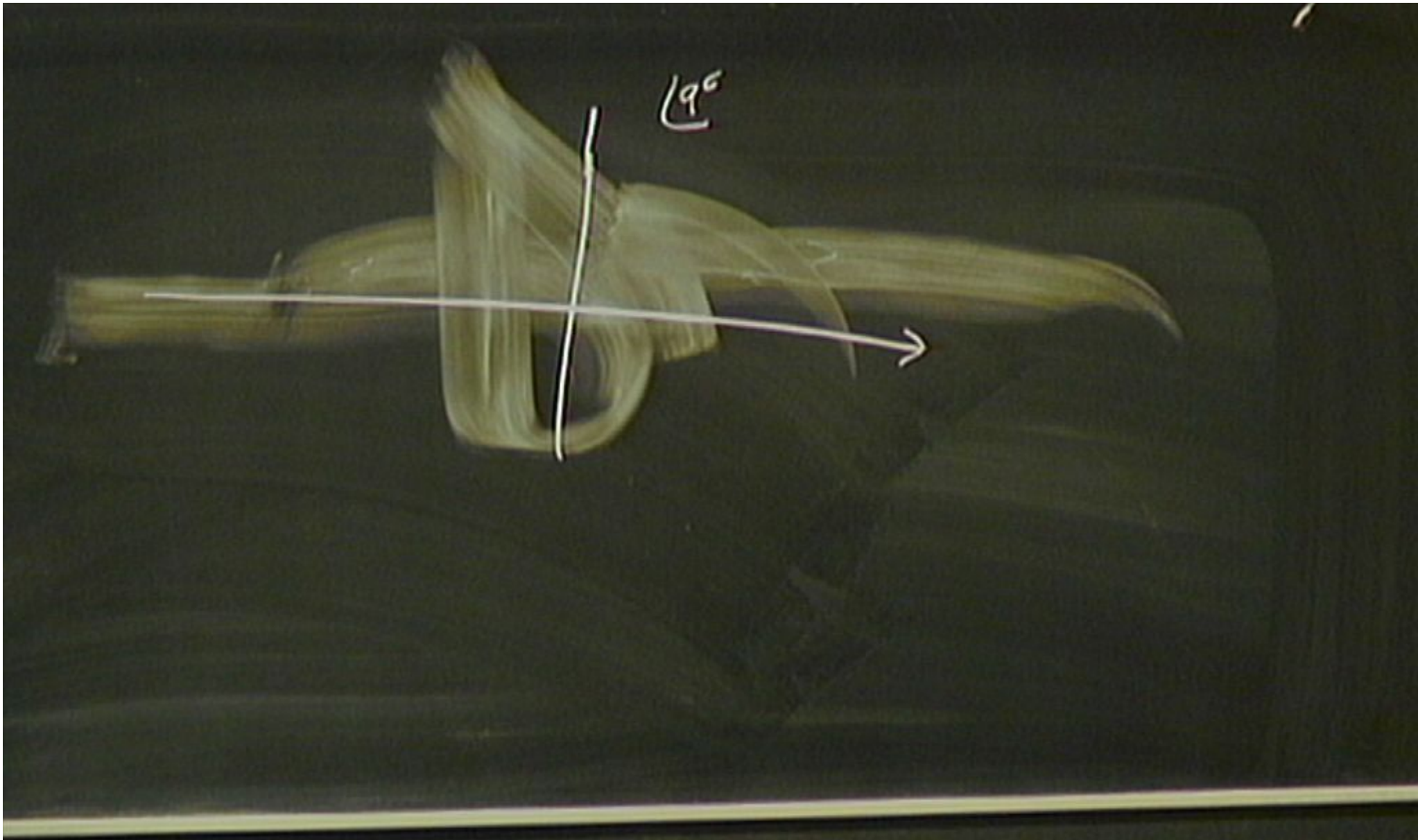
$$A_n^{\text{1-loop}}(p_1, \dots, p_n) = \int \frac{d^3 q}{(2\pi)^3 2E_q}$$

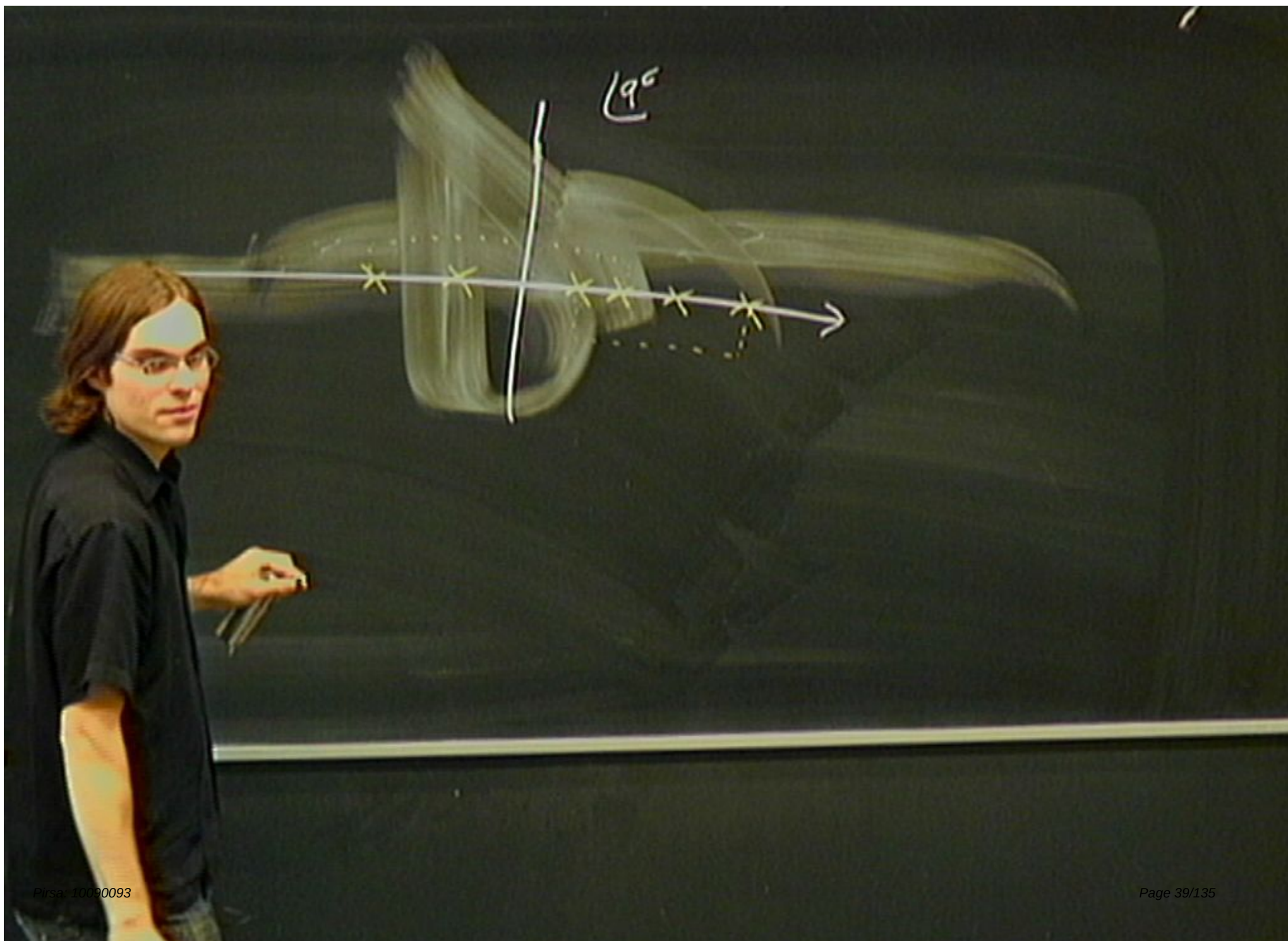
o. 677.)

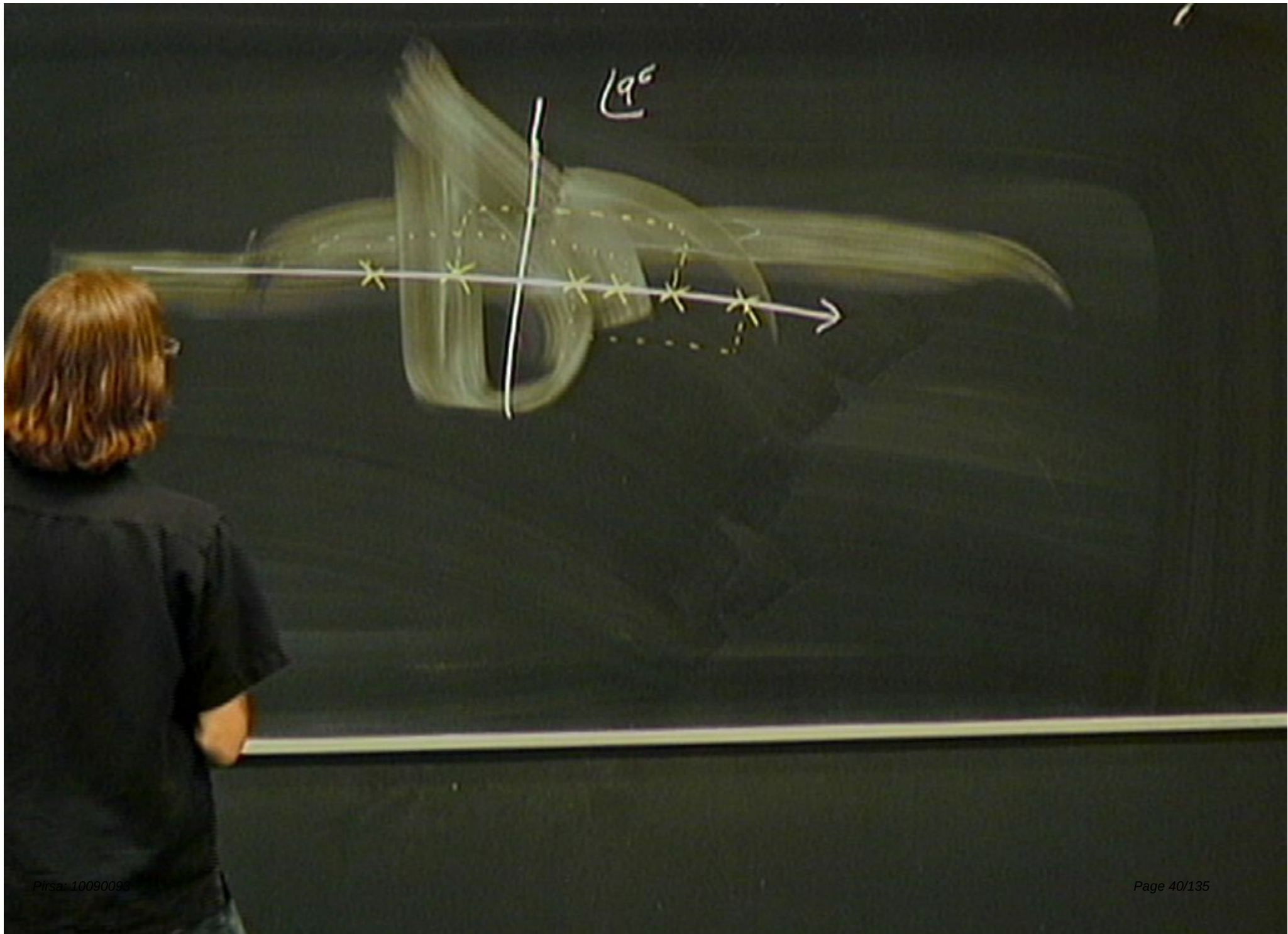
$$A_n^{\text{loop}}(p_1, \dots, p_n) = \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \dots, p_n, q, -q).$$

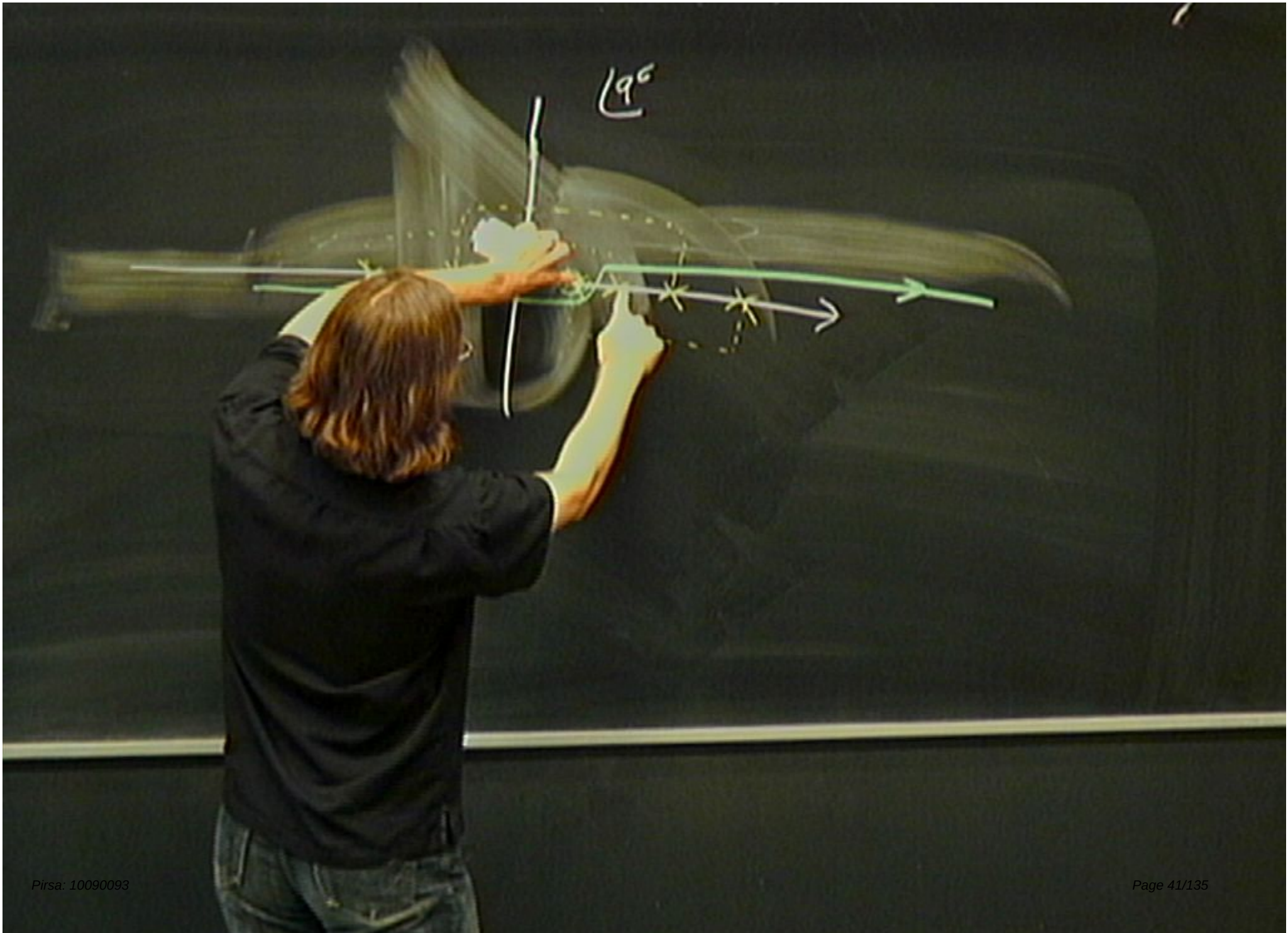


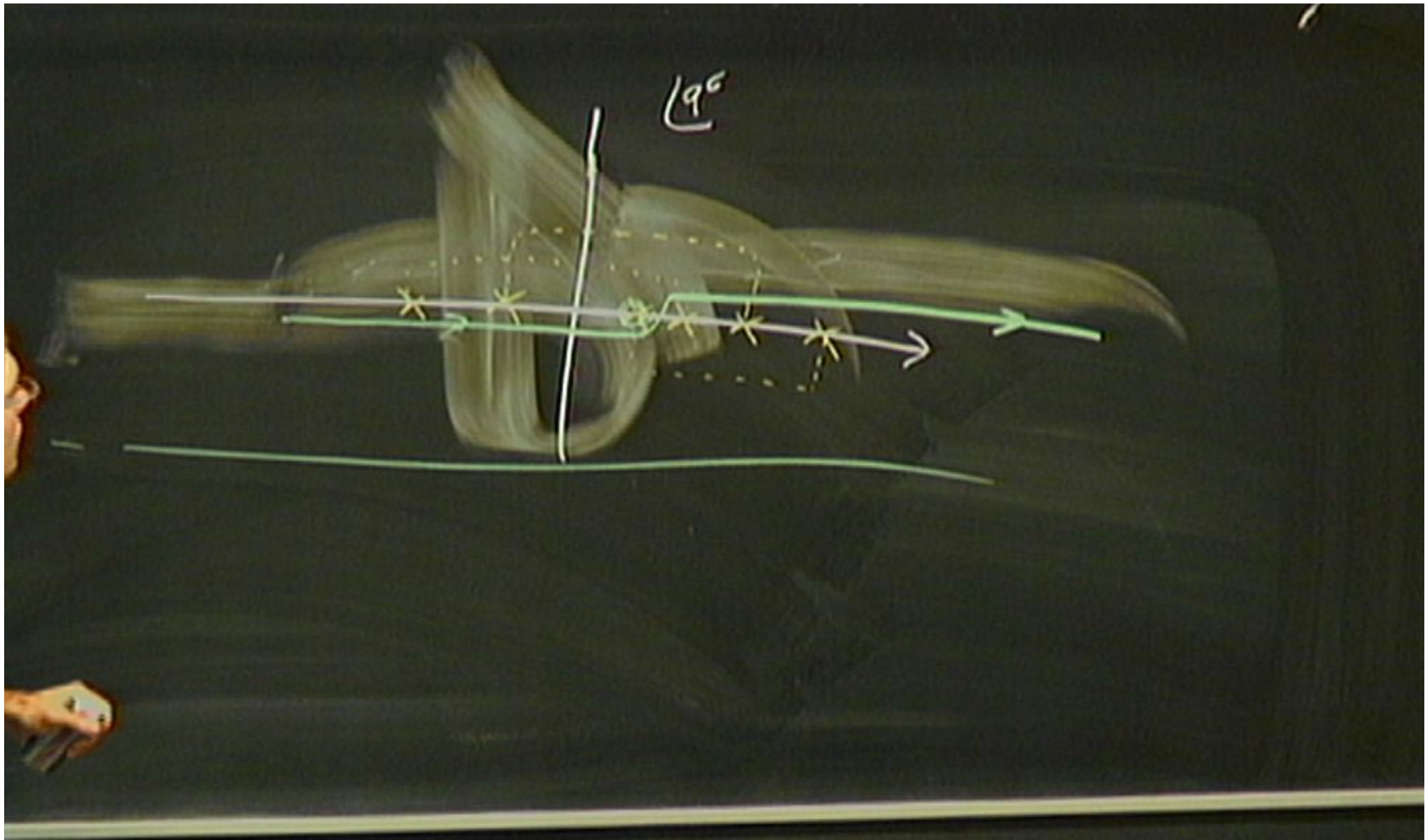


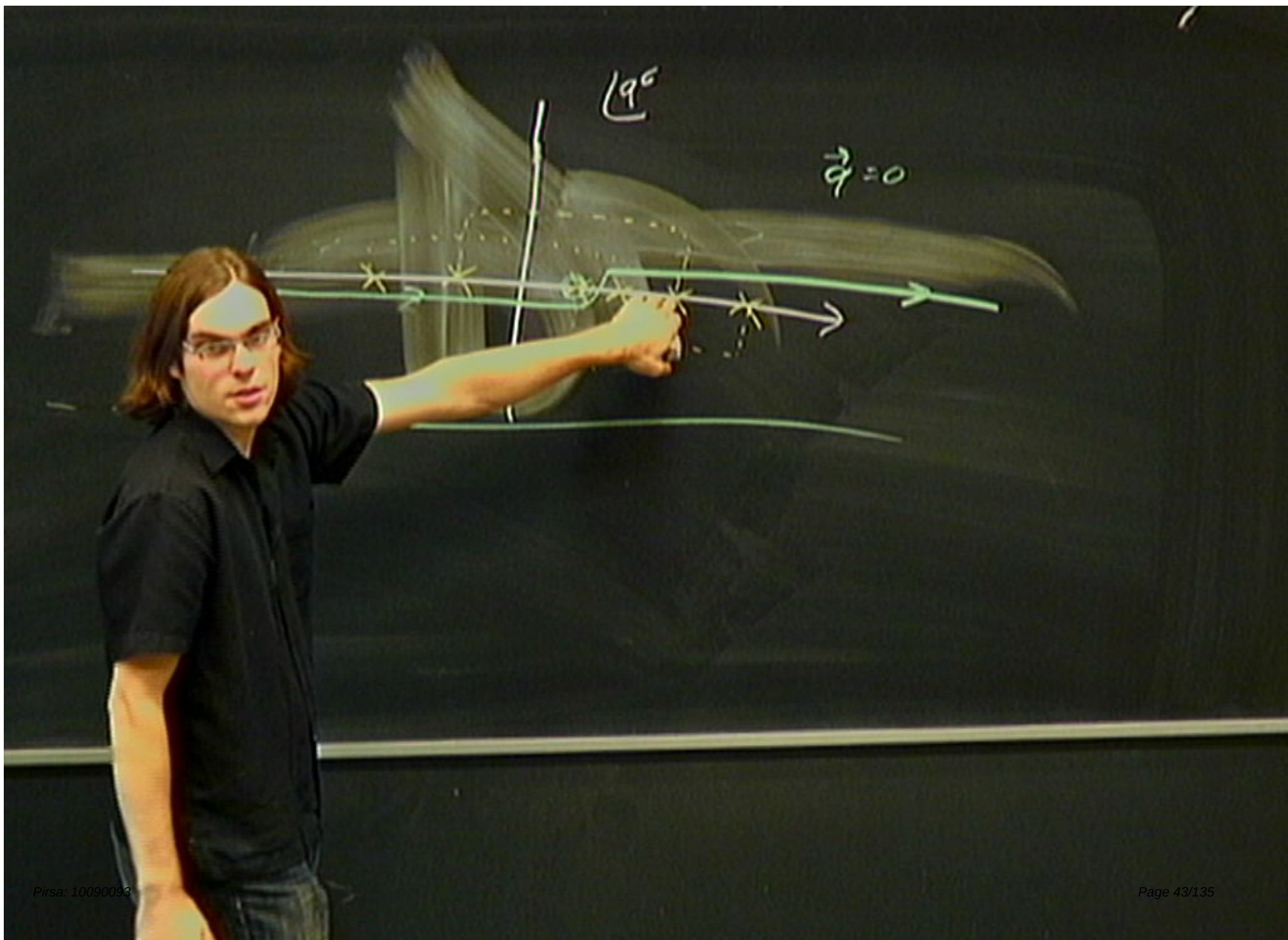


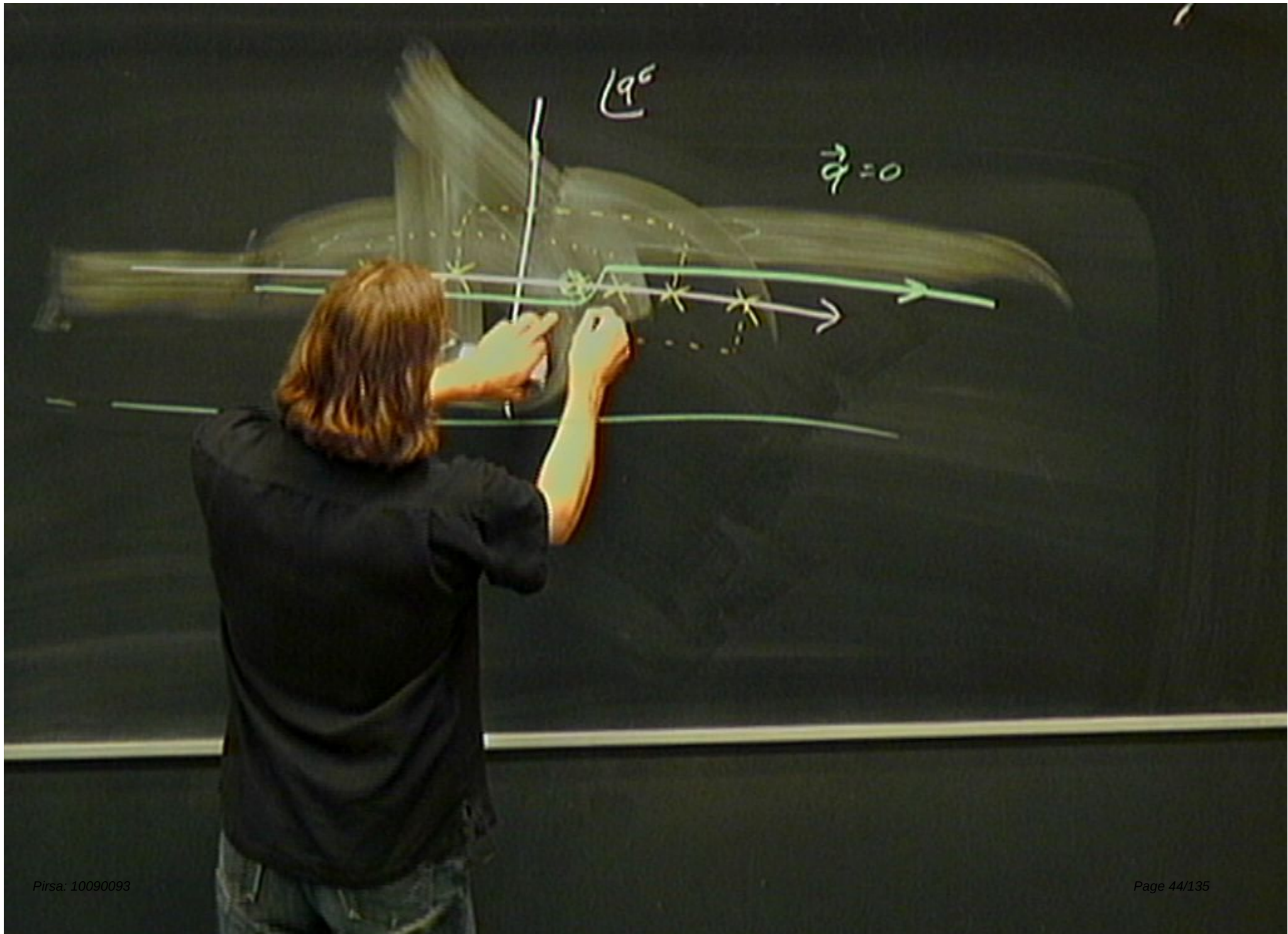


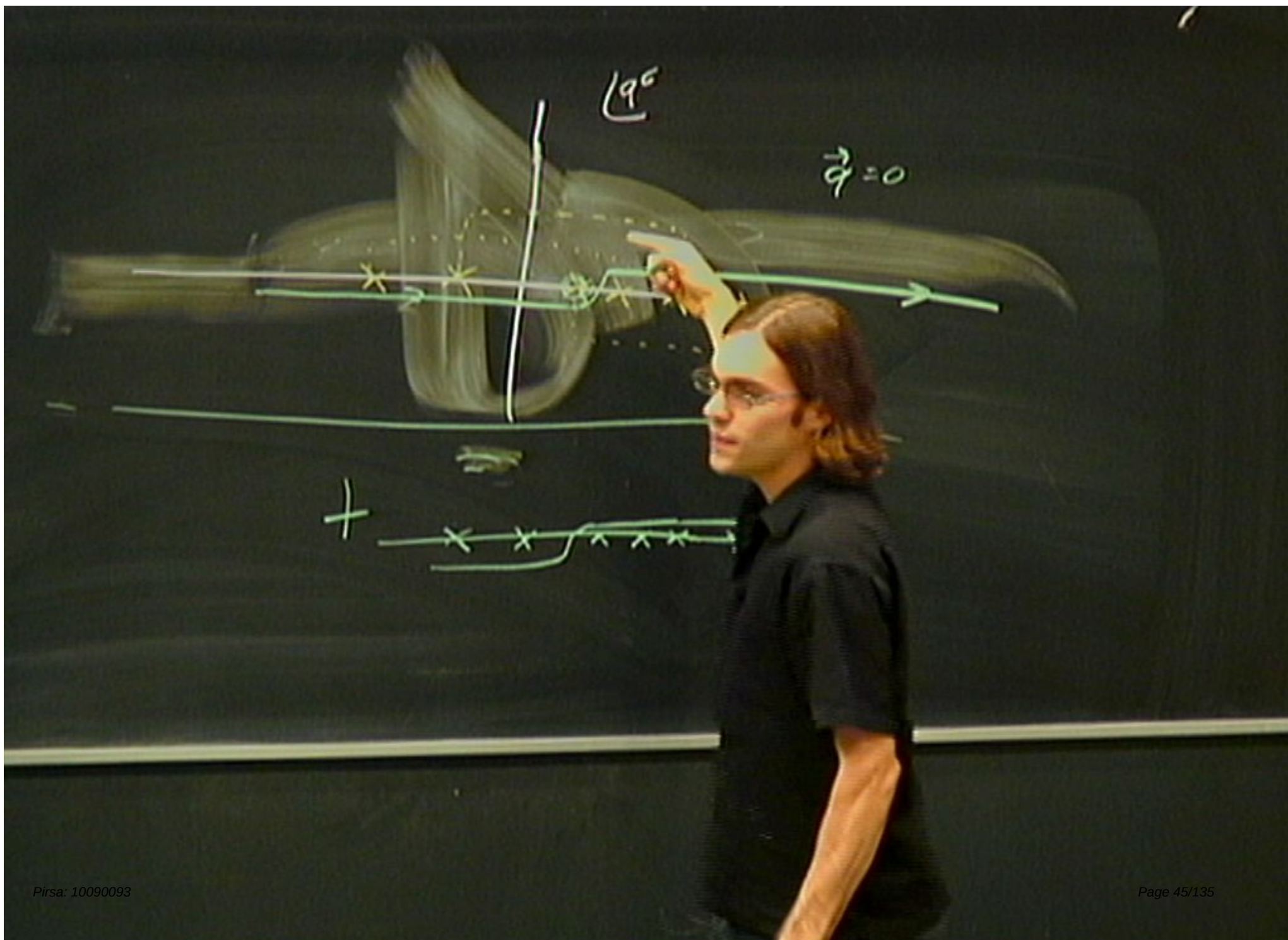


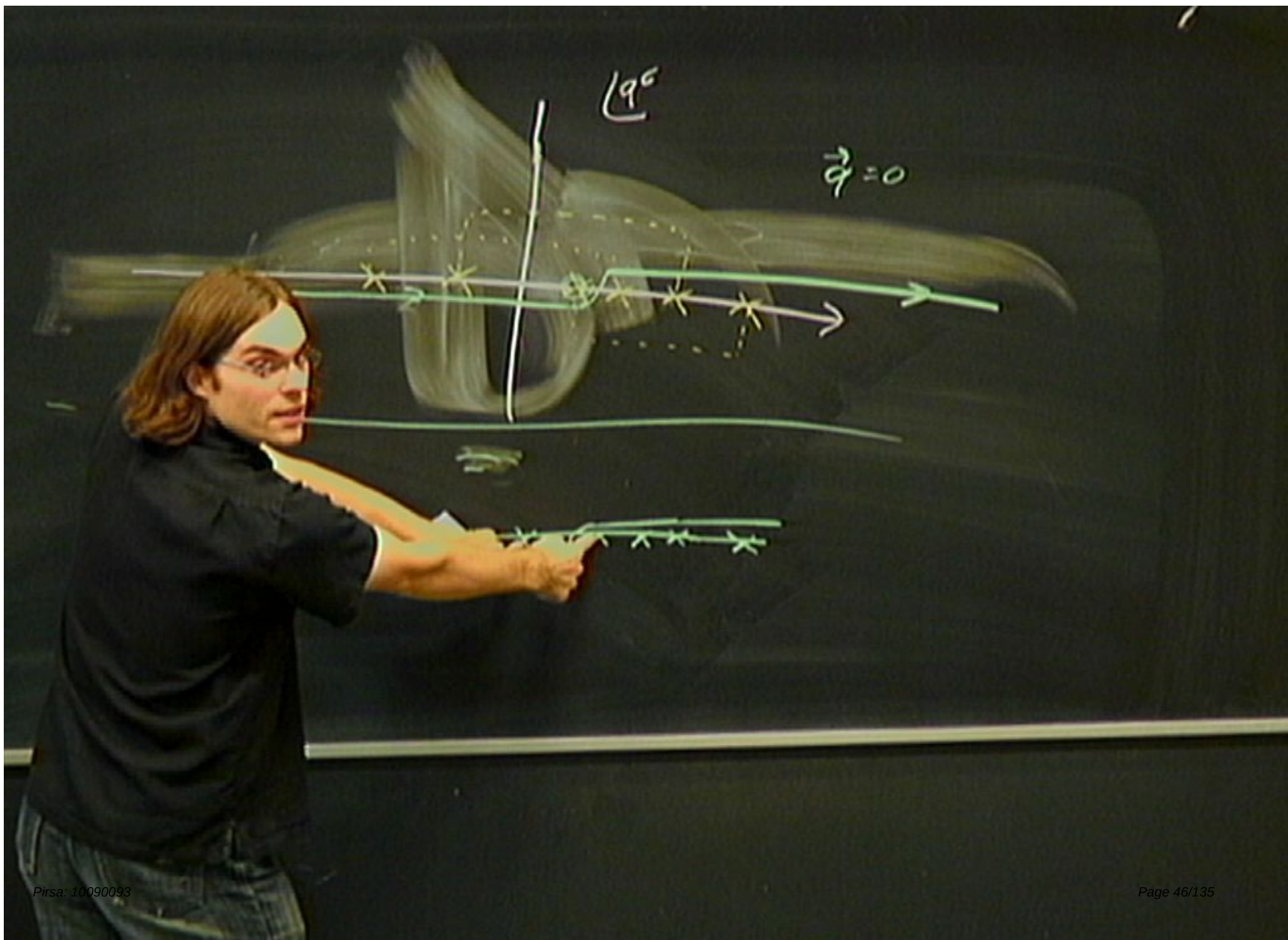


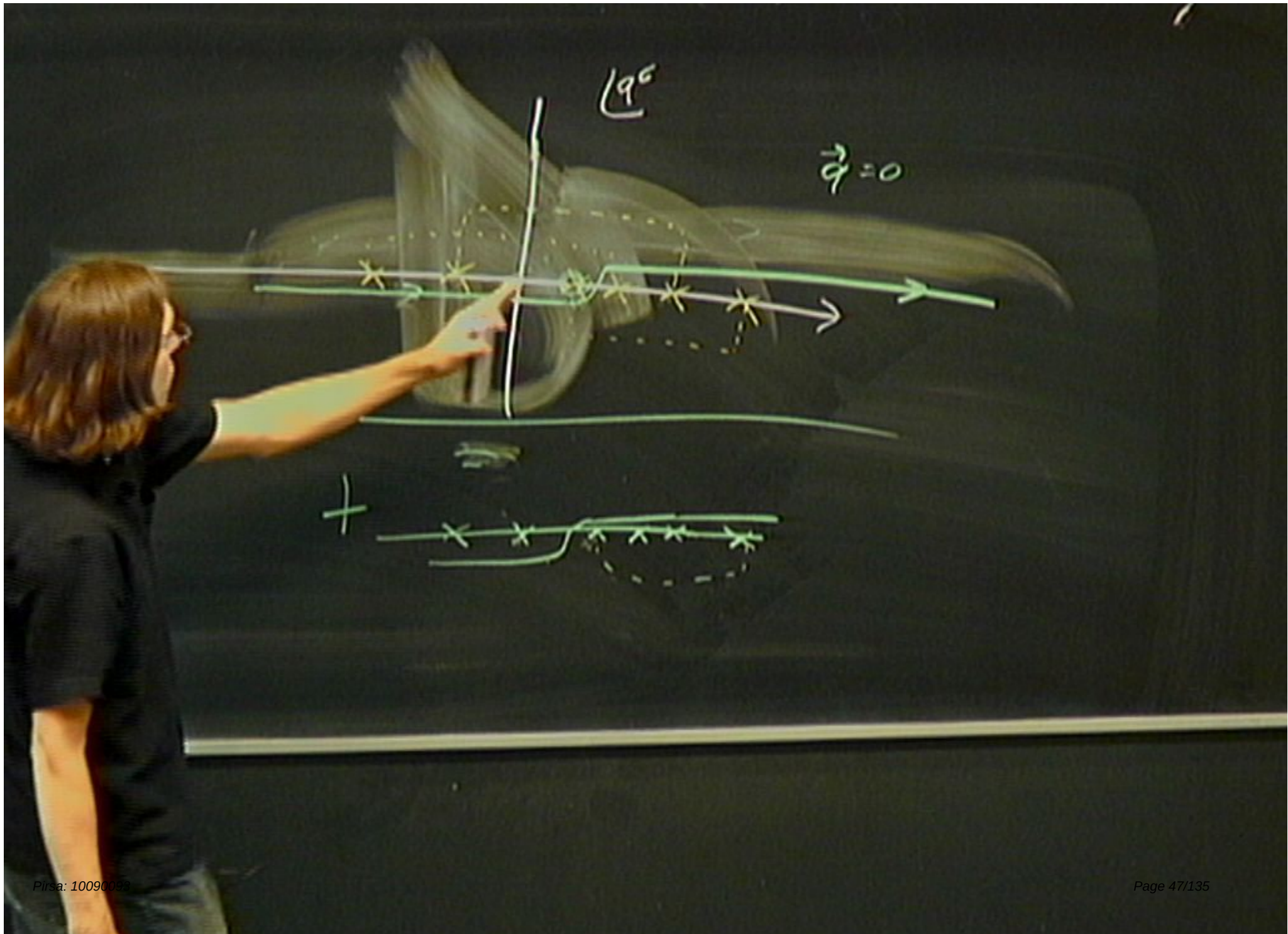


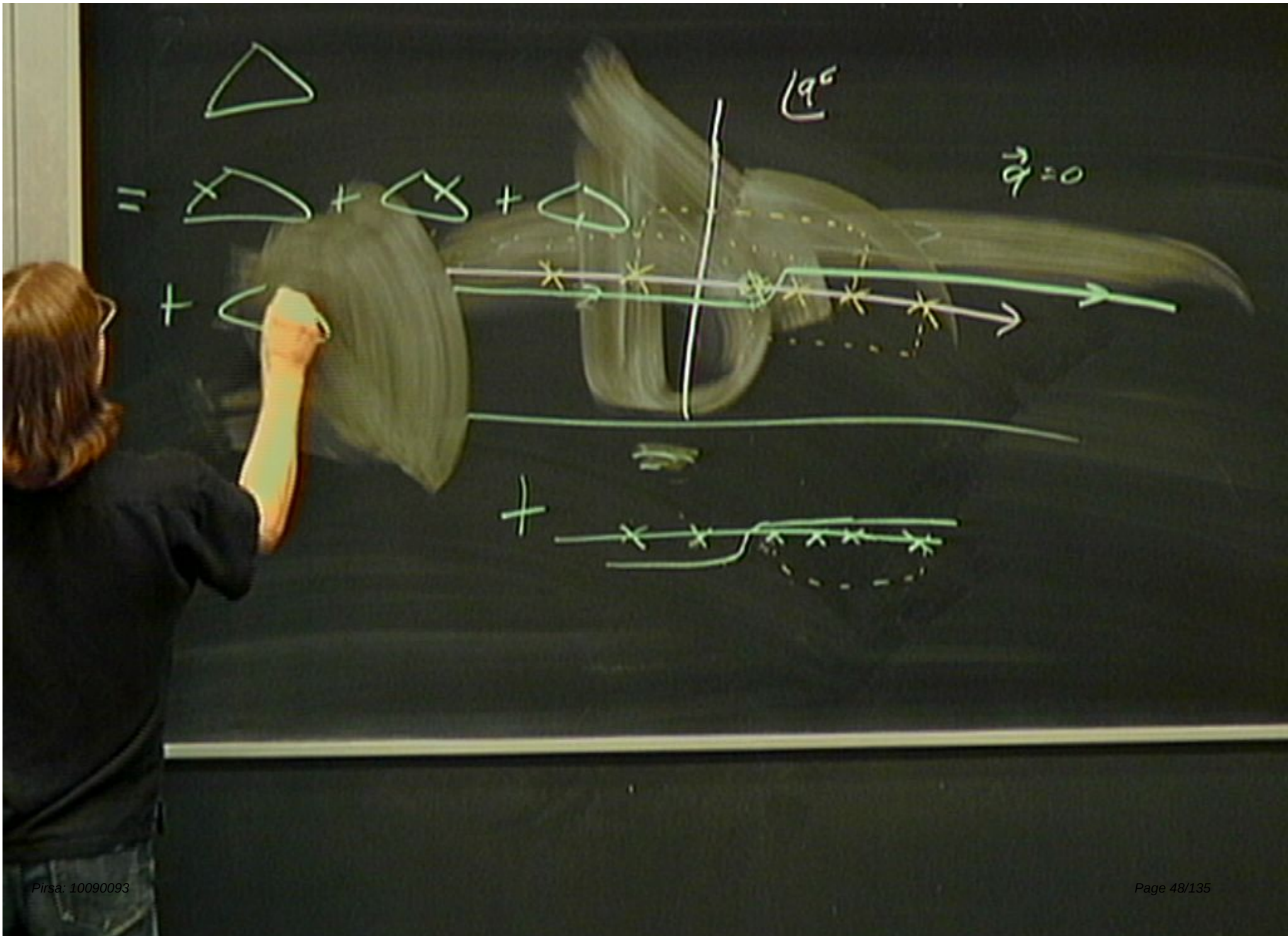


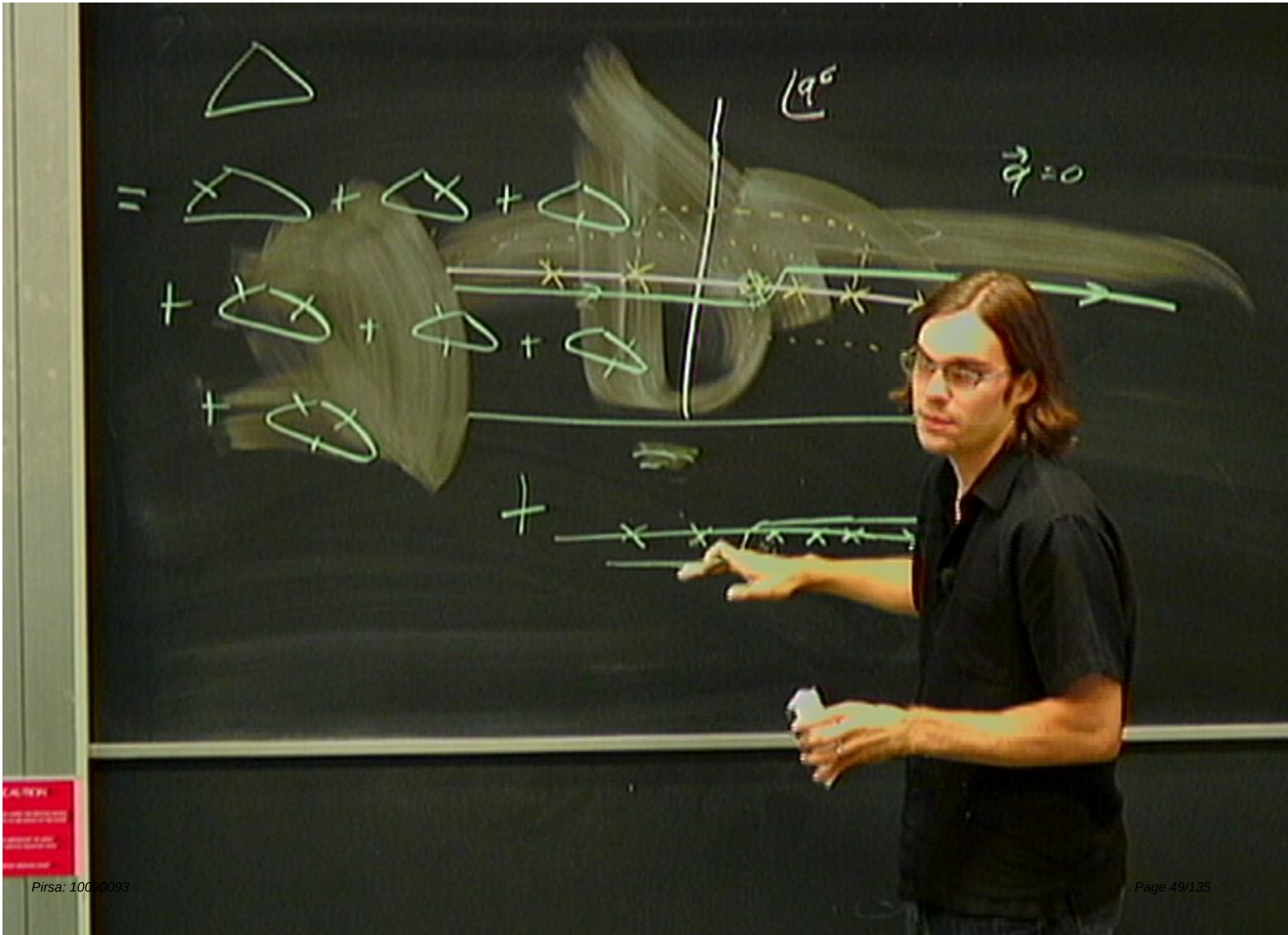












(e.o. 677.)

$$A_n^{\text{loop}}(p_1 \dots p_n) = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \dots, p_n, q, -q).$$

Consider

19.0.677.

$$A_n^{\text{loop}}(p_1 \dots p_n) = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \dots, p_n, q, -q).$$

Consider instead net. amplitude.

(4.0.677)

$$A_n^{\text{loop}}(p_1 \dots p_n) = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \dots, p_n, q, -q).$$

Consider instead net. amplitude.

$$\langle 0 | [O_1(x), O_2(y)]$$

(4.0.677)

$$A_n^{\text{loop}}(p_1 \dots p_n) = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \dots, p_n, q, -q).$$

Consider instead net. amplitude,

$$\langle 0 | [O_1(x), O_2(y)] | 0 \rangle \propto \theta(x-y)$$

14.0.677

$$A_n^{\text{loop}}(p_1 \dots p_n) = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \dots, p_n, q, -q).$$

Consider instead net. amplitude.

$$\langle 0 | [O_1(x), O_2(y)] | 0 \rangle \propto \theta(x-y)$$

$$e^{i\theta(x)} |0\rangle$$

(4.0.677)

$$A_n^{\text{loop}}(p_1 \dots p_n) = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \dots, p_n, q, -q).$$

Consider instead net. amplitude $\langle 0 | e^{i q_1(x)} e^{i q_2(y)} | 0 \rangle$

$$\langle 0 | [O_1(x), O_2(y)] | 0 \rangle \quad O(x-y)$$

(4.0.677)

$$A_n^{\text{loop}}(p_1 \dots p_n) = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \dots, p_n, q, -q).$$

Consider instead net. amplitude $\langle 0 | e^{iQ_1(x)} e^{iQ_2(y)} | 0 \rangle$

$$\langle 0 | [Q_1(x), Q_2(y)] | 0 \rangle \propto \delta(x-y)$$

[

$$\langle 0 | e^{i \dots} Q_1(x) e^{-i \int dx(x)} | 0 \rangle$$

(c.o. 677)

$$A_n^{\text{loop}}(p_1 \dots p_n) = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \dots, p_n, q, -q).$$

Consider instead net. amplitude $\langle 0 | e^{i q(x)} O_2 e^{-i q(x)} | 0 \rangle$

$$\langle 0 | [O_1(x), O_2(y)] | 0 \rangle \propto \delta(x-y)$$

$$[\dots \langle 0 | e^{i \dots} O_2(y) e^{-i \int dx(x)} | 0 \rangle$$

14.0.677

$$A_n^{\text{loop}}(p_1 \dots p_n) = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \dots, p_n, q, -q).$$

Consider instead vet. amplitude $\langle 0 | e^{i q(x)} O_2 e^{i O_1(x)} | 0 \rangle$

$$\langle 0 | [O_1(x), O_2(y)] | 0 \rangle \propto \delta(x-y)$$

$$\langle 0 | e^{i \dots} O_2(y) e^{-i \int^y \delta x(x)} | 0 \rangle$$

$$= \frac{-i}{p^2 - i\epsilon} \rightarrow \frac{-i}{p^2 - i\epsilon p^0}$$



$\mathcal{L}_q$

$\vec{q}=0$

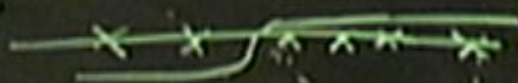
$$= \triangle_x + \triangle_x + \triangle_x$$

$$+ \triangle_x + \triangle_x + \triangle_x$$

$$+ \triangle_x$$

$\mathcal{L}_R$

+





$1q^6$

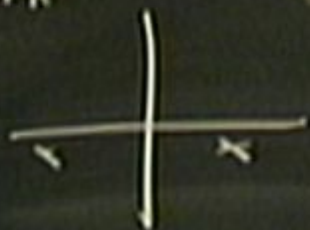
$\vec{q}=0$

$$= \text{triangle with } x \text{ on left} + \text{triangle with } x \text{ on right} + \text{triangle with } x \text{ on top}$$

$$+ \text{triangle with } x \text{ on left} + \text{triangle with } x \text{ on right} + \text{triangle with } x \text{ on top}$$

$$+ \text{triangle with } x \text{ on left}$$

$4R$



$$+ \text{line with } x \text{ marks and a dashed arc below it}$$

(4.0.677)

$$A_n^{\text{loop}}(p_1 \dots p_n) = \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4 i} A_{n+2}^{\text{tree}}(p_1, \dots, p_n, q, q).$$

Consider 1 instad vert. amplitude $\langle 0 | T \{ \phi(x) \phi(y) \} | 0 \rangle$

$$\langle 0 | [O_1(x), O_2(y)] | 0 \rangle \propto \delta(x-y)$$

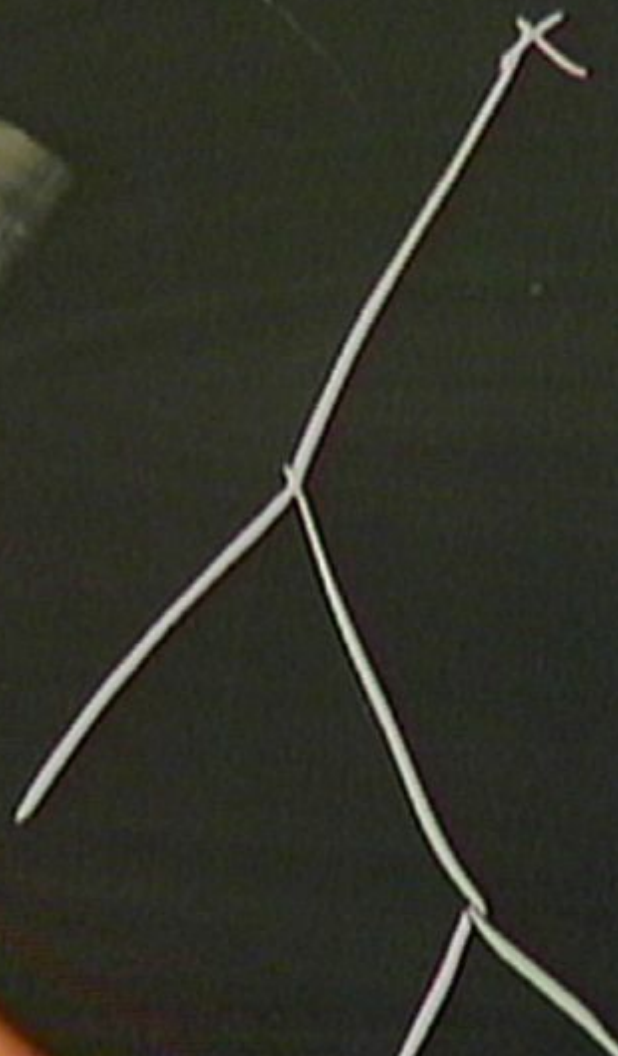
$$\langle 0 | e^{i \int \phi(x) \phi(y)} | 0 \rangle$$

$$= \frac{i}{p^2 - i\epsilon} \rightarrow \frac{i}{p^2 - i\epsilon}$$

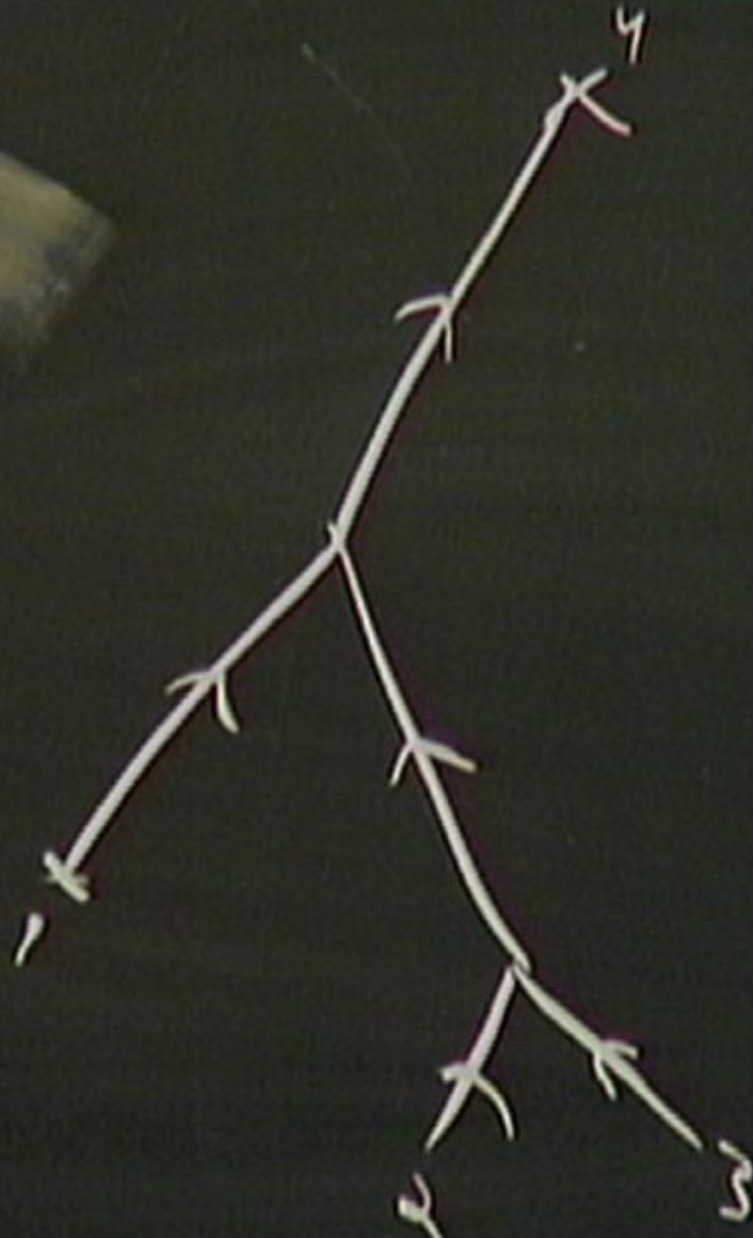
Rel. n-pt let.: one special las



Red. n-pt int.: one special las

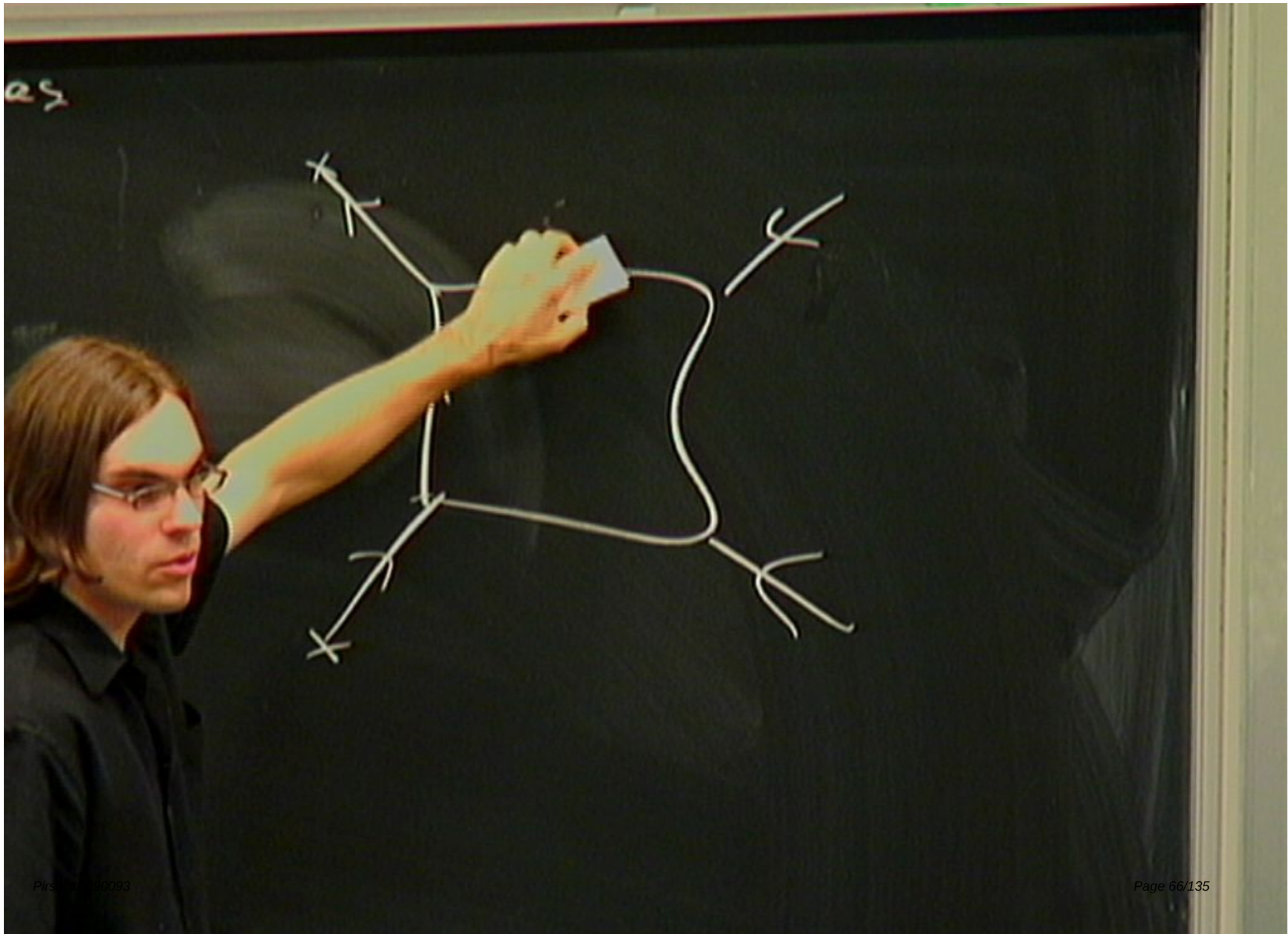


Rel. n-pt let.: one special las

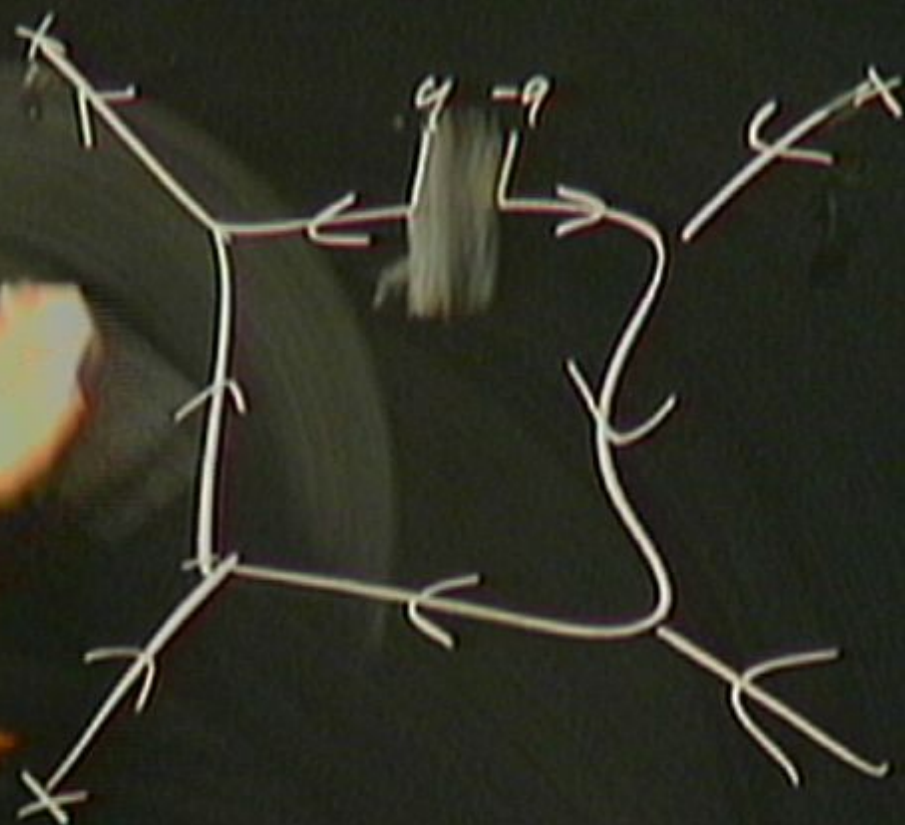


Rel. n-pt let.: one special lag

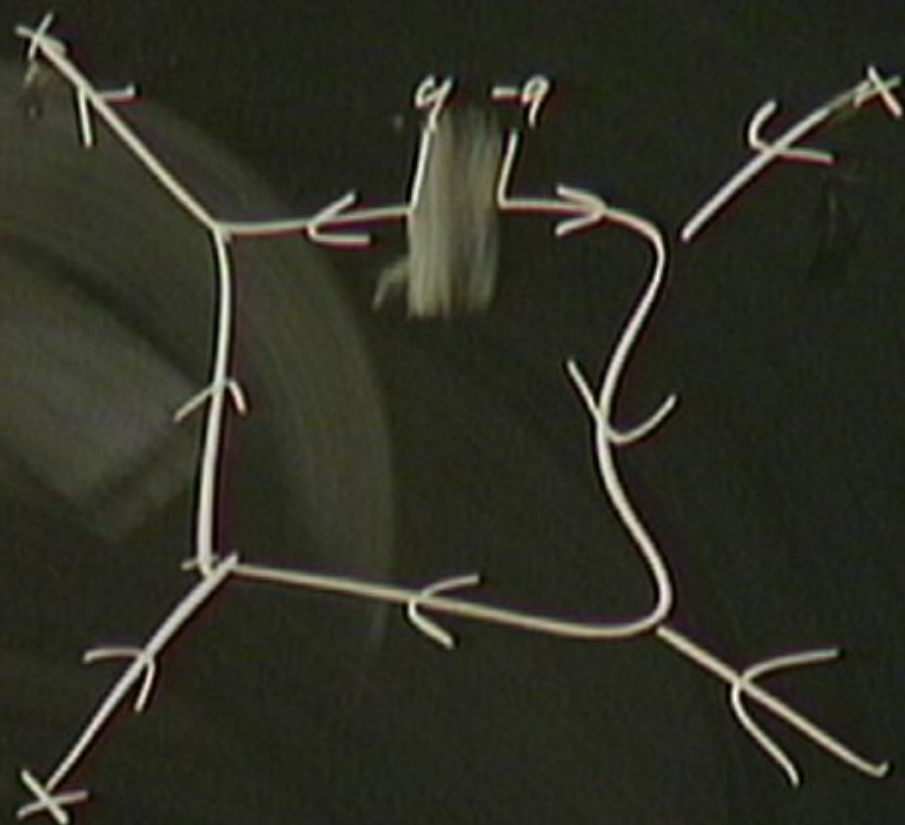




Q2



Q2



$$A_n^{\text{loop}}(p_1 \dots p_n) = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \dots, p_n, q, -q).$$

Consider instead net. amplitude $\langle 0 | e^{i\phi_1(x)} e^{i\phi_2(y)} | 0 \rangle$

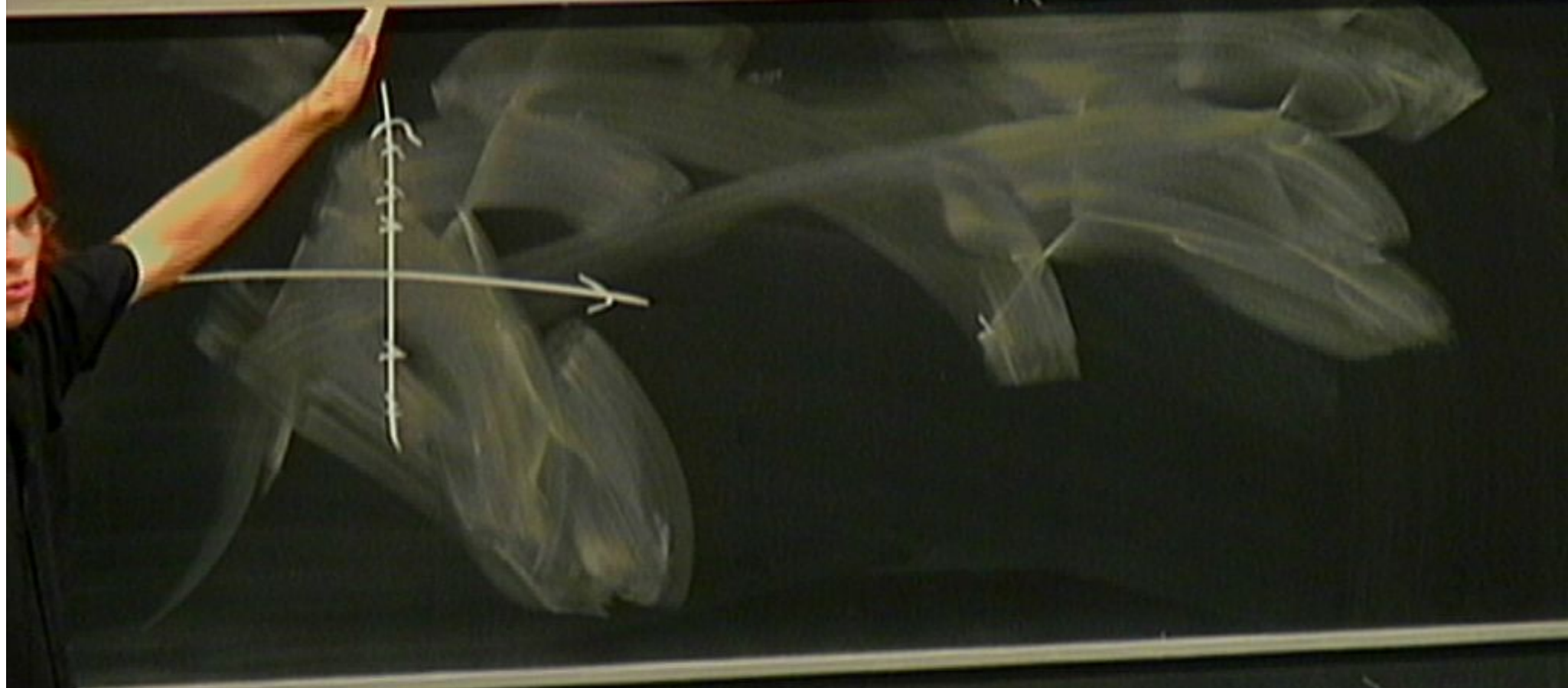
$$\langle 0 | [\phi_1(x), \phi_2(y)] | 0 \rangle \propto \theta(x-y)$$

$$\langle 0 | e^{i\phi_1(x)} e^{i\phi_2(y)} | 0 \rangle$$

$$\frac{-i}{p^2 - i\epsilon} \rightarrow \frac{-i}{p^2 - i\epsilon p^0}$$



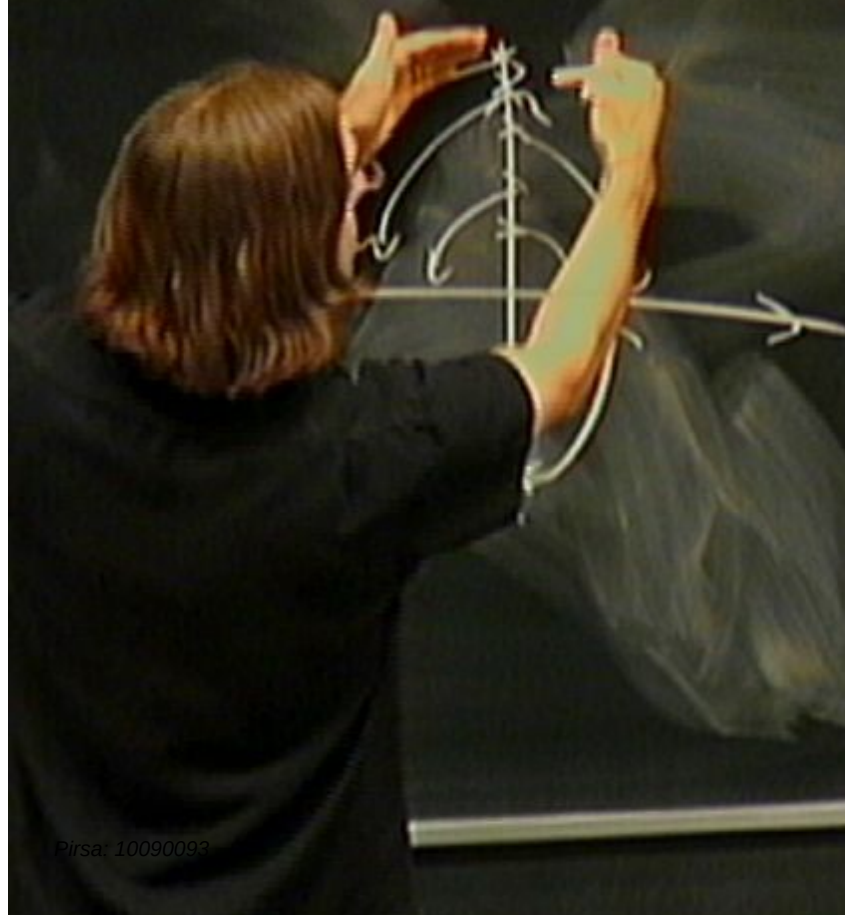
$$= \frac{-i}{p^2 - i\epsilon} \rightarrow \frac{-i}{p^2 - i\epsilon p^0}$$





=

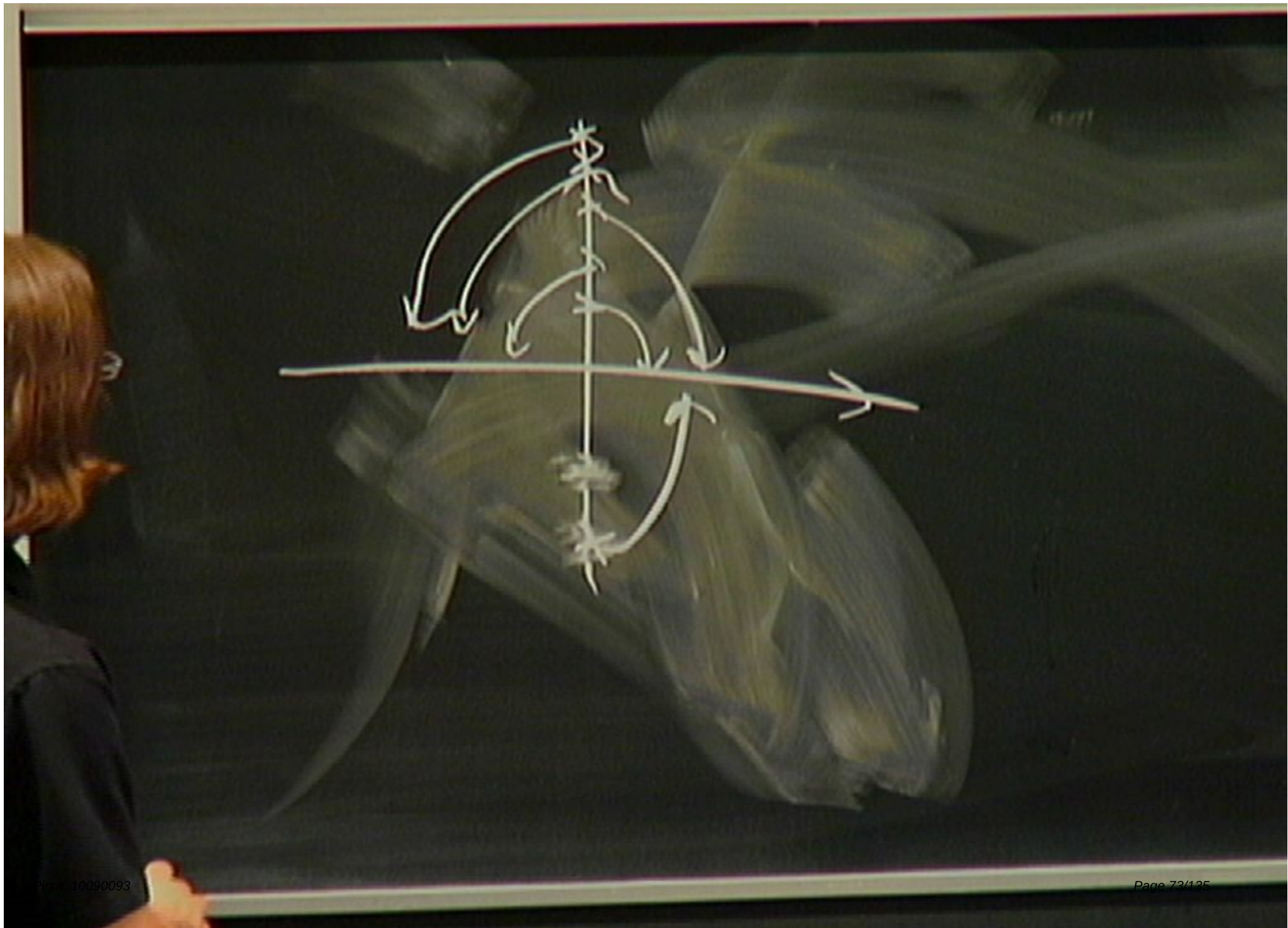
$$\frac{-i}{p^2 - i\epsilon} \rightarrow \frac{-i}{p^2 - i\epsilon p^0}$$

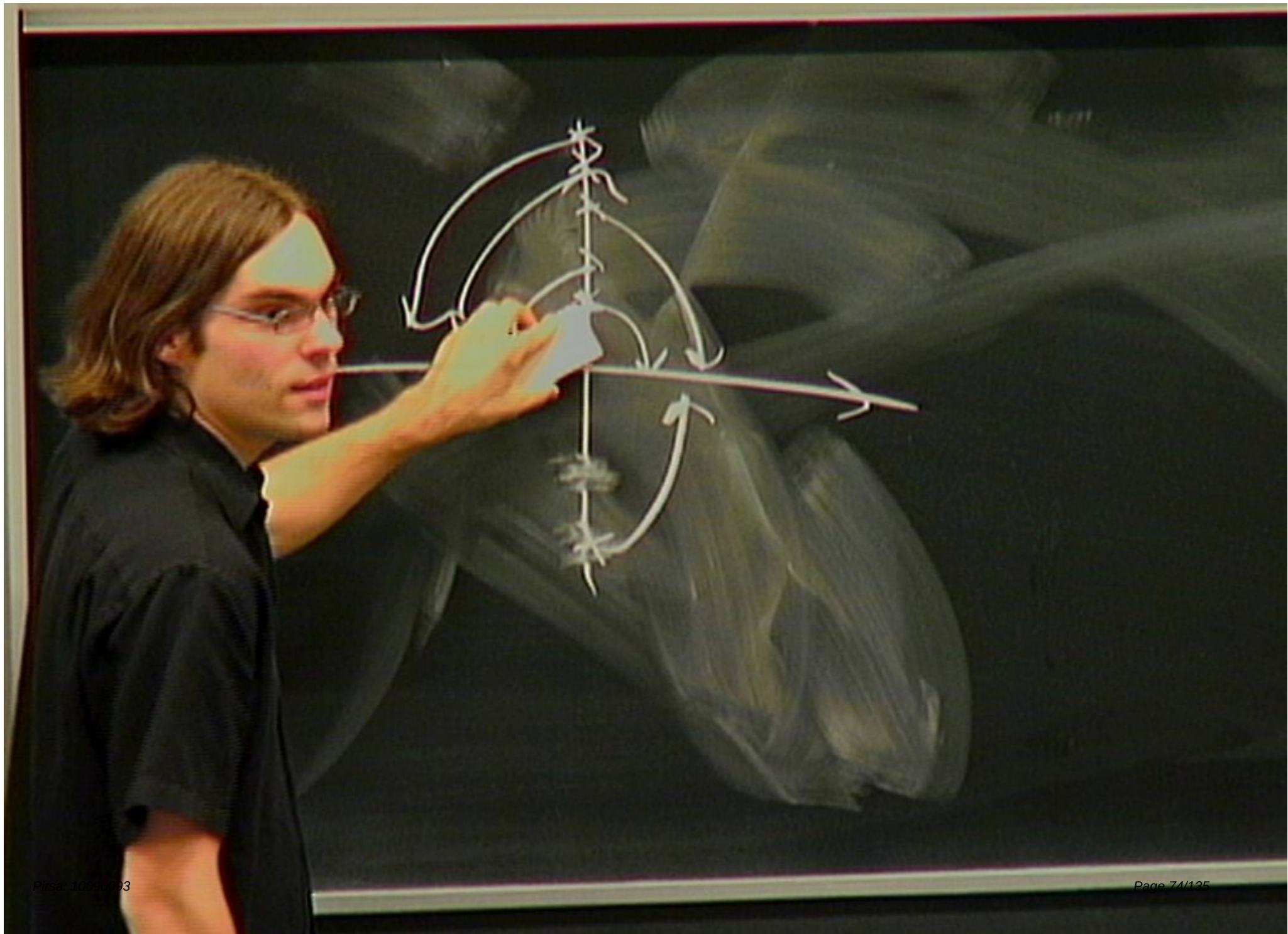


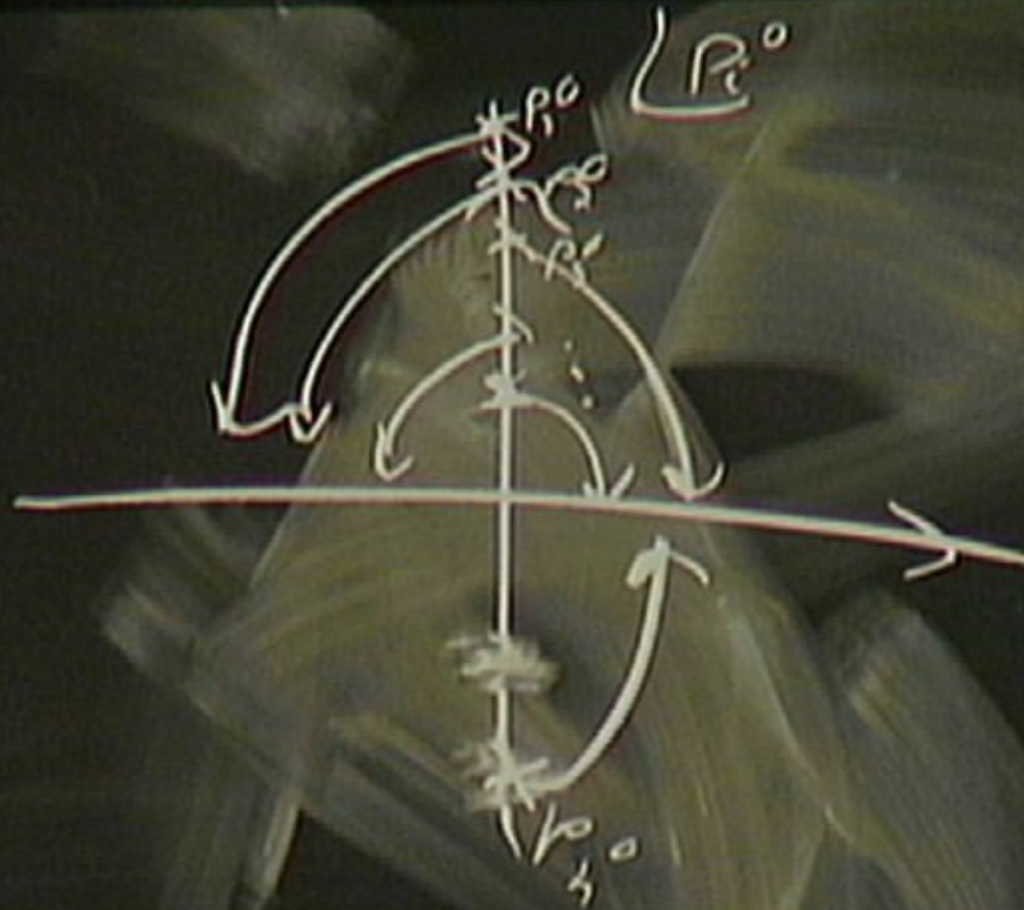


$$= \frac{-i}{p^2 - i\epsilon} \rightarrow \frac{-i}{p^2 - i\epsilon p^0}$$



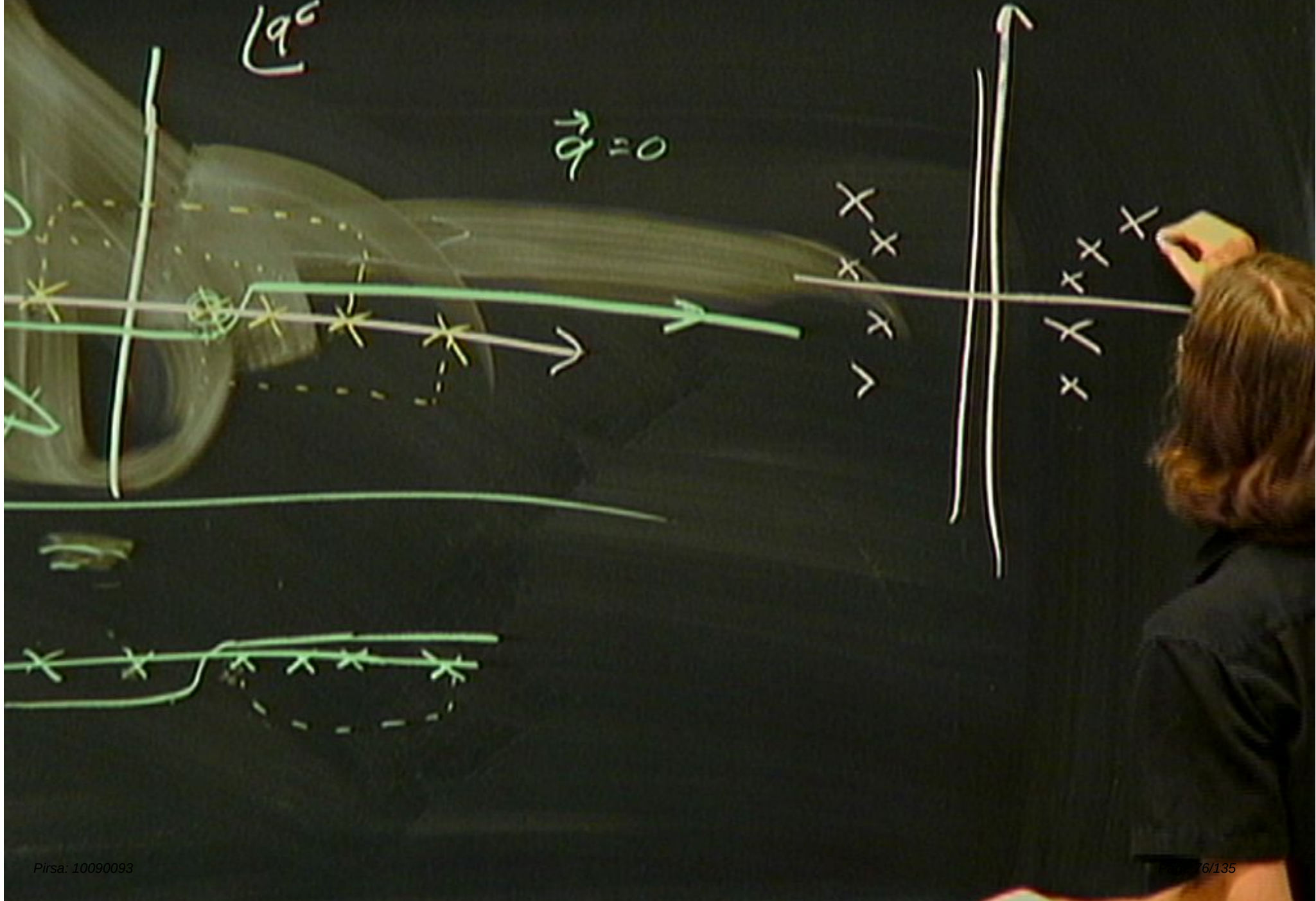






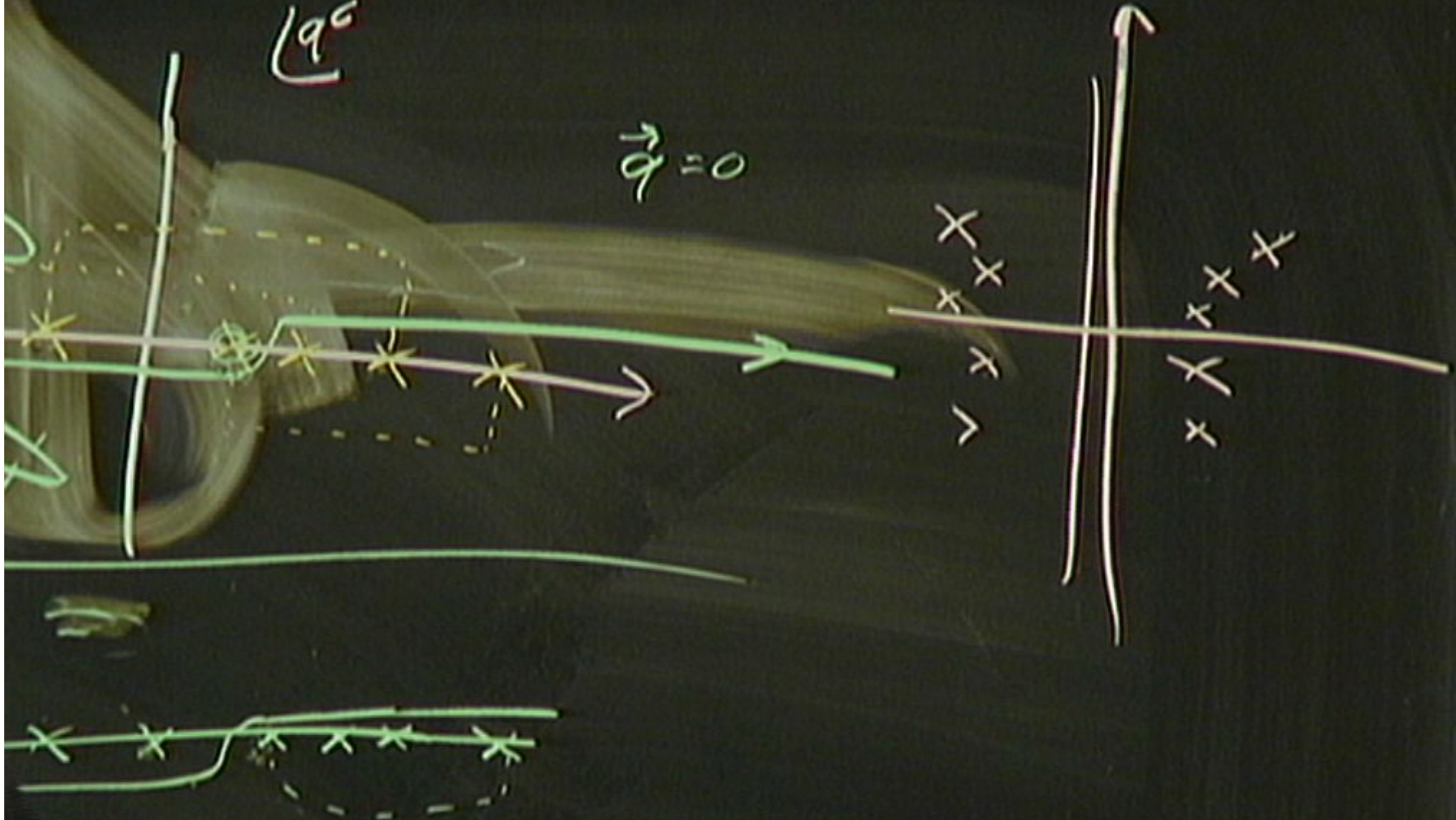
19°

$$\vec{q} = 0$$



$\langle q^c \rangle$

$$\vec{q} = 0$$



$$A_n^{\text{1-loop}}(p_1 \dots p_n)_R = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \dots, p_n, q, -q)_R$$

Consider instead net. amplitude $\langle 0 | e^{i q(x)} \phi_2(x) e^{i q(y)} \phi_1(y) | 0 \rangle$

$$\langle 0 | [\phi_1(x), \phi_2(y)] | 0 \rangle \propto \theta(x-y)$$

$$\langle 0 | e^{i \dots} \phi_2(y) e^{-i \int^y \delta x(x)} \phi_1(x) | 0 \rangle$$

$$= \frac{-i}{p^2 - i\epsilon} \rightarrow \frac{-i}{p^2 - i\epsilon p^0}$$

$$A_n^{\text{loop}}(p_1, \dots, p_n)_R = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \dots, p_n, q, -q)_R$$

Consider instead ret. amplitude

$$\langle 0 | [O_1(x), O_2(y)] | 0 \rangle \sim O(x-y)$$

$$\langle 0 | e^{i \dots} O_2(y) e^{-i \int^y \delta x(x)} | 0 \rangle$$

$$= \frac{-i}{p^2 - i\epsilon} \rightarrow \frac{-i}{p^2 - i\epsilon p^0}$$

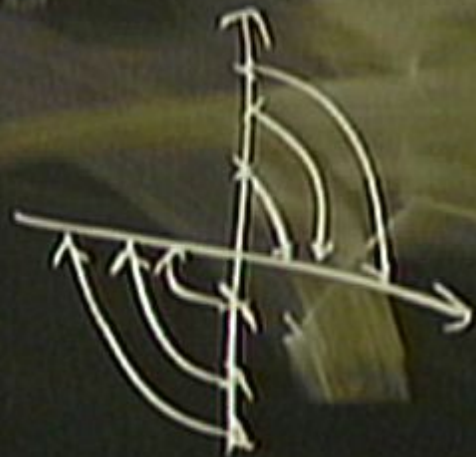
$$= \frac{-i}{p^2 - i\epsilon} \rightarrow \frac{-i}{p^2 - i\epsilon p^0}$$

(0)

Ret.

$\mathcal{L}_{R^0}$

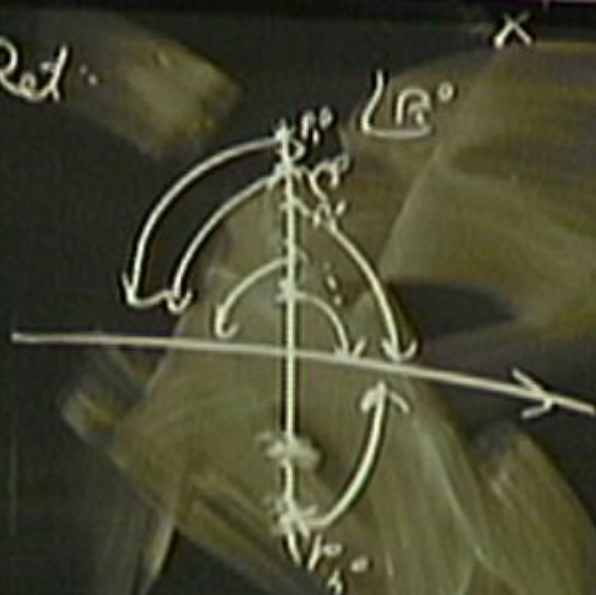
$y = y$



$$= \frac{-i}{p^2 - i} \rightarrow \frac{-i}{p^2 - i \epsilon p^0}$$

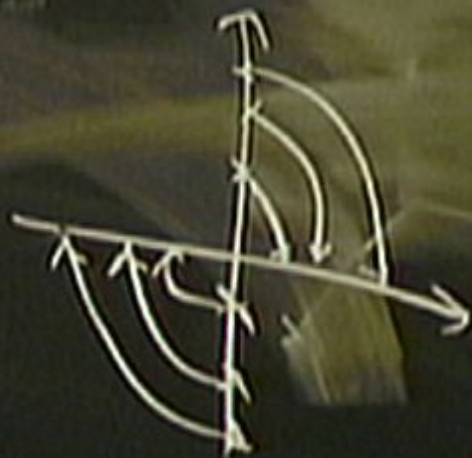
(0)

Ret.



$\mathbb{R}^0$

$y = 9$



15.0.677

$$A^{loop}|_T$$

$$A_n^{loop}(p_1 \dots p_n)_R = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+1}^{tree}(p_1 \dots p_n, q, -q)_R$$

Consider instead net. amplitude

$$\langle 0 | [\phi_1(x), \phi_2(y)] | 0 \rangle \propto \theta(x-y)$$

$$\langle 0 | \phi_1(x) \phi_2(y) | 0 \rangle$$

$$\langle 0 | \phi_1(x) \phi_2(y) e^{-i \int d^4 x \mathcal{H}(x)} | 0 \rangle$$

$$= \frac{-i}{p^2 - i\epsilon} \rightarrow \frac{-i}{p^2 + i\epsilon}$$

19.0.677.

$$A^{loop}|_T$$

$$A_n^{loop}(p_1 \dots p_n) = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{tree}(p_1, \dots, p_n, q, -q)$$

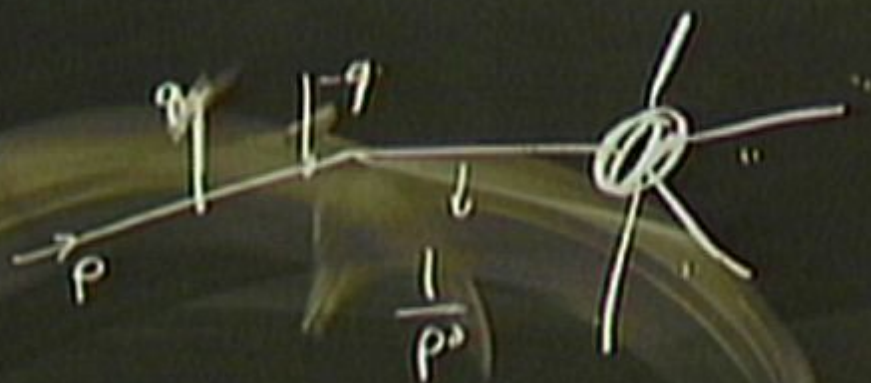
True off-shell

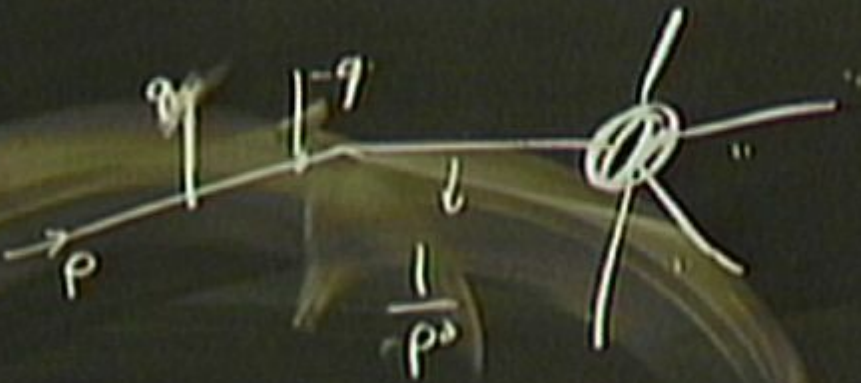
Consider instead net. amplitude

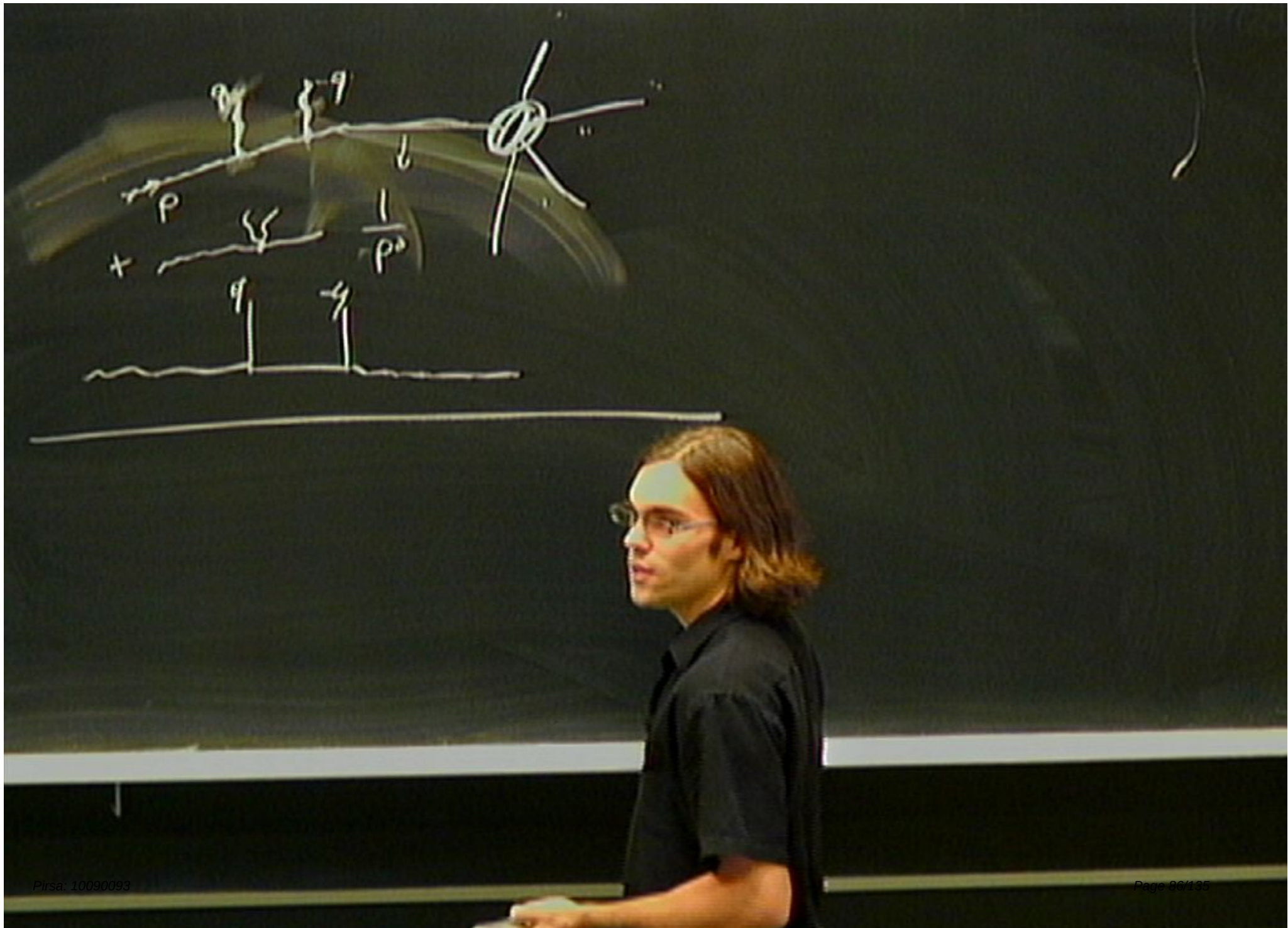
$$\langle 0 | [\phi_1(x), \phi_2(y)] | 0 \rangle \propto \langle 0 | \phi_1(x) \phi_2(y) | 0 \rangle$$

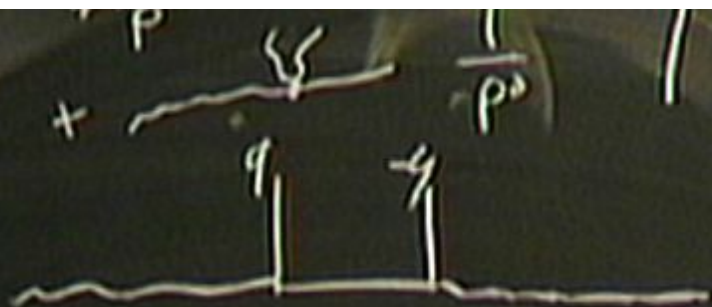
$$\langle 0 | e^{i \dots} \phi_1(x) e^{-i \dots} \phi_2(y) | 0 \rangle$$

$$= \frac{-i}{p^2 - i\epsilon} \rightarrow \frac{-i}{p^2 + i\epsilon}$$

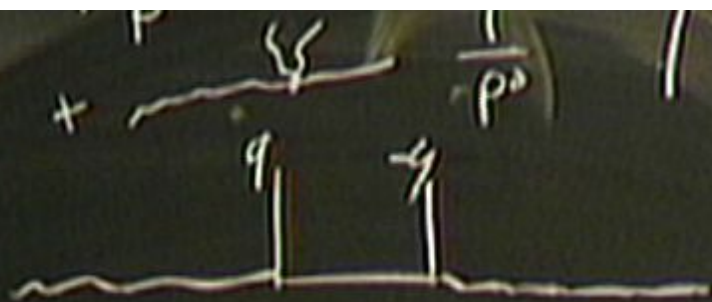




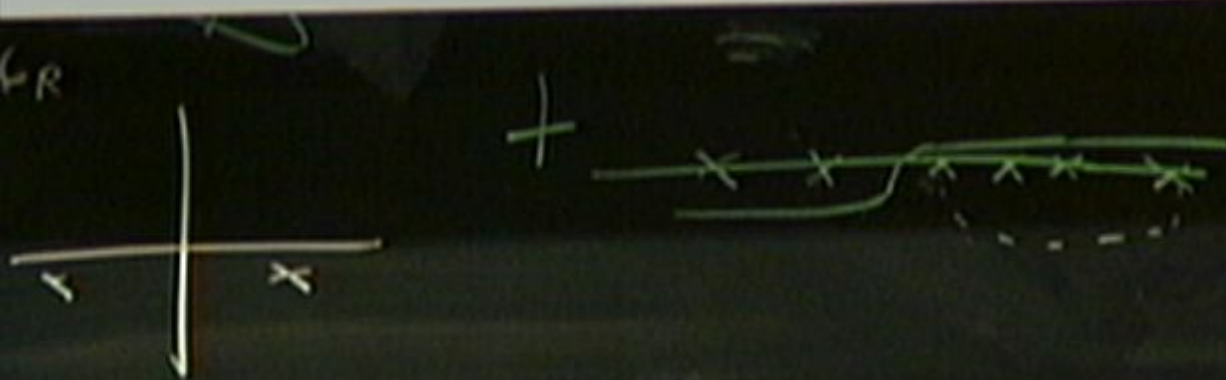




$N=1$ SUSY, for a massless



$N=1$ SUSY, for a massless particle



$$A_n^{\text{loop}}(p_1 \dots p_n)_R = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1 \dots p_n, q, -q)_R$$

Consider instead net. amplitude

$$| [O_1(x), O_2(y)] | 0 \rangle \sim O(x-y)$$

$$\langle 0 | e^{i \dots} O_2(y) e^{-i \int^y \delta H(x)} | 0 \rangle$$

$$= \frac{-i}{p^2 - i\epsilon} \rightarrow \frac{-i}{p^2 - i\epsilon p^0}$$

15.0.677.

$$A^{1-loop} | \tau$$

$$A_n^{1-loop}(p_1 \dots p_n) = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{tree}(p_1, \dots, p_n, q, -q)$$

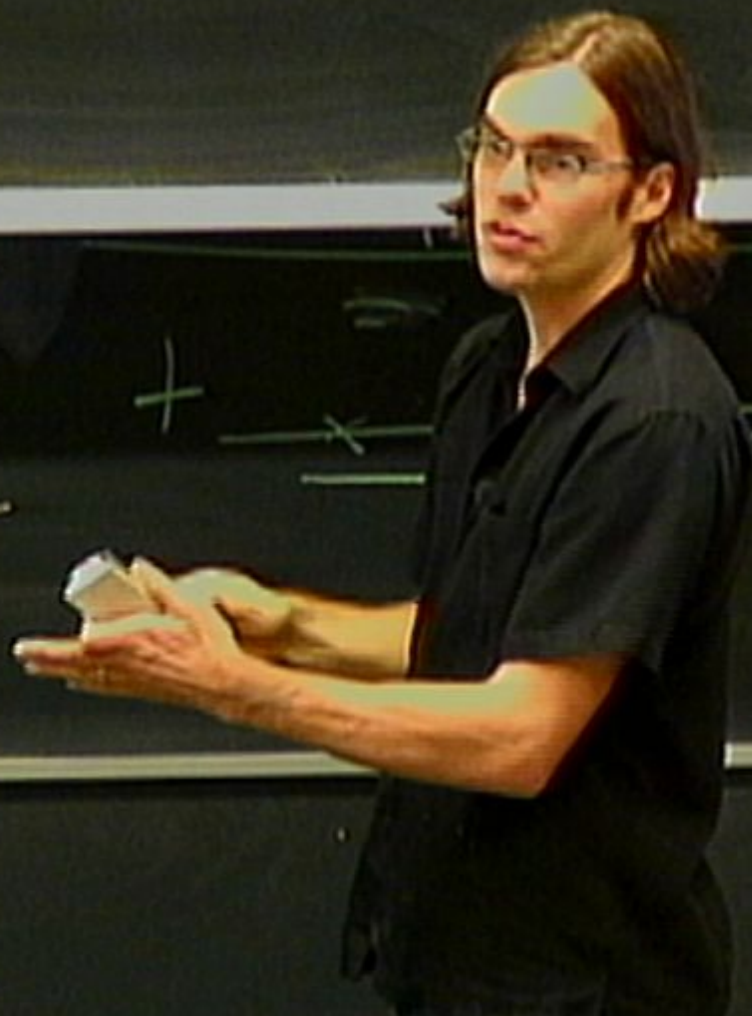
True off-shell

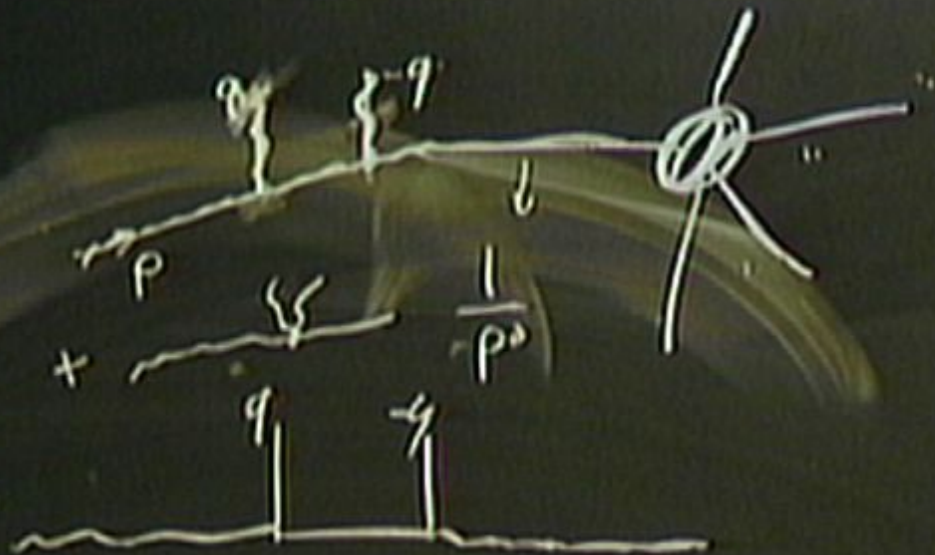
Consider instead net. amplitude $\langle q e^{i q(x)} q e^{i q(y)} | 0 \rangle$
 $\langle [O_1(x), O_2(y)] | 0 \rangle \propto O(x-y)$

$$\langle 0 | e^{i \dots} O_2(y) e^{-i \int^y \partial \mu(x)} | 0 \rangle$$

$$= \frac{-i}{p^2 - i\epsilon} \rightarrow \frac{-i}{p^2 - i\epsilon p^0}$$

$N=1$ SUSY, for a massless particle
 $N=2$ SUSY, massive





$N=1$ SUSY, for a massless particle
 $N=2$ SUSY, massive

eq. 6.77

$$A_n^{\text{loop}}|_{\pi}$$

$$A_n^{\text{loop}}(p_1 \dots p_n)_R = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1 \dots p_n, q, -q)_R$$

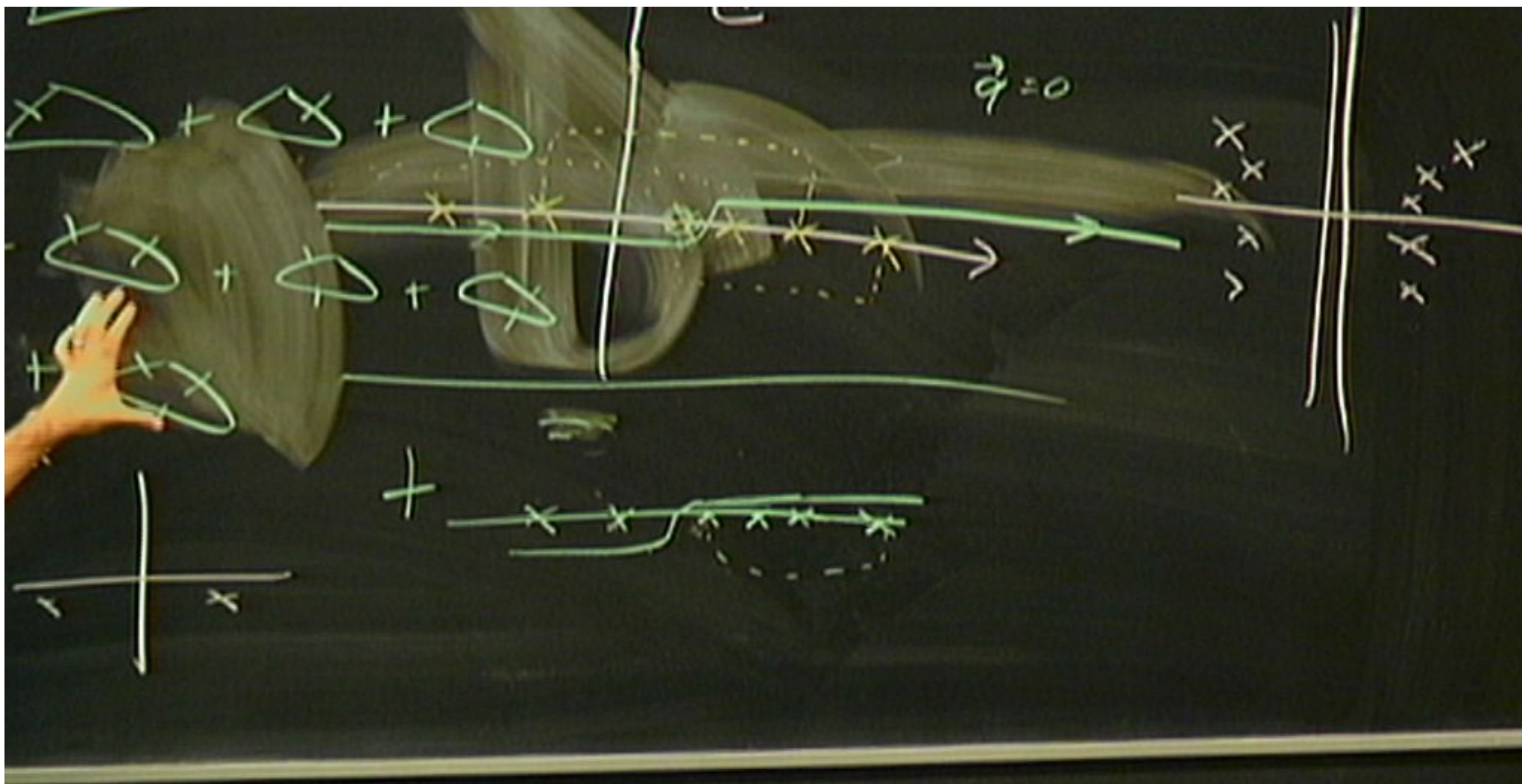
True off-shell

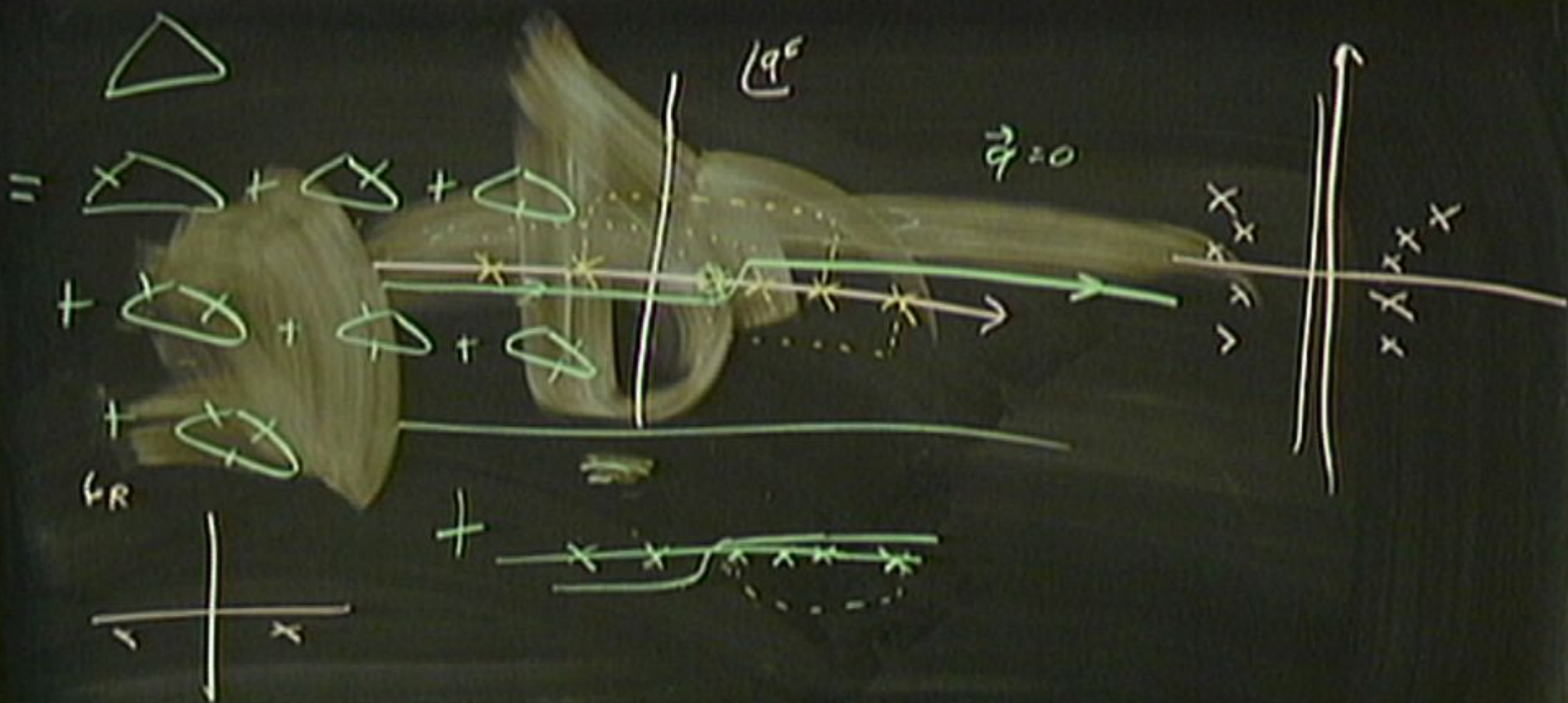
Consider instead ret. amplitude

$$\langle 0 | [O_1(x), O_2(y)] | 0 \rangle \sim \langle 0 | e^{i q(x)} O_2(y) e^{-i q(x)} | 0 \rangle$$

$$= \frac{-i}{p^2 - i\epsilon} \rightarrow \frac{-i}{p^2 + i\epsilon}$$

$$\langle 0 | e^{i \dots} O_2(y) e^{-i \int^y \partial \chi(x)} | 0 \rangle$$





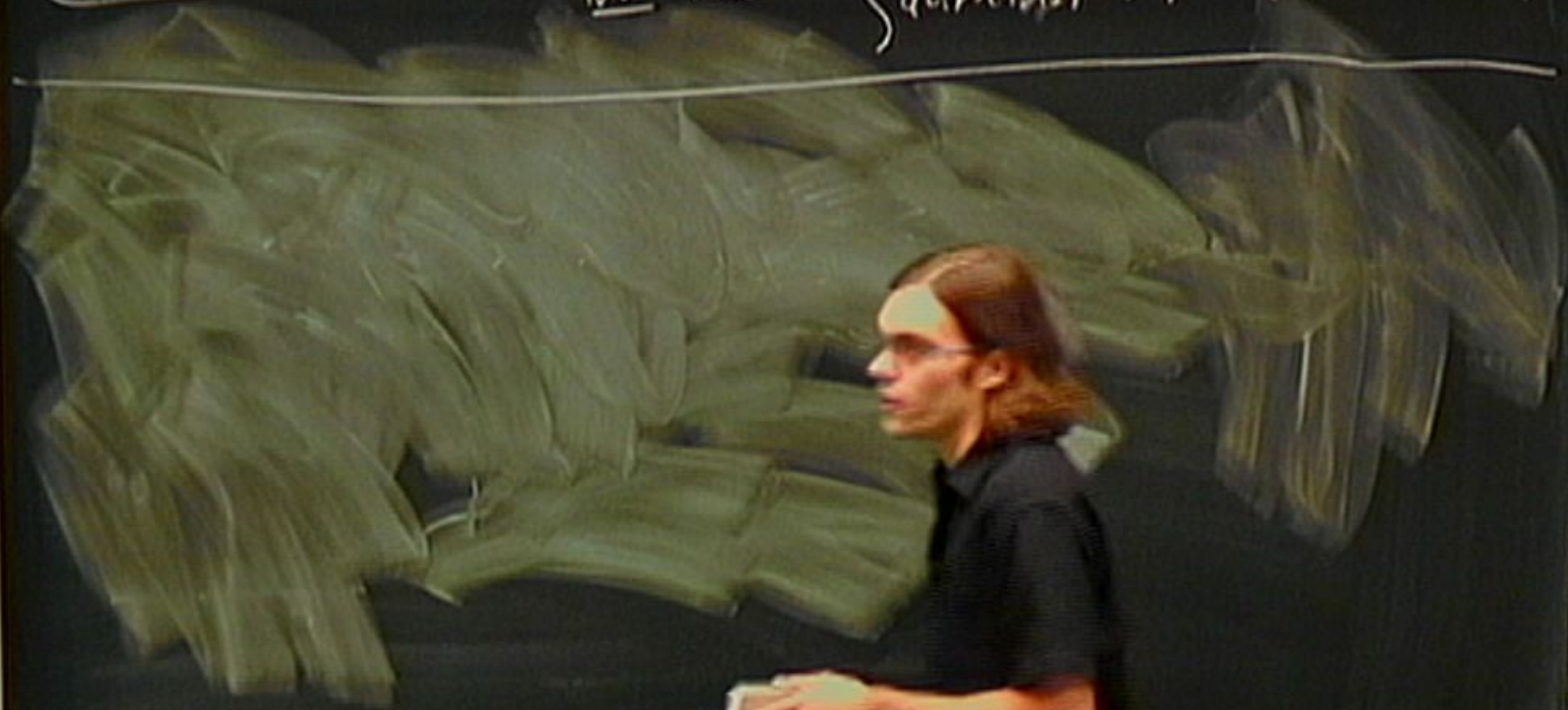
(2-bops). In general, NOT true

(dli)

(2-bops), In general, NOT true $\int d\vec{p}_1 d\vec{p}_2 A^{(2)}(p_1, p_2)$

(2-loops), In general, NOT true $\int d\vec{p}_1 d\vec{p}_2 A^{\text{tree}}(p_1, \dots, p_n, q_1, q_2, q_3, q_4)$

(2-loops). In general, NOT true $\int d\vec{q} \delta(q) A^{\text{tree}}(p_1, \dots, p_n, q, -q, q_3, -q_3)$



(2-bps), In general, NOT true $\int d\vec{r} \psi(\vec{r}) A^{\text{top}}(p_1 \dots p_n, q_1, q_2, q_3, q_4)$



(2-loops). In general, NOT true $\int d\Omega_{\text{loop}} A^{\text{loop}}(p_1, \dots, p_n, q_1, q_2, q_3, q_4)$



(2-loops). In general, NOT true $\int d^4k \delta(k^2) A^{\text{tree}}(p_1, \dots, p_n, q_1, q_2, q_3, q_4)$



(2-loop). In general, NOT true $\int d^4x \delta(x) A^{\text{tree}}(p_1, \dots, p_n, q_1, q_2, q_3, q_4)$



0+1 dim.



(2-loops). In general, NOT true $\int d\Omega(p) A^{\text{tree}}(p_1, \dots, p_n, q_1, q_2, q_3, q_4)$



$O(1) \text{ dim.}$

$$\frac{1}{w^2 - (e_1 + e_2 + e_3)^2}$$

$=$

(2-loop). In general, NOT true $\int d\text{Lips}(q, \dots) A^{\text{tree}}(p_1, \dots, p_n, q_1, \dots, q_{3-n})$

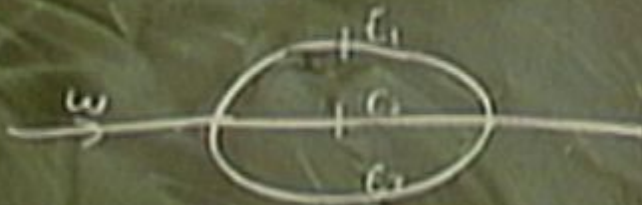


$O(1) \text{ dim.}$

$$\frac{1}{w^2 - (\epsilon_1 + \epsilon_2 + \epsilon_3)^2}$$

$$= \int \frac{dw_1 dw_2}{(w_1^2 - \epsilon_1^2)(w_2^2 - \epsilon_2^2)((w_1 + w_2 - w)^2 - \epsilon_3^2)}$$

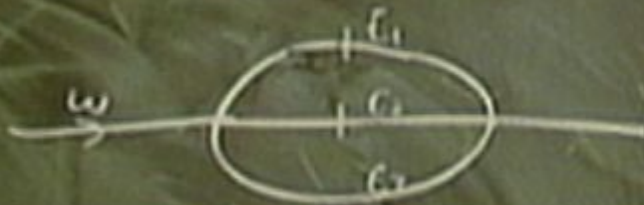
(2-loop). In general, NOT true $\int d\text{Lips}(q, \dots) A^{\text{tree}}(p_1, \dots, p_n, q_1, \dots, q_{3-n})$



0+1 dim.

$$= \int \frac{dw_1 dw_2}{(w_1^2 - \epsilon_1^2)(w_2^2 - \epsilon_2^2)((w_1 + w_2 - \omega)^2)}$$

(2-loop). In general, NOT true $\int d\text{Lips}(q, \dots) A^{\text{tree}}(p_1, \dots, p_n, q_1, \dots, q_{n-2})$



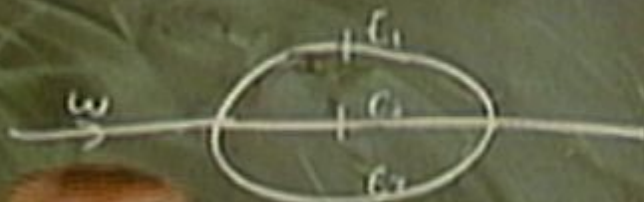
0+1 dim.

$$\frac{1}{\omega^2 - (\epsilon_1 + \epsilon_2 + \epsilon_3)^2}$$



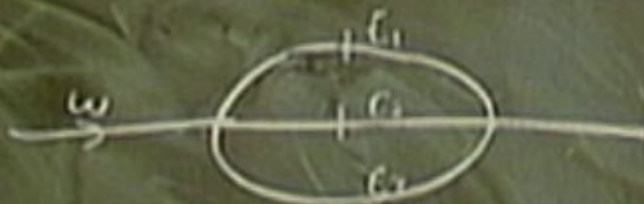
$$= \int \frac{dw_1 dw_2}{(w_1^2 - \epsilon_1^2)(w_2^2 - \epsilon_2^2)((w_1 + w_2 - \omega)^2 - \epsilon_3^2)}$$

(2-loops). In general, NOT true $\int d\text{Lips}(q, \dots) A^{\text{tree}}(p_1, \dots, p_n, q_1, \dots, q_{n-2})$



$$\frac{(\epsilon_1 + \epsilon_2 + \epsilon_3)^2}{w_1 dw_2} \frac{(w_1^2 - \epsilon_1^2)(w_2^2 - \epsilon_2^2)((w_1 + w_2 - w)^2 - \epsilon_3^2)^2}{(w_1^2 - \epsilon_1^2)(w_2^2 - \epsilon_2^2)((w_1 + w_2 - w)^2 - \epsilon_3^2)^2}$$

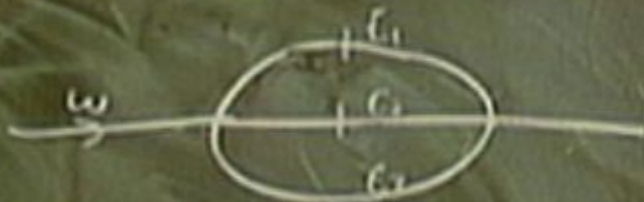
(2-loops). In general, NOT true $\int d\Omega_{\text{loop}} A^{\text{loop}}(p_1, \dots, p_n, q_1, q_2, q_3, \dots)$



0+1 dim.

$$= \int \frac{dw_1 dw_2}{(w_1^2 - \epsilon_1^2)(w_2^2 - \epsilon_2^2)((w_1 + w_2 - w)^2 - \epsilon_3^2)}$$

(2-loop). In general, NOT true $\int d^4x \delta(x) / \sum_{i=1}^3 (p_i \dots p_i, q_i, q_i, q_i - q_i)$

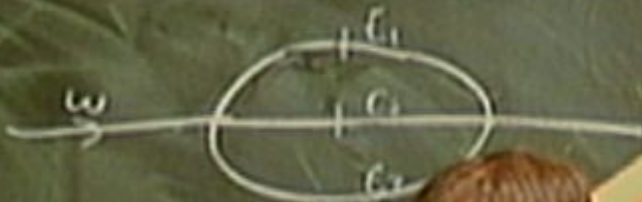


0+1 dim.

$$= \int \frac{dw_1 dw_2}{(w_1^2 - e_1^2)(w_2^2 - e_2^2)((w_1 + w_2 - w)^2 - e_3^2)}$$

(2-loop). In general, NOT true

$$\int d\vec{q}_1 d\vec{q}_2 \left| \sum_{\text{diagrams}} (p_1 \dots p_n, q_1, q_2, q_3, q_4) \right|$$



0+1 dim.



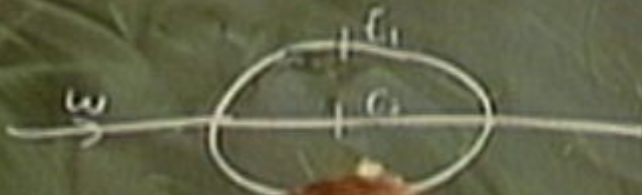
$$= \int \frac{dw_1 dw_2}{(w_1^2 + \epsilon_1)(w_2^2 + \epsilon_2)}$$

$$((w_1 + w_2 - w)^2 - \epsilon_3)^2$$

(2-loop). In general, NOT true

$$\int d^4x \delta(x) \left| \sum_{\text{diagrams}} (p_1 \dots p_n, q_1, -q_1, q_2, -q_2) \right.$$

$$\times \begin{cases} 4, & q_1 + q_2 \text{ add} \\ 3, & q_1 - q_2 \text{ subtract} \\ 2, \\ 1, \end{cases}$$



0+1 dim.

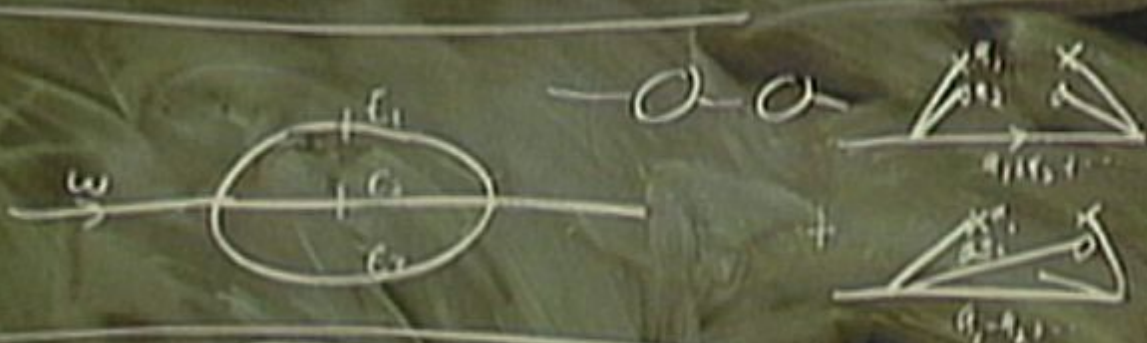
$$(\epsilon_2 + \epsilon_3)^2$$

$$(\epsilon_2^2) ((\omega_1 + \omega_2 - \omega)^2 - \epsilon_3^2)^2$$

(2-loop). In general, NOT true

$$\int d^4x \delta(x) \left| \sum_{\text{diagrams}} (p_1 \dots p_n, q_1, q_2, q_3, q_4) \right.$$

$$\times \begin{cases} 4, & q_1, q_2 \text{ add} \\ 3, & q_1, q_2 \text{ subtract} \\ 2, & q_1, q_2 \text{ don't interact} \end{cases}$$



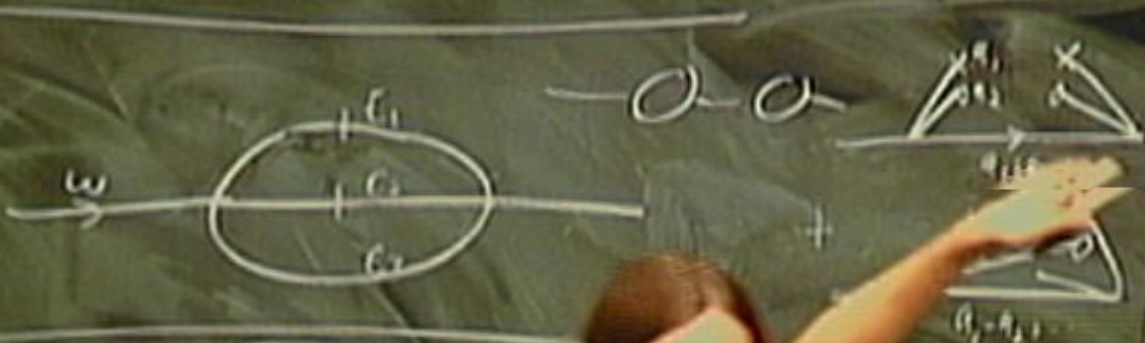
0+1 dim.

$$= \int \frac{dw_1 dw_2}{(w_1^2 - \epsilon_1^2)(w_2^2 - \epsilon_2^2)((w_1 + w_2 - w)^2 - \epsilon_3^2)}$$

(2-loop). In general, NOT true

$$\int d^4x \delta(x) \left| \sum_{\text{diagrams}} (p_1 \dots p_n, q_1, q_2, q_3, q_4) \right|$$

$\times \begin{cases} 4, & q_1, q_2 \text{ add} \\ 3, & q_1, q_2 \text{ subtract} \\ 1, & q_1, q_2 \text{ don't intersect} \end{cases}$



0+1 dim.

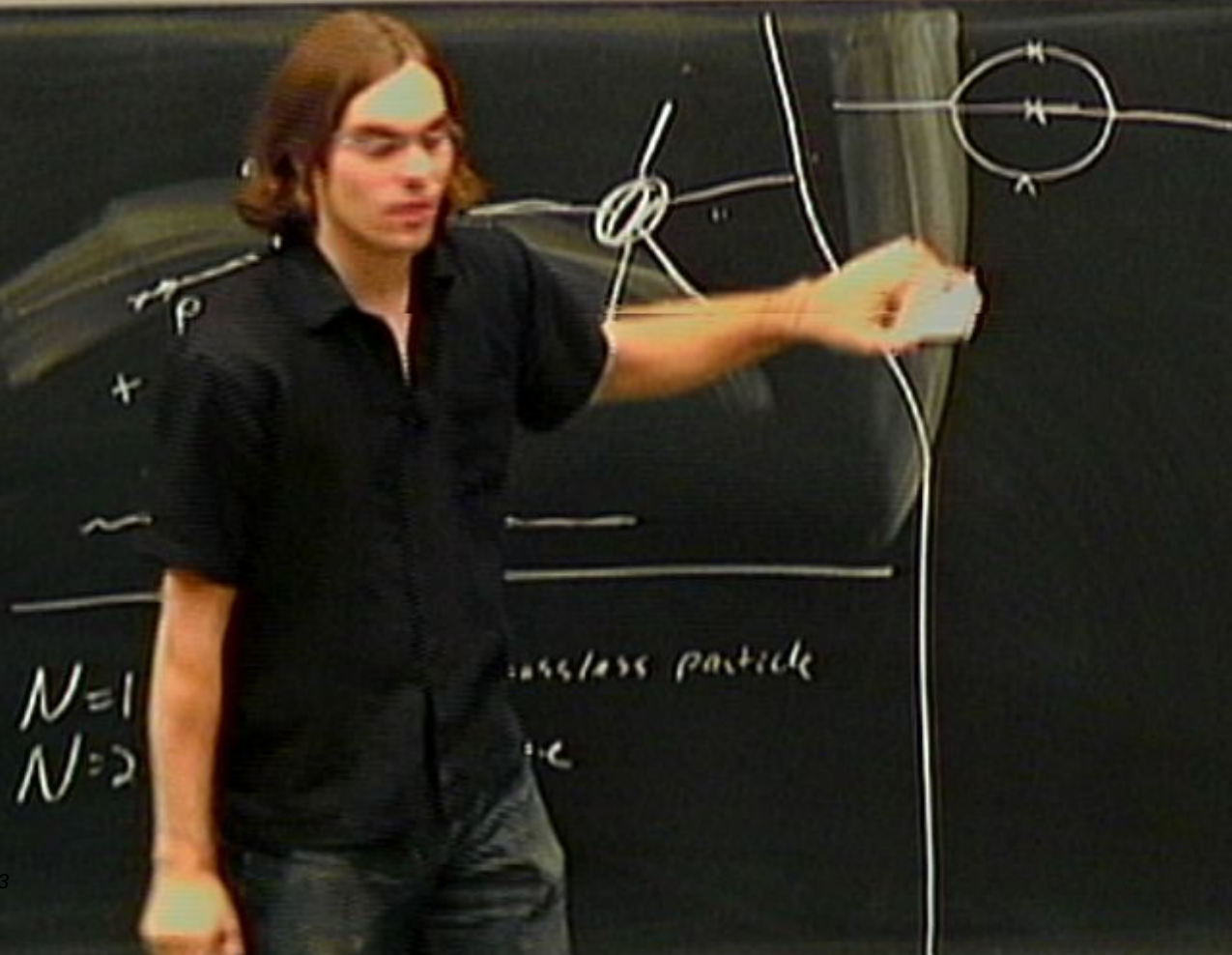
$$= \int \frac{dw_1 dw_2}{(w_1^2 - \epsilon_1)(w_2^2 - \epsilon_2)}$$



(2-loops), In general, NOT true

$$\int d^4x \delta(q) \left| \sum_{\text{signs}} (p_1 \dots p_i, q_1, q_2, q_3, q_4) \right|$$

$$\times \begin{cases} 4, & q_1, q_2 \text{ add} \\ 3, & q_1, q_2 \text{ subtract} \\ 2, & q_1, q_2 \text{ don't intersect} \\ 1, & \end{cases}$$



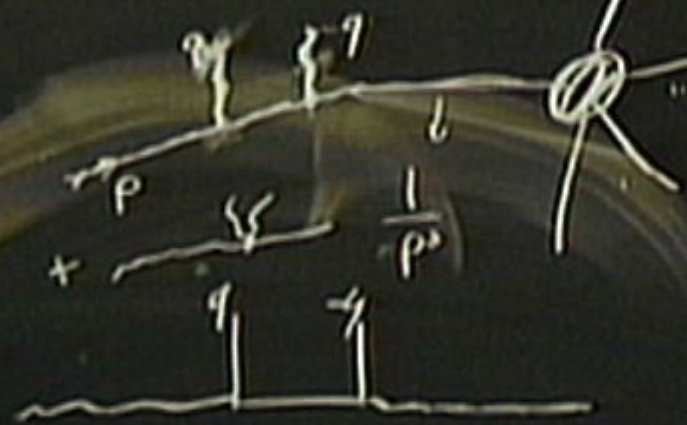
$N=1$
 $N=2$

massless particle

(2-loops), In general, NOT true

$$\int d^4x \langle \dots \rangle \left| \sum_{\text{diagrams}} (p_1 \dots p_n, q_1, -q_1, q_2, -q_2) \right.$$

$$\times \begin{cases} 4, & q_1, q_2 \text{ add} \\ 3, & q_1, -q_2 \text{ subtract} \\ 2, & q_1, q_2 \text{ don't intersect} \\ 1, & \end{cases}$$



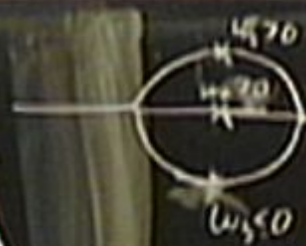
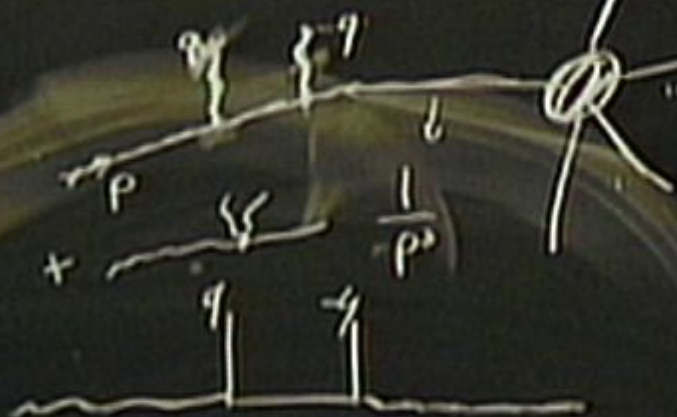
$N=1$ SUSY, for a massless particle
 $N=2$ SUSY, massive



(2-loops), In general, NOT true

$$\left(\int d^4x \delta(x) \right) \left(\sum_{\text{diagrams}} (p_1 \dots p_n, q_1, q_2, q_3, \dots) \right)$$

$\times \begin{cases} 4, & q_1, q_2 \text{ add} \\ 3, & q_1, q_2 \text{ subtract} \\ 2, & q_1, q_2 \text{ don't intersect} \end{cases}$

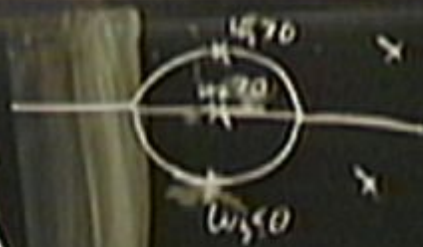
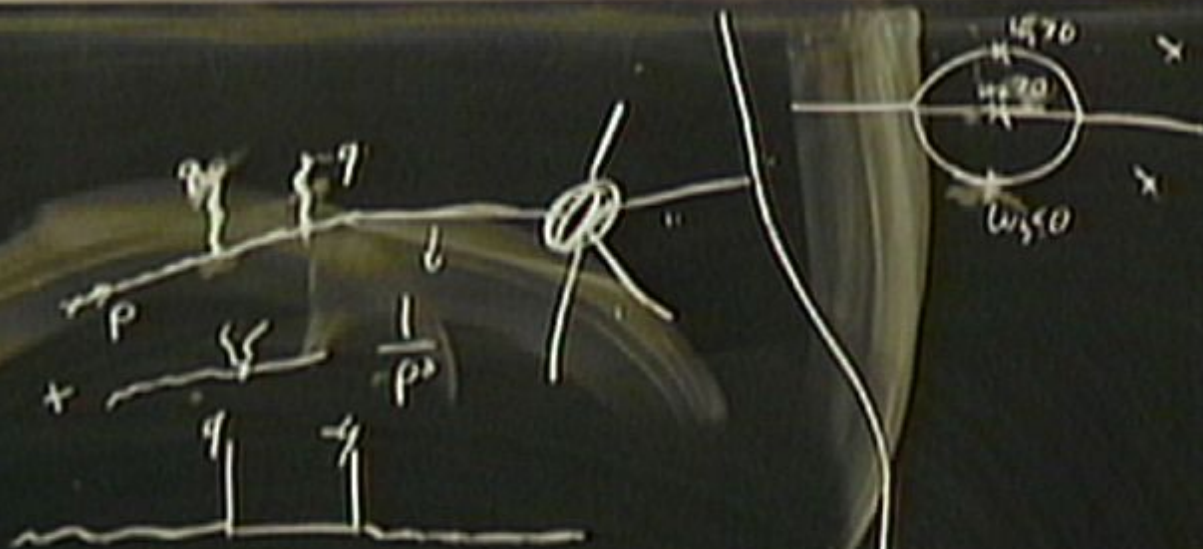


$N=1$ SUSY, for a massless particle
 $N=2$ SUSY, massive

(2-loops). In general, NOT true

$$\left(\sum_{\text{diagrams}} \right) / \left(\sum_{\text{diagrams}} (p_1 \dots p_n, q_1, q_2, q_3, \dots) \right)$$

$\times \begin{cases} 4, & q_1, q_2 \text{ odd} \\ 3, & q_1, q_2 \text{ subtracted} \\ 1, & q_1, q_2 \text{ don't intersect} \end{cases}$



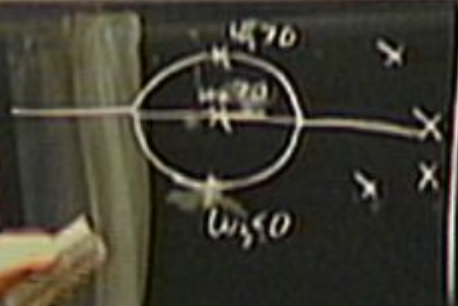
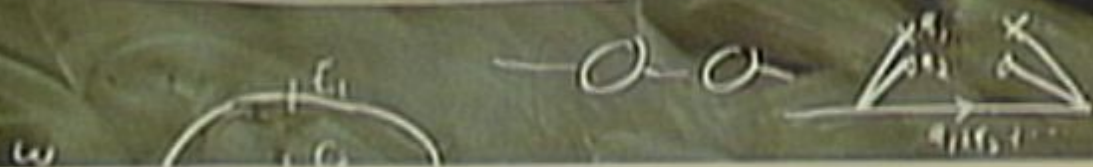
$$\frac{4}{3} \rightarrow \frac{2}{3}$$

$N=1$ SUSY, for a massless particle
 $N=2$ SUSY, massive

(2-loops). In general, NOT true

$$\left(\sum_{\text{diagrams}} \right) / \left(\sum_{\text{diagrams}} (p_1 \dots p_n, q_1, q_2, q_3, \dots) \right)$$

$$\times \begin{cases} 4, & q_1, q_2 \text{ add} \\ 3, & q_1, q_2 \text{ subtract} \\ 1, & q_1, q_2 \text{ don't intersect} \end{cases}$$



$$\frac{4}{3} \times (-) + \frac{2}{3} + \frac{2}{3} = 0.$$

$$N=1 \quad S_L$$

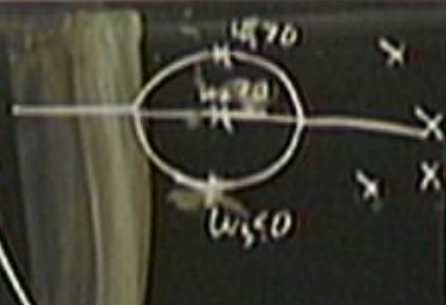
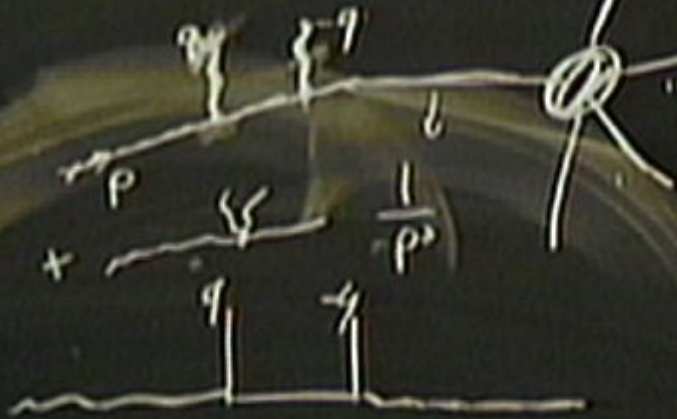
$$N=2 \quad S_L$$

massless particle

i.e.

(2-loops). In general, NOT true

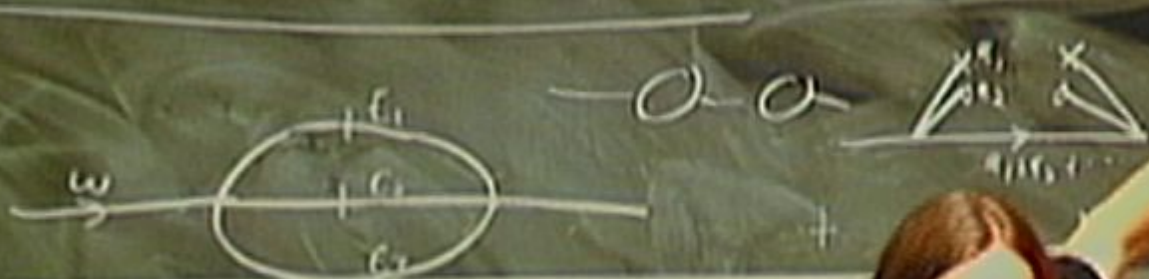
$$\left(\frac{d}{ds} \right) \left(\sum_{\text{diagrams}} (p_1 \dots p_n, q_1, q_2, q_3, \dots) \right) \times \begin{cases} 4, & q_1, q_2 \text{ odd} \\ 3, & q_1, q_2 \text{ even} \\ 2, & q_1, q_2 \text{ subtract} \end{cases}$$



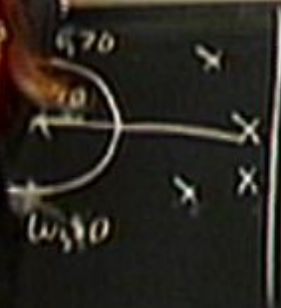
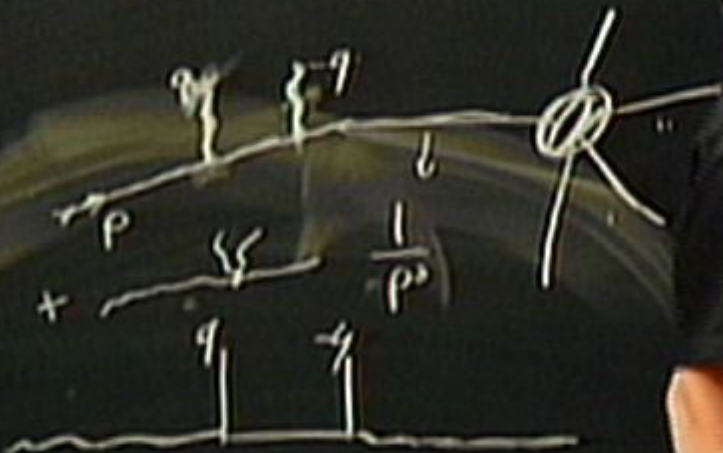
$$\frac{4}{3}(-1) + \frac{2}{3} + \frac{2}{3} = 0$$

$N=1$ SUSY, for a massless particle
 $N=2$ SUSY, massive

(2-loops). In general, NOT true $\int d^4x \mathcal{L}(x) \neq \sum_{\text{diagrams}} (p_1 \dots p_n, q_1, -q_1, q_2, -q_2)$



$\times$ $\frac{4}{3}$, q_1, q_2 odd
 q_1, q_2 subtract
 q_1, q_2 don't interact



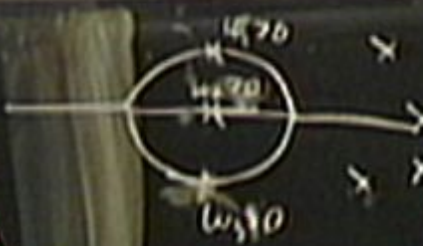
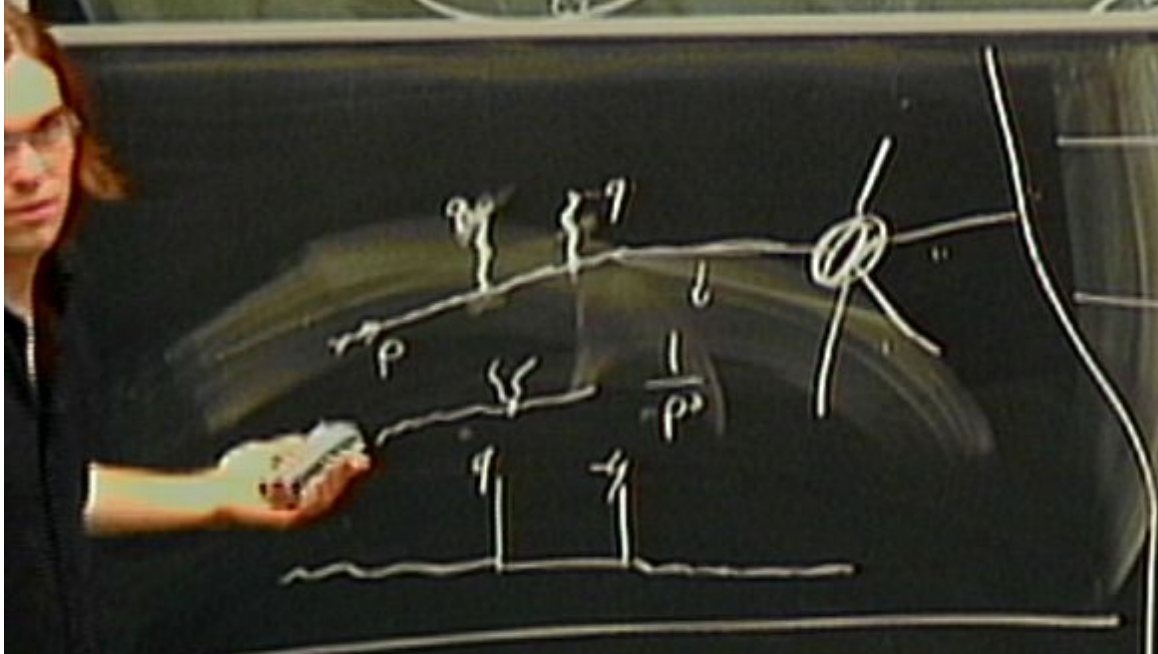
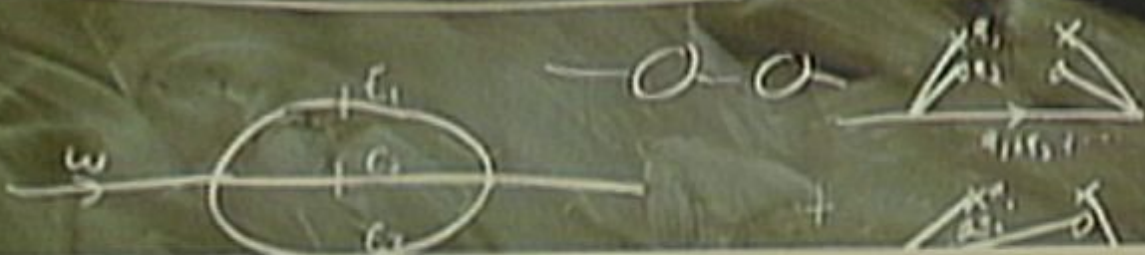
$$\frac{4}{3} \times (-) + \frac{2}{3} + \frac{2}{3} = 0 \checkmark$$

$N=1$ SUSY, for a massless P
 $N=2$ SUSY, massive

(2-loops), In general, NOT true

$\int d^4x \delta(x)$

$\sum_{j=1}^n (p_1 \dots p_i, q_1, q_2, q_3, q_4)$
 $\times \begin{cases} 4, & q_1, q_2 \text{ add} \\ 3, & q_1, q_2 \text{ subtract} \\ 2, & q_1, q_2 \text{ don't intersect} \\ 1, & \end{cases}$



$$\frac{4}{3} \times (-1) + \frac{2}{3} + \frac{2}{3} = 0 \checkmark$$

The learn: NOT the phys. tree diag

$N=1$ SUSY, for a massless particle
 $N=2$ SUSY, massive

$N=1$ SUSY, mass particle
 $N=2$ SUSY,

We learn: NOT the phys. tree diag

Solⁿ: $N_c \rightarrow \infty$.



(2-loops). In general, NOT true

$\int d\text{Lips}(q, A)$

$\sum_{\text{signs}} (p_1 \dots p_n, q_1, q_2, q_3, q_4)$
 $\times \begin{cases} 4, & q_1, q_2 \text{ add} \\ 3, & q_1, q_2 \text{ subtract} \\ 2, & q_1, q_2 \text{ don't intersect} \\ 1, & \end{cases}$



$$((w_2^2 - E_2^2) ((w_1 + w_2 - w)^2 - E_3^2)^2$$

(2-loop). In general, NOT true

$\int d^4 p \delta(p^2)$

$$\sum_{\text{signs}} (p_1 \dots p_n, q_1, q_2, q_3, \dots)$$

$\times \begin{cases} 4, & q_1, q_2 \text{ add} \\ 3, & q_1, q_2 \text{ subtract} \\ 2, & q_1, q_2 \text{ don't intersect} \\ 1, & \end{cases}$



0+1 dim.

$$w^2 - (e_1 + e_2 + e_3)^2$$

$$= \int \frac{dw_1 dw_2}{(w_1^2 - e_1^2)(w_2^2 - e_2^2)((w_1 + w_2 - w)^2 - e_3^2)}$$



(2-loops). In general, NOT true

$\{q_i\}$

$$\sum_{j=1}^n (p_1 \dots p_i, q_1, -q_2, q_3, \dots, q_n)$$

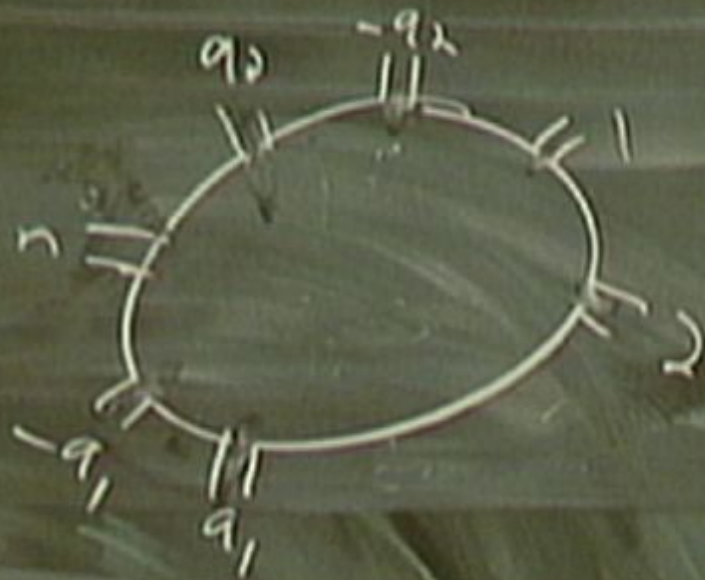
$$\times \begin{cases} 4, & q_1 + q_2 \text{ add} \\ 3, & q_1 - q_2 \text{ subtract} \\ 2, & q_1, q_2 \text{ don't intersect} \\ 1, & \end{cases}$$



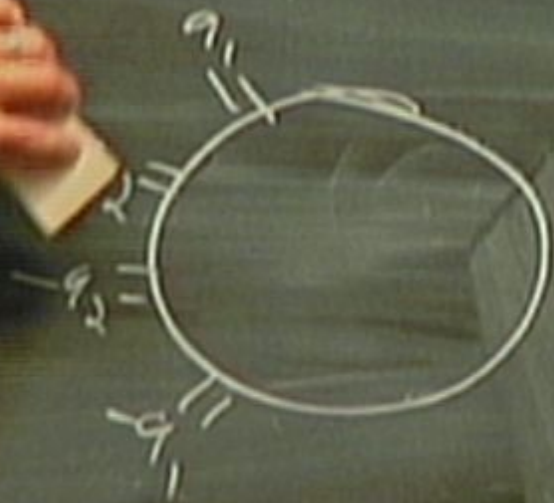
$\Rightarrow$ "subtract"



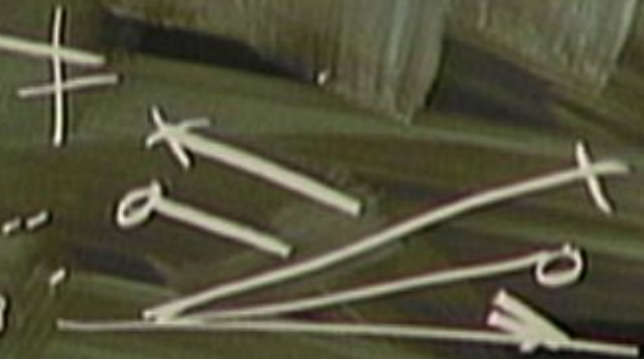
$$(q_1 - q_2)^2$$



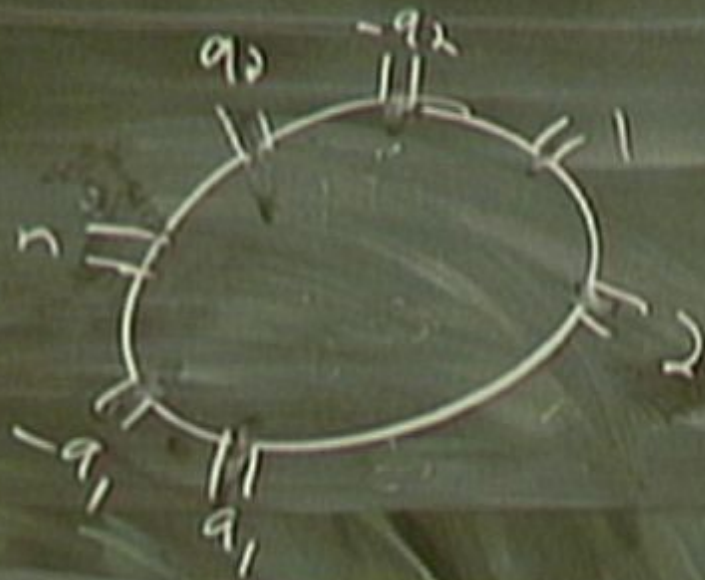
⇒ "Subtract"



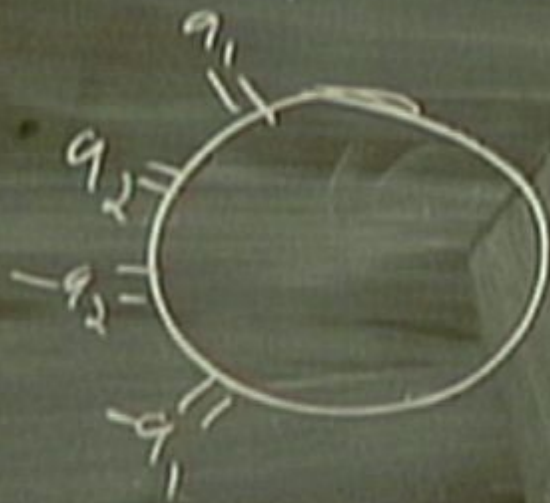
⇒ "Add category"



$$(w)^2 - (E_3)^2$$



⇒ "Subtract"



⇒ "Add category"



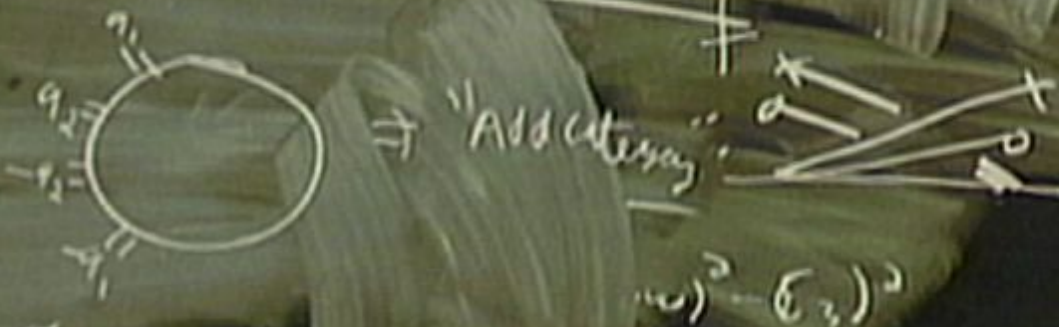
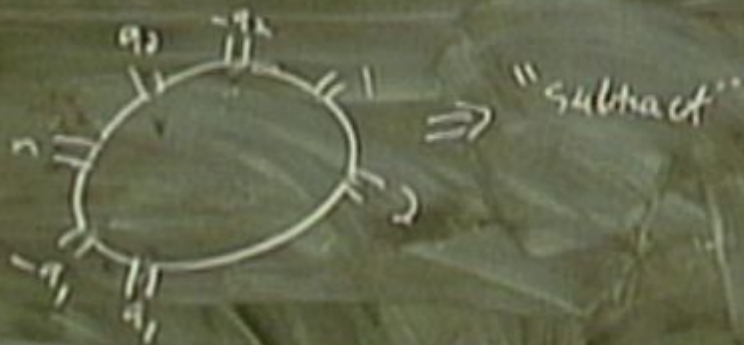
$$(\omega)^2 - (\epsilon_3)^2$$

loops], In general, NOT true

$$\left(\text{dLips}(q, A) \right)$$

$$\sum_{\text{regions}} (P_1 \dots P_n, q_1, -q_1, q_2, -q_2)$$

$$\times \begin{cases} 4, & q_1 + q_2 \text{ add} \\ 3, & q_1 - q_2 \text{ subtract} \\ 2, & \\ 1, & q_1, q_2 \text{ don't intersect} \end{cases}$$



$$(u)^2 - (v)^2$$

loops], In general, NOT true

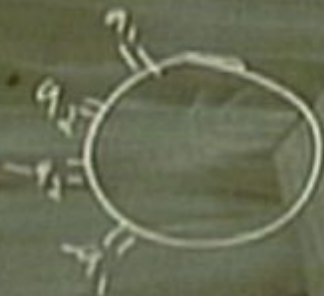
$\{ \text{drips}(q, A) \}$

$$\sum_{\text{regions}} (p_1 \dots p_n, q_1, -q_1, q_2, -q_2)$$

$$\times \begin{cases} 4, & q_1 + q_2 \text{ add} \\ 3, & q_1 - q_2 \text{ subtract} \\ 2, & \\ 1, & q_1, q_2 \text{ don't intersect} \end{cases}$$



"subtract"



"Add"



True off-q

$$A_n^{\text{loop}}(p_1 \dots p_n)_R = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \dots, p_n, q, -q)_R$$

Consider instead ret. amplitude $\langle 0 | e^{iQ_1(x)} e^{iQ_2(y)} | 0 \rangle$

$$\langle 0 | [Q_1(x), Q_2(y)] | 0 \rangle \propto \theta(x-y)$$

$$\langle 0 | e^{i \dots} Q_2(y) e^{-i \int^y \delta K(x)} | 0 \rangle$$



$$= \frac{-i}{p^2 - i\epsilon} \rightarrow \frac{-i}{p^2 - i\epsilon p}$$

$$\downarrow A_n(p_1, \dots, p_n)_R = \frac{1}{2} \left(\frac{1}{(2\pi)^3 2E_n} A_{n+2}(p_1, \dots, p_n, q, -q)_R \right)$$

Consider instead net. amplitude $\langle 0 | e^{iQ_1(x)} e^{iQ_2(y)} | 0 \rangle$

$$\langle 0 | [Q_1(x), Q_2(y)] | 0 \rangle \propto \Theta(x-y)$$

$$\langle 0 | e^{i \dots} Q_2(y) e^{-i \int^y \delta H(x)} | 0 \rangle$$



$$= \frac{-i}{p^2 - i\epsilon} \rightarrow \frac{-i}{p^2 - i\epsilon p^0}$$

→ SUSY, Planar th, ,

True off-shell

$$A_n^{\text{loop}}(p_1 \dots p_n) = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \dots, p_n, q, -q)$$

Consider instead vet. amplitude

$$\langle 0 | [O_1(x), O_2(y)] | 0 \rangle \propto \theta(x-y)$$



$$= \frac{-i}{p^2 - i\epsilon} \rightarrow \frac{-i}{p^2 - i\epsilon}$$

$$\langle 0 | e^{i \int \dots} O(x) e^{-i \int \dots} \delta x(x) | 0 \rangle$$

→ SUSY, Planck th, $A^{\text{tree}} = \sum \text{diagrams} \times A^{\text{tree}}$

$$A_n^{\text{loop}}(p_1, \dots, p_n) = \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3 E_q} A_{n+2}^{\text{tree}}(p_1, \dots, p_n, q, -q)$$

Tree off-shell

Consider instead net. amplitude

$$\langle 0 | [O_1(x), O_2(y)] | 0 \rangle \sim O(x-y)$$



$$\langle 0 | e^{i \int \dots} O_1(x) e^{-i \int \dots} O_2(y) | 0 \rangle$$

$$\rightarrow \text{SUSY, Planar th. } A^{\text{loop}} = \sum \text{diagrams } A^{\text{tree}}$$

SCH 1007.3224
 MA-Hetal. 1008.2958
 R. Boels 1008.3101

Plan today :-

- i.e.-ology
- Two-loops (planar vs. non-planar)
- Computing the "single cuts".
- Example

0810.2764

- A technique to do num. algebra in QFT.

- Build on knowledge of tree amps.

- BCFW for loop integrands

- $N=4$ application

Application to ^{planar} 2-loop 5pt

