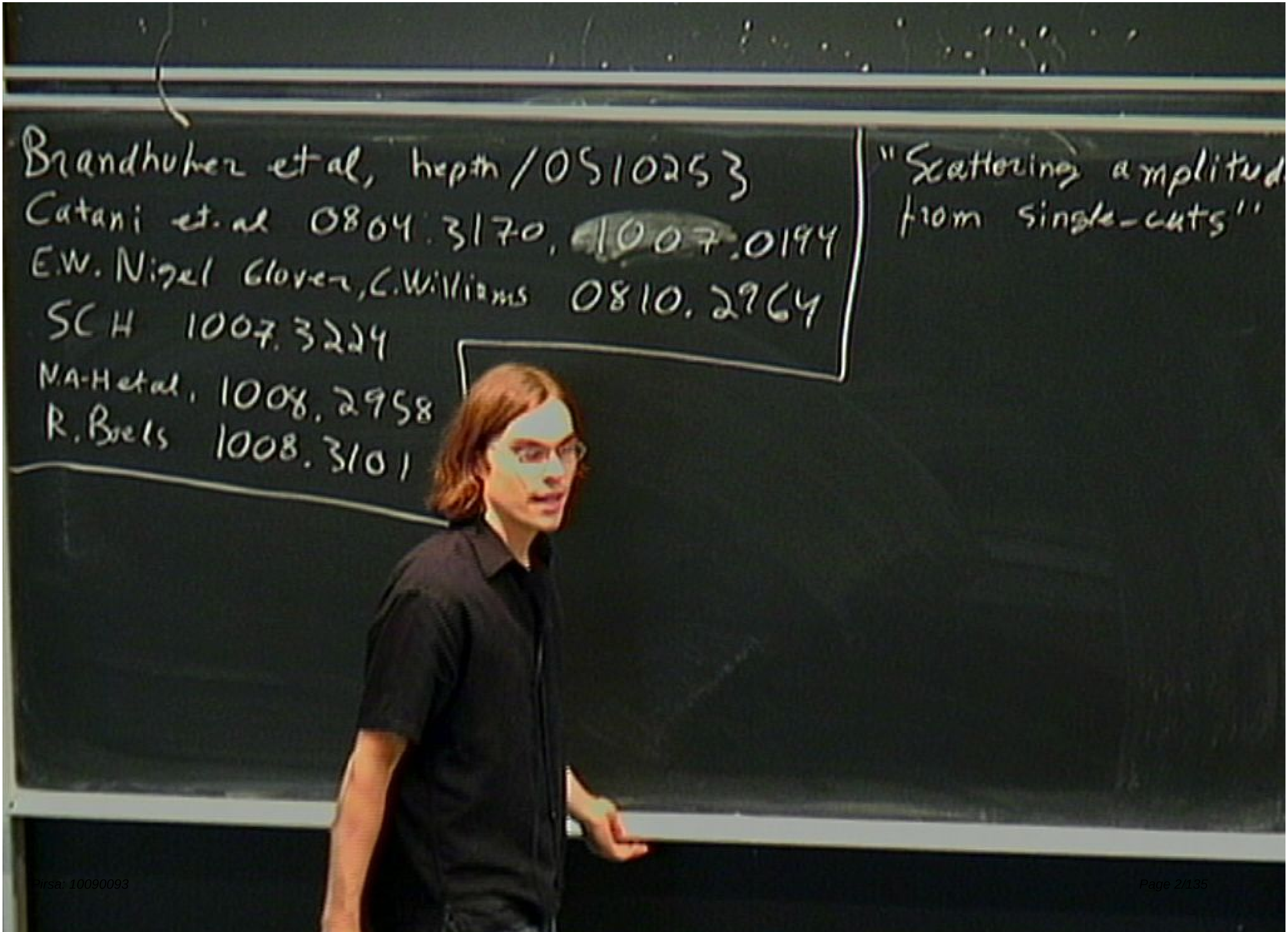
Title: Scattering Amplitudes from Single-Cuts: Lecture 1

Date: Sep 13, 2010 02:00 PM

URL: http://pirsa.org/10090093

Abstract: On-shell methods provide a powerful tool for the perturbative computation of scattering amplitudes in gauge theories, such as QCD. In these lectures I will focus on such methods which can be developed without invoking unitarity, but rather by setting one propagator by loop level on-shell ("single-cuts"). After summarizing the present status of applying these ideas to QCD, I will discuss their recent successful application to planar N=4 super Yang-Mills, leading to on-shell recursion relations for loop integrands.


Brandhuber et al, hepth/0510253
Catani et. al 0804.3170, 1007.0194
E.W. Nigel Glover, C. Williams 0810.2964
SCH 1007.3224
N.A-H et al. 1008.2958
R. Boels 1008.3101
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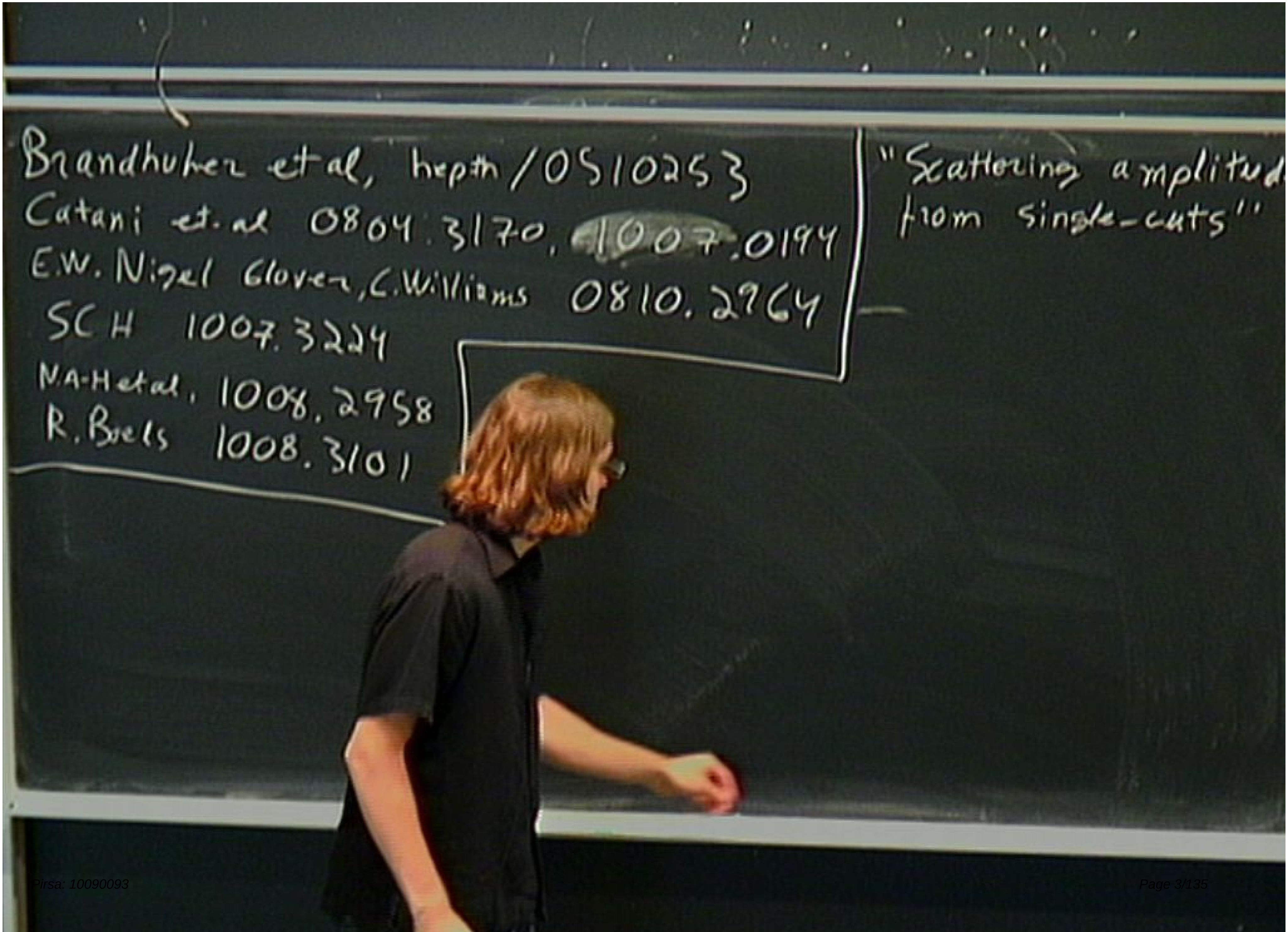
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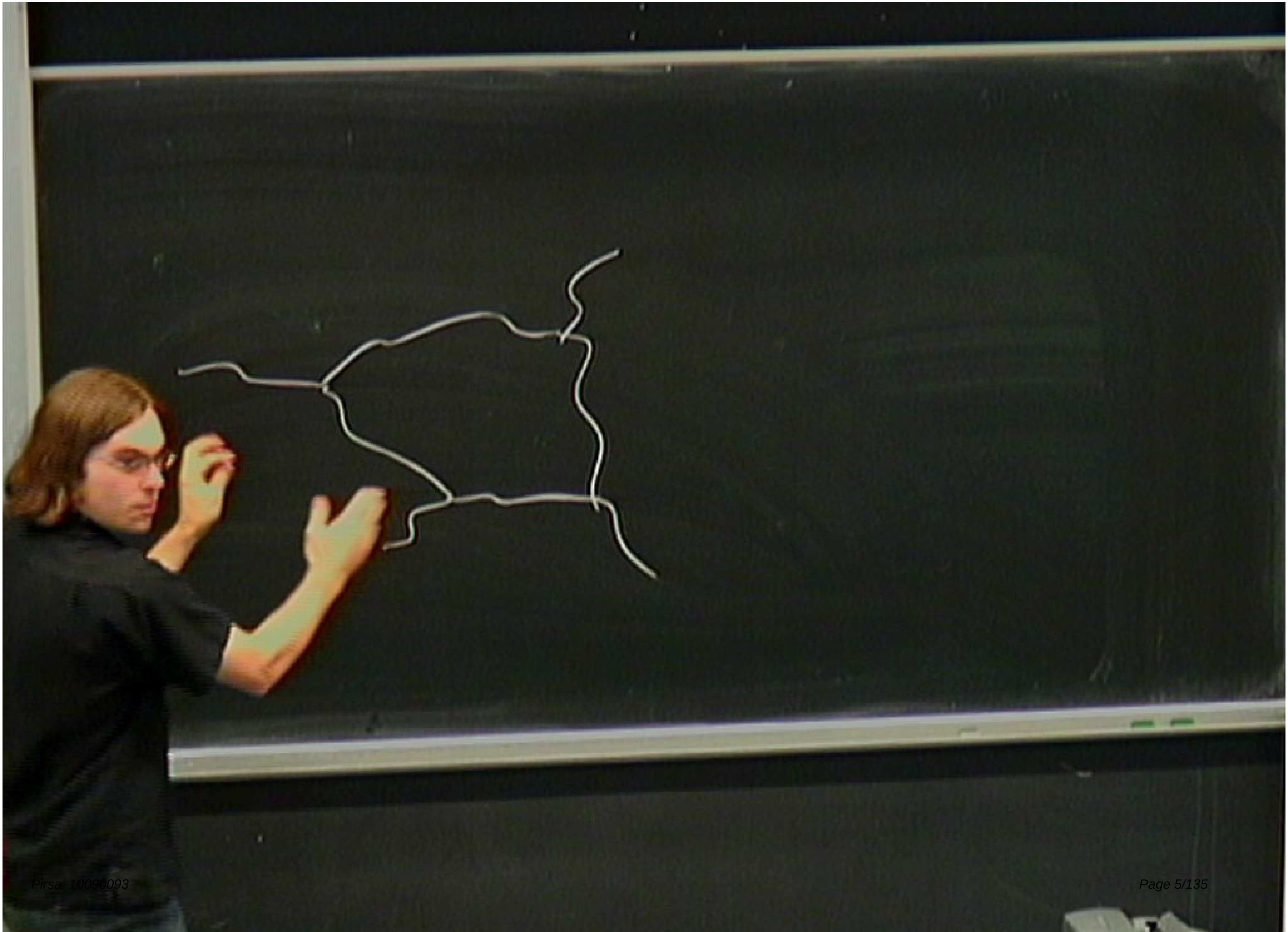

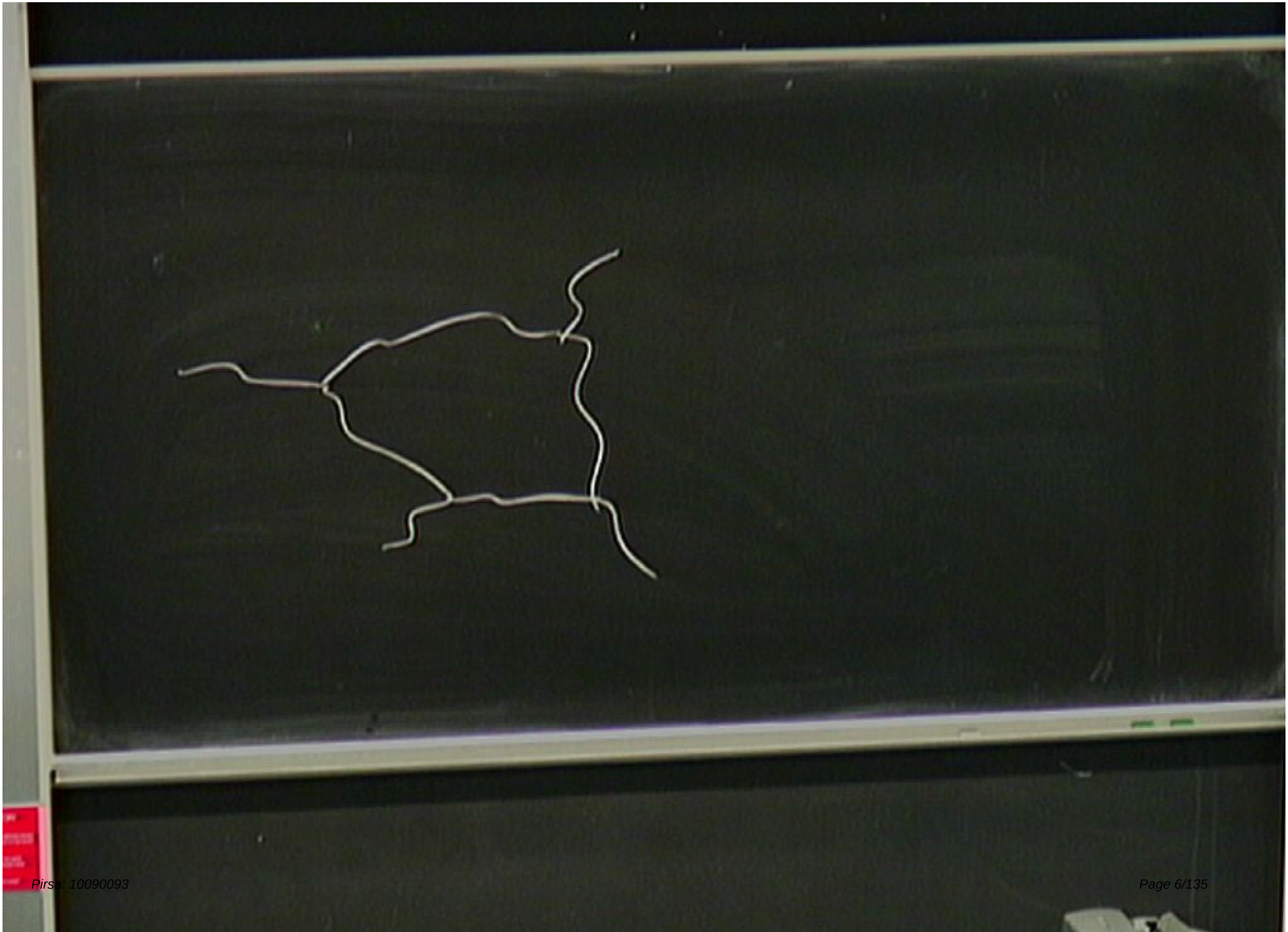

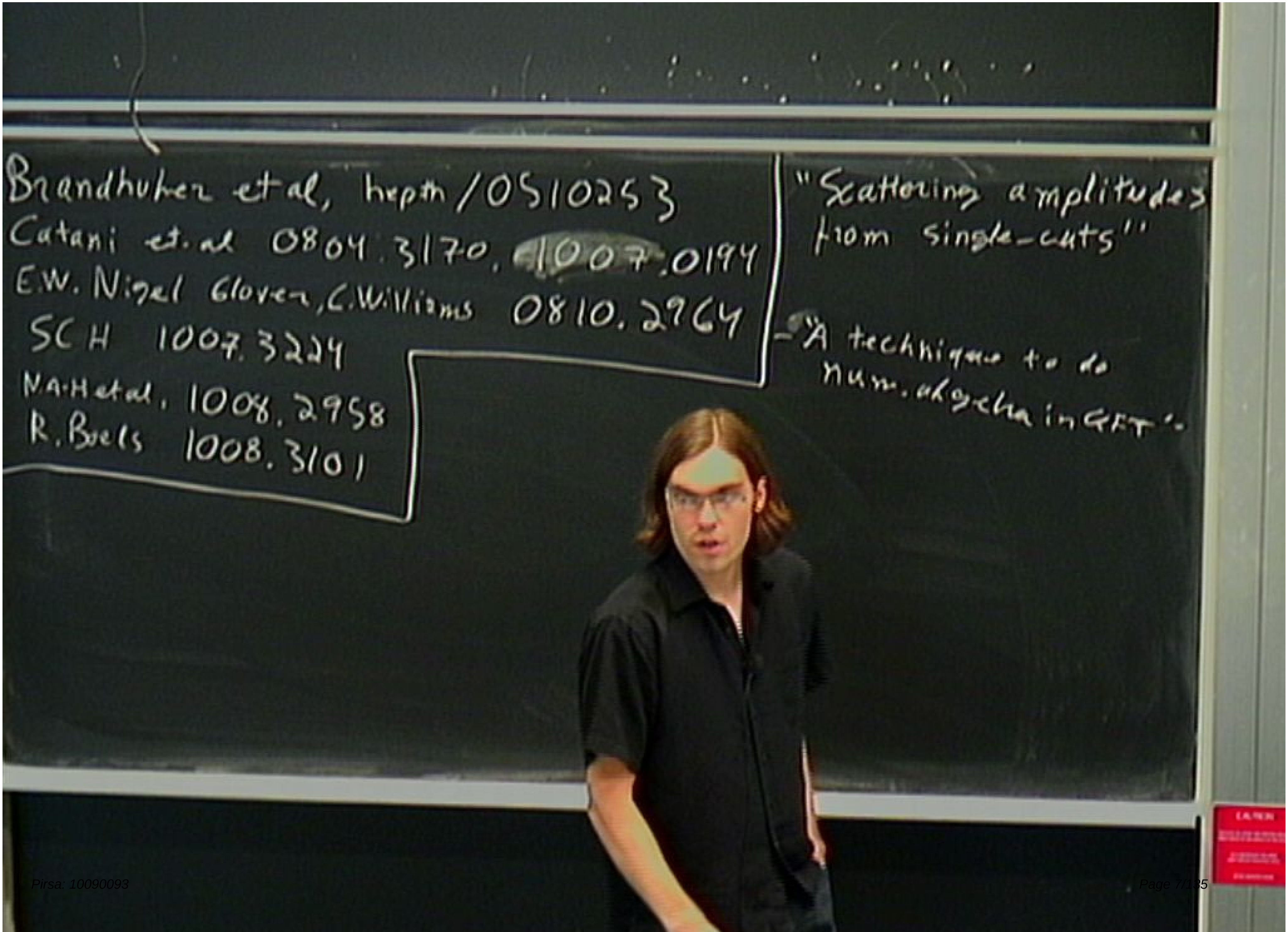
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- "A technique to do num. algebra in QFT".
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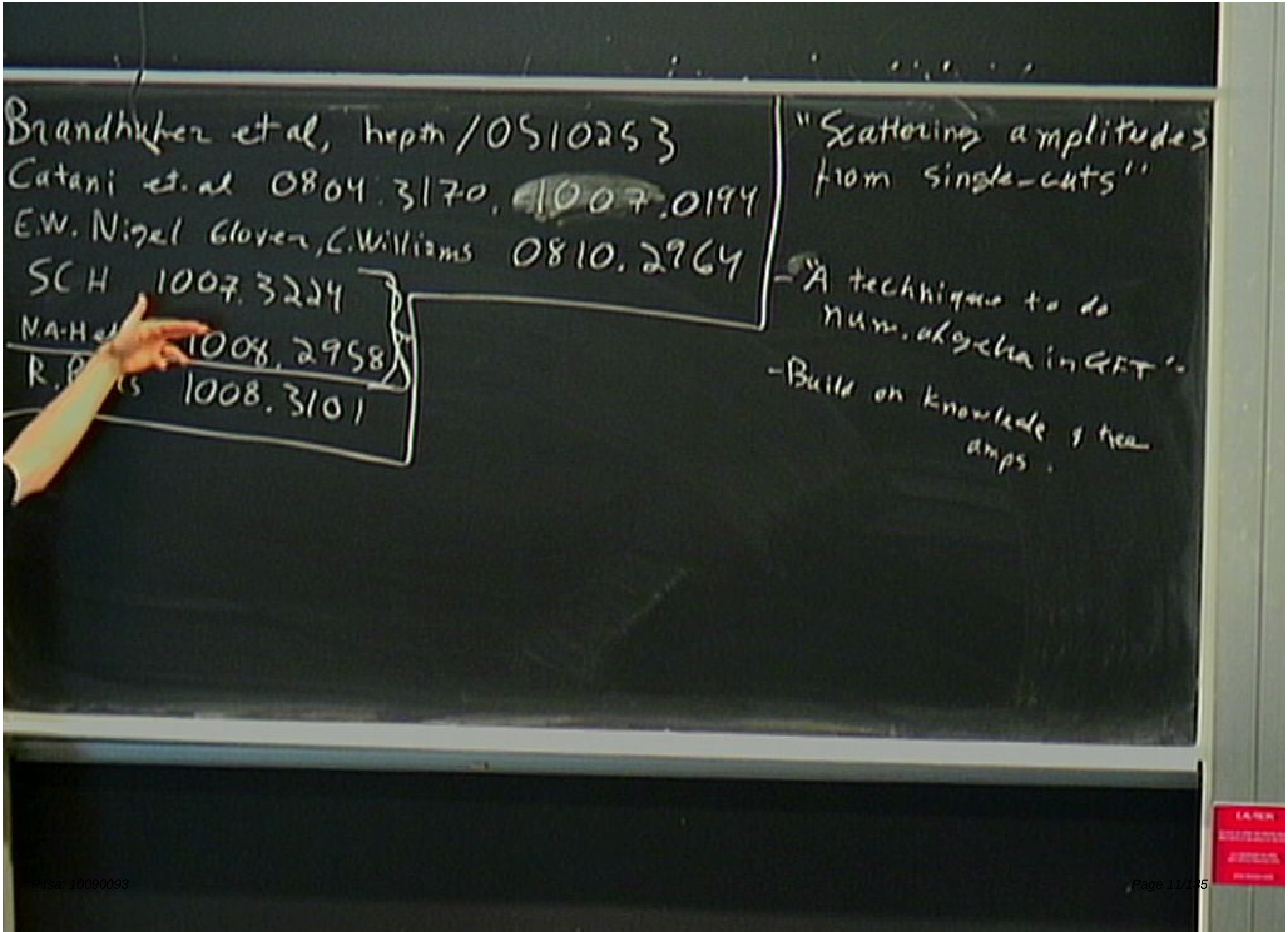
- "A technique to do num. algebra in QFT"
- Build on knowledge of tree amps.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Application to planar $N=4$ SYM

"Scattering amplitudes from single-cuts"

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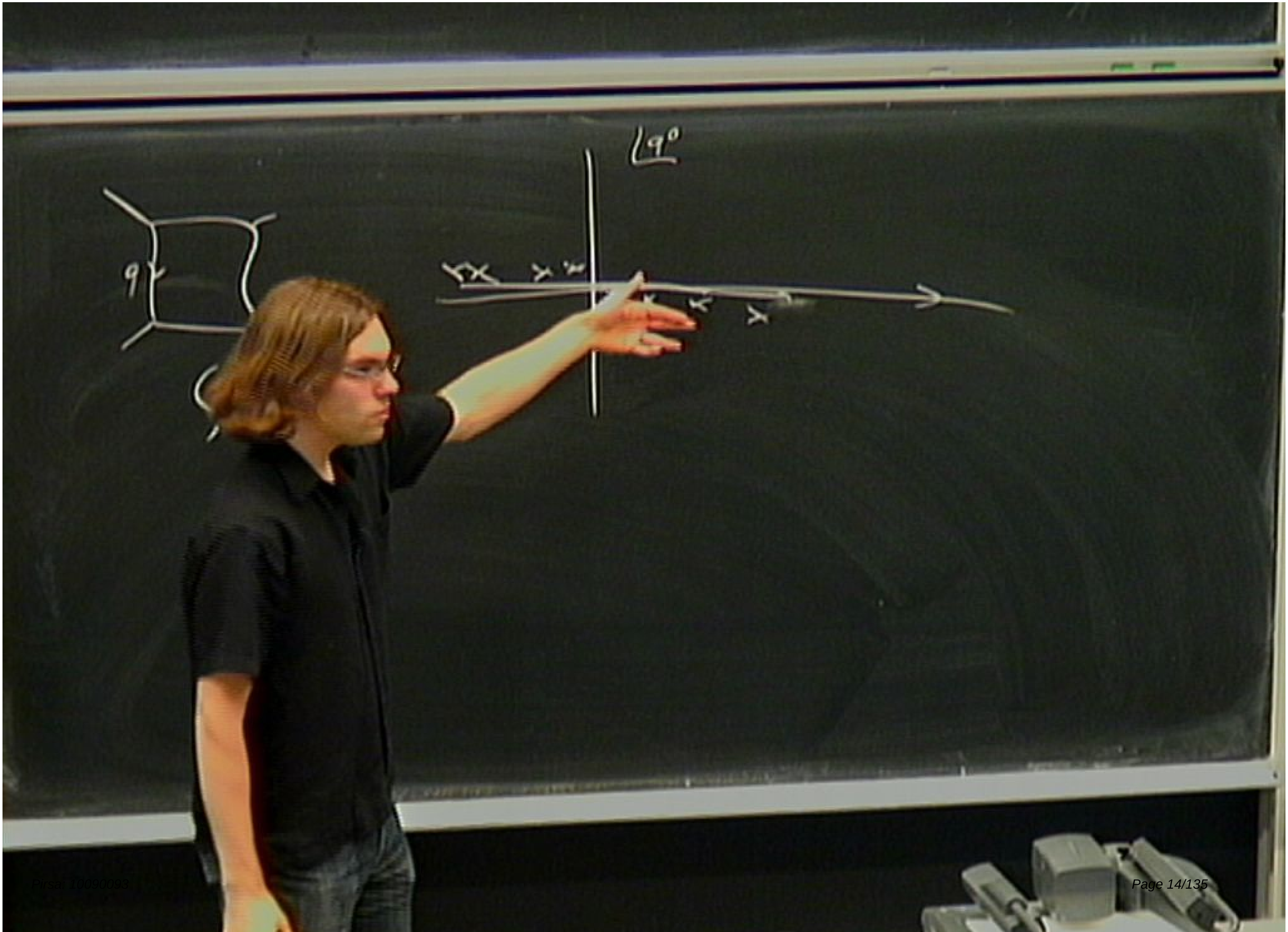
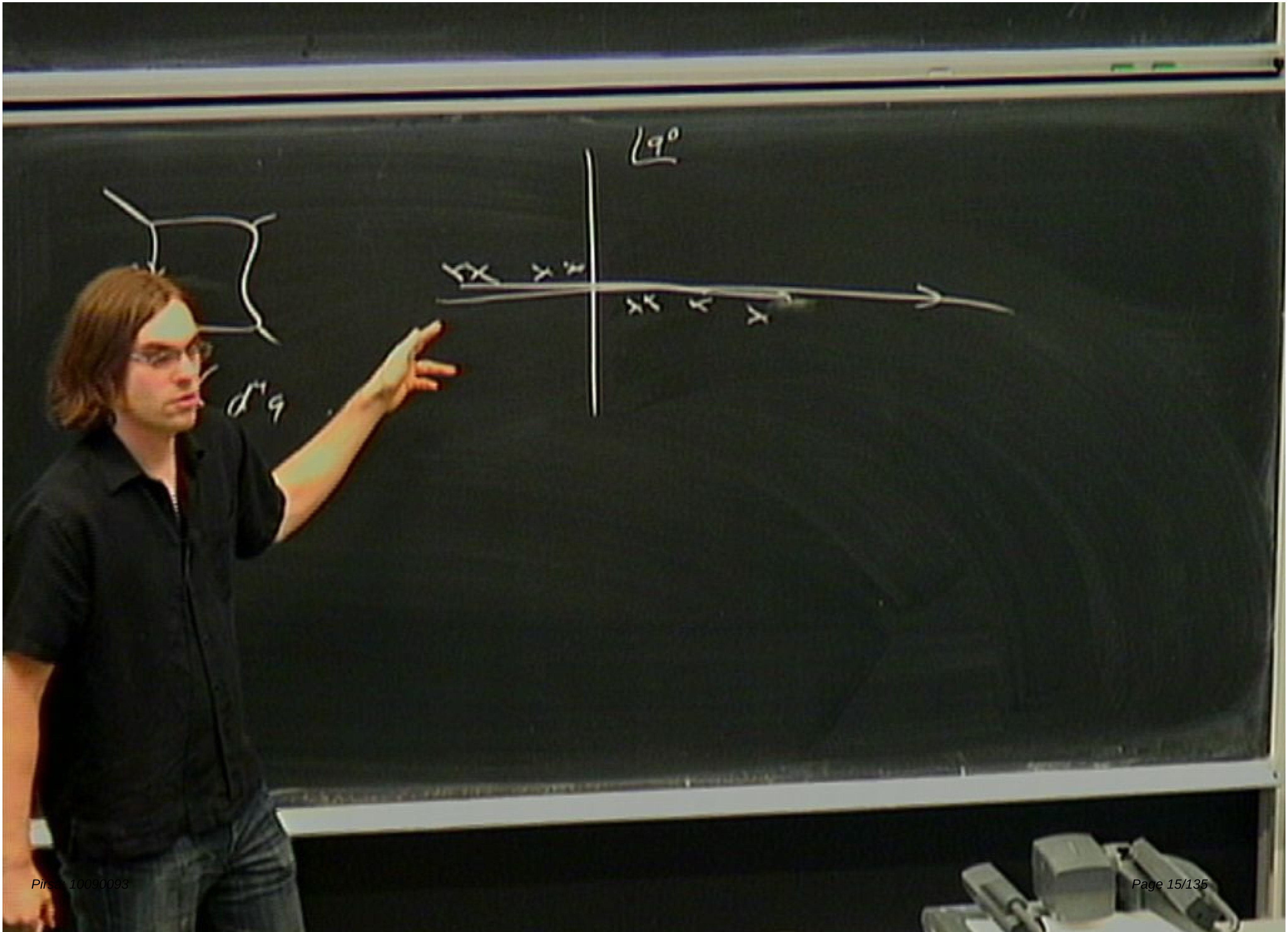
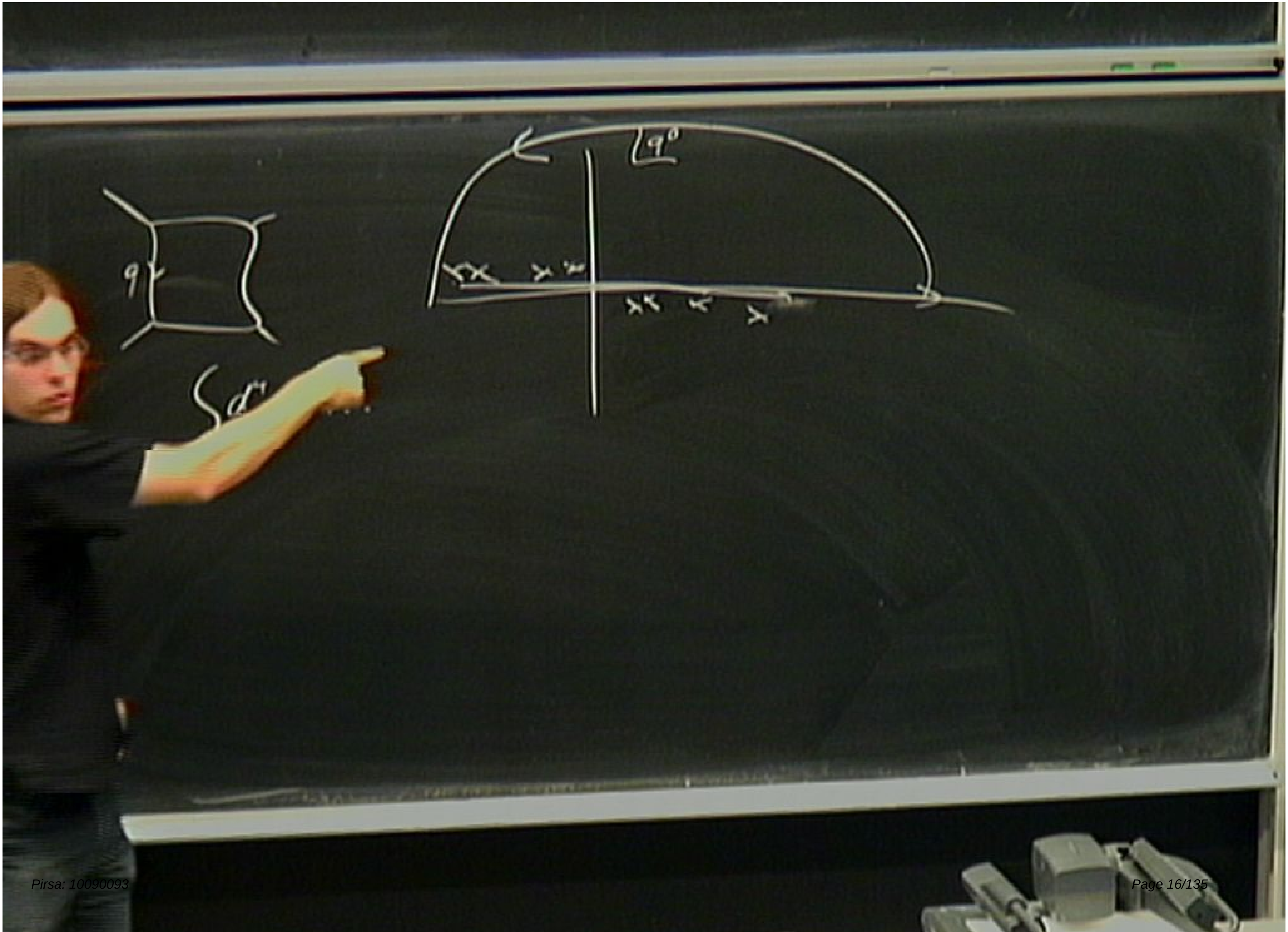

$\mathcal{L}q^0$


q^0
d^4q


q
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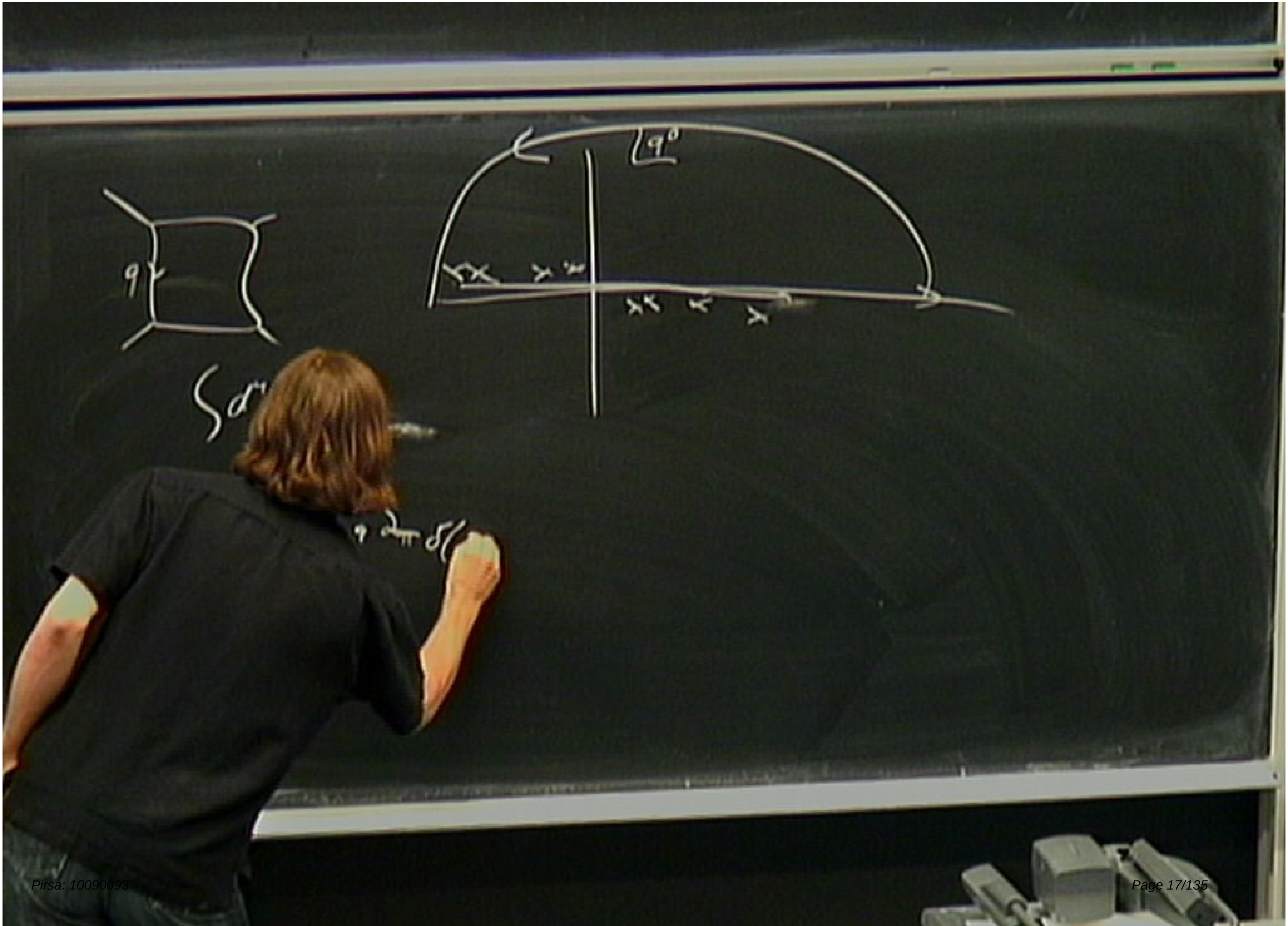
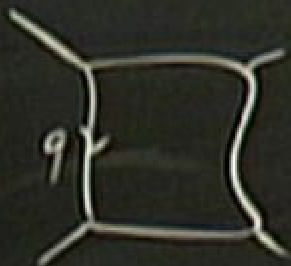
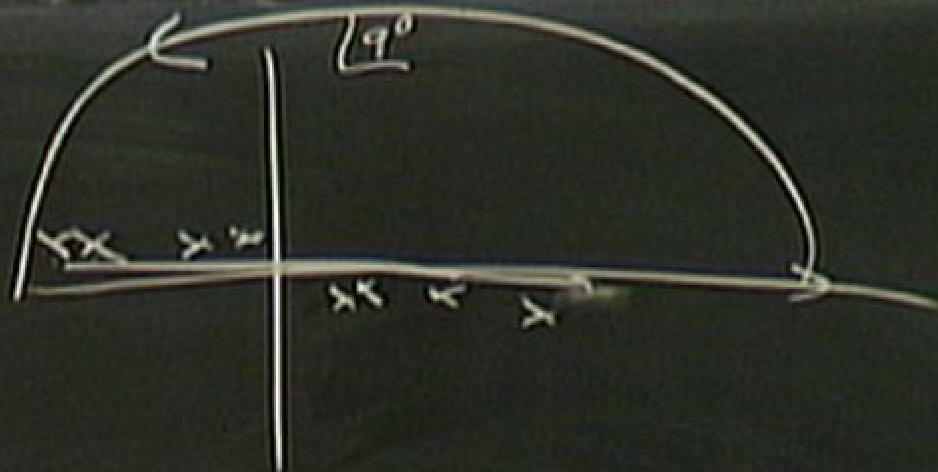
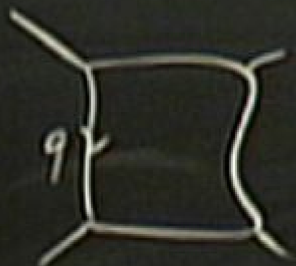


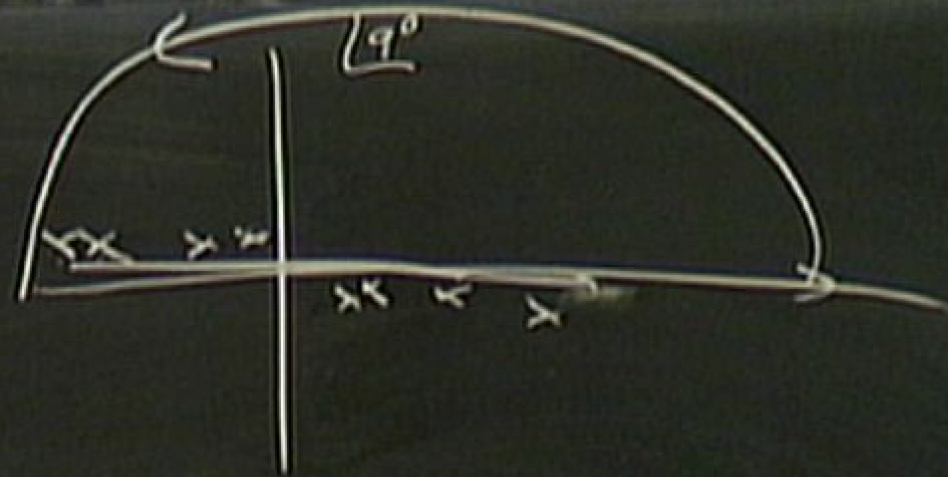




$$\int d^4q \; \frac{1}{q^2}$$

$$\rightarrow \int d^4q \; 2\pi\,\delta(q^2)$$





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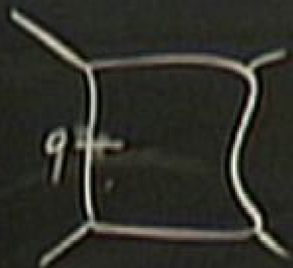

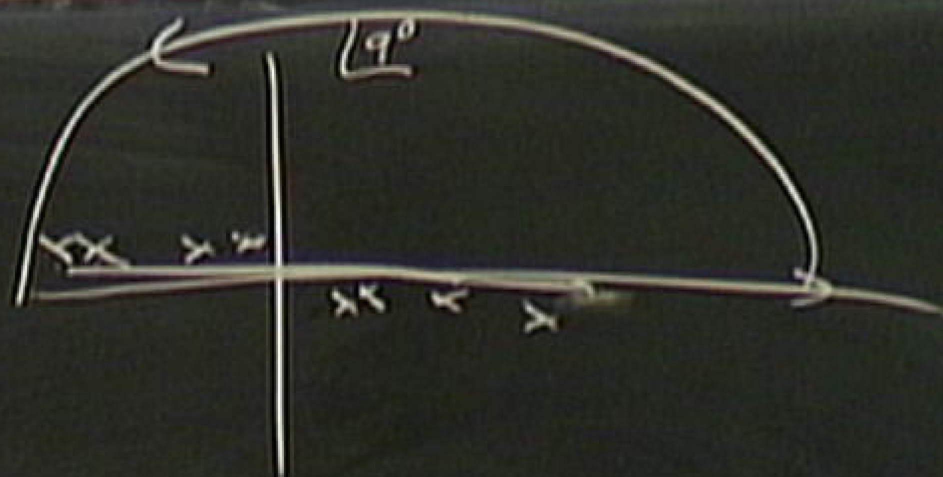

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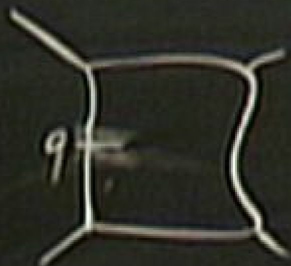

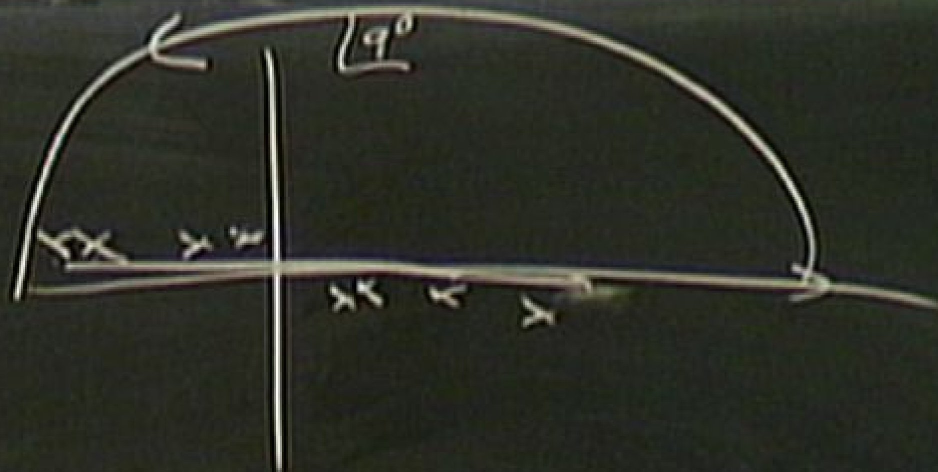

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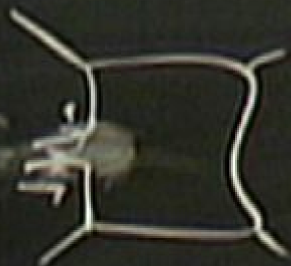

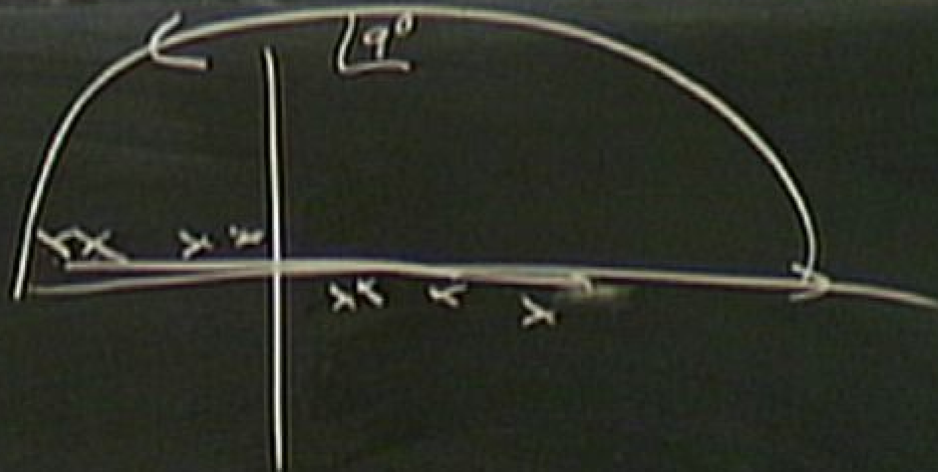

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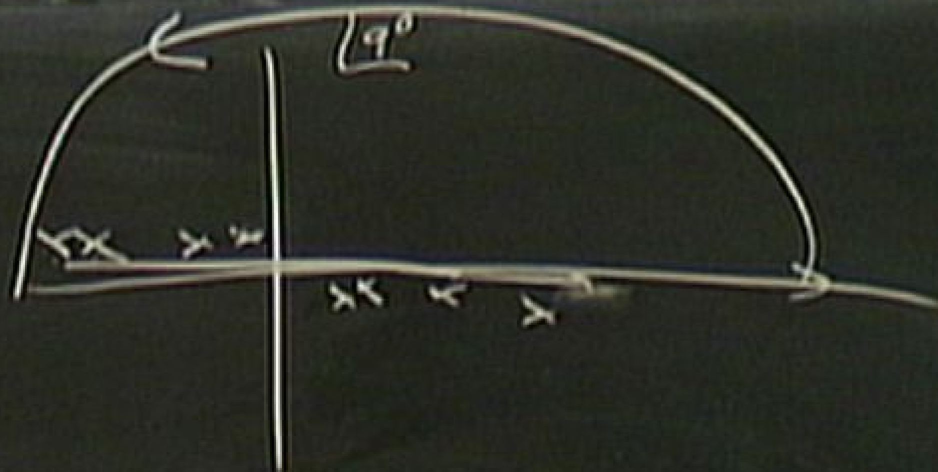

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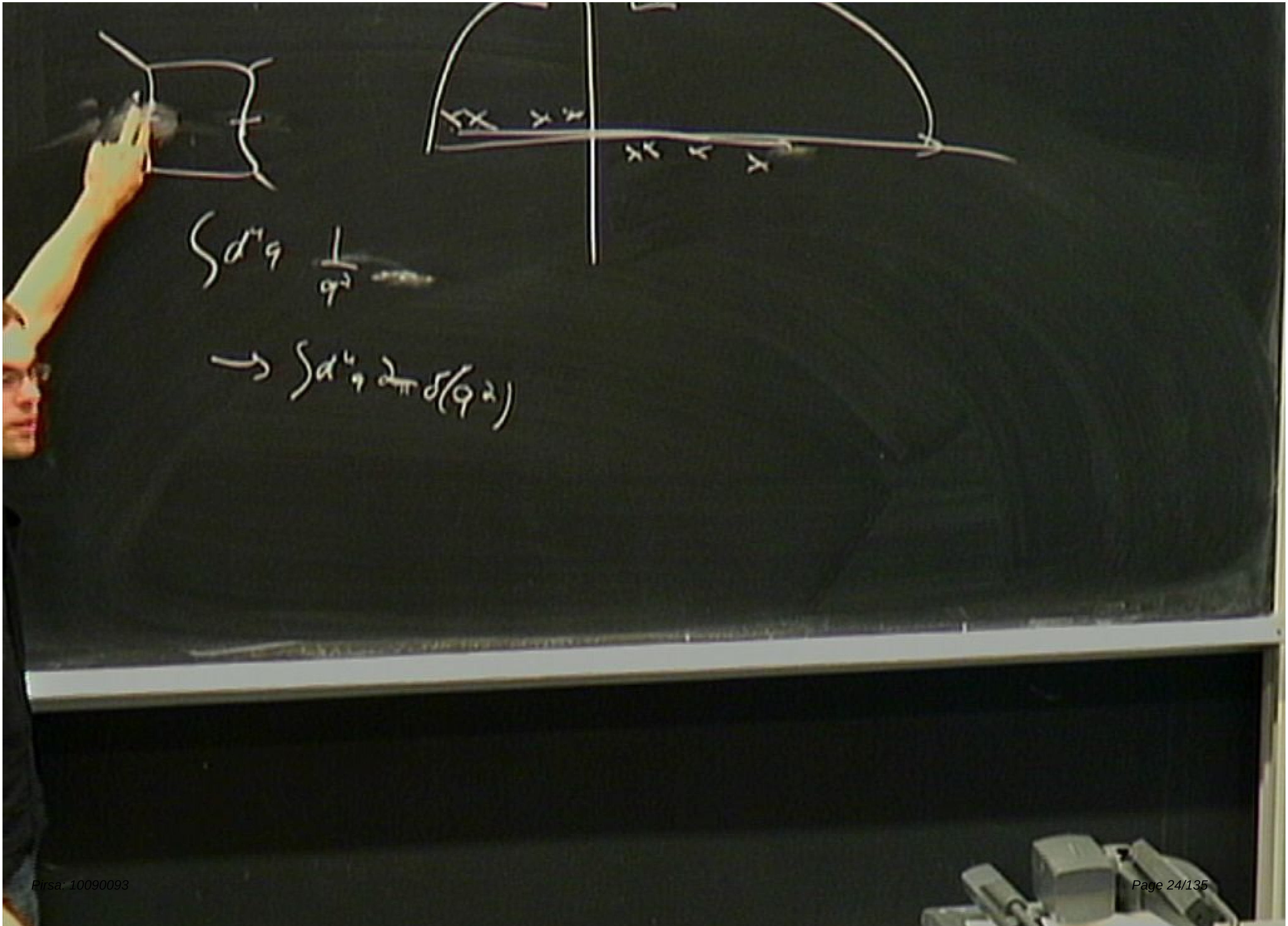
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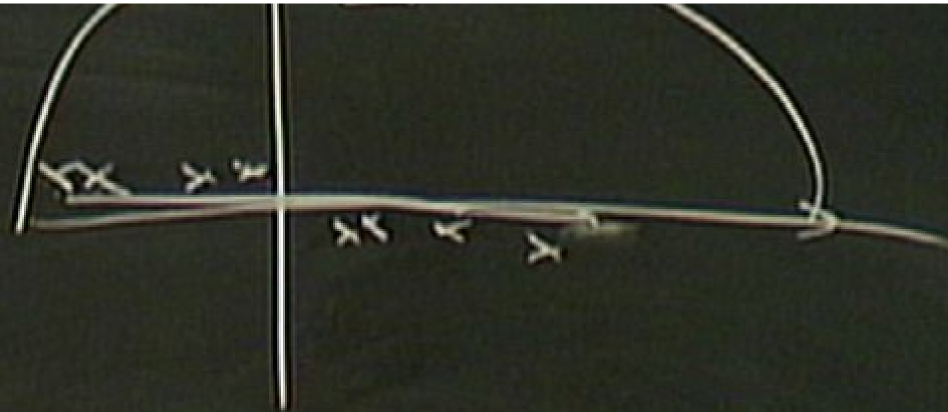

$$\int d^4q \, \frac{1}{q^2}$$

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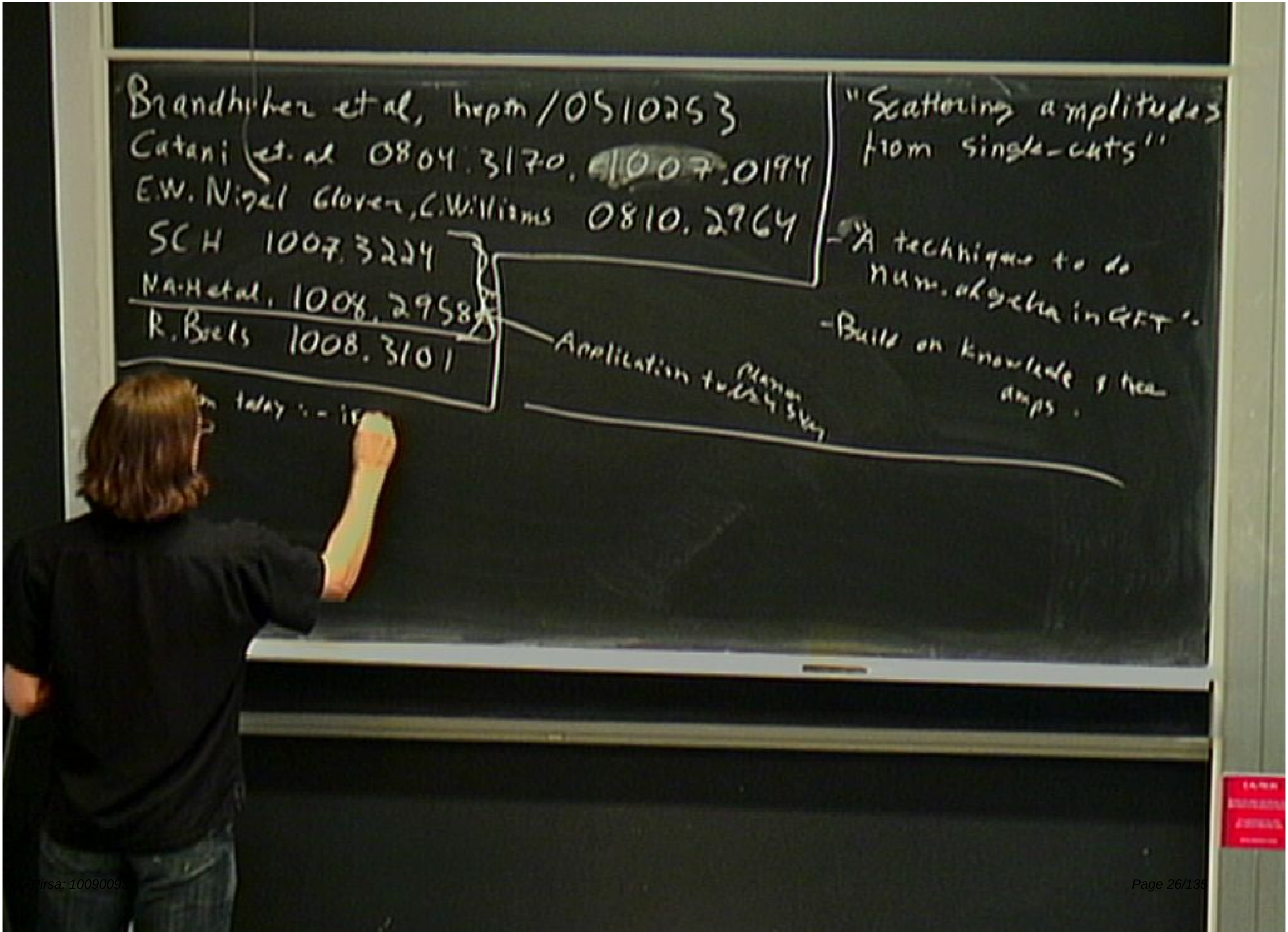
$$\longrightarrow \int d^4q \, 2\pi\,\delta(q^2)$$



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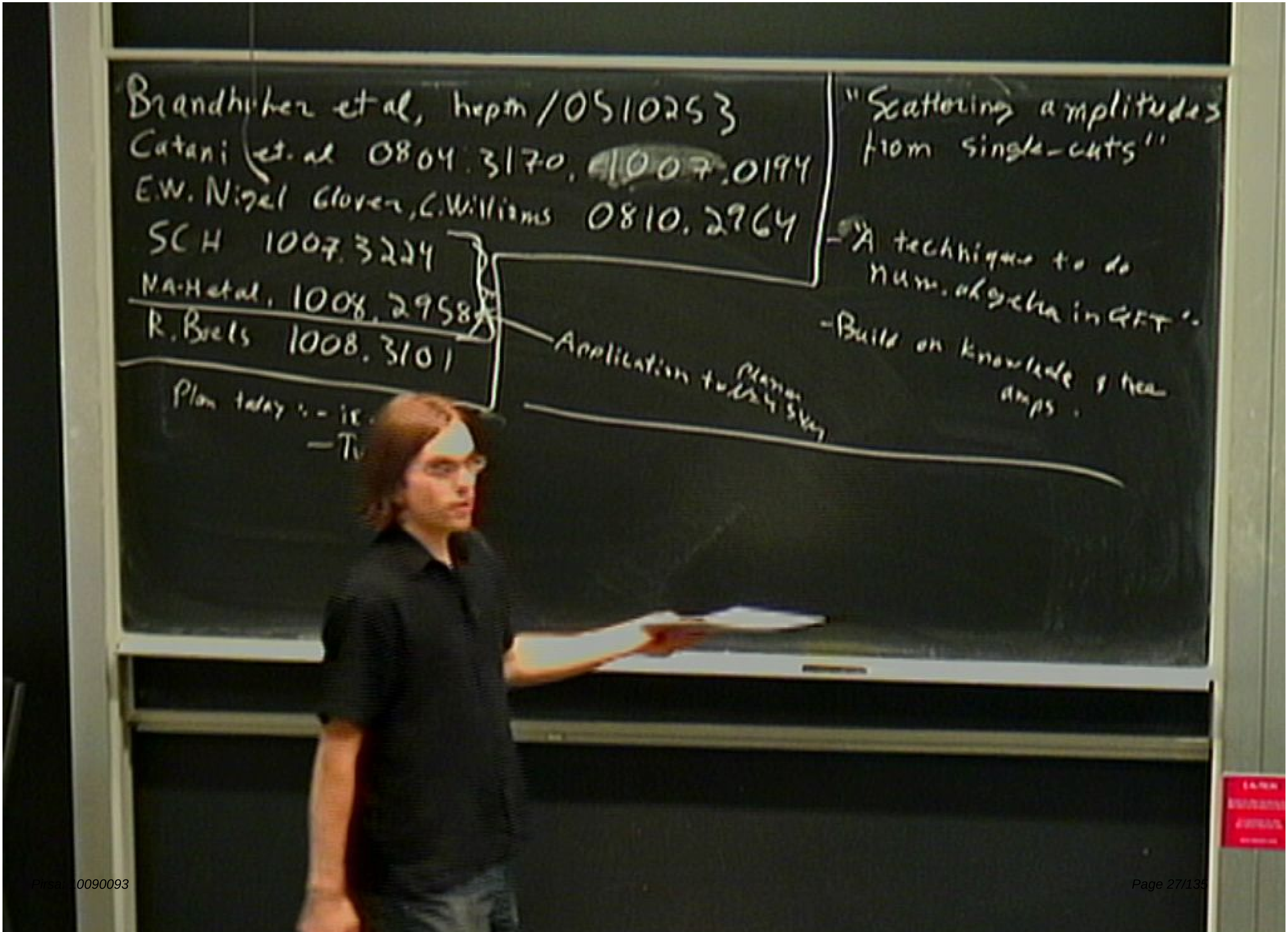

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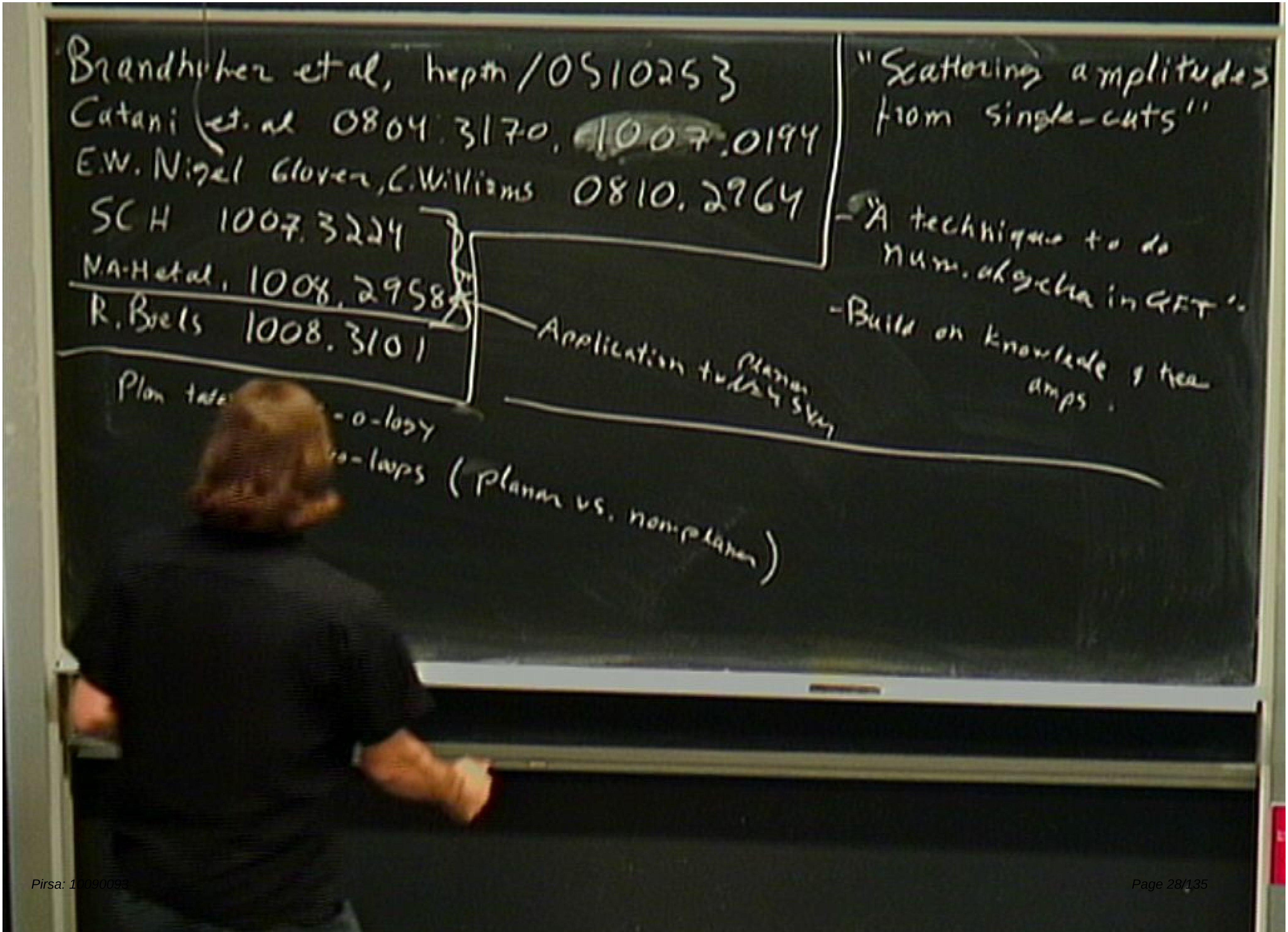
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Plan
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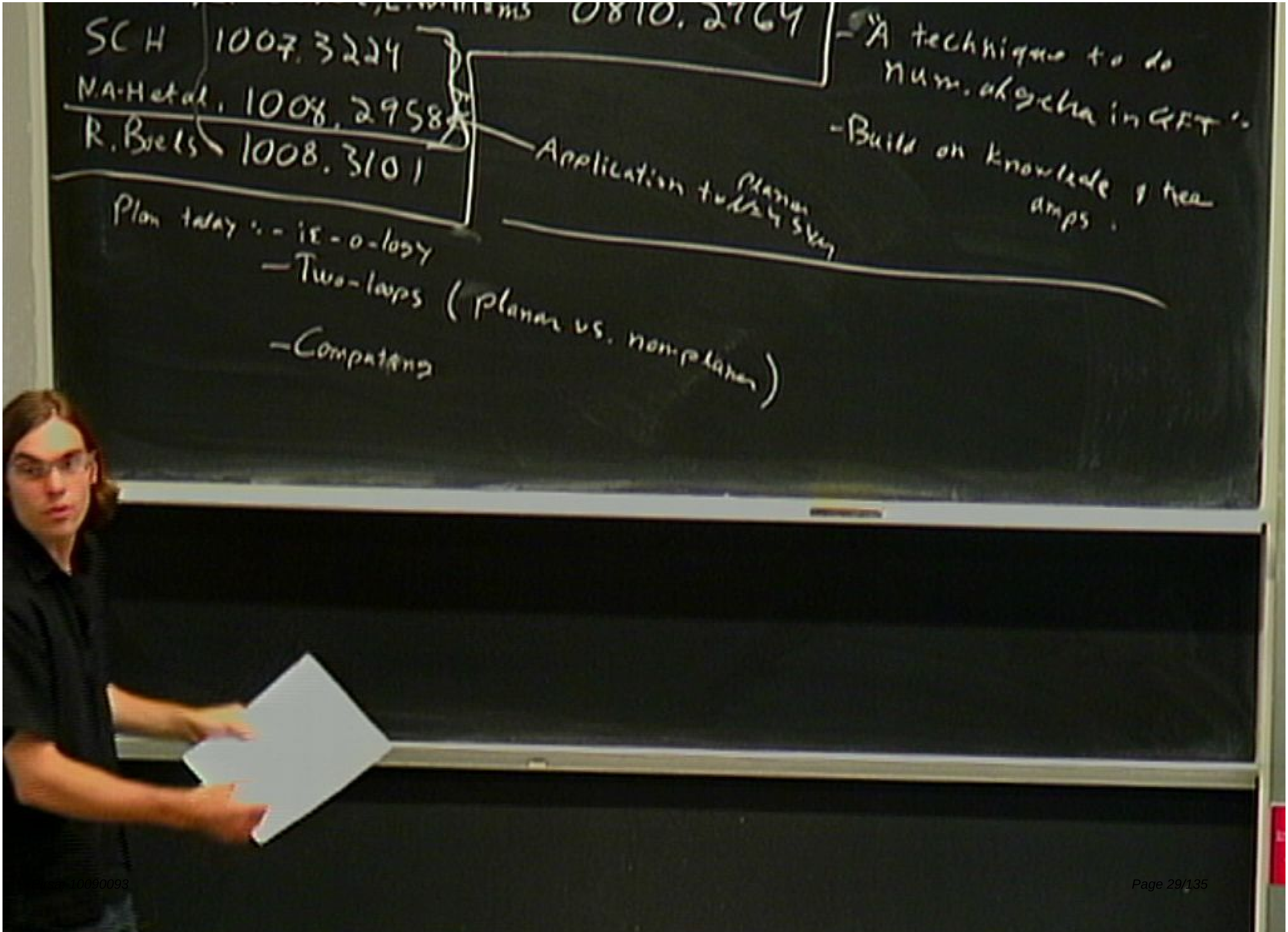
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Application to planar 4d SYM
Plan today: - it-o-logy
-Two-loops (planar vs. non-planar)
-Computing

owledge of tree amps.

Planar
$N=4$ SYM

- BCFW for loop integrands
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amps.

Planar
$\mathcal{N}=4$ SYM

- BCFW for loop integrands
- $\mathcal{N}=4$ application

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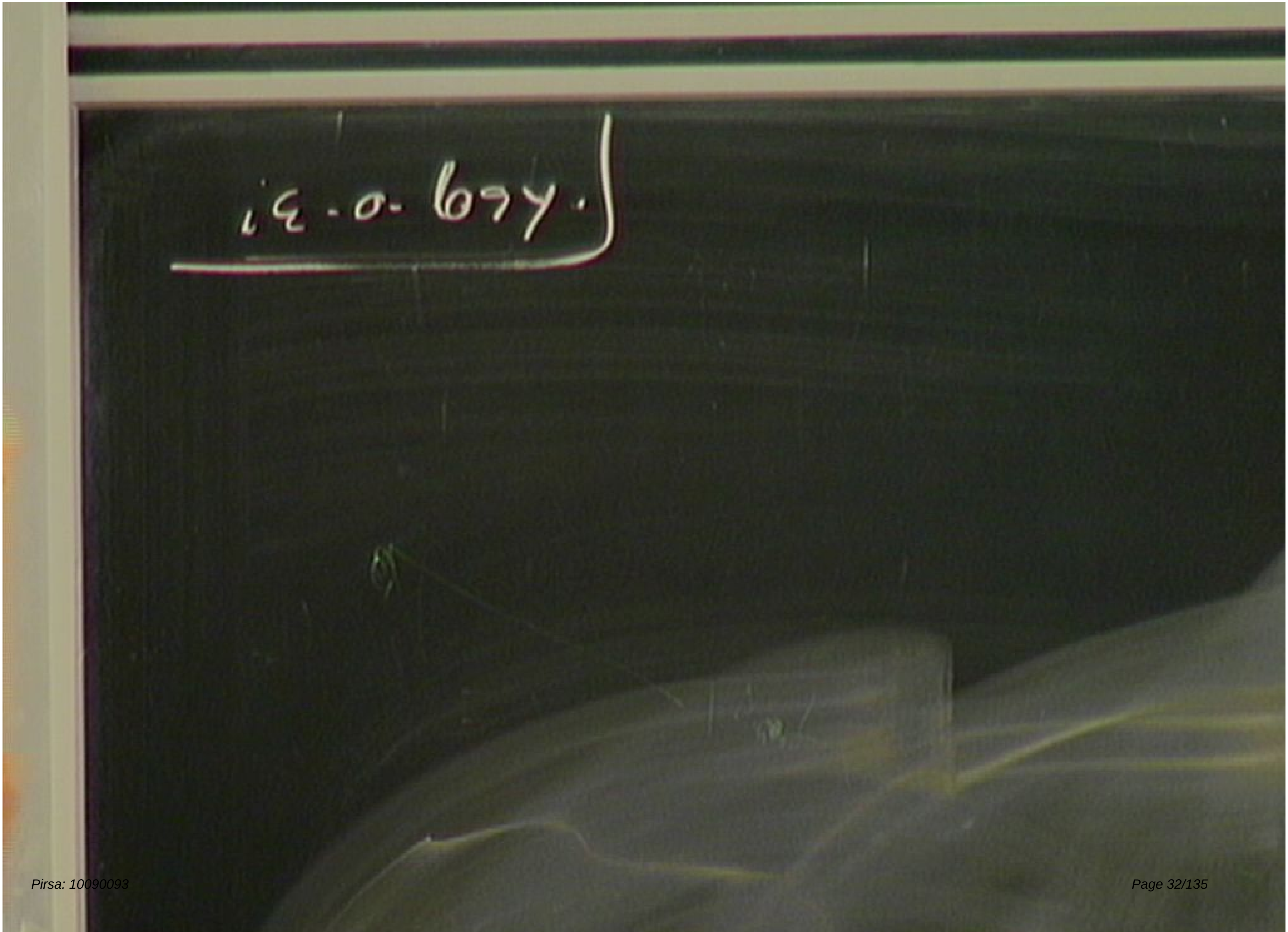

$$i\varepsilon \cdot \sigma \cdot b q y \cdot$$

$$A_n^{\text{1-loop}}(p_1 \dots p_n) =$$

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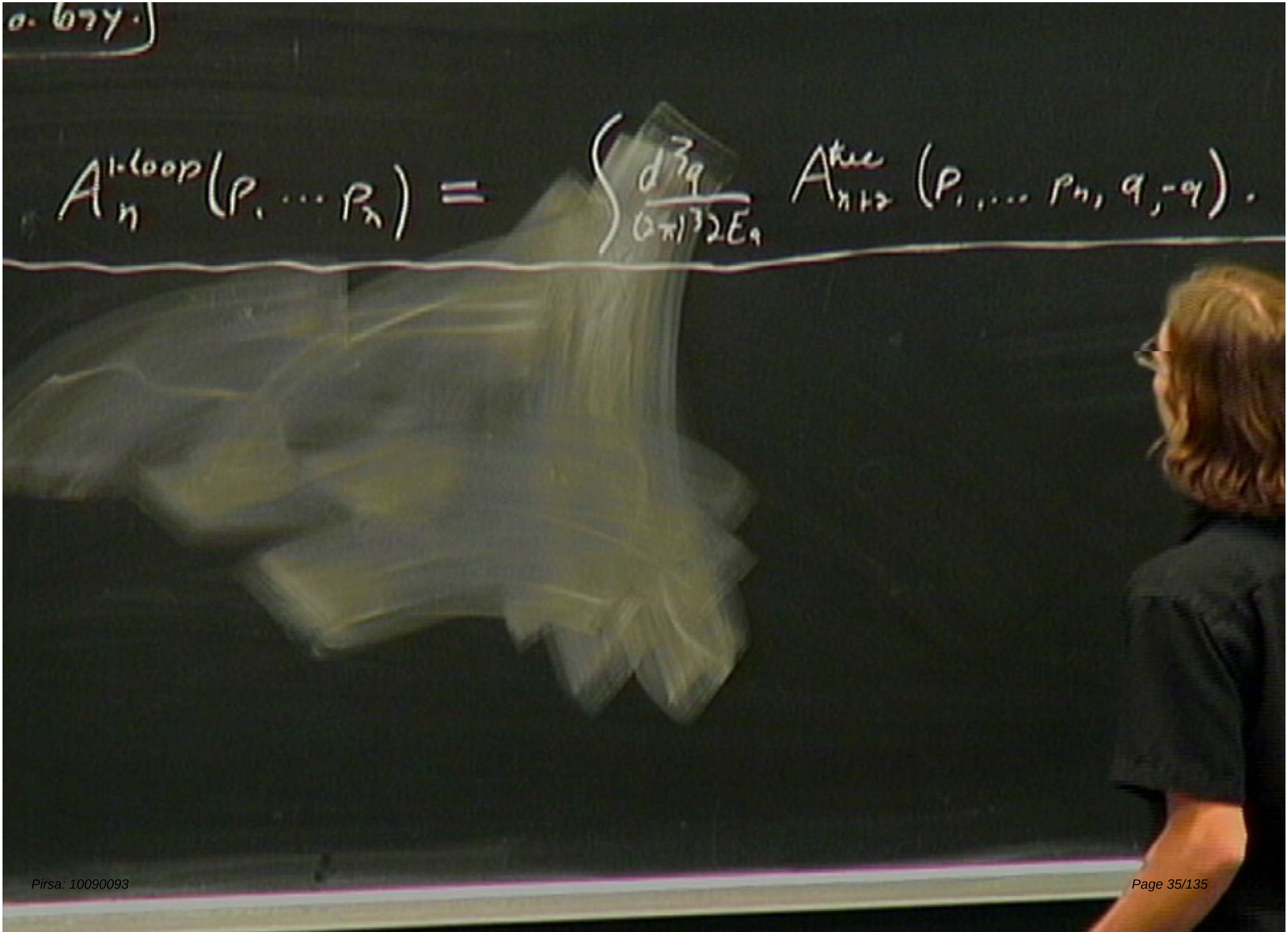
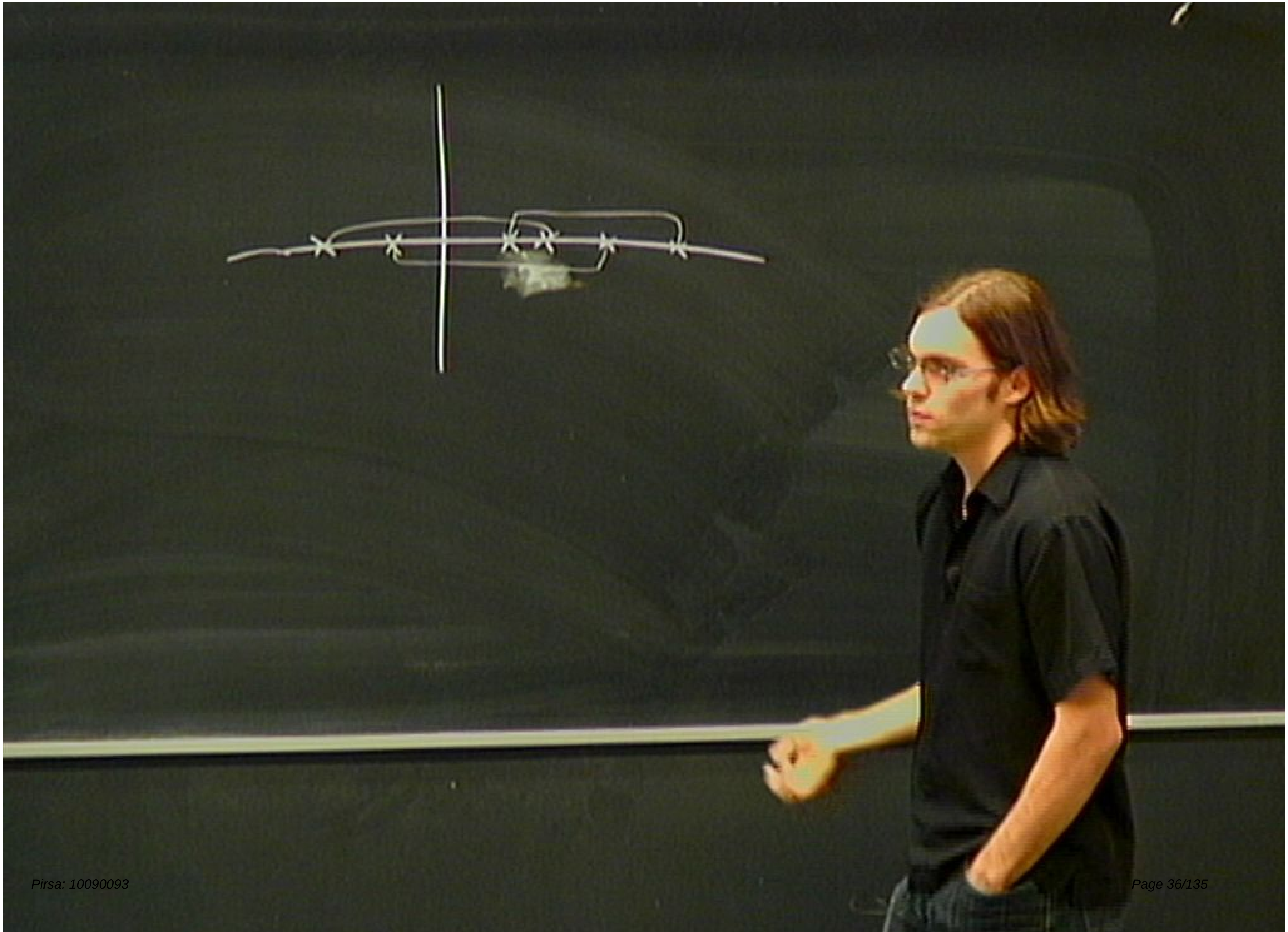
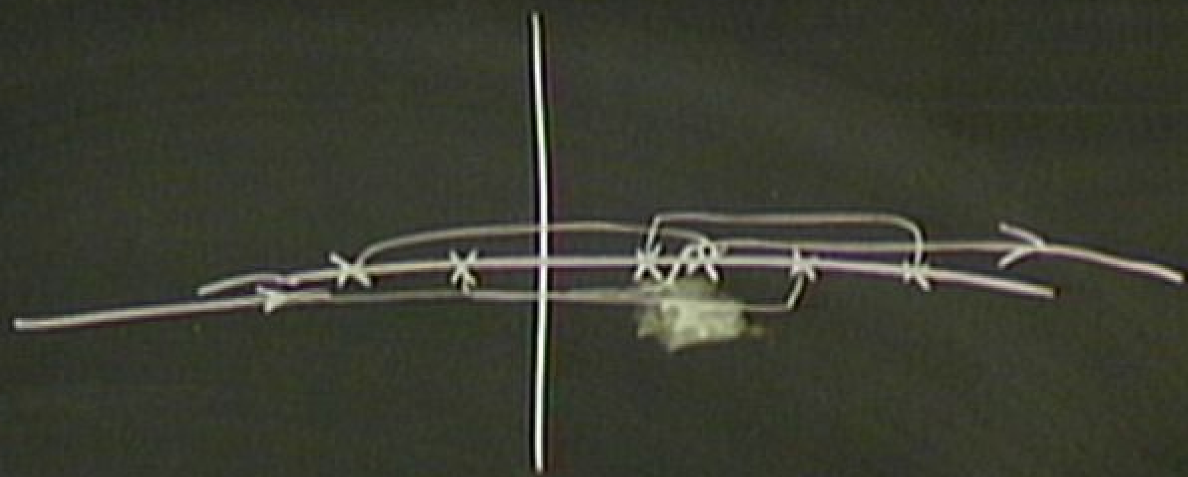
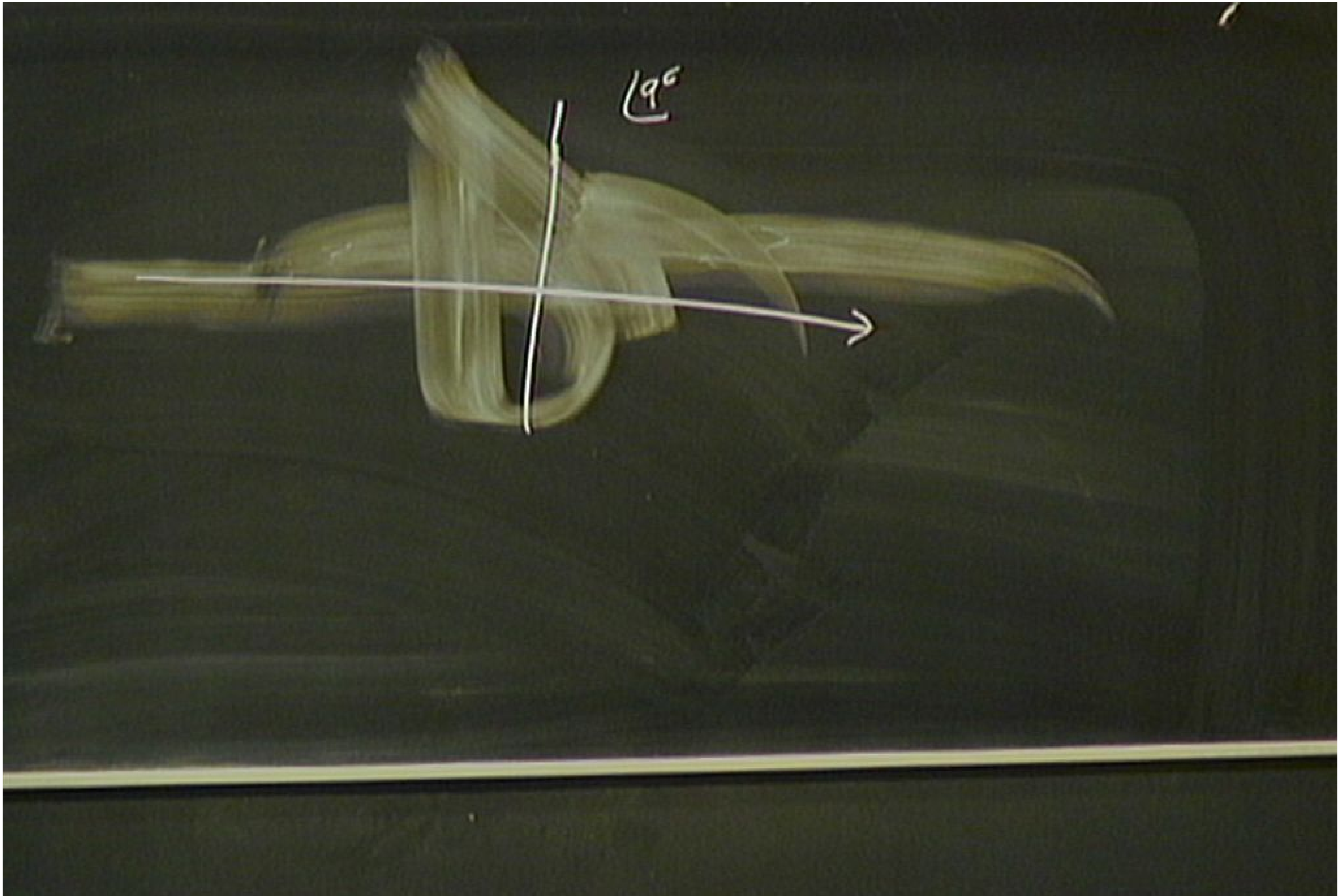
$$A_n^{1\text{-loop}}(p_1 \cdots p_n) = \int \frac{d^3 q}{(2\pi)^3 \, 2E_q}$$


$A_n^{1\text{-loop}}(p_1 \ldots p_n) = \int \frac{d^3 q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \ldots p_n, q, -q).$



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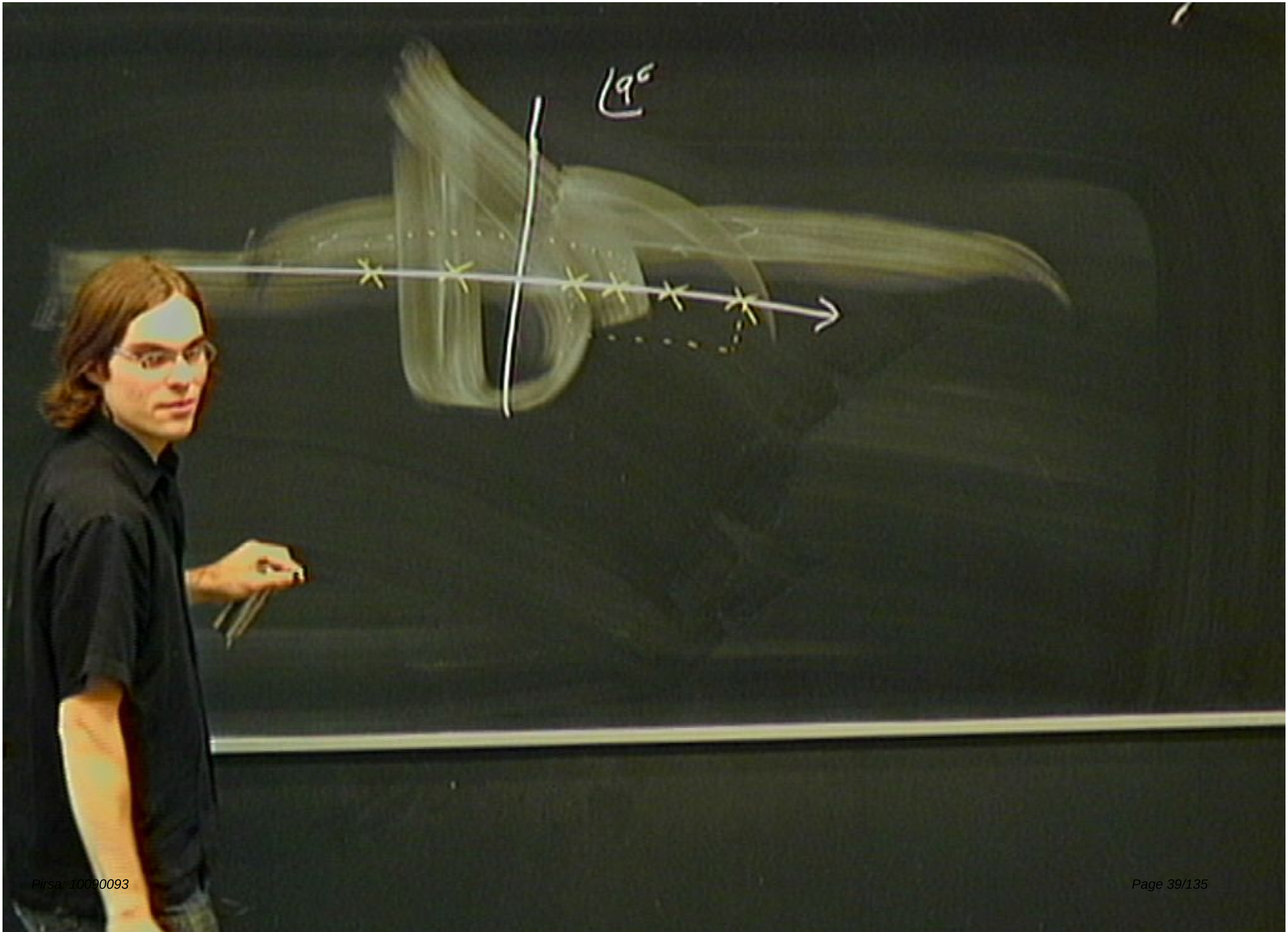
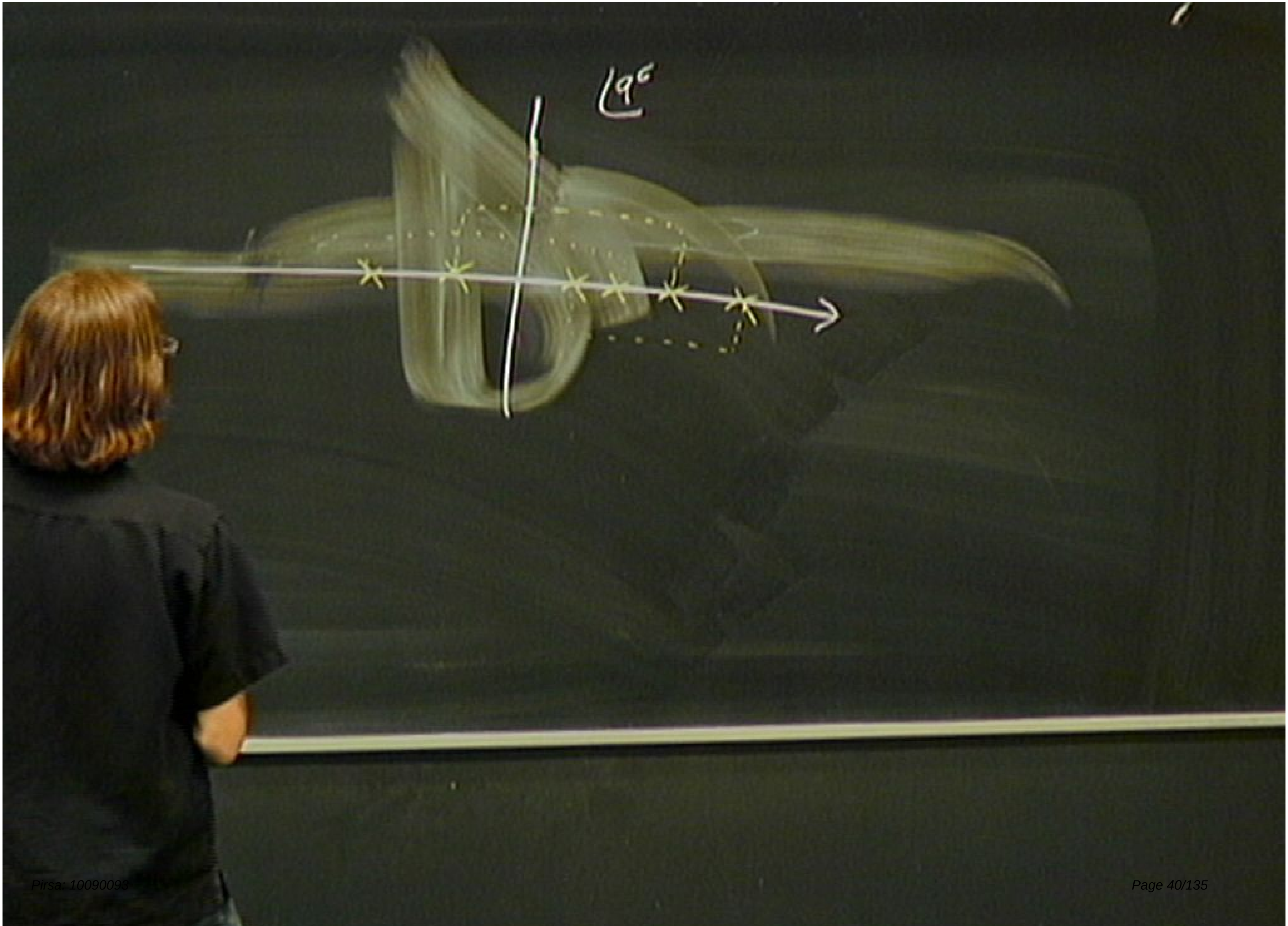
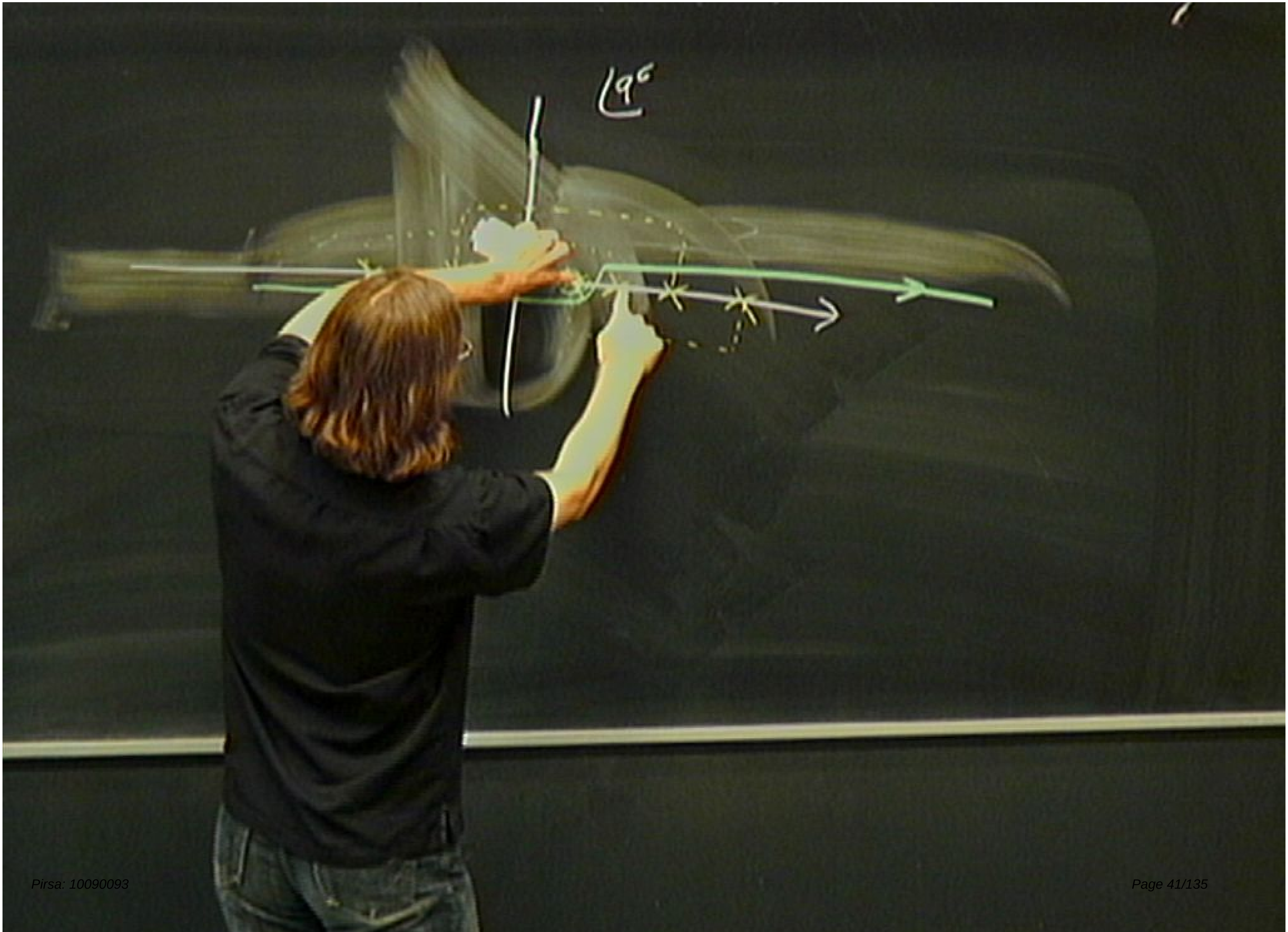
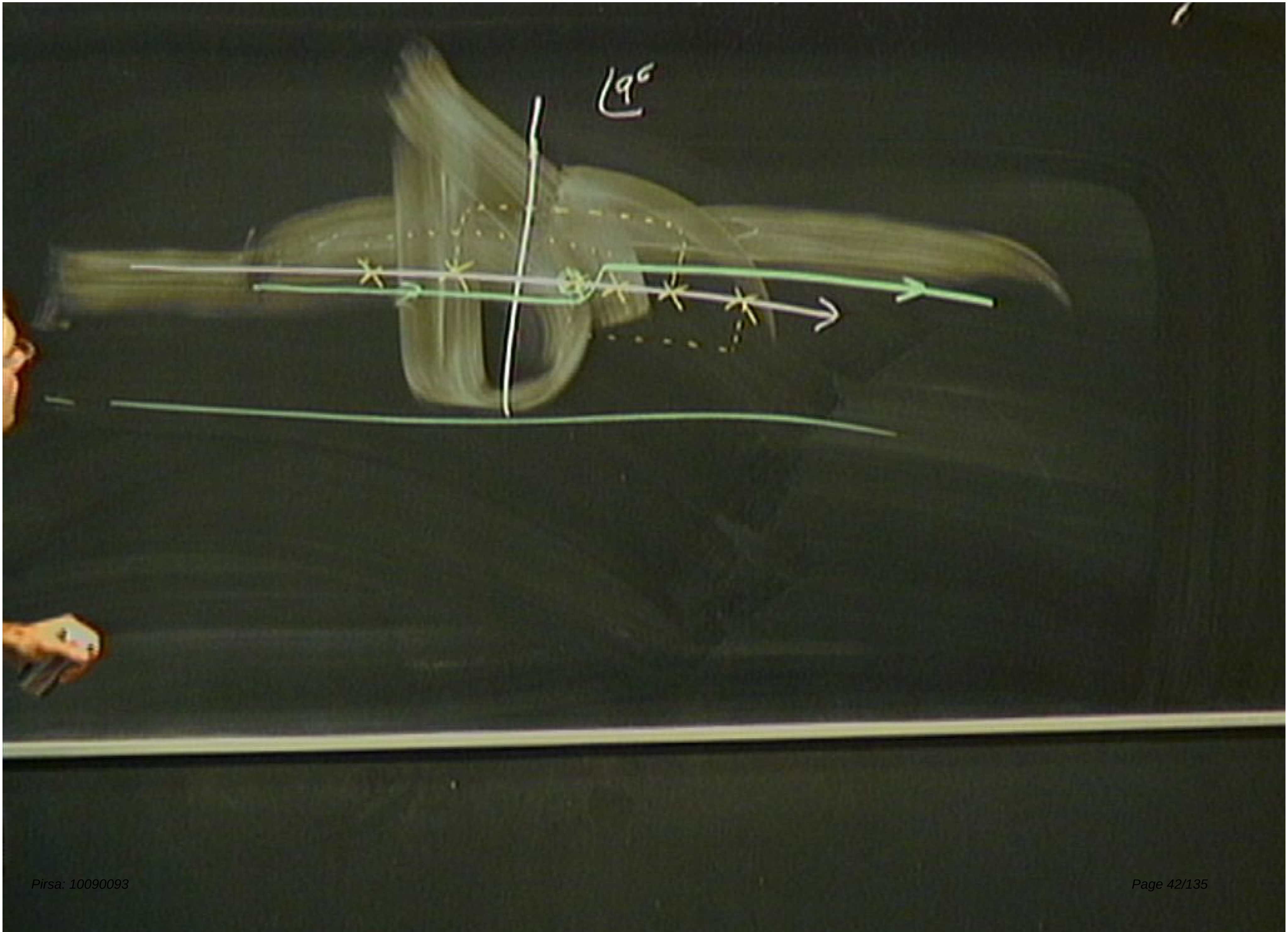
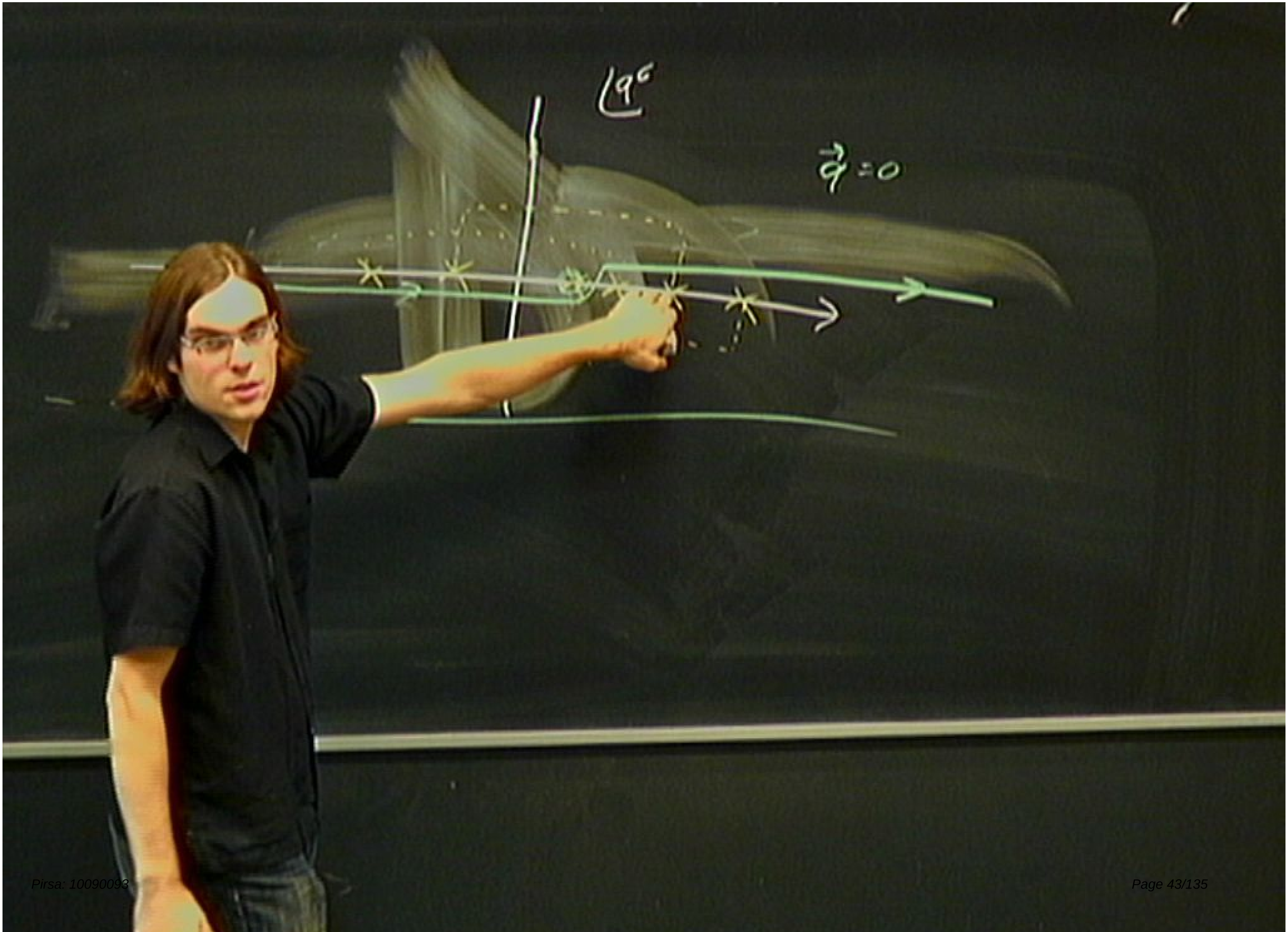
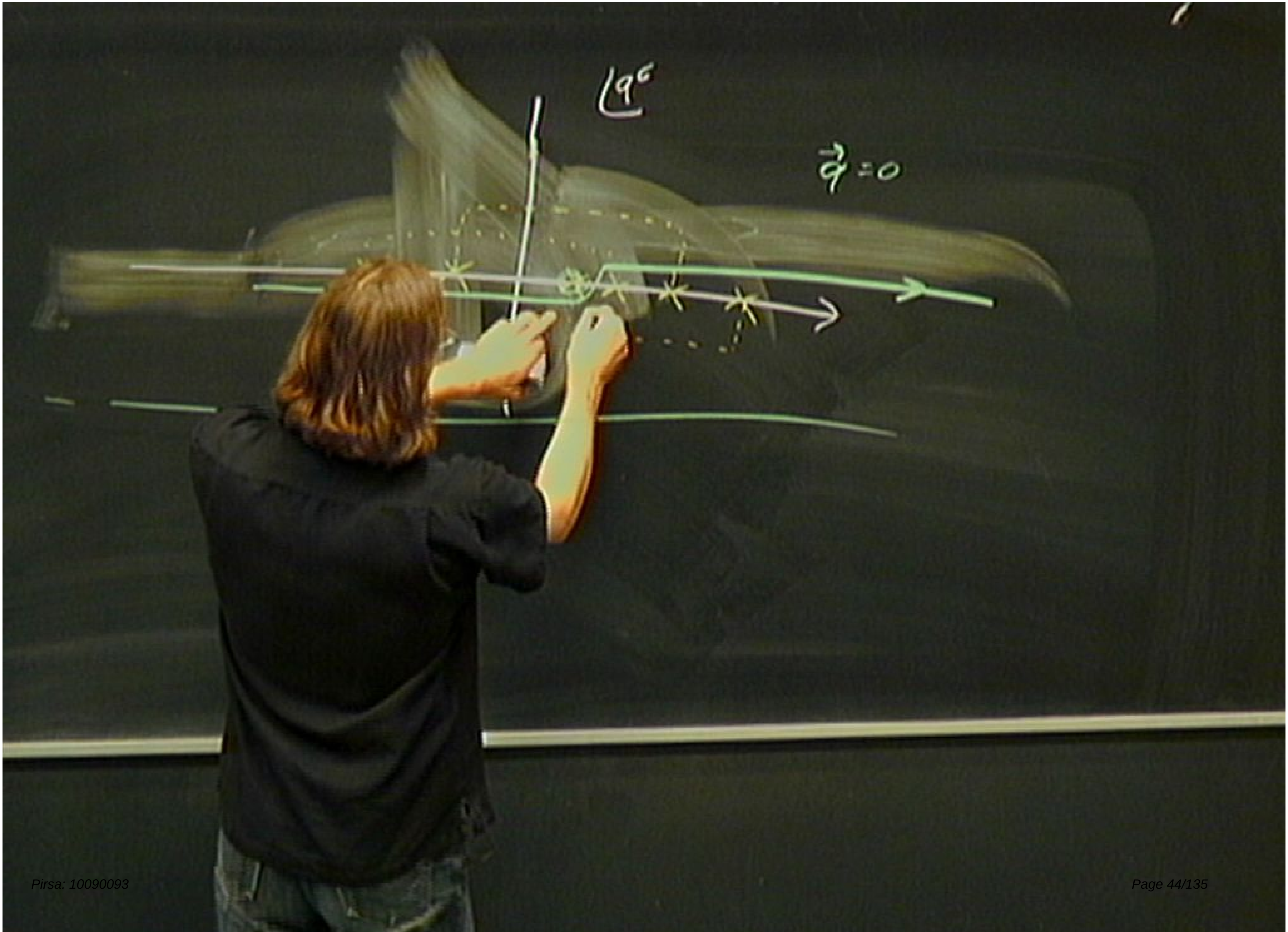









$\vec{q}=0$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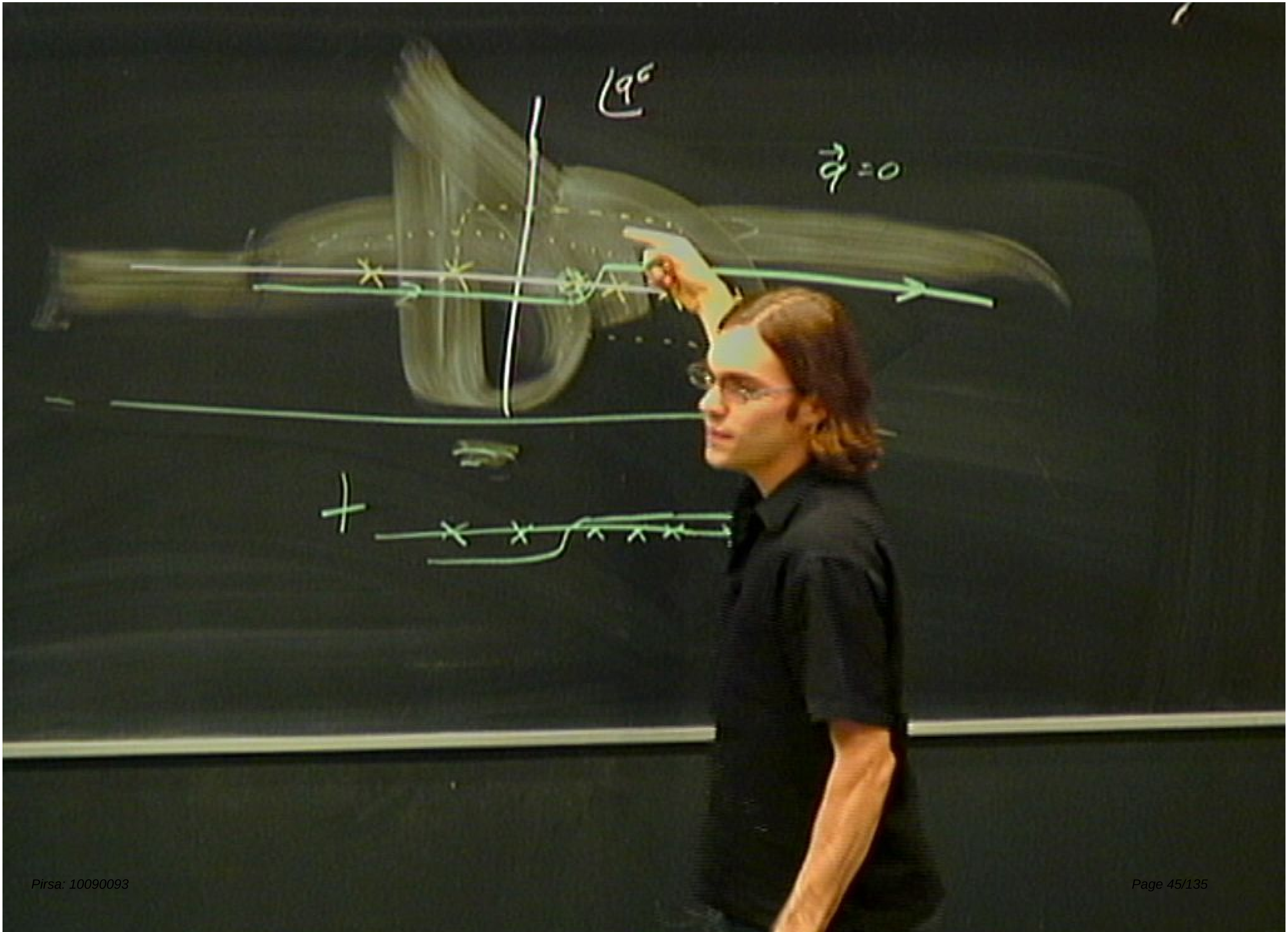
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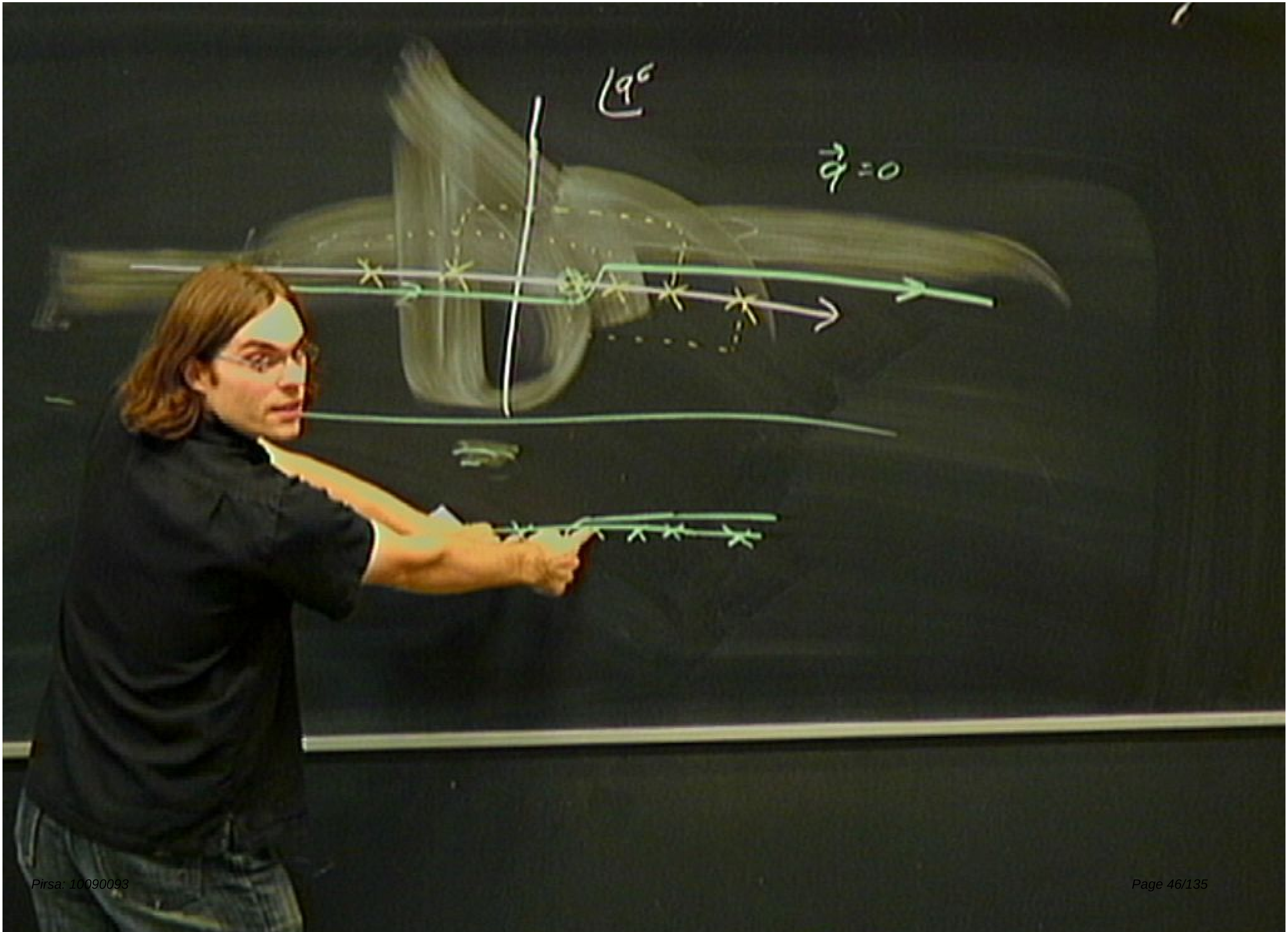
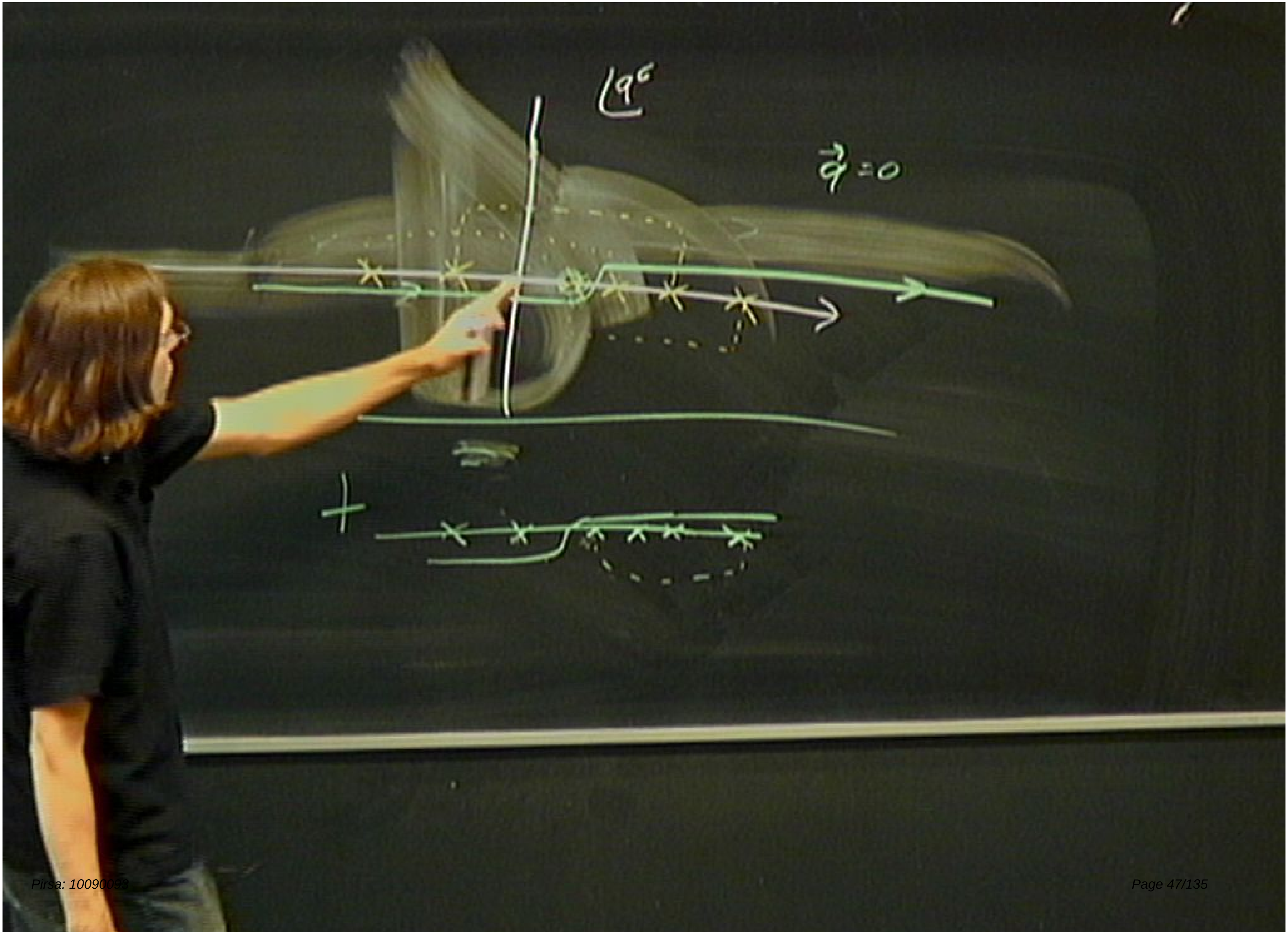


$\vec{q} = 0$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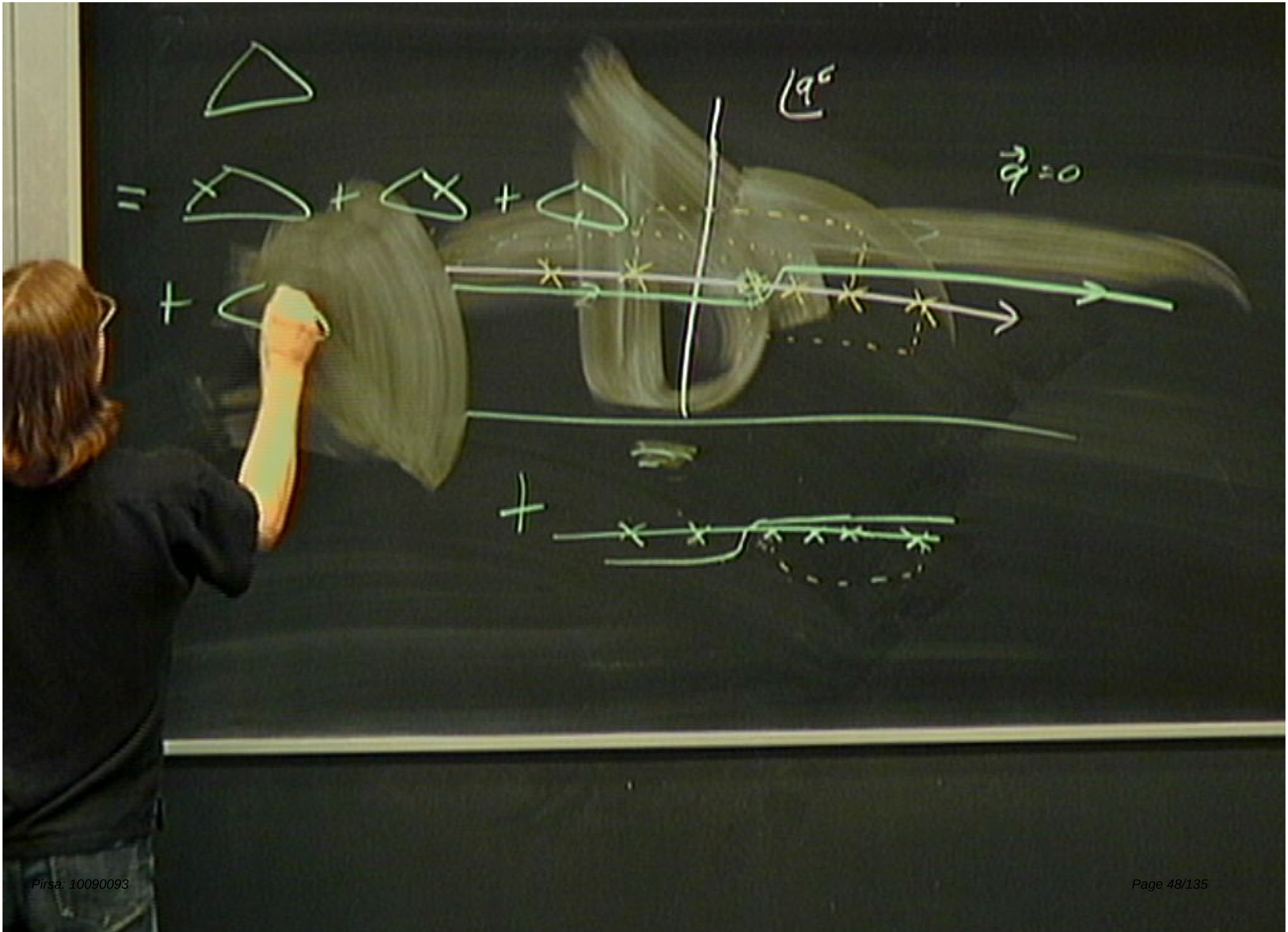

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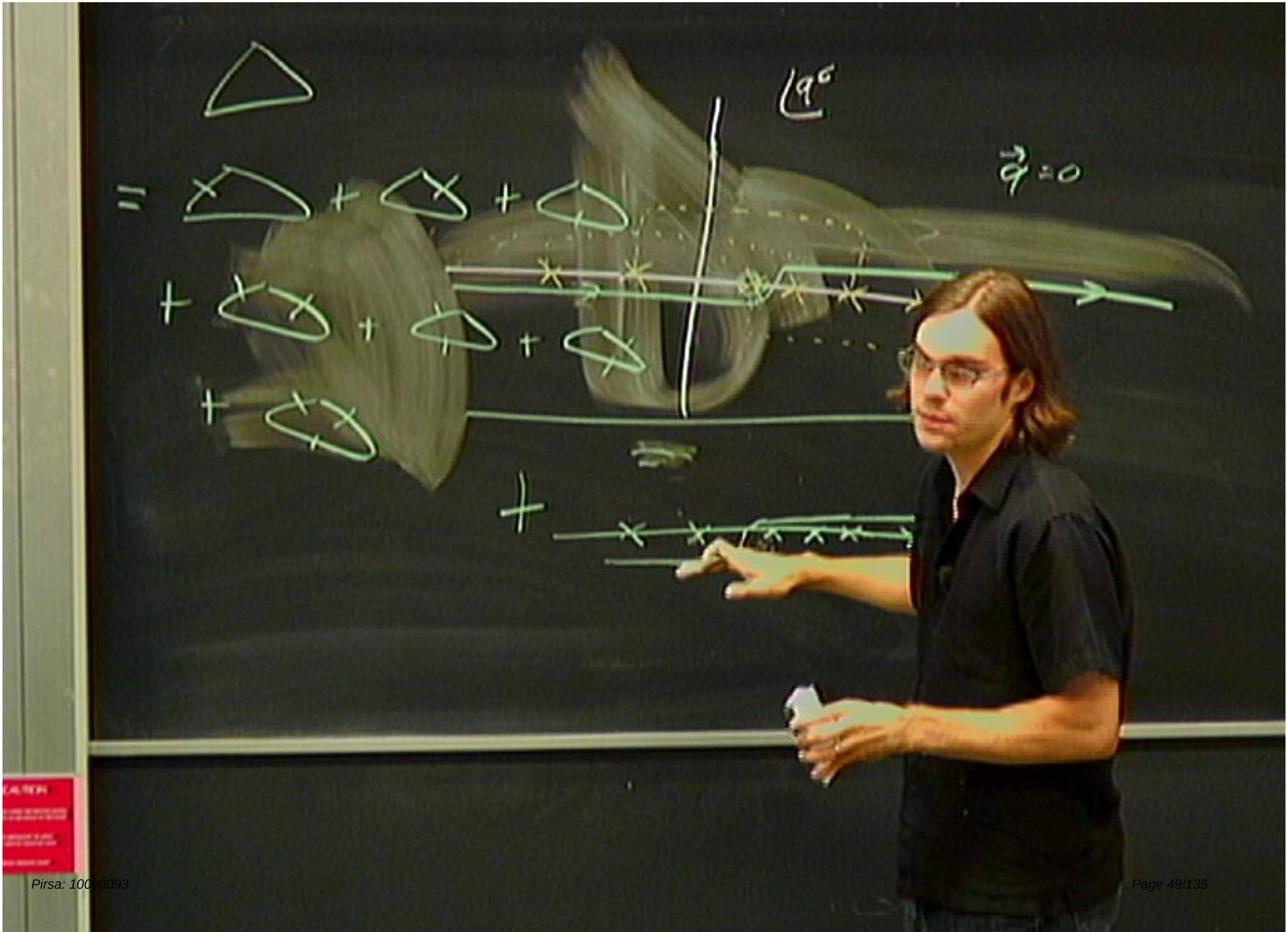

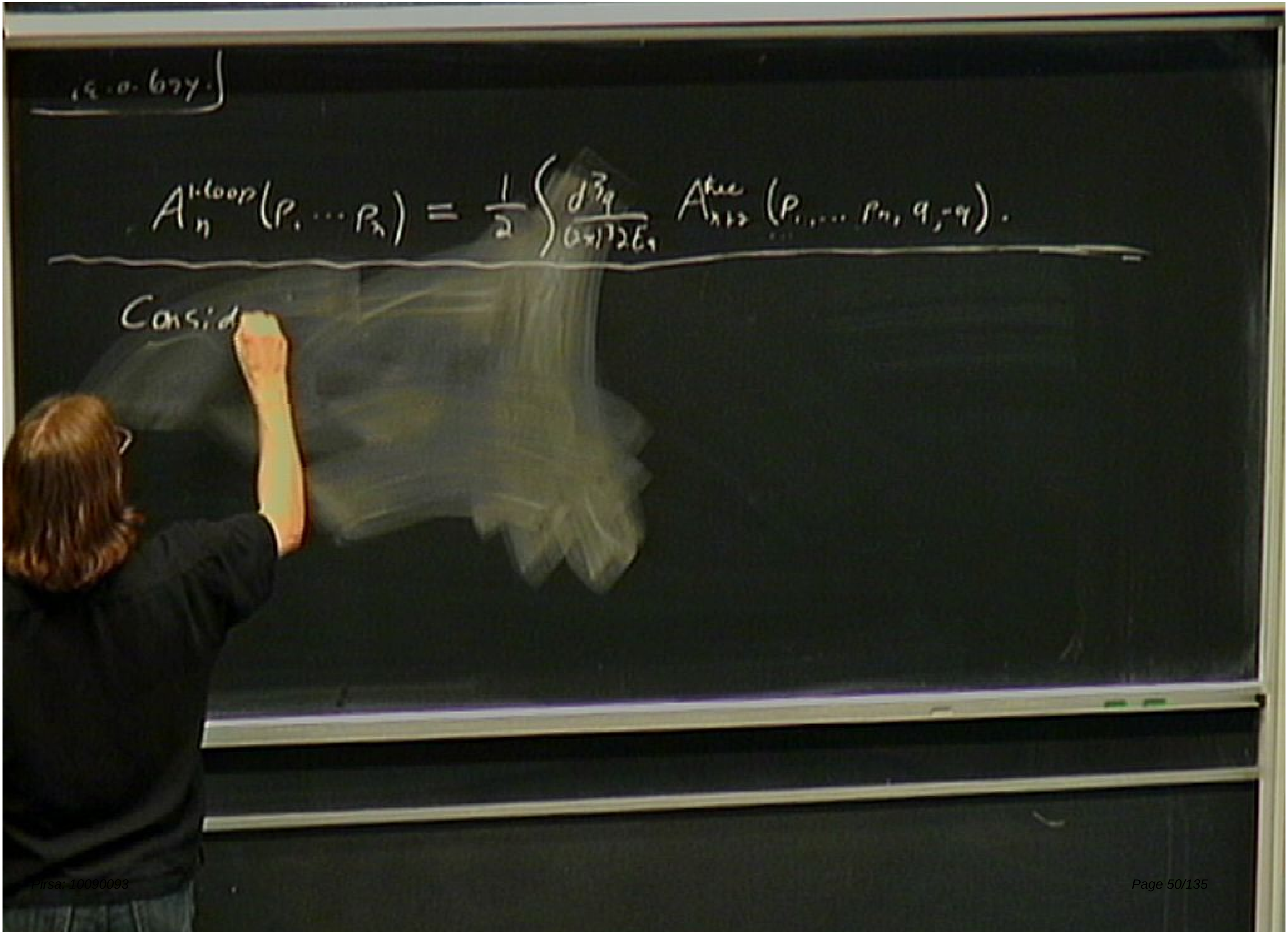
$A_n^{1\text{-loop}}(p_1 \cdots p_n) = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3 2E_q} A^{tree}_{n+2}(p_1, \dots p_n, q, -q)$.
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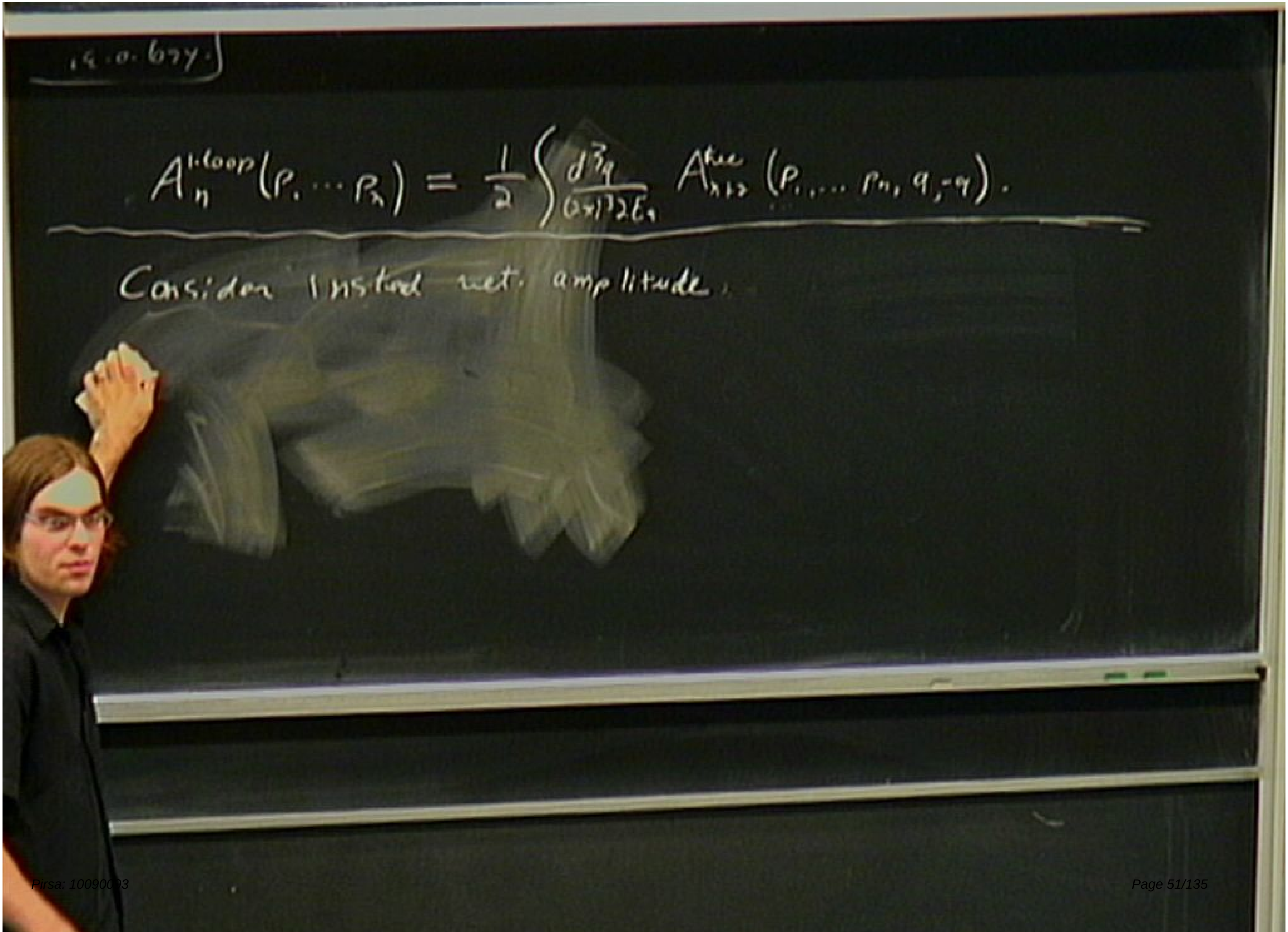
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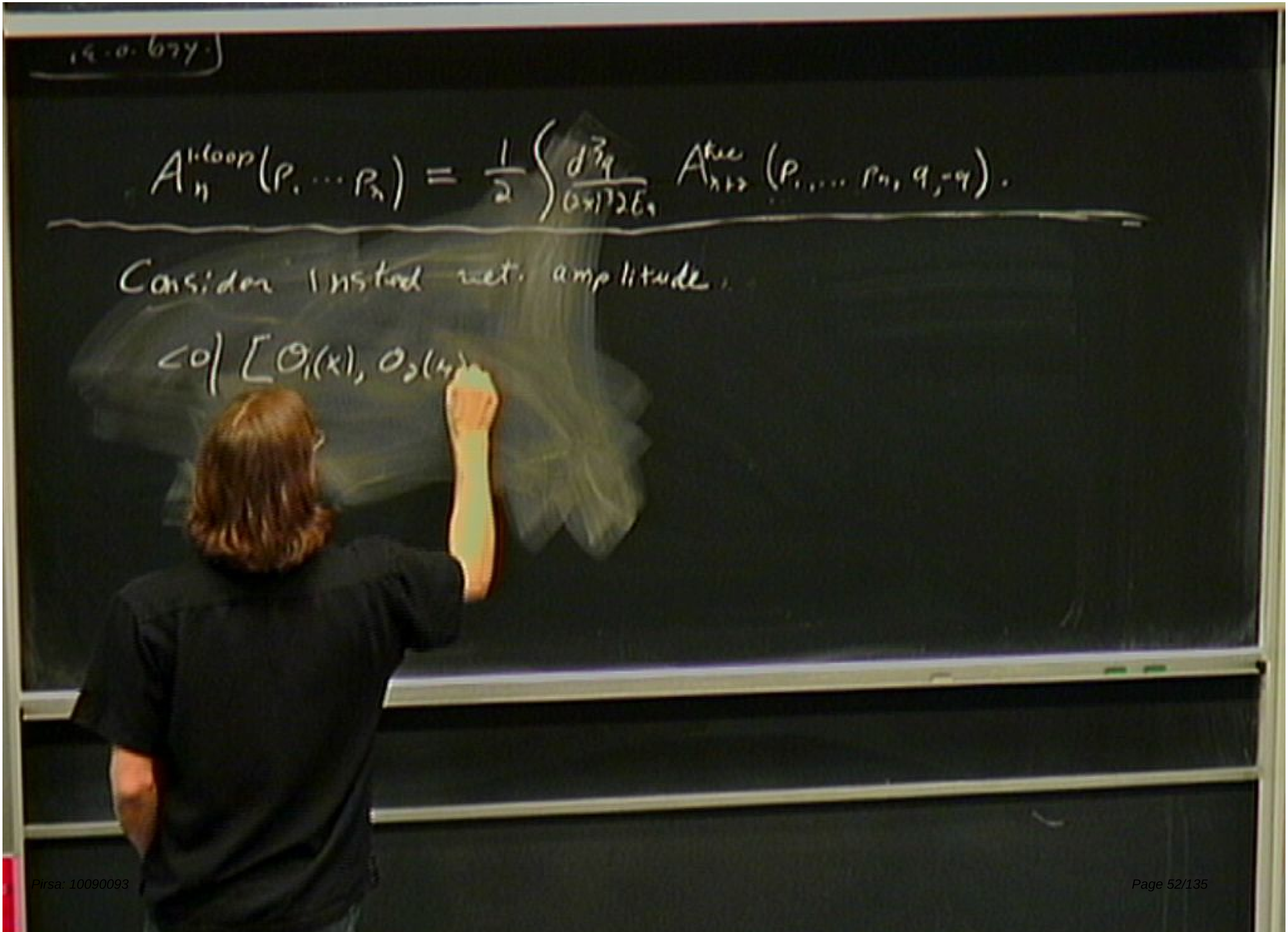
Consider instead ret. amplitude.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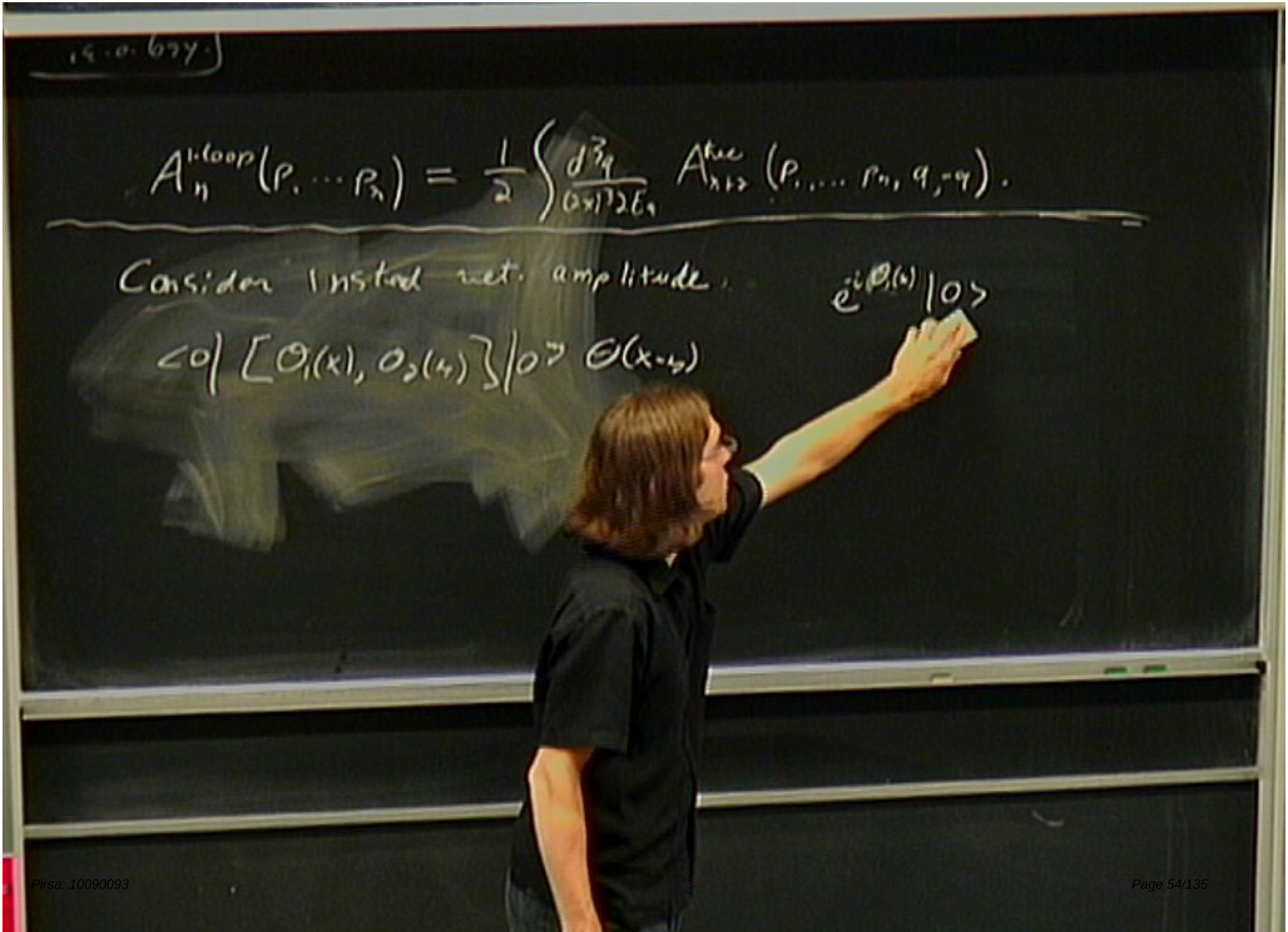
$\langle 0| [O_1(x), O_2(y)$

(e.q. 677.)

$$A_n^{1\text{-loop}}(p_1 \cdots p_n) = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \ldots p_n, q, -q).$$

Consider instead ret. amplitude.

$$\langle 0| [O_1(x), O_2(y)] |0\rangle \, \Theta(x-y)$$



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Consider instead ret. amplitude.
$e^{i\mathcal{O}_1(x)}|0\rangle$
$\langle 0|[\mathcal{O}_1(x), \mathcal{O}_2(y)]|0\rangle\,\Theta(x-y)$

(9.0.6yy.)

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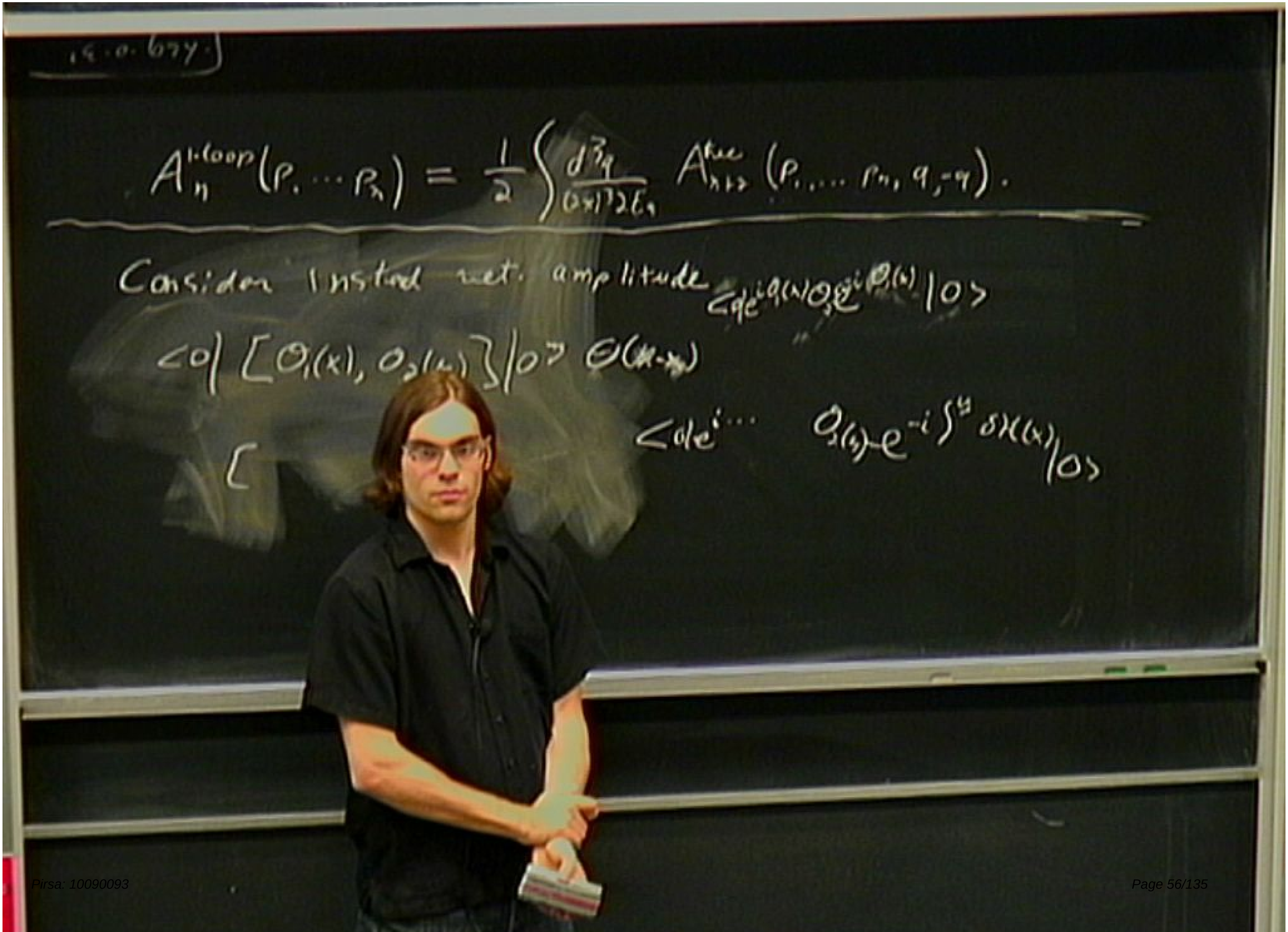
Consider instead ret. amplitude $\langle 0|e^{iO_1(x)} O_2 e^{-iO_1(x)}|0\rangle$

$$\langle 0|\,[O_1(x), O_2(y)]\,|0\rangle\, \theta(x-y)$$


$A_n^{1\text{-loop}}(p_1 \cdots p_n) = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3 2E_q} A^{\text{tree}}_{n+2}(p_1, \ldots p_n, q, -q).$
Consider instead ret. amplitude
$\langle 0|\,[\mathcal{O}_1(x), \mathcal{O}_2(y)]\,|0\rangle\, \Theta(x-y)$
$|0\rangle$

(5.0.677.)

$$A_n^{1\text{-loop}}(p_1 \cdots p_n) = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \ldots p_n, q, -q).$$

Consider instead ret. amplitude $\langle 0| e^{i\,Q(x)} O_2 e^{i\,Q_1(x)} |0\rangle$

$\langle 0| [O_1(x), O_2(y)] |0\rangle\, \Theta(x_0 - y_0)$

$[$

$\langle 0| e^{i\cdots} \quad O_2(y)\, e^{-i\int^y \delta H(x)} |0\rangle$

(4.0.677)

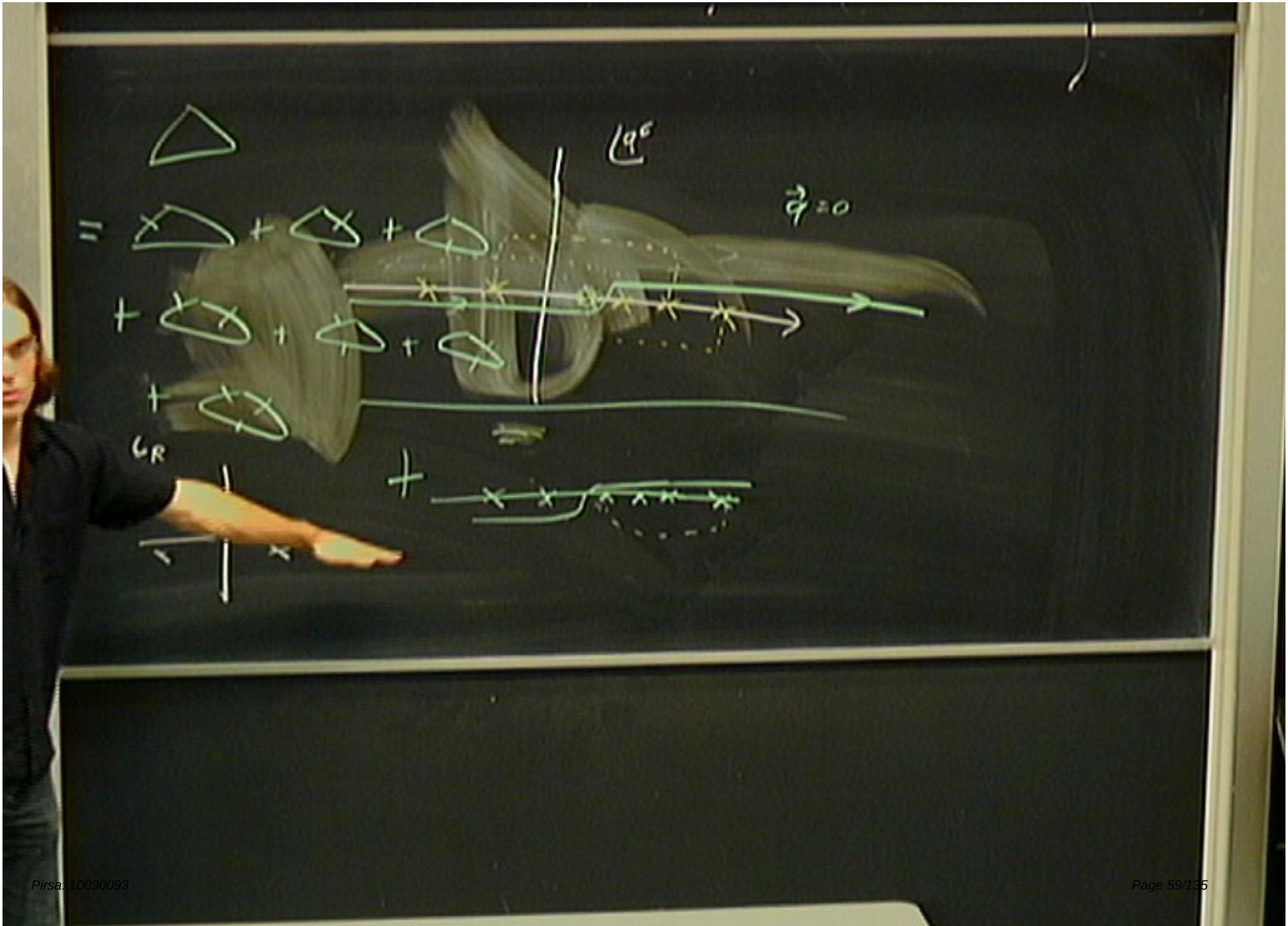
$$A_n^{1\text{-loop}}(p_1 \cdots p_n) = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \ldots p_n, q, -q).$$

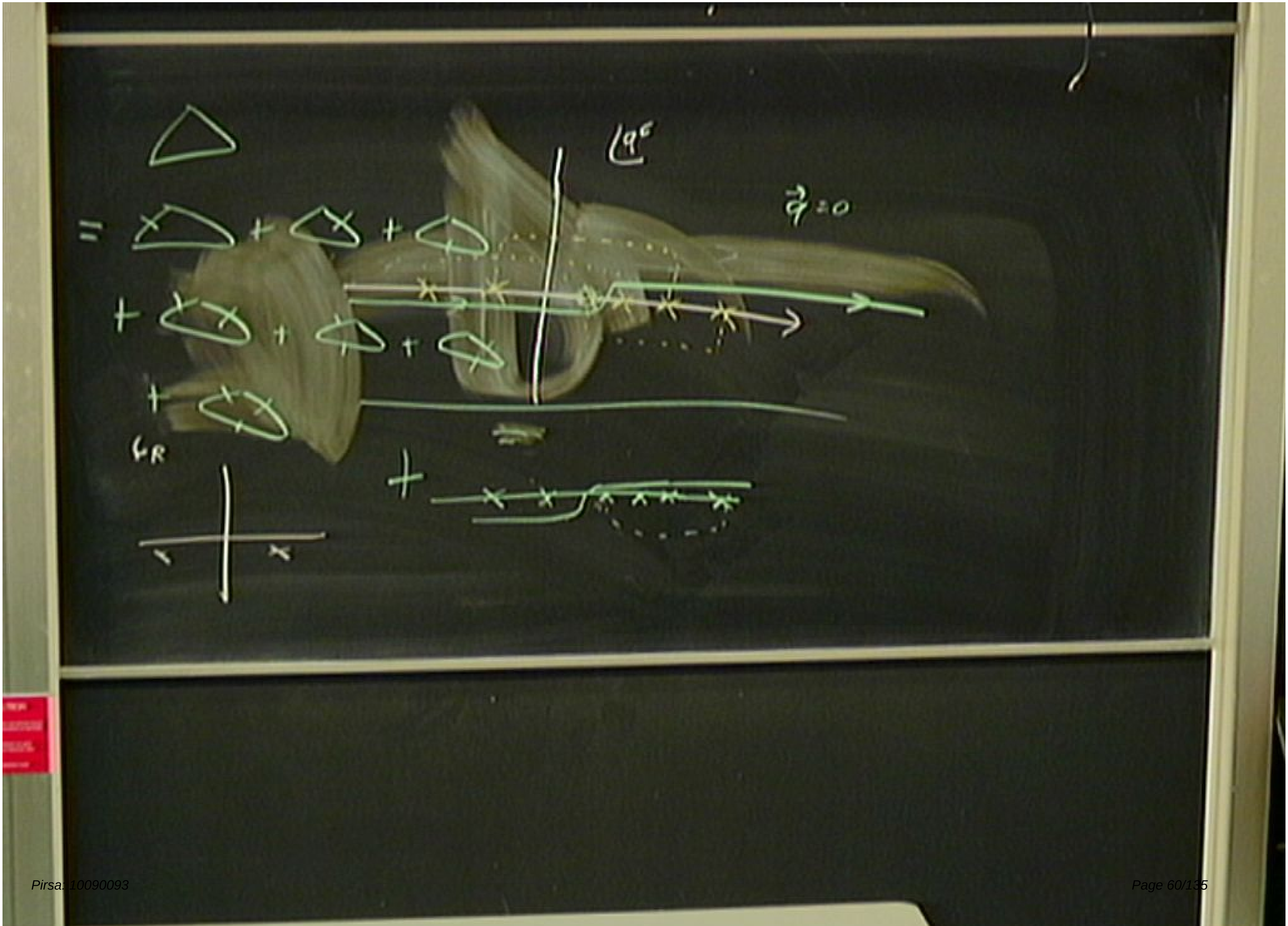
Consider instead ret. amplitude $\langle 0| e^{iO_1(x)} O_2 e^{iO_1(x)} |0\rangle$

$$\langle 0| [O_1(x), O_2(y)] |0\rangle\, \Theta(x_0 - y_0)$$

$$\langle 0| e^{i\cdots}\ O_2(y)\, e^{-i\int^y \delta H(x)} |0\rangle$$

 $= \dfrac{-i}{p^2 - i\epsilon} \to \dfrac{-i}{p^2 - i\epsilon p^0}$




q^0
$\vec{q}=0$
G_R

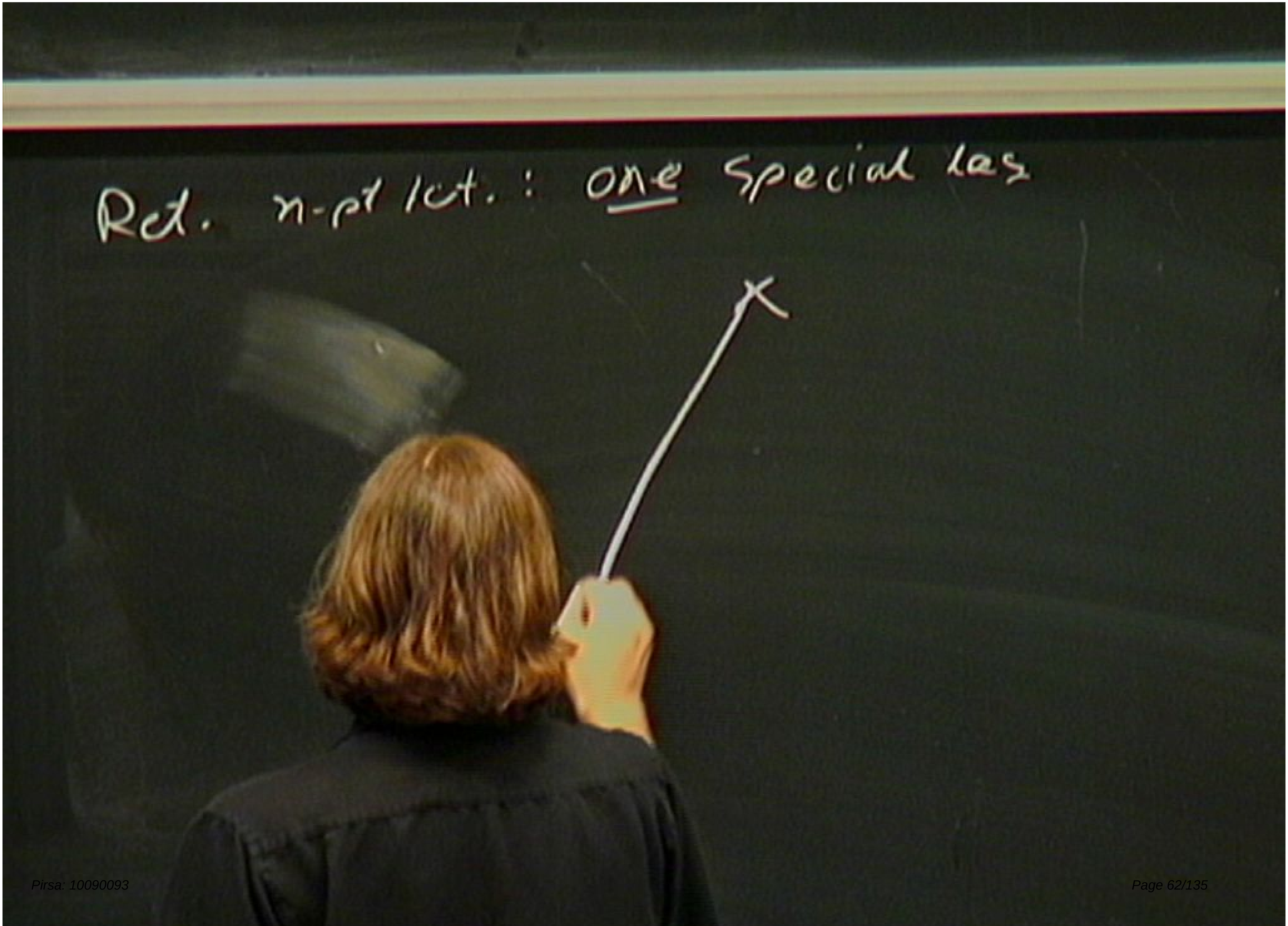
$$A_n^{1\text{loop}}(p_1 \cdots p_n) = \frac{1}{2}\int \frac{d^dq}{(2\pi)^d\, 2\ell_q}\, A_{n+2}^{\text{tree}}(p_1, \ldots, p_n, q, -q).$$

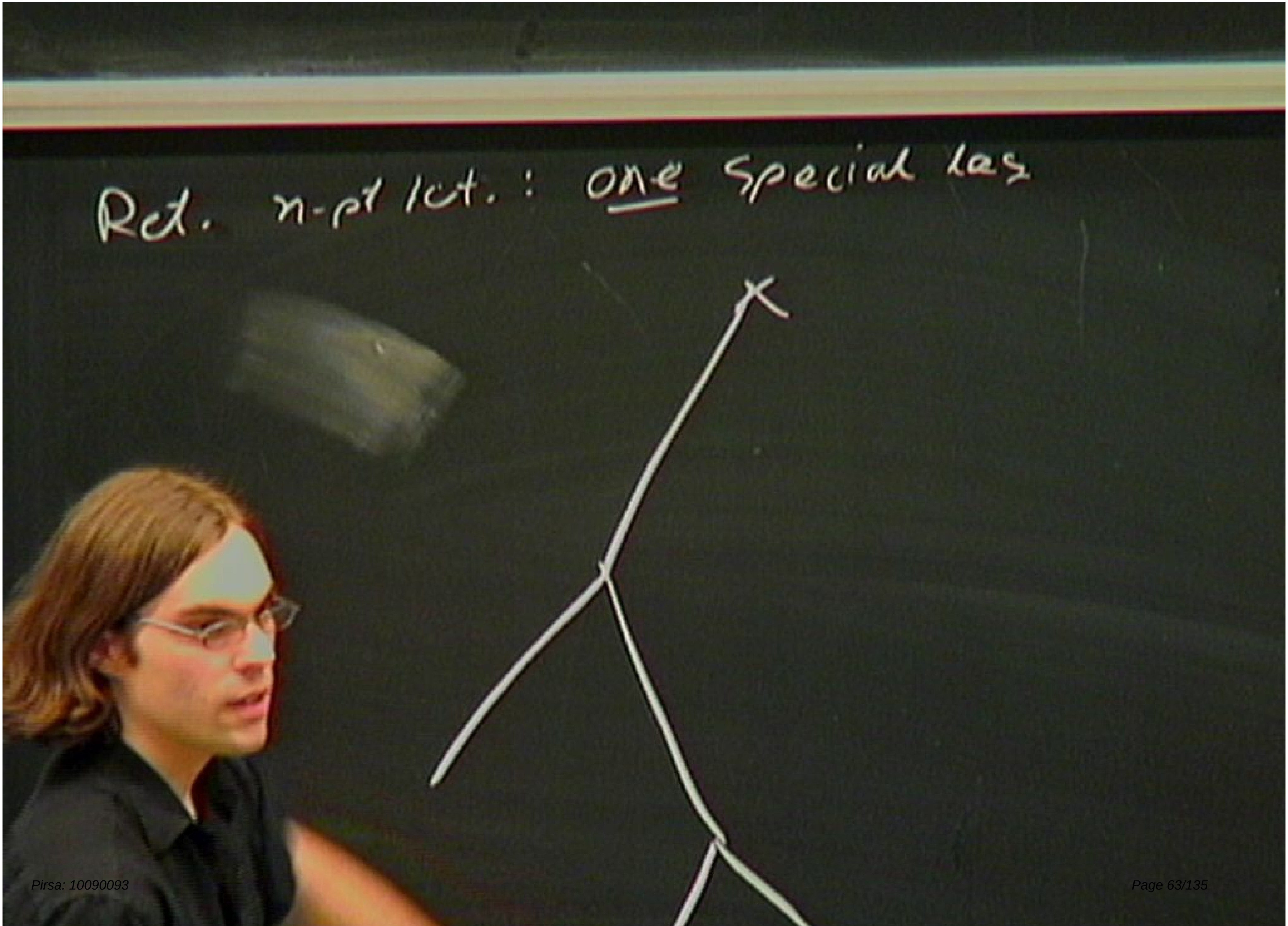
Consider instead ret. amplitude $\langle 0|e^{i\cdots}\,\mathcal{O}\,e^{i\cdots}|0\rangle$

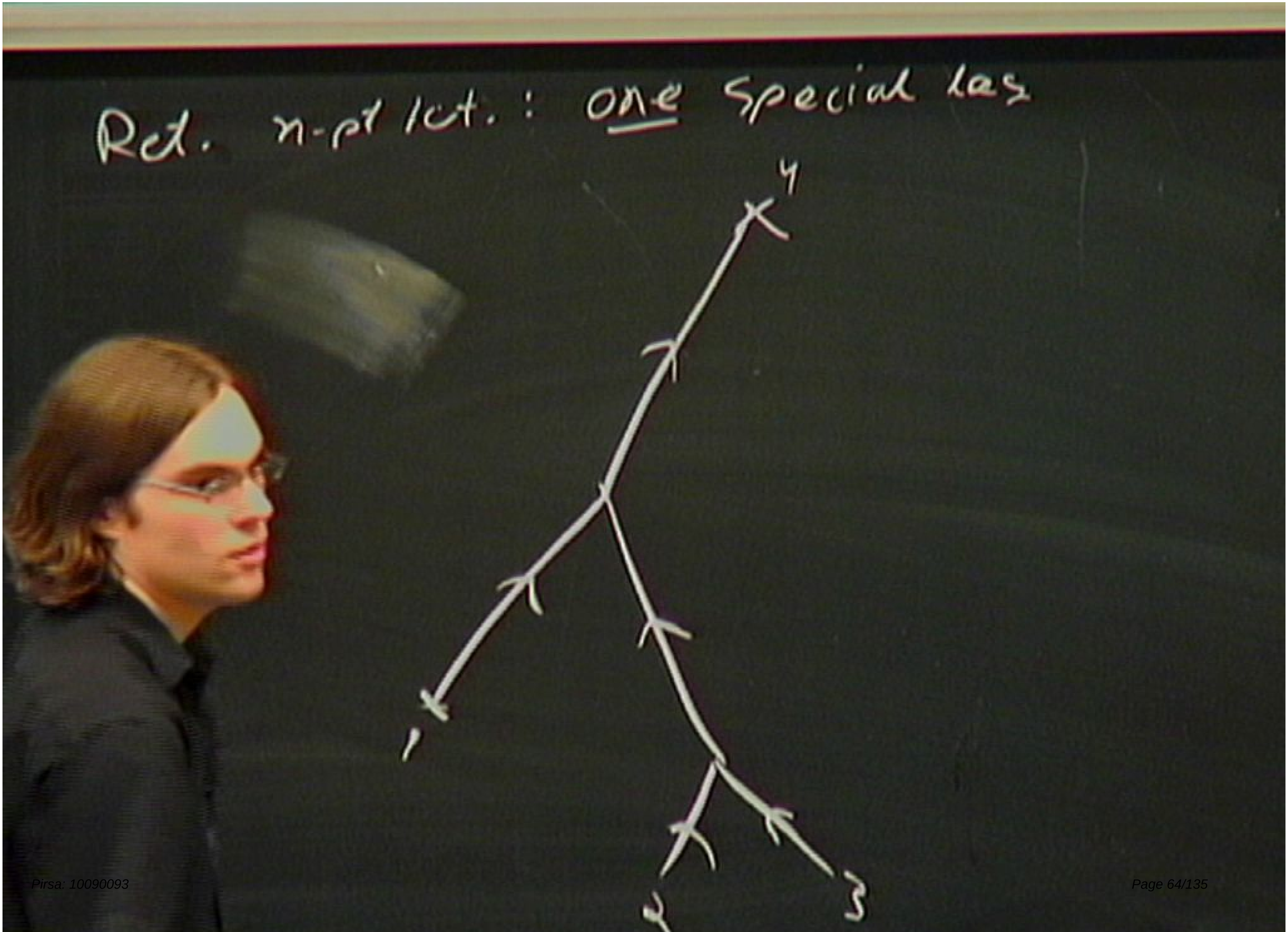
$$\langle 0|\,[\mathcal{O}_1(x), \mathcal{O}_2(y)]\,|0\rangle\, \theta(x-y)$$

$$\langle 0|e^{i\cdots}\quad \mathcal{O}(y)\, e^{-i\int^y \delta H(t)}|0\rangle$$

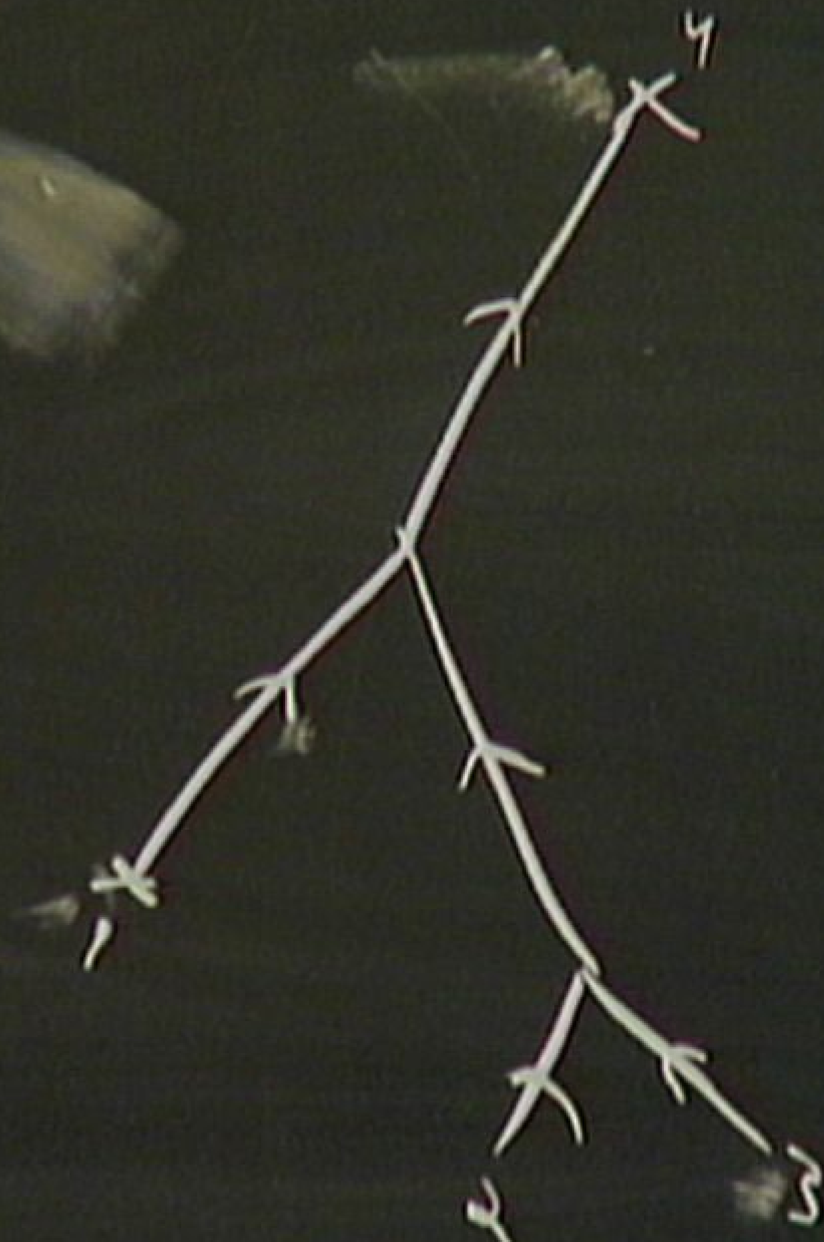
$$= \frac{-i}{p^2 - i\epsilon} \to \frac{-i}{p^2 \cdots}$$

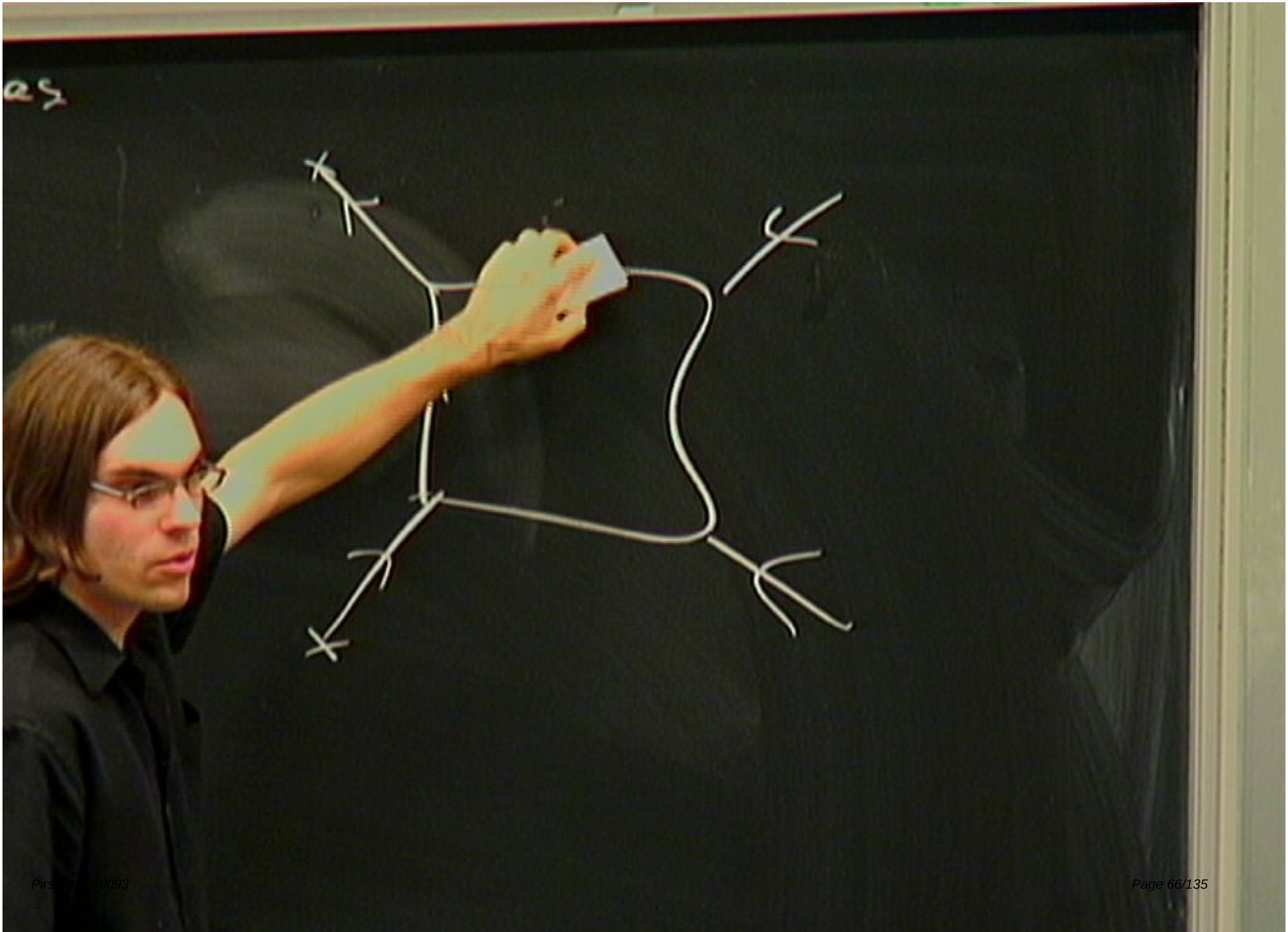

Rat. n-pt int. : one special leg

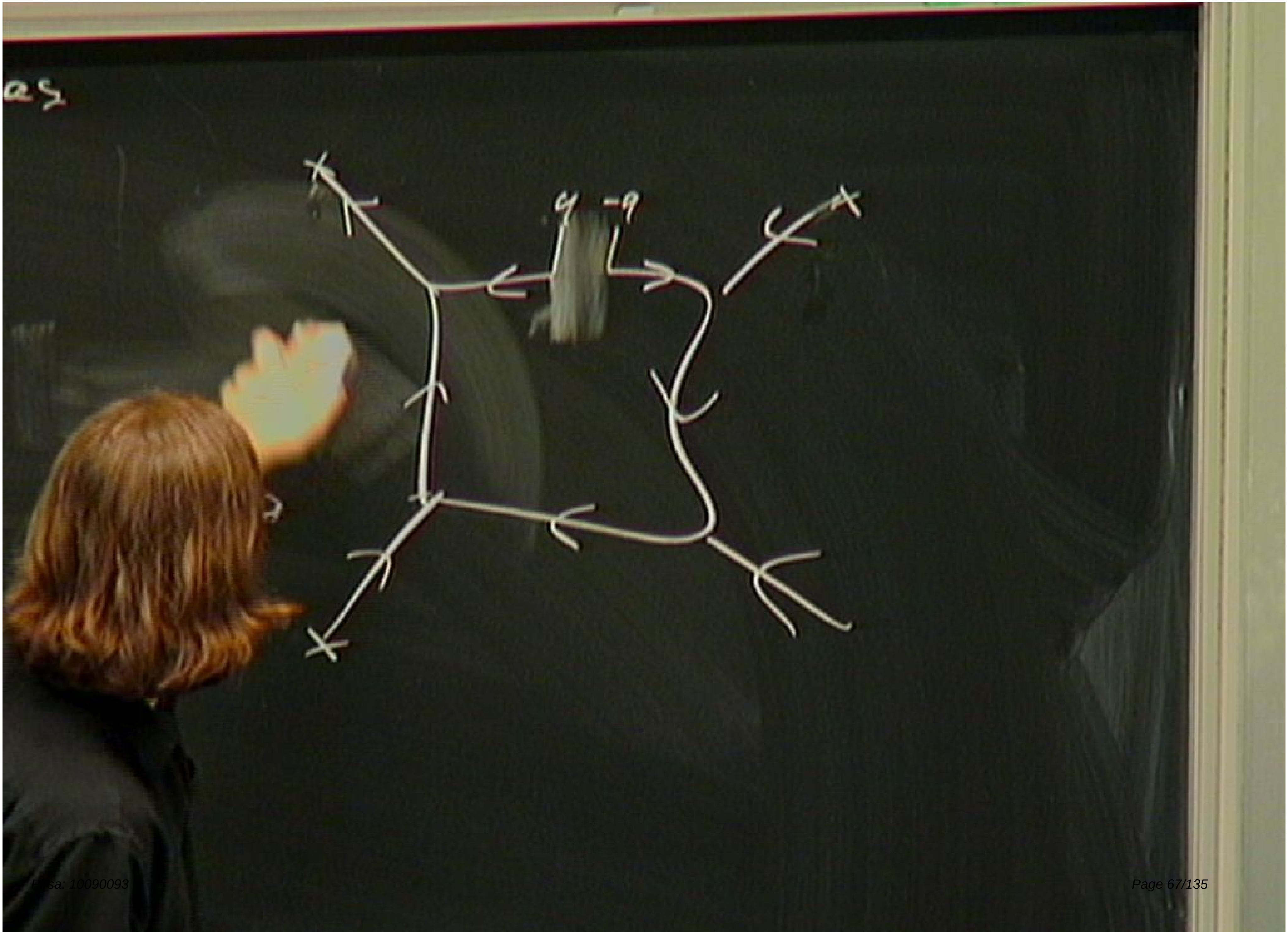

Rct. n-pt lct. : one special leg

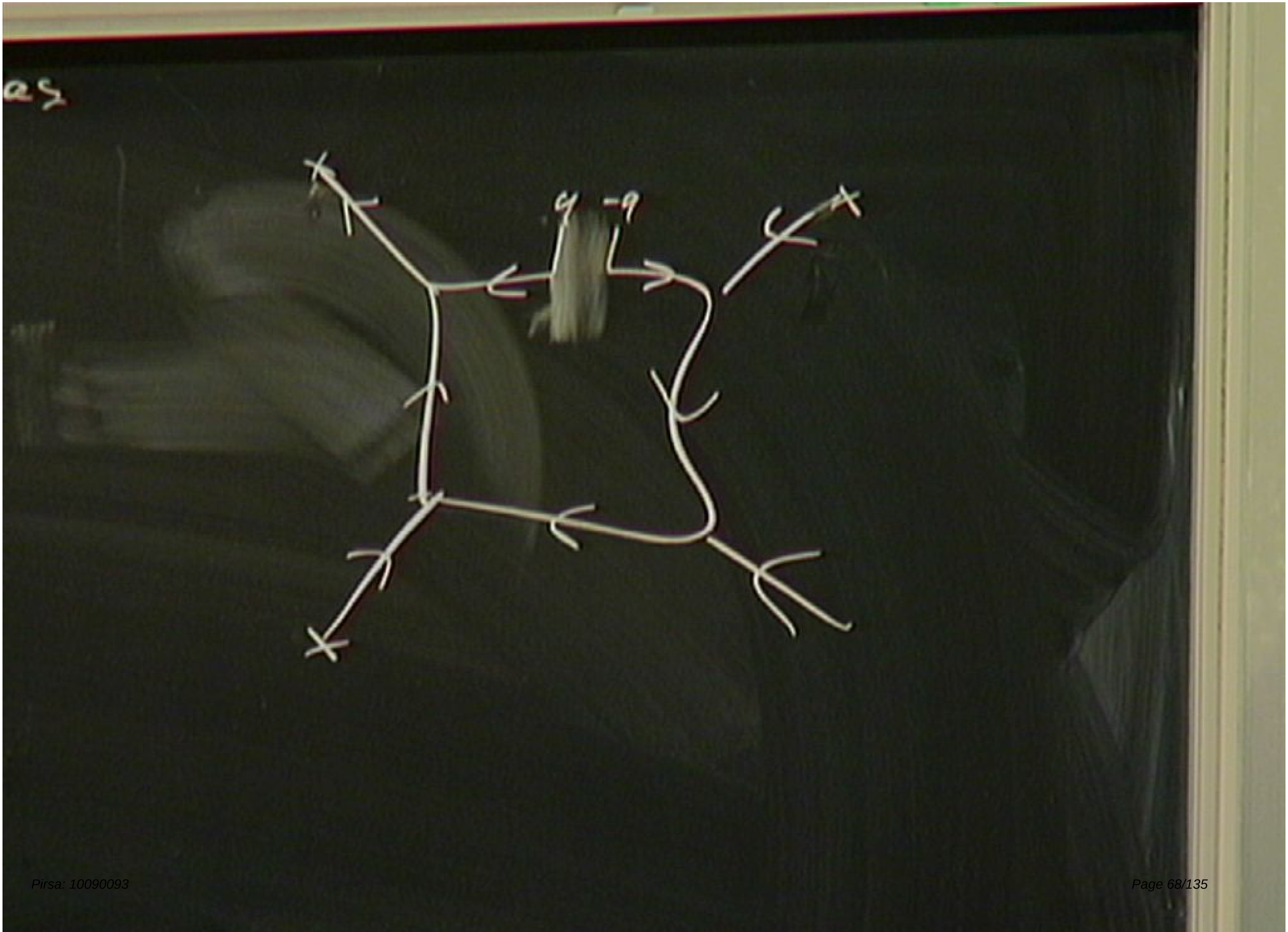

Rel. n-pt fct. : one special leg
4
1
2
3

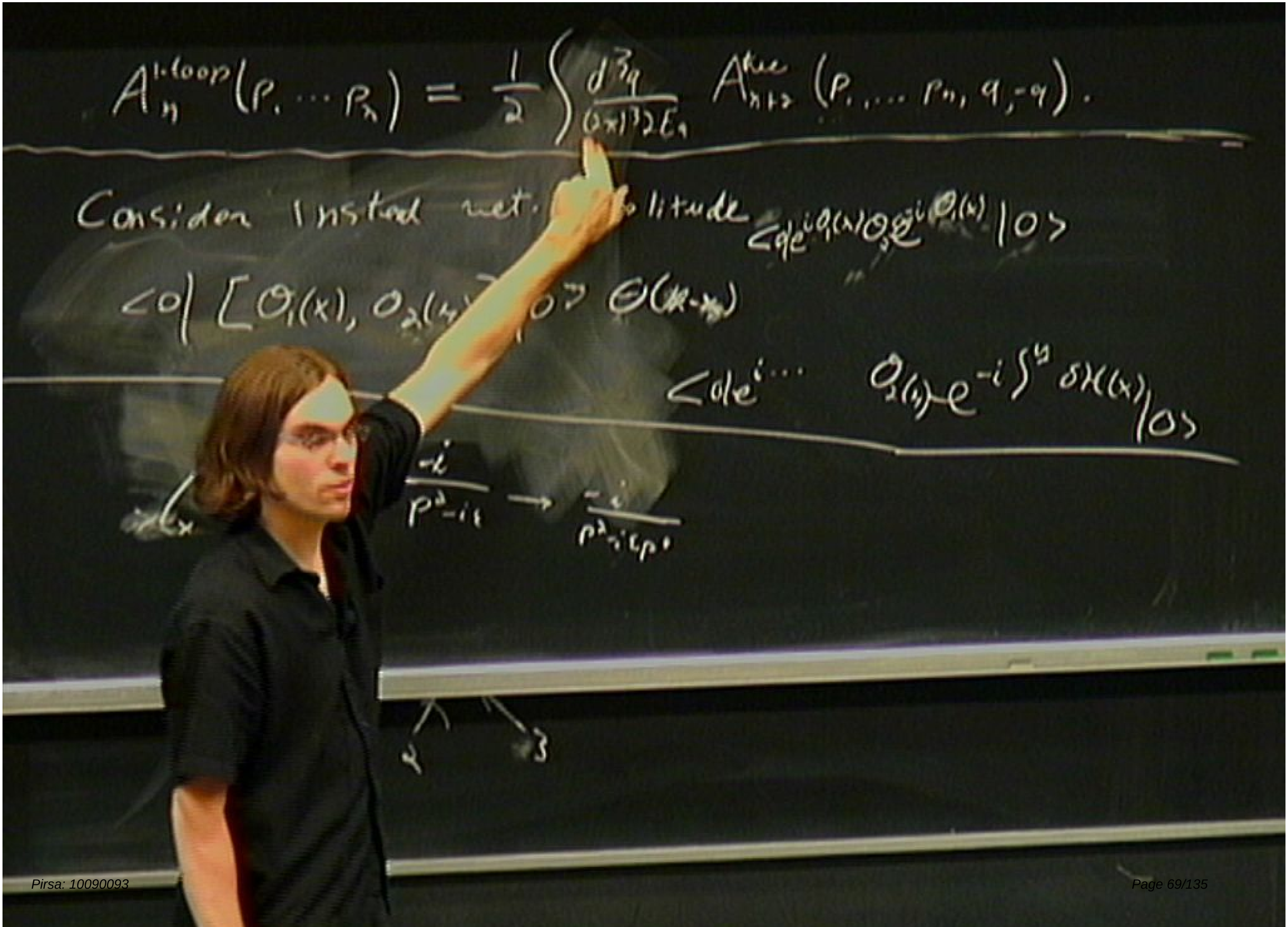
Rat. n-pt fct. : <u>one</u> special leg

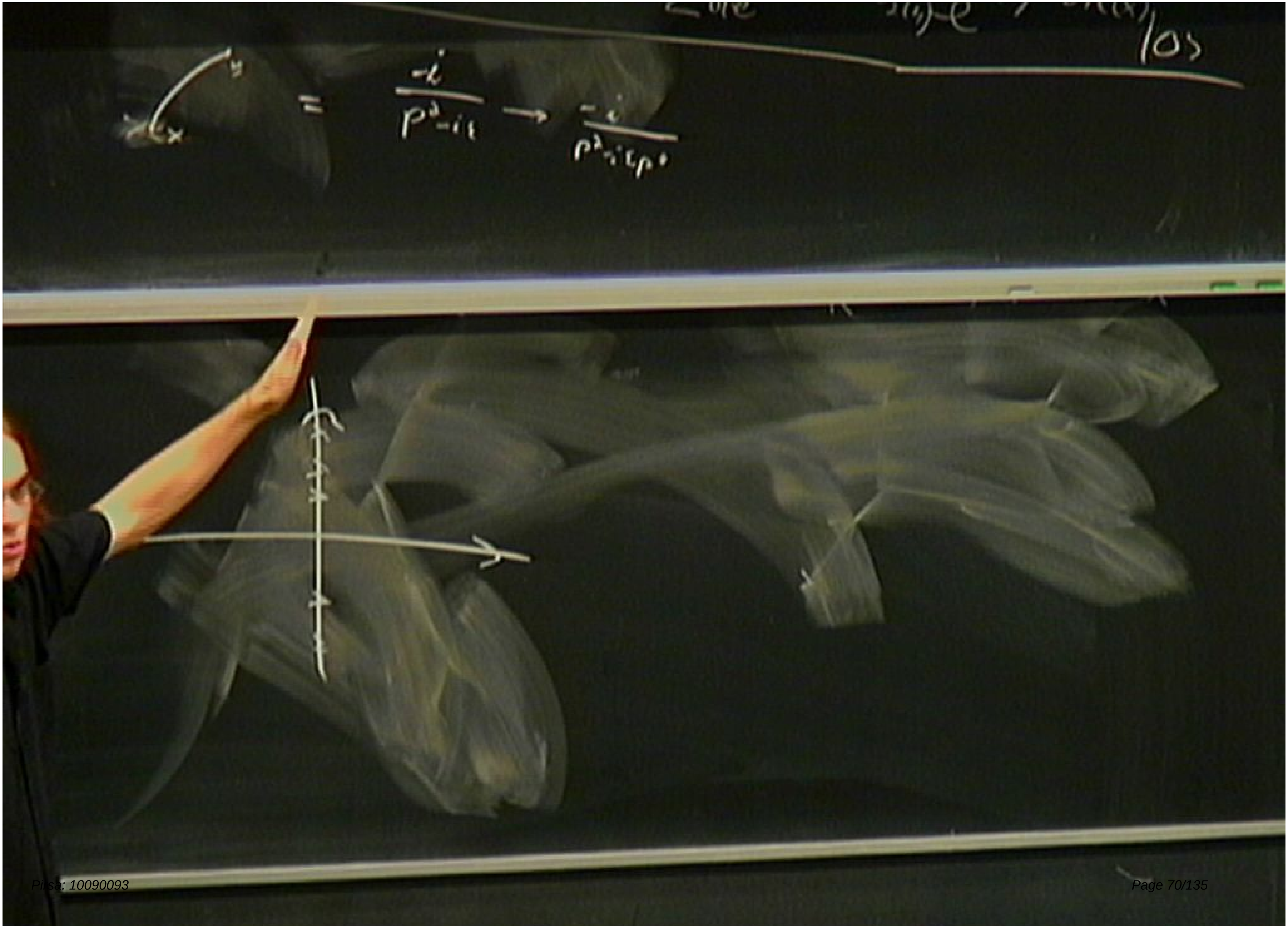


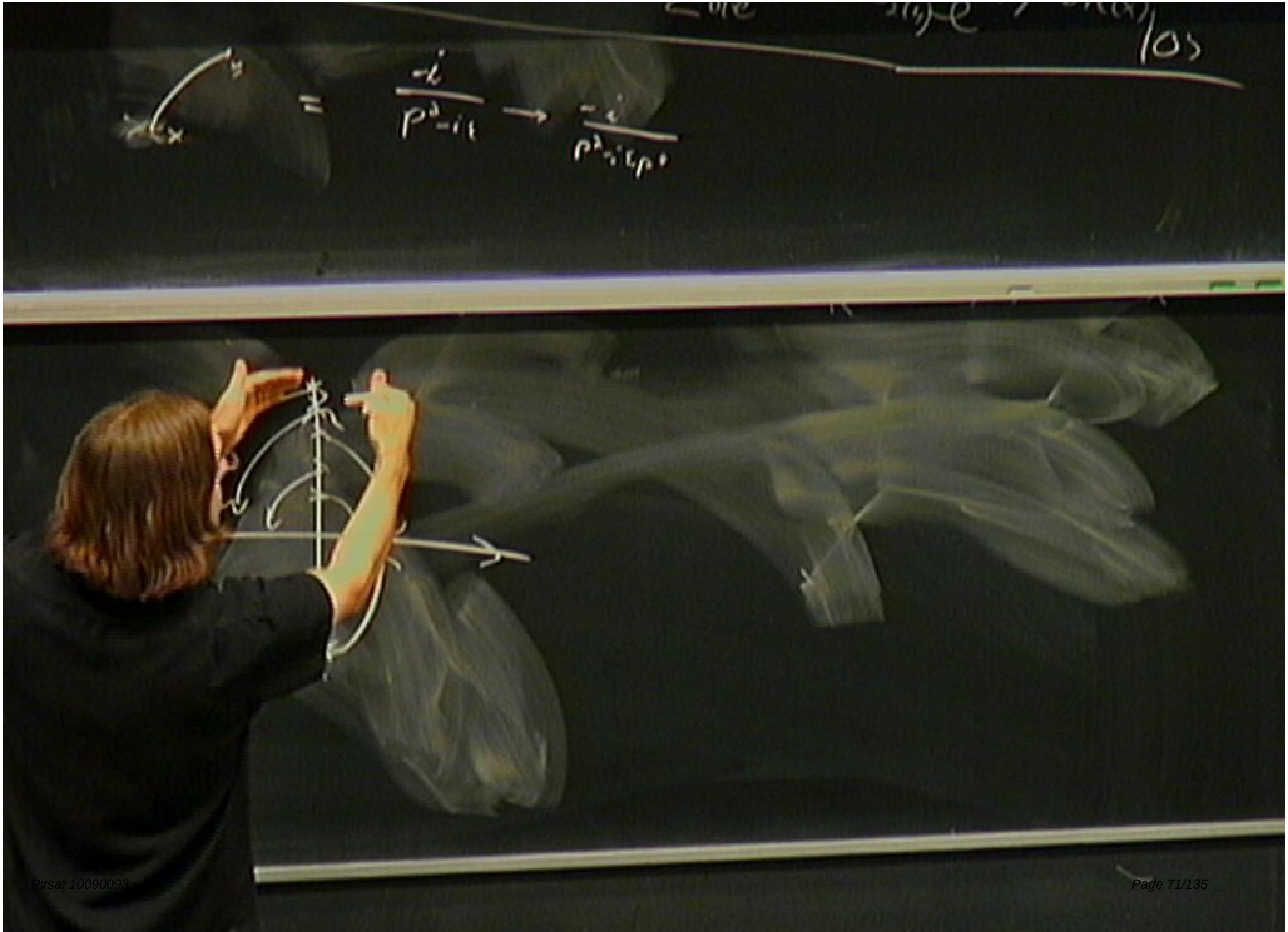


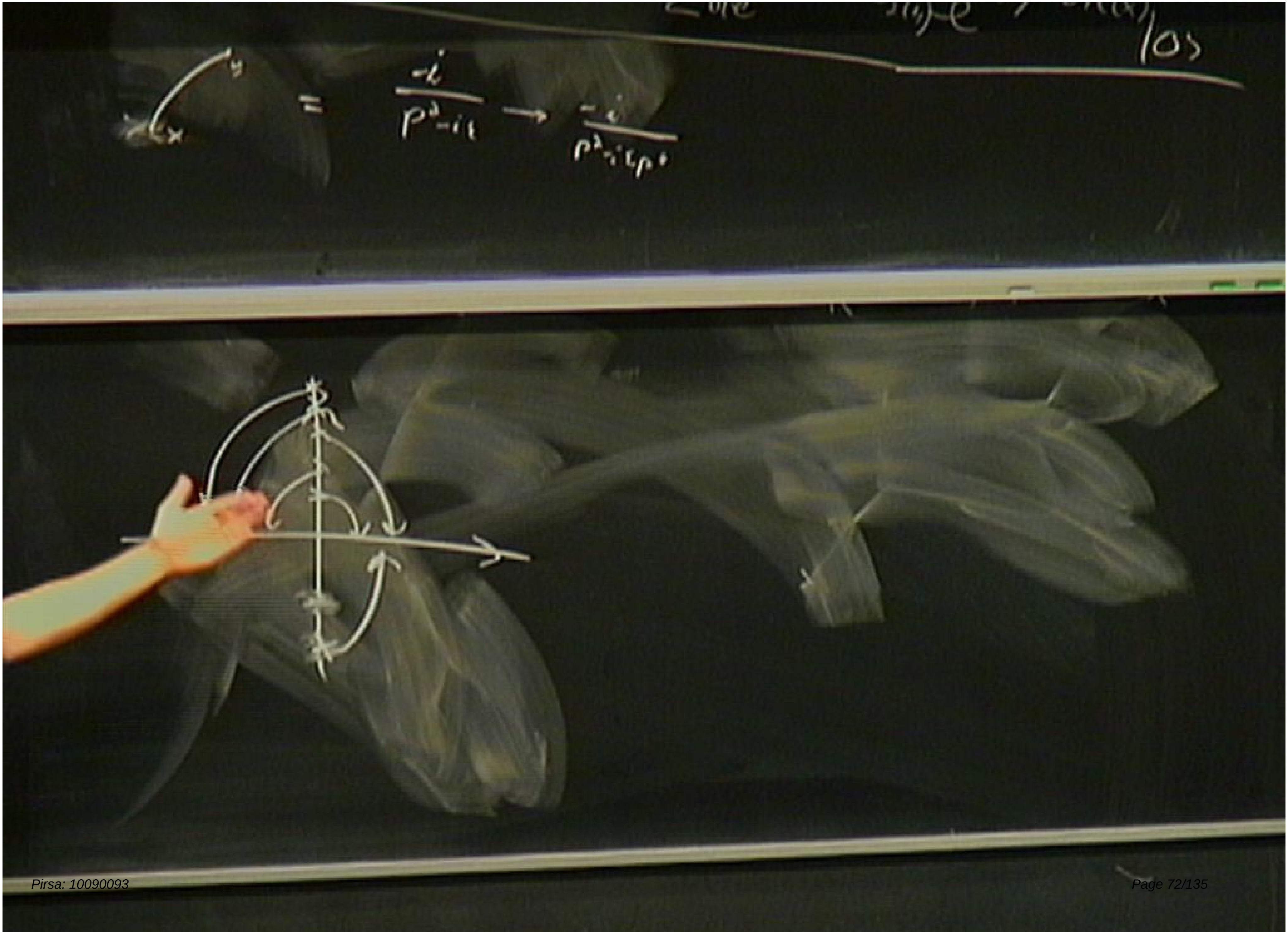

q
-q


q
-q

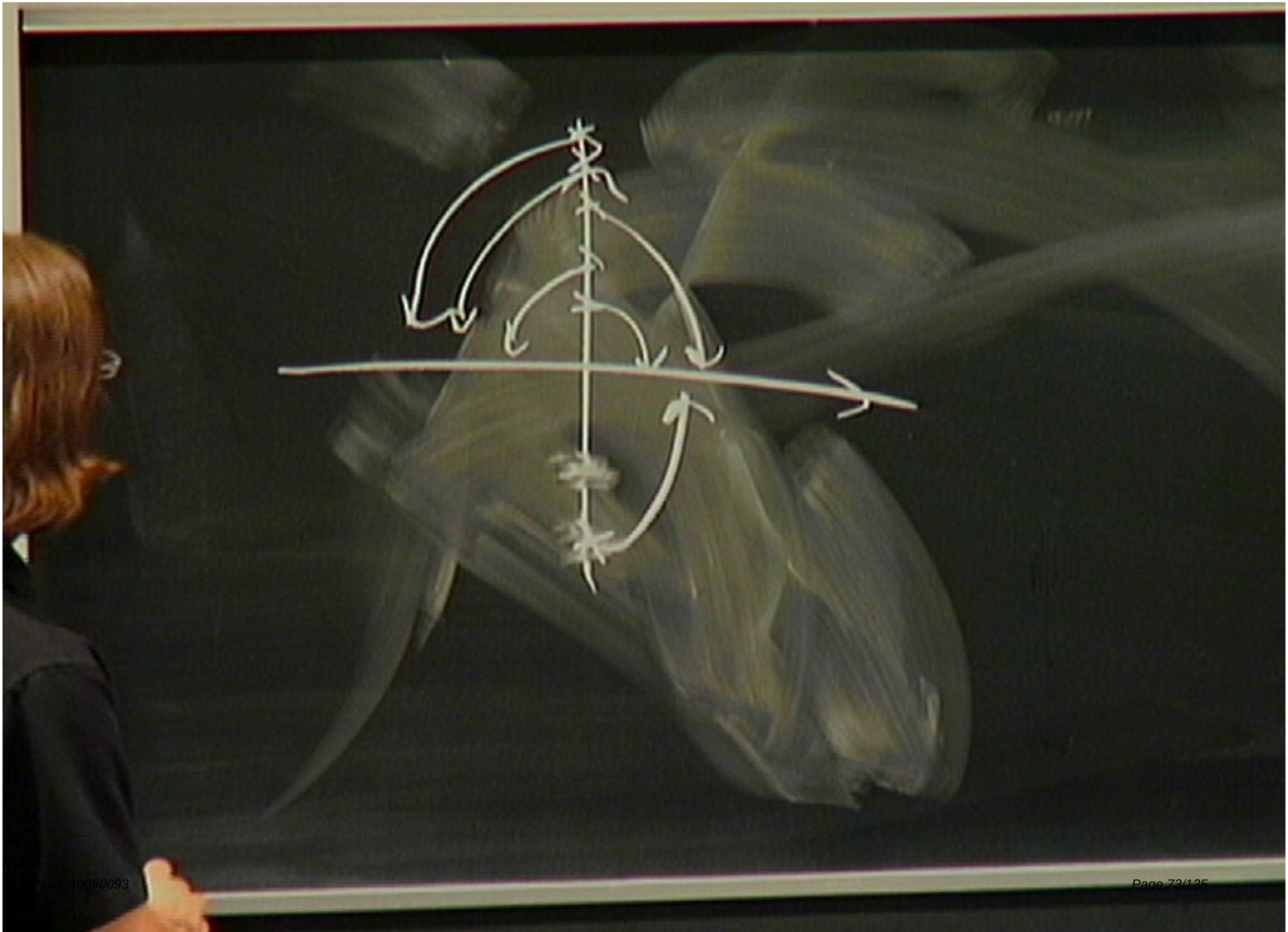

A_n^{1-loop}(p_1 ... p_n) = 1/2 ∫ d^3q/((2π)^3 2E_q) A^{tree}_{n+2}(p_1, ... p_n, q, -q).
Consider instead

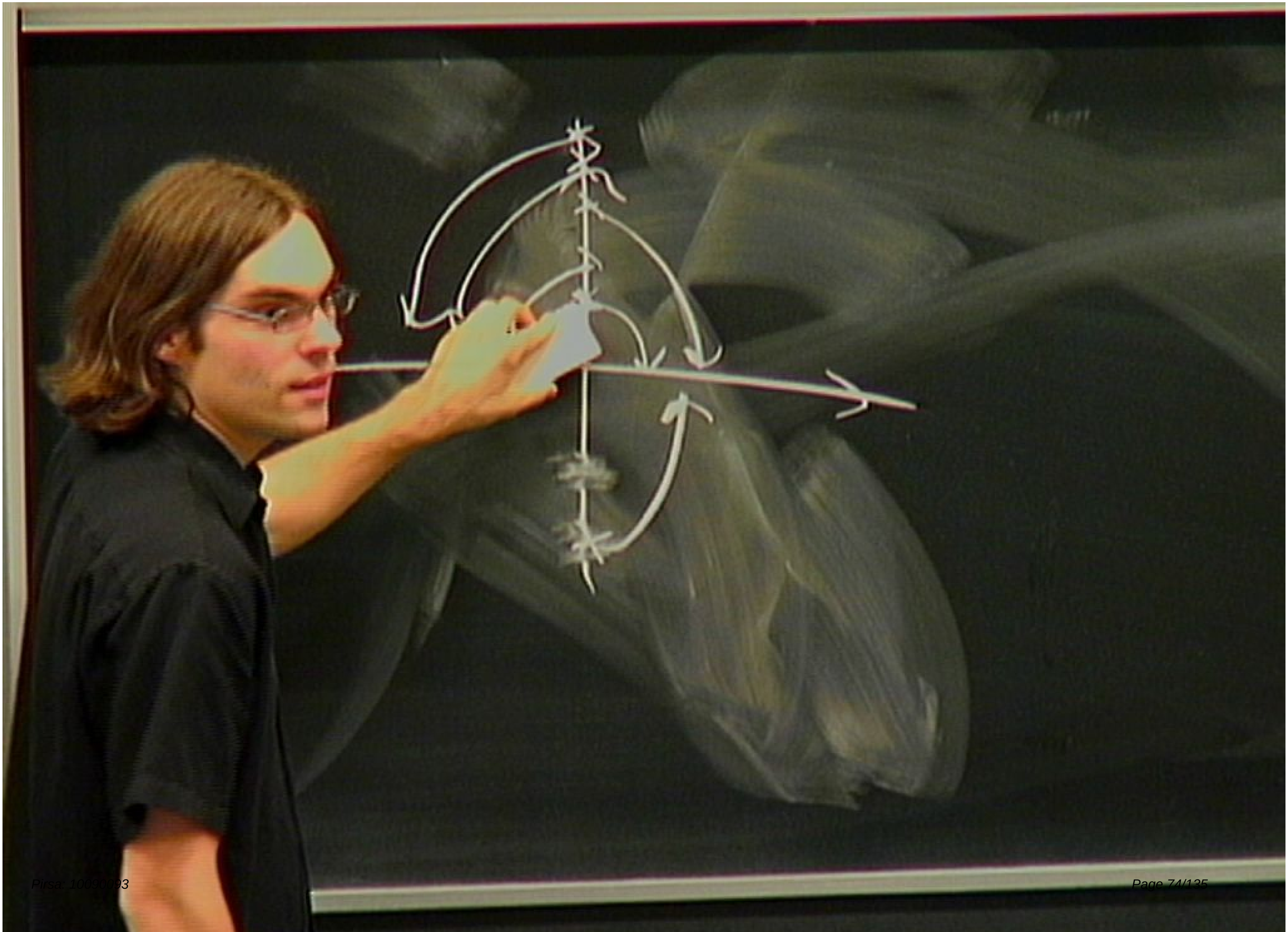


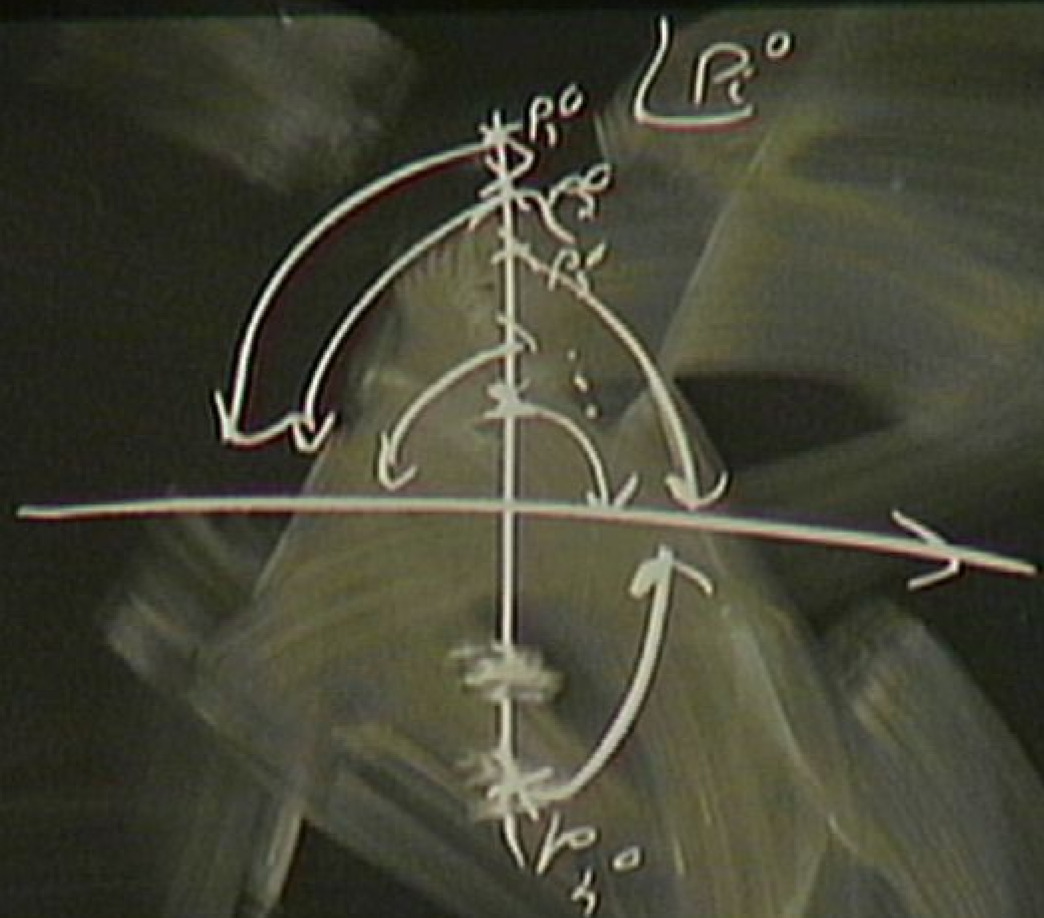


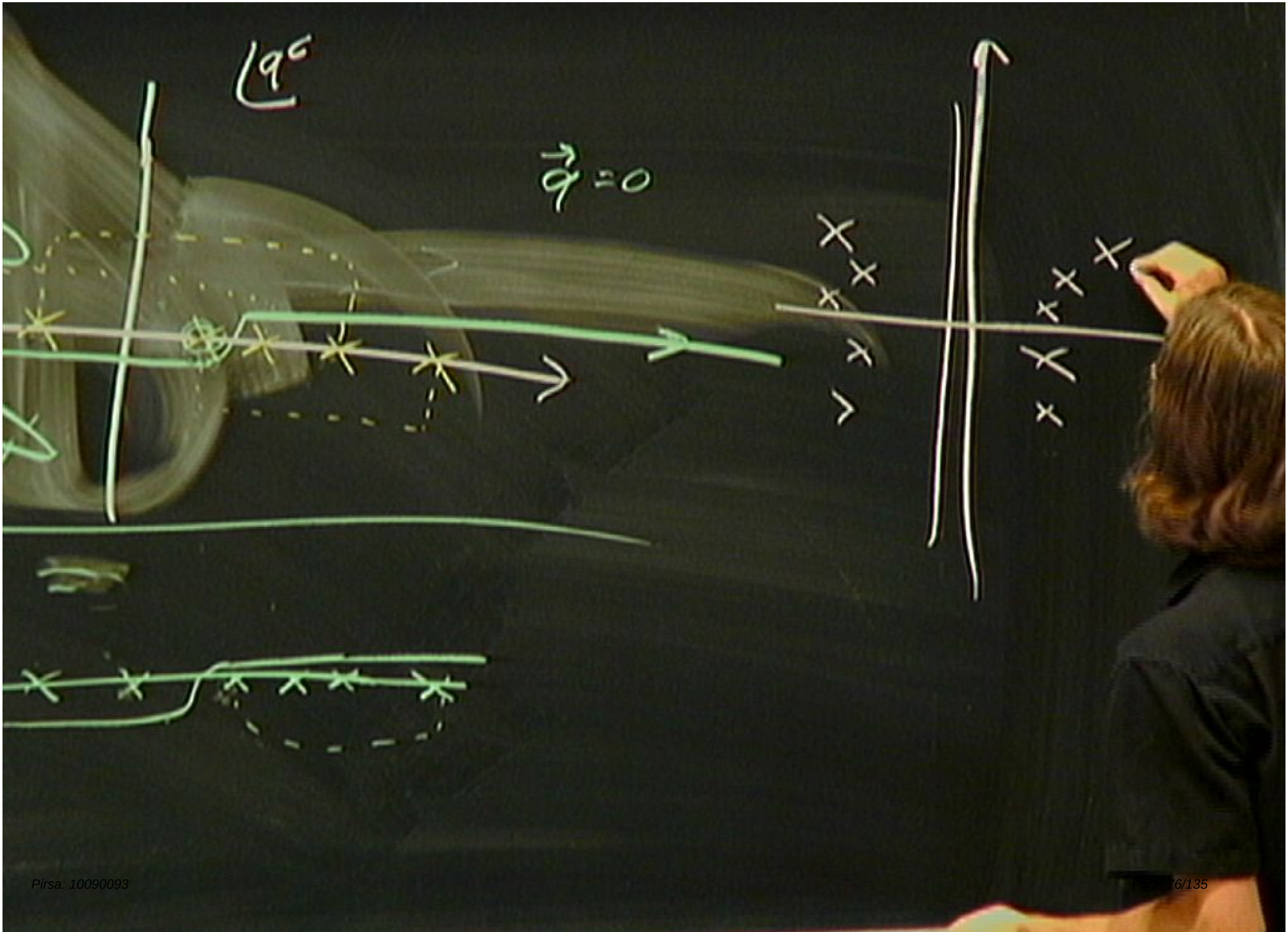


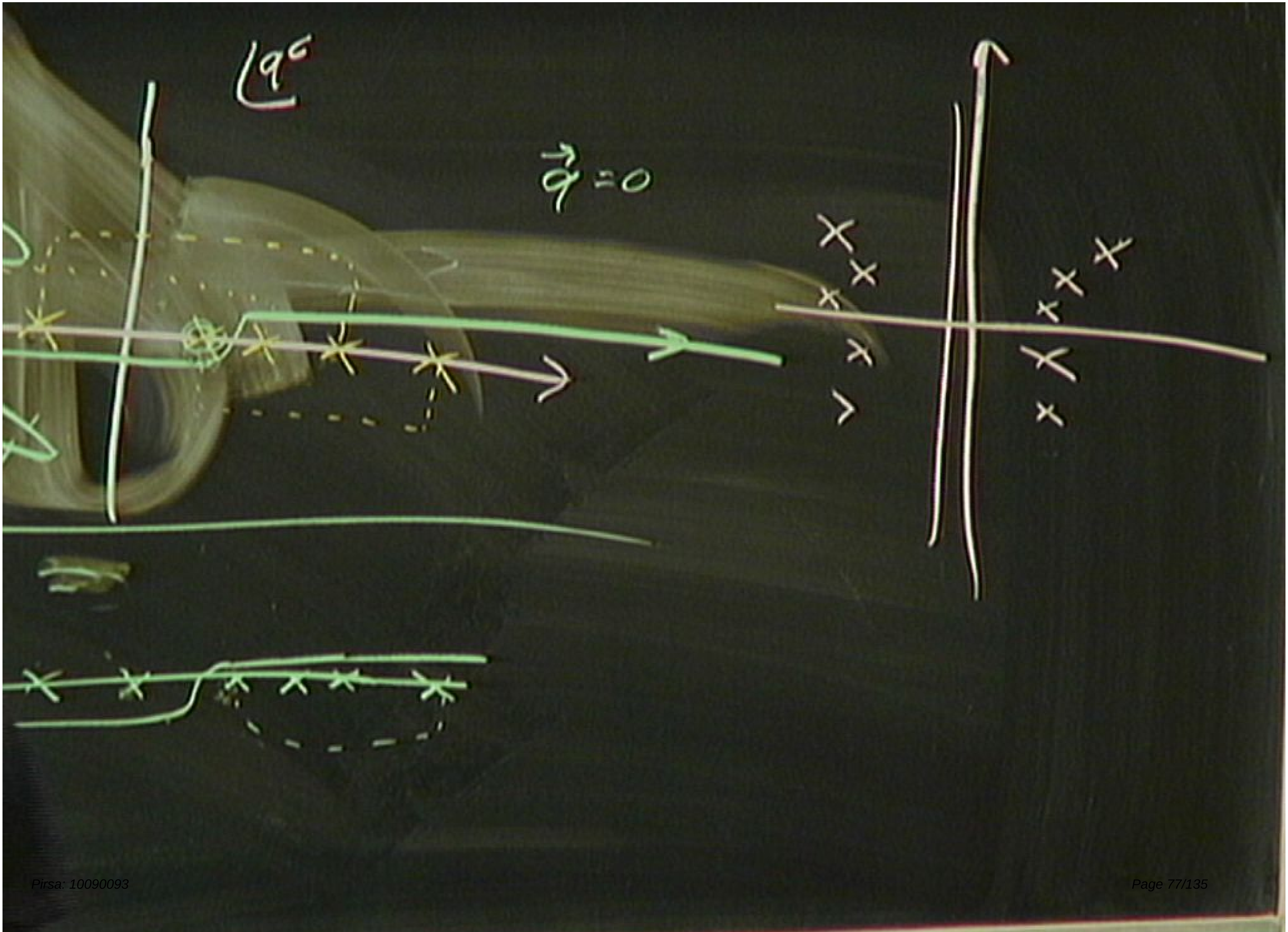
$= \frac{-i}{p^2-i\epsilon} \to \frac{-i}{p^2-i\epsilon p^0}$








$\vec{q} = 0$



ℓq^0
$\vec{q}=0$

$$A_n^{1\text{-loop}}(p_1 \cdots p_n)_R = \frac{1}{2}\int \frac{d^3 q}{(2\pi)^3\, 2E_q}\, A^{\text{tree}}_{n+2}(p_1, \dots p_n, q, -q)_{R'}$$

Consider instead ret. amplitude

$$\langle 0 | e^{iQ(x)} O_2\, e^{iQ(x)} | 0 \rangle$$

$$\langle 0 | [O_1(x), O_2(y)] | 0 \rangle\, \theta(x-y)$$

$$\langle 0 | e^{i\cdots} \quad O_2(y)\, e^{-i\int^y \delta H(x)} | 0 \rangle$$

$$x \to y \; = \frac{-i}{p^2 - i\epsilon} \to \frac{-i}{p^2 - i\epsilon p^0}$$

$\to A^{\text{tree}}|_T$

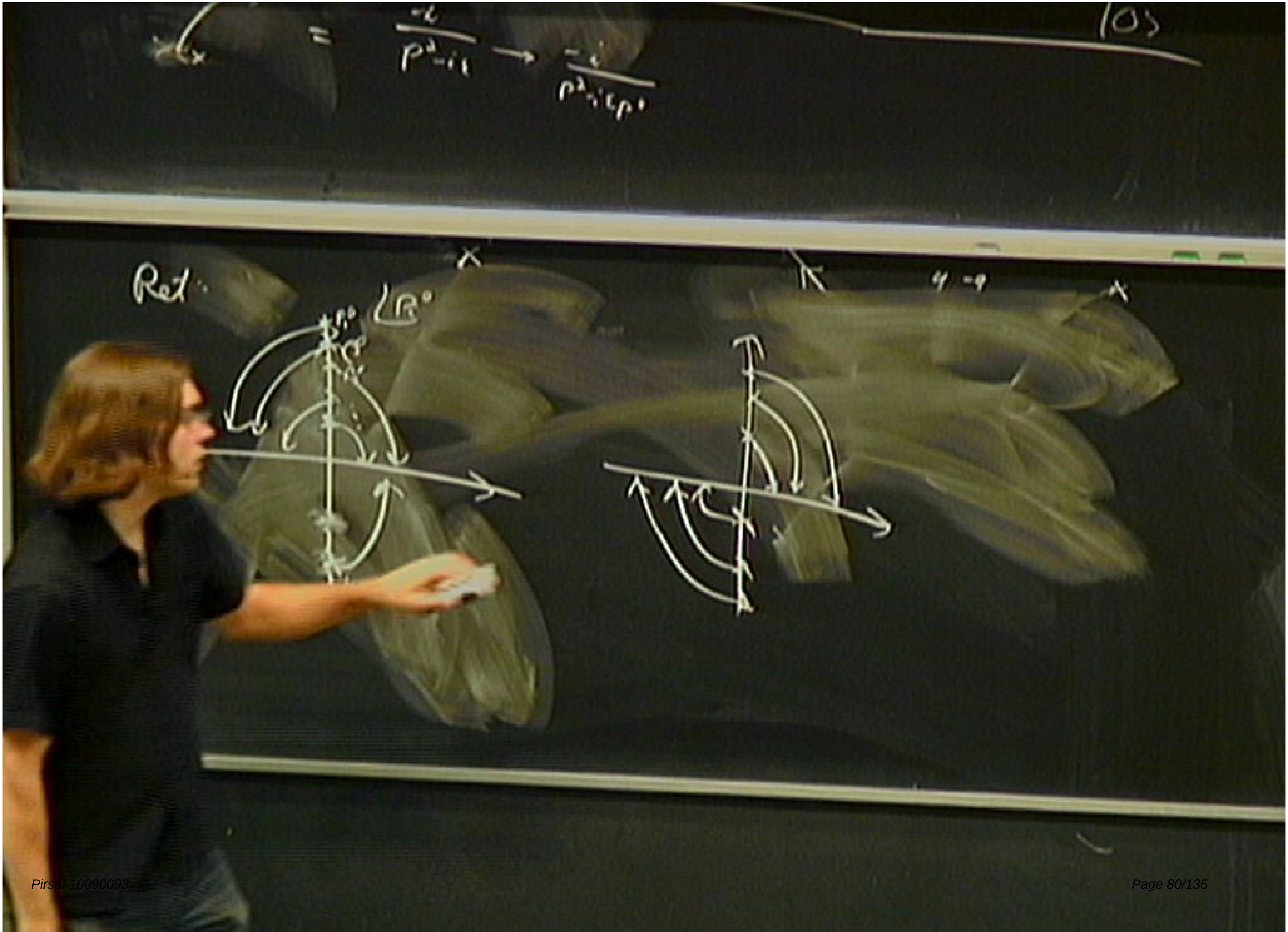
$$A_n^{1\text{-loop}}(p_1 \cdots p_n)_R = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \dots p_n, q, -q)_{R'}$$

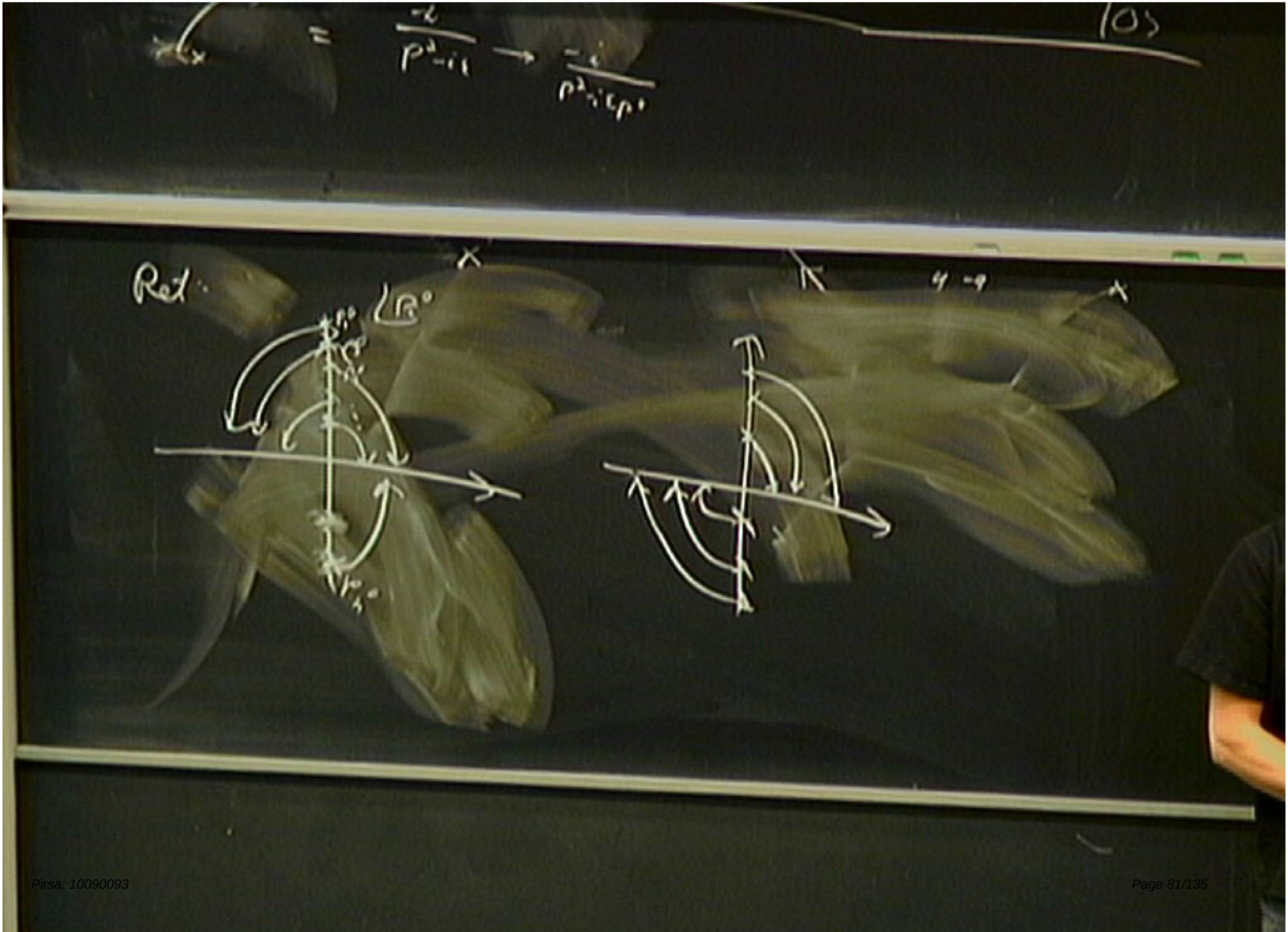
Consider instead ret. amplitude $\langle q e^{iQ(x)} O_2 e^{iQ(x)} | 0 \rangle$

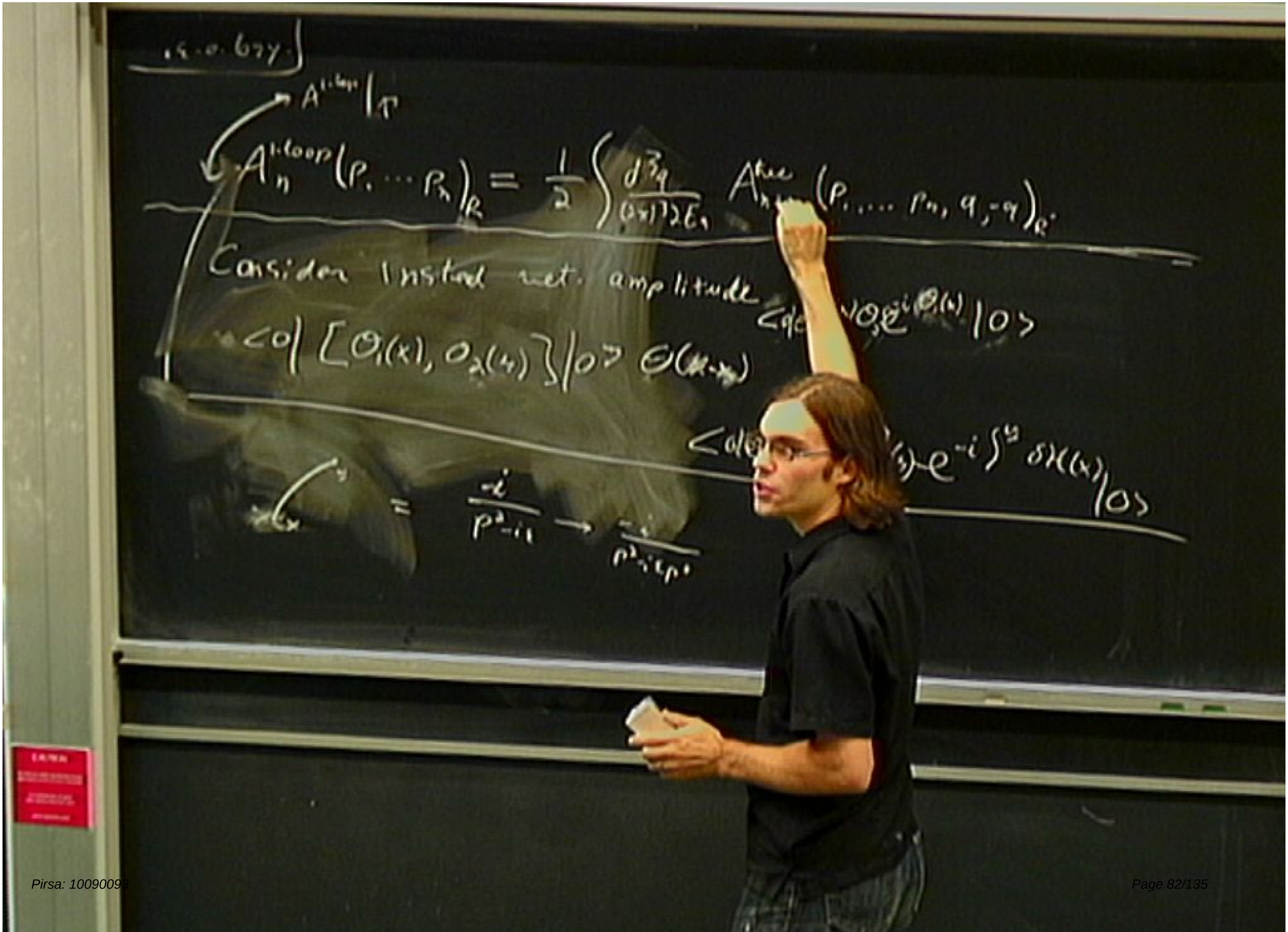
$$\langle 0 | [O_1(x), O_2(y)] | 0 \rangle \, \Theta(x-y)$$

$$\langle 0 | e^{i \cdots} \quad O_2(y) e^{-i\int^y \delta H(x)} | 0 \rangle$$

$$= \frac{-i}{p^2 - i\epsilon} \to \frac{-i}{p^2 - i\epsilon p^0}$$




Ret:



$A_n^{1\text{-loop}}(p_1 \cdots p_n)_R = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \ldots p_n, q, -q)_R$
Consider instead ret. amplitude
$\langle 0| [O_1(x), O_2(y)] |0\rangle \Theta(x-y)$
$= \frac{-i}{p^2 - i\epsilon} \to \frac{-i}{p^2 - i\epsilon p^0}$

$\mathcal{A}^{1\text{-loop}}\big|_T$

$$A_n^{1\text{loop}}(p_1 \cdots p_n)_R = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \ldots p_n, q, -q)_R$$

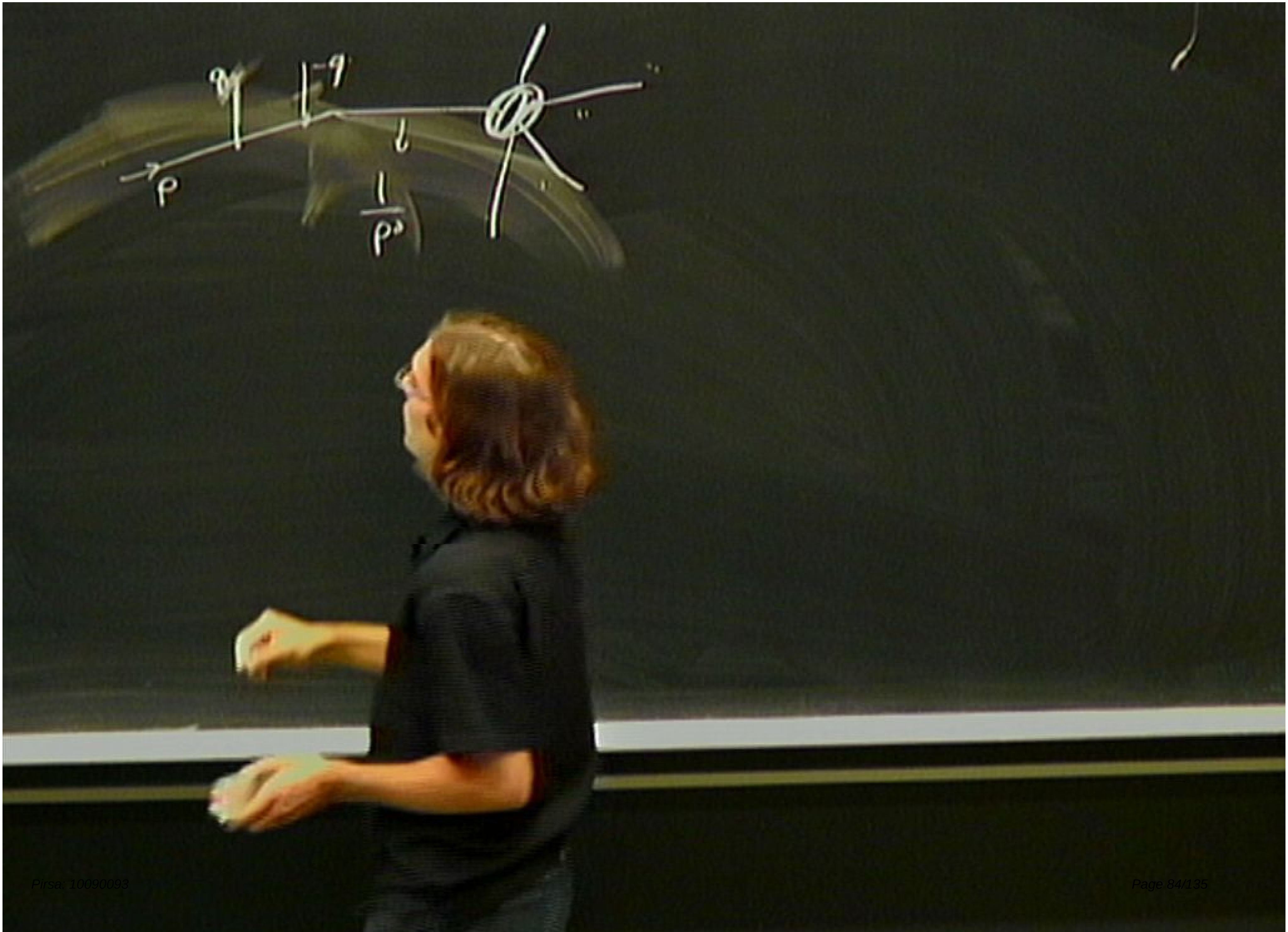
True off-she[ll]

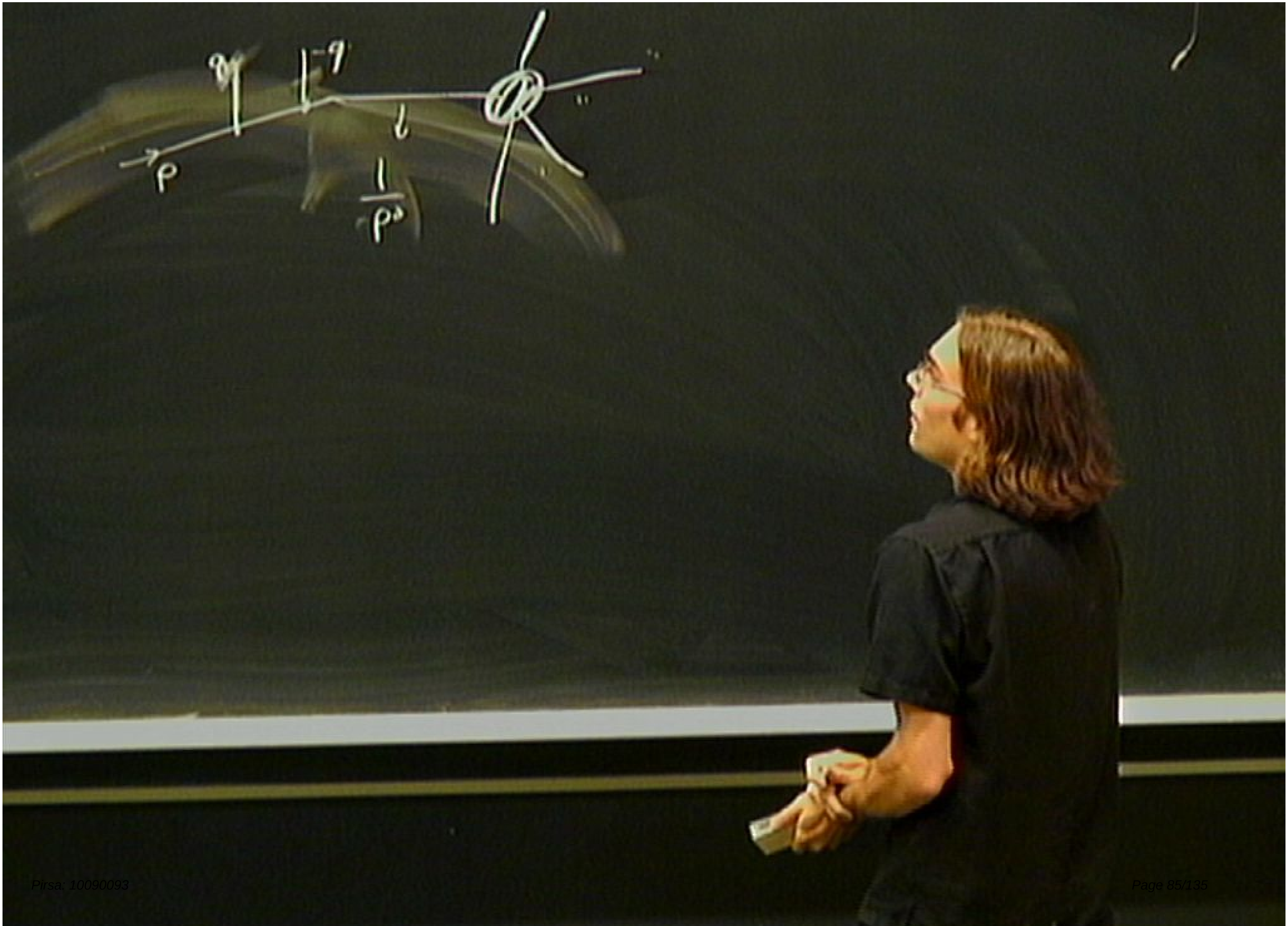
Consider instead ret. amplitude $\langle q | e^{iq(x)} O_2 e^{-iQ(x)} |0\rangle$

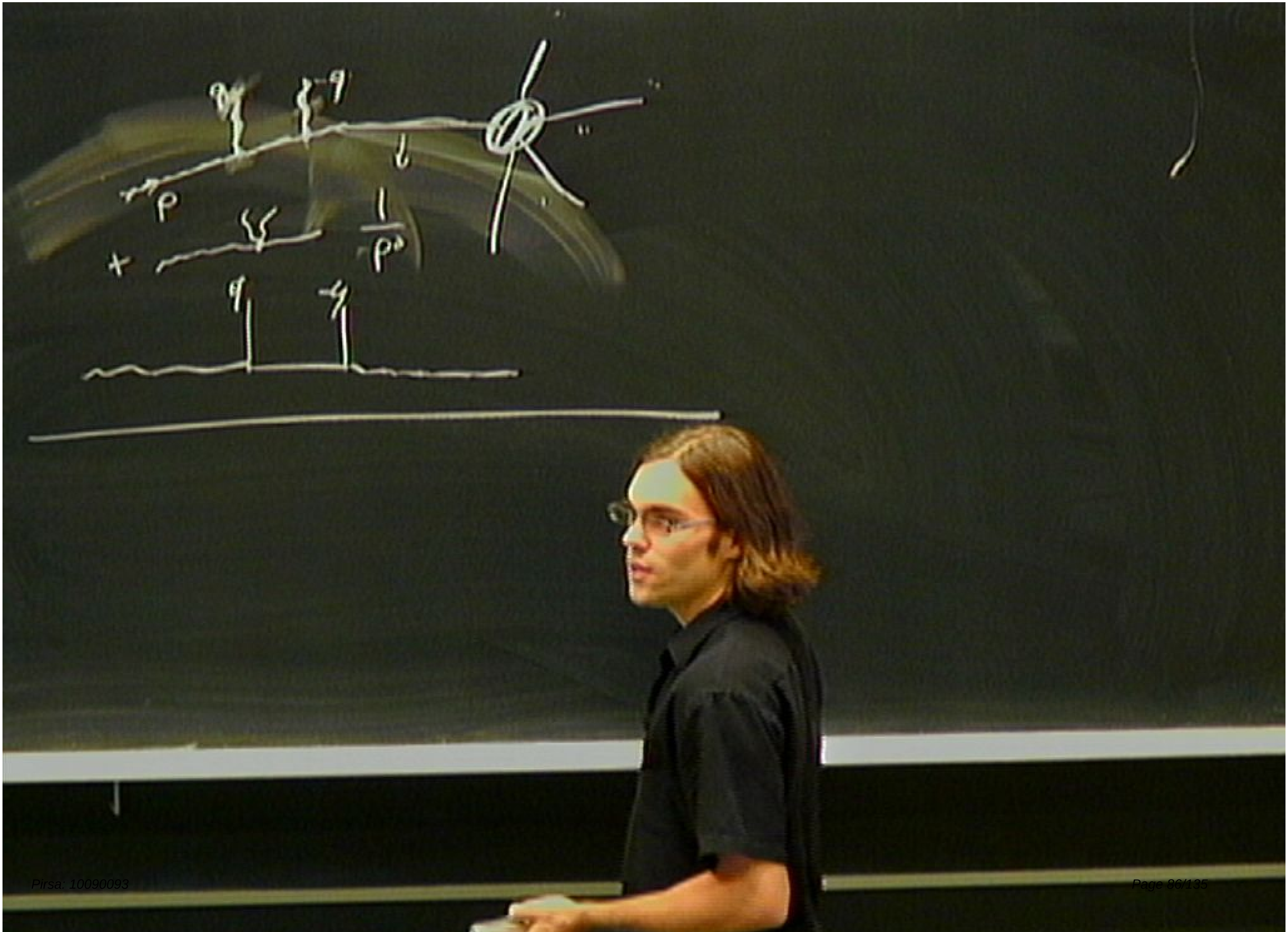
$\langle 0 | [O_1(x), O_2(y)] |0\rangle \, \Theta(x-y)$

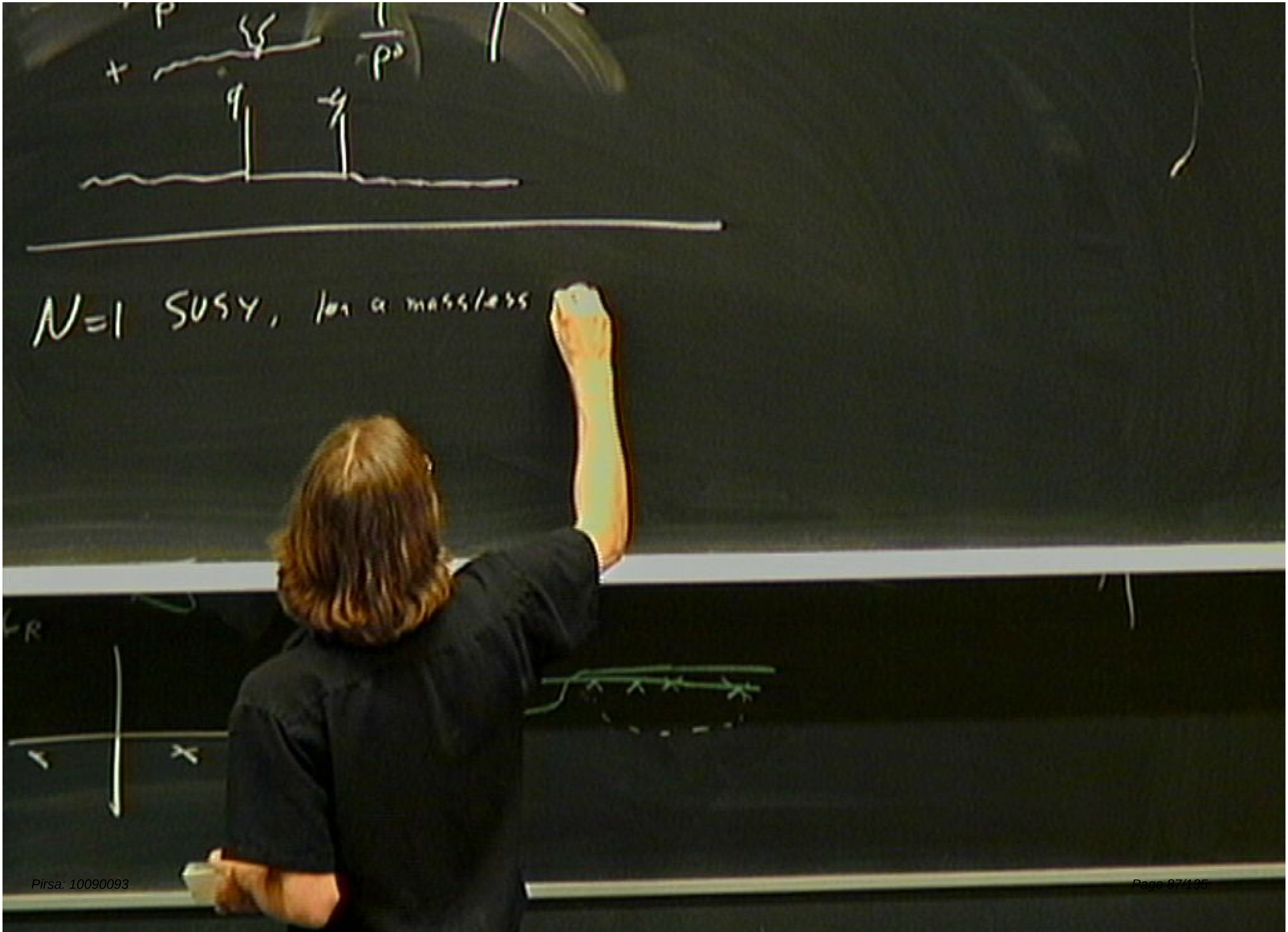
$\langle 0 | e^{i \cdots} \; O_2(y) e^{-i\int^y \delta H(x)} |0\rangle$

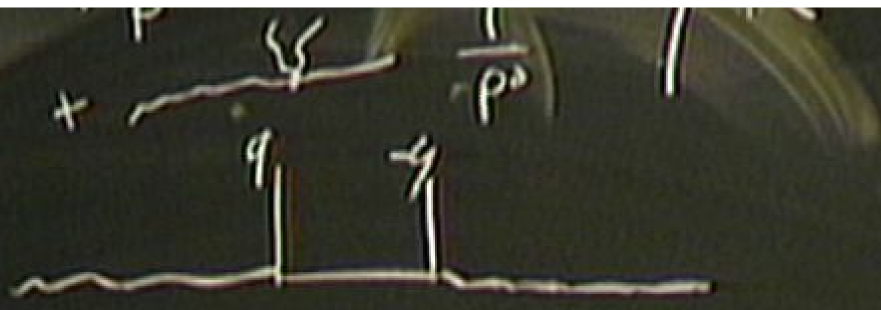
$$= \frac{i}{p^2 - i\epsilon} \rightarrow \frac{i}{p^2 - i\epsilon p^0}$$



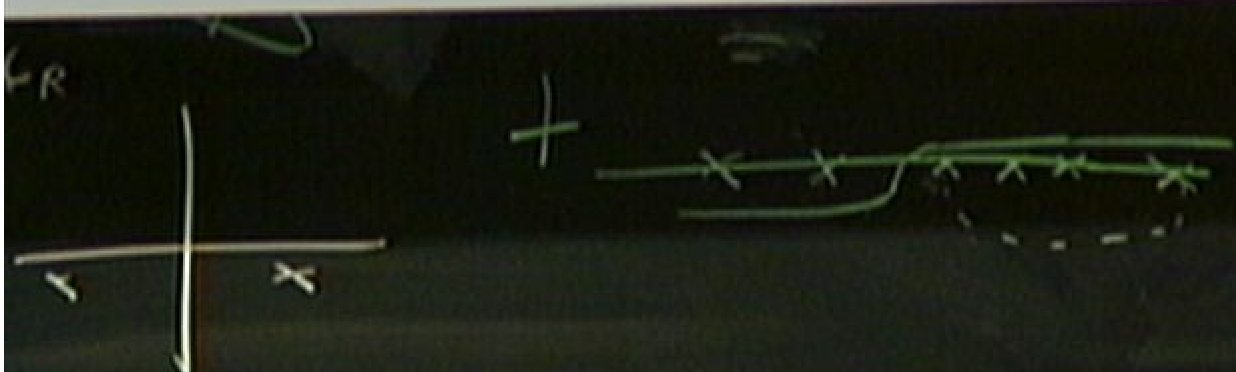


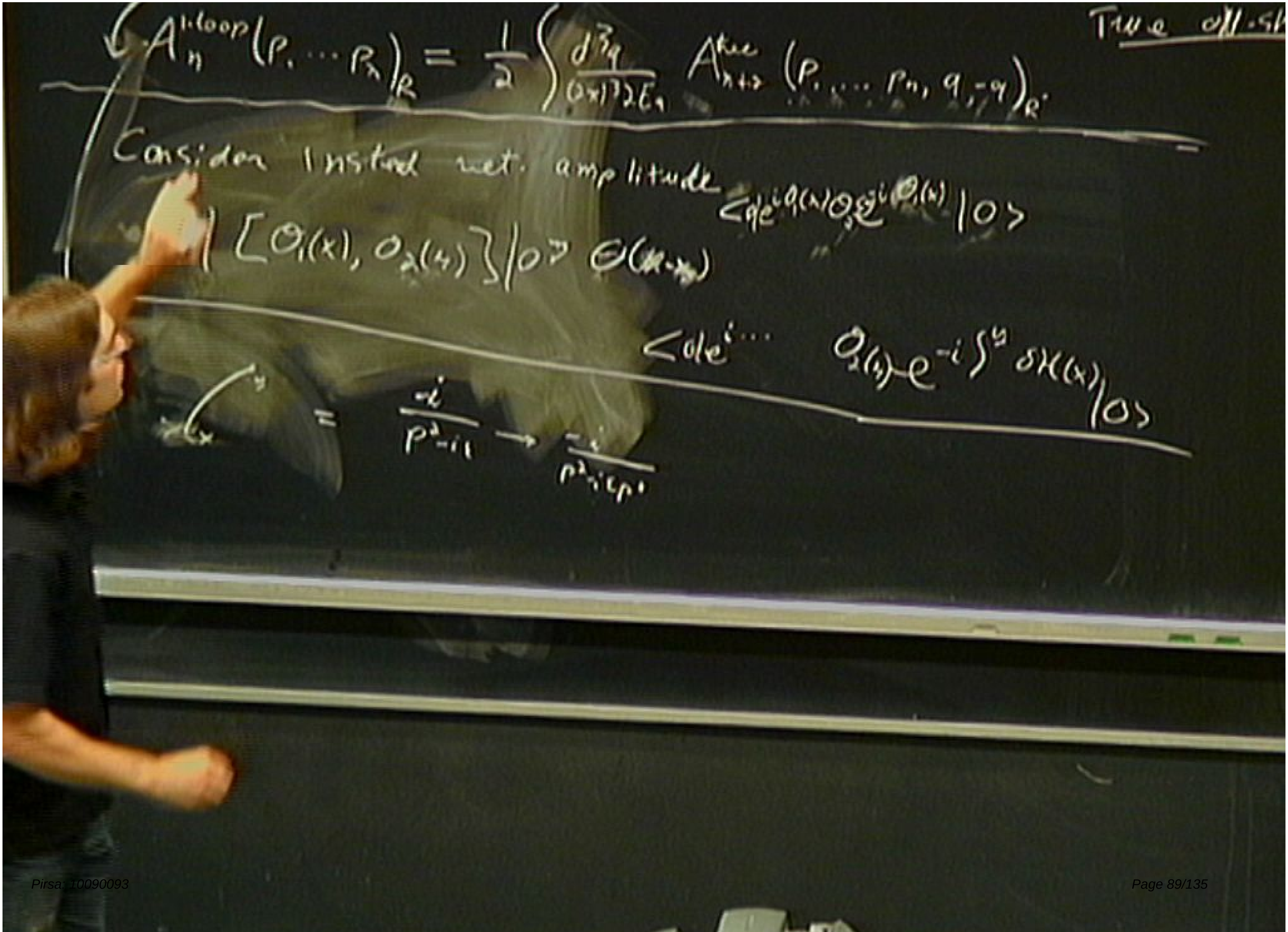



N=1 SUSY, for a massless



N=1 SUSY, for a massless particle




True off-sh
$A_n^{1\text{-loop}}(p_1 \cdots p_n)_R = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \ldots, p_n, q, -q)_R$
Consider instead ret. amplitude
$[O_1(x), O_2(y)]|0\rangle$
$\langle 0|e^{i\cdots} \quad O_2(y)\, e^{-i\int^y \delta H(x)}|0\rangle$
$= \frac{-i}{p^2-i\epsilon} \rightarrow \frac{-i}{p^2-i\epsilon p^0}$

$$A^{1\text{-loop}}\big|_{\Gamma}$$

True off-sh

$$A_n^{1\text{-loop}}(p_1 \cdots p_n)_R = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3 2E_q} A_{n+2}^{\text{tree}}(p_1, \ldots, p_n, q, -q)_{R'}$$

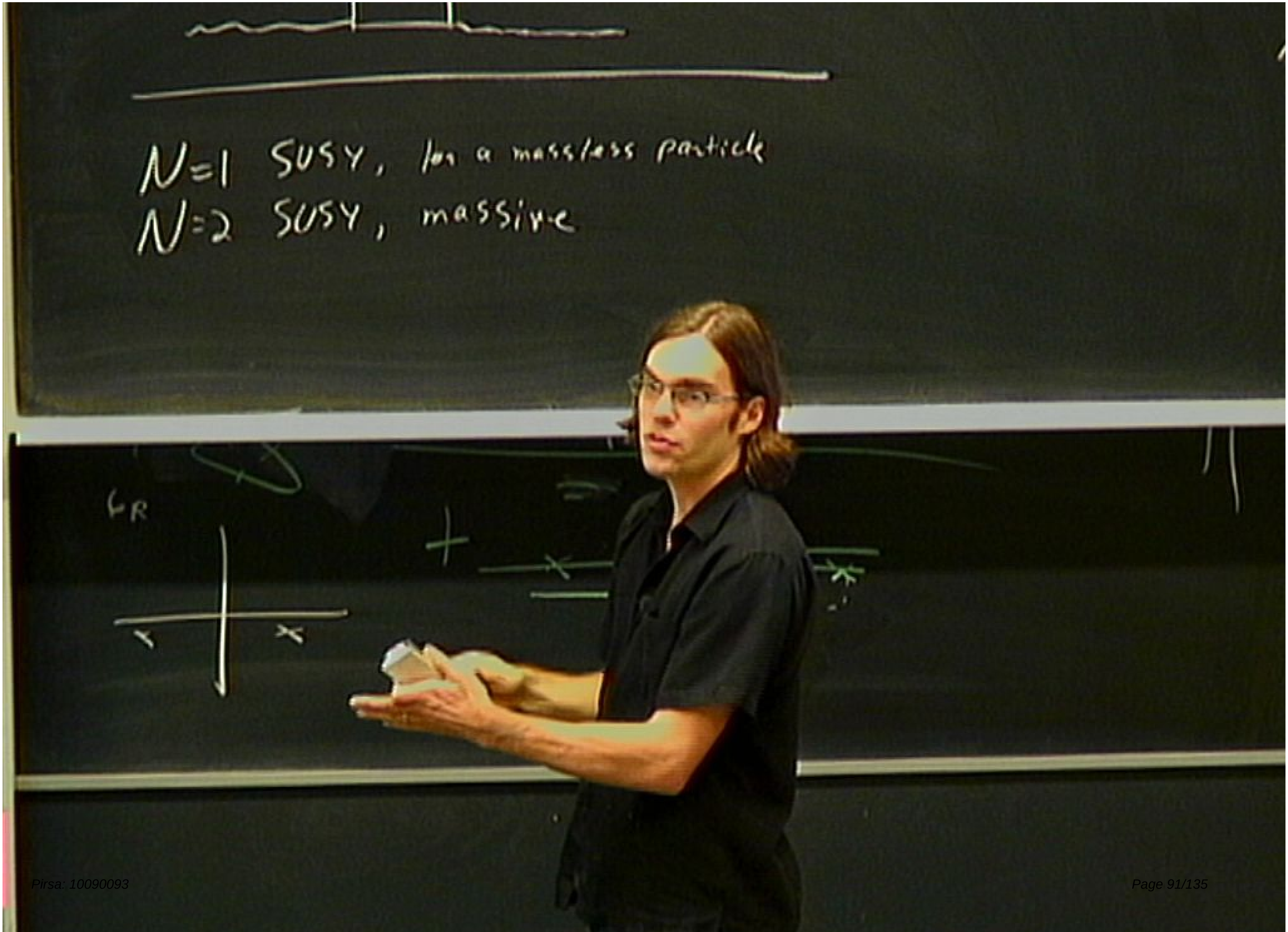
Consider instead ret. amplitude

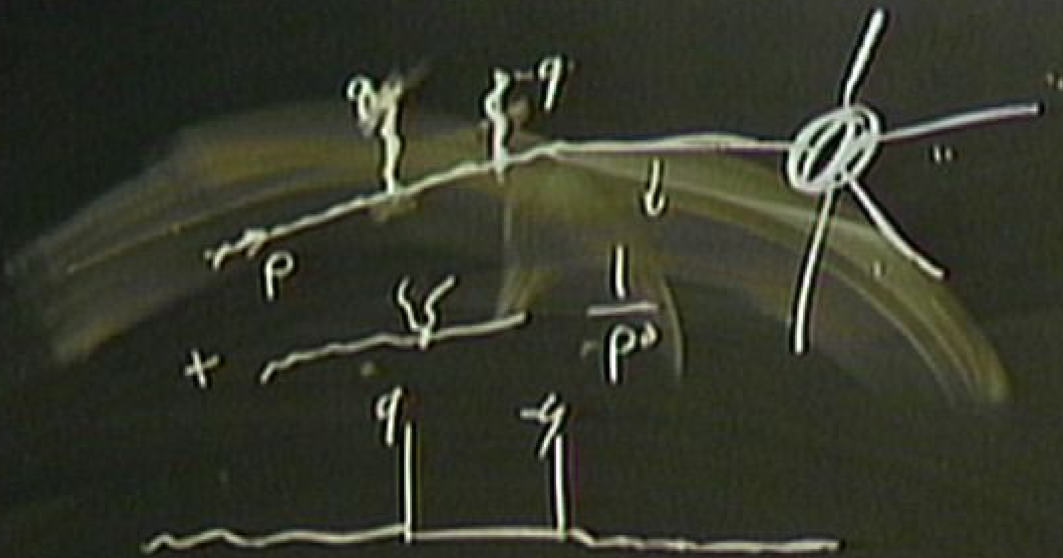
$$\langle 0|[O_1(x), O_2(y)]|0\rangle\, \Theta(x-y)$$

$$\langle 0|e^{iO_1(x)} O_2 e^{-iO_1(x)}|0\rangle$$

$$\langle 0|e^{i\cdots}\; O_2(y) e^{-i\int^y \delta H(x)}|0\rangle$$

$$= \frac{-i}{p^2 - i\epsilon} \to \frac{-i}{p^2 - i\epsilon p^0}$$


N=1 SUSY, for a massless particle
N=2 SUSY, massive



$\mathcal{N}=1$ SUSY, for a massless particle

$\mathcal{N}=2$ SUSY, massive

i.e. o. bay.

$A^{1\text{-loop}}\big|_T$

True off-shell

$$A_n^{1\text{-loop}}(p_1 \cdots p_n)_R = \frac{1}{2}\int \frac{d^3 q}{(2\pi)^3 2E_q} A^{\text{tree}}_{n+2}(p_1, \ldots p_n, q, -q)_{R'}$$

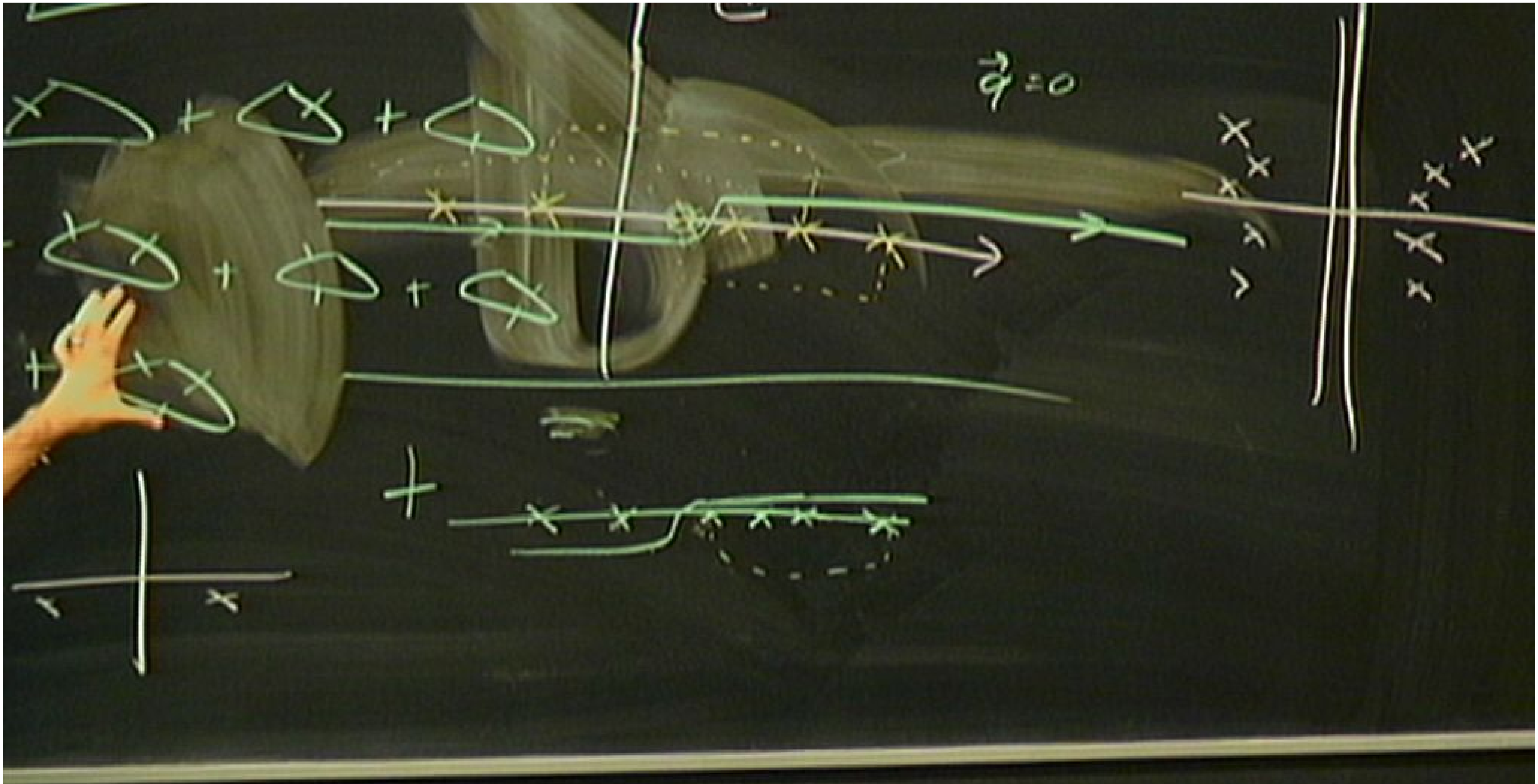
Consider instead ret. amplitude

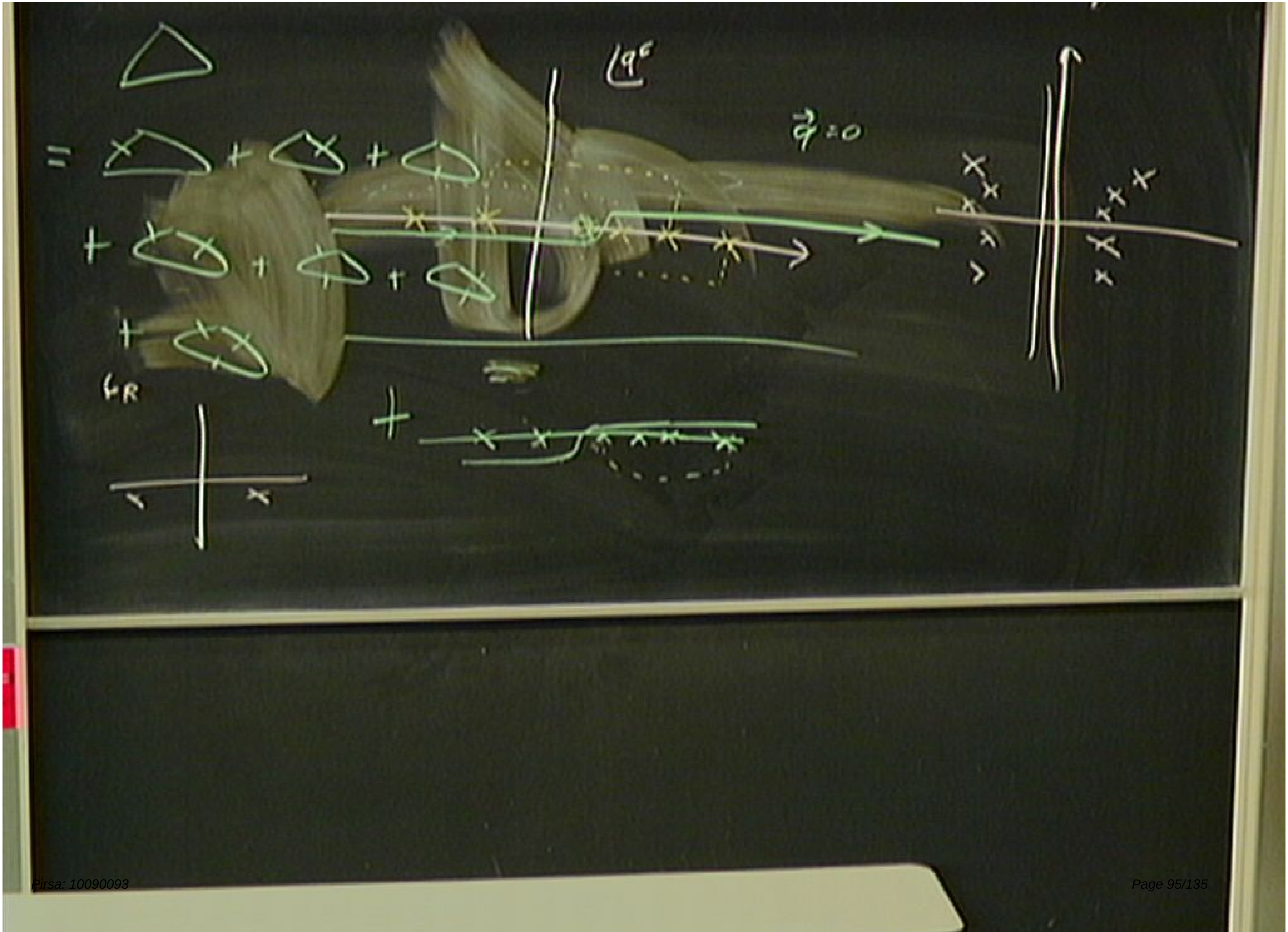
$\langle 0| [O_1(x), O_2(y)] |0\rangle \, \Theta(x-y)$

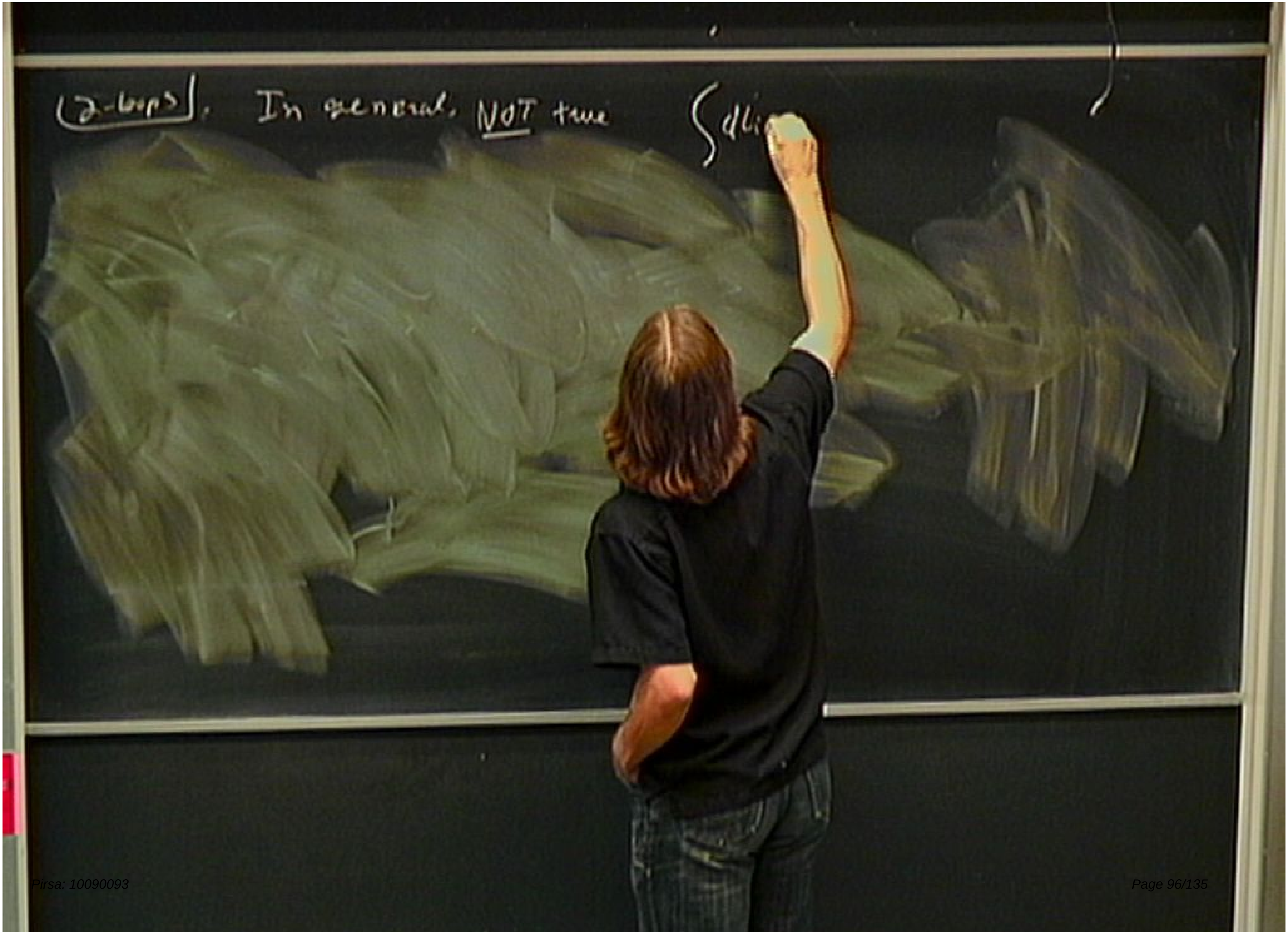
$\langle 0| e^{i\theta_1(x)} O_2 e^{i\theta_1(x)} |0\rangle$

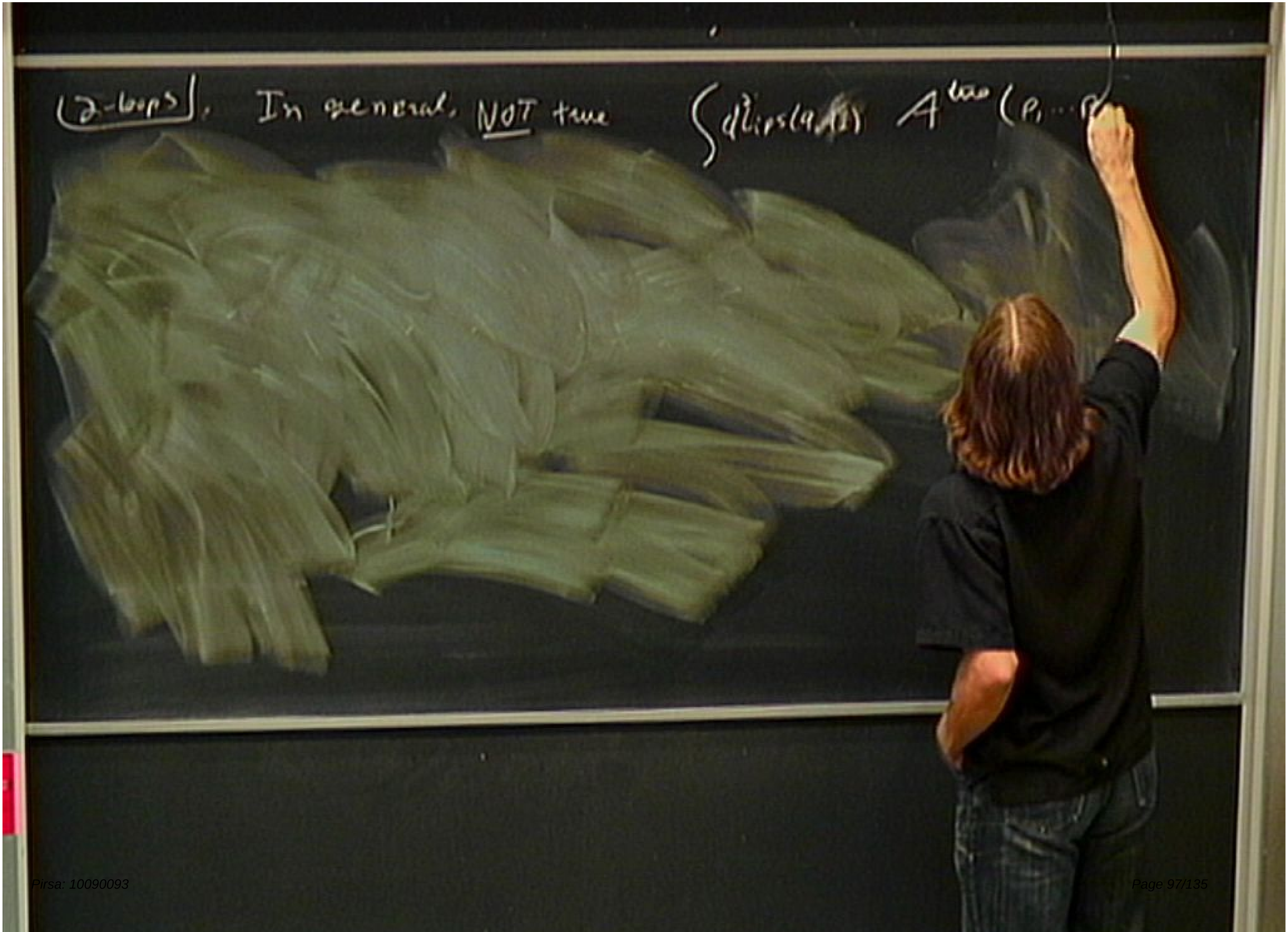
$\langle 0| e^{i\cdots} \quad O_2(y) e^{-i\int^y \delta H(x)} |0\rangle$

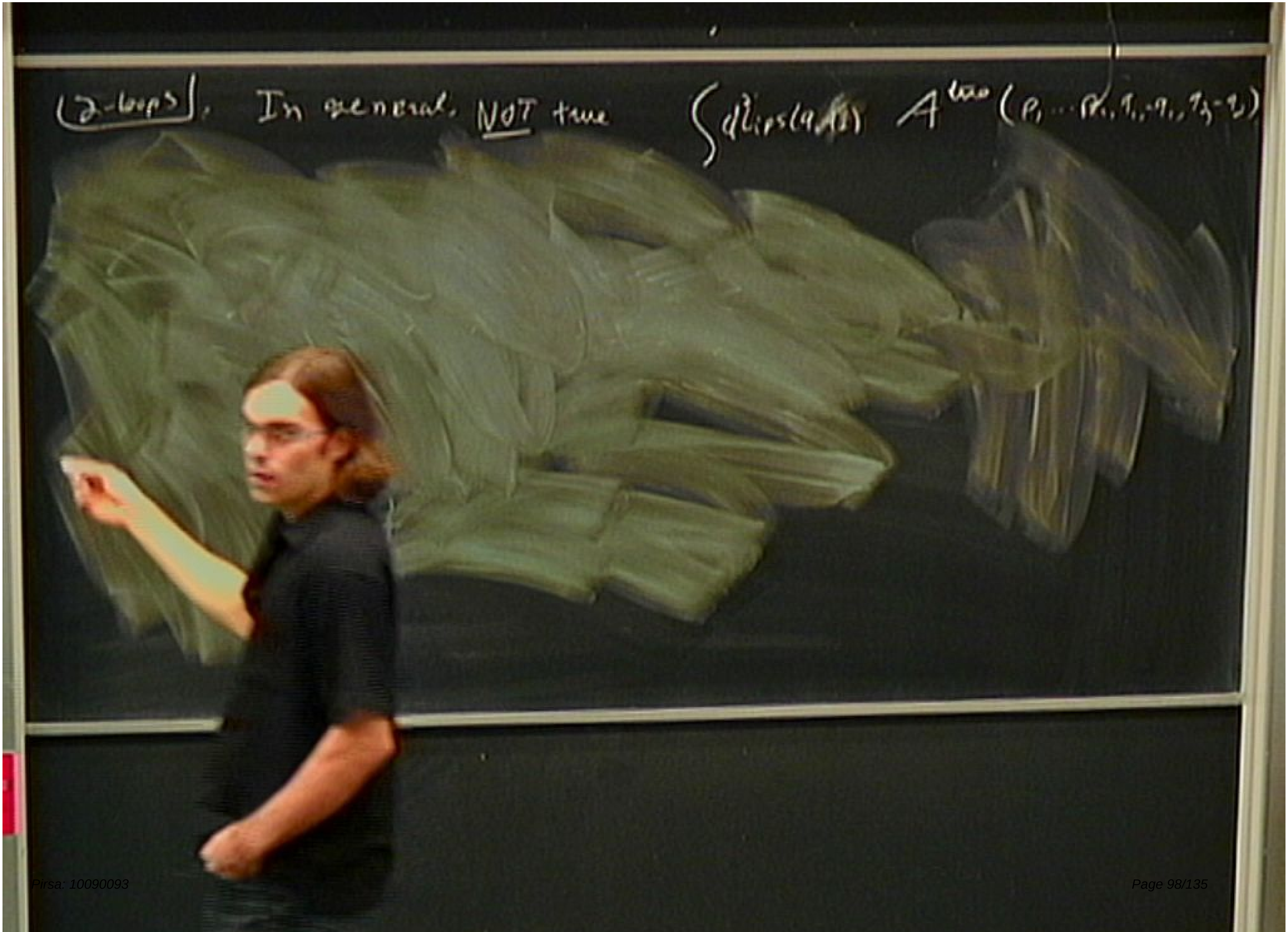
$$= \frac{-i}{p^2 - i\epsilon} \to \frac{-i}{p^2 - i\epsilon p^0}$$

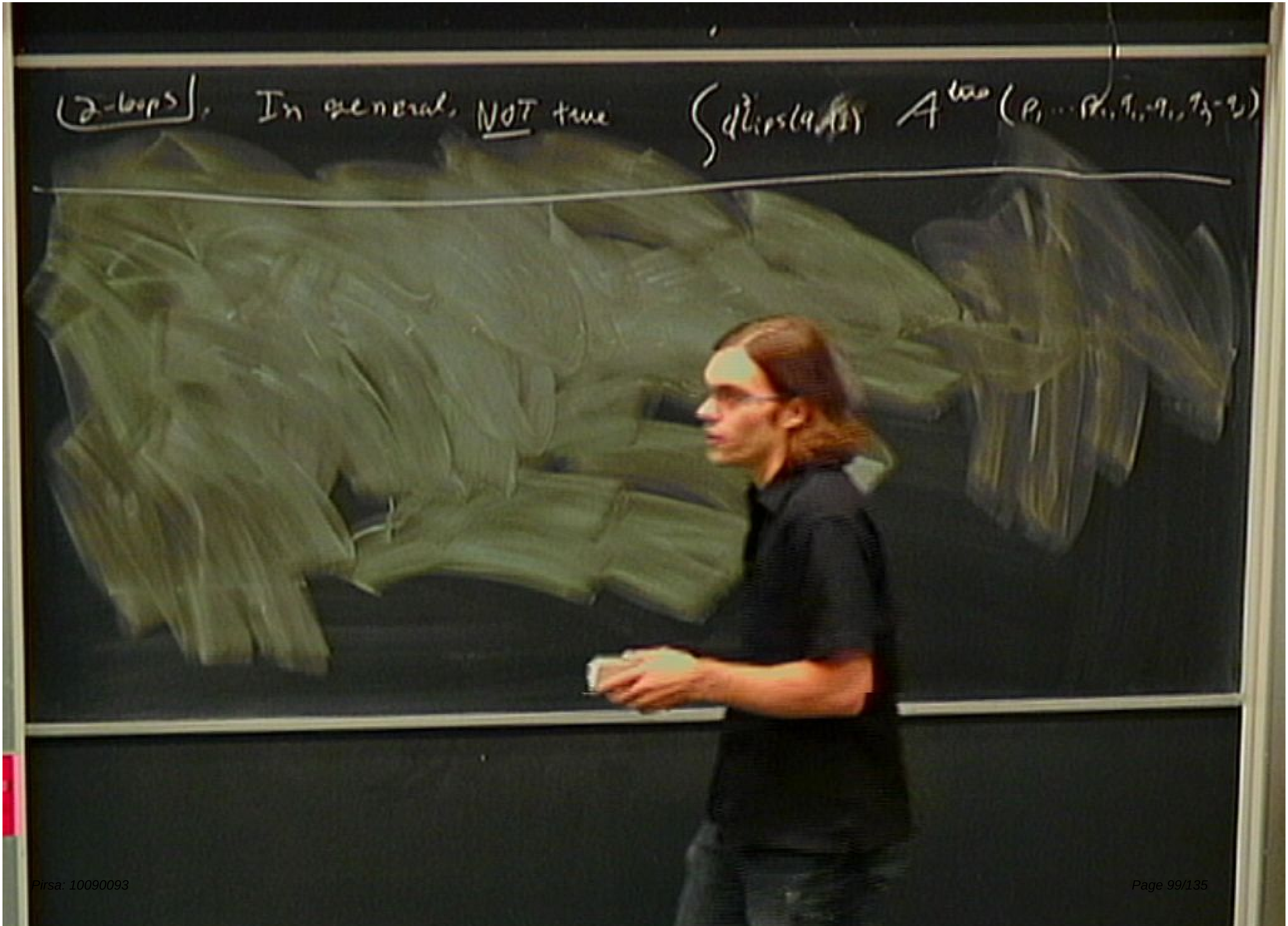

$\vec{q} = 0$


$\vec{q}=0$

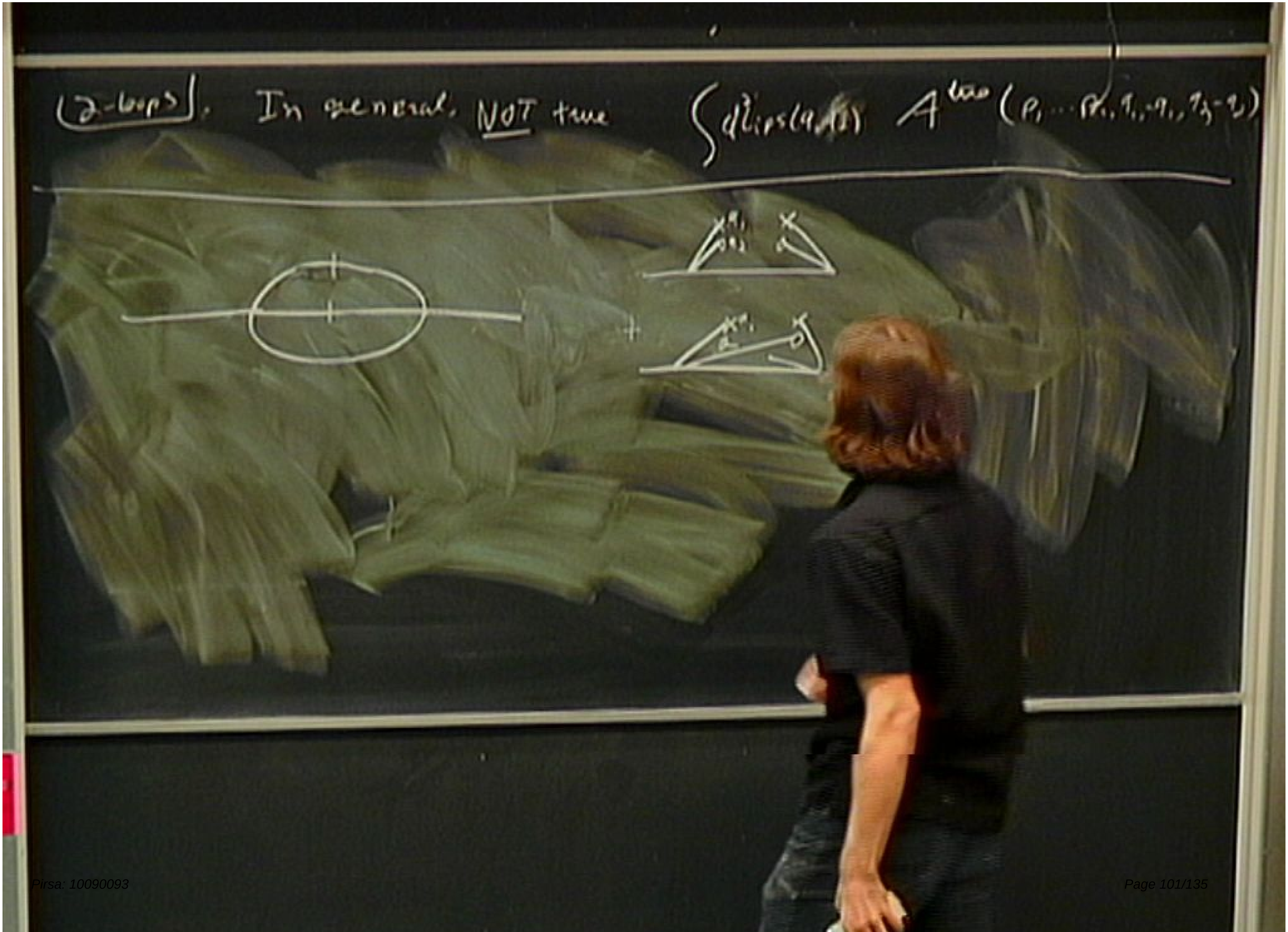

(2-loops). In general, NOT true

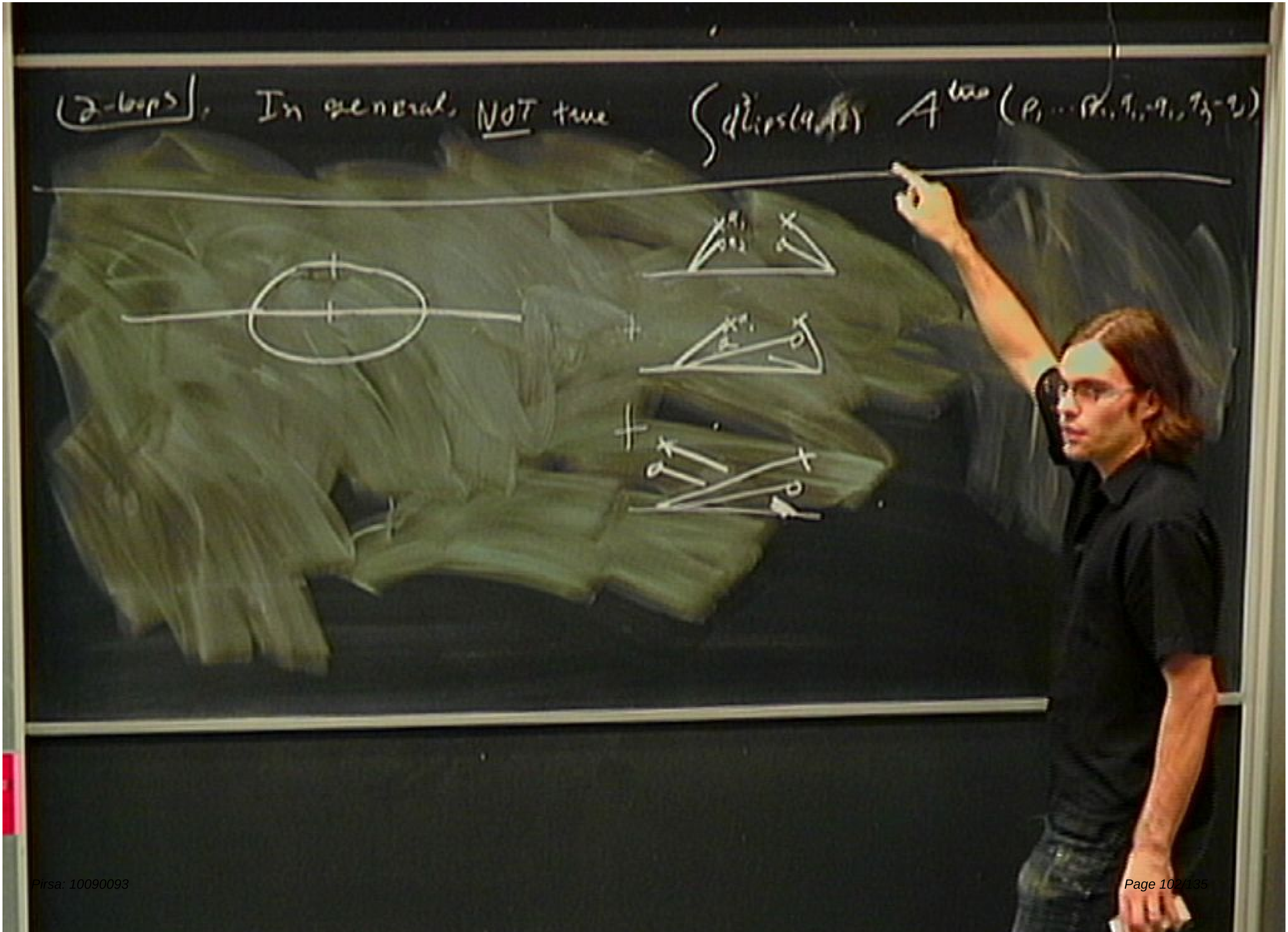

(2-loops). In general, NOT true

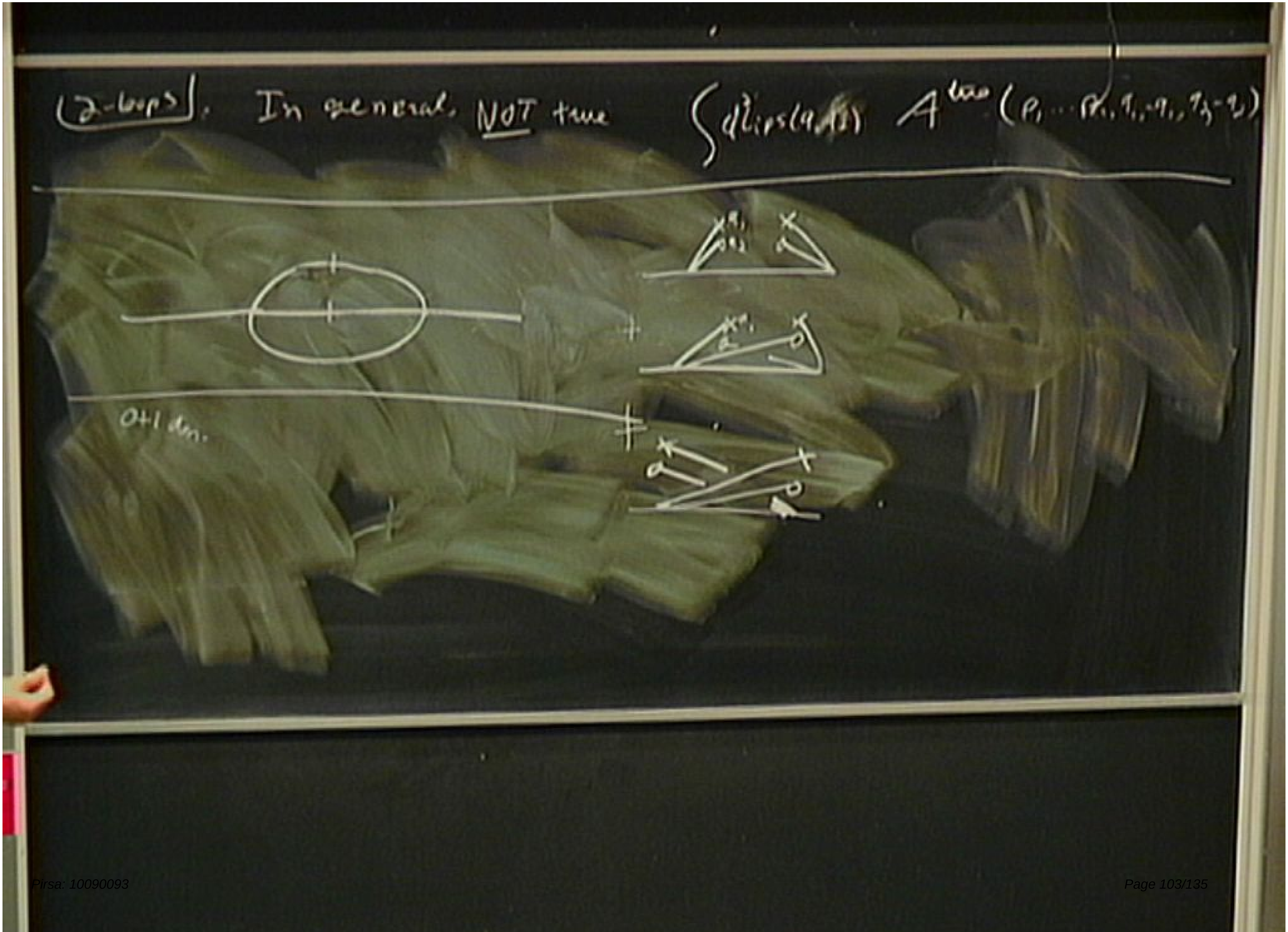

(2-loops). In general, NOT true

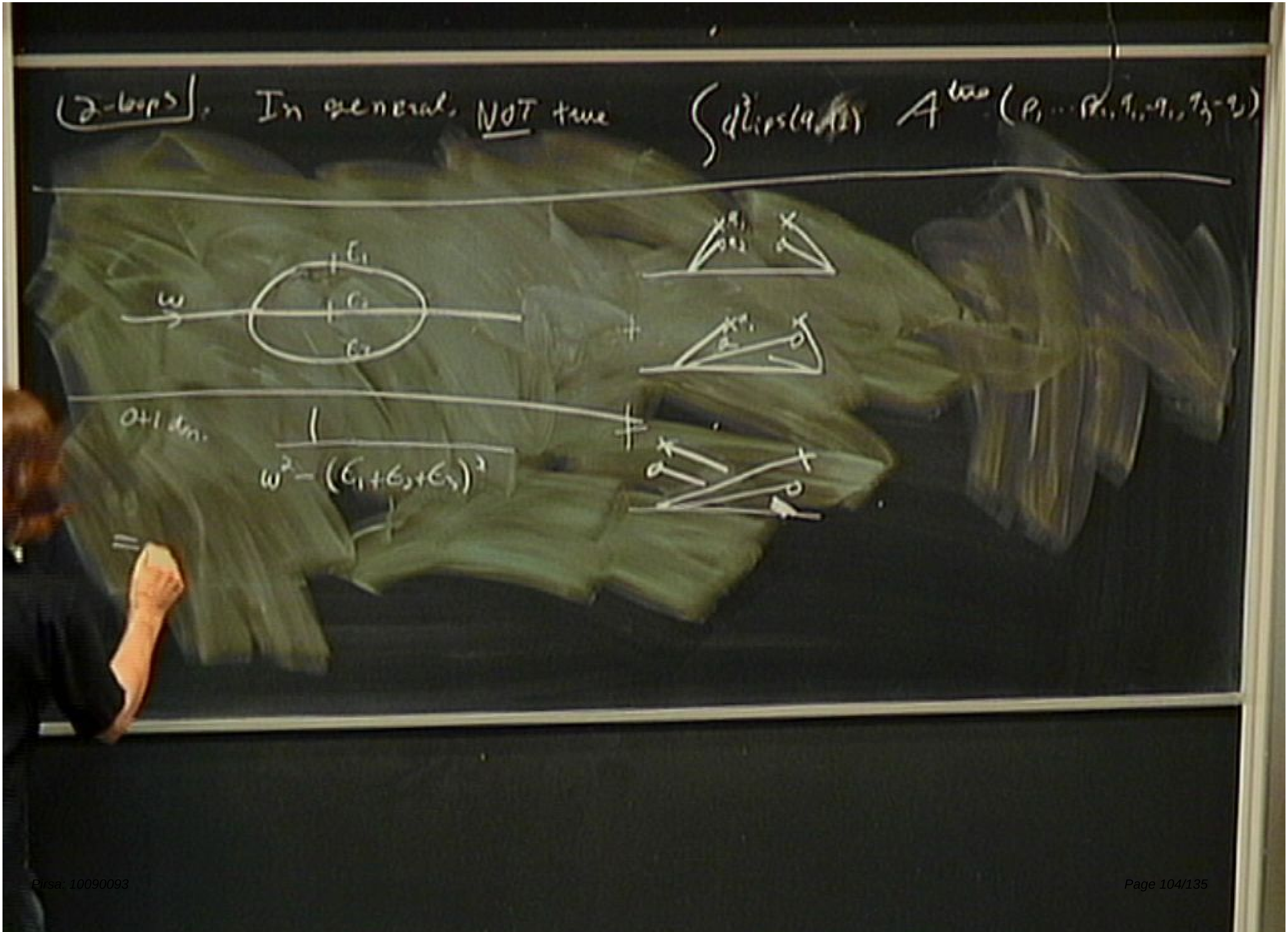

(2-loops). In general, NOT true

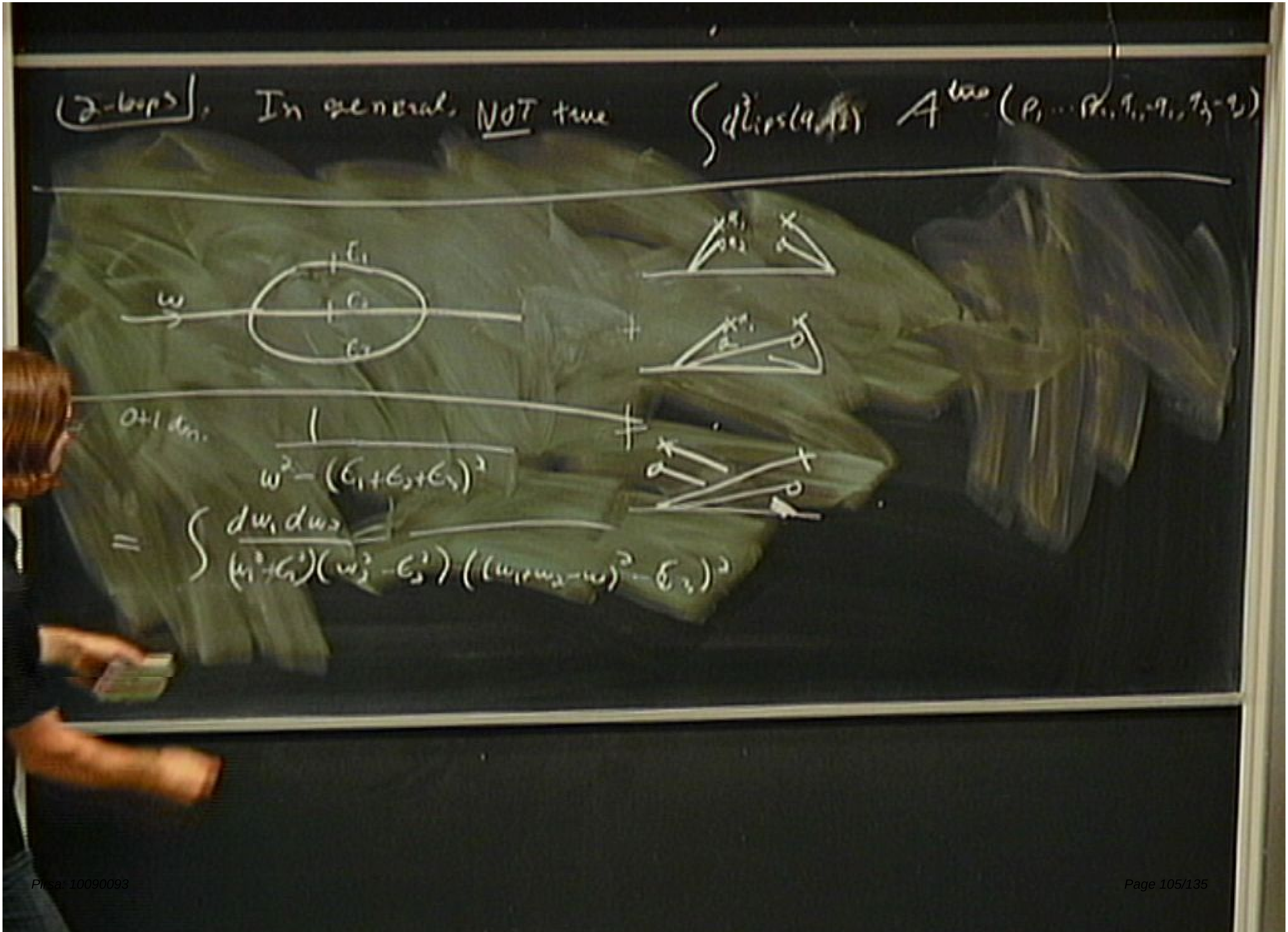
(2-loops). In general, NOT true $\int d\text{LIPS}(q_1, q_2)\, A^{\text{tree}}(p_1 \ldots p_n, q_1, -q_1, q_2, -q_2)$

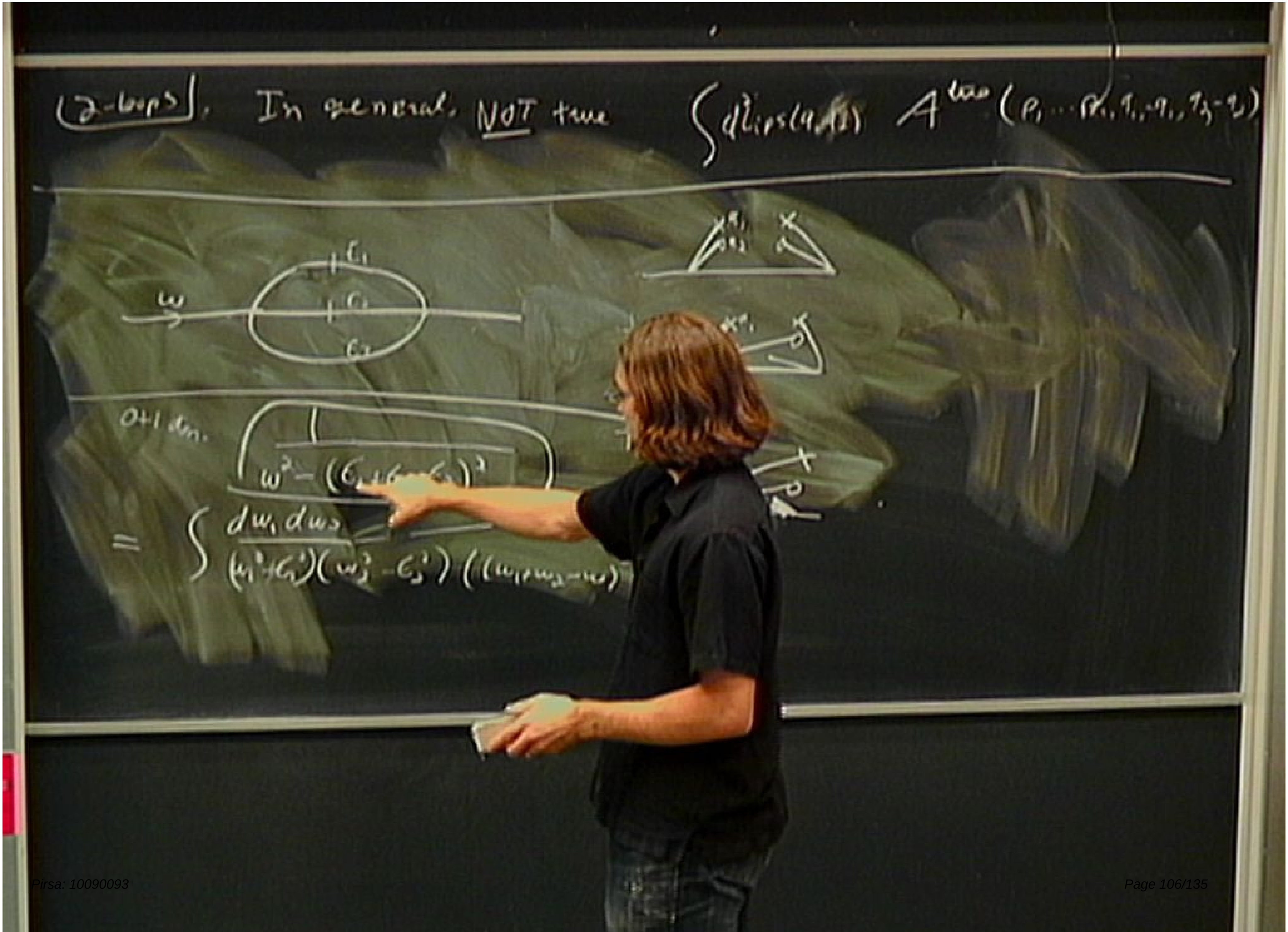

(2-loops). In general, NOT true

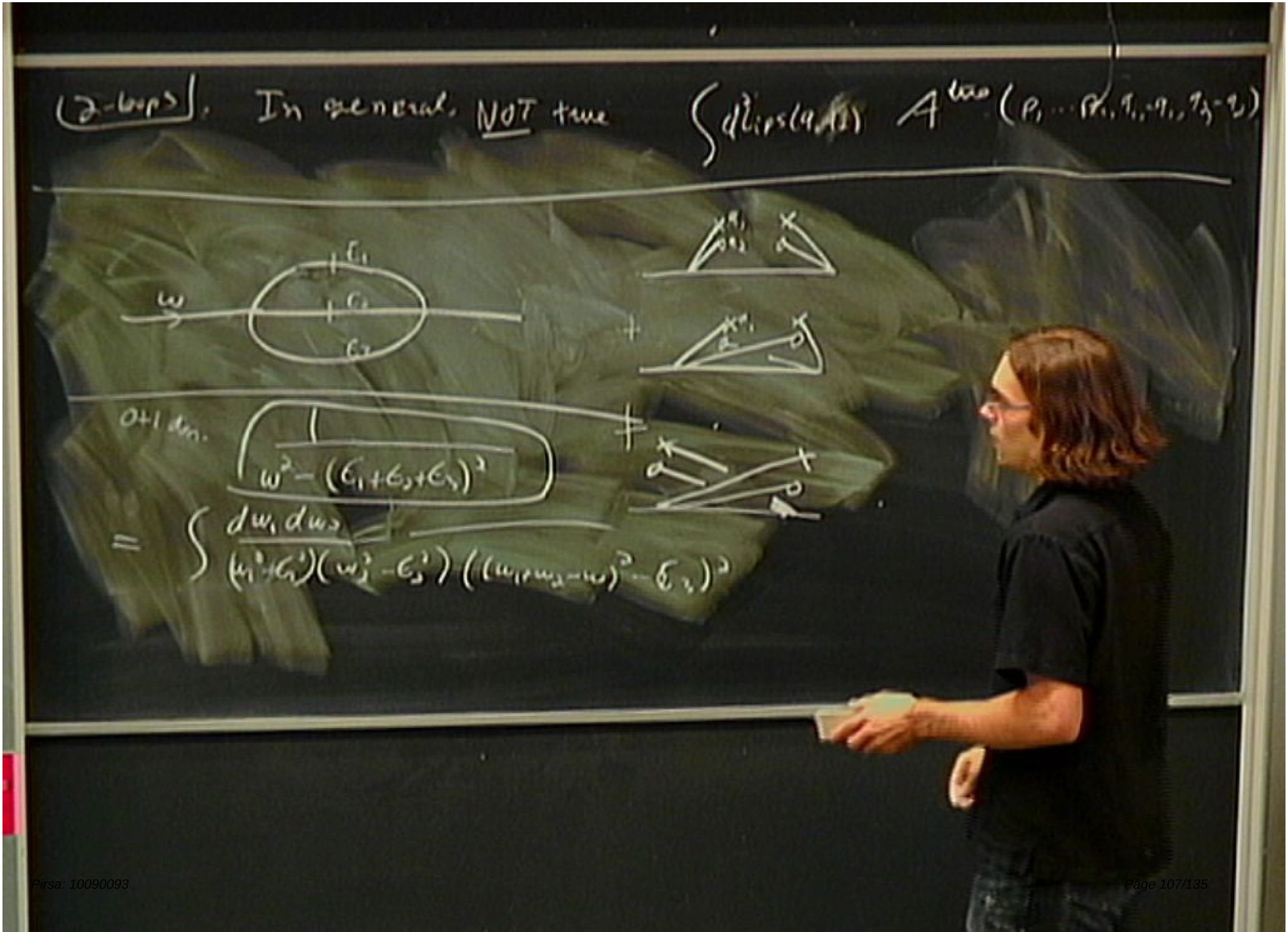

(2-loops). In general, NOT true

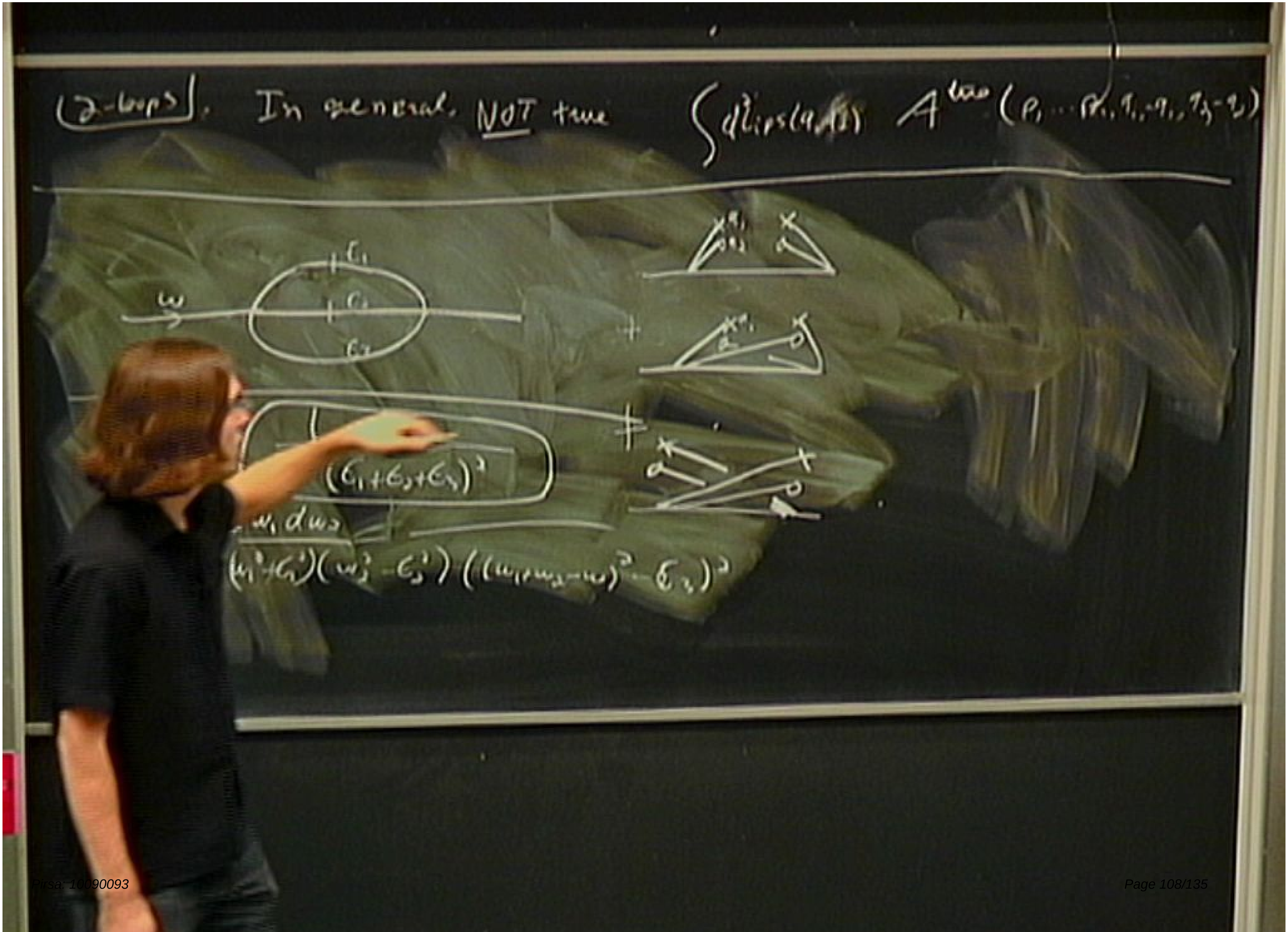

(2-loops). In general, NOT true
0+1 dim.


(2-loops).
In general, NOT true
0+1 dim.

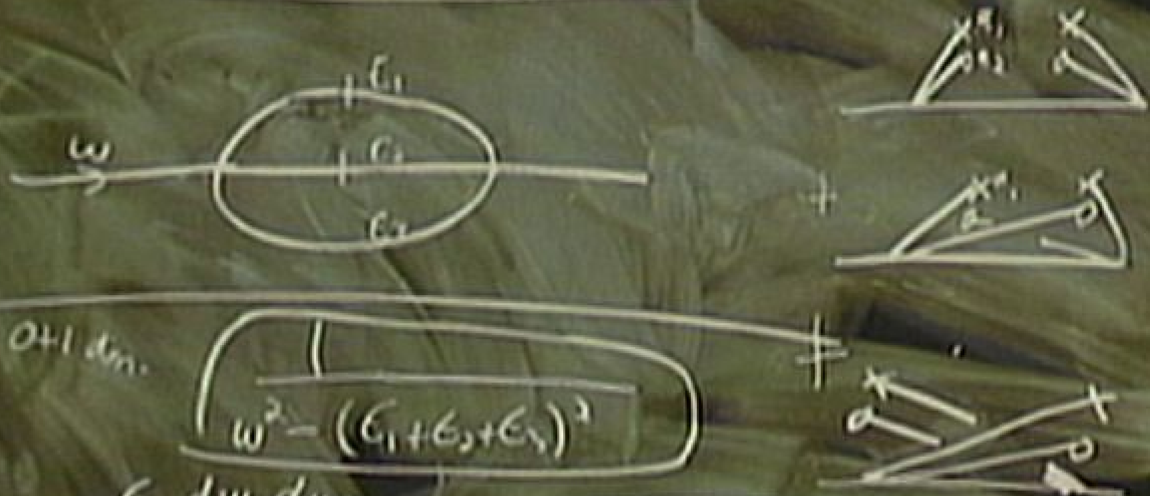

(2-loops). In general, NOT true
0+1 dim.


2-loops
In general, NOT true
0+1 dim.


(2-loops). In general, NOT true
0+1 dim.
ω² − (ε₁+ε₂+ε₃)²
= ∫ dω₁ dω₂ / (ω₁²−ε₁²)(ω₂²−ε₂²)((ω₁+ω₂−ω)²−ε₃)²


(2-loops). In general, NOT true

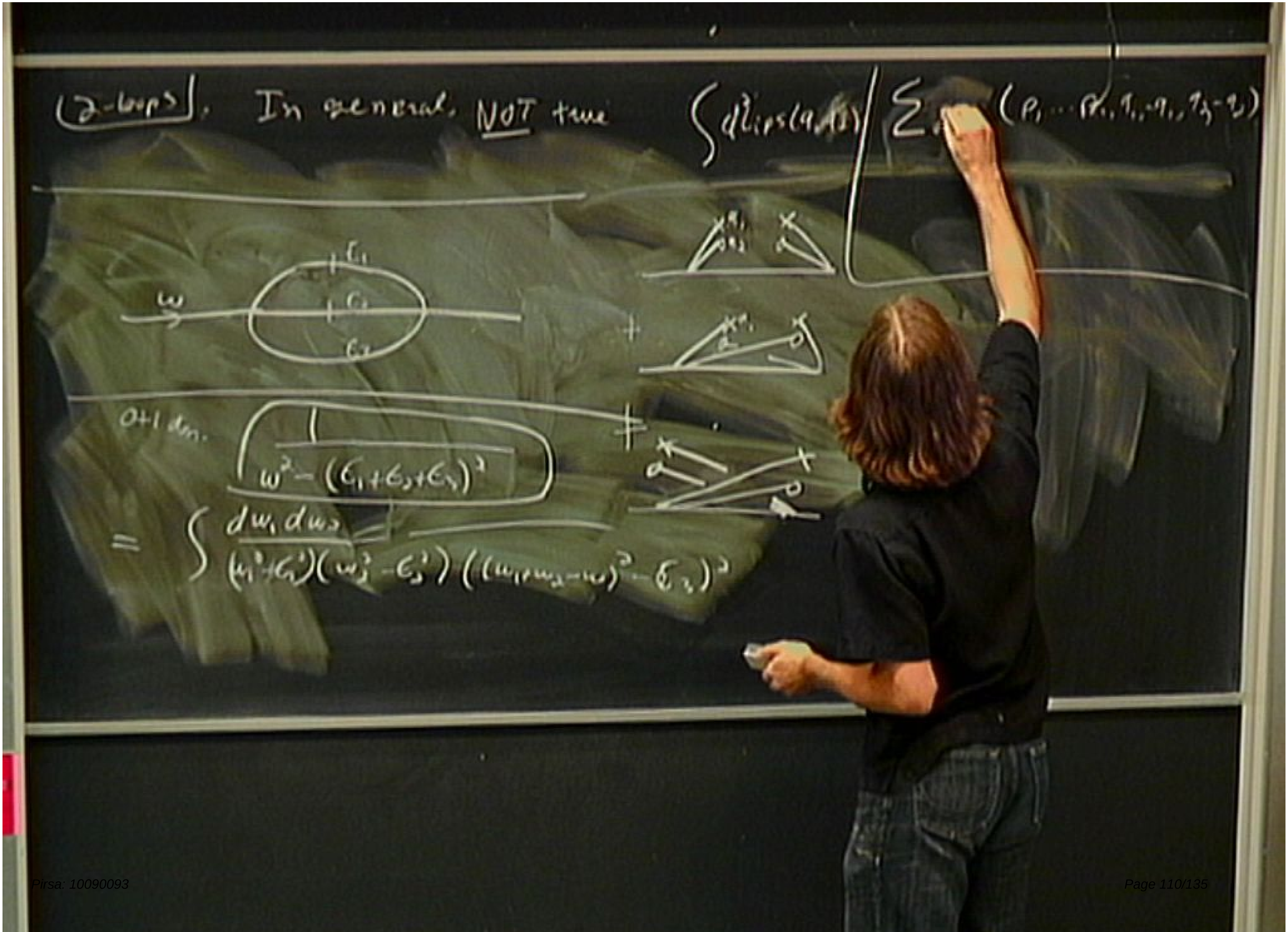
(2-loops). In general, NOT true $\int d^2 Lips(q_1, q_2)\ A^{tree}(p_1 \ldots p_n, q_1, -q_1, q_2, -q_2)$

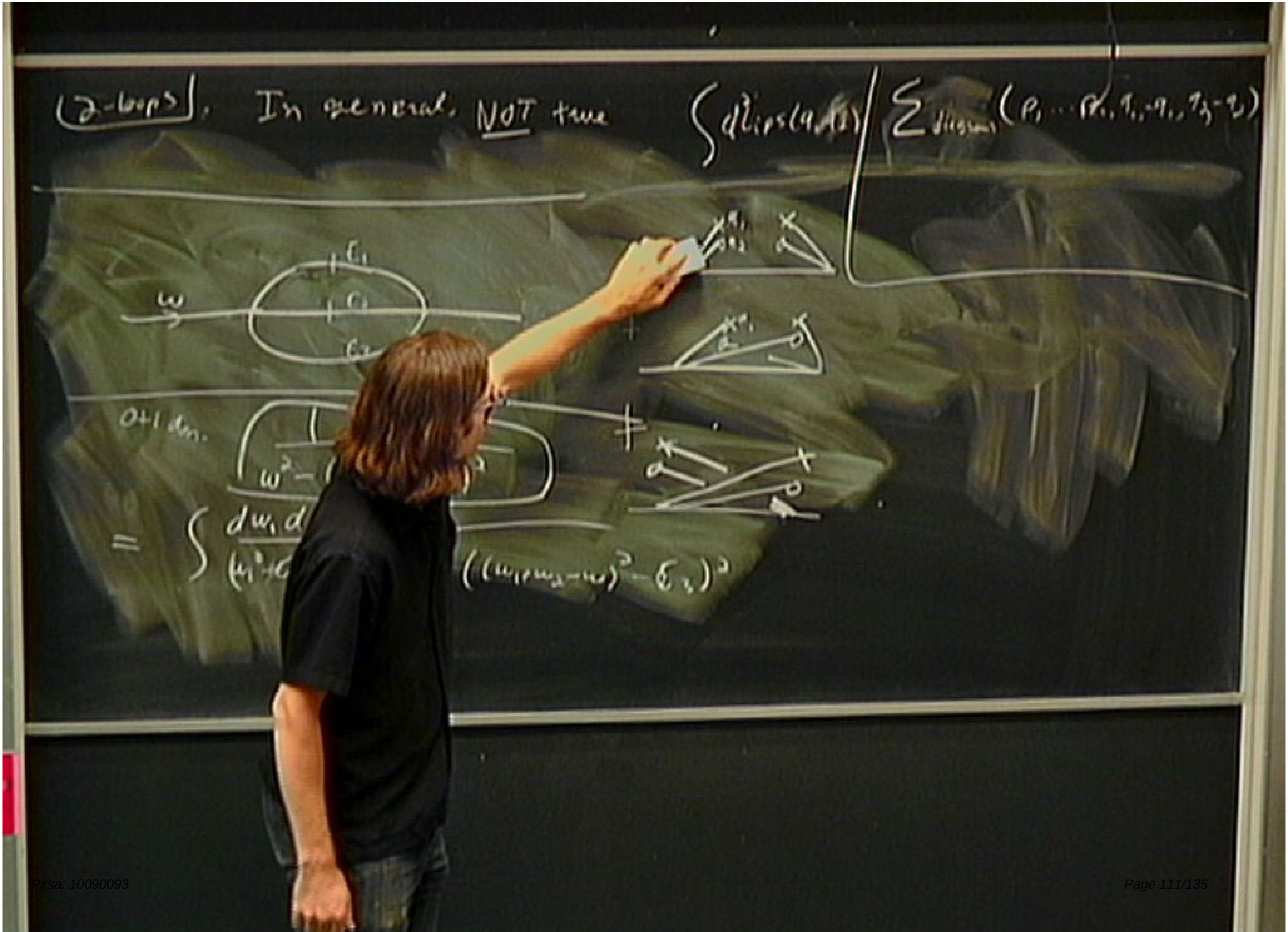


0+1 dim.

$$\frac{(\ \ )}{\omega^2 - (\epsilon_1+\epsilon_2+\epsilon_3)^2}$$

$$= \int \frac{d\omega_1\, d\omega_2}{(\omega_1^2+\epsilon_1^2)(\omega_2^2-\epsilon_2^2)((\omega_1+\omega_2-\omega)^2-\epsilon_3)^2}$$


(2-loops). In general, NOT true
ω
ϵ_1
ϵ_2
ϵ_3
0+1 dim.
$\omega^2 - (\epsilon_1+\epsilon_2+\epsilon_3)^2$


(2-loops).
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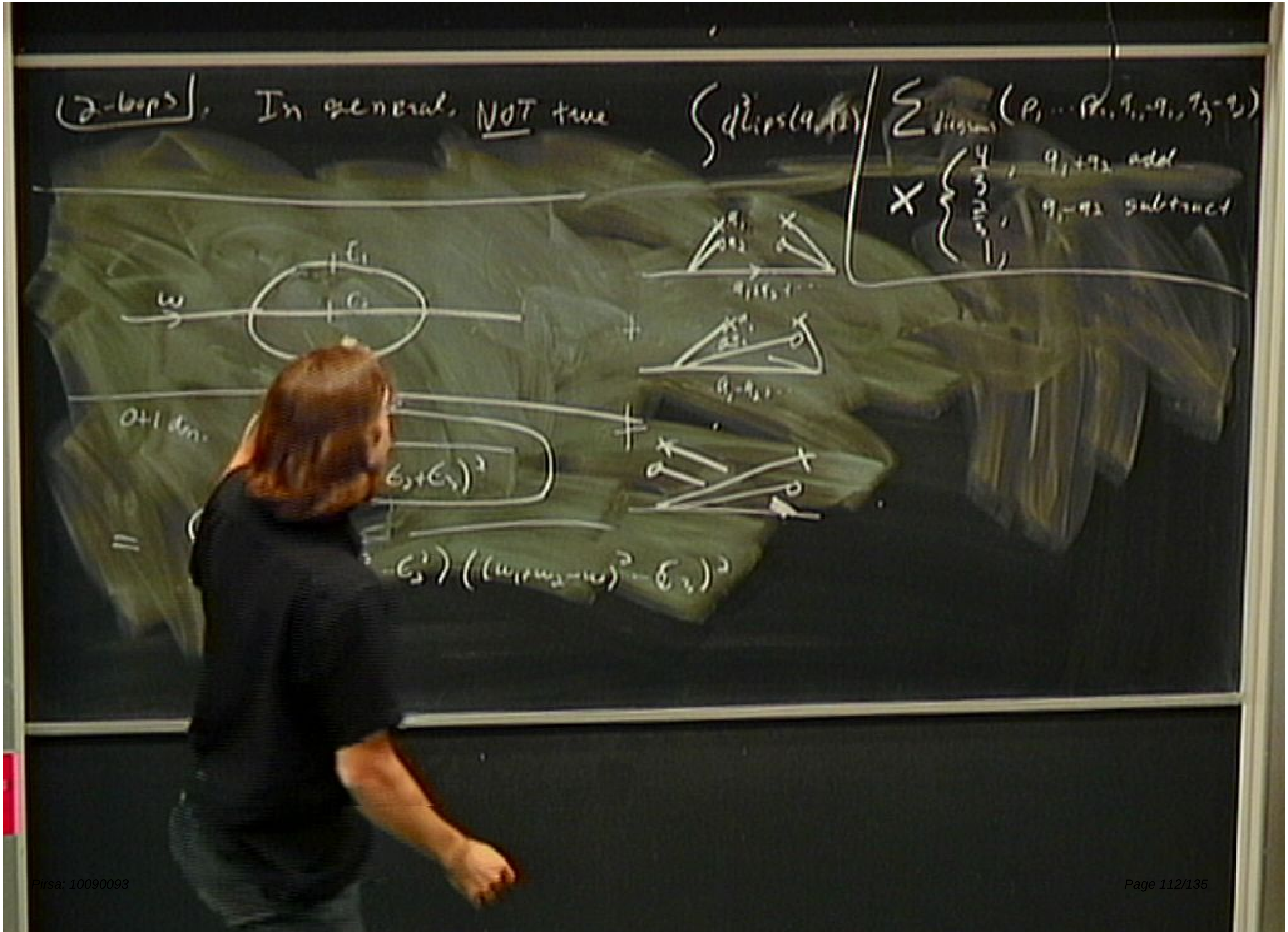
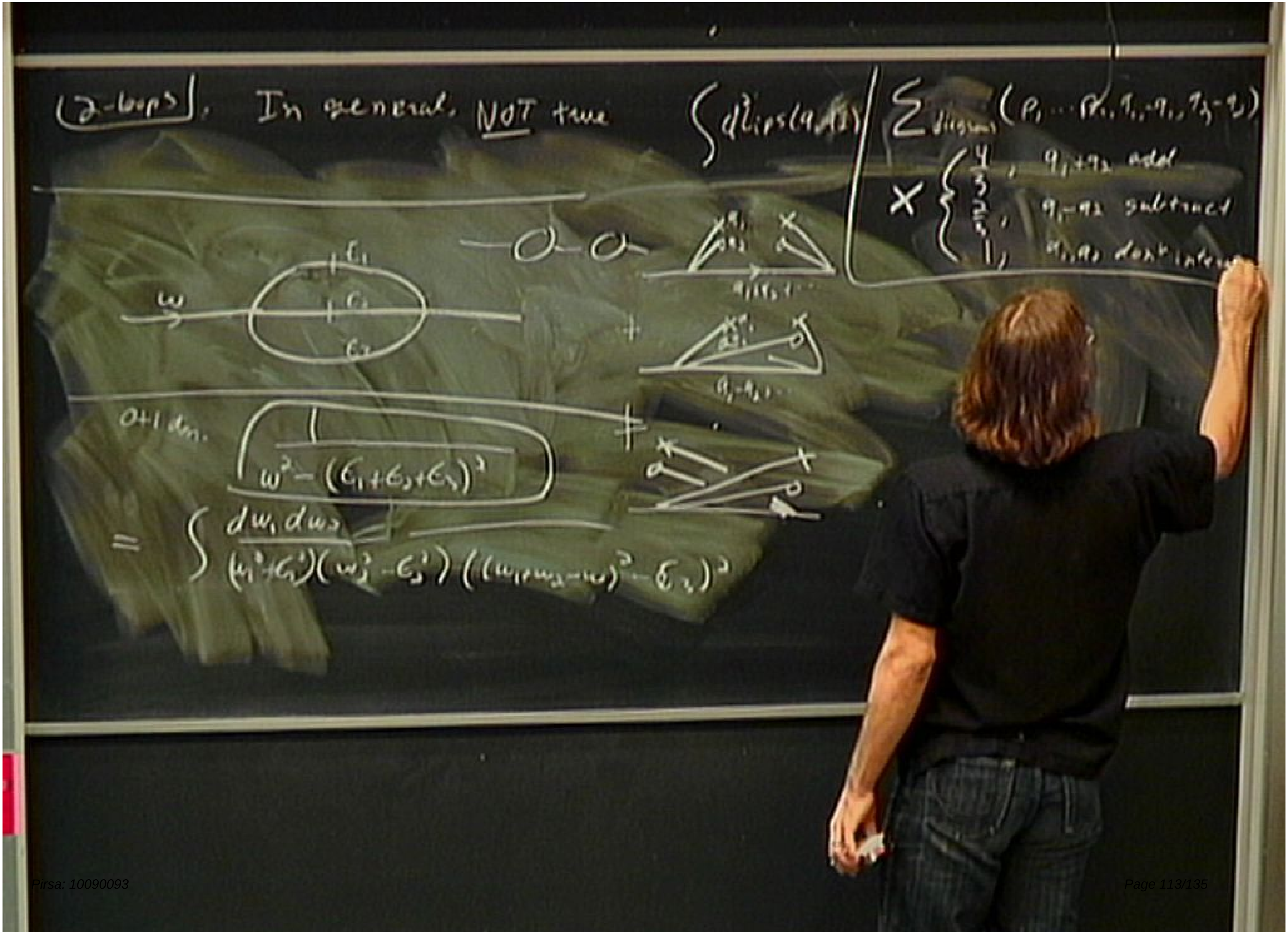
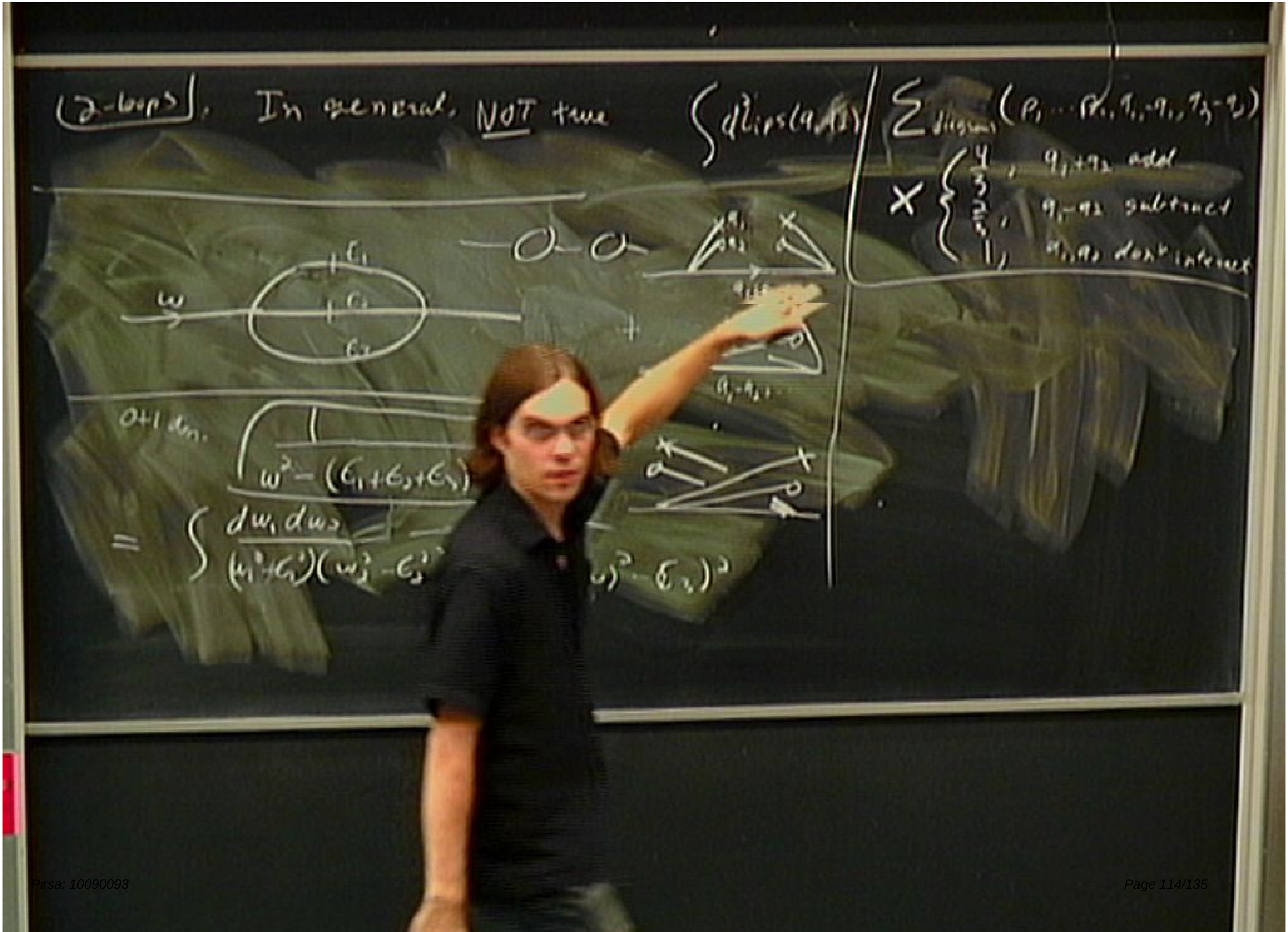
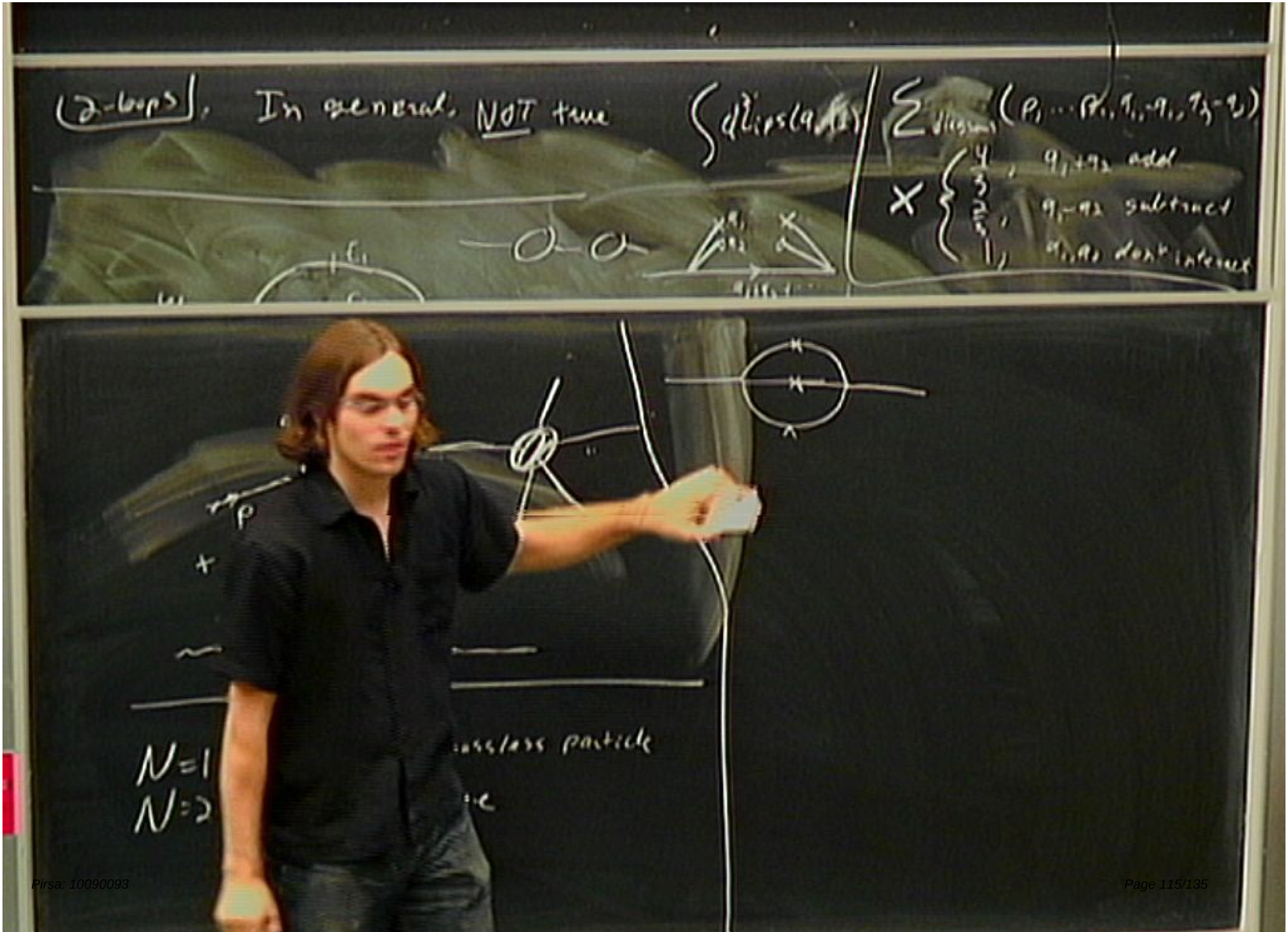
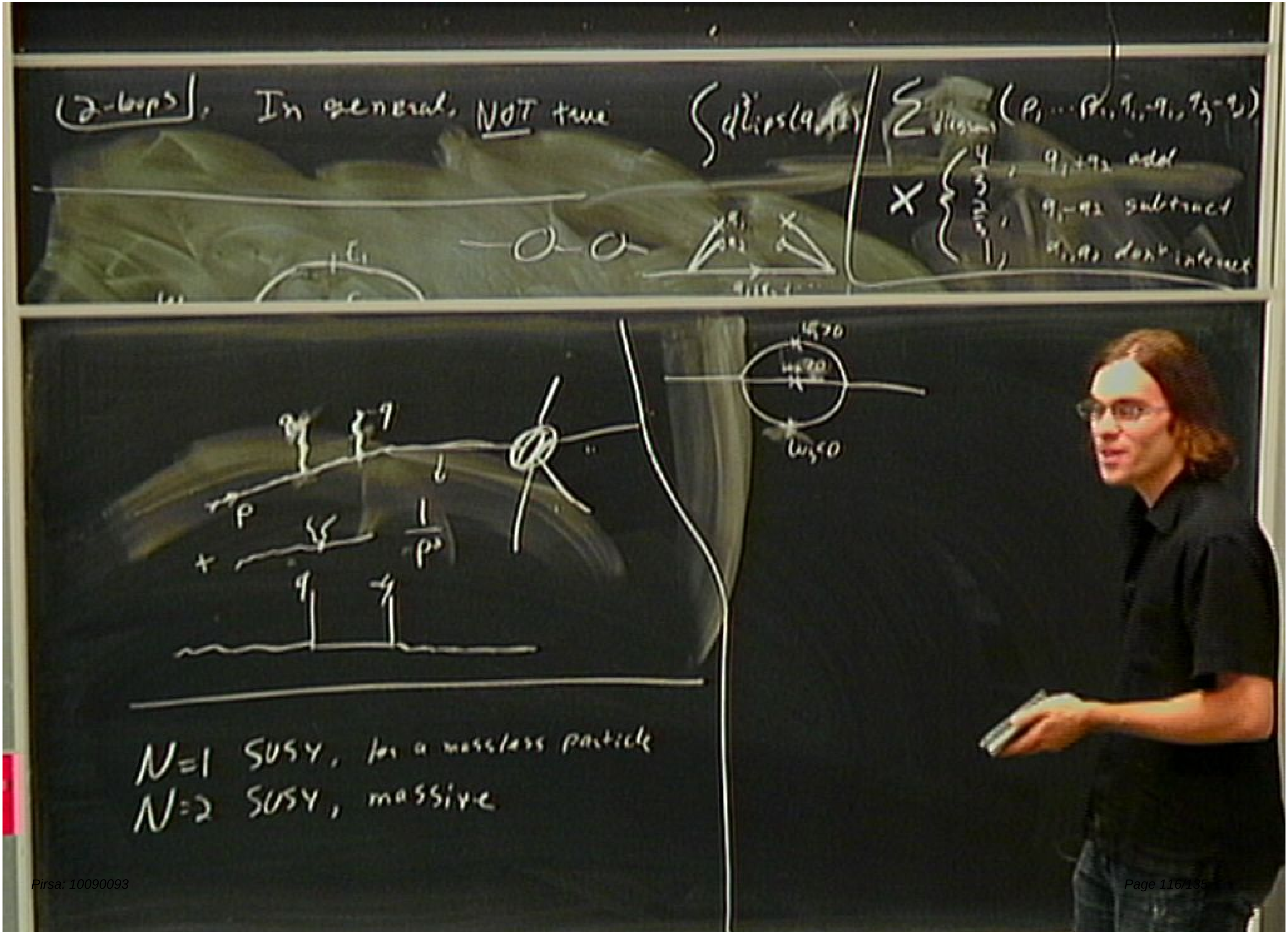
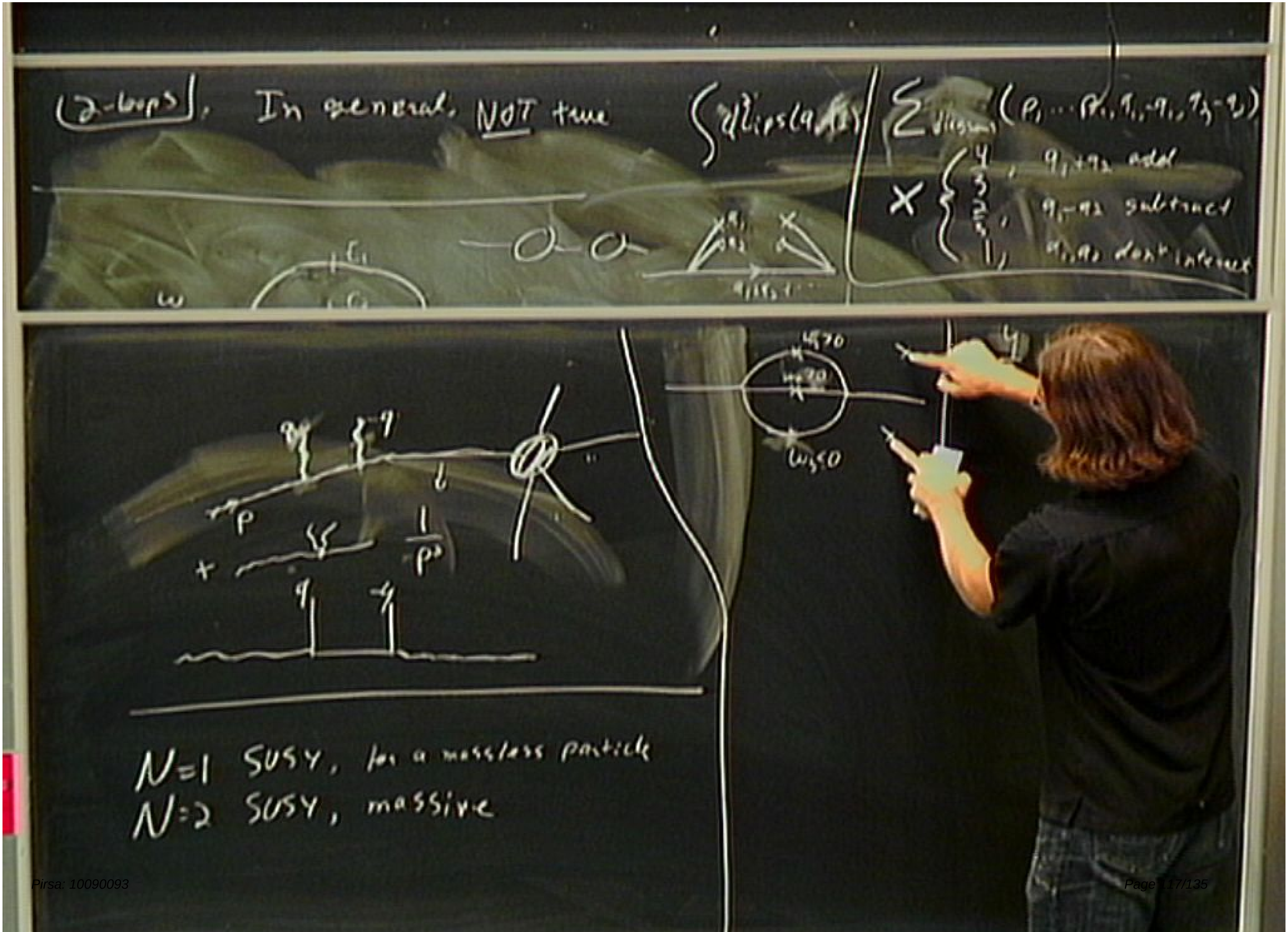

(2-loops). In general, NOT true
0+1 dim.


(2-loops). In general, NOT true
q1+q2 add
q1-q2 subtract
q1, q2 don't interf
0+1 dim.
$\omega^2 - (\epsilon_1+\epsilon_2+\epsilon_3)^2$
$= \int \frac{d\omega_1 d\omega_2}{(\omega_1^2-\epsilon_1^2)(\omega_2^2-\epsilon_2^2)((\omega_1+\omega_2-\omega)^2-\epsilon_3^2)}$


(2-loops).
In general, NOT true
q_1+q_2 add
q_1-q_2 subtract
q_1, q_2 don't interact
0+1 dim.


(2-loops). In general, NOT true
× { 4, q1+q2 add
2, q1−q2 subtract
1, q1,q2 don't interact
N=1
N=2


(2-loops). In general, NOT true
q1+q2 odd
q1-q2 subtract
q1,q2 don't interact
N=1 SUSY, for a massless particle
N=2 SUSY, massive


(2-loops). In general, NOT true
q_1+q_2 add
q_1-q_2 subtract
q_1, q_2 don't interact
N=1 SUSY, for a massless particle
N=2 SUSY, massive

(2-loops). In general, NOT true

$\int \mathcal{U}_{\text{loops}}(q_1, q_2) \Big/ \sum_{\text{diagrams}} (p_1 \cdots p_n, q_1, -q_1, q_2, -q_2)$

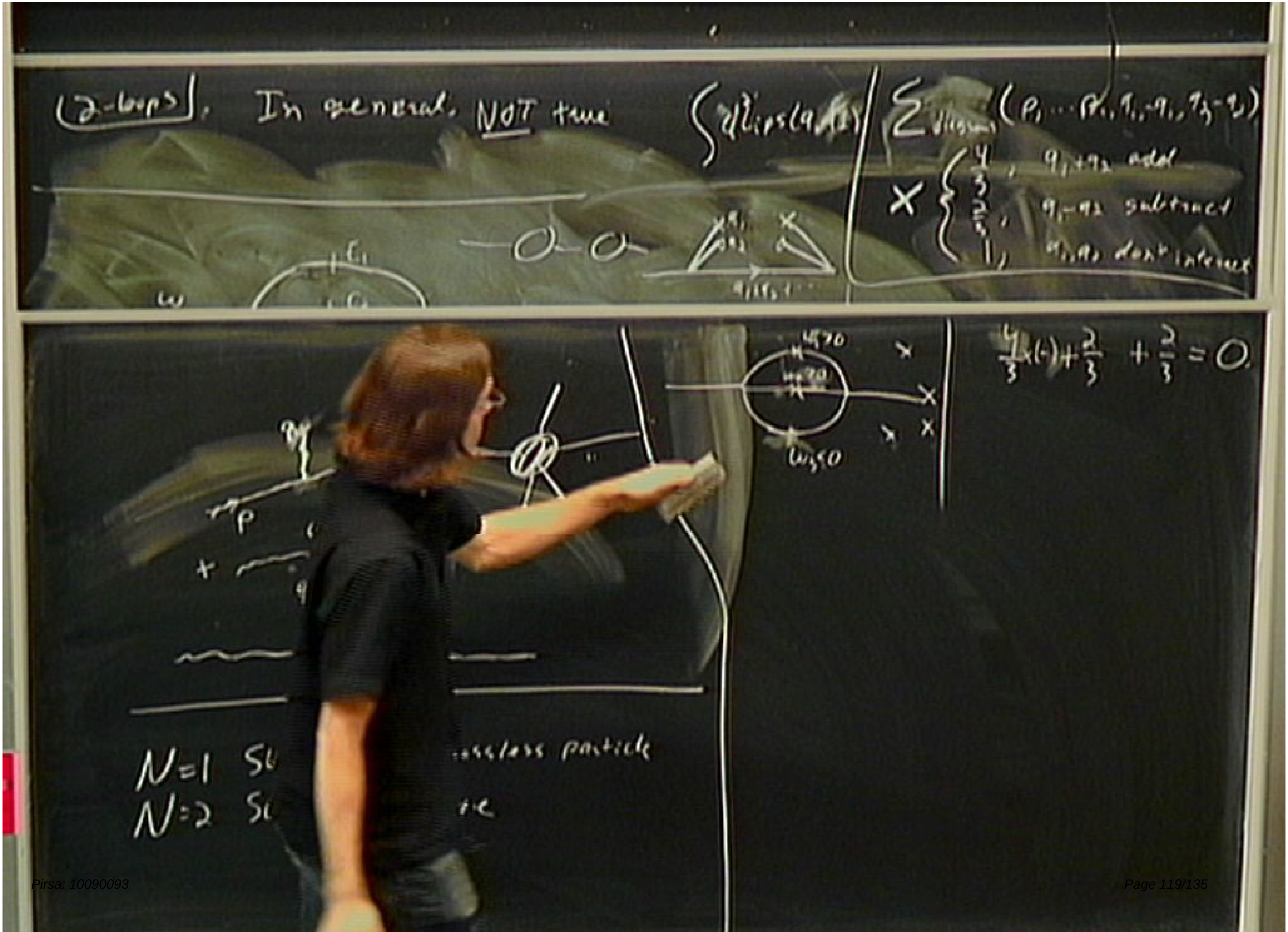
$\times \begin{cases} \frac{4}{3}, & q_1 + q_2 \text{ add} \\ \frac{2}{3}, & q_1 - q_2 \text{ subtract} \\ 1, & q_1, q_2 \text{ don't interact} \end{cases}$

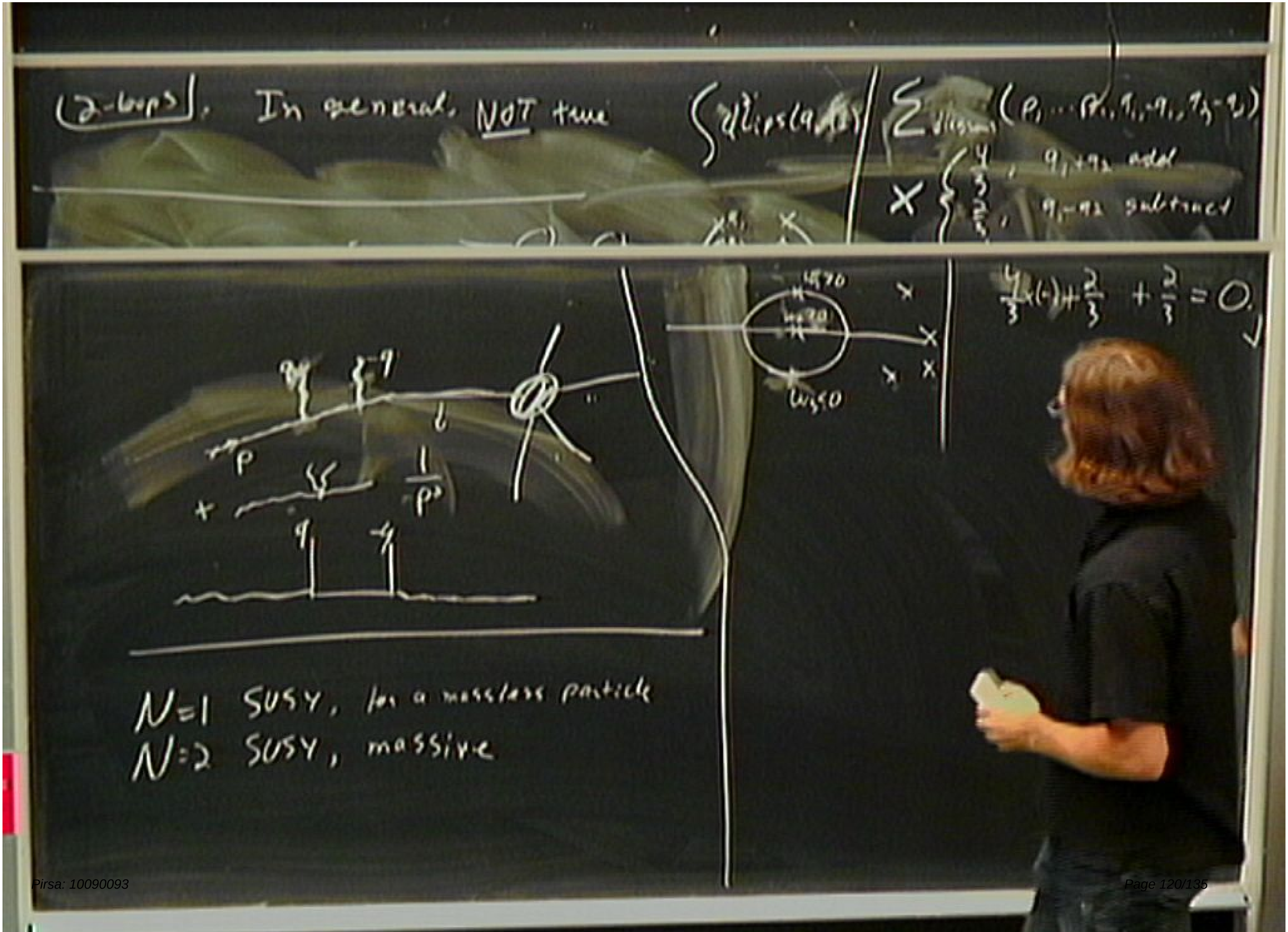
$\frac{4}{3}$ $\frac{2}{3}$

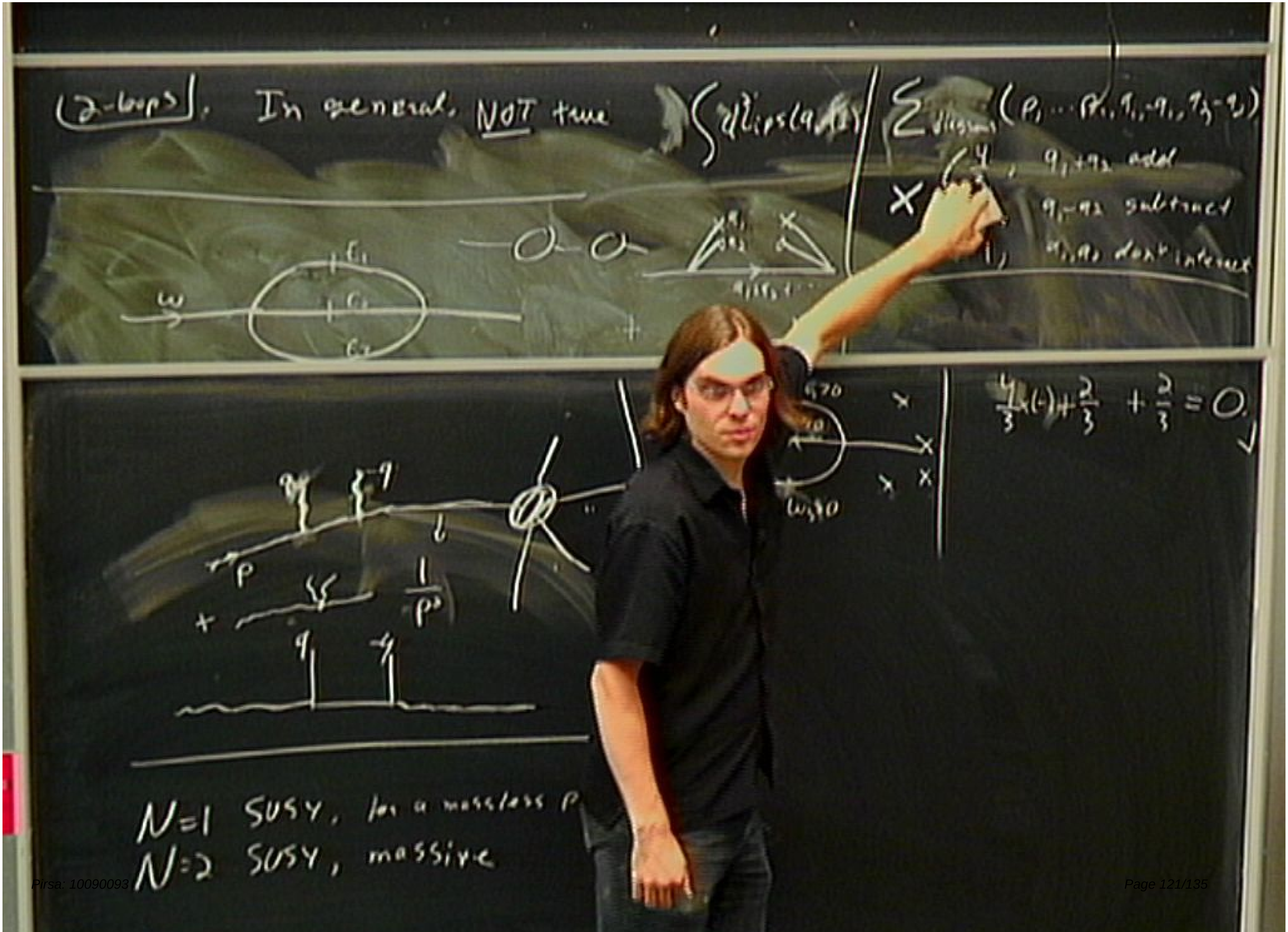
$\frac{1}{p^2}$

N=1 SUSY, for a massless particle

N=2 SUSY, massive


(2-loops). In general, NOT true
q1+q2 add
q1-q2 subtract
q1, q2 don't interact
N=1
N=2
massless particle

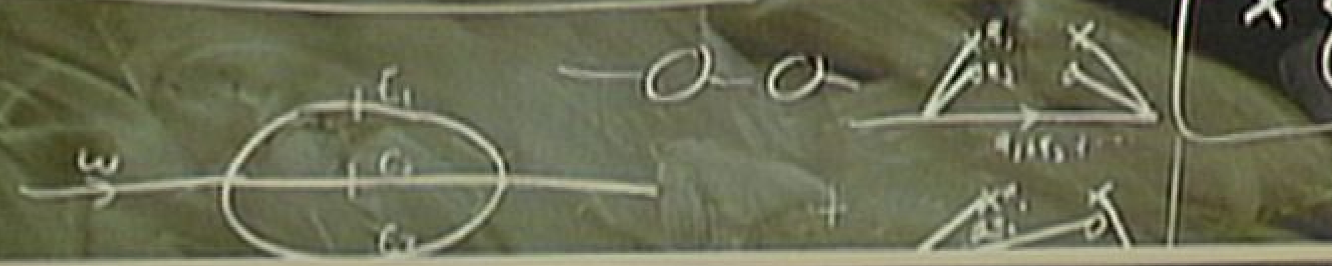

(2-loops). In general, NOT true
N=1 SUSY, for a massless particle
N=2 SUSY, massive
4/3 (-1) + 2/3 + 2/3 = 0



(2-loops). In general, NOT true
q1+q2 add
q1-q2 subtract
q1,q2 don't interact
N=1 SUSY, for a massless p
N=2 SUSY, massive

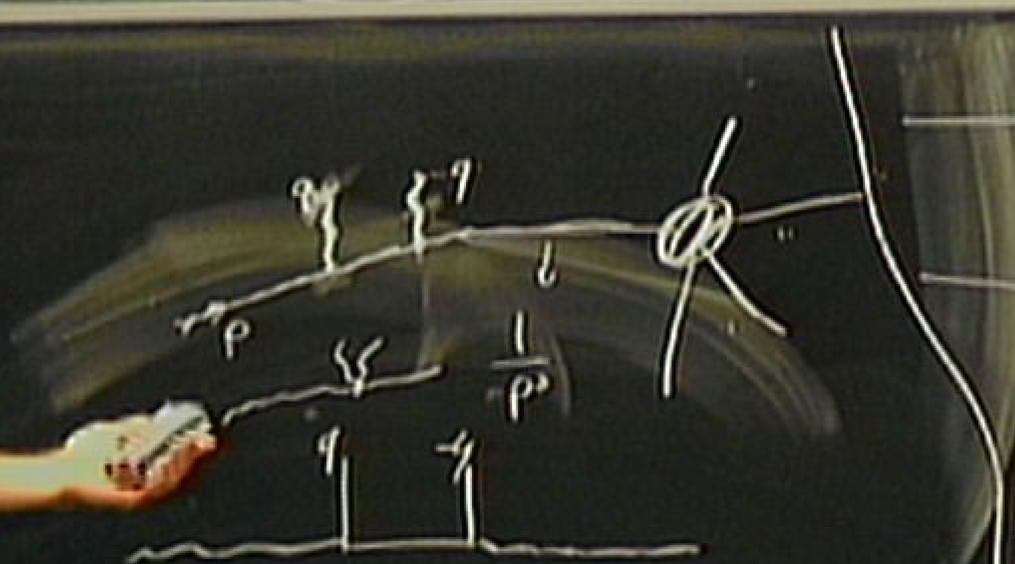
(2-loops). In general, NOT true $\int d^dl_1 \ldots$ $\sum_{\text{diagrams}} (p_1 \ldots p_n, q_1, -q_1, q_2, -q_2)$

$\times \begin{cases} \frac{4}{3}, & q_1+q_2 \text{ add} \\ \frac{2}{3}, & q_1-q_2 \text{ subtract} \\ 1, & q_1, q_2 \text{ don't interact} \end{cases}$



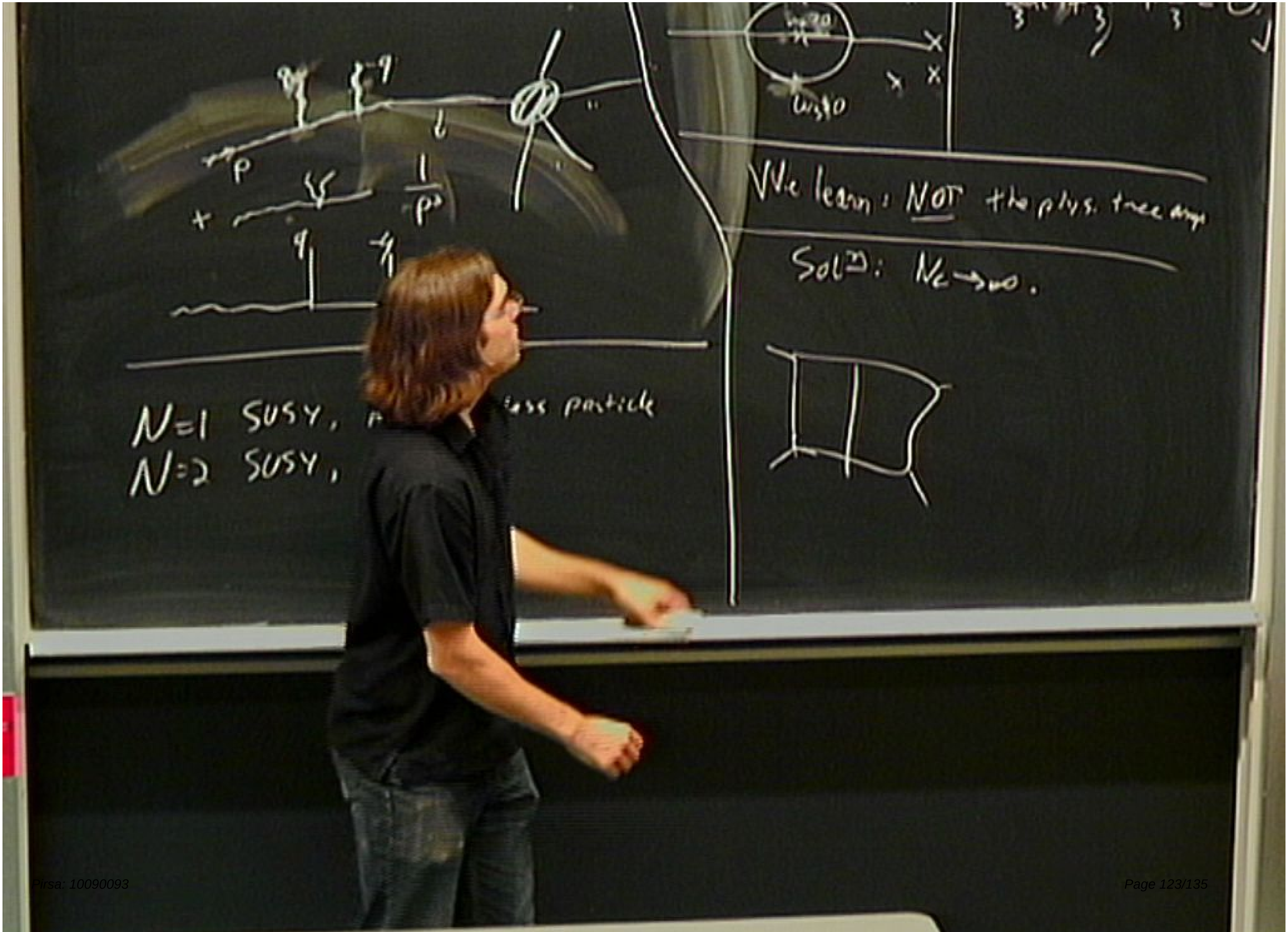
$\frac{4}{3}(-) + \frac{2}{3} + \frac{2}{3} = 0$ ✓

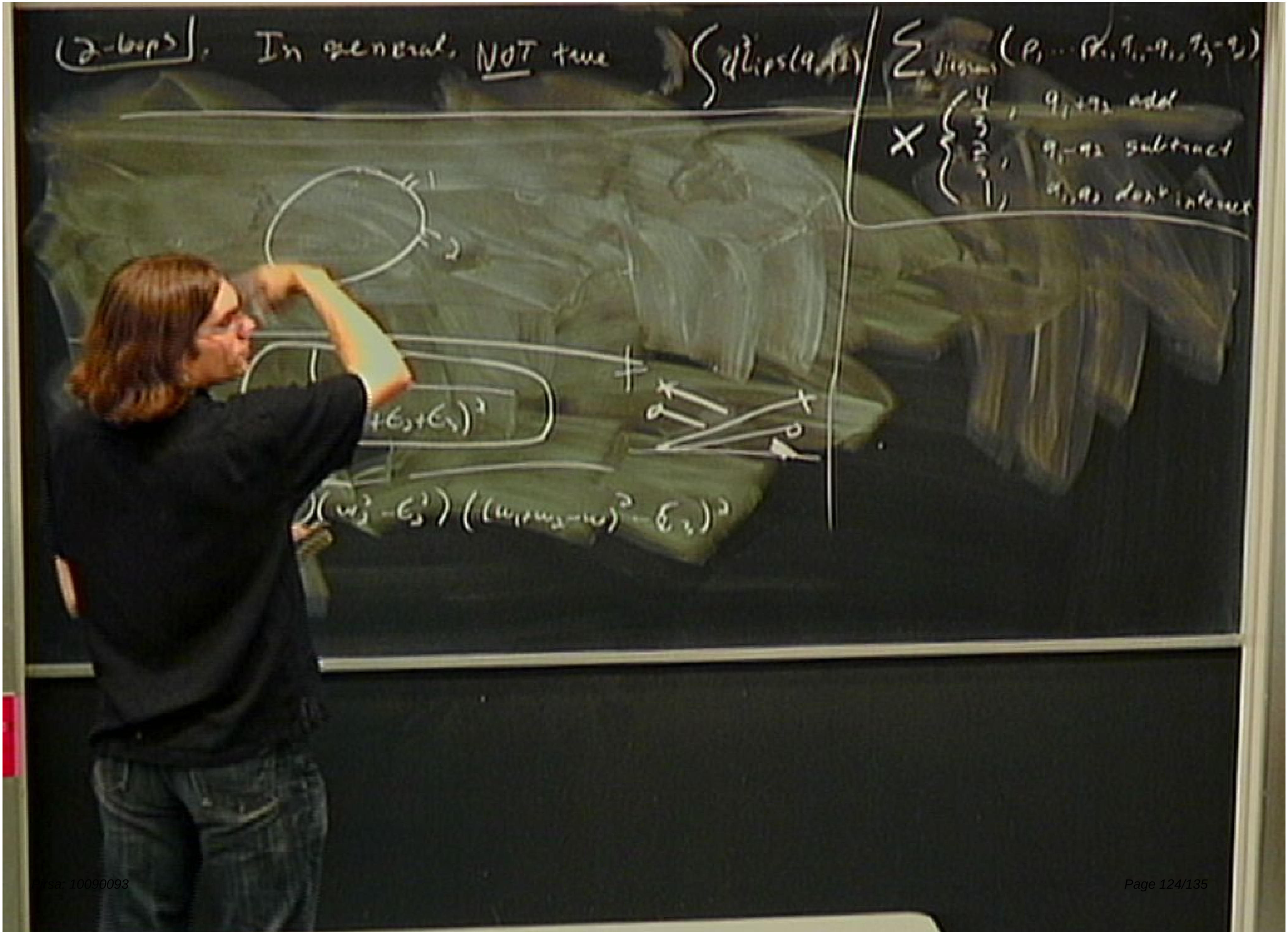
We learn: NOT the phys. tree amps



$N=1$ SUSY, for a massless particle

$N=2$ SUSY, massive


We learn: NOT the phys. tree amps
N=1 SUSY,
N=2 SUSY,



2-loops. In general, NOT true
q1+q2 add
q1-q2 subtract
q1,q2 don't interact

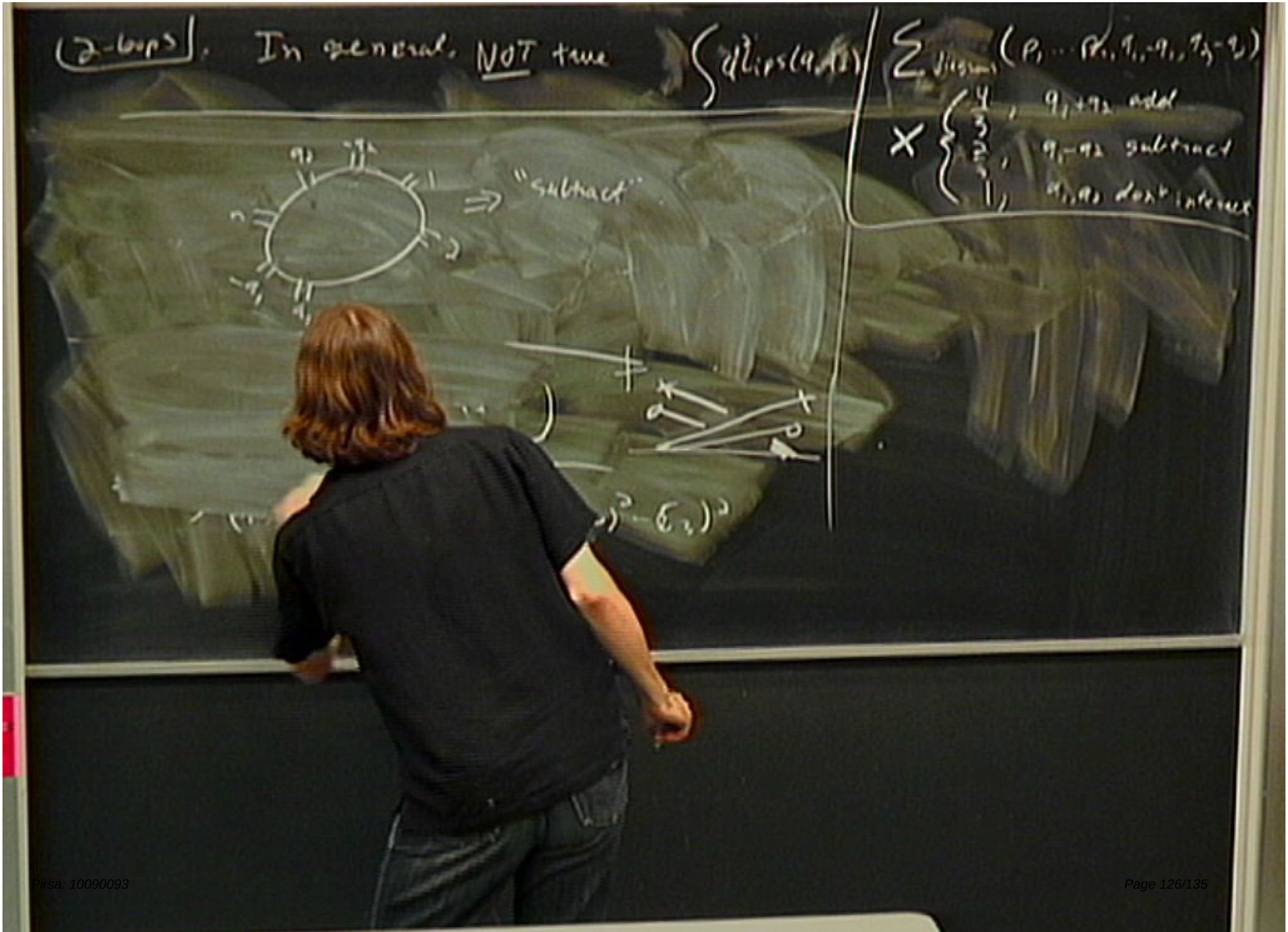
(2-loops). In general, NOT true $\int d^2 l_{ips}(q_1, q_2) \,\Big|\, \sum_{\text{diagrams}} (p_1 \cdots p_n, q_1, -q_1, q_2, -q_2)$

$$\times \begin{cases} \frac{4}{3}, & q_1 + q_2 \text{ add} \\ \frac{2}{3}, & q_1 - q_2 \text{ subtract} \\ 1, & q_1, q_2 \text{ don't interact} \end{cases}$$

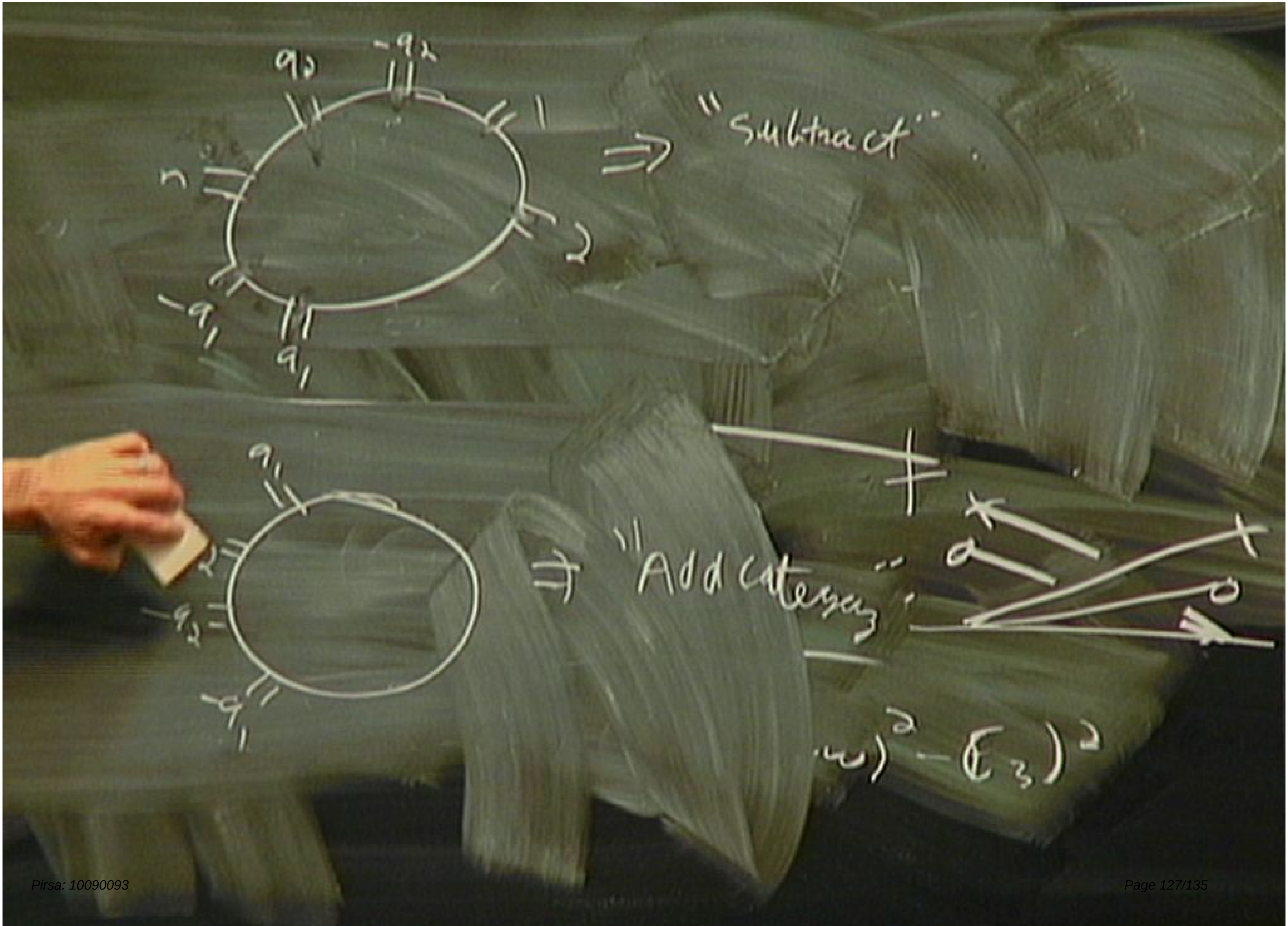
0+1 dim.

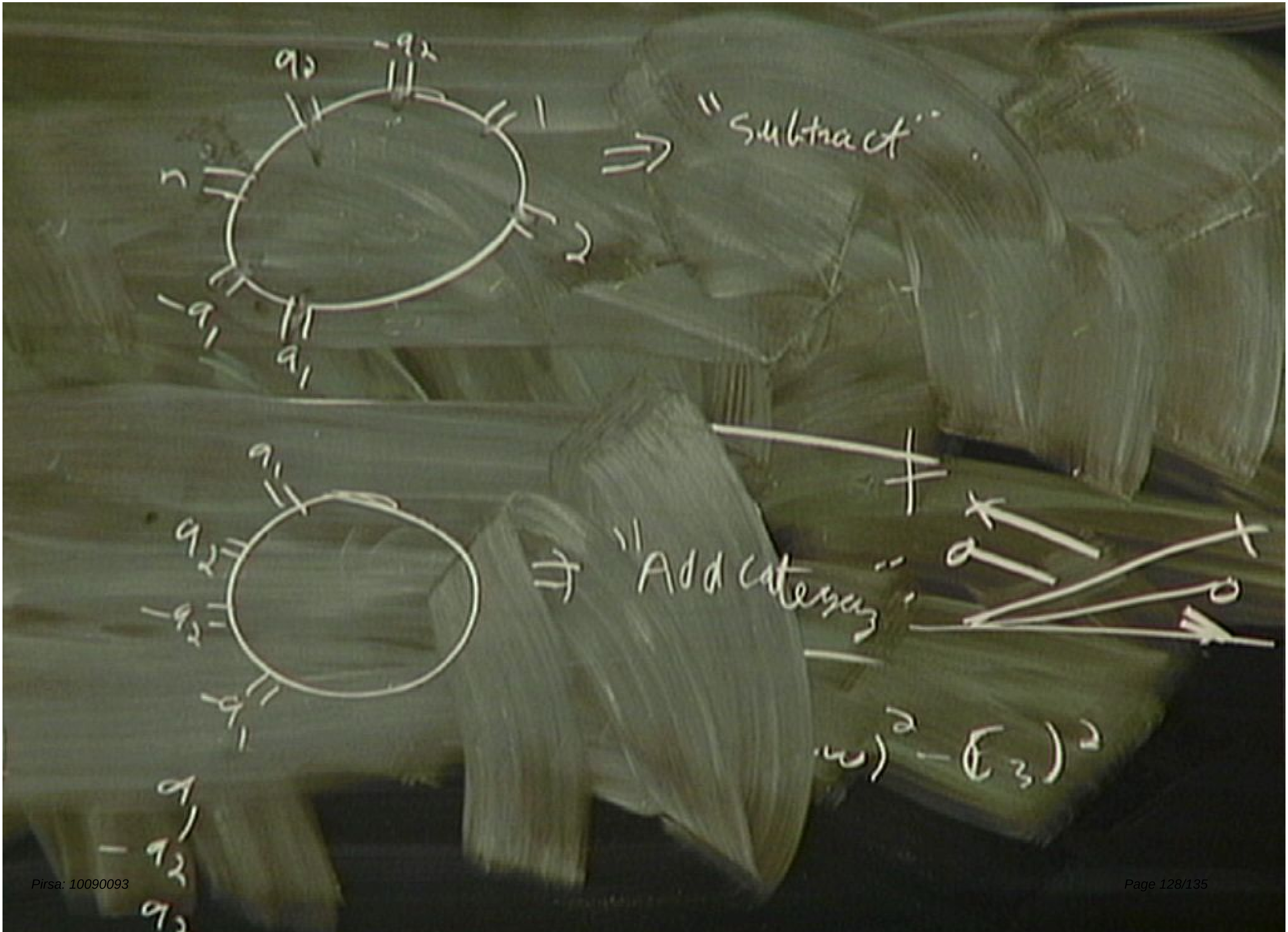
$$\boxed{\int \frac{}{\omega^2 - (\epsilon_1 + \epsilon_2 + \epsilon_3)^2}} \neq$$

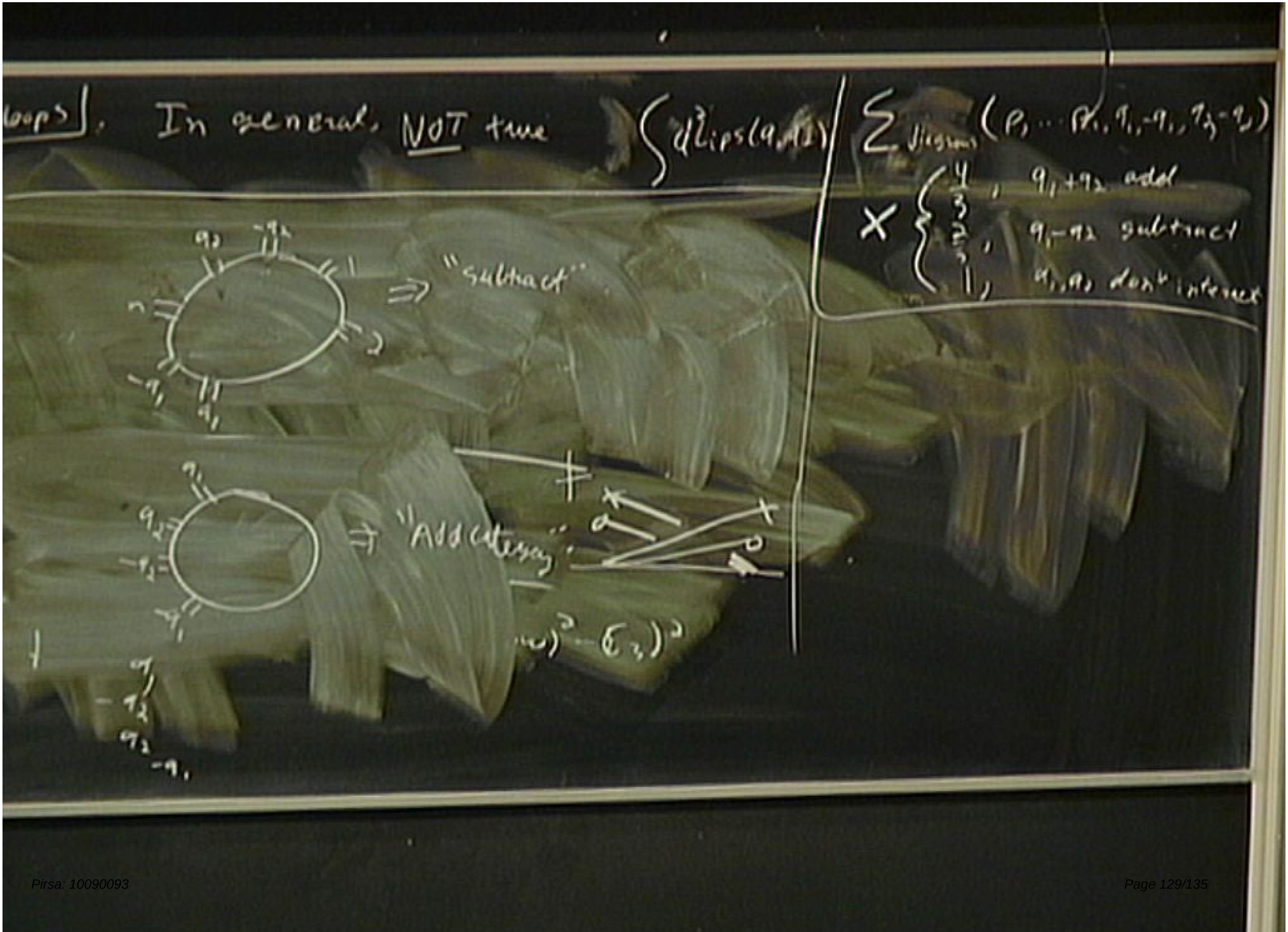
$$= \int \frac{d\omega_1 \, d\omega_2}{(\omega_1^2 - \epsilon_1^2)(\omega_2^2 - \epsilon_2^2)\left((\omega_1 + \omega_2 - \omega)^2 - \epsilon_3^2\right)}$$



(2-loops). In general, NOT true
q1+q2 add
q1-q2 subtract
q1,q2 don't interact
"subtract"


"subtract"
"Add category"
q_2
$-q_2$
$-q_1$
q_1
$(\mathcal{E}_3)^2$

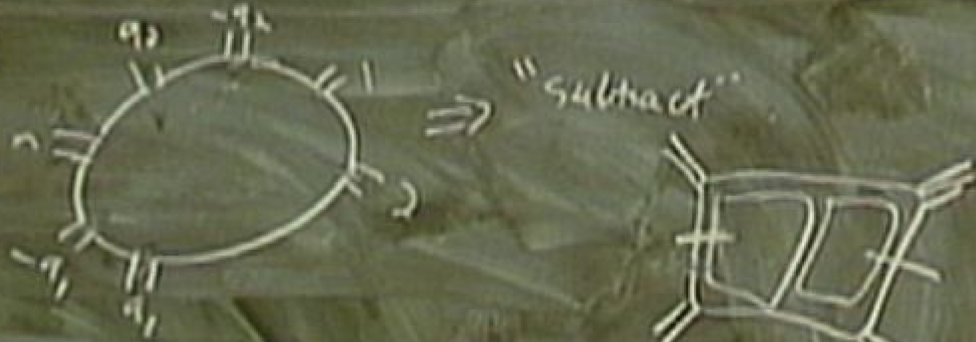

"subtract"
"Add category"
q_2
$-q_2$
q_1
$-q_1$
n
1
2

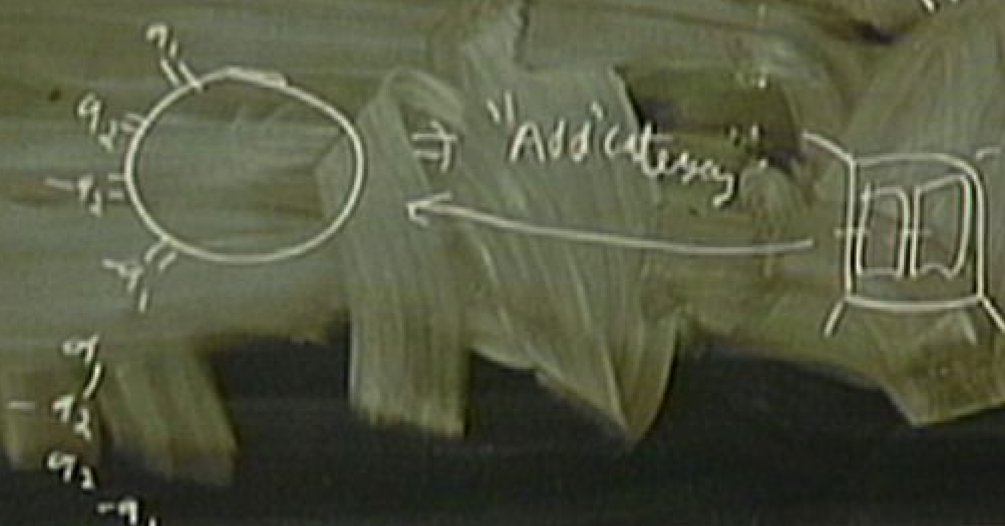

loops]. In general, NOT true
∫ d²Lips(q₁,q₂)
Σ diagrams (p₁ … pₙ, q₁, −q₁, q₂, −q₂)
q₁+q₂ add
q₁−q₂ subtract
q₁,q₂ don't interact
"subtract"

loops]. In general, NOT true $\int d^2 l_{ips}(q, \ldots)$

$$\sum_{\text{diagrams}} (p_1 \cdots p_n, q_1, -q_1, q_2, -q_2)$$

$$\times \begin{cases} \frac{4}{3}, & q_1 + q_2 \text{ add} \\ \frac{2}{3}, & q_1 - q_2 \text{ subtract} \\ 1, & q_1, q_2 \text{ don't interact} \end{cases}$$





$A^{\text{1-loop}}|_{T}$

True off-s

$$A_n^{\text{1-loop}}(p_1 \cdots p_n)_R = \frac{1}{2}\int \frac{d^3 q}{(2\pi)^3 2E_q} A^{\text{tree}}_{n+2}(p_1, \ldots p_n, q, -q)_R$$

Consider instead ret. amplitude $\langle 0| e^{iQ_1(x)} O_2 e^{-iQ_1(x)} |0\rangle$

$$\langle 0| [O_1(x), O_2(y)] |0\rangle \; \theta(x^0 - y^0)$$

$$\langle 0| e^{i\cdots} \; O_2(y) e^{-i\int^y \delta H(x)} |0\rangle$$

$$= \frac{-i}{p^2 - i\epsilon} \rightarrow \frac{-i}{p^2 - i\epsilon p^0}$$

$$A_n(p_1 \cdots p_n)_R = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3 2E_q} A_{n+2}(p_1, \ldots p_n, q, -q)_R$$

Consider instead ret. amplitude $\langle q| e^{iQ_1(x)} O_2 e^{-iQ_1(x)} |0\rangle$

$\langle 0| [O_1(x), O_2(y)] |0\rangle\, \theta(x - y)$

$\langle 0| e^{i\cdots}\ O_2(y)\, e^{-i\int^y \delta H(x)} |0\rangle$

$$= \frac{-i}{p^2 - i\epsilon} \rightarrow \frac{-i}{p^2 - i\epsilon p^0}$$

→ SUSY, Planar thy,

Tree off-shell

$$A_n^{1\text{-loop}}(p_1 \cdots p_n)_R = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3 2E_q} A^{tree}_{n+2}(p_1, \dots, p_n, q, -q)_{R'}$$

Consider instead ret. amplitude

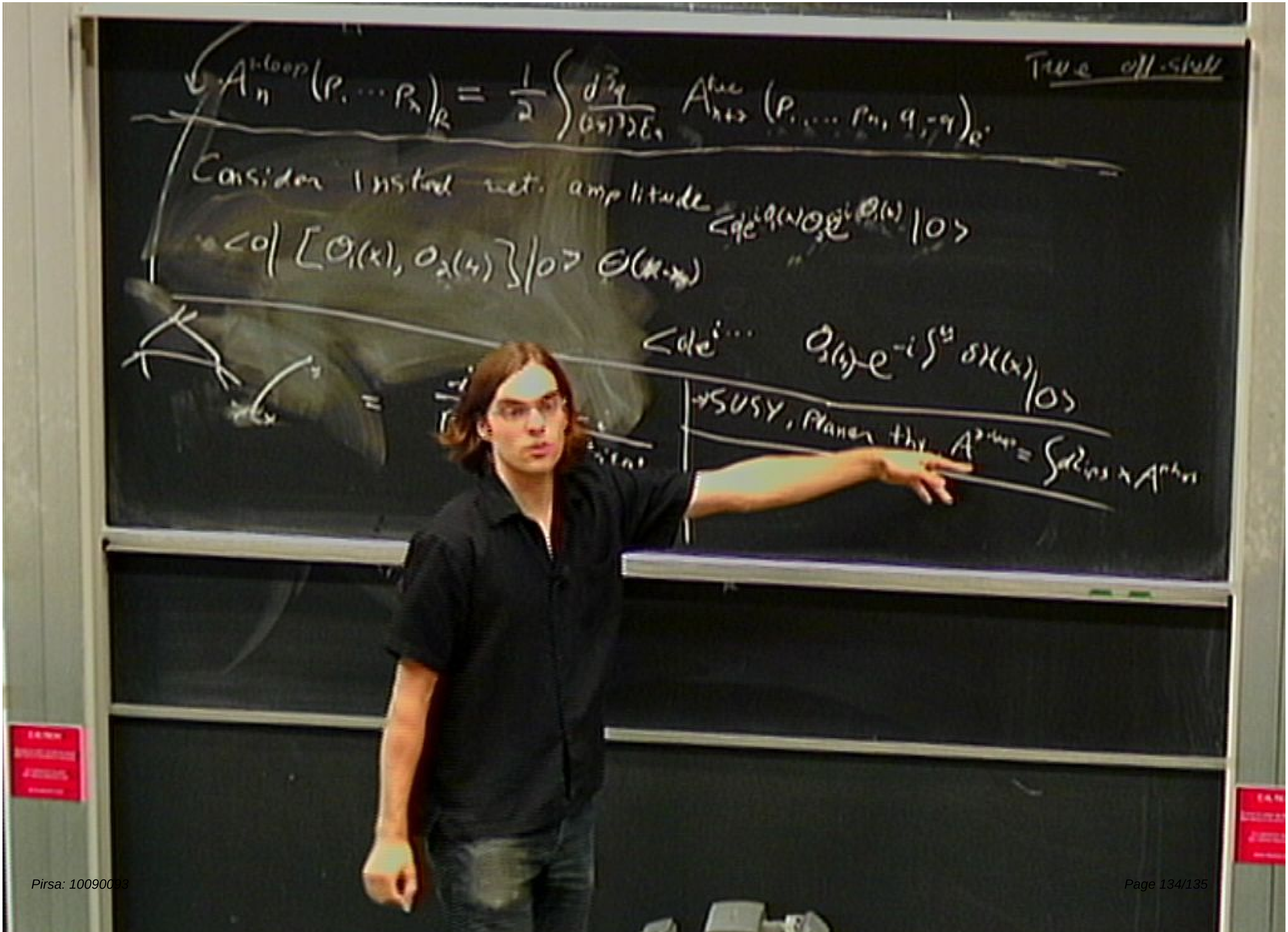
$$\langle 0| [O_1(x), O_2(y)] |0\rangle \,\Theta(x-y)$$

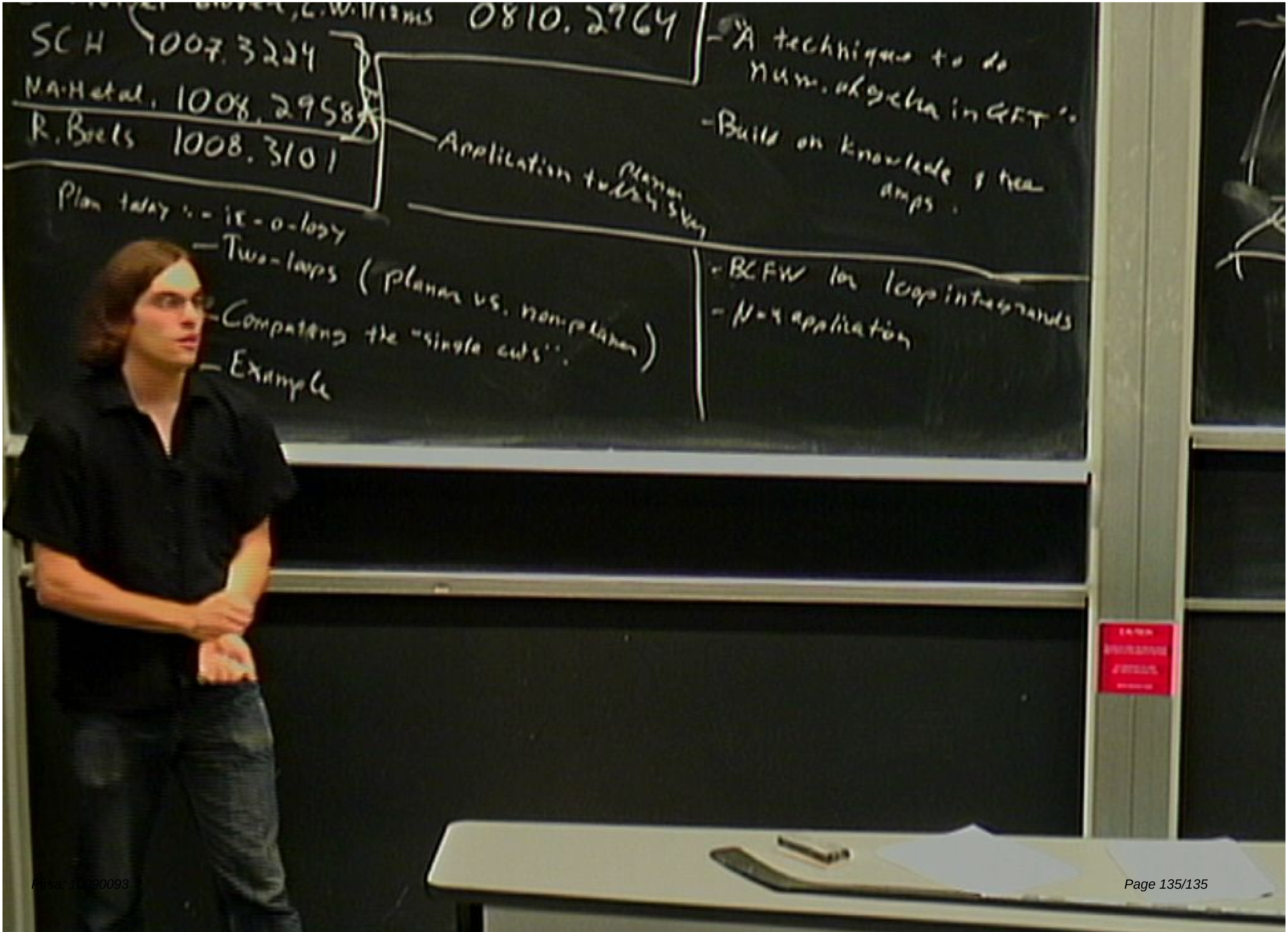
$$\langle 0| e^{i\cdots} O_2 e^{i\cdots} |0\rangle$$

$$\langle 0| e^{i\cdots} \; O_2(y) e^{-i\int^y \delta H(x)} |0\rangle$$

$$= \frac{-i}{p^2 - i\epsilon} \to \frac{-i}{p^2 - i\epsilon p^0}$$

+SUSY, Planar thy, $A^{2\text{-loop}} = \int \cdots A^{tree}$


True off-shell
Consider instead ret. amplitude
SUSY, Planar thy


C.Williams 0810.2764
SCH 1007.3224
N.A.H et al. 1008.2958
R.Boels 1008.3101
Application to
Plan today: - IR-o-logy
— Two-loops (planar vs. non-planar)
— Computing the "single cuts".
— Example
- "A technique to do num. algebra in QFT".
- Build on knowledge of tree amps.
- BCFW for loop integrands
- N=4 application