Title: An invitation to an invitation to causal sets

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Abstract: A brief review of some recent work on the causal set approach to quantum gravity. Causal sets are a discretisation of spacetime that allow the symmetries of GR to be preserved in the continuum approximation. One proposed application of causal sets is to use them as the histories in a quantum sum-over-histories, i.e. to construct a quantum theory of spacetime. It is expected by many that quantum gravity will introduce some kind of fuzziness uncertainty and perhaps discreteness into spacetime, and generic effects of this fuzziness are currently being sought. Applied as a model of discrete spacetime, causal sets can be used to construct simple phenomenological models which allow us to understand some of the consequences of this general expectation.


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Discreteness without symmetry breaking

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Discovering the Discrete Universe

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Comments: 24 pages, 4 figures. Based on a proceedings article for the "Foundations of Space and Time" conference.

Subjects: **General Relativity and Quantum Cosmology (gr-qc)**

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Joe Henson: Causal sets

Back to the rough ground

Example: Lord Rayleigh in Darjeeling

$$n = \frac{32\pi^3(\mu - 1)^2}{3\lambda^4\beta},$$

where there are *n* molecules per m3, λ is the wavelength of the light, μ is the refractive index of air and $1/\beta$ is the distance over which the light is attenuated by $1/e$.

By noting that Everest was visible at a distance of 160 km, Rayleigh estimated 1/β to be 160km and thus obtained a value for Avagadro's constant as around 4 x 10^23, from casual observation.

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100 years later, it is not the **atomicity of matter** which is in question, but the **atomicity of spacetime.** Again, we are equipped with a discrete kinematical picture, but lack a definitive dynamics. Can we make progress by proceeding in the spirit of Rayleigh's calculation?

Joe Henson: Causal sets

Questions for this talk

- What
- Why
- How
- Where

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- Where

… can we look for signals of spacetime discreteness?

Beyond the continuum

- Common idea: Lorentzian manifolds may not be a good description of spacetime near the Planck scale.
- Several “clues” from current theory (infinities in GR, QFT, BH entropy) suggest that the replacement should be discrete. But what?

An intriguing result

Given the causal structure and the conformal factor of a manifold with Lorentzian metric, one can recover the dimension, differential structure, topology and metric of that manifold.

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Causal structure + volumes = geometry

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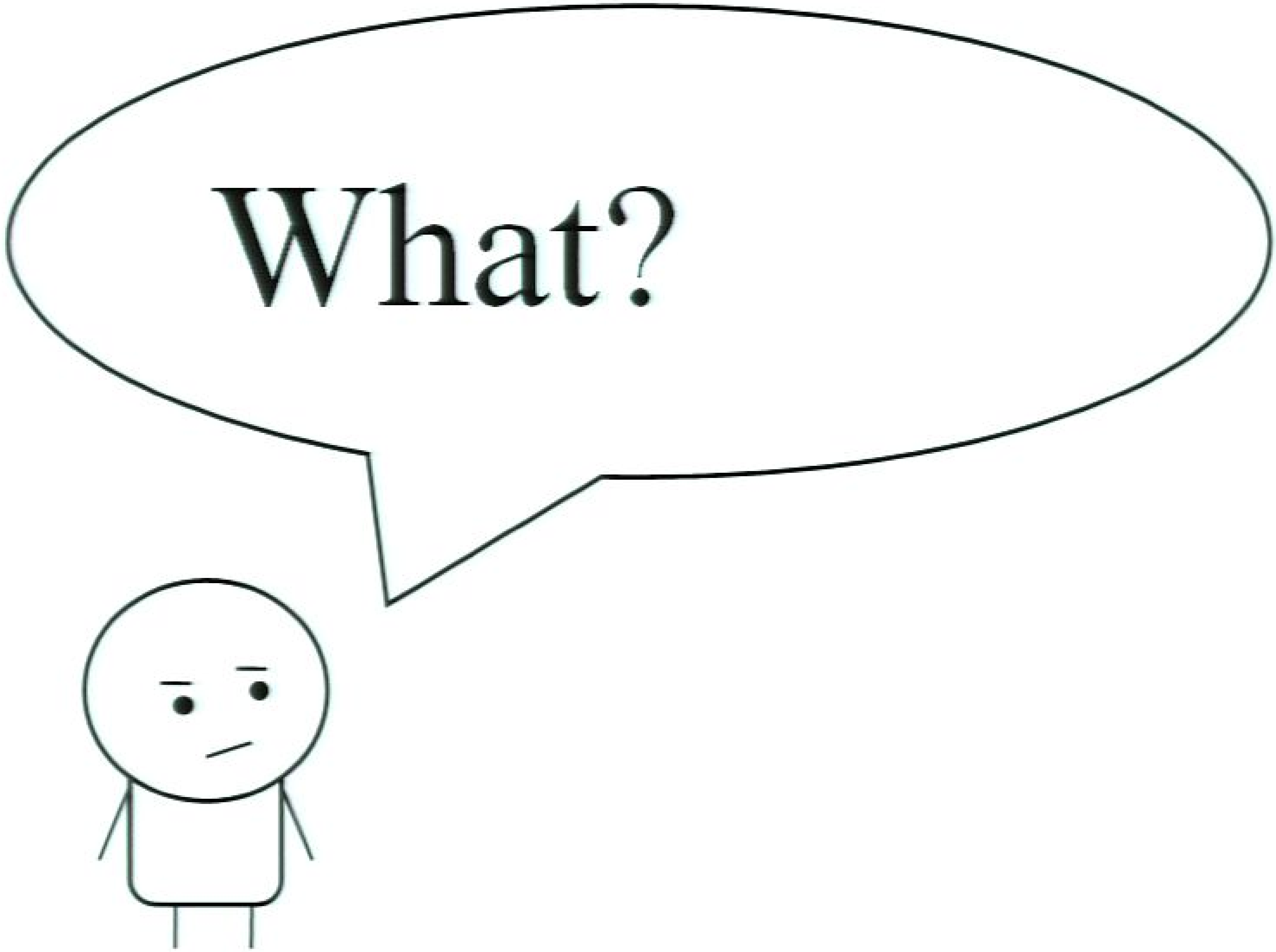

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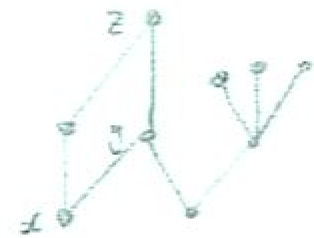
Causal sets

The causal order of a spacetime is a partial order $\prec$ on the set of points C, meaning:

(i) transitivity: $(\forall x, y, z \in C)(x \prec y \prec z \Rightarrow x \prec z)$

(ii) irreflexivity: $(\forall x \in C)(x \not\prec x)$



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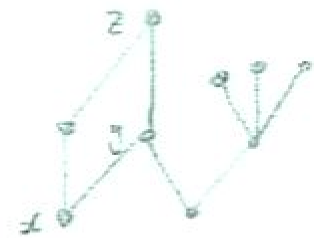

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Order ⇔ Causal structure

Number ⇔ Volume

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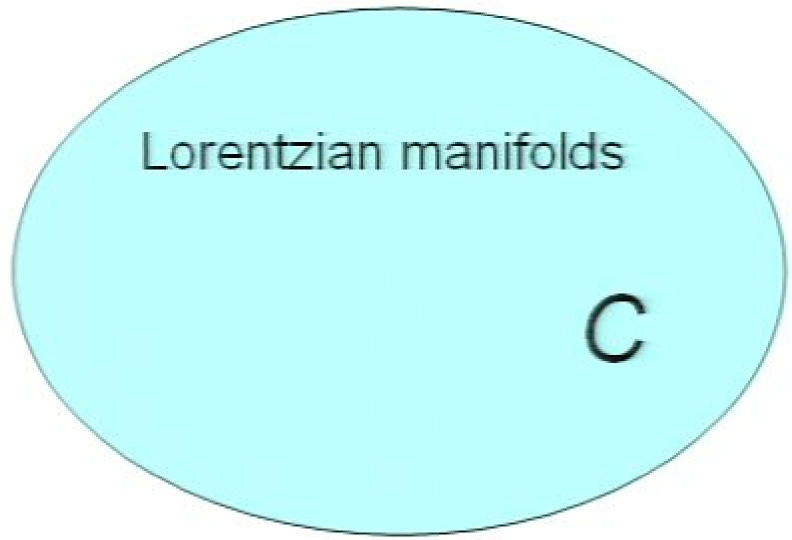

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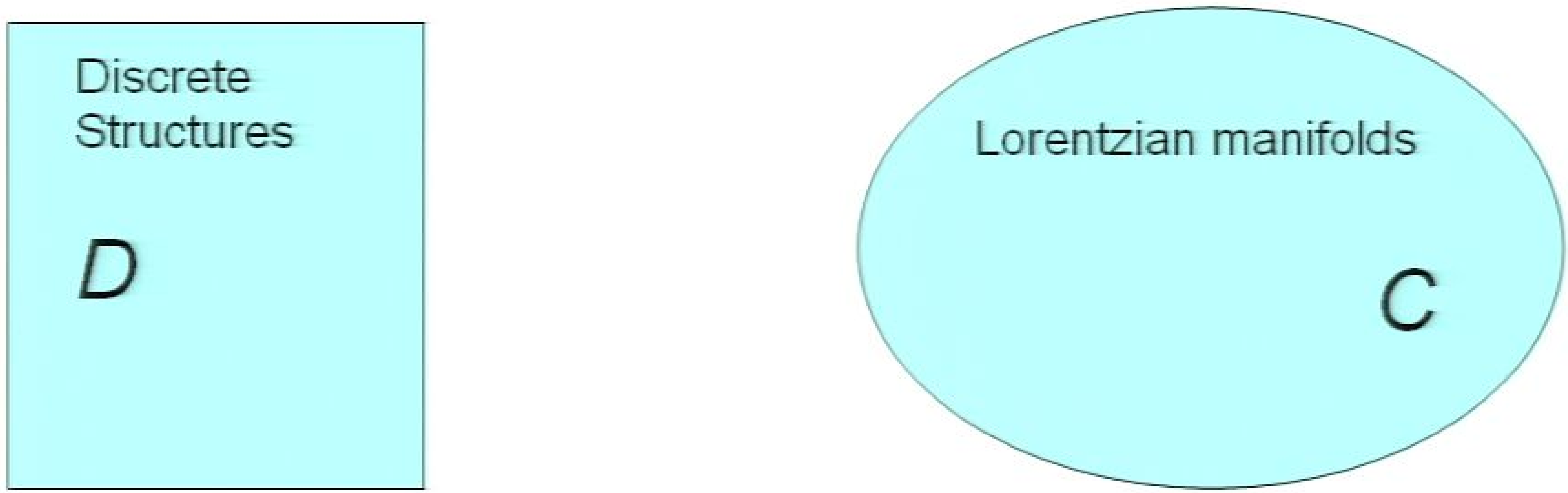

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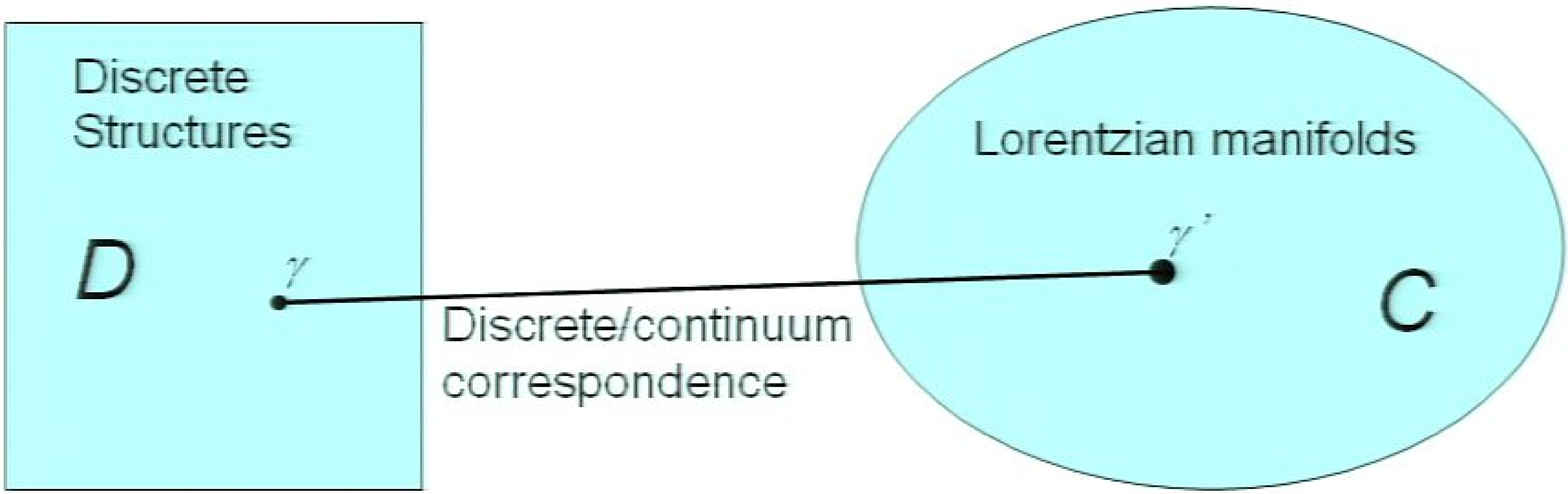

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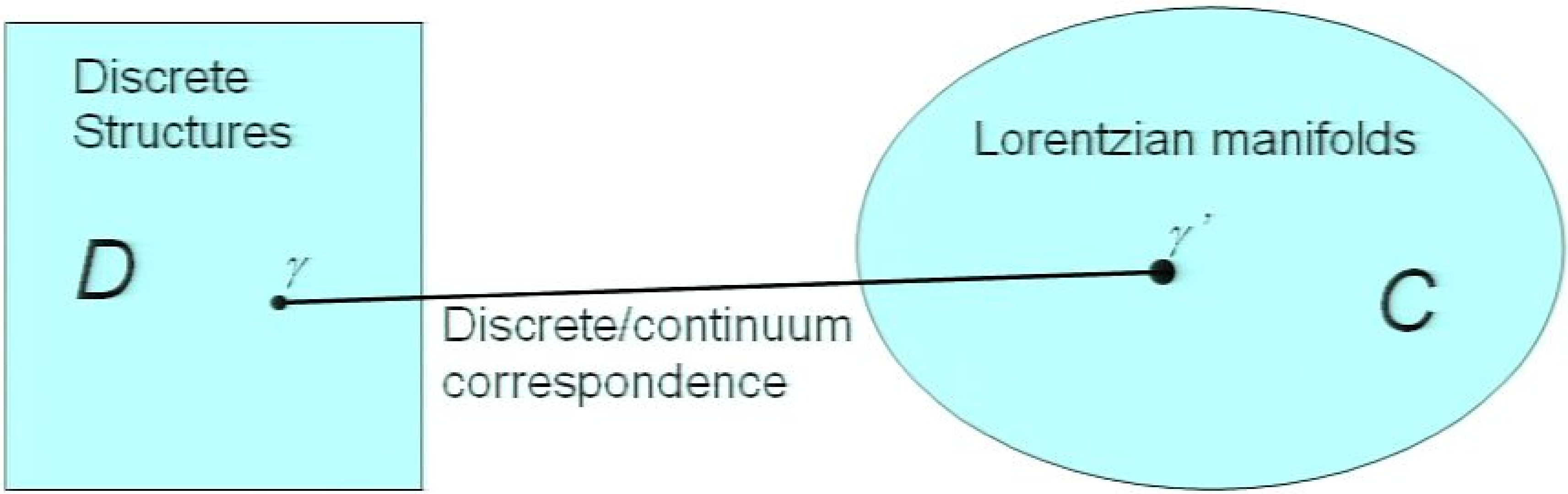
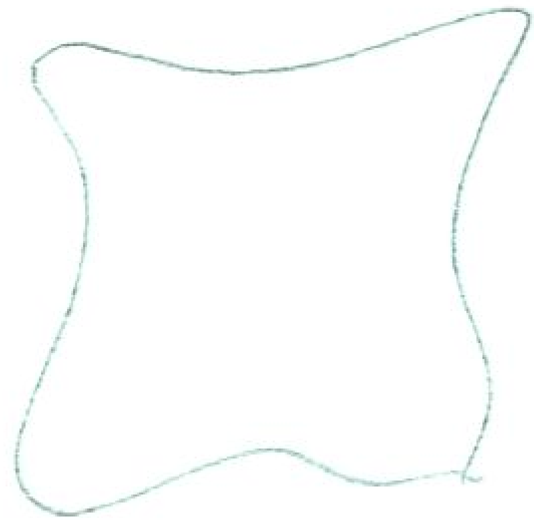


Binary relation $\gamma \sim \gamma'$

$\sim$ must be surjective;
If $\gamma \sim \gamma_1'$ and $\gamma \sim \gamma_2'$ then γ_1' and γ_2' must be physically indistinguishable.

Spacetime as approximation

We must recover the spacetimes of GR as approximations to some of these causal sets. When does a spacetime (M,g) approximate to (C, $\prec$)?



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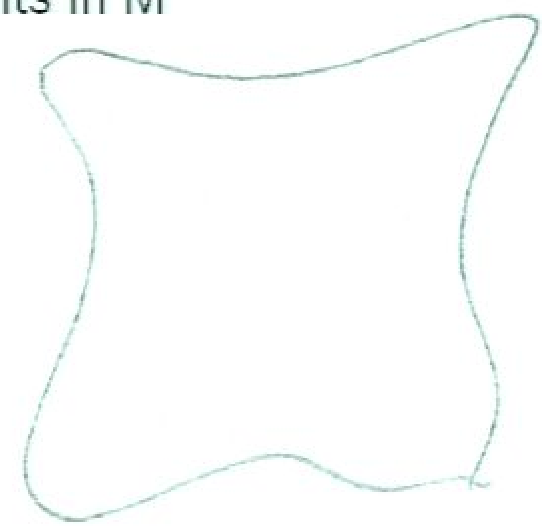

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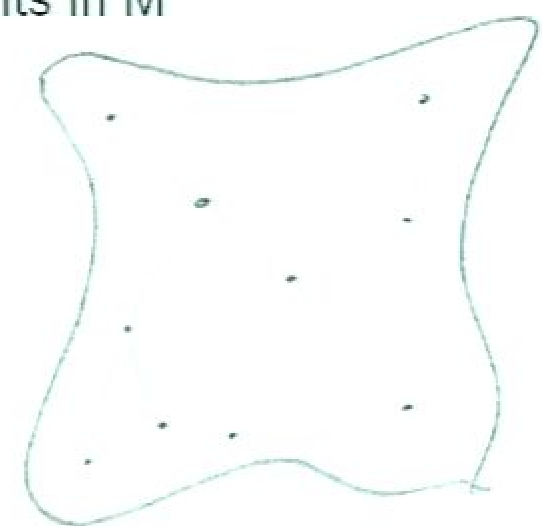

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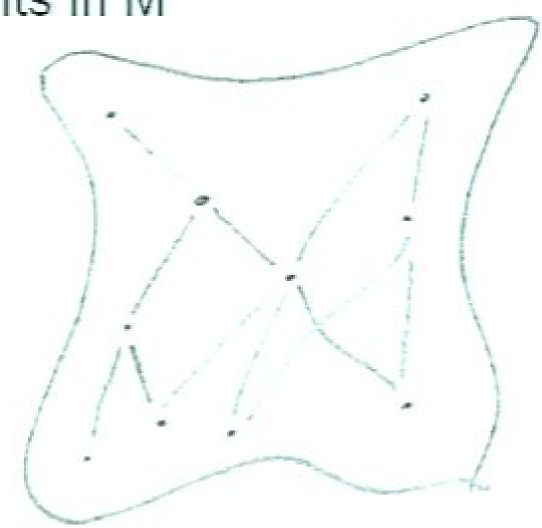

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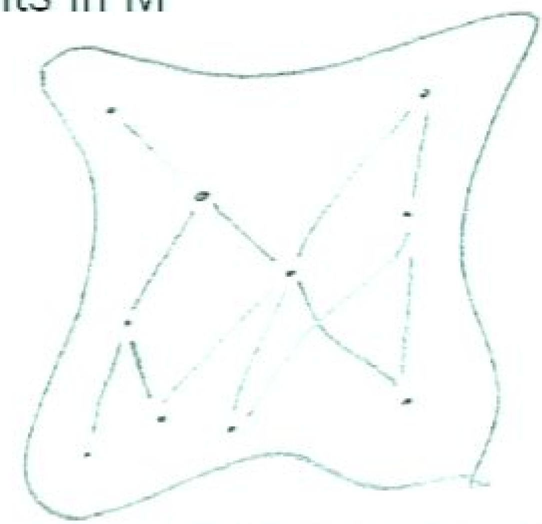

It is "**faithfully embeddable**" if that set of points could have arisen, with relatively high probability, from "sprinkling":

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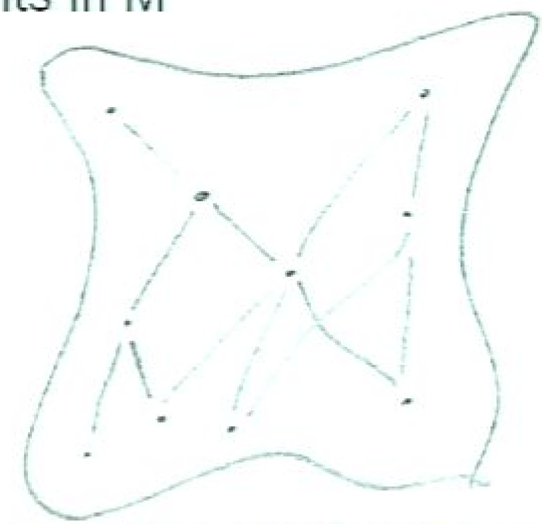

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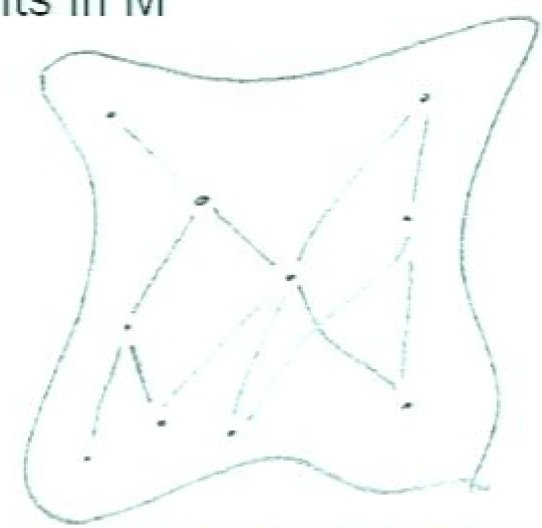

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This defines the discrete/continuum approximation.

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The causal set program

- Kinematics
  - Find methods to recover geometry from causal sets
  - Prove the fundamental conjecture
- Dynamics
  - Find a Sum-Over-Histories QM theory with causal sets as the histories, as a candidate QG theory
  - Use "principle approach", or generalize the gravitational path integral
- Phenomenology
  - Use in the spirit of Rayleigh's calculation as a model of discrete spacetime
  - Consistent with observation? New predictions?

Recovering Geometry

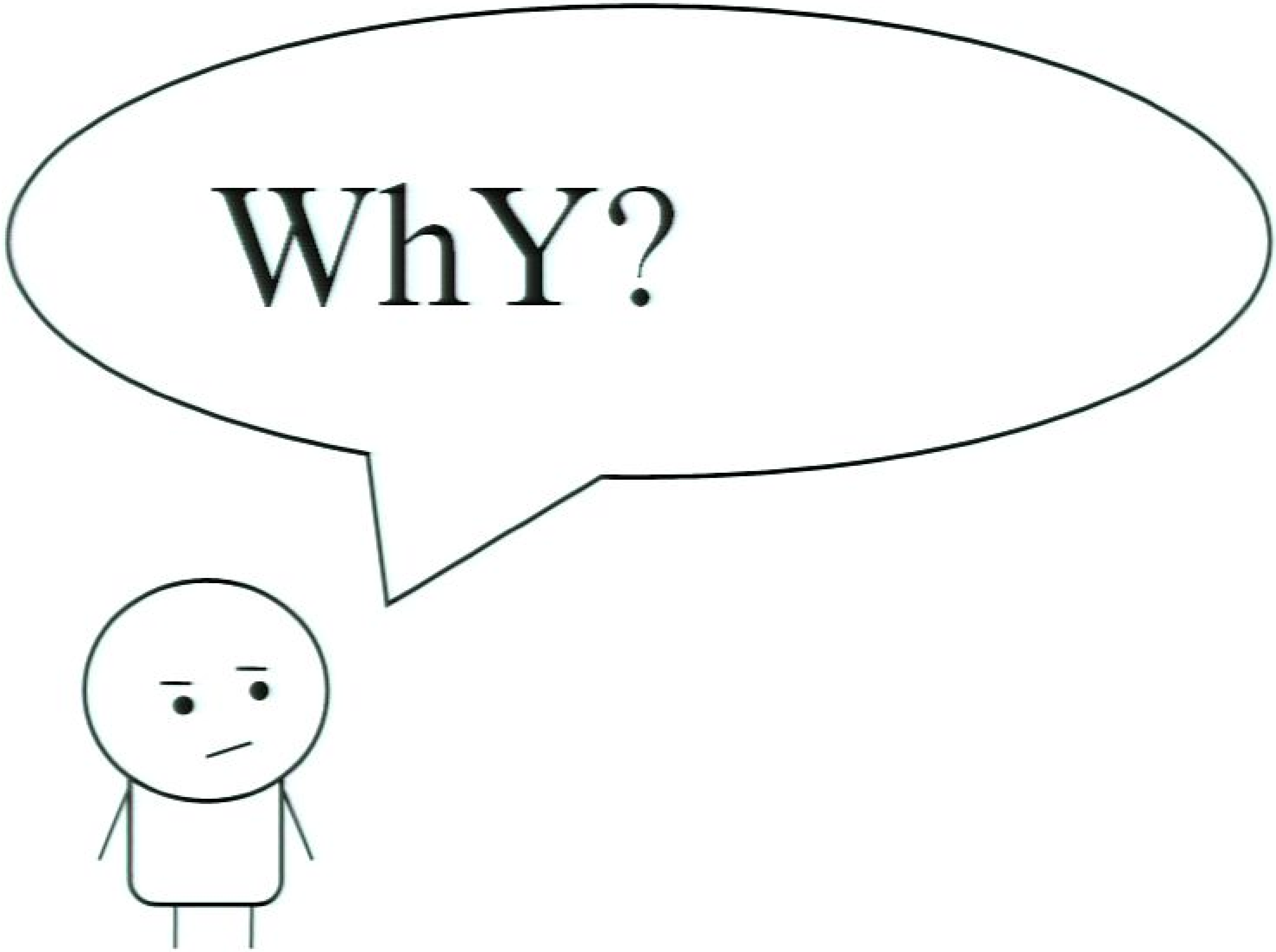
We know how to recover continuum geometry in principle; but given a causal set, how do we work out the properties of its continuum approximation (if it has one)?

An example: recovering dimension as a function of the causal set. If $(M,g) \sim (C, \prec)$, what dimension does M have?

The fraction of pairs of points in M that are causally related is a function of the dimension. This relationship can be reversed and applied to the causal set to make $D\,(C, \prec)$, the "Myrheim-Meyer" dimension.

"Manifoldlike" causal sets have integer valued, matching dimension estimators.

Similar results for lengths, topology...



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Lorentz invariance?

A fertile ground for phenomenology, and a problem for most discrete structures. Does discreteness imply Lorentz violation?

We can only talk about continuum symmetries when there is a continuum! They have no meaning at the fundamental level but only when there is a continuum approximation. Consider the case of Minkowski space.

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To ensure Lorentz invariance our discrete/continuum correspondence, and our criterion of "physical indistinguishablility", must be invariant.

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The trouble with Lattices

What if D was some set of lattices? How do we compare continuum geometry with a lattice?

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Consider Minkowski space and a regular lattice. There are many such embeddings related by symmetries; each one picks out a direction.

Large regions contain no embedded points; no symmetric discrete/continuum correspondence known.

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2) Causal information is Lorentz invariant.

3) *Outcomes* of the sprinkling process do not pick a direction.

Theorem: There is no "equivariant" map between outcomes of sprinklings and directions in Minkowski.

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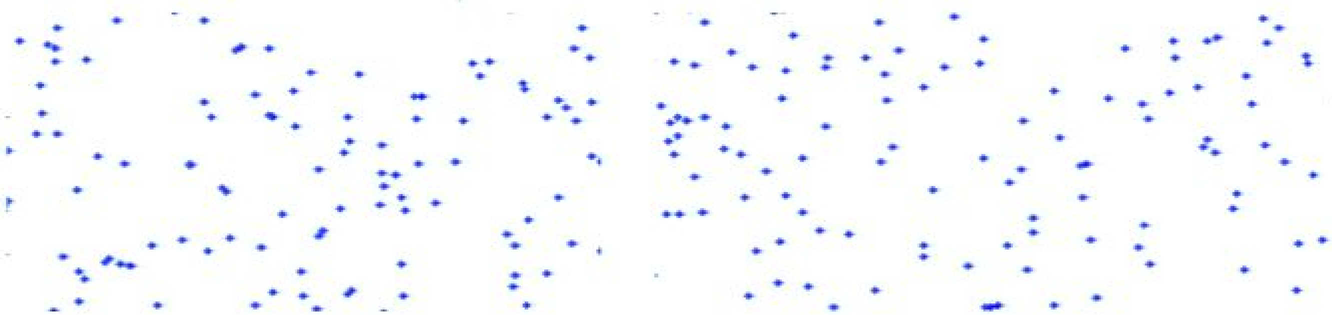
Lorentz invariance!

Random discreteness saves symmetry

E.g. gas, liquid, glass.

The causal set gives an equally good approximation of all causal intervals;



not so a regular lattice.

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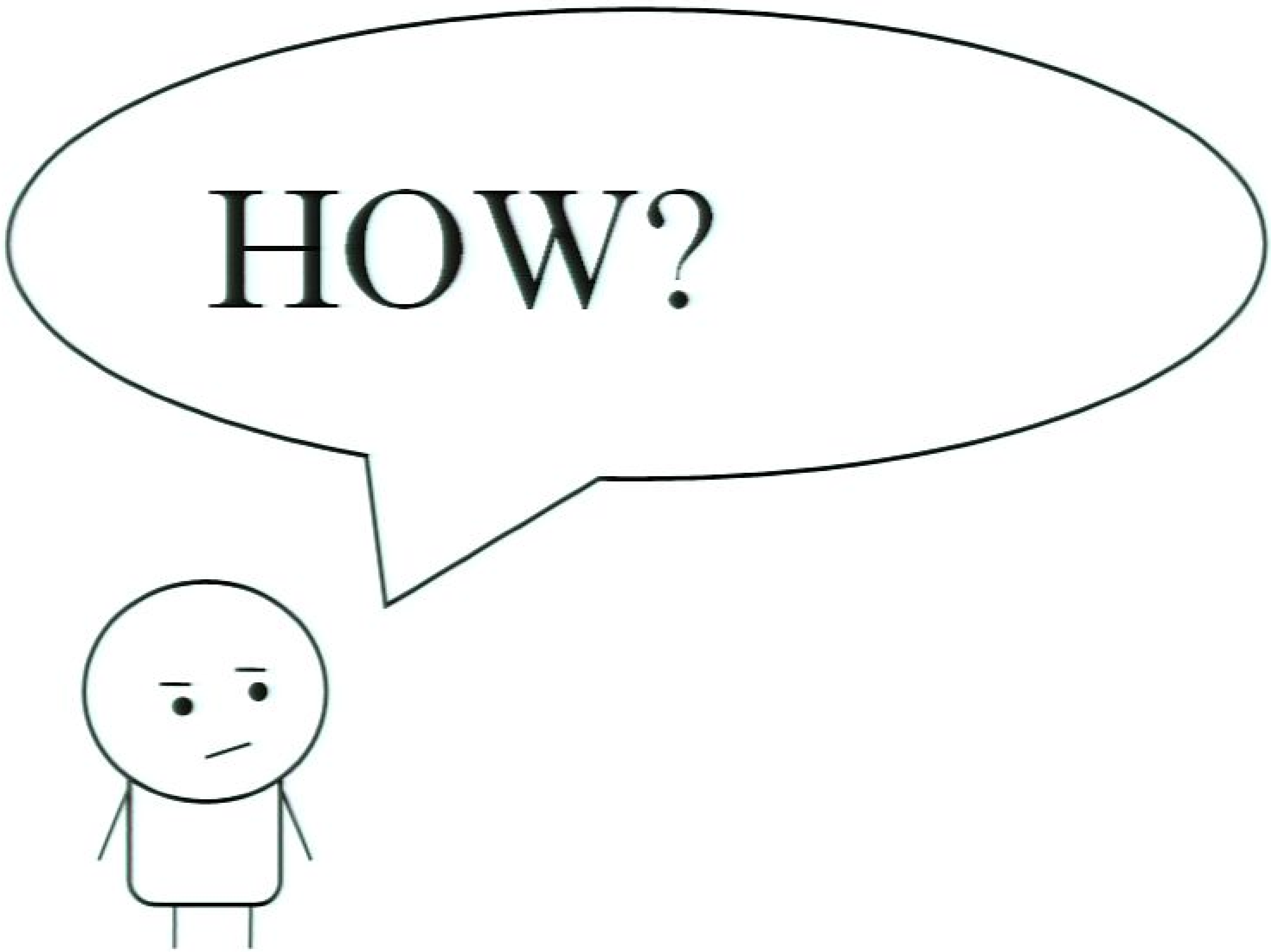

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Recovering spacetime

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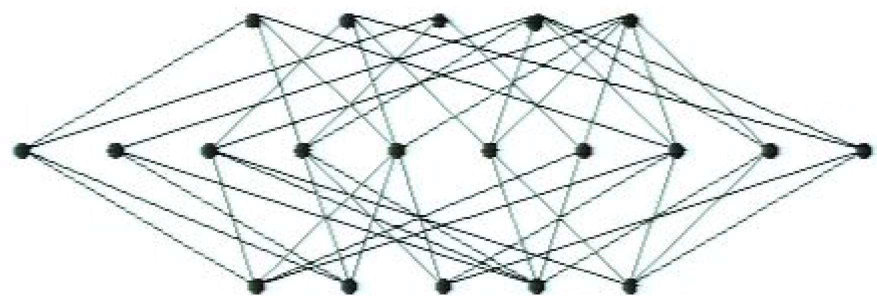

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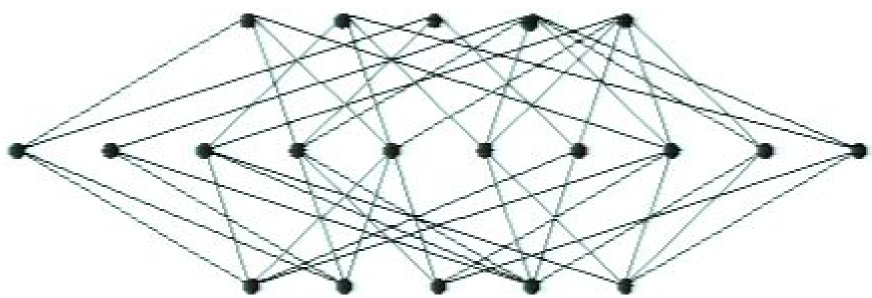


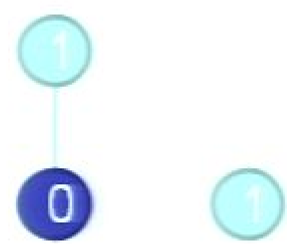
If we consider a statistical sum over causal sets, who could manifoldlike ones dominate, if the number of them only grows exponentially? With a local action, this seems impossible.

But causal sets are not local! E.g. exp(#links) grows faster than exponentially in N for manifoldlike causal sets.

Classical Sequential Growth

A quantum SOH can be seen as a generalization of a stochastic theory; we can test the principle approach in a stochastic setting.

Sequential growth: the causal set, starting from one element, is “grown” by randomly adding elements to the future (or spacelike) to existing elements:



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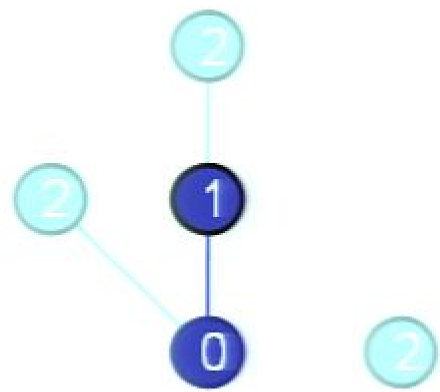

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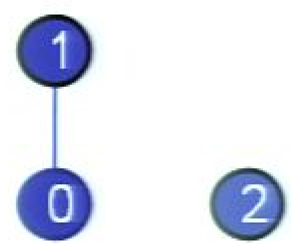

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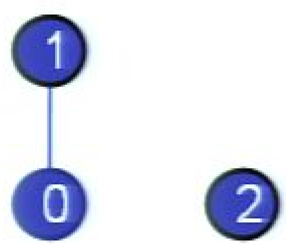
Defining all the "transition probabilities" gives a probability measure on infinite causal sets.



Constrain these probabilities by physical principles:
Causality
General covariance

One parameter remains per element.

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Classical Sequential Growth

- Solution to an analog of the "problem of time": the set of covariant "questions" can be defined and interpreted.

- Interesting "bouncing cosmologies" that select parameters.

- some manifoldlike properties, but unlikely that CSG reproduces manifoldlike causal sets.

- Some math tools developed will be relevant to Quantum case.

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Where?

Fields on causal sets

Is this discreteness consistent with observation?

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The problem is also relevant to dynamics. How do we recover effective locality from causal sets?

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Scalar fields on Minkowski

We need to make some **approximation to the local, Lorentz invariant D'Alembertian operator**. A lattice provides an easy way to recover locality, but breaks Lorentz invariance. On the other hand, the Lorentz invariant causal set discretisation makes it more difficult to recover locality.

On a light-cone lattice:

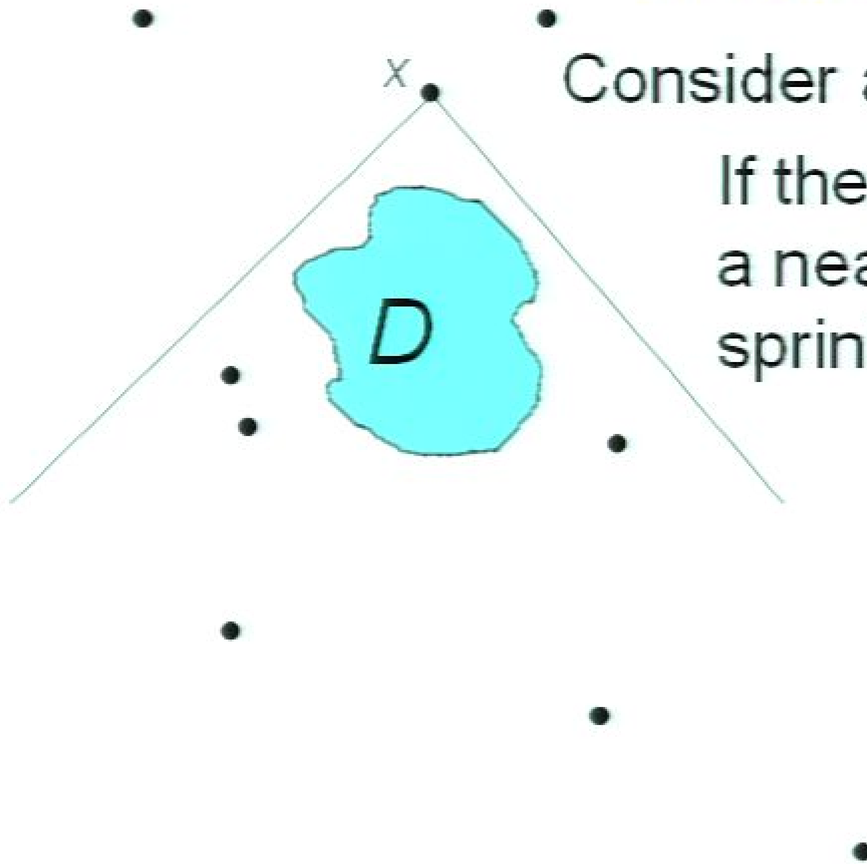
$$\Box\phi(u,v) = \frac{\partial^2\phi(u,v)}{\partial u \partial v} \approx \frac{\phi(u,v) - \phi(u-a,v) - \phi(u,v-a) + \phi(u-a,v-a)}{a^2}.$$

A weighted sum of field values at a finite set of "near neighbours".

But in a truly Lorentzian discretisation, there can be no such finite set.

E.g.: how many "links" to a given element are there in a sprinkling of Minkowski?

Lorentz invariance or locality: a choice



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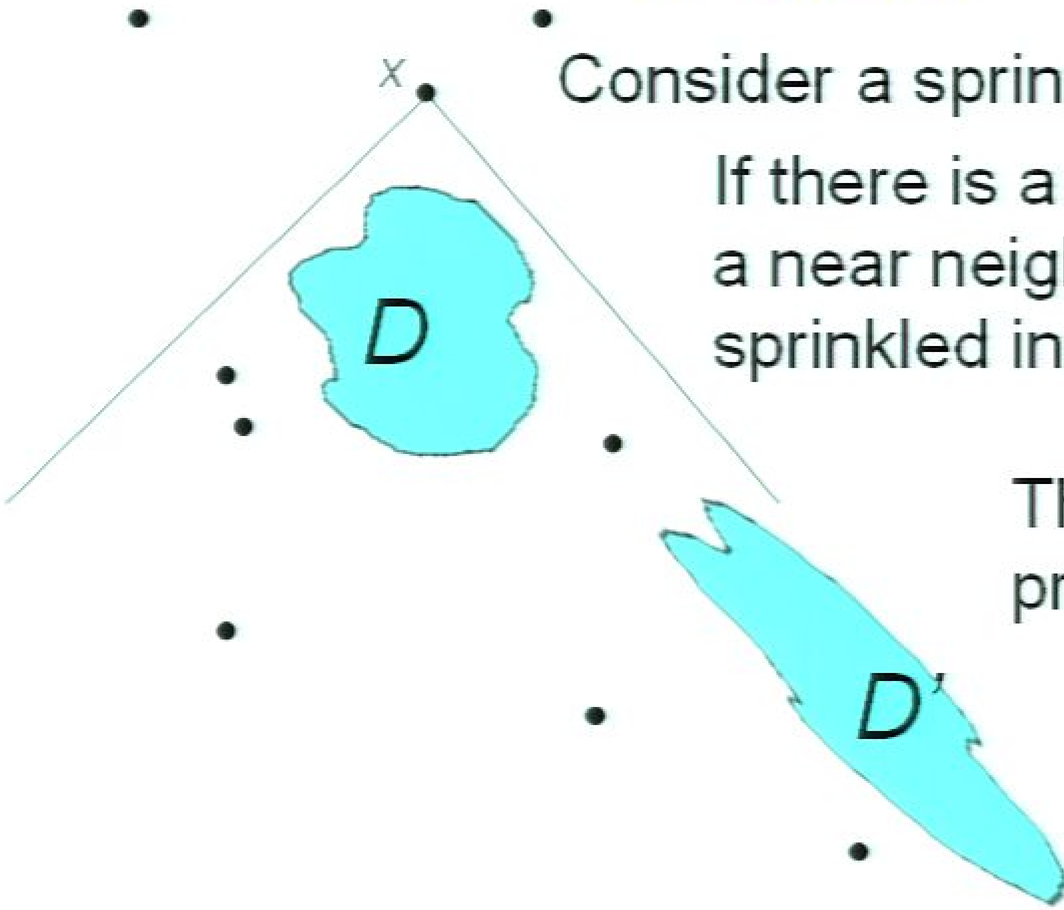

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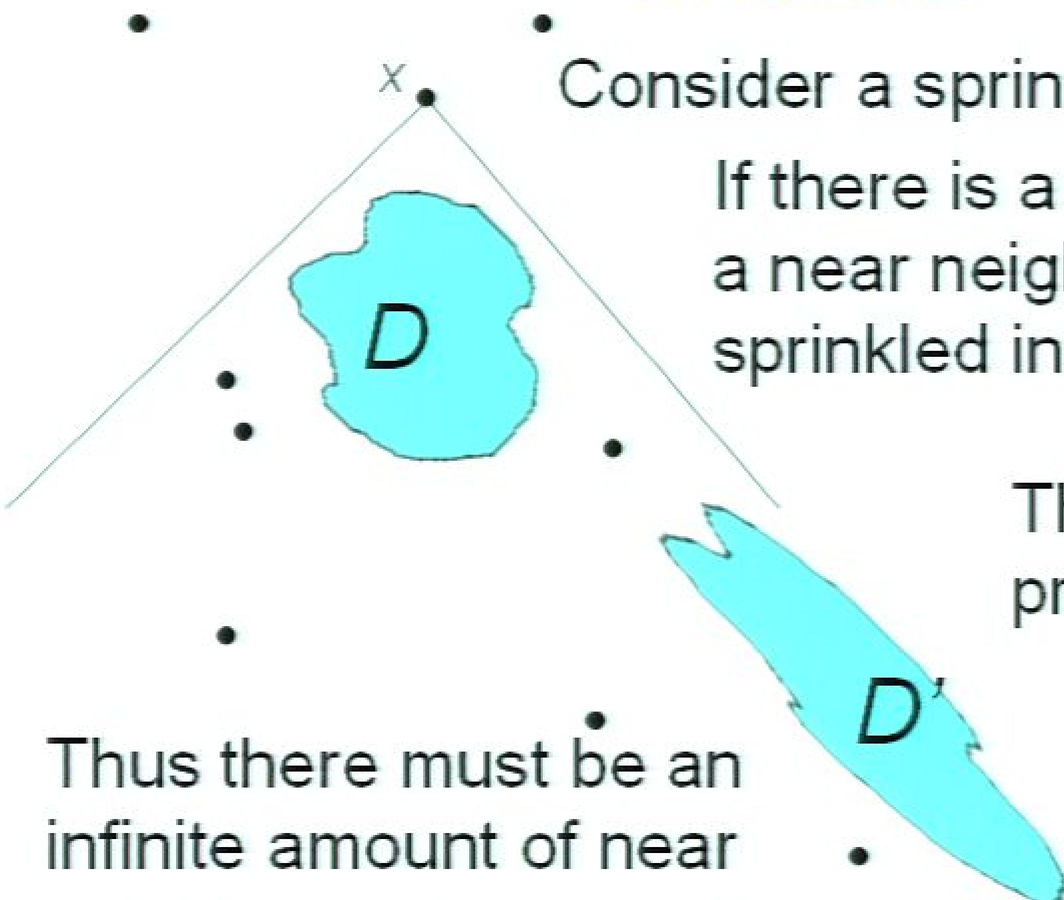

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Thus there must be an infinite amount of near neighbours, however they are defined.

Approximating Green's functions

Equivalently to the d'Alembertian, the field theory can be defined by the Green's function of the d'Alembertian: $\Box G(x, y) = \delta(x - y)$

In 4-dimensional Minkowski space, the Retarded Greens' function is given by

$$G(x, y) = \begin{cases} \frac{1}{2\pi}\delta(|x-y|^2) & \text{if } y \text{ is in the causal future of } x \\ 0 & \text{otherwise} \end{cases}$$
$$= \frac{1}{4\pi r}\delta(y^0 - x^0 - r),$$

A delta function on the future light-cone of *x*.

This function is defined using purely causal information.

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Approximating Green's functions

We have seen that the links from one element "hug the light-cone". Consider following function on pairs of causal set elements:

$$L(e_i, e_j) = \begin{cases} 1 & \text{if } e_i \prec e_j \text{ and } |I(x,y)| = 0 \\ 0 & \text{otherwise} \end{cases}$$

In the limit of dense sprinkling (with suitable normalisation) this function goes to the delta-function on the future light-cone, *G(x,y)*.

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$$L(e_i, e_j) = \begin{cases} 1 & \text{if } e_i \prec e_j \text{ and } |I(x,y)| = 0 \\ 0 & \text{otherwise} \end{cases}$$

In the limit of dense sprinkling (with suitable normalisation) this function goes to the delta-function on the future light-cone, *G(x,y)*.

This can be used to define the propagation of a scalar field on the causal set:

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$$\Box G(x,y) = \delta(x-y) \quad \Longrightarrow \quad \Box_{ij} L_{jk} = \delta_{ik}$$

This method has some problems, but can be used to give a model of a scalar field propagating from source to detector. This helps us to see whether causal set discreteness is consistent with the coherence of light travelling over long distances, and gives an example of Lorentz invariant discrete dynamics.

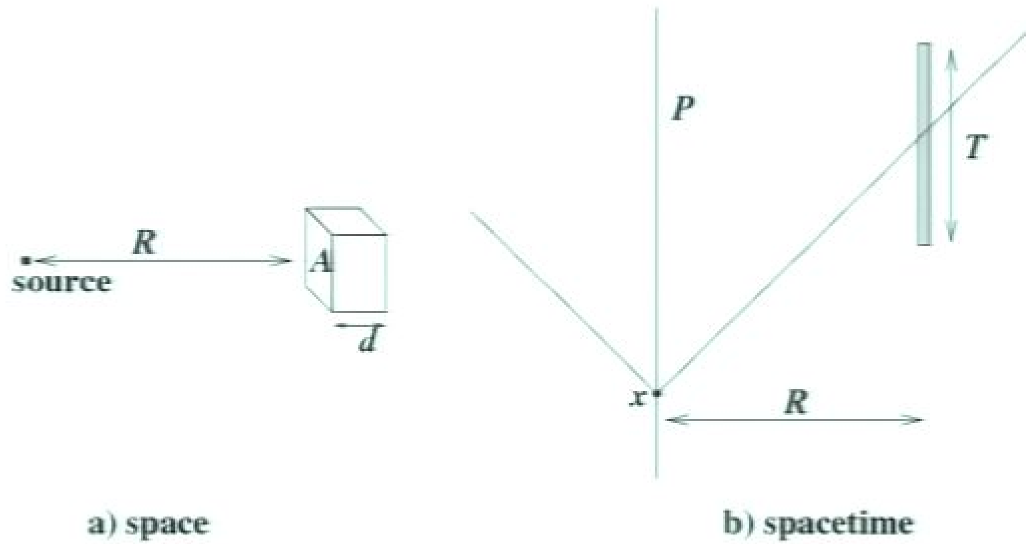
A Model of Propagation

We can model propagation from a small source to a distant detector and compare the standard model with the causal set model. We define the signal *F* as follows:

$$F = \int_{\mathcal{D}} d^4y\, \phi(y)$$

$$\phi(y) = q \int_P ds\, G(x(s), y),$$

$$F \approx \frac{q}{4\pi R} T A d.$$



In the continuum we are finding the measure of the set of pairs of points (one in source, one in detector) that are null related.

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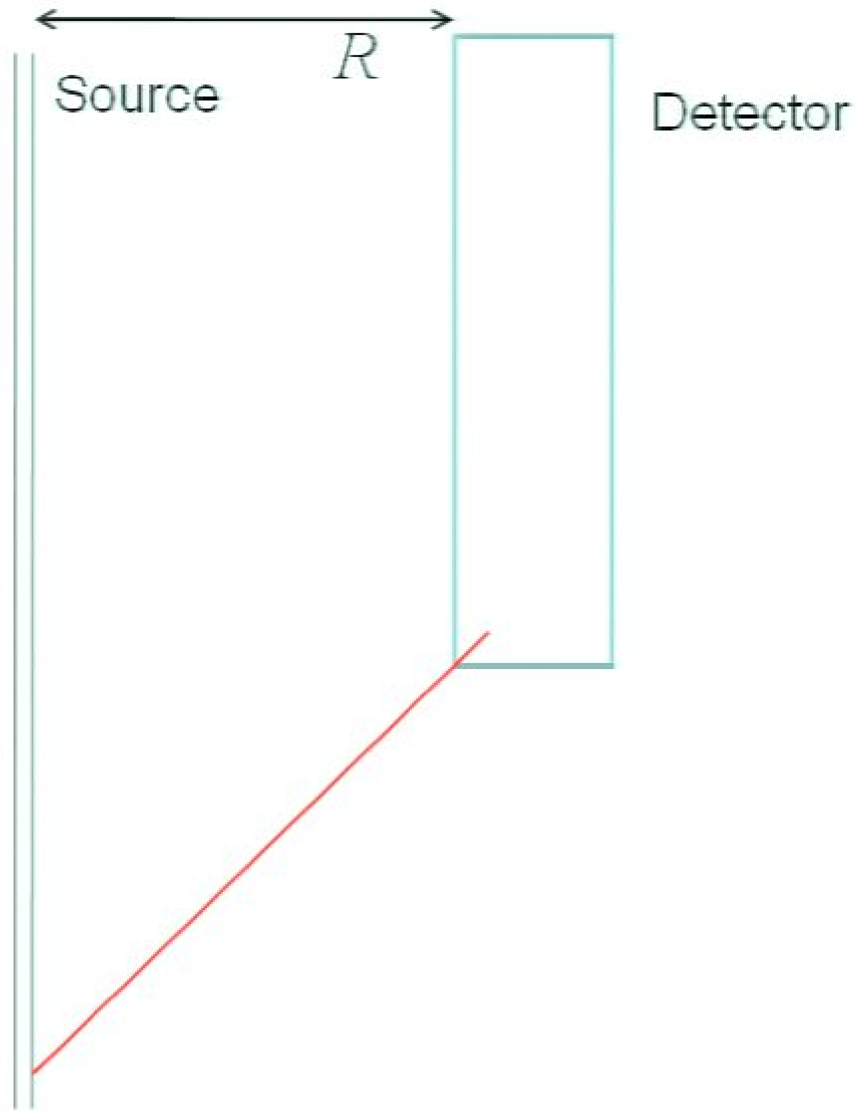

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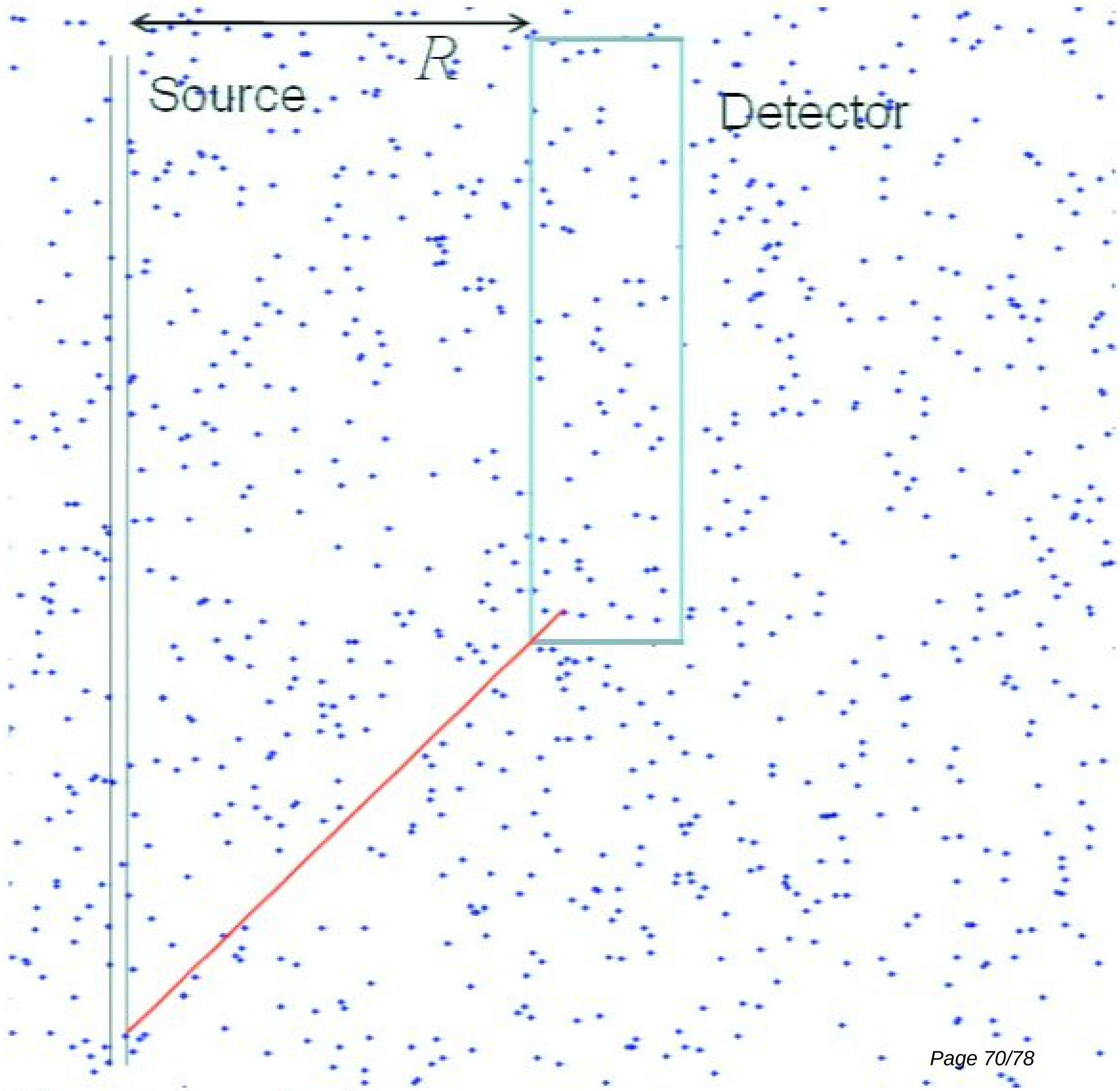

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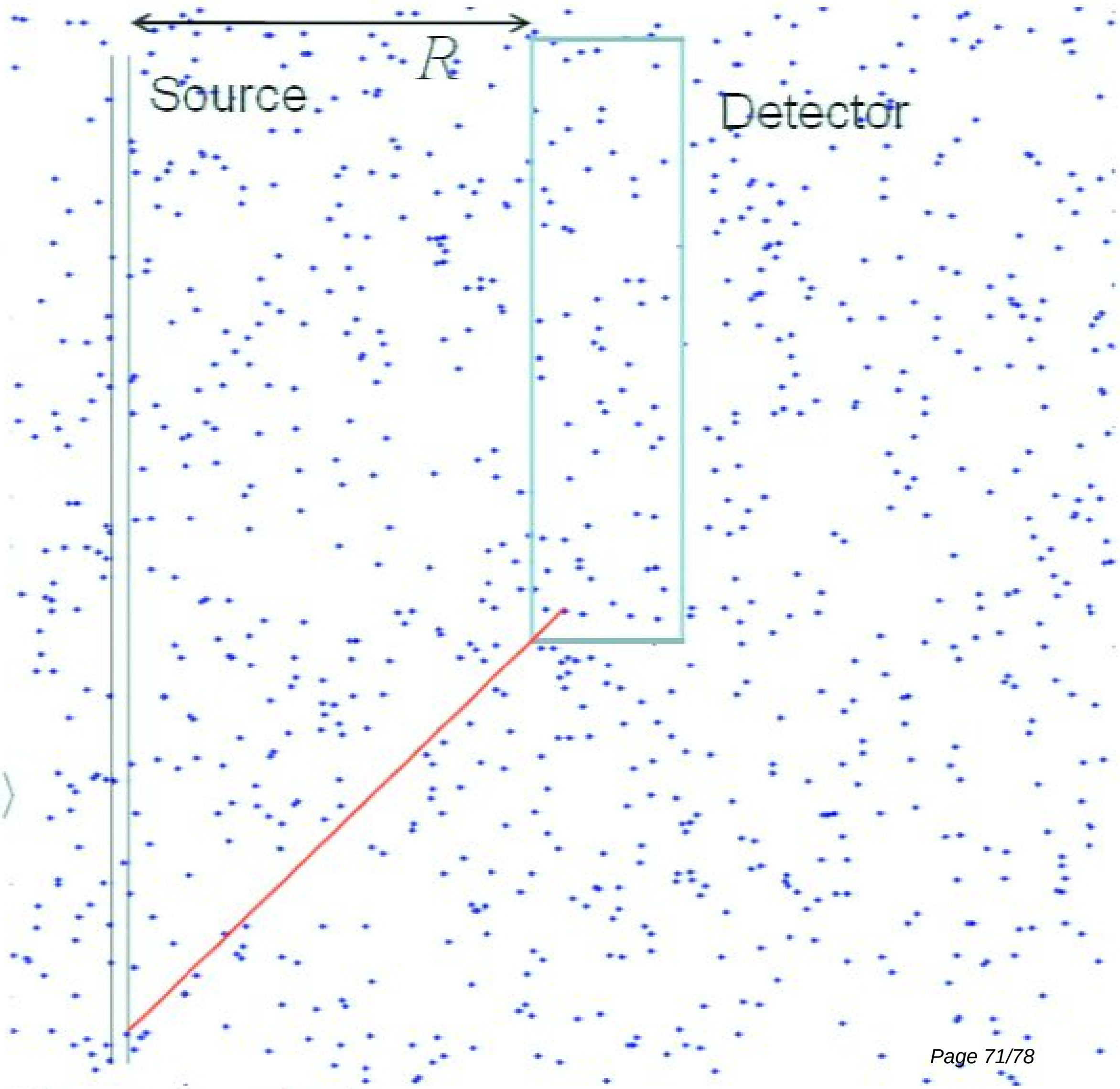
Discrete version:

$$\langle \tilde{F} \rangle = \sum_{e_j^* \in \tilde{\mathcal{D}}} \langle \tilde{\phi}(j) \rangle$$

$$\langle \tilde{\phi}(j) \rangle = q \sum_{e_i \in \tilde{P}} \langle L(e_i, e_j^*) \rangle$$

$$\langle \tilde{F} \rangle \approx \frac{q}{4\pi R} T A d.$$



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A Model of Propagation

In the causal set case, to find the detector signal we counted the number of links between the source and detector region for a typical causal set approximating to Minkowski space. The result is the same, with negligible corrections.

The signal varies with the strength of the source just as in the continuum. No significant random or systematic effects come in, *e.g.* to change the phase of a propagating wave.

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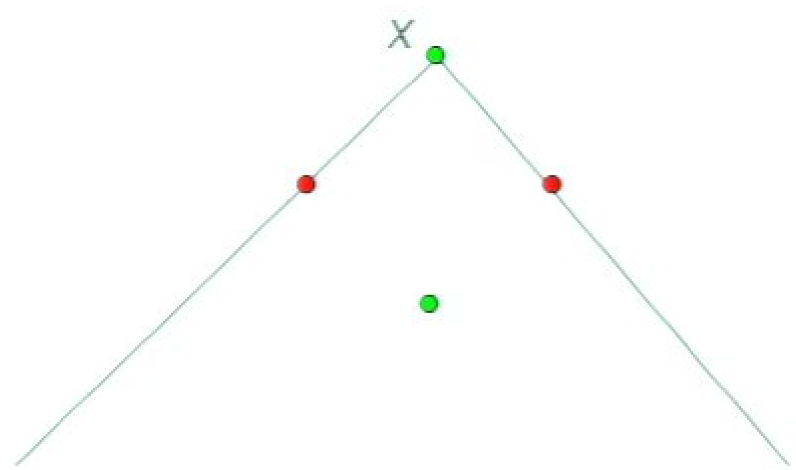
Spacetime "fluctuations" $\not\rightarrow$ loss of coherence

A non-local, causal “d’Alembertian”

- The Green’s function method is limited to flat space, and seems to give rise to unacceptable random fluctuations in the value of the d’Alembertian.

Another idea: in the lattice we had a weighted sum of the field values at the nearest neighbours.



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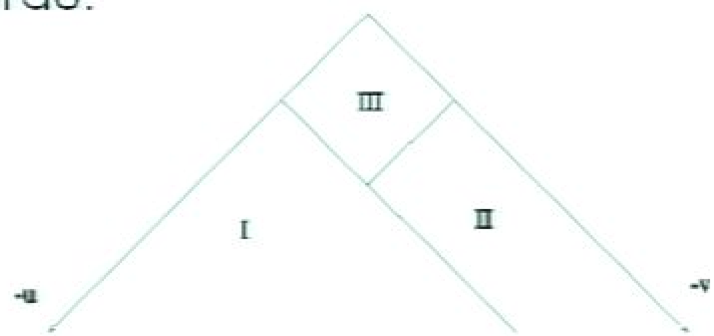
A potential problem

Let us assume that the field is of **compact support**, and **slowly varying in some frame** (for now), and work in that frame. We are in 2D, so in light-cone co-ords:



$$I(k) = \int_{-\infty}^{0} du \int_{-\infty}^{0} dv \, e^{-kuv} \, \phi(u,v) \, .$$

Work in units where ***k* >> 1.**

$$I_1(k) =$$

$$\int_{-\infty}^{-1} du \int_{-\infty}^{0} dv \, e^{-kuv} \left(\phi(u,0) + \frac{\partial \phi(u,v)}{\partial v} \bigg|_{v=0} v + ... \right)$$

$$= \int_{-\infty}^{-1} \frac{du}{-ku} \phi(u,0) + O(\frac{1}{k^2}) + O(e^{-k}) \, . \quad (10)$$

Using the previous equations, which get rid of terms of order k^ -1 and k^-2, we see that the **contribution to *J*(*k*) is small** -- of order k^-3. This shows how the above relations help to regain approximate locality. Thus the only significant contribution is from region 3:

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A non-local, causal "d'Alembertian"

This approximation of the d'Alembertian can easily be discretised on a causal set, and is interesting in its own right.

Properties: it is **Lorentz-invariant**, **causal**, but **non-local** (in time).

These ideas can be extended to 4D.

Because on the non-locality in time it is challenging to do QFT using this approximation.

The next step is to look for phenomenology of this non-local d'Alembertian.

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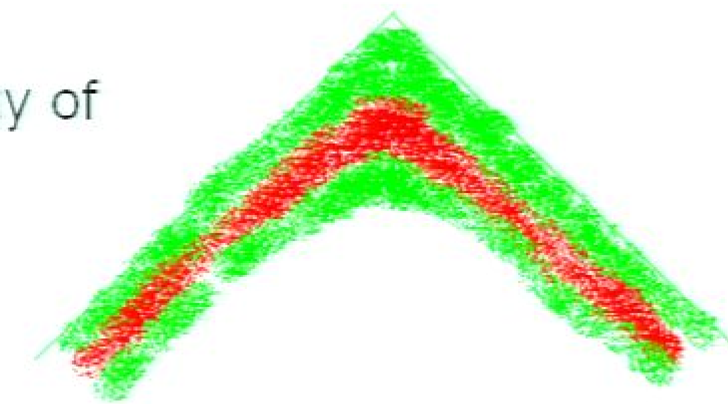

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Conclusions

- The causal set offers a discretisation of spacetime consistent with the symmetries we observe
- Fields can be described in causal sets, if one allows causal non-locality.
- Lorentz violation and loss of coherence of light from distant sources can be avoided in discrete spacetime models.

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