Title: Introduction to Complex Analysis/Computer Skills - Lecture 8B

Date: Sep 09, 2010 10:30 AM

URL: http://pirsa.org/10090088

Abstract:

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Untitled-25

In[15]:=
```
Hs = Table[h[L], {L, 2, max}] // N;
```

In[16]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[16]=
```
{8.987602, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

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(*
Eigenvalues[Hs[[3]],1]
  (tb=Table[{L,Eigenvalues[Hs[[L-1]],1][[1]]},{L,2,max}])//AbsoluteTiming
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In[17]:=
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Hs2 = SparseArray /@ Hs;
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  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[22]:=
```
Eigenvalues[Hs[[3]]]
```

Out[22]=
```
{-0.404006, -0.239277, -0.239277, -0.239277,
 0.1875, 0.1875, 0.1875, 0.1875, 0.1875, 0.114277,
 0.114277, 0.114277, -0.0625, -0.0625, -0.0625, 0.0290064}
```

```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
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```
Eigenvalues[Hs[[3]],1]
```

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Out[23]=
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{-0.404006}
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(tb = Table[{L, Hs⟦L - 1⟧ // Eigenvalues // Sort // First}, {L, 2, max}]) //
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Eigenvalues[Hs⟦3⟧, 1]⟦1⟧
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Out[24]=
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```
(tb =
   Table[{L, Eigenvalues[Hs⟦L - 1⟧, 1]⟦1⟧ // Eigenvalues // Sort // First},
    {L, 2, max}]) // AbsoluteTiming
```

```
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Eigenvalues[Hs⟦3⟧,1]
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Out[24]=
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 AbsoluteTiming
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In[24]:=
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Eigenvalues[Hs[[3]], 1][[1]]
```

Out[24]=
```
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In[24]:=
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Eigenvalues[Hs[[3]], 1][[1]]
```

Out[24]=
```
-0.404006
```

In[25]:=
```
(tb = Table[{L, Eigenvalues[Hs[[L - 1]], 1][[1]]}, {L, 2, max}]) //
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```

Out[25]=
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{9.060123, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
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In[24]:=
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Eigenvalues[Hs[[3]], 1][[1]]
```

Out[24]=
```
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In[25]:=
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(tb = Table[{L, Eigenvalues[Hs[[L - 1]], 1][[1]]}, {L, 2, max}]) //
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```

In[24]:=
```
Eigenvalues[Hs[[3]], 1][[1]]
```

Out[24]=
```
-0.404006
```

In[25]:=
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(tb = Table[{L, Eigenvalues[Hs[[L - 1]], 1][[1]]}, {L, 2, max}]) //
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In[24]:=
```
Eigenvalues[Hs[[3]], 1][[1]]
```

Out[24]=
```
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```

In[26]:=
```
Hs2 = SparseArray /@ Hs;
```

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Untitled-25

Out[16]=
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In[24]:=
```
Eigenvalues[Hs[[3]], 1][[1]]
```

Out[24]=
```
-0.404006
```

In[25]:=
```
(tb = Table[{L, Eigenvalues[Hs[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[25]=
```
{9.060123, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[26]:=
```
Hs2 = SparseArray /@ Hs;
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[16]=
```
{8.987602, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[24]:=
```
Eigenvalues[Hs[[3]], 1][[1]]
```

Out[24]=
```
-0.404006
```

In[25]:=
```
(tb = Table[{L, Eigenvalues[Hs[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[25]=
```
{9.060123, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[26]:=
```
Hs2 = SparseArray /@ Hs;
```

```
(tb = Table[{L, Eigenvalues[Hs[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

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Untitled-25

In[25]:=
```
(tb = Table[{L, Eigenvalues[Hs[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[25]=
```
{9.060123, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[26]:=
```
Hs2 = SparseArray /@ Hs;
```

```
(tb = Table[{L, Eigenvalues[Hs[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

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Untitled-25

In[25]:=
```
(tb = Table[{L, Eigenvalues[Hs[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[25]=
```
{9.060123, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[26]:=
```
Hs2 = SparseArray /@ Hs;
```

```
(tb = Table[{L, Eigenvalues[Hs2[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[25]:=
```
(tb = Table[{L, Eigenvalues[Hs[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[25]=
```
{9.060123, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[26]:=
```
Hs2 = SparseArray /@ Hs;
```

In[27]:=
```
(tb = Table[{L, Eigenvalues[Hs2[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[27]=
```
{0.010763, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

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Untitled-25

In[25]:=
```
(tb = Table[{L, Eigenvalues[Hs[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[25]=
```
{9.060123, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
f /@ {|
```

In[26]:=
```
Hs2 = SparseArray /@ Hs;
```

In[27]:=
```
(tb = Table[{L, Eigenvalues[Hs2[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[27]=
```
{0.010763, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

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Untitled-25

In[25]:=
```
(tb = Table[{L, Eigenvalues[Hs[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[25]=
```
{9.060123, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
f /@ {1, 2}
f[1]
```

In[26]:=
```
Hs2 = SparseArray /@ Hs;
```

In[27]:=
```
(tb = Table[{L, Eigenvalues[Hs2[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[27]=
```
{0.010763, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

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Untitled-25

In[25]:=
```
(tb = Table[{L, Eigenvalues[Hs[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[25]=
```
{9.060123, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[28]:=
```
f /@ {1, 2}
f@[1, 2]
```

Out[28]=
```
{f[1], f[2]}
```

In[26]:=
```
Hs2 = SparseArray /@ Hs;
```

In[27]:=
```
(tb = Table[{L, Eigenvalues[Hs2[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[27]=
```
{0.010763, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

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Untitled-25

In[25]:=
```
(tb = Table[{L, Eigenvalues[Hs[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[25]=
```
{9.060123, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
f /@ {1, 2}
f@{1, 2}
```

Out[28]=
```
{f[1], f[2]}
```

In[26]:=
```
Hs2 = SparseArray /@ Hs;
```

In[27]:=
```
(tb = Table[{L, Eigenvalues[Hs2[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[27]=
```
{0.010763, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

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Untitled-25

In[25]:=
```
(tb = Table[{L, Eigenvalues[Hs[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[25]=
```
{9.060123, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[29]:=
```
f /@ {1, 2}
f@{1, 2}
```

Out[29]=
```
{f[1], f[2]}
```

Out[30]=
```
f[{1, 2}]
```

In[26]:=
```
Hs2 = SparseArray /@ Hs;
```

In[27]:=
```
(tb = Table[{L, Eigenvalues[Hs2[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```


Out[27]=
```
{0.010763, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
```

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Untitled-25

```
f@{1, 2}
```

Out[29]= {f[1], f[2]}

Out[30]= f[{1, 2}]

In[26]:=
```
Hs2 = SparseArray /@ Hs;
```

In[27]:=
```
(tb = Table[{L, Eigenvalues[Hs2[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[27]=
```
{0.010763, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

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Untitled-25

```
f@{1, 2}
```

Out[29]=
```
{f[1], f[2]}
```

Out[30]=
```
f[{1, 2}]
```

In[26]:=
```
Hs2 = SparseArray /@ Hs;
```

In[27]:=
```
(tb = Table[{L, Eigenvalues[Hs2[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[27]=
```
{0.010763, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
ListPlot[|
```

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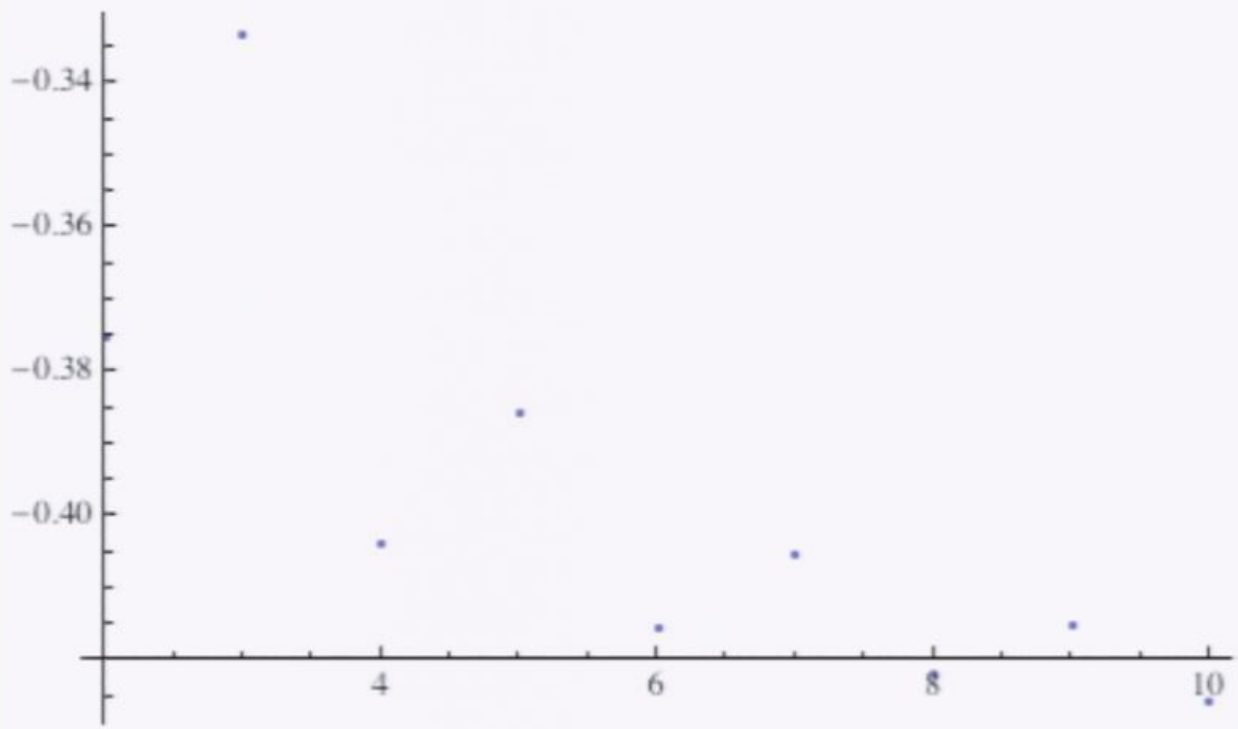
Untitled-25

Out[27]=
```
{0.010763, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[31]:=
```
ListPlot[tb]
```

Out[31]=



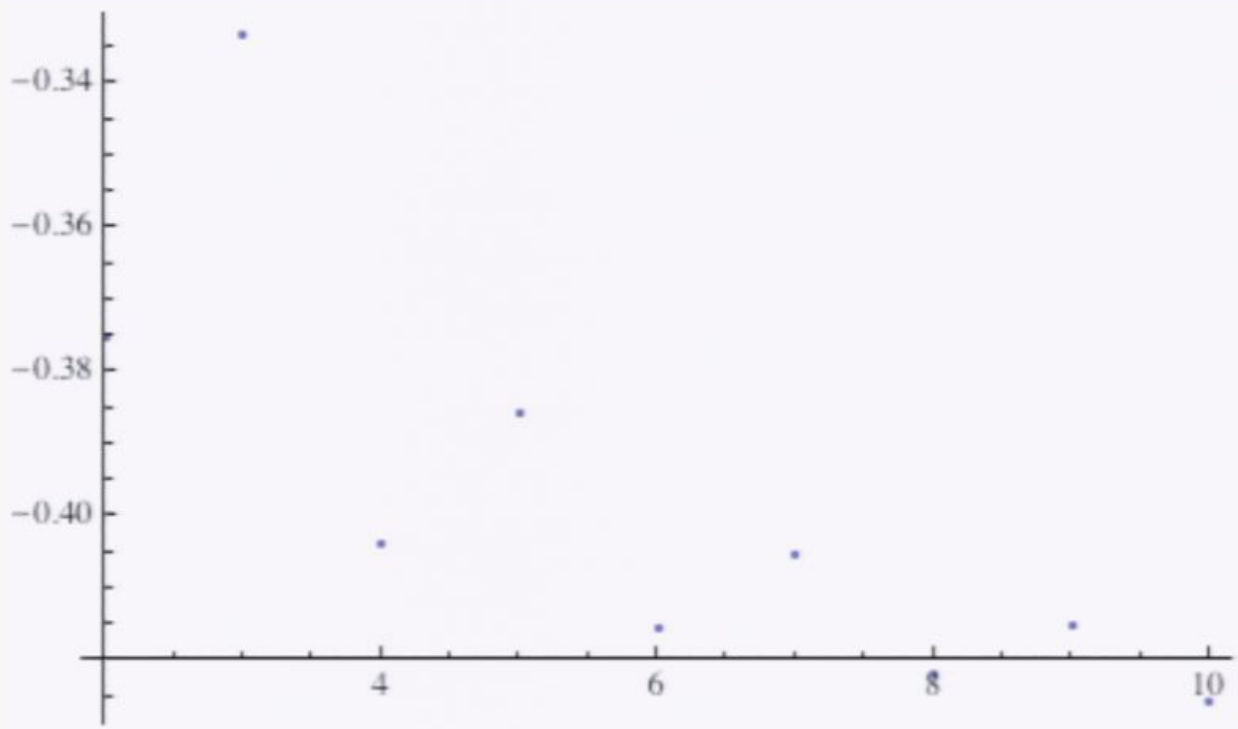
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[27]=
```
{0.010763, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
ListPlot[tb, PlotStyle ->]
```

Out[31]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

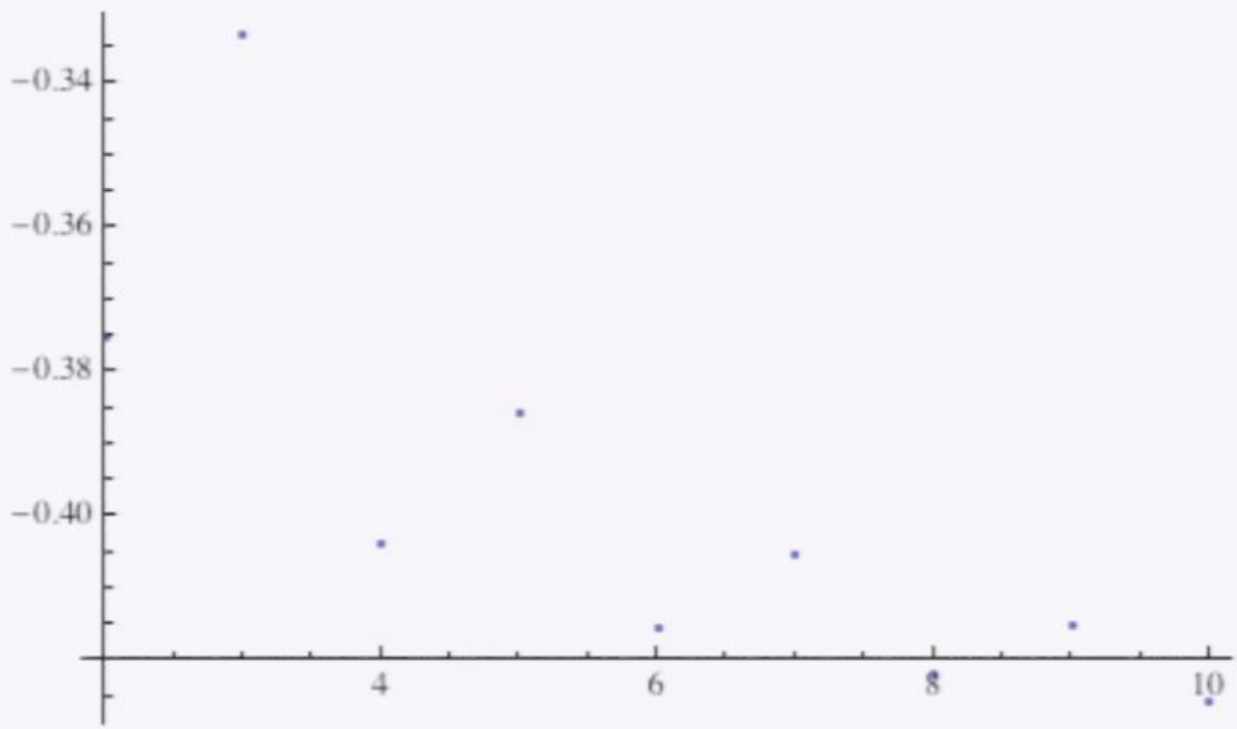
```
AbsoluteTiming
```

Out[27]=

```
{0.010763, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
ListPlot[tb, PlotStyle → {PointSize → 0.}]
```

Out[31]=



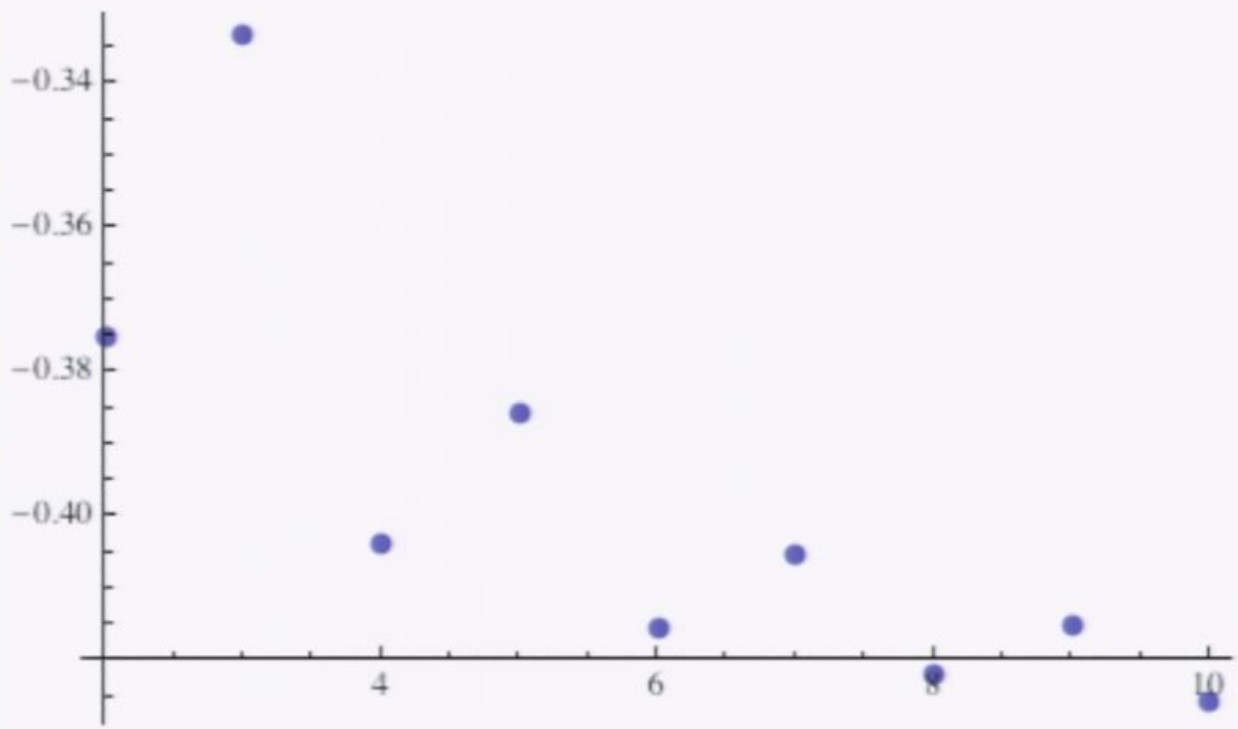
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[27]=
```
{0.010763, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[32]:=
```
ListPlot[tb, PlotStyle → {PointSize → 0.02}]
```

Out[32]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
AbsoluteTiming
```

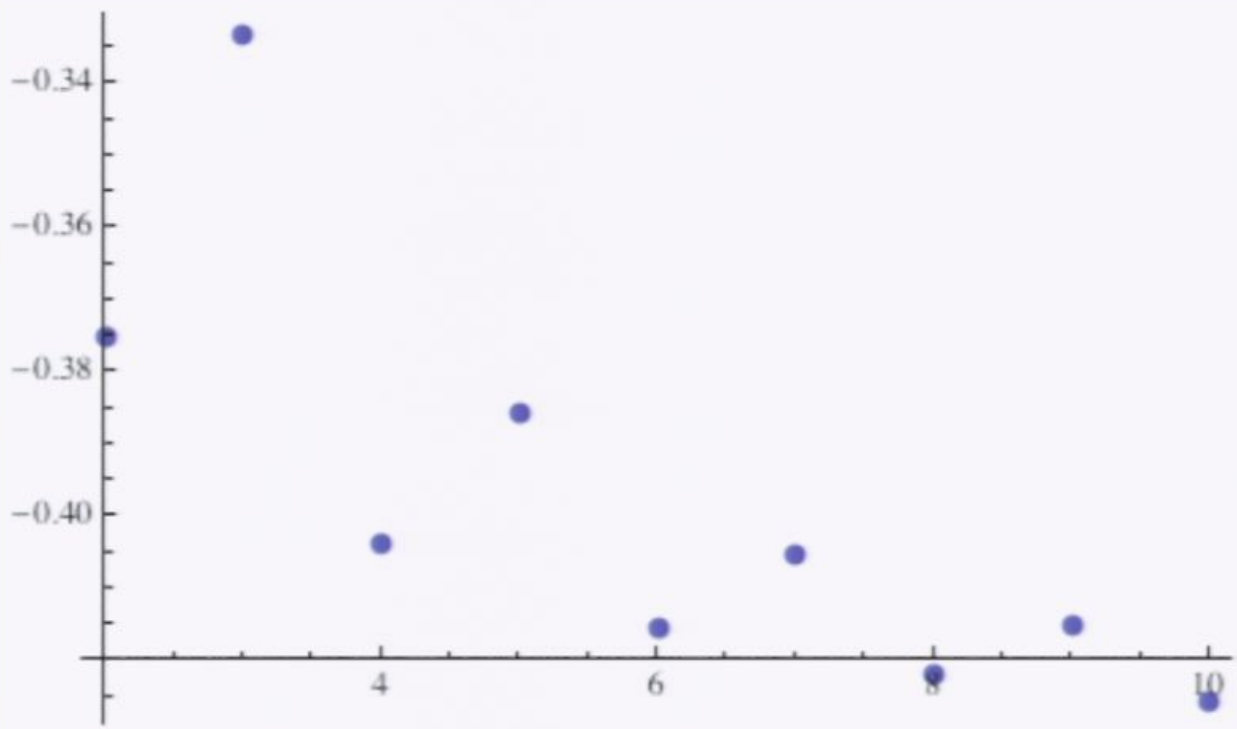
Out[27]=

```
{0.010763, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[33]:=

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02}]
```

Out[33]=



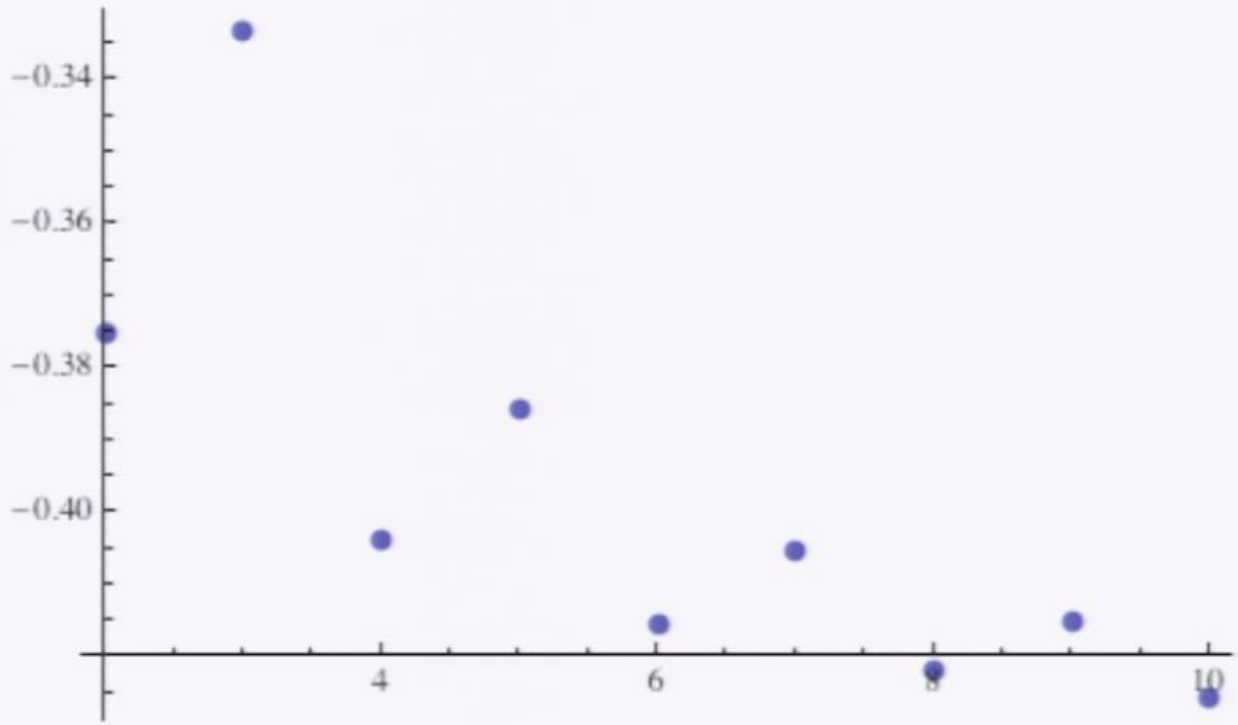
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[27]=
```
{0.010763, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[33]:=
```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02}]
```

Out[33]=




```
FullForm
```

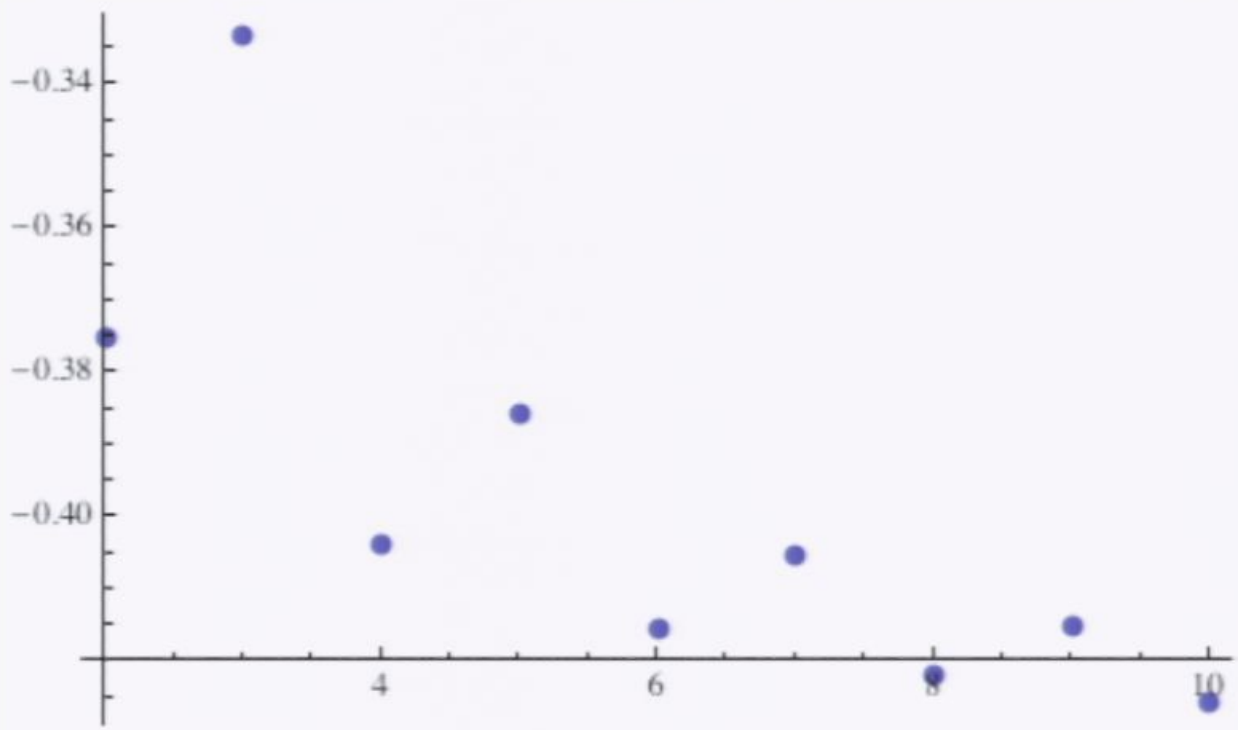
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[27]=
```
{0.010763, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[33]:=
```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02}]
```

Out[33]=

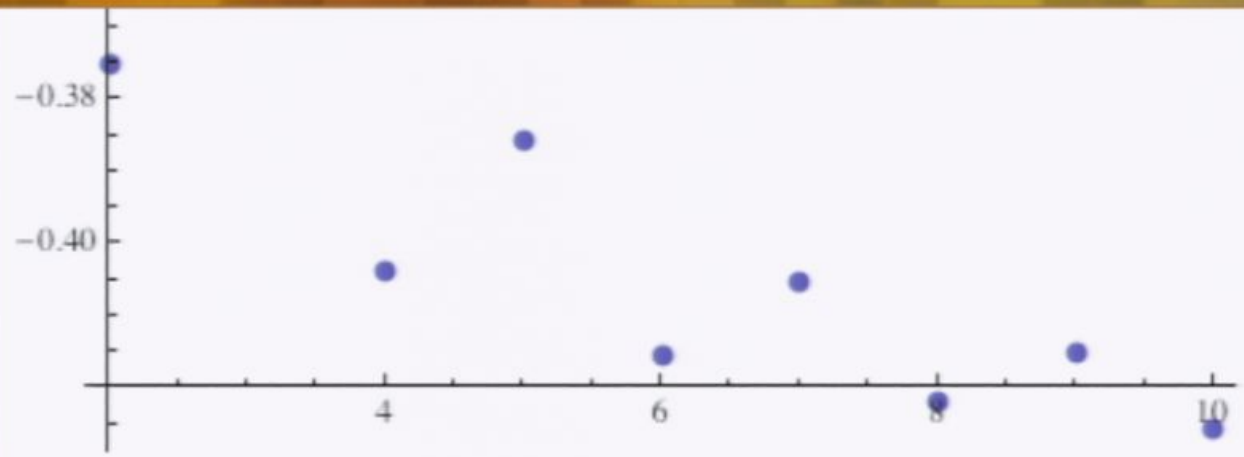



```
FullForm[lp]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[33]=



In[34]:=

```
FullForm[lp]
```

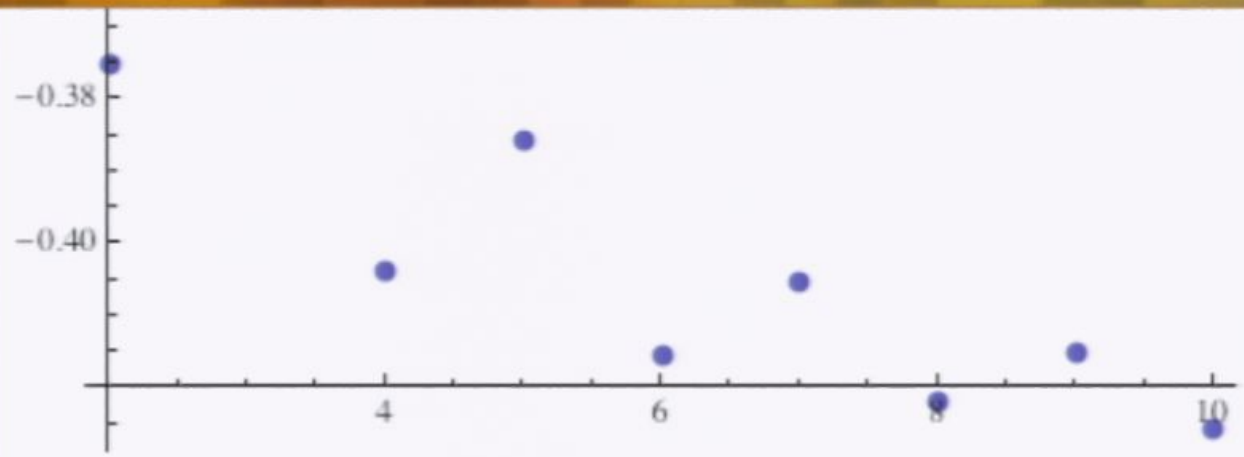
t[34]//FullForm=

```
Graphics[List[Hue[0.67`, 0.6`, 0.6`], Rule[PointSize, 0.02`],
  Point[List[List[2.`, -0.375`], List[3.`, -0.3333333333333334`],
    List[4.`, -0.40400635094610965`], List[5.`, -0.3855772506635989`],
    List[6.`, -0.41559618898132095`], List[7.`, -0.4051770972409499`],
    List[8.`, -0.42186657483598594`], List[9.`,
     -0.41514685626436837`], List[10.`, -0.42580352072828803`]]]],
 List[Rule[AspectRatio, Power[GoldenRatio, -1]],
  Rule[Axes, True], Rule[PlotRange, Automatic],
  Rule[PlotRangeClipping, True]]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[33]=




```
lp
```

```
FullForm[lp]
```

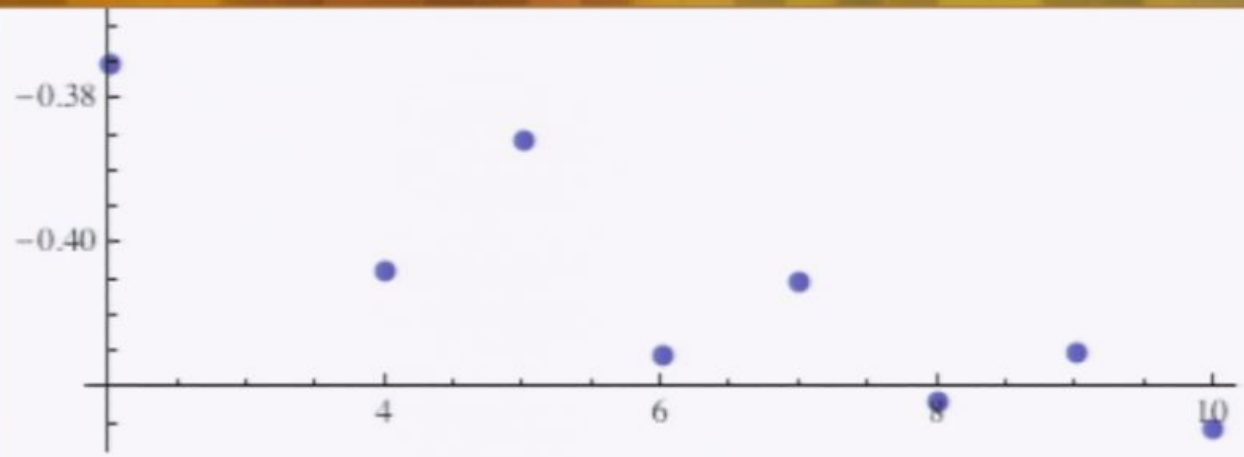
t[34]//FullForm=

```
Graphics[List[Hue[0.67`, 0.6`, 0.6`], Rule[PointSize, 0.02`],
  Point[List[List[2.`, -0.375`], List[3.`, -0.3333333333333334`],
    List[4.`, -0.40400635094610965`], List[5.`, -0.3855772506635989`],
    List[6.`, -0.41559618898132095`], List[7.`, -0.4051770972409499`],
    List[8.`, -0.42186657483598594`], List[9.`,
     -0.41514685626436837`], List[10.`, -0.42580352072828803`]]]],
 List[Rule[AspectRatio, Power[GoldenRatio, -1]],
  Rule[Axes, True], Rule[PlotRange, Automatic],
  Rule[PlotRangeClipping, True]]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[33]=




```
lp /. Hue[
```

```
FullForm[lp]
```

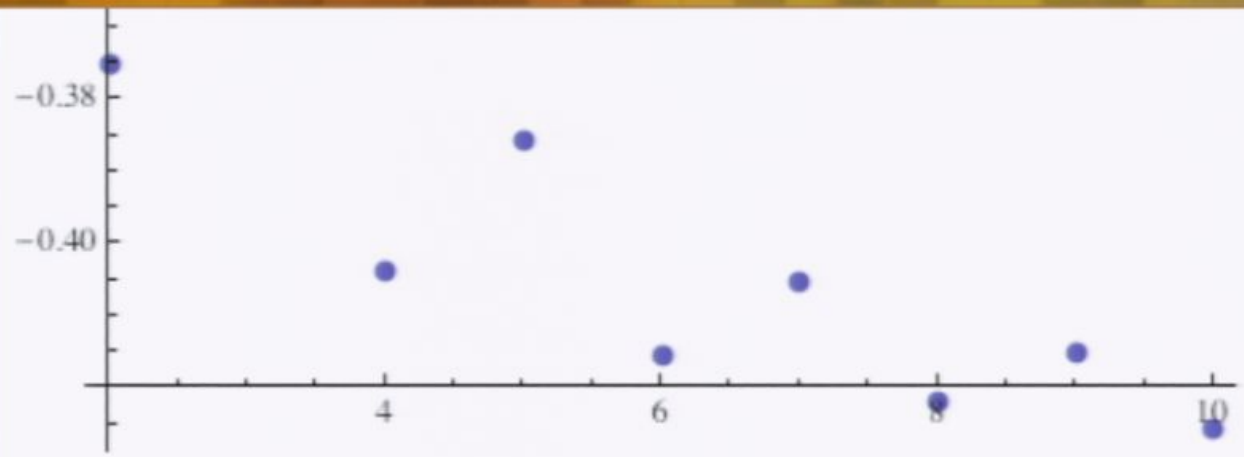
t[34]//FullForm=

```
Graphics[List[Hue[0.67`, 0.6`, 0.6`], Rule[PointSize, 0.02`],
  Point[List[List[2.`, -0.375`], List[3.`, -0.3333333333333334`],
    List[4.`, -0.40400635094610965`], List[5.`, -0.3855772506635989`],
    List[6.`, -0.41559618898132095`], List[7.`, -0.4051770972409499`],
    List[8.`, -0.42186657483598594`], List[9.`,
     -0.41514685626436837`], List[10.`, -0.42580352072828803`]]]],
 List[Rule[AspectRatio, Power[GoldenRatio, -1]],
  Rule[Axes, True], Rule[PlotRange, Automatic],
  Rule[PlotRangeClipping, True]]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[33]=




```
lp /. Hue[a__]
```

```
FullForm[lp]
```

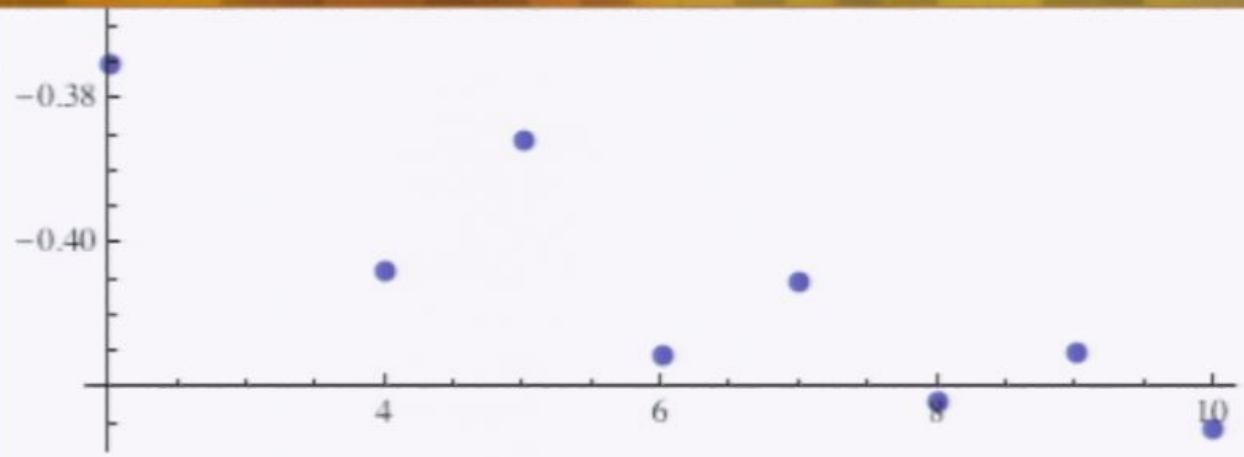
t[34]//FullForm=

```
Graphics[List[Hue[0.67`, 0.6`, 0.6`], Rule[PointSize, 0.02`],
  Point[List[List[2.`, -0.375`], List[3.`, -0.3333333333333334`],
    List[4.`, -0.40400635094610965`], List[5.`, -0.3855772506635989`],
    List[6.`, -0.41559618898132095`], List[7.`, -0.4051770972409499`],
    List[8.`, -0.42186657483598594`], List[9.`,
     -0.41514685626436837`], List[10.`, -0.42580352072828803`]]]],
 List[Rule[AspectRatio, Power[GoldenRatio, -1]],
  Rule[Axes, True], Rule[PlotRange, Automatic],
  Rule[PlotRangeClipping, True]]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[33]=




```
lp /. Hue[a__] → Red
```

```
FullForm[lp]
```

t[34]//FullForm=

```
Graphics[List[Hue[0.67`, 0.6`, 0.6`], Rule[PointSize, 0.02`],
  Point[List[List[2.`, -0.375`], List[3.`, -0.3333333333333334`],
    List[4.`, -0.40400635094610965`], List[5.`, -0.3855772506635989`],
    List[6.`, -0.41559618898132095`], List[7.`, -0.4051770972409499`],
    List[8.`, -0.42186657483598594`], List[9.`,
     -0.41514685626436837`], List[10.`, -0.42580352072828803`]]]],
 List[Rule[AspectRatio, Power[GoldenRatio, -1]],
  Rule[Axes, True], Rule[PlotRange, Automatic],
  Rule[PlotRangeClipping, True]]]
```

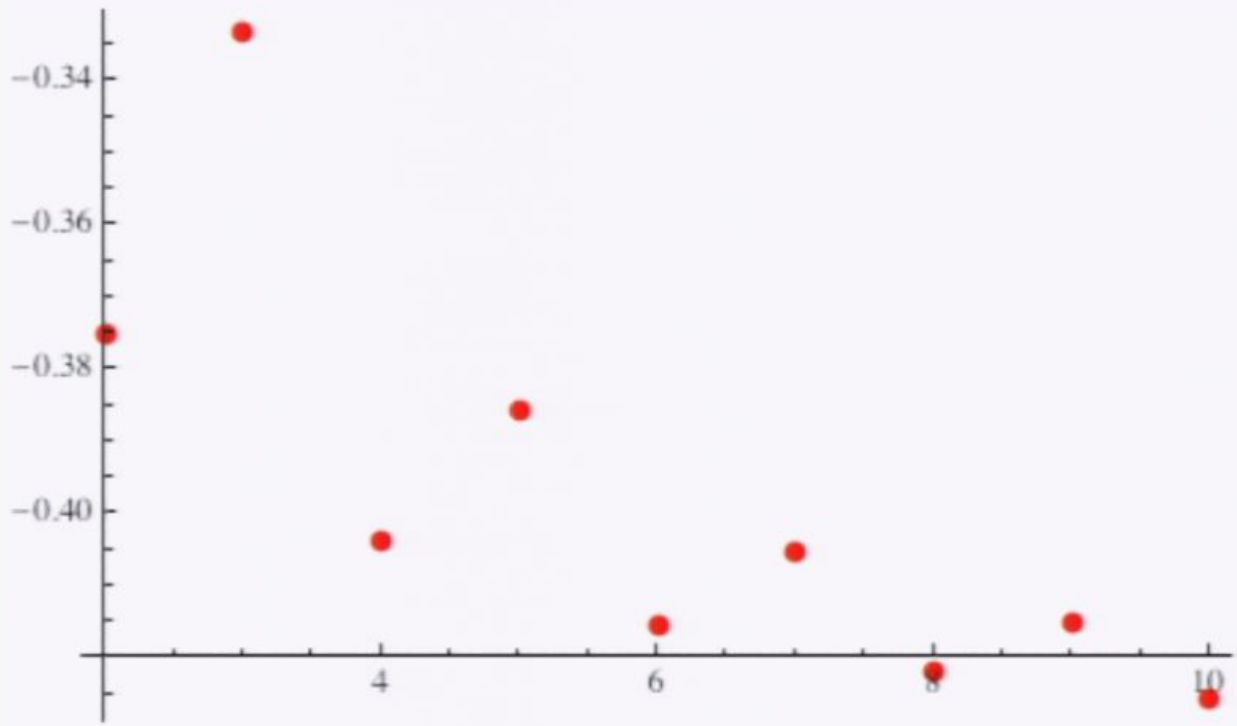
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[35]:=

```
lp /. Hue[a__] → Red
```

Out[35]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[35]:=
```
lp /. Hue[a__] → Red
```

```
FullForm[lp]
```

t[34]//FullForm=

```
Graphics[List[Hue[0.67`, 0.6`, 0.6`], Rule[PointSize, 0.02`],
  Point[List[List[2.`, -0.375`], List[3.`, -0.3333333333333334`],
    List[4.`, -0.40400635094610965`], List[5.`, -0.3855772506635989`],
    List[6.`, -0.41559618898132095`], List[7.`, -0.4051770972409499`],
    List[8.`, -0.42186657483598594`], List[9.`,
     -0.41514685626436837`], List[10.`, -0.42580352072828803`]]]],
 List[Rule[AspectRatio, Power[GoldenRatio, -1]],
  Rule[Axes, True], Rule[PlotRange, Automatic],
  Rule[PlotRangeClipping, True]]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25


```
lp /. {Hue[a__] → Red
```

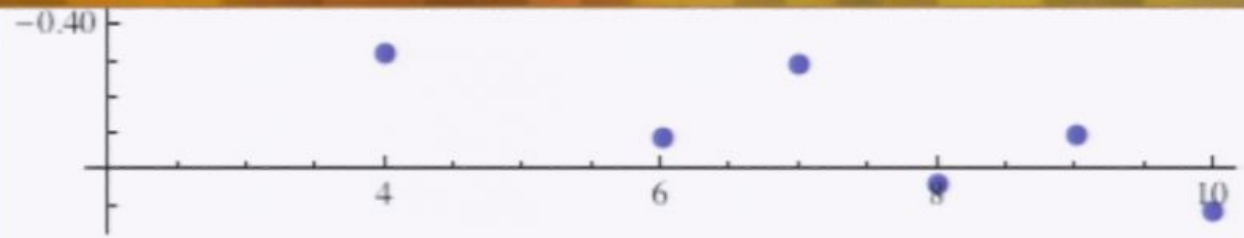
```
FullForm[lp]
```

t[34]//FullForm=

```
Graphics[List[Hue[0.67`, 0.6`, 0.6`], Rule[PointSize, 0.02`],
  Point[List[List[2.`, -0.375`], List[3.`, -0.3333333333333334`],
    List[4.`, -0.40400635094610965`], List[5.`, -0.3855772506635989`],
    List[6.`, -0.41559618898132095`], List[7.`, -0.4051770972409499`],
    List[8.`, -0.42186657483598594`], List[9.`,
     -0.41514685626436837`], List[10.`, -0.42580352072828803`]]]],
 List[Rule[AspectRatio, Power[GoldenRatio, -1]],
  Rule[Axes, True], Rule[PlotRange, Automatic],
  Rule[PlotRangeClipping, True]]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25




```
lp /. {Hue[a__] → Red, Rule[Poin
```

```
FullForm[lp]
```

t[34]//FullForm=

```
Graphics[List[Hue[0.67`, 0.6`, 0.6`], Rule[PointSize, 0.02`],
  Point[List[List[2.`, -0.375`], List[3.`, -0.3333333333333334`],
    List[4.`, -0.40400635094610965`], List[5.`, -0.3855772506635989`],
    List[6.`, -0.41559618898132095`], List[7.`, -0.4051770972409499`],
    List[8.`, -0.42186657483598594`], List[9.`,
     -0.41514685626436837`], List[10.`, -0.42580352072828803`]]]],
 List[Rule[AspectRatio, Power[GoldenRatio, -1]],
  Rule[Axes, True], Rule[PlotRange, Automatic],
  Rule[PlotRangeClipping, True]]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25


```
lp /. {Hue[a__] → Red, Rule[Poi
```

```
FullForm[lp]
```

t[34]//FullForm=

```
Graphics[List[Hue[0.67`, 0.6`, 0.6`], Rule[PointSize, 0.02`],
  Point[List[List[2.`, -0.375`], List[3.`, -0.3333333333333334`],
    List[4.`, -0.40400635094610965`], List[5.`, -0.3855772506635989`],
    List[6.`, -0.41559618898132095`], List[7.`, -0.4051770972409499`],
    List[8.`, -0.42186657483598594`], List[9.`,
     -0.41514685626436837`], List[10.`, -0.42580352072828803`]]]],
 List[Rule[AspectRatio, Power[GoldenRatio, -1]],
  Rule[Axes, True], Rule[PlotRange, Automatic],
  Rule[PlotRangeClipping, True]]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25


```
lp /. {Hue[a__] → Red, Rule[PointSize, b]}
```

```
FullForm[lp]
```

t[34]//FullForm=

```
Graphics[List[Hue[0.67`, 0.6`, 0.6`], Rule[PointSize, 0.02`],
  Point[List[List[2.`, -0.375`], List[3.`, -0.3333333333333334`],
    List[4.`, -0.40400635094610965`], List[5.`, -0.3855772506635989`],
    List[6.`, -0.41559618898132095`], List[7.`, -0.4051770972409499`],
    List[8.`, -0.42186657483598594`], List[9.`,
     -0.41514685626436837`], List[10.`, -0.42580352072828803`]]]],
 List[Rule[AspectRatio, Power[GoldenRatio, -1]],
  Rule[Axes, True], Rule[PlotRange, Automatic],
  Rule[PlotRangeClipping, True]]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25


```
lp /. {Hue[a__] → Red, Rule[PointSize, b_] -> Rule[PointSize, b_]}
```

```
FullForm[lp]
```

t[34]//FullForm=

```
Graphics[List[Hue[0.67`, 0.6`, 0.6`], Rule[PointSize, 0.02`],
  Point[List[List[2.`, -0.375`], List[3.`, -0.3333333333333334`],
    List[4.`, -0.40400635094610965`], List[5.`, -0.3855772506635989`],
    List[6.`, -0.41559618898132095`], List[7.`, -0.4051770972409499`],
    List[8.`, -0.42186657483598594`], List[9.`,
     -0.41514685626436837`], List[10.`, -0.42580352072828803`]]]],
 List[Rule[AspectRatio, Power[GoldenRatio, -1]],
  Rule[Axes, True], Rule[PlotRange, Automatic],
  Rule[PlotRangeClipping, True]]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25


```
lp /. {Hue[a__] → Red, Rule[PointSize, b_] -> Rule[PointSize, 0.]}
```

```
FullForm[lp]
```

t[34]//FullForm=

```
Graphics[List[Hue[0.67`, 0.6`, 0.6`], Rule[PointSize, 0.02`],
  Point[List[List[2.`, -0.375`], List[3.`, -0.3333333333333334`],
    List[4.`, -0.40400635094610965`], List[5.`, -0.3855772506635989`],
    List[6.`, -0.41559618898132095`], List[7.`, -0.4051770972409499`],
    List[8.`, -0.42186657483598594`], List[9.`,
     -0.41514685626436837`], List[10.`, -0.42580352072828803`]]]],
 List[Rule[AspectRatio, Power[GoldenRatio, -1]],
  Rule[Axes, True], Rule[PlotRange, Automatic],
  Rule[PlotRangeClipping, True]]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25


```
lp /. {Hue[a__] → Red, Rule[PointSize, b_] -> Rule[PointSize, 0.04]}
```

```
FullForm[lp]
```

t[34]//FullForm=

```
Graphics[List[Hue[0.67`, 0.6`, 0.6`], Rule[PointSize, 0.02`],
  Point[List[List[2.`, -0.375`], List[3.`, -0.3333333333333334`],
    List[4.`, -0.40400635094610965`], List[5.`, -0.3855772506635989`],
    List[6.`, -0.41559618898132095`], List[7.`, -0.4051770972409499`],
    List[8.`, -0.42186657483598594`], List[9.`,
     -0.41514685626436837`], List[10.`, -0.42580352072828803`]]]],
 List[Rule[AspectRatio, Power[GoldenRatio, -1]],
  Rule[Axes, True], Rule[PlotRange, Automatic],
  Rule[PlotRangeClipping, True]]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25


```
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

```
FullForm[lp]
```

t[34]//FullForm=

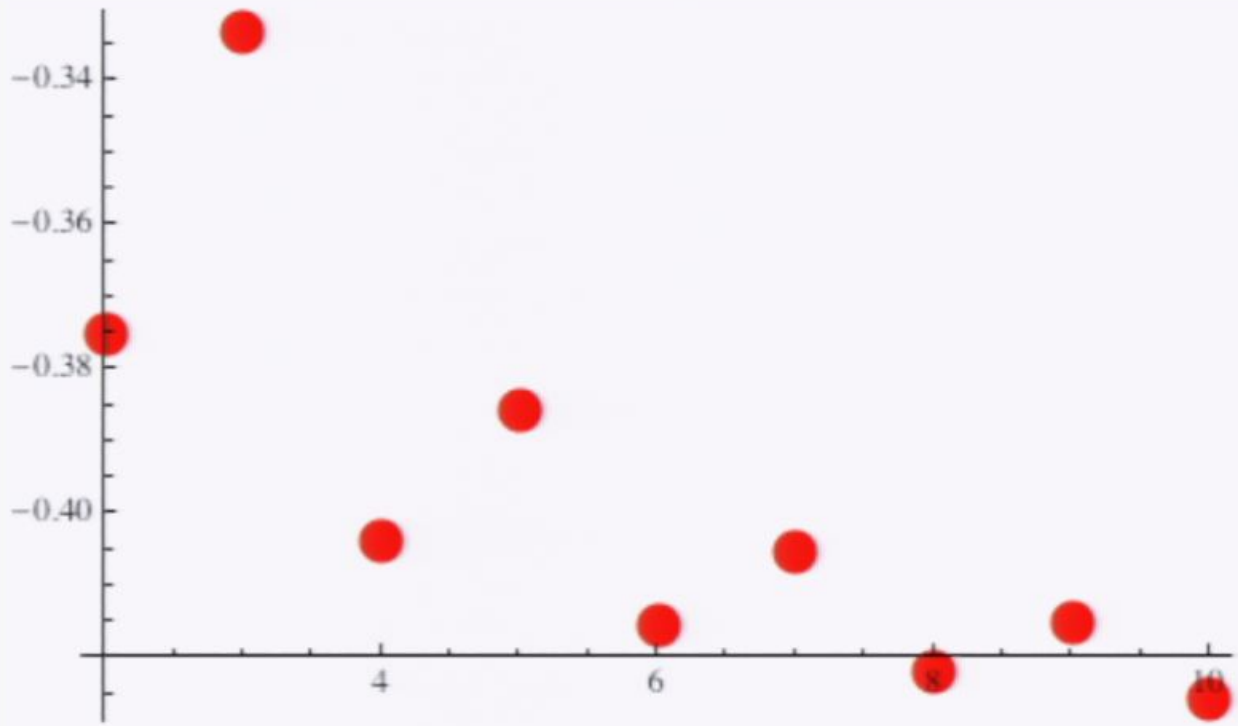
```
Graphics[List[Hue[0.67`, 0.6`, 0.6`], Rule[PointSize, 0.02`],
  Point[List[List[2.`, -0.375`], List[3.`, -0.3333333333333334`],
    List[4.`, -0.40400635094610965`], List[5.`, -0.3855772506635989`],
    List[6.`, -0.41559618898132095`], List[7.`, -0.4051770972409499`],
    List[8.`, -0.42186657483598594`], List[9.`,
     -0.41514685626436837`], List[10.`, -0.42580352072828803`]]]],
 List[Rule[AspectRatio, Power[GoldenRatio, -1]],
  Rule[Axes, True], Rule[PlotRange, Automatic],
  Rule[PlotRangeClipping, True]]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[36]:=
```
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

Out[36]=

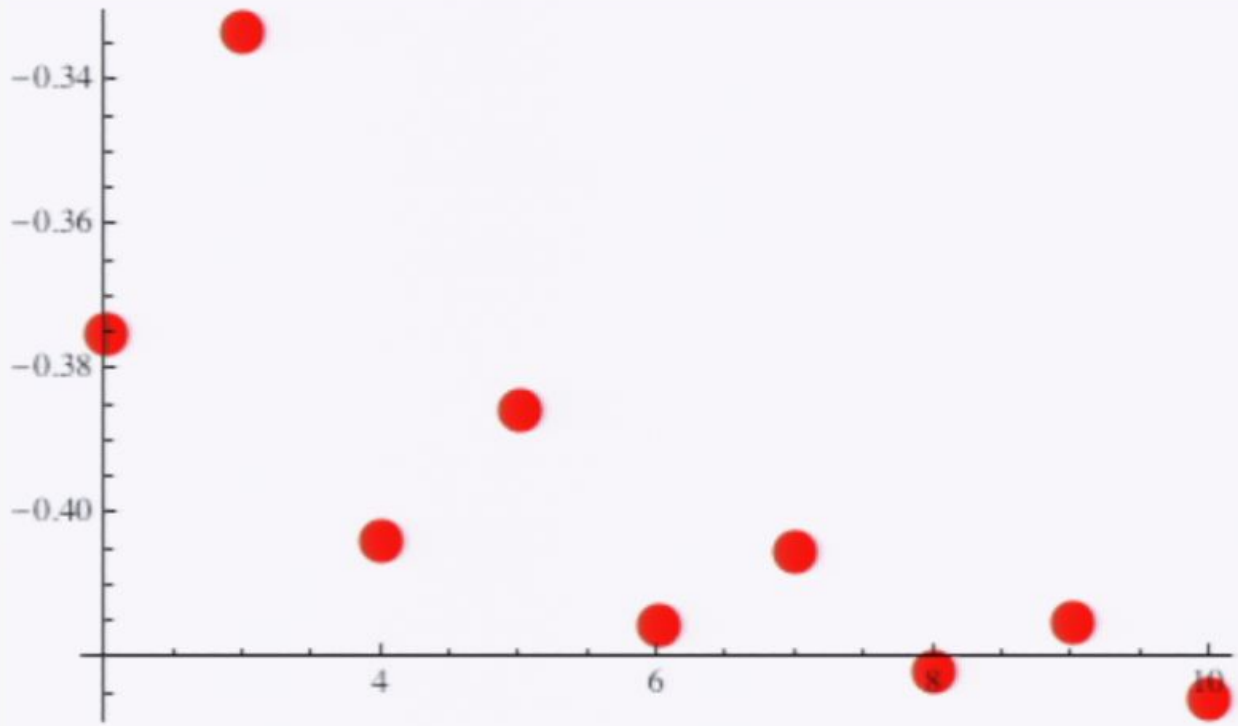


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[36]:=
```
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

Out[36]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[36]:=
```
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

```
FullForm[lp]
```

t[34]//FullForm=

```
Graphics[List[Hue[0.67`, 0.6`, 0.6`], Rule[PointSize, 0.02`],
  Point[List[List[2.`, -0.375`], List[3.`, -0.3333333333333334`],
    List[4.`, -0.40400635094610965`], List[5.`, -0.3855772506635989`],
    List[6.`, -0.41559618898132095`], List[7.`, -0.4051770972409499`],
    List[8.`, -0.42186657483598594`], List[9.`,
     -0.41514685626436837`], List[10.`, -0.42580352072828803`]]]],
 List[Rule[AspectRatio, Power[GoldenRatio, -1]],
  Rule[Axes, True], Rule[PlotRange, Automatic],
  Rule[PlotRangeClipping, True]]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[36]:=
```
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

```
FullForm[lp]
```

t[34]//FullForm=

```
Graphics[List[Hue[0.67`, 0.6`, 0.6`], Rule[PointSize, 0.02`],
  Point[List[List[2.`, -0.375`], List[3.`, -0.3333333333333334`],
    List[4.`, -0.40400635094610965`], List[5.`, -0.3855772506635989`],
    List[6.`, -0.41559618898132095`], List[7.`, -0.4051770972409499`],
    List[8.`, -0.42186657483598594`], List[9.`,
     -0.41514685626436837`], List[10.`, -0.42580352072828803`]]]],
 List[Rule[AspectRatio, Power[GoldenRatio, -1]],
  Rule[Axes, True], Rule[PlotRange, Automatic],
  Rule[PlotRangeClipping, True]]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[36]:=
```
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

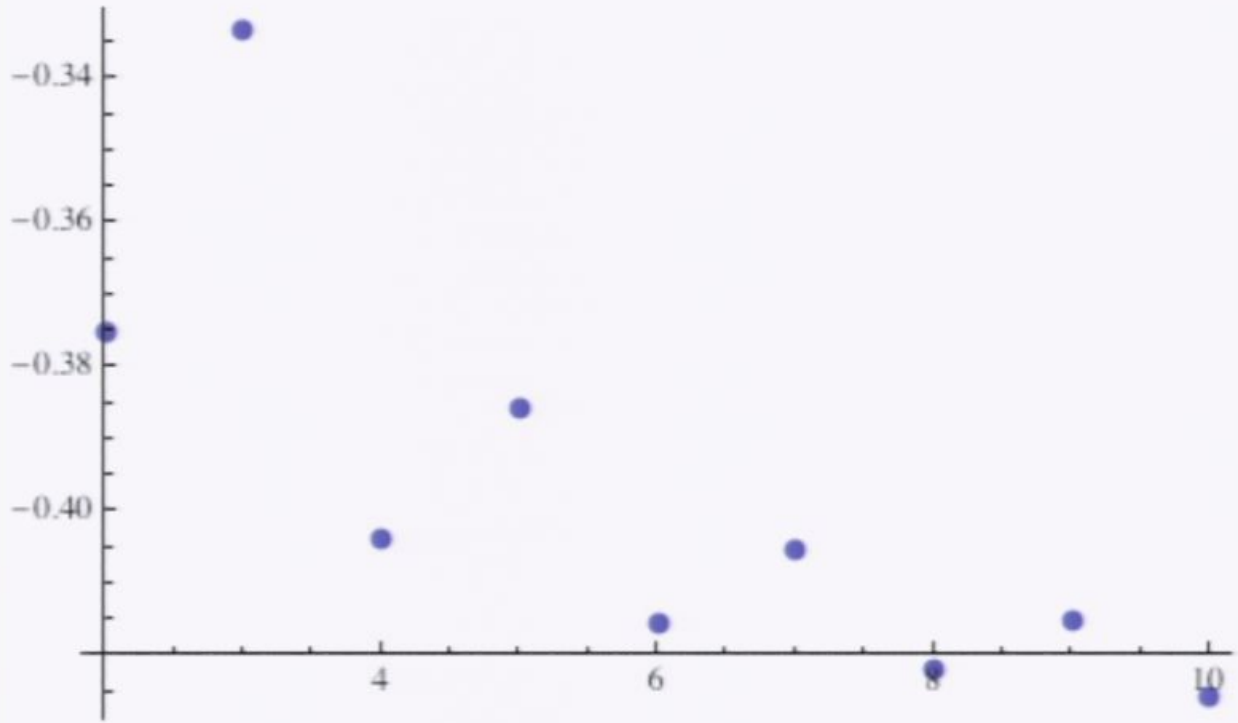

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[33]:=
```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02}]
```

Out[33]=



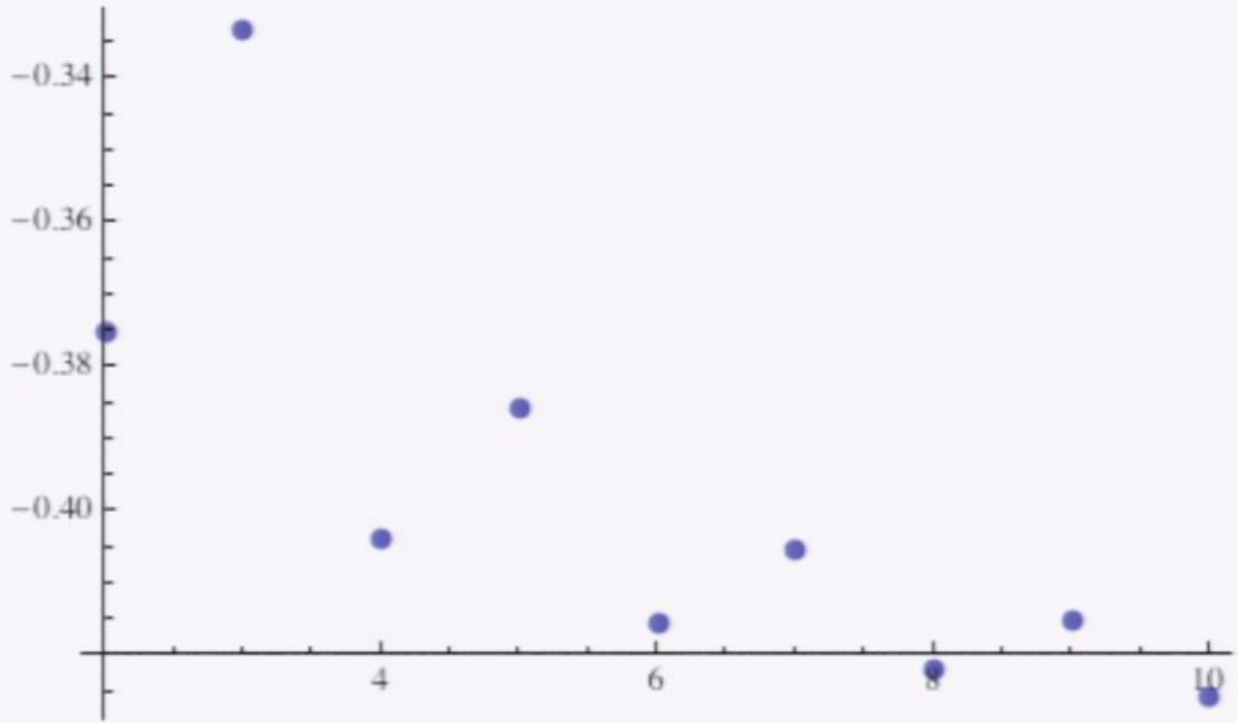

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[33]:=
```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02}]
```

Out[33]=



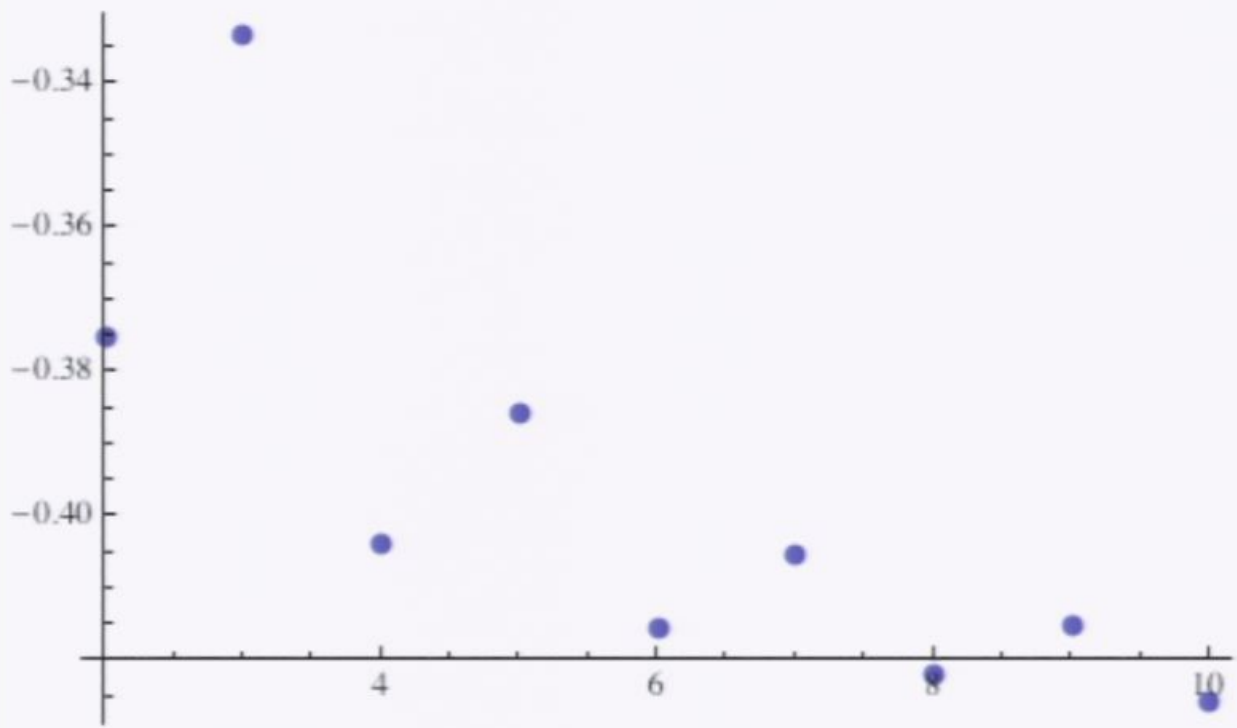

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[33]:=
```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02}]
```

Out[33]=




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

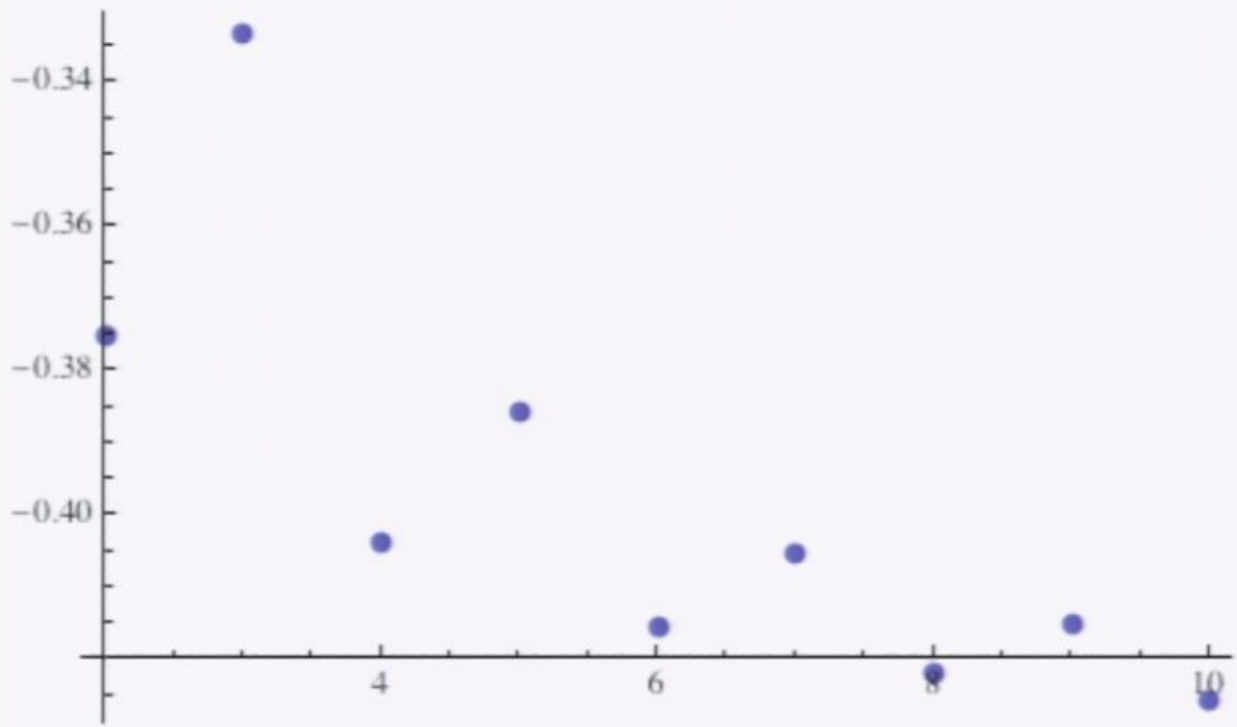
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02}, Plot]
```

Out[33]=




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

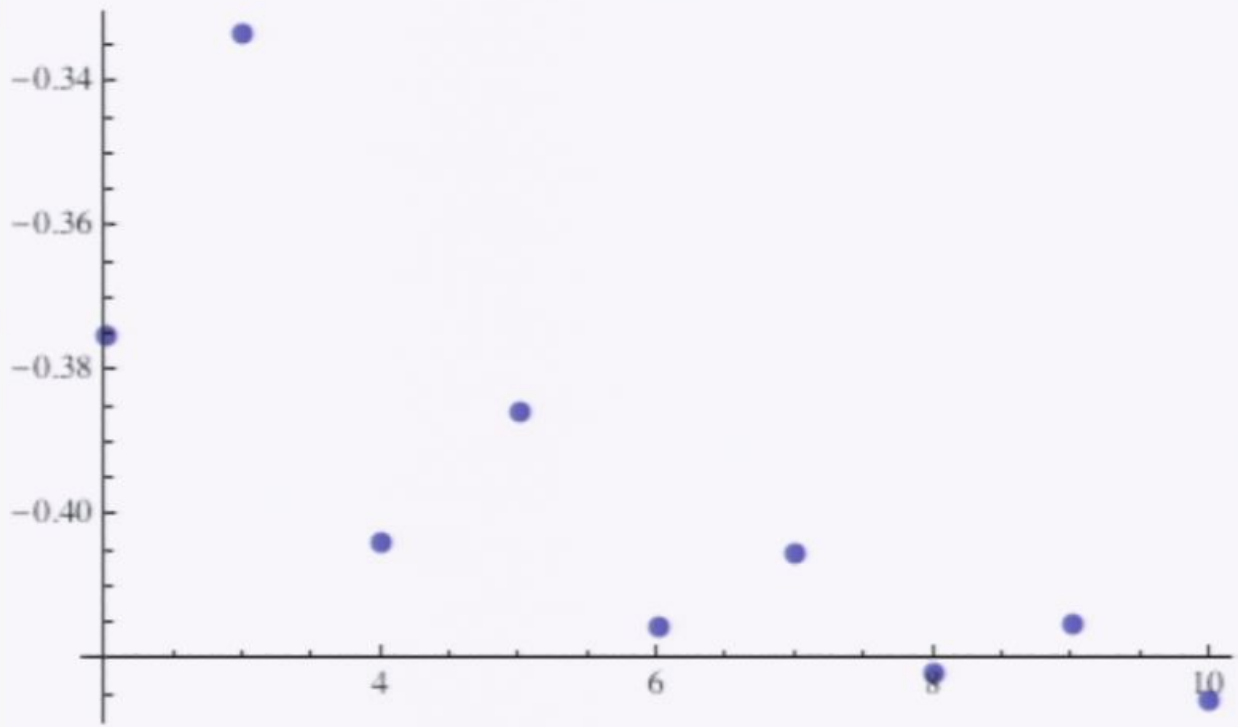
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02}, PlotLabel]
```

Out[33]=




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

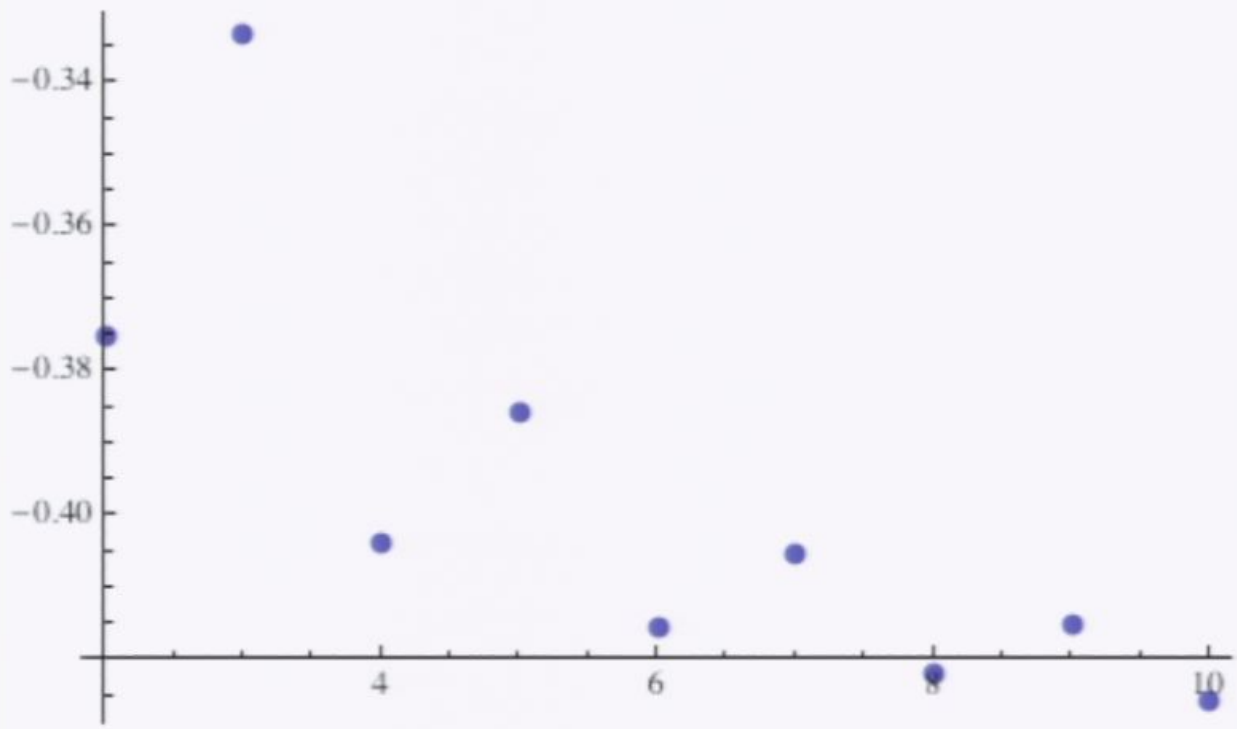
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02}, PlotLabel → "bal]
```

Out[33]=




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

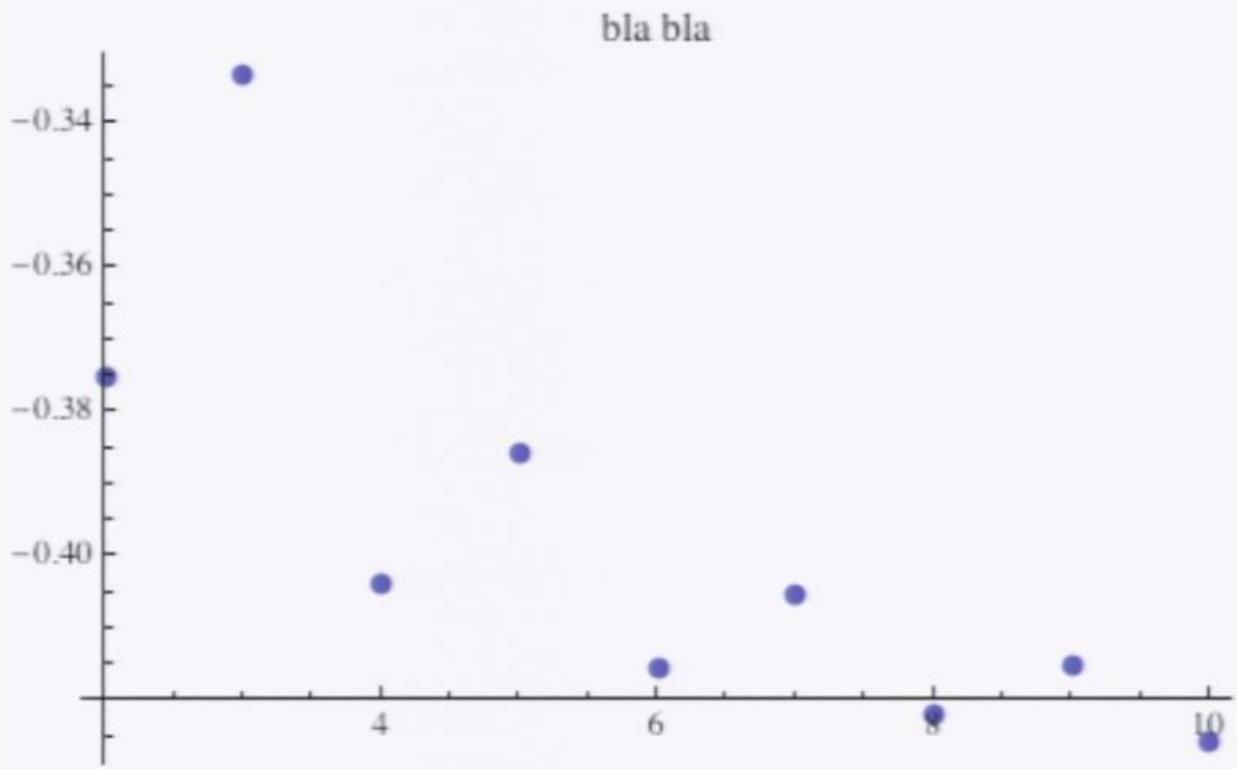
Untitled-25

```
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[37]:=

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02}, PlotLabel → "bla bla"]
```

Out[37]=




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

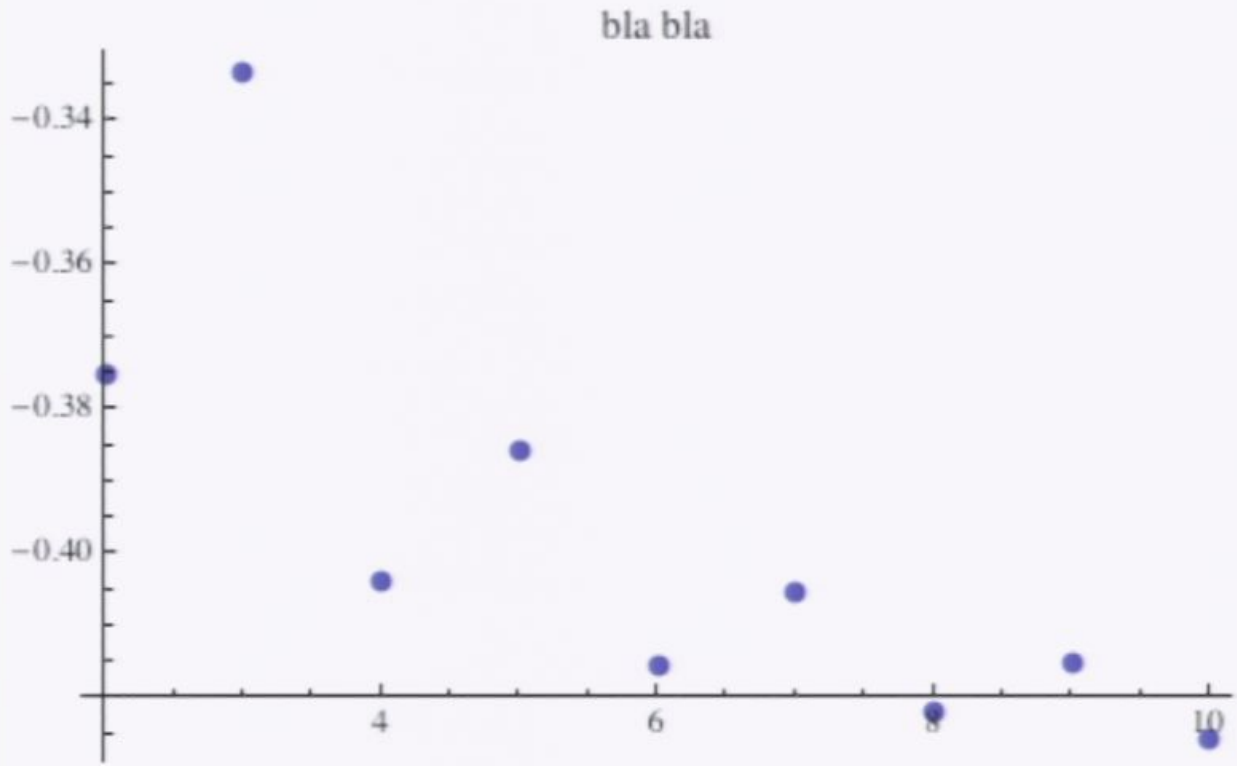
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
  PlotLabel → Style["bla bla"]]
```

Out[37]=




```
FullForm[lp]
lp /. {Hue[a_] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

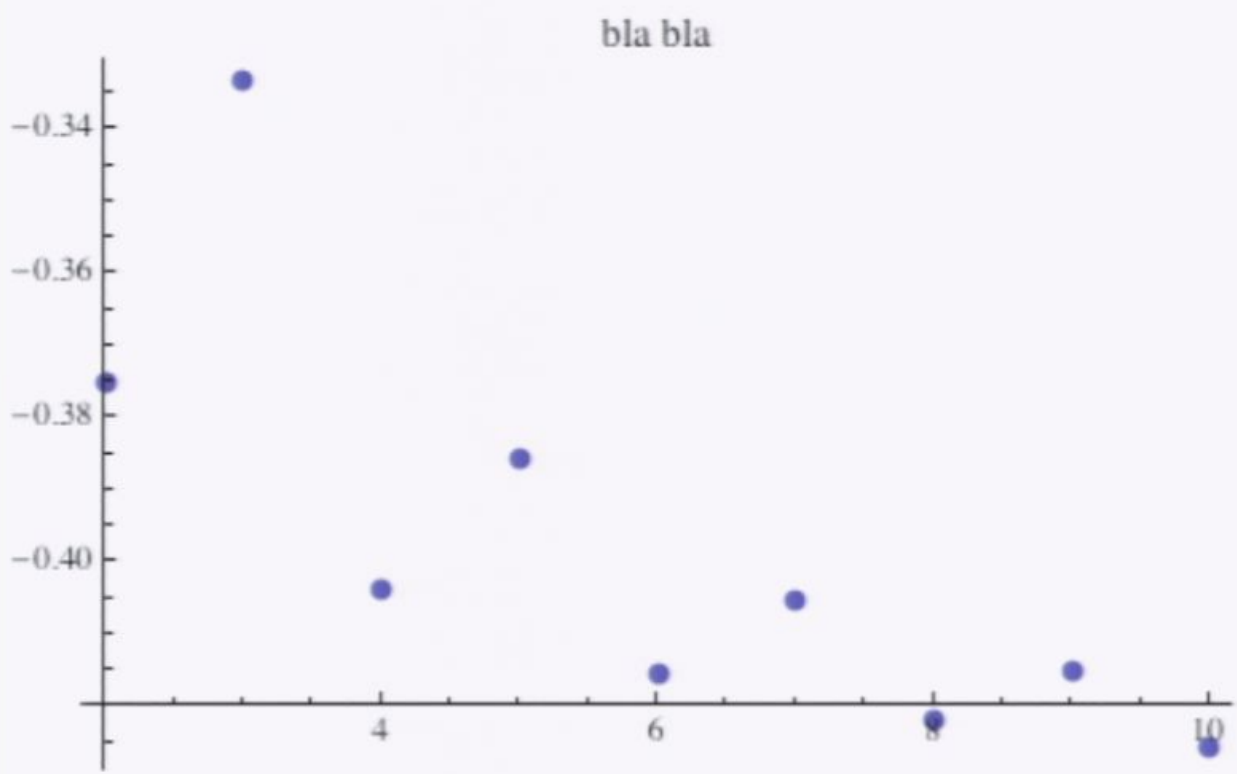
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
{5, -0.385577}, {6, -0.415596}, {7, -0.405177},
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
  PlotLabel → Style["bla bla",]]
```

Out[37]=




```
FullForm[lp]
```

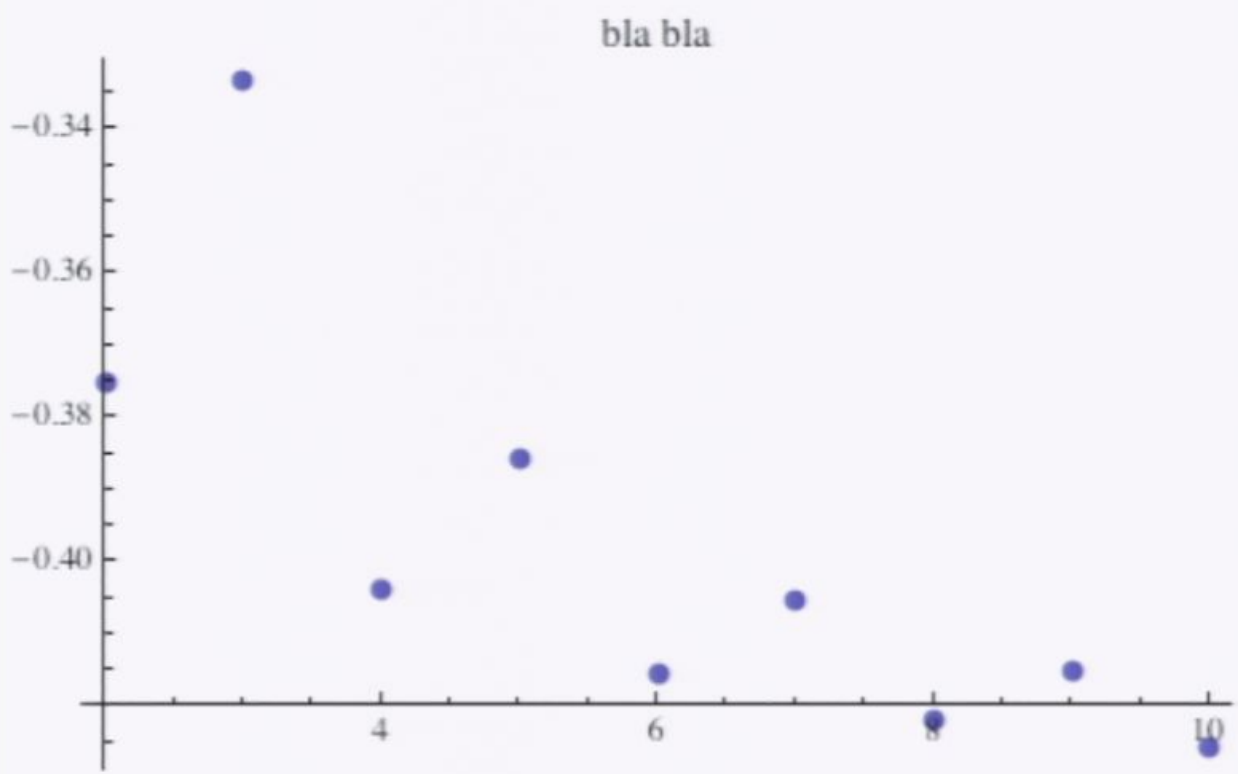
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
{5, -0.385577}, {6, -0.415596}, {7, -0.405177},
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
   PlotLabel → Style["bla bla", {R]
```

Out[37]=




```
FullForm[lp]
```

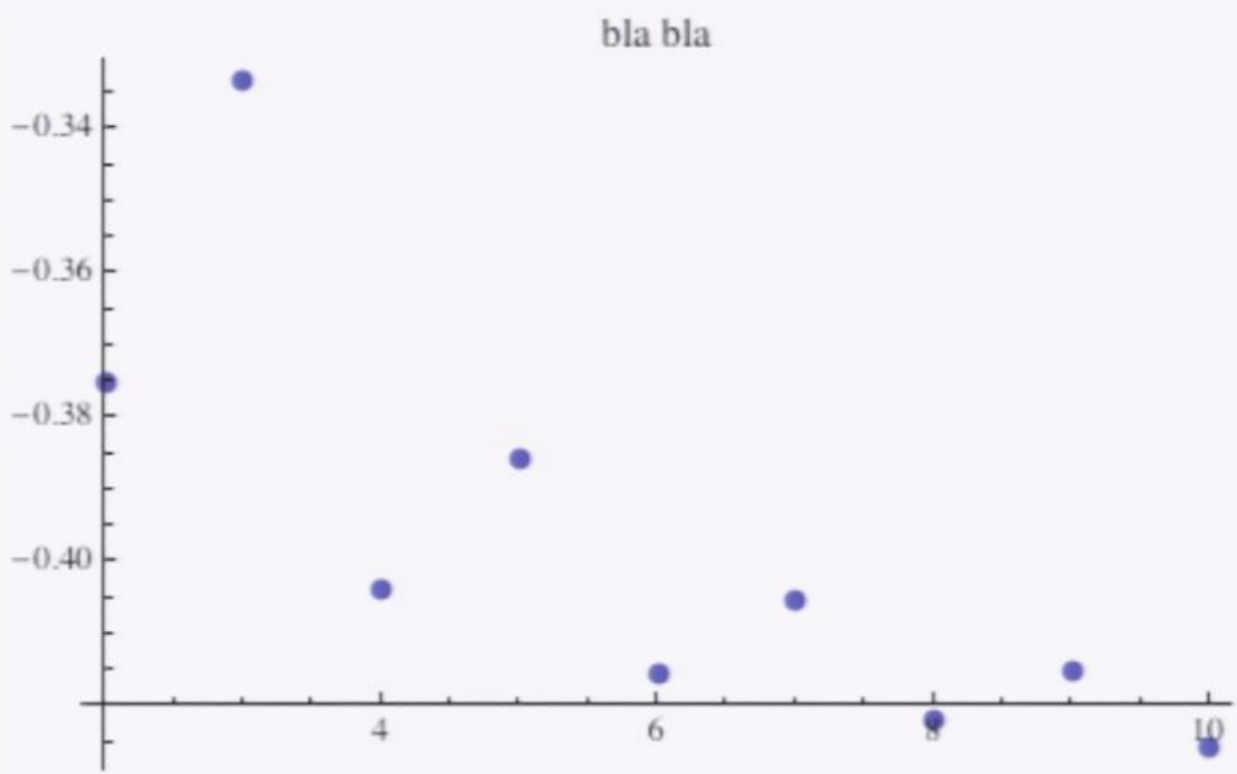
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
{5, -0.385577}, {6, -0.415596}, {7, -0.405177},
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
  PlotLabel → Style["bla bla", {Red, Italic}]]
```

Out[37]=




```
FullForm[lp]
```

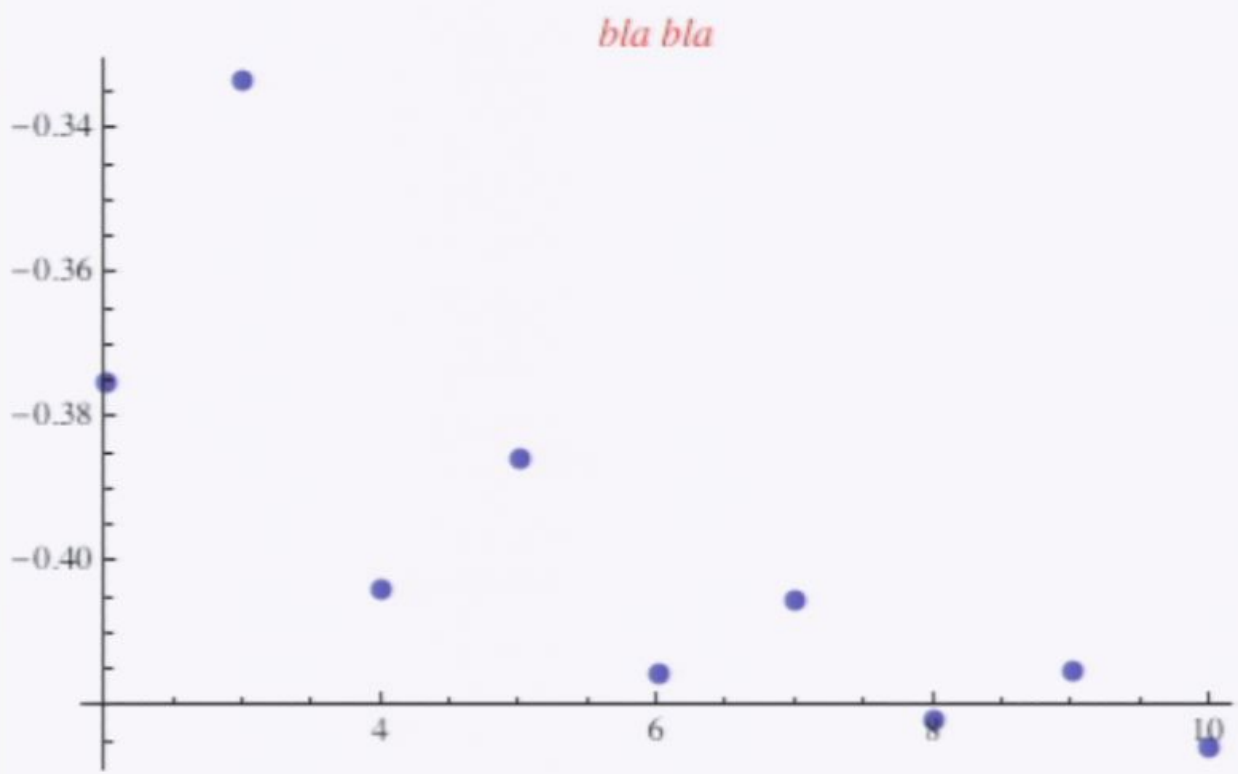
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
{5, -0.385577}, {6, -0.415596}, {7, -0.405177},
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
  PlotLabel → Style["E0| bla", {Red, Italic}]]
```

Out[38]=




```
FullForm[lp]
```

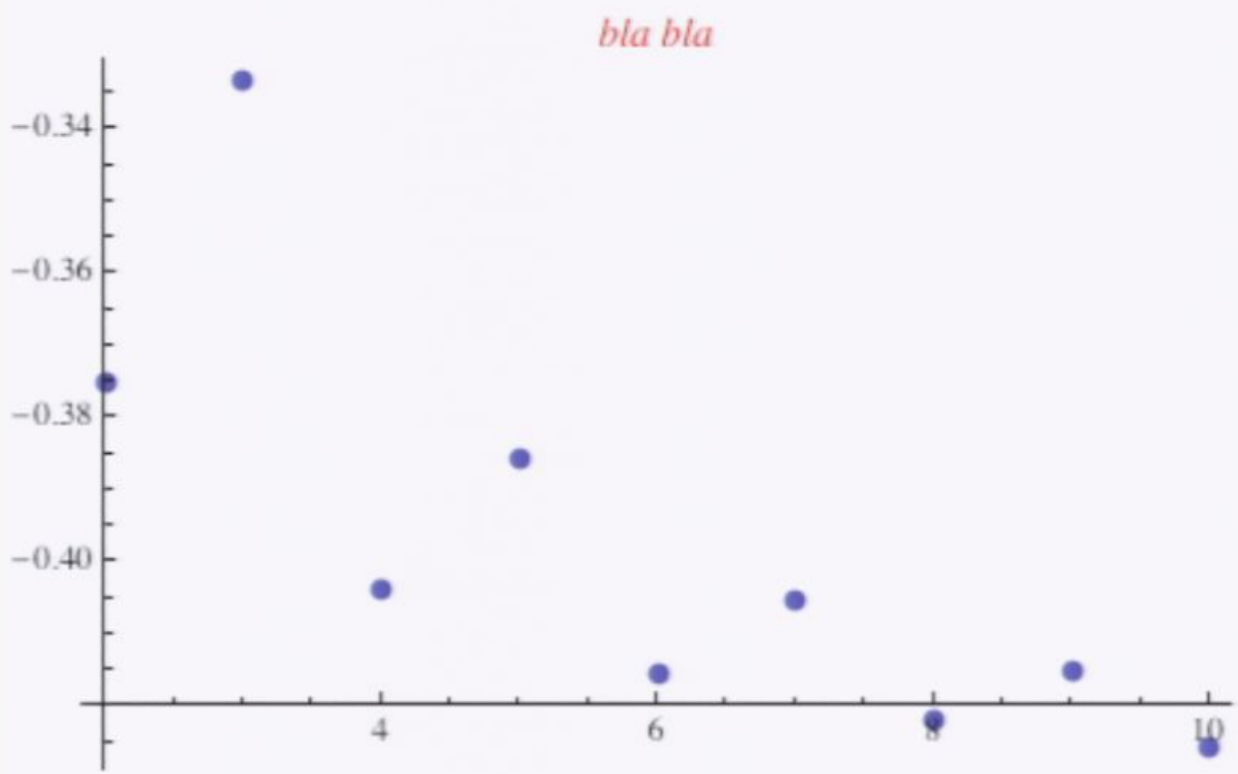
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
{5, -0.385577}, {6, -0.415596}, {7, -0.405177},
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
  PlotLabel → Style["E0 of the Heisen bla", {Red, Italic}]]
```

Out[38]=




```
FullForm[lp]
```

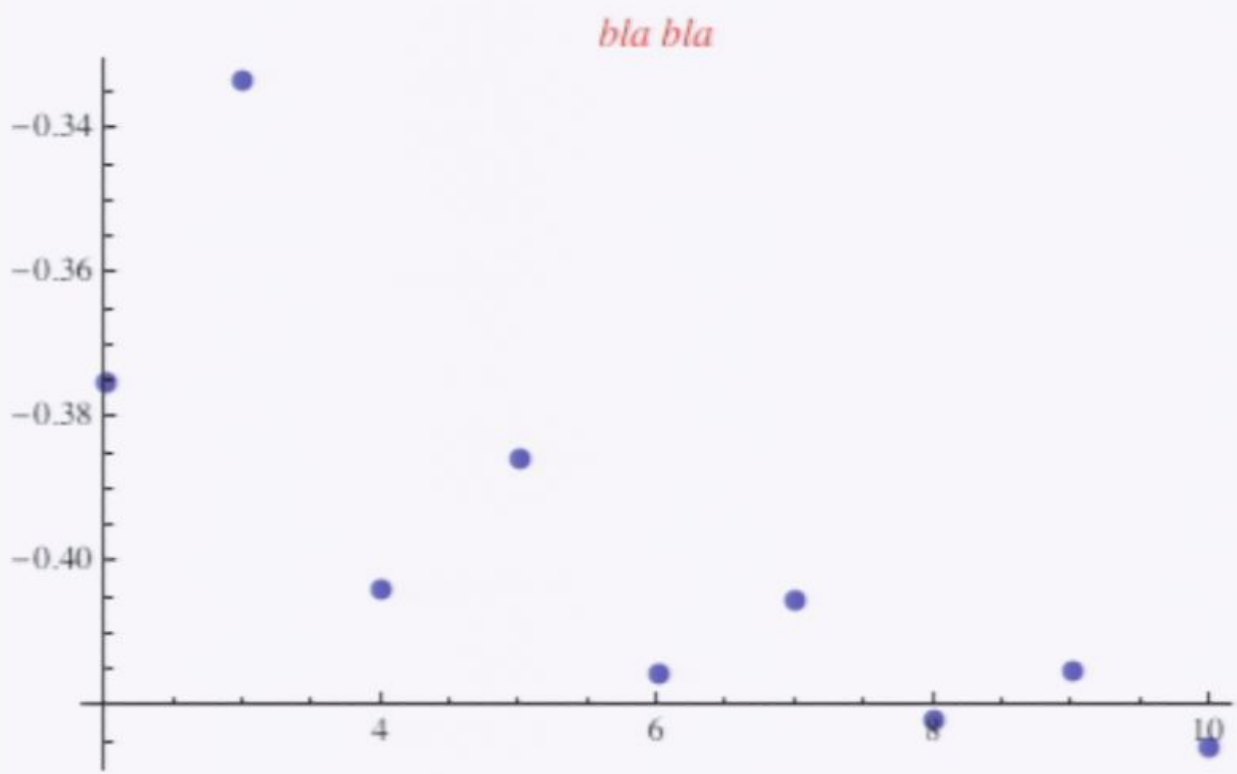
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
{5, -0.385577}, {6, -0.415596}, {7, -0.405177},
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
  PlotLabel → Style["E0 of the Heisenberg | bla", {Red, Italic}]]
```

Out[38]=




```
FullForm[lp]
```

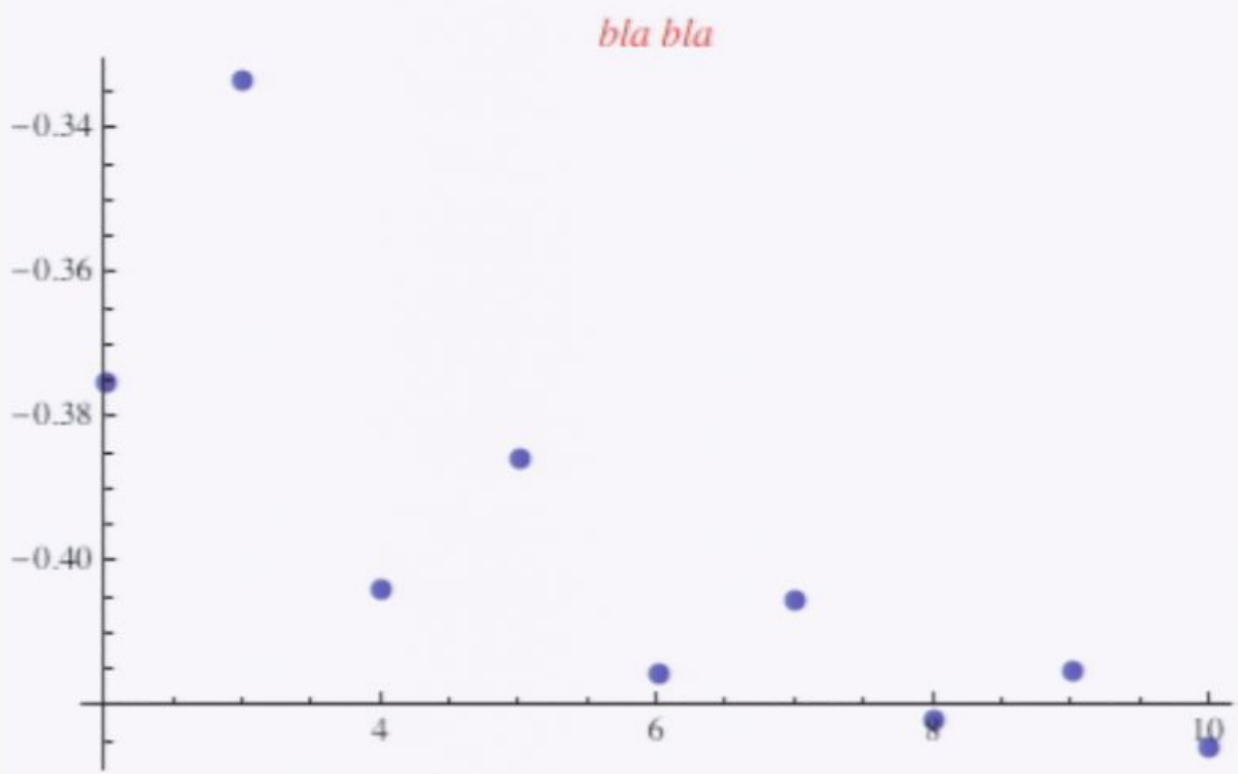
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
{5, -0.385577}, {6, -0.415596}, {7, -0.405177},
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
  PlotLabel → Style["E0 of the Heisenberg chain | bla", {Red, Italic}]]
```

Out[38]=




```
FullForm[lp]
```

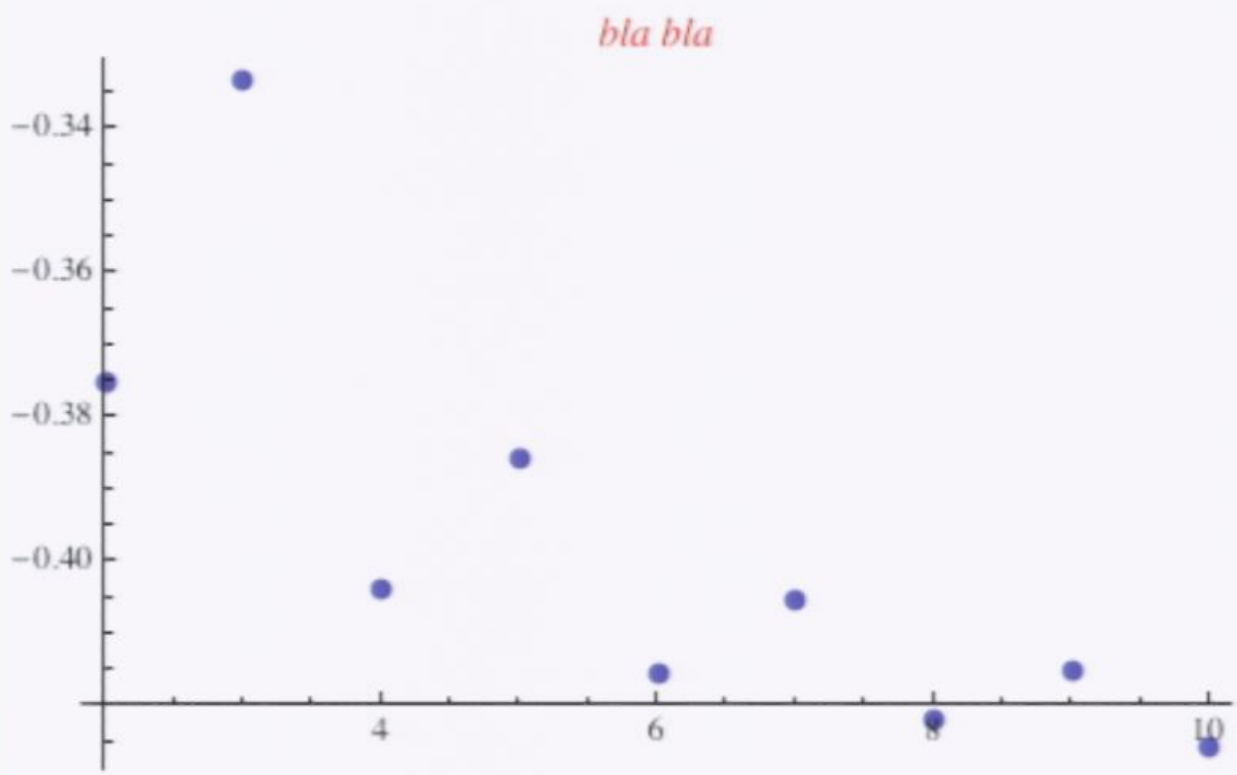
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
{5, -0.385577}, {6, -0.415596}, {7, -0.405177},
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
  PlotLabel → Style["E0 of the Heisenberg chain for L=| bla",
    {Red, Italic}]]
```

Out[38]=



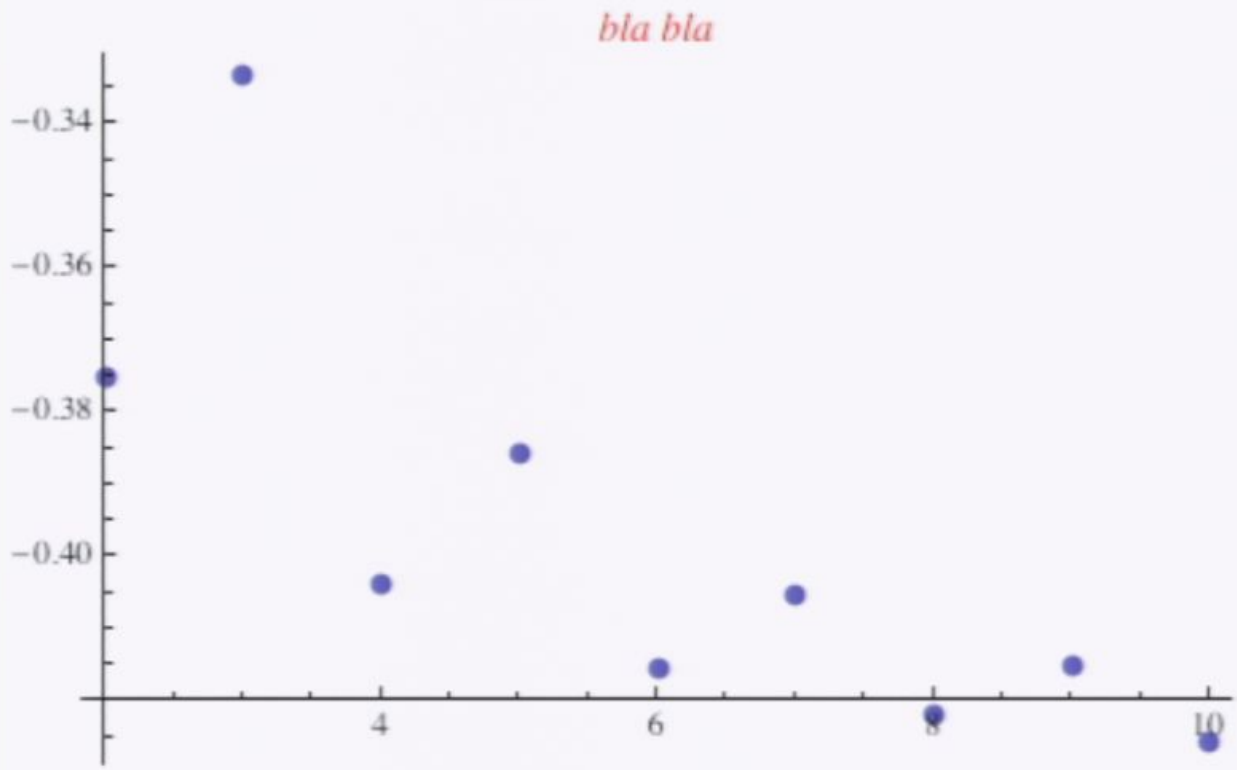
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
{5, -0.385577}, {6, -0.415596}, {7, -0.405177},
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
  PlotLabel → Style["E0 of the Heisenberg chain for L=2,...,",
    {Red, Italic}]]
```

Out[38]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[27]:=
```
(tb = Table[{L, Eigenvalues[Hs2[[L - 1]], 1][[1]]}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[27]=
```
{0.010763, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
  PlotLabel → Style["E0 of the Heisenberg chain for L=2,...,",
    {Red, Italic}]]
```

Out[38]=

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

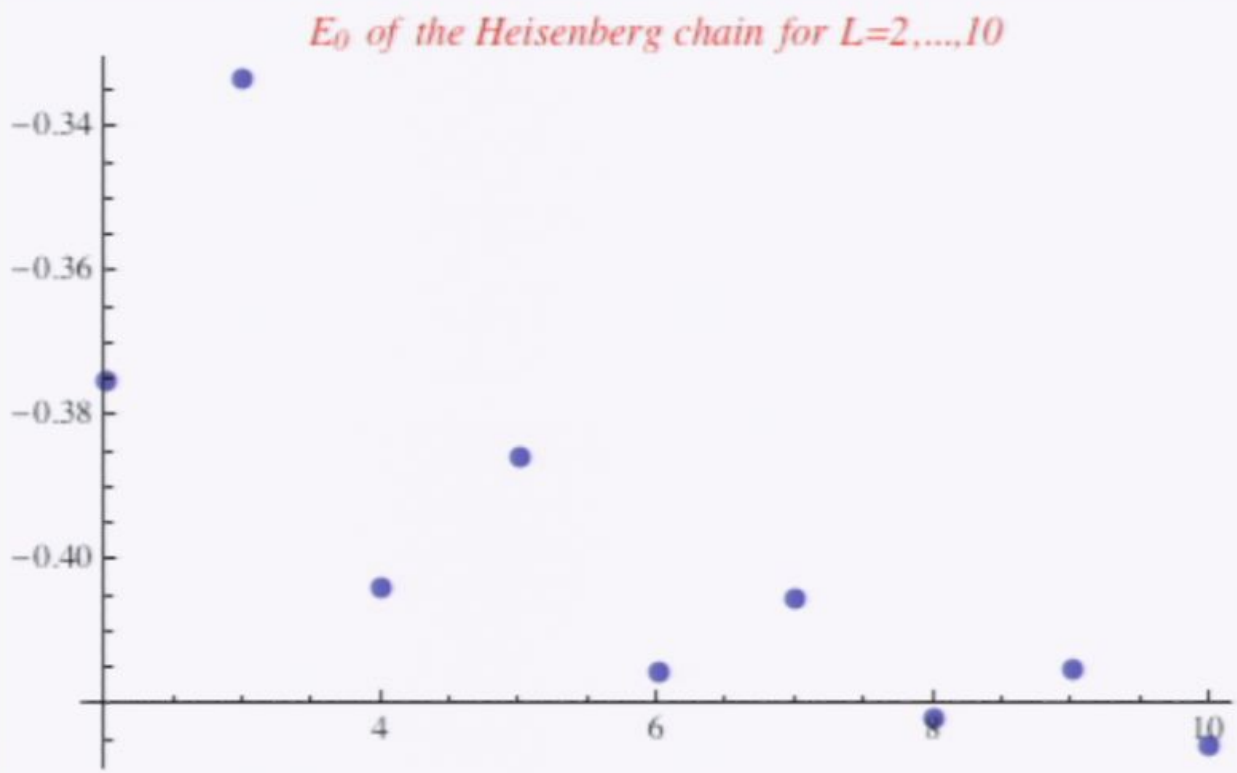
Untitled-25

```
{5, -0.385377}, {6, -0.415596}, {7, -0.405177},
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[39]:=

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
  PlotLabel → Style["E0 of the Heisenberg chain for L=2,...,10",
    {Red, Italic}]]
```

Out[39]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

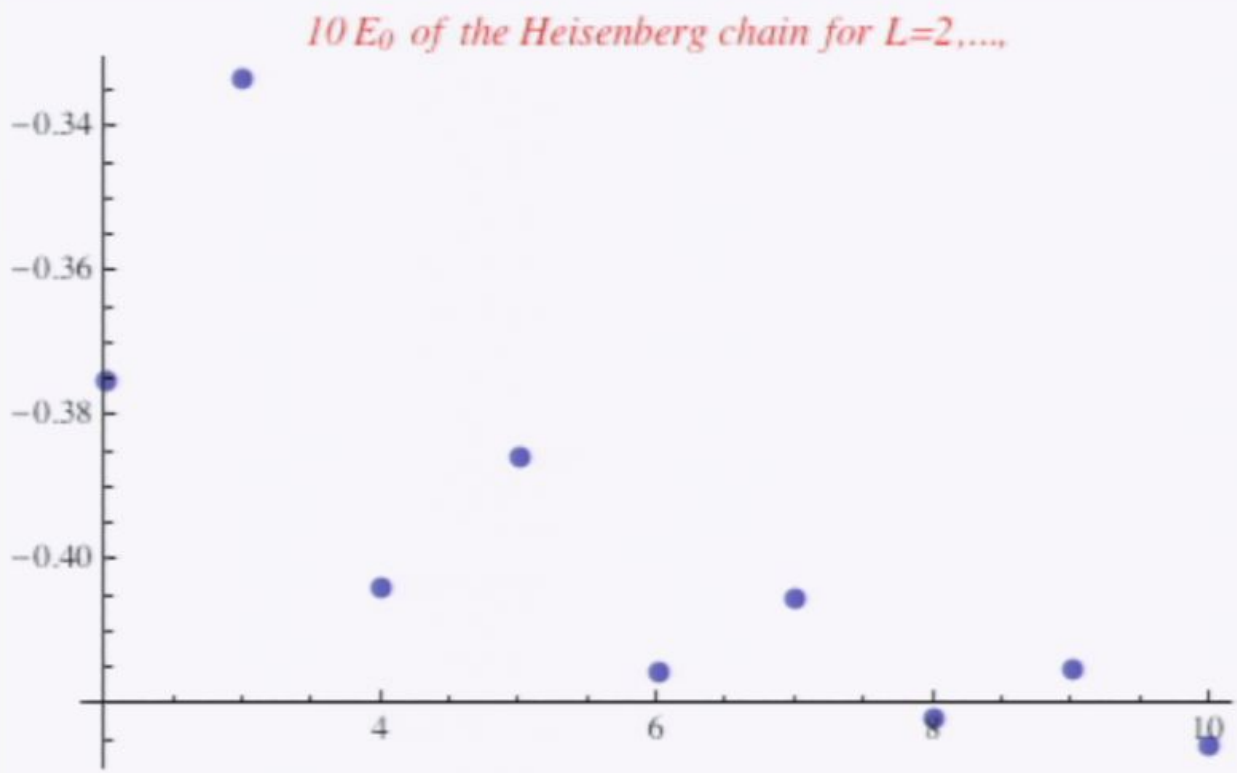
Untitled-25

```
{5, -0.385377}, {6, -0.415396}, {7, -0.405177},
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[40]:=

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
  PlotLabel → Style["E0 of the Heisenberg chain for L=2,...," max,
    {Red, Italic}]]
```

Out[40]=



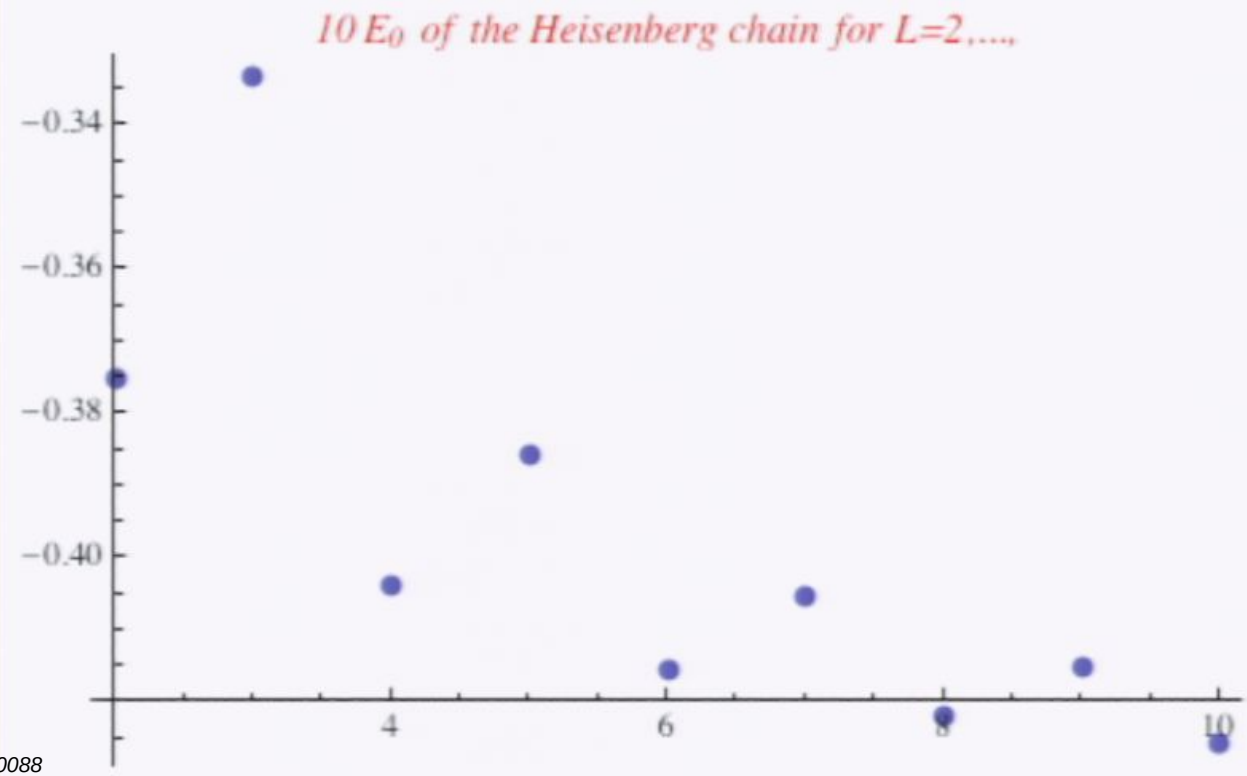
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
{5, -0.385377}, {6, -0.415596}, {7, -0.405177},
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
  PlotLabel →
   Style["E0 of the Heisenberg chain for L=2,...,"
     ToString[max, {Red, Italic}]]
```

Out[40]=



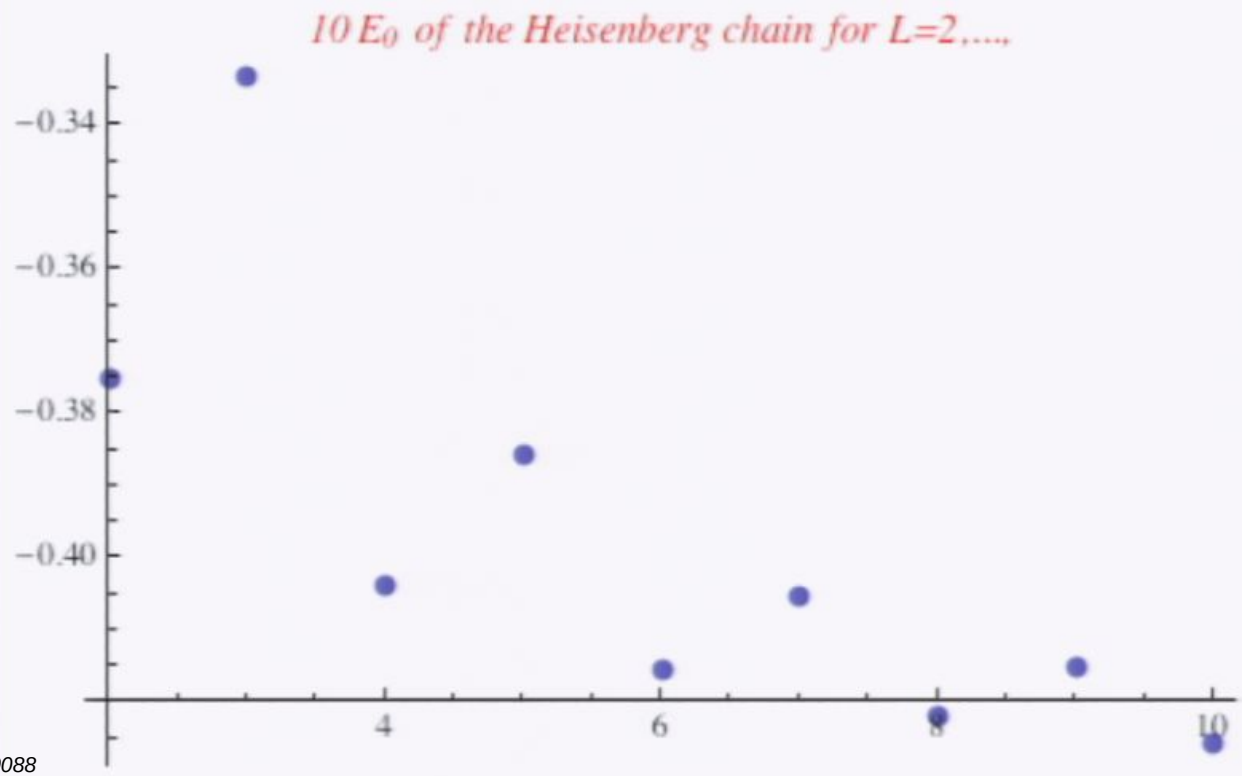
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
{5, -0.385377}, {6, -0.415596}, {7, -0.405177},
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
  PlotLabel →
   Style["E0 of the Heisenberg chain for L=2,...," ToString[max],
    {Red, Italic}]]
```

Out[40]=



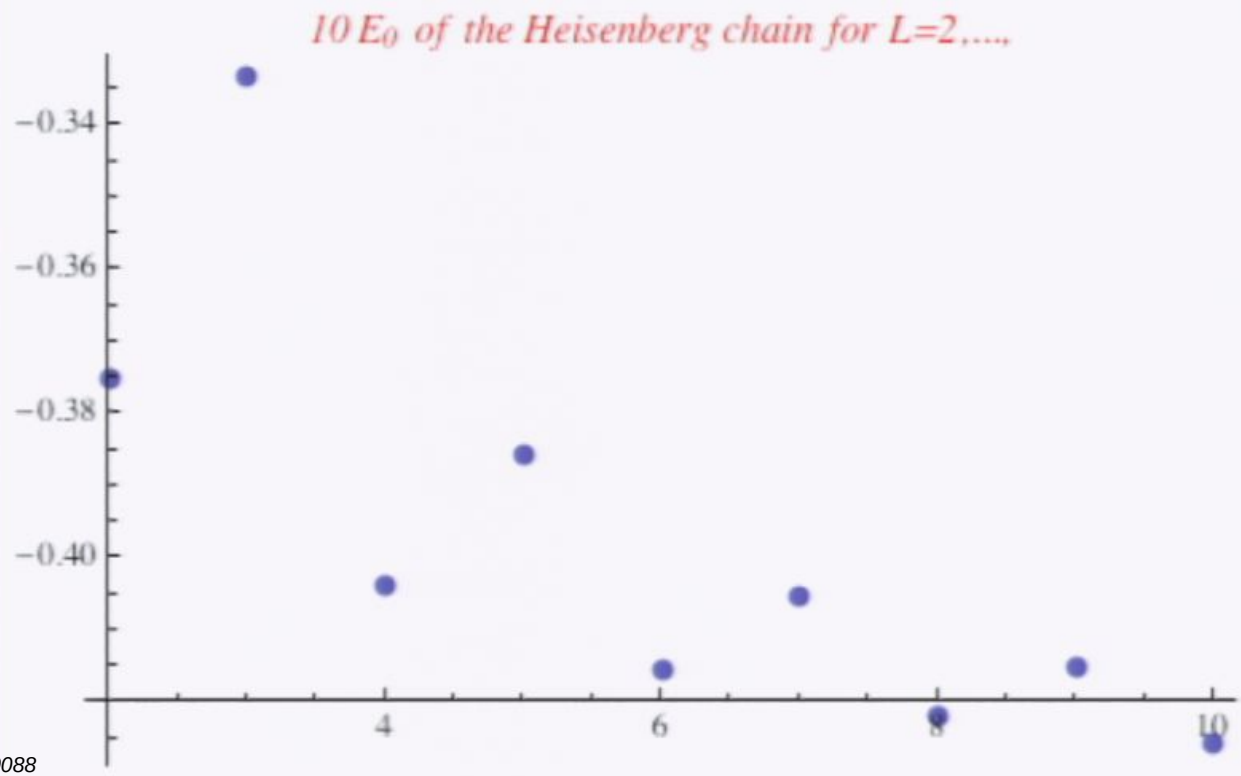
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
{5, -0.385377}, {6, -0.415396}, {7, -0.405177},
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
  PlotLabel →
   Style["E0 of the Heisenberg chain for L=2,...," <>ToString[max],
    {Red, Italic}]]
```

Out[40]=



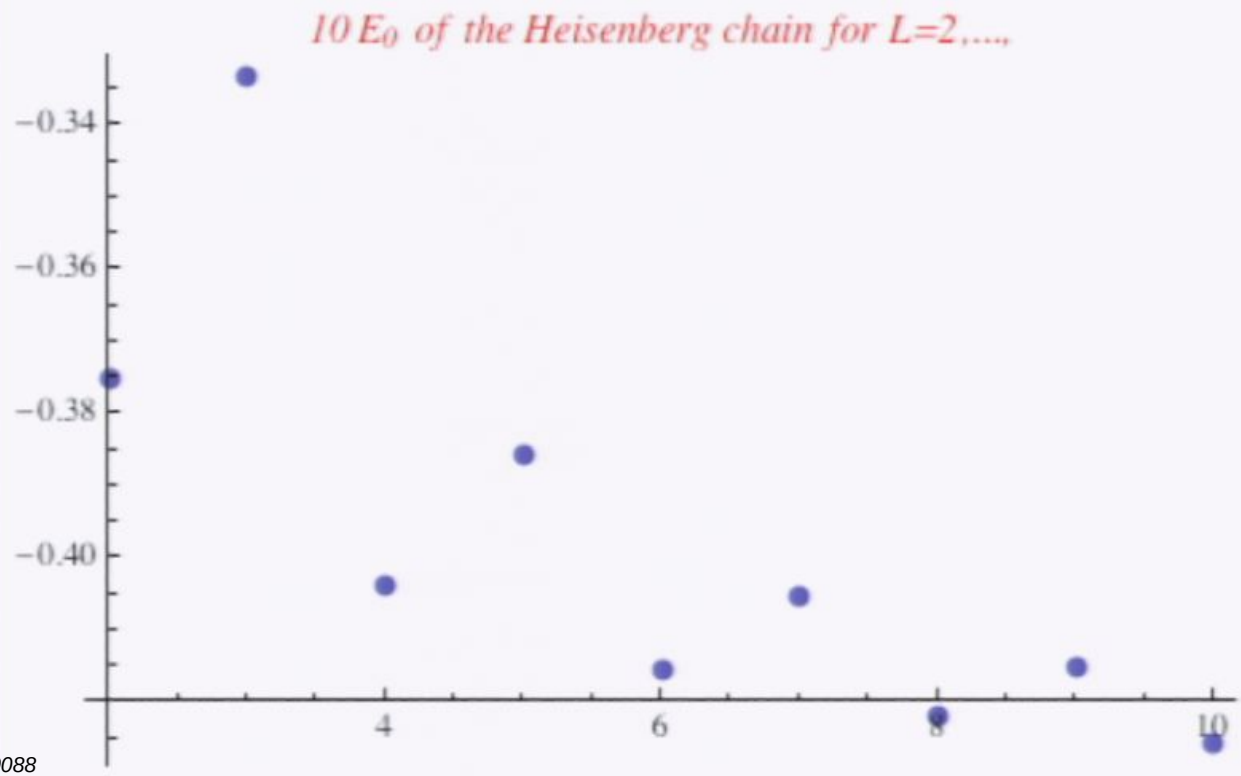
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
{5, -0.385377}, {6, -0.415396}, {7, -0.405177},
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
  PlotLabel →
   Style["E0 of the Heisenberg chain for L=2,...," <> ToString[max],
    {Red, Italic}]]
```

Out[40]=



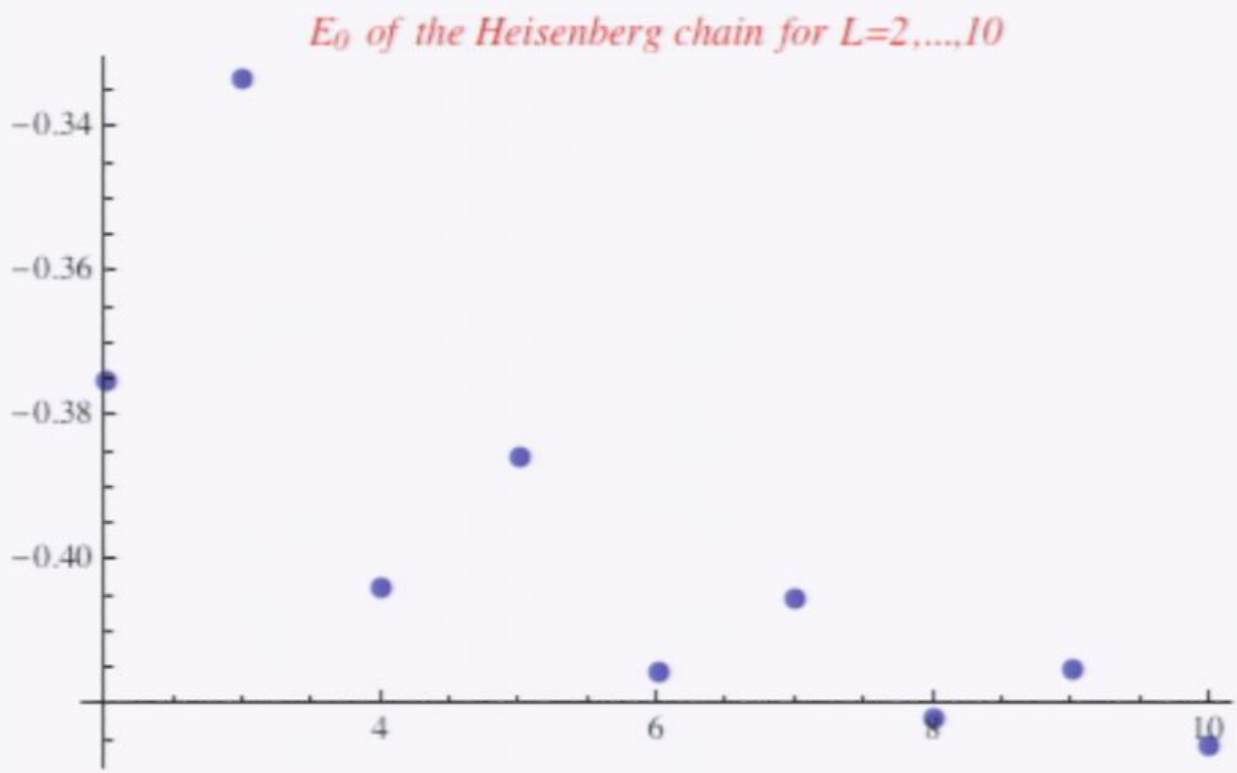
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[41]:=

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
  PlotLabel →
   Style["E0 of the Heisenberg chain for L=2,...," <> ToString[max],
    {Red, Italic}]]
```

Out[41]=

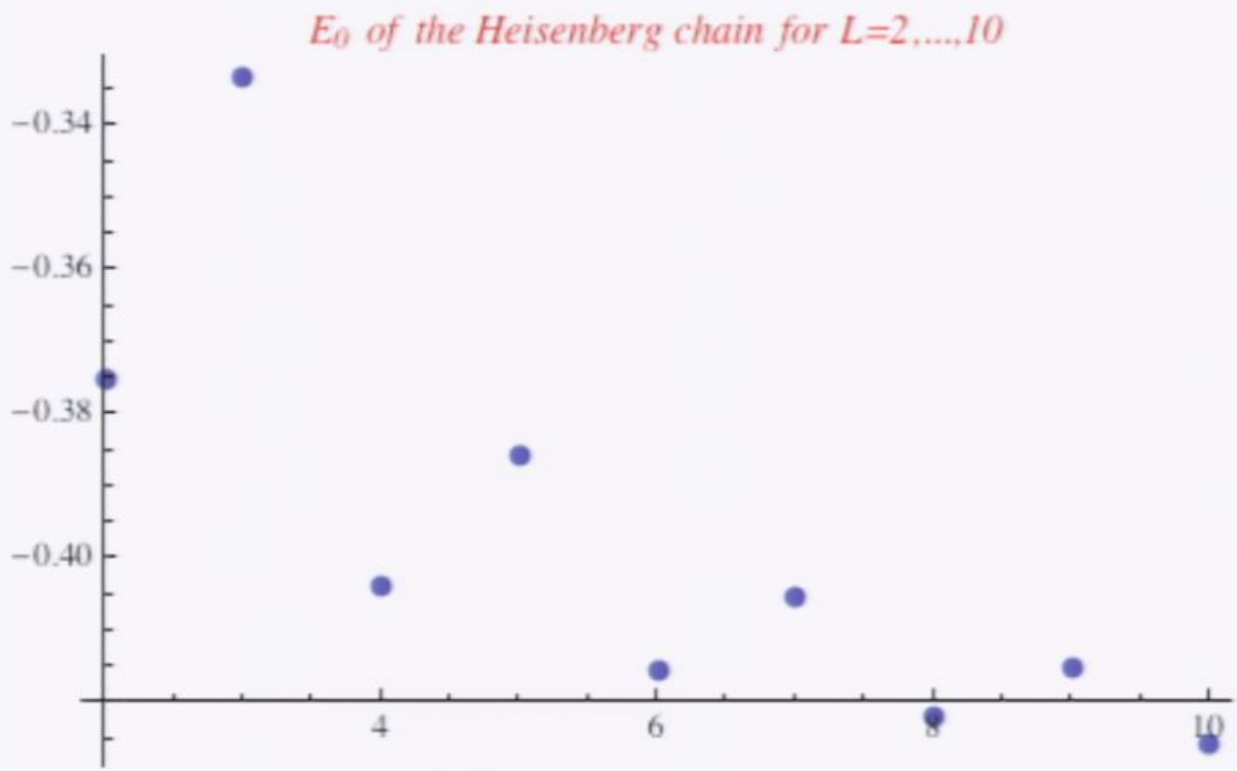


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
PlotLabel →
 Style["E0 of the Heisenberg chain for L=2,...," <> ToString[max],
  {Red, Italic}]]
```

Out[41]=



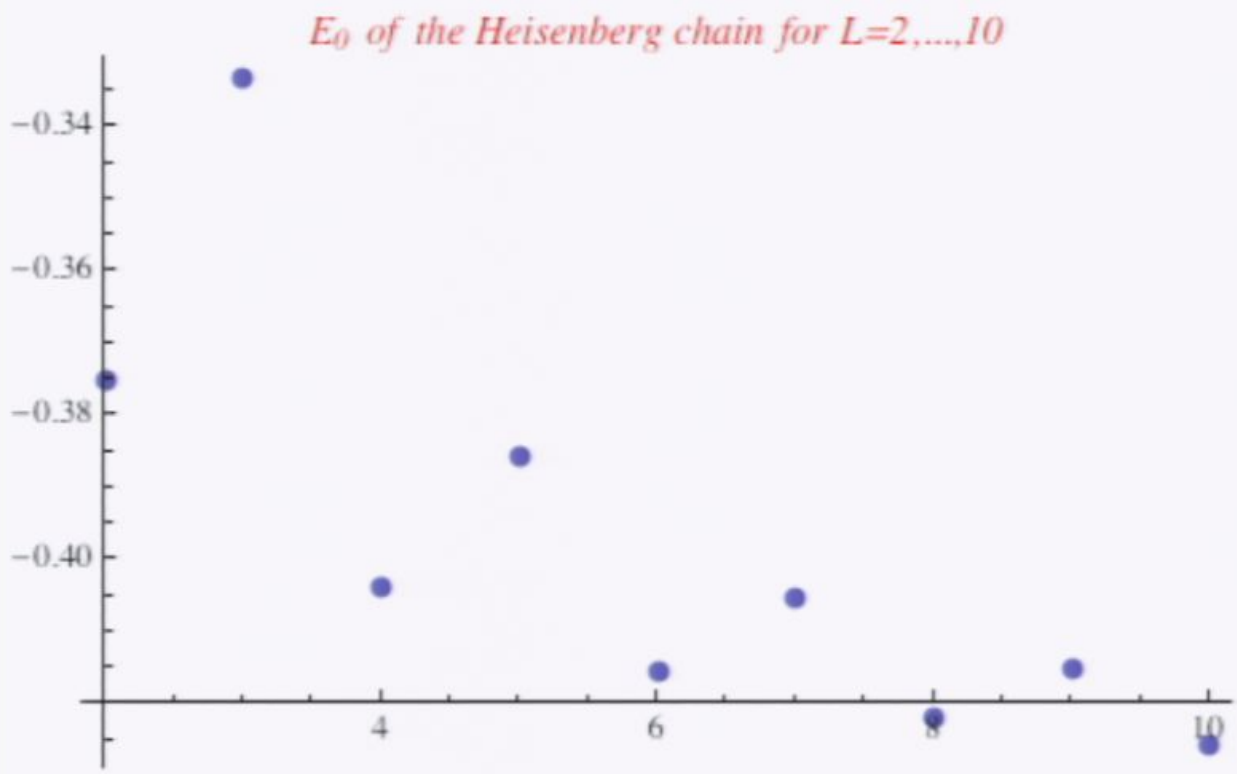

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
{Red, Italic}]]
```

Out[41]=



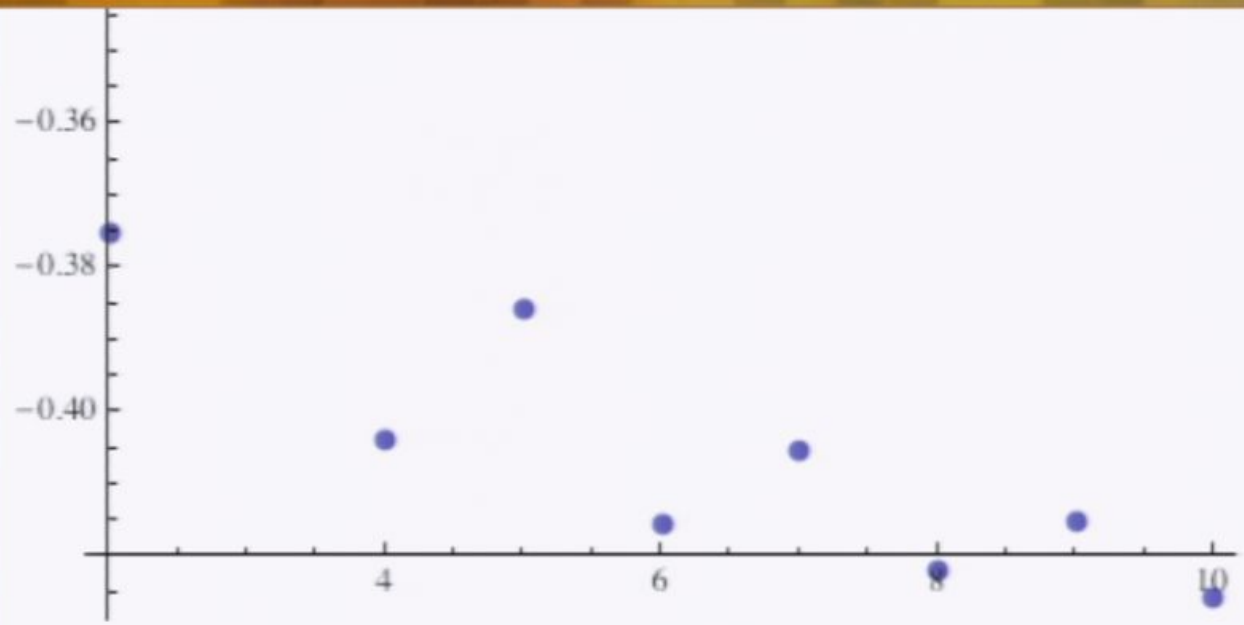

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

```
Fit[
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[41]=




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[42]:=

```
tb
```

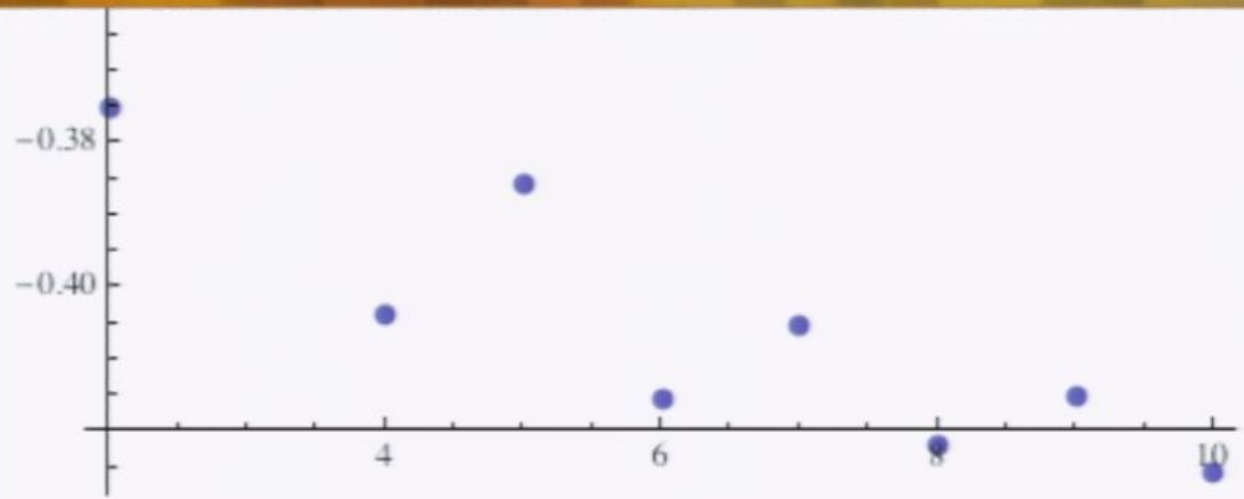
Out[42]=

```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[41]=




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[42]:=

```
tb
```

Out[42]=

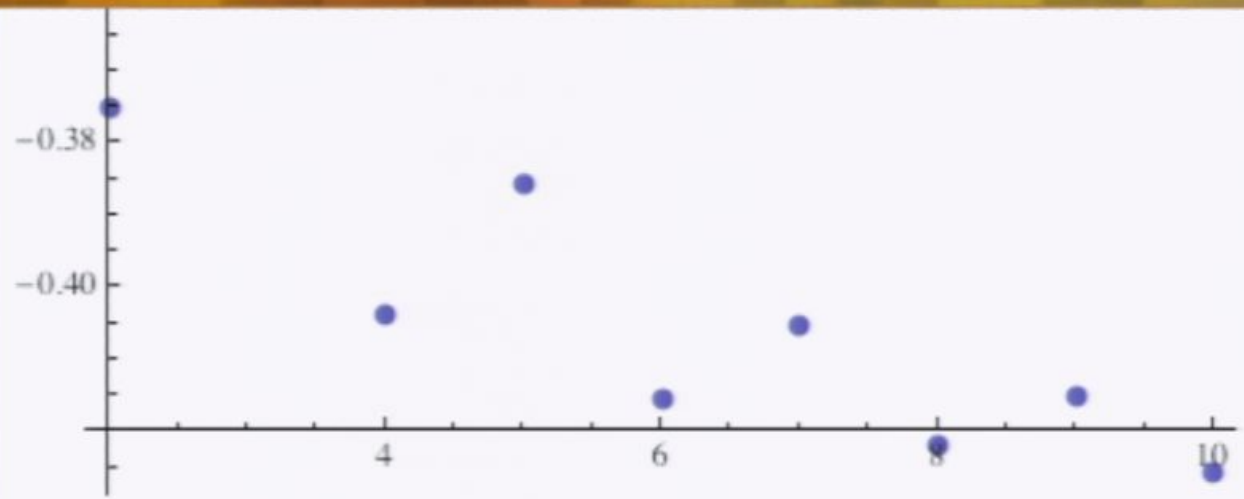
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

```
Fit[
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[41]=




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[42]:=

```
tb
```

Out[42]=

```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

```
Fit[tb, {1, 1/L, 1/L^2}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[42]:=
```
tb
```

Out[42]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

```
Fit[tb, {1, 1/L, 1/L^2}, L]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[42]:=
```
tb
```

Out[42]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[43]:=
```
Fit[tb, {1, 1/L, 1/L^2}, L]
```

Out[43]=

$$-0.485173 - \frac{0.812302}{L^2} + \frac{0.63849}{L}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[42]:=
```
tb
```

Out[42]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[44]:=
```
fit = Fit[tb, {1, 1/L, 1/L^2}, L]
```

Out[44]=

$$-0.485173 - \frac{0.812302}{L^2} + \frac{0.63849}{L}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[42]:=
```
tb
```

Out[42]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[44]:=
```
fit = Fit[tb, {1, 1/L, 1/L^2}, L]
```

Out[44]=

$$-0.485173 - \frac{0.812302}{L^2} + \frac{0.63849}{L}$$

```
Plot[
```

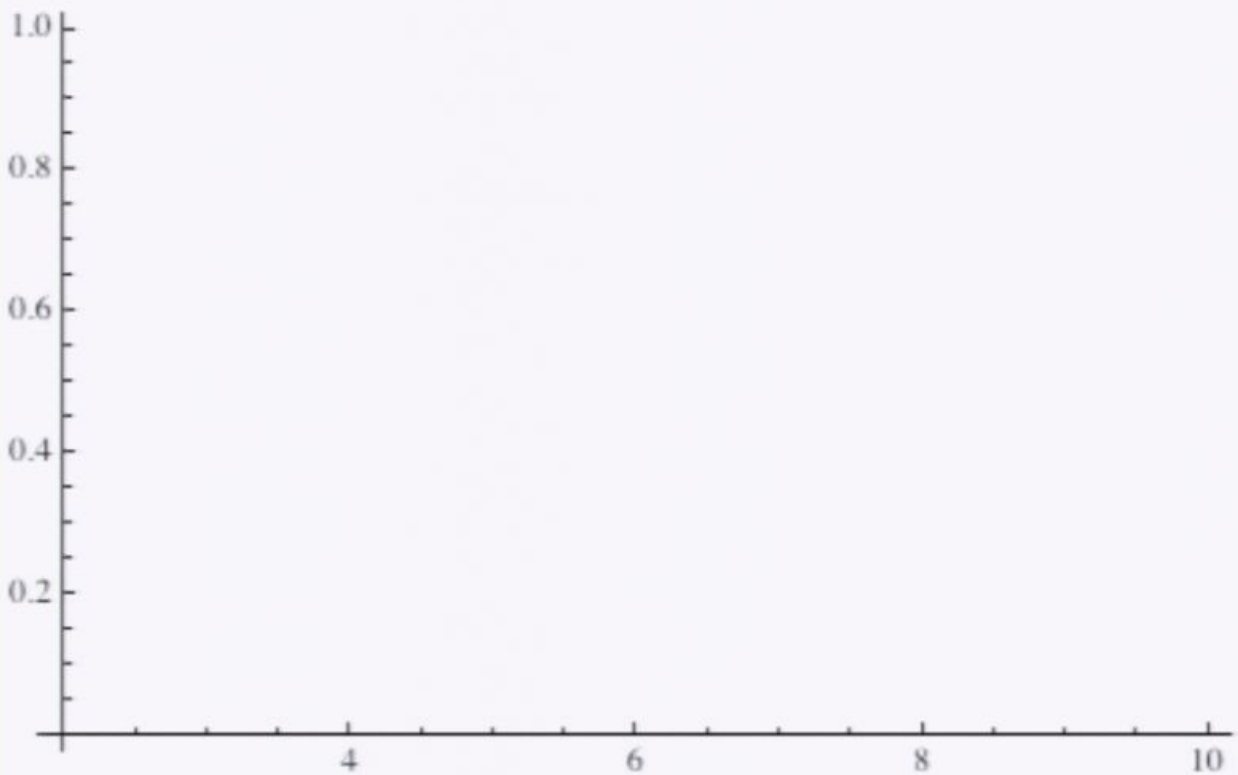
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[44]= $-0.485173 - \frac{0.812302}{L^2} + \frac{0.63849}{L}$

In[45]:=
```
Plot[fit, {x, 2, 10}]
```

Out[45]=



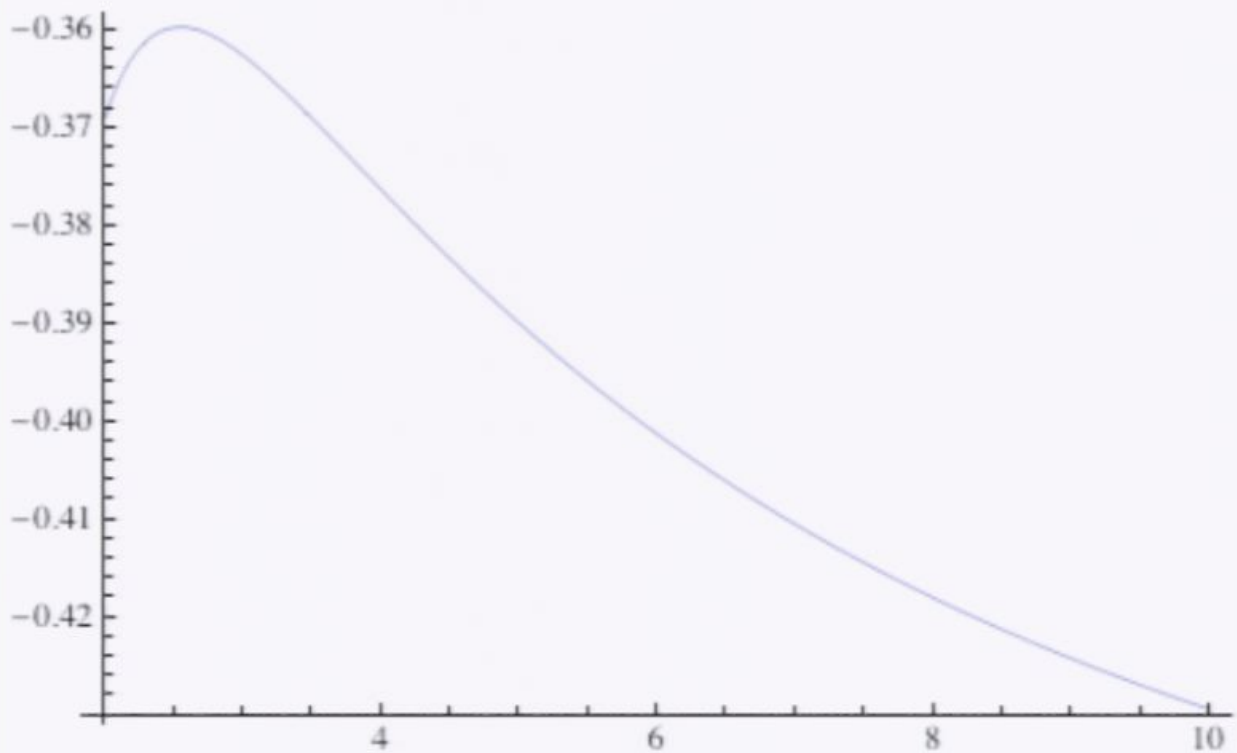
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[44]= $-0.485173 - \frac{0.812302}{L^2} + \frac{0.63849}{L}$

In[46]:=
```
Plot[fit, {L, 2, 10}]
```

Out[46]=



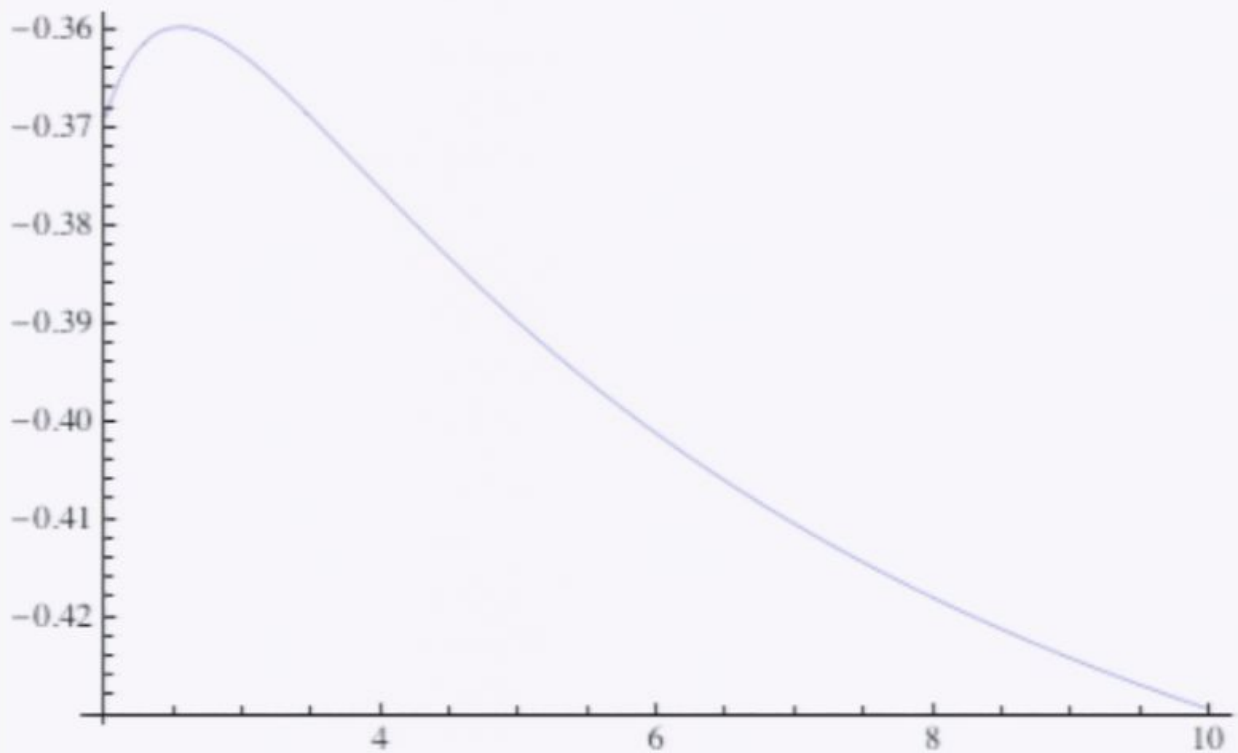
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[44]= $-0.485173 - \frac{0.812302}{L^2} + \frac{0.63849}{L}$

```
Plot[Plot[fit, {L, 2, 10}]
```

Out[46]=



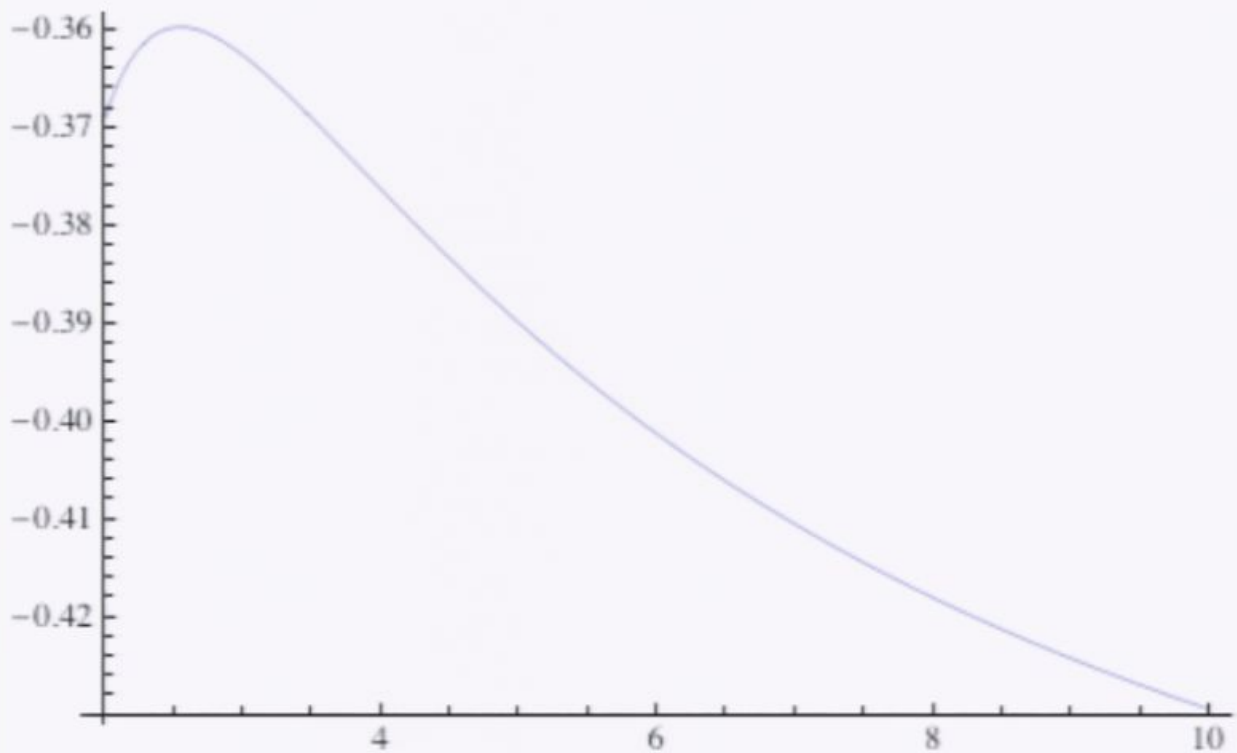
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[44]= $-0.485173 - \frac{0.812302}{L^2} + \frac{0.63849}{L}$

```
Show[Plot[fit, {L, 2, 10}]
```

Out[46]=



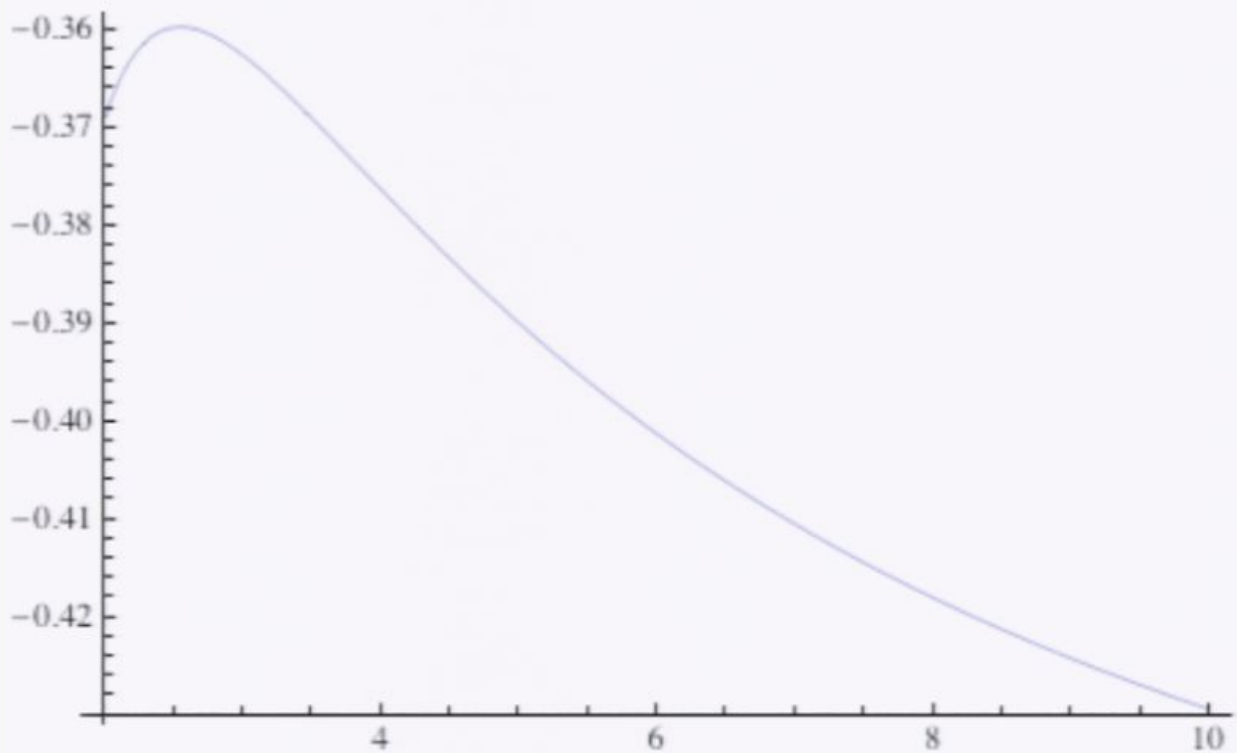
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[44]= $-0.485173 - \frac{0.812302}{L^2} + \frac{0.63849}{L}$

```
Show[lp, Plot[fit, {L, 2, 10}]]
```

Out[46]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

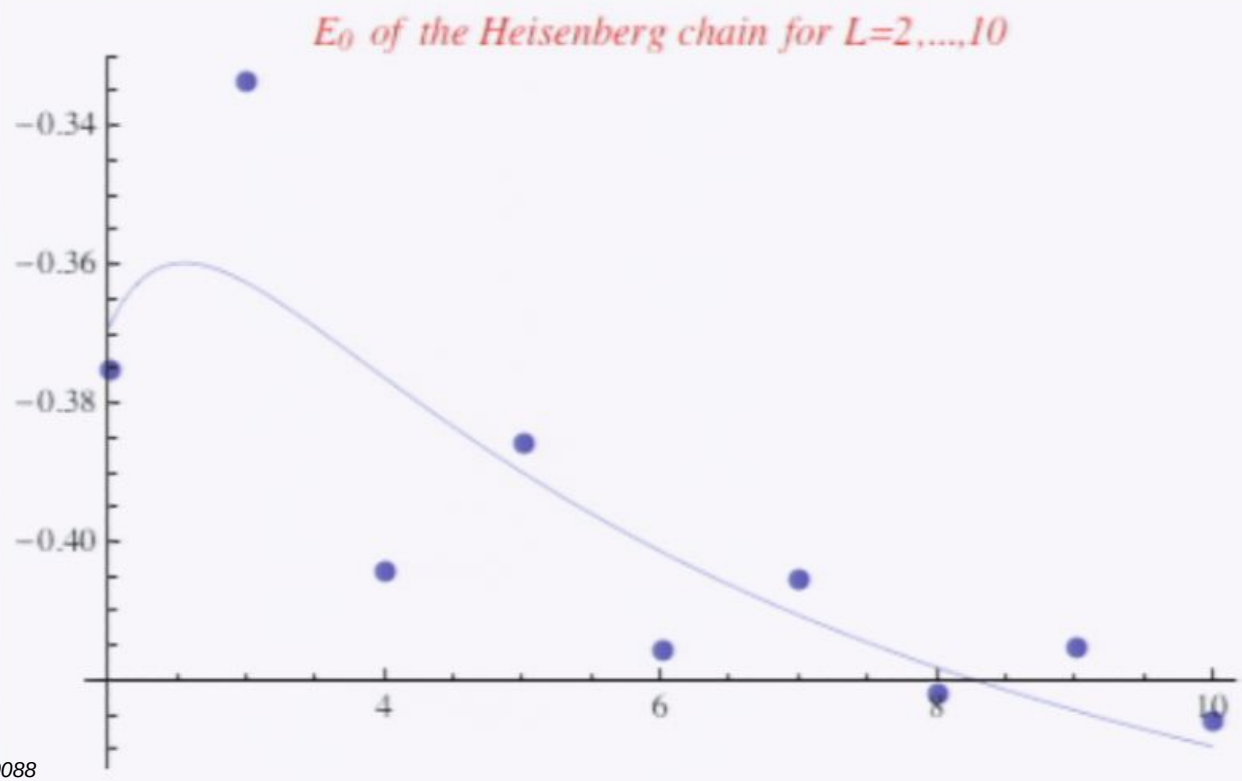
Untitled-25

In[44]:=
```
fit = Fit[tb, {1, 1/L, 1/L^2}, L]
```

Out[44]=
$$-0.485173 - \frac{0.812302}{L^2} + \frac{0.63849}{L}$$

In[47]:=
```
Show[lp, Plot[fit, {L, 2, 10}]]
```

Out[47]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

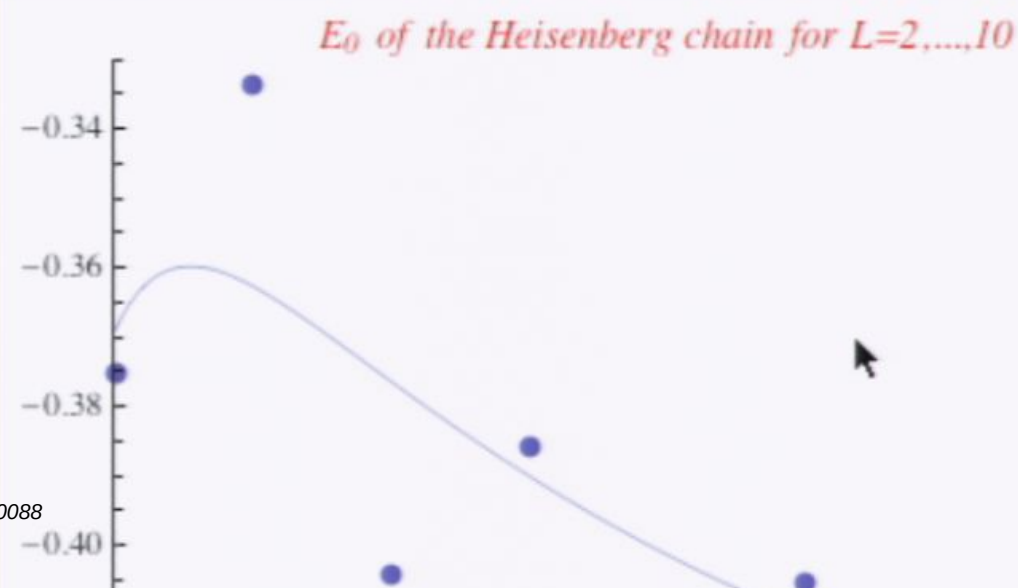
Out[42]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[44]:=
```
fit = Fit[tb, {1, 1/L, 1/L^2}, L]
```

Out[44]=
$$-0.485173 - \frac{0.812302}{L^2} + \frac{0.63849}{L}$$

In[47]:=
```
Show[lp, Plot[fit, {L, 2, 10}]]
```

Out[47]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

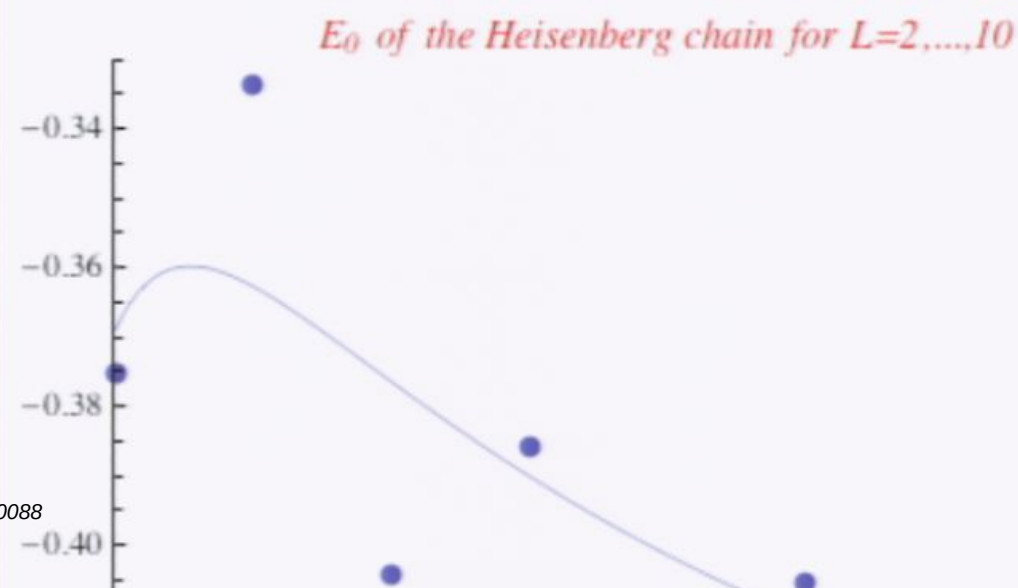
Out[42]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[44]:=
```
fit = Fit[tb, {1, 1/L, 1/L^2}, L]
```

Out[44]=
$$-0.485173 - \frac{0.812302}{L^2} + \frac{0.63849}{L}$$

In[47]:=
```
Show[lp, Plot[fit, {L, 2, 10}]]
```

Out[47]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25


```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[42]:=
```
tb
```

Out[42]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[44]:=
```
fit = Fit[tb, {1, 1/L, 1/L^2}, L]
```

Out[44]=
$$-0.485173 - \frac{0.812302}{L^2} + \frac{0.63849}{L}$$

In[47]:=
```
Show[lp, Plot[fit, {L, 2, 10}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

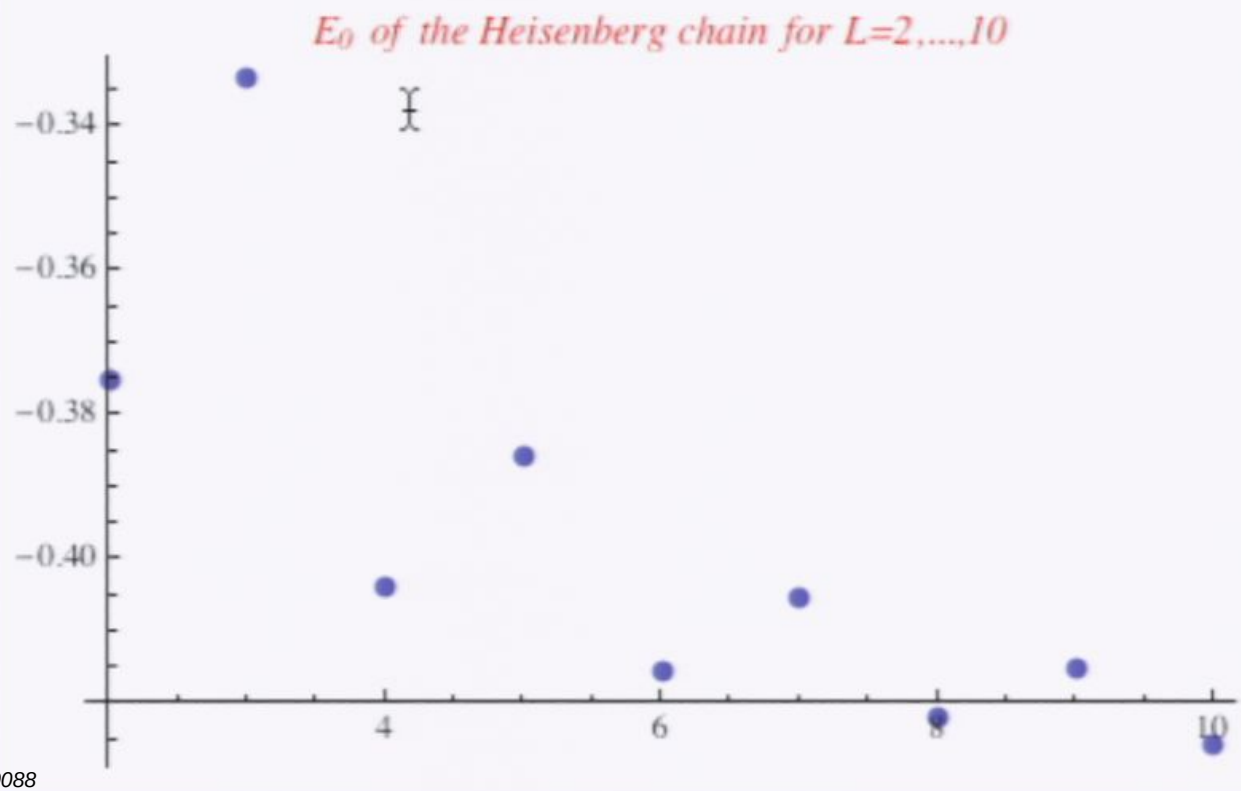
Untitled-25

```
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

In[41]:=

```
lp = ListPlot[tb, PlotStyle → {PointSize → 0.02},
  PlotLabel →
   Style["E0 of the Heisenberg chain for L=2,...," <> ToString[max],
    {Red, Italic}]]
```

Out[41]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

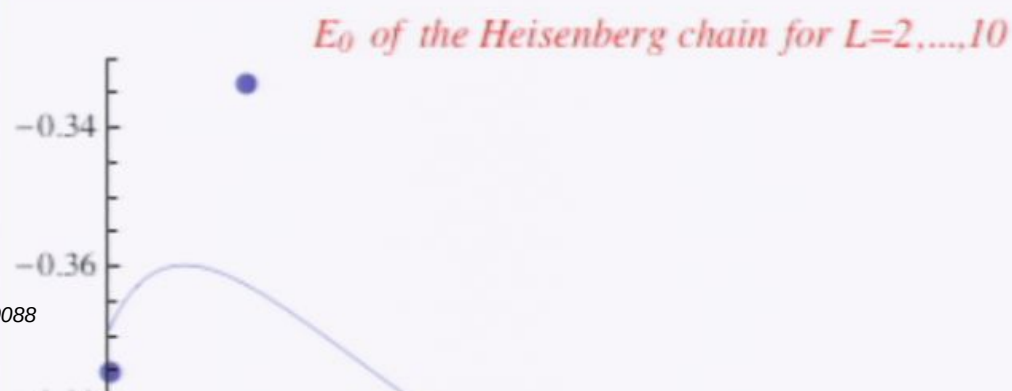
In[42]:=
```
tb
```

Out[42]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[44]:=
```
fit = Fit[tb, {1, 1/L, 1/L^2}, L]
```

Out[44]=
$$-0.485173 - \frac{0.812302}{L^2} + \frac{0.63849}{L}$$

In[47]:=
```
Show[lp, Plot[fit, {L, 2, 10}]]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

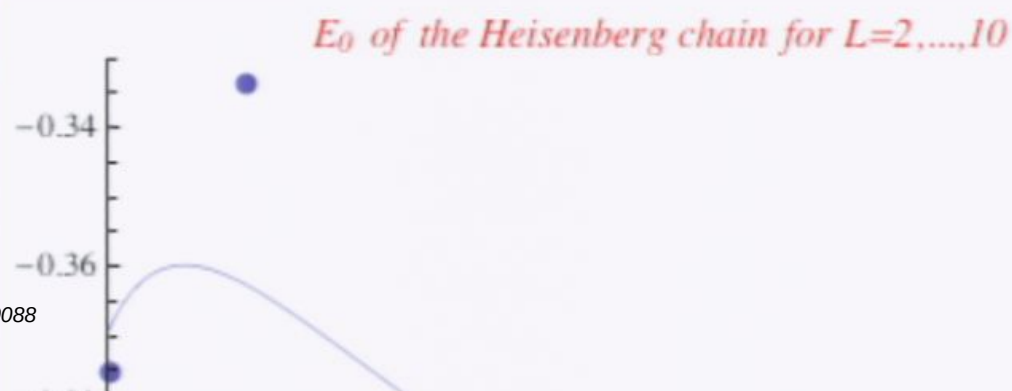
In[42]:=
```
tb
```

Out[42]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[48]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[48]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

In[47]:=
```
Show[lp, Plot[fit, {L, 2, 10}]]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

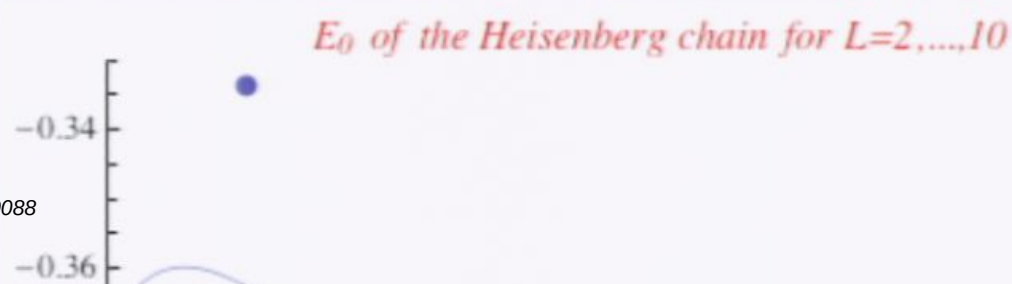
In[42]:=
```
tb
```

Out[42]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[48]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[48]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

In[47]:=
```
Show[lp, Plot[fit, {L, 2, 10}]]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

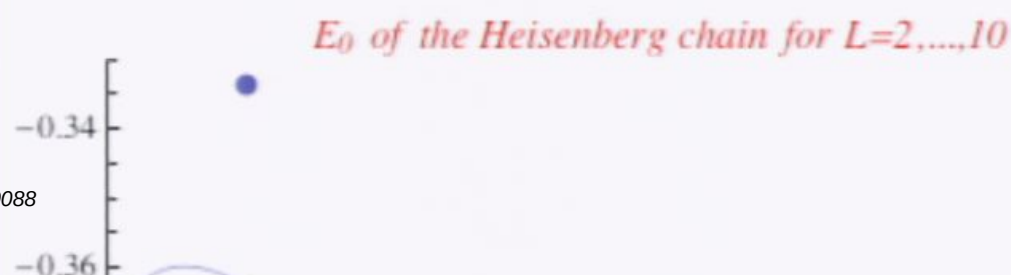
In[42]:=
```
tb
```

Out[42]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[48]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[48]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

In[47]:=
```
Show[lp, Plot[fit, {L, 2, 10}]]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

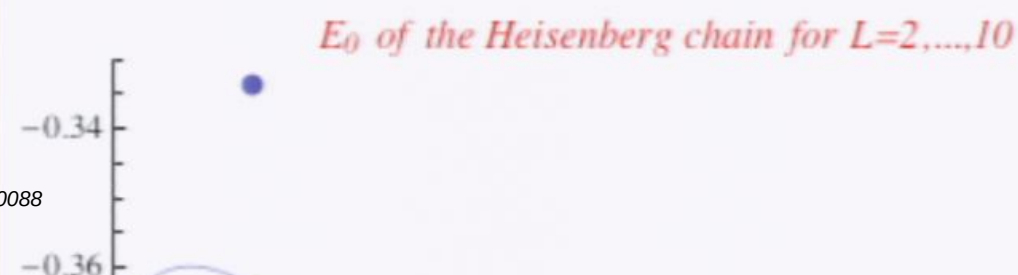
In[42]:=
```
tb
```

Out[42]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[49]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[49]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

In[47]:=
```
Show[lp, Plot[fit, {L, 2, 10}]]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

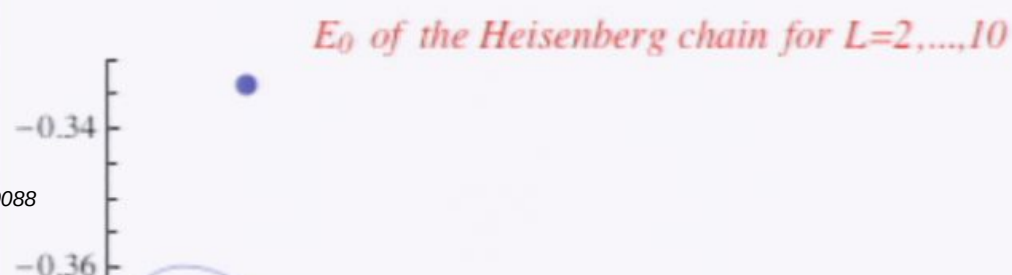
In[42]:=
```
tb
```

Out[42]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[49]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[49]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

In[47]:=
```
Show[lp, Plot[fit, {L, 2, 10}]]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

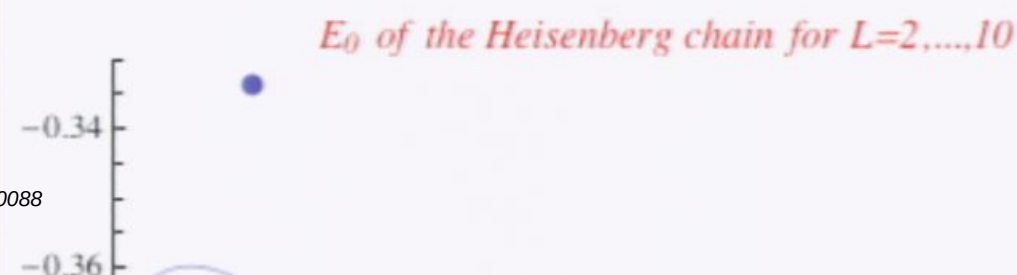
In[42]:=
```
tb
```

Out[42]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[49]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[49]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

In[47]:=
```
Show[lp, Plot[fit, {L, 2, 10}]]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

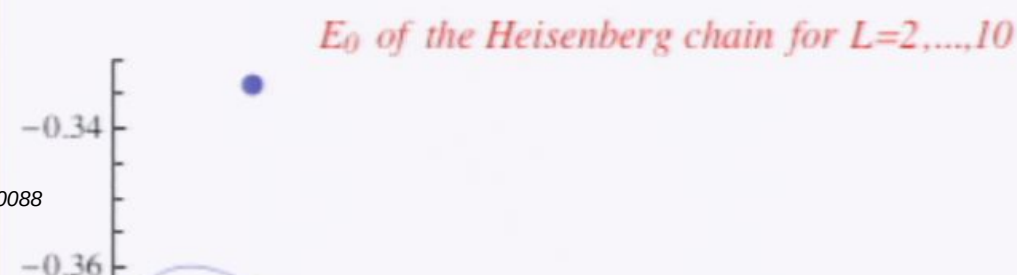
In[50]:=
```
tb
```

Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

In[47]:=
```
Show[lp, Plot[fit, {L, 2, 10}]]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

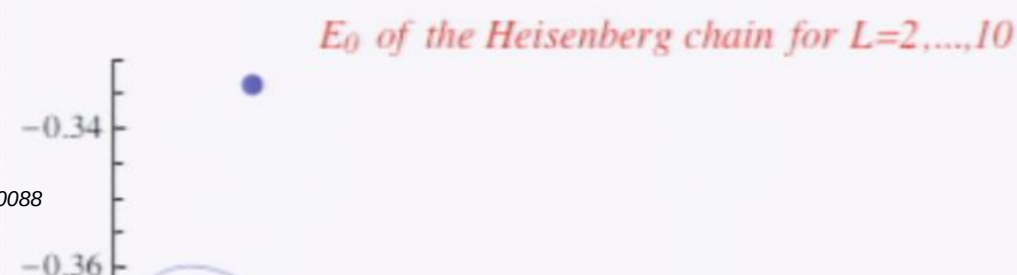
In[50]:=
```
tb
```

Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

In[47]:=
```
Show[lp, Plot[fit, {L, 2, 10}]]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[50]:=
```
tb
```

Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

```
Fit[|
```

In[47]:=
```
Show[lp, Plot[fit, {L, 2, 10}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[50]:=
```
tb
```

Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

```
Fit[{{2, -0.375`}, {3, -0.3333333333333334`},
  {4, -0.40400635094610965`}, {5, -0.3855772506635989`},
  {6, -0.41559618898132095`}, {7, -0.4051770972409499`},
  {8, -0.42186657483598594`}, {9, -0.41514685626436837`},
  {10, -0.42580352072828803`}}
```



```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[50]:=
```
tb
```

Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

```
Fit[{{2, -0.375`}, {3, -0.3333333333333334`},
  {4, -0.40400635094610965`}, {5, -0.3855772506635989`},
  {6, -0.41559618898132095`}, {7, -0.4051770972409499`},
  {8, -0.42186657483598594`}, {9, -0.41514685626436837`},
  {10, -0.42580352072828803`}}, {
```



```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[50]:=
```
tb
```

Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

```
Fit[{{2, -0.375`}, {3, -0.3333333333333334`},
  {4, -0.4040063509461096 5`}, {5, -0.3855772506635989`},
  {6, -0.4155961889813209 5`}, {7, -0.4051770972409499`},
  {8, -0.4218665748359859 4`}, {9, -0.4151468562643683 7`},
  {10, -0.4258035207282880 3`}}, {1, 1/x, 1/x^2}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[50]:=
```
tb
```

Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

In[52]:=
```
Fit[{{2, -0.375`}, {3, -0.3333333333333334`},
  {4, -0.4040063509461096 5`}, {5, -0.3855772506635989`},
  {6, -0.4155961889813209 5`}, {7, -0.4051770972409499`},
  {8, -0.4218665748359859 4`}, {9, -0.4151468562643683 7`},
  {10, -0.4258035207282880 3`}}, {1, 1/x, 1/x^2}, x]
```

Out[52]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[50]:=
```
tb
```

Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

In[52]:=
```
Fit[{{2, -0.375`}, {3, -0.3333333333333334`},
  {4, -0.4040063509461096 5`}, {5, -0.3855772506635989`},
  {6, -0.4155961889813209 5`}, {7, -0.4051770972409499`},
  {8, -0.4218665748359859 4`}, {9, -0.4151468562643683 7`},
  {10, -0.4258035207282880 3`}}, {1, 1/x, 1/x^2}, x]
```

Out[52]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[50]:=
```
tb
```

Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

In[52]:=
```
Fit[{{2, -0.375`}, {3, -0.3333333333333334`},
  {4, -0.40400635094610965`}, {5, -0.3855772506635989`},
  {6, -0.41559618898132095`}, {7, -0.4051770972409499`},
  {8, -0.42186657483598594`}, {9, -0.41514685626436837`},
  {10, -0.42580352072828803`}}, {1, 1/x, 1/x^2}, x]
```

Out[52]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$


```
Show[ln, Plot[fit, {x, 2, 10}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[50]:=
```
tb
```

Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

In[52]:=
```
Fit[{{2, -0.375`}, {3, -0.3333333333333334`},
  {4, -0.40400635094610965`}, {5, -0.3855772506635989`},
  {6, -0.41559618898132095`}, {7, -0.4051770972409499`},
  {8, -0.42186657483598594`}, {9, -0.41514685626436837`},
  {10, -0.42580352072828803`}}, {1, 1/x, 1/x^2}, x]
```

Out[52]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$


```
Show[ln, Plot[fit, {x, 2, 10}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[50]:=
```
tb
```

Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

In[52]:=
```
Fit[{{2, -0.375`}, {3, -0.3333333333333334`},
  {4, -0.4040063509461096 5`}, {5, -0.3855772506635989`},
  {6, -0.4155961889813209 5`}, {7, -0.4051770972409499`},
  {8, -0.4218665748359859 4`}, {9, -0.4151468562643683 7`},
  {10, -0.4258035207282880 3`}}, {1, 1/x, 1/x^2}, x]
```

Out[52]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[50]:=
```
tb
```

Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

In[52]:=
```
Fit[{{2, -0.375`}, {3, -0.33333333333333334`},
  {4, -0.40400635094610965`}, {5, -0.3855772506635989`},
  {6, -0.41559618898132095`}, {7, -0.40517709724094 99`},
  {8, -0.42186657483598594`}, {9, -0.41514685626436837`},
  {10, -0.42580352072828803`}}, {1, 1/x, 1/x^2}, x]
```

Out[52]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[50]:=
```
tb
```

Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

```
Fit[{{2, -0.375`}, {3, -0.3333333333333334`},
  {4, -0.40400635094610965`}, {5, -0.3855772506635989`},
  {6, -0.41559618898132095`}, {7, -0.40517709724094 99`},
  {8, -0.42186657483598594`}, {9, -0.41514685626436837`},
  {10, -0.42580352072828803`}}, {1, 1/x, 1/x^2}, x]
```

Out[52]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[50]:=
```
tb
```

Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
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In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
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In[53]:=
```
Fit[{{2, -0.375`}, {3, -0.3333333333333334`},
  {4, -0.4040063509461096 5`}, {5, -0.3855772506635989`},
  {6, -0.4155961889813209 5`}, {7, -0.4051770972409499`},
  {8, -0.4218665748359859 4`}, {9, -0.4151468562643683 7`},
  {10, -0.4258035207282880 3`}}, {1, 1/x, 1/x^2}, x]
```

Out[53]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[50]:=
```
tb
```

Out[50]=
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{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
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In[51]:=
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fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

```
Fit
```

In[53]:=
```
Fit[{{2, -0.375`}, {3, -0.33333333333333334`},
  {4, -0.40400635094610965`}, {5, -0.3855772506635989`},
  {6, -0.41559618898132095`}, {7, -0.4051770972409499`},
  {8, -0.42186657483598594`}, {9, -0.41514685626436837`},
  {10, -0.42580352072828803`}}, {1, 1/x, 1/x^2}, x]
```

Out[53]=
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```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

In[54]:=
```
Fit[{{1, 2}, {2, 3}}, {1, x}, x]
```

Out[54]=
```
1. + 1. x
```

In[53]:=
```
Fit[{{2, -0.375`}, {3, -0.3333333333333334`},
  {4, -0.4040063509461096 5`}, {5, -0.3855772506635989`},
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  {8, -0.4218665748359859 4`}, {9, -0.4151468562643683 7`},
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In[50]:=
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In[51]:=
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fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

```
Fit[{{1, 2}, {2, 3}, {5, 3}}, {1, x}, x]
```

Out[54]=
```
1. + 1. x
```

In[53]:=
```
Fit[{{2, -0.375`}, {3, -0.3333333333333334`},
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FindFit[{{2, -0.375`}, {3, -0.3333333333333334`},
  {4, -0.40400635094610965`}, {5, -0.3855772506635989`},
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FindFit – Wolfram Mathematica

SEARCH FindFit

Mathematica > Data Manipulation > Numerical Data > Curve Fitting & Approximate Functions >
Mathematica > Data Manipulation > Statistics > Curve Fitting & Approximate Functions >
Mathematica > Mathematics and Algorithms > Statistics > Curve Fitting & Approximate Functions >

BUILT-IN MATHEMATICA SYMBOL

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FindFit

FindFit[*data*, *expr*, *pars*, *vars*]
finds numerical values of the parameters *pars* that make *expr* give a best fit to *data* as a function of *vars*. The data can have the form $\{\{x_1, y_1, \ldots, f_1\}, \{x_2, y_2, \ldots, f_2\}, \ldots\}$, where the number of coordinates $x, y, \ldots$ is equal to the number of variables in the list *vars*. The data can also be of the form $\{f_1, f_2, \ldots\}$, with a single coordinate assumed to take values 1, 2, ...

FindFit[*data*, {*expr*, *cons*}, *pars*, *vars*]
finds a best fit subject to the parameter constraints *cons*.

MORE INFORMATION

EXAMPLES

Basic Examples (1)

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FindFit – Wolfram Mathematica

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Search for all pages containing FindFit.

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BUILT-IN MATHEMATICA SYMBOL

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FindFit

FindFit[*data*, *expr*, *pars*, *vars*]

finds numerical values of the parameters *pars* that make *expr* give a best fit to *data* as a function of *vars*. The data can have the form $\{\{x_1, y_1, \ldots, f_1\}, \{x_2, y_2, \ldots, f_2\}, \ldots\}$, where the number of coordinates $x, y, \ldots$ is equal to the number of variables in the list *vars*. The data can also be of the form $\{f_1, f_2, \ldots\}$, with a single coordinate assumed to take values 1, 2, ...

Fit the best-fit parameters a, b, and c:

```
In[2]:= FindFit[%, a x Log[b + c x], {a, b, c}, x]

Out[2]= {a → 1.42076, b → 1.65558, c → 0.534645}
```

Evaluate the fitted function:

```
In[3]:= Table[a x Log[b + c x] /. %, {x, 10}]

Out[3]= {1.11388, 2.84839, 5.03621, 7.5781,
  10.4091, 13.4837, 16.7682, 20.2371, 23.87, 27.6508}
```

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FindFit – Wolfram Mathematica

SEARCH FindFit

Search for all pages containing FindFit.

EXAMPLES

Basic Examples (1)

Find a nonlinear fit to a list of primes:

```
In[1]:= Table[Prime[x], {x, 20}]
Out[1]= {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71}
```

Fit the best-fit parameters a, b, and c:

```
In[2]:= FindFit[%, a x Log[b + c x], {a, b, c}, x]
Out[2]= {a → 1.42076, b → 1.65558, c → 0.534645}
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Evaluate the fitted function:

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Out[3]= {1.11388, 2.84839, 5.03621, 7.5781,
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```

▶ Scope (7)

▶ Options (7)

▶ Applications (7)

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Untitled-25

In[50]:=
```
tb
```

Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
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In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
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FindFit[{{2, -0.375`}, {3, -0.3333333333333334`},
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Out[56]=
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Untitled-25

In[50]:=
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  {4, -0.40400635094610965`}, {5, -0.3855772506635989`},
  {6, -0.41559618898132095`}, {7, -0.4051770972409499`},
  {8, -0.42186657483598594`}, {9, -0.41514685626436837`},
  {10, -0.42580352072828803`}}, a + b/x + c/x^2, {1, 1/x, 1/x^2}, x]
```

Out[56]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[50]:=
```
tb
```

Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=
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fit = Fit[tb, {1, 1/x, 1/x^2}, x]
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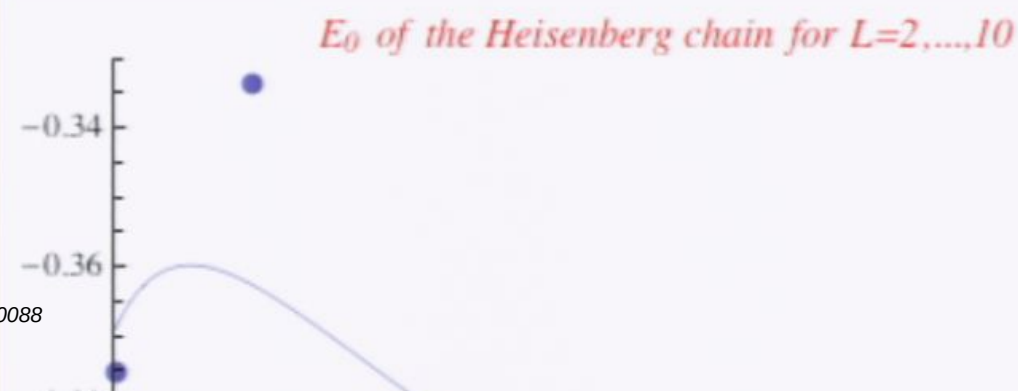

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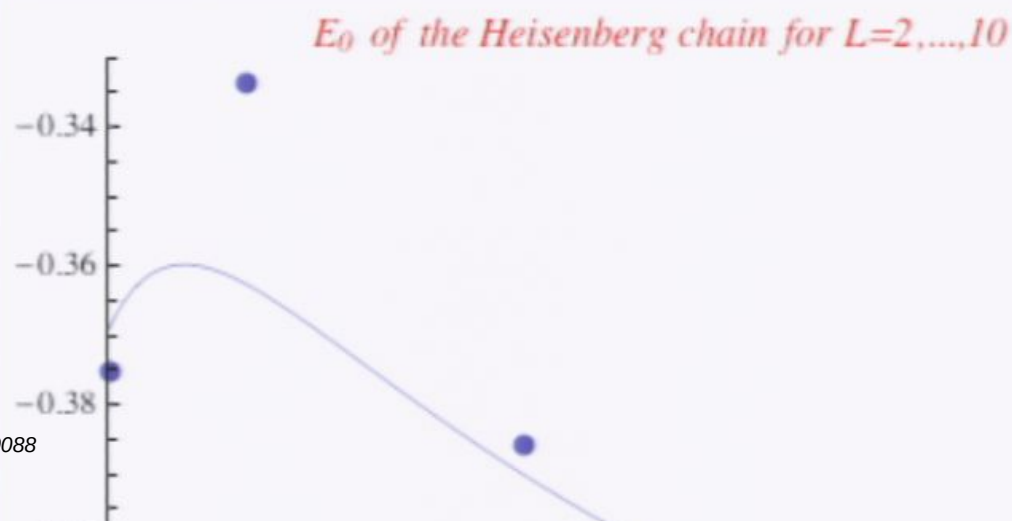

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Fit
```

In[57]:=
```
FindFit[{{2, -0.375`}, {3, -0.33333333333333334`},
  {4, -0.40400635094610965`}, {5, -0.3855772506635989`},
  {6, -0.41559618898132095`}, {7, -0.4051770972409499`},
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Fit
Find
```

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FindFit[{{2, -0.375`}, {3, -0.3333333333333334`},
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Fit
FindFit
```

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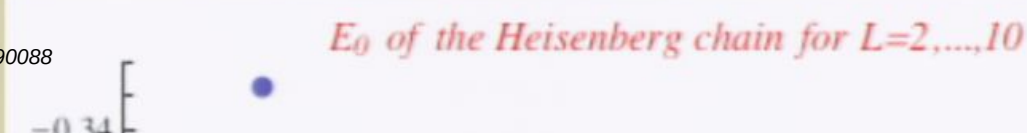

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E_0 of the Heisenberg chain for L=2,....,10

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FindFit – Wolfram Mathematica

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Mathematica > Mathematics and Algorithms > Statistics > Curve Fitting & Approximate Functions >

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FindFit

The data can have the form {{x1, y1, ...}, {x2, y2, ...}, ...}, where the number of coordinates $x, y, \ldots$ is equal to the number of variables in the list *vars*. The data can also be of the form $\{f_1, f_2, \ldots\}$, with a single coordinate assumed to take values 1, 2, ...

FindFit[*data*, {*expr*, *cons*}, *pars*, *vars*]
finds a best fit subject to the parameter constraints *cons*.

MORE INFORMATION

EXAMPLES

Basic Examples (1)

Find a nonlinear fit to a list of primes:

```
In[1]:= Table[Prime[x], {x, 20}]
```

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FindFit – Wolfram Mathematica

SEARCH FindFit

Search for all pages containing FindFit.

```
In[3]:= Table[a x Log[b + c x] /. 4, {x, 10}]

Out[3]= {1.11388, 2.84839, 5.03621, 7.5781,
         10.4091, 13.4837, 16.7682, 20.2371, 23.87, 27.6508}
```

- **Scope** (7)
- **Options** (7)
- **Applications** (7)
- **Properties & Relations** (4)
- **Possible Issues** (1)

SEE ALSO

FindMinimum · **Fit** · **NMinimize** · **LeastSquares** · **Interpolation** · **FindCurvePath** · **ChiSquareDistribution**

TUTORIALS

- Manipulating Numerical Data
- Curve Fitting
- Constrained Optimization
- Unconstrained Optimization

RELATED LINKS

- Demonstrations with FindFit (Wolfram Demonstrations Project)
- Implementation notes: Numerical and Related Functions

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Untitled-25

```
{8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
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In[51]:=

```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
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Out[51]= $-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$

```
FindFit[{{2, -0.375`}, {3, -0.3333333333333334`},
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Out[58]= {a → -0.485173, b → 0.63849, c → -0.812302}

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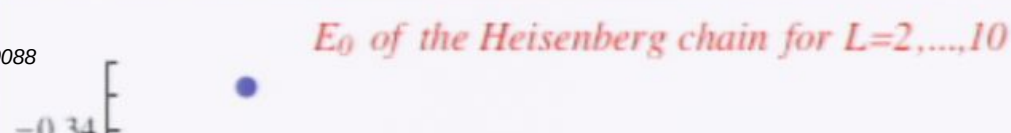

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E_0 of the Heisenberg chain for L=2,....,10

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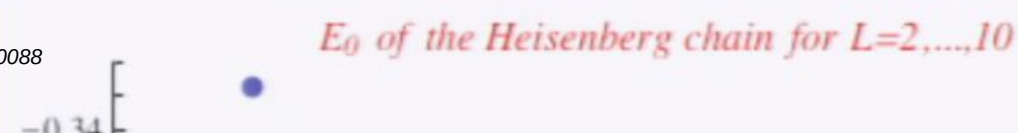

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E₀ of the Heisenberg chain for L=2,...,10

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Out[59]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{101}{250}\right\}, \left\{5, -\frac{123}{319}\right\}, \left\{6, -\frac{357}{859}\right\}, \left\{7, -\frac{47}{116}\right\}, \left\{8, -\frac{27}{64}\right\}, \left\{9, -\frac{137}{330}\right\}, \left\{10, -\frac{66}{155}\right\}\right\}$

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In[60]:=
```
TB = {{2, -0.375`}, {3, -0.3333333333333334`},
   {4, -0.40400635094610965`}, {5, -0.3855772506635989`},
   {6, -0.41559618898132095`}, {7, -0.4051770972409499`},
   {8, -0.42186657483598594`}, {9, -0.41514685626436837`},
   {10, -0.42580352072828803`}} // Rationalize[#, 10^-5] &
```

Out[60]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{101}{250}\right\}, \left\{5, -\frac{123}{319}\right\}, \left\{6, -\frac{357}{859}\right\}, \left\{7, -\frac{47}{116}\right\}, \left\{8, -\frac{27}{64}\right\}, \left\{9, -\frac{137}{330}\right\}, \left\{10, -\frac{66}{155}\right\}\right\}$$

```
FindFit[, a + b/x + c/x^2, {a, b, c}, x]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]= $-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$

In[60]:=
```
TB = {{2, -0.375`}, {3, -0.3333333333333334`},
   {4, -0.40400635094610965`}, {5, -0.3855772506635989`},
   {6, -0.41559618898132095`}, {7, -0.4051770972409499`},
   {8, -0.42186657483598594`}, {9, -0.41514685626436837`},
   {10, -0.42580352072828803`}} // Rationalize[#, 10^-5] &
```

Out[60]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{101}{250}\right\}, \left\{5, -\frac{123}{319}\right\}, \left\{6, -\frac{357}{859}\right\}, \left\{7, -\frac{47}{116}\right\}, \left\{8, -\frac{27}{64}\right\}, \left\{9, -\frac{137}{330}\right\}, \left\{10, -\frac{66}{155}\right\}\right\}$

```
FindFit[TB, a + b/x + c/x^2, {a, b, c}, x]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[51]= $-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$

In[60]:=
```
TB = {{2, -0.375`}, {3, -0.3333333333333334`},
   {4, -0.40400635094610965`}, {5, -0.3855772506635989`},
   {6, -0.41559618898132095`}, {7, -0.4051770972409499`},
   {8, -0.42186657483598594`}, {9, -0.41514685626436837`},
   {10, -0.42580352072828803`}} // Rationalize[#, 10^-5] &
```

Out[60]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{101}{250}\right\}, \left\{5, -\frac{123}{319}\right\}, \left\{6, -\frac{357}{859}\right\}, \left\{7, -\frac{47}{116}\right\}, \left\{8, -\frac{27}{64}\right\}, \left\{9, -\frac{137}{330}\right\}, \left\{10, -\frac{66}{155}\right\}\right\}$

In[61]:= $\text{FindFit}\left[\text{TB}, a + \frac{b}{x} + \frac{c}{x^2}, \{a, b, c\}, x\right]$

Out[61]= $\{a \to -0.485184, b \to 0.638572, c \to -0.812422\}$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[51]= $-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$

In[60]:=
```
TB = {{2, -0.375`}, {3, -0.3333333333333334`},
   {4, -0.40400635094610965`}, {5, -0.3855772506635989`},
   {6, -0.41559618898132095`}, {7, -0.4051770972409499`},
   {8, -0.42186657483598594`}, {9, -0.41514685626436837`},
   {10, -0.42580352072828803`}} // Rationalize[#, 10^-5] &
```

Out[60]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{101}{250}\right\}, \left\{5, -\frac{123}{319}\right\}, \left\{6, -\frac{357}{859}\right\}, \left\{7, -\frac{47}{116}\right\}, \left\{8, -\frac{27}{64}\right\}, \left\{9, -\frac{137}{330}\right\}, \left\{10, -\frac{66}{155}\right\}\right\}$

In[61]:= $\text{FindFit}\left[\text{TB}, a + \frac{b}{x} + \frac{c}{x^2}, \{a, b, c\}, x\right]$

Out[61]= $\{a \to -0.485184, b \to 0.638572, c \to -0.812422\}$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[51]= $-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$

In[60]:=
```
TB = {{2, -0.375`}, {3, -0.3333333333333334`},
   {4, -0.40400635094610965`}, {5, -0.3855772506635989`},
   {6, -0.41559618898132095`}, {7, -0.4051770972409499`},
   {8, -0.42186657483598594`}, {9, -0.41514685626436837`},
   {10, -0.42580352072828803`}} // Rationalize[#, 10^-5] &
```

Out[60]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{101}{250}\right\}, \left\{5, -\frac{123}{319}\right\}, \left\{6, -\frac{357}{859}\right\}, \left\{7, -\frac{47}{116}\right\}, \left\{8, -\frac{27}{64}\right\}, \left\{9, -\frac{137}{330}\right\}, \left\{10, -\frac{66}{155}\right\}\right\}$

In[61]:= $\text{FindFit}\left[\text{TB}, a + \frac{b}{x} + \frac{c}{x^2}, \{a, b, c\}, x\right]$

Out[61]= $\{a \to -0.485184, b \to 0.638572, c \to -0.812422\}$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[51]= $-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$

In[60]:=
```
TB = {{2, -0.375`}, {3, -0.3333333333333334`},
   {4, -0.40400635094610965`}, {5, -0.3855772506635989`},
   {6, -0.41559618898132095`}, {7, -0.4051770972409499`},
   {8, -0.42186657483598594`}, {9, -0.41514685626436837`},
   {10, -0.42580352072828803`}} // Rationalize[#, 10^-5] &
```

Out[60]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{101}{250}\right\}, \left\{5, -\frac{123}{319}\right\}, \left\{6, -\frac{357}{859}\right\}, \left\{7, -\frac{47}{116}\right\}, \left\{8, -\frac{27}{64}\right\}, \left\{9, -\frac{137}{330}\right\}, \left\{10, -\frac{66}{155}\right\}\right\}$

In[61]:= $\text{FindFit}\left[\text{TB}, a + \frac{b}{x} + \frac{c}{x^2}, \{a, b, c\}, x\right]$

Out[61]= $\{a \to -0.485184, b \to 0.638572, c \to -0.812422\}$

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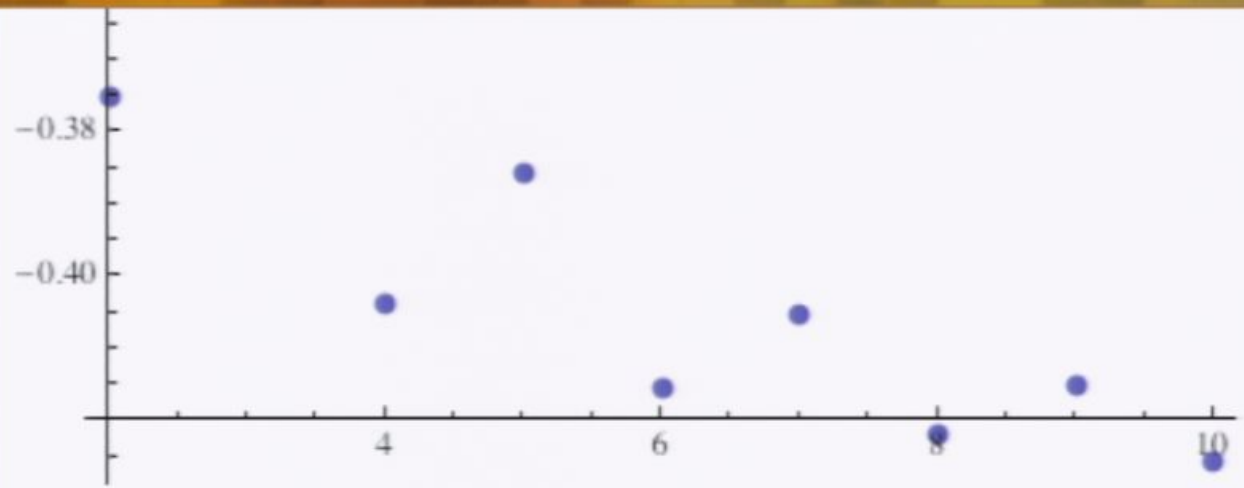
Untitled-25

Out[41]=




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[50]:=

```
tb
```

Out[50]=

```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

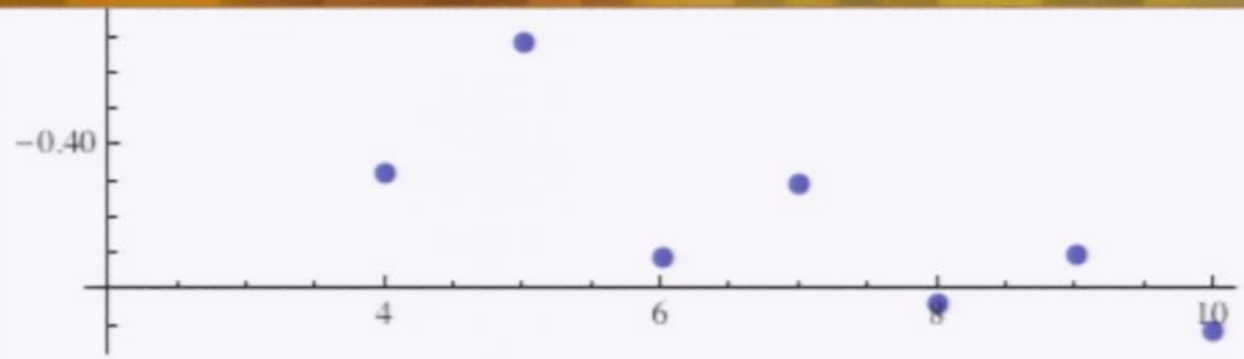
In[51]:=

```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

0.812302 0.63849

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[50]:=
```
tb
```

Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

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Untitled-25


```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

```
tbfit = Rationali[tb
```

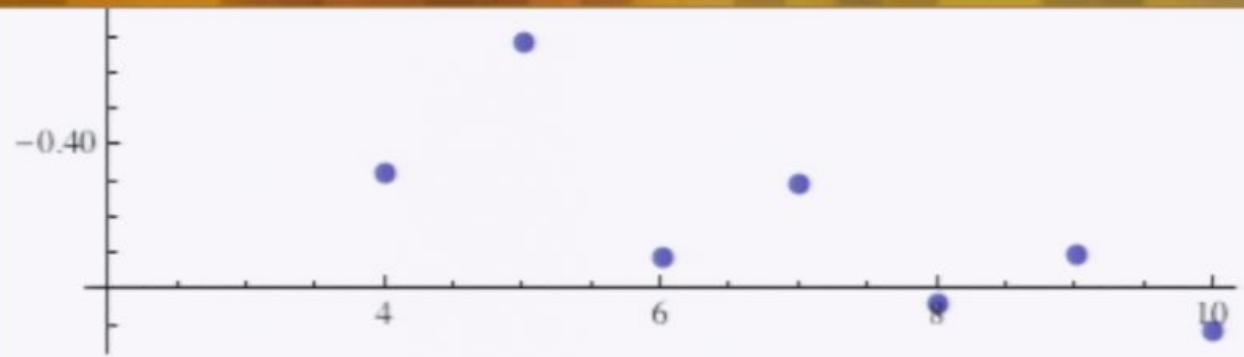
Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

```
tbfit = Rationalize[tb,
```

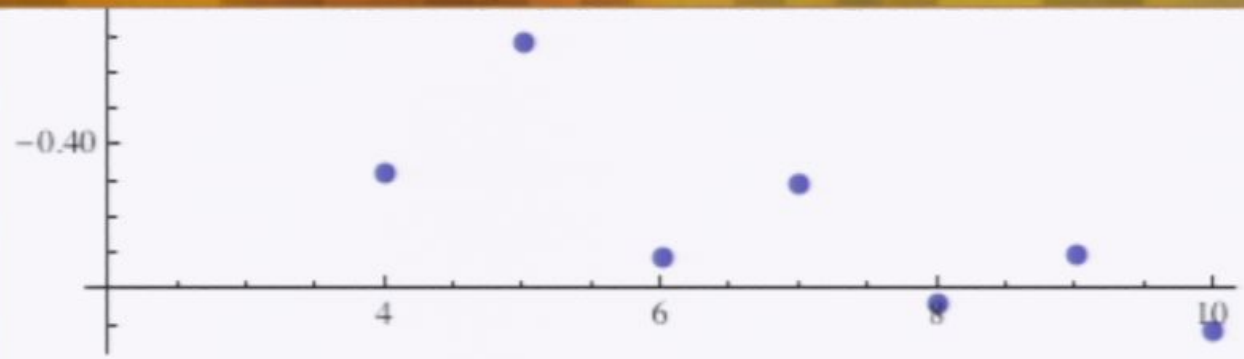
Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

```
tbfit = Rationalize[tb, 10^
```

Out[50]=

```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=

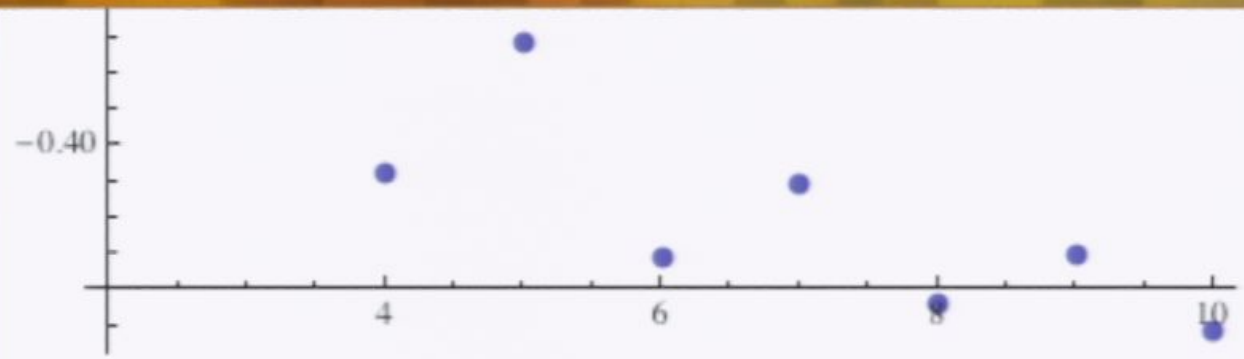
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=

$$-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

```
tbfit = Rationalize[tb, 10^-7]
```

Out[50]=
```
{{2, -0.375}, {3, -0.333333}, {4, -0.404006},
 {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
 {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}
```

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]= $-0.485173 - \frac{0.812302}{x^2} + \frac{0.63849}{x}$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25


```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[62]:=
```
tbfit = Rationalize[tb, 10^-7]
```

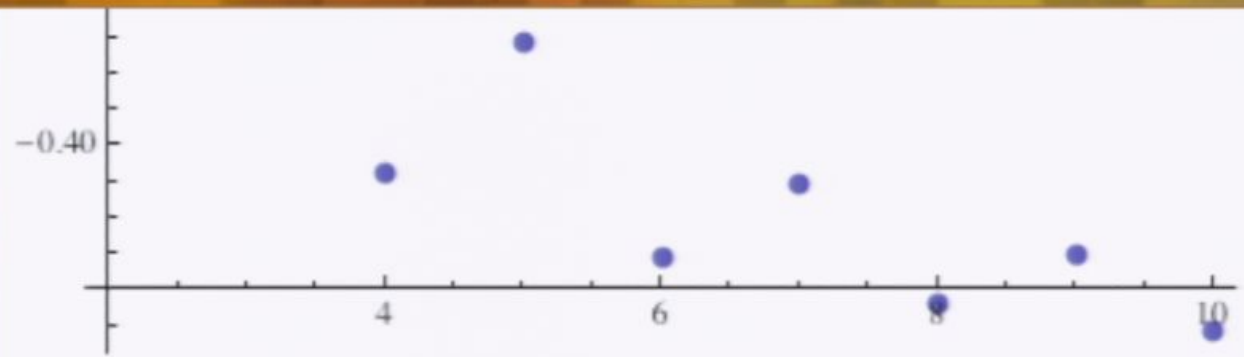
Out[62]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{\ }+\frac{0.63849}{\ }$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[62]:=
```
tbfit = Rationalize[tb, 10^-7]
```

Out[62]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[51]:=
```
fit = Fit[tb, {1, 1/x, 1/x^2}, x]
```

Out[51]=
$$-0.485173 - \frac{0.812302}{\ }+\frac{0.63849}{\ }$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25


```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[62]:=
```
tbfit = Rationalize[tb, 10^-7]
```

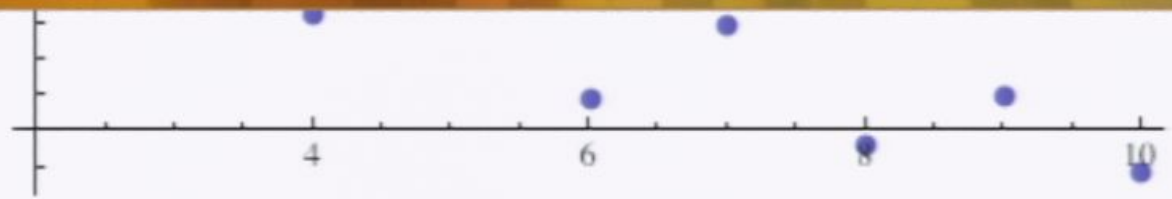
Out[62]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$

In[63]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[63]= $-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[62]:=
```
tbfit = Rationalize[tb, 10^-7]
```

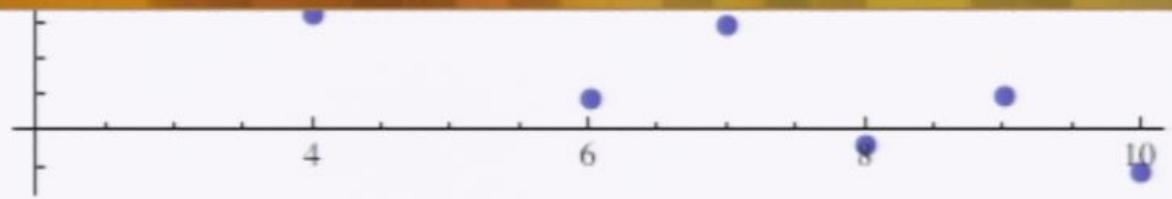
Out[62]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[63]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[63]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[62]:=
```
tbfit = Rationalize[tb, 10^-7]
```

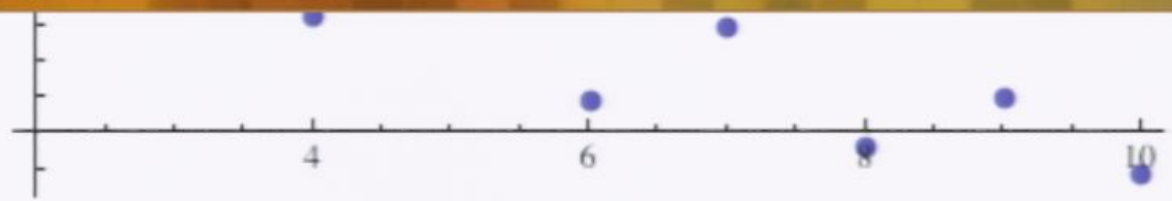
Out[62]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[63]:=
```
fit = Fit[tbfit, {1, 1 / x, 1 / x^2}, x]
```

Out[63]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[62]:=
```
tbfit = Rationalize[tb, 10^-7]
```

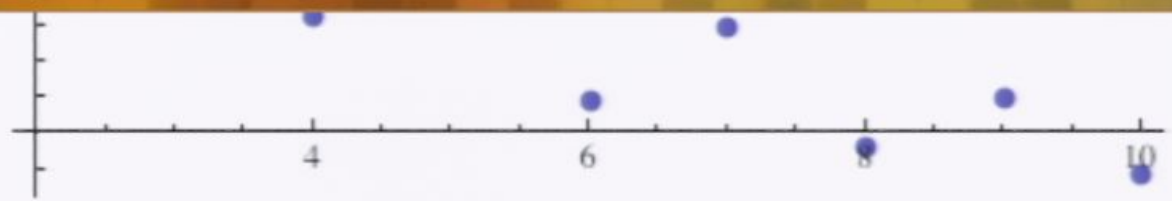
Out[62]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[63]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[63]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[62]:=
```
tbfit = Rationalize[tb, 10^-7]
```

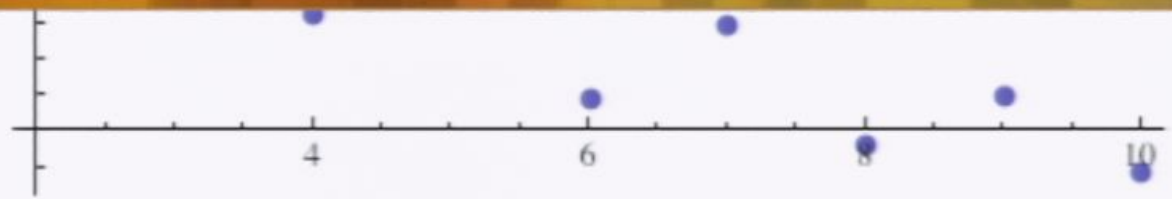
Out[62]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[63]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[63]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[62]:=
```
tbfit = Rationalize[tb, 10^-7]
```

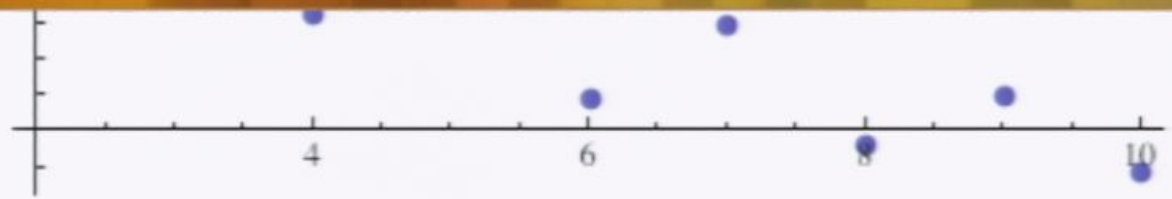
Out[62]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[63]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[63]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25




```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[62]:=
```
tbfit = Rationalize[tb, 10^-7]
```

Out[62]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[64]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[64]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

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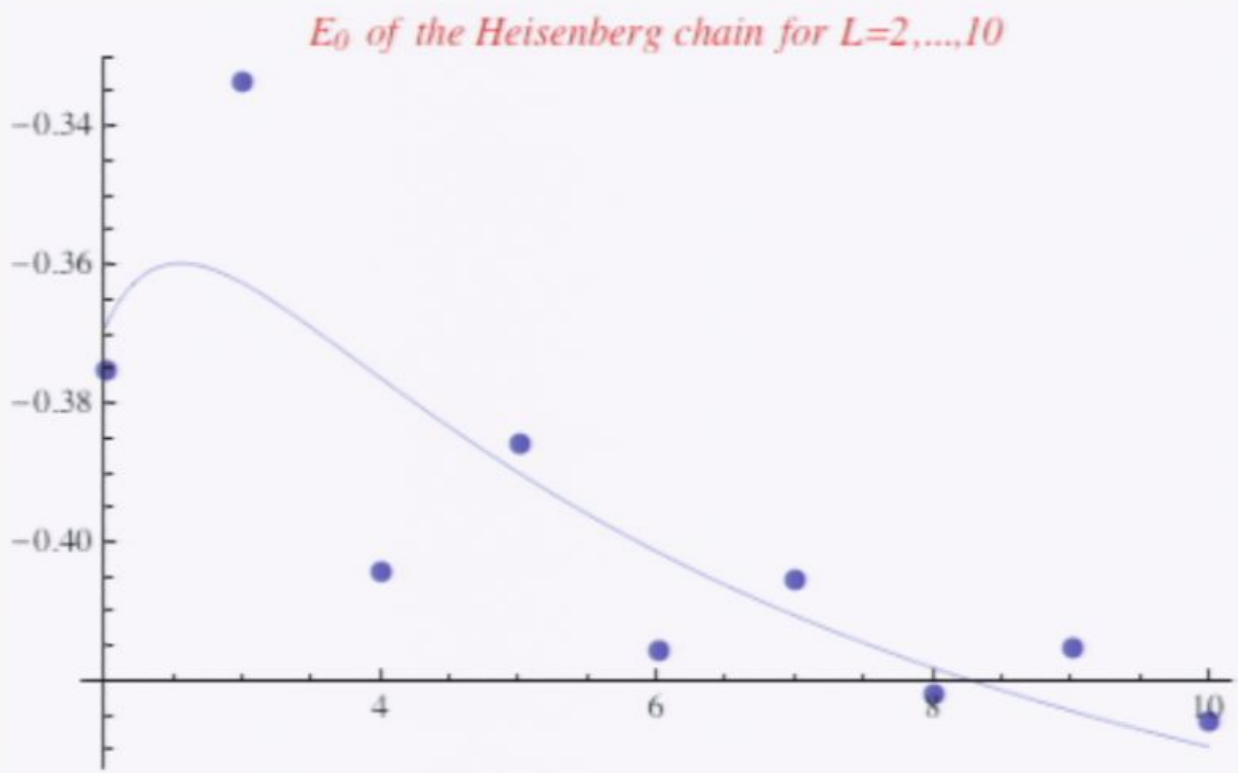
Untitled-25

In[61]:=
```
FindFit[TB, a + b/x + c/x^2, {a, b, c}, x]
```

Out[61]=
```
{a → -0.485184, b → 0.638572, c → -0.812422}
```

In[47]:=
```
Show[lp, Plot[fit, {L, 2, 10}]]
```

Out[47]=



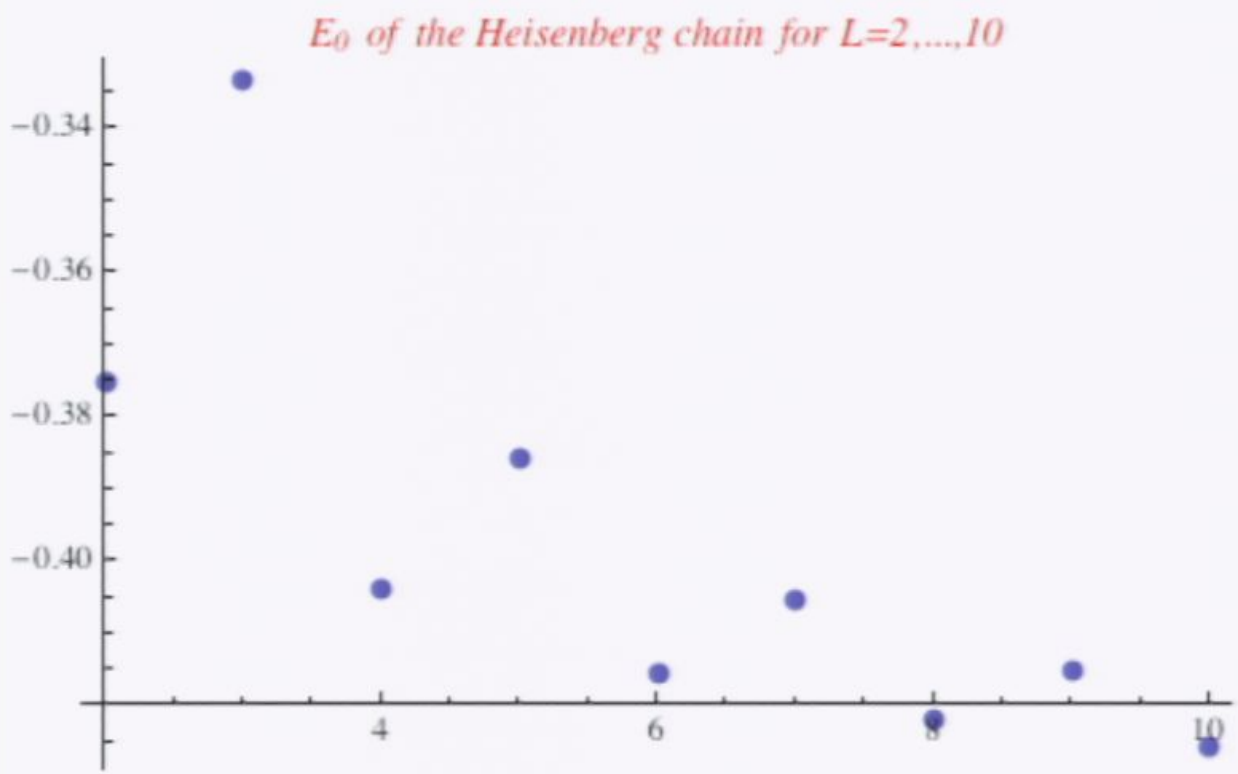
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[64]= $-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$

In[65]:=
```
Show[lp, Plot[fit, {L, 2, 10}]]
```

Out[65]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

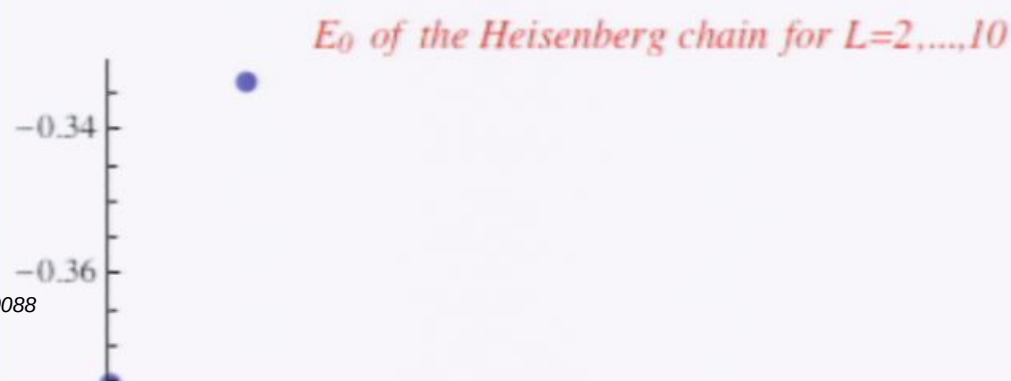
In[62]:=
```
tbfit = Rationalize[tb, 10^-7]
```

Out[62]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[64]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[64]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

In[65]:=
```
Show[lp, Plot[fit, {L, 2, 10}]]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

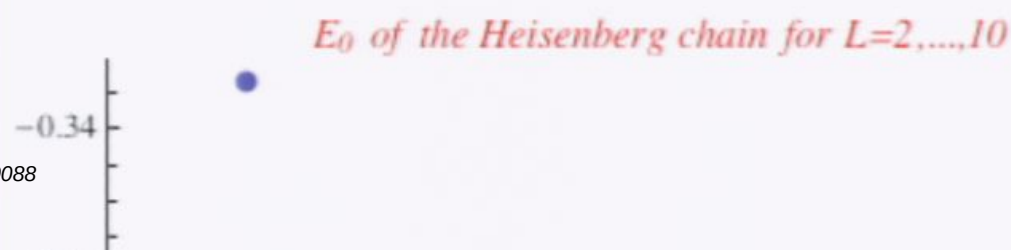
In[62]:=
```
tbfit = Rationalize[tb, 10^-7]
```

Out[62]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[64]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[64]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

In[65]:=
```
Show[lp, Plot[fit, {L, 2, 10}]]
```



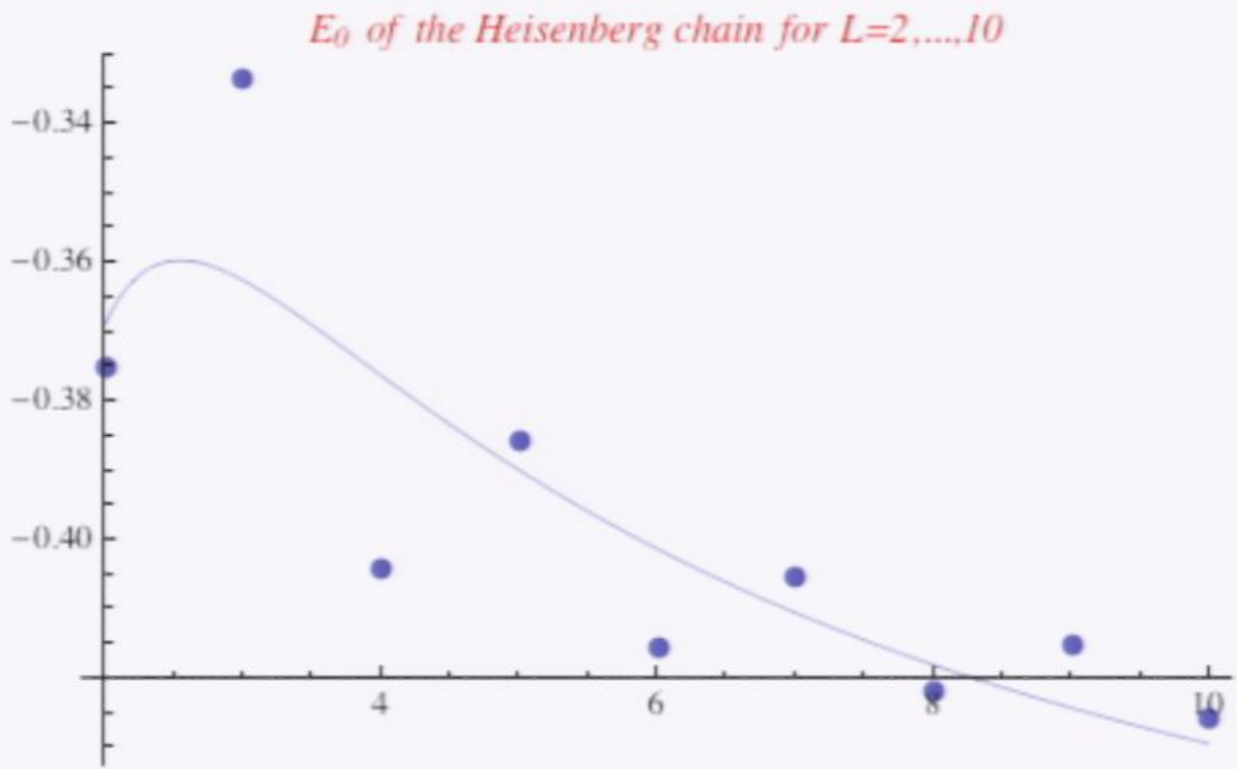
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[64]= $-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Out[66]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

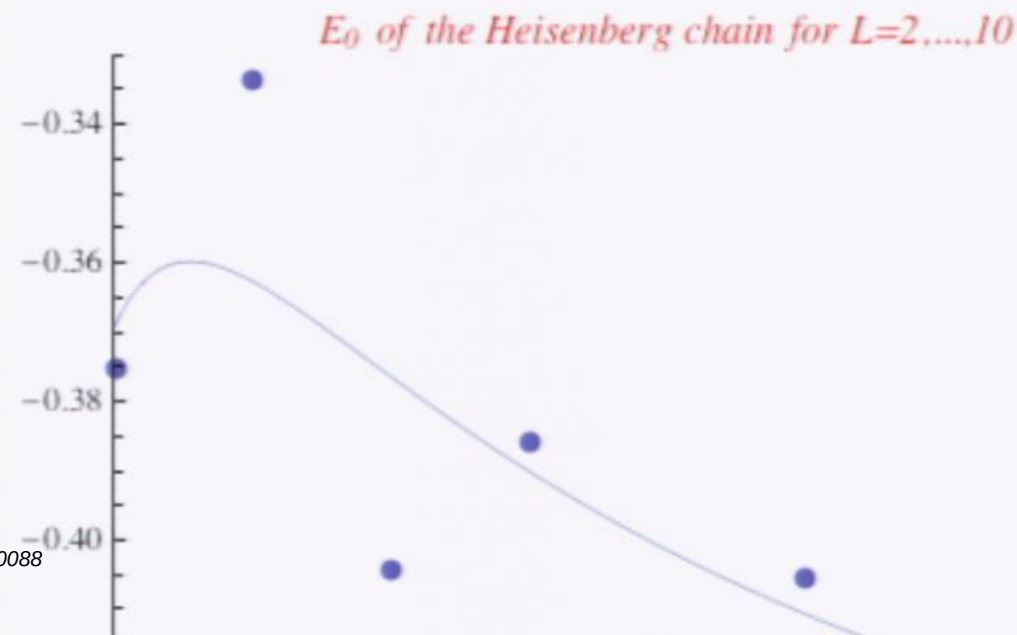
$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

In[64]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[64]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Out[66]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[62]:= tbfit = Rationalize[tb, 10^-6]

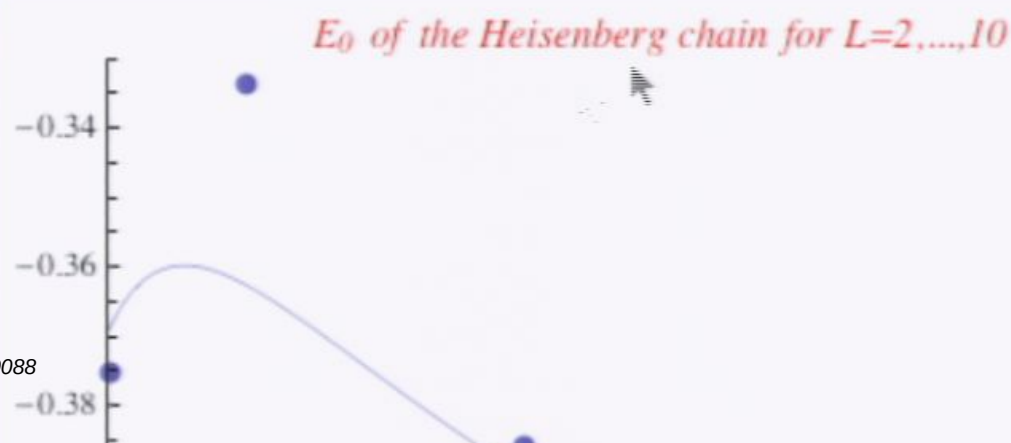
Out[62]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$

In[64]:= fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]

Out[64]= $-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$

In[66]:= Show[lp, Plot[fit, {x, 2, 10}]]

Out[66]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

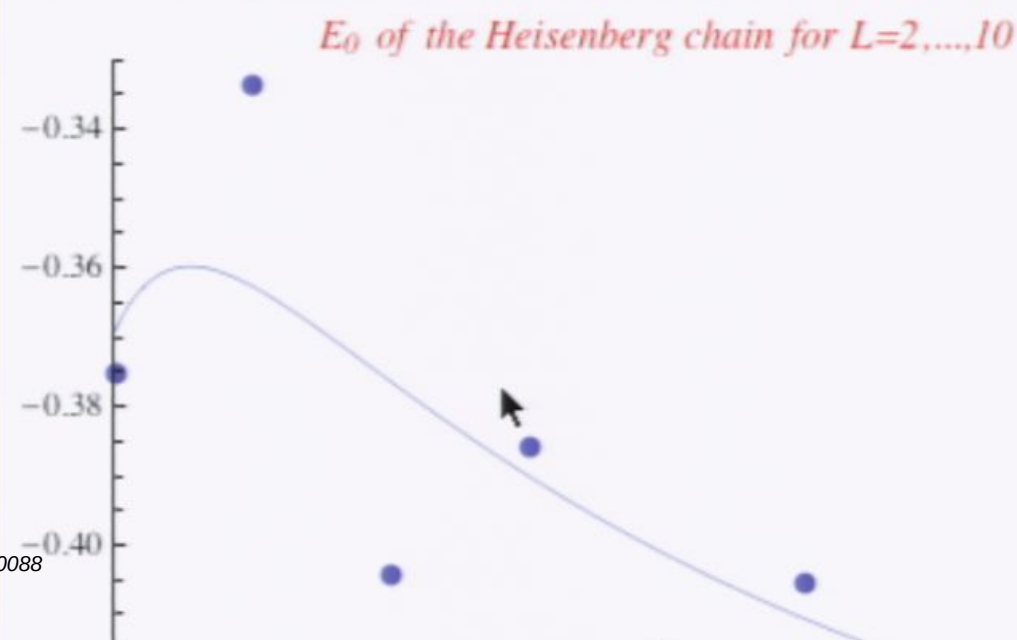
$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

In[64]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[64]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Out[66]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

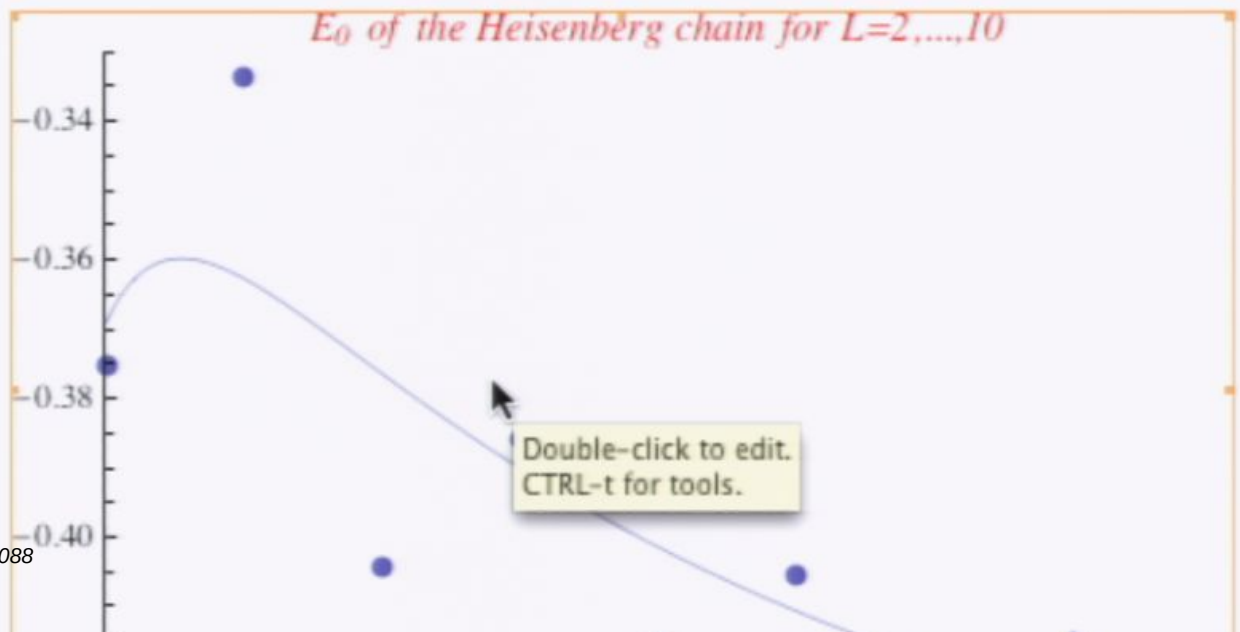
$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

In[64]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[64]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Out[66]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

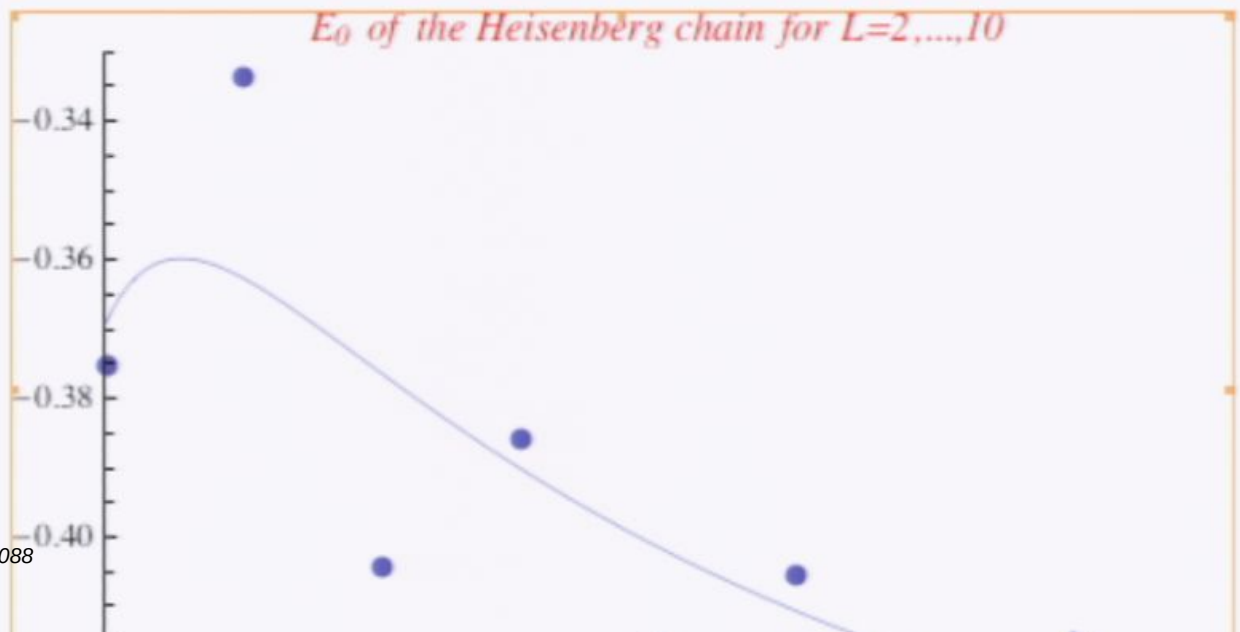
$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

In[64]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[64]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Out[66]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

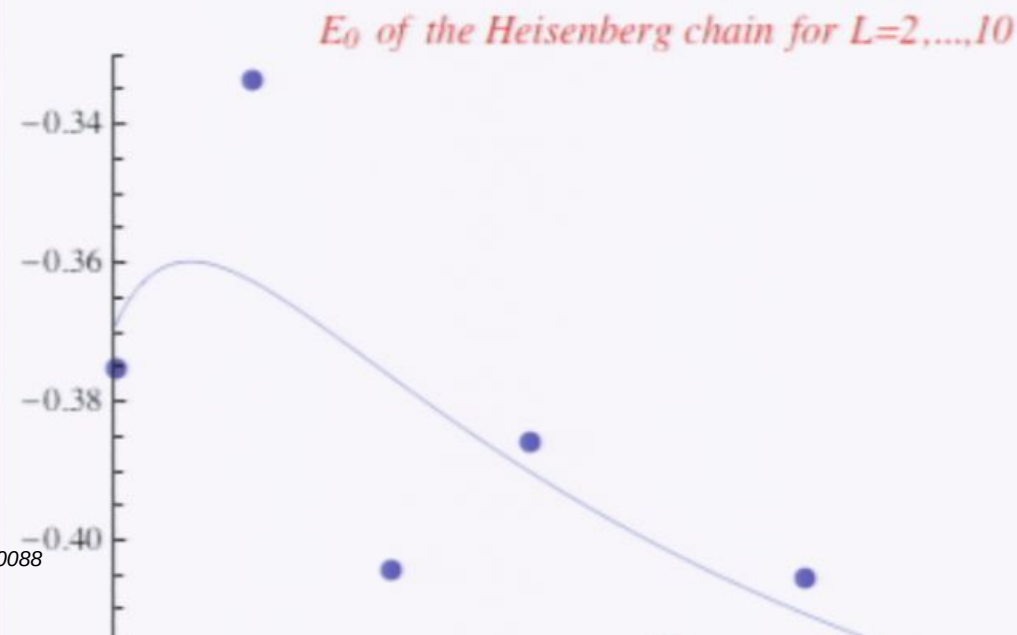
$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

In[64]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[64]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Out[66]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

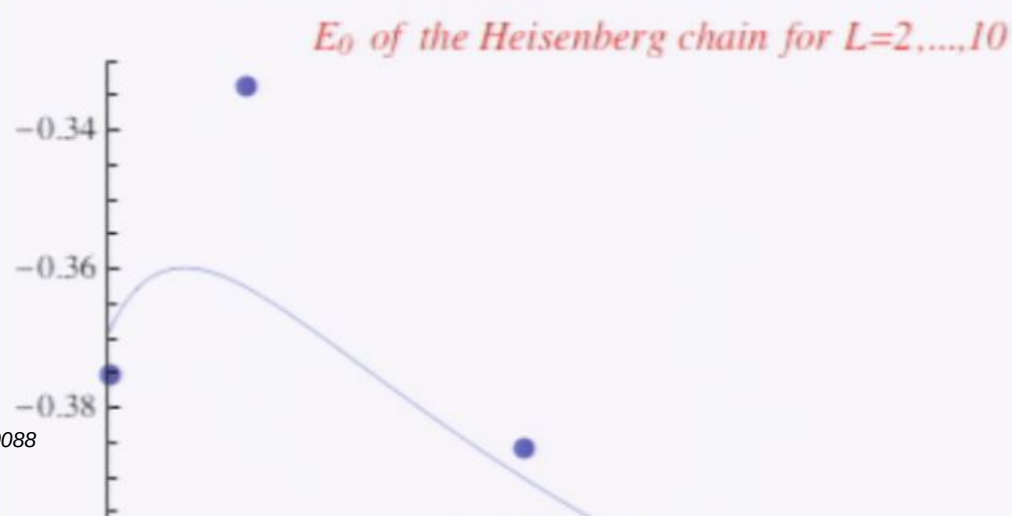
In[64]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[64]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

```
fit
```

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Out[66]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

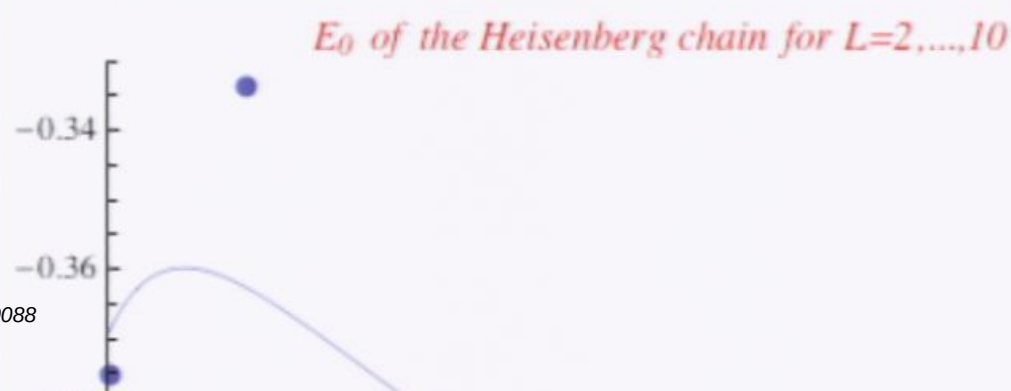
In[64]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[64]= $-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$

In[67]:=
```
fit /. x → ∞
```

Out[67]= -0.485173

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```



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Untitled-25

$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

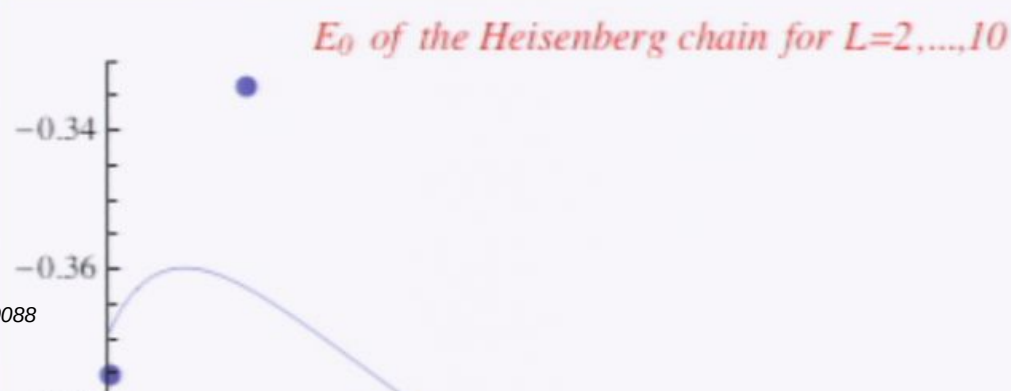
In[64]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[64]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

```
prediction fit /. x → ∞
```

Out[67]=
$$-0.485173$$

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```



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Untitled-25

$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

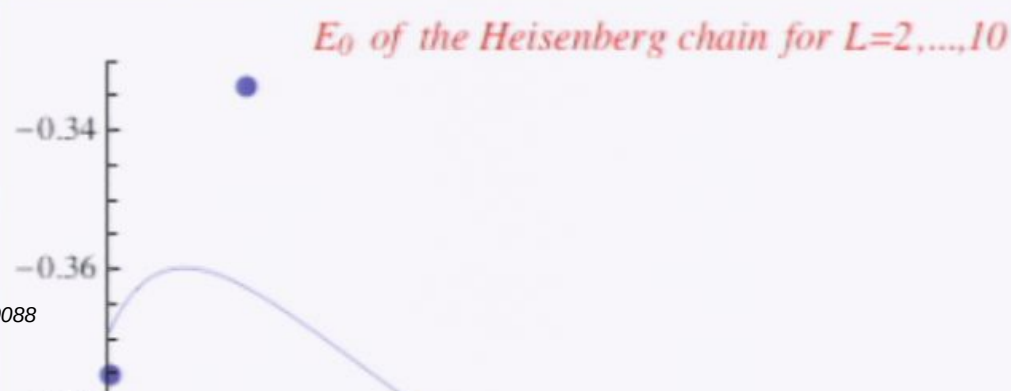
In[64]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[64]= $-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$

In[68]:=
```
prediction = fit /. x → ∞
```

Out[68]= -0.485173

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[62]:=
```
tbfit = Rationalize[tb, 10^-7]
```

Out[62]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[64]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[64]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

In[68]:=
```
prediction = fit /. x → ∞
```

Out[68]=
$$-0.485173$$

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

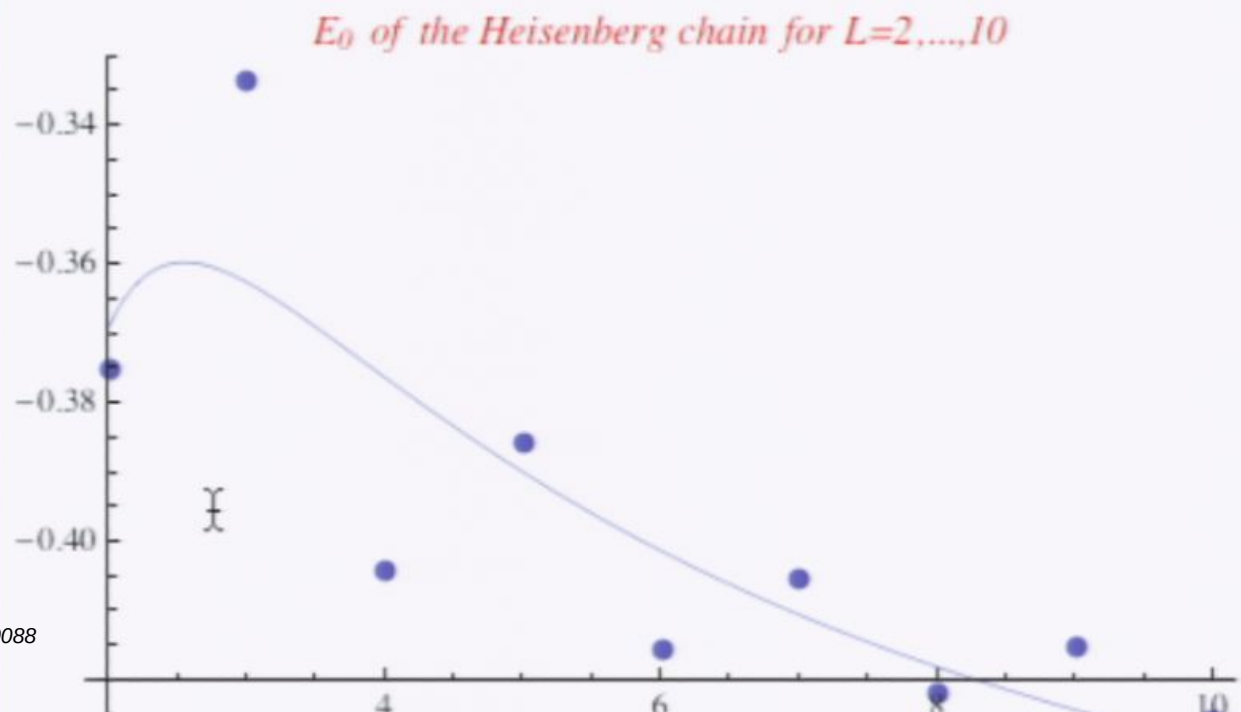
Out[64]= $-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$

In[68]:=
```
prediction = fit /. x → ∞
```

Out[68]= -0.485173

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Out[66]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

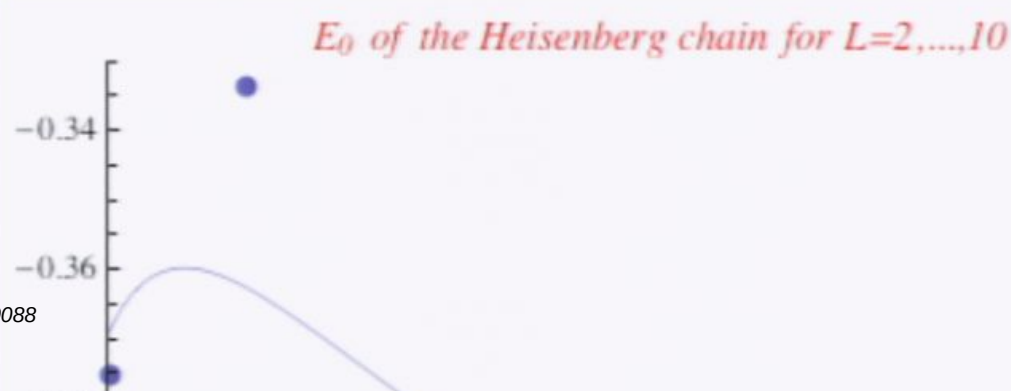
In[64]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[64]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

In[68]:=
```
prediction = fit /. x → ∞
```

Out[68]=
$$-0.485173$$

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

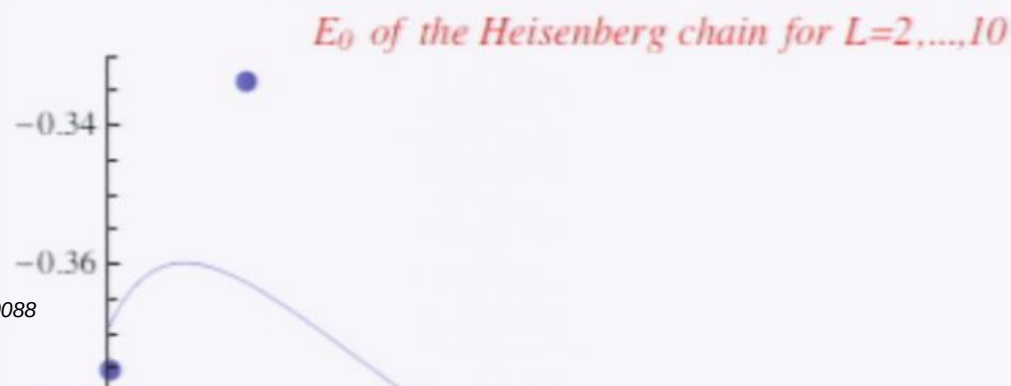
In[64]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[64]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

In[68]:=
```
prediction = fit /. x → ∞
```

Out[68]=
$$-0.485173$$

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

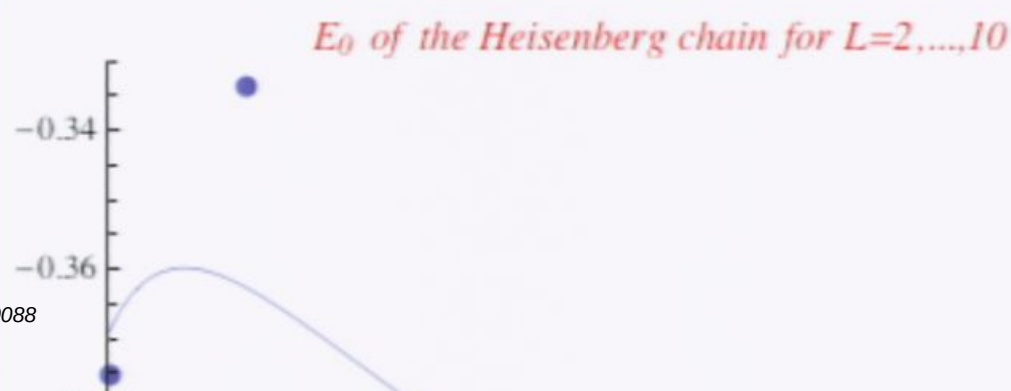
In[64]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2}, x]
```

Out[64]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

In[68]:=
```
prediction = fit /. x → ∞
```

Out[68]=
$$-0.485173$$

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

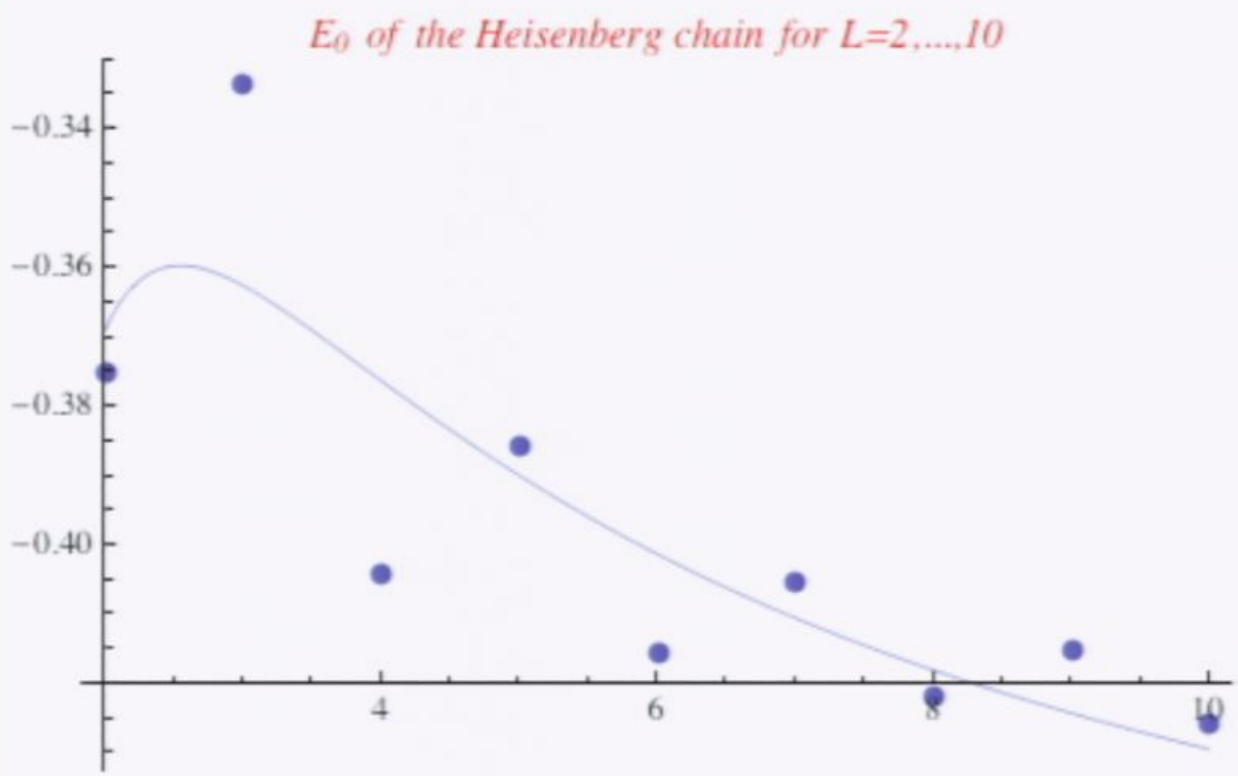
Untitled-25

In[68]:=
```
prediction = fit /. x → ∞
```

Out[68]=
```
-0.485173
```

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Out[66]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

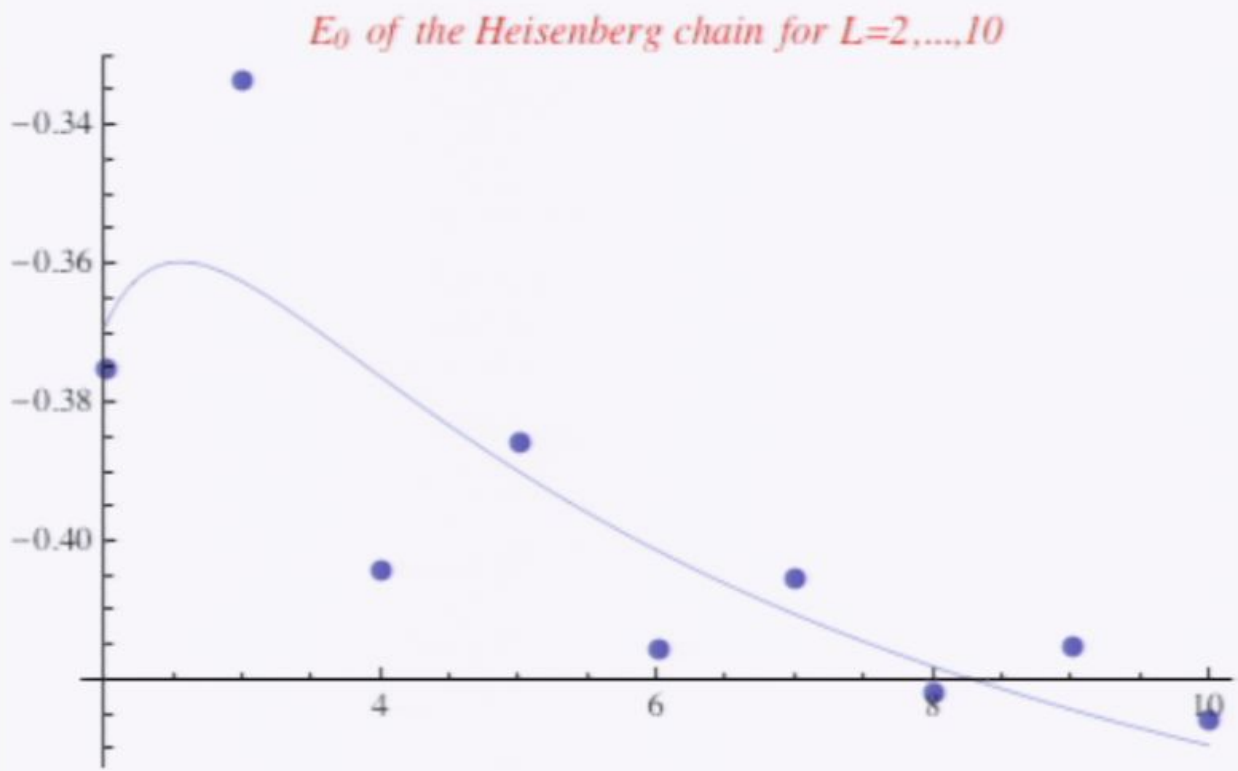
Untitled-25

In[68]:=
```
prediction = fit /. x → ∞
```

Out[68]=
```
-0.485173
```

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Out[66]=



Lets spl

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

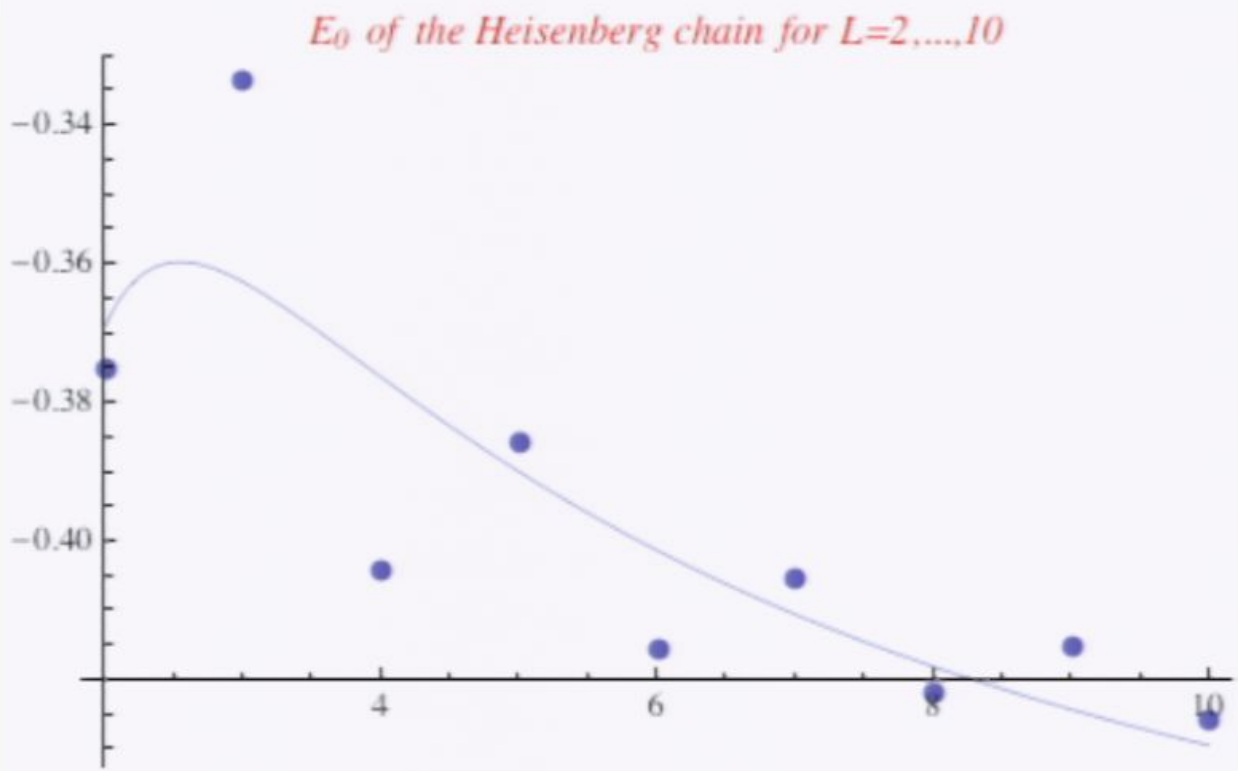
Untitled-25

In[68]:= `prediction = fit /. x → ∞`

Out[68]= -0.485173

In[66]:= `Show[lp, Plot[fit, {x, 2, 10}]]`

Out[66]=



Lets split into odd and even to

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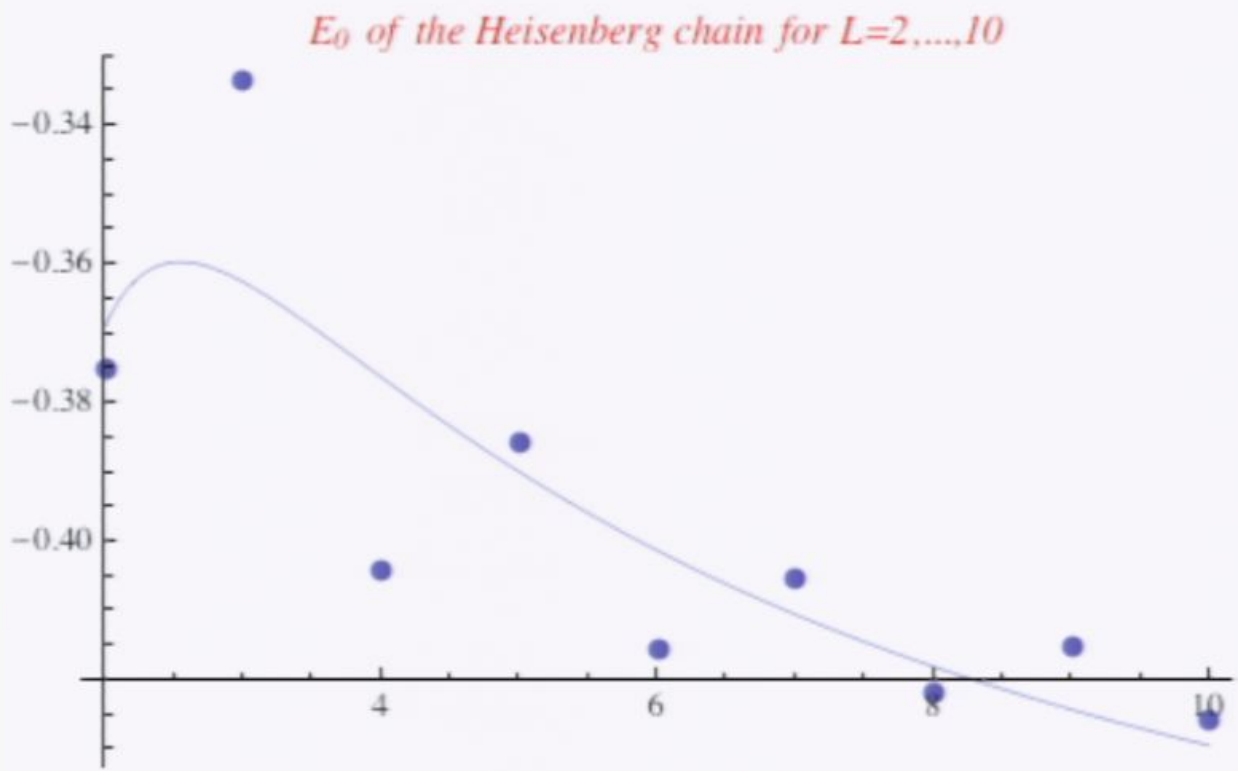
Untitled-25

In[68]:=
```
prediction = fit /. x → ∞
```

Out[68]=
```
-0.485173
```

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Out[66]=

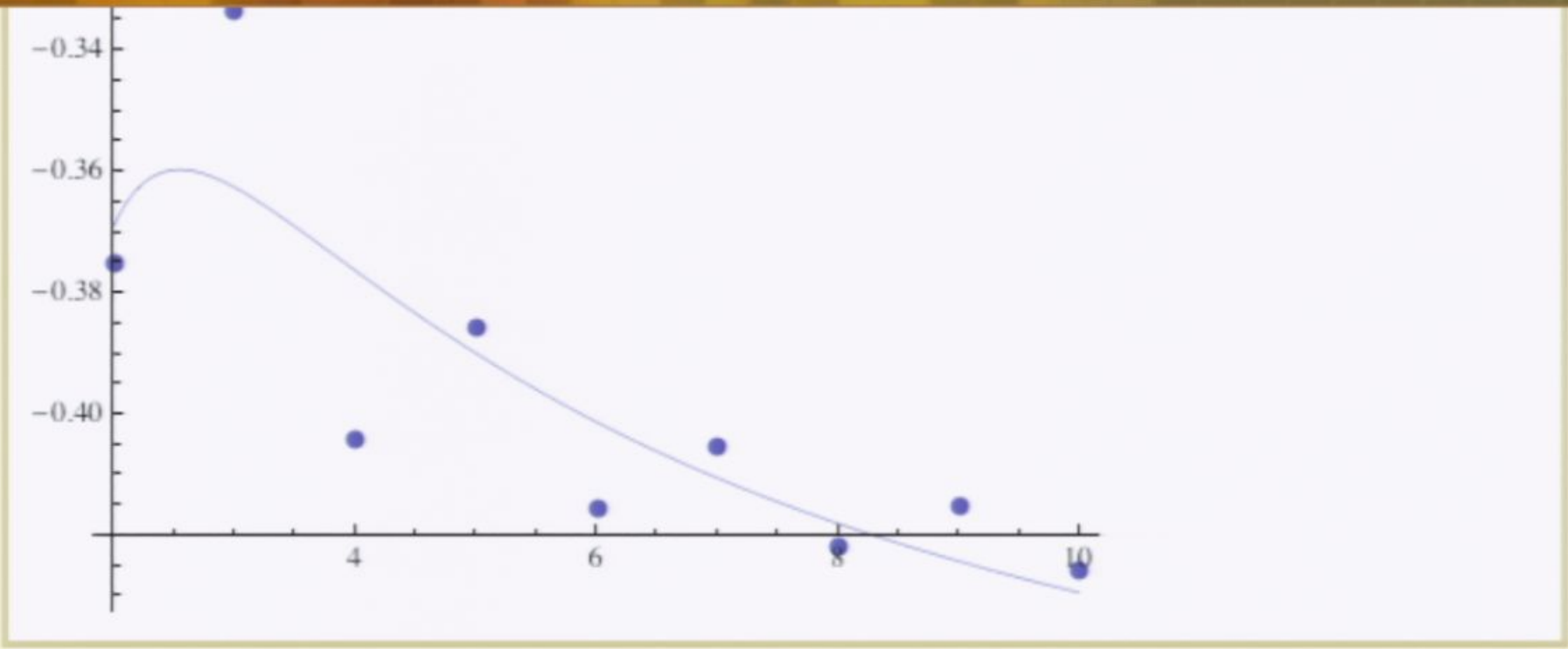


Lets split into odd and even to improve our prediction:

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[66]=

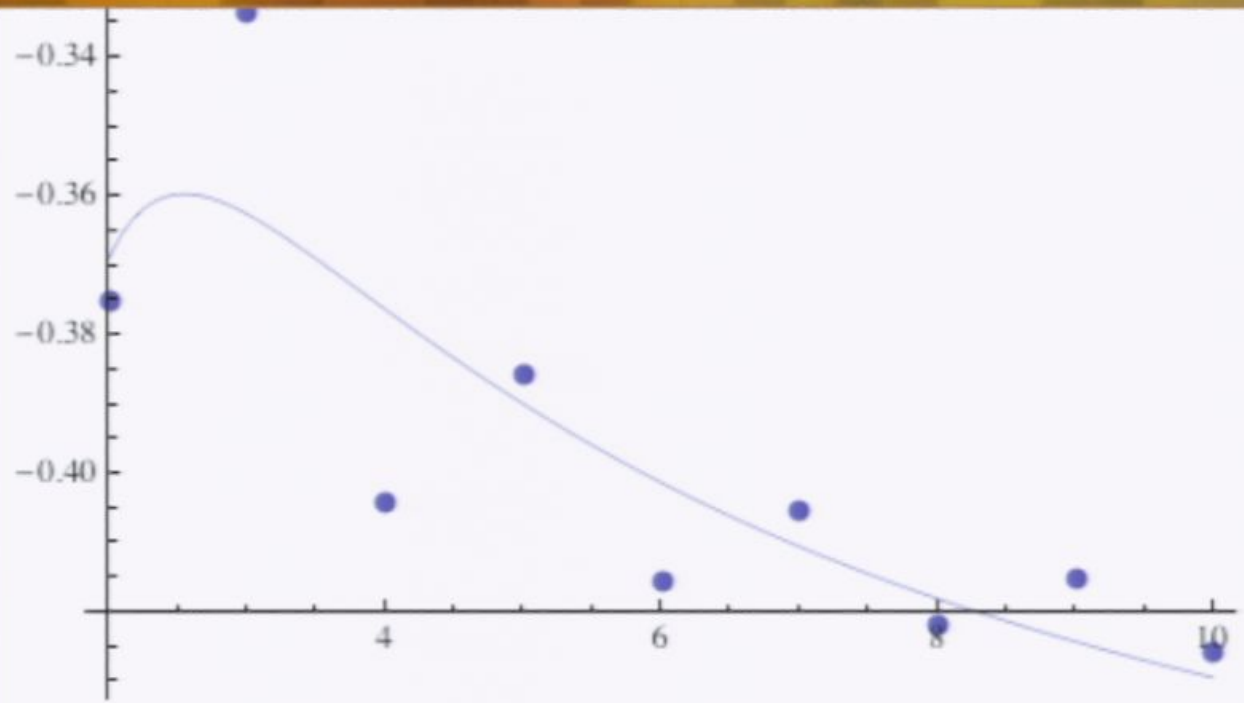


Lets split into odd and even to improve our prediction:

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[66]=



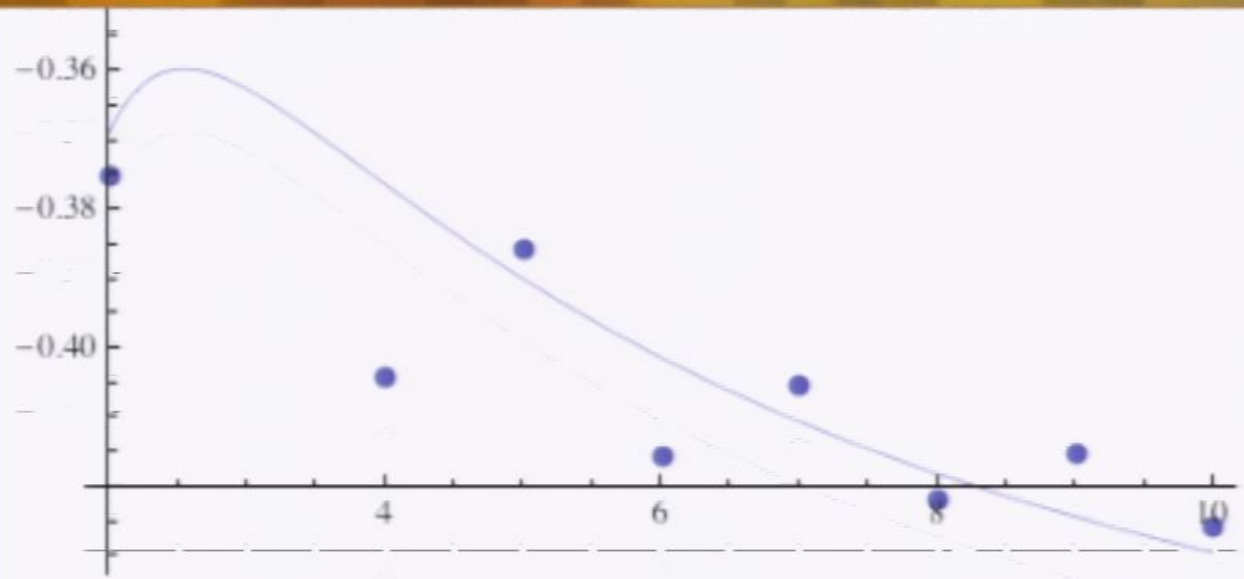
Lets split into odd and even to improve our prediction:

```
odd
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[66]=



Lets split into odd and even to improve our prediction:

In[69]:=

```
tbfit
```

Out[69]=

$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

```
od
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Lets split into odd and even to improve our prediction:

In[69]:=
```
tbfit
```

Out[69]=

$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

```
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```

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Untitled-25

Lets split into odd and even to improve our prediction:

In[69]:=
```
tbfit
```

Out[69]=

$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

```
odd = Table[tbf
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Lets split into odd and even to improve our prediction:

In[69]:=
```
tbfit
```

Out[69]=

$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

```
odd = Table[tbfit[[j]],
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[69]:=
```
tbfit
```

Out[69]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[70]:=
```
odd = Table[tbfit[[j]], {j, 2, max, 2}]
```

Part::partw :

Part 10 of $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \ll 3 \gg, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$ does not exist. ≫

Out[70]=
$$\left\{\left\{3, -\frac{1}{3}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}[[10]]\right\}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[69]:=
```
tbfit
```

Out[69]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}]
```

Part::partw :

Part 10 of $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \ll 3 \gg, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$ does not exist. ≫

Out[70]=
$$\left\{\left\{3, -\frac{1}{3}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}[[10]]\right\}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[69]:=
```
tbfit
```

Out[69]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[71]:=
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}]
```

Out[71]=
$$\left\{\left\{3, -\frac{1}{3}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{9, -\frac{1343}{3235}\right\}\right\}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

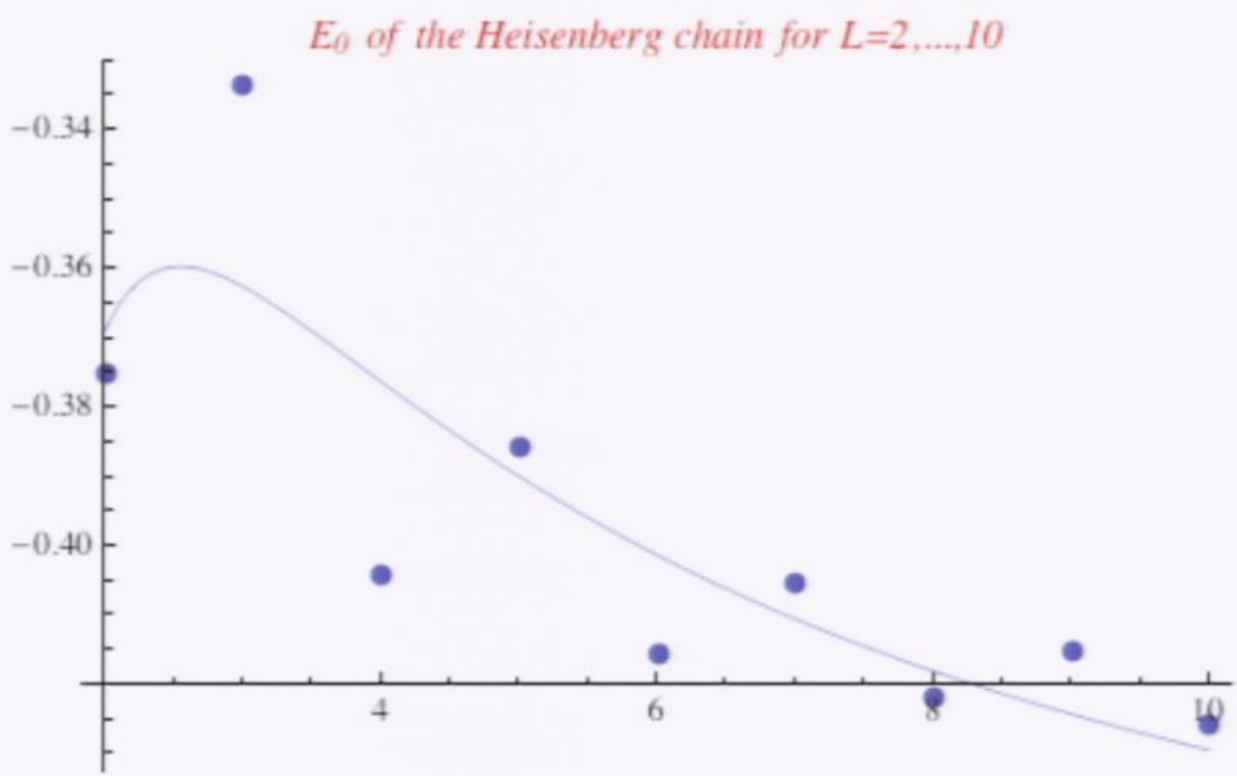
Untitled-25

In[68]:= prediction = fit /. x → ∞

Out[68]= -0.485173

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Out[66]=



Lets split into odd and even to improve our prediction:

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[68]:= prediction = fit /. x → ∞

Out[68]= -0.485173

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Lets split into odd and even to improve our prediction:

In[69]:=
```
tbfit
```

Out[69]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$

In[71]:=
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}]
```

Out[71]= $\left\{\left\{3, -\frac{1}{3}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{9, -\frac{1343}{3235}\right\}\right\}$

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In[68]:= prediction = fit /. x → ∞

Out[68]= -0.485173

In[66]:=
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```

Lets split into odd and even to improve our prediction:

In[69]:=
```
tbfit
```

Out[69]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$

In[71]:=
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}]
```

Out[71]= $\left\{\left\{3, -\frac{1}{3}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{9, -\frac{1343}{3235}\right\}\right\}$

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Untitled-25

In[68]:= prediction = fit /. x → 8

Out[68]= -0.485173

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Lets split into odd and even to improve our prediction:

In[69]:=
```
tbfit
```

Out[69]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$

In[71]:=
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}];
```

Out[71]= $\left\{\left\{3, -\frac{1}{3}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{9, -\frac{1343}{3235}\right\}\right\}$

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Untitled-25

In[68]:= prediction = fit /. x → ∞

Out[68]= -0.485173

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Lets split into odd and even to improve our prediction:

In[69]:=
```
tbfit
```

Out[69]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$

```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] // Fit[#, {1, 1 / x, 1
```

Out[71]= $\left\{\left\{3, -\frac{1}{3}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{9, -\frac{1343}{3235}\right\}\right\}$

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In[66]:=
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```

Lets split into odd and even to improve our prediction:

In[69]:=
```
tbfit
```

Out[69]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$

```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] // Fit[#, {1, 1/x, 1/x^2,
```

Out[71]= $\left\{\left\{3, -\frac{1}{3}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{9, -\frac{1343}{3235}\right\}\right\}$

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```
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```

Out[69]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$

```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1 /|
```

Out[71]= $\left\{\left\{3, -\frac{1}{3}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{9, -\frac{1343}{3235}\right\}\right\}$

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```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}
```

Out[71]= $\left\{\left\{3, -\frac{1}{3}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{9, -\frac{1343}{3235}\right\}\right\}$

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Untitled-25

In[68]:= prediction = fit /. x → ∞

Out[68]= -0.485173

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Lets split into odd and even to improve our prediction:

In[69]:=
```
tbfit
```

Out[69]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$

```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
```

Out[71]= $\left\{\left\{3, -\frac{1}{3}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{9, -\frac{1343}{3235}\right\}\right\}$

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In[69]:=
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tbfit
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Out[69]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$

In[72]:=
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
```

Out[72]= $-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[68]:= prediction = fit /. x → ∞

Out[68]= -0.485173

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

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```
tbfit
```

Out[69]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$

In[72]:=
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
```

Out[72]= $-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$

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Untitled-25

In[66]:=
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Show[lp, Plot[fit, {x, 2, 10}]]
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Lets split into odd and even to improve our prediction:

In[69]:=
```
tbfit
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Out[69]=
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In[72]:=
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
```

Out[72]=
$$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

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Untitled-25

In[69]:=
```
tbfit
```

Out[69]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[72]:=
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
```

Out[72]=
$$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[69]:=
```
tbfit
```

Out[69]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[72]:=
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
```

Out[72]=
$$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[69]:=
```
tbfit
```

Out[69]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[72]:=
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
```

Out[72]=
$$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

```
Show[lp, Plot[{fit, odd}, {x, 2, 10}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[69]:=
```
tbfit
```

Out[69]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[72]:=
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
```

Out[72]=
$$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

```
Show[lp, Plot[{fit, odd}, {x, 2, 10}]]
```

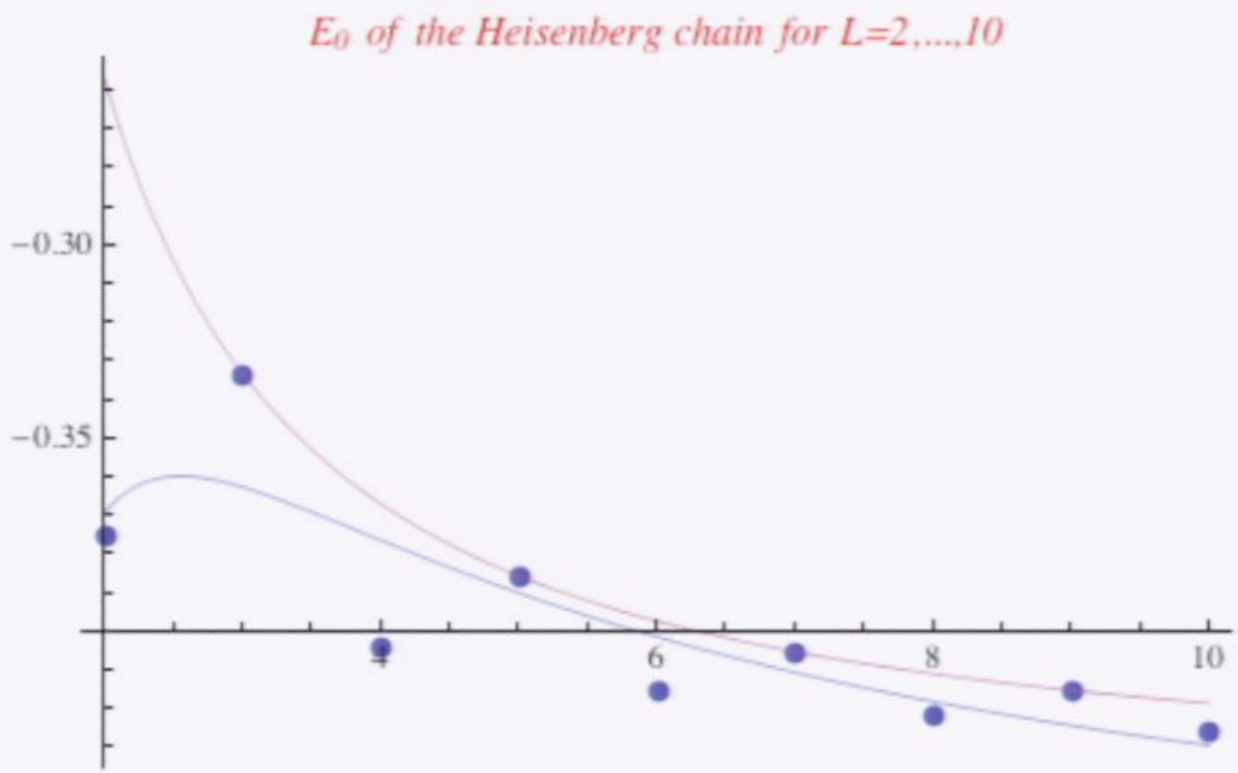
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[72]= $-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$

In[73]:=
```
Show[lp, Plot[{fit, odd}, {x, 2, 10}]]
```

Out[73]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[69]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1793}{4443}\right\}, \left\{5, -\frac{923}{2399}\right\}, \left\{6, -\frac{1811}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$

In[72]:=
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
```

Out[72]= $-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$

In[73]:=
```
Show[lp, Plot[{fit, odd}, {x, 2, 10}]]
```

Out[73]=

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

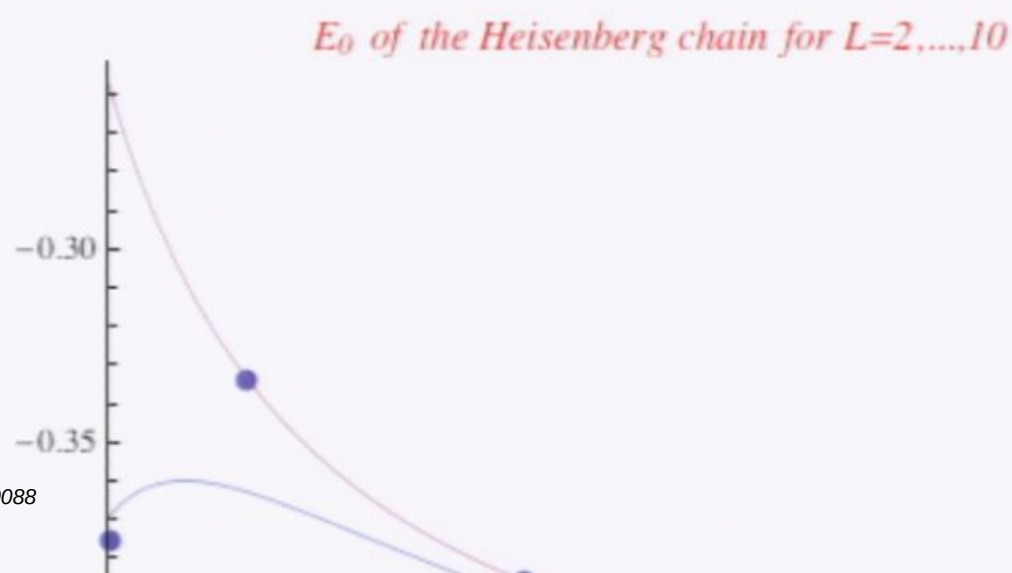
$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

In[72]:=
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
```

Out[72]=
$$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

In[73]:=
```
Show[lp, Plot[{fit, odd}, {x, 2, 10}]]
```

Out[73]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

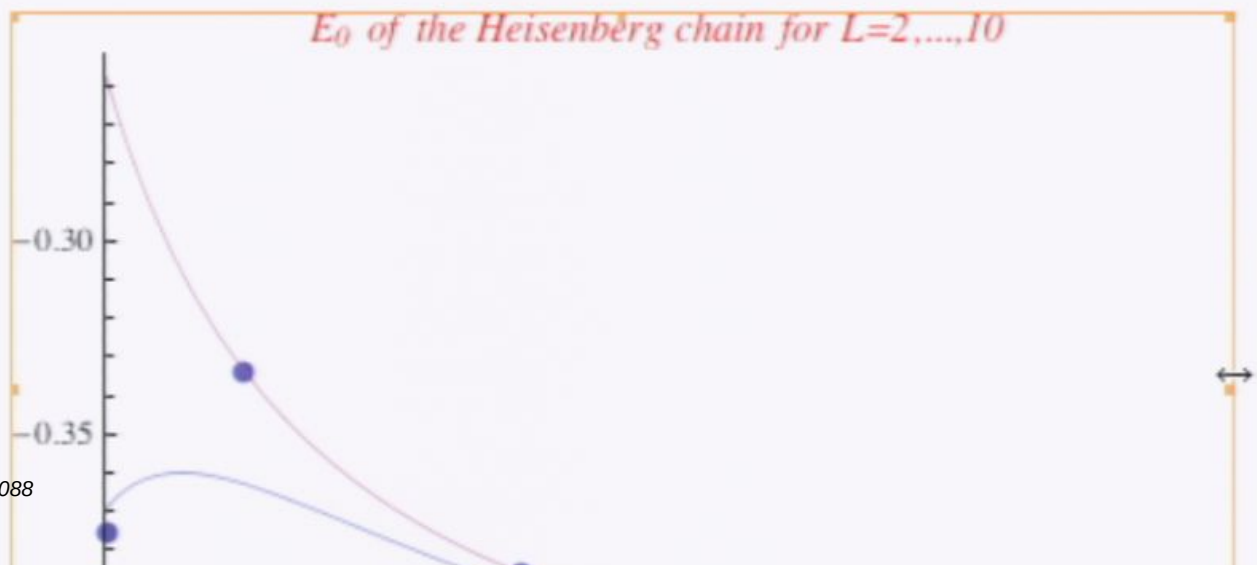
$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

In[72]:=
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
```

Out[72]=
$$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

In[73]:=
```
Show[lp, Plot[{fit, odd}, {x, 2, 10}]]
```

Out[73]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

In[72]:=
```
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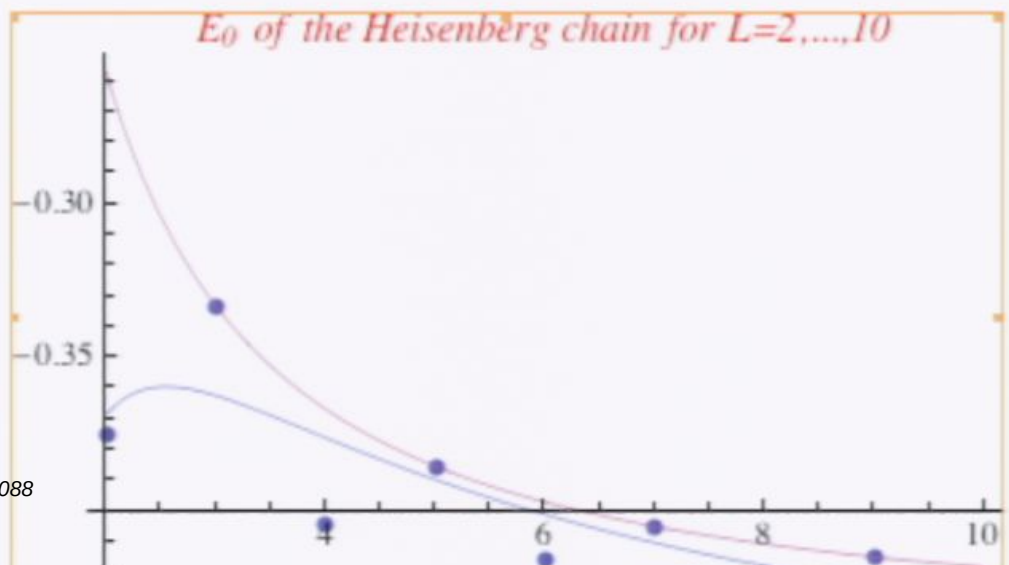
```

Out[72]=
$$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

In[73]:=
```
Show[lp, Plot[{fit, odd}, {x, 2, 10}]]
```

Out[73]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

In[72]:=
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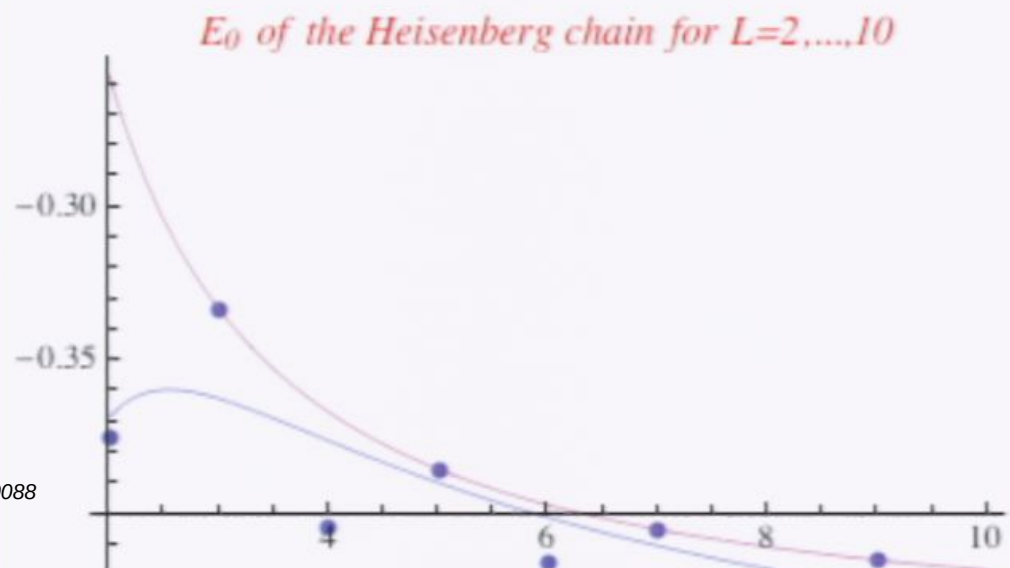
```

Out[72]=
$$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

In[73]:=
```
Show[lp, Plot[{fit, odd}, {x, 2, 10}]]
```

Out[73]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
```

Out[72]= $$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

In[73]:=
```
Show[lp, Plot[{fit, odd}, {x, 2, 10}]]
```

Out[73]=

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

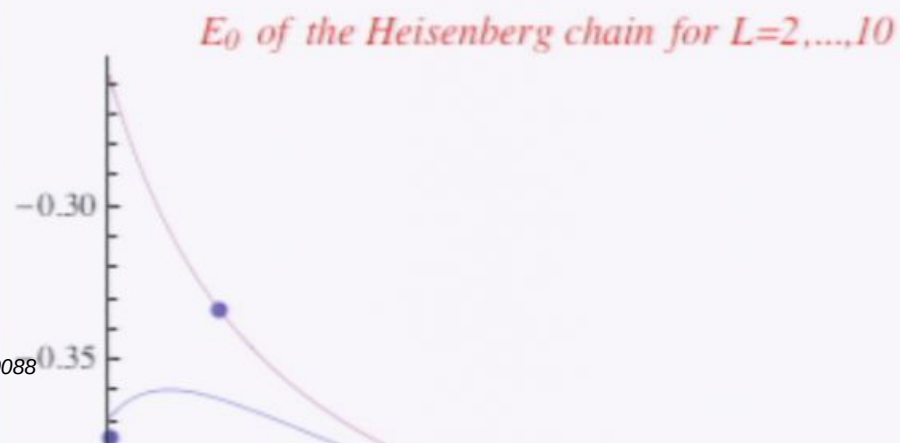
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
even = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
```

Out[72]= $$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

In[73]:=

```
Show[lp, Plot[{fit, odd}, {x, 2, 10}]]
```

Out[73]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

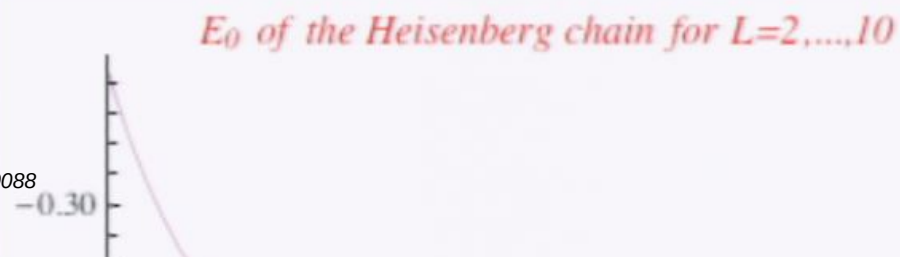
$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

In[74]:=
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
even = Table[tbfit[[j]], {j, 1, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
```

Out[74]= $-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$

Out[75]= $-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$

In[73]:=
```
Show[lp, Plot[{fit, odd}, {x, 2, 10}]]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

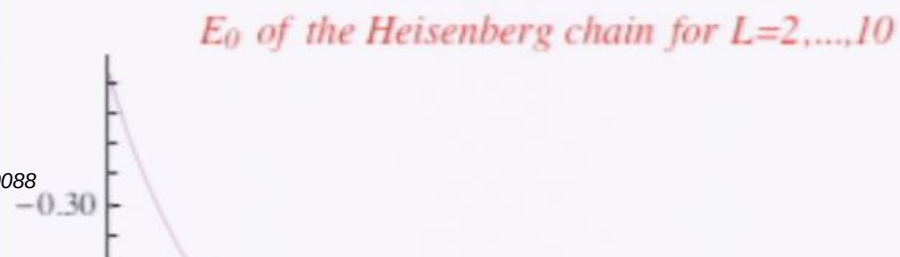
$$\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\}$$

In[74]:=
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
even = Table[tbfit[[j]], {j, 1, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
```

Out[74]=
$$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

Out[75]=
$$-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$$

```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10}]]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

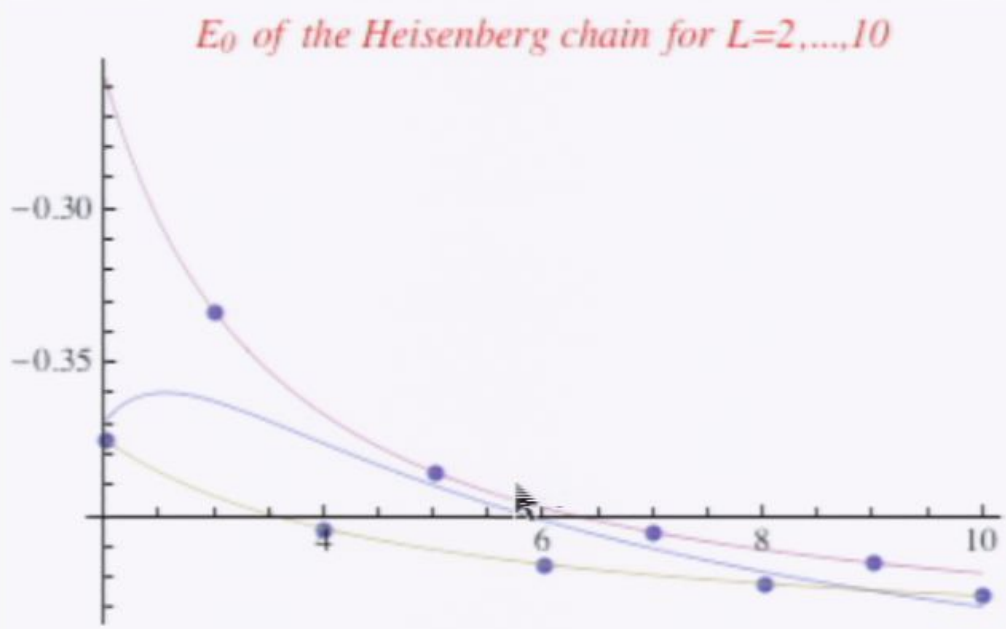
Untitled-25

Out[74]= $$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

Out[75]= $$-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$$

In[76]:=
```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10}]]
```

Out[76]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

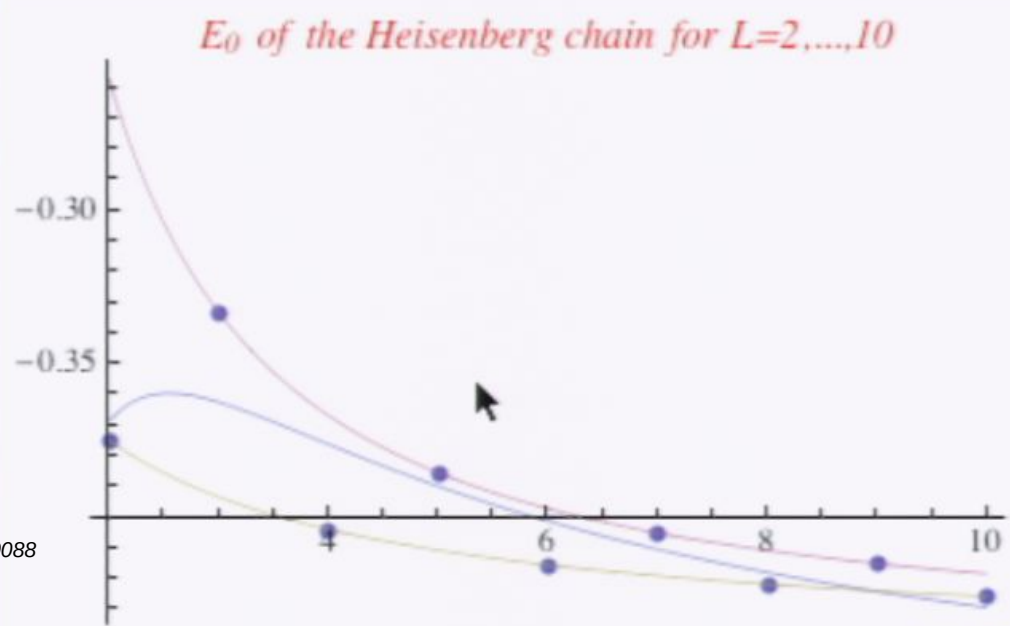
```
even = Table[tbfit[[j]], {j, 1, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
```

Out[74]= $-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$

Out[75]= $-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$

In[76]:=
```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10}]]
```

Out[76]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

x^2 x

In[68]:=
```
prediction = fit /. x → ∞
```

Out[68]= -0.485173

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Lets split into odd and even to improve our prediction:

In[69]:=
```
tbfit
```

Out[69]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$

In[74]:=
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
even = Table[tbfit[[j]], {j, 1, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
```

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Untitled-25

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[62]:=
```
tbfit = Rationalize[tb, 10^-7]
```

Out[62]=

$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[64]:=
```
fit = Fit[tbfit, {1, 1 / x, 1 / x^2}, x]
```

Out[64]=

$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

In[68]:=
```
prediction = fit /. x → ∞
```

Out[68]=

$$-0.485173$$


```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[62]:=
```
tbfit = Rationalize[tb, 10^-7]
```

Out[62]=
$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

```
fit = Fit[tbfit, {1, 1/x, 1/x^2, 1/x^2}, x]
```

Out[64]=
$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

In[68]:=
```
prediction = fit /. x → ∞
```

Out[68]=
$$-0.485173$$


```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[62]:=

```
tbfit = Rationalize[tb, 10^-7]
```

Out[62]=

$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

```
fit = Fit[tbfit, {1, 1/x, 1/x^2, 1/x^3}, x]
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Out[64]=

$$-0.485173 - \frac{0.812303}{x^2} + \frac{0.638491}{x}$$

In[68]:=

```
prediction = fit /. x → ∞
```

Out[68]=

-0.485173


```
Show[lp, Plot[fit, {x, 2, 10}]]
```

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Untitled-25

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FullForm[lp]
lp /. {Hue[a__] → Red, Rule[PointSize, b_] → Rule[PointSize, 0.04]}
```

In[62]:=

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tbfit = Rationalize[tb, 10^-7]
```

Out[62]=

$$\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[77]:=

```
fit = Fit[tbfit, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x]
```

Out[77]=

$$-0.697758 - \frac{100.148}{x^4} + \frac{104.171}{x^3} - \frac{36.6017}{x^2} + \frac{5.4223}{x}$$

In[68]:=

```
prediction = fit /. x → ∞
```

Out[68]=

-0.485173


```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[62]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$

In[77]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x]
```

Out[77]= $-0.697758 - \frac{100.148}{x^4} + \frac{104.171}{x^3} - \frac{36.6017}{x^2} + \frac{5.4223}{x}$

In[68]:=
```
prediction = fit /. x → ∞
```

Out[68]= -0.485173

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Lets split into odd and even to improve our prediction:

In[69]:=
```
tbfit
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[62]= $\left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1795}{4443}\right\}, \left\{5, -\frac{925}{2399}\right\}, \left\{6, -\frac{1844}{4437}\right\}, \left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$

In[77]:=
```
fit = Fit[tbfit, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x]
```

Out[77]= $-0.697758 - \frac{100.148}{x^4} + \frac{104.171}{x^3} - \frac{36.6017}{x^2} + \frac{5.4223}{x}$

In[68]:=
```
prediction = fit /. x → ∞
```

Out[68]= -0.485173

In[66]:=
```
Show[lp, Plot[fit, {x, 2, 10}]]
```

Lets split into odd and even to improve our prediction:

In[69]:=
```
tbfit
```

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Untitled-25

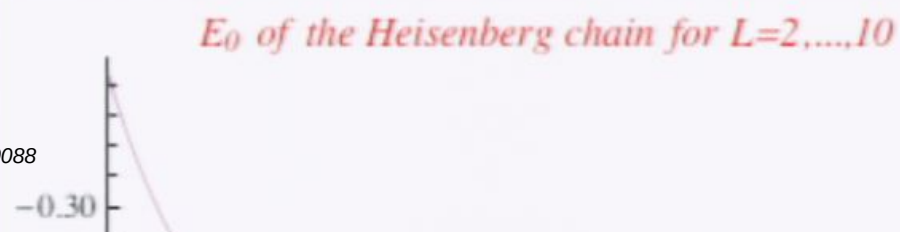
$$\left\{\left\{7, -\frac{1487}{3670}\right\}, \left\{8, -\frac{791}{1875}\right\}, \left\{9, -\frac{1343}{3235}\right\}, \left\{10, -\frac{967}{2271}\right\}\right\}$$

In[74]:=
```
odd = Table[tbfit[[j]], {j, 2, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
even = Table[tbfit[[j]], {j, 1, max - 1, 2}] //
  Fit[#, {1, 1/x, 1/x^2, 1/x^3, 1/x^4}, x] &
```

Out[74]= $-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$

Out[75]= $-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$

In[76]:=
```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10}]]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

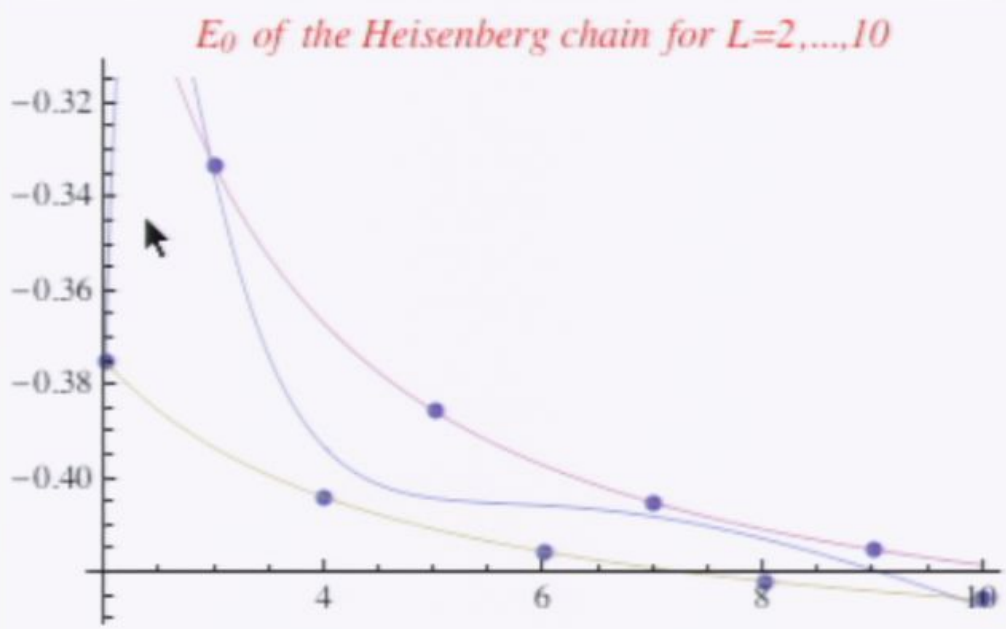
Untitled-25

Out[74]= $$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

Out[75]= $$-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$$

In[78]:=
```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10}]]
```

Out[78]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

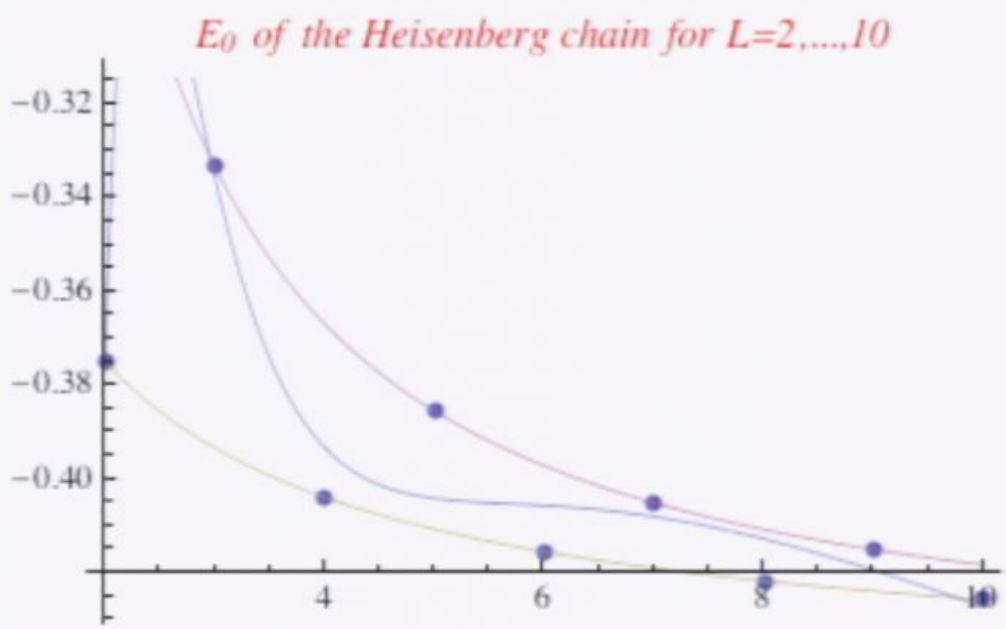
Untitled-25

Out[74]= $$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

Out[75]= $$-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$$

```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10}, PlotStyle ->]]
```

Out[78]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

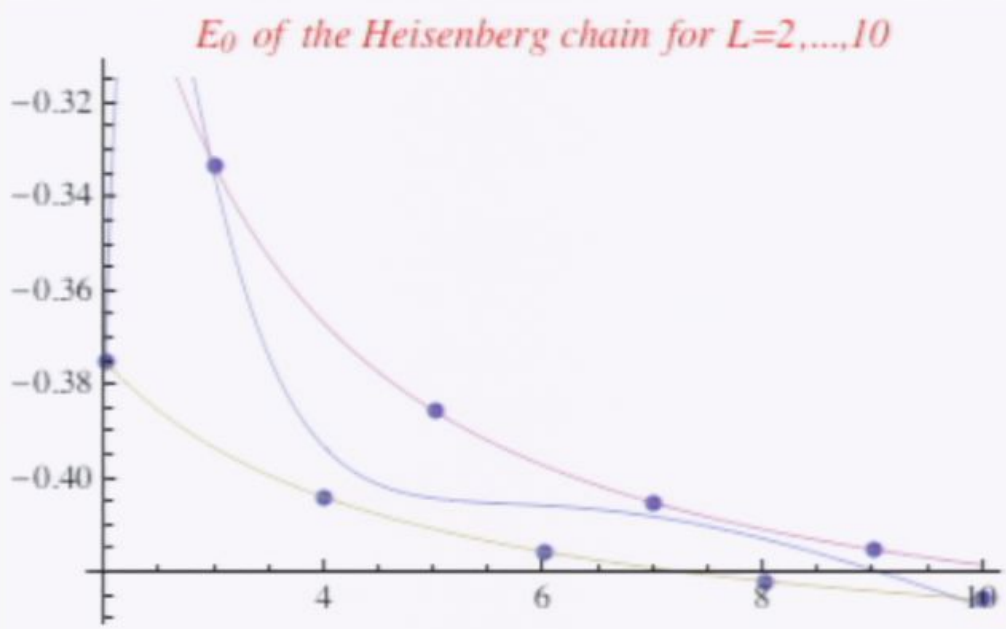
Untitled-25

Out[74]= $$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

Out[75]= $$-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$$

```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10}, PlotStyle → {|, , }]]
```

Out[78]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

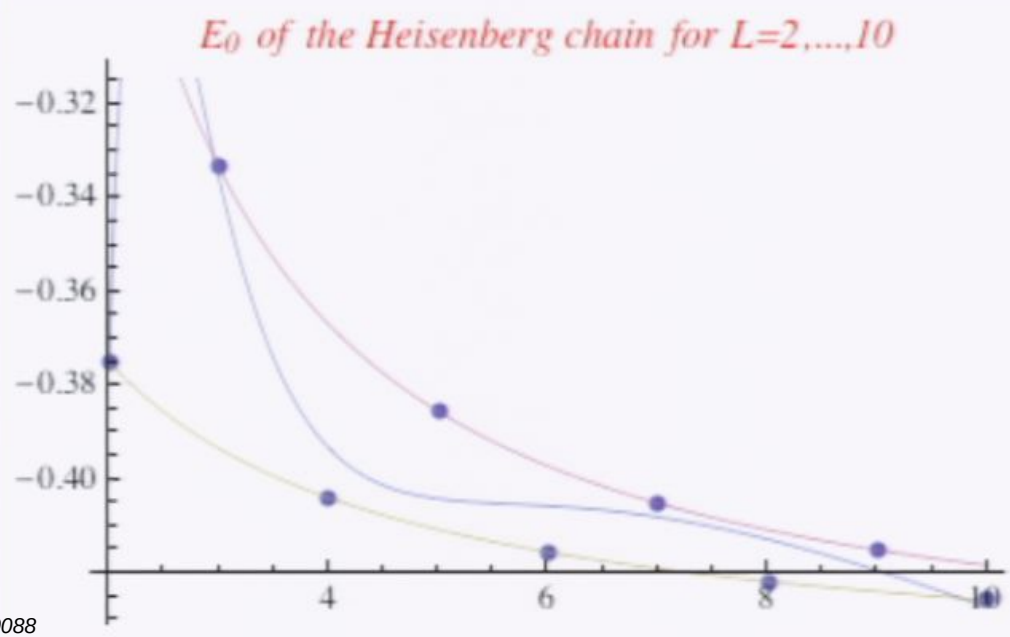
Untitled-25

Out[74]= $$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

Out[75]= $$-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$$

```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
  PlotStyle → {Thick, Dashed,}]]
```

Out[78]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

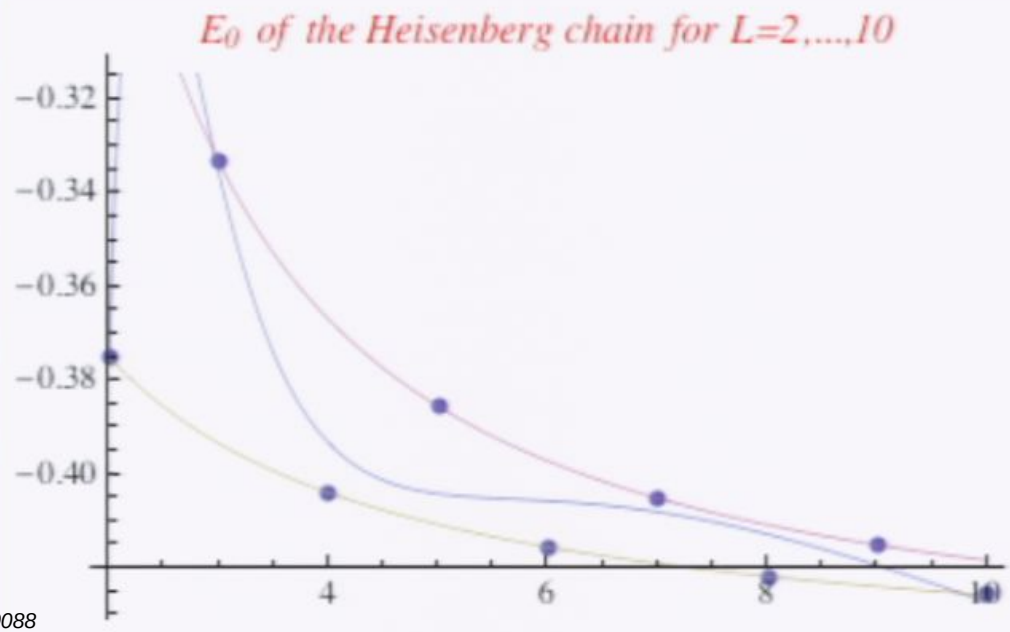
Untitled-25

Out[74]= $$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

Out[75]= $$-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$$

```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
    PlotStyle → {Thick, {Dashed,}}]]
```

Out[78]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

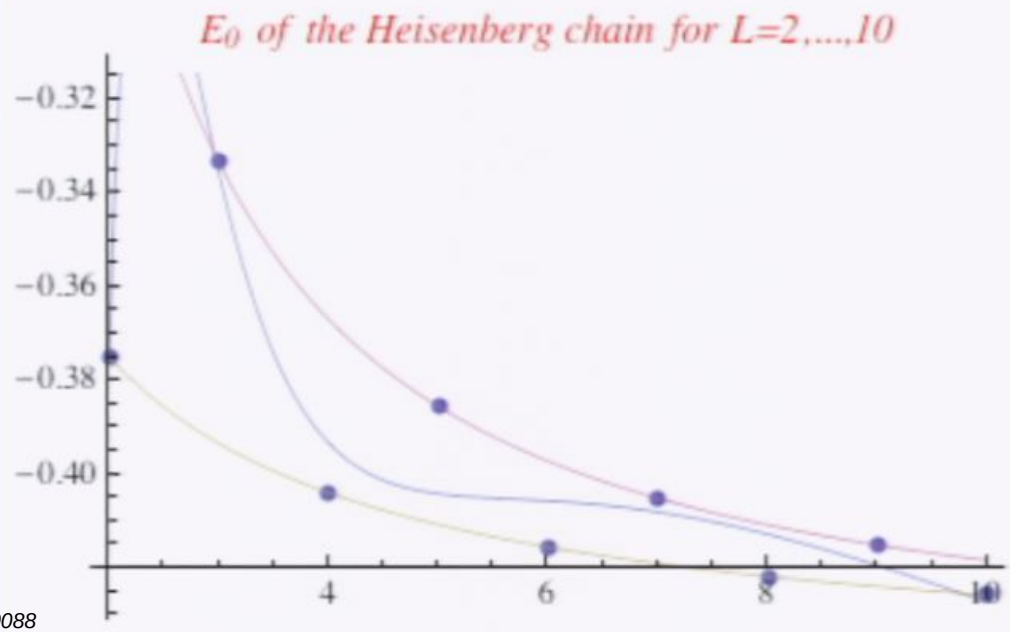
Untitled-25

Out[74]= $$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

Out[75]= $$-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$$

```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
  PlotStyle → {Thick, {Red, Dashed},}]]
```

Out[78]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

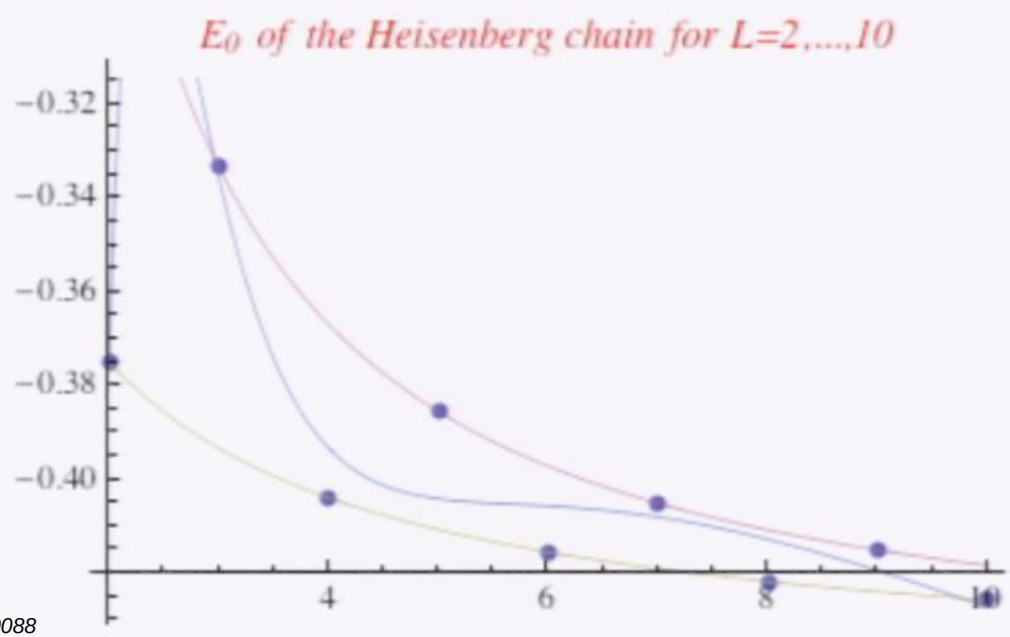
Untitled-25

Out[74]= $$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

Out[75]= $$-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$$

```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
  PlotStyle → {Thick, {Red, Dashed}, {Red, Dashed}}]]
```

Out[78]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

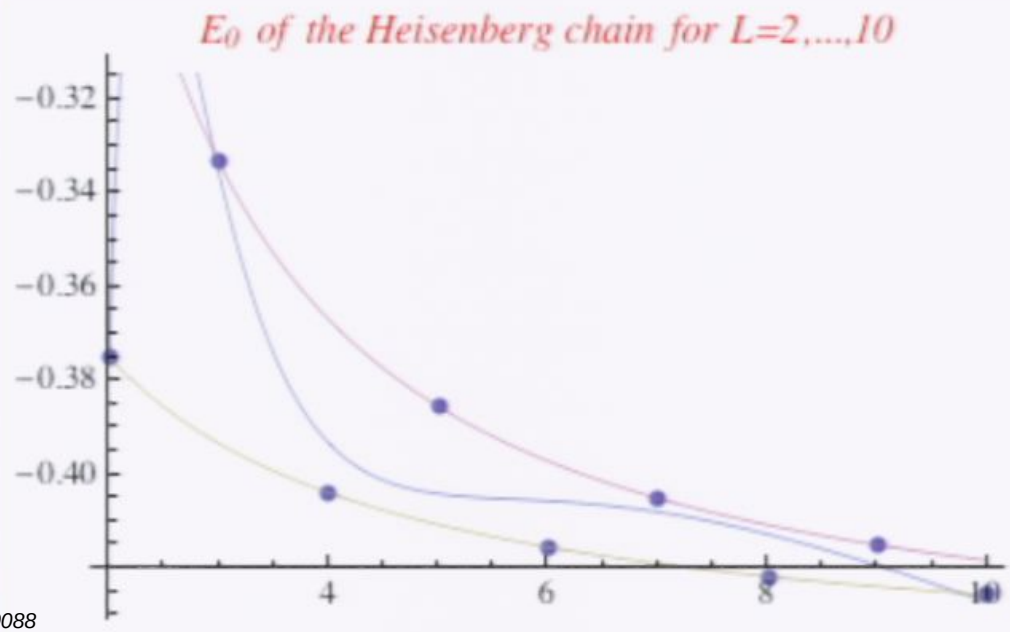
Untitled-25

Out[74]= $$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

Out[75]= $$-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$$

```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
    PlotStyle → {{BkThick, {Red, Dashed}, {Red, Dashed}}]]
```

Out[78]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

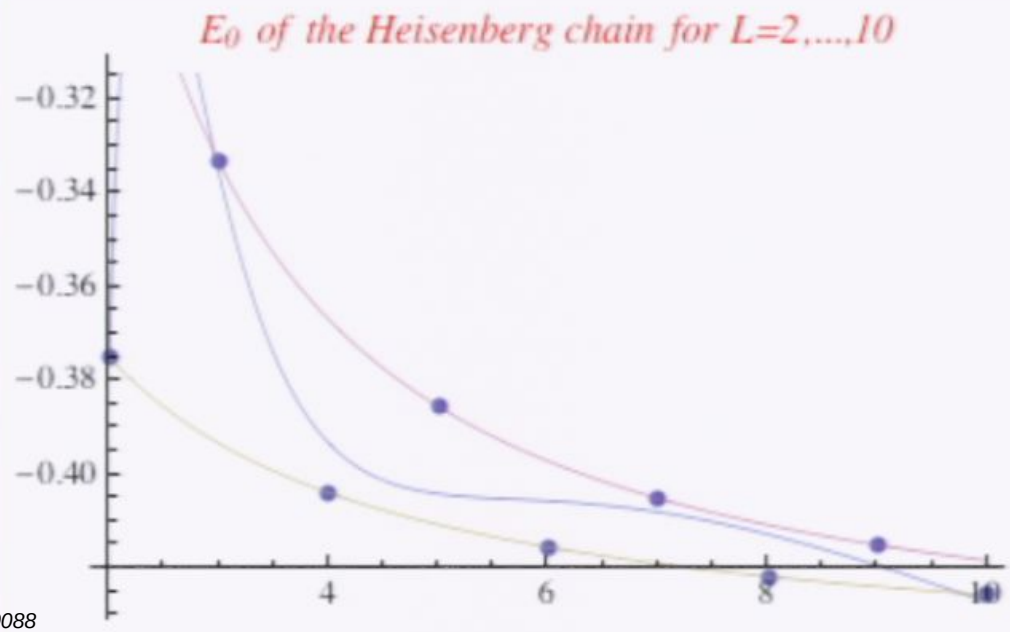
Untitled-25

Out[74]= $$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

Out[75]= $$-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$$

```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
    PlotStyle → {{Blue, Thick, {Red, Dashed}, {Red, Dashed}}]]
```

Out[78]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

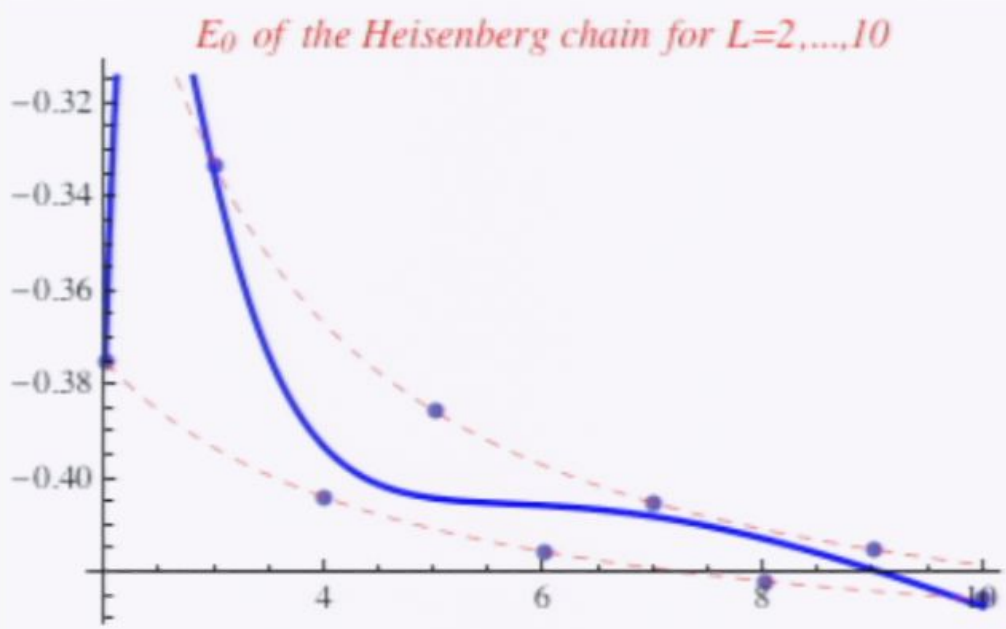
Untitled-25

Out[74]= $$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

Out[75]= $$-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$$

In[79]:=
```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
  PlotStyle → {{Blue, Thick}, {Red, Dashed}, {Red, Dashed}}]]
```

Out[79]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

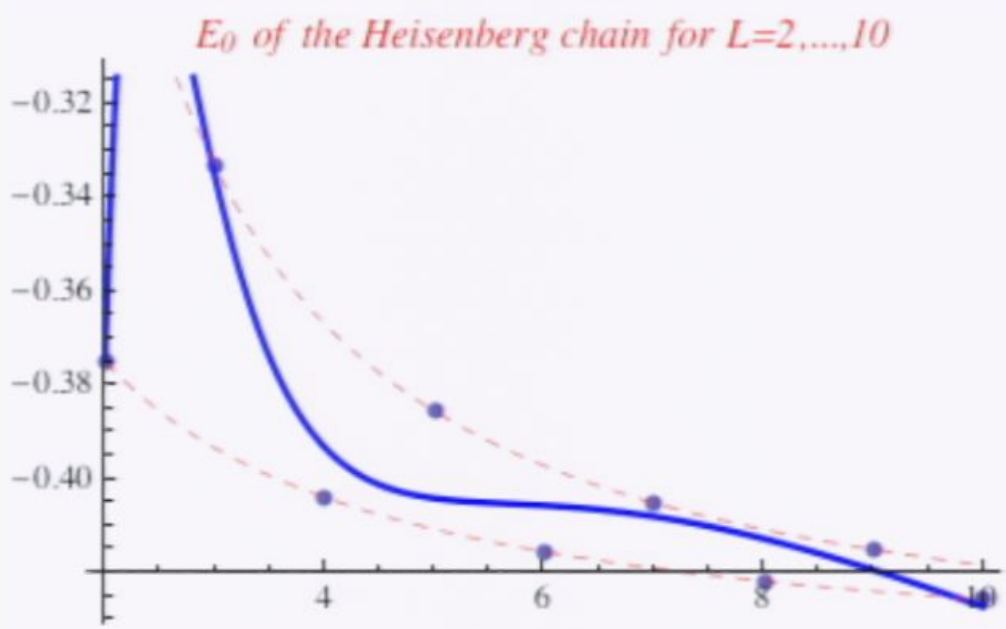
Untitled-25

Out[74]= $-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$

Out[75]= $-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$

```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
  PlotStyle → {{Blue, Dash}, {Red, Dashed}, {Red, Dashed}}]]
```

Out[79]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

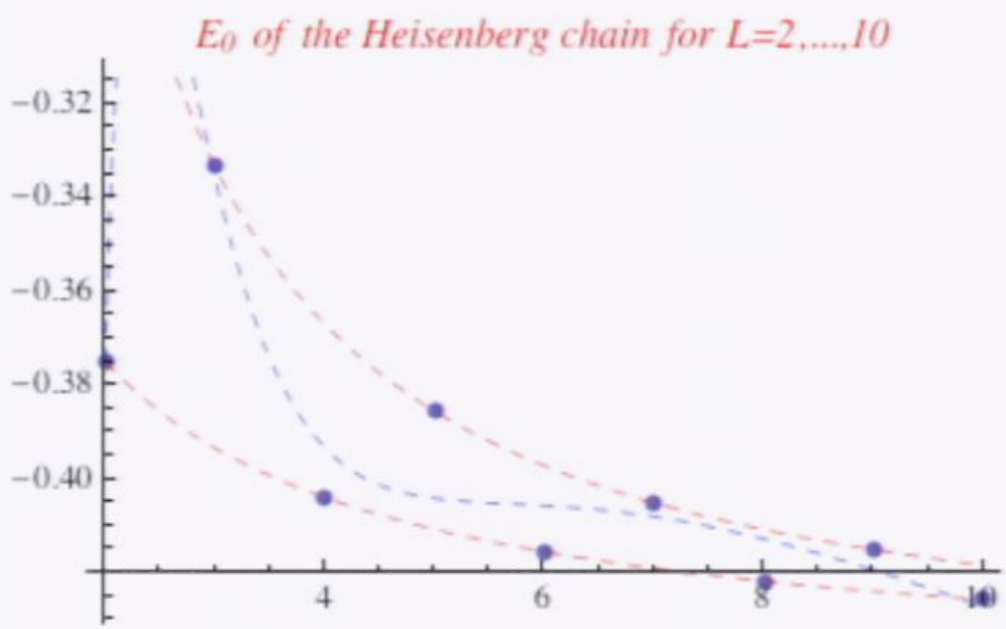
Untitled-25

Out[74]= $$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

Out[75]= $$-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$$

In[80]:=
```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
  PlotStyle → {{Blue, Dashed}, {Red, Dashed}, {Red, Dashed}}]]
```

Out[80]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

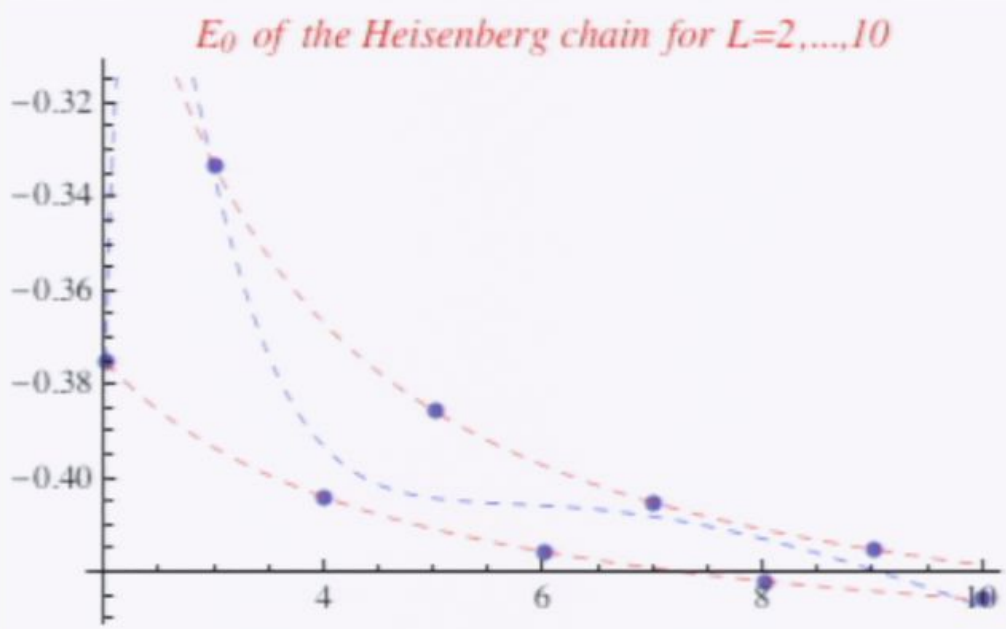
Untitled-25

Out[74]= $$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

Out[75]= $$-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$$

```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
  PlotStyle → {{Blue, Dashed}, {Re}, {Red, Dashed}}]]
```

Out[80]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

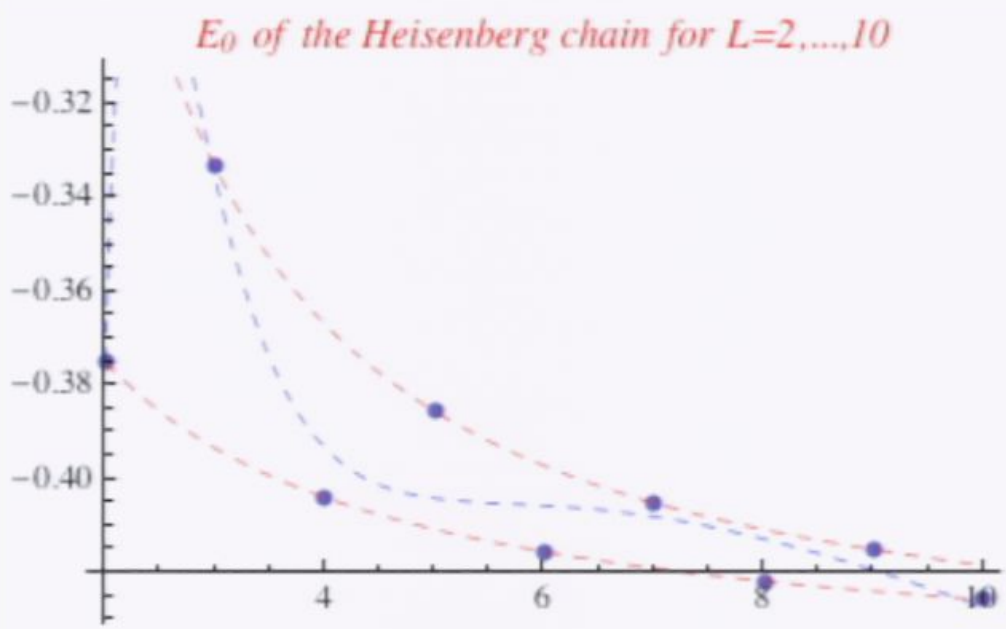
Untitled-25

Out[74]= $$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

Out[75]= $$-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$$

```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
  PlotStyle → {{Blue, Dashed}, {Red}, {Red}}]]
```

Out[80]=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

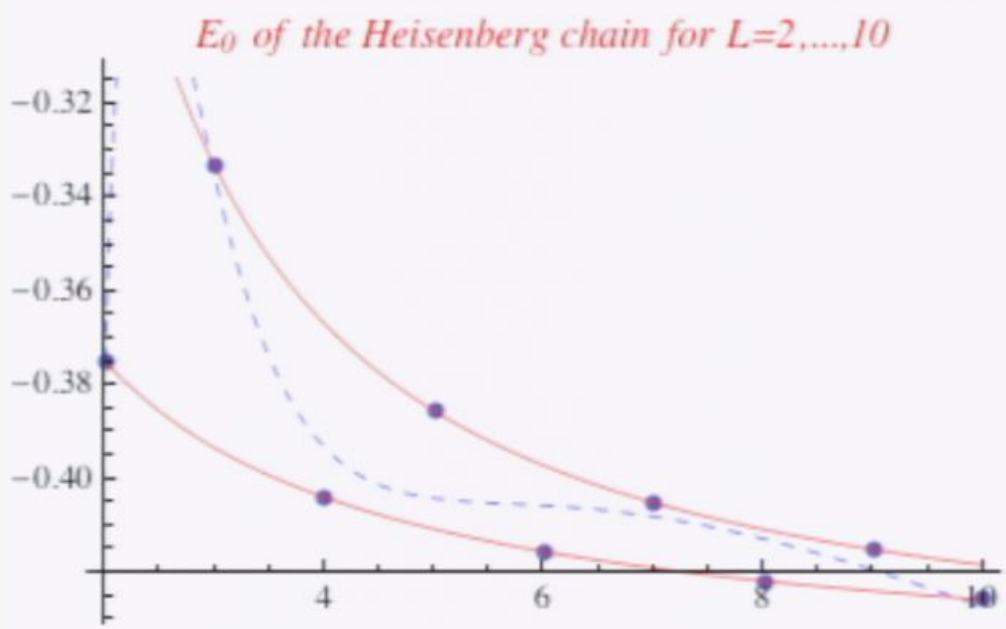
Untitled-25

Out[74]= $$-0.443297 + \frac{0.551602}{x^4} - \frac{0.746182}{x^3} + \frac{0.587527}{x^2} + \frac{0.196529}{x}$$

Out[75]= $$-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$$

In[81]:=
```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
  PlotStyle → {{Blue, Dashed}, {Red}, {Red}}]]
```

Out[81]=



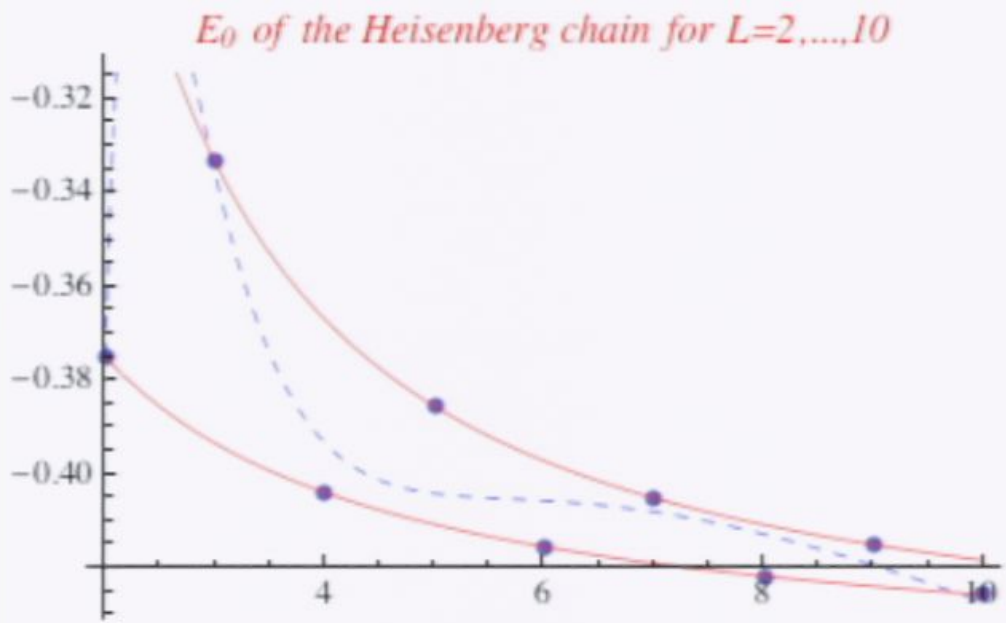
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[75]= $-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$

In[81]:=
```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
  PlotStyle → {{Blue, Dashed}, {Red}, {Red}}]]
```

Out[81]=




```
predict
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

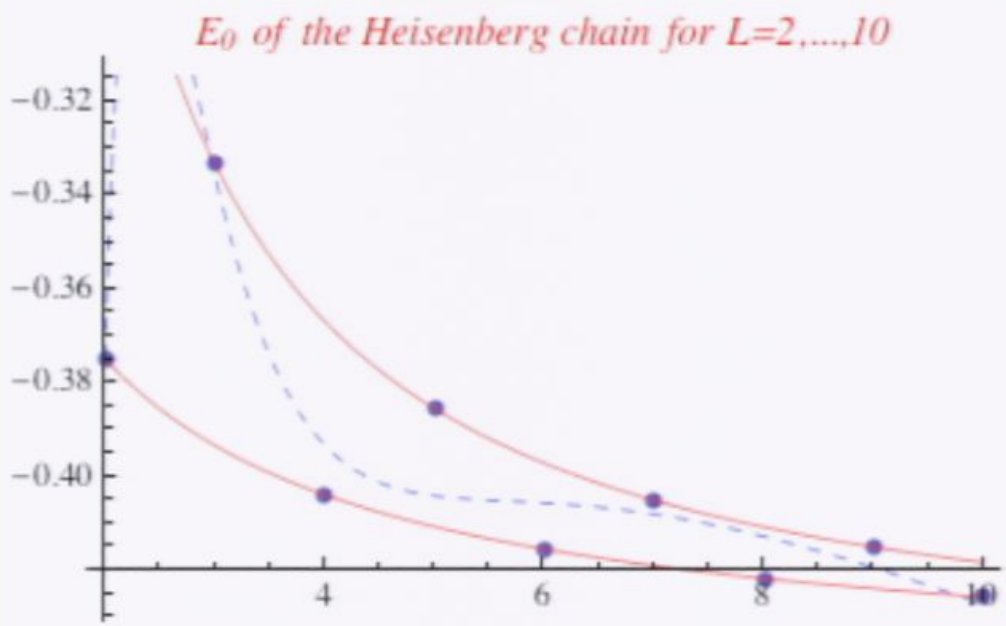
Out[75]=

$$-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$$

In[81]:=

```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
  PlotStyle → {{Blue, Dashed}, {Red}, {Red}}]]
```

Out[81]=




```
prediction = odd + even
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

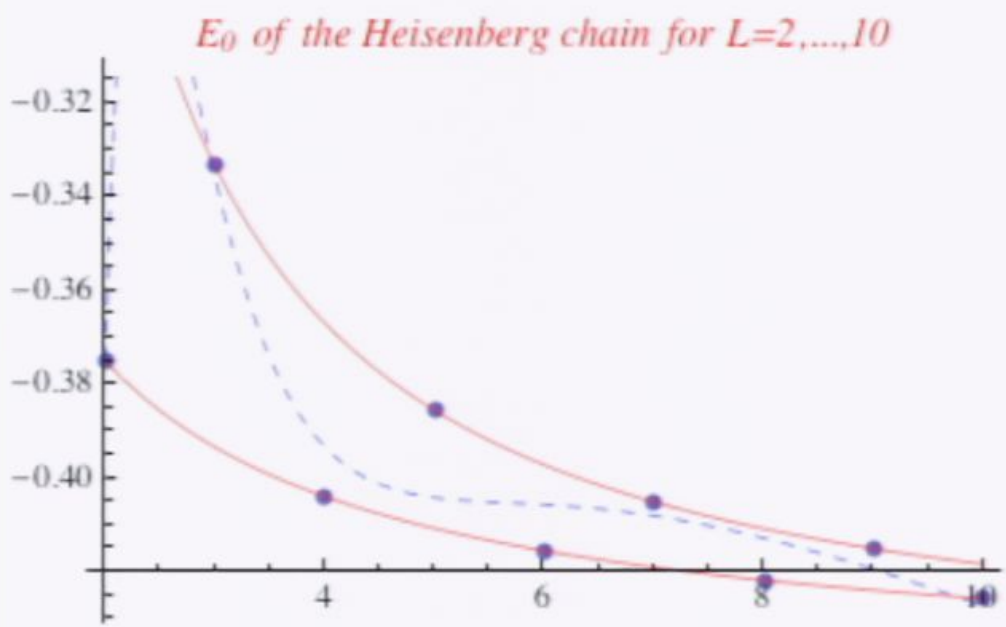
Out[75]=

$$-0.443088 - \frac{0.0764015}{x^4} + \frac{0.138688}{x^3} - \frac{0.15119}{x^2} + \frac{0.186649}{x}$$

In[81]:=

```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
  PlotStyle → {{Blue, Dashed}, {Red}, {Red}}]]
```

Out[81]=



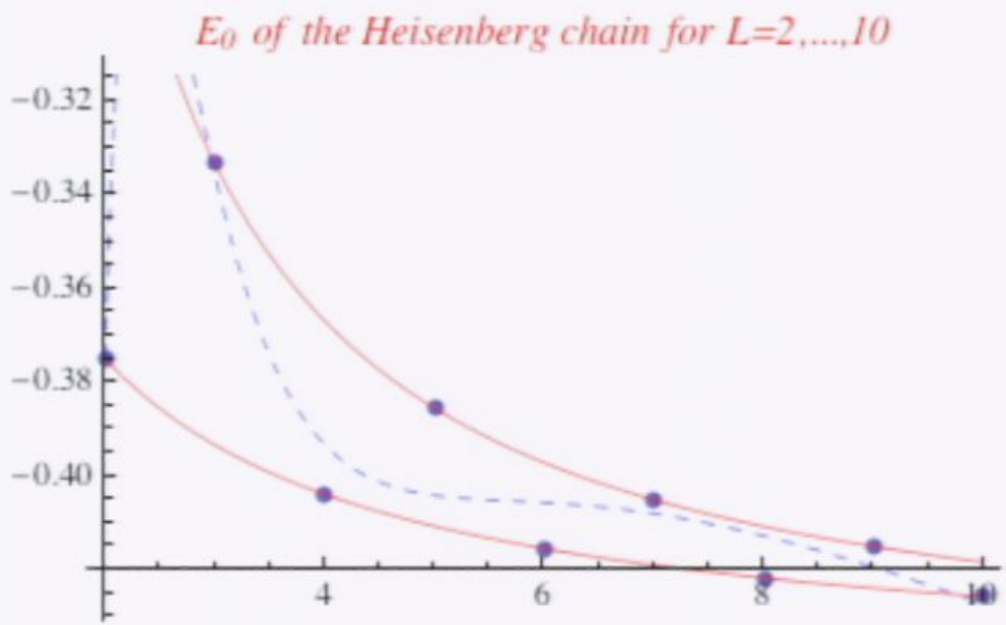
$$\text{prediction} = \frac{\text{odd} + \text{even}}{2}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[81]:=
```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
  PlotStyle → {{Blue, Dashed}, {Red}, {Red}}]]
```

Out[81]=



In[82]:=
```
prediction = (odd + even)/2 /. x → ∞
```

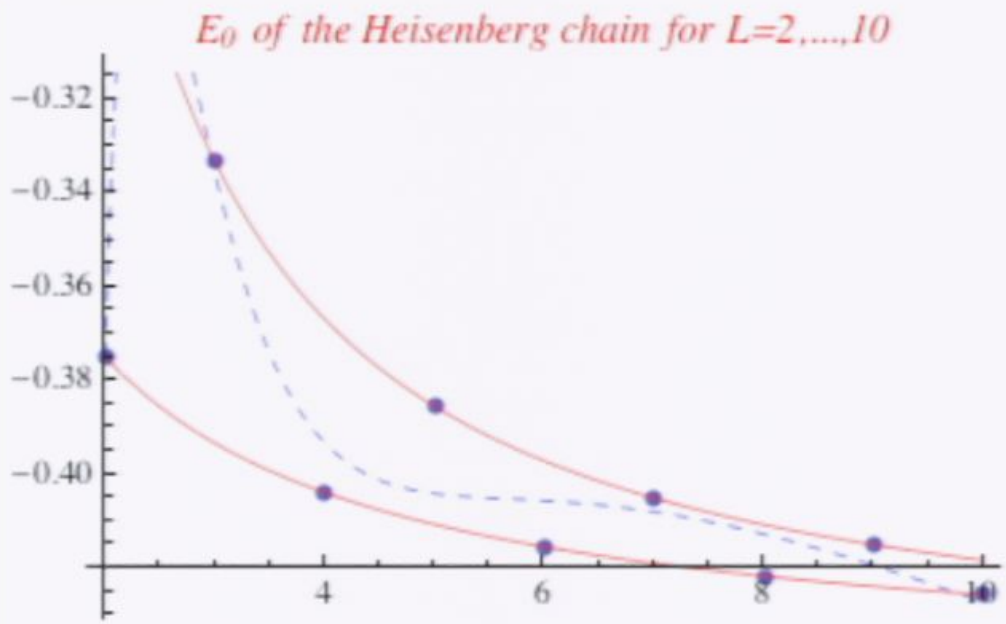
Out[82]=
```
-0.443192
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[81]:=
```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
  PlotStyle → {{Blue, Dashed}, {Red}, {Red}}]]
```

Out[81]=



In[82]:=
```
prediction = (odd + even)/2 /. x → ∞
```

Out[82]=
```
-0.443192
```

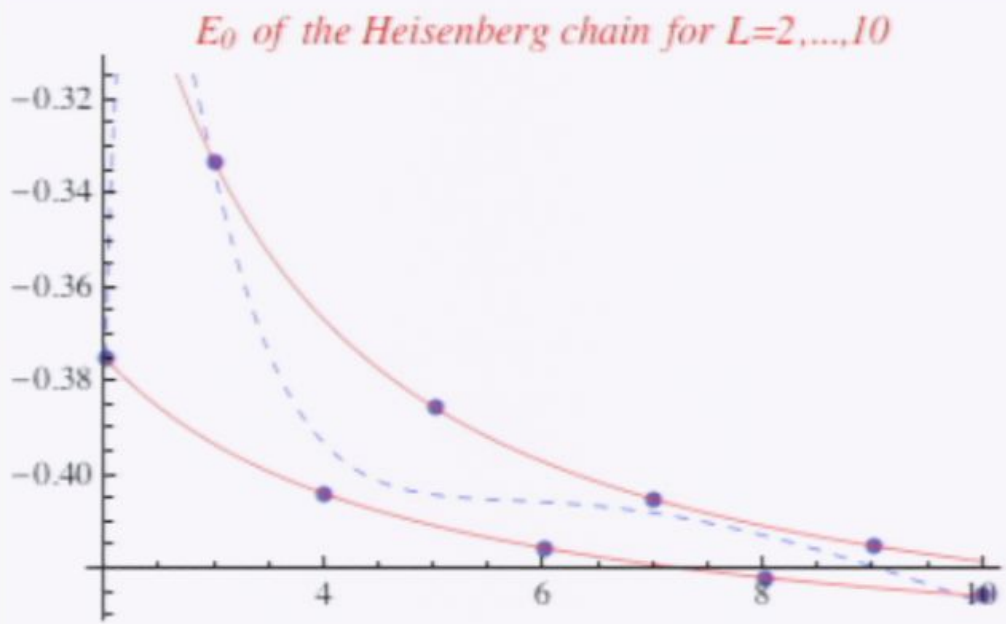
```
exact
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[81]:=
```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
  PlotStyle → {{Blue, Dashed}, {Red}, {Red}}]]
```

Out[81]=



In[82]:=
```
prediction = (odd + even)/2 /. x → ∞
```

Out[82]=
```
-0.443192
```

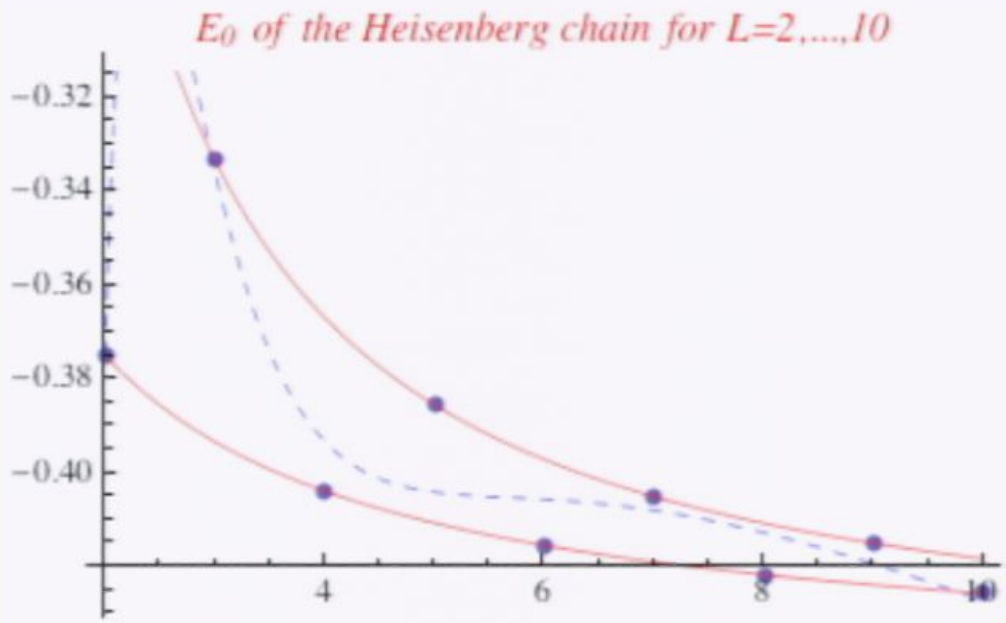
```
Bethe
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[81]:=
```
Show[lp, Plot[{fit, odd, even}, {x, 2, 10},
  PlotStyle → {{Blue, Dashed}, {Red}, {Red}}]]
```

Out[81]=



In[82]:=
```
prediction = (odd + even)/2 /. x → ∞
```

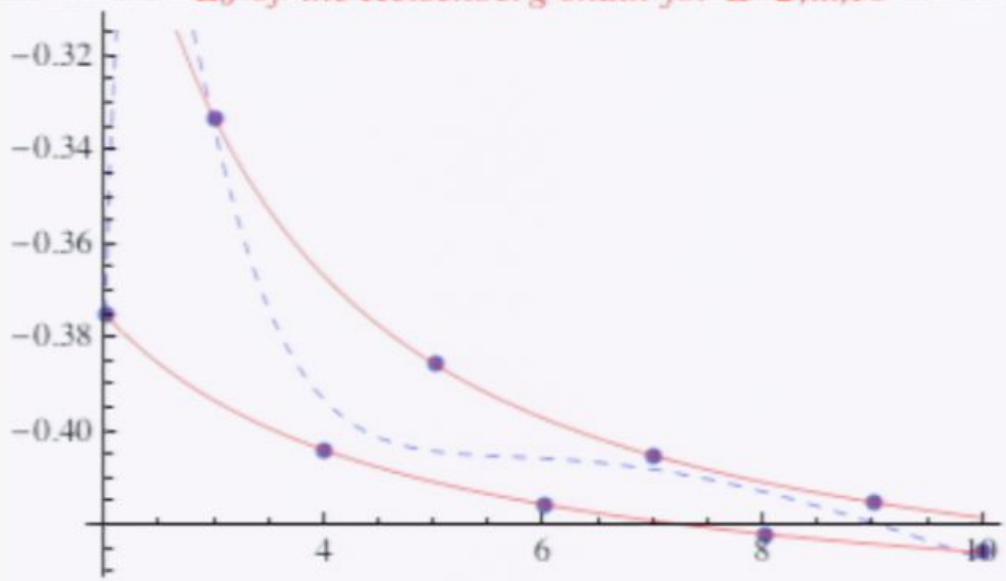
Out[82]=
```
-0.443192
```

```
Bethe = -Log[2] + 1 /|
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[81]=



In[82]:=

```
prediction = (odd + even)/2 /. x → ∞
```

Out[82]= -0.443192

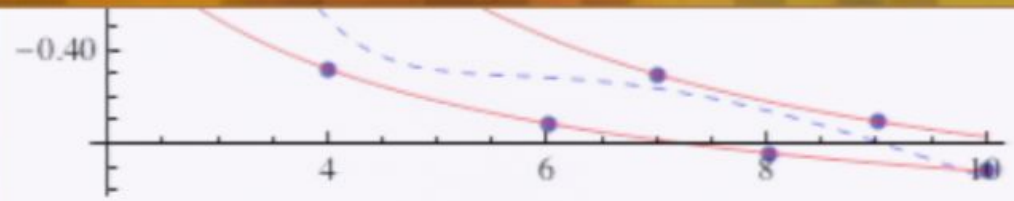
In[83]:=

```
Bethe = -Log[2] + 1/4 // N
```

Out[83]= -0.443147

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25



In[82]:=
```
prediction = (odd + even)/2 /. x → ∞
```

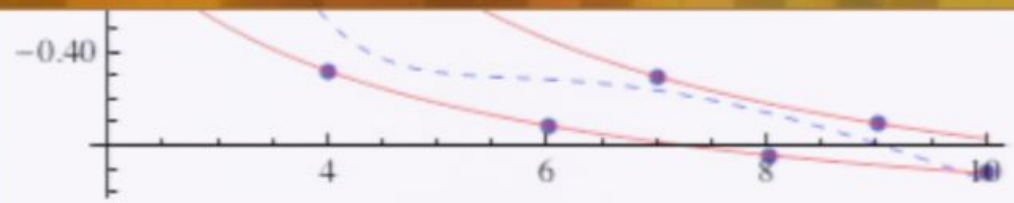
Out[82]=
```
-0.443192
```

In[83]:=
```
Bethe = -Log[2] + 1/4 // N
```

Out[83]=
```
-0.443147
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25



In[82]:=
```
prediction = (odd + even)/2 /. x → ∞
```

Out[82]=
```
-0.443192
```

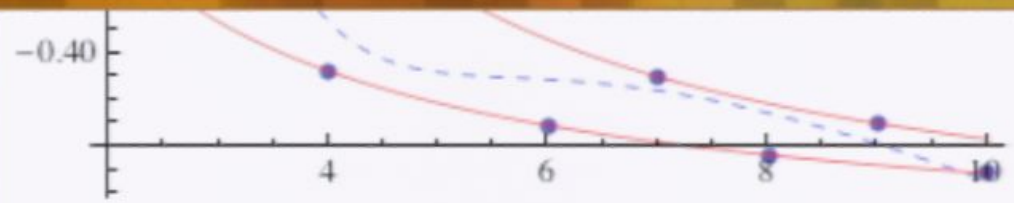
In[83]:=
```
Bethe = -Log[2] + 1/4 // N
```

Out[83]=
```
-0.443147
```

```
prediction
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25



In[82]:=
```
prediction = (odd + even)/2 /. x → ∞
```

Out[82]=
```
-0.443192
```

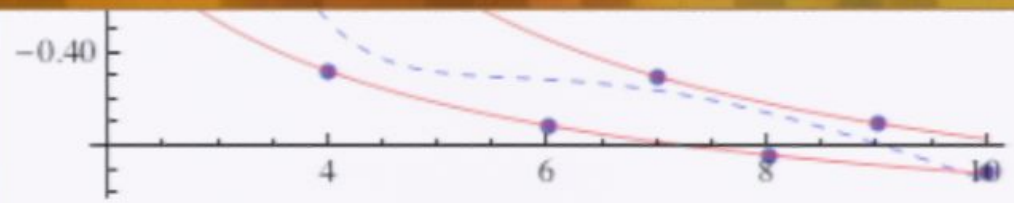
In[83]:=
```
Bethe = -Log[2] + 1/4 // N
```

Out[83]=
```
-0.443147
```

```
prediction - Bethe
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25



In[82]:=
```
prediction = (odd + even)/2 /. x → ∞
```

Out[82]=
```
-0.443192
```

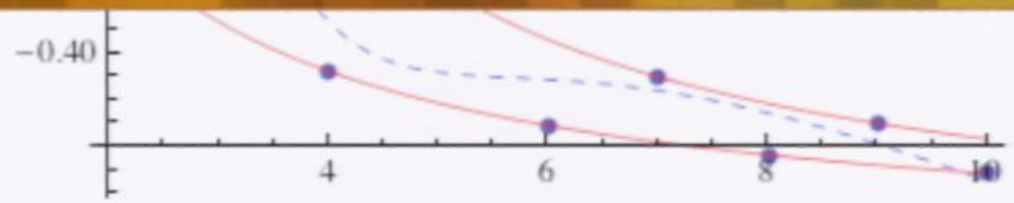
In[83]:=
```
Bethe = -Log[2] + 1/4 // N
```

Out[83]=
```
-0.443147
```

```
prediction - Bethe
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25



In[82]:=
```
prediction = (odd + even)/2 /. x → ∞
```

Out[82]= -0.443192

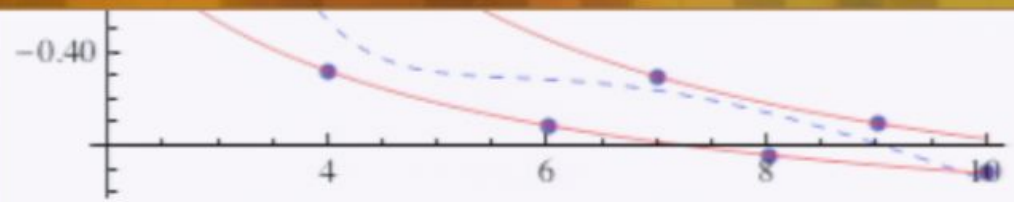
In[83]:=
```
Bethe = -Log[2] + 1/4 // N
```

Out[83]= -0.443147

```
prediction - Bethe
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25



In[82]:=
```
prediction = (odd + even)/2 /. x → ∞
```

Out[82]=
```
-0.443192
```

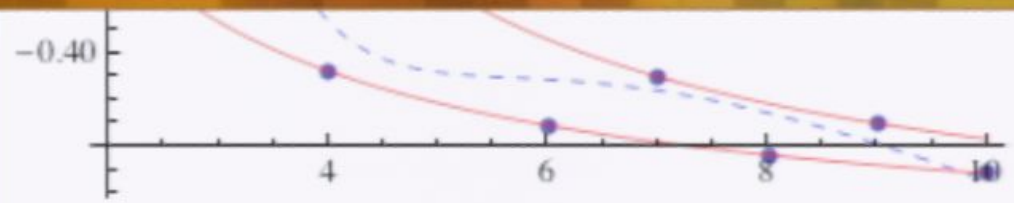
In[83]:=
```
Bethe = -Log[2] + 1/4 // N
```

Out[83]=
```
-0.443147
```

```
prediction - Bethe
prediction - Bethe
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25



In[82]:=
```
prediction = (odd + even)/2 /. x → ∞
```

Out[82]= -0.443192

In[83]:=
```
Bethe = -Log[2] + 1/4 // N
```

Out[83]= -0.443147

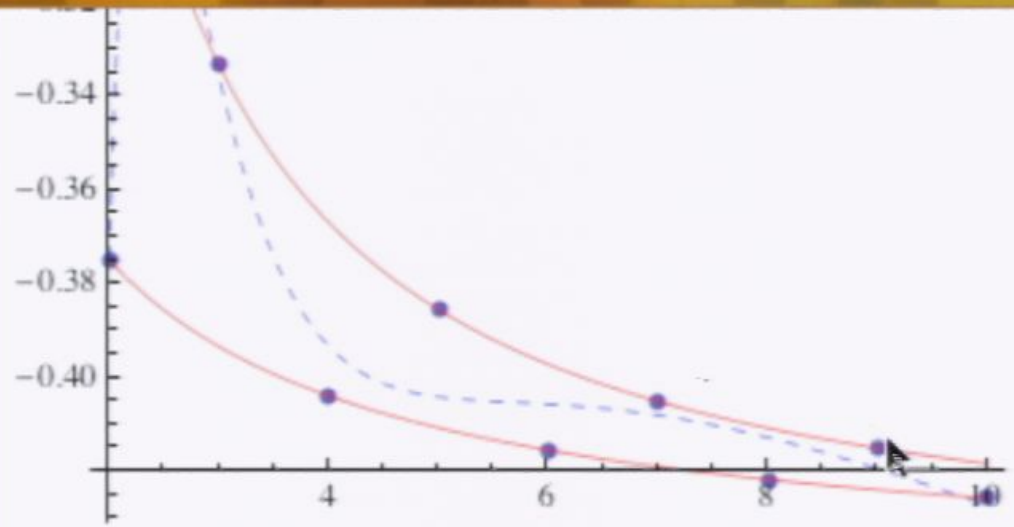
In[84]:=
```
(prediction - Bethe)/(prediction + Bethe)
```

Out[84]= 0.0000510871

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[81]=



In[82]:=
```
prediction = (odd + even)/2 /. x → ∞
```

Out[82]= -0.443192

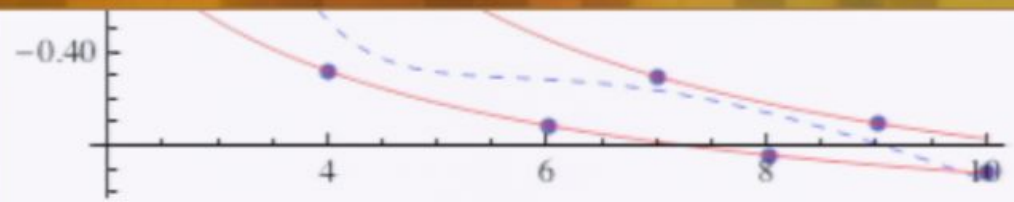
In[83]:=
```
Bethe = -Log[2] + 1/4 // N
```

Out[83]= -0.443147

In[84]:=
```
prediction - Bethe
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25



In[82]:=
```
prediction = (odd + even)/2 /. x → ∞
```

Out[82]= -0.443192

In[83]:=
```
Bethe = -Log[2] + 1/4 // N
```

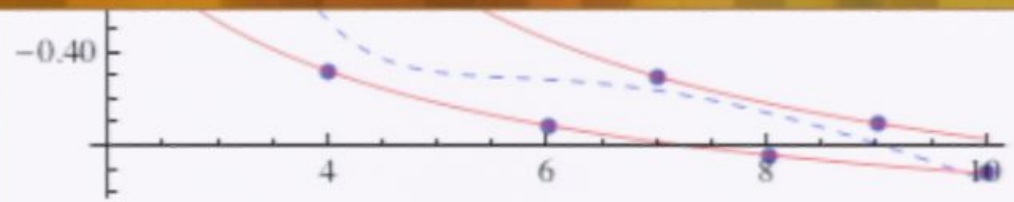
Out[83]= -0.443147

In[84]:=
```
(prediction - Bethe)/(prediction + Bethe)
```

Out[84]= 0.0000510871

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25



In[82]:=
```
prediction = (odd + even)/2 /. x → ∞
```

Out[82]= -0.443192

In[83]:=
```
Bethe = -Log[2] + 1/4 // N
```

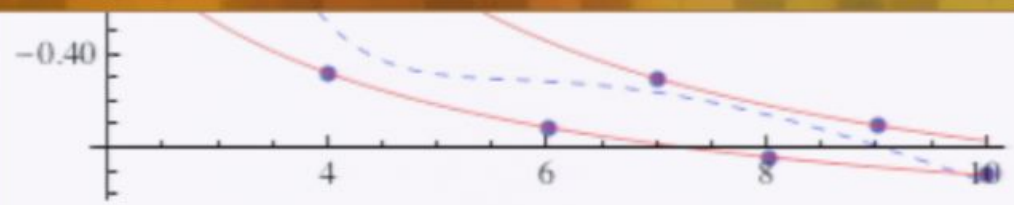
Out[83]= -0.443147

```
prediction - Bethe
------------------
       Bethe
```

Out[84]= 0.0000510871

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25



In[82]:=
```
prediction = (odd + even)/2 /. x → ∞
```

Out[82]= -0.443192

In[83]:=
```
Bethe = -Log[2] + 1/4 // N
```

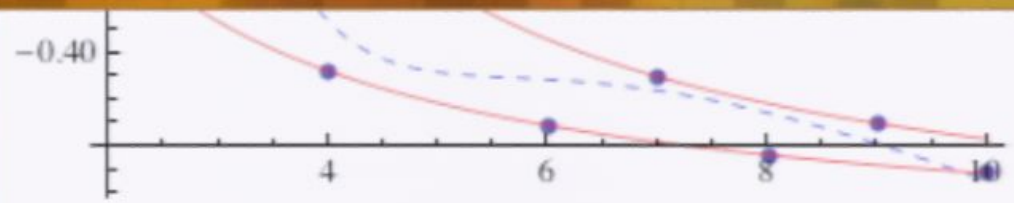
Out[83]= -0.443147

In[85]:=
```
(prediction - Bethe)/Bethe
```

Out[85]= 0.000102179

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25




```
odd + even
----------- /. x → ∞
     2
```

Out[82]= -0.443192

In[83]:=
```
Bethe = -Log[2] + 1 / 4 // N
```

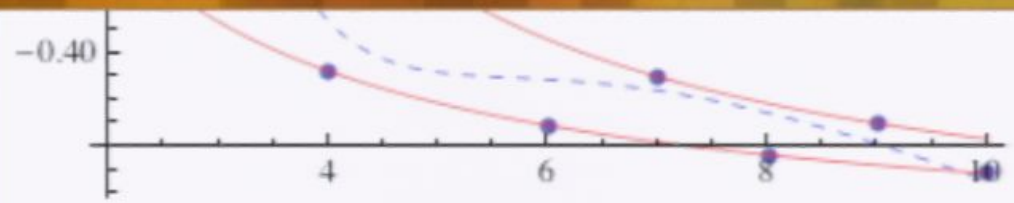
Out[83]= -0.443147

In[85]:=
```
prediction - Bethe
------------------
       Bethe
```

Out[85]= 0.000102179

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25




```
(odd + even)/2 /. x → ∞
```

Out[82]= -0.443192

```
-Log[2] + 1/4 // N
```

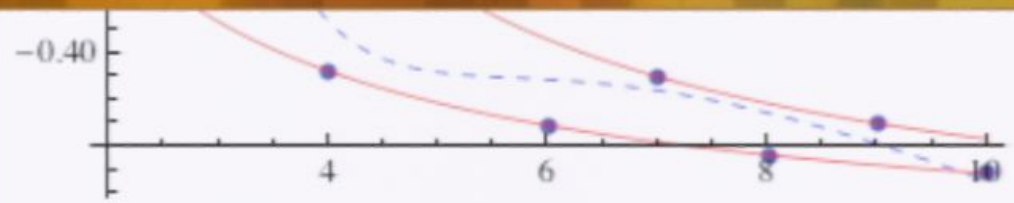
Out[83]= -0.443147

In[85]:=
```
(prediction - Bethe)/Bethe
```

Out[85]= 0.000102179

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25



In[86]:=
```
odd + even
----------- /. x → ∞
     2
```

Out[86]= -0.443192

```
-Log[2] + 1 / 4 // N
```

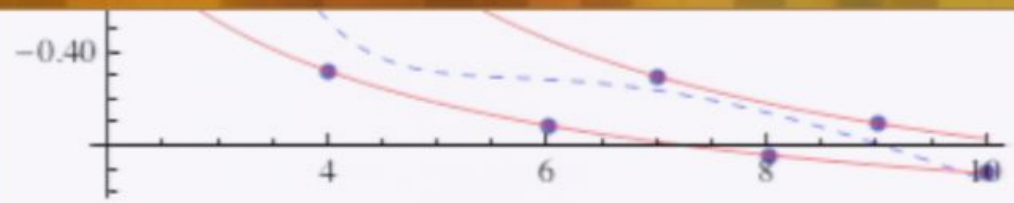
Out[83]= -0.443147

In[85]:=
```
prediction - Bethe
------------------
       Bethe
```

Out[85]= 0.000102179

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25



In[86]:=
```
odd + even
----------- /. x → ∞
     2
```

Out[86]= -0.443192

In[87]:=
```
-Log[2] + 1 / 4 // N
```

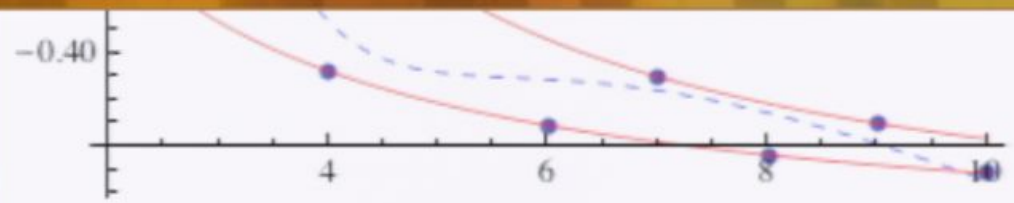
Out[87]= -0.443147

In[85]:=
```
prediction - Bethe
------------------
       Bethe
```

Out[85]= 0.000102179

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25



In[86]:=
```
odd + even
----------- /. x → ∞
     2
```

Out[86]= -0.443192

In[87]:=
```
-Log[2] + 1 / 4 // N
```

Out[87]= -0.443147

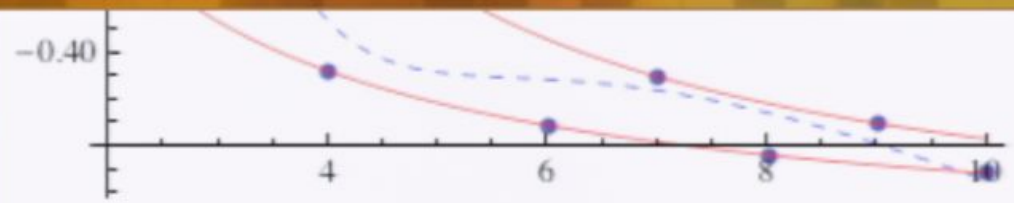
```
%
```

In[85]:=
```
prediction - Bethe
------------------
      Bethe
```

Out[85]= 0.000102179

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25



In[86]:=
```
odd + even
----------- /. x → ∞
     2
```

Out[86]= -0.443192

In[87]:=
```
-Log[2] + 1 / 4 // N
```

Out[87]= -0.443147

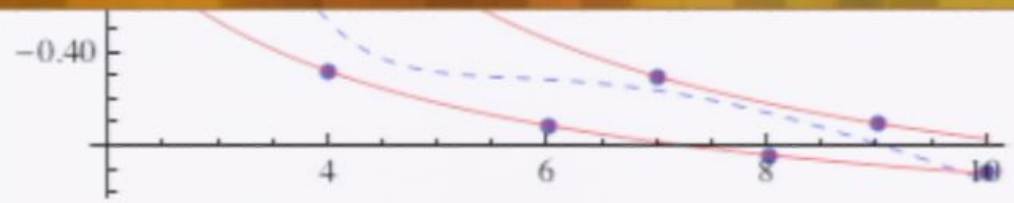
```
% - %%
```

In[85]:=
```
prediction - Bethe
------------------
       Bethe
```

Out[85]= 0.000102179

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25



In[86]:=
```
odd + even
----------- /. x → ∞
     2
```

Out[86]= -0.443192

In[87]:=
```
-Log[2] + 1 / 4 // N
```

Out[87]= -0.443147

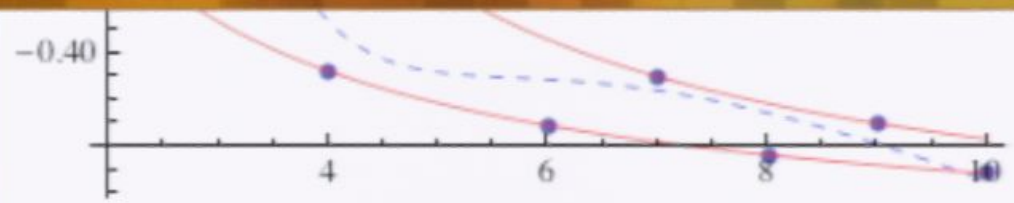
```
% - %%
------
  %
```

In[85]:=
```
prediction - Bethe
------------------
       Bethe
```

Out[85]= 0.000102179

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25



In[86]:=
```
odd + even
----------- /. x → ∞
     2
```

Out[86]= -0.443192

In[87]:=
```
-Log[2] + 1 / 4 // N
```

Out[87]= -0.443147

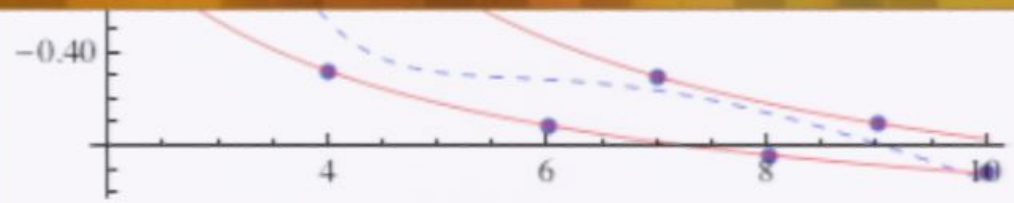
In[88]:=
```
% - %%
------
  %
```

Out[88]= -0.000102179

In[85]:=
```
prediction - Bethe
------------------
       Bethe
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25



In[86]:=
```
odd + even
----------- /. x → ∞
     2
```

Out[86]= -0.443192

In[87]:=
```
-Log[2] + 1 / 4 // N
```

Out[87]= -0.443147

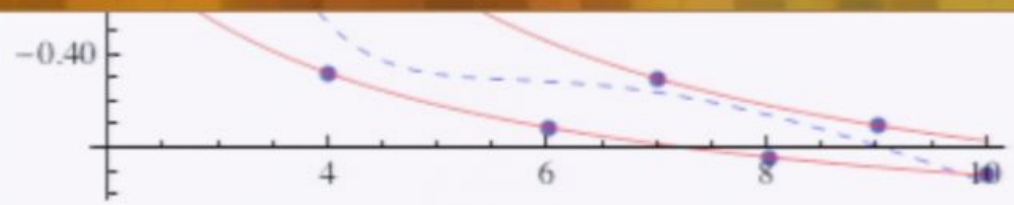
In[89]:=
```
%86 - %87
---------
   %87
```

Out[89]= 0.000102179

In[85]:=
```
prediction - Bethe
------------------
       Bethe
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25



In[86]:=
```
odd + even
----------- /. x → ∞
     2
```

Out[86]= -0.443192

In[87]:=
```
-Log[2] + 1 / 4 // N
```

Out[87]= -0.443147

In[89]:=
```
%86 - %87
---------
   %87
```

Out[89]= 0.000102179

In[85]:=
```
prediction - Bethe
------------------
       Bethe
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[86]:=
```
odd + even
----------- /. x → ∞
     2
```

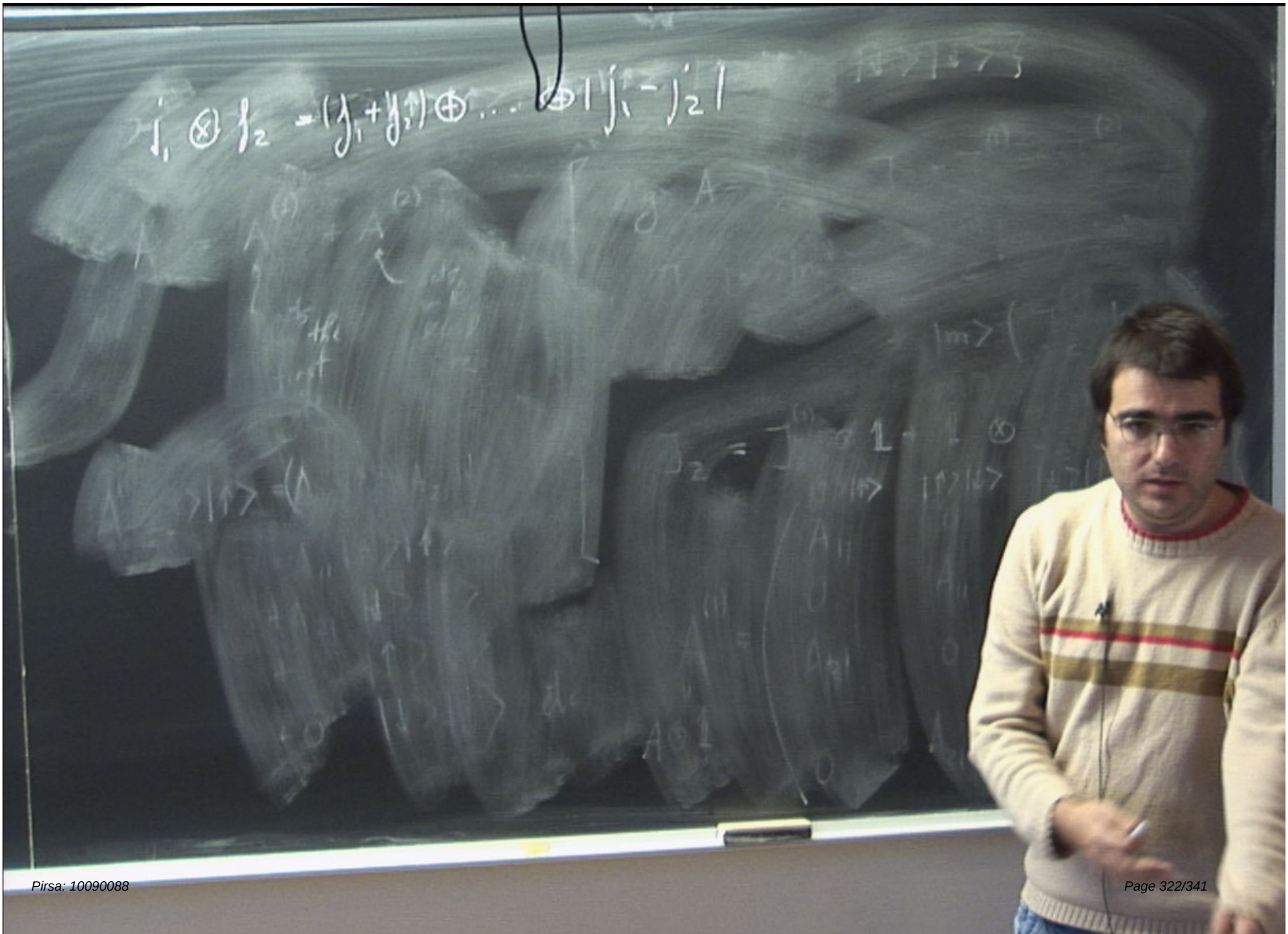
Out[86]= -0.443192

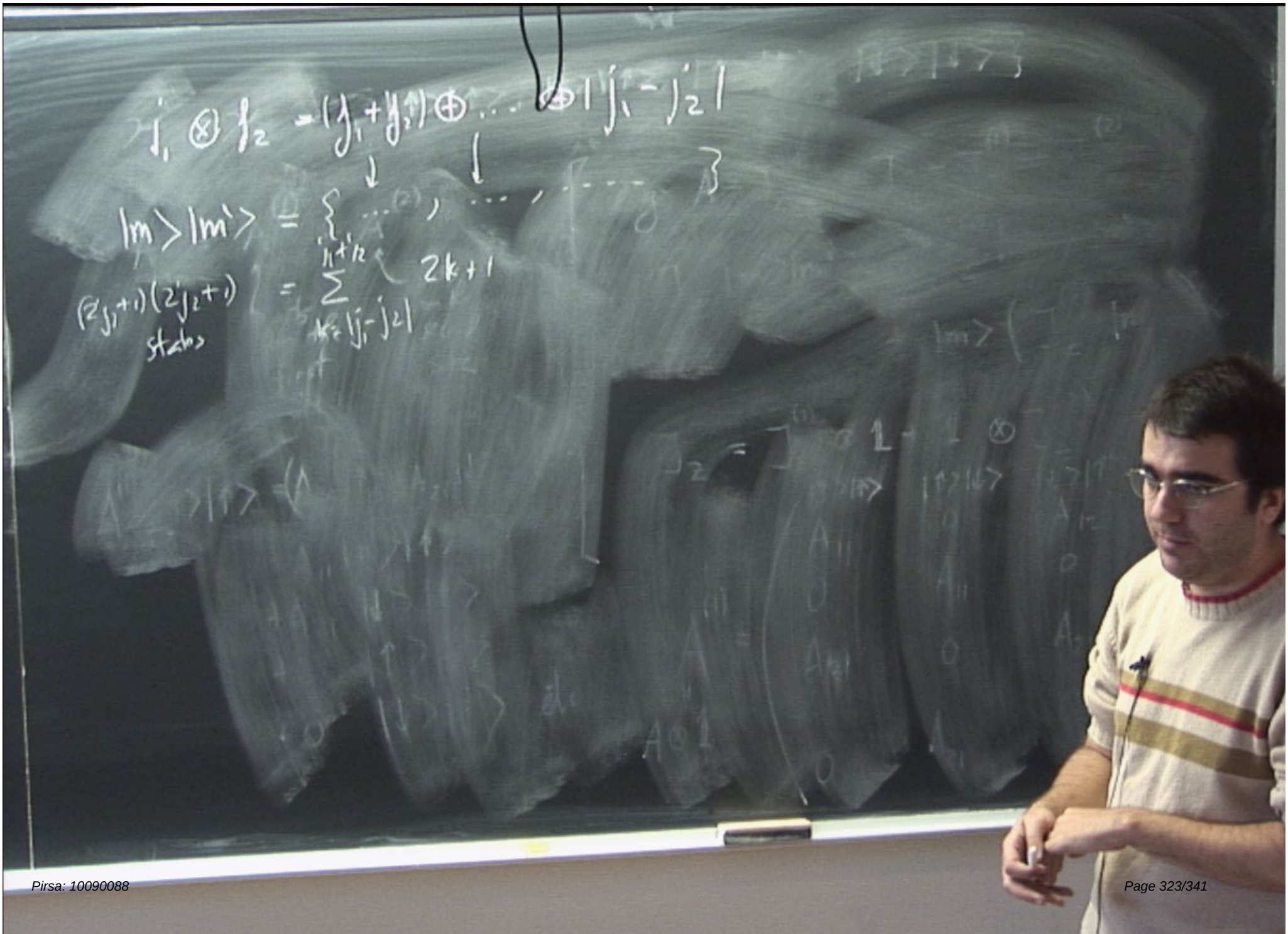
In[87]:=
```
-Log[2] + 1 / 4 // N
```

Out[87]= -0.443147

In[89]:=
```
%86 - %87
---------
   %87
```

Out[89]= 0.000102179


$j_1 \otimes j_2 = (j_1+j_2) \oplus \ldots \oplus |j_1-j_2|$


$j_1 \otimes j_2 = (j_1+j_2) \oplus \dots \oplus |j_1 - j_2|$
$|m\rangle|m'\rangle = \{ \dots, \dots, \dots \}$
$(2j_1+1)(2j_2+1) = \sum_{k=|j_1-j_2|}^{j_1+j_2} 2k+1$
states

$$j_1 \otimes j_2 = (j_1+j_2) \oplus \dots \oplus |j_1-j_2|$$

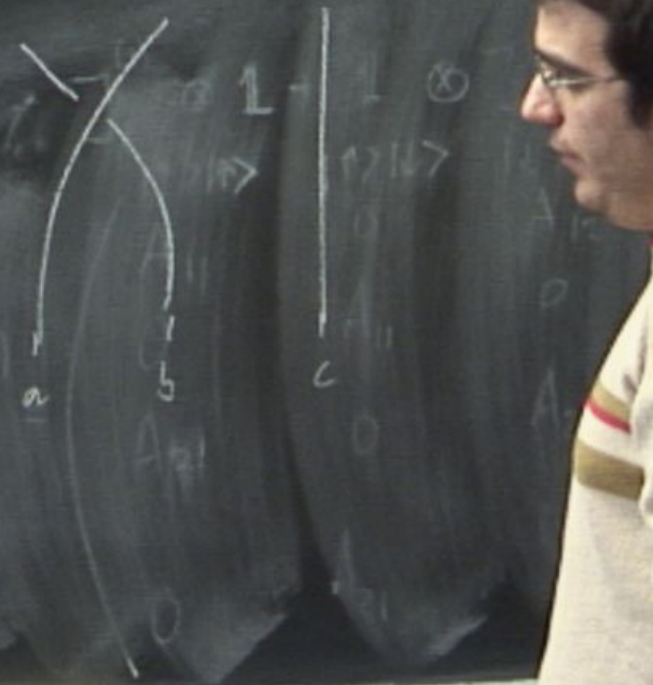
$$|m\rangle|m'\rangle = \{ \dots, \dots, \dots \}$$

$$(2j_1+1)(2j_2+1) = \sum_{k=|j_1-j_2|}^{j_1+j_2} 2k+1$$

states

S^N permutes (a, b, c) $(N=3)$

$$(2,1,3) \circ (a,b,c) = (b,a,c)$$



$$j_1 \otimes j_2 = (j_1 + j_2) \oplus \dots \oplus |j_1 - j_2|$$

$$|m\rangle |m'\rangle = \{ \dots, \dots, \dots \}$$

$$(2j_1+1)(2j_2+1) = \sum_{k=|j_1-j_2|}^{j_1+j_2} 2k+1$$

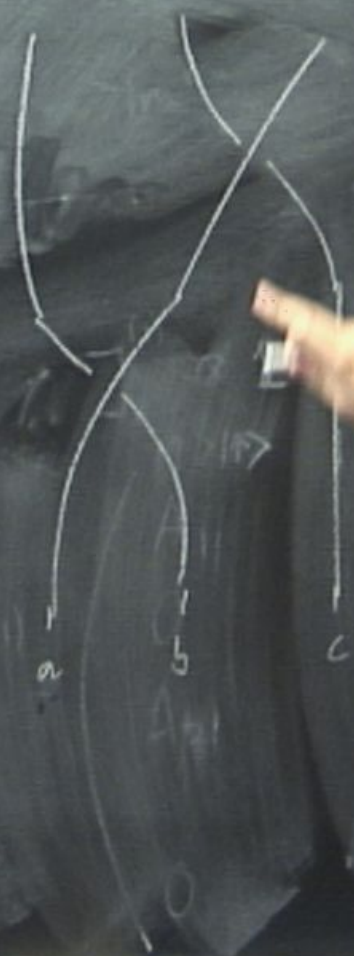
states

S^N permutes (a, b, c) $(N=3)$

$$(2,1,3) \circ (a,b,c) = (b,a,c)$$

$$(1,3,2) \circ (a,b,c) = (a,c,b)$$

$$\big((2,1,3) \circ (1,3,2)\big) \circ (a,b,c) = (3,$$



$$j_1 \otimes j_2 = (j_1+j_2) \oplus \dots \oplus |j_1 - j_2|$$

$$|m\rangle|m'\rangle = \{ \dots, \dots, \dots \}$$

$$(2j_1+1)(2j_2+1) = \sum_{k=|j_1-j_2|}^{j_1+j_2} 2k+1$$

states

S^N permutes (a, b, c) . $(N=3)$

$$(2,1,3) \circ (a,b,c) = (b,a,c)$$

$$(1,3,2) \circ (a,b,c) = (a,c,b)$$

$$\big((2,13) \circ (1,3,2)\big) \circ (a,b,c) = (3,1,2) \circ (a,b,c)$$

$\circ$	(132)
(213)	(312)

etc

$$j_1 \otimes j_2 = (j_1 + j_2) \oplus \dots \oplus |j_1 - j_2|$$

$$|m\rangle|m'\rangle = \{ \dots, \dots, \dots \}$$

$$(2j_1+1)(2j_2+1) = \sum_{k=|j_1-j_2|}^{j_1+j_2} 2k+1$$

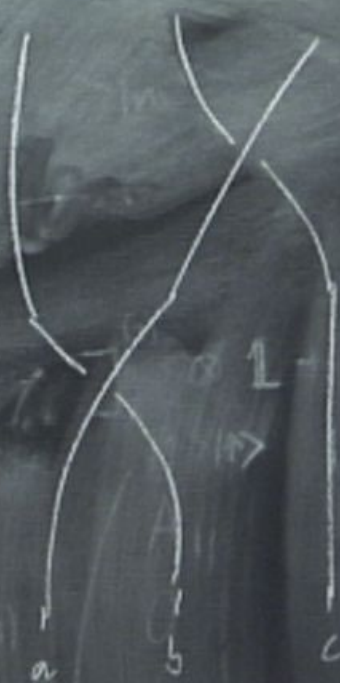
states

S^N permutes (a, b, c) $(N=3)$

$$(2,1,3) \circ (a,b,c) = (b,a,c)$$

$$(1,3,2) \circ (a,b,c) = (a,c,b)$$

$$\big((2,13) \circ (1,3,2)\big) \circ (a,b,c) = (3,1,2) \circ (a,b,c)$$



$\circ$	(132)
(213)	(312)

etc

$$j_1 \otimes j_2 = (j_1+j_2) \oplus \dots \oplus |j_1 - j_2|$$

$$|m\rangle|m'\rangle = \{ \dots (2) , \dots , \dots \}$$

$$(2j_1+1)(2j_2+1) = \sum_{k=|j_1-j_2|}^{j_1+j_2} 2k+1$$

states

S^N permutes (a, b, c) $(N=3)$

$$(2,1,3) \circ (a,b,c) = (b,a,c)$$

$$(1,3,2) \circ (a,b,c) = (a,c,b)$$

$$\big((1,3,2) \circ (2,1,3)\big) \circ (a,b,c) = (3,1,2) \circ (a,b,c)$$

$\circ$	(213)
(132)	(312)

etc

$\mathbb{1} = (1, 2, 3)$

$$j_1 \otimes j_2 = (j_1 + j_2) \oplus \cdots \oplus |j_1 - j_2|$$

$$|m\rangle |m'\rangle = \{ \cdots, \cdots, \cdots \}$$

$$(2j_1+1)(2j_2+1) = \sum_{k=|j_1-j_2|}^{j_1+j_2} 2k+1$$

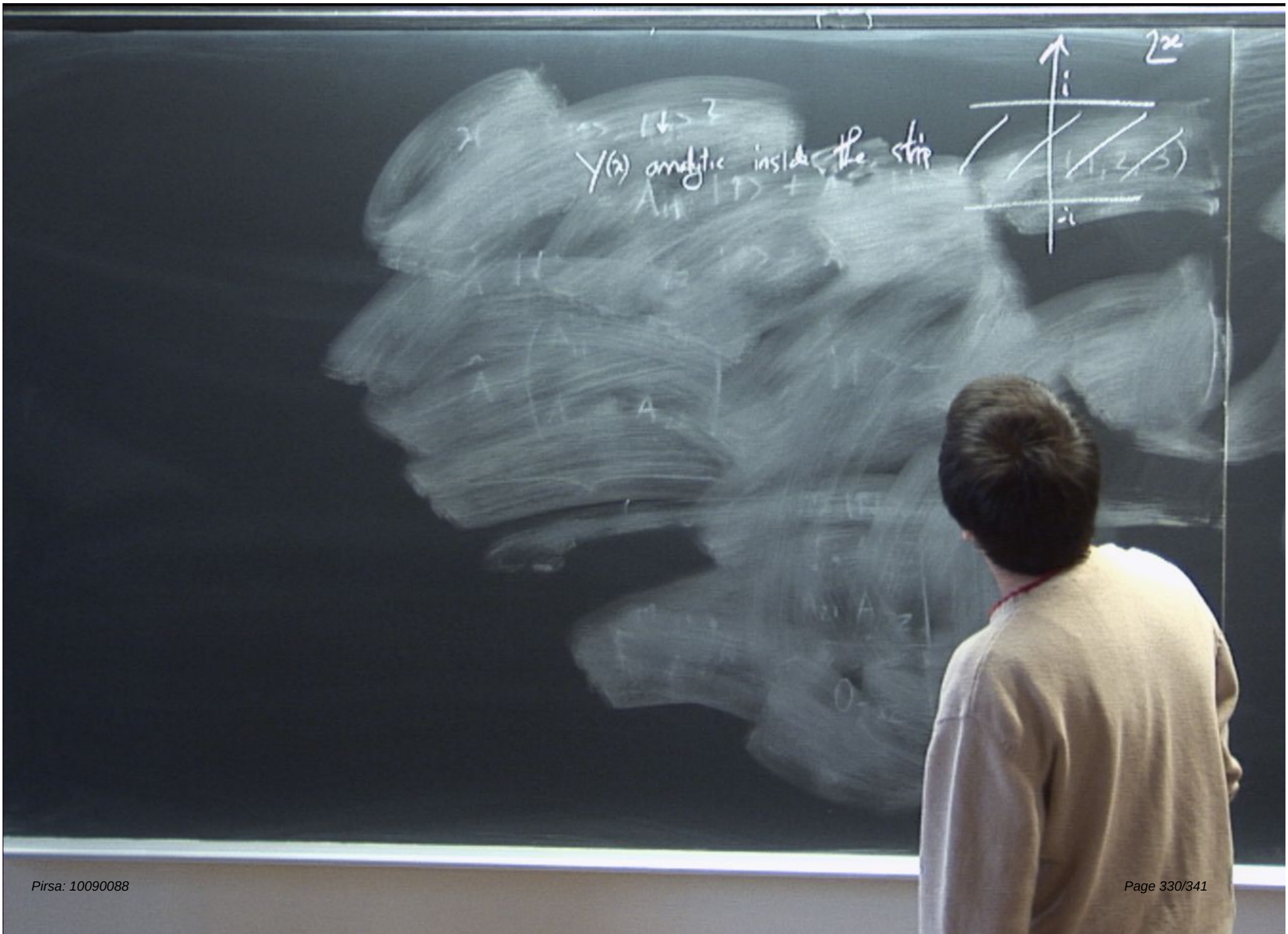
states

S^N permutes (a, b, c) $(N=3)$

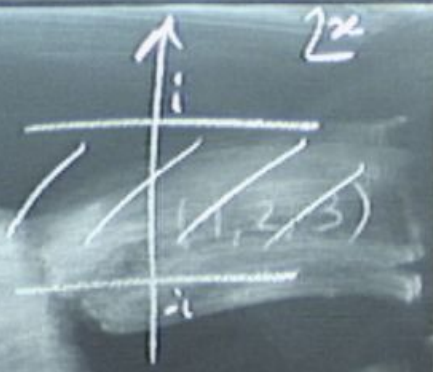
$$(2, 1, 3) \circ (a, b, c) = (b, a, c)$$

$$(1, 3, 2) \circ (a, b, c) = (a, c, b)$$

$$\big((1, 3, 2) \circ (2, 1, 3)\big) \circ (a, b, c) = (3, 1, 2) \circ (a, b, c)$$

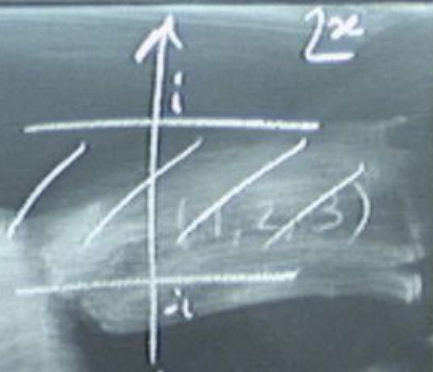

Y(x) analytic inside the strip
2x
i
-i

$Y(x)$ analytic inside the strip



$\log Y(x)$ also analytic

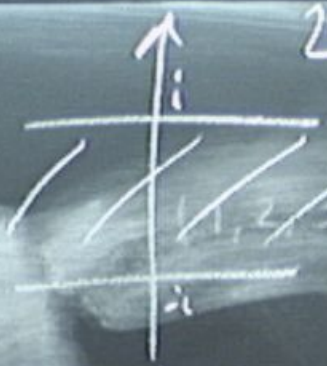
$Y(x)$ analytic inside the strip



by $Y(x)$ also analytic

at ∞ inside the strip $Y(x) \simeq e^{-m \cosh(x)}$

Z^{∞}



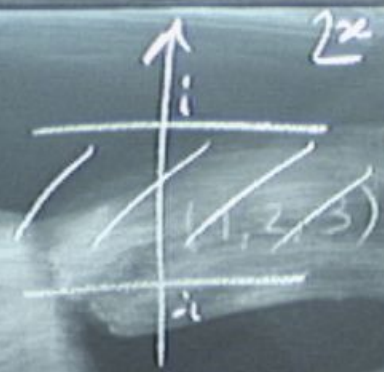
$Y(x)$ analytic inside the strip

by $Y(x)$ also analytic

at ∞ inside the strip $Y(x) \simeq e^{-m \cosh(x)}$

$m \in \mathbb{R}^{+}$

$Y(x)$ analytic inside the strip



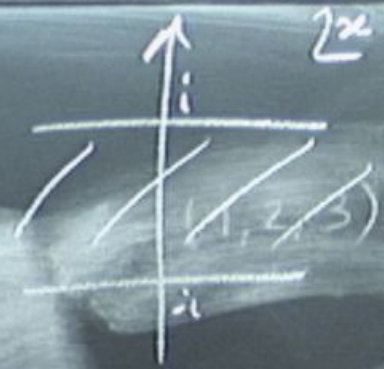
by $Y(x)$ also analytic

at ∞ inside the strip $Y(x) \simeq e^{-m \cosh(x)}$

$m \in \mathbb{R}^+$

$$Y\left(x+\tfrac{i}{2}\right)Y\left(x-\tfrac{i}{2}\right) = 1 + Y(x)$$

$Y(x)$ analytic inside the strip



$\log Y(x)$ also analytic

at ∞ inside the strip $Y(x) \simeq e^{-m \cosh(x)}$

$m \in \mathbb{R}^+$

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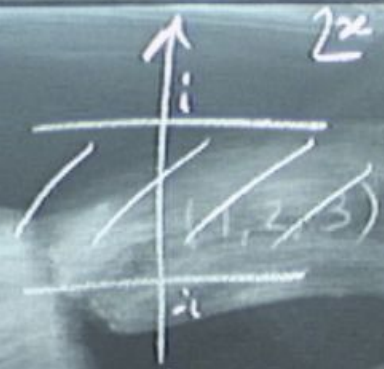
ex: plot $Y(x)$ for real x

$Y(x)$ analytic inside the strip



log $Y(x)$ also analytic

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functional equation

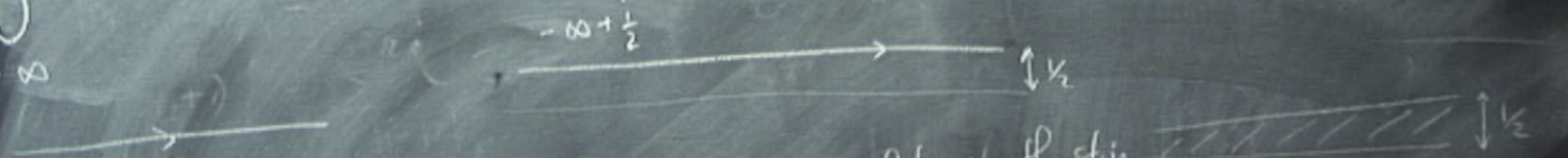
$$Y\left(x+\tfrac{i}{2}\right)Y\left(x-\tfrac{i}{2}\right) = 1 + Y(x)$$

ex: plot $Y(x)$ for real x

$$\int_{-\infty}^{+\infty} dx \; f(x+i/2)\, e^{i\omega x} = \int_{-\infty+\frac{i}{2}}^{\infty+\frac{i}{2}} dx' \; f(x')\, e^{i\omega(x'-i/2)}$$

$\updownarrow i/2$

$$\int_{-\infty}^{+\infty} dx \; f(x+i/2)\, e^{i\omega x} = \int_{-\infty+\frac{i}{2}}^{\infty+\frac{i}{2}} dx' \; f(x')\, e^{i\omega(x'-i/2)}$$



if the function decays at ∞ and its analytic in the strip

$$= \int_{-\infty}^{+\infty} dx' \; f(x')\, e^{i\omega(x'-i/2)} = e^{\omega/2}\, \mathcal{F}(f(x)) = \mathcal{F}(f(x+i/2))$$

find $T(x)$ and $Q(x)$, such that

polynomial

pol of degree N

$$\sum T(u) = \frac{(u+i/2)^L Q(u+i) + (u-i/2)^L Q(u-i)}{Q(u)}$$

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$$Q(u) = \prod (u-u_j)$$ — fix u_j bec

T is pol