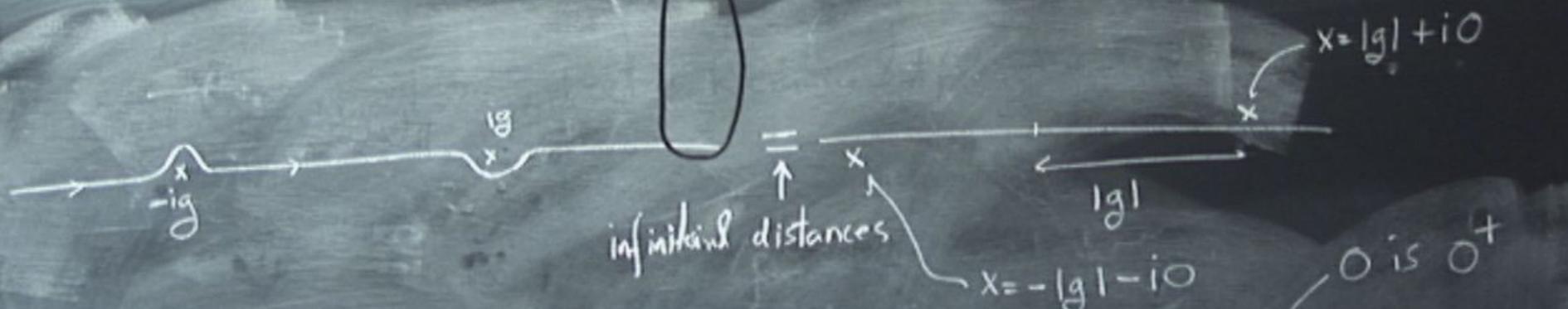


Title: Introduction to Complex Analysis/Computer Skills - Lecture 3B

Date: Sep 01, 2010 10:30 AM

URL: <http://pirsa.org/10090080>

Abstract:



$$I(g) = \frac{1}{g} \int_{-\infty}^{\infty} \frac{dx}{\pi} \frac{g}{x^2 + g^2}$$

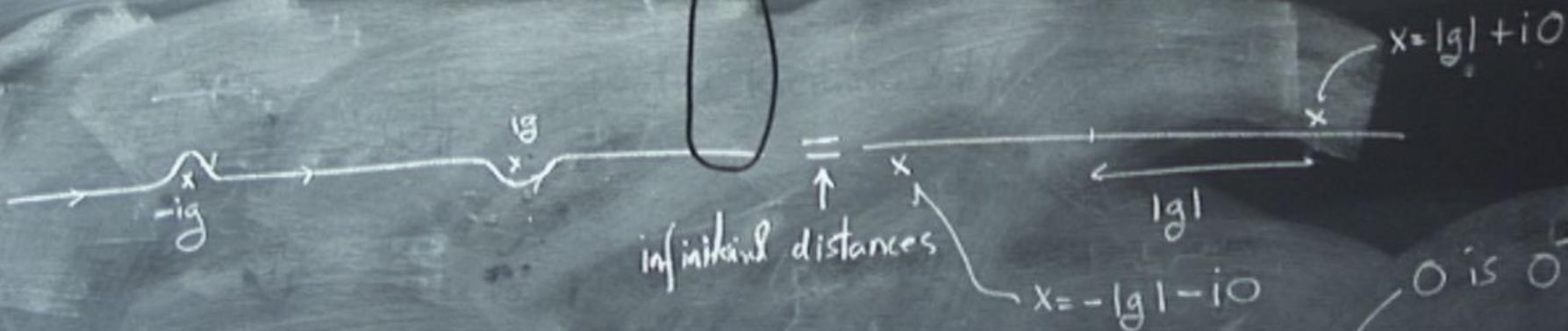
$$x^2 + g^2 = (x - i0)(x + i0)$$

$$= (x - i0)^2$$

When g hits real axis

$$I(g) = \frac{1}{g} \int_{-\infty}^{\infty} \frac{dx}{\pi} \frac{g}{x^2 + g^2 - i0}$$

MAKES SENSE FOR $g =$



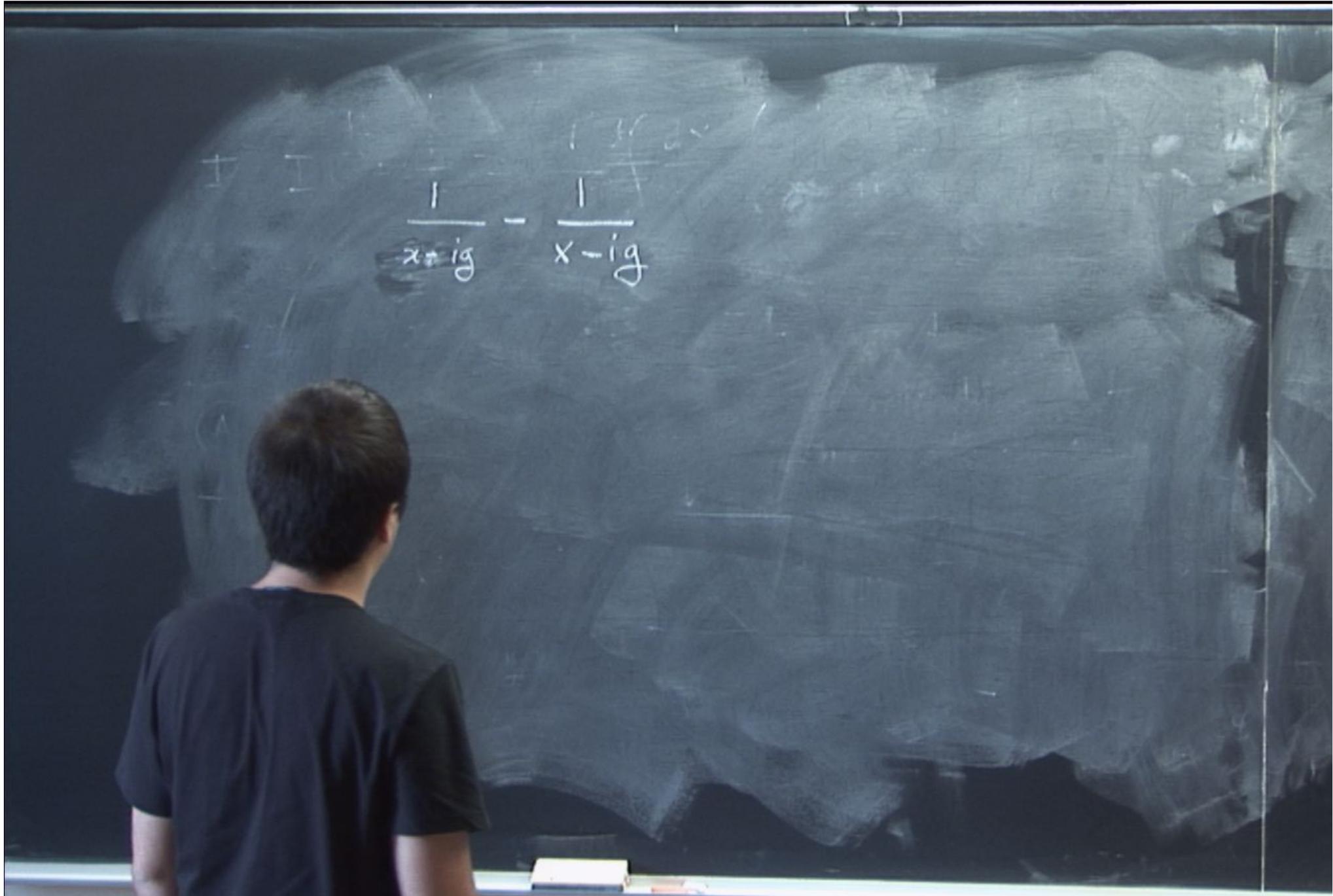
$$I(g) = \frac{1}{g} \int_{-\infty}^{\infty} \frac{dx}{\pi} \frac{g}{x^2 + g^2}$$

When g hits real axis

$$I(g) = \frac{1}{g} \int_{-\infty}^{\infty} \frac{dx}{\pi} \frac{g}{x^2 + g^2 - i0}$$

MAKES SENSE FOR $g = -|g|$

$$\begin{aligned} x^2 + g^2 &= (x - |g| - i0)(x + |g| + i0) \\ &= x^2 - (|g| + i0)^2 \\ &= x^2 - |g|^2 - 2i0|g| - i0^2 \end{aligned}$$

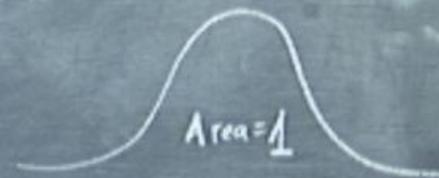


$$\frac{1}{x+ig} - \frac{1}{x-ig}$$

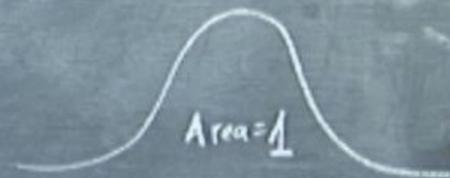
$$\frac{1}{\pi} \left(\frac{1}{x+ig} - \frac{1}{x-ig} \right) = \frac{1}{\pi} \frac{g}{x^2+g^2}$$

$$\frac{1}{2\pi i} \left(\frac{1}{x+ig} - \frac{1}{x-ig} \right) = \frac{1}{\pi} \frac{g}{x^2+g^2}$$

$$\frac{1}{2\pi i} \left(\frac{1}{x+ig} - \frac{1}{x-ig} \right) = \frac{1}{\pi} \frac{g}{x^2+g^2}$$

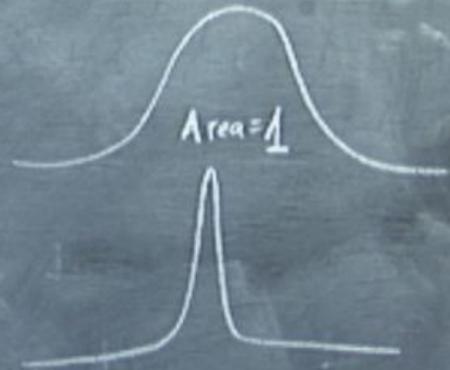


$$\frac{1}{2\pi i} \left(\frac{1}{x+ig} - \frac{1}{x-ig} \right) = \frac{1}{\pi} \frac{g}{x^2+g^2}$$



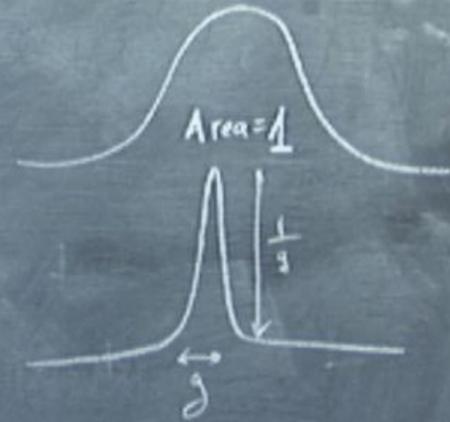
$$\frac{1}{2\pi i} \left(\frac{1}{x+ig} - \frac{1}{x-ig} \right) = \frac{1}{\pi} \frac{g}{x^2+g^2}$$

$g \rightarrow 0$



$$\frac{1}{2\pi i} \left(\frac{1}{x+ig} - \frac{1}{x-ig} \right) = \frac{1}{\pi} \frac{g}{x^2+g^2}$$

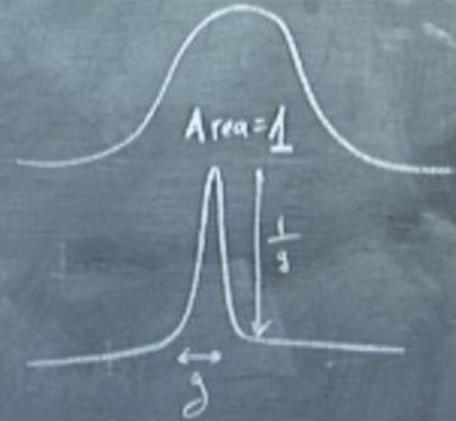
$g \rightarrow 0$



$$\frac{1}{2\pi i} \left(\frac{1}{x+ig} - \frac{1}{x-ig} \right) = \frac{1}{\pi} \frac{g}{x^2+g^2}$$

$\downarrow g \rightarrow 0$
 $\delta(x)$

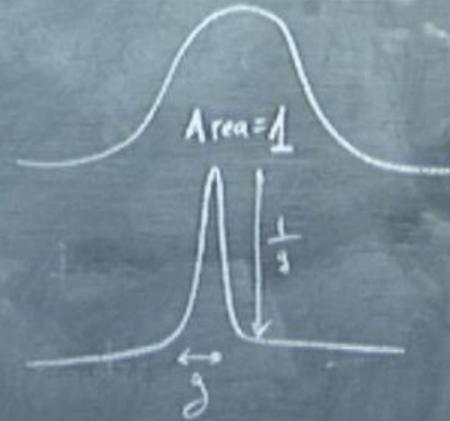
$g \rightarrow 0$



$$\frac{1}{2\pi i} \left(\frac{1}{x+ig} - \frac{1}{x-ig} \right) = \frac{1}{\pi} \frac{g}{x^2+g^2}$$

$\downarrow g \rightarrow 0$
 $\delta(x)$

$g \rightarrow 0$

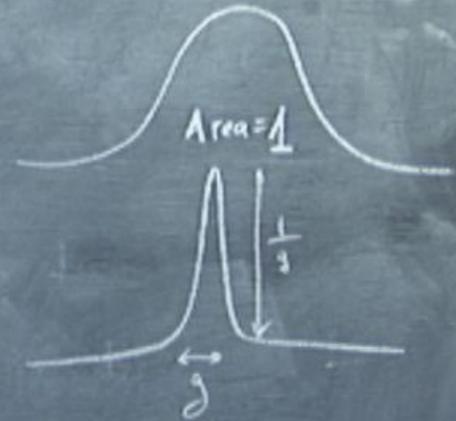


$$\int \frac{dx}{2\pi i} \left(\frac{1}{x-i0} - \frac{1}{x+i0} \right) \frac{x^2+1}{x^4+2x^2+2}$$

$$\frac{1}{2\pi i} \left(\frac{1}{x+ig} - \frac{1}{x-ig} \right) = \frac{1}{\pi} \frac{g}{x^2+g^2}$$

$\downarrow g \rightarrow 0$
 $\delta(x)$

$g \rightarrow 0$



$$\int \frac{dx}{2\pi i} \left(\frac{1}{x-i0} - \frac{1}{x+i0} \right) \frac{x^2+1}{x^4+2x^2+2} = \frac{1}{2}$$

$\frac{1}{2}$ at $x=0$

$$\int dx \delta(x) f(x)$$

$$\int dx \delta(x) f(x)$$

analytic in the real axis

$$\equiv \int \frac{dx}{2\pi i} \left(\frac{f(x)}{x-i0} - \frac{f(x)}{x+i0} \right)$$

$$\int dx \delta(x) f(x)$$

analytic in the real axis

$$\int \frac{dx}{2\pi i} \left(\frac{f(x)}{x-i0} - \frac{f(x)}{x+i0} \right)$$

function with pole at $+i0$



$$\int dx \delta(x) f(x)$$

analytic in the real axis

$$\int \frac{dx}{2\pi i} \left(\frac{f(x)}{x-i0} - \frac{f(x)}{x+i0} \right) = \int \frac{f(x)}{x}$$

function with pole at $+i0$

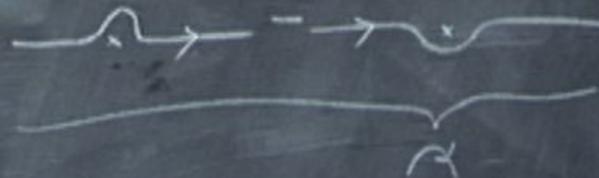


$$\int dx \delta(x) f(x)$$

↑ analytic in the real axis

$$\int \frac{dx}{2\pi i} \left(\frac{f(x)}{x-i0} - \frac{f(x)}{x+i0} \right) = \int \frac{f(x)}{x}$$

function with pole at $+i0$

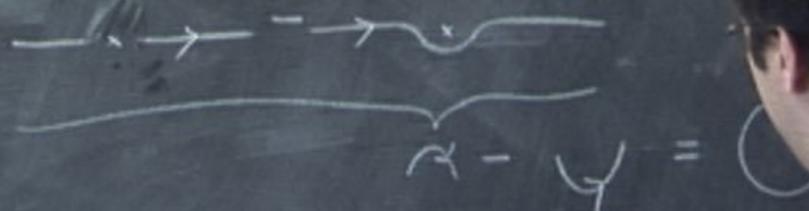


$$\int dx \delta(x) f(x)$$

analytic in the real axis

$$\int \frac{dx}{2\pi i} \left(\frac{f(x)}{x-i0} - \frac{f(x)}{x+i0} \right) = \int \frac{f(x)}{x}$$

function with pole at $+i0$



$$\int dx \delta(x) f(x)$$

analytic in the real axis

$$\int \frac{dx}{2\pi i} \left(\frac{f(x)}{x-i0} - \frac{f(x)}{x+i0} \right) = \int \frac{f(x)}{x}$$

function with pole at $+i0$



$$\int dx \delta(x) f(x)$$

↑ analytic in the real axis

$$\int \frac{dx}{2\pi i} \left(\frac{f(x)}{x-i0} - \frac{f(x)}{x+i0} \right) = \int \frac{f(x)}{x} = \int_{|x|=0} \frac{f(x)}{x} = f(0)$$

function with pole at $-i0$

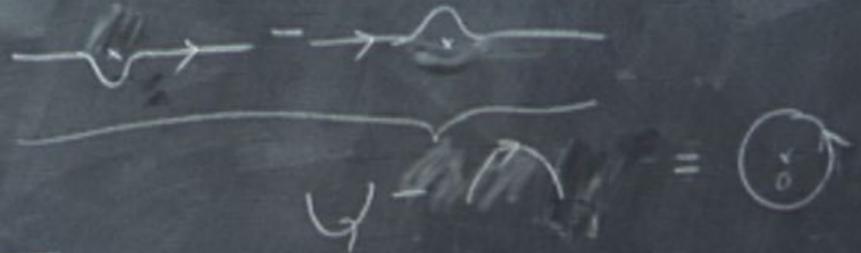


$$\int dx \delta(x) f(x)$$

analytic in the real axis

$$\int \frac{dx}{2\pi i} \left(\frac{f(x)}{x-i0} - \frac{f(x)}{x+i0} \right) = \int \frac{f(x)}{x} = \int_{|x|=0} \frac{f(x)}{x} = f(0)$$

function with pole at $+i0$



$$\frac{1}{2\pi i} \left(\frac{1}{x-io} - \frac{1}{x+io} \right) = \delta(x)$$

$$\frac{1}{2\pi i} \left(\frac{1}{x+io} - \frac{1}{x-io} \right)$$

$$\frac{1}{2\pi i} \left(\frac{1}{x-io} - \frac{1}{x+io} \right) = \delta(x)$$

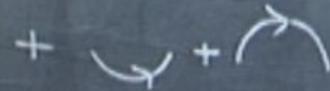
distributions

$$\int \frac{1}{2\pi i} \left(\frac{1}{x-io} + \frac{1}{x+io} \right) f(x) = \int \frac{f(x)}{x}$$



$$\frac{1}{2\pi i} \left(\frac{1}{x-io} - \frac{1}{x+io} \right) = \delta(x) \quad \leftarrow \text{distributions}$$

$$\int \frac{1}{2\pi i} \left(\frac{1}{x-io} + \frac{1}{x+io} \right) f(x) = \int \frac{f(x)}{x}$$



$$\frac{1}{2\pi i} \left(\frac{1}{x-io} - \frac{1}{x+io} \right) = \delta(x)$$

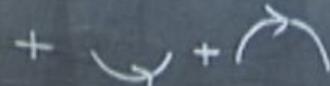
distributions

$$\int \frac{1}{2\pi i} \left(\frac{1}{x-io} + \frac{1}{x+io} \right) f(x) = \int \frac{f(x)}{x}$$



$$\int \frac{f(x)}{x}$$

$$= 2 \rightarrow$$

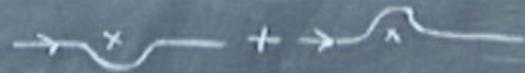


$$= 2$$

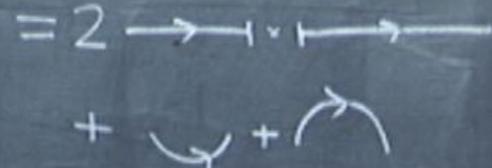
$$\frac{1}{2\pi i} \left(\frac{1}{x-i0} - \frac{1}{x+i0} \right) = \delta(x)$$

distributions

$$\int \frac{1}{2\pi i} \left(\frac{1}{x-i0} + \frac{1}{x+i0} \right) f(x) = \int \frac{f(x)}{x}$$



$$\int \frac{f(x)}{x} = f(0) \int \frac{1}{x}$$



$$= 2$$

$$\frac{1}{2\pi i} \left(\frac{1}{x-i0} - \frac{1}{x+i0} \right) = \delta(x)$$

distributions

$$\int \frac{1}{2\pi i} \left(\frac{1}{x-i0} + \frac{1}{x+i0} \right) f(x) = \int \frac{f(x)}{x}$$



$$\frac{1}{2\pi i} \int \frac{f(x)}{x} = f(0) \int \frac{dx}{x} \frac{1}{2\pi i}$$

$$= 2 \rightarrow \text{---} \text{---} \text{---}$$



$$= f(0) \int \frac{d(e^{i0})}{e^{i0}} \frac{1}{2\pi i} = 2$$

ide

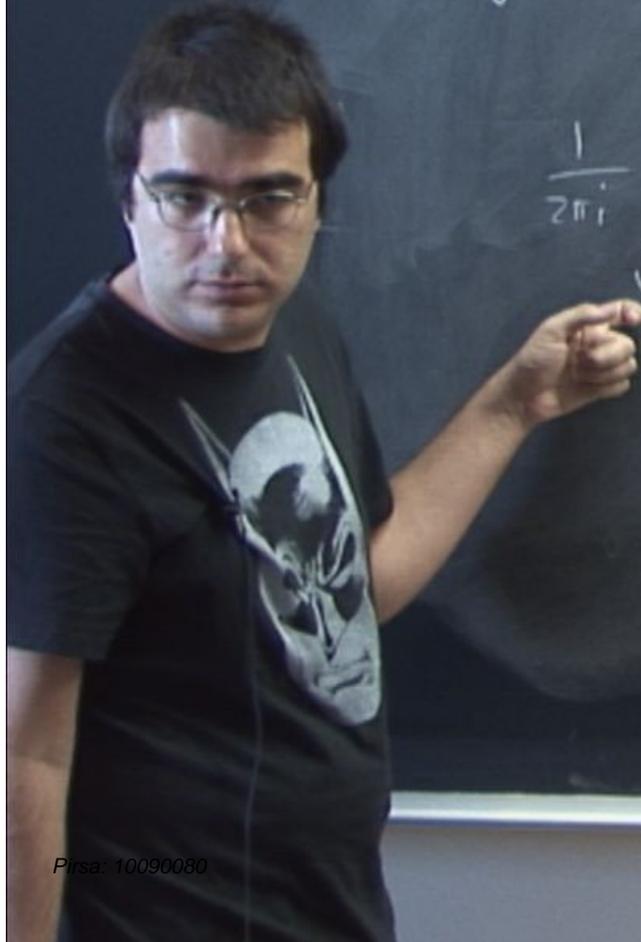
$$\frac{1}{2\pi i} \left(\frac{1}{x-i0} - \frac{1}{x+i0} \right) = \delta(x) \quad \leftarrow \text{distributions}$$

$$\int \frac{1}{2\pi i} \left(\frac{1}{x-i0} + \frac{1}{x+i0} \right) f(x) = \int \frac{f(x)}{x}$$

$$\frac{1}{2\pi i} \int \frac{f(x)}{x} = f(0) \int \frac{dx}{x} \frac{1}{2\pi i} = 2 \rightarrow \text{---} \leftarrow \leftarrow \leftarrow$$

$$= f(0) \int \frac{d(e^{i\theta})}{e^{i\theta}} \frac{1}{2\pi i} = 2$$

$$= \frac{1}{2} f(0) \text{ id}$$



$$\frac{1}{2\pi i} \left(\frac{1}{x-i0} - \frac{1}{x+i0} \right) = \delta(x) \quad \leftarrow \text{distributions}$$

$$\int \frac{1}{2\pi i} \left(\frac{1}{x-i0} + \frac{1}{x+i0} \right) f(x) = \int \frac{f(x)}{x}$$



$$\begin{aligned} \frac{1}{2\pi i} \int \frac{f(x)}{x} &= f(0) \int \frac{dx}{x} \frac{1}{2\pi i} = 2 \rightarrow \text{---} \text{---} \text{---} \\ &= f(0) \int \frac{d(e^{i\theta})}{e^{i\theta}} \frac{1}{2\pi i} = \underbrace{\quad}_{=0} \\ &= \frac{1}{2} f(0) \text{ id} \end{aligned}$$

$$\frac{1}{2\pi i} \left(\frac{1}{x-i0} - \frac{1}{x+i0} \right) = \delta(x)$$

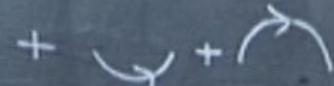
distributions

$$\int \frac{1}{2\pi i} \left(\frac{1}{x-i0} + \frac{1}{x+i0} \right) f(x) = \int \frac{f(x)}{x}$$



$$\frac{1}{2\pi i} \int \frac{f(x)}{x} = f(0) \int \frac{dx}{x} \frac{1}{2\pi i}$$

$$= 2 \rightarrow \text{---} \cdot \text{---} \rightarrow$$



$$= f(0) \int \frac{d(e^{i0})}{e^{i0}} \frac{1}{2\pi i}$$

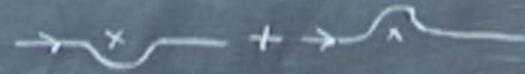
$$= 2 \rightarrow \text{---} \cdot \text{---} \rightarrow$$

$$= \frac{1}{2} f(0) \text{ id}$$

distributions

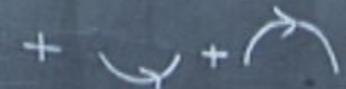
$$\frac{1}{2\pi i} \left(\frac{1}{x-i0} - \frac{1}{x+i0} \right) = \delta(x)$$

$$\int \frac{1}{2\pi i} \left(\frac{1}{x-i0} + \frac{1}{x+i0} \right) f(x) = \int \frac{f(x)}{x}$$



$$\frac{1}{2\pi i} \int \frac{f(x)}{x} = f(0) \int \frac{dx}{x} \frac{1}{2\pi i}$$

$$= 2 \rightarrow \text{---} \text{---} \text{---}$$



$$= f(0) \int \frac{d(e^{i\theta})}{e^{i\theta}} \frac{1}{2\pi i}$$

$$= 2 \rightarrow \text{---} \text{---} \text{---}$$

$$= \frac{1}{2} f(0) \text{ id}$$

$$\frac{1}{x-i0} = \mathcal{P} \frac{1}{x} + i\pi\delta(x)$$

$$\frac{1}{x+i0} = \mathcal{P} \frac{1}{x} - i\pi\delta(x)$$

$$\frac{1}{x-i0} = \mathcal{P} \frac{1}{x} + i\pi\delta(x)$$

$$\frac{1}{x+i0} = \mathcal{P} \frac{1}{x} - i\pi\delta(x)$$

principal part integration

$$\log \frac{z-a}{z-b} = \int$$

$$\log \frac{z-a}{z-b} = \int_a^b \frac{dw}{w-z}$$

\int



$$\log \frac{z-b}{z-a} = \int_a^b \frac{dw}{w-z}$$

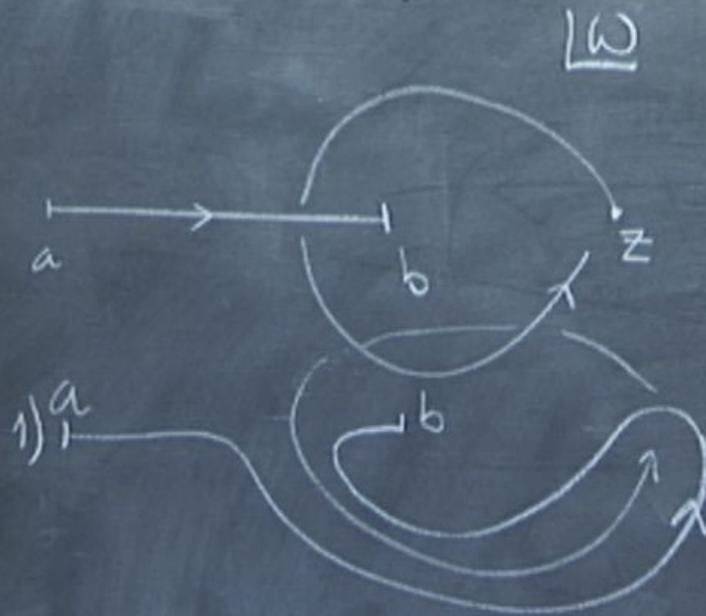
analytic cont. in z

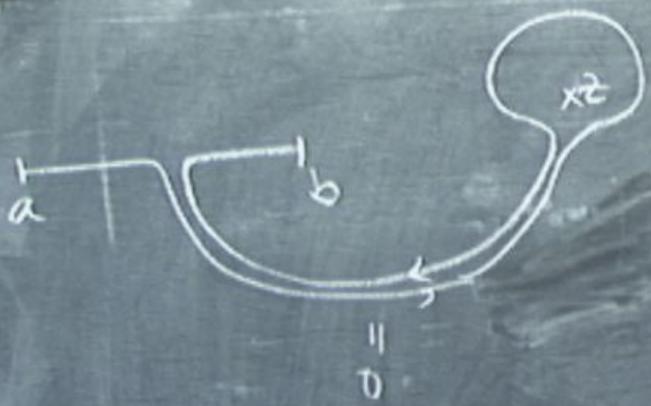
\int_a^b



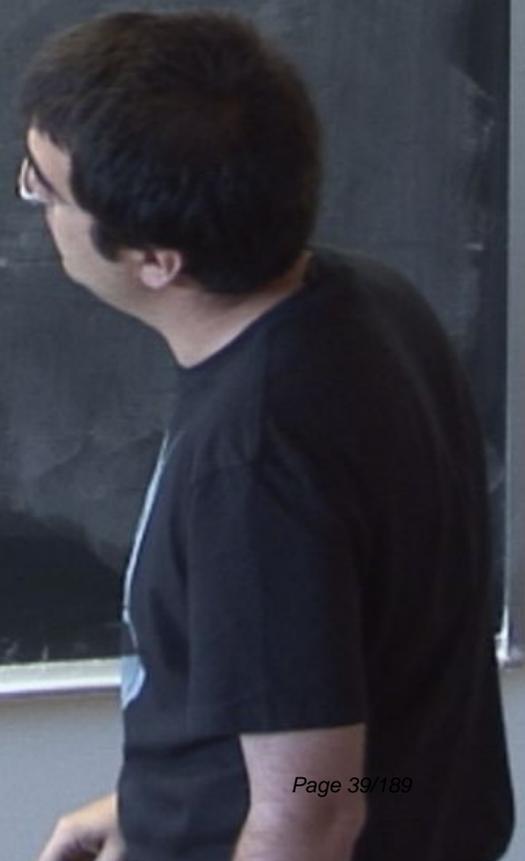
$$f(z) = \log \frac{z-b}{z-a} = \int_a^b \frac{dw}{w-z}$$

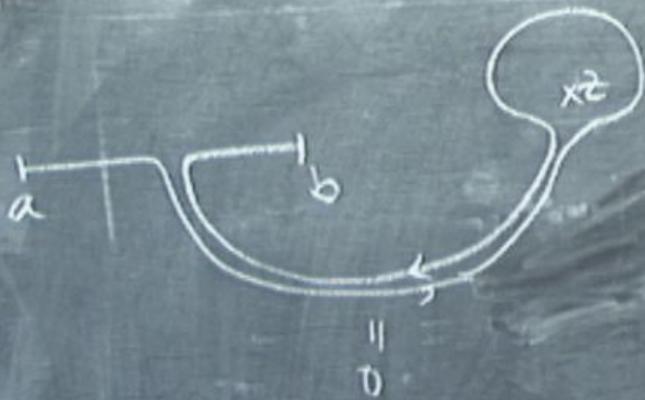
analytic cont in z





2





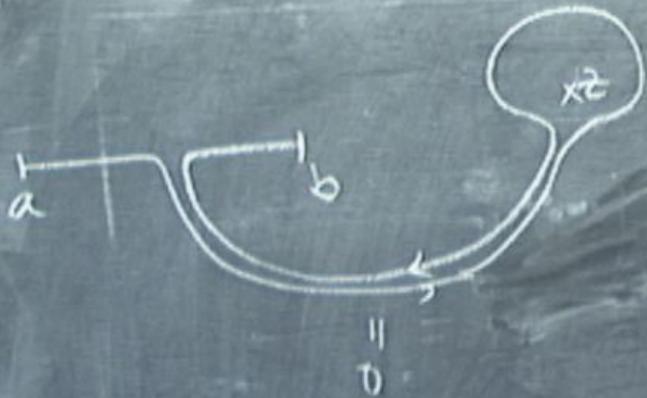
2



+

$$\oint \frac{dw}{z-w} = 2\pi i$$

$$\epsilon = |z-w|$$



2

as z goes around a

$$f(z) \rightarrow f(z) + 2\pi i$$

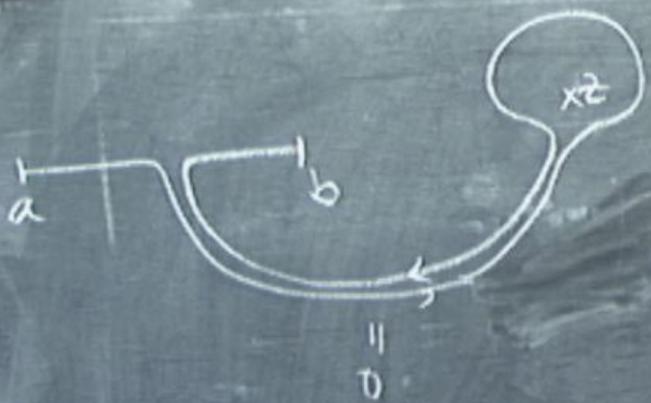


+



$$\oint \frac{dw}{z-w} = 2\pi i$$

$$\epsilon = |z-w|$$

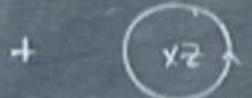


\sqrt{z}

as z goes around b

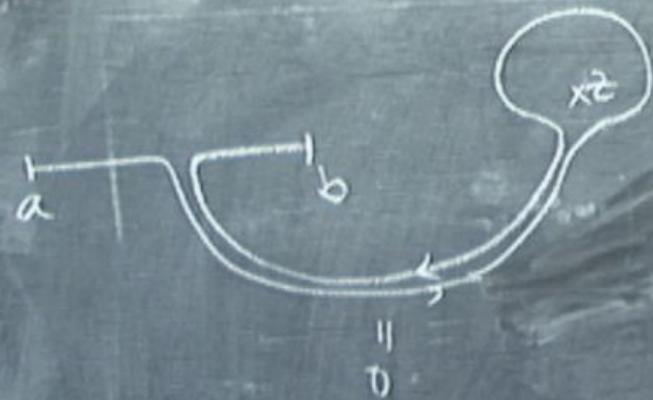
$$f(z) \rightarrow f(z) + 2\pi i$$

i.e. b is a log branch point!



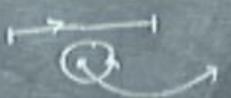
$$\oint \frac{dw}{z-w} = 2\pi i$$

$$\epsilon = |z-w|$$



$\int \frac{dw}{z-w}$

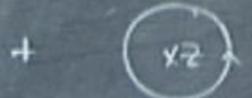
We can "pick the pole" before the end



as z goes around b

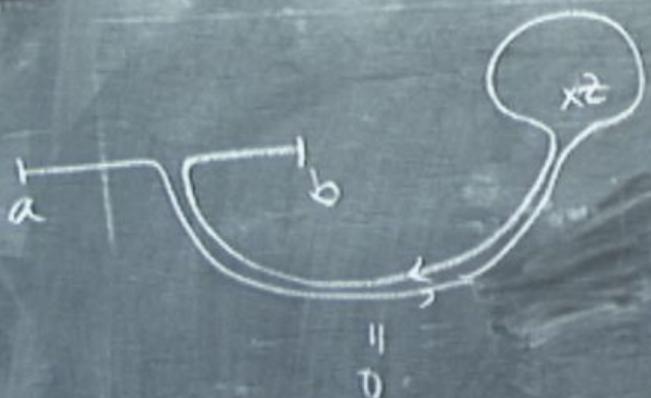
$$f(z) \rightarrow f(z) + 2\pi i$$

i.e. b is a log branch point!



$$\oint \frac{dw}{z-w} = 2\pi i$$

$$\epsilon = |z-w|$$



$\int \frac{dw}{z-w}$

We can "pick the pole" before the end

A diagram showing a horizontal line with a pole at z (indicated by a circle with a dot). A contour is drawn around the pole, consisting of a small circle and a larger loop that extends to the right and then back to the left, crossing the line above and below the pole.

as z goes around b

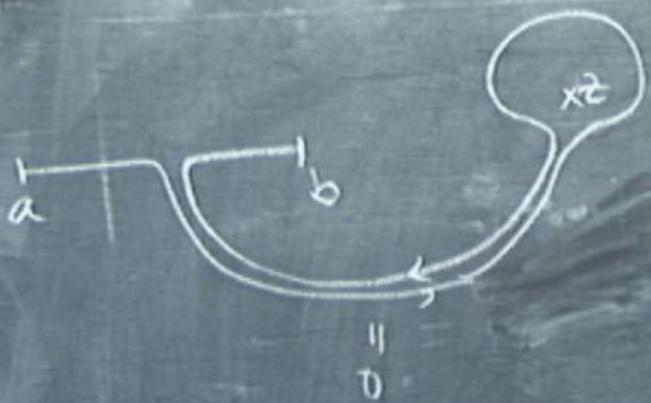
$$f(z) \rightarrow f(z) + 2\pi i$$

i.e. b is a log branch point!



$$\oint \frac{dw}{z-w} = 2\pi i$$

$$\epsilon = |z-w|$$



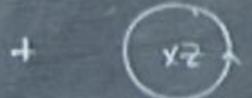
$\int \frac{dw}{z-w}$

We can "pick the pole" before the end

as z goes around b

$$f(z) \rightarrow f(z) + 2\pi i$$

i.e. b is a log branch point!



$$\oint \frac{dw}{z-w} = 2\pi i$$

$$\epsilon = |z-w|$$

$$f(z) = \int_0^1 \frac{dw}{(w-z)(w-a)} = \frac{1}{z-a} \log \left(\frac{1-z}{z} \frac{a}{1-a} \right)$$

$\rightarrow 0$
 $z \rightarrow a$

$$f(z) = \int_0^1 \frac{dw}{(w-z)(w-a)} = \frac{1}{z-a} \log \left(\frac{1-z}{z} \frac{a}{1-a} \right)$$

$\rightarrow 0$

$z \rightarrow a$

regular when $z \rightarrow a$
in some sheet \rightarrow



ω
• a

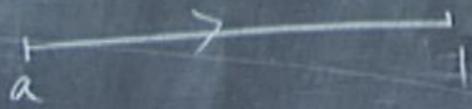
• z

ω
 $\cdot a$

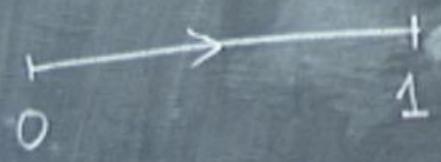


$\cdot z$

lit Zapprocher



\int_0^1
 $\cdot a$



$\cdot z$

lit Zapprocher



$$f(z) = \int_0^1 \frac{dw}{(w-z)(w-a)} = \frac{1}{z-a} \log \left(\frac{1-z}{z} \frac{a}{1-a} \right)$$

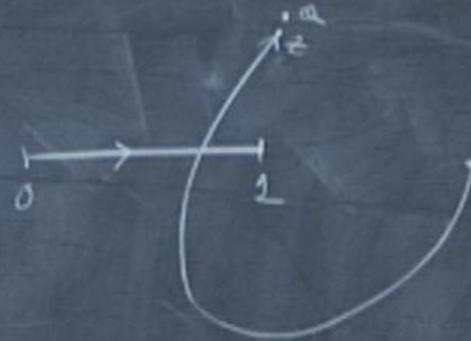
$\rightarrow 0$

$z \rightarrow a$

when $z \rightarrow a$

in some sheet

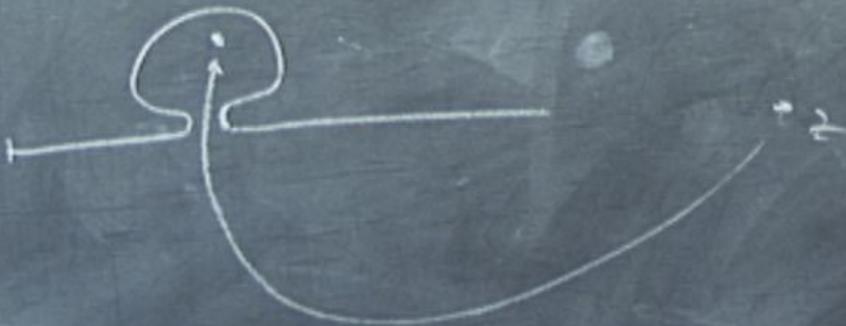
let z approach a as



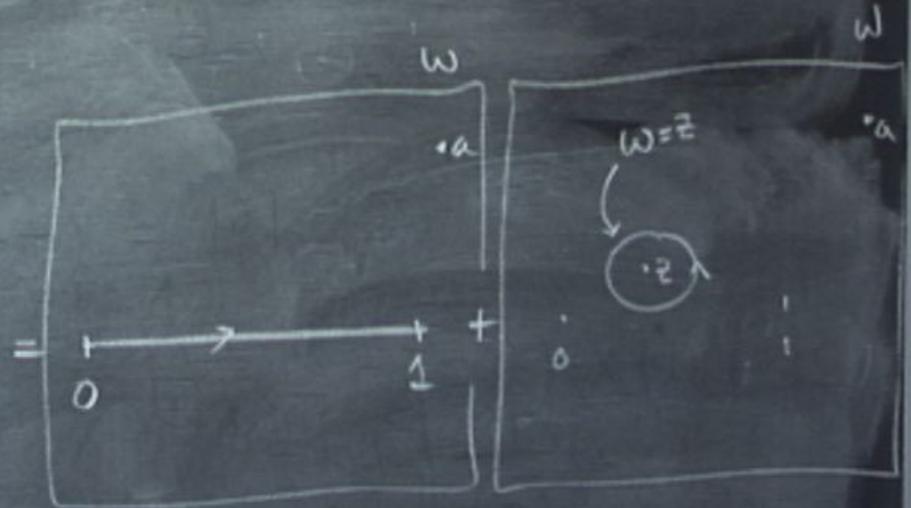
as we cross the contour
'a



as we cross the contour
a



as we cross the contour
at a



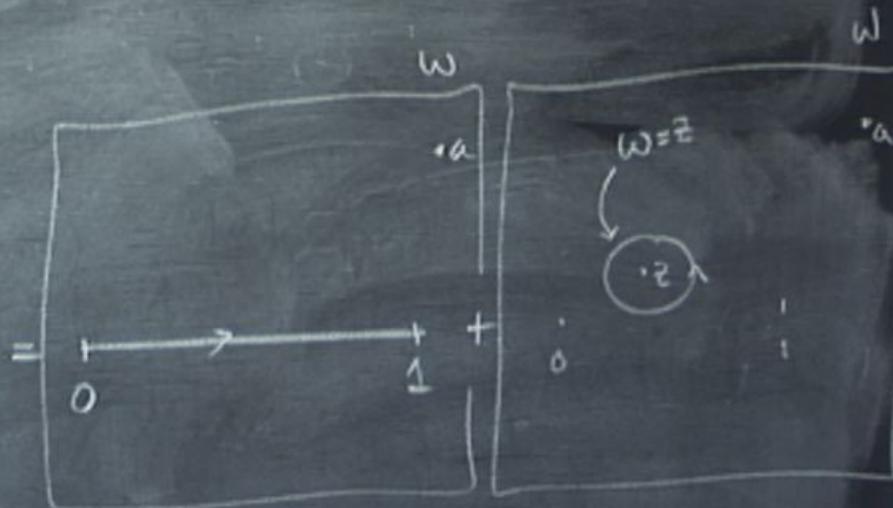
$2\pi i$ Residue (integrand) at $w=z$

$$= 2\pi i \frac{1}{z-a}$$

as we cross the contour
 a



$$= \int_0^1 + \frac{1}{z-a} 2\pi i$$



$2\pi i$ Residue (integrand) at $w=z$

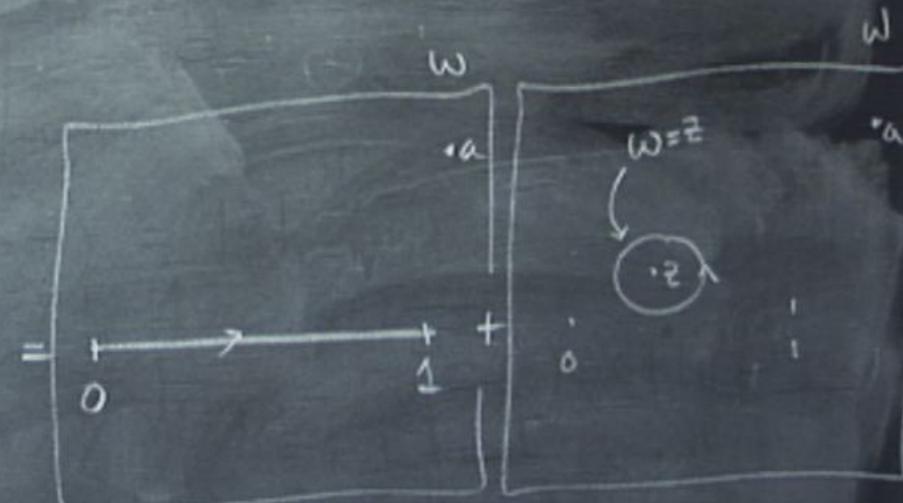
$$= 2\pi i \frac{1}{z-a}$$

as we cross the contour
 a



Now there is a pole!

$$+ \frac{1}{z-a} 2\pi i$$



$2\pi i$ Residue (integrand) at $w=z$

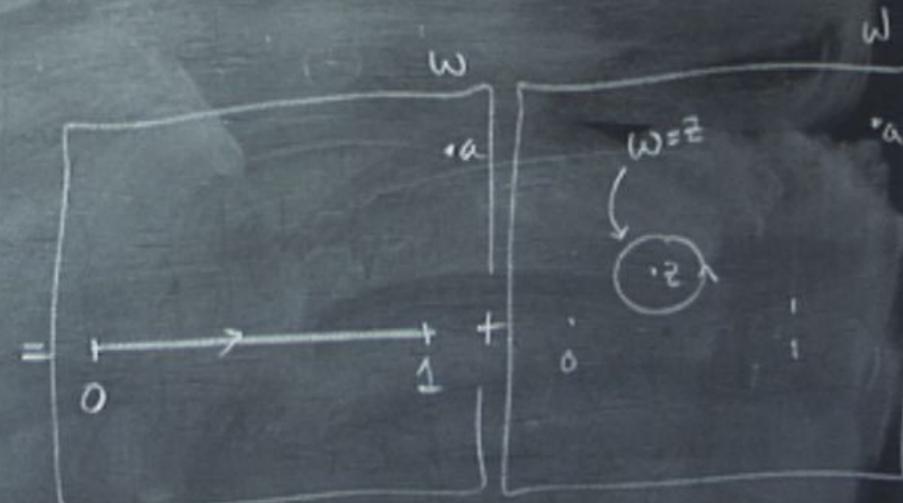
$$= 2\pi i \frac{1}{z-a}$$

as we cross the contour
 a



now there
 is a pole!

$$+ \frac{1}{z-a} 2\pi i$$



$2\pi i$ Residue (integrand) at $w=z$

$$= 2\pi i \frac{1}{z-a}$$

$$\frac{dw}{(w-z)(w-a)} = \frac{1}{z-a} \log \left(\frac{1-z}{z} \frac{a}{1-a} \right)$$

$\rightarrow 0$
 $z \rightarrow a$

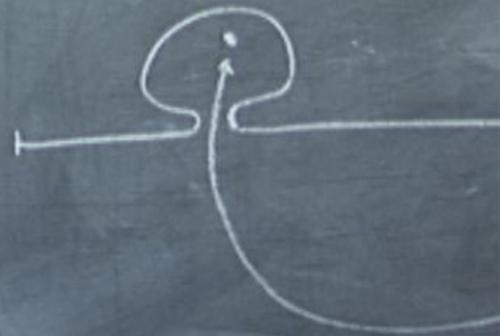
regular when $z \rightarrow a$
in some sheet

deed
when we go around
 $z=1$ we go from

Sheet where $\log(1)=0$

to the sheet where $\log(1)=2\pi i$; so that we have a pole!

when we cross the cut



$$= \frac{1}{z-a}$$

$$\text{Li}_2(z)$$

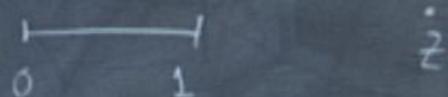
$$Li_2(z)$$

$$\text{Li}_2(z) = - \int_0^1 \frac{dt}{t} \log(1-zt)$$

$$\text{Li}_2(z) = - \int_0^1 \frac{dt}{t} \log(1-zt)$$



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$$\text{Li}_2(z) = - \int_0^1 \frac{dt}{t} \log(1-zt)$$



Γ -function

Γ -function

$$\Gamma(u) = \int_0^{\infty} e^{-t} t^{u-1} dt$$

Γ -function

$$\Gamma(u) = \int_0^{\infty} e^{-t} t^{u-1} dt$$

Γ -function

$$\Gamma(u) = \int_0^{\infty} e^{-t} t^{u-1} dt \quad \text{Re}(u) > 0$$

Γ -function

$$\Gamma(u) = \int_0^{\infty} e^{-t} t^{u-1} dt$$

$\text{Re}(u) > 0$

Γ -function

$$\Gamma(u) = \int_0^{\infty} e^{-t} t^{u-1} dt$$

$$\text{Re}(u) > 0$$

$$\Gamma(u) = \frac{1}{u} \int_0^{\infty} e^{-t} \frac{d}{dt} t^u$$

Γ -function

$$\Gamma(u) = \int_0^{\infty} e^{-t} t^{u-1} dt$$

$\text{Re}(u) > 0$

$$\Gamma(u) = \frac{1}{u} \int_0^{\infty} e^{-t} \frac{d}{dt} t^u dt$$

$$= \frac{1}{u} \int_0^{\infty} e^{-t} t^{-u} dt$$

Γ -function

$$\Gamma(u) = \int_0^{\infty} e^{-t} t^{u-1} dt$$

$\text{Re}(u) > 0$

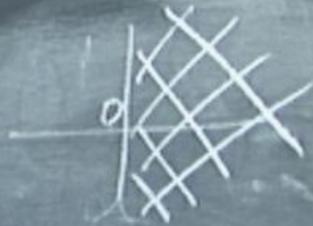
$$\Gamma(u) = \frac{1}{u} \int_0^{\infty} e^{-t} \frac{d}{dt} t^u dt$$

$$= \frac{1}{u} \int_0^{\infty} e^{-t} t^u dt$$

valid for
 $\text{Re}(u) > -1$!

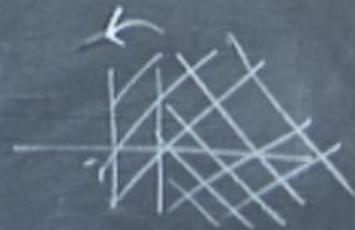
Γ -function

$$\Gamma(u) = \int_0^{\infty} e^{-t} t^{u-1} dt$$



$\text{Re}(u) > 0$

$$\Gamma(u) = \frac{1}{u} \int_0^{\infty} e^{-t} \frac{d}{dt} t^u dt$$

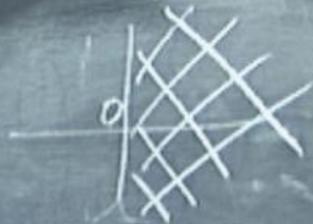


$$= \frac{1}{u} \int_0^{\infty} e^{-t} t^{u+1} dt$$

valid for
 $\text{Re}(u) > -1$!

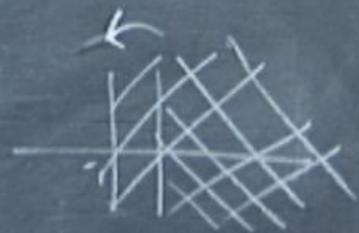
Γ -function

$$\Gamma(u) = \int_0^{\infty} e^{-t} t^{u-1} dt$$



$\text{Re}(u) > 0$

$$\Gamma(u) = \frac{1}{u} \int_0^{\infty} e^{-t} \frac{d}{dt} t^u$$



$$= \frac{1}{u} \int_0^{\infty} e^{-t} t^u$$

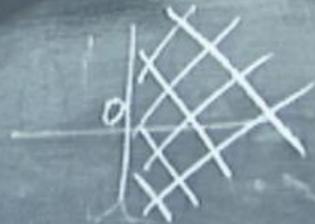
valid for
 $\text{Re}(u) > -1$!

$$= \frac{1}{u(u+1)} \int_0^{\infty} e^{-t} t^{u+1}$$

$\text{Re}(u) > -2$
etc

Γ -function

$$\Gamma(u) = \int_0^{\infty} e^{-t} t^{u-1} dt$$



$\text{Re}(u) > 0$

$$\Gamma(u) = \frac{1}{u} \int_0^{\infty} e^{-t} \frac{d}{dt} t^u$$



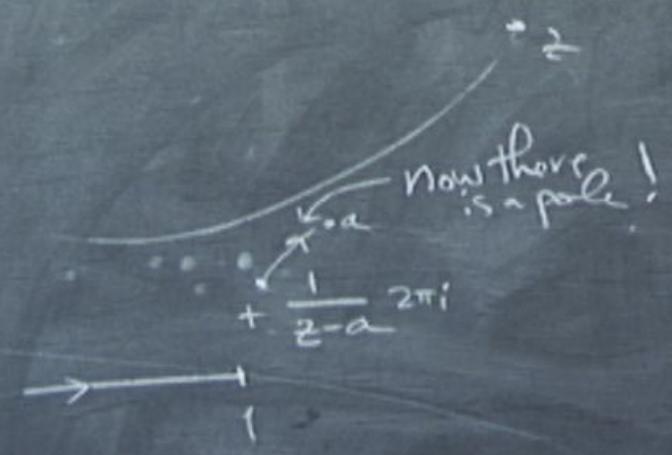
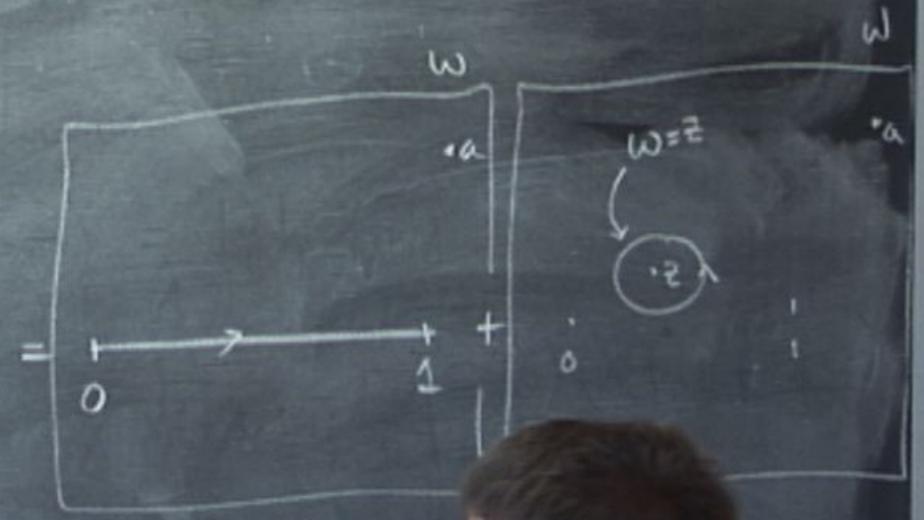
$$= \frac{1}{u} \int_0^{\infty} e^{-t} t^{u-1} dt$$

valid for
 $\text{Re}(u) > -1$!

$$= \frac{1}{u(u+1)} \int_0^{\infty} e^{-t} t^{u+1} dt$$

$\text{Re}(u) > -2$
etc

$$\int_0^1 t^{u-1} dt \sim t^u$$

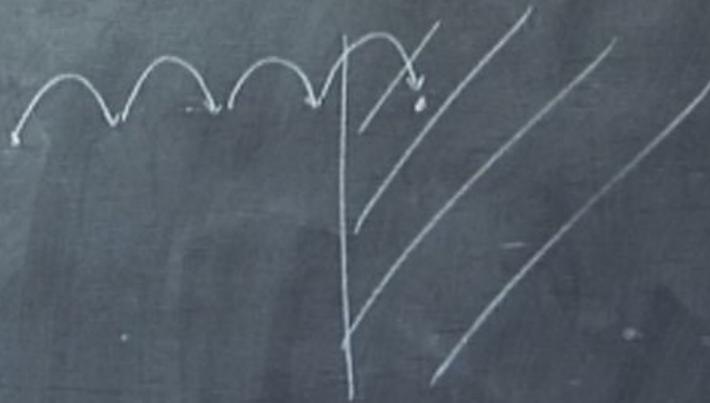


$2\pi i$ Residue (integr)

$$= 2\pi i \frac{1}{z-a}$$

$$\int_0^{\infty} t^{u-1} e^{-t} dt \sim t^u \rightarrow 0 \quad \text{Re}(u) > 0$$

$$\Gamma(u+1) = u \Gamma(u)$$



$$\int_0^1 t^{u-1} \sim t^u \rightarrow 0 \quad \text{Re}(u) > 0$$

$$\Gamma(u+1) = u \Gamma(u)$$

$$\frac{\Gamma(u+1)}{u \Gamma(u)} = 1$$

↳ holds every!



■ New version

In[993]:= $F[z_] = z + 1/z$

Out[993]= $\frac{1}{z} + z$

■ Airplane wings

■ New version

In[993]:= $F[z_] = z + 1/z$

Out[993]= $\frac{1}{z} + z$

$\text{Solve}[F[x] == z, x]$

■ Airplane wings

■ New version

In[993]:= $F[z_] = z + 1/z$

Out[993]= $\frac{1}{z} + z$

In[997]:= $z == F[x]$

Out[997]= $z == \frac{1}{x} + x$

In[995]:= z

Out[995]= $\frac{1}{x} + x$

■ New version

In[993]:= $F[z_] = z + 1/z$

Out[993]= $\frac{1}{z} + z$

In[999]:= $\text{equation} = z == F[x]$

Out[999]= $z == \frac{1}{x} + x$

$\text{Solve}[z == F[x], x]$

■ Airplane wings

■ New version

In[993]:= `F[z_] = z + 1/z`

Out[993]= $\frac{1}{z} + z$

In[1001]:= `equation = z == F[x];`

In[1005]:= `{a, b, c}[[2]]`

Out[1005]= `b`

`Solve[equation, x][[1]]`

In[993]:= `F[z_] = z + 1/z`

Out[993]= $\frac{1}{z} + z$

In[1001]:= `equation = z == F[x];`

In[1005]:= `{a, b, c}[[2]]`

Out[1005]=
b

In[1006]:= `{a, b, c}[[2]]`

Out[1006]=
b

In[993]:= `F[z_] = z + 1/z`

Out[993]= $\frac{1}{z} + z$

In[1001]:= `equation = z == F[x];`

In[1007]:= `{a, b, c}[[2]]`

Out[1007]=
b

In[1006]:= `|{a, b, c}[[2]]`

Out[1006]=
b

Out[993]= $\frac{1}{z} + z$

In[1001]:= `equation = z == F[x];`

In[1007]:= `{a, b, c}[[2]]`

Out[1007]= b

In[1006]:= `{a, b, c}[[2]]`

Out[1006]= b

`{a, b, c}[[2]]`

In[1001]:= `equation = z == F[x];`

In[1007]:= `{a, b, c}[[2]]`

Out[1007]=
b

In[1006]:= `{a, b, c}[[2]]`

Out[1006]=
b

In[1008]:= `Solve[equation, x][[1]]`

Out[1008]=
 $\left\{ x \rightarrow \frac{1}{2} \left(z - \sqrt{-4 + z^2} \right) \right\}$

In[1001]:= `equation = z == F[x];`

In[1007]:= `{a, b, c}[[2]]`

Out[1007]=
b

In[1006]:= `{a, b, c}[[2]]`

Out[1006]=
b

`solution =`

In[1009]:= `x /. Solve[equation, x][[1]]`

Out[1009]=

$$\frac{1}{2} \left(z - \sqrt{-4 + z^2} \right)$$

In[1006]:= `{a, b, c}[[2]]`

Out[1006]=

b

In[1010]:= `solution = Solve[equation, x][[1]]`

Out[1010]=

$$\left\{ x \rightarrow \frac{1}{2} \left(z - \sqrt{-4 + z^2} \right) \right\}$$

In[1011]:= `x /. solution`

Out[1011]=

$$\frac{1}{2} \left(z - \sqrt{-4 + z^2} \right)$$

In[1001]:= `equation = z == F[x];`

In[1010]:= `solution = Solve[equation, x][[1]]`

Out[1010]=

$$\left\{ x \rightarrow \frac{1}{2} \left(z - \sqrt{-4 + z^2} \right) \right\}$$

Out[1011]= `ToPlot = x /. solution`

$$\frac{1}{2} \left(z - \sqrt{-4 + z^2} \right)$$

■ Airplane wings

```
In[1001]:= equation = z == F[x];
```

```
In[1010]:= solution = Solve[equation, x][[1]]
```

```
Out[1010]=
```

$$\left\{ x \rightarrow \frac{1}{2} \left(z - \sqrt{-4 + z^2} \right) \right\}$$

```
Out[1012]=
```

```
ToPlot = x /. solution /. z -> x + I y // Im
```

$$\frac{1}{2} \left(x - \sqrt{-4 + (x + i y)^2} + i y \right)$$

■ Airplane wings

```
In[1001]:= equation = z == F[x];
```

```
In[1010]:= solution = Solve[equation, x][[1]]
```

Out[1010]=

$$\left\{ x \rightarrow \frac{1}{2} \left(z - \sqrt{-4 + z^2} \right) \right\}$$

```
In[1013]:= ToPlot = x /. solution /. z -> x + I y // Im
```

Out[1013]=

$$\frac{1}{2} \left(\text{Im} \left[x - \sqrt{-4 + (x + i y)^2} \right] + \text{Re} [y] \right)$$

■ Airplane wings

```
In[1001]:= equation = z == F[x];
```

```
In[1010]:= solution = Solve[equation, x][[1]]
```

```
Out[1010]=
```

$$\left\{ x \rightarrow \frac{1}{2} \left(z - \sqrt{-4 + z^2} \right) \right\}$$

```
In[1013]:= ToPlot = x /. solution /. z -> x + I y // Im
```

```
Out[1013]=
```

$$\frac{1}{2} \left(\text{Im} \left[x - \sqrt{-4 + (x + i y)^2} \right] + \text{Re}[y] \right)$$

```
Plot3D[|
```

```
In[1001]:= equation = z == F[x];
```

```
In[1010]:= solution = Solve[equation, x][[1]]
```

```
Out[1010]=
```

$$\left\{ x \rightarrow \frac{1}{2} \left(z - \sqrt{-4 + z^2} \right) \right\}$$

```
In[1013]:= ToPlot = x /. solution /. z -> x + I y // Im
```

```
Out[1013]=
```

$$\frac{1}{2} \left(\text{Im} \left[x - \sqrt{-4 + (x + i y)^2} \right] + \text{Re} [y] \right)$$

```
Plot3D[ToPlot, {x, -3, 3}, {y, -3, 3}]
```

In[1001]:= `equation = z == F[x];`

In[1010]:= `solution = Solve[equation, x][[1]]`

Out[1010]=

$$\left\{ x \rightarrow \frac{1}{2} \left(z - \sqrt{-4 + z^2} \right) \right\}$$

In[1013]:= `ToPlot = x /. solution /. z -> x + I y // Im`

Out[1013]=

$$\frac{1}{2} \left(\text{Im} \left[x - \sqrt{-4 + (x + i y)^2} \right] + \text{Re}[y] \right)$$

`Plot3D[ToPlot, {x, -3, 3}, {y, -3, 3}]`

■ Airplane wings

```
in[1001]:= equation = z == F[x];
```

```
in[1010]:= solution = Solve[equation, x] [[1]]
```

```
out[1010]=
```

$$\left\{ x \rightarrow \frac{1}{2} \left(z - \sqrt{-4 + z^2} \right) \right\}$$

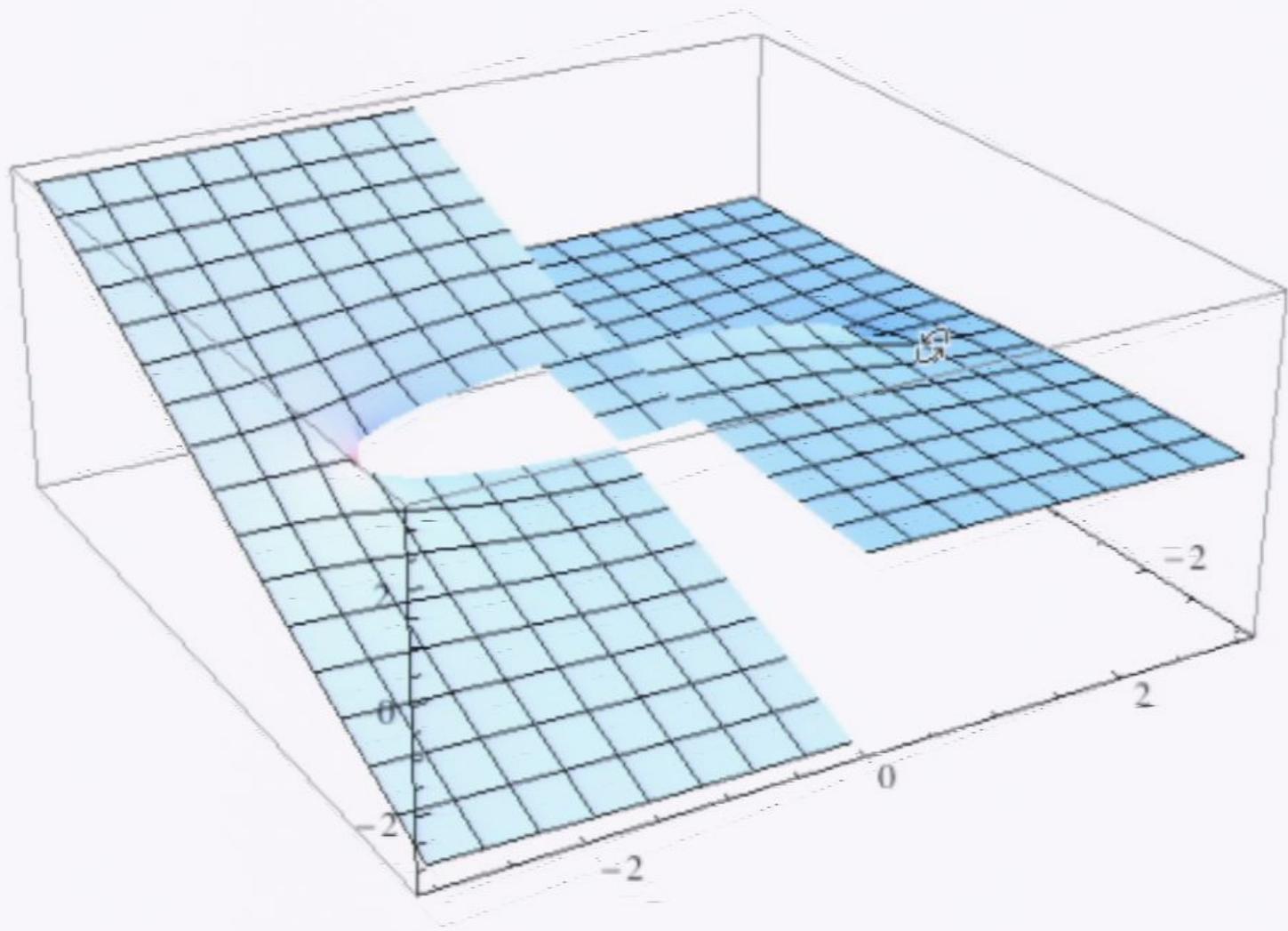
```
in[1013]:= ToPlot = x /. solution /. z -> x + I y // Im
```

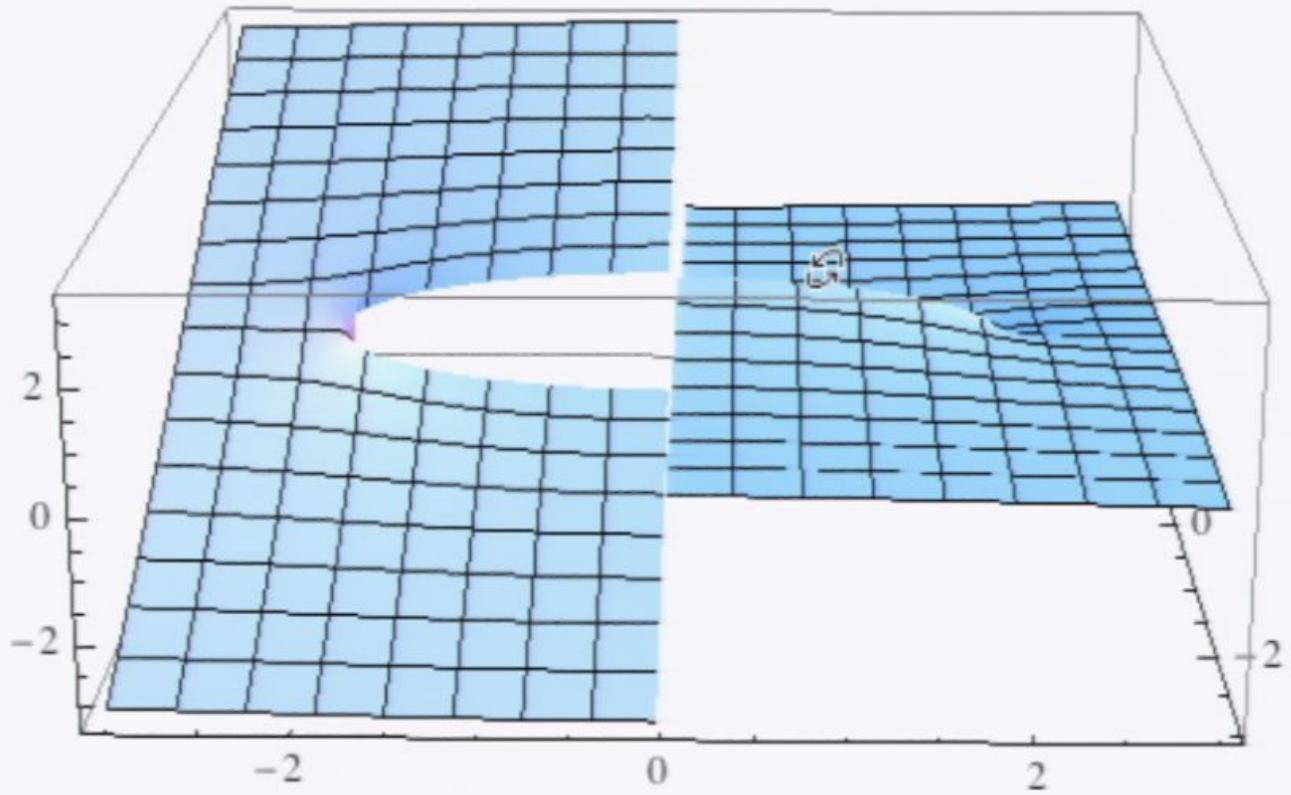
```
out[1013]=
```

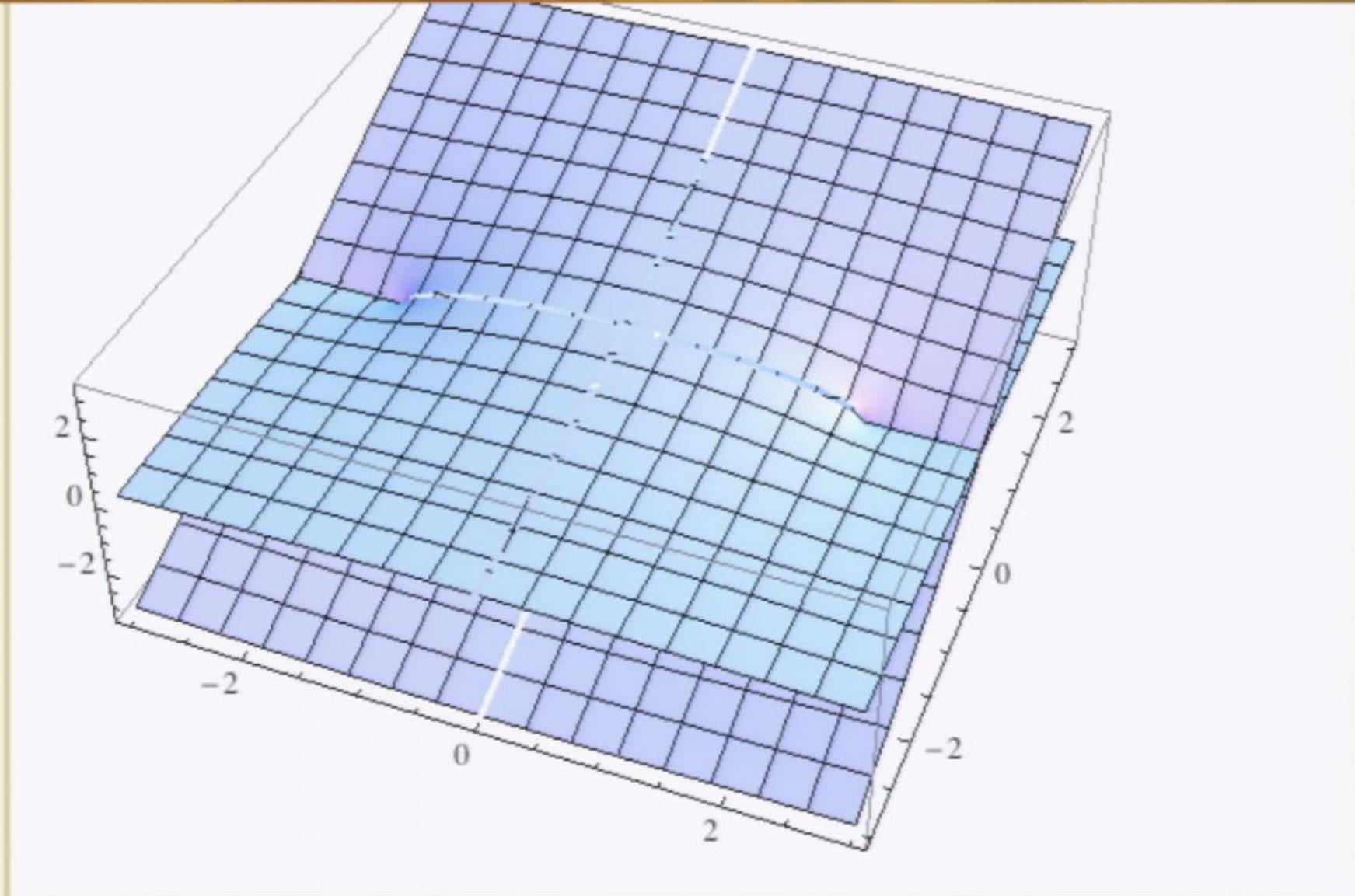
$$\frac{1}{2} \left(\text{Im} \left[x - \sqrt{-4 + (x + i y)^2} \right] + \text{Re} [y] \right)$$

```
Plot3D[ToPlot, {x, -3, 3}, {y, -3, 3}]
```

■ Airplane wings



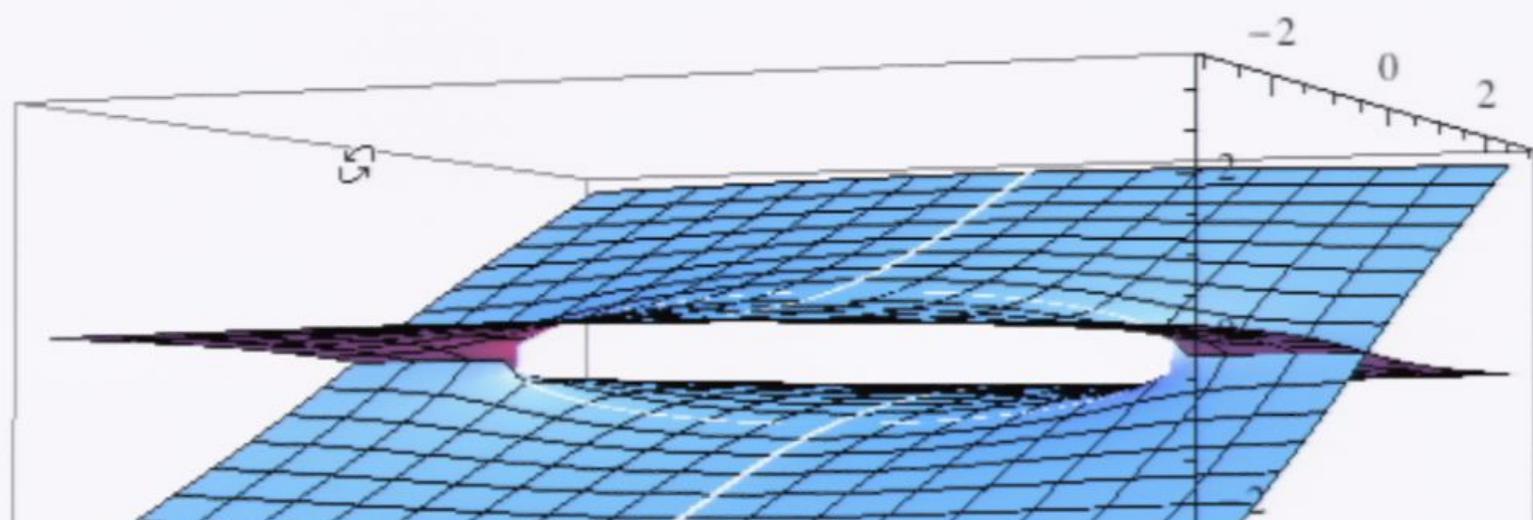




$$\left\{ \frac{1}{2} \left(\operatorname{Im} \left[x - \sqrt{-4 + (x + i y)^2} \right] + \operatorname{Re} [y] \right), \right. \\ \left. \frac{1}{2} \left(\operatorname{Im} \left[x + \sqrt{-4 + (x + i y)^2} \right] + \operatorname{Re} [y] \right) \right\}$$

In[1017]:=
Out[1017]=

```
Plot3D[ToPlot, {x, -3, 3}, {y, -3, 3}]
```

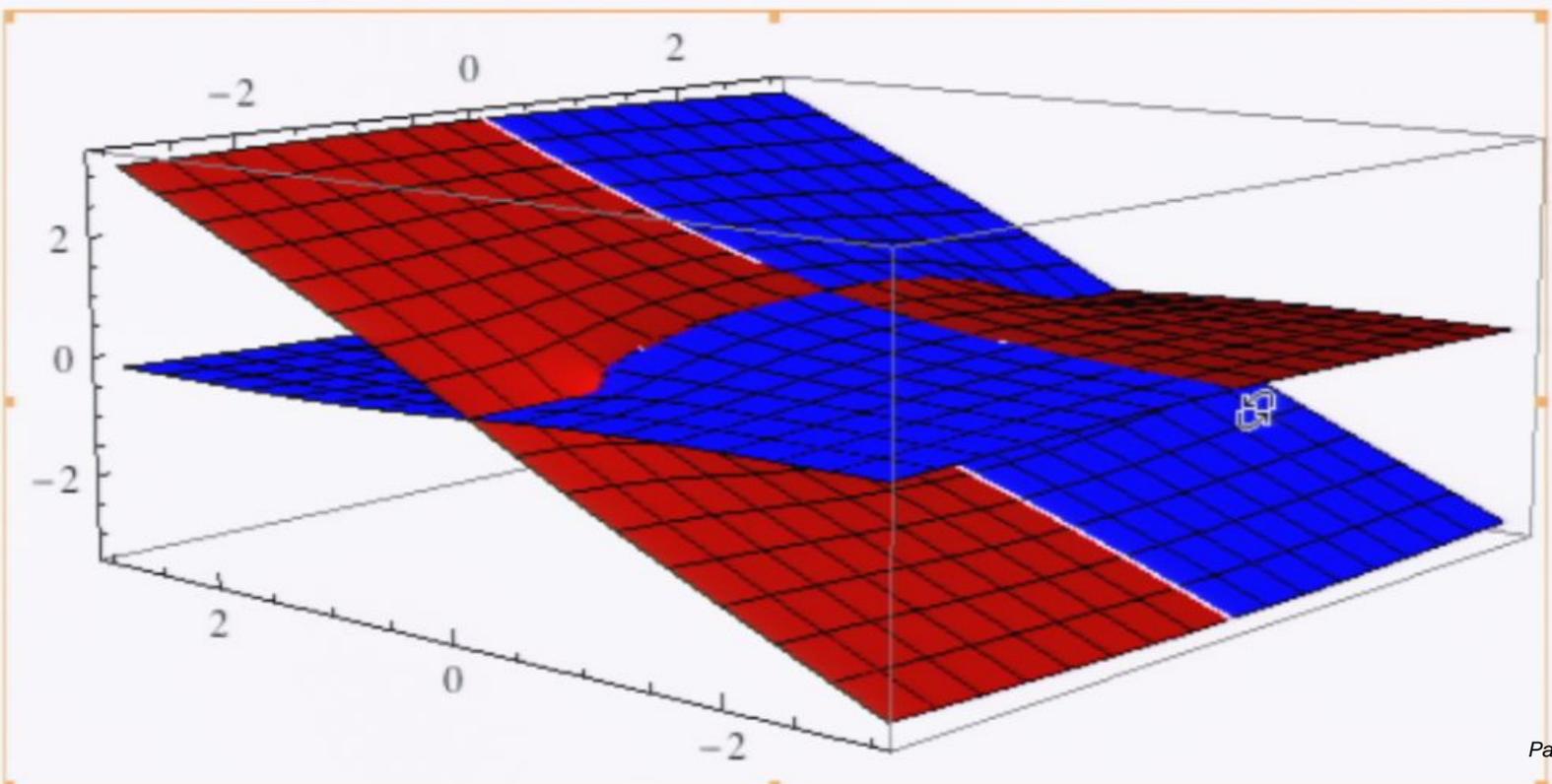


$$\frac{1}{2} \left(\operatorname{Im} \left[x + \sqrt{-4 + (x + 1) y} \right] + \operatorname{Re} [y] \right)$$

In[1018]:=

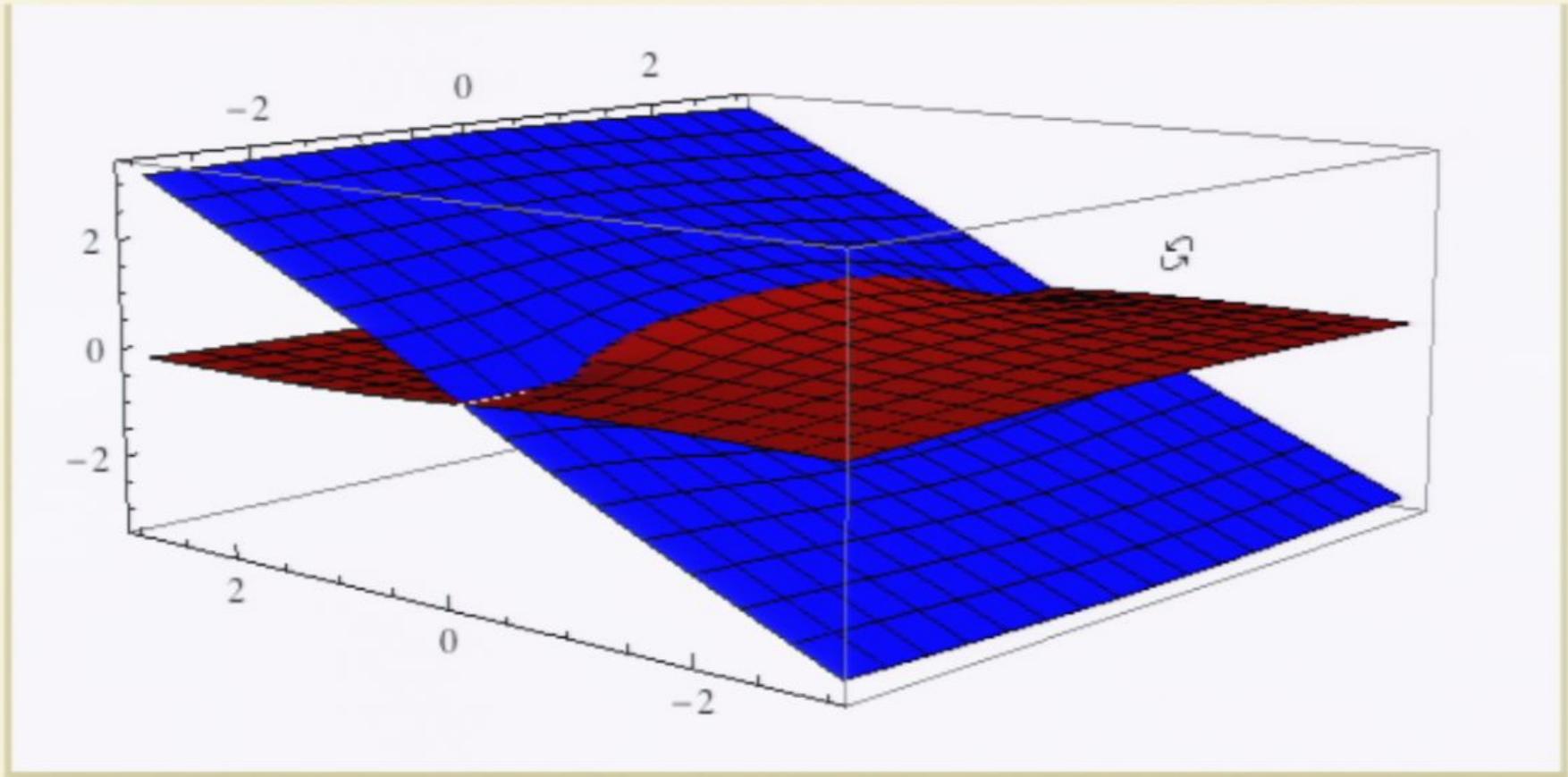
```
Plot3D[ToPlot, {x, -3, 3}, {y, -3, 3},  
PlotStyle -> {Red, Blue}]
```

Out[1018]=



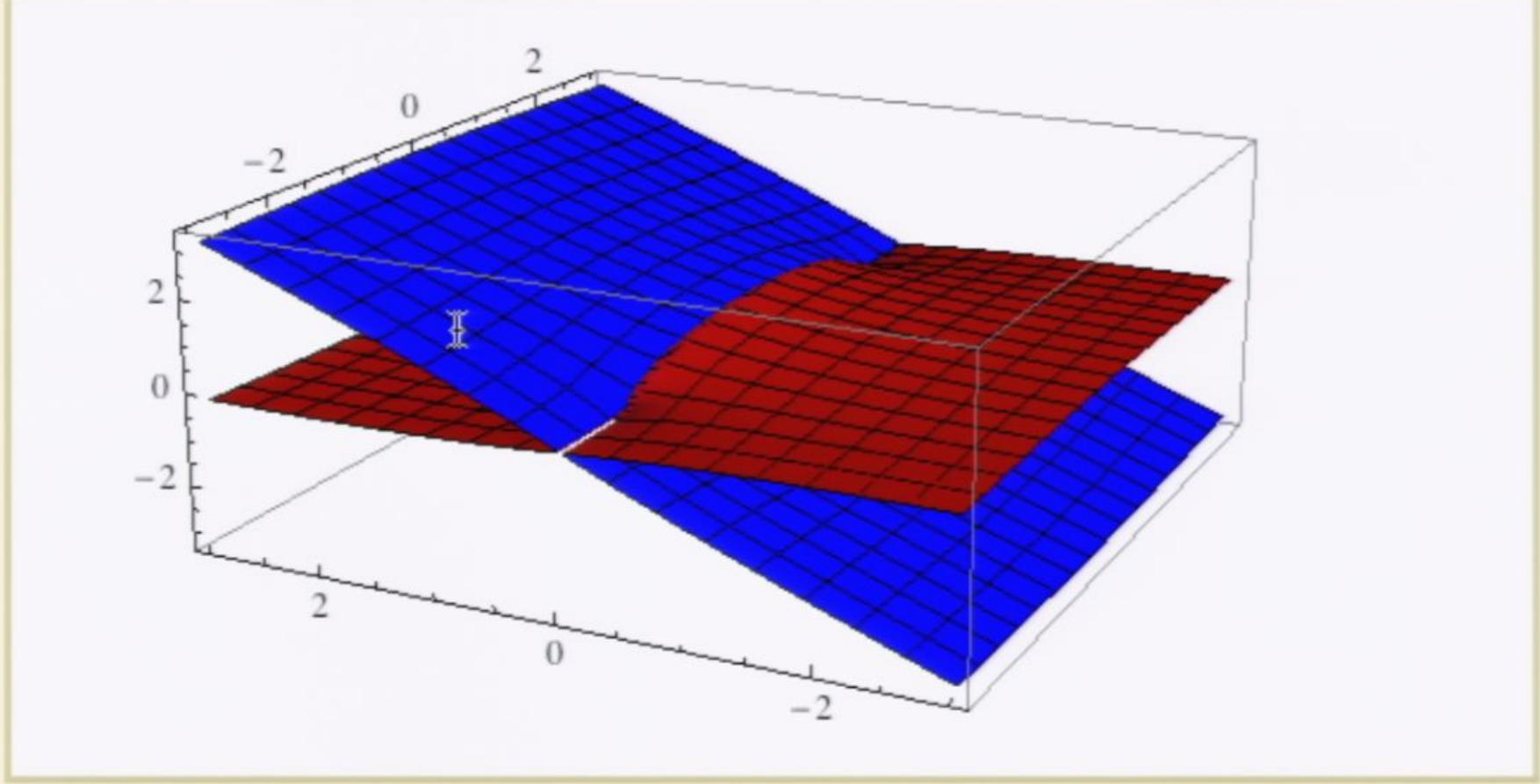
```
In[1023]:= Plot3D[ToPlot, {x, -3, 3}, {y, -3, 3},  
PlotStyle -> {Red, Blue}]
```

Out[1023]=



```
In[1023]:= Plot3D[ToPlot, {x, -3, 3}, {y, -3, 3},  
PlotStyle -> {Red, Blue}]
```

Out[1023]=



■ New version

```
In[1019]:= F[z_] = z + 1. / z
```

```
Out[1019]=
```

$$\frac{1.}{z} + z$$

```
In[1020]:= equation = z == F[x];
```

```
In[1021]:= solution = Solve[equation, x]
```

```
Out[1021]=
```

$$\left\{ \left\{ x \rightarrow 0.5 \left(z - 1. \sqrt{-2. + z} \sqrt{2. + z} \right) \right\}, \right. \\ \left. \left\{ x \rightarrow 0.5 \left(z + 1. \sqrt{-2. + z} \sqrt{2. + z} \right) \right\} \right\}$$

```
In[1022]:= ToPlot = x /. solution /. z -> x + I y // Im
```

■ New version

In[1019]:= `F[z_] = z + 1. / z`

Out[1019]=
$$\frac{1.}{z} + z$$

In[1020]:= `equation = z == F[x];`

In[1021]:= `solution = Solve[equation, x]`

Out[1021]=
$$\left\{ \left\{ x \rightarrow 0.5 \left(z - 1. \sqrt{-2. + z} \sqrt{2. + z} \right) \right\}, \right. \\ \left. \left\{ x \rightarrow 0.5 \left(z + 1. \sqrt{-2. + z} \sqrt{2. + z} \right) \right\} \right\}$$

In[1022]:= `ToPlot = x /. solution /. z -> x + I y // Im`

■ New version

In[1019]:= `F[z_] = z + 1. / z`

Out[1019]=
$$\frac{1.}{z} + z$$

In[1020]:= `equation = z == F[x];`

In[1021]:= `solution = Solve[equation, x]`

Out[1021]=
$$\left\{ \left\{ x \rightarrow 0.5 \left(z - 1. \sqrt{-2. + z} \sqrt{2. + z} \right) \right\}, \right. \\ \left. \left\{ x \rightarrow 0.5 \left(z + 1. \sqrt{-2. + z} \sqrt{2. + z} \right) \right\} \right\}$$

In[1022]:= `Plot = x /. solution /. z -> x + I y // Im`

- New version
- New version

In[1019]:= `F[z_] = z + 1. / z`

Out[1019]=
$$\frac{1.}{z} + z$$

In[1020]:= `equation = z == F[x];`

In[1021]:= `solution = Solve[equation, x]`

Out[1021]=
$$\left\{ \left\{ x \rightarrow 0.5 \left(z - 1. \sqrt{-2. + z} \sqrt{2. + z} \right) \right\}, \right. \\ \left. \left\{ x \rightarrow 0.5 \left(z + 1. \sqrt{-2. + z} \sqrt{2. + z} \right) \right\} \right\}$$

- New version
- New version 2

In[1028]:=

```
F[z_] = z + 1/z;  
iF[z_] =  $\frac{z + \sqrt{z-2} \sqrt{z+2}}{2}$ ;  
(* human square roots *)
```

In[1030]:=

```
F[iF[z]] // Simplify
```

Out[1030]=

```
z
```

In[1020]:=

```
equation = z == F[x];
```

In[1021]:=

```
solution = Solve[equation, x]
```

$$\text{iF}[z_] = \frac{z + \sqrt{z-2} \sqrt{z+2}}{2};$$

(* human square roots *)

```
In[1030]:= F[iF[z]] // Simplify
```

```
Out[1030]=
```

z

```
In[1020]:= equation = z == F[x];
```

```
In[1021]:= solution = Solve[equation, x]
```

```
Out[1021]=
```

$$\left\{ \left\{ x \rightarrow 0.5 \left(z - 1. \sqrt{-2. + z} \sqrt{2. + z} \right) \right\}, \right. \\ \left. \left\{ x \rightarrow 0.5 \left(z + 1. \sqrt{-2. + z} \sqrt{2. + z} \right) \right\} \right\}$$

```
IF[z_] =  $\frac{\quad}{2}$ ;
```

```
(* human square roots *)
```

```
In[990]:=
```

```
wing[p_, r_] :=
```

```
ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r}(-p + e^{i\phi})$ ],  
{ $\phi$ , 0, 2\pi}, PlotStyle -> {Thickness[0.015], Red}]
```

■ Airplane wings

```
IF[z_] =  $\frac{\quad}{2}$ ;
```

```
(* human square roots *)
```

We want a function to map $5+i$ to $\{5,6\}$

```
In[990]:= wing[p_, r_] :=
  ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r}(-p + e^{i\phi})$ ],
    { $\phi$ , 0, 2\pi}, PlotStyle -> {Thickness[0.015], Red}]
```

■ Airplane wings

■ New version 2

```
In[1028]:= F[z_] = z + 1/z;
iF[z_] =  $\frac{z + \sqrt{z-2} \sqrt{z+2}}{2}$ ;
(* human square roots *)
```

We want a function to map $5+i6$ to $\{5,6\}$

```
In[990]:= wing[p_, r_] :=
  ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r}(-p + e^{i\phi})$ ],
    {phi, 0, 2 pi}, PlotStyle -> {Thickness[0.015], Red}]
```

■ Airplane wings

■ New version 2

In[1028]:=

```
F[z_] = z + 1 / z;
iF[z_] =  $\frac{z + \sqrt{z - 2} \sqrt{z + 2}}{2}$ ;
(* human square roots *)
```

We want a function to map $5+i6$ to $\{5,6\}$

In[990]:=

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wing[p_, r_] :=
  ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r} (-p + e^{i\phi})$ ],
    {phi, 0, 2 pi}, PlotStyle -> {Thickness[0.015], Red}]
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■ Airplane wings

■ New version 2

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In[1028]:= F[z_] = z + 1/z;
iF[z_] =  $\frac{z + \sqrt{z-2} \sqrt{z+2}}{2}$ ;
(* human square roots *)
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We want a function to map $5+i6$ to $\{5,6\}$

```
F[z_]
```

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In[990]:= wing[p_, r_] :=
  ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r}(-p + e^{i\phi})$ ],
    {phi, 0, 2 pi}, PlotStyle -> {Thickness[0.015], Red}]
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■ Airplane wings

■ New version 2

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In[1028]:= F[z_] = z + 1/z;
iF[z_] =  $\frac{z + \sqrt{z-2} \sqrt{z+2}}{2}$ ;
(* human square roots *)
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We want a function to map $5+i6$ to $\{5,6\}$

```
F[z_] = |
```

```
In[990]:= wing[p_, r_] :=
  ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r}(-p + e^{i\phi})$ ],
    { $\phi$ , 0, 2\pi}, PlotStyle -> {Thickness[0.015], Red}]
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■ Airplane wings

■ New version 2

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We want a function to map $5+i6$ to $\{5,6\}$

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F[z_] = {Re[z], Im[z]}
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In[990]:= wing[p_, r_] :=
  ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r}(-p + e^{i\phi})$ ],
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■ Airplane wings

■ New version 2

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iF[z_] =  $\frac{z + \sqrt{z - 2} \sqrt{z + 2}}{2}$ ;
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We want a function to map $5+i6$ to $\{5,6\}$

```
H[z_] = {Re[z], Im[z]}
H[5 + I 6]
```

In[990]:=

```
wing[p_, r_] :=
  ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r} (-p + e^{i\phi})$ ],
    {phi, 0, 2 pi}, PlotStyle -> {Thickness[0.015], Red}]
```

■ New version 2

```

In[1028]:= F[z_] = z + 1/z;
            iF[z_] =  $\frac{z + \sqrt{z-2} \sqrt{z+2}}{2}$ ;
            (* human square roots *)

```

We want a function to map $5+i6$ to $\{5,6\}$

```

In[1031]:= H[z_] = {Re[z], Im[z]}
            H[5 + I 6]

```

```

Out[1031]= {Re[z], Im[z]}

```

```

Out[1032]= {5, 6}

```

■ New version 2

```
In[1028]:= F[z_] = z + 1/z;
iF[z_] =  $\frac{z + \sqrt{z-2} \sqrt{z+2}}{2}$ ;
(* human square roots *)
```

We want a function to map $5+i6$ to $\{5,6\}$

```
In[1033]:= H[z_] = {Re[z], Im[z]};
H[5 + I 6]
```

```
Out[1034]= {5, 6}
```

```
In[990]:= wing[p_, r_] :=
ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r} (-p + e^{i\phi})$ ],
```

■ New version 2

```
In[1028]:= F[z_] = z + 1/z;
iF[z_] =  $\frac{z + \sqrt{z-2} \sqrt{z+2}}{2}$ ;
(* human square roots *)
```

We want a function to map $5+i6$ to $\{5,6\}$

```
In[1033]:= H[z_] = {Re[z], Im[z]};
H[5 + I 6]
```

```
Out[1034]= {5, 6}
```

```
In[990]:= wing[p_, r_] :=
ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r} (-p + e^{i\phi})$ ],
```

```

In[1028]:= F[z_] = z + 1/z;
           iF[z_] =  $\frac{z + \sqrt{z-2} \sqrt{z+2}}{2}$ ;
           (* human square roots *)

```

We want a function to map $5+i6$ to $\{5,6\}$

```

In[1033]:= H[z_] = {Re[z], Im[z]};
           H[5 + I 6]

```

```

Out[1034]= {5, 6}

```

```

ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r} (-p + e^{i\phi})$ ],
               { $\phi$ , 0, 2\pi}, PlotStyle -> {Thickness[0.015], Red}]

```

```

In[1028]:= F[z_] = z + 1/z;
            iF[z_] =  $\frac{z + \sqrt{z-2} \sqrt{z+2}}{2}$ ;
            (* human square roots *)

```

We want a function to map $5+i6$ to $\{5,6\}$

```

In[1033]:= H[z_] = {Re[z], Im[z]};
            H[5 + I 6]

```

```

Out[1034]= {5, 6}

```

```

ParametricPlot [
  { Re [ F [  $\frac{1}{r} (-p + e^{i \phi})$  ] ], {  $\phi, 0, 2 \pi$  },

```

```

  PlotStyle -> {Thickness[0.015], Red} ]

```

```
In[1033]:= H[z_] = {Re[z], Im[z]};
           H[5 + I 6]
```

```
Out[1034]= {5, 6}
```

```
ParametricPlot [
  {Re [F [1/r (-p + ei φ) ]], {φ, 0, 2 π},
  PlotStyle → {Thickness [0.015], Red} ]
```

```
In[990]:= wing [p_, r_] :=
  ParametricPlot [ {Re [#], Im [#]} &@F [1/r (-p + ei φ) ],
  {φ, 0, 2 π}, PlotStyle → {Thickness [0.015], Red} ]
```

```
In[1033]:= H[z_] = {Re[z], Im[z]};
           H[5 + I 6]
```

```
Out[1034]= {5, 6}
```

```
ParametricPlot[
  {Re[F[10/8 (-1 + ei φ)], {φ, 0, 2 π},
  PlotStyle → {Thickness[0.015], Red}]
```

```
In[990]:= wing[p_, r_] :=
  ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r} (-p + e^{i \phi})$ ],
  {φ, 0, 2 π}, PlotStyle → {Thickness[0.015], Red}]
```

```
In[1033]:= H[z_] = {Re[z], Im[z]};
           H[5 + I 6]
```

```
Out[1034]= {5, 6}
```

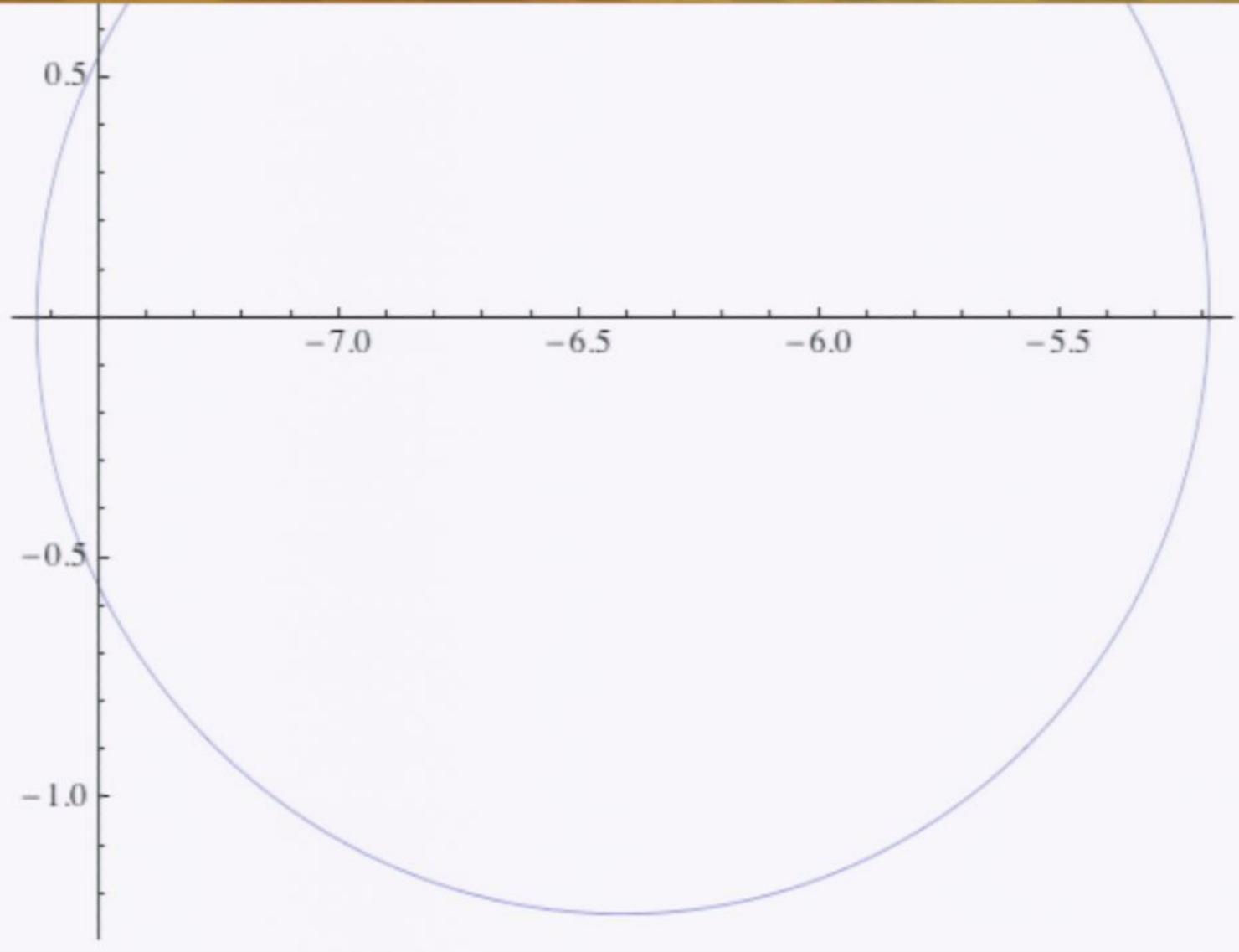
```
ParametricPlot[
  {Re[F[10/8 (-10 + eiϕ)]], {ϕ, 0, 2π},
  PlotStyle → {Thickness[0.015], Red]}
```

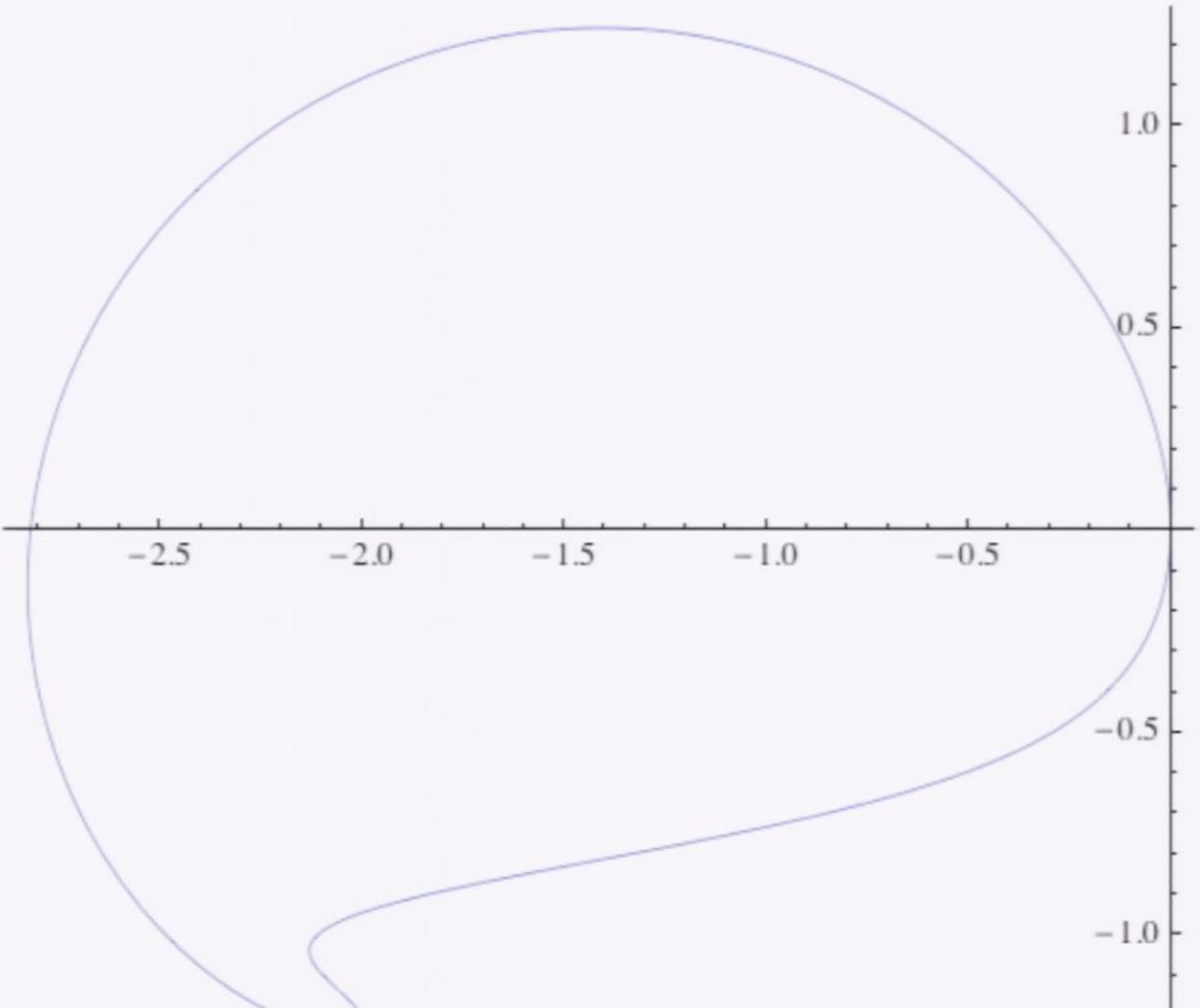
```
In[990]:= wing[p_, r_] :=
  ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r}$  (-p + eiϕ)],
  {ϕ, 0, 2π}, PlotStyle → {Thickness[0.015], Red}]
```

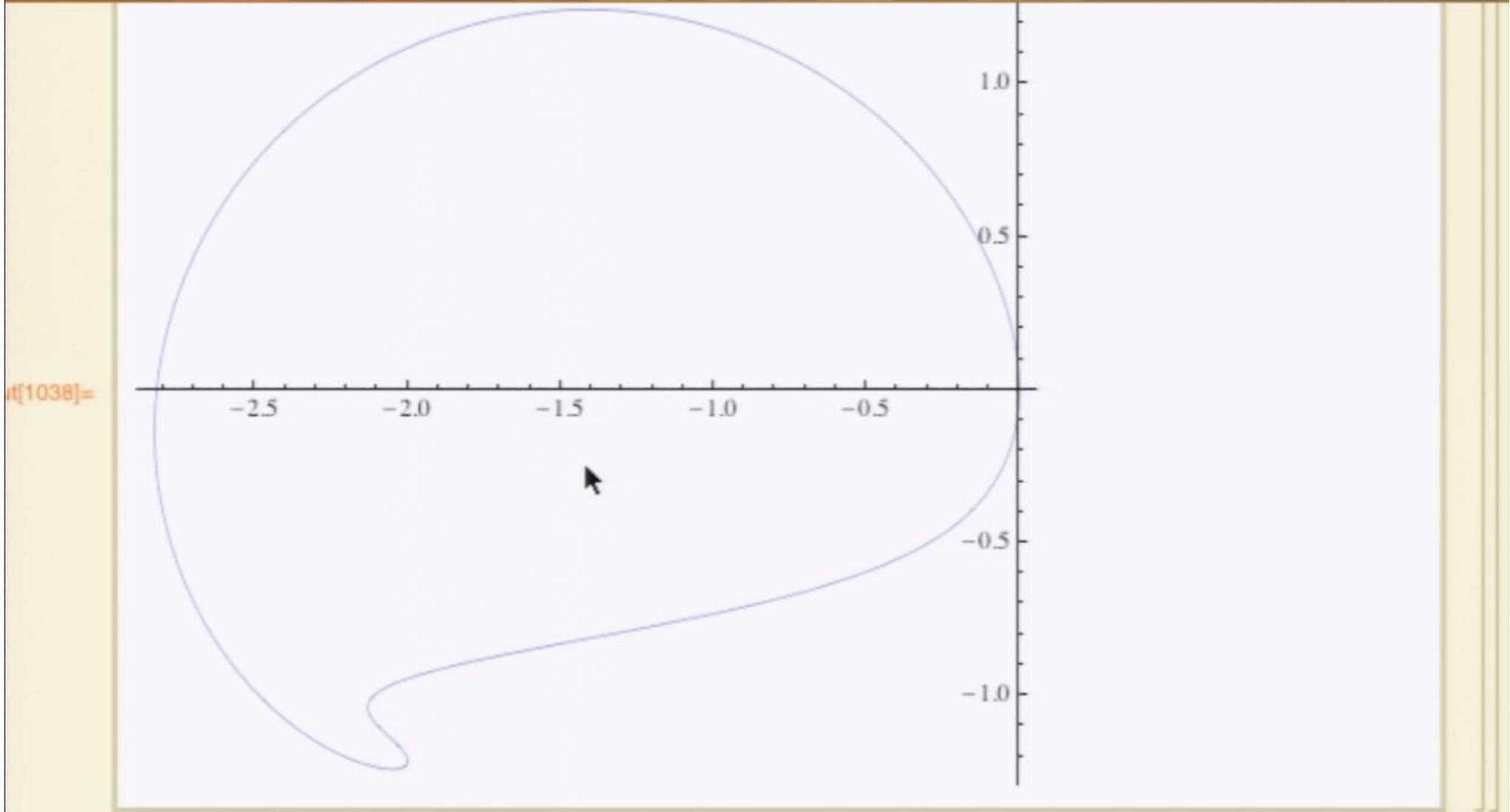
In[990]:=

```
wing[p_, r_] :=  
  ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r}(-p + e^{i\phi})$ ],  
  { $\phi$ , 0, 2  $\pi$ }, PlotStyle -> {Thickness[0.015], Red}]
```

■ Airplane wings







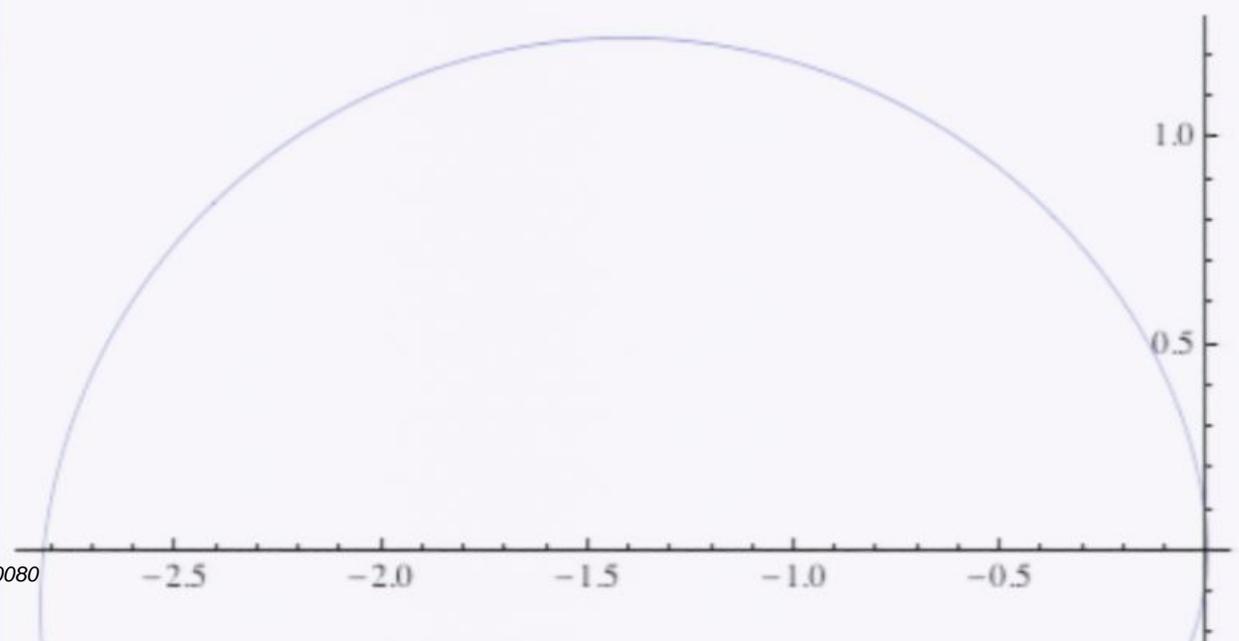
in[1038]=

We want a function to map $5+i 6$ to $\{5,6\}$

```
In[1033]:= H[z_] = {Re[z], Im[z]};  
H[5 + I 6]
```

```
Out[1034]= {5, 6}
```

```
In[1038]:= ParametricPlot[{Re[F[10/8 (-1 + I + ei φ)]], Im[F[10/8 (-10 + ei φ)]]},  
{φ, 0, 2 π}]
```



We want a function to map $5+i 6$ to $\{5,6\}$

```
In[1033]:= H[z_] = {Re[z], Im[z]};  
H[5 + I 6]
```

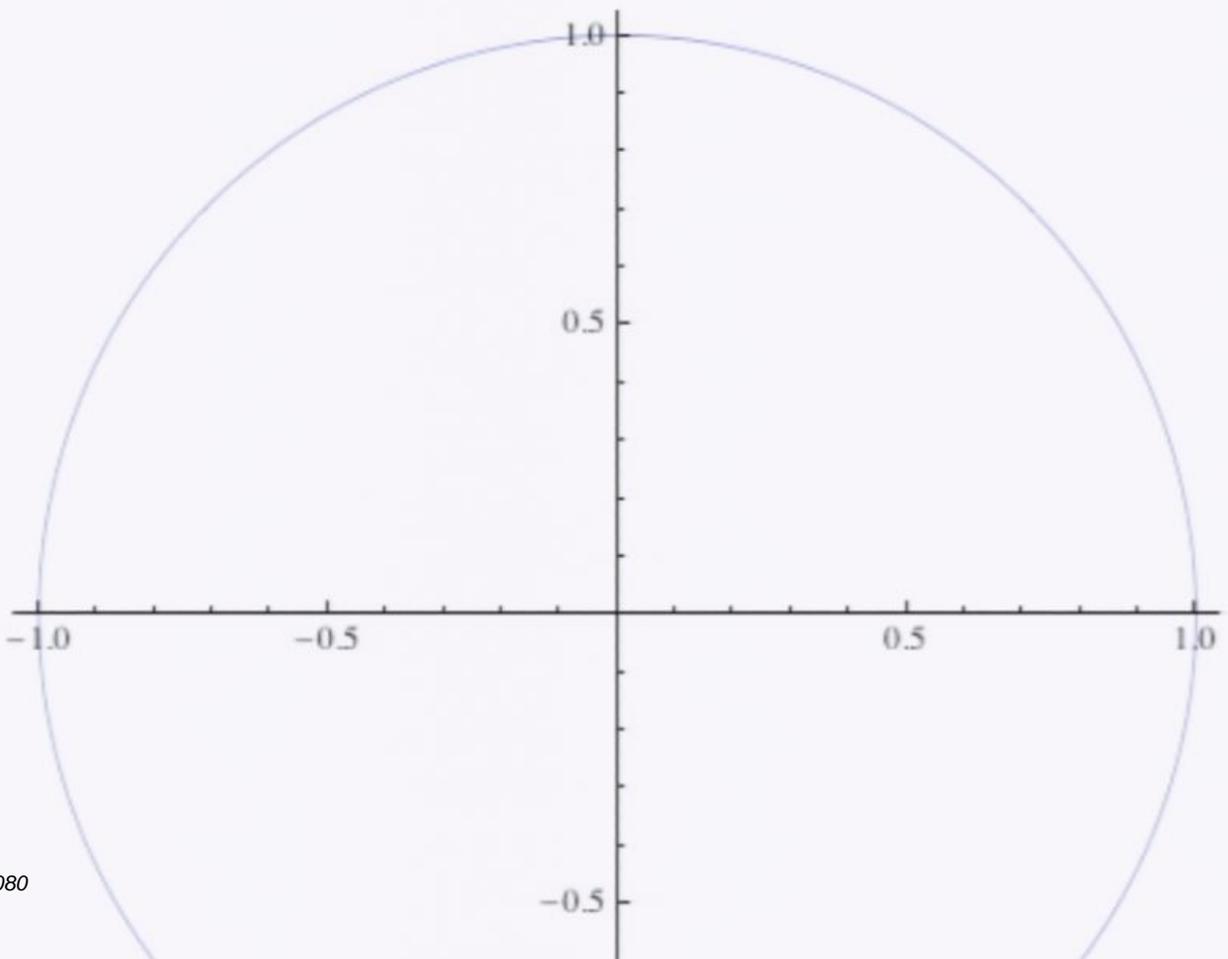
```
Out[1034]= {5, 6}
```

```
In[1038]:= ParametricPlot[{Re[F[10/8 (-1 + I + ei φ)]], Im[F[10/8 (-10 + ei φ)]]},  
{φ, 0, 2 π}]
```



In[1034]:= {5, 6}

In[1041]:= ParametricPlot[{Re[e^{I φ}], Im[e^{I φ}]}, {φ, 0, 2 π}]



In[1041]=

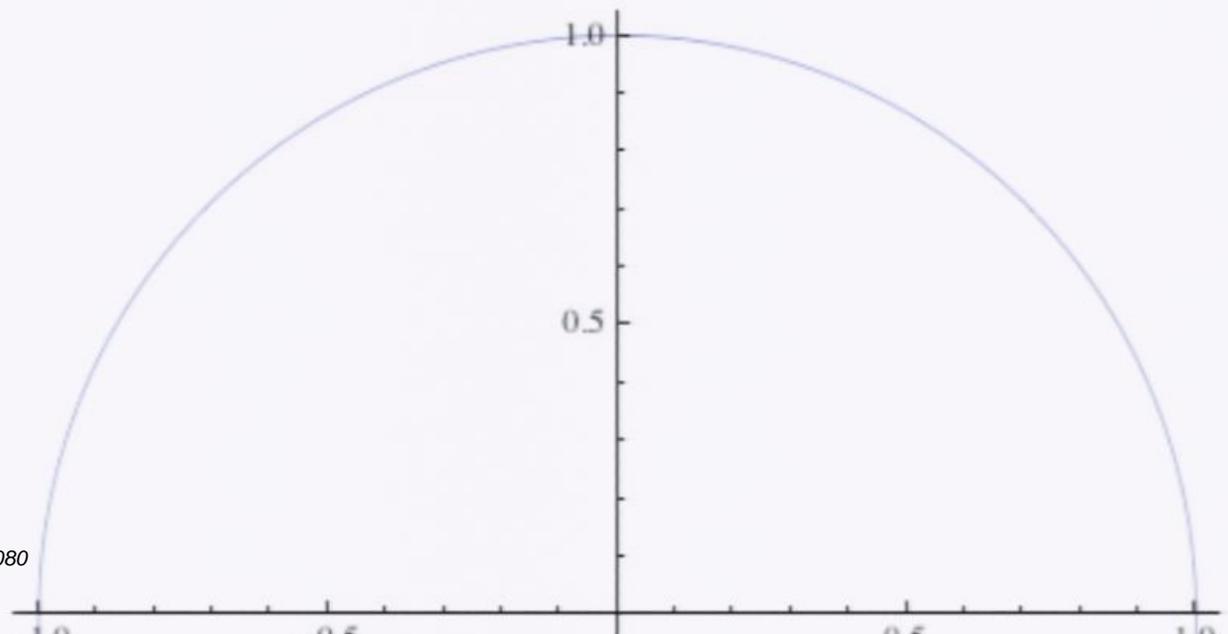
(* human square roots *)

We want a function to map $5+i 6$ to $\{5,6\}$

```
[1033]:= H[z_] = {Re[z], Im[z]};  
H[5 + I 6]
```

```
[1034]= {5, 6}
```

```
[1041]:= ParametricPlot[{Re[eI φ], Im[eI φ]}, {φ, 0, 2 π}]
```

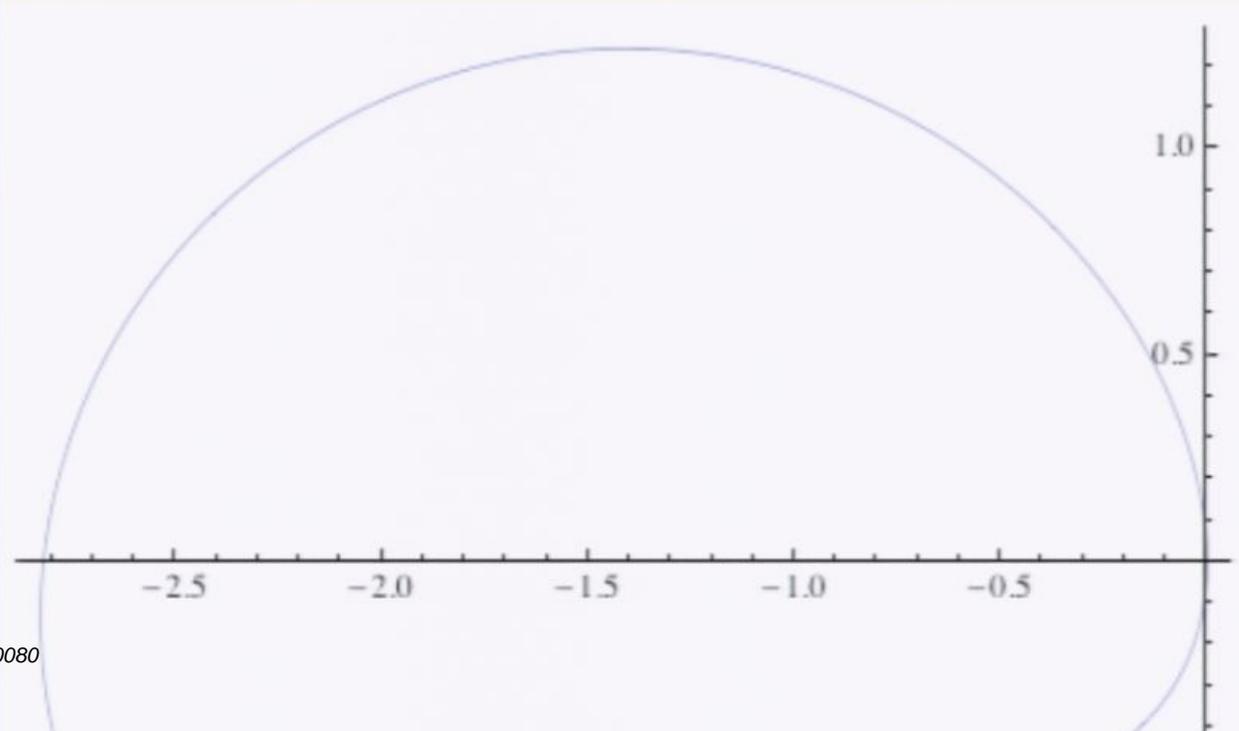


```
[1041]=
```

in[1034]= {5, 6}

ParametricPlot[{Re[e^{I φ}], Im[e^{I φ}]}, {φ, 0, 2 π}]

in[1038]= ParametricPlot[{Re[F[10 / 8 (-1 + I + e^{i φ})]], Im[F[10 / 8 (-10 + e^{i φ})]]}], {φ, 0, 2 π}]



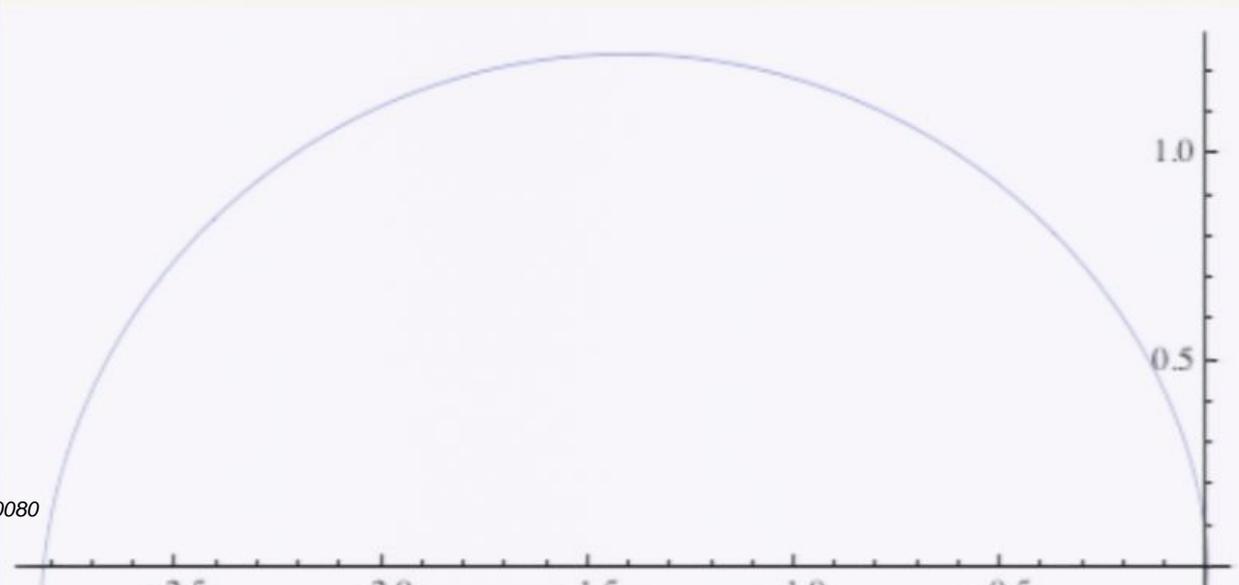
in[1038]=

in[1034]= {5, 6}

```
ParametricPlot[{Re[eI φ], Im[eI φ]}, {φ, 0, 2 π}];
```

```
ParametricPlot[{{Re[eI φ], Im[eI φ]}, {φ, 0, 2 π}]
```

in[1038]= ParametricPlot[{Re[F[10 / 8 (-1 + I + e^{i φ})]], Im[F[10 / 8 (-10 + e^{i φ})]]}, {φ, 0, 2 π}]



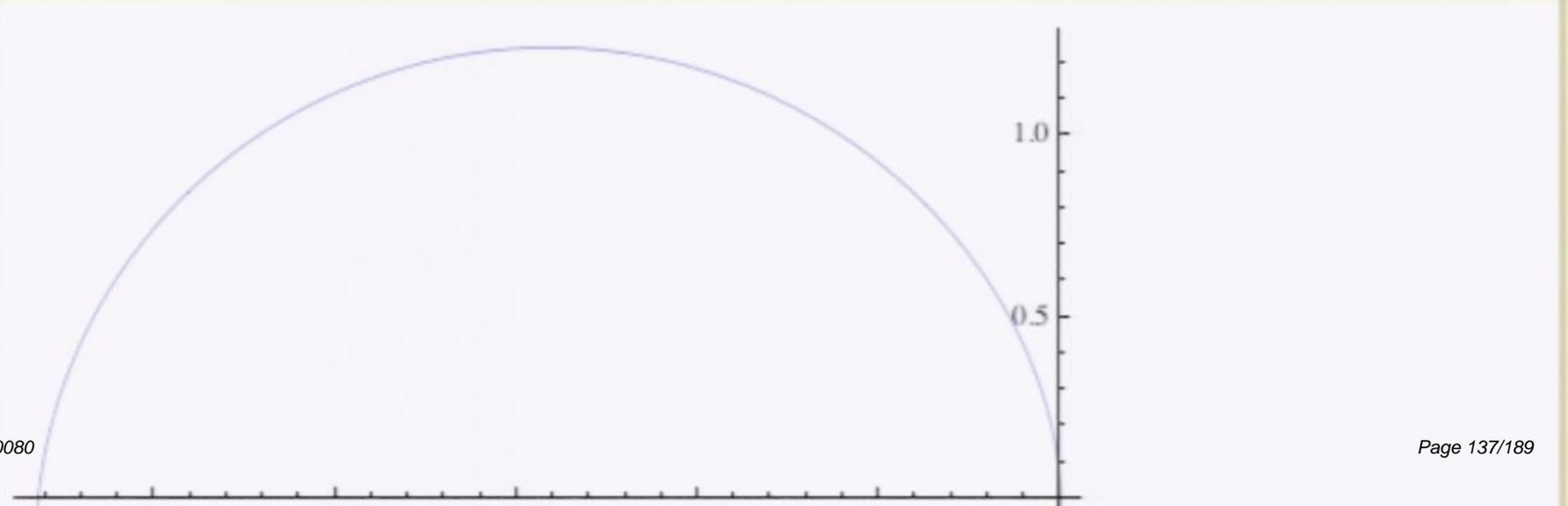
in[1038]=

it[1034]= {5, 6}

```
ParametricPlot[{Re[eI φ], Im[eI φ]}, {φ, 0, 2 π}];
```

```
ParametricPlot[H[eI φ], {φ, 0, 2 π}]
```

it[1038]= ParametricPlot[{Re[F[10 / 8 (-1 + I + e^{i φ})]], Im[F[10 / 8 (-10 + e^{i φ})]]}, {φ, 0, 2 π}]



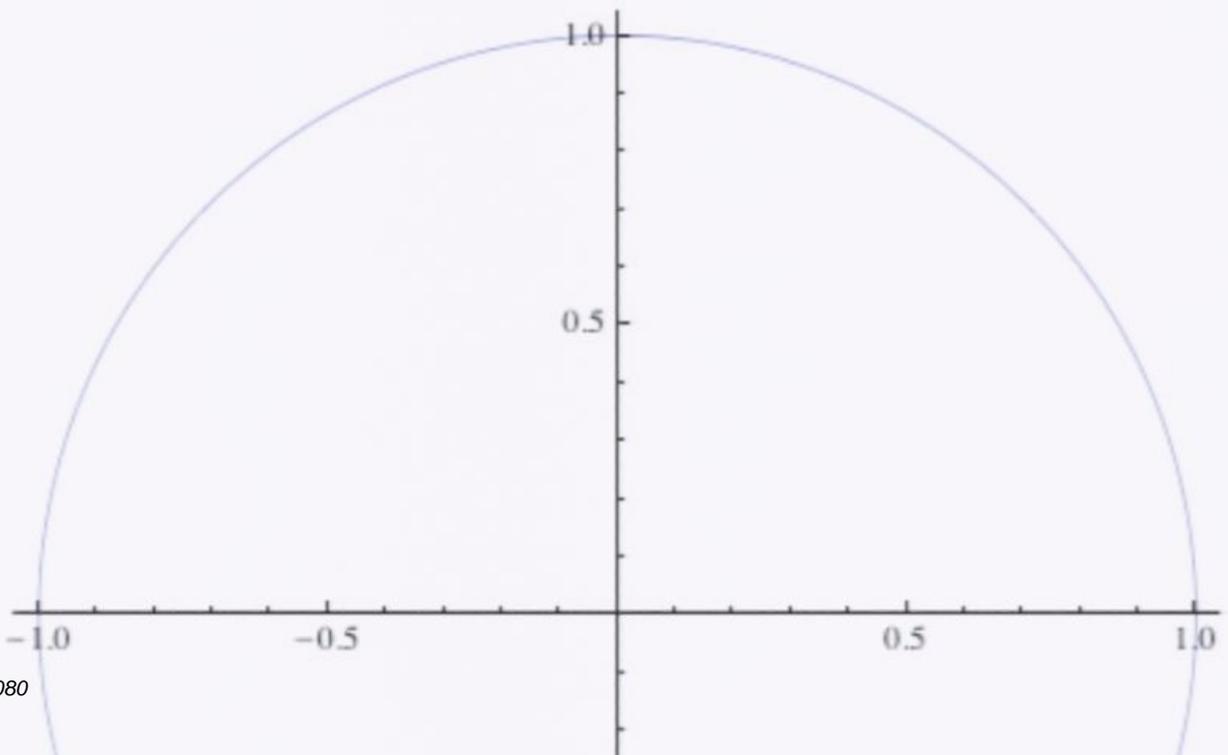
it[1038]=

```
In[1034]:= H[5 + I 6]
```

{5, 6}

```
ParametricPlot[{Re[eI φ], Im[eI φ]}, {φ, 0, 2 π}];
```

```
In[1042]:= ParametricPlot[H[eI φ], {φ, 0, 2 π}]
```

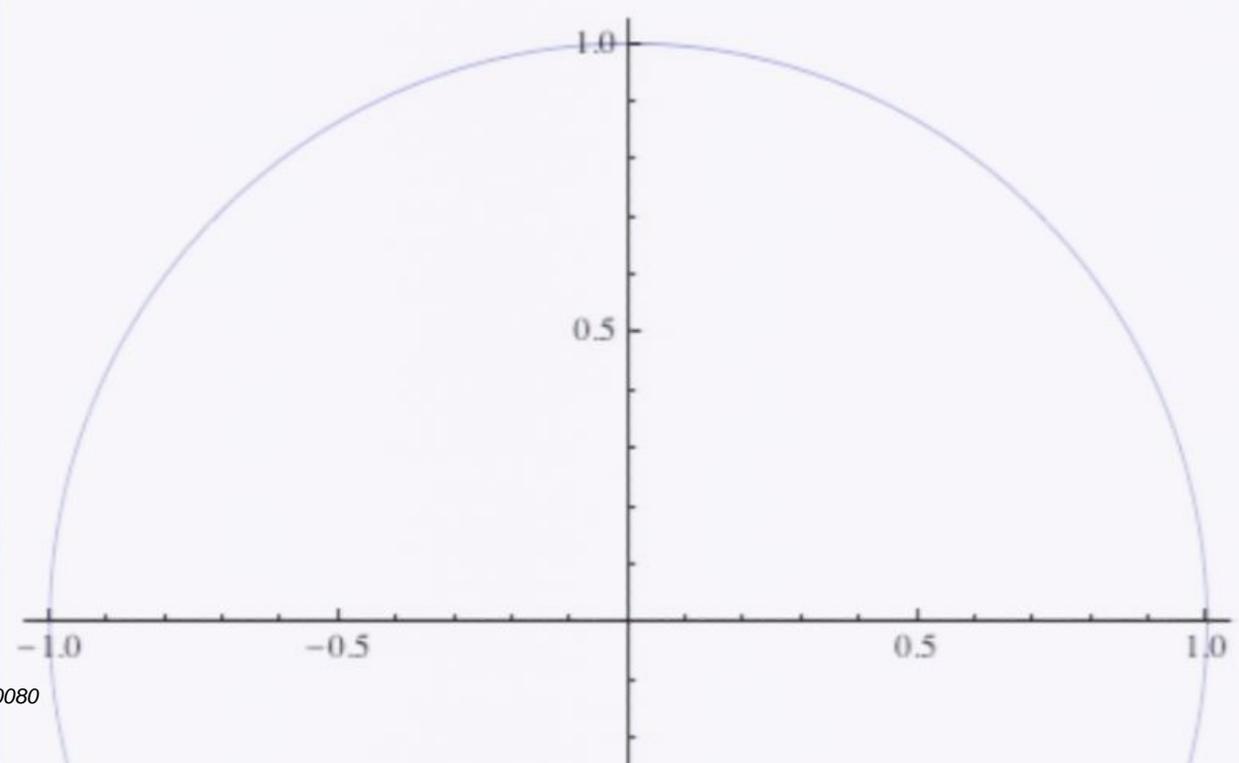


`H[5 + I 6]`

`In[1034]:=` `{5, 6}`

`ParametricPlot[{Re[eI φ], Im[eI φ]}, {φ, 0, 2 π}];`

`In[1042]:=` `ParametricPlot[H[eI φ], {φ, 0, 2 π}]`



`In[1042]=`

```
ParametricPlot[{Re[eI φ], Im[eI φ]}, {φ, 0, 2 π}];
```

```
ParametricPlot[H[eI φ], {φ, 0, 2 π}];
```

Another notation is

```
H = {Re[#], Im[#]} &
```

```
[1038]:= ParametricPlot[{Re[F[10 / 8 (-1 + I + ei φ)]], Im[F[10 / 8 (-10 + ei φ)]]}, {φ, 0, 2 π}]
```



```
ParametricPlot[{Re[eI φ], Im[eI φ]}, {φ, 0, 2 π}];
```

```
ParametricPlot[H[eI φ], {φ, 0, 2 π}];
```

Another notation is

```
[1044]:= H = {Re[#], Im[#]} &;
```

```
[1038]:= ParametricPlot[{Re[F[10 / 8 (-1 + I + ei φ)]], Im[F[10 / 8 (-10 + ei φ)]]},  
{φ, 0, 2 π}]
```



```
ParametricPlot[{Re[eI φ], Im[eI φ]}, {φ, 0, 2 π}];
```

```
ParametricPlot[H[eI φ], {φ, 0, 2 π}];
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Another notation is

```
[1044]:= H = {Re[#], Im[#]} &;
```

```
{Re[#], Im[#]} &|
```

```
[1038]:= ParametricPlot[{Re[F[10 / 8 (-1 + I + ei φ)]], Im[F[10 / 8 (-10 + ei φ)]]},  
{φ, 0, 2 π}]
```



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ParametricPlot[{Re[eI φ], Im[eI φ]}, {φ, 0, 2 π}];
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Another notation is

```
[1044]:= H = {Re[#], Im[#]} &;
```

```
[1045]:= {Re[#], Im[#]} &[5 + 6 I]
```

```
Out[1045]= {5, 6}
```

```
[1038]:= ParametricPlot[{Re[F[10 / 8 (-1 + I + ei φ)]], Im[F[10 / 8 (-10 + ei φ)]]}, {φ, 0, 2 π}]
```



```
ParametricPlot[{Re[eI φ], Im[eI φ]}, {φ, 0, 2 π}];
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ParametricPlot[H[eI φ], {φ, 0, 2 π}];
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[1044]:= H = {Re[#], Im[#]} &;
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[1038]:= ParametricPlot[{Re[F[10 / 8 (-1 + I + ei φ)]], Im[F[10 / 8 (-10 + ei φ)]]}, {φ, 0, 2 π}]
```



```
{1038}:= ParametricPlot[{Re[F[10/8 (-1 + I + ei φ)]], Im[F[10/8 (-10 + ei φ)]]},  
  {φ, 0, 2 π}]
```

```
ParametricPlot[{Re[F[10/8 (-1 + I + ei φ)]], Im[F[10/8 (-10 + ei φ)]]},  
  {φ, 0, 2 π}]
```

```
In[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r} (-p + e^{i \phi})$ ],  
  {φ, 0, 2 π}, PlotStyle → {Thickness[0.015], Red}]
```

■ Airplane wings

```
{1038}:= ParametricPlot[{Re[F[10/8 (-1 + I + ei φ)]], Im[F[10/8 (-10 + ei φ)]]},  
  {φ, 0, 2 π}]
```

```
ParametricPlot[{Re[#], Im[F[10/8 (-10 + ei φ)]]}, {φ, 0, 2 π}]
```

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In[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r}$  (-p + ei φ)],  
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{1038}:= ParametricPlot[{Re[F[10/8 (-1 + I + ei φ)]], Im[F[10/8 (-10 + ei φ)]]},  
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  {φ, 0, 2 π}]
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In[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} & @ F[ $\frac{1}{r}$  (-p + ei φ)],  
  {φ, 0, 2 π}, PlotStyle → {Thickness[0.015], Red}]
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■ Airplane wings

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{1038}:= ParametricPlot[{Re[F[10/8 (-1 + I + ei φ)]], Im[F[10/8 (-10 + ei φ)]]},  
  {φ, 0, 2 π}]
```

```
ParametricPlot[{Re[#], Im[#]} & [F[10/8 (-10 + ei φ)]], {φ, 0, 2 π}]
```

```
in[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} & @ F[ $\frac{1}{r}$  (-p + ei φ)],  
  {φ, 0, 2 π}, PlotStyle → {Thickness[0.015], Red}]
```

■ Airplane wings

```
ParametricPlot[H[ei φ], {φ, 0, 2 π}];
```

Another notation is

```
(1044):= H = {Re[#], Im[#]} &;
```

```
(1045):= {Re[#], Im[#]} &[5 + 6 I]
```

```
(1045):= {5, 6}
```

```
(1038):= ParametricPlot[{Re[F[10/8 (-1 + I + ei φ)]], Im[F[10/8 (-10 + ei φ)]]}, {φ, 0, 2 π}]
```

```
ParametricPlot[{Re[#], Im[#]} &[F[10/8 (-10 + ei φ)]], {φ, 0, 2 π}]
```

```
(990):= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[1/r (-p + ei φ)], {φ, 0, 2 π}, PlotStyle -> {Thickness[0.015], Red}]
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ParametricPlot[H[ei φ], {φ, 0, 2 π}];
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Another notation is

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[1044]:= H = {Re[#], Im[#]} &;
```

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[1045]:= {Re[#], Im[#]} &[5 + 6 I]
```

```
Out[1045]= {5, 6}
```

```
[1038]:= ParametricPlot[{Re[F[10/8 (-1 + I + ei φ)]], Im[F[10/8 (-10 + ei φ)]]}, {φ, 0, 2 π}]
```

```
ParametricPlot[{Re[#], Im[#]} &[F[10/8 (-10 + ei φ)]], {φ, 0, 2 π}]
```

```
ParametricPlot[H[F[10/8 (-10 + ei φ)]], {φ, 0, 2 π}]
```

```
In[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[1/r (-p + ei φ)],
```

```
{φ, 0, 2 π}, PlotStyle -> {Thickness[0.015], Red}]
```

```
ParametricPlot[H[e^{i \phi}], {\phi, 0, 2 \pi}];
```

Another notation is

```
[1044]:= H = {Re[#], Im[#]} &;
```

```
[1045]:= {Re[#], Im[#]} &[5 + 6 I]
```

```
[1045]:= {5, 6}
```

```
[1038]:= ParametricPlot[{Re[F[10/8 (-1 + I + e^{i \phi})]], Im[F[10/8 (-10 + e^{i \phi})]]}, {\phi, 0, 2 \pi}]
```

```
ParametricPlot[{Re[#], Im[#]} &[F[10/8 (-10 + e^{i \phi})]], {\phi, 0, 2 \pi}]
```

```
ParametricPlot[H[F[10/8 (-10 + e^{i \phi})]], {\phi, 0, 2 \pi}]
```

```
[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[1/r (-p + e^{i \phi})],
```

```
{\phi, 0, 2 \pi}, PlotStyle -> {Thickness[0.015], Red}]
```

```
ParametricPlot[H[ei φ], {φ, 0, 2 π}];
```

Another notation is

```
[1044]:= H = {Re[#], Im[#]} &;
```

```
[1046]:= {Re[#], Im[#]} & [5 + 6 I]
```

```
Out[1046]= {5, 6}
```

```
#2 &
```

```
[1038]:= ParametricPlot[{Re[F[10/8 (-1 + I + ei φ)]], Im[F[10/8 (-10 + ei φ)]]}, {φ, 0, 2 π}]
```

```
ParametricPlot[{Re[#], Im[#]} & [F[10/8 (-10 + ei φ)]], {φ, 0, 2 π}]
```

```
ParametricPlot[H[F[10/8 (-10 + ei φ)]], {φ, 0, 2 π}]
```

Another notation is

```
[1044]:= H = {Re[#], Im[#]} &;
```

```
[1046]:= {Re[#], Im[#]} &[5 + 6 I]
```

```
Out[1046]= {5, 6}
```

```
[1048]:= #^2 &[5]  
Sin[5]
```

```
Out[1048]= 25
```

```
Out[1049]= Sin[5]
```

```
[1050]:= Sin@5
```

```
Out[1050]= Sin[5]
```

```
[1044]:= H = {Re[#], Im[#]} &;
```

```
[1046]:= {Re[#], Im[#]} &[5 + 6 I]
```

```
Out[1046]= {5, 6}
```

```
[1048]:= #^2 &[5]  
Sin[5]
```

```
Out[1048]= 25
```

```
Out[1049]= Sin[5]
```

```

$$\frac{1}{(x + I y)^2} // \text{Re} // \text{Com}$$

```

```
Out[1051]= 
$$\text{Re} \left[ \frac{1}{(x + i y)^2} \right]$$

```

In[1046]:= {Re[#], Im[#]} &[5 + 6 I]

Out[1046]= {5, 6}

In[1048]:= #^2 &[5]
Sin[5]

Out[1048]= 25

In[1049]= Sin[5]

In[1053]:= $\frac{1}{(x + I y)^2}$ // Re // ComplexExpand // Simplify

Out[1053]= $\frac{x^2 - y^2}{(x^2 + y^2)^2}$

In[1053]:= $\frac{1}{(x + I y)^2}$ // Re // ComplexExpand // Simplify

In[1048]:= 25

In[1049]:= Sin[5]

In[1053]:= $\frac{1}{(x + I y)^2}$ // Re // ComplexExpand // Simplify

In[1053]:= $\frac{x^2 - y^2}{(x^2 + y^2)^2}$

In[1054]:= Simplify[ComplexExpand[Re[$\frac{1}{(x + I y)^2}$]]]

In[1054]:= $\frac{x^2 - y^2}{(x^2 + y^2)^2}$

Simplify@

```
[1053]:=  $\frac{1}{(x + I y)^2}$  // Re // ComplexExpand // Simplify
```

```
Out[1053]=  $\frac{x^2 - y^2}{(x^2 + y^2)^2}$ 
```

```
[1054]:= Simplify[ComplexExpand[Re[ $\frac{1}{(x + I y)^2}$ ]]]
```

```
Out[1054]=  $\frac{x^2 - y^2}{(x^2 + y^2)^2}$ 
```

```
[1055]:= Simplify@ComplexExpand@Re@ $\frac{1}{(x + I y)^2}$ 
```

```
Out[1055]=  $\frac{x^2 - y^2}{(x^2 + y^2)^2}$ 
```

Out[1054]=
$$\frac{x^2 - y^2}{(x^2 + y^2)^2}$$

In[1055]:= `Simplify@ComplexExpand@Re@`
$$\frac{1}{(x + I y)^2}$$

Out[1055]=
$$\frac{x^2 - y^2}{(x^2 + y^2)^2}$$

In[1050]:= `Sin@5`

Out[1050]= `Sin[5]`

In[1038]:= `ParametricPlot[{Re[F[10/8 (-1 + I + ei φ)]], Im[F[10/8 (-10 + ei φ)]]}, {φ, 0, 2 π}]`

`ParametricPlot[{Re[#], Im[#]} & [F[10/8 (-10 + ei φ)]], {φ, 0, 2 π}]`

`[1055]:= Simplify@ComplexExpand@Re@` $\frac{1}{(x + I y)^2}$

`Out[1055]=` $\frac{x^2 - y^2}{(x^2 + y^2)^2}$

`[1050]:= Sin@5`

`Out[1050]= Sin[5]`

`[1038]:= ParametricPlot[{Re[F[10/8 (-1 + I + ei φ)]], Im[F[10/8 (-10 + ei φ)]]}, {φ, 0, 2 π}]`

`ParametricPlot[{Re[#], Im[#]} & [F[10/8 (-10 + ei φ)]], {φ, 0, 2 π}]`

`ParametricPlot[{Re[#], Im[#]} & @F[10/8 (-10 + ei φ)], {φ, 0, 2 π}]`

In[1050]:= Sin@5

Out[1050]:= Sin[5]

In[1038]:= ParametricPlot[{Re[F[10/8(-1 + I + e^{iϕ})]], Im[F[10/8(-10 + e^{iϕ})]]}, {ϕ, 0, 2π}]

ParametricPlot[{Re[#], Im[#]} & [F[10/8(-10 + e^{iϕ})]], {ϕ, 0, 2π}]

ParametricPlot[{Re[#], Im[#]} & @F[10/8(-10 + e^{iϕ})], {ϕ, 0, 2π}]

ParametricPlot[H[F[10/8(-10 + e^{iϕ})]], {ϕ, 0, 2π}]

In[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} & @F[$\frac{1}{r}(-p + e^{i\phi})$], {ϕ, 0, 2π}, PlotStyle → {Thickness[0.015], Red}]

In[1050]:= Sin[5]

In[1038]:= ParametricPlot[{Re[F[10/8 (-1 + I + e^{i φ})]], Im[F[10/8 (-10 + e^{i φ})]]}, {φ, 0, 2 π}]

ParametricPlot[{Re[#], Im[#]} & [F[10/8 (-10 + e^{i φ})]], {φ, 0, 2 π}]

ParametricPlot[{Re[#], Im[#]} & @F[10/8 (-10 + e^{i φ})], {φ, 0, 2 π}]

ParametricPlot[H[F[10/8 (-10 + e^{i φ})]], {φ, 0, 2 π}]

In[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} & @F[$\frac{1}{r} (-p + e^{i \phi})$], {φ, 0, 2 π}, PlotStyle → {Thickness[0.015], Red}]

wing[

```
ParametricPlot[H[F[10/8 (-10 + ei φ)]], {φ, 0, 2 π}]
```

}

```
in[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r} (-p + e^{i \phi})$ ],
    {φ, 0, 2 π}, PlotStyle → {Thickness[0.015], Red}]
```

```
wing[
```

■ Airplane wings

```
in[992]:= F[z_] = z +  $\frac{1}{z}$ ; iF[z_] =  $\frac{z + \sqrt{z-2} \sqrt{z+2}}{2}$ ;
```

```
in[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r} (-p + e^{i \phi})$ ],
    {φ, 0, 2 π}, PlotStyle → {Thickness[0.015], Red}]
```

```
in[991]:= Manipulate[du = Re@{∂x #, ∂y #} &@F[a + I b + r iF[x + I y]];
    sp = StreamPlot[du, {x, -4, 4}, {y, -2, 2}];
```

```
ParametricPlot[H[F[10/8 (-10 + ei φ)]], {φ, 0, 2 π}]
```

```
in[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r} (-p + e^{i \phi})$ ],  
{φ, 0, 2 π}, PlotStyle → {Thickness[0.015], Red}]
```

```
wing[.2 + I .1]
```

■ Airplane wings

```
in[992]:= F[z_] = z +  $\frac{1}{z}$ ; iF[z_] =  $\frac{z + \sqrt{z - 2} \sqrt{z + 2}}{2}$ ;
```

```
in[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r} (-p + e^{i \phi})$ ],  
{φ, 0, 2 π}, PlotStyle → {Thickness[0.015], Red}]
```

```
in[991]:= Manipulate[du = Re@{∂x #, ∂y #} &@F[a + I b + r iF[x + I y]];  
sp = StreamPlot[du, {x, -4, 4}, {y, -3, 3}];  
Show[sp, wing[a + I b, r], PlotRange → All, AspectRatio → Automatic],
```

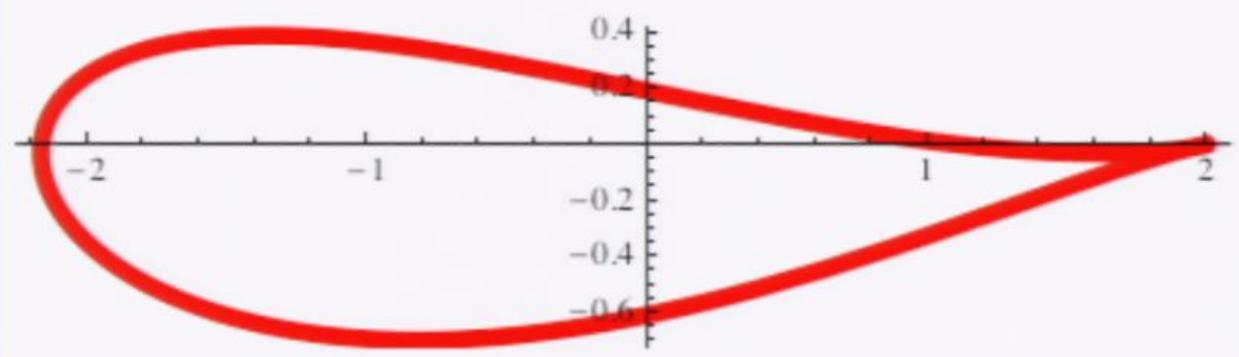
```
ParametricPlot[H[F[10/8 (-10 + eiϕ)]], {ϕ, 0, 2π}]
```

```
in[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[1/r (-p + eiϕ)],  

    {ϕ, 0, 2π}, PlotStyle -> {Thickness[0.015], Red}]
```

```
in[1056]:= wing[.2 + I .1, .8]
```

out[1056]=



■ Airplane wings

```
in[992]:= F[z_] = z + 1/z; iF[z_] = (z + sqrt(z-2) sqrt(z+2))/2;
```

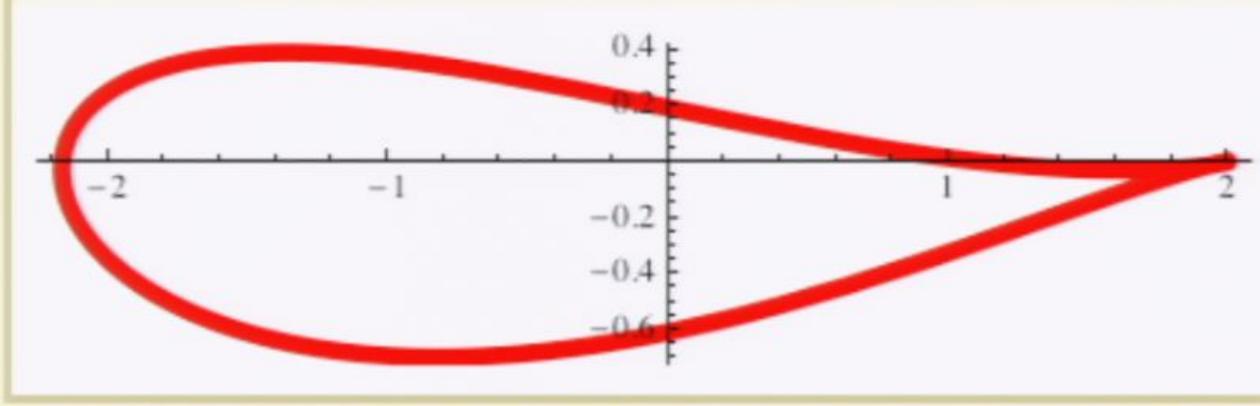
```
ParametricPlot[{Re[#], Im[#]} &@F[10/8 (-10 + ei φ)], {φ, 0, 2 π}]
```

```
ParametricPlot[H[F[10/8 (-10 + ei φ)]], {φ, 0, 2 π}]
```

```
in[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r} (-p + e^{i \phi})$ ],  
{φ, 0, 2 π}, PlotStyle → {Thickness[0.015], Red}]
```

```
out[1056]:= wing[.2 + I .1, .8]
```

out[1056]=



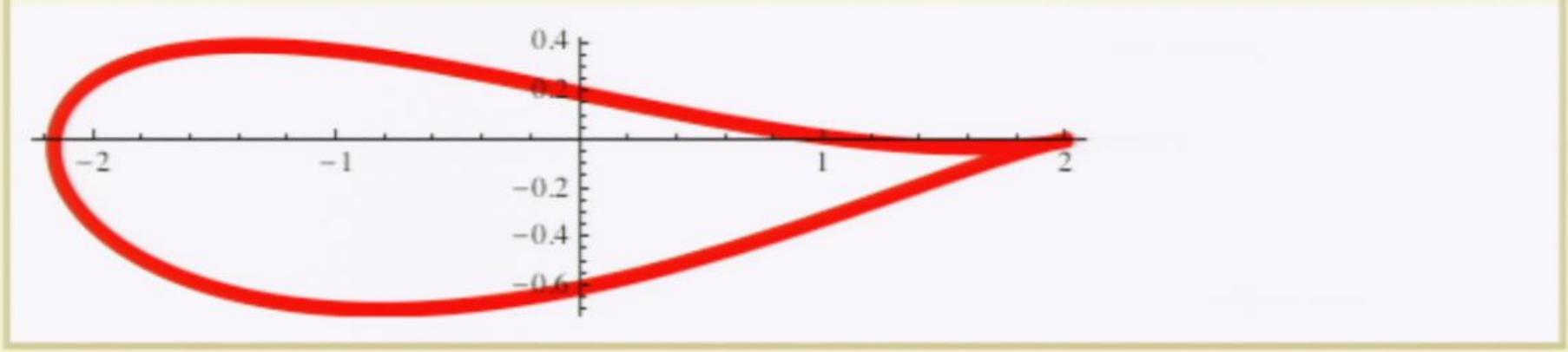
```
ParametricPlot[{Re[#], Im[#]} &@F[10/8 (-10 + ei φ)], {φ, 0, 2 π}]
```

```
ParametricPlot[H[F[10/8 (-10 + ei φ)]], {φ, 0, 2 π}]
```

```
In[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r} (-p + e^{i \phi})$ ],  
{φ, 0, 2 π}, PlotStyle → {Thickness[0.015], Red}]
```

```
Out[1056]:= wing[.2 + I .1, .8]
```

Out[1056]=



```
(1038):= ParametricPlot[{Re[F[10/8 (-1 + I + ei φ)]], Im[F[10/8 (-10 + ei φ)]]},
{φ, 0, 2 π}]
```

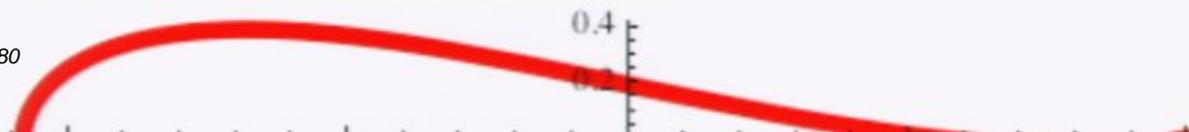
```
ParametricPlot[{Re[#], Im[#]} & [F[10/8 (-10 + ei φ)]], {φ, 0, 2 π}]
```

```
ParametricPlot[{Re[#], Im[#]} & @F[10/8 (-10 + ei φ)], {φ, 0, 2 π}]
```

```
ParametricPlot[H[F[10/8 (-10 + ei φ)]], {φ, 0, 2 π}]
```

```
(990):= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} & @F[ $\frac{1}{r} (-p + e^{i \phi})$ ],
{φ, 0, 2 π}, PlotStyle → {Thickness[0.015], Red}]
```

```
(1056):= wing[.2 + I .1, .8]
```



- New version
- New version 2
- Airplane wings

```
In[992]:= F[z_] = z + 1/z; iF[z_] = (z + Sqrt[z - 2] Sqrt[z + 2])/2;
```

```
In[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[1/r (-p + e^i phi)],
    {phi, 0, 2 pi}, PlotStyle -> {Thickness[0.015], Red}]
```

```
In[991]:= Manipulate[du = Re@{Dx#, Dy#} &@F[a + I b + r iF[x + I y]];
    sp = StreamPlot[du, {x, -4, 4}, {y, -3, 3}];
    Show[sp, wing[a + I b, r], PlotRange -> All, AspectRatio -> Automatic],
    {a, .2, .4}, {b, -.2, .3}, {r, .5, .9}]
```

- Small subtlety

- New version
- New version 2
- Airplane wings

```
in[992]:= F[z_] = z + 1/z; iF[z_] = (z + Sqrt[z - 2] Sqrt[z + 2])/2;
```

```
in[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[1/r (-p + e^i phi)],
    {phi, 0, 2 pi}, PlotStyle -> {Thickness[0.015], Red}]
```

```
in[991]:= Manipulate[du = Re@{Dx#, Dy#} &@F[a + I b + r iF[x + I y]];
    sp = StreamPlot[du, {x, -4, 4}, {y, -3, 3}];
    Show[sp, wing[a + I b, r], PlotRange -> All, AspectRatio -> Automatic],
    {a, .2, .4}, {b, -.2, .3}, {r, .5, .9}]
```

- Small subtlety

- New version
- New version 2
- Airplane wings

```
in[992]:= F[z_] = z + 1/z; iF[z_] = (z + Sqrt[z - 2] Sqrt[z + 2])/2;
```

```
in[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[1/r (-p + e^i phi)],
  {phi, 0, 2 pi}, PlotStyle -> {Thickness[0.015], Red}]
```

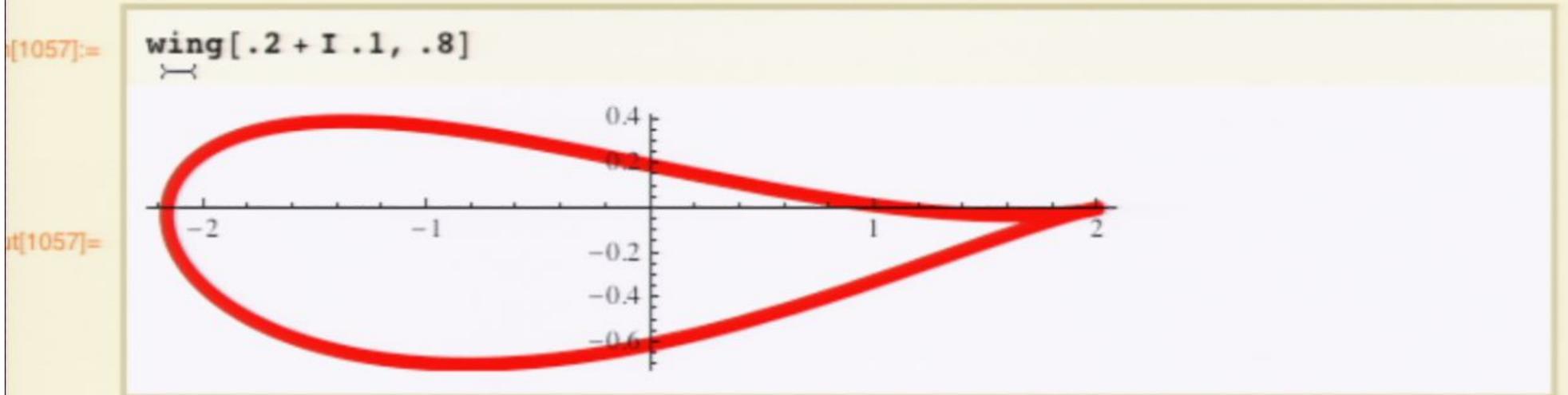
```
wing[.2 + I .1, .8]
```

```
in[991]:= Manipulate[du = Re@{Dx#, Dy#} &@F[a + I b + r iF[x + I y]];
  sp = StreamPlot[du, {x, -4, 4}, {y, -3, 3}];
  Show[sp, wing[a + I b, r], PlotRange -> All, AspectRatio -> Automatic],
  {a, .2, .4}, {b, -.2, .3}, {r, .5, .9}]
```

- New version 2
- Airplane wings

```
In[992]:= F[z_] = z + 1/z; iF[z_] = (z + Sqrt[z - 2] Sqrt[z + 2])/2;
```

```
In[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[1/r (-p + e^i phi)],  
  {phi, 0, 2 pi}, PlotStyle -> {Thickness[0.015], Red}]
```

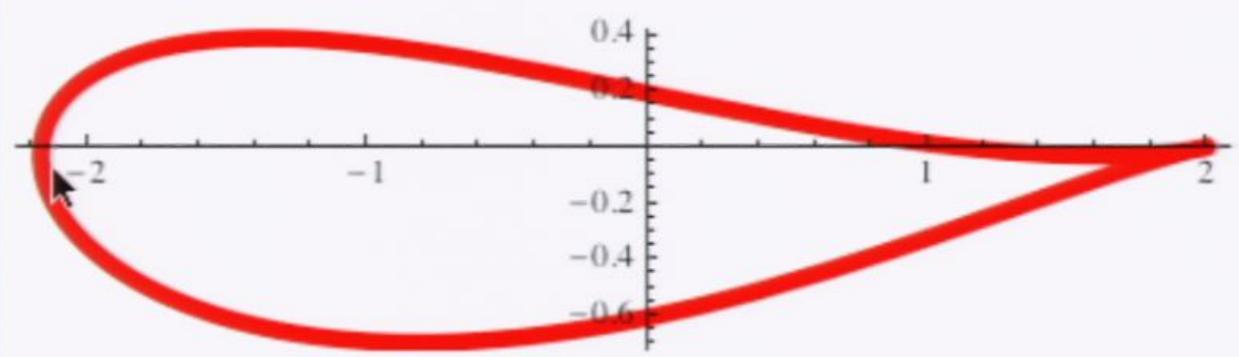


```
In[992]:= F[z_] = z + 1/z; iF[z_] = (z + Sqrt[z - 2] Sqrt[z + 2])/2;
```

```
In[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[1/r (-p + e^i phi)],
{phi, 0, 2 pi}, PlotStyle -> {Thickness[0.015], Red}]
```

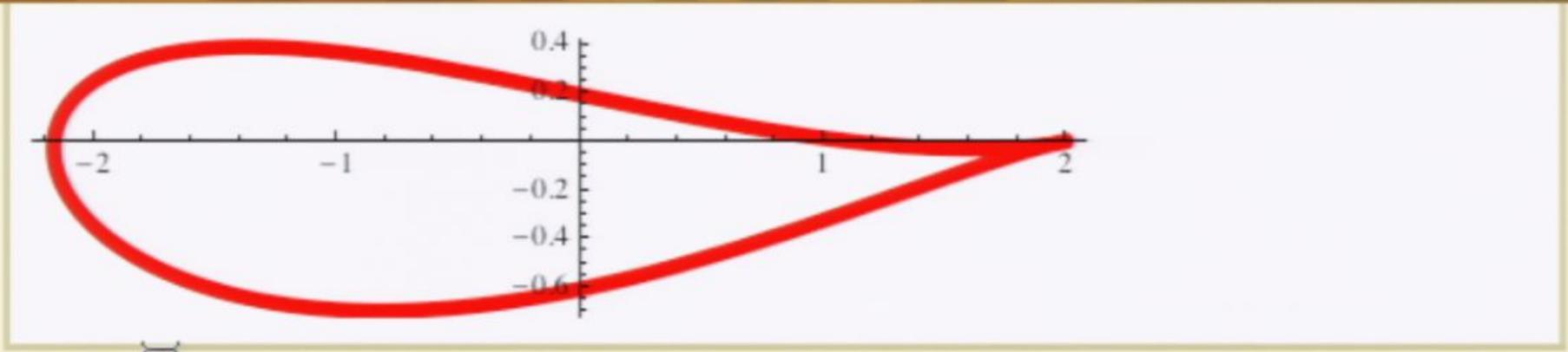
```
In[1057]:= wing[.2 + I .1, .8]
```

Out[1057]=



```
In[991]:= Manipulate[du = Re@{Dx#, Dy#} &@F[a + I b + r iF[x + I y]];
sp = StreamPlot[du, {x, -4, 4}, {y, -3, 3}];
Show[sp, wing[a + I b, r], PlotRange -> All, AspectRatio -> Automatic],
{a, .2, .4}, {b, -.2, .3}, {r, .5, .9}]
```

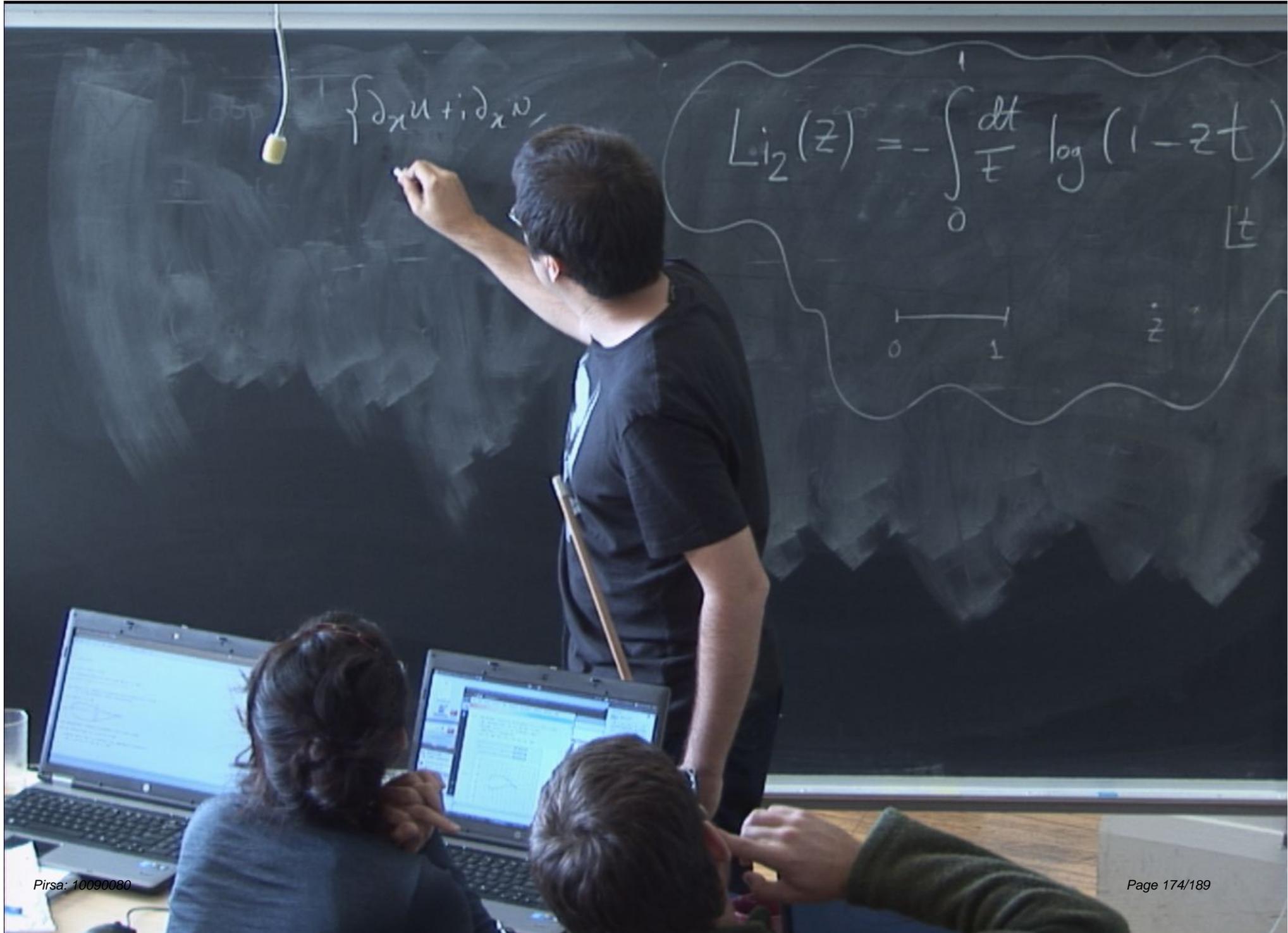
in[1057]=



in[991]=

```
Manipulate[du = Re@{∂x #, ∂y #} &@F[a + I b + r iF[x + I y]];  
sp = StreamPlot[du, {x, -4, 4}, {y, -3, 3}];  
Show[sp, wing[a + I b, r], PlotRange → All, AspectRatio → Automatic],  
{a, .2, .4}, {b, -.2, .3}, {r, .5, .9}]
```

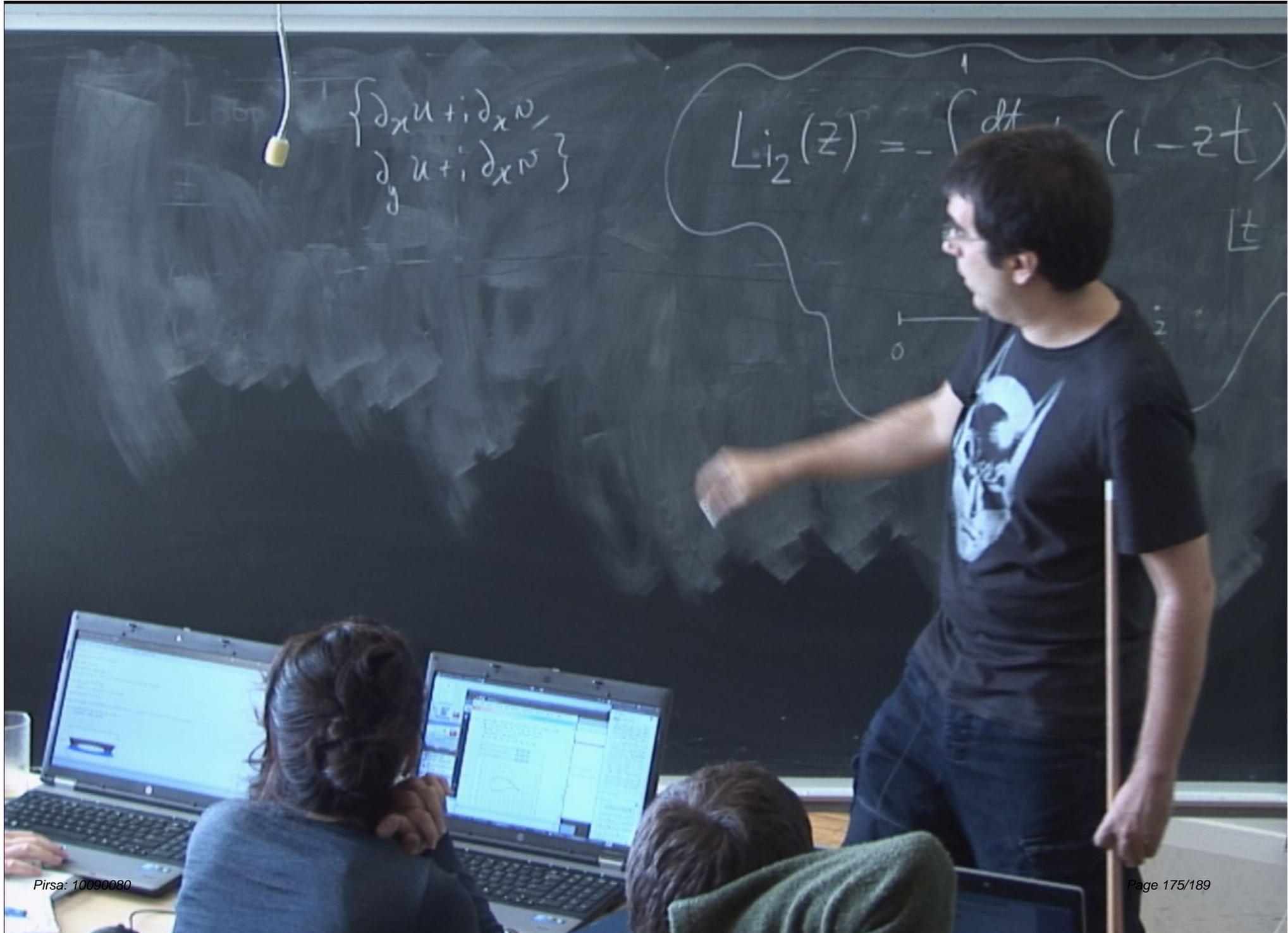
▣ Small subtlety



$$\{ \partial_x u + i \partial_x v,$$

$$Li_2(z) = - \int_0^1 \frac{dt}{t} \log(1-zt)$$

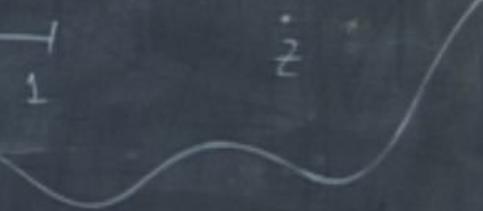




$$\text{Re} \left\{ \begin{array}{l} \partial_x u + i \partial_x v \\ \partial_y u + i \partial_y v \end{array} \right\}$$

real functions

$$\log(z) = - \int_0^1 \frac{dt}{t} \log(1 - zt)$$

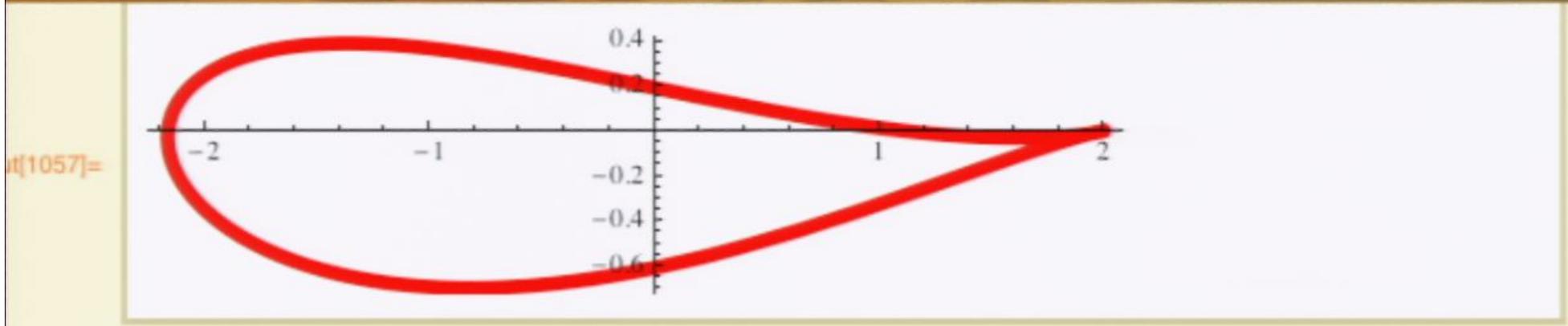


$\text{Re} \left\{ \begin{array}{l} \partial_x u + i \partial_x v \\ \partial_y u + i \partial_y v \end{array} \right\}$
 \parallel
 $\left\{ \partial_x u, \partial_y u \right\}$

real functions

$$\text{Li}_2(z) = - \int_0^1 \frac{dt}{t} \log(1-zt)$$



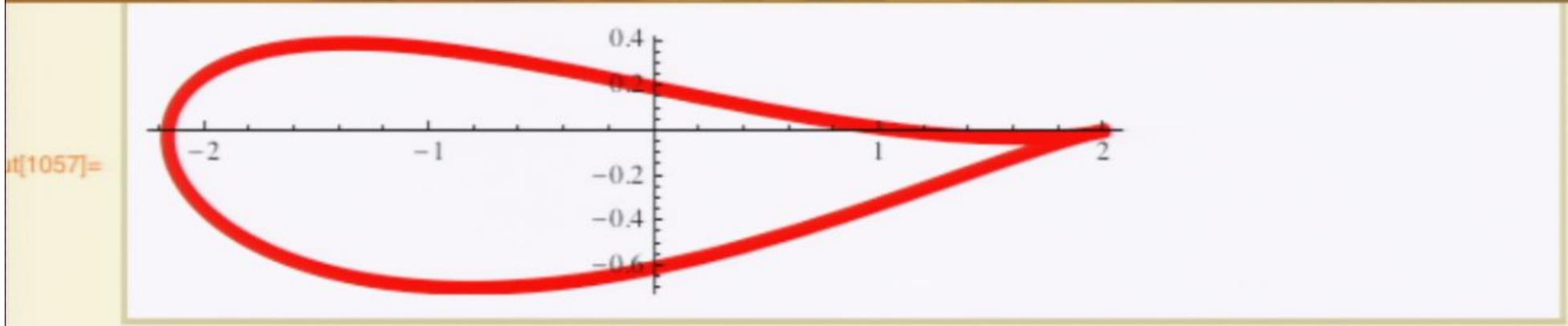


in[1058]:= `D[Re[x], x]`

out[1058]= `Re'[x]`

in[991]:= `Manipulate[du = Re@{∂x#, ∂y#} &@F[a + I b + r iF[x + I y]];
 sp = StreamPlot[du, {x, -4, 4}, {y, -3, 3}];
 Show[sp, wing[a + I b, r], PlotRange → All, AspectRatio → Automatic],
 {a, .2, .4}, {b, -.2, .3}, {r, .5, .9}]`

▣ Small subtlety



```

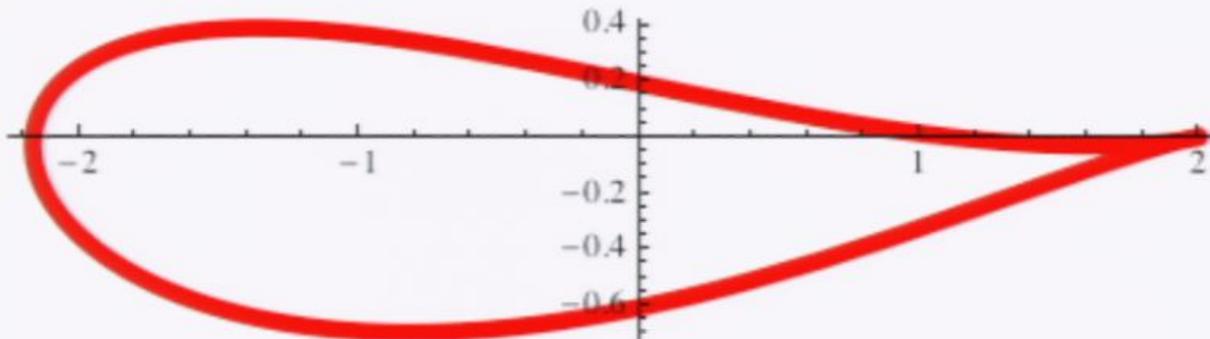
In[1057]:=
In[1058]:= D[Re[x], x]
Out[1058]= Re'[x]
    
```

```

In[991]:= Manipulate[du = Re@{∂x#, ∂y#} &@F[a + I b + r iF[x + I y]];
           sp = StreamPlot[du, {x, -4, 4}, {y, -3, 3}];
           Show[sp, wing[a + I b, r], PlotRange → All, AspectRatio → Automatic],
           {a, .2, .4}, {b, -.2, .3}, {r, .5, .9}]
    
```

Small subtlety

In[1057]:=



In[1058]:=

`D[Re[x], x]`

In[1058]:=

`Re'[x]`

In[991]:=

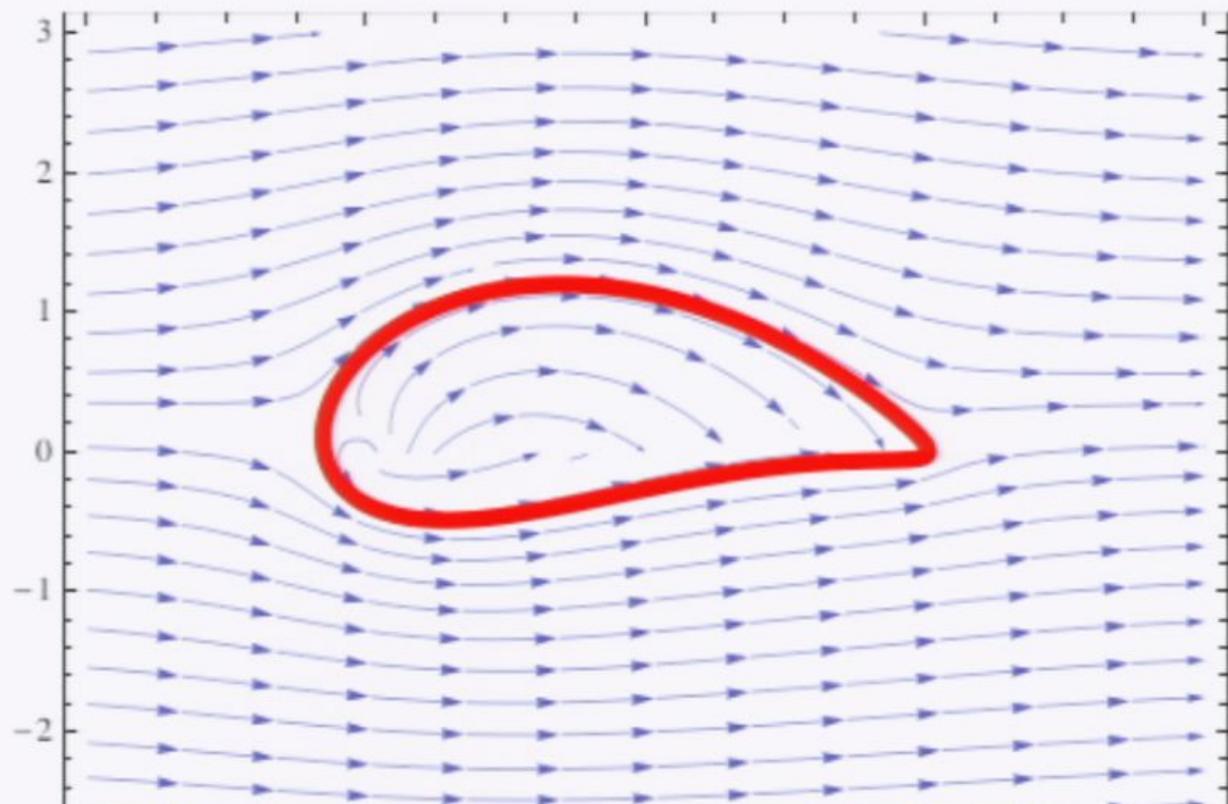
```
Manipulate[du = Re@{∂x#, ∂y#} &@F[a + I b + r iF[x + I y]];
  sp = StreamPlot[du, {x, -4, 4}, {y, -3, 3}];
  Show[sp, wing[a + I b, r], PlotRange → All, AspectRatio → Automatic],
  {a, .2, .4}, {b, -.2, .3}, {r, .5, .9}]
```

▣ Small subtlety

a  +

b  +

r  +

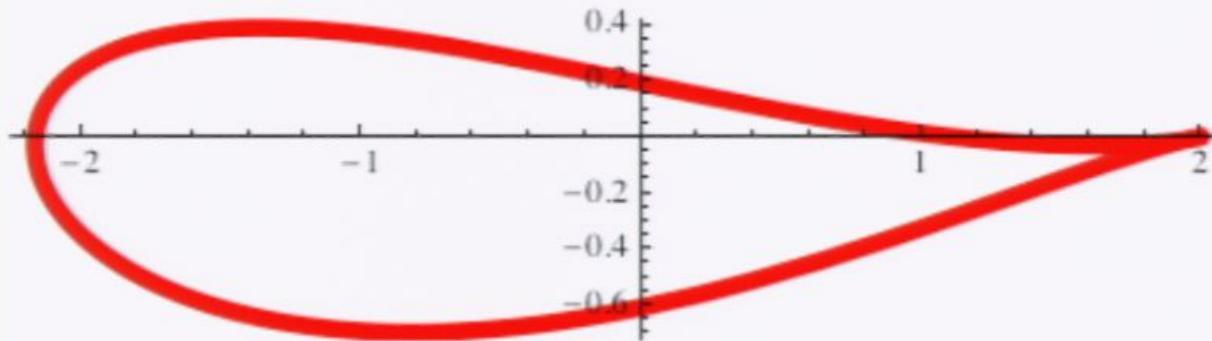


it[1059]=

```
in[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[ $\frac{1}{r}(-p + e^{i\phi})$ ],
    {phi, 0, 2 pi}, PlotStyle -> {Thickness[0.015], Red}]
```

```
in[1057]:= wing[.2 + I .1, .8]
```

```
out[1057]=
```



```
in[1059]:= Manipulate[du = Re@{Dx#, Dy#} &@F[a + I b + r iF[x + I y]];
    sp = StreamPlot[du, {x, -4, 4}, {y, -3, 3}];
    Show[sp, wing[a + I b, r], PlotRange -> All, AspectRatio -> Automatic],
    {a, .2, .4}, {b, -.2, .3}, {r, .5, .9}]
```

- New version
- New version 2
- Airplane wings

In[992]:=

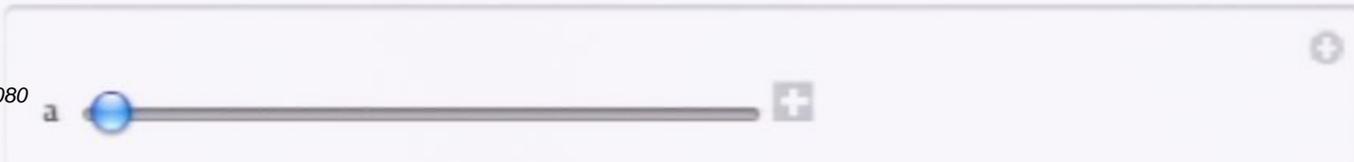
$$F[z_] = z + \frac{1}{z}; \quad iF[z_] = \frac{z + \sqrt{z-2} \sqrt{z+2}}{2};$$

In[990]:=

```
wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[1/r (-p + e^i phi)],
  {phi, 0, 2 pi}, PlotStyle -> {Thickness[0.015], Red}]
```

In[1059]:=

```
Manipulate[du = Re@{Dx#, Dy#} &@F[a + I b + r iF[x + I y]];
  sp = StreamPlot[du, {x, -4, 4}, {y, -3, 3}];
  Show[sp, wing[a + I b, r], PlotRange -> All, AspectRatio -> Automatic],
  {a, .2, .4}, {b, -.2, .3}, {r, .5, .9}]
```



- New version
- New version 2
- Airplane wings

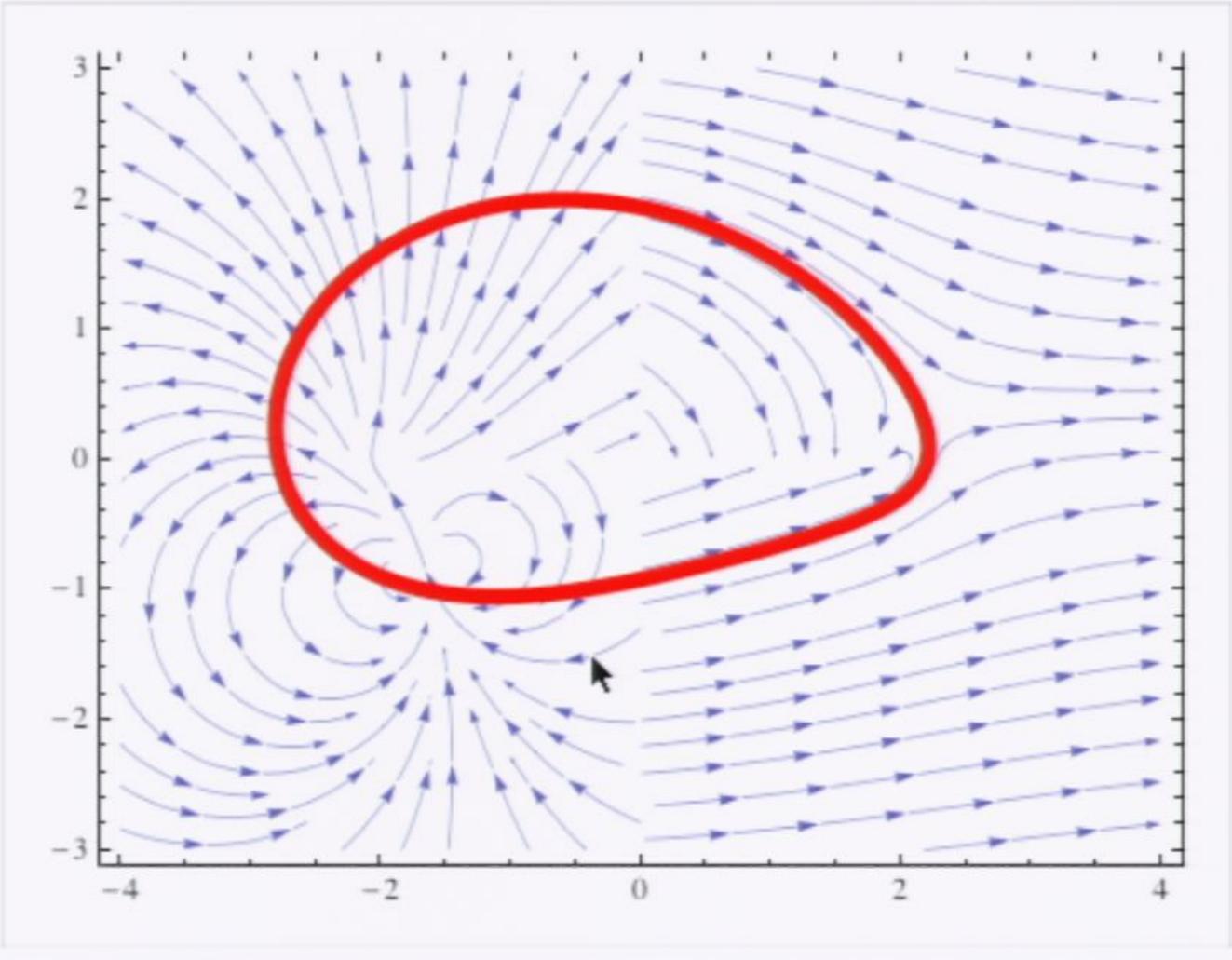
```
In[992]:= F[z_] = z + 1/z; iF[z_] = (z + Sqrt[z - 2] Sqrt[z + 2])/2;
```

```
In[990]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[1/r (-p + e^i phi)],
  {phi, 0, 2 pi}, PlotStyle -> {Thickness[0.015], Red}]
```

```
In[1059]:= Manipulate[du = Re@{Dx#, Dy#} &@F[a + I b + r iF[x + I y]];
  sp = StreamPlot[du, {x, -4, 4}, {y, -3, 3}];
  Show[sp, wing[a + I b, r], PlotRange -> All, AspectRatio -> Automatic],
  {a, .2, .4}, {b, -.2, .3}, {r, .5, .9}]
```

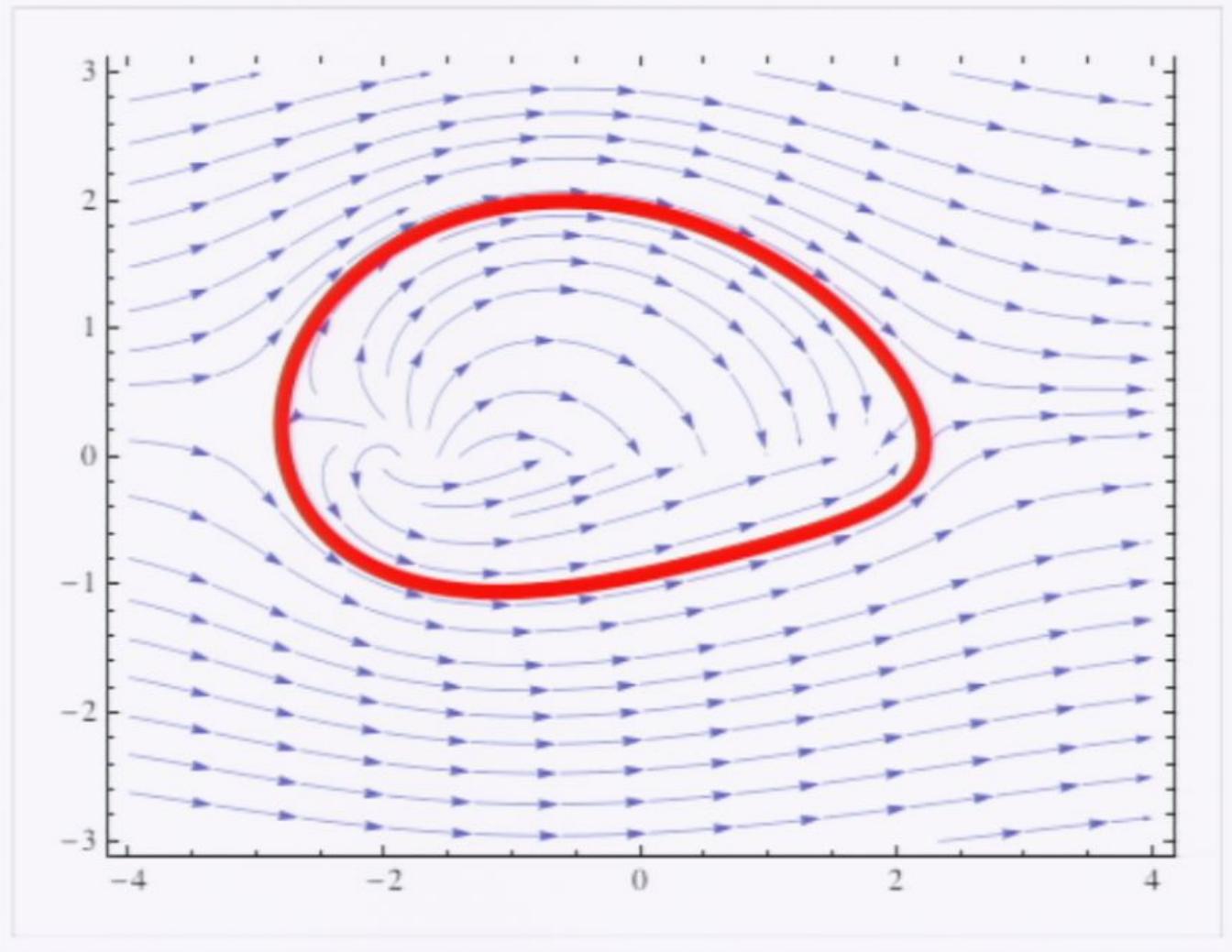


r 



it[1062]=

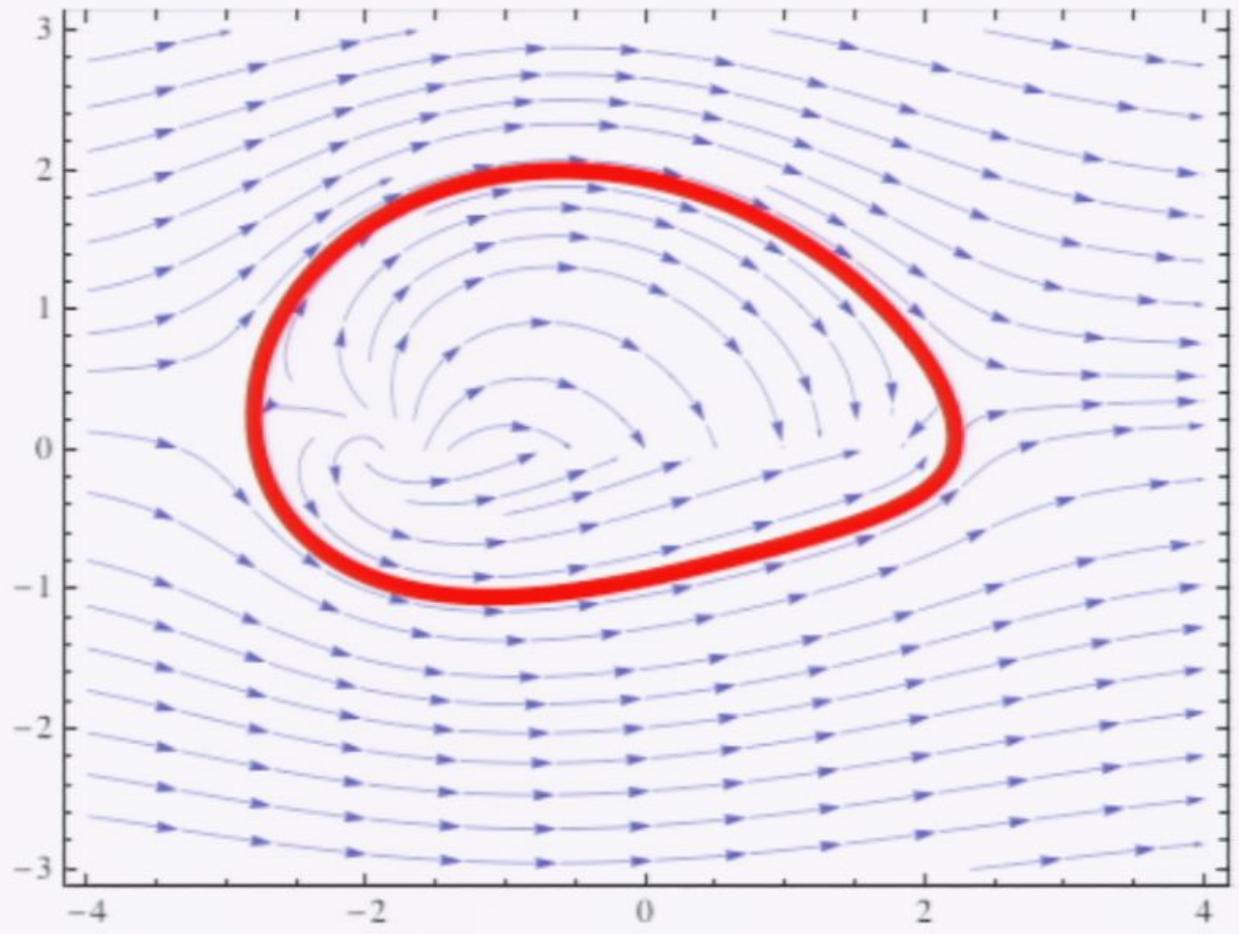
r  



at[1065]=

b +

r +



it[1065]=

```
{1063}:= F[z_] = z + 1/z; iF[z_] = (z + Sqrt[z - 2] Sqrt[z + 2])/2;
```

```
{1064}:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[1/r (-p + e^i phi)],
    {phi, 0, 2 pi}, PlotStyle -> {Thickness[0.015], Red}]
```

```
{1065}:= Manipulate[du = Re@{Dx#, Dy#} &@F[a + I b + r iF[x + I y]];
    sp = StreamPlot[du, {x, -4, 4}, {y, -3, 3}];
    Show[sp, wing[a + I b, r], PlotRange -> All, AspectRatio -> Automatic],
    {a, .2, .4}, {b, -.2, .3}, {r, .5, .9}]
```

The image shows the interactive Manipulate interface for the Mathematica code. It features three horizontal sliders:

- Slider 'a' with a blue knob, ranging from approximately 0.1 to 0.5.
- Slider 'b' with a blue knob, ranging from approximately -0.4 to 0.4.
- Slider 'r' with a blue knob, ranging from approximately 0.3 to 1.1.

 Each slider has a small square button with a plus sign to its right, used for zooming in on the parameter value.



```
[1063]:= F[z_] = z + 1/z; iF[z_] = (z + Sqrt[z - 2] Sqrt[z + 2])/2;
```

```
[1064]:= wing[p_, r_] := ParametricPlot[{Re[#], Im[#]} &@F[1/r (-p + e^i phi)],
{phi, 0, 2 pi}, PlotStyle -> {Thickness[0.015], Red}]
```

```
[1065]:= Manipulate[du = Re@{Dx#, Dy#} &@F[a + I b + r iF[x + I y]];
sp = StreamPlot[du, {x, -4, 4}, {y, -3, 3}];
Show[sp, wing[a + I b, r], PlotRange -> All, AspectRatio -> Automatic],
{a, .2, .4}, {b, -.2, .3}, {r, .5, .9}]
```

The image shows the control interface for the Manipulate function. It features three horizontal sliders, each with a blue circular knob and a plus sign button to its right. The sliders are labeled 'a', 'b', and 'r' from top to bottom. The 'a' slider is positioned at approximately 0.2, the 'b' slider at approximately -0.2, and the 'r' slider at approximately 0.5.

