Title: Introduction to Complex Analysis/Computer Skills - Lecture 8A

Date: Sep 09, 2010 09:00 AM

URL: http://pirsa.org/10090078

Abstract:

$\mathcal{H} = \{|\uparrow\rangle, |\downarrow\rangle\}$

$$\mathcal{H} : \{|\uparrow\rangle, |\downarrow\rangle\}$$

$$\hat{A}\,|\uparrow\rangle = A_{11}\,|\uparrow\rangle + A_{21}\,|\downarrow\rangle$$

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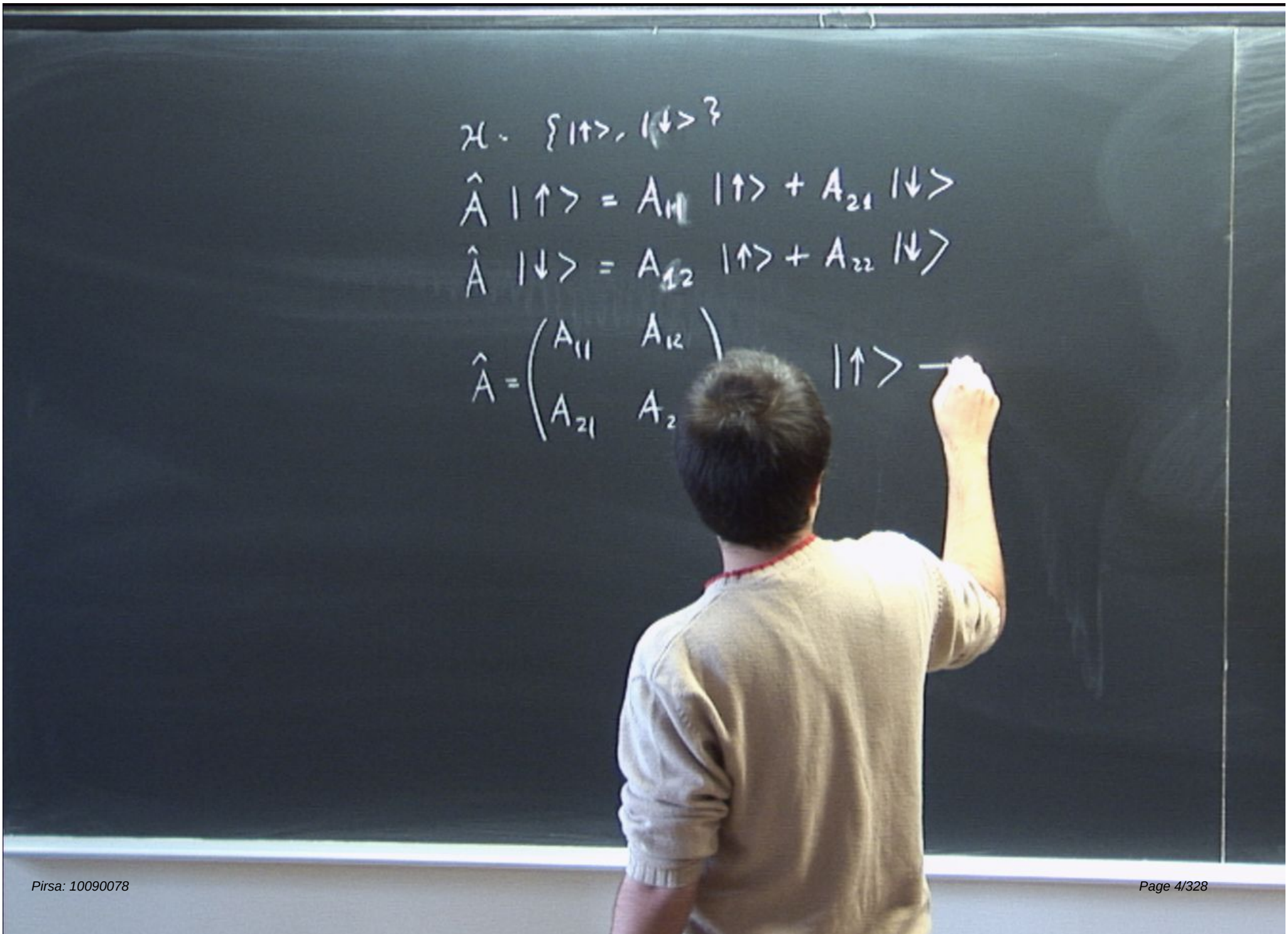
$\mathcal{H} = \{|\uparrow\rangle, |\downarrow\rangle\}$
$\hat{A}\,|\uparrow\rangle = A_{11}\,|\uparrow\rangle + A_{21}\,|\downarrow\rangle$
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$\hat{A} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{2} \end{pmatrix}$
$|\uparrow\rangle$

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$$\mathcal{H}_1 \otimes \mathcal{H}_2 = \{ |\uparrow\rangle|\uparrow\rangle, |\downarrow\rangle|\uparrow\rangle, |\uparrow\rangle|\downarrow\rangle, |\downarrow\rangle|\downarrow\rangle \}$$

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$$+ 0\ |\uparrow\rangle|\downarrow\rangle$$

$$+ 0\ |$$

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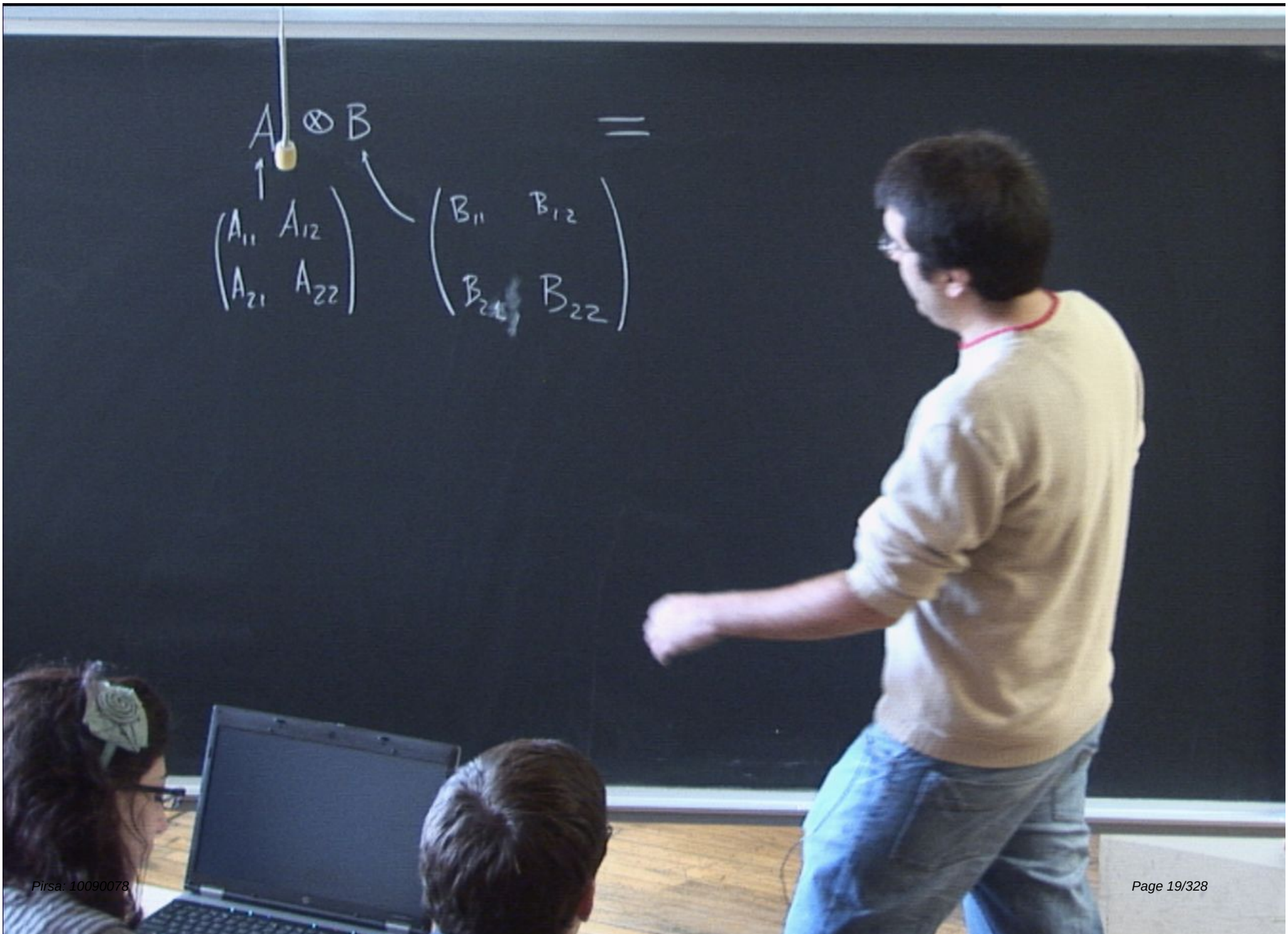
$$A^{(1)} = A \otimes \mathbb{1}$$


$A \otimes B =$
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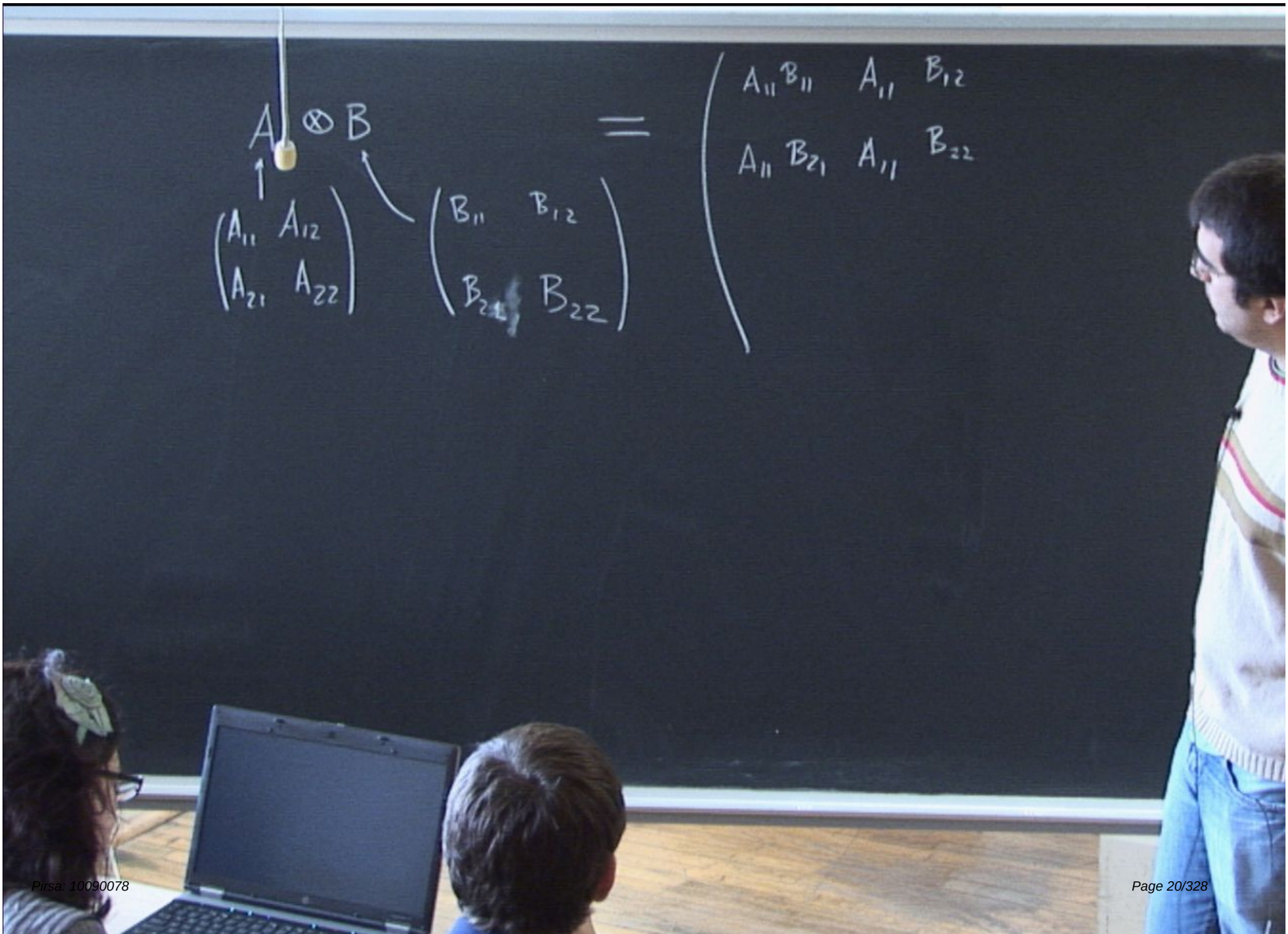
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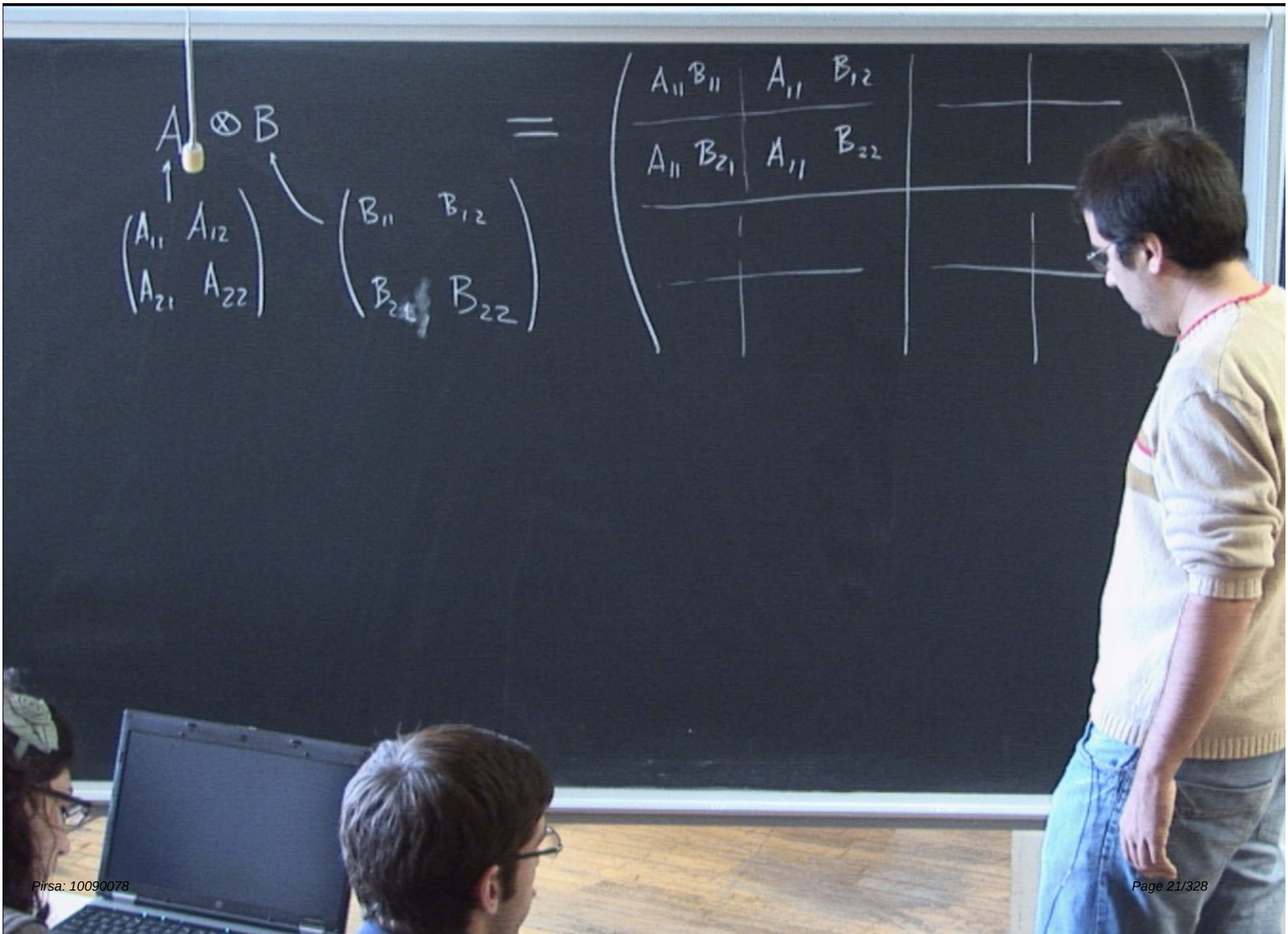
A ⊗ B =
A11 A12
A21 A22
B11 B12
B21 B22
A11 B11 A11 B12
A11 B21 A11 B22

$$A \otimes B = \left(\begin{array}{cc|cc} A_{11}B_{11} & A_{11}B_{12} & & \\ A_{11}B_{21} & A_{11}B_{22} & & \\ \hline & & & A_{22}B_{12} \\ & & & \end{array}\right)$$

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$$\begin{aligned} &= A_{11}|\uparrow\rangle|\uparrow\rangle \\ &+ A_{21}|\downarrow\rangle|\uparrow\rangle \\ &+ 0\ |\uparrow\rangle|\downarrow\rangle \\ &+ 0\ |\downarrow\rangle|\downarrow\rangle \end{aligned}$$

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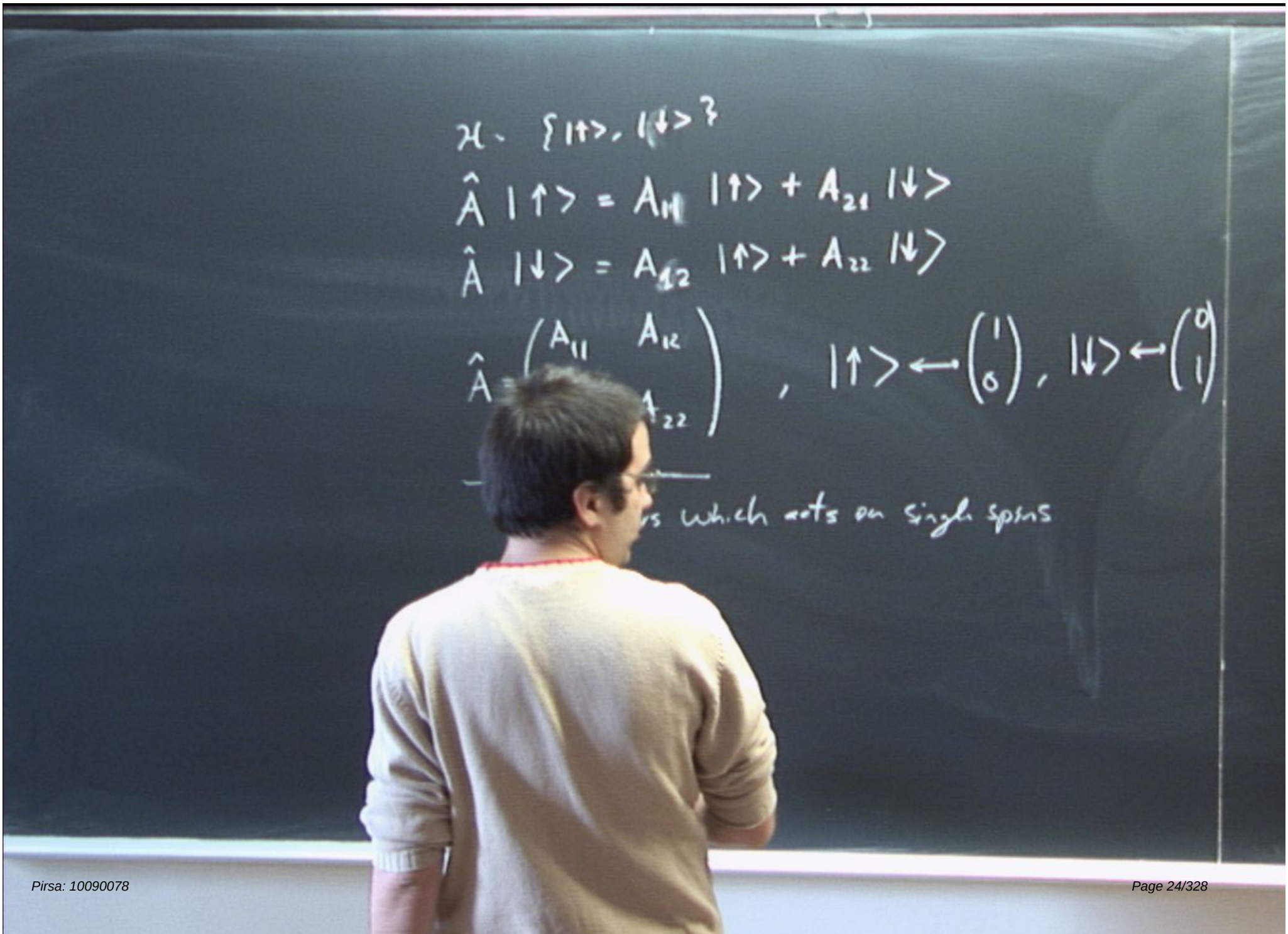
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$\hat{A} = \begin{pmatrix} A_{11} & A_{12} \\ & A_{22} \end{pmatrix}$, $|\uparrow\rangle \leftrightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|\downarrow\rangle \leftrightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
which acts on single spins

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$$A^{(1)} = \mathbb{1} \otimes A = \left(\begin{array}{c|c} \begin{matrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{matrix} & 0_{2\times} \\ \hline 0_{2\times 2} & \begin{matrix} A_{11} & A_{21} \\ A_{21} & A_{22} \end{matrix} \end{array}\right)$$

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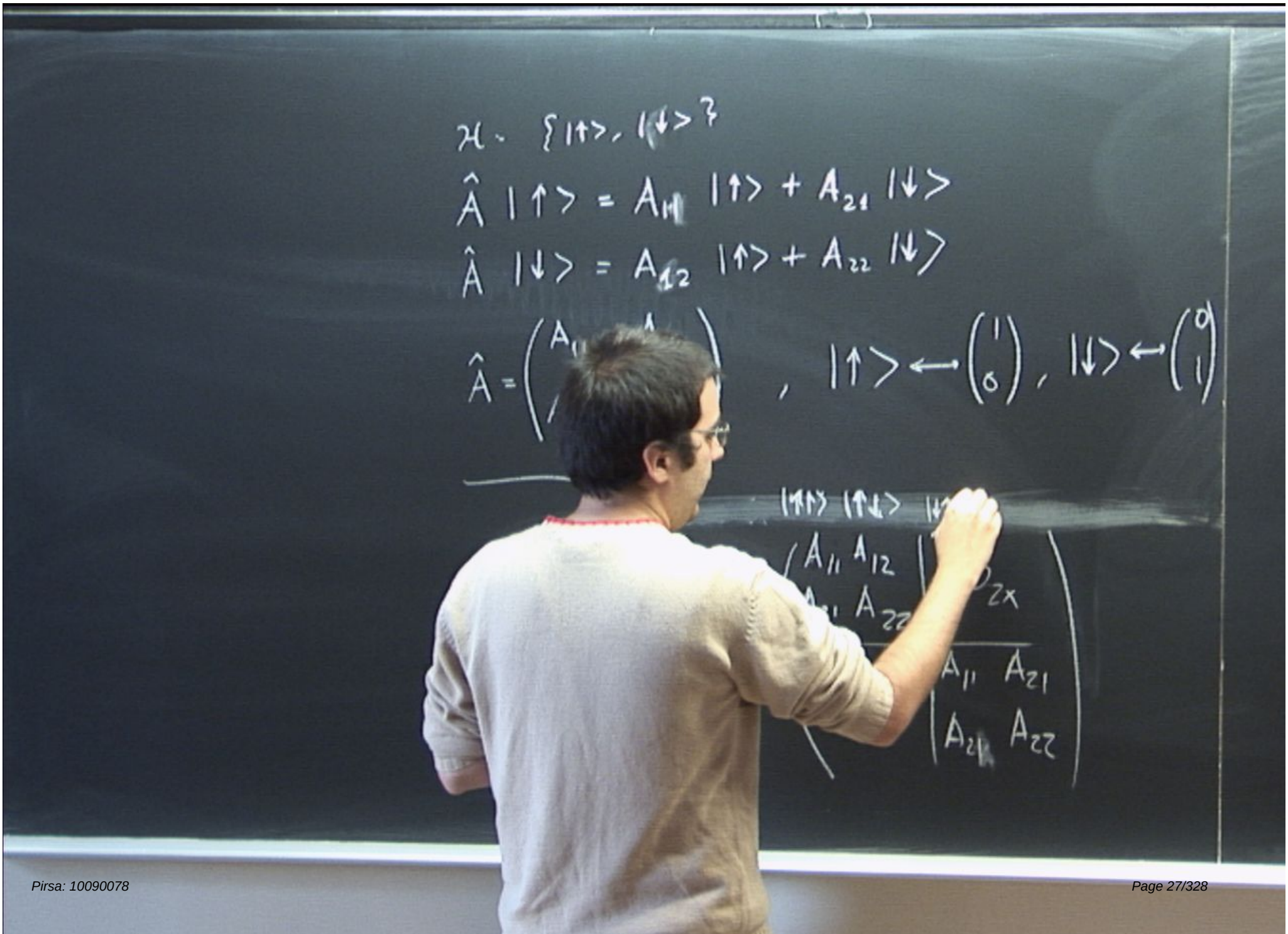
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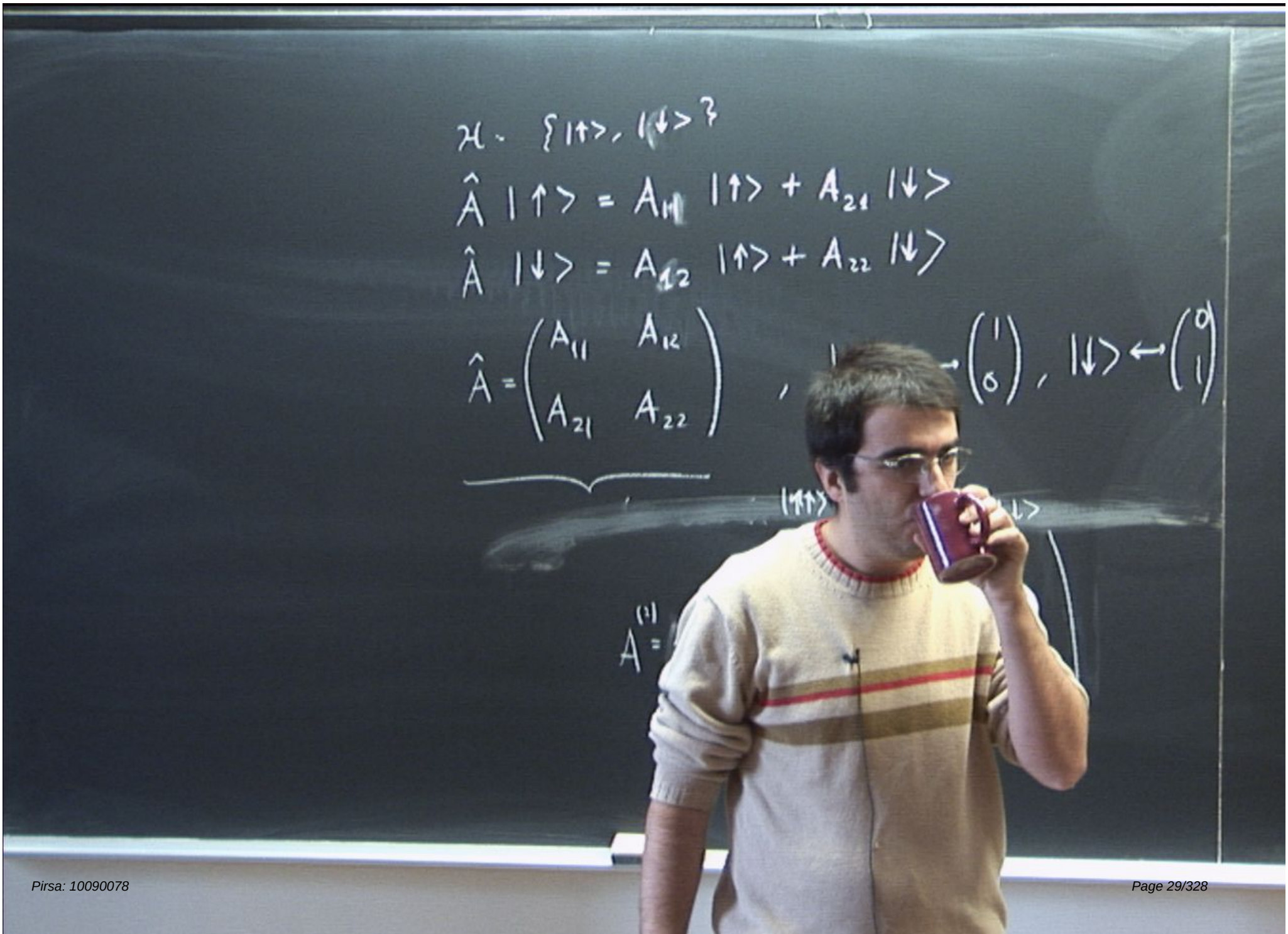
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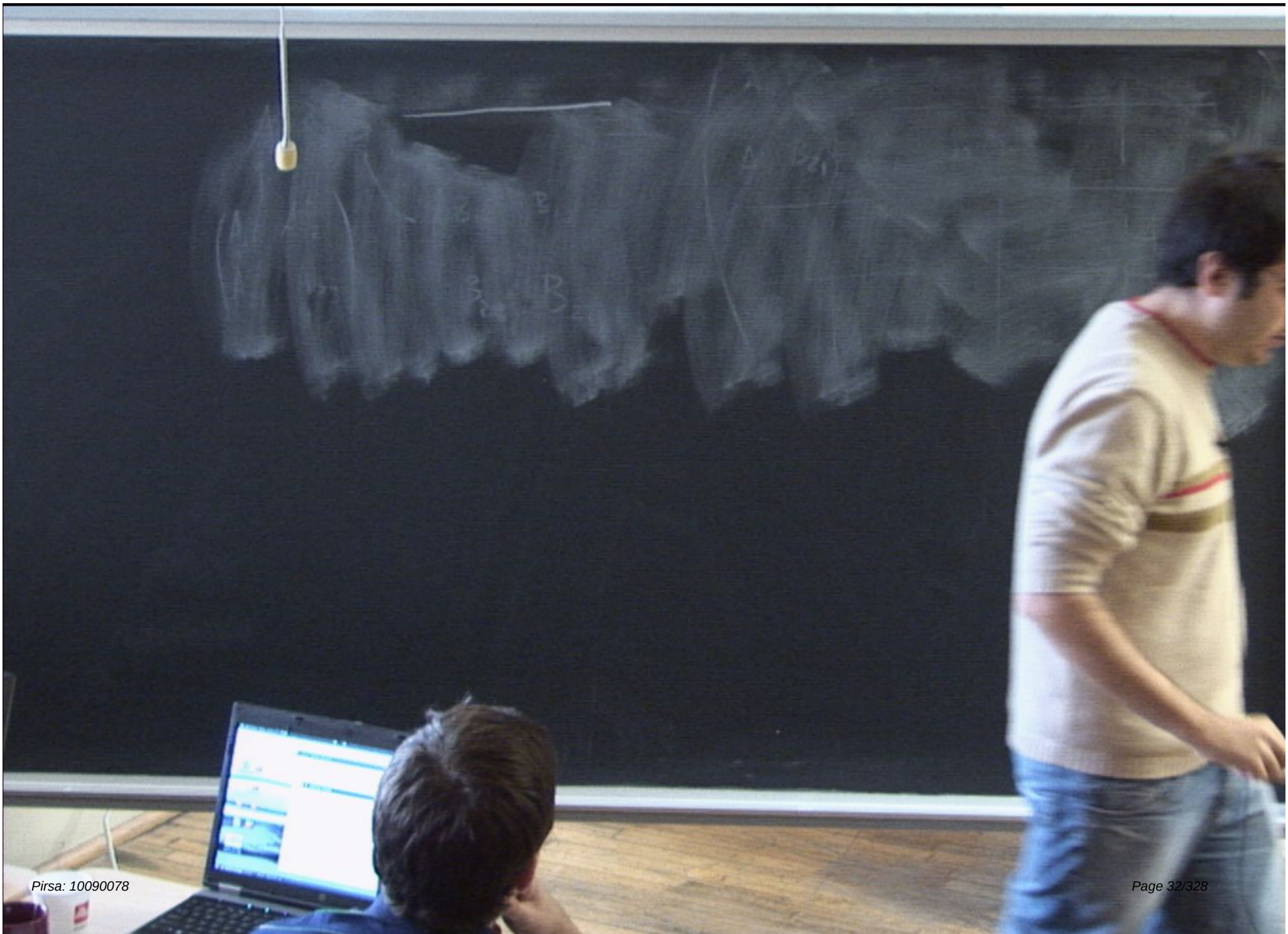
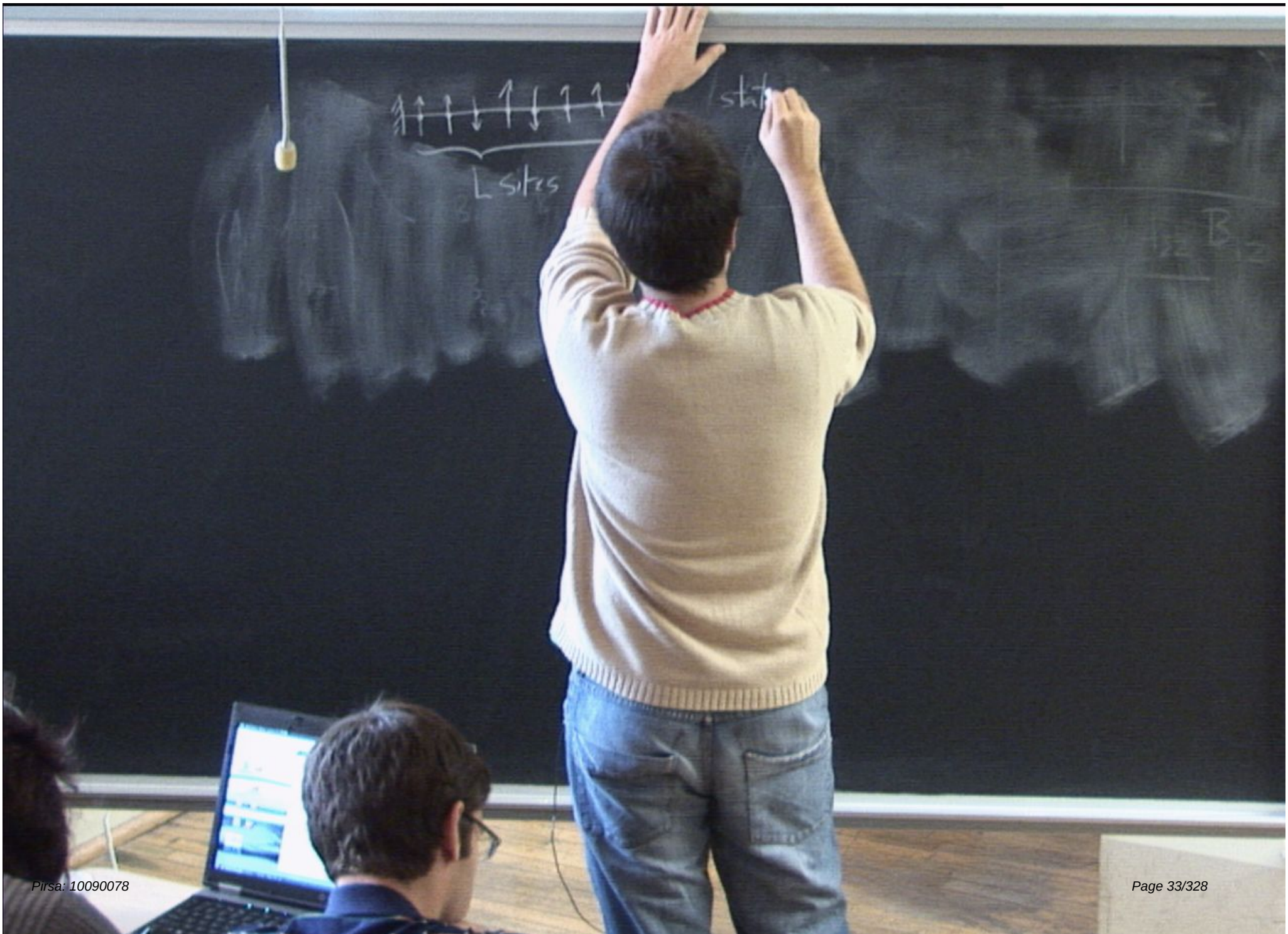
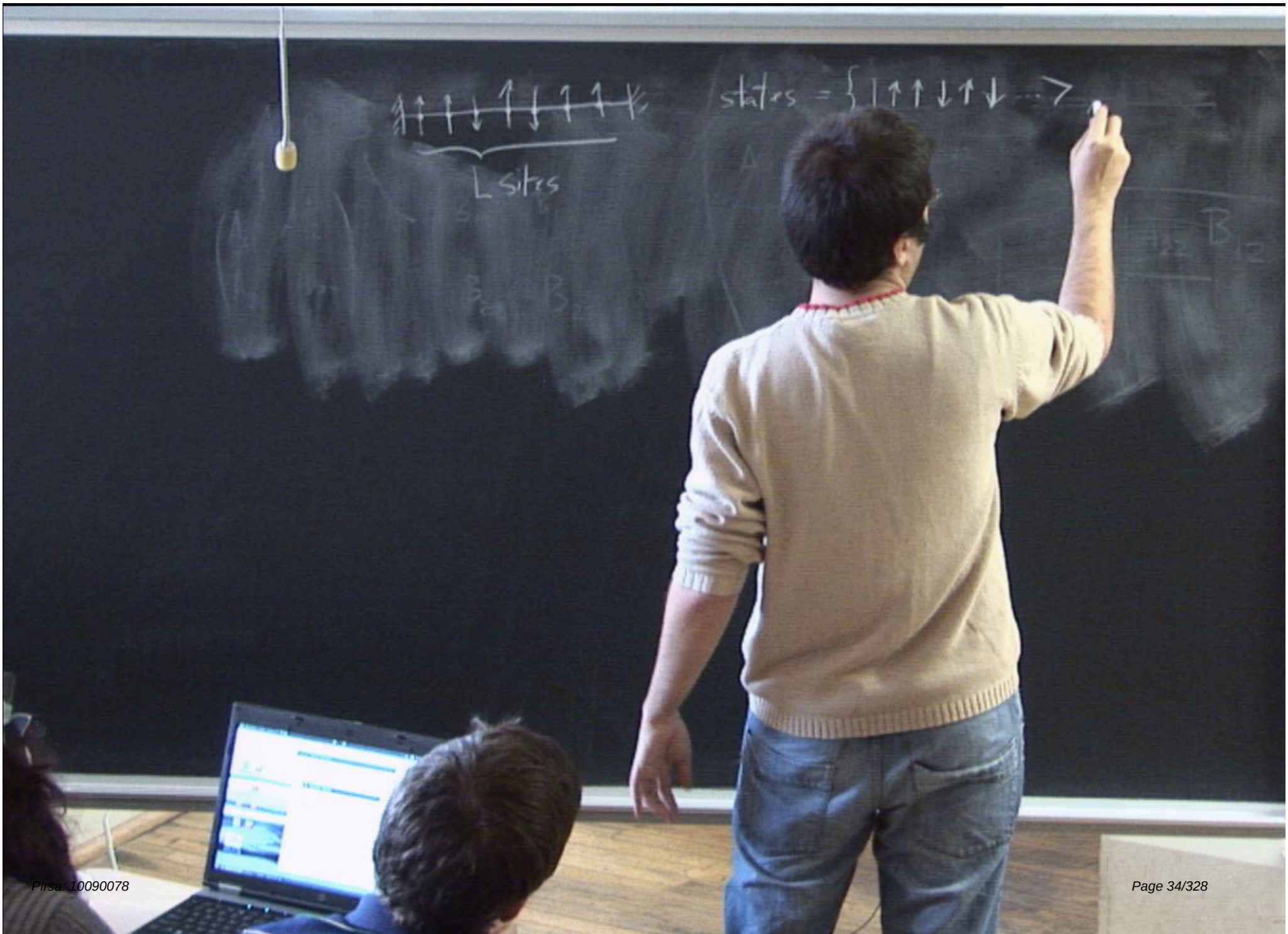
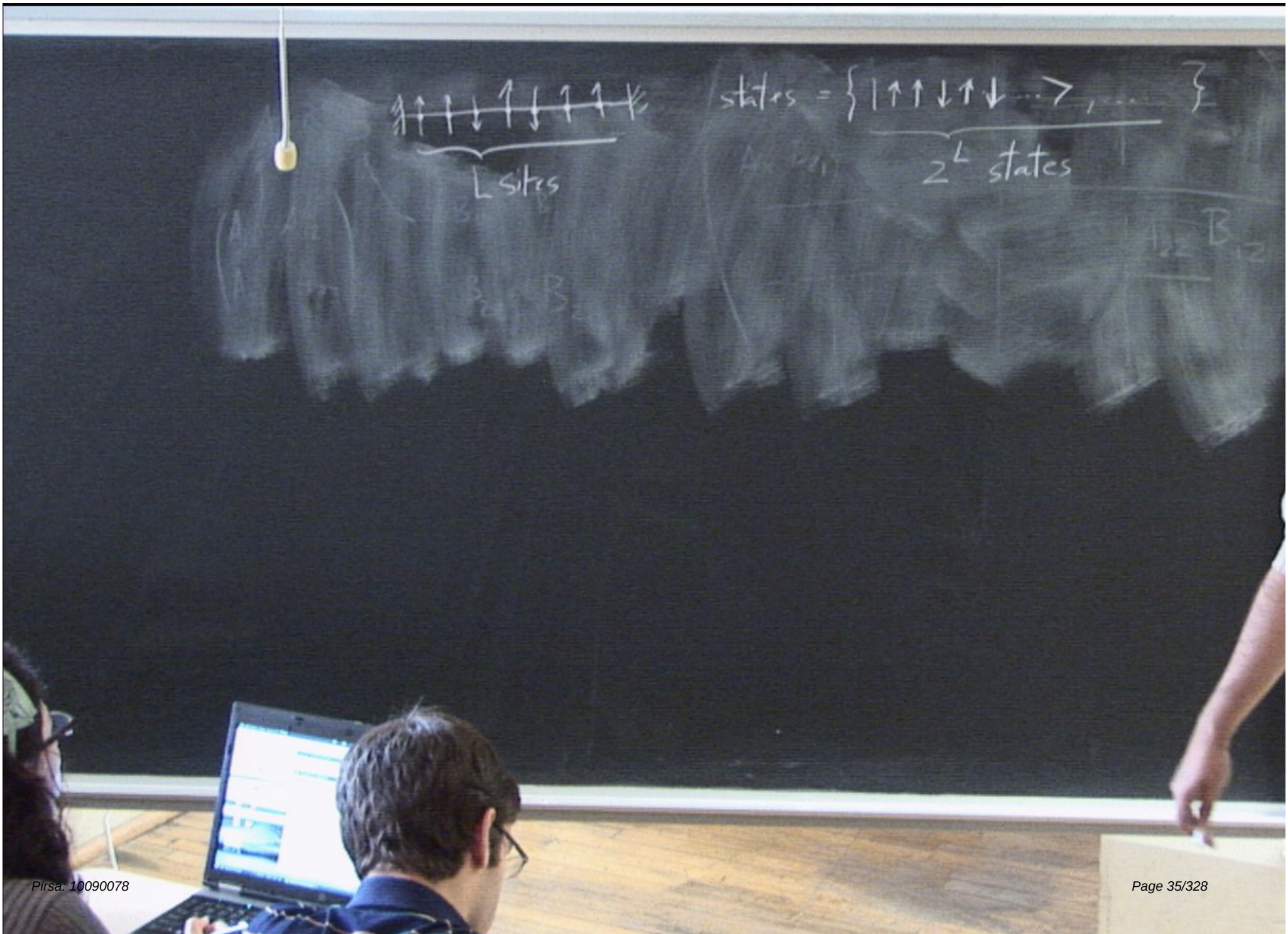
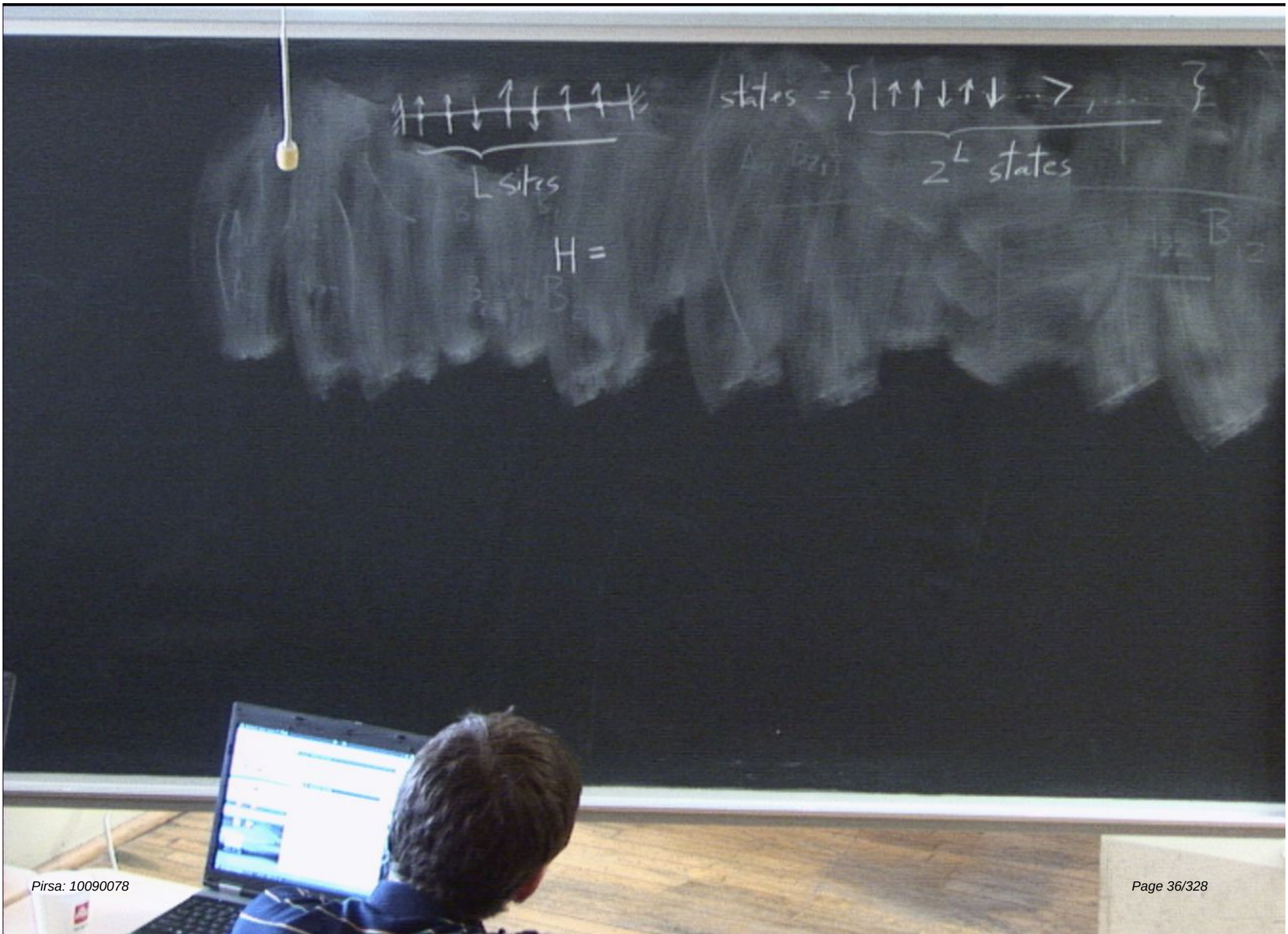
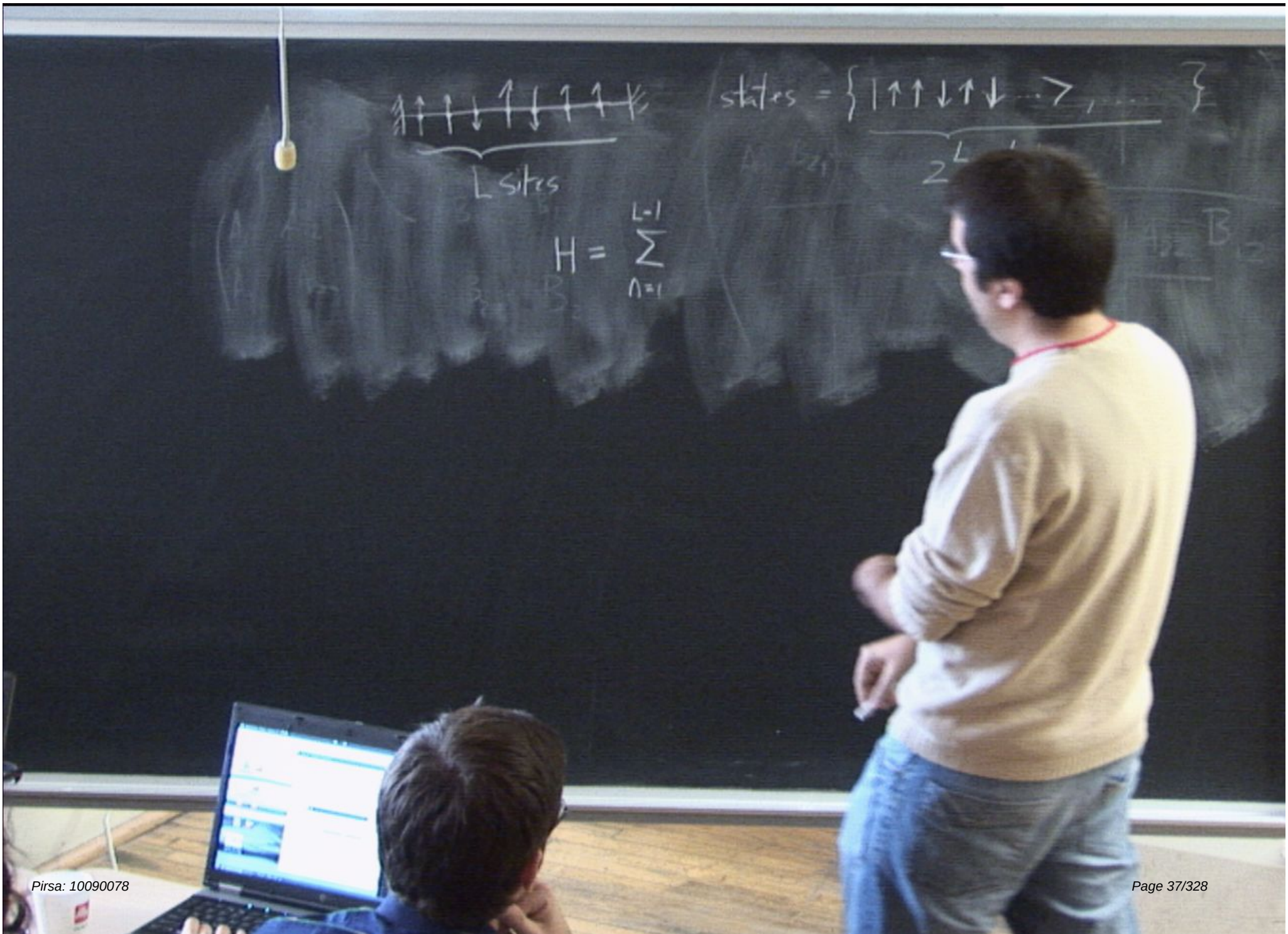


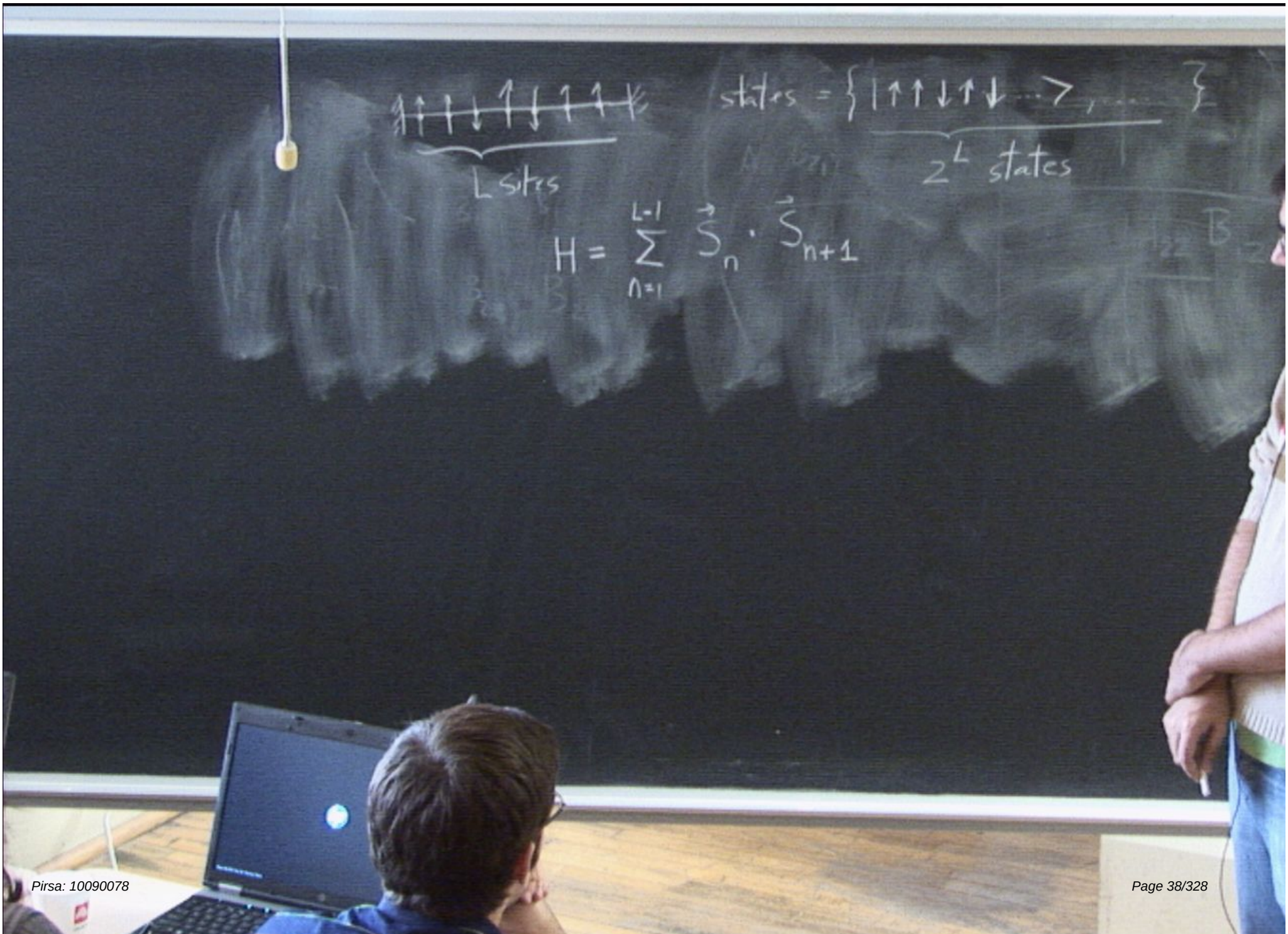

L sites

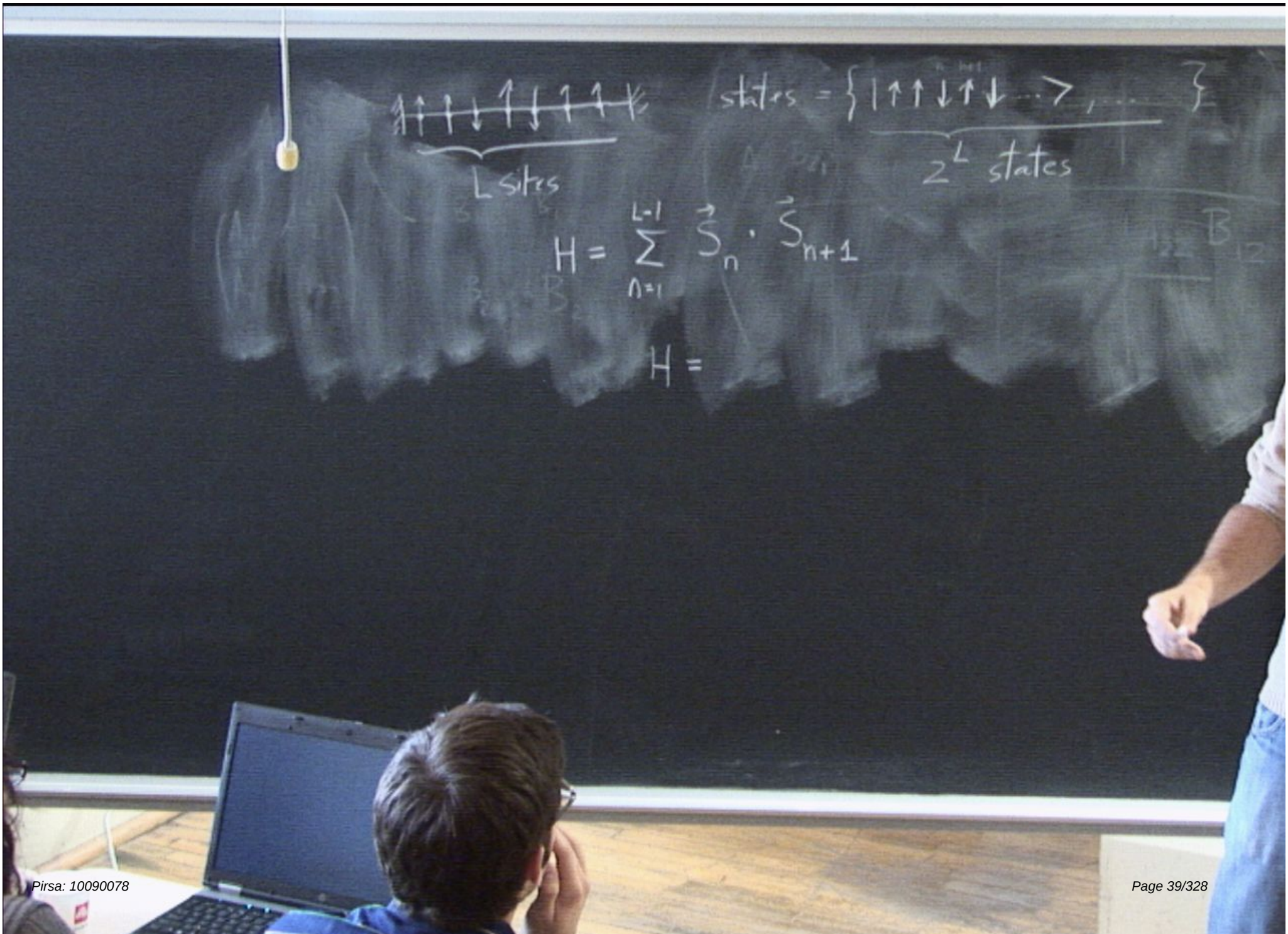

L sites
states = { |↑↑↓↑↓ ... >

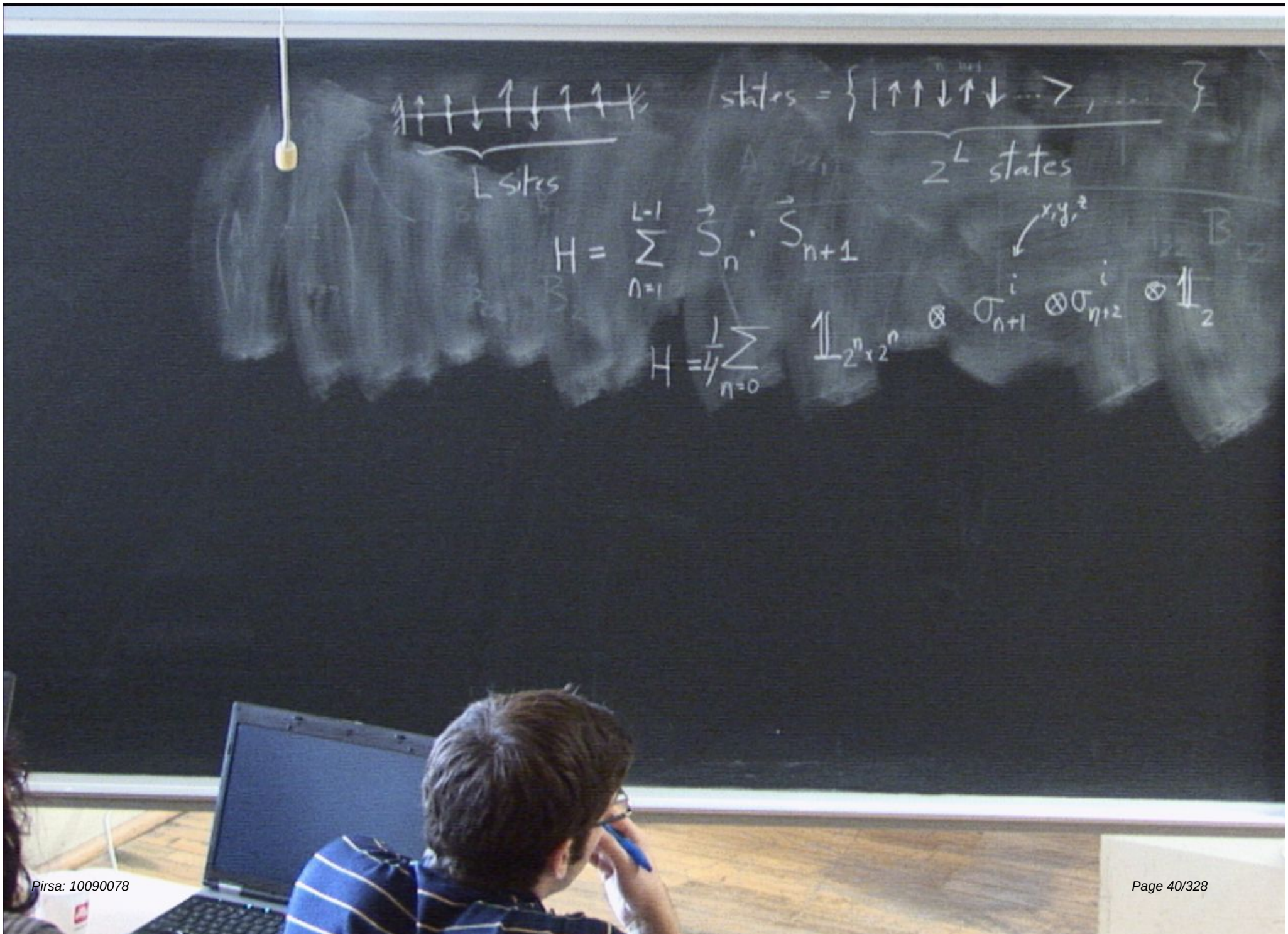

L sites
states = { |↑↑↓↑↓ ... >, ... }
2^L states

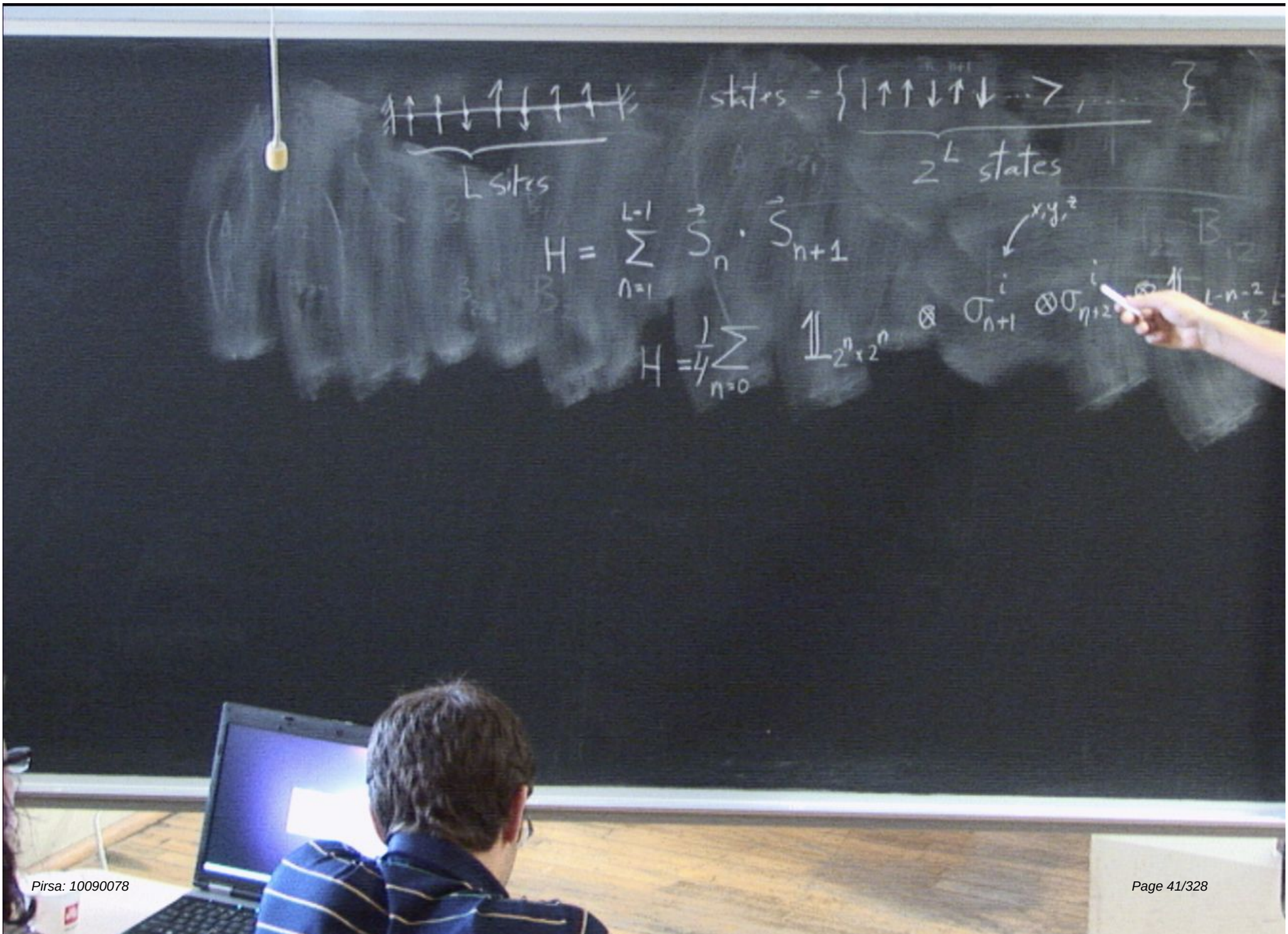

L sites
states = { |↑↑↓↑↓ ... >, ... }
2^L states
H =

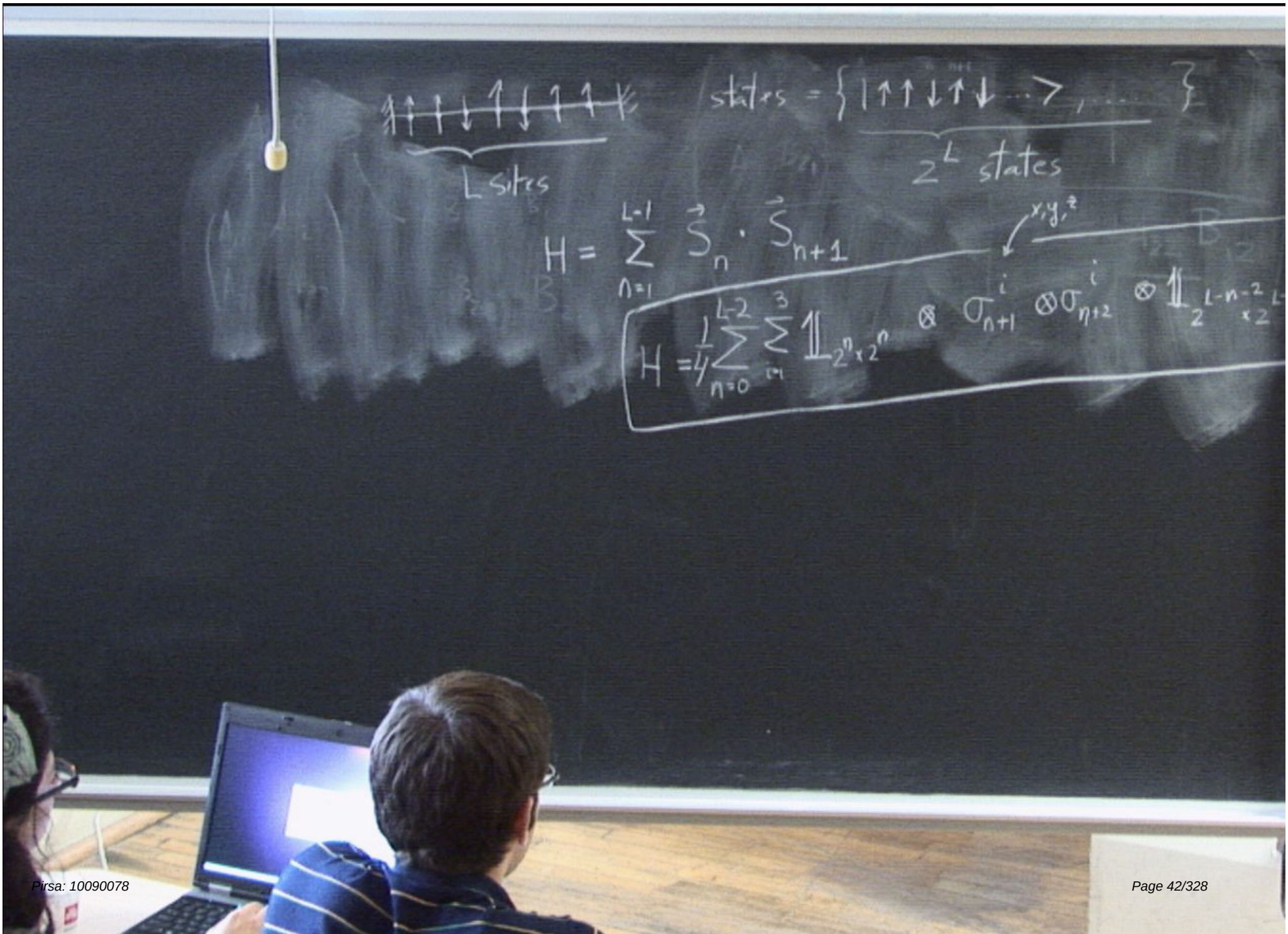

L sites
states = { |↑↑↓↑↓ ... >, ... }
2^L
H = Σ
n=1
L-1

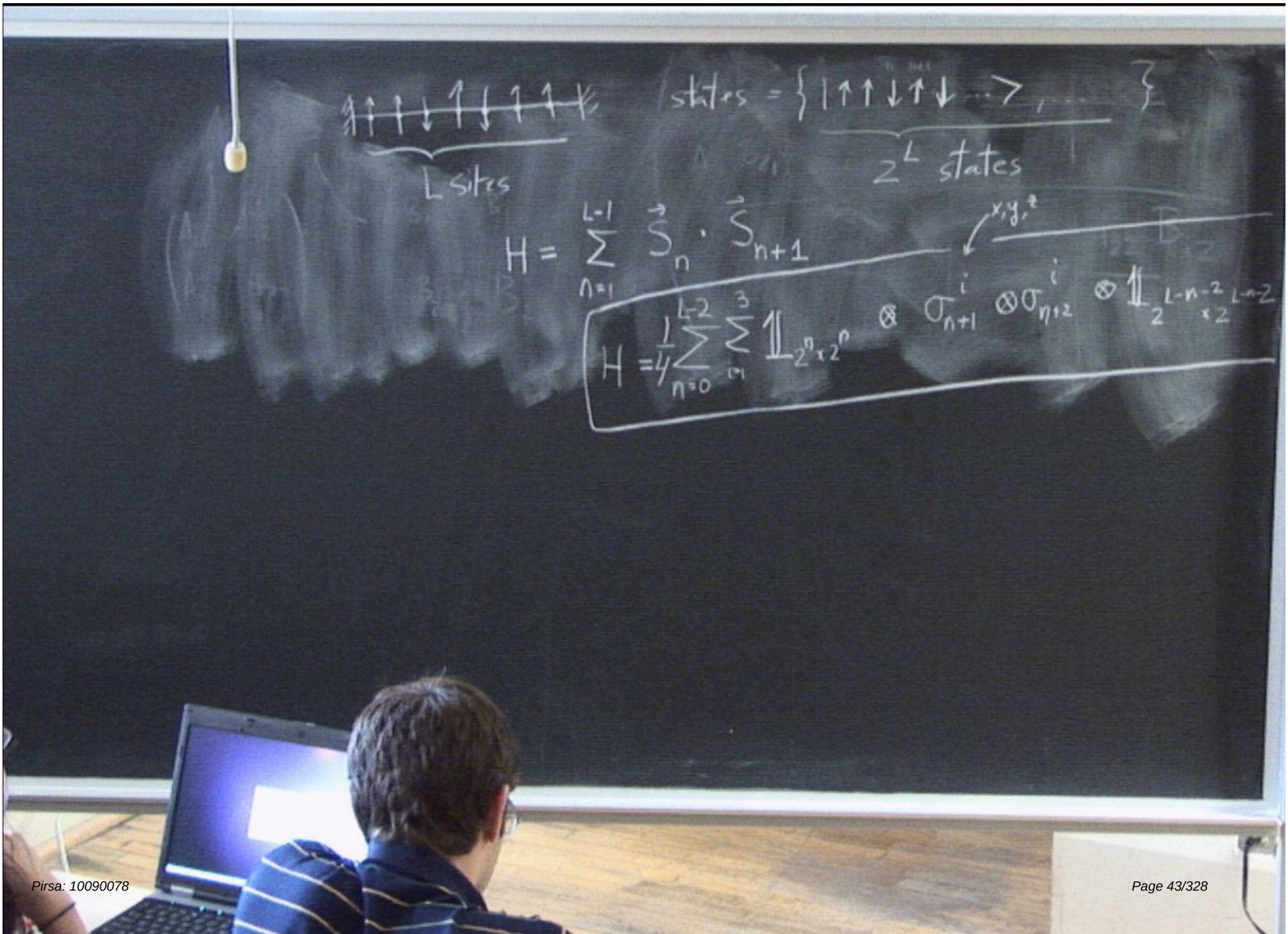

states = { |↑↑↓↑↓ ... >, ... }
L sites
2^L states
H = Σ_{n=1}^{L-1} S_n · S_{n+1}

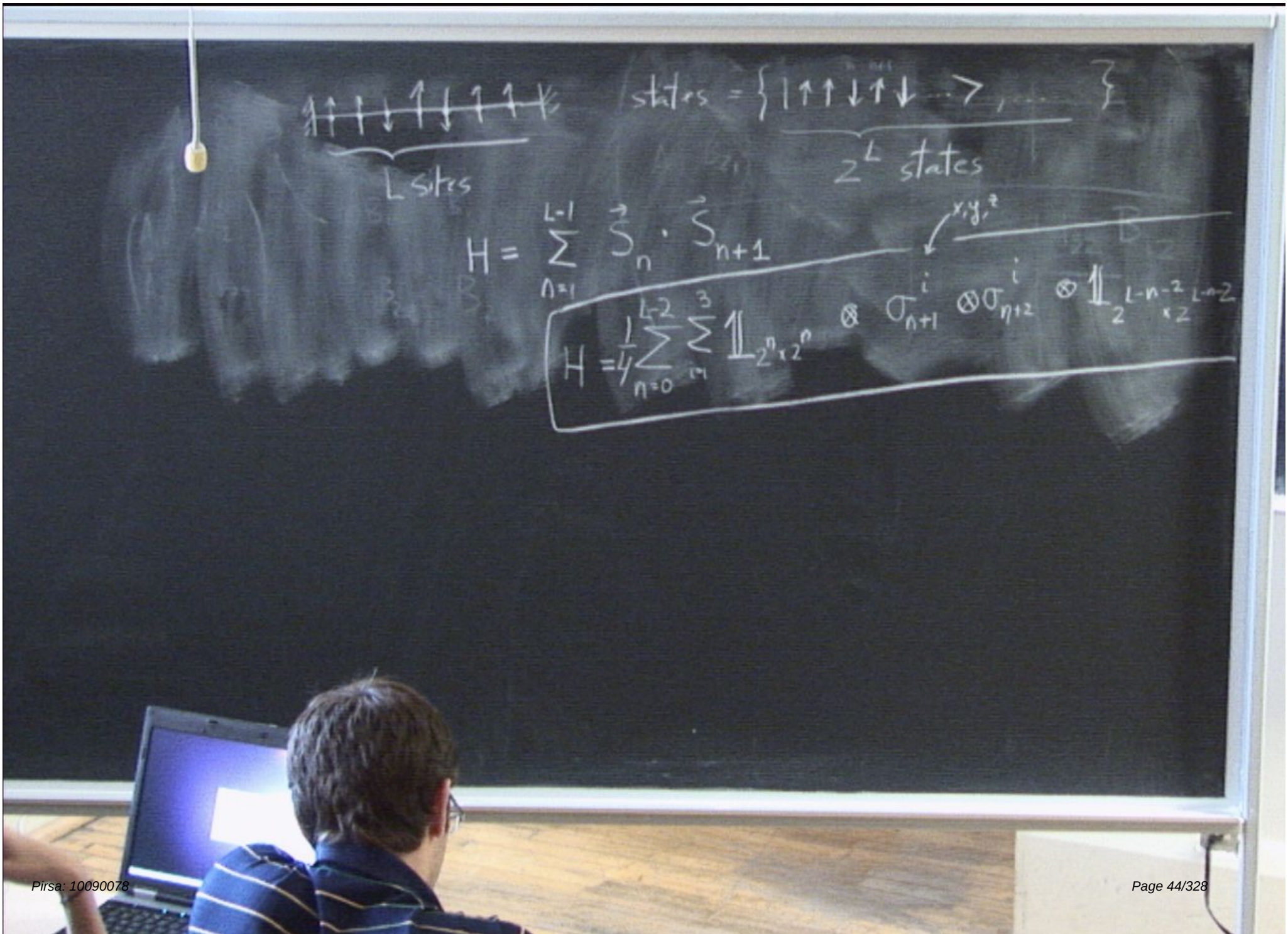

L sites
states = { |↑↑↓↑↓ ... > , ... }
2^L states
H = Σ_{n=1}^{L-1} S_n · S_{n+1}
H =

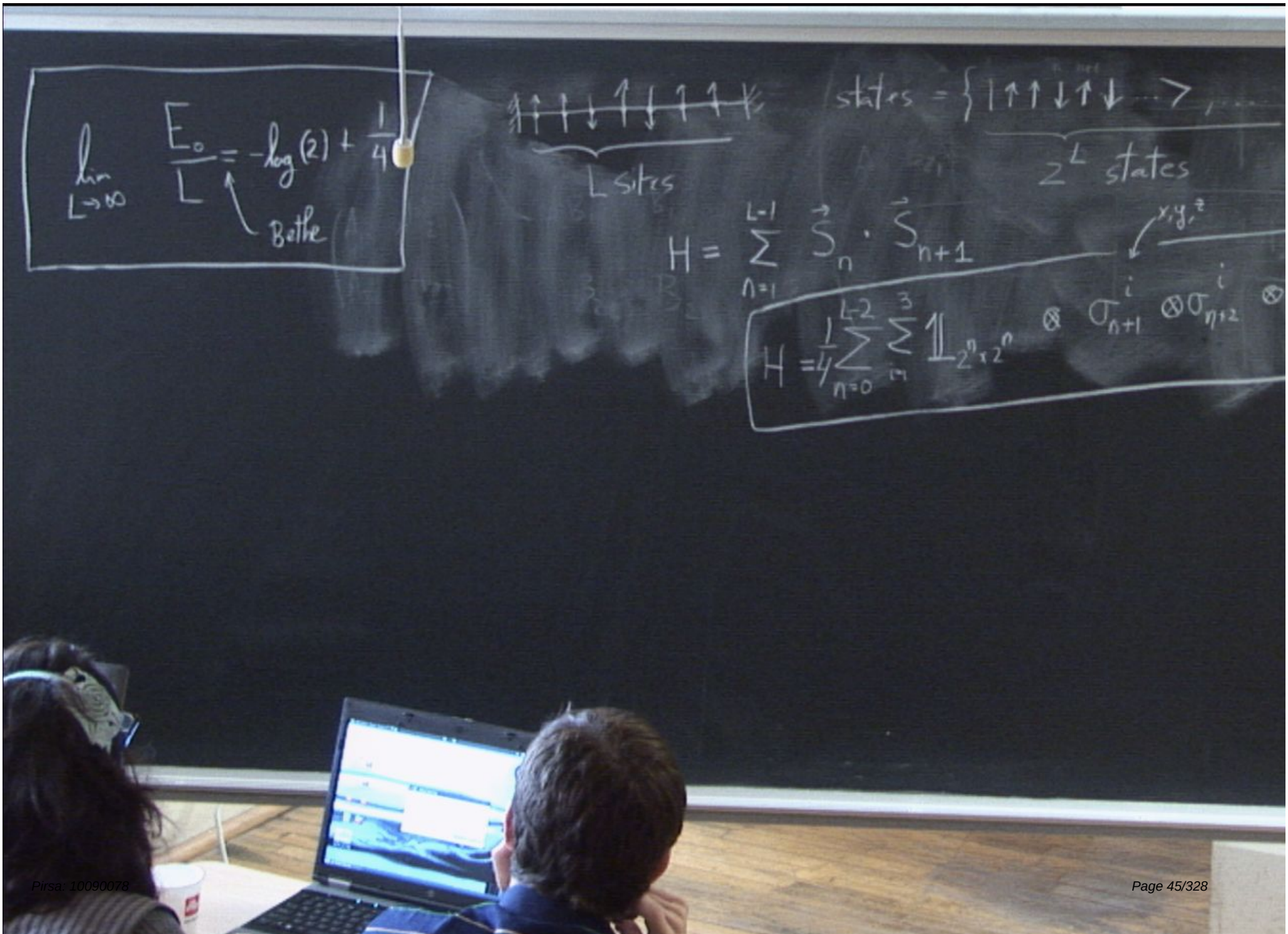

L sites
states = { |↑↑↓↑↓ ... > , ... }
2^L states
H = Σ_{n=1}^{L-1} S_n · S_{n+1}
x, y, z
H = 1/4 Σ_{n=0} 𝟙_{2^n×2^n} ⊗ σ^i_{n+1} ⊗ σ^i_{n+2} ⊗ 𝟙_2

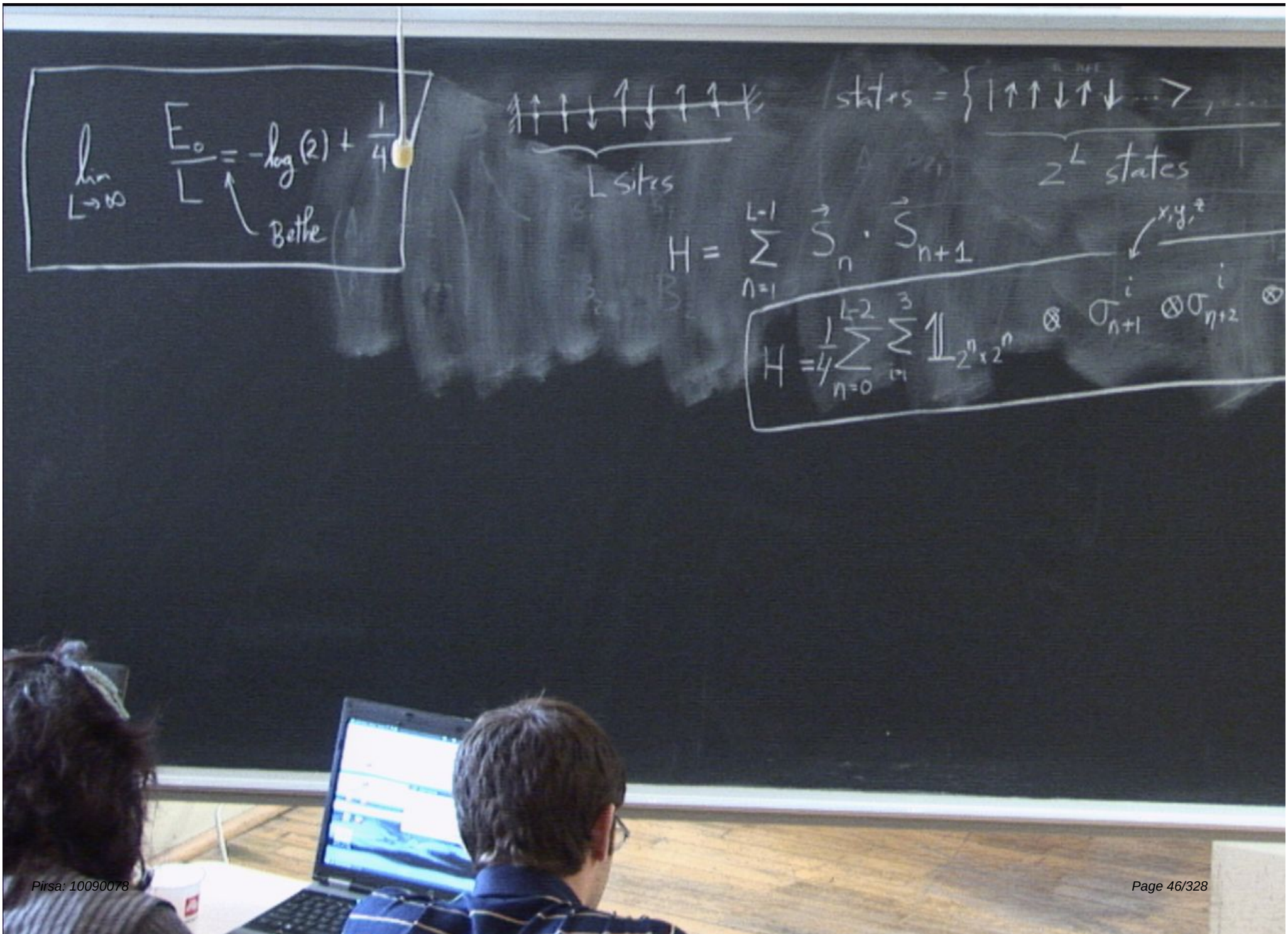

states = { |↑↑↓↑↓ ... >, ... }
L sites
2^L states
H = Σ_{n=1}^{L-1} S_n · S_{n+1}
x, y, z
H = 1/4 Σ_{n=0} 𝟙_{2^n × 2^n} ⊗ σ^i_{n+1} ⊗ σ^i_{n+2}


states = { |↑↑↓↑↓ ... >, ... }
L sites
2^L states
x,y,z
H = Σ_{n=1}^{L-1} S_n · S_{n+1}
H = 1/4 Σ_{n=0}^{L-2} Σ_{i=1}^{3} 𝟙_{2^n x 2^n} ⊗ σ^i_{n+1} ⊗ σ^i_{n+2} ⊗ 𝟙_{2^{L-n-2} x 2^L}


states = { |↑↑↓↑↓ ... >, ... }
L sites
2^L states
x, y, z
H = Σ_{n=1}^{L-1} S_n · S_{n+1}
H = 1/4 Σ_{n=0}^{L-2} Σ_{i=1}^{3} 𝟙_{2^n × 2^n} ⊗ σ^i_{n+1} ⊗ σ^i_{n+2} ⊗ 𝟙_{2^{L-n-2} × 2^{L-n-2}}


states = { |↑↑↓↑↓ ... >, ... }
L sites
2^L states
x, y, z
H = Σ_{n=1}^{L-1} S_n · S_{n+1}
H = 1/4 Σ_{n=0}^{L-2} Σ_{i=1}^{3} 𝟙_{2^n×2^n} ⊗ σ^i_{n+1} ⊗ σ^i_{n+2} ⊗ 𝟙_{2^{L-n-2}×2^{L-n-2}}


$\lim_{L\to\infty} \frac{E_0}{L} = -\log(2) + \frac{1}{4}$
Bethe
L sites
states = { |↑↑↓↑↓ ...> , ...
2^L states
$H = \sum_{n=1}^{L-1} \vec{S}_n \cdot \vec{S}_{n+1}$
x, y, z
$H = \frac{1}{4} \sum_{n=0}^{L-2} \sum_{i=1}^{3} \mathbb{1}_{2^n \times 2^n} \otimes \sigma^i_{n+1} \otimes \sigma^i_{n+2} \otimes$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Bethe
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states = { |↑↑↓↑↓ ... > , ...
2^L states
$H = \sum_{n=1}^{L-1} \vec{S}_n \cdot \vec{S}_{n+1}$
x, y, z
$H = \frac{1}{4} \sum_{n=0}^{L-2} \sum_{i=1}^{3} \mathbb{1}_{2^n \times 2^n} \otimes \sigma^i_{n+1} \otimes \sigma^i_{n+2} \otimes$

$$\lim_{L\to\infty} \frac{E_0}{L} = -\log(2) + \frac{1}{4}$$

Bethe

$\underbrace{\uparrow\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow\uparrow}_{L \text{ sites}}$

states $= \{ \underbrace{|\uparrow\uparrow\downarrow\uparrow\downarrow\cdots\rangle}_{2^L \text{ states}}, \ldots$

$$H = \sum_{n=1}^{L-1} \vec{S}_n \cdot \vec{S}_{n+1}$$

x, y, z

$$H = \frac{1}{4} \sum_{n=0}^{L-2} \sum_{i=1}^{3} \mathbb{1}_{2^n \times 2^n} \otimes \sigma^i_{n+1} \otimes \sigma^i_{n+2} \otimes$$

$$\lim_{L\to\infty} \frac{E_0}{L} = -\log(2) + \frac{1}{4}$$

Bethe

$$\underbrace{\uparrow\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow}_{L \text{ sites}}$$

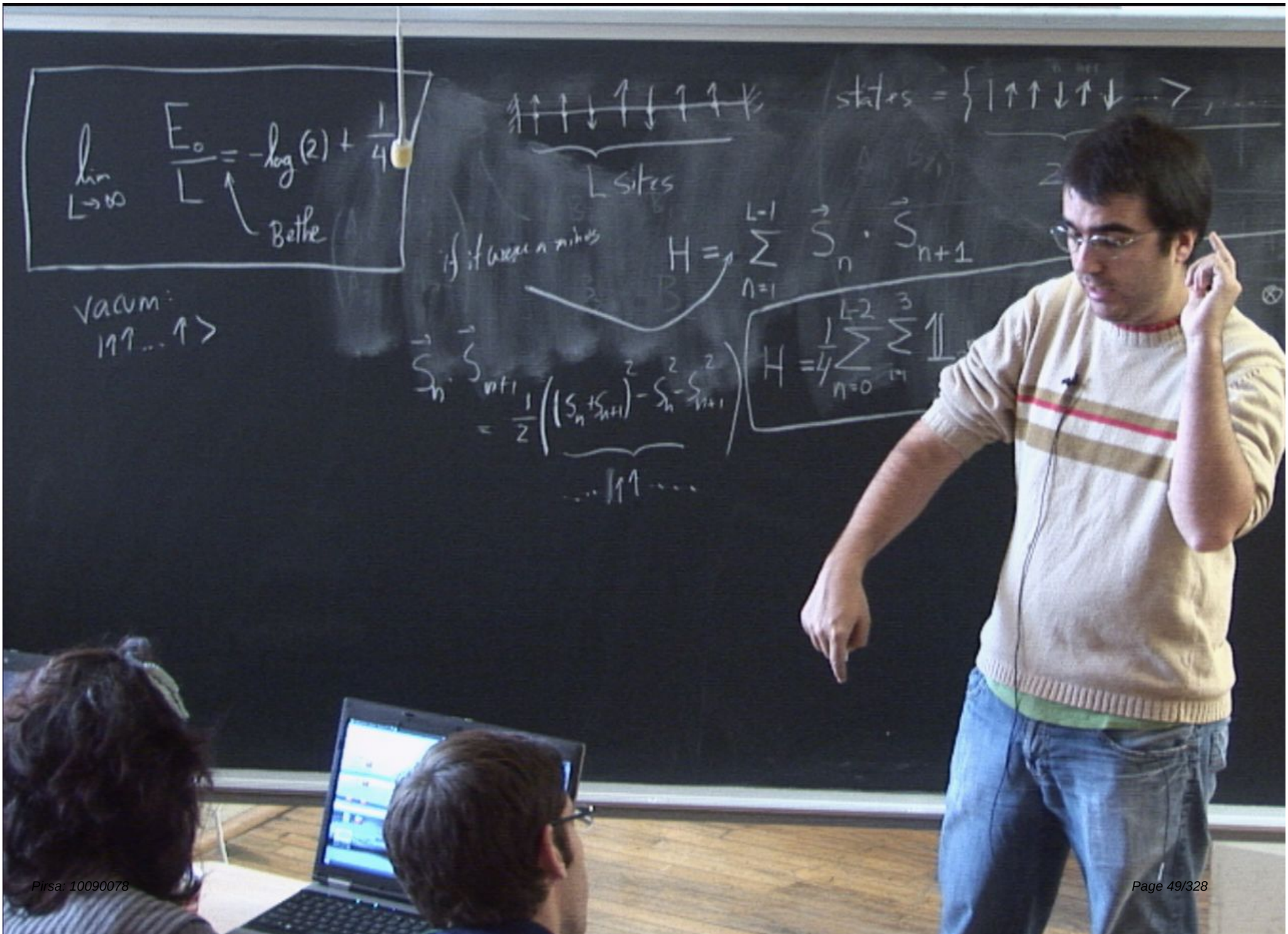
states $= \{ |\uparrow\uparrow\downarrow\uparrow\downarrow \cdots\rangle, \ldots$

2^L states

$$H = \sum_{n=1}^{L-1} \vec{S}_n \cdot \vec{S}_{n+1}$$

x, y, z

$$H = \frac{1}{4} \sum_{n=0}^{L-2} \sum_{i=1}^{3} \mathbb{1}_{2^n \times 2^n} \otimes \sigma^i_{n+1} \otimes \sigma^i_{n+2} \otimes$$


$\lim_{L\to\infty} \frac{E_0}{L} = -\log(2) + \frac{1}{4}$
Bethe
L sites
states = {|↑↑↓↑↓...>, ...
$H = \sum_{n=1}^{L-1} \vec{S}_n \cdot \vec{S}_{n+1}$
vacum:
|↑↑...↑>
$\vec{S}_n \cdot \vec{S}_{n+1} = \frac{1}{2}\left((S_n + S_{n+1})^2 - S_n^2 - S_{n+1}^2\right)$
...|↑↑....
$H = \frac{1}{4}\sum_{n=0}^{L-2}\sum_{m=1}^{3} \mathbb{1}$

$$\lim_{L\to\infty} \frac{E_0}{L} = -\log(2) + \frac{1}{4}$$

Bethe

vacum:
$|\uparrow\uparrow\ldots\uparrow\rangle$

$\underbrace{|\uparrow\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow\uparrow\rangle}_{L \text{ sites}}$

states $= \{ \underbrace{|\uparrow\uparrow\downarrow\uparrow\downarrow\ldots\rangle, \ldots}_{2^L \text{ states}}$

if it wasn't a mistake

$$H = \sum_{n=1}^{L-1} \vec{S}_n \cdot \vec{S}_{n+1}$$

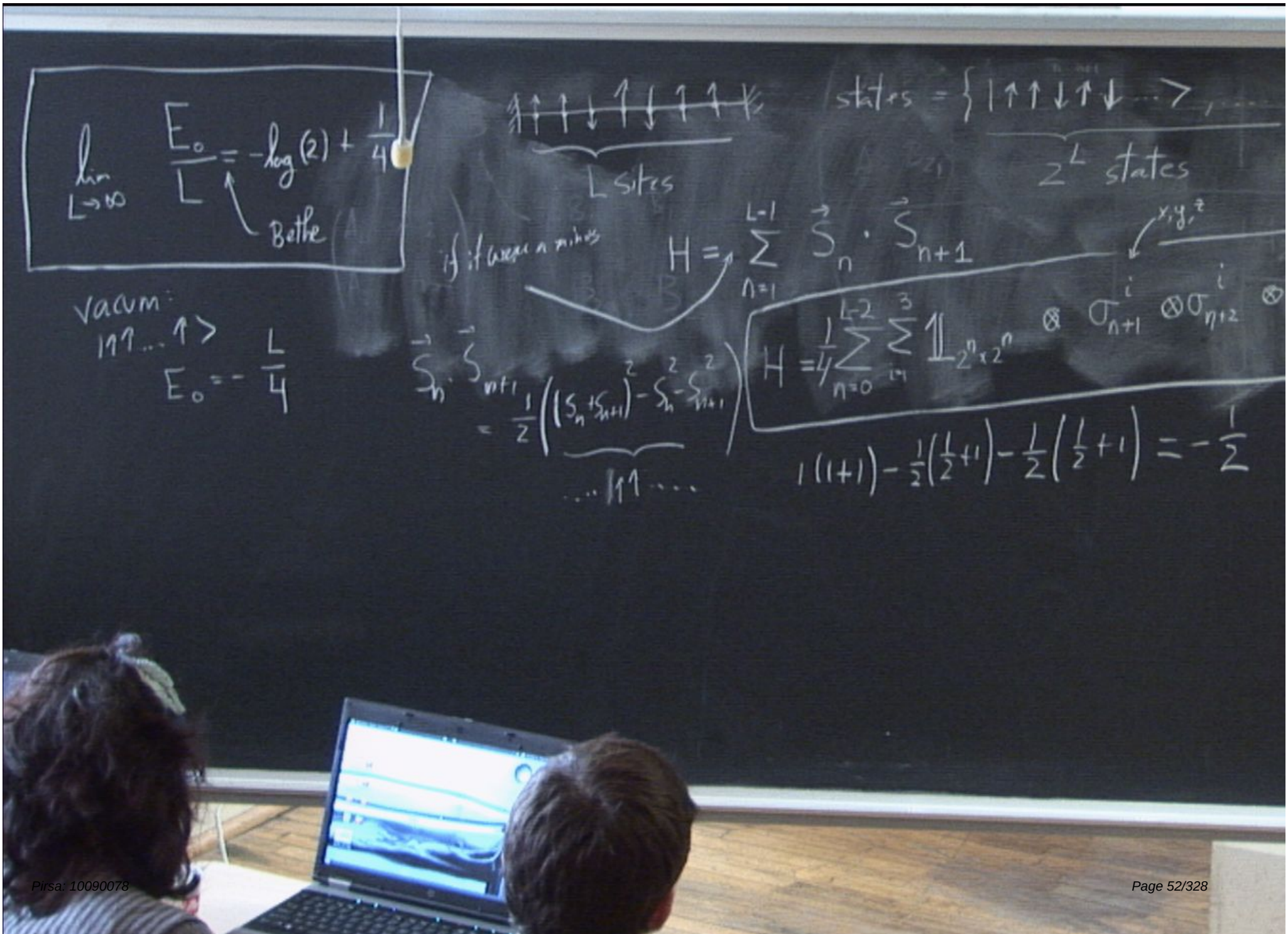
x, y, z

$$\vec{S}_n \cdot \vec{S}_{n+1} = \frac{1}{2}\Big(\underbrace{(S_n + S_{n+1})^2}_{\ldots|\uparrow\uparrow\ldots} - S_n^2 - S_{n+1}^2 \Big)$$

$$\boxed{H = \frac{1}{4} \sum_{n=0}^{L-2} \sum_{i=1}^{3} \mathbb{1}_{2^n \times 2^n} \otimes \sigma^i_{n+1} \otimes \sigma^i_{n+2} \otimes}$$

$$1(1+1) - \frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right) =$$

$\lim_{L\to\infty} \frac{E_0}{L} = -\log(2) + \frac{1}{4}$
Bethe
states = $\{|\uparrow\uparrow\downarrow\uparrow\downarrow\ldots\rangle, \ldots$
2^L states
$\sum_{n=1}^{L-1} \vec{S}_n \cdot \vec{S}_{n+1}$
x, y, z
vacum:
$|\uparrow\uparrow\ldots\uparrow\rangle$
$= \frac{1}{4}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{1}_{2^n\times 2^n} \otimes \sigma^i_{n+1} \otimes \sigma^i_{n+2} \otimes$
$(1+1) - \frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right) = -\frac{1}{2}$


$\lim_{L\to\infty} \frac{E_0}{L} = -\log(2) + \frac{1}{4}$
Bethe
L sites
states = { |↑↑↓↑↓ ...> , ...
2^L states
x, y, z
$H = \sum_{n=1}^{L-1} \vec{S}_n \cdot \vec{S}_{n+1}$
vacum:
|↑↑...↑>
$E_0 = -\frac{L}{4}$
$\vec{S}_n \cdot \vec{S}_{n+1} = \frac{1}{2}\left((S_n + S_{n+1})^2 - S_n^2 - S_{n+1}^2\right)$
...|↑↑....
$H = \frac{1}{4}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{1}_{2^n\times 2^n} \otimes \sigma^i_{n+1} \otimes \sigma^i_{n+2} \otimes$
$1(1+1) - \frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right) = -\frac{1}{2}$

$$\lim_{L\to\infty} \frac{E_0}{L} = -\log(2) + \frac{1}{4}$$

Bethe

$\underbrace{\uparrow\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow\uparrow}_{L \text{ sites}}$

states $= \{ \underbrace{|\uparrow\uparrow\downarrow\uparrow\downarrow\cdots\rangle, \ldots}_{2^L \text{ states}}$

if it wasn't a mistake

$$H = \sum_{n=1}^{L-1} \vec{S}_n \cdot \vec{S}_{n+1}$$

x, y, z

vacum:

$|\uparrow\uparrow\ldots\uparrow\rangle$

$E_0 = -\frac{L}{4}$

$$\vec{S}_n \cdot \vec{S}_{n+1} = \frac{1}{2}\Big(\underbrace{(S_n + S_{n+1})^2}_{\ldots|\uparrow\uparrow\ldots} - S_n^2 - S_{n+1}^2 \Big)$$

$$H = \frac{1}{4} \sum_{n=0}^{L-2} \sum_{i=1}^{3} \mathbb{1}_{2^n \times 2^n} \otimes \sigma^i_{n+1} \otimes \sigma^i_{n+2} \otimes$$

$$1(1+1) - \frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right) = -\frac{1}{2}$$

$$\frac{E_0}{L} = -\log(2) + \frac{1}{4}$$

Bethe

$$\underbrace{\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow}_{L \text{ sites}}$$

states $= \{ \underbrace{|\uparrow\uparrow\downarrow\uparrow\downarrow\cdots\rangle, \ldots}_{2^L \text{ states}} \}$

if it takes n steps

$$H = \sum_{n=1}^{L-1} \vec{S}_n \cdot \vec{S}_{n+1}$$

x, y, z

$$H = \frac{1}{4} \sum_{n=0}^{L-2} \sum_{i=1}^{3} \mathbb{1}_{2^n \times 2^n} \otimes \sigma^i_{n+1} \otimes \sigma^i_{n+2} \otimes \mathbb{1}_{2^{L-n-2} \times 2^{L-n-2}}$$

$$\vec{S}_n \cdot \vec{S}_{n+1} = \frac{1}{2}\left(\underbrace{(S_n + S_{n+1})^2}_{\cdots\uparrow\uparrow\cdots} - S_n^2 - S_{n+1}^2 \right)$$

$$1(1+1) - \frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right) = -\frac{1}{2}$$

$$|\uparrow\rangle$$

$$E_0 = -\frac{L}{4}$$

$$\frac{E_0}{L} = -\log(2) + \frac{1}{4}$$

Bethe

$$\underbrace{\uparrow\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow\uparrow}_{L \text{ sites}}$$

states $= \{ \underbrace{|\uparrow\uparrow\downarrow\uparrow\downarrow \cdots\rangle, \ldots}_{2^L \text{ states}} \}$

if it wasn't a ring

$$H = \sum_{n=1}^{L-1} \vec{S}_n \cdot \vec{S}_{n+1}$$

x, y, z

$$H = \frac{1}{4} \sum_{n=0}^{L-2} \sum_{i=1}^{3} \mathbb{1}_{2^n \times 2^n} \otimes \sigma^i_{n+1} \otimes \sigma^i_{n+2} \otimes \mathbb{1}_{2^{L-n-2} \times 2^{L-n-2}}$$

$$\vec{S}_n \cdot \vec{S}_{n+1} = \frac{1}{2}\Big(\underbrace{(S_n + S_{n+1})^2}_{\cdots\uparrow\uparrow\cdots} - S_n^2 - S_{n+1}^2 \Big)$$

$$1(1+1) - \frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right) = -\frac{1}{2}$$

$$|\uparrow\rangle$$

$$E_0 = -\frac{L}{4}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Exact diagonalization of the XXX spin spin chain

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Exact diagonalization of the XXX spin spin chain

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Exact diagonalization of the XXX Heisespin spin chain

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Exact diagonalization of the XXX Heisenberg spin spin chain

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Exact diagonalization of the XXX Heisenberg spin chain

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Exact diagonalization of the XXX Heisenberg spin 1/2 chain

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Exact diagonalization of the XXX (sotro

Heisenberg spin 1/2 chain

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Exact diagonalization of the XXX (sotropic) Heisenberg spin 1/2 chain

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Exact diagonalization of the XXX (isotropic) Heisenberg spin 1/2 chain

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Exact diagonalization of the XXX (isotropic) Heisenberg spin 1/2 chain

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Exact diagonalization of the XXX (isotropic) Heisenberg spin 1/2 chain

Spin operators

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Exact diagonalization of the XXX (isotropic) Heisenberg spin 1/2 chain

Spin operators

```
Clear
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Exact diagonalization of the XXX (isotropic) Heisenberg spin 1/2 chain

Spin operators

```
ClearAll[S]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Exact diagonalization of the XXX (isotropic) Heisenberg spin 1/2 chain

Spin operators

```
ClearAll[S]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

```
ClearAll[S]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

```
ClearAll[S]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

```
S[a
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

```
S[i_] := PauliMatrix[i]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

```
S[i_] := PauliMatrix[i] / 2;
```

```
dim :=
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

```
S[i_] := PauliMatrix[i] / 2;
```

```
dim := Length[S[1]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[141]:=
```
dim := Length[S[1]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[142]:=
```
Length[S[1]]
```

Out[142]=
```
2
```

In[141]:=
```
dim := Length[S[1]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[142]:=
```
Length[S[1]]
```

Out[142]=
```
2
```

```
S[
```

In[141]:=
```
dim := Length[S[1]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[142]:=
```
Length[S[1]]
```

Out[142]= 2

In[143]:=
```
S[1]
```

Out[143]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

In[141]:=
```
dim := Length[S[1]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
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In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
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In[142]:=
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Length[S[1]]
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Out[142]= 2

In[143]:=
```
S[1]
```

Out[143]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

```
Length
```

In[141]:=
```
dim := Length[S[1]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[142]:=
```
Length[S[1]]
```

Out[142]= 2

In[143]:=
```
S[1]
```

Out[143]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

In[144]:=
```
Length[{a, b}]
```

Out[144]= 2

In[141]:=
```
dim := Length[S[1]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[139]:=
```
ClearAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[142]:=
```
Length[S[1]]
```

Out[142]= 2

In[143]:=
```
S[1]
```

Out[143]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

In[144]:=
```
Length[{a, b}]
```

Out[144]= 2

In[141]:=
```
dim := Length[S[1]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[139]:=
```
ClearAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[142]:=
```
Length[S[1]]
```

Out[142]=
```
2
```

In[143]:=
```
S[1]
```

Out[143]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

In[144]:=
```
Length[{a, b}]
```

Out[144]=
```
2
```

In[141]:=
```
dim := Length[S[1]]
```

```
spin :=
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[142]:=
```
Length[S[1]]
```

Out[142]= 2

In[143]:=
```
S[1]
```

Out[143]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

In[144]:=
```
Length[{a, b}]
```

Out[144]= 2

In[141]:=
```
dim := Length[S[1]]
```

$$\text{spin} := \frac{\text{dim} - 1}{2}$$

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Untitled-25

In[142]:=
```
Length[S[1]]
```
Out[142]= 2

In[143]:=
```
S[1]
```
Out[143]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

In[144]:=
```
Length[{a, b}]
```
Out[144]= 2

In[141]:=
```
dim := Length[S[1]]
```

In[145]:=
```
spin := (dim - 1)/2
```

```
S[1]
```

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Untitled-25

In[142]:=
```
Length[S[1]]
```
Out[142]= 2

In[143]:=
```
S[1]
```
Out[143]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

In[144]:=
```
Length[{a, b}]
```
Out[144]= 2

In[141]:=
```
dim := Length[S[1]]
```

In[145]:= $\text{spin} := \frac{\text{dim} - 1}{2}$

```
S[1].S[2]
```

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Untitled-25

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[142]:=
```
Length[S[1]]
```

Out[142]= 2

In[143]:=
```
S[1]
```

Out[143]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

In[144]:=
```
Length[{a, b}]
```

Out[144]= 2

In[141]:=
```
dim := Length[S[1]]
```

In[145]:= $\text{spin} := \frac{\text{dim} - 1}{2}$

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Untitled-25

In[139]:=
```
CircuAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[141]:=
```
dim := Length[S[1]]
```

In[145]:=
```
spin := (dim - 1)/2
```

```
S[1].S[2]
```

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[141]:=
```
dim := Length[S[1]]
```

In[145]:=
```
spin := (dim - 1)/2
```

```
S[1].S[2]
```

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[141]:=
```
dim := Length[S[1]]
```

In[145]:=
$$\text{spin} := \frac{\text{dim} - 1}{2}$$

```
S[1].S[2] - S[2].S[1]
```

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[141]:=
```
dim := Length[S[1]]
```

In[145]:=
```
spin := (dim - 1)/2
```

```
S[1].S[2] - S[2].S[1] == I S[3]
```

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[141]:=
```
dim := Length[S[1]]
```

In[145]:=
```
spin := (dim - 1)/2
```

```
S[1].S[2] - S[2].S[1] == I S[3],
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[141]:=
```
dim := Length[S[1]]
```

In[145]:=
```
spin := (dim - 1)/2
```

```
S[1].S[2] - S[2].S[1] == I S[3],
S[1].S[2] - S[2].S[1] == I S[3],
S[1].S[2] - S[2].S[1] == I S[3]
```

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[141]:=
```
dim := Length[S[1]]
```

In[145]:=
```
spin := (dim - 1)/2
```

```
S[1].S[2] - S[2].S[1] == I S[3],
S[2].S[3] - S[3].S[2] == I S[1],
S[3].S[1] - S[1].S[3] == I S[3]
```

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[141]:=
```
dim := Length[S[1]]
```

In[145]:=
```
spin := (dim - 1)/2
```

```
And[S[1].S[2] - S[2].S[1] == I S[3],
 S[2].S[3] - S[3].S[2] == I S[1],
 S[3].S[1] - S[1].S[3] == I S[2]
```

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[141]:=
```
dim := Length[S[1]]
```

In[145]:=
```
spin := (dim - 1)/2
```

```
And[
 S[1].S[2] - S[2].S[1] == I S[3],
 S[2].S[3] - S[3].S[2] == I S[1],
 S[3].S[1] - S[1].S[3] == I S[2]]
```

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

In[141]:=
```
dim := Length[S[1]]
```

In[145]:=
$$\text{spin} := \frac{\text{dim} - 1}{2}$$

```
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]]
```

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

```
S
```

In[141]:=
```
dim := Length[S[1]]
```

In[145]:=
```
spin := (dim - 1)/2
```

```
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]]
```

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[140]:=
```
S[i_] := PauliMatrix[i] / 2;
```

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dim := Length[S[1]]
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In[145]:=
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spin := (dim - 1)/2
```

```
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]]
```

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
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S[i_] = PauliMatrix[i] / 2;
```

In[141]:=
```
dim := Length[S[1]]
```

In[145]:=
$$\text{spin} := \frac{\text{dim} - 1}{2}$$

```
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]]
```

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[146]:=
```
S[i_] = PauliMatrix[i] / 2
```

Out[146]=

$$\frac{\text{PauliMatrix}[i]}{2}$$

In[141]:=
```
dim := Length[S[1]]
```

In[145]:=

$$\text{spin} := \frac{\text{dim} - 1}{2}$$

```
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]]
```

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[146]:=
```
S[i_] = PauliMatrix[i] / 2
```

Out[146]=

$$\frac{\text{PauliMatrix}[i]}{2}$$

In[141]:=
```
dim := Length[S[1]]
```

In[145]:=

$$\text{spin} := \frac{\text{dim} - 1}{2}$$

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check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]]
```

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Untitled-25

Spin operators

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```

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dim := Length[S[1]]
```

In[145]:=
```
spin := (dim - 1)/2
```

```
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]]
```

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Untitled-25

Spin operators

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In[145]:=
$$\text{spin} := \frac{\text{dim} - 1}{2}$$

In[149]:=
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  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]]
```

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In[139]:=
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```
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```
dim := Length[S[1]]
```

In[145]:=
$$\text{spin} := \frac{\text{dim} - 1}{2}$$

In[149]:=
```
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]]
```

```
dim
```

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[148]:=
```
S[i_] = PauliMatrix[i] / 2;
```

In[141]:=
```
dim := Length[S[1]]
```

In[145]:=
```
spin := (dim - 1)/2
```

In[149]:=
```
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]]
```

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Untitled-25

Spin operators

In[139]:=
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In[148]:=
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In[141]:=
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dim := Length[S[1]]
```

In[145]:=
```
spin := (dim - 1)/2
```

In[149]:=
```
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]]
```

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[148]:=
```
S[i_] = PauliMatrix[i] / 2;
```

In[141]:=
```
dim := Length[S[1]]
```

In[145]:=
$$\text{spin} := \frac{\text{dim} - 1}{2}$$

In[149]:=
```
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]]
```

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[148]:=
```
S[i_] = PauliMatrix[i] / 2;
```

In[141]:=
```
dim := Length[S[1]]
```

In[145]:=
```
spin := (dim - 1)/2
```

In[149]:=
```
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]]
```

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[148]:=
```
S[i_] = PauliMatrix[i] / 2;
```

```
dim := Length[S[1]]
```

$$\text{spin} := \frac{\text{dim} - 1}{2}$$

In[149]:=
```
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]]
```

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[148]:=
```
S[i_] = PauliMatrix[i] / 2;
```

```
dim := Length[S[1]]
```

$$\text{spin} := \frac{\text{dim} - 1}{2}$$

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Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[148]:=
```
S[i_] = PauliMatrix[i] / 2;
```

```
dim := Length[S[1]]
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]]
```

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Untitled-25

In[139]:=
```
ClearAll[S]
```

In[148]:=
```
S[i_] = PauliMatrix[i] / 2;
```

```
dim := Length[S[1]]
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

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Untitled-25

In[139]:=
```
ClearAll[S]
```

In[148]:=
```
S[i_] = PauliMatrix[i] / 2;
```

In[150]:=
```
dim := Length[S[1]]
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

```
dim
```

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Untitled-25

In[139]:=
```
ClearAll[S]
```

In[148]:=
```
S[i_] = PauliMatrix[i] / 2;
```

In[150]:=
```
dim := Length[S[1]]
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```

Out[153]=
```
2
```

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Untitled-25

In[148]:=
```
S[i_] = PauliMatrix[i] / 2;
```

In[150]:=
```
dim := Length[S[1]]
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```

Out[153]= 2

In[154]:=
```
spin
```

Out[154]= $\frac{1}{2}$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[150]:=
```
dim := Length[S[1]]
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```

Out[153]= 2

In[154]:=
```
spin
```

Out[154]= $\frac{1}{2}$

```
check
```

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Untitled-25

```
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```
Out[153]=
```
2
```

In[154]:=
```
spin
```
Out[154]=

$$\frac{1}{2}$$

In[155]:=
```
check
```
Out[155]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[150]:=
```
dim := Length[S[1]]
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```

Out[153]= 2

In[154]:=
```
spin
```

Out[154]= $\frac{1}{2}$

In[155]:=
```
check
```

Out[155]= True

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Untitled-25

```
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```
Out[153]=
```
2
```

In[154]:=
```
spin
```
Out[154]=

$$\frac{1}{2}$$

In[155]:=
```
check
```
Out[155]=
```
True
```

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Untitled-25

```
2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```
Out[153]=
```
2
```

In[154]:=
```
spin
```
Out[154]=

$$\frac{1}{2}$$

In[155]:=
```
check
```
Out[155]=
```
True
```

Suppos

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Untitled-25

```
                2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```
Out[153]=
```
2
```

In[154]:=
```
spin
```
Out[154]=

$$\frac{1}{2}$$

In[155]:=
```
check
```
Out[155]=
```
True
```

Suppose we want spin 1

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Untitled-25

```
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```

Out[153]=
```
2
```

In[154]:=
```
spin
```

Out[154]= $\frac{1}{2}$

In[155]:=
```
check
```

Out[155]=
```
True
```

Suppose we want spin 1

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```

Out[153]=
```
2
```

In[154]:=
```
spin
```

Out[154]= $\frac{1}{2}$

In[155]:=
```
check
```

Out[155]=
```
True
```

Suppose we want spin 1

```
a[a__]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```

Out[153]=
```
2
```

In[154]:=
```
spin
```

Out[154]= $\frac{1}{2}$

In[155]:=
```
check
```

Out[155]=
```
True
```

Suppose we want spin 1

```
s[a__] := Signature[{a}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```

Out[153]=
```
2
```

In[154]:=
```
spin
```

Out[154]=
$$\frac{1}{2}$$

In[155]:=
```
check
```

Out[155]=
```
True
```

Suppose we want spin 1

```
ϵ[a__] := Signature[{a}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```

Out[153]=
```
2
```

In[154]:=
```
spin
```

Out[154]=
$$\frac{1}{2}$$

In[155]:=
```
check
```

Out[155]=
```
True
```

Suppose we want spin 1

```
ϵ[a__] := Signature[{a}]
S[I
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```

Out[153]=
```
2
```

In[154]:=
```
spin
```

Out[154]=

$$\frac{1}{2}$$

In[155]:=
```
check
```

Out[155]=
```
True
```

Suppose we want spin 1

```
ϵ[a__] := Signature[{a}]
S[i_] :=
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```

Out[153]=
```
2
```

In[154]:=
```
spin
```

Out[154]=

$$\frac{1}{2}$$

In[155]:=
```
check
```

Out[155]=
```
True
```

Suppose we want spin 1

```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j|
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```

Out[153]=
```
2
```

In[154]:=
```
spin
```

Out[154]=

$$\frac{1}{2}$$

In[155]:=
```
check
```

Out[155]=
```
True
```

Suppose we want spin 1

```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```

Out[153]=
```
2
```

In[154]:=
```
spin
```

Out[154]= $\frac{1}{2}$

In[155]:=
```
check
```

Out[155]=
```
True
```

Suppose we want spin 1

In[156]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[154]= $\frac{1}{2}$

In[155]:=
```
check
```

Out[155]= True

Suppose we want spin 1

In[156]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[154]= $\frac{1}{2}$

In[155]:=
```
check
```

Out[155]= True

Suppose we want spin 1

In[156]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

In[158]:=
```
S[1]
```

Out[158]= {{0, 0, 0}, {0, 0, -i}, {0, i, 0}}

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[154]= $\frac{1}{2}$

In[155]:=
```
check
```

Out[155]= True

Suppose we want spin 1

In[156]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

In[158]:=
```
S[1]
```

Out[158]= {{0, 0, 0}, {0, 0, -i}, {0, i, 0}}

```
dim
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[148]:=
```
S[i_] = PauliMatrix[i] / 2;
```

In[150]:=
```
dim := Length[S[1]]
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```

Out[153]= 2

In[154]:=
```
spin
```

Out[154]= $\frac{1}{2}$

In[155]:=
```
check
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[154]= $\frac{1}{2}$

In[155]:=
```
check
```

Out[155]= True

Suppose we want spin 1

In[156]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

In[158]:=
```
S[1]
```

Out[158]= {{0, 0, 0}, {0, 0, -i}, {0, i, 0}}

```
dim
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[154]= $\frac{1}{2}$

In[155]:=
```
check
```

Out[155]= True

Suppose we want spin 1

In[156]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

In[158]:=
```
S[1]
```

Out[158]= {{0, 0, 0}, {0, 0, -i}, {0, i, 0}}

In[159]:=
```
dim
```

Out[159]= 3

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
2
```

In[155]:=
```
check
```

Out[155]=
```
True
```

Suppose we want spin 1

In[156]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

In[158]:=
```
S[1]
```

Out[158]=
```
{{0, 0, 0}, {0, 0, -i}, {0, i, 0}}
```

In[159]:=
```
dim
```

Out[159]=
```
3
```

```
spin
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[155]:=
```
check
```
Out[155]=
```
True
```

Suppose we want spin 1

In[156]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

In[158]:=
```
S[1]
```
Out[158]=
```
{{0, 0, 0}, {0, 0, -i}, {0, i, 0}}
```

In[159]:=
```
dim
```
Out[159]=
```
3
```

In[160]:=
```
spin
```
Out[160]=
```
1
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[155]= True

Suppose we want spin 1

In[156]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

In[158]:=
```
S[1]
```

Out[158]= {{0, 0, 0}, {0, 0, -i}, {0, i, 0}}

In[159]:=
```
dim
```

Out[159]= 3

In[160]:=
```
spin
```

Out[160]= 1

```
check
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[156]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

In[158]:=
```
S[1]
```

Out[158]=
```
{{0, 0, 0}, {0, 0, -i}, {0, i, 0}}
```

In[159]:=
```
dim
```

Out[159]=
```
3
```

In[160]:=
```
spin
```

Out[160]=
```
1
```

In[161]:=
```
check
```

Out[161]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Suppose we want spin 1

In[156]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

In[158]:=
```
S[1]
```

Out[158]=
```
{{0, 0, 0}, {0, 0, -i}, {0, i, 0}}
```

In[159]:=
```
dim
```

Out[159]=
```
3
```

In[160]:=
```
spin
```

Out[160]=
```
1
```

In[161]:=
```
check
```

Out[161]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Suppose we want spin 1

In[162]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

In[158]:=
```
S[1]
```

Out[158]=
```
{{0, 0, 0}, {0, 0, -i}, {0, i, 0}}
```

In[159]:=
```
dim
```

Out[159]=
```
3
```

In[160]:=
```
spin
```

Out[160]=
```
1
```

In[161]:=
```
check
```

Out[161]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[162]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

In[158]:=
```
S[1]
```

Out[158]=
```
{{0, 0, 0}, {0, 0, -i}, {0, i, 0}}
```

In[159]:=
```
dim
```

Out[159]=
```
3
```

In[160]:=
```
spin
```

Out[160]=
```
1
```

In[164]:=
```
check
```

Out[164]=
```
False
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Suppose we want spin 1

In[165]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

In[158]:=
```
S[1]
```

Out[158]=
```
{{0, 0, 0}, {0, 0, -i}, {0, i, 0}}
```

In[159]:=
```
dim
```

Out[159]=
```
3
```

In[160]:=
```
spin
```

Out[160]=
```
1
```

In[164]:=
```
check
```

Out[164]=
```
False
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[165]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

In[167]:=
```
S[1]
```

Out[167]=
```
{{0, 0, 0}, {0, 0, -i}, {0, i, 0}}
```

In[168]:=
```
dim
```

Out[168]=
```
3
```

In[169]:=
```
spin
```

Out[169]=
```
1
```

In[170]:=
```
check
```

Out[170]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Suppose we want spin 1

In[165]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

In[167]:=
```
S[1]
```

Out[167]=
```
{{0, 0, 0}, {0, 0, -i}, {0, i, 0}}
```

In[168]:=
```
dim
```

Out[168]=
```
3
```

In[169]:=
```
spin
```

Out[169]=
```
1
```

In[170]:=
```
check
```

Out[170]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[155]:=
```
check
```
Out[155]=
```
True
```

Suppose we want spin 1

In[165]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

In[167]:=
```
S[1]
```
Out[167]=
```
{{0, 0, 0}, {0, 0, -i}, {0, i, 0}}
```

In[168]:=
```
dim
```
Out[168]=
```
3
```

In[169]:=
```
spin
```
Out[169]=
```
1
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[155]:=
```
check
```
Out[155]=
```
True
```

Suppose we want spin 1

In[165]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

In[167]:=
```
S[1]
```
Out[167]=
```
{{0, 0, 0}, {0, 0, -i}, {0, i, 0}}
```

In[168]:=
```
dim
```
Out[168]=
```
3
```

In[169]:=
```
spin
```
Out[169]=
```
1
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
spin := 1/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

In[153]:=
```
dim
```
Out[153]=
```
2
```

In[154]:=
```
spin
```
Out[154]=

$$\frac{1}{2}$$

In[155]:=
```
check
```
Out[155]=
```
True
```

Suppose we want spin 1

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
spin := 1/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[165]:=
```
ϵ[a__] := Signature[{a}]
S[i_] := -I Table[ϵ[i, j, k], {j, 3}, {k, 3}]
```

In[167]:=
```
S[1]
```

Out[167]=
```
{{0, 0, 0}, {0, 0, -i}, {0, i, 0}}
```

In[168]:=
```
dim
```

Out[168]=
```
3
```

In[169]:=
```
spin
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
spin := 1/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

```
(*ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}]
```

In[167]:=
```
S[1]
```

Out[167]=
```
{{0, 0, 0}, {0, 0, -i}, {0, i, 0}}
```

In[168]:=
```
dim
```

Out[168]=
```
3
```

In[169]:=
```
spin
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
spin := 1/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

In[168]:=
```
dim
```

Out[168]=
```
3
```

In[169]:=
```
spin
```

Out[169]=
```
1
```

In[170]:=
```
check
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[139]:=
```
ClearAll[S]
```

In[148]:=
```
S[i_] = PauliMatrix[i] / 2;
```

In[150]:=
```
dim := Length[S[1]]
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

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Untitled-25

In[172]:=
```
S[i_] = PauliMatrix[i] / 2;
```

In[173]:=
```
dim := Length[S[1]]
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

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Untitled-25

In[172]:=
```
S[i_] = PauliMatrix[i] / 2;
```

In[173]:=
```
dim := Length[S[1]]
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

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Untitled-25

```
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

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Untitled-25

```
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

Tens

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Untitled-25

```
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

Tensor product

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

Tensor product

```
C
```

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Untitled-25

```
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

Tensor product

```
CircleTimes = KroneckerPR
```

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Untitled-25

```
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
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In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

```
S[1]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
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(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

```
S[1] ⊗ S[1] + S[1] ⊗ S[1]
```

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Untitled-25

```
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
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(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
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In[176]:=
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S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

```
S[1] ⊗ S[1] + S[1] ⊗ S[1] + S[1] ⊗ S[1]
```

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Untitled-25

In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

```
S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

```
:c
```

```
S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]
```

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Untitled-25

In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

```
c*
```

```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

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Untitled-25

In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

```
c*
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

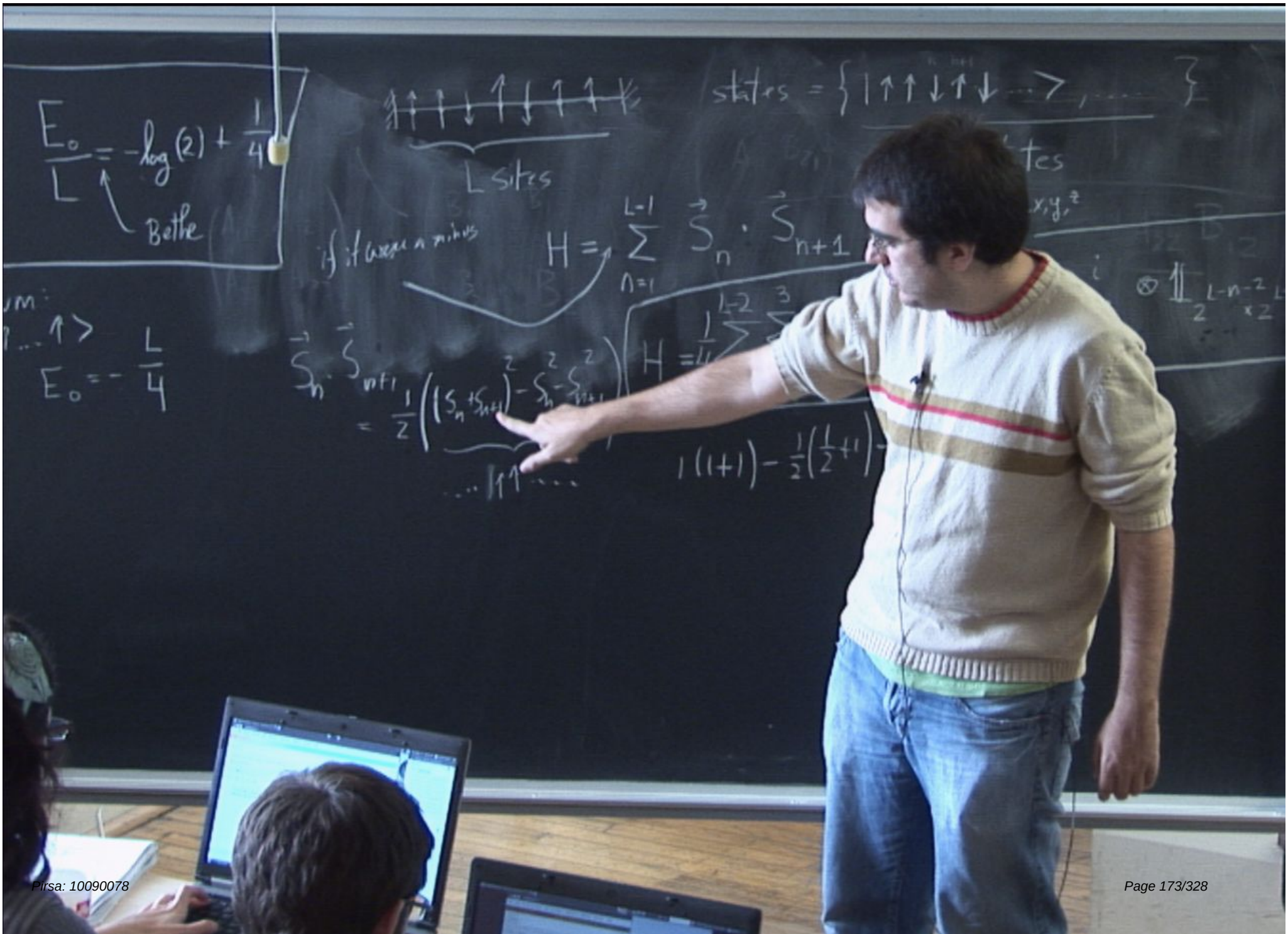
Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

```
c*
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$


E₀/L = -log(2) + 1/4
Bethe
L sites
states = { |↑↑↓↑↓ ...> , ... }
H = Σ_{n=1}^{L-1} S_n · S_{n+1}
E₀ = - L/4

$$\frac{E_0}{L} = -\log(2) + \frac{1}{4}$$

Bethe

um:

$|\uparrow \ldots \uparrow\rangle$

$$E_0 = -\frac{L}{4}$$

$\underbrace{\uparrow\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow\uparrow}_{L \text{ sites}}$

states $= \{ \underbrace{|\uparrow\uparrow\downarrow\uparrow\downarrow \ldots\rangle, \ldots}_{2^L \text{ states}} \}$

if it takes n minus

$$H = \sum_{n=1}^{L-1} \vec{S}_n \cdot \vec{S}_{n+1}$$

x, y, z

$$\vec{S}_n \cdot \vec{S}_{n+1} = \frac{1}{2}\Big(\underbrace{(S_n + S_{n+1})^2}_{\ldots\uparrow\uparrow\ldots} - S_n^2 - S_{n+1}^2 \Big)$$

$$H = \frac{1}{4} \sum_{n=0}^{L-2} \sum_{i=1}^{3} \mathbb{1}_{2^n \times 2^n} \otimes \sigma^i_{n+1} \otimes \sigma^i_{n+2} \otimes \mathbb{1}_{2^{L-n-2} \times 2^{\ldots}}$$

$$1(1+1) - \frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right) = -\frac{1}{2}$$

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Untitled-25

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

```
c* , dsI
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Hamiltonian

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Untitled-25

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

```
c* , dsI
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Hamiltonian

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Untitled-25

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

```
c* , dsI
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Hamiltonian

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Untitled-25

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

```
c* , dsI
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Hamiltonian

```
I[
```

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Untitled-25

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

```
c* , dsI
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Hamiltonian

```
I[n_]
```

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Untitled-25

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

```
c* , dsI
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Hamiltonian

```
I[n_] := IdentityMatrix
```

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Untitled-25

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

```
c* , dsI
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Hamiltonian

```
I[n_] := IdentityMatrix[2^n]
```

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Untitled-25

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

```
c* , dsI
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

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Untitled-25

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

```
c* , dsI
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

```
ds
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
c* , dsI
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[180]:=
```
I[3]
```

Out[180]=
```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}
```

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Untitled-25

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
```

Out[181]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
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In[181]:=
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I[3] // MatrixForm
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Out[181]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
ds
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Hamiltonian

In[179]:=
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I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
```

Out[181]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

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Untitled-25

Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
```

Out[181]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

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Untitled-25

Hamiltonian

In[179]:=
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I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
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Out[181]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
I[|
```

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Untitled-25

Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
```

Out[181]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
I[n] ⊗ S[i] ⊗ S[i] ⊗ I[
```

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Untitled-25

In[177]:=
```
CircleTimes = KroneckerProduct;
```

```
c* , dsI
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]=
$$\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$$

Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
```

Out[181]//MatrixForm=
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
```

Out[181]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
I[n] ⊗ S[i] ⊗ S[i] ⊗ I[L - n - 2]
```

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Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
```

Out[181]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
c* , dsI
```

```
I[n] ⊗ S[i] ⊗ S[i] ⊗ I[L - n - 2]
```

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Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
```

Out[181]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
c* , dsI ,
```

$$\sum I[n] \otimes S[i] \otimes S[i] \otimes I[L - n - 2]$$

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Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
```

Out[181]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
c* , dsI ,
```

$$\sum_{i=1} I[n] \otimes S[i] \otimes S[i] \otimes I[L - n - 2]$$

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Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
```

Out[181]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
c* , dsI ,
```

$$\sum_{i=1}^{n} I[n] \otimes S[i] \otimes S[i] \otimes I[L - n - 2]$$

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Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
```

Out[181]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
c* , dsI ,
```

$$\sum_{i=1}^{3} I[n] \otimes S[i] \otimes S[i] \otimes I[L - n - 2]$$

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Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
```

Out[181]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
c* , dsI ,
```

$$\sum_{i=1}^{3} I[n] \otimes S[i] \otimes S[i] \otimes I[L - n - 2]$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
```

Out[181]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
c* , dsI ,
```

$$\sum \sum_{i=1}^{3} I[n] \otimes S[i] \otimes S[i] \otimes I[L - n - 2]$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
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I[3] // MatrixForm
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```
c* , dsI ,
```

$$\sum_{\square}\sum_{i=1}^{3} \mathbb{I}[n] \otimes S[i] \otimes S[i] \otimes \mathbb{I}[L - n - 2]$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
```

Out[181]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
c* , dsI ,
```

$$\sum_{n=0}^{} \sum_{i=1}^{3} \mathbb{I}[n] \otimes S[i] \otimes S[i] \otimes \mathbb{I}[L - n - 2]$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
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I[3] // MatrixForm
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```
c* , dsI ,
```

$$\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2]$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[179]:=
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I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
```

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```
c* , dsI ,
```

$$\text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2]\right]$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

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In[179]:=
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I[n_] := IdentityMatrix[dim^n];
```

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```
I[3] // MatrixForm
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```
c* , dsI ,
```

```
max = 1
```

$$\text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2],\ \{L,\ 2,\ \text{max}\}\right]$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
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```
c* , dsI ,
```

```
max = 10;
```

$$\text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2],\ \{L,\ 2,\ \text{max}\}\right]$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
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Out[181]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
c* , dsI ,
```

```
max = 10;
```

$$\text{HTable}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2],\ \{L,\ 2,\ \text{max}\}\right]$$

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Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
```

Out[181]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
c* , dsI ,
```

```
max = 10;
```

$$Hs = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2],\ \{L,\ 2,\ max\}\right]$$

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Untitled-25

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]=

$$\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$$

Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
```

Out[181]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
c* , dsI ,
```

```
max = 10;
```

$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n] \otimes S[i] \otimes S[i] \otimes \mathbb{I}[L-n-2],\ \{L, 2, \text{max}\}\right]$$

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Untitled-25

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
c* , dsI ,
```

In[182]:=
```
max = 10;
```

$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n] \otimes S[i] \otimes S[i] \otimes \mathbb{I}[L-n-2],\ \{L, 2, \text{max}\}\right];$$

```
2
```

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Untitled-25

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
c* , dsI ,
```

In[182]:=
```
max = 10;
```

$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,\ 2,\ \text{max}\}\right];$$

In[183]:= 2^{10}

Out[183]= 1024

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Running...Untitled-25

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
c* , dsI ,
```

In[182]:=
```
max = 10;
```

In[184]:=

$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,2,\text{max}\}\right];$$

In[183]:= 2^{10}

Out[183]= 1024

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Untitled-25

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
c* , dsI ,
```

In[182]:=
```
max = 10;
```

In[184]:=

$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n] \otimes S[i] \otimes S[i] \otimes \mathbb{I}[L-n-2],\ \{L, 2, \text{max}\}\right];$$

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Untitled-25

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
c* , dsI ,
```

In[182]:=
```
max = 10;
```

In[184]:=
```
Hs = Table[1/L Sum[Sum[I[n] ⊗ S[i] ⊗ S[i] ⊗ I[L - n - 2], {i, 1, 3}], {n, 0, L-2}], {L, 2, max}];
```

```
Hs
```

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Untitled-25

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
c* , dsI ,
```

In[182]:=
```
max = 10;
```

In[184]:=

$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n] \otimes S[i] \otimes S[i] \otimes \mathbb{I}[L-n-2],\ \{L, 2, \text{max}\}\right];$$

In[185]:=
```
Hs[[1]]
```

Out[185]=

$$\left\{\left\{\frac{1}{8}, 0, 0, 0\right\}, \left\{0, -\frac{1}{8}, \frac{1}{4}, 0\right\}, \left\{0, \frac{1}{4}, -\frac{1}{8}, 0\right\}, \left\{0, 0, 0, \frac{1}{8}\right\}\right\}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
c* , dsI ,
```

In[182]:=
```
max = 10;
```

In[184]:=

$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,\ 2,\ \text{max}\}\right];$$

In[186]:=
```
Hs[[2]]
```

Out[186]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

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Untitled-25

In[181]:=
```
I[3] // MatrixForm
```

In[182]:=
```
max = 10;
```

In[184]:=

$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,\ 2,\ \text{max}\}\right];$$

In[186]:=
```
Hs[[2]]
```

Out[186]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

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Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
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I[3] // MatrixForm
```

In[182]:=
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max = 10;
```

In[184]:=

$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2],\ \{L,\ 2,\ \text{max}\}\right];$$

In[186]:=
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Out[186]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

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Untitled-25

Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[181]:=
```
I[3] // MatrixForm
```

In[182]:=
```
max = 10;
```

In[184]:=

$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2],\ \{L,\ 2,\ \text{max}\}\right];$$

In[186]:=
```
Hs[[2]]
```

Out[186]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \right.$$

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Untitled-25

Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[182]:=
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max = 10;
```

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$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2],\ \{L,\ 2,\ \text{max}\}\right];$$

In[186]:=
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Hs[[2]]
```

Out[186]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\},\right.$$
$$\left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\},$$
$$\left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\},$$
$$\left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\}$$

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Untitled-25

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[182]:=
```
max = 10;
```

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Untitled-25

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

Tensor product

In[177]:=
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CircleTimes = KroneckerProduct;
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Hamiltonian

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Untitled-25

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Tensor product

In[177]:=
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CircleTimes = KroneckerProduct;
```

In[178]:=
```
Eigenvalues[S[1]
```
[3]]

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

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Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

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Untitled-25

In[172]:=
```
S[i_] = PauliMatrix[i] / 2;
```

In[173]:=
```
dim := Length[S[1]]
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

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Untitled-25

In[172]:=
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In[173]:=
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  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

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```

In[176]:=
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S[1]
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Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

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Untitled-25

In[172]:=
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  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

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```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
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In[176]:=
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S[1]
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Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

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Untitled-25

In[172]:=
```
S[i_] = PauliMatrix[i] / 2;
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In[173]:=
```
dim := Length[S[1]]
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

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Untitled-25

```
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

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Untitled-25

```
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

In[176]:=
```
S[1]
```

Out[176]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

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Untitled-25

```
 S[3].S[1] - S[1].S[3] == I S[2]
]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

In[187]:=
```
S[1]
dim
```

Out[187]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

Out[188]= 2

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

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Untitled-25

Out[188]= 2

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

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Untitled-25

In[182]:=
```
max = 10;
```

In[184]:=
```
Hs = Table[1/L Sum[Sum[I[n] ⊗ S[i] ⊗ S[i] ⊗ I[L - n - 2], {i, 1, 3}], {n, 0, L-2}], {L, 2, max}];
```

In[186]:=
```
Hs[[2]]
```

Out[186]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

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Untitled-25

In[182]:=
```
max = 10;
```

In[184]:=
```
Hs = Table[1/L Sum[Sum[I[n] ⊗ S[i] ⊗ S[i] ⊗ I[L - n - 2], {i, 1, 3}], {n, 0, L-2}], {L, 2, max}];
```

In[186]:=
```
Hs[[2]]
```

```
tb =
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[182]:=
```
max = 10;
```

In[184]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n] \otimes S[i] \otimes S[i] \otimes \mathbb{I}[L-n-2],\ \{L, 2, \text{max}\}\right];$$

In[186]:=
```
Hs[[2]]
```

```
tb = Table[{L, Hs[[L - 1]]
```

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Untitled-25

In[182]:=
```
max = 10;
```

In[184]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n] \otimes S[i] \otimes S[i] \otimes \mathbb{I}[L-n-2],\ \{L, 2, \text{max}\}\right];$$

In[186]:=
```
Hs[[2]]
```

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues
```

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Untitled-25

In[182]:=
```
max = 10;
```

In[184]:=
```
Hs = Table[1/L Sum[Sum[I[n] ⊗ S[i] ⊗ S[i] ⊗ I[L - n - 2], {i, 1, 3}], {n, 0, L-2}], {L, 2, max}];
```

In[186]:=
```
Hs[[2]]
```

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First
```

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Untitled-25

In[182]:=
```
max = 10;
```

In[184]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n] \otimes S[i] \otimes S[i] \otimes \mathbb{I}[L-n-2],\ \{L, 2, \text{max}\}\right];$$

In[186]:=
```
Hs[[2]]
```

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

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Untitled-25

In[182]:=
```
max = 10;
```

In[184]:=
```
Hs = Table[1/L Sum[Sum[I[n] ⊗ S[i] ⊗ S[i] ⊗ I[L - n - 2], {i, 1, 3}], {n, 0, L-2}], {L, 2, max}];
```

In[186]:=
```
Hs[[2]]
```

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[182]:=
```
max = 10;
```

In[184]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n] \otimes S[i] \otimes S[i] \otimes \mathbb{I}[L-n-2],\ \{L, 2, \text{max}\}\right];$$

In[186]:=
```
Hs[[2]]
```

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

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Untitled-25

In[182]:=
```
max = 10;
```

In[184]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,\ 2,\ \text{max}\}\right];$$

In[189]:=
```
Hs[[2]]
```

Out[189]=
$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

$$\left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\},$$

$$\left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\}$$

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[189]:=
```
Hs[[2]]
```

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[182]:=
```
max = 10;
```

In[184]:=
```
Hs = Table[1/L Sum[Sum[I[n] ⊗ S[i] ⊗ S[i] ⊗ I[L - n - 2], {i, 1, 3}], {n, 0, L-2}], {L, 2, max}];
```

In[189]:=
```
Hs[[2]]
```

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[189]:=
```
Hs[[2]]
```

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[189]:=
```
Hs[[2]]
```

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

```
Fac
```

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[189]:=
```
Hs[[2]]
```

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

```
FactorInteger[1
```

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

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Untitled-25

In[189]:=
```
Hs[[2]]
```

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

```
FactorInteger[10
```

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

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Untitled-25

In[189]:=
```
Hs[[2]]
```

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

```
FactorInteger[10^100]
```

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

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Untitled-25

In[189]:=
```
Hs[[2]]
```

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

In[190]:=
```
FactorInteger[10^100]
```

Out[190]=

$$\{\{2, 100\}, \{5, 100\}\}$$

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

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Running...Untitled-25

In[189]:=
```
Hs[[2]]
```

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

In[191]:=
```
FactorInteger[10^100 + 2]
```

Out[190]=
```
{{2, 100}, {5, 100}}
```

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

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Untitled-25

In[189]:=
```
Hs[[2]]
```

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

```
FactorInteger[10^21 + 2]
```

Out[191]=
```
$Aborted
```

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

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Untitled-25

In[189]:=
```
Hs[[2]]
```

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

In[192]:=
```
FactorInteger[10^20 + 2]
```

Out[192]=

$$\{\{2, 1\}, \{3, 1\}, \{155\,977\,777, 1\}, \{106\,852\,828\,571, 1\}\}$$

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

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Untitled-25

In[189]:=
```
Hs[[2]]
```

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

In[193]:=
```
FactorInteger[10^40 + 2]
```

Out[193]=

{{2, 1}, {3, 1}, {5 049 577 699, 1},
{506 774 895 097, 1}, {651 296 280 594 275 489, 1}}

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

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Untitled-25

In[189]:=
```
Hs[[2]]
```

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

In[194]:=
```
FactorInteger[10^70 + 2]
```

Out[194]=

{{2, 1}, {3, 1}, {2417, 1},
{728 771, 1}, {331 844 753, 1}, {1 315 441 529, 1},
{2 167 576 895 034 805 670 716 583 728 798 525 811 120 513, 1}}

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

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Untitled-25

In[189]:=
```
Hs[[2]]
```

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

In[195]:=
```
FactorInteger[10^70 + 2] // Timing
```

Out[195]=
```
{0.294993, {{2, 1}, {3, 1}, {2417, 1},
  {728771, 1}, {331844753, 1}, {1315441529, 1},
  {2167576895034805670716583728798525811120513, 1}}}
```

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[189]:=
```
Hs[[2]]
```

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

```
fiFactorInteger[10^70 + 2] // Timing
```

Out[195]=

```
{0.294993, {{2, 1}, {3, 1}, {2417, 1},
  {728771, 1}, {331844753, 1}, {1315441529, 1},
  {2167576895034805670716583728798525811120513, 1}}}
```

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[189]:=
```
Hs[[2]]
```

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

```
fi = FactorInteger[10^70 + 2] // Timing
```

Out[195]=

```
{0.294993, {{2, 1}, {3, 1}, {2417, 1},
  {728771, 1}, {331844753, 1}, {1315441529, 1},
  {2167576895034805670716583728798525811120513, 1}}}
```

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[189]:=
```
Hs[[2]]
```

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

In[196]:=
```
fi = FactorInteger[10^70 + 2] // Timing
```

Out[196]=
```
{0.298136, {{2, 1}, {3, 1}, {2417, 1},
  {728771, 1}, {331844753, 1}, {1315441529, 1},
  {2167576895034805670716583728798525811120513, 1}}}
```

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

In[196]:=

```
fi = FactorInteger[10^70 + 2] // Timing
```

Out[196]=

```
{0.298136, {{2, 1}, {3, 1}, {2417, 1},
  {728771, 1}, {331844753, 1}, {1315441529, 1},
  {2167576895034805670716583728798525811120513, 1}}}
```

In[197]:=

```
fi
```

Out[197]=

```
{0.298136, {{2, 1}, {3, 1}, {2417, 1},
  {728771, 1}, {331844753, 1}, {1315441529, 1},
  {2167576895034805670716583728798525811120513, 1}}}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

```
(fi = FactorInteger[10^70 + 2]) // Timing
```

Out[196]=

```
{0.298136, {{2, 1}, {3, 1}, {2417, 1},
  {728771, 1}, {331844753, 1}, {1315441529, 1},
  {2167576895034805670716583728798525811120513, 1}}}
```

In[197]:=

```
fi
```

Out[197]=

```
{0.298136, {{2, 1}, {3, 1}, {2417, 1},
  {728771, 1}, {331844753, 1}, {1315441529, 1},
  {2167576895034805670716583728798525811120513, 1}}}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

In[198]:=

```
(fi = FactorInteger[10^70 + 2]) // Timing
```

Out[198]=

```
{0.297229, {{2, 1}, {3, 1}, {2417, 1},
  {728 771, 1}, {331 844 753, 1}, {1 315 441 529, 1},
  {2 167 576 895 034 805 670 716 583 728 798 525 811 120 513, 1}}}
```

In[199]:=

```
fi
```

Out[199]=

```
{{2, 1}, {3, 1}, {2417, 1},
 {728 771, 1}, {331 844 753, 1}, {1 315 441 529, 1},
 {2 167 576 895 034 805 670 716 583 728 798 525 811 120 513, 1}}
```

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Untitled-25

Out[189]=

$$\left\{\left\{\frac{}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{}{6}, 0, 0, 0, 0, 0\right\},\right.$$
$$\left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\},$$
$$\left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\},$$
$$\left.\left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

In[198]:=

```
(fi = FactorInteger[10^70 + 2]) // Timing
```

In[199]:=

```
fi
```

Out[199]=

```
{{2, 1}, {3, 1}, {2417, 1},
 {728 771, 1}, {331 844 753, 1}, {1 315 441 529, 1},
 {2 167 576 895 034 805 670 716 583 728 798 525 811 120 513, 1}}
```

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

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Untitled-25

Out[189]=

$$\left\{\left\{\frac{}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{}{6}, 0, 0, 0, 0, 0\right\},\right.$$
$$\left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\},$$
$$\left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\},$$
$$\left.\left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

In[198]:=

```
(fi = FactorInteger[10^70 + 2]) // Timing
```

In[199]:=

```
fi
```

Out[199]=

```
{{2, 1}, {3, 1}, {2417, 1},
 {728 771, 1}, {331 844 753, 1}, {1 315 441 529, 1},
 {2 167 576 895 034 805 670 716 583 728 798 525 811 120 513, 1}}
```

```
tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[189]=

$$\left\{\left\{\frac{}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{}{6}, 0, 0, 0, 0, 0\right\},\right.$$
$$\left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\},$$
$$\left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\},$$
$$\left.\left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

In[198]:=

```
(fi = FactorInteger[10^70 + 2]) // Timing
```

In[199]:=

```
fi
```

Out[199]=

```
{{2, 1}, {3, 1}, {2417, 1},
 {728 771, 1}, {331 844 753, 1}, {1 315 441 529, 1},
 {2 167 576 895 034 805 670 716 583 728 798 525 811 120 513, 1}}
```

```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

In[198]:=

```
(fi = FactorInteger[10^70 + 2]) // Timing
```

In[199]:=

```
fi
```

```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\},\right.$$
$$\left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\},$$
$$\left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\},$$
$$\left.\left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

```
(fi = FactorInteger[10^70 + 2]) // Timing
fi
```

```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\},\right.$$
$$\left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\},$$
$$\left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\},$$
$$\left.\left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

```
(fi = FactorInteger[10^70 + 2]) // Timing
fi
```

```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

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Running...Untitled-25

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\},\right.$$
$$\left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\},$$
$$\left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\},$$
$$\left.\left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

```
(fi = FactorInteger[10^70 + 2]) // Timing
fi
```

In[200]:=

```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

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Running...Untitled-25

```
(fi = FactorInteger[10^70 + 2]) // Timing
fi
```

In[200]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Running...Untitled-25

```
(fi = FactorInteger[10^70 + 2]) // Timing
fi
```

In[200]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Running...Untitled-25

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

```
(fi = FactorInteger[10^70 + 2]) // Timing
fi
```

In[200]:=

```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Running...Untitled-25

Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[182]:=
```
max = 10;
```

In[184]:=

$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,\ 2,\ \text{max}\}\right];$$

In[189]:=
```
Hs[[2]]
```

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

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Running...Untitled-25

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

```
(fi = FactorInteger[10^70 + 2]) // Timing
fi
```

In[200]:=

```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[189]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

```
(fi = FactorInteger[10^70 + 2]) // Timing
fi
```

In[200]:=

```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[200]=

```
$Aborted
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[182]:=
```
max = 10;
```

In[184]:=
```
Hs = Table[1/L Sum[Sum[I[n] ⊗ S[i] ⊗ S[i] ⊗ I[L - n - 2], {i, 1, 3}], {n, 0, L-2}], {L, 2, max}];
```

In[189]:=
```
Hs[[2]]
```

Out[189]= $\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\},\right.$

$\left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\},$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Running...Untitled-25

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[201]:=
```
max = 9;
```

In[202]:=
```
Hs = Table[1/L Sum[Sum[I[n] ⊗ S[i] ⊗ S[i] ⊗ I[L - n - 2], {i, 1, 3}], {n, 0, L-2}], {L, 2, max}];
```

In[189]:=
```
Hs[[2]]
```

Out[189]= $\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\},\right.$

$\left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\},$

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Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[201]:=
```
max = 9;
```

In[202]:=
```
Hs = Table[1/L Sum[Sum[I[n] ⊗ S[i] ⊗ S[i] ⊗ I[L - n - 2], {i, 1, 3}], {n, 0, L-2}], {L, 2, max}];
```

In[203]:=
```
Hs[[2]]
```

Out[203]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

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Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[201]:=
```
max = 9;
```

In[202]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,\ 2,\ \text{max}\}\right];$$

In[203]:=
```
Hs[[2]]
```

```
(fi = FactorInteger[10^70 + 2]) // Timing
fi
```

In[200]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[200]=
```
$Aborted
```

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Running...Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim];
```

In[201]:=
```
max = 9;
```

In[202]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,\ 2,\ \text{max}\}\right];$$

In[203]:=
```
Hs[[2]]
```

In[204]:=
$$\left(\text{fi} = \text{FactorInteger}\left[10^{70}+2\right]\right)\ //\ \text{Timing}$$
```
fi
```

Out[204]=
```
{0.297902, {{2, 1}, {3, 1}, {2417, 1},
  {728 771, 1}, {331 844 753, 1}, {1 315 441 529, 1},
  {2 167 576 895 034 805 670 716 583 728 798 525 811 120 513, 1}}}
```

Out[205]=
```
{{2, 1}, {3, 1}, {2417, 1},
 {728 771, 1}, {331 844 753, 1}, {1 315 441 529, 1},
 {2 167 576 895 034 805 670 716 583 728 798 525 811 120 513, 1}}
```

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Running...Untitled-25

In[204]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing
fi
```

Out[204]=
```
{0.297902, {{2, 1}, {3, 1}, {2417, 1},
  {728 771, 1}, {331 844 753, 1}, {1 315 441 529, 1},
  {2 167 576 895 034 805 670 716 583 728 798 525 811 120 513, 1}}}
```

Out[205]=
```
{{2, 1}, {3, 1}, {2417, 1},
 {728 771, 1}, {331 844 753, 1}, {1 315 441 529, 1},
 {2 167 576 895 034 805 670 716 583 728 798 525 811 120 513, 1}}
```

In[206]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[200]=
```
$Aborted
```

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Untitled-25

In[204]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing
fi
```

Out[204]=
```
{0.297902, {{2, 1}, {3, 1}, {2417, 1},
  {728 771, 1}, {331 844 753, 1}, {1 315 441 529, 1},
  {2 167 576 895 034 805 670 716 583 728 798 525 811 120 513, 1}}}
```

Out[205]=
```
{{2, 1}, {3, 1}, {2417, 1},
 {728 771, 1}, {331 844 753, 1}, {1 315 441 529, 1},
 {2 167 576 895 034 805 670 716 583 728 798 525 811 120 513, 1}}
```

In[206]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[206]=
```
$Aborted
```

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Untitled-25

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[201]:=
```
max = 19;
```

In[202]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n] \otimes S[i] \otimes S[i] \otimes \mathbb{I}[L-n-2], \{L, 2, \text{max}\}\right];$$

In[203]:=
```
Hs[[2]]
```

In[204]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing
fi
```

Out[204]= {0.297902, {{2, 1}, {3, 1}, {2417, 1},

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Running...Untitled-25

$$\left\{0, \frac{}{6}, -\frac{}{6}, 0, \frac{}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{}{6}, 0, 0\right\},$$

$$\left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\},$$

$$\left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\}$$

In[211]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing
fi
```

Out[211]=
```
{0.297501, {{2, 1}, {3, 1}, {2417, 1},
  {728 771, 1}, {331 844 753, 1}, {1 315 441 529, 1},
  {2 167 576 895 034 805 670 716 583 728 798 525 811 120 513, 1}}}
```

Out[212]=
```
{{2, 1}, {3, 1}, {2417, 1},
 {728 771, 1}, {331 844 753, 1}, {1 315 441 529, 1},
 {2 167 576 895 034 805 670 716 583 728 798 525 811 120 513, 1}}
```

In[213]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[206]=
```
$Aborted
```

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Untitled-25

$$\left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\}$$

In[211]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing
fi
```

Out[211]=
```
{0.297501, {{2, 1}, {3, 1}, {2417, 1},
  {728 771, 1}, {331 844 753, 1}, {1 315 441 529, 1},
  {2 167 576 895 034 805 670 716 583 728 798 525 811 120 513, 1}}}
```

Out[212]=
```
{{2, 1}, {3, 1}, {2417, 1},
 {728 771, 1}, {331 844 753, 1}, {1 315 441 529, 1},
 {2 167 576 895 034 805 670 716 583 728 798 525 811 120 513, 1}}
```

In[213]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[213]=

$$\left\{2.031738, \left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1}{16}\right\}, \left\{5, \frac{1}{5}\right\}, \left\{6, -\frac{1}{8}\right\}\right\}\right\}$$

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Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[208]:=
```
max = 6;
```

In[209]:=

$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,\ 2,\ \text{max}\}\right];$$

In[210]:=
```
Hs[[2]]
```

Out[210]=

$$\left\{\left\{\frac{1}{6}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{6}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, \frac{1}{6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{6}, 0, -\frac{1}{6}, \frac{1}{6}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{6}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{6}\right\}\right\}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[208]:=
```
max = 6;
```

In[214]:=

$$Hs = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2],\ \{L,\ 2,\ max\}\right]\ //\ N;$$

In[215]:=
```
Hs[[2]]
```

Out[215]=
```
{{0.166667, 0., 0., 0., 0., 0., 0., 0.},
 {0., 0., 0.166667, 0., 0., 0., 0., 0.},
 {0., 0.166667, -0.166667, 0., 0.166667, 0., 0., 0.},
 {0., 0., 0., 0., 0., 0.166667, 0., 0.},
 {0., 0., 0.166667, 0., 0., 0., 0., 0.},
 {0., 0., 0., 0.166667, 0., -0.166667, 0.166667, 0.},
 {0., 0., 0., 0., 0., 0.166667, 0., 0.},
 {0., 0., 0., 0., 0., 0., 0., 0.166667}}
```


In[211]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing
```

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Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[208]:=
```
max = 6;
```

In[214]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,\ 2,\ \text{max}\}\right]\ //\ N;$$

In[211]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing
fi
```

Out[211]=
```
{0.297501, {{2, 1}, {3, 1}, {2417, 1},
  {728 771, 1}, {331 844 753, 1}, {1 315 441 529, 1},
  {2 167 576 895 034 805 670 716 583 728 798 525 811 120 513, 1}}}
```

Out[212]=
```
{{2, 1}, {3, 1}, {2417, 1},
 {728 771, 1}, {331 844 753, 1}, {1 315 441 529, 1},
 {2 167 576 895 034 805 670 716 583 728 798 525 811 120 513, 1}}
```

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Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[208]:=
```
max = 6;
```

In[214]:=
```
Hs = Table[1/L Sum[Sum[I[n] ⊗ S[i] ⊗ S[i] ⊗ I[L - n - 2], {i, 1, 3}], {n, 0, L-2}], {L, 2, max}] // N;
```

```
(fi = FactorInteger[10^70 + 2]) // Timing;
fi
```

In[213]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[213]=
$$\left\{2.031738, \left\{\left\{2, -\frac{3}{8}\right\}, \left\{3, -\frac{1}{3}\right\}, \left\{4, -\frac{1}{16}\right\}, \left\{5, \frac{1}{5}\right\}, \left\{6, -\frac{1}{8}\right\}\right\}\right\}$$

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Running...Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[216]:=
```
max = 10;
```

In[217]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,\ 2,\ \max\}\right]\ //\ N;$$

```
(fi = FactorInteger[10^70 + 2]) // Timing;
fi;
```

In[213]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[213]=
$$\left\{2.031738,\ \left\{\left\{2,\ -\frac{3}{8}\right\},\ \left\{3,\ -\frac{1}{3}\right\},\ \left\{4,\ -\frac{1}{16}\right\},\ \left\{5,\ \frac{1}{5}\right\},\ \left\{6,\ -\frac{1}{8}\right\}\right\}\right\}$$

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Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[216]:=
```
max = 10;
```

In[217]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,\ 2,\ \max\}\right]\ //\ N;$$

In[218]:=
$$\left(\text{fi} = \text{FactorInteger}\left[10^{70}+2\right]\right)\ //\ \text{Timing};$$
```
fi;
```

In[220]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[220]=
```
{8.995014, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[216]:=
```
max = 10;
```

In[217]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,\ 2,\ \max\}\right]\ //\ N;$$

In[218]:=
$$\left(\text{fi} = \text{FactorInteger}\left[10^{70}+2\right]\right)\ //\ \text{Timing};$$
```
fi;
```

In[220]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[220]=
```
{8.995014, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

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Running...Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[222]:=
```
max = 11;
```

In[223]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,\ 2,\ \text{max}\}\right] \text{ // N};$$

In[218]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing;
fi;
```

In[220]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[220]=
```
{8.995014, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Running...Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[222]:=
```
max = 11;
```

In[223]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,2,\text{max}\}\right] \text{ // N};$$

In[218]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing;
fi;
```

In[220]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[220]=
```
{8.995014, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Running...Untitled-25

In[179]:=
```
𝕀[n_] := IdentityMatrix[dim^n];
```

```
max = 10;
```

In[223]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,2,\max\}\right] \text{ // N};$$

In[224]:=
$$\left(\text{fi} = \text{FactorInteger}\left[10^{70}+2\right]\right) \text{ // Timing};$$
```
fi;
```

In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[220]=
```
{8.995014, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Running...Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

```
max = 10;
```

In[223]:=
```
Hs = Table[1/L Sum[Sum[I[n] ⊗ S[i] ⊗ S[i] ⊗ I[L - n - 2], {i, 1, 3}], {n, 0, L-2}], {L, 2, max}] // N;
```

In[224]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing;
fi;
```

In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[220]=
```
{8.995014, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Running...Untitled-25

In[179]:=
```
𝕀[n_] := IdentityMatrix[dim^n];
```

```
max = 10;
```

In[223]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,2,\text{max}\}\right] \text{ // N};$$

In[224]:=
$$\left(\text{fi} = \text{FactorInteger}\left[10^{70}+2\right]\right) \text{ // Timing};$$
```
fi;
```

In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[220]=
```
{8.995014, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Running...Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

```
max = 10;
```

In[223]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,2,\text{max}\}\right] \text{ // N;}$$

In[224]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing;
fi;
```

In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[220]=
```
{8.995014, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Running...Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

```
max = 10;
```

In[223]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L, 2, \max\}\right] \text{ // N;}$$

In[224]:=
$$(\text{fi} = \text{FactorInteger}[10^{70}+2]) \text{ // Timing;}$$
```
fi;
```

In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[220]=
```
{8.995014, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

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Running...Untitled-25

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

```
max = 10;
```

In[223]:=
```
Hs = Table[1/L Sum[Sum[I[n] ⊗ S[i] ⊗ S[i] ⊗ I[L - n - 2]
```

In[224]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing;
fi;
```

In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort
 AbsoluteTiming
```

Out[220]=
```
{8.995014, {{2, -0.375}, {3, -0.333333}, {4, -
  {5, -0.385577}, {6, -0.415596}, {7, -0.4051
  {8, -0.421867}, {9, -0.415147}, {10, -0.425
```

Evaluation menu:

- Evaluate Cells ⇧↵
- Evaluate in Place ⌘↵
- Evaluate in Subsession ⌥⇧↵
- Evaluate Notebook
- Evaluate Initialization Cells
- ✓ Dynamic Updating Enabled
- Convert Dynamic to Literal
- Debugger
- Debugger Controls ▶
- Interrupt Evaluation... ⌥⌘.
- Abort Evaluation ⌘.
- Remove from Evaluation Queue ⇧⌘.
- Find Currently Evaluating Cell
- Kernel Configuration Options...
- Parallel Kernel Configuration...
- Parallel Kernel Status...
- Default Kernel ▶
- Notebook's Kernel ▶
- Notebook's Default Context ▶
- Start Kernel ▶
- Quit Kernel ▶

Local
The other

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Untitled-25

In[179]:=
```
𝕀[n_] := IdentityMatrix[dim^n];
```

```
max = 10;
```

In[223]:=
```
Hs = Table[1/L Sum[Sum[𝕀[n] ⊗ S[i] ⊗ S[i] ⊗ 𝕀[L - n - 2], {i, 1, 3}], {n, 0, L-2}], {L, 2, max}] // N;
```

In[224]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing;
fi;
```

In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[220]=
```
{8.995014, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[172]:=
```
S[i_] = PauliMatrix[i] / 2;
```

In[173]:=
```
dim := Length[S[1]]
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[1
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[172]:=
```
S[i_] = PauliMatrix[i] / 2;
```

In[173]:=
```
dim := Length[S[1]]
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[1
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

In[139]:=
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ClearAll[S]
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In[172]:=
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S[i_] = PauliMatrix[i] / 2;
```

In[173]:=
```
dim := Length[S[1]]
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[1
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Running...Untitled-25

Spin operators

In[139]:=
```
ClearAll[S]
```

In[172]:=
```
S[i_] = PauliMatrix[i] / 2;
```

In[173]:=
```
dim := Length[S[1]]
spin := (dim - 1)/2
check := And[
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[1
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

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Untitled-25

```
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

In[187]:=
```
S[1]
dim
```

Out[187]= $\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0\right\}\right\}$

Out[188]= 2

Tensor product

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

```
  S[1].S[2] - S[2].S[1] == I S[3],
  S[2].S[3] - S[3].S[2] == I S[1],
  S[3].S[1] - S[1].S[3] == I S[2]
 ]
```

Suppose we want spin 1

In[171]:=
```
(* ϵ[a__]:=Signature[{a}]
   S[i_]:=-I Table[ϵ[i,j,k],{j,3},{k,3}] *)
```

Tensor product

In[177]:=
```
CircleTimes = KroneckerProduct;
```

In[178]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[178]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

chain

Spin operators

Tensor product

In[6]:=
```
CircleTimes = KroneckerProduct;
```

In[7]:=
```
Eigenvalues[S[1] ⊗ S[1] + S[2] ⊗ S[2] + S[3] ⊗ S[3]]
```

Out[7]= $\left\{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

Hamiltonian

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

Tensor product

Hamiltonian

In[179]:=
```
I[n_] := IdentityMatrix[dim^n];
```

```
max = 10;
```

In[223]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \mathbb{I}[n]\otimes S[i]\otimes S[i]\otimes \mathbb{I}[L-n-2],\ \{L,2,\text{max}\}\right] \text{ // N;}$$

In[224]:=
$$(\text{fi} = \text{FactorInteger}[10^{70}+2]) \text{ // Timing;}$$
```
fi;
```

In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

Tensor product

Hamiltonian

In[8]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[9]:=
```
max = 10;
```

In[223]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \text{I}[n]\otimes\text{S}[i]\otimes\text{S}[i]\otimes\text{I}[L-n-2],\ \{L,\ 2,\ \text{max}\}\right]\ //\ \text{N};$$

In[224]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing;
fi;
```

In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

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Untitled-25

Spin operators

Tensor product

Hamiltonian

In[8]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[9]:=
```
max = 10;
```

```
h
```

In[223]:=
$$\text{Hs} = \text{Table}\left[\frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} \text{I}[n]\otimes\text{S}[i]\otimes\text{S}[i]\otimes\text{I}[L-n-2],\ \{L,\ 2,\ \text{max}\}\right]\ //\ \text{N};$$

In[224]:=
$$\left(\text{fi} = \text{FactorInteger}\left[10^{70}+2\right]\right)\ //\ \text{Timing};$$
```
fi;
```


In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
```

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Untitled-25

Spin operators

Tensor product

Hamiltonian

In[8]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[9]:=
```
max = 10;
```

```
h[L_] := h[L] =
```

In[223]:=
$$Hs = \text{Table}\left[\frac{1}{2}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2],\ \{L,\ 2,\ max\}\right]\ //\ N;$$

In[224]:=
$$\left(fi = \text{FactorInteger}\left[10^{70}+2\right]\right)\ //\ \text{Timing};$$
```
fi;
```


In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

Tensor product

Hamiltonian

In[8]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[9]:=
```
max = 10;
```

```
h[L_] := h[L] =
```

```
Hs = Table[, {L, 2, max}] // N;
```

In[224]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing;
fi;
```

In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

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Untitled-25

Spin operators

Tensor product

Hamiltonian

In[8]:=
```
I[n_] := IdentityMatrix[dim^n];
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In[9]:=
```
max = 10;
```

$$h[L_] := h[L] = \frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2]$$

```
Hs = Table[h[L], {L, 2, max}] // N;
```

In[224]:=
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(fi = FactorInteger[10^70 + 2]) // Timing;
fi;
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In[226]:=
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(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
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Untitled-25

Spin operators

Tensor product

Hamiltonian

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Hs = Table[h[L], {L, 2, max}] // N;
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(fi = FactorInteger[10^70 + 2]) // Timing;
fi;
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(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
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Untitled-25

Spin operators

Tensor product

Hamiltonian

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$$h[L_] := h[L] = \frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2]$$

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Hs = Table[h[L], {L, 2, max}] // N;
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In[224]:=
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(fi = FactorInteger[10^70 + 2]) // Timing;
fi;
```

In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
```

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Untitled-25

Spin operators

Tensor product

Hamiltonian

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```
I[n_] := IdentityMatrix[dim^n];
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In[9]:=
```
max = 10;
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$$h[L_] := h[L] = \frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2]$$

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Hs = Table[h[L], {L, 2, max}] // N;
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In[224]:=
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(fi = FactorInteger[10^70 + 2]) // Timing;
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In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
```

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Untitled-25

Spin operators

Tensor product

Hamiltonian

In[8]:=
```
I[n_] := IdentityMatrix[dim^n];
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In[9]:=
```
max = 10;
```

$$h[L_] := h[L] = \frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2]$$

```
Hs = Table[h[L], {L, 2, max}] // N;
```

In[224]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing;
fi;
```

In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
```

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Running...Untitled-25

Spin operators

Tensor product

Hamiltonian

In[8]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[9]:=
```
max = 10;
```

In[10]:=

$$h[L_] := h[L] = \frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2]$$

In[11]:=
```
Hs = Table[h[L], {L, 2, max}] // N;
```

In[224]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing;
fi;
```

In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Spin operators

Tensor product

Hamiltonian

In[8]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[9]:=
```
max = 10;
```

In[10]:=
$$h[L_] := h[L] = \frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2]$$

In[11]:=
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Hs = Table[h[L], {L, 2, max}] // N;
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In[224]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing;
fi;
```


In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
```

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Untitled-25

Spin operators

Tensor product

Hamiltonian

In[8]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[9]:=
```
max = 10;
```

In[10]:=
$$h[L_] := h[L] = \frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2]$$

In[15]:=
```
Hs = Table[h[L], {L, 2, max}] // N;
```

In[224]:=
```
(fi = FactorInteger[10^70 + 2]) // Timing;
fi;
```


In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
```

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Untitled-25

Spin operators

Tensor product

Hamiltonian

In[8]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[9]:=
```
max = 10;
```

In[10]:=

$$h[L_] := h[L] = \frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2]$$

In[15]:=
```
Hs = Table[h[L], {L, 2, max}] // N;
```

In[226]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[220]=
```
{8.995014, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Running...Untitled-25

Spin operators

Tensor product

Hamiltonian

In[8]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[9]:=
```
max = 10;
```

In[10]:=

$$h[L_] := h[L] = \frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2]$$

In[15]:=
```
Hs = Table[h[L], {L, 2, max}] // N;
```

In[16]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[220]=
```
{8.995014, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Hamiltonian

In[8]:=
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I[n_] := IdentityMatrix[dim^n];
```

In[9]:=
```
max = 10;
```

In[10]:=

$$h[L_] := h[L] = \frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n] \otimes S[i] \otimes S[i] \otimes I[L-n-2]$$

In[15]:=
```
Hs = Table[h[L], {L, 2, max}] // N;
```

In[16]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[16]=
```
{8.987602, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Hamiltonian

In[8]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[9]:=
```
max = 10;
```

In[10]:=

$$h[L_] := h[L] = \frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2]$$

In[15]:=
```
Hs = Table[h[L], {L, 2, max}] // N;
```

In[16]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[16]=
```
{8.987602, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
Hs2
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Hamiltonian

In[8]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[9]:=
```
max = 10;
```

In[10]:=

$$h[L_] := h[L] = \frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n] \otimes S[i] \otimes S[i] \otimes I[L-n-2]$$

In[15]:=
```
Hs = Table[h[L], {L, 2, max}] // N;
```

In[16]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[16]=
```
{8.987602, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
Hs2 = Hs
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Hamiltonian

In[8]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[9]:=
```
max = 10;
```

In[10]:=

$$h[L_] := h[L] = \frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2]$$

In[15]:=
```
Hs = Table[h[L], {L, 2, max}] // N;
```

In[16]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[16]=
```
{8.987602, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
Hs2 = SparseArray[Hs
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

Hamiltonian

In[8]:=
```
I[n_] := IdentityMatrix[dim^n];
```

In[9]:=
```
max = 10;
```

In[10]:=

$$h[L_] := h[L] = \frac{1}{L}\sum_{n=0}^{L-2}\sum_{i=1}^{3} I[n]\otimes S[i]\otimes S[i]\otimes I[L-n-2]$$

In[15]:=
```
Hs = Table[h[L], {L, 2, max}] // N;
```

In[16]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[16]=
```
{8.987602, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
Hs2 = SparseArray /@ Hs
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-25

In[15]:=
```
Hs = Table[h[L], {L, 2, max}] // N;
```

In[16]:=
```
(tb = Table[{L, Hs[[L - 1]] // Eigenvalues // Sort // First}, {L, 2, max}]) //
 AbsoluteTiming
```

Out[16]=
```
{8.987602, {{2, -0.375}, {3, -0.333333}, {4, -0.404006},
  {5, -0.385577}, {6, -0.415596}, {7, -0.405177},
  {8, -0.421867}, {9, -0.415147}, {10, -0.425804}}}
```

```
Hs2 = SparseArray /@ Hs
```