Title: Introduction to Complex Analysis/Computer Skills - Lecture 7A

Date: Sep 08, 2010 09:00 AM

URL: http://pirsa.org/10090077

Abstract:

Let $V(x) = x^L$ with $L \in \mathbb{Z}^+$

Let $Q(x)$ be a polynomial of degree N

Let $T(x)$ be a polynomial.

How to find $T(x)$ and $Q(x)$ (there are finitely many solutions)

knowing that

$$T(x) = \frac{V(x+i/2)Q(x+i) + V(x-i/2)Q(x-i)}{Q(x)}$$

Hint: take $L=2$ and small N first.

Let $V(x) = x^L$ with $L \in \mathbb{Z}^+$

Let $Q(x)$ be a polynomial of degree N

Let $T(x)$ be a polynomial.

How to find $T(x)$ and $Q(x)$ (there are finitely many solutions)

knowing that

$$T(x) = \frac{V(x+i/2)Q(x+i) + V(x-i/2)Q(x-i)}{Q(x)}$$

Hint: Take $L=2$ and small N first.

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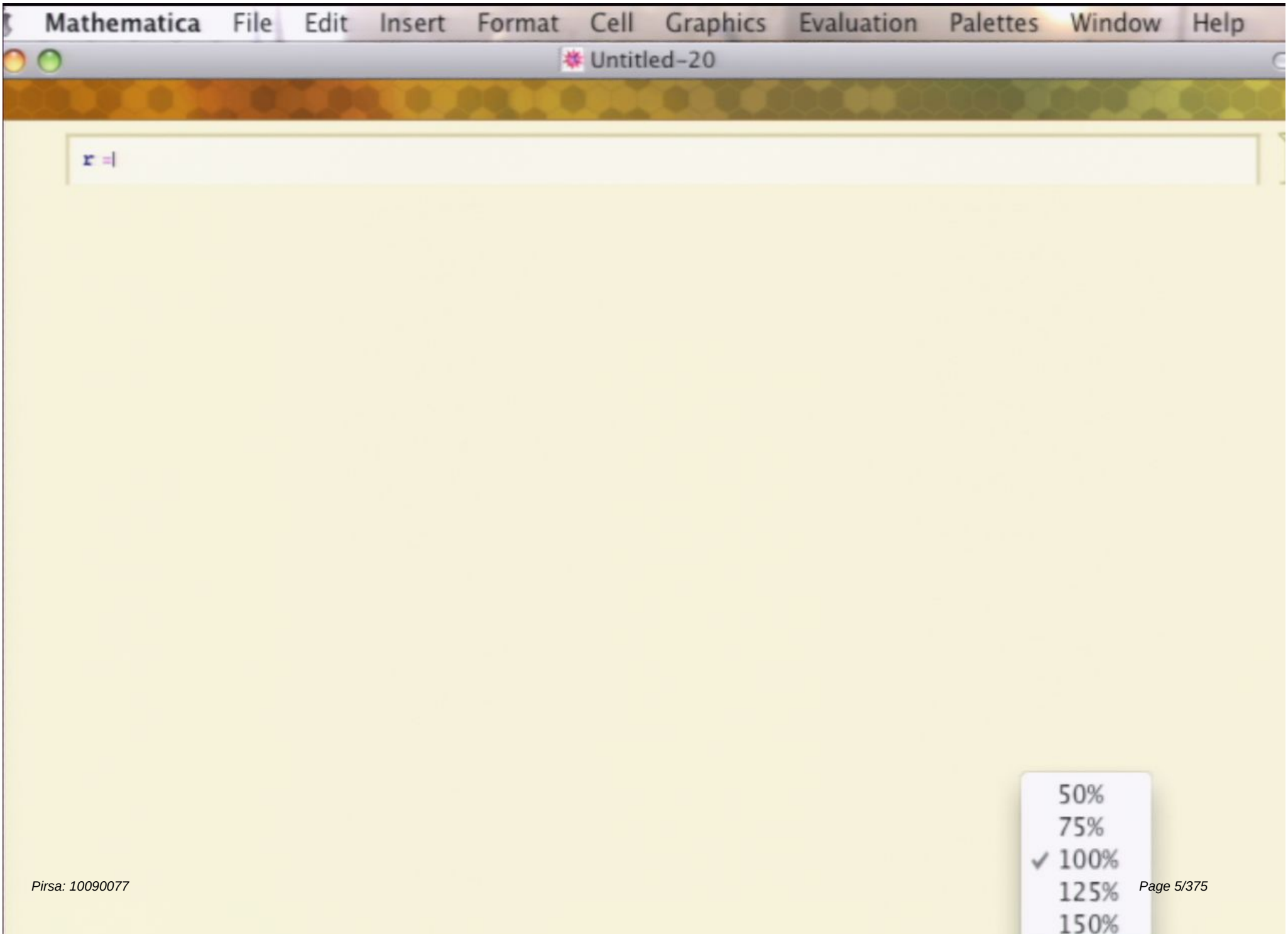
Untitled-20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Untitled-20
r =
50%
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Untitled-20

In[1]:=

```
r = Sqrt[x^2 + y^2 + z^2];
xs = {x, y, z};
```

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Untitled-20

In[1]:=
```
r = Sqrt[x^2 + y^2 + z^2];
xs = {x, y, z};
```

In[3]:=
```
Signature[{1, 3, 2}]
```

Out[3]=
```
-1
```

In[4]:=
```
Signature[{1, 2, 3}]
```

Out[4]=
```
1
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[1]:=
```
r = Sqrt[x^2 + y^2 + z^2];
xs = {x, y, z};
```

In[3]:=
```
Signature[{1, 3, 2}]
```

Out[3]= -1

In[4]:=
```
Signature[{1, 2, 3}]
```

Out[4]= 1

In[5]:=
```
Signature[{1, 2, 2}]
```

Out[5]= 0

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[1]:=
```
r = Sqrt[x^2 + y^2 + z^2];
xs = {x, y, z};
```

In[3]:=
```
Signature[{1, 3, 2}]
```

Out[3]= -1

In[4]:=
```
Signature[{1, 2, 3}]
```

Out[4]= 1

In[5]:=
```
Signature[{1, 2, 2}]
```

Out[5]= 0

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[1]:=
```
r = √(x^2 + y^2 + z^2);
xs = {x, y, z};
```

In[3]:=
```
Signature[{1, 3, 2}]
```

Out[3]=
```
-1
```

In[4]:=
```
Signature[{1, 2, 3}]
```

Out[4]=
```
1
```

In[5]:=
```
Signature[{1, 2, 2}]
```

Out[5]=
```
0
```

```
ϵ[a_,
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[1]:=
```
r = √(x^2 + y^2 + z^2);
xs = {x, y, z};
```

In[3]:=
```
Signature[{1, 3, 2}]
```

Out[3]=
```
-1
```

In[4]:=
```
Signature[{1, 2, 3}]
```

Out[4]=
```
1
```

In[5]:=
```
Signature[{1, 2, 2}]
```

Out[5]=
```
0
```

```
ϵ[a_, b_, c_] = Signature[{a, b, c}]
```

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Untitled-20

In[1]:=
```
r = √(x^2 + y^2 + z^2);
xs = {x, y, z};
```

In[3]:=
```
Signature[{1, 3, 2}]
```

Out[3]=
```
-1
```

In[4]:=
```
Signature[{1, 2, 3}]
```

Out[4]=
```
1
```

In[5]:=
```
Signature[{1, 2, 2}]
```

Out[5]=
```
0
```

```
ϵ[a_, b_, c_] := Signature[{a, b, c}]
```

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Untitled-20

In[3]:=
```
Signature[{1, 3, 2}]
```
Out[3]=
```
-1
```

In[4]:=
```
Signature[{1, 2, 3}]
```
Out[4]=
```
1
```

In[5]:=
```
Signature[{1, 2, 2}]
```
Out[5]=
```
0
```

In[6]:=
```
ϵ[a_, b_, c_] := Signature[{a, b, c}]
```

```
ϵ[|
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[3]= -1

In[4]:=
```
Signature[{1, 2, 3}]
```

Out[4]= 1

In[5]:=
```
Signature[{1, 2, 2}]
```

Out[5]= 0

In[6]:=
```
ϵ[a_, b_, c_] := Signature[{a, b, c}]
```

In[8]:=
```
ϵ[1, 3, 2]
```

Out[8]= -1

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[3]= -1

In[4]:=
```
Signature[{1, 2, 3}]
```

Out[4]= 1

In[5]:=
```
Signature[{1, 2, 2}]
```

Out[5]= 0

In[6]:=
```
ϵ[a_, b_, c_] := Signature[{a, b, c}]
```

In[8]:=
```
ϵ[1, 3, 2]
```

Out[8]= -1

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[4]:=
```
Signature[{1, 2, 3}]
```
Out[4]=
```
1
```

In[5]:=
```
Signature[{1, 2, 2}]
```
Out[5]=
```
0
```

In[6]:=
```
ϵ[a_, b_, c_] := Signature[{a, b, c}]
```

```
ClearAll[ϵ];
ϵ[a__] := Signature[{a}]
```

In[11]:=
```
ϵ[1, 3, 2]
```
Out[11]=
```
ϵ[1, 3, 2]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[4]:=
```
Signature[{1, 2, 3}]
```
Out[4]=
```
1
```

In[5]:=
```
Signature[{1, 2, 2}]
```
Out[5]=
```
0
```

In[6]:=
```
ϵ[a_, b_, c_] := Signature[{a, b, c}]
```

In[12]:=
```
ClearAll[ϵ];
ϵ[a__] := Signature[{a}]
```

In[14]:=
```
ϵ[1, 3, 2]
```
Out[14]=
```
-1
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[3]:=
```
Signature[{1, 3, 2}]
```
Out[3]=
```
-1
```

In[4]:=
```
Signature[{1, 2, 3}]
```
Out[4]=
```
1
```

In[5]:=
```
Signature[{1, 2, 2}]
```
Out[5]=
```
0
```

In[6]:=
```
ϵ[a_, b_, c_] := Signature[{a, b, c}]
```

In[12]:=
```
ClearAll[ϵ];
ϵ[a__] := Signature[{a}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[1]:=
```
x = Sqrt[x^2 + y^2 + z^2];
xs = {x, y, z};
```

In[3]:=
```
Signature[{1, 3, 2}]
```

Out[3]=
```
-1
```

In[4]:=
```
Signature[{1, 2, 3}]
```

Out[4]=
```
1
```

In[5]:=
```
Signature[{1, 2, 2}]
```

Out[5]=
```
0
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[6]:=
```
ϵ[a_, b_, c_] := Signature[{a, b, c}]
```

In[12]:=
```
ClearAll[ϵ];
ϵ[a__] := Signature[{a}]
```

In[14]:=
```
ϵ[1, 3, 2]
```

Out[14]=
```
-1
```

In[15]:=
```
r = Sqrt[x^2 + y^2 + z^2];
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

```
Signature[1, 2,
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}]
```

In[14]:=
```
ϵ[1, 3, 2]
```

Out[14]=
```
-1
```

In[15]:=
```
r = Sqrt[x^2 + y^2 + z^2];
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

In[18]:=
```
Signature[1, 2, 3]
```

Signature::argt :
Signature called with 3 arguments; 1 or 2 arguments are expected. >>

Out[18]=
```
Signature[1, 2, 3]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}]
```

In[14]:=
```
ϵ[1, 3, 2]
```

Out[14]=
```
-1
```

In[15]:=
```
r = Sqrt[x^2 + y^2 + z^2];
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

In[19]:=
```
Signature[{1, 2, 3}]
```

Out[19]=
```
1
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}]
```

In[14]:=
```
ϵ[1, 3, 2]
```

Out[14]=
```
-1
```

In[15]:=
```
r = Sqrt[x^2 + y^2 + z^2];
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

```
p[
```

In[19]:=
```
Signature[{1, 2, 3}]
```

Out[19]=
```
1
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}]
```

In[14]:=
```
ϵ[1, 3, 2]
```

Out[14]=
```
-1
```

In[15]:=
```
r = Sqrt[x^2 + y^2 + z^2];
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

```
p[i_
```

In[19]:=
```
Signature[{1, 2, 3}]
```

Out[19]=
```
1
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}]
```

In[14]:=
```
ϵ[1, 3, 2]
```

Out[14]=
```
-1
```

In[15]:=
```
r = Sqrt[x^2 + y^2 + z^2];
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

```
p[i_][F_] = 1/I
```

In[19]:=
```
Signature[{1, 2, 3}]
```

Out[19]=
```
1
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}]
```

In[14]:=
```
ϵ[1, 3, 2]
```

Out[14]=
```
-1
```

In[15]:=
```
r = Sqrt[x^2 + y^2 + z^2];
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

```
p[i_][F_] = 1/I
```

In[19]:=
```
Signature[{1, 2, 3}]
```

Out[19]=
```
1
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}]
```

In[14]:=
```
ϵ[1, 3, 2]
```

Out[14]=
```
-1
```

In[15]:=
```
r = Sqrt[x^2 + y^2 + z^2];
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[19]:=
```
Signature[{1, 2, 3}]
```

Out[19]=
```
1
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[14]:=
```
ϵ[1, 3, 2]
```

Out[14]=
```
-1
```

In[15]:=
```
r = Sqrt[x^2 + y^2 + z^2];
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[20]:=
```
xs
```

Out[20]=
```
{x, y, z}
```

```
xs
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[14]:=
```
ϵ[1, 3, 2]
```

Out[14]=
```
-1
```

In[15]:=
```
r = Sqrt[x^2 + y^2 + z^2];
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[20]:=
```
xs
```

Out[20]=
```
{x, y, z}
```

```
xs :[[
```

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Untitled-20

In[15]:=
```
r = Sqrt[x^2 + y^2 + z^2];
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[20]:=
```
xs
```

Out[20]= {x, y, z}

In[21]:=
```
xs[[1]]
```

Out[21]= x

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[15]:=
```
r = Sqrt[x^2 + y^2 + z^2];
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[20]:=
```
xs
```

Out[20]= {x, y, z}

In[21]:=
```
xs[[1]]
```

Out[21]= x

```
xs
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[15]:=
```
r = Sqrt[x^2 + y^2 + z^2];
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

```
p[i_][F_] := 1/I ∂_xs[[i]] F
```

In[20]:=
```
xs
```

Out[20]= {x, y, z}

In[21]:=
```
xs[[1]]
```

Out[21]= x

```
xs[[1]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
e[a__] := Signature[{a}];
```

```
p[i_][F_] := 1/I ∂xs[[i]] F
```

In[20]:=
```
xs
```

Out[20]=
```
{x, y, z}
```

In[21]:=
```
xs[[1]]
```

Out[21]=
```
x
```

In[22]:=
```
xs[[1]]
```

Out[22]=
```
x
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
e[a__] := Signature[{a}];
```

```
p[i_][F_] := 1/I ∂xs[[i]] F
```

In[20]:=
```
xs
```

Out[20]=
```
{x, y, z}
```

In[21]:=
```
xs[[1]]
```

Out[21]=
```
x
```

In[22]:=
```
xs[[1]]
```

Out[22]=
```
x
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
e[a__] := Signature[{a}];
```

```
p[i_][F_] := 1/I ∂xs[[i]] F
```

In[20]:=
```
xs
```

Out[20]=
```
{x, y, z}
```

In[21]:=
```
xs[[1]]
```

Out[21]=
```
x
```

In[22]:=
```
xs[[1]]
```

Out[22]=
```
x
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

$$\mathbf{p}[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[20]:=
```
xs
```

Out[20]=
```
{x, y, z}
```

In[21]:=
```
xs[[1]]
```

Out[21]=
```
x
```

In[22]:=
```
xs[[1]]
```

Out[22]=
```
x
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[20]:=
```
xs
```

Out[20]=
```
{x, y, z}
```

In[21]:=
```
xs[[1]]
```

Out[21]=
```
x
```

In[22]:=
```
xs[[1]]
```

Out[22]=
```
x
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[20]:=
```
xs
```

Out[20]=
```
{x, y, z}
```

In[21]:=
```
xs[[1]]
```

Out[21]=
```
x
```

In[22]:=
```
xs[[1]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

```
p[i_][F_] := 1/I ∂xs[[i]] F
P[i_,|F_] := 1/I ∂xs[[i]] F
```

In[20]:=
```
xs
```

Out[20]=
```
{x, y, z}
```

In[21]:=
```
xs[[1]]
```

Out[21]=
```
x
```

In[22]:=
```
xs[[1]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

In[23]:=

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

```
f =
```

In[20]:=

```
xs
```

Out[20]=

```
{x, y, z}
```

In[21]:=

```
xs[[1]]
```

Out[21]=

```
x
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

In[23]:=

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[25]:=

```
f = g[x, y, z];
```

In[20]:=

```
xs
```

Out[20]=

```
{x, y, z}
```

In[21]:=

```
xs[[1]]
```

Out[21]=

```
x
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

In[23]:=

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

In[25]:=

```
f = g[x, y, z];
```

```
p[
```

In[20]:=

```
xs
```

Out[20]=

```
{x, y, z}
```


In[21]:=

```
xs[[1]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

In[23]:=

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[25]:=

```
f = g[x, y, z];
```

In[26]:=

```
p[1][f]
```

Out[26]= $-i\, g^{(1,0,0)}[x, y, z]$

In[20]:=

```
xs
```

Out[20]= {x, y, z}

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

In[23]:=

```
p[i_][F_] := 1/I ∂xs[[i]] F
P[i_, F_] := 1/I ∂xs[[i]] F
```

In[25]:=

```
f = g[x, y, z];
```

```
p[1][f]
p[1|f]
```

Out[26]= $-i\, g^{(1,0,0)}[x, y, z]$

In[20]:=

```
xs
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

In[23]:=

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

In[25]:=

```
f = g[x, y, z];
```

In[27]:=

```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Out[28]= $-i\, g^{(1,0,0)}[x, y, z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[23]:=

```
p[i_][F_] := 1/I ∂xs[[i]] F
P[i_, F_] := 1/I ∂xs[[i]] F
```

In[25]:=

```
f = g[x, y, z];
```

In[27]:=

```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Out[28]= $-i\, g^{(1,0,0)}[x, y, z]$

```
p
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[23]:=

```
p[i_][F_] := 1/I ∂xs[[i]] F
P[i_, F_] := 1/I ∂xs[[i]] F
```

In[25]:=

```
f = g[x, y, z];
```

In[27]:=

```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Out[28]= $-i\, g^{(1,0,0)}[x, y, z]$

```
p[1]
```

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Untitled-20

```
P[i_, F_] := 1/I ∂xs[[i]] F
```

In[25]:=
```
f = g[x, y, z];
```

In[27]:=
```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Out[28]= $-i\, g^{(1,0,0)}[x, y, z]$

In[29]:=
```
p[1]@f
```

Out[29]= $-i\, g^{(1,0,0)}[x, y, z]$

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Untitled-20

In[23]:=

```
p[i_][F_] := 1/I ∂xs[[i]] F

P[i_, F_] := 1/I ∂xs[[i]] F
```

In[25]:=

```
f = g[x, y, z];
```

In[27]:=

```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Out[28]= $-i\, g^{(1,0,0)}[x, y, z]$

In[29]:=

```
p[1]@f
```

Out[29]= $-i\, g^{(1,0,0)}[x, y, z]$

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Untitled-20

```
e[a__] := Signature[{a}];
```

In[23]:=

```
p[i_][F_] := 1/I ∂xs[[i]] F
P[i_, F_] := 1/I ∂xs[[i]] F
```

In[25]:=

```
f = g[x, y, z];
```

In[27]:=

```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Out[28]= $-i\, g^{(1,0,0)}[x, y, z]$

In[29]:=

```
p[1]@f
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
e[a__] := Signature[{a}];
```

In[23]:=

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

In[25]:=

```
f = g[x, y, z];
```

In[27]:=

```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Out[28]= $-i\, g^{(1,0,0)}[x, y, z]$

In[29]:=

```
p[1]@f
```

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Untitled-20

```
e[a__] := Signature[{a}];
```

In[23]:=

```
p[i_][F_] := 1/I ∂xs[[i]] F
P[i_, F_] := 1/I ∂xs[[i]] F
```

In[25]:=

```
f = g[x, y, z];
```

In[27]:=

```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Out[28]= $-i\, g^{(1,0,0)}[x, y, z]$

In[29]:=

```
p[1]@f
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

In[23]:=

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[25]:=

```
f = g[x, y, z];
```

In[27]:=

```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Out[28]= $-i\, g^{(1,0,0)}[x, y, z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

In[23]:=

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

In[25]:=

```
f = g[x, y, z];
```

In[27]:=

```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Out[28]= $-i\, g^{(1,0,0)}[x, y, z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

In[23]:=

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[25]:=

```
f = g[x, y, z];
```

In[27]:=

```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Out[28]= $-i\, g^{(1,0,0)}[x, y, z]$

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Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

In[23]:=

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

In[25]:=

```
f = g[x, y, z];
```

In[27]:=

```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Out[28]= $-i\, g^{(1,0,0)}[x, y, z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

In[23]:=

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]|}F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

In[25]:=

```
f = g[x, y, z];
```

In[27]:=

```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Out[28]= $-i\, g^{(1,0,0)}[x, y, z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]}\,\#\,\&$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[25]:=
```
f = g[x, y, z];
```

In[27]:=
```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Out[28]= $-i\, g^{(1,0,0)}[x, y, z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
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ϵ[a__] := Signature[{a}];
```

$$p[i_][\ := \frac{1}{I}\,\partial_{xs[[i]]}\#\,\&$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

In[25]:=
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In[27]:=
```
p[1][f]
P[1, f]
```

Out[27]= $-i\,g^{(1,0,0)}[x, y, z]$

Out[28]= $-i\,g^{(1,0,0)}[x, y, z]$

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```
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ϵ[a__] := Signature[{a}];
```

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$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[25]:=
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In[27]:=
```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Out[28]= $-i\, g^{(1,0,0)}[x, y, z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

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```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

In[25]:=
```
f = g[x, y, z];
```

In[27]:=
```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

In[25]:=
```
f = g[x, y, z];
```

In[27]:=
```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[25]:=
```
f = g[x, y, z];
```

In[27]:=
```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Out[28]= $-i\, g^{(1,0,0)}[x, y, z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

```
Clear[
```

In[25]:=
```
f = g[x, y, z];
```

In[27]:=
```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

```
ClearAll[p]
```

In[25]:=
```
f = g[x, y, z];
```

In[27]:=
```
p[1][f]
P[1, f]
```

Out[27]= $-i\, g^{(1,0,0)}[x, y, z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
xs = {x, y, z};
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[30]:=

```
ClearAll[p]
```

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]}\,\#\,\&$$

In[25]:=

```
f = g[x, y, z];
```

In[27]:=

```
p[1][f]
P[1, f]
```


$$i\,g^{(1,0,0)}[x, y, z]$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[30]:=

```
ClearAll[p]
```

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]}\,\#\,\&$$

In[25]:=

```
f = g[x, y, z];
```

In[32]:=

```
p[1][f]
```

Out[32]= $-i\,g^{(1,0,0)}[x, y, z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

```
ClearAll[p]
```

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]}\,\#\,\&$$

In[25]:=
```
f = g[x, y, z];
```

In[32]:=
```
p[1][f]
```

Out[32]= $-i\,g^{(1,0,0)}[x, y, z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

$$P[i_,\ F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

```
ClearAll[p]
```

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]}\,\#\ \&$$

$$H = -\frac{1}{2\,m}$$

In[25]:=
```
f = g[x, y, z];
```

In[32]:=
```
p[1][f]
```

Out[32]= $-i\,g^{(1,0,0)}[x,\ y,\ z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

```
ClearAll[p]
```

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]}\#\,\&$$

$$H = -\frac{1}{2\,m}\,\partial_{\{xs[[i]],2\}}F$$

In[25]:=
```
f = g[x, y, z];
```

In[32]:=
```
p[1][f]
```

Out[32]= $-i\,g^{(1,0,0)}[x, y, z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

$$P[i_,\ F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

```
ClearAll[p]
```

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]}\,\#\ \&$$

$$H = -\frac{1}{2\,m}\sum\partial_{\{xs[[i]],2\}}\,\#$$

In[25]:=
```
f = g[x, y, z];
```

In[32]:=
```
p[1][f]
```

Out[32]=

$$-i\,g^{(1,0,0)}[x,\ y,\ z]$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}F$$

```
ClearAll[p]
```

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]}\#\,\&$$

$$H = -\frac{1}{2\,m}\sum_{\square}\partial_{\{xs[[i]],2\}}\#$$

In[25]:=
```
f = g[x, y, z];
```

In[32]:=
```
p[1][f]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}];
```

$$\mathbf{p}[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

$$\mathbf{P}[i_,\ F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

```
ClearAll[p]
```

$$\mathbf{p}[i_] := \frac{1}{I}\,\partial_{xs[[i]]}\,\#\ \&$$

$$\mathbf{H} = -\frac{1}{2\,m}\sum_{i=1}\partial_{\{xs[[i]],2\}}\,\#$$

In[25]:=
```
f = g[x, y, z];
```

In[32]:=
```
p[1][f]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

```
ClearAll[p]
```

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]}\,\#\,\&$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{\square}\partial_{\{xs[[i]],2\}}\,\#$$

In[25]:=
```
f = g[x, y, z];
```

In[32]:=
```
p[1][f]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

$$P[i_,\ F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

```
ClearAll[p]
```

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]}\,\#\ \&$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\,\#$$

In[25]:=
```
f = g[x, y, z];
```

In[32]:=
```
p[1][f]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}];
```

$$\mathbf{p}[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

$$\mathbf{P}[i_,\ F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

```
ClearAll[p]
```

$$\mathbf{p}[i_] := \frac{1}{I}\,\partial_{xs[[i]]}\,\#\ \&$$

$$\mathbf{H} = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\,\# - \frac{e^2}{r}\Big|\&$$

In[25]:=
```
f = g[x, y, z];
```

In[32]:=
```
p[1][f]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}];
```

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

$$P[i_,\ F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[33]:=

```
ClearAll[p]
```

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]}\,\#\ \&$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\,\#\ -\ \frac{e^2}{r}\,\#\ \&$$

Out[35]=

$$-\frac{\sum_{i=1}^{3}\partial_{(xs[i],2)}\,\#1}{2\,m}\ -\ \frac{e^2\,\#1}{r}\ \&$$

In[25]:=

```
f = q[x, y, z];
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}];
```

```
p[i_][F_] := 1/I ∂xs[[i]] F

P[i_, F_] := 1/I ∂xs[[i]] F
```

In[33]:=

```
ClearAll[p]

p[i_] := 1/I ∂xs[[i]] # &

H = -1/(2 m) Sum[∂{xs[[i]],2} # - e^2/r # &, {i, 1, 3}]
```

Out[35]=

$$-\frac{\sum_{i=1}^{3} \partial_{\{xs[[i]],2\}} \#1}{2\,m} - \frac{e^2\,\#1}{r}\ \&$$

In[25]:=

```
f = g[x, y, z];
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}];
```

```
p[i_][F_] := 1/I ∂xs[[i]] F

P[i_, F_] := 1/I ∂xs[[i]] F
```

```
ClearAll[p]

p[i_] := 1/I ∂xs[[i]] # &

H = -1/(2 m) Sum[∂{xs[[i]],2} # - e^2/r # &, {i, 1, 3}];
```

In[25]:=
```
f = g[x, y, z];
```

In[32]:=
```
p[1][f]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}];
```

```
p[i_][F_] := 1/I ∂xs[[i]] F
P[i_, F_] := 1/I ∂xs[[i]] F
```

```
ClearAll[p]
p[i_] := 1/I ∂xs[[i]] # &;
H = -1/(2 m) Sum[∂{xs[[i]],2} # - e^2/r # &, {i, 1, 3}];
```

In[25]:=
```
f = g[x, y, z];
```

In[32]:=
```
p[1][f]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
ϵ[a__] := Signature[{a}];
```

```
p[i_][F_] := 1/I ∂xs[[i]] F

P[i_, F_] := 1/I ∂xs[[i]] F
```

In[36]:=

```
ClearAll[p]

p[i_] := 1/I ∂xs[[i]] # &;

H = -1/(2 m) Sum[∂{xs[[i]],2} # - e^2/r # &, {i, 1, 3}];
```

```
H@
```

In[25]:=

```
f = g[x, y, z];
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
P[i_, F_] := 1/I ∂xs[[i]] F
```

In[36]:=

```
ClearAll[p]
p[i_] := 1/I ∂xs[[i]] # &;
H = -1/(2 m) Sum[∂{xs[[i]],2} # , {i, 1, 3}] - e^2/r # &;
```

In[39]:=

```
H@f
```

Out[39]=

$$-\frac{e^2\, g[x, y, z]}{\sqrt{x^2+y^2+z^2}} - \frac{g^{(0,0,2)}[x, y, z] + g^{(0,2,0)}[x, y, z] + g^{(2,0,0)}[x, y, z]}{2\, m}$$

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Untitled-20

In[36]:=

```
ClearAll[p]
```

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]}\#\,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\#-\frac{e^2}{r}\#\,\&;$$

In[39]:=

```
H@f
```

Out[39]=

$$-\frac{e^2\,g[x,y,z]}{\sqrt{x^2+y^2+z^2}}-\frac{g^{(0,0,2)}[x,y,z]+g^{(0,2,0)}[x,y,z]+g^{(2,0,0)}[x,y,z]}{2\,m}$$

```
sum
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[36]:=

```
ClearAll[p]
```

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]}\#\,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\#-\frac{e^2}{r}\#\,\&;$$

In[39]:=

```
H@f
```

Out[39]=

$$-\frac{e^2\,g[x,y,z]}{\sqrt{x^2+y^2+z^2}}-\frac{g^{(0,0,2)}[x,y,z]+g^{(0,2,0)}[x,y,z]+g^{(2,0,0)}[x,y,z]}{2\,m}$$

$\sum$

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Untitled-20

In[36]:=

```
ClearAll[p]
p[i_] := 1/I ∂xs[[i]] # &;
H = -1/(2 m) Sum[∂{xs[[i]],2} # - e^2/r # &, {i, 1, 3}];
```

In[39]:=

```
H@f
```

Out[39]=

$$-\frac{e^2\, g[x, y, z]}{\sqrt{x^2 + y^2 + z^2}} - \frac{g^{(0,0,2)}[x, y, z] + g^{(0,2,0)}[x, y, z] + g^{(2,0,0)}[x, y, z]}{2\, m}$$

In[25]:=

```
f = g[x, y, z];
```

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Untitled-20

In[36]:=

```
ClearAll[p]
```

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]}\#\,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\#-\frac{e^2}{r}\#\,\&;$$

In[39]:=

```
H@f
```

Out[39]=

$$-\frac{e^2\,g[x,y,z]}{\sqrt{x^2+y^2+z^2}}-\frac{g^{(0,0,2)}[x,y,z]+g^{(0,2,0)}[x,y,z]+g^{(2,0,0)}[x,y,z]}{2\,m}$$

$\sum$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[36]:=

```
ClearAll[p]
p[i_] := 1/I ∂xs[[i]] # &;
H = -1/(2 m) Sum[∂{xs[[i]],2} # - e^2/r # &, {i, 1, 3}];
```

In[39]:=

```
H@f
```

Out[39]=

$$-\frac{e^2\, g[x, y, z]}{\sqrt{x^2 + y^2 + z^2}} - \frac{g^{(0,0,2)}[x, y, z] + g^{(0,2,0)}[x, y, z] + g^{(2,0,0)}[x, y, z]}{2\, m}$$

$\sum_{\square}$

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Untitled-20

In[36]:=

```
ClearAll[p]
```

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]}\#\,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\#-\frac{e^2}{r}\#\,\&;$$

In[39]:=

```
H@f
```

Out[39]=

$$-\frac{e^2\,g[x,y,z]}{\sqrt{x^2+y^2+z^2}}-\frac{g^{(0,0,2)}[x,y,z]+g^{(0,2,0)}[x,y,z]+g^{(2,0,0)}[x,y,z]}{2\,m}$$

In[25]:=

```
f = g[x, y, z];
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[36]:=

```
ClearAll[p]
```

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]}\#\,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\#-\frac{e^2}{r}\#\,\&;$$

In[39]:=

```
H@f
```

Out[39]=

$$-\frac{e^2\,g[x,y,z]}{\sqrt{x^2+y^2+z^2}}-\frac{g^{(0,0,2)}[x,y,z]+g^{(0,2,0)}[x,y,z]+g^{(2,0,0)}[x,y,z]}{2\,m}$$

$$\sum_{\square=\square}^{\square}\square$$

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Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

In[36]:=

```
ClearAll[p]
```

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]} \# \,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \,\&;$$

In[39]:=

```
H@f
```

Out[39]=

$$-\frac{e^2\, g[x, y, z]}{\sqrt{x^2 + y^2 + z^2}} - \frac{g^{(0,0,2)}[x, y, z] + g^{(0,2,0)}[x, y, z] + g^{(2,0,0)}[x, y, z]}{2\,m}$$

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Untitled-20

```
P[i_, F_] := 1/I ∂xs[[i]] F
```

In[36]:=

```
ClearAll[p]
p[i_] := 1/I ∂xs[[i]] # &;
H = -1/(2 m) Sum[∂{xs[[i]],2} # , {i,1,3}] - e^2/r # &;
```

In[39]:=

```
H@f
```

Out[39]=

$$-\frac{e^2\, g[x, y, z]}{\sqrt{x^2+y^2+z^2}} - \frac{g^{(0,0,2)}[x, y, z] + g^{(0,2,0)}[x, y, z] + g^{(2,0,0)}[x, y, z]}{2\, m}$$

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Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]} \# \,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \,\&;$$

In[39]:= `H@f`

Out[39]=

$$-\frac{e^2\, g[x, y, z]}{\sqrt{x^2 + y^2 + z^2}} - \frac{g^{(0,0,2)}[x, y, z] + g^{(0,2,0)}[x, y, z] + g^{(2,0,0)}[x, y, z]}{2\,m}$$

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Untitled-20

```
P[i_, F_] := 1/I ∂xs[[i]] F
```

```
p[i_] := 1/I ∂xs[[i]] # &;
```

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\#-\frac{e^2}{r}\#\&;$$

```
L
```

In[39]:=

```
H@f
```

Out[39]=

$$-\frac{e^2\,g[x,y,z]}{\sqrt{x^2+y^2+z^2}}-\frac{g^{(0,0,2)}[x,y,z]+g^{(0,2,0)}[x,y,z]+g^{(2,0,0)}[x,y,z]}{2\,m}$$

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$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]} \# \,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \,\&;$$

L[i_] :|

In[39]:= H@f

Out[39]=

$$-\frac{e^2\, g[x, y, z]}{\sqrt{x^2 + y^2 + z^2}} - \frac{g^{(0,0,2)}[x, y, z] + g^{(0,2,0)}[x, y, z] + g^{(2,0,0)}[x, y, z]}{2\,m}$$

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$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

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$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \,\&;$$

$$L[i_] := \frac{1}{I}$$

In[39]:= H@f

Out[39]=

$$-\frac{e^2\, g[x, y, z]}{\sqrt{x^2 + y^2 + z^2}} - \frac{g^{(0,0,2)}[x, y, z] + g^{(0,2,0)}[x, y, z] + g^{(2,0,0)}[x, y, z]}{2\,m}$$

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$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]} \# \,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \,\&;$$

$$L[i_] := \frac{1}{I}\,\epsilon[i, j, k]\; xs[[j]]$$

In[39]:= `H@f`

Out[39]=

$$-\frac{e^2\, g[x, y, z]}{\sqrt{x^2 + y^2 + z^2}} - \frac{g^{(0,0,2)}[x, y, z] + g^{(0,2,0)}[x, y, z] + g^{(2,0,0)}[x, y, z]}{2\,m}$$

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$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \,\&;$$

$$L[i_] := \frac{1}{I}\,\epsilon[i, j, k]\; xs[[j]]\; \partial_{xs[[k]]} \# \,\&$$

In[39]:= H@f

Out[39]=

$$-\frac{e^2\, g[x, y, z]}{\sqrt{x^2 + y^2 + z^2}} - \frac{g^{(0,0,2)}[x, y, z] + g^{(0,2,0)}[x, y, z] + g^{(2,0,0)}[x, y, z]}{2\,m}$$

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$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]} \# \,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \,\&;$$

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In[39]:= H@f

Out[39]=

$$-\frac{e^2\, g[x, y, z]}{\sqrt{x^2 + y^2 + z^2}} - \frac{g^{(0,0,2)}[x, y, z] + g^{(0,2,0)}[x, y, z] + g^{(2,0,0)}[x, y, z]}{}$$

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Untitled-20

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$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]} \# \,\&;$$

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$$L[i_] := \frac{1}{I}\sum_{j=1}^{3}\epsilon[i, j, k]\; xs[[j]]\;\partial_{xs[[k]]} \# \,\&$$

In[39]:= H@f

Out[39]=

$$-\frac{e^2\, g[x, y, z]}{\sqrt{x^2 + y^2 + z^2}} - \frac{g^{(0,0,2)}[x, y, z] + g^{(0,2,0)}[x, y, z] + g^{(2,0,0)}[x, y, z]}{}$$

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Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

$$p[i_] := \frac{1}{I}\,\partial_{xs[[i]]} \# \,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \,\&;$$

$$L[i_] := \frac{1}{I}\sum_{j=1}^{3}\left(\sum_{j=1}^{3}\epsilon[i, j, k]\; xs[[j]]\;\partial_{xs[[k]]} \# \,\&\right)$$

In[39]:= H@f

Out[39]=

$$-\frac{e^2\, g[x, y, z]}{\sqrt{x^2 + y^2 + z^2}} - \frac{g^{(0,0,2)}[x, y, z] + g^{(0,2,0)}[x, y, z] + g^{(2,0,0)}[x, y, z]}{}$$

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Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

$$p[i_] = \frac{1}{I}\,\partial_{xs[[i]]} \# \,\&;$$

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$$L[i_] := \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i, j, k]\; xs[[j]]\;\partial_{xs[[k]]} \# \,\&;$$

In[39]:= H@f

Out[39]=

$$-\frac{e^2\, g[x, y, z]}{\sqrt{x^2 + y^2 + z^2}} - \frac{g^{(0,0,2)}[x, y, z] + g^{(0,2,0)}[x, y, z] + g^{(2,0,0)}[x, y, z]}{}$$

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Untitled-20

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$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i, j, k]\; xs[[j]]\;\partial_{xs[[k]]} \# \,\&;$$

In[39]:= H@f

Out[39]=

$$-\frac{e^2\, g[x, y, z]}{\sqrt{x^2 + y^2 + z^2}} - \frac{g^{(0,0,2)}[x, y, z] + g^{(0,2,0)}[x, y, z] + g^{(2,0,0)}[x, y, z]}{}$$

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Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

```
Clea
```

$$p[i_] = \frac{1}{I}\,\partial_{xs[[i]]} \# \&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i, j, k]\; xs[[j]]\;\partial_{xs[[k]]} \# \&;$$

In[39]:= `H@f`

Out[39]=

$$-\frac{e^2\, g[x, y, z]}{\sqrt{x^2 + y^2 + z^2}} -$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

```
ClearAll[p]
```

$$p[i_] = \frac{1}{I}\,\partial_{xs[[i]]} \# \,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \,\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i, j, k]\; xs[[j]]\;\partial_{xs[[k]]} \# \,\&;$$

In[39]:= `H@f`

Out[39]=

$$-\frac{e^2\, g[x, y, z]}{\sqrt{x^2 + y^2 + z^2}} -$$

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Untitled-20

$$P[i_, F_] := \frac{1}{I} \partial_{xs[[i]]} F$$

In[43]:=

```
ClearAll[p]
```

$$p[i_] = \frac{1}{I} \partial_{xs[[i]]} \# \&;$$

$$H = -\frac{1}{2\,m} \sum_{i=1}^{3} \partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \&;$$

$$L[i_] = \frac{1}{I} \sum_{j=1}^{3} \sum_{k=1}^{3} \epsilon[i, j, k]\, xs[[j]]\, \partial_{xs[[k]]} \# \&;$$

```
p[1]
```

In[39]:=

```
H@f
```

$$e^2\, q[x, y, z]$$

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Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

In[43]:=

```
ClearAll[p]
```

$$p[i_] = \frac{1}{I}\,\partial_{xs[[i]]}\,\#\,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\,\# - \frac{e^2}{r}\,\#\,\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i, j, k]\; xs[[j]]\;\partial_{xs[[k]]}\,\#\,\&;$$

```
p[1]@g
```

In[39]:=

```
H@f
```

$$e^2\, g[x, y, z]$$

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Untitled-20

```
P[i_, F_] := 1/I ∂xs[[i]] F
```

In[43]:=

```
ClearAll[p]
p[i_] = 1/I ∂xs[[i]] # &;
H = -1/(2 m) Sum[∂{xs[[i]],2} # , {i, 1, 3}] - e^2/r # &;
L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # , {k, 1, 3}], {j, 1, 3}] &;
```

In[47]:=

```
p[1]@f
```

Out[47]=

$-i\, g^{(1,0,0)}[x, y, z]$

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Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

$$p[i_] = \frac{1}{I}\,\partial_{xs[[i]]} \# \;\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \;\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i, j, k]\; xs[[j]]\;\partial_{xs[[k]]} \# \;\&;$$

In[47]:=
```
p[1]@f
```

Out[47]= $-i\, g^{(1,0,0)}[x, y, z]$

In[39]:=
```
H@f
```

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Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

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In[47]:=
```
p[1]@f
```

Out[47]= $-i\, g^{(1,0,0)}[x, y, z]$

In[39]:=
```
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```

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Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

$$p[i_] = \frac{1}{I}\,\partial_{xs[[i]]} \#|\,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r}\,\#\,\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i, j, k]\; xs[[j]]\;\partial_{xs[[k]]}\,\#\,\&;$$

In[47]:=
```
p[1]@f
```

Out[47]= $-i\, g^{(1,0,0)}[x, y, z]$

In[39]:=
```
H@f
```

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Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

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In[47]:=
```
p[1]@f
```

Out[47]= $-i\, g^{(1,0,0)}[x, y, z]$

In[39]:=
```
H@f
```

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Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

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$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \,\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i, j, k]\; xs[[j]]\;\partial_{xs[[k]]} \# \,\&;$$

In[47]:=
```
p[1]@f
```

Out[47]= $-i\, g^{(1,0,0)}[x, y, z]$

In[39]:=
```
H@f
```

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Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

$$p[i_] = \frac{1}{I}\,\partial_{xs[[i]]} \# \,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \,\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i, j, k]\; xs[[j]]\;\partial_{xs[[k]]} \# \,\&;$$

$$p[i_] = \frac{1}{I}\,\partial_{xs[[i]]} \# \,\&$$

In[47]:=

```
p[1]@f
```

Out[47]=

$$-i\,\sigma^{(1,0,0)}[x, y, z]$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

$$p[i_] = \frac{1}{I}\,\partial_{xs[[i]]} \# \,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \,\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i, j, k]\; xs[[j]]\;\partial_{xs[[k]]} \# \,\&;$$

$$p[i_]\,[F] := \frac{1}{I}\,\partial_{xs[[i]]} \# \,\&$$

In[47]:=

```
p[1]@f
```

Out[47]=

$$-i\,\partial^{(1,0,0)}[x, y, z]$$

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Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

$$p[i_] = \frac{1}{I}\,\partial_{xs[[i]]} \#\,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r}\,\#\,\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i, j, k]\; xs[[j]]\;\partial_{xs[[k]]} \#\,\&;$$

$$p[i_][F_] := \frac{1}{I}\,\partial_{xs[[i]]} \#\,\&$$

In[47]:=

```
p[1]@f
```

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Out[47]=

$$-i\,\partial^{(1,0,0)}[x, y, z]$$

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Untitled-20

$$P[i_, F_] := \frac{1}{I} \partial_{xs[[i]]} F$$

$$p[i_] = \frac{1}{I} \partial_{xs[[i]]} \# \&;$$

$$H = -\frac{1}{2\,m} \sum_{i=1}^{3} \partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \&;$$

$$L[i_] = \frac{1}{I} \sum_{j=1}^{3} \sum_{k=1}^{3} \epsilon[i, j, k]\, xs[[j]]\, \partial_{xs[[k]]} \# \&;$$

```
Clear
```

$$p[i_][F_] = \frac{1}{I} \partial_{xs[[i]]} F$$

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Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

$$p[i_] = \frac{1}{I}\,\partial_{xs[[i]]} \# \,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \,\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i, j, k]\; xs[[j]]\;\partial_{xs[[k]]} \# \,\&;$$

$$p[i_][F_] = \frac{1}{I}\,\partial_{xs[[i]]} F$$

In[47]:=

```
p[1]@f
```


Out[47]=

$$-i\,\partial^{(1,0,0)}[x, y, z]$$

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Untitled-20

$$P[i_, F_] := \frac{}{I}\,\partial_{xs[[i]]} F$$

$$p[i_] = \frac{1}{I}\,\partial_{xs[[i]]} \# \,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \,\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i, j, k]\; xs[[j]]\;\partial_{xs[[k]]} \# \,\&;$$

In[49]:= $$p[i_][F_] = \frac{1}{I}\,\partial_{xs[[i]]} F$$

Part::pspec : Part specification i is neither an integer nor a list of integers. ≫

Out[49]= 0

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Untitled-20

$$P[i_, F_] := \frac{}{I}\,\partial_{xs[[i]]} F$$

$$p[i_] = \frac{1}{I}\,\partial_{xs[[i]]} \# \,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \,\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i, j, k]\; xs[[j]]\;\partial_{xs[[k]]} \# \,\&;$$

In[47]:=
```
p[1]@f
```

Out[47]= $-i\, g^{(1,0,0)}[x, y, z]$

In[39]:=
```
H@f
```

$e^2\, g[x, y, z]$

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Untitled-20

```
P[i_, F_] := 1/I ∂xs[[i]] F
```

```
p[i_] = 1/I ∂xs[[i]] # &;

H = -1/(2 m) Sum[∂{xs[[i]],2} # - e^2/r # &, {i, 1, 3}];

L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1, 3}], {j, 1, 3}];
```

In[47]:=
```
p[1]@f
```

Out[47]=
```
-i g^(1,0,0)[x, y, z]
```

In[39]:=
```
H@f
```

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Untitled-20

```
P[i_, F_] := 1/I ∂xs[[i]] F
```

```
p[i_] = 1/I ∂xs[[i]] # &;

H = -1/(2 m) Sum[∂{xs[[i]],2} # , {i, 1, 3}] - e^2/r # &;

L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # , {k, 1, 3}], {j, 1, 3}] &;
```

In[47]:=
```
p[1]@f
```

Out[47]=
```
-i g^(1,0,0)[x, y, z]
```

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Untitled-20

```
P[i_, F_] := 1/I ∂xs[[i]] F
```

```
p[i_] = 1/I ∂xs[[i]] # &;

H = -1/(2 m) Sum[∂{xs[[i]],2} # , {i, 1, 3}] - e^2/r # &;

L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # , {k, 1, 3}], {j, 1, 3}] &;
```

M|

In[47]:=
```
p[1]@f
```

Out[47]= $-i\, g^{(1,0,0)}[x, y, z]$

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Untitled-20

$$P[i_, F_] := \frac{1}{I} \partial_{xs[[i]]} F$$

$$p[i_] = \frac{1}{I} \partial_{xs[[i]]} \# \&;$$

$$H = -\frac{1}{2\,m} \sum_{i=1}^{3} \partial_{\{xs[[i]],2\}} \# - \frac{e^2}{r} \# \&;$$

$$L[i_] = \frac{1}{I} \sum_{j=1}^{3} \sum_{k=1}^{3} \epsilon[i, j, k]\, xs[[j]]\, \partial_{xs[[k]]} \# \&;$$

M was the vector M

In[47]:=

```
p[1]@f
```

Out[47]= $-i\, g^{(1,0,0)}[x, y, z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
P[i_, F_] := 1/I ∂xs[[i]] F
```

```
p[i_] = 1/I ∂xs[[i]] # &;

H = -1/(2 m) Sum[∂{xs[[i]],2} # - e^2/r # &, {i, 1, 3}];

L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1, 3}], {j, 1, 3}];
```

M was the vector $M = \frac{1}{2\,m}$

In[47]:=
```
p[1]@f
```

Out[47]=
$$-i\, g^{(1,0,0)}[x, y, z]$$

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Untitled-20

```
P[i_, F_] := 1/I ∂xs[[i]] F
```

```
p[i_] = 1/I ∂xs[[i]] # &;

H = -1/(2 m) Sum[∂{xs[[i]],2} # , {i, 1, 3}] - e^2/r # &;

L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # , {k, 1, 3}], {j, 1, 3}] &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}$

In[47]:=

```
p[1]@f
```


Out[47]=

$-i\, g^{(1,0,0)}[x, y, z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]} F$$

$$p[i_] = \frac{1}{I}\,\partial_{xs[[i]]} \# \,\&;$$

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$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i, j, k]\; xs[[j]]\;\partial_{xs[[k]]} \# \,\&;$$

M was the vector $M = \frac{1}{2\,m}(p\times L - L\times p) - \frac{e^2}{r}\,r$

In[47]:=

```
p[1]@f
```

Out[47]=

$$-i\,g^{(1,0,0)}[x, y, z]$$

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In[47]:=

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p[1]@f
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Out[47]=

$$-i\,g^{(1,0,0)}[x, y, z]$$

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```
P[i_, F_] := 1/I ∂xs[[i]] F
```

```
p[i_] = 1/I ∂xs[[i]] # &;

H = -1/(2 m) Sum[∂{xs[[i]],2} # , {i, 1, 3}] - e^2/r # &;

L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # , {k, 1, 3}], {j, 1, 3}] &;

M[i_]
```

M was the vector $M = \frac{1}{2\,m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[47]:=

```
p[1]@f
```


Out[47]=

$-i\, g^{(1,0,0)}[x, y, z]$

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L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # , {k, 1, 3}], {j, 1, 3}] &;

M[i_] = 1/(2 m)
```

M was the vector $M = \frac{1}{2\,m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

```
In[47]:= p[1]@f
```

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Untitled-20

```
P[i_, F_] := 1/I ∂xs[[i]] F
```

```
p[i_] = 1/I ∂xs[[i]] # &;

H = -1/(2 m) Sum[∂{xs[[i]],2} # - e^2/r # &, {i, 1, 3}];

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M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

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In[47]:= p[1]@f
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Untitled-20

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M was the vector $M = \frac{1}{2\,m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

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In[47]:= p[1]@f
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Untitled-20

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p[i_] = 1/I ∂xs[[i]] # &;

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L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1, 3}], {j, 1, 3}];

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```

M was the vector $M = \frac{1}{2\,m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$


```
In[47]:= p[1]@f
```

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Untitled-20

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L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1, 3}], {j, 1, 3}];

M[i_] = 1/(2 m) Sum[Sum[ϵ[i, j, k]|
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$


```
In[47]:= p[1]@f
```

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Untitled-20

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L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1, 3}], {j, 1, 3}];

M[i_] = 1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]|
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$


```
In[47]:= p[1]@f
```

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Untitled-20

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M[i_] = 1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@#, {k, 1, 3}], {j, 1, 3}]
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$


```
In[47]:= p[1]@f
```

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Untitled-20

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L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # , {k, 1, 3}], {j, 1, 3}] &;

M[i_] = 1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - p[j]@L[k]@#
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$


```
In[47]:= p[1]@f
```

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Untitled-20

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M[i_] = 1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#) &, {k, 1, 3}], {j, 1, 3}]
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$


```
p[1]@f
```

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Untitled-20

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p[i_] = 1/I ∂xs[[i]] # &;

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M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#) , {k, 1, 3}], {j, 1, 3}] - e|&
```

M was the vector $M = \frac{1}{2\,m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

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Untitled-20

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M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1, 3}], {j, 1, 3}] - e^2/r | &
```

M was the vector $M = \frac{1}{2\,m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

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M[i_] = 1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1, 3}], {j, 1, 3}] -
  e^2/r xs[[i]]&
```


M was the vector $M = \frac{1}{\ }(p \times L - L \times p) - \frac{e^2}{\ }\vec{r}$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
P[i_, F_] := 1/I ∂xs[[i]] F
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M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#) , {k, 1, 3}], {j, 1, 3}] -
    e^2/r xs[[i]]|&
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

$$H = -\frac{}{2\,m}\sum_{i=1} \partial_{\{xs[[i]],2\}} \# - \frac{}{r} \# \&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3} \epsilon[i, j, k]\; xs[[j]]\; \partial_{xs[[k]]} \# \&;$$

$$M[i_] = \frac{1}{2\,m}\sum_{j=1}^{3}\sum_{k=1}^{3} \epsilon[i, j, k]\; (p[j]@L[k]@\# - L[j]@p[k]@\#) - \frac{e^2}{r}\, xs[[i]]\; \# \&$$

M was the vector $M = \frac{1}{2\,m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[47]:=
```
p[1]@f
```

Out[47]=
$$-i\, g^{(1,0,0)}[x, y, z]$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

$$H = -\frac{}{2\,m}\sum_{i=1} \partial_{\{xs[[i]],2\}} \# - \frac{}{r} \# \&;$$

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M was the vector $M = \frac{1}{2\,m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

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Untitled-20

$$H = -\frac{}{2\,m}\sum_{i=1} \partial_{\{xs[[i]],2\}} \# - \frac{}{r} \# \&;$$

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Out[56]= $$\frac{\sum_{j=1}^{3}\sum_{k=1}^{3} \epsilon[i, j, k]\; (p[j][L[k][\#1]] - L[j][p[k][\#1]])}{2\,m} - \frac{e^2\, xs[[i]]\; \#1}{r} \&$$

M was the vector $M = \frac{1}{}(p \times L - L \times p) - \frac{e^2}{}\vec{r}$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

$$H = -\frac{}{2\,m}\sum_{i=1} \partial_{\{xs[[i]],2\}} \# - \frac{}{r} \# \&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3} \epsilon[i, j, k]\; xs[[j]]\; \partial_{xs[[k]]} \# \&;$$

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M was the vector $M = \frac{1}{2\,m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[47]:=
```
p[1]@f
```

Out[47]= $-i\, g^{(1,0,0)}[x, y, z]$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

$$H = -\frac{}{2\,m}\sum_{i=1}\partial_{\{xs[[i]],2\}}\#-\frac{}{r}\#\,\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,j,k]\;xs[[j]]\;\partial_{xs[[k]]}\#\,\&;$$

$$M[i_] = \frac{1}{2\,m}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,j,k]\;(p[j]@L[k]@\#-L[j]@p[k]@\#)-\frac{e^2}{r}\,xs[[i]]\;\#\,\&;$$

M was the vector $M=\frac{1}{2\,m}(p\times L-L\times p)-\frac{e^2}{r}\vec{r}$

In[47]:=
```
p[1]@f
```

Out[47]= $-i\,g^{(1,0,0)}[x,y,z]$

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125%
✓ 150%

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Untitled-20

In[57]:=

$$p[i_] = \frac{1}{I}\,\partial_{xs[[i]]}\,\#\,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\,\# - \frac{e^2}{r}\,\#\,\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,\,j,\,k]\;xs[[j]]\;\partial_{xs[[k]]}\,\#\,\&;$$

$$M[i_] = \frac{1}{2\,m}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,\,j,\,k]\;(p[j]\,@\,L[k]\,@\,\# - L[j]\,@\,p[k]\,@\,\#) - \frac{e^2}{r}\,xs[[i]]\,\#\,\&;$$

M was the vector $M = \frac{1}{2m}(p\times L - L\times p) - \frac{e^2}{r}\vec{r}$

In[47]:=

```
p[1]@f
```

Out[47]= $-i\,g^{(1,0,0)}[x,\,y,\,z]$

In[39]:=

```
H@f
```

Out[39]= $-\frac{e^2\,g[x,\,y,\,z]}{\ldots} - \frac{g^{(0,0,2)}[x,\,y,\,z] + g^{(0,2,0)}[x,\,y,\,z] + g^{(2,0,0)}[x,\,y,\,z]}{\ldots}$

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Untitled-20

In[57]:=

$$p[i_] = \frac{1}{I}\,\partial_{xs[[i]]}\,\#\,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\,\# - \frac{e^2}{r}\,\#\,\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,\,j,\,k]\;xs[[j]]\;\partial_{xs[[k]]}\,\#\,\&;$$

$$M[i_] = \frac{1}{2\,m}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,\,j,\,k]\;(p[j]\,@\,L[k]\,@\,\# - L[j]\,@\,p[k]\,@\,\#) - \frac{e^2}{r}\,xs[[i]]\,\#\,\&;$$

M was the vector $M = \frac{1}{2m}(p\times L - L\times p) - \frac{e^2}{r}\vec{r}$

In[47]:=

```
p[1]@f
```

Out[47]= $-i\,g^{(1,0,0)}[x,\,y,\,z]$

In[55]:=

```
H@f
```

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Untitled-20

```
P[i_, F_] := 1/I ∂xs[[i]] F
```

In[57]:=

```
p[i_] = 1/I ∂xs[[i]] # &;

H = -1/(2 m) Sum[∂{xs[[i]],2} # - e^2/r # &, {i, 1, 3}];

L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1, 3}], {j, 1, 3}];

M[i_] = 1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#) - e^2/r xs[[i]] # &, {k, 1, 3}], {j, 1, 3}];
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[47]:=

```
p[1]@f
```

Out[47]= $i\, g^{(1,0,0)}[x, y, z]$

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Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[57]:=

$$p[i_] = \frac{1}{I}\,\partial_{xs[[i]]}\,\#\,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\,\# - \frac{e^2}{r}\,\#\,\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,\,j,\,k]\;xs[[j]]\;\partial_{xs[[k]]}\,\#\,\&;$$

$$M[i_] = \frac{1}{2\,m}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,\,j,\,k]\;(p[j]@L[k]@\# - L[j]@p[k]@\#) - \frac{e^2}{r}\,xs[[i]]\,\#\,\&;$$

f =

M was the vector $M = \frac{1}{2m}(p\times L - L\times p) - \frac{e^2}{r}\vec{r}$

In[47]:= p[1]@f

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Untitled-20

$$P[i_, F_] := \frac{1}{I}\,\partial_{xs[[i]]}\,F$$

In[57]:=

$$p[i_] = \frac{1}{I}\,\partial_{xs[[i]]}\,\#\,\&;$$

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\,\# - \frac{e^2}{r}\,\#\,\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,\,j,\,k]\;xs[[j]]\;\partial_{xs[[k]]}\,\#\,\&;$$

$$M[i_] = \frac{1}{2\,m}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,\,j,\,k]\;(p[j]@L[k]@\# - L[j]@p[k]@\#) - \frac{e^2}{r}\,xs[[i]]\,\#\,\&;$$

M was the vector $M = \frac{1}{2m}(p\times L - L\times p) - \frac{e^2}{r}\vec{r}$

In[61]:=

```
f = g[x, y, z];
```

In[47]:=

```
p[1]@f
```

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Untitled-20

```
P[i_, F_] := 1/I ∂xs[[i]] F
```

In[57]:=

```
p[i_] = 1/I ∂xs[[i]] # &;

H = -1/(2 m) Sum[∂{xs[[i]],2} # - e^2/r # &, {i, 1, 3}];

L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1, 3}], {j, 1, 3}];

M[i_] = 1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#) - e^2/r xs[[i]] # &, {k, 1, 3}], {j, 1, 3}];
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=

```
f = g[x, y, z];
```

In[47]:=

```
p[1]@f
```

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Untitled-20

```
P[i_, F_] := 1/I ∂xs[[i]] F
```

In[57]:=

```
p[i_] = 1/I ∂xs[[i]] # &;

H = -1/(2 m) Sum[∂{xs[[i]],2} # , {i, 1, 3}] - e^2/r # &;

L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # , {k, 1, 3}], {j, 1, 3}] &;

M[i_] = 1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1, 3}], {j, 1, 3}] - e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=

```
f = g[x, y, z];
```

In[47]:=

```
p[1]@f
```

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Untitled-20

```
P[i_, F_] := 1/I ∂xs[[i]] F
```

In[57]:=

```
p[i_] = 1/I ∂xs[[i]] # &;

H = -1/(2 m) Sum[∂{xs[[i]],2} # , {i, 1, 3}] - e^2/r # &;

L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # , {k, 1, 3}], {j, 1, 3}] &;

M[i_] = 1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#) , {k, 1, 3}], {j, 1, 3}] - e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=

```
f = g[x, y, z];
```

```
H
```

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Untitled-20

```
P[i_, F_] := 1/I ∂xs[[i]] F
```

In[57]:=

```
p[i_] = 1/I ∂xs[[i]] # &;

H = -1/(2 m) Sum[∂{xs[[i]],2} # , {i, 1, 3}] - e^2/r # &;

L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # , {k, 1, 3}], {j, 1, 3}] &;

M[i_] = 1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#) , {k, 1, 3}], {j, 1, 3}] - e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=

```
f = g[x, y, z];
```

```
H@L[1]
```

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Untitled-20

```
P[i_, F_] := 1/I ∂xs[[i]] F
```

In[57]:=

```
p[i_] = 1/I ∂xs[[i]] # &;

H = -1/(2 m) Sum[∂{xs[[i]],2} # , {i, 1, 3}] - e^2/r # &;

L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # , {k, 1, 3}], {j, 1, 3}] &;

M[i_] = 1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#) , {k, 1, 3}], {j, 1, 3}] - e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=

```
f = g[x, y, z];
```

```
H@L[1]@f - L[1]@H
```

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Untitled-20

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\#-\frac{e^2}{r}\#\,\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,j,k]\;xs[[j]]\;\partial_{xs[[k]]}\#\,\&;$$

$$M[i_] = \frac{1}{2\,m}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,j,k]\;(p[j]@L[k]@\#-L[j]@p[k]@\#)-\frac{e^2}{r}\,xs[[i]]\;\#\,\&;$$

M was the vector $M = \frac{1}{2\,m}(p\times L - L\times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

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Untitled-20

```
L[i_] = -1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1}], {j, 1}];

M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1, 3}], {j, 1, 3}] -
   e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

```
H@L[1]@f - L[1]@H@f
```

In[47]:=
```
p[1]@f
```

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Untitled-20

$$z\, g^{(0,1,2)}[x, y, z]) - i\, (2\, g^{(0,1,1)}[x, y, z] + y\, g^{(0,2,1)}[x, y, z] - z\, g^{(0,3,0)}[x, y, z]) - i\, (y\, g^{(2,0,1)}[x, y, z] - z\, g^{(2,1,0)}[x, y, z]))$$

$$+ i \left(y \left(\frac{e^2\, z\, g[x, y, z]}{(x^2 + y^2 + z^2)^{3/2}} - \frac{e^2\, g^{(0,0,1)}[x, y, z]}{\sqrt{x^2 + y^2 + z^2}} - \frac{g^{(0,0,3)}[x, y, z] + g^{(0,2,1)}[x, y, z] + g^{(2,0,1)}[x, y, z]}{2\, m} \right) - z \left(\frac{e^2\, y\, g[x, y, z]}{(x^2 + y^2 + z^2)^{3/2}} - \frac{e^2\, g^{(0,1,0)}[x, y, z]}{\sqrt{x^2 + y^2 + z^2}} - \frac{g^{(0,1,2)}[x, y, z] + g^{(0,3,0)}[x, y, z] + g^{(2,1,0)}[x, y, z]}{2\, m} \right) \right)$$

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Untitled-20

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[62]:=
```
H@L[1]@f - L[1]@H@f
```

Out[62]=

$$\frac{i\, e^2 \left(y\, g^{(0,0,1)}[x, y, z] - z\, g^{(0,1,0)}[x, y, z]\right)}{\sqrt{x^2 + y^2 + z^2}} - \frac{1}{2m}\Big(-i\left(y\, g^{(0,0,3)}[x, y, z] - 2\, g^{(0,1,1)}[x, y, z] - z\, g^{(0,1,2)}[x, y, z]\right) - i\left(2\, g^{(0,1,1)}[x, y, z] + y\, g^{(0,2,1)}[x, y, z] - z\, g^{(0,3,0)}[x, y, z]\right) - i\left(y\, g^{(2,0,1)}[x, y, z] - z\, g^{(2,1,0)}[x, y, z]\right)\Big) + i\left(y\left(\frac{e^2 z\, g[x, y, z]}{(x^2 + y^2 + z^2)^{3/2}} - \frac{e^2\, g^{(0,0,1)}[x, y, z]}{\sqrt{x^2 + y^2 + z^2}} - \right.\right.$$

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Untitled-20

```
M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1, 3}], {j, 1, 3}] -
   e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2\,m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[62]:=
```
H@L[1]@f - L[1]@H@f
```

Out[62]=

$$\frac{i\,e^2\left(y\,g^{(0,0,1)}[x,y,z] - z\,g^{(0,1,0)}[x,y,z]\right)}{\sqrt{x^2+y^2+z^2}} - \frac{1}{2\,m}\Big(-i\,\big(y\,g^{(0,0,3)}[x,y,z] - 2\,g^{(0,1,1)}[x,y,z] -$$

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Untitled-20

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i, j, k]\ xs[[j]]\ \partial_{xs[[k]]}\#\ \&;$$

$$M[i_] = \frac{1}{2\,m}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i, j, k]\ (p[j]@L[k]@\# - L[j]@p[k]@\#) - \frac{e^2}{r}\ xs[[i]]\ \#\ \&;$$

M was the vector $M = \frac{1}{2\,m}(p\times L - L\times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[62]:=
```
H@L[1]@f - L[1]@H@f
```

$$i\,e^2\left(y\,g^{(0,0,1)}[x, y, z] - z\,g^{(0,1,0)}[x, y, z]\right)$$

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Untitled-20

```
L[i_] = -1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1}], {j, 1}];

M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1}], {j, 1}] -
    e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[63]:=
```
H@L[1]@f - L[1]@H@f // Simplify
```

Out[63]= 0

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Untitled-20

```
L[i_] = -1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1}], {j, 1}];

M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1}], {j, 1}] -
    e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

```
H@L[1]@f - L[1]@H@f // Simplify
```

Out[63]= 0

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Untitled-20

```
L[i_] = -1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1}], {j, 1}];

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  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1}], {j, 1}] -
    e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
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f = g[x, y, z];
```

```
H@L[1]@f - L[1]@H@f // Simplify
```

Out[63]= 0

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

$$H = -\frac{1}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\#-\frac{e^2}{r}\#\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,j,k]\;xs[[j]]\;\partial_{xs[[k]]}\#\&;$$

$$M[i_] = \frac{1}{2\,m}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,j,k]\;(p[j]@L[k]@\#-L[j]@p[k]@\#)-\frac{e^2}{r}\,xs[[i]]\;\#\&;$$

M was the vector $M=\frac{1}{2\,m}(p\times L-L\times p)-\frac{e^2}{r}\vec{r}$

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Untitled-20

```
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M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#) -
      e^2/r xs[[i]] # &, {k, 1, 3}], {j, 1, 3}];
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

```
In[61]:= f = g[x, y, z];
```

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Untitled-20

```
H = -1/(2 m) Sum[∂{xs[[i]],2} # - e^2/r # &, {i, 1, 3}];

L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1, 3}], {j, 1, 3}];

M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1, 3}], {j, 1, 3}] -
   e^2/r xs[[i]] # &;
```

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In[61]:=

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```

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Untitled-20

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L[i_] = 1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1, 3}], {j, 1, 3}];

M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1, 3}], {j, 1, 3}] -
   e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=

```
f = g[x, y, z];
```

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Untitled-20

$$H = -\frac{\hbar^2}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\#-\frac{e^2}{r}\#\,\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,j,k]\;xs[[j]]\;\partial_{xs[[k]]}\#\,\&;$$

$$M[i_] = \frac{1}{2\,m}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,j,k]\;(p[j]@L[k]@\#-L[j]@p[k]@\#)-\frac{e^2}{r}\,xs[[i]]\,\#\,\&;$$

M was the vector $M=\frac{1}{2\,m}(p\times L-L\times p)-\frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

```
H@L[i]@f - L[l]@H@f // Simplify
```

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Untitled-20

$$H = -\frac{\hbar^2}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\,\#\;-\;\frac{e^2}{r}\,\#\;\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,\,j,\,k]\;xs[[j]]\;\partial_{xs[[k]]}\,\#\;\&;$$

$$M[i_] = \frac{1}{2\,m}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,\,j,\,k]\;(p[j]@L[k]@\#\,-\,L[j]@p[k]@\#)\;-\;\frac{e^2}{r}\;xs[[i]]\;\#\;\&;$$

M was the vector $M = \frac{1}{2\,m}(p\times L - L\times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

```
Table[H@L[i]@f - L[i]@H@f // Simplify, {
```

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Untitled-20

$$H = -\frac{\hbar^2}{2\,m}\sum_{i=1}^{3}\partial_{\{xs[[i]],2\}}\#-\frac{e^2}{r}\#\,\&;$$

$$L[i_] = \frac{1}{I}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,j,k]\;xs[[j]]\;\partial_{xs[[k]]}\#\,\&;$$

$$M[i_] = \frac{1}{2\,m}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i,j,k]\;(p[j]@L[k]@\#-L[j]@p[k]@\#)-\frac{e^2}{r}\,xs[[i]]\,\#\,\&;$$

M was the vector $M=\frac{1}{2\,m}(p\times L-L\times p)-\frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

```
Table[H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

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Untitled-20

```
L[i_] = -1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1}], {j, 1}];

M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1}], {j, 1}] -
    e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[64]:=
```
Table[H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[64]=
```
{0, 0, 0}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
L[i_] = -1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1}], {j, 1}];

M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1}], {j, 1}] -
    e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

```
Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[64]=
```
{0, 0, 0}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
L[i_] = -/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1}], {j, 1}];

M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1}], {j, 1}] -
    e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[65]:=
```
Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[65]=
```
{True, True, True}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
L[i_] = -1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1}], {j, 1}];

M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1}], {j, 1}] -
    e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2\,m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

```
Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}] //
 Full
```

Out[65]=
```
{True, True, True}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
L[i_] = -/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1}], {j, 1}];

M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1}], {j, 1}] -
    e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

```
Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}] //
 FullForm
```

Out[65]=
```
{True, True, True}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
L[i_] = -1/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1}], {j, 1}];

M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1}], {j, 1}] -
    e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

```
Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[65]=
```
{True, True, True}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
L[i_] = -/I Sum[Sum[ϵ[i, j, k] xs[[j]] ∂xs[[k]] # &, {k, 1}], {j, 1}];

M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1}], {j, 1}] -
    e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[66]:=
```
table = Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[66]=
```
{True, True, True}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1, 3}], {j, 1, 3}] -
   e^2/r xs[[i]] # &;
```

$$M[i_] = \frac{1}{2\,m}\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon[i, j, k]\,(p[j]@L[k]@\# - L[j]@p[k]@\#) - \frac{e^2}{r}\,xs[[i]]\,\#\,\&;$$

M was the vector $M = \frac{1}{2\,m}(p\times L - L\times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[66]:=
```
table = Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[66]=
```
{True, True, True}
```

```
Ful
```

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Untitled-20

```
M[i_] =
  1/(2 m) Sum[Sum[ϵ[i, j, k] (p[j]@L[k]@# - L[j]@p[k]@#), {k, 1, 3}], {j, 1, 3}] -
   e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2\,m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[66]:=
```
table = Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[66]=
```
{True, True, True}
```

```
FullForm@
```

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Untitled-20

```
j=1 k=1

e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[66]:=
```
table = Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[66]=
```
{True, True, True}
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[66]:=
```
table = Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[66]=
```
{True, True, True}
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

```
FullForm@table
```

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Untitled-20

```
— xs[[i]] # &;
r
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[66]:=
```
table = Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[66]=
```
{True, True, True}
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

```
TreeForm@table
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
1/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[66]:=
```
table = Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[66]=
```
{True, True, True}
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

```
TreeForm@table
```

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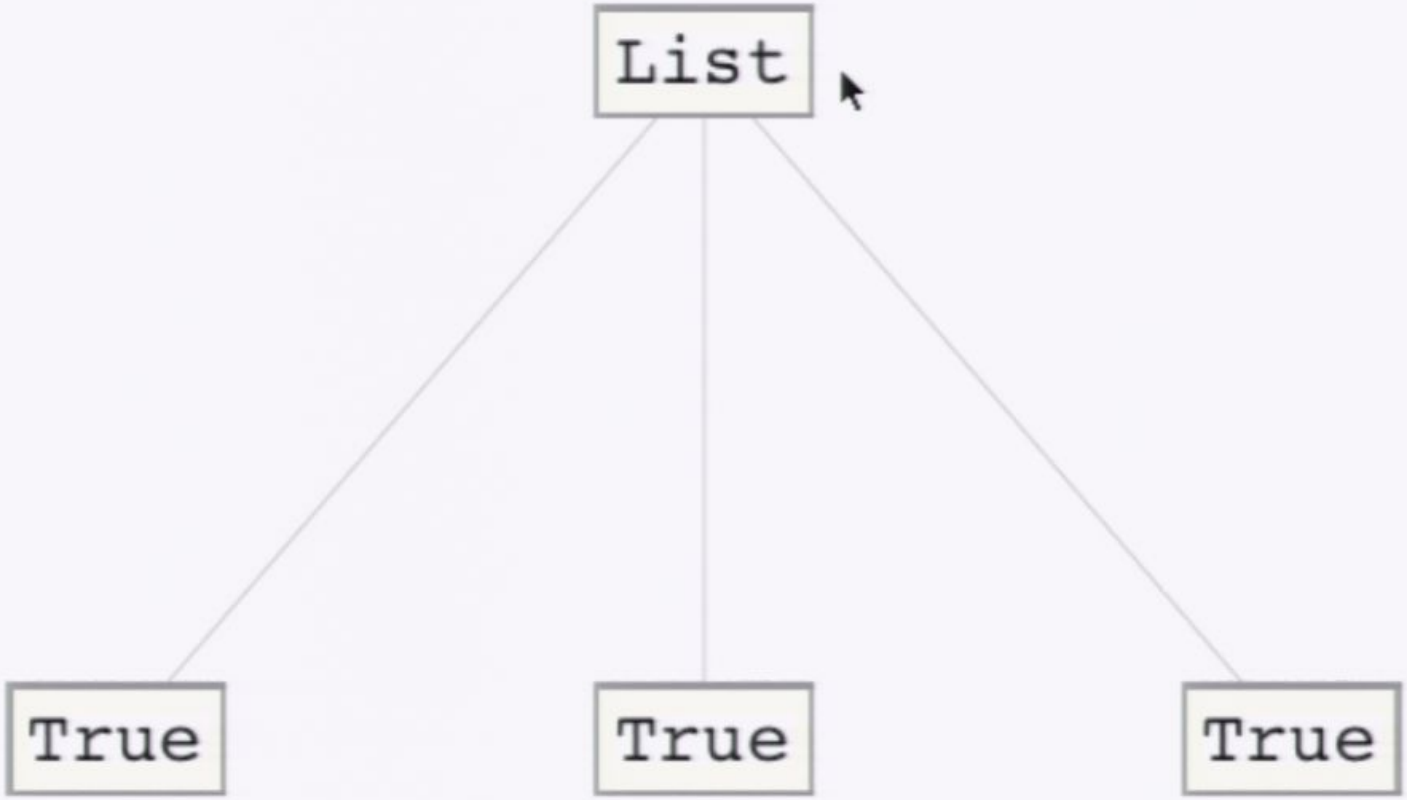
Untitled-20

In[68]:=

```
TreeForm@table
```

ut[68]//TreeForm=



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Untitled-20

In[66]:=
```
table = Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

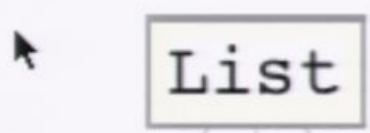
Out[66]=
```
{True, True, True}
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

In[68]:=
```
TreeForm@table
```

ut[68]//TreeForm=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
In[66]:= table = Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]

Out[66]= {True, True, True}

In[67]:= FullForm@table

Out[67]//FullForm=

List[True, True, True]

In[68]:= TreeForm@table

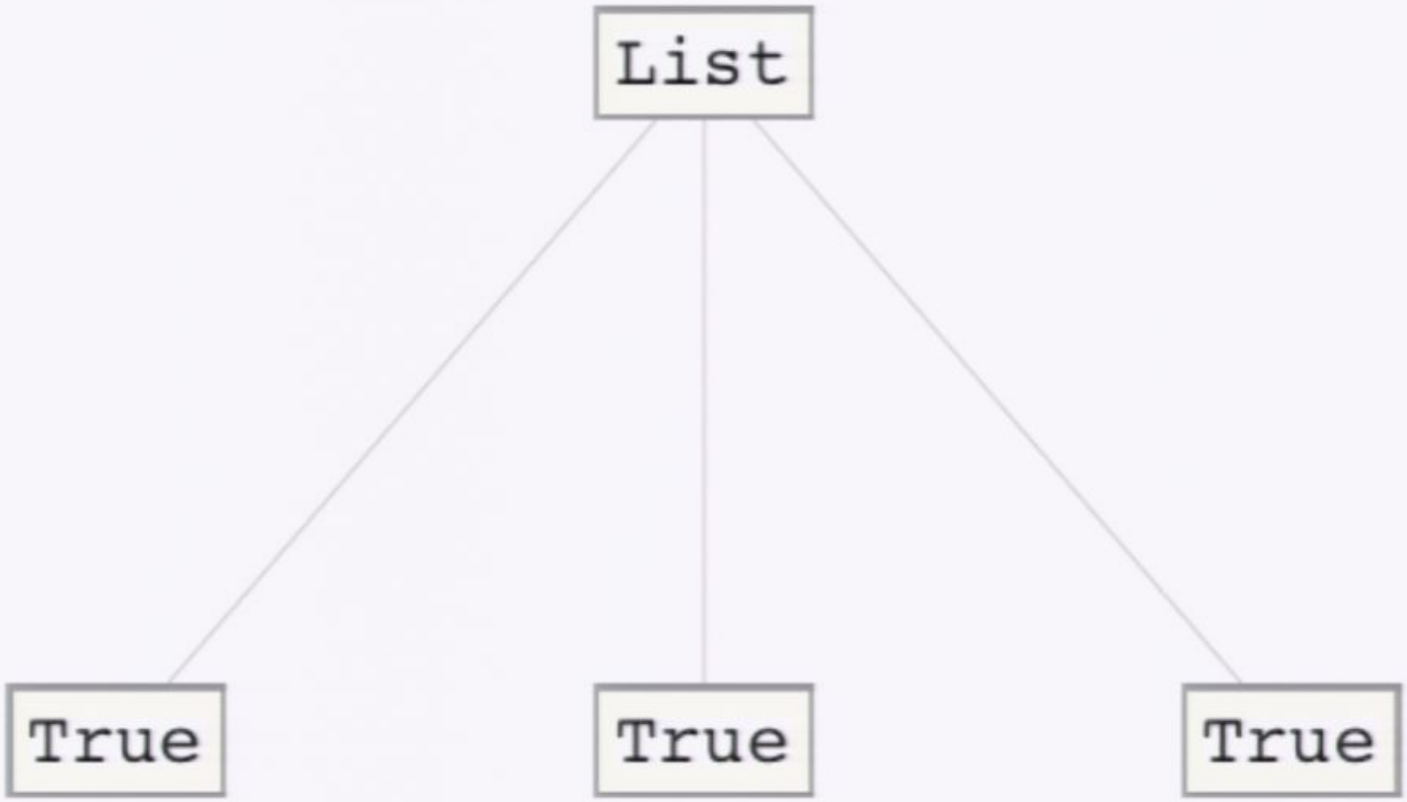
Out[68]//TreeForm=
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[68]:= `TreeForm@table`

Out[68]//TreeForm=



`And`

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[66]:=
```
table = Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[66]=
```
{True, True, True}
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

In[68]:=
```
TreeForm@table
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[66]:=
```
table = Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[66]=
```
{True, True, True}
```

```
And@
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[66]:=
```
table = Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[66]=
```
{True, True, True}
```

```
And @@ table
```

In[67]:=
```
FullForm@table
```

Out[67]//FullForm=
```
List[True, True, True]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
e^2/r xs[[i]] # &;
```

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[66]:=
```
table = Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[66]=
```
{True, True, True}
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

In[68]:=
```
TreeForm@table
```

ut[68]//TreeForm=

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[66]:=
```
table = Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[66]=
```
{True, True, True}
```

In[69]:=
```
And @@ table
```

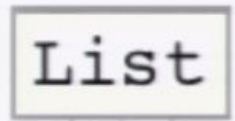
Out[69]=
```
True
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

In[68]:=
```
TreeForm@table
```

ut[68]//TreeForm=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[66]:=
```
table = Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[66]=
```
{True, True, True}
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

```
And [|
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

In[68]:=
```
TreeForm@table
```

ut[68]//TreeForm=

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[66]:=
```
table = Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[66]=
```
{True, True, True}
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

In[70]:=
```
And[1 == 1, 2 == 2]
```

Out[70]=
```
True
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[66]:=
```
table = Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[66]=
```
{True, True, True}
```

In[69]:=
```
And @@ table
```

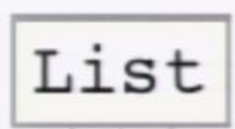
Out[69]=
```
True
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

In[68]:=
```
TreeForm@table
```

ut[68]//TreeForm=



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[66]:=
```
table = Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[66]=
```
{True, True, True}
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

In[68]:=
```
TreeForm@table
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

```
AndTable[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[66]=
```
{True, True, True}
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

In[68]:=
```
TreeForm@table
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[66]=
```
{True, True, True}
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

In[68]:=
```
TreeForm@table
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

In[68]:=
```
TreeForm@table
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

In[68]:=
```
TreeForm@table
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

In[68]:=
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TreeForm@table
```

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Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

```
L
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

In[67]:=
```
FullForm@table
```

Out[67]//FullForm=

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

```
L[i]@L[j]@f
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

In[67]:=
```
FullForm@table
```

Out[67]//FullForm=

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

```
L[i]@L[j]@f - L[j]@L[i]
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
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In[67]:=
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FullForm@table
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Out[67]//FullForm=

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Untitled-20

In[61]:=
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f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

```
L[i]@L[j]@f - L[j]@L[i]@f == I ϵ[i, k, k]
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
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In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

```
L[i]@L[j]@f - L[j]@L[i]@f == I ϵ[i, j, k]
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
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In[67]:=
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FullForm@table
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ut[67]//FullForm=

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In[61]:=
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f = g[x, y, z];
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In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

```
L[i]@L[j]@f - L[j]@L[i]@f == I ϵ[i, j, k] L[
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

```
L[i]@L[j]@f - L[j]@L[i]@f == I ϵ[i, j, k] L[k]@f
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

```
L[i]@L[j]@f - L[j]@L[i]@f == ∑I ϵ[i, j, k] L[k]@f
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

In[67]:=
```
FullForm@table
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
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f = g[x, y, z];
```

In[71]:=
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And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

```
L[i]@L[j]@f - L[j]@L[i]@f == ∑_□ I ϵ[i, j, k] L[k]@f
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```


In[67]:=
```
FullForm@table
```

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Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1} I\,\epsilon[i, j, k]\, L[k]@f$$

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```


In[67]:=
```
FullForm@table
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1} I\,\epsilon[i, j, k]\, L[k]@f$$

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```


In[67]:=
```
FullForm@table
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\,\epsilon[i, j, k]\, L[k]@f$$

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

```
Table[L[i]@L[j]@f - L[j]@L[i]@f == ∑_{k=1}^{3} I ϵ[i, j, k] L[k]@f
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

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Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

```
Table[L[i]@L[j]@f - L[j]@L[i]@f == ∑_{k=1}^{3} I ϵ[i, j, k] L[k]@f,
 {i,
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

```
Table[L[i]@L[j]@f - L[j]@L[i]@f == ∑_{k=1}^{3} I ϵ[i, j, k] L[k]@f,
 {i, 3}, {j,
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

$$\text{Table}\left[L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\,\epsilon[i, j, k]\, L[k]@f,\ \{i, 3\}, \{j, 3\}\right]$$

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

$$\text{Table}\left[L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\,\epsilon[i, j, k]\, L[k]@f \;//\; \text{Simplify}, \{i, 3\}, \{j, 3\}\right]$$

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

In[72]:=
```
Table[L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[72]=
```
{{True, True, True}, {True, True, True}, {True, True, True}}
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```


In[69]:=
```
And @@ table
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

```
FlatteTable[
 L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[72]=
```
{{True, True, True}, {True, True, True}, {True, True, True}}
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

```
Flatten@
 Table[
  L[i]@L[j]@f - L[j]@L[i]@f == ∑_{k=1}^{3} I ϵ[i, j, k] L[k]@f //
   Simplify, {i, 3}, {j, 3}]
```

Out[72]=
```
{{True, True, True}, {True, True, True}, {True, True, True}}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

In[73]:=
```
Flatten@
 Table[
  L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
   Simplify, {i, 3}, {j, 3}]
```

Out[73]=
```
{True, True, True, True, True, True, True, True, True}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

```
And@Flatten@
 Table[
  L[i]@L[j]@f - L[j]@L[i]@f == ∑_{k=1}^{3} I ϵ[i, j, k] L[k]@f //
   Simplify, {i, 3}, {j, 3}]
```

Out[73]=
```
{True, True, True, True, True, True, True, True, True}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

```
And @@
 Flatten@
  Table[
   L[i]@L[j]@f - L[j]@L[i]@f == ∑_{k=1}^{3} I ϵ[i, j, k] L[k]@f //
    Simplify, {i, 3}, {j, 3}]
```

Out[73]=
```
{True, True, True, True, True, True, True, True, True}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

In[74]:=
```
And @@
 Flatten@
  Table[
   L[i]@L[j]@f - L[j]@L[i]@f == ∑_{k=1}^{3} I ϵ[i, j, k] L[k]@f //
    Simplify, {i, 3}, {j, 3}]
```

Out[74]=
```
True
```

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Untitled-20

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

In[74]:=
```
And @@
 Flatten@
  Table[
   L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
    Simplify, {i, 3}, {j, 3}]
```

Out[74]=
```
True
```

{|

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Untitled-20

Out[71]= True

In[74]:=
```
And @@
 Flatten@
  Table[
   L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[75]:=
```
F@{a, b}
```

Out[75]= F[{a, b}]

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Untitled-20

Out[71]=
```
True
```

In[74]:=
```
And @@
 Flatten@
  Table[
   L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
    Simplify, {i, 3}, {j, 3}]
```

Out[74]=
```
True
```

```
F@{a, b}
F|@{a, b}
```

Out[75]=
```
F[{a, b}]
```

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Untitled-20

In[74]:=

```
Flatten@
 Table[
  L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
   Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[76]:=

```
F@{a, b}
F @@ {a, b}
```

Out[76]= F[{a, b}]

Out[77]= F[a, b]

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Untitled-20

```
Table[
 L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[76]:=
```
F@{a, b}
F @@ {a, b}
```

Out[76]= F[{a, b}]

Out[77]= F[a, b]

```
{a,
```

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Untitled-20

```
Table[
 L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[76]:=
```
F@{a, b}
F @@ {a, b}
```

Out[76]= F[{a, b}]

Out[77]= F[a, b]

```
F[a, b, c}
```

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Untitled-20

```
Table[
 L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[76]:=
```
F@{a, b}
F @@ {a, b}
```

Out[76]= F[{a, b}]

Out[77]= F[a, b]

```
F[a, b, c]
```

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Untitled-20

```
k=1

    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

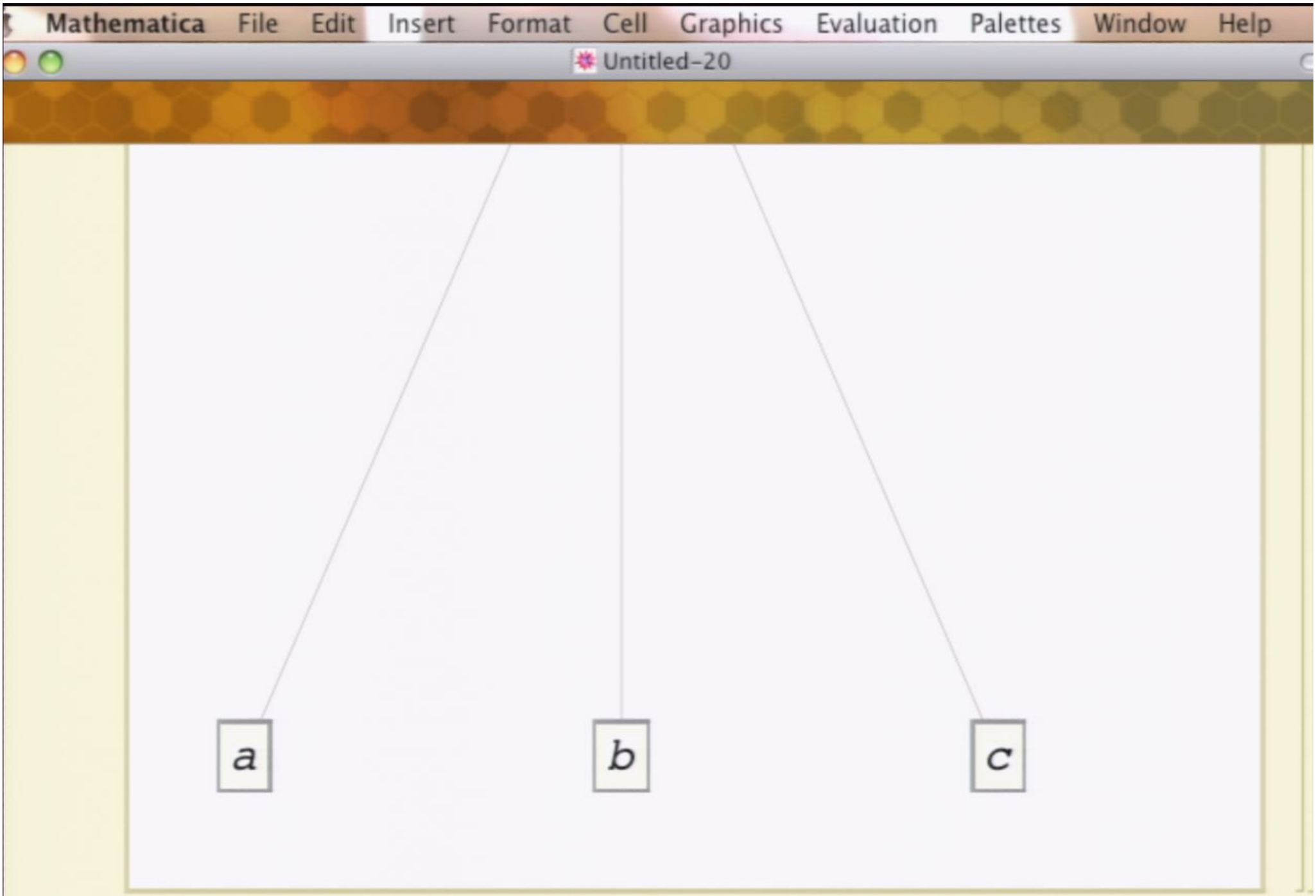
In[76]:=
```
F@{a, b}
F@@{a, b}
```

Out[76]= F[{a, b}]

Out[77]= F[a, b]

In[78]:=
```
F[a, b, c]
```

Out[78]= F[a, b, c]


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Untitled-20
a
b
c

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[77]=
```
F[a, b]
```

In[79]:=
```
F[a, b, c] // TreeForm
```

ut[79]//TreeForm=

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[77]=
```
F[a, b]
```

```
Times @@ F[a, b, c] // TreeForm
```

ut[79]//TreeForm=

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[77]=
```
F[a, b]
```

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]=
```
a b c
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

In[67]:=
```
FullForm@table
```

ut[67]//FullForm=
```
List[True, True, True]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[76]:=
```
F@{a, b}
F @@ {a, b}
```

Out[76]= F[{a, b}]

Out[77]= F[a, b]

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]= a b c

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[76]:=
```
F@{a, b}
F @@ {a, b}
```

Out[76]= F[{a, b}]

Out[77]= F[a, b]

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]= a b c

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Untitled-20

```
L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]= a b c

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]= True

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[71]=
```
True
```

In[74]:=
```
And @@
 Flatten@
  Table[
   L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
    Simplify, {i, 3}, {j, 3}]
```

Out[74]=
```
True
```

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]=
```
a b c
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[71]= True

In[74]:=

```
And @@
 Flatten@
  Table[
   L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[80]:=

```
Times @@ F[a, b, c]
```

Out[80]= a b c

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
Table[
 L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

```
And @@
 Flatten@
  Table[
   L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
    Simplify, {i, 3}, {j, 3}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
Table[
 L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

```
check[
 SAnd @@
  Flatten@
   Table[L[i]@L[j]@f - L[j]@L[i]@f ==
     Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] // Simplify, {i, 3}, {j, 3}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
Table[
 L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

```
check[
 SAnd @@
  Flatten@
   Table[L[i]@L[j]@f - L[j]@L[i]@f ==
     Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] // Simplify, {i, 3}, {j, 3}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
Table[
 L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

```
check[S_] :=
 And @@
  Flatten@
   Table[L[i]@L[j]@f - L[j]@L[i]@f ==
     Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] // Simplify, {i, 3}, {j, 3}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
Table[
 L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

```
check[S_] :=
 And @@
  Flatten@
   Table[L[i]@L[j]@f - L[j]@L[i]@f ==
     Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] // Simplify, {i, 3}, {j, 3}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
Table[
 L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]= a b c

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
Table[
 L[i]@L[j]@f - L[j]@L[i]@f == ∑_{k=1}^{3} I ϵ[i, j, k] L[k]@f //
  Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[81]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]= a b c

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
Table[
 L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[81]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
check
```

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]= a b c

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
Table[
 L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
   Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[81]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
check
```

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]= a b c

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Untitled-20

```
Table[
 L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[81]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
check
```

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]= a b c

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
Table[
 L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[81]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
check
```

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]= a b c

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
Table[
 L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[81]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
check[
```

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]= a b c

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
Table[
 L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
  Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[81]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
check[L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]]
```

In[86]:=
```
Times @@ F[a, b, c]
```

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Untitled-20

```
(y - z) g^(1,0,0)[x, y, z] == 0 &&
(x - y) g^(0,0,1)[x, y, z] + (-x + z) g^(0,1,0)[x, y, z] +
  (y - z) g^(1,0,0)[x, y, z] == 0 &&
(x - y) g^(0,0,1)[x, y, z] + (-x + z) g^(0,1,0)[x, y, z] +
  (y - z) g^(1,0,0)[x, y, z] == 0 &&
(x - y) g^(0,0,1)[x, y, z] + (-x + z) g^(0,1,0)[x, y, z] +
  (y - z) g^(1,0,0)[x, y, z] == 0 &&
(x - y) g^(0,0,1)[x, y, z] + (-x + z) g^(0,1,0)[x, y, z] +
  (y - z) g^(1,0,0)[x, y, z] == 0 &&
(x - y) g^(0,0,1)[x, y, z] + (-x + z) g^(0,1,0)[x, y, z] +
  (y - z) g^(1,0,0)[x, y, z] == 0 &&
(x - y) g^(0,0,1)[x, y, z] + (-x + z) g^(0,1,0)[x, y, z] +
  (y - z) g^(1,0,0)[x, y, z] == 0 &&
(x - y) g^(0,0,1)[x, y, z] + (-x + z) g^(0,1,0)[x, y, z] +
  (y - z) g^(1,0,0)[x, y, z] == 0 &&
(x - y) g^(0,0,1)[x, y, z] + (-x + z) g^(0,1,0)[x, y, z] +
  (y - z) g^(1,0,0)[x, y, z] == 0
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[82]:=

$$\text{check}\left[L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{j} I\,\epsilon[i, j, k]\, L[k]@f\right]$$

Out[82]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0\ \&\&$$

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0\ \&\&$$

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0\ \&\&$$

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0\ \&\&$$

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0\ \&\&$$

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0\ \&\&$$

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0\ \&\&$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[81]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[82]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]]
```

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]=
```
a b c
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[81]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
check[L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]] //
 Simpli
```

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]=
```
a b c
```

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

In[69]:=
```
And @@ table
```

Out[69]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[81]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
   Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] // Simplify
```

Out[83]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]=

a b c

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[81]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
   Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]] // Simplify
```

Out[83]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]=

a b c

```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

In[81]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
   Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]] // Simplify
```

Out[83]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]=
```
a b c
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

In[81]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
   Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]] // Simplify
```

Out[83]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]=
```
a b c
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

In[81]:=
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check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
   Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]] // Simplify
```

Out[83]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]=
```
a b c
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]= True

In[81]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
   Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]] // Simplify
```

Out[83]= $(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]= a b c

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

In[81]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
   Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]] // Simplify
```

Out[83]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]=
```
a b c
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

In[81]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
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In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
   Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]] // Simplify
```

Out[83]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]=
```
a b c
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

In[81]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[83]:=

$$\text{check}\left[L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\,\epsilon[i, j, k]\, L[k]@f\right] \text{ // Simplify}$$

Out[83]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]=
```
a b c
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

```
Atr
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
   Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]] // Simplify
```

Out[83]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

In[80]:=
```
Times @@ F[a, b, c]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

```
Attributes
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
   Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]] // Simplify
```

Out[83]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

In[80]:=
```
Times @@ F[a, b, c]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

```
Attributes[chec
   check[S_] :=
  And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
   Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]] // Simplify
```

Out[83]=
$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

In[80]:=
```
Times @@ F[a, b, c]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

```
Attributes[check]
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
   Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]] // Simplify
```

Out[83]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

In[80]:=
```
Times @@ F[a, b, c]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

```
Attributes[check] = {
  check[S_] :=
   And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
   Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]] // Simplify
```

Out[83]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

In[80]:=
```
Times @@ F[a, b, c]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

```
Attributes[check] = {HoldF
    check[S_] :=
   And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
   Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]] // Simplify
```

Out[83]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

In[80]:=
```
Times @@ F[a, b, c]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

```
Attributes[check] = {HoldFirst}
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
   Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]] // Simplify
```

Out[83]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

In[80]:=
```
Times @@ F[a, b, c]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

In[84]:=
```
Attributes[check] = {HoldFirst}
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

Out[84]=
```
{HoldFirst}
```

In[83]:=

$$\text{check}\left[L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f\right] \text{ // Simplify}$$

Out[83]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

In[86]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
   Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] // Simplify
   Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] // Simplify
```

Out[83]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

In[86]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
? che
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
   Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]] // Simplify
```

Out[83]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

In[86]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[88]:=
```
? check
```

```
Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
    Sum[I ∈[i, j, k] L[k]@f, {I, 1, 3}]] // Simplify
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

In[86]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[88]:=
```
? check
```

```
Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
    Sum[I ϵ[i, j, k] L[k]@f, {k, 3}]] // Simplify
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

In[89]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
Attributes[check] = {HoldFirst};
```

In[88]:=
```
? check
```

```
Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

In[89]:=
```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
Attributes[check] = {HoldFirst};
```

In[91]:=
```
? check
```

```
Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]= True

```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
Attributes[check] = {HoldFirst};
```

In[91]:= ? check

Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]

In[83]:= $\text{check}\left[L[i]@L[j]@f - L[j]@L[i]@f == \sum_{I \in [i,j,k]}^{3} L[k]@f\right] \text{ // Simplify}$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]= True

```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:= ? check

Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]

In[83]:= $\text{check}\left[L[i]@L[j]@f - L[j]@L[i]@f == \sum_{I \in [i,j,k]}^{3} L[k]@f\right] \text{ // Simplify}$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

In[92]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:=
```
? check
```

```
Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]= True

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:=

```
? check
```

Global`check

```
Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```

In[83]:=

$$\text{check}\left[L[i]@L[j]@f - L[j]@L[i]@f == \sum_{I \in [i,j,k]}^{3} L[k]@f\right] \text{ // Simplify}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

Out[74]=
```
True
```

In[94]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:=
```
? check
```

```
Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```

In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
    ∑_I^3 I ϵ[i, j, k] L[k]@f] // Simplify
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

```
Clear
```

In[94]:=

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:=

```
? check
```

```
Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[96]:=
```
Clear[check]
```

In[94]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:=
```
? check
```

```
Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[96]:=
```
Clear[check]
```

```
? chec
```

In[94]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:=
```
? check
```

Global`check

Attributes[check] = {HoldFirst}

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[96]:=
```
Clear[check]
```

In[97]:=
```
? check
```

Global`check

Attributes[check] = {HoldFirst}

In[94]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```


In[91]:=
```
? check
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[98]:=
```
ClearAll[check]
```

In[99]:=
```
? check
```

Global`check

In[94]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:=
```
? check
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[98]:=
```
ClearAll[check]
```

In[94]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:=
```
? check
```

```
Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

```
Attributes[check] = {HoldFirst};
```

```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:=

```
? check
```

```
Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:= ? check

```
Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:=
```
? check
```

```
Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```


In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:=

```
? check
```

Global`check

```
Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```


In[83]:=

```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:=
```
? check
```

```
Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```


In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:=

```
? check
```

```
Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```


In[83]:=

```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:=
```
? check
```

```
Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```


In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:=

```
? check
```

Global`check

```
Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```

In[83]:=

```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:=
```
? check
```

```
Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```


In[83]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f ==
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[91]:= `? check`

```
Global`check

Attributes[check] = {HoldFirst}

check[S_] := And @@ Flatten[Table[Simplify[S], {i, 3}, {j, 3}]]
```


In[83]:= `check[L[i]@L[j]@f - L[j]@L[i]@f ==`

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[101]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[83]:=

$$\text{check}\left[L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f\right] \text{ // Simplify}$$

Out[83]=

$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[101]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

$$\text{check}\left[L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\,\epsilon[i, j, k]\, L[k]@f\right]$$

Out[83]=
$$(x - y)\, g^{(0,0,1)}[x, y, z] + (-x + z)\, g^{(0,1,0)}[x, y, z] + (y - z)\, g^{(1,0,0)}[x, y, z] == 0$$

In[80]:=
```
Times @@ F[a, b, c]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[101]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[103]:=

$$\text{check}\left[L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\,\epsilon[i, j, k]\, L[k]@f\right]$$

Out[103]= True

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]= a b c

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Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[101]:=

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f]$$

In[103]:=

$$\text{check}\left[L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f\right]$$

Out[103]= True

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[101]:=

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f$$

In[103]:=

$$\text{check}\left[L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f\right]$$

Out[103]= True

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[101]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := L[i]@L[j]@f - L[j]@L[i]@f ==
```

$$\sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f$$

In[103]:=

$$\text{check}\left[L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f\right]$$

Out[103]= True

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[101]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck :=
 And[L[i]@L[j]@f - L[j]@L[i]@f == ∑_{k=1}^{3} I ϵ[i, j, k] L[k]@f
```

In[103]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f == ∑_{k=1}^{3} I ϵ[i, j, k] L[k]@f]
```


Out[103]= True

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[101]:=

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f$$

In[103]:=

$$\text{check}\left[L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f\right]$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
    Simplify, {i, 3}, {j, 3}]
```

Out[74]= True

In[101]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

In[103]:=

$$\text{check}\left[L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f\right]$$

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Untitled-20

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

In[74]:=
```
And @@
 Flatten@
  Table[
   L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
    Simplify, {i, 3}, {j, 3}]
```

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Untitled-20

M was the vector $M = \frac{1}{2m}(p \times L - L \times p) - \frac{e^2}{r}\vec{r}$

In[61]:=
```
f = g[x, y, z];
```

In[71]:=
```
And @@ Table[0 == H@L[i]@f - L[i]@H@f // Simplify, {i, 3}]
```

Out[71]=
```
True
```

In[74]:=
```
And @@
 Flatten@
  Table[
   L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}] //
    Simplify, {i, 3}, {j, 3}]
```

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Untitled-20

In[101]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[

  L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}],

  |
```

In[103]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]]
```

Out[103]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[101]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
  L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}],
  0 == H@L[i]@f - L[i]@H@f
```

In[103]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]]
```

Out[103]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[101]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
  L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}],
  0 == H@L[i]@f - L[i]@H@]
```

In[103]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]]
```

Out[103]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[101]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[104]:=

$$\text{tocheck} := \text{And}\Big[\; L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\,\epsilon[i, j, k]\; L[k]@f,\quad 0 == H@L[i]@f - L[i]@H\Big]$$

```
check[]
```

In[103]:=

$$\text{check}\Big[L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\,\epsilon[i, j, k]\; L[k]@f\Big]$$

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Untitled-20

$$y\, g^{(1,0,2)}[x, y, z] - y\, g^{(1,2,0)}[x, y, z] + x\, g^{(2,1,0)}[x, y, z] - y\, g^{(3,0,0)}[x, y, z]\Big) == 0 \;\&\&\; \frac{1}{2}\, i \Bigg(2\,(x - y)\,(0\,\&) + \frac{2\, e^2 \left(x\, g^{(0,1,0)}[x, y, z] - y\, g^{(1,0,0)}[x, y, z]\right)}{\sqrt{x^2 + y^2 + z^2}} + \frac{1}{m}\Big(x\, g^{(0,1,2)}[x, y, z] + x\, g^{(0,3,0)}[x, y, z] - y\, g^{(1,0,2)}[x, y, z] - y\, g^{(1,2,0)}[x, y, z] + x\, g^{(2,1,0)}[x, y, z] - y\, g^{(3,0,0)}[x, y, z]\Big)\Bigg) == 0$$

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Untitled-20

In[105]:=

```
check[tocheck]
```

Out[105]=

$$\frac{1}{2} i \left(2 (y - z) (0 \&) + \frac{2 e^2 \left(y g^{(0,0,1)}[x, y, z] - z g^{(0,1,0)}[x, y, z]\right)}{\sqrt{x^2 + y^2 + z^2}} + \frac{1}{m}\left(y g^{(0,0,3)}[x, y, z] - z g^{(0,1,2)}[x, y, z] + y g^{(0,2,1)}[x, y, z] - z g^{(0,3,0)}[x, y, z] + y g^{(2,0,1)}[x, y, z] - z g^{(2,1,0)}[x, y, z]\right)\right) ==$$

$$0 \&\& \frac{1}{2} i \left(2 (y - z) (0 \&) +\right.$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}],
0 == H@L[i]@f - L[i]@H]
```

In[105]:=
```
check[tocheck]
```

In[103]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]]
```

Out[103]=
```
True
```

In[80]:=
```
Times @@ F[a, b, c]
```

Out[80]=
```
a b c
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[104]:=

```
tocheck := And[
  L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}],
  0 == H@L[i]@f - L[i]@H]
```

In[105]:=

```
check[tocheck]
```

In[103]:=

```
check[L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]]
```

Out[103]=

```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
  L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}],
  0 == H@L[i]@f - L[i]@H@f]
```

In[105]:=
```
check[tocheck]
```

In[103]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]]
```

Out[103]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[106]:=

```
tocheck := And[
  L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}],
  0 == H@L[i]@f - L[i]@H@f]
```

In[107]:=

```
check[tocheck]
```

Out[107]= True

In[103]:=

```
check[L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[101]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[106]:=
```
tocheck := And[
  L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}],
  0 == H@L[i]@f - L[i]@H@f]
```

In[107]:=
```
check[tocheck]
```

Out[107]=
```
True
```

In[103]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[101]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[106]:=

$$\text{tocheck} := \text{And}\Big[\; L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\,\epsilon[i, j, k]\, L[k]@f,\quad 0 == H@L[i]@f - L[i]@H@f\Big]$$

In[107]:=
```
check[tocheck]
```

Out[107]= True

In[103]:=

$$\text{check}\Big[L[i]@L[j]@f - L[j]@L[i]@f == \sum^{3} I\,\epsilon[i, j, k]\, L[k]@f\Big]$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[108]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[110]:=
```
tocheck := And[
  L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}],
  0 == H@L[i]@f - L[i]@H@f]
```

In[111]:=
```
check[tocheck]
```

Out[111]=
```
True
```

In[103]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[108]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[110]:=

$$\text{tocheck} := \text{And}\Big[\; L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\,\epsilon[i, j, k]\; L[k]@f,\quad 0 == H@L[i]@f - L[i]@H@f \Big]$$

In[111]:=
```
check[tocheck]
```

Out[111]= True

In[103]:=

$$\text{check}\Big[L[i]@L[j]@f - L[j]@L[i]@f == \sum^{3} I\,\epsilon[i, j, k]\; L[k]@f\Big]$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[108]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[110]:=
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tocheck := And[
  L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}],
  0 == H@L[i]@f - L[i]@H@f]
```

In[111]:=
```
check[tocheck]
```

Out[111]= True

In[103]:=
```
check[L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[108]:=

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
  L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}],
  0 == H@L[i]@f - L[i]@H@f,
 ]
```

In[111]:=

```
check[tocheck]
```

Out[111]=

```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[108]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\,\epsilon[i, j, k]\, L[k]@f,$$

```
  0 == H@L[i]@f - L[i]@H@f,
  0 == H@L[i]@f - L[i]@H@f]
```

In[111]:=
```
check[tocheck]
```

Out[111]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[108]:=
```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
  L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}],
  0 == H@L[i]@f - L[i]@H@f,
 ]
```

In[111]:=
```
check[tocheck]
```

Out[111]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[108]:=

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
  L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}],
  |
  0 == H@L[i]@f - L[i]@H@f,
 ]
```

In[111]:=

```
check[tocheck]
```

Out[111]=

```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[108]:=

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

```
 0 == H@L[i]@f - L[i]@H@f,
]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[108]:=

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

```
  0 == H@L[i]@f - L[i]@H@f,
 ]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[108]:=

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

$$L[i]@p[j]@f - p[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

```
  0 == H@L[i]@f - L[i]@H@f,
 ]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[108]:=

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

$$L[i]@p[j]@f - p[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, p[k]@f,$$

$$L[i]@M[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

```
  0 == H@L[i]@f - L[i]@H@f,
 ]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[108]:=

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

$$L[i]@p[j]@f - p[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, p[k]@f,$$

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]|[k]@f,$$

```
  0 == H@L[i]@f - L[i]@H@f,
 ]
```

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Untitled-20

In[108]:=

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

$$L[i]@p[j]@f - p[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, p[k]@f,$$

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

```
  0 == H@L[i]@f - L[i]@H@f,
 ]
```

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Untitled-20

In[108]:=

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

$$L[i]@p[j]@f - p[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, p[k]@f,$$

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

```
  0 == H@L[i]@f - L[i]@H@f
 ]
```

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Untitled-20

In[108]:=

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

$$L[i]@p[j]@f - p[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, p[k]@f,$$

$$f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

$$@f - L[i]@H@f$$

- ✓ Miguel Sousa Costa
- Skype™
- Contact List
- Change Status ▶
- Remove from Dock
- ✓ Open at Login
- Show in Finder
- Hide
- Quit

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Untitled-20

In[108]:=

```
Attributes[check] = {HoldFirst};
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

$$L[i]@p[j]@f - p[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, p[k]@f,$$

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

```
  0 == H@L[i]@f - L[i]@H@f
 ]
```

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Untitled-20

In[112]:=

```
tocheck := And[
  L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}],
  L[i]@p[j]@f - p[j]@L[i]@f == Sum[I ϵ[i, j, k] p[k]@f, {k, 1, 3}],
  L[i]@M[j]@f - M[j]@L[i]@f == Sum[I ϵ[i, j, k] M[k]@f, {k, 1, 3}],
  0 == H@L[i]@f - L[i]@H@f
 ]
```

$$\sum_{k=1}^{3} I\,\epsilon[i, j, k]\, L[k]@f$$

$$\sum_{k=1}^{3} I\,\epsilon[i, j, k]\, p[k]@f$$

$$\sum_{k=1}^{3} I\,\epsilon[i, j, k]\, M[k]@f$$

In[113]:=

```
check[tocheck]
```

Out[113]=

```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

In[112]:=

```
tocheck := And[
  L[i]@L[j]@f - L[j]@L[i]@f == Sum[I ϵ[i, j, k] L[k]@f, {k, 1, 3}],
  L[i]@p[j]@f - p[j]@L[i]@f == Sum[I ϵ[i, j, k] p[k]@f, {k, 1, 3}],
  L[i]@M[j]@f - M[j]@L[i]@f == Sum[I ϵ[i, j, k] M[k]@f, {k, 1, 3}],
  0 == H@L[i]@f - L[i]@H@f
 ]
```

$$\sum_{k=1}^{3} I\,\epsilon[i, j, k]\, L[k]@f$$

$$\sum_{k=1}^{3} I\,\epsilon[i, j, k]\, p[k]@f$$

$$\sum_{k=1}^{3} I\,\epsilon[i, j, k]\, M[k]@f$$

In[113]:=

```
check[tocheck]
```

Out[113]=

```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[112]:=

```
tocheck := And[
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

$$L[i]@p[j]@f - p[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, p[k]@f,$$

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

```
  0 == H@L[i]@f - L[i]@H@f
 ]
```

In[113]:=

```
check[tocheck]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[112]:=

```
tocheck := And[
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

$$L[i]@p[j]@f - p[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, p[k]@f,$$

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

```
  0 == H@L[i]@f - L[i]@H@f
 ]
```

In[113]:=

```
check[tocheck]
```

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Untitled-20

```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

In[112]:=

```
tocheck := And[
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

$$L[i]@p[j]@f - p[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, p[k]@f,$$

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

```
  0 == H@L[i]@f - L[i]@H@f
 ]
```

In[113]:=

```
check[tocheck]
```

10:08 AM

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Untitled-20

```
check[S_] :=
 And @@ Flatten@Table[S // Simplify, {i, 3}, {j, 3}]
```

```
tocheck := And[
```

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, L[k]@f,$$

$$L[i]@p[j]@f - p[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, p[k]@f,$$

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

```
  0 == H@L[i]@f - L[i]@H@f,
 ]
```

```
In[1]:= check[tocheck]
```

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Untitled-20

$$k=1$$

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

$$0 == H@L[i]@f - L[i]@H@f,$$

$$0 == H@M[i]@f - M[i]@H@f,$$

$$0 == \sum_{k=1}^{3} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f ==$$

$$\sum_{k=1}^{3} I\epsilon[i, j, k] \left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3} (M$$

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Untitled-20

$$k=1$$

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

$$0 == H@L[i]@f - L[i]@H@f,$$

$$0 == H@M[i]@f - M[i]@H@f,$$

$$0 == \sum_{k=1}^{3} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f ==$$

$$\sum_{k=1}^{3} I\epsilon[i, j, k] \left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3} (M[k]|$$

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Untitled-20

$$k=1$$

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

$$0 == H@L[i]@f - L[i]@H@f,$$

$$0 == H@M[i]@f - M[i]@H@f,$$

$$0 == \sum_{k=1}^{3} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f ==$$

$$\sum_{k=1}^{3} I\epsilon[i, j, k] \left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3} (M[k]@M[k]|$$

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Untitled-20

$$k=1$$

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

$$0 == H@L[i]@f - L[i]@H@f,$$

$$0 == H@M[i]@f - M[i]@H@f,$$

$$0 == \sum_{k=1}^{3} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f ==$$

$$\sum_{k=1}^{3} I\epsilon[i, j, k] \left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3} \left(M[k]@M[k]@f - \frac{2}{m} H@\right.$$

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Untitled-20

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

$$0 == H@L[i]@f - L[i]@H@f,$$

$$0 == H@M[i]@f - M[i]@H@f,$$

$$0 == \sum_{k=1}^{3} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f ==$$

$$\sum_{k=1}^{3} I\epsilon[i, j, k] \left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3} \left(M[k]@M[k]@f - \frac{2}{m} H@L[k\right.$$

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Untitled-20

$$k=1$$

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\in[i, j, k]\, M[k]@f,$$

$$0 == H@L[i]@f - L[i]@H@f,$$

$$0 == H@M[i]@f - M[i]@H@f,$$

$$0 == \sum_{k=1}^{3} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f ==$$

$$\sum_{k=1}^{3} I\in[i, j, k]\left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3}\left(M[k]@M[k]@f - \frac{2}{m} H@L[k]@\right.$$

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Untitled-20

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

$$0 == H@L[i]@f - L[i]@H@f,$$

$$0 == H@M[i]@f - M[i]@H@f,$$

$$0 == \sum_{k=1}^{3} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f ==$$

$$\sum_{k=1}^{3} I\epsilon[i, j, k] \left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3} \left(M[k]@M[k]@f - \frac{2}{m} H@L[k]@L[k]\right.$$

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Untitled-20

$$k=1$$

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

$$0 == H@L[i]@f - L[i]@H@f,$$

$$0 == H@M[i]@f - M[i]@H@f,$$

$$0 == \sum_{k=1}^{3} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f ==$$

$$\sum_{k=1}^{3} I\epsilon[i, j, k] \left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3} \left(M[k]@M[k]@f - \frac{2}{m} H@L[k]@L[k]@f\right) - 1$$

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Untitled-20

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

$$0 == H@L[i]@f - L[i]@H@f,$$

$$0 == H@M[i]@f - M[i]@H@f,$$

$$0 == \sum_{k=1}^{3} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f ==$$

$$\sum_{k=1}^{3} I\epsilon[i, j, k] \left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3} \left(M[k]@M[k]@f - \frac{2}{m} H@L[k]@L[k]@f\right) - \frac{1}{\square}$$

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Untitled-20

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

$$0 == H@L[i]@f - L[i]@H@f,$$

$$0 == H@M[i]@f - M[i]@H@f,$$

$$0 == \sum_{k=1}^{3} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f ==$$

$$\sum_{k=1}^{3} I\epsilon[i, j, k] \left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3} \left(M[k]@M[k]@f - \frac{2}{m} H@L[k]@L[k]@f\right) - \frac{2}{m} H@f$$

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Untitled-20

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

$$0 == H@L[i]@f - L[i]@H@f,$$

$$0 == H@M[i]@f - M[i]@H@f,$$

$$0 == \sum_{k=1}^{3} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f ==$$

$$\sum_{k=1}^{3} I\epsilon[i, j, k] \left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3} \left(M[k]@M[k]@f - \frac{2}{m} H@L[k]@L[k]@f\right) - \frac{2}{m} H@f$$

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Untitled-20

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

$$0 == H@L[i]@f - L[i]@H@f,$$

$$0 == H@M[i]@f - M[i]@H@f,$$

$$0 == \sum_{k=1}^{3} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f ==$$

$$\sum_{k=1}^{3} I\epsilon[i, j, k] \left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3} \left(M[k]@M[k]@f - \frac{2}{m} H@L[k]@L[k]@f\right) - \frac{2}{m} H@f ==$$

]

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Untitled-20

$$0 == H@M[i]@f - M[i]@H@f,$$

$$0 == \sum_{k=1}^{3} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k] \left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3} \left(M[k]@M[k]@f - \frac{2}{m} H@L[k]@L[k]@f\right) - \frac{2}{m} H@f == e^4 f$$

]

In[115]:=

```
check[tocheck]
```

Out[115]=

```
True
```

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Running...Untitled-20

$$0 == H@M[i]@f - M[i]@H@f,$$

$$0 == \sum_{k=1}^{3} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k] \left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3} \left(M[k]@M[k]@f - \frac{2}{m} H@L[k]@L[k]@f\right) - \frac{2}{m} H@f == e^4 f$$

]

In[117]:=

```
check[tocheck]
```

Out[115]=

```
True
```

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Untitled-20

$$0 == \sum_{k=1} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k] \left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3} \left(M[k]@M[k]@f - \frac{2}{m} H@L[k]@L[k]@f\right) - \frac{2}{m} H@f == e^4 f$$

]

In[117]:=
```
check[tocheck]
```

Out[117]=
```
True
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-20

```
0 == H@L[i]@f - L[i]@H@f,
0 == H@M[i]@f - M[i]@H@f,
```

$$0 == \sum_{k=1}^{3} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k] \left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3} \left(M[k]@M[k]@f - \frac{2}{m} H@L[k]@L[k]@f\right) - \frac{2}{m} H@f == e^4 f$$

```
]
```

In[ ]:=

```
check[tocheck]
```

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Untitled-20

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

$$0 == H@L[i]@f - L[i]@H@f,$$

$$0 == H@M[i]@f - M[i]@H@f,$$

$$0 == \sum_{k=1}^{3} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f ==$$

$$\sum_{k=1}^{3} I\epsilon[i, j, k] \left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3} \left(M[k]@M[k]@f - \frac{2}{m} H@L[k]@L[k]@f\right) - \frac{2}{m} H@f == e^4 f$$

]

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Untitled-20

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

$$0 == H@L[i]@f - L[i]@H@f,$$

$$0 == H@M[i]@f - M[i]@H@f,$$

$$0 == \sum_{k=1}^{3} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f ==$$

$$\sum_{k=1}^{3} I\epsilon[i, j, k] \left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3} \left(M[k]@M[k]@f - \frac{2}{m} H@L[k]@L[k]@f\right) - \frac{2}{m} H@f == e$$

50%
75%
100%
125%
150%

]

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Untitled-20

$$L[i]@L[j]@f - L[j]@L[i]@f == \sum_{k=1} I\epsilon[i, j, k]\, L[k]@f,$$

$$L[i]@p[j]@f - p[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, p[k]@f,$$

$$L[i]@M[j]@f - M[j]@L[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k]\, M[k]@f,$$

$$0 == H@L[i]@f - L[i]@H@f,$$

$$0 == H@M[i]@f - M[i]@H@f,$$

$$0 == \sum_{k=1}^{3} L[k]@M[k]@f,$$

$$0 == \sum_{k=1}^{3} M[k]@L[k]@f,$$

$$M[i]@M[j]@f - M[j]@M[i]@f == \sum_{k=1}^{3} I\epsilon[i, j, k] \left(-\frac{2}{m}\right) H@L[k]@f,$$

$$\sum_{k=1}^{3} \left(M[k]@M[k]@f - \frac{2}{m} H@L[k]@L[k]@f\right) - \frac{2}{m} H@f == e^4 f$$

SU3.nb

Let $V(x) = x^L$ with $L \in \mathbb{Z}^+$

Let $Q(x)$ be a polynomial of degree N

Let $T(x)$ be a polynomial.

How to find $T(x)$ and $Q(x)$ (there are finitely many solutions)

knowing that

$$T(x) = \frac{V(x+i/2)Q(x+i) + V(x-i/2)Q(x-i)}{Q(x)}$$

Hint: take $L=2$ and small N first.