Title: Introduction to Complex Analysis/Computer Skills - Lecture 4A

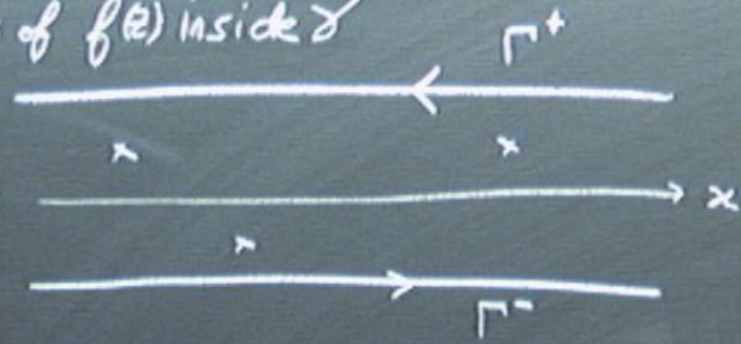
Date: Sep 02, 2010 09:00 AM

URL: http://pirsa.org/10090074

Abstract:

$$\int \frac{dz}{2\pi i} f(z) = \sum \text{Residues of } f(z) \text{ inside } \gamma$$

$$\Gamma = \Gamma^+ \cup \Gamma^-$$



function decreasing as fast as $1/z^2$ at ∞ inside Γ

$$\int_\Gamma \frac{dz}{2\pi i} f(z) = \sum \text{Residues of } f(z) \text{ inside the strip } \Gamma$$

$$\int \frac{dz}{2\pi i} f(z) = \sum \text{Residues of } f(z) \text{ inside } \gamma$$

Γ^+

Γ =

Γ^-

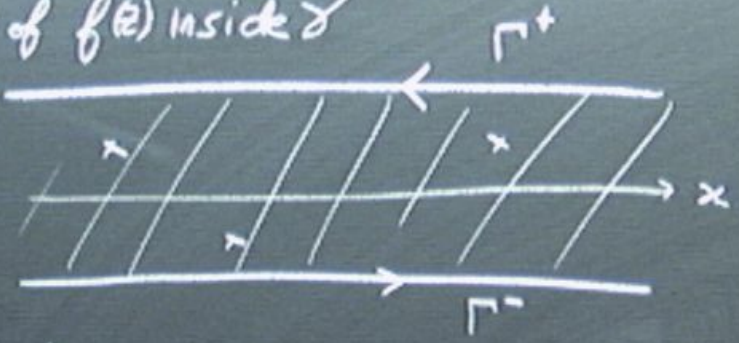
$= \Gamma^+ \cup \Gamma^-$

function decreasing as fast as $1/z^2$ at ∞ inside Γ

$$\int_\Gamma \frac{dz}{2\pi i} f(z) = \sum \text{Residues of } f(z) \text{ inside the strip } \Gamma$$

$$\int \frac{dz}{2\pi i} f(z) = \sum \text{Residues of } f(z) \text{ inside } \gamma$$

Γ =



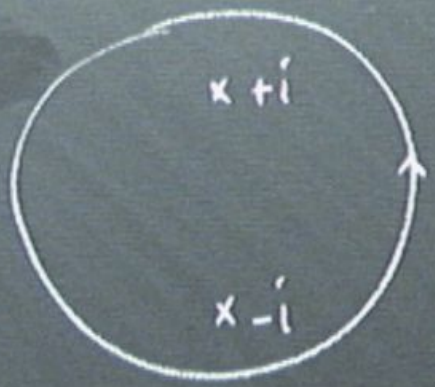
$= \Gamma^+ \cup \Gamma^-$

function decreasing as fast as $1/z^2$ at ∞ inside Γ

$$\int_\Gamma \frac{dz}{2\pi i} f(z) = \sum \text{Residues of } f(z) \text{ inside the strip } \Gamma$$

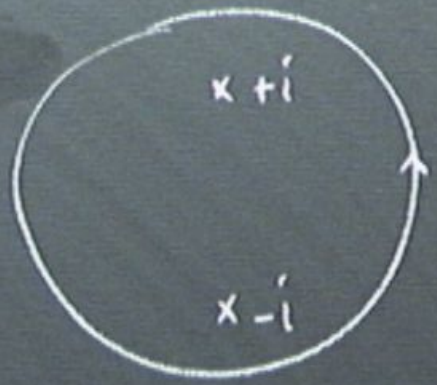
$$\int \frac{dx}{2\pi i} \frac{1}{x^2+1}$$

$|z|=2$



$$\int \frac{dx}{2\pi i} \underbrace{\frac{1}{x^2+1}}_{dt+i}$$

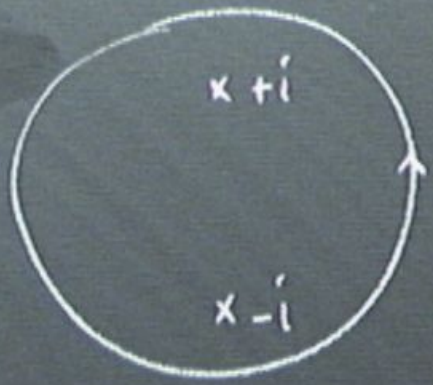
$|z|=2$



$$\frac{1}{x^2+1} = \frac{1}{(x-i)(x+i)}$$

$$\int \frac{dx}{2\pi i} \frac{1}{x^2+1}$$

$|z|=2$



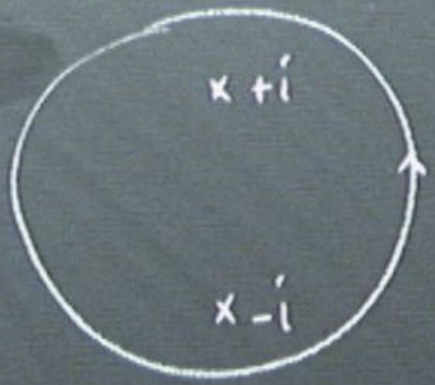
$$\underbrace{\qquad\qquad}_{dt+i}$$

$$\frac{1}{x^2+1} = \frac{1}{(x-i)(x+i)}$$

$$\underbrace{\qquad}_{2i}$$

$$\int \frac{dx}{2\pi i} \underbrace{\frac{1}{x^2+1}}_{at+i}$$

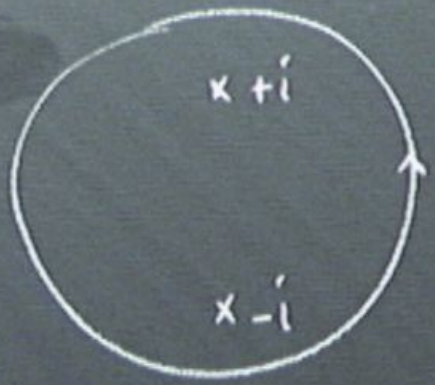
$|z|=2$



$$\frac{1}{x^2+1} = \frac{1}{(x-i)\underbrace{(x+i)}_{2i}}$$

$$\text{Res}_f \text{ at } x=i = \frac{1}{2i}$$

$$\int_{|z|=2} \frac{dx}{2\pi i} \underbrace{\frac{1}{x^2+1}}_{at+i}$$

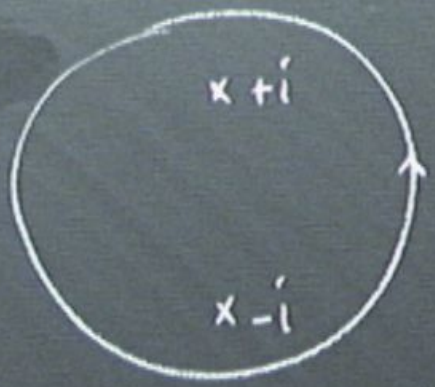


$$\frac{1}{x^2+1} = \frac{1}{(x-i)(x+i)}$$

$$\left(\text{Res } f \text{ at } x=i\right) =$$

$$\left(\text{Res } f \text{ at } x=-i\right) =$$

$$\int_{|z|=2} \frac{dx}{2\pi i} \underbrace{\frac{1}{x^2+1}}_{at+i}$$



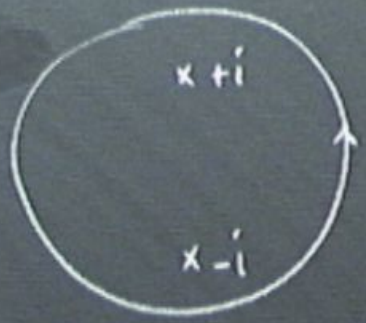
$$\frac{1}{x^2+1} = \frac{1}{(x-i)\underbrace{(x+i)}_{2i}}$$

$$\left(\text{Res } f \text{ at } x=i\right) = \frac{1}{2i}$$

$$\left(\text{Res } f \text{ at } x=-i\right) = -\frac{1}{2i}$$

$$\int_{|z|=2} \frac{dx}{2\pi i} \underbrace{\frac{1}{x^2+1}}_{dt+i} = 0$$

- $P(z) \,/\!/\, \log \,/\!/\, \partial$
- discontinuity function
- Residue at ∞



$$\frac{1}{x^2+1} = \frac{1}{(x-i)(x+i)}$$

$$\underbrace{\quad}_{2i}$$

$$f \simeq \frac{1}{2i} \frac{1}{x-i}$$

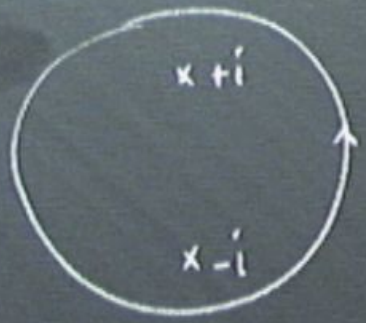
$$f \simeq -\frac{1}{2i} \frac{1}{x+i}$$

$$(\text{Res } f \text{ at } x=i) = \frac{1}{2i}$$

$$(\text{Res } f \text{ at } x=-i) = -\frac{1}{2i}$$

$$\int_{|z|=2} \frac{dx}{2\pi i} \underbrace{\frac{1}{x^2+1}}_{\text{at } +i} = 0$$

- P(z) // log // ∂
- discontinuity function
- Residue at ∞



$$\frac{1}{x^2+1} = \underbrace{\frac{1}{(x-i)(x+i)}}_{2i}$$

$$f \simeq \frac{1}{2i}\,\frac{1}{x-i}$$

$$f \simeq -\frac{1}{2i}\,\frac{1}{x+i}$$

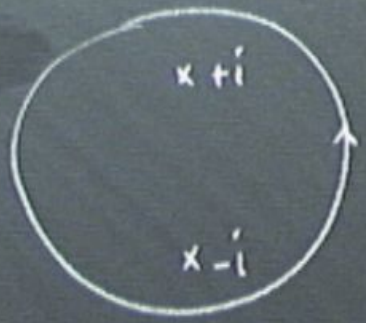
$$(\text{Res } f \text{ at } x=i) = \frac{1}{2i}$$

$$(\text{Res } f \text{ at } x=-i) = -\frac{1}{2i}$$

$$\frac{1}{x^2+1} = \left(\frac{1}{x-i} - \frac{1}{x+i}\right)\frac{1}{2i}$$

$$\int_{|z|=2} \frac{dx}{2\pi i} \underbrace{\frac{1}{x^2+1}}_{dt+i} = 0$$

- $P(z) \,/\!/\, \log \,/\!/\, \partial$
- discontinuity function
- Residue at ∞



$$\frac{1}{x^2+1} = \underbrace{\frac{1}{(x-i)(x+i)}}_{2i}$$

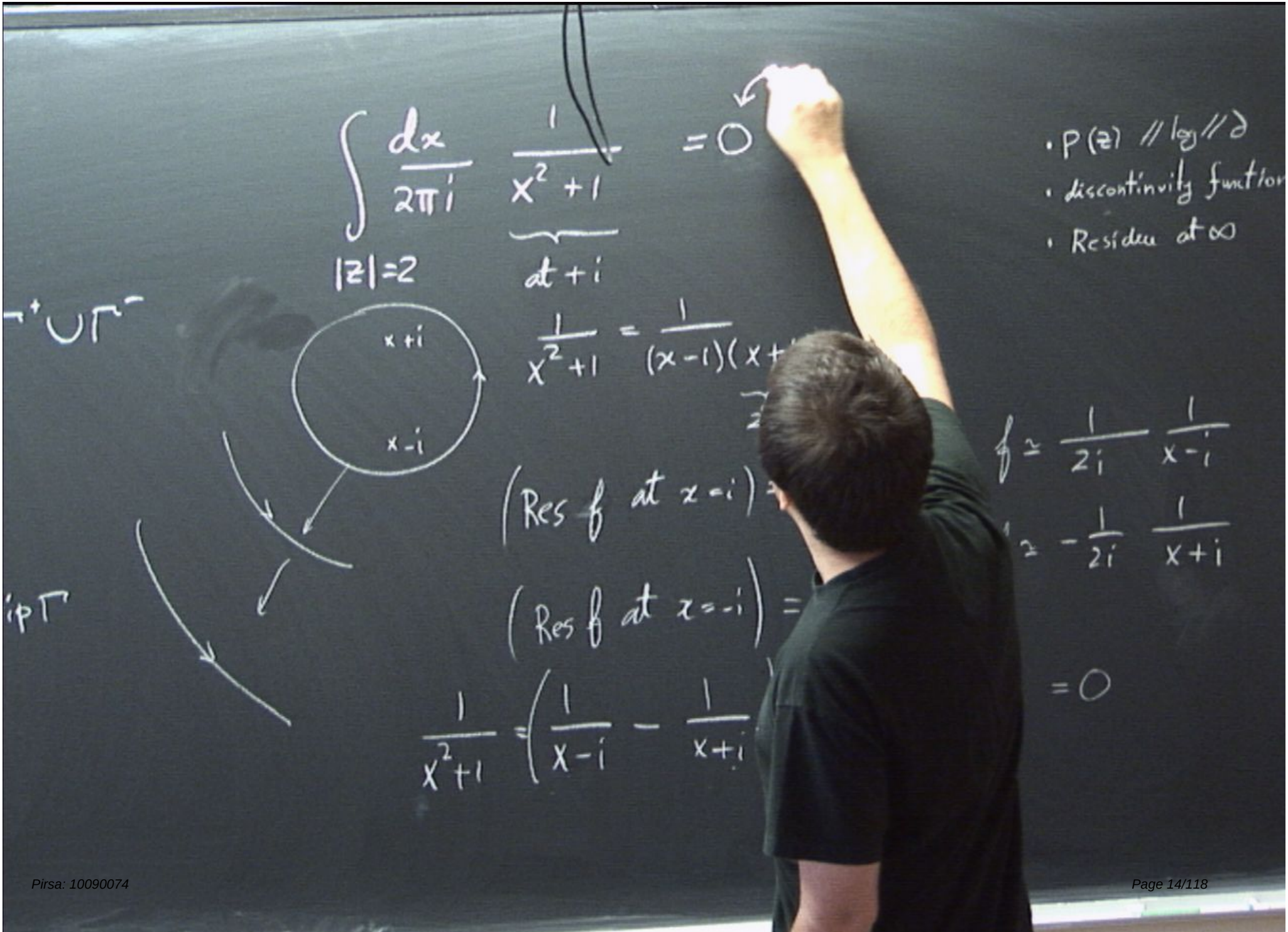
$$(\text{Res } f \text{ at } x=i) = \frac{1}{2i}$$

$$(\text{Res } f \text{ at } x=-i) = -\frac{1}{2i}$$

$$f = \frac{1}{2i}\frac{1}{x-i}$$

$$f = -\frac{1}{2i}\frac{1}{x+i}$$

$$\frac{1}{x^2+1} = \left(\frac{1}{x-i} - \frac{1}{x+i}\right)\frac{1}{2i} \rightarrow \int_\gamma f = 0$$


$\int \frac{dx}{2\pi i} \frac{1}{x^2+1} = 0$
$|z|=2$
$\frac{1}{at+i}$
$\frac{1}{x^2+1} = \frac{1}{(x-1)(x+$
x+i
x-i
(Res f at x=i)
(Res f at x=-i) =
$\frac{1}{x^2+1} = \left(\frac{1}{x-i} - \frac{1}{x+i}\right.$
$f \simeq \frac{1}{2i} \frac{1}{x-i}$
$\simeq -\frac{1}{2i} \frac{1}{x+i}$
$= 0$
• P(z) // log // ∂
• discontinuity function
• Residue at ∞

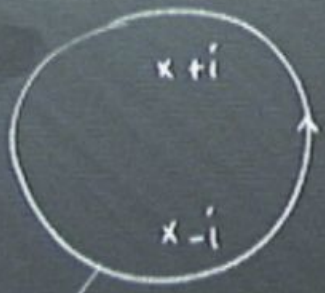
$$\int \frac{dx}{2\pi i} \underbrace{\frac{1}{x^2+1}}_{at\ +i} = 0$$

because we can close γ to ∞

$|z|=2$

- $P(z)$ // log // ∂
- discontinuity function
- Residue at ∞

$\Gamma^+ \cup \Gamma^-$



$$\frac{1}{x^2+1} = \underbrace{\frac{1}{(x-1)(x+i)}}_{2i}$$

$$f = \frac{1}{2i}\frac{1}{x-i}$$

$$f = -\frac{1}{2i}\frac{1}{x+i}$$

$$(\text{Res } f \text{ at } x=i) = \frac{1}{2i}$$

$$(\text{Res } f \text{ at } x=-i) = -\frac{1}{2i}$$

$ip\Gamma$

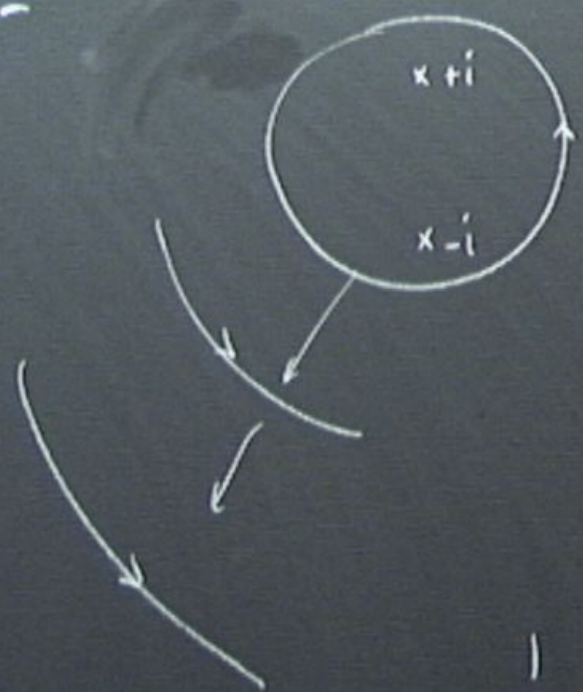
$$\frac{1}{x^2+1} = \left(\frac{1}{x-i} - \frac{1}{x+i}\right)\frac{1}{2i} \rightarrow \oint_\gamma f = 0$$

$$\int_{|z|=2} \frac{dx}{2\pi i} \underbrace{\frac{1}{x^2+1}}_{\text{at } +i} = 0$$

because we can blow γ to ∞

- $P(z)$ // log // ∂
- discontinuity function
- Residue at ∞

$\Gamma^+ \cup \Gamma^-$



$$\frac{1}{x^2+1} = \frac{1}{(x-i)\underbrace{(x+i)}_{2i}}$$

$$(\text{Res } f \text{ at } x=i) = \frac{1}{2i}$$

$$(\text{Res } f \text{ at } x=-i) = -\frac{1}{2i}$$

$$f \simeq \frac{1}{2i}\frac{1}{x-i}$$

$$f \simeq -\frac{1}{2i}\frac{1}{x+i}$$

$\text{ip}\,\Gamma$

$$\frac{1}{x^2+1} = \left(\frac{1}{x-i} - \frac{1}{x+i}\right)\frac{1}{2i} \rightarrow \int_\gamma f = 0$$

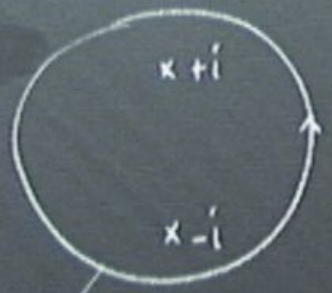
$$\int \frac{dx}{2\pi i} \underbrace{\frac{1}{x^2+1}}_{\text{at } +i} = 0$$

because we can [illegible] γ to ∞

$|z| = 2$

- $P(z)$ // log // ∂
- discontinuity function
- Residue at ∞

$\Gamma^+ \cup \Gamma^-$



Γ

$$\frac{1}{x^2+1} = \underbrace{\frac{1}{(x-i)(x+i)}}_{2i}$$

$$(\text{Res } f \text{ at } x=i) = \frac{1}{2i}$$

$$(\text{Res } f \text{ at } x=-i) = -\frac{1}{2i}$$

$$f = \frac{1}{2i}\frac{1}{x-i}$$

$$f = -\frac{1}{2i}\frac{1}{x+i}$$

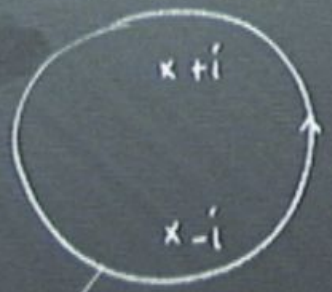
$$\frac{1}{x^2+1} = \left(\frac{1}{x-i} - \frac{1}{x+i}\right)\frac{1}{2i} \rightarrow \int_\gamma f = 0$$

$$\int_{|z|=2} \frac{dx}{2\pi i} \underbrace{\frac{1}{x^2+1}}_{at\ +i} = 0$$

because we can close γ to ∞

- $P(z) \,//\, \log \,//\, \partial$
- discontinuity function
- Residue at ∞

$\Gamma^+ \cup \Gamma^-$



Γ

$$\frac{1}{x^2+1} = \underbrace{\frac{1}{(x-i)(x+i)}}_{2i}$$

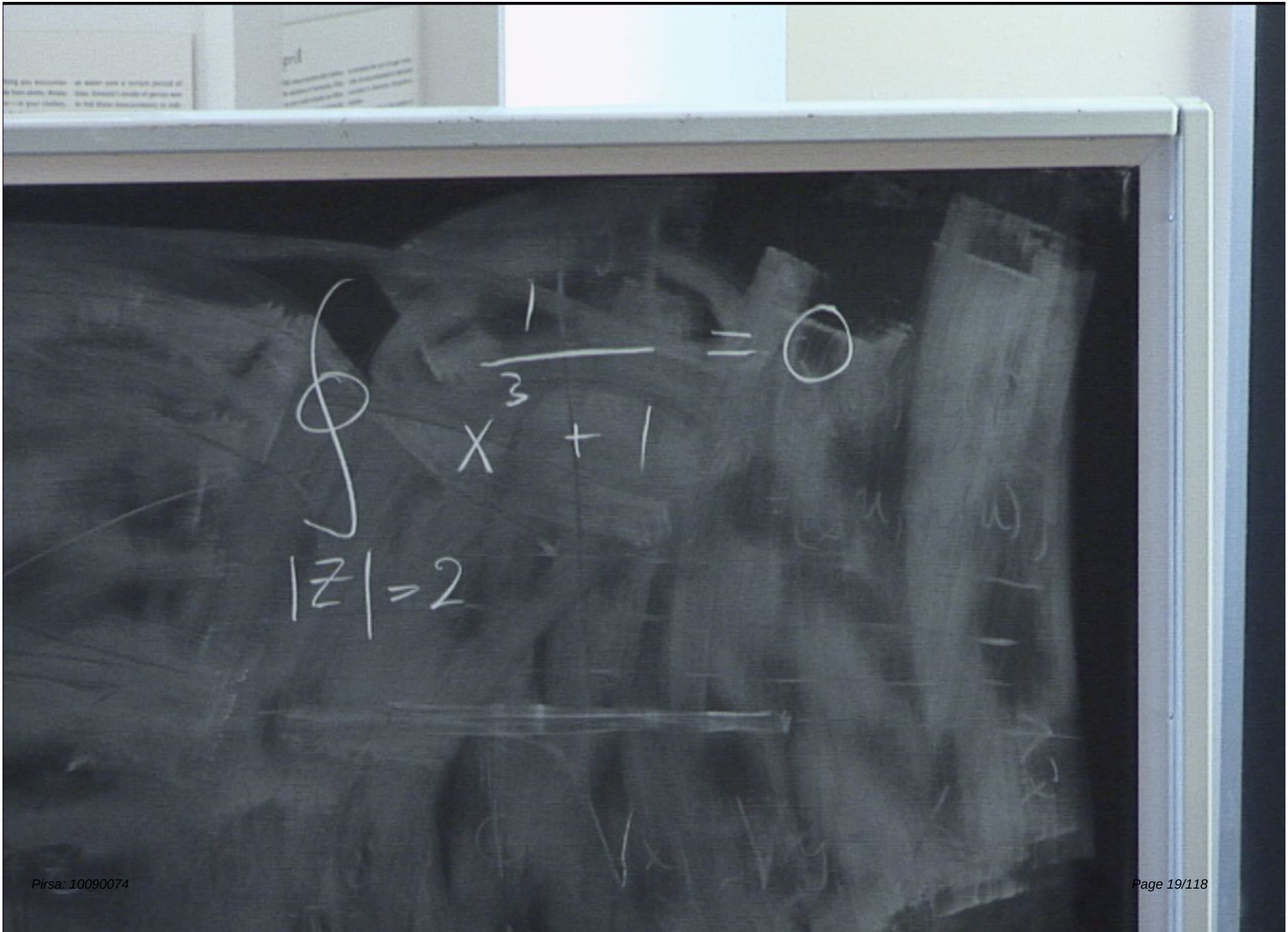
$$(\text{Res } f \text{ at } x=i) = \frac{1}{2i}$$

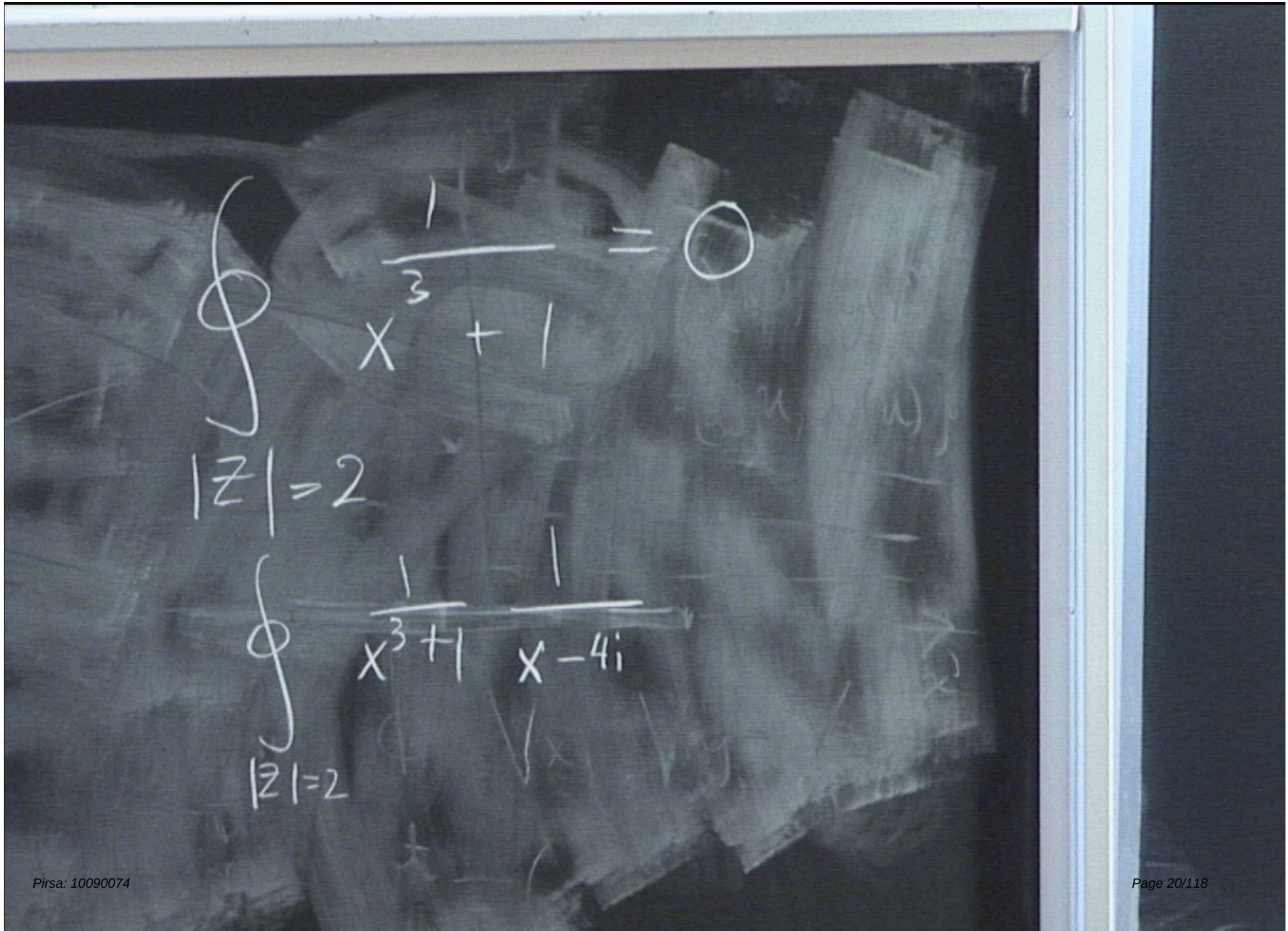
$$(\text{Res } f \text{ at } x=-i) = -\frac{1}{2i}$$

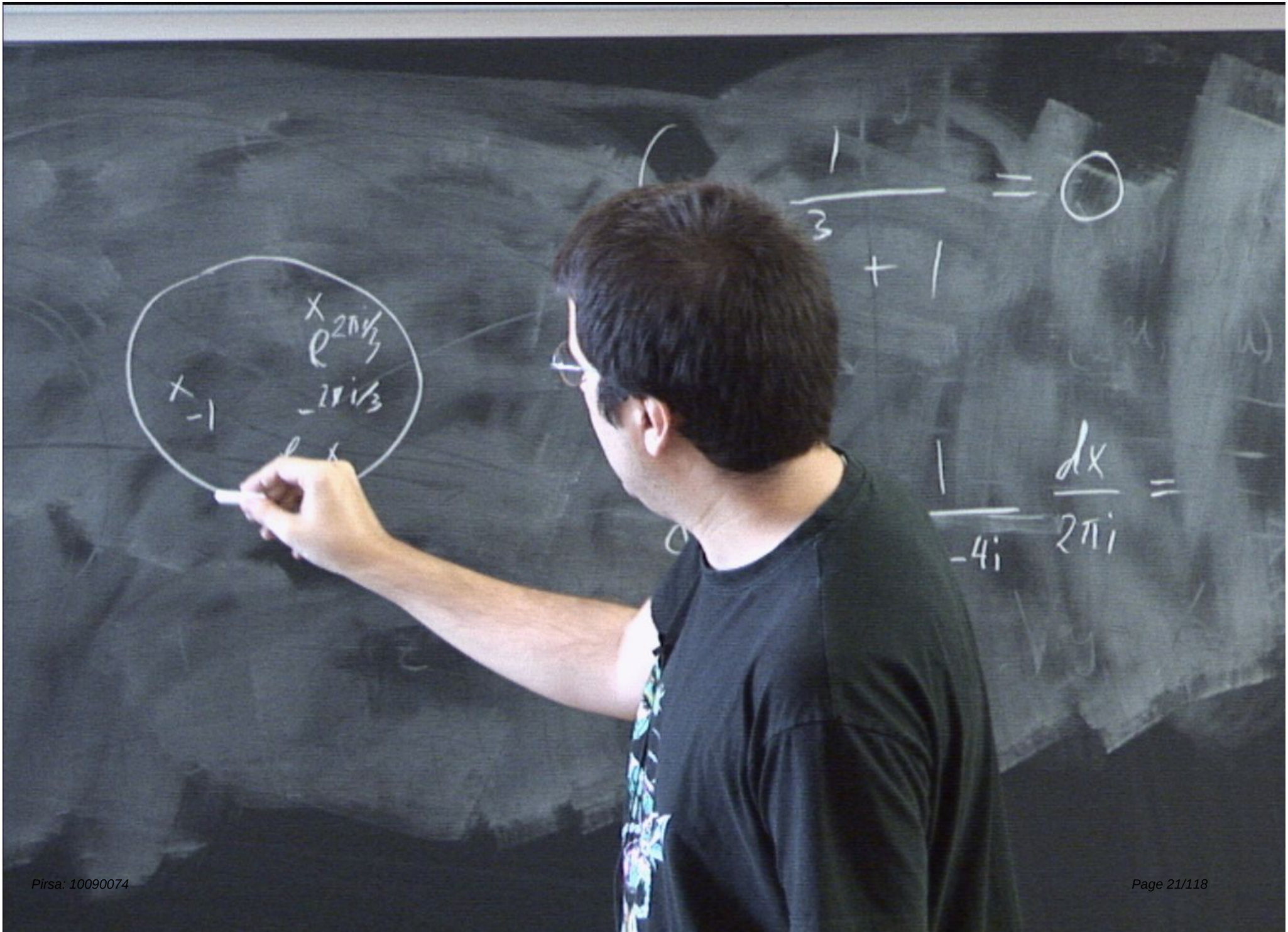
$$f = \frac{1}{2i}\frac{1}{x-i}$$

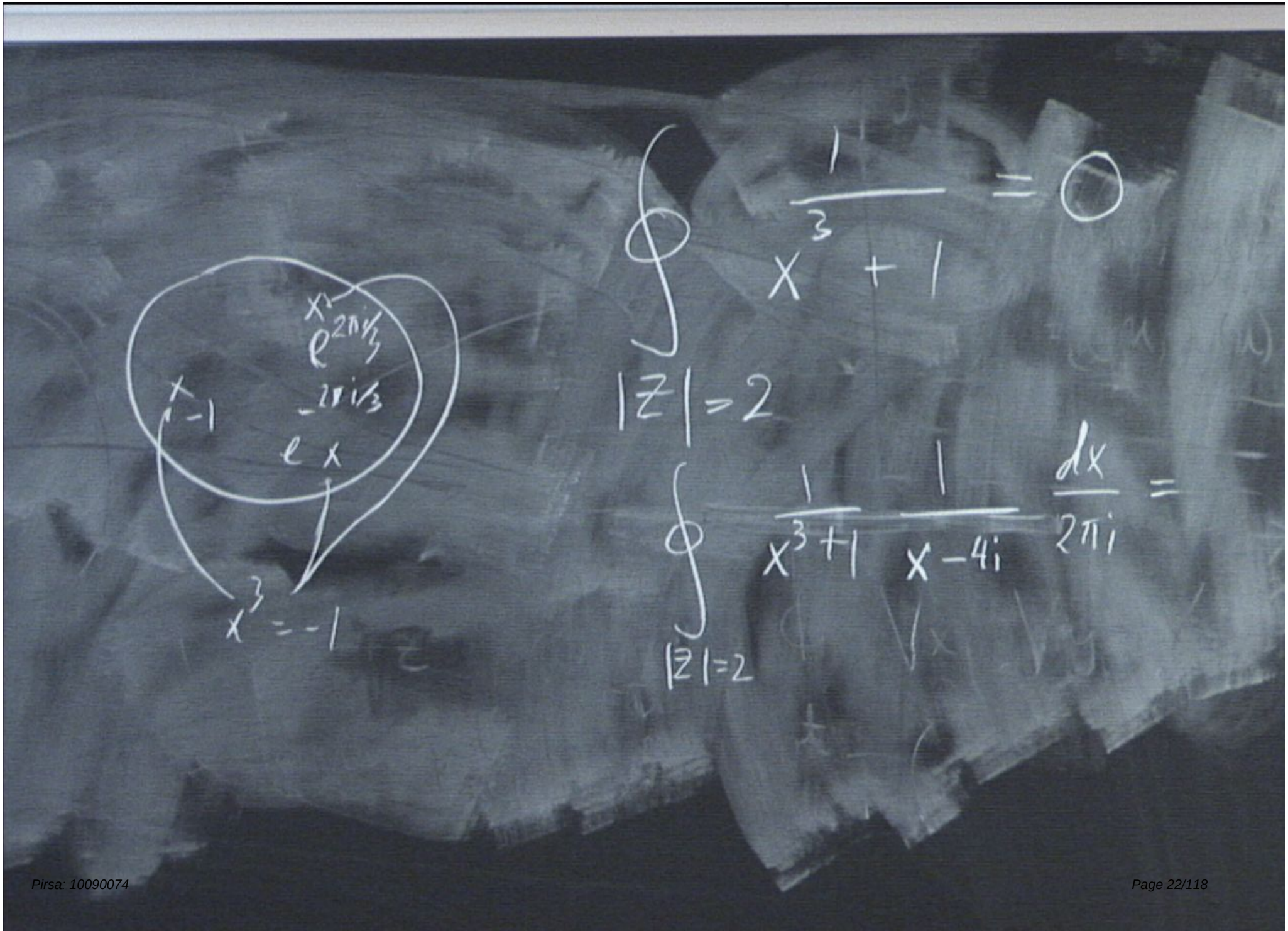
$$f = -\frac{1}{2i}\frac{1}{x+i}$$

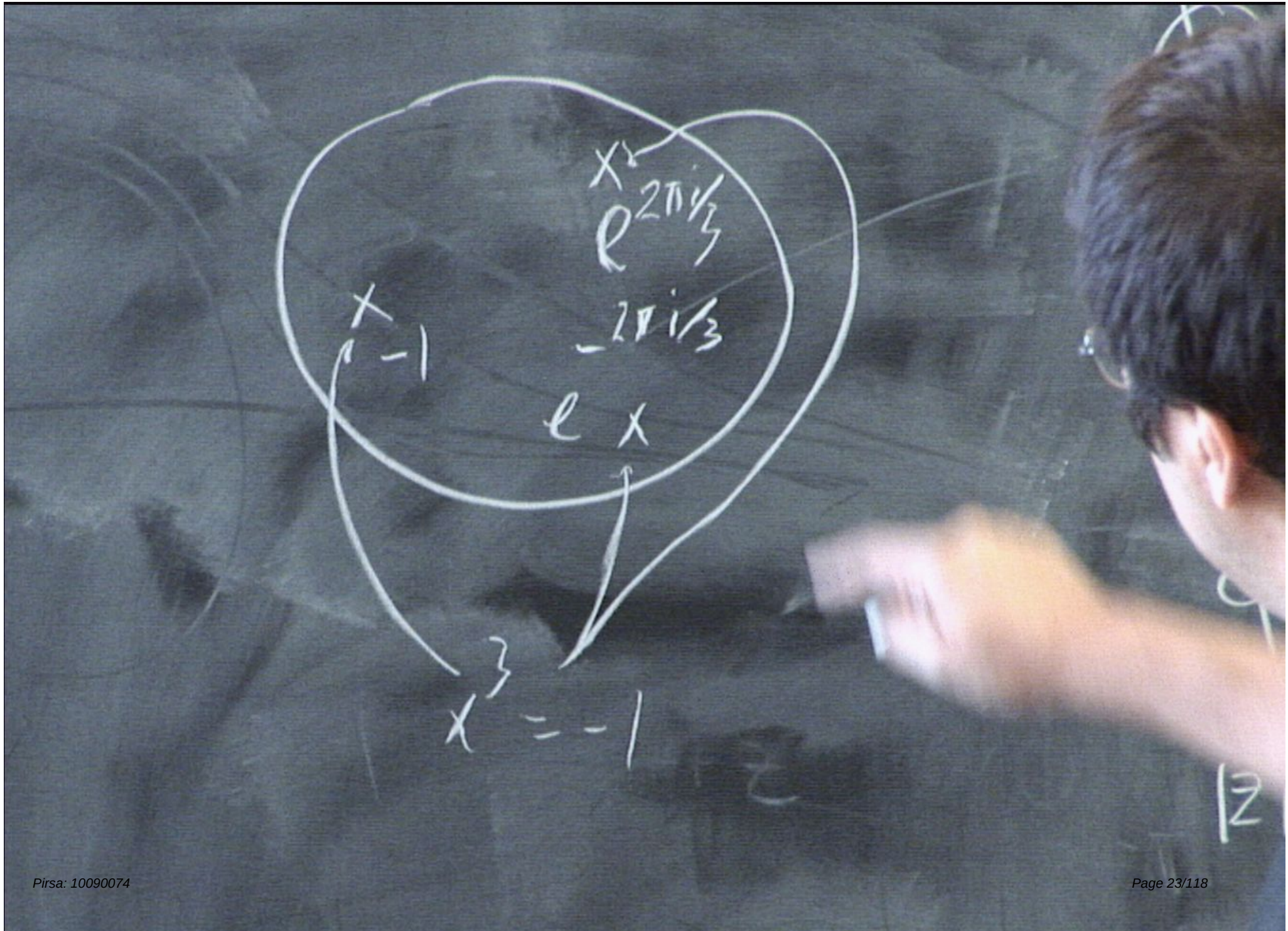
$$\frac{1}{x^2+1} = \left(\frac{1}{x-i} - \frac{1}{x+i}\right)\frac{1}{2i} \rightarrow \oint_\gamma f = 0$$

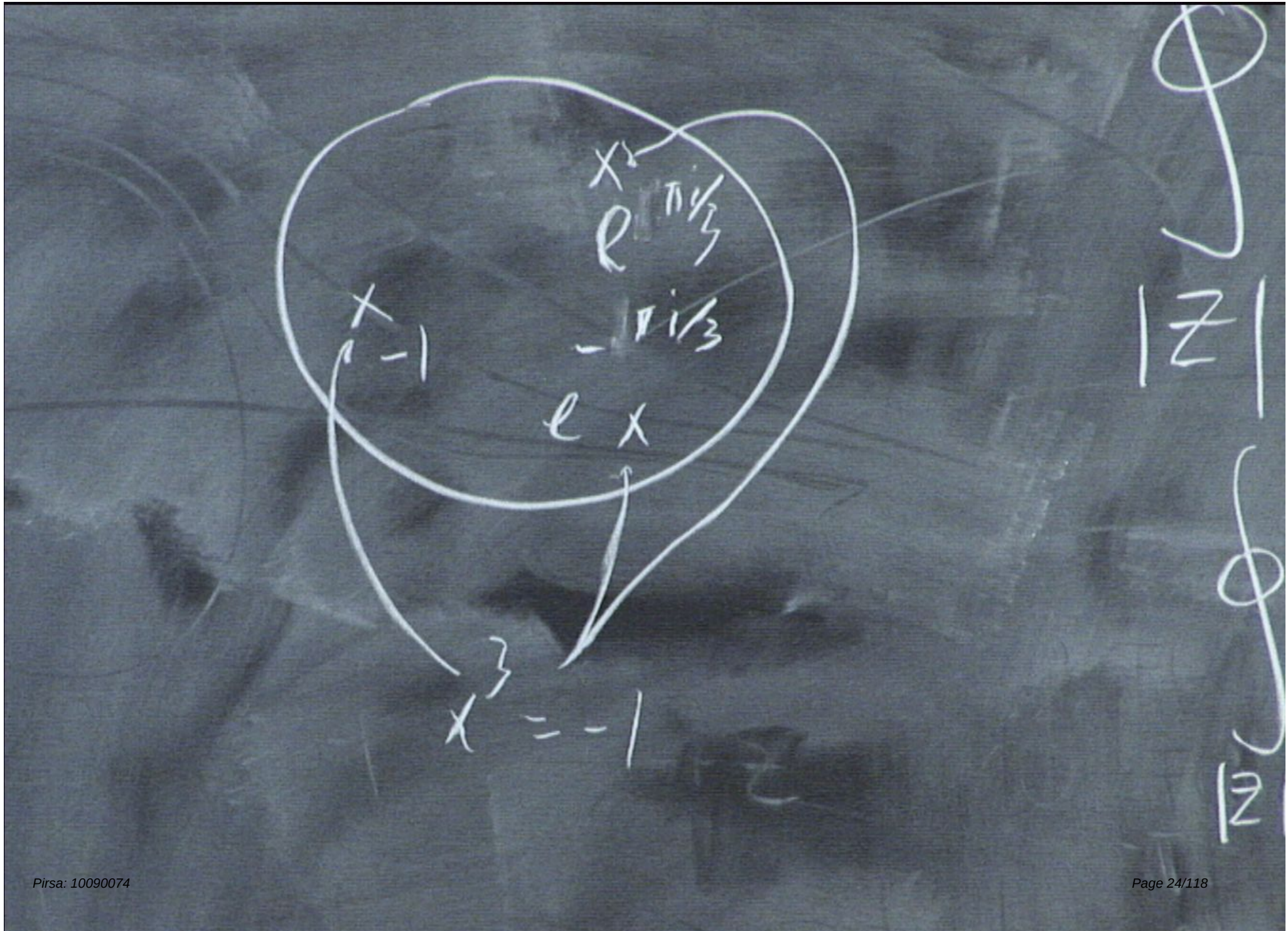

$\oint \frac{1}{x^3+1} = 0$
$|z|=2$

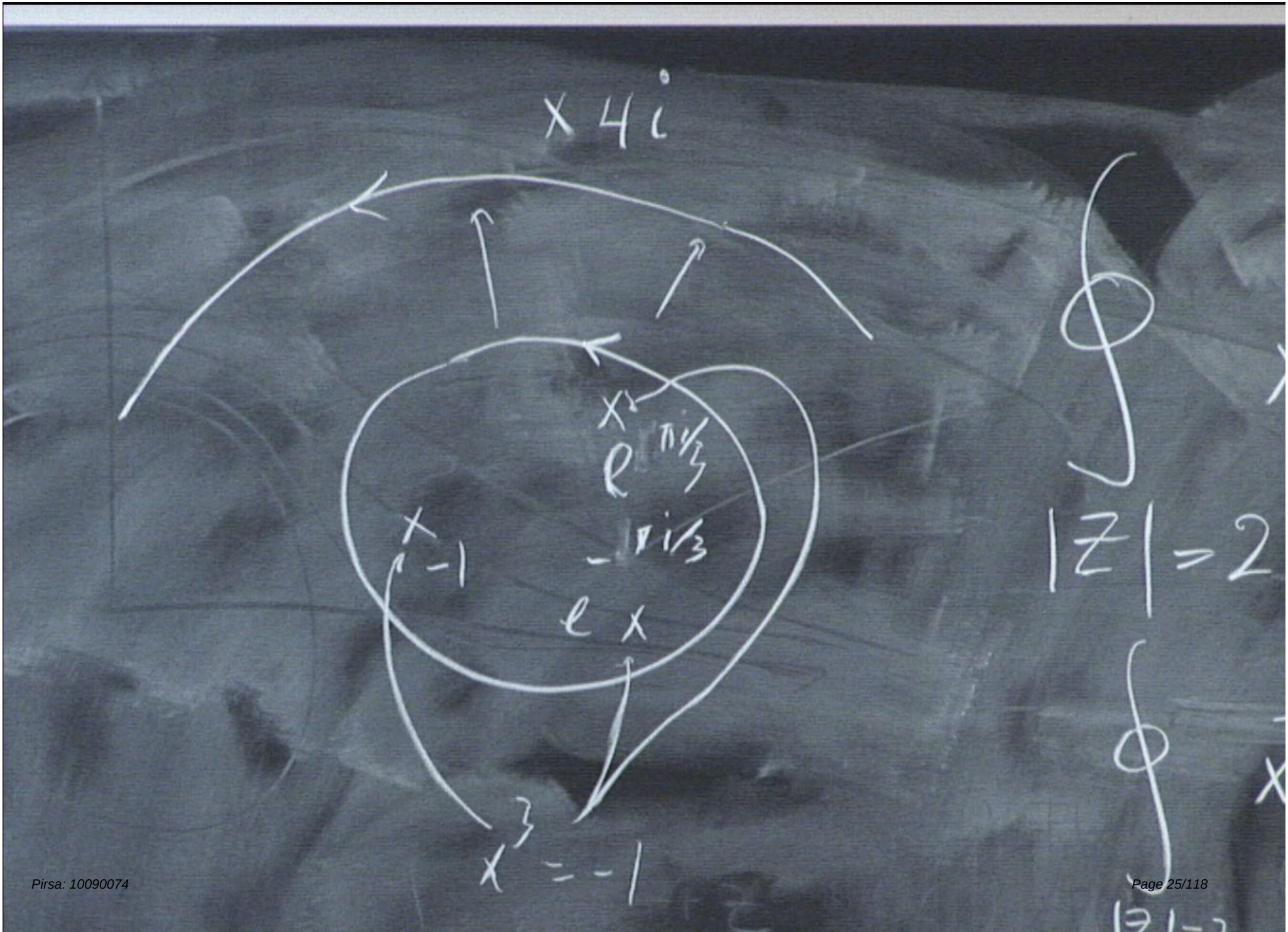

$\oint \frac{1}{x^3+1} = 0$
$|z|=2$
$\oint \frac{1}{x^3+1} \frac{1}{x-4i}$
$|z|=2$

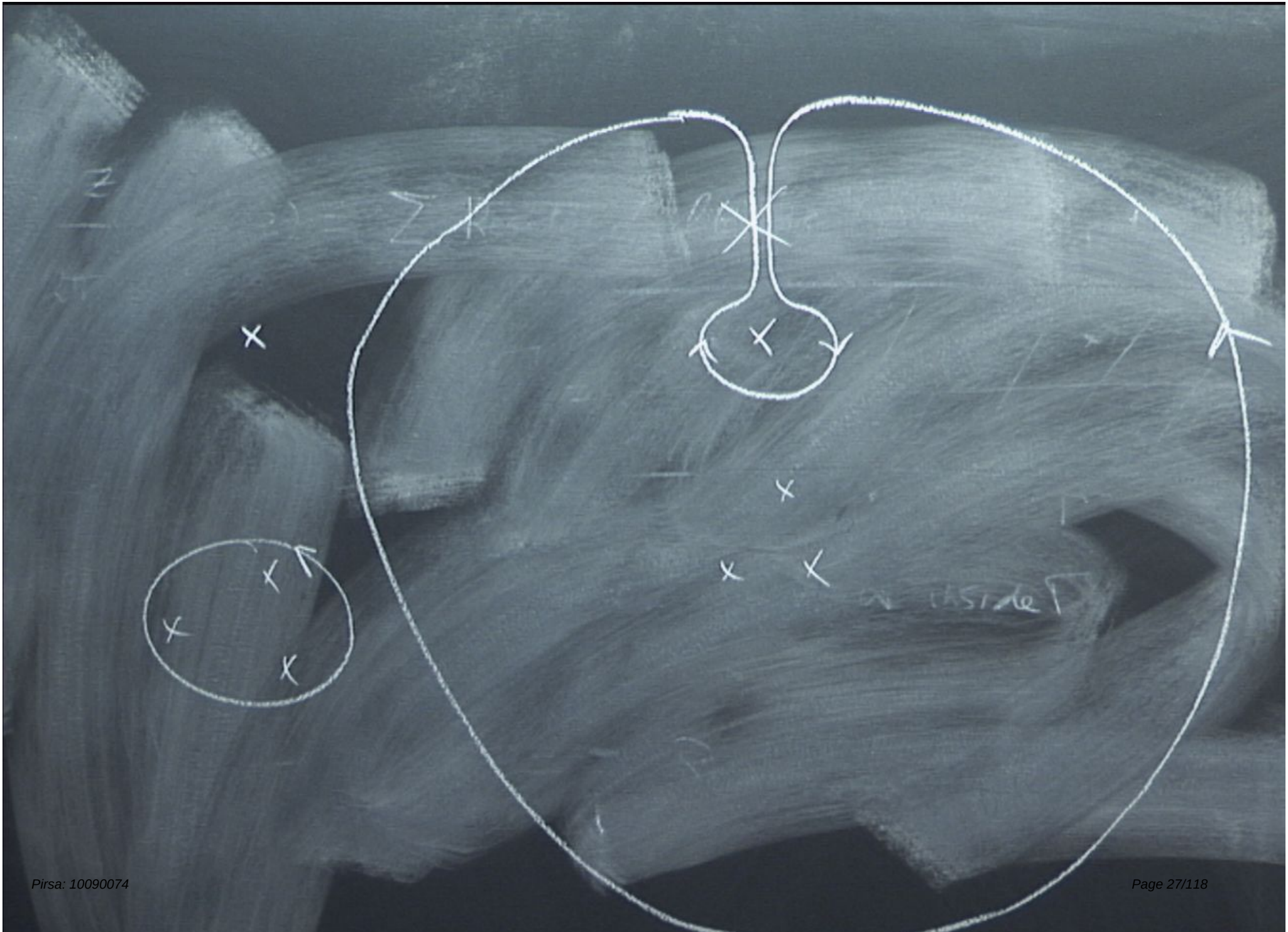

$\frac{1}{3} = 0$
$+1$
$e^{2\pi i/3}$
-1
$-2\pi i/3$
$\frac{1}{-4i} \frac{dx}{2\pi i} =$

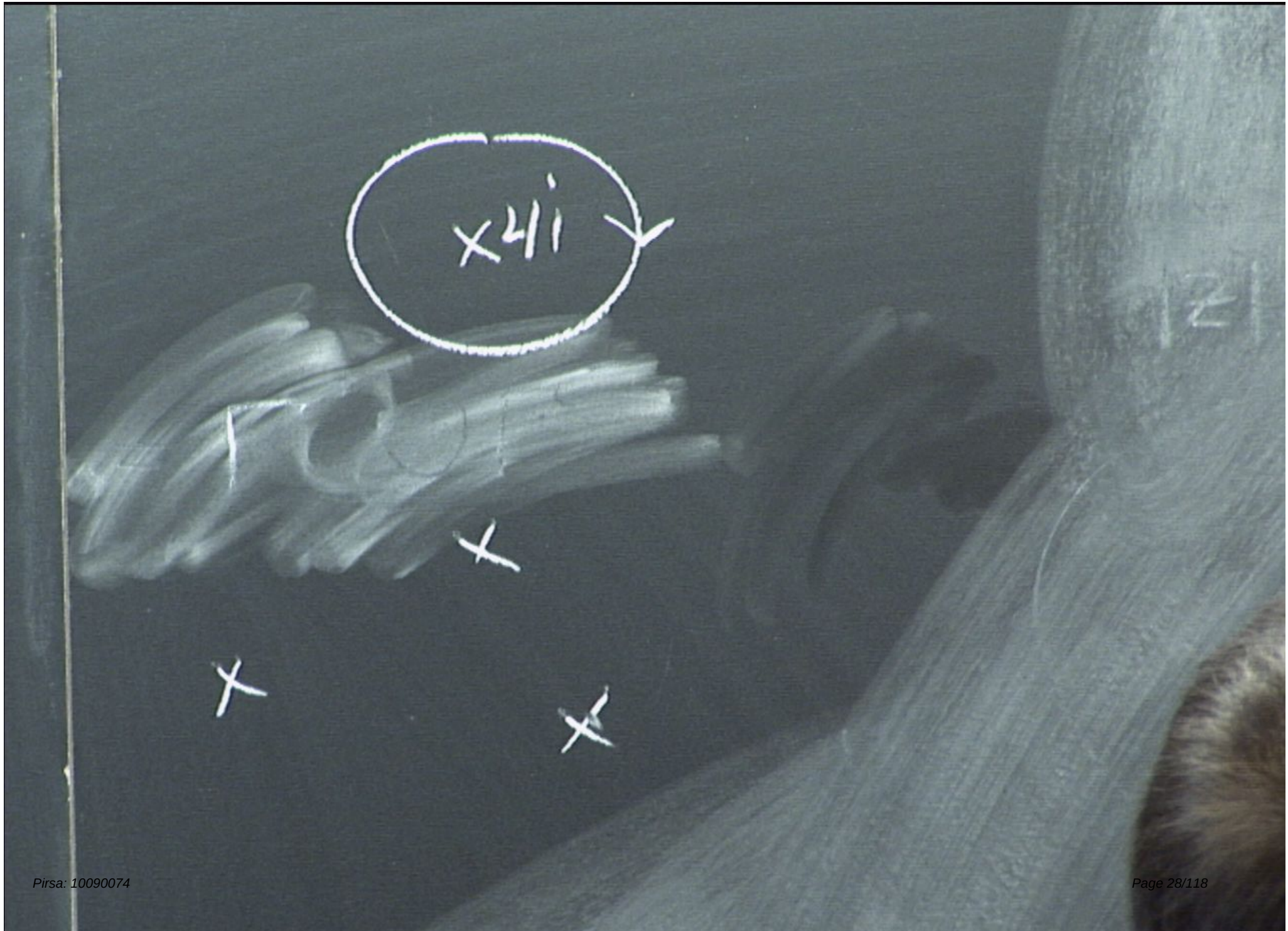

$\oint_{|z|=2} \frac{1}{x^3+1} = 0$
$\oint_{|z|=2} \frac{1}{x^3+1} \frac{1}{x-4i} \frac{dx}{2\pi i} =$
$x^3 = -1$
$x = -1$
$x = e^{2\pi i/3}$
$x = e^{-2\pi i/3}$

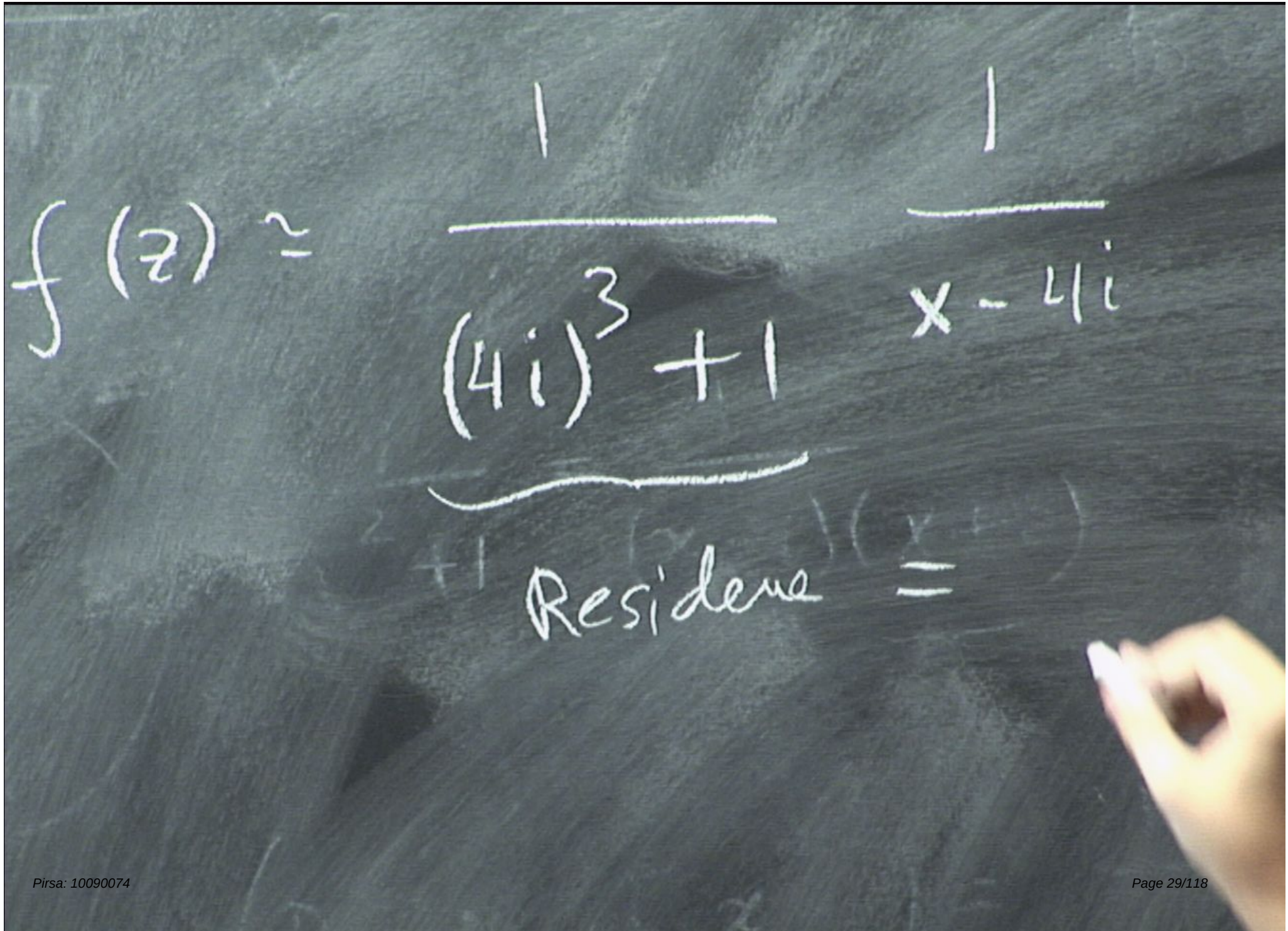

x^2
$e^{2\pi i/3}$
$-2\pi i/3$
x^{-1}
e x
$x^3 = -1$

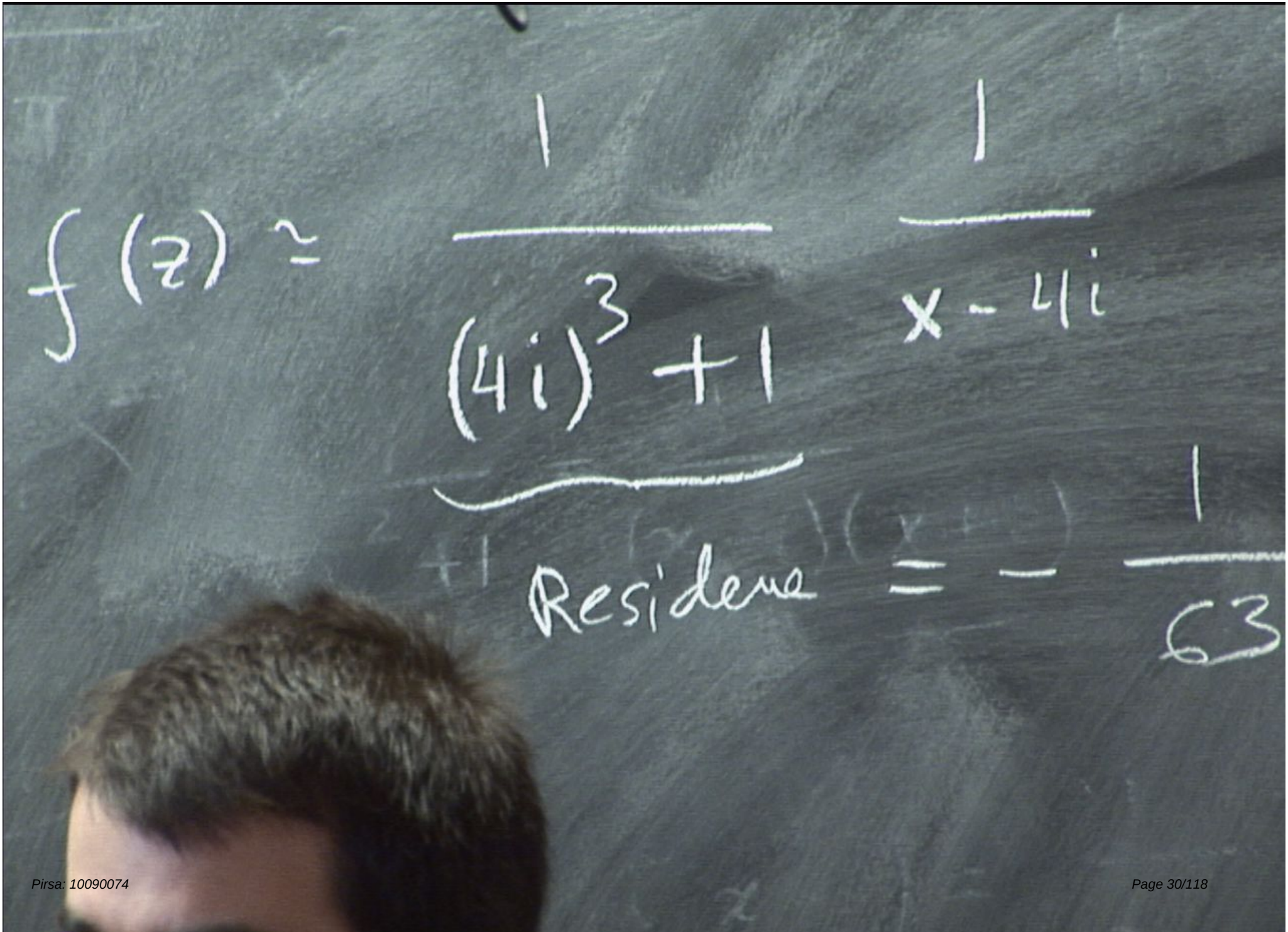

$x^3 = -1$
$|z|$
$|z|$


x 4i
$x^3 = -1$
$e^{\pi i/3}$
$e^{-\pi i/3}$
$e^{\pi i}$
$|z| = 2$



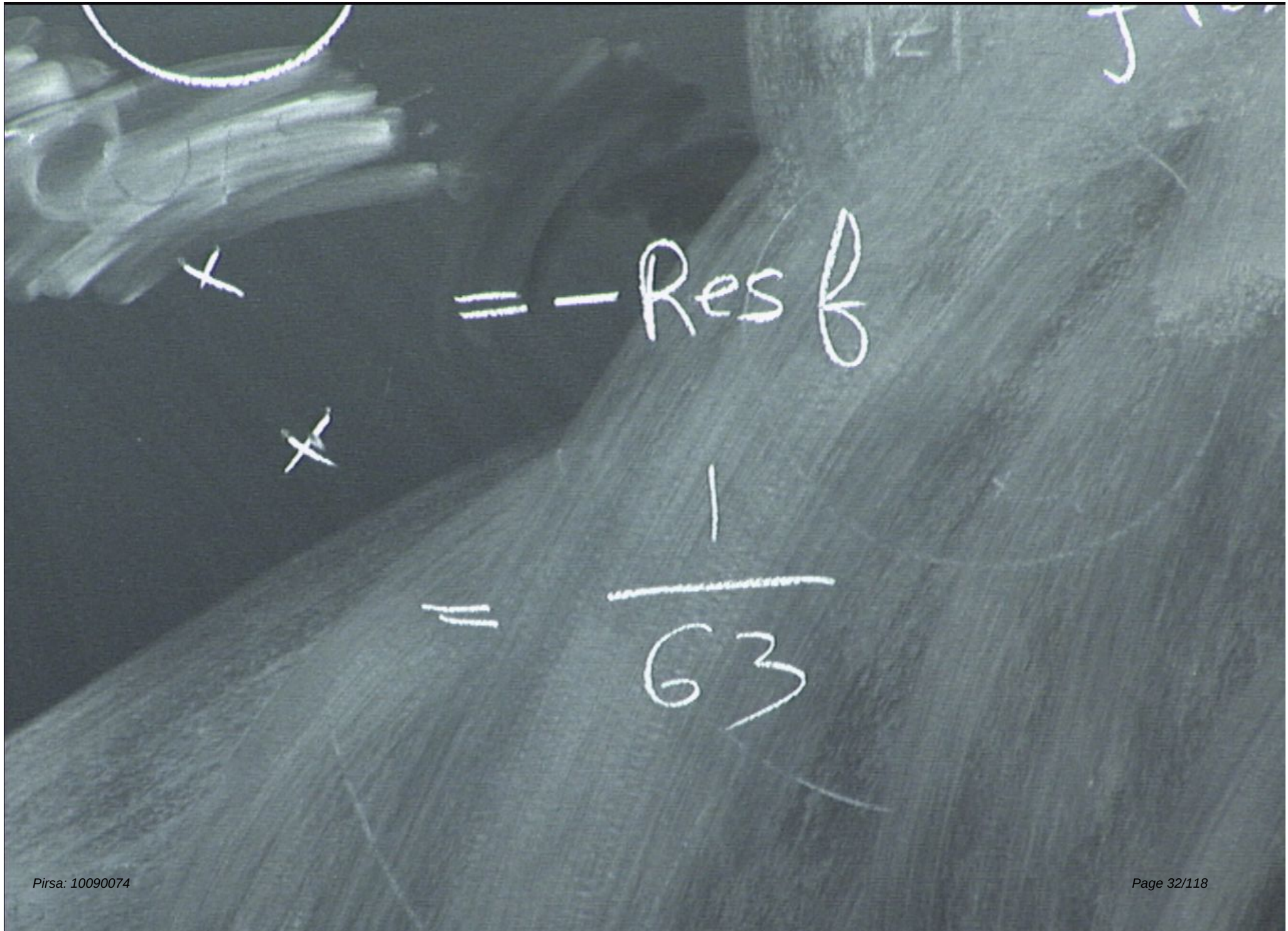

x4i

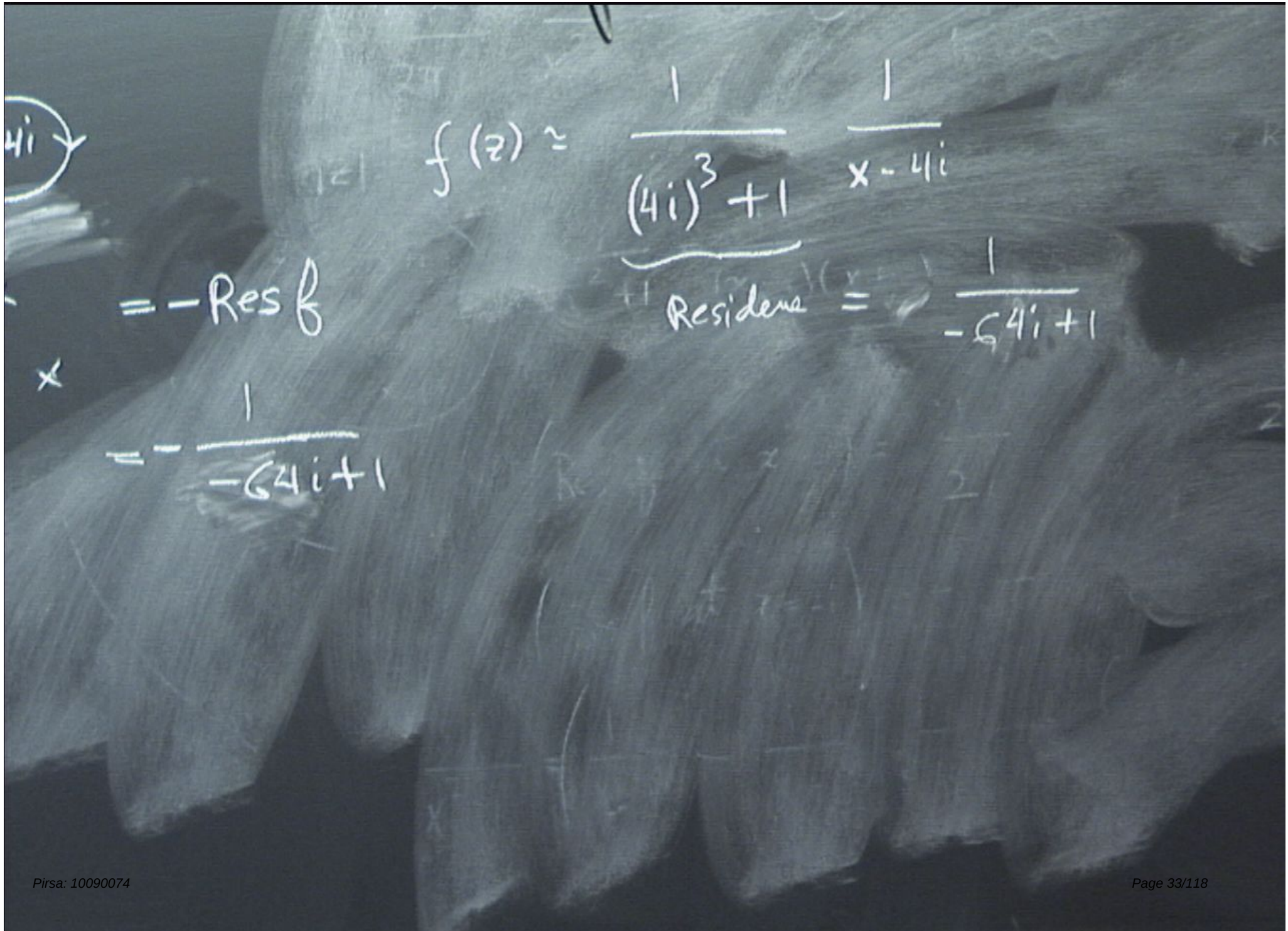

f(z) ≃ 1/(4i)^3 + 1 · 1/(x − 4i)
Residue =


f(z) ≃ 1 / (4i)^3 + 1 · 1 / (x − 4i)
Residue = − 1 / 63

$$f(z) \simeq \frac{1}{(4i)^3 + 1} \cdot \frac{1}{x - 4i}$$

$$\text{Residue} = -\frac{1}{63}$$


= − Res f
= 1/63



$4i$

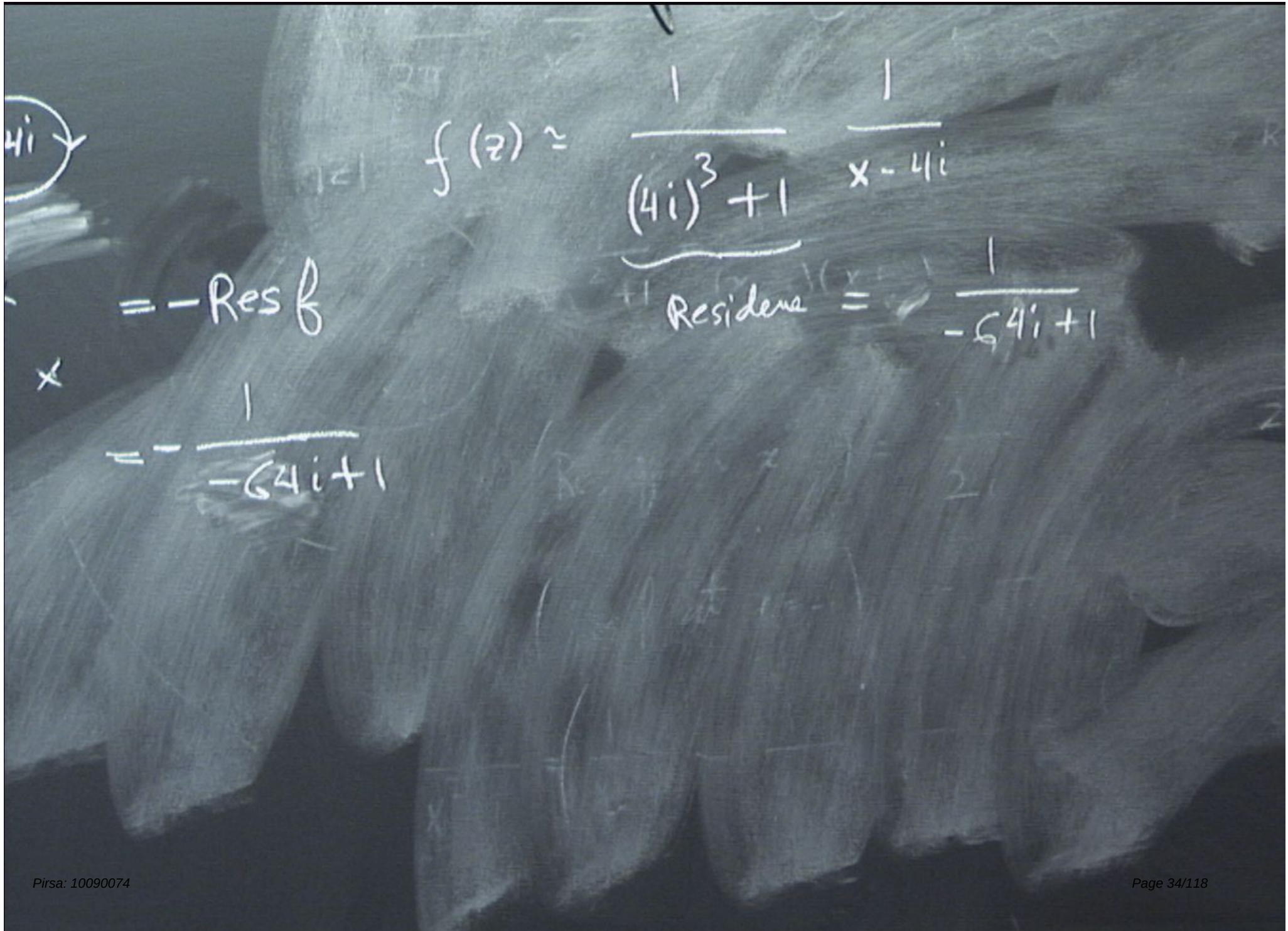
$$f(z) \simeq \underbrace{\frac{1}{(4i)^3+1}}_{\text{Residue}} \frac{1}{x-4i}$$

$$= -\text{Res}\, f$$

$$\text{Residue} = \frac{1}{-64i+1}$$

x

$$= -\frac{1}{-64i+1}$$



$4i$

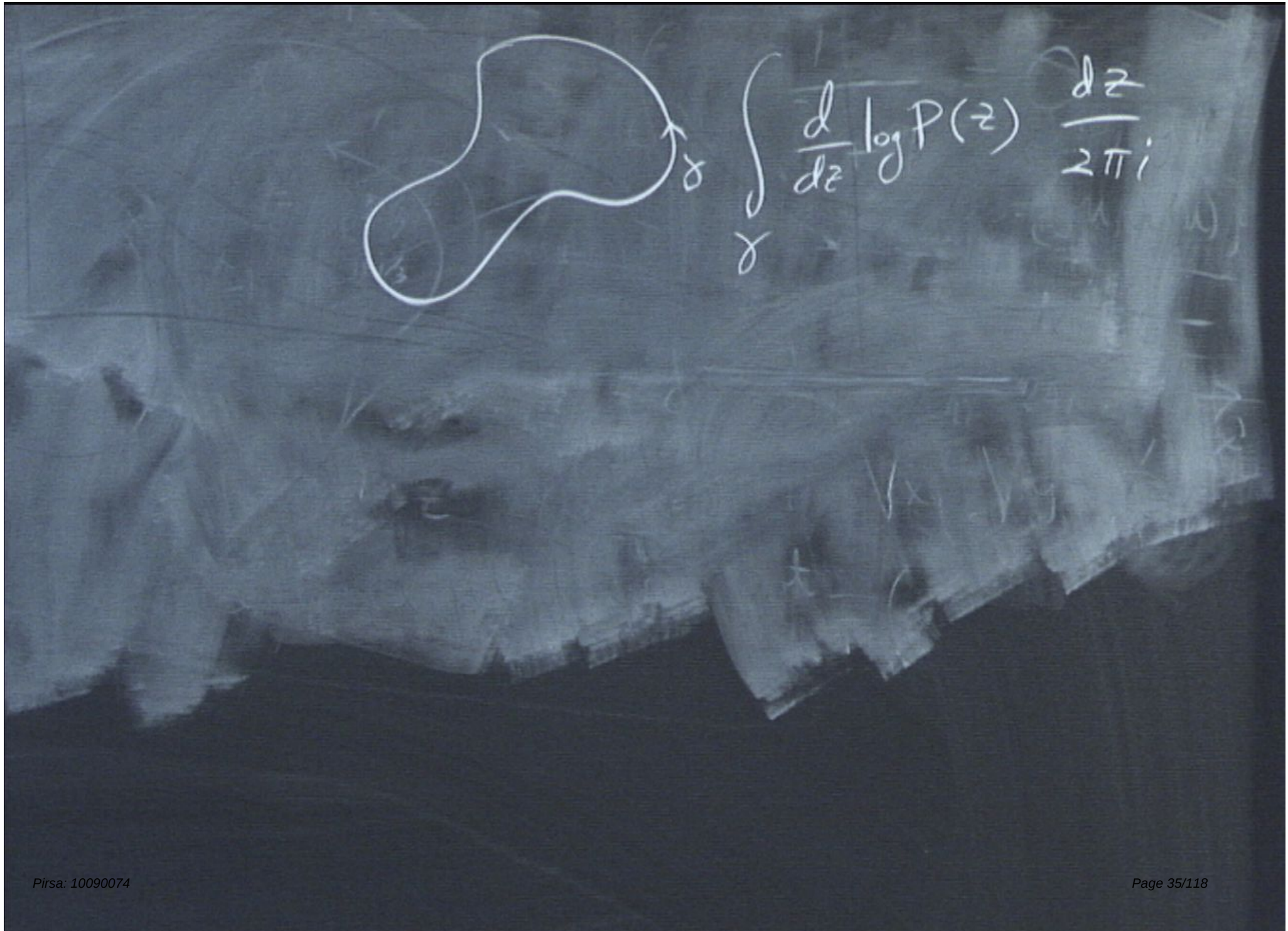
$$f(z) \simeq \underbrace{\frac{1}{(4i)^3+1}}_{\text{Residue}} \frac{1}{x-4i}$$

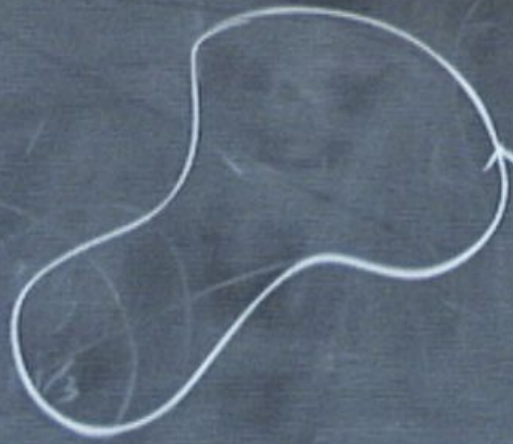
$$= -\text{Res}\, f$$

$$\text{Residue} = \frac{1}{-64i+1}$$

x

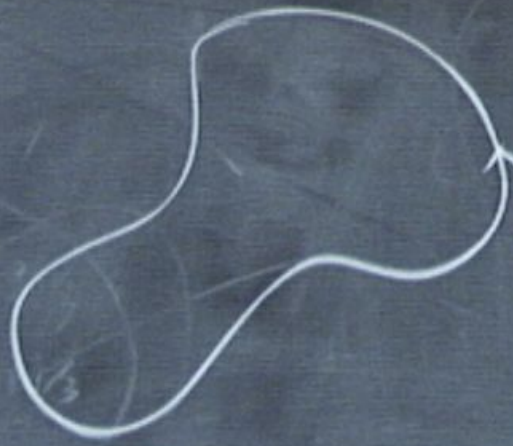
$$= -\frac{1}{-64i+1}$$


γ
$\int_\gamma \frac{d}{dz} \log P(z) \frac{dz}{2\pi i}$



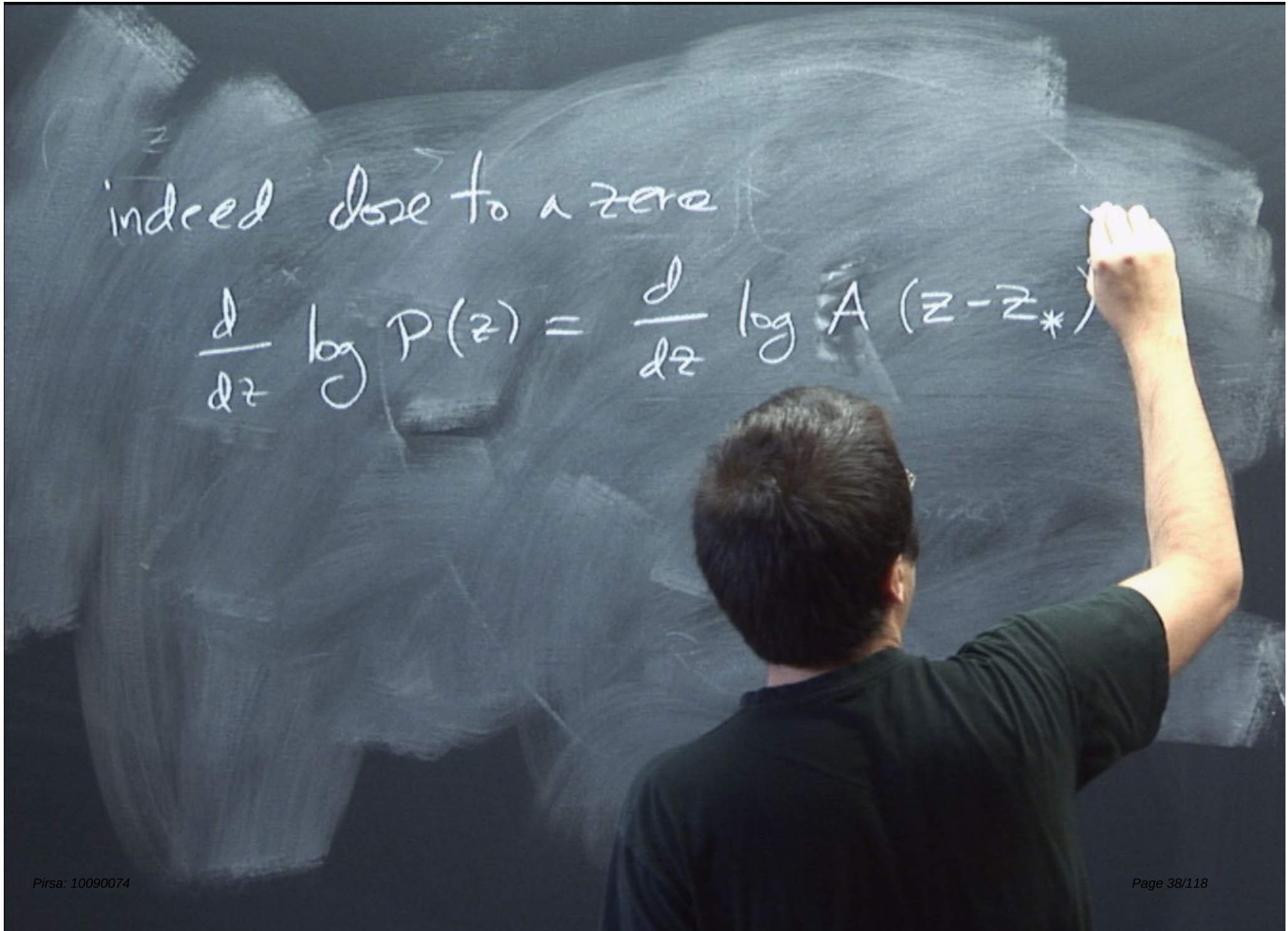
$$\int_\gamma \frac{d}{dz} \log P(z) \, \frac{dz}{2\pi i}$$

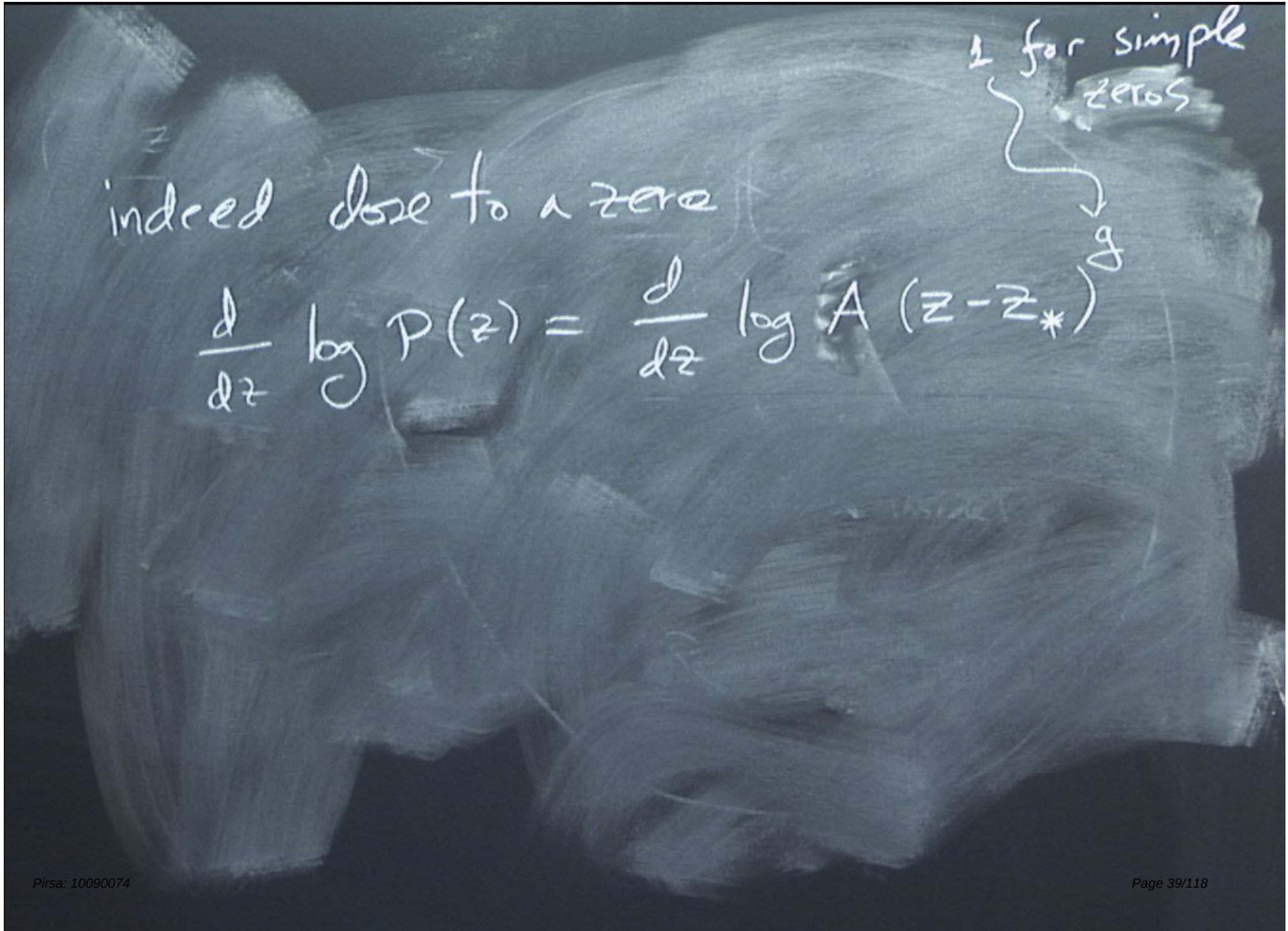
= number of zeros of $P(z)$ (with degeneracy) inside γ

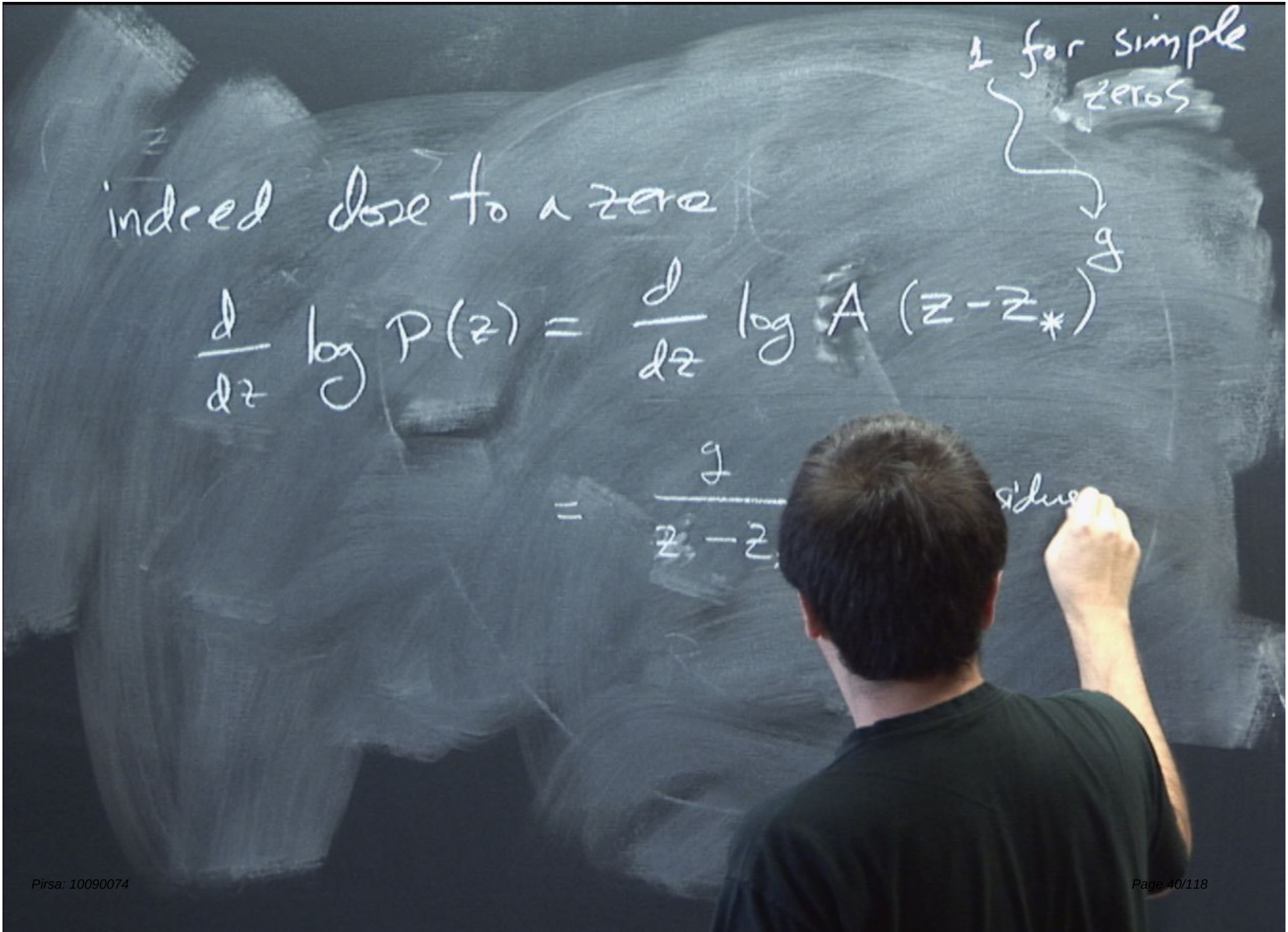


$$\gamma \qquad \int_\gamma \frac{d}{dz} \log P(z) \, \frac{dz}{2\pi i}$$

$$= \text{number of zeros of } P(z) \text{ (with degeneracy) inside } \gamma$$


indeed close to a zero
$\frac{d}{dz} \log P(z) = \frac{d}{dz} \log A(z - z_*)$


1 for simple zeros
indeed close to a zero
$\frac{d}{dz} \log P(z) = \frac{d}{dz} \log A (z - z_*)^g$


1 for simple zeros
indeed close to a zero
$\frac{d}{dz}\log P(z) = \frac{d}{dz}\log A(z-z_*)^g$
$= \frac{g}{z-z_*}$ + ...

1 for simple zeros

indeed close to a zero

$$\frac{d}{dz}\log P(z) = \frac{d}{dz}\log A(z-z_*)^g$$

$$= \frac{g}{z-z_*} \Rightarrow \text{Residue} = g$$

1 for simple zeros

indeed close to a zero

$$\frac{d}{dz}\log P(z) = \frac{d}{dz}\log A\,(z - z_*)^g$$

$$= \frac{g}{z - z_*} \Rightarrow \text{Residue} = g$$

$$N = \sum_{z_*} g(z_*)$$

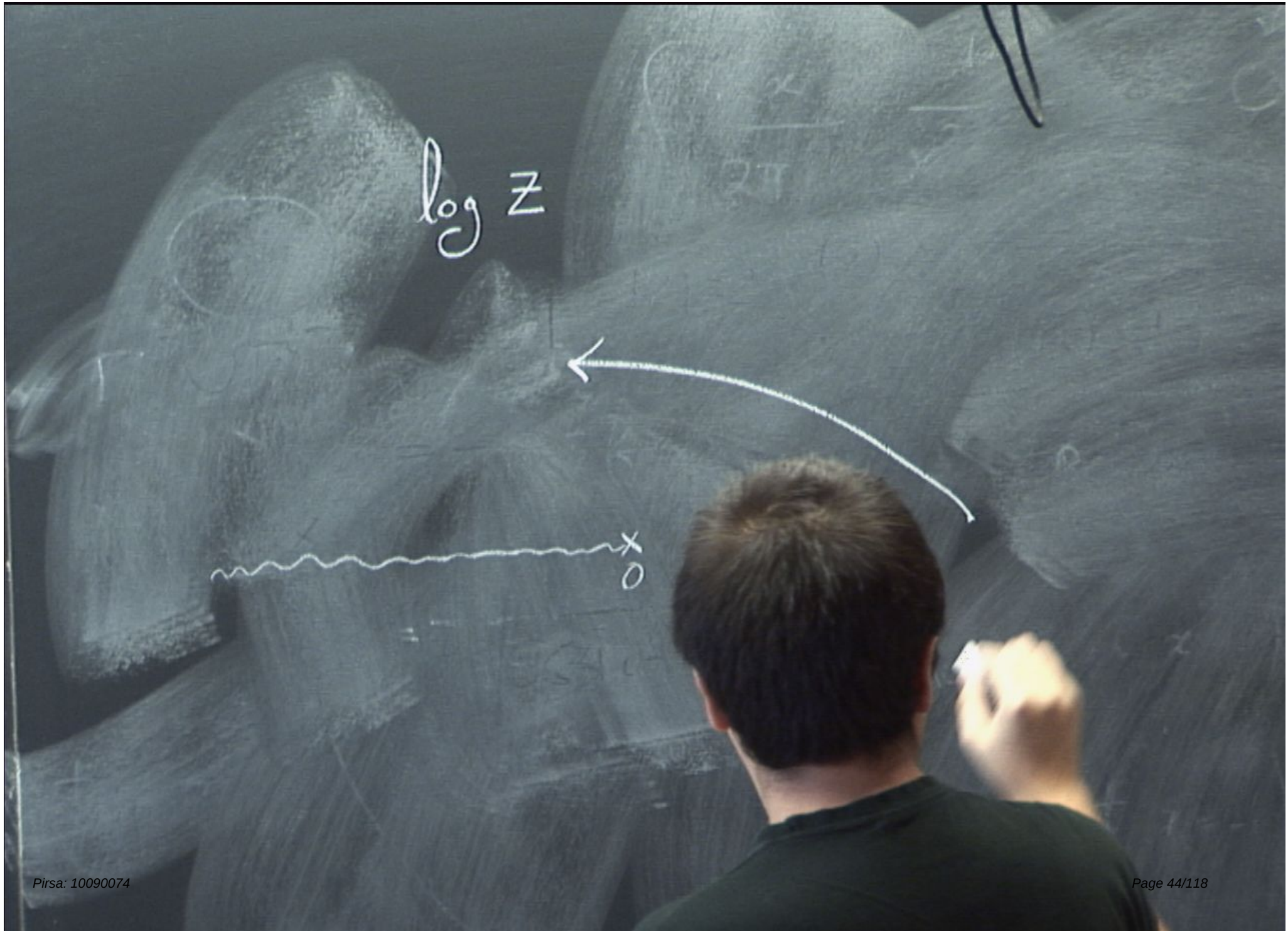
1 for simple zeros

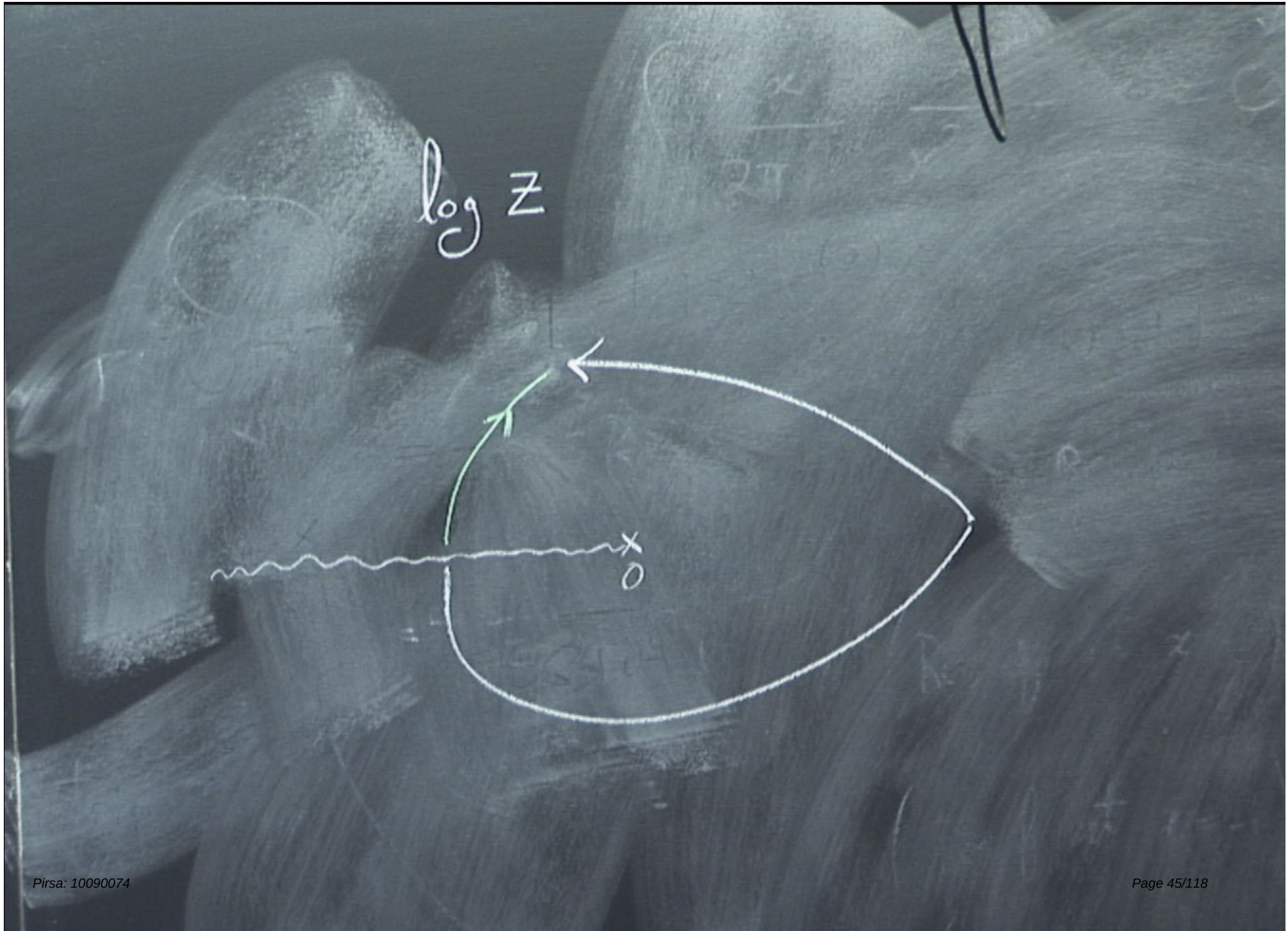
indeed close to a zero

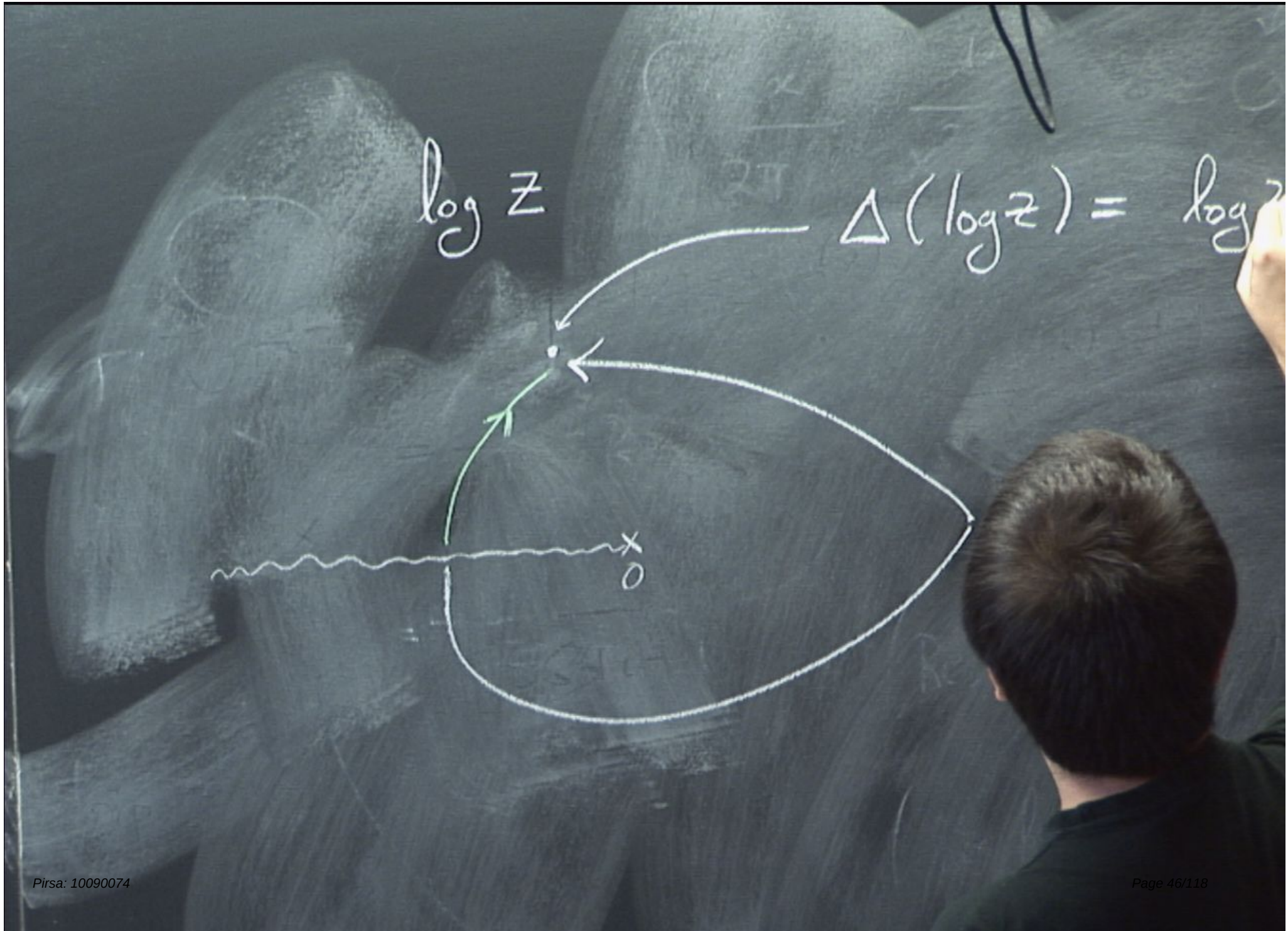
$$\frac{d}{dz}\log P(z) = \frac{d}{dz}\log A\,(z-z_*)^g$$

$$= \frac{g}{z-z_*} \Rightarrow \text{Residue} = g$$

$$N = \sum_{z_*} g(z_*)$$

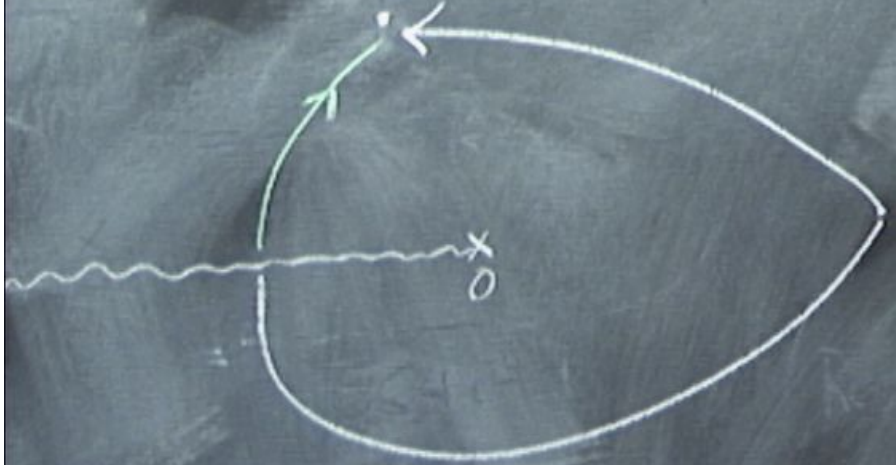

log z
0


log z
0


log z
Δ(log z) =
0

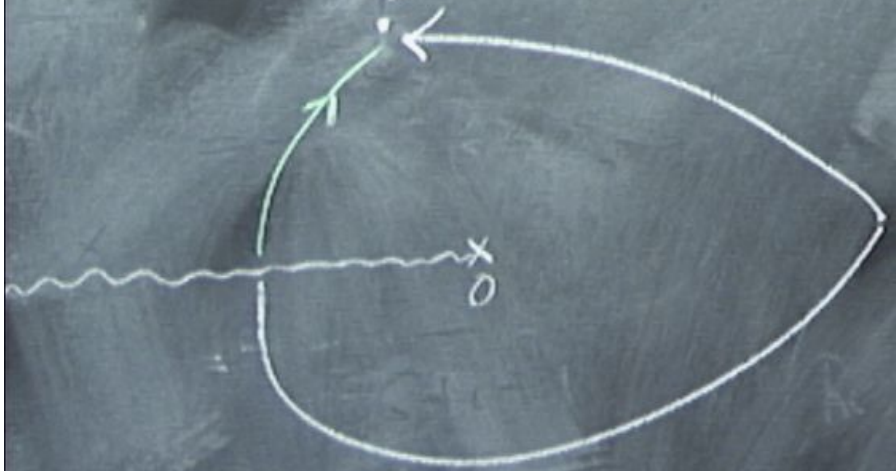
$\log z$

$\Delta(\log z) = \left(\log z \right) - \left(\log z \right)$



$\log z$

$$\Delta(\log z) = \left(\log z\right) - \left(\log z\right)$$



$$= 2\pi i$$

1 for simple zeros

indeed close to a zero

$$\frac{d}{dz}\log P(z) = \frac{d}{dz}\log A\,(z-z_*)^g$$

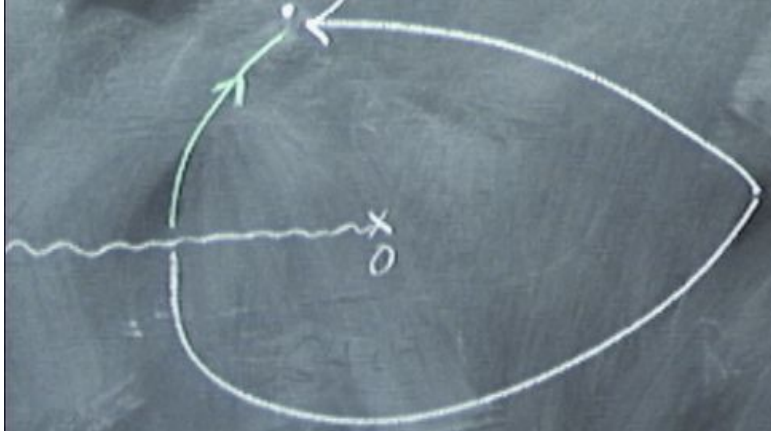
$$= \frac{g}{z-z_*} \Rightarrow \text{Residue} = g$$

$$N = \sum_{z_*} g(z_*)$$

if $P(z) \longrightarrow R(z)$ (↑ rational) N = number of zeros − number of poles

$\log z$

$$\Delta(\log z) = \left(\log z \;\nwarrow\right) - \left(\log z \;\circlearrowright\right)$$



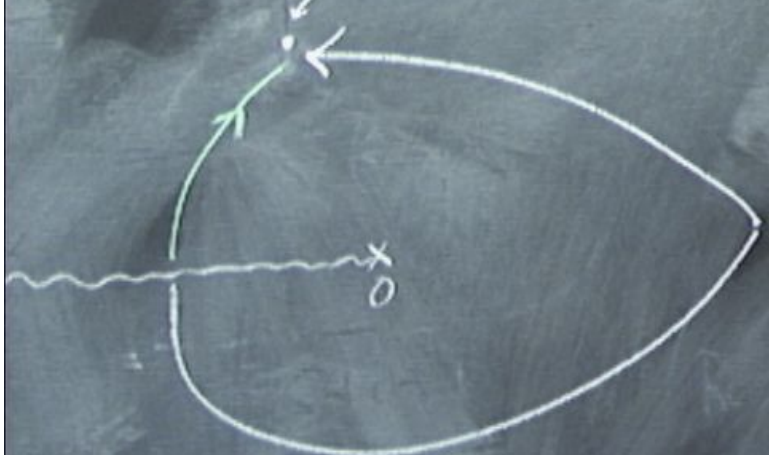
$$= 2\pi i = \Delta(z)$$

defined everywhere!

not only on

$\log z$

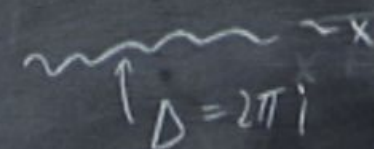
$$\Delta(\log z) = \left(\log z \;\curvearrowleft\right) - \left(\log z \;\curvearrowright\right)$$

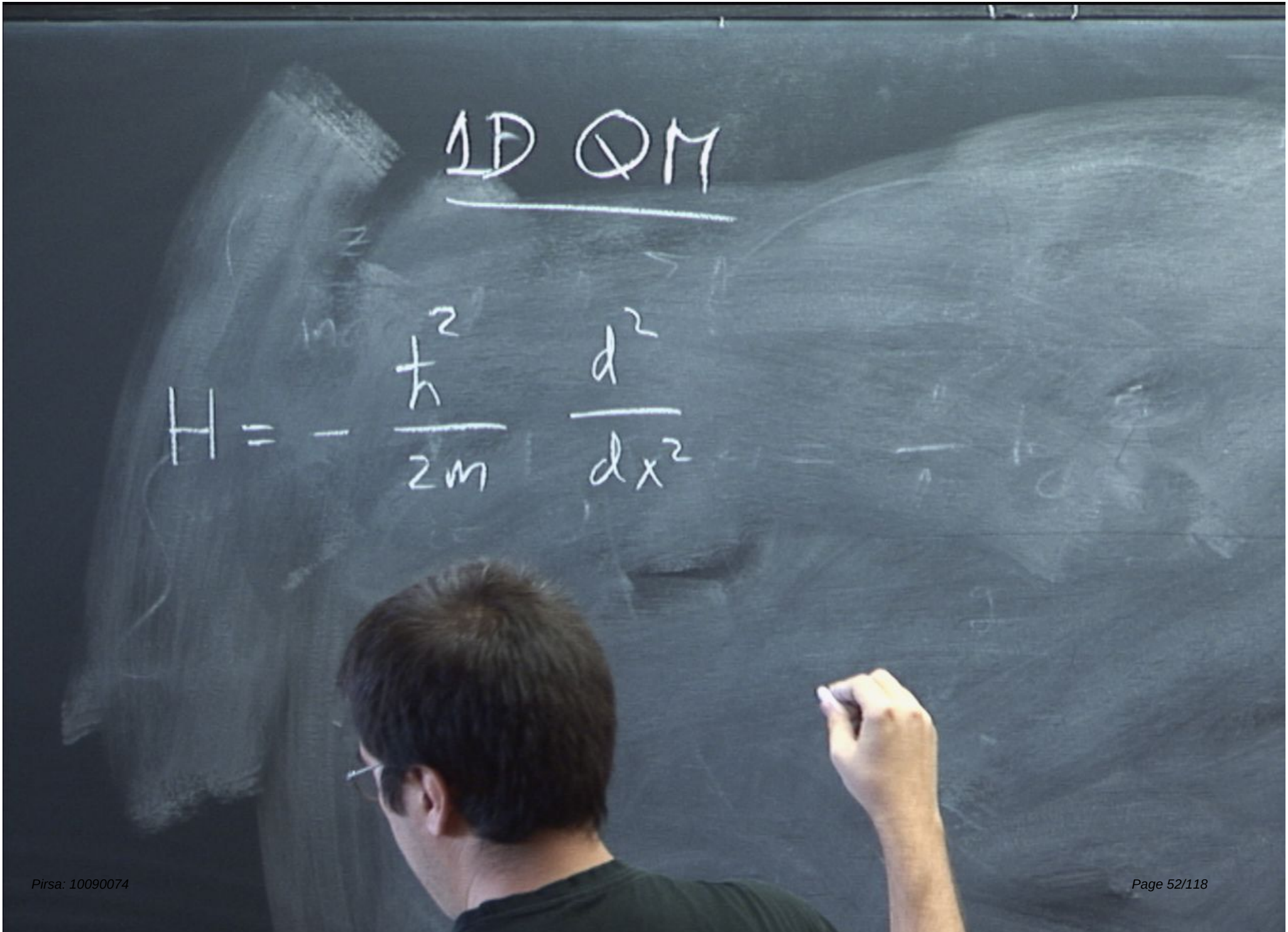


$$= 2\pi i = \Delta(z)$$

defined everywhere!

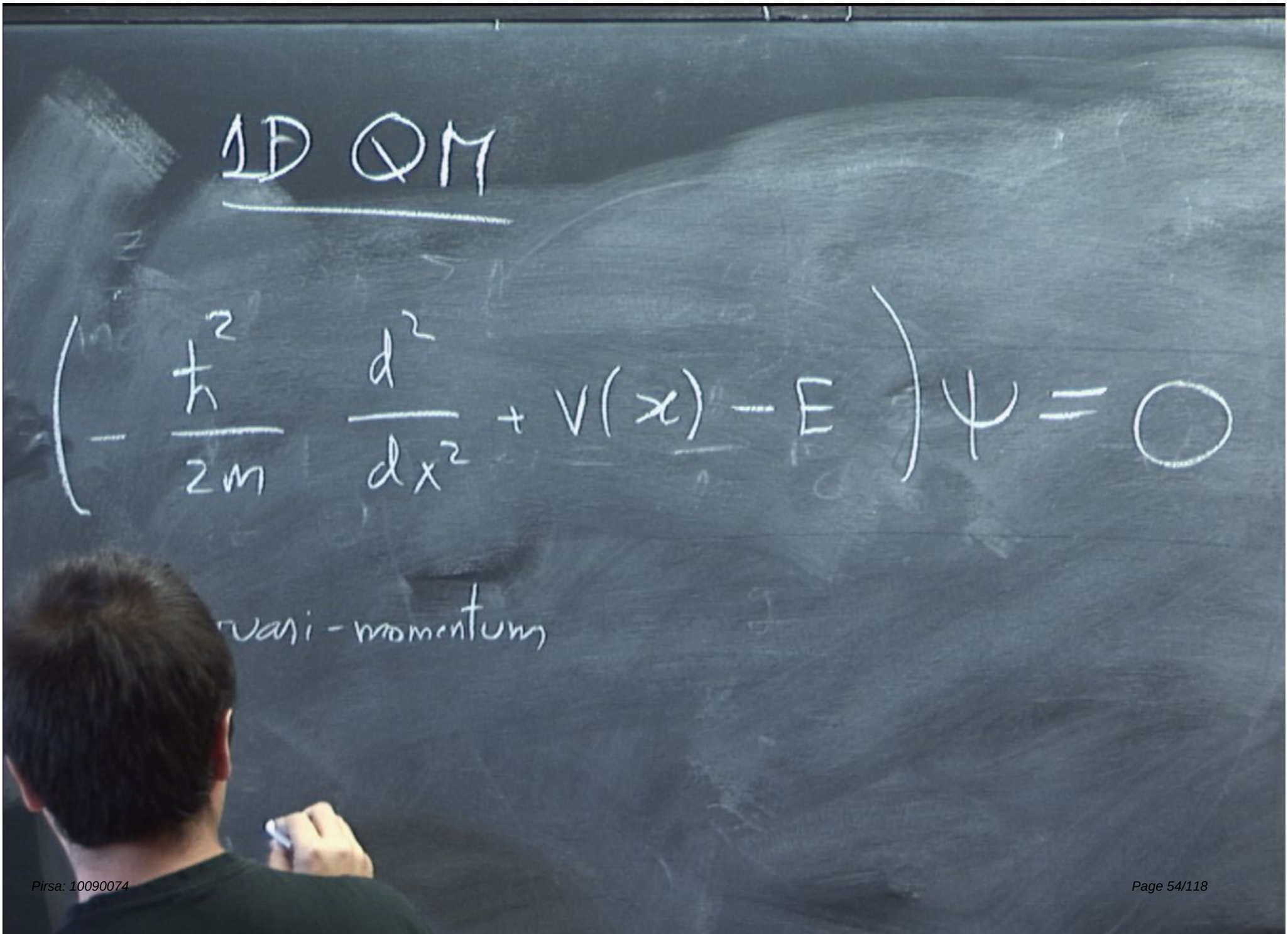
not only on

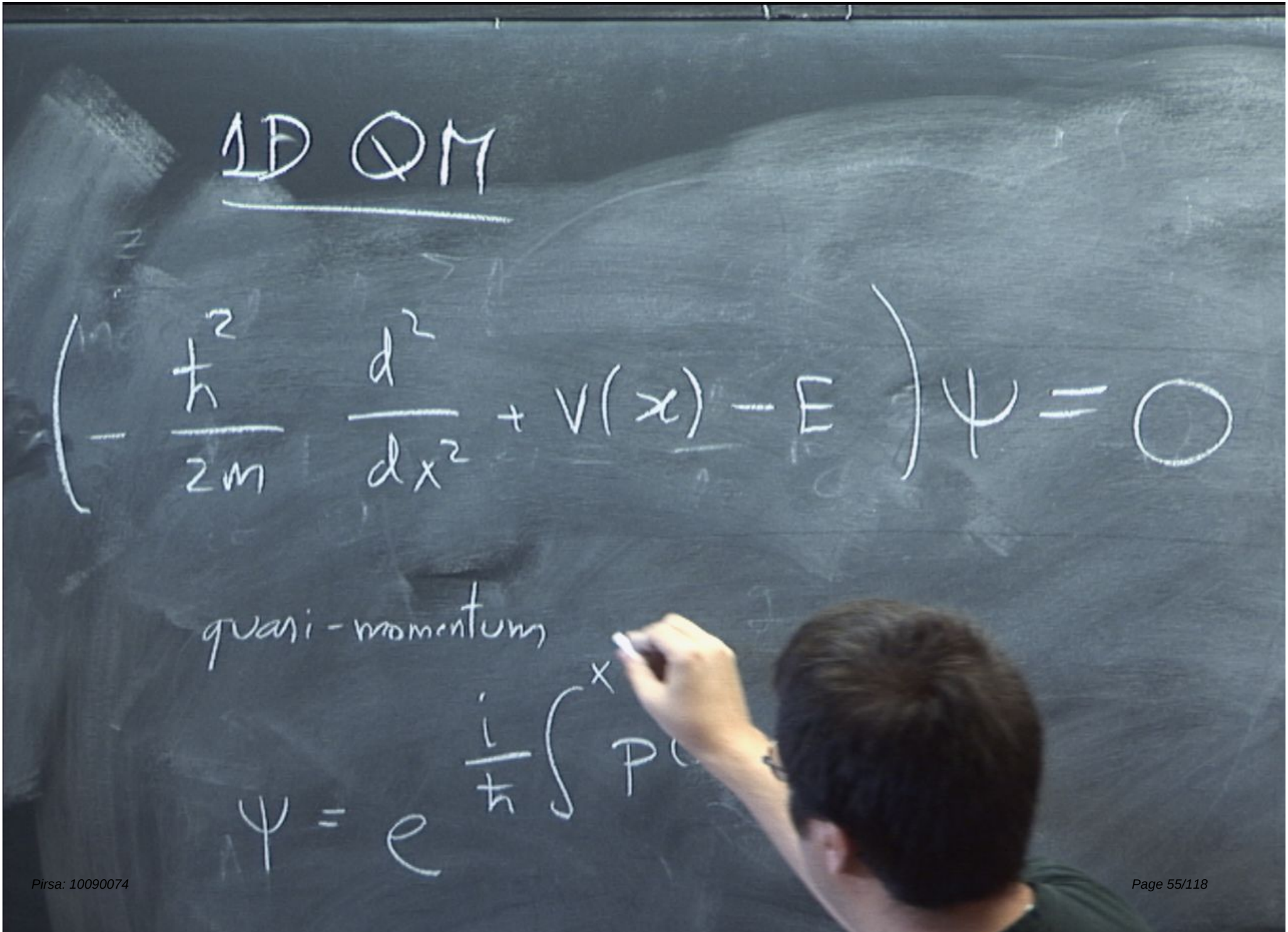



1D QM
$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

1D QM

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x) - E \right)\psi = 0$$


1D QM
$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)-E\right)\psi=0$
wavi-momentum


1D QM
$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)-E\right)\psi=0$
quasi-momentum
$\psi = e^{\frac{i}{\hbar}\int^{x}$

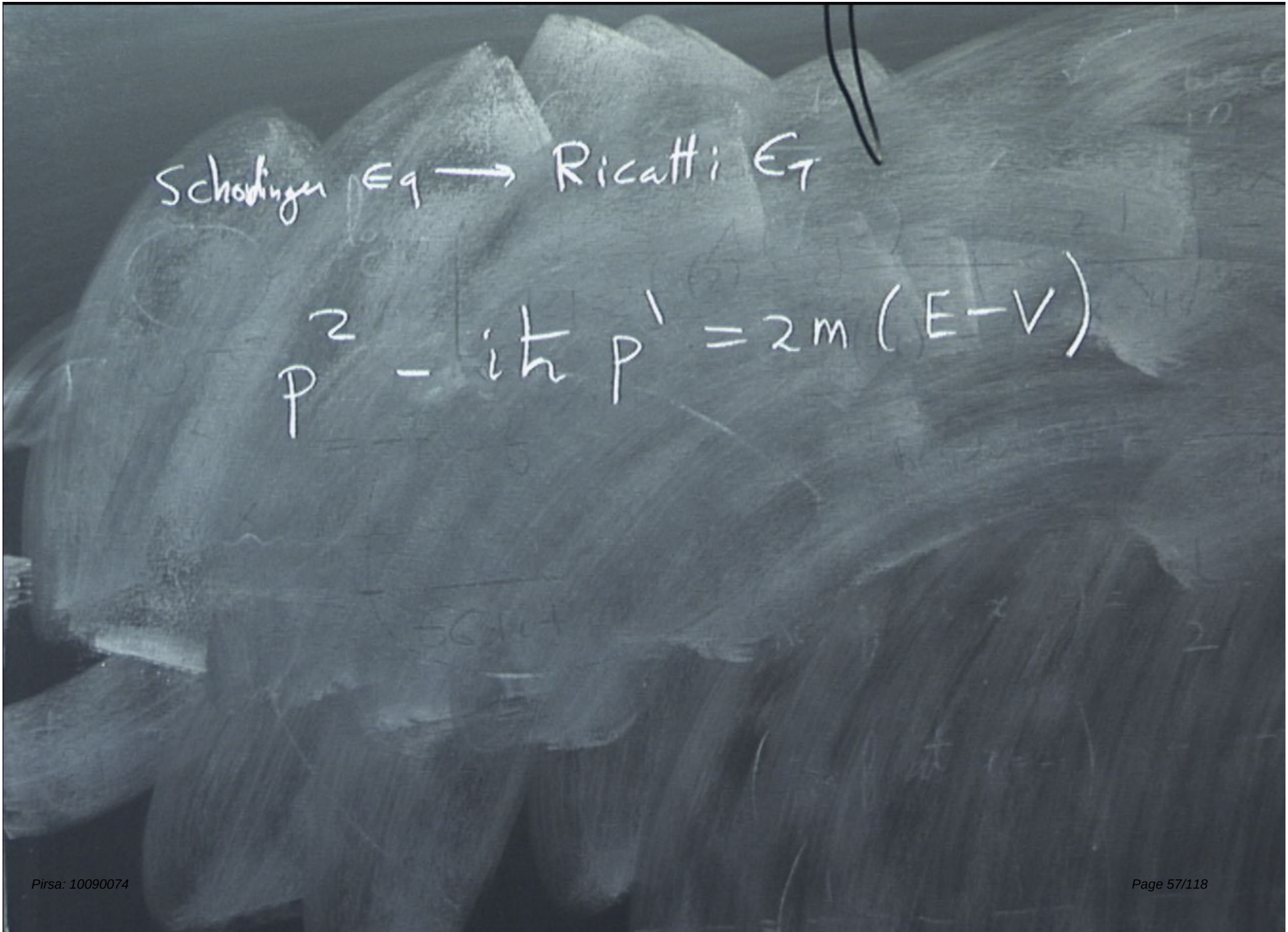
1D QM

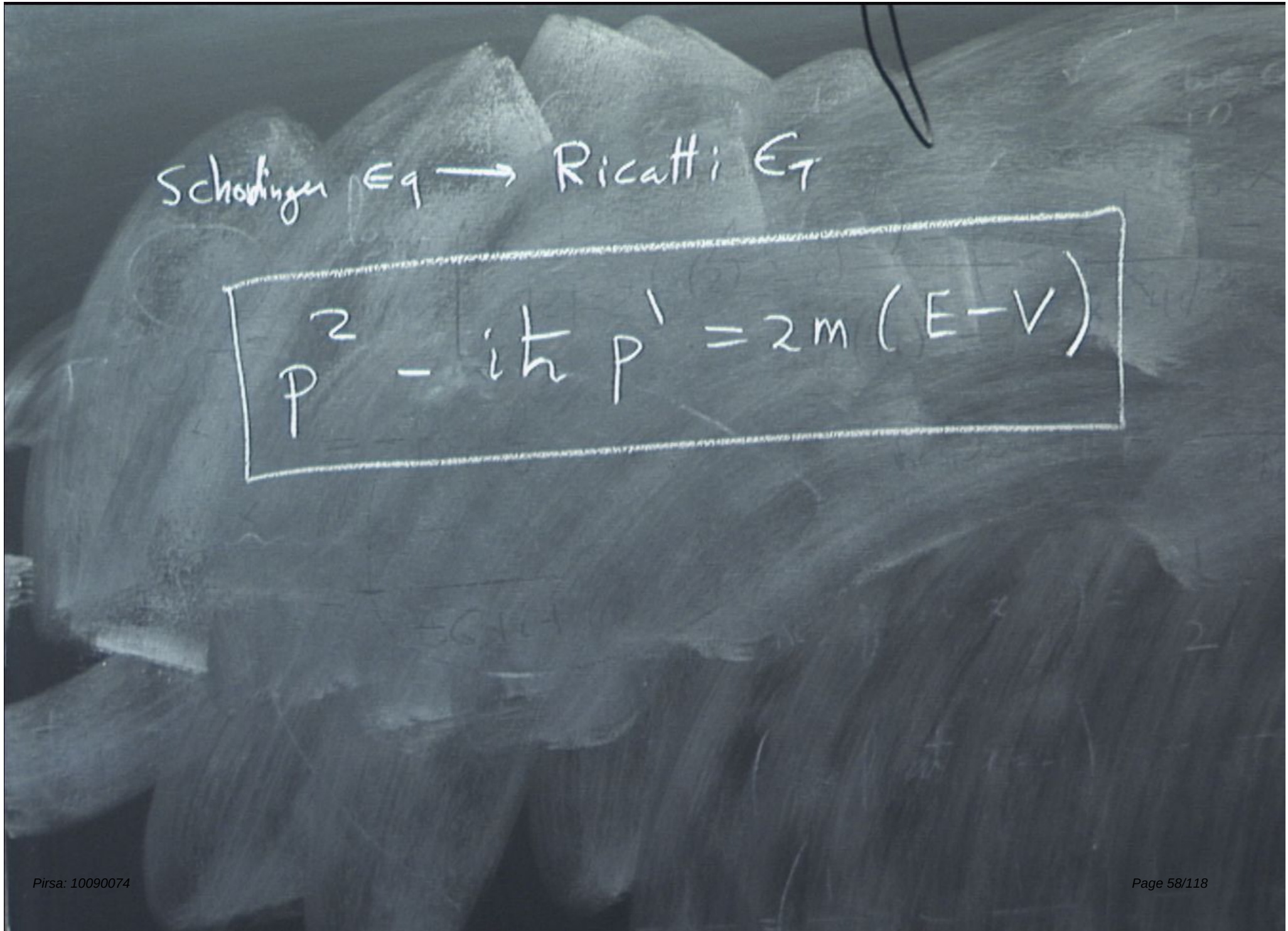
$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x) - E\right)\psi = 0$$

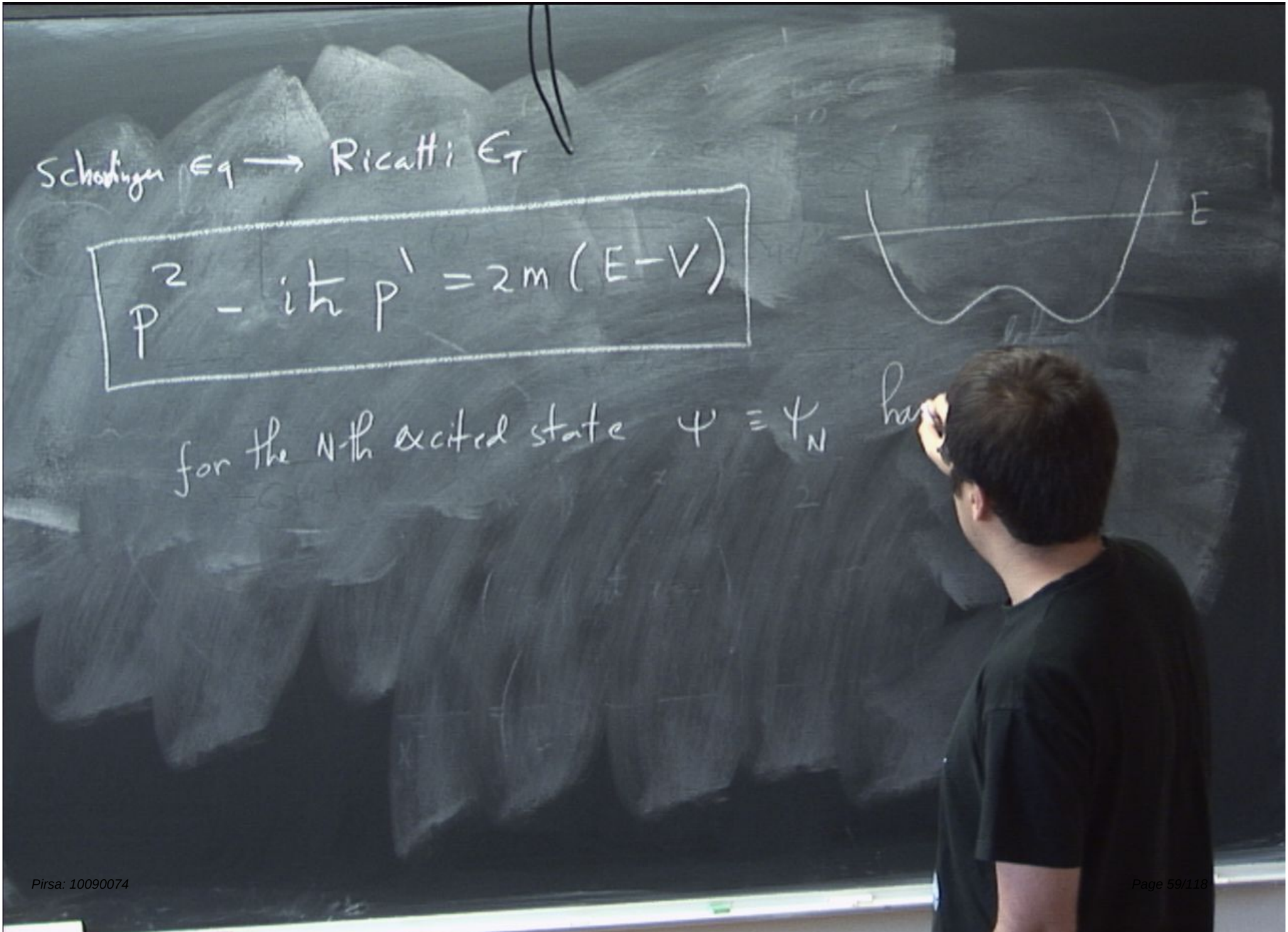
quasi-momentum $p(x)$

$$\psi = e^{\frac{i}{\hbar}\int^x p(x)\,dx}$$

$$p(x) = \frac{\hbar}{i}\frac{\psi'}{\psi} = \frac{\hbar}{i}\left(\partial_x \log \psi\right)$$


Schrodinger Eq ⟶ Riccati Eq
$p^2 - i\hbar p' = 2m(E-V)$


Schrodinger Eq ⟶ Riccati Eq
$p^2 - i\hbar p' = 2m(E-V)$


Schrödinger Eq ⟶ Riccati Eq
$p^2 - i\hbar p' = 2m(E-V)$
E
for the N-th excited state $\psi \equiv \psi_N$ ha

Schrödinger Eq ⟶ Riccati Eq

$$p^2 - i\hbar p' = 2m(E-V)$$

* for the N-th excited state $\psi \equiv \psi_N$ has N zeros

Schrödinger Eq ⟶ Riccati Eq

$$p^2 - i\hbar p' = 2m(E-V)$$

* for the N-th excited state $\psi = \psi_N$ has N zeros in the real axis

1D QM

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x) - E\right)\psi = 0$$

quasi-momentum $p(x)$

$$\psi = e^{\frac{i}{\hbar}\int^x p(x)\,dx}$$

$$p(x) = \frac{\hbar}{i}\frac{\psi'}{\psi} = \frac{\hbar}{i}\left(\partial_x \log\psi\right)$$

Schrödinger eq ⟶ Riccati eq

$$p^2 - i\hbar p' = 2m(E-V)$$

* for the N-th excited state $\psi = \psi_N$ has N zeros in the real axis

$p(x)$ has N poles with residue =

Schrödinger Eq ⟶ Riccati Eq

$$p^2 - i\hbar p' = 2m(E-V)$$

* for the N-th excited state $\psi = \psi_N$ has N zeros in the real axis

$p(x)$ has N poles with residue $= \hbar/i$

(close to a zero $p(x) \simeq \frac{\hbar}{i}\partial_x \underbrace{(\log A(z-z_*))}_{\psi}$

Schrödinger Eq ⟶ Riccati Eq

$$p^2 - i\hbar p' = 2m(E-V)$$

* for the N-th excited state $\psi = \psi_N$ has N zeros in the real axis

$p(x)$ has N poles with residue $= \hbar/i$

(close to a zero $p(x) \simeq \frac{\hbar}{i}\partial_x \underbrace{\left(\log A(z-z_*)\right)}_{\psi} \simeq \frac{\hbar/i}{z-z_*}$

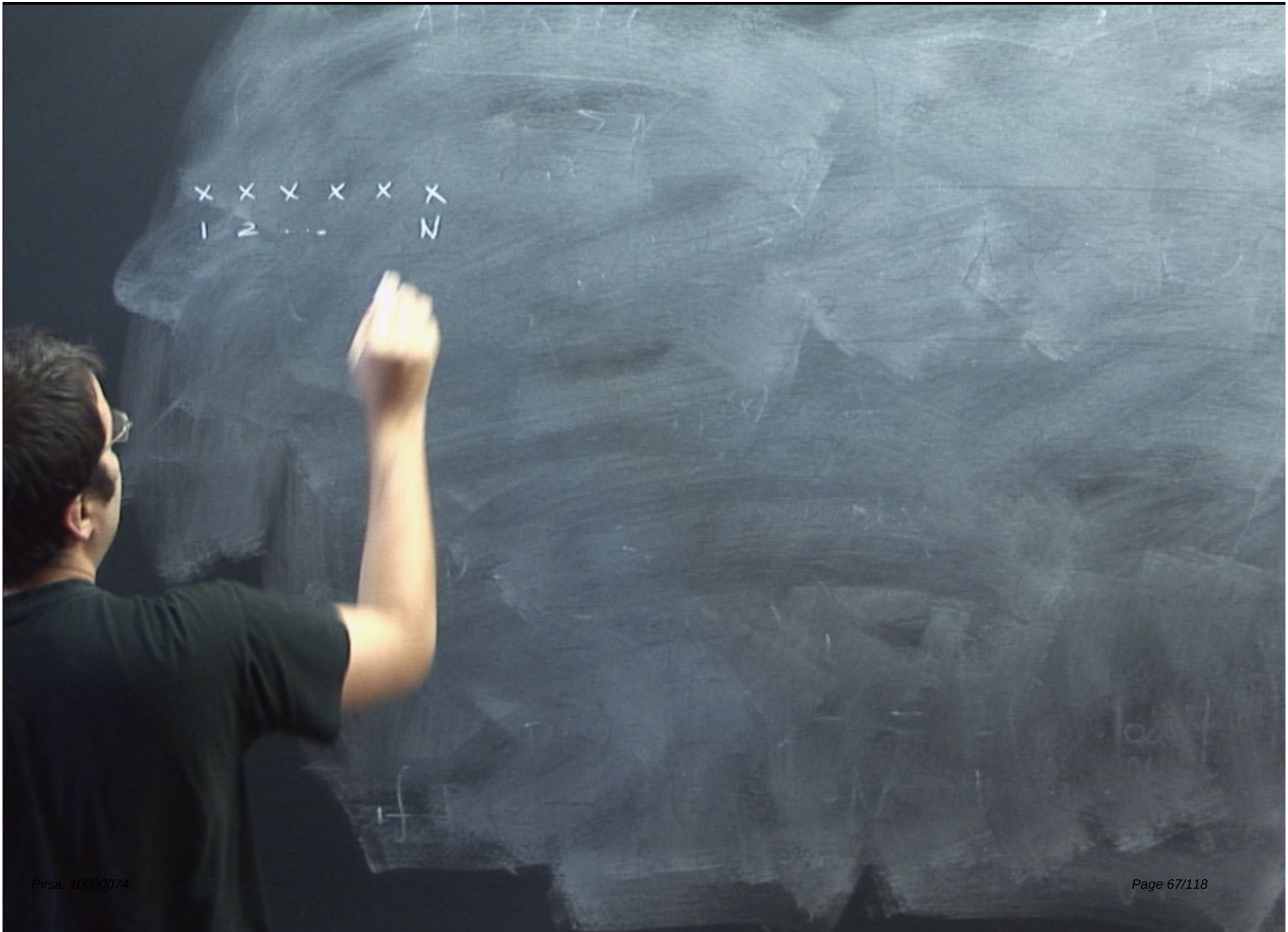
Schrödinger Eq ⟶ Riccati Eq

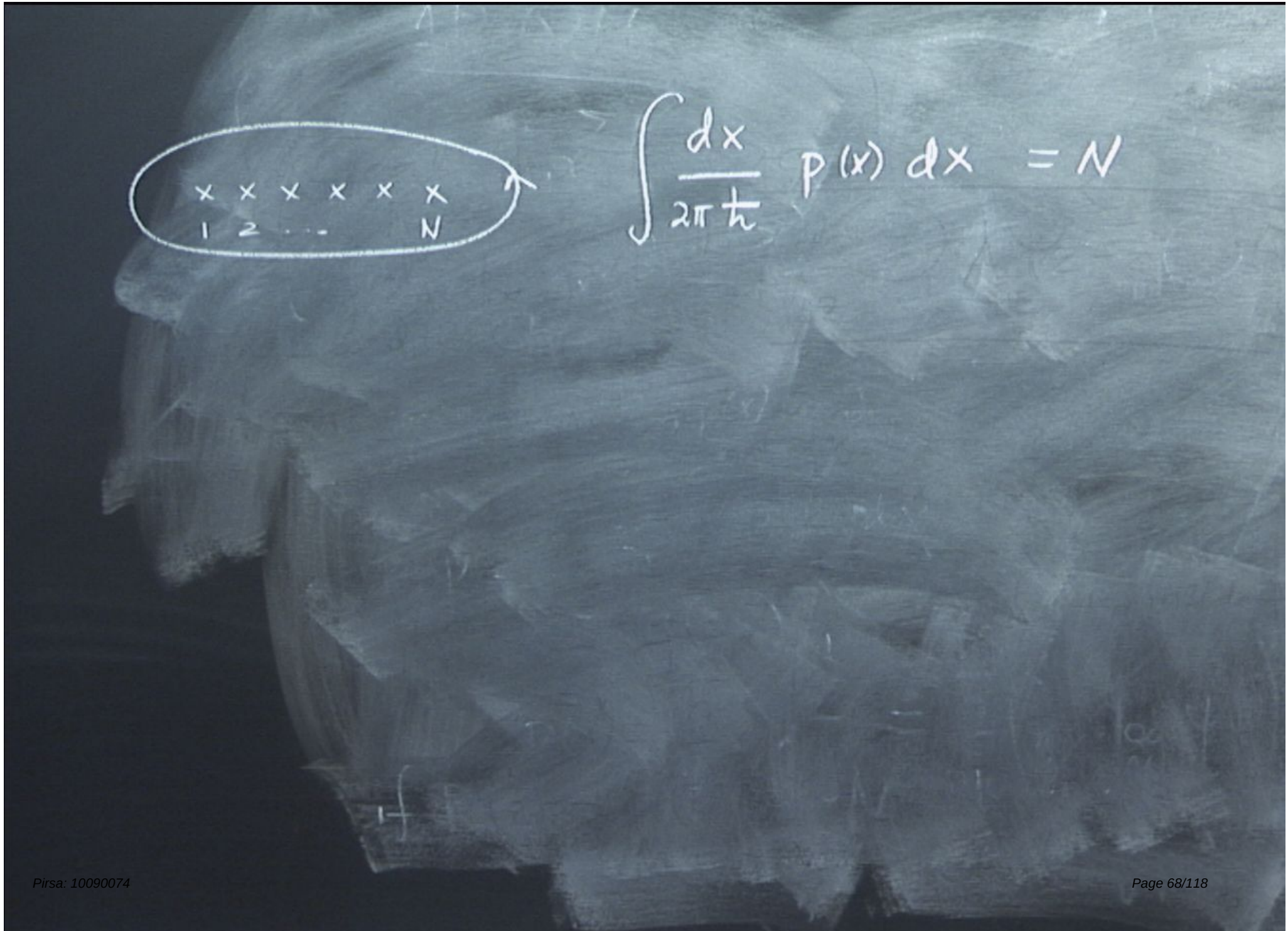
$$p^2 - i\hbar p' = 2m(E-V)$$

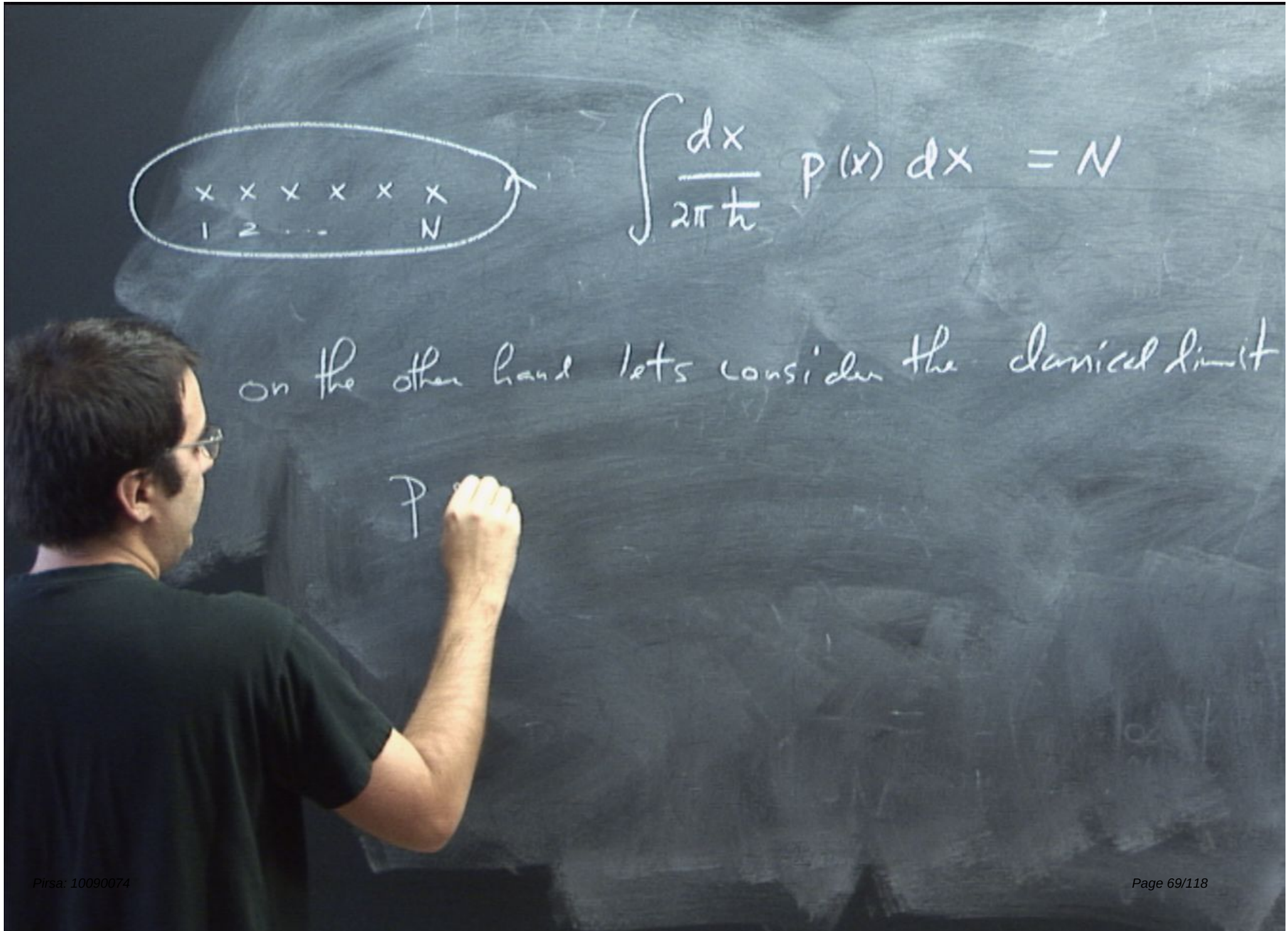
* for the N-th excited state $\psi = \psi_N$ has N zeros in the real axis

$p(x)$ has N poles with residue $= \hbar/i$

(close to a zero $p(x) \simeq \frac{\hbar}{i}\partial_x \underbrace{\left(\log A(z-z_*)\right)}_{\psi} \simeq \frac{\hbar/i}{z-z_*}$


X X X X X X
1 2 · · · N


x x x x x x
1 2 . . . N
$\int \frac{dx}{2\pi\hbar}\, p(x)\, dx = N$


x x x x x x
1 2 . . . N
$\int \frac{dx}{2\pi\hbar} p(x)\, dx = N$
on the other hand lets consider the classical limit
P

$$\int \frac{dx}{2\pi\hbar}\, p(x)\, dx = N$$

on the other hand lets consider the classical limit $\hbar \to 0$

$$p \simeq p_{cl} \equiv \sqrt{2m(E-V)}$$

$$\int \frac{dx}{2\pi\hbar}\, p(x)\, dx = N$$

on the other hand lets consider the classical limit $\hbar \to 0$

$$p \simeq p_{cl} \equiv \sqrt{2m(E-V)}$$

poles

$$\int \frac{dx}{2\pi\hbar}\, p(x)\, dx = N$$

on the other hand lets consider the classical limit $\hbar \to 0$

$$p \simeq p_{cl} \equiv \sqrt{2m(E-V)}$$

cut

poles

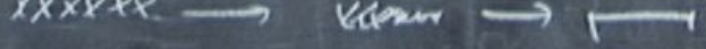
$$\int \frac{dx}{2\pi\hbar}\, p(x)\, dx = N$$

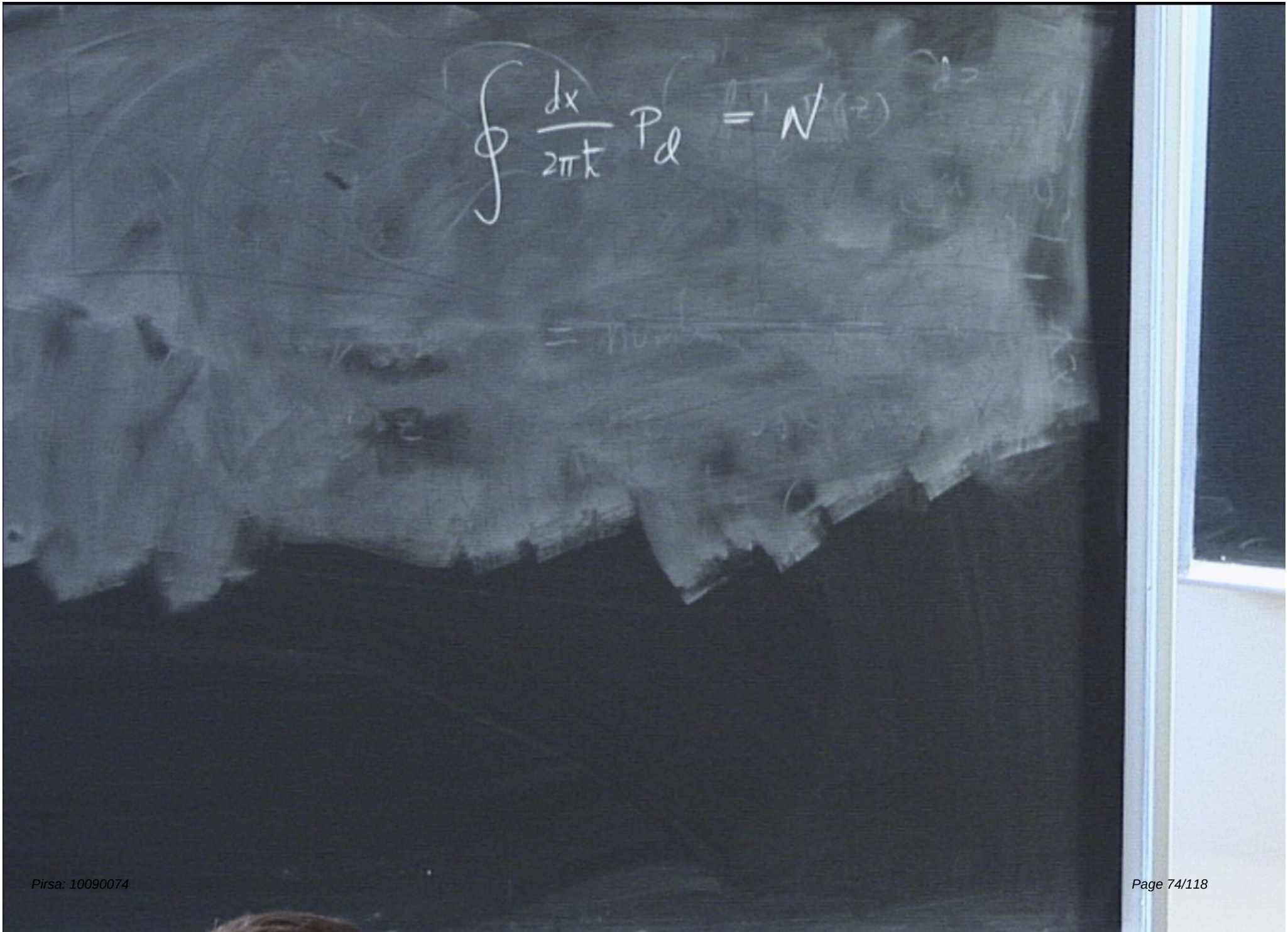
on the other hand lets consider the classical limit $\hbar \to 0$

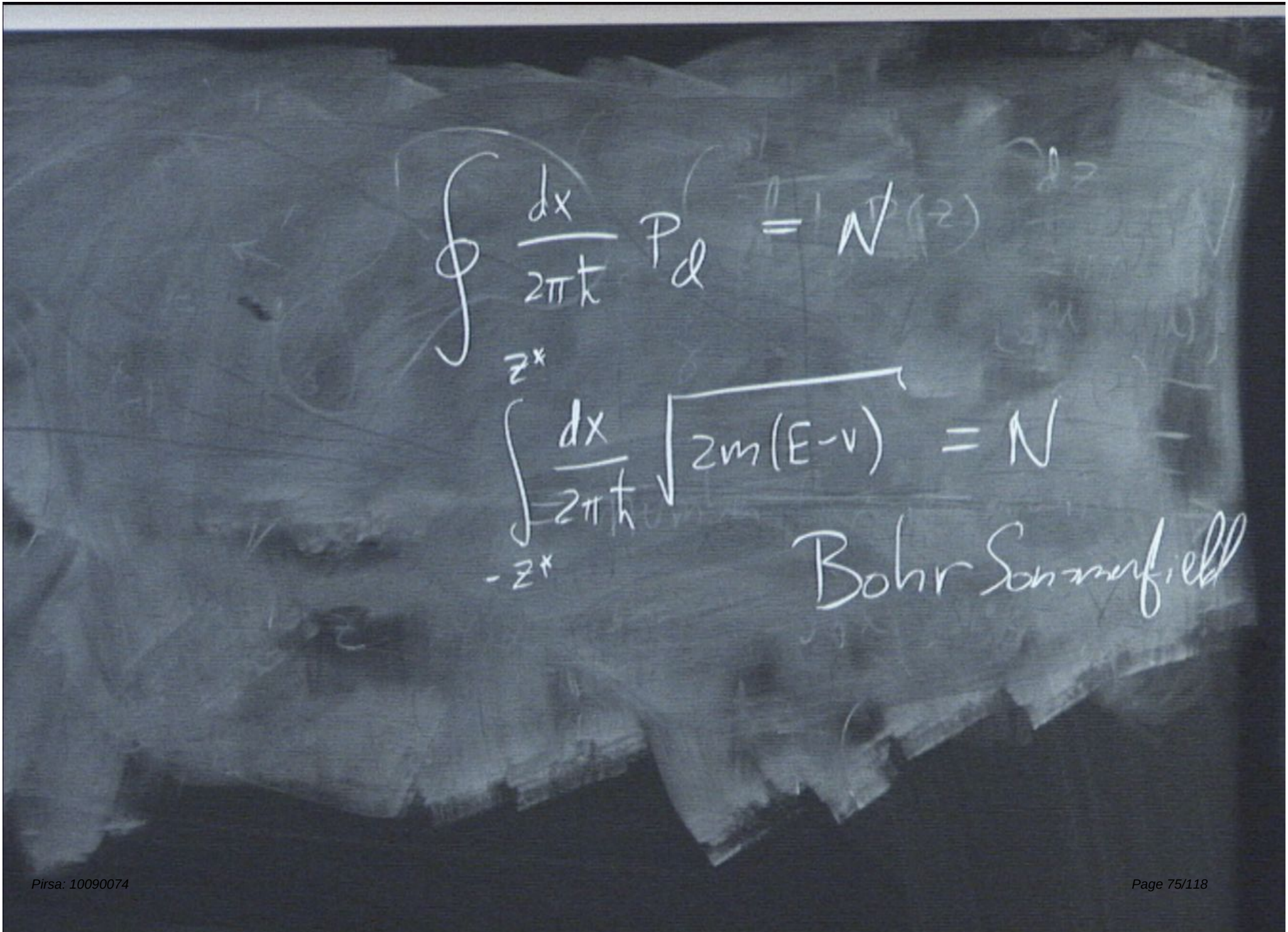
$$p \simeq p_{cl} \equiv \sqrt{2m(E-V)}$$

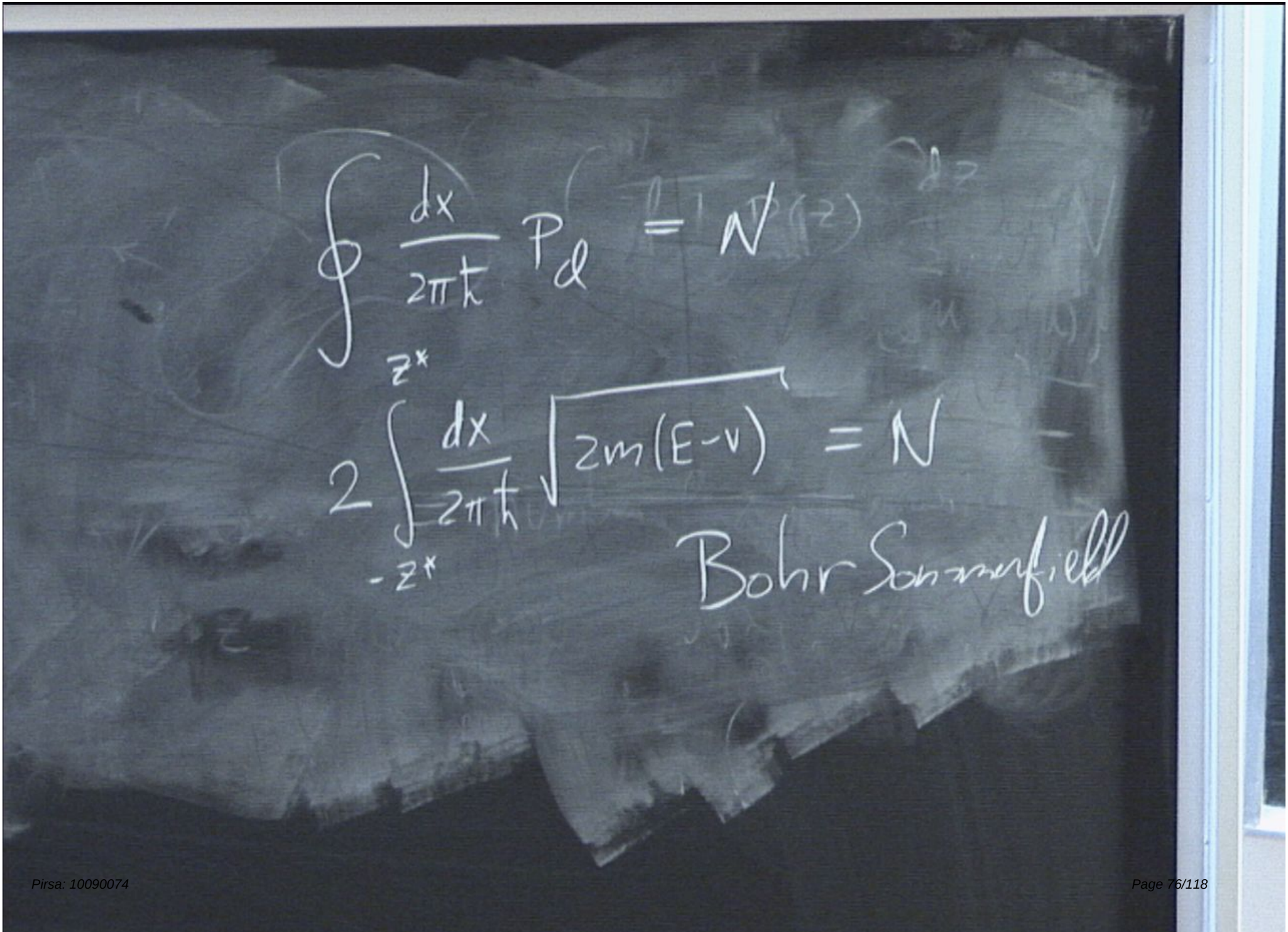
what happened was

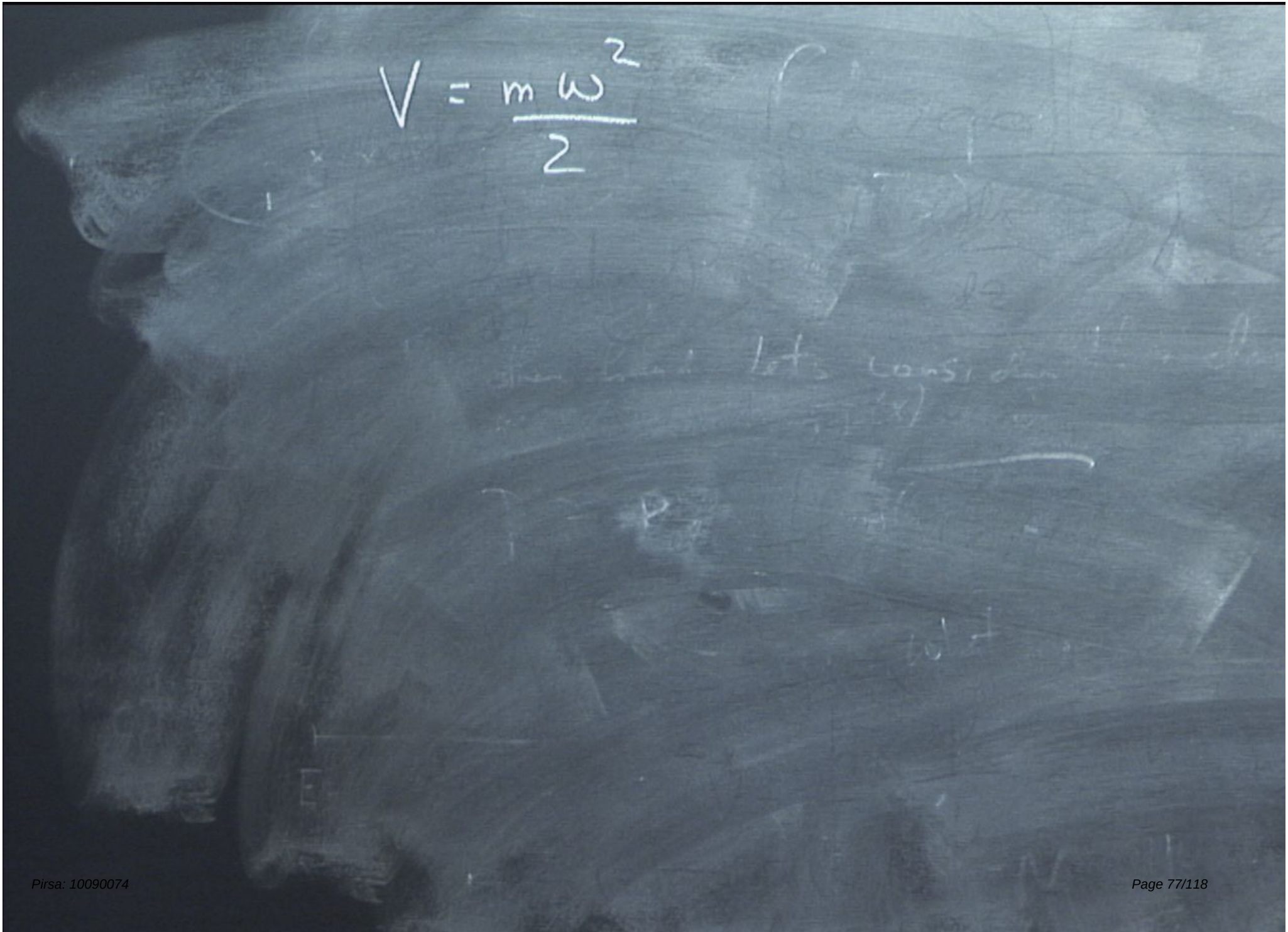
cut




$\oint \frac{dx}{2\pi i} P_\alpha = N$

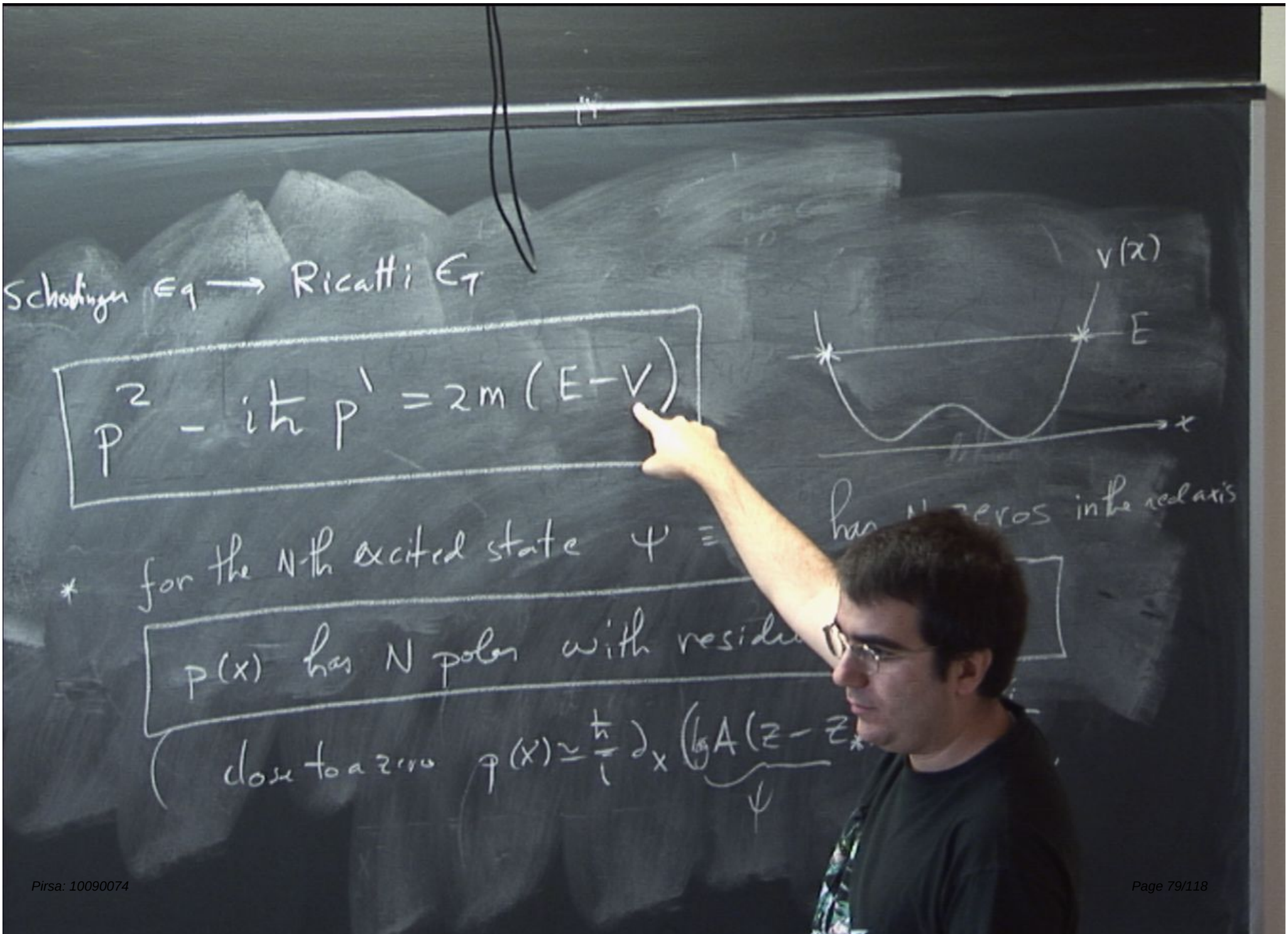

$\oint \frac{dx}{2\pi\hbar} p_q = N$
$\int_{-z^*}^{z^*} \frac{dx}{2\pi\hbar} \sqrt{2m(E-V)} = N$
Bohr Sommerfeld


$\oint \frac{dx}{2\pi\hbar} p_q = N$
z^*
$2\int_{-z^*} \frac{dx}{2\pi\hbar} \sqrt{2m(E-V)} = N$
Bohr Sommerfeld


$V = \frac{m\omega^2}{2}$

$$V = \frac{m\omega^2}{2} x^2$$

$p(x)$ at large $x = ?$


Schrodinger Eq → Ricatti Eq
$p^2 - i\hbar p' = 2m(E-V)$
V(x)
E
x
* for the N-th excited state ψ ≡
has N zeros in the real axis
p(x) has N poles with residu
close to a zero $p(x) \simeq \frac{\hbar}{i}\partial_x(\log A(z-z_*$
ψ

Schrödinger Eq ⟶ Riccati Eq

$$\# \quad \boxed{p^2 - i\hbar p' = 2m(E-V)}$$

* for the N-th excited state $\psi = \psi_N$ has N zeros in the real axis

$$\boxed{p(x) \text{ has N poles with residue} = \hbar/i}$$

$$\left(\text{close to a zero } p(x) \simeq \frac{\hbar}{i}\partial_x \underbrace{\left(\text{const}\, A(z-z_*)\right)}_{\psi} \simeq \frac{\hbar/i}{z-z_*}\right)$$

$$V = \frac{m\omega^2}{2} x^2$$

$p(x)$ at large x = ?

$$\text{Rhs}(\#) = -(m\omega x)^2 + 2mE$$

Schrodinger Eq $\longrightarrow$ Ricatti Eq

$$\# \quad \boxed{p^2 - i\hbar\, p' = 2m(E-V)}$$

* for the N-th excited state $\psi \equiv \psi_N$ has N zeros in the real axis

$$\boxed{p(x) \text{ has N poles with residue } = \hbar/i}$$

(close to a zero $p(x) \simeq \frac{\hbar}{i}\partial_x \big(\underbrace{\log A(z-z_*)}_{\psi}\big) \simeq \frac{\hbar/i}{z-z_*}$

$$V = \frac{m\omega^2}{2} x^2$$

$p(x)$ at large x = ?

$$\text{Rhs}(\#) = -(m\omega x)^2 + 2mE$$

$$p(x) \simeq im\omega x$$

$$V = \frac{m\omega^2}{2} x^2$$

$p(x)$ at large x = ?

$$\text{Rhs}(\#) = -(m\omega x)^2 + 2mE$$

$$p(x) \simeq im\omega x + O(\ )$$

$$V = \frac{m\omega^2}{2} x^2$$

$p(x)$ at large x = ?

$$\text{Rhs}(\#) = -(m\omega x)^2 + 2mE$$

$$p(x) \simeq im\omega x + O(\ ) \qquad x \to \infty$$

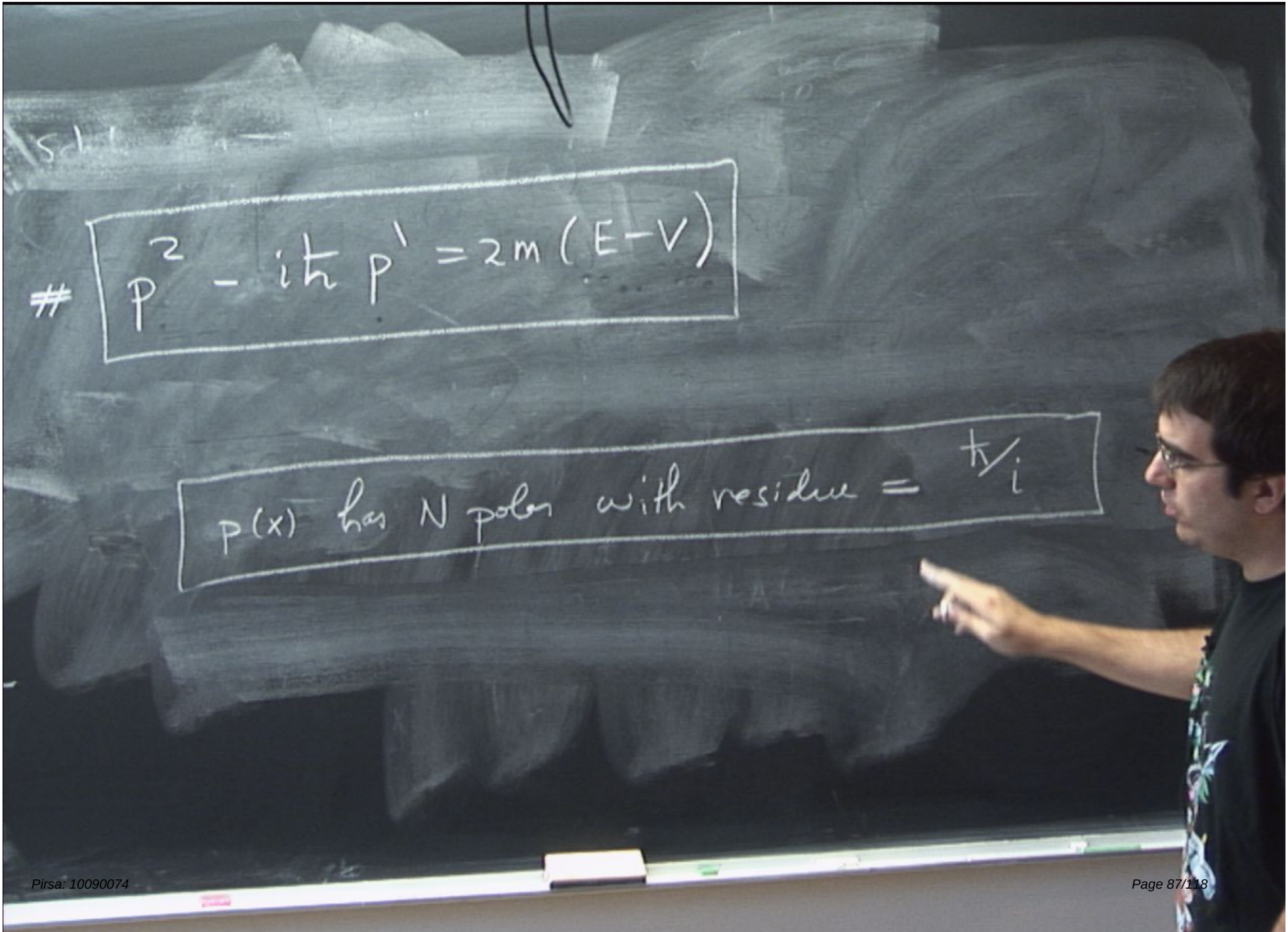
$$V = \frac{m\omega^2}{2} x^2$$

$p(x)$ at large x = ?

$$\text{Rhs}(\#) = -(m\omega x)^2 + 2mE$$

$$p(x) \simeq im\omega x + O\left(\frac{1}{x}\right) \qquad x \to \infty$$

no $O(1)$ term otherwise
p^2 would contain linear ter


$p^2 - i\hbar p' = 2m(E-V)$
p(x) has N poles with residue = $\hbar/i$

$$V = \frac{m\omega^2}{2} x^2$$

$p(x)$ at large x = ?

$$\text{Rhs}(\#) = -(m\omega x)^2 + 2mE$$

$$\boxed{p(x) \simeq im\omega x + O\left(\frac{1}{x}\right)} \qquad x \to \infty$$

no $O(1)$ term otherwise
p^2 would contain linear term

#

therefore $p(x) = im\omega x + \sum_{j=1}^{N} \frac{\hbar}{i} \frac{1}{x - x_j}$

x_j: zeros of ψ

$p^2 - i\hbar p' = 2m(E - V)$

$p(x)$ has N poles with residue $= \hbar/i$

therefore $p(x) = im\omega x + \sum_{j=1}^{N} \frac{\hbar}{i} \frac{1}{x - x_j}$

x_j zeros of ψ

$$\# \quad \boxed{p^2 - i\hbar p' = 2m(E - V)}$$

match constant term at $x \to \infty$ of #

$p(x)$ has N poles with residues $= \hbar/i$

therefore $p(x) = im\omega x + \sum_{j=1}^{N} \frac{\hbar}{i} \frac{1}{x - x_j} \simeq im\omega x + \ldots$

x_j: zeros of ψ

$$\# \quad \boxed{p^2 - i\hbar p' = 2m(E - V)}$$

match constant term at $x \to \infty$ of $\#$:

$$\boxed{p(x) \text{ has } N \text{ poles with residue} = \hbar/i}$$

ter

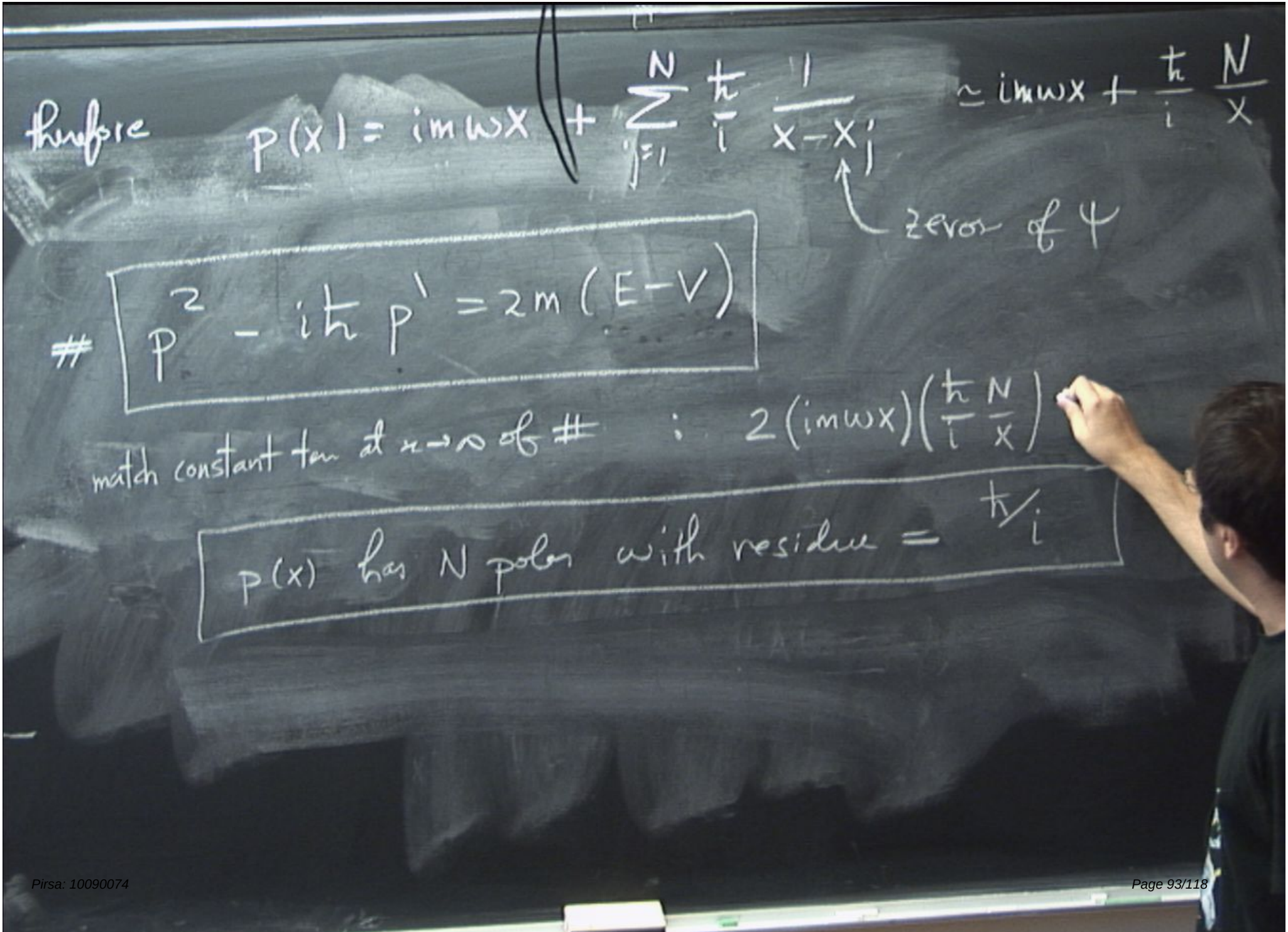
therefore $p(x) = im\omega x + \sum_{j=1}^{N} \frac{\hbar}{i} \frac{1}{x - x_j'} \simeq im\omega x + \frac{\hbar}{i} \frac{N}{x}$

x_j' : zeros of ψ

$$\# \quad \boxed{p^2 - i\hbar p' = 2m(E - V)}$$

match constant term at $x \to \infty$ of #

$$\boxed{p(x) \text{ has } N \text{ poles with residue} = \hbar/i}$$


therefore $p(x) = im\omega x + \sum_{j=1}^{N} \frac{\hbar}{i} \frac{1}{x - x_j'} \simeq im\omega x + \frac{\hbar}{i} \frac{N}{x}$
zeros of ψ
$p^2 - i\hbar p' = 2m(E-V)$
match constant term at $x \to \infty$ of # : $2(im\omega x)\left(\frac{\hbar}{i}\frac{N}{x}\right)$
$p(x)$ has N poles with residue $= \hbar/i$

therefore $p(x) = im\omega x + \sum_{j=1}^{N} \frac{\hbar}{i} \frac{1}{x - x_j'} \simeq im\omega x + \frac{\hbar}{i} \frac{N}{x}$

x_j' : zeros of ψ

$$\# \quad \boxed{p^2 - i\hbar p' = 2m(E - V)}$$

match constant term at $x \to \infty$ of # : $2(im\omega x)\left(\frac{\hbar}{i}\frac{N}{x}\right) - i\hbar(im\omega)$

$\boxed{p(x) \text{ has } N \text{ poles with residue} = \hbar/i}$

therefore $p(x) = im\omega x + \sum_{j=1}^{N} \frac{\hbar}{i} \frac{1}{x - x_j} \simeq im\omega x + \frac{\hbar}{i} \frac{N}{x}$

x_j: zeros of ψ

$$\# \quad \boxed{p^2 - i\hbar p' = 2m(E - V)}$$

p^2 $\quad -i\hbar p'$ $\quad 2mE$

$$2(im\omega x)\left(\frac{\hbar}{i}\frac{N}{x}\right) - i\hbar(im\omega) = 2mE$$

match constant term at $x \to \infty$ of #

$p(x)$ has N poles with residue $= \hbar/i$

therefore $p(x) = im\omega x + \sum_{j=1}^{N} \frac{\hbar}{i} \frac{1}{x - x_j} \simeq im\omega x + \frac{\hbar}{i} \frac{N}{x}$

x_j: zeros of ψ

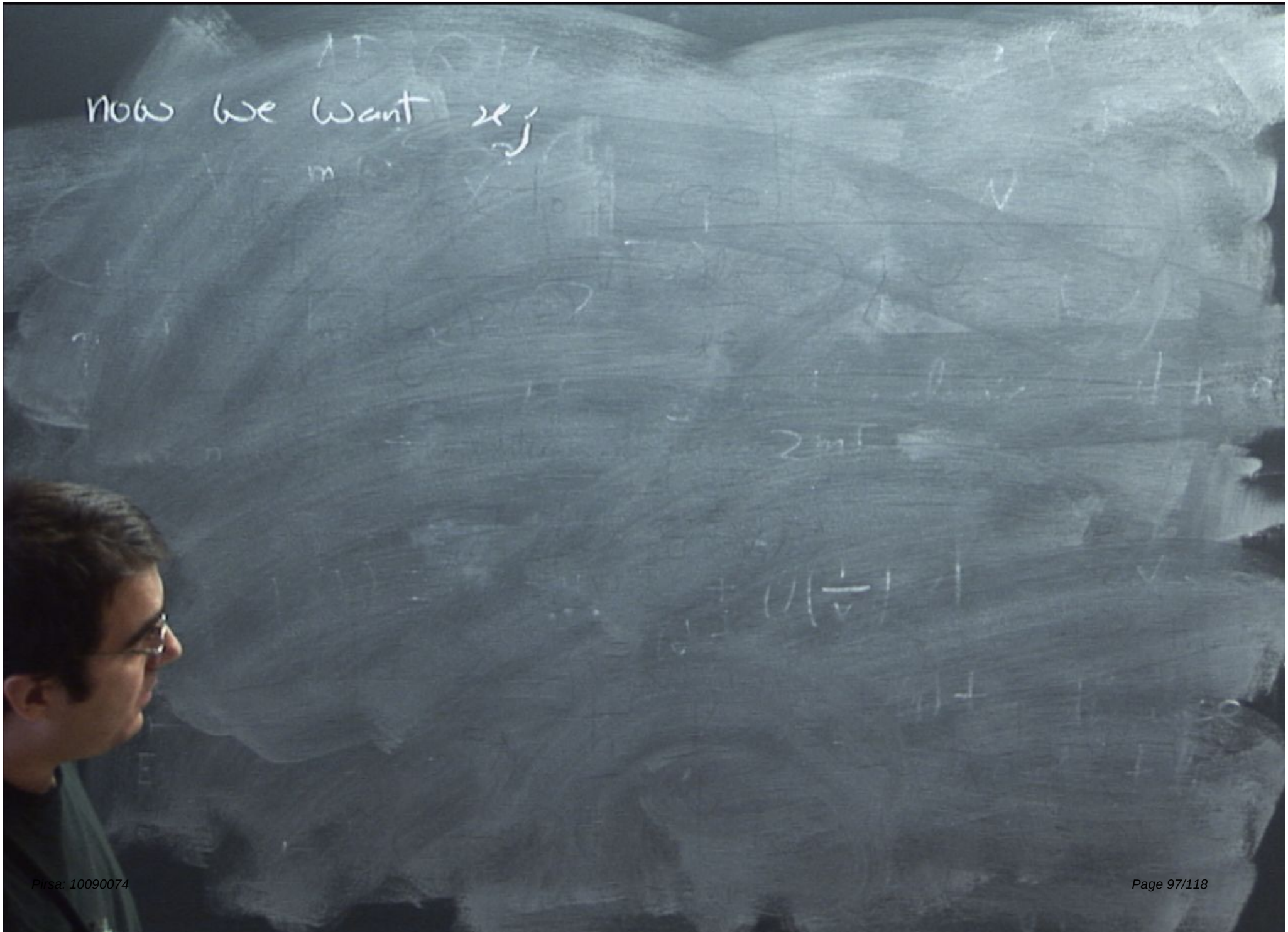
$$\# \quad \boxed{p^2 - i\hbar p' = 2m(E - V)}$$

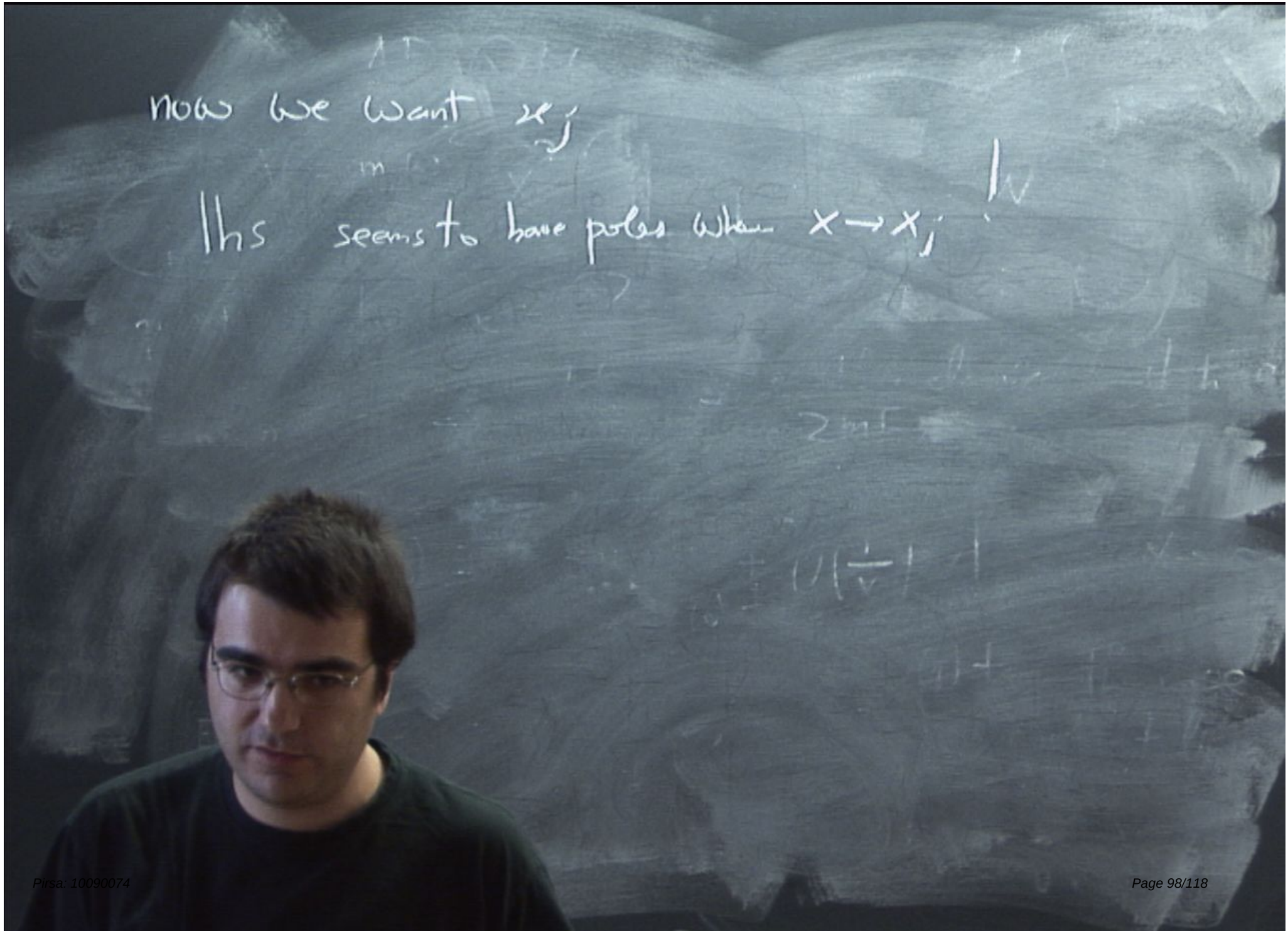
p^2 : $2(im\omega x)\left(\frac{\hbar}{i}\frac{N}{x}\right)$; $-i\hbar p'$: $-i\hbar(im\omega) = 2mE$; $2mE$

match constant term at $x \to \infty$ of #

$p(x)$ has N poles with residue $= \hbar/i$

$$E = \hbar\omega\left(N + \frac{1}{2}\right)$$


now we want


now we want x_j
in
lhs seems to have poles when $x \to x_j$

Now we want x_j

lhs seems to have poles when $x \to x_j$

Residue at x_j of lhs $= 0$

$\Updownarrow$

Now we want x_j

lhs seems to have poles when $x \to x_j$!

$\therefore$ Residue at x_j of lhs $= 0$

$$\Updownarrow$$

$$x_j = \frac{\hbar}{2m} \sum_{k \neq j}^{N} \frac{1}{x_i - x_j}$$

Now we want x_j

lhs seems to have poles when $x \to x_j$

? Residue at x_j of lhs $= 0$

$$\Updownarrow$$

$$x_j = \frac{\hbar}{2m} \sum_{k \neq j}^{N} \frac{1}{x_j - x_k}$$

Now we want x_j

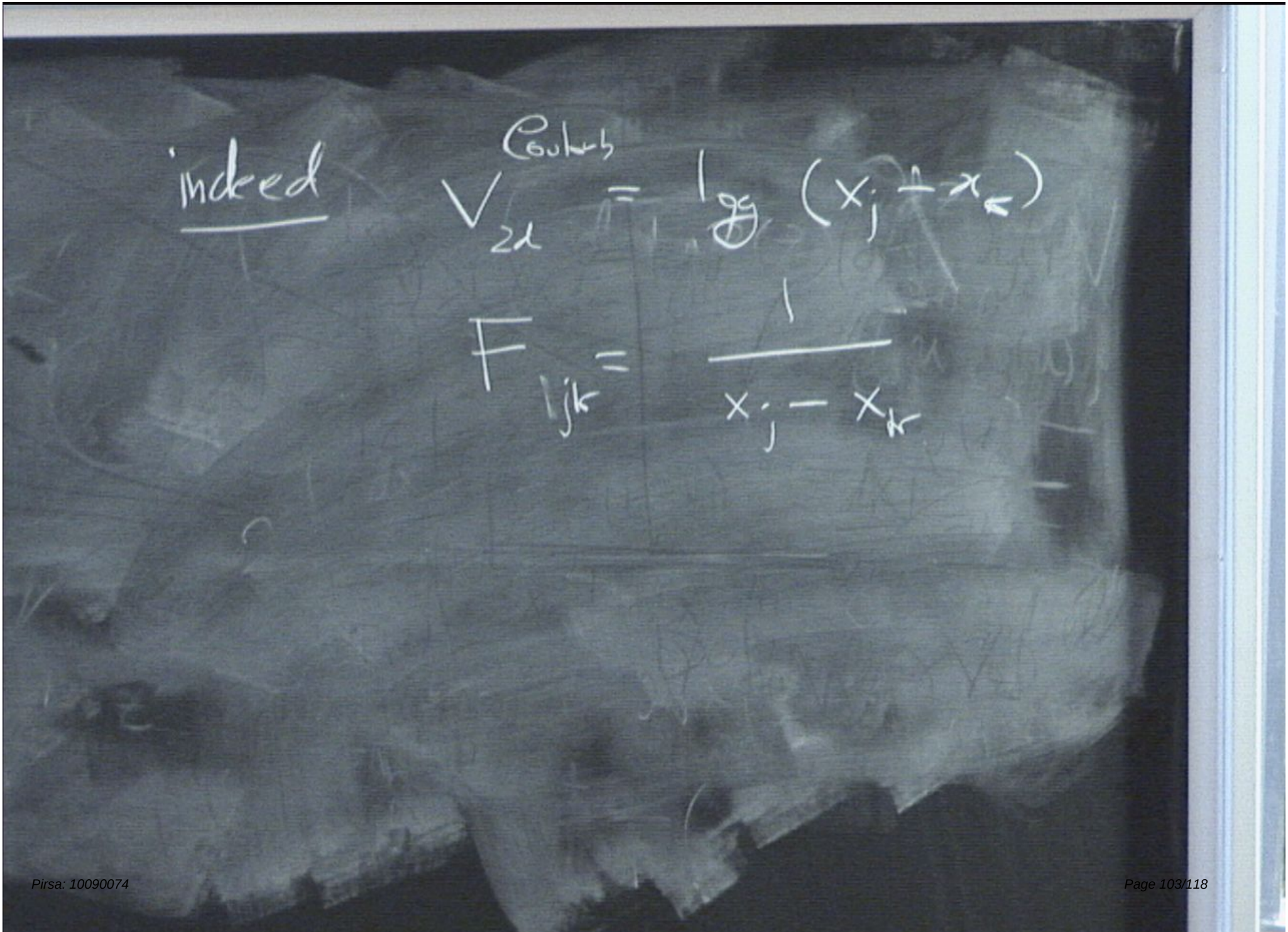
lhs seems to have poles when $x \to x_j$

Residue at x_j of lhs $= 0$

$$\Updownarrow$$

$$x_j = \frac{\hbar}{2m} \sum_{k \neq j}^{N} \frac{1}{x_j - x_k}$$

↑ looks like the electrostatic equil. condition for N 2d charges in Elliptical x^2 potential


Indeed
$V_{2d}^{Coulomb} = \log(x_j - x_k)$
$F_{ijk} = \frac{1}{x_j - x_k}$

Indeed $V_{2d}^{\text{Coulomb}} = \log(x_j - x_k)$

$$F_{ijk} = \frac{1}{x_j - x_k}$$

$$V_{ext} = x^2$$

$$F_{ext} = x$$

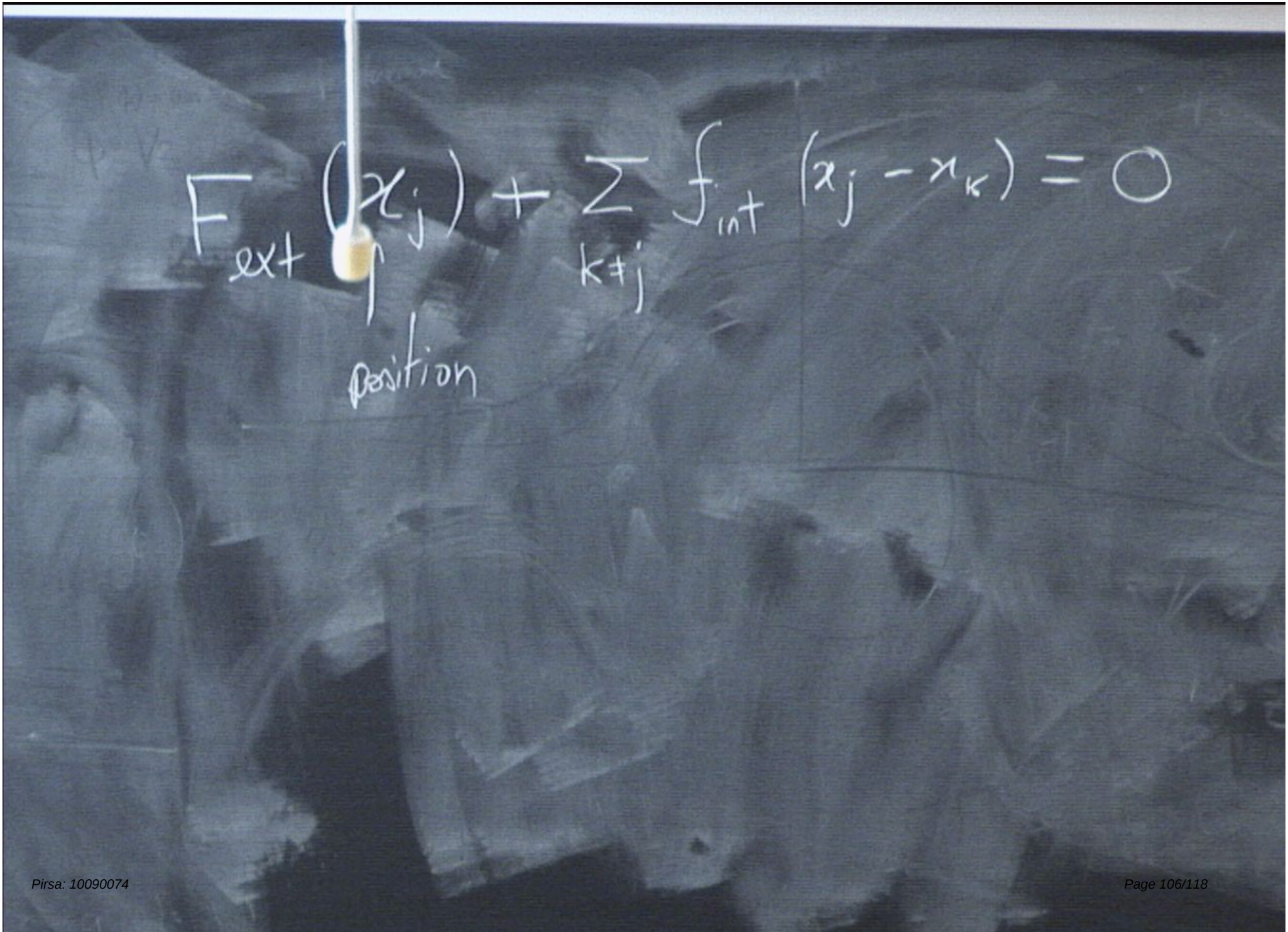
Indeed

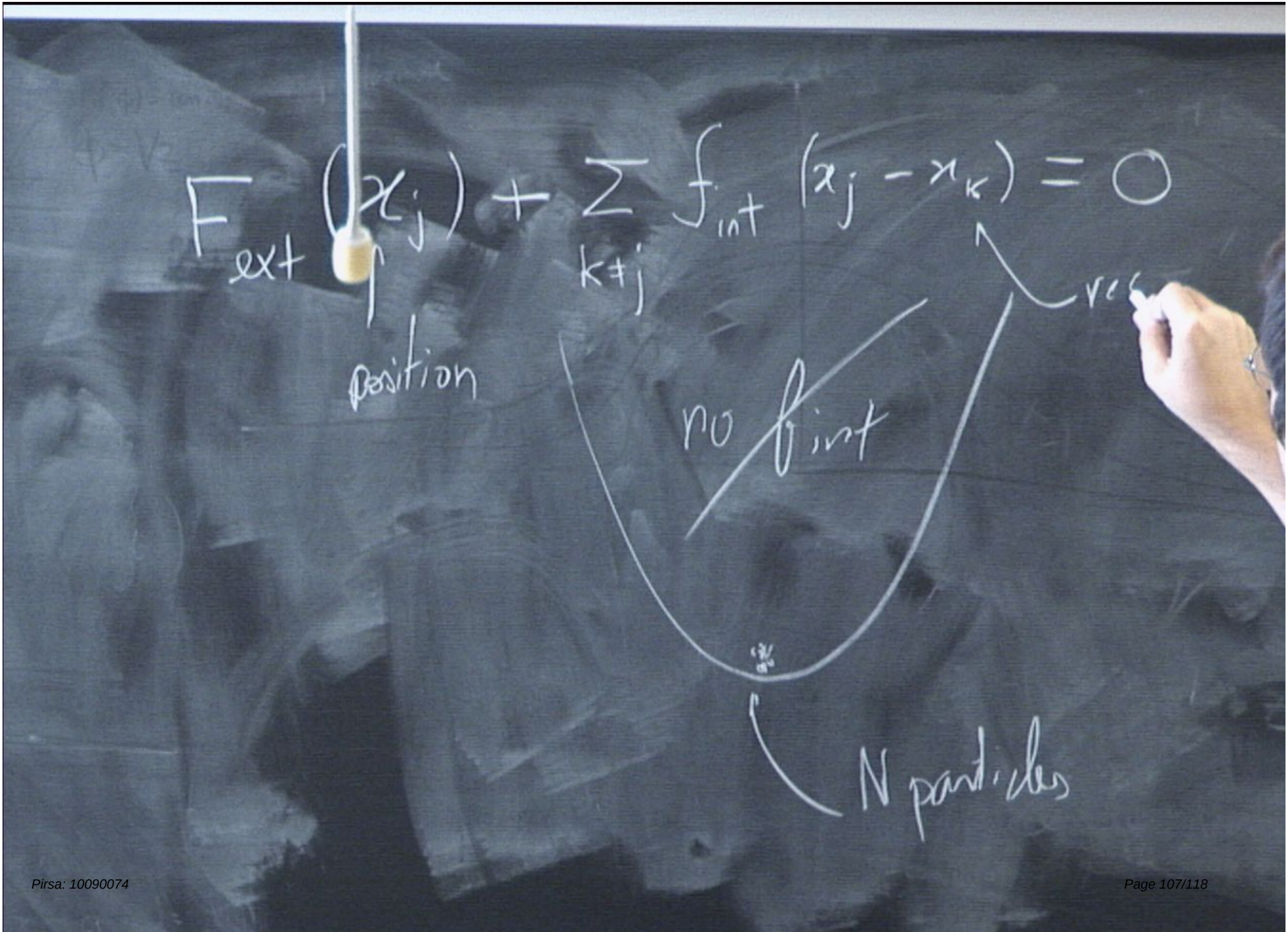
$$V_{2d}^{\text{Coulomb}} = \log(x_j - x_k)$$

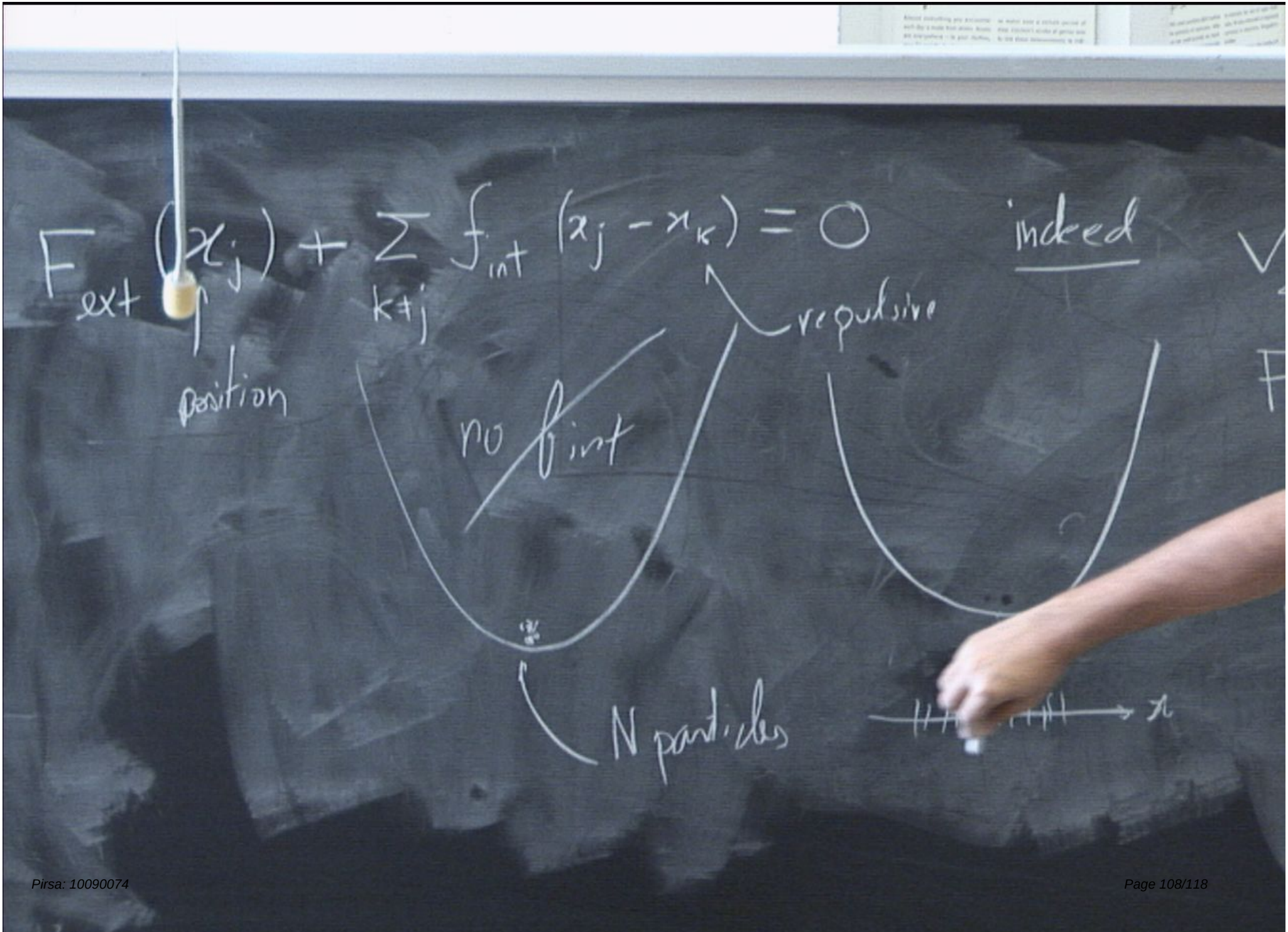
$$F_{ijk} = \frac{1}{x_j - x_k}$$

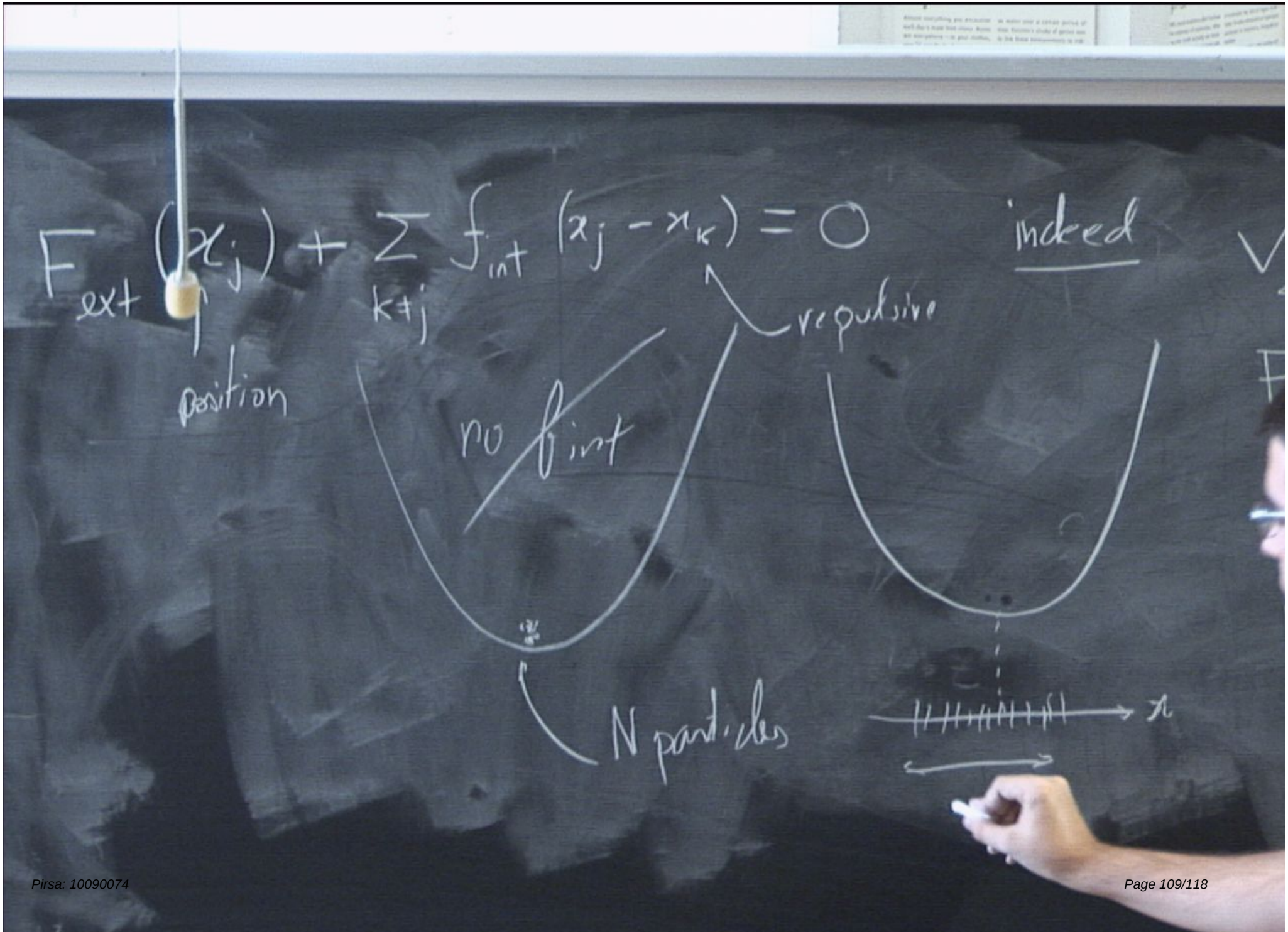
$$V_{ext} = x^2/2$$

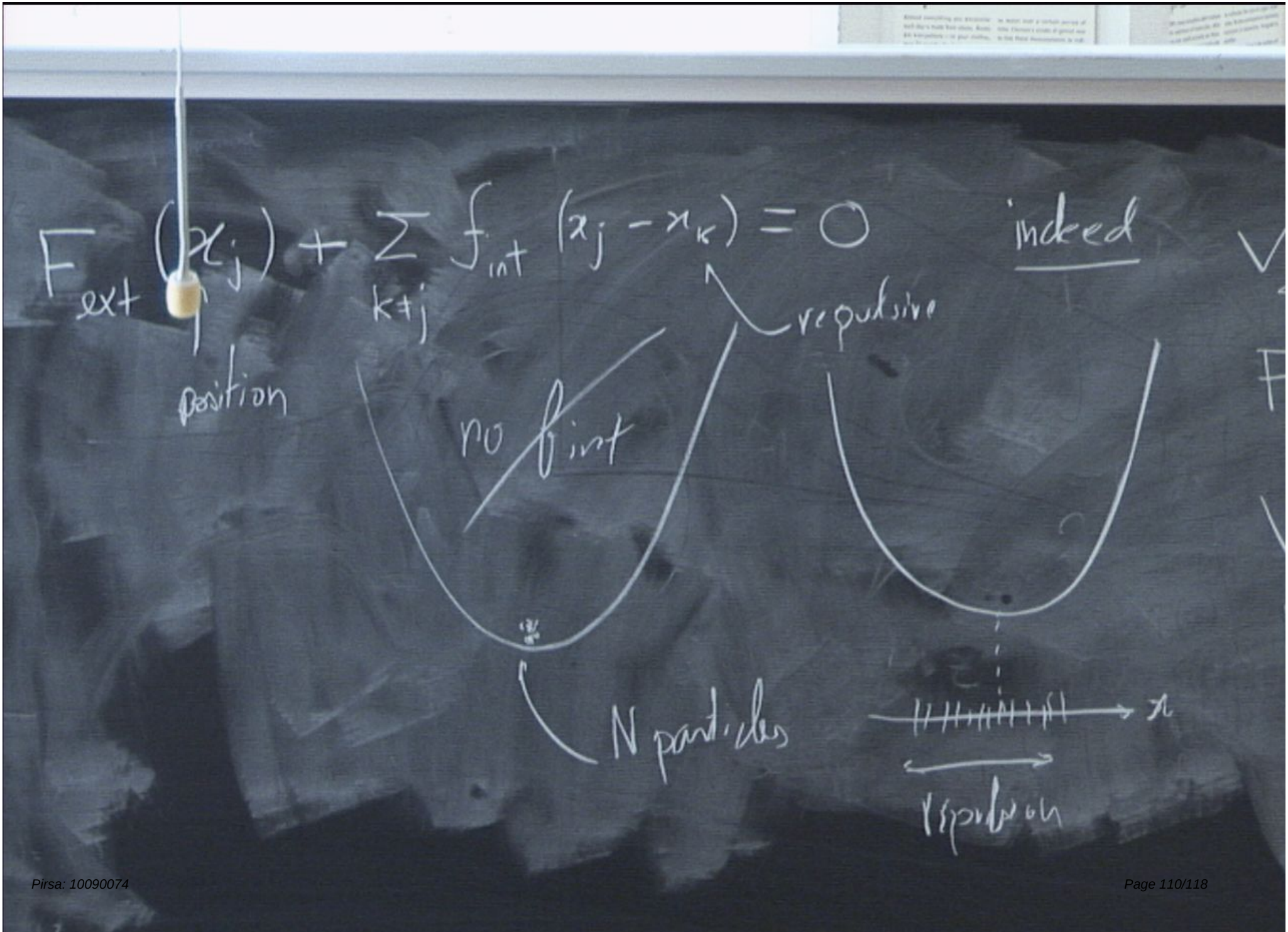
$$F_{ext} = x$$

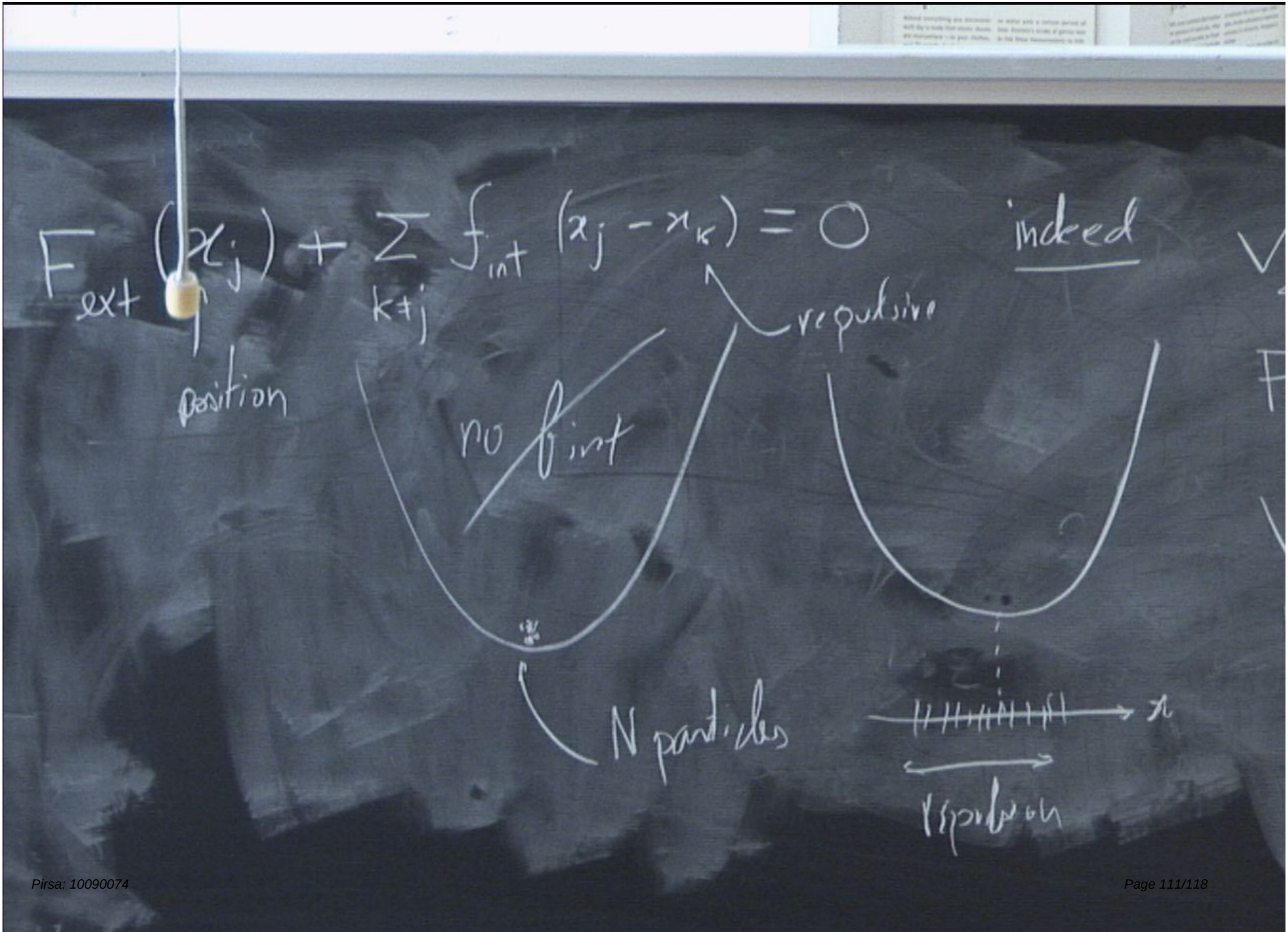

$F_{ext}(x_j) + \sum_{k \neq j} f_{int}(x_j - x_k) = 0$
position


$F_{ext}(x_j) + \sum_{k \neq j} f_{int}(x_j - x_k) = 0$
position
no f_{int}
N particles


$F_{ext}(x_j) + \sum_{k \neq j} f_{int}(x_j - x_k) = 0$
indeed
position
repulsive
no f_{int}
N particles
x


$F_{ext}(x_j) + \sum_{k \neq j} f_{int}(x_j - x_k) = 0$
indeed
position
repulsive
no f_{int}
N particles
x


$F_{ext}(x_j) + \sum_{k \neq j} f_{int}(x_j - x_k) = 0$
indeed
position
repulsive
no f_{int}
N particles
x


$F_{ext}(x_j) + \sum_{k \neq j} f_{int}(x_j - x_k) = 0$
indeed
position
repulsive
no f_{int}
N particles
x

now we want x_j

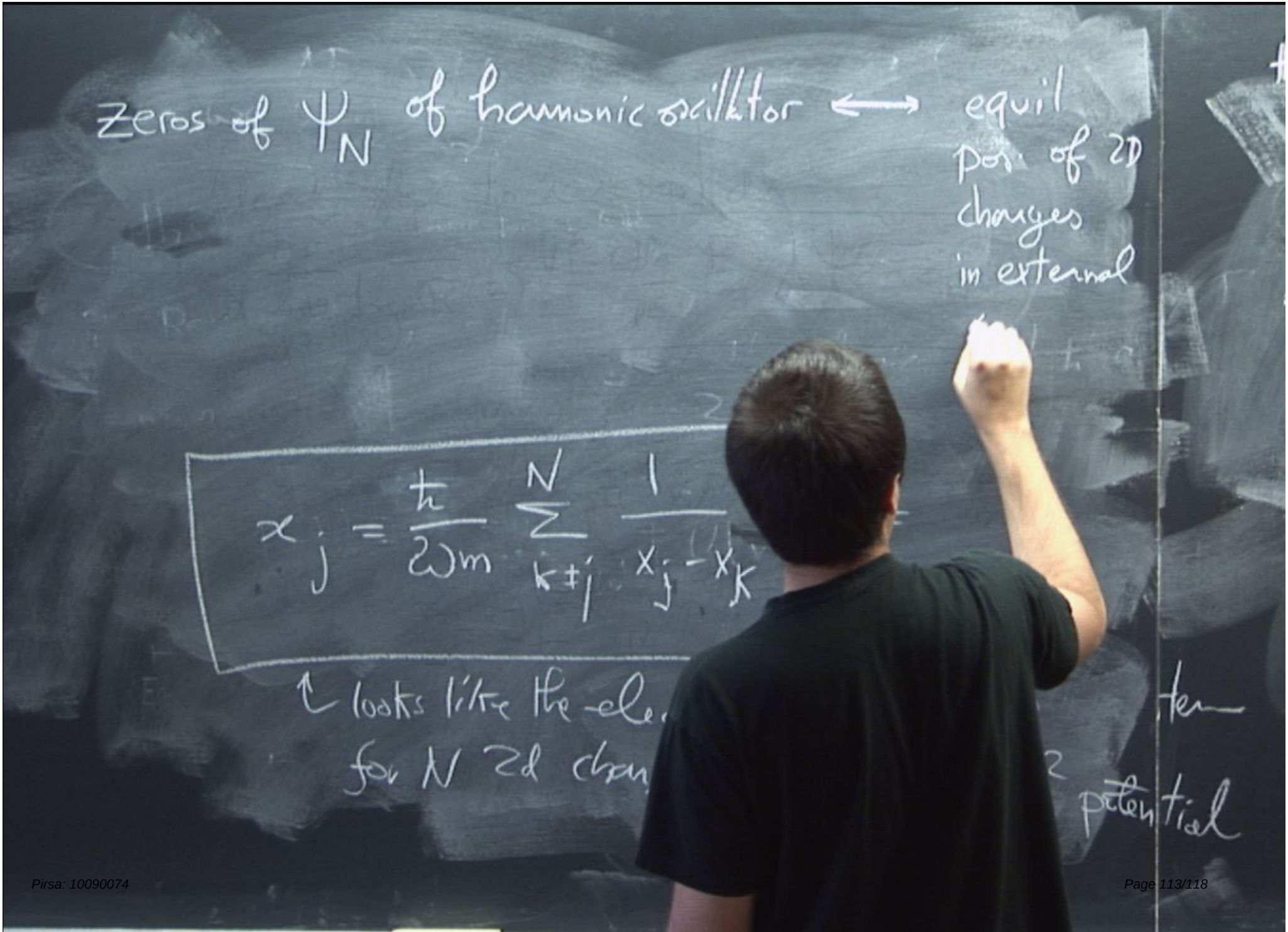
lhs seems to have poles when $x \to x_j$!

Residue at x_j of lhs $= 0$

$\Updownarrow$

$$x_j = \frac{\hbar}{2m} \sum_{k \neq j}^{N} \frac{1}{x_j - x_k} \quad \#$$

↳ looks like the electrostatic equil. cond for N 2d charges in Elliptical X^2 potentia


Zeros of ψ_N of harmonic oscillator ⟷ equil pos. of 2D charges in external
$x_j = \frac{\hbar}{2m} \sum_{k \neq j}^{N} \frac{1}{x_j - x_k}$
↑ looks like the ele
for N 2d chan
potential

Zeros of Ψ_N of harmonic oscillator $\longleftrightarrow$ equil pos of 2D charges in external harmonic x^2 potential

$$\boxed{x_j = \frac{\hbar}{2\omega m} \sum_{k \neq j}^{N} \frac{1}{x_j - x_k}} \quad \#$$

↑ looks like the electrostatic equil. load ... term for N 2d charges in external x^2 potential

Zeros of ψ_N of harmonic oscillator $\longleftrightarrow$ equil pos. of 2D charges in external harmonic x^2 potential

$$x_j = \frac{\hbar}{2m} \sum_{k \neq j}^{N} \frac{1}{x_j - x_k} \quad \#$$

↑ looks like the electrostatic equil. cond. for N 2d charges in External x^2 potential

Zeros of Ψ_N of harmonic oscillator $\longleftrightarrow$ equil pos of 2D charges in external harmonic x^2 potential

has exact solution!

$$\boxed{x_j = \frac{\hbar}{2\omega m} \sum_{k \neq j}^{N} \frac{1}{x_j - x_k}} \quad \#$$

↑ looks like the electrostatic equil. Load ... tem for N 2d charges in External x^2 potential

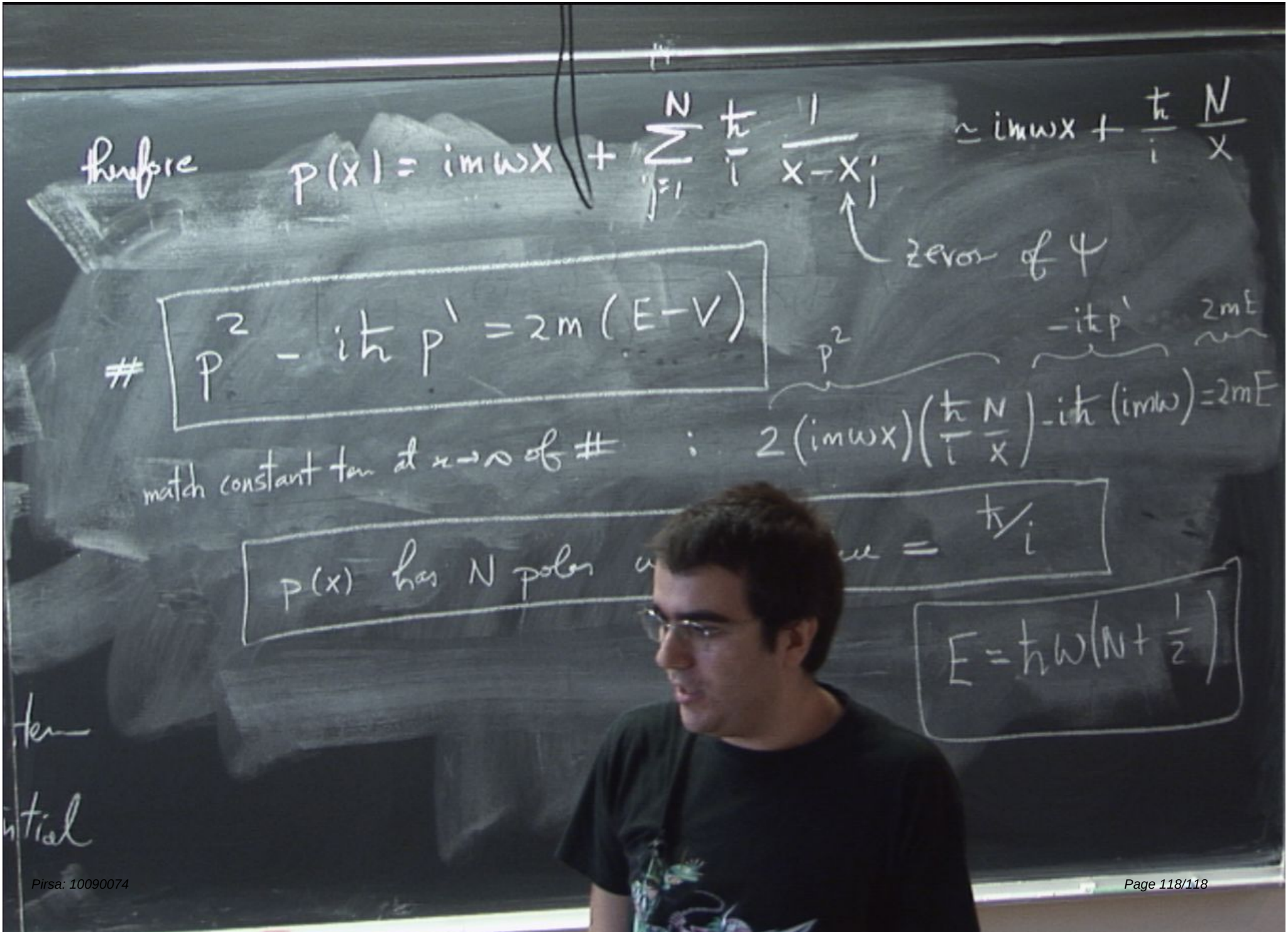
Zeros of ψ_N of harmonic oscillator $\longleftrightarrow$ equil pos. of 2D charges in external harmonic x^2 potential

has exact solution!

$$H_N\left(\sqrt{\frac{m\omega}{\hbar}}\, x_i\right) = 0$$

$$x_j = \frac{\hbar}{2m} \sum_{k \neq j}^{N} \frac{1}{x_j - x_k} \quad \#$$

↳ looks like the electrostatic equil. cond. for N 2d charges in External x^2 potential


therefore $p(x) = im\omega x + \sum_{j=1}^{N} \frac{\hbar}{i} \frac{1}{x - x_j} \simeq im\omega x + \frac{\hbar}{i} \frac{N}{x}$
zeros of ψ
$p^2 - i\hbar p' = 2m(E-V)$
p^2 $-i\hbar p'$ $2mE$
match constant term at $x \to \infty$ of # : $2(im\omega x)\left(\frac{\hbar}{i}\frac{N}{x}\right) - i\hbar(im\omega) = 2mE$
$p(x)$ has N poles
$= \hbar/i$
$E = \hbar\omega\left(N + \frac{1}{2}\right)$