Title: The flavor puzzle : Why neutrinos are different ?

Date: Sep 24, 2010 02:30 PM

URL: http://pirsa.org/10090071

Abstract: Large mixing angles and a mild mass hierarchy are observed in neutrino oscillations, in stark contrast with the quarks and charged leptons sectors where very hierarchical masses come along with small mixings.
We review and discuss the neutrino mass patterns that are technically natural, in the context of the seesaw mechanism and with a quark-lepton unification perspective.
We show that a seesaw in six dimensions offers an elegant and unique solution to the flavor puzzle. An explicit model is constructed, with a vortex background on a sphere. It offers an explanation for the replication of families in the Standard Model, and predicts suppressed flavour violating interactions.

THE FLAVOUR PUZZLE : WHY NEUTRINOS ARE DIFFERENT ?

Fu-Sin Ling

Perimeter Institute - September 24th 2010

J.-M. Frère, M. Libanov, FSL, arXiv : 1006.5196
FSL, arXiv : 1009.2371

OUTLINE & SUMMARY

- The flavour puzzle

  Neutrinos : tiny masses + large mixings ?

  Quarks : hierarchical masses + small mixings

- Neutrino mass matrix reconstruction

  Statistics favour inverted hierarchy ?

- Origin of an inverted hierarchy

  Origin of the large mixing angles

  See-saw mechanism

  Majorana mass in six dimensions ?

Outline & Summary

- A flavour model with a vortex background in 6D

  1 family in 6D $\longleftrightarrow$ 3 (?) chiral families in 4D

  See-saw in 6D : the neutrino mass matrix is automatically off-diagonal

$$M_\nu \sim \begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \end{pmatrix}$$

  Constraints on the size of xdim come from flavour violating processes. Processes with a change of the family number are automatically suppressed.

The Flavour Puzzle in the New Standard Model (νSM)

- **Why three families ?**
  - Same gauge interactions
  - Hierarchical masses
    I < II < III
  - Small mixing angles
- **Neutrinos**
  - Hierarchy ?
  - Some mixing angles large
  - Lepton number violation ?

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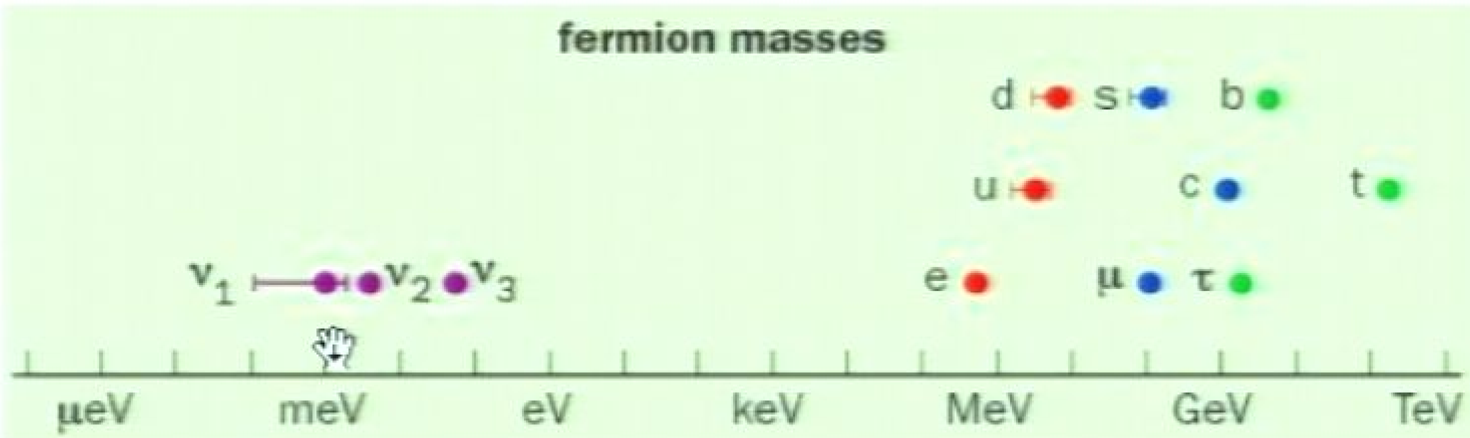
  - Lepton number violation ?

HIERARCHICAL MASSES



- Intra-generation hierarchy
- Inter-generation hierarchy

Standard Parameterization of the Mixing Matrix

$$
\begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\mathrm{CP}}} \\
-s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\mathrm{CP}}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\mathrm{CP}}} & c_{13}s_{23} \\
s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\mathrm{CP}}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\mathrm{CP}}} & c_{13}c_{23}
\end{pmatrix}
$$

$$s_{ij} = \sin\theta_{ij},\ c_{ij} = \cos\theta_{ij}$$

THE CKM MIXING MATRIX IN THE QUARK SECTOR

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & c_{13}c_{23} \end{pmatrix}$$

$$\theta_{12} = 13.04 \pm 0.05^\circ,\ \theta_{13} = 0.201 \pm 0.011^\circ,\ \theta_{23} = 2.38 \pm 0.06^\circ$$

$$\delta_{CP} = 1.20 \pm 0.08$$

THE PMNS MIXING MATRIX IN THE NEUTRINO SECTOR

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & c_{13}c_{23} \end{pmatrix}$$

3σ

$\tan^2\theta_{12} = 0.47^{+0.14}_{-0.10}$ *Solar angle*

$\tan^2\theta_{23} = 0.9^{+1.0}_{-0.4}$ *Atmospheric angle*

$\sin^2\theta_{13} \le 0.05$ *Reactor angle*

δ_{CP} ??

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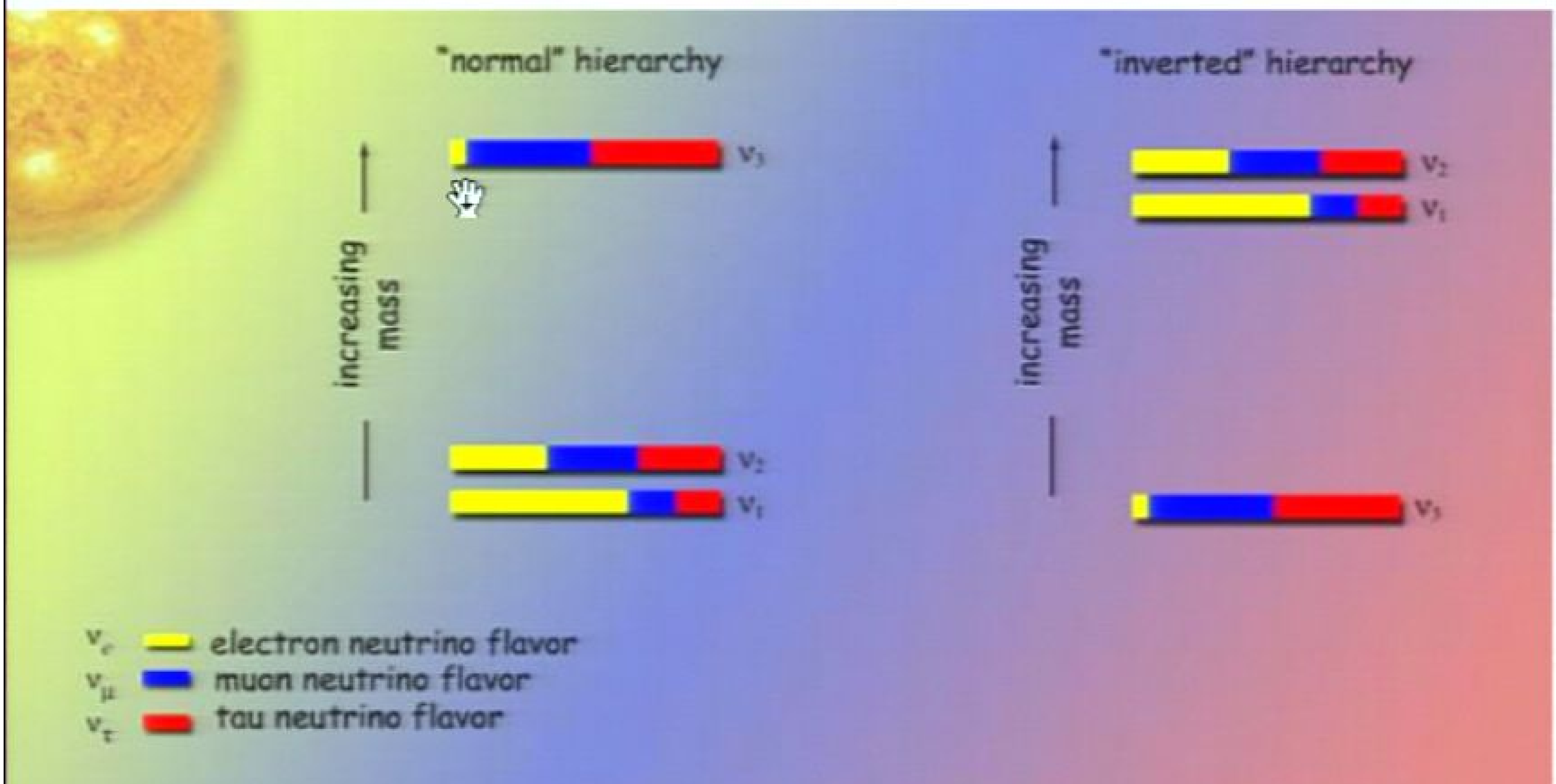
δ_{CP} ??

→ ***Two large angles !!***

Neutrino mass hierarchy

- Only $\Delta m_{ij}^2 = m_{\nu_i}^2 - m_{\nu_j}^2$ are measured by oscillation experiments !



NEUTRINO MASS HIERARCHY

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Solar neutrinos

$$\Delta m_{21}^2 = 7.6 \pm 0.7 \times 10^{-5}\ \mathrm{eV}^2$$

Atmospheric neutrinos

$$\Delta m_{31}^2 = 2.46 \pm 0.37 \times 10^{-3}\ \mathrm{eV}^2 \qquad NH$$

$$\Delta m_{31}^2 = -2.36 \pm 0.37 \times 10^{-3}\ \mathrm{eV}^2 \qquad IH$$

CAN ALL THIS FIT TOGETHER IN A UNIFIED PICTURE ?

We need to reconstruct the mass matrices...

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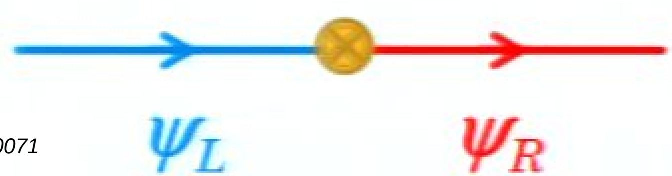
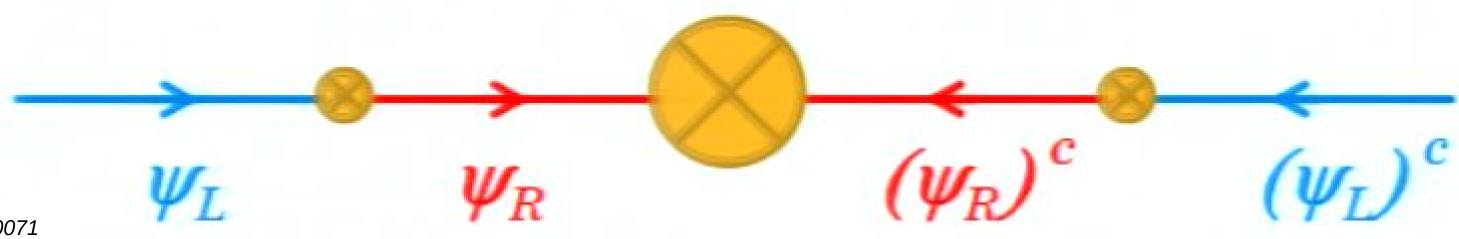
NEUTRINOS : DIRAC OR MAJORANA ?



SEE-SAW MECHANISM (TYPE I)

- Might explain the smallness of neutrino masses
- Might explain the presence of large mixing angles
- $M_\nu = M_D \cdot M_R^{-1} \cdot M_D^t$



QUARK SECTOR

- Wolfenstein parameterization of the CKM matrix

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\lambda \equiv \lambda_C = \sin\theta_C = 0.2257^{+0.0009}_{-0.0010}$$

- Empirical relations advocate for a connection between hierarchical masses and small mixing angles

$$\frac{m_d}{m_s} \simeq |V_{us}|^2 \qquad \frac{m_u}{m_c} \simeq \frac{|V_{ub}|^2}{|V_{cb}|^2}$$

LEPTON SECTOR

- Hierarchical charged lepton masses

$$\frac{m_\tau}{m_t} \sim \frac{m_b}{m_t} \qquad \frac{m_\mu}{m_\tau} \sim \frac{m_s}{m_b} \qquad \frac{m_e}{m_\tau} \sim \frac{m_d}{m_b}$$

→ *Grand Unification ??*

- Hierarchical neutrinos masses ??

$$\frac{\Delta m^2_{21}}{|\Delta m^2_{31}|} \simeq 3.2\,\% \equiv \delta^2$$

$$\delta \simeq \lambda_C$$

→ *Hidden hierarchy ??*

- Quark -- Lepton complementarity

$$\frac{\pi}{4} - \theta_\odot \simeq \lambda_C$$

→ *QL unification ??*

Neutrino mass matrix reconstruction

- (Effective) Majorana mass matrix

$$M_\nu = U_\nu \tilde{M}_\nu U_\nu^t$$

NH $\quad (m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \sim m_{\nu_3} \cdot (\delta^\alpha, \delta, 1)$

IH $\quad |m_{\nu_2}| \stackrel{\sim}{=} |m_{\nu_1}|(1 + a\delta^2) \qquad a > 0 \sim \mathcal{O}(1)$

$\quad m_{\nu_3} \sim \delta^\alpha$

- Charged lepton contribution

$$U_{MNS} = U_l^\dagger U_\nu$$

If U_l contains 0 large angle $\qquad U_l = U(13.0°, 0.2°, 2.4°, 0)$

If U_l contains 1 large angle $\qquad U_l = U(13.0°, 0.2°, 45.0°, 0)$

Neutrino mass matrix reconstruction

- 2 x 2 case

$$\tilde{M}_\nu = \begin{pmatrix} m_{\nu_1} & \\ & m_{\nu_2} \end{pmatrix} \qquad U_\nu = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \qquad s \equiv \sin\theta,\ c \equiv \cos\theta$$

$$M_\nu = \begin{pmatrix} c^2 m_{\nu_1} + s^2 m_{\nu_2} & sc(m_{\nu_2} - m_{\nu_1}) \\ sc(m_{\nu_2} - m_{\nu_1}) & s^2 m_{\nu_1} + c^2 m_{\nu_2} \end{pmatrix}$$

→ *2 cases with a large mixing angle*

NH $\quad m_{\nu_1} \ll m_{\nu_2} \quad \rightarrow \quad M_{\nu,11} \sim M_{\nu,22} \sim M_{\nu,12}$

IH $\quad \begin{cases} m_{\nu_1} \simeq -m_{\nu_2} \\ \theta \simeq \pi/4 \end{cases} \quad \rightarrow \quad M_{\nu,11}\,,\ M_{\nu,22} \ll M_{\nu,12}$

Neutrino mass matrix reconstruction

- 3 × 3 case

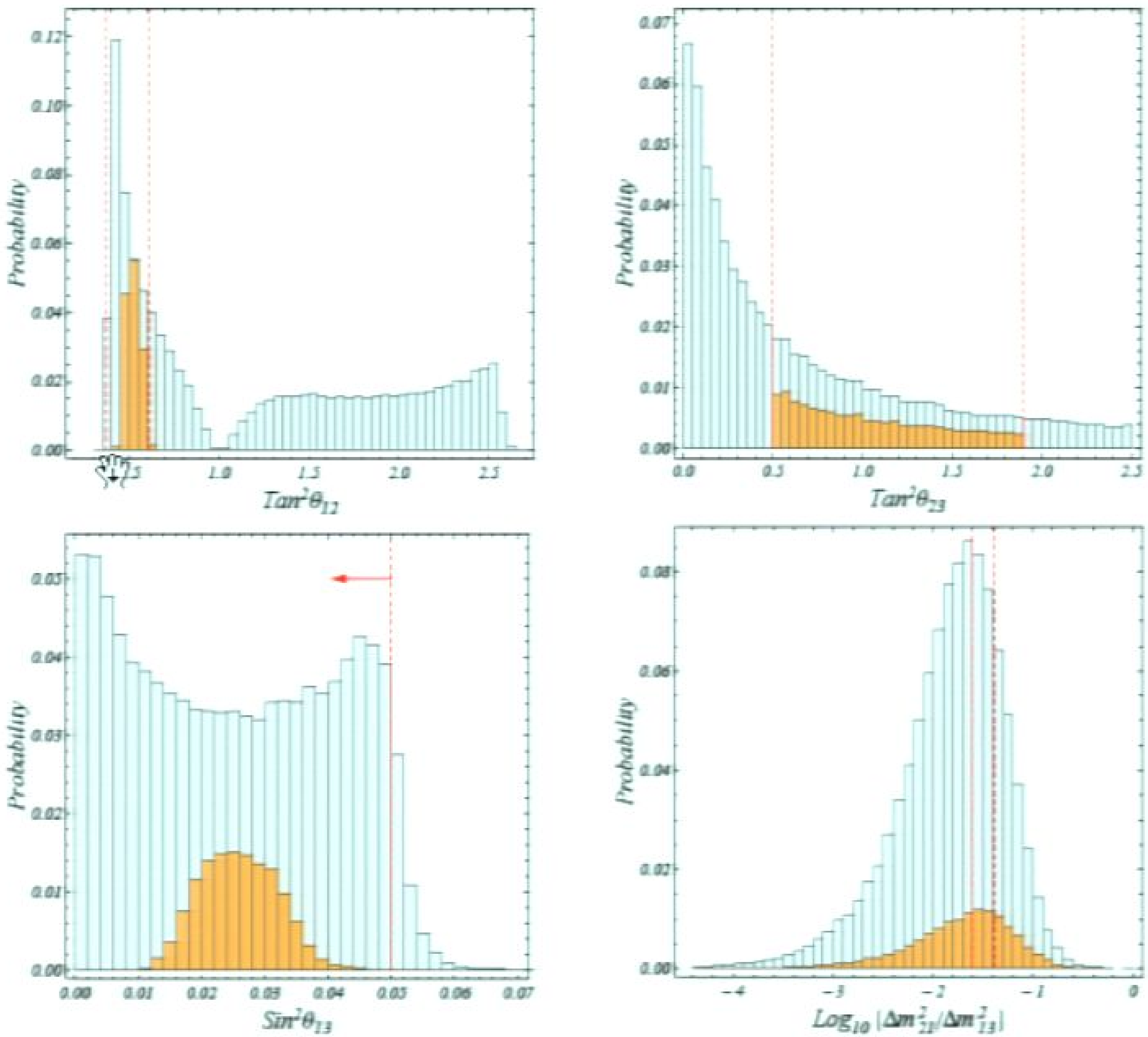
NH
$$\begin{pmatrix} \delta & \delta & \delta \\ \delta & 1 & 1 \\ \delta & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} \delta^2 & \delta & \delta \\ \delta & \delta^2 & \delta \\ \delta & \delta & 1 \end{pmatrix} \quad \begin{pmatrix} \delta^2 & \delta & \delta \\ \delta & 1 & \delta \\ \delta & \delta & \delta^2 \end{pmatrix}$$

IH
$$\begin{pmatrix} \delta^2 & 1 & 1 \\ 1 & \delta^2 & \delta^2 \\ 1 & \delta^2 & \delta^2 \end{pmatrix} \quad \begin{pmatrix} \delta^2 & 1 & \delta \\ 1 & \delta^2 & \delta \\ \delta & \delta & \delta \end{pmatrix} \quad \begin{pmatrix} \delta^2 & \delta & 1 \\ \delta & \delta & \delta \\ 1 & \delta & \delta^2 \end{pmatrix}$$

→ *Put order one coefficients for a numerical analysis*

$$\begin{pmatrix} \delta^2 & 1 & 1 \\ 1 & \delta^2 & \delta^2 \\ 1 & \delta^2 & \delta^2 \end{pmatrix}$$

$l \simeq V_{CKM}$



Conclusions from the numerical & statistical analysis

- Key ingredient to have large angles :
  dominant off diagonal elements

- Inverted hierarchy preferred

→ *Two important questions :*

- Can we get IH patterns with the see-saw ?

- Is the IH compatible with hierarchy ?

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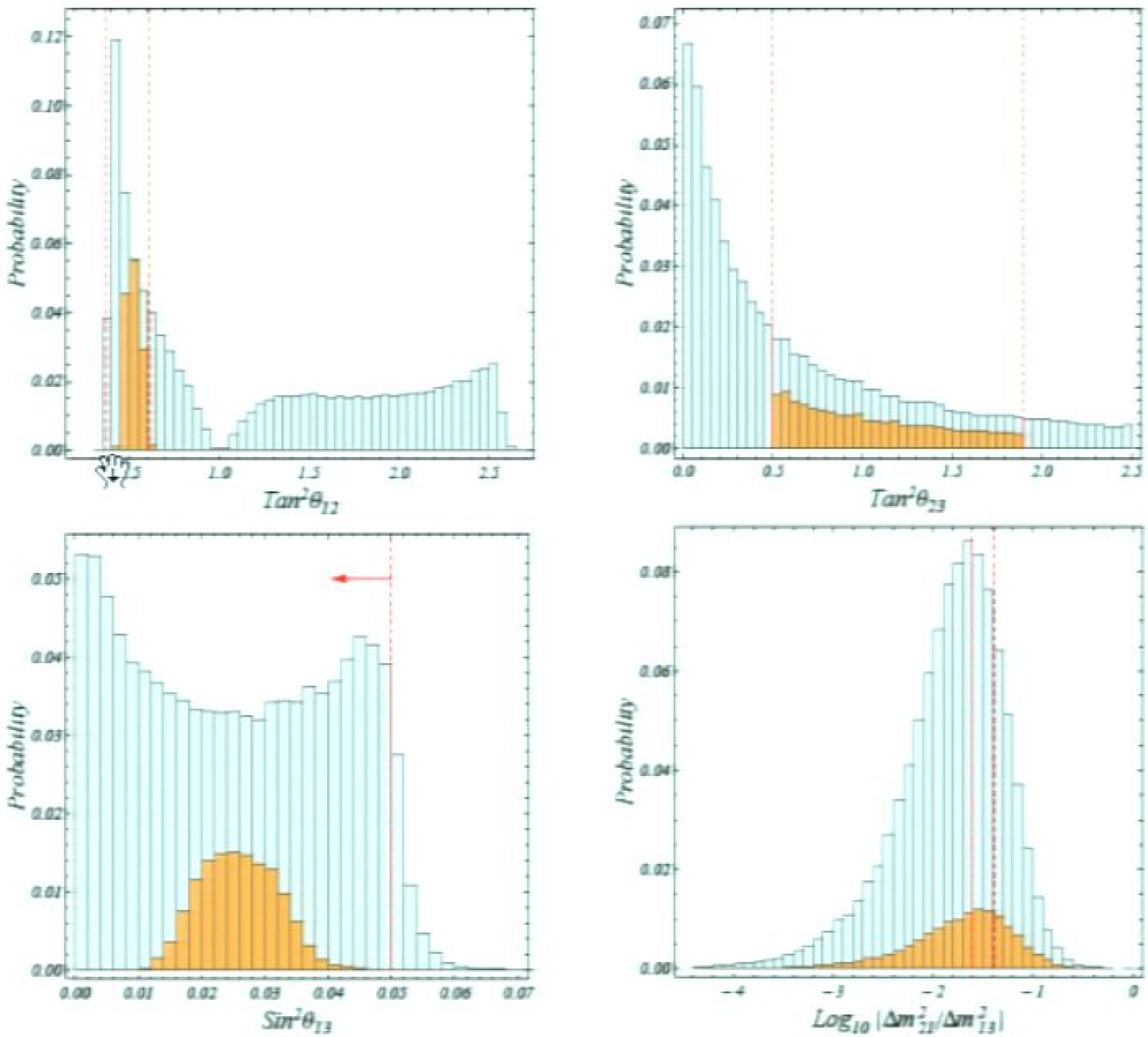

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- 2 x 2 case

$$M_\nu = M_D \cdot M_R^{-1} \cdot M_D^t$$

$$M_D = U_D \cdot \tilde{M}_D \cdot V_D^t$$

$$\tilde{M}_D \sim m \begin{pmatrix} \delta^\alpha & \\ & 1 \end{pmatrix} \qquad V_D \sim \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \qquad M_R \sim \begin{pmatrix} M_1 & \\ & M_2 \end{pmatrix}$$

$$\tilde{M}_D \cdot V_D^t \cdot M_R^{-1} \cdot V_D^* \cdot \tilde{M}_D = m^2 \begin{pmatrix} \delta^{2\alpha}\left(\frac{c^2}{M_1} + \frac{s^2}{M_2}\right) & \delta^\alpha sc\left(\frac{1}{M_1} - \frac{1}{M_2}\right) \\ \delta^\alpha sc\left(\frac{1}{M_1} - \frac{1}{M_2}\right) & \frac{s^2}{M_1} + \frac{c^2}{M_2} \end{pmatrix}$$

$$\boxed{\tan^2\theta = -\frac{M_1}{M_2}\left(1 + o(\delta^\alpha)\right)}$$

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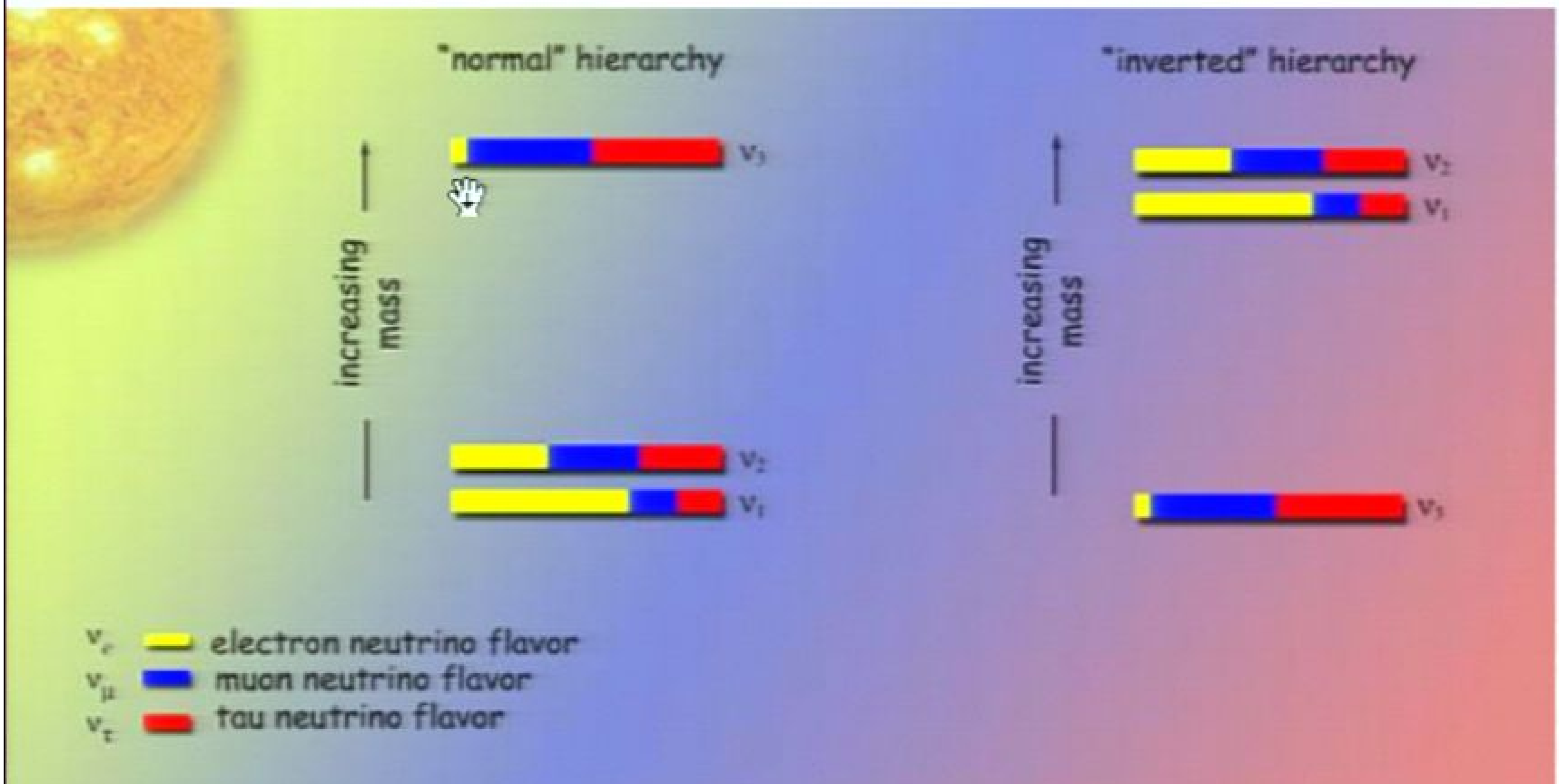

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- Key ingredient to have large angles :
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$$M_D = U_D \cdot \tilde{M}_D \cdot V_D^t$$

$$\tilde{M}_D \sim m \begin{pmatrix} \delta^\alpha & \\ & 1 \end{pmatrix} \qquad V_D \sim \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \qquad M_R \sim \begin{pmatrix} M_1 & \\ & M_2 \end{pmatrix}$$

$$\tilde{M}_D \cdot V_D^t \cdot M_R^{-1} \cdot V_D^* \cdot \tilde{M}_D = m^2 \begin{pmatrix} \delta^{2\alpha}\left(\frac{c^2}{M_1} + \frac{s^2}{M_2}\right) & \delta^\alpha sc\left(\frac{1}{M_1} - \frac{1}{M_2}\right) \\ \delta^\alpha sc\left(\frac{1}{M_1} - \frac{1}{M_2}\right) & \frac{s^2}{M_1} + \frac{c^2}{M_2} \end{pmatrix}$$

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SEE-SAW & HIERARCHY

- 3 x 3 case

$$M_D = U_D \cdot \tilde{M}_D \cdot V_D^t$$

$$\sim m \begin{pmatrix} \delta^{\alpha+\beta} & & \\ & \delta^{\alpha} & \\ & & 1 \end{pmatrix} \quad U_D \sim V_D \sim \begin{pmatrix} 1 & \delta^{\alpha_1} & \delta^{\alpha_3} \\ \delta^{\alpha_1} & 1 & \delta^{\alpha_2} \\ \delta^{\alpha_3} & \delta^{\alpha_2} & 1 \end{pmatrix} \qquad M_R^{-1} = M^{-1} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \delta^{\gamma} \end{pmatrix}$$

$$= 1\,,\ \beta = 1\,,\ \gamma = 2 \quad ; \quad \alpha_1 = 2\,,\ \alpha_2 = 1\,,\ \alpha_3 = 2 \qquad \begin{pmatrix} \delta^2 & \delta & \delta \\ \delta & \delta^2 & \delta \\ \delta & \delta & 1 \end{pmatrix}$$

$$= 1\,,\ \beta = 1\,,\ \gamma = 4 \quad ; \quad \alpha_1 = 3\,,\ \alpha_2 = 2\,,\ \alpha_3 = 3 \qquad \begin{pmatrix} \delta^2 & 1 & \delta \\ 1 & \delta^2 & \delta \\ \delta & \delta & \delta \end{pmatrix}$$

$$= 1\,,\ \beta = 1\,,\ \gamma = 5 \quad ; \quad \alpha_1 = 3\,,\ \alpha_2 = 1\,,\ \alpha_3 = 4 \qquad \begin{pmatrix} \delta^2 & 1 & 1 \\ 1 & \delta^2 & \delta^2 \\ 1 & \delta^2 & \delta^2 \end{pmatrix}$$

INVERTED HIERARCHY ?

→ *Two important questions :*

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INVERTED HIERARCHY ?

→ *Two important questions :*

- Can we get IH patterns with the see-saw ? YES
- Is the IH compatible with hierarchy ? YES

<u>But</u> *it requires a Majorana mass matrix with a Dirac-like pattern !!*

→ *What is the origin of such a strange pattern ??*

MAJORANA MASS IN 6D

- A Dirac spinor in 6D has 8 components, and can be written as 4 4D Weyl spinors, which have different 4D or 6D chiralities

$$\Psi = \begin{pmatrix} \psi_{+R} \\ \psi_{+L} \\ \psi_{-L} \\ \psi_{-R} \end{pmatrix} \qquad \begin{matrix} \pm \;\rightarrow\; \dfrac{1 \pm \Gamma_7}{2} \\ L.R \;\rightarrow\; \dfrac{1 \pm \tilde{\Gamma}_5}{2} \end{matrix}$$

$$\Gamma_7 = -\Gamma_0 \ldots \Gamma_5 \qquad \tilde{\Gamma}_5 = i\Gamma_0 \ldots \Gamma_3$$

Majorana mass in 6D

- A Majorana mass term always connects different 6D chiralities

$$\bar{\Psi}^c\Psi + \mathrm{h.c.} = 2\psi_{-R}\psi_{+R} - 2\psi_{-L}\psi_{+L} + \mathrm{h.c.}$$

$$\Psi^c = C\Gamma^0\Psi^* \qquad C = \Gamma_0\Gamma_2\Gamma_4$$

$$\psi_R\psi_R \equiv \psi_R^t(i\sigma_2)\psi_R = \epsilon^{ij}\psi_{Ri}\psi_{Rj}$$

$$\psi_L\psi_L \equiv \psi_L^t(-i\sigma_2)\psi_L = -\epsilon^{ij}\psi_{Li}\psi_{Lj}$$

Majorana mass matrix in 6D

- If we also have a Dirac mass for Ψ (could arise from dimensional compactification)

$$\frac{M}{2}(\bar{\Psi}^c\Psi + \text{h.c.}) + \Lambda\bar{\Psi}\Psi =$$

$$M(\chi_-\chi_+ - \xi_-\xi_+) + \Lambda(\chi_-\xi_+ + \chi_+\xi_-) + \text{h.c.}$$

$$\begin{array}{c} \\ \chi_- \\ \xi_+ \\ \chi_+ \\ \xi_- \end{array} \begin{array}{cccc} \chi_- & \xi_+ & \chi_+ & \xi_- \\ \end{array}$$

$$\begin{array}{c} \chi_- \\ \xi_+ \\ \chi_+ \\ \xi_- \end{array} \begin{pmatrix} & \Lambda & M & \\ \Lambda & & & -M \\ M & & & \Lambda \\ & -M & \Lambda & \end{pmatrix} \qquad \begin{array}{l} \xi_\pm \equiv \psi_{\pm L} \\ \chi_\pm \equiv (i\sigma_2)\psi^*_{\pm R} \end{array}$$

➔ *The Majorana matrix is completely off-diagonal*

SEE-SAW IN 6D

- With the previous Majorana mass matrix, and SM neutrinos connecting only to $\xi_\pm$, the see-saw masses are always proportional to M
- For example, with one flavour, if $M_D = (a\ 0\ b\ 0)$

$$M_\nu = \frac{abM}{\Lambda^2 + M^2}$$

➔ ***In 6D, we can have a small neutrino mass from a small Majorana mass***

In 4D, this is similar to a double see-saw mechanism

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$$\begin{array}{c|cccc} & \chi_- & \xi_+ & \chi_+ & \xi_- \\ \hline \chi_- & & \Lambda & M & \\ \xi_+ & \Lambda & & & -M \\ \chi_+ & M & & & \Lambda \\ \xi_- & & -M & \Lambda & \end{array}$$

$$\xi_\pm \equiv \psi_{\pm L}$$

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A MODEL OF FAMILY REPLICATION WITH A VORTEX IN 6D

Frère, Libanov, FSL, Nugaev, Troitsky

- The basic idea is to have a topological defect in 6D (vortex) made of a $U(1)_g$ gauge field A and a scalar field Φ
- The interaction of a single fermion family with the vortex leads to several chiral zero-modes, as a consequence of the index theorem
- Family number in 4D corresponds to winding number in extradim

ABIKOSOV-NIELSEN-OLESEN VORTEX

- Abelian "Higgs" Lagrangian (here on $M^4 \times S^2$)

$$= \sqrt{-\det(g_{AB})}\left(-\frac{1}{4}F_{AB}F^{AB} + (D^A\Phi)^\dagger D_A\Phi - \frac{\lambda}{2}(|\Phi|^2 - v^2)^2\right)$$

$$ds^2 = g_{AB}dx^A dx^B = g_{\mu\nu}dx^\mu dx^\nu - R^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

- Look for a static solution

$$A_\varphi = \frac{1}{e}A(\theta)\,, \quad A_\theta = 0\,, \quad \Phi = F(\theta)e^{i\varphi}.$$

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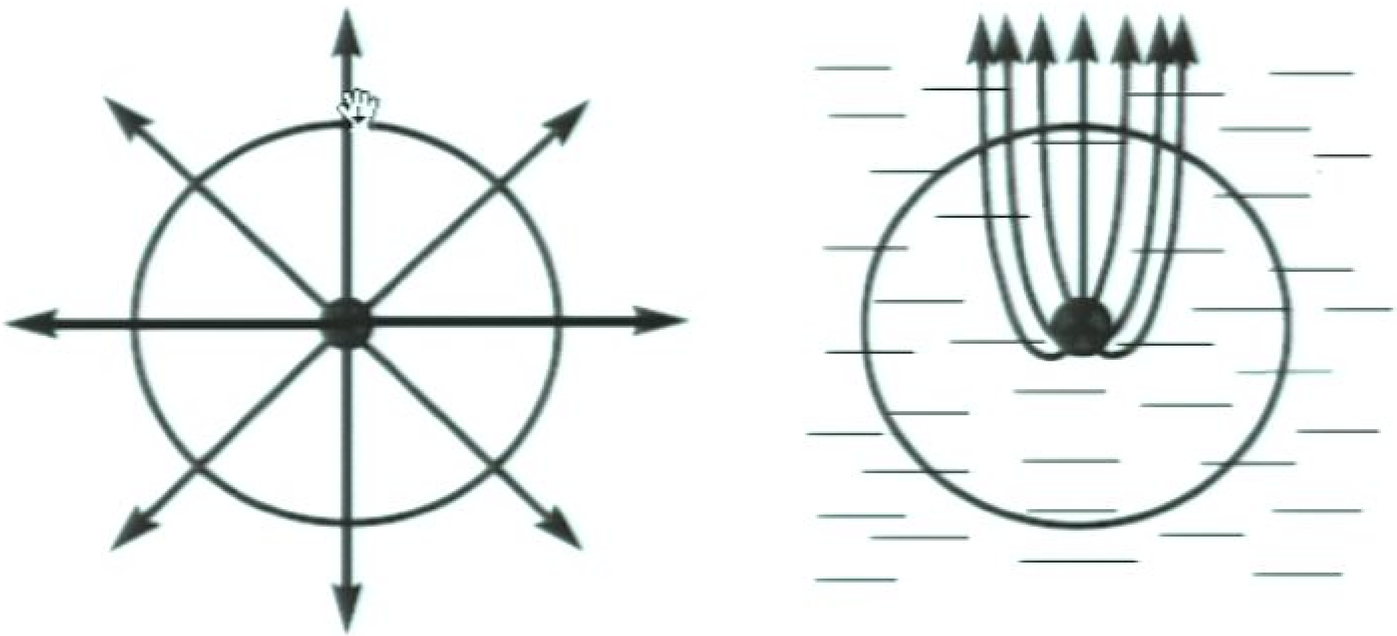

FIELD CONTENT OF THE MODEL

fields		charges		representations	
		$U(1)_g$	$U(1)_Y$	$SU(2)_W$	$SU(3)_C$
scalar	Φ	$+1$	0	$\mathbf{1}$	$\mathbf{1}$
scalar	X	$+1$	0	$\mathbf{1}$	$\mathbf{1}$
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CHIRAL ZERO-MODES

Frère et al. hep-ph/0304117

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→ *Index theorem : k chiral zero-modes*

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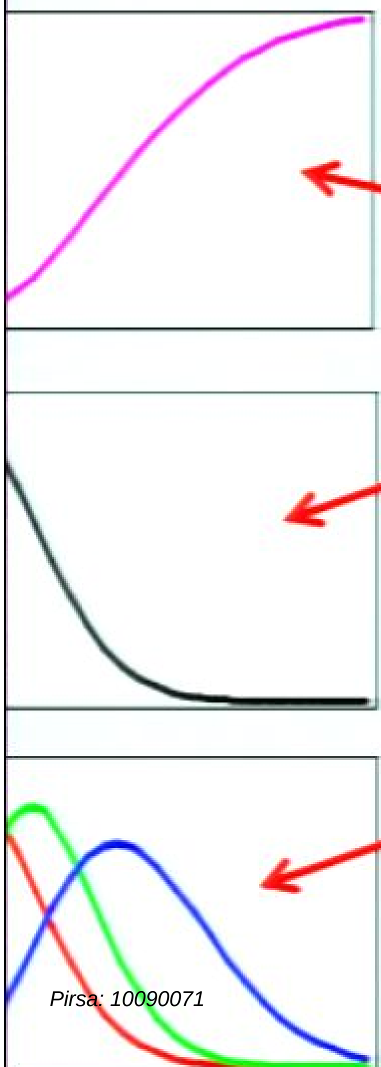

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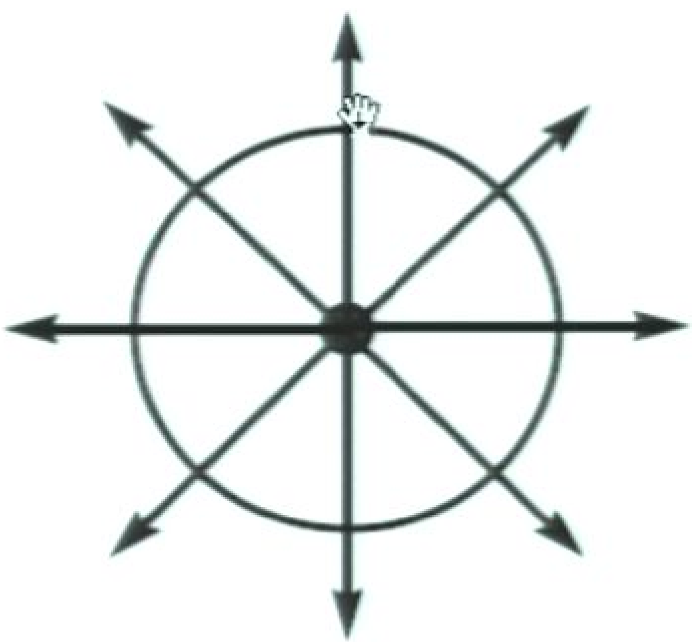

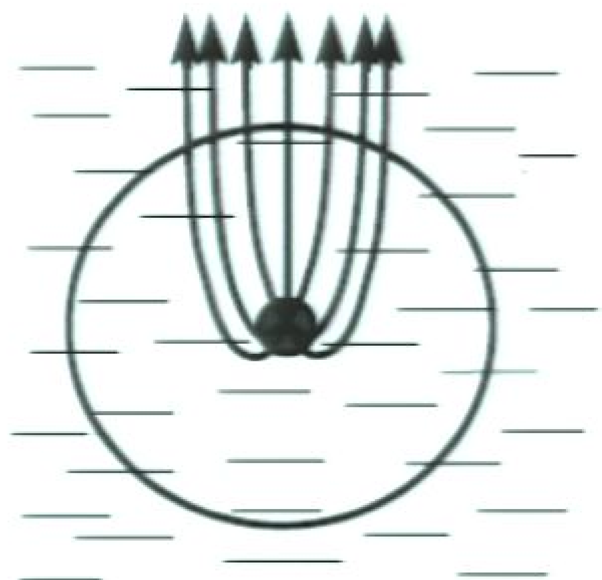

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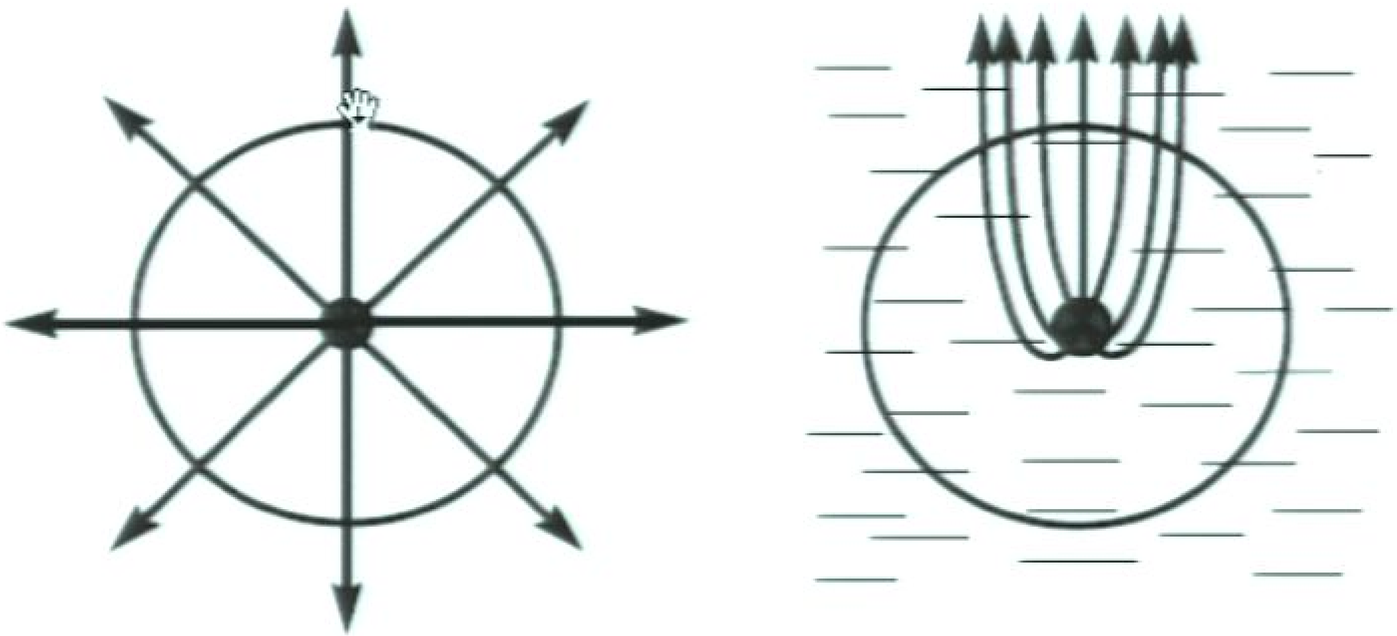
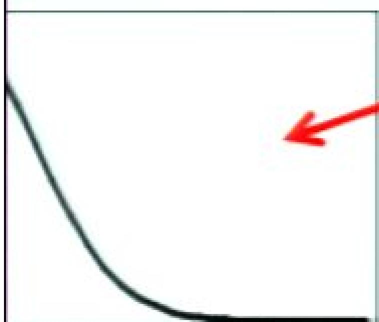
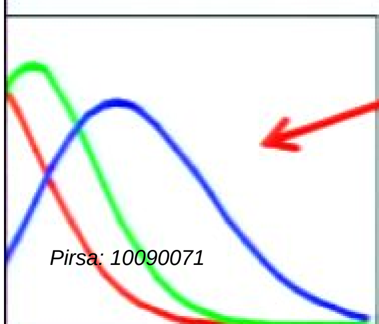


Field content of the model





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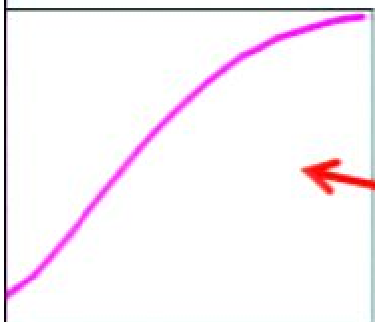

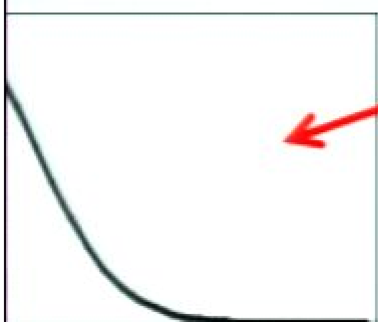

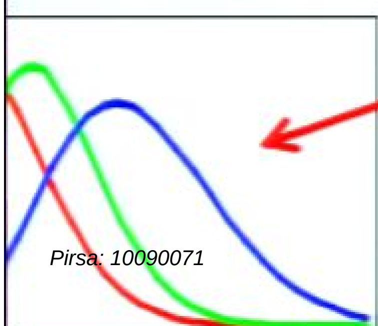

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STRUCTURE OF THE ZERO-MODES

$$\Psi = \begin{pmatrix} \psi_{+R} \\ \psi_{+L} \\ \psi_{-L} \\ \psi_{-R} \end{pmatrix}$$

- Left-handed zero-modes

$$L_n(\theta, \phi, x^\mu) = \begin{pmatrix} 0 \\ e^{-i\phi(n-7/2)} f_2(n, \theta)\, l_n(x^\mu) \\ e^{-i\phi(n-1/2)} f_3(n, \theta)\, l_n(x^\mu) \\ 0 \end{pmatrix}$$

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$$f_2 \sim a_0 \theta^{k-n} \qquad f_3 \sim b_0 \theta^{n-1} \qquad 0 < n \le k$$

Hierarchical Dirac masses and small interfamily mixing angles

- Simplest case for charged lepton mass matrix

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$$M_l \sim \begin{pmatrix} \delta^4 & \delta^3 & \\ & \delta^2 & \delta \\ & & 1 \end{pmatrix}$$

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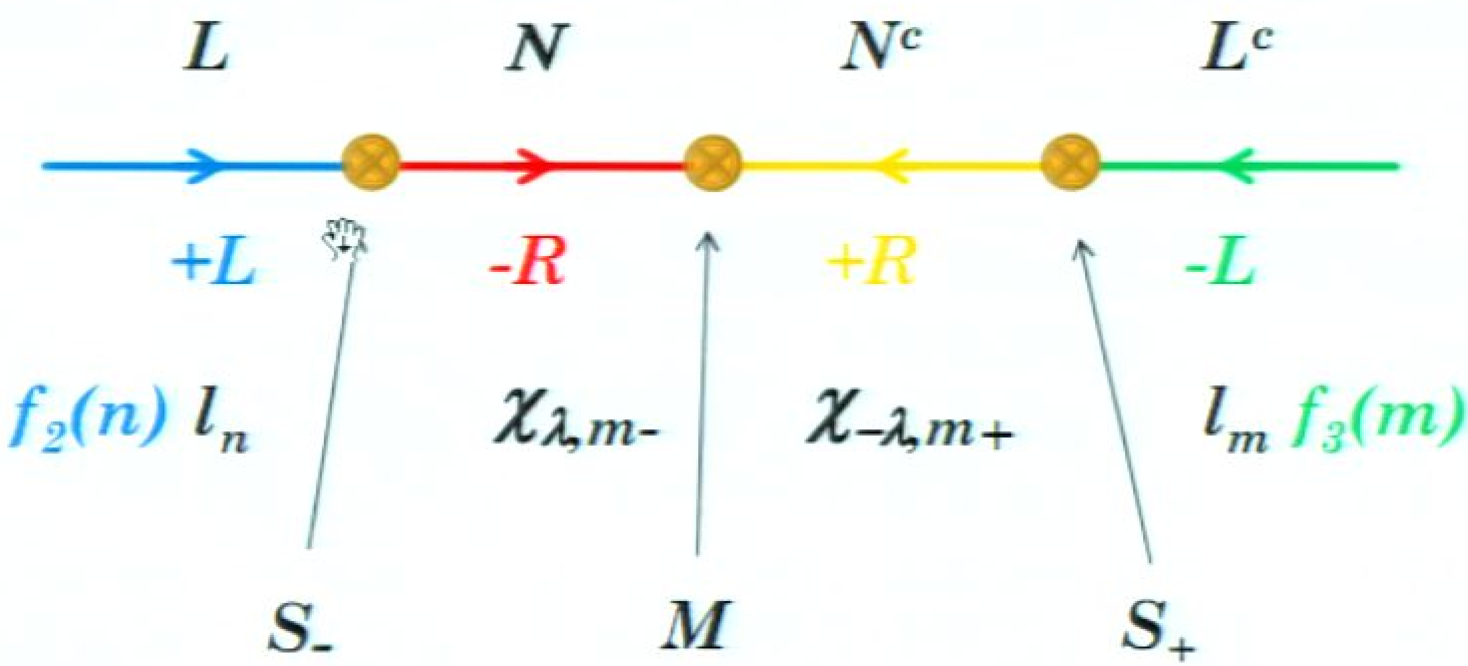

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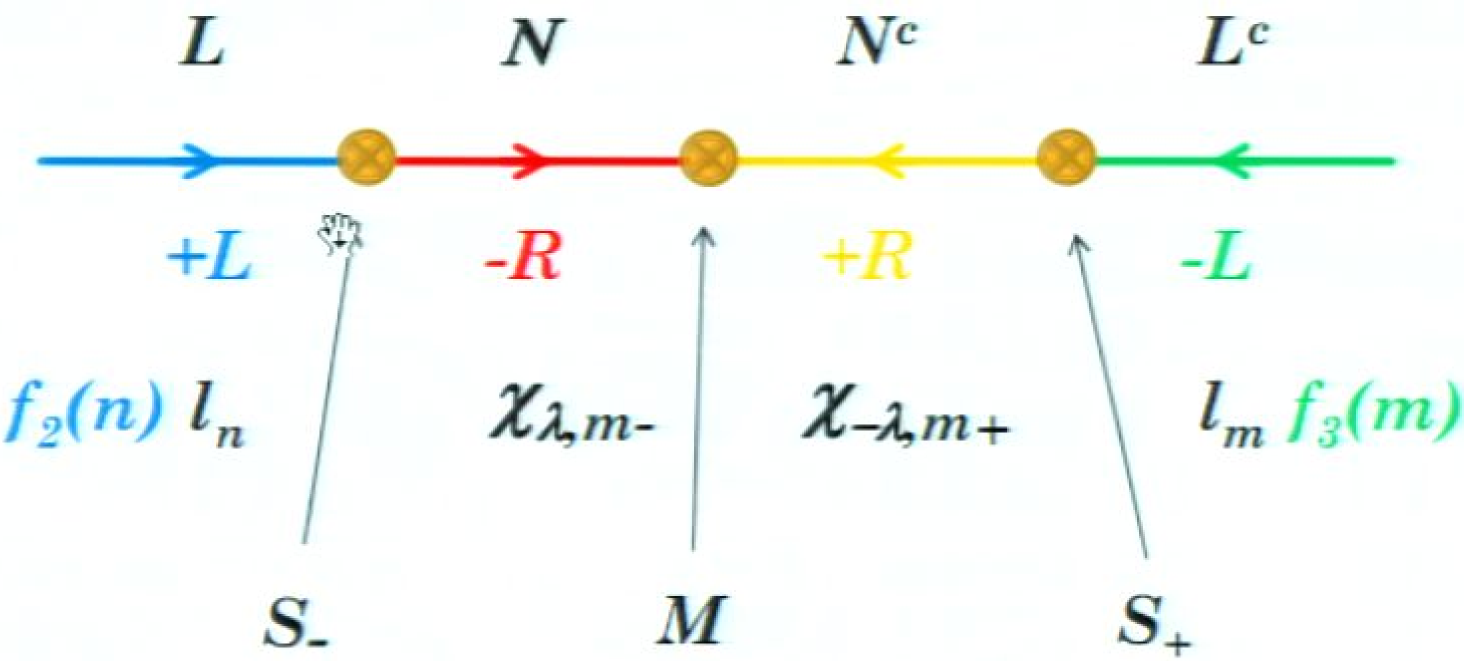

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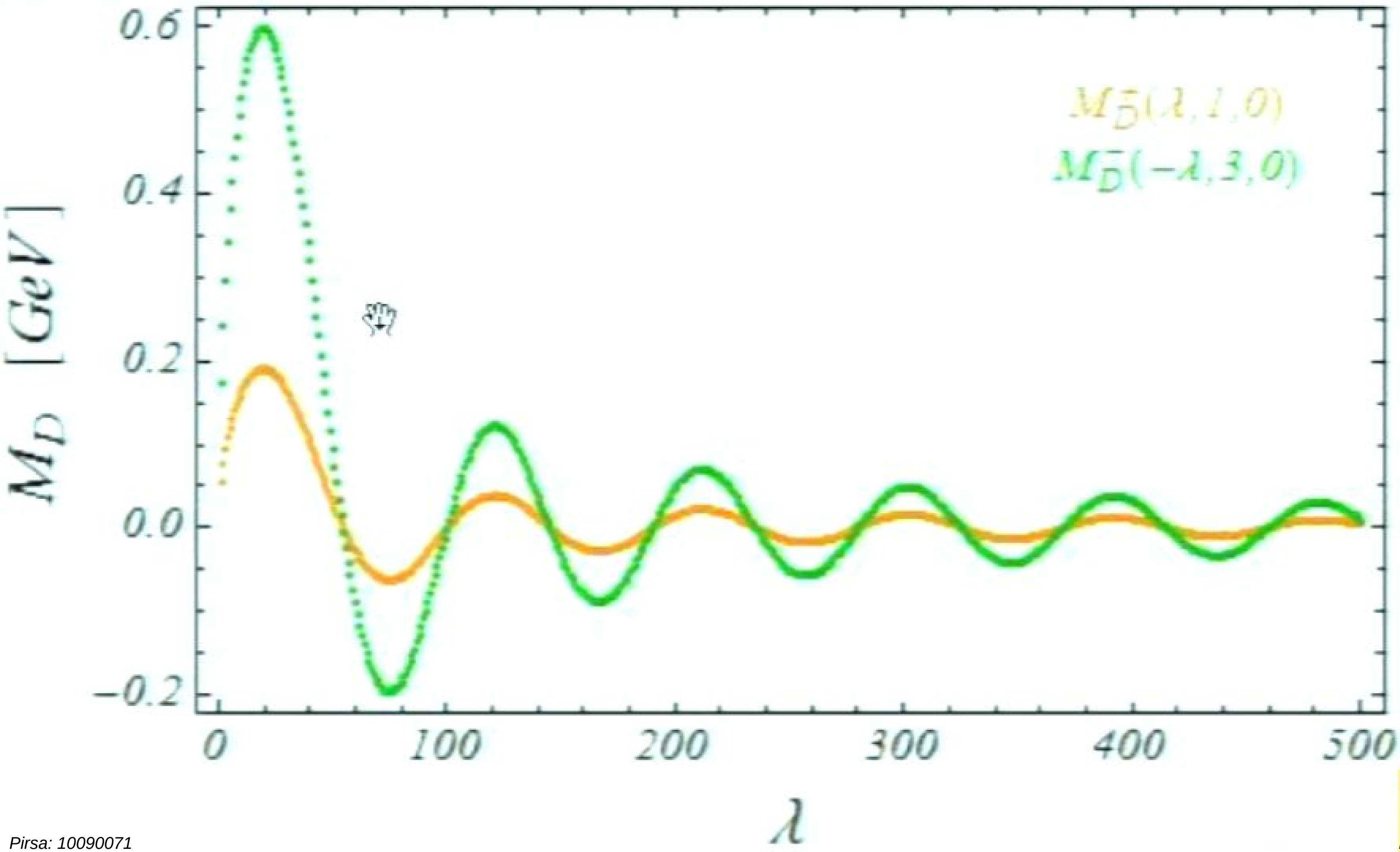

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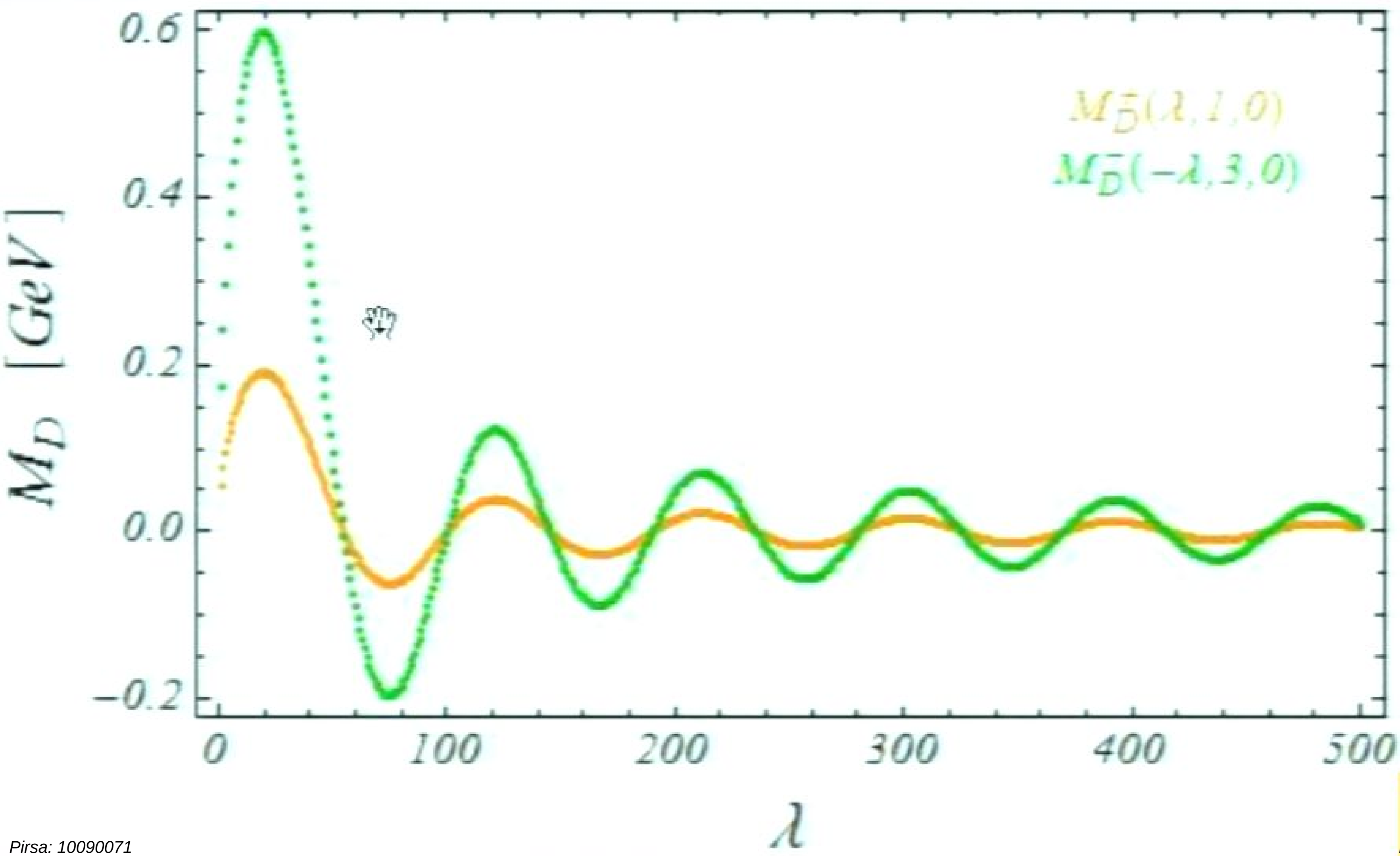

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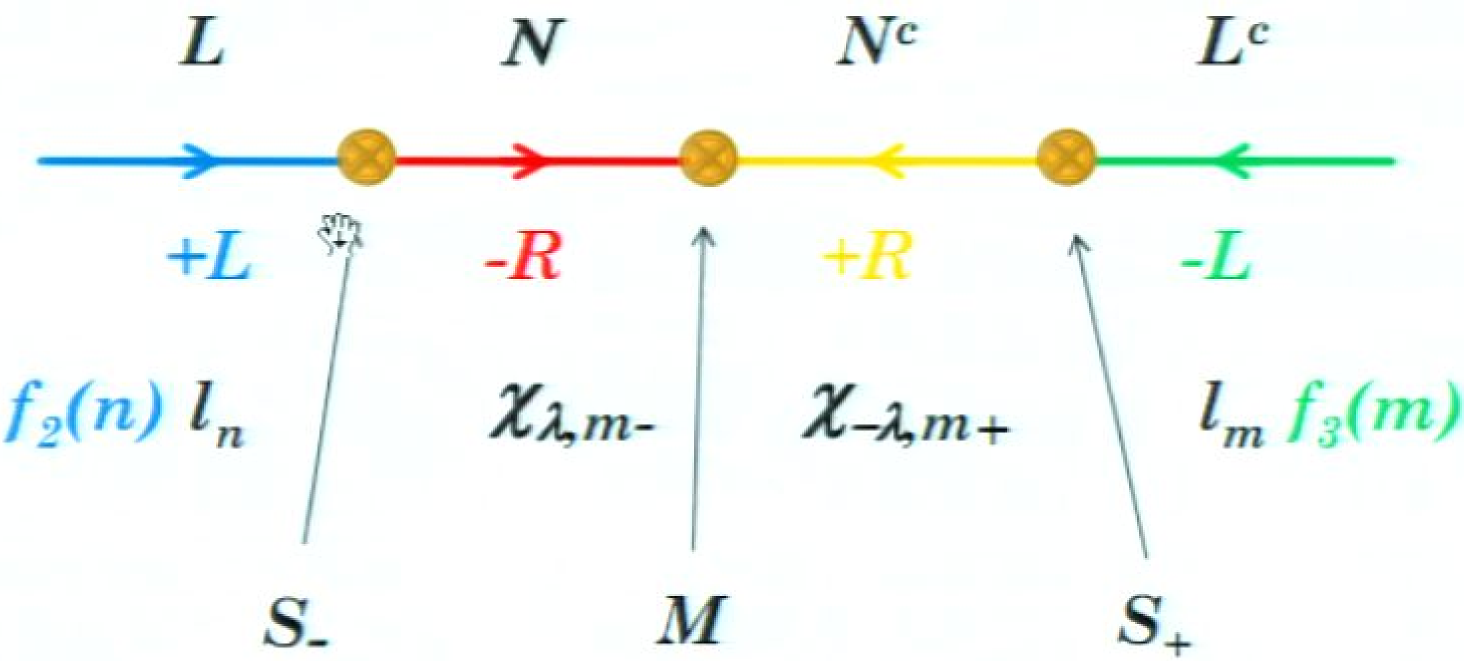
$$L_k \sim (0,\; f_2(k)\,l_k,\; f_3(k)\,l_k,\; 0)^t$$



$m_+ = -m_-$ → $n + m + s_+ + s_- = 4$

NEUTRINO SEE-SAW MASSES

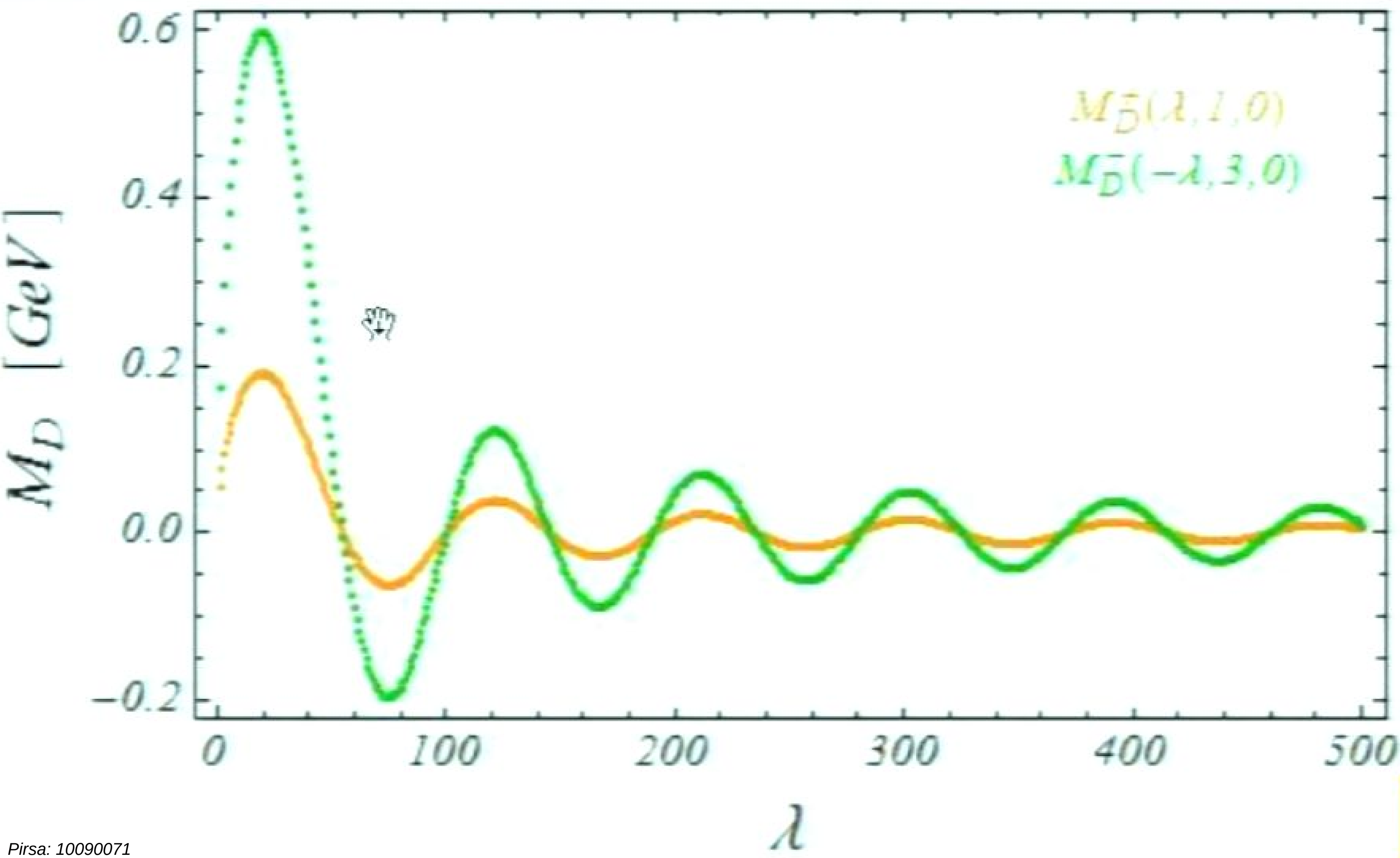
$$\frac{-M_D^+(\lambda, n, s_+) M_D^-(-\lambda, m, s_-) M (-1)^{l-(1/2-n-s_+)} \epsilon(\lambda)}{M^2 + \lambda^2/R^2} + n \leftrightarrow m$$

$$f_2(n) \sim \theta^{3-n}$$

$$f_3(m) \sim \theta^{m-1}$$

$$\rightarrow \qquad M_\nu \sim \begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \end{pmatrix}$$

NEUTRINO DIRAC MASSES



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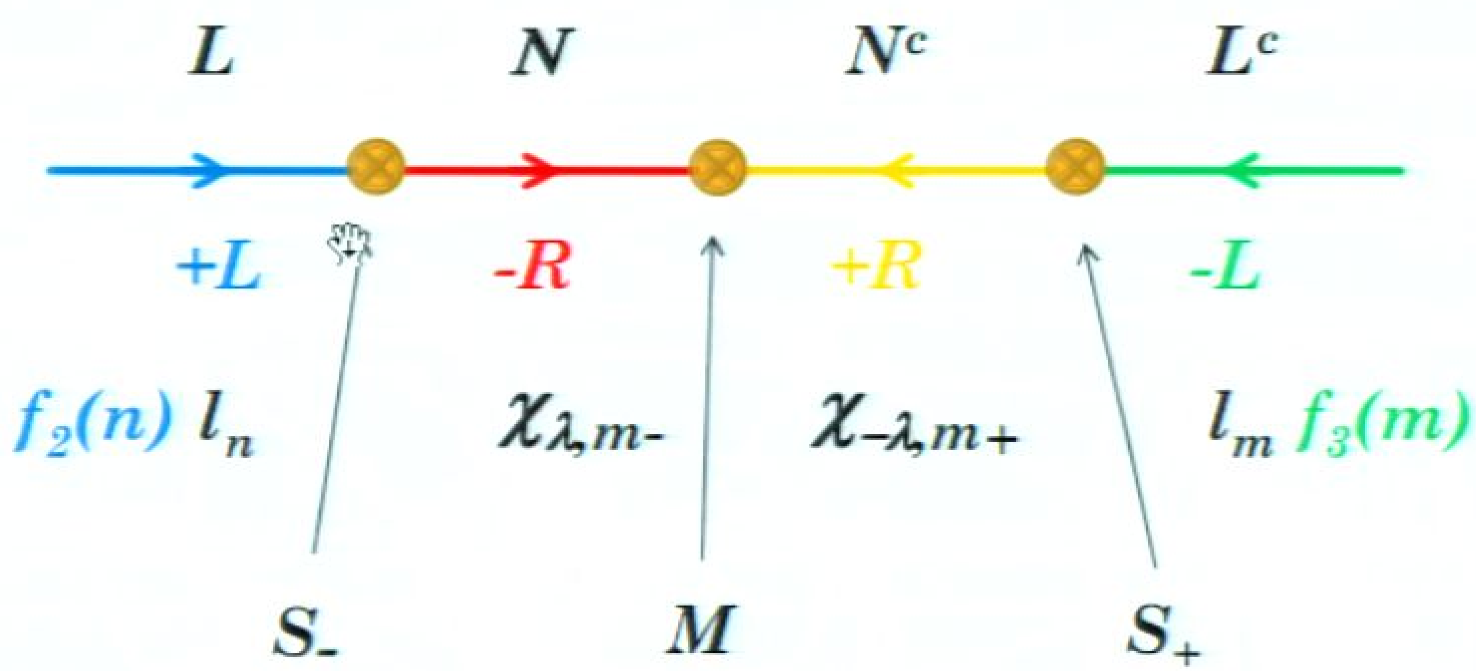

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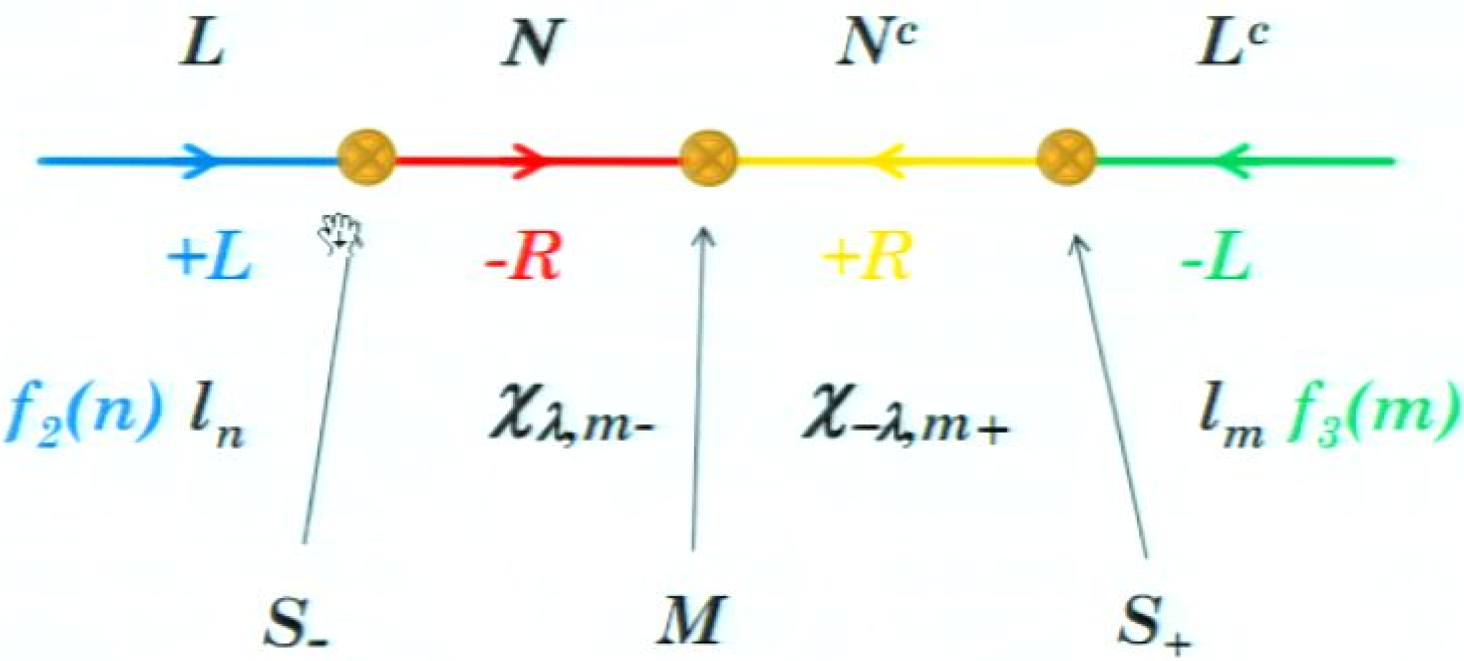

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$\longrightarrow$ ***Possibility to have a bimaximal mixing***

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$$S_- = X^2, \boxed{X\Phi}, \Phi^2, \ \ldots$$

$\tilde{Y}_\nu^+ = y_\nu \{1, 1.7\}$ $\qquad$ $y_\nu = 2.8 \cdot 10^{-2}$

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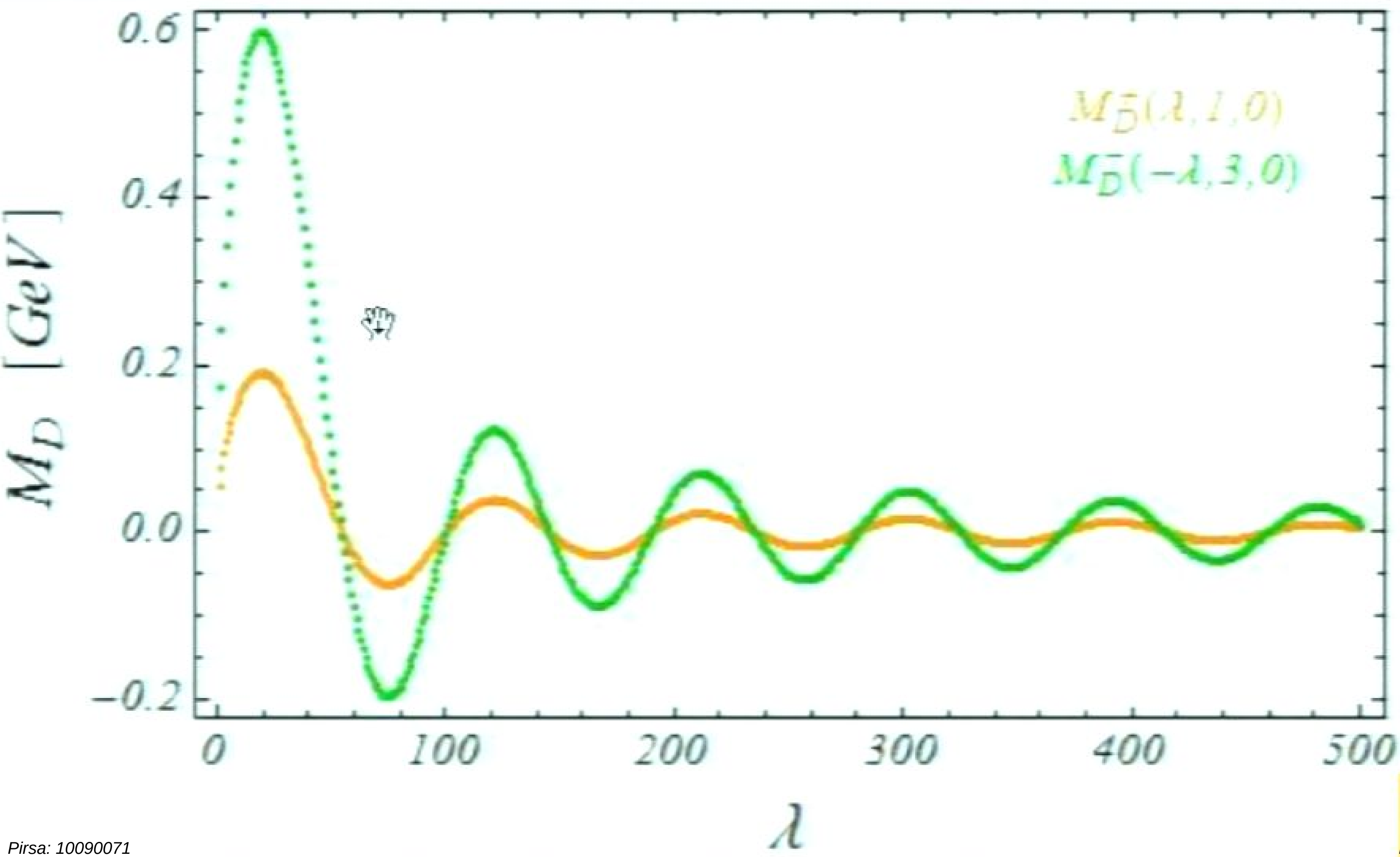

Fermion singlet on $M^4 \times S^2$

- Decomposition in spherical modes

$$\int d\theta\, d\phi\, \mathcal{L}_N = \sum_{\lambda,m} \chi_{\lambda,m} i\partial\bar{\chi}_{\lambda,m} + \bar{\xi}_{\lambda,m} i\bar{\partial}\xi_{\lambda,m}$$

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Hierarchical Dirac masses and small interfamily mixing angles

- Simplest case for charged lepton mass matrix

$$Y_X H X \bar{L} \frac{1-\Gamma_7}{2} E + Y_\Phi H \Phi \bar{L} \frac{1-\Gamma_7}{2} E$$

$$M_l \sim \begin{pmatrix} \delta^4 & \delta^3 & \\ & \delta^2 & \delta \\ & & 1 \end{pmatrix}$$

Structure of the zero-modes

$$\Psi = \begin{pmatrix} \psi_{+R} \\ \psi_{+L} \\ \psi_{-L} \\ \psi_{-R} \end{pmatrix}$$

- Left-handed zero-modes

$$L_n(\theta,\phi,x^\mu) = \begin{pmatrix} 0 \\ e^{-i\phi(n-7/2)} f_2(n,\theta)\, l_n(x^\mu) \\ e^{-i\phi(n-1/2)} f_3(n,\theta)\, l_n(x^\mu) \\ 0 \end{pmatrix}$$

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Chiral zero-modes

Frère et al. hep-ph/0304117

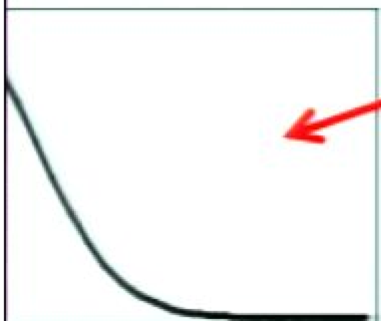
- Coupling of fermions with axial charges to the vortex background

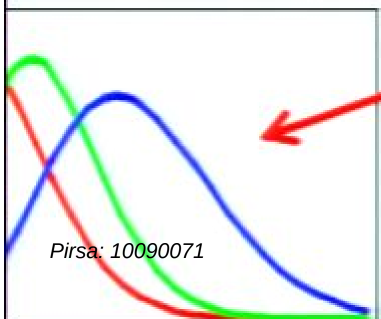
$$\Psi' = e^{i\frac{(1+\Gamma_7)}{2}k\alpha}\Psi$$

$$\mathcal{L}_\Psi = \sqrt{-\det(g_{AB})}\left\{ i\bar{\Psi}h_a^A\Gamma^a\left(\nabla_A - iek\frac{1+\Gamma_7}{2}A_A\right)\Psi - g\Phi^k\bar{\Psi}\frac{1-\Gamma_7}{2}\Psi - g\Phi^{*k}\bar{\Psi}\frac{1+\Gamma_7}{2}\Psi\right\}$$

→ ***Index theorem : k chiral zero-modes***

FIELD CONTENT OF THE MODEL

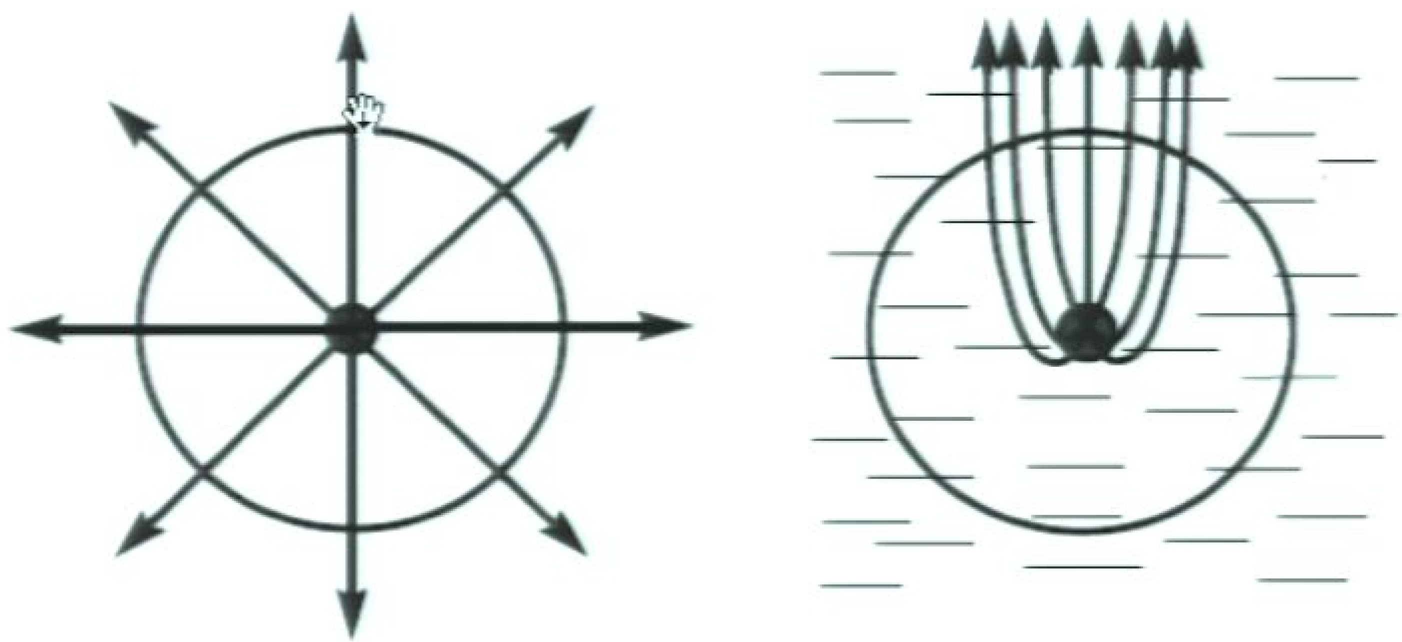




fields		charges		representations	
		$U(1)_g$	$U(1)_Y$	$SU(2)_W$	$SU(3)_C$
scalar	Φ	$+1$	0	$\mathbf{1}$	$\mathbf{1}$
scalar	X	$+1$	0	$\mathbf{1}$	$\mathbf{1}$
scalar	H	-1	$+1/2$	$\mathbf{2}$	$\mathbf{1}$
fermion	L_+, L_-	$(3, 0)$	$-1/2$	$\mathbf{2}$	$\mathbf{1}$
fermion	E_+, E_-	$(0, 3)$	-1	$\mathbf{1}$	$\mathbf{1}$
fermion	N	0	0	$\mathbf{1}$	$\mathbf{1}$

Abikosov-Nielsen-Olesen vortex

- The vortex on the sphere is in fact like a magnetic monopole in 3D



ABIKOSOV-NIELSEN-OLESEN VORTEX

- Abelian "Higgs" Lagrangian (here on $M^4 \times S^2$)

$$= \sqrt{-\det(g_{AB})}\left(-\frac{1}{4}F_{AB}F^{AB} + (D^A\Phi)^\dagger D_A\Phi - \frac{\lambda}{2}(|\Phi|^2 - v^2)^2\right)$$

$$ds^2 = g_{AB}dx^A dx^B = g_{\mu\nu}dx^\mu dx^\nu - R^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

- Look for a static solution

$$A_\varphi = \frac{1}{e}A(\theta)\,, \quad A_\theta = 0\,, \quad \Phi = F(\theta)e^{i\varphi}.$$

Here the winding number of Φ is equal to 1

A MODEL OF FAMILY REPLICATION WITH A VORTEX IN 6D

Frère, Libanov, FSL, Nugaev, Troitsky

- The basic idea is to have a topological defect in 6D (vortex) made of a $U(1)_g$ gauge field A and a scalar field Φ
- The interaction of a single fermion family with the vortex leads to several chiral zero-modes, as a consequence of the index theorem
- Family number in 4D corresponds to winding number in extradim

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Neutrino Dirac masses

- We need Dirac masses with both projectors $\frac{1 \pm \Gamma_7}{2}$ for a successful see-saw

$$\frac{\mathcal{L}_D}{\sqrt{-\det g_{AB}}} = \sum_{S_+} Y_\nu^+(S_+) \tilde{H} S_+ \bar{L} \frac{1+\Gamma_7}{2} N$$

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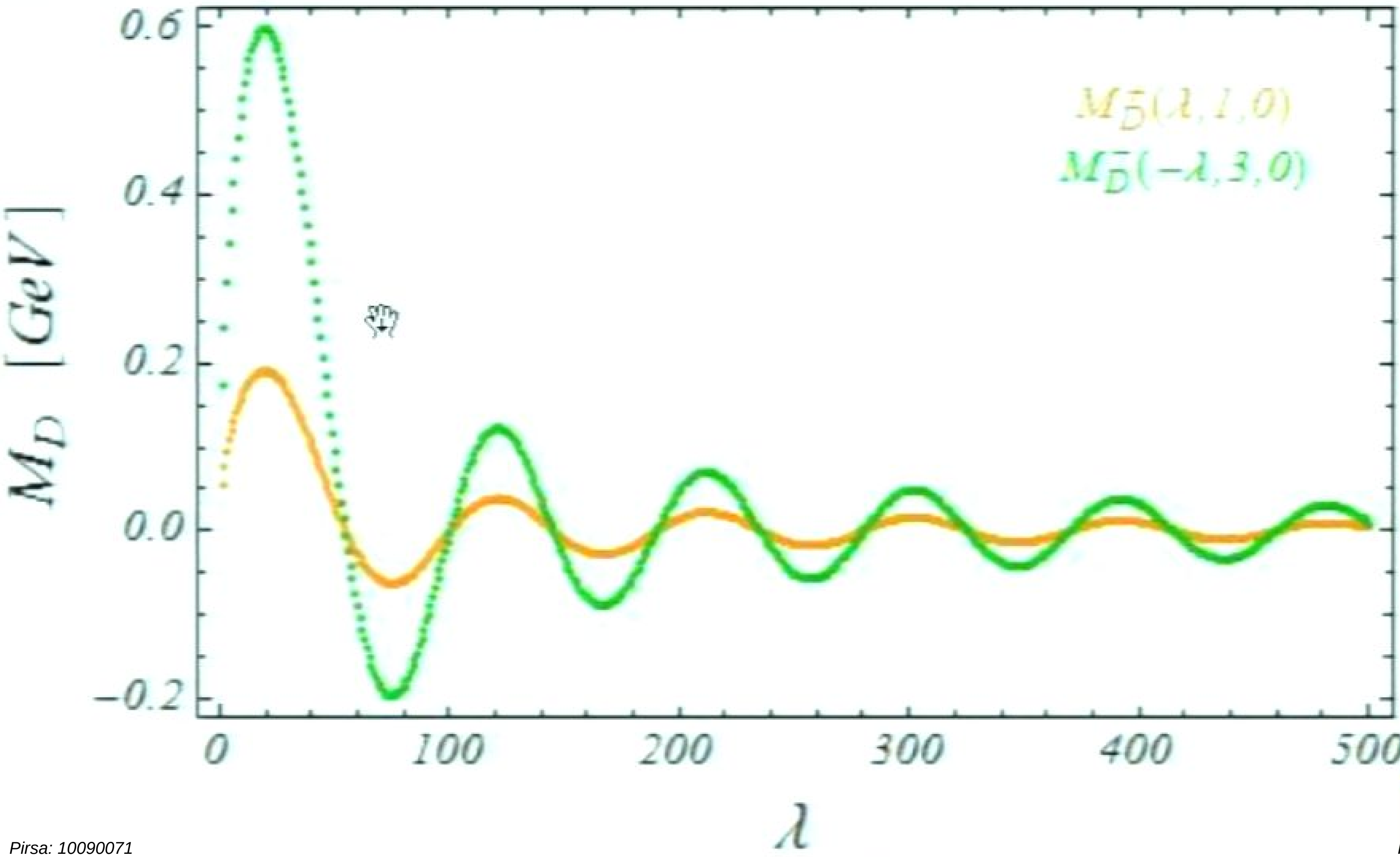

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$$M_\nu = \begin{pmatrix} 0 & 3.62\cdot 10^{-2} & 3.50\cdot 10^{-2} \\ 3.62\cdot 10^{-2} & 1.46\cdot 10^{-3} & 0 \\ 3.50\cdot 10^{-2} & 0 & 0 \end{pmatrix} \quad [\mathrm{eV}]$$

$$\Delta m^2_{21} = 7.63\times 10^{-5}\ \mathrm{eV}^2$$
$$\Delta m^2_{13} = 2.50\times 10^{-3}\ \mathrm{eV}^2$$
$$\rightarrow \quad \Delta m^2_{21}/\Delta m^2_{13} = 3.05\%$$

$$M_l = \begin{pmatrix} 4.21\cdot 10^{-4} & 1.08\cdot 10^{-3} & 0 \\ 0 & 4.19\cdot 10^{-3} & 5.98\cdot 10^{-2} \\ 0 & 0 & 1.71 \end{pmatrix} \quad [\mathrm{GeV}]$$

$$U_l^\dagger M_l V_l = D_l = \mathrm{diag}\{4.07\cdot 10^{-4}, 4.33\cdot 10^{-3}, 1.71\} \quad [\mathrm{GeV}]$$

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$$U_{MNS} = \begin{pmatrix} 0.808 & 0.555 & 0.196 \\ -0.286 & 0.662 & -0.693 \\ -0.514 & 0.504 & 0.694 \end{pmatrix}$$

$\tan^2\theta_{12} = 0.471 \quad \tan^2\theta_{23} = 0.997 \quad \sin^2\theta_{13} = 3.85 \cdot 10^{-2}$

- Consequence for 0νββ decay

$$\langle m_{\beta\beta}\rangle = \sum_i m_i U_{ei}^2$$

$$|\langle m_{\beta\beta}\rangle| = 17.0 \quad \text{meV}$$

➔ *Partially suppressed effective Majorana mass*

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FLAVOUR VIOLATION

Frère et al. hep-ph/0309014

- Like in the UED, vector bosons can travel in the bulk of space. From the 4D point of view :

1 massless vector boson in 6D =

1 massless vector boson in 4D (zero-mode)

\+ KK tower of massive vector bosons

\+ KK tower of massive scalar bosons

- KK scalar modes do not interact with fermion zero-modes

FLAVOUR VIOLATION

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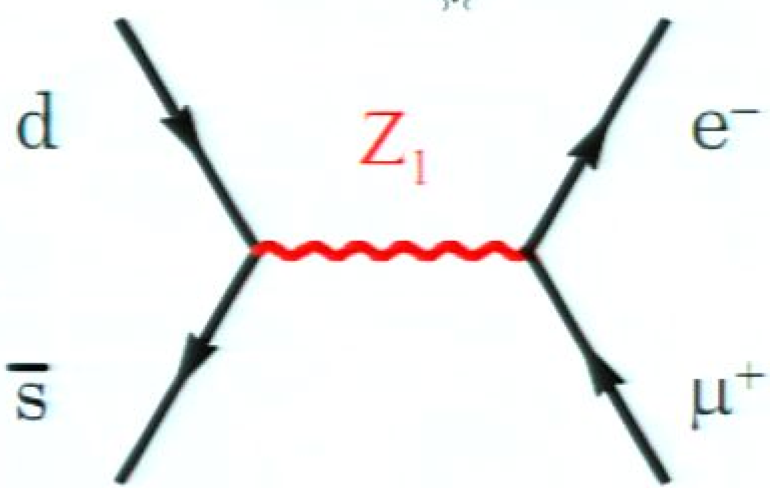
- KK vector modes carry a family number = winding number. In the absence of fermion mixings, family number is an exactly conserved quantity
- Example: <u>FCNC with *ΔG=0*</u>



$K_L \to \mu^+ e^-$ or $\mu^- e^+$

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→ *Family conserving*

$$B.R. < 10^{-12} \quad \to \quad R^{-1} \geq 65 \text{ TeV}$$

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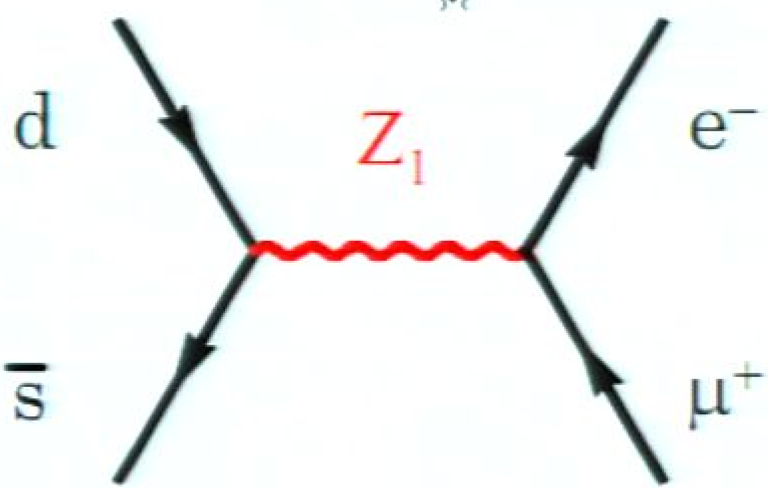

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$\underline{\Delta G = 1}$ $\mu^- \to e^- e^- e^+$

$\mu^- \to e^- \gamma$

$\mu^- \to e^-$ on nuclei

$\underline{\Delta G = 2}$ $K_L - K_S$ mass difference and CP violation

→ *Less constraining !*

Conclusions

- In the context of hierarchical fermions, the most plausible scenario in the neutrino sector is:
  - Off-diagonal dominance
  - Inverted hierarchy
- Going to 6D may solve or give an explanation to
  - The hierarchy problem
  - The replication of families in the SM
  - The flavour puzzle, including the presence of large angles in the neutrino sector and the suppression of flavour violating processes

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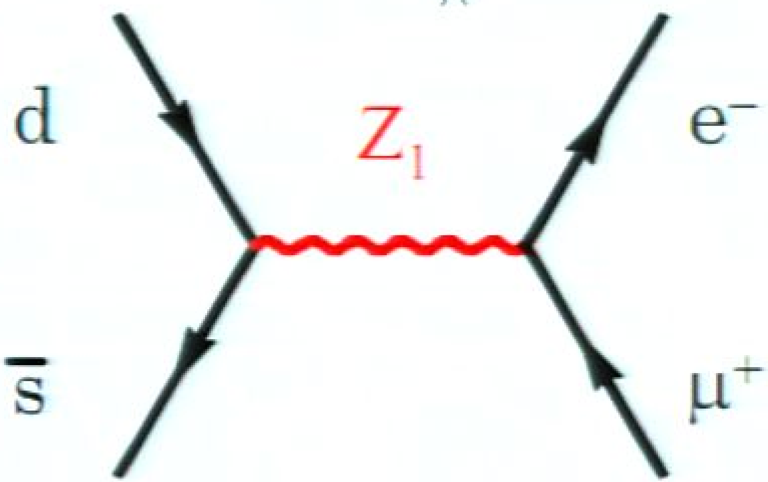
FLAVOUR VIOLATION

Frère et al. hep-ph/0309014

- KK vector modes carry a family number = winding number. In the absence of fermion mixings, family number is an exactly conserved quantity
- Example: <u>FCNC with *ΔG=0*</u>



$K_L \to \mu^+ e^-$ or $\mu^- e^+$

→ *Flavour violating*

→ *Family conserving*

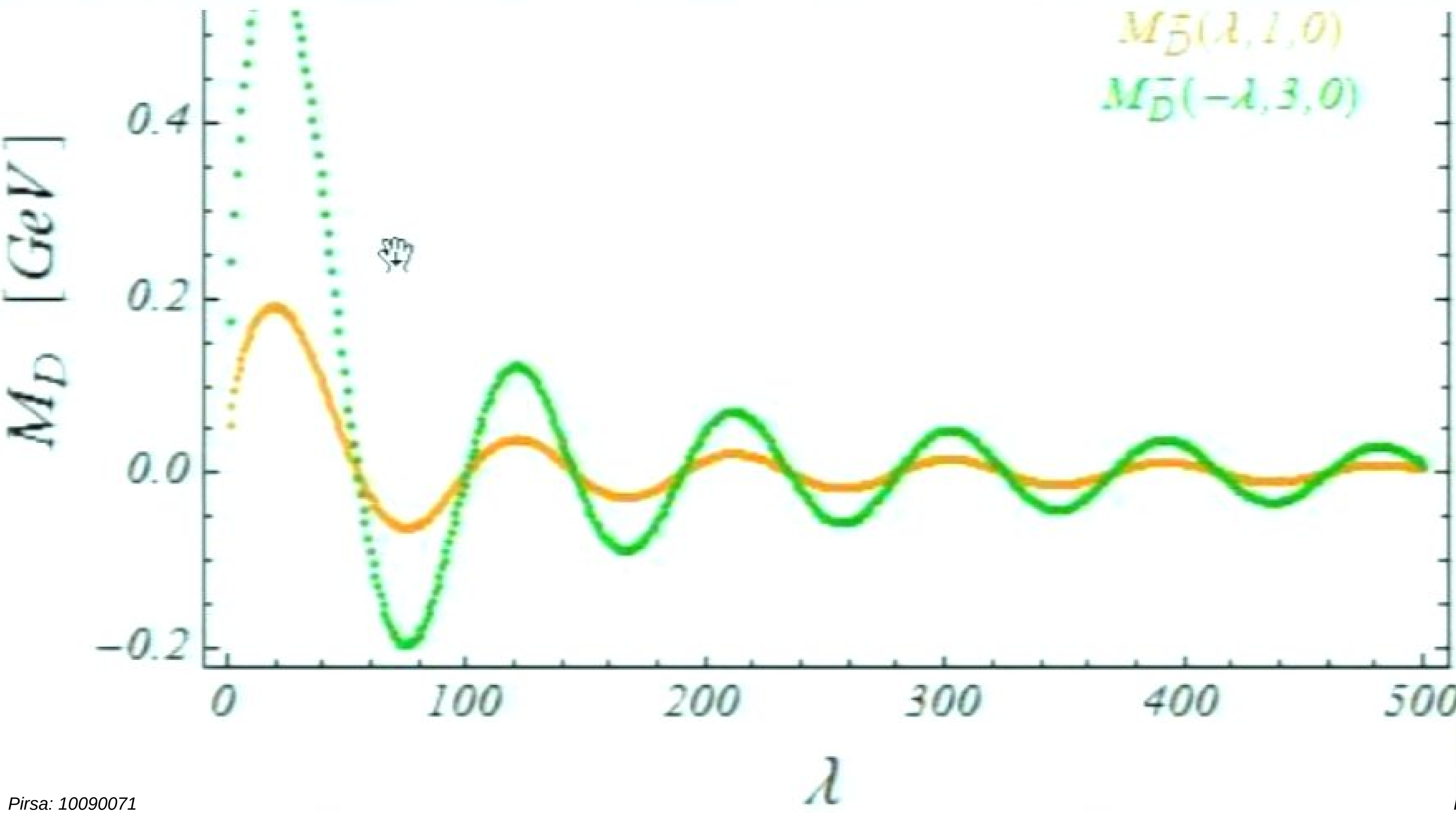
$$B.R. < 10^{-12} \quad \to \quad R^{-1} \geq 65 \text{ TeV}$$

CONCLUSIONS

- In the context of hierarchical fermions, the most plausible scenario in the neutrino sector is:
  - Off-diagonal dominance
  - Inverted hierarchy
- Going to 6D may solve or give an explanation to
  - The hierarchy problem
  - The replication of families in the SM
  - The flavour puzzle, including the presence of large angles in the neutrino sector and the suppression of flavour violating processes

NEUTRINO SEE-SAW MASSES $\bar{L^c}(A + B\,\Gamma_7)L \;+ \text{h.c.}$

$$L_k \sim (0,\ f_2(k)\,l_k,\ f_3(k)\,l_k,\ 0)^t$$



Fermion singlet on $M^4 \times S^2$

- Decomposition in spherical modes

$$\int d\theta\, d\phi\, \mathcal{L}_N = \sum_{\lambda,m} \chi_{\lambda,m} i\partial\bar{\chi}_{\lambda,m} + \bar{\xi}_{\lambda,m} i\bar{\partial}\xi_{\lambda,m}$$

$$-\frac{\lambda}{R}\chi_{\lambda,m}\xi_{-\lambda,m} + \frac{\tilde{M}}{2}(\xi_{\lambda,m}\xi_{-\lambda,-m} - \chi_{\lambda,m}\chi_{-\lambda,-m}) + \text{h.c.}$$

$$\tilde{M} = \epsilon(\lambda)(-1)^{l-m} M$$

→ *Modes are connected by groups of four*

→ $|\lambda| \geq 1$: *Due to the compactification on a sphere, there are no massless modes !*

Fermion singlet on $M^4 \times S^2$

- Decomposition in spherical modes

$$\frac{\mathcal{L}_N}{\sqrt{-\det g_{AB}}} = i\bar{N}\partial_\mu \Gamma^\mu N + \bar{N}\frac{\hat{D}}{R}N - \frac{M}{2}(\bar{N}^c N + \bar{N} N^c)$$

$$N(\theta, \phi, x^\mu) = \sum_{\lambda, m} \frac{e^{im\phi}}{\sqrt{2\pi} R} \begin{pmatrix} S^{-\epsilon}_{d,lm}(\theta)\, e^{i\pi/4}\, \bar{\chi}_{\lambda,m}(x^\mu) \\ S^{\epsilon}_{u,lm}(\theta)\, e^{-i\pi/4}\, \xi_{\lambda,m}(x^\mu) \\ S^{\epsilon}_{d,lm}(\theta)\, e^{-i\pi/4}\, \xi_{\lambda,m}(x^\mu) \\ S^{-\epsilon}_{u,lm}(\theta)\, e^{i\pi/4}\, \bar{\chi}_{\lambda,m}(x^\mu) \end{pmatrix}$$

$$\lambda = \pm(l + \tfrac{1}{2}),\ l = \tfrac{1}{2}, \tfrac{3}{2}, \ldots,\ m = \pm\tfrac{1}{2}, \pm\tfrac{3}{2}, \ldots,\ |m| \leq l$$

Hierarchical Dirac masses and small interfamily mixing angles

- Simplest case for charged lepton mass matrix

$$Y_X H X \bar{L} \frac{1-\Gamma_7}{2} E + Y_\Phi H \Phi \bar{L} \frac{1-\Gamma_7}{2} E$$

$$M_l \sim \begin{pmatrix} \delta^4 & \delta^3 & \\ & \delta^2 & \delta \\ & & 1 \end{pmatrix}$$

CHIRAL ZERO-MODES

Frère et al. hep-ph/0304117

- Coupling of fermions with axial charges to the vortex background

$$\Psi' = e^{i\frac{(1+\Gamma_7)}{2}k\alpha}\Psi$$

$$\mathcal{L}_\Psi = \sqrt{-\det(g_{AB})}\left\{i\bar{\Psi}h_a^A\Gamma^a\left(\nabla_A - iek\frac{1+\Gamma_7}{2}A_A\right)\Psi - g\Phi^k\bar{\Psi}\frac{1-\Gamma_7}{2}\Psi - g\Phi^{*k}\bar{\Psi}\frac{1+\Gamma_7}{2}\Psi\right\}$$

→ *Index theorem : k chiral zero-modes*

ABIKOSOV-NIELSEN-OLESEN VORTEX

- Abelian "Higgs" Lagrangian (here on $M^4 \times S^2$)

$$= \sqrt{-\det(g_{AB})}\left(-\frac{1}{4}F_{AB}F^{AB} + (D^A\Phi)^\dagger D_A\Phi - \frac{\lambda}{2}(|\Phi|^2 - v^2)^2\right)$$

$$ds^2 = g_{AB}dx^A dx^B = g_{\mu\nu}dx^\mu dx^\nu - R^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

- Look for a static solution

$$A_\varphi = \frac{1}{e}A(\theta)\,, \quad A_\theta = 0\,, \quad \Phi = F(\theta)e^{i\varphi}.$$

Here the winding number of Φ is equal to 1

A MODEL OF FAMILY REPLICATION WITH A VORTEX IN 6D

Frère, Libanov, FSL, Nugaev, Troitsky

- The basic idea is to have a topological defect in 6D (vortex) made of a $U(1)_g$ gauge field A and a scalar field Φ
- The interaction of a single fermion family with the vortex leads to several chiral zero-modes, as a consequence of the index theorem
- Family number in 4D corresponds to winding number in extradim