Title: An apology for DRS

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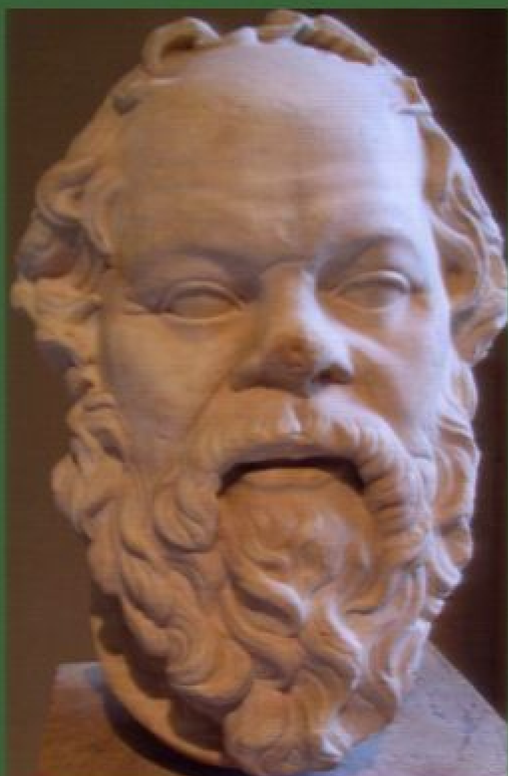
Abstract: In my talk I would like to discuss the present status of Doubly Special Relativity. DSR is an extension of Special Relativity aimed at describing kinematics of particles and fields in the regime where (quantum) gravity effects might become relevant. I will discuss an interplay between DSR physics and mathematics of Hopf algebras.

Απολογια ΔΣΡ

(An apology for DSR)

PI, September 2010

Apology



- Socrates, one of the greatest minds in the history of mankind, was found guilty of corrupting the minds of the youth of Athens and sentenced to death.

What is DSR?

- DSR is based on 2+1 postulates:

❖ **Relativity principle**: All inertial frames are totally equivalent for the performance of all physical experiments. The difference of outcomes of the experiments made by two observers depend only on their relative, uniform motion.

❖ **Observer independent scales**: There are two observer-independent scales: one of velocity, identified with the speed of light, and the second of mass (or length), identified with Planck scale.

What is DSR?

- **Transformations**: There exists spacetime transformations, which tell how the observations made by two inertial observers are related. They are parametrized by ten parameters (in 4D) corresponding to translations, rotations, and boosts (generalized Poincare transformations). It follows that the two scales must be present as parameters in these transformations.

What is DSR – comments

- These postulates are quite vague; surprisingly it is not easy to satisfy them.
- These are the postulates of a fundamental theory and *NOT* of a phenomenology-motivated test theory.
- Such fundamental theory should be DERIVED from a more fundamental theory of (quantum) gravity.

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DSR

- A real challenge is to find a theory which is both mathematically consistent and physically sensible that can be confronted with experiments.
- It seems that, except in 3D, such theory has not been found yet.

Naive DSR

- The simplest possibility is to replace momenta of SR with

$$p_\mu \to P_\mu = P_\mu(p;\kappa)$$

$$p^2 - m^2 = 0 \to C(P;\kappa) - m^2 = 0$$

Lorentz transfs. leaving C inv. contain κ

- Question: This is just a change of variables, so isn't that equivalent to the original SR?

Naive DSR

- Answer: Depends. Changing variables is not everything, we still must tell how the new momenta add up for many particles and what `minus momentum' means.
- In other words we must specify what is the co-product and the antipode in terms of the new variables.

Naive DSR

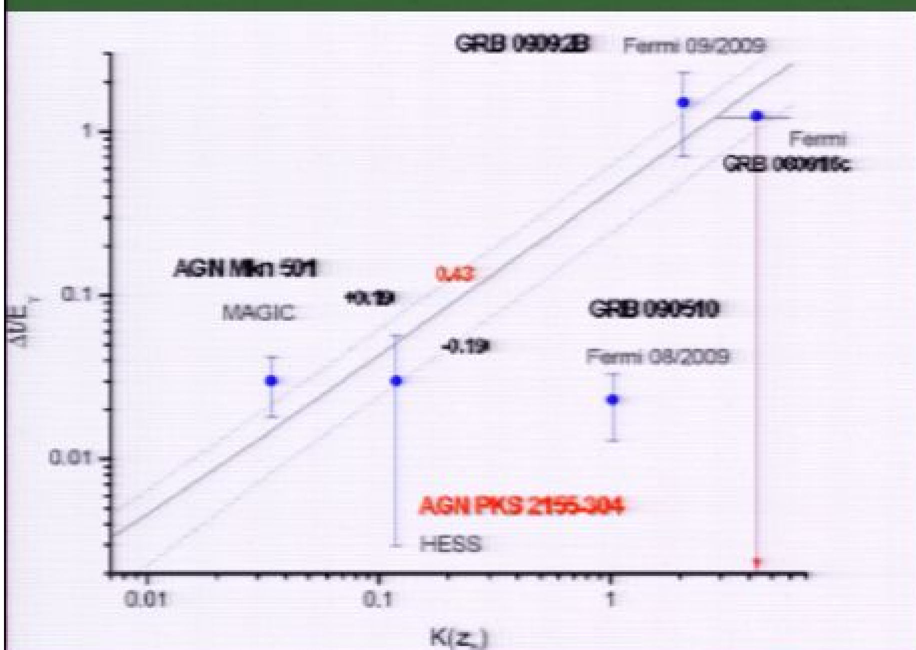
- P and p with primitive co-products and antipodes describe different theories.

$$L = \sum_i p^{(i)}{}_\mu \dot{x}_{(i)}{}^\mu + \lambda\left(p^{(i)2} - m_{(i)}{}^2\right)$$

$$L' = \sum_i P^{(i)}{}_\mu \dot{X}_{(i)}{}^\mu + \lambda\left(C(P^{(i)};\kappa) - m_{(i)}{}^2\right)$$

- The speed of light in the second theory is energy-dependent.

Phenomenology: Lazy photons



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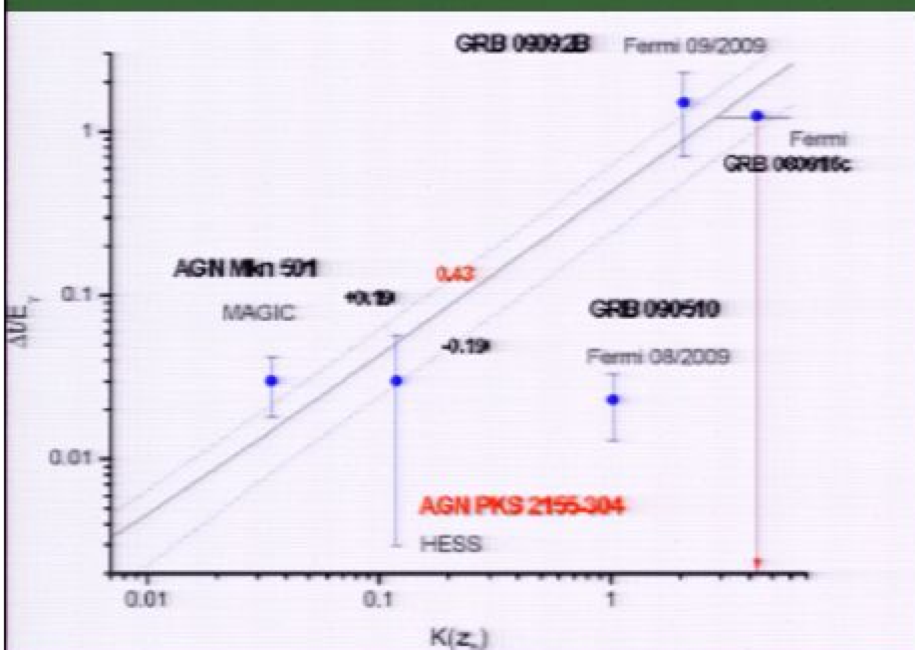

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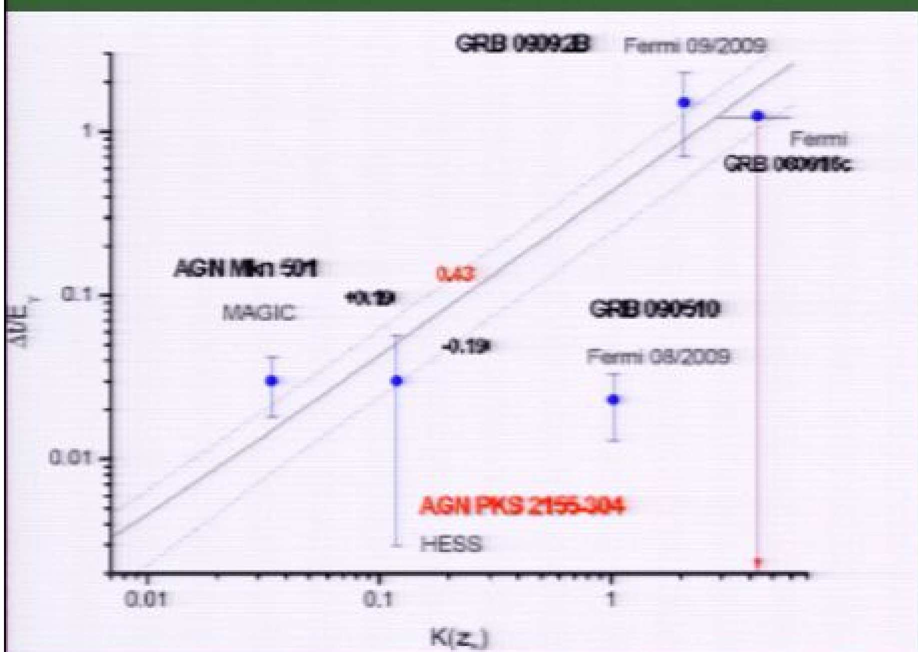

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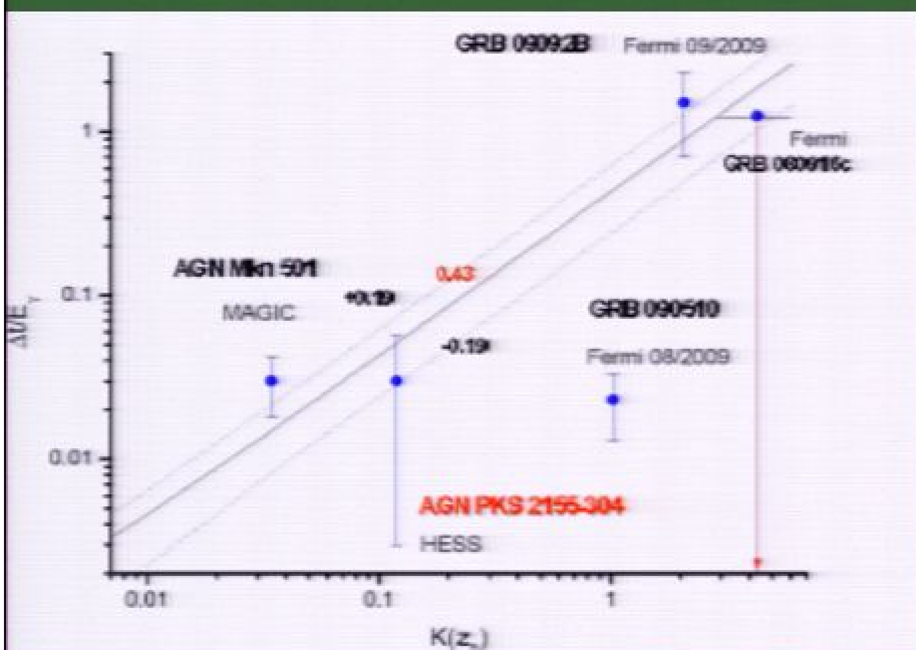

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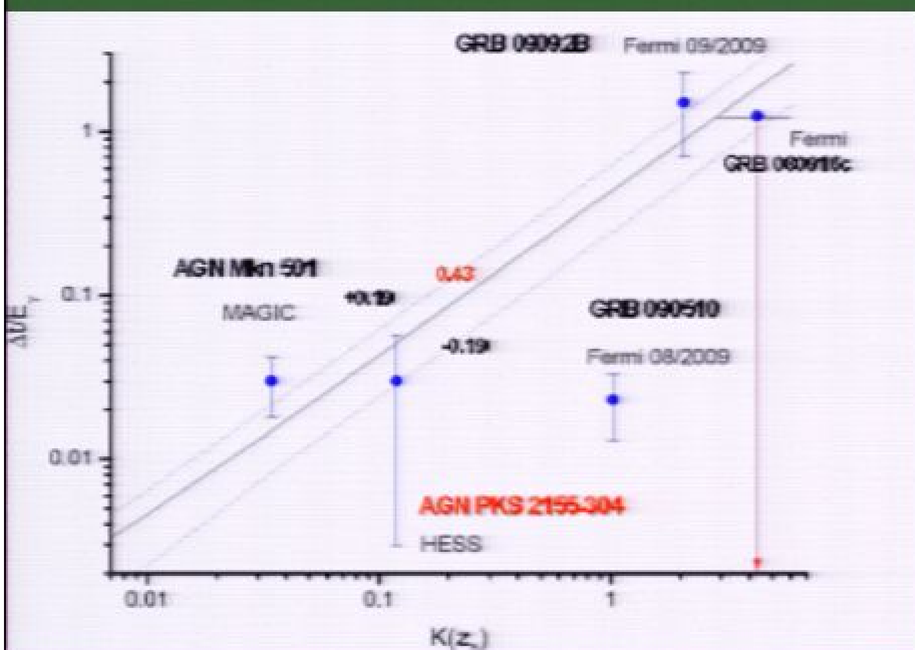

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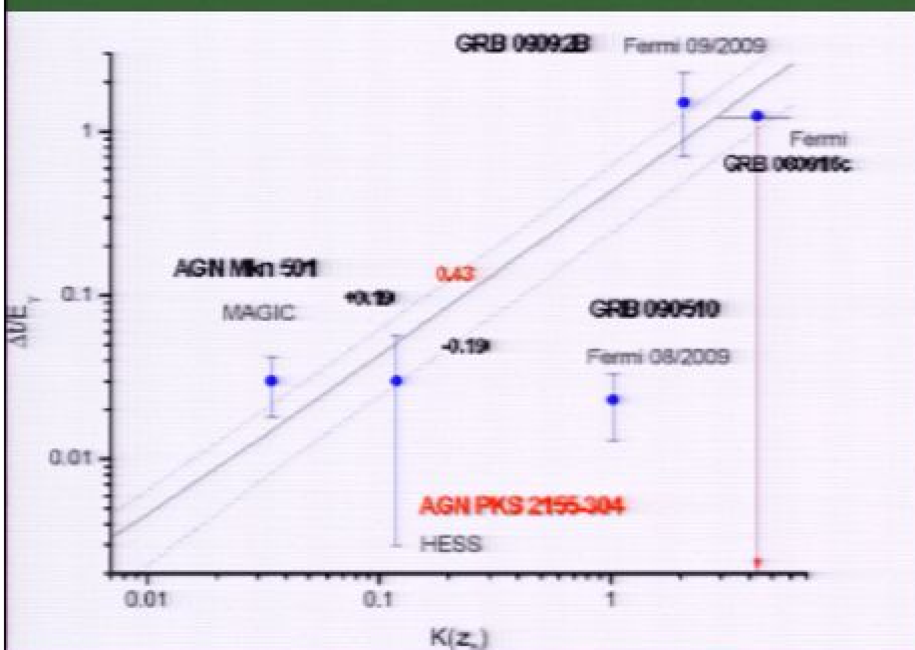

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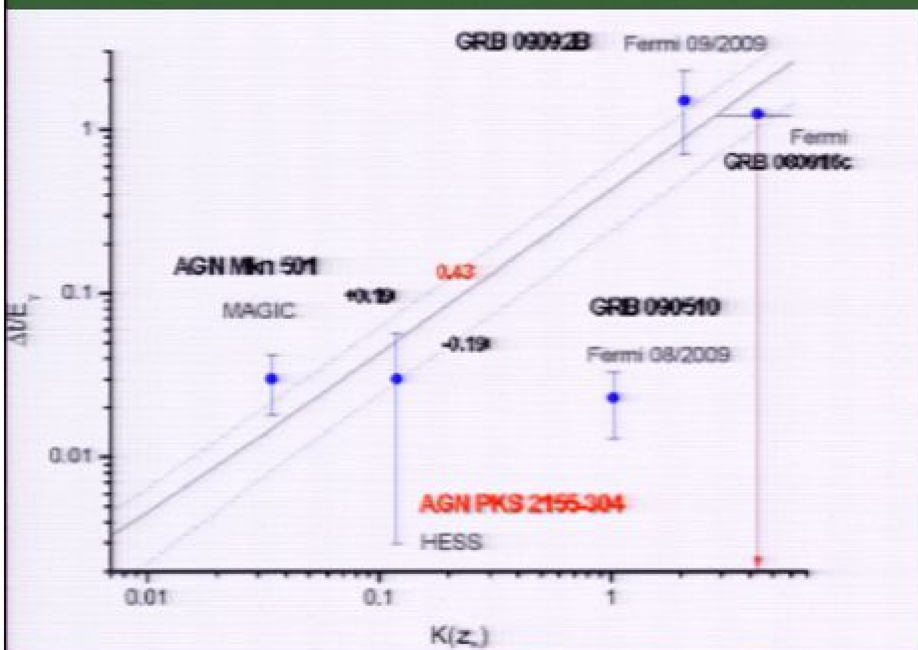

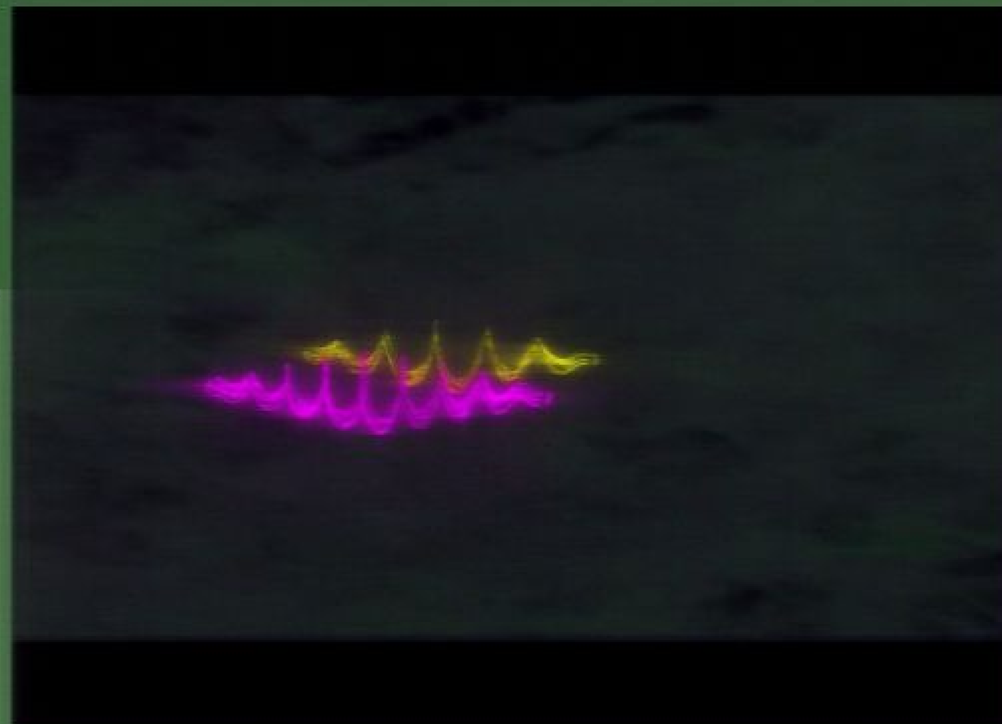

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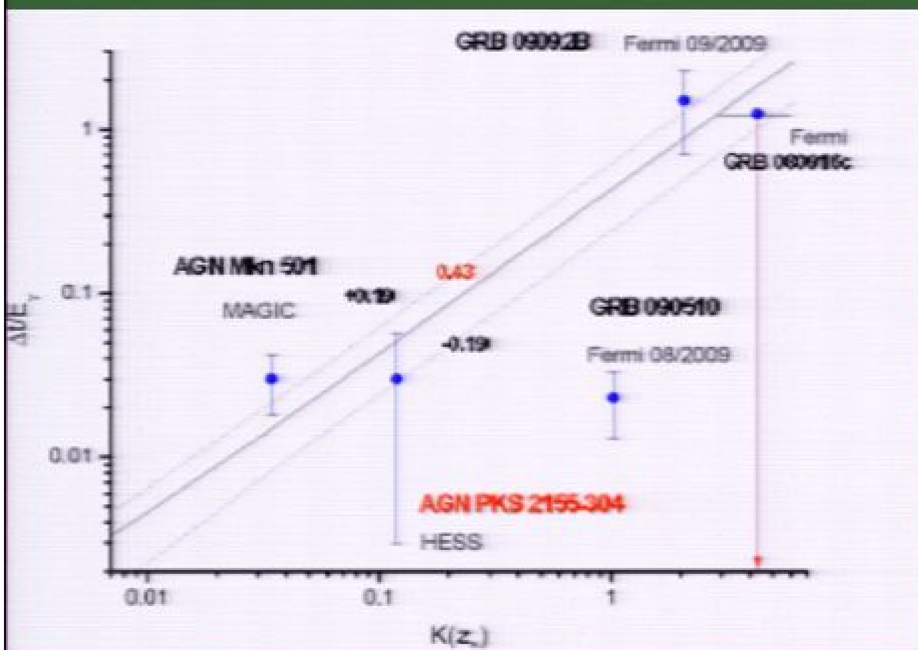

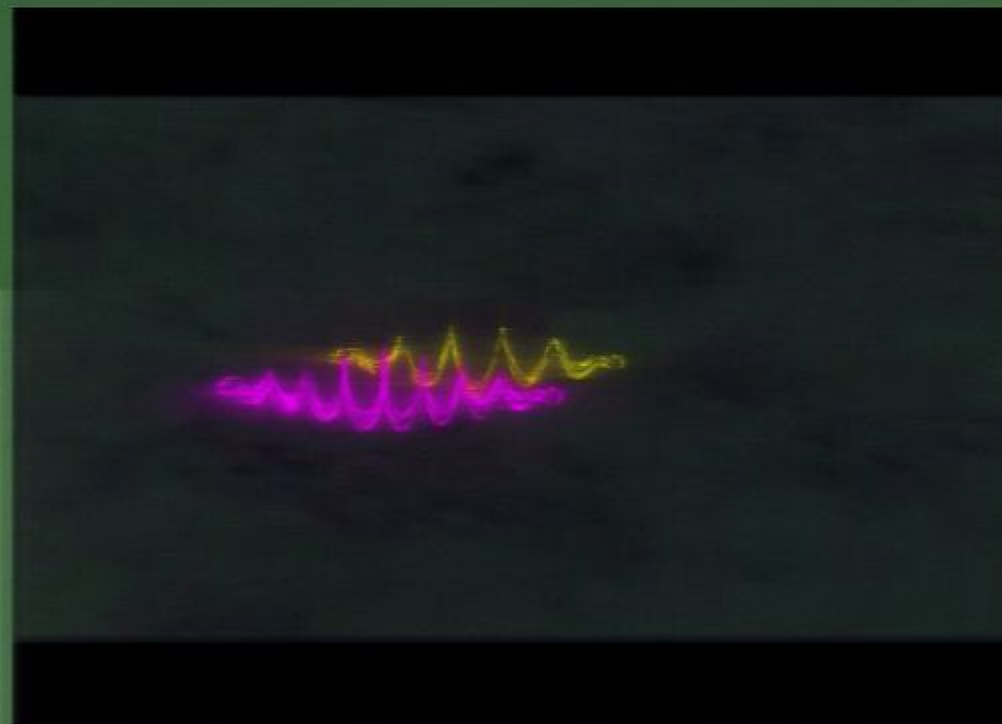

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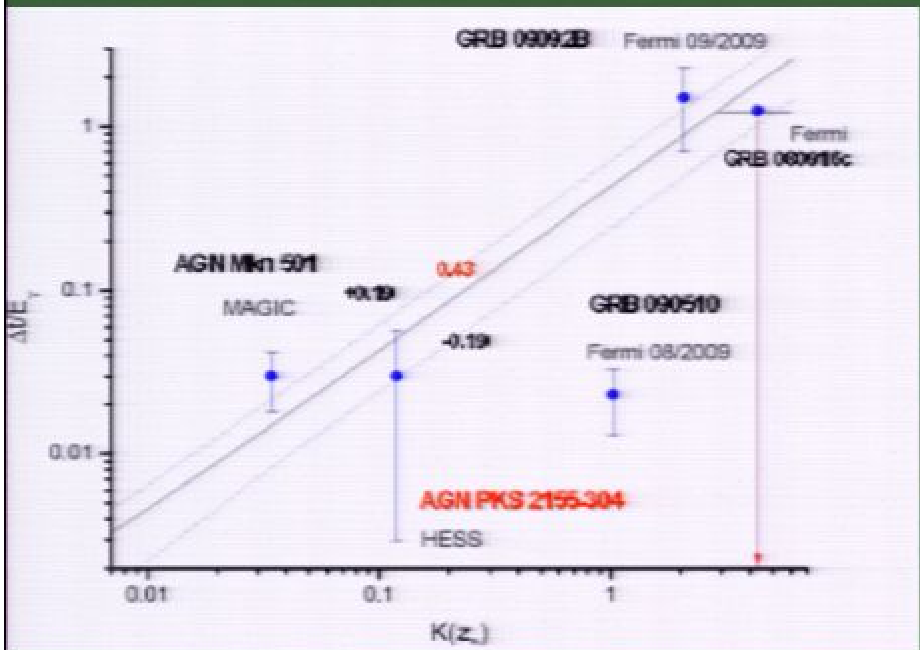

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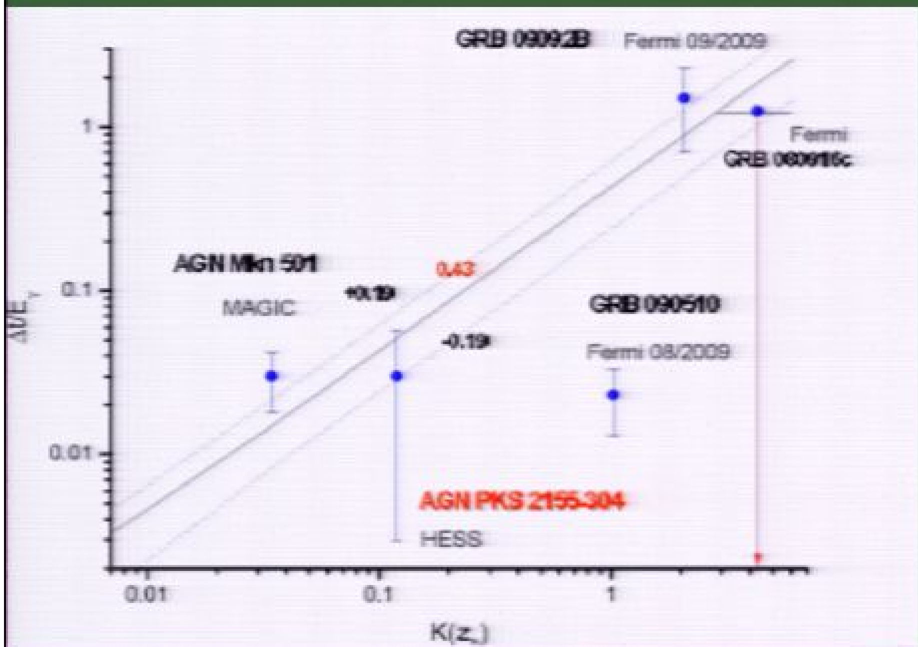

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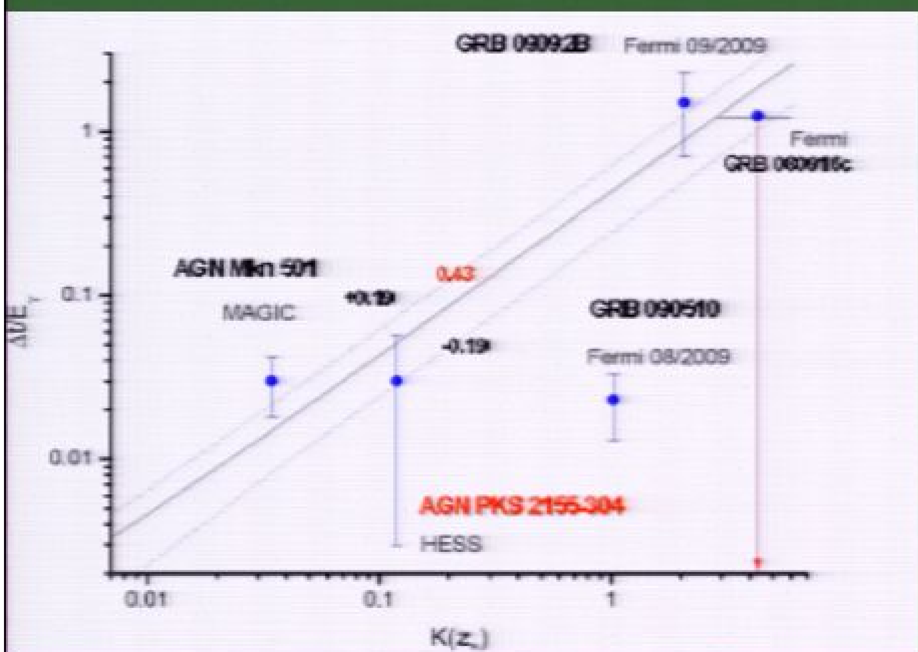

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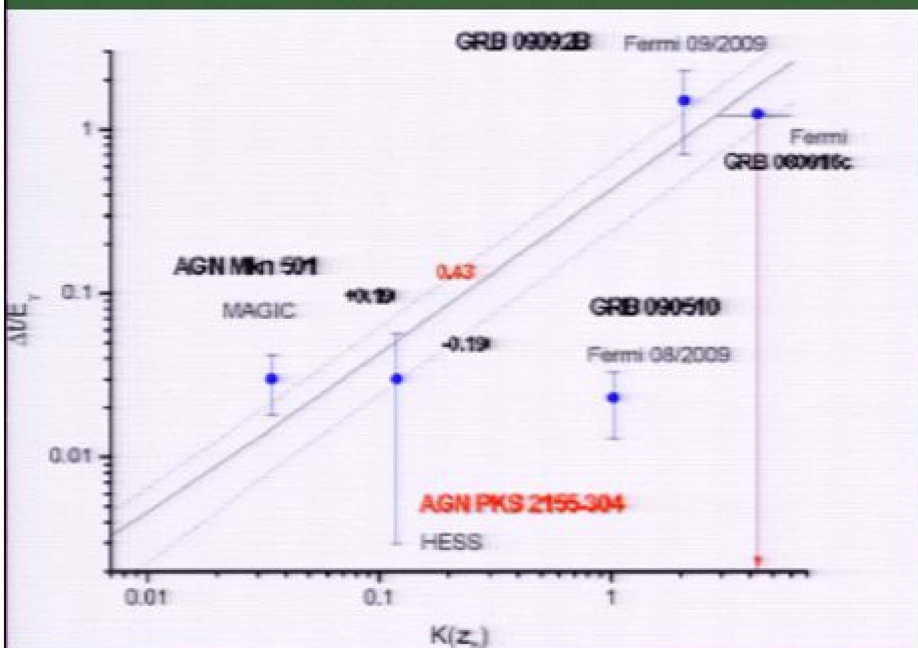

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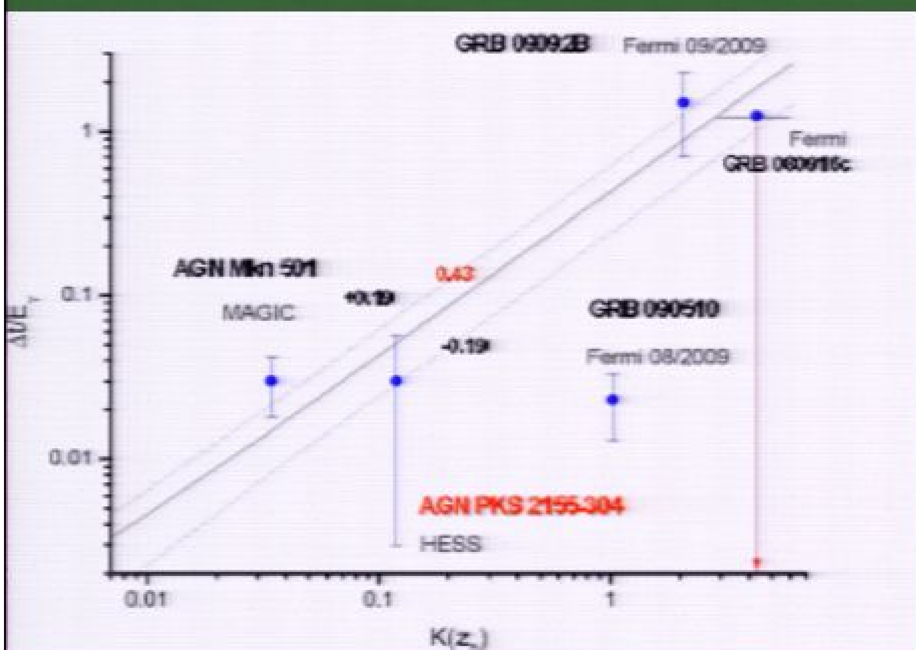

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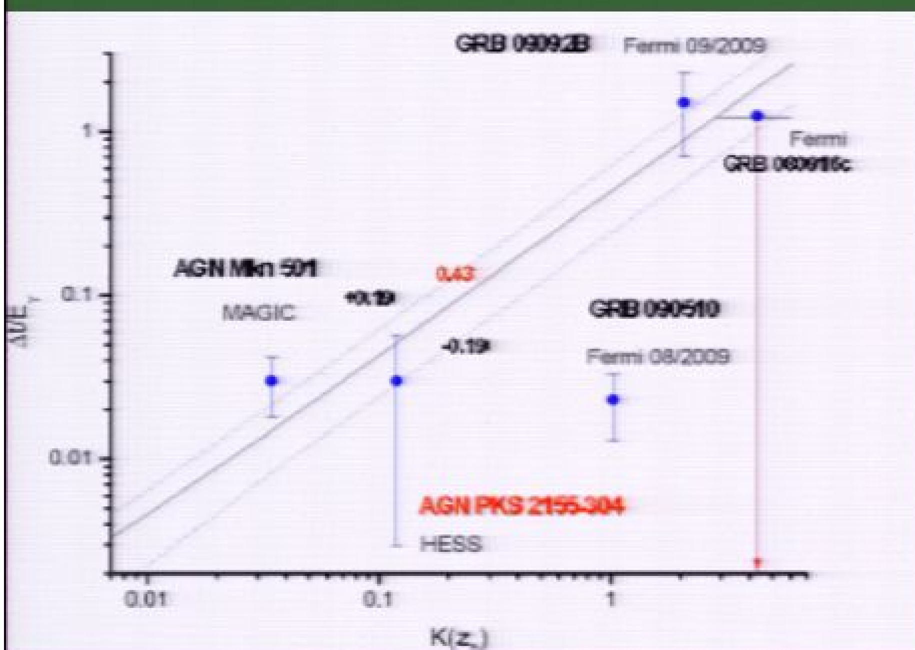

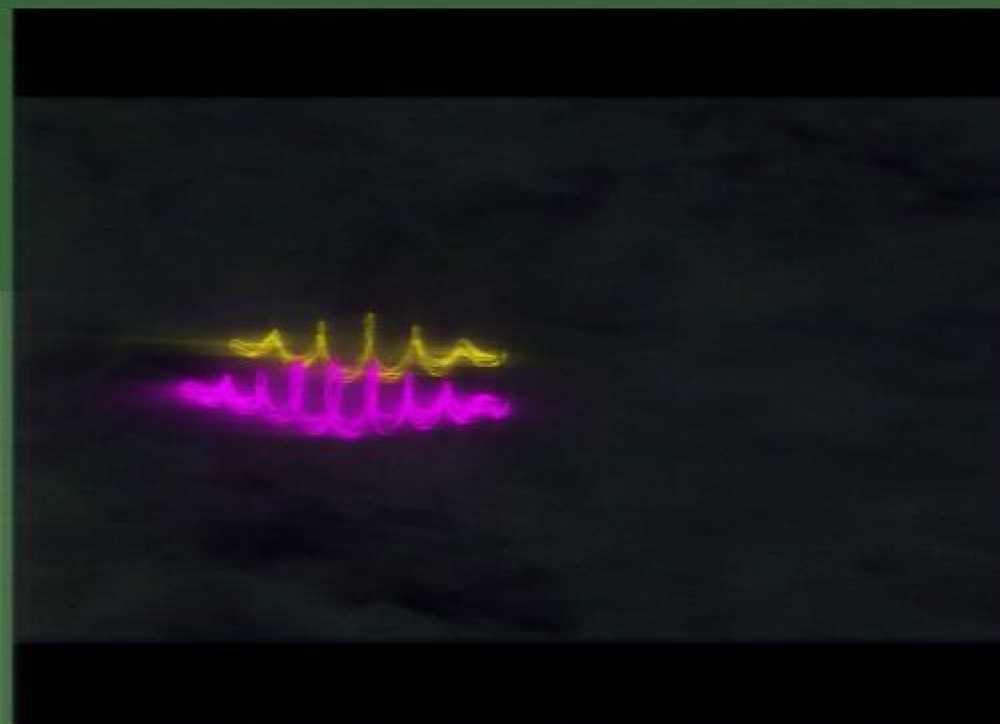

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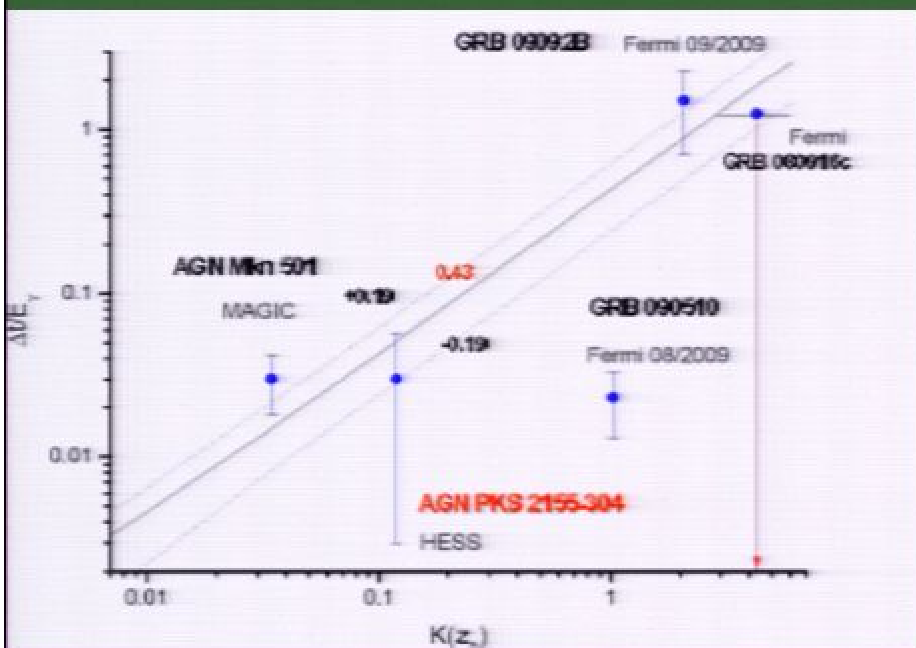

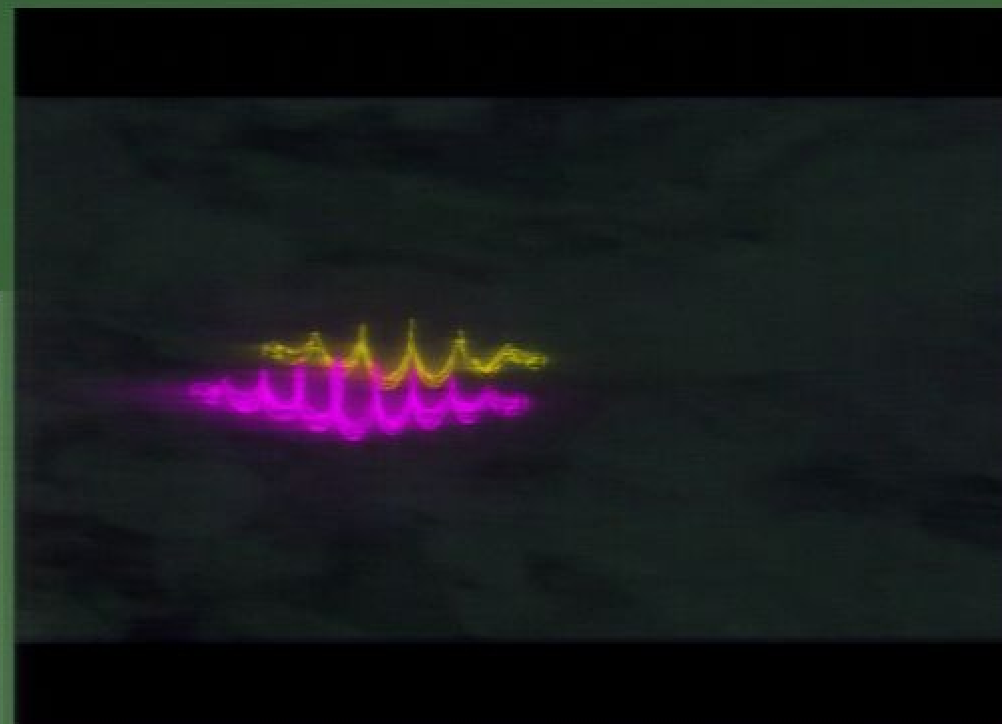

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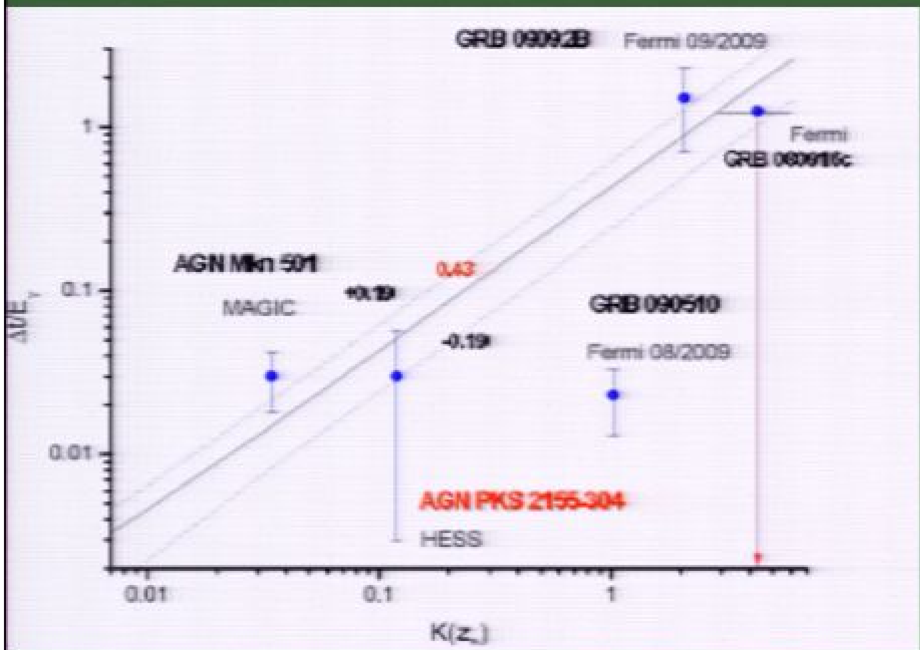

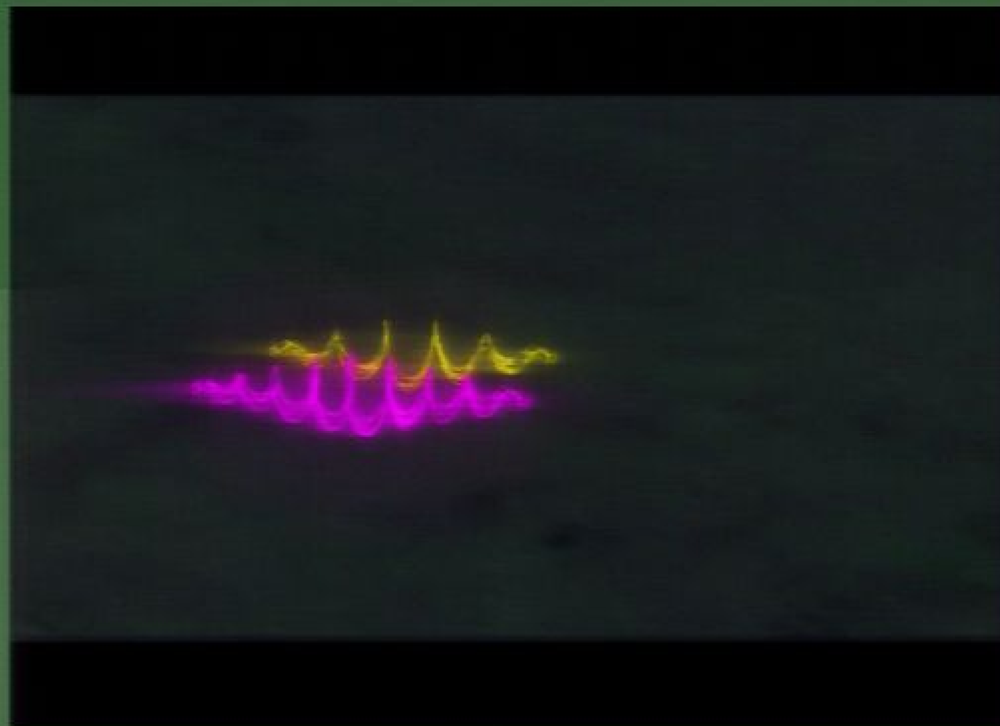

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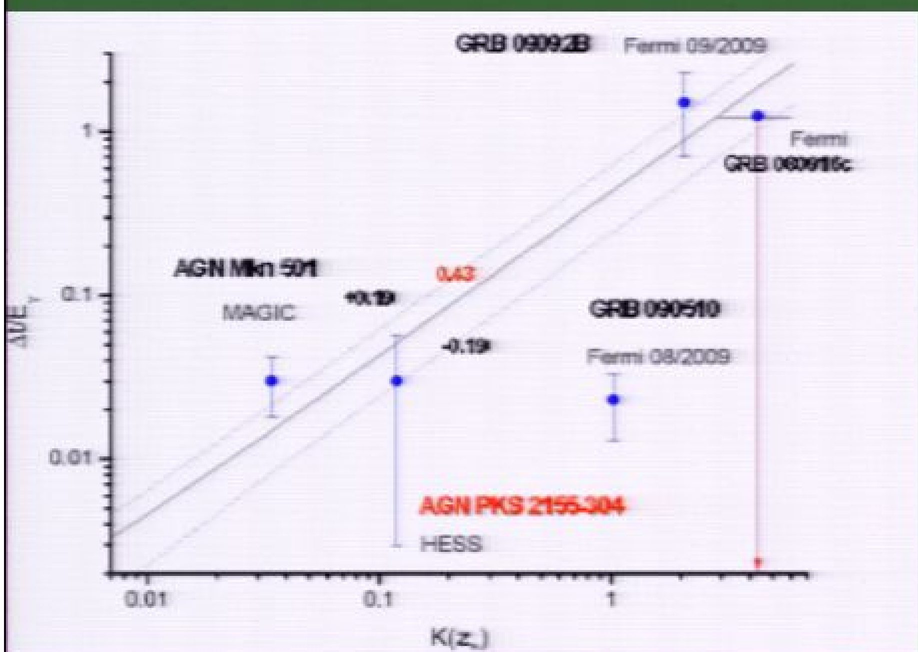

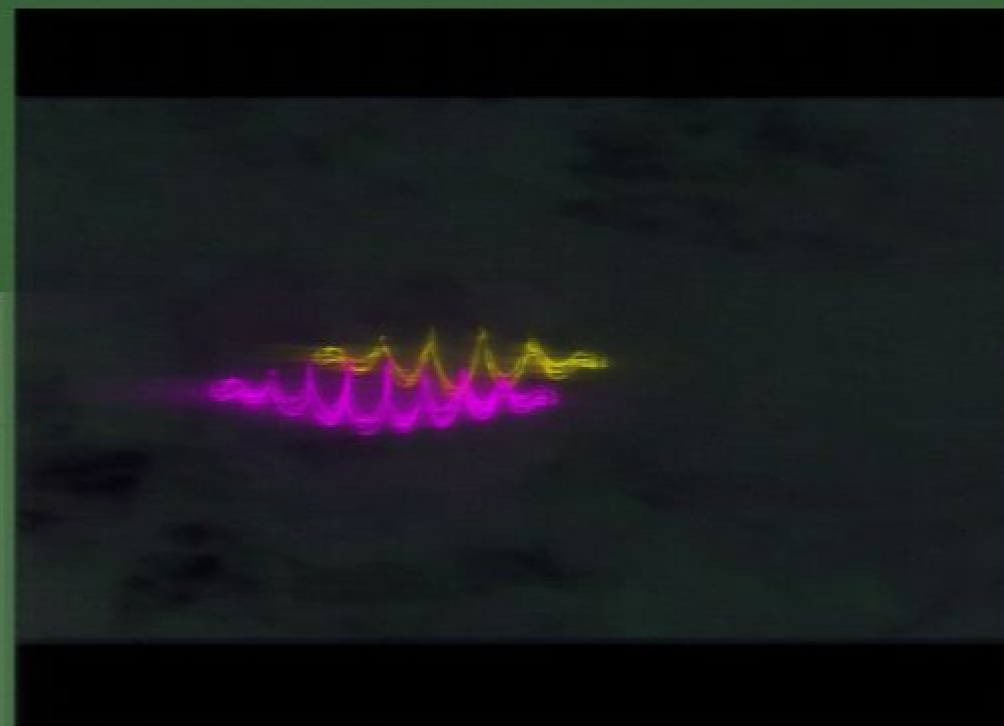

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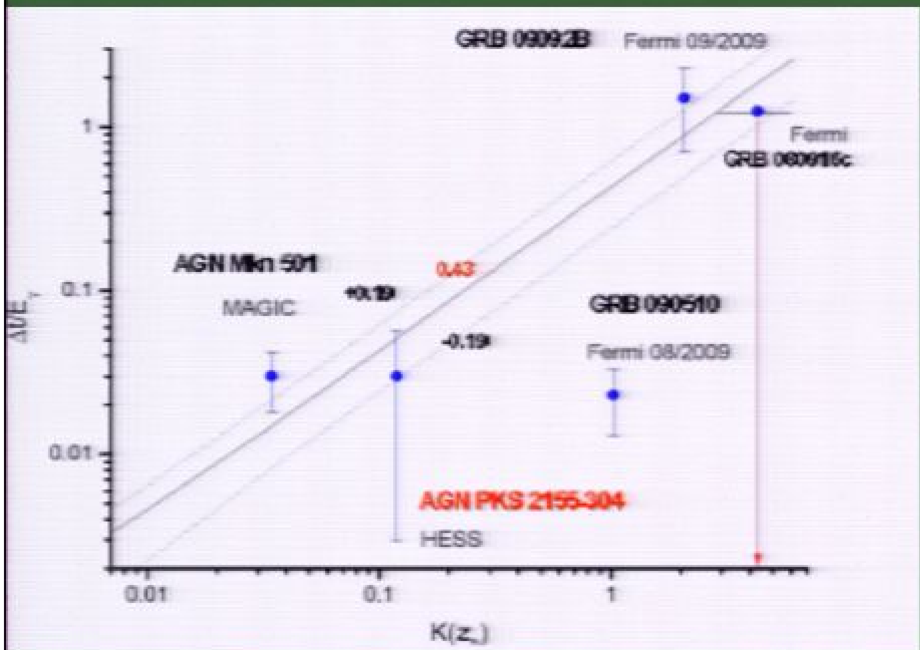

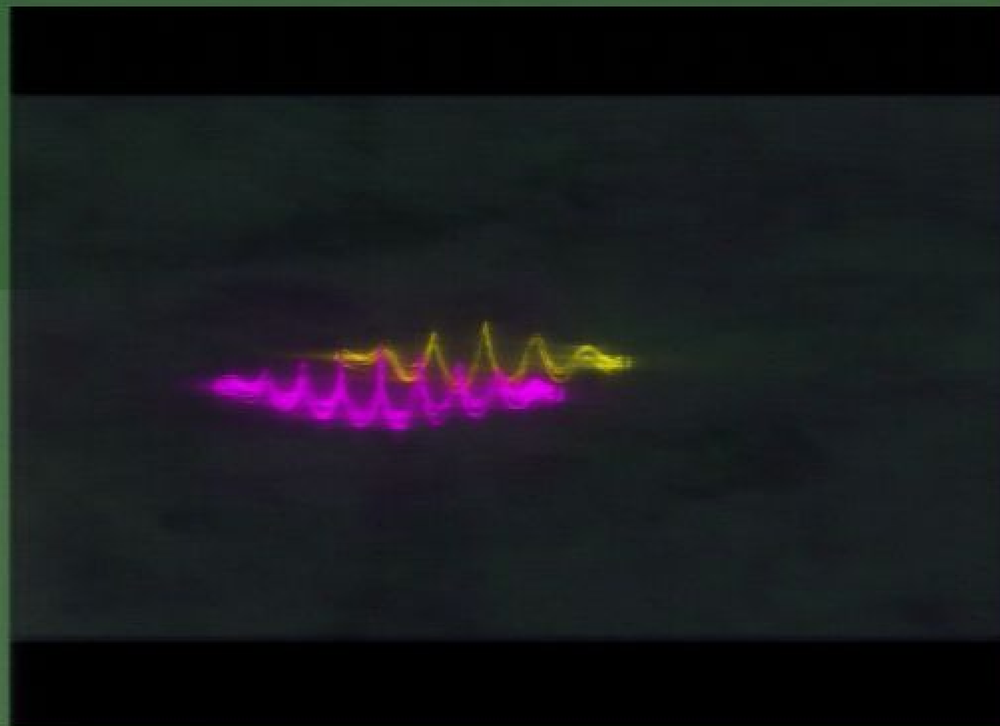

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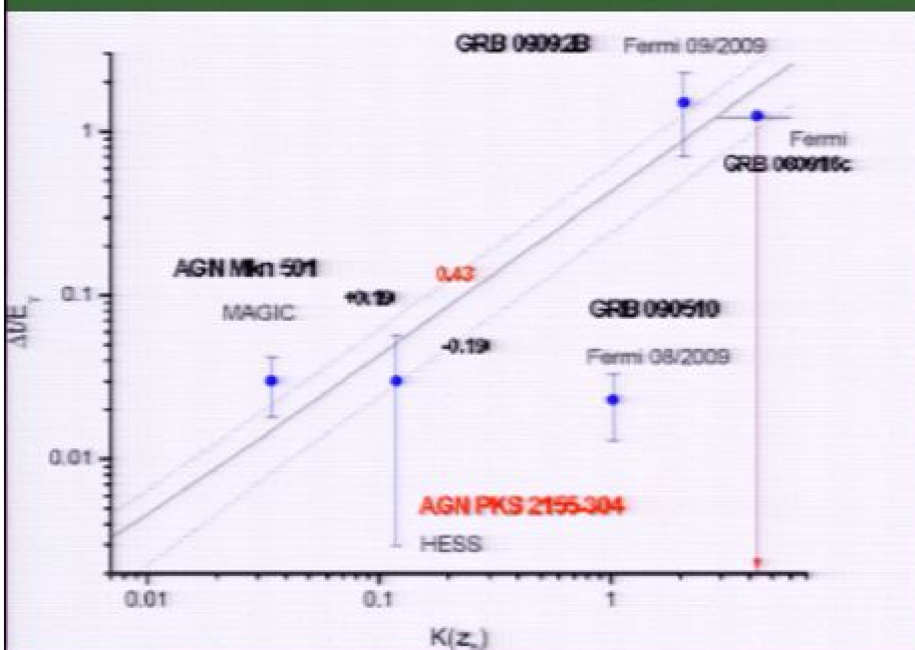

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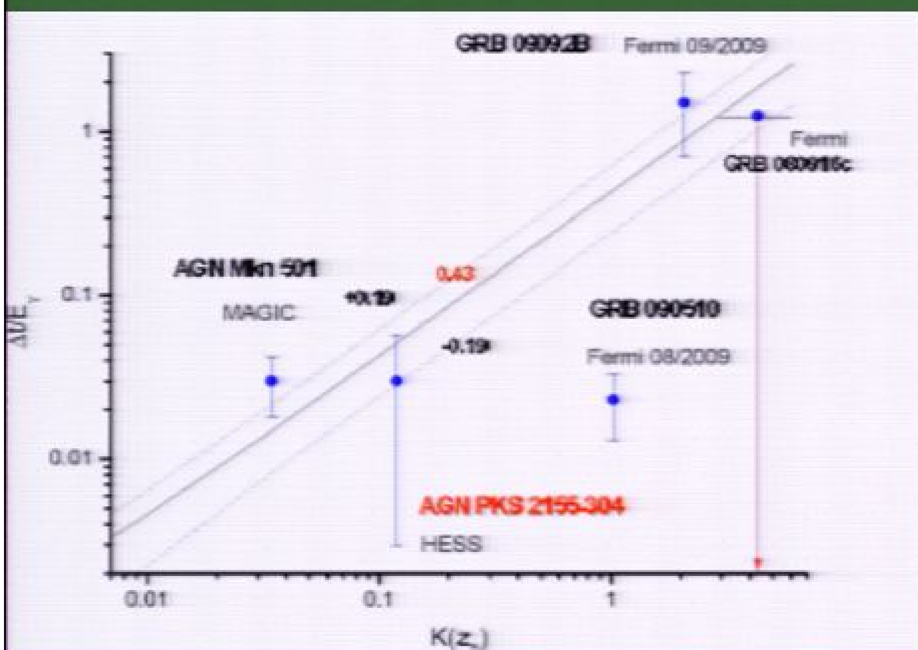

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GZK cutoff condition

$$p+\gamma \rightarrow p+\pi_0$$

- For protons with E > 10^{20} eV the free path is of order of 150 Mpc.
- If we find such protons coming from more distant source this would mean deviation from the standard relativistic kinematics.

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From gravity to DSR

- DSR cannot be fundamental; if correct it must arise as a ``no gravity'' limit of gravity.
- We must therefore:
  - Find the formulation of gravity, in which this limit can be naturally taken.
  - Derive the effective deformed dynamics of particles.

Gravity as a constrained BF

- The action for pure gravity

Topological

Gravitational dynamics

$$L = B^{IJ} \wedge F_{IJ} - \frac{\beta}{2} B^{IJ} \wedge B_{IJ} - \frac{\alpha}{4} B^{IJ} \wedge B^{KL} \epsilon_{IJKL4}$$

$$A_\mu{}^{a4} = \frac{1}{2\ell} e_\mu{}^a, \quad A_\mu{}^{ab} = \omega_\mu{}^{ab}$$

- This action leads to vacuum Einstein eqs.

GR from constrained BF

$$S = S_{GR} + \text{Topological invariants}$$

$$S_{GR} = \frac{1}{2G}\int R^{ij}(\omega)\wedge e^k \wedge e^l \epsilon_{ijkl}$$

$$-\frac{\Lambda}{12G}\int e^i \wedge e^j \wedge e^k \wedge e^l \epsilon_{ijkl} + \frac{1}{G\gamma}\int R^{ij}(\omega)\wedge e_i \wedge e_j$$

$$\Lambda = \frac{3}{\ell^2}, G = \ell^2 \frac{\alpha^2+\beta^2}{\alpha}, \gamma = \frac{\beta}{\alpha}$$

Particles coupling

- The gauge degrees of freedom of gravity are promoted to dynamical degree of freedom.

Gravity coupling

Kinetic term

$$L(z,h;A) = -\mathrm{tr}(C\,A_\tau) - \mathrm{tr}(h^{-1}\dot{h}\,D)$$

$$D \equiv m\ell T^{04} + sT^{23}$$

$$C \equiv hDh^{-1} = \ell p_a T^{a4} + s_{ab}T^{ab}$$

h completely describes particle kinematics

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Particles + gravity

- From this action one finds correct Einstein-Cartan equations with masses and spins as (point) sources for curvature and torsion and correct equations of motion for particles (Mathisson-Papapetrou eq. with torsion).

The topological limit

- In the limit $\alpha \to 0$ the theory becomes topological, with no local gravitational degrees of freedom.
- It can be shown that in this limit the partition function for quantum gravity becomes the partition function for Chern-Simons theory, on a spherical boundary, with punctures corresponding to particles' insertion.

The topological limit - CS action

$$S = \frac{1}{\beta}\int_R dx^0 \int_{S_n^2} \langle \partial_0 A_S \wedge A_S \rangle - \int_R dx^0 \sum_{i=1}^{n} \langle D_i, h_i^{-1}\partial_0 h_i \rangle$$

with the constraint

$$F_S \sim \sum_{i=1}^{n} D_i \delta^{(3)}(x - x_{(i)})$$

$$A = A_0 dx^0 + A_S,\ F_S = dA_S + A_S \wedge A_S,$$

$$h_i \in SO(4,1),\ D_i = m_i \ell T^{04} + sT^{23}$$

The (reduced) phase space

- It can be shown that:
- The phase space of this theory is finite dimensional, and can be expressed in terms of n group elements g_i, i=1,…,n.
- The Poisson brackets on this phase space are defined with the help of r–matrix associated with Chern–Simons action.
- $U_q(SO(4,1))$ should describe the symmetries of the system.

The r matrix for CS

- The relevant r-matrix must satisfy:

$$r = C + r_A$$

- where C is the Casimir of the gauge group SO(4,1) corresponding to the CS action;
- r satisfies the (classical) Yang-Baxter eqn. for SO(4,1).

The phase space

- Knowing r-matrix, one can calculate Poisson brackets on the phase space.
- Sklyanin Poisson-Lie structure is defined via the bi-vector

$$B_S = \frac{1}{2} r^{\alpha\beta} \left(X_\alpha^R \wedge X_\beta^R - X_\alpha^L \wedge X_\beta^L \right)$$

$$r = r^{\alpha\beta} X_\alpha \otimes X_\beta$$

- $X_\alpha^{R/L}$ are right (left) inv. vector fields on the group

Sklyanin bracket and κ-Poincare group

- One can compute Sklyanin bracket for SO(4,1), which after taking the contraction limit ($\Lambda \to 0$) reproduces the κ-Poincare group.

$$\left\{S^{a}{}_{b}, S^{c}{}_{d}\right\} = 0,$$

$$\left\{q^{a}, q^{b}\right\} = \frac{1}{\kappa}\left(\delta_{0}^{a}\, q^{b} - \delta_{0}^{b}\, q^{a}\right)$$

$$\left\{S^{a}{}_{b}, q^{c}\right\} = -\frac{1}{\kappa}\left(\left(S^{a}{}_{0} - \delta_{0}^{a}\right) S^{c}{}_{b} - \eta^{ac}\left(S^{0}{}_{b} - \delta_{b}^{0}\right)\right)$$

κ-Minkowski space

For κ-Minkowski space the coordinates do not commute (have vanishing PB), but instead they satisfy

$$\left\{q^0, q^i\right\} = \frac{1}{\kappa} q^i, \quad \left\{q^i, q^j\right\} = 0$$

These brackets define an_3 Lie algebra associated with the Iwasawa decomposition $SO(4,1)=SO(3,1)\,AN_3$

The group AN_3

- We can parametrize AN_3 group element as follows ('non-commutative plane-wave')

$$\mathcal{P} = e^{X^i p_i/\kappa} e^{X^0 p_0/\kappa}$$

$$\left[X^0, X^i\right] = X^i, \quad \left[X^i, X^j\right] = 0$$

- With κ being a scale of dimension of mass.

κ-Poincaré particle

- The Lagrangian for one particle reads

$$L = \left\langle \mathcal{P}^{-1} \frac{d}{d\tau} \mathcal{P}, q \right\rangle - \lambda\left(C(p,\kappa) - m^2 \right)$$

$$= p_0 \dot{q}^0 + p_i \dot{q}^i - \frac{1}{\kappa} p_i q^i \dot{p}_0 - \lambda\left(C(p,\kappa) - m^2 \right)$$

$$C(p,\kappa) = \kappa^2 \cosh\left(\frac{p_0}{\kappa} \right) - \frac{p^2}{2} e^{p_0/\kappa} - \kappa^2$$

κ-Poincaré particle – comments

- The PB of positions is identical with the Sklyanin bracket of CS;

$$\{q^0, q^i\} = \frac{1}{\kappa} q^i, \quad \{q^i, q^j\} = 0$$

- The infinitesimal symmetries form κ-Poincaré algebra;
- The particle is free i.e., it moves uniformly.

The speed of light

- The speed of light is (formally) defined to be

$$c = \left.\frac{|\dot{x}^i|}{\dot{x}^0}\right|_{m=0} \equiv \left.\frac{\left|\{x^i, C(p,\kappa)\}\right|}{\{x^0, C(p,\kappa)\}}\right|_{m=0} = 1$$

- and thus it is energy-independent.

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Symmetries of positions

- Rotations

$$\delta_\rho q^0 = \rho^i \left\{M_i, q^0\right\} = 0, \quad \delta^M q^j = \rho^i \left\{M_i, q^j\right\} = \rho^i \epsilon_{ijk} q^k$$

- Boosts

$$\delta_\xi q^i = \xi^j \left\{N_j, q^i\right\} = -\xi^i q^0 + \frac{1}{\kappa} q^i \vec{\xi} \cdot \vec{p} - \frac{1}{\kappa} p^i \vec{\xi} \cdot \vec{q}$$

$$\delta_\xi q^0 = \xi^j \left\{N_j, q^0\right\} = -\vec{\xi} \cdot \vec{q} + \frac{1}{\kappa} x^0 \vec{\xi} \cdot \vec{p} + \vec{\xi} \cdot \vec{p} \left[\frac{1}{2}\left(1 - e^{-2p_0/\kappa}\right) + \frac{\vec{p}^2}{2\kappa^2}\right]$$

Symmetries of positions

- Translations

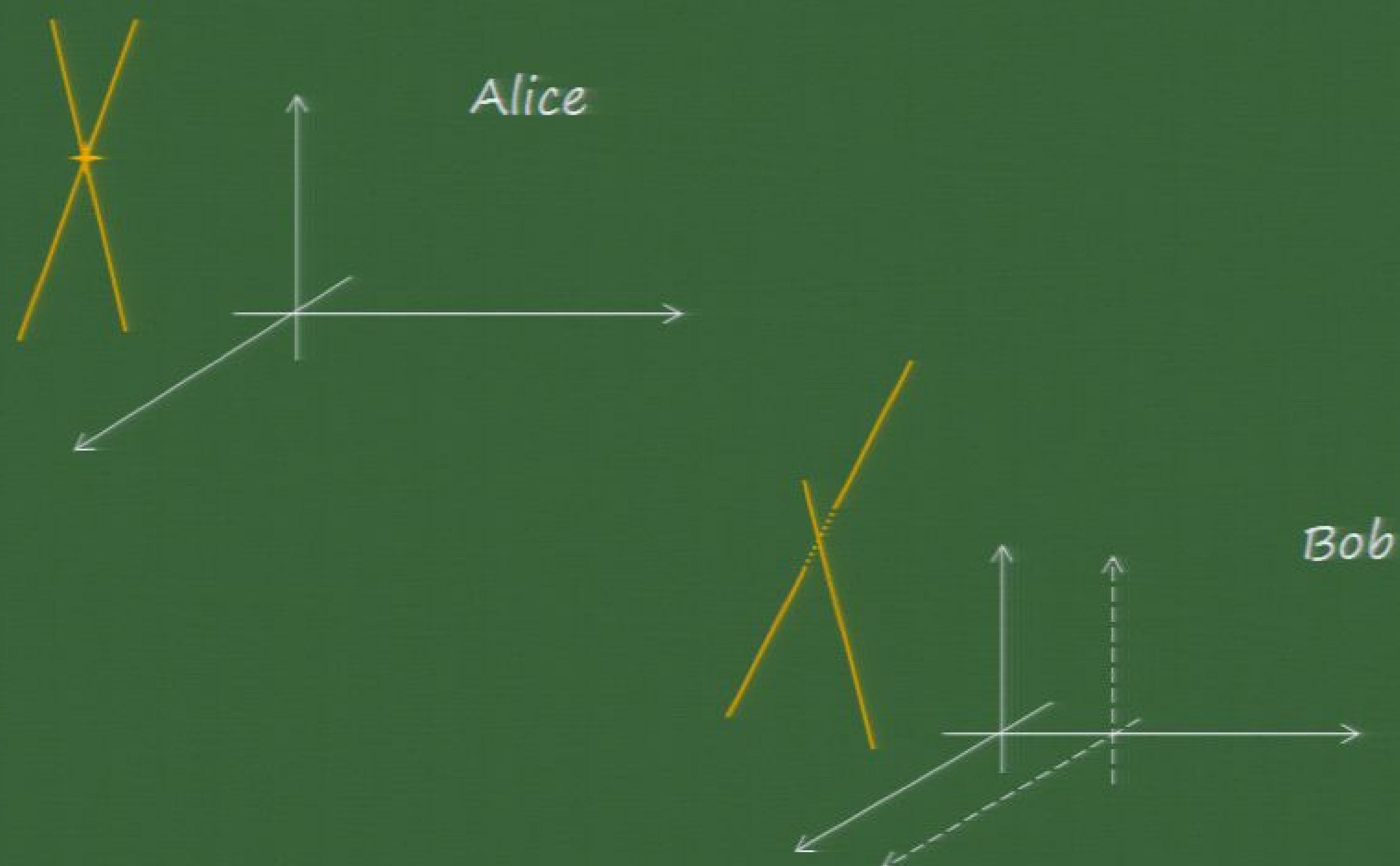
$$\delta q^0 = \{a^\mu p_\mu, q^0\} = -a^0 - \frac{1}{\kappa} p_i a^i,$$
$$\delta q^i = \{a^\mu p_\mu, q^i\} = -a^i$$

- Notice that (contrary to SR) both the translation and the boost of position depend on momentum carried by the particle.

The problem

- Take worldlines of two particles that cross for one observer, Alice.
- Then for another (translated, boosted) observer, Bob, the worldlines will miss each other.
- This is a disturbing problem.
- It is a direct consequence of the fact that transformations of positions depend on momentum.

Alice vs Bob



Size of the effect

- The effect is small and phenomenologically not relevant:
- $\Delta t \approx 10^{-3}$ s for the distance of 10^9 ly, and for 1GeV photon.
- However it poses a conceptual problem.

Many particles

- In the argument we implicitly assumed that the action of translation on two worldlines is independent; i.e., we assumed implicitly primitive `co-product' (Noether charges of particles are sums of the individual ones).
- Perhaps choosing non-trivial co-product helps?

Many particles

- In SR we have no real choice:

$$L = \sum_i L_i$$

- Here P is group valued and we have more possibilities.

Two particles Lagrangian

$$L_{(1+2)} = \left\langle \mathcal{P}_{(1)}{}^{-1} \frac{d}{d\tau} \mathcal{P}_{(1)}, q_{(1)} \right\rangle + \left\langle \left(\mathcal{P}_{(1)}\mathcal{P}_{(2)}\right)^{-1} \frac{d}{d\tau} \left(\mathcal{P}_{(1)}\mathcal{P}_{(2)}\right) - \mathcal{P}_{(1)}{}^{-1} \frac{d}{d\tau} \mathcal{P}_{(1)}, q_{(2)} \right\rangle$$

$$-\lambda_{(1)} \left(C(p_{(1)}, \kappa) - m_{(1)}{}^2 \right) - \lambda_{(2)} \left(C(p_{(2)}, \kappa) - m_{(2)}{}^2 \right)$$

$$= p_{(1)0} \dot{q}_{(1)}{}^0 + p_{(1)i} \dot{q}_{(1)}{}^i - \frac{1}{\kappa} p_{(1)i} q_{(1)}{}^i p_{(1)0}$$

$$+ p_{(2)0} \dot{q}_{(2)}{}^0 + e^{-p_{(1)0}/\kappa} p_{(2)i} \dot{q}_{(2)}{}^i - \frac{1}{\kappa} e^{-p_{(1)0}/\kappa} p_{(2)i} q_{(2)}{}^i p_{(2)0}$$

$$-\lambda_{(1)} \left(C(p_{(1)}, \kappa) - m_{(1)}{}^2 \right) - \lambda_{(2)} \left(C(p_{(2)}, \kappa) - m_{(2)}{}^2 \right)$$

Translational invariance

- The lagrangian is invariant under

$$\delta q_{(1)}{}^{0} = a^{0} - \frac{1}{\kappa}\vec{p}_{(1)}\vec{a},\ \delta q_{(2)}{}^{0} = a^{0} - \frac{1}{\kappa}e^{-p_{(1)0}/\kappa}\vec{p}_{(2)}\vec{a}$$

$$\delta q_{(1)}{}^{i} = a^{i},\ \delta q_{(2)}{}^{i} = a^{i}$$

- With conserved charges being

$$p_0^{tot} = p_{(1)0} + p_{(1)0},\quad \vec{p}^{tot} = \vec{p}_{(1)} + e^{-p_{(1)0}/\kappa}\vec{p}_{(2)}$$

- Which reflects κ–Poincaré co-product

$$\boxed{\Delta p_0 = p_0 \otimes 1 + 1 \otimes p_0,\quad \Delta\vec{p} = \vec{p}\otimes 1 + e^{-p_0/\kappa}\otimes\vec{p}}$$

κ-Poincaré particles

- This is nice, but clearly does not solve the worldlines problem.
- What can be done?

The wordlines problem

- p and q form useful parametrization of the phase space of the particles, but q is not directly related to position measurements in the spacetime. One should construct another variable x that does the job.
- But then the statement concerning the velocity of light is completely meaningless, of course.

The worldline problem

- Notice that we considered a universe consisting just of two particles. This would be OK. in SR where particles are really free.
- Here it might be necessary to average over all the rest of the `spectator' particles somehow.
- Perhaps one should consider quantum fields, not particles.

Summary

- The charges against DSR are very serious, and the current verdict seems `guilty'. But perhaps when we understand physics of DSR better, the situation will change.
- An important lesson is that algebra is not enough; to do physics we must understand physics.