Title: Modal quantum theory

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Abstract: In this talk we will explore a "toy model" of quantum theory that is similar to actual quantum theory, but uses scalars drawn from a finite field. The set of possible states of a system is discrete and finite. Our theory does not have a quantitative notion of probability, but only makes the "modal" distinction between possible and impossible measurement results. Despite its very simple structure, our toy model nevertheless includes many of the key phenomena of actual quantum systems: interference, complementarity, entanglement, nonlocality, and the impossibility of cloning.

Modal Quantum Theory

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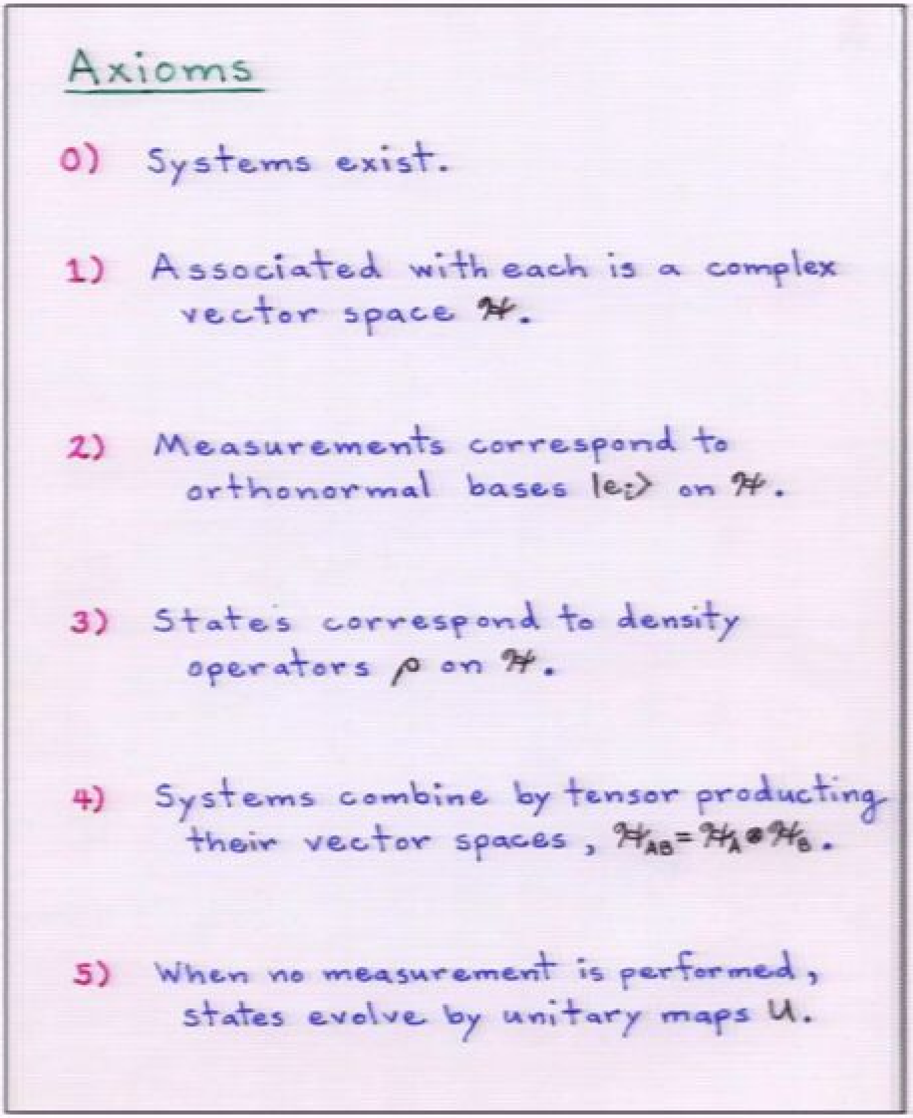
Actual quantum theory

Hilbert space in AQT

- Vectors describe pure states and basis states for measurements.
- Operators describe mixed states, generalized measurements, numerical observables, time evolution, etc.
- Probabilities are calculated from complex probability amplitudes

$$p(a) = |\langle a|\psi\rangle|^2$$



Axioms

0) Systems exist.

1) Associated with each is a complex vector space $\mathcal{H}$.

2) Measurements correspond to orthonormal bases $|e_i\rangle$ on $\mathcal{H}$.

3) States correspond to density operators ρ on $\mathcal{H}$.

4) Systems combine by tensor producting their vector spaces, $\mathcal{H}_{AB}=\mathcal{H}_A\otimes\mathcal{H}_B$.

5) When no measurement is performed, states evolve by unitary maps $\mathcal{U}$.

Chris Fuchs, "*The Oyster and the Quantum*"

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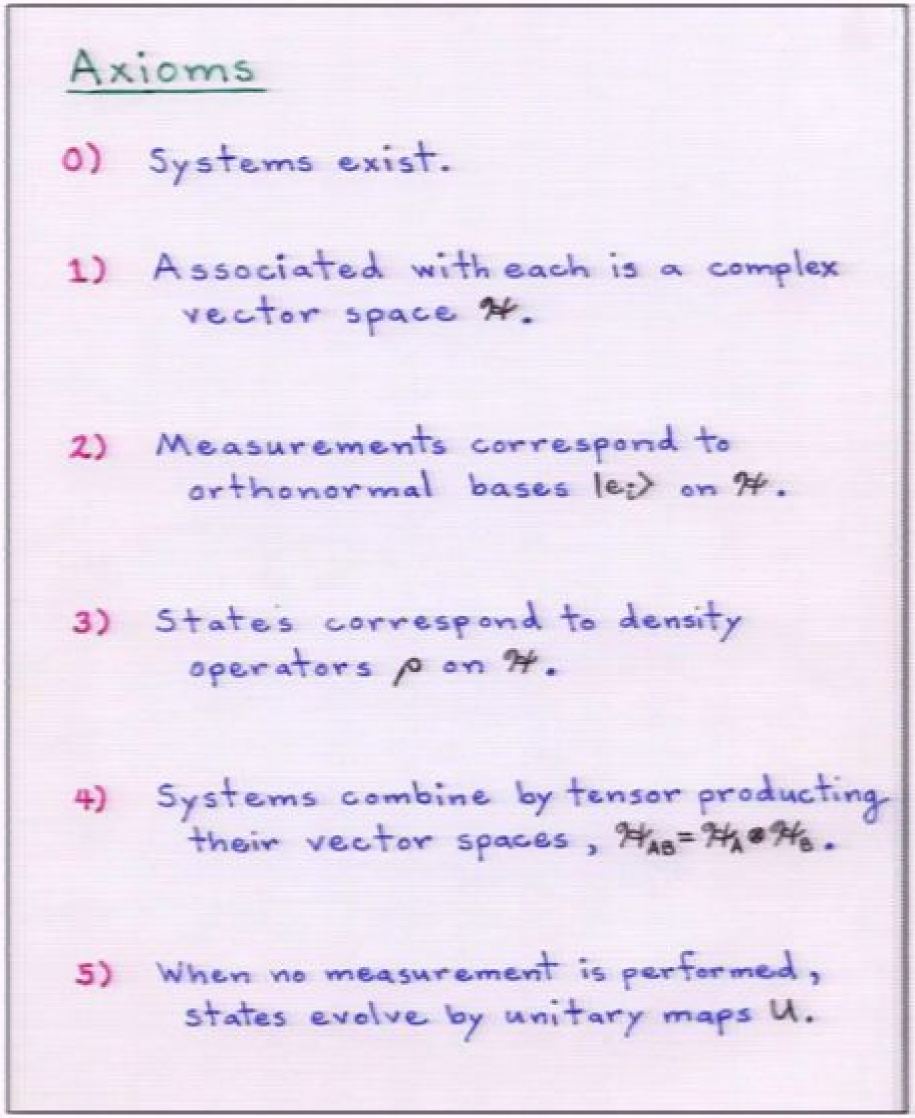

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But what if things were otherwise . . . ?

Probable versus possible

Probability

Set of events $\{\alpha\}$ with a probability measure p

$$0 \leq p(\alpha) \leq 1$$

$$\sum_{\alpha} p(\alpha) = 1$$

Events are more or less likely, depending on the value of $p(\alpha)$.

Possibility

Set of events $\{\alpha\}$ with a non-empty subset P

$$\alpha \in P \quad \text{(possible)}$$

$$\alpha \notin P \quad \text{(impossible)}$$

No additional concept of likelihood beyond "impossible", "possible", and "necessary".

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(Weak) connection to modal logic

Modal quantum theory

States

A system is described by a vector space $\mathcal{V}$ over a field $\mathcal{F}$.

- $\mathcal{F}$ might be finite.
- $\mathcal{V}$ need have no norm or inner product.
- The state of a system may be any non-zero $|\phi) \in \mathcal{V}$.

Basic measurements

The results $\{a\}$ of a measurement are associated with elements $\{|a)\}$ of a basis for $\mathcal{V}$.

$$|\phi) = \sum_a \phi_a |a).$$

If we expand a state $|\phi)$ in this basis, then $a \in P$ iff $\phi_a \neq 0$.

Linear evolution

In the absence of measurement, the state evolves by an invertible linear operator T.

$$|\phi(t)) = T(t,0)\,|\phi(0))$$

Composite systems $\mathcal{V}^{(12)} = \mathcal{V}^{(1)} \otimes \mathcal{V}^{(2)}$

"Mobits"

$1+1=0$

Simplest possible situation: $\mathcal{F} = Z_2$ and $\dim \mathcal{V} = 2$

Only three possible states $|0)$, $|1)$, $|\sigma) = |0) + |1)$

Three basis sets (X, Y, Z)

$$|+_x) = |1) \qquad |+_y) = |\sigma) \qquad |+_z) = |0)$$

$$|-_x) = |\sigma) \qquad |-_y) = |0) \qquad |-_z) = |1)$$

A cautionary tale: Given state $|\sigma)$, measure in a basis that includes $|0)$. Is this outcome possible?

Basis Z:

$$|\sigma) = |+_z) + |-_z)$$

$|+_z) = |0)$ possible!

Basis Y:

$$|\sigma) = |+_y)$$

$|-_y) = |0)$ not possible!

The dual view

A better way: Think about the dual space $\mathcal{V}^*$.

$$\{|a)\} \leftrightarrow \{(a|\}$$

$\mathcal{V}$ basis

$\mathcal{V}^*$ basis

$$(a|\phi) = \phi_a$$

in $|\phi) = \sum_a \phi_a |a)$

NB: Correspondence $|a) \leftrightarrow (a|$ depends on the *entire* basis.

- A measurement is associated with a basis for $\mathcal{V}^*$. (This always corresponds to a basis for $\mathcal{V}$ itself.)
- Each measurement result a is represented by a dual vector $(a|$ -- the "effect functional".
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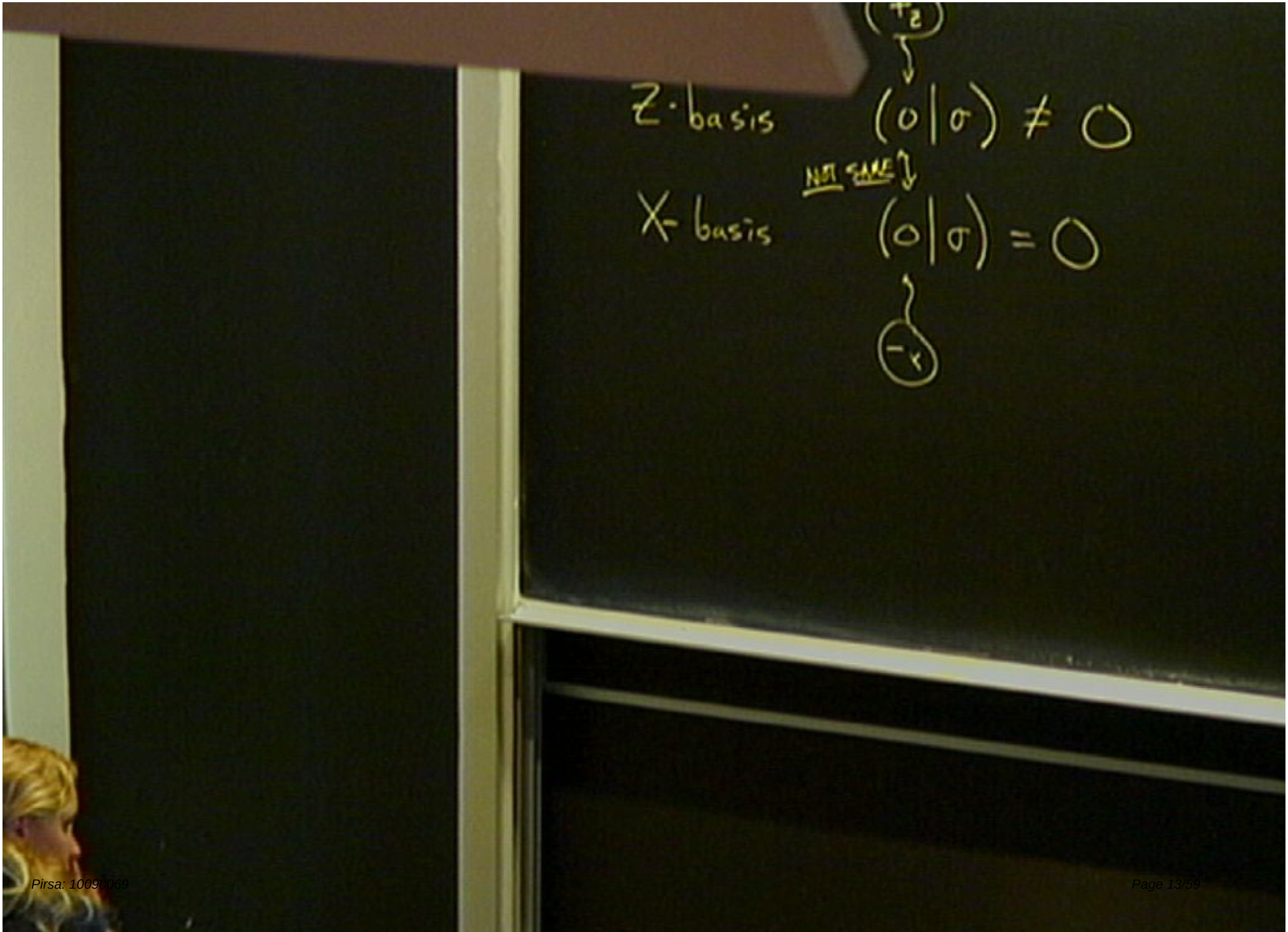


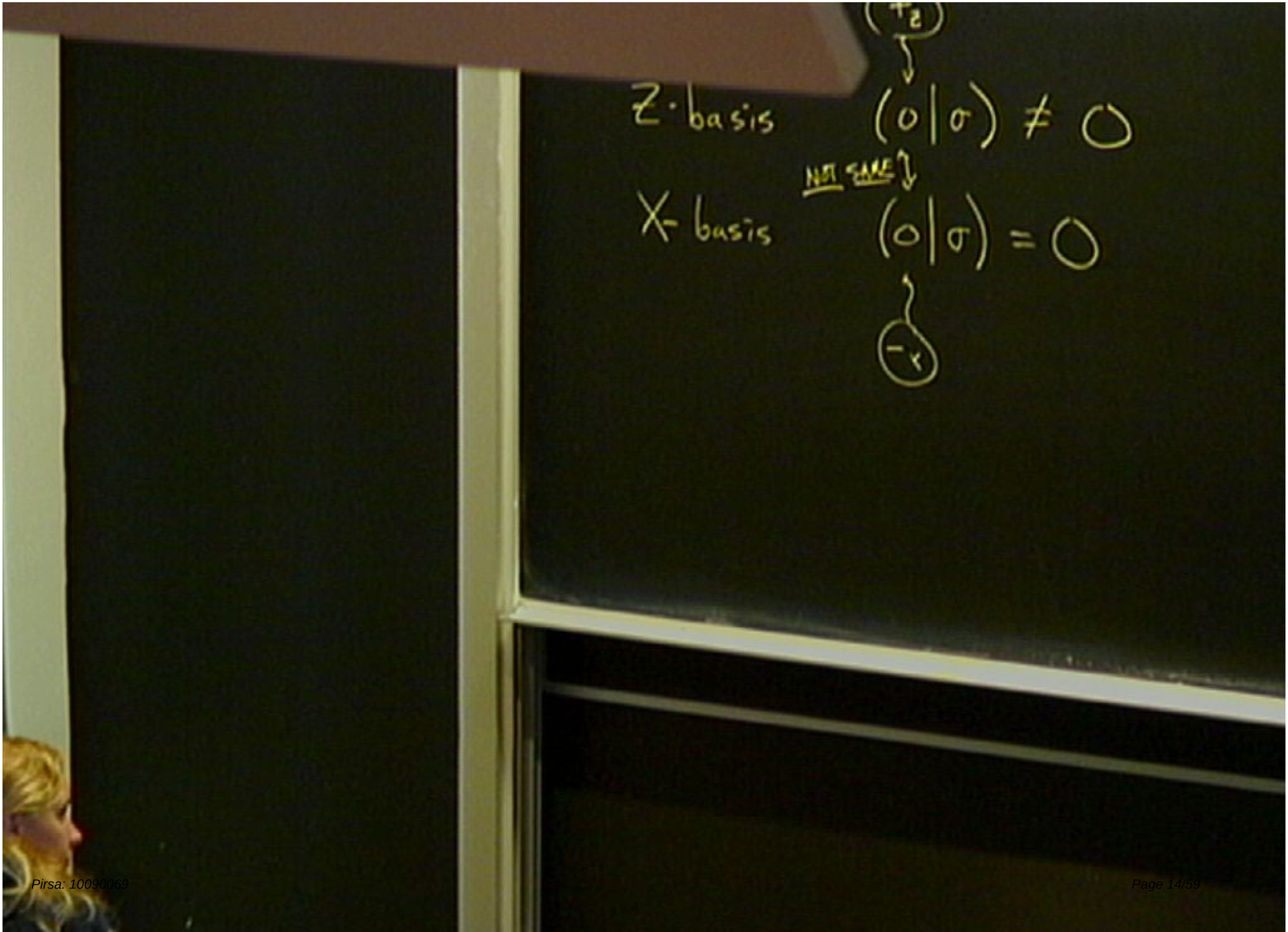
$+_z$

Z-basis $(0|\sigma) \neq 0$

NOT SAME

X-basis $(0|\sigma) = 0$

$-_x$



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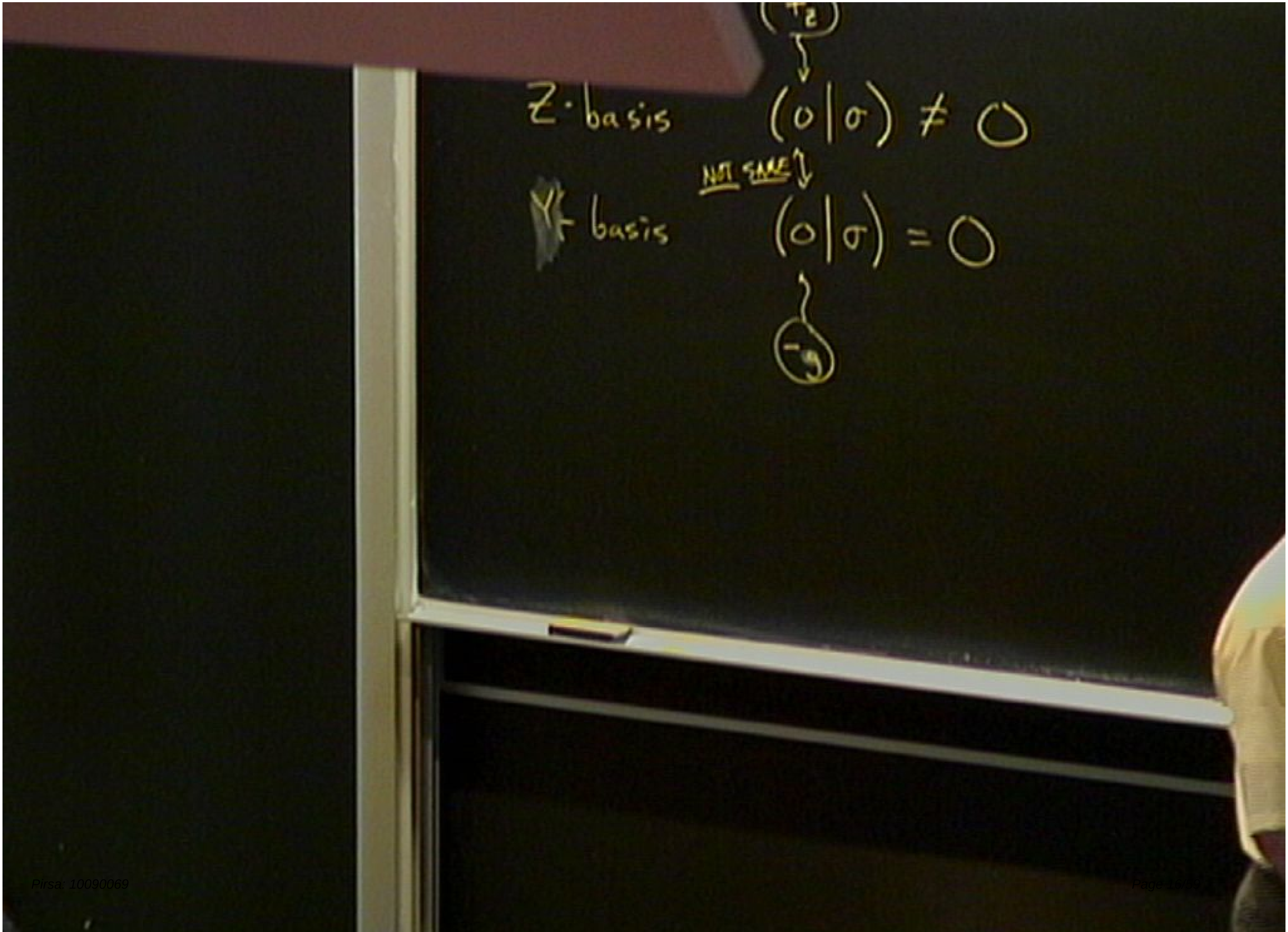

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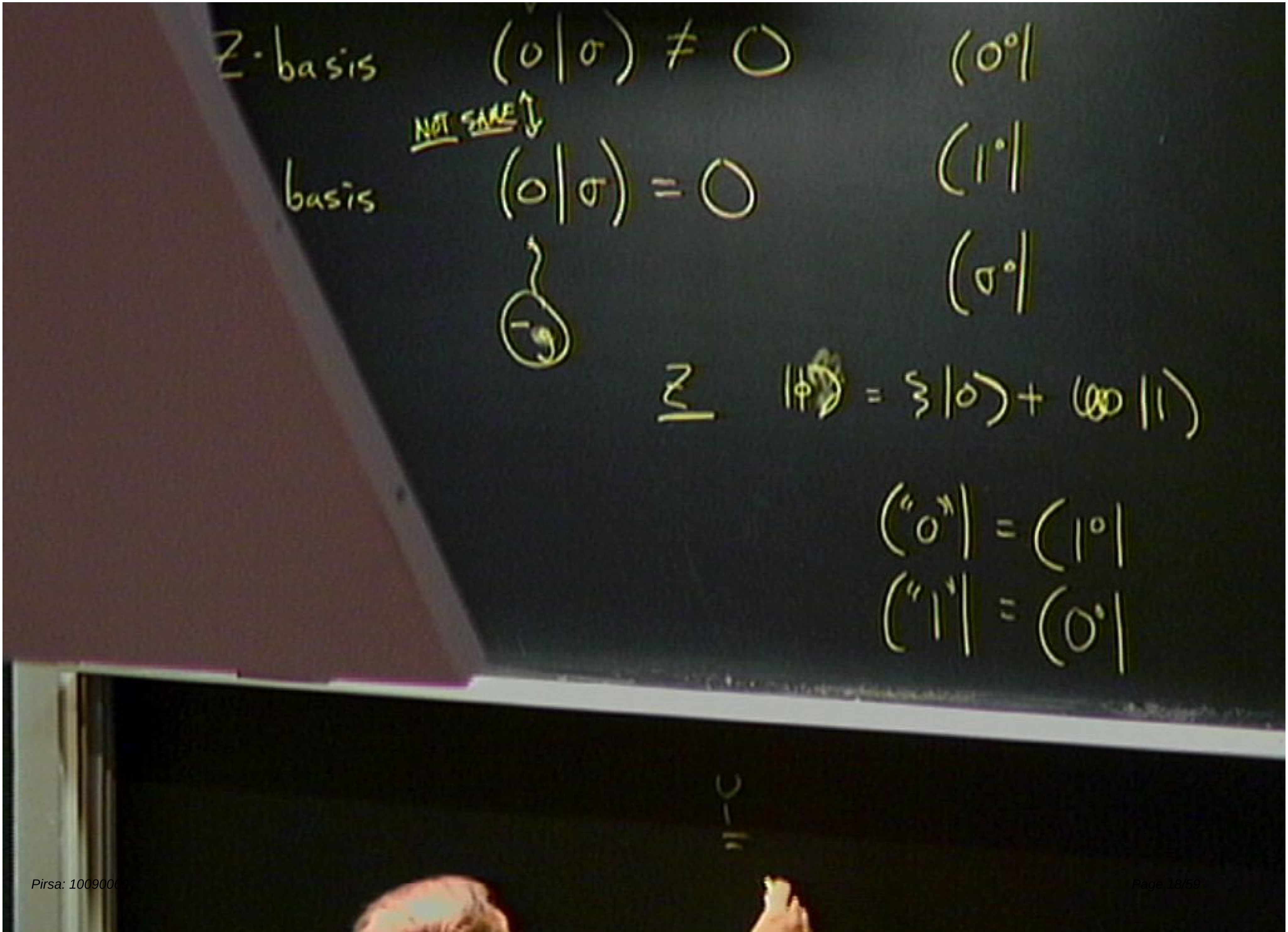


Z-basis $(0|\sigma) \neq 0$ $(0^\circ|$

NOT SAME

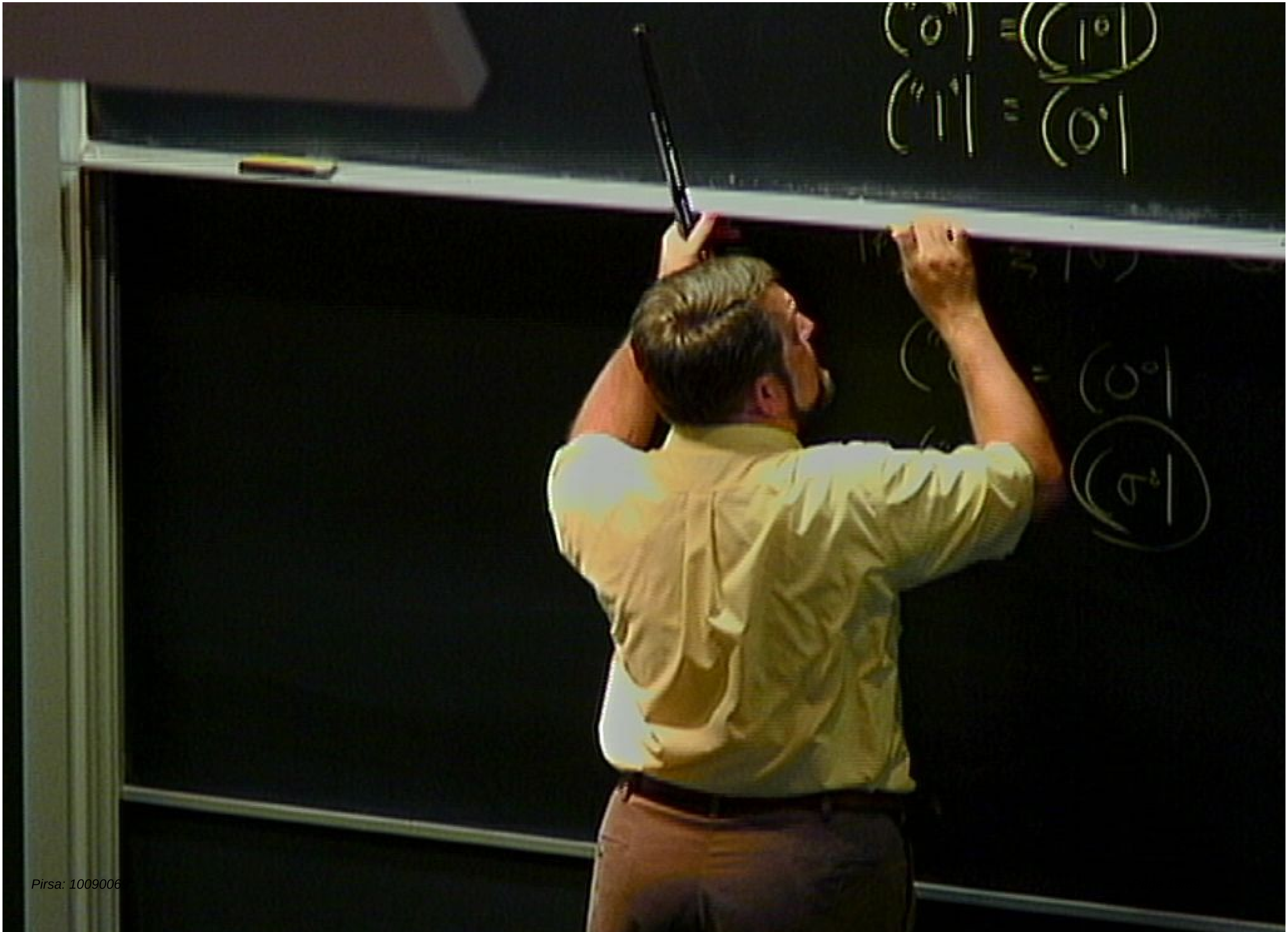
basis $(0|\sigma) = 0$ $(1^\circ|$

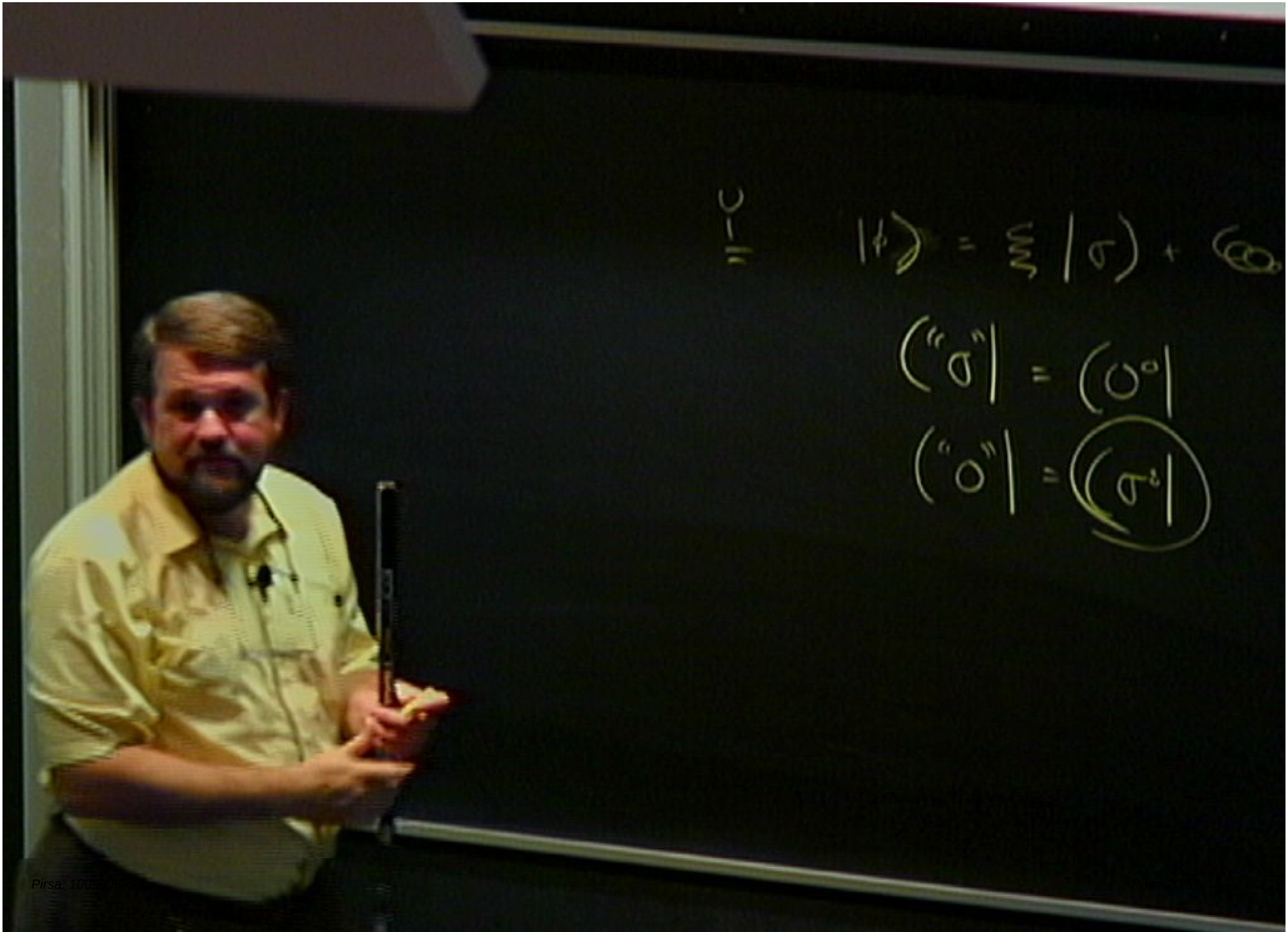
$(\sigma^\circ|$

$\underline{Z}$ $|\phi\rangle = \xi|0\rangle +$ [illegible] $|1\rangle$

$(``0"| = (1^\circ|$

$(``1"| = (0^\circ|$

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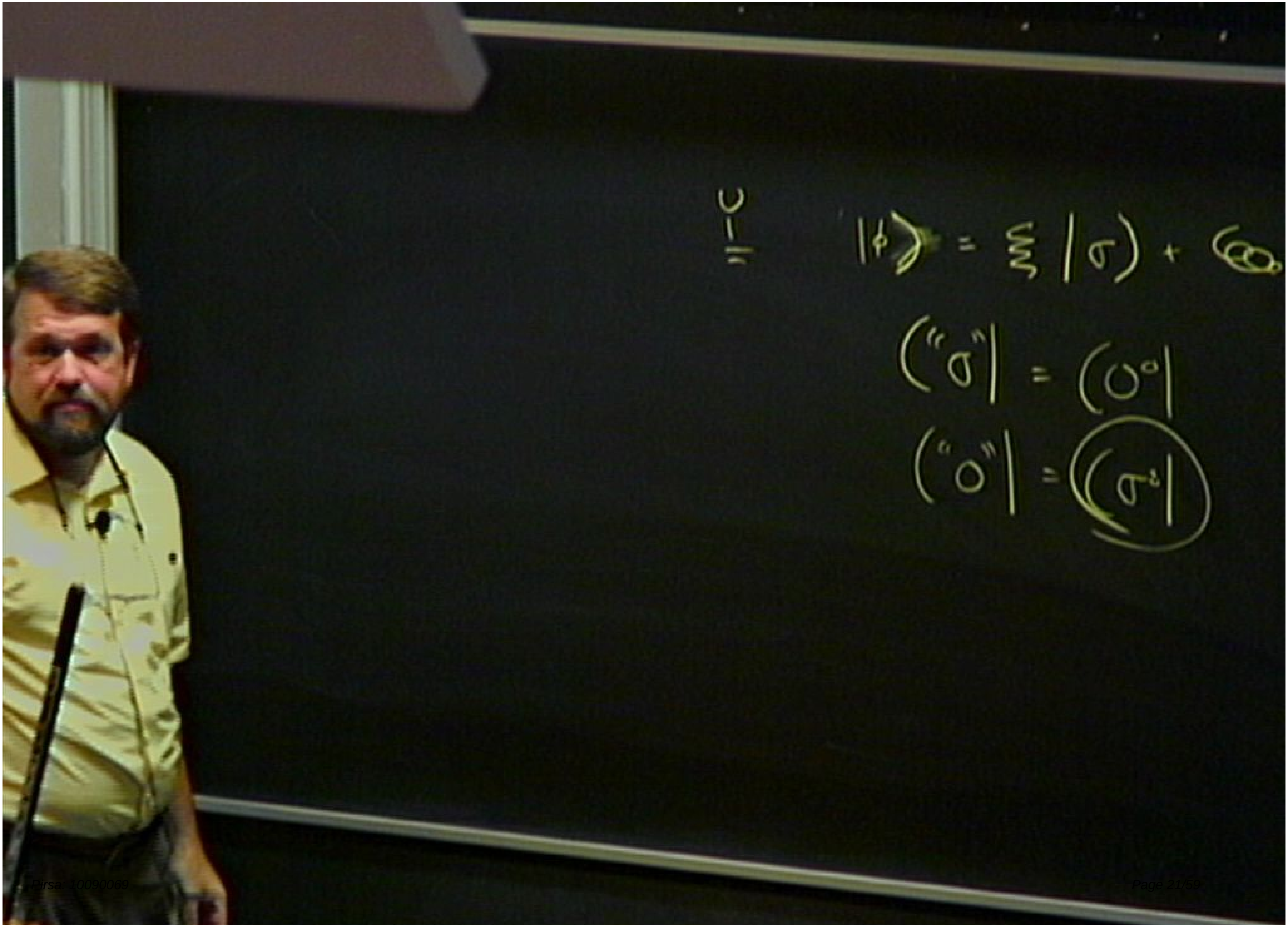
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This looks similar to AQT.

- In AQT, the result α is possible provided $\langle\alpha|\psi\rangle \neq 0$.
- However, the Hilbert space inner product in AQT fixes a natural correspondence $\langle\alpha| \leftrightarrow |\alpha\rangle$.

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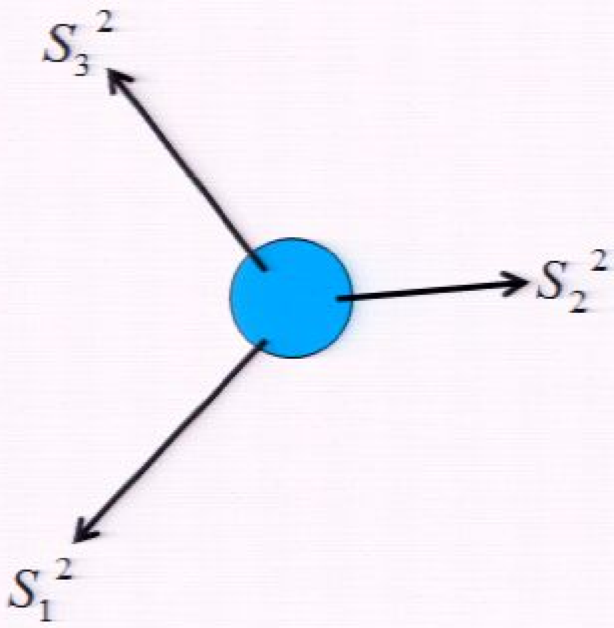
Contextuality in AQT



The Kochen-Specker theorem (AQT)

Spin-1 system

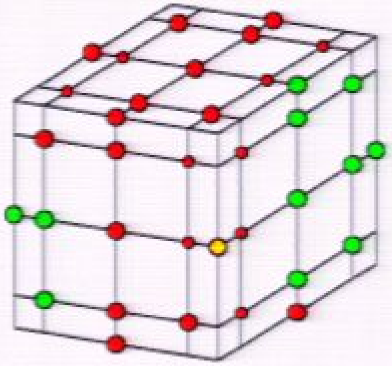
We can simultaneously measure S_i^2 for three orthogonal spin components.

We always get results like (+1,+1,0).

Two ideas

Hidden variables: The results of all measurements are predetermined

Non-contextuality: The result of one particular S_i^2 measurement does not depend on which other two orthogonal directions are also part of the measurement.



33 directions for Asher Peres's simplified version of the KS theore
(they used 117)

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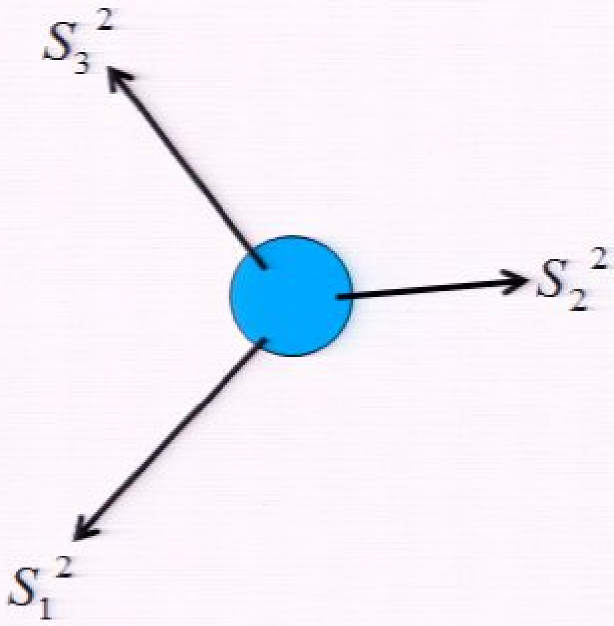

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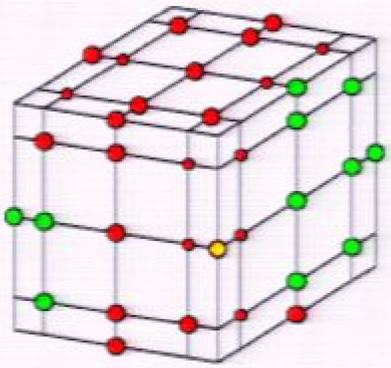

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KS theorem No theory of non-contextual hidden variables is consistent with spin-1 properties.

Contextuality in MQT

Mobit in Z_2-MQT

Three bases for V* (X, Y, Z):

$$\boxed{\begin{aligned} X &= \{(+_x|, (-_x|\} \\ Y &= \{(+_y|, (-_y|\} \\ Z &= \{(+_z|, (-_z|\} \end{aligned}} \Rightarrow \begin{aligned} (+_x| &= (-_y| \\ (+_y| &= (-_z| \\ (+_z| &= (-_x| \end{aligned}$$

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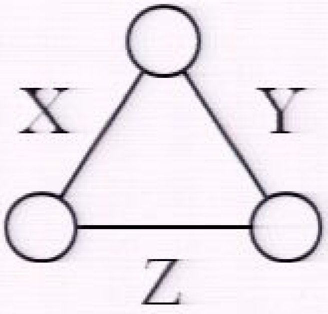

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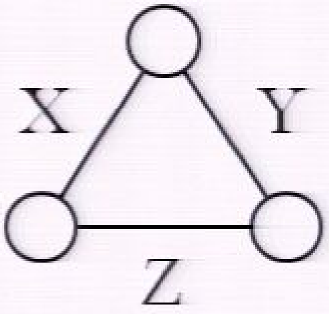

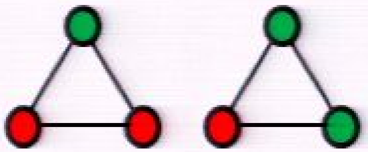

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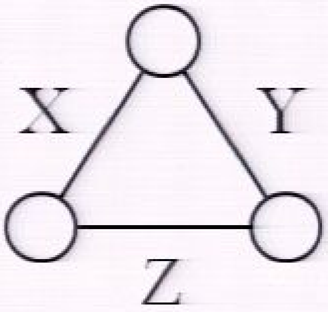

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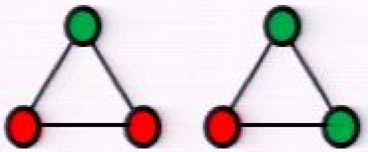
Hidden variables: The results of all measurements are predetermined (only one possible result).

Non-contextuality: Whether an effect $(a|$ actually occurs does not depend on which other effects are part of the measurement.

KS theorem (a la MQT): No theory of non-contextual hidden variables is consistent with mobit properties.



Entanglement

Composite systems in MQT have both product and entangled states.

Two mobits in Z_2-MQT:

$\mathcal{V} \otimes \mathcal{V}$ contains 16 vectors (15 states), including

- 9 product states $|a,b) = |a) \otimes |b)$
- 6 entangled states e.g., $|S) = |0,1) + |1,0)$

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In AQT, Bell's theorem states that the statistical properties of entangled quantum states are inconsistent with any theory of local hidden variables. (Q-violation of some Bell inequality.)

Can we exclude local hidden variables in MQT?

Bell approach won't work -- no probabilities, no mean values, etc.

Hardy's theorem (AQT)

A pair of qubits (#1 and #2)

- State $|\Psi\rangle$ is a non-maximal entangled state.
- Binary (0/1) measurements A and B on each qubit.
- (x,y|X,Y) = joint result (x,y) for the joint measurement (X,Y) on the pair.

Hardy: We can find A and B such that

- (0,0|A,B) and (0,0|B,A) are both impossible ($p = 0$). (All other results are possible.)
- (0,0|B,B) is possible ($p > 0$).
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Under LHV assumptions:

(0,0|B,B) possible,
but not (0,0|A,B)
or (0,0|B,A)

⇓

For those cases,
(1,0|A,B) and
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⇓

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Contradicts QM!

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No local hidden variable theory is consistent with this set of possibilities and impossibilities.

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MQT "singlet"

Consider |S) = |0,1) + |1,0)

Measure (X,X) , (Y,Y) , or (Z,Z):

$$\begin{aligned} |S\,) &= |+_z,-_z\,)+|-_z,+_z\,) \\ &= |+_x,-_x\,)+|-_x,+_x\,) \\ &= |+_y,-_y\,)+|-_y,+_y\,) \end{aligned}$$

Results of "parallel" measurements must always disagree!

Measure (Z,X) , etc.:

$$|S\,) = |+_z,+_x\,)+|-_z,+_x\,)+|-_z,-_x\,)$$

One joint result -- in this case (+,–) -- is always impossible.

Local hidden variables: In each run of our experiment, joint outcomes (x y z : x' y' z') exist for all 6 possible measurements.

State |S) corresponds to some collection of possible 6-tuples.

LHV for |S) in MQT?

What 6-tuples (x y z : x' y' z') could appear in the possible set for |S) ?

Since parallel measurements (X,X), (Y,Y) and (Z,Z) must always disagree, there are only 8 candidates:

$$
\begin{gathered}
(+++:---)\\
(++-:--+)\\
(+-+:-+-)\\
(+--:-++)\\
(-++:+--)\\
(-+-:+-+)\\
(--+:+++)\\
(---:+++)
\end{gathered}
$$

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(+ + − : − − +)
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(+ − − : − + +)
(− + + : + − −)
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(− − + : + + +)
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Measure (Z,X): (+,–) impossible.
We cannot have (+ y + : – y' –).

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(X,Z): no (− y − : + y' +)
(X,Y): no (+ + z : − − z')
(Y,X): no (− − z : + + z')
(Y,Z): no (x + + : x' − −)
(Z,Y): no (x − − : x' + +)

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(Z,X): no (+ y + : − y' −)
(X,Z): no (− y − : + y' +)
(X,Y): no (+ + z : − − z')
(Y,X): no (− − z : + + z')
(Y,Z): no (x + + : x' − −)
(Z,Y): no (x − − : x' + +)

No local hidden variable theory is consistent with this set of possibilities and impossibilities.

Superdense coding

Single-mobit evolution operators

$$G|0) = |1) \qquad K|0) = |0)$$

$$G|1) = |0) \qquad K|1) = |0) + |1)$$

Basis of entangled mobit states

$$|R) = |0,0) + |1,1)$$

$$|S) = |0,1) + |1,0)$$

$$|U) = |0,0) + |1,0) + |1,1)$$

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Transformations of states

$$|S) = G^{(1)}|R)$$

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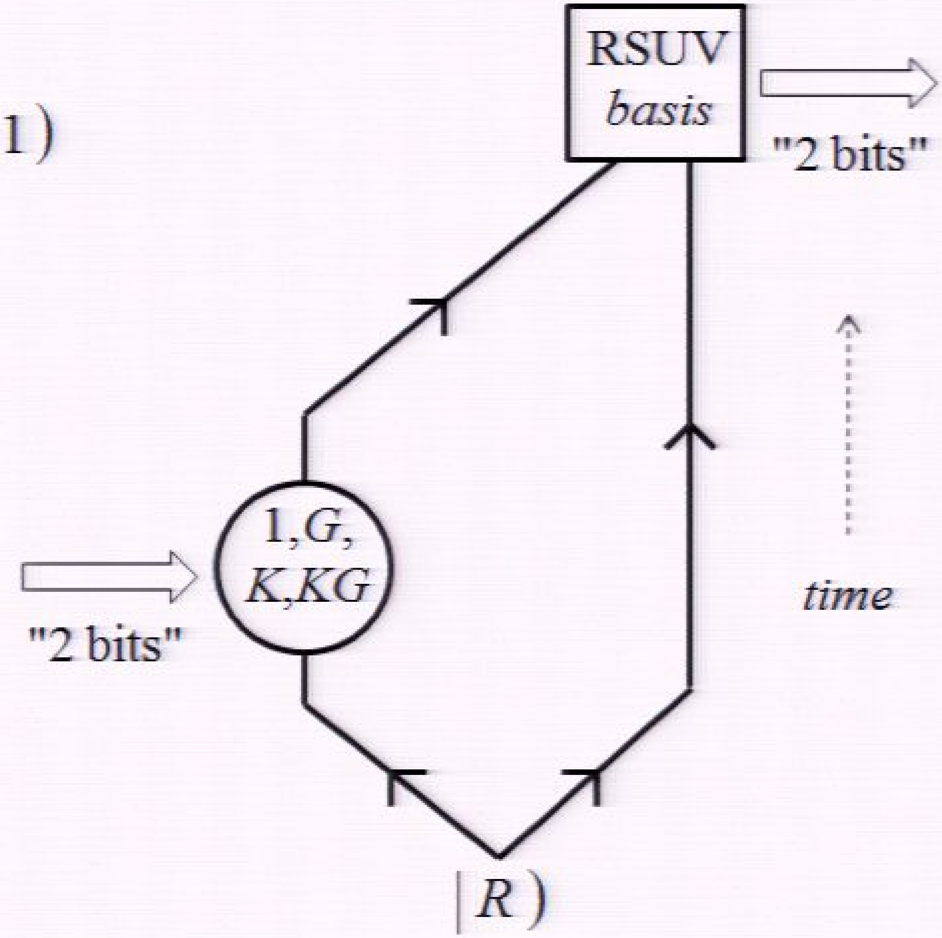

Superdense coding

Single-mobit evolution operators

$$G|0) = |1) \qquad K|0) = |0)$$
$$G|1) = |0) \qquad K|1) = |0)+|1)$$

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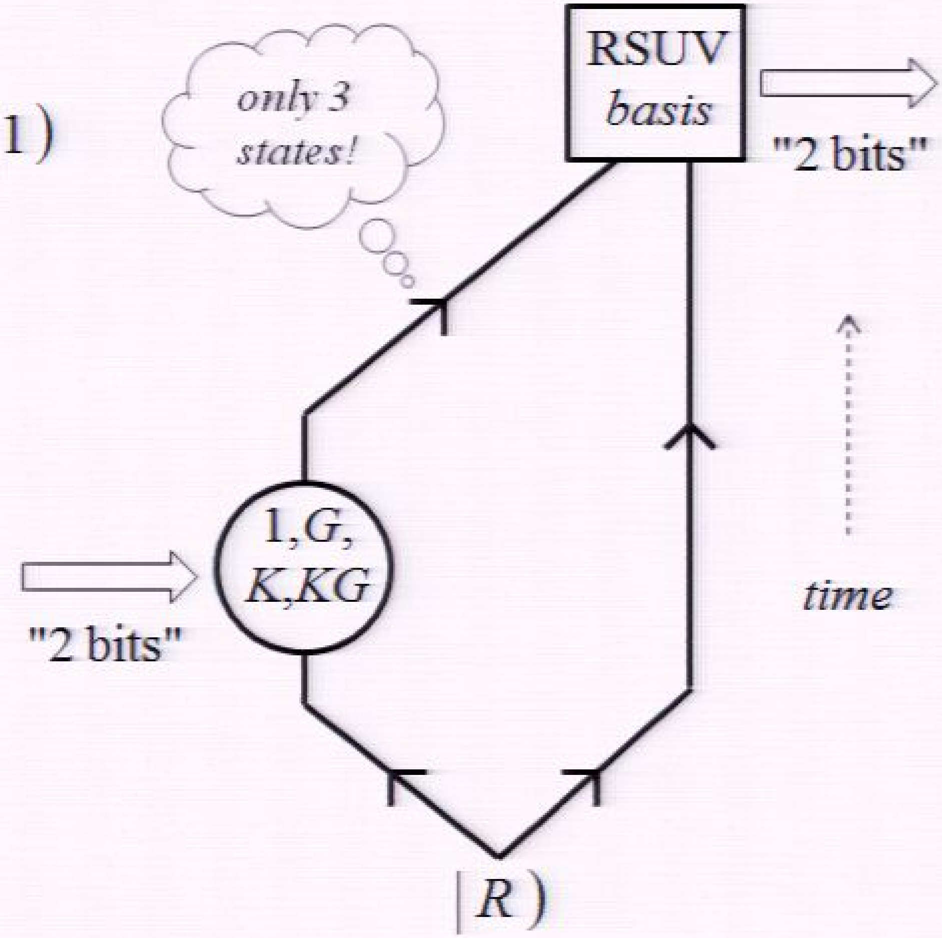

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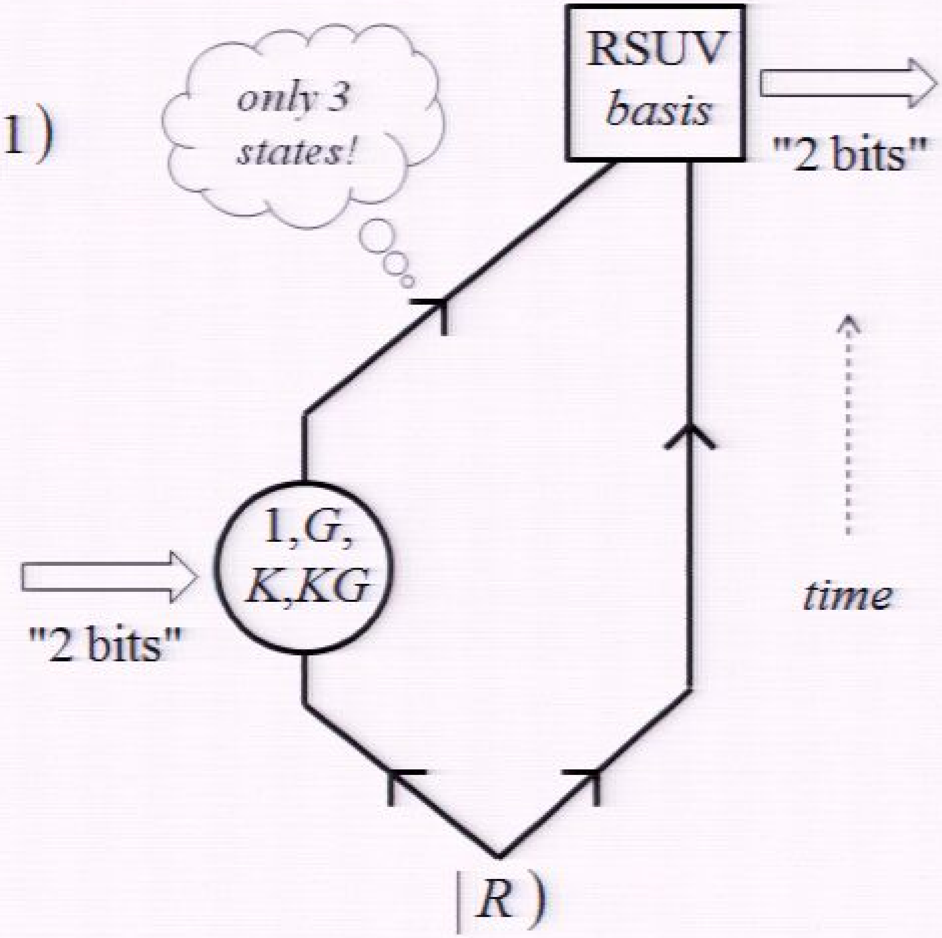


We can also do teleportation in MOT

Mixed states

Mixture = collection of possible states: $M = \{\, |a),\, |b),\, \ldots \}$

Mixtures M and M' are equivalent iff they span the same subspace. A mixed state $\langle \mathrm{M} \rangle$ is a subspace.

Entangled state $\left|\Phi^{(12)}\right) = \sum_a \left|a^{(1)}\right) \otimes \left|\phi_a^{(2)}\right)$

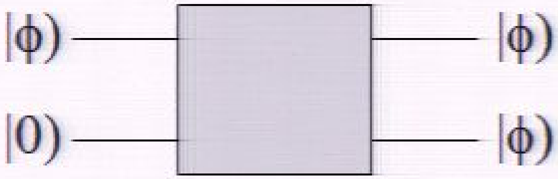
$\{|a)\}$-basis for system 1

System 2 is in the mixed state $\left\langle \left\{ \left|\phi_a^{(2)}\right) \right\} \right\rangle$

This is a mixture of conditional states given an $\{|a)\}$-basis measurement on system 1.

By choosing this basis, we can "steer" the mixture (but not the overall mixed state) for system 2.

No cloning in MQT!



Assume machine works on distinct input states $|a)$ and $|b)$:

$$|a,0,\mu_0) \rightarrow |a,a,\mu_a)$$

$$|b,0,\mu_0) \rightarrow |b,b,\mu_b)$$

Overall time evolution is linear. Consider input $|c) = |a) + |b)$

$$|c,0,\mu_0) \rightarrow |a,a,\mu_a) + |b,b,\mu_b) \neq |c,c,\mu_c)$$

Final state of original (or clone): $\langle \{ |a), |b) \} \rangle \neq \langle \{ |c) \} \rangle$

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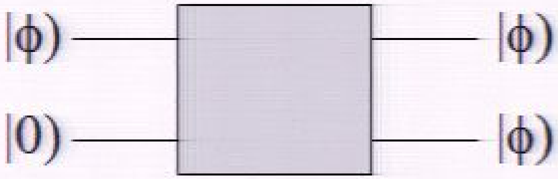

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No cloning machine in MQT
can work for all input states.

Bugs and features

What MQT **does not** have

- Probabilities, expectations
- (Finite $\mathcal{F}$) Continuous sets of states and observables, or continuous time evolution
- Inner product, outer product, orthogonality
- Convexity
- Hermitian conjugation ($\dagger$)
- Effect operators
- Density operators

What MQT **does** have

- "Classical" versus "quantum" theories
- Superposition, interference
- Complementarity
- Linear dynamics
- Measurement theory, effect functionals
- Entanglement
- Superdense coding
- Mixed states
- Steering of ensembles
- No local hidden variables!
- No cloning!
- Nonclassical models of computation

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... and both lists are very incomplete!

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Advertisement

Open systems in modal quantum theory

Mike Westmoreland

Denison University

Tuesday, October 6 at 4:00 pm

Room 301

Entanglement

Composite systems in MQT have both product and entangled states.

Two mobits in Z_2-MQT:

$\mathcal{V} \otimes \mathcal{V}$ contains 16 vectors (15 states), including

- 9 product states $|a,b) = |a) \otimes |b)$
- 6 entangled states e.g., $|S) = |0,1) + |1,0)$

Probable versus possible

Probability

Set of events $\{\alpha\}$ with a probability measure p

$$0 \le p(\alpha) \le 1$$

$$\sum_{\alpha} p(\alpha) = 1$$

Events are more or less likely, depending on the value of $p(\alpha)$.

Possibility

Set of events $\{\alpha\}$ with a non-empty subset P

$$\alpha \in P \quad \text{(possible)}$$

$$\alpha \notin P \quad \text{(impossible)}$$

No additional concept of likelihood beyond "impossible", "possible", and "necessary".

Modal quantum theory

States

A system is described by a vector space $\mathcal{V}$ over a field $\mathcal{F}$.

- $\mathcal{F}$ might be finite.
- $\mathcal{V}$ need have no norm or inner product.
- The state of a system may be any non-zero $|\phi) \in \mathcal{V}$.

Basic measurements

The results $\{a\}$ of a measurement are associated with elements $\{|a)\}$ of a basis for $\mathcal{V}$.

$$|\phi) = \sum_a \phi_a |a).$$

If we expand a state $|\phi)$ in this basis, then $a \in P$ iff $\phi_a \neq 0$.

Linear evolution

In the absence of measurement, the state evolves by an invertible linear operator T.

$$|\phi(t)) = T(t,0)\,|\phi(0))$$

Composite systems $\mathcal{V}^{(12)} = \mathcal{V}^{(1)} \otimes \mathcal{V}^{(2)}$

"Mobits"

$1+1=0$

Simplest possible situation: $\mathcal{F}=Z_2$ and $\dim\mathcal{V}=2$

Only three possible states $|0),\ |1),\ |\sigma)=|0)+|1)$

Three basis sets (X, Y, Z)

$$|+_x) = |1) \qquad |-_x) = |\sigma)$$

$$|+_y) = |\sigma) \qquad |-_y) = |0)$$

$$|+_z) = |0) \qquad |-_z) = |1)$$

A cautionary tale: Given state $|\sigma)$, measure in a basis that includes $|0)$. Is this outcome possible?

Basis Z:

$$|\sigma) = |+_z)+|-_z)$$

$|+_z) = |0)$ possible!

Basis Y:

$$|\sigma) = |+_y)$$

$|-_y) = |0)$ not possible!

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In AQT, Bell's theorem states that tl
entangled quantum states are ir
local hidden variables. (Q-viol

Can we exclude local hidden variab

Bell approach won't work -- no prol

Hardy's theorem (AQT)

A pair of qubits (#1 and #2)

- State $|\Psi\rangle$ is a non-maximal entangled state.
- Binary (0/1) measurements A and B on each qubit.
- (x,y|X,Y) = joint result (x,y) for the joint measurement (X,Y) on the pair.

Hardy: We can find A and B such that

- (0,0|A,B) and (0,0|B,A) are both impossible (p = 0). (All other results are possible.)
- (0,0|B,B) is possible (p > 0).
- (1,1|A,A) is impossible (p = 0).

No local hidden variable theory is consistent with this set of possibilities and impossibilities.

Under LHV assumptions:

(0,0|B,B) possible, but not (0,0|A,B) or (0,0|B,A)

⇓

For those cases, (1,0|A,B) and (0,1|B,A)

⇓

(1,1|A,A)

Contradicts QM!

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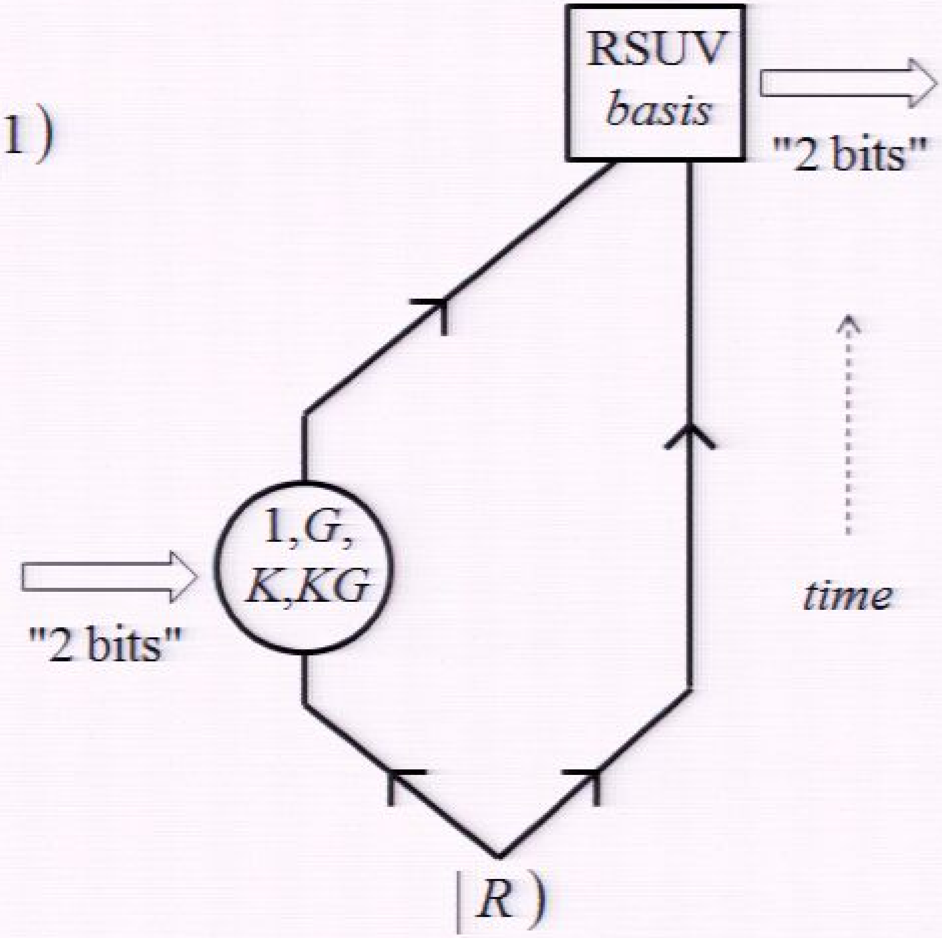

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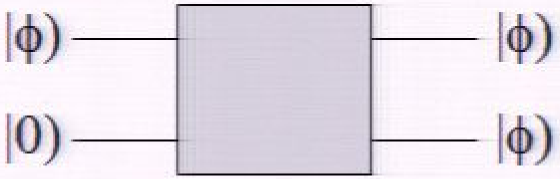

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