

Title: Relativity (PHYS 604) - Lecture 12

Date: Sep 28, 2010 10:30 AM

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Abstract:



Reparameterisation Invariance
= General Coordinate invariance
on the world line



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$$S = -m \int d\lambda \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}$$



Reparameterisation Invariance

= General Coordinate invariance
the world line

$$S = -m \int d\lambda \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}$$

$$\rightarrow x'^\mu(x)$$

$$S' = -m \int d\lambda \sqrt{-g'_{\mu\nu}(x') \frac{dx'^\mu}{d\lambda} \frac{dx'^\nu}{d\lambda}}$$

$$\lambda = x$$



Reparameterisation Invariance

= General Coordinate invariance
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$$S' = -m \int d\lambda \sqrt{-g'_{\mu\nu}(x') \frac{dx'^\mu}{d\lambda} \frac{dx'^\nu}{d\lambda}}$$

$$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$$



Reparameterisation Invariance

= General Coordinate invariance
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$$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$$

$$\frac{\partial x^\alpha}{\partial x'^\mu} \frac{dx'^\mu}{d\lambda} = \frac{dx^\alpha}{d\lambda}$$

$$\Rightarrow S'' = S'$$

$$\lambda \rightarrow \lambda'(\lambda)$$



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$$\Rightarrow \frac{d\lambda'}{d\lambda} \neq 0$$

$$W: \lambda \rightarrow \tilde{\lambda}(\lambda)$$



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$$| - | \Rightarrow \frac{d\tilde{\lambda}}{d\lambda} \neq 0$$

ie. x^{μ} transforms as a scalar under reparameterisations of W .

$$\text{e.g. } (x, y) = (\cos \lambda, \sin \lambda)$$

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$$\tilde{x}^{\mu}(\tilde{\lambda}) = x^{\mu}(\lambda)$$



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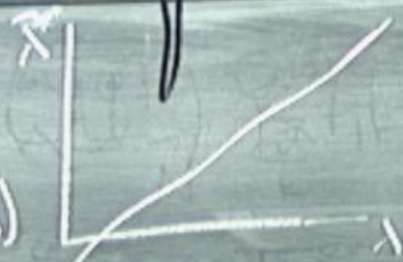
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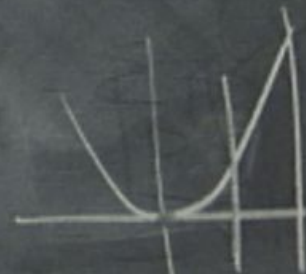
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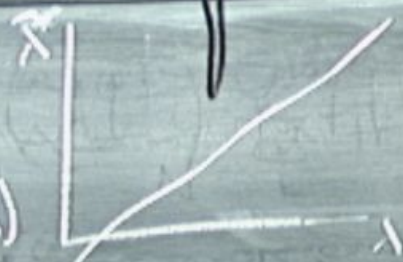
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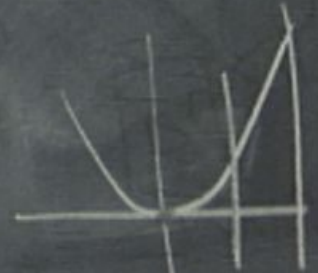
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geodesic eq.

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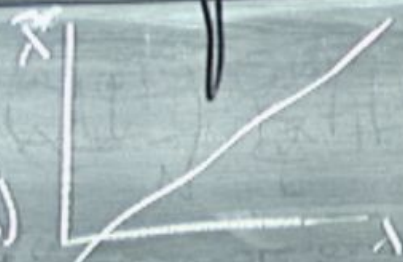
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geodesic eqⁿ following from δ involved

$$\frac{1}{\sqrt{-g_{\mu\nu} \frac{dx^{\mu}}{d\lambda} \frac{dx^{\nu}}{d\lambda}}} \frac{dx^{\mu}}{d\lambda}$$

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$$\frac{1}{\sqrt{-g_{\mu\nu} \frac{dx^{\mu}}{d\lambda} \frac{dx^{\nu}}{d\lambda}}} \frac{dx^{\mu}}{d\lambda} \Rightarrow x^{\mu}(\lambda)$$

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$$\frac{1}{\sqrt{-g_{\mu\nu} \frac{dx^{\mu}}{d\lambda} \frac{dx^{\nu}}{d\lambda}}} \frac{dx^{\mu}}{d\lambda} \Rightarrow \text{if } X^{\mu}(\lambda) \text{ is a sol. then } \tilde{X}^{\mu}(\tilde{\lambda})$$

$\lambda \rightarrow \tilde{\lambda}(\lambda)$
 $W: \tilde{x}^{\mu}(\tilde{\lambda}) = x^{\mu}(\lambda)$

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$\tilde{\lambda} > 0$

geodesic eqⁿ following from $\mathcal{L}$ involved

$$\frac{1}{\sqrt{-g_{\mu\nu} \frac{dx^{\mu}}{d\lambda} \frac{dx^{\nu}}{d\lambda}}} \frac{dx^{\mu}}{d\lambda}$$

$\Rightarrow$ if $x^{\mu}(\lambda)$ is a solⁿ. then $\tilde{x}^{\mu}(\tilde{\lambda}) = x^{\mu}(\lambda(\tilde{\lambda}))$



$$\frac{1}{\sqrt{-\tilde{g}_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}} \frac{d}{d\lambda}$$

$$\frac{1}{\sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}} \frac{d}{d\lambda}$$

$$x^\mu \rightarrow x'^\mu(x)$$

$$S' = -m \int d\lambda \sqrt{-g'_{\mu\nu}(x') \frac{dx'^\mu}{d\lambda} \frac{dx'^\nu}{d\lambda}}$$

$$g_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$$

$$\frac{\partial x^\alpha}{\partial x'^\mu} \frac{dx'^\mu}{d\lambda} = \frac{dx^\alpha}{d\lambda}$$

$$\Rightarrow S' = S$$



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$$= \frac{1}{\sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}} \frac{dx^\mu}{d\lambda}$$

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$$P_m = i\hbar \frac{\partial}{\partial x}$$

$$P_m = i\hbar \frac{\partial}{\partial m}$$

$$P_m = -i\hbar \frac{\partial}{\partial x_m}$$

$$\rightarrow H =$$

$$P_\mu = i\hbar \frac{\partial}{\partial x^\mu}$$

$$\mathbb{H}\Psi = \left[-\frac{\hbar^2}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \frac{\partial}{\partial x^\nu}) + m^2 \right] \Psi = 0$$

$$P_\mu = i\hbar \frac{\partial}{\partial x^\mu}$$

Bryce de Witt
 Rev. Mod. Phys.
 late '50s

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↑
 $+ \Xi R$
 -

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When we do QFT, this equation becomes
 the field equation for the quantum field ϕ

What is the action for φ ?

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$$S_{\varphi} = \int d^4x \sqrt{-g} ($$

φ

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$$S_{\varphi} = \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} \partial_{\mu} \varphi \partial_{\nu} \varphi - \frac{1}{2} m^2 \varphi^2 \right)$$

φ

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$$\frac{\delta S_\varphi}{\delta \varphi} = 0 \Rightarrow H\varphi = 0.$$

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$$\delta \varphi \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi)$$

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$$\int d^4x \delta\varphi \left[\partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi) - m^2 \varphi \right] = 0 \quad \forall \delta\varphi$$

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$$\times \delta\varphi \left[\underbrace{\partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi)}_{\sqrt{-g} H} - \sqrt{-g} m^2 \varphi \right] = 0 \quad \forall \delta\varphi$$

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What is the action for φ ?

$$(set \hbar=1) \quad S_{\varphi} = \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} \partial_{\mu} \varphi \partial_{\nu} \varphi - \frac{1}{2} m^2 \varphi^2 \right)$$

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$$\delta S_{\varphi} = \int d^4x \sqrt{-g} \left(-g^{\mu\nu} \partial_{\mu} \delta \varphi \partial_{\nu} \varphi - m^2 \varphi \delta \varphi \right)$$

$$\int d^4x \underbrace{\delta \varphi \left[\underbrace{(\sqrt{-g} g^{\mu\nu} \partial_{\nu} \varphi)}_{-\sqrt{-g} H \varphi} - \sqrt{-g} m^2 \varphi \right]}_{=0 \quad \forall \delta \varphi}$$

What is the action for φ ?

$$\left(\text{set } \frac{\hbar}{c} = 1\right) \quad S_{\varphi} = \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} \partial_{\mu} \varphi \partial_{\nu} \varphi - \frac{1}{2} m^2 \varphi^2 \right)$$

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$$\delta S_{\varphi} = \int d^4x \sqrt{-g} \left(-g^{\mu\nu} \partial_{\mu} \delta \varphi \partial_{\nu} \varphi - m^2 \varphi \delta \varphi \right)$$

$$\int d^4x \underbrace{\delta \varphi \left[\underbrace{\partial_{\mu} (\sqrt{-g} g^{\mu\nu} \partial_{\nu} \varphi)}_{-\sqrt{-g} H \varphi} - \sqrt{-g} m^2 \varphi \right]}_{-\sqrt{-g} H \varphi} = 0 \quad \forall \delta \varphi$$

What is the action for φ ?

(set $\hbar=1$
 $c=1$)

$$p = \hbar \frac{\partial}{\partial x} = m$$

$$S_\varphi = \int d^4x \sqrt{-g} \left(-\frac{\hbar^2}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} m^2 \varphi^2 \right)$$

$$\frac{\delta S_\varphi}{\delta \varphi} = 0$$

$$p = 0.$$

$$\delta S_\varphi = \int d^4x \sqrt{-g} \left(g^{\mu\nu} \partial_\mu \delta \varphi \partial_\nu \varphi - m^2 \varphi \delta \varphi \right)$$

$$\left[\partial_\mu \varphi \partial^\mu \varphi - \sqrt{-g} m^2 \varphi \right] = 0 \quad \forall \delta \varphi$$

What is the action for φ ?

(set $\hbar=1$)
c=

$$p = \hbar \frac{\partial}{\partial x^m} = m$$

$$S_\varphi = \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} \frac{m^2}{\hbar^2} \varphi^2 \right)$$

$$\varphi = \tilde{\varphi}/\hbar$$

$$\frac{\delta S_\varphi}{\delta \varphi} = 0 \Rightarrow H\varphi = 0.$$

$$\delta S_\varphi = \int d^4x \sqrt{-g} \left(-g^{\mu\nu} \partial_\mu \delta\varphi \partial_\nu \varphi - m^2 \varphi \delta\varphi \right)$$

$$\int d^4x \delta\varphi \left[\underbrace{\partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi)}_{-\sqrt{-g} H\varphi} - \sqrt{-g} m^2 \varphi \right] = 0 \quad \forall \delta\varphi$$

What is the action for φ ?

"kinetic"

(set $\hbar=1$)
 $c=$

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$$T_{\varphi}^{mv} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\varphi}}{\delta g_{mv}}$$

$$T_{\varphi}^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\varphi}}{\delta g_{\mu\nu}} = g^{\mu\nu} \left(-\frac{1}{2} g^{\mu\nu} \partial_{\mu} \varphi \partial_{\nu} \varphi - \frac{1}{2} m^2 \varphi^2 \right) + \partial^{\mu} \varphi \partial^{\nu} \varphi$$

$$T_{\varphi}^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\varphi}}{\delta g_{\mu\nu}} = g^{\mu\nu} \left(-\frac{1}{2} g^{\mu\nu} \partial_{\mu} \varphi \partial_{\nu} \varphi - \frac{1}{2} m^2 \varphi^2 \right) + \partial^{\mu} \varphi \partial^{\nu} \varphi$$

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$$T_{\varphi}^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\varphi}}{\delta g_{\mu\nu}} = g^{\mu\nu} \left(-\frac{1}{2} g^{\alpha\beta} \partial_{\alpha} \varphi \partial_{\beta} \varphi - \frac{1}{2} m^2 \varphi^2 \right) + \partial^{\mu} \varphi \partial^{\nu} \varphi$$

$$\delta \sqrt{-g} = \frac{1}{2} \sqrt{-g} g^{\mu\nu} \delta g_{\mu\nu}$$

$$\partial^{\mu} \varphi = g^{\mu\alpha} \frac{\partial \varphi}{\partial x^{\alpha}}$$

$$g =$$

What is the action for φ ?

(set $\hbar=1$)
 $c=$

$$p = \hbar \frac{\partial}{\partial x} = m$$

"kinetic"

$$\int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} \frac{m^2}{\hbar^2} \varphi^2 \right)$$

$$\varphi = \tilde{\varphi} / \hbar$$

$$\Rightarrow H\varphi = 0.$$

$$\partial_\mu \delta\varphi \partial_\nu \varphi - m^2 \varphi \delta\varphi$$

$$\left[\partial_\mu \delta\varphi \partial_\nu \varphi - \sqrt{-g} g^{\mu\nu} \partial_\nu \varphi \right] = 0 \quad \forall \delta\varphi$$

$H\varphi$

What is the action for φ ?

(set $\hbar=1$)
c=

$$p = \hbar \frac{\partial}{\partial x^m} = m$$

$$S_\varphi = \int d^4x \sqrt{-g} \left(\overset{\text{kinetic}}{\downarrow \textcircled{0}} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} \frac{m^2}{\hbar^2} \varphi^2 \right) \right)$$

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$$\delta S_\varphi = \int d^4x \sqrt{-g} \left(-g^{\mu\nu} \partial_\mu \delta\varphi \partial_\nu \varphi - m^2 \varphi \delta\varphi \right)$$

$$\int d^4x \delta\varphi \underbrace{\left[\partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi) - \sqrt{-g} m^2 \varphi \right]}_{-\sqrt{-g} H\varphi} = 0 \quad \forall \delta\varphi$$

$$T_{\varphi}^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\varphi}}{\delta g_{\mu\nu}} = g^{\mu\nu} \left(-\frac{1}{2} g^{\alpha\beta} \partial_{\alpha} \varphi \partial_{\beta} \varphi - \frac{1}{2} m^2 \varphi^2 \right) + \partial^{\mu} \varphi \partial^{\nu} \varphi$$

$$\delta \sqrt{-g} = \frac{1}{2} \sqrt{-g} g^{\mu\nu} \delta g_{\mu\nu}$$

$$\delta g^{\mu\nu} = -g^{\mu\kappa} \delta g_{\kappa\beta} g^{\beta\nu}$$

$$\partial^{\mu} \varphi = g^{\mu\kappa} \frac{\partial \varphi}{\partial x^{\kappa}}$$

$$T_{\varphi}^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\varphi}}{\delta g_{\mu\nu}} = g^{\mu\nu} \left(-\frac{1}{2} g^{\alpha\beta} \partial_{\alpha} \varphi \partial_{\beta} \varphi - \frac{1}{2} m^2 \varphi^2 \right) + \partial^{\mu} \varphi \partial^{\nu} \varphi$$

$$\left. \begin{aligned} \delta \sqrt{-g} &= \frac{1}{2} \sqrt{-g} g^{\mu\nu} \delta g_{\mu\nu} \\ \delta g^{\mu\nu} &= -g^{\mu\kappa} \delta g_{\alpha\beta} g^{\beta\nu} \end{aligned} \right\}$$

$$\partial^{\mu} \varphi = g^{\mu\kappa} \frac{\partial \varphi}{\partial x^{\kappa}}$$

$$T_{\varphi}^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\varphi}}{\delta g_{\mu\nu}} = g^{\mu\nu} \left(-\frac{1}{2} g^{\alpha\beta} \partial_{\alpha} \varphi \partial_{\beta} \varphi - \frac{1}{2} m^2 \varphi^2 \right) + \partial^{\mu} \varphi \partial^{\nu} \varphi$$

$$\partial^{\mu} \varphi = g^{\mu\alpha} \frac{\partial \varphi}{\partial x^{\alpha}}$$

$$\delta \sqrt{-g} = -\frac{1}{2} \sqrt{-g} g^{\mu\nu} \delta g_{\mu\nu}$$

$$\delta g^{\mu\nu} = -g^{\mu\alpha} g^{\nu\beta} \delta g_{\alpha\beta}$$

$$\nabla_{\mu} (A^{\mu\nu}) = 0$$

$$T_{\varphi}^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\varphi}}{\delta g_{\mu\nu}} = g^{\mu\nu} \left(-\frac{1}{2} g^{\alpha\beta} \partial_{\alpha} \varphi \partial_{\beta} \varphi - \frac{1}{2} m^2 \varphi^2 \right) + \partial^{\mu} \varphi \partial^{\nu} \varphi$$

$$\delta \sqrt{-g} = \frac{1}{2} \sqrt{-g} g^{\mu\nu} \delta g_{\mu\nu}$$

$$\delta g^{\mu\nu} = -g^{\mu\kappa} \delta g_{\kappa\alpha} g^{\alpha\nu}$$

$$\partial^{\mu} \varphi = g^{\mu\kappa} \frac{\partial \varphi}{\partial x^{\kappa}}$$

$$\nabla_{\mu} \left(\begin{matrix} A^{\mu\nu} \\ + D^{\mu\nu} \end{matrix} \right) = 0$$

$$\nabla_{\mu} D^{\mu\nu} = 0 \text{ identically.}$$

$$T_{\varphi}^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\varphi}}{\delta g_{\mu\nu}} = g^{\mu\nu} \left(-\frac{1}{2} g^{\alpha\beta} \partial_{\alpha} \varphi \partial_{\beta} \varphi - \frac{1}{2} m^2 \varphi^2 \right) + \partial^{\mu} \varphi \partial^{\nu} \varphi$$

$$\delta \sqrt{-g} = \frac{1}{2} \sqrt{-g} g^{\mu\nu} \delta g_{\mu\nu}$$

$$\delta g^{\mu\nu} = -g^{\mu\kappa} \delta g_{\alpha\beta} g^{\beta\nu}$$

$$\partial^{\mu} \varphi = g^{\mu\kappa} \frac{\partial \varphi}{\partial x^{\kappa}}$$

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$$S = S_G + S_m$$

$$= \int \sqrt{-g} \frac{R}{16\pi G} + S_m$$

$$\delta S = \int \sqrt{-g} \left(\frac{\delta g_{\mu\nu}}{16\pi G} \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) \right) + \frac{\delta S_m}{\delta g_{\mu\nu}} \delta g_{\mu\nu}$$

$$S = S_G + S_m$$

$$= \int \sqrt{-g} \frac{R}{16\pi G} + S_m$$

$$= \int \sqrt{-g} \left(\frac{\delta g_{\mu\nu}}{16\pi G} \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) \right) + \frac{\delta S_m}{\delta g_{\mu\nu}} \delta g_{\mu\nu}$$

$$\Rightarrow R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 8\pi G \cdot 2$$

$$S = S_G + S_m$$

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$$\Rightarrow R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 8\pi G \frac{\delta S_m}{\delta g_{\mu\nu}}$$

$\equiv T_{\mu\nu}$

$$S = S_G + S_m$$

$$= \int \sqrt{-g} \frac{R}{16\pi G} + S_m$$

$$\delta S = \int \sqrt{-g} \left(\frac{\delta g_{\mu\nu}}{16\pi G} \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) + \frac{\delta S_m}{\delta g_{\mu\nu}} \right) \delta g^{\mu\nu}$$

$$\Rightarrow R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 8\pi G \cdot \frac{\delta S_m}{\sqrt{-g} \delta g_{\mu\nu}}$$

$$\Rightarrow \nabla_\mu T^{\mu\nu} = 0$$

$$\parallel T_{\mu\nu}$$

$$T_{\varphi}^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\varphi}}{\delta g_{\mu\nu}} = g^{\mu\nu} \left(-\frac{1}{2} g^{\alpha\beta} \partial_{\alpha} \varphi \partial_{\beta} \varphi - \frac{1}{2} m^2 \varphi^2 \right) + \partial^{\mu} \varphi \partial^{\nu} \varphi \quad (1)$$

$$\delta \sqrt{-g} = \frac{1}{2} \sqrt{-g} g^{\mu\nu} \delta g_{\mu\nu}$$

$$\delta g^{\mu\nu} = -g^{\mu\alpha} \delta g_{\alpha\beta} g^{\beta\nu}$$

$$\text{EM: } S = \int d^4x \sqrt{-g} \left(-\frac{1}{4} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma} \right)$$

$$T_{\varphi}^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\varphi}}{\delta g_{\mu\nu}} = g^{\mu\nu} \left(-\frac{1}{2} g^{\alpha\beta} \partial_{\alpha} \varphi \partial_{\beta} \varphi - \frac{1}{2} m^2 \varphi^2 \right) + \partial^{\mu} \varphi \partial^{\nu} \varphi$$

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$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$$

$$EM: S = \int d^4x \sqrt{-g} \left(-\frac{1}{4} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma} \right)$$

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EM

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$$

$$S[A_{\mu}, g_{\mu\nu}] = \int d^4x \sqrt{-g} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} g^{\mu\rho} g^{\nu\sigma} \right)$$

$$T_{\varphi}^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\varphi}}{\delta g_{\mu\nu}} = g^{\mu\nu} \left(-\frac{1}{2} g^{\alpha\beta} \partial_{\alpha} \varphi \partial_{\beta} \varphi - \frac{1}{2} m^2 \varphi^2 \right) + \partial^{\mu} \varphi \partial^{\nu} \varphi$$

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EM

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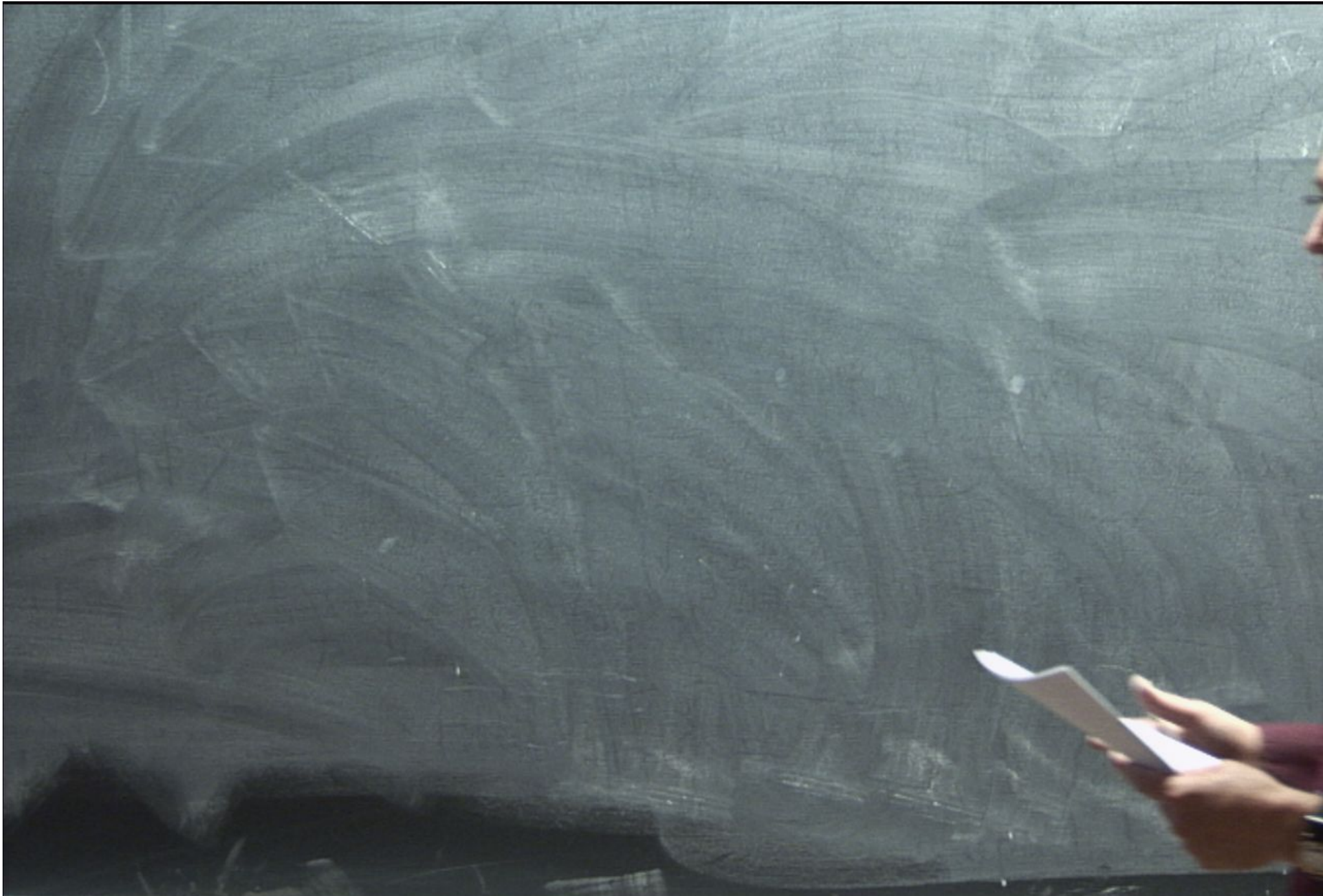
$$S[A_{\mu}, g_{\mu\nu}] = \int d^4x \sqrt{-g} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} g^{\mu\rho} g^{\nu\sigma} \right)$$

$$\frac{\delta S}{\delta A} \rightarrow \text{Maxwell}$$

$$\frac{\delta S}{\delta g} \Rightarrow T_{\mu\nu}^{\text{Maxwell}}$$

Extending the Schwarzschild Metric

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Extending the Schwarzschild Metric

(Schwarzschild) S metric: $ds^2 = -\left(1 - \frac{r_s}{r}\right) dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$
goes singular at $r = r_s$.

Extending the Schwarzschild Metric

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goes singular at $r = r_s$.

$$d\theta^2 + \sin^2\theta d\varphi^2$$

↑

$$d\Omega^2$$

Extending the Schwarzschild Metric

metric: $ds^2 = -\left(1 - \frac{r_s}{r}\right) dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$

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r is a space-like coordinate
time-like coordinate t

Extending the Schwarzschild Metric

(Sch) S metric: $ds^2 = -\left(1 - \frac{r_s}{r}\right) dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 +$

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$$g = \begin{pmatrix} -(1 - \frac{r_s}{r}) & & & \\ & + (1 - \frac{r_s}{r})^{-1} & & \\ & & r^2 & \\ & & & r^2 \sin^2 \theta \end{pmatrix}$$

r is a space-like coordinate
time-like coordinate t
 t is opposite.

Extending the Schwarzschild Metric

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r is a space-like coordinate $r > r_s$

timelike coordinate $r < r_s$

t is opposite.

$$d\theta^2 + \sin^2 \theta d\phi^2$$

Consider radial null rays.

$$r^2 d\Omega^2 \uparrow d\theta^2 + \sin^2 \theta d\phi^2$$

$> r_s$

$< r_s$

Consider radial null rays.

$$r^2 d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$$

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$ds^2 = 0$ allows you to solve very simply.

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Consider radial null rays.

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$$dt = \pm \frac{1}{\left(1 - \frac{r_s}{r}\right)} dr$$

$$= \pm dr \left(1 + \frac{r_s}{r - r_s}\right)$$

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$$\begin{array}{c} d\theta^2 + \sin^2\theta d\varphi^2 \\ \uparrow \\ r^2 d\Omega^2 \end{array}$$

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$$d\theta^2 + \sin^2\theta d\phi^2$$

↑
 Ω^2

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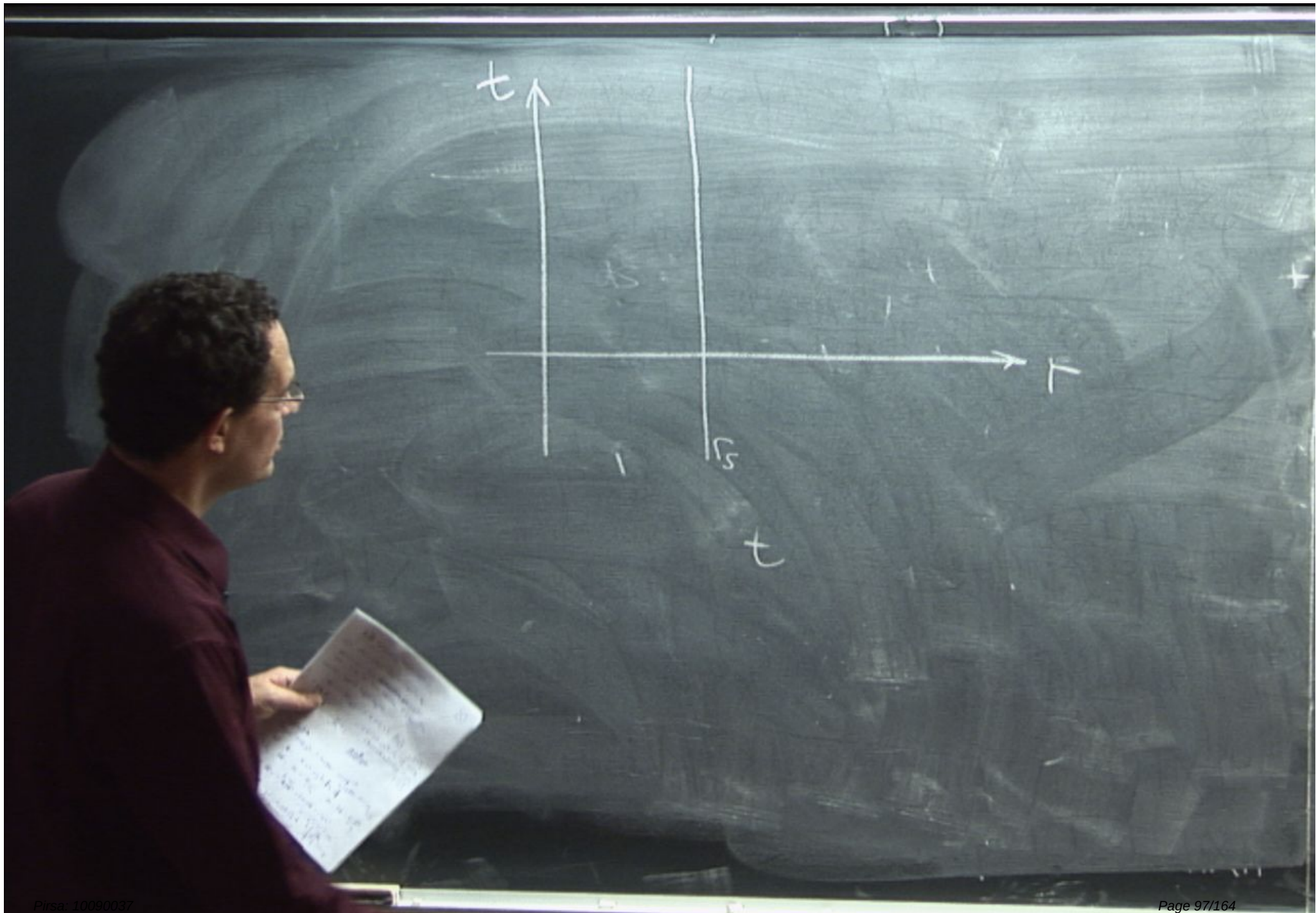
$$dt = \pm \frac{1}{(1 - \frac{r_s}{r})} dr$$

$$\frac{r}{r - r_s}$$

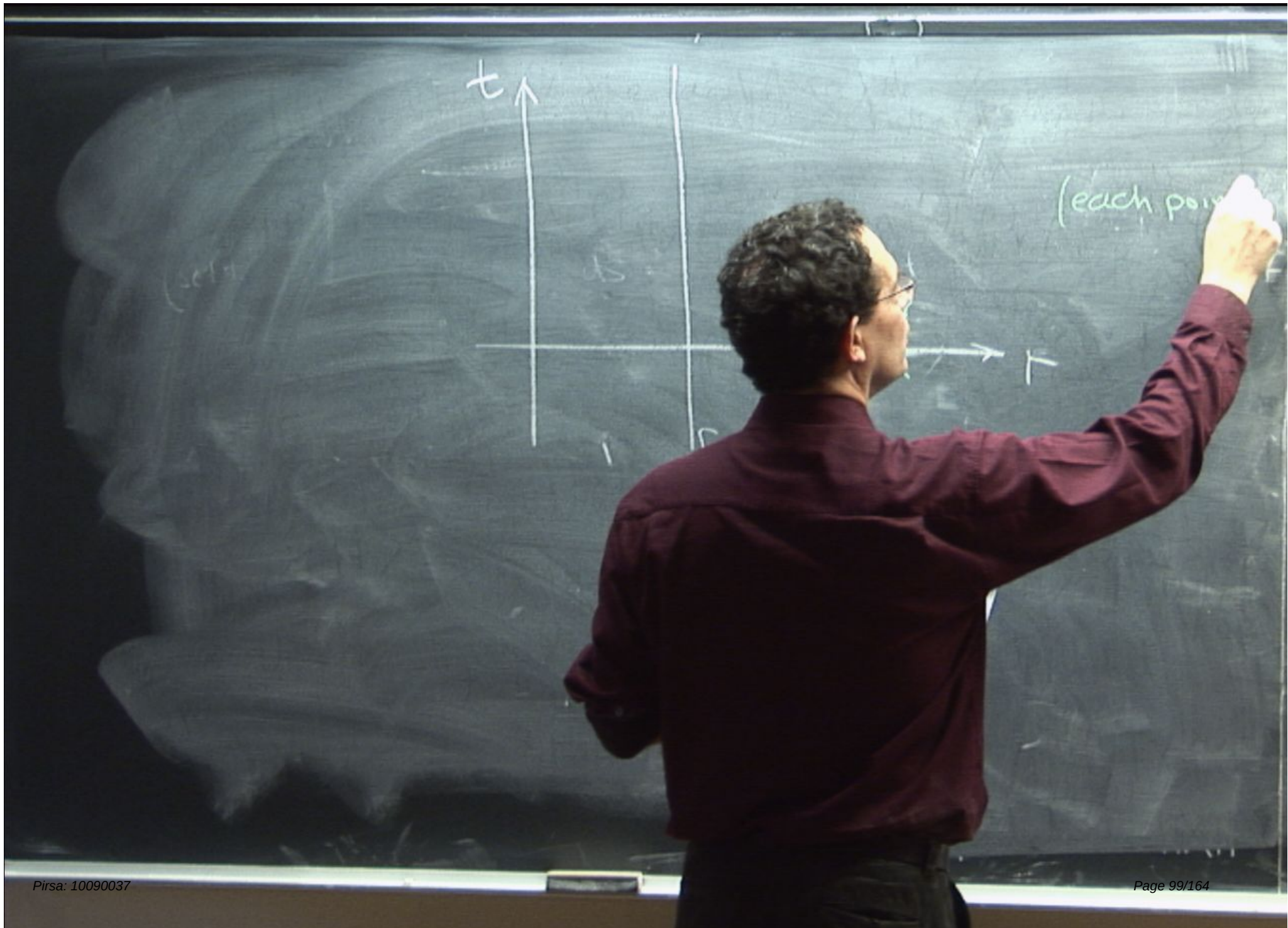
$$= \pm dr \left(1 + \frac{r_s}{r - r_s} \right)$$

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t



(each point
on diagram
is a S^2 : θ, ϕ)

t



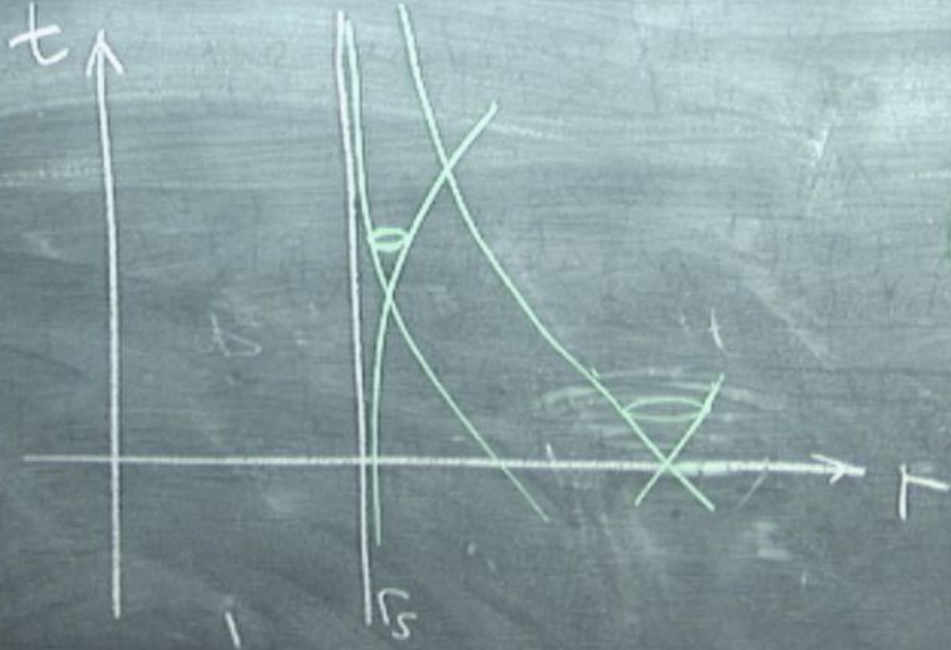
(each point
on diagram
is a S^2 : θ, ϕ)

t

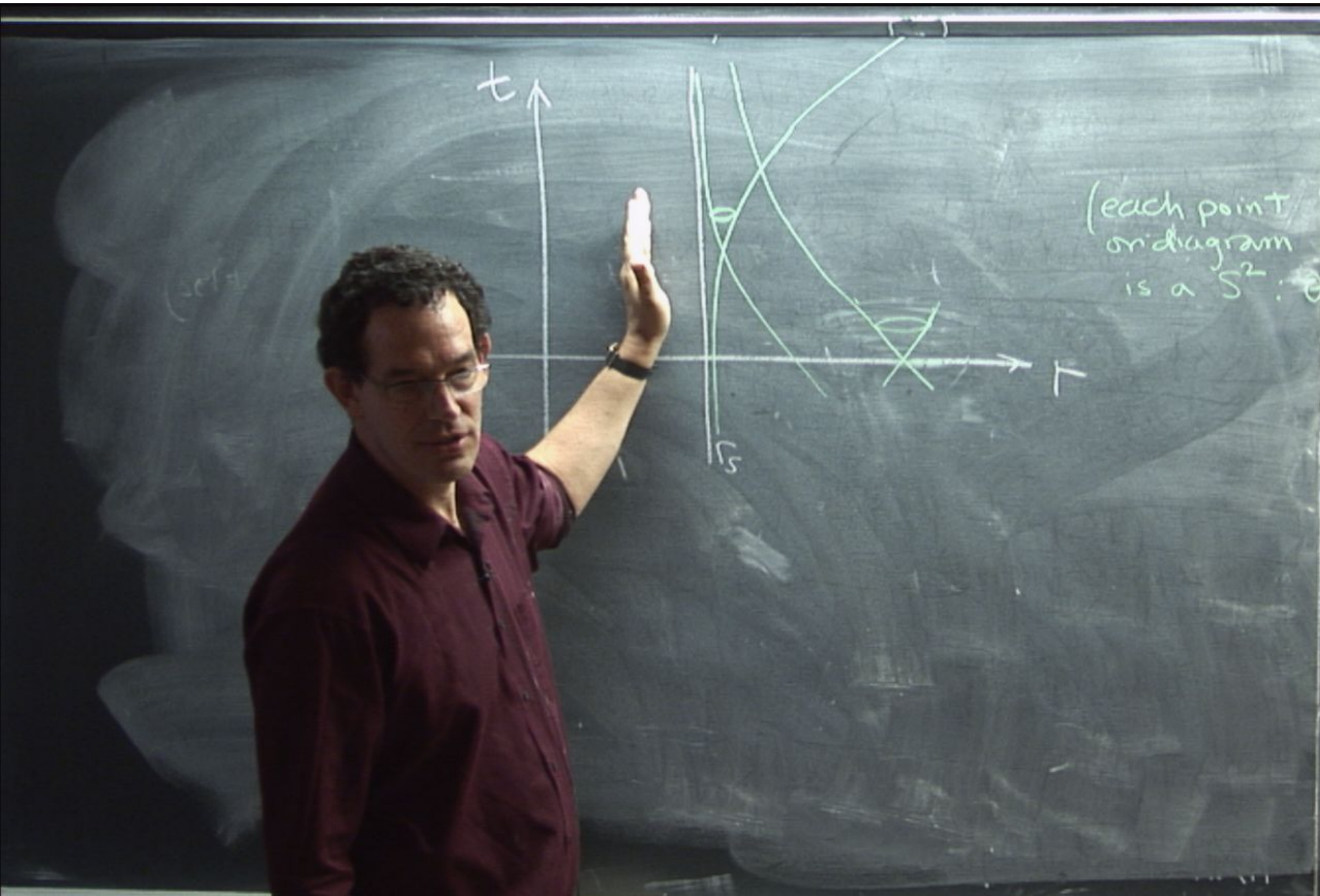
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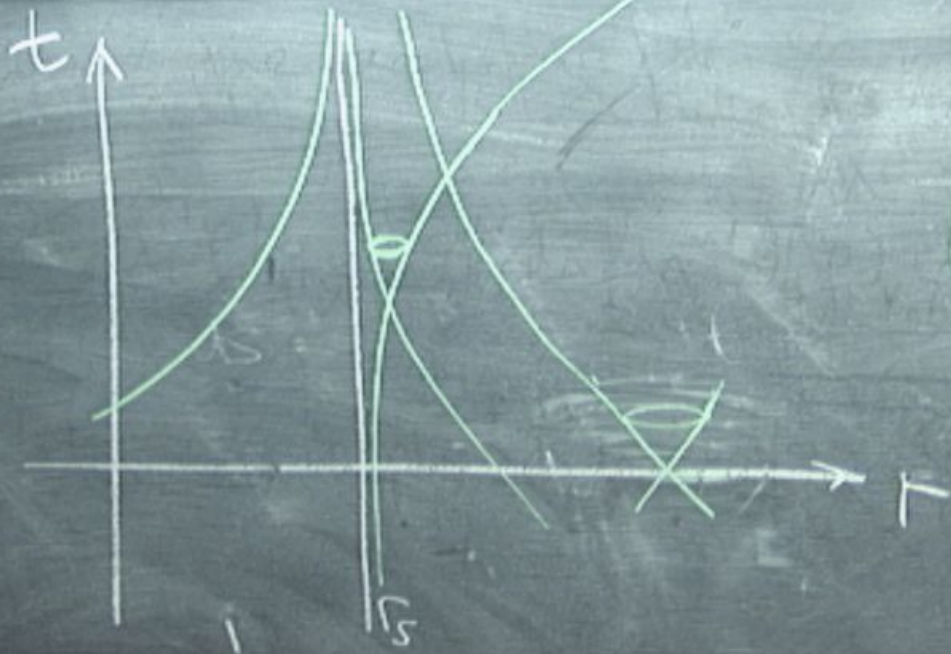


(each point
on diagram
is a $S^2 : \theta, \phi$)

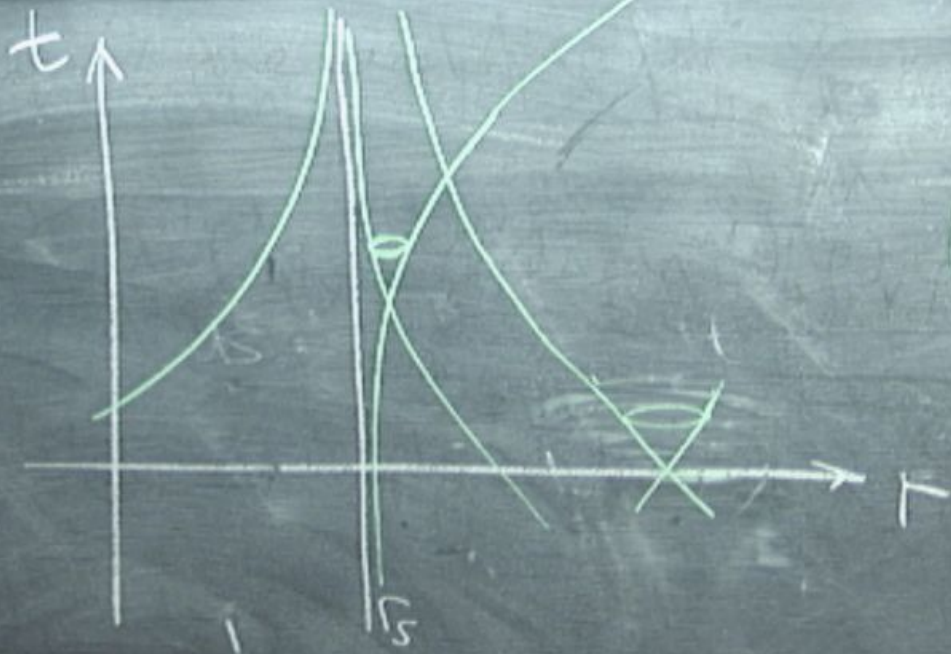


(each point
on diagram
is a S^2 : θ)





(each point
on diagram
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$$= \pm dr \left(1 + \frac{r_s}{r - r_s}\right)$$

$$\rightarrow t = \pm \left(r + r_s \ln \left(\frac{r}{r_s} - 1 \right) \right) + \text{const} \quad r > r_s$$

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$$\pm \left(r - r_s \ln \left(1 - \frac{r_s}{r} \right) \right) + \text{const}$$

Consider radial null rays.

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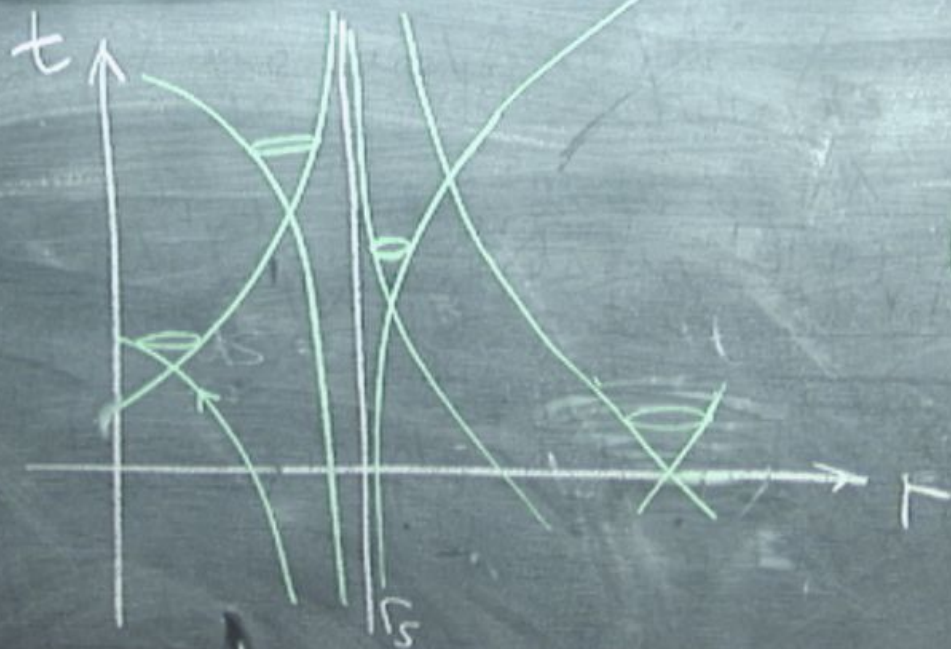
$$dt = \pm \frac{1}{\left(1 - \frac{r_s}{r}\right)} dr$$

$$\frac{r}{r - r_s}$$

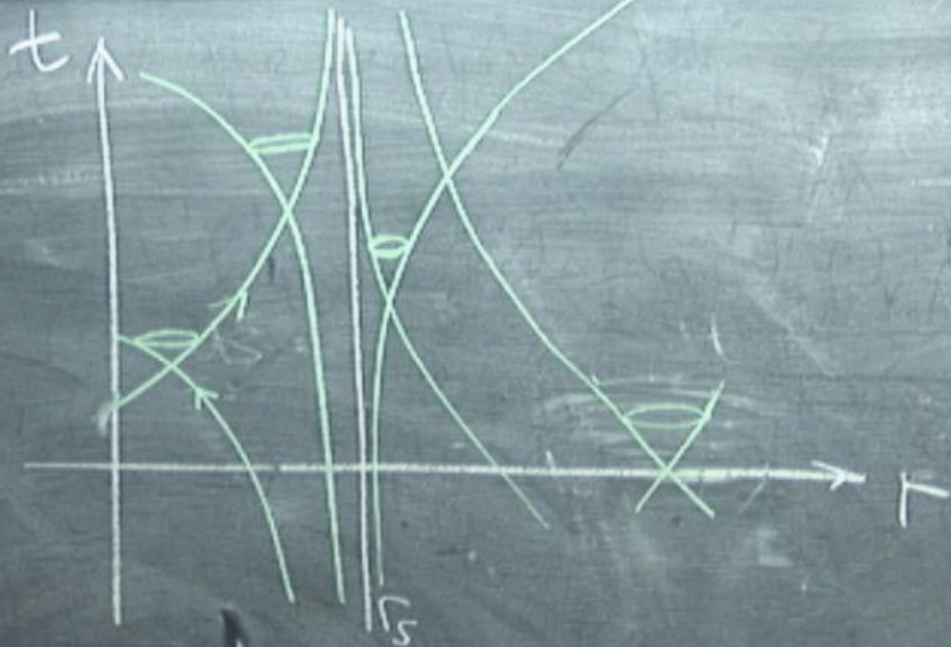
$$= \pm dr \left(1 + \frac{r_s}{r - r_s}\right)$$

$$\rightarrow t = \pm \left(r + r_s \ln \left(\frac{r}{r_s} - 1 \right) \right) + \text{const} \quad r > r_s$$

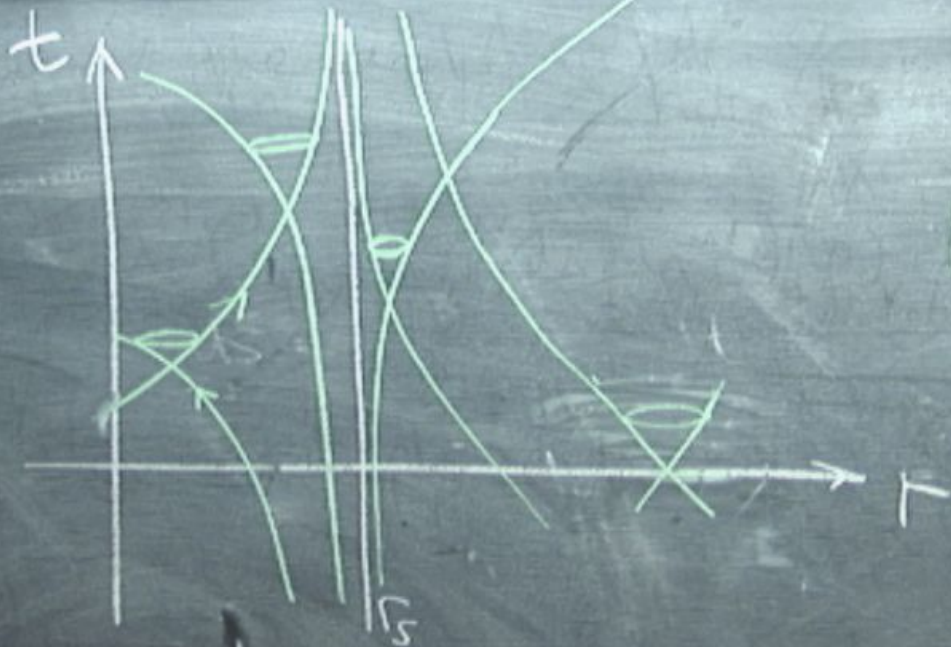
$$\pm \left(r - r_s \ln \left(1 - \frac{r_s}{r} \right) \right) + \text{const} \quad \underline{r < r_s}$$



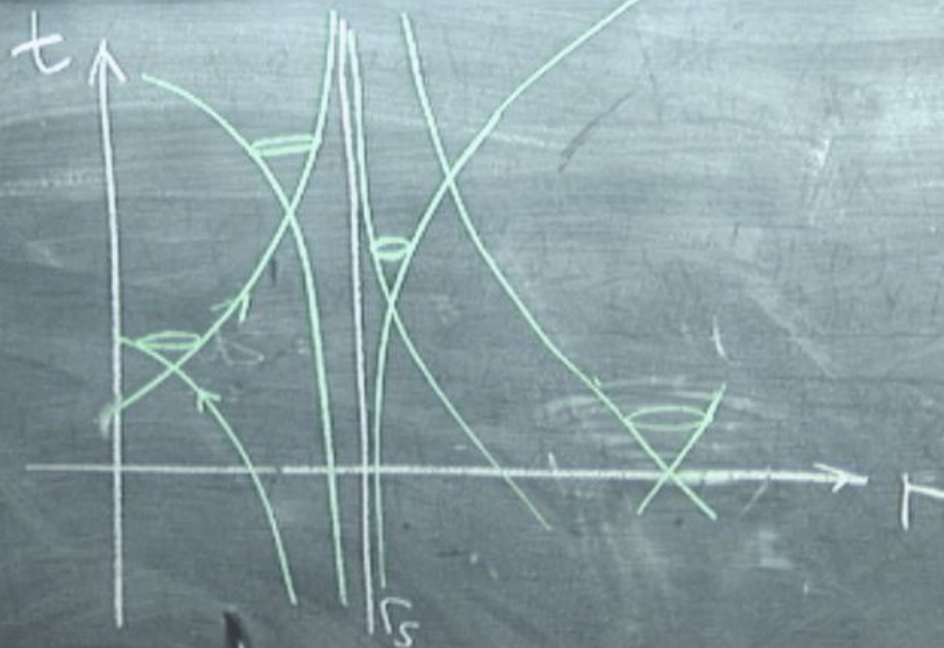
(each point
on diagram
is a S^2 : θ)



(each point
on diagram
is a S^2 : θ)

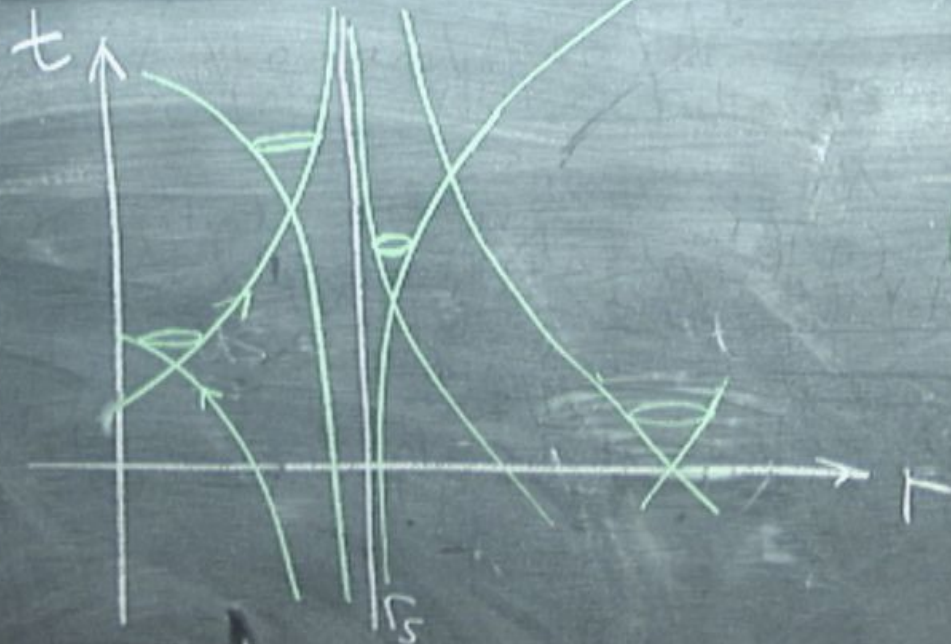


(each point
on diagram
is a S^2 : θ)



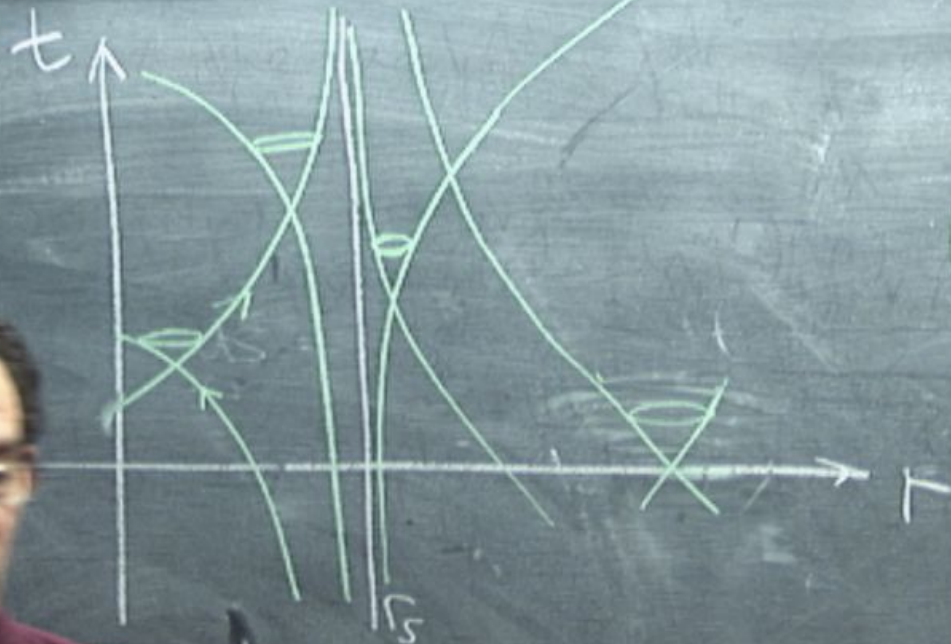
(each point
on diagram
is a S^2 : ∂

New coordinates



(each point
on diagram
is a $S^2 : \theta, \phi$)

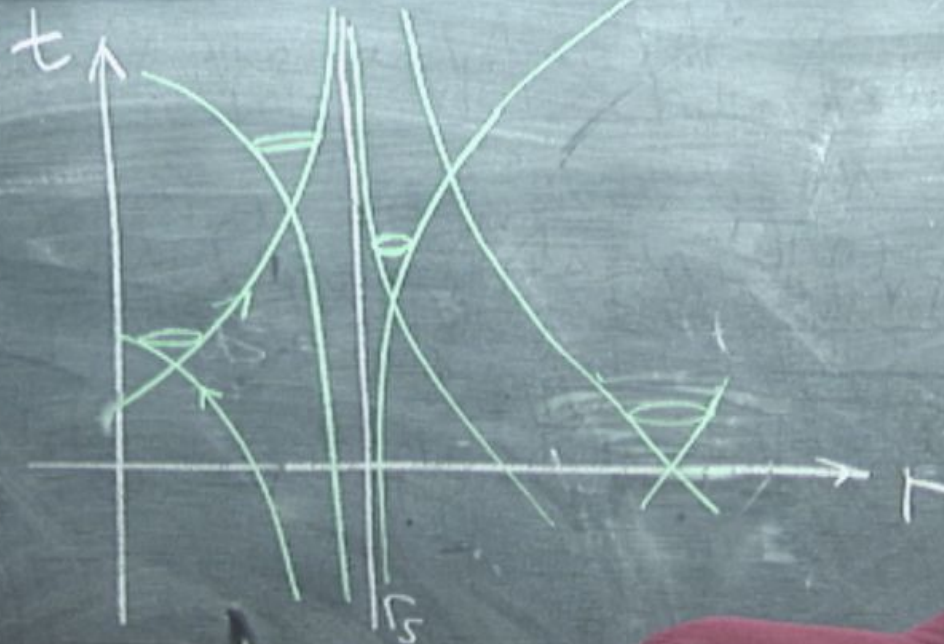
New coordinates : ingoing Eddington-Finkelstein
coord S .



(each point
on diagram
is a $S^2 : \theta, \phi$)

coordinates : ingoing Eddington-Finkelstein
coord S .

$$u = t + r + r_s \ln \left| \frac{r}{r_s} - 1 \right|$$



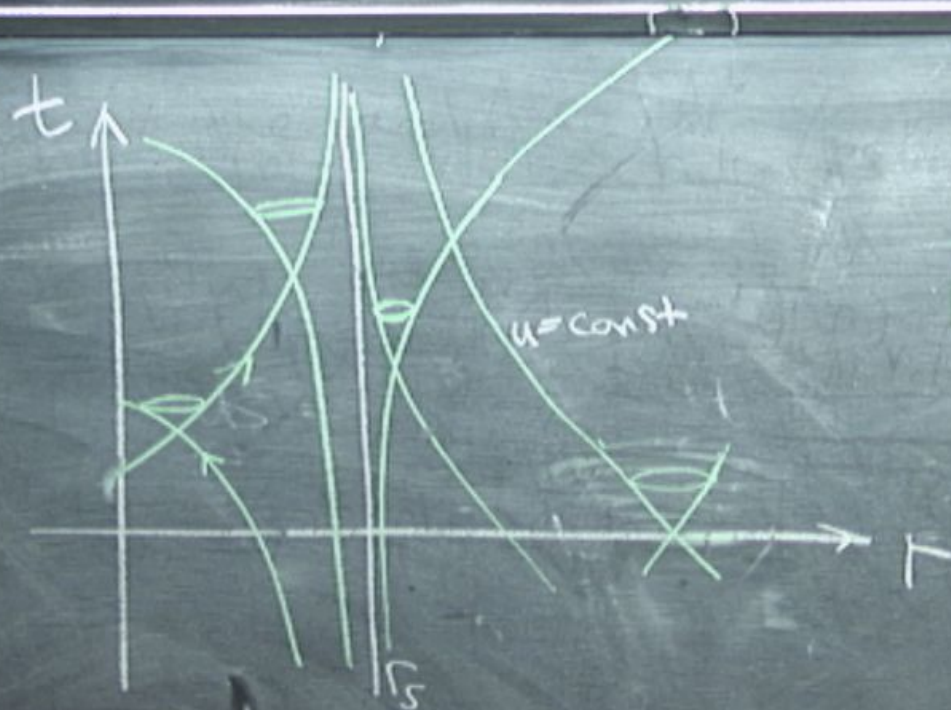
(each point
on diagram
is a $S^2 : \theta, \phi$)

New coordinates :

let $u =$

Finkelstein
coord S .

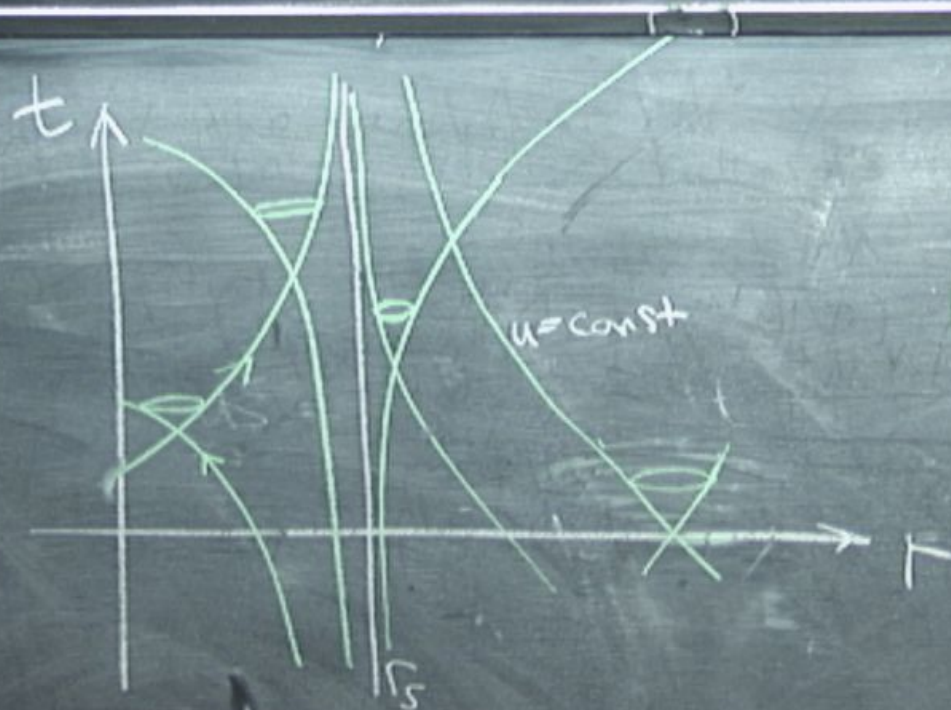
inst on
going null ra



(each point
on diagram
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New coordinates : ingoing Eddington-Finkelstein
coord u .

$$\text{let } u = t + r + r_s \ln \left| \frac{r}{r_s} - 1 \right| = \text{const on ingoing null rays}$$



(each point
on diagram
is a $S^2 : \theta, \phi$)

New coordinates : ingoing Eddington-Finkelstein
coord S .

$$\text{let } u = t + r + r_s \ln \left| \frac{r}{r_s} - 1 \right| = \text{const on ingoing null rays}$$

$$\Rightarrow du = dt + dr + \frac{dr \cdot r_s}{r - r_s}$$

$$= dt + \frac{r dr}{r - r_s} = dt + \frac{dr}{(1 - \frac{r_s}{r})}$$

stein
s.
on
null rays

$$\Rightarrow du = dt + dr + \frac{dr \cdot r_s}{r - r_s}$$

$$= dt + \frac{r dr}{r - r_s} = dt + \frac{dr}{(1 - \frac{r_s}{r})}$$

$$\Rightarrow dt = du - \frac{dr}{(1 - \frac{r_s}{r})}$$

$$-(1 - \frac{r_s}{r}) dt^2 + (1 - \frac{r_s}{r})^{-1} dr^2 + r^2 d\Omega^2$$

=

stein
s.
on
null rays

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$$-(1 - \frac{r_s}{r}) dt^2 + (1 - \frac{r_s}{r})^{-1} dr^2 + r^2$$

$$= -(1 - \frac{r_s}{r}) du^2 + 2 dr$$

stein
s.
on
null rays

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$$g_{ru} = 1 = g_{ur}$$

$$= - \underbrace{(1 - \frac{r_s}{r})}_{0} du^2 + \underbrace{2 dr du}_{0} + r^2 d\Omega^2$$

$r \rightarrow r_s$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{r^2} \begin{pmatrix} \theta \\ \phi \end{pmatrix}$$

stein
s.
on
null rays

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$$g_{ru} = 1 = g_{ur}$$

$$= - \underbrace{(1 - \frac{r_s}{r})}_{0} du^2 + \underbrace{2 dr du + r^2 d\Omega^2}_{r^2 \sin^2 \theta}$$

$r \rightarrow r_s$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{r^2 \sin^2 \theta}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

→ metric is completely nonsingular at $r=r_s$!

at $r=r_s$!

$$u = t + r + r_s \ln\left(\frac{r}{r_s} - 1\right)$$

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$r \rightarrow r_s$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{r^2}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

at $r=r_s$!

$$\Rightarrow du = dt + dr + \frac{dr}{r-r_s}$$

$$u = t + r + r_s \ln\left(\frac{r}{r_s} - 1\right)$$

$$= dt + \frac{r dr}{r-r_s} = dt + \frac{dr}{1 - \frac{r_s}{r}}$$

Singular
coordinate
transformation!

$$\Rightarrow dt = du - \frac{dr}{(1 - \frac{r_s}{r})}$$

$$-(1 - \frac{r_s}{r}) dt^2 + (1 - \frac{r_s}{r})^{-1} dr^2 + r^2 d\Omega^2$$

$$g_{ru} = 1$$

$$= -\underbrace{(1 - \frac{r_s}{r})}_{0} du^2 + \underbrace{2 dr du + r^2 d\Omega^2}_{r^2 d\Omega^2}$$

$r \rightarrow r_s$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{r^2}$$

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Singular
coordinate
transformation!
 $\frac{du}{dr} \rightarrow \infty$

$$\Rightarrow dt = du - \frac{dr}{1 - \frac{r_s}{r}}$$

$$-(1 - \frac{r_s}{r}) dt^2 + (1 - \frac{r_s}{r})^{-1} dr^2 + r^2 d\Omega^2$$

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$r \rightarrow r_s$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{r^2}$$

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at $r=r_s$!

$$u = t + r + r_s \ln\left(\frac{r}{r_s} - 1\right)$$

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at $r=r_s$!

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$$-(1 - \frac{r_s}{r}) dt^2 + (1 - \frac{r_s}{r})^{-1} dr^2 + r^2 d\Omega^2$$

$$= - \underbrace{(1 - \frac{r_s}{r})}_{0} du^2 + 2 dr du + r^2 d\Omega^2$$

$r \rightarrow r_s$

$$g_{rr} = 1$$

at $r=r_s$!

$$\Rightarrow du = dt + dr + \frac{dr}{r-r_s}$$

$$u = t + r + r_s \ln\left(\frac{r}{r_s} - 1\right)$$

$$= dt + \frac{r dr}{r-r_s} = dt + \frac{dr}{1 - \frac{r_s}{r}}$$

Singular
coordinate
transformation!
 $\frac{du}{dr} \rightarrow \infty$

$$\Rightarrow dt = du - \frac{dr}{1 - \frac{r_s}{r}}$$

$$-(1 - \frac{r_s}{r}) dt^2 + (1 - \frac{r_s}{r})^{-1} dr^2 + r^2 d\Omega^2$$

$$g_{ru} = 1$$

$$= -\underbrace{(1 - \frac{r_s}{r})}_{0} du^2 + \underbrace{2 dr du + r^2 d\Omega^2}_{r^2 d\Omega^2}$$

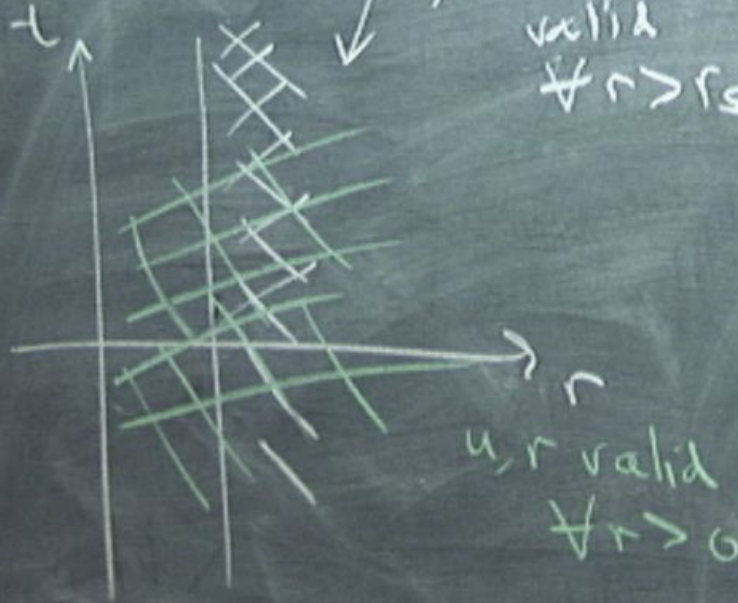
$r \rightarrow r_s$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{r^2}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

→ metric is completely nonsingular at

t, r coords
valid
 $\forall r > r_s$



u, r valid
 $\forall r > 0$

→ metric is completely nonsingular at

t, r coords
valid
 $\forall r > r_s$

u, r valid
 $\forall r > 0$

completely nonsingular at $r=r_s$!

follow ingoing null ray

$$u = t + r + r_s \ln\left(\frac{r}{r_s} - 1\right)$$

Singular
coordinate
transformation!

$$\frac{du}{dr} \rightarrow \infty$$

$$\Rightarrow du = dt + dr$$

$$= dt + dr$$

$$\Rightarrow dt = du - dr$$

$$-(1 - \frac{r_s}{r}) dt^2 + (1 - \frac{r_s}{r})^{-1} dr^2$$

$$= -\underbrace{(1 - \frac{r_s}{r})}_{0} du^2 + 2 du dr$$

$$r \rightarrow r_s$$

$r=0$ is real singularity

$R_{\mu\nu\rho\lambda}$

ing

completely nonsingular at $r=r_s$!

follow ingoing null ray

$$R_{\mu\nu}=0$$

R

$$R_{\mu\nu\rho\lambda} R^{\mu\nu\rho\lambda}$$

ing

$$u = t + r + r_s \ln\left(\frac{r}{r_s} - 1\right)$$

Singular
coordinate
transformation!

$$\frac{du}{dr} \rightarrow \infty$$

Scalar

$r=0$ is real singularity

$$\Rightarrow du = dt + dr$$

$$= dt +$$

$$\Rightarrow dt = du -$$

$$-(1-\frac{r_s}{r})dt^2 + (1-\frac{r_s}{r})^{-1}dr^2$$

$$= -\underbrace{(1-\frac{r_s}{r})}_{0} du^2 + 2dr du$$

$$r \rightarrow r_s$$

completely nonsingular at $r=r_s$!

follow ingoing null ray

$$\begin{aligned} G_{\mu\nu} &= 0 \\ R_{\mu\nu} &= 0 \\ R &= 0 \end{aligned}$$

$$\boxed{R_{\mu\nu\rho\lambda}} R^{\mu\nu\rho\lambda}$$

ing

$$u = t + r + r_s \ln\left(\frac{r}{r_s} - 1\right)$$

Singular
coordinate
transformation!
 $\frac{du}{dr} \rightarrow \infty$

Scalar

$r=0$ is real singularity

$$\Rightarrow du = dt + dr$$

$$= dt + dr$$

$$\Rightarrow dt = du - dr$$

$$-(1 - \frac{r_s}{r}) dt^2 + (1 - \frac{r_s}{r})^{-1} dr^2$$

$$= - \underbrace{(1 - \frac{r_s}{r})}_{0} du^2 + 2 du dr$$

$$r \rightarrow r_s$$

singular at $r=r_s$!

all rays

$G_{\mu\nu} = 0$
 $R_{\mu\nu} = 0$
 R

$R_{\mu\nu\rho\lambda}$

$R_{\mu\nu\rho\lambda}$

$= \frac{G}{r^4}$

$u = t + r + r_s \ln\left(\frac{r}{r_s} - 1\right)$

Singular coordinate transformation

$\frac{du}{dr} \rightarrow \infty$

$r=0$ is

$\Rightarrow du = dt + dr + \frac{dr \cdot r_s}{r - r_s}$

$= dt + \frac{r dr}{r - r_s} = dt + \frac{dr}{(1 - \frac{r_s}{r})}$

$t = du - \frac{dr}{(1 - \frac{r_s}{r})}$

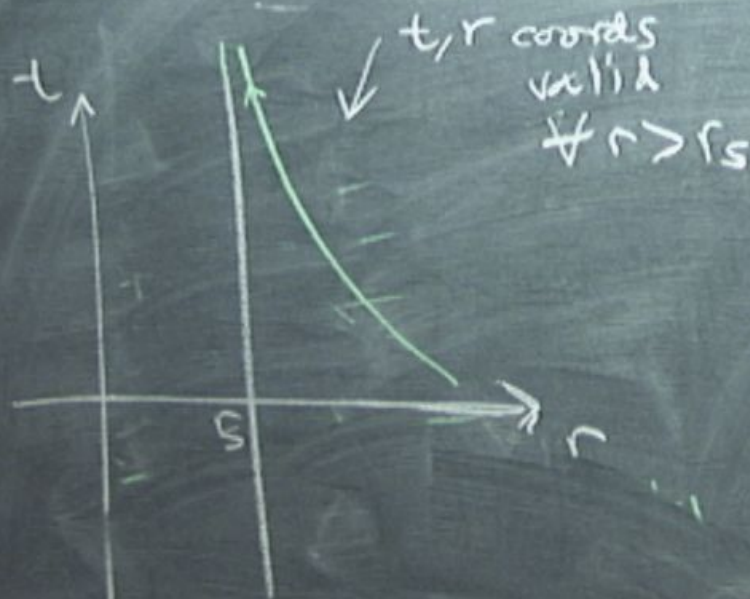
$+ (1 - \frac{r_s}{r})^{-1} dr^2 + r^2 d\Omega^2$

$g_{ru} =$

$-du^2 + 2drdu + r^2 d\Omega^2$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{r^2}$
 $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{r^2 \sin^2 \theta}$

→ metric is completely nonsingular at



follow ingoing null ray
e.g. $u=0$

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follow ingoing null ray

e.g. $u=0$

$$(u = t + r + \ln|\frac{r}{r_s} - 1|)$$

→ metric is completely nonsingular at

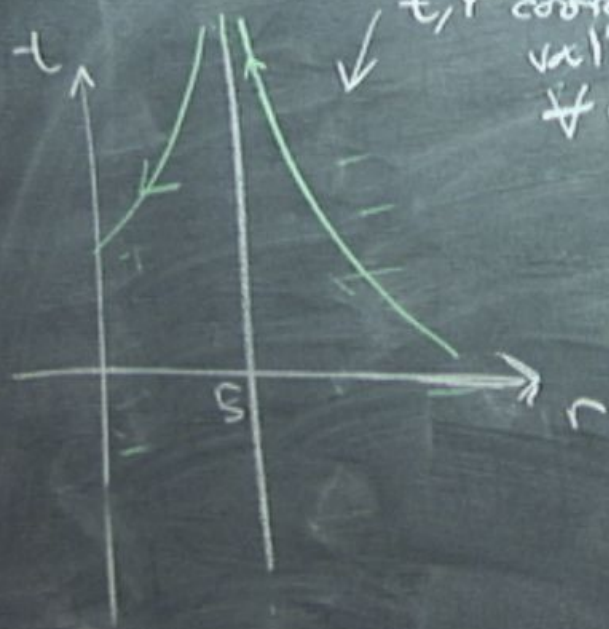
t, r coords
valid
 $\forall r > r_s$

follow ingoing null ray

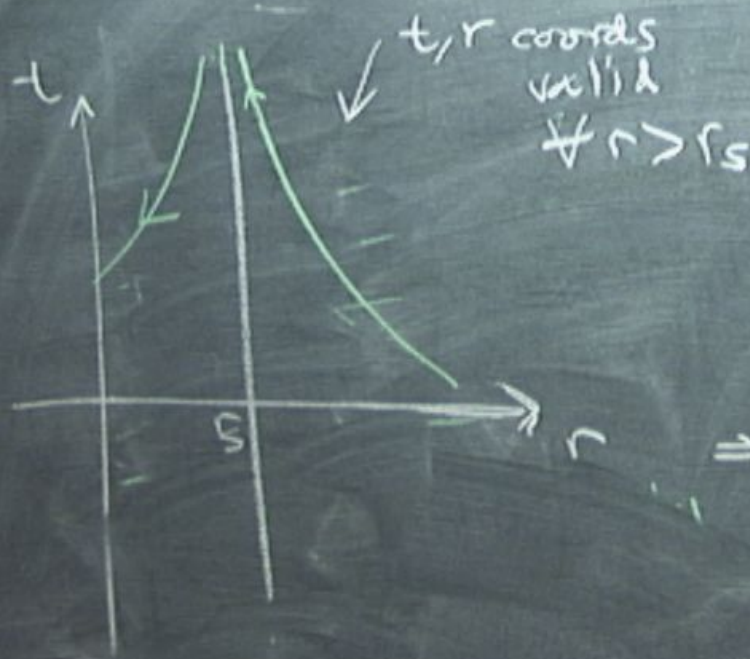
e.g. $u=0$

$$u = t + r + \ln \left| \frac{r}{r_s} - 1 \right|$$

⇒ direction of r i.e. "time"



→ metric is completely nonsingular at

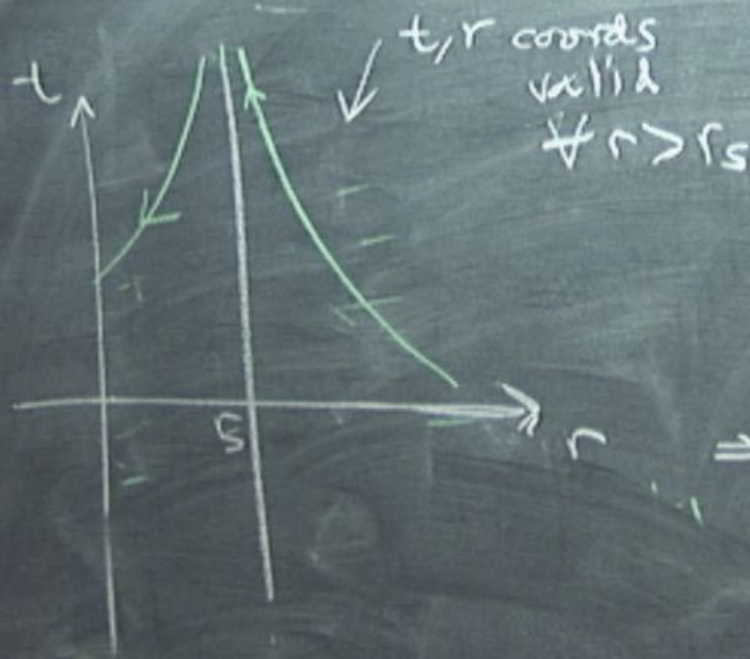


follow ingoing null ray
e.g. $u=0$

$$(u = t + r + \ln|\frac{r}{r_s} - 1|)$$

⇒ direction of r i.e. "time"
is decreasing.

→ metric is completely nonsingular at



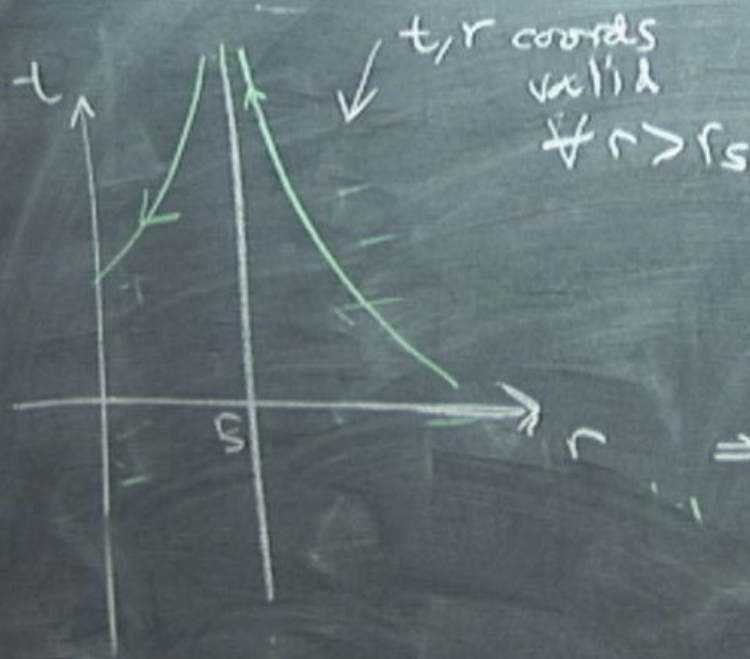
follow ingoing null ray

e.g. $u=0$

$$(u = t + r + \ln|\frac{r}{r_s} - 1|)$$

⇒ direction of r i.e. "time" is decreasing.

→ metric is completely nonsingular at



follow ingoing null ray

e.g. $u=0$

$$(u = t + r + \ln|\frac{r}{r_s} - 1|)$$

⇒ direction of r i.e. "time" is decreasing.

$r=r_s!$

Let's set $u = t' + r$

$r=r_s!$

Let's set $u = t' + r$

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) du^2 + 2 du dr + r^2 d\Omega^2$$

$$= -\left(1 - \frac{r_s}{r}\right) dt'^2 - \left(1 - \frac{r_s}{r}\right) dr^2 - 2\left(1 - \frac{r_s}{r}\right) dr dt'$$

$r=r_s!$

Let's set $u = t' + r$

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) du^2 + 2 du dr + r^2 d\Omega^2$$

$$= -\left(1 - \frac{r_s}{r}\right) dt'^2 - \left(1 - \frac{r_s}{r}\right) dr^2 - 2\left(1 - \frac{r_s}{r}\right) dr dt' + 2(dt' + dr)dr + r^2 d\Omega^2$$

$$= -\left(1 - \frac{r_s}{r}\right) dt'^2 + \left(1 + \frac{r_s}{r}\right) dr^2 + \frac{2r_s}{r} dr dt' + r^2 d\Omega^2$$

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null rays are $dt' = -dr$ (ingoing)

$$dt' = \frac{(r - r_s)}{(r + r_s)} dr \quad (\text{outgoing})$$

$r=r_s!$

Let's set $u = t' + r$

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) du^2 + 2 du dr + r^2 d\Omega^2$$

$$= -\left(1 - \frac{r_s}{r}\right) dt'^2 - \left(1 - \frac{r_s}{r}\right) dr^2 - 2\left(1 - \frac{r_s}{r}\right) dr dt' + 2(dt' + dr)dr + r^2 d\Omega^2$$

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Lets set $u = t' + r$

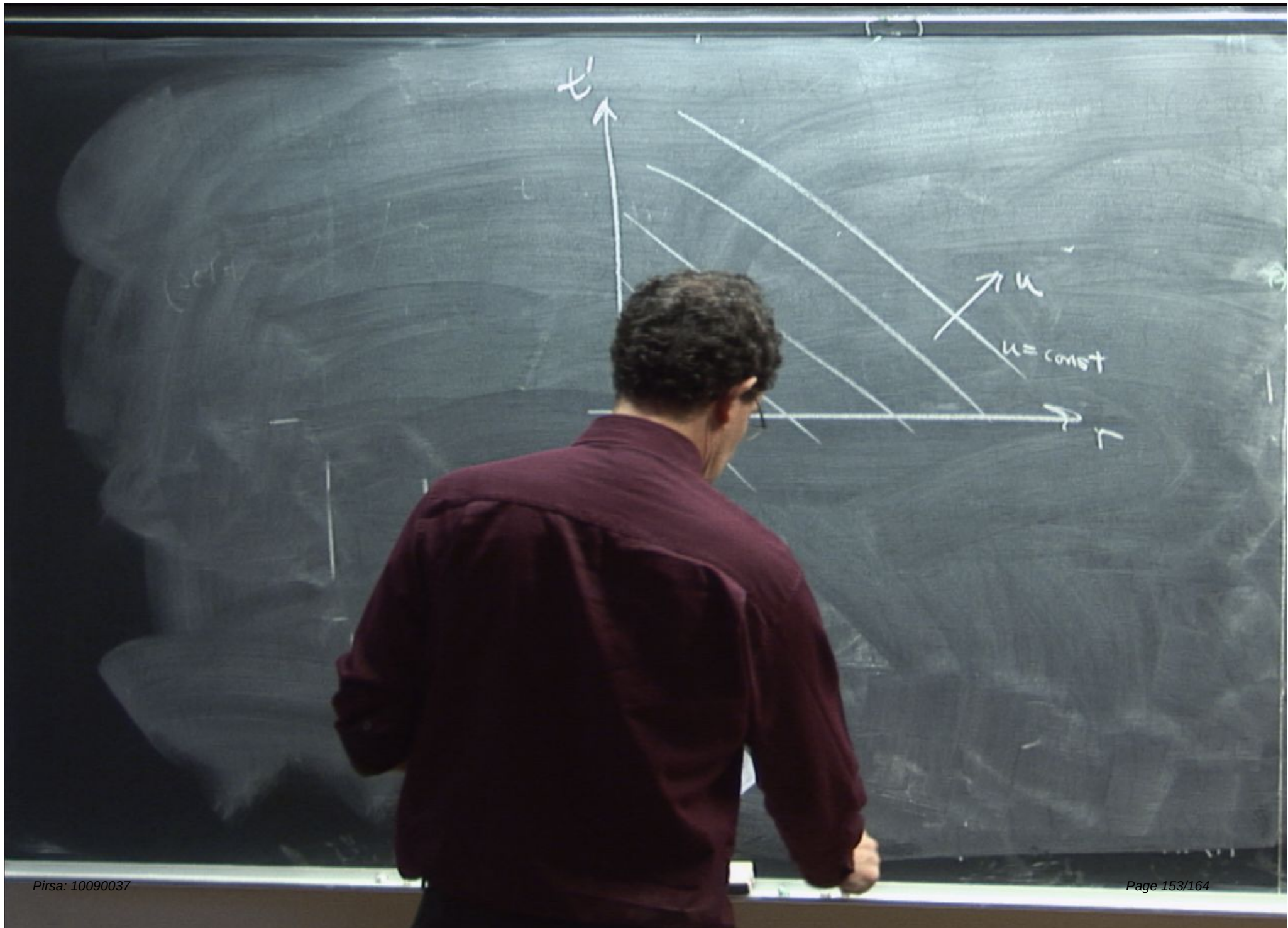
$$ds^2 = -\left(1 - \frac{r_s}{r}\right) du^2 + 2 du dr + r^2 d\Omega^2$$

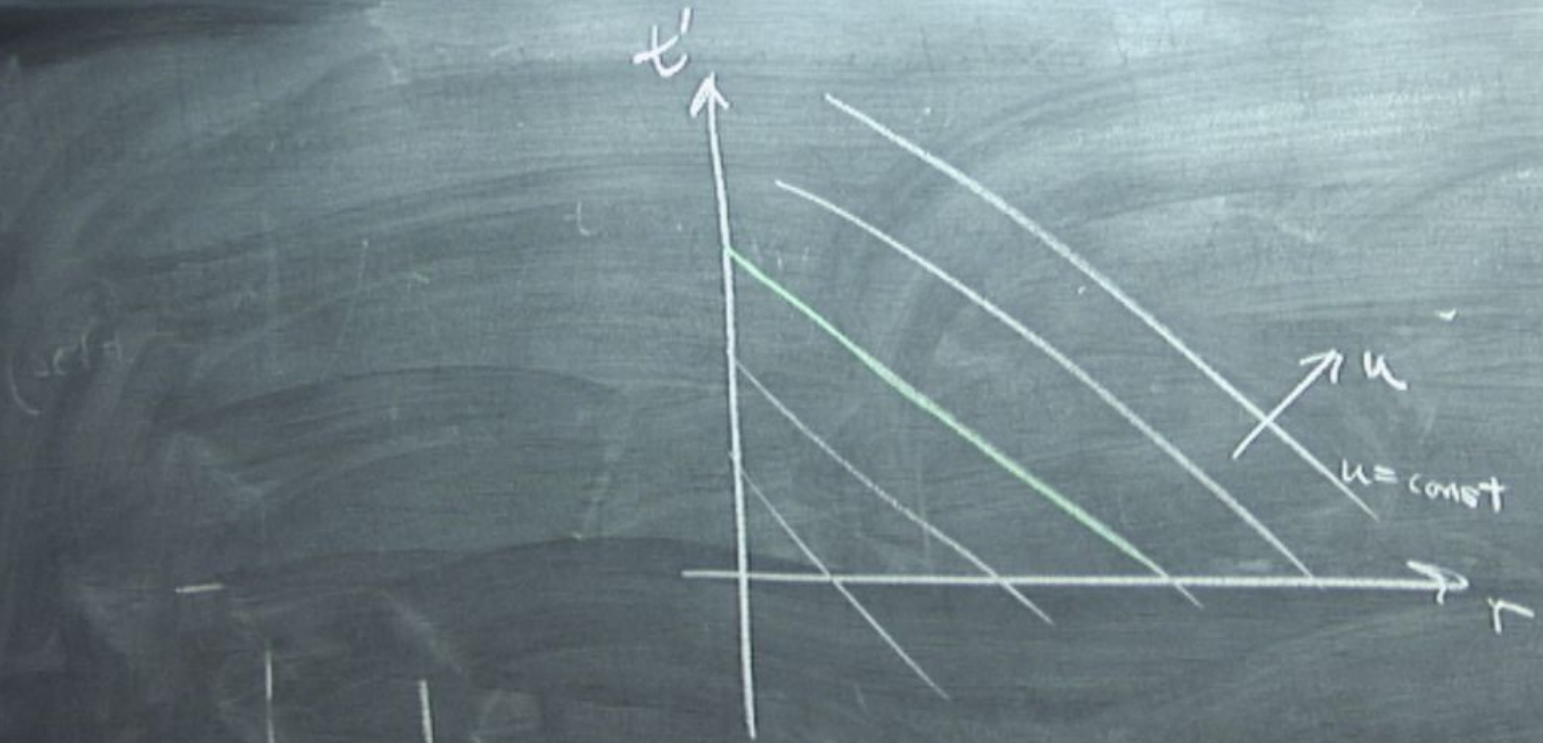
$$= -\left(1 - \frac{r_s}{r}\right) dt'^2 - \left(1 - \frac{r_s}{r}\right) dr^2 - 2\left(1 - \frac{r_s}{r}\right) dr dt' + 2(dt' + dr)dr + r^2 d\Omega^2$$

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Lets set $u = t' + r$

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) du^2 + 2 du dr + r^2 d\Omega^2$$

$$= -\left(1 - \frac{r_s}{r}\right) dt'^2 - \left(1 - \frac{r_s}{r}\right) dr^2 - 2\left(1 - \frac{r_s}{r}\right) dr dt' + 2(dt' + dr)dr + r^2 d\Omega^2$$

$$= -\left(1 - \frac{r_s}{r}\right) dt'^2 + \left(1 + \frac{r_s}{r}\right) dr^2 + 2\frac{r_s}{r} dr dt' + r^2 d\Omega^2$$

null rays are $dt' = -dr$ (ingoing)

$$dt' = \frac{(r - r_s)}{(r + r_s)} dr \text{ (outgoing)}$$

