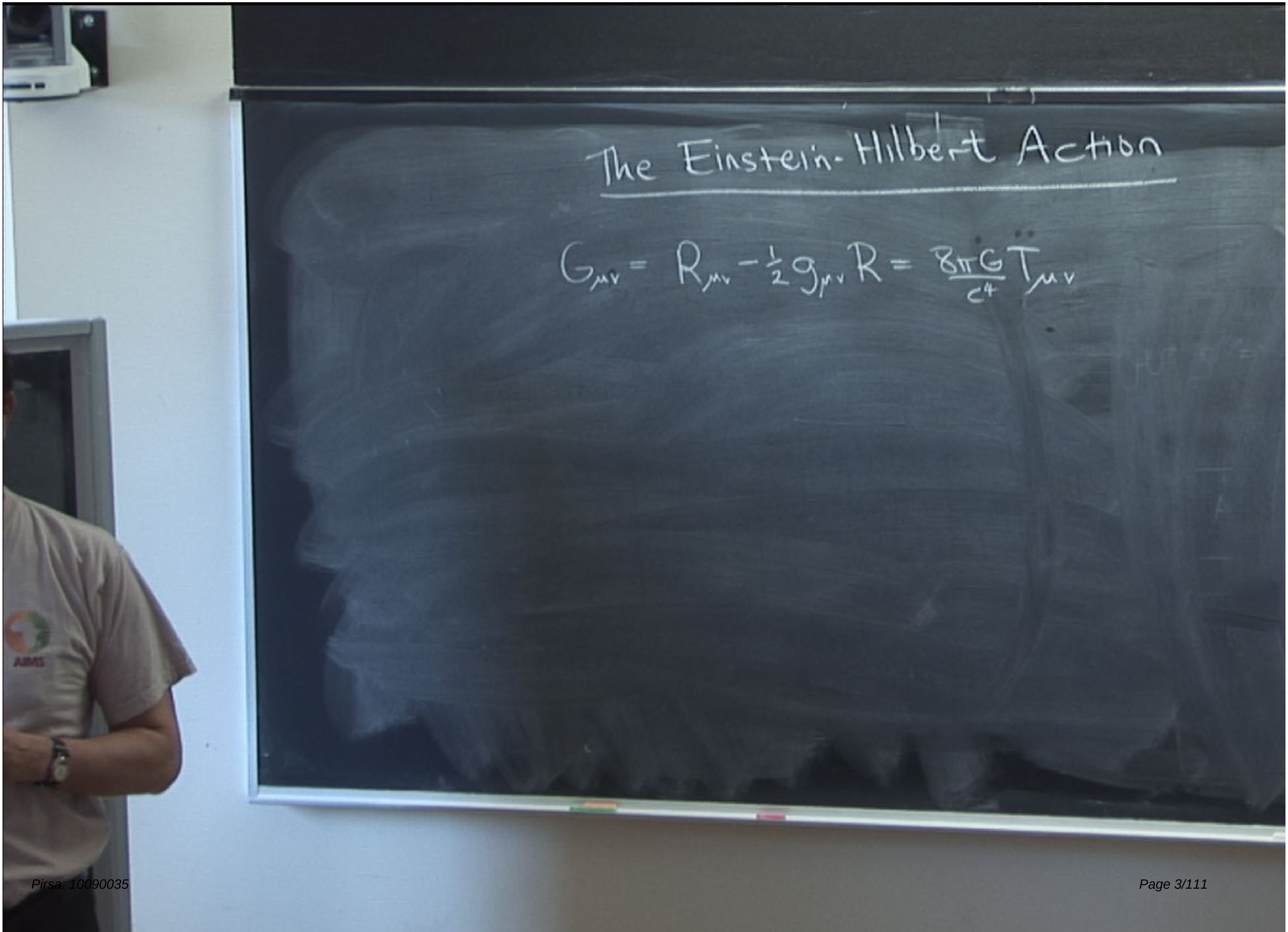
Title: Relativity (PHYS 604) - Lecture 10

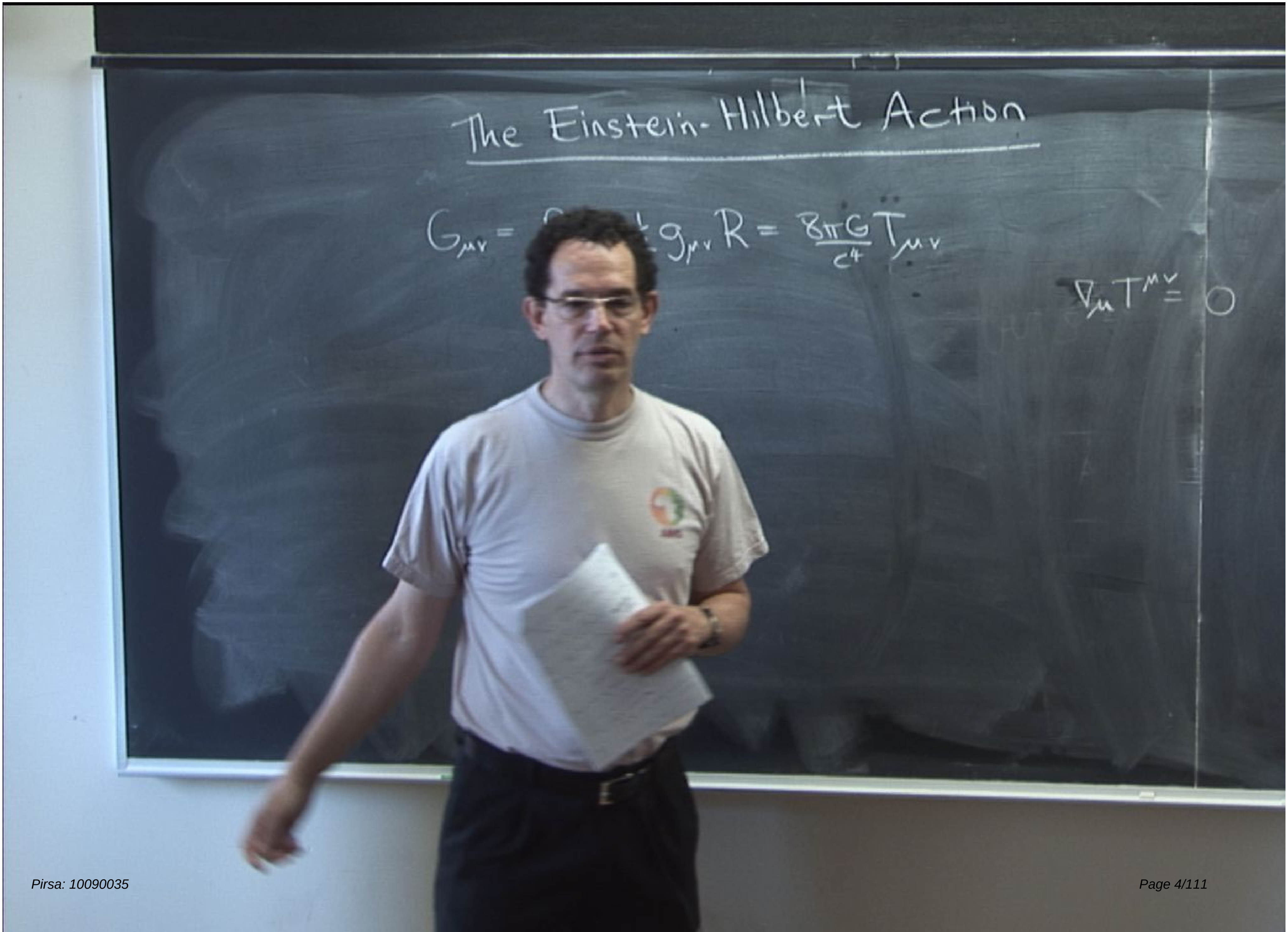
Date: Sep 24, 2010 10:30 AM

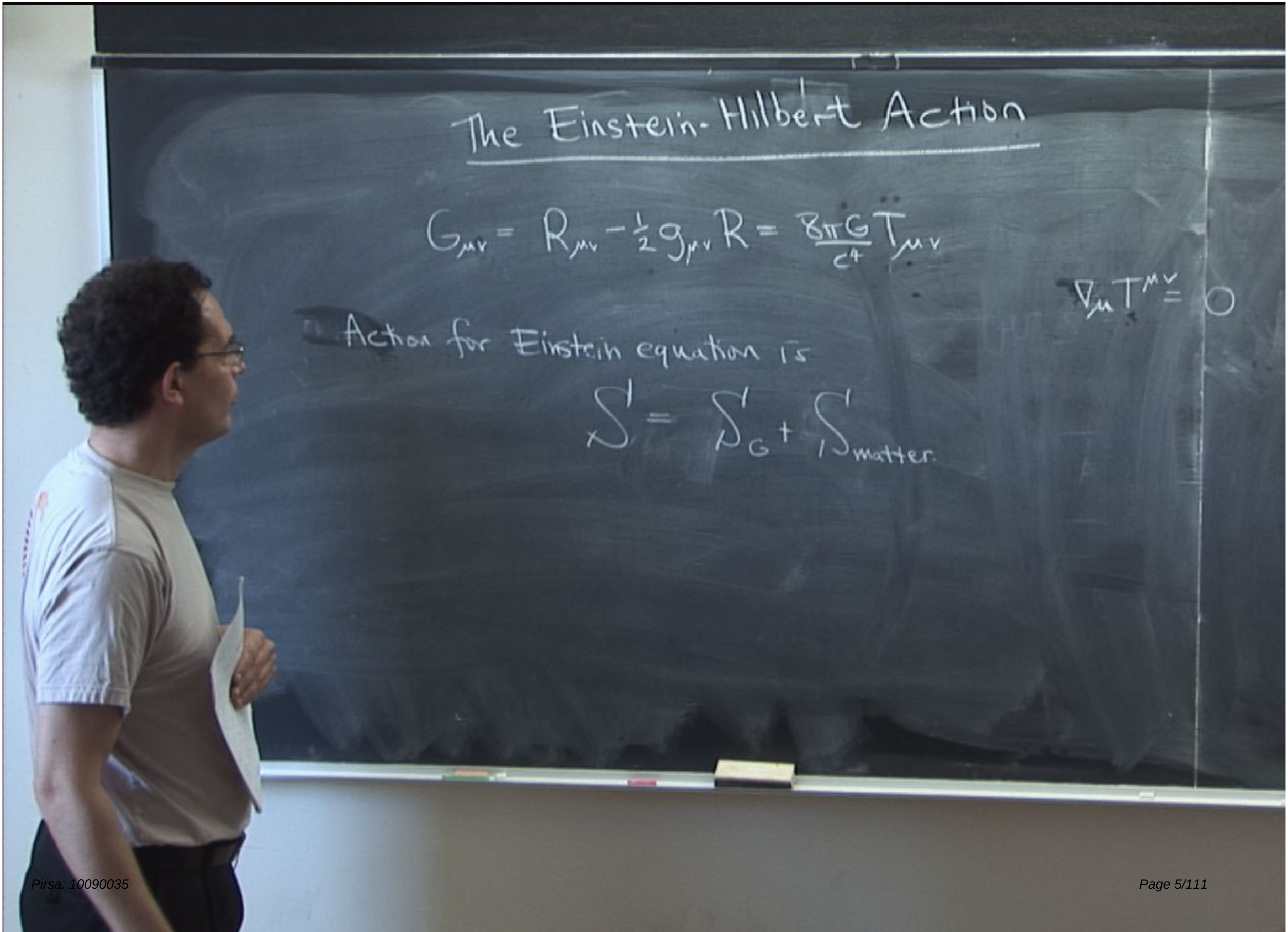
URL: http://pirsa.org/10090035

Abstract:

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS


The Einstein-Hilbert Action
$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$
AIMS


The Einstein-Hilbert Action
$G_{\mu\nu} =$
$g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$
$\nabla_\mu T^{\mu\nu} = 0$


The Einstein-Hilbert Action
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$\nabla_\mu T^{\mu\nu} = 0$
Action for Einstein equation is
$S = S_G + S_{matter}$

The Einstein-Hilbert Action

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\nabla_\mu T^{\mu\nu} \overset{?}{=} 0$$

Action for Einstein equation is

$$S = S_G + S_{matter}$$

$$S_G = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, R$$

The Einstein-Hilbert Action

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Action for Einstein equation is

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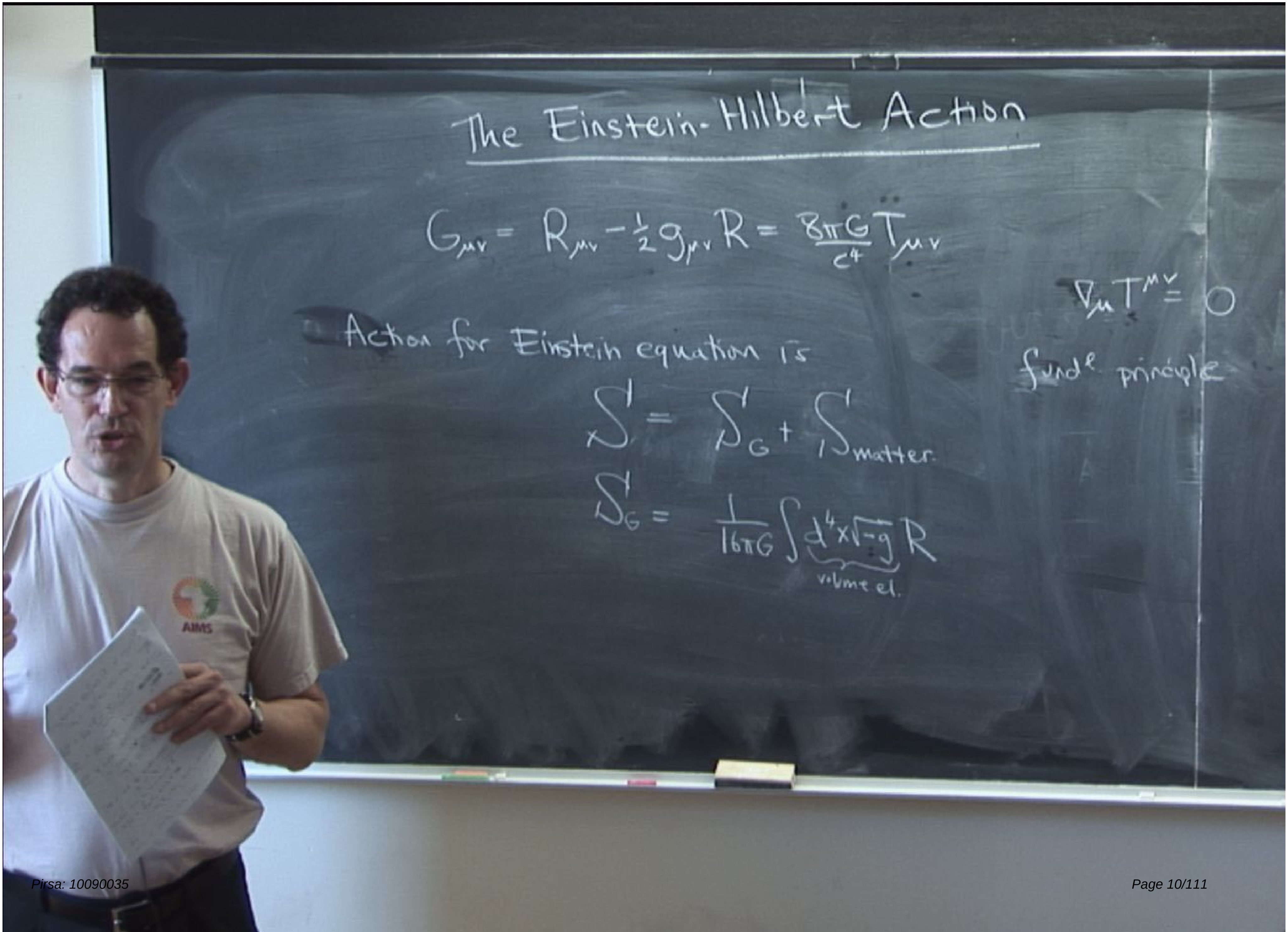
$$S_G = \frac{1}{16\pi G} \int \underbrace{d^4x \sqrt{-g}}_{\text{volume el.}} R$$


The Einstein-Hilbert Action
$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$
$\nabla_\mu T^{\mu\nu} = 0$
fund^e principle
Action for Einstein equation is
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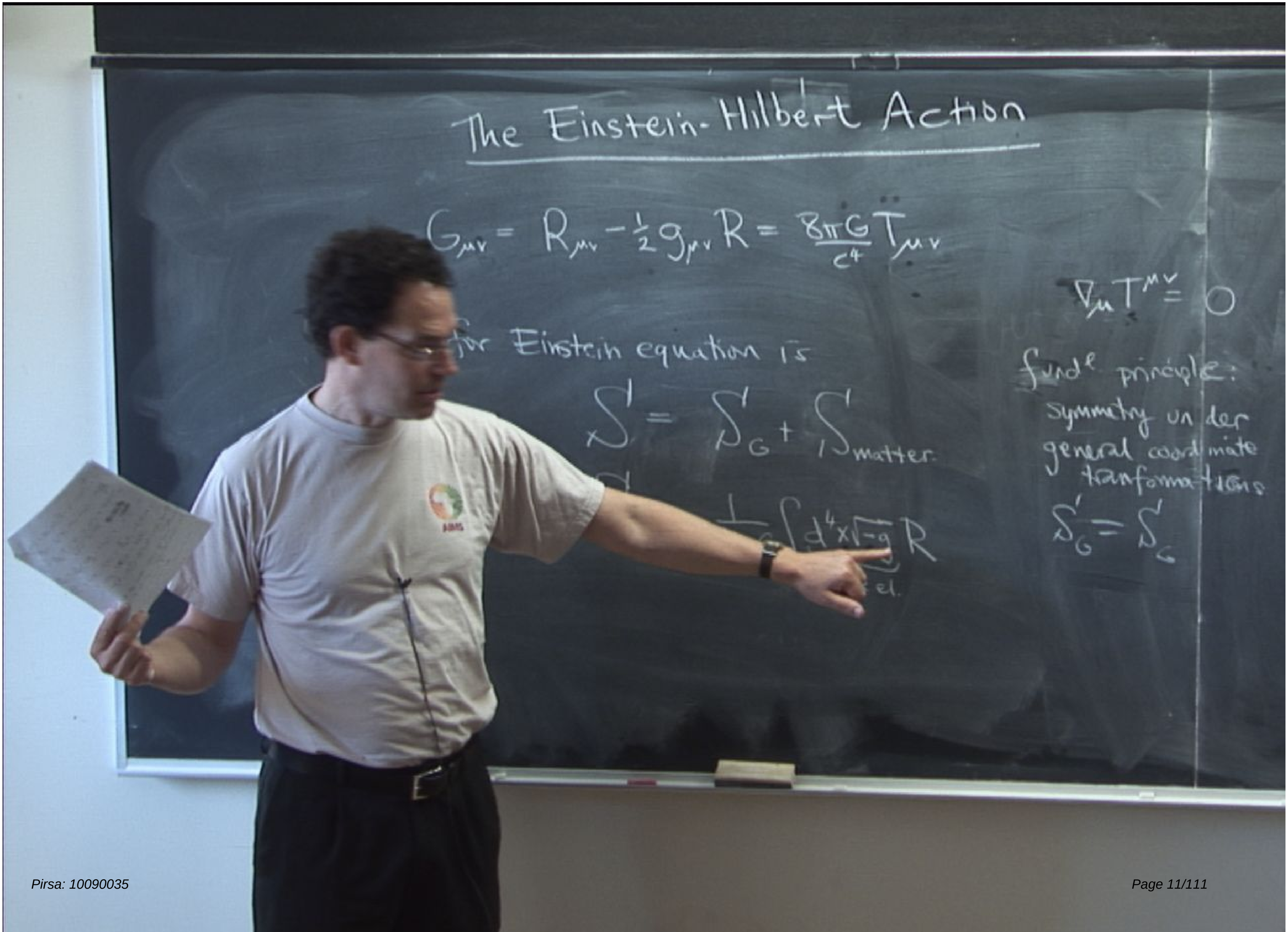
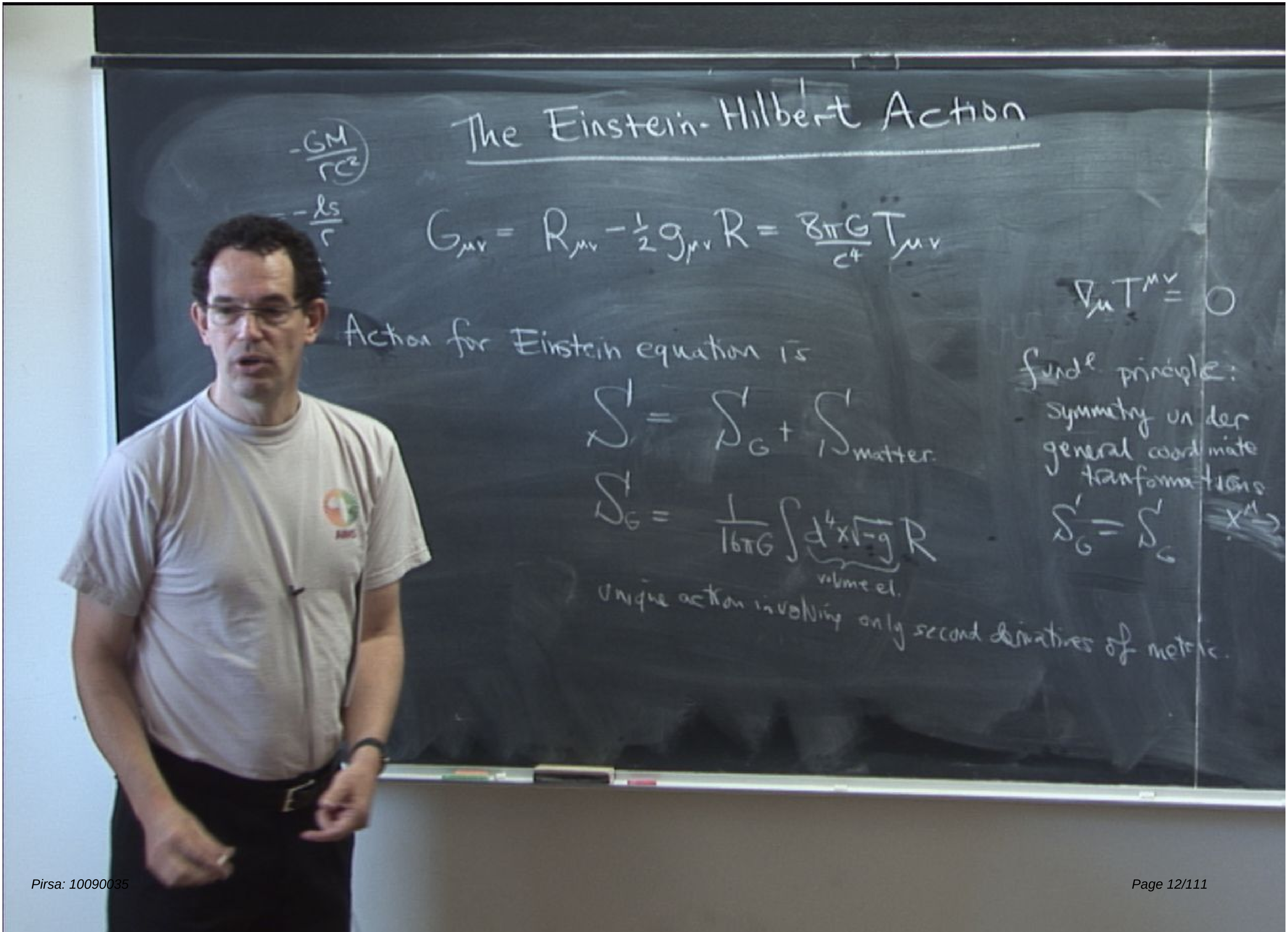
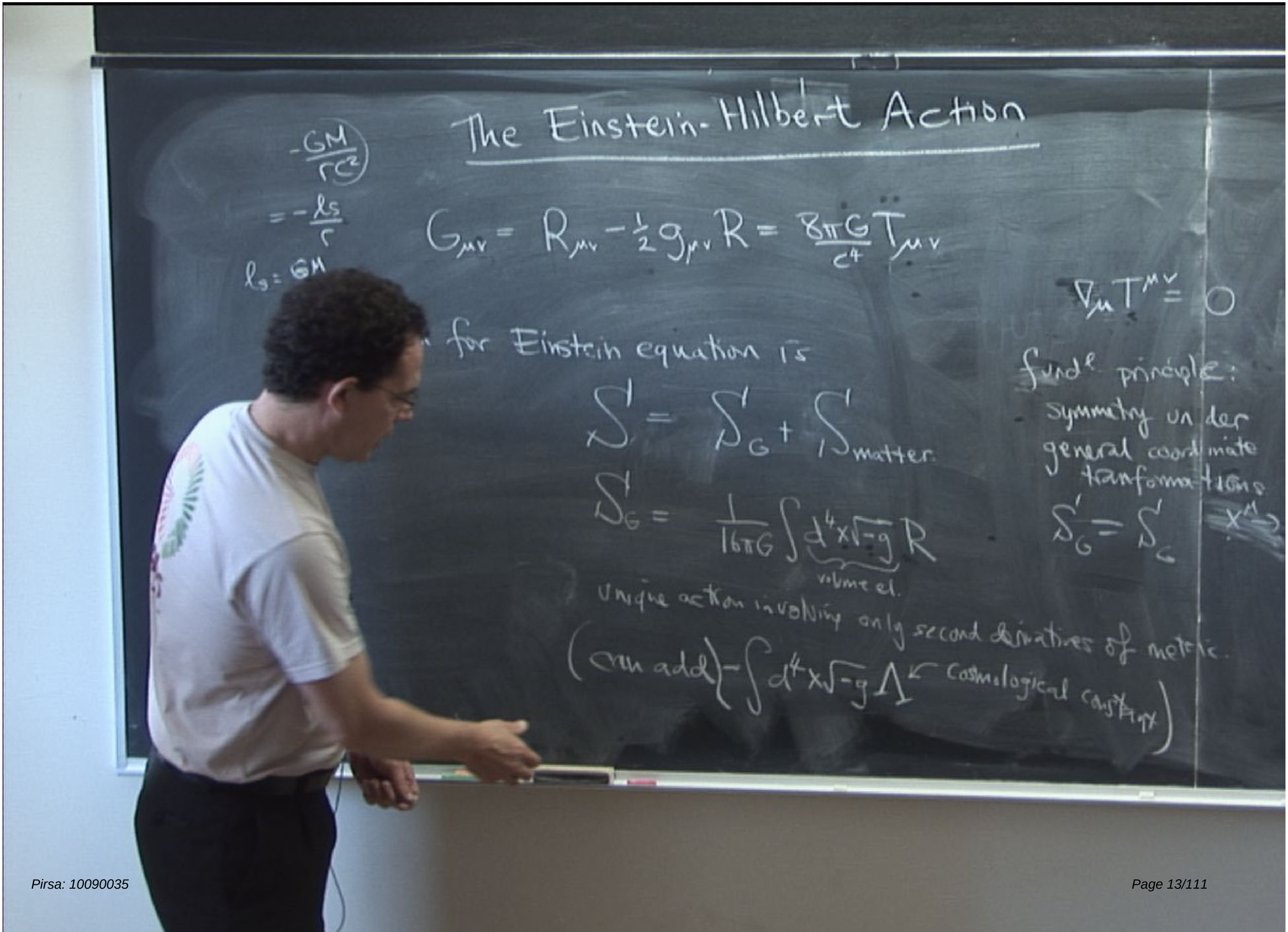
AIMS



The Einstein-Hilbert Action
$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$
$\nabla_\mu T^{\mu\nu} = 0$
for Einstein equation is
$S = S_G + S_{matter}$
$\int d^4x \sqrt{-g}\, R$
fund^e principle:
symmetry under
general coordinate
tranformations
$S'_G = S_G$


The Einstein-Hilbert Action
$-\frac{GM}{rc^2}$
$-\frac{\ell s}{r}$
$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$
$\nabla_\mu T^{\mu\nu} = 0$
Action for Einstein equation is
$S = S_G + S_{matter}$
$S_G = \frac{1}{16\pi G} \int \underbrace{d^4x\sqrt{-g}}_{\text{volume el.}} R$
unique action involving only second derivatives of metric.
fund^l principle:
symmetry under
general coordinate
transformations
$S_G' = S_G$
$x^\mu \rightarrow$


The Einstein-Hilbert Action
$-\frac{GM}{rc^2}$
$= -\frac{l_s}{r}$
$l_g = GM$
$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$
$\nabla_\mu T^{\mu\nu} = 0$
for Einstein equation is
$S = S_G + S_{matter}$
$S_G = \frac{1}{16\pi G} \int \underbrace{d^4x\sqrt{-g}}_{\text{volume el.}} R$
fund^e principle:
symmetry under
general coordinate
transformations
$S'_G = S_G$
unique action involving only second derivatives of metric.
(can add $\int d^4x\sqrt{-g}\,\Lambda$ ← cosmological constant)

Useful formulae

$$\delta(\sqrt{-g}) = \frac{1}{2}\sqrt{-g}\,\delta g_{\alpha\beta}\, g^{\alpha\beta}$$

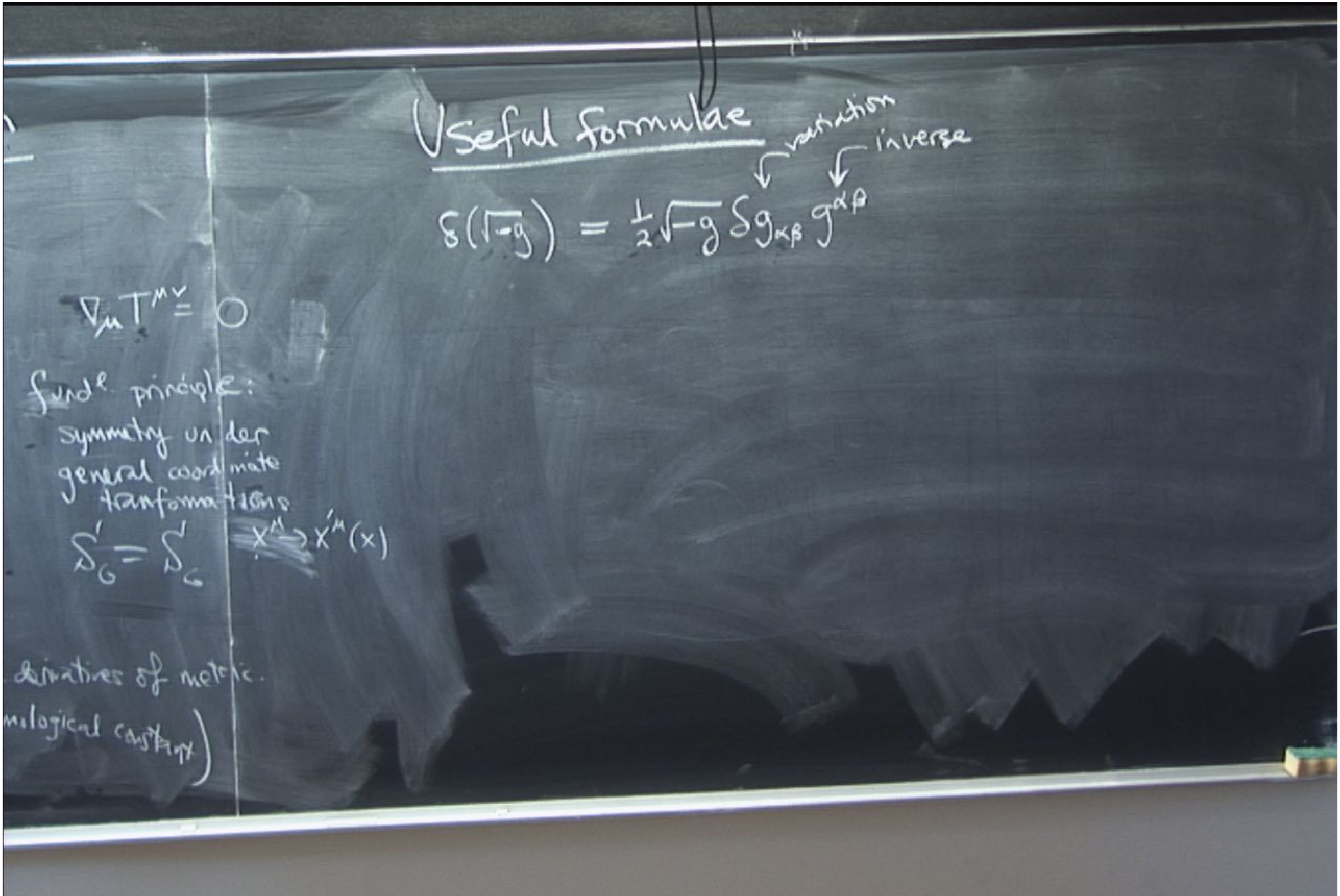
$\nabla_\mu T^{\mu\nu} = 0$

fund^l principle:
symmetry under
general coordinate
transformations

$S'_G = S_G \qquad x^\mu \to x'^\mu(x)$

derivatives of metric.

mological constant)


Useful formulae
variation
inverse
$\delta(\sqrt{-g}) = \frac{1}{2}\sqrt{-g}\,\delta g_{\alpha\beta}\, g^{\alpha\beta}$
$\nabla_\mu T^{\mu\nu} = 0$
fund^{l} principle:
symmetry under
general coordinate
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$S'_G = S_G$
$x^\mu \to x'^\mu(x)$
derivatives of metric.
mological constant)

$\nabla_\mu T^{\mu\nu} \stackrel{?}{=} 0$

fund^{l} principle:
symmetry under
general coordinate
transformations

$S_G' = S_G \qquad x^\mu \to x'^\mu(x)$

derivatives of metric.

...mological constant)

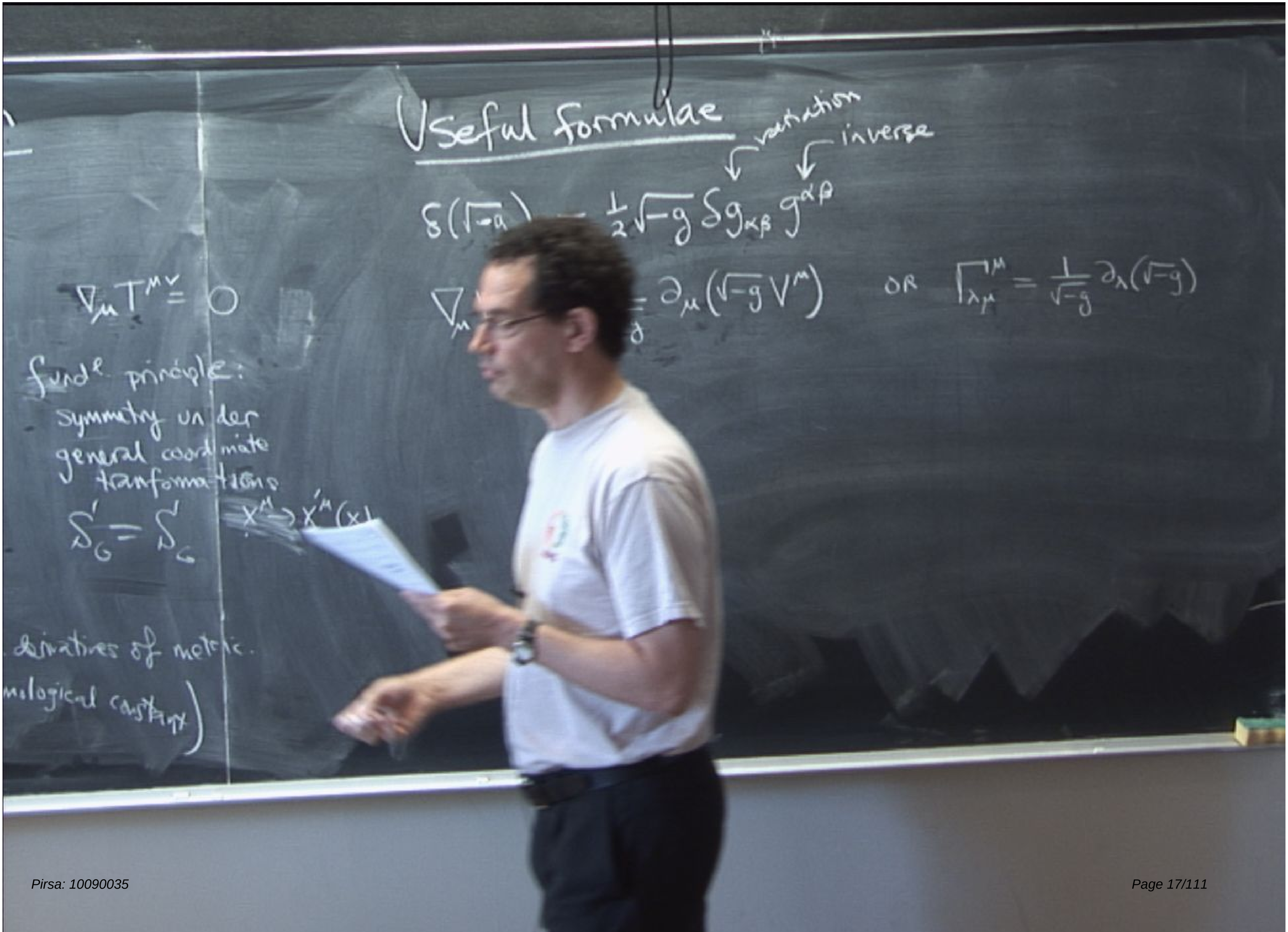
Useful formulae

variation

inverse

$$\delta(\sqrt{-g}) = \frac{1}{2}\sqrt{-g}\,\delta g_{\alpha\beta}\, g^{\alpha\beta}$$

$$\nabla_\mu V^\mu = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,V^\mu\right)$$


Useful formulae
variation
inverse
$\frac{1}{2}\sqrt{-g}\,\delta g_{\alpha\beta}\, g^{\alpha\beta}$
$\nabla_\mu T^{\mu\nu} = 0$
fund^{l} principle:
symmetry under
general coordinate
tranformations
$S'_G = S_G$
$x^\mu \to x'^\mu(x)$
$\partial_\mu(\sqrt{-g}\,V^\mu)$
OR
$\Gamma^\mu_{\lambda\mu} = \frac{1}{\sqrt{-g}}\partial_\lambda(\sqrt{-g})$
derivatives of metric.

$\nabla_\mu T^{\mu\nu} \stackrel{?}{=} 0$

fund^e principle:
symmetry under
general coordinate
tranformations

$S_G' = S_G$ $\quad x^\mu \to x'^\mu(x)$

derivatives of metric.

mological constant)

Useful formulae

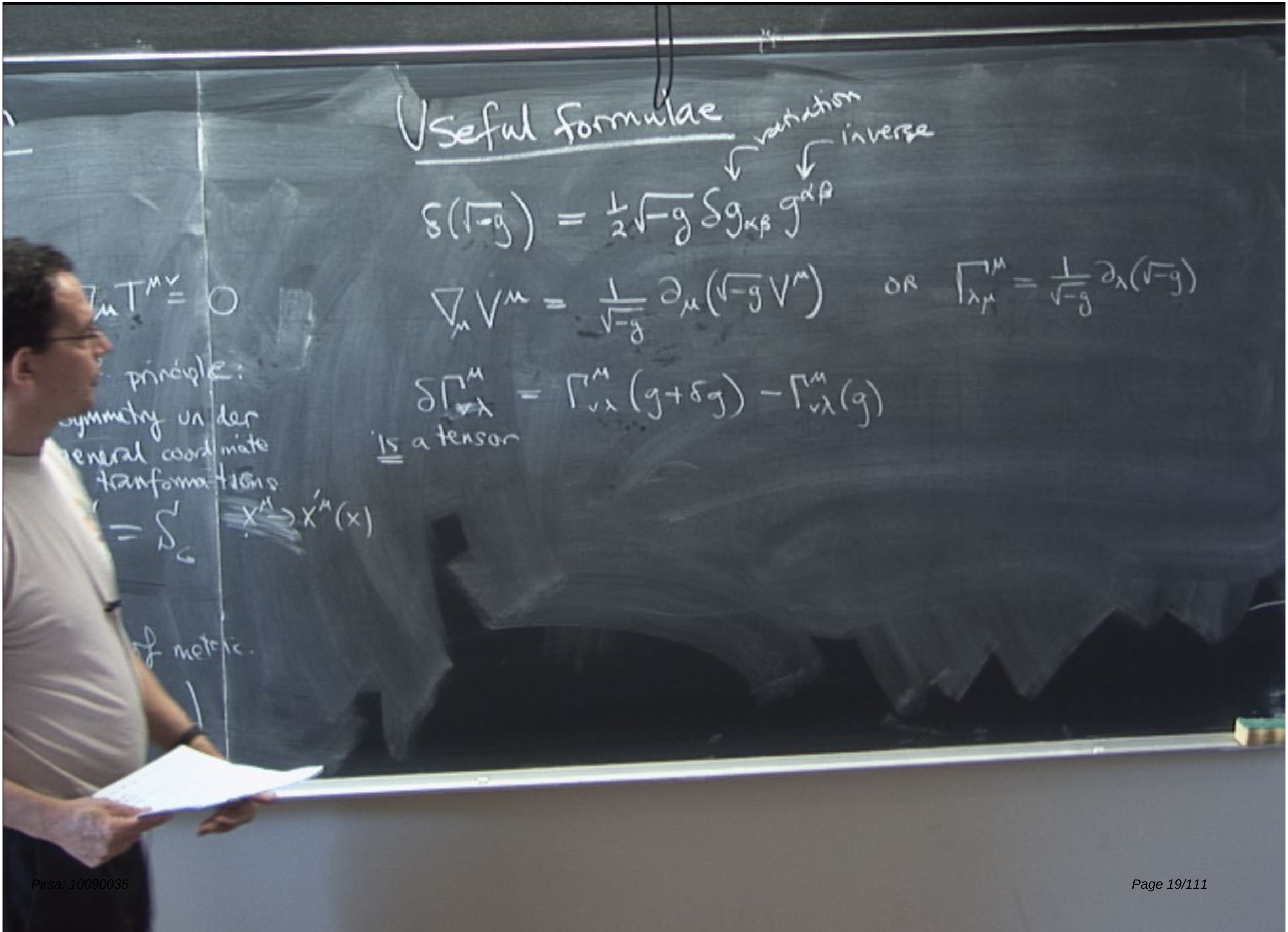
variation inverse

$$\delta(\sqrt{-g}) = \frac{1}{2}\sqrt{-g}\,\delta g_{\alpha\beta}\, g^{\alpha\beta}$$

$$\nabla_\mu V^\mu = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\,V^\mu) \quad \text{OR} \quad \Gamma^{\mu}_{\lambda\mu} = \frac{1}{\sqrt{-g}}\partial_\lambda(\sqrt{-g})$$

$\delta\Gamma^{\mu}_{\nu\lambda}$

is a tensor


Useful formulae
variation
inverse
$\delta(\sqrt{-g}) = \frac{1}{2}\sqrt{-g}\,\delta g_{\alpha\beta}\, g^{\alpha\beta}$
$\nabla_\mu V^\mu = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\,V^\mu)$ OR $\Gamma^\mu_{\lambda\mu} = \frac{1}{\sqrt{-g}}\partial_\lambda(\sqrt{-g})$
$\delta\Gamma^\mu_{\nu\lambda} = \Gamma^\mu_{\nu\lambda}(g+\delta g) - \Gamma^\mu_{\nu\lambda}(g)$
is a tensor
$\nabla_\mu T^{\mu\nu} = 0$
principle:
symmetry under
general coordinate
tranformations
$x^\mu \to x'^\mu(x)$
of metric.

$\nabla_\mu T^{\mu\nu} = 0$

fund^e principle:
symmetry under general coordinate tranformations

$S_G' = S_G \qquad x^\mu \to x'^\mu(x)$

derivatives of metric.

mological constant)

Useful formulae

variation   inverse

$$\delta(\sqrt{-g}) = \tfrac{1}{2}\sqrt{-g}\,\delta g_{\alpha\beta}\, g^{\alpha\beta}$$

$$\nabla_\mu V^\mu = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,V^\mu\right) \quad \text{OR} \quad \Gamma^\mu_{\lambda\mu} = \frac{1}{\sqrt{-g}}\partial_\lambda\left(\sqrt{-g}\right)$$

$$\delta\Gamma^\mu_{\nu\lambda} = \Gamma^\mu_{\nu\lambda}(g+\delta g) - \Gamma^\mu_{\nu\lambda}(g)$$

is a tensor

$$= \tfrac{1}{2} g^{\mu\alpha}\left(\delta g_{\nu\alpha;\lambda} + \delta g_{\lambda\alpha;\nu} - \delta g_{\nu\lambda;\alpha}\right)$$

Useful formulae

variation; inverse

$$\delta(\sqrt{-g}) = \frac{1}{2}\sqrt{-g}\,\delta g_{\alpha\beta}\, g^{\alpha\beta}$$

$$\nabla_\mu T^{\mu\nu} \stackrel{?}{=} 0$$

$$\nabla_\mu V^\mu = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,V^\mu\right) \quad \text{OR} \quad \Gamma^\mu_{\lambda\mu} = \frac{1}{\sqrt{-g}}\partial_\lambda(\sqrt{-g})$$

fund^{l} principle:
symmetry under
general coordinate
tranformations

$$S'_G = S_G \qquad x^\mu \to x'^\mu(x)$$

$$\delta\Gamma^\mu_{\nu\lambda} = \Gamma^\mu_{\nu\lambda}(g+\delta g) - \Gamma^\mu_{\nu\lambda}(g)$$

<u>is</u> a tensor

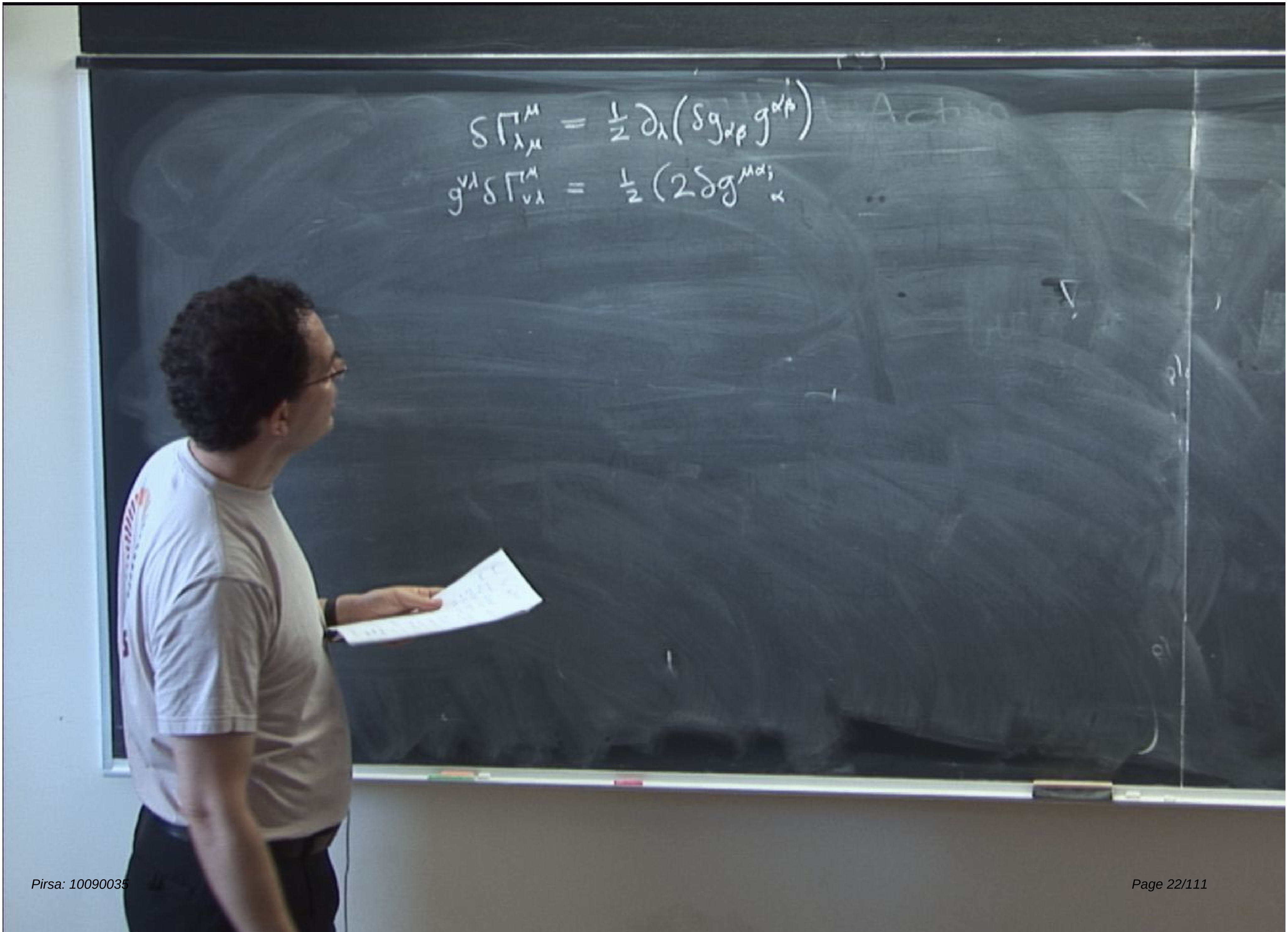
$$= \frac{1}{2} g^{\mu\alpha}\left(\delta g_{\nu\alpha;\lambda} + \delta g_{\lambda\alpha;\nu} - \delta g_{\nu\lambda;\alpha}\right)$$

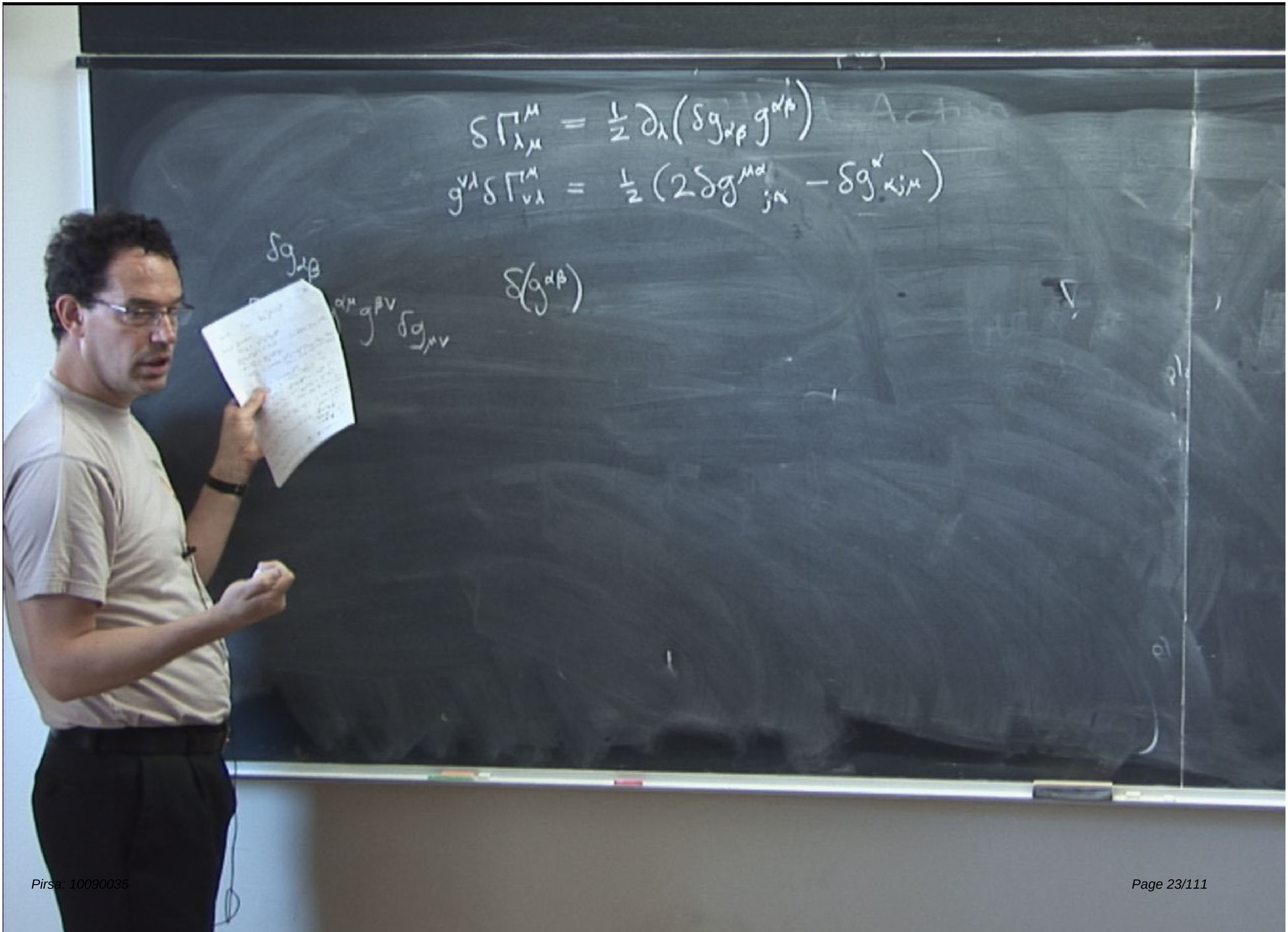
covariant deriv.

$$\delta g_{\nu\lambda;\alpha} = \delta g_{\nu\lambda,\alpha} - \Gamma\delta g - \Gamma\delta g$$

derivatives of metric.

(…mological constant)


$\delta\Gamma^{\mu}_{\lambda\mu} = \frac{1}{2}\partial_\lambda\left(\delta g_{\alpha\beta}\, g^{\alpha\beta}\right)$
$g^{\nu\lambda}\delta\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2}\left(2\delta g^{\mu\alpha}{}_{;\alpha}\right.$



$\delta\Gamma^{\mu}_{\lambda\mu} = \frac{1}{2}\partial_\lambda(\delta g_{\alpha\beta}\, g^{\alpha\beta})$
$g^{\nu\lambda}\delta\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2}(2\delta g^{\mu\alpha}{}_{;\alpha} - \delta g^{\alpha}{}_{\alpha;\mu})$
$\delta g_{\alpha\beta}$
$g^{\alpha\mu} g^{\beta\nu} \delta g_{\mu\nu}$
$\delta(g^{\alpha\beta})$

$$\delta \Gamma^{\mu}_{\lambda\mu} = \frac{1}{2} \partial_\lambda \left(\delta g_{\alpha\beta}\, g^{\alpha\beta} \right)$$

$$g^{\nu\lambda} \delta \Gamma^{\mu}_{\nu\lambda} = \frac{1}{2} \left(2 \delta g^{\mu\alpha}{}_{;\alpha} - \delta g^{\alpha}{}_{\alpha;\mu} \right)$$

$\delta g_{\alpha\beta}$

$$\delta g^{\alpha\beta} = g^{\alpha\mu} g^{\beta\nu} \delta g_{\mu\nu}$$

$$\delta(g^{\alpha\beta}) = - g^{\alpha\mu} \delta g_{\mu\nu} g^{\beta\nu} = - \delta g^{\alpha\beta} \quad !$$

$$\delta\Gamma^{\mu}_{\lambda\mu} = \frac{1}{2}\partial_\lambda\left(\delta g_{\alpha\beta}\, g^{\alpha\beta}\right)$$

$$g^{\nu\lambda}\delta\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2}\left(2\delta g^{\mu\alpha}{}_{;\alpha} - \delta g^{\alpha}{}_{\alpha}{}^{;\mu}\right)$$

$\delta g_{\alpha\beta}$

$\delta g^{\alpha\beta} = g^{\alpha\mu} g^{\beta\nu}\delta g_{\mu\nu}$

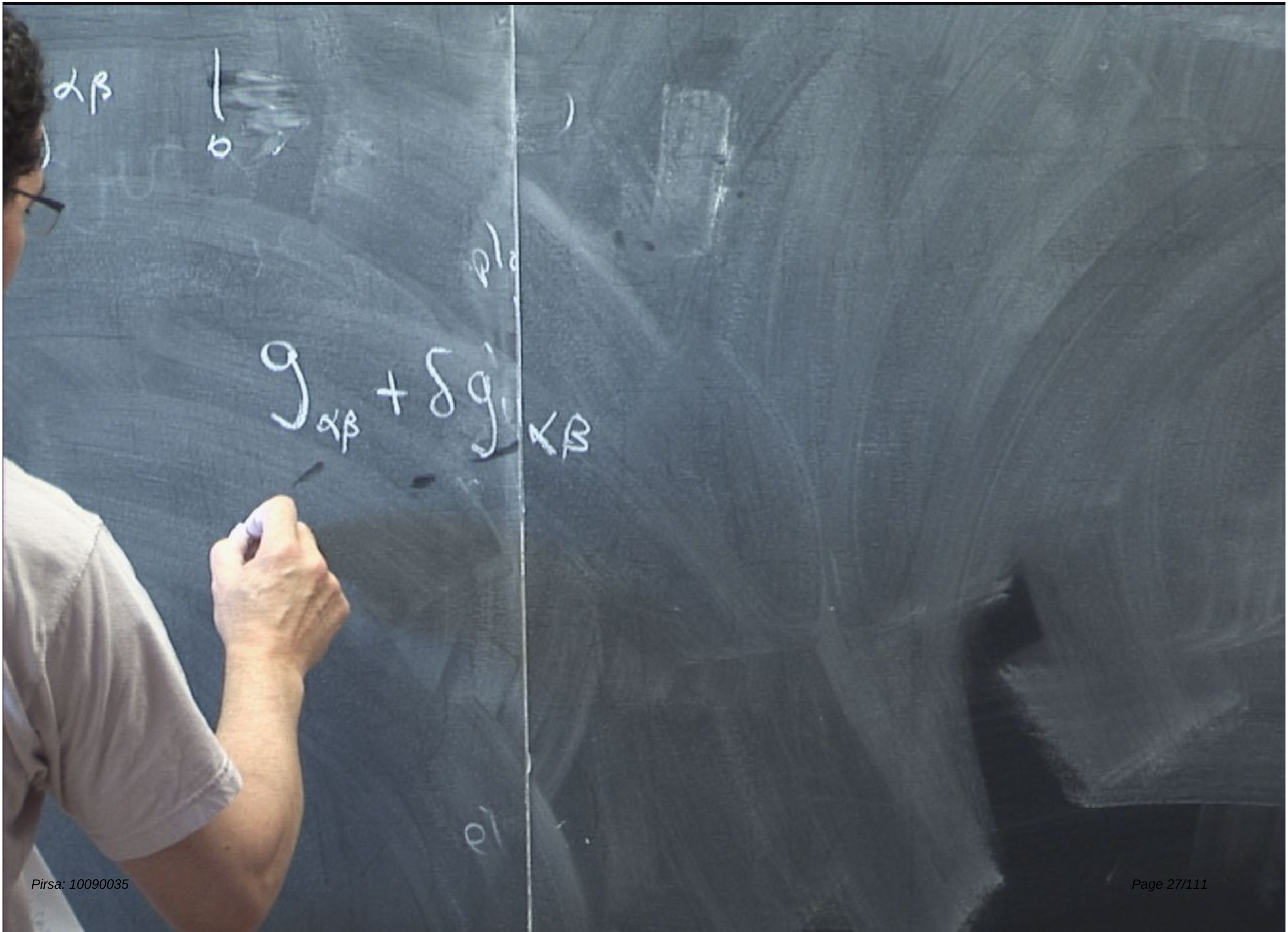
$$\delta(g^{\alpha\beta}) = -g^{\alpha\mu}\delta g_{\mu\nu}\, g^{\beta\nu} = -\delta g^{\alpha\beta} \quad !$$

$$g^{\alpha\beta} g_{\beta\nu} = \delta^{\alpha}_{\nu}$$

$$\delta(g^{\alpha\beta})\, g_{\beta\nu} + g^{\alpha\beta}\delta g_{\beta\nu} = 0$$

$$\Rightarrow \delta(g^{\alpha\beta}) = -g^{\alpha\mu}\delta g_{\mu\nu}\, g^{\beta\nu}$$

$\delta(\sqrt{-g}\,R)$


$g_{\alpha\beta} + \delta g_{\alpha\beta}$

$g^{\alpha\beta}$

$$\delta g_{\alpha\beta} = \left(g_{\alpha\beta} + \delta g_{\alpha\beta}\right) - g_{\alpha\beta}$$

$g^{\alpha\beta}$

$$\delta g_{\alpha\beta} = \left(g_{\alpha\beta} + \delta g_{\alpha\beta}\right) - g_{\alpha\beta}$$

$\delta\left(g^{\alpha\beta}\right) =$

$$g^{\alpha\beta}$$

$$\delta g_{\alpha\beta} = \left(g_{\alpha\beta} + \delta g_{\alpha\beta}\right) - g_{\alpha\beta}$$

$$\delta\left(g^{\alpha\beta}\right) = \left(g_{\alpha\beta} + \delta g_{\alpha\beta}\right)^{-1} - g^{-1}_{\alpha\beta}$$

$$= -g^{\alpha\mu}\,\delta g_{\mu\nu}\, g^{\nu\beta} = \delta g^{\alpha\beta}$$

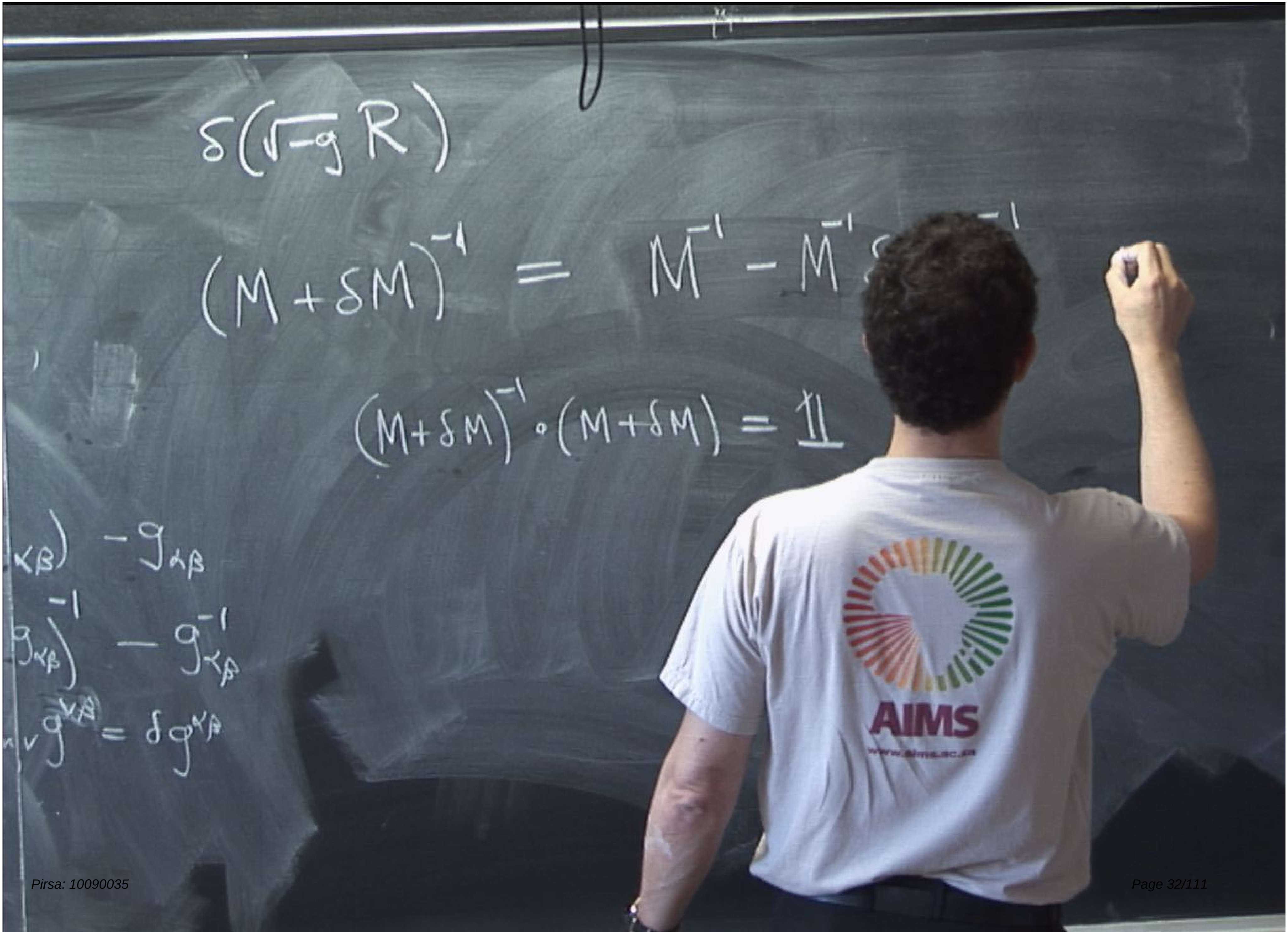
$$\delta(\sqrt{-g}\,R)$$

$$(M+\delta M)^{-1} =$$

$$\alpha\beta) - g_{\alpha\beta}$$

$$g_{\alpha\beta})^{-1} - g^{-1}_{\alpha\beta}$$

$$\nu g^{\nu\beta} = \delta g^{\alpha\beta}$$


$\delta(\sqrt{-g}\,R)$
$(M+\delta M)^{-1} = M^{-1} - M^{-1}$
$(M+\delta M)^{-1}\cdot(M+\delta M) = \mathbb{1}$

$$\delta(\sqrt{-g}\,R)$$

$$(M+\delta M)^{-1} = M^{-1} - M^{-1}\delta M M^{-1} + o(\delta M^2)$$

$$\underbrace{(M+\delta M)^{-1}}_{(M - M^{-1}\delta M M^{-1})} \cdot (M+\delta M) = \mathbb{1}$$

$$(M - M^{-1}\delta M M^{-1})(M+\delta M)$$

$$\alpha\beta) - g_{\alpha\beta}$$

$$g_{\alpha\beta})^{-1} - g^{-1}_{\alpha\beta}$$

$$\nu\, g^{\nu\beta} = \delta g^{\alpha\beta}$$

$$\delta\left(\sqrt{-g}\,R\right)$$

$$(M+\delta M)^{-1} = M^{-1} - M^{-1}\,\delta M\, M^{-1} + o(\delta M^2)$$

$$\underbrace{(M+\delta M)^{-1}} \circ (M+\delta M) = \mathbb{1}$$

$$\alpha\beta) - g_{\alpha\beta} \qquad \left(M^{-1} - M^{-1}\delta M M^{-1}\right)(M+\delta M)$$

$$g_{\alpha\beta})^{-1} - g^{-1}_{\alpha\beta} \qquad \mathbb{1} - M^{-1}\delta M + M^{-1}\delta M = \mathbb{1} + o(\delta M^2)$$

$$g^{\nu\beta} = \delta g^{\nu\beta}$$

$$\frac{1}{1+x} = 1 - x + o(x^2)$$

$$\delta(\sqrt{-g}\,R)$$

$$(M+\delta M)^{-1} = \underbrace{M^{-1} - M^{-1}\delta M M^{-1}} + o(\delta M^2)$$

$$\underbrace{(M+\delta M)^{-1}} \cdot (M+\delta M) = \mathbb{1}$$

$$\kappa\beta) - g_{\alpha\beta} \qquad (M^{-1} - M^{-1}\delta M M^{-1})(M+\delta M)$$

$$g_{\alpha\beta})^{-1} - g^{-1}_{\alpha\beta} \qquad \mathbb{1} - M^{-1}\delta M + M^{-1}\delta M = \mathbb{1} + o(\delta M^2)$$

$$g^{\nu\beta} = \delta g^{\alpha\beta}$$

$$\delta\left(\sqrt{-g}\,R\right)$$

$$\frac{1}{1+x} = 1 - x + o(x^2)$$

$$g = \det(g_{\alpha\beta})$$

$$= \left(\sqrt{-(g+\delta g)}\right)$$

$$\alpha\beta) - g_{\alpha\beta}$$

$$g_{\alpha\beta})^{-1} - g^{-1}_{\alpha\beta}$$

$$g^{\alpha\beta} = \delta g^{\alpha\beta}$$

$$\frac{1}{1+x} = 1 - x + o(x^2)$$

$$g = \det(g_{\alpha\beta})$$

$$\delta(\sqrt{-g}\,R)$$

$$= \sqrt{-(g+\delta g)}\,R(g+\delta g) - \sqrt{-g}\,R(g)$$

$$\ldots_{\alpha\beta}) - g_{\alpha\beta}$$

$$\ldots g_{\alpha\beta})^{-1} - g^{-1}_{\alpha\beta}$$

$$\ldots g^{\alpha\beta}$$

$$\delta(\sqrt{-g}\,R) \qquad \frac{1}{1+x} = 1 - x + o(x^2) \qquad g = \det(g_{\alpha\beta})$$

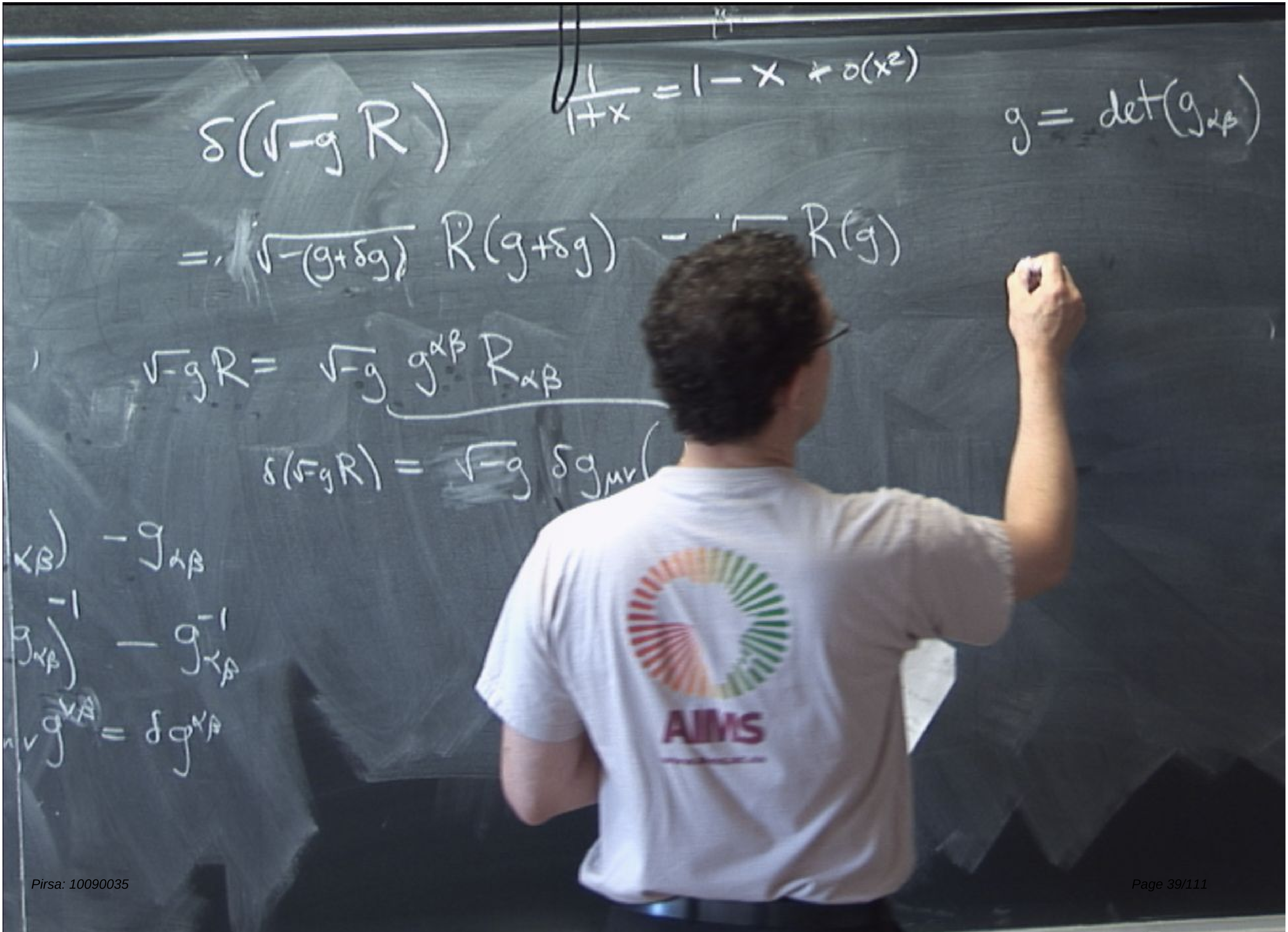
$$= \sqrt{-(g+\delta g)}\; R(g+\delta g) - \sqrt{-g}\,R(g)$$

$$\sqrt{-g}\,R = \sqrt{-g}\; g^{\alpha\beta} R_{\alpha\beta}$$

$$\alpha\beta) - g_{\alpha\beta}$$

$$g_{\alpha\beta})^{-1} - g^{-1}_{\alpha\beta}$$

$$g^{\alpha\beta} = \delta g^{\alpha\beta}$$


$\delta(\sqrt{-g}R)$
$\frac{1}{1+x} = 1 - x + o(x^2)$
$g = \det(g_{\alpha\beta})$
$= \sqrt{-(g+\delta g)}\, R(g+\delta g) - \quad R(g)$
$\sqrt{-g}R = \sqrt{-g}\, g^{\alpha\beta} R_{\alpha\beta}$
$\delta(\sqrt{-g}R) = \sqrt{-g}\, \delta g_{\mu\nu}($
$\alpha\beta) - g_{\alpha\beta}$
$g_{\alpha\beta})^{-1} - g^{-1}_{\alpha\beta}$
$g^{\nu\beta} = \delta g^{\alpha\beta}$
AIMS

$$\delta(\sqrt{-g}\,R)$$

$$\frac{1}{1+x} = 1 - x + o(x^2)$$

$$g = \det(g_{\alpha\beta})$$

$$= \sqrt{-(g+\delta g)}\, R(g+\delta g) - \sqrt{-g}\, R(g)$$

$$\delta\sqrt{-g} = \tfrac{1}{2}\sqrt{-g}\, g^{\alpha}$$

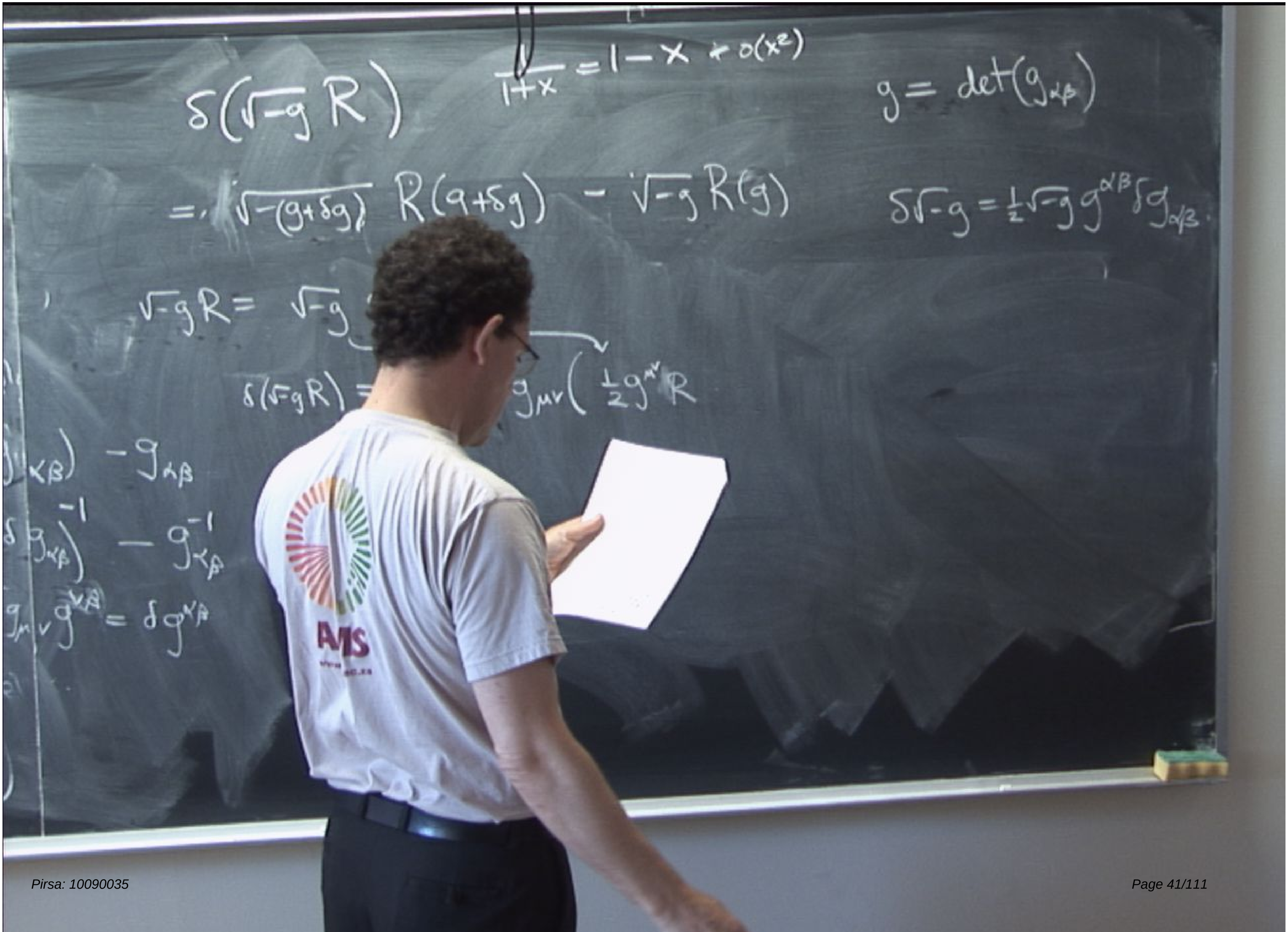
$$\sqrt{-g}\,R = \sqrt{-g}\, g^{\alpha\beta} R_{\alpha\beta}$$

$$\delta(\sqrt{-g}\,R) = \sqrt{-g}\,\delta g_{\mu\nu}\Big(\tfrac{1}{2} g^{\mu\nu} R$$

$${}_{\alpha\beta}) - g_{\alpha\beta}$$

$${}^{-1} - g^{-1}_{\alpha\beta}$$

$$= \delta g^{\alpha\beta}$$



$\delta(\sqrt{-g}\,R)$
$\frac{1}{1+x} = 1 - x + o(x^2)$
$g = \det(g_{\alpha\beta})$
$= \sqrt{-(g+\delta g)}\,R(g+\delta g) - \sqrt{-g}\,R(g)$
$\delta\sqrt{-g} = \frac{1}{2}\sqrt{-g}\,g^{\alpha\beta}\delta g_{\alpha\beta}$
$\sqrt{-g}R = \sqrt{-g}$
$\delta(\sqrt{-g}R) =$
$g_{\mu\nu}(\frac{1}{2}g^{\mu\nu}R$
$- g_{\alpha\beta}$
$- g^{-1}_{\alpha\beta}$
$= \delta g^{\alpha\beta}$

$$\frac{1}{1+x} = 1 - x + o(x^2)$$

$$\delta(\sqrt{-g}\,R) \qquad\qquad g = \det(g_{\alpha\beta})$$

$$= \sqrt{-(g+\delta g)}\,R(g+\delta g) - \sqrt{-g}\,R(g) \qquad \delta\sqrt{-g} = \tfrac{1}{2}\sqrt{-g}\,g^{\alpha\beta}\delta g_{\alpha\beta}$$

$$\sqrt{-g}\,R = \sqrt{-g}\,g^{\alpha\beta}R_{\alpha\beta}$$

$$\delta(\sqrt{-g}\,R) = \sqrt{-g}\,\delta g_{\mu\nu}\left(\tfrac{1}{2}g^{\mu\nu}R - R^{\mu\nu}\right) + \sqrt{-g}\,g^{\mu\nu}\delta R_{\mu\nu}$$

$$g_{\alpha\beta}) - g_{\alpha\beta}$$

$$\delta g_{\alpha\beta})^{-1} - g^{-1}_{\alpha\beta}$$

$$g_{\mu\nu}\,g^{\nu\beta} = \delta g^{\alpha\beta}$$

$$\frac{1}{1+x} = 1 - x + o(x^2)$$

$$\delta(\sqrt{-g}\,R) \qquad g = \det(g_{\alpha\beta})$$

$$= \sqrt{-(g+\delta g)}\; R(g+\delta g) - \sqrt{-g}\, R(g) \qquad \delta\sqrt{-g} = \tfrac{1}{2}\sqrt{-g}\, g^{\alpha\beta}\,\delta g_{\alpha\beta}$$

$$\sqrt{-g}\,R = \sqrt{-g}\; g^{\alpha\beta} R_{\alpha\beta}$$

$$\delta(\sqrt{-g}\,R) = \sqrt{-g}\;\delta g_{\mu\nu}\left(\tfrac{1}{2} g^{\mu\nu} R - R^{\mu\nu}\right) + \sqrt{-g}\, g^{\mu\nu}\,\delta R_{\mu\nu}$$

$$(g_{\alpha\beta}) - g_{\alpha\beta}$$

$$\delta R_{\mu\nu} = -\,\delta\Gamma^{\lambda}_{\mu\lambda;\nu} + \delta\Gamma^{\lambda}_{\mu\nu;\lambda}$$

$$(\ \delta g_{\alpha\beta})^{-1} - g^{-1}_{\alpha\beta}$$

(follows from $R_{\mu\nu} = -\Gamma^{\lambda}_{\mu\lambda,\nu} + \Gamma^{\lambda}_{\mu\nu,\lambda} + \Gamma^2$ terms.

$$g_{\mu\nu}\, g^{\nu\beta} = \delta g^{\alpha\beta}$$

$$\frac{1}{1+x} = 1 - x + o(x^2)$$

$$\delta(\sqrt{-g}\,R)$$

$$g = \det(g_{\alpha\beta})$$

$$= \sqrt{-(g+\delta g)}\; R(g+\delta g) - \sqrt{-g}\, R(g)$$

$$\delta\sqrt{-g} = \tfrac{1}{2}\sqrt{-g}\, g^{\alpha\beta}\,\delta g_{\alpha\beta}.$$

$$\sqrt{-g}\,R = \sqrt{-g}\; g^{\alpha\beta} R_{\alpha\beta}$$

$$\delta(\sqrt{-g}\,R) = \sqrt{-g}\;\delta g_{\mu\nu}\left(\tfrac{1}{2} g^{\mu\nu} R - R^{\mu\nu}\right) + \sqrt{-g}\, g^{\mu\nu}\,\delta R_{\mu\nu}$$

$$(g_{\alpha\beta}) - g_{\alpha\beta}$$

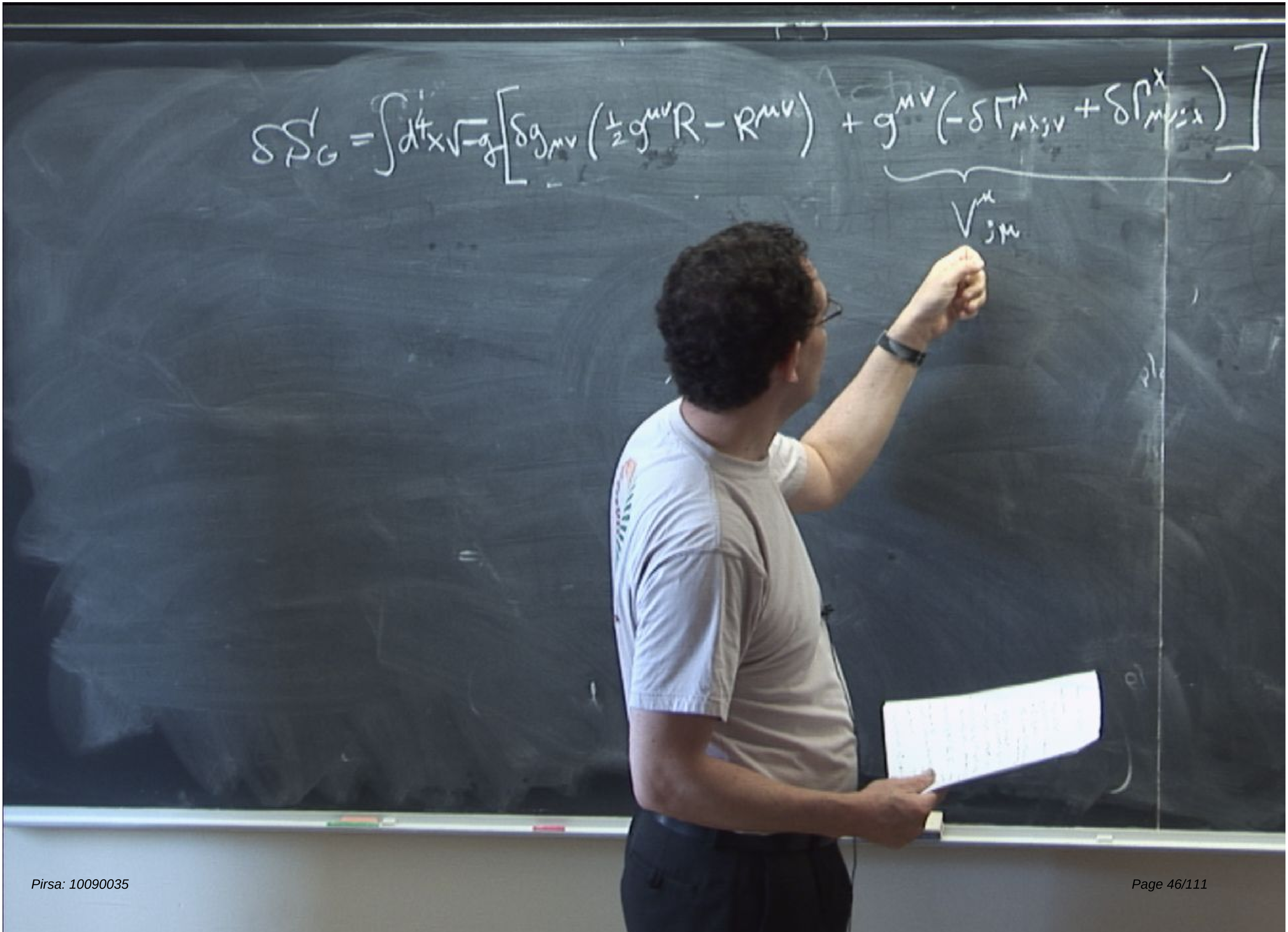
$$\delta R_{\mu\nu} = -\,\delta\Gamma^{\lambda}_{\mu\lambda;\nu} + \delta\Gamma^{\lambda}_{\mu\nu;\lambda}$$

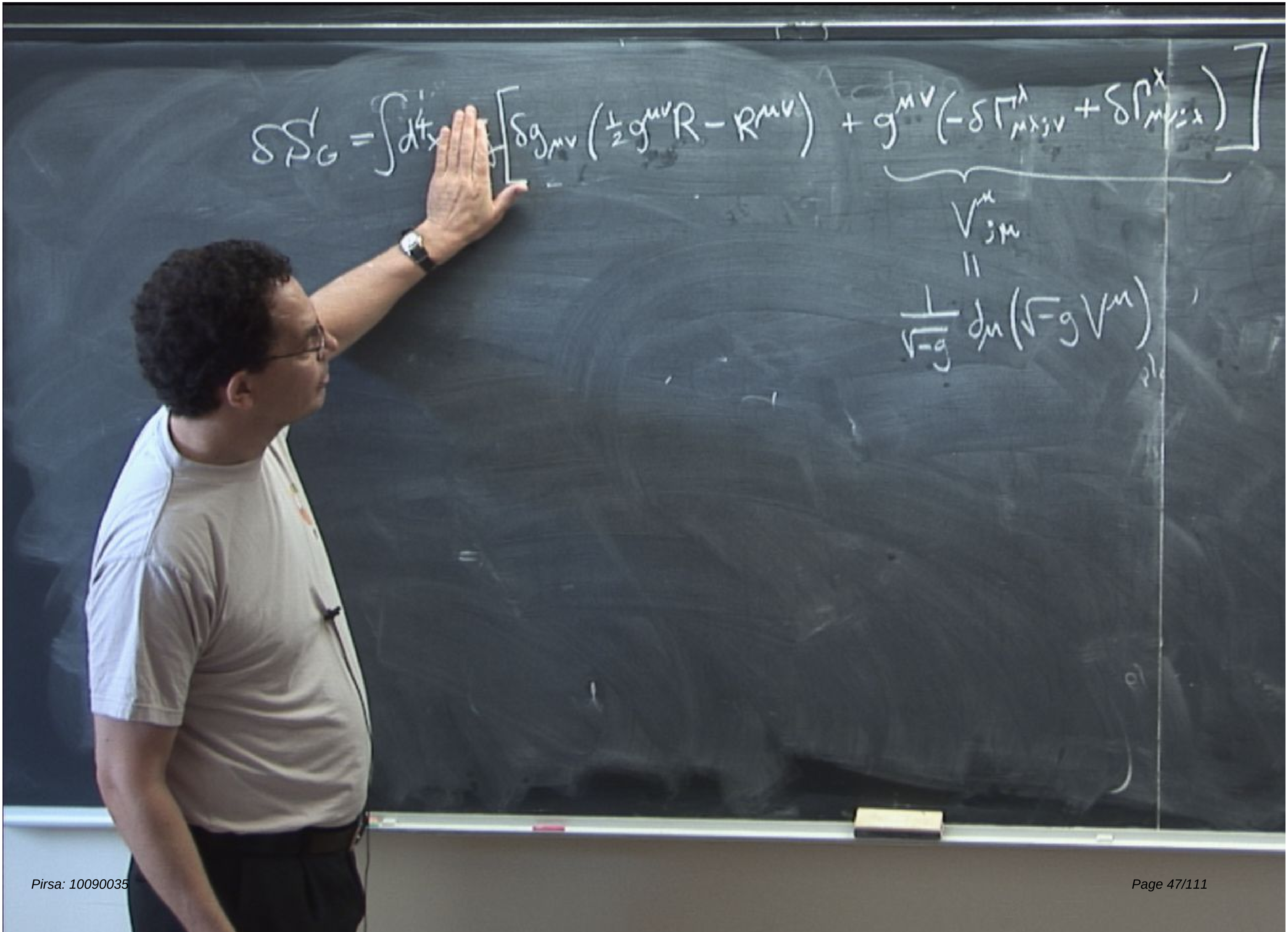
$$(\delta g_{\alpha\beta})^{-1} - g^{-1}_{\alpha\beta}$$

(follows from $R_{\mu\nu} = -\Gamma^{\lambda}_{\mu\lambda,\nu} + \Gamma^{\lambda}_{\mu\nu,\lambda} + \Gamma^2$ terms.)

$$g_{\mu\nu}\, g^{\nu\beta} = \delta g^{\alpha\beta}$$

$$\delta S_G = \int d^4x \sqrt{-g}\left[\delta g_{\mu\nu}\left(\tfrac{1}{2}g^{\mu\nu}R - R^{\mu\nu}\right) + g^{\mu\nu}\left(-\delta\Gamma^{\lambda}_{\mu\lambda;\nu} + \delta\Gamma^{\lambda}_{\mu\nu;\lambda}\right)\right]$$


$\delta S_G = \int d^4x \sqrt{-g}\left[\delta g_{\mu\nu}\left(\frac{1}{2}g^{\mu\nu}R - R^{\mu\nu}\right) + \underbrace{g^{\mu\nu}\left(-\delta\Gamma^{\lambda}_{\mu\lambda;\nu} + \delta\Gamma^{\lambda}_{\mu\nu;\lambda}\right)}_{V^{\mu}_{;\mu}}\right]$


$\delta S'_G = \int d^4x \, [\delta g_{\mu\nu}(\frac{1}{2} g^{\mu\nu} R - R^{\mu\nu}) + g^{\mu\nu}(-\delta \Gamma^{\lambda}_{\mu\lambda;\nu} + \delta \Gamma^{\lambda}_{\mu\nu;\lambda})]$
$V^{\mu}_{;\mu}$
$= \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} V^\mu)$

$$\delta S_G = \int d^4x \sqrt{-g}\left[\delta g_{\mu\nu}\left(\tfrac{1}{2} g^{\mu\nu}R - R^{\mu\nu}\right) + \underbrace{g^{\mu\nu}\left(-\delta\Gamma^{\lambda}_{\mu\lambda;\nu} + \delta\Gamma^{\lambda}_{\mu\nu;\lambda}\right)}_{V^{\mu}{}_{;\mu}}\right]$$

$$V^{\mu}{}_{;\mu} = \frac{1}{\sqrt{-g}}\,\partial_\mu\left(\sqrt{-g}\,V^{\mu}\right)$$

$\Rightarrow$ total divergence

$\Rightarrow$ zero if $\delta g_{\alpha\beta}$ vanishes on surface of region of spacetime being considered.

$\sqrt{-g}\,R =$

$$\delta S'_G = \int d^4x \sqrt{-g}\left[\delta g_{\mu\nu}\left(\tfrac{1}{2}g^{\mu\nu}R - R^{\mu\nu}\right) + \underbrace{g^{\mu\nu}\left(-\delta\Gamma^{\lambda}_{\mu\lambda;\nu} + \delta\Gamma^{\lambda}_{\mu\nu;\lambda}\right)}_{V^{\mu}_{;\mu}}\right]$$

$$V^{\mu}_{;\mu} = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,V^{\mu}\right)$$

$\Rightarrow$ total divergence

$\Rightarrow$ zero if $\delta g_{\alpha\beta}$ vanishes on surface of region of spacetime being considered.

Now consider

$$\delta S'_{matter} = \int d^4x \sqrt{-g}\,\delta g_{\mu\nu}$$

$\delta(\sqrt{-}$

$= \int\sqrt{-}$

$\sqrt{-g}R =$

$\delta(\sqrt{-}$

$$\delta S_G = \int d^4x \sqrt{-g}\left[\delta g_{\mu\nu}\left(\tfrac{1}{2}g^{\mu\nu}R - R^{\mu\nu}\right) + \underbrace{g^{\mu\nu}\left(-\delta\Gamma^{\lambda}_{\mu\lambda;\nu} + \delta\Gamma^{\lambda}_{\mu\nu;\lambda}\right)}_{V^{\mu}_{;\mu}}\right]$$

$$V^{\mu}_{;\mu} = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,V^\mu\right)$$

$\Rightarrow$ total divergence

$\Rightarrow$ zero if $\delta g_{\alpha\beta}$ vanishes on surface of region of spacetime being considered.

Now consider

defn. of coeff. of $\delta g_{\mu\nu}$

$$\delta S_{matter} = \int d^4x \sqrt{-g}\,\delta g_{\mu\nu}\left(\frac{T^{\mu\nu}}{2}\right)$$

$\delta(\sqrt{-}$

$= \sqrt{-}$

$\sqrt{-g}R =$

$\delta(\sqrt{-}$

$$\delta_1 S_G = \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\left[\delta g_{\mu\nu}\left(\tfrac{1}{2}g^{\mu\nu}R - R^{\mu\nu}\right) + \underbrace{g^{\mu\nu}\left(-\delta\Gamma^{\lambda}_{\mu\lambda;\nu} + \delta\Gamma^{\lambda}_{\mu\nu;\lambda}\right)}_{V^{\mu}{}_{;\mu}}\right]$$

$$V^{\mu}{}_{;\mu} = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,V^{\mu}\right)$$

$\Rightarrow$ total divergence

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$\delta(\sqrt{-}$

$= \sqrt{-}$

$\sqrt{-g}\,R =$

$\delta(\sqrt{-}$

$$\delta_1 S_G = \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\Big[\delta g_{\mu\nu}\left(\tfrac{1}{2}g^{\mu\nu}R - R^{\mu\nu}\right) + \underbrace{g^{\mu\nu}\left(-\delta\Gamma^\lambda_{\mu\lambda;\nu} + \delta\Gamma^\lambda_{\mu\nu;\lambda}\right)}_{V^\mu_{;\mu}}\Big]$$

$$V^\mu_{;\mu} = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,V^\mu\right)$$

$\Rightarrow$ total divergence

$\Rightarrow$ zero if $\delta g_{\alpha\beta}$ vanishes on surface of region of spacetime being considered.

Now consider

defn. of coeff. of $\delta g_{\mu\nu}$

$$\delta_1 S_{matter} = \int d^4x\sqrt{-g}\,\delta g_{\mu\nu}\left(\frac{T^{\mu\nu}}{2}\right)$$

$$\delta_1 S_G + \delta_1 S_{matter} = 0$$

$\Rightarrow$

$\delta(\sqrt{-}$

$= \sqrt{-}$

$\sqrt{-g}R =$

$\delta(\sqrt{-}$

$$\delta_1 S_G = \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\Big[\delta g_{\mu\nu}\left(\tfrac{1}{2}g^{\mu\nu}R - R^{\mu\nu}\right) + \underbrace{g^{\mu\nu}\left(-\delta\Gamma^{\lambda}_{\mu\lambda;\nu} + \delta\Gamma^{\lambda}_{\mu\nu;\lambda}\right)}_{V^{\mu}_{;\mu}}\Big]$$

$$V^{\mu}_{;\mu} = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,V^\mu\right)$$

$\Rightarrow$ total divergence

$\Rightarrow$ zero if $\delta g_{\alpha\beta}$ vanishes on surface of region of spacetime being considered.

Now consider

defn. of coeff. of $\delta g_{\mu\nu}$

$$\delta_1 S_{matter} = \int d^4x\sqrt{-g}\,\delta g_{\mu\nu}\left(\frac{T^{\mu\nu}}{2}\right)$$

$$\delta_1 S_G + \delta S_{matter} = 0$$

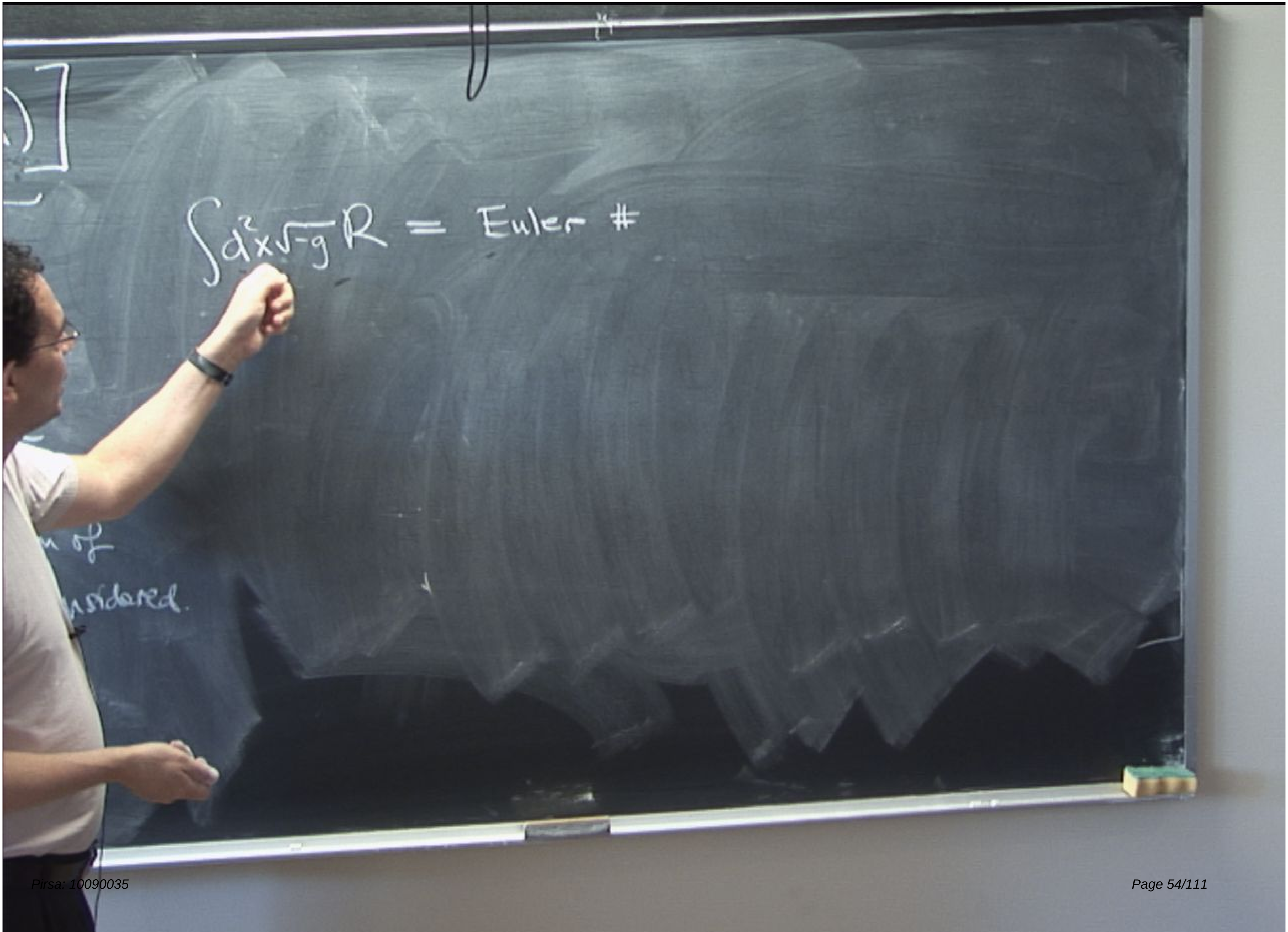
$$\Rightarrow \boxed{R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}}$$

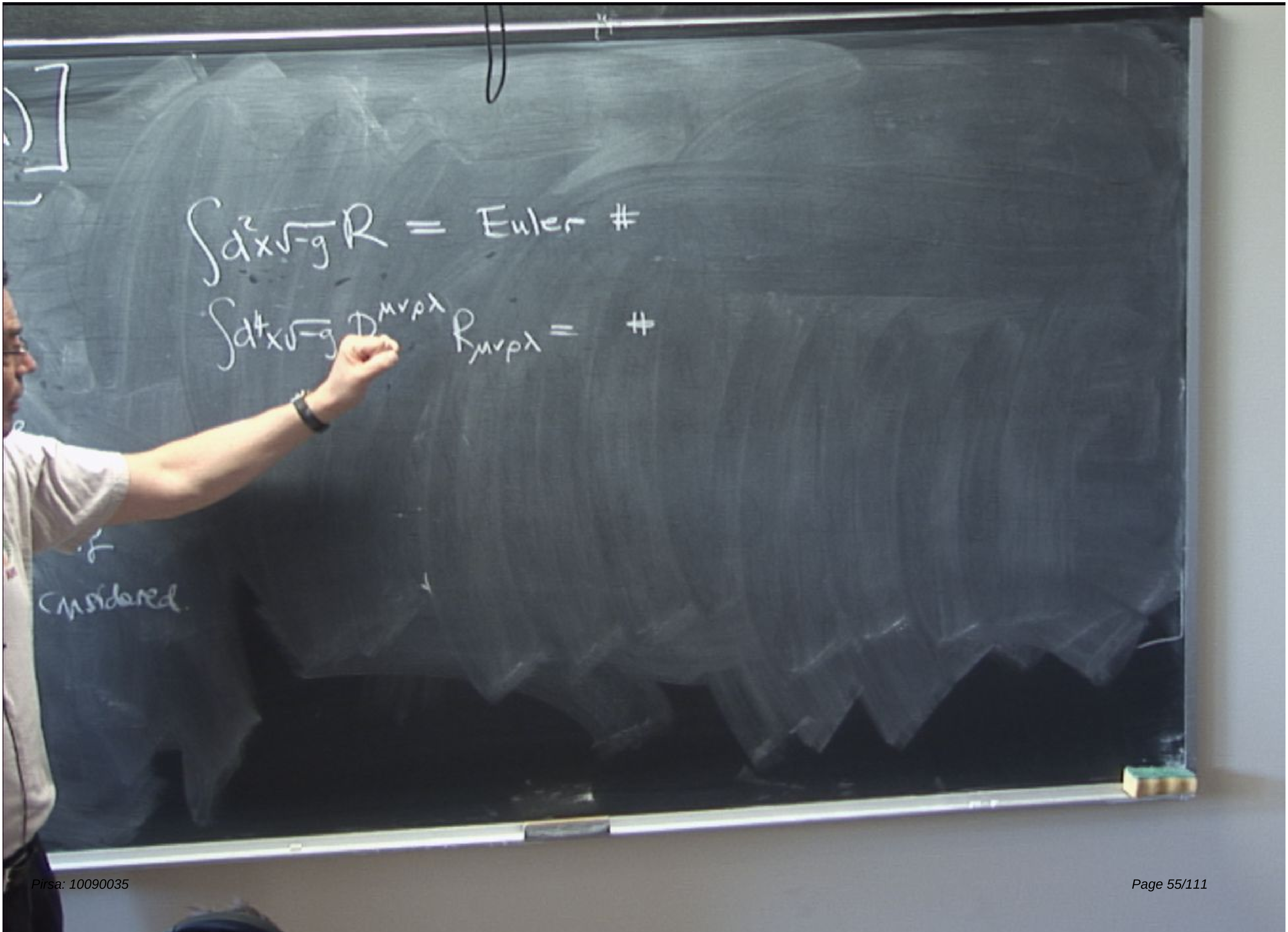
$\delta(\sqrt{-\ldots}$

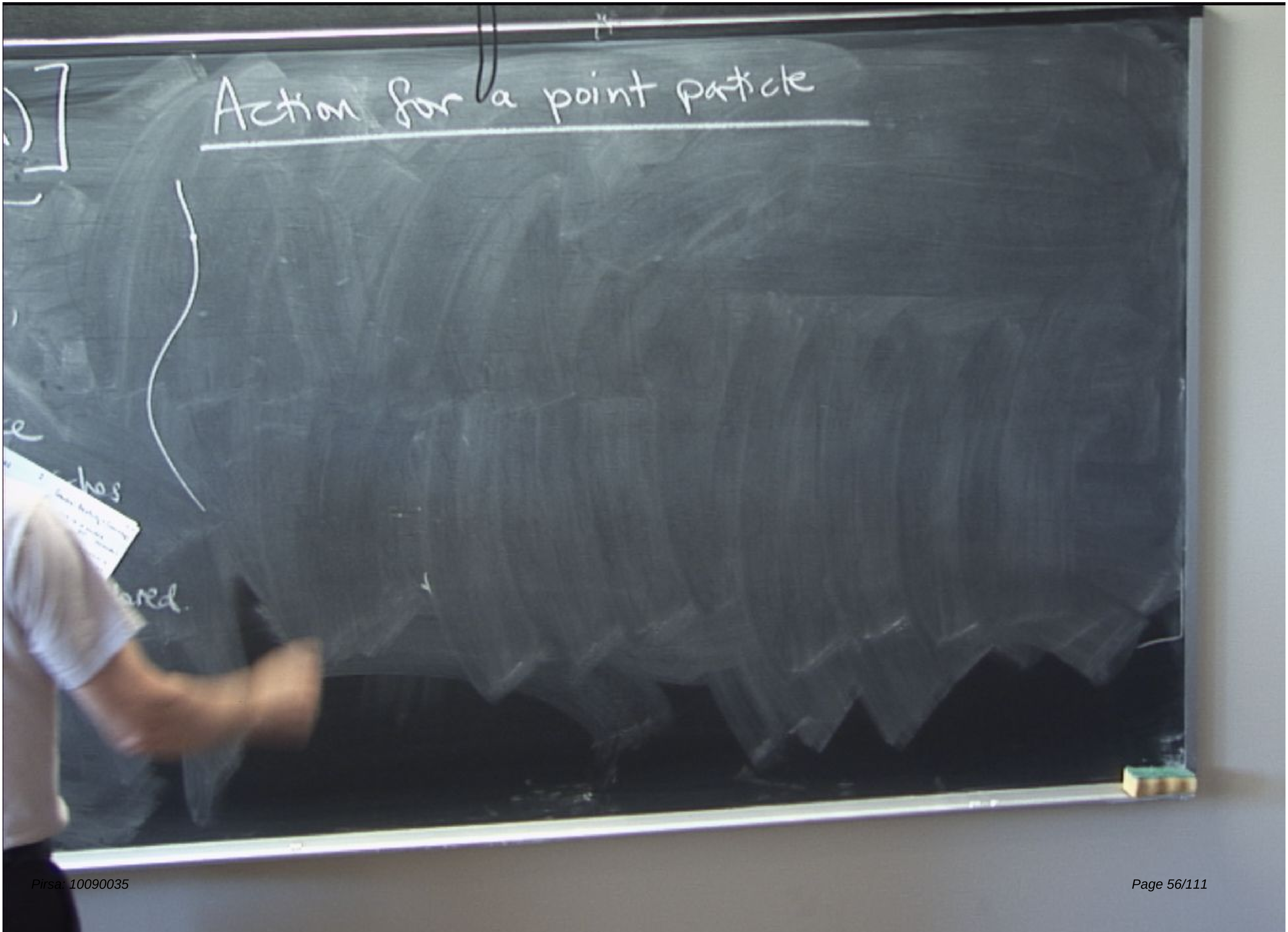
$= \sqrt{-\ldots}$

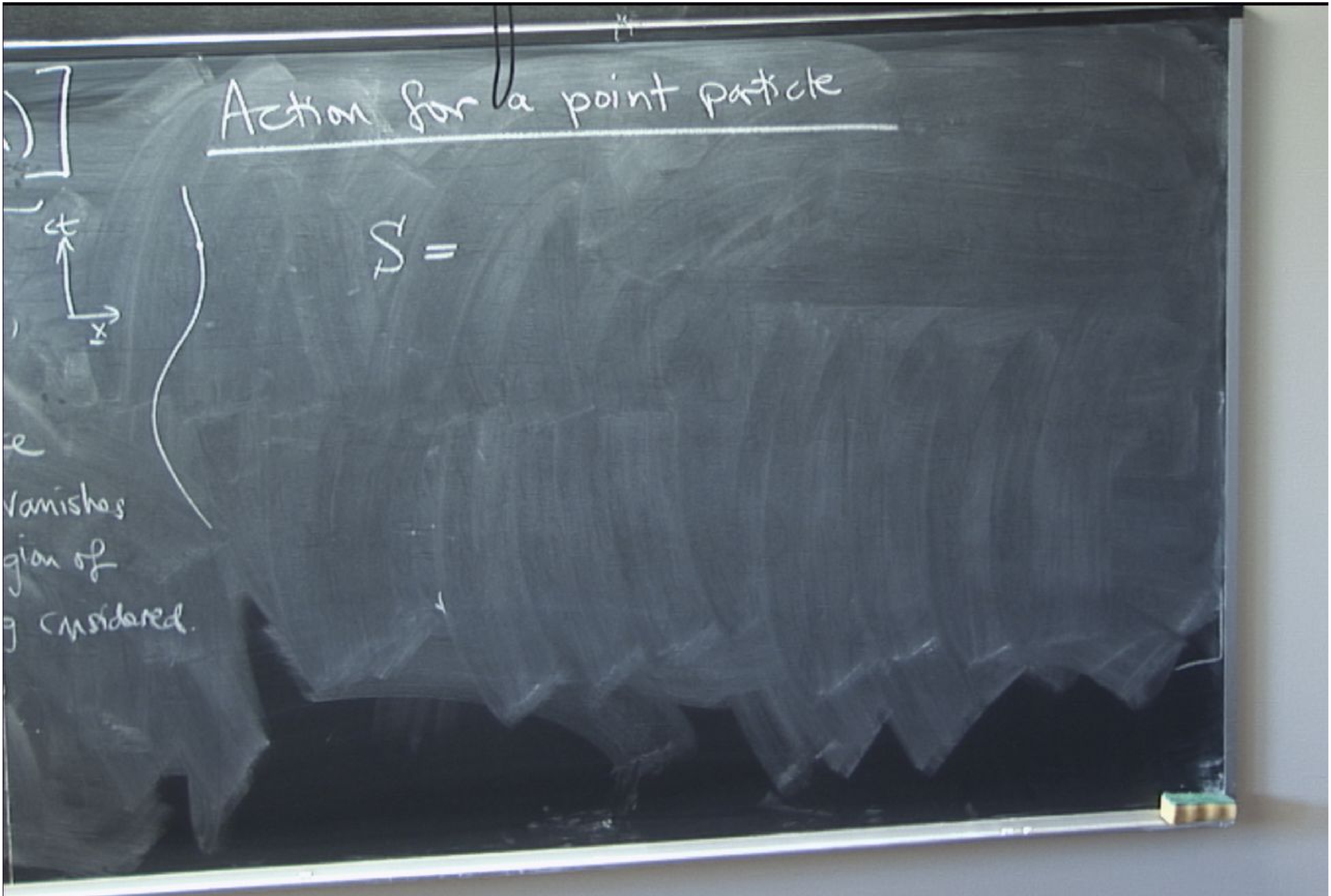
$\sqrt{-g}R =$

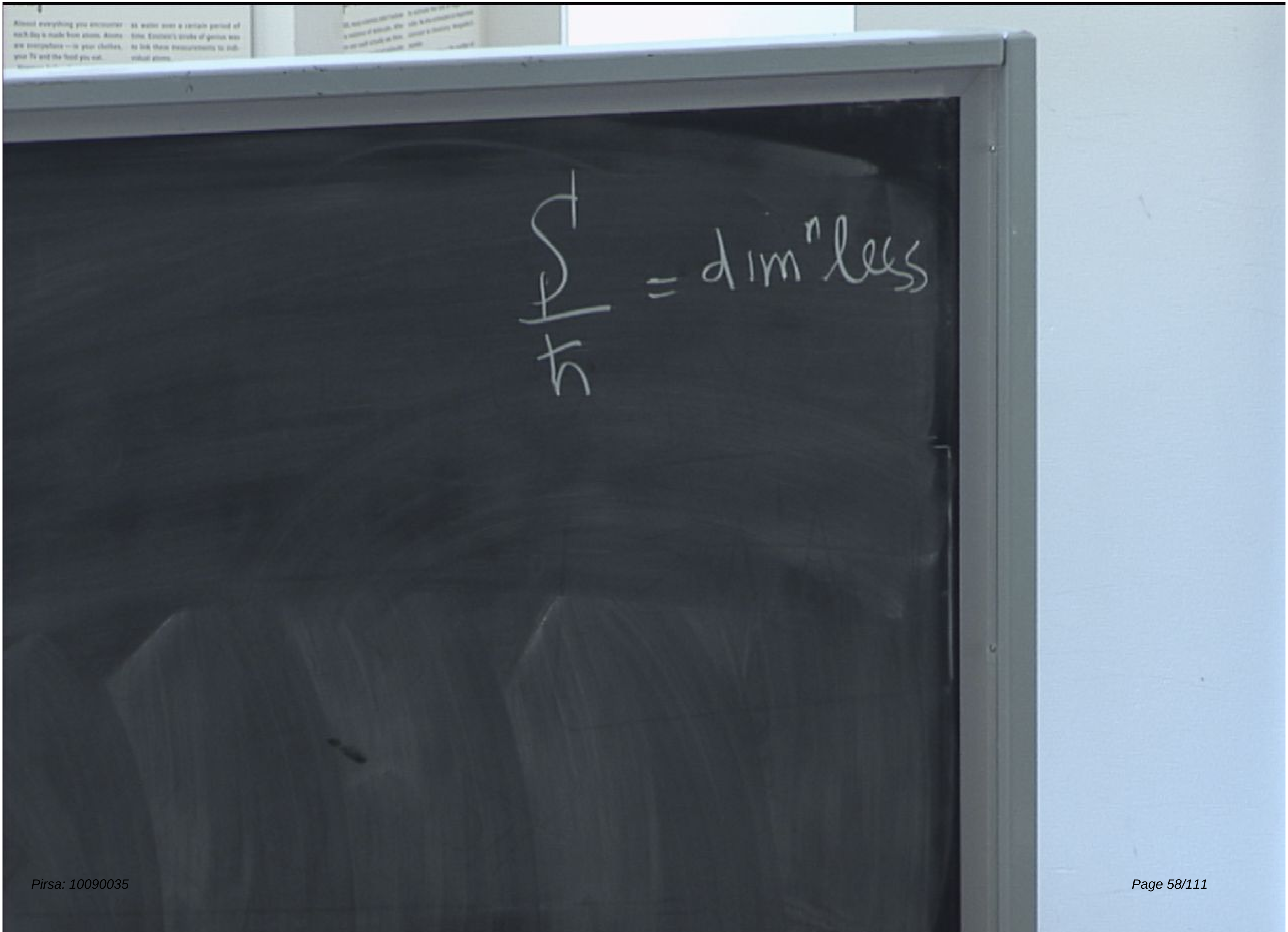
$\delta(\sqrt{-\ldots}$


$\int d^2x \sqrt{-g}\, R =$ Euler #
n of
nsidered.


$\int d^2x\sqrt{-g}\,R = $ Euler #
$\int d^4x\sqrt{-g}\,D^{\mu\nu\rho\lambda}R_{\mu\nu\rho\lambda} = $ #
considered

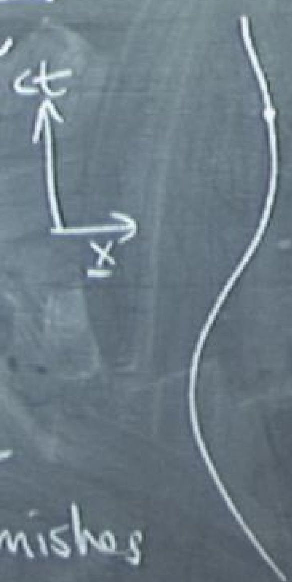

Action for a point particle


Action for a point particle
S =
ct
x
vanishes



$\frac{S}{\hbar}$ = dim'less

$\Gamma^{\chi}_{\mu\nu;x}$)]



Action for a point particle

"rest mass"

$$S = -m\int$$

... vanishes

... of region of

being considered.

$\Gamma^{x}_{\mu\nu;x}\,)\,]$

m)

gence

vanishes

$\alpha\beta$

f region of

being considered.

Action for a point particle

"rest mass" m_0

$$S = -m\int d\tau = -m\int\sqrt{-g_{\mu\nu}\,dx^{\mu}dx^{\nu}}$$

$\Gamma^{x}_{\mu\nu;x})\Big]$

$\mu)$

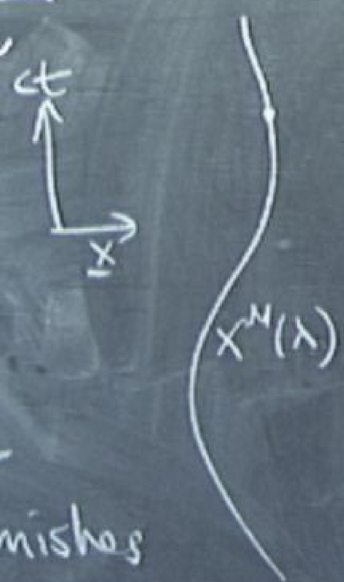
...ance

...vanishes

...of region of

being considered.

Action for a point particle



"rest mass" m_0

$$S = -m\int d\tau = -m\int\sqrt{-g_{\mu\nu}dx^{\mu}dx^{\nu}}$$

$$= -m\int d\lambda\,\sqrt{-g_{\mu\nu}\frac{dx^{\mu}}{d\lambda}\frac{dx^{\nu}}{d\lambda}}$$

$\Gamma^{\alpha}_{\mu\nu;\alpha})\Big]$

$\mu)$

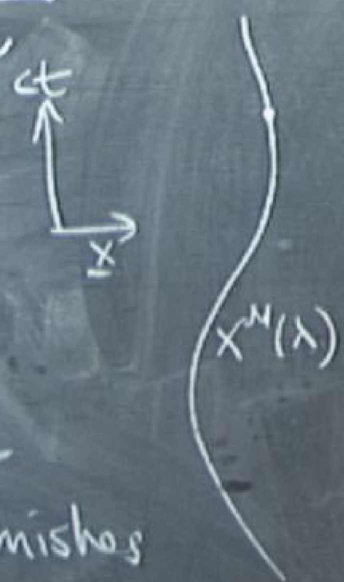
ence

vanishes

$\alpha\beta$

f region of

being considered.



Action for a point particle

"rest mass" m_0

$$S = -m\int d\tau = -m\int\sqrt{-g_{\mu\nu}dx^\mu dx^\nu}$$

$$= -m\int d\lambda\sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}$$

$\Gamma^{x}_{\mu\nu;x})]$

$m)$

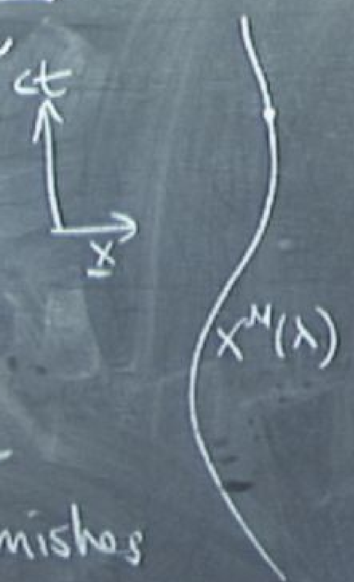
ence

vanishes

of region of

being considered.

Action for a point particle



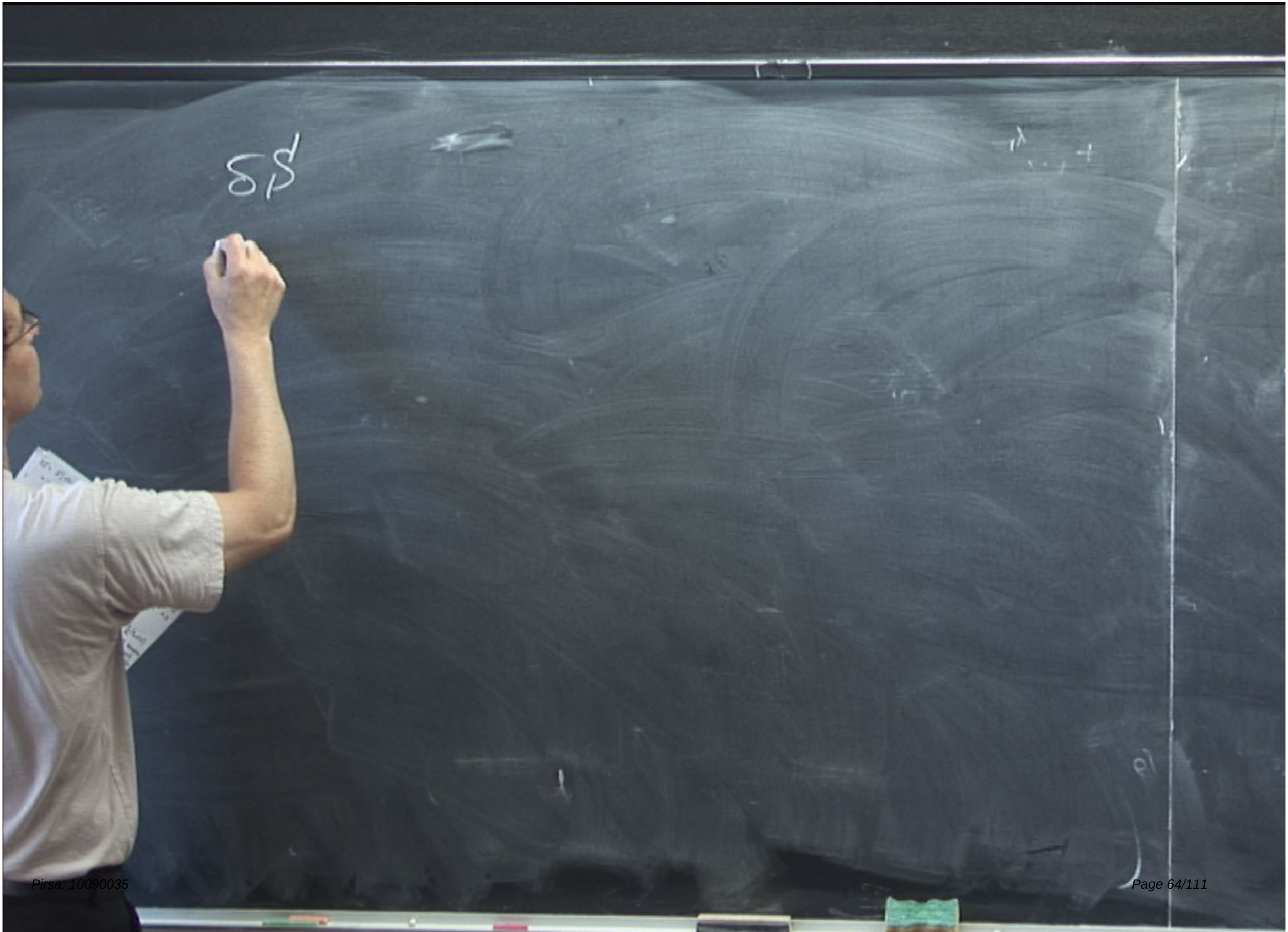
"rest mass" m_0

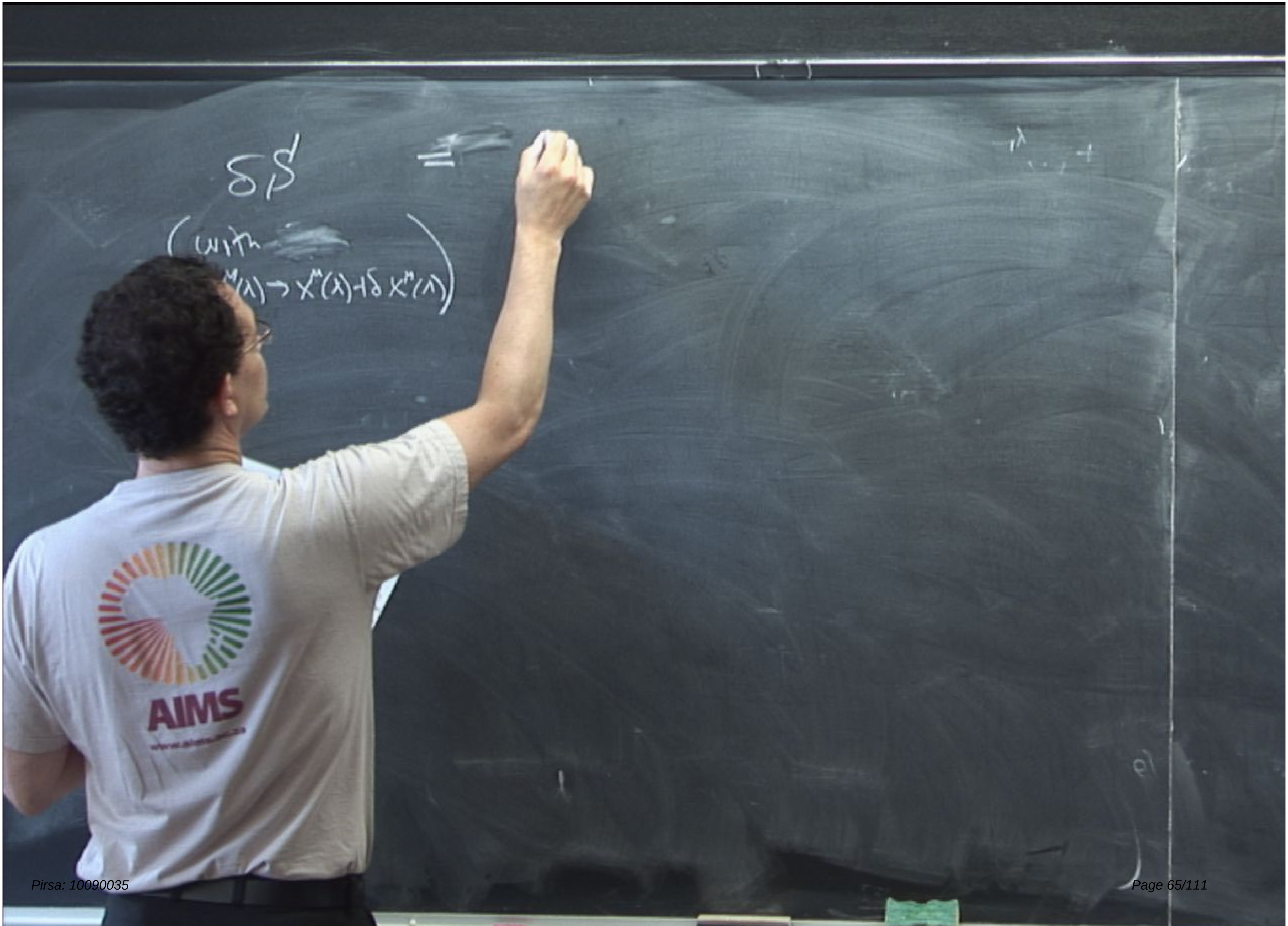
$$S = -m\int d\tau = -m\int\sqrt{-g_{\mu\nu}\,dx^\mu dx^\nu}$$

$$= -m\int d\lambda\,\sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}$$

$$\int d\lambda'\,\sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\lambda'}\frac{dx^\nu}{d\lambda'}}$$

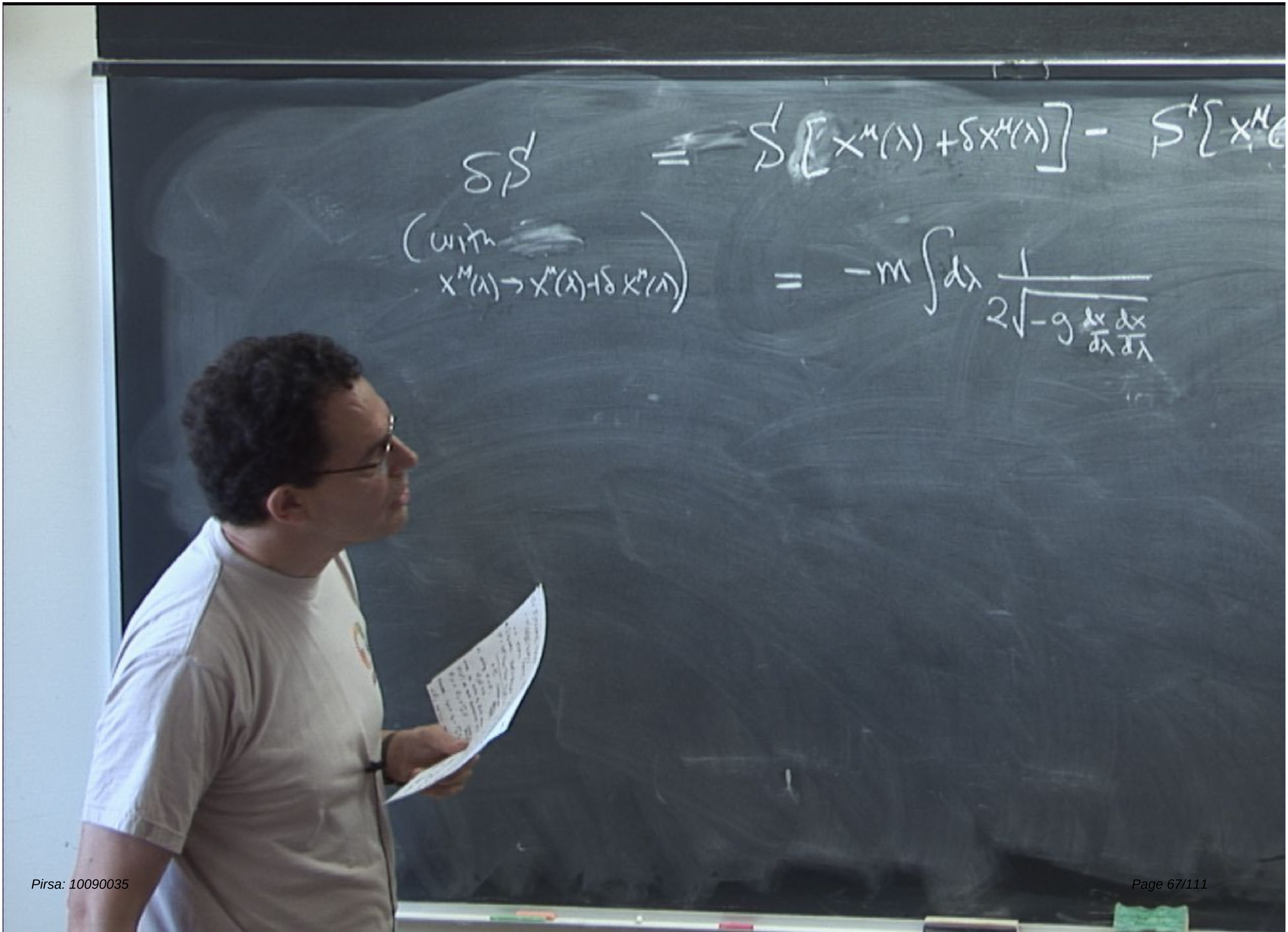
- reparameterisation invariant.


δS'


δS =
(with
(λ) → x^μ(λ)+δx^μ(λ))
AIMS

$$\delta S' = S'[x^\mu(\lambda) + \delta x^\mu(\lambda)] - S'[x^\mu(\lambda)]$$

$\left(\text{with } x^\mu(\lambda) \to x^\mu(\lambda) + \delta x^\mu(\lambda)\right)$


δS' = S'[x^μ(λ) + δx^μ(λ)] − S'[x^μ
(with x^μ(λ) → x^μ(λ) + δx^μ(λ))
= −m ∫dλ 1 / 2√(−g dx/dλ dx/dλ)

$$\delta S' = S'[x^\mu(\lambda) + \delta x^\mu(\lambda)] - S'[x^\mu(\lambda)]$$

(with $x^\mu(\lambda) \to x^\mu(\lambda) + \delta x^\mu(\lambda)$)

$$= -m \int d\lambda \frac{1}{2\sqrt{-g \frac{dx}{d\lambda}\frac{dx}{d\lambda}}} \left(-g_{\mu\nu,\alpha}\,\delta x^\alpha\, \dot{x}^\mu \dot{x}^\nu \right.$$

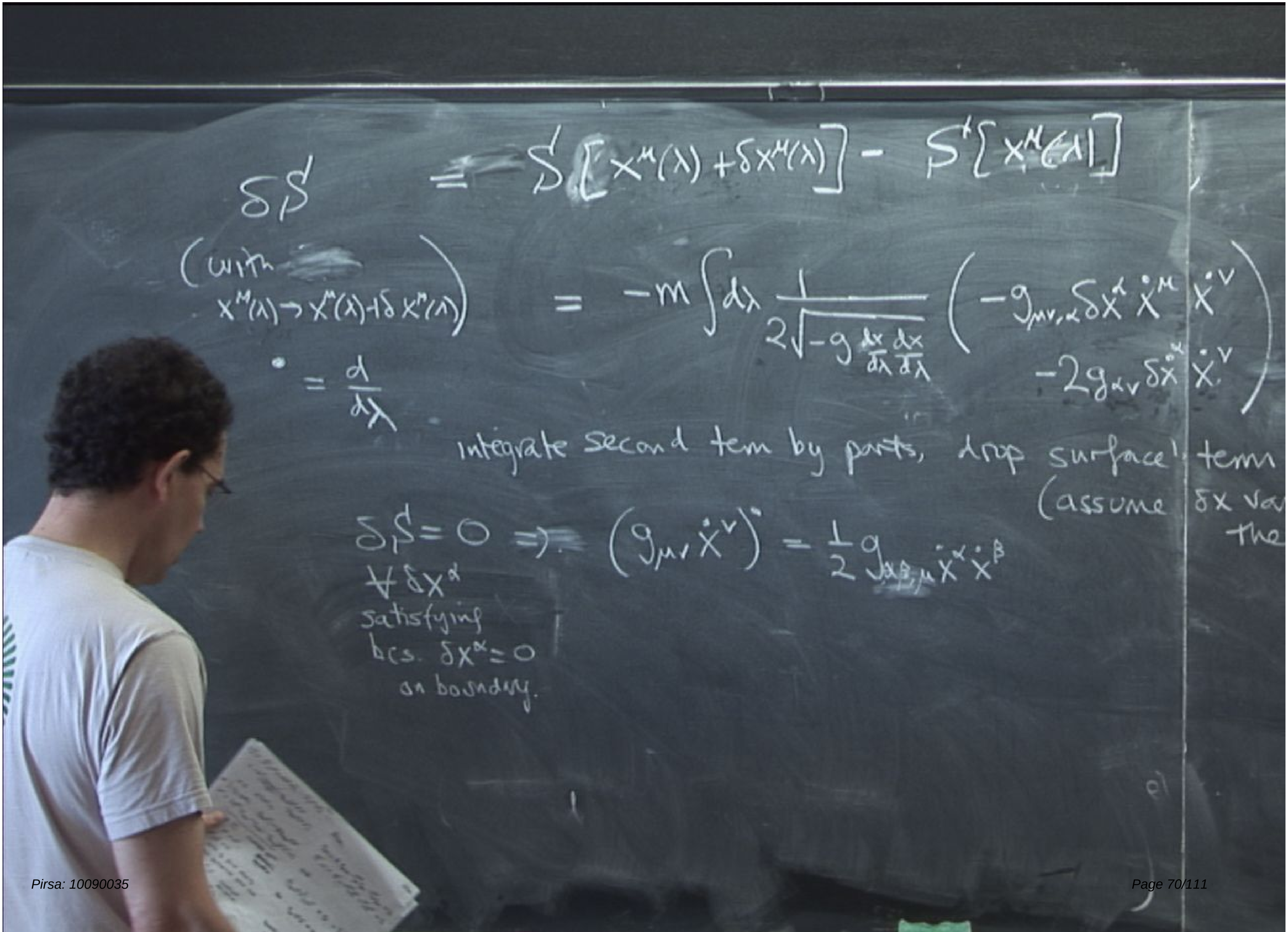
$$\dot{} = \frac{d}{d\lambda}$$

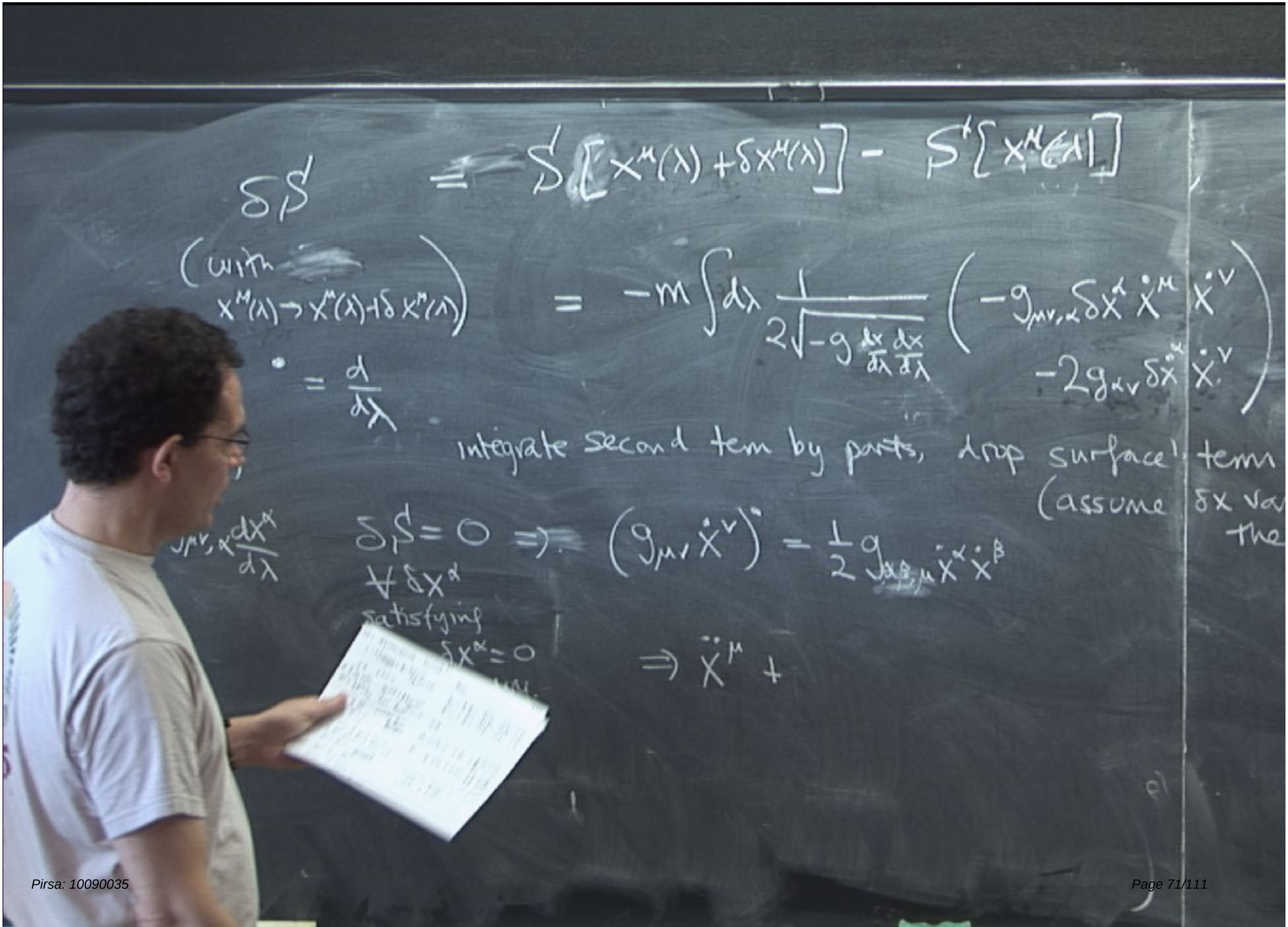
$$\delta S' = S'[x^\mu(\lambda) + \delta x^\mu(\lambda)] - S'[x^\mu(\lambda)]$$

(with $x^\mu(\lambda) \to x^\mu(\lambda) + \delta x^\mu(\lambda)$)

$$= -m \int d\lambda \frac{1}{2\sqrt{-g \frac{dx}{d\lambda}\frac{dx}{d\lambda}}} \left(-g_{\mu\nu,\alpha}\, \delta x^\alpha \dot{x}^\mu \dot{x}^\nu - 2 g_{\alpha\nu}\, \delta \dot{x}^\alpha \dot{x}^\nu \right)$$

$$\dot{} = \frac{d}{d\lambda}$$


$\delta S' = S'[x^\mu(\lambda) + \delta x^\mu(\lambda)] - S'[x^\mu(\lambda)]$
(with $x^\mu(\lambda) \to x^\mu(\lambda) + \delta x^\mu(\lambda)$)
$= -m \int d\lambda \frac{1}{2\sqrt{-g \frac{dx}{d\lambda}\frac{dx}{d\lambda}}} \left(-g_{\mu\nu,\alpha} \delta x^\alpha \dot{x}^\mu \dot{x}^\nu - 2 g_{\alpha\nu} \delta \dot{x}^\alpha \dot{x}^\nu \right)$
$\dot{} = \frac{d}{d\lambda}$
integrate second term by parts, drop 'surface' term
(assume δx va
the
$\delta S' = 0 \Rightarrow (g_{\mu\nu} \dot{x}^\nu)^{\cdot} = \frac{1}{2} g_{\alpha\beta,\mu} \dot{x}^\alpha \dot{x}^\beta$
$\forall \delta x^\alpha$
satisfying
bcs. $\delta x^\alpha = 0$
on boundary.


$\delta S' = S'[x^\mu(\lambda) + \delta x^\mu(\lambda)] - S'[x^\mu(\lambda)]$
(with $x^\mu(\lambda) \to x^\mu(\lambda) + \delta x^\mu(\lambda)$)
$= -m \int d\lambda \frac{1}{2\sqrt{-g \frac{dx}{d\lambda}\frac{dx}{d\lambda}}} \left(-g_{\mu\nu,\alpha} \delta x^\alpha \dot{x}^\mu \dot{x}^\nu - 2 g_{\alpha\nu} \delta \dot{x}^\alpha \dot{x}^\nu \right)$
$\dot{} = \frac{d}{d\lambda}$
integrate second term by parts, drop surface term
(assume δx va
the
$\delta S' = 0 \Rightarrow (g_{\mu\nu}\dot{x}^\nu)^{\cdot} - \frac{1}{2} g_{\alpha\beta,\mu} \dot{x}^\alpha \dot{x}^\beta$
$\forall \delta x^\alpha$
satisfying
$\delta x^\alpha = 0$
$\Rightarrow \ddot{x}^\mu +$

$$\delta S' = S'[x^\mu(\lambda) + \delta x^\mu(\lambda)] - S'[x^\mu(\lambda)]$$

(with $x^\mu(\lambda) \to x^\mu(\lambda) + \delta x^\mu(\lambda)$)

$$= -m \int d\lambda \frac{1}{2\sqrt{-g \frac{dx}{d\lambda}\frac{dx}{d\lambda}}} \left(-g_{\mu\nu,\alpha}\, \delta x^\alpha \dot{x}^\mu \dot{x}^\nu - 2 g_{\alpha\nu}\, \delta \dot{x}^\alpha \dot{x}^\nu \right)$$

$\dot{} = \frac{d}{d\lambda}$

$\frac{d}{d\lambda}(g_{\mu\nu}) = g_{\mu\nu,\alpha}\frac{dx^\alpha}{d\lambda}$

integrate second term by parts, drop "surface" term (assume δx va... the...

$\delta S = 0 \Rightarrow (g_{\mu\nu}\dot{x}^\nu)^{\cdot} = \frac{1}{2} g_{\alpha\beta,\mu}\dot{x}^\alpha \dot{x}^\beta$

$\forall\, \delta x^\alpha$ satisfying b.c.s. $\delta x^\alpha = 0$ on boundary.

$$\Rightarrow \ddot{x}^\mu + \underbrace{\tfrac{1}{2} g^{\mu\beta}(g_{\lambda\beta,\nu} + g_{\nu\beta,\lambda} - g_{\lambda\nu,\beta})}_{\Gamma^\mu_{\nu\lambda}} \dot{x}^\lambda$$

$$\Longrightarrow \ddot{x}^{\mu} + \Gamma^{\mu}_{\nu\lambda}\dot{x}^{\nu}\dot{x}^{\lambda} = 0$$

$x^{\lambda}\dot{x}^{\mu}\ \dot{x}^{\nu}$

$g_{\alpha\nu}\delta\dot{x}^{\mu}\ \dot{x}^{\nu}$

$x^{\mu}(\lambda)$

-face' term

-sume δx vanishes there

$\lambda - g_{\lambda\nu,\beta})\dot{x}^{\lambda}\dot{x}^{\nu} = 0$

$$\delta_1 S = S_1[x^\mu(\lambda)+\delta x^\mu(\lambda)] - S_1[x^\mu(\lambda)]$$

with $x^\mu(\lambda) \to x^\mu(\lambda)+\delta x^\mu(\lambda)$

$$= -m\int d\lambda \frac{1}{2\sqrt{-g\frac{dx}{d\lambda}\frac{dx}{d\lambda}}}\left(-g_{\mu\nu,\alpha}\delta x^\alpha \frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} - 2g_{\alpha\nu}\frac{d\delta x^\alpha}{d\lambda}\frac{dx^\nu}{d\lambda}\right)$$

$\dot{} = \frac{d}{d\lambda}$

integrate second term by parts, drop "surface" term (assume δx vanishes there)

$x^\mu(\lambda)$

$\delta_1 S = 0 \Rightarrow$ $\forall\, \delta x^\alpha$ satisfying b.c.s. $\delta x^\alpha = 0$ on boundary.

$$\frac{d}{d\lambda}\left(\frac{1}{\sqrt{-g\frac{dx}{d\lambda}\frac{dx}{d\lambda}}}\frac{dx^\mu}{d\lambda}g_{\mu\nu}\right) = \frac{\frac{1}{2}g_{\alpha\beta,\mu}\frac{dx}{d\lambda}\frac{dx}{d\lambda}}{\sqrt{-g\frac{dx}{d\lambda}\frac{dx}{d\lambda}}}$$

$$\Rightarrow \ddot{x}^\mu + \underbrace{\frac{1}{2}g^{\mu\beta}\left(g_{\lambda\beta,\nu}+g_{\nu\beta,\lambda}-g_{\lambda\nu,\beta}\right)}_{\Gamma^\mu_{\nu\lambda}}\dot{x}^\lambda\dot{x}^\nu = 0$$

$$\Longrightarrow \ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}\dot{x}^\nu \dot{x}^\lambda = 0$$

$$d\tau = \sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}\, d\lambda$$

$x^\mu(\lambda)$

λ

$${}_{,\alpha}\delta x^\alpha \frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}$$

$$-2g_{\alpha\nu}\frac{d\delta x^\alpha}{d\lambda}\frac{dx^\nu}{d\lambda}$$

"surface" term
(assume δx vanishes there)

$${}_{\alpha\beta,\mu}\frac{dx}{d\lambda}\frac{dx}{d\lambda}$$
$$\sqrt{-g\frac{dx}{d\lambda}\frac{dx}{d\lambda}}$$

$$g_{\nu\beta,\lambda} - g_{\lambda\nu,\beta})\dot{x}^\lambda\dot{x}^\nu = 0$$

$$\frac{dx}{d\lambda}\ \frac{dx}{d\lambda}$$

...ate second term by parts, drop "surface" term

(assume δx ...

$$\Rightarrow \frac{1}{\sqrt{-g\frac{dx}{d\lambda}\frac{dx}{d\lambda}}}\frac{d}{d\lambda}\left(\underbrace{\frac{1}{\sqrt{-g\frac{dx}{d\lambda}\frac{dx}{d\lambda}}}\frac{dx^\mu}{d\lambda}}_{d\tau} g_{\mu\nu}\right) = \frac{\frac{1}{2} g_{\alpha\beta,\mu}\frac{dx}{d\lambda}\frac{dx}{d\lambda}}{\sqrt{-g\frac{dx}{d\lambda}\frac{dx}{d\lambda}}}$$

$$\Rightarrow \ddot{x}^\mu + \frac{1}{2} g^{\mu\beta}\underbrace{\left(g_{\lambda\beta,\nu} + g_{\nu\beta,\lambda} - g_{\lambda\nu,\beta}\right)}_{\Gamma^\mu_{\nu\lambda}}$$

$$\dot{\ } = \frac{d}{d\tau}$$

$$\Longrightarrow \ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}\dot{x}^\nu \dot{x}^\lambda = 0$$

Geodesic equation in affine parameterisation.

$$d\tau = \sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}\; d\lambda$$

$x^\mu(\lambda)$

...em

...x vanishes there

$\frac{1}{\sqrt{\;}}$

$\ldots{}_{\nu,\beta})\dot{x}^\lambda \dot{x}^\nu = 0$

$$\delta S' = S'[x^\mu(\lambda) + \delta x^\mu(\lambda)] - S'[x^\mu(\lambda)]$$

(with $x^\mu(\lambda) \to x^\mu(\lambda) + \delta x^\mu(\lambda)$)

$$= -m \int d\lambda \frac{1}{2\sqrt{-g \frac{dx}{d\lambda}\frac{dx}{d\lambda}}} \Big(-g_{\mu\nu,\alpha}\delta x^\alpha \ldots$$

$$-2g_{\alpha\nu} \ldots$$

$$\frac{\partial L}{\partial x} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)$$

integrate second term by parts, drop surfa…

(assum…

$$\delta S' = 0 \Rightarrow \frac{1}{\sqrt{-g \frac{dx}{d\lambda}\frac{dx}{d\lambda}}} \frac{d}{d\lambda}\Bigg(\underbrace{\frac{1}{\sqrt{-g \frac{dx}{d\lambda}\frac{dx}{d\lambda}}} \frac{dx^\mu}{d\lambda}}_{d\tau} g_{\mu\nu} \Bigg) = \frac{1}{2} g_{\lambda\beta,\mu} \frac{dx}{d\lambda} \ldots \frac{}{\sqrt{-g}\ldots}$$

$\forall \delta x^\alpha$ satisfying bcs. $\delta x^\alpha = 0$ on boundary.

$$\Rightarrow \ddot{x}^\mu + \frac{1}{2} g^{\mu\beta} \underbrace{(g_{\lambda\beta,\nu} + g_{\nu\beta,\lambda} - \ldots}_{\Gamma^\mu_{\nu\lambda}}$$

$$\cdot = \frac{d}{d\tau}$$

$$\delta S' = S'[x^\mu(\lambda)+\delta x^\mu(\lambda)] - S'[x^\mu(\lambda)]$$

(with $x^\mu(\lambda) \to x^\mu(\lambda) + \delta x^\mu(\lambda)$)

$$= -m\int d\lambda \frac{1}{2\sqrt{-g\frac{dx}{d\lambda}\frac{dx}{d\lambda}}}\left(-g_{\mu\nu,\alpha}\delta x^\alpha \ldots \right.$$
$$-2g_{\alpha\nu}\ldots$$

$$\frac{\partial L}{\partial x} = \frac{d}{dt}\left(\frac{\partial L}{\partial x'}\right) \qquad x' = \frac{dx}{d\lambda}$$

$$L = \sqrt{-g x' x'}$$

integrate second term by parts, drop surfa…

(assum…

$$\delta_1 S = 0 \Rightarrow \frac{1}{\sqrt{-g\frac{dx}{d\lambda}\frac{dx}{d\lambda}}}\frac{d}{d\lambda}\left(\frac{1}{\sqrt{-g\frac{dx}{d\lambda}\frac{dx}{d\lambda}}}\frac{dx^\mu}{d\lambda}g_{\mu\nu}\right) = \frac{1}{2}g_{\lambda\beta,\mu}\frac{dx}{d\lambda}\ldots$$

$d\tau$

$\forall\, \delta x^\alpha$ satisfying bcs. $\delta x^\alpha = 0$ on boundary.

$$\Rightarrow \ddot{x}^\mu + \frac{1}{2}g^{\mu\beta}\underbrace{(g_{\lambda\beta,\nu}+g_{\nu\beta,\lambda} - \ldots}_{\Gamma^\mu_{\nu\lambda}}$$

$$\cdot = \frac{d}{d\tau}$$

$$\Longrightarrow \ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}\dot{x}^\nu \dot{x}^\lambda = 0$$

Geodesic equation in affine parameterisation.

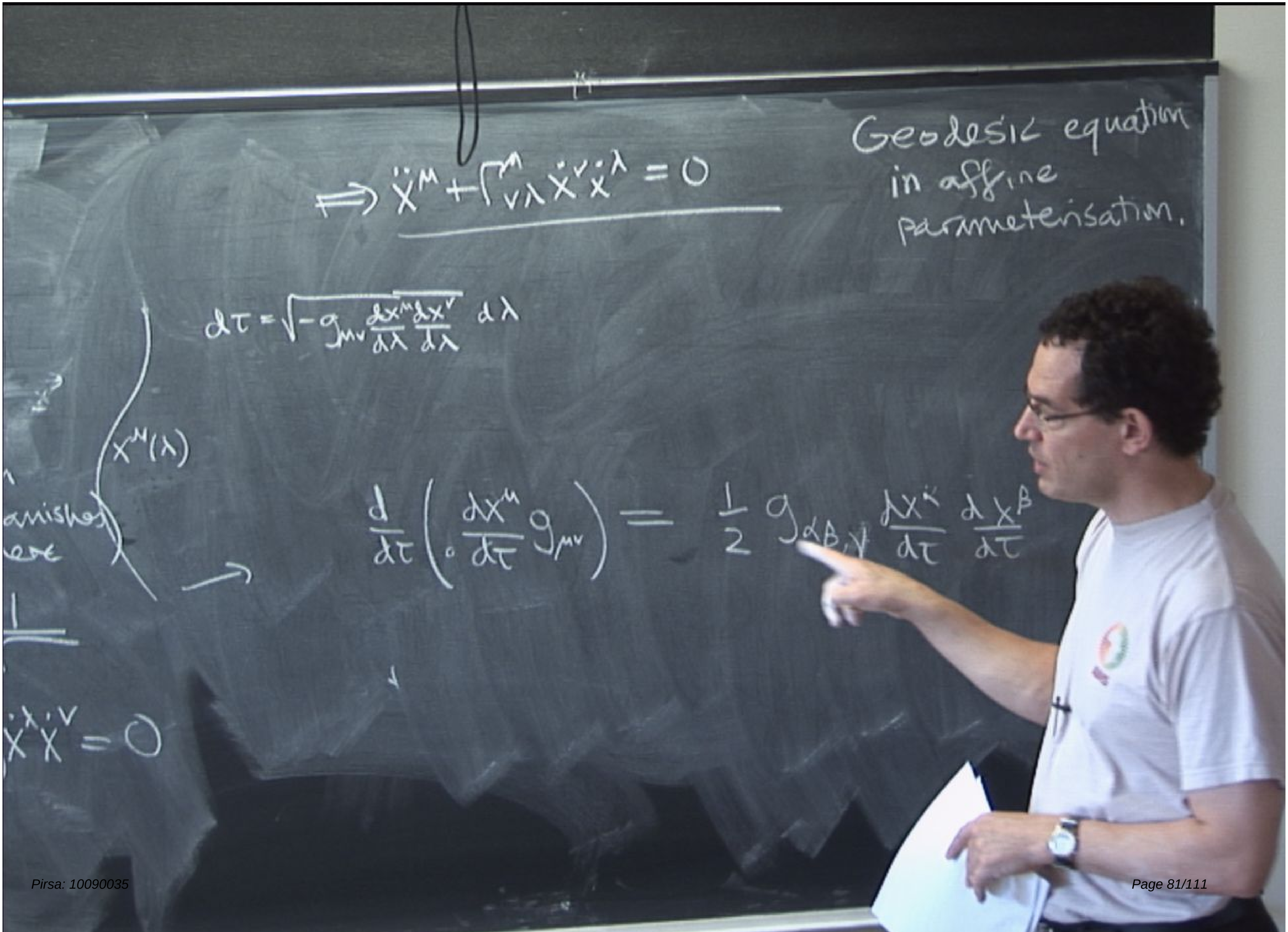
$$d\tau = \sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}\, d\lambda$$

$x^\mu(\lambda)$

anishes
ere

$$\frac{d}{d\tau}\left(\frac{dx^\mu}{d\tau} g_{\mu\nu}\right) = \frac{1}{2} g_{\alpha\beta,\mu}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}$$

$$\dot{x}^\lambda \dot{x}^\nu = 0$$


$\Longrightarrow \ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}\dot{x}^\nu\dot{x}^\lambda = 0$
Geodesic equation in affine parameterisation.
$d\tau = \sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}\, d\lambda$
$x^\mu(\lambda)$
$\frac{d}{d\tau}\left(\frac{dx^\mu}{d\tau}g_{\mu\nu}\right) = \frac{1}{2} g_{\alpha\beta,\gamma}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}$
$\dot{x}^\lambda\dot{x}^\nu = 0$

$$\Rightarrow \ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}\dot{x}^\nu\dot{x}^\lambda = 0$$

Geodesic equation in affine parameterisation.

$$d\tau = \sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}\, d\lambda$$

$x^\mu(\lambda)$

anishes

ere

$(x(\tau))$

$$\frac{d}{d\tau}\left(\frac{dx^\mu}{d\tau} g_{\mu\nu}\right) = \frac{1}{2} g_{\alpha\beta,\nu}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}$$

$$\frac{d^2x^\mu}{d\tau^2} g_{\mu\nu} + \frac{dx^\mu}{d\tau} g_{\mu\nu,\alpha}\frac{dx^\alpha}{d\tau}$$

$$\dot{x}^\lambda\dot{x}^\nu = 0$$

$$\Longrightarrow \ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}\dot{x}^\nu\dot{x}^\lambda = 0$$

Geodesic equation in affine parameterisation.

$$d\tau = \sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}\, d\lambda$$

$x^\mu(\lambda)$

anished

ere

$$\frac{d}{d\tau}\left(\frac{dx^\mu}{d\tau} g_{\mu\nu}(x(\tau))\right) = \frac{1}{2} g_{\alpha\beta,\nu}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}$$

$$\frac{d^2x^\mu}{d\tau^2} g_{\mu\nu} + \frac{dx^\mu}{d\tau} g_{\mu\nu,\alpha}\frac{dx^\alpha}{d\tau} = \frac{1}{2} g_{\alpha\beta,\nu}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}$$

$$\dot{x}^\lambda\dot{x}^\nu = 0$$

Geodesic equation in affine parameterisation

$$\Rightarrow \ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}\dot{x}^\nu \dot{x}^\lambda = 0$$

$$d\tau = \sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}\, d\lambda$$

$x^\mu(\lambda)$

vanishes there

$(x(\tau))$

$$\frac{d}{d\tau}\left(\frac{dx^\mu}{d\tau} g_{\mu\nu}\right) = \frac{1}{2} g_{\alpha\beta,\nu}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}$$

$$\frac{d^2x^\mu}{d\tau^2} g_{\mu\nu} + \frac{dx^\mu}{d\tau} g_{\mu\nu,\alpha}\frac{dx^\alpha}{d\tau} = \frac{1}{2} g_{\alpha\beta,\nu}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}$$

$$\frac{d^2x^\mu}{d\tau^2} \quad g^{\mu\nu}\left(\frac{1}{2} g_{\alpha\beta,\nu}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} - g\right.$$

$\dot{x}^\nu = 0$

Geodesic equation in affine parameterisation

$$\Longrightarrow \ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}\dot{x}^\nu\dot{x}^\lambda = 0$$

$$d\tau = \sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}\, d\lambda$$

$x^\mu(\lambda)$

'' term

δx vanishes there

$$\frac{d}{d\tau}\left(\frac{dx^\mu}{d\tau} g_{\mu\nu}\right) = \frac{1}{2} g_{\alpha\beta,\nu}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}$$

$(x(\tau))$

$$\frac{d^2x^\mu}{d\tau^2} g_{\mu\nu} + \frac{dx^\gamma}{d\tau} g_{\gamma\nu,\alpha}\frac{dx^\alpha}{d\tau} = \frac{1}{2} g_{\alpha\beta,\nu}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}$$

$$\frac{d^2x^\mu}{d\tau^2} + g^{\mu\gamma}\left(g_{\beta\gamma,\alpha} - \frac{1}{2} g_{\alpha\beta,\gamma}\right)\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}$$

$$\frac{1}{\sqrt{\ }}$$

$$g_{\lambda\nu,\beta})\dot{x}^\lambda\dot{x}^\nu = 0$$

Improved formulation of action for point particle

term

δx vanishes there

$\frac{1}{\sqrt{\ }}$

$\lambda\nu,\beta)\dot{X}^{\lambda}\dot{X}^{\nu} = 0$

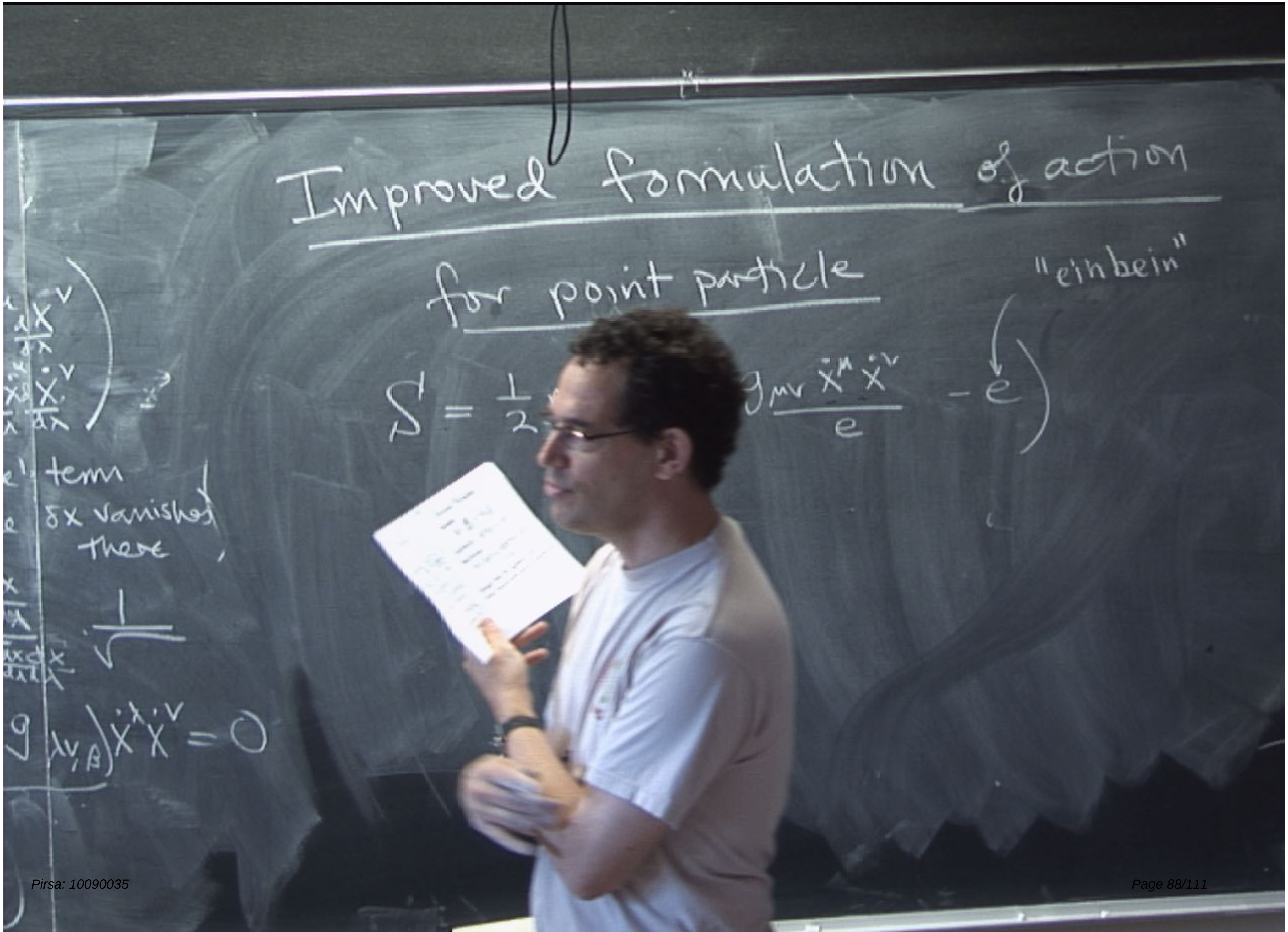
'' term

δx vanishes there

$g_{\lambda\nu,\beta})\dot{X}^\lambda \dot{X}^\nu = 0$

Improved formulation of action for point particle

$$S' = \frac{1}{2} m \int d\lambda \left(\frac{g_{\mu\nu}\dot{X}^\mu \dot{X}^\nu}{e} - e \right)$$


Improved formulation of action
for point particle
"einbein"
term
δx vanishes there

Improved formulation of action

for point particle

"einbein"

$$e'(\lambda') = \frac{d\lambda}{d\lambda'} e(\lambda)$$

$$S' = \frac{1}{2} m \int d\lambda \left(\frac{g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}{e} - e \right)$$

term

δx vanishes there

$$g_{\lambda\nu,\beta}) \dot{x}^\lambda \dot{x}^\nu = 0$$

Improved formulation of action

for point particle

"einbein"

$$e'(\lambda') = \frac{d\lambda}{d\lambda'}\, e(\lambda)$$

"tensor density"

$$S' = \frac{1}{2} m \int d\lambda \left(\frac{g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}{e} - e \right)$$

cf $\sqrt{-g'} = \left|\frac{\partial x}{\partial x'}\right| \sqrt{-g}$

...m

vanishes

there

$\frac{1}{\sqrt{\ }}$

$\beta) \dot{x}^\lambda \dot{x}^\nu = 0$

Improved formulation of action

for point particle

"einbein"

$$e'(\lambda') = \frac{d\lambda}{d\lambda'}\, e(\lambda)$$

"tensor density"

$$S' = \frac{1}{2} m \int d\lambda \left(\frac{g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}{e} - e \right)$$

cf. $\sqrt{-g'} = \left|\frac{\partial x}{\partial x'}\right| \sqrt{-g}$

e^2 = 1d metric on world line

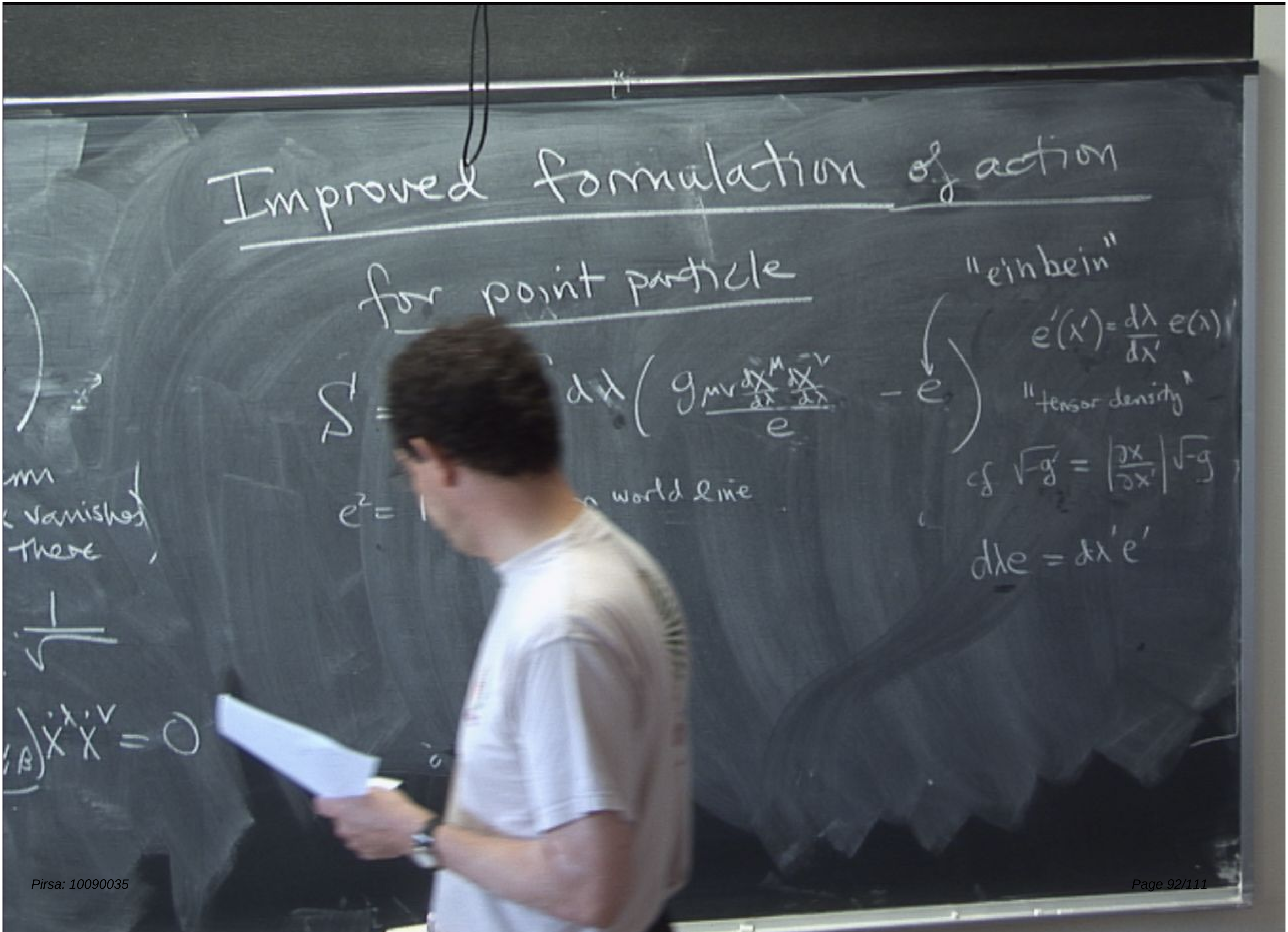
...m

... vanishes

there

$\frac{1}{\sqrt{\ }}$

$\ldots_{\beta})\dot{x}^\lambda \dot{x}^\nu = 0$


Improved formulation of action
for point particle
"einbein"
e'(λ') = dλ/dλ' e(λ)
"tensor density"
cf √-g' = |∂x/∂x'| √-g
dλe = dλ'e'
world line
vanishes
there

Improved formulation of action for point particle

$$S' = \frac{1}{2} m \int d\lambda \left(g_{\mu\nu} \frac{\frac{dX^\mu}{d\lambda} \frac{dX^\nu}{d\lambda}}{e} - e \right)$$

$e^2 =$ 1d metric on world line

– reparam. invariant

"einbein"

$$e'(\lambda') = \frac{d\lambda}{d\lambda'} e(\lambda)$$

"tensor density"

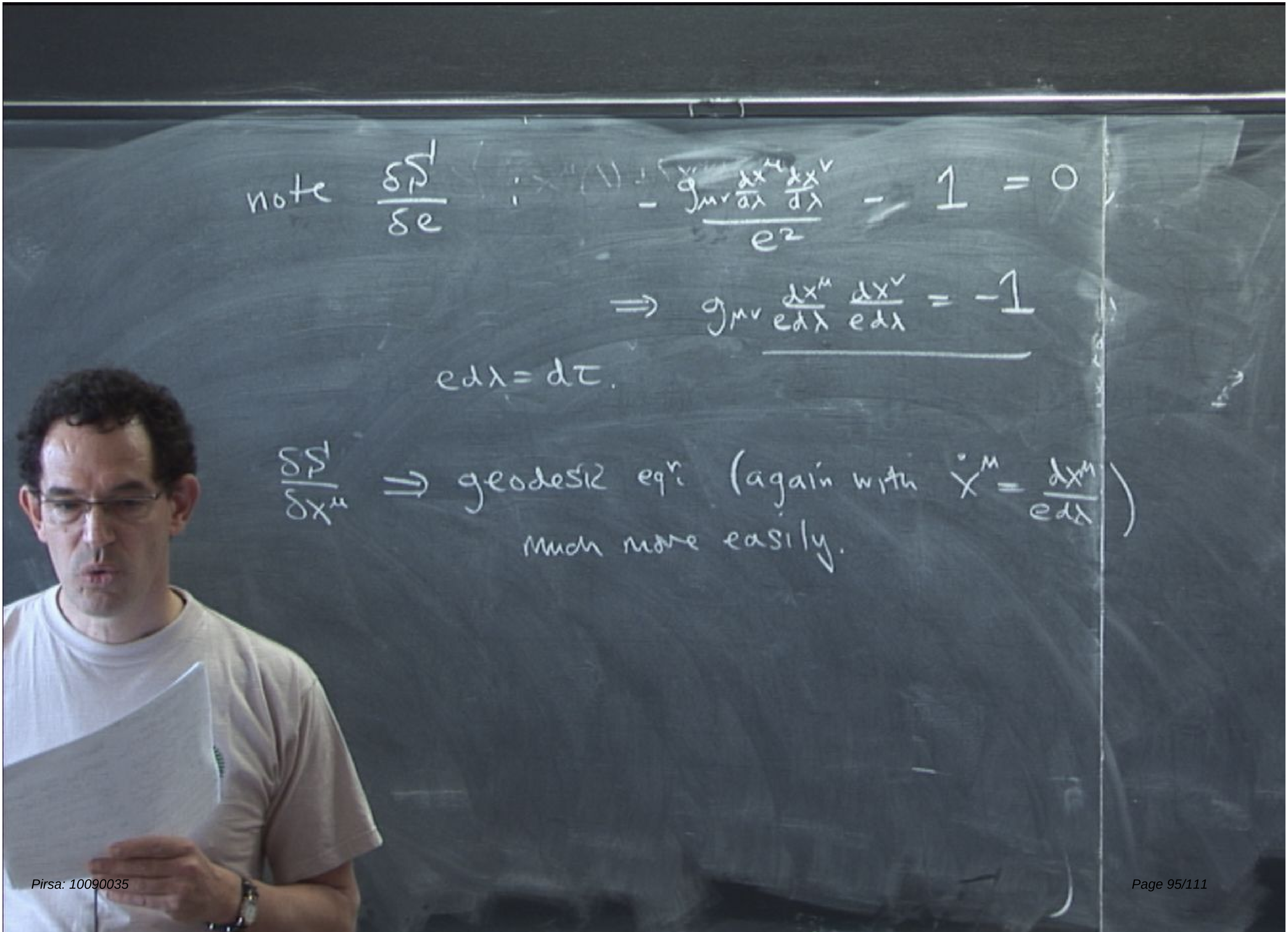
cf $\sqrt{-g'} = \left|\frac{\partial x}{\partial x'}\right| \sqrt{-g}$

$$d\lambda e = d\lambda' e'$$

note $\frac{\delta S'}{\delta e}$: $-\frac{g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}{e^2} - 1 = 0$

$$\Longrightarrow \quad g_{\mu\nu}\frac{dx^\mu}{e\,d\lambda}\frac{dx^\nu}{e\,d\lambda} = -1$$

$e\,d\lambda = d\tau.$


note $\frac{\delta S'}{\delta e}$: $-\frac{g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}{e^2} - 1 = 0$
$\Rightarrow g_{\mu\nu}\frac{dx^\mu}{e\,d\lambda}\frac{dx^\nu}{e\,d\lambda} = -1$
$e\,d\lambda = d\tau$.
$\frac{\delta S'}{\delta x^\mu} \Rightarrow$ geodesic eqⁿ (again with $\dot{x}^\mu = \frac{dx^\mu}{e\,d\lambda}$)
much more easily.

note $\frac{\delta S'}{\delta e}$: $-\frac{g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}{e^2} - 1 = 0$

$$\Rightarrow \underline{g_{\mu\nu}\frac{dx^\mu}{e\,d\lambda}\frac{dx^\nu}{e\,d\lambda} = -1}$$

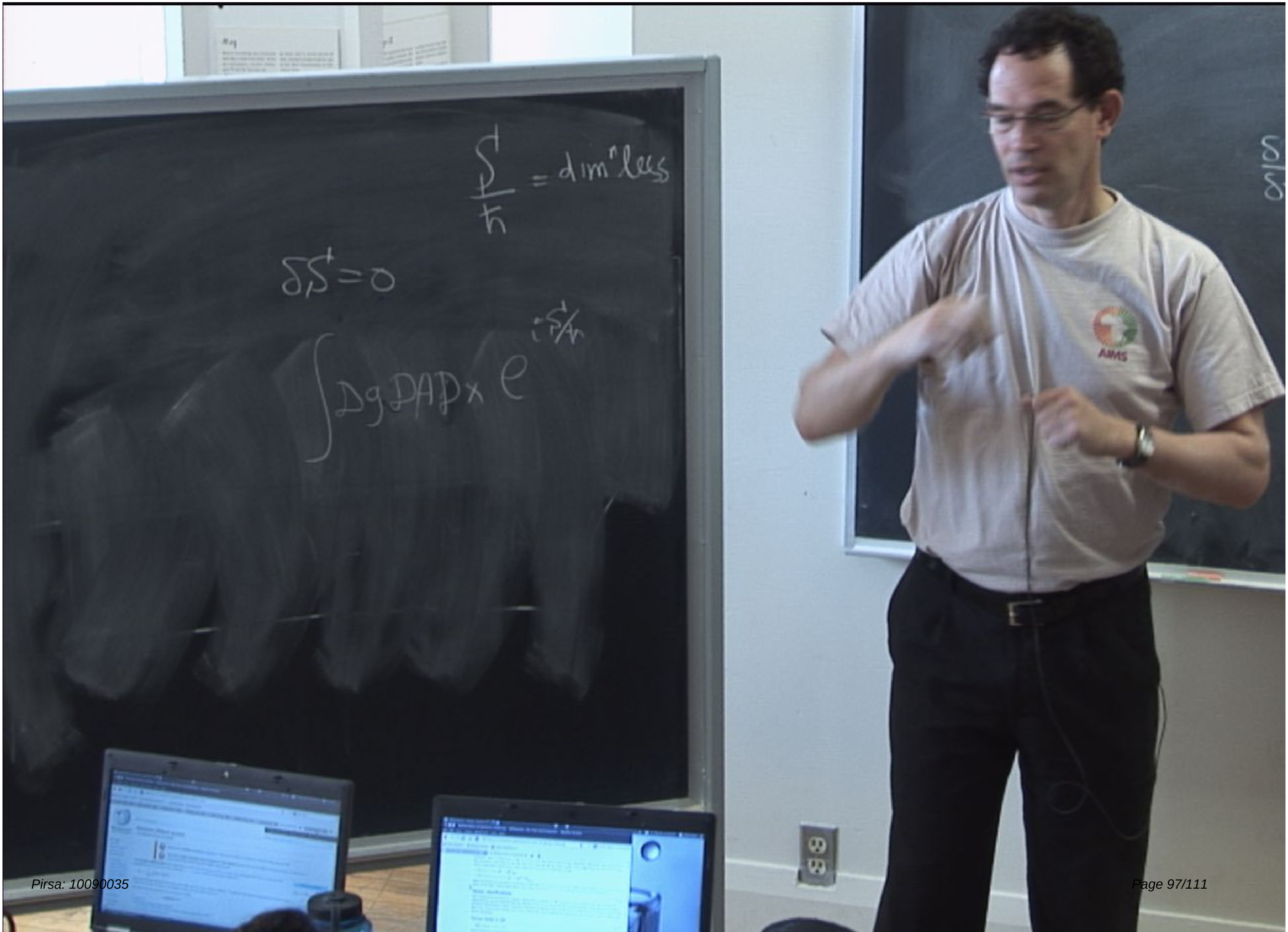
$e\,d\lambda = d\tau$.

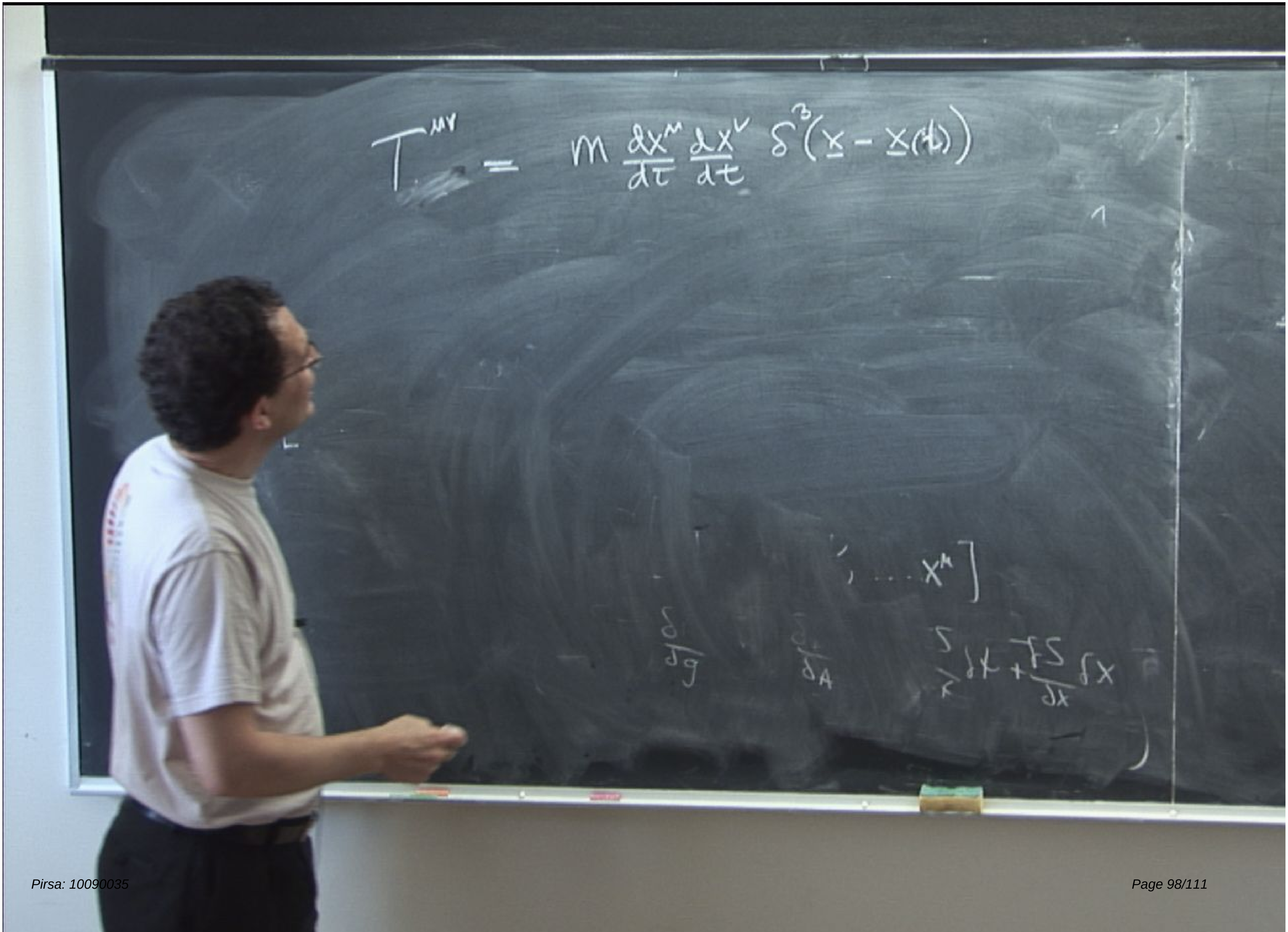
$\frac{\delta S'}{\delta x^\mu} \Rightarrow$ geodesic eq^n (again with $\dot{x}^\mu = \frac{dx^\mu}{e\,d\lambda}$)

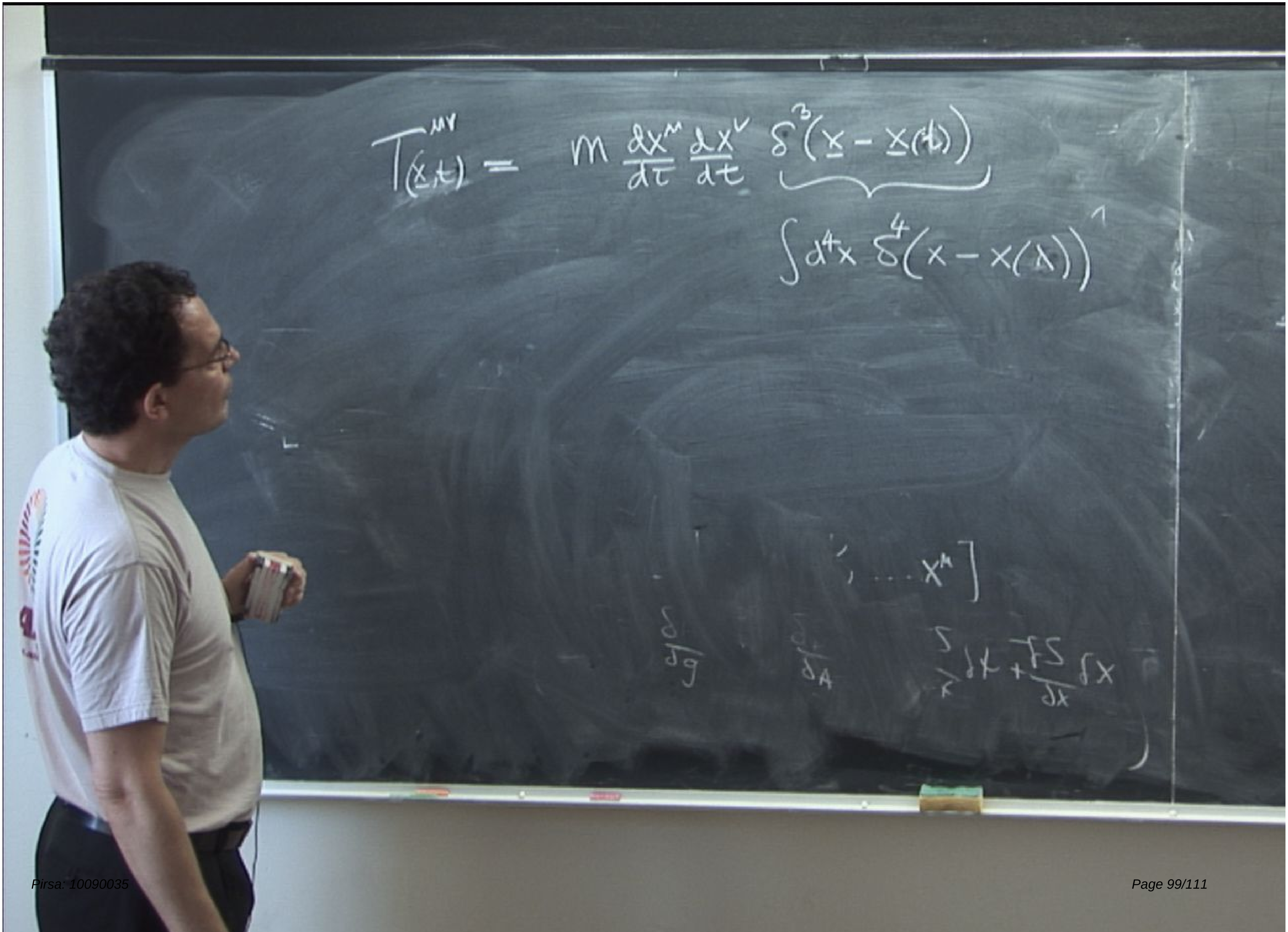
much more easily.

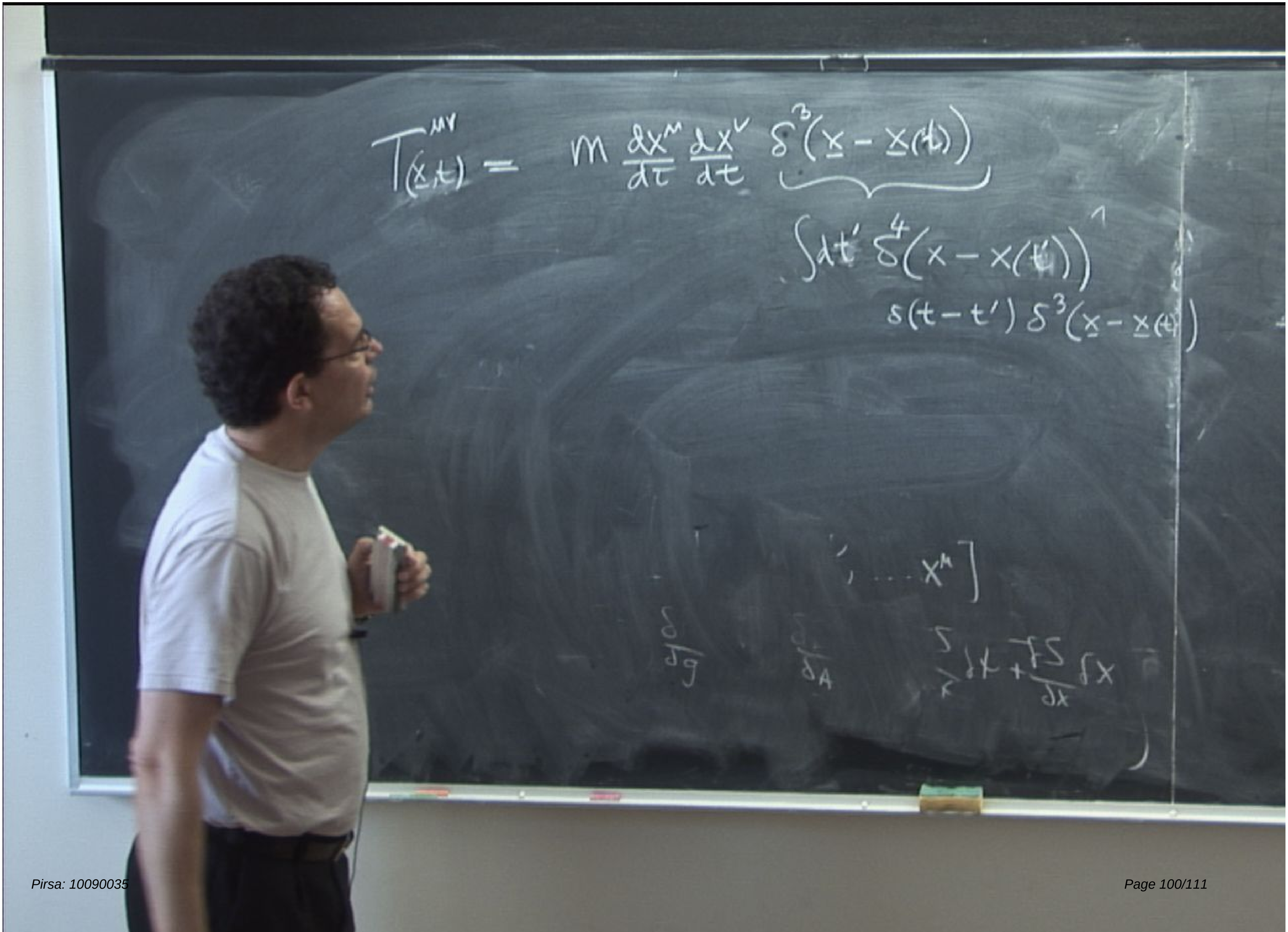
$$S'[g, A_\mu, \psi, \ldots x^\mu]$$

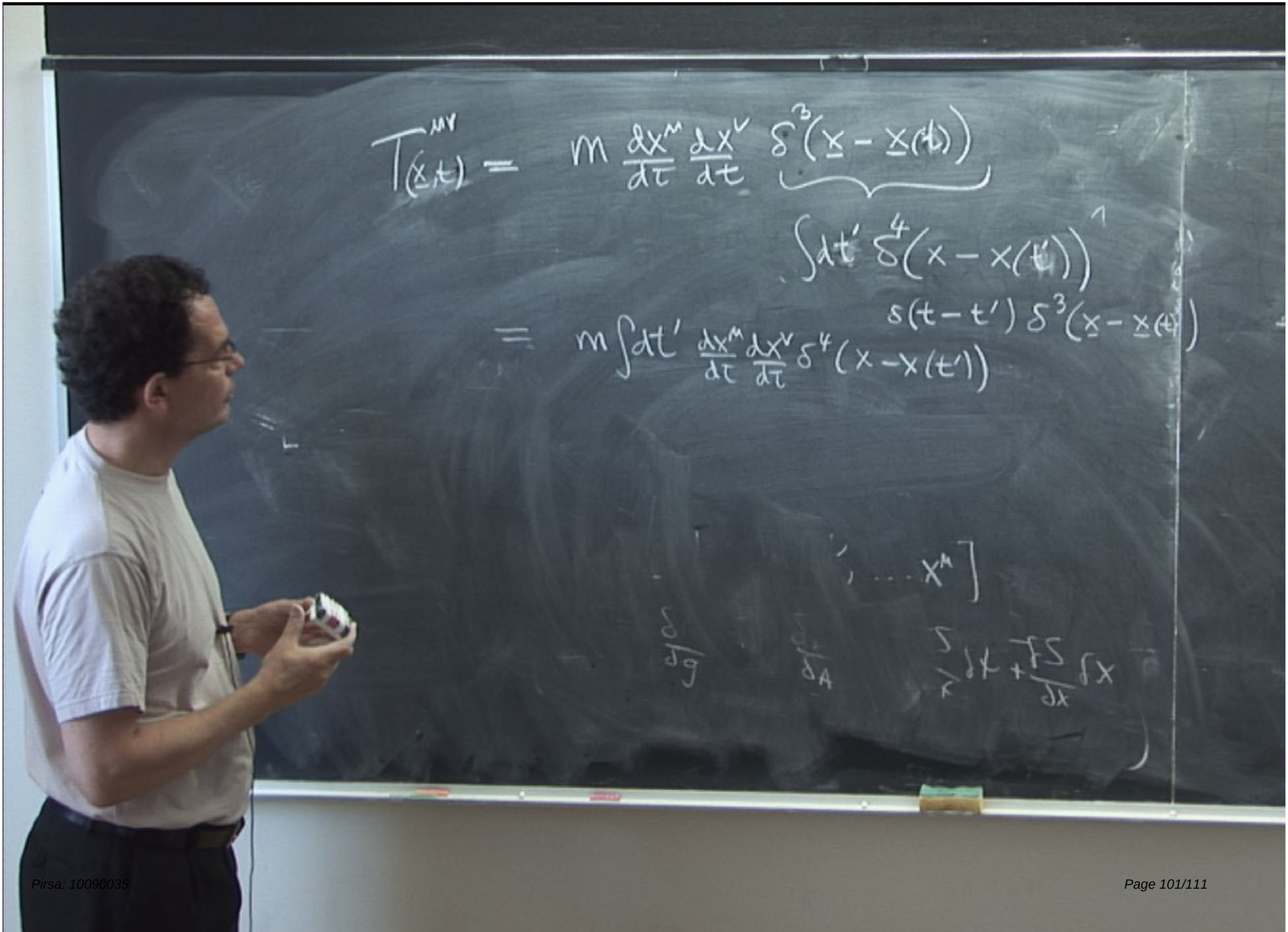
$$\delta S' = \frac{\delta S'}{\delta g}\delta g + \frac{\delta S}{\delta A}\delta A + \frac{\delta S}{\delta \psi}\delta \psi + \frac{\delta S}{\delta x}\delta x$$


$\frac{S}{\hbar}$ = dim^{n}less
$\delta S = 0$
$\int DgDADx \, e^{iS/\hbar}$
AIMS

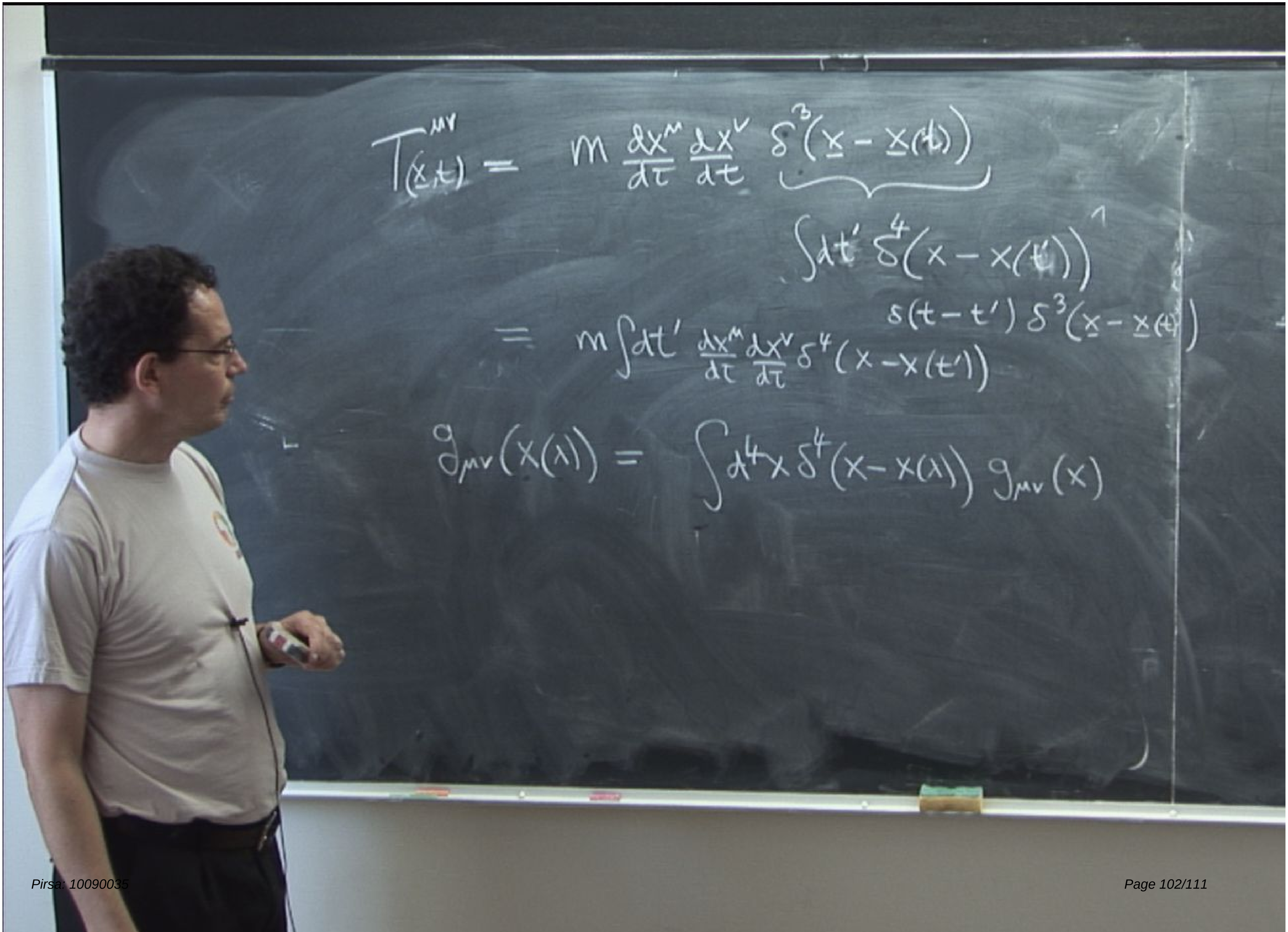

$T^{\mu\nu} = m \frac{dx^\mu}{d\tau} \frac{dx^\nu}{dt} \delta^3(\underline{x} - \underline{x}(t))$

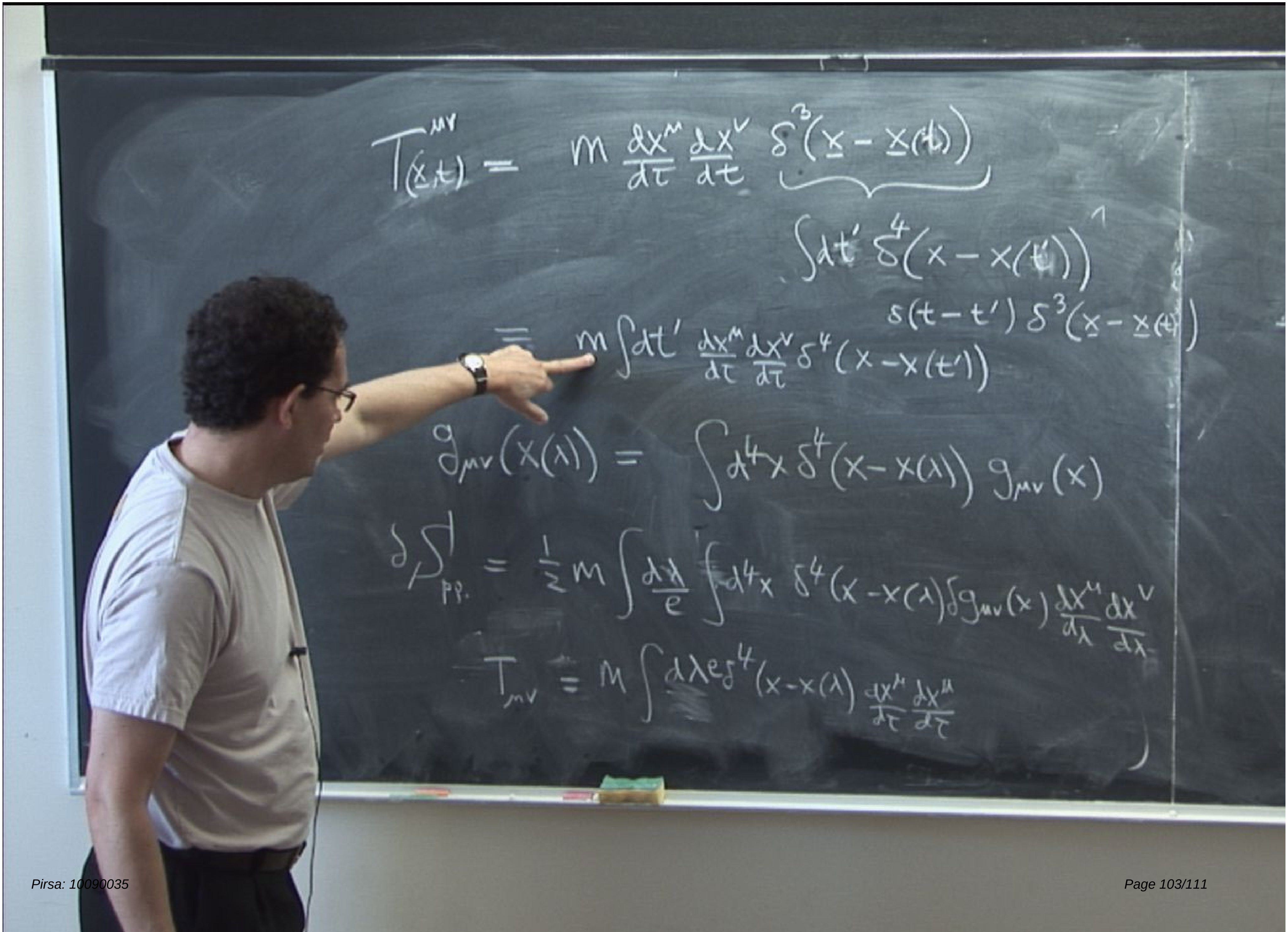

$T^{\mu\nu}(\underline{x},t) = m \frac{dx^\mu}{d\tau}\frac{dx^\nu}{dt}\,\delta^3(\underline{x}-\underline{x}(t))$
$\int d^4x\, \delta^4(x - x(\lambda))$

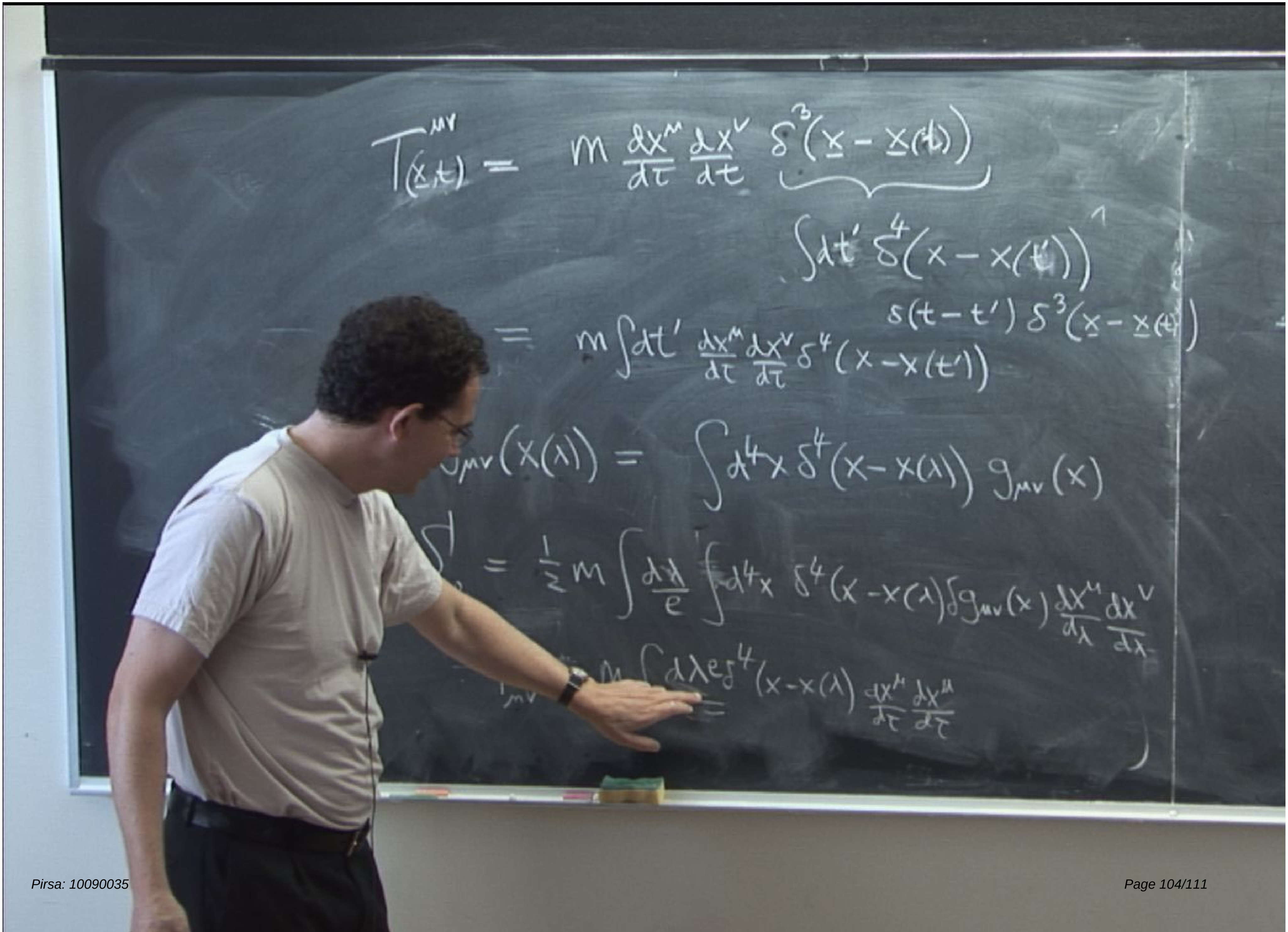

$T^{\mu\nu}(\underline{x},t) = m \frac{dx^\mu}{d\tau}\frac{dx^\nu}{dt}\,\delta^3(\underline{x}-\underline{x}(t))$
$\int dt'\,\delta^4(x - x(t'))$
$\delta(t-t')\,\delta^3(\underline{x}-\underline{x}(t))$



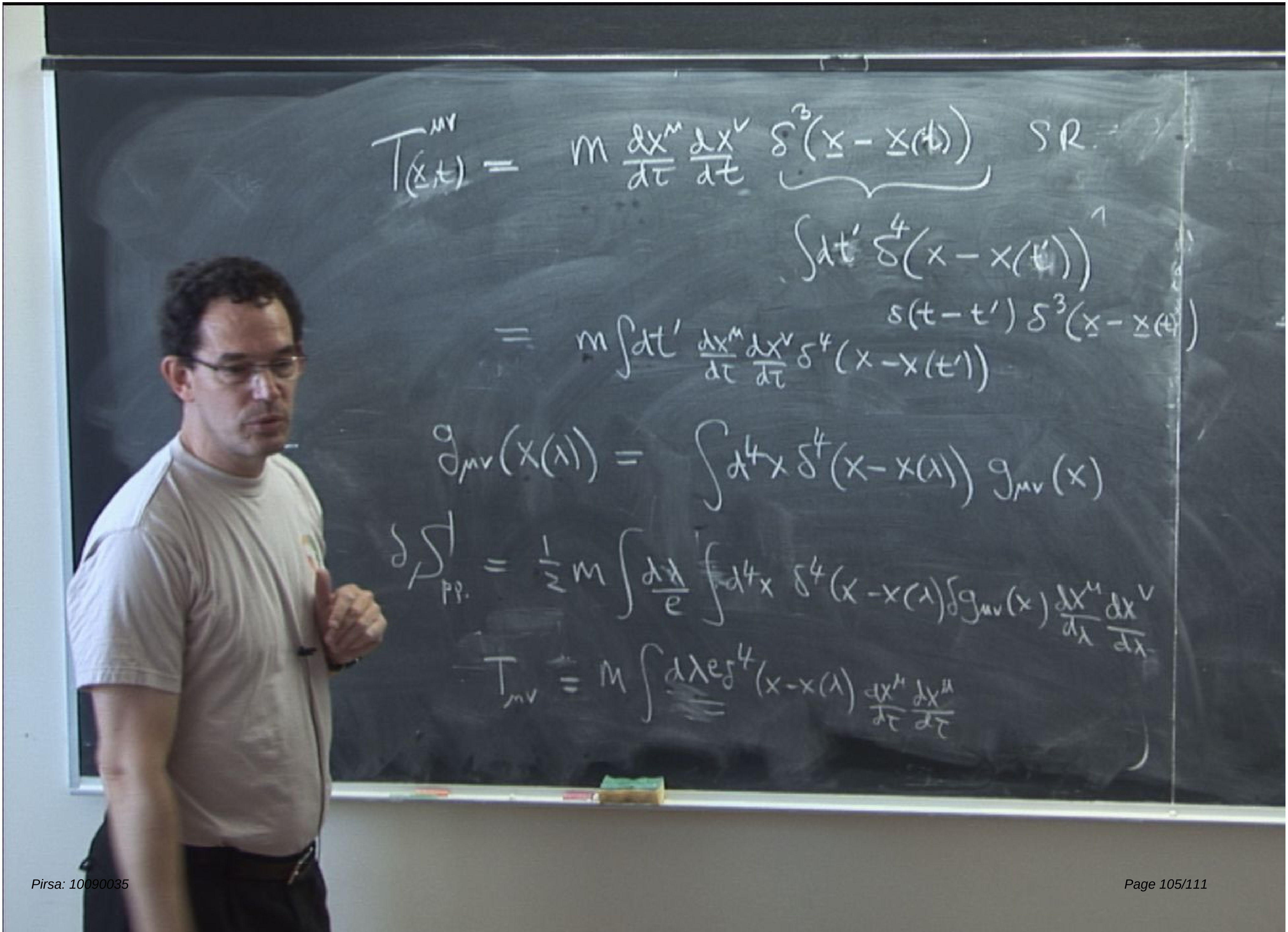
$T^{\mu\nu}(\underline{x},t) = m \frac{dx^\mu}{d\tau}\frac{dx^\nu}{dt}\underbrace{\delta^3(\underline{x}-\underline{x}(t))}_{\int dt'\,\delta^4(x-x(t'))}$
$\delta(t-t')\,\delta^3(\underline{x}-\underline{x}(t))$
$= m\int dt'\,\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\delta^4(x-x(t'))$
$\dots x^\mu]$
$\frac{\delta}{\delta g}$
$\frac{\delta}{\delta A}$


$T^{\mu\nu}(\underline{x},t) = m \frac{dx^\mu}{d\tau}\frac{dx^\nu}{dt} \underbrace{\delta^3(\underline{x}-\underline{x}(t))}_{\int dt' \delta^4(x-x(t'))}$
$\delta(t-t')\,\delta^3(\underline{x}-\underline{x}(t'))$
$= m\int dt' \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\delta^4(x-x(t'))$
$g_{\mu\nu}(x(\lambda)) = \int d^4x\, \delta^4(x-x(\lambda))\, g_{\mu\nu}(x)$


$T^{\mu\nu}(\underline{x},t) = m \frac{dx^\mu}{d\tau}\frac{dx^\nu}{dt}\, \delta^3(\underline{x}-\underline{x}(t))$
$\int dt'\, \delta^4(x - x(t'))$
$\delta(t-t')\,\delta^3(\underline{x}-\underline{x}(t))$
$= m\int dt' \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\delta^4(x-x(t'))$
$g_{\mu\nu}(x(\lambda)) = \int d^4x\, \delta^4(x-x(\lambda))\, g_{\mu\nu}(x)$
$\delta S_{pp.} = \frac{1}{2} m\int \frac{d\lambda}{e}\int d^4x\, \delta^4(x-x(\lambda))\,\delta g_{\mu\nu}(x)\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}$
$T_{\mu\nu} = m\int d\lambda\, e\, \delta^4(x-x(\lambda))\frac{dx^\mu}{d\tau}\frac{dx^\mu}{d\tau}$



$T^{\mu\nu}(\underline{x},t) = m \frac{dx^\mu}{d\tau}\frac{dx^\nu}{dt} \delta^3(\underline{x}-\underline{x}(t))$
$\int dt' \delta^4(x - x(t'))$
$\delta(t-t')\delta^3(\underline{x}-\underline{x}(t))$
$= m\int dt' \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\delta^4(x-x(t'))$
$g_{\mu\nu}(x(\lambda)) = \int d^4x\, \delta^4(x-x(\lambda))\, g_{\mu\nu}(x)$
$S = \frac{1}{2}m\int \frac{d\lambda}{e}\int d^4x\, \delta^4(x-x(\lambda))\, g_{\mu\nu}(x)\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}$
$m\int d\lambda\, e\, \delta^4(x-x(\lambda))\frac{dx^\mu}{d\tau}\frac{dx^\mu}{d\tau}$


$T^{\mu\nu}(\underline{x},t) = m \frac{dx^\mu}{d\tau}\frac{dx^\nu}{dt} \underbrace{\delta^3(\underline{x} - \underline{x}(t))}$ SR.
$\int dt' \, \delta^4(x - x(t'))$
$\delta(t-t')\,\delta^3(\underline{x} - \underline{x}(t))$
$= m \int dt' \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} \delta^4(x - x(t'))$
$g_{\mu\nu}(x(\lambda)) = \int d^4x \, \delta^4(x - x(\lambda))\, g_{\mu\nu}(x)$
$\delta S'_{pp.} = \frac{1}{2} m \int \frac{d\lambda}{e} \int d^4x \, \delta^4(x - x(\lambda))\, \delta g_{\mu\nu}(x) \frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}$
$T_{\mu\nu} = m \int d\lambda\, e\, \delta^4(x - x(\lambda)) \frac{dx^\mu}{d\tau}\frac{dx^\mu}{d\tau}$

$$T^{\mu\nu}(\underline{x},t) = m \frac{dx^\mu}{d\tau}\frac{dx^\nu}{dt} \underbrace{\delta^3(\underline{x} - \underline{x}(t))}_{\int dt' \, \delta^4(x - x(t'))} \quad SR.$$

$$\delta(t-t')\,\delta^3(\underline{x}-\underline{x}(t))$$

$$= m \int dt' \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\delta^4(x - x(t'))$$

$$g_{\mu\nu}(x(\lambda)) = \int d^4x \, \delta^4(x - x(\lambda))\, g_{\mu\nu}(x)$$

$$\delta S_{pp.} = \frac{1}{2} m \int \frac{d\lambda}{e} \int d^4x \, \delta^4(x - x(\lambda)\,\delta g_{\mu\nu}(x) \frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}$$

$$T_{\mu\nu} = m \int d\lambda \, e\, \delta^4(x - x(\lambda) \frac{dx^\mu}{d\tau}\frac{dx^\mu}{d\tau}$$

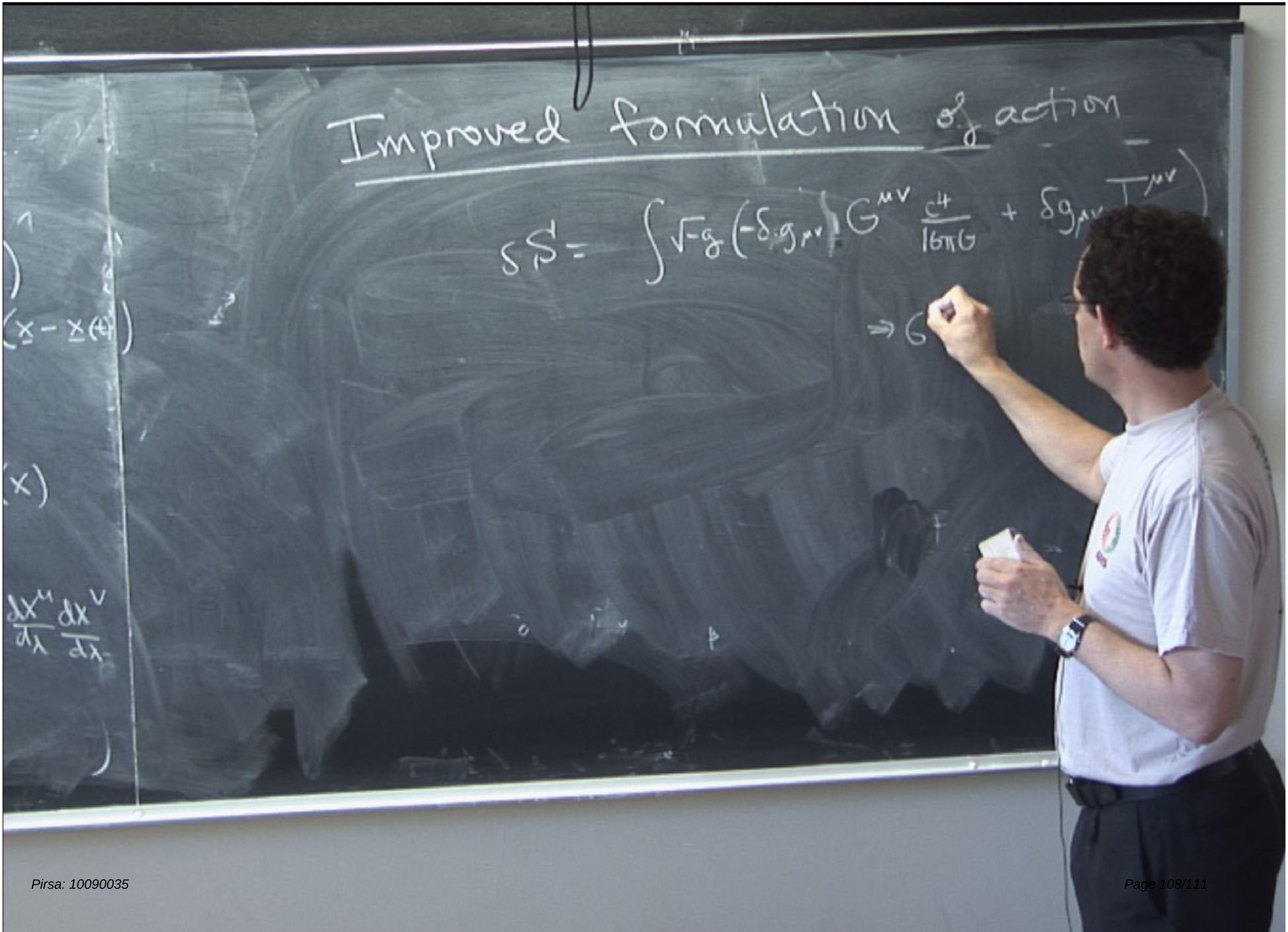
$(\underline{x} - \underline{x}(t))$

(x)

$\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}$

Improved formulation of action

$$\delta S' = \int \sqrt{-g}\,(-\delta g_{\mu\nu})\, G^{\mu\nu} \frac{c^4}{16\pi G}$$


Improved formulation of action
δS = ∫√-g (-δg_μν G^μν c^4/16πG + δg_μν T^μν)
⇒ G

)

$(\underline{x} - \underline{x}(t))$

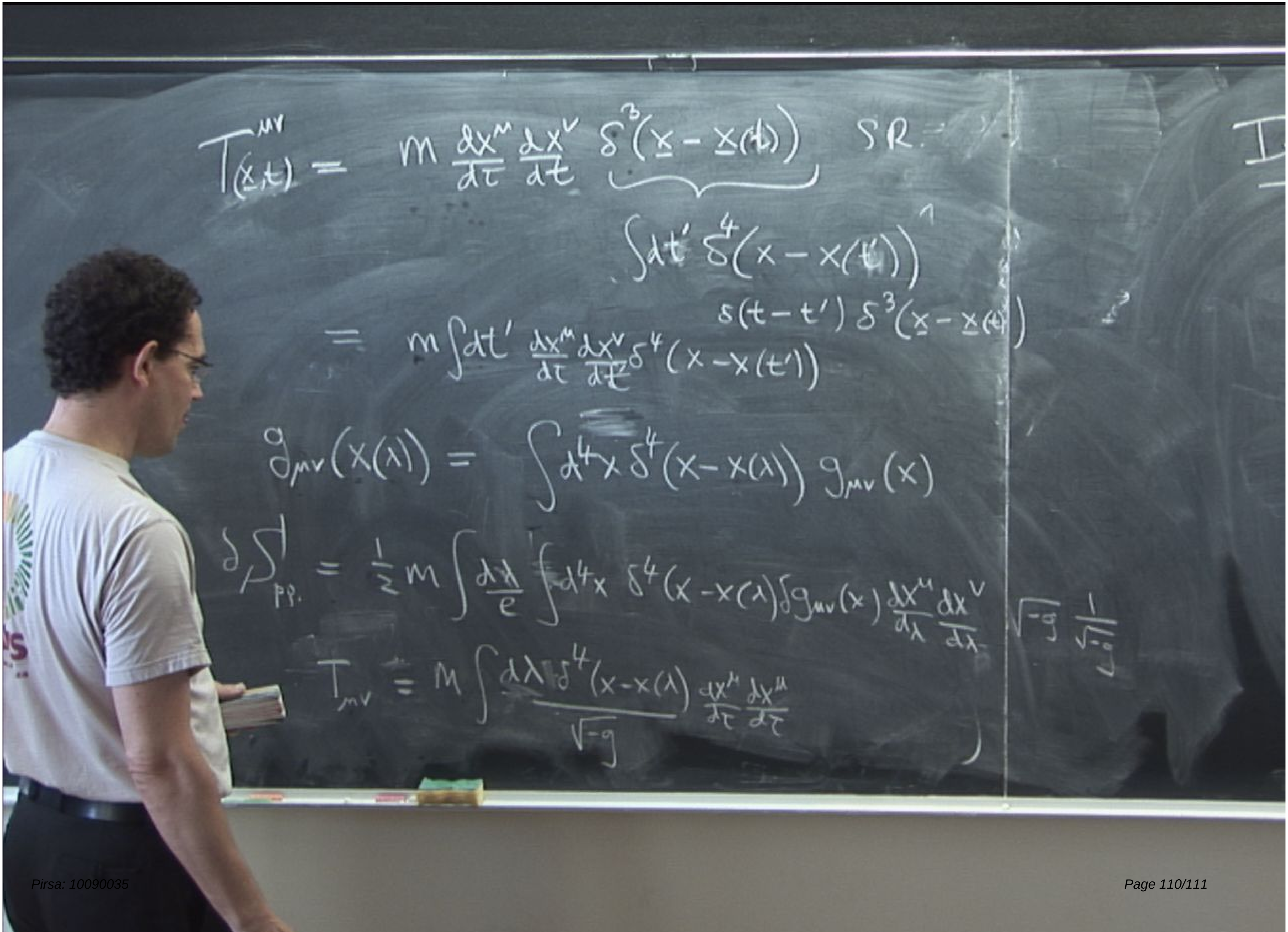
(x)

$\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}$

Improved formulation of action

$$\delta S' = \int \sqrt{-g}\left(-\delta g_{\mu\nu} G^{\mu\nu}\frac{c^4}{16\pi G} + \delta g_{\mu\nu}\frac{T^{\mu\nu}}{2}\right)$$

$$\Rightarrow G^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$


$T^{\mu\nu}(\underline{x},t) = m \frac{dx^\mu}{d\tau}\frac{dx^\nu}{dt} \delta^3(\underline{x}-\underline{x}(t))$ SR.
$\int dt' \delta^4(x - x(t'))$
$\delta(t-t')\delta^3(\underline{x}-\underline{x}(t))$
$= m\int dt' \frac{dx^\mu}{d\tau}\frac{dx^\nu}{dt'}\delta^4(x-x(t'))$
$g_{\mu\nu}(x(\lambda)) = \int d^4x\, \delta^4(x-x(\lambda))\, g_{\mu\nu}(x)$
$\delta S_{pp.} = \frac{1}{2} m \int \frac{d\lambda}{e} \int d^4x\, \delta^4(x-x(\lambda))\delta g_{\mu\nu}(x) \frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}$ $\sqrt{-g}\frac{1}{\sqrt{-g}}$
$T_{\mu\nu} = m\int d\lambda \frac{\delta^4(x-x(\lambda))}{\sqrt{-g}} \frac{dx^\mu}{d\tau}\frac{dx^\mu}{d\tau}$

$$T^{\mu\nu}(\underline{x},t) = m \frac{dx^\mu}{d\tau}\frac{dx^\nu}{dt} \underbrace{\delta^3(\underline{x}-\underline{x}(t))}_{\int dt' \, \delta^4(x - x(t'))} \quad SR$$

$$\delta(t-t')\,\delta^3(\underline{x}-\underline{x}(t))$$

$e\,d\lambda = d\tau$

$$= m\int d\lambda \, \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\lambda}\delta^4(x - x(t'))$$

$$g_{\mu\nu}(X(\lambda)) = \int d^4x \, \delta^4(x - x(\lambda))\, g_{\mu\nu}(x)$$

$$\delta S_{pp.} = \frac{1}{2} m \int \frac{d\lambda}{e} \int d^4x \, \delta^4(x - x(\lambda)) \delta g_{\mu\nu}(x) \frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} \quad \sqrt{-g}\,\frac{1}{\sqrt{-g}}$$

$$T_{\mu\nu} = m \int \frac{d\lambda \, \delta^4(x - x(\lambda))}{\sqrt{-g}} \frac{dx^\mu}{d\tau}\frac{dx^\mu}{d\lambda}$$

I