

Title: Relativity (PHYS 604) - Lecture 7

Date: Sep 21, 2010 10:30 AM

URL: <http://pirsa.org/10090032>

Abstract:

Recall

$$R^{\gamma}_{\alpha\mu\nu} = - \left(\Gamma^{\gamma}_{\alpha\mu,\nu} + \Gamma^{\beta}_{\alpha\mu} \Gamma^{\gamma}_{\beta\nu} - \Gamma^{\gamma}_{\alpha\nu,\mu} - \Gamma^{\beta}_{\alpha\nu} \Gamma^{\gamma}_{\beta\mu} \right)$$

Recall

$$R^{\alpha}_{\beta\gamma\delta} = - \left(\Gamma^{\alpha}_{\beta\gamma,\delta} + \Gamma^{\alpha}_{\beta\delta}\Gamma^{\gamma}_{\delta\gamma} - \Gamma^{\alpha}_{\gamma\delta}\Gamma^{\gamma}_{\beta\gamma} - \Gamma^{\alpha}_{\gamma\gamma}\Gamma^{\gamma}_{\beta\delta} \right)$$

Trajectories of particles in spacetime:



Recall

$$R^{\gamma}_{\alpha\mu\nu} = -\left(\Gamma^{\gamma}_{\alpha\mu,\nu} + \Gamma^{\beta}_{\alpha\mu}\Gamma^{\gamma}_{\beta\nu} - \Gamma^{\gamma}_{\alpha\nu,\mu} - \Gamma^{\beta}_{\alpha\nu}\Gamma^{\gamma}_{\beta\mu}\right)$$

Trajectories of particles in spacetime:
geodesic equation.



Recall

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Trajectories of particles in spacetime:
geodesic equation.



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Trajectories of particles in spacetime:
geodesic equation.



Recall

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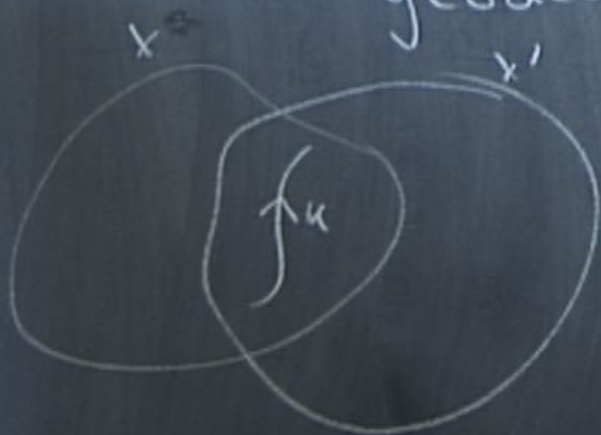
Trajectories of particles in spacetime:
geodesic equation.



Recall

$$R^{\gamma}_{\alpha\mu\nu} = - \left(\Gamma^{\gamma}_{\alpha\mu,\nu} + \Gamma^{\beta}_{\alpha\mu} \Gamma^{\gamma}_{\beta\nu} - \Gamma^{\gamma}_{\alpha\nu,\mu} - \Gamma^{\beta}_{\alpha\nu} \Gamma^{\gamma}_{\beta\mu} \right)$$

Trajectories of particles in spacetime:
geodesic equation.

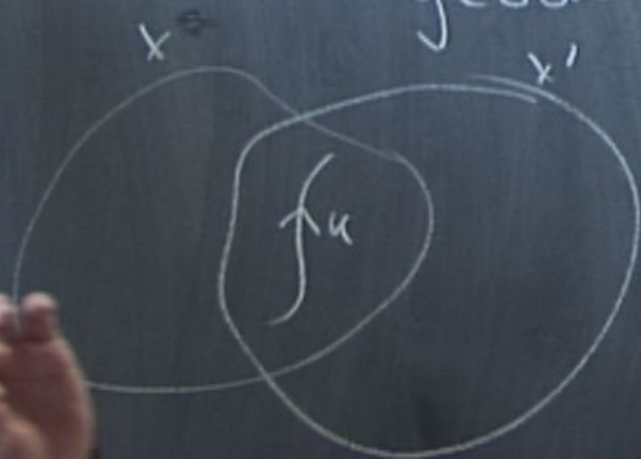


curve on manifold M $x^{\mu}(u)$

Recall

$$R^{\gamma}_{\alpha\mu\nu} = -\left(\Gamma^{\gamma}_{\alpha\mu,\nu} + \Gamma^{\beta}_{\alpha\mu}\Gamma^{\gamma}_{\beta\nu} - \Gamma^{\gamma}_{\alpha\nu,\mu} - \Gamma^{\beta}_{\alpha\nu}\Gamma^{\gamma}_{\beta\mu}\right)$$

Trajectories of particles in spacetime:
geodesic equation.



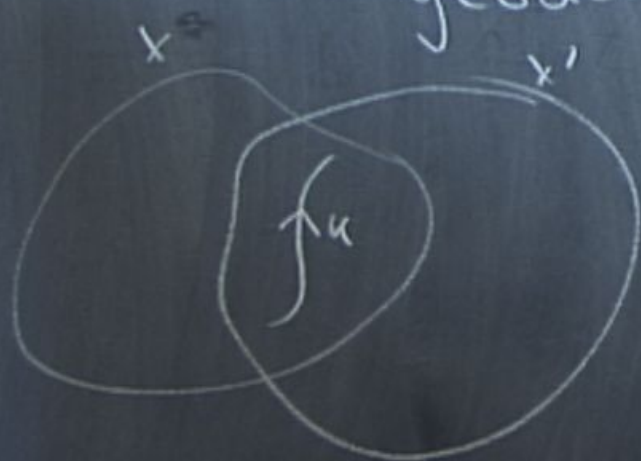
curve on manifold M

parameter
 $x^{\mu}(u)$

Recall

$$R^{\gamma}_{\alpha\mu\nu} = -\left(\Gamma^{\gamma}_{\alpha\mu,\nu} + \Gamma^{\beta}_{\alpha\mu}\Gamma^{\gamma}_{\beta\nu} - \Gamma^{\gamma}_{\alpha\nu,\mu} - \Gamma^{\beta}_{\alpha\nu}\Gamma^{\gamma}_{\beta\mu}\right)$$

Trajectories of particles in spacetime:
geodesic equation.



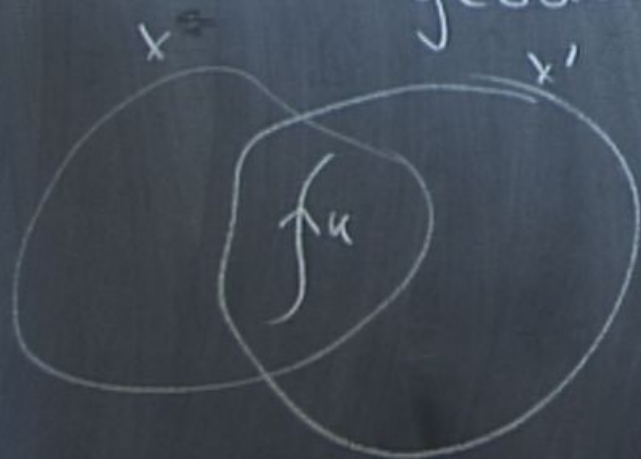
curve on manifold M

parameter
 $x^{\mu}(u)$ or $\dot{x}^{\mu}(u)$

Recall

$$R^{\gamma}_{\alpha\mu\nu} = -\left(\Gamma^{\gamma}_{\alpha\mu,\nu} + \Gamma^{\beta}_{\alpha\mu}\Gamma^{\gamma}_{\beta\nu} - \Gamma^{\gamma}_{\alpha\nu,\mu} - \Gamma^{\beta}_{\alpha\nu}\Gamma^{\gamma}_{\beta\mu}\right)$$

Trajectories of particles in spacetime:
geodesic equation.



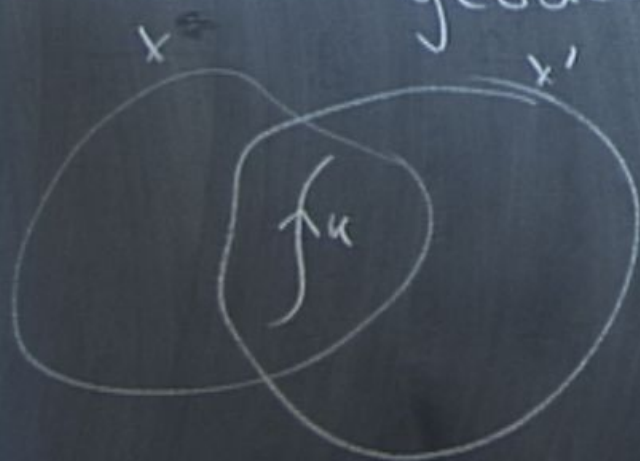
curve on manifold M

parameter
 $x^{\mu}(u)$ or $x^{\mu}(u)$

Recall

$$R^{\gamma}_{\alpha\mu\nu} = - \left(\Gamma^{\gamma}_{\alpha\mu,\nu} + \Gamma^{\beta}_{\alpha\mu} \Gamma^{\gamma}_{\beta\nu} - \Gamma^{\gamma}_{\alpha\nu,\mu} - \Gamma^{\beta}_{\alpha\nu} \Gamma^{\gamma}_{\beta\mu} \right)$$

Trajectories of particles in spacetime:
geodesic equation.



curve on manifold M

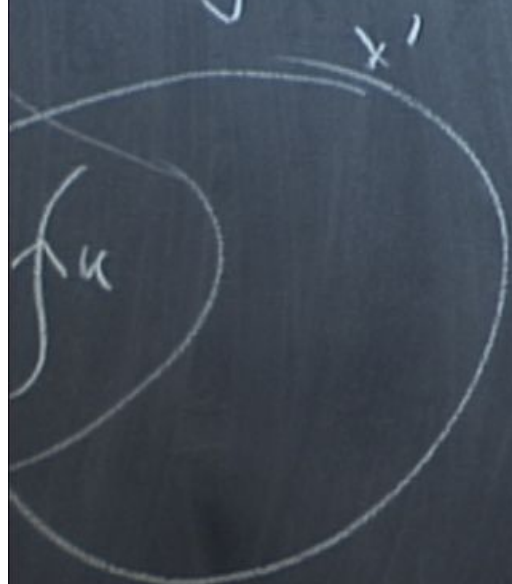
parameter
 $x^{\mu}(u)$ or $x^{\mu}(u)$

tangent vector $\frac{dx^{\mu}}{du} = \dot{x}^{\mu}(u)$ or $\frac{dx'^{\mu}}{du} = \dot{x}'^{\mu}(u)$

Recall

$$R^{\gamma}_{\alpha\mu\nu} = -\left(\Gamma^{\gamma}_{\alpha\mu,\nu} + \Gamma^{\beta}_{\alpha\mu}\Gamma^{\gamma}_{\beta\nu} - \Gamma^{\gamma}_{\alpha\nu,\mu} - \Gamma^{\beta}_{\alpha\nu}\Gamma^{\gamma}_{\beta\mu}\right)$$

Trajectories of particles in spacetime:
geodesic equation.



curve on manifold M

param $x^{\mu}(u)$ or

tangent vector $\frac{dx^{\mu}}{du} \equiv \dot{x}^{\mu}(u)$ or $\frac{d}{du}$

but curve $\dot{x}^{\mu}(u) = \dot{x}^{\mu}(x^{\alpha}(u))$

Recall

$$R^{\gamma}_{\alpha\mu\nu} = -\left(\Gamma^{\gamma}_{\alpha\mu,\nu} + \Gamma^{\beta}_{\alpha\mu}\Gamma^{\gamma}_{\beta\nu} - \Gamma^{\gamma}_{\alpha\nu,\mu} - \Gamma^{\beta}_{\alpha\nu}\Gamma^{\gamma}_{\beta\mu}\right)$$

Trajectories of particles in spacetime:
geodesic equation.



curve on manifold M

parameter
 $x^{\mu}(u)$ or x^{μ}

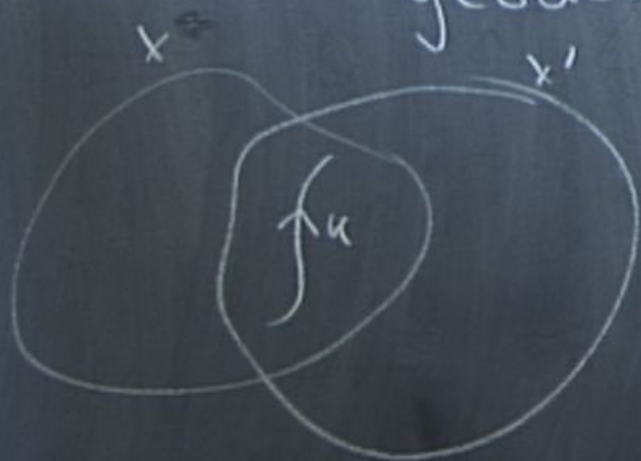
tangent vector $\frac{dx^{\mu}}{du} \equiv \dot{x}^{\mu}(u)$ or $\frac{dx^{\mu}}{du} \equiv \dot{x}^{\mu}(u)$

but curve $\dot{x}^{\mu}(u) = \dot{x}^{\mu}(x^{\alpha}(u))$
curve is x coordinate

Recall

$$R^{\gamma}_{\alpha\mu\nu} = - \left(\Gamma^{\gamma}_{\alpha\mu,\nu} + \Gamma^{\beta}_{\alpha\mu} \Gamma^{\gamma}_{\beta\nu} - \Gamma^{\gamma}_{\alpha\nu,\mu} - \Gamma^{\beta}_{\alpha\nu} \Gamma^{\gamma}_{\beta\mu} \right)$$

Trajectories of particles in spacetime:
geodesic equation.



curve on manifold M

parameter
 $x^{\mu}(u)$ or $x^{\mu}(u)$

tangent vector $\frac{dx^{\mu}}{du} \equiv \dot{x}^{\mu}(u)$ or $\frac{dx'^{\mu}}{du} \equiv \dot{x}'^{\mu}(u)$

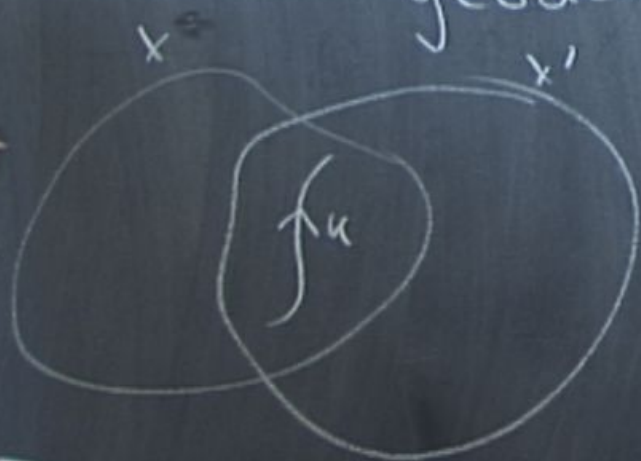
but curve $\dot{x}^{\mu}(u) = \dot{x}^{\mu}(x^{\alpha}(u))$ curve is x roads

$$\dot{x}^{\mu} = \frac{dx^{\mu}}{du} =$$

Recall

$$R^{\gamma}_{\alpha\mu\nu} = -\left(\Gamma^{\gamma}_{\alpha\mu,\nu} + \Gamma^{\beta}_{\alpha\mu}\Gamma^{\gamma}_{\beta\nu} - \Gamma^{\gamma}_{\alpha\nu,\mu} - \Gamma^{\beta}_{\alpha\nu}\Gamma^{\gamma}_{\beta\mu}\right)$$

Trajectories of particles in spacetime:
geodesic equation.



curve on manifold M

parameter
 $x^{\mu}(u)$ or $x^{\mu}(u)$

tangent vector $\frac{dx^{\mu}}{du} \equiv \dot{x}^{\mu}(u)$ or $\frac{dx^{\mu}}{du} \equiv \dot{x}^{\mu}(u)$

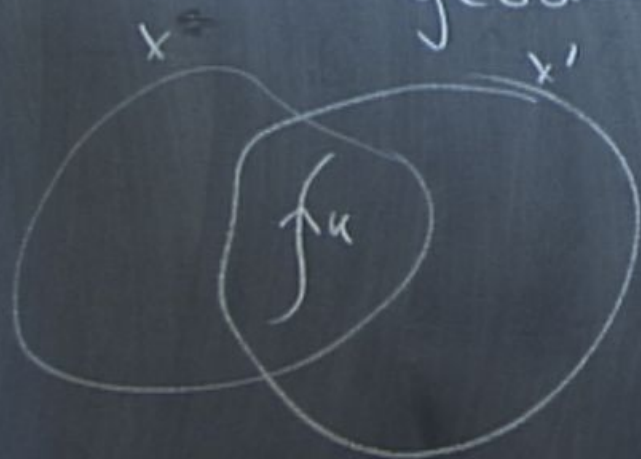
but curve $\dot{x}^{\mu}(u) = \dot{x}^{\mu}(x^{\alpha}(u))$ curve in x coords

$$\dot{x}^{\mu} = \frac{dx^{\mu}}{du} = \frac{dx^{\beta}}{du} \frac{\partial x^{\mu}}{\partial x^{\beta}}$$

Recall

$$R^{\gamma}_{\alpha\mu\nu} = -\left(\Gamma^{\gamma}_{\alpha\mu,\nu} + \Gamma^{\beta}_{\alpha\mu}\Gamma^{\gamma}_{\beta\nu} - \Gamma^{\gamma}_{\alpha\nu,\mu} - \Gamma^{\beta}_{\alpha\nu}\Gamma^{\gamma}_{\beta\mu}\right)$$

Trajectories of particles in spacetime:
geodesic equation.



curve on manifold M

parameter $x^{\mu}(u)$ or $\frac{dx^{\mu}}{du}$

tangent vector $\frac{dx^{\mu}}{du} = \dot{x}^{\mu}(u)$ or

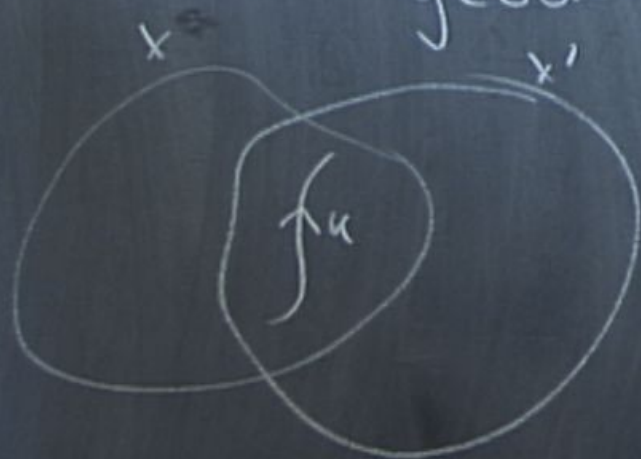
but curve $\dot{x}^{\mu}(u) = \dot{x}^{\mu}(x^{\alpha}(u))$

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Recall

$$R^{\gamma}_{\alpha\mu\nu} = -\left(\Gamma^{\gamma}_{\alpha\mu,\nu} + \Gamma^{\beta}_{\alpha\mu}\Gamma^{\gamma}_{\beta\nu} - \Gamma^{\gamma}_{\alpha\nu,\mu} - \Gamma^{\beta}_{\alpha\nu}\Gamma^{\gamma}_{\beta\mu}\right)$$

Trajectories of particles in spacetime:
geodesic equation.



curve on manifold M

parameter
 $x^{\mu}(u)$ or $x^{\mu}(u)$

tangent vector $\frac{dx^{\mu}}{du} = \dot{x}^{\mu}(u)$ or $\frac{dx^{\mu}}{du} = \dot{x}^{\mu}(u)$

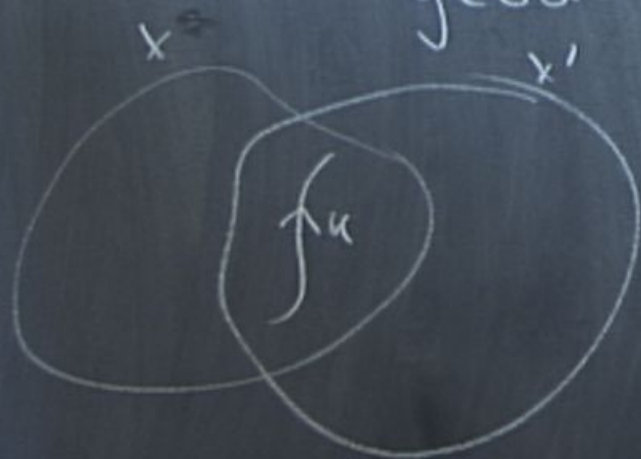
but curve $\dot{x}^{\mu}(u) = \dot{x}^{\mu}(x^{\alpha}(u))$ curve in x coords

$$\dot{x}^{\mu} = \frac{dx^{\mu}}{du} = \frac{dx^{\beta}}{du} \frac{\partial x^{\mu}}{\partial x^{\beta}} = \frac{\partial x^{\mu}}{\partial x^{\beta}} \dot{x}^{\beta}$$

Recall

$$R^{\gamma}_{\alpha\mu\nu} = -\left(\Gamma^{\gamma}_{\alpha\mu,\nu} + \Gamma^{\beta}_{\alpha\mu}\Gamma^{\gamma}_{\beta\nu} - \Gamma^{\gamma}_{\alpha\nu,\mu} - \Gamma^{\beta}_{\alpha\nu}\Gamma^{\gamma}_{\beta\mu}\right)$$

Trajectories of particles in spacetime:
geodesic equation.



curve on manifold M

parameter
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tangent vector $\frac{dx^{\mu}}{du} = \dot{x}^{\mu}(u)$ or $\frac{dx'^{\mu}}{du} = \dot{x}'^{\mu}(u)$

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$$\dot{x}^{\mu} = \frac{dx^{\mu}}{du} = \frac{dx^{\beta}}{du} \frac{\partial x^{\mu}}{\partial x^{\beta}} = \frac{\partial x^{\mu}}{\partial x^{\beta}} \dot{x}^{\beta}$$

$\Rightarrow \xi^M$ is a contravariant vector.

but $\frac{d\xi^M}{du}$

(u)

$\Rightarrow \xi^M$ is a contravariant vector.

but $\frac{d\xi^M}{du}$ not a contravariant vector.

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So define $\frac{D\xi^M}{du} \equiv$

(u)

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but $\frac{d\xi^M}{du}$ not a contravariant vector.

So define $\frac{D\xi^M}{du} \equiv \frac{d\xi^M}{du} + \Gamma_{\nu\lambda}^M \xi^\nu \frac{dx^\lambda}{du}$

$$\frac{\partial V^\alpha}{\partial x^\mu} + \Gamma_{\beta\mu}^\alpha V^\beta$$

$\Rightarrow \xi^M$ is a contravariant vector.

but $\frac{d\xi^M}{du}$ not a contravariant vector.

So
$$\frac{D\xi^M}{du} \equiv \frac{d\xi^M}{du} + \Gamma_{\nu\lambda}^M V^\nu \frac{dx^\lambda}{du} \left(\frac{\partial V^\alpha}{\partial x^\mu} + \Gamma_{\beta\mu}^\alpha V^\beta \right)$$

$\rightarrow \xi^M$ is a contravariant vector. Consider a vector $V^\alpha(u)$ on a curve.

then $\frac{dV^\alpha}{du}$ not a contravariant vector.

So define $\frac{D\xi^M}{du} \equiv \frac{d\xi^M}{du} + \Gamma_{\nu\lambda}^M V^\nu \frac{dx^\lambda}{du}$ $\left(\frac{\partial V^\alpha}{\partial x^\mu} + \Gamma_{\beta\mu}^\alpha V^\beta \right)$

$\rightarrow \xi^M$ is a contravariant vector. Consider a ^{cov.} vector $V^\alpha(u)$ on a curve.

then $\frac{dV^\mu}{du}$ not a contravariant vector.

So define $\frac{DV^\mu}{du} \equiv \frac{dV^\mu}{du} + \Gamma_{\nu\lambda}^\mu V^\nu \frac{dx^\lambda}{du}$ $\left(\frac{\partial V^\alpha}{\partial x^\mu} + \Gamma_{\beta\mu}^\alpha V^\beta \right)$

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if $\frac{DV^M}{du} = 0$, we say $V^M(u)$ is "parallel transported" along a curve.

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$$\text{So define } \frac{DV^M}{du} \equiv \frac{dV^M}{du} + \Gamma_{\nu\lambda}^M V^\nu \frac{dx^\lambda}{du}$$

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if $\frac{DV^M}{du} = 0$, we say $V^M(u)$ is "parallel transported"

along a curve.

$$\frac{\partial V^\alpha}{\partial x^\mu} + \Gamma_{\beta\mu}^\alpha V^\beta$$

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if $\frac{DV^M}{du} = 0$, we say $V^M(u)$ is "parallel transported" along the curve $x^\mu(u)$

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So define $\frac{DV^M}{du} \equiv \frac{dV^M}{du} + \Gamma_{\nu\lambda}^M V^\nu \frac{dx^\lambda}{du}$

$$\frac{\partial V^\alpha}{\partial x^\mu} + \Gamma_{\beta\mu}^\alpha V^\beta$$

which is a contravariant vector.

if $\frac{DV^M}{du} = 0$, we say $V^M(u)$ is "parallel transported" along the curve $x^\mu(u)$

$$V'^\mu = \frac{\partial x'^\mu}{\partial x^\alpha} V^\alpha$$

$\rightarrow \xi^\mu$ is a contravariant vector. Consider a ^{cov.} vector $V^\alpha(u)$ on a curve.

then $\frac{dV^\mu}{du}$ not a contravariant vector.

So define $\frac{DV^\mu}{du} \equiv \frac{dV^\mu}{du} + \Gamma_{\nu\lambda}^\mu V^\nu \frac{dx^\lambda}{du}$

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if $\frac{DV^\mu}{du} = 0$, we say $V^\mu(u)$ is "parallel transported"

along the curve $x^\mu(u)$

$$V'^\mu = \underbrace{\frac{\partial x'^\mu}{\partial x^\alpha}}_{(u)} V^\alpha(u)$$

$\rightarrow \xi^M$ is a contravariant vector. Consider a contravariant vector $V^\alpha(u)$ on a curve.

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So define $\frac{DV^\mu}{du} \equiv \frac{dV^\mu}{du} + \Gamma_{\nu\lambda}^\mu V^\nu \frac{dx^\lambda}{du}$

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which is a contravariant vector.

if $\frac{DV^M}{du} = 0$, we say $V^M(u)$ is "parallel transported"

along the curve. $x^\mu(u)$

$$\dot{V}^\mu = \underbrace{\frac{\partial x'^\mu}{\partial x^\alpha}}_{(u)} V^\alpha(u)$$

$\frac{D}{Du}$ is a covariant derivative along the curve.

$\rightarrow \xi^M$ is a contravariant vector. Consider a contravariant vector $V^\alpha(u)$ on a curve.

then $\frac{dV^M}{du}$ not a contravariant vector.

So define $\frac{DV^M}{du} \equiv \frac{dV^M}{du} + \Gamma_{\nu\lambda}^M V^\nu \frac{dx^\lambda}{du}$

$$\frac{\partial V^\alpha}{\partial x^\mu} + \Gamma_{\beta\mu}^\alpha V^\beta$$

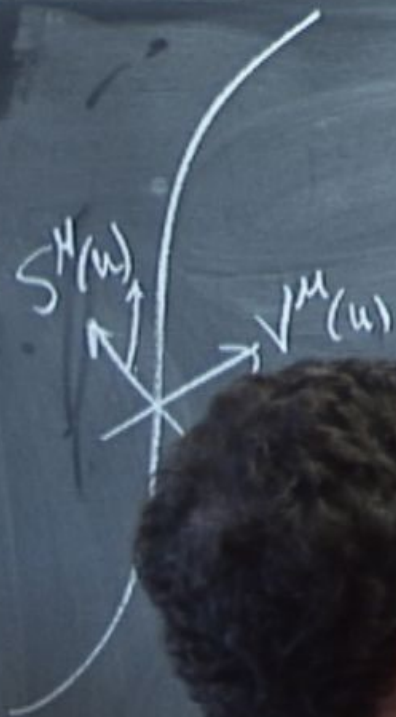
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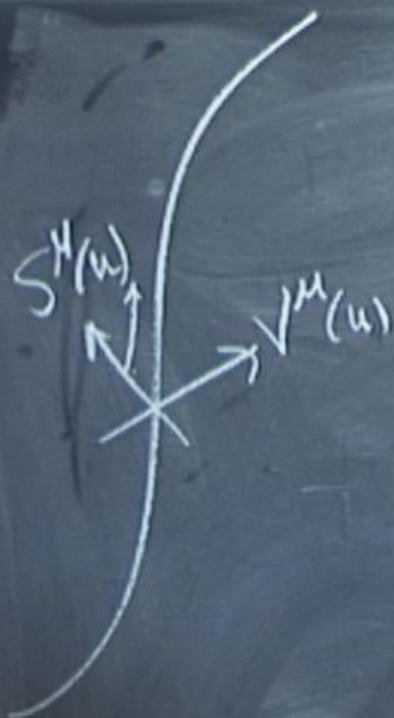
$$V'^\mu = \frac{\partial x'^\mu}{\partial x^\alpha} V^\alpha(u)$$

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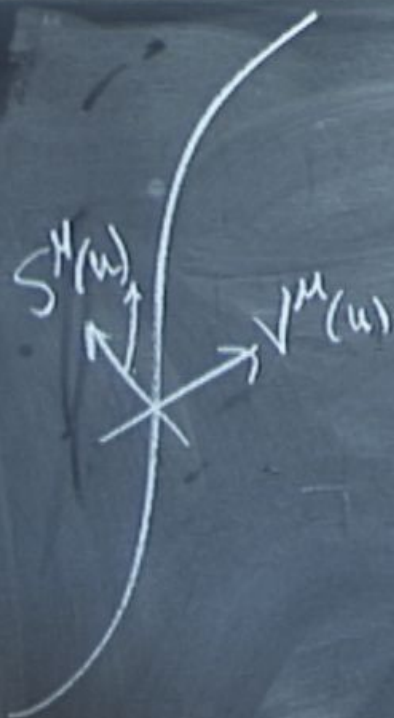


"angle between S^M and V^M "

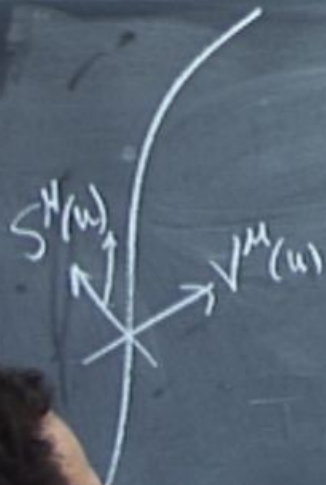
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"angle between S^μ and V^μ "
$$= g_{\mu\nu} V^\mu S^\nu$$



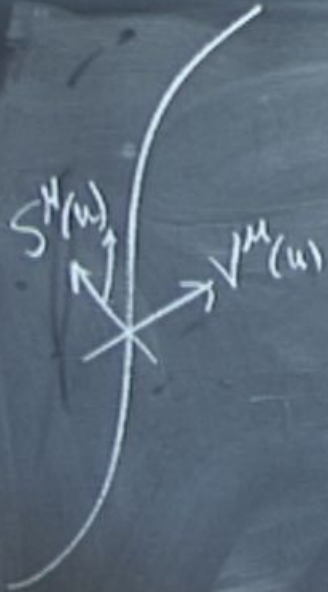
"angle between S^μ and V^μ "
$$= g_{\mu\nu} V^\mu S^\nu$$



"angle between S^M and V^M "

$$= g_{\mu\nu} V^{\mu} S^{\nu} = \cos \theta |V| |S|$$

$$|V|^2 = g_{\mu\nu} V^{\mu} V^{\nu}$$

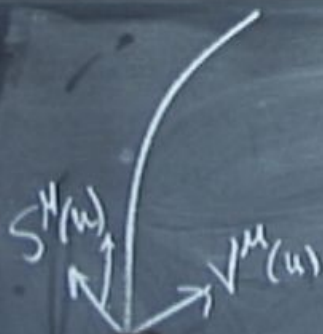


"angle between S^M and V^M "

$$= g_{\mu\nu} V^{\mu} S^{\nu} = \cos \theta |V| |S|$$

e.g. $|V| = |S| = 1$

$$|V|^2 = g_{\mu\nu} V^{\mu} V^{\nu}$$



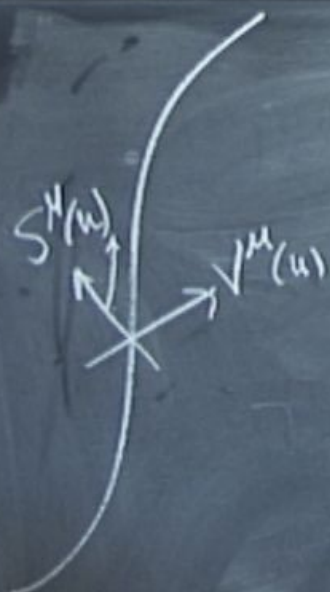
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$$|V|^2 = g_{\mu\nu} V^{\mu} V^{\nu}$$

e.g. $|V| = |S| = 1$

$$\begin{aligned} \frac{D}{du} (V_{\mu} S^{\mu}) &= \frac{dV_{\mu}}{du} S^{\mu} + V_{\mu} \frac{dS^{\mu}}{du} \\ &\quad - \Gamma_{\mu\alpha}^{\kappa} V_{\kappa} S^{\mu} \frac{dx^{\alpha}}{du} \end{aligned}$$



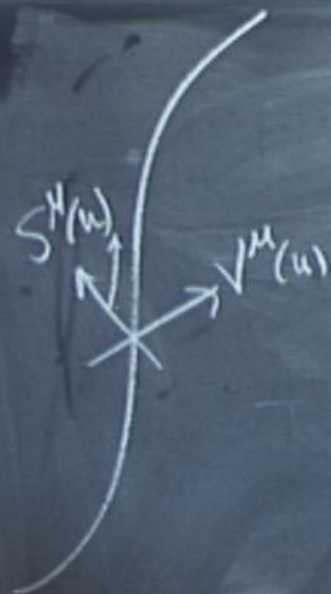
"angle between S^M and V^M "

$$= g_{\mu\nu} V^{\mu} S^{\nu} = \cos \theta |V| |S|$$

$$|V|^2 = g_{\mu\nu} V^{\mu} V^{\nu}$$

e.g. $|V| = |S| = 1$

$$\begin{aligned} \frac{D}{du} (V_{\mu} S^{\mu}) &= \frac{dV_{\mu}}{du} S^{\mu} + V_{\mu} \frac{dS^{\mu}}{du} \\ &= \Gamma^{\kappa}_{\mu\lambda} V_{\kappa} S^{\mu} \frac{dx^{\lambda}}{du} + V_{\mu} \Gamma^{\mu}_{\kappa\lambda} S^{\kappa} \frac{dx^{\lambda}}{du} \end{aligned}$$



"angle between S^M and V^M "

$$= g_{\mu\nu} V^\mu S^\nu = \cos\theta |V| |S|$$

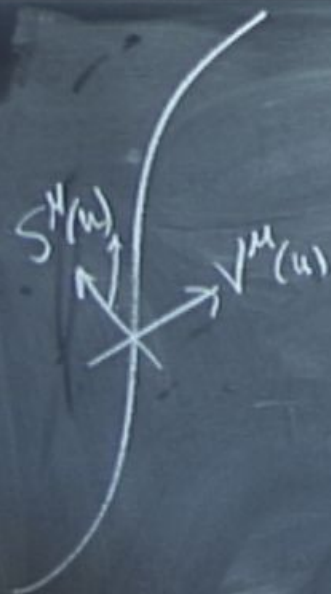
$$|V|^2 = g_{\mu\nu} V^\mu V^\nu$$

e.g. $|V| = |S| = 1$

$$\frac{D}{du} (V_\mu S^\mu) = \frac{dV_\mu}{du} S^\mu + V_\mu \frac{dS^\mu}{du}$$

$$= \cancel{\Gamma_{\lambda\mu}^\alpha V^\lambda S^\mu} \frac{dx^\beta}{du} + V_\mu \cancel{\Gamma_{\lambda\mu}^\alpha} S^\lambda \frac{dx^\nu}{du}$$

=



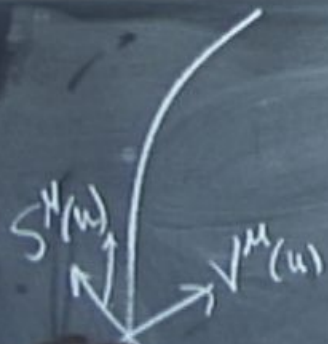
"angle between S^M and V^M "

$$= g_{\mu\nu} V^\mu S^\nu = \cos\theta |V| |S|$$

$$|V|^2 = g_{\mu\nu} V^\mu V^\nu$$

e.g. $|V| = |S| = 1$

$$\begin{aligned} \frac{D}{du}(V_\mu S^\mu) &= \frac{dV_\mu}{du} S^\mu + V_\mu \frac{dS^\mu}{du} \\ &= \cancel{\Gamma_{\lambda\mu}^\kappa V_\kappa S^\mu \frac{dx^\lambda}{du}} + V_\mu \cancel{\Gamma_{\lambda\kappa}^\mu S^\kappa \frac{dx^\lambda}{du}} \\ &= \frac{d}{du}(V_\mu S^\mu) \end{aligned}$$



"angle between S^M and V^M "

$$= g_{\mu\nu} V^\mu S^\nu = \cos\theta |V| |S|$$

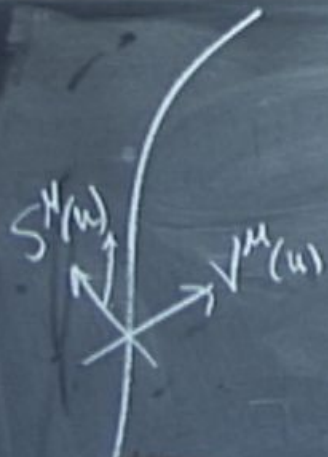
$$|V|^2 = g_{\mu\nu} V^\mu V^\nu$$

e.g. $|V| = |S| = 1$

$$\frac{D}{du}(V_\mu S^\mu) = \frac{dV_\mu}{du} S^\mu + V_\mu \frac{dS^\mu}{du}$$

$$\cancel{\Gamma_{\lambda\mu}^\alpha V^\lambda S^\mu \frac{dx^\beta}{du}} + \cancel{V_\mu \Gamma_{\lambda\mu}^\alpha S^\lambda \frac{dx^\beta}{du}}$$

$$= \frac{d}{du}(V_\mu S^\mu)$$



"angle between S^M and V^M "

$$= g_{\mu\nu} V^\mu S^\nu = \cos \theta |V| |S|$$

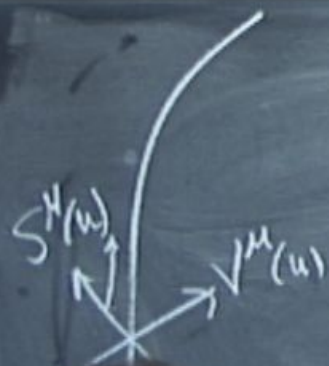
$$|V|^2 = g_{\mu\nu} V^\mu V^\nu$$

e.g. $|V| = |S| = 1$

$$\frac{D}{du} (V_\mu S^\mu) = \frac{dV_\mu}{du} S^\mu + V_\mu \frac{dS^\mu}{du}$$

$$= \cancel{\Gamma_{\alpha\mu}^\nu V^\alpha S^\mu \frac{dx^\beta}{du}} + \cancel{V_\mu \Gamma_{\alpha\mu}^\nu S^\alpha \frac{dx^\nu}{du}}$$

$$= \frac{d}{du} (V_\mu S^\mu)$$



"angle between S^M and V^M "

$$= g_{\mu\nu} V^{\mu} S^{\nu} = \cos \theta |V| |S|$$

$$|V|^2 = g_{\mu\nu} V^{\mu} V^{\nu}$$

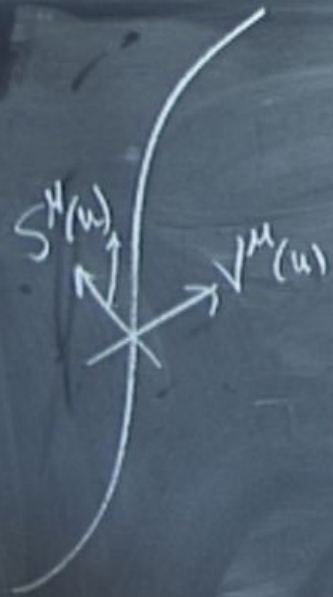
e.g. $|V| = |S| = 1$

$$\begin{aligned} \frac{D}{du} (V_{\mu} S^{\mu}) &= \frac{dV_{\mu}}{du} S^{\mu} + V_{\mu} \frac{dS^{\mu}}{du} \\ &= \cancel{\Gamma^{\alpha}_{\mu\nu} V^{\nu} S^{\mu} \frac{dx^{\beta}}{du}} + \cancel{V_{\mu} \Gamma^{\mu}_{\alpha\nu} S^{\alpha} \frac{dx^{\nu}}{du}} \end{aligned}$$

$$= \frac{d}{du} (V_{\mu} S^{\mu})$$

// transported

$$= 0$$



"angle between S^M and V^M "

$$= g_{\mu\nu} V^M S^\nu = \cos\theta |V| |S|$$

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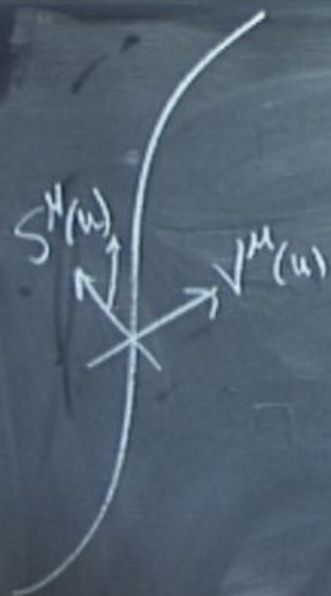
e.g. $|V| = |S| = 1$

$$\begin{aligned} \frac{D}{du}(V_\mu S^\mu) &= \frac{dV_\mu}{du} S^\mu + V_\mu \frac{dS^\mu}{du} \\ &= \cancel{\Gamma_{\lambda\mu}^\alpha V^\lambda S^\mu \frac{dx^\beta}{du}} + V_\mu \cancel{\Gamma_{\lambda\mu}^\alpha S^\lambda \frac{dx^\beta}{du}} \end{aligned}$$

$$= \frac{d}{du}(V_\mu S^\mu)$$

If V^M, S^V // transported

$$= 0$$



"angle between S^M and V^M "

$$= g_{\mu\nu} V^{\mu} S^{\nu} = \cos \theta |V| |S|$$

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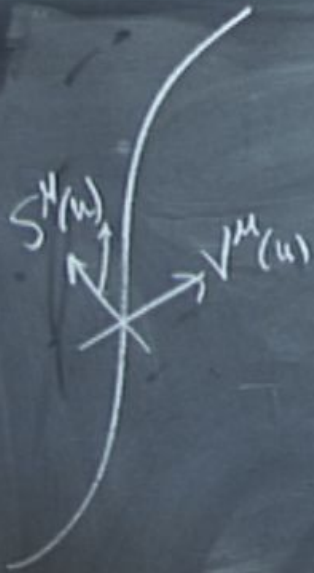
e.g. $|V| = |S| = 1$

$$\begin{aligned} \frac{D}{du} (V_{\mu} S^{\mu}) &= \frac{dV_{\mu}}{du} S^{\mu} + V_{\mu} \frac{dS^{\mu}}{du} \\ &\quad - \Gamma^{\alpha}_{\lambda\mu} V^{\lambda} S^{\mu} \frac{dx^{\alpha}}{du} + V_{\mu} \Gamma^{\alpha}_{\lambda\nu} S^{\lambda} \frac{dx^{\nu}}{du} \end{aligned}$$

$$= \frac{d}{du} (V_{\mu} S^{\mu})$$

If V^{μ}, S^{ν} // transported

$$= 0$$



"angle between S^M and V^M "

$$= g_{\mu\nu} V^\mu S^\nu = \cos\theta |V| |S|$$

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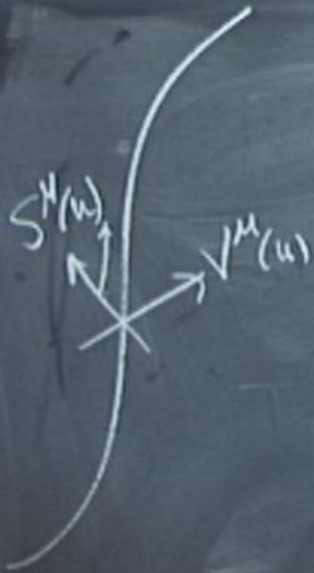
e.g. $|V| = |S| = 1$

$$\begin{aligned} \nabla_\mu (T^{\alpha\beta} S_{\alpha\beta}) \\ \stackrel{(i)}{=} \frac{\partial}{\partial x^\mu} (T^{\alpha\beta} S_{\alpha\beta}) \\ (\nabla_\mu T^{\alpha\beta}) S_{\alpha\beta} \\ + T^{\alpha\beta} \nabla_\mu S_{\alpha\beta} \end{aligned}$$

$$\begin{aligned} \frac{D}{du} (V_\mu S^\mu) &= \frac{dV_\mu}{du} S^\mu + V_\mu \frac{dS^\mu}{du} \\ &= \cancel{\Gamma^\kappa_{\lambda\mu} V^\lambda S^\mu \frac{dx^\beta}{du}} + V_\mu \cancel{\Gamma^\mu_{\lambda\kappa} S^\lambda \frac{dx^\kappa}{du}} \\ &= \frac{d}{du} (V_\mu S^\mu) \end{aligned}$$

If V^μ, S^ν // transported

$$= 0$$



"angle between S^M and V^M "

$$= g_{\mu\nu} V^\mu S^\nu = \cos\theta |V| |S|$$

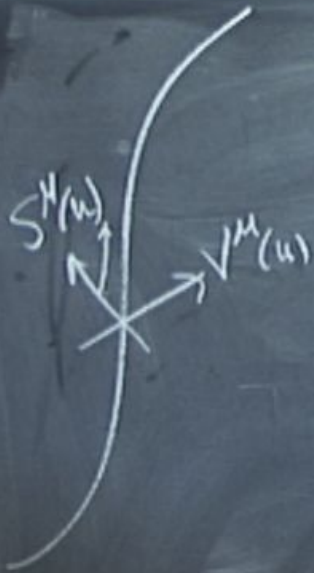
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$$\begin{aligned} &= \frac{d}{du} (V_\mu S^\mu) \\ \text{If } V^\mu, S^\nu \text{ // transported} \\ &= 0 \end{aligned}$$



"angle between S^M and V^M "

$$= g_{\mu\nu} V^\mu S^\nu = \cos\theta |V| |S|$$

$$|V|^2 = g_{\mu\nu} V^\mu V^\nu$$

e.g. $|V| = |S| = 1$

$$\begin{aligned} \nabla_\mu (T^{\alpha\beta} S_{\alpha\beta}) \\ \stackrel{(i)}{=} \frac{\partial}{\partial x^\mu} (T^{\alpha\beta} S_{\alpha\beta}) \\ = (\nabla_\mu T^{\alpha\beta}) S_{\alpha\beta} \\ + T^{\alpha\beta} \nabla_\mu S_{\alpha\beta} \end{aligned}$$

$$\frac{D}{du} (V_\mu S^\mu) = \frac{dV_\mu}{du} S^\mu + V_\mu \frac{dS^\mu}{du}$$

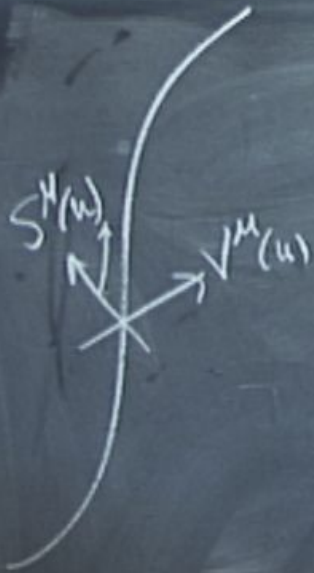
$$\boxed{\frac{DV^\mu}{Du} = \frac{DS^\mu}{Du} = 0}$$

$$= \cancel{\Gamma^\lambda_{\mu\nu} V^\mu S^\nu \frac{dx^\beta}{du}} + \cancel{V_\mu \Gamma^\mu_{\lambda\nu} S^\lambda \frac{dx^\nu}{du}}$$

$$\uparrow = \frac{d}{du} (V_\mu S^\mu)$$

If V^μ, S^ν // transported

$$= 0$$



"angle between S^M and V^M "

$$= g_{\mu\nu} V^\mu S^\nu = \cos\theta |V| |S|$$

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If V^μ, S^ν // transported

$$= \cancel{\Gamma^\lambda_{\mu\nu} V^\mu S^\nu \frac{dx^\beta}{du}} + \cancel{V_\mu \Gamma^\mu_{\lambda\nu} S^\nu \frac{dx^\lambda}{du}}$$

$$= \frac{d}{du} (V_\mu S^\mu)$$

$$= 0$$

$\Rightarrow \xi^M$ is a contravariant vector. Consider a contravariant vector $V^\alpha(u)$ on a curve.

then $\frac{dV^\mu}{du}$ not a contravariant vector.

So define $\frac{DV^\mu}{du} \equiv \frac{dV^\mu}{du} + \Gamma_{\nu\lambda}^\mu V^\lambda \frac{dx^\nu}{du}$

which is a contravariant vector.

if $\frac{DV^\mu}{du} = 0$, we say V

along the curve $x^\mu(u)$

$\frac{D}{Du}$ is a covariant derivative along

$$+ \Gamma_{\beta\mu}^\alpha V^\beta$$

"spoke"

$$\frac{\partial x^\mu}{\partial x^\alpha} V^\alpha(u)$$

ector. Consider a contravariant vector V^α .

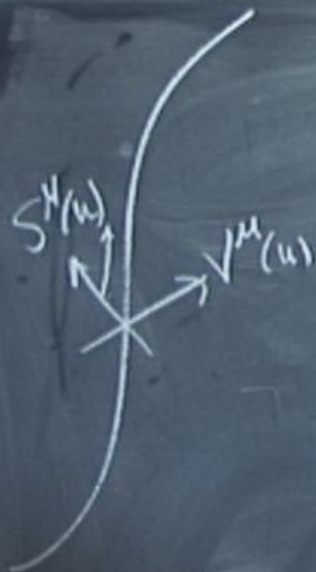
5. The covariant derivative of V^α is

$$\frac{D V^\alpha}{D u} \equiv \frac{d V^\alpha}{d u} + \Gamma^\alpha_{\beta\mu} V^\beta \frac{d x^\mu}{d u}$$

which is zero if $\frac{D V^\alpha}{D u} = 0$, we say $V^\alpha(u)$ is "parallel transported" along curve $x^\mu(u)$.

$$\dot{V}^\alpha = \frac{\partial x^\alpha}{\partial x^\mu} V^\mu(u)$$

$\frac{D}{D u}$ is a covariant derivative along the curve.



"angle between S^M and V^M "

$$= g_{\mu\nu} V^{\mu} S^{\nu} = \cos \theta |V| |S|$$

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e.g. $|V| = |S| = 1$

$$\begin{aligned} \nabla_{\mu} (T^{\alpha\beta} S_{\alpha\beta}) \\ \stackrel{(1)}{=} \frac{\partial}{\partial x^{\mu}} (T^{\alpha\beta} S_{\alpha\beta}) \\ = (\nabla_{\mu} T^{\alpha\beta}) S_{\alpha\beta} + T^{\alpha\beta} \nabla_{\mu} S_{\alpha\beta} \end{aligned}$$

$$\frac{D}{du} (V_{\mu} S^{\mu}) = \frac{dV_{\mu}}{du} S^{\mu} + V_{\mu} \frac{dS^{\mu}}{du}$$

$$\boxed{\frac{DV^{\mu}}{Du} = \frac{DS^{\mu}}{Du} = 0}$$

$$= \Gamma^{\alpha}_{\mu\nu} V^{\mu} S^{\nu} \frac{dx^{\alpha}}{du} + \dots$$

$$= \frac{d}{du} (V_{\mu} S^{\mu})$$

If V^{μ}, S^{ν} // transported

$$= 0$$

$$dV^\mu$$

$$\cos \theta |V| |S|$$

$$|V|^2 = g_{\mu\nu} V^\mu V^\nu$$

$$\frac{dS^\mu}{du}$$

$$\frac{dx^\beta}{du} + V_\mu \frac{dx^\mu}{du} S^\alpha \frac{dx^\nu}{du}$$

$$\frac{d}{du} (V_\mu S^\mu)$$

$$= \frac{D}{Du} (V_\mu S^\mu)$$

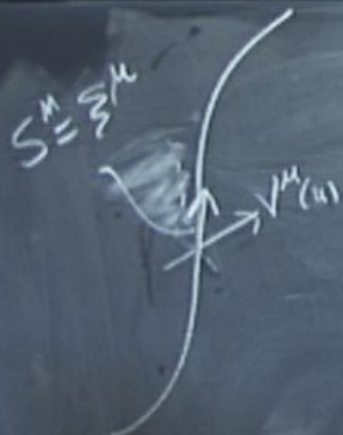
$$= \frac{DV_\mu S^\mu}{Du} + V_\mu \frac{DS^\mu}{du}$$

$$= 0 + 0$$

we

$$\lambda^\mu(u)$$

mit den



"angle between S^M and V^M "

$$= g_{\mu\nu} V^{\mu} S^{\nu} = \cos \theta |V| |S|$$

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$$\frac{D}{du} (V_{\mu} S^{\mu}) = \frac{dV_{\mu}}{du} S^{\mu} + V_{\mu} \frac{dS^{\mu}}{du}$$

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$$\begin{aligned} \text{If } V^{\mu}, S^{\nu} \text{ // transported} \\ \frac{d}{du} (V_{\mu} S^{\mu}) = 0 \end{aligned}$$

$$\frac{d}{du} (V_{\mu} S^{\mu})$$

$$= \frac{D}{du} (V_{\mu} S^{\mu})$$

$$= \frac{dV_{\mu}}{du} S^{\mu} + V_{\mu} \frac{dS^{\mu}}{du}$$

$$= 0 + 0$$

$$\frac{d}{du} (V_{\mu} V^{\mu}) = 0$$

Geodesic curves

$x^M(u)$ is geodesic if $\frac{D\xi^M}{du}$

$\frac{S^M}{du}$

Geodesic curves

$x^M(u)$ is geodesic if

$$\frac{D\xi^M}{du} = \lambda(u)\xi^M$$

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$$\left(\xi^M = \frac{dx^M}{du}\right)$$

e.. $\frac{d^2x^M}{du^2}$

Geodesic curves

$x^M(u)$ is geodesic if

$$\frac{D\xi^M}{du} = \lambda(u)\xi^M$$

i.e.
$$\frac{d^2 x^M}{du^2} + \Gamma_{\nu\lambda}^M \frac{dx^\nu}{du} \frac{dx^\lambda}{du} = \lambda(u) \frac{dx^M}{du}$$

$$\left(\xi^M = \frac{dx^M}{du} \right)$$

Geodesic curves

$x^M(u)$ is geodesic if $\frac{D\xi^M}{du} = \lambda(u)\xi^M$

i.e. $\frac{d^2 x^M}{du^2} + \Gamma_{\nu\lambda}^M \frac{dx^\nu}{du} \frac{dx^\lambda}{du} = \lambda(u) \frac{dx^M}{du}$ $\left(\xi^M = \frac{dx^M}{du} \right)$

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Geodesic curves

$x^M(u)$ is geodesic if $\frac{D\xi^M}{du} = \lambda(u)\xi^M$

ie. $\frac{d^2 x^M}{du^2} + \Gamma_{\nu\lambda}^M \frac{dx^\nu}{du} \frac{dx^\lambda}{du} = \lambda(u) \frac{dx^M}{du}$ $(\xi^M = \frac{dx^M}{du})$

Can always choose an affine parametrisation for the curve to cancel term on rhs.

$X^M(u)$: change parameterisation
by $u \rightarrow \sigma(u)$

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$$\frac{dx^M}{du} = \frac{d\sigma}{du} \frac{dx^M}{d\sigma}$$

$x^M(u)$: change parameterisation
by $u \rightarrow \sigma(u)$

$x^M(\sigma(u))$

$$\frac{dx^M}{du} = \frac{d\sigma}{du} \frac{dx^M}{d\sigma}$$

at $in(*)$

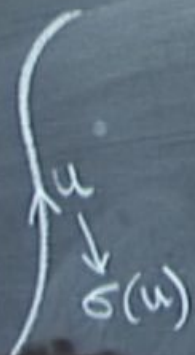
$$\frac{d^2\sigma}{du^2}$$

$x^M(u)$: change parameterisation
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$x^M(\sigma(u))$

$$\frac{dx^M}{du} = \frac{d\sigma}{du} \frac{dx^M}{d\sigma}$$

$$\frac{d^2\sigma}{du^2}$$



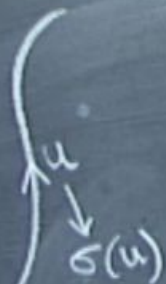
$x^M(u)$: change parameterisation
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$x^M(\sigma(u))$

$$\frac{dx^M}{du} = \frac{d\sigma}{du} \frac{dx^M}{d\sigma}$$

insert in (*)

$$\frac{d^2\sigma}{du^2} \frac{dx^M}{d\sigma} + \left(\frac{d\sigma}{du} \right)^2 \left(\frac{d^2x^M}{d\sigma^2} \right)$$



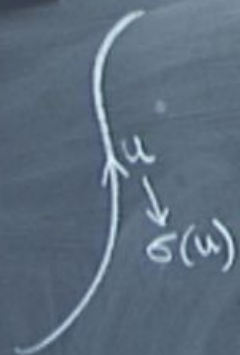
$x^M(u)$: change parameterisation
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$$\frac{dx^M}{du} = \frac{d\sigma}{du} \frac{dx^M}{d\sigma}$$

insert in (*)

$$\frac{d^2\sigma}{du^2} \frac{dx^M}{d\sigma} + \left(\frac{d\sigma}{du}\right)^2 \left(\frac{d^2x^M}{d\sigma^2} + \Gamma_{\nu\lambda}^M \frac{dx^\nu}{d\sigma} \frac{dx^\lambda}{d\sigma} \right) = \lambda(u) \frac{d\sigma}{du} \frac{dx^M}{d\sigma}$$



$x^M(u)$: change parameterisation
by $u \rightarrow \sigma(u)$

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$$\frac{dx^M}{du} = \frac{d\sigma}{du} \frac{dx^M}{d\sigma}$$

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$\rightarrow$ set $\frac{d^2\sigma}{du^2} = \lambda(u) \frac{d\sigma}{du}$

$x^M(u)$: change parameterisation
by $u \rightarrow \sigma(u)$

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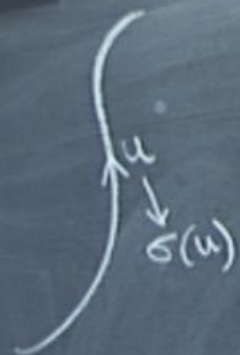
$$\frac{dx^M}{du} = \frac{d\sigma}{du} \frac{dx^M}{d\sigma}$$

set in (*)

$$\frac{d^2\sigma}{du^2} \frac{dx^M}{d\sigma} + \left(\frac{d\sigma}{du}\right)^2 \left(\frac{d^2x^M}{d\sigma^2} + \Gamma^M_{\nu\lambda} \frac{dx^\nu}{d\sigma} \frac{dx^\lambda}{d\sigma} \right) = \lambda(u) \frac{d\sigma}{du} \frac{dx^M}{d\sigma}$$

set

$$\frac{d^2\sigma}{du^2} = \lambda(u) \frac{d\sigma}{du}$$



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by $u \rightarrow \sigma(u)$

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$$\frac{dx^M}{du} = \frac{d\sigma}{du} \frac{dx^M}{d\sigma}$$

insert in (*)

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→ set $\frac{d^2\sigma}{du^2} = \lambda(u) \frac{d\sigma}{du}$

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$$\frac{dx^M}{du} = \frac{d\sigma}{du} \frac{dx^M}{d\sigma}$$

insert in (*)

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→ set

$$\frac{d^2\sigma}{du^2} = \lambda(u) \frac{d\sigma}{du}$$

$u \rightarrow \sigma(u)$

$x^M(u)$: change parameterisation
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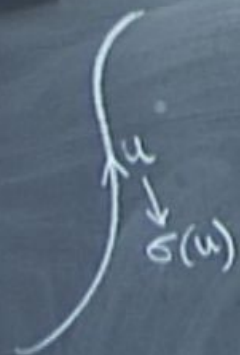
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insert in (*)

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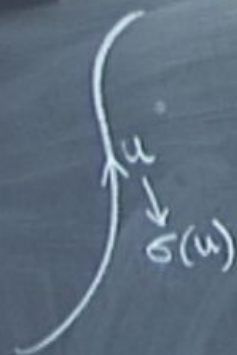
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insert in (*)

$$\frac{d^2\sigma}{du^2} \frac{dx^M}{d\sigma} + \left(\frac{d\sigma}{du}\right)^2 \left(\frac{d^2x^M}{d\sigma^2} + \Gamma^M_{\nu\lambda} \frac{dx^\nu}{d\sigma} \frac{dx^\lambda}{d\sigma} \right) = \lambda(u) \frac{d\sigma}{du} \frac{dx^M}{d\sigma}$$

→ set

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$$\frac{d^2\sigma}{du^2} \frac{dx^M}{d\sigma} + \left(\frac{d\sigma}{du}\right)^2 \left(\frac{d^2x^M}{d\sigma^2} + \Gamma^M_{\nu\lambda} \frac{dx^\nu}{d\sigma} \frac{dx^\lambda}{d\sigma} \right) = \lambda(u) \frac{d\sigma}{du} \frac{dx^M}{d\sigma}$$

→ set

$$\frac{d^2\sigma}{du^2} = \lambda(u) \frac{d\sigma}{du} \quad ; \quad \Rightarrow \frac{d\sigma}{du} = e^{\int \lambda(u) du}$$

$$\frac{d}{du} \left(\frac{d\sigma}{du} \right) = \lambda(u)$$

→ can always choose an affine parameterisation
for a geodesic, in which

$$\frac{dx^\mu}{ds^2} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{ds} \frac{dx^\lambda}{ds} = 0$$

i.e. D

$$\lambda(u) \frac{d}{ds}$$

→ can always choose an affine parametrisation for a geodesic, in which

$$\frac{d^2 x^m}{ds^2} + \Gamma_{\nu\lambda}^m \frac{dx^\nu}{ds} \frac{dx^\lambda}{ds} = 0$$

$$\frac{D}{Ds} \left(\frac{dx^m}{ds} \right) = \frac{D}{Ds} (\xi^m) = 0$$

tangent

→ can always choose an affine parameterisation
for a geodesic, in which

$$\frac{dx^m}{ds} + \Gamma_{\nu\lambda}^m \frac{dx^\nu}{ds} \frac{dx^\lambda}{ds} = 0$$

$$\text{i.e. } \frac{D}{Ds} \left(\frac{dx^m}{ds} \right) = \frac{D}{Ds} (\xi^m) = 0$$

i.e. tangent vector is // transported along curve.

$$\frac{ds}{du} \frac{dx^m}{ds}$$

→ can always choose an affine parameterisation for a geodesic, in which

$$\frac{dx^m}{ds} + \Gamma_{\nu\lambda}^m \frac{dx^\nu}{ds} \frac{dx^\lambda}{ds} = 0$$

$$\text{i.e. } \frac{D}{Ds} \left(\frac{dx^m}{ds} \right) = \frac{D}{Ds} (\xi^m) = 0$$

i.e. tangent vector is // transported along curve.

How unique is affine parameterisation? s unique up to $s \rightarrow s' = as + b$.

→ can always choose an affine parameterisation for a geodesic, in which

$$\frac{dx^m}{ds} + \Gamma_{\nu\lambda}^m \frac{dx^\nu}{ds} \frac{dx^\lambda}{ds} = 0$$

$$\text{i.e. } \frac{D}{Ds} \left(\frac{dx^m}{ds} \right) = \frac{D}{Ds} (\xi^m) = 0$$

i.e. tangent vector is // transported along curve.

How unique is affine parameterisation? σ unique up to $\sigma \rightarrow \sigma' = a\sigma + b$.

$$\left(\text{soln } \sigma \text{ of } \frac{d^2\sigma}{du^2} = \lambda(u) \frac{d\sigma}{du} \right)$$

$$\Rightarrow \text{if } \frac{D}{D\sigma} V^M = 0, \text{ then } \frac{D}{D\sigma} (V^M \xi_M) = 0$$

$$\text{in particular, } \frac{D}{D\sigma} (\xi^M \xi_M) = 0$$

$$\Rightarrow \text{if } \frac{D}{D\sigma} V^M = 0, \text{ then } \frac{D}{D\sigma} (V^M \xi_M) = 0$$

In particular, $\frac{D}{D\sigma} (\xi^M \xi_M) = 0$

$$= \frac{d}{d\sigma} (\xi^M \xi_M) = \frac{d}{d\sigma} (g_{\mu\nu} \xi^\mu \xi^\nu)$$

ie. $\xi^M \xi_M g_{\mu\nu}$ is a constant of the motion

by

$$\Rightarrow \text{if } \frac{D}{D\sigma} V^M = 0, \text{ then } \frac{D}{D\sigma} (V^M \xi_\mu) = 0$$

$$\text{In particular, } \frac{D}{D\sigma} (\xi^M \xi_\mu) = 0$$

$$= \frac{d}{d\sigma} (\xi^M \xi_\mu) = \frac{d}{d\sigma} (g_{\mu\nu} \xi^\mu \xi^\nu)$$

i.e. $\xi^\mu \xi^\nu g_{\mu\nu}$ is a constant of the motion

by linear rescaling of σ , can choose this to be ± 1 or 0 .

$$\Rightarrow \text{if } \frac{D}{D\sigma} V^M = 0, \text{ then } \frac{D}{D\sigma} (V^M \xi_\mu) = 0$$

$$\text{In particular, } \frac{D}{D\sigma} (\xi^\mu \xi_\mu) = 0$$

$$= \frac{d}{d\sigma} (\xi^\mu \xi_\mu) = \frac{d}{d\sigma} (g_{\mu\nu} \xi^\mu \xi^\nu)$$

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$$\Rightarrow \text{if } \frac{D}{D\sigma} V^M = 0, \text{ then } \frac{D}{D\sigma} (V^M \xi_M) = 0$$

timelike

$$g_{\mu\nu} \xi^\mu \xi^\nu = -1$$

$$\text{in particular, } \frac{D}{D\sigma} (\xi^M \xi_M) = 0$$

$$= \frac{d}{d\sigma} (\xi^M \xi_M) = \frac{d}{d\sigma} (g_{\mu\nu} \xi^\mu \xi^\nu)$$

ie. $\xi^M \xi_M$ is a constant of the motion

by linear rescaling of σ , can choose this to be ± 1 or 0 .

$$\sigma' = a\sigma + b \Rightarrow \text{if } \frac{D}{D\sigma} V^M = 0, \text{ then } \frac{D}{D\sigma} (V^M \xi_M) = 0$$

take

$$g^{\mu\nu} \xi_\mu \xi_\nu = -1$$

In particular, $\frac{D}{D\sigma} (\xi^M \xi_M) = 0$
 $= \frac{d}{d\sigma} (\xi^M \xi_M) = \frac{d}{d\sigma} (g_{\mu\nu} \xi^\mu \xi^\nu)$
 i.e. $\xi^\mu \xi^\nu g_{\mu\nu}$ is a constant of the motion

by linear rescaling of σ , can choose this to be ± 1 or 0 .

$$\sigma' = a\sigma + b \Rightarrow \text{if } \frac{D}{D\sigma} V^M = 0, \text{ then } \frac{D}{D\sigma} (V^M \xi_M) = 0$$

timelike

$$g_{\mu\nu} \xi^\mu \xi^\nu = -1$$

$$\begin{aligned} \text{In particular, } \frac{D}{D\sigma} (\xi^M \xi_M) &= 0 \\ &= \frac{d}{d\sigma} (\xi^M \xi_M) = \frac{d}{d\sigma} (g_{\mu\nu} \xi^\mu \xi^\nu) \end{aligned}$$

ie. $\xi^\mu \xi^\nu g_{\mu\nu}$ is a constant of the motion

by linear rescaling of σ , can choose this to be ± 1 or 0 .

$$\sigma' = a\sigma + b \Rightarrow \text{if } \frac{D}{D\sigma} V^M = 0, \text{ then } \frac{D}{D\sigma} (V^M \xi_M) = 0$$

timelike

$$g_{\mu\nu} \xi^\mu \xi^\nu = -1$$

null

$$g_{\mu\nu} \xi^\mu \xi^\nu = 0$$

In particular, $\frac{D}{D\sigma} (\xi^\mu \xi_\mu) = 0$

$$= \frac{d}{d\sigma} (\xi^\mu \xi_\mu) = \frac{d}{d\sigma} (g_{\mu\nu} \xi^\mu \xi^\nu)$$

ie. $\xi^\mu \xi^\nu g_{\mu\nu}$ is a constant of the motion

by linear rescaling of σ , can choose this to be ± 1 or 0 .

$$\sigma' = a\sigma + b \Rightarrow \text{if } \frac{D}{D\sigma} V^M = 0, \text{ then } \frac{D}{D\sigma} (V^M \xi_M) = 0$$

timelike

$$g_{\mu\nu} \xi^\mu \xi^\nu = -1$$

null

$$g_{\mu\nu} \xi^\mu \xi^\nu = 0$$

in particular, $\frac{D}{D\sigma} (\xi^M \xi_M) = 0$
 $= \frac{d}{d\sigma} (\xi^M \xi_M) = \frac{d}{d\sigma} (g_{\mu\nu} \xi^\mu \xi^\nu)$

ie. $\xi^\mu \xi^\nu g_{\mu\nu}$ is a constant of the motion

by linear rescaling of σ , can choose this to be ± 1 or 0 .

$$\sigma' = a\sigma + b \Rightarrow \text{if } \frac{D}{D\sigma} V^M = 0, \text{ then } \frac{D}{D\sigma} (V^M \xi_M) = 0$$

timelike

$$g_{\mu\nu} \xi^\mu \xi^\nu = -1$$

null

$$g_{\mu\nu} \xi^\mu \xi^\nu = 0$$

in particular, $\frac{D}{D\sigma} (\xi^M \xi_M) = 0$

$$= \frac{d}{d\sigma} (\xi^M \xi_M) = \frac{d}{d\sigma} (g_{\mu\nu} \xi^\mu \xi^\nu)$$

ie. $\xi^\mu \xi^\nu g_{\mu\nu}$ is a constant of the motion

by linear rescaling of σ , can choose this to be ± 1 or 0 .

by using
 $\underline{\sigma' = a\sigma + b}$

$\Rightarrow$ if $\frac{D}{D\sigma} V^M = 0$, then $\frac{D}{D\sigma} (V^M \xi_\mu) = 0$

timelike

$\Rightarrow g_{\mu\nu} \xi^\mu \xi^\nu = -1$

null

$(g_{\mu\nu} \xi^\mu \xi^\nu = 0)$

spacelike

$g_{\mu\nu} \xi^\mu \xi^\nu = +1$

In particular, $\frac{D}{D\sigma} (\xi^\mu \xi_\mu) = 0$

$= \frac{d}{d\sigma} (\xi^\mu \xi_\mu) = \frac{d}{d\sigma} (g_{\mu\nu} \xi^\mu \xi^\nu)$

is $(\xi^\mu \xi^\nu g_{\mu\nu})$ is a constant of the motion

by linear rescaling of σ , can choose this to be ± 1 or 0

$$\frac{dx^\mu}{ds}$$

$$g_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = -1$$

$$\frac{dx^\mu}{d\sigma}$$

$$g_{\mu\nu} \frac{dx^\mu}{d\sigma} \frac{dx^\nu}{d\sigma} = -1$$

$$\frac{dx^\mu}{ds}$$

$$g_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = -1$$



$$\frac{dx^\mu}{ds}$$

$$g_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = -1$$

↓ local inertial coords at P

$$\eta_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} = -1 \implies dt = dt \sqrt{1 - \left(\frac{dx}{dt}\right)^2}$$

A man with glasses, wearing a maroon long-sleeved shirt, is standing on the left side of a large chalkboard. He is holding a piece of chalk in his right hand and is in the process of writing on the board. The chalkboard is dark green and has some faint, circular chalk marks from previous use. The man is looking towards the board as he writes.
$$\nabla_{\mu} T^{\mu}$$

Equation of Geodesic Deviation

Equation of Geodesic Deviation



Equation of Geodesic Deviation

Spin S^M

4-velocity $\xi^M = \frac{dx^M}{ds}$

$g_{\mu\nu}$



Equation of Geodesic Deviation

Spin S^M

4-velocity $\xi^M = \frac{dx^M}{ds}$

$g_{\mu\nu} \xi^\mu \xi^\nu = 1$



Equation of Geodesic Deviation

Spin S^M

4-velocity $\xi^M = \frac{dx^M}{ds}$

$$g_{\mu\nu} \xi^M \frac{dx^M}{ds} = 0$$

Equation of Geodesic Deviation

Spin S^M

locally in frame

$$4\text{-velocity } \xi^M = \frac{dx^M}{ds} = (1, \underline{0})$$

$$g_{\mu\nu} \xi^M \frac{dx^M}{ds} = 0$$

Equation of Geodesic Deviation

Spin S^M

locally in frame

4-velocity $\xi^M = \frac{dx^M}{d\sigma} = (1, \underline{0})$

$$\partial_{\mu\nu} \xi^M \frac{dx^M}{d\sigma} = 0$$

$$g_{\mu\nu} \xi^M \xi^N = -1$$

Equation of Geodesic Deviation

Spin S^M

locally in frame

4-velocity $\xi^M = \frac{dx^M}{ds} = (1, \underline{0})$

$$g_{\mu\nu} \xi^M \frac{dx^M}{ds} = 0$$

$$g_{\mu\nu} \xi^M \xi^V = -1$$

Equation of Geodesic Deviation

Spin S^M

locally in frame

$$\text{4-velocity } \xi^M = \frac{dx^M}{d\sigma} = (1, \underline{0})$$

$$g_{\mu\nu} S^{\mu} \frac{dx^{\nu}}{d\sigma} = 0$$

$$\Rightarrow S^0 = 0$$

$$g_{\mu\nu} \xi^{\mu} \xi^{\nu} =$$

Equation of Geodesic Deviation

Spin S^M

locally in frame

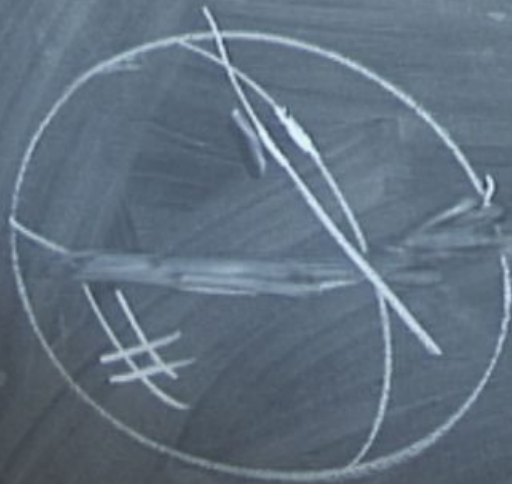
4-velocity $\xi^M = \frac{dx^M}{d\sigma} = (1, \underline{0})$

$$g_{\mu\nu} \xi^M \frac{dx^M}{d\sigma} = 0$$

$$\Rightarrow S^0 = 0$$

$$g_{\mu\nu} \xi^M \xi^V = -1$$

Equation of Geodesic Deviation



Equation of Geodesic Deviation



$$\frac{d^2 x^M}{d\tau^2} + \Gamma_{\nu\lambda}^M \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

Equation of Geodesic Deviation

geodesic
eqr.

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

I consider a nearby geodesic, coords $x^\mu + \delta x^\mu(\tau)$

Equation of Geodesic Deviation

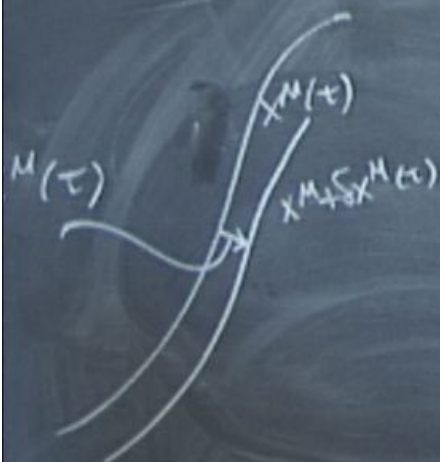
$x^M(\tau)$
 $x^M + \delta x^M(\tau)$

geodesic
eq^r

$$\frac{d^2 x^M}{d\tau^2} + \Gamma^M_{\nu\lambda} \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

I consider a nearby geodesic, coords $x^M + \delta x^M(\tau)$

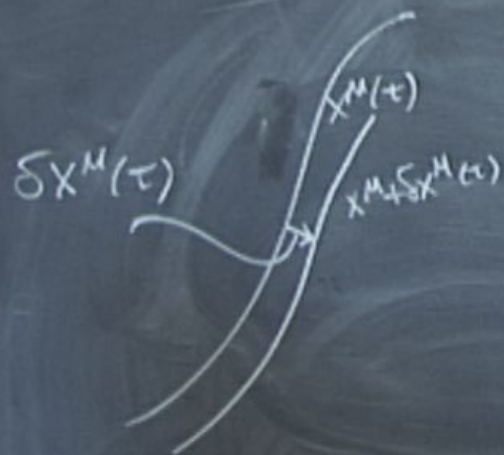
Equation of Geodesic Deviation



geodesic eqr.
$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\lambda} \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

I consider a nearby geodesic, coords $x^\mu + \delta x^\mu(\tau)$,

Equation of Geodesic Deviation



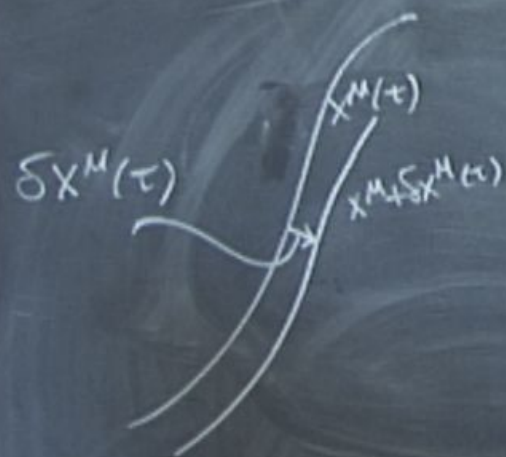
geodesic eq.

$$\frac{d^2 x^M}{d\tau^2} + \Gamma^M_{\nu\lambda} \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

consider a nearby geodesic, coords $x^M + \delta x^M(\tau)$

obeys:
$$\frac{d^2}{d\tau^2} [x^M + \delta x^M] + \Gamma^M_{\nu\lambda} \frac{d}{d\tau} (x^\nu + \delta x^\nu) \frac{d}{d\tau} (x^\lambda + \delta x^\lambda) = 0$$

Equation of Geodesic Deviation

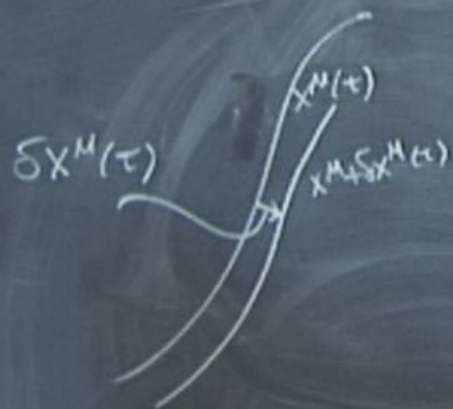


geodesic eq^r $\frac{d^2 x^M}{d\tau^2} + \Gamma_{\nu\lambda}^M \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$

I consider a nearby geodesic, coords $x^M + \delta x^M(\tau)$

obeys: $\frac{d^2}{d\tau^2} [x^M + \delta x^M] + \Gamma_{\nu\lambda}^M \frac{d}{d\tau} (x^\nu + \delta x^\nu) \frac{d}{d\tau} (x^\lambda + \delta x^\lambda) =$

Equation of Geodesic Deviation

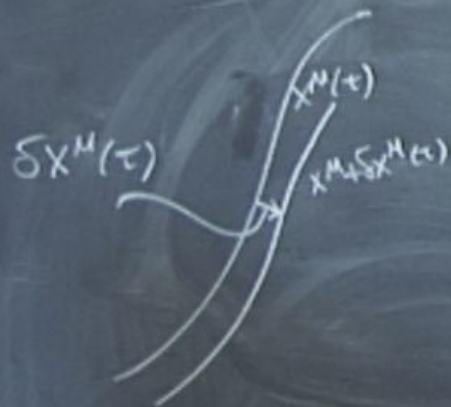


geodesic eq.
$$\frac{d^2 x^M}{d\tau^2} + \Gamma_{\nu\lambda}^M \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

I consider a nearby geodesic, coords $x^M + \delta x^M(\tau)$

obeys:
$$\frac{d^2}{d\tau^2} [x^M + \delta x^M] + \Gamma_{\nu\lambda}^M \frac{d}{d\tau} (x^\nu + \delta x^\nu) \frac{d}{d\tau} (x^\lambda + \delta x^\lambda) = 0$$

Equation of Geodesic Deviation

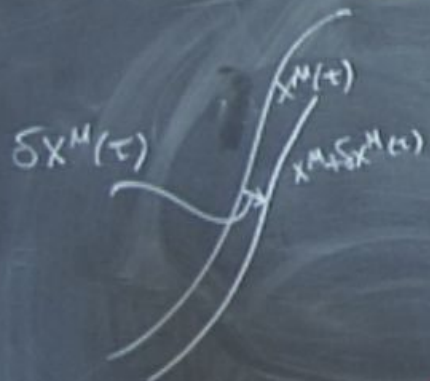


geodesic eq.
$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

I consider a nearby geodesic, coords $x^\mu + \delta x^\mu(\tau)$

obeys:
$$\frac{d^2}{d\tau^2} [x^\mu + \delta x^\mu] + \Gamma_{\nu\lambda}^\mu \frac{d}{d\tau} (x^\nu + \delta x^\nu) \frac{d}{d\tau} (x^\lambda + \delta x^\lambda) = 0$$

Equation of Geodesic Deviation

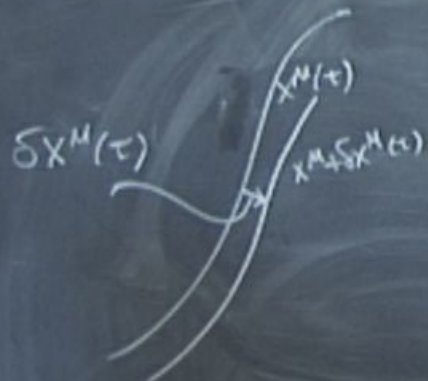


geodesic:
$$\frac{d^2 X^M}{d\tau^2} + \Gamma_{\nu\lambda}^M \frac{dX^\nu}{d\tau} \frac{dX^\lambda}{d\tau} = 0 \quad (1)$$

x nearby geodesic, coords $X^M + \delta X^M(\tau)$

$$\frac{d^2}{d\tau^2} [X^M + \delta X^M] + \Gamma_{\nu\lambda}^M \frac{d}{d\tau} (X^\nu + \delta X^\nu) \frac{d}{d\tau} (X^\lambda + \delta X^\lambda) = 0 \quad (2)$$

Equation of Geodesic Deviation



geodesic eqr.
$$\frac{d^2 x^M}{d\tau^2} + \Gamma_{\nu\lambda}^M \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0 \quad (1)$$

I consider a nearby geodesic, coords $x^M + \delta x^M(\tau)$

obeys:
$$\frac{d^2}{d\tau^2} [x^M + \delta x^M] + \Gamma_{\nu\lambda}^M \left| \frac{d}{d\tau} (x^\nu + \delta x^\nu) \right| \frac{d}{d\tau} (x^\lambda + \delta x^\lambda) = 0 \quad (2)$$

(2)-(1)
to first order
in δx^M

Equation of Geodesic Deviation

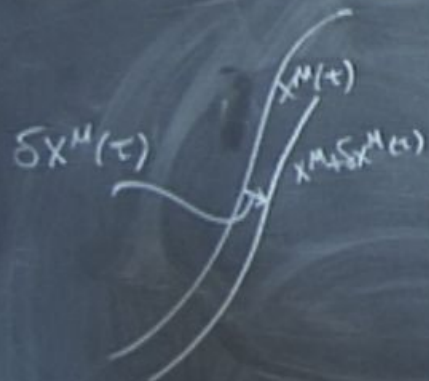
geodesic eqr. $\frac{d^2 x^M}{d\tau^2} + \Gamma_{\nu\lambda}^M \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0 \quad (1)$

I consider a nearby geodesic, coords $x^M_{(0)} + \delta x^M(\tau)$

obeys: $\frac{d^2}{d\tau^2} [x^M + \delta x^M] + \Gamma_{\nu\lambda}^M \frac{d}{d\tau} (x^\nu + \delta x^\nu) \frac{d}{d\tau} (x^\lambda + \delta x^\lambda) = 0$

(1) $\frac{d^2}{d\tau^2} \delta x^M$

Equation of Geodesic Deviation



geodesic

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0 \quad (1)$$

Consider a nearby geodesic, coords $x^\mu + \delta x^\mu(\tau)$

$$\frac{d^2}{d\tau^2} [x^\mu + \delta x^\mu] + \Gamma_{\nu\lambda}^\mu \frac{d}{d\tau} (x^\nu + \delta x^\nu) \frac{d}{d\tau} (x^\lambda + \delta x^\lambda) = 0 \quad (2)$$

$$\frac{d^2}{d\tau^2} \delta x^\mu + \Gamma_{\nu\lambda, \rho}^\mu \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau}$$

Equation of Geodesic Deviation

geodesic eqr. $\frac{d^2 x^M}{d\tau^2} + \Gamma_{\nu\lambda}^M \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0 \quad (1)$

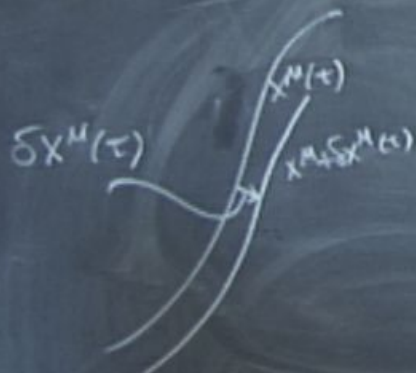
I consider a nearby geodesic, coords $x^M + \delta x^M(\tau)$

obeys: $\frac{d^2}{d\tau^2} [x^M + \delta x^M] + \Gamma_{\nu\lambda}^M \frac{d}{d\tau} (x^\nu + \delta x^\nu) \frac{d}{d\tau} (x^\lambda + \delta x^\lambda) = 0$

(2)-(1)
to first order
in δx^M

$$\frac{d^2}{d\tau^2} \delta x^M + \Gamma_{\nu\lambda, \rho}^M \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + 2 \Gamma_{\nu\lambda}^M \frac{d}{d\tau} \delta x^\nu \frac{dx^\lambda}{d\tau}$$

Equation of Geodesic Deviation



geodesic eq.
$$\frac{d^2 X^M}{d\tau^2} + \Gamma_{\nu\lambda}^M \frac{dX^\nu}{d\tau} \frac{dX^\lambda}{d\tau} = 0 \quad (1)$$

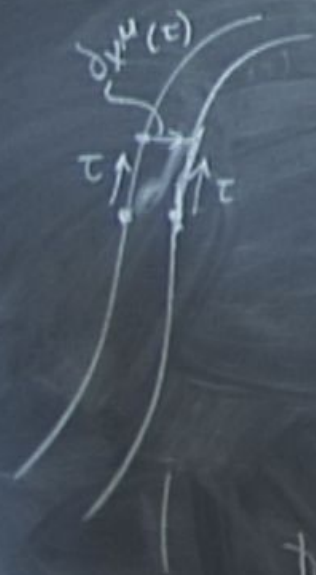
I consider a nearby geodesic, coords $X_{(1)}^M + \delta X^M(\tau)$

obeys:
$$\frac{d^2}{d\tau^2} [X^M + \delta X^M] + \Gamma_{\nu\lambda}^M \frac{d}{d\tau} (X^\nu + \delta X^\nu) \frac{d}{d\tau} (X^\lambda + \delta X^\lambda) = 0 \quad (2)$$

(2)-(1)
to first order
in δX^M

$$\frac{d^2}{d\tau^2} \delta X^M + \Gamma_{\nu\lambda, \rho}^M \delta X^\rho \frac{dX^\nu}{d\tau} \frac{dX^\lambda}{d\tau} + 2 \Gamma_{\nu\lambda}^M \frac{d\delta X^\nu}{d\tau} \frac{dX^\lambda}{d\tau} = 0$$

Equation of Geodesic Deviation



geodesic eqr.
$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0 \quad (1)$$

consider a nearby geodesic, coords $x^\mu + \delta x^\mu(\tau)$

obeys:
$$\frac{d^2}{d\tau^2} [x^\mu + \delta x^\mu] + \Gamma_{\nu\lambda}^\mu \frac{d}{d\tau} (x^\nu + \delta x^\nu) \frac{d}{d\tau} (x^\lambda + \delta x^\lambda) = 0 \quad (2)$$

(2)-(1)
to first order
in δx^μ

$$\frac{d^2 \delta x^\mu}{d\tau^2} + \Gamma_{\nu\lambda, \rho}^\mu \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + 2 \Gamma_{\nu\lambda}^\mu \frac{d}{d\tau} \delta x^\nu \frac{dx^\lambda}{d\tau} = 0$$

Equation of Geodesic Deviation

$$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$$

geodesic egr.

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0 \quad (1)$$

consider a nearby geodesic, coords $x^\mu + \delta x^\mu(\tau)$

obeys:

$$\frac{d^2}{d\tau^2} [x^\mu + \delta x^\mu] + \Gamma_{\nu\lambda}^\mu \frac{d}{d\tau} (x^\nu + \delta x^\nu) \frac{d}{d\tau} (x^\lambda + \delta x^\lambda) = 0 \quad (2)$$

order

$$\frac{d^2}{d\tau^2} \delta x^\mu + \Gamma_{\nu\lambda,\rho}^\mu \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + 2 \Gamma_{\nu\lambda}^\mu \frac{d}{d\tau} \delta x^\nu \frac{dx^\lambda}{d\tau} = 0$$

$$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1 \quad \text{try} \quad \frac{D^2 \delta x^\mu}{D\tau^2} = \frac{D}{D\tau} \left[\frac{d \delta x^\mu}{d\tau} + \Gamma_{\nu\lambda}^\mu \delta x^\nu \frac{dx^\lambda}{d\tau} \right]$$

$$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1 \quad \text{from} \quad \frac{D^2 \delta x^\mu}{D\tau^2} = \frac{D}{D\tau} \left[\frac{d \delta x^\mu}{d\tau} + \Gamma_{\nu\lambda}^\mu \delta x^\nu \frac{dx^\lambda}{d\tau} \right]$$

$$\frac{d^2 \delta x^\mu}{d\tau^2}$$

$$\delta x^\mu(\tau)$$

$$(x^\lambda + \delta x^\lambda) = 0$$

$$\frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} =$$

$$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1 \quad \text{try} \quad \frac{D^2 \delta x^M}{D\tau^2} = \frac{D}{D\tau} \left[\frac{d \delta x^M}{d\tau} + \Gamma_{\nu\lambda}^M \delta x^\nu \frac{dx^\lambda}{d\tau} \right]$$

$$= \frac{d^2 \delta x^M}{d\tau^2} + \Gamma_{\nu\lambda, \alpha}^M \frac{dx^\alpha}{d\tau} \delta x^\nu \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \frac{d^2 x^\lambda}{d\tau^2} \delta x^\nu$$

$$\delta \dot{x}^M(\tau)$$

$$(x^\lambda + \delta x^\lambda) = 0 \quad (2)$$

$$\frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0 \quad \left. \vphantom{\frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau}} \right\} \text{not tensorial.}$$

$$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1 \quad \text{try} \quad \frac{D^2 \delta x^M}{D\tau^2} = \frac{D}{D\tau} \left[\frac{d \delta x^M}{d\tau} + \Gamma_{\nu\lambda}^M \delta x^\nu \frac{dx^\lambda}{d\tau} \right]$$

$$= \frac{d^2 \delta x^M}{d\tau^2} + \Gamma_{\nu\lambda, \alpha}^M \frac{dx^\alpha}{d\tau} \delta x^\nu \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \frac{d^2 x^\lambda}{d\tau^2} \delta x^\nu$$

$$\delta \dot{x}^M(\tau)$$

$$x^\lambda + \delta x^\lambda = 0 \quad (2)$$

$$+ \Gamma_{\alpha\beta}^M \frac{d \delta x^\alpha}{d\tau} \frac{dx^\beta}{d\tau}$$

$$\left(\delta x^\nu \frac{dx^\lambda}{d\tau} = 0 \right) \text{ not tensorial.}$$

$$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1 \quad \text{try} \quad \frac{D^2 \delta x^M}{D\tau^2} = \frac{D}{D\tau} \left[\frac{d \delta x^M}{d\tau} + \Gamma_{\nu\lambda}^M \delta x^\nu \frac{dx^\lambda}{d\tau} \right]$$

$$= \frac{d^2 \delta x^M}{d\tau^2} + \Gamma_{\nu\lambda, \alpha}^M \frac{dx^\alpha}{d\tau} \delta x^\nu \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \frac{d^2 x^\lambda}{d\tau^2} \delta x^\nu$$

$$+ \delta x^M(\tau)$$

$$(x^\lambda + \delta x^\lambda) = 0 \quad (2)$$

$$+ \Gamma_{\alpha\beta}^M \frac{d \delta x^\alpha}{d\tau} \frac{dx^\beta}{d\tau} + \Gamma_{\alpha\beta}^M \Gamma_{\nu\lambda}^\alpha \delta x^\nu \frac{dx^\lambda}{d\tau} \frac{dx^\beta}{d\tau}$$

$$\frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0 \quad \left. \vphantom{\frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0} \right\} \text{not tensorial.}$$

(1)

$D\tau^2$

$$= \frac{d^2 \delta x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\lambda} \frac{d\delta x^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau}$$

$$- \Gamma^\mu_{\nu\lambda, \rho} \delta x^\rho \frac{d\delta x^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau} - 2 \Gamma^\mu_{\nu\lambda} \frac{d\delta x^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau}$$

$$+ \Gamma^\mu_{\alpha\beta} \frac{d\delta x^\alpha}{d\tau} \frac{d\delta x^\beta}{d\tau}$$

$$+ 2 \Gamma^\mu_{\nu\lambda} \frac{d\delta x^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau} = 0 \quad \left. \vphantom{\frac{d\delta x^\nu}{d\tau}} \right\} \text{not tensorial.}$$

Equation of Geodesic Deviation

geodesic eq. $\frac{d^2 X^M}{d\tau^2} + \Gamma_{\nu\lambda}^M \frac{dX^\nu}{d\tau} \frac{dX^\lambda}{d\tau} = 0 \quad (1)$

I consider a nearby geodesic

obeys: $\frac{d^2}{d\tau^2} [X^M + \delta X^M]$

(2) = (1)
to first order
in δX^M

$$\frac{d^2 \delta X^M}{d\tau^2} + \Gamma_{\nu\lambda,\rho}^M \delta X^\rho \frac{dX^\nu}{d\tau} \frac{dX^\lambda}{d\tau} + 2\Gamma_{\nu\lambda}^M \frac{dX^\nu}{d\tau} \frac{d\delta X^\lambda}{d\tau} = 0 \quad \text{not tensorial.}$$

$$g_{\mu\nu} \frac{dX^\mu}{d\tau} \frac{dX^\nu}{d\tau} = -1 \quad \text{try} \quad \frac{D^2 \delta X^M}{D\tau^2} = \frac{D}{D\tau} \left[\frac{d}{d\tau} \delta X^M + \Gamma_{\nu\lambda}^M \delta X^\nu \frac{dX^\lambda}{d\tau} \right]$$

$$\begin{aligned} &= \frac{d^2 \delta X^M}{d\tau^2} + \Gamma_{\nu\lambda,\rho}^M \delta X^\rho \frac{dX^\nu}{d\tau} \frac{dX^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \frac{d\delta X^\nu}{d\tau} \frac{dX^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \delta X^\nu \frac{d^2 X^\lambda}{d\tau^2} \\ &\quad - \Gamma_{\nu\lambda,\rho}^M \delta X^\rho \frac{dX^\nu}{d\tau} \frac{dX^\lambda}{d\tau} - 2\Gamma_{\nu\lambda}^M \frac{d\delta X^\nu}{d\tau} \frac{dX^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \frac{d\delta X^\nu}{d\tau} \frac{dX^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \delta X^\nu \frac{d^2 X^\lambda}{d\tau^2} \end{aligned}$$

Equation of Geodesic Deviation

geodesic eq. $\frac{d^2 x^M}{d\tau^2} + \Gamma_{\nu\lambda}^M \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$ (1)

I consider a nearby geodesic

obeys: $\frac{d^2}{d\tau^2} [x^M + \delta x^M]$

(2)-(1)
to first order
in δx^M

$$\frac{d^2 \delta x^M}{d\tau^2} + \Gamma_{\nu\lambda, \rho}^M \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + 2\Gamma_{\nu\lambda}^M \frac{dx^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau} = 0$$

not
tensorial.

$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$ try $\frac{D^2 \delta x^M}{D\tau^2} = \frac{D}{D\tau} \left[\frac{d}{d\tau} \delta x^M + \Gamma_{\nu\lambda}^M \delta x^\nu \frac{dx^\lambda}{d\tau} \right]$

$$= \frac{d^2 \delta x^M}{d\tau^2} + \Gamma_{\nu\lambda, \rho}^M \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \frac{d\delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \delta x^\nu \frac{d^2 x^\lambda}{d\tau^2}$$

Equation of Geodesic Deviation

geodesic eq: $\frac{d^2 x^M}{d\tau^2} + \Gamma_{\nu\lambda}^M \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$ (1)

I consider a nearby geodesic

obeys: $\frac{d^2}{d\tau^2} [x^M + \delta x^M]$

(2)-(1)
to first order
in δx^M

$$\frac{d^2 \delta x^M}{d\tau^2} + \Gamma_{\nu\lambda, \rho}^M \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + 2\Gamma_{\nu\lambda}^M \frac{dx^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau} = 0$$

not
tensorial.

$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$ try $\frac{D^2 \delta x^M}{D\tau^2} = \frac{D}{D\tau} \left[\frac{d}{d\tau} \delta x^M + \Gamma_{\nu\lambda}^M \delta x^\nu \frac{dx^\lambda}{d\tau} \right]$

$$= \frac{d^2 \delta x^M}{d\tau^2} + \Gamma_{\nu\lambda, \rho}^M \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + 2\Gamma_{\nu\lambda}^M \frac{dx^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau}$$

Equation of Geodesic Deviation

geodesic eq. $\frac{d^2 x^M}{d\tau^2} + \Gamma_{\nu\lambda}^M \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$ (1)

I consider a nearby geodesic

obeys: $\frac{d^2}{d\tau^2} [x^M + \delta x^M]$

(2)-(1)
to first order
in δx^M

$$\frac{d^2 \delta x^M}{d\tau^2} + \Gamma_{\nu\lambda, \rho}^M \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + 2\Gamma_{\nu\lambda}^M \frac{d\delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$ try $\frac{D^2 \delta x^M}{D\tau^2} = \frac{D}{D\tau} \left[\frac{d}{d\tau} \delta x^M + \Gamma_{\nu\lambda}^M \delta x^\nu \frac{dx^\lambda}{d\tau} \right]$

$$\begin{aligned} &= \frac{d^2 \delta x^M}{d\tau^2} + \Gamma_{\nu\lambda, \rho}^M \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \frac{d\delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \delta x^\nu \frac{d^2 x^\lambda}{d\tau^2} \\ &\quad - \Gamma_{\nu\lambda, \rho}^M \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} - 2\Gamma_{\nu\lambda}^M \frac{d\delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \frac{d\delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \delta x^\nu \frac{d^2 x^\lambda}{d\tau^2} \end{aligned}$$

$$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1 \quad \text{try} \quad \frac{D^2 \delta x^\mu}{D\tau^2} = \frac{D}{D\tau} \left[\frac{d \delta x^\mu}{d\tau} + \Gamma_{\nu\lambda}^\mu \delta x^\nu \frac{dx^\lambda}{d\tau} \right]$$

$$\begin{aligned}
 &= \cancel{\frac{d^2 \delta x^\mu}{d\tau^2}} + \Gamma_{\nu\lambda, \alpha}^\mu \frac{dx^\alpha}{d\tau} \delta x^\nu \frac{dx^\lambda}{d\tau} + \cancel{\Gamma_{\nu\lambda}^\mu \frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau}} + \Gamma_{\nu\lambda}^\mu \frac{d^2 x^\lambda}{d\tau^2} \\
 &\quad - \Gamma_{\nu\lambda, \rho}^\mu \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} - \cancel{2 \Gamma_{\nu\lambda}^\mu \frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau}} + \Gamma_{\alpha\beta}^\mu \frac{d \delta x^\alpha}{d\tau} \frac{dx^\beta}{d\tau} + \Gamma_{\alpha\beta}^\mu \Gamma_{\nu\lambda}^\alpha \delta x^\nu \frac{dx^\lambda}{d\tau} \frac{dx^\beta}{d\tau}
 \end{aligned}$$

$$\frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0 \quad \left. \begin{array}{l} \text{not} \\ \text{tensorial.} \end{array} \right\}$$

$$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1 \quad \text{try} \quad \frac{D^2 \delta x^\mu}{D\tau^2} = \frac{D}{D\tau} \left[\frac{d \delta x^\mu}{d\tau} + \Gamma_{\nu\lambda}^\mu \delta x^\nu \frac{dx^\lambda}{d\tau} \right]$$

$$= \frac{d^2 \delta x^\mu}{d\tau^2} + \Gamma_{\nu\lambda,\alpha}^\mu \frac{dx^\alpha}{d\tau} \delta x^\nu \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^\mu \frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^\mu \frac{d^2 x^\lambda}{d\tau^2} \delta x^\nu$$

$$- \Gamma_{\nu\lambda,\rho}^\mu \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} - 2 \Gamma_{\nu\lambda}^\mu \frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\alpha\beta}^\mu \frac{d \delta x^\alpha}{d\tau} \frac{dx^\beta}{d\tau} + \Gamma_{\alpha\beta}^\mu \Gamma_{\nu\lambda}^\alpha \delta x^\nu \frac{dx^\lambda}{d\tau} \frac{dx^\beta}{d\tau}$$

$$\frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0 \quad \left. \vphantom{\frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau}} \right\} \text{not tensorial.}$$

$$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1 \quad \text{try} \quad \frac{D^2 \delta x^\mu}{D\tau^2} = \frac{D}{D\tau} \left[\frac{d \delta x^\mu}{d\tau} + \Gamma_{\nu\lambda}^\mu \delta x^\nu \frac{dx^\lambda}{d\tau} \right]$$

$$= \frac{d^2 \delta x^\mu}{d\tau^2} + \Gamma_{\nu\lambda,\alpha}^\mu \frac{dx^\alpha}{d\tau} \delta x^\nu \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^\mu \frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^\mu \frac{d^2 x^\lambda}{d\tau^2} \delta x^\nu$$

$$- \Gamma_{\nu\lambda,\rho}^\mu \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} - 2 \Gamma_{\nu\lambda}^\mu \frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\alpha\beta}^\mu \frac{d \delta x^\alpha}{d\tau} \frac{dx^\beta}{d\tau} + \Gamma_{\alpha\beta}^\mu \Gamma_{\nu\lambda}^\alpha \delta x^\nu \frac{dx^\lambda}{d\tau} \frac{dx^\beta}{d\tau}$$

$$\frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0 \quad \left. \vphantom{\frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau}} \right\} \text{not tensorial.}$$

$$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1 \quad \text{try}$$

$$\frac{D^2 \delta x^\mu}{D\tau^2} = \frac{D}{D\tau} \left[\frac{d \delta x^\mu}{d\tau} + \Gamma_{\nu\lambda}^\mu \delta x^\nu \frac{dx^\lambda}{d\tau} \right]$$

$$\begin{aligned}
 &= \cancel{\frac{d^2 \delta x^\mu}{d\tau^2}} + \Gamma_{\nu\lambda,\alpha}^\mu \cancel{\frac{dx^\alpha}{d\tau}} \delta x^\nu \frac{dx^\lambda}{d\tau} + \cancel{\Gamma_{\nu\lambda}^\mu \frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau}} \\
 &\quad - \Gamma_{\nu\lambda,\rho}^\mu \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} - 2 \Gamma_{\nu\lambda}^\mu \frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^\mu \frac{d^2 x^\lambda}{d\tau^2} \\
 &\quad + \Gamma_{\alpha\beta}^\mu \frac{d \delta x^\alpha}{d\tau} \frac{dx^\beta}{d\tau} + \Gamma_{\alpha\beta}^\mu \Gamma_{\nu\lambda}^\alpha \delta x^\nu \frac{dx^\lambda}{d\tau} \frac{dx^\beta}{d\tau}
 \end{aligned}$$

$$\frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0 \quad \left. \begin{array}{l} \text{not} \\ \text{tensorial.} \end{array} \right\}$$

$$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1 \quad \text{try} \quad \frac{D^2 \delta x^\mu}{D\tau^2} = \frac{D}{D\tau} \left[\frac{d \delta x^\mu}{d\tau} + \Gamma_{\nu\lambda}^\mu \delta x^\nu \frac{dx^\lambda}{d\tau} \right]$$

$$\begin{aligned}
 &= \frac{d^2 \delta x^\mu}{d\tau^2} + \Gamma_{\nu\lambda, \alpha}^\mu \frac{dx^\alpha}{d\tau} \delta x^\nu \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^\mu \frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} \\
 &\quad + \Gamma_{\nu\lambda}^\mu \frac{d^2 x^\lambda}{d\tau^2} \delta x^\nu \\
 &\quad - \Gamma_{\nu\lambda, \rho}^\mu \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} - 2 \Gamma_{\nu\lambda}^\mu \frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} \\
 &\quad + \Gamma_{\alpha\beta}^\mu \frac{d \delta x^\alpha}{d\tau} \frac{dx^\beta}{d\tau} + \Gamma_{\alpha\beta}^\mu \Gamma_{\nu\lambda}^\alpha \delta x^\nu \frac{dx^\lambda}{d\tau} \frac{dx^\beta}{d\tau}
 \end{aligned}$$

$$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1 \quad \text{try} \quad \frac{D^2 \delta x^M}{D\tau^2} = \frac{D}{D\tau} \left[\frac{d \delta x^M}{d\tau} + \Gamma_{\nu\lambda}^M \delta x^\nu \frac{dx^\lambda}{d\tau} \right]$$

$$= \frac{d^2 \delta x^M}{d\tau^2} + \Gamma_{\nu\lambda, \alpha}^M \frac{dx^\alpha}{d\tau} \delta x^\nu \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \frac{d^2 x^\lambda}{d\tau^2} \delta x^\nu$$

$$- \Gamma_{\nu\lambda, \rho}^M \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} - 2 \Gamma_{\nu\lambda}^M \frac{d \delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\alpha\beta}^M \frac{d \delta x^\alpha}{d\tau} \frac{dx^\beta}{d\tau} + \Gamma_{\alpha\beta}^M \Gamma_{\nu\lambda}^\alpha \delta x^\nu \frac{dx^\lambda}{d\tau} \frac{dx^\beta}{d\tau} \quad (1)$$

$$\Gamma_{\nu\lambda}^M \left(- \Gamma_{\alpha\beta}^\lambda \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} \right) \delta x^\nu$$

$$= \left(-\Gamma^{\mu}_{\nu\lambda,\rho} + \Gamma^{\mu}_{\rho\nu,\lambda} \right) \delta x^{\rho} \dot{x}^{\nu} \dot{x}^{\lambda}$$

$$= \left(- \overset{\textcircled{1}}{\Gamma^{\mu}_{\nu\lambda,\rho}} + \Gamma^{\mu}_{\rho\nu,\lambda} \right) \delta x^{\rho} \dot{x}^{\nu} \ddot{x}^{\lambda}$$

$$\left(\overset{\textcircled{1}}{-\Gamma^{\mu}_{\nu\lambda,\rho}} + \overset{\textcircled{2}}{\Gamma^{\mu}_{\rho\nu,\lambda}} \right) \delta x^{\rho} \dot{x}^{\nu} \dot{x}^{\lambda}$$

$$-\Gamma^{\mu}_{\rho\sigma} \Gamma^{\sigma}_{\nu\lambda}$$

$$\begin{aligned}
 & \textcircled{1} \quad - \Gamma_{\nu\lambda,\rho}^{\mu} + \Gamma_{\rho\nu,\lambda}^{\mu} \\
 & \textcircled{2} \quad \delta x^{\rho} \ddot{x}^{\nu} \dot{x}^{\lambda} \\
 & \textcircled{3} \quad - \Gamma_{\rho\gamma}^{\mu} \Gamma_{\nu\lambda}^{\gamma} + \Gamma_{\gamma\lambda}^{\mu} \Gamma_{\rho\nu}^{\gamma}
 \end{aligned}$$

$$= \left(\begin{array}{c} \textcircled{1} \quad \textcircled{2} \\ -\Gamma_{\nu\lambda,\rho}^{\mu} + \Gamma_{\rho\nu,\lambda}^{\mu} \\ \textcircled{3} \quad -\Gamma_{\rho\sigma}^{\mu} \Gamma_{\nu\lambda}^{\sigma} + \Gamma_{\sigma\lambda}^{\mu} \Gamma_{\rho\nu}^{\sigma} \end{array} \right) \delta x^{\rho} \ddot{x}^{\nu} \dot{x}^{\lambda}$$

$$\boxed{\frac{D^2 \delta x^{\mu}}{D\tau^2} = R^{\mu}_{\nu\lambda\rho} \delta x^{\rho} \frac{dx^{\nu}}{d\tau} \frac{dx^{\lambda}}{d\tau}}$$

$$= \left(\begin{array}{c} \textcircled{1} \quad \textcircled{2} \\ -\Gamma_{\nu\lambda,\rho}^{\mu} + \Gamma_{\rho\nu,\lambda}^{\mu} \\ \textcircled{3} \\ -\Gamma_{\rho\sigma}^{\mu} \Gamma_{\nu\lambda}^{\sigma} + \Gamma_{\sigma\lambda}^{\mu} \Gamma_{\rho\nu}^{\sigma} \end{array} \right) \delta x^{\rho} \ddot{x}^{\nu} \dot{x}^{\lambda}$$

$$\frac{D^2 \delta x^{\mu}}{D\tau^2} = R^{\mu}_{\nu\lambda\rho} \delta x^{\rho} \frac{dx^{\nu}}{d\tau} \frac{dx^{\lambda}}{d\tau}$$

Recovering Newton

Recovering Newton

1 slow-moving particle

$$\left| \frac{dx}{d\tau} \right| \ll \left| \frac{dt}{d\tau} \right|$$

Recovering Newton

1. slow-moving particle

$$\left| \frac{dx}{d\tau} \right| \ll \left| \frac{d(ct)}{d\tau} \right|$$

$$|\underline{v}| = \left| \frac{dx}{dt} \right| \ll c$$

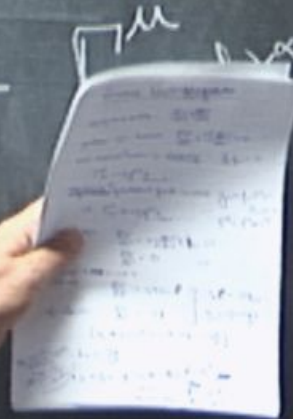
Recovering Newton

$$|v| = \left| \frac{dx}{dt} \right| \ll c$$

slowly moving particle

$$\left| \frac{dx}{d\tau} \right| \ll \left| \frac{d(ct)}{d\tau} \right|$$

→ geo. q becomes $\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}$



Recovering Newton

$$|\underline{v}| = \left| \frac{d\underline{x}}{dt} \right| \ll c$$

slow-moving particle

$$\left| \frac{d\underline{x}}{d\tau} \right| \ll \left| \frac{d(ct)}{d\tau} \right|$$

geodesic eq becomes $\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\alpha\beta}^{\mu} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}$

covering Newton

$$|\underline{v}| = \left| \frac{d\underline{x}}{dt} \right|$$

moving particle

$$\left| \frac{d\underline{x}}{d\tau} \right| < \left| \frac{d(ct)}{d\tau} \right|$$

basic eq becomes $\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{00}^\mu \frac{dx^0}{d\tau} \frac{dx^0}{d\tau} = 0$

$x^0 =$

Recovering Newton

$$|\vec{v}| = \left| \frac{d\vec{x}}{dt} \right| \ll c$$

$$\left| \frac{d\vec{x}}{d\tau} \right| \ll \left| \frac{d(ct)}{d\tau} \right|$$

slow particle

→ geodesic becomes $\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0$ $x^0 = ct$

metric is stationary $\frac{\partial}{\partial t} g_{\mu\nu} = 0$

$$g_{\mu\alpha}$$

Recovering Newton

$$|\vec{v}| = \left| \frac{dx}{dt} \right| \ll c$$

1) slow-moving particle

$$\left| \frac{dx}{d\tau} \right| \ll \left| \frac{d(ct)}{d\tau} \right|$$

→ geodesic eq becomes $\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0$ $x^0 = ct$

assume 1) metric is stationary $\frac{\partial}{\partial t} g_{\mu\nu} = 0$

$$\Gamma_{00}^\mu = \frac{1}{2} g^{\mu\alpha}$$

Recovering Newton

$$|\underline{v}| = \left| \frac{dx}{dt} \right| \ll c$$

1) slow-moving particle

$$\left| \frac{dx}{d\tau} \right| \ll \left| \frac{d(ct)}{d\tau} \right|$$

→ geodesic eq becomes $\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{00}^\mu \frac{dx^0}{d\tau} \frac{dx^0}{d\tau} = 0$ $x^0 = ct$

assume 1) metric is stationary $\frac{\partial}{\partial t} g_{\mu\nu} = 0$

$$\Gamma_{00}^\mu = \frac{1}{2} g^{\mu\alpha} (g_{0\alpha,0} + g_{0\alpha,0} - g_{00,\alpha})$$

Recovering Newton

$$|\vec{v}| = \left| \frac{d\vec{x}}{dt} \right| \ll c$$

slow-moving particle

$$\left| \frac{d\vec{x}}{d\tau} \right| \ll \left| \frac{d(ct)}{d\tau} \right|$$

→ geodesic eq becomes $\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{00}^\mu \frac{dx^0}{d\tau} \frac{dx^0}{d\tau} = 0$ $x^0 = ct$

assume 1) metric is stationary $\frac{\partial}{\partial t} g_{\mu\nu} = 0$

$$\Gamma_{00}^\mu = \frac{1}{2} g^{\mu\alpha} (\cancel{g_{0\alpha,0}} + \cancel{g_{0\alpha,0}} - g_{00,\alpha}) = -\frac{1}{2} g^{\mu\alpha} g_{00,\alpha}$$

Recovering Newton

$$|\underline{v}| = \left| \frac{d\underline{x}}{dt} \right| \ll c$$

slow-moving particle

$$\left| \frac{d\underline{x}}{d\tau} \right| \ll \left| \frac{d(ct)}{d\tau} \right|$$

→ geodesic eq becomes $\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{00}^\mu \frac{dx^0}{d\tau} \frac{dx^0}{d\tau} = 0$ $x^0 = ct$

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$$(g^{\alpha\beta} \simeq \eta^{\alpha\beta} - h^{\alpha\beta} + o(h^2)) \quad |h_{\mu\nu}| \ll 1$$

$$\Rightarrow \Gamma_{00}^M = -\frac{1}{2} \eta^{\mu\alpha} h_{00,\alpha} + o(h^2)$$

geodesic
equation

$$\frac{d^2 x^i}{d\tau^2} = -\Gamma_{00}^i \left(\frac{d(ct)}{d\tau} \right)^2$$

$$\Rightarrow \Gamma_{00}^{\mu} = -\frac{1}{2} \eta^{\mu\alpha} h_{00,\alpha} + o(h^2)$$

geodesic equation $\mu=i$ $\frac{d^2 x^i}{d\tau^2} = -\Gamma_{00}^i \left(\frac{d(ct)}{d\tau} \right)^2 = \frac{1}{2} \nabla_i h_{00} \left(\frac{d(ct)}{d\tau} \right)^2$

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e.g spherical mass, $\phi = -\frac{GM}{r}$

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$$g_{00} = \eta_{00} + h_{00}$$

$$h_{00} = -2\frac{\phi}{c^2}, \quad g_{00} = -\left(1 + \frac{2\phi}{c^2}\right)$$

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