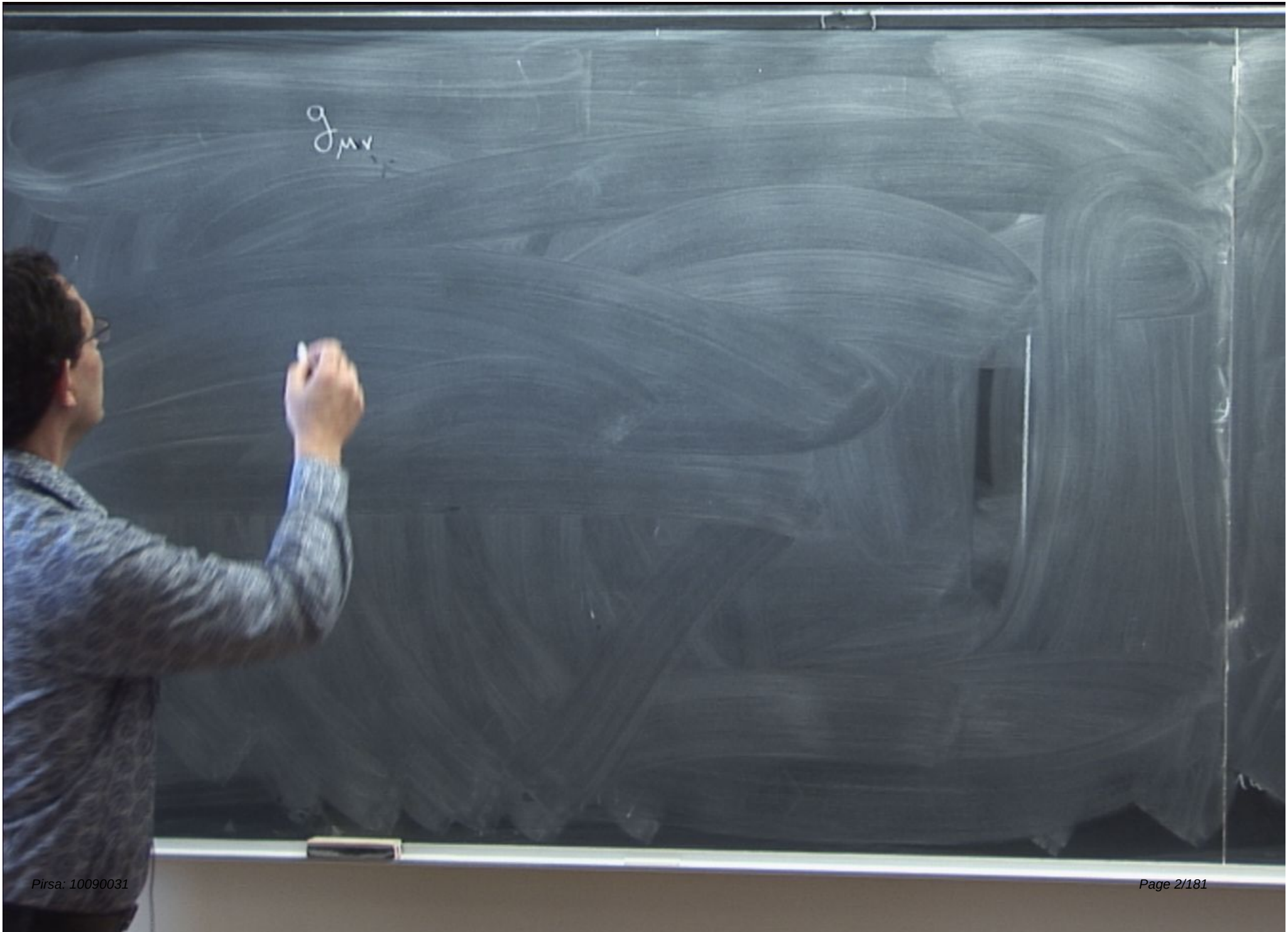
Title: Relativity (PHYS 604) - Lecture 6

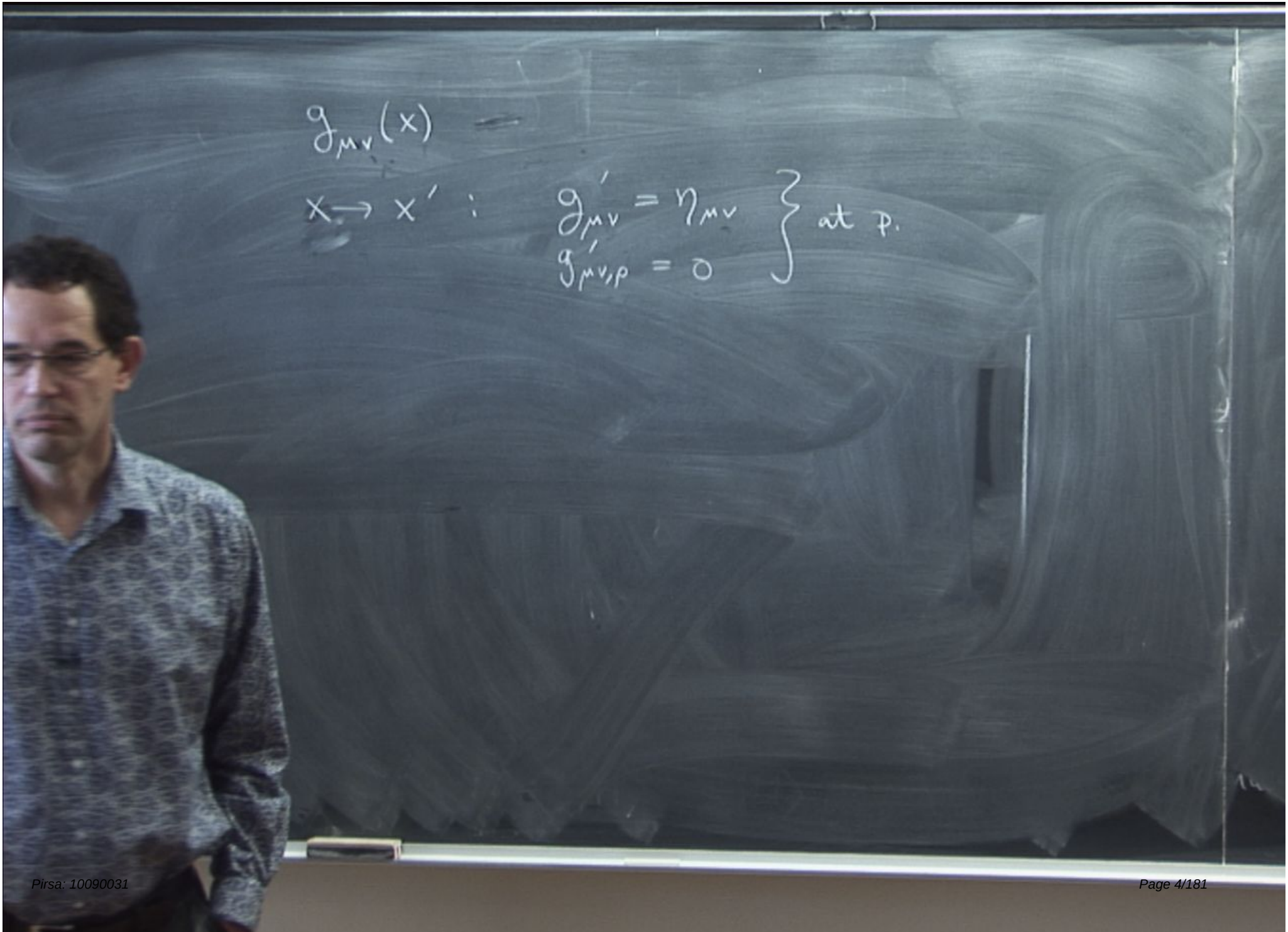
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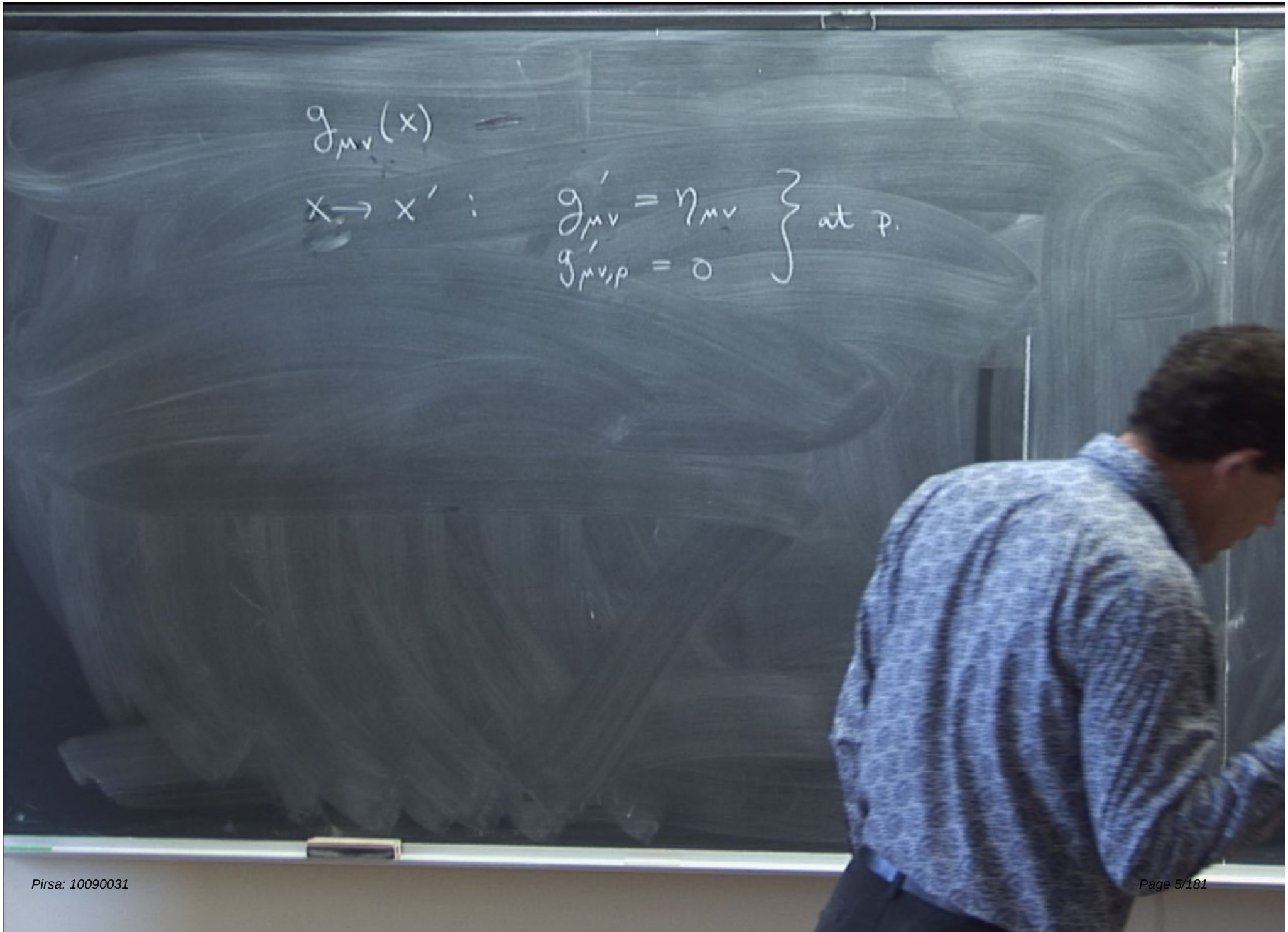
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Abstract:


$g_{\mu\nu}$

$g_{\mu\nu}(x)$ —


$g_{\mu\nu}(x)$
$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$ } at p.
$g'_{\mu\nu,\rho} = 0$ }


$g_{\mu\nu}(x)$
$x \longrightarrow x'$:
$g'_{\mu\nu} = \eta_{\mu\nu}$
$g'_{\mu\nu,\rho} = 0$
at P.

$g_{\mu\nu}(x)$ —

$x \longrightarrow x'$: $\left.\begin{aligned} g'_{\mu\nu} &= \eta_{\mu\nu} \\ g'_{\mu\nu,\rho} &= 0 \end{aligned}\right\}$ at P.

$$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$$

$g_{\mu\nu}(x)$ —

$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$ } at P.
$g'_{\mu\nu,\rho} = 0$ }

$x'^{\mu}($

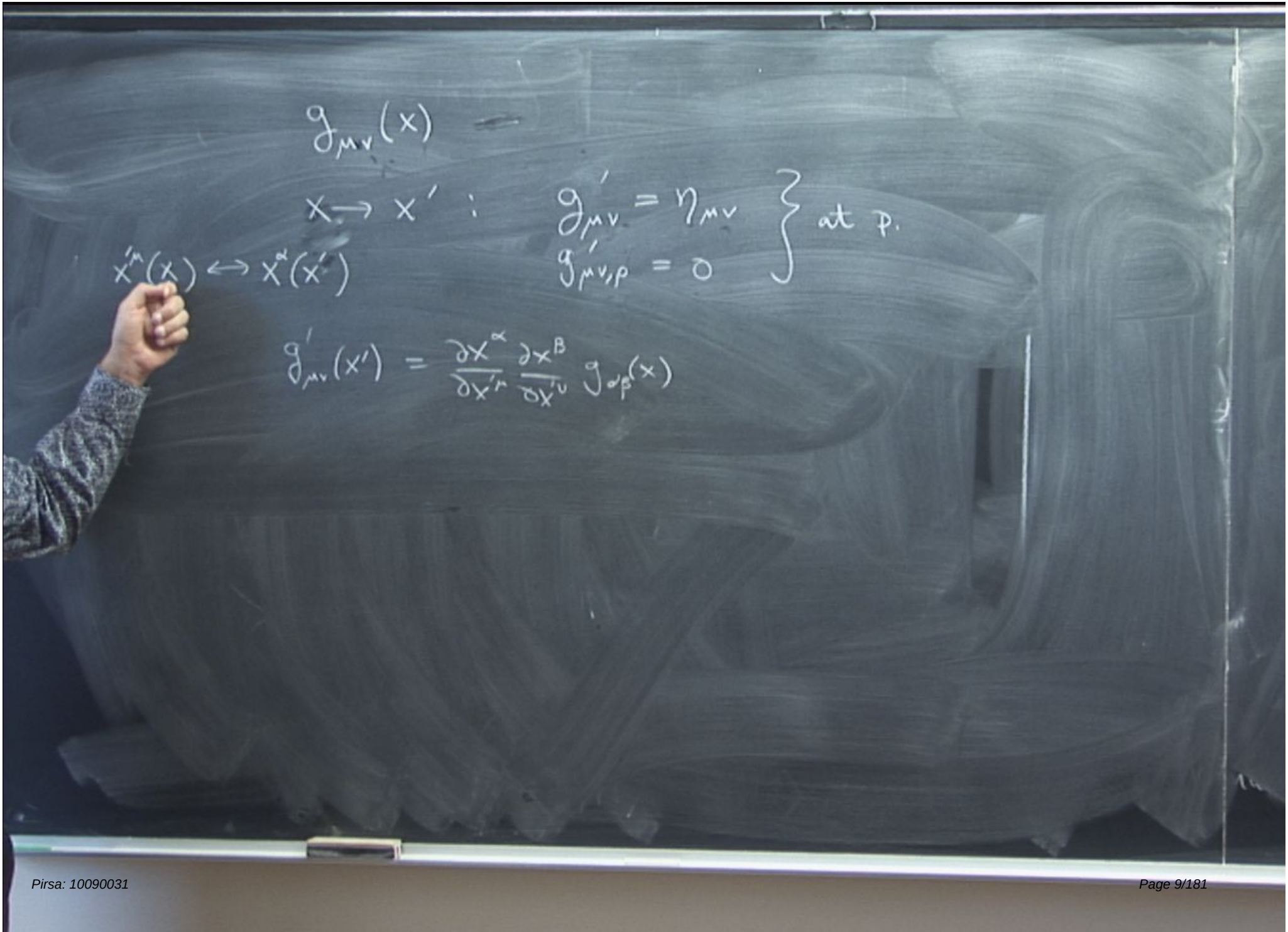
$$g'_{\mu\nu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$$

$g_{\mu\nu}(x)$ —

$x \longrightarrow x'$: $\left.\begin{array}{l} g'_{\mu\nu} = \eta_{\mu\nu} \\ g'_{\mu\nu,\rho} = 0 \end{array}\right\}$ at p.

$x'^{\mu}(x)$

$$g'_{\mu\nu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$$

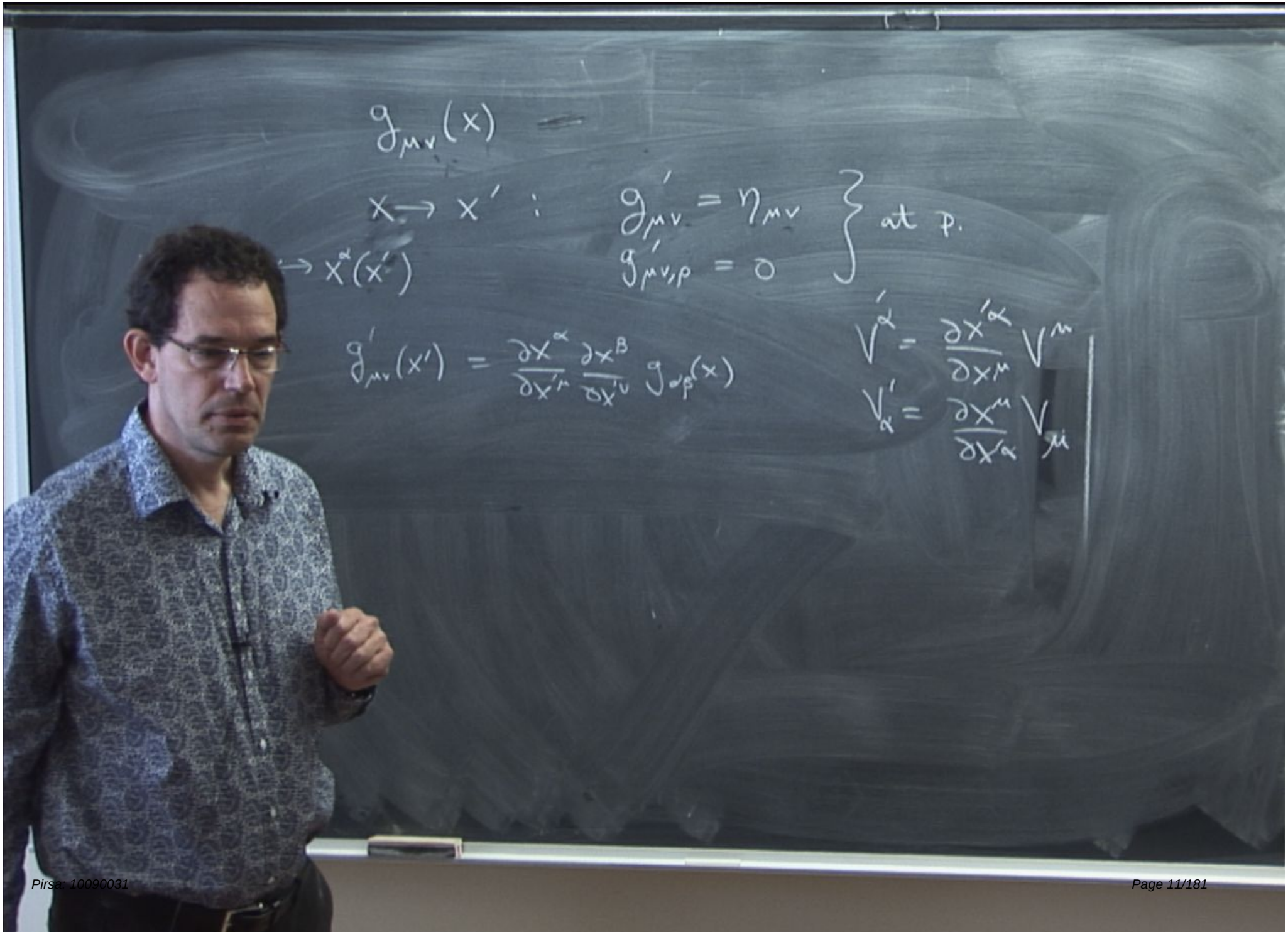

$g_{\mu\nu}(x)$
$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$ } at p.
$g'_{\mu\nu,\rho} = 0$
$x'^{\mu}(x) \leftrightarrow x^{\alpha}(x')$
$g'_{\mu\nu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$

$g_{\mu\nu}(x)$

$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$ } at P.
$g'_{\mu\nu,\rho} = 0$ }

$x'^{\mu}(x') \leftrightarrow x^{\alpha}(x')$

$$g'_{\mu\nu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$$


$g_{\mu\nu}(x)$
$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$ } at p.
$g'_{\mu\nu,\rho} = 0$
$\to x^\alpha(x')$
$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$
$V'^\alpha = \frac{\partial x'^\alpha}{\partial x^\mu} V^\mu$
$V'_\alpha = \frac{\partial x^\mu}{\partial x'^\alpha} V_\mu$

$g_{\mu\nu}(x)$

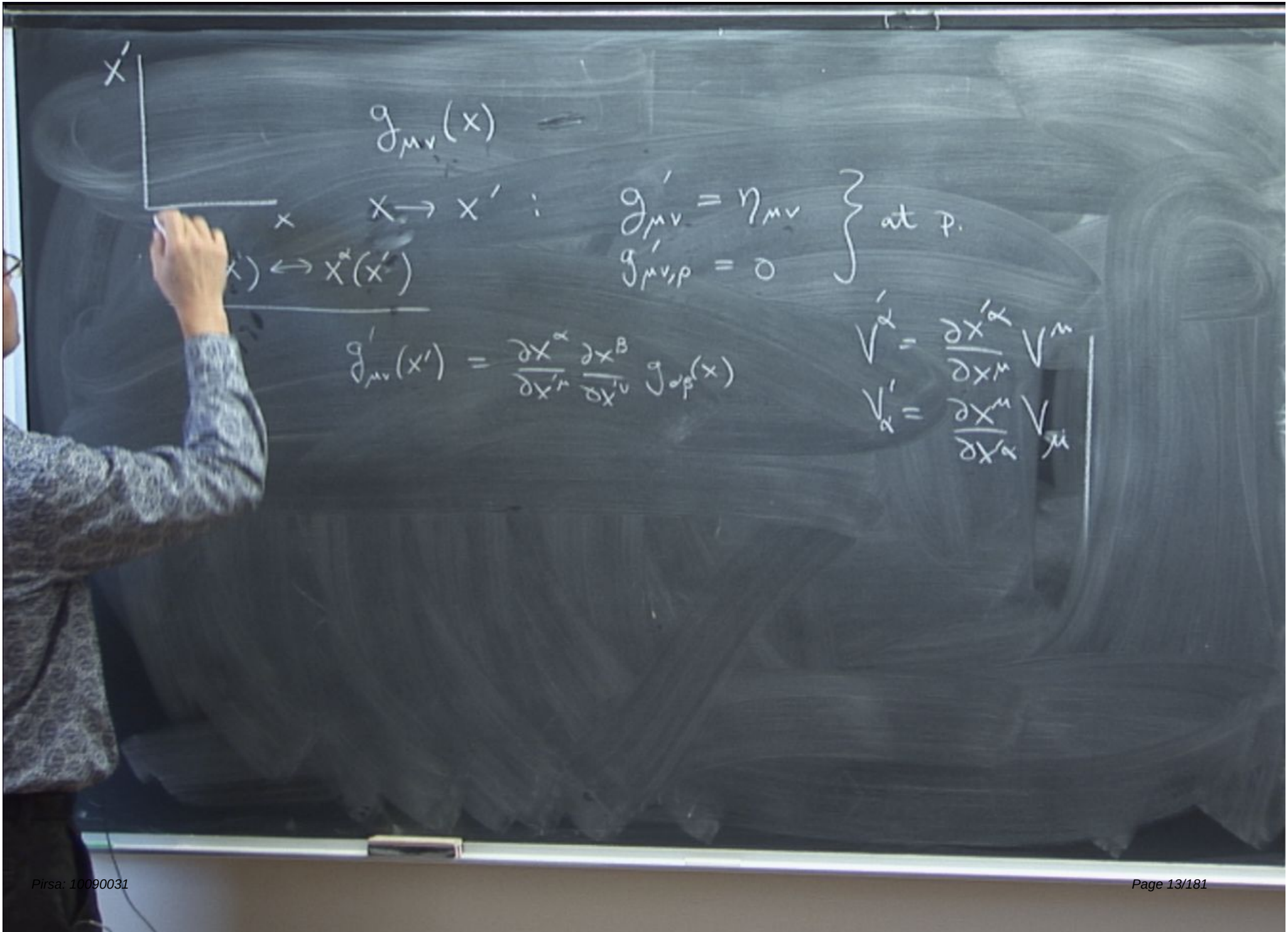
$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$ } at P.
$g'_{\mu\nu,\rho} = 0$ }

$x'^{\mu}(x') \leftrightarrow x^{\alpha}(x')$

$$g'_{\mu\nu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$$

$$V'^{\alpha} = \frac{\partial x'^{\alpha}}{\partial x^{\mu}} V^{\mu}$$

$$V'_{\alpha} = \frac{\partial x^{\mu}}{\partial x'^{\alpha}} V_{\mu}$$


x'
x
$g_{\mu\nu}(x)$
$x \to x'$:
$g'_{\mu\nu} = \eta_{\mu\nu}$
$g'_{\mu\nu,\rho} = 0$
at P.
$x^\alpha(x')$
$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$
$V'^\alpha = \frac{\partial x'^\alpha}{\partial x^\mu} V^\mu$
$V'_\alpha = \frac{\partial x^\mu}{\partial x'^\alpha} V_\mu$



$g_{\mu\nu}(x)$ —

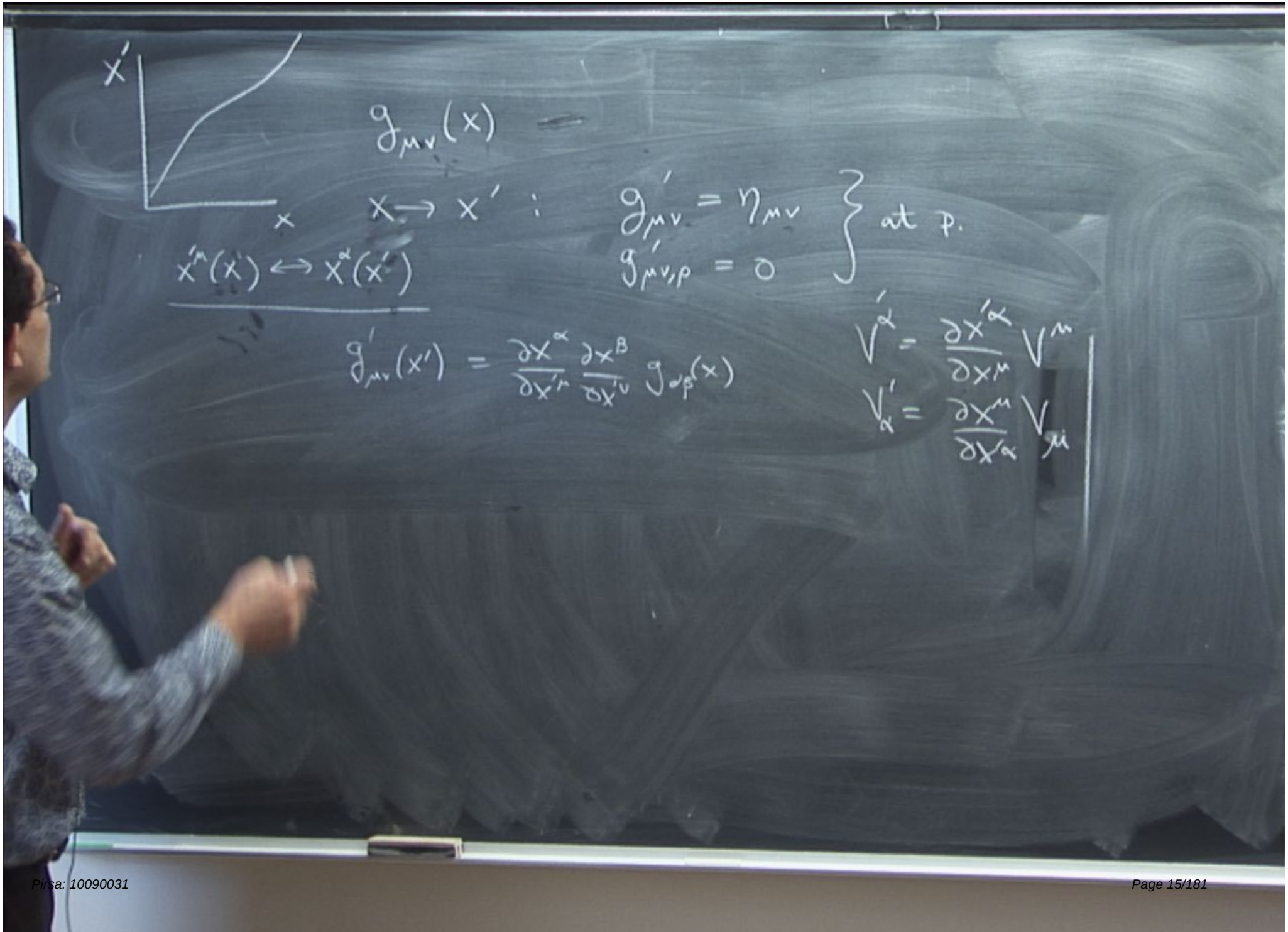
$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$ } at P.
$g'_{\mu\nu,\rho} = 0$ }

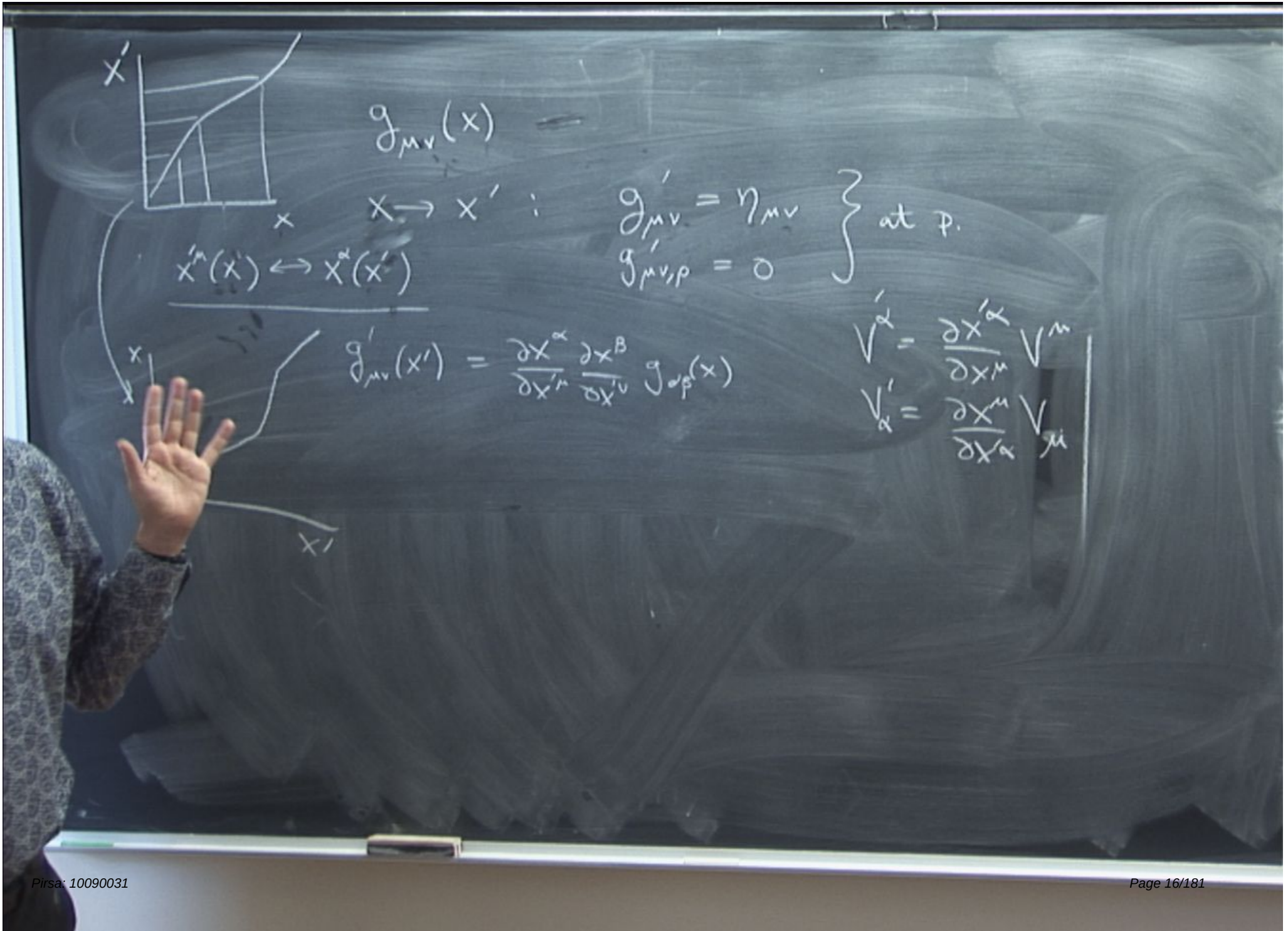
$x'^{\mu}(x') \leftrightarrow x^{\alpha}(x')$

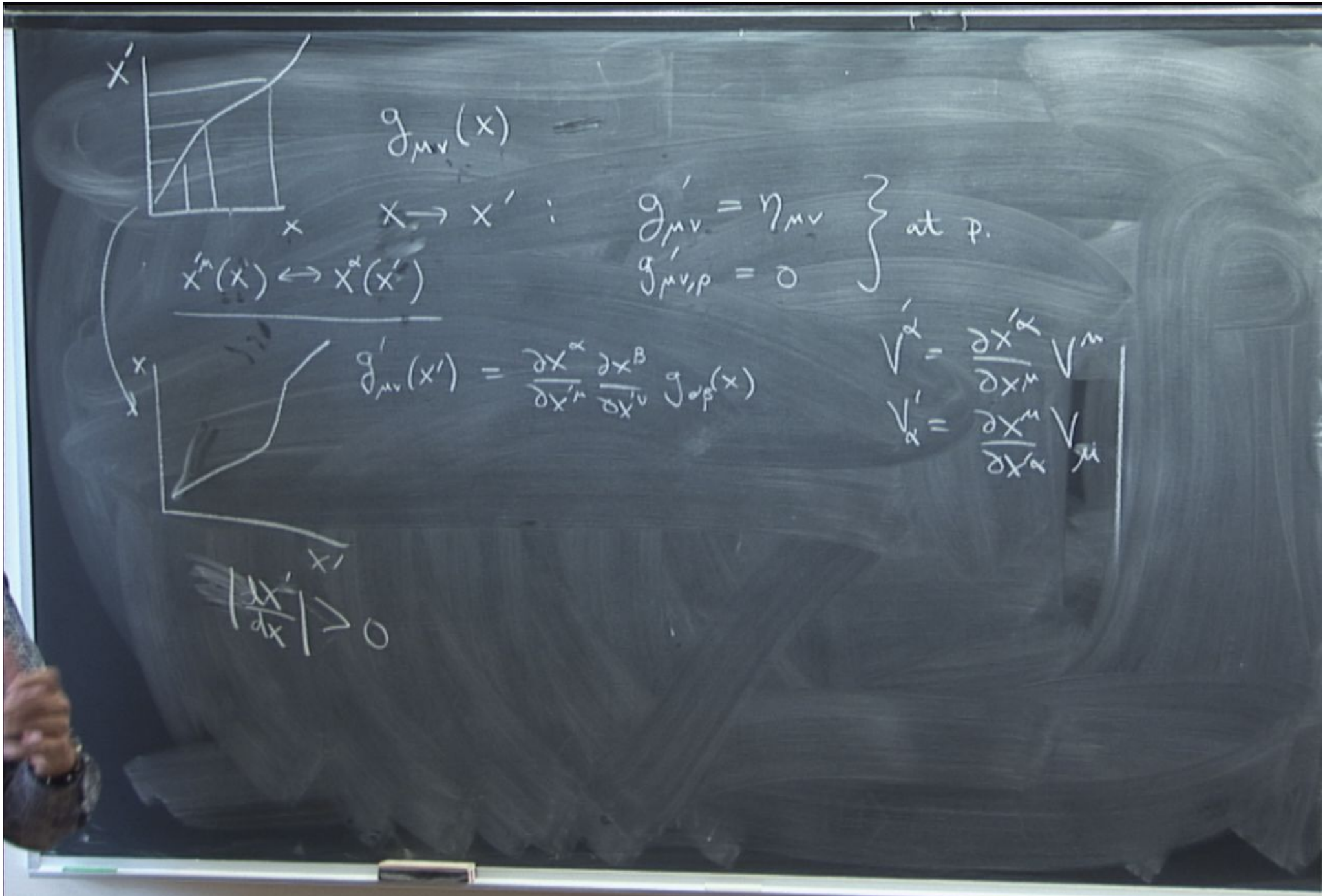
$$g'_{\mu\nu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$$

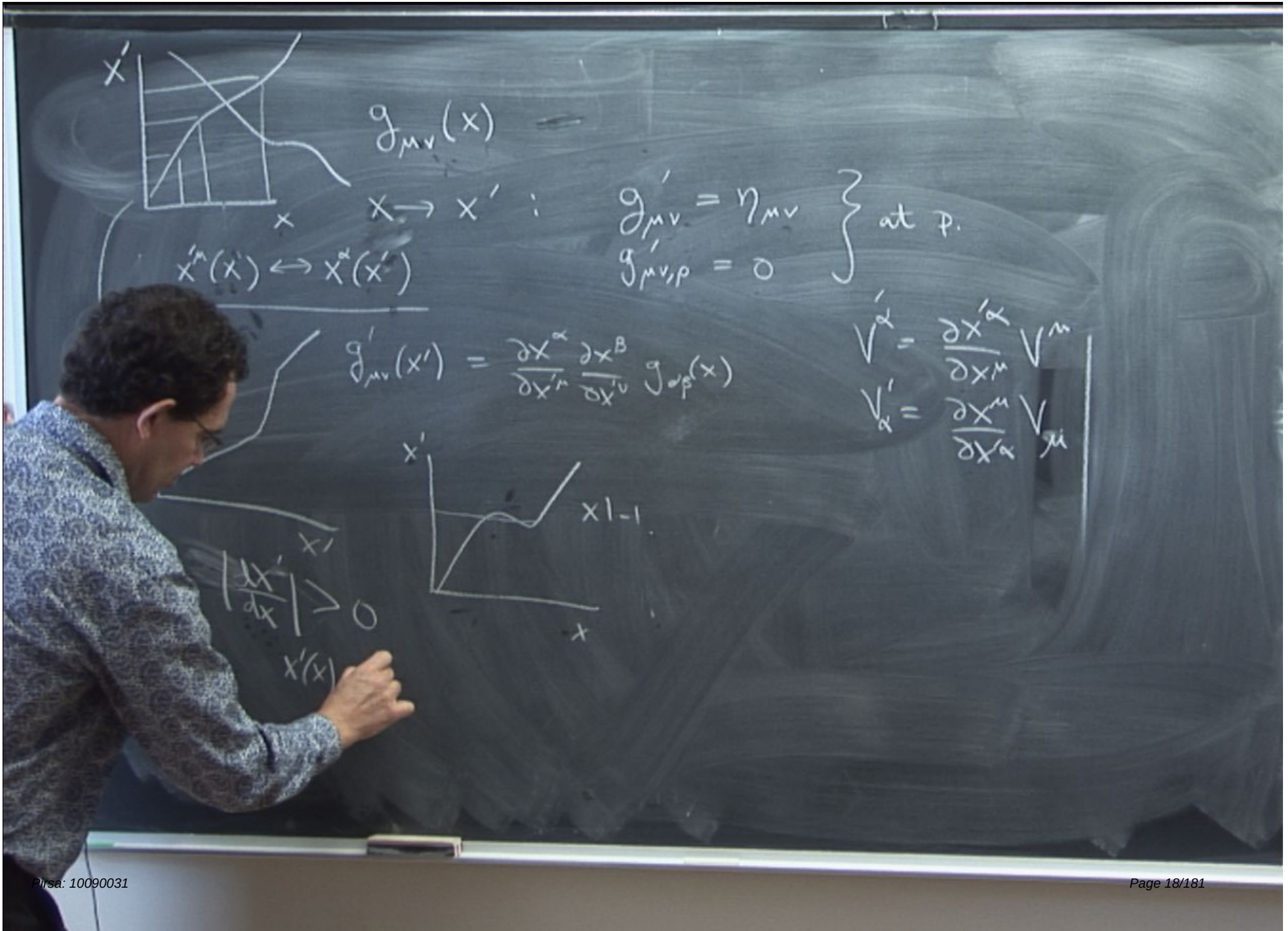
$$V'^{\alpha} = \frac{\partial x'^{\alpha}}{\partial x^{\mu}} V^{\mu}$$

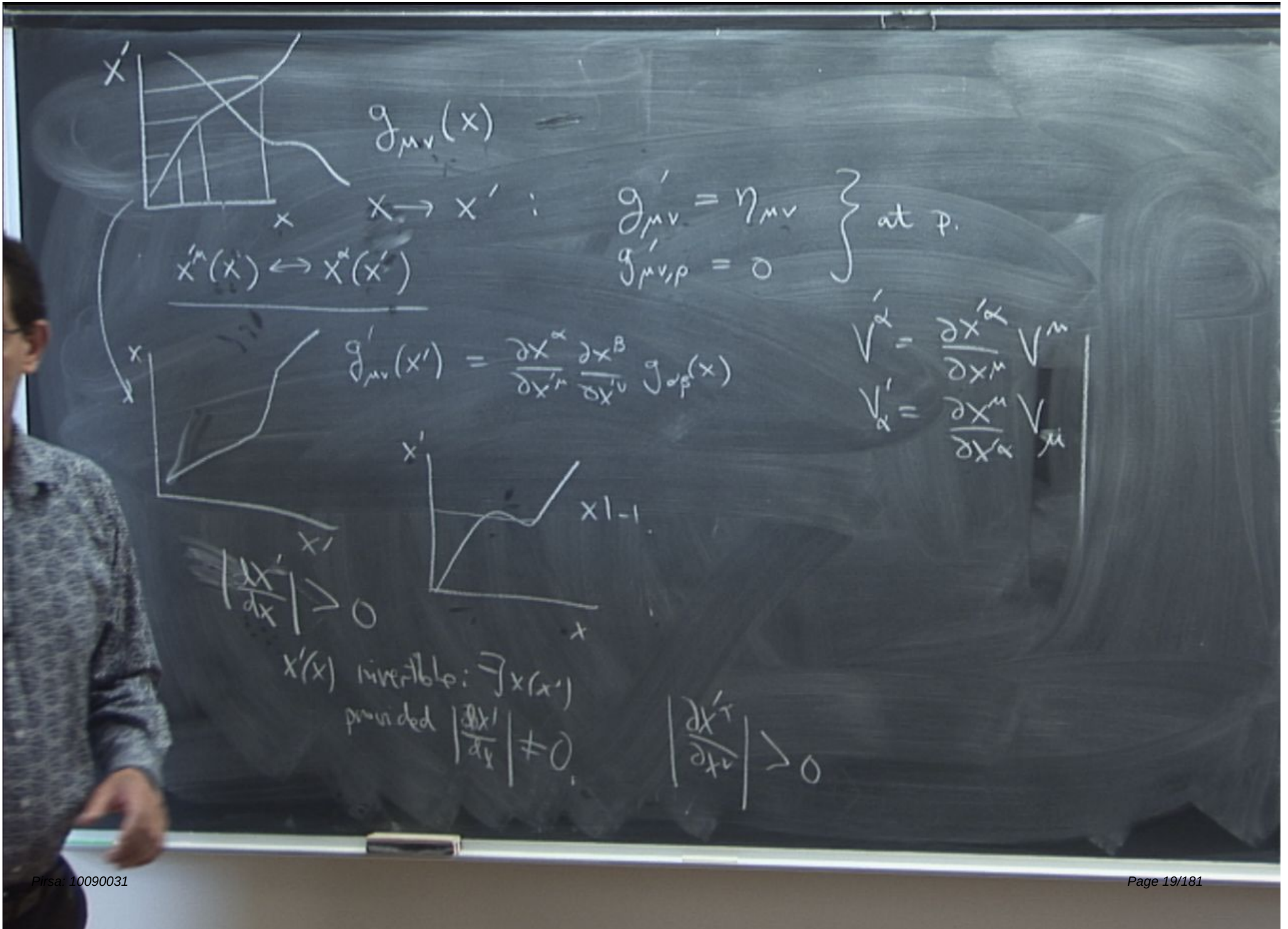
$$V'_{\alpha} = \frac{\partial x^{\mu}}{\partial x'^{\alpha}} V_{\mu}$$


x'
x
$g_{\mu\nu}(x)$
$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$ } at p.
$g'_{\mu\nu,\rho} = 0$
$x'^{\mu}(x') \leftrightarrow x^{\alpha}(x')$
$g'_{\mu\nu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$
$V'^{\alpha} = \frac{\partial x'^{\alpha}}{\partial x^{\mu}} V^{\mu}$
$V'_{\alpha} = \frac{\partial x^{\mu}}{\partial x'^{\alpha}} V_{\mu}$


x'
x
$g_{\mu\nu}(x)$
$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$ } at p.
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$x'^{\mu}(x') \leftrightarrow x^{\alpha}(x')$
$g'_{\mu\nu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$
$V'^{\alpha} = \frac{\partial x'^{\alpha}}{\partial x^{\mu}} V^{\mu}$
$V'_{\alpha} = \frac{\partial x^{\mu}}{\partial x'^{\alpha}} V_{\mu}$
x
x'


x'
x
$g_{\mu\nu}(x)$
$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$ } at p.
$g'_{\mu\nu,\rho} = 0$
$x'^{\mu}(x') \leftrightarrow x^{\alpha}(x')$
$g'_{\mu\nu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$
$V'^{\alpha} = \frac{\partial x'^{\alpha}}{\partial x^{\mu}} V^{\mu}$
$V'_{\alpha} = \frac{\partial x^{\mu}}{\partial x'^{\alpha}} V_{\mu}$
x
x'
$\left|\frac{\partial x'}{\partial x}\right| > 0$


$g_{\mu\nu}(x)$
$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$ at p.
$g'_{\mu\nu,\rho} = 0$
$x'^{\mu}(x') \leftrightarrow x^{\alpha}(x')$
$g'_{\mu\nu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$
$V'^{\alpha} = \frac{\partial x'^{\alpha}}{\partial x^{\mu}} V^{\mu}$
$V'_{\alpha} = \frac{\partial x^{\mu}}{\partial x'^{\alpha}} V_{\mu}$
x'
x
x1-1
$\left|\frac{\partial x'}{\partial x}\right| > 0$
x'(x)


$g_{\mu\nu}(x)$
$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$ at p.
$g'_{\mu\nu,\rho} = 0$
$x'^{\mu}(x') \leftrightarrow x^{\alpha}(x')$
$g'_{\mu\nu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$
$V'^{\alpha} = \frac{\partial x'^{\alpha}}{\partial x^{\mu}} V^{\mu}$
$V'_{\alpha} = \frac{\partial x^{\mu}}{\partial x'^{\alpha}} V_{\mu}$
x
x'
x'
x
x 1-1
$\left|\frac{\partial x'}{\partial x}\right| > 0$
$x'(x)$ invertible: $\exists x(x')$
provided $\left|\frac{\partial x'}{\partial x}\right| \neq 0$.
$\left|\frac{\partial x'^{T}}{\partial x^{\nu}}\right| > 0$

$X'^{\mu}(x)$

$\left|\frac{\partial x'^{\mu}}{\partial x^{\alpha}}\right|$

$g'_{\mu\nu} = \eta_{\mu\nu}$ } at p.

$g'_{\mu\nu,\rho} = 0$

$V'^{\alpha} = \frac{\partial x'^{\alpha}}{\partial x^{\mu}} V^{\mu}$

$V'_{\alpha} = \frac{\partial x^{\mu}}{\partial x'^{\alpha}} V_{\mu}$

$\frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$

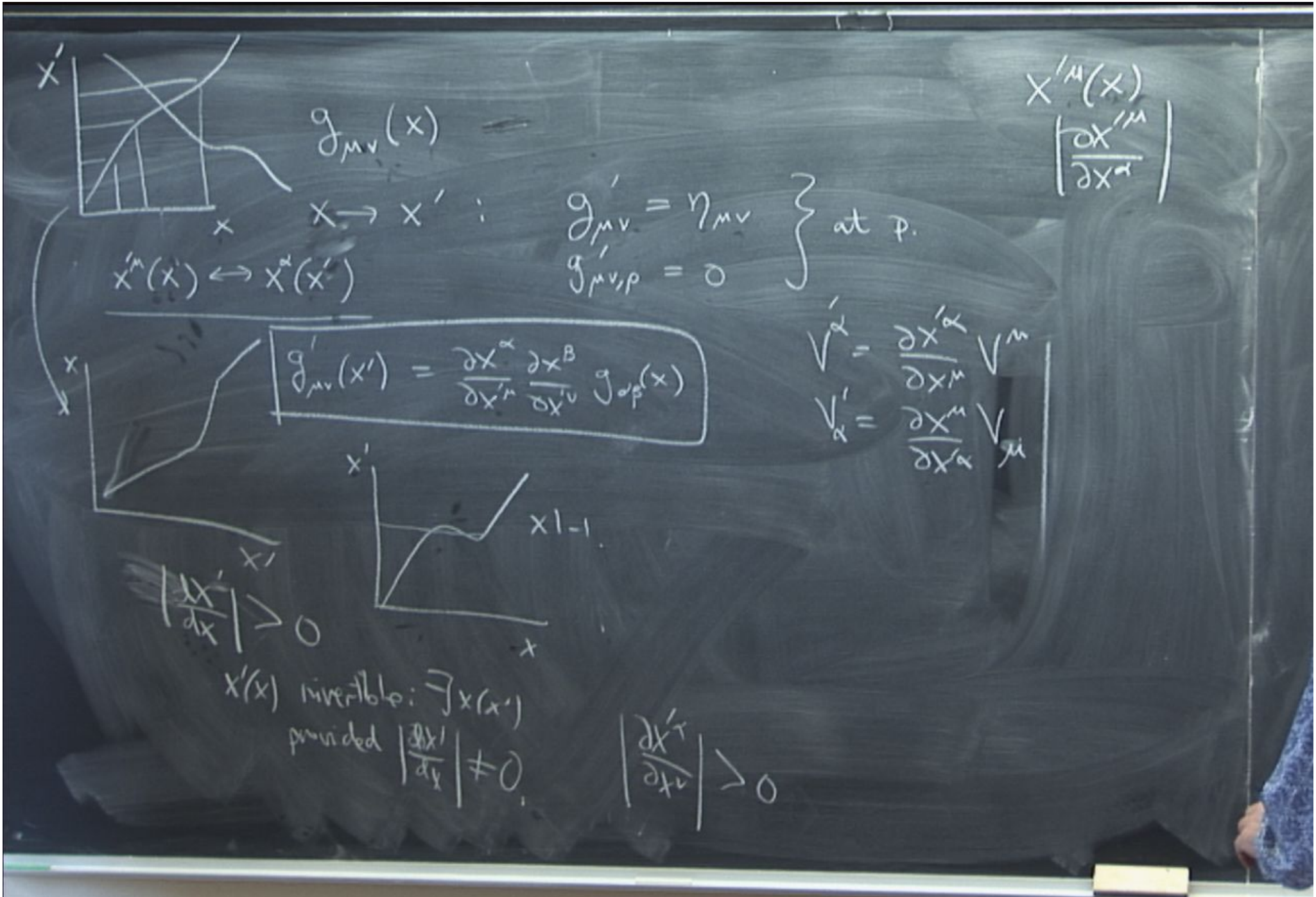
$\times 1 \times -1$

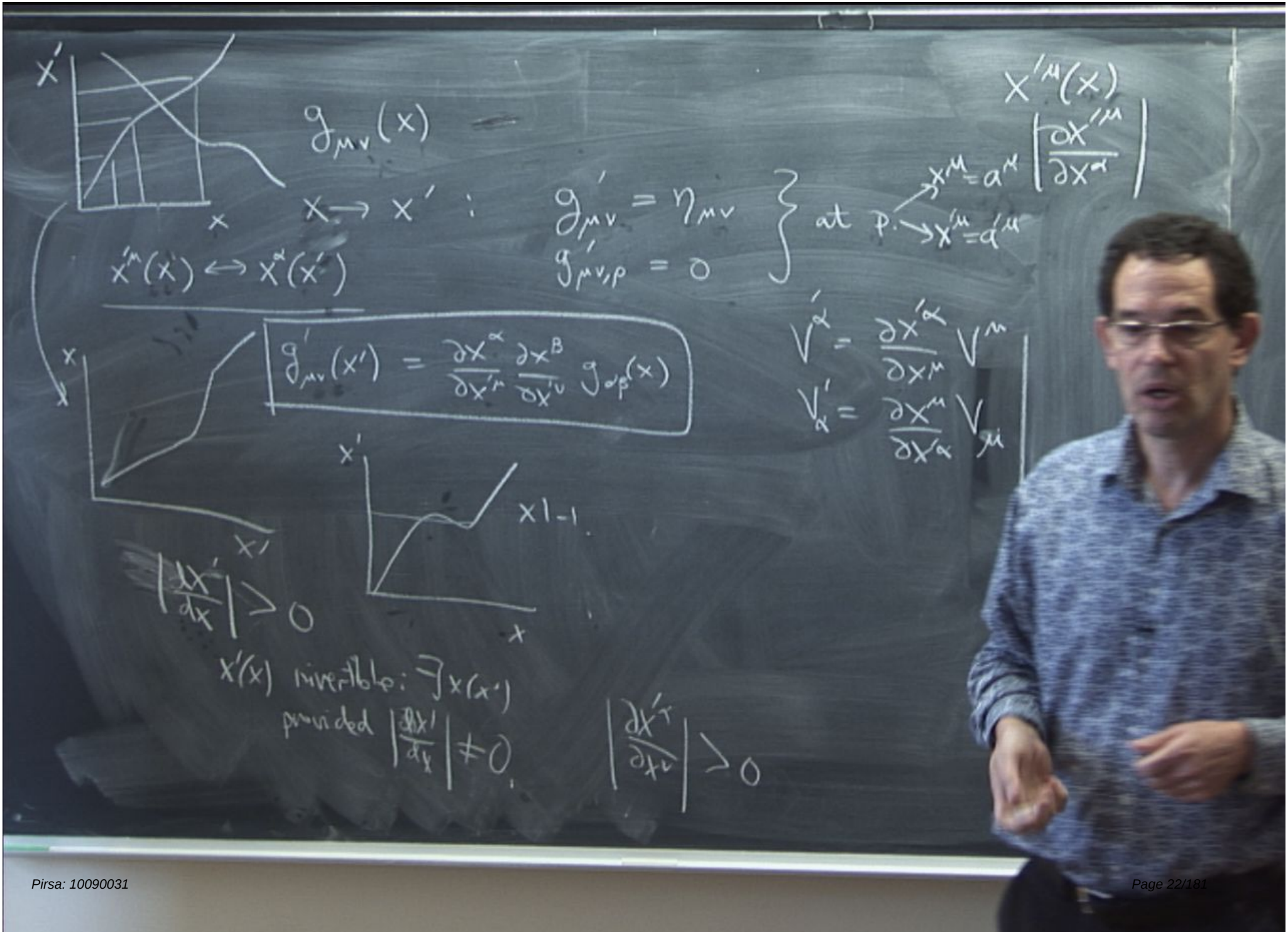
x

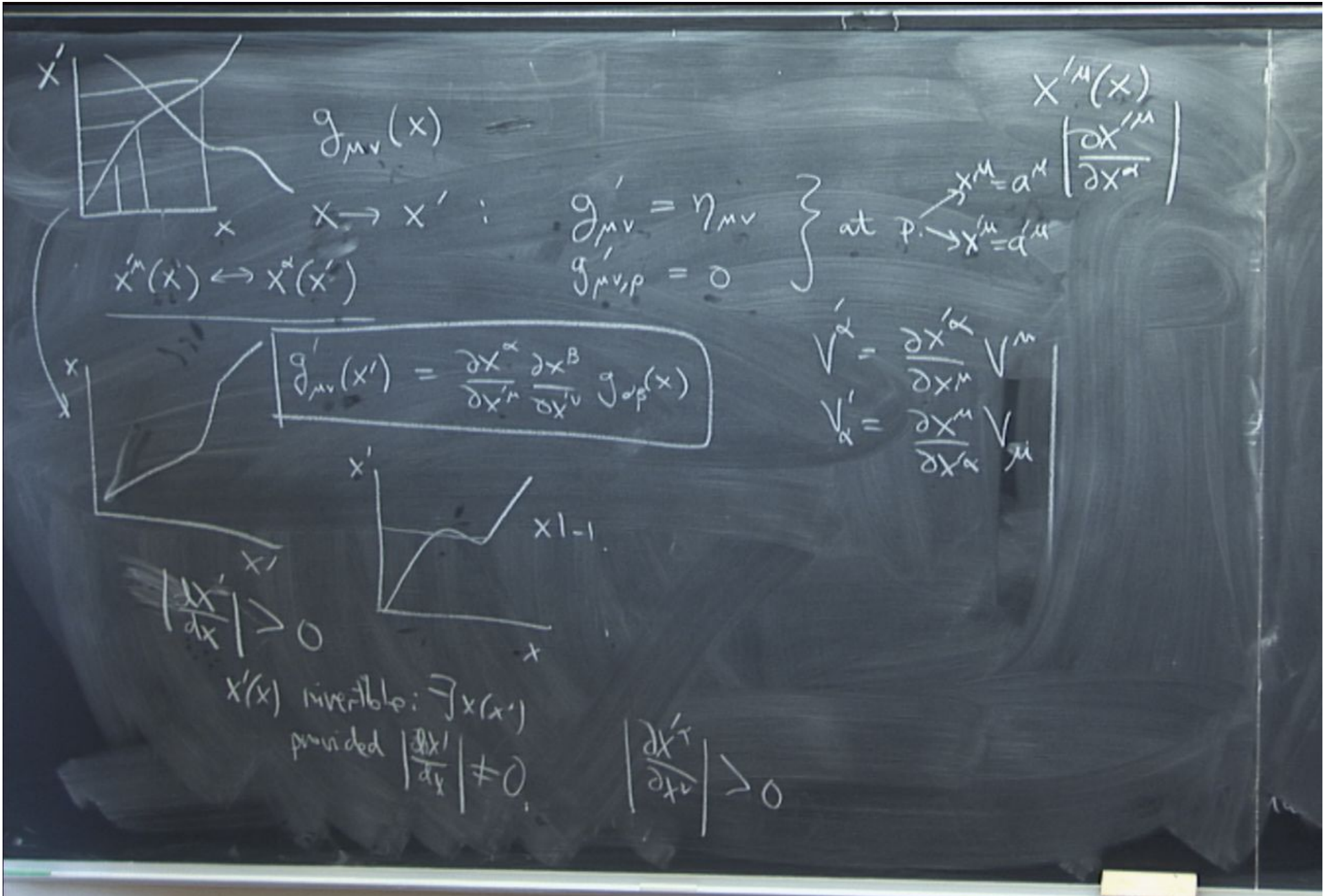
$x(x')$

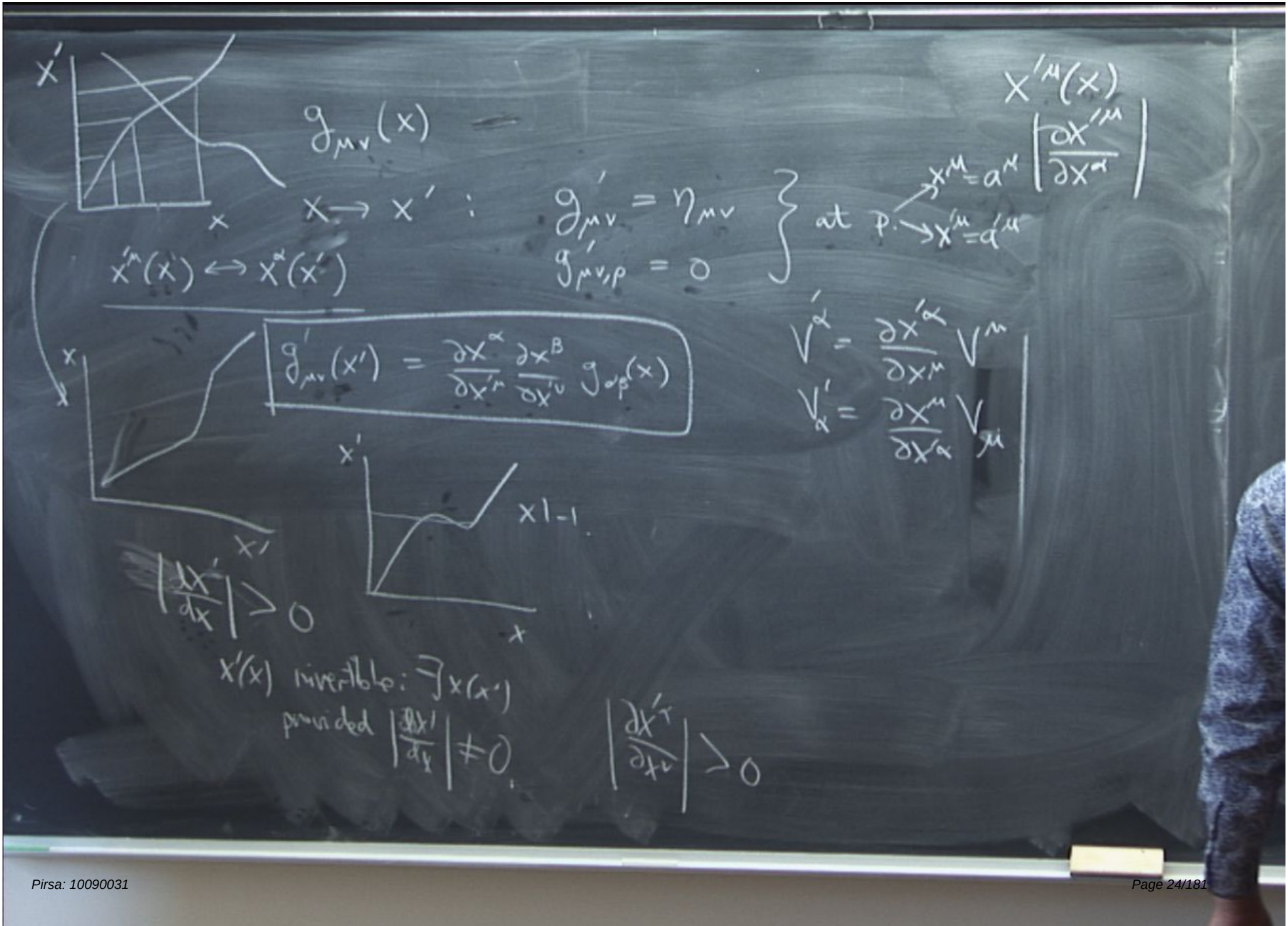
$\neq 0$

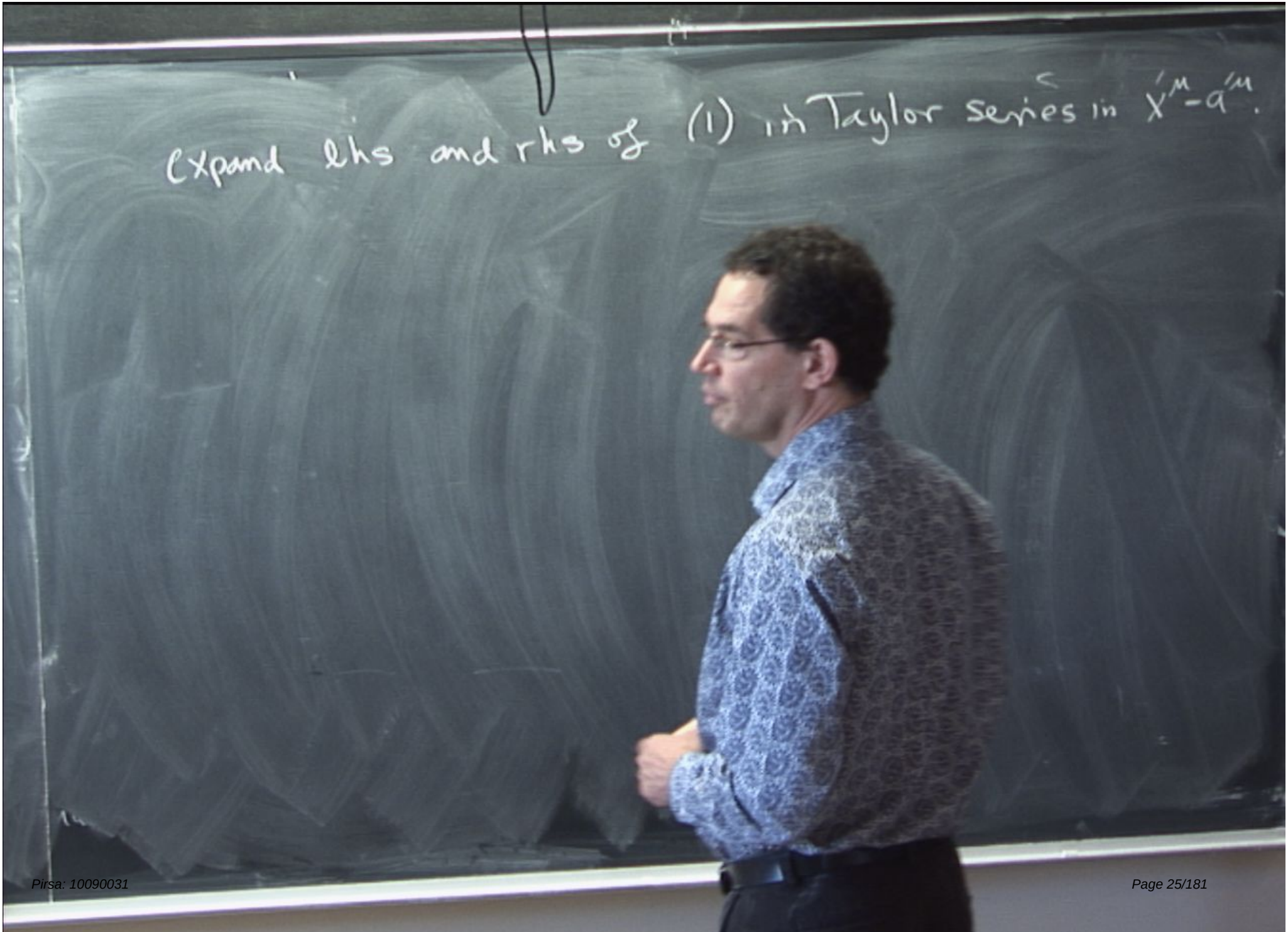
$\left|\frac{\partial x'^{\tau}}{\partial x^{\nu}}\right| > 0$


$g_{\mu\nu}(x)$
$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$ } at p.
$g'_{\mu\nu,\rho} = 0$
$x'^{\mu}(x') \leftrightarrow x^{\alpha}(x')$
$x'^{\mu}(x)$
$\left|\frac{\partial x'^{\mu}}{\partial x^{\alpha}}\right|$
$g'_{\mu\nu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$
$V'^{\alpha} = \frac{\partial x'^{\alpha}}{\partial x^{\mu}} V^{\mu}$
$V'_{\alpha} = \frac{\partial x^{\mu}}{\partial x'^{\alpha}} V_{\mu}$
x
x'
x'
x
x1-1.
$\left|\frac{\partial x'}{\partial x}\right| > 0$
x'(x) invertible: $\exists x(x')$
provided $\left|\frac{\partial x'}{\partial x}\right| \neq 0$.
$\left|\frac{\partial x'^{\tau}}{\partial x^{\nu}}\right| > 0$


$g_{\mu\nu}(x)$
$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$
$g'_{\mu\nu,\rho} = 0$
at P. $x^\mu = a^\mu$, $x'^\mu = a'^\mu$
$x'^\mu(x)$
$\left|\frac{\partial x'^\mu}{\partial x^\alpha}\right|$
$x'^\mu(x) \leftrightarrow x^\alpha(x')$
$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$
$V'^\alpha = \frac{\partial x'^\alpha}{\partial x^\mu} V^\mu$
$V'_\alpha = \frac{\partial x^\mu}{\partial x'^\alpha} V_\mu$
x1-1
$\left|\frac{\partial x'}{\partial x}\right| > 0$
$x'(x)$ invertible: $\exists x(x')$
provided $\left|\frac{\partial x'}{\partial x}\right| \neq 0$.
$\left|\frac{\partial x'^\tau}{\partial x^\nu}\right| > 0$


$g_{\mu\nu}(x)$
$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$
$g'_{\mu\nu,\rho} = 0$
at P. $x'^\mu = a^\mu$, $x'^\mu = q'^\mu$
$x'^\mu(x) \leftrightarrow x^\alpha(x')$
$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$
$V'^\alpha = \frac{\partial x'^\alpha}{\partial x^\mu} V^\mu$
$V'_\alpha = \frac{\partial x^\mu}{\partial x'^\alpha} V_\mu$
1-1
$\left|\frac{\partial x'}{\partial x}\right| > 0$
$x'(x)$ invertible: $\exists x(x')$
provided $\left|\frac{\partial x'}{\partial x}\right| \neq 0$.
$\left|\frac{\partial x'^\tau}{\partial x^\nu}\right| > 0$


$g_{\mu\nu}(x)$
$x \to x'$:
$g'_{\mu\nu} = \eta_{\mu\nu}$
$g'_{\mu\nu,\rho} = 0$
at P. $\to x'^{\mu} = a'^{\mu}$
$x^{\mu} = a^{\mu}$
$x'^{\mu}(x)$
$\left|\frac{\partial x'^{\mu}}{\partial x^{\alpha}}\right|$
$x'^{\mu}(x') \leftrightarrow x^{\alpha}(x')$
$g'_{\mu\nu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$
$V'^{\alpha} = \frac{\partial x'^{\alpha}}{\partial x^{\mu}} V^{\mu}$
$V'_{\alpha} = \frac{\partial x^{\mu}}{\partial x'^{\alpha}} V_{\mu}$
1-1
$\left|\frac{\partial x'}{\partial x}\right| > 0$
$x'(x)$ invertible: $\exists\, x(x')$
provided $\left|\frac{\partial x'}{\partial x}\right| \neq 0$.
$\left|\frac{\partial x'^{\tau}}{\partial x^{\nu}}\right| > 0$


Expand lhs and rhs of (1) in Taylor series in $x'^{\mu} - q'^{\mu}$.

Expand lhs and rhs of (1) in Taylor series in $x'^{\mu} - a'^{\mu}$.

some constr

$$g'_{\mu\nu} = \eta_{\mu\nu} + D_{\mu\nu\rho\lambda}(x'-a')^{\rho}(x'-a')^{\lambda}$$

Expand lhs and rhs of (1) in Taylor series in $x'^{\mu} - a'^{\mu}$.

some constr

$$g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{g'_{\mu\nu,\rho\lambda}|_P} (x'-a')^{\rho}(x'-a')^{\lambda}$$

Expand lhs and rhs of (1) in Taylor series in $x'^{\mu} - a'^{\mu}$.

some constr

$$\text{lhs } g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{g'_{\mu\nu,\rho\lambda}|_P} (x'-a')^{\rho}(x'-a')^{\lambda} + O\left((x'-a')^3\right)$$

rhs x^{α}

Expand lhs and rhs of (1) in Taylor series in $x'^{\mu} - a'^{\mu}$.

some constr

lhs $g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{g'_{\mu\nu,\rho\lambda}|_P} (x'-a')^{\rho}(x'-a')^{\lambda} + O((x'-a')^3)$

rhs $x^{\alpha} = a^{\alpha}$

Expand lhs and rhs of (1) in Taylor series in $x'^\mu - a'^\mu$.

some constr

$$\text{lhs } g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{g'_{\mu\nu,\rho\lambda}\big|_P}(x'-a')^\rho(x'-a')^\lambda + O\left((x'-a')^3\right)$$

$$\text{rhs } x^\alpha = a^\alpha + \frac{\partial x^\alpha}{\partial x'^\mu}(x'^\mu - a'^\mu)$$

Expand lhs and rhs of (1) in Taylor series in $x'^\mu - a'^\mu$.

some constr

lhs $g' = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{g'_{\mu\nu,\rho\lambda}|_P} (x'-a')^\rho (x'-a')^\lambda + O((x'-a')^3)$

$x^\kappa(x')$

rhs $x^\alpha = a^\alpha + \frac{\partial x^\alpha}{\partial x'^\mu}(x'^\mu - a'^\mu) + \frac{\partial^2 x^\kappa}{\partial x'^\mu \partial x'^\nu}(x'^\mu - a'^\mu)(x'^\nu - a'^\nu) + \cdots$

Expand lhs and rhs of (1) in Taylor series in $x'^\mu - a'^\mu$.

some constr

$$\text{lhs } g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{g'_{\mu\nu,\rho\lambda}\big|_p}(x'-a')^\rho(x'-a')^\lambda + \mathcal{O}\left((x'-a')^3\right)$$

$x^\kappa(x')$

$$\text{rhs } x^\alpha = a^\alpha + \left.\frac{\partial x^\alpha}{\partial x'^\mu}\right|_p (x'^\mu - a'^\mu) + \left.\frac{\partial^2 x^\kappa}{\partial x'^\mu \partial x'^\nu}\right|_p (x'^\mu - a'^\mu)(x'^\nu - a'^\nu) + \cdots$$

$$\frac{\partial x^\kappa}{\partial x'^\mu} =$$

Expand lhs and rhs of (1) in Taylor series in $x'^\mu - a'^\mu$.

some constr

$$\text{lhs } g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{g'_{\mu\nu,\rho\lambda}\big|_p}(x'-a')^\rho(x'-a')^\lambda + O\left((x'-a')^3\right)$$

$x^\alpha(x')$

$$\text{rhs } x^\alpha = a^\alpha + \left.\frac{\partial x^\alpha}{\partial x'^\mu}\right|_p (x'^\mu - a'^\mu) + \frac{1}{2}\left.\frac{\partial^2 x^\alpha}{\partial x'^\mu \partial x'^\nu}\right|_p (x'^\mu - a'^\mu)(x'^\nu - a'^\nu) + \cdots$$

$$\frac{\partial x^\alpha}{\partial x'^\mu} = ?$$

Expand lhs and rhs of (1) in Taylor series in $x'^\mu - a'^\mu$.

some constr

lhs $g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{\frac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_P} (x'-a')^\rho (x'-a')^\lambda + O((x'-a')^3)$

$x^\kappa(x')$

rhs $x^\alpha = a^\alpha + \left.\frac{\partial x^\alpha}{\partial x'^\mu}\right|_P (x'^\mu - a'^\mu) + \frac{1}{2}\left.\frac{\partial^2 x^\kappa}{\partial x'^\mu \partial x'^\nu}\right|_P (x'^\mu - a'^\mu)(x'^\nu - a'^\nu) + \cdots$

$\frac{\partial x^\kappa}{\partial x'^\mu} = ?$

Expand lhs and rhs of (1) in Taylor series in $x'^\mu - a'^\mu$.

some constr

$$\text{lhs } g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{\frac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_p} (x'-a')^\rho (x'-a')^\lambda + O\left((x'-a')^3\right)$$

$x^\kappa(x')$

$$\text{rhs } x^\alpha = a^\alpha + \left.\frac{\partial x^\alpha}{\partial x'^\mu}\right|_p (x'^\mu - a'^\mu) + \frac{1}{2}\left.\frac{\partial^2 x^\kappa}{\partial x'^\mu \partial x'^\nu}\right|_p (x'^\mu - a'^\mu)(x'^\nu - a'^\nu) + \cdots$$

$$\frac{\partial x^\kappa(x')}{\partial x'^\mu} = \left.\frac{\partial x^\kappa}{\partial x'^\mu}\right|_p (x'-a') + \left.\frac{\partial^2 x}{\partial x' \partial x'}\right|_p (x'-a)$$

Expand lhs and rhs of (1) in Taylor series in $x'^\mu - a'^\mu$.

some constr

lhs $g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{\frac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_p} (x'-a')^\rho (x'-a')^\lambda + O((x'-a')^3)$

$x')$

$x^\alpha = a^\alpha + \left.\frac{\partial x^\alpha}{\partial x'^\mu}\right|_p (x'^\mu - a'^\mu) + \frac{1}{2}\left.\frac{\partial^2 x^\alpha}{\partial x'^\mu \partial x'^\nu}\right|_p (x'^\mu - a'^\mu)(x'^\nu - a'^\nu) + \cdots$

$\frac{\partial x^\alpha}{\partial x'^\mu}(x') = \left.\frac{\partial x^\alpha}{\partial x'^\mu}\right|_p + \left.\frac{\partial^2 x}{\partial x' \partial x'}\right|_p (x'-a)$

Expand lhs and rhs of (1) in Taylor series in $x'^\mu - a'^\mu$.

some constr

lhs $g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{\frac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_p} (x'-a')^\rho (x'-a')^\lambda + \mathcal{O}((x'-a')^3)$

$x^\kappa(x')$

rhs $x^\alpha = a^\alpha + \left.\frac{\partial x^\alpha}{\partial x'^\mu}\right|_p (x'^\mu - a'^\mu) + \frac{1}{2}\left.\frac{\partial^2 x^\kappa}{\partial x'^\mu \partial x'^\nu}\right|_p (x'^\mu - a'^\mu)(x'^\nu - a'^\nu) + \cdots$

$$\frac{\partial x^\alpha(x')}{\partial x'^\mu} = \left.\frac{\partial x^\alpha}{\partial x'^\mu}\right|_p + \left.\frac{\partial^2 x}{\partial x' \partial x'}\right|_p (x'-a)$$

$$g_{\alpha\beta}(x) = g_{\alpha\beta}\big|_p$$

Expand lhs and rhs of (1) in Taylor series in $x'^\mu - a'^\mu$.

some constr

lhs $g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{\frac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_p} (x'-a')^\rho (x'-a')^\lambda + O((x'-a')^3)$

$x^\kappa(x')$

rhs $x^\alpha = a^\alpha + \left.\frac{\partial x^\alpha}{\partial x'^\mu}\right|_p (x'^\mu - a'^\mu) + \frac{1}{2}\left.\frac{\partial^2 x^\kappa}{\partial x'^\mu \partial x'^\nu}\right|_p (x'^\mu - a'^\mu)(x'^\nu - a'^\nu) + \cdots$

$\frac{\partial x^\kappa(x')}{\partial x'^\mu} = \left.\frac{\partial x^\kappa}{\partial x'^\mu}\right|_p + \left.\frac{\partial^2 x}{\partial x' \partial x'}\right|_p (x'-a)$

$g_{\alpha\beta}(x) = g_{\alpha\beta}\big|_p + g_{\alpha\beta,\rho}\big|_p (x^\rho - a^\rho) + \frac{1}{2} g_{\alpha\beta,\rho\delta}\big|_p (x^\rho - a^\rho)(x^\delta - a^\delta) + \cdots$

Expand lhs and rhs of (1) in Taylor series in $x'^\mu - a'^\mu$.

some constr

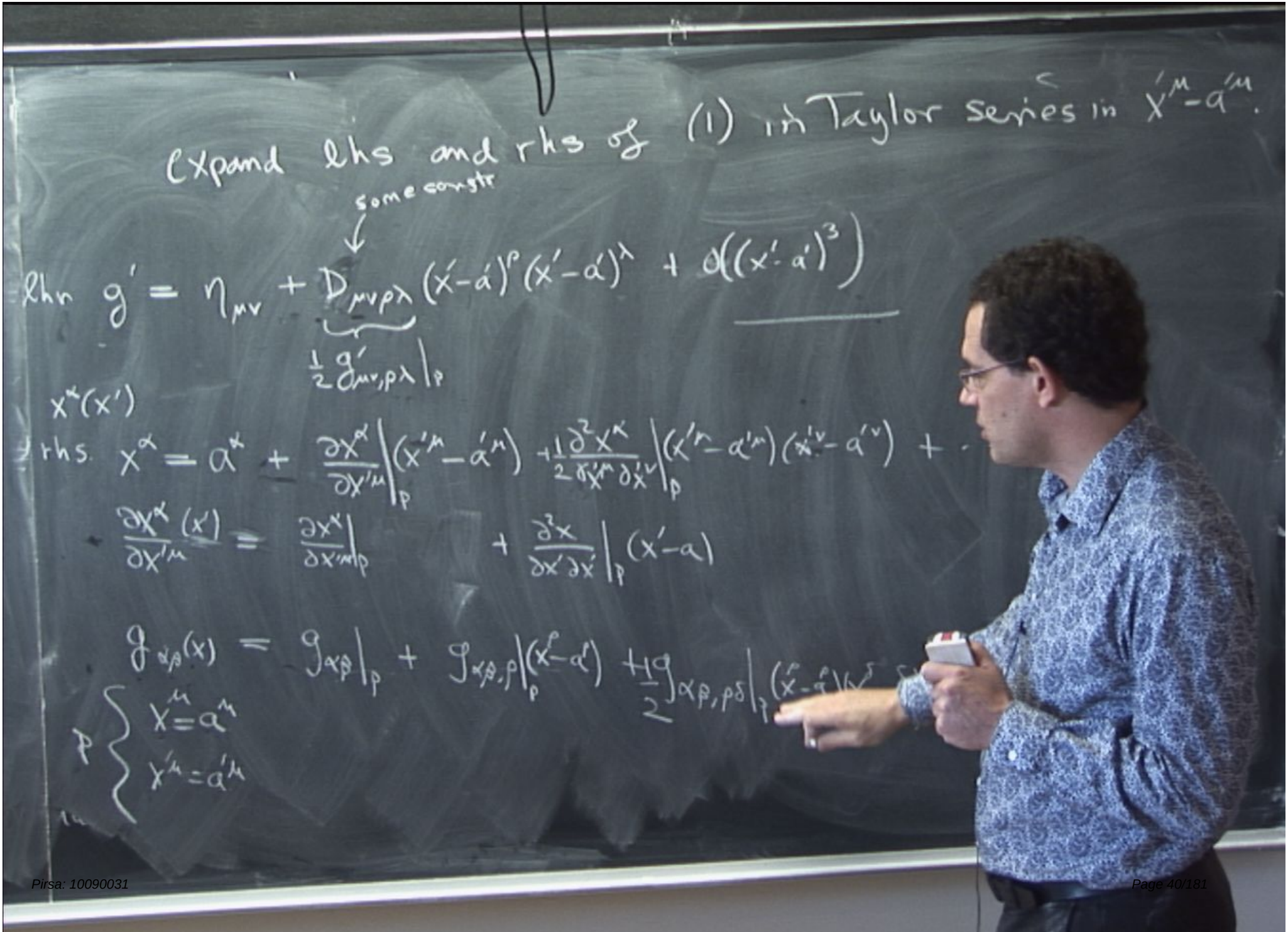
lhs $g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{\frac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_p} (x'-a')^\rho (x'-a')^\lambda + \mathcal{O}((x'-a')^3)$

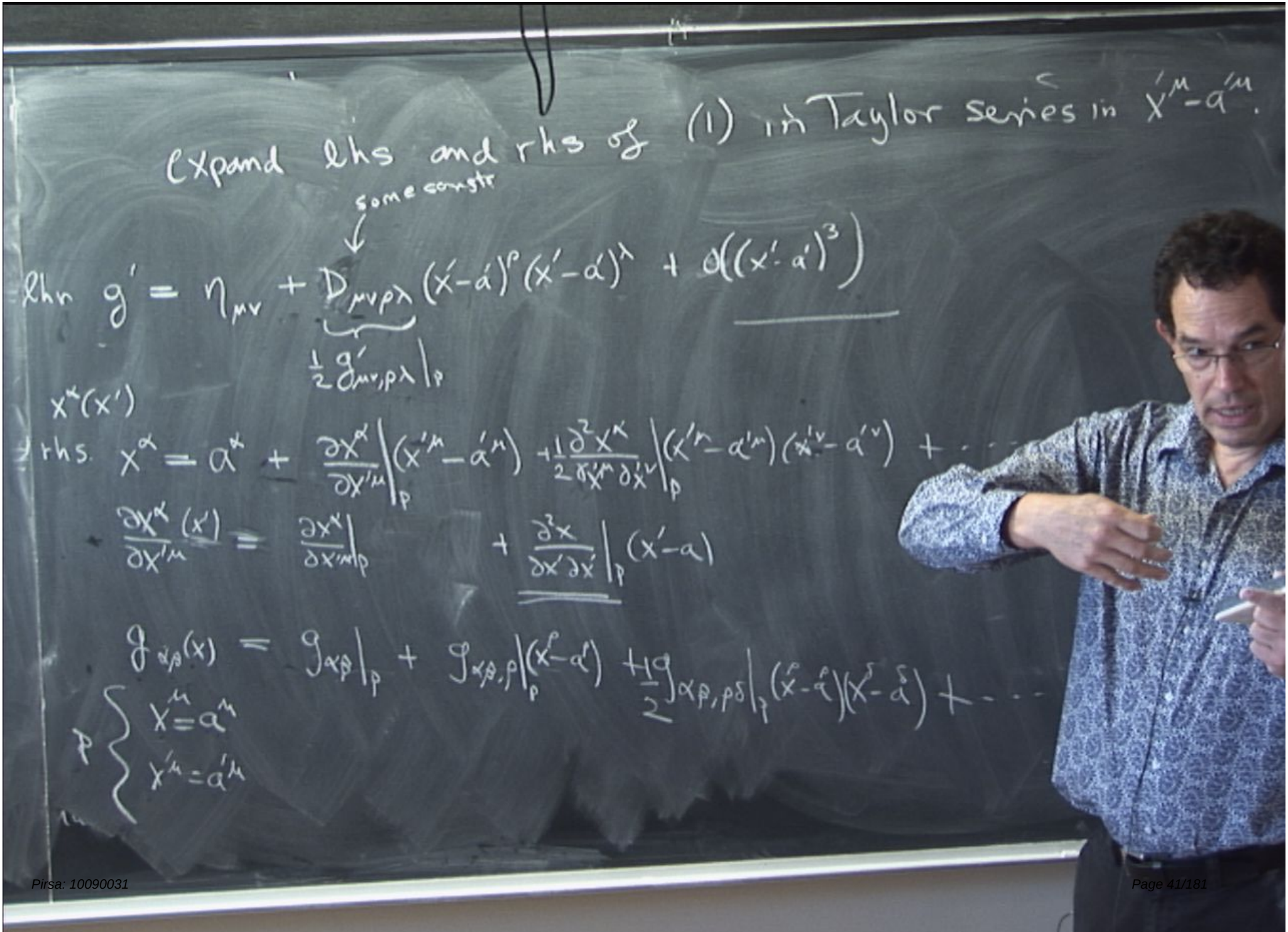
$x^\kappa(x')$

rhs $x^\alpha = a^\alpha + \left.\frac{\partial x^\alpha}{\partial x'^\mu}\right|_p (x'^\mu - a'^\mu) + \frac{1}{2}\left.\frac{\partial^2 x^\alpha}{\partial x'^\mu \partial x'^\nu}\right|_p (x'^\mu - a'^\mu)(x'^\nu - a'^\nu) + \cdots$

$\frac{\partial x^\kappa(x')}{\partial x'^\mu} = \left.\frac{\partial x^\kappa}{\partial x'^\mu}\right|_p + \left.\frac{\partial^2 x}{\partial x' \partial x'}\right|_p (x'-a)$

$g_{\alpha\beta}(x) = g_{\alpha\beta}\big|_p + g_{\alpha\beta,\rho}\big|_p (x^\rho - a^\rho) + \frac{1}{2} g_{\alpha\beta,\rho\delta}\big|_p (x^\rho - a^\rho)(x^\delta - a^\delta) + \cdots$


Expand lhs and rhs of (1) in Taylor series in $x'^\mu - a'^\mu$.
some constr
lhs $g' = \eta_{\mu\nu} + D_{\mu\nu\rho\lambda}(x'-a')^\rho(x'-a')^\lambda + \mathcal{O}((x'-a')^3)$
$\frac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_p$
$x^\alpha(x')$
rhs $x^\alpha = a^\alpha + \frac{\partial x^\alpha}{\partial x'^\mu}\big|_p (x'^\mu - a'^\mu) + \frac{1}{2}\frac{\partial^2 x^\alpha}{\partial x'^\mu \partial x'^\nu}\big|_p (x'^\mu - a'^\mu)(x'^\nu - a'^\nu) +$
$\frac{\partial x^\alpha}{\partial x'^\mu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\big|_p + \frac{\partial^2 x}{\partial x' \partial x'}\big|_p (x'-a)$
$g_{\alpha\beta}(x) = g_{\alpha\beta}\big|_p + g_{\alpha\beta,\rho}\big|_p (x^\rho - a^\rho) + \frac{1}{2} g_{\alpha\beta,\rho\delta}\big|_p$
p
$x^\mu = a^\mu$
$x'^\mu = a'^\mu$


Expand lhs and rhs of (1) in Taylor series in $x'^\mu - a'^\mu$.
some constr
lhs $g'_{\mu\nu}(x') = \eta_{\mu\nu} + D_{\mu\nu\rho\lambda}(x'-a')^\rho(x'-a')^\lambda + O((x'-a')^3)$
$\frac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_p$
$x^\kappa(x')$
rhs $x^\alpha = a^\alpha + \frac{\partial x^\alpha}{\partial x'^\mu}\Big|_p (x'^\mu - a'^\mu) + \frac{1}{2}\frac{\partial^2 x^\alpha}{\partial x'^\mu \partial x'^\nu}\Big|_p (x'^\mu - a'^\mu)(x'^\nu - a'^\nu) + \cdots$
$\frac{\partial x^\alpha(x')}{\partial x'^\mu} = \frac{\partial x^\alpha}{\partial x'^\mu}\Big|_p + \frac{\partial^2 x}{\partial x' \partial x'}\Big|_p (x'-a)$
$g_{\alpha\beta}(x) = g_{\alpha\beta}\big|_p + g_{\alpha\beta,\rho}\big|_p (x^\rho - a^\rho) + \frac{1}{2} g_{\alpha\beta,\rho\delta}\big|_p (x^\rho - a^\rho)(x^\delta - a^\delta) + \cdots$
p
$x^\mu = a^\mu$
$x'^\mu = a'^\mu$

Expand lhs and rhs of (1) in Taylor series in $x'^\mu - a'^\mu$.

some constr

$$\text{lhs}\quad g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{\frac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_P} (x'-a')^\rho (x'-a')^\lambda + \underline{\mathcal{O}\left((x'-a')^3\right)}$$

$x^\kappa(x')$

$$\text{rhs}\quad x^\alpha = a^\alpha + \left.\frac{\partial x^\alpha}{\partial x'^\mu}\right|_P (x'^\mu - a'^\mu) + \frac{1}{2}\left.\frac{\partial^2 x^\alpha}{\partial x'^\mu \partial x'^\nu}\right|_P (x'^\mu - a'^\mu)(x'^\nu - a'^\nu) + \cdots$$

$$\frac{\partial x^\alpha(x')}{\partial x'^\mu} = \left.\frac{\partial x^\alpha}{\partial x'^\mu}\right|_P + \underline{\underline{\left.\frac{\partial^2 x}{\partial x' \partial x'}\right|_P (x'-a)}}$$

$$g_{\alpha\beta}(x) = g_{\alpha\beta}\big|_P + g_{\alpha\beta,\rho}\big|_P (x^\rho - a^\rho) + \frac{1}{2} g_{\alpha\beta,\rho\delta}\big|_P (x^\rho - a^\rho)(x^\delta - a^\delta) + \cdots$$

P: $x^\mu = a^\mu$, $x'^\mu = a'^\mu$



Expand lhs and rhs of (1) in Taylor series in $x'^\mu - a'^\mu$.

some constr

lhs $g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{\frac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_p} (x'-a')^\rho (x'-a')^\lambda + \mathcal{O}((x'-a')^3)$

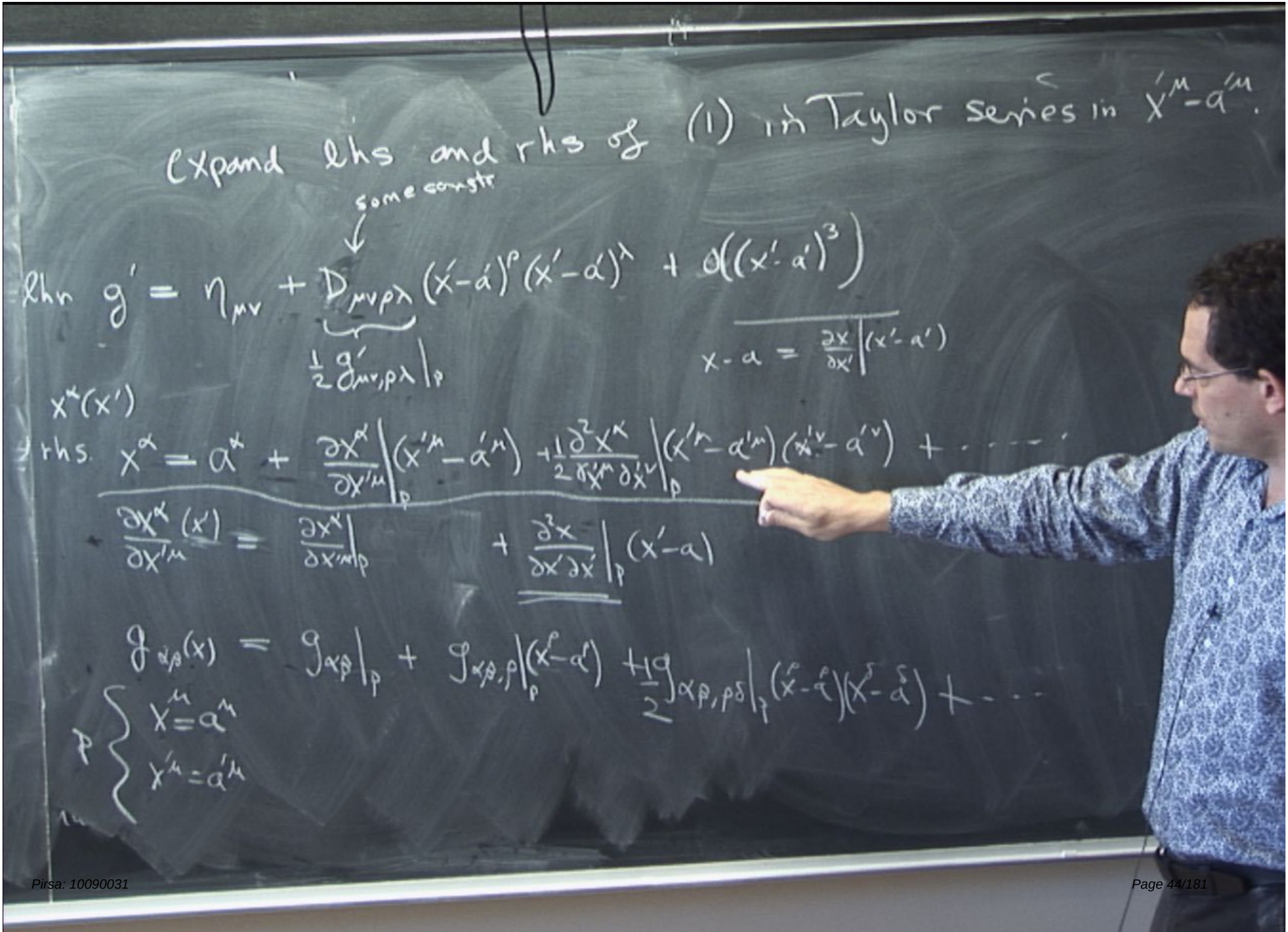
$x^\kappa(x')$

rhs $x^\alpha = a^\alpha + \left.\frac{\partial x^\alpha}{\partial x'^\mu}\right|_p (x'^\mu - a'^\mu) + \frac{1}{2}\left.\frac{\partial^2 x^\alpha}{\partial x'^\mu \partial x'^\nu}\right|_p (x'^\mu - a'^\mu)(x'^\nu - a'^\nu) + \cdots$

$+ \left.\frac{\partial^2 x}{\partial x' \partial x'}\right|_p (x'-a)$

$g_{\alpha\beta}(x) = g_{\alpha\beta}\big|_p + g_{\alpha\beta,\rho}\big|_p (x^\rho - a^\rho) + \frac{1}{2} g_{\alpha\beta,\rho\delta}\big|_p (x^\rho - a^\rho)(x^\delta - a^\delta) + \cdots$

p: $x^\mu = a^\mu$, $x'^\mu = a'^\mu$


Expand lhs and rhs of (1) in Taylor series in $x'^\mu - a'^\mu$.
some constr
lhs $g' = \eta_{\mu\nu} + D_{\mu\nu\rho\lambda}(x'-a')^\rho(x'-a')^\lambda + \mathcal{O}((x'-a')^3)$
$\frac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_p$
$x - a = \frac{\partial x}{\partial x'}\big|(x'-a')$
$x^\alpha(x')$
rhs $x^\alpha = a^\alpha + \frac{\partial x^\alpha}{\partial x'^\mu}\big|_p (x'^\mu - a'^\mu) + \frac{1}{2}\frac{\partial^2 x^\alpha}{\partial x'^\mu \partial x'^\nu}\big|_p (x'^\mu - a'^\mu)(x'^\nu - a'^\nu) + \cdots$
$\frac{\partial x^\alpha}{\partial x'^\mu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\big|_p + \frac{\partial^2 x}{\partial x' \partial x'}\big|_p (x'-a)$
$g_{\alpha\beta}(x) = g_{\alpha\beta}\big|_p + g_{\alpha\beta,\rho}\big|_p (x^\rho - a^\rho) + \frac{1}{2} g_{\alpha\beta,\rho\delta}\big|_p (x^\rho - a^\rho)(x^\delta - a^\delta) + \cdots$
p
$x^\mu = a^\mu$
$x'^\mu = a'^\mu$

Expand lhs and rhs of (1) in Taylor series in $x'^\mu - a'^\mu$.

some constr

lhs $g'_{\mu\nu} = \eta_{\mu\nu} + D_{\mu\nu\rho\lambda}(x'-a')^\rho(x'-a')^\lambda + O((x'-a')^3)$

$D_{\mu\nu\rho\lambda} = \frac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_p$

$x - a = \left.\frac{\partial x}{\partial x'}\right|_p (x'-a')$

$x^\alpha(x')$

rhs $x^\alpha = a^\alpha + \left.\frac{\partial x^\alpha}{\partial x'^\mu}\right|_p (x'^\mu - a'^\mu) + \frac{1}{2}\left.\frac{\partial^2 x^\alpha}{\partial x'^\mu \partial x'^\nu}\right|_p (x'^\mu - a'^\mu)(x'^\nu - a'^\nu) + \cdots$

$\frac{\partial x^\alpha(x')}{\partial x'^\mu} = \left.\frac{\partial x^\alpha}{\partial x'^\mu}\right|_p + \left.\frac{\partial^2 x}{\partial x' \partial x'}\right|_p (x'-a)$

$g_{\alpha\beta}(x) = g_{\alpha\beta}|_p + g_{\alpha\beta,\rho}\big|_p (x^\rho - a^\rho) + \frac{1}{2} g_{\alpha\beta,\rho\delta}\big|_p (x^\rho - a^\rho)(x^\delta - a^\delta) + \cdots$

$p \begin{cases} x^\mu = a^\mu \\ x'^\mu = a'^\mu \end{cases}$

Expand lhs and rhs of (1) in Taylor series in $x'^\mu - a'^\mu$.

some constr

lhs $g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{\frac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_p} (x'-a')^\rho (x'-a')^\lambda + \mathcal{O}\left((x'-a')^3\right)$

$x^\kappa(x')$

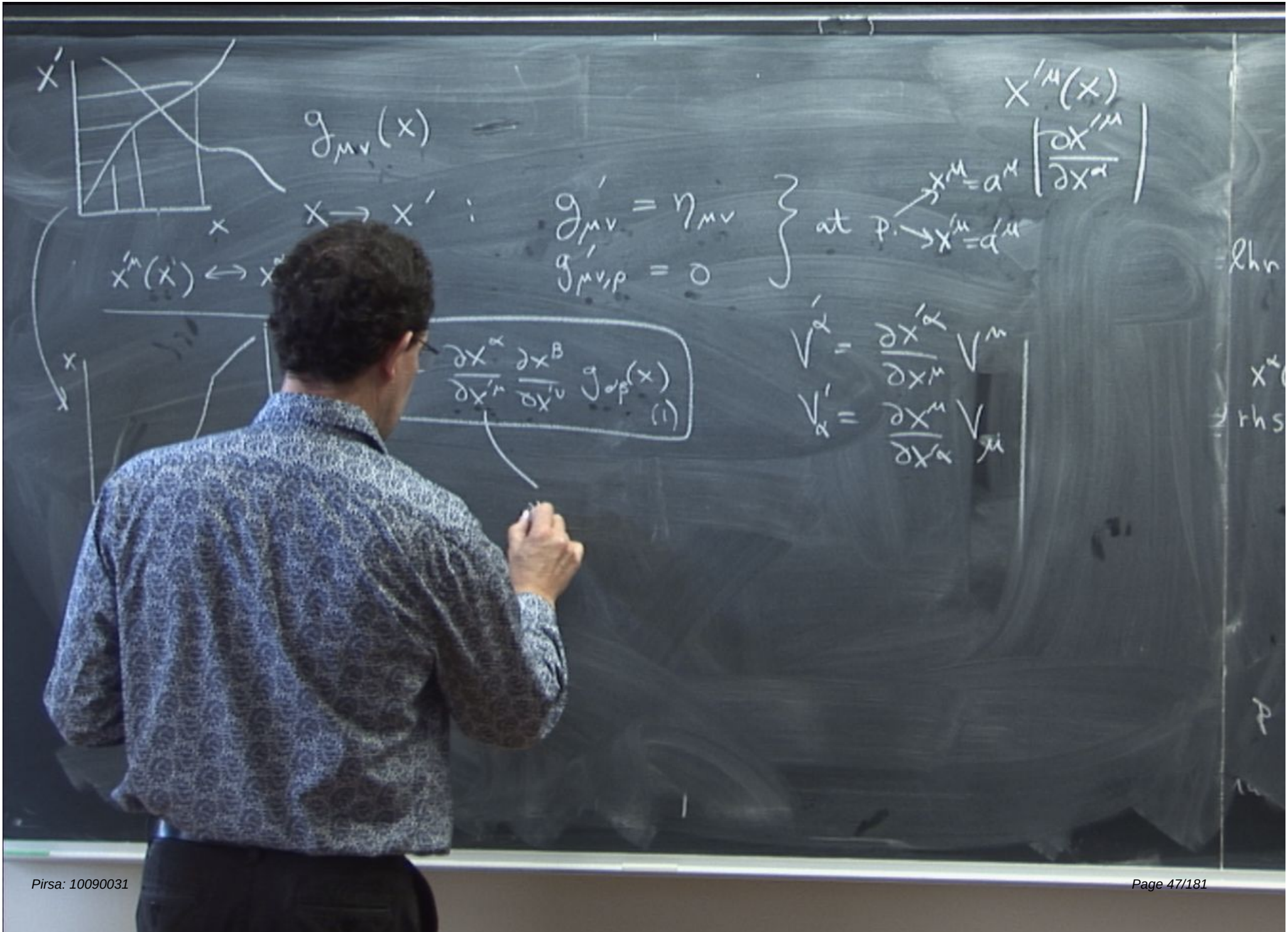
$x - a = \left.\frac{\partial x}{\partial x'}\right|_p (x'-a')$

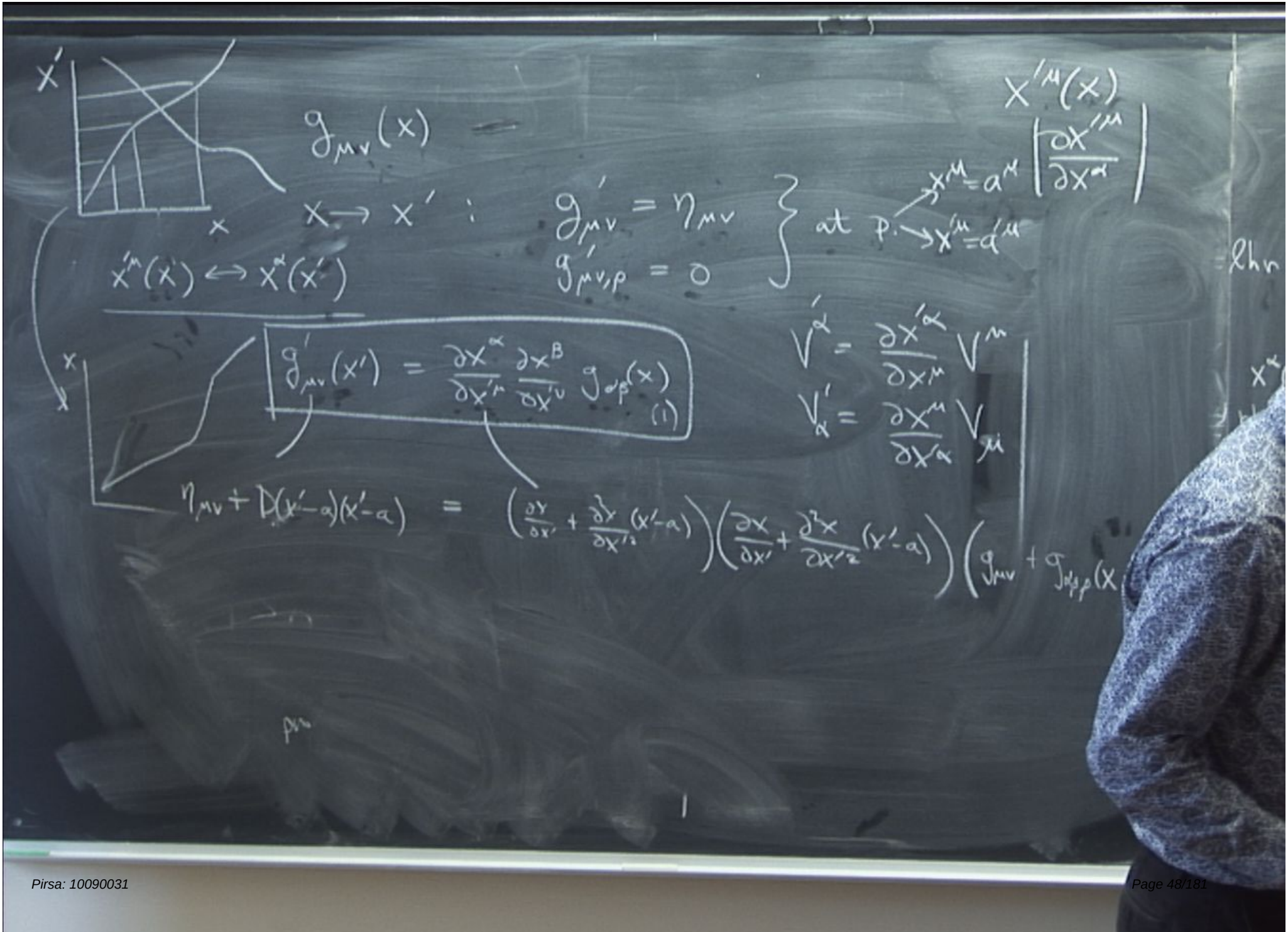
rhs $x^\alpha = a^\alpha + \left.\frac{\partial x^\alpha}{\partial x'^\mu}\right|_p (x'^\mu - a'^\mu) + \frac{1}{2}\left.\frac{\partial^2 x^\kappa}{\partial x'^\mu \partial x'^\nu}\right|_p (x'^\mu - a'^\mu)(x'^\nu - a'^\nu) + \cdots$

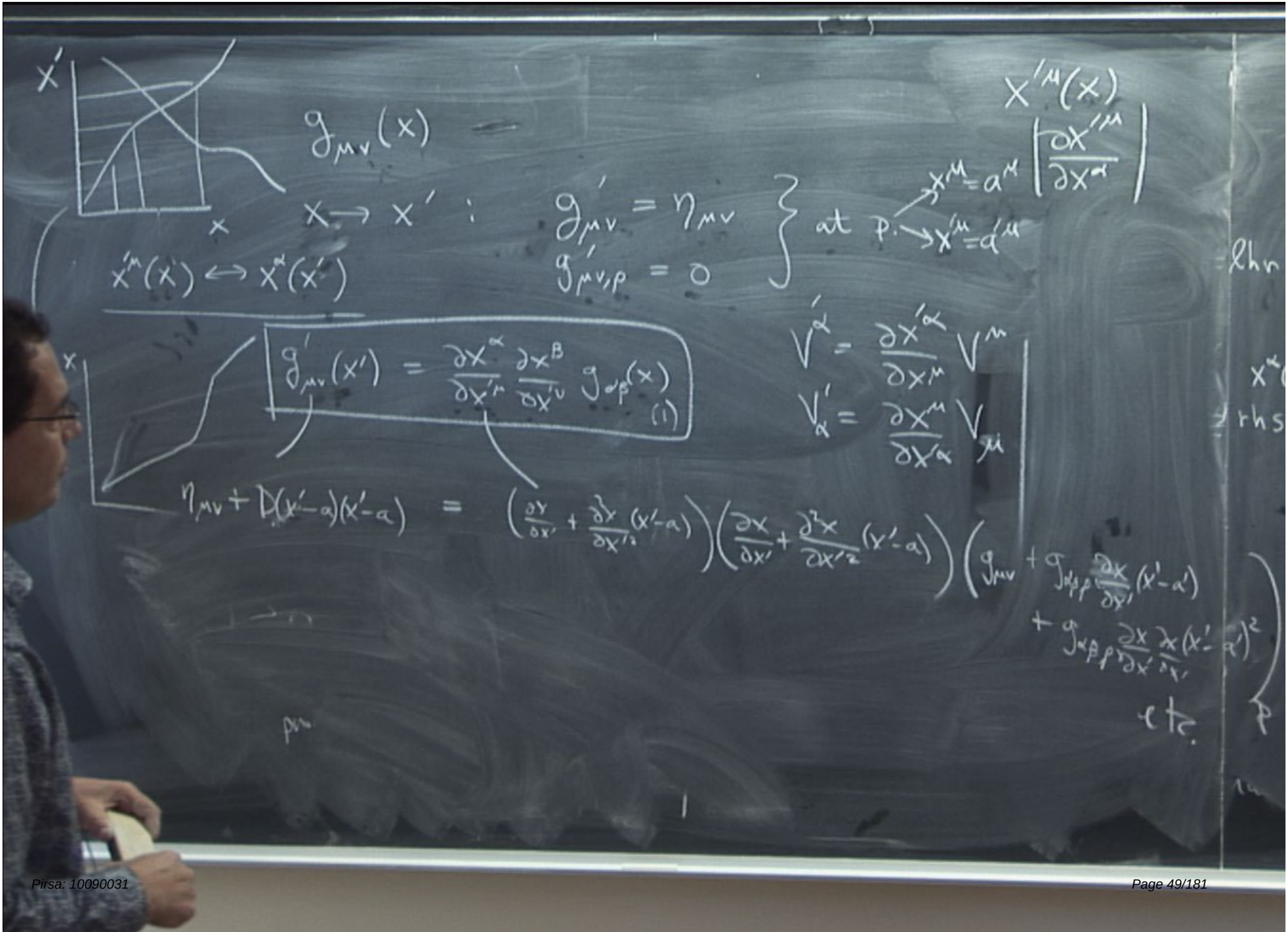
$\frac{\partial x^\alpha(x')}{\partial x'^\mu} = \left.\frac{\partial x^\alpha}{\partial x'^\mu}\right|_p + \left.\frac{\partial^2 x}{\partial x' \partial x'}\right|_p (x'-a)$

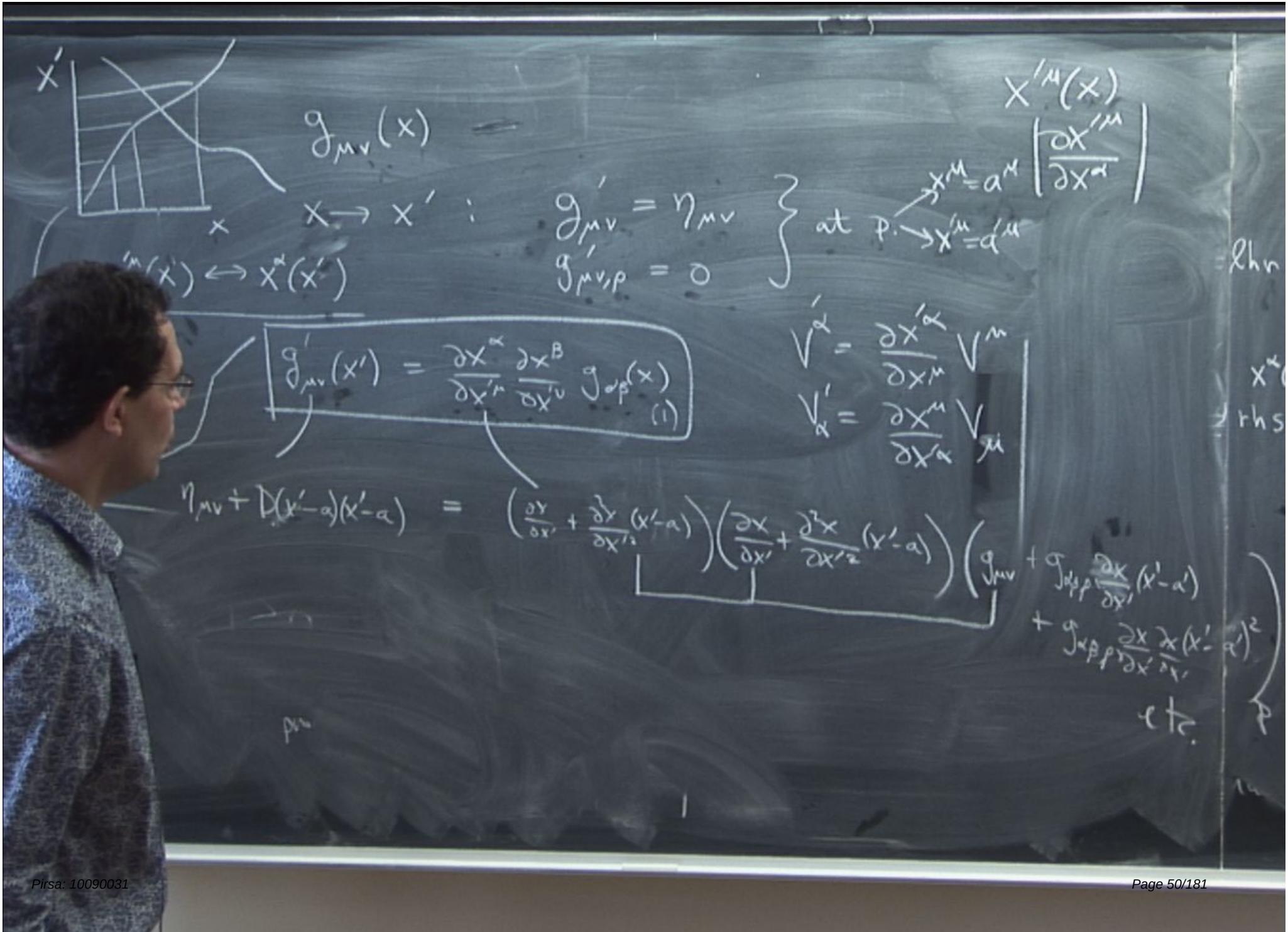
$g_{\alpha\beta}(x) = g_{\alpha\beta}\big|_p + g_{\alpha\beta,\rho}\big|_p (x^\rho - a^\rho) + \frac{1}{2} g_{\alpha\beta,\rho\delta}\big|_p (x^\rho - a^\rho)(x^\delta - a^\delta) + \cdots$

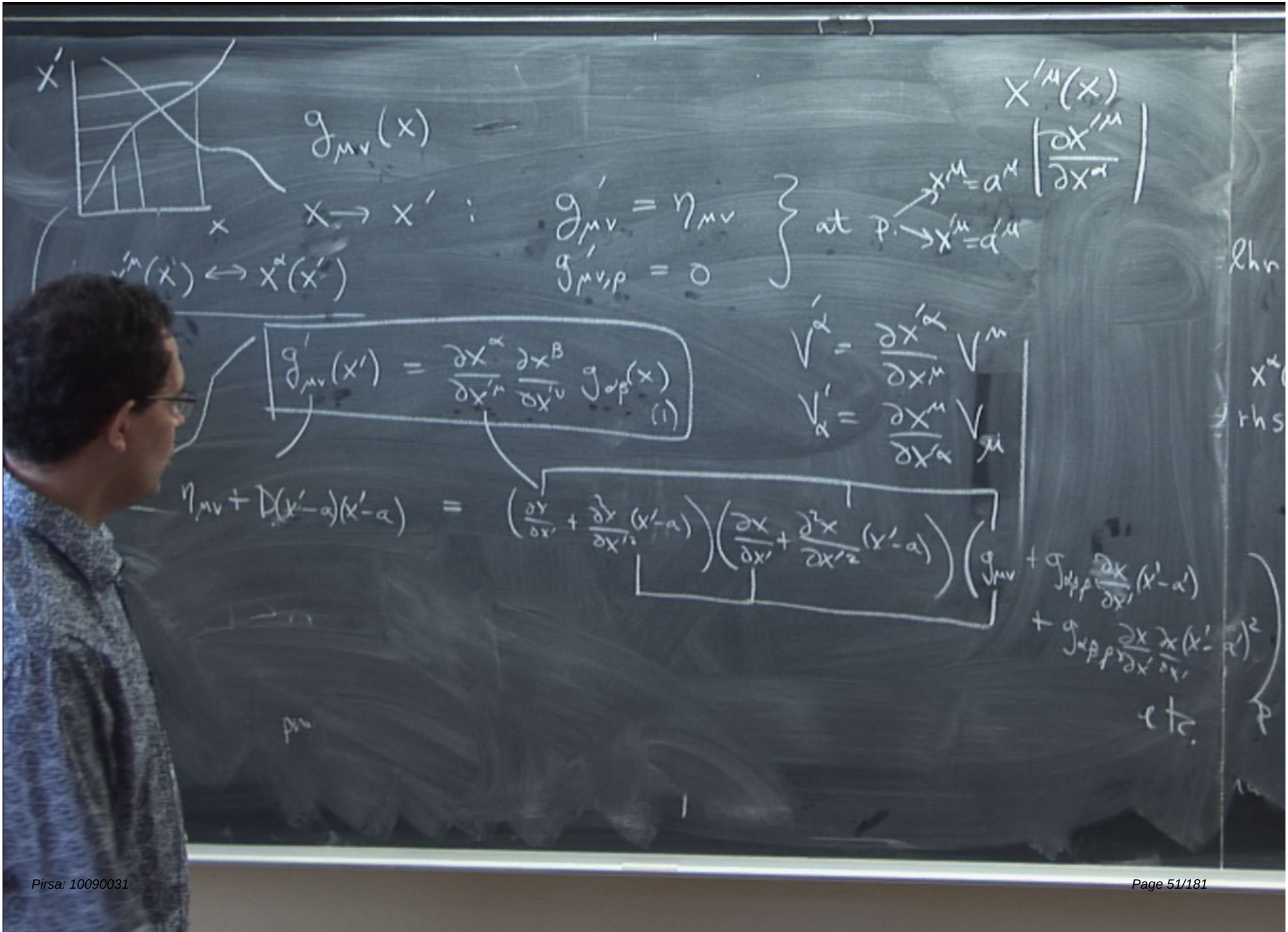
P $\begin{cases} x^\mu = a^\mu \\ x'^\mu = a'^\mu \end{cases}$

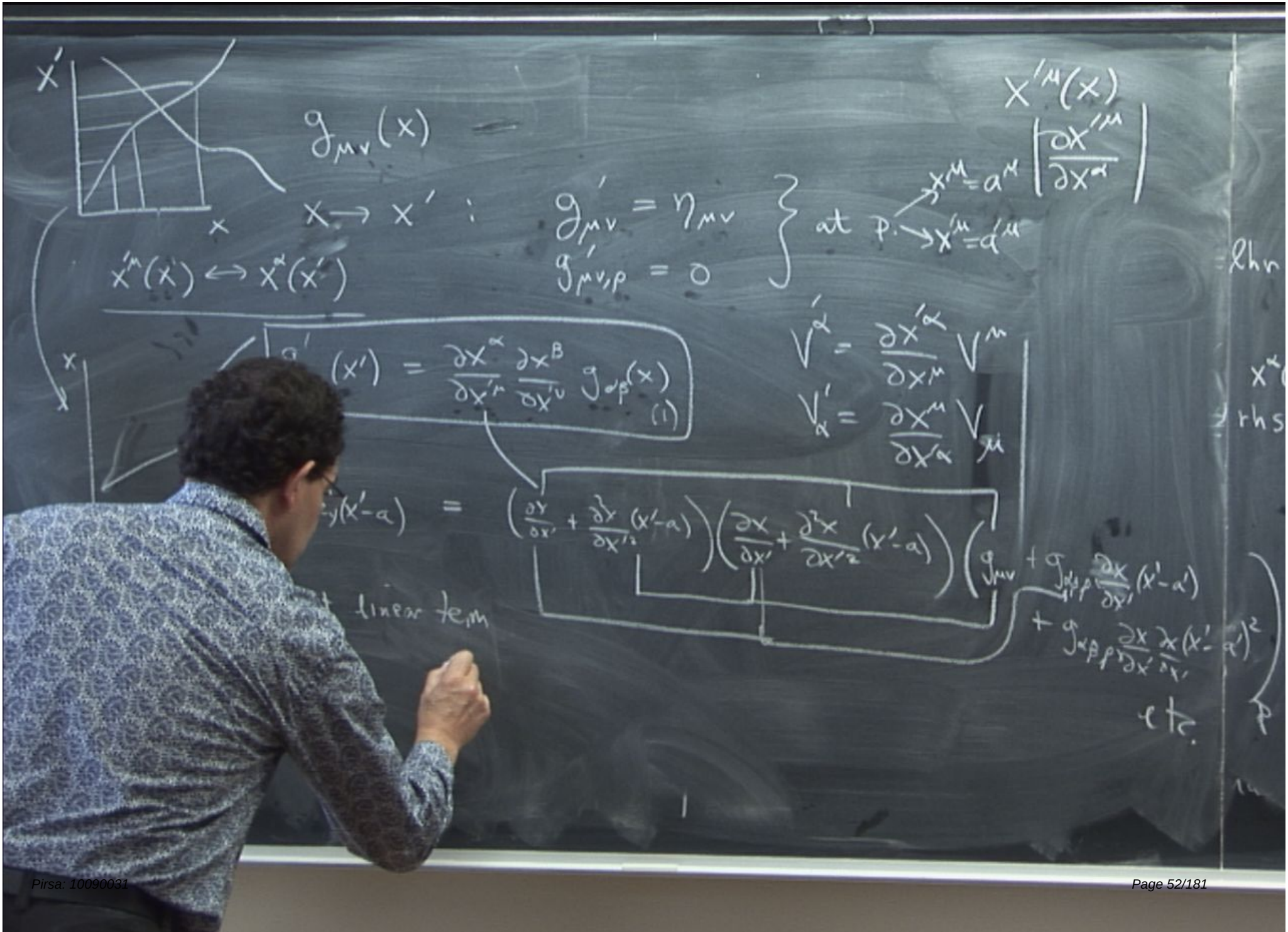

$g_{\mu\nu}(x)$
$x \to x'$:
$g'_{\mu\nu} = \eta_{\mu\nu}$
$g'_{\mu\nu,\rho} = 0$
at P.
$x^\mu = a^\mu$
$x'^\mu = a'^\mu$
$x'^\mu(x)$
$\left|\frac{\partial x'^\mu}{\partial x^\alpha}\right|$
$x'^\mu(x') \leftrightarrow x^\mu$
$\frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$ (1)
$V'^\alpha = \frac{\partial x'^\alpha}{\partial x^\mu} V^\mu$
$V'_\alpha = \frac{\partial x^\mu}{\partial x'^\alpha} V_\mu$

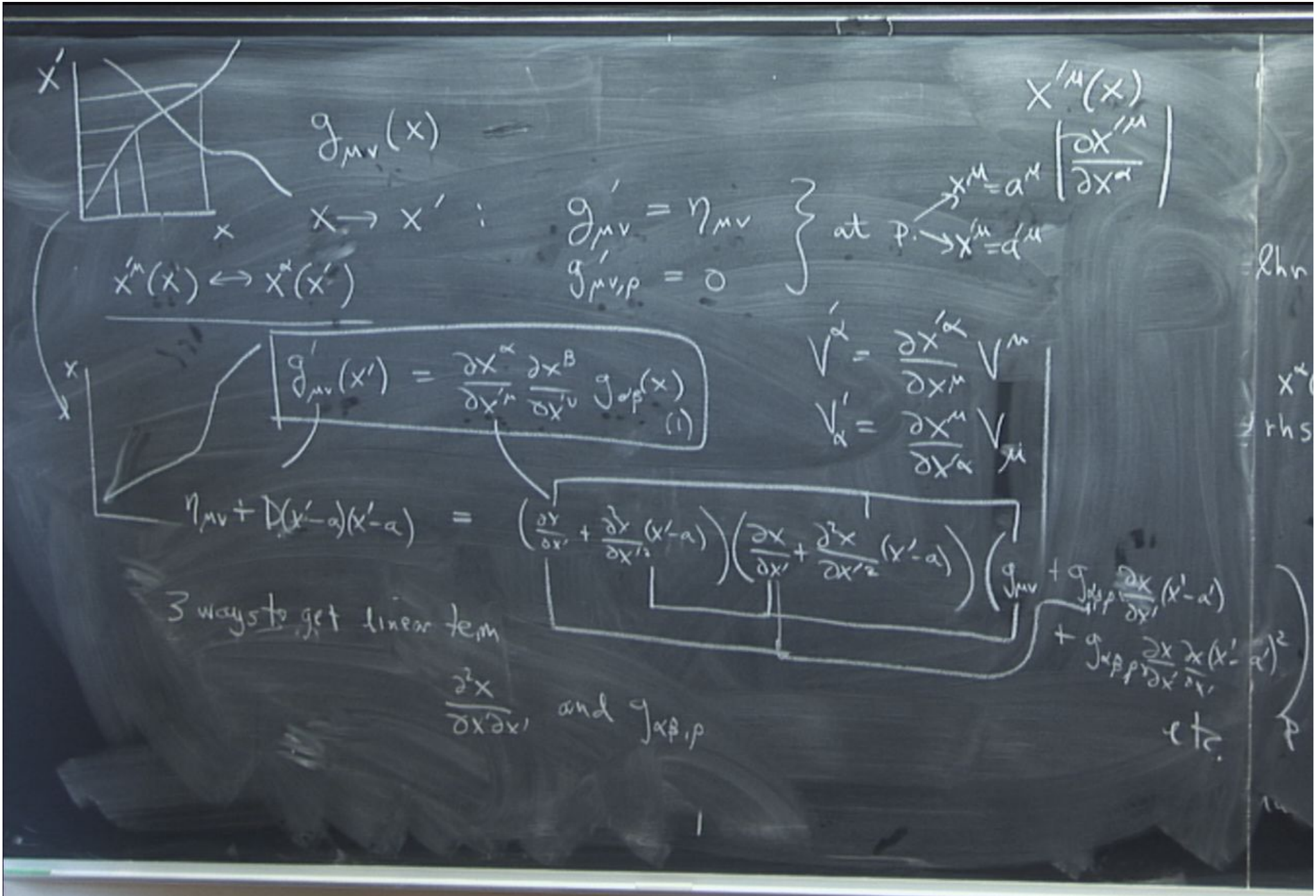

$g_{\mu\nu}(x)$
$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$, $g'_{\mu\nu,\rho} = 0$ at P.
$x'^{\mu}(x)$
$x^{\mu} = a^{\mu}$
$x'^{\mu} = a'^{\mu}$
$x'^{\mu}(x') \leftrightarrow x^{\alpha}(x')$
$g'_{\mu\nu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$ (1)
$V'^{\alpha} = \frac{\partial x'^{\alpha}}{\partial x^{\mu}} V^{\mu}$
$V'_{\alpha} = \frac{\partial x^{\mu}}{\partial x'^{\alpha}} V_{\mu}$
$\eta_{\mu\nu} + D(x'-a)(x'-a) = \left(\frac{\partial x}{\partial x'} + \frac{\partial^2 x}{\partial x'^2}(x'-a)\right)\left(\frac{\partial x}{\partial x'} + \frac{\partial^2 x}{\partial x'^2}(x'-a)\right)\left(g_{\mu\nu} + g_{\alpha\beta,\rho}(x\right.$

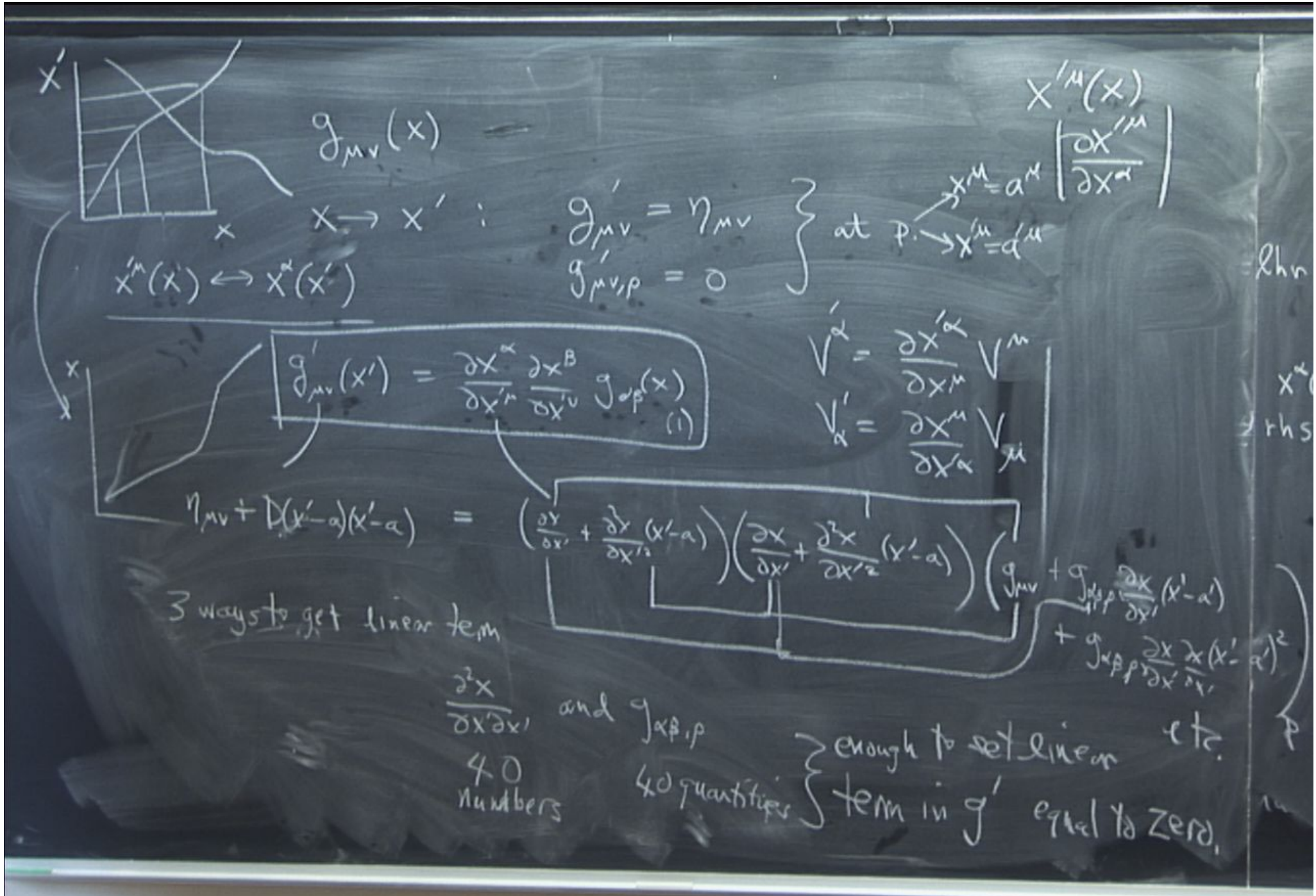

$g_{\mu\nu}(x)$
$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$, $g'_{\mu\nu,\rho} = 0$ at P.
$x'^{\mu}(x)$
$x^{\mu} = a^{\mu}$, $x'^{\mu} = a'^{\mu}$
$\left|\frac{\partial x'^{\mu}}{\partial x^{\alpha}}\right|$
$x'^{\mu}(x') \leftrightarrow x^{\alpha}(x')$
$g'_{\mu\nu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$ (1)
$V'^{\alpha} = \frac{\partial x'^{\alpha}}{\partial x^{\mu}} V^{\mu}$
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$\eta_{\mu\nu} + D(x'-a)(x'-a) = \left(\frac{\partial x}{\partial x'} + \frac{\partial^2 x}{\partial x'^2}(x'-a)\right)\left(\frac{\partial x}{\partial x'} + \frac{\partial^2 x}{\partial x'^2}(x'-a)\right)\left(g_{\mu\nu} + g_{\alpha\beta,\rho}\frac{\partial x}{\partial x'}(x'-a)\right.$
$\left. + g_{\alpha\beta,\rho\gamma}\frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'}(x'-a)^2 \right)$
etc.
= lhs
= rhs

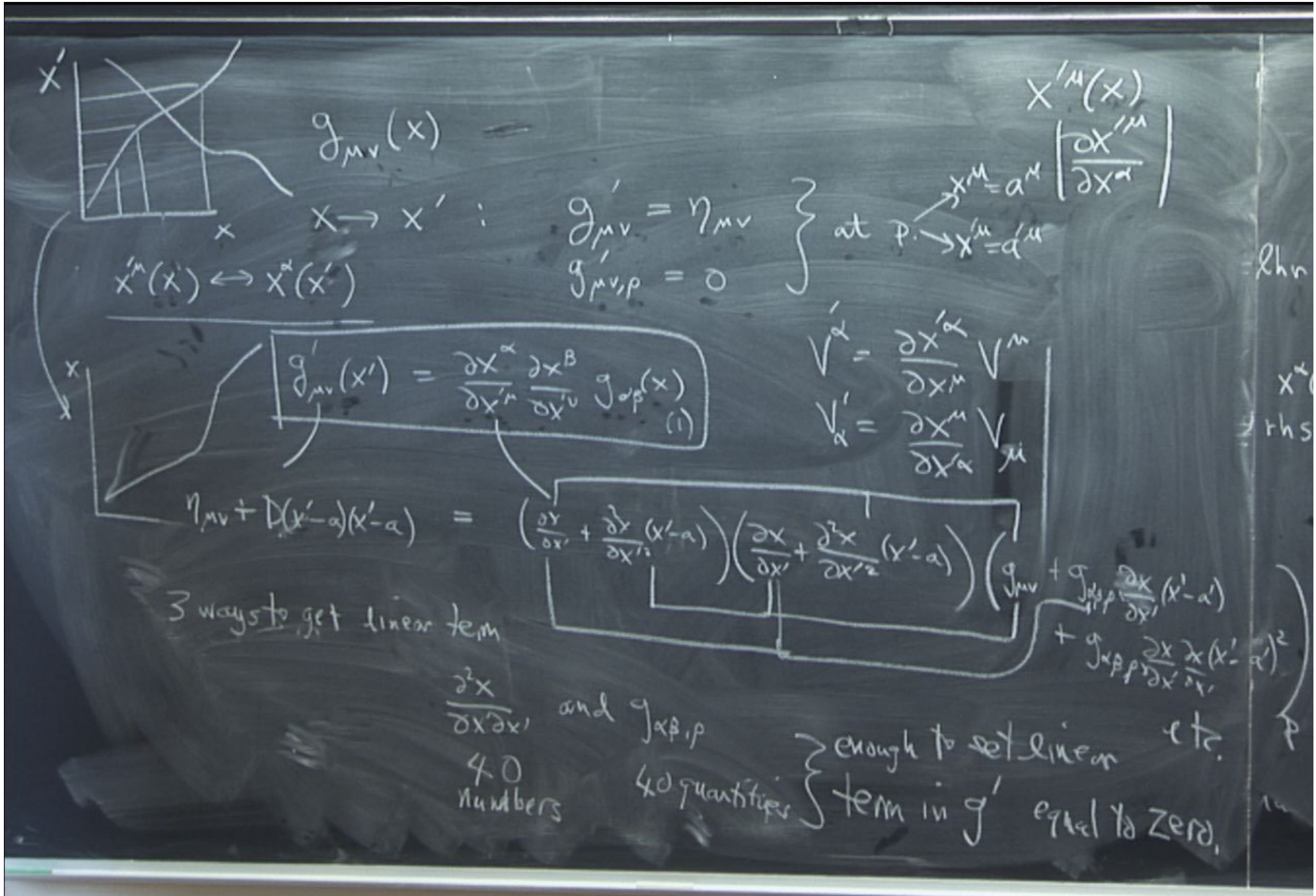

$g_{\mu\nu}(x)$
$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$, $g'_{\mu\nu,\rho} = 0$ at P: $x^\mu = a^\mu$, $x'^\mu = a'^\mu$
$x'^\mu(x)$
$\left|\frac{\partial x'^\mu}{\partial x^\alpha}\right|$
$x'^\mu(x') \leftrightarrow x^\alpha(x')$
$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$ (1)
$V'^\alpha = \frac{\partial x'^\alpha}{\partial x^\mu} V^\mu$
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$+ g_{\alpha\beta,\rho\gamma}\frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'}(x'-a)^2$
$\left.+ \text{etc.}\right)$

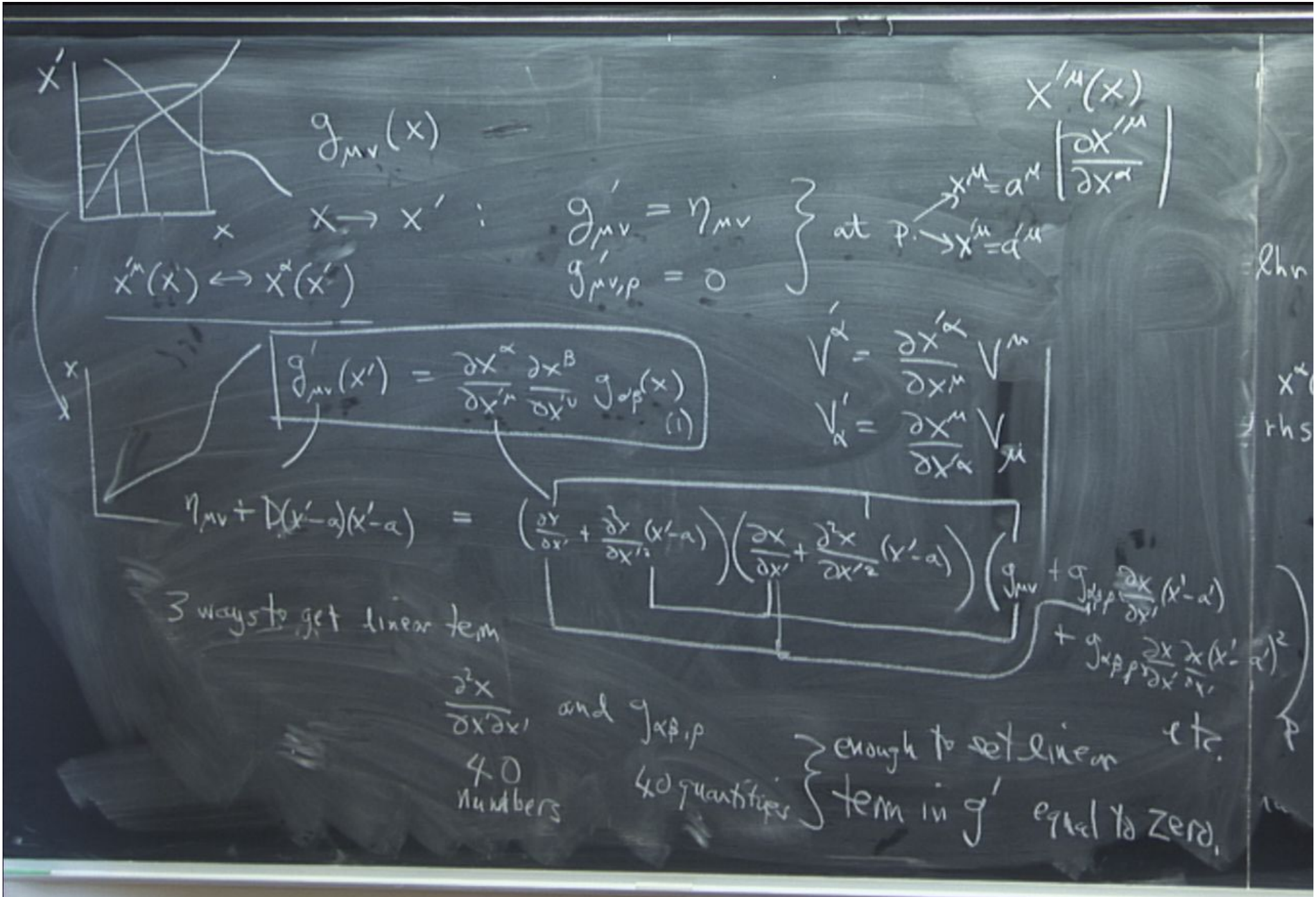

$g_{\mu\nu}(x)$
$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$, $g'_{\mu\nu,\rho} = 0$ at $P: x^\mu = a^\mu \to x'^\mu = a'^\mu$
$x'^\mu(x)$
$x'^\mu(x') \leftrightarrow x^\alpha(x')$
$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$ (1)
$V'^\alpha = \frac{\partial x'^\alpha}{\partial x^\mu} V^\mu$
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$\eta_{\mu\nu} + D(x'-a)(x'-a) = \left(\frac{\partial x}{\partial x'} + \frac{\partial^2 x}{\partial x'^2}(x'-a)\right)\left(\frac{\partial x}{\partial x'} + \frac{\partial^2 x}{\partial x'^2}(x'-a)\right)\left(g_{\mu\nu} + g_{\alpha\beta,\rho}\frac{\partial x}{\partial x'}(x'-a)\right.$
$+ g_{\alpha\beta,\rho\gamma}\frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'}(x'-a)^2$
$+ \text{etc.}\left.\right)$

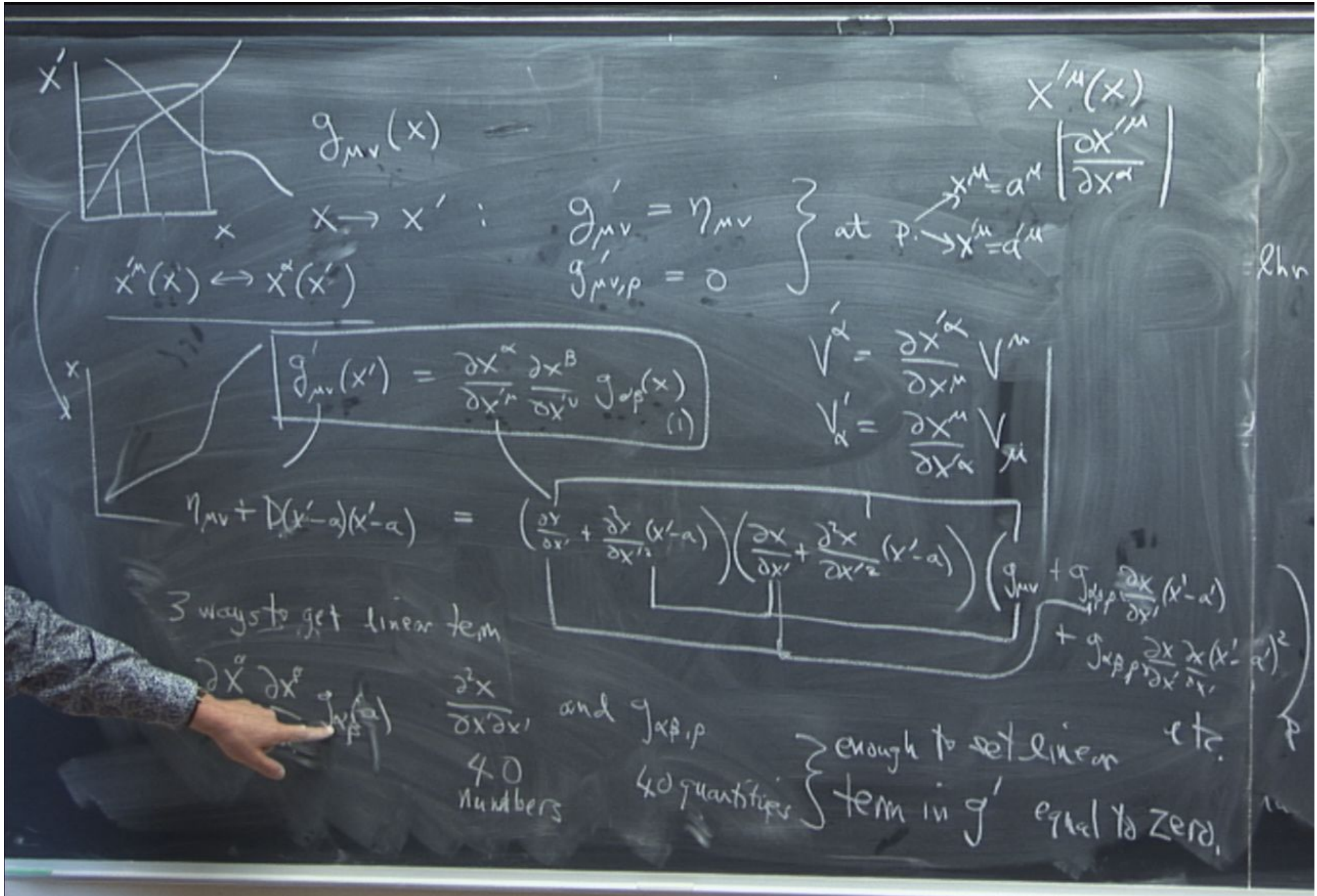

x'
x
$g_{\mu\nu}(x)$
$x \to x'$:
$g'_{\mu\nu} = \eta_{\mu\nu}$
$g'_{\mu\nu,\rho} = 0$
at P : $x^\mu = a^\mu$, $x'^\mu = a'^\mu$
$X'^\mu(x)$
$\left|\frac{\partial x'^\mu}{\partial x^\alpha}\right|$
$x'^\mu(x') \leftrightarrow x^\alpha(x')$
$g'_{\alpha\beta}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$ (1)
$V'^\alpha = \frac{\partial x'^\alpha}{\partial x^\mu} V^\mu$
$V'_\alpha = \frac{\partial x^\mu}{\partial x'^\alpha} V_\mu$
$\left(\frac{\partial x}{\partial x'} + \frac{\partial^2 x}{\partial x'^2}(x'-a)\right)\left(\frac{\partial x}{\partial x'} + \frac{\partial^2 x}{\partial x'^2}(x'-a)\right)\left(g_{\mu\nu} + g_{\mu\nu,\rho}\frac{\partial x}{\partial x'}(x'-a') + g_{\mu\nu,\rho\sigma}\frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'}(x'-a')^2\right)$
linear term
etc.
lhs
rhs

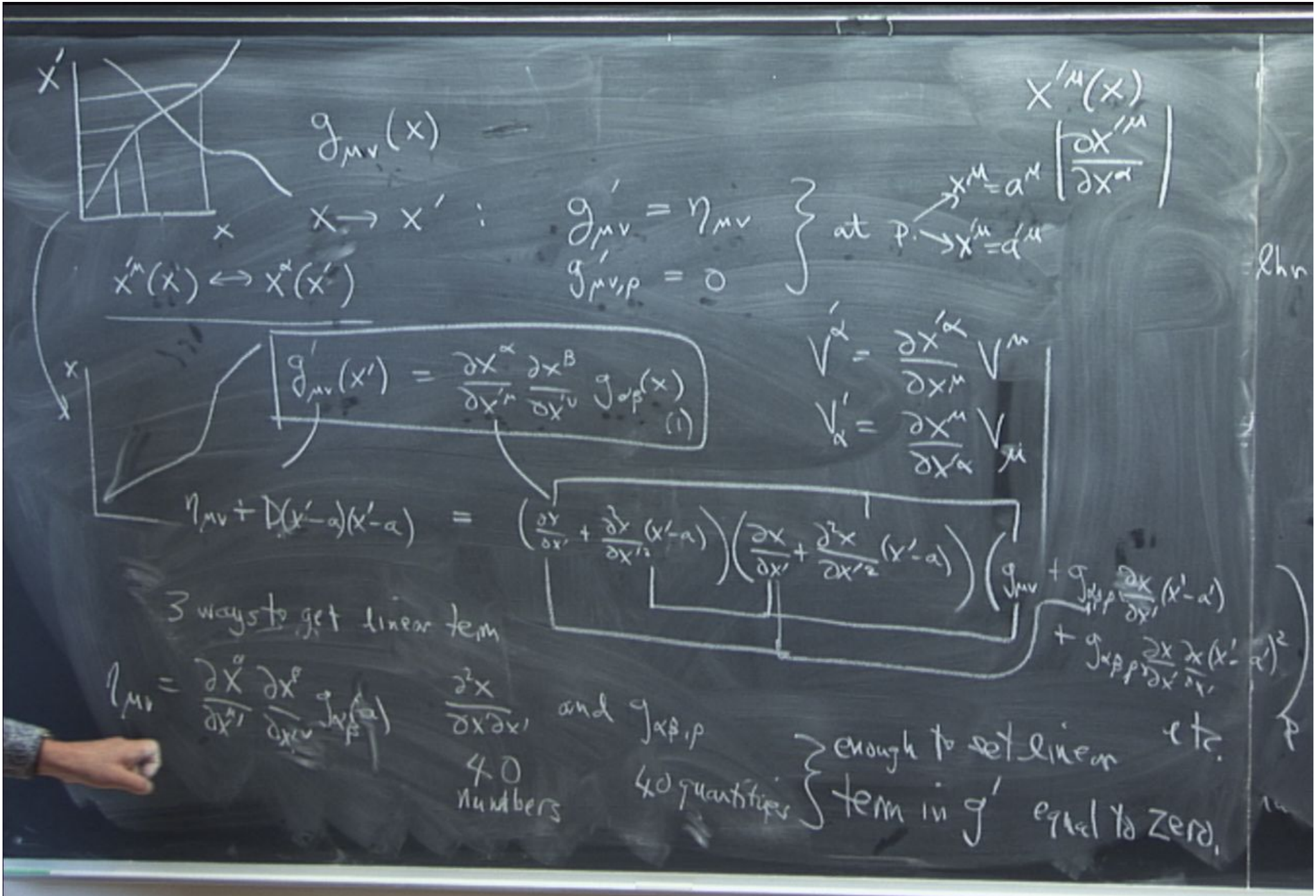

$g_{\mu\nu}(x)$
$x \to x'$: $g'_{\mu\nu} = \eta_{\mu\nu}$, $g'_{\mu\nu,\rho} = 0$ at P
$x'^{\mu}(x') \leftrightarrow x^{\alpha}(x')$
$x'^{\mu}(x)$
$x^{\mu} = a^{\mu}$
$x'^{\mu} = a'^{\mu}$
$g'_{\mu\nu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$ (1)
$V'^{\alpha} = \frac{\partial x'^{\alpha}}{\partial x^{\mu}} V^{\mu}$
$V'_{\alpha} = \frac{\partial x^{\mu}}{\partial x'^{\alpha}} V_{\mu}$
$\eta_{\mu\nu} + D(x'-a)(x'-a) = \left(\frac{\partial x}{\partial x'} + \frac{\partial^2 x}{\partial x'^2}(x'-a)\right)\left(\frac{\partial x}{\partial x'} + \frac{\partial^2 x}{\partial x'^2}(x'-a)\right)\left(g_{\mu\nu} + g_{\alpha\beta,\rho}\frac{\partial x}{\partial x'}(x'-a) + g_{\alpha\beta,\rho}\frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'}(x'-a)^2 + \text{etc.}\right)$
3 ways to get linear term
$\frac{\partial^2 x}{\partial x' \partial x'}$ and $g_{\alpha\beta,\rho}$
lhs
rhs

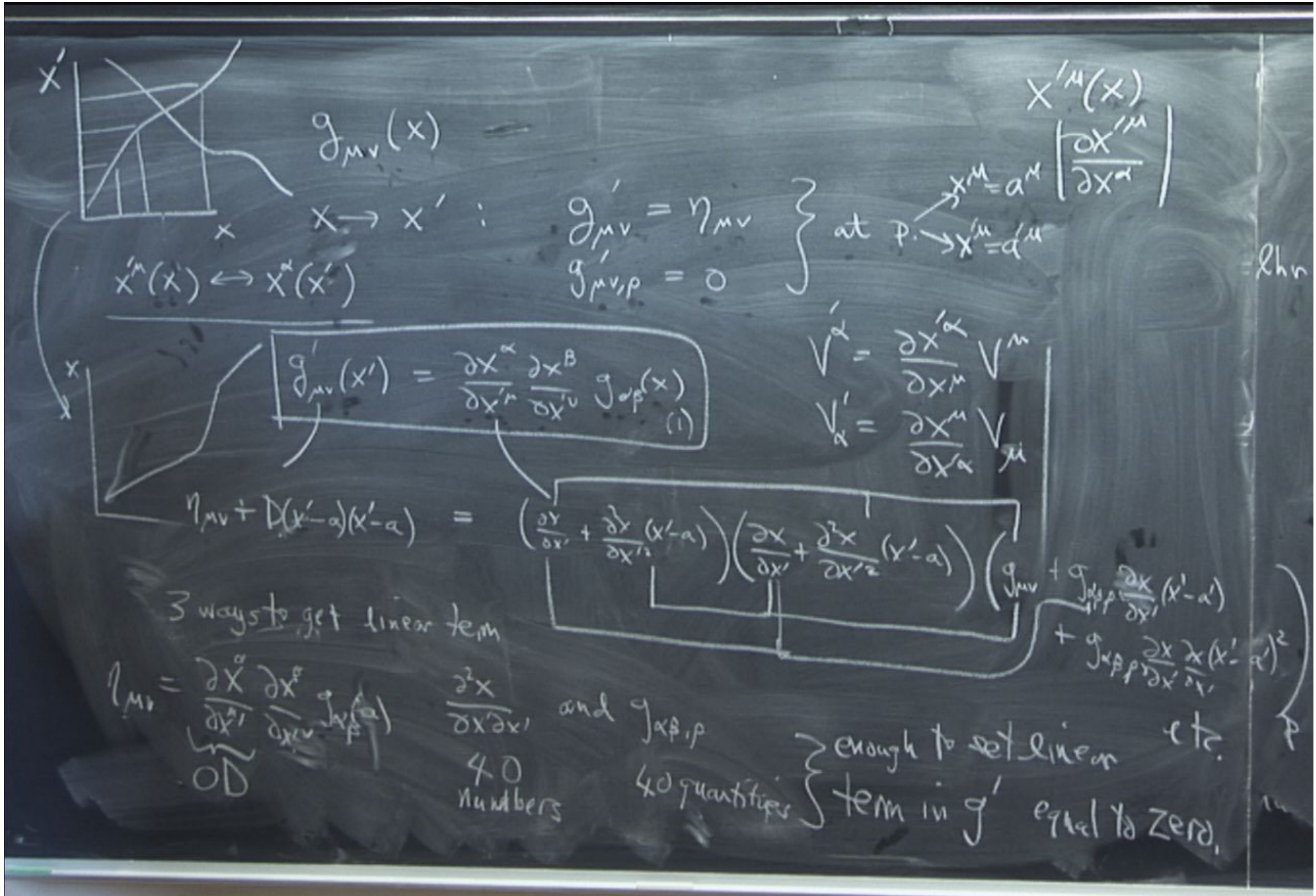

$g_{\mu\nu}(x)$
$x \to x'$:
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at P : $x^\mu = a^\mu$ → $x'^\mu = a'^\mu$
$x'^\mu(x)$
$\left|\frac{\partial x'^\mu}{\partial x^\alpha}\right|$
$x'^\mu(x') \leftrightarrow x^\alpha(x')$
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3 ways to get linear term
$\frac{\partial^2 x}{\partial x' \partial x'}$ and $g_{\alpha\beta,\rho}$
40 numbers
40 quantities
} enough to set linear term in g' equal to zero.
= lhs
x^α
= rhs

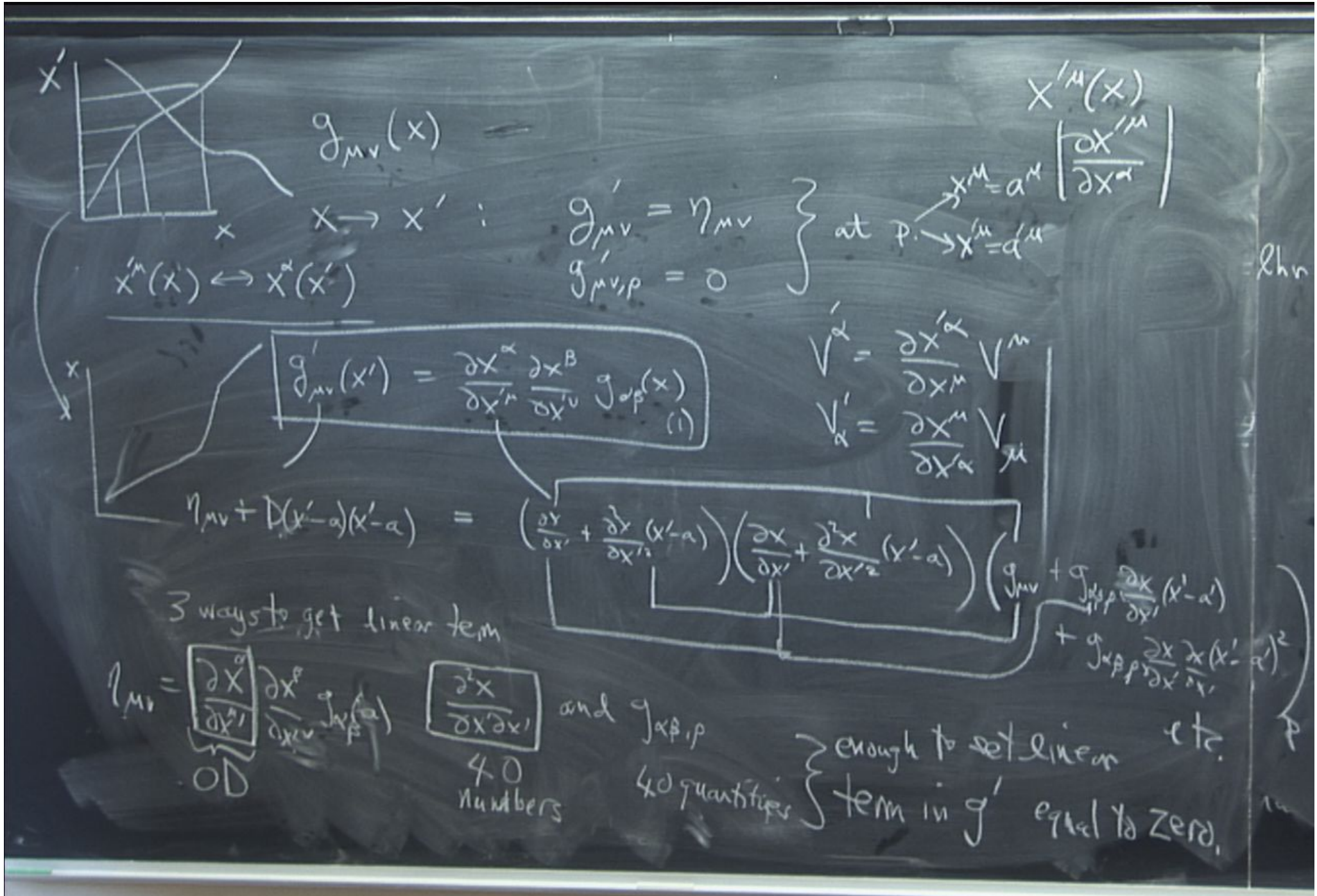

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lhs


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OD
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40 Numbers
and $g_{\alpha\beta,\rho}$
40 quantities
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etc.
$= \text{lhs}$

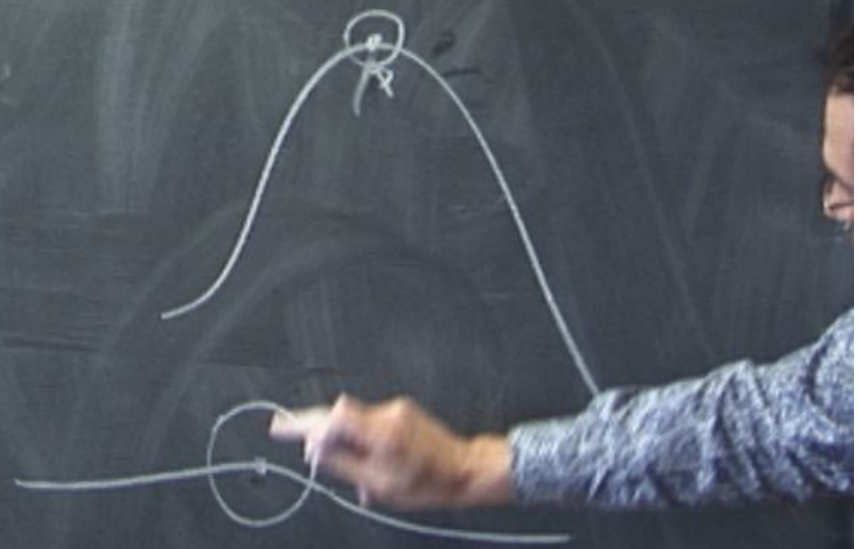

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$\frac{\partial x^\alpha}{\partial x'^\mu}$ 0D
$\frac{\partial^2 x}{\partial x' \partial x'}$ 40 numbers
and $g_{\alpha\beta,\rho}$ 40 quantities
enough to set linear term in g' equal to zero.
etc.
= lhs

$$\text{Then } g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{\frac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_P} (x'-a')^\rho (x'-a')^\lambda + O\left((x'-a')^3\right) + \cdots$$

$$\text{Rhn}\quad g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{\frac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_P} (x'-a')^\rho (x'-a')^\lambda + O\left((x'-a')^3\right) + \cdots$$



$$\text{Rhs } g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{\frac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_P} (x'-a')^\rho (x'-a')^\lambda + O\left((x'-a')^3\right) + \cdots$$



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Can we cancel this too?

$$\frac{\partial^3 x}{\partial x' \partial x' \partial x'}$$

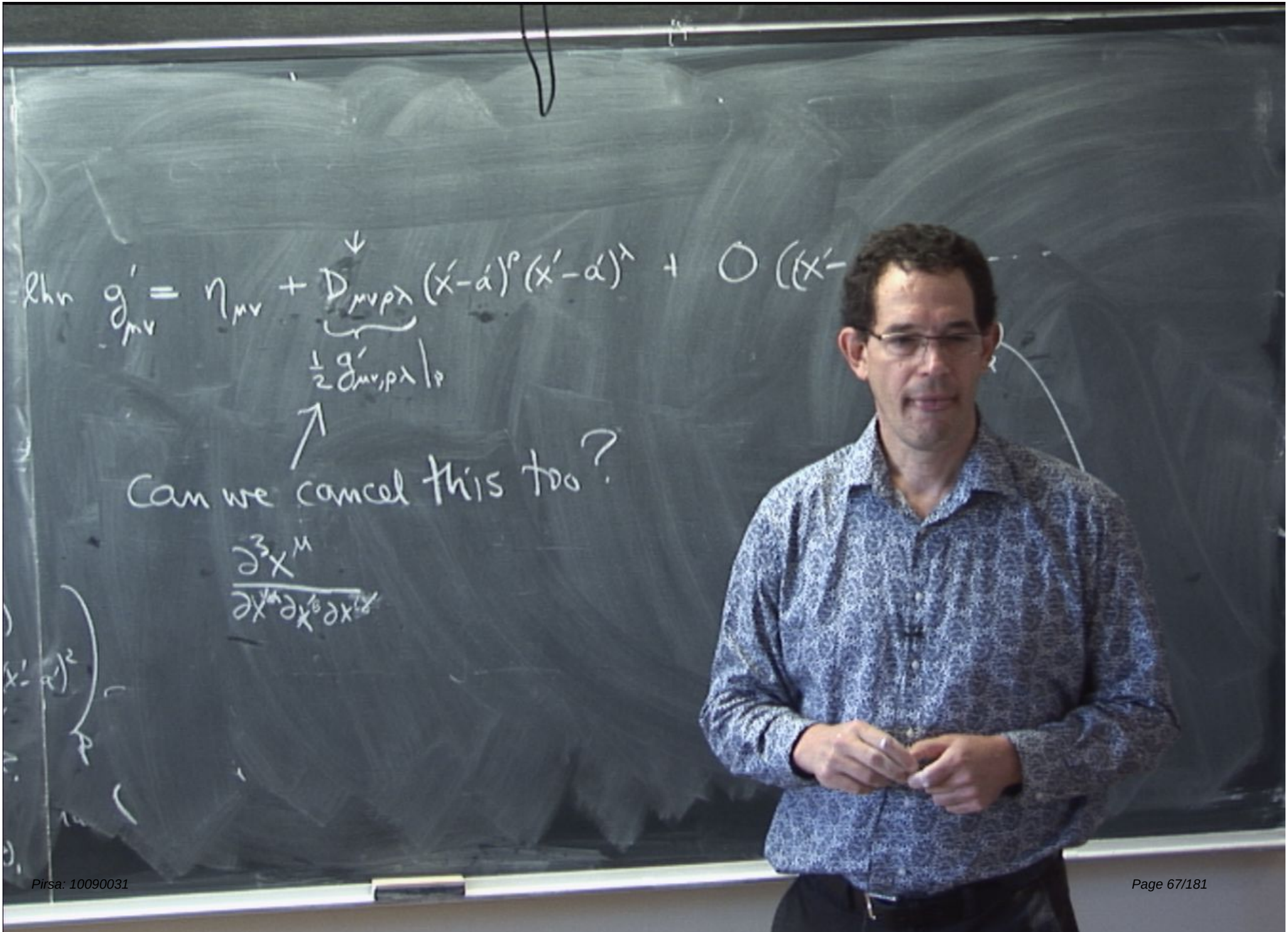


$$\text{Rhs } g'_{\mu\nu} = \eta_{\mu\nu} + \underbrace{D_{\mu\nu\rho\lambda}}_{\frac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_P} (x'-a')^\rho (x'-a')^\lambda + O\left((x'-a')^3\right) + \cdots$$

Can we cancel this too?

$$\frac{\partial^3 x^\mu}{\partial x'^\alpha \partial x'^\beta \partial x'^\gamma}$$

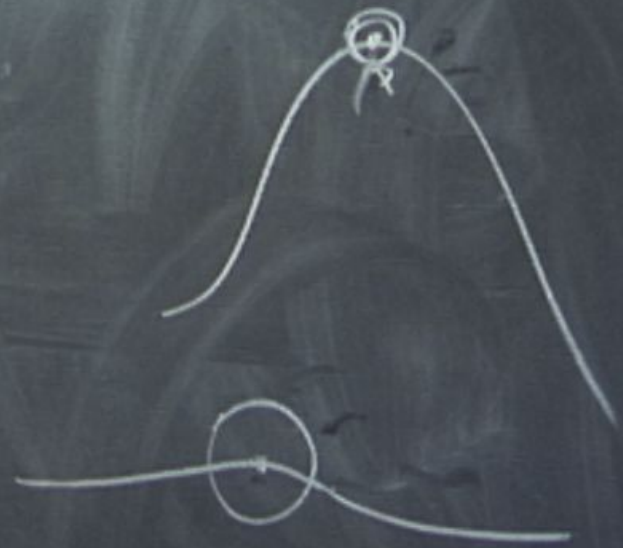



$g'_{\mu\nu} = \eta_{\mu\nu} + D_{\mu\nu\rho\lambda}(x'-a')^\rho(x'-a')^\lambda + O((x'-$
$\frac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_p$
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Can we cancel this too?

$$\frac{\partial^3 x^\mu}{\partial x'^\alpha \partial x'^\beta \partial x'^\gamma} < 4^4$$



$$\text{Then } g'_{\mu\nu} = \eta_{\mu\nu} + D_{\mu\nu\rho\lambda}(x'-a')^\rho(x'-a')^\lambda + O((x'-a')^3) + \cdots$$

$$D_{\mu\nu\rho\lambda} = \frac{1}{2} g'_{\mu\nu,\rho\lambda}\Big|_P$$

Can we cancel this too?

$$\frac{\partial^3 x^\mu}{\partial x'^\alpha \partial x'^\beta \partial x'^\delta} < 4^4$$



$$A_{\alpha\beta\gamma} = A_{(\alpha\beta\gamma)}$$

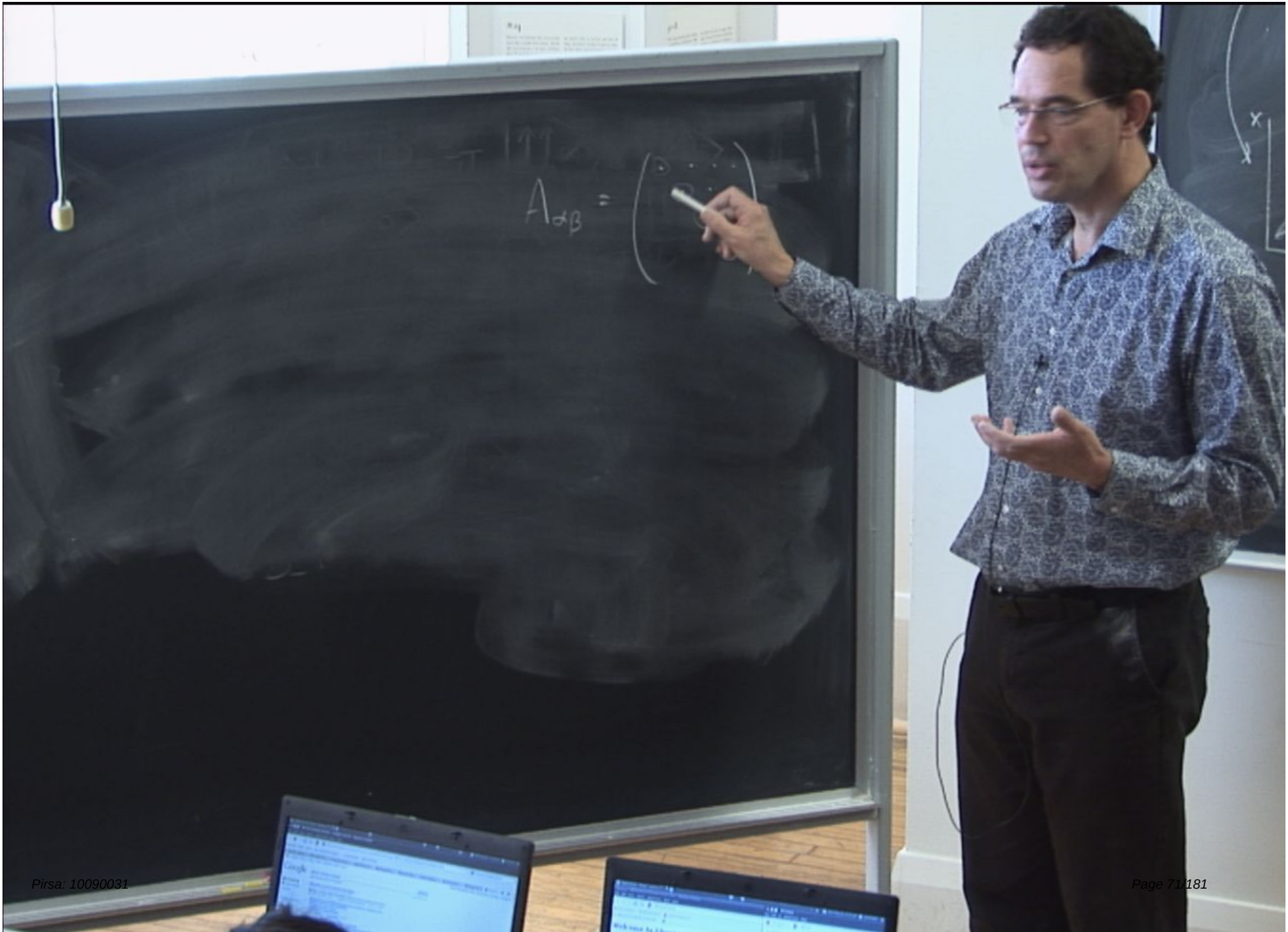
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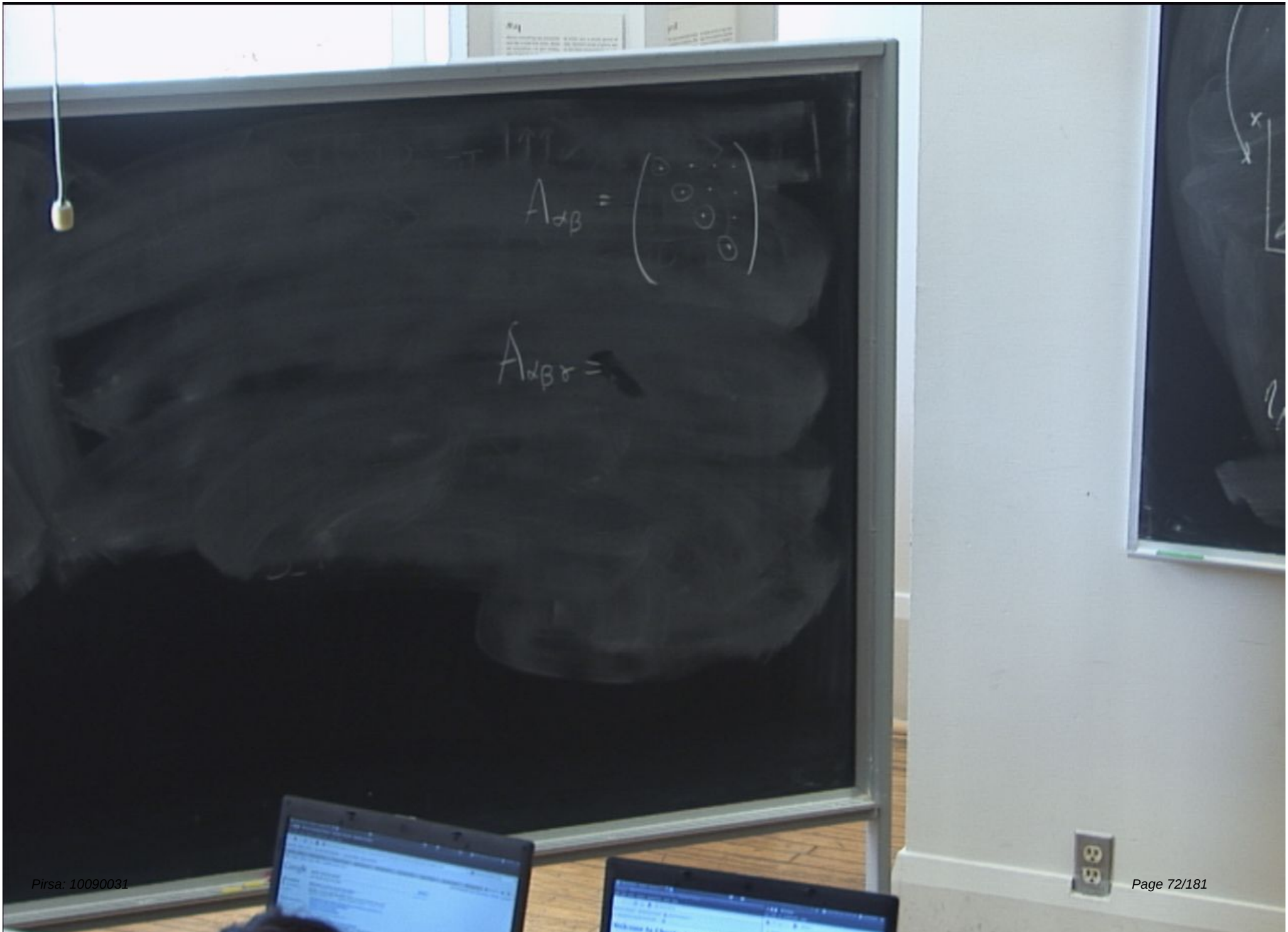
$$D_{\mu\nu\rho\lambda} = \tfrac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_P$$

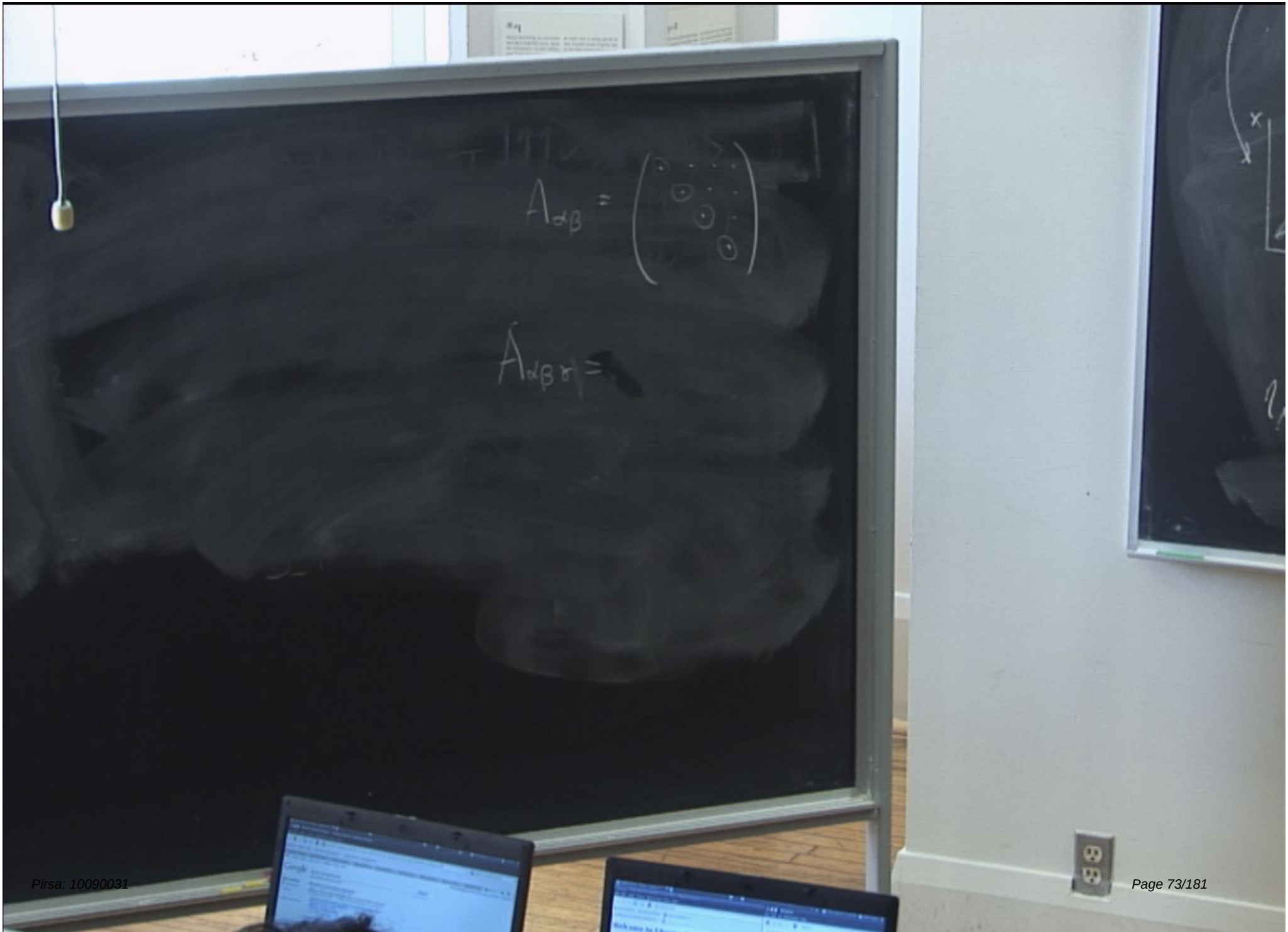
Can we cancel this too?

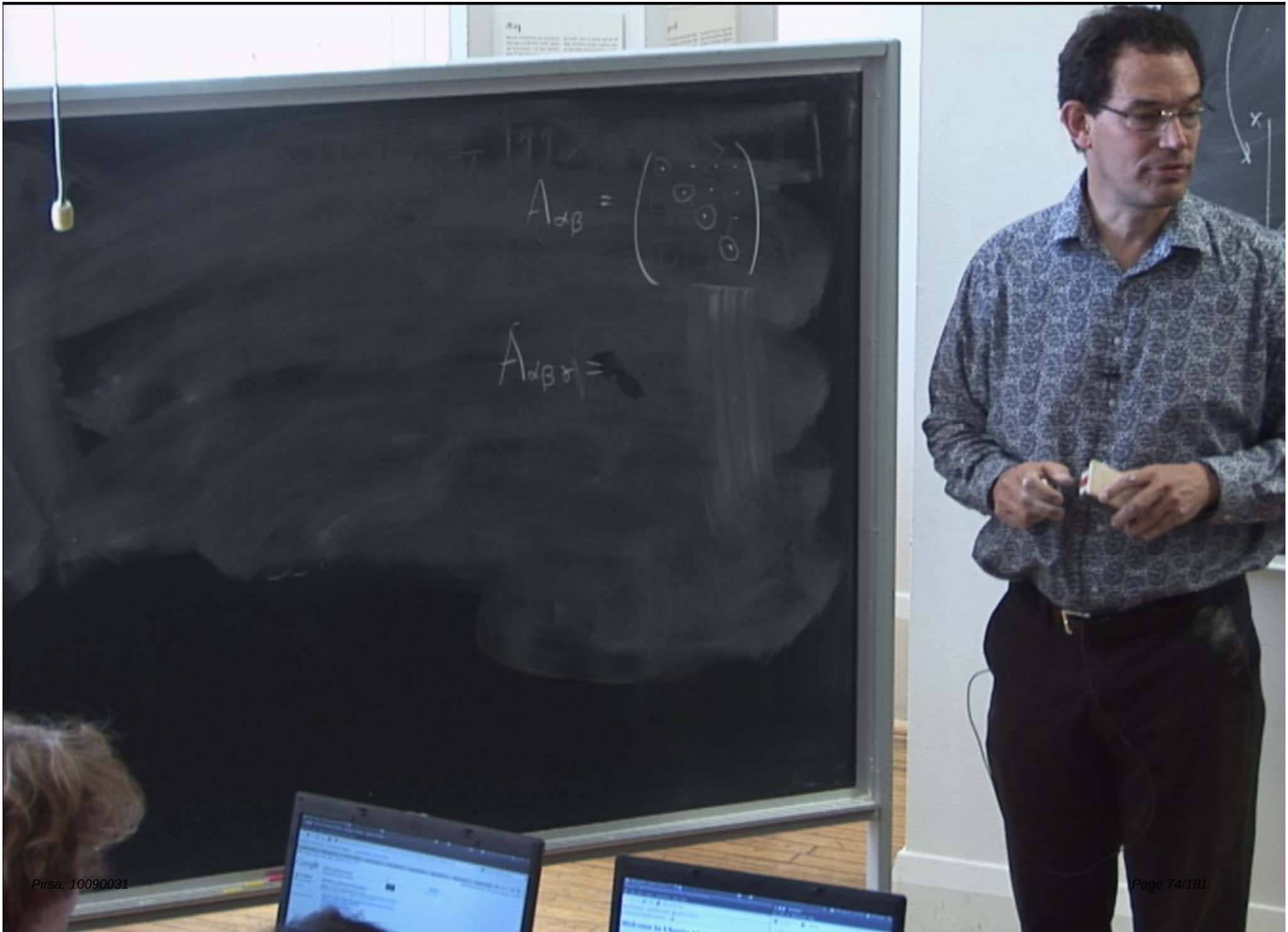
$$\frac{\partial^3 x^{\mu}}{\partial x'^{\alpha}\partial x'^{\beta}\partial x'^{\gamma}} < 4^4$$

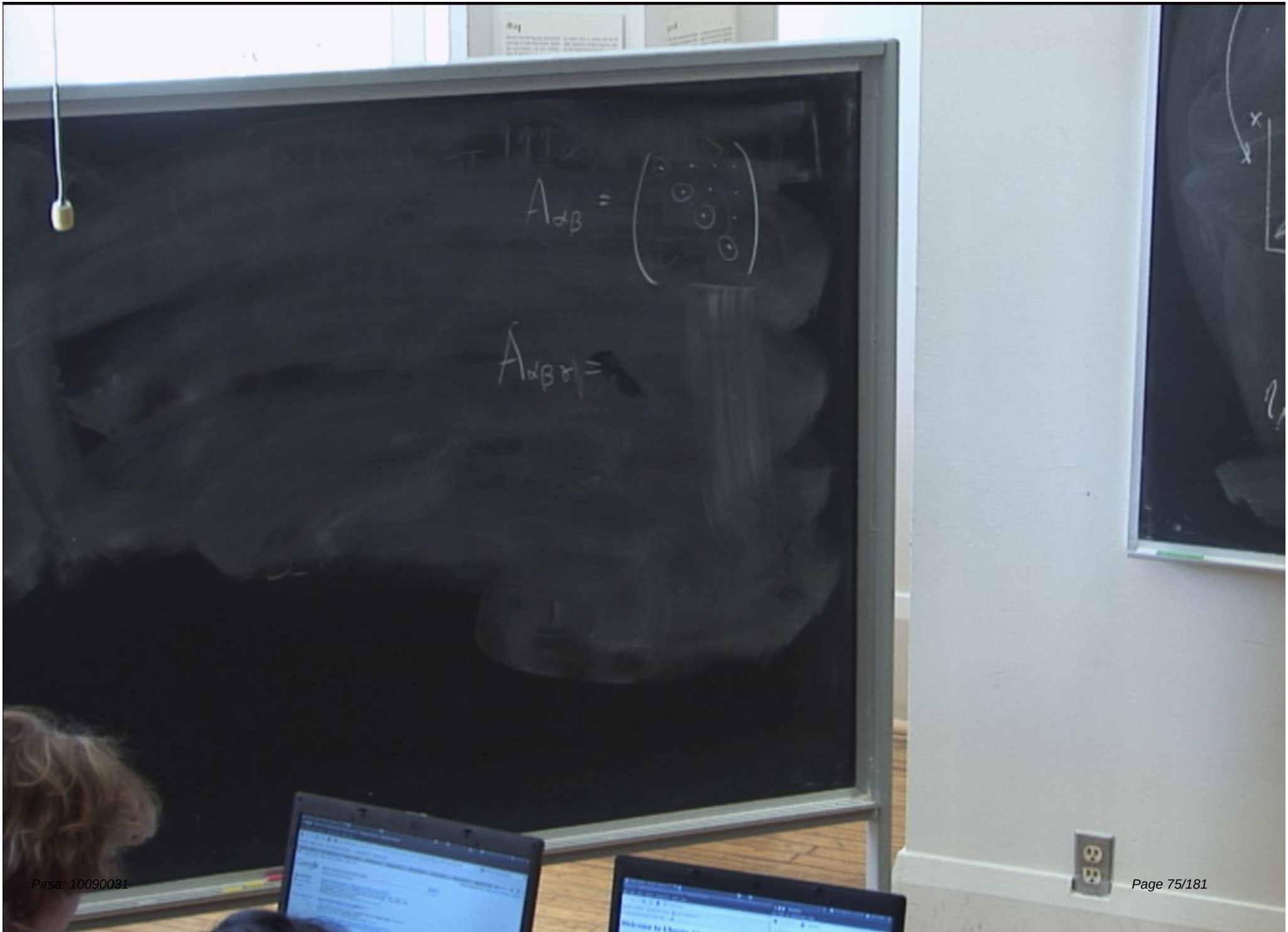


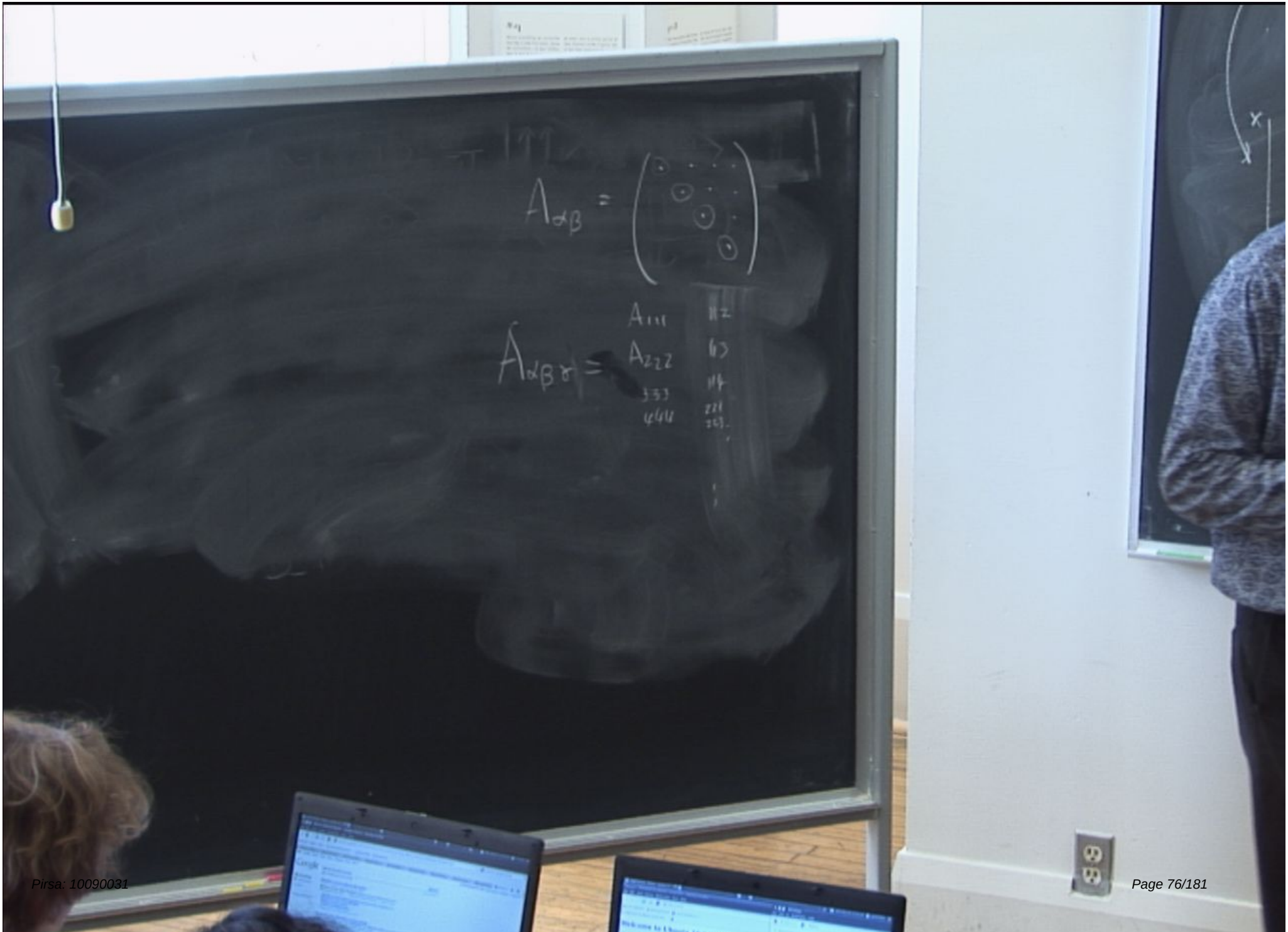

$A_{\alpha\beta} =$

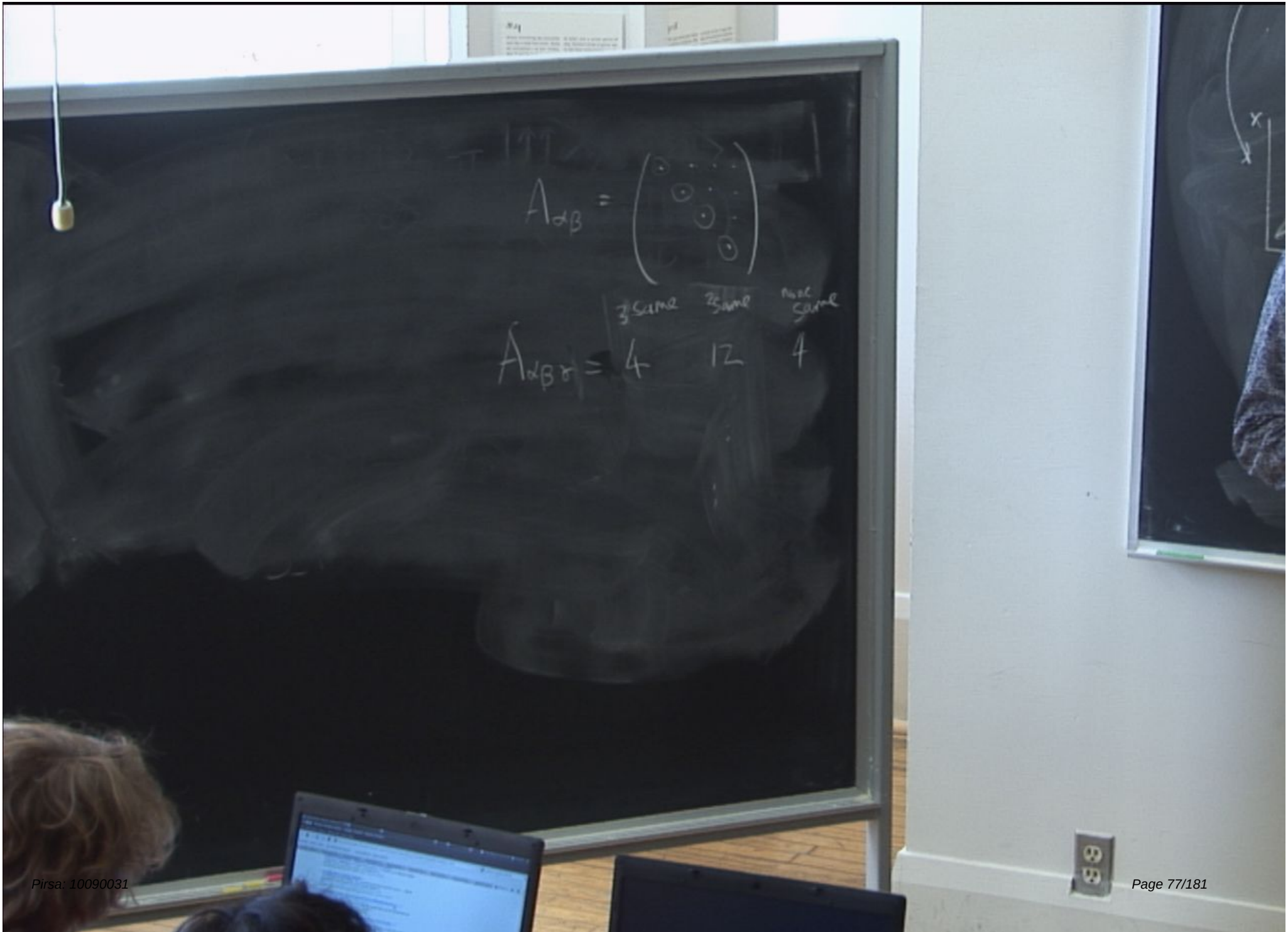

$A_{\alpha\beta} =$
$A_{\alpha\beta\gamma} =$

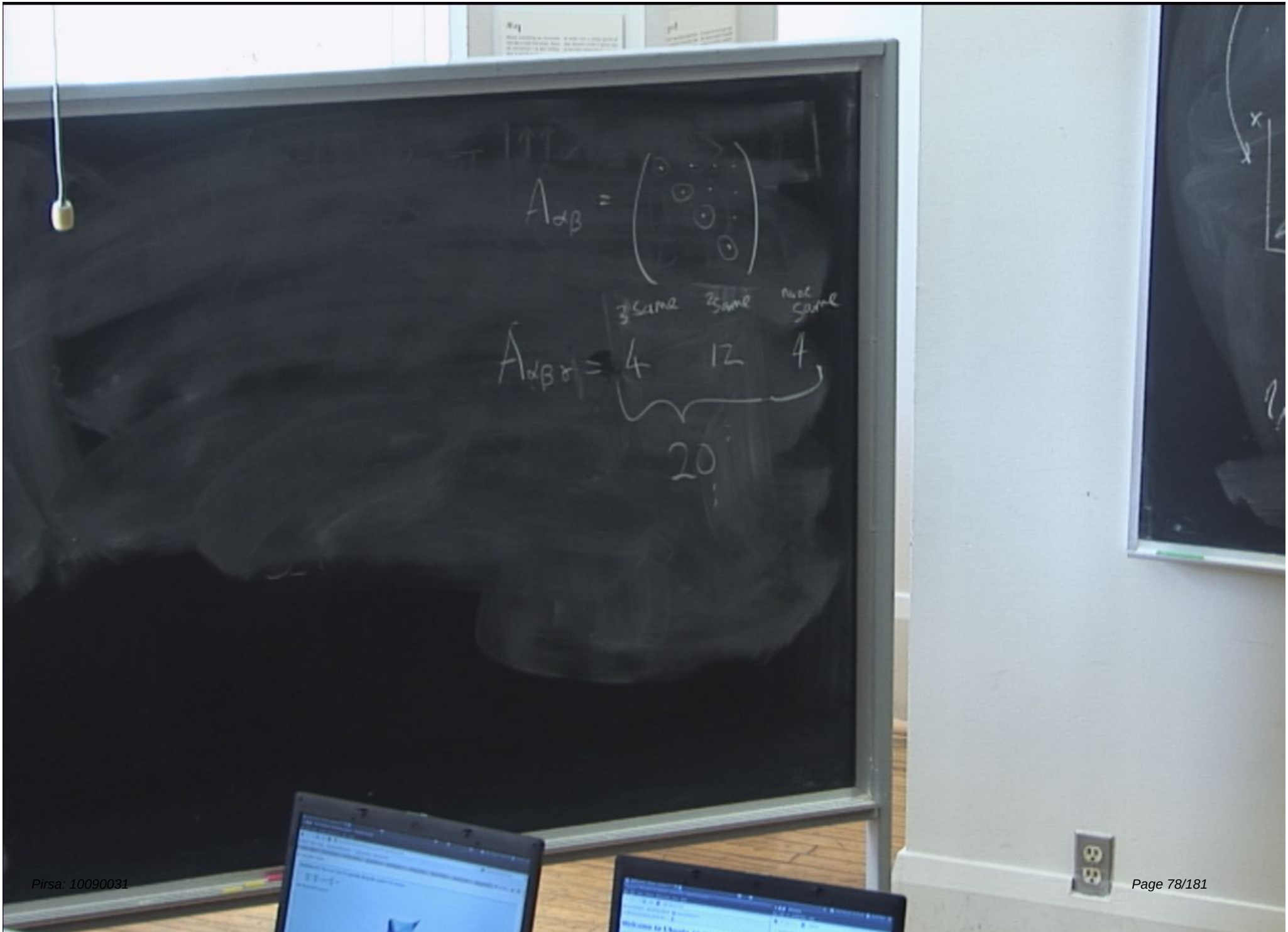

$A_{\alpha\beta} =$
$A_{\alpha\beta\gamma} =$


$A_{\alpha\beta} =$
$A_{\alpha\beta\gamma} =$


$A_{\alpha\beta} =$
$A_{\alpha\beta\gamma} =$


$A_{\alpha\beta} =$
$A_{\alpha\beta\gamma} =$
A_{111}
A_{222}
333
444
112
113
114
221
223
X


$A_{\alpha\beta} =$
3 same
2 same
none same
$A_{\alpha\beta\gamma} =$ 4 12 4


$A_{\alpha\beta}$ =
3 same
2 same
none same
$A_{\alpha\beta\gamma}$ = 4 12 4
20

$$A_{\alpha\beta\gamma} = A_{(\alpha\beta\gamma)}$$

$$\text{Then } g'_{\mu\nu} = \eta_{\mu\nu} + D_{\mu\nu\rho\lambda}(x'-a')^\rho(x'-a')^\lambda + O((x'-a')^3) + \cdots$$

$$D_{\mu\nu\rho\lambda} = \tfrac{1}{2} g'_{\mu\nu,\rho\lambda}\big|_P$$

Can we cancel this too?

$$\left(\frac{\partial^3 x^\mu}{\partial x'^\alpha \partial x'^\beta \partial x'^\gamma}\right) < 4^4$$

# of indept comp^{n}ts = 80.



$A_{\alpha\beta\gamma} = A_{(\alpha\beta\gamma)}$

$\ln g'_{\mu\nu} = \eta_{\mu\nu} + D_{\mu\nu\rho\lambda}(x'-a')^{\rho}(x'-a')^{\lambda} + O((x'-a')^3) + \cdots$

$\frac{1}{2} g'_{\mu\nu}$

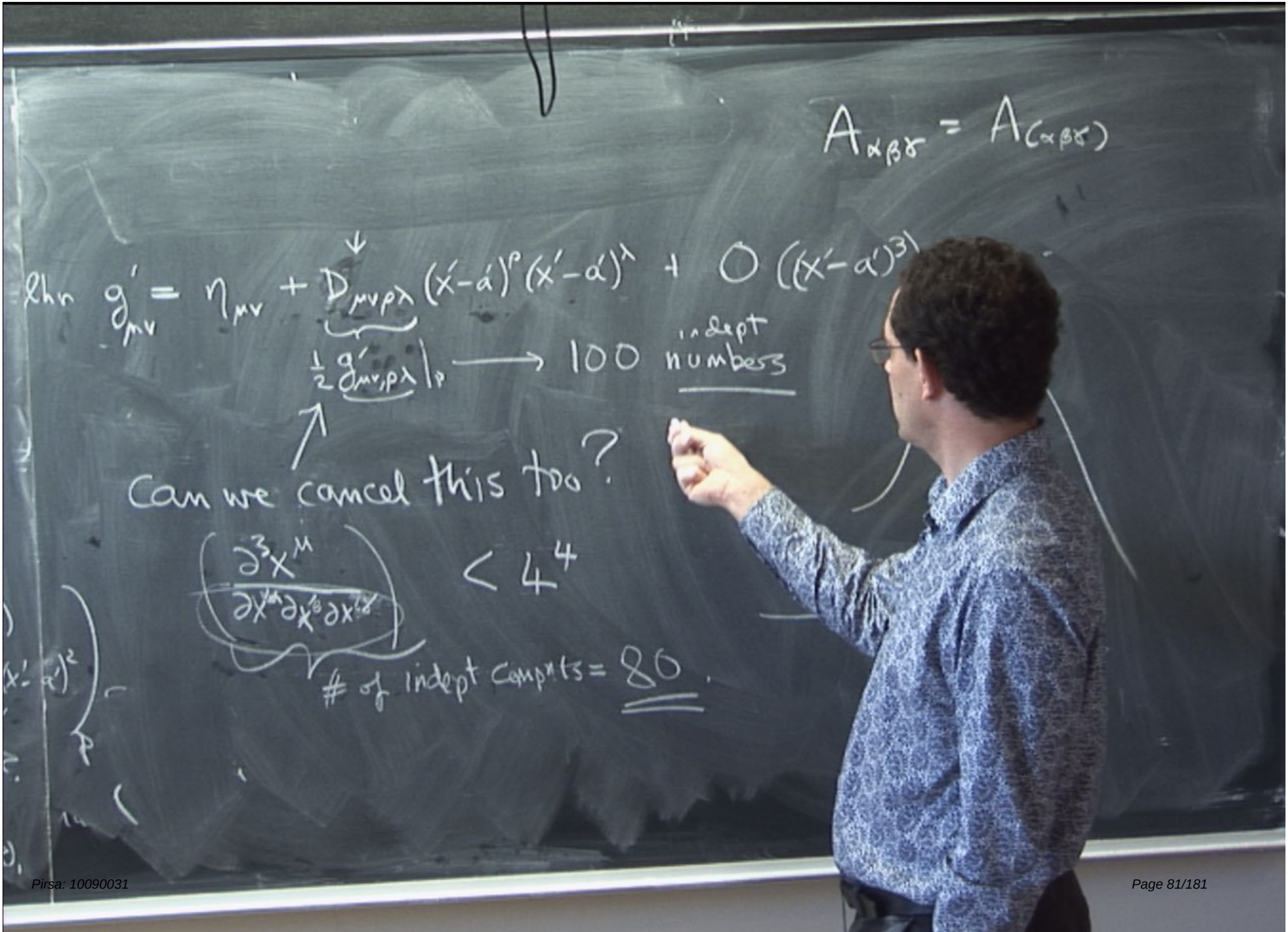
cancel this too?

$< 4^4$

$\frac{\partial^3 x^{\mu}}{\partial x'^{\alpha} \partial x'^{\beta} \partial x'^{\gamma}}$

# of indept comp'ts = 80.




$A_{\alpha\beta\gamma} = A_{(\alpha\beta\gamma)}$
$g'_{\mu\nu} = \eta_{\mu\nu} + D_{\mu\nu\rho\lambda}(x'-a')^\rho(x'-a')^\lambda + O((x'-a')^3)$
$\frac{1}{2} g'_{\mu\nu,\rho\lambda}|_p \longrightarrow 100$ indept numbers
Can we cancel this too?
$\frac{\partial^3 x^\mu}{\partial x'^\alpha \partial x'^\beta \partial x'^\gamma} < 4^4$
# of indept comp'ts = 80

$A_{\alpha\beta\gamma} = A_{(\alpha\beta\gamma)}$

$$g'_{\mu\nu} = \eta_{\mu\nu} + D_{\mu\nu\rho\lambda}(x'-a')^\rho(x'-a')^\lambda + O((x'-a')^3) + \cdots$$

$\frac{1}{2} g'_{\mu\nu,\rho\lambda}|_p \longrightarrow$ 100 indept numbers

Can we cancel this too?

$$\left(\frac{\partial^3 x^\mu}{\partial x'^\alpha \partial x'^\beta \partial x'^\gamma}\right) < 4^4$$

# of indept comp'ts = 80.



$\exists$ a tensor with 20 independent components which encode the second derivatives of the metric which cannot be set zero by a coordinate transformation: Riemann tensor. $R^l{}_{hrs}$

$A_{\alpha\beta\gamma} = A_{(\alpha\beta\gamma)}$

$R_{\mu\nu\rho\lambda}$

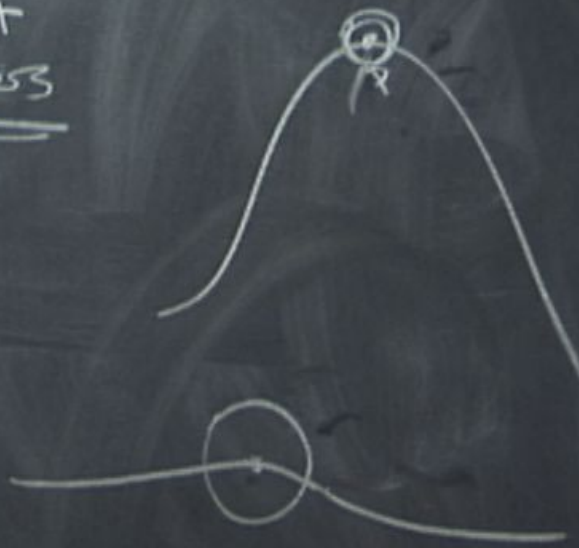
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$\frac{1}{2} g'_{\mu\nu,\rho\lambda}|_p \longrightarrow 100$ indept numbers

Can we cancel this too?

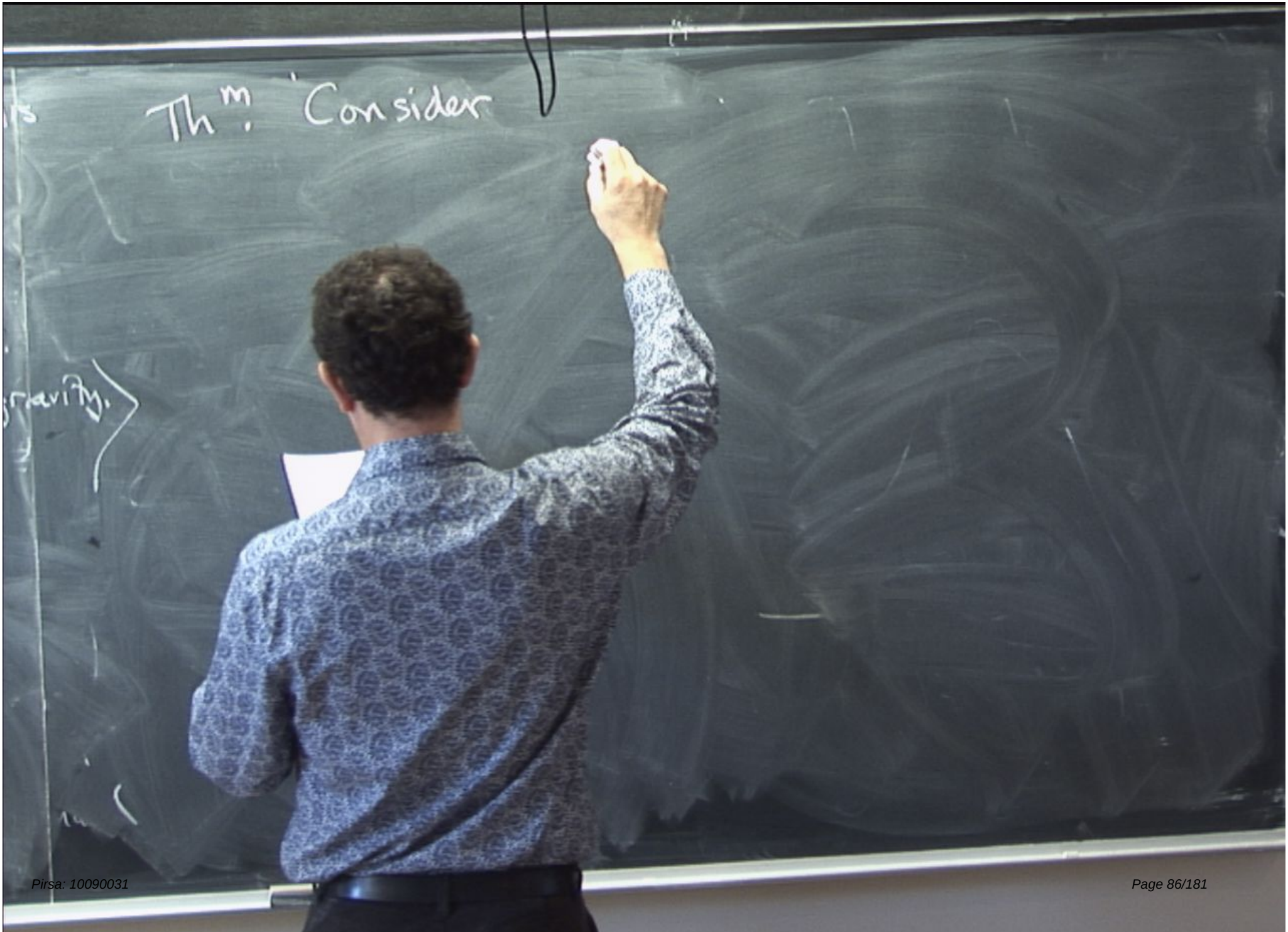
$\frac{\partial^3 x^\mu}{\partial x'^\alpha \partial x'^\beta \partial x'^\gamma} < 4^4$

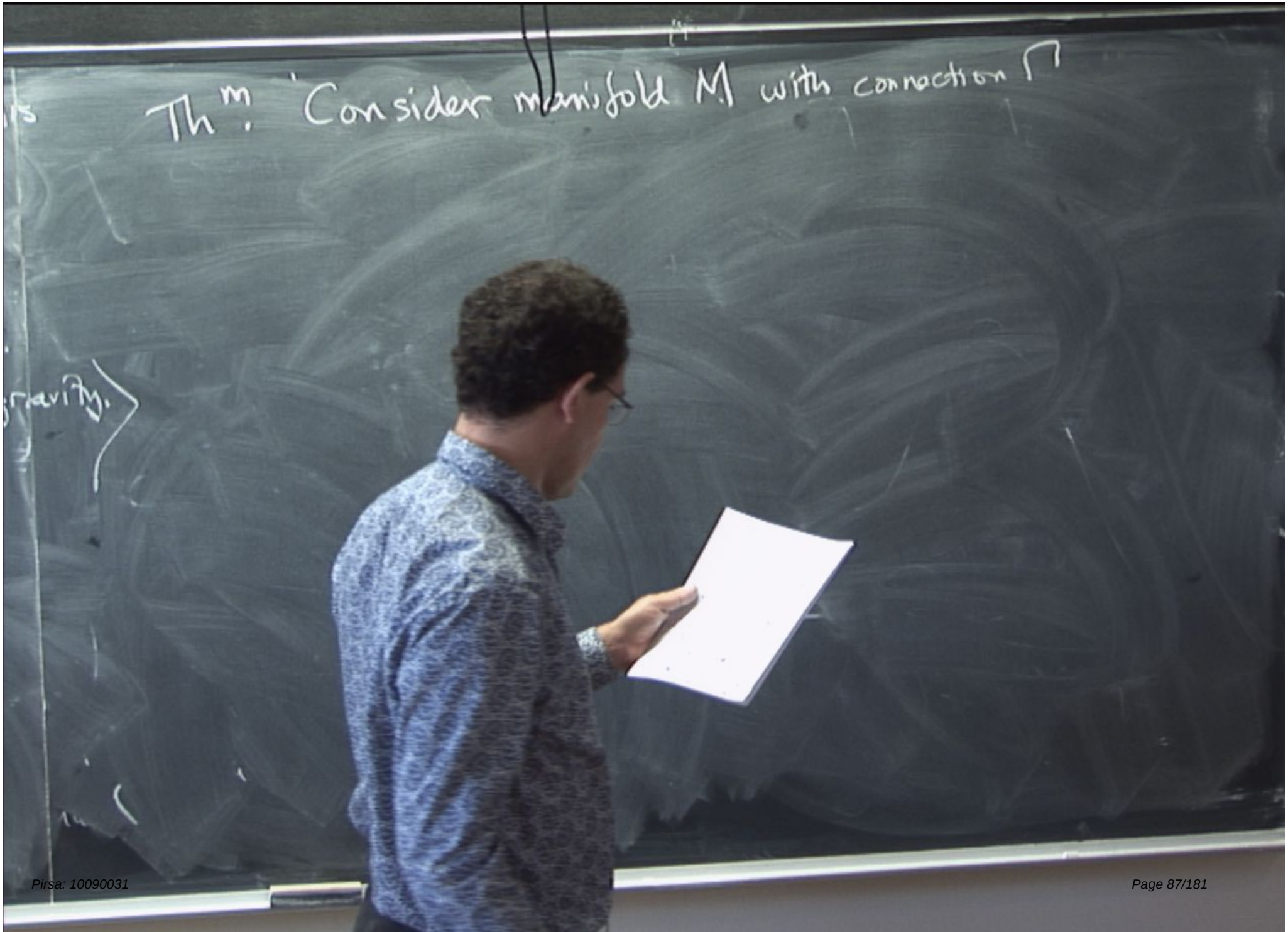
# of indept compn'ts = 80.



∃ a tensor with 20 independent components which encode the second derivatives of the metric which cannot be set zero by a coordinate transformation; Riemann tensor.

→ represents the curvature of spacetime ⇔ gravit


Thm. Consider
gravity.


Thm. Consider manifold M with connection Γ
gravity.

Th^m. Consider manifold M with connection Γ which is differentiable in each coordinate patch.

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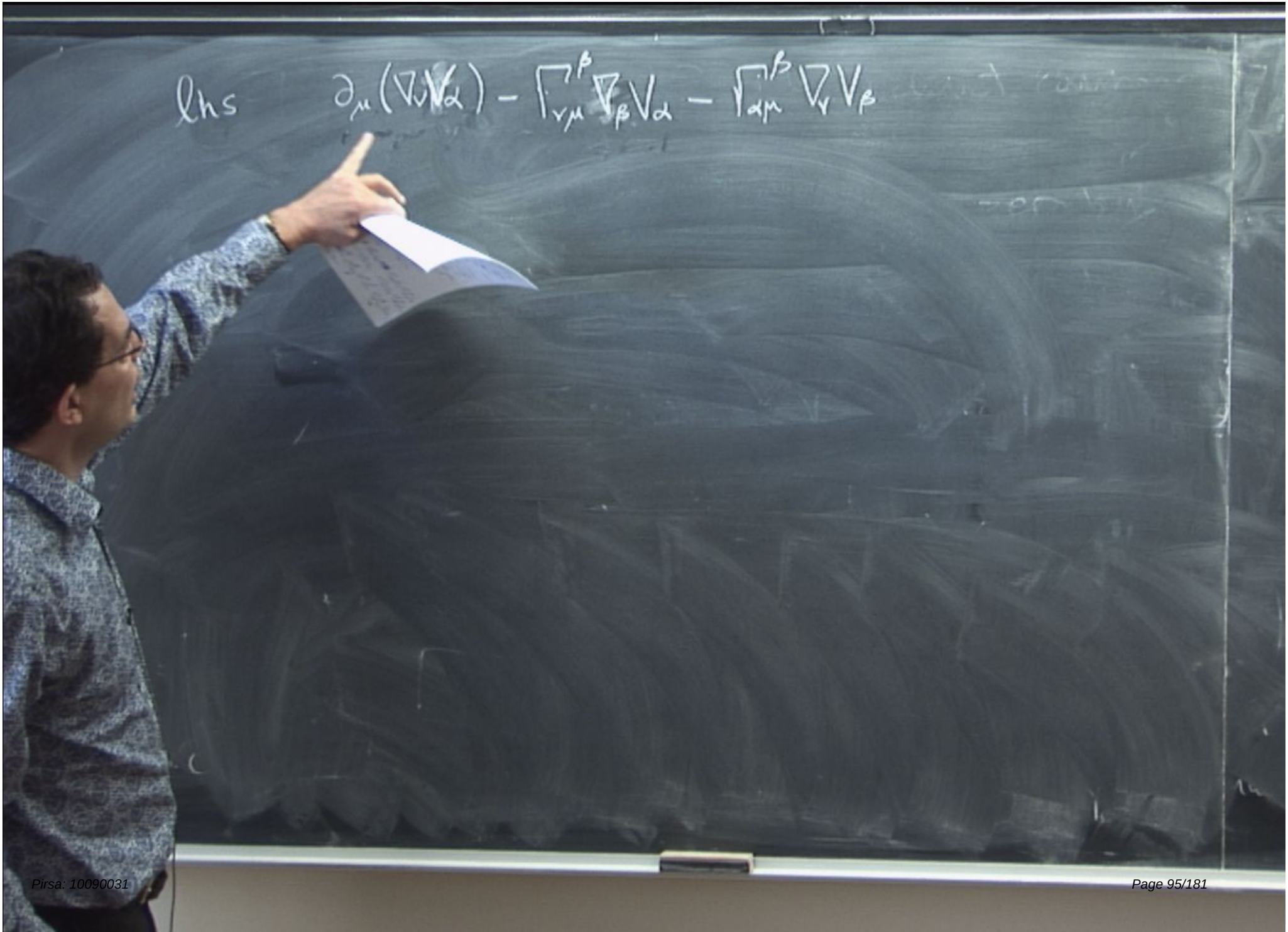
$\nabla_\mu(R_\nu g_{\alpha\beta})$
$= 0$

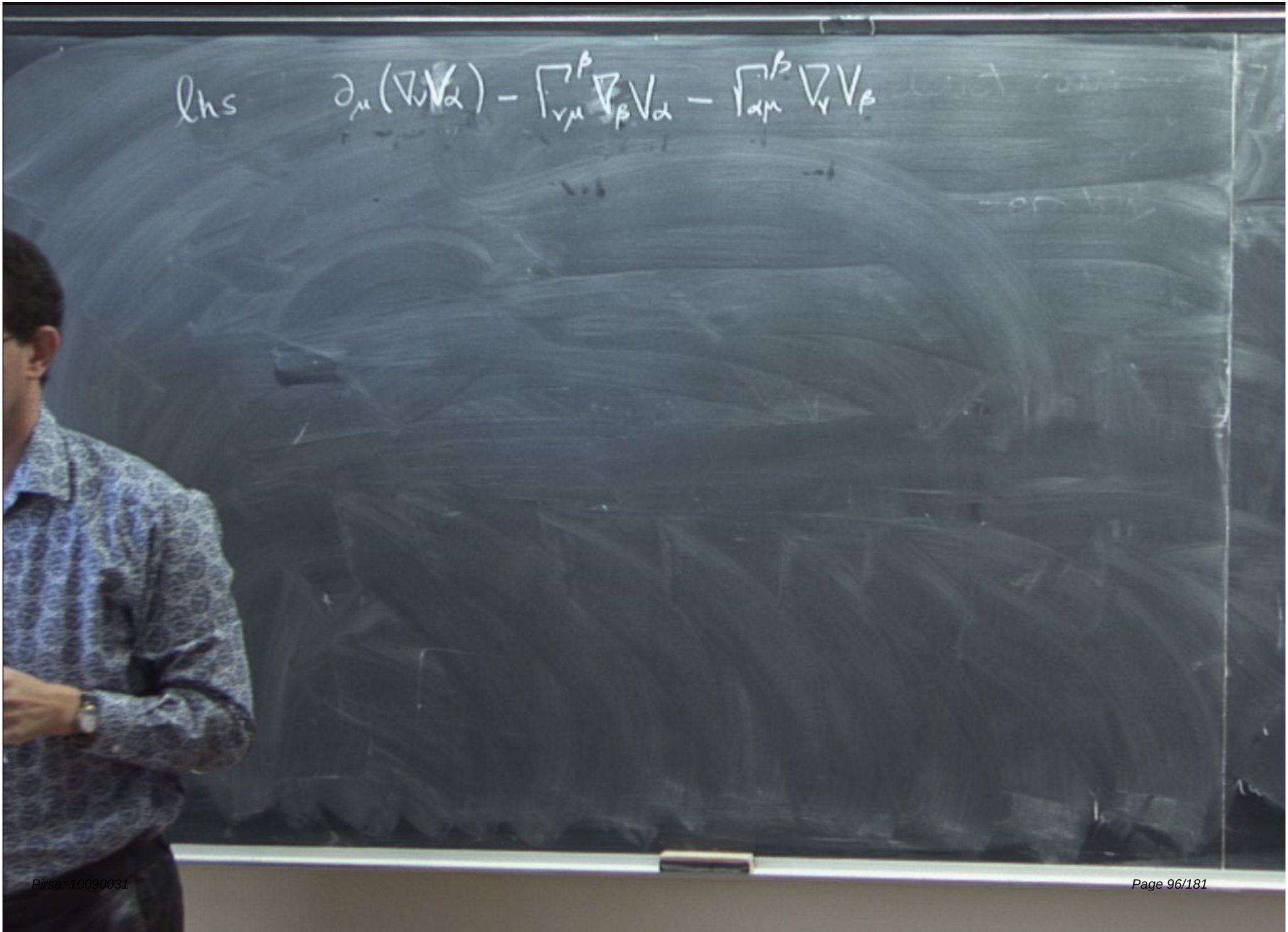
gravity.

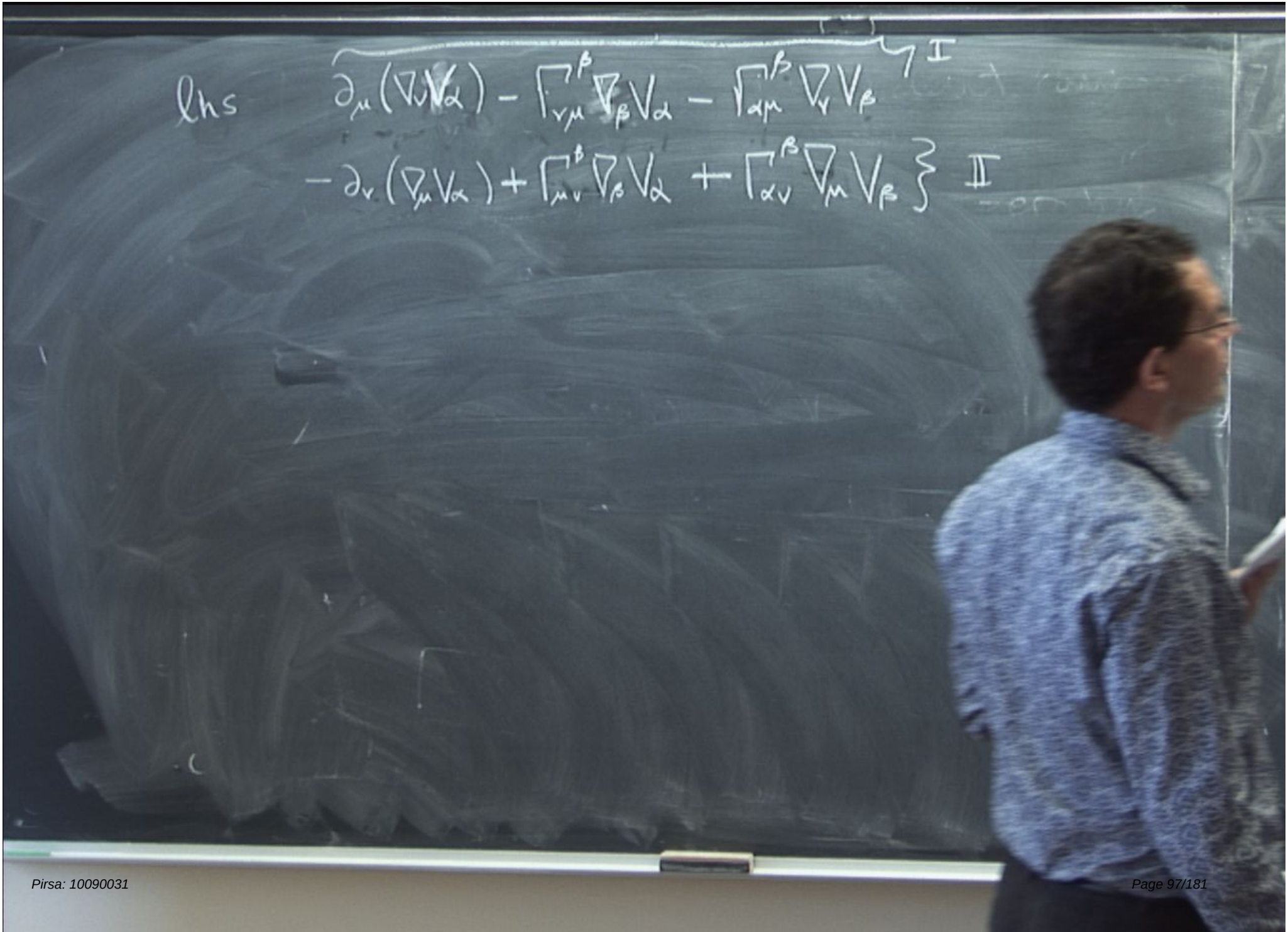
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$\forall$ covariant vectors V_α


lhs
$\partial_\mu(\nabla_\nu V_\alpha) - \Gamma^\beta_{\nu\mu}\nabla_\beta V_\alpha - \Gamma^\beta_{\alpha\mu}\nabla_\nu V_\beta$


lhs $\partial_\mu(\nabla_\nu V_\alpha) - \Gamma^\beta_{\nu\mu}\nabla_\beta V_\alpha - \Gamma^\beta_{\alpha\mu}\nabla_\nu V_\beta$

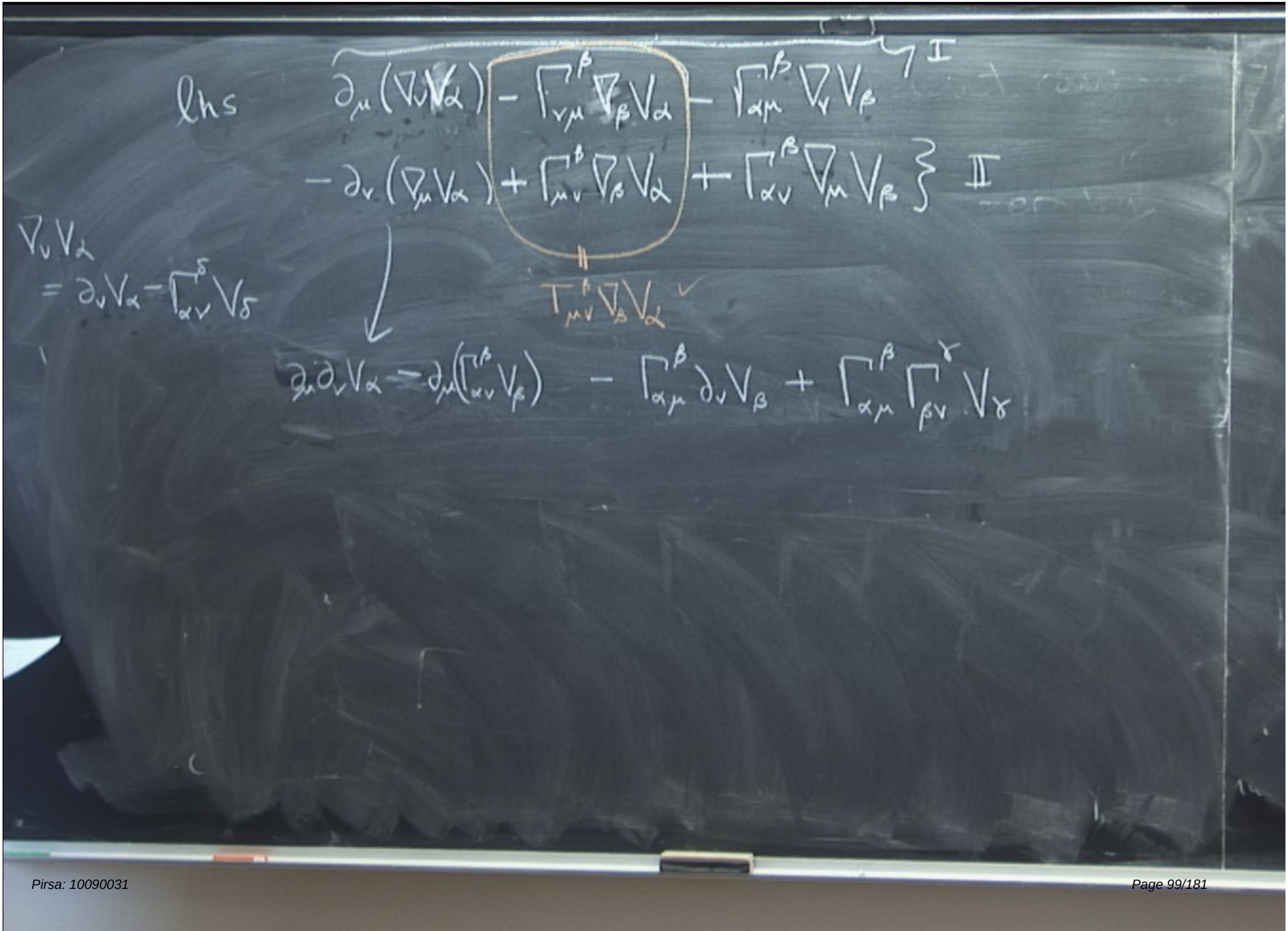

lhs
$\partial_\mu(\nabla_\nu V_\alpha) - \Gamma^\beta_{\nu\mu}\nabla_\beta V_\alpha - \Gamma^\beta_{\alpha\mu}\nabla_\nu V_\beta$ I
$-\partial_\nu(\nabla_\mu V_\alpha) + \Gamma^\beta_{\mu\nu}\nabla_\beta V_\alpha + \Gamma^\beta_{\alpha\nu}\nabla_\mu V_\beta\}$ II

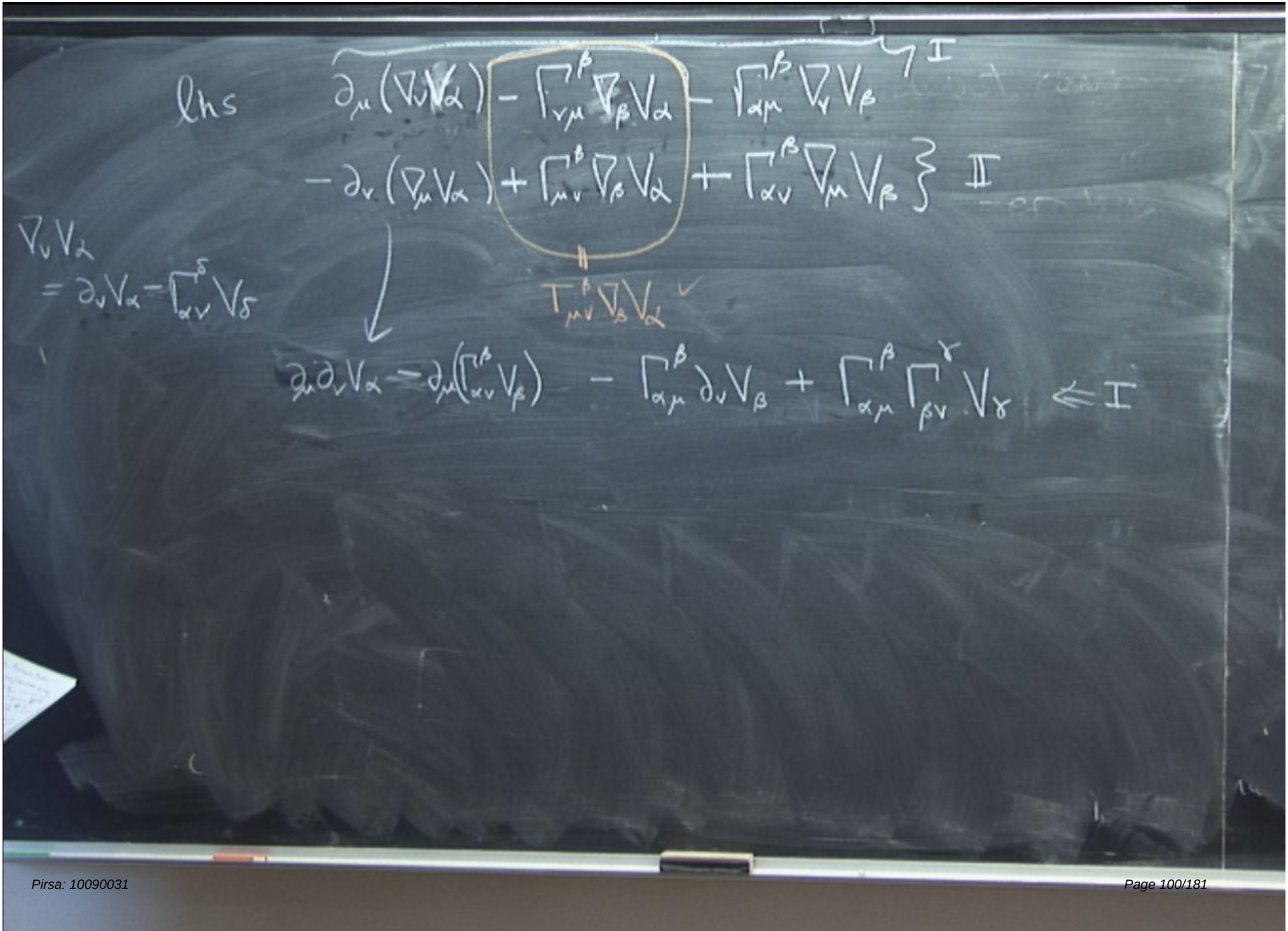
lhs $\overbrace{\partial_\mu(\nabla_\nu V_\alpha) - \Gamma^\beta_{\nu\mu}\nabla_\beta V_\alpha - \Gamma^\beta_{\alpha\mu}\nabla_\nu V_\beta}^{\text{I}}$

$-\partial_\nu(\nabla_\mu V_\alpha) + \Gamma^\beta_{\mu\nu}\nabla_\beta V_\alpha + \Gamma^\beta_{\alpha\nu}\nabla_\mu V_\beta \}$ II

$\nabla_\nu V_\alpha = \partial_\nu V_\alpha - \Gamma^\delta_{\alpha\nu} V_\delta$

$T^\beta_{\mu\nu}\nabla_\beta V_\alpha$ ✓


lhs
$\partial_\mu(\nabla_\nu V_\alpha) - \Gamma^\beta_{\nu\mu}\nabla_\beta V_\alpha - \Gamma^\beta_{\alpha\mu}\nabla_\nu V_\beta$ I
$-\partial_\nu(\nabla_\mu V_\alpha) + \Gamma^\beta_{\mu\nu}\nabla_\beta V_\alpha + \Gamma^\beta_{\alpha\nu}\nabla_\mu V_\beta$ II
$\nabla_\nu V_\alpha = \partial_\nu V_\alpha - \Gamma^\delta_{\alpha\nu} V_\delta$
$T^\beta_{\mu\nu}\nabla_\beta V_\alpha$
$\partial_\mu\partial_\nu V_\alpha - \partial_\mu(\Gamma^\beta_{\alpha\nu} V_\beta) - \Gamma^\beta_{\alpha\mu}\partial_\nu V_\beta + \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu} V_\gamma$


lhs
$\partial_\mu(\nabla_\nu V_\alpha) - \Gamma^\beta_{\nu\mu}\nabla_\beta V_\alpha - \Gamma^\beta_{\alpha\mu}\nabla_\nu V_\beta$ I
$-\partial_\nu(\nabla_\mu V_\alpha) + \Gamma^\beta_{\mu\nu}\nabla_\beta V_\alpha + \Gamma^\beta_{\alpha\nu}\nabla_\mu V_\beta$ II
$\nabla_\nu V_\alpha = \partial_\nu V_\alpha - \Gamma^\delta_{\alpha\nu} V_\delta$
$T^\beta_{\mu\nu}\nabla_\beta V_\alpha$
$\partial_\mu\partial_\nu V_\alpha - \partial_\mu(\Gamma^\beta_{\alpha\nu} V_\beta) - \Gamma^\beta_{\alpha\mu}\partial_\nu V_\beta + \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu} V_\gamma$ ⇐ I

lhs $\partial_\mu(\nabla_\nu V_\alpha) - \Gamma^\beta_{\nu\mu}\nabla_\beta V_\alpha - \Gamma^\beta_{\alpha\mu}\nabla_\nu V_\beta$ } I

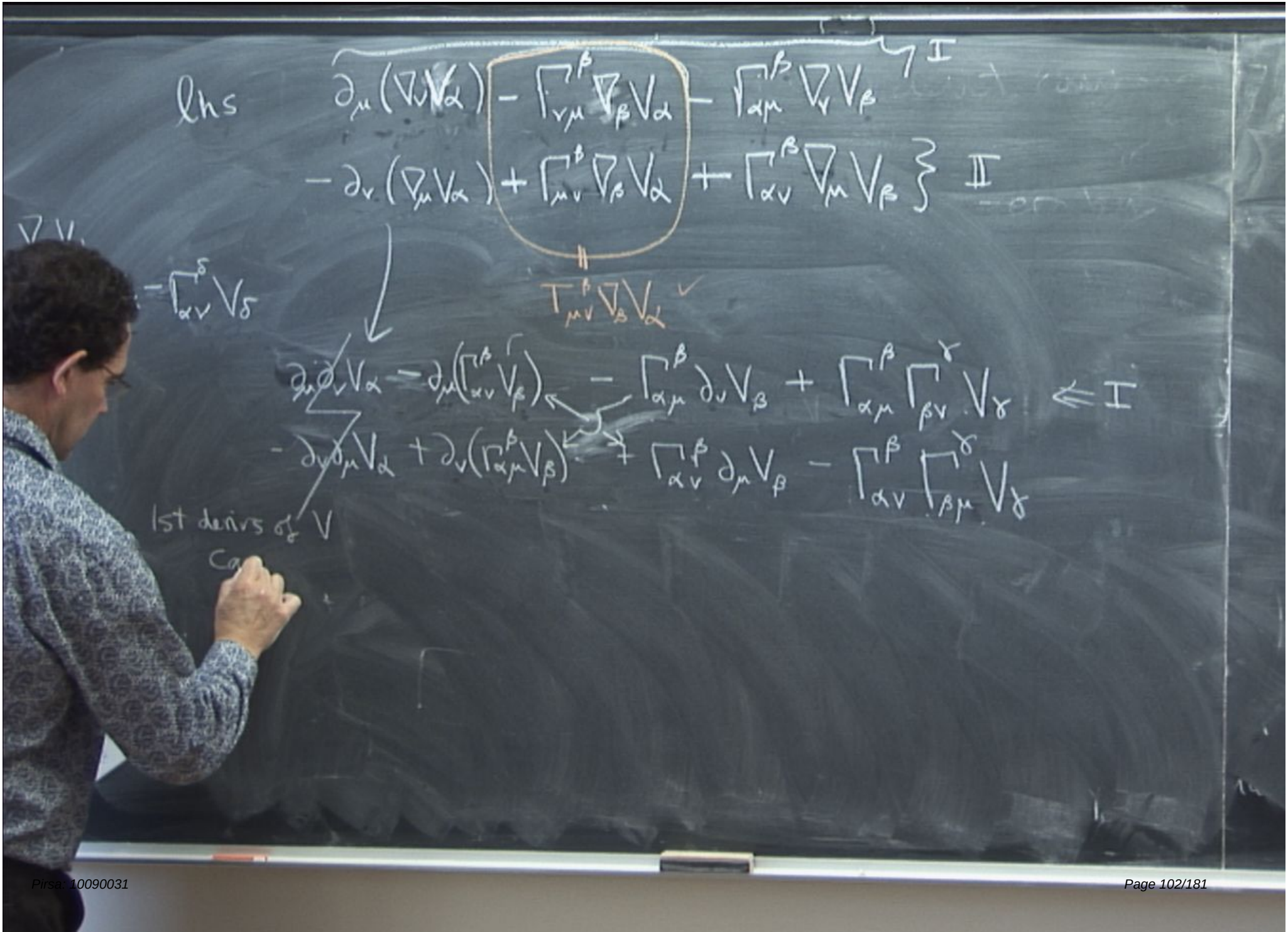
$-\partial_\nu(\nabla_\mu V_\alpha) + \Gamma^\beta_{\mu\nu}\nabla_\beta V_\alpha + \Gamma^\beta_{\alpha\nu}\nabla_\mu V_\beta$ } II

$\nabla_\nu V_\alpha = \partial_\nu V_\alpha - \Gamma^\delta_{\alpha\nu} V_\delta$

$= T^\beta_{\mu\nu}\nabla_\beta V_\alpha$ ✓

$\partial_\mu\partial_\nu V_\alpha - \partial_\mu(\Gamma^\beta_{\alpha\nu} V_\beta) - \Gamma^\beta_{\alpha\mu}\partial_\nu V_\beta + \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu} V_\gamma$ ⇐ I

$-\partial_\nu\partial_\mu V_\alpha + \partial_\nu(\Gamma^\beta_{\alpha\mu} V_\beta) + \Gamma^\beta_{\alpha\nu}\partial_\mu V_\beta - \Gamma^\beta_{\alpha\nu}\Gamma^\gamma_{\beta\mu} V_\gamma$


lhs
$\partial_\mu(\nabla_\nu V_\alpha) - \Gamma^\beta_{\nu\mu}\nabla_\beta V_\alpha - \Gamma^\beta_{\alpha\mu}\nabla_\nu V_\beta$ I
$-\partial_\nu(\nabla_\mu V_\alpha) + \Gamma^\beta_{\mu\nu}\nabla_\beta V_\alpha + \Gamma^\beta_{\alpha\nu}\nabla_\mu V_\beta$ II
$T^\beta_{\mu\nu}\nabla_\beta V_\alpha$
$\partial_\mu\partial_\nu V_\alpha - \partial_\mu(\Gamma^\beta_{\alpha\nu}V_\beta) - \Gamma^\beta_{\alpha\mu}\partial_\nu V_\beta + \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu}V_\gamma$ ⇐ I
$-\partial_\nu\partial_\mu V_\alpha + \partial_\nu(\Gamma^\beta_{\alpha\mu}V_\beta) + \Gamma^\beta_{\alpha\nu}\partial_\mu V_\beta - \Gamma^\beta_{\alpha\nu}\Gamma^\gamma_{\beta\mu}V_\gamma$
1st derivs of V

lhs $\partial_\mu(\nabla_\nu V_\alpha) - \Gamma^\beta_{\nu\mu}\nabla_\beta V_\alpha - \Gamma^\beta_{\alpha\mu}\nabla_\nu V_\beta$ } I

$-\partial_\nu(\nabla_\mu V_\alpha) + \Gamma^\beta_{\mu\nu}\nabla_\beta V_\alpha + \Gamma^\beta_{\alpha\nu}\nabla_\mu V_\beta$ } II

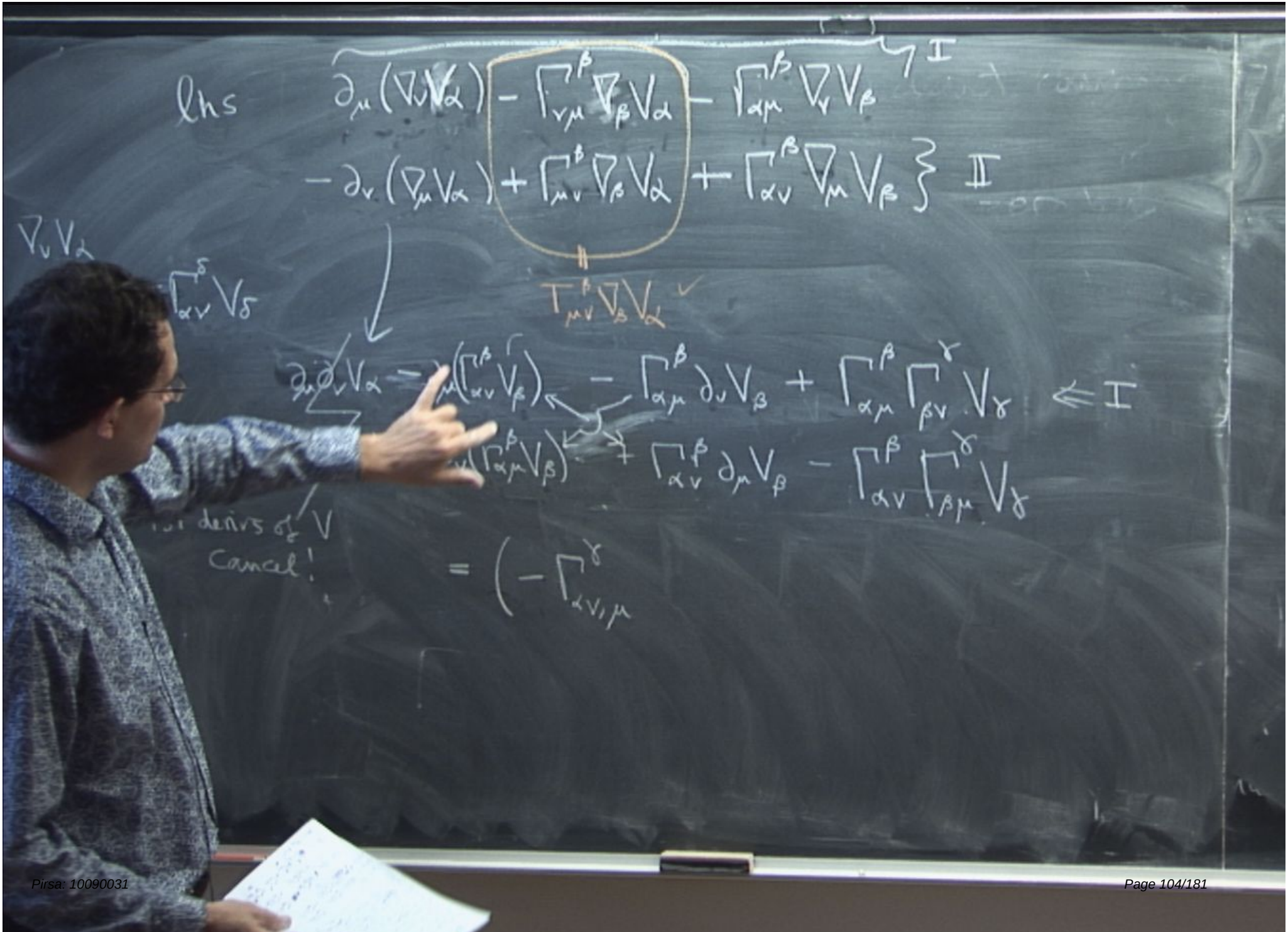
$\nabla_\nu V_\alpha = \partial_\nu V_\alpha - \Gamma^\delta_{\alpha\nu} V_\delta$

$T^\beta_{\mu\nu}\nabla_\beta V_\alpha$ ✓

$\partial_\mu\partial_\nu V_\alpha - \partial_\mu(\Gamma^\beta_{\alpha\nu} V_\beta) - \Gamma^\beta_{\alpha\mu}\partial_\nu V_\beta + \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu} V_\gamma$ ⇐ I

$-\partial_\nu\partial_\mu V_\alpha + \partial_\nu(\Gamma^\beta_{\alpha\mu} V_\beta) + \Gamma^\beta_{\alpha\nu}\partial_\mu V_\beta - \Gamma^\beta_{\alpha\nu}\Gamma^\gamma_{\beta\mu} V_\gamma$

1st derivs of V cancel!


lhs
$\partial_\mu(\nabla_\nu V_\alpha) - \Gamma^\beta_{\nu\mu}\nabla_\beta V_\alpha - \Gamma^\beta_{\alpha\mu}\nabla_\nu V_\beta$ I
$-\partial_\nu(\nabla_\mu V_\alpha) + \Gamma^\beta_{\mu\nu}\nabla_\beta V_\alpha + \Gamma^\beta_{\alpha\nu}\nabla_\mu V_\beta$ II
$T^\beta_{\mu\nu}\nabla_\beta V_\alpha$
$\nabla_\nu V_\alpha$
$\Gamma^\delta_{\alpha\nu} V_\delta$
$\partial_\mu\partial_\nu V_\alpha - \partial_\mu(\Gamma^\beta_{\alpha\nu} V_\beta) - \Gamma^\beta_{\alpha\mu}\partial_\nu V_\beta + \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu} V_\gamma$ ⇐ I
$+ \Gamma^\beta_{\alpha\nu}\partial_\mu V_\beta - \Gamma^\beta_{\alpha\nu}\Gamma^\gamma_{\beta\mu} V_\gamma$
2nd derivs of V
Cancel!
$= \left(-\Gamma^\gamma_{\alpha\nu,\mu}\right.$

lhs $\partial_\mu(\nabla_\nu V_\alpha) - \Gamma^\beta_{\nu\mu}\nabla_\beta V_\alpha - \Gamma^\beta_{\alpha\mu}\nabla_\nu V_\beta$ } I

$-\partial_\nu(\nabla_\mu V_\alpha) + \Gamma^\beta_{\mu\nu}\nabla_\beta V_\alpha + \Gamma^\beta_{\alpha\nu}\nabla_\mu V_\beta$ } II

$\nabla_\nu V_\alpha = \partial_\nu V_\alpha - \Gamma^\delta_{\alpha\nu} V_\delta$

$T^\beta_{\mu\nu}\nabla_\beta V_\alpha$ ✓

$\partial_\mu\partial_\nu V_\alpha - \partial_\mu(\Gamma^\beta_{\alpha\nu} V_\beta) - \Gamma^\beta_{\alpha\mu}\partial_\nu V_\beta + \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu} V_\gamma$ ⇐ I

$-\partial_\nu\partial_\mu V_\alpha + \partial_\nu(\Gamma^\beta_{\alpha\mu} V_\beta) + \Gamma^\beta_{\alpha\nu}\partial_\mu V_\beta - \Gamma^\beta_{\alpha\nu}\Gamma^\gamma_{\beta\mu} V_\gamma$

1st derivs of V cancel!

$= \left(-\Gamma^\gamma_{\alpha\nu,\mu} + \Gamma^\gamma_{\alpha\mu,\nu} + \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu} - \Gamma^\beta_{\alpha\nu}\Gamma^\gamma_{\beta\mu}\right) V_\gamma$

lhs $\partial_\mu(\nabla_\nu V_\alpha) - \Gamma^\beta_{\nu\mu}\nabla_\beta V_\alpha - \Gamma^\beta_{\alpha\mu}\nabla_\nu V_\beta$ } I

$-\partial_\nu(\nabla_\mu V_\alpha) + \Gamma^\beta_{\mu\nu}\nabla_\beta V_\alpha + \Gamma^\beta_{\alpha\nu}\nabla_\mu V_\beta$ } II

$\nabla_\nu V_\alpha = \partial_\nu V_\alpha - \Gamma^\delta_{\alpha\nu} V_\delta$

$= T^\beta_{\mu\nu}\nabla_\beta V_\alpha$ ✓

$\partial_\mu\partial_\nu V_\alpha - \partial_\mu(\Gamma^\beta_{\alpha\nu} V_\beta) - \Gamma^\beta_{\alpha\mu}\partial_\nu V_\beta + \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu} V_\gamma$ ⇐ I

$-\partial_\nu\partial_\mu V_\alpha + \partial_\nu(\Gamma^\beta_{\alpha\mu} V_\beta) + \Gamma^\beta_{\alpha\nu}\partial_\mu V_\beta - \Gamma^\beta_{\alpha\nu}\Gamma^\gamma_{\beta\mu} V_\gamma$

1st derivs of V cancel!

$= \left(-\Gamma^\gamma_{\alpha\nu,\mu} + \Gamma^\gamma_{\alpha\mu,\nu} + \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu} - \Gamma^\beta_{\alpha\nu}\Gamma^\gamma_{\beta\mu}\right) V_\gamma$

Thm: Consider manifold M with connection Γ which is differentiable in each coordinate patch.
Then there exists a unique tensor $R^{\mu}{}_{\nu\alpha\beta}$ s.t.

$$\underbrace{(\nabla_\mu \nabla_\nu}_{\text{I}} - \underbrace{\nabla_\nu \nabla_\mu)}_{\text{II}} V_\alpha = T^{\beta}_{\mu\nu} \nabla_\beta V_\alpha - R^{\gamma}{}_{\alpha\mu\nu} V_\gamma$$

$\forall$ covariant vectors V_α

$$\Rightarrow R^{\gamma}{}_{\alpha\mu\nu} = -\Gamma^{\gamma}_{\alpha\mu,\nu}$$

Thm: Consider manifold M with connection Γ which is differentiable in each coordinate patch.
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$\forall$ covariant vectors V_α

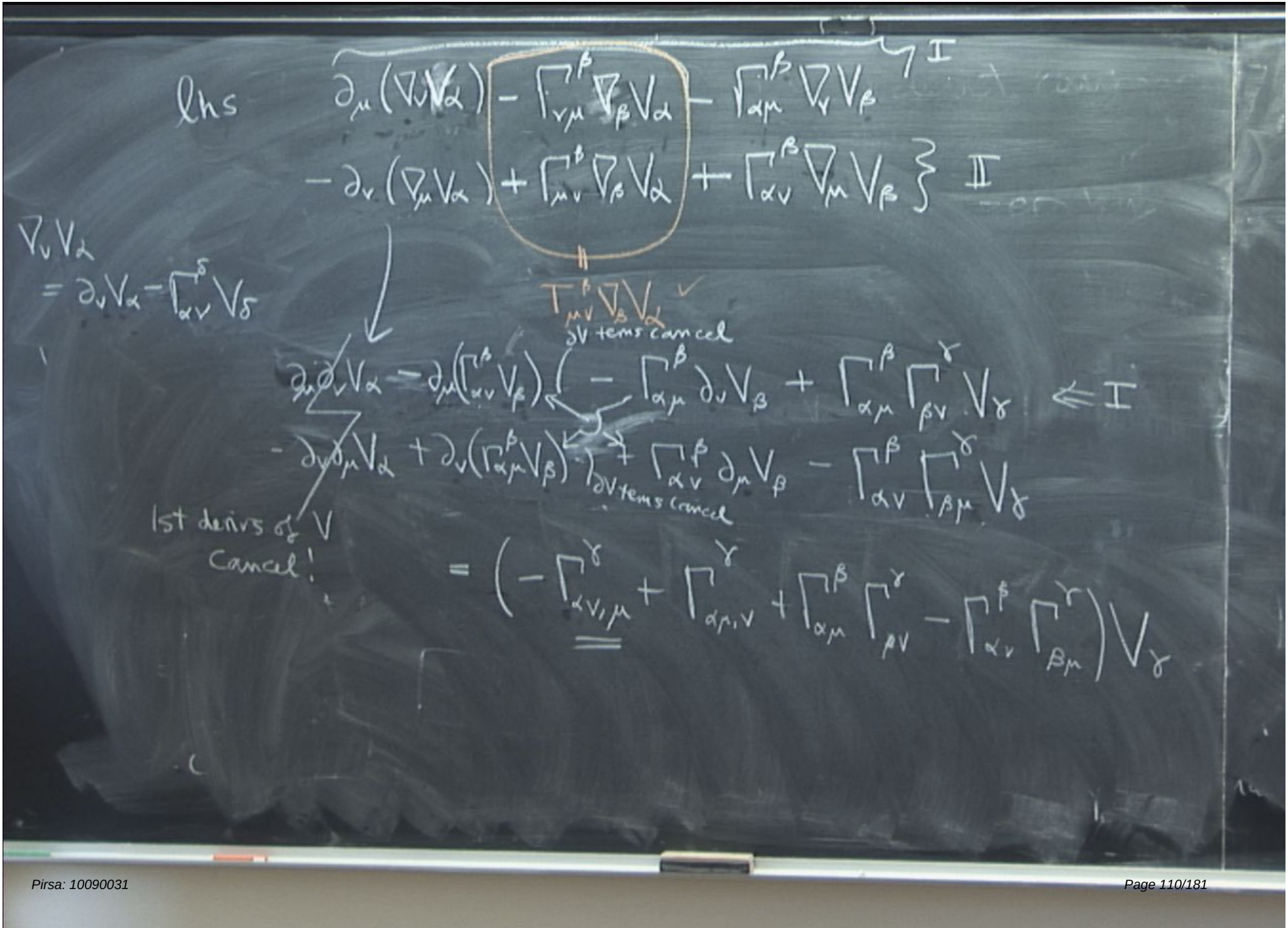
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$$\Rightarrow R^{\gamma}{}_{\alpha\mu\nu} = -\Gamma^{\gamma}_{\alpha\mu,\nu} + \Gamma^{\gamma}_{\alpha\mu,} - \Gamma^{\beta}_{\alpha\mu}\Gamma^{\gamma}_{\beta\nu} + \Gamma^{\beta}_{\alpha\nu}\Gamma^{\gamma}_{\beta\mu}$$


lhs
$\partial_\mu(\nabla_\nu V_\alpha) - \Gamma^\beta_{\nu\mu}\nabla_\beta V_\alpha - \Gamma^\beta_{\alpha\mu}\nabla_\nu V_\beta$ I
$-\partial_\nu(\nabla_\mu V_\alpha) + \Gamma^\beta_{\mu\nu}\nabla_\beta V_\alpha + \Gamma^\beta_{\alpha\nu}\nabla_\mu V_\beta$ II
$\nabla_\nu V_\alpha = \partial_\nu V_\alpha - \Gamma^\delta_{\alpha\nu} V_\delta$
$T^\beta_{\mu\nu}\nabla_\beta V_\alpha$
∂V terms cancel
$\partial_\mu\partial_\nu V_\alpha - \partial_\mu(\Gamma^\beta_{\alpha\nu}V_\beta)$ $-\Gamma^\beta_{\alpha\mu}\partial_\nu V_\beta + \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu}V_\gamma$ ⇐ I
$-\partial_\nu\partial_\mu V_\alpha + \partial_\nu(\Gamma^\beta_{\alpha\mu}V_\beta)$ $+\Gamma^\beta_{\alpha\nu}\partial_\mu V_\beta - \Gamma^\beta_{\alpha\nu}\Gamma^\gamma_{\beta\mu}V_\gamma$
∂V terms cancel
1st derivs of V cancel!
$= \left(-\Gamma^\gamma_{\alpha\nu,\mu} + \Gamma^\gamma_{\alpha\mu,\nu} + \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu} - \Gamma^\beta_{\alpha\nu}\Gamma^\gamma_{\beta\mu}\right)V_\gamma$

Thm: Consider manifold M with connection Γ which is differentiable in each coordinate patch.
Then there exists a unique tensor $R^{\mu}{}_{\nu\alpha\beta}$ s.t.

$$\underbrace{(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu)}_{\text{I}\quad\text{II}} V_\alpha = T^{\beta}_{\mu\nu} \nabla_\beta V_\alpha - R^{\gamma}{}_{\alpha\mu\nu} V_\gamma$$

$\forall$ covariant vectors V_α

$$\Rightarrow R^{\gamma}{}_{\alpha\mu\nu} = -\Gamma^{\gamma}_{\alpha\mu,\nu} + \Gamma^{\gamma}_{\alpha\mu} - \Gamma^{\beta}_{\alpha\mu}\Gamma^{\gamma}_{\beta\nu} + \Gamma^{\beta}_{\alpha\nu}\Gamma^{\gamma}_{\beta\mu} \qquad \Gamma_{\sigma,\alpha\beta} \equiv g_{\sigma\gamma}\Gamma^{\gamma}_{\alpha\beta}$$

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Then there exists a unique tensor $R^{\mu}{}_{\nu\alpha\beta}$ s.t.

$$\underbrace{(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu)}_{\text{I}\quad\text{II}} V_\alpha = T^{\beta}_{\mu\nu} \nabla_\beta V_\alpha - R^{\gamma}{}_{\alpha\mu\nu} V_\gamma$$

$\forall$ covariant vectors V_α

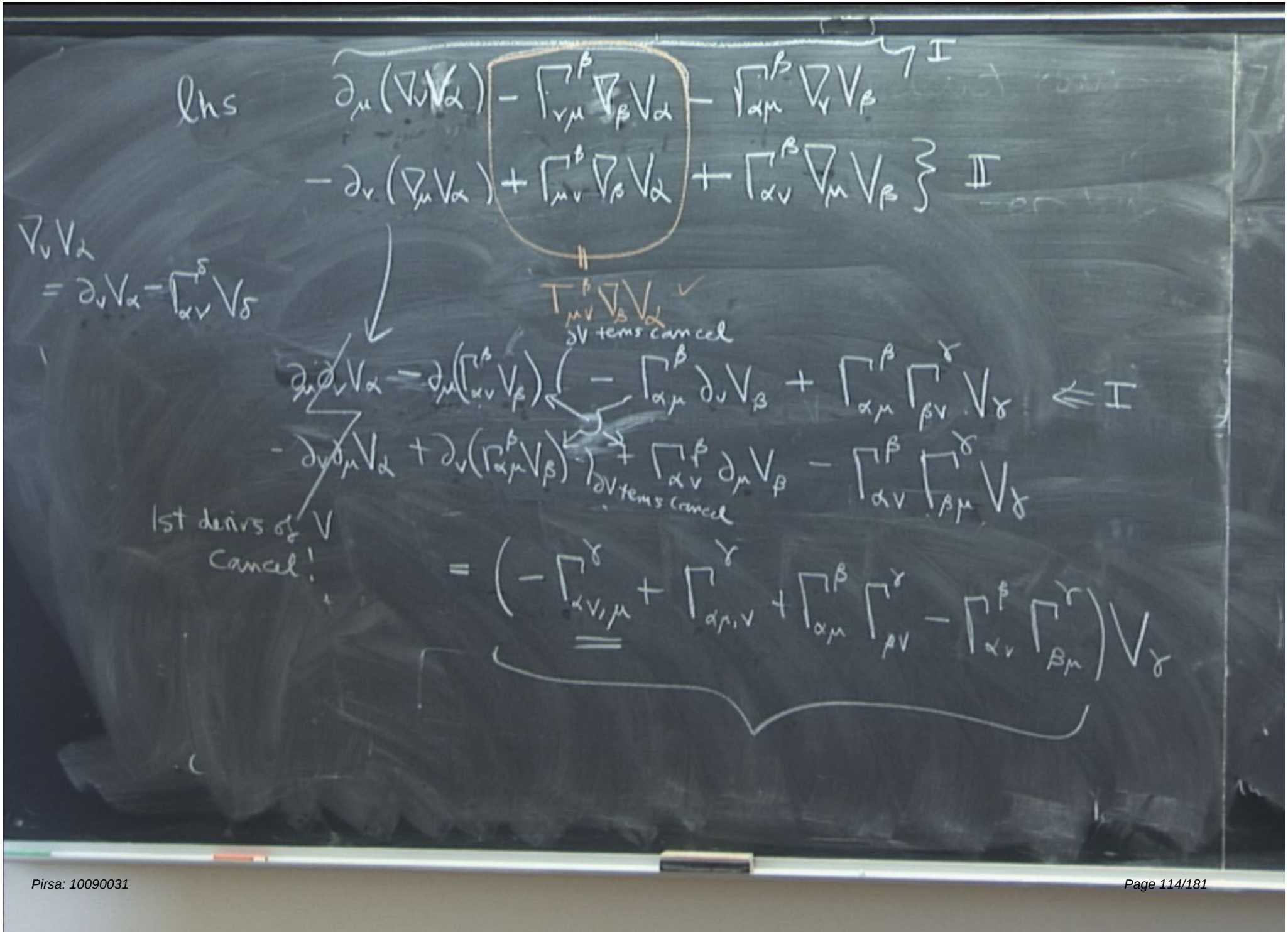
$$\Rightarrow R^{\gamma}{}_{\alpha\mu\nu} = -\Gamma^{\gamma}_{\alpha\mu,\nu} + \Gamma^{\gamma}_{\alpha\nu,\mu} - \Gamma^{\beta}_{\alpha\mu}\Gamma^{\gamma}_{\beta\nu} + \Gamma^{\beta}_{\alpha\nu}\Gamma^{\gamma}_{\beta\mu} \qquad \Gamma_{\sigma,\alpha\beta} \equiv g_{\gamma\sigma}\Gamma^{\gamma}_{\alpha\beta}$$

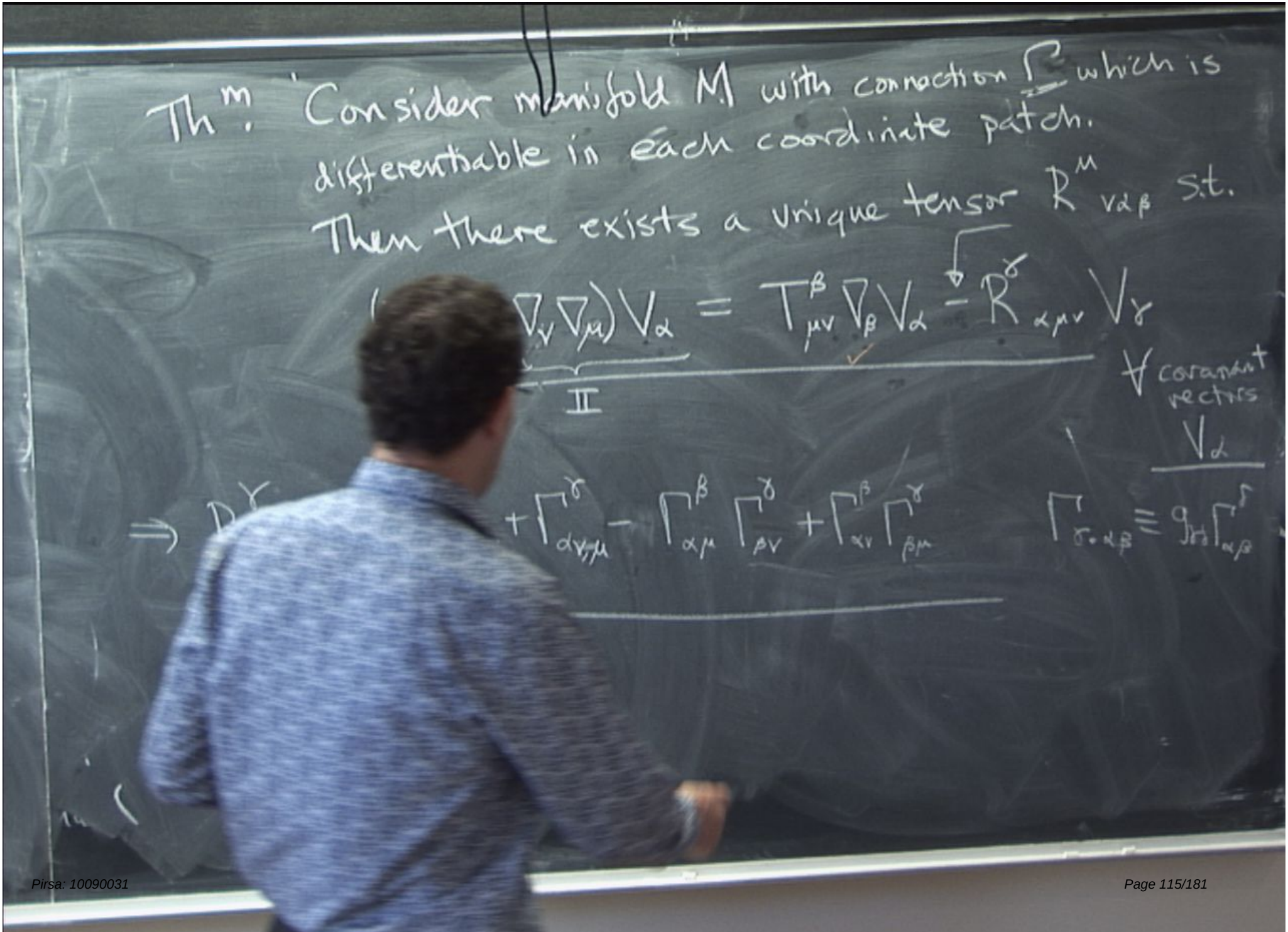
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$\forall$ covariant vectors V_α

$$\Rightarrow R^\gamma{}_{\alpha\mu\nu} = -\Gamma^\gamma_{\alpha\mu,\nu} + \Gamma^\gamma_{\alpha\nu,\mu} - \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu} + \Gamma^\beta_{\alpha\nu}\Gamma^\gamma_{\beta\mu} \qquad \Gamma_{\sigma,\alpha\beta} \equiv g_{\sigma\gamma}\Gamma^\gamma_{\alpha\beta}$$


lhs
$\partial_\mu(\nabla_\nu V_\alpha) - \Gamma^\beta_{\nu\mu}\nabla_\beta V_\alpha - \Gamma^\beta_{\alpha\mu}\nabla_\nu V_\beta$ I
$-\partial_\nu(\nabla_\mu V_\alpha) + \Gamma^\beta_{\mu\nu}\nabla_\beta V_\alpha + \Gamma^\beta_{\alpha\nu}\nabla_\mu V_\beta$ II
$\nabla_\nu V_\alpha = \partial_\nu V_\alpha - \Gamma^\delta_{\alpha\nu} V_\delta$
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∂V terms cancel
$\partial_\mu\partial_\nu V_\alpha - \partial_\mu(\Gamma^\beta_{\alpha\nu}V_\beta)$ $-\Gamma^\beta_{\alpha\mu}\partial_\nu V_\beta + \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu}V_\gamma$ ⇐I
$-\partial_\nu\partial_\mu V_\alpha + \partial_\nu(\Gamma^\beta_{\alpha\mu}V_\beta)$ $+\Gamma^\beta_{\alpha\nu}\partial_\mu V_\beta - \Gamma^\beta_{\alpha\nu}\Gamma^\gamma_{\beta\mu}V_\gamma$
∂V terms cancel
1st derivs of V cancel!
$= \left(-\Gamma^\gamma_{\alpha\nu,\mu} + \Gamma^\gamma_{\alpha\mu,\nu} + \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu} - \Gamma^\beta_{\alpha\nu}\Gamma^\gamma_{\beta\mu}\right)V_\gamma$


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$(\nabla_\nu\nabla_\mu)V_\alpha = T^{\beta}_{\mu\nu}\nabla_\beta V_\alpha - R^{\gamma}{}_{\alpha\mu\nu}V_\gamma$
II
$\forall$ covariant vectors V_α
$\Rightarrow D^\gamma$
$+\Gamma^{\gamma}_{\alpha\nu,\mu} - \Gamma^{\beta}_{\alpha\mu}\Gamma^{\gamma}_{\beta\nu} + \Gamma^{\beta}_{\alpha\nu}\Gamma^{\gamma}_{\beta\mu}$
$\Gamma_{\sigma,\alpha\beta} \equiv g_{\sigma\gamma}\Gamma^{\gamma}_{\alpha\beta}$

Thm: Consider manifold M with connection Γ which is differentiable in each coordinate patch.
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$$\Rightarrow R^\gamma{}_{\alpha\mu\nu} = -\Gamma^\gamma_{\alpha\mu,\nu} + \Gamma^\gamma_{\alpha\nu,\mu} - \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu} + \Gamma^\beta_{\alpha\nu}\Gamma^\gamma_{\beta\mu} \qquad \Gamma_{\sigma,\alpha\beta} \equiv g_{\sigma\gamma}\Gamma^\gamma_{\alpha\beta}$$

$$= -\left(\Gamma^\gamma_{\alpha\mu,\nu} + \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu} - (\mu \leftrightarrow \nu)\right)$$

recall $\Gamma \sim g_{\alpha\beta,\gamma}$ $\rightarrow R \sim g_{\alpha\beta,\gamma\delta}$
(T=0)

Th^m: Consider manifold M with connection Γ which is differentiable in each coordinate patch.
Then there exists a unique tensor $R^{\mu}{}_{\nu\alpha\beta}$ s.t.

$$\underbrace{(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu)}_{\text{I}\quad\text{II}} V_\alpha = T^{\beta}_{\mu\nu} \nabla_\beta V_\alpha = R^{\gamma}{}_{\alpha\mu\nu} V_\gamma$$

$\forall$ covariant vectors V_α

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$\forall$ covariant vectors V_α

4×4.

$$\Rightarrow R^{\gamma}{}_{\alpha\mu\nu} = -\Gamma^{\gamma}_{\alpha\mu,\nu} + \Gamma^{\gamma}_{\alpha\nu,\mu} - \Gamma^{\beta}_{\alpha\mu}\Gamma^{\gamma}_{\beta\nu} + \Gamma^{\beta}_{\alpha\nu}\Gamma^{\gamma}_{\beta\mu} \qquad \Gamma_{\sigma,\alpha\beta} \equiv g_{\sigma\gamma}\Gamma^{\gamma}_{\alpha\beta}$$

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Th^m. Consider manifold M with connection Γ which is differentiable in each coordinate patch.
Then there exists a unique tensor $R^{\mu}{}_{\nu\alpha\beta}$ s.t.

$$\underbrace{(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu)}_{\text{I}\quad\text{II}} V_\alpha = T^\beta_{\mu\nu} \nabla_\beta V_\alpha - R^\gamma{}_{\alpha\mu\nu} V_\gamma$$

$\forall$ covariant vectors V_α

4×4×6

$$\Rightarrow R^\gamma{}_{\alpha\mu\nu} = -\Gamma^\gamma_{\alpha\mu,\nu} + \Gamma^\gamma_{\alpha\nu,\mu} - \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu} + \Gamma^\beta_{\alpha\nu}\Gamma^\gamma_{\beta\mu} \qquad \Gamma_{\sigma,\alpha\beta} \equiv g_{\sigma\gamma}\Gamma^\gamma_{\alpha\beta}$$

$$= -\left(\Gamma^\gamma_{\alpha\mu,\nu} + \Gamma^\beta_{\alpha\mu}\Gamma^\gamma_{\beta\nu} - (\mu \leftrightarrow \nu)\right)$$

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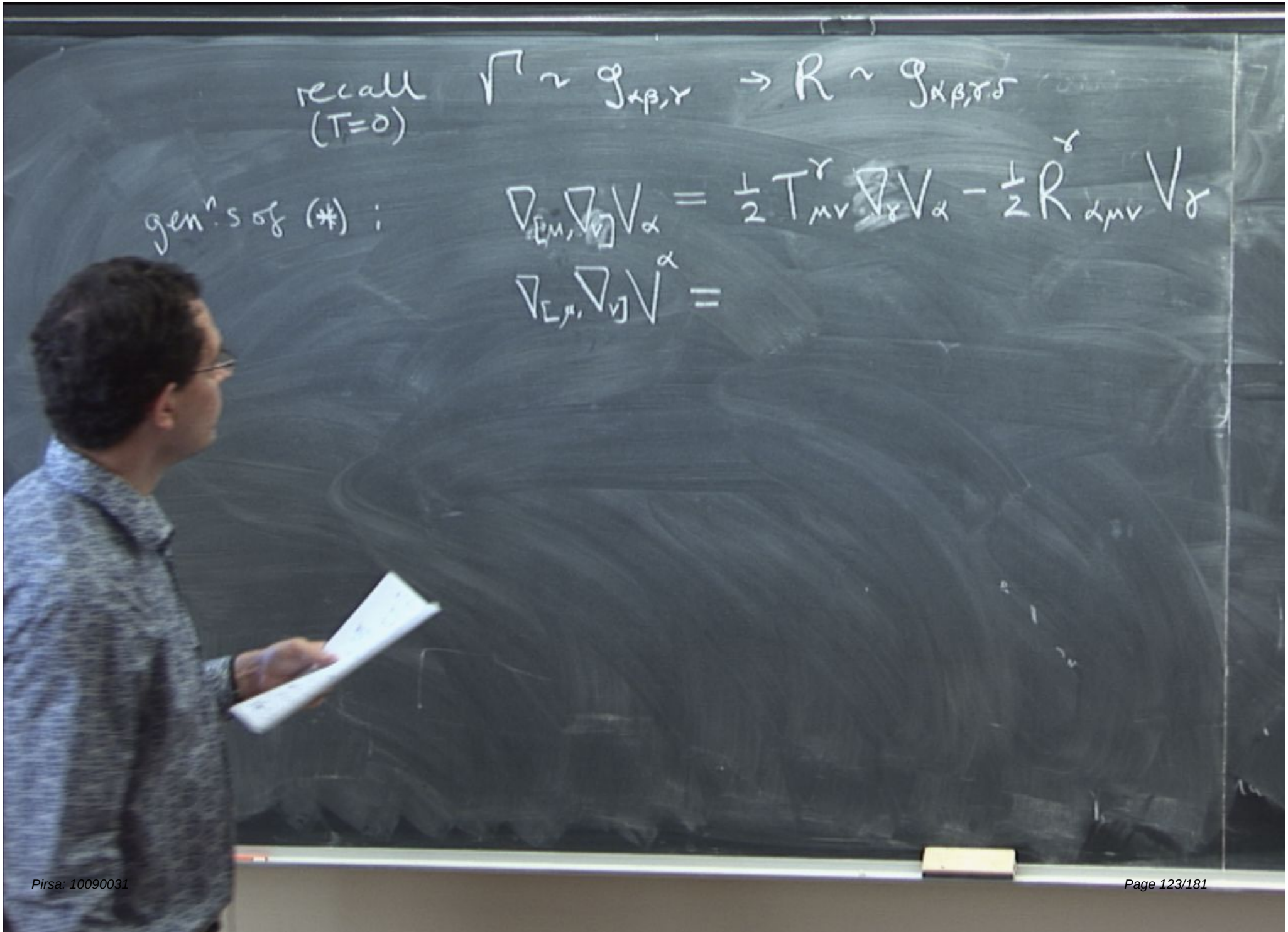
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recall (T=0) $\Gamma \sim g_{\alpha\beta,\gamma} \rightarrow R \sim g_{\alpha\beta,\gamma\delta}$

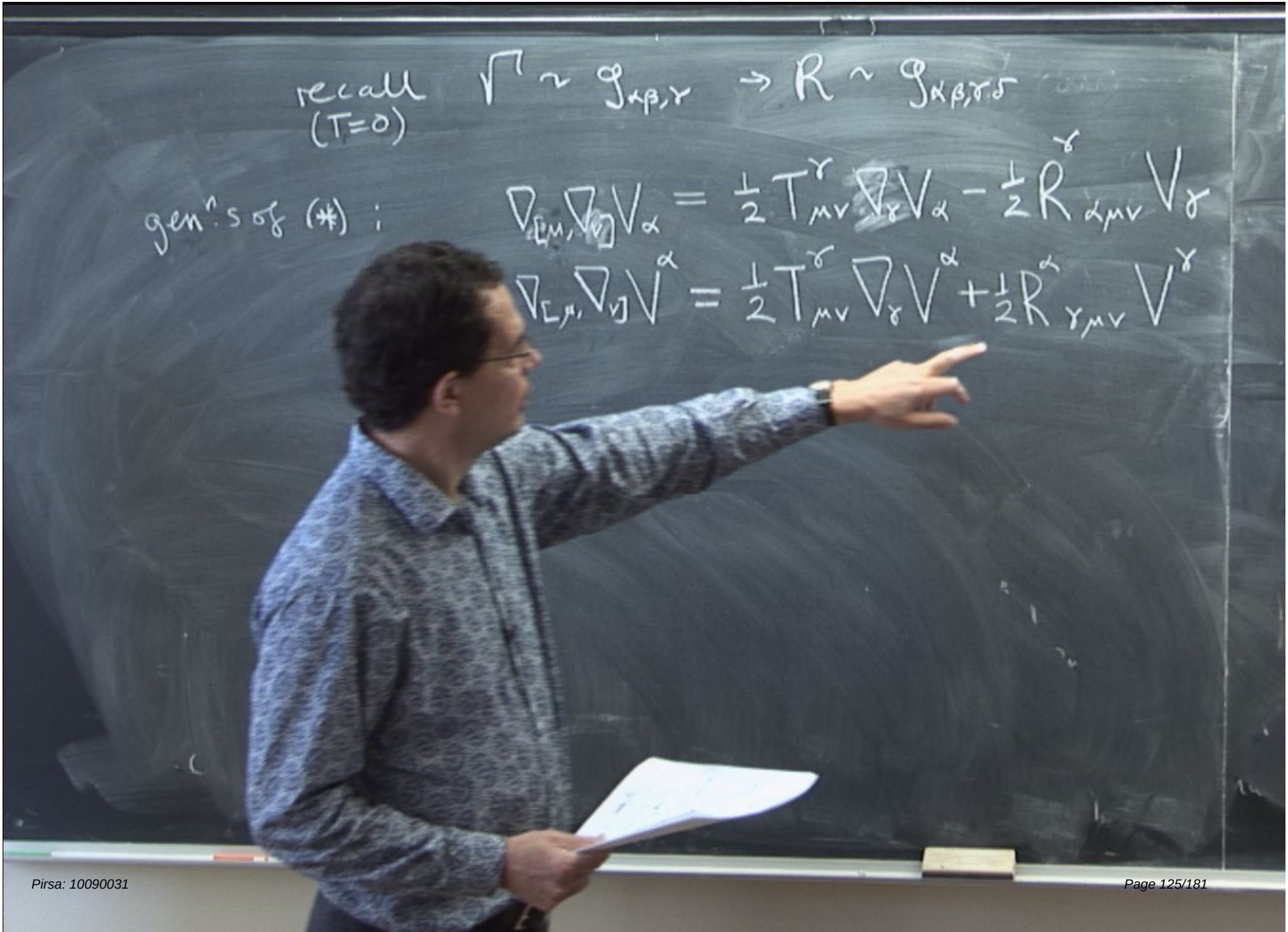
gen'n of (*) ; $\nabla_{[\mu,\nu]} V_\alpha = \frac{1}{2} T^{\gamma}_{\mu\nu} \nabla_\gamma V_\alpha - \frac{1}{2} R^{\gamma}_{\ \alpha\mu\nu} V_\gamma$

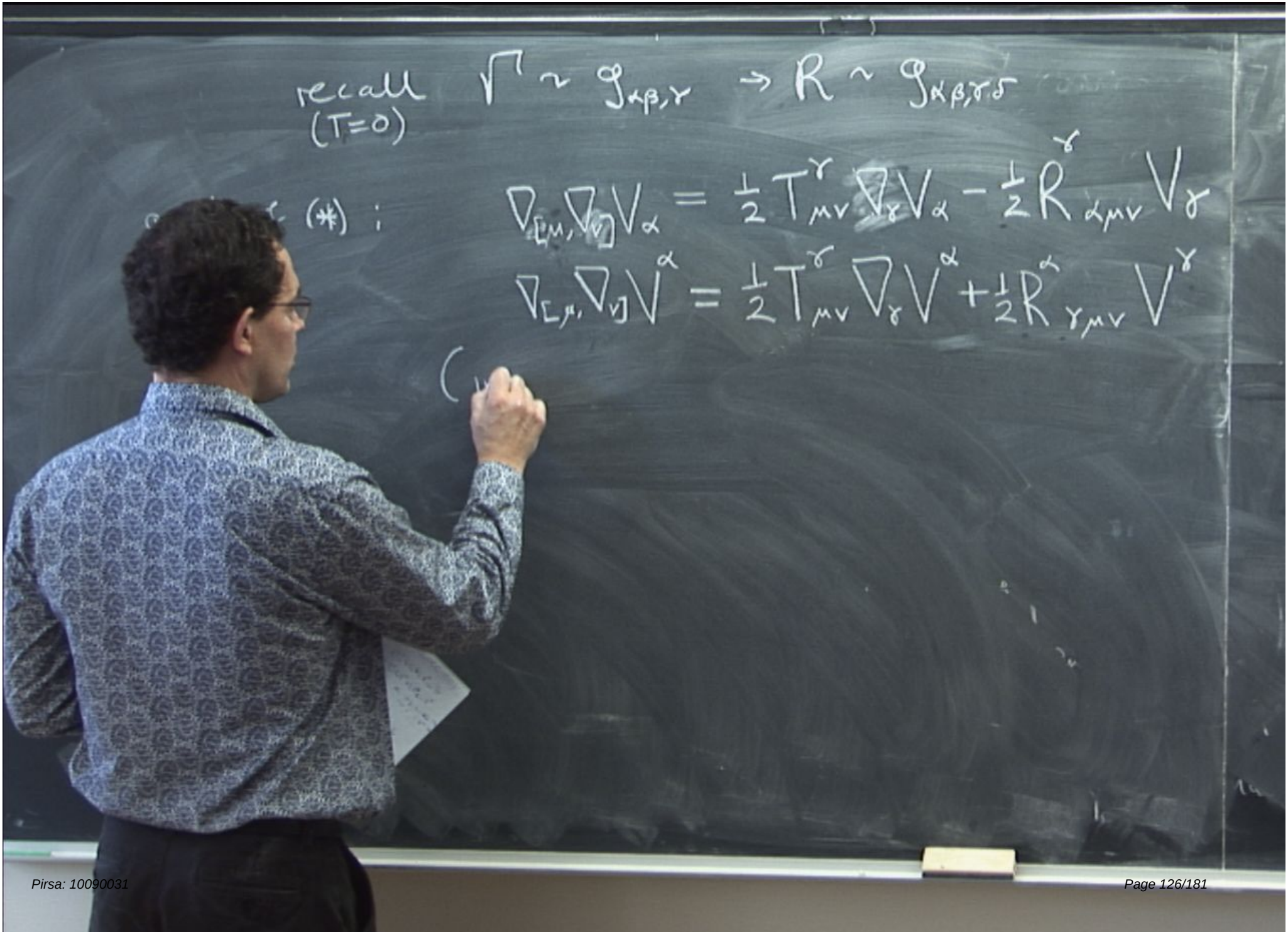

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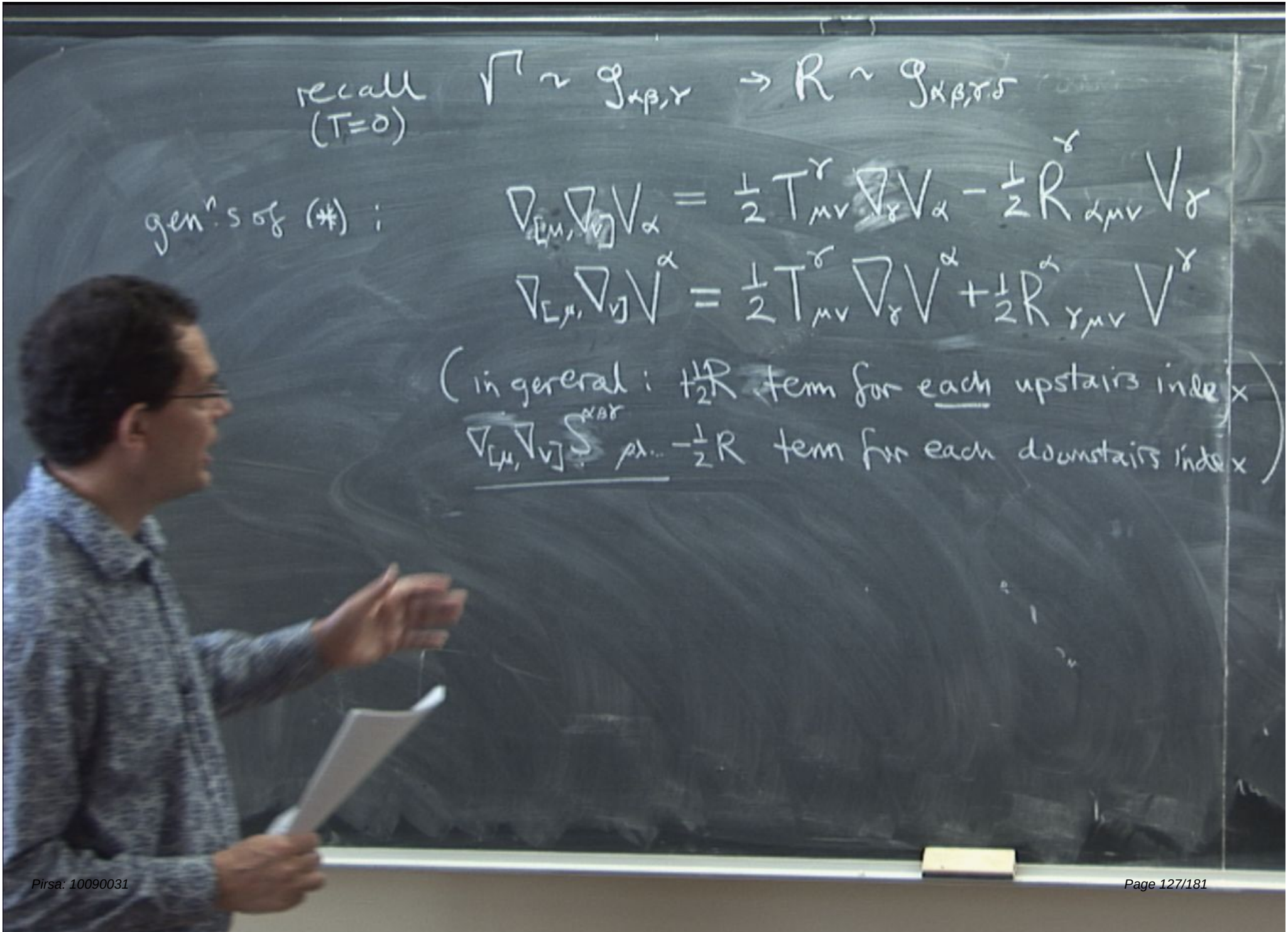
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$\nabla_{[\mu}\nabla_{\nu]}V^\alpha = \frac{1}{2}T^\sigma_{\mu\nu}\nabla_\sigma V^\alpha + \frac{1}{2}R^\alpha{}_{\gamma\mu\nu}V^\gamma$
(in gereral : $+\frac{1}{2}R$ term for each upstairs index
$\nabla_{[\mu}\nabla_{\nu]}S^{\alpha\beta\gamma}{}_{\rho\lambda\ldots}$ $-\frac{1}{2}R$ term for each downstairs index)

recall ($T=0$) $\Gamma \sim g_{\alpha\beta,\gamma} \to R \sim g_{\alpha\beta,\gamma\delta}$

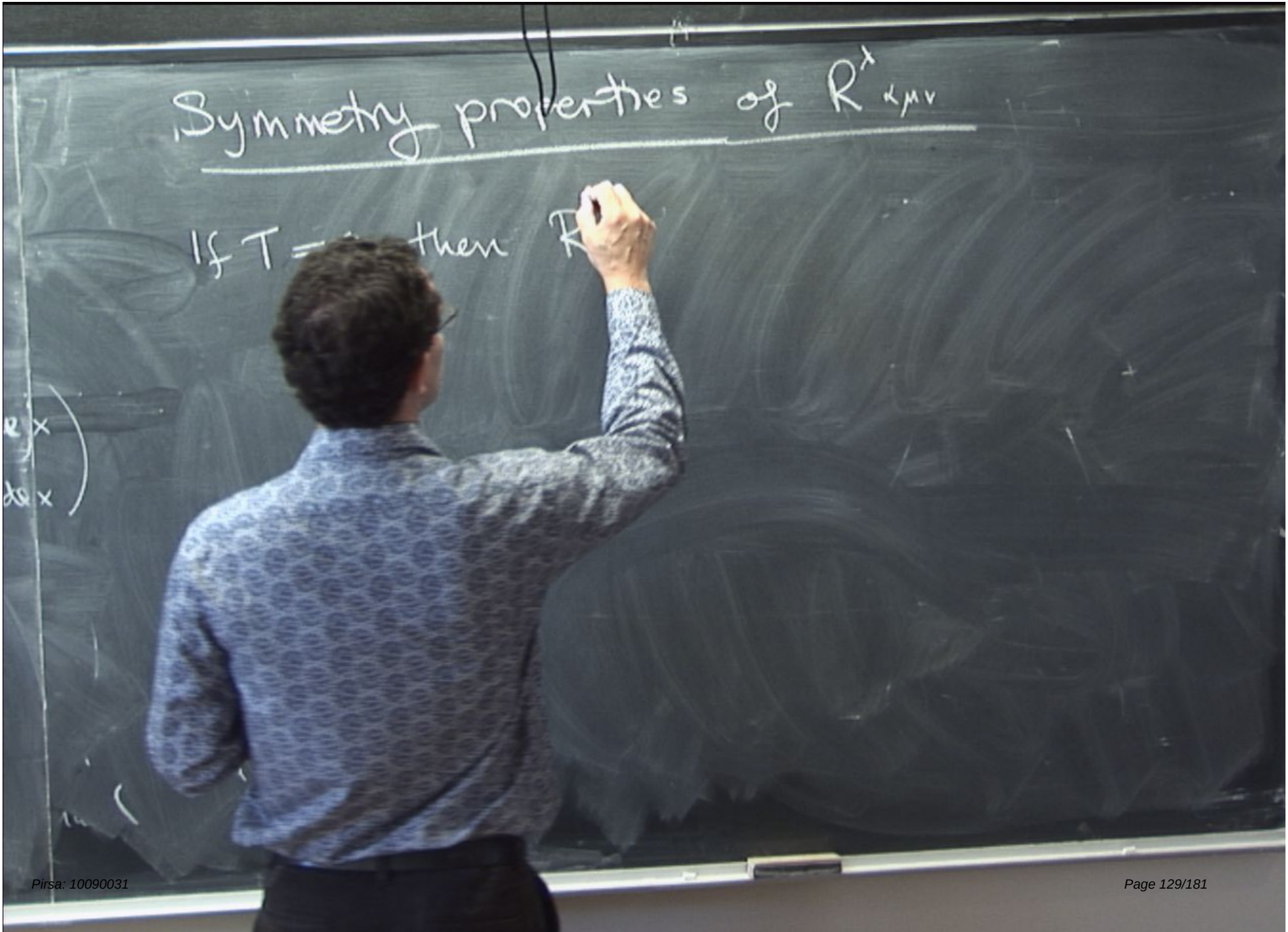
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Symmetry properties of $R^{\lambda}{}_{\kappa\mu\nu}$
If $T =$ then R

Symmetry properties of $R^{\lambda}{}_{\alpha\mu\nu}$

If $T=0$, then $R^{\gamma}{}_{[\alpha\mu\nu]} = 0$

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Symmetry properties of $R^{\lambda}{}_{\alpha\mu\nu}$

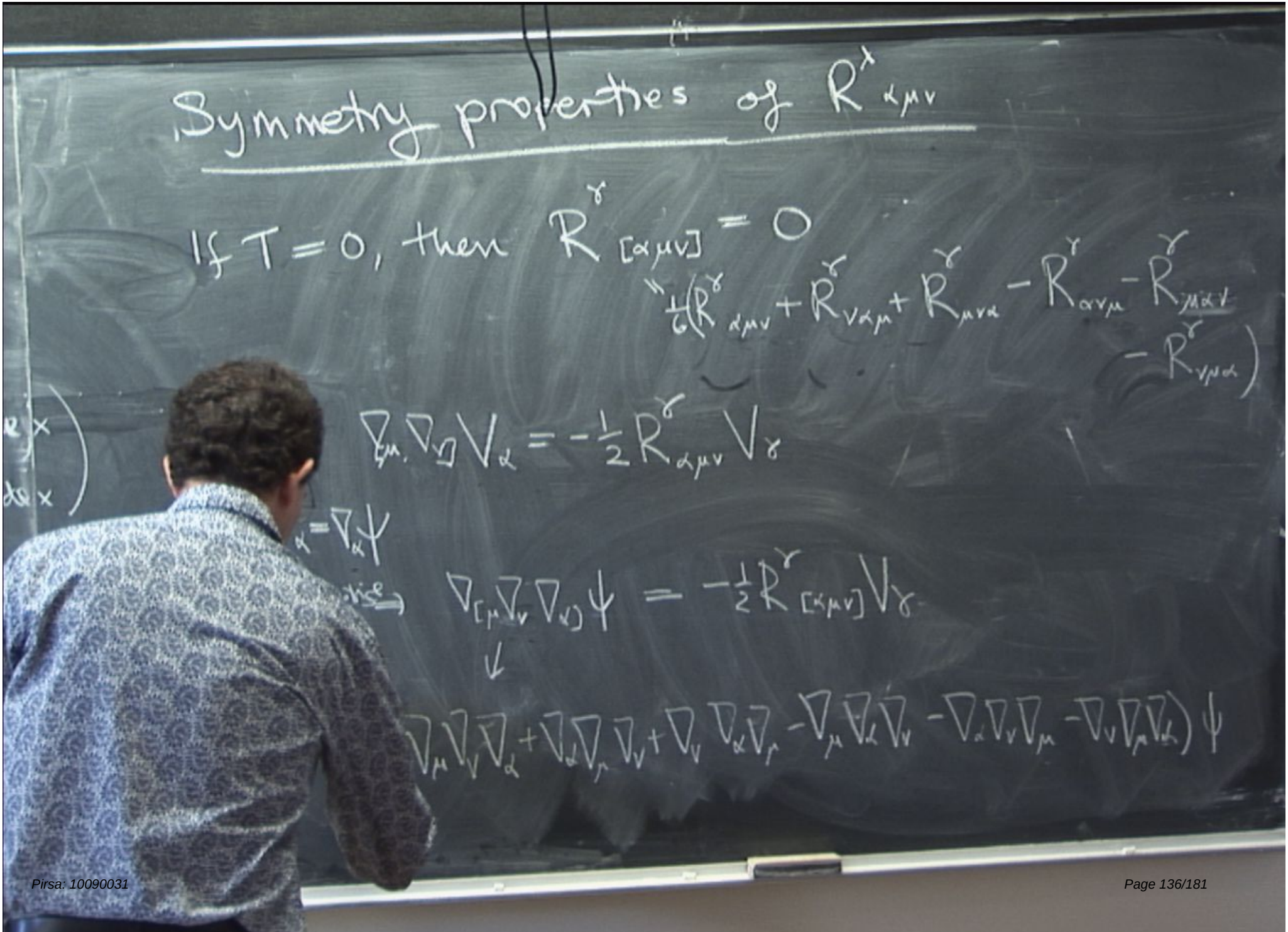
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If $V_{\alpha} = \nabla_{\alpha}\psi$

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$$\frac{1}{6}\left(\nabla_{\mu}\nabla_{\nu}\nabla_{\alpha} + \nabla_{\alpha}\nabla_{\mu}\nabla_{\nu} + \nabla_{\nu}\nabla_{\alpha}\nabla_{\mu} - \nabla_{\mu}\nabla_{\alpha}\nabla_{\nu} - \nabla_{\alpha}\nabla_{\nu}\nabla_{\mu} - \nabla_{\nu}\nabla_{\mu}\nabla_{\alpha}\right)\psi$$

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$$\frac{1}{6}\left(\nabla_{\mu}\nabla_{\nu}\nabla_{\alpha}+\nabla_{\alpha}\nabla_{\mu}\nabla_{\nu}+\nabla_{\nu}\nabla_{\alpha}\nabla_{\mu}-\nabla_{\mu}\nabla_{\alpha}\nabla_{\nu}-\nabla_{\alpha}\nabla_{\nu}\nabla_{\mu}-\nabla_{\nu}\nabla_{\mu}\nabla_{\alpha}\right)\psi$$

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$$\nabla_{\mu}\nabla_{\nu}V_{\alpha} = -\frac{1}{2}R^{\gamma}_{\alpha\mu\nu}V_{\gamma}$$

If $V_{\alpha} = \nabla_{\alpha}\psi$

If $T=0$

$\nabla_{\mu}\nabla_{\nu}\psi = \nabla_{\nu}\nabla_{\mu}\psi$

$\nabla_{\mu}\partial_{\nu}\psi$

$= \left(\partial_{\mu}\partial_{\nu} - \Gamma^{\alpha}_{\nu\mu}\partial_{\alpha}\right)\psi$

antisymmetrise $\Rightarrow$ $\nabla_{[\mu}\nabla_{\nu}\nabla_{\alpha]}\psi = -\frac{1}{2}R^{\gamma}_{[\alpha\mu\nu]}V_{\gamma}$

$$\frac{1}{6}\left(\nabla_{\mu}\nabla_{\nu}\nabla_{\alpha} + \nabla_{\alpha}\nabla_{\mu}\nabla_{\nu} + \nabla_{\nu}\nabla_{\alpha}\nabla_{\mu} - \nabla_{\mu}\nabla_{\alpha}\nabla_{\nu} - \nabla_{\alpha}\nabla_{\nu}\nabla_{\mu} - \nabla_{\nu}\nabla_{\mu}\nabla_{\alpha}\right)\psi$$

Symmetry properties of $R^{\lambda}{}_{\kappa\mu\nu}$

If $T=0$, then $R^{\gamma}{}_{[\alpha\mu\nu]} = 0$

$$= \frac{1}{6}\left(R^{\gamma}{}_{\alpha\mu\nu} + R^{\gamma}{}_{\nu\alpha\mu} + R^{\gamma}{}_{\mu\nu\alpha} - R^{\gamma}{}_{\alpha\nu\mu} - R^{\gamma}{}_{\mu\alpha\nu} - R^{\gamma}{}_{\nu\mu\alpha}\right)$$

$$\nabla_{[\mu}\nabla_{\nu]} V_{\alpha} = -\frac{1}{2} R^{\gamma}{}_{\alpha\mu\nu} V_{\gamma}$$

If $V_{\alpha} = \nabla_{\alpha}\psi$

If $T=0$

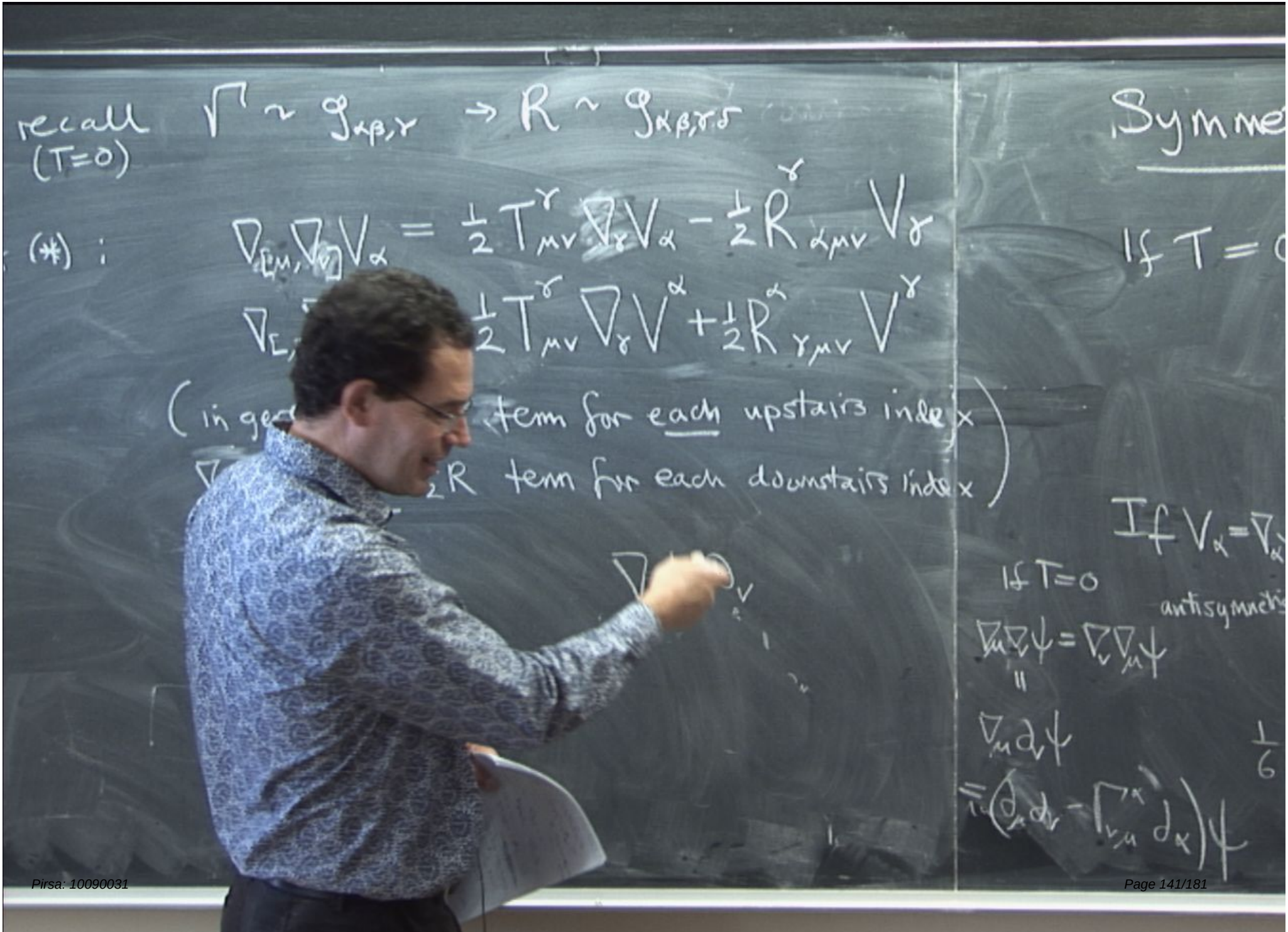
$\nabla_{\mu}\nabla_{\nu}\psi = \nabla_{\nu}\nabla_{\mu}\psi$

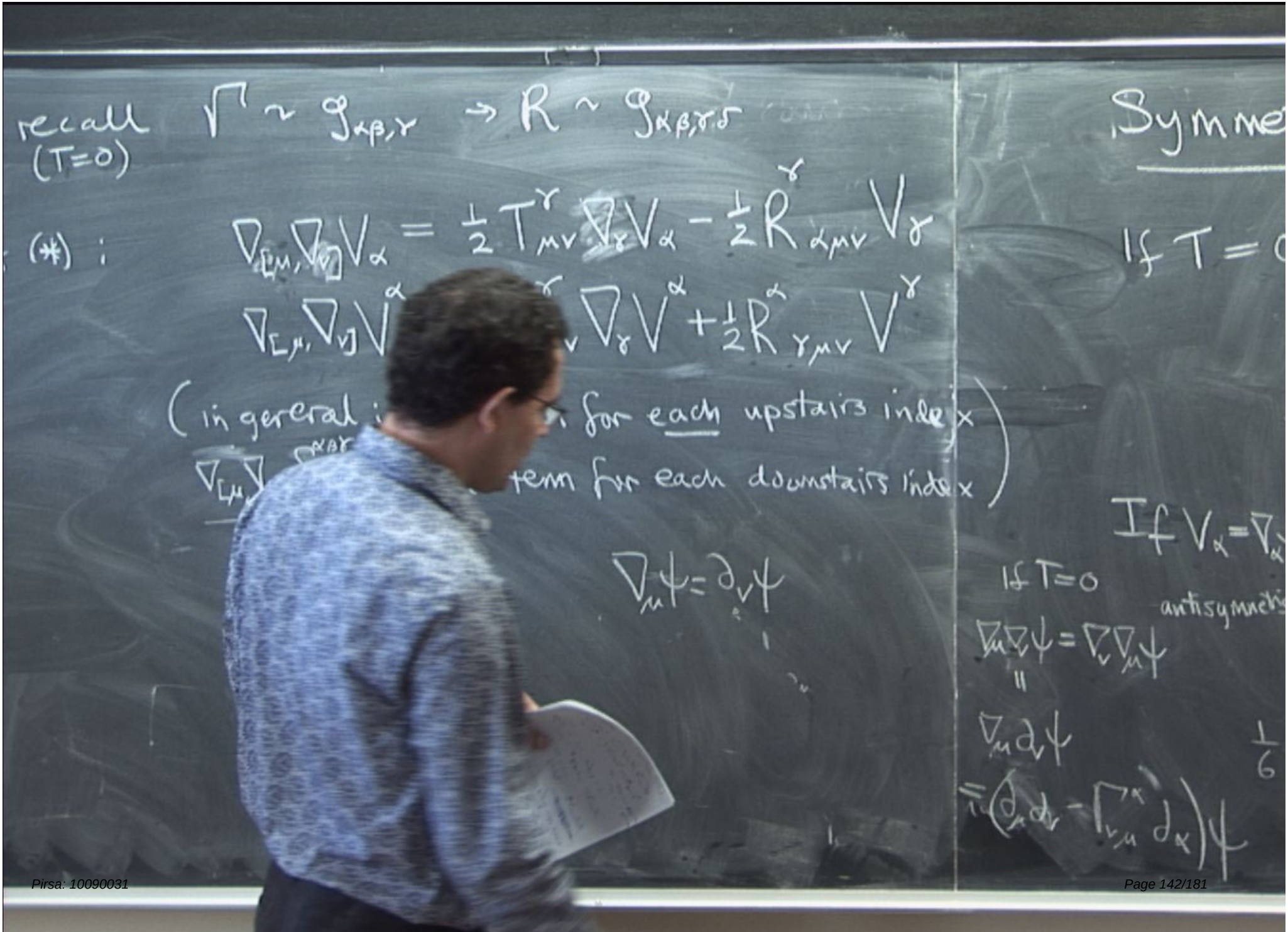
$\nabla_{\mu}\partial_{\nu}\psi$

$= \left(\partial_{\mu}\partial_{\nu} - \Gamma^{\kappa}_{\nu\mu}\partial_{\kappa}\right)\psi$

antisymmetrise $\Rightarrow$ $\nabla_{[\mu}\nabla_{\nu}\nabla_{\alpha]}\psi = -\frac{1}{2} R^{\gamma}{}_{[\kappa\mu\nu]} V_{\gamma}$

$$\frac{1}{6}\left(\nabla_{\mu}\nabla_{\nu}\nabla_{\alpha} + \nabla_{\alpha}\nabla_{\mu}\nabla_{\nu} + \nabla_{\nu}\nabla_{\alpha}\nabla_{\mu} - \nabla_{\mu}\nabla_{\alpha}\nabla_{\nu} - \nabla_{\alpha}\nabla_{\nu}\nabla_{\mu} - \nabla_{\nu}\nabla_{\mu}\nabla_{\alpha}\right)\psi$$


recall (T=0)
$\Gamma \sim g_{\alpha\beta,\gamma} \to R \sim g_{\alpha\beta,\gamma\delta}$
(*) :
$\nabla_{[\mu}\nabla_{\nu]}V_\alpha = \frac{1}{2}T^\gamma_{\mu\nu}\nabla_\gamma V_\alpha - \frac{1}{2}R^\gamma{}_{\alpha\mu\nu}V_\gamma$
$\frac{1}{2}T^\gamma_{\mu\nu}\nabla_\gamma V^\alpha + \frac{1}{2}R^\alpha{}_{\gamma\mu\nu}V^\gamma$
(in ge tem for each upstairs inde
R tem for each downstairs inde
Symme
If T=0
If T=0
antisymmetri
$\nabla_\mu\nabla_\nu\psi = \nabla_\nu\nabla_\mu\psi$
$\nabla_\mu\partial_\nu\psi$
$= (\partial_\mu\partial_\nu - \Gamma^\alpha_{\nu\mu}\partial_\alpha)\psi$
1/6


recall (T=0)
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(in gereral ... for each upstairs index
... term for each downstairs index)
$\nabla_\mu\psi = \partial_\mu\psi$
Symme
If T = 0
If $V_\alpha = \nabla_\alpha$
if T=0
antisymmetric
$\nabla_\mu\nabla_\nu\psi = \nabla_\nu\nabla_\mu\psi$
$\nabla_\mu\partial_\nu\psi$
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1/6

Symmetry properties of $R^{\lambda}{}_{\alpha\mu\nu}$

If $T=0$, then $R^{\gamma}{}_{[\alpha\mu\nu]}=0$

$$= \frac{1}{6}\left(R^{\gamma}{}_{\alpha\mu\nu} + R^{\gamma}{}_{\nu\alpha\mu} + R^{\gamma}{}_{\mu\nu\alpha} - R^{\gamma}{}_{\alpha\nu\mu} - R^{\gamma}{}_{\mu\alpha\nu} - R^{\gamma}{}_{\nu\mu\alpha}\right)$$

$$\nabla_{[\mu}\nabla_{\nu]}V_{\alpha} = -\frac{1}{2}R^{\gamma}{}_{\alpha\mu\nu}V_{\gamma}$$

If $V_{\alpha}=\nabla_{\alpha}\psi$

If $T=0$

$\nabla_{\mu}\nabla_{\nu}\psi = \nabla_{\nu}\nabla_{\mu}\psi$

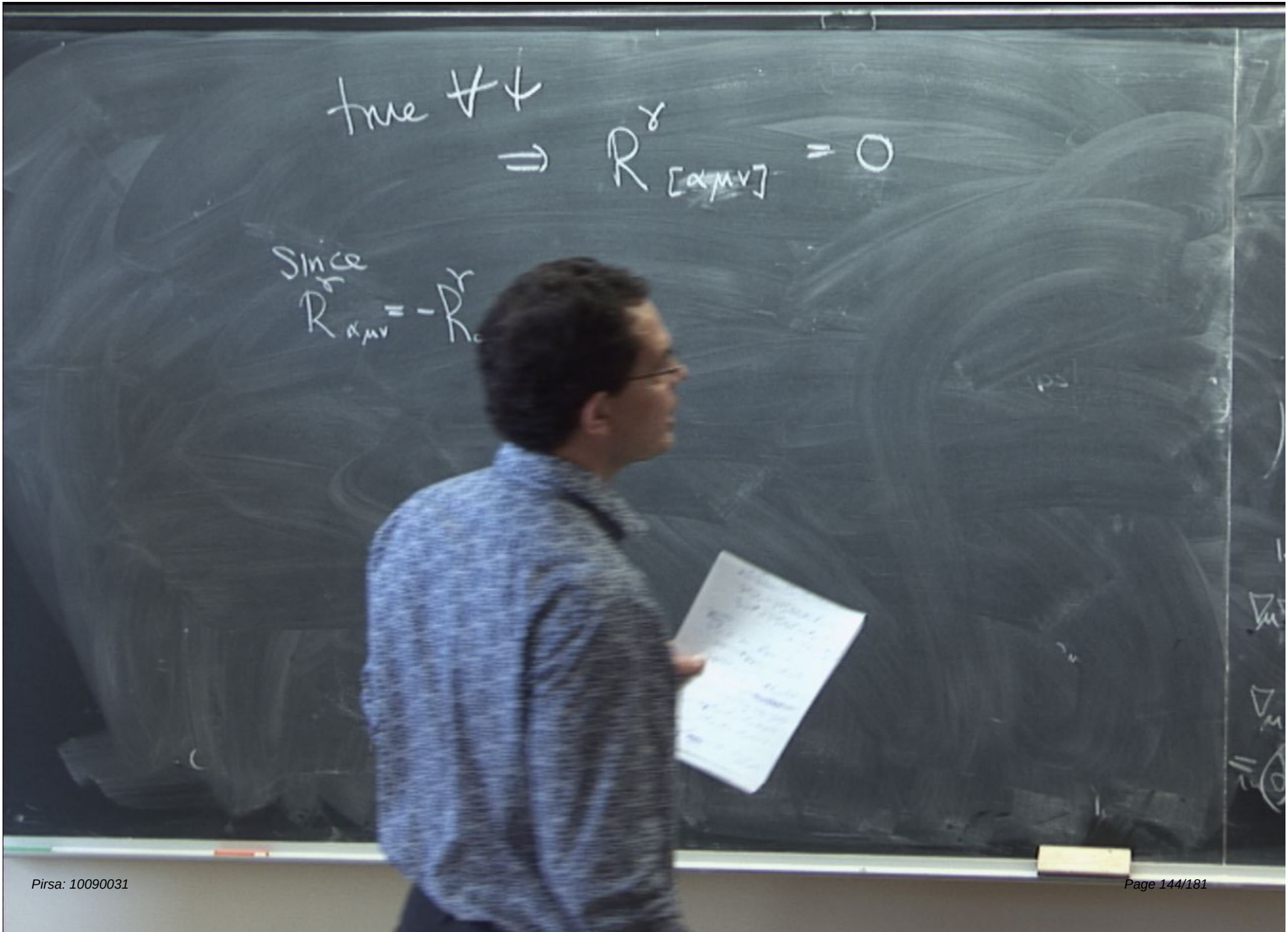
$\nabla_{\mu}\partial_{\nu}\psi$

$= (\partial_{\mu}\partial_{\nu} - \Gamma^{\alpha}_{\nu\mu}\partial_{\alpha})\psi$

antisymmetrise $\Rightarrow$ $\nabla_{[\mu}\nabla_{\nu}\nabla_{\alpha]}\psi = -\frac{1}{2}R^{\gamma}{}_{[\alpha\mu\nu]}V_{\gamma}$

$= 0.$

$$\frac{1}{6}\left(\nabla_{\mu}\nabla_{\nu}\nabla_{\alpha} + \nabla_{\alpha}\nabla_{\mu}\nabla_{\nu} + \nabla_{\nu}\nabla_{\alpha}\nabla_{\mu} - \nabla_{\mu}\nabla_{\alpha}\nabla_{\nu} - \nabla_{\alpha}\nabla_{\nu}\nabla_{\mu} - \nabla_{\nu}\nabla_{\mu}\nabla_{\alpha}\right)\psi$$


true $\forall \psi$
$\Rightarrow R^{\gamma}_{[\alpha\mu\nu]} = 0$
Since
$R^{\gamma}_{\alpha\mu\nu} = -R^{\gamma}$

true $\forall \psi$

$$\Rightarrow R^{\gamma}_{[\alpha\mu\nu]} = 0$$

Since $R^{\gamma}_{\alpha\mu\nu} = -R^{\gamma}_{\alpha\nu\mu}$

$$\Rightarrow R^{\gamma}_{\alpha\mu\nu} + R^{\gamma}_{\nu\alpha\mu} + R^{\gamma}_{\mu\nu\alpha} = 0$$

true $\forall \psi$

$$\Rightarrow R^{\gamma}_{[\alpha\mu\nu]} = 0$$

Since $R^{\gamma}_{\alpha\mu\nu} = -R^{\gamma}_{\alpha\nu\mu}$ $\Rightarrow$ $R^{\gamma}_{\alpha\mu\nu} + R^{\gamma}_{\nu\alpha\mu} + R^{\gamma}_{\mu\nu\alpha} = 0$

All symmetries:

true $\forall \psi$

$$\Rightarrow R^{\gamma}_{[\alpha\mu\nu]} = 0$$

Since $R^{\gamma}_{\alpha\mu\nu} = -R^{\gamma}_{\alpha\nu\mu}$ $\Rightarrow$ $R^{\gamma}_{\alpha\mu\nu} + R^{\gamma}_{\nu\alpha\mu} + R^{\gamma}_{\mu\nu\alpha} = 0$

All symmetries of Riemann: define $R_{\delta\alpha\mu\nu} = g_{\delta\sigma} R^{\sigma}_{\alpha\mu\nu}$

$$R_{\alpha\beta\mu\nu} = R_{\alpha\beta[\mu\nu]}$$

$$R_{\alpha[\beta\mu\nu]} = 0$$

$$R_{\alpha\beta\mu\nu} = R_{[\alpha\beta]\mu\nu}$$

$$R_{\alpha\beta\mu\nu} = R_{\mu\nu\alpha\beta}$$

true $\forall \psi$

$$\Rightarrow R^{\gamma}_{[\alpha\mu\nu]} = 0$$

Since $R^{\gamma}_{\alpha\mu\nu} = -R^{\gamma}_{\alpha\nu\mu}$ $\Rightarrow$ $R^{\gamma}_{\alpha\mu\nu} + R^{\gamma}_{\nu\alpha\mu} + R^{\gamma}_{\mu\nu\alpha} = 0$

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$$R_{\alpha\beta\mu\nu} = R_{\mu\nu\alpha\beta}$$

$\nabla_{[\mu}\nabla_{\nu]} g_{\alpha\beta} = 0$

true $\forall \psi$

$$\Rightarrow R^{\gamma}_{[\alpha\mu\nu]} = 0$$

Since $R^{\gamma}_{\alpha\mu\nu} = -R^{\gamma}_{\alpha\nu\mu}$

$$\Rightarrow R^{\gamma}_{\alpha\mu\nu} + R^{\gamma}_{\nu\alpha\mu} + R^{\gamma}_{\mu\nu\alpha} = 0$$

All symmetries of Riemann: define $R_{\delta\alpha\mu\nu} = g_{\delta\sigma} R^{\sigma}_{\alpha\mu\nu}$

reduce # of indept Cmpnts to 20!

$$\begin{cases} R_{\alpha\beta\mu\nu} = R_{\alpha\beta[\mu\nu]} \\ R_{\alpha[\beta\mu\nu]} = 0 \\ R_{\alpha\beta\mu\nu} = R_{[\alpha\beta]\mu\nu} \\ R_{\alpha\beta\mu\nu} = R_{\mu\nu\alpha\beta} \end{cases}$$

$\leftarrow \nabla_{[\mu}\nabla_{\nu]} g_{\alpha\beta} = 0$

true $\forall \psi$

$$\Rightarrow R^{\gamma}_{[\alpha\mu\nu]} = 0$$

Since $R^{\gamma}_{\alpha\mu\nu} = -R^{\gamma}_{\alpha\nu\mu}$

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All symmetries of Riemann: define $R_{\delta\alpha\mu\nu} = g_{\delta\sigma} R^{\sigma}_{\alpha\mu\nu}$

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$$\begin{cases} R_{\alpha\beta\mu\nu} = R_{\alpha\beta[\mu\nu]} \\ R_{\alpha[\beta\mu\nu]} = 0 \\ R_{\alpha\beta\mu\nu} = R_{[\alpha\beta]\mu\nu} \\ R_{\alpha\beta\mu\nu} = R_{\mu\nu\alpha\beta} \end{cases}$$

$\leftarrow \nabla_{[\mu}\nabla_{\nu]} g_{\alpha\beta} = 0$

true $\forall \psi$

$$\Rightarrow R^{\gamma}_{[\alpha\beta\mu\nu]} = 0$$

Since $R^{\gamma}_{\alpha\beta\mu\nu} = -R^{\gamma}_{\alpha\nu\beta\mu}$

$$\Rightarrow R^{\gamma}_{\alpha\beta\mu\nu} + R^{\gamma}_{\nu\alpha\beta\mu} + R^{\gamma}_{\mu\nu\alpha\beta} = 0$$

All symmetries of Riemann: define $R_{\delta\alpha\beta\mu\nu} = g_{\delta\gamma} R^{\gamma}_{\alpha\beta\mu\nu}$ (assuming $T=0$)

reduce # of indept Cmpnts to 20!

$$\begin{cases} R_{\alpha\beta\mu\nu} = R_{\alpha\beta[\mu\nu]} \\ R_{\alpha[\beta\mu\nu]} = 0 \\ R_{\alpha\beta\mu\nu} = R_{[\alpha\beta]\mu\nu} \\ R_{\alpha\beta\mu\nu} = R_{\mu\nu\alpha\beta} \end{cases}$$

$\leftarrow \nabla_{[\mu}\nabla_{\nu]} g_{\alpha\beta} = 0$

Contractions of Riemann:

$$R^{\alpha}{}_{\beta\alpha\nu}$$

Contractions of Riemann:

$R^{\alpha}{}_{\alpha\mu\nu}$ $\quad$ $R^{\alpha}{}_{\beta\alpha\nu}$

Contractions of Riemann:

$0 = R^{\alpha}_{\beta\mu\nu}$ $\quad R^{\alpha}_{\beta\alpha\nu}$

Contractions of Riemann:

$R^{\alpha}{}_{\beta\alpha\nu} \equiv R_{\beta\nu}$ Ricci

$\theta_i = R^{\alpha}{}_{\beta\mu\nu}$

Contractions of Riemann:

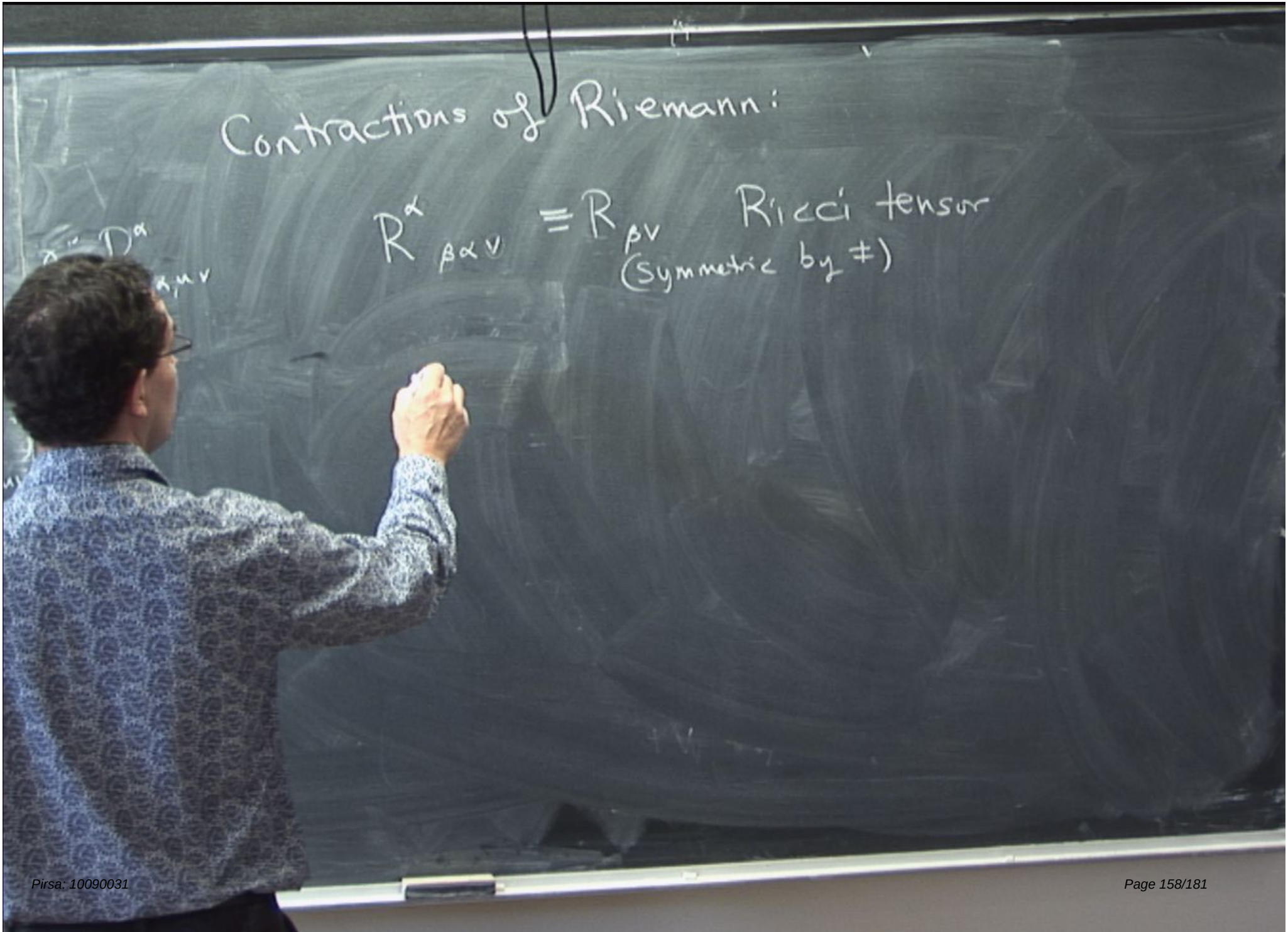
$0 = R^{\alpha}{}_{\beta\mu\nu}$

$R^{\alpha}{}_{\beta\alpha\nu} \equiv R_{\beta\nu}$ Ricci

Contractions of Riemann:

$R^{\alpha}{}_{\beta\alpha\nu} \equiv R_{\beta\nu}$ Ricci tensor

$0 = R^{\alpha}{}_{\beta\mu\nu}$


Contractions of Riemann:
$R^{\alpha}{}_{\beta\alpha\nu} \equiv R_{\beta\nu}$ Ricci tensor
(symmetric by $\neq$)

Contractions of Riemann:

$0 = R^{\alpha}_{\beta\mu\nu}$

$R^{\alpha}_{\beta\alpha\nu} \equiv R_{\beta\nu}$ Ricci tensor
(Symmetric by $\neq$)

$R^{\alpha}_{\alpha} = R$ Ricci scalar
"intrinsic curvature"

Contractions of Riemann:

$0 = R^{\alpha}{}_{\beta\mu\nu}$

$R^{\alpha}{}_{\beta\alpha\nu} \equiv R_{\beta\nu}$ Ricci tensor
(Symmetric by $\neq$)

$R^{\alpha}{}_{\alpha} = R$ Ricci scalar
"intrinsic curvature"

Bianchi Identity.

Bianchi Identity

Gen.d Bianchi $\nabla_{\alpha} R^{\gamma}{}_{\mu\nu}$

Bianchi Identity

Gen.d Bianchi $\nabla_{[\lambda} R^{\gamma}{}_{|\alpha|\mu\nu]}$

α does not participate in the antisymmetrisation.

Bianchi Identity

Gen.d Bianchi $\quad \nabla_{[\alpha} R^{\gamma}{}_{|\alpha|\mu\nu]} = 0$

α does not participate in the antisymmetrisation.

Bianchi Identity

Gen.d Bianchi $\nabla_{[\lambda} R^{\gamma}{}_{|\alpha|\mu\nu]} = 0$

Proof Hughston + Tod p. 59.

α does not participate in the antisymmetrisation.

Bianchi Identity.

Gen.d Bianchi $\nabla_{[\rho} R^{\gamma}{}_{|\alpha|\mu\nu]} = 0$

Proof Hughston + Tod p. 59.

α does not participate in the antisymmetrisation.

$\nabla_\alpha(g_{\beta\gamma}) = 0$

$\Rightarrow \nabla_{[\rho} R_{|\gamma\alpha|\mu\nu]} = 0$

Bianchi Identity

Gen.d Bianchi $\nabla_{[\rho} R^{\gamma}{}_{|\alpha|\mu\nu]} = 0$

Proof Hughston + Tod p. 59.

α does not participate in the antisymmetrisation.

$\nabla_\alpha(g_{\beta\delta}) = 0$

$\Rightarrow \nabla_{[\rho} R_{|\delta\alpha|\mu\nu]} = 0$

Bianchi Identity

Gen.d Bianchi $\nabla_{[\rho} R^{\gamma}{}_{|\alpha|\mu\nu]} = 0$

Proof Hughston + Tod p. 59.

α does not participate in the antisymmetrisation.

$\nabla_\alpha(g_{\beta\gamma}) = 0$

$\Rightarrow \nabla_{[\rho} R_{|\gamma\alpha|\mu\nu]} = 0$

Bianchi Identity

Gen.d Bianchi $\quad \nabla_{[\rho} R^{\gamma}{}_{|\alpha|\mu\nu]} = 0$

Proof Hughston + Tod p. 59.

α does not participate in the antisymmetrisation.

$\nabla_\alpha(g_{\beta\delta}) = 0$

$\Rightarrow \quad \nabla_{[\rho} R_{|\delta\alpha|\mu\nu]} = 0$

$= \nabla_\rho R_{\tau\alpha\mu\nu} + \nabla_\mu R_{\tau\alpha\nu\rho} + \nabla_\nu R_{\tau\alpha\rho\mu} = 0$

(because $R_{|\delta\alpha|\mu\nu} = -R_{|\delta\alpha|\nu\mu}$)

Bianchi Identity

Gen.d Bianchi $\nabla_{[\rho} R^{\gamma}{}_{|\alpha|\mu\nu]} = 0$

Proof Hughston + Tod p. 59.

$|\alpha|$ does not participate in the antisymmetrisation.

$\nabla_\alpha(g_{\beta\delta}) = 0$

$\Rightarrow \nabla_{[\rho} R_{|\delta\alpha|\mu\nu]} = 0$

(because $R_{|\delta\alpha|\nu\mu} = -R_{|\delta\alpha|\mu\nu}$)

$= \nabla_\rho R_{\gamma\alpha\mu\nu} + \nabla_\mu R_{\gamma\alpha\nu\rho} + \nabla_\nu R_{\gamma\alpha\rho\mu} = 0$

$g^{\gamma\mu} g^{\alpha\nu}$

Bianchi Identity

Gen.d Bianchi $\nabla_{[\rho} R^{\gamma}{}_{|\alpha|\mu\nu]} = 0$

Proof Hughston + Tod p. 59.

α does not participate in the antisymmetrisation.

$\nabla_\alpha(g_{\beta\delta}) = 0$

$\Rightarrow \nabla_{[\rho} R_{|\delta\alpha|\mu\nu]} = 0$

$= \nabla_\rho R_{\delta\alpha\mu\nu} + \nabla_\mu R_{\delta\alpha\nu\rho} + \nabla_\nu R_{\delta\alpha\rho\mu} = 0$

(because $R_{|\delta\alpha|\mu\nu}$ $\times g^{\delta\mu} g^{\rho\alpha}$)

$\nabla^\rho R_{\rho\nu} + \nabla^\mu R_{\mu\nu}$

Bianchi Identity

Gen.d Bianchi $\nabla_{[\rho} R^{\gamma}{}_{|\alpha|\mu\nu]} = 0$

Proof Hughston + Tod p. 59.

α does not participate in the antisymmetrisation.

$\nabla_\alpha(g_{\beta\delta}) = 0$

$\Rightarrow \nabla_{[\rho} R_{|\delta\alpha|\mu\nu]} = 0$

(because $R_{|\delta\alpha|\mu\nu} = R_{|\delta\alpha|\nu\mu}$) $\times g^{\gamma\mu} g^{\rho\alpha}$

$= \nabla_\rho R_{\gamma\alpha\mu\nu} + \nabla_\mu R_{\gamma\alpha\nu\rho} + \nabla_\nu R_{\gamma\alpha\rho\mu} = 0$

$\nabla^\rho R_{\gamma\rho\nu} + \nabla^\mu R_{\mu\nu} - \nabla_\nu R = 0$

Bianchi Identity

Gen.d Bianchi $\nabla_{[\rho} R^{\gamma}{}_{|\alpha|\mu\nu]} = 0$

Proof Hughston + Tod p. 59.

α does not participate in the antisymmetrisation.

$\nabla_\alpha(g_{\beta\gamma}) = 0$

$\Rightarrow \nabla_{[\rho} R_{|\gamma\alpha|\mu\nu]} = 0$

(because $R_{|\gamma\alpha|\mu\nu} = -R_{|\gamma\alpha|\nu\mu}$ $\times g^{\gamma\mu} g^{\alpha\nu}$)

$= \nabla_\rho R_{\gamma\alpha\mu\nu} + \nabla_\mu R_{\gamma\alpha\nu\rho} + \nabla_\nu R_{\gamma\alpha\rho\mu} = 0$

$\nabla^\rho R_{\rho\nu} + \nabla^\mu R_{\mu\nu} - \nabla_\nu R = 0$

Contractions of Riemann:

$R^{\alpha}{}_{\beta\alpha\nu} \equiv R_{\beta\nu}$ Ricci tensor
(Symmetric by $\neq$)

$R^{\alpha}{}_{\alpha} = R$ Ricci scalar
"intrinsic curvature"

$\rightarrow \nabla^{\rho}\left(R_{\rho\nu} - \frac{1}{2} g_{\rho\nu} R\right) = 0$

(contracted)

Bianchi Identity

$$\nabla^\rho \left(R_{\rho\nu} - \tfrac{1}{2} g_{\rho\nu} R \right) = 0$$

$$G_{\mu\nu} = T_{\mu\nu}$$

$$\nabla_\mu T^{\mu\nu} = 0$$

(contracted)

Bianchi Identity

$$\rightarrow \nabla^\rho \underbrace{\left(R_{\rho\nu} - \tfrac{1}{2} g_{\rho\nu} R\right)}_{G_{\rho\nu}} = 0$$

$$G_{\mu\nu} = (8\pi G) T_{\mu\nu}$$

$$\nabla_\mu T^{\mu\nu} = 0$$

(contracted)

Bianchi Identity

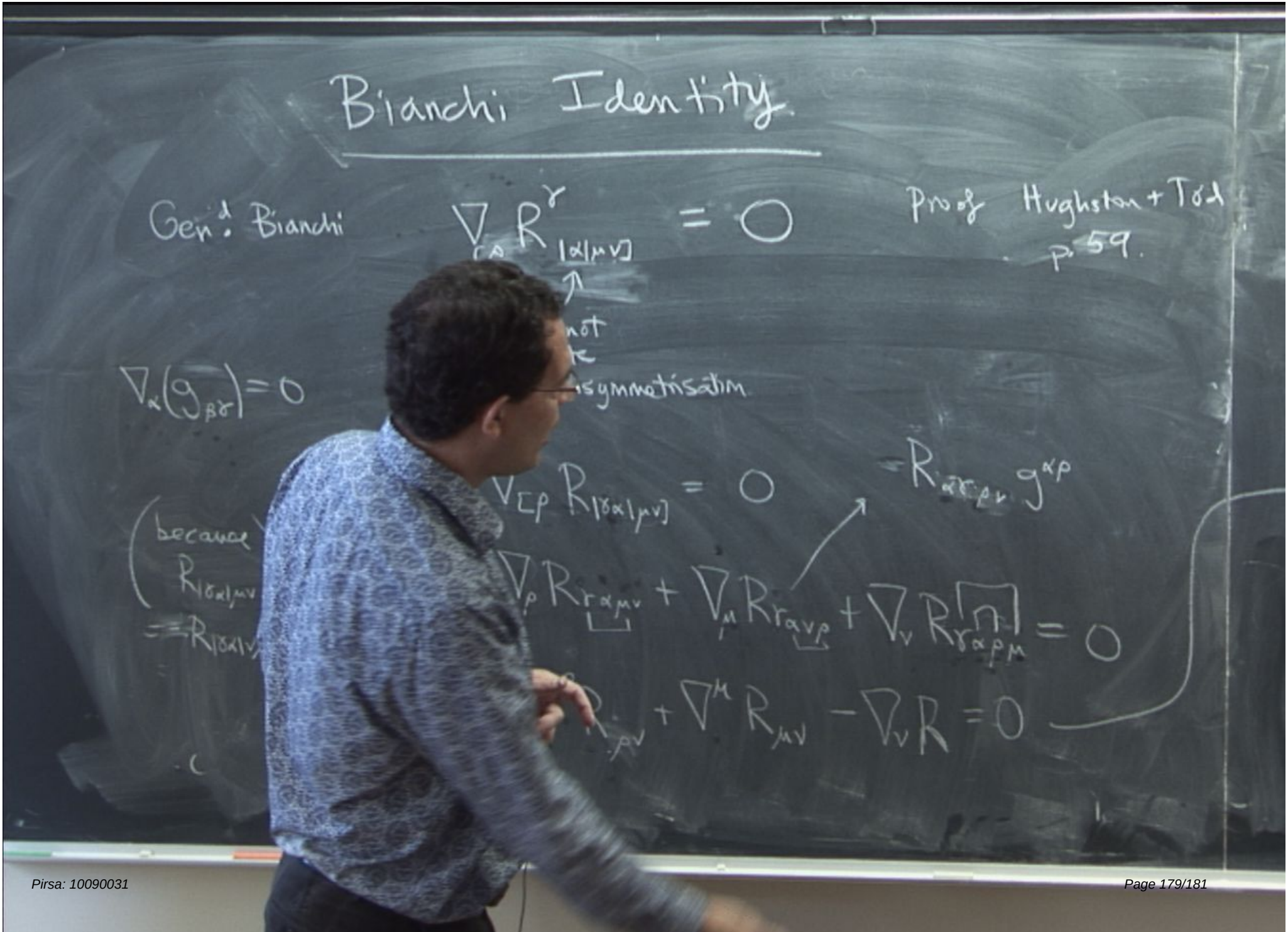
$$\rightarrow \nabla^\rho \underbrace{\left(R_{\rho\nu} - \tfrac{1}{2} g_{\rho\nu} R\right)}_{G_{\rho\nu}} = 0$$

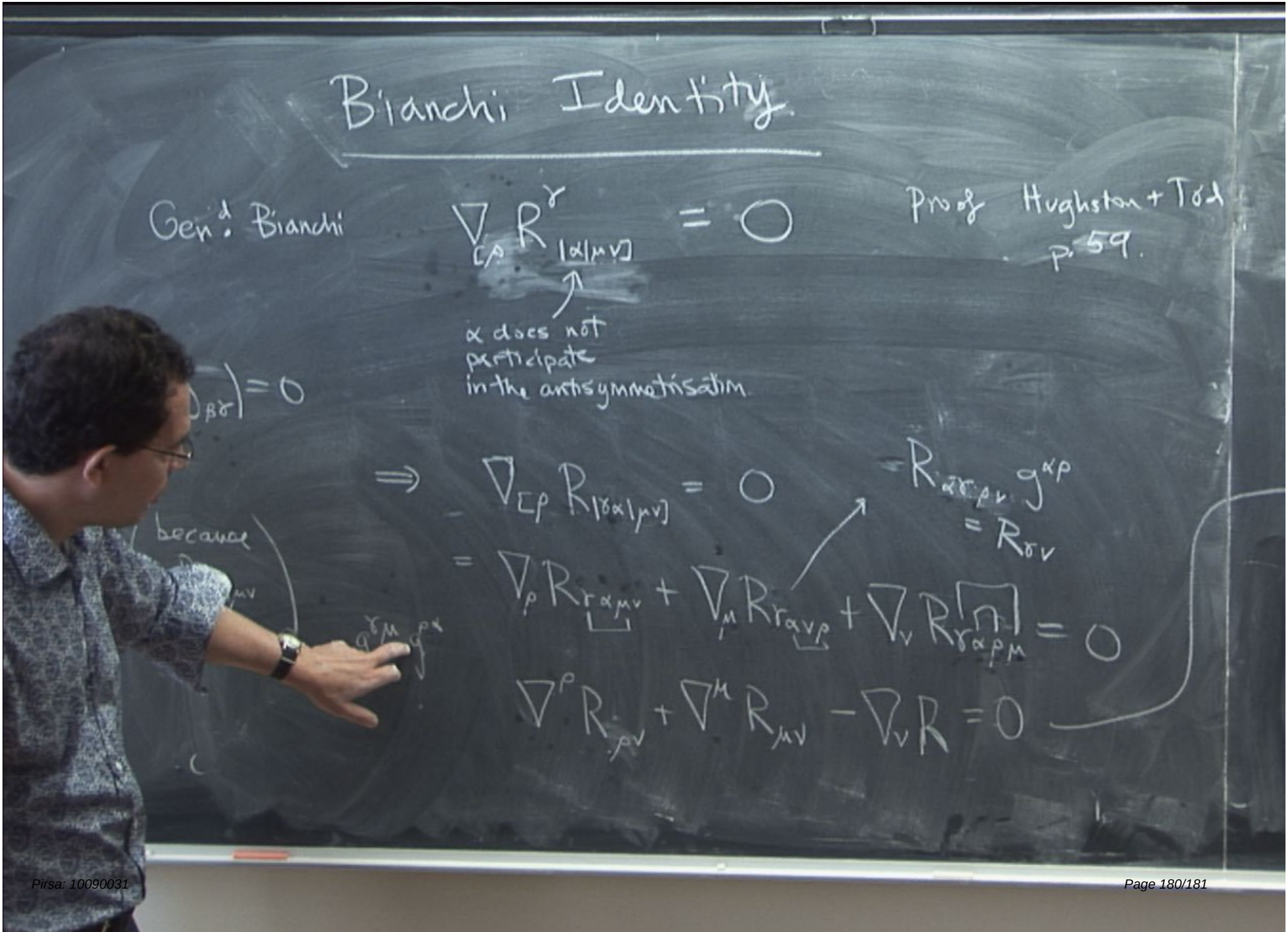
$$G_{\mu\nu} = (8\pi G) T_{\mu\nu}$$

$$\nabla_\mu T^{\mu\nu} = 0$$

(contracted)
Bianchi Identity

$$\rightarrow \nabla^\rho \underbrace{\left(R_{\rho\nu} - \tfrac{1}{2} g_{\rho\nu} R\right)}_{G_{\rho\nu}} = 0$$


Bianchi Identity
Gen.d Bianchi
Proof Hughston + Tod
p. 59.
$\nabla_{[\alpha} R^{\gamma}{}_{|\alpha|\mu\nu]} = 0$
$\nabla_\alpha(g_{\beta\gamma}) = 0$
$\nabla_{[\rho} R_{|\beta\alpha|\mu\nu]} = 0$
$= R_{\alpha\beta\rho\nu} g^{\alpha\rho}$
$\nabla_\rho R_{\gamma\alpha\mu\nu} + \nabla_\mu R_{\gamma\alpha\nu\rho} + \nabla_\nu R_{\gamma\alpha\rho\mu} = 0$
$\nabla^\mu R_{\mu\nu} - \nabla_\nu R = 0$


Bianchi Identity
Gen.d Bianchi
$\nabla_{[\rho} R^{\gamma}{}_{|\alpha|\mu\nu]} = 0$
Proof Hughston + Tod p. 59.
α does not participate in the antisymmetrisation.
$\Rightarrow \nabla_{[\rho} R_{|\gamma\alpha|\mu\nu]} = 0$
because
$= \nabla_\rho R_{\gamma\alpha\mu\nu} + \nabla_\mu R_{\gamma\alpha\nu\rho} + \nabla_\nu R_{\gamma\alpha\rho\mu} = 0$
$-R_{\alpha\gamma\rho\nu} g^{\alpha\rho} = R_{\gamma\nu}$
$\nabla^\rho R_{\gamma\rho\nu} + \nabla^\mu R_{\mu\nu} - \nabla_\nu R = 0$

Bianchi Identity

Gen.d Bianchi $\nabla_{[\rho} R^{\gamma}{}_{|\alpha|\mu\nu]} = 0$

Proof Hughston + Tod p. 59.

α does not participate in the antisymmetrisation.

$\nabla_\alpha(g_{\beta\gamma}) = 0$

$\nabla^\gamma R_{\gamma\nu}$

$\Rightarrow \nabla_{[\rho} R_{|\gamma\alpha|\mu\nu]} = 0$

$= R_{\alpha\gamma\rho\nu} g^{\alpha\rho} = R_{\gamma\nu}$

(because $R_{|\gamma\alpha|\nu\mu} = -R_{|\gamma\alpha|\mu\nu}$) $\times g^{\gamma\mu} g^{\alpha\nu}$

$= \nabla_\rho R_{\gamma\alpha\mu\nu} + \nabla_\mu R_{\gamma\alpha\nu\rho} + \nabla_\nu R_{\gamma\alpha\rho\mu} = 0$

$\nabla^\rho R_{\rho\nu} + \nabla^\mu R_{\mu\nu} - \nabla_\nu R = 0$