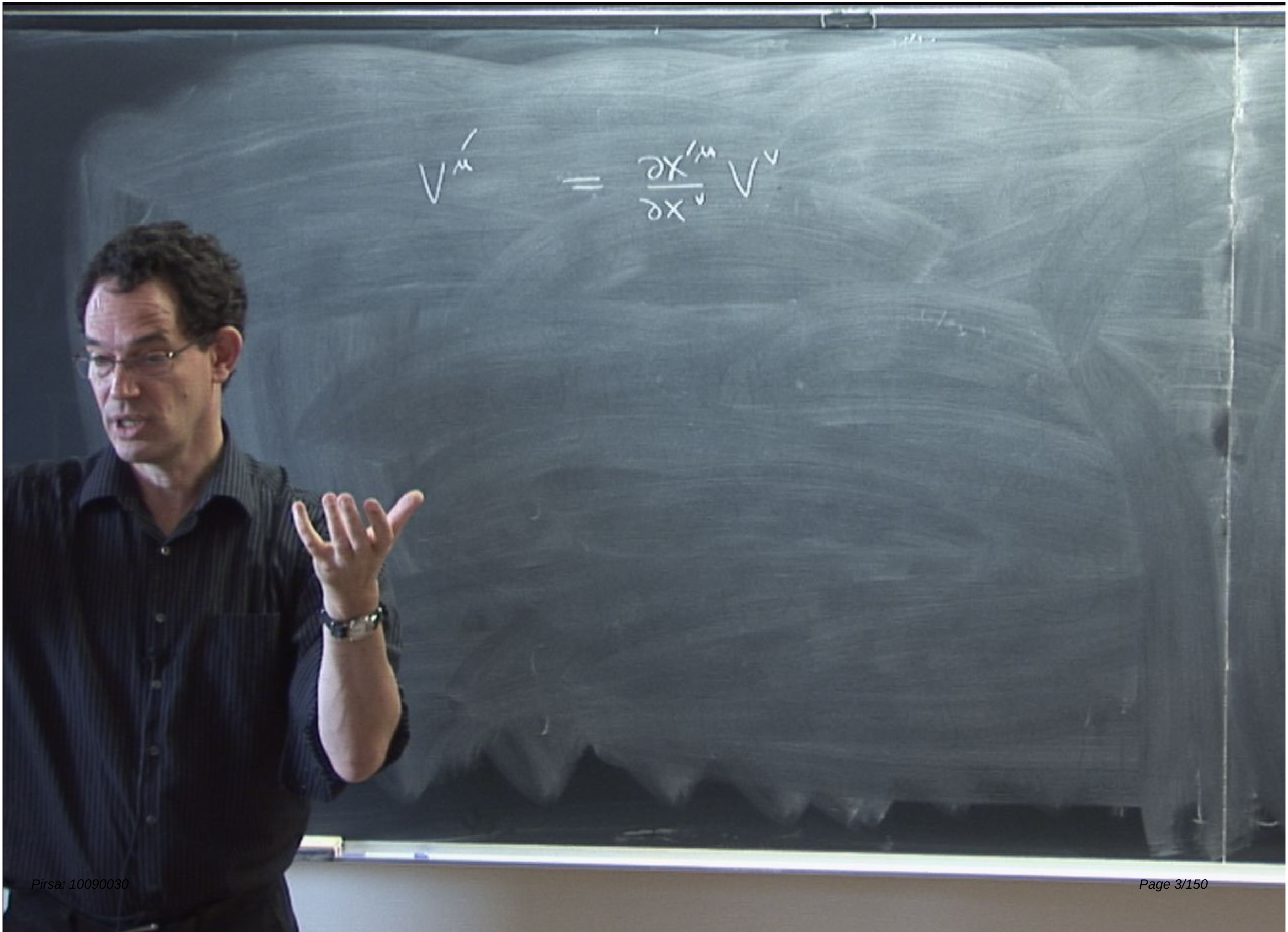
Title: Relativity (PHYS 604) - Lecture 5

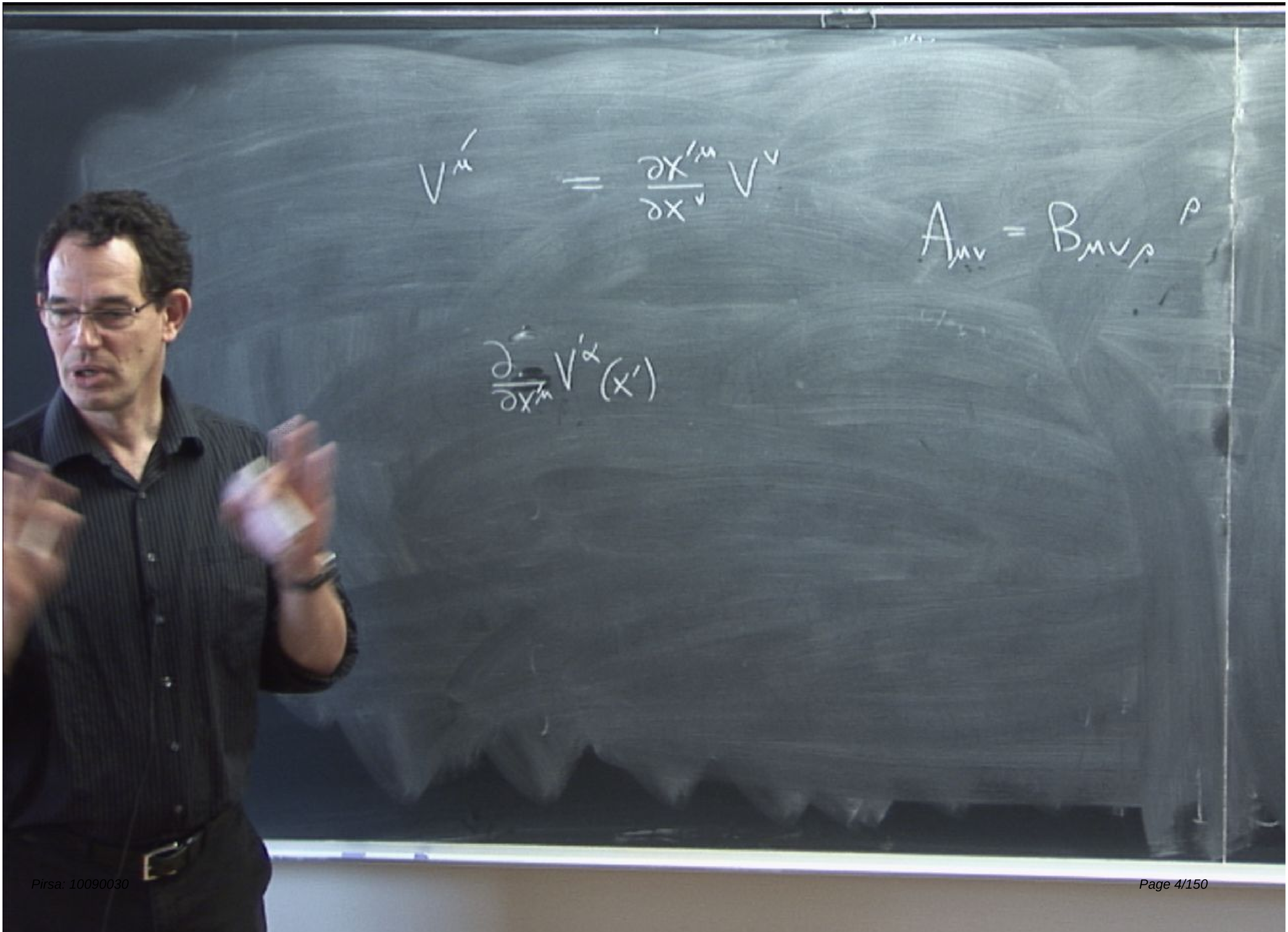
Date: Sep 17, 2010 10:30 AM

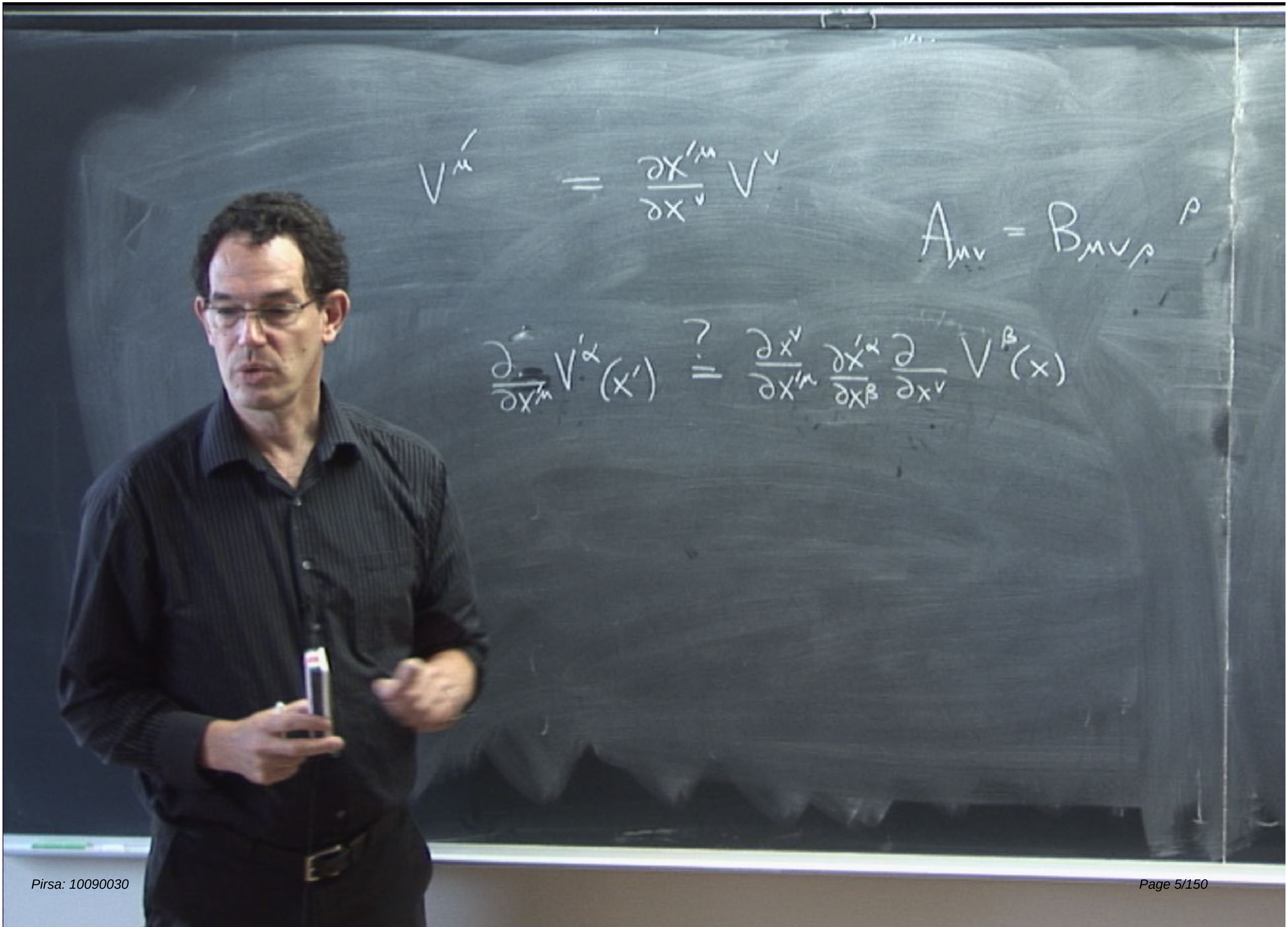
URL: http://pirsa.org/10090030

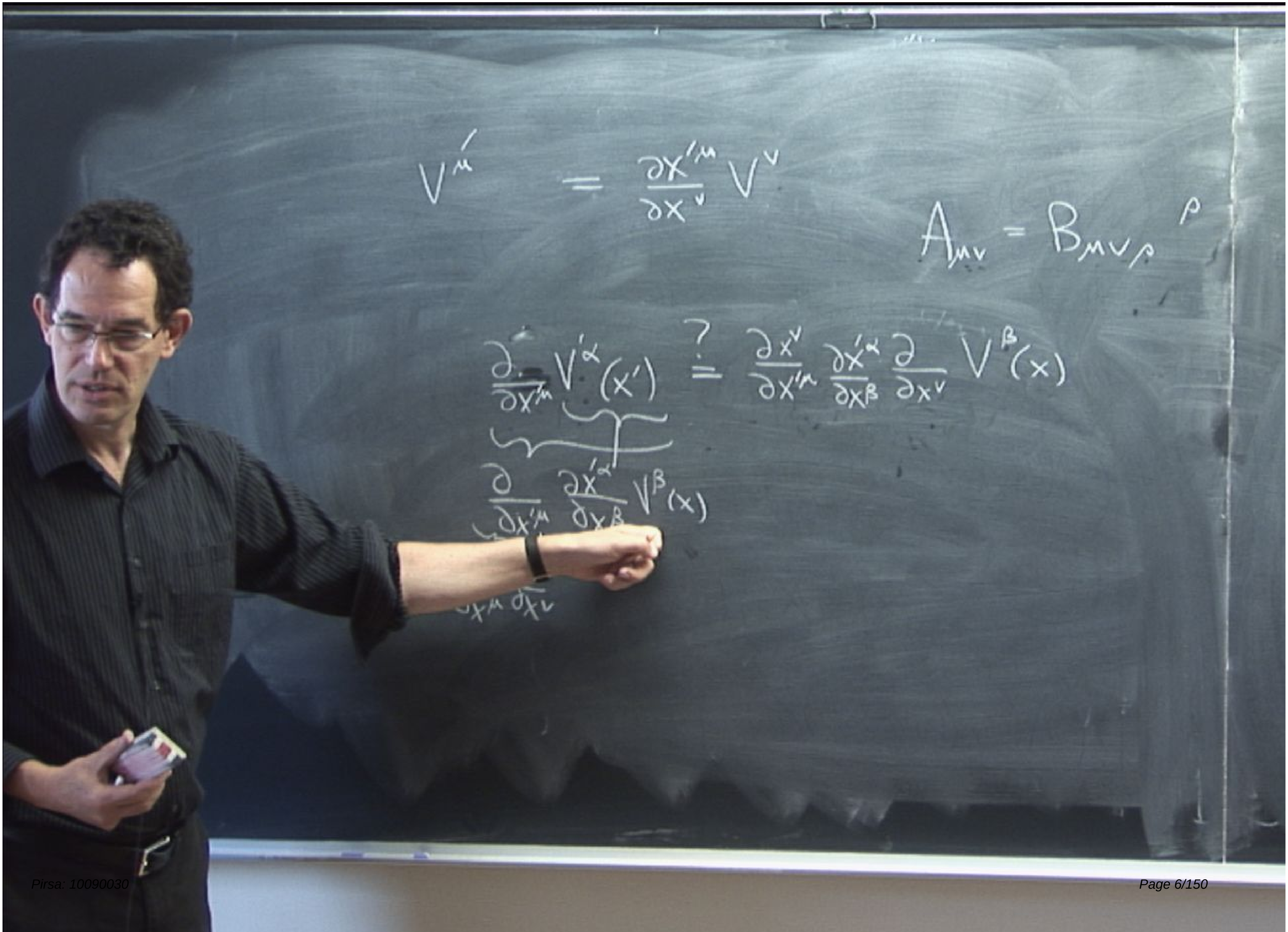
Abstract:

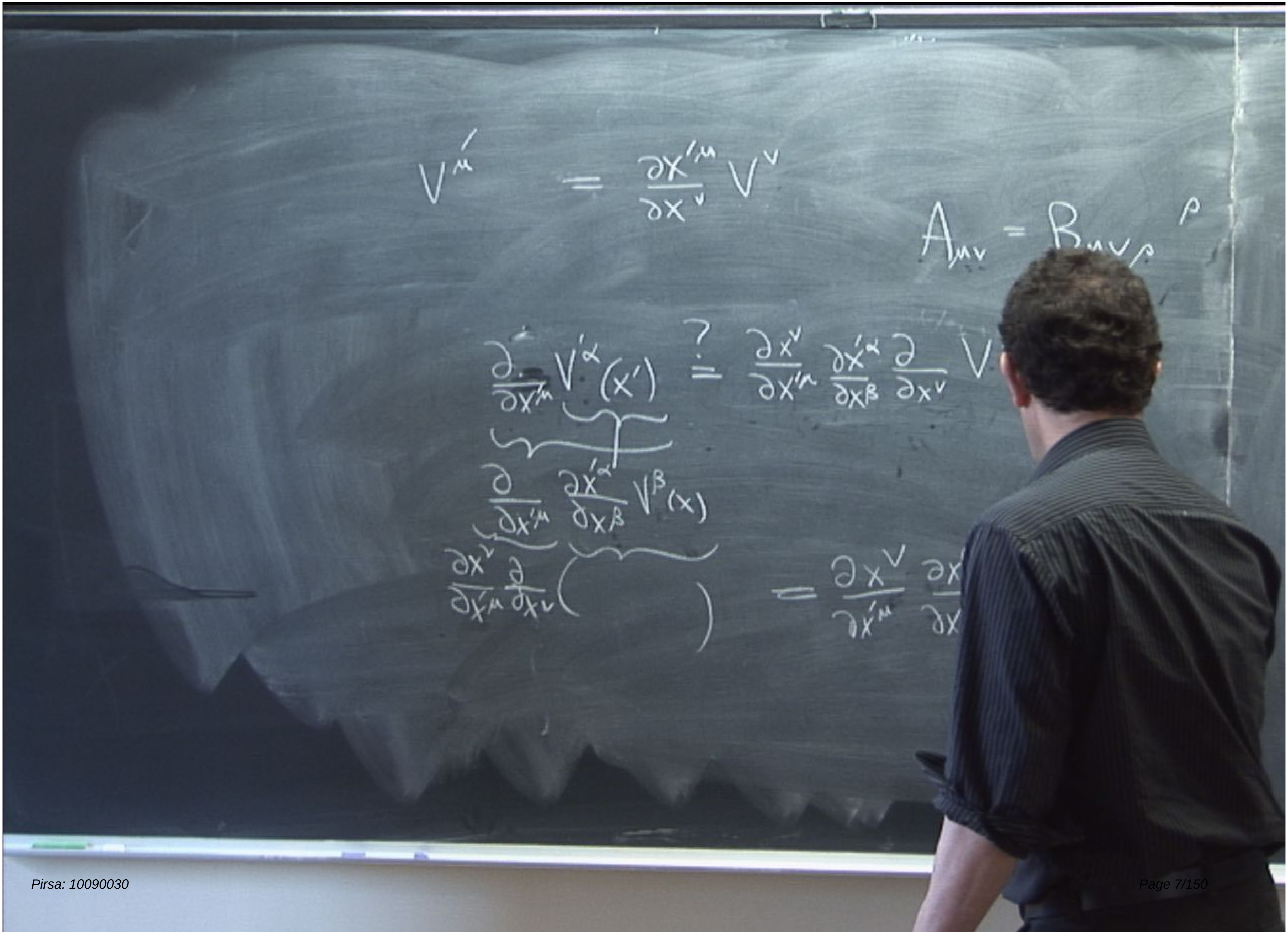
V^μ $\qquad$ $\dfrac{\partial x'^\mu}{\partial x^\nu}$

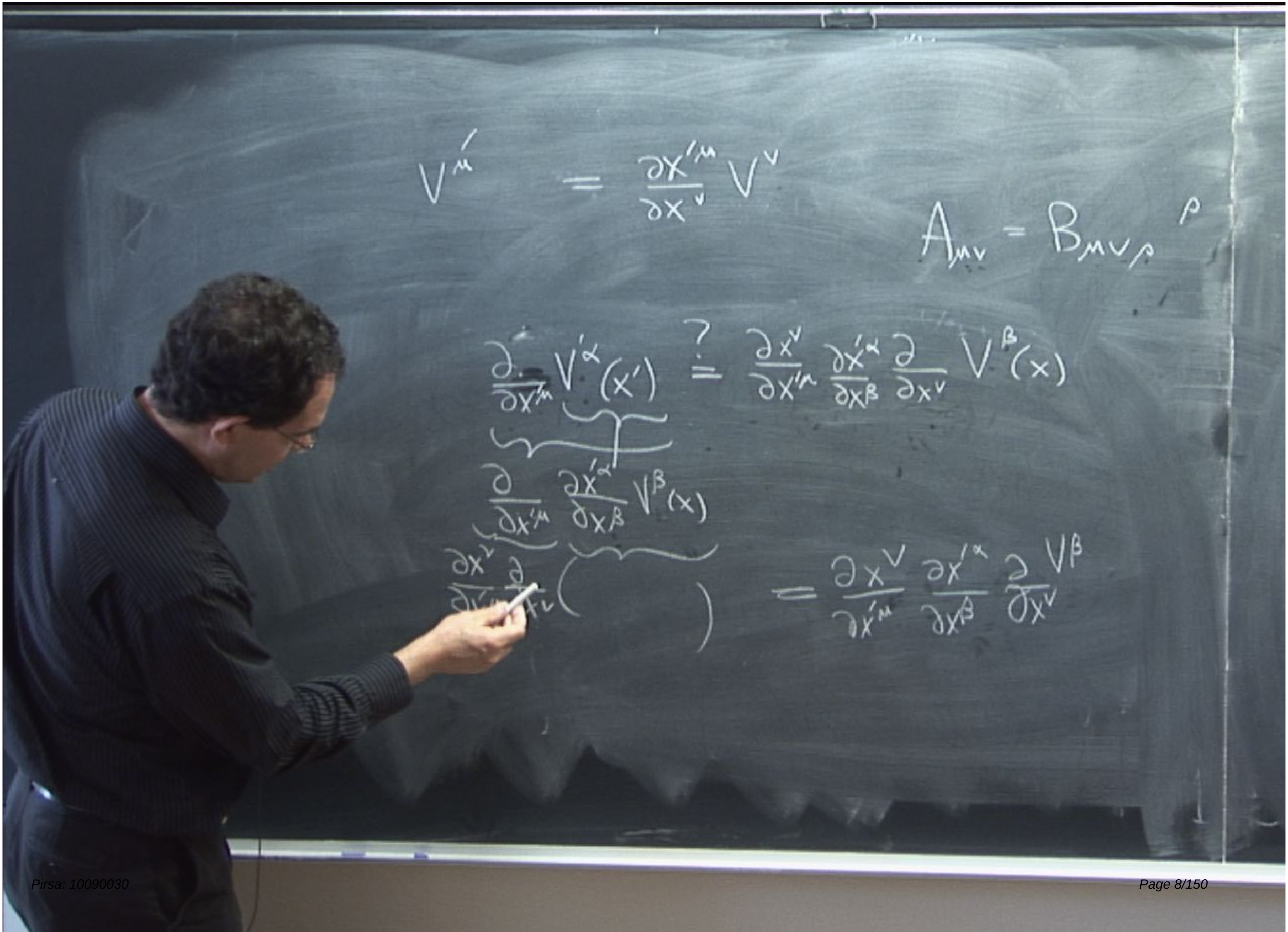

$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} V^{\nu}$


$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} V^{\nu}$
$A_{\mu\nu} = B_{\mu\nu\rho}{}^{\rho}$
$\frac{\partial}{\partial x'^{\mu}} V'^{\alpha}(x')$


$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} V^{\nu}$
$A_{\mu\nu} = B_{\mu\nu\rho}{}^{\rho}$
$\frac{\partial}{\partial x'^{\mu}} V'^{\alpha}(x') \overset{?}{=} \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} \frac{\partial}{\partial x^{\nu}} V^{\beta}(x)$


$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} V^{\nu}$
$A_{\mu\nu} = B_{\mu\nu\rho}{}^{\rho}$
$\frac{\partial}{\partial x'^{\mu}} V'^{\alpha}(x') \overset{?}{=} \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} \frac{\partial}{\partial x^{\nu}} V^{\beta}(x)$
$\frac{\partial}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} V^{\beta}(x)$


$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} V^{\nu}$
$A_{\mu\nu} = B_{\mu\nu\rho}{}^{\rho}$
$\frac{\partial}{\partial x'^{\mu}} V'^{\alpha}(x') \stackrel{?}{=} \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} \frac{\partial}{\partial x^{\nu}} V$
$\frac{\partial}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} V^{\beta}(x)$
$\frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial}{\partial x^{\nu}} (\quad) = \frac{\partial x^{\nu}}{\partial x'^{\mu}}$


$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} V^{\nu}$
$A_{\mu\nu} = B_{\mu\nu\rho}{}^{\rho}$
$\frac{\partial}{\partial x'^{\mu}} V'^{\alpha}(x') \stackrel{?}{=} \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} \frac{\partial}{\partial x^{\nu}} V^{\beta}(x)$
$\frac{\partial}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} V^{\beta}(x)$
$\frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial}{\partial x^{\nu}} \left(\quad \right)$
$= \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} \frac{\partial}{\partial x^{\nu}} V^{\beta}$

$$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} V^{\nu}$$

$$A_{\mu\nu} = B_{\mu\nu\rho}{}^{\rho}$$

$$\frac{\partial}{\partial x'^{\mu}} V'^{\alpha}(x') \stackrel{?}{=} \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x_{\beta}} \frac{\partial}{\partial x^{\nu}} V^{\beta}(x)$$

$$\frac{\partial}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x_{\beta}} V^{\beta}(x)$$

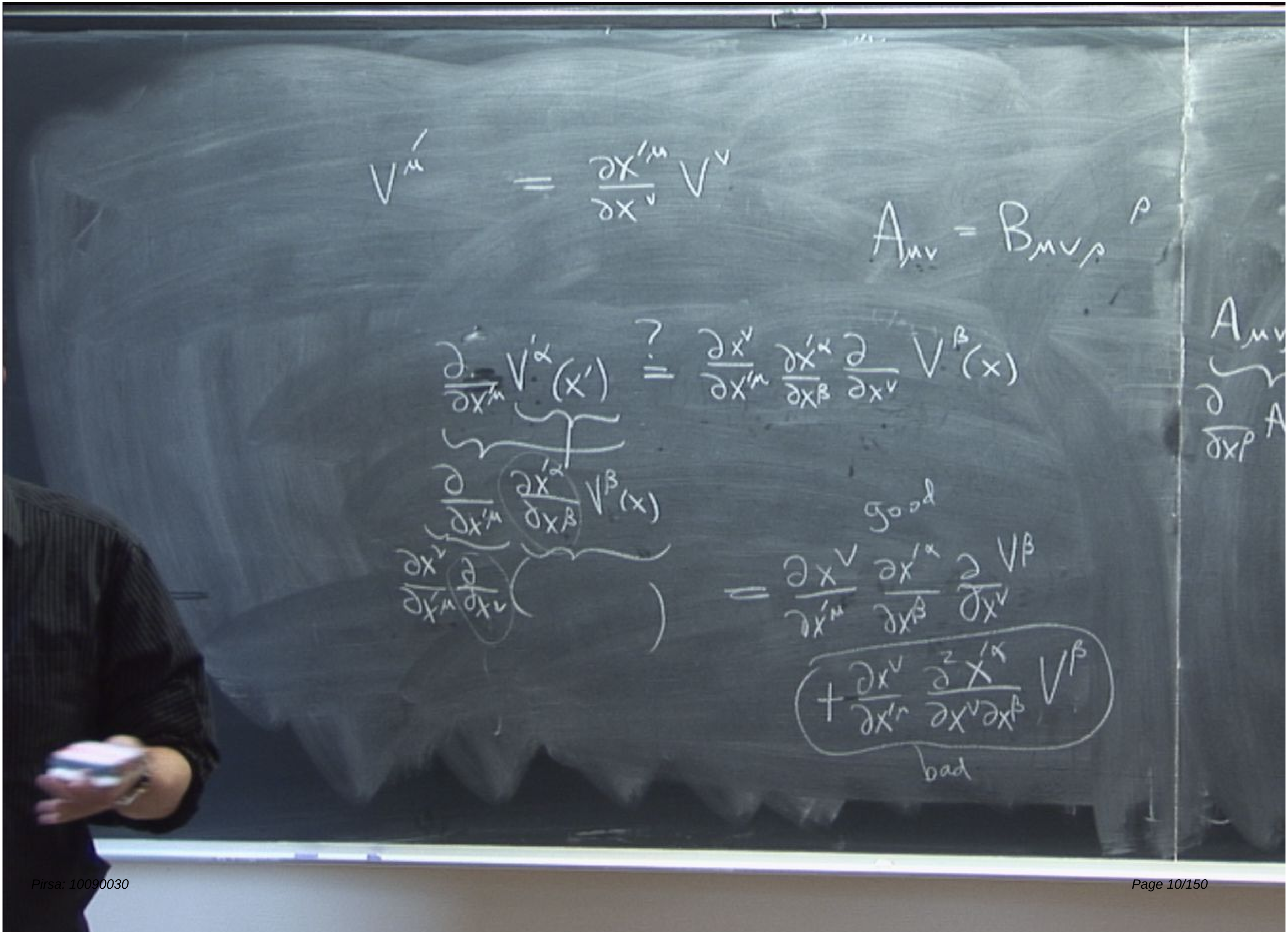
$$\frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial}{\partial x_{\nu}} \left(\quad \right)$$

good

$$= \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} \frac{\partial}{\partial x^{\nu}} V^{\beta}$$

$$+ \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial^2 x'^{\alpha}}{\partial x^{\nu} \partial x^{\beta}} V^{\beta}$$

bad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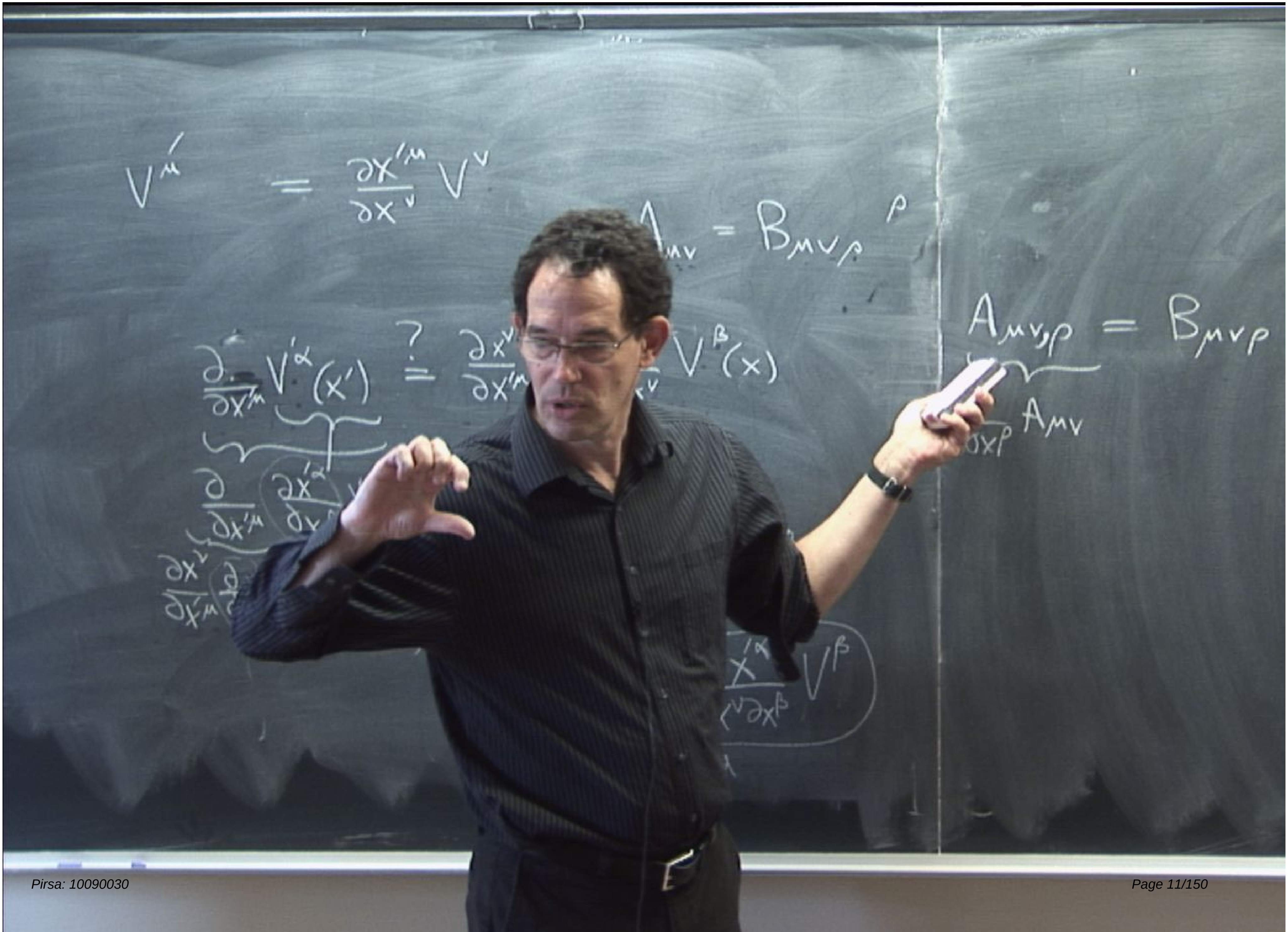
$\frac{\partial}{\partial x'^{\mu}} V'^{\alpha}(x') \stackrel{?}{=} \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} \frac{\partial}{\partial x^{\nu}} V^{\beta}(x)$
$\frac{\partial}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} V^{\beta}(x)$
$\frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial}{\partial x^{\nu}} (\quad)$
good
$= \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} \frac{\partial}{\partial x^{\nu}} V^{\beta}$
$+ \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial^2 x'^{\alpha}}{\partial x^{\nu} \partial x^{\beta}} V^{\beta}$
bad


V'^μ = ∂x'^μ/∂x^ν V^ν
A_μν = B_μνρ ρ
A_μν,ρ = B_μνρ

$$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} V^{\nu}$$

$$A_{\mu\nu} = B_{\mu\nu\rho}{}^{\rho}$$

$$\frac{\partial}{\partial x'^{\mu}} V'^{\alpha}(x') \overset{?}{=} \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} \frac{\partial}{\partial x^{\nu}} V^{\beta}(x)$$

$$\frac{\partial}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} V^{\beta}(x)$$

$$\frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial}{\partial x^{\nu}} \left(\quad \right)$$

good

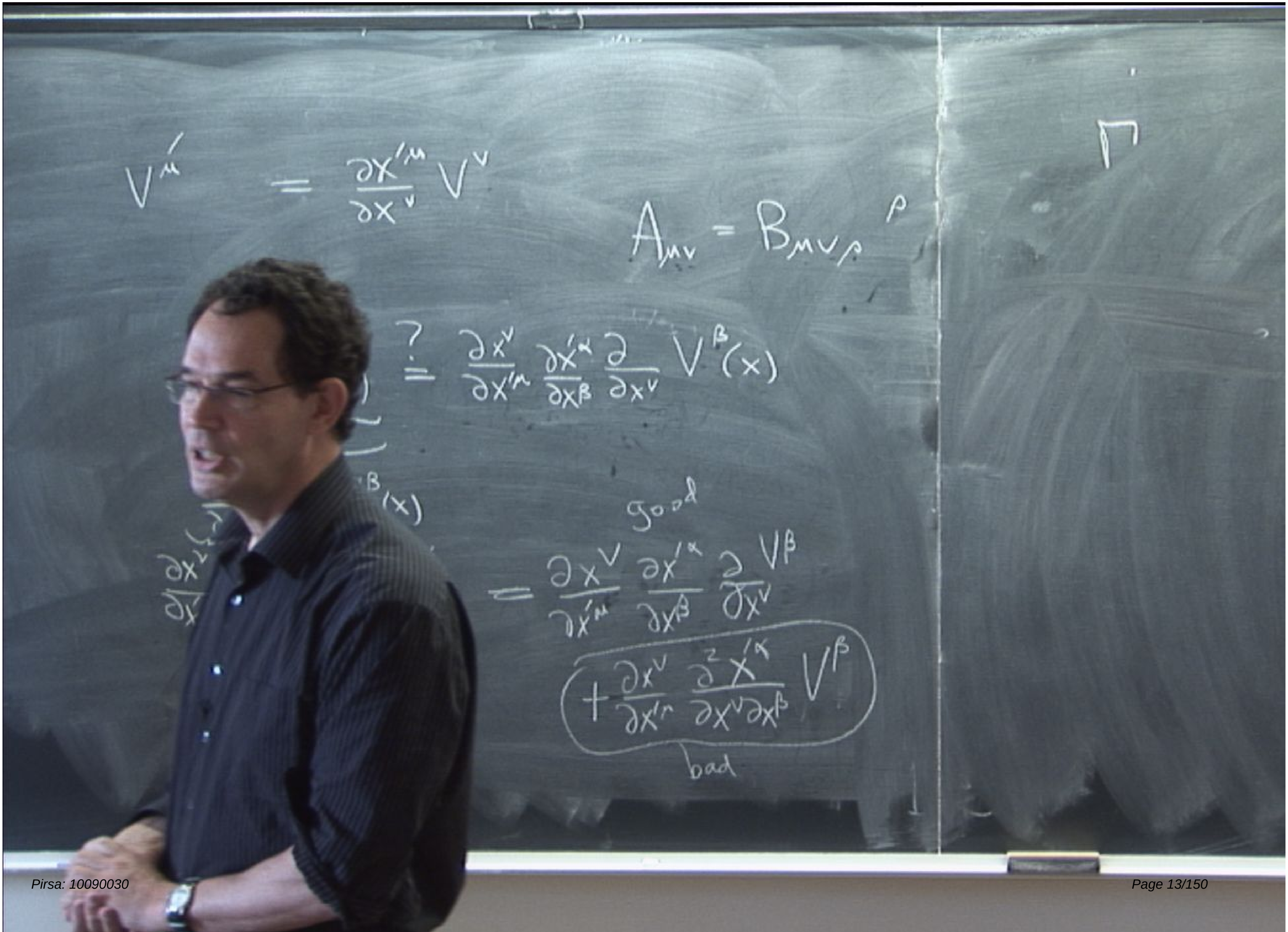
$$= \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} \frac{\partial V^{\beta}}{\partial x^{\nu}}$$

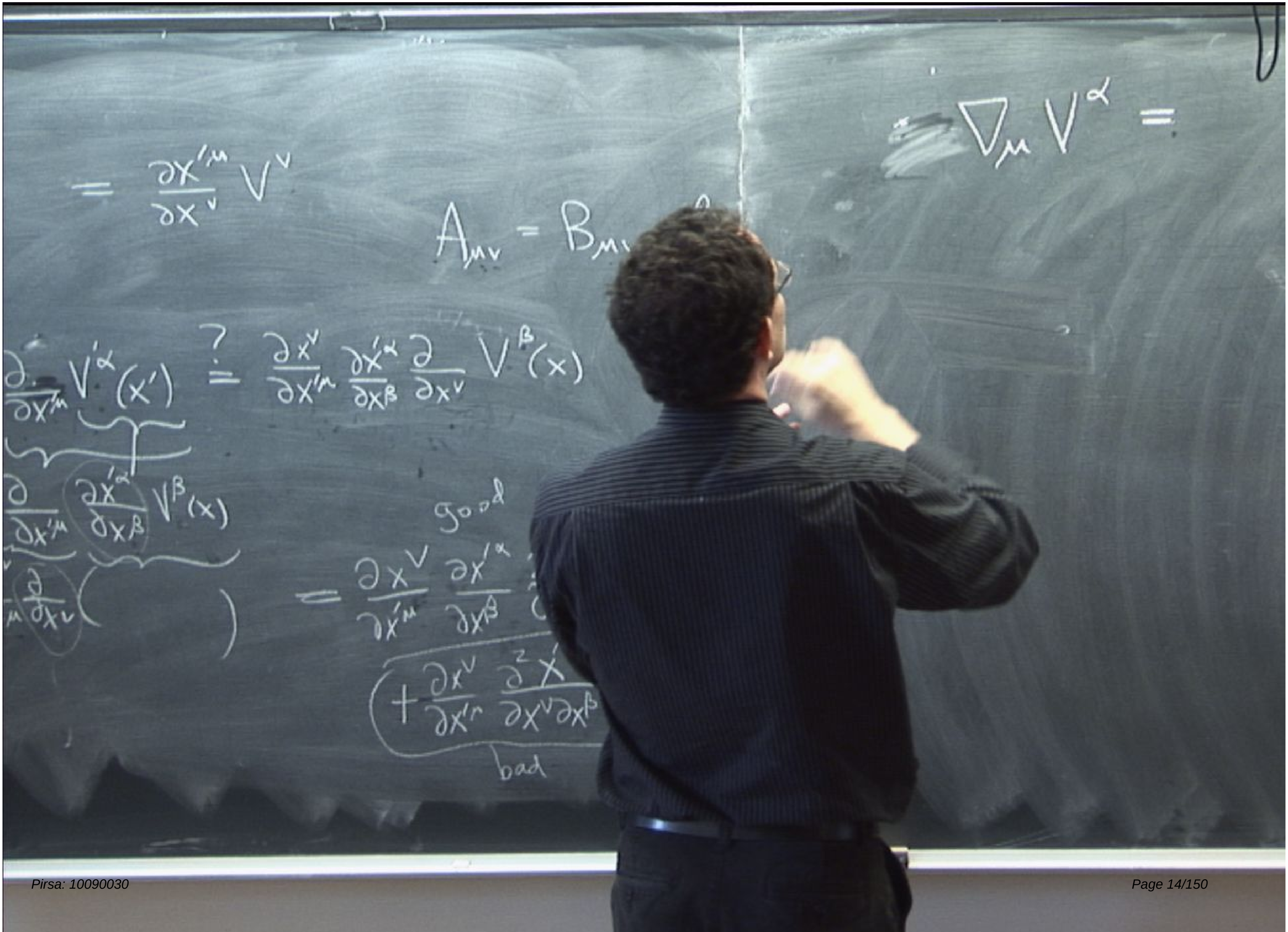
$$+ \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial^2 x'^{\alpha}}{\partial x^{\nu} \partial x^{\beta}} V^{\beta}$$

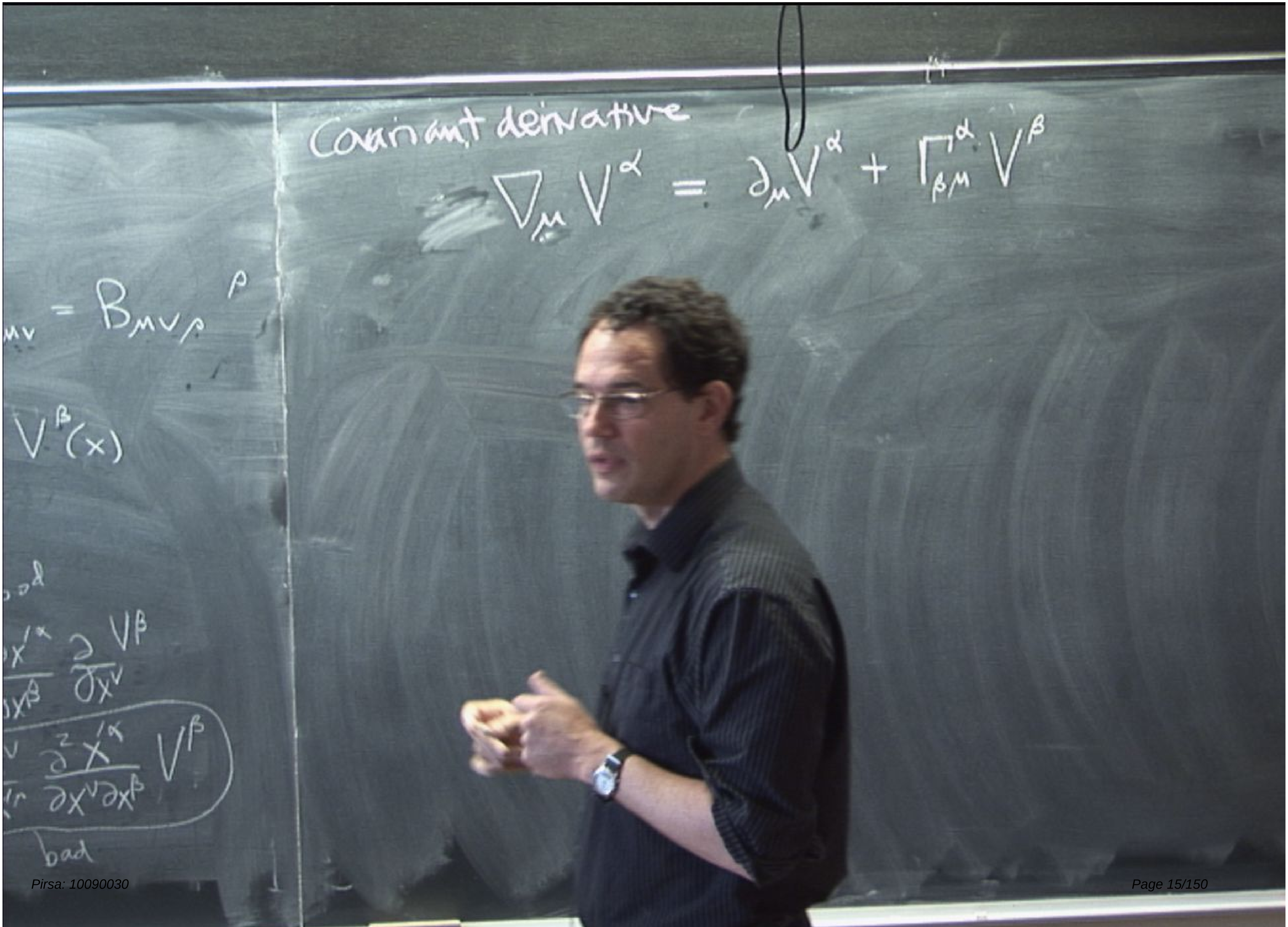
bad

$$A_{\mu\nu,\rho} = B_{\mu\nu\rho}$$

$$A_{\mu\nu,\rho} = \frac{\partial}{\partial x^{\rho}} A_{\mu\nu}$$


$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} V^{\nu}$
$A_{\mu\nu} = B_{\mu\nu\rho}{}^{\rho}$
good
bad


= ∂x'^μ/∂x^ν V^ν
A_μν = B_μν
∇_μ V^α =
∂/∂x'^μ V'^α(x') ?= ∂x^ν/∂x'^μ ∂x'^α/∂x^β ∂/∂x^ν V^β(x)
good
bad

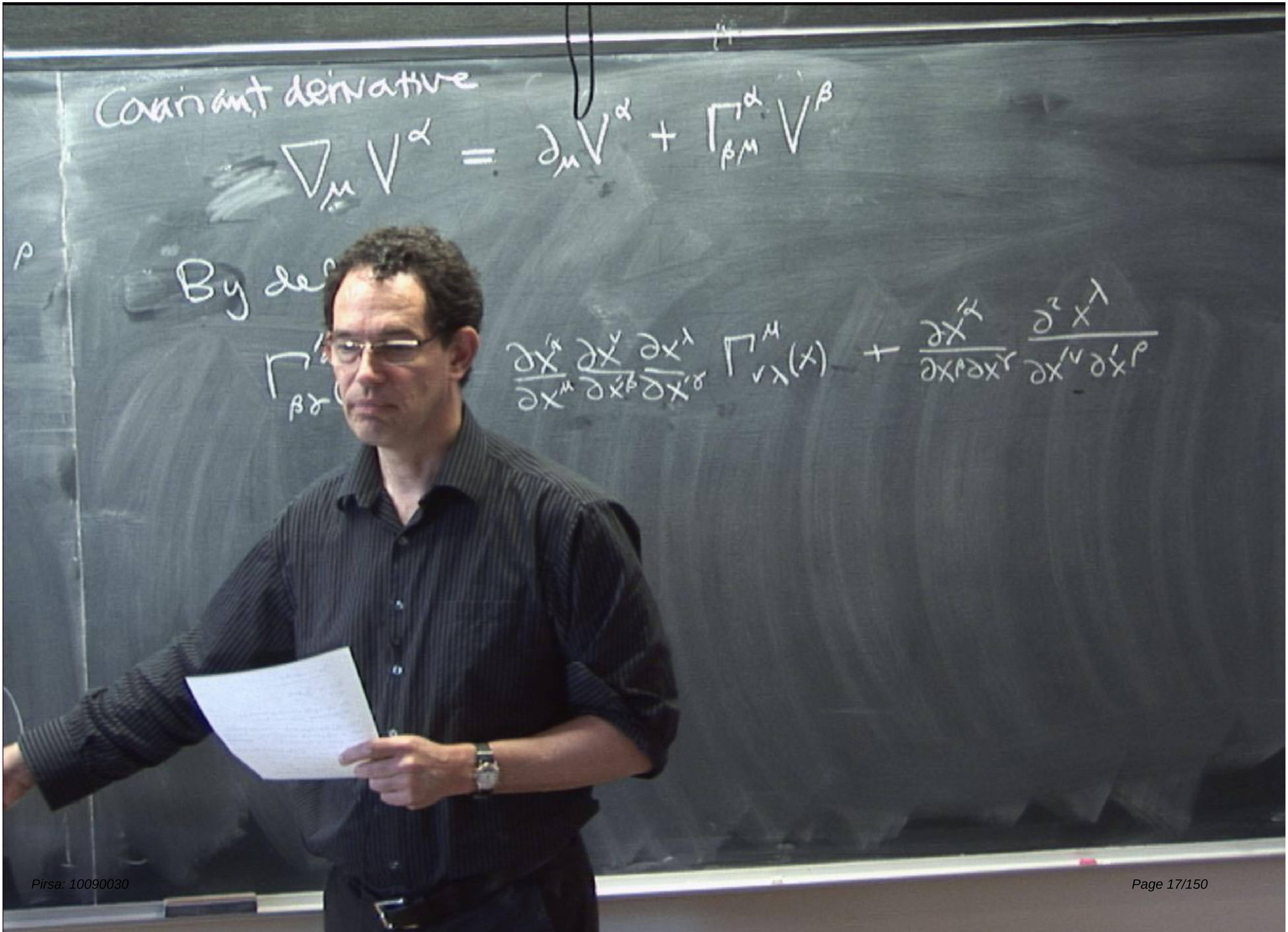

Covariant derivative
$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$
$= B_{\mu\nu\rho}{}^{\rho}$
$V^\beta(x)$
bad

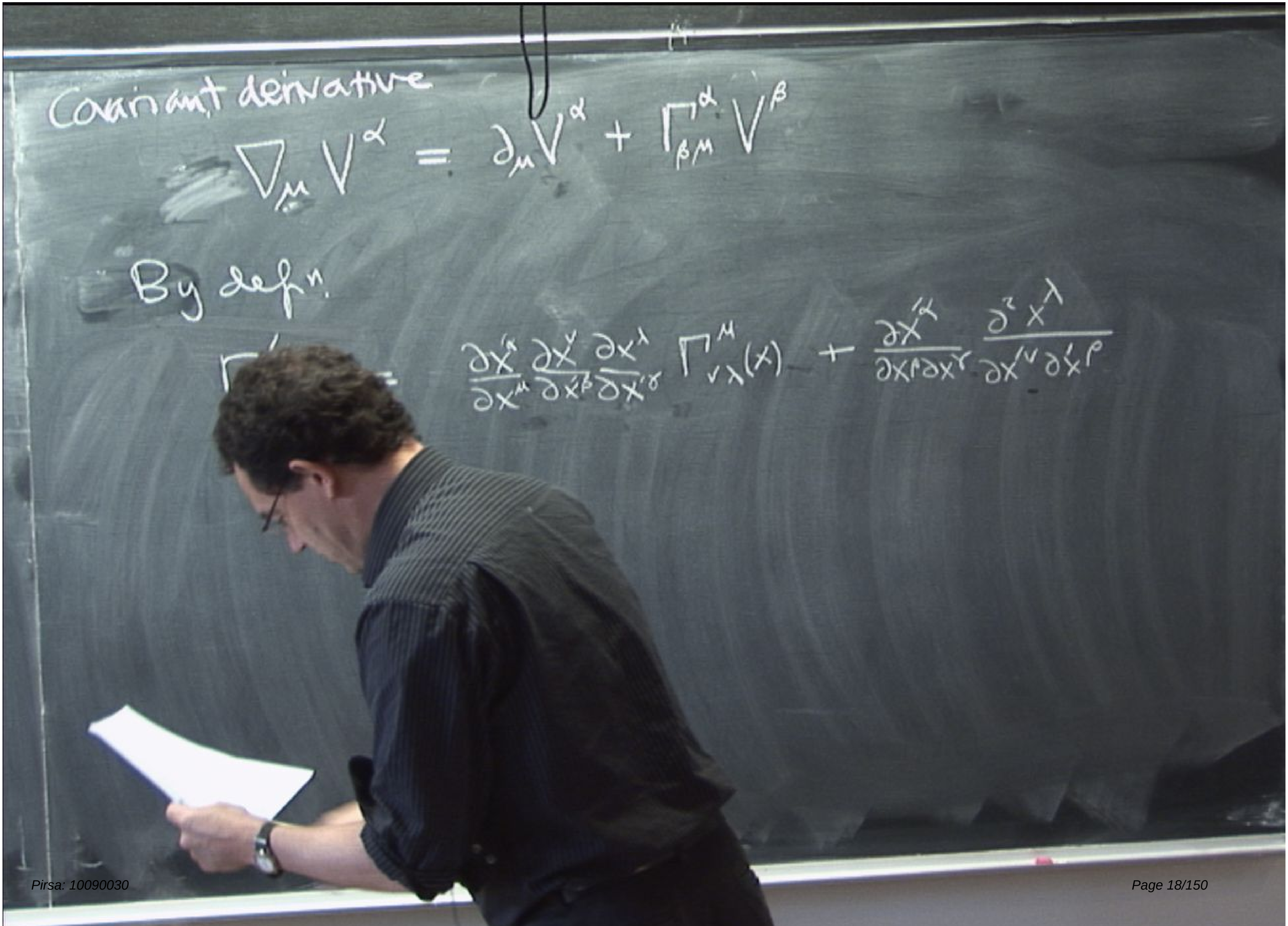
Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$$

By def^n:

$$\Gamma'^\alpha_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial x'^\beta}\frac{\partial x^\lambda}{\partial x'^\gamma}\Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\rho \partial x^\gamma}\frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$$


Covariant derivative
$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$
By de
$\Gamma'^\alpha_{\beta\gamma}($
$\frac{\partial x'^\alpha}{\partial x^\mu} \frac{\partial x^\nu}{\partial x'^\beta} \frac{\partial x^\lambda}{\partial x'^\gamma} \Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\rho \partial x^\gamma} \frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$


Covariant derivative
$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$
By defn:

Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$$

By defn:

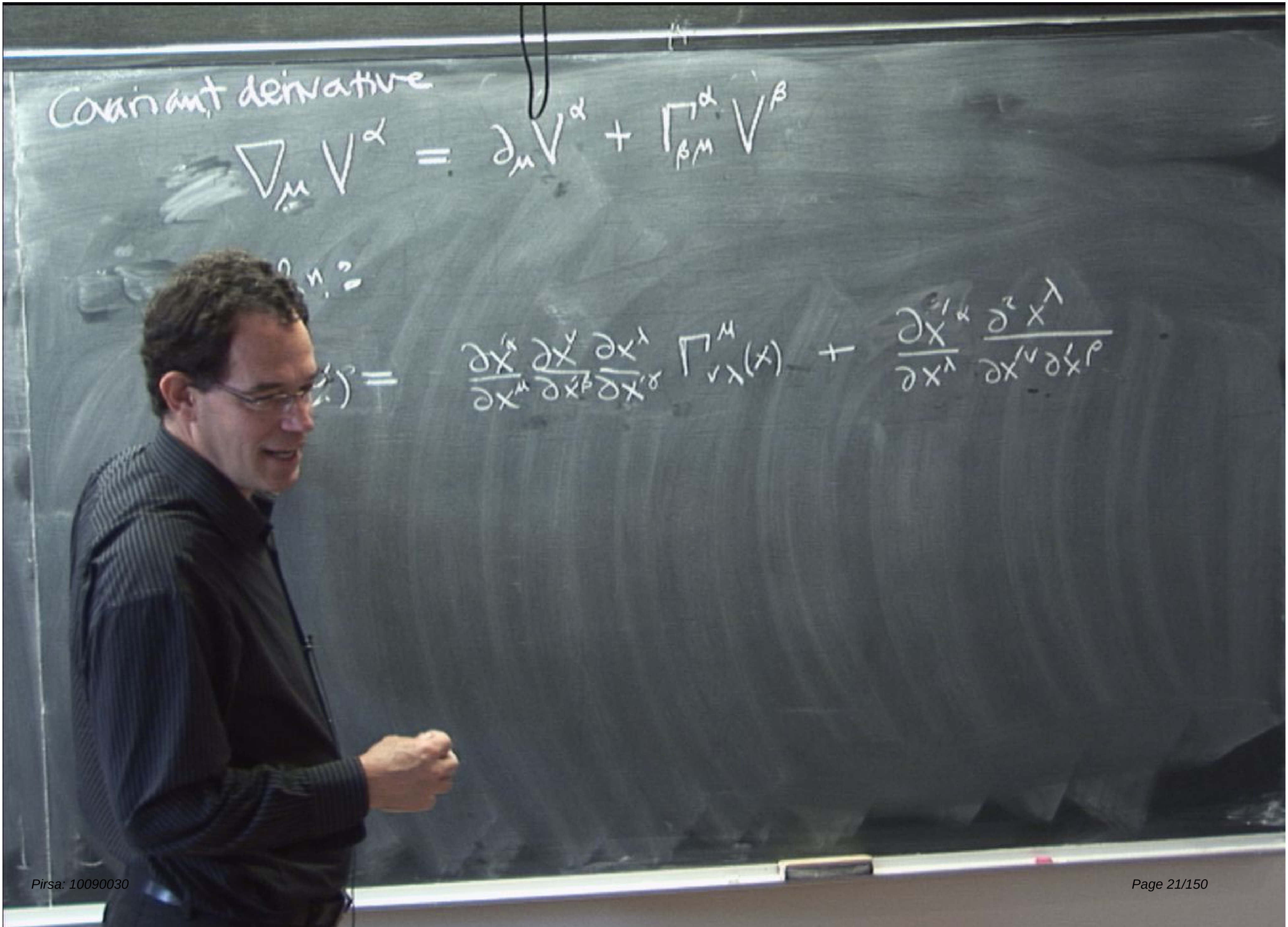
$$\Gamma'^\alpha_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial x'^\beta}\frac{\partial x^\lambda}{\partial x'^\gamma}\,\Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda}\frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$$

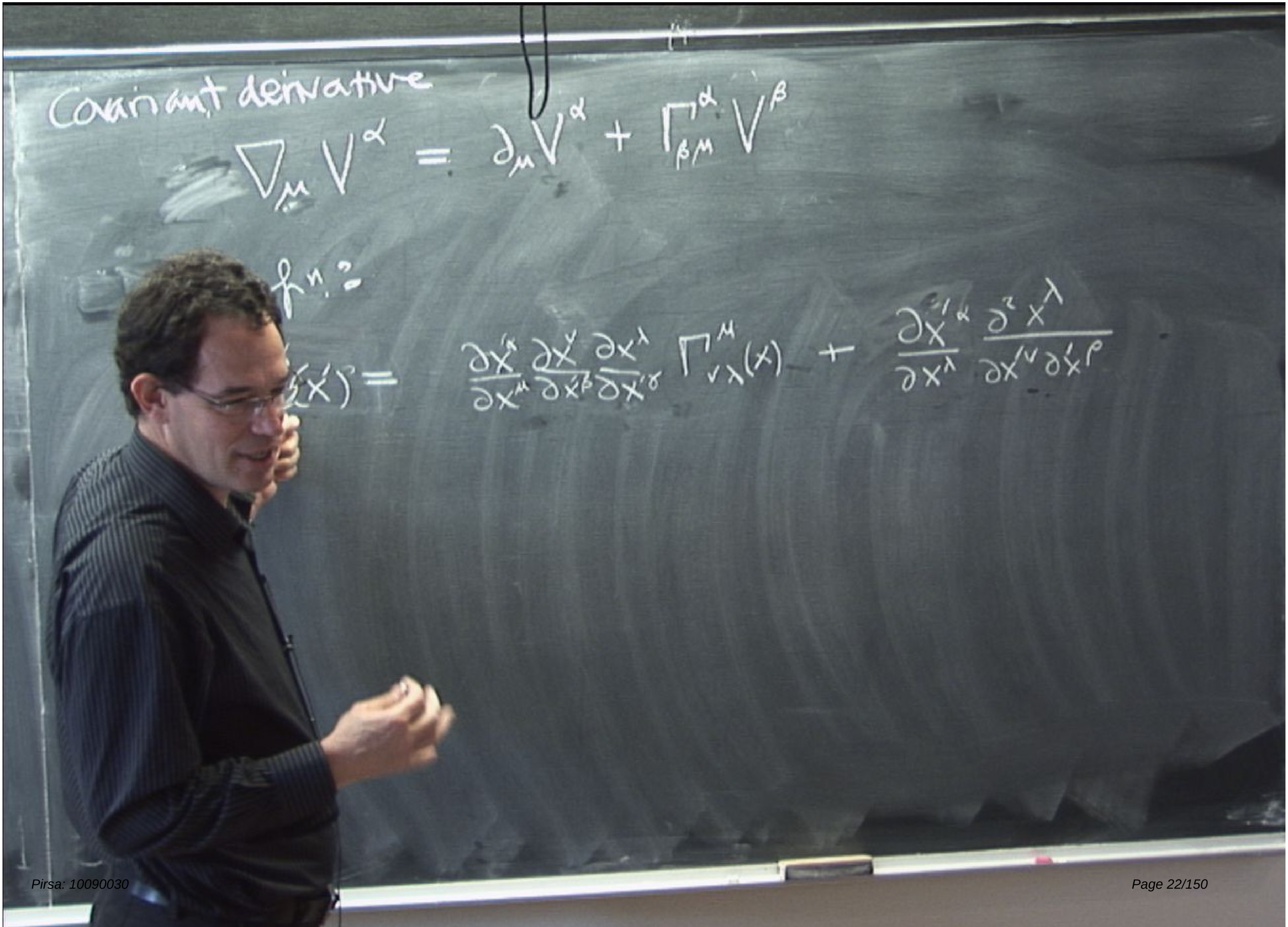
Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$$

By defn.

$$\Gamma'^\alpha_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial x'^\beta}\frac{\partial x^\lambda}{\partial x'^\gamma}\Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda}\frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$$


Covariant derivative
$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$
$\frac{\partial x'^\alpha}{\partial x^\mu} \frac{\partial x^\nu}{\partial x'^\beta} \frac{\partial x^\lambda}{\partial x'^\gamma} \Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda} \frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$


Covariant derivative
$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$

Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$$

Defn:

$$\Gamma'^\alpha_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial x'^\beta}\frac{\partial x^\lambda}{\partial x'^\gamma}\Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda}\frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$$

Consequence

$$(\nabla_\mu V^\alpha)' = \frac{\partial x^\lambda}{\partial x'^\mu}\frac{\partial x'^\alpha}{\partial x^\gamma}\nabla_\lambda V^\gamma(x)$$

Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$$

Defn.:

$$\Gamma'^\alpha_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial x'^\beta}\frac{\partial x^\lambda}{\partial x'^\gamma}\Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda}\frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$$

Consequence

$$(\nabla_\mu V^\alpha)' = \frac{\partial x^\lambda}{\partial x'^\mu}\frac{\partial x'^\alpha}{\partial x^\gamma}\nabla_\lambda V^\gamma(x)$$

Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$$

Defn.:

$$\Gamma'^\alpha_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial x'^\beta}\frac{\partial x^\lambda}{\partial x'^\gamma}\Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda}\frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$$

$$\frac{\partial x'^\mu}{\partial x^\lambda}\frac{\partial x^\lambda}{\partial x'^\rho} = \delta^\mu_\rho$$

Consequence

$$(\nabla_\mu V^\alpha)' = \frac{\partial x^\lambda}{\partial x'^\mu}\frac{\partial x'^\alpha}{\partial x^\gamma}\nabla_\lambda V^\gamma(x)$$

Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$$

Defn.:

$$\Gamma'^\alpha_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial x'^\beta}\frac{\partial x^\lambda}{\partial x'^\gamma}\Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda}\frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$$

$$\frac{\partial x'^\mu}{\partial x^\lambda}\frac{\partial x^\lambda}{\partial x'^\rho} = \delta^\mu_\rho$$

Consequence

$$(\nabla_\mu V^\alpha)' = \frac{\partial x^\lambda}{\partial x'^\mu}\frac{\partial x'^\alpha}{\partial x^\gamma}\nabla_\lambda V^\gamma(x)$$

$$\frac{\partial}{\partial x'^\nu} \quad \frac{\partial x^\lambda}{\partial \ldots} \quad \frac{\partial x'^\mu}{\ldots}$$

Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$$

Defn.:

$$\Gamma'^{\alpha}_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial x'^\beta}\frac{\partial x^\lambda}{\partial x'^\gamma}\Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda}\frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$$

$$\frac{\partial x'^\mu}{\partial x^\lambda}\frac{\partial x^\lambda}{\partial x'^\rho} = \delta^\mu_\rho$$

Consequence

$$(\nabla_\mu V^\alpha)' = \frac{\partial x^\lambda}{\partial x'^\mu}\frac{\partial x'^\alpha}{\partial x^\gamma}\nabla_\lambda V^\gamma(x)$$

$$\frac{\partial}{\partial x'^\nu} \quad \frac{\partial x^\lambda}{\partial x'^\rho \partial x'^\nu}\frac{\partial x'^\mu}{\partial x^\nu} + \frac{\partial x^\beta}{\partial x'^\nu}\frac{\partial}{\partial x^\beta}$$

Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$$

Defn.:

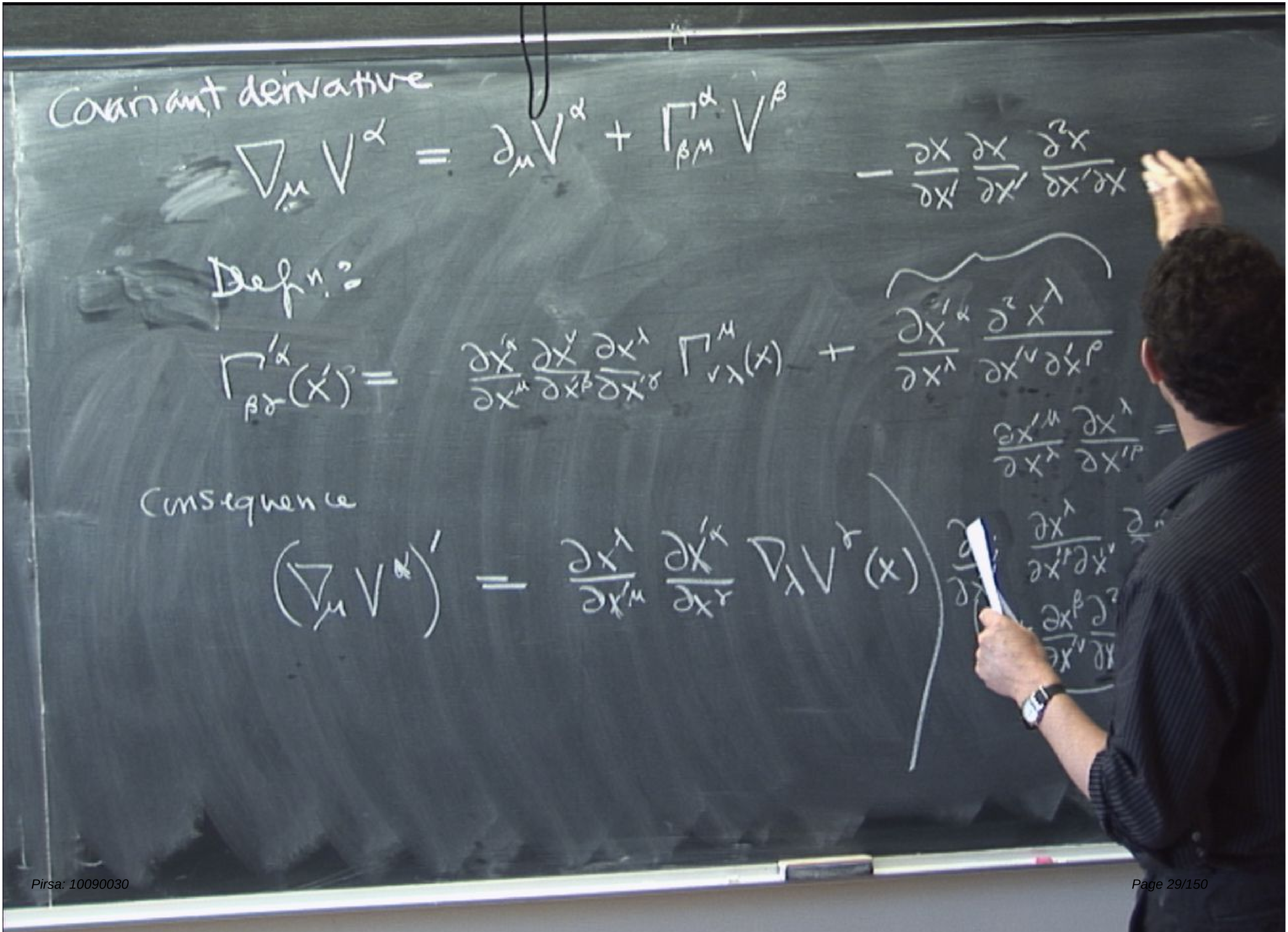
$$\Gamma'^\alpha_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial x'^\beta}\frac{\partial x^\lambda}{\partial x'^\gamma}\Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda}\frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$$

$$\frac{\partial x'^\mu}{\partial x^\lambda}\frac{\partial x^\lambda}{\partial x'^\rho} = \delta^\mu_\rho$$

Consequence

$$(\nabla_\mu V^\alpha)' = \frac{\partial x^\lambda}{\partial x'^\mu}\frac{\partial x'^\alpha}{\partial x^\gamma}\nabla_\lambda V^\gamma(x)$$

$$\frac{\partial}{\partial x'^\nu}: \quad \frac{\partial x^\lambda}{\partial x'^\rho \partial x^\nu}\frac{\partial x'^\mu}{\partial x^\nu} + \frac{\partial x^\beta}{\partial x'^\nu}\frac{\partial^2 x'^\mu}{\partial x^\beta \partial x^\lambda} = 0$$


Covariant derivative
$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$
$- \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} \frac{\partial^2 x}{\partial x' \partial x}$
Defn.:
$\Gamma'^\alpha_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu} \frac{\partial x^\nu}{\partial x'^\beta} \frac{\partial x^\lambda}{\partial x'^\gamma} \Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda} \frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$
$\frac{\partial x'^\mu}{\partial x^\lambda} \frac{\partial x^\lambda}{\partial x'^\rho} =$
Consequence
$(\nabla_\mu V^\alpha)' = \frac{\partial x^\lambda}{\partial x'^\mu} \frac{\partial x'^\alpha}{\partial x^\gamma} \nabla_\lambda V^\gamma(x)$
$\frac{\partial x^\lambda}{\partial x'^\rho \partial x'^\nu}$

Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$$

$$- \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} \frac{\partial^2 x'}{\partial x \partial x}$$

Def^{n}:

$$\Gamma'^\alpha_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu} \frac{\partial x^\nu}{\partial x'^\beta} \frac{\partial x^\lambda}{\partial x'^\gamma} \Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda} \frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$$

$$\frac{\partial x'^\mu}{\partial x^\lambda} \frac{\partial x^\lambda}{\partial x'^\rho} = \delta^\mu_\rho$$

Consequence

$$(\nabla_\mu V^\alpha)' = \frac{\partial x^\lambda}{\partial x'^\mu} \frac{\partial x'^\alpha}{\partial x^\gamma} \nabla_\lambda V^\gamma(x)$$

$$\frac{\partial}{\partial x'^\nu} : \quad \frac{\partial x^\lambda}{\partial x'^\rho \partial x'^\nu} \frac{\partial x'^\mu}{\partial x^\lambda} + \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial^2 x'^\mu}{\partial x^\beta \partial x^\lambda} \frac{\partial x^\lambda}{\partial x'^\rho} = 0$$

Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$$

$$- \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} \frac{\partial^2 x'}{\partial x \, \partial x}$$

Defn.:

$$\Gamma'^\alpha_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu} \frac{\partial x^\nu}{\partial x'^\beta} \frac{\partial x^\lambda}{\partial x'^\gamma} \Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda} \frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$$

$$\frac{\partial x'^\mu}{\partial x^\lambda} \frac{\partial x^\lambda}{\partial x'^\rho} = \delta^\mu_\rho$$

Consequence

$$(\nabla_\mu V^\alpha)' = \frac{\partial x^\lambda}{\partial x'^\mu} \frac{\partial x'^\alpha}{\partial x^\gamma} \nabla_\lambda V^\gamma(x)$$

$$\frac{\partial}{\partial x'^\nu} : \quad \frac{\partial x^\lambda}{\partial x'^\rho \partial x'^\nu} \frac{\partial x'^\mu}{\partial x^\lambda} + \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial^2 x'^\mu}{\partial x^\beta \partial x^\lambda} \frac{\partial x^\lambda}{\partial x'^\rho} = 0$$

Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta \qquad - \frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'}\frac{\partial^2 x'^\alpha}{\partial x\,\partial x}$$

Defn.:

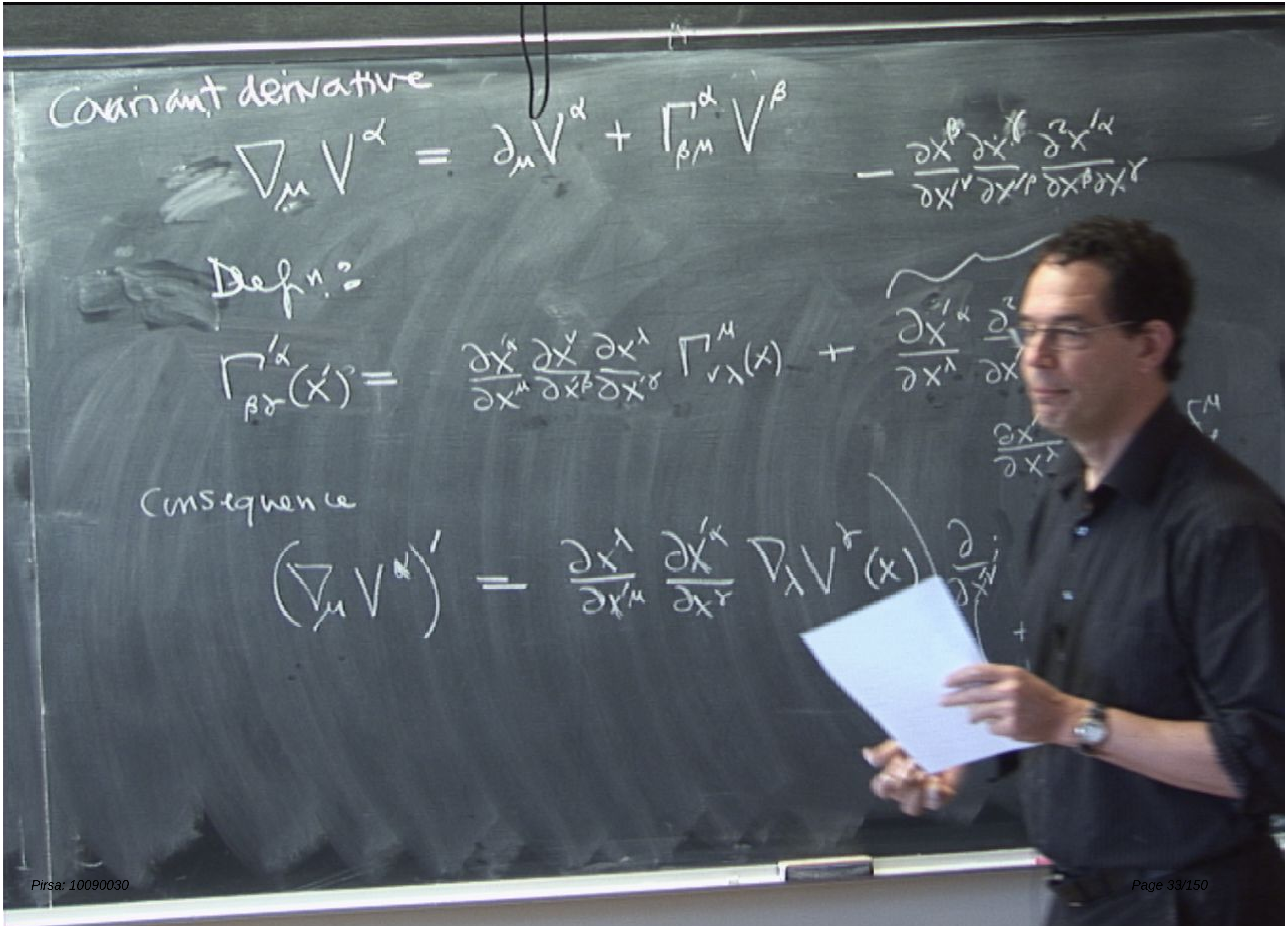
$$\Gamma'^\alpha_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial x'^\beta}\frac{\partial x^\lambda}{\partial x'^\gamma}\Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda}\frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$$

$$\frac{\partial x'^\mu}{\partial x^\lambda}\frac{\partial x^\lambda}{\partial x'^\rho} = \delta^\mu_\rho$$

Consequence

$$(\nabla_\mu V^\alpha)' = \frac{\partial x^\lambda}{\partial x'^\mu}\frac{\partial x'^\alpha}{\partial x^\gamma}\nabla_\lambda V^\gamma(x)$$

$$\frac{\partial}{\partial x'^\nu}: \quad \frac{\partial x^\lambda}{\partial x'^\rho \partial x'^\nu}\frac{\partial x'^\mu}{\partial x^\lambda} + \frac{\partial x^\beta}{\partial x'^\nu}\frac{\partial^2 x'^\mu}{\partial x^\beta \partial x^\lambda}\frac{\partial x^\lambda}{\partial x'^\rho} = 0$$


Covariant derivative
$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$
$- \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial x^\gamma}{\partial x'^\rho} \frac{\partial^2 x'^\alpha}{\partial x^\beta \partial x^\gamma}$
Defn.:
$\Gamma'^\alpha_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu} \frac{\partial x^\nu}{\partial x'^\beta} \frac{\partial x^\lambda}{\partial x'^\gamma} \Gamma^\mu_{\nu\lambda}(x) +$
Consequence
$(\nabla_\mu V^\alpha)' = \frac{\partial x^\lambda}{\partial x'^\mu} \frac{\partial x'^\alpha}{\partial x^\gamma} \nabla_\lambda V^\gamma(x)$

Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$$

$$- \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial x'^\gamma}{\partial x'^\rho} \frac{\partial^2 x'^\alpha}{\partial x^\beta \partial x^\gamma}$$

Defn.:

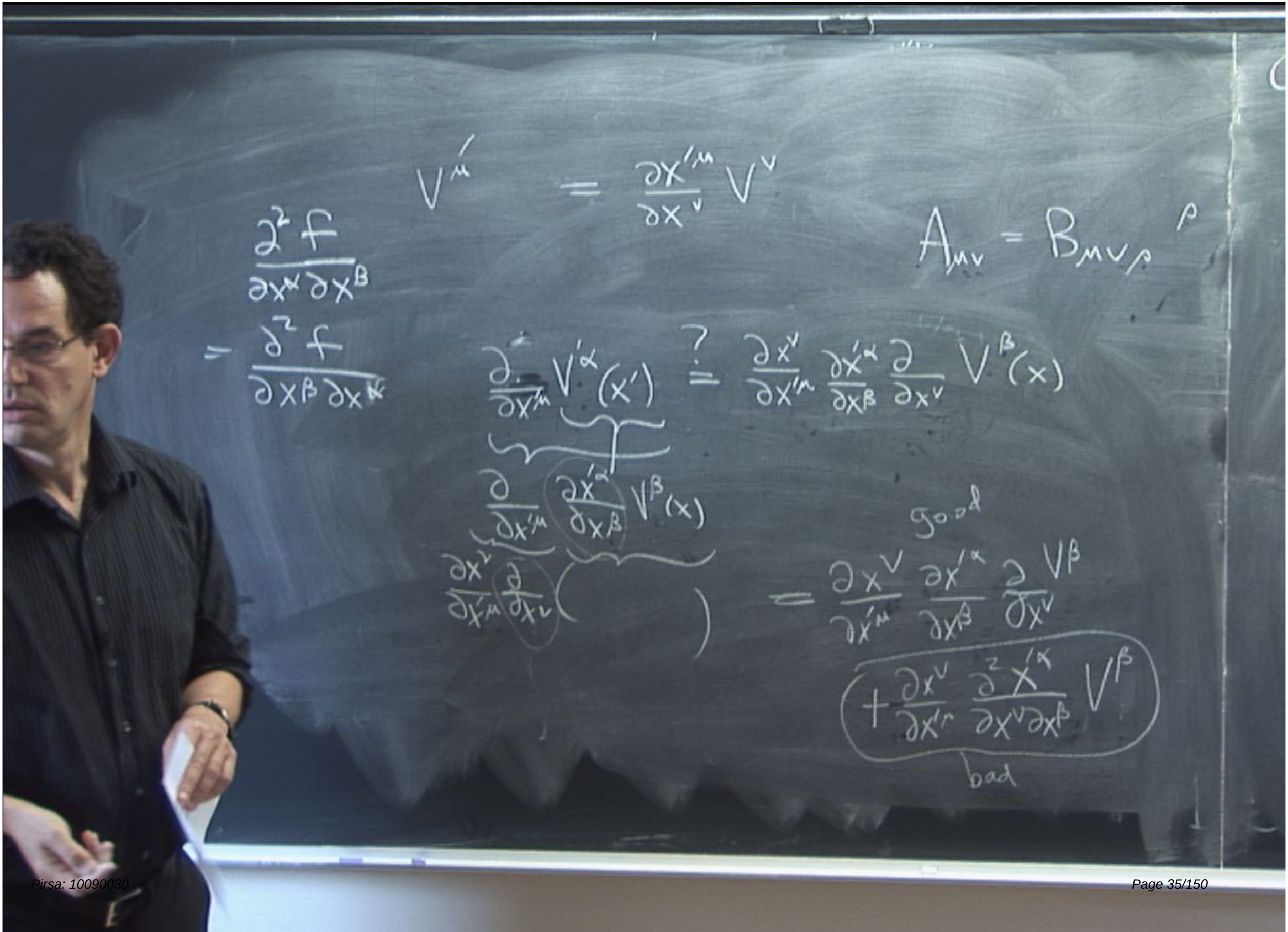
$$\Gamma'^\alpha_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu} \frac{\partial x^\nu}{\partial x'^\beta} \frac{\partial x^\lambda}{\partial x'^\gamma} \Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda} \frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$$

$$\frac{\partial x'^\mu}{\partial x^\lambda} \frac{\partial x^\lambda}{\partial x'^\rho} = \delta^\mu_\rho$$

Consequence

$$(\nabla_\mu V^\alpha)' = \frac{\partial x^\lambda}{\partial x'^\mu} \frac{\partial x'^\alpha}{\partial x^\gamma} \nabla_\lambda V^\gamma(x)$$

$$\frac{\partial}{\partial x'^\nu}: \quad \frac{\partial x^\lambda}{\partial x'^\rho \partial x'^\nu} \frac{\partial x'^\mu}{\partial x^\lambda} + \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial^2 x'^\mu}{\partial x^\beta \partial x^\lambda} \frac{\partial x^\lambda}{\partial x'^\rho} = 0$$


$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} V^{\nu}$
$\frac{\partial^2 f}{\partial x^\alpha \partial x^\beta} = \frac{\partial^2 f}{\partial x^\beta \partial x^\alpha}$
$A_{\mu\nu} = B_{\mu\nu\rho}{}^{\rho}$
$\frac{\partial}{\partial x'^{\mu}} V'^{\alpha}(x') \overset{?}{=} \frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial x'^\alpha}{\partial x^\beta} \frac{\partial}{\partial x^\nu} V^\beta(x)$
$\frac{\partial}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} V^\beta(x)$
$\frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial}{\partial x^\nu} (\quad)$
good
$= \frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial x'^\alpha}{\partial x^\beta} \frac{\partial}{\partial x^\nu} V^\beta$
$+ \frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial^2 x'^\alpha}{\partial x^\nu \partial x^\beta} V^\beta$
bad

$$\frac{\partial^2 f}{\partial x^\alpha \partial x^\beta} = \frac{\partial^2 f}{\partial x^\beta \partial x^\alpha}$$

$$V'^\mu = \frac{\partial x'^\mu}{\partial x^\nu} V^\nu$$

$$A_{\mu\nu} = B_{\mu\nu\rho}{}^{\rho}$$

$$\frac{\partial}{\partial x'^\mu} V'^\alpha(x') \overset{?}{=} \frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial x'^\alpha}{\partial x^\beta} \frac{\partial}{\partial x^\nu} V^\beta(x)$$

$$\frac{\partial}{\partial x'^\mu} \frac{\partial x'^\alpha}{\partial x^\beta} V^\beta(x)$$

$$\frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial}{\partial x^\nu} (\quad)$$

good

$$= \frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial x'^\alpha}{\partial x^\beta} \frac{\partial}{\partial x^\nu} V^\beta + \frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial^2 x'^\alpha}{\partial x^\nu \partial x^\beta} V^\beta$$

bad

$$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} V^{\nu}$$

$$A_{\mu\nu} = B_{\mu\nu\rho}{}^{\rho}$$

$$\frac{\partial^2 f}{\partial x^\alpha \partial x^\beta} = \frac{\partial^2 f}{\partial x^\beta \partial x^\alpha}$$

$$\left.\frac{\partial}{\partial x}\right|_y \left.\frac{\partial}{\partial y}\right|_x f = \left.\frac{\partial}{\partial y}\right|_x \left.\frac{\partial}{\partial x}\right|_y f$$

$$\frac{\partial}{\partial x'^{\mu}} V'^{\alpha}(x') \stackrel{?}{=} \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} \frac{\partial}{\partial x^{\nu}} V^{\beta}(x)$$

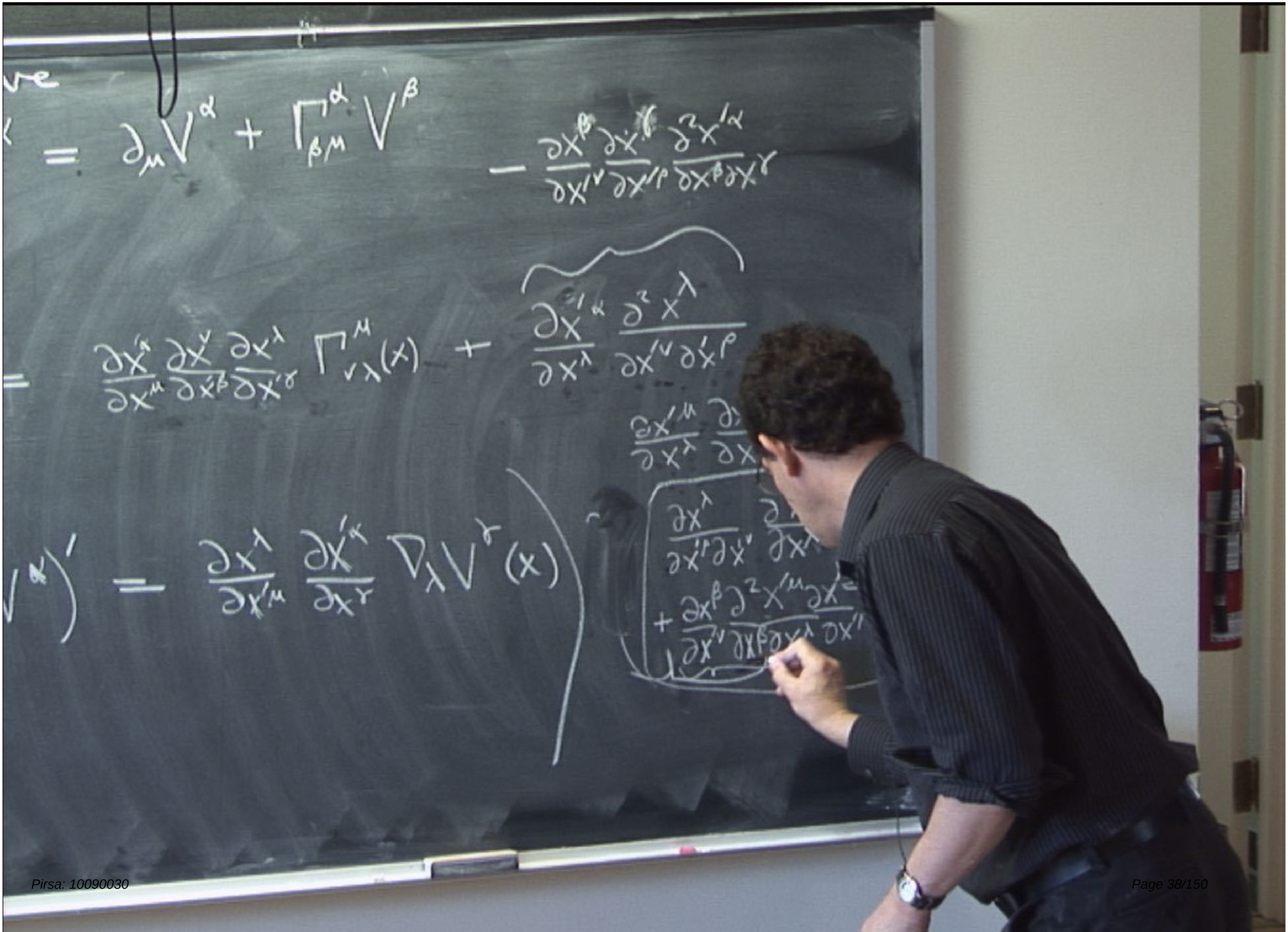
$$\frac{\partial}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} V^{\beta}(x)$$

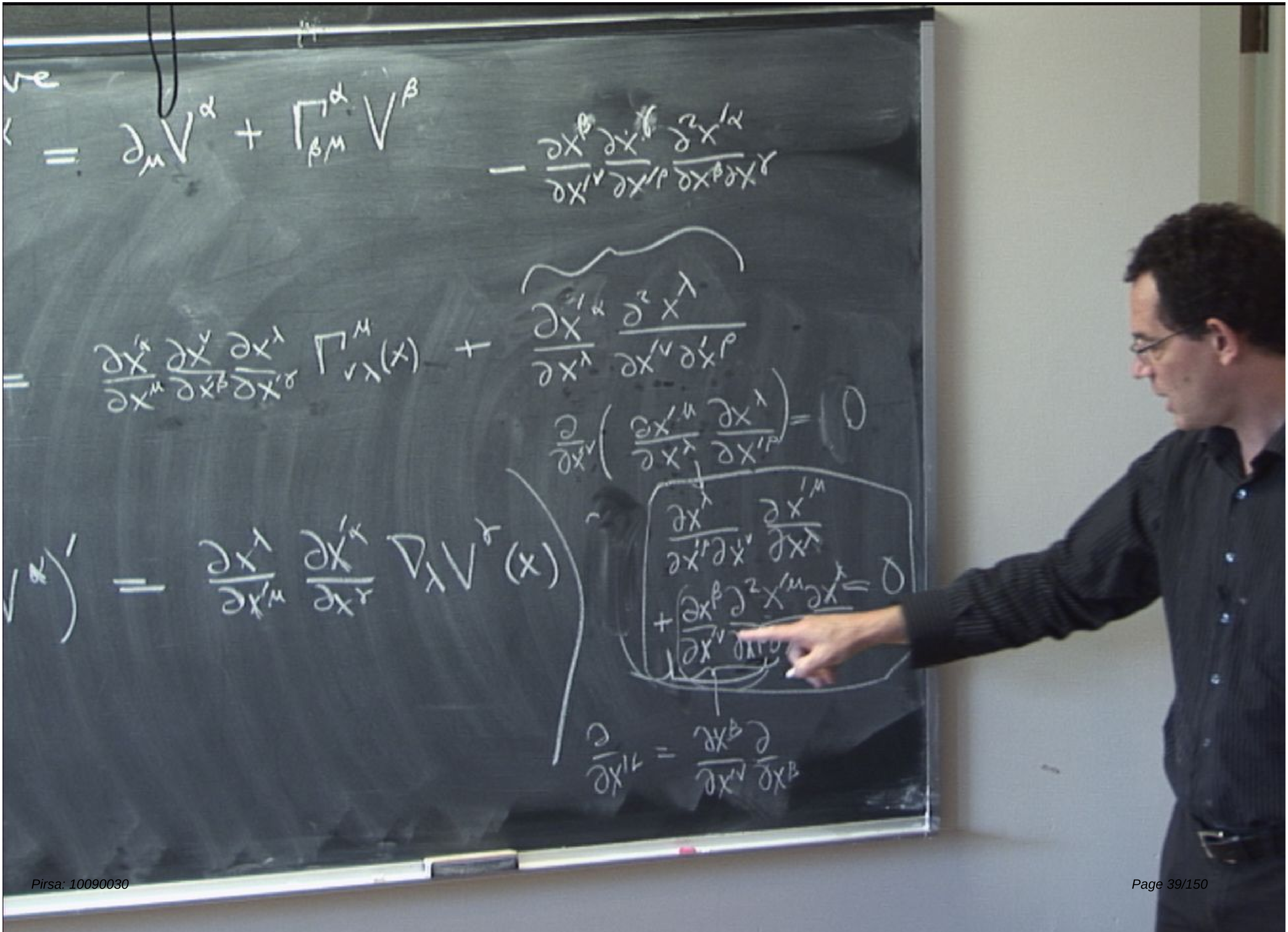
$$\frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial}{\partial x^{\nu}} \left(\quad \right)$$

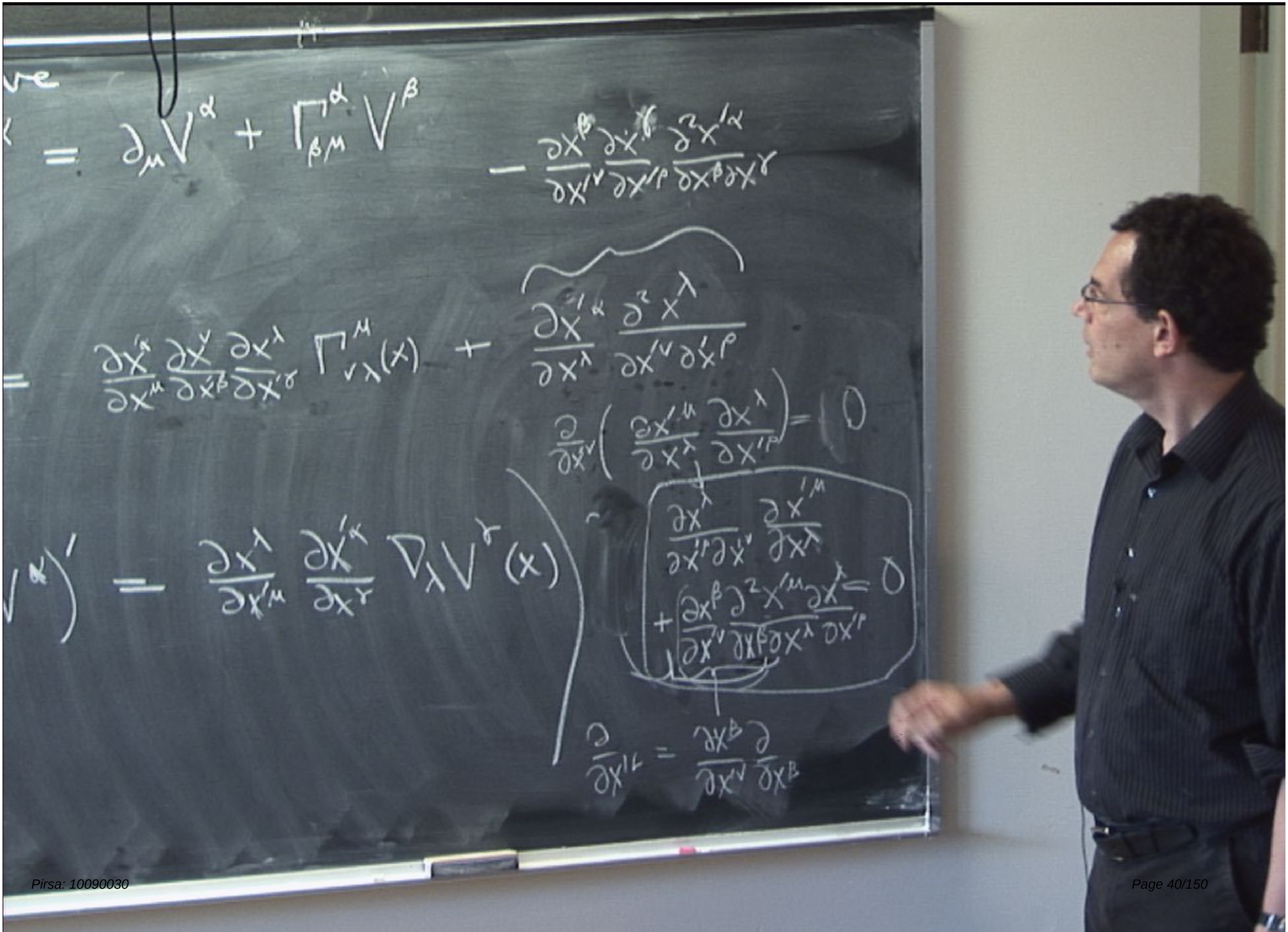
good

$$= \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} \frac{\partial}{\partial x^{\nu}} V^{\beta} + \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial^2 x'^{\alpha}}{\partial x^{\nu} \partial x^{\beta}} V^{\beta}$$

bad


ve
$= \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$
$- \frac{\partial x^\beta}{\partial x'^\nu}\frac{\partial x^\gamma}{\partial x'^\rho}\frac{\partial^2 x'^\alpha}{\partial x^\beta \partial x^\gamma}$
$= \frac{\partial x'^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial x'^\beta}\frac{\partial x^\lambda}{\partial x'^\gamma}\Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda}\frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$
$(\nabla V^\alpha)' = \frac{\partial x^\lambda}{\partial x'^\mu}\frac{\partial x'^\alpha}{\partial x^\gamma}\nabla_\lambda V^\gamma(x)$





$= \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$
$- \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial x^\gamma}{\partial x'^\rho} \frac{\partial^2 x'^\alpha}{\partial x^\beta \partial x^\gamma}$
$= \frac{\partial x'^\alpha}{\partial x^\mu} \frac{\partial x^\nu}{\partial x'^\beta} \frac{\partial x^\lambda}{\partial x'^\gamma} \Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda} \frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$
$\frac{\partial}{\partial x'^\nu}\left(\frac{\partial x'^\mu}{\partial x^\lambda} \frac{\partial x^\lambda}{\partial x'^\rho} \right) = 0$
$\frac{\partial^2 x^\lambda}{\partial x'^\rho \partial x'^\nu} \frac{\partial x'^\mu}{\partial x^\lambda} + \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial^2 x'^\mu}{\partial x^\beta \partial x^\lambda} \frac{\partial x^\lambda}{\partial x'^\rho} = 0$
$= \frac{\partial x^\lambda}{\partial x'^\mu} \frac{\partial x'^\alpha}{\partial x^\gamma} \nabla_\lambda V^\gamma(x)$
$\frac{\partial}{\partial x'^\nu} = \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial}{\partial x^\beta}$

Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$$

$$- \frac{\partial x^\beta}{\partial x'^\nu}\frac{\partial x^\gamma}{\partial x'^\rho}\frac{\partial^2 x'^\alpha}{\partial x^\beta \partial x^\gamma}$$

Defn.:

$$\Gamma'^\alpha_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial x'^\beta}\frac{\partial x^\lambda}{\partial x'^\gamma}\Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda}\frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$$

$$\frac{\partial}{\partial x'^\nu}\left(\frac{\partial x'^\mu}{\partial x^\lambda}\frac{\partial x^\lambda}{\partial x'^\rho}\right) = 0$$

$$\frac{\partial x^\lambda}{\partial x'^\rho \partial x'^\nu}\frac{\partial x'^\mu}{\partial x^\lambda} + \frac{\partial x^\beta}{\partial x'^\nu}\frac{\partial^2 x'^\mu}{\partial x^\beta \partial x^\lambda}\frac{\partial x^\lambda}{\partial x'^\rho} = 0$$

$$\frac{\partial}{\partial x'^\nu} = \frac{\partial x^\beta}{\partial x'^\nu}\frac{\partial}{\partial x^\beta}$$

Consequence

$$(\nabla_\mu V^\alpha)' = \frac{\partial x^\lambda}{\partial x'^\mu}\frac{\partial x'^\alpha}{\partial x^\gamma}\nabla_\lambda V^\gamma(x)$$

$$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} V^{\nu}$$

$$A_{\mu\nu} = B_{\mu\nu\rho}{}^{\rho}$$

$$\frac{\partial^2 f}{\partial x'^{\alpha}\partial x^{\beta}} = \frac{\partial^2 f}{\partial x^{\beta}\partial x'^{\alpha}}$$

$$\frac{\partial}{\partial x'}\Big|_{y'} \frac{\partial}{\partial y}\Big|_{x} f = \frac{\partial}{\partial y}\Big|_{x} \frac{\partial}{\partial x}\Big|_{y} f$$

$$\frac{\partial}{\partial x'^{\mu}} V'^{\alpha}(x') \stackrel{?}{=} \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} \frac{\partial}{\partial x^{\nu}} V^{\beta}(x)$$

$$\frac{\partial}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} V^{\beta}(x)$$

$$\frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial}{\partial x^{\nu}} \left(\quad \right)$$

good

$$= \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} \frac{\partial}{\partial x^{\nu}} V^{\beta}$$

$$+ \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial^2 x'^{\alpha}}{\partial x^{\nu}\partial x^{\beta}} V^{\beta}$$

bad

Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$$

$$- \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial x^\gamma}{\partial x'^\rho} \frac{\partial^2 x'^\alpha}{\partial x^\beta \partial x^\gamma}$$

Defn.:

$$\Gamma'^\alpha_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu} \frac{\partial x^\nu}{\partial x'^\beta} \frac{\partial x^\lambda}{\partial x'^\gamma} \Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda} \frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$$

$$\frac{\partial}{\partial x'^\nu}\left(\frac{\partial x'^\mu}{\partial x^\lambda} \frac{\partial x^\lambda}{\partial x'^\rho} \right) = 0$$

$$\frac{\partial x^\lambda}{\partial x'^\rho \partial x'^\nu} \frac{\partial x'^\mu}{\partial x^\lambda} + \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial^2 x'^\mu}{\partial x^\beta \partial x^\lambda} \frac{\partial x^\lambda}{\partial x'^\rho} = 0$$

$$\frac{\partial}{\partial x'^\nu} = \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial}{\partial x^\beta}$$

Consequence

$$(\nabla_\mu V^\alpha)' = \frac{\partial x^\lambda}{\partial x'^\mu} \frac{\partial x'^\alpha}{\partial x^\gamma} \nabla_\lambda V^\gamma(x)$$

Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$$

$$- \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial x^\gamma}{\partial x'^\rho} \frac{\partial^2 x'^\alpha}{\partial x^\beta \partial x^\gamma}$$

Defn.:

$$\Gamma'^\alpha_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu} \frac{\partial x^\nu}{\partial x'^\beta} \frac{\partial x^\lambda}{\partial x'^\gamma} \Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda} \frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$$

$$\frac{\partial}{\partial x'^\nu}\left(\frac{\partial x'^\mu}{\partial x^\lambda} \frac{\partial x^\lambda}{\partial x'^\rho} \right) = 0$$

$$\frac{\partial x^\lambda}{\partial x'^\rho \partial x'^\nu} \frac{\partial x'^\mu}{\partial x^\lambda} + \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial^2 x'^\mu}{\partial x^\beta \partial x^\lambda} \frac{\partial x^\lambda}{\partial x'^\rho} = 0$$

$$\frac{\partial}{\partial x'^\nu} = \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial}{\partial x^\beta}$$

Consequence

$$(\nabla_\mu V^\alpha)' = \frac{\partial x^\lambda}{\partial x'^\mu} \frac{\partial x'^\alpha}{\partial x^\gamma} \nabla_\lambda V^\gamma(x)$$

Likewise

$$\nabla_\mu V_\alpha = \partial_\mu V_\alpha - \Gamma^\lambda_{\alpha\mu} V_\lambda$$

$$(\nabla_\mu V_\alpha)' = \frac{\partial x}{\partial x} \frac{\partial x}{\partial x} (\nabla_\mu V_\alpha)$$

Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\beta\mu} V^\beta$$

$$- \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial x^\gamma}{\partial x'^\rho} \frac{\partial^2 x'^\alpha}{\partial x^\beta \partial x^\gamma}$$

Defn.:

$$\Gamma'^\alpha_{\beta\gamma}(x') = \frac{\partial x'^\alpha}{\partial x^\mu} \frac{\partial x^\nu}{\partial x'^\beta} \frac{\partial x^\lambda}{\partial x'^\gamma} \Gamma^\mu_{\nu\lambda}(x) + \frac{\partial x'^\alpha}{\partial x^\lambda} \frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho}$$

$$\frac{\partial}{\partial x'^\nu}\left(\frac{\partial x'^\mu}{\partial x^\lambda} \frac{\partial x^\lambda}{\partial x'^\rho} \right) = 0$$

$$\frac{\partial x^\lambda}{\partial x'^\rho \partial x'^\nu} \frac{\partial x'^\mu}{\partial x^\lambda} + \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial^2 x'^\mu}{\partial x^\beta \partial x^\lambda} \frac{\partial x^\lambda}{\partial x'^\rho} = 0$$

$$\frac{\partial}{\partial x'^\nu} = \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial}{\partial x^\beta}$$

Consequence

$$(\nabla_\mu V^\alpha)' = \frac{\partial x^\lambda}{\partial x'^\mu} \frac{\partial x'^\alpha}{\partial x^\gamma} \nabla_\lambda V^\gamma(x)$$

likewise

$$\nabla_\mu V_\alpha = \partial_\mu V_\alpha - \Gamma^\lambda_{\alpha\mu} V_\lambda$$

$$(\nabla_\mu V_\alpha)' = \frac{\partial x^\beta}{\partial x'^\mu} \frac{\partial x^\gamma}{\partial x'^\alpha} (\nabla_\beta V_\gamma)$$

$$T^{\mu\nu\rho\lambda} \nabla_\mu \nabla_\nu \nabla_\rho \nabla_\lambda A^{\alpha\beta}$$

$$\frac{\partial}{\partial x'^\mu} V'^\alpha(x') \stackrel{?}{=} \frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial x'^\alpha}{\partial x^\beta} \frac{\partial}{\partial x^\nu} V^\beta(x)$$

$$\frac{\partial x'}{\partial y}\Big|_x \frac{\partial y}{\partial x}\Big|_y = \frac{\partial}{\partial y}\Big|_x \frac{\partial}{\partial x}\Big|_y f$$

$$\frac{\partial x}{\partial x'^\mu} \frac{\partial}{\partial x^\nu}\left(\frac{\partial x'^\alpha}{\partial x^\beta} V^\beta(x) \right)$$

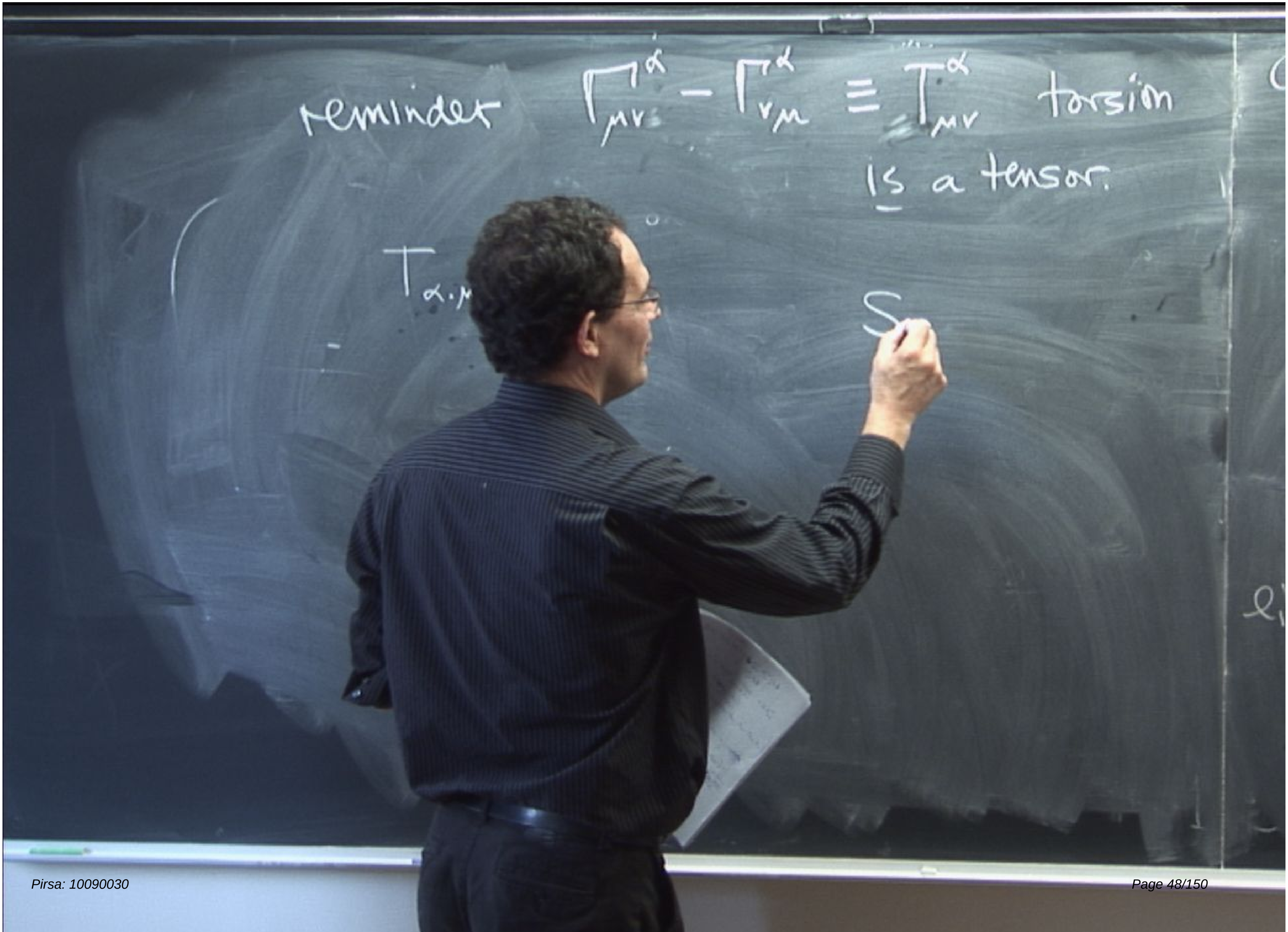
good

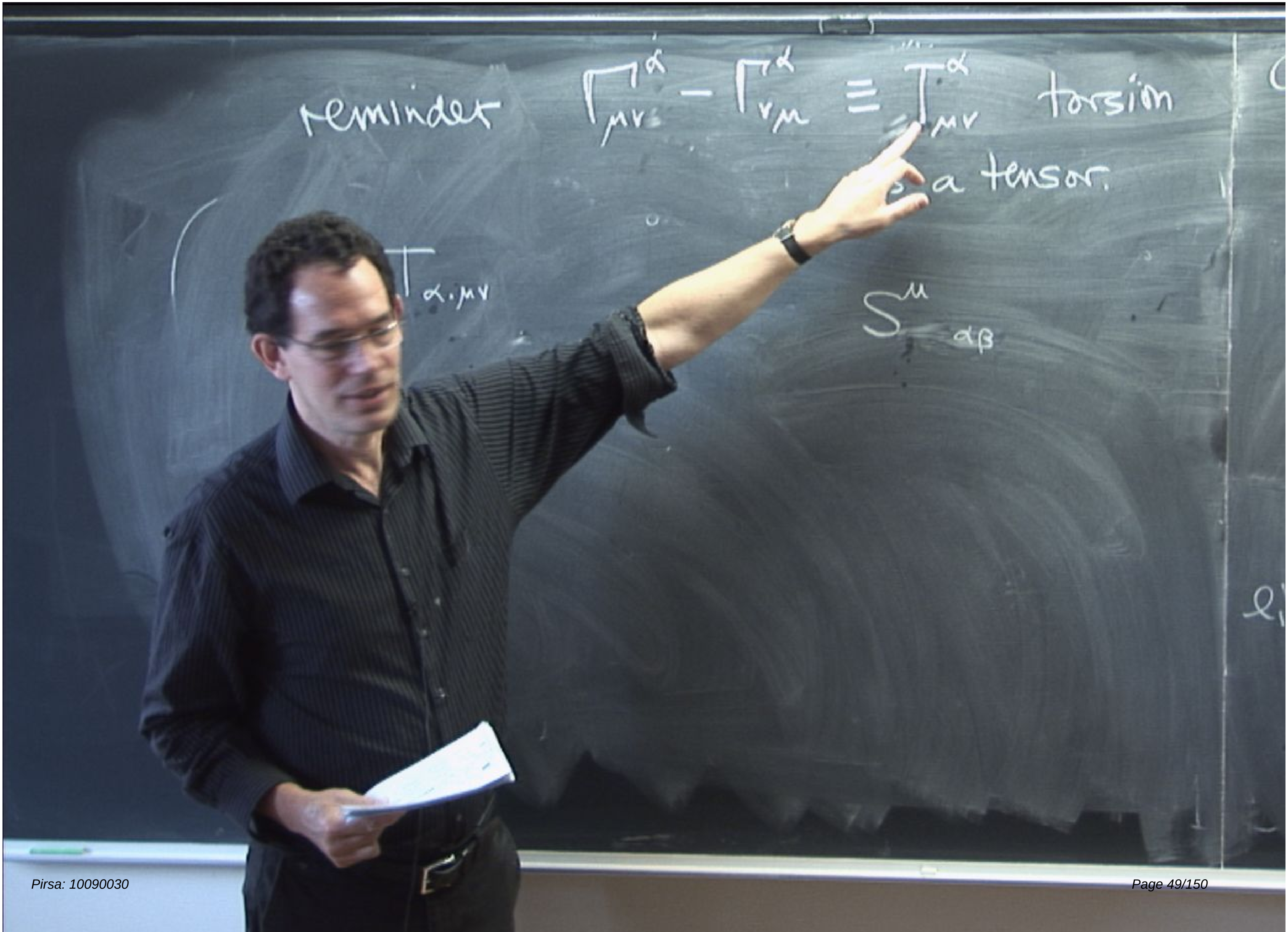
$$= \frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial x'^\alpha}{\partial x^\beta} \frac{\partial}{\partial x^\nu} V^\beta + \frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial^2 x'^\alpha}{\partial x^\nu \partial x^\beta} V^\beta$$

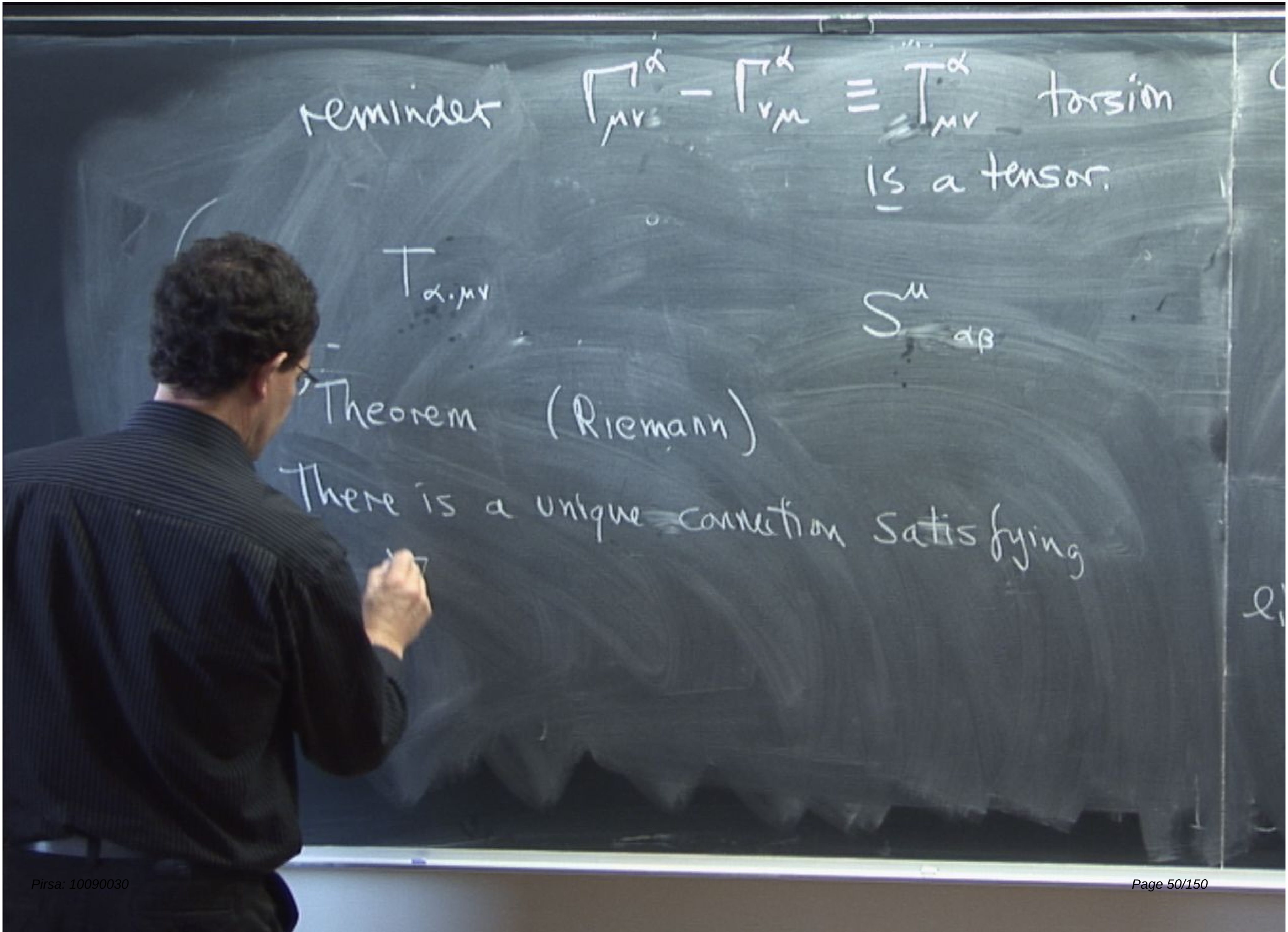
bad

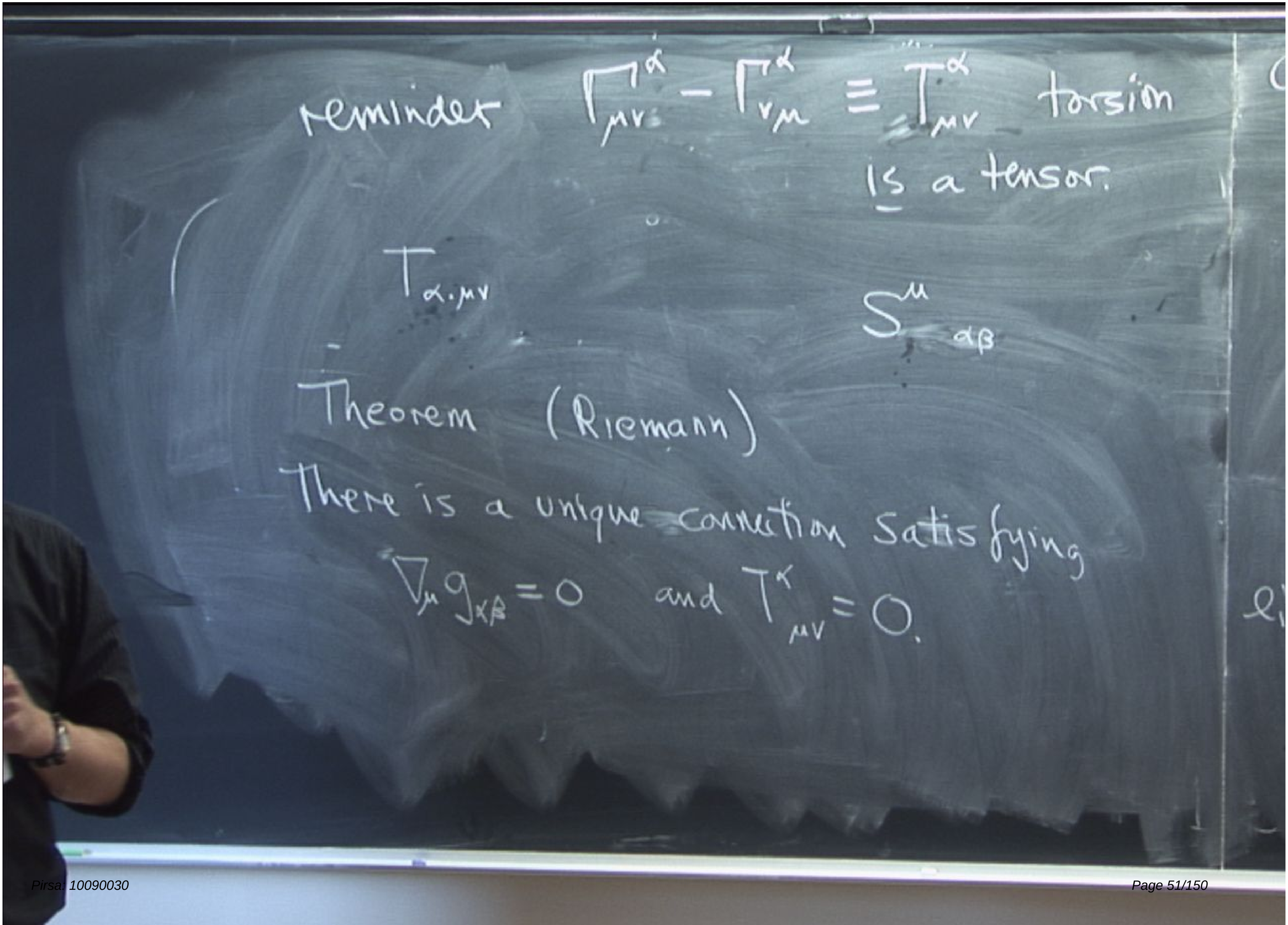
reminder $\Gamma_{\mu\nu}^{\alpha} - \Gamma_{\nu\mu}^{\alpha} \equiv T_{\mu\nu}^{\alpha}$ torsion

is a tensor.


reminder $\Gamma^{\alpha}_{\mu\nu} - \Gamma^{\alpha}_{\nu\mu} \equiv T^{\alpha}_{\mu\nu}$ torsion
is a tensor.
$T_{\alpha.\mu}$
S


reminder
$\Gamma_{\mu\nu}^{\alpha} - \Gamma_{\nu\mu}^{\alpha} \equiv T_{\mu\nu}^{\alpha}$ torsion
a tensor.
$T_{\alpha.\mu\nu}$
$S^{\mu}_{\alpha\beta}$


reminder $\Gamma_{\mu\nu}^{\alpha} - \Gamma_{\nu\mu}^{\alpha} \equiv T_{\mu\nu}^{\alpha}$ torsion
is a tensor.
$T_{\alpha.\mu\nu}$
$S^{\mu}_{\alpha\beta}$
Theorem (Riemann)
There is a unique connection satisfying


reminder $\Gamma^{\alpha}_{\mu\nu} - \Gamma^{\alpha}_{\nu\mu} \equiv T^{\alpha}_{\mu\nu}$ torsion
is a tensor.
$T_{\alpha.\mu\nu}$
$S^{\mu}{}_{\alpha\beta}$
Theorem (Riemann)
There is a unique connection satisfying
$\nabla_\mu g_{\alpha\beta} = 0$ and $T^{\alpha}_{\mu\nu} = 0$.

reminder $\Gamma^{\alpha}_{\mu\nu} - \Gamma^{\alpha}_{\nu\mu} \equiv T^{\alpha}_{\mu\nu}$ torsion

is a tensor.

$T_{\alpha.\mu\nu}$

$S^{\mu}{}_{\alpha\beta}$

Theorem (Riemann)

There is a unique connection satisfying

$$\nabla_\mu g_{\alpha\beta} = 0 \quad \text{and} \quad T^{\alpha}_{\mu\nu} = 0.$$

reminder $\Gamma^{\alpha}_{\mu\nu} - \Gamma^{\alpha}_{\nu\mu} \equiv T^{\alpha}_{\mu\nu}$ torsion

is a tensor.

$T_{\alpha.\mu\nu}$

$S^{\mu}{}_{\alpha\beta}$

Theorem (Riemann)

There is a unique connection satisfying

$$\nabla_\mu g_{\alpha\beta} = 0 \quad \text{and} \quad T^{\alpha}_{\mu\nu} = 0.$$

$$\nabla_\mu g_{\alpha\beta} = \frac{\partial}{\partial x^\mu} - \Gamma^{\lambda}_{\alpha\mu} g_{\lambda\beta} - \Gamma^{\lambda}_{\beta\mu} g_{\alpha\lambda}$$

reminder $\Gamma^{\alpha}_{\mu\nu} - \Gamma^{\alpha}_{\nu\mu} \equiv T^{\alpha}_{\mu\nu}$ torsion

is a tensor.

$T_{\alpha.\mu\nu}$ $S^{\mu}{}_{\alpha\beta}$

Theorem (Riemann)

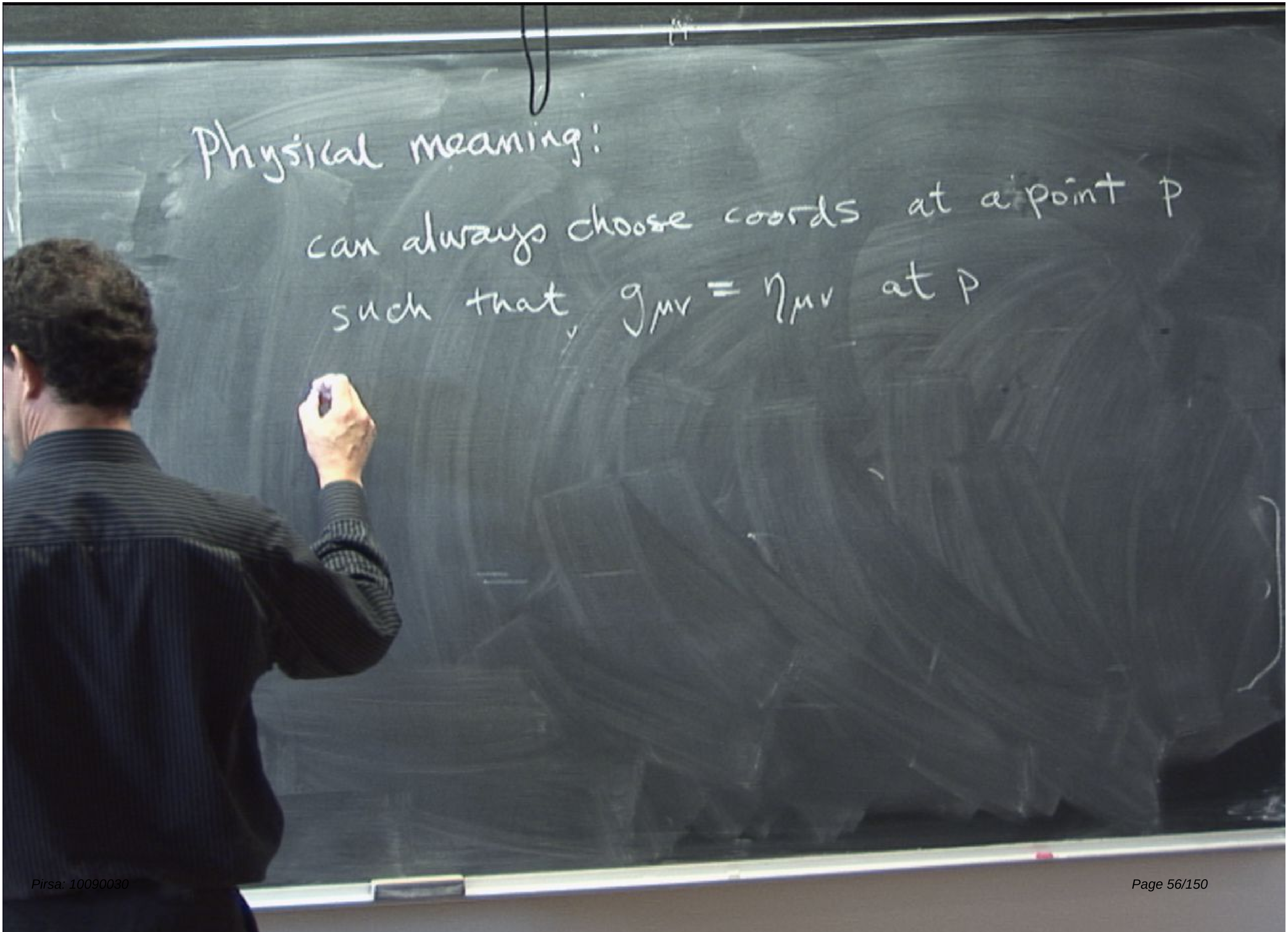
There is a unique connection satisfying

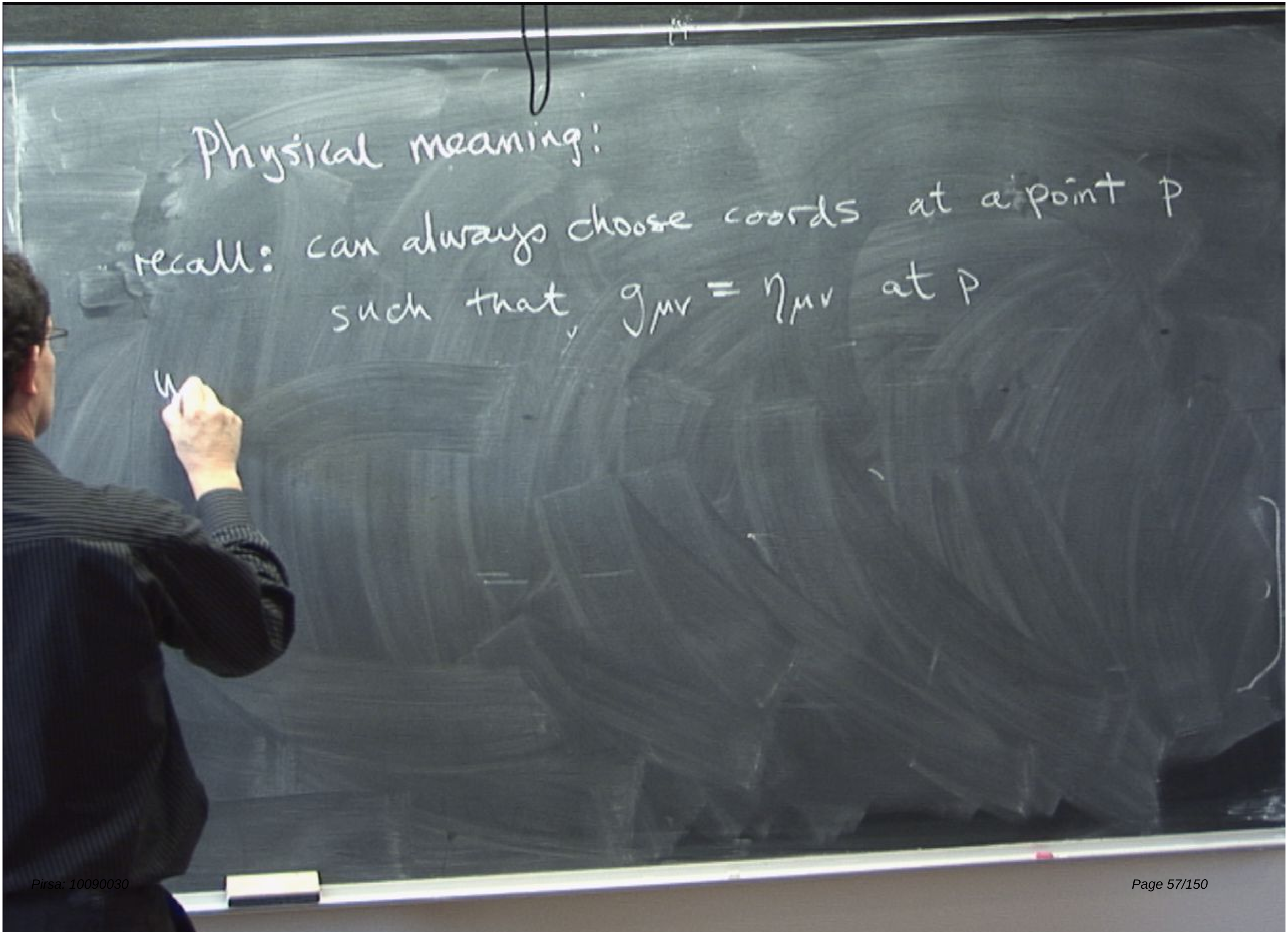
$\nabla_\mu g_{\alpha\beta} = 0$ and $T^{\alpha}_{\mu\nu} = 0.$

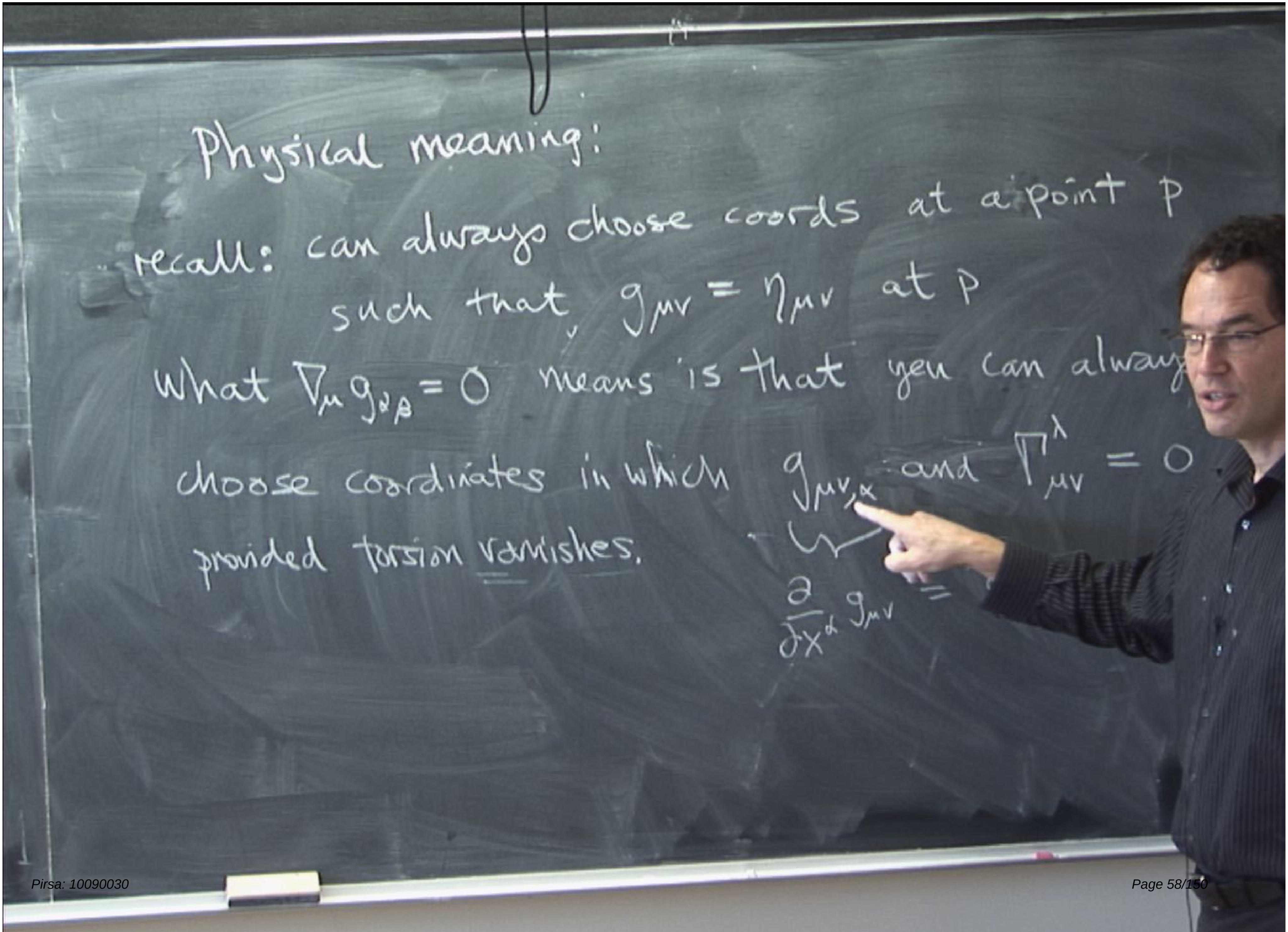
$$\frac{\partial}{\partial x^\mu} - \Gamma^{\lambda}_{\alpha\mu} g_{\lambda\beta} - \Gamma^{\lambda}_{\beta\mu} g_{\alpha\lambda}$$

Physical meaning:

can always choose coords at a point such that $g_{\mu\nu} = \eta_{\mu\nu}$


Physical meaning:
can always choose coords at a point p
such that $g_{\mu\nu} = \eta_{\mu\nu}$ at p


Physical meaning:
recall: can always choose coords at a point p
such that $g_{\mu\nu} = \eta_{\mu\nu}$ at p


Physical meaning:
recall: can always choose coords at a point p
such that $g_{\mu\nu} = \eta_{\mu\nu}$ at p
What $\nabla_\mu g_{\alpha\beta} = 0$ means is that you can alway
choose coordinates in which $g_{\mu\nu,\alpha}$ and $\Gamma^\lambda_{\mu\nu} = 0$
provided torsion vanishes.
$\frac{\partial}{\partial x^\alpha} g_{\mu\nu} =$

Physical meaning:

recall: can always choose coords at a point p such that $g_{\mu\nu} = \eta_{\mu\nu}$ at p

What $\nabla_\mu g_{\alpha\beta} = 0$ means is that you can always choose coordinates in which $g_{\mu\nu,\alpha}$ and $\Gamma^\lambda_{\mu\nu} = 0$ provided torsion vanishes.

$$\frac{\partial}{\partial x^\alpha} g_{\mu\nu} = \partial_\alpha g_{\mu\nu}$$

$T^\alpha_{\nu\mu} \equiv T^\alpha_{\mu\nu}$ torsion

is a tensor.

$S^\mu{}_{\alpha\beta}$

$g_{\alpha\beta} = g_{\beta\alpha}$

...ection satisfying

$T^\alpha{}_{\mu\nu} = 0.$

$\nabla_\mu V_\alpha \equiv V_{\alpha;\mu}$

Physical meaning:

recall: can always choose coor...

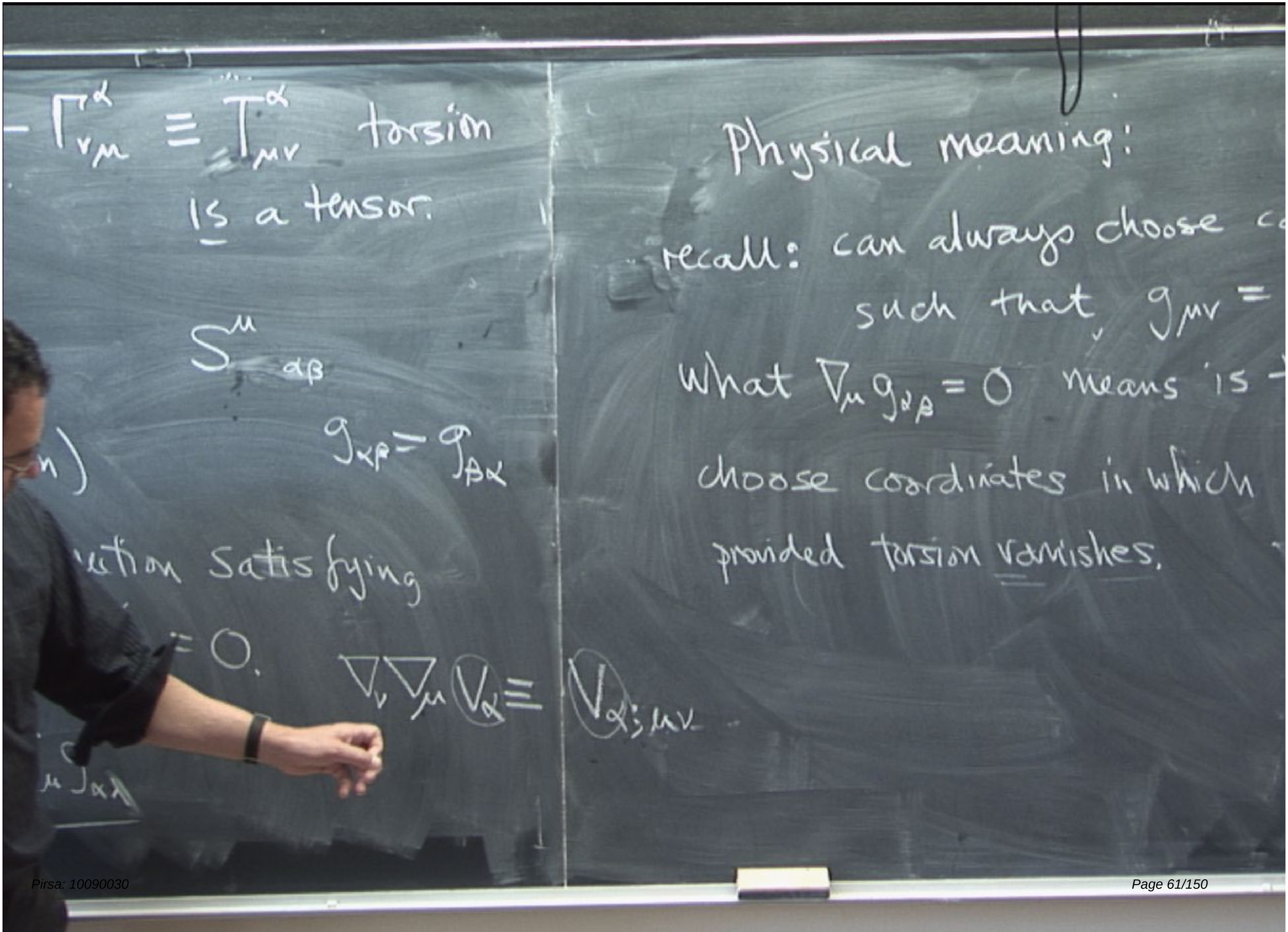
such that $g_{\mu\nu} = \eta_{\mu...}$

What $\nabla_\mu g_{\alpha\beta} = 0$ means is tha...

choose coordinates in which g...

provided torsion vanishes.

$\frac{\partial}{\partial x}$...



$- \Gamma^{\alpha}_{\nu\mu} \equiv T^{\alpha}_{\mu\nu}$ torsion
is a tensor.
$S^{\mu}{}_{\alpha\beta}$
$g_{\alpha\beta} = g_{\beta\alpha}$
satisfying
$= 0.$
$\nabla_\nu \nabla_\mu V_\alpha \equiv V_{\alpha;\mu\nu}$
Physical meaning:
recall: can always choose
such that $g_{\mu\nu} =$
What $\nabla_\mu g_{\alpha\beta} = 0$ means is
choose coordinates in which
provided torsion vanishes.

Physical meaning:

recall: can always choose coords at a point p such that $g_{\mu\nu} = \eta_{\mu\nu}$ at p

What $\nabla_\mu g_{\alpha\beta} = 0$ means is that you can always choose coordinates in which $g_{\mu\nu,\alpha}$ and $\Gamma^\lambda_{\mu\nu} = 0$

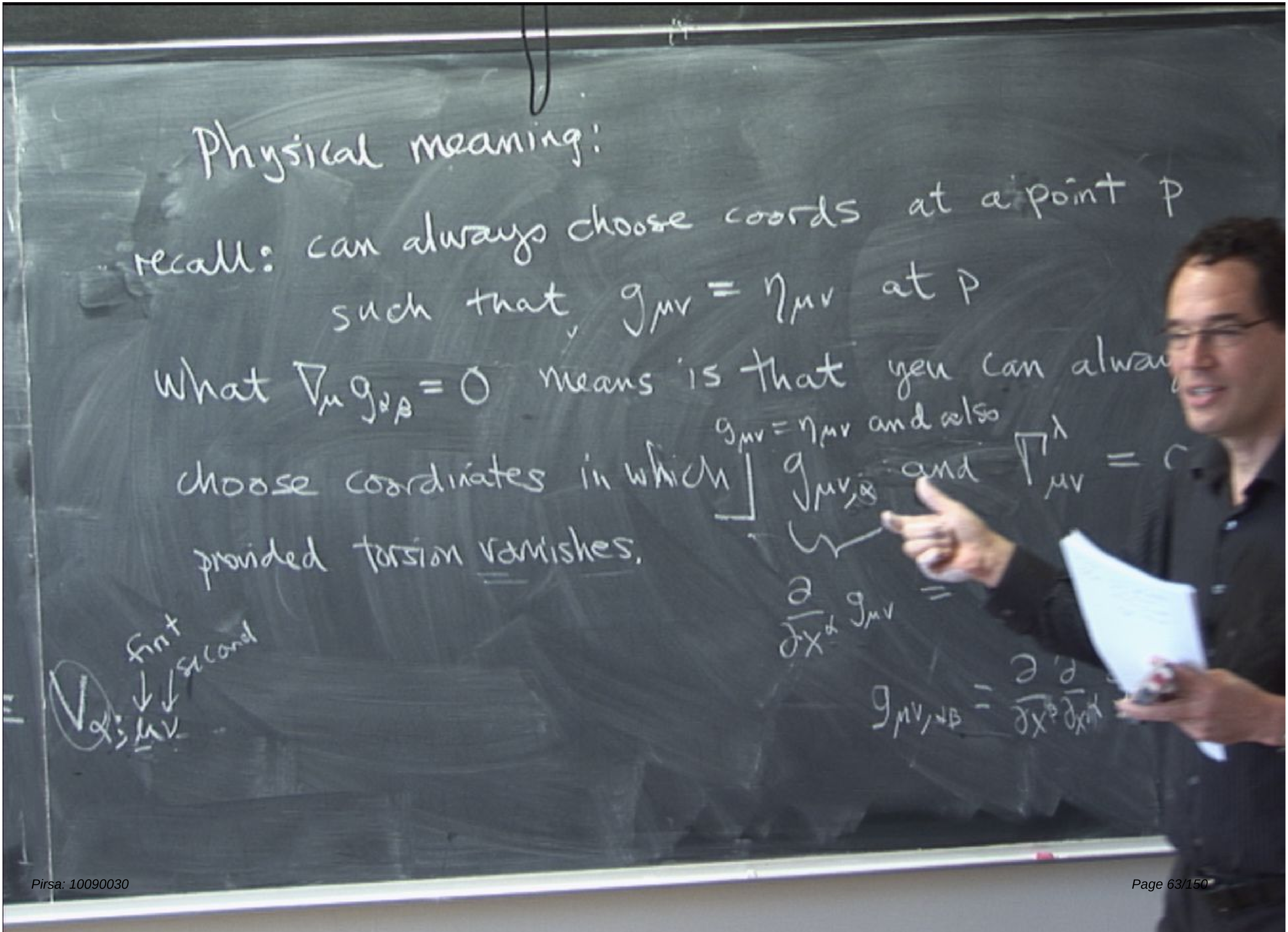
provided torsion vanishes.

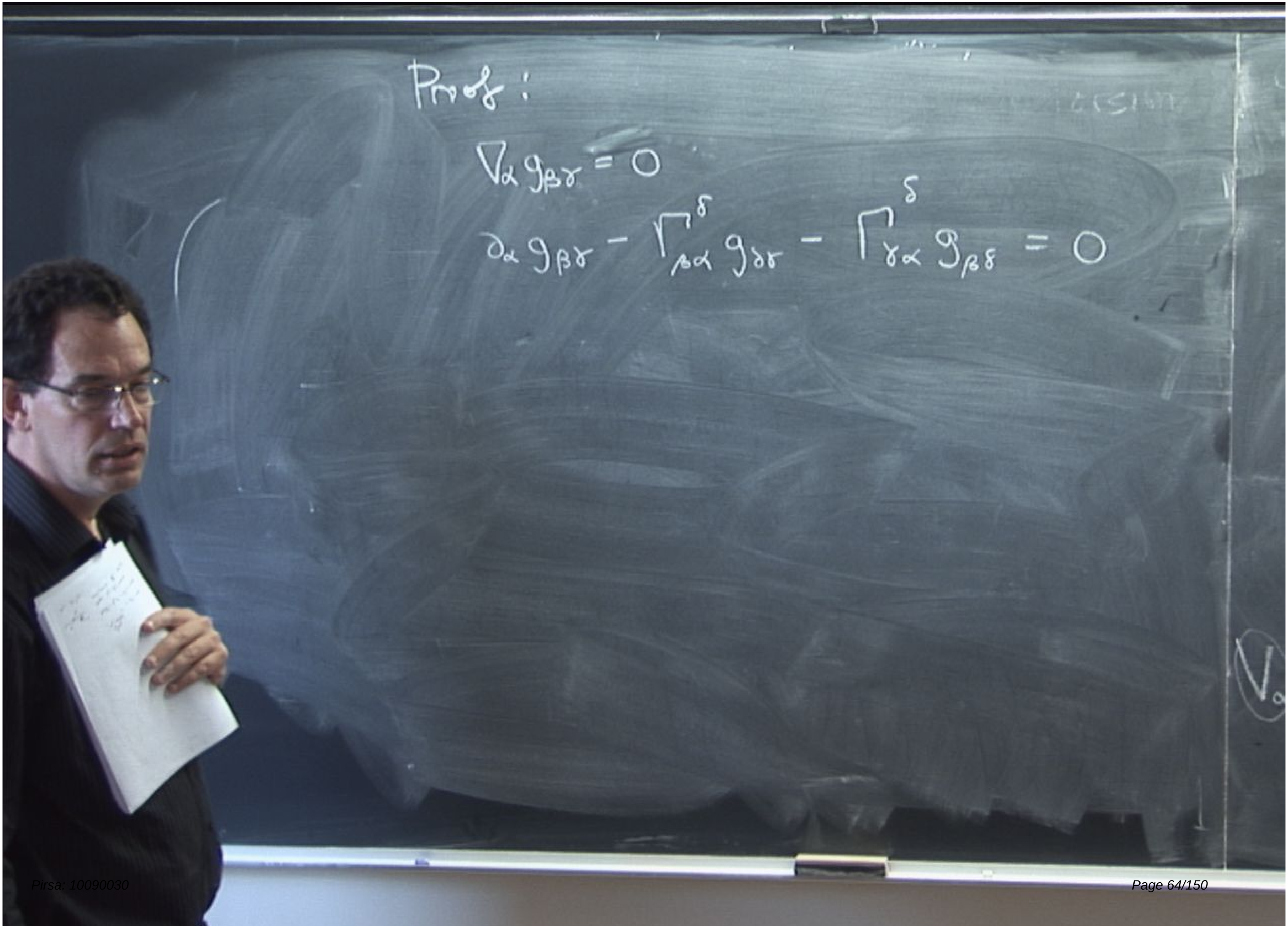
$$\frac{\partial}{\partial x^\alpha} g_{\mu\nu} = \partial_\alpha g_{\mu\nu}$$

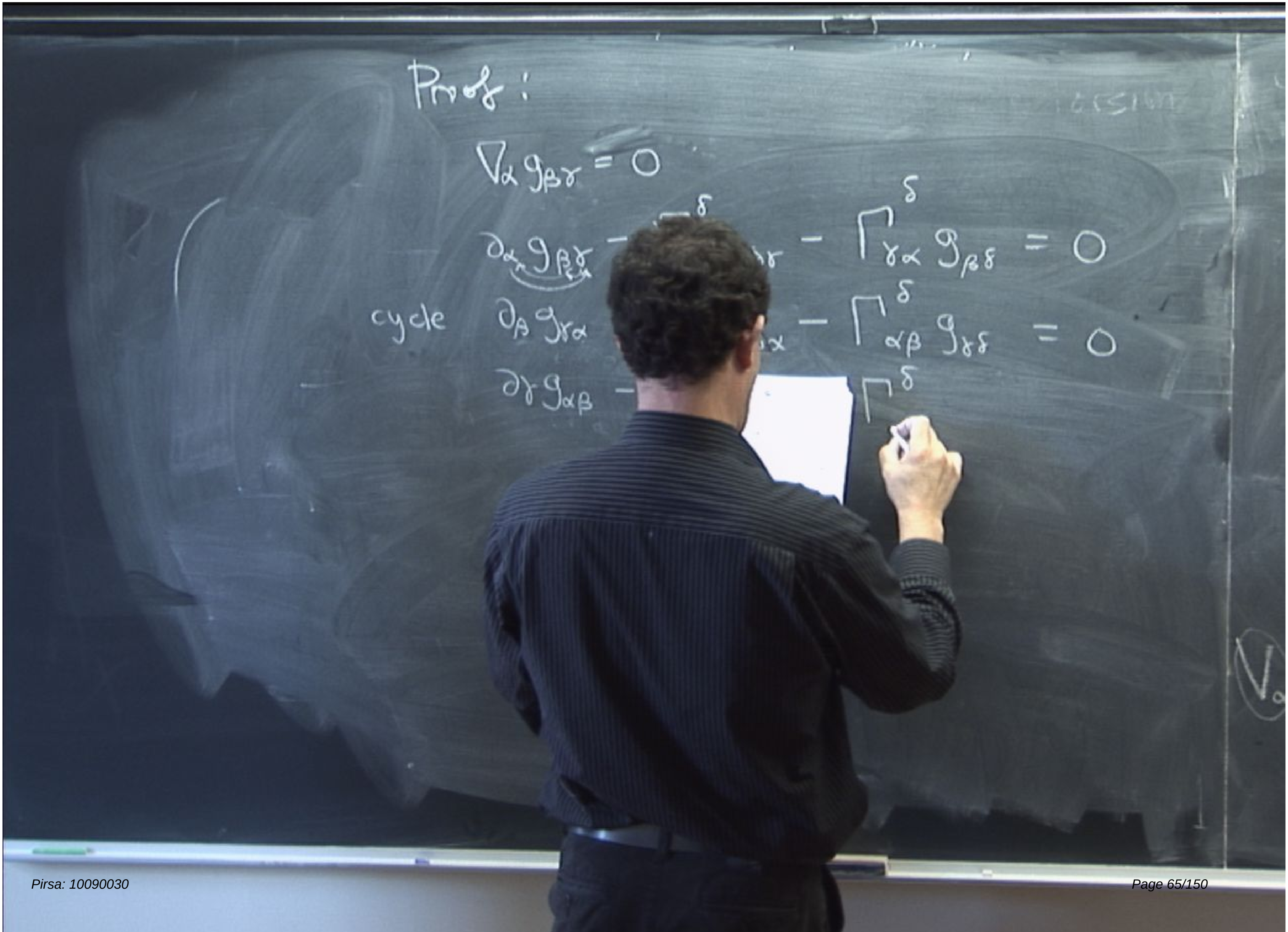
$$g_{\mu\nu,\alpha\beta} = \frac{\partial}{\partial x^\beta}\frac{\partial}{\partial x^\alpha} g_{\mu\nu}$$

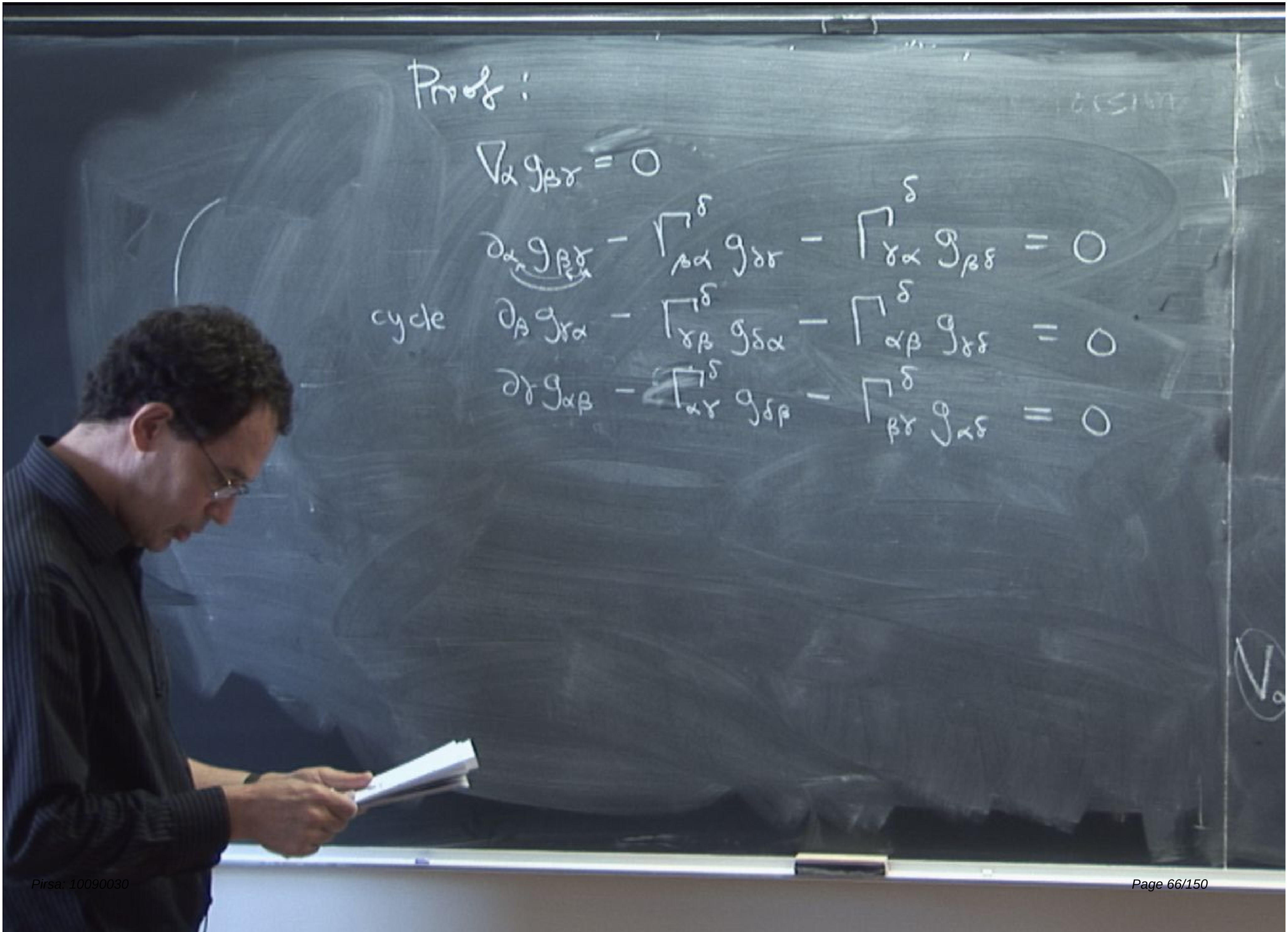
$$V_{\alpha;\mu\nu}$$

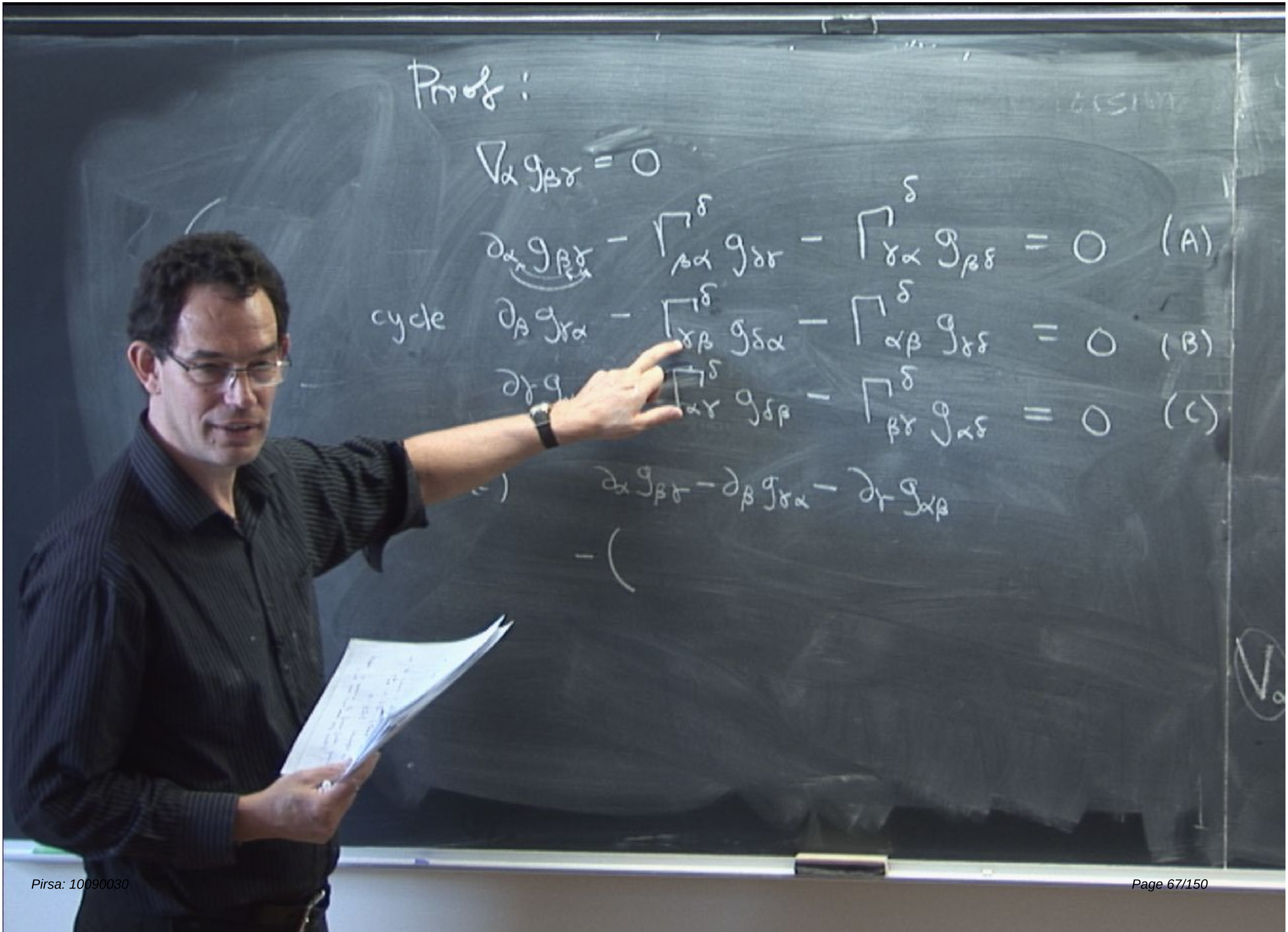
first (μ), second (ν)


Physical meaning:
recall: can always choose coords at a point p
such that $g_{\mu\nu} = \eta_{\mu\nu}$ at p
What $\nabla_\mu g_{\alpha\beta} = 0$ means is that you can alway
choose coordinates in which $g_{\mu\nu} = \eta_{\mu\nu}$ and also $g_{\mu\nu,\alpha}$ and $\Gamma^\lambda_{\mu\nu} =$
provided torsion vanishes.
$\frac{\partial}{\partial x^\alpha} g_{\mu\nu} =$
$g_{\mu\nu,\alpha\beta} = \frac{\partial}{\partial x^\beta}\frac{\partial}{\partial x^\alpha}$
first
second
$V_{\alpha;\mu\nu}$


Proof:
$\nabla_\alpha g_{\beta\gamma} = 0$
$\partial_\alpha g_{\beta\gamma} - \Gamma^\delta_{\beta\alpha} g_{\delta\gamma} - \Gamma^\delta_{\gamma\alpha} g_{\beta\delta} = 0$


Proof :
∇α gβγ = 0
cycle


Proof:
$\nabla_\alpha g_{\beta\gamma} = 0$
$\partial_\alpha g_{\beta\gamma} - \Gamma^\delta_{\beta\alpha} g_{\delta\gamma} - \Gamma^\delta_{\gamma\alpha} g_{\beta\delta} = 0$
cycle $\partial_\beta g_{\gamma\alpha} - \Gamma^\delta_{\gamma\beta} g_{\delta\alpha} - \Gamma^\delta_{\alpha\beta} g_{\gamma\delta} = 0$
$\partial_\gamma g_{\alpha\beta} - \Gamma^\delta_{\alpha\gamma} g_{\delta\beta} - \Gamma^\delta_{\beta\gamma} g_{\alpha\delta} = 0$


Proof:
$\nabla_\alpha g_{\beta\gamma} = 0$
$\partial_\alpha g_{\beta\gamma} - \Gamma^\delta_{\beta\alpha} g_{\delta\gamma} - \Gamma^\delta_{\gamma\alpha} g_{\beta\delta} = 0$ (A)
cycle $\partial_\beta g_{\gamma\alpha} - \Gamma^\delta_{\gamma\beta} g_{\delta\alpha} - \Gamma^\delta_{\alpha\beta} g_{\gamma\delta} = 0$ (B)
$\Gamma^\delta_{\alpha\gamma} g_{\delta\beta} - \Gamma^\delta_{\beta\gamma} g_{\alpha\delta} = 0$ (C)
$\partial_\alpha g_{\beta\gamma} - \partial_\beta g_{\gamma\alpha} - \partial_\gamma g_{\alpha\beta}$
$-($

Proof:

$$\nabla_\alpha g_{\beta\gamma} = 0$$

$$g_{\mu\nu} \Gamma^{\nu}_{\alpha\beta} = \Gamma_{\mu\cdot\alpha\beta}$$

$$\partial_\alpha g_{\beta\gamma} - \Gamma^{\delta}_{\beta\alpha} g_{\delta\gamma} - \Gamma^{\delta}_{\gamma\alpha} g_{\beta\delta} = 0 \quad (A)$$

cycle

$$\partial_\beta g_{\gamma\alpha} - \Gamma^{\delta}_{\gamma\beta} g_{\delta\alpha} - \Gamma^{\delta}_{\alpha\beta} g_{\gamma\delta} = 0 \quad (B)$$

$$\partial_\gamma g_{\alpha\beta} - \Gamma^{\delta}_{\alpha\gamma} g_{\delta\beta} - \Gamma^{\delta}_{\beta\gamma} g_{\alpha\delta} = 0 \quad (C)$$

(A) − (B) − (C)

$$\partial_\alpha g_{\beta\gamma} - \partial_\beta g_{\gamma\alpha} - \partial_\gamma g_{\alpha\beta}$$

$$-\,($$

Proof:

$$\nabla_\alpha g_{\beta\gamma} = 0$$

$g_{\mu\nu}\Gamma^{\nu}_{\alpha\beta} = \Gamma_{\mu\cdot\alpha\beta}$

$$\partial_\alpha g_{\beta\gamma} - \Gamma^{\delta}_{\beta\alpha} g_{\delta\gamma} - \Gamma^{\delta}_{\gamma\alpha} g_{\beta\delta} = 0 \qquad (A)$$

cycle

$$\partial_\beta g_{\gamma\alpha} - \Gamma^{\delta}_{\gamma\beta} g_{\delta\alpha} - \Gamma^{\delta}_{\alpha\beta} g_{\gamma\delta} = 0 \qquad (B)$$

$$\partial_\gamma g_{\alpha\beta} - \Gamma^{\delta}_{\alpha\gamma} g_{\delta\beta} - \Gamma^{\delta}_{\beta\gamma} g_{\alpha\delta} = 0 \qquad (C)$$

(A) − (B) − (C)

$$\partial_\alpha g_{\beta\gamma} - \partial_\beta g_{\gamma\alpha} - \partial_\gamma g_{\alpha\beta}$$

$$-\left(\Gamma_{\gamma\cdot\beta\alpha} - \Gamma_{\gamma\cdot\alpha\beta}\right) - \left(\Gamma_{\beta\cdot\gamma\alpha} - \Gamma_{\beta\cdot\alpha\gamma}\right)$$

$$+\left(\Gamma_{\alpha\cdot\gamma\beta} + \Gamma_{\alpha\cdot\beta\gamma}\right) = 0$$

∇_α

Proof:

$$\nabla_\alpha g_{\beta\gamma} = 0$$

$g_{\mu\nu}\Gamma^{\nu}_{\alpha\beta} \equiv \Gamma_{\mu\cdot\alpha\beta}$

$$\partial_\alpha g_{\beta\gamma} - \Gamma^{\delta}_{\beta\alpha} g_{\delta\gamma} - \Gamma^{\delta}_{\gamma\alpha} g_{\beta\delta} = 0 \quad (A)$$

cycle
$$\partial_\beta g_{\gamma\alpha} - \Gamma^{\delta}_{\gamma\beta} g_{\delta\alpha} - \Gamma^{\delta}_{\alpha\beta} g_{\gamma\delta} = 0 \quad (B)$$

$$\partial_\gamma g_{\alpha\beta} - \Gamma^{\delta}_{\alpha\gamma} g_{\delta\beta} - \Gamma^{\delta}_{\beta\gamma} g_{\alpha\delta} = 0 \quad (C)$$

(A) − (B) − (C)
$$\partial_\alpha g_{\beta\gamma} - \partial_\beta g_{\gamma\alpha} - \partial_\gamma g_{\alpha\beta}$$
$$- \left(\Gamma_{\gamma\cdot\beta\alpha} - \Gamma_{\gamma\cdot\alpha\beta}\right) - \left(\Gamma_{\beta\cdot\gamma\alpha} - \Gamma_{\beta\cdot\alpha\gamma}\right)$$
$$+ \left(\Gamma_{\alpha\cdot\gamma\beta} + \Gamma_{\alpha\cdot\beta\gamma}\right) = 0$$

Proof:

$$\nabla_\alpha g_{\beta\gamma} = 0$$

$$g_{\mu\nu}\Gamma^\nu_{\alpha\beta} \equiv \Gamma_{\mu\cdot\alpha\beta}$$

$$\partial_\alpha g_{\beta\gamma} - \Gamma^\delta_{\beta\alpha} g_{\delta\gamma} - \Gamma^\delta_{\gamma\alpha} g_{\beta\delta} = 0 \qquad (A)$$

cycle

$$\partial_\beta g_{\gamma\alpha} - \Gamma^\delta_{\gamma\beta} g_{\delta\alpha} - \Gamma^\delta_{\alpha\beta} g_{\gamma\delta} = 0 \qquad (B)$$

$$\partial_\gamma g_{\alpha\beta} - \Gamma^\delta_{\alpha\gamma} g_{\delta\beta} - \Gamma^\delta_{\beta\gamma} g_{\alpha\delta} = 0 \qquad (C)$$

(A) − (B) − (C)

$$\partial_\alpha g_{\beta\gamma} - \partial_\beta g_{\gamma\alpha} - \partial_\gamma g_{\alpha\beta} - \left(\Gamma_{\gamma\cdot\beta\alpha} - \Gamma_{\gamma\cdot\alpha\beta}\right) - \left(\Gamma_{\beta\cdot\gamma\alpha} - \Gamma_{\beta\cdot\alpha\gamma}\right) + \left(\Gamma_{\alpha\cdot\gamma\beta} + \Gamma_{\alpha\cdot\beta\gamma}\right) = 0$$

Proof:

$$\nabla_\alpha g_{\beta\gamma} = 0$$

$$\left(g_{\mu\nu} \Gamma^{\nu}_{\alpha\beta} \equiv \Gamma_{\mu\cdot\alpha\beta} \right.$$

$$\partial_\alpha g_{\beta\gamma} - \overset{①}{\Gamma}{}^{\delta}_{\beta\alpha} g_{\delta\gamma} - \overset{②}{\Gamma}{}^{\delta}_{\gamma\alpha} g_{\beta\delta} = 0 \qquad (A)$$

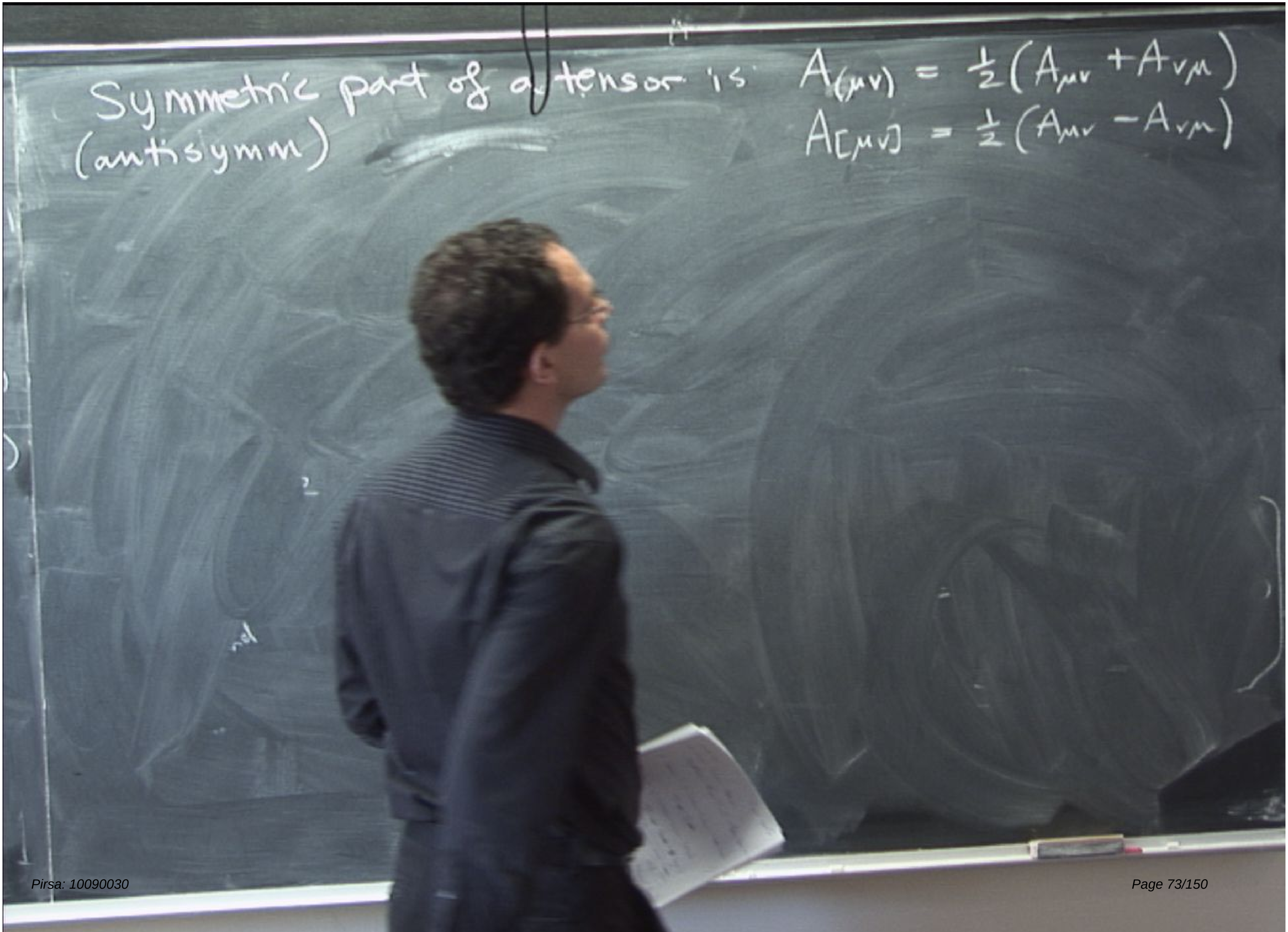
cycle

$$\partial_\beta g_{\gamma\alpha} - \overset{③}{\Gamma}{}^{\delta}_{\gamma\beta} g_{\delta\alpha} - \overset{④}{\Gamma}{}^{\delta}_{\alpha\beta} g_{\gamma\delta} = 0 \qquad (B)$$

$$\partial_\gamma g_{\alpha\beta} - \overset{⑤}{\Gamma}{}^{\delta}_{\alpha\gamma} g_{\delta\beta} - \overset{⑥}{\Gamma}{}^{\delta}_{\beta\gamma} g_{\alpha\delta} = 0 \qquad (C)$$

(A) − (B) − (C)

$$\partial_\alpha g_{\beta\gamma} - \partial_\beta g_{\gamma\alpha} - \partial_\gamma g_{\alpha\beta}$$
$$- \left(\overset{①}{\Gamma}_{\gamma\cdot\beta\alpha} - \overset{④}{\Gamma}_{\gamma\cdot\alpha\beta} \right) - \left(\overset{②}{\Gamma}_{\beta\cdot\gamma\alpha} - \overset{⑤}{\Gamma}_{\beta\cdot\alpha\gamma} \right)$$
$$+ \left(\overset{③}{\Gamma}_{\alpha\cdot\gamma\beta} + \overset{⑥}{\Gamma}_{\alpha\cdot\beta\gamma} \right) = 0$$


Symmetric part of a tensor is $A_{(\mu\nu)} = \frac{1}{2}(A_{\mu\nu} + A_{\nu\mu})$
(antisymm) $A_{[\mu\nu]} = \frac{1}{2}(A_{\mu\nu} - A_{\nu\mu})$

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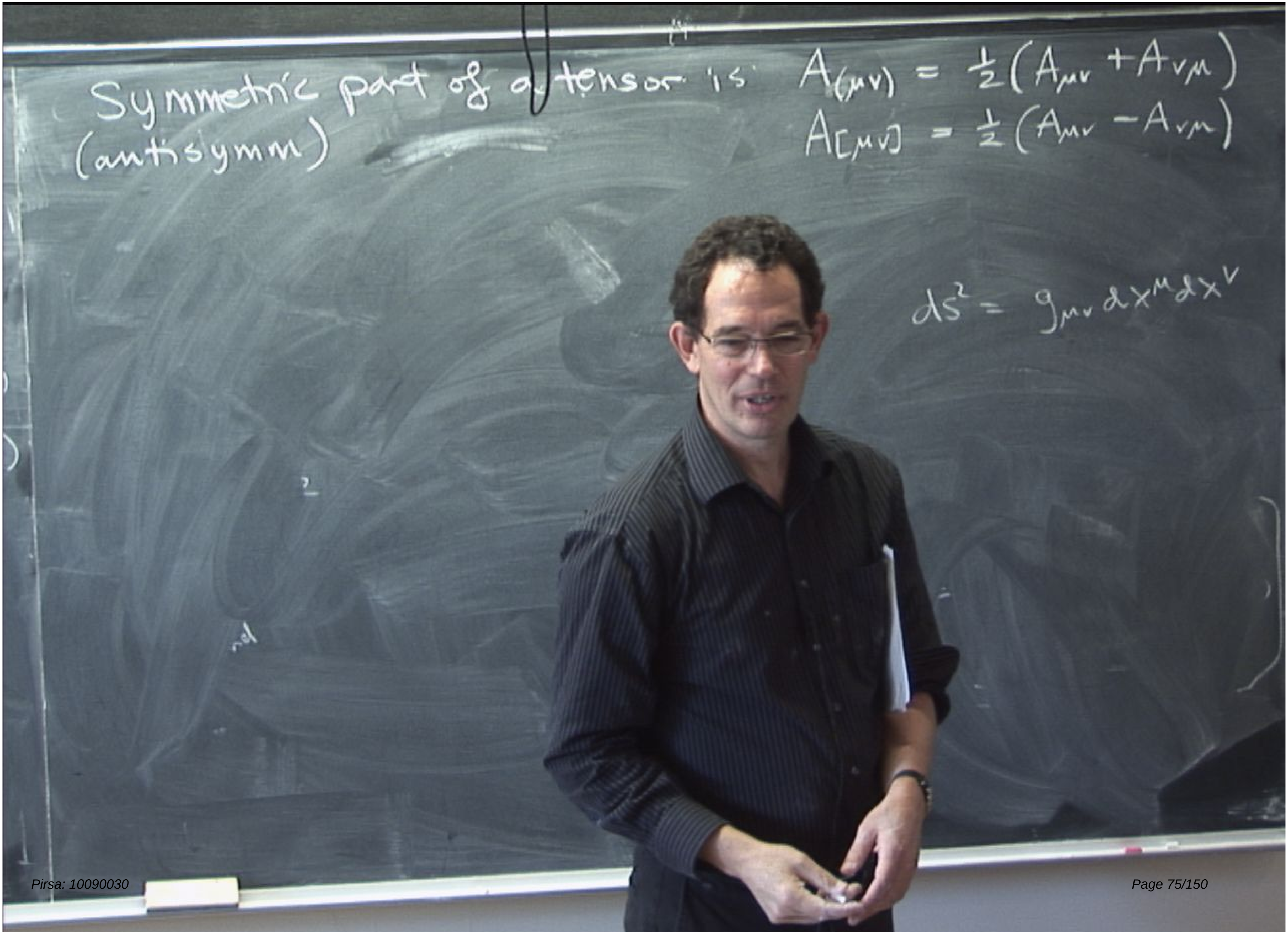
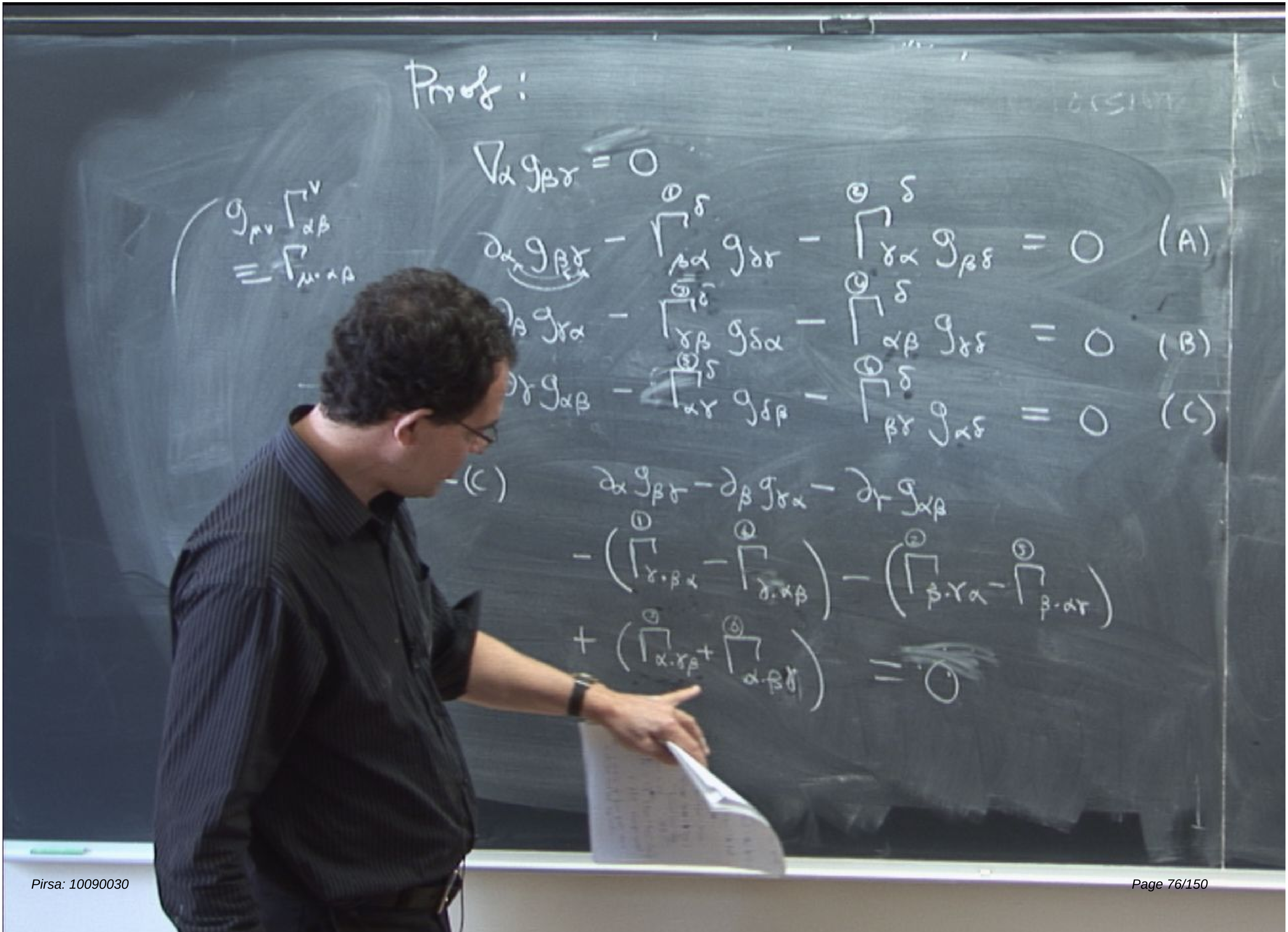
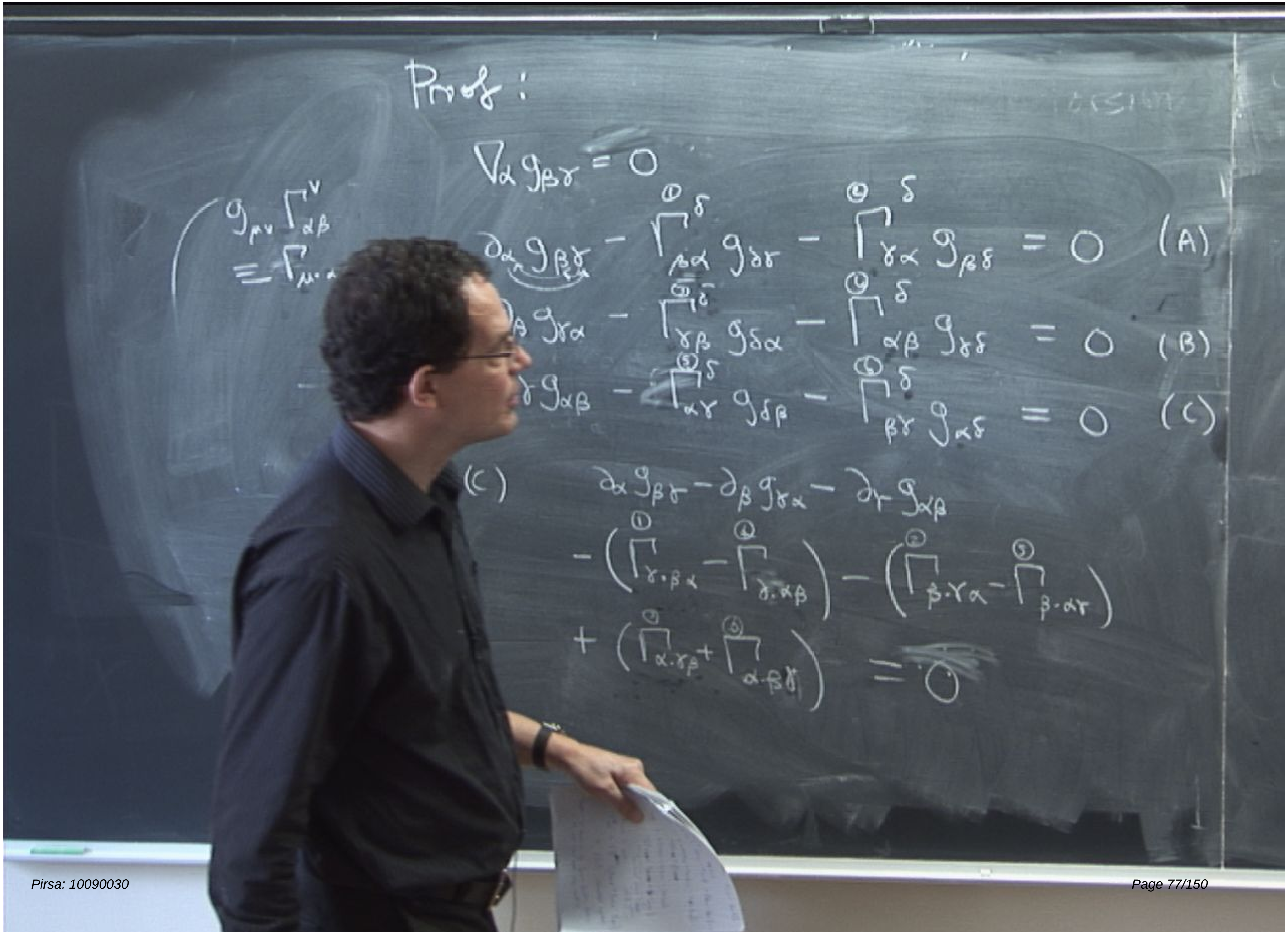

Symmetric part of a tensor is $A_{(\mu\nu)} = \frac{1}{2}(A_{\mu\nu} + A_{\nu\mu})$
(antisymm) $A_{[\mu\nu]} = \frac{1}{2}(A_{\mu\nu} - A_{\nu\mu})$
$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$


Proof:
$\nabla_\alpha g_{\beta\gamma} = 0$
$g_{\mu\nu}\Gamma^\nu_{\alpha\beta} = \Gamma_{\mu\cdot\alpha\beta}$
$\partial_\alpha g_{\beta\gamma} - \Gamma^\delta_{\beta\alpha} g_{\delta\gamma} - \Gamma^\delta_{\gamma\alpha} g_{\beta\delta} = 0$ (A)
$\partial_\beta g_{\gamma\alpha} - \Gamma^\delta_{\gamma\beta} g_{\delta\alpha} - \Gamma^\delta_{\alpha\beta} g_{\gamma\delta} = 0$ (B)
$\partial_\gamma g_{\alpha\beta} - \Gamma^\delta_{\alpha\gamma} g_{\delta\beta} - \Gamma^\delta_{\beta\gamma} g_{\alpha\delta} = 0$ (C)
-(C)
$\partial_\alpha g_{\beta\gamma} - \partial_\beta g_{\gamma\alpha} - \partial_\gamma g_{\alpha\beta}$
$-(\Gamma_{\gamma\cdot\beta\alpha} - \Gamma_{\gamma\cdot\alpha\beta}) - (\Gamma_{\beta\cdot\gamma\alpha} - \Gamma_{\beta\cdot\alpha\gamma})$
$+ (\Gamma_{\alpha\cdot\gamma\beta} + \Gamma_{\alpha\cdot\beta\gamma}) = 0$



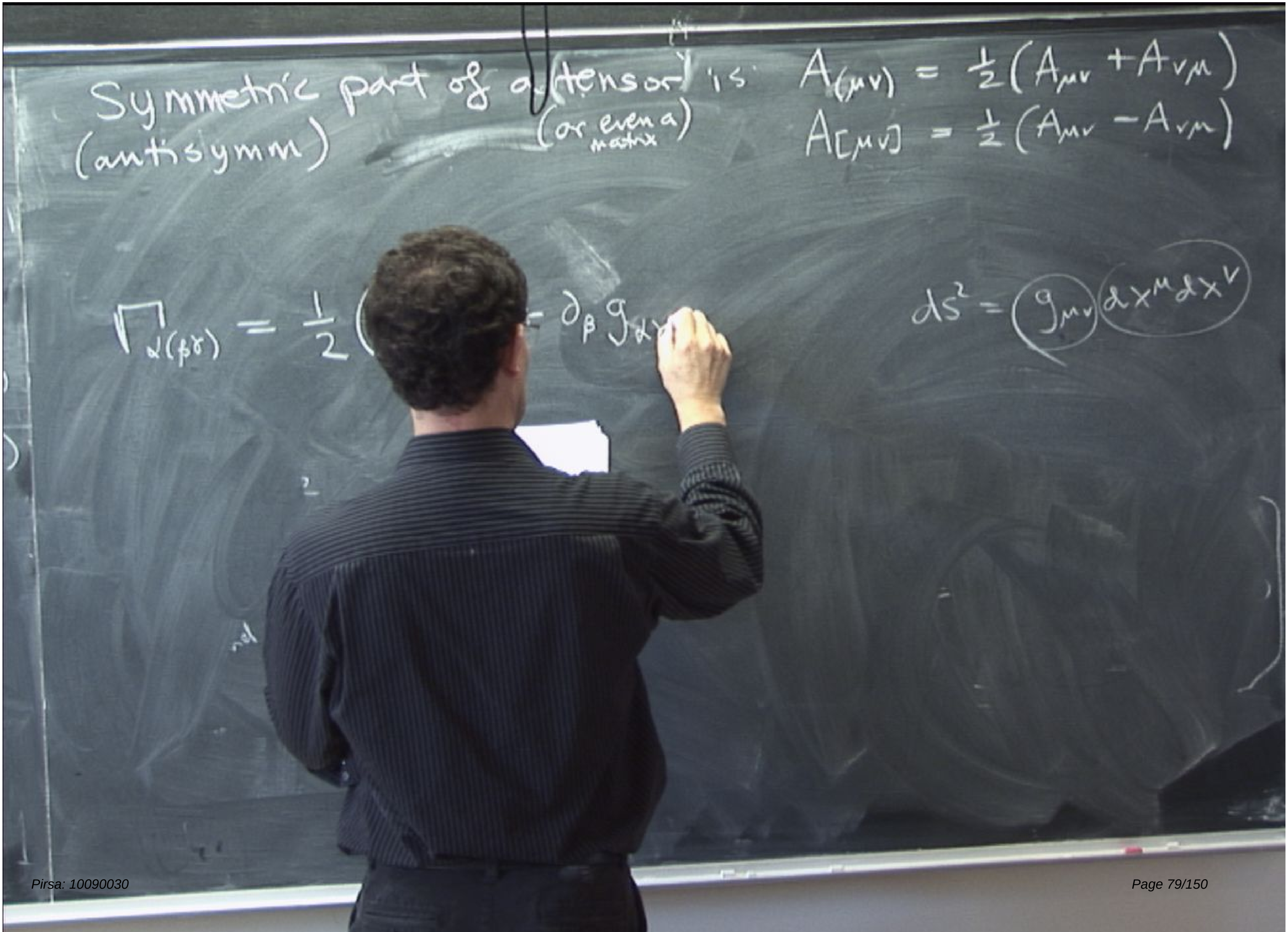
Proof:
∇_α g_βγ = 0
g_μν Γ^ν_αβ = Γ_μ·αβ
∂_α g_βγ − Γ^δ_βα g_δγ − Γ^δ_γα g_βδ = 0 (A)
∂_β g_γα − Γ^δ_γβ g_δα − Γ^δ_αβ g_γδ = 0 (B)
∂_γ g_αβ − Γ^δ_αγ g_δβ − Γ^δ_βγ g_αδ = 0 (C)
∂_α g_βγ − ∂_β g_γα − ∂_γ g_αβ
− (Γ_γ·βα − Γ_γ·αβ) − (Γ_β·γα − Γ_β·αγ)
+ (Γ_α·γβ + Γ_α·βγ) = 0

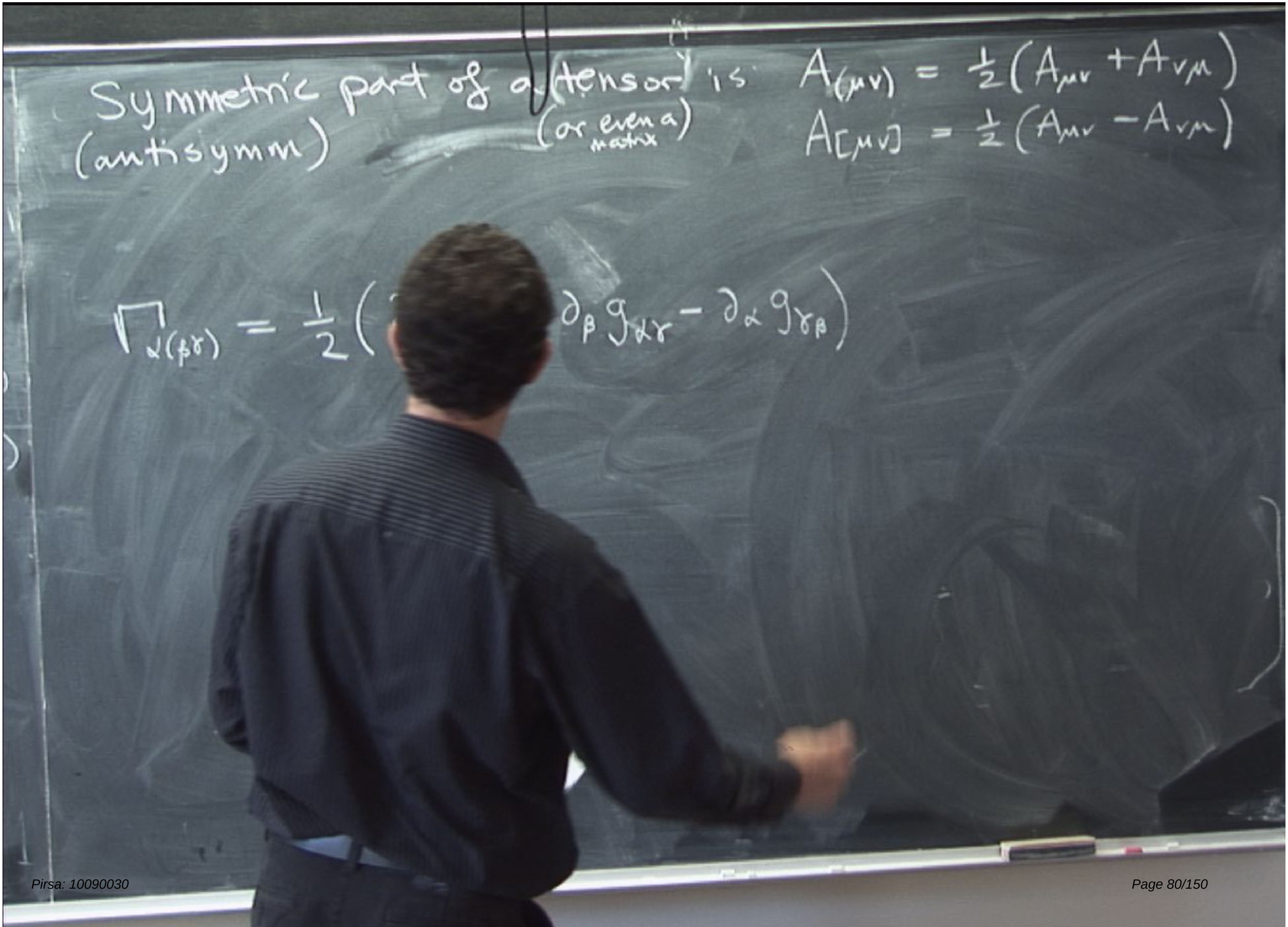
Symmetric part of a tensor (or even a matrix) is: $A_{(\mu\nu)} = \frac{1}{2}(A_{\mu\nu} + A_{\nu\mu})$

(antisymm) $A_{[\mu\nu]} = \frac{1}{2}(A_{\mu\nu} - A_{\nu\mu})$

$\Gamma_{\alpha(\beta\gamma)} = \frac{1}{2}$

$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$


Symmetric part of a tensor is:
(antisymm)
(or even a matrix)
$A_{(\mu\nu)} = \frac{1}{2}(A_{\mu\nu} + A_{\nu\mu})$
$A_{[\mu\nu]} = \frac{1}{2}(A_{\mu\nu} - A_{\nu\mu})$
$\Gamma_{\alpha(\beta\gamma)} = \frac{1}{2}($
$\partial_\beta g_{\alpha\gamma}$
$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$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$\Gamma_{\alpha(\beta\gamma)} = \frac{1}{2}(\ldots \partial_\beta g_{\alpha\gamma} - \partial_\alpha g_{\gamma\beta})$

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$$\Gamma_{\alpha(\beta\gamma)} = \frac{1}{2}\left(\partial_\gamma g_{\alpha\beta} + \partial_\beta g_{\alpha\gamma} - \partial_\alpha g_{\gamma\beta}\right) + \frac{1}{2} T_{\gamma\cdot\beta\alpha} + \frac{1}{2} T_{\beta\cdot\gamma\alpha}$$

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$$\Gamma_{\alpha(\beta\gamma)} = \frac{1}{2}\left(\partial_\gamma g_{\alpha\beta} + \partial_\beta g_{\alpha\gamma} - \partial_\alpha g_{\gamma\beta}\right) + \frac{1}{2}T_{\gamma\cdot\beta\alpha} + \frac{1}{2}T_{\beta\cdot\gamma\alpha}$$

$$\Gamma_{\alpha\cdot\beta\gamma} = \frac{1}{2}\left(g_{\alpha\beta,\gamma} + g_{\alpha\gamma,\beta} - g_{\gamma\beta,\alpha}\right) + \frac{1}{2}\left(T_{\gamma\cdot\beta\alpha} + T_{\beta\cdot\gamma\alpha} + T_{\alpha\cdot\gamma\beta}\right)$$

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$$\Gamma_{\alpha(\beta\gamma)} = \frac{1}{2}\left(\partial_\gamma g_{\alpha\beta} + \partial_\beta g_{\alpha\gamma} - \partial_\alpha g_{\gamma\beta}\right)$$

$$+ \frac{1}{2} T_{\gamma\cdot\beta\alpha} + \frac{1}{2} T_{\beta\cdot\gamma\alpha}$$

$$\boxed{\Gamma_{\alpha\cdot\beta\gamma} = \frac{1}{2}\left(g_{\alpha\beta,\gamma} + g_{\alpha\gamma,\beta} - g_{\gamma\beta,\alpha}\right) + \frac{1}{2}\left(T_{\gamma\cdot\beta\alpha} + T_{\beta\cdot\gamma\alpha} + T_{\alpha\cdot\gamma\beta}\right)}$$

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$$\Gamma_{\alpha(\beta\gamma)} = \frac{1}{2}\left(\partial_\gamma g_{\alpha\beta} + \partial_\beta g_{\alpha\gamma} - \partial_\alpha g_{\gamma\beta}\right) + \frac{1}{2} T_{\gamma\cdot\beta\alpha} + \frac{1}{2} T_{\beta\cdot\gamma\alpha}$$

contorsion

$$\Gamma_{\alpha\cdot\beta\gamma} = \frac{1}{2}\left(g_{\alpha\beta,\gamma} + g_{\alpha\gamma,\beta} - g_{\gamma\beta,\alpha}\right) + \frac{1}{2}\left(T_{\gamma\cdot\beta\alpha} + T_{\beta\cdot\gamma\alpha} + T_{\alpha\cdot\gamma\beta}\right)$$

If $T^{\mu}_{\alpha\beta} = 0$

then

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\xi}\left(g_{\xi\beta,\gamma} + g_{\xi\gamma,\beta} - g_{\alpha\beta,\xi}\right)$$

$$\equiv \left\{ {\alpha \atop \beta\gamma} \right\} \quad \text{"Christoffel symbol".}$$

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$$\Gamma_{\alpha(\gamma\beta)} = \frac{1}{2}\left(\partial_\gamma g_{\alpha\beta} + \partial_\beta g_{\alpha\gamma} - \partial_\alpha g_{\gamma\beta}\right)$$

$$+ \frac{1}{2} T_{\gamma\cdot\beta\alpha} + \frac{1}{2} T_{\beta\cdot\gamma\alpha}$$

contorsion

$$\boxed{\Gamma_{\alpha\cdot\gamma\beta} = \frac{1}{2}\left(g_{\alpha\beta,\gamma} + g_{\alpha\gamma,\beta} - g_{\gamma\beta,\alpha}\right) + \frac{1}{2}\left(T_{\gamma\cdot\beta\alpha} + T_{\beta\cdot\gamma\alpha} + T_{\alpha\cdot\gamma\beta}\right)}$$

If $T^{\mu}_{\alpha\beta} = 0$

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$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\xi}\left(g_{\xi\beta,\gamma} + g_{\xi\gamma,\beta} - g_{\alpha\beta,\xi}\right)$$

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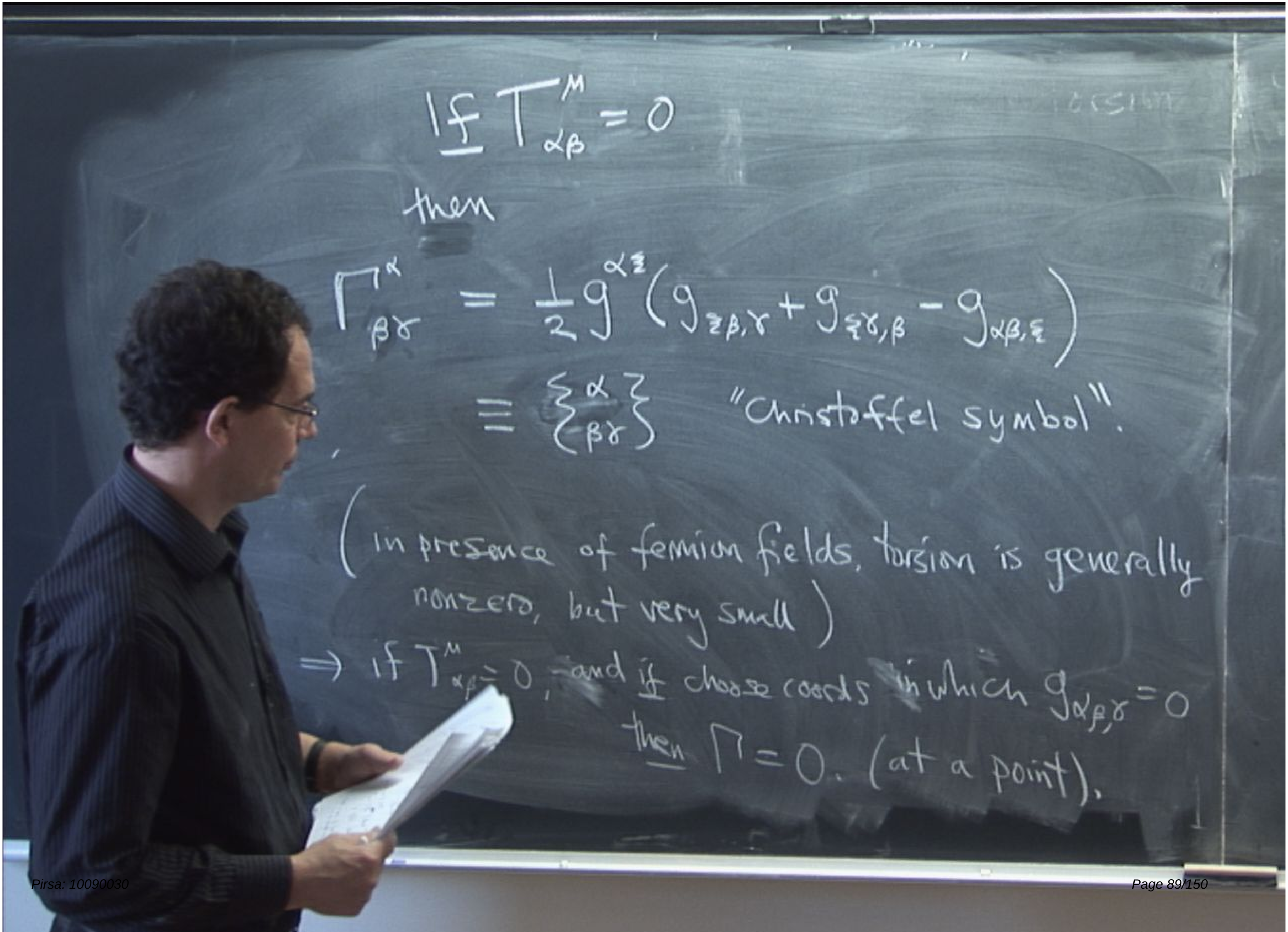
(in presence of fermion fields, torsion is generally nonzero, but very small)


If $T^{\mu}_{\alpha\beta} = 0$
then
$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\xi}(g_{\xi\beta,\gamma} + g_{\xi\gamma,\beta} - g_{\alpha\beta,\xi})$
$\equiv \left\{ {\alpha \atop \beta\gamma} \right\}$ "Christoffel symbol".
(in presence of fermion fields, torsion is generally nonzero, but very small)
$\Rightarrow$ if $T^{\mu}_{\alpha\beta} = 0$, and if choose coords in which $g_{\alpha\beta,\gamma} = 0$
then $\Gamma = 0$. (at a point).

If $T^{\mu}_{\alpha\beta} = 0$

then

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\xi}\left(g_{\xi\beta,\gamma} + g_{\xi\gamma,\beta} - g_{\alpha\beta,\xi}\right)$$

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$T_{\mu\nu} =$

(in presence of fermion fields, torsion is generally nonzero, but very small)

$\Rightarrow$ if $T^{\mu}_{\alpha\beta} = 0$, and if choose coords in which $g_{\alpha\beta,\gamma} = 0$

then $\Gamma = 0$. (at a point).

$\underline{\text{If}}\ T^{\mu}_{\alpha\beta} = 0$

then

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\xi}\left(g_{\xi\beta,\gamma} + g_{\xi\gamma,\beta} - g_{\alpha\beta,\xi}\right)$$

$$\equiv \left\{ {\alpha \atop \beta\gamma} \right\} \quad \text{"Christoffel symbol".}$$

$T_{\alpha\mu\nu} = \epsilon_{\alpha\mu\nu\lambda} J^{\lambda}_5$

(in presence of fermion fields, torsion is generally nonzero, but very small)

$\Rightarrow$ if $T^{\mu}_{\alpha\beta} = 0$, and $\underline{\text{if}}$ choose coords in which $g_{\alpha\beta,\gamma} = 0$

$\underline{\text{then}}$ $\Gamma = 0$. (at a point).

$\underline{\text{If}}\ T^{\mu}_{\alpha\beta} = 0$

then

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\xi}\left(g_{\xi\beta,\gamma} + g_{\xi\gamma,\beta} - g_{\alpha\beta,\xi}\right)$$

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$$T_{\alpha\mu\nu} = \epsilon_{\alpha\mu\nu\lambda} J^{\lambda}_{5}$$

(in presence of fermion fields, torsion is generally nonzero, but very small)

$\Rightarrow$ if $T^{\mu}_{\alpha\beta} = 0$, and $\underline{\text{if}}$ choose coords in which $g_{\alpha\beta,\gamma} = 0$

$\underline{\text{then}}$ $\Gamma = 0$. (at a point).

$\underline{\text{If}}\ T^{\mu}_{\alpha\beta} = 0$ (everywhere)

then

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\xi}\left(g_{\xi\beta,\gamma} + g_{\xi\gamma,\beta} - g_{\alpha\beta,\xi}\right)$$

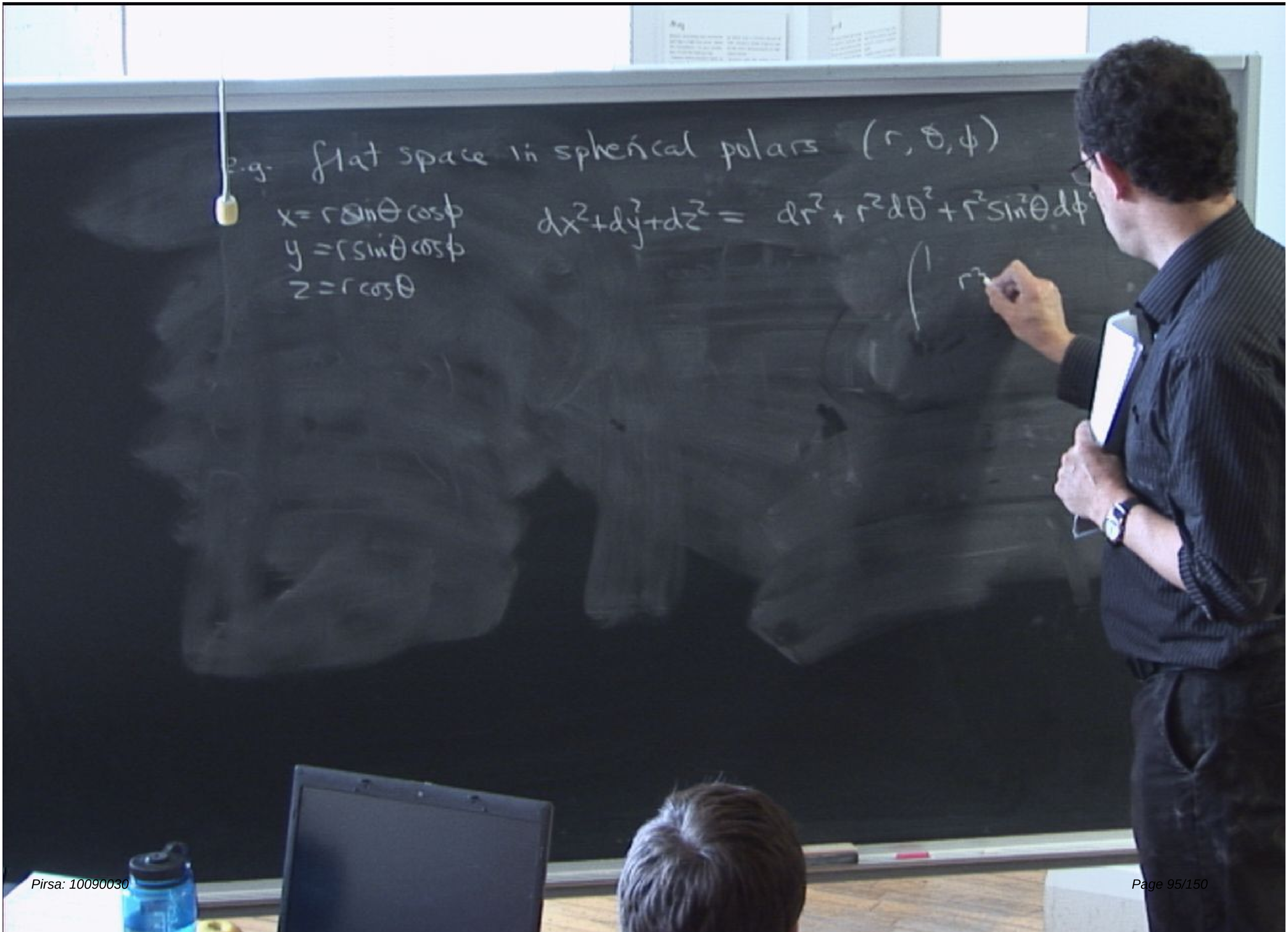
$$\equiv \left\{ {\alpha \atop \beta\gamma} \right\} \quad \text{"Christoffel symbol".}$$

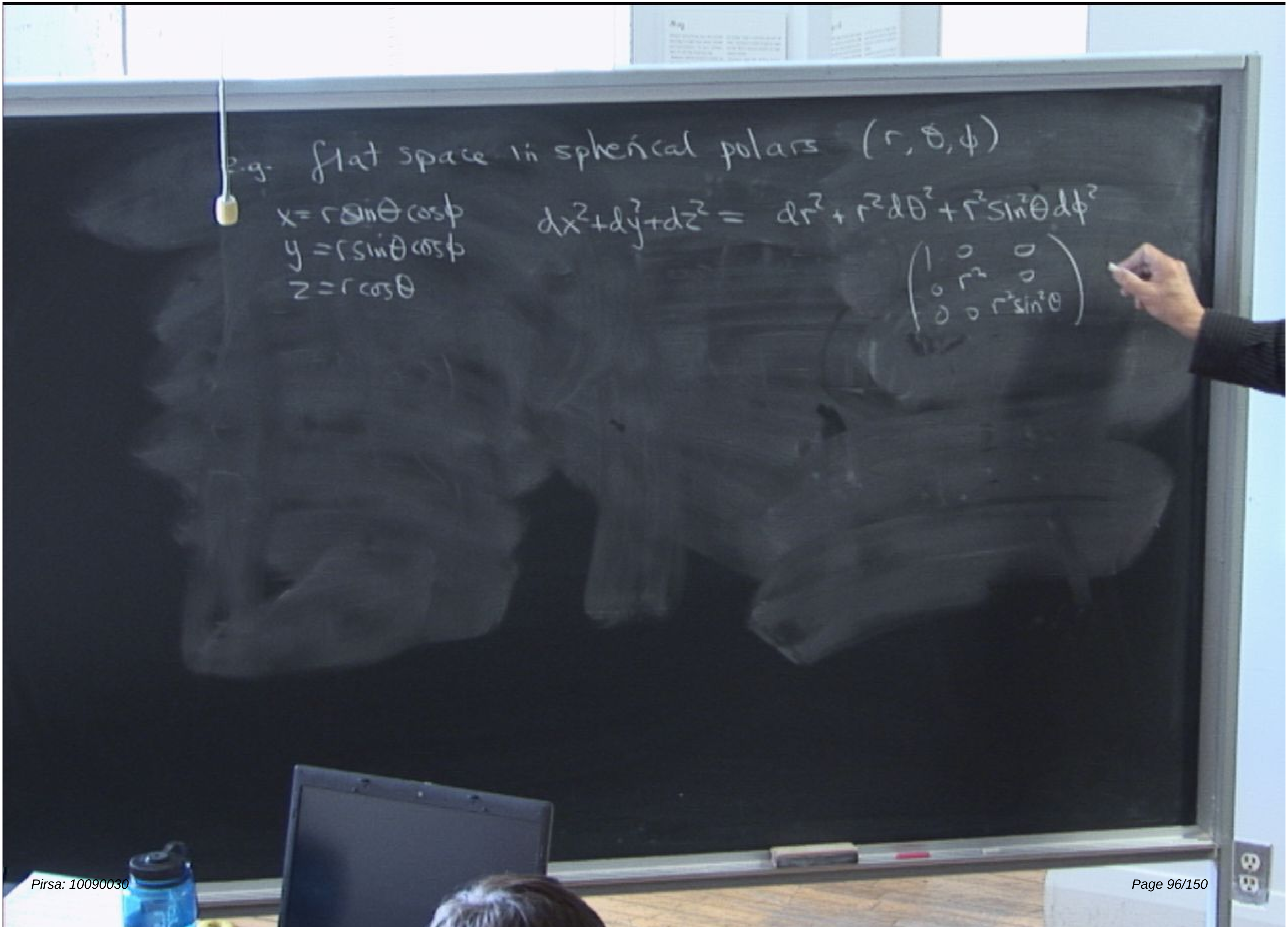
$T_{\alpha\mu\nu} = \epsilon_{\alpha\mu\nu\lambda} J_5^{\lambda}$

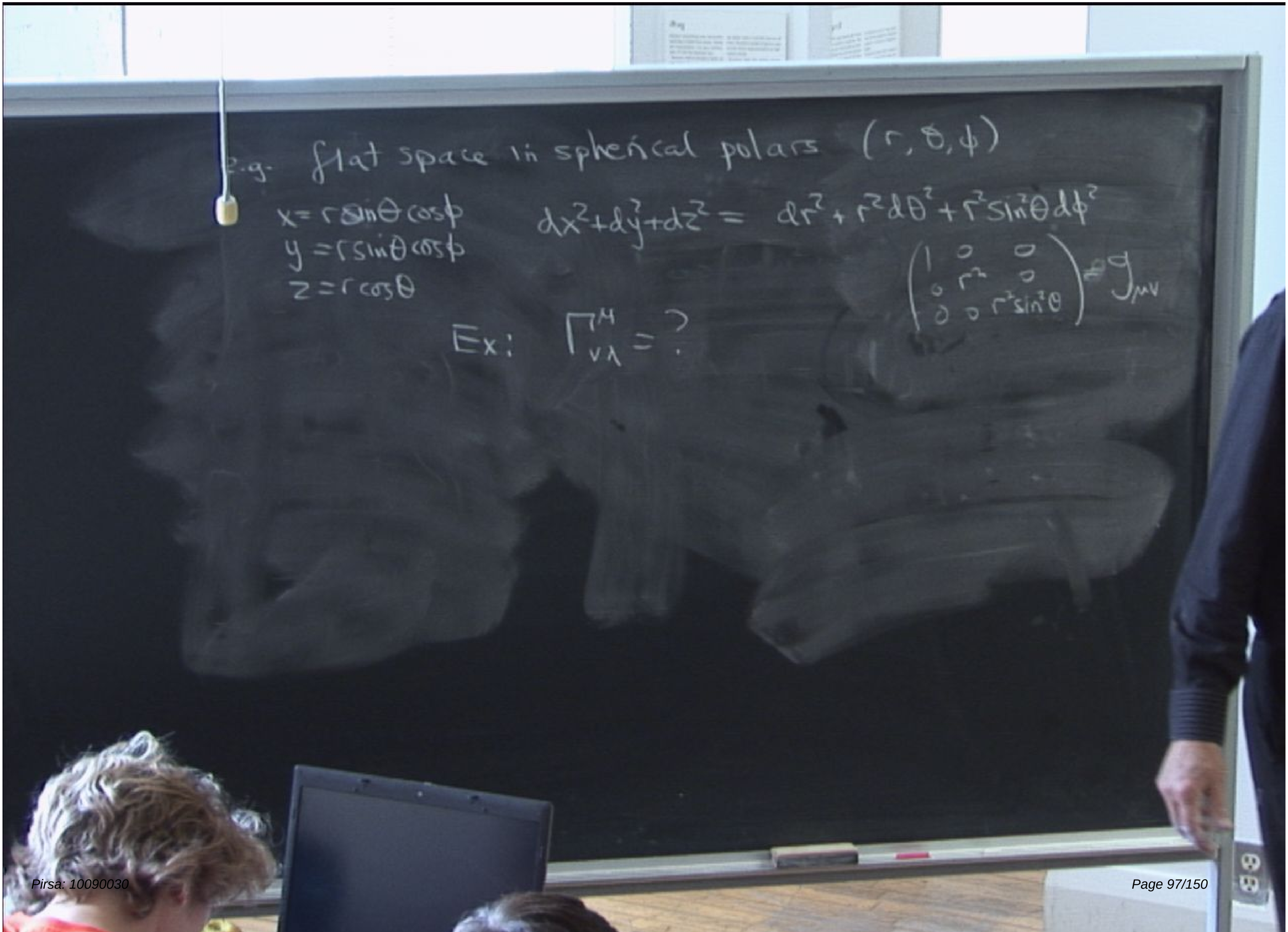
(in presence of fermion fields, torsion is generally nonzero, but very small)

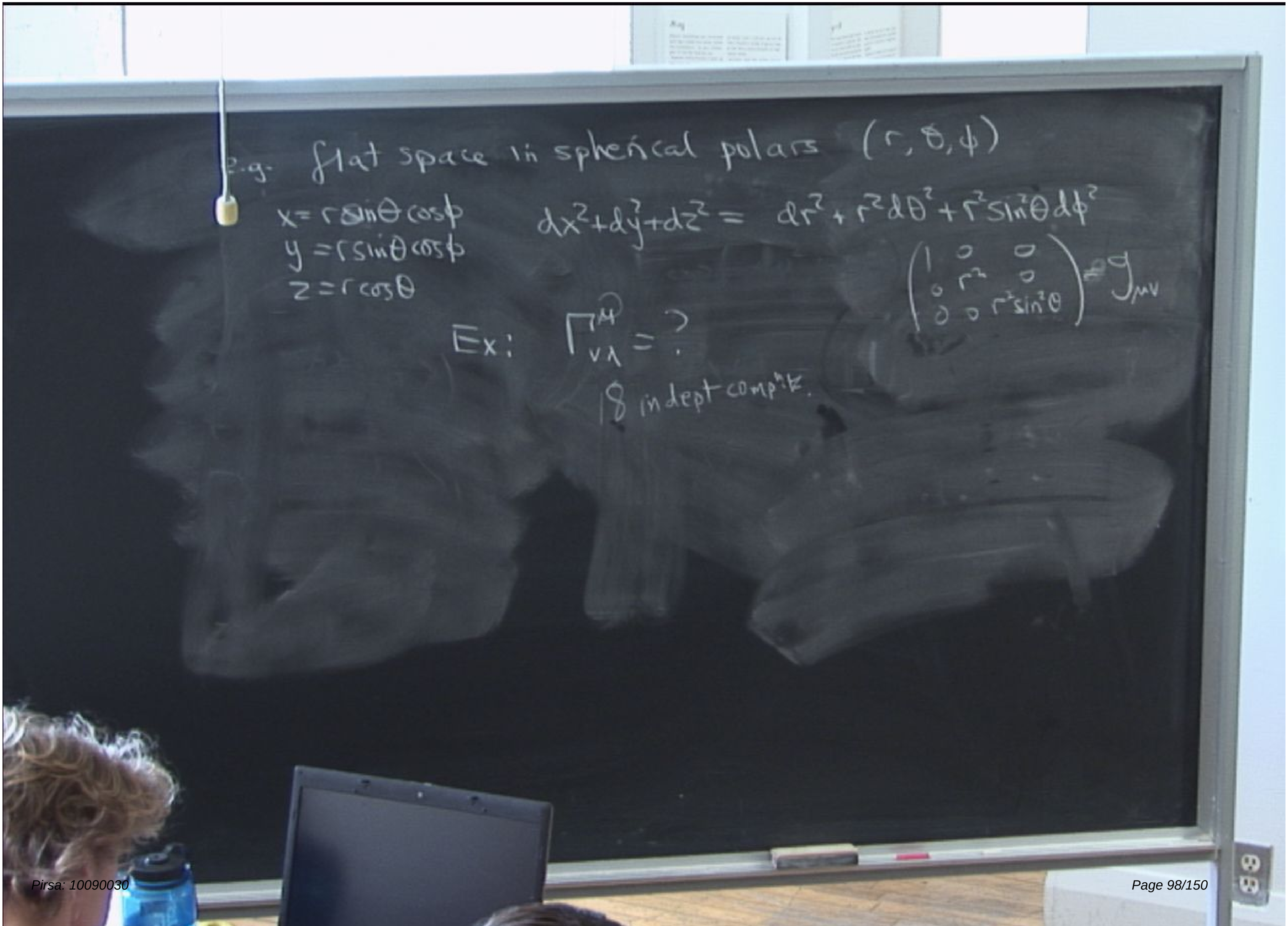
$\Rightarrow$ if $T^{\mu}_{\alpha\beta} = 0$, and $\underline{\text{if}}$ choose coords in which $g_{\alpha\beta,\gamma} = 0$

$\underline{\text{then}}$ $\Gamma = 0$. (at a point).


e.g. flat space in spherical polars (r, θ, ϕ)
$x = r\sin\theta\cos\phi$
$y = r\sin\theta\cos\phi$
$z = r\cos\theta$
$dx^2 + dy^2 + dz^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2$


e.g. flat space in spherical polars (r, θ, φ)
x = r sin θ cos φ
y = r sin θ cos φ
z = r cos θ
dx² + dy² + dz² = dr² + r²dθ² + r² sin²θ dφ²
1 0 0
0 r² 0
0 0 r² sin²θ


e.g. flat space in spherical polars (r, θ, φ)
x = r sinθ cosφ
y = r sinθ cosφ
z = r cosθ
dx² + dy² + dz² = dr² + r²dθ² + r² sin²θ dφ²
(1 0 0; 0 r² 0; 0 0 r² sin²θ) = g_μν
Ex: Γ^μ_νλ = ?


e.g. flat space in spherical polars (r, θ, ϕ)
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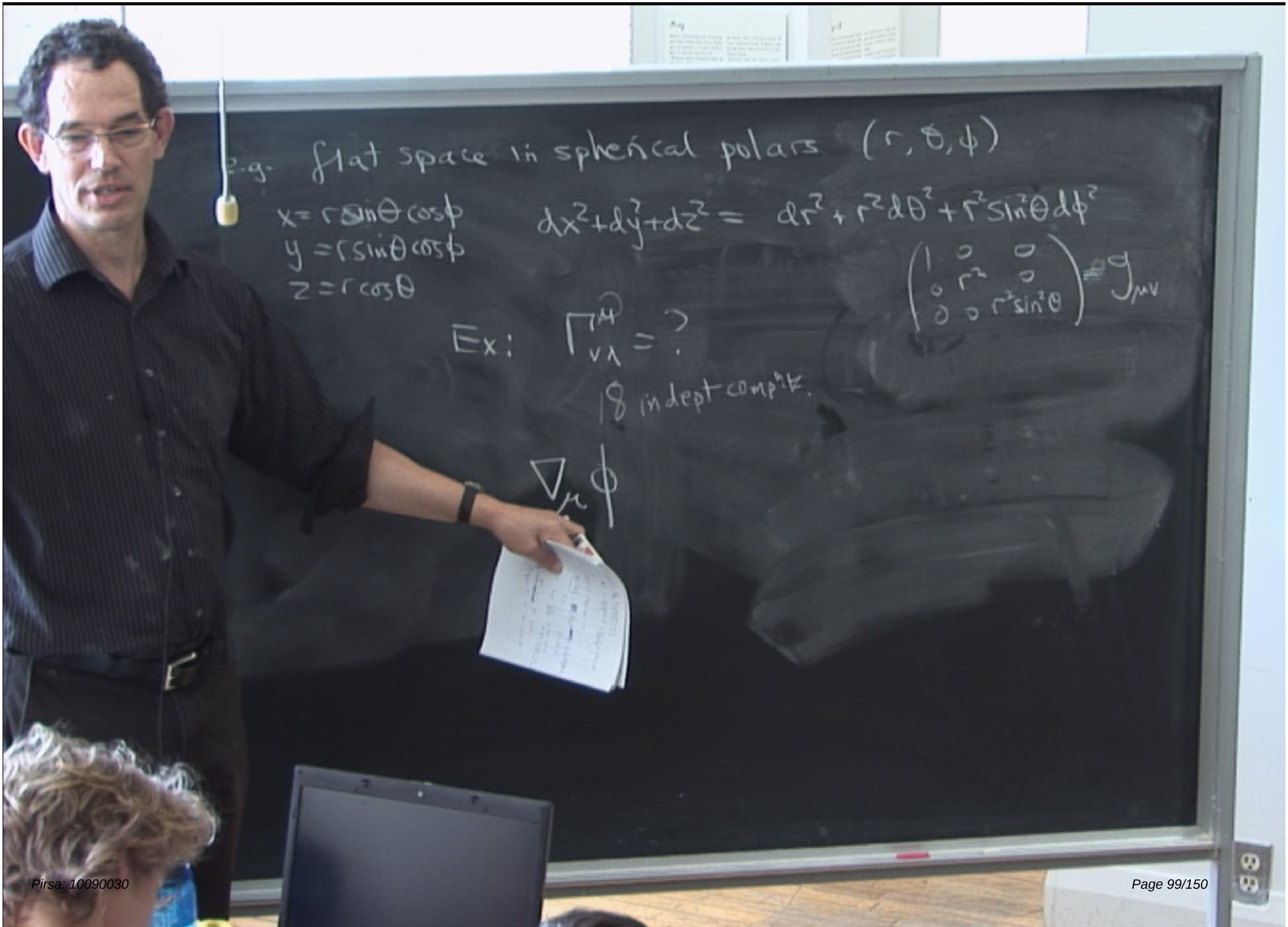
$y = r\sin\theta\cos\phi$
$z = r\cos\theta$
$dx^2 + dy^2 + dz^2 = dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\, d\phi^2$
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2\sin^2\theta \end{pmatrix} = g_{\mu\nu}$
Ex: $\Gamma^{\mu}_{\nu\lambda} = ?$
18 indept compnts.


e.g. flat space in spherical polars (r, θ, ϕ)
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$\begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2\sin^2\theta \end{pmatrix} = g_{\mu\nu}$
Ex: $\Gamma^{\mu}_{\nu\lambda} = ?$
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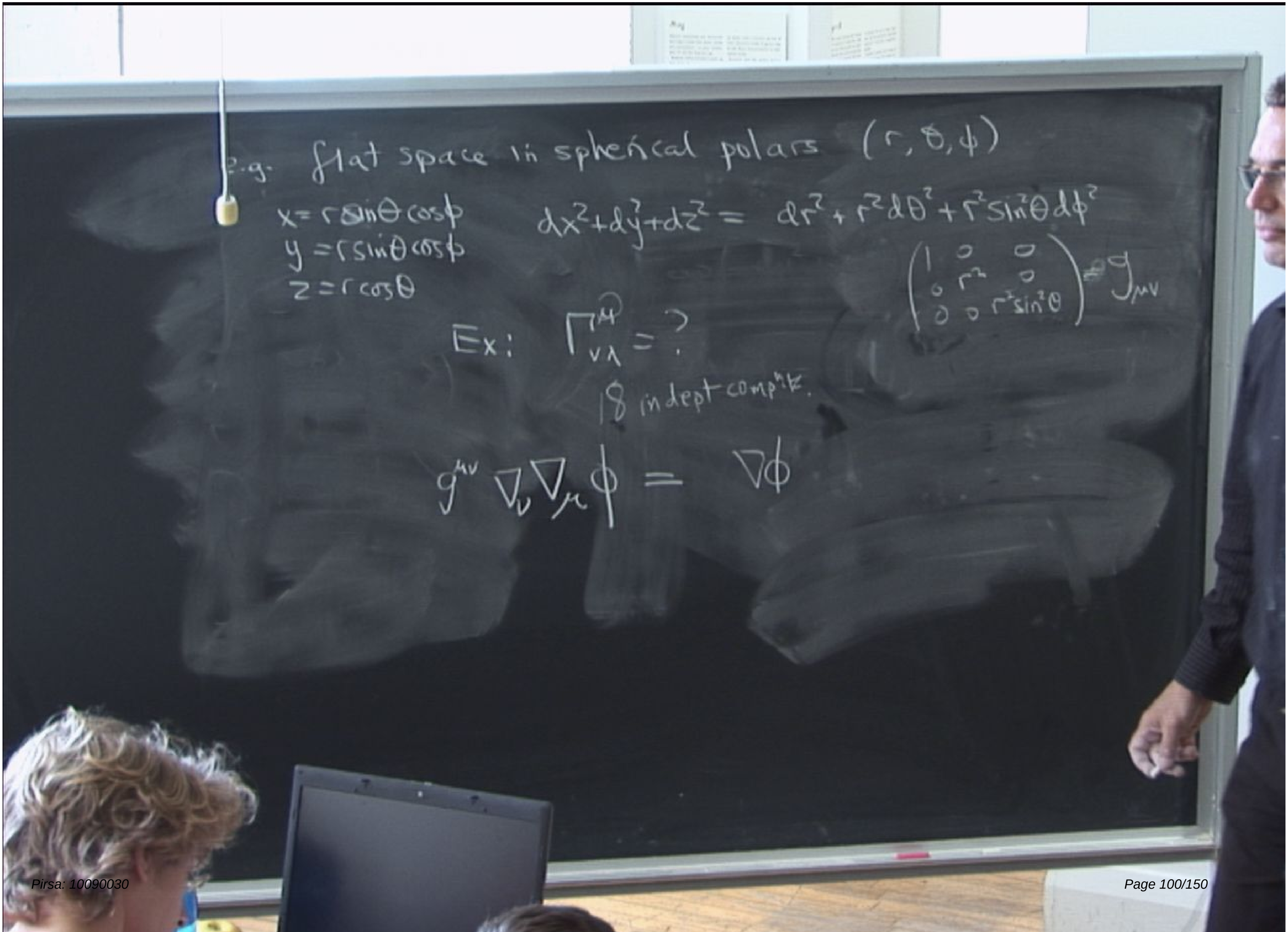
$\nabla_\mu \phi$


e.g. flat space in spherical polars (r, θ, φ)
x = r sinθ cosφ
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z = r cosθ
dx² + dy² + dz² = dr² + r² dθ² + r² sin²θ dφ²
(1 0 0 / 0 r² 0 / 0 0 r² sin²θ) = g_μν
Ex: Γ^μ_νλ = ?
18 indept compnts.
g^μν ∇_ν ∇_μ φ = ∇φ

e.g. flat space in spherical polars (r, θ, ϕ)

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\cos\phi$$
$$z = r\cos\theta$$

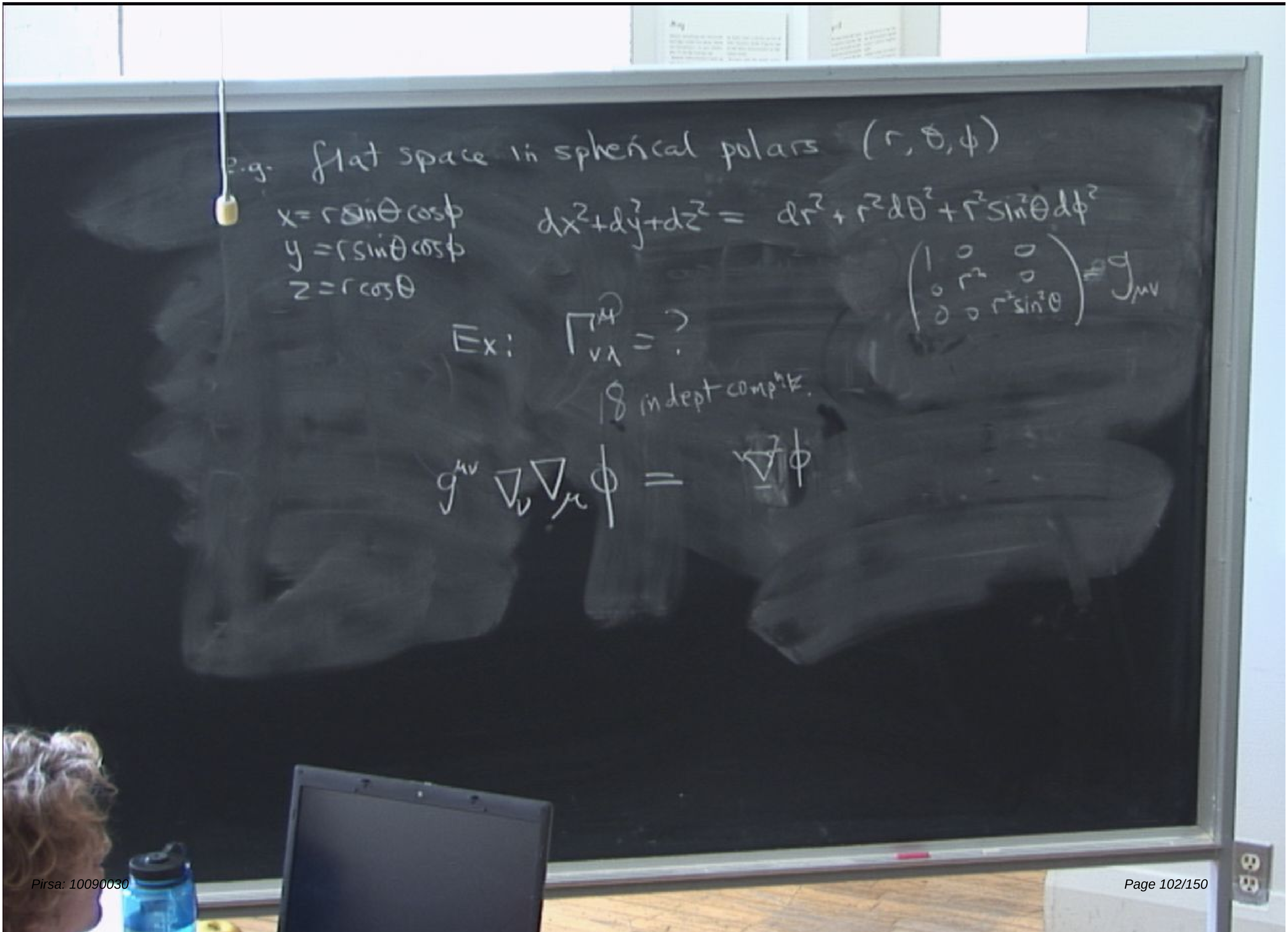
$$dx^2 + dy^2 + dz^2 = dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\, d\phi^2$$

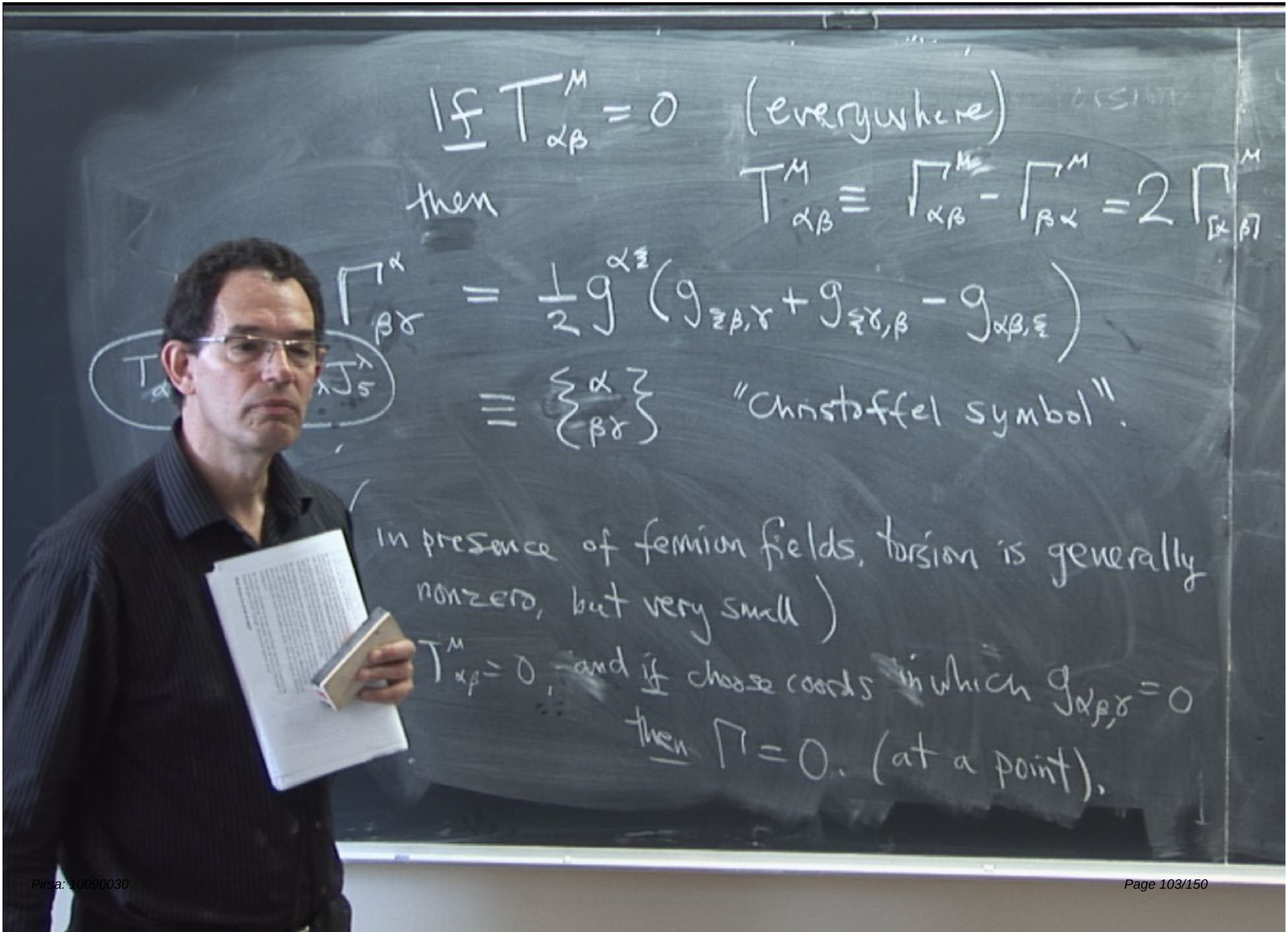
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2\sin^2\theta \end{pmatrix} = g_{\mu\nu}$$

Ex: $\Gamma^{\mu}_{\nu\lambda} = ?$

18 indept compnts.

$$g^{\mu\nu} \nabla_\nu \nabla_\mu \phi = \nabla^2 \phi$$


e.g. flat space in spherical polars (r, θ, φ)
x = r sinθ cosφ
y = r sinθ cosφ
z = r cosθ
dx² + dy² + dz² = dr² + r²dθ² + r²sin²θ dφ²
(1 0 0 / 0 r² 0 / 0 0 r²sin²θ) = g_μν
Ex: Γ^μ_νλ = ?
18 indep comp'ts.
g^μν ∇_ν ∇_μ φ = ∇²φ


If $T^{\mu}_{\alpha\beta} = 0$ (everywhere)
then $T^{\mu}_{\alpha\beta} \equiv \Gamma^{\mu}_{\alpha\beta} - \Gamma^{\mu}_{\beta\alpha} = 2\Gamma^{\mu}_{[\alpha\beta]}$
$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\xi}(g_{\xi\beta,\gamma} + g_{\xi\gamma,\beta} - g_{\alpha\beta,\xi})$
$\equiv \left\{ {\alpha \atop \beta\gamma} \right\}$ "Christoffel symbol".
in presence of fermion fields, torsion is generally nonzero, but very small)
$T^{\mu}_{\alpha\beta} = 0$, and if choose coords in which $g_{\alpha\beta,\gamma} = 0$
then $\Gamma = 0$. (at a point).

<u>If</u> $T^{\mu}_{\alpha\beta} = 0$ (everywhere)

then $\boxed{U^{\alpha\beta} V_{\mu} T^{\mu}_{\alpha\beta}} \equiv \Gamma^{\mu}_{\alpha\beta} - \Gamma^{\mu}_{\beta\alpha} = 2\Gamma^{\mu}_{[\alpha\beta]}$

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\xi} \left(g_{\xi\beta,\gamma} + g_{\xi\gamma,\beta} - g_{\alpha\beta,\xi} \right)$$

$\boxed{T_{\alpha\mu\nu} = \epsilon_{\alpha\mu\nu\lambda} J^{\lambda}_{5}}$

$$\equiv \left\{ {\alpha \atop \beta\gamma} \right\} \quad \text{"Christoffel symbol".}$$

(in presence of fermion fields, torsion is generally nonzero, but very small)

$\Rightarrow$ if $T^{\mu}_{\alpha\beta} = 0$, and <u>if</u> choose coords in which $g_{\alpha\beta,\gamma} = 0$

<u>then</u> $\Gamma = 0$. (at a point).

If $T^{\mu}_{\alpha\beta} = 0$ (everywhere)

then $T^{\mu}_{\alpha\beta} \equiv \Gamma^{\mu}_{\alpha\beta} - \Gamma^{\mu}_{\beta\alpha} = 2\Gamma^{\mu}_{[\alpha\beta]}$

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\xi}\left(g_{\xi\beta,\gamma} + g_{\xi\gamma,\beta} - g_{\beta\gamma,\xi}\right)$$

$$T_{\alpha\mu\nu} = \epsilon_{\alpha\mu\nu\lambda} J^{\lambda}_{5}$$

$$\equiv \left\{ {\alpha \atop \beta\gamma} \right\}$$ "Christoffel symbol".

(in presence of fermion fields, torsion is generally nonzero, but very small)

$\Rightarrow$ if $T^{\mu}_{\alpha\beta} = 0$, and if choose coords in which $g_{\alpha\beta,\gamma} = 0$

then $\Gamma = 0$. (at a point).

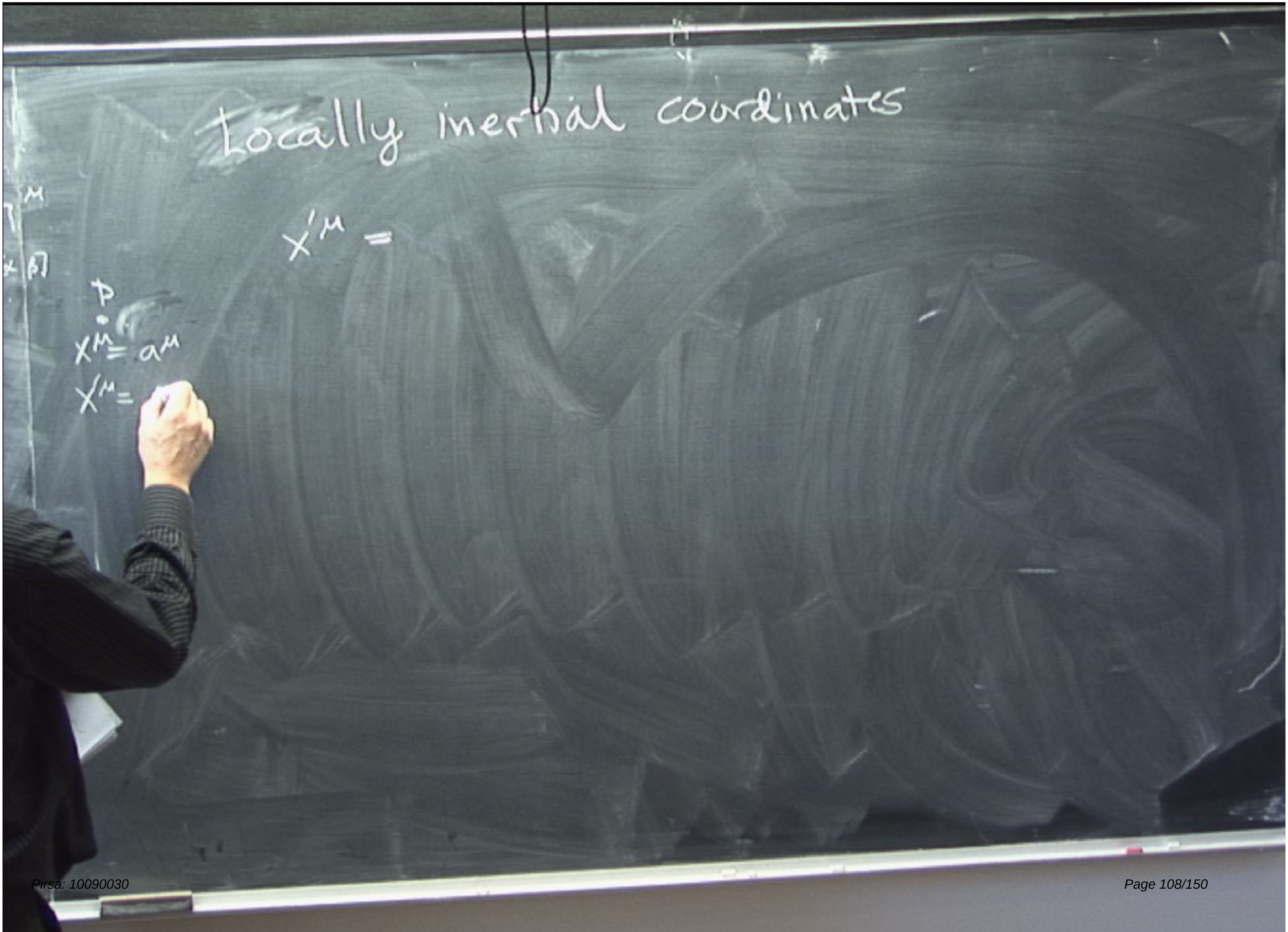

Locally inertial coordinates

Locally inertial coordinates

$x'^{\mu} =$

P

$x^{\mu} = a^{\mu}$


Locally inertial coordinates
x'^μ =
p
x^μ = a^μ
x'^μ =

Locally inertial coordinates

P

$x^\mu = a^\mu$

$x'^\mu = a'^\mu$

$x'^\mu =$

Locally inertial coordinates

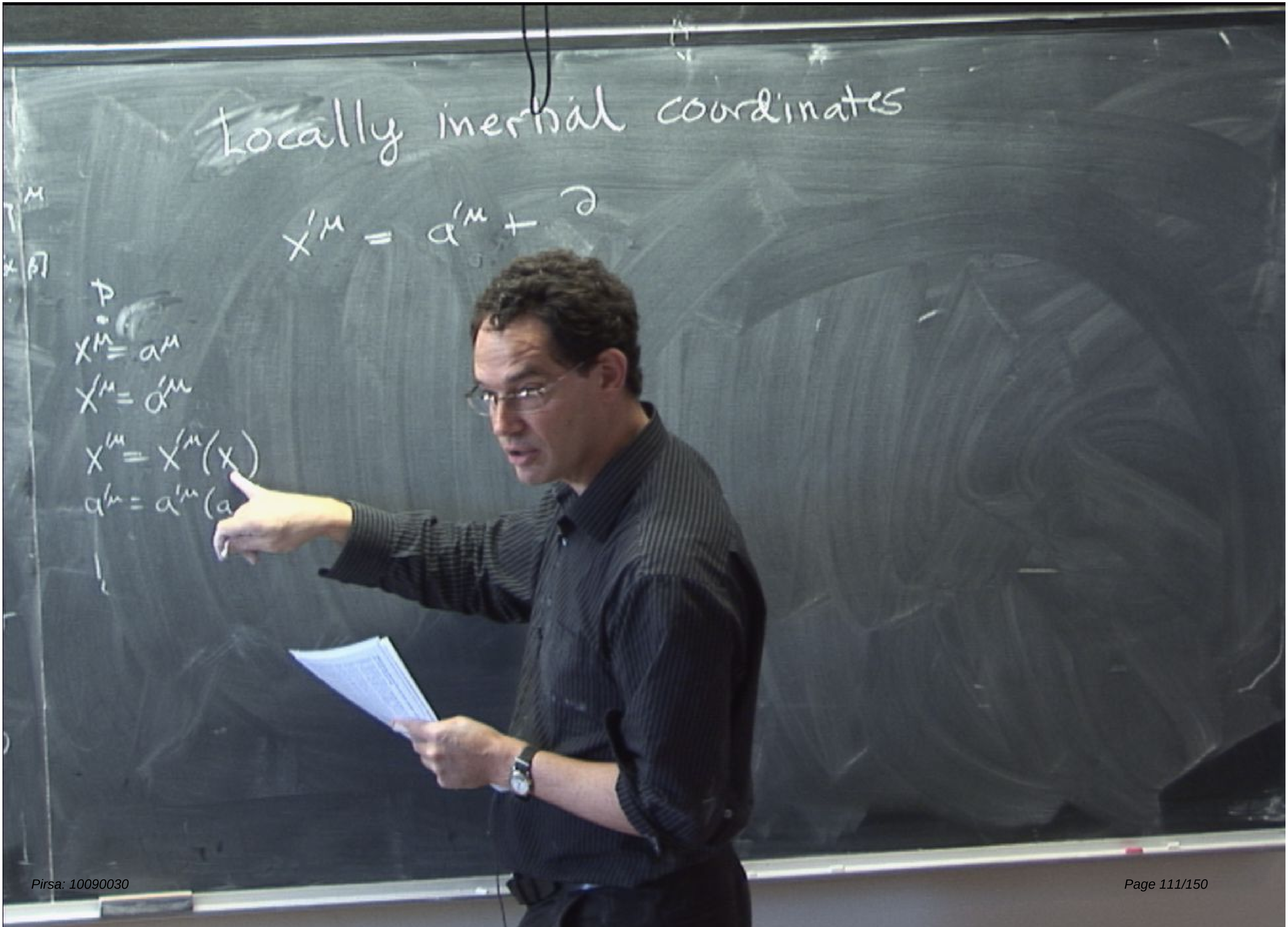
$$x'^{\mu} = a'^{\mu}$$

P

$x^{\mu} = a^{\mu}$

$x'^{\mu} = a'^{\mu}$

$x'^{\mu} = x'^{\mu}(x)$


Locally inertial coordinates
x'μ = a'μ + ∂
P
xμ = aμ
x'μ = a'μ
x'μ = x'μ(x)
a'μ = a'μ(a

Locally inertial coordinates

Taylor

$$x'^{\mu} = a'^{\mu} + \left.\frac{\partial x'^{\mu}}{\partial x^{\nu}}\right|_{p} (x-a)^{\nu} + \frac{\partial^2 x'^{\mu}}{\partial x^{\nu}\partial x^{\alpha}} (x-a)^{\nu}(x-a)^{\alpha} + \cdots$$

p

$x^{\mu} = a^{\mu}$

$x'^{\mu} = a'^{\mu}$

$x'^{\mu} = x'^{\mu}(x)$

$a'^{\mu} = a'^{\mu}(a)$

Locally inertial coordinates

Taylor

$$x'^{\mu} = a'^{\mu} + \left.\frac{\partial x'^{\mu}}{\partial x^{\nu}}\right|_{p} (x-a)^{\nu} + \frac{1}{2}\frac{\partial^2 x'^{\mu}}{\partial x^{\nu}\partial x^{\alpha}} (x-a)^{\nu}(x-a)^{\alpha}$$

$+ \ldots$

p

$x^{\mu} = a^{\mu}$

$x'^{\mu} = a'^{\mu}$

$x'^{\mu} = x'^{\mu}(x)$

$a'^{\mu} = a'^{\mu}(a)$

$\Longrightarrow g'$

Locally inertial coordinates

Taylor

$$x'^{\mu} = a'^{\mu} + \left.\frac{\partial x'^{\mu}}{\partial x^{\nu}}\right|_{P} (x-a)^{\nu} + \frac{1}{2}\frac{\partial^2 x'^{\mu}}{\partial x^{\nu}\partial x^{\alpha}} (x-a)^{\nu}(x-a)^{\alpha} + \cdots$$

Simy

$$x^{\mu} = a^{\mu} + \frac{\partial x^{\mu}}{\partial x'^{\nu}} (x'-a')^{\nu} + \cdots$$

P

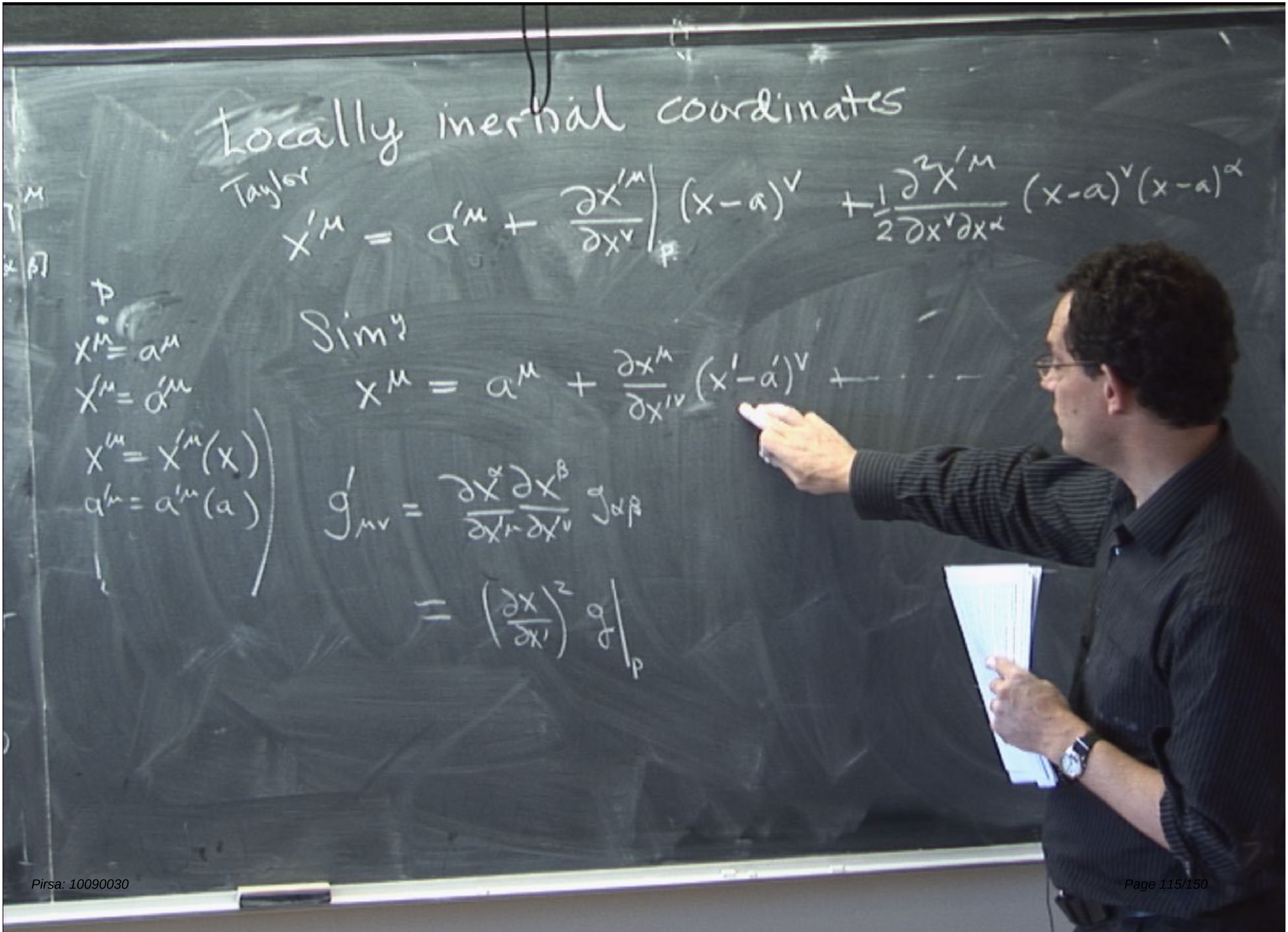
$x^{\mu} = a^{\mu}$

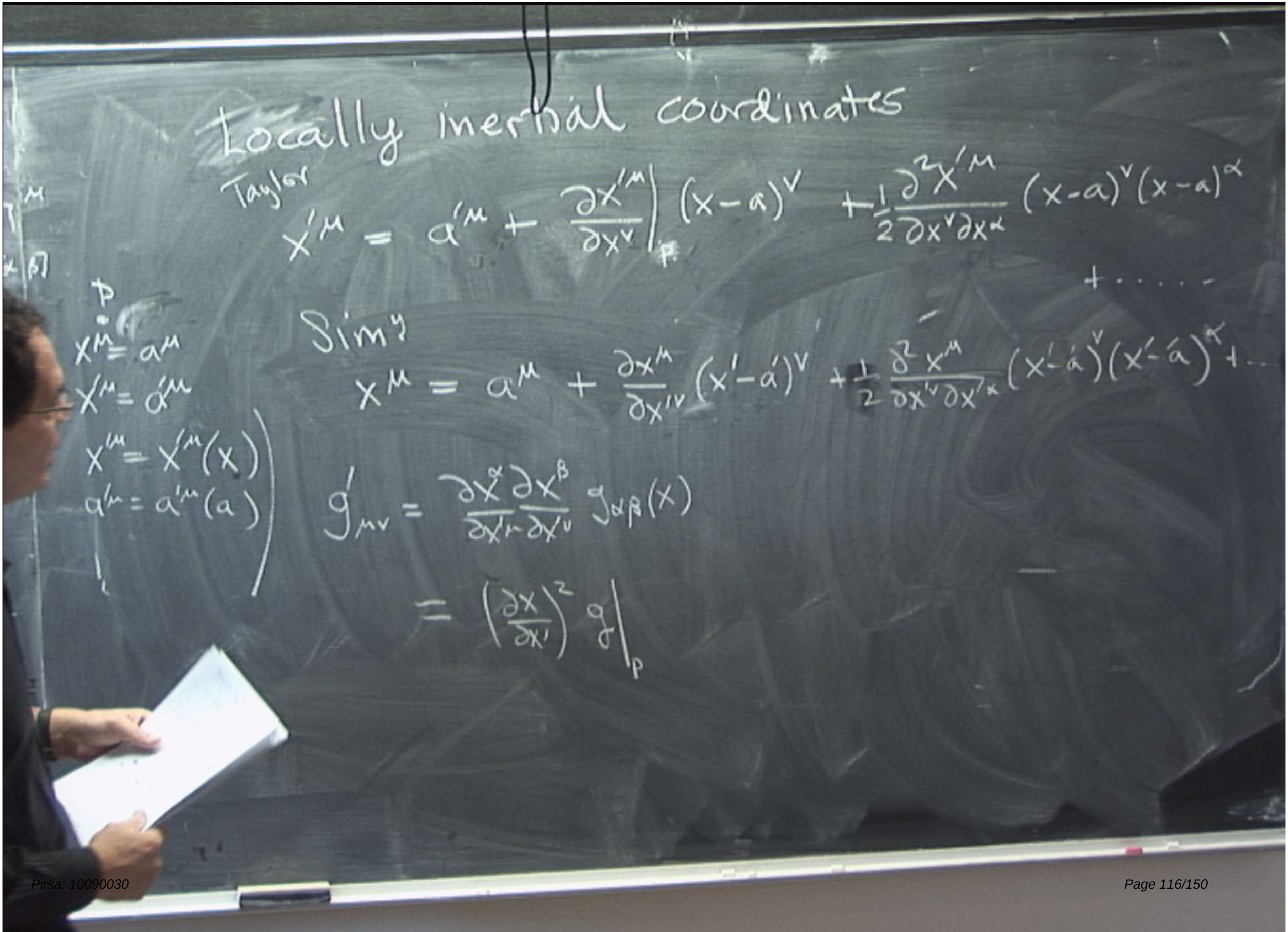
$x'^{\mu} = a'^{\mu}$

$x'^{\mu} = x'^{\mu}(x)$

$a'^{\mu} = a'^{\mu}(a)$

$$g'_{\mu\nu} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}}\frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}$$


Locally inertial coordinates
Taylor
$x'^{\mu} = a'^{\mu} + \frac{\partial x'^{\mu}}{\partial x^{\nu}}\Big|_{p} (x-a)^{\nu} + \frac{1}{2}\frac{\partial^2 x'^{\mu}}{\partial x^{\nu}\partial x^{\alpha}} (x-a)^{\nu}(x-a)^{\alpha}$
Sim.
$x^{\mu} = a^{\mu} + \frac{\partial x^{\mu}}{\partial x'^{\nu}} (x'-a')^{\nu} + \cdots$
p
$x^{\mu} = a^{\mu}$
$x'^{\mu} = a'^{\mu}$
$x'^{\mu} = x'^{\mu}(x)$
$a'^{\mu} = a'^{\mu}(a)$
$g'_{\mu\nu} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}}\frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}$
$= \left(\frac{\partial x}{\partial x'}\right)^2 g\Big|_{p}$


Locally inertial coordinates
Taylor
$x'^{\mu} = a'^{\mu} + \frac{\partial x'^{\mu}}{\partial x^{\nu}}\Big|_{p} (x-a)^{\nu} + \frac{1}{2}\frac{\partial^2 x'^{\mu}}{\partial x^{\nu}\partial x^{\alpha}} (x-a)^{\nu}(x-a)^{\alpha} + \cdots$
Simy
$x^{\mu} = a^{\mu} + \frac{\partial x^{\mu}}{\partial x'^{\nu}} (x'-a')^{\nu} + \frac{1}{2}\frac{\partial^2 x^{\mu}}{\partial x'^{\nu}\partial x'^{\alpha}} (x'-a')^{\nu}(x'-a')^{\alpha} + \cdots$
p
$x^{\mu} = a^{\mu}$
$x'^{\mu} = a'^{\mu}$
$x'^{\mu} = x'^{\mu}(x)$
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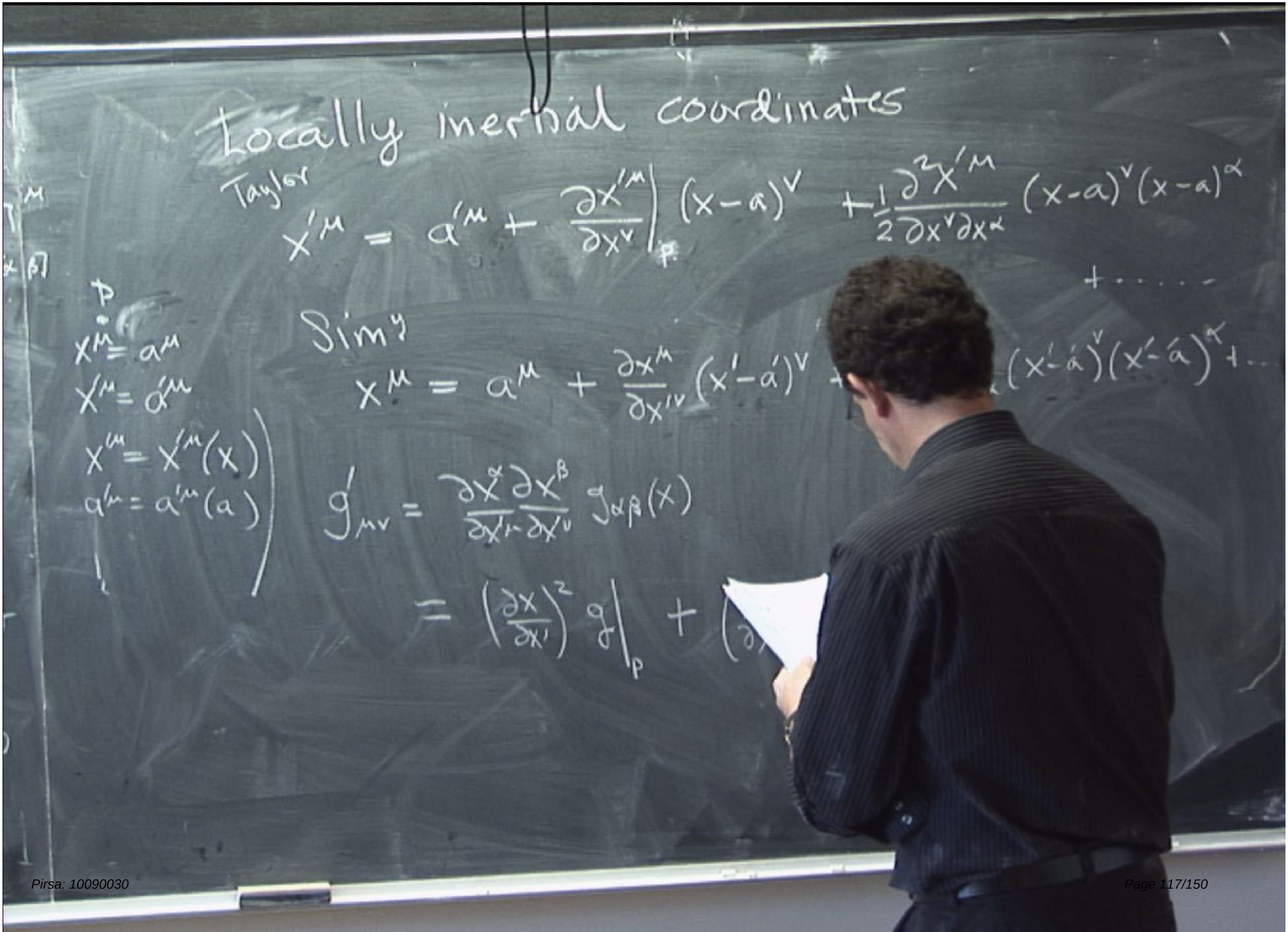
Locally inertial coordinates
Taylor
$x'^\mu = a'^\mu + \frac{\partial x'^\mu}{\partial x^\nu}\Big|_p (x-a)^\nu + \frac{1}{2}\frac{\partial^2 x'^\mu}{\partial x^\nu \partial x^\alpha}(x-a)^\nu (x-a)^\alpha + \dots$
Simy
$x^\mu = a^\mu + \frac{\partial x^\mu}{\partial x'^\nu}(x'-a')^\nu$
$(x'-a')^\nu (x'-a')^\alpha + \dots$
$x^\mu = a^\mu$
$x'^\mu = a'^\mu$
$x'^\mu = x'^\mu(x)$
$a'^\mu = a'^\mu(a)$
$g'_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$
$= \left(\frac{\partial x}{\partial x'}\right)^2 g\Big|_p +$

Locally inertial coordinates

Taylor

$$x'^{\mu} = a'^{\mu} + \frac{\partial x'^{\mu}}{\partial x^{\nu}}\bigg|_p (x-a)^{\nu} + \frac{1}{2}\frac{\partial^2 x'^{\mu}}{\partial x^{\nu}\partial x^{\alpha}}(x-a)^{\nu}(x-a)^{\alpha} + \cdots$$

p

$x^{\mu} = a^{\mu}$

$x'^{\mu} = a'^{\mu}$

$x'^{\mu} = x'^{\mu}(x)$

$a'^{\mu} = a'^{\mu}(a)$

Sim'y

$$x^{\mu} = a^{\mu} + \frac{\partial x^{\mu}}{\partial x'^{\nu}}(x'-a')^{\nu} + \frac{1}{2}\frac{\partial^2 x^{\mu}}{\partial x'^{\nu}\partial x'^{\alpha}}(x'-a')^{\nu}(x'-a')^{\alpha} + \cdots$$

$$g'_{\mu\nu} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}}\frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$$

$$= \left(\frac{\partial x}{\partial x'}\right)^2 g\bigg|_p + \left(\frac{\partial x}{\partial x'}\frac{\partial^2 x}{\partial x'\partial x'} g + \frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'}\partial g\right)\bigg|_p (x-a)$$

$$+ \cdots$$

Locally inertial coordinates

Taylor

$$x'^{\mu} = a'^{\mu} + \frac{\partial x'^{\mu}}{\partial x^{\nu}}\bigg|_p (x-a)^{\nu} + \frac{1}{2}\frac{\partial^2 x'^{\mu}}{\partial x^{\nu}\partial x^{\alpha}}(x-a)^{\nu}(x-a)^{\alpha} + \dots$$

p

$x^{\mu} = a^{\mu}$

$x'^{\mu} = a'^{\mu}$

$x'^{\mu} = x'^{\mu}(x)$

$a'^{\mu} = a'^{\mu}(a)$

Simy

$$x^{\mu} = a^{\mu} + \frac{\partial x^{\mu}}{\partial x'^{\nu}}(x'-a')^{\nu} + \frac{1}{2}\frac{\partial^2 x^{\mu}}{\partial x'^{\nu}\partial x'^{\alpha}}(x'-a')^{\nu}(x'-a')^{\alpha} + \dots$$

$$g'_{\mu\nu} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}}\frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x)$$

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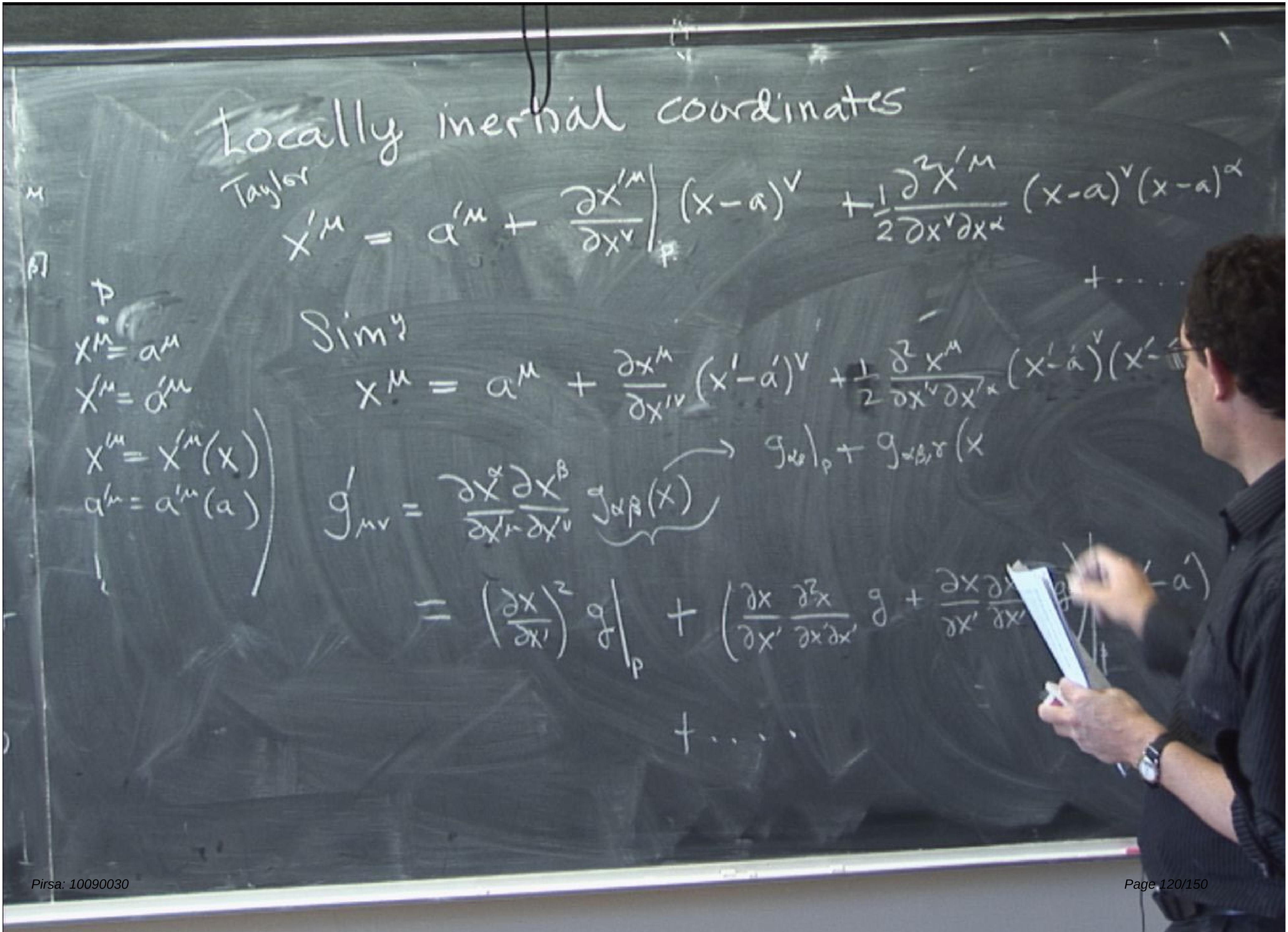
$$+ \dots$$


Locally inertial coordinates
Taylor
x'^μ = a'^μ + ∂x'^μ/∂x^ν |_p (x − a)^ν + 1/2 ∂²x'^μ/∂x^ν∂x^α (x − a)^ν (x − a)^α
+ ....
Simy
x^μ = a^μ + ∂x^μ/∂x'^ν (x' − a')^ν + 1/2 ∂²x^μ/∂x'^ν∂x'^α (x' − a')^ν (x' −
p
x^μ = a^μ
x'^μ = a'^μ
x'^μ = x'^μ(x)
a'^μ = a'^μ(a)
g'_μν = ∂x^α/∂x'^μ ∂x^β/∂x'^ν g_αβ(x)
g_αβ|_p + g_αβ,γ (x
= (∂x/∂x')² g|_p + (∂x/∂x' ∂²x/∂x'∂x' g + ∂x/∂x' ∂x/∂x'
+ ....

Locally inertial coordinates

Taylor

$$x'^{\mu} = a'^{\mu} + \frac{\partial x'^{\mu}}{\partial x^{\nu}}\bigg|_p (x-a)^{\nu} + \frac{1}{2}\frac{\partial^2 x'^{\mu}}{\partial x^{\nu}\partial x^{\alpha}}(x-a)^{\nu}(x-a)^{\alpha}$$

$$+ \dots$$

p

$x^{\mu} = a^{\mu}$

$x'^{\mu} = a'^{\mu}$

$x'^{\mu} = x'^{\mu}(x)$

$a'^{\mu} = a'^{\mu}(a)$

Sim'y

$$x^{\mu} = a^{\mu} + \frac{\partial x^{\mu}}{\partial x'^{\nu}}(x'-a')^{\nu} + \frac{1}{2}\frac{\partial^2 x^{\mu}}{\partial x'^{\nu}\partial x'^{\alpha}}(x'-a')^{\nu}(x'-a')^{\alpha} + \dots$$

$$g'_{\mu\nu} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}}\frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x) \quad \to \quad g_{\alpha\beta}\big|_p + g_{\alpha\beta,\gamma}(x-a)$$

$$= \left(\frac{\partial x}{\partial x'}\right)^2 g\bigg|_p + \left(\frac{\partial x}{\partial x'}\frac{\partial^2 x}{\partial x' \partial x'} g + \frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'}\partial g\right)\bigg|_p (x'-a')$$

$$+ \dots$$

Locally inertial coordinates

Taylor

$$x'^{\mu} = a'^{\mu} + \left.\frac{\partial x'^{\mu}}{\partial x^{\nu}}\right|_{p} (x-a)^{\nu} + \frac{1}{2}\frac{\partial^2 x'^{\mu}}{\partial x^{\nu}\partial x^{\alpha}} (x-a)^{\nu}(x-a)^{\alpha}$$

$$+ \cdots$$

p

$x^{\mu} = a^{\mu}$

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$x'^{\mu} = x'^{\mu}(x)$

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Sim'y

$$x^{\mu} = a^{\mu} + \frac{\partial x^{\mu}}{\partial x'^{\nu}} (x'-a')^{\nu} + \frac{1}{2}\frac{\partial^2 x^{\mu}}{\partial x'^{\nu}\partial x'^{\alpha}} (x'-a')^{\nu}(x'-a')^{\alpha} + \cdots$$

$$g'_{\mu\nu} = \frac{\partial x^{\alpha}\partial x^{\beta}}{\partial x'^{\mu}\partial x'^{\nu}} g_{\alpha\beta}(x)$$

$$g_{\alpha\beta} \Big|_p + g_{\alpha\beta,\gamma}(x'-a')\frac{\partial x'}{\partial x}$$

$$= \left(\frac{\partial x}{\partial x'}\right)^2 g\Big|_p + \left(\frac{\partial x}{\partial x'}\frac{\partial^2 x}{\partial x\partial x'} g + \frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'}\partial g\right)\Big|_p (x'-a')$$

$$+ \cdots$$

Locally inertial coordinates

Taylor

$$x'^{\mu} = a'^{\mu} + \frac{\partial x'^{\mu}}{\partial x^{\nu}}\Big|_p (x-a)^{\nu} + \frac{1}{2}\frac{\partial^2 x'^{\mu}}{\partial x^{\nu}\partial x^{\alpha}} (x-a)^{\nu}(x-a)^{\alpha} + \ldots$$

P

$x^{\mu} = a^{\mu}$

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$$g'_{\mu\nu} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}}\frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x) \longrightarrow g_{\alpha\beta}\big|_p + g_{\alpha\beta,\gamma}(x'-a')\frac{\partial x}{\partial x'}$$

$$= \left(\frac{\partial x}{\partial x'}\right)^2 g\Big|_p + \left(\frac{\partial x}{\partial x'}\frac{\partial^2 x}{\partial x \partial x'} g + \frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'}\partial g\right)\Big|_p (x'-a')$$

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Locally inertial coordinates

Taylor

$$x'^{\mu} = a'^{\mu} + \frac{\partial x'^{\mu}}{\partial x^{\nu}}\bigg|_{p} (x-a)^{\nu} + \frac{1}{2}\frac{\partial^2 x'^{\mu}}{\partial x^{\nu}\partial x^{\alpha}} (x-a)^{\nu}(x-a)^{\alpha}$$

$$+ \dots$$

p

$x^{\mu} = a^{\mu}$

$x'^{\mu} = a'^{\mu}$

$x'^{\mu} = x'^{\mu}(x)$

$a'^{\mu} = a'^{\mu}(a)$

Sim

$$x^{\mu} = a^{\mu} + \frac{\partial x^{\mu}}{\partial x'^{\nu}} (x'-a')^{\nu} + \frac{1}{2}\frac{\partial^2 x^{\mu}}{\partial x'^{\nu}\partial x'^{\alpha}} (x'-a')^{\nu}(x'-a')^{\alpha} + \dots$$

$$g'_{\mu\nu} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}}\frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x) \quad \longrightarrow \quad g_{\alpha\beta}\big|_p + g_{\alpha\beta,\gamma}(x-a)\frac{\partial x}{\partial x'}$$

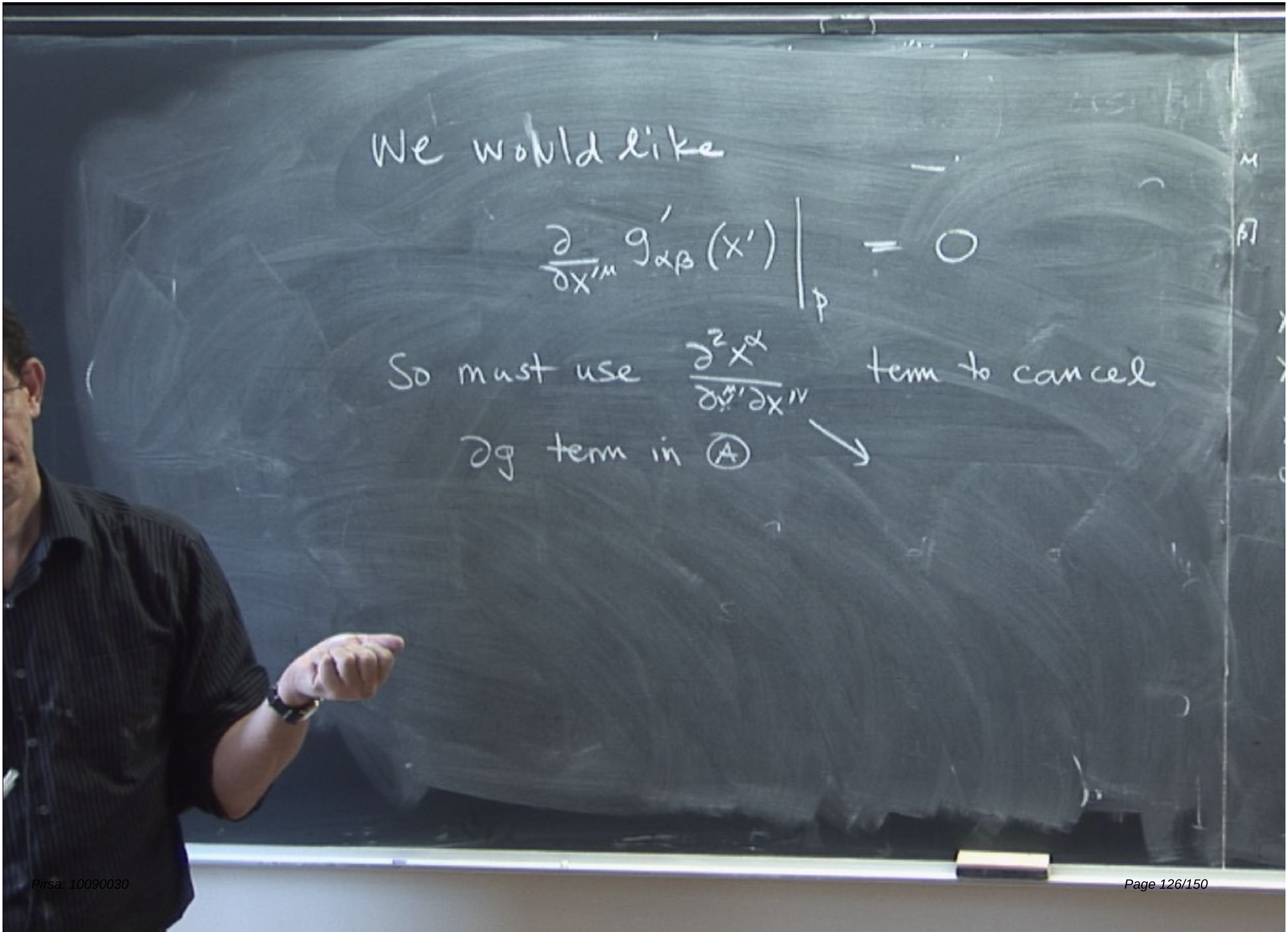
$$= \left(\frac{\partial x}{\partial x'}\right)^2 g\bigg|_p + \left(\frac{\partial x}{\partial x'}\frac{\partial^2 x}{\partial x'\partial x'} g + \frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'}\partial g\right)\bigg|_p (x'-a')$$

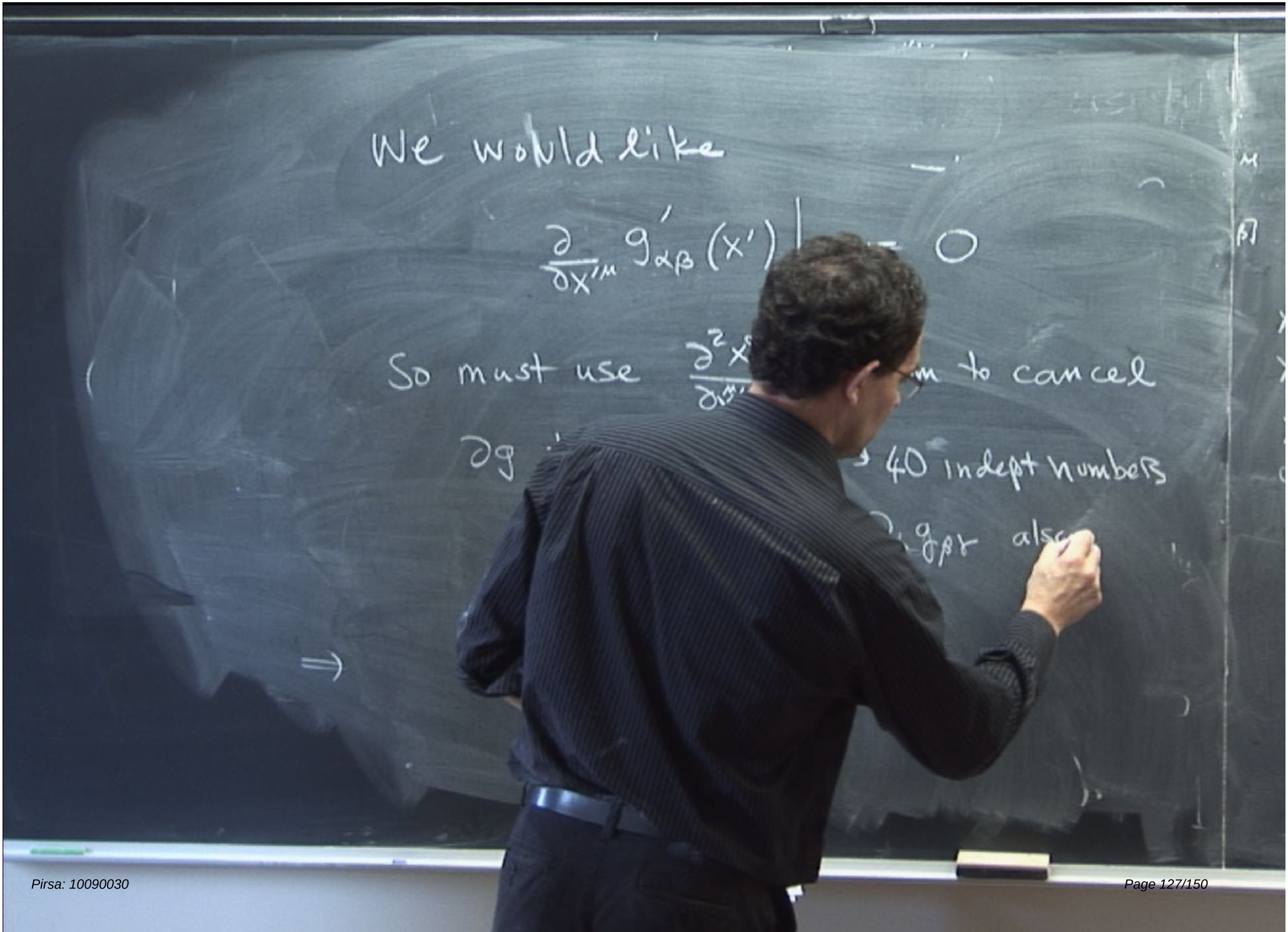
$$+ \dots$$

We would like

$$\frac{\partial}{\partial x'^{\mu}} g'_{\alpha\beta}(x') \Big|_{p} = 0$$

$\Rightarrow$


We would like
$\frac{\partial}{\partial x'^\mu} g'_{\alpha\beta}(x') \Big|_p = 0$
So must use $\frac{\partial^2 x^\alpha}{\partial x'^\mu \partial x'^\nu}$ term to cancel
∂g term in (A)


We would like
$\frac{\partial}{\partial x'^{\mu}} g'_{\alpha\beta}(x') \Big| = 0$
So must use
to cancel
40 indept numbers
also

We would like

$$\frac{\partial}{\partial x'^{\mu}} g'_{\alpha\beta}(x') \Big|_p = 0$$

So must use $\frac{\partial^2 x^\alpha}{\partial x^{\mu\prime} \partial x'^\nu}$ term to cancel

∂g term in (A) → 40 indept numbers

have to cancel $\partial_\alpha g_{\beta\gamma}$ also 40 numbers

So it is possible.

⟹

We would like

$$\frac{\partial}{\partial x'^{\mu}} g'_{\alpha\beta}(x')\Big|_p = 0$$

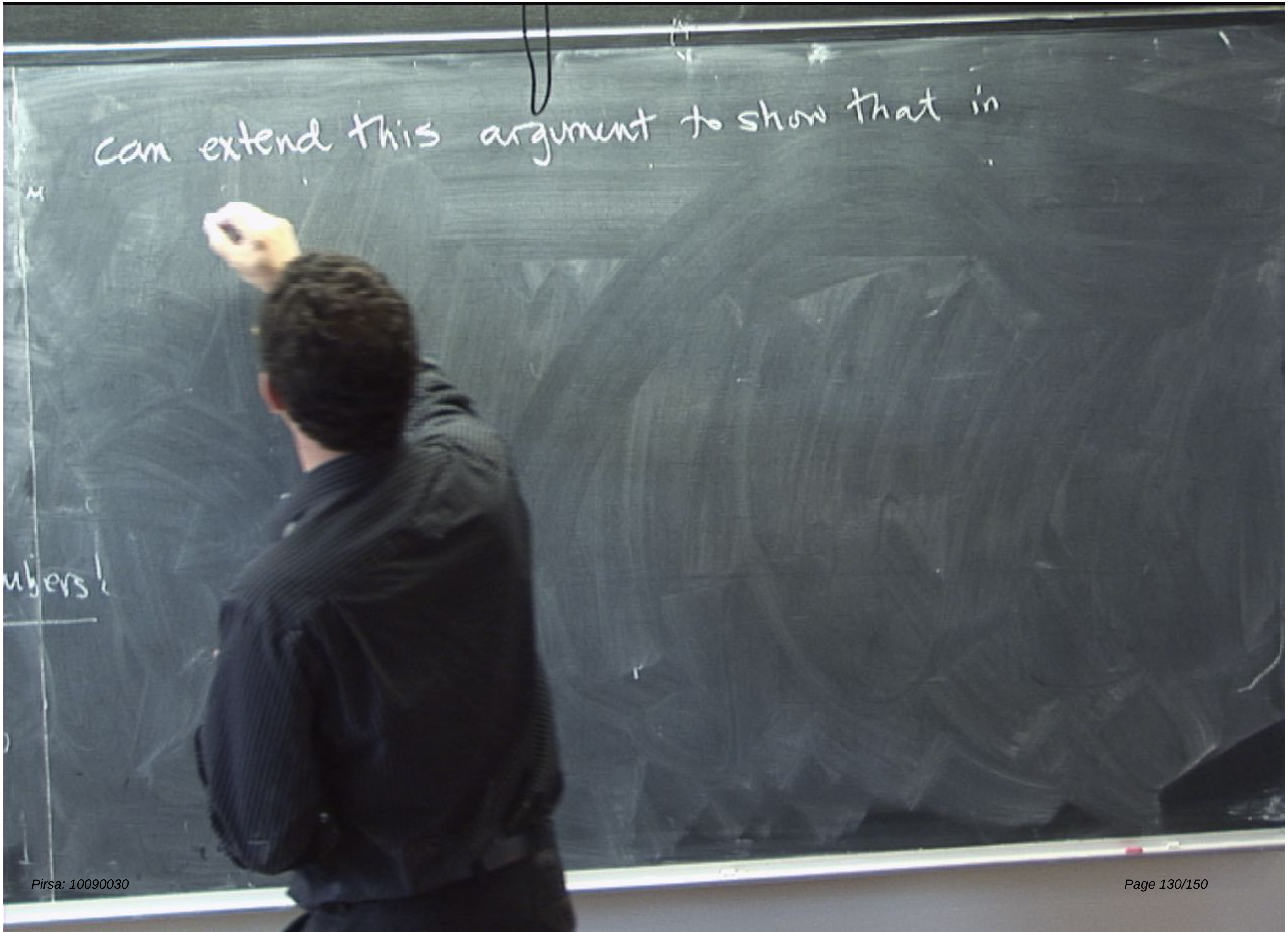
So must use $\dfrac{\partial^2 x^\alpha}{\partial x^{\mu\prime}\partial x'^{\nu}}$ term to cancel

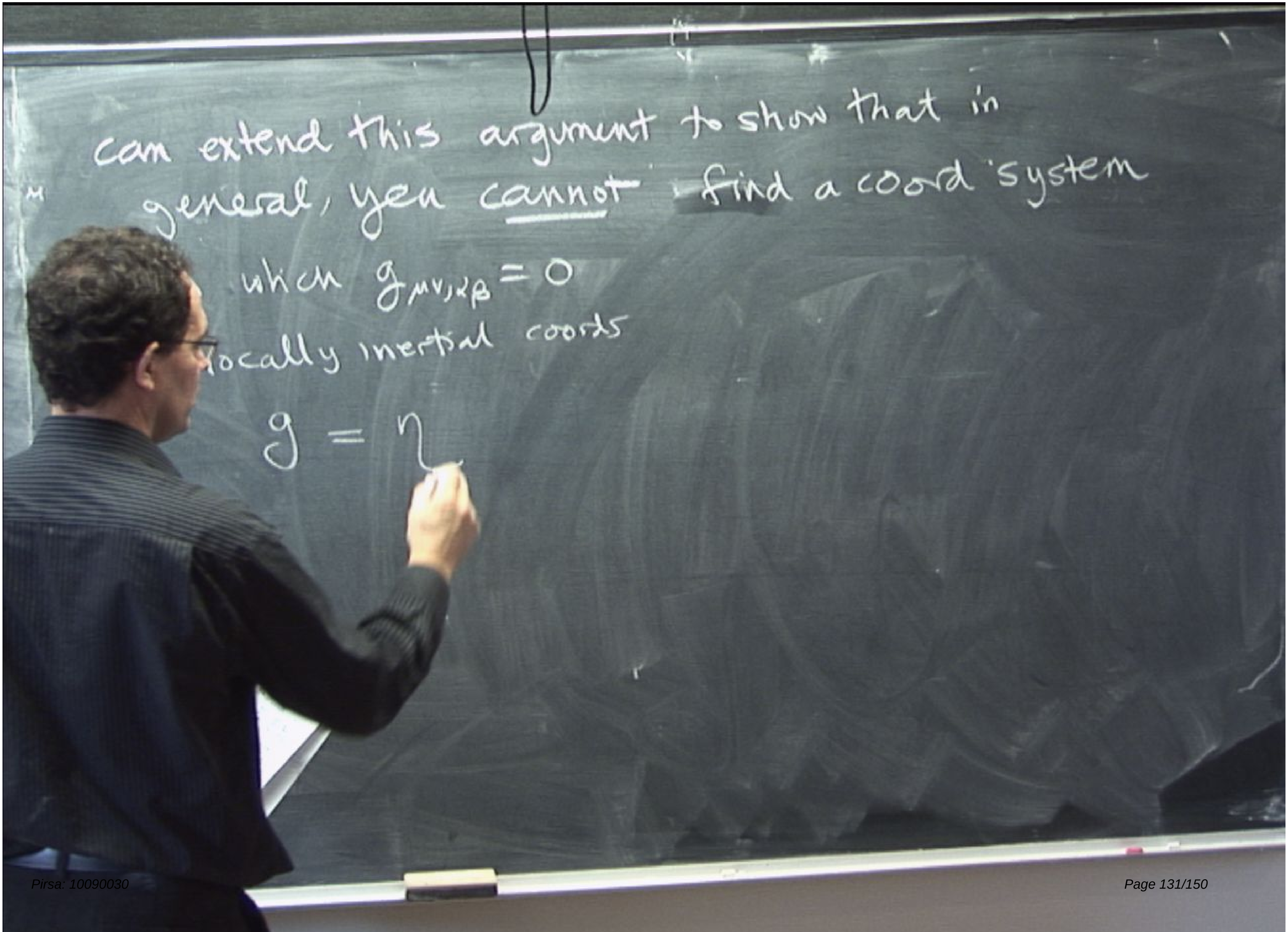
∂g term in (A) → 40 indept numbers

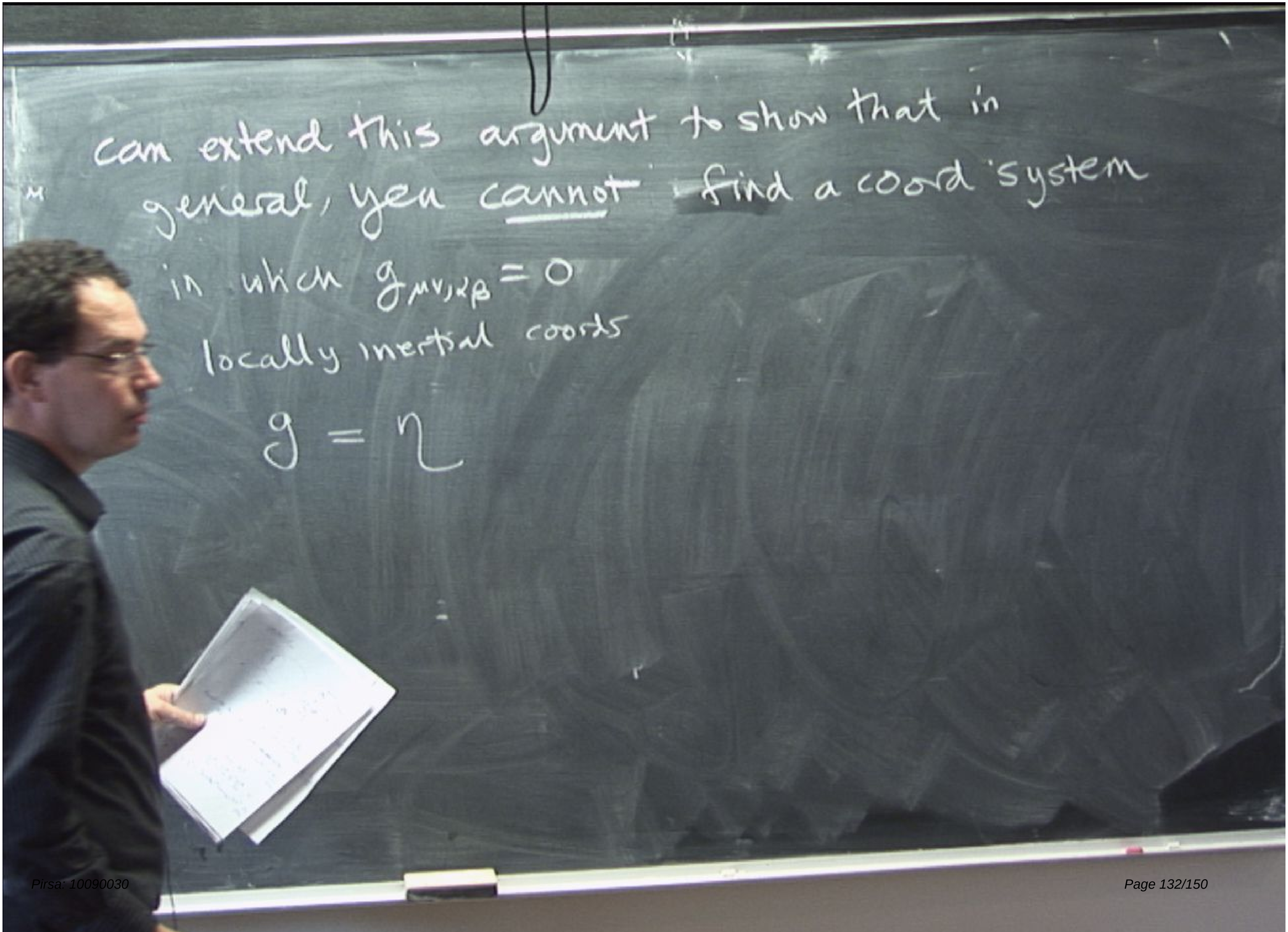
have to cancel $\partial_\alpha g_{\beta\gamma}$ also 40 numbers

So it is possible.

$\Rightarrow$


can extend this argument to show that in


can extend this argument to show that in
general, you cannot find a coord system
which $g_{\mu\nu,\alpha\beta} = 0$
locally inertial coords
$g = \eta$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locally inertial coords

$$g_{\mu\nu} = \eta_{\mu\nu} + R_{\mu\nu\rho\lambda}(x-a)^{\nu}(x-a)^{\lambda}$$

can extend this argument to show that in general, you cannot find a coord system in which $g_{\mu\nu,\alpha\beta} = 0$

in general, can choose locally inertial coords at any point p, with coords $x^\mu = a^\mu$

$$g_{\mu\nu} = \eta_{\mu\nu} + R_{\mu\nu\rho\lambda}(x-a)^\nu (x-a)^\lambda$$

ubers!

can extend this argument to show that in general, you <u>cannot</u> find a coord system in which $g_{\mu\nu,\alpha\beta} = 0$

in general, can choose locally inertial coords at any point p, with coords $x^\mu = a^\mu$

$$g_{\mu\nu} = \eta_{\mu\nu} + R_{\mu\nu\rho\lambda}(x-a)^\nu(x-a)^\lambda$$

for some $R_{\mu\nu\rho\lambda}$.

mbers!

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for some $R_{\mu\nu\rho\lambda}$.

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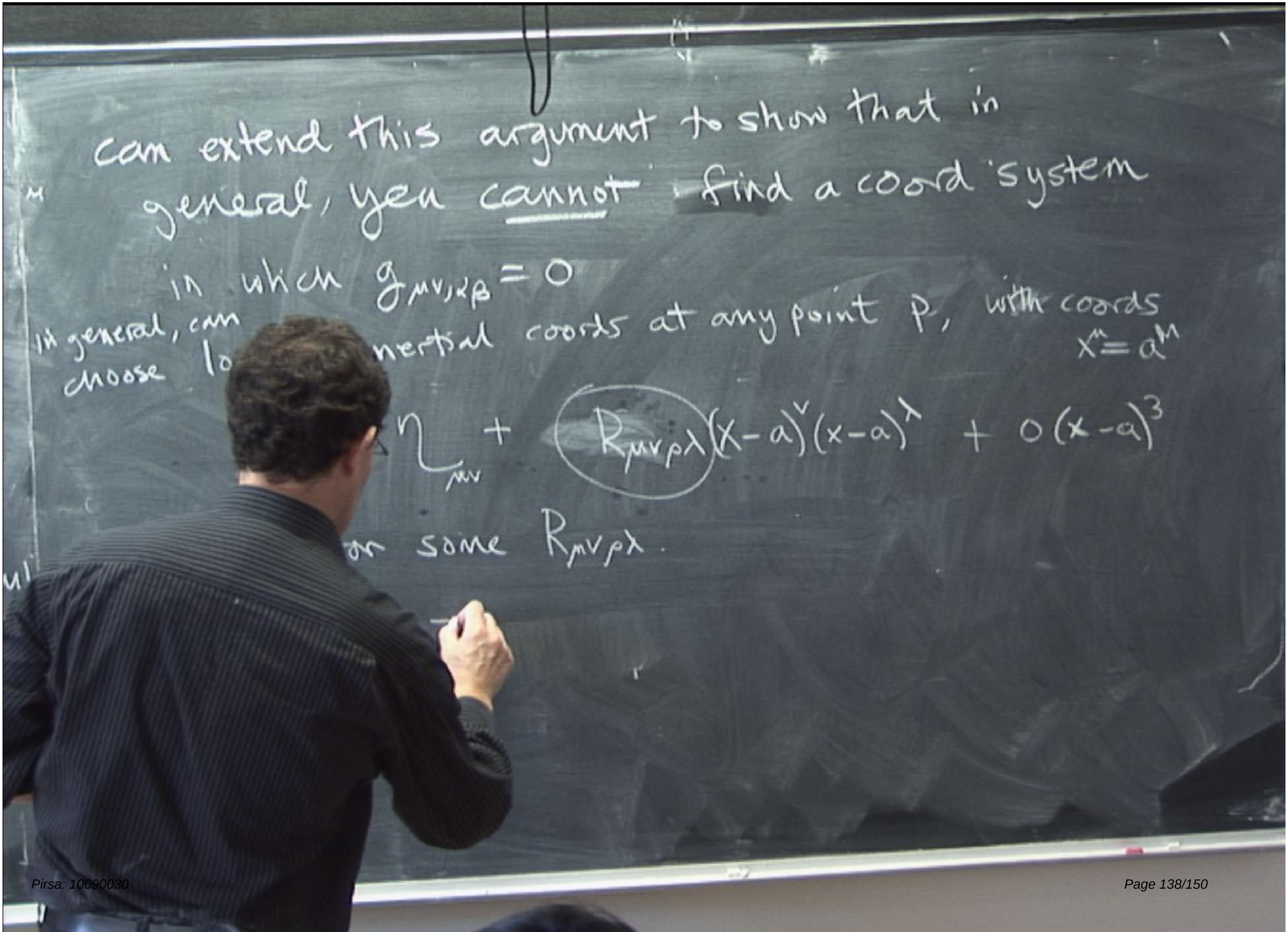
for some $R_{\mu\nu\rho\lambda}$.

ubers!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general, you cannot find a coord system
in which $g_{\mu\nu,\alpha\beta} = 0$
in general, can choose lo[cal] inertial coords at any point p, with coords $x^\mu = a^\mu$
$\eta_{\mu\nu}$ + $R_{\mu\nu\rho\lambda}(x-a)^\nu(x-a)^\lambda$ + $O(x-a)^3$
for some $R_{\mu\nu\rho\lambda}$.

can extend this argument to show that in general, you cannot find a coord system in which $g_{\mu\nu,\alpha\beta} = 0$

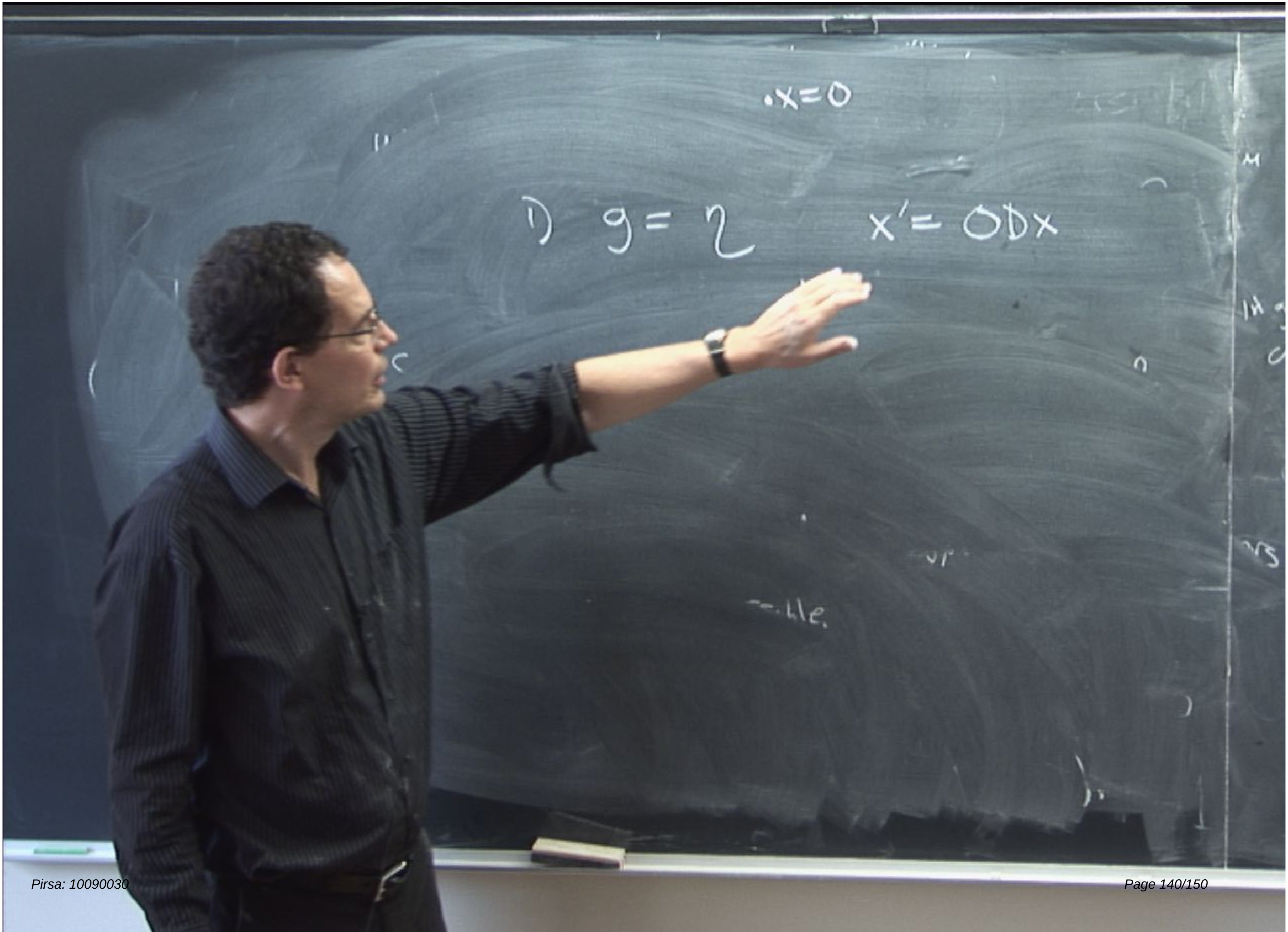
in general, can choose locally inertial coords at any point p, with coords $x^\mu = a^\mu$

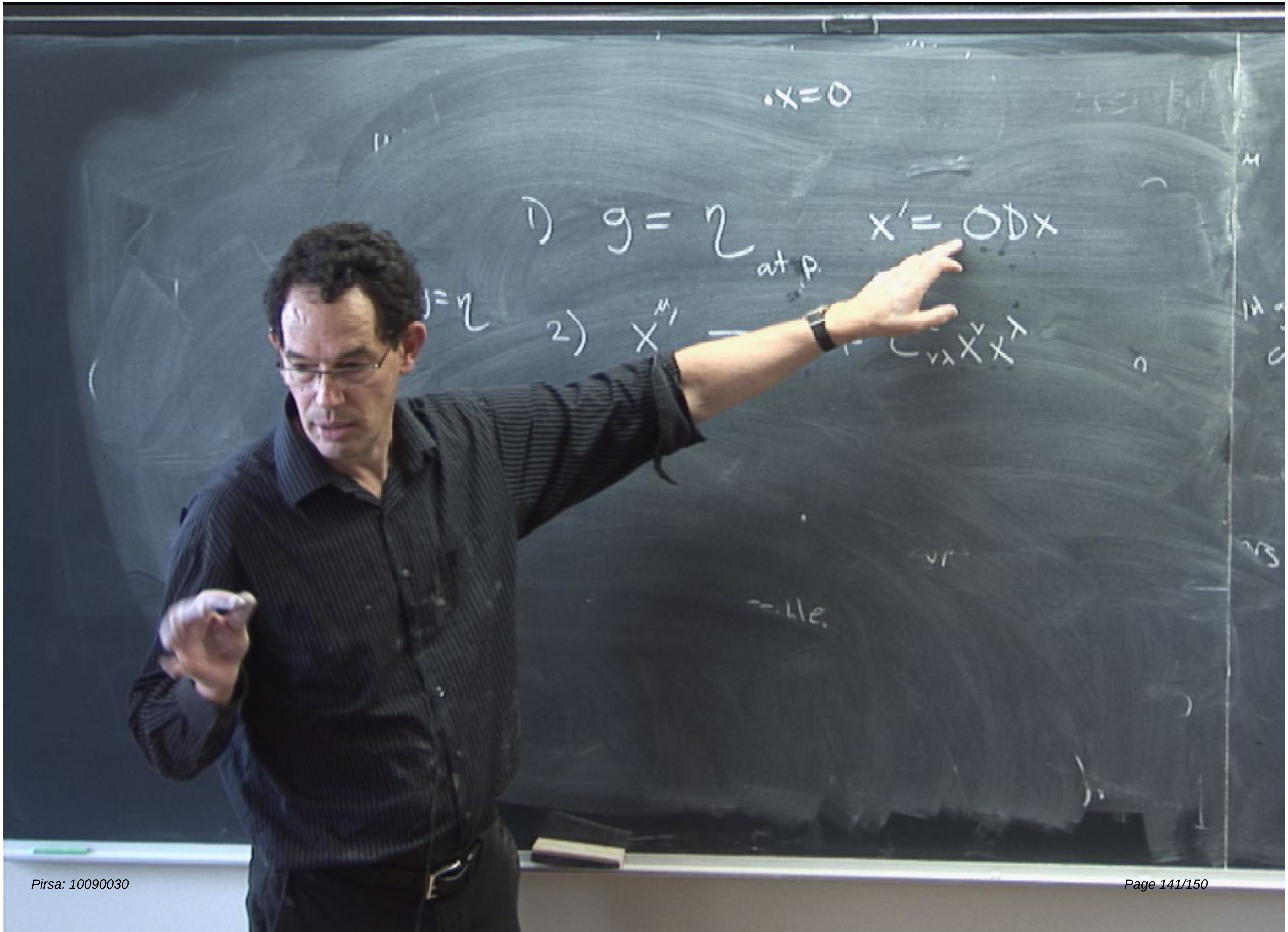
$$g_{\mu\nu} = \eta_{\mu\nu} + R_{\mu\nu\rho\lambda}(x-a)^\nu (x-a)^\lambda + O(x-a)^3$$

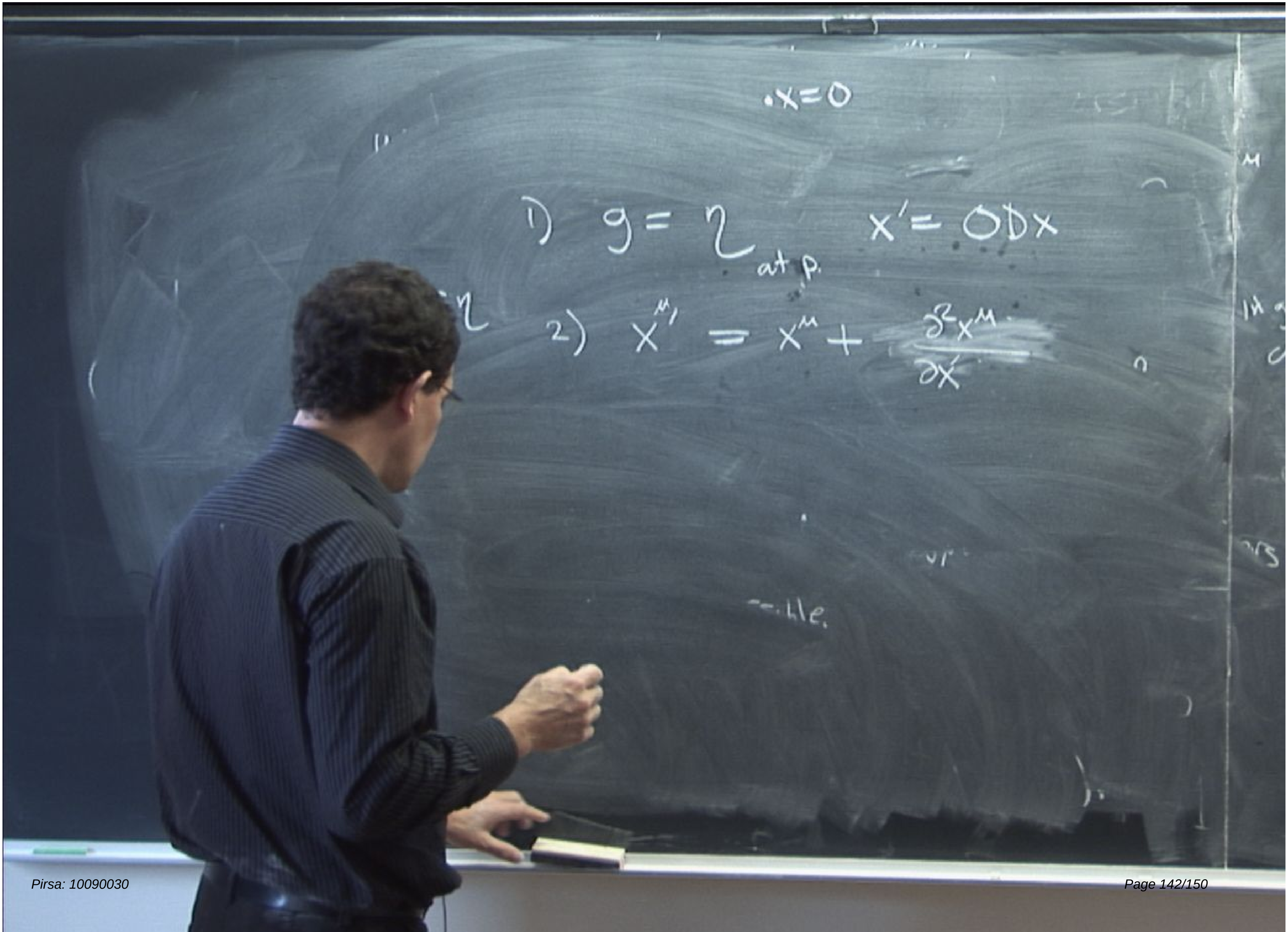
for some $R_{\mu\nu\rho\lambda}$.

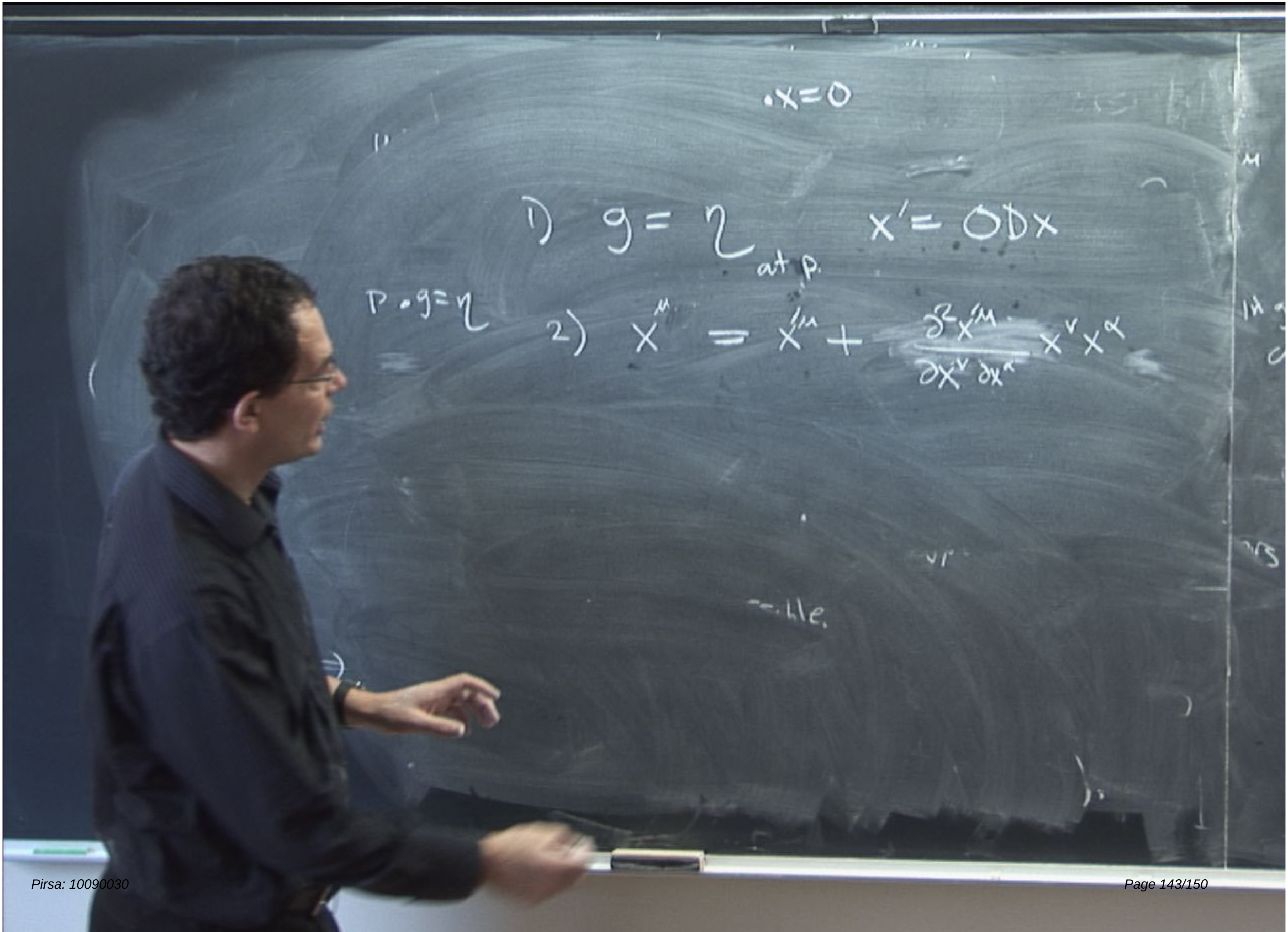
$$\frac{\partial}{\partial x^\beta} g_{\mu\nu} = R_{\mu\nu\rho\lambda}\,\delta^\nu_\beta (x-a)^\lambda + (\quad)$$

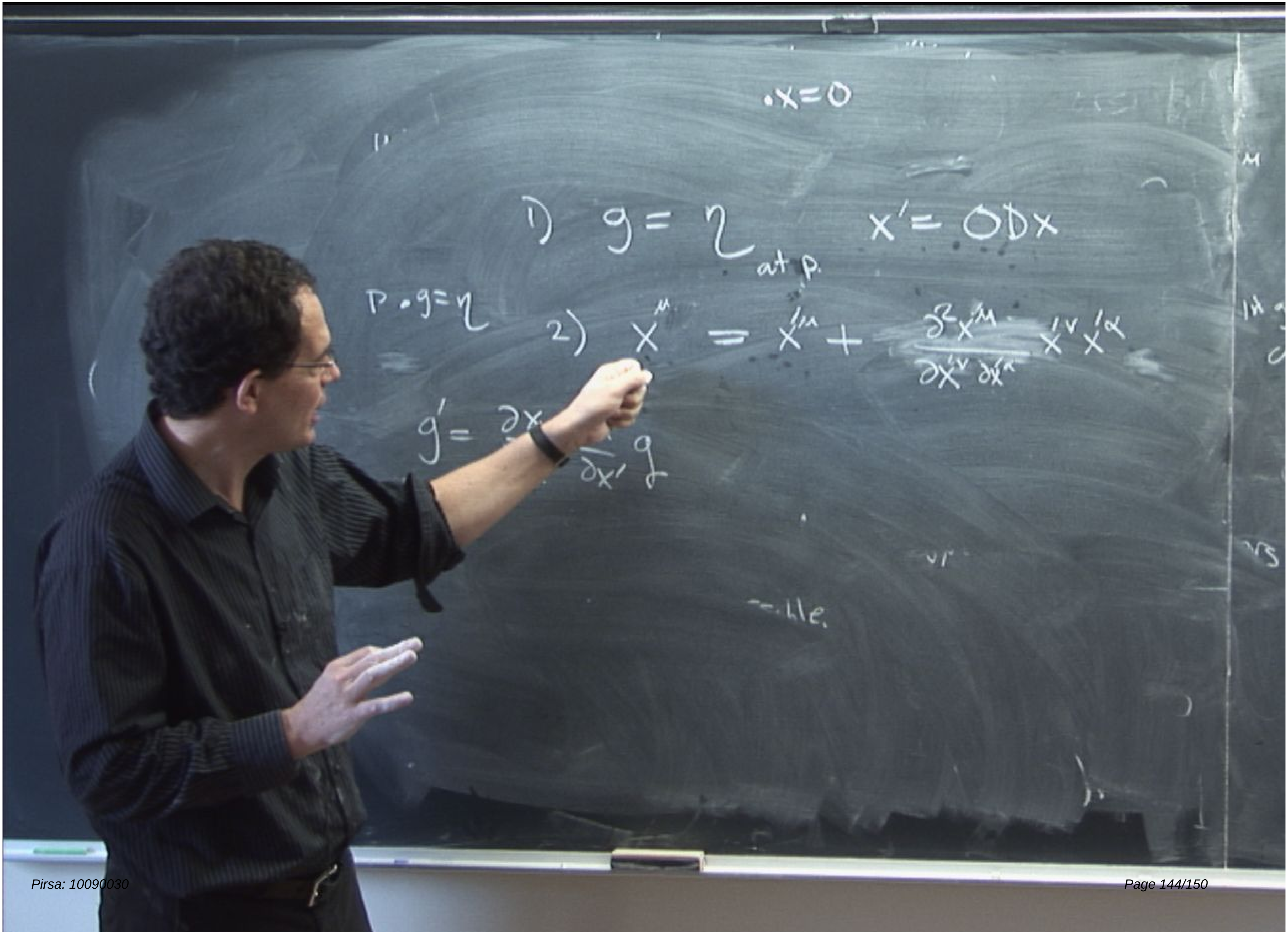
ubers!

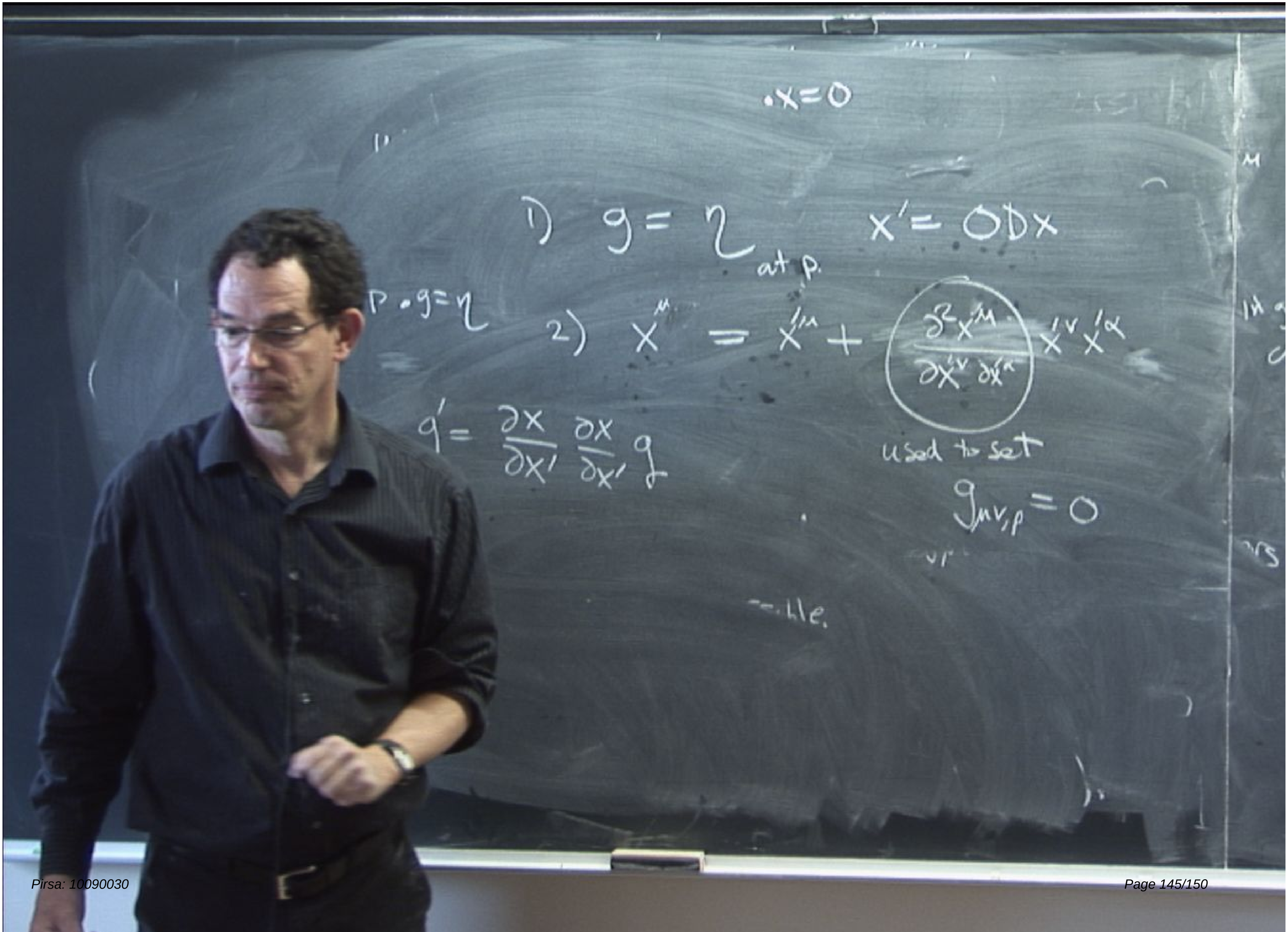

.x=0
1) g = η
x' = ODx

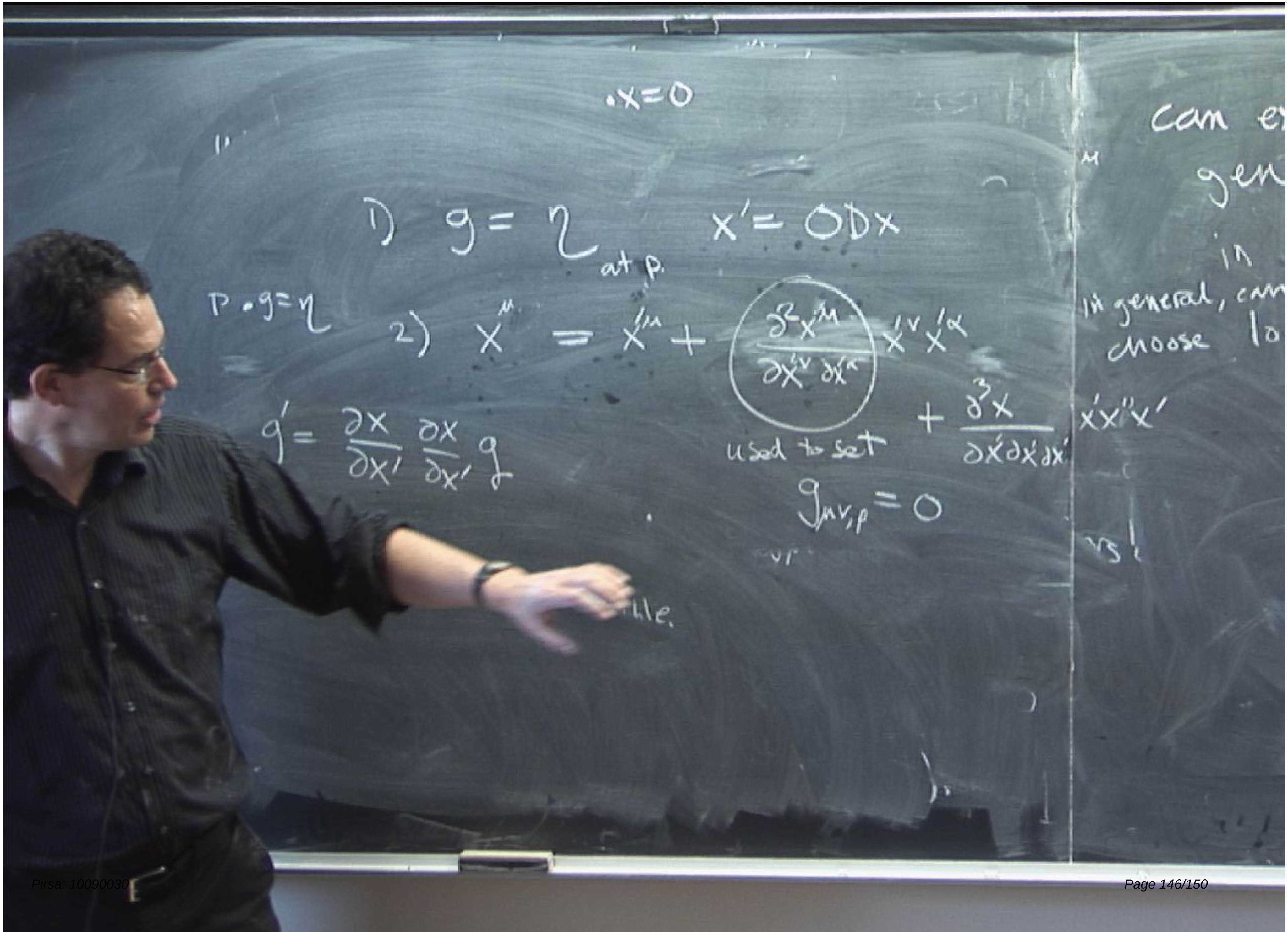

•x=0
1) g = η at p.
x′ = ODx

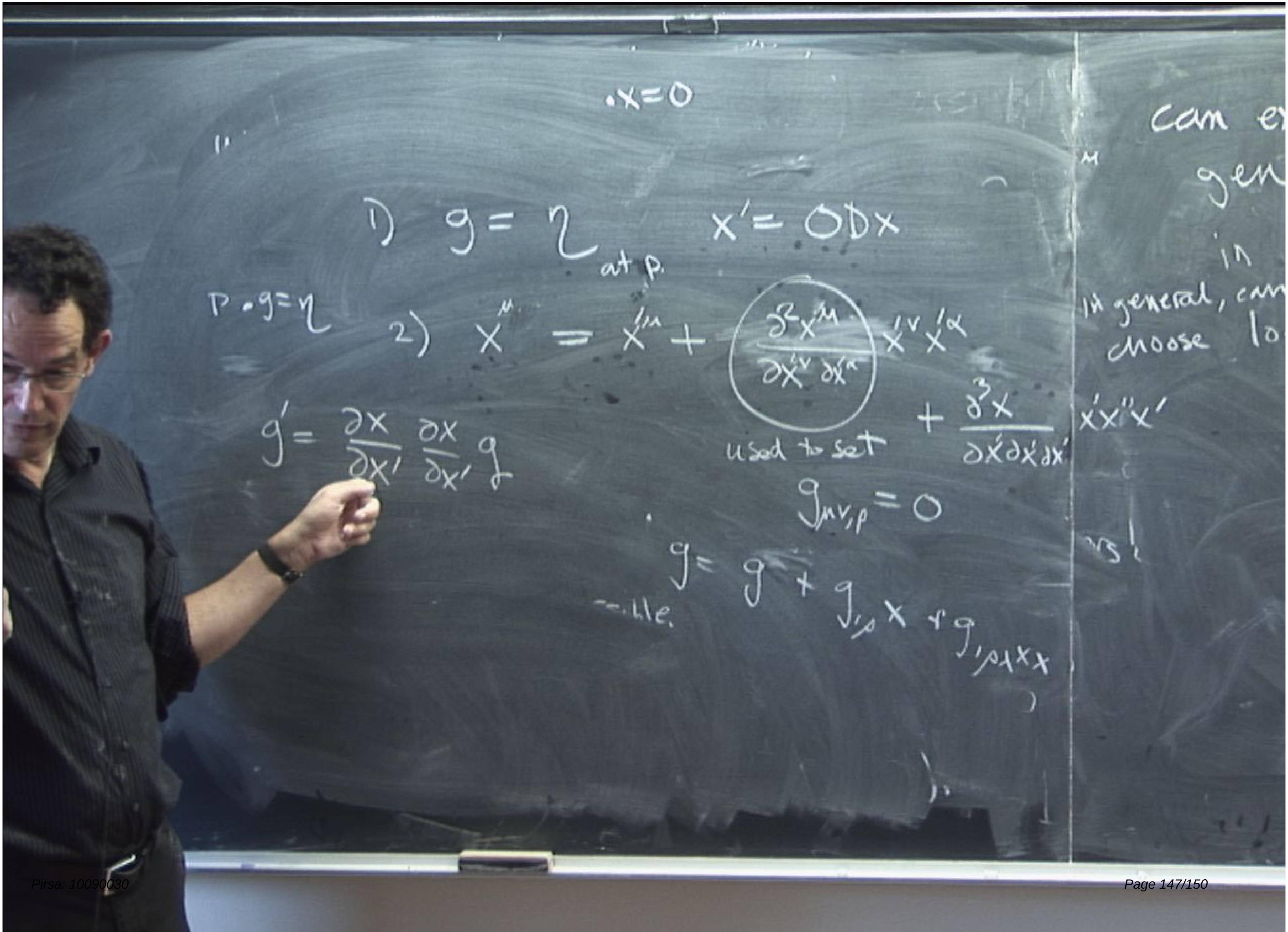

x=0
1) g= η at p.
x′= ODx
2) x^μ′ = x^μ + ∂²x^μ / ∂x′


.x=0
1) g = η at p.   x' = ODx
p . g=η
2) x^μ = x'^μ + ∂²x'^μ / ∂x^ν ∂x^α x^ν x^α

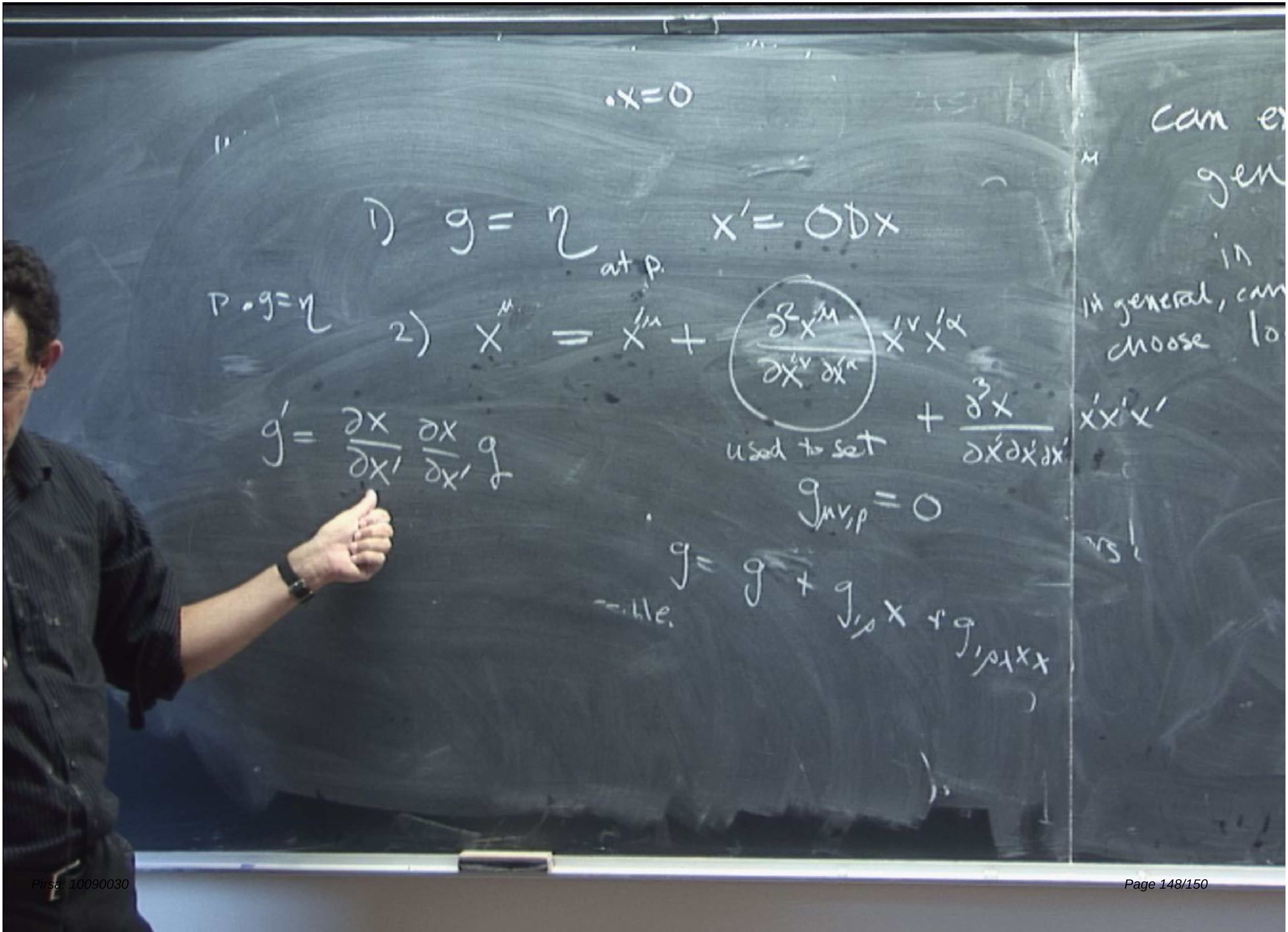

x=0
1) g = η at p.
x′ = ODx
p g=η
2) x^μ = x′^μ + ∂²x^μ/∂x′^ν∂x′^α x′^ν x′^α
g′ = ∂x/∂x′ g

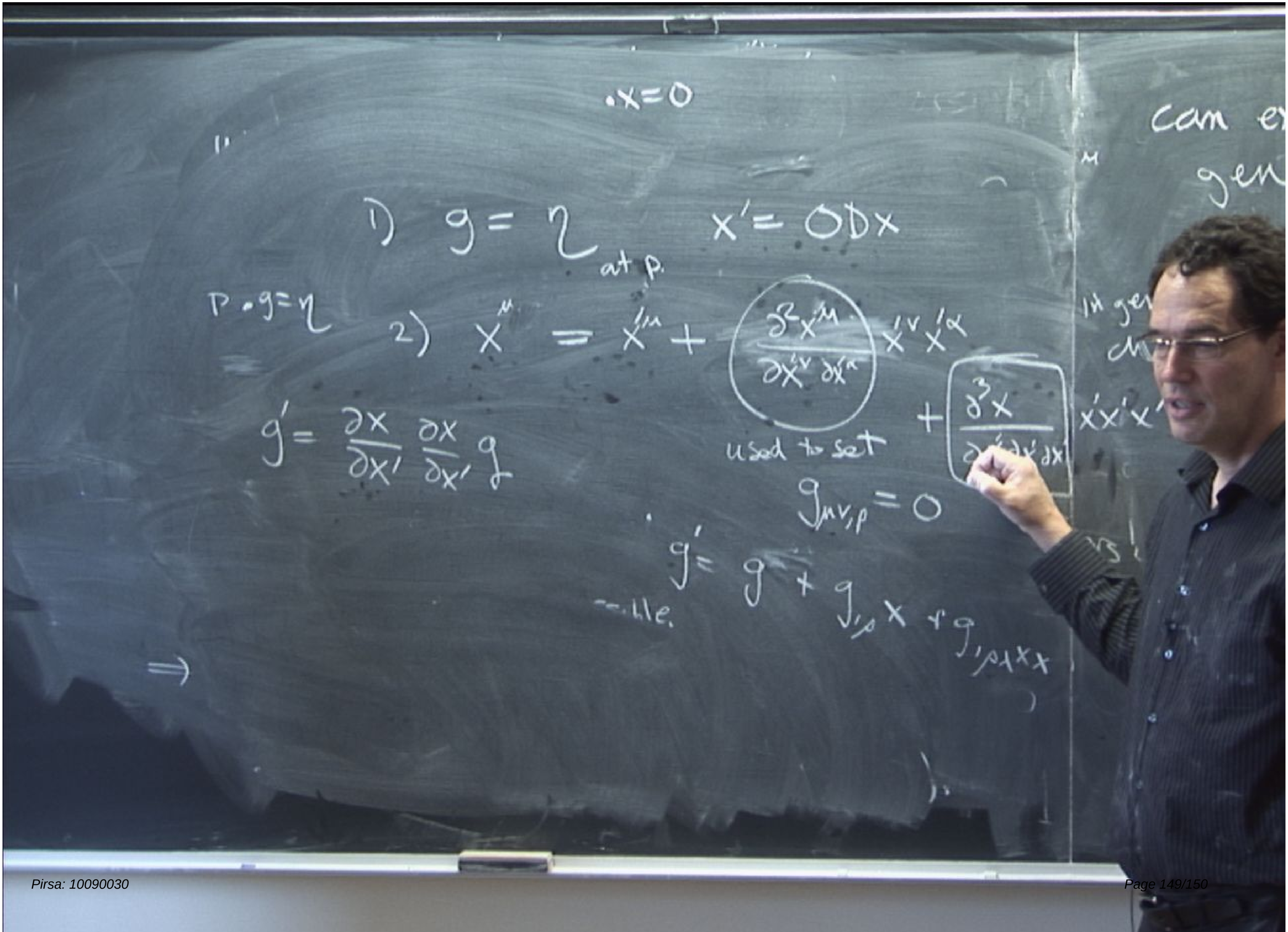

x=0
1) g = η at p.   x' = O D x
p g = η
2) x^μ = x'^μ + ∂²x^μ/∂x'^ν ∂x'^α x'^ν x'^α
g' = ∂x/∂x' ∂x/∂x' g
used to set
g_μν,ρ = 0


x=0
can ex
gen
1) g = η at p.
x′ = ODx
in
p g=η
in general, can
choose lo
2) x^μ = x′^μ + ∂²x^μ/∂x′^ν∂x′^α x′^ν x′^α + ∂³x/∂x′∂x′∂x′ x′x′x′
g′ = ∂x/∂x′ ∂x/∂x′ g
used to set
g_μν,ρ = 0



x=0
1) g = η at p.
x′ = ODx
p g = η
2) x = x′ + ∂²x / ∂x′ ∂x′ x′ x′
used to set
g′ = ∂x/∂x′ ∂x/∂x′ g
+ ∂³x / ∂x′∂x′∂x′ x′x′x′
g_{μν,ρ} = 0
g = g + g,ρ x + g,ρλ x x
can e
gen
in
in general, can
choose lo


x=0
1) g= η at p. x′= O D x
p g=η
2) x^μ = x′^μ + ∂²x^μ/∂x′^ν ∂x′^α x′^ν x′^α + ∂³x/∂x′∂x′∂x′ x′x′x′
used to set
g′= ∂x/∂x′ ∂x/∂x′ g
g_μν,ρ = 0
g= g + g,ρ x + g,ρλ x x
can ex
gen
in
in general, can
choose lo


x=0
can ex
gen
1) g = η at p.
x′ = ODx
p g = η
2) x^μ = x′^μ + ∂²x^μ / ∂x′^ν ∂x′^α x′^ν x′^α + ∂³x / ∂x′∂x′∂x x′x′x′
g′ = ∂x/∂x′ ∂x/∂x′ g
used to set
g_μν,ρ = 0
g′ = g + g,ρ x + g,ρλ x x

$\cdot x = 0$

can ex
gen

in

1) $g = \eta$ at p. $\quad x' = ODx$

p $\cdot g = \eta$

2) $x^{\mu} = x'^{\mu} + \frac{\partial^2 x^{\mu}}{\partial x'^{\nu} \partial x'^{\alpha}} x'^{\nu} x'^{\alpha} + \frac{\partial^3 x}{\partial x' \partial x' \partial x} x' x' x'$

used to set

$g_{\mu\nu,\rho} = 0$

$g' = \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} g$

$g' = g + g_{,\rho} x + g_{,\rho\lambda} x x$

in general, can
choose lo

...ble.

$\Longrightarrow$