

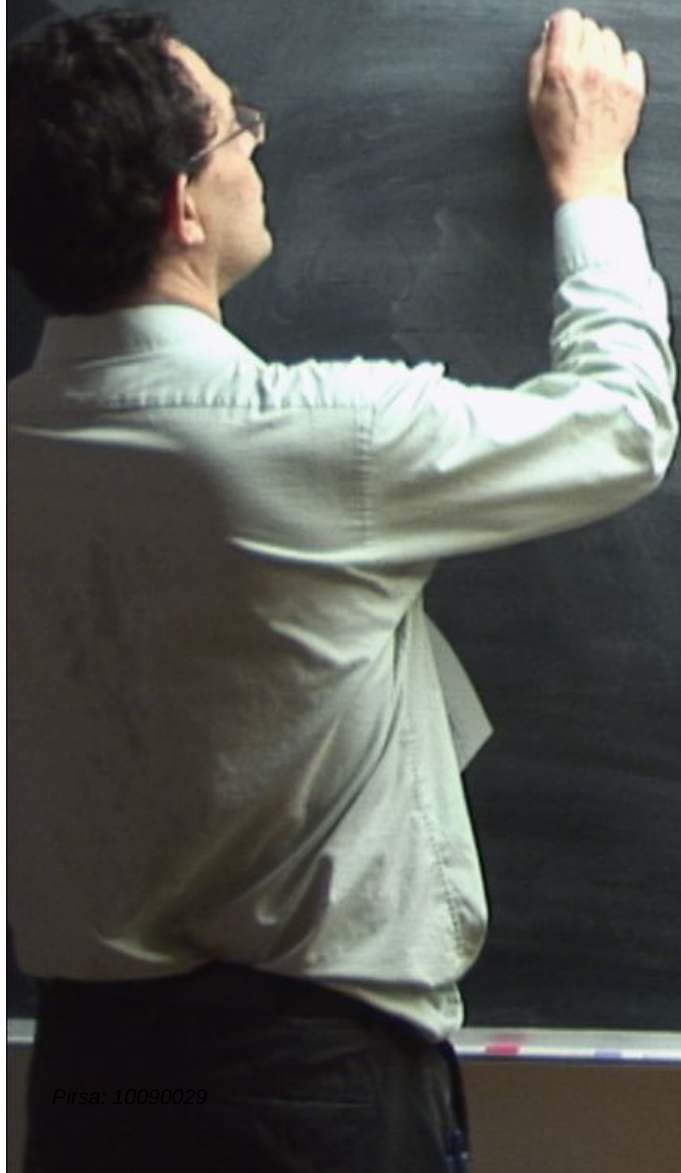
Title: Relativity (PHYS 604) - Lecture 4

Date: Sep 16, 2010 10:30 AM

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Abstract:

The Principle of Equivalence



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At every spacetime point, it is possible to choose locally inertial (or "free fall") coordinates such that within a sufficiently small neighbourhood, the laws of physics take the same special relativistic form as they do in Minkowski spacetime.

Minkowski spacetime M^4 has a metric

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$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

$$= \eta_{\mu\nu} dx^\mu dx^\nu$$

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curved spacetime has a metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

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arbitrary 4×4 symmetric, invertible matrix, with
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$$\begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \lambda_3 & \\ & & & \lambda_4 \end{pmatrix}$$

$$\det = \lambda_1 \lambda_2 \lambda_3 \lambda_4$$

Minkowski spacetime M^4 has a metric

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$
$$= \eta_{\mu\nu} dx^\mu dx^\nu$$

Curved spacetime has a metric $\det \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \lambda_3 & \\ & & & \lambda_4 \end{pmatrix}$

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

$$\det g = \lambda_1 \lambda_2 \lambda_3 \lambda_4$$

arbitrary 4×4 symmetric, invertible matrix, with
1 negative and 3 positive eigenvalues: signature $(-+++)$

"pseudo-Riemannian"

At any point P ,

$$g_{\mu\nu} dx^\mu dx^\nu$$

$$dx^\mu =$$

At any point P ,

$$g_{\mu\nu} dx^\mu dx^\nu$$

$$dx^\mu = \underbrace{O^\mu_\alpha}_{\text{orthogonal}} \underbrace{D^\alpha_\beta}_{\text{diagonal (non-singular)}} dy^\beta$$

orthogonal

diagonal
(non-singular)

At any point P ,

$$g_{\mu\nu} dx^\mu dx^\nu = dx^\mu g_{\mu\nu} dx^\nu = dy^\alpha D^\alpha D^\beta g_{\alpha\beta} dy^\beta$$

$\underbrace{D^\alpha}_{\text{orthogonal}} \underbrace{D^\beta}_{\text{diagonal (non-singular)}}$

At any point P,

$$g_{\mu\nu} dx^\mu dx^\nu = dx^\top g dx$$
$$dx^\mu = \underbrace{O^\mu_\alpha}_{\text{orthogonal}} \underbrace{D^\alpha_\beta}_{\text{diagonal (non-singular)}} dy^\beta = dy^\top \underbrace{D O^\top g O D}_{\begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \lambda_3 & \\ & & & \lambda_4 \end{pmatrix}} dy$$

At any point P ,

$$g_{\mu\nu} dx^\mu dx^\nu = dx^T g dx$$
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Choose O to diagonalise g

$$D = \begin{pmatrix} d_1 & & & \\ & d_2 & & \\ & & d_3 & \\ & & & d_4 \end{pmatrix}$$

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$$D = \begin{pmatrix} d_1 & & & \\ & d_2 & & \\ & & d_3 & \\ & & & d_4 \end{pmatrix}$$

$$dy^T \begin{pmatrix} \lambda_1 d_1^2 & & & \\ & \lambda_2 d_2^2 & & \\ & & \lambda_3 d_3^2 & \\ & & & \lambda_4 d_4^2 \end{pmatrix} dy$$

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Choose O to diagonalise g

$$D = \begin{pmatrix} d_1 & & & \\ & d_2 & & \\ & & d_3 & \\ & & & d_4 \end{pmatrix}$$

$$d_1^2 = \frac{1}{|\lambda_1|}$$

$$dy^T \begin{pmatrix} \lambda_1 d_1^2 & & & \\ & \lambda_2 d_2^2 & & \\ & & \lambda_3 d_3^2 & \\ & & & \lambda_4 d_4^2 \end{pmatrix} dy$$

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Choose O to diagonalise g

$$D = \begin{pmatrix} d_1 & & & \\ & d_2 & & \\ & & d_3 & \\ & & & d_4 \end{pmatrix}$$

$$d_1^2 = \frac{1}{|\lambda_1|} \text{ etc.}$$

$$dy^T \begin{pmatrix} \lambda_1 d_1^2 & & & \\ & \lambda_2 d_2^2 & & \\ & & \lambda_3 d_3^2 & \\ & & & \lambda_4 d_4^2 \end{pmatrix} dy$$

$$= dy^T \eta dy$$

Coordinate transformation properties of metric

Coordinate transformation properties of

metric

$$\begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$\left(\begin{array}{c} d_1^2 \\ d_2^2 \\ d_3^2 \\ d_4^2 \end{array} \right) dy$$

Coordinate transformation properties of
metric

$$\begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$$

$$\left(\begin{array}{c} d_1^2 \\ d_2^2 \\ d_3^2 \\ d_4^2 \end{array} \right) dy$$

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$$\det g' =$$

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$$\begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$$

$$\underbrace{\det g'}_{<0} = \left(\det \left(\frac{\partial x}{\partial x'} \right) \right)^2 \det g$$

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$$\sqrt{-\det g'} = \left| \frac{\partial x}{\partial x'} \right|$$

dy
d3

Coordinate transformation properties of
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$$\begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

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$$\Rightarrow \int d^4x$$

$$\left(\begin{matrix} d_1^2 \\ d_2^2 \\ d_3^2 \\ d_4^2 \end{matrix} \right) dy$$

Coordinate transformation properties of
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$$\begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

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$$\Rightarrow \int d^4x \sqrt{-\det g}$$

$$\left(d_3^2 d_4^2 \right) dy$$

Coordinate transformation properties of
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Scalar or coord-inv't quantity.

Coordinate transformation properties of
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$$\Rightarrow \underbrace{\int d^4 x' \sqrt{-\det g'}}_{\text{scalar or coord-inv't quantity}} = \int d^4 x \sqrt{-\det g}$$

scalar or coord-inv't quantity.

$$\int dx \sqrt{-g} ($$

Einstein

$$\int d^4x \sqrt{-g} \left(-\frac{R}{16\pi G} \right)$$

Einstein

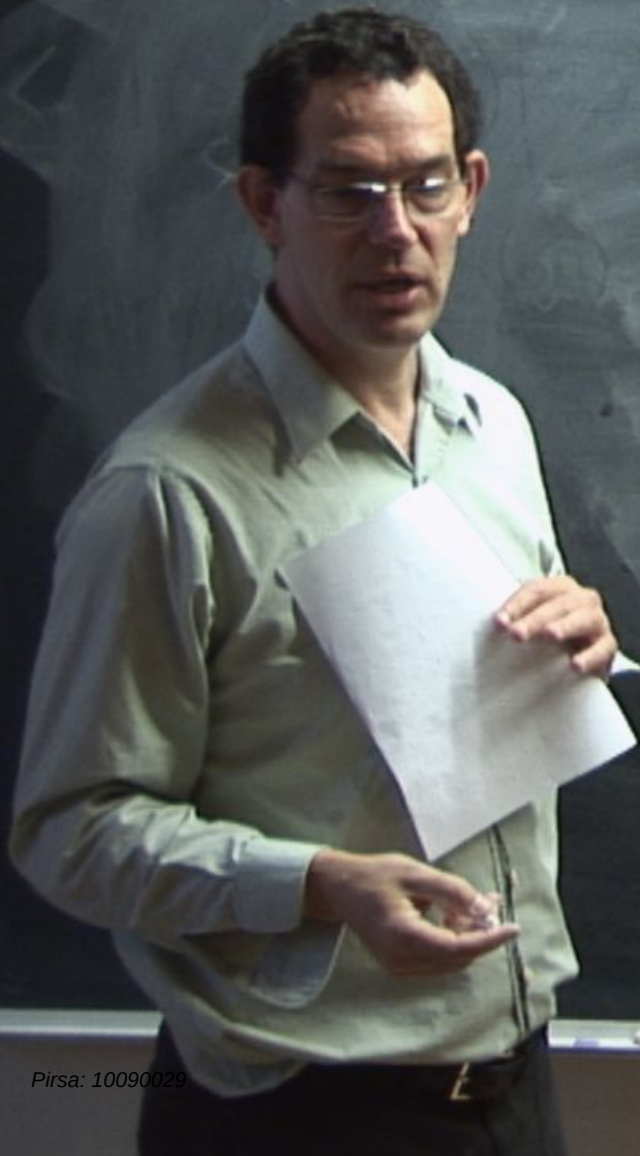
$$\int d^4x \sqrt{-g} \left(-\frac{R}{16\pi G} - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} \right)$$

Einstein Maxwell

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Einstein Maxwell

$$\int d^4x \sqrt{-g} \left(-\frac{R}{16\pi G} - \frac{1}{4} F_{\alpha\beta}^2 F^{\alpha\beta} \right)$$



Einstein Maxwell

$$\int d^4x \sqrt{-g} \left(-\frac{R}{16\pi G} - \frac{1}{4} F_{\alpha\beta}^2 F^{\alpha\beta} \right)$$

$U(1) \times SU(2) \times SU(3)$
electroweak strong force
grand unified force

Einstein Maxwell

$$\int d^4x \sqrt{-g} \left(-\frac{R}{16\pi G} - \frac{1}{4} F_{\alpha\beta}^a F^{\alpha\beta}_a - i\bar{\Psi} \not{\partial} \Psi \right)$$

$U(1) \times SU(2) \times SU(3)$
 electroweak strong force
 grand unified force

$$\int d^4x \sqrt{-g} \left(\overset{\text{Einstein}}{-\frac{R}{16\pi G}} - \overset{\text{Maxwell}}{\frac{1}{4} F_{\alpha\beta}^2} - \overset{\text{Dirac}}{i\bar{\psi}\not{D}\psi} \right)$$

$U(1) \times SU(2) \times SU(3)$
 electroweak strong force
 grand unified force

$$\int d^4x \sqrt{-g} \left(-\frac{R}{16\pi G} - \frac{1}{4} F_{\alpha\beta}^a F^{\alpha\beta}_a - i \bar{\psi} \not{\partial} \psi - \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 - \lambda \phi^4 \right)$$

Einstein Maxwell Dirac Yukawa
 Higgs

$U(1) \times SU(2) \times SU(3)$
 electroweak strong force
 grand unified force

$$\int d^4x \sqrt{-g} \left(-\frac{R}{16\pi G} - \frac{1}{4} F_{\alpha\beta}^a F^{\alpha\beta}_a - (\bar{\Psi} i \not{D} \Psi) - \frac{1}{2} (D_\mu \phi)^\dagger D^\mu \phi - V(\phi) \right)$$

Einstein Maxwell Dirac Yukawa
 (under $F_{\alpha\beta}^a F^{\alpha\beta}_a$)
 $U(1) \times SU(2) \times SU(3)$
 electroweak strong force
 grand unified force
 Higgs

$$\begin{array}{cccc}
 \text{Einstein} & \text{Maxwell} & \text{Dirac} & \text{Yukawa} \\
 \int d^4x \sqrt{-g} \left(-\frac{R}{16\pi G} - \frac{1}{4} F_{\alpha\beta}^a F^{\alpha\beta}_a \right) & - (\bar{\psi} i \not{\partial} \psi) & - \lambda |\phi|^2 \bar{\psi} \psi & \\
 \downarrow & & \text{Higgs} & \\
 \underbrace{U(1) \times SU(2) \times SU(3)}_{\text{electroweak strong force}} & & + |D\phi|^2 - V(\phi) & \\
 \text{grand unified force} & & &
 \end{array}$$

$$\int d^4x \sqrt{-g} \left(\underbrace{-\frac{R}{16\pi G}}_{\text{Einstein}} - \underbrace{\frac{1}{4} F_{\alpha\beta}^a F^{\alpha\beta}_a}_{\substack{\text{Maxwell} \\ \downarrow \\ \text{electroweak} \times \text{strong force}}} - \underbrace{\bar{\psi} i \not{D} \psi}_{\text{Dirac}} - \underbrace{\lambda (\phi^\dagger \phi)^2}_{\substack{\text{Yukawa} \\ \downarrow \\ \text{Higgs}}} + |D\phi|^2 - V(\phi) \right)$$



$$\int d^4x \sqrt{-g} \left(\overset{\text{Einstein}}{-\frac{R}{16\pi G}} - \overset{\text{Maxwell}}{\frac{1}{4} F_{\alpha\beta}^a F^{\alpha\beta}_a} - \overset{\text{Dirac}}{i\bar{\Psi}\not{D}\Psi} - \overset{\text{Yukawa}}{\lambda\bar{\Psi}\Psi} \right.$$

$\underbrace{U(1) \times SU(2) \times SU(3)}_{\text{electroweak strong force}}$
 "internal" grand unified force

$$\left. + |D\phi|^2 - V(\phi) \right)$$

$\left. \begin{matrix} F^a_{\mu\nu} & F^a_{\mu\nu} \end{matrix} \right\}$

$$\int d^4x \sqrt{-g} \left(-\frac{R}{16\pi G} - \frac{1}{4} F_{\alpha\beta}^a F^{\alpha\beta}_a - (\bar{\psi} i \not{\partial} \psi - \lambda \phi \bar{\psi} \psi) - |D\phi|^2 - V(\phi) \right)$$

Einstein Maxwell Dirac Yukawa
 (1) x SU(2) x SU(3)
 electroweak strong force
 "internal" grand unified force

$$A^\mu A^\mu \leftarrow N$$

$$\begin{matrix} F^a_{\mu\nu} & F^a_{\mu\nu} \end{matrix}$$

$$\int d^4x \sqrt{-g} \left(-\frac{R}{16\pi G} - \frac{1}{4} F_{\alpha\beta}^a F^{\alpha\beta}_a - (\bar{\Psi} i \not{\partial} \Psi - \lambda \bar{\Psi} \Psi) - |D\phi|^2 - V(\phi) \right)$$

Einstein Maxwell Dirac Yukawa

$\int d^4x$ $\sqrt{-g}$ $-\frac{R}{16\pi G}$ $-\frac{1}{4} F_{\alpha\beta}^a F^{\alpha\beta}_a$ $-(\bar{\Psi} i \not{\partial} \Psi - \lambda \bar{\Psi} \Psi)$ $-|D\phi|^2 - V(\phi)$

$\underbrace{U(1) \times SU(2) \times SU(3)}_{\text{electroweak strong force}}$

"internal" $F^a_{\mu\nu}$ $F^a_{\mu\nu}$

grand unified force

Higgs

Einstein

Maxwell

Dirac

Yukawa

$$\int d^4x \sqrt{-g} \left(-\frac{R}{16\pi G} \right)$$

$$-\frac{1}{4} F_{\alpha\beta}^a F^{\alpha\beta a}$$

$$-i\bar{\psi}\gamma^\mu\partial_\mu\psi$$

$$-\lambda\phi^4$$

Higgs

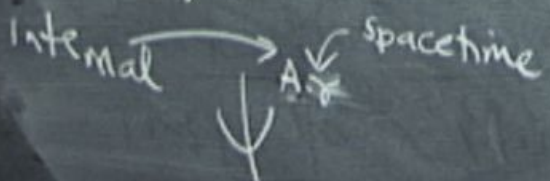
$$+|D\phi|^2 - V(\phi)$$

$$A^\mu A^\mu \leftarrow N$$

$U(1) \times SU(2) \times SU(3)$
electroweak strong force
grand unified force

"internal"

$$F_{\mu\nu}^a \quad F_{\mu\nu}^a$$



$$S = \int d^4x \sqrt{-g} \left(\overset{\text{Einstein}}{-\frac{R}{16\pi G}} - \overset{\text{Maxwell}}{\frac{1}{4} F_{\alpha\beta}^a F^{\alpha\beta}_a} - \overset{\text{Dirac}}{i \bar{\Psi} \not{D} \Psi} - \overset{\text{Yukawa}}{\lambda \bar{\Psi} \Psi \phi} + \overset{\text{Higgs}}{|D\phi|^2 - V(\phi)} \right)$$

$$A^\mu A^\mu \leftarrow N$$

$U(1) \times SU(2) \times SU(3)$
 electromagnetic strong force
 "internal" field force

$$F_{\mu\nu}^a$$

$$\delta S = 0 \Rightarrow \text{classical field equations}$$

$$S = \int d^4x \sqrt{-g} \left(\overset{\text{Einstein}}{-\frac{R}{16\pi G}} - \overset{\text{Maxwell}}{\frac{1}{4} F_{\alpha\beta}^2} - \overset{\text{Dirac}}{i\bar{\Psi}\not{D}\Psi} - \overset{\text{Yukawa}}{\bar{\Psi}\Phi\Psi} + \overset{\text{Higgs}}{|D\Phi|^2 - V(\Phi)} \right)$$

$U(1) \times SU(2) \times SU(3)$
 electromagnetic strong force
 grand unified force



$\delta S = 0 \Rightarrow$ classical field equations

Amplitude $\text{in} \rightarrow \text{out} = \int D(\text{fields}) e^{i\frac{S}{\hbar}}$

$$S = \int d^4x \sqrt{-g} \left(\overset{\text{Einstein}}{-\frac{R}{16\pi G}} - \overset{\text{Maxwell}}{\frac{1}{4} F_{\alpha\beta}^2} - \overset{\text{Dirac}}{\bar{\psi} i \not{D} \psi} - \overset{\text{Yukawa}}{\lambda \phi \bar{\psi} \psi} + |D\phi|^2 - V(\phi) \right)$$

$$\delta S = 0 \Rightarrow \text{classical field equations}$$

$$\text{Amplitude}_{\text{in} \rightarrow \text{out}} = \int_{\text{in}}^{\text{out}} D(\text{fields}) e^{iS/\hbar}$$

Invertible: inverse $g^{\alpha\beta} g_{\beta\gamma} = \delta^\alpha_\gamma$

$$g'_{\mu\nu} = \frac{\partial x'^\mu}{\partial x^\alpha} \frac{\partial x'^\nu}{\partial x^\beta} g^{\alpha\beta}$$

Invertible: inverse $g^{\alpha\beta} g_{\beta\gamma} = \delta^\alpha_\gamma$

$$g'^{\mu\nu} = \frac{\partial x'^\mu}{\partial x^\alpha} \frac{\partial x'^\nu}{\partial x^\beta} g^{\alpha\beta}$$

$$e^{\frac{i\hbar}{k}}$$

Invertible: inverse $g^{\alpha\beta} g_{\beta\gamma} = \delta^\alpha_\gamma$

$$g'_{\mu\nu} = \frac{\partial x'^\mu}{\partial x^\alpha} \frac{\partial x'^\nu}{\partial x^\beta} g^{\alpha\beta}$$

can use $g^{\alpha\beta}$ or $g_{\mu\nu}$ to raise or lower indices

$$T^\alpha_{\mu\nu} = g^{\alpha\beta} T_{\beta\mu\nu}$$

$$e^{\frac{i\hbar}{k}}$$

Invertible: inverse $g^{\alpha\beta} g_{\beta\gamma} = \delta^\alpha_\gamma$

$$g'_{\mu\nu} = \frac{\partial x'^\mu}{\partial x^\alpha} \frac{\partial x'^\nu}{\partial x^\beta} g^{\alpha\beta}$$

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can use $g^{\alpha\beta}$ or $g_{\mu\nu}$ to raise or lower indices

$$T^\alpha_{\mu\nu} = g^{\alpha\beta} T_{\beta\mu\nu}$$

$\frac{t_2}{t_1}$

Invertible: inverse $g^{\alpha\beta} g_{\beta\gamma} = \delta^\alpha_\gamma$

$$g'_{\mu\nu}(x') = \left(\frac{\partial x'^\mu}{\partial x^\alpha} \right) \left(\frac{\partial x'^\nu}{\partial x^\beta} \right) g^{\alpha\beta}(x)$$

$$g \leftrightarrow \eta$$

$$\frac{\partial x'}{\partial x} \leftrightarrow \Lambda$$

can use $g^{\alpha\beta}$ or $g_{\mu\nu}$ to raise or lower indices

$$T^\alpha_{\mu\nu} = g^{\alpha\beta} T_{\beta\mu\nu}$$

$$e^{\frac{i\hbar}{\hbar}}$$

Invertible inverse $g^{\alpha\beta} g_{\beta\gamma} = \delta^\alpha_\gamma$

$$g'_{\mu\nu}(x') = \left(\frac{\partial x'^\mu}{\partial x^\alpha} \right) \left(\frac{\partial x'^\nu}{\partial x^\beta} \right) g^{\alpha\beta}(x)$$

$$\begin{array}{ccc} g & \longleftrightarrow & \eta \\ \frac{\partial x'}{\partial x} & \longleftrightarrow & \Lambda \end{array}$$

can use $g^{\alpha\beta}$ or $g_{\mu\nu}$ to raise or lower indices

$$T^\alpha_{\mu\nu} = g^{\alpha\beta} T_{\beta\mu\nu}$$

$$e^{\frac{i\sigma_k}{2}} = \exp\left(\frac{i\sigma_k}{2}\right)$$

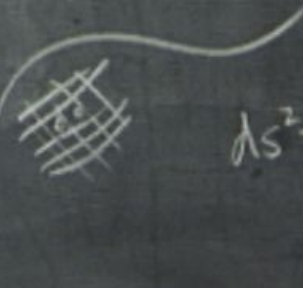
Invertible: inverse $g^{\alpha\beta} g_{\beta\gamma} = \delta^\alpha_\gamma$

$$g'_{\mu\nu}(x') = \left(\frac{\partial x'^\mu}{\partial x^\alpha} \right) \left(\frac{\partial x'^\nu}{\partial x^\beta} \right) g^{\alpha\beta}(x)$$

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$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$e^{\frac{i\hbar}{\hbar}}$$

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if $g = \eta$ $\frac{\partial x'}{\partial x} = \Lambda$ $x' = \Lambda x$

$$\eta' = \frac{\partial x'}{\partial x} \frac{\partial x'}{\partial x} \eta$$

$$= \eta$$

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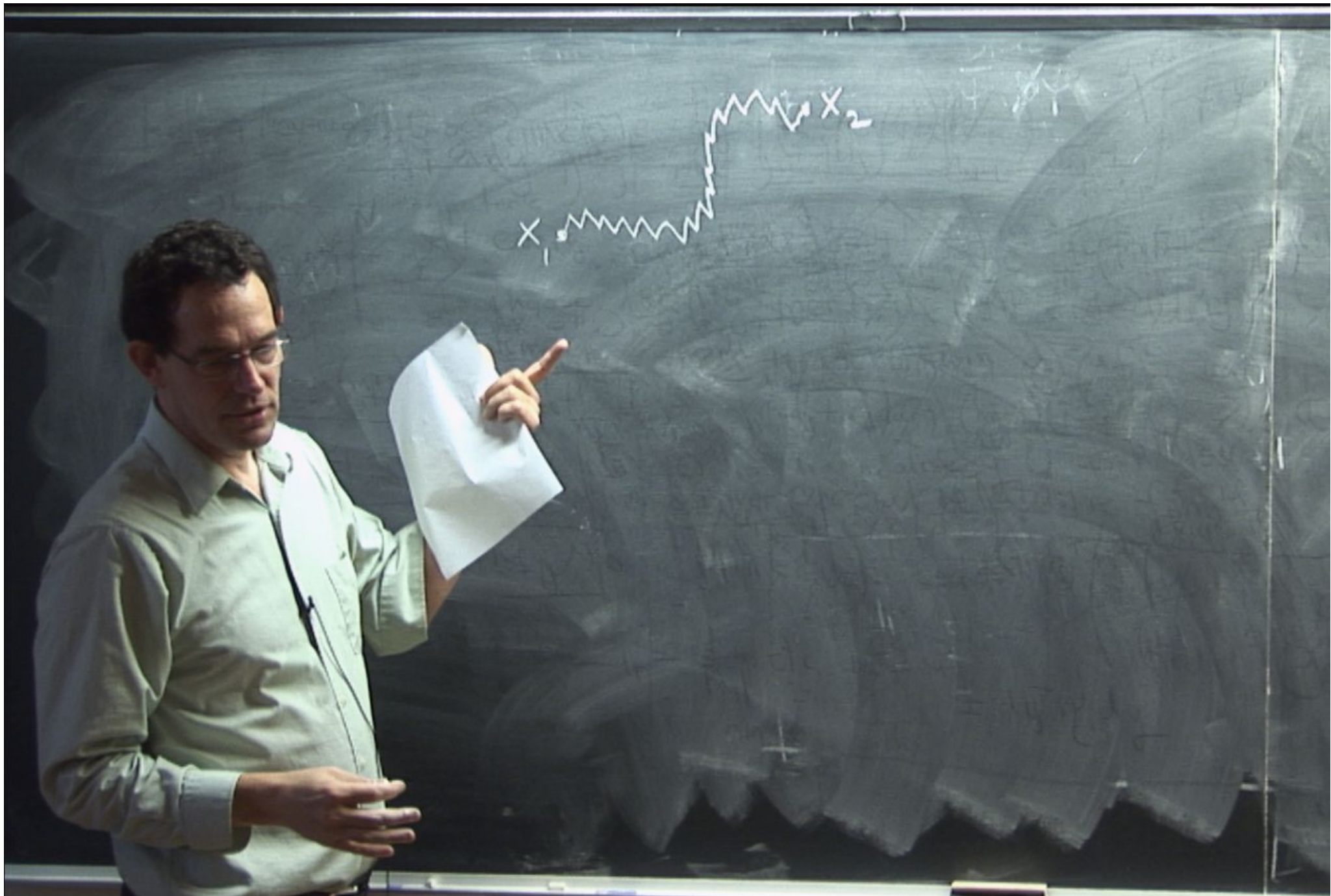
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Example

$$S^2 \in \mathbb{E}^3$$

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metric

$$\left. \begin{aligned} x &= \sin \theta \cos \phi \\ y &= \sin \theta \sin \phi \\ z &= \cos \theta \end{aligned} \right\} \text{embedding}$$

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$$dx^2 + dy^2 + dz^2 = d\theta^2 + \sin^2 \theta d\phi^2$$

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$$dx = \cos \theta d\theta \cos \phi - \sin \theta \sin \phi d\phi$$

etc.

$$\Rightarrow g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & \sin^2\theta \end{pmatrix}$$

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The connection

$$\frac{\partial}{\partial x'} \phi(x') = \frac{\partial x}{\partial x'} \frac{\partial}{\partial x}$$

Scalar $\phi(x)$ $\rightarrow$ $\frac{\partial}{\partial x^m} \phi$
 $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} . \\ . \end{bmatrix}$

isometric

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isometrical

The connection

$$\frac{\partial}{\partial x'} \phi(x') = \underbrace{\frac{\partial x}{\partial x'} \frac{\partial}{\partial x}}_{\text{correct}} \phi(x)$$

$$\text{Scalar } \phi(x) \rightarrow \frac{\partial}{\partial x^m} \phi(x)$$
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

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but $\frac{\partial V^m}{\partial x^v}(x)$

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 $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

but

$$\frac{\partial V^{\mu'}(x')}{\partial x'^{\nu}} = \frac{\partial x^{\alpha}}{\partial x'^{\nu}} \frac{\partial}{\partial x^{\alpha}}$$

isometrical

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Scalar $\phi(x)$
[]

$$\frac{\partial}{\partial x^m} \phi(x)$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

but

$$= \frac{\partial x^\mu}{\partial x'^\nu} \frac{\partial}{\partial x^\mu}$$

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but
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 $[0]$

but

$$= \frac{\partial x^\alpha}{\partial x'^\nu} \frac{\partial}{\partial x^\alpha} \left(\frac{\partial x'^m}{\partial x^\beta} V^\beta(x) \right)$$

$$= \underbrace{\frac{\partial x^\alpha}{\partial x'^\nu} \frac{\partial x'^m}{\partial x^\beta} \frac{\partial V^\beta}{\partial x^\alpha}}_{\text{good}} + \underbrace{\frac{\partial x^\alpha}{\partial x'^\nu} \frac{\partial^2 x'^m}{\partial x^\alpha \partial x^\beta}}_{\text{bad}} V^\beta$$

The connection

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alar $\phi(x) \rightarrow \frac{\partial}{\partial x^m} \phi(x)$
 $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$+ \frac{\partial V^{M'}}{\partial x'^{\nu}}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\nu}} \frac{\partial}{\partial x^{\alpha}} \left(\frac{\partial x'^M}{\partial x^{\beta}} V^{\beta}(x) \right)$$

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introduce new object to cancel bad term.

$p(x)$

$\begin{array}{c|c} X & M \\ \hline X & \partial x B \end{array}$

introduce new object to cancel bad term.

This is the connection, allows tensors of any valence to be differentiated to produce new tensors.

transformation law

$$\Gamma'^{\mu}_{\nu\rho} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} \frac{\partial x^{\alpha}}{\partial x'^{\nu}} \frac{\partial x^{\beta}}{\partial x'^{\rho}} \Gamma^{\alpha}_{\beta\gamma}$$

$$\frac{\partial x'^{\mu}}{\partial x^{\alpha}} \frac{\partial x^{\alpha}}{\partial x'^{\nu}} \frac{\partial x^{\beta}}{\partial x'^{\rho}} \Gamma^{\alpha}_{\beta\gamma}$$

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note:
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note

$$\frac{\partial x^\lambda}{\partial x'^\rho} \frac{\partial x'^\mu}{\partial x^\lambda} = \frac{\partial x'^\mu}{\partial x'^\rho} = \delta_\rho^\mu$$

$$\Rightarrow \frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\mu}$$

note

$$\frac{\partial x^\lambda}{\partial x'^\rho} \frac{\partial x'^\mu}{\partial x^\lambda} = \frac{\partial x'^\mu}{\partial x'^\rho} = \delta_\rho^\mu$$

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note

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$$\Rightarrow \frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho} \frac{\partial x'^\mu}{\partial x^\lambda} = - \frac{\partial x^\lambda}{\partial x'^\rho} \left(\frac{\partial x^\lambda}{\partial x'^\nu} \frac{\partial^2 x'^\mu}{\partial x^\lambda \partial x^\lambda} \right)$$

$\frac{\partial}{\partial x'^\nu}$

note

$$\frac{\partial x^\lambda}{\partial x'^\rho} \frac{\partial x'^\mu}{\partial x^\lambda} = \frac{\partial x'^\mu}{\partial x'^\rho} = \delta_\rho^\mu$$

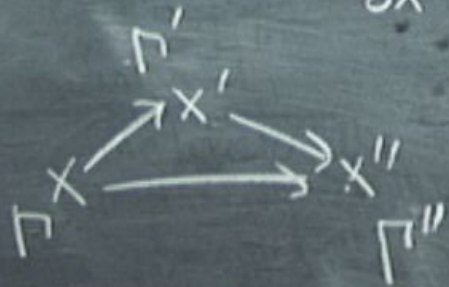
$$\Rightarrow \frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho} \frac{\partial x'^\mu}{\partial x^\lambda} = - \frac{\partial x^\lambda}{\partial x'^\rho} \left(\frac{\partial x^\lambda}{\partial x'^\nu} \frac{\partial^2 x'^\mu}{\partial x^\lambda \partial x^\lambda} \right)$$

$\frac{\partial}{\partial x'^\nu}$

note

$$\frac{\partial x^\lambda}{\partial x'^\rho} \frac{\partial x'^\mu}{\partial x^\lambda} = \frac{\partial x'^\mu}{\partial x'^\rho} = \delta_\rho^\mu$$

$$\Rightarrow \frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho} \frac{\partial x'^\mu}{\partial x^\lambda} = - \frac{\partial x^\lambda}{\partial x'^\rho} \left(\frac{\partial x^\lambda}{\partial x'^\nu} \frac{\partial^2 x'^\mu}{\partial x^\lambda \partial x^\lambda} \right)$$



Hughston + Tod

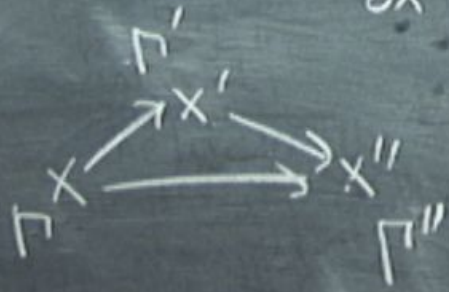
Get same Γ'' for $x' \rightarrow x' \rightarrow x''$
as for $x \rightarrow x''$ directly.

note

$$\frac{\partial x^\lambda}{\partial x'^\rho} \frac{\partial x'^\mu}{\partial x^\lambda} = \frac{\partial x'^\mu}{\partial x'^\rho} = \delta_\rho^\mu$$

$$\Rightarrow \frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho} \frac{\partial x'^\mu}{\partial x^\lambda} = - \frac{\partial x^\lambda}{\partial x'^\rho} \left(\frac{\partial x^\lambda}{\partial x'^\nu} \frac{\partial^2 x'^\mu}{\partial x^\lambda \partial x^\lambda} \right)$$

$\frac{\partial}{\partial x'^\nu}$



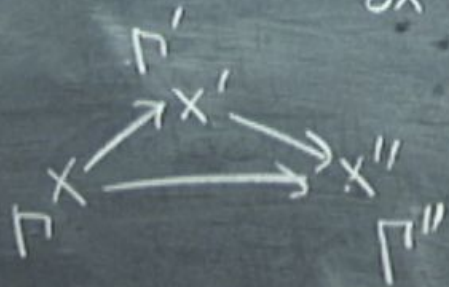
Hughston + Tod

Get same Γ'' for $x' \rightarrow x' \rightarrow x''$
as for $x \rightarrow x''$ directly.

note

$$\frac{\partial x^\lambda}{\partial x'^\rho} \frac{\partial x'^\mu}{\partial x^\lambda} = \frac{\partial x'^\mu}{\partial x'^\rho} = \delta^\mu_\rho$$

$$\Rightarrow \frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho} \frac{\partial x'^\mu}{\partial x^\lambda} = - \frac{\partial x^\lambda}{\partial x'^\rho} \left(\frac{\partial x^\lambda}{\partial x'^\nu} \frac{\partial^2 x'^\mu}{\partial x^\lambda \partial x^\lambda} \right) \frac{\partial}{\partial x'^\nu}$$



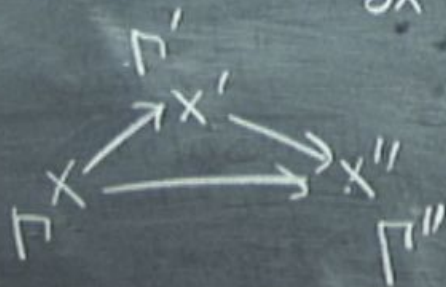
Hughston + Tod

Get same Γ'' for $x' \rightarrow x' \rightarrow x''$
as for $x \rightarrow x''$ directly.

note

$$\frac{\partial x^\lambda}{\partial x'^\rho} \frac{\partial x'^\mu}{\partial x^\lambda} \Rightarrow \frac{\partial x'^\mu}{\partial x'^\rho} = \delta_\rho^\mu$$

$$\Rightarrow \frac{\partial^2 x^\lambda}{\partial x'^\nu \partial x'^\rho} \frac{\partial x'^\mu}{\partial x^\lambda} = - \frac{\partial x^\lambda}{\partial x'^\rho} \left(\frac{\partial x^\lambda}{\partial x'^\nu} \frac{\partial^2 x'^\mu}{\partial x^\lambda \partial x'^\nu} \right)$$



Hughston + Tod
p. 53

Get same Γ'' for $x' \rightarrow x''$
as for $x \rightarrow x''$ directly.

Differentiating Tensors to get new Tensors

$$\frac{dx^m}{dx^k}$$

Differentiating Tensors to get new Tensors

$$\Gamma' = \frac{\partial x'}{\partial x} \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} \Gamma + \frac{\partial x'}{\partial x} \frac{\partial^2 x}{\partial x \partial x'}$$

$x' \rightarrow x$
 $x \rightarrow x'$

$x' \rightarrow x' \rightarrow x''$
rectly.

Differentiating Tensors to get new Tensors

$$\Gamma' = \frac{\partial x'}{\partial x} \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} \Gamma + \underbrace{\frac{\partial x'}{\partial x} \frac{\partial^2 x}{\partial x \partial x}}_{- \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} \frac{\partial^2 x'}{\partial x \partial x}}$$

Differentiating Tensors to get new Tensors

$$\frac{x'^{\mu}}{x^{\alpha} \partial x^{\lambda}}$$

$$\Gamma'^{\lambda} = \frac{\partial x'^{\lambda}}{\partial x^{\alpha}} \frac{\partial x^{\alpha}}{\partial x'^{\beta}} \frac{\partial x^{\beta}}{\partial x^{\gamma}} \Gamma^{\gamma} + \underbrace{\frac{\partial x'^{\lambda}}{\partial x^{\alpha}} \frac{\partial^2 x^{\alpha}}{\partial x'^{\beta} \partial x'^{\gamma}}}_{= \frac{\partial x^{\alpha}}{\partial x'^{\beta}} \frac{\partial x^{\beta}}{\partial x'^{\gamma}} \frac{\partial^2 x'^{\lambda}}{\partial x^{\alpha} \partial x^{\beta}}}$$

$$\frac{\partial x'^{\alpha}}{\partial x^{\beta}} = J^{\alpha}_{\beta}$$

$$\frac{\partial x^{\alpha}}{\partial x'^{\beta}} = \tilde{J}^{\alpha}_{\beta}$$

$x \rightarrow x' \rightarrow x''$
rectly.

$$\partial'_{\mu} V^{\nu'} = \tilde{J}^{\alpha}_{\mu} \partial_{\alpha} (J^{\nu'}_{\beta} V^{\beta})$$

Differentiating Tensors to get new Tensors

$$\Gamma'^{\alpha} = \frac{\partial x'^{\alpha}}{\partial x^{\beta}} \frac{\partial x^{\gamma}}{\partial x'^{\mu}} \Gamma^{\mu} + \underbrace{\frac{\partial x'^{\alpha}}{\partial x^{\beta}} \frac{\partial^2 x^{\gamma}}{\partial x^{\mu} \partial x^{\nu}}}_{-\underbrace{\frac{\partial x^{\gamma}}{\partial x'^{\mu}} \frac{\partial x^{\mu}}{\partial x'^{\alpha}} \frac{\partial^2 x'^{\alpha}}{\partial x^{\beta} \partial x^{\gamma}}}_{\tilde{\Gamma}^{\alpha}_{\beta\gamma}}}$$

$$\Gamma'^{\alpha} = \tilde{\Gamma}^{\alpha}_{\beta\gamma} \Gamma^{\gamma}$$

$$\partial'_{\mu} V'^{\alpha} = \tilde{\Gamma}^{\alpha}_{\beta\gamma} \partial (J^{\beta}_{\mu} V^{\gamma})$$

$$= \tilde{\Gamma}^{\alpha}_{\beta\gamma} J^{\beta}_{\mu} \partial V^{\gamma} + \tilde{\Gamma}^{\alpha}_{\beta\gamma} V^{\gamma} \partial J^{\beta}_{\mu}$$

Differentiating Tensors to get new Tensors

$$\frac{\partial x'^{\mu}}{\partial x^{\alpha}}$$

$$\Gamma'^{\lambda}_{\mu\nu} = \frac{\partial x'^{\lambda}}{\partial x^{\alpha}} \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\alpha}}{\partial x'^{\nu}} \Gamma^{\lambda}_{\alpha\beta} + \underbrace{\frac{\partial x'^{\lambda}}{\partial x^{\alpha}} \frac{\partial^2 x^{\alpha}}{\partial x'^{\mu} \partial x'^{\nu}}}_{-\frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\alpha}}{\partial x'^{\nu}} \frac{\partial^2 x'^{\lambda}}{\partial x^{\alpha} \partial x^{\beta}}}$$

$$\frac{\partial x'^{\mu}}{\partial x^{\alpha}} = J^{\mu}_{\alpha}$$

$$\frac{\partial x^{\alpha}}{\partial x'^{\mu}} = \tilde{J}^{\alpha}_{\mu}$$

$x \rightarrow x' \rightarrow x''$
rectly.

$$\partial'_{\mu} V^{\nu'} = \tilde{J}^{\alpha}_{\mu} \partial_{\alpha} (J^{\nu'}_{\beta} V^{\beta})$$

$$= \tilde{J}^{\alpha}_{\mu} J^{\nu'}_{\beta} \partial_{\alpha} V^{\beta} + \tilde{J}^{\alpha}_{\mu} V^{\beta} \partial_{\alpha} J^{\nu'}_{\beta}$$

$$\Gamma'^{\lambda}_{\mu\nu} V^{\nu'} = J^{\lambda}_{\alpha} \tilde{J}^{\alpha}_{\mu} \tilde{J}^{\alpha}_{\nu} \Gamma^{\lambda}_{\alpha\beta}$$

Differentiating Tensors to get new Tensors

x'^M
 $x \rightarrow \partial x$

$x \rightarrow x' \rightarrow x''$
 rectly.

$$\Gamma' = \frac{\partial x'}{\partial x} \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} \Gamma + \underbrace{\frac{\partial x'}{\partial x} \frac{\partial^2 x}{\partial x \partial x}}_{\tilde{J} J \partial J} - \underbrace{\frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} \frac{\partial^2 x'}{\partial x \partial x}}_{\tilde{J} J \partial J}$$

$$\frac{\partial x'^\alpha}{\partial x^\beta} = J^\alpha_\beta$$

$$\frac{\partial x^\alpha}{\partial x'^\beta} = \tilde{J}^\alpha_\beta$$

$$\partial'_\mu V^{\nu'} = \tilde{J}^\alpha_\mu \partial (J^\nu_\alpha V)$$

$$= \tilde{J}^\alpha_\mu J^\nu_\alpha \partial V + \tilde{J}^\alpha_\mu V \partial J^\nu_\alpha$$

$$\Gamma^{\nu'}_{\alpha\mu} V^{\alpha'} = J^\nu_\alpha \tilde{J}^{\alpha'}_\mu \Gamma^\alpha_{\nu\beta} V^\beta - \tilde{J}^{\alpha'}_\mu \partial J^\nu_\alpha J^\alpha_{\nu'} V$$

Differentiating Tensors to get new Tensors

$$x'^{\mu}$$

$$x \rightarrow \partial x^{\lambda}$$

$x \rightarrow x' \rightarrow x''$
re. chg.

$$\Gamma'^{\lambda}_{\mu\nu} = \frac{\partial x'^{\lambda}}{\partial x^{\alpha}} \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\alpha}}{\partial x'^{\nu}} \Gamma^{\alpha}_{\beta\gamma} + \underbrace{\frac{\partial x'^{\lambda}}{\partial x^{\alpha}} \frac{\partial^2 x^{\alpha}}{\partial x'^{\mu} \partial x'^{\nu}}}_{-\frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\alpha}}{\partial x'^{\nu}} \frac{\partial^2 x'^{\lambda}}{\partial x^{\alpha} \partial x^{\beta}}}$$

$$\frac{\partial x'^{\mu}}{\partial x^{\alpha}} = J^{\mu}_{\alpha}$$

$$\frac{\partial x^{\alpha}}{\partial x'^{\mu}} = \tilde{J}^{\alpha}_{\mu}$$

$$V'^{\nu} = \frac{\partial x'^{\nu}}{\partial x^{\alpha}} V^{\alpha}$$

$$\partial'_{\mu} V'^{\nu} = \tilde{J}^{\alpha}_{\mu} \partial (J^{\nu}_{\beta} V^{\beta})$$

$$= \tilde{J}^{\alpha}_{\mu} J^{\nu}_{\beta} \partial V^{\beta} + \tilde{J}^{\alpha}_{\mu} V^{\beta} \partial J^{\nu}_{\beta}$$

$$\Gamma^{\nu}_{\alpha\mu} V^{\alpha} = J^{\nu}_{\beta} \tilde{J}^{\alpha}_{\mu} \partial V^{\beta} - \tilde{J}^{\alpha}_{\mu} \partial J^{\nu}_{\beta} J^{\beta}_{\gamma} V^{\gamma}$$

Differentiating Tensors to get new Tensors

$$x'^{\mu}$$

$$\frac{\partial x'^{\mu}}{\partial x^{\alpha}}$$

$x' \rightarrow x' \rightarrow x''$
re ctly.

$$\Gamma' = \frac{\partial x'}{\partial x} \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} \Gamma + \frac{\partial x'}{\partial x} \frac{\partial^2 x}{\partial x \partial x'} - \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} \frac{\partial^2 x'}{\partial x \partial x}$$

$$\frac{\partial x'^{\mu}}{\partial x^{\alpha}} = J^{\mu}_{\alpha}$$

$$\frac{\partial x'^{\mu}}{\partial x^{\alpha}} = \tilde{J}^{\mu}_{\alpha}$$

$$\frac{\partial x'^{\mu}}{\partial x^{\alpha}} = \tilde{J}^{\mu}_{\alpha}$$

$$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} V^{\alpha}$$

$$\partial_{\mu} V'^{\mu} = \tilde{J} \partial (J V)$$

$$= \tilde{J} J \partial V + \tilde{J} V \partial J$$

$$\Gamma^{\mu}_{\alpha\mu} V^{\alpha} = \tilde{J} \tilde{J} \tilde{J} \Gamma J V - \tilde{J} \tilde{J} \partial \Gamma$$

Differentiating Tensors to get new Tensors

$$x'^{\mu}$$

$$\frac{\partial x'^{\mu}}{\partial x^{\alpha}}$$

$x \rightarrow x' \rightarrow x''$
rectly.

$$\Gamma'^{\lambda}_{\mu\nu} = \frac{\partial x'^{\lambda}}{\partial x^{\alpha}} \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\alpha}}{\partial x'^{\nu}} \Gamma^{\alpha}_{\beta\gamma} + \frac{\partial x'^{\lambda}}{\partial x^{\alpha}} \frac{\partial^2 x^{\alpha}}{\partial x'^{\mu} \partial x'^{\nu}}$$

$$\frac{\partial x'^{\mu}}{\partial x^{\alpha}} = J^{\mu}_{\alpha}$$

$$\frac{\partial x^{\alpha}}{\partial x'^{\mu}} = \tilde{J}^{\alpha}_{\mu}$$

$$- \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} \frac{\partial^2 x^{\gamma}}{\partial x^{\alpha} \partial x^{\beta}}$$

$$\tilde{J} \tilde{J} \partial J$$

$$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} V^{\alpha}$$

$$J$$

$$\partial'_{\mu} V'^{\nu} = \tilde{J}^{\alpha}_{\mu} \partial_{\alpha} (J^{\nu}_{\beta} V^{\beta})$$

$$= \tilde{J}^{\alpha}_{\mu} J^{\nu}_{\beta} \partial_{\alpha} V^{\beta} + \tilde{J}^{\alpha}_{\mu} V^{\beta} \partial_{\alpha} J^{\nu}_{\beta}$$

$$\Gamma^{\gamma}_{\alpha\mu} V^{\alpha} = \tilde{J}^{\gamma}_{\delta} \tilde{J}^{\alpha}_{\mu} \tilde{J}^{\beta}_{\nu} \Gamma^{\gamma}_{\alpha\beta} V^{\nu} - \tilde{J}^{\gamma}_{\delta} \tilde{J}^{\alpha}_{\mu} \partial_{\alpha} J^{\nu}_{\beta} J^{\beta}_{\nu} V^{\gamma}$$

$$\downarrow \delta \quad \downarrow \delta$$

Differentiating Tensors to get new Tensors

$$x'^{\mu}$$

$$\frac{\partial x'^{\mu}}{\partial x^{\alpha}}$$

$$\Gamma'^{\lambda}_{\mu\nu} = \frac{\partial x'^{\lambda}}{\partial x^{\alpha}} \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\alpha}}{\partial x'^{\nu}} \Gamma^{\lambda}_{\alpha\beta} + \underbrace{\frac{\partial x'^{\lambda}}{\partial x^{\alpha}} \frac{\partial^2 x^{\alpha}}{\partial x'^{\mu} \partial x'^{\nu}}}_{-\frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\alpha}}{\partial x'^{\nu}} \frac{\partial^2 x'^{\lambda}}{\partial x^{\alpha} \partial x^{\beta}}}$$

$$\frac{\partial x'^{\mu}}{\partial x^{\alpha}} = J^{\mu}_{\alpha}$$

$$\frac{\partial x^{\alpha}}{\partial x'^{\mu}} = \tilde{J}^{\alpha}_{\mu}$$

$$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} V^{\alpha}$$

$x' \rightarrow x' \rightarrow x''$
rectly.

$$\partial'_{\mu} V'^{\nu} = \tilde{J}^{\alpha}_{\mu} \partial_{\alpha} (J^{\nu}_{\beta} V^{\beta})$$

$$= \tilde{J}^{\alpha}_{\mu} J^{\nu}_{\beta} \partial_{\alpha} V^{\beta} + \tilde{J}^{\alpha}_{\mu} V^{\beta} \partial_{\alpha} J^{\nu}_{\beta} \quad \swarrow \text{cancel.}$$

$$\Gamma'^{\nu}_{\mu\alpha} V'^{\alpha} = \underbrace{\tilde{J}^{\alpha}_{\mu} \tilde{J}^{\beta}_{\alpha} \Gamma^{\nu}_{\beta\gamma}}_{\delta^{\nu}_{\mu}} J^{\gamma}_{\delta} V^{\delta} - \underbrace{\tilde{J}^{\alpha}_{\mu} \tilde{J}^{\beta}_{\alpha} \partial_{\beta} J^{\nu}_{\gamma}}_{\delta^{\nu}_{\mu}} J^{\gamma}_{\delta} V^{\delta}$$

Differentiating Tensors to get new Tensors

$$\Gamma' = \underbrace{\tilde{J} \tilde{J} \tilde{J}}_{\tilde{J} \tilde{J} \partial \tilde{J}} \frac{\partial x'}{\partial x} \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} \Gamma + \frac{\partial x'}{\partial x} \frac{\partial^2 x}{\partial x \partial x'} - \underbrace{\frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} \frac{\partial^2 x'}{\partial x \partial x}}_{\tilde{J} \tilde{J} \partial \tilde{J}}$$

$$\frac{\partial x'}{\partial x} = J^\alpha_\beta$$

$$\frac{\partial x}{\partial x'} = \tilde{J}^\alpha_\beta$$

$$V'^\nu = \underbrace{\frac{\partial x'}{\partial x^\alpha}}_{\tilde{J}} V^\alpha$$

$$\partial'_\mu V'^\nu = \tilde{J} \partial (J V)$$

$$= \tilde{J} J \partial V + \tilde{J} V \partial J \quad \swarrow \text{cancel.}$$

$$\Gamma^\gamma_{\alpha\mu} V^\alpha = \underbrace{\tilde{J} \tilde{J} \tilde{J} \Gamma}_{\downarrow \delta} J V - \tilde{J} \tilde{J} \underbrace{\partial J J}_{\downarrow \delta} V$$

$$\Rightarrow \partial'_m V'^\alpha + \Gamma'_{\alpha m} V'^\alpha =$$

$$\Rightarrow \partial'_m V'^\mu + \Gamma^{\mu}_{\alpha m} V'^\alpha = () () () () (\partial_m V^\mu + \Gamma^{\mu}_{\alpha m} V^\alpha)$$

i.e.,

$$\Rightarrow \partial'_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha = () () () (\partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha)$$

$$V^\nu = \partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha$$

is a $[1]$ tensor.

$$U_\nu = \partial_\mu U_\nu - \Gamma$$

$$\Rightarrow \partial'_\mu V'^\nu + \Gamma_{\alpha\mu}^\nu V'^\alpha = (\quad)(\quad)(\quad) (\partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha)$$

i.e. $\nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha$

$[1]$ tensor.

$$= \partial_\mu U_\nu - \Gamma_{\nu\mu}^\alpha U_\alpha$$

$[0]$ te

$$\Rightarrow \partial'_\mu V'^\nu + \Gamma'^\nu_{\alpha\mu} V'^\alpha = () () () (\partial_\mu V^\nu + \Gamma^\nu_{\alpha\mu} V^\alpha)$$

$$\text{i.e. } \nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\alpha\mu} V^\alpha$$

is a $[1]$ tensor.

$$\text{Since } \nabla_\mu U_\nu = \partial_\mu U_\nu - \Gamma^\alpha_{\nu\mu} U_\alpha$$

is a $[0, 1]$ tensor.

$$\Rightarrow \partial'_\mu V'^\nu + \Gamma'^\nu_{\alpha\mu} V'^\alpha = () () () (\partial_\mu V^\nu + \Gamma^\nu_{\alpha\mu} V^\alpha)$$

$$\text{i.e. } \nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\alpha\mu} V^\alpha$$

is a $[1]$ tensor.

$$\text{Since } \nabla_\mu U_\nu = \partial_\mu U_\nu - \Gamma^\alpha_{\nu\mu} U_\alpha$$

is a $[0]$ tensor.

$$+ \Gamma_{\mu}^{\nu} V^{\alpha}$$

Differentiating Tensors to get new Tensors

$$\Gamma' = \underbrace{\frac{\partial x'}{\partial x} \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'}}_{\tilde{J} \tilde{J} \tilde{J}} \Gamma + \underbrace{\frac{\partial x'}{\partial x} \frac{\partial^2 x}{\partial x \partial x'}}_{\tilde{J} \tilde{J} \partial \tilde{J}} - \underbrace{\frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} \frac{\partial^2 x'}{\partial x \partial x}}_{\tilde{J} \tilde{J} \partial \tilde{J}}$$

$$V'^{\nu} = \frac{\partial x'^{\nu}}{\partial x^{\alpha}} V^{\alpha}$$

$$\frac{\partial x'^{\mu}}{\partial x^{\alpha}} = J^{\mu}_{\alpha}$$

$$\frac{\partial x^{\alpha}}{\partial x'^{\mu}} = \tilde{J}^{\alpha}_{\mu}$$

$$\partial'_{\mu} V'^{\nu} = \tilde{J}^{\alpha}_{\mu} \partial (J^{\nu}_{\alpha} V^{\alpha})$$

$$= \tilde{J}^{\alpha}_{\mu} J^{\nu}_{\alpha} \partial V^{\alpha} + \tilde{J}^{\alpha}_{\mu} V^{\alpha} \partial J^{\nu}_{\alpha} \quad \swarrow \text{cancel.}$$

$$\Gamma^{\nu}_{\mu} V^{\alpha} = \underbrace{\tilde{J}^{\alpha}_{\mu} \tilde{J}^{\beta}_{\alpha} \Gamma^{\nu}_{\beta}}_{\delta^{\nu}_{\mu}} V - \underbrace{\tilde{J}^{\alpha}_{\mu} \tilde{J}^{\beta}_{\alpha} \partial J^{\nu}_{\beta}}_{\delta^{\nu}_{\mu}} J^{\nu}_{\alpha} V$$

$$+ \left(\Gamma_{\alpha\mu}^{\nu} V^{\alpha} \right)$$

$$\frac{x' \partial^2 x}{\partial x \partial x' \partial x}$$

$$- \frac{\frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} \frac{\partial^2 x'}{\partial x \partial x}}{\tilde{J} \tilde{J} \partial \tilde{J}}$$

$$\mu V^{\alpha'} = \tilde{J} \partial (J V)$$

$$= \tilde{J} J \partial V + \tilde{J} V$$

$$\Gamma_{\alpha\mu}^{\nu} V^{\alpha} = \tilde{J} \tilde{J} \tilde{\Gamma}_{\alpha\mu}^{\nu} V - \tilde{\delta}^{\nu}_{\mu}$$

general rule

$$\nabla_\mu (V_\nu^{\alpha\beta}) = \partial_\mu V_\nu^{\alpha\beta} + \Gamma_\mu^\alpha$$

general rule

$$\nabla_\mu (V_\nu^{\alpha\beta}) = \partial_\mu V_\nu^{\alpha\beta} + \Gamma_{\sigma\mu}^\alpha V_\nu^{\sigma\beta} + \Gamma_{\sigma\mu}^\beta V_\nu^{\alpha\sigma} - \Gamma_{\mu\nu}^\sigma V_\sigma^{\alpha\beta}$$

general rule

$$\nabla_\mu (V_\nu^{\alpha\beta}) = \partial_\mu V_\nu^{\alpha\beta} + \Gamma_{\mu\nu}^\alpha V_\nu^{\gamma\beta} + \Gamma_{\mu\nu}^\beta V_\nu^{\alpha\gamma} - \Gamma_{\nu\mu}^\sigma V_\sigma^{\alpha\beta}$$

i.e. + Γ f stairs

general rule

$$\nabla_\mu (V_\nu^{\alpha\beta}) = \partial_\mu V_\nu^{\alpha\beta} + \Gamma_{\sigma\mu}^\alpha V_\nu^{\sigma\beta} + \Gamma_{\sigma\mu}^\beta V_\nu^{\alpha\sigma} - \Gamma_{\nu\mu}^\sigma V_\sigma^{\alpha\beta}$$

i.e. $+\Gamma$ for each upstairs index
 $-\Gamma$ " " downstairs index.

$$+ \Gamma_{\mu}^{\nu} V^{\alpha}$$

general rule

$$\nabla_{\mu}(V_{\nu}^{\alpha\beta}) = \partial_{\mu} V_{\nu}^{\alpha\beta} + \Gamma_{\sigma\mu}^{\alpha} V_{\nu}^{\sigma\beta} + \Gamma_{\sigma\mu}^{\beta} V_{\nu}^{\alpha\sigma} - \Gamma_{\nu\mu}^{\sigma} V_{\sigma}^{\alpha\beta}$$

i.e. $+\Gamma$ for each upstairs index
 $-\Gamma$ " " downstairs index.

$$+ \Gamma_{\mu}^{\nu} V^{\alpha}$$

general rule

$$\nabla_{\mu}(V_{\nu}^{\alpha\beta}) = \partial_{\mu} V_{\nu}^{\alpha\beta} + \Gamma_{\mu}^{\alpha} V_{\nu}^{\gamma\beta} + \Gamma_{\mu}^{\beta} V_{\nu}^{\alpha\gamma} - \Gamma_{\nu}^{\mu} V_{\gamma}^{\alpha\beta}$$

i.e. $+\Gamma$ for each upstairs index
 $-\Gamma$ " " downstairs index.

$$+ \Gamma_{\mu}^{\nu} V^{\alpha}$$

general rule

$$\nabla_{\mu}(V_{\nu}^{\alpha\beta}) = \partial_{\mu} V_{\nu}^{\alpha\beta} + \Gamma_{\sigma\mu}^{\alpha} V_{\nu}^{\sigma\beta} + \Gamma_{\sigma\mu}^{\beta} V_{\nu}^{\alpha\sigma} - \Gamma_{\nu\mu}^{\sigma} V_{\sigma}^{\alpha\beta}$$

i.e. $+\Gamma$ for each upstairs index
 $-\Gamma$ " " downstairs index.

general rule

$$\nabla_{(\mu} (V_{\nu}^{\alpha\beta}) = \partial_{\mu} V_{\nu}^{\alpha\beta} + \Gamma_{\gamma\mu}^{\alpha} V_{\nu}^{\gamma\beta} - \Gamma_{\gamma\mu}^{\nu} V_{\nu}^{\alpha\gamma}$$

i.e. $+\Gamma$ for each upper index
 $-\Gamma$ " " lower index.

general rule

$$\nabla_{\mu}(V_{\nu}^{\alpha\beta}) = \partial_{\mu}V_{\nu}^{\alpha\beta} + \Gamma_{\mu}^{\alpha} V_{\nu}^{\delta\beta} + \Gamma_{\mu}^{\beta} V_{\nu}^{\alpha\delta} - \Gamma_{\nu\mu}^{\gamma} V_{\gamma}^{\alpha\beta}$$

i.e. $+\Gamma$ for each upstairs index
 $-\Gamma$ " " downstairs index.

general rule

$$\nabla_{\mu}(V_{\nu}^{\alpha\beta}) = \partial_{\mu}V_{\nu}^{\alpha\beta} + \Gamma_{\mu}^{\alpha} V_{\nu}^{\delta\beta} + \Gamma_{\mu}^{\beta} V_{\nu}^{\alpha\delta} - \Gamma_{\nu\mu}^{\gamma} V_{\gamma}^{\alpha\beta}$$

i.e. $+\Gamma$ for each upstairs index
 $-\Gamma$ " " downstairs index.

$$\Rightarrow \partial'_\mu V'^\nu + \Gamma'^\nu_{\alpha\mu} V'^\alpha = () () () (\partial_\mu V^\nu + \Gamma^\nu_{\alpha\mu} V^\alpha)$$

$$\text{i.e. } \nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\alpha\mu} V^\alpha$$

is a $[1]$ tensor.

$$\text{Since } \nabla_\mu U_\nu = \partial_\mu U_\nu - \Gamma^\alpha_{\nu\mu} U_\alpha$$

is a $[2]$ tensor.

genera

$$\nabla_\mu (V_\nu)$$

i.e. $+\Gamma$

$$V^\mu U^\nu g_{\mu\nu}$$

general rule

$$\nabla_{\mu}(V_{\nu}^{\alpha\beta}) = \partial_{\mu}V_{\nu}^{\alpha\beta} + \Gamma_{\mu}^{\alpha} V_{\nu}^{\delta\beta} + \Gamma_{\mu}^{\beta} V_{\nu}^{\alpha\delta} - \Gamma_{\nu\mu}^{\delta} V_{\delta}^{\alpha\beta}$$

i.e. $+\Gamma$ for each upstairs index
 $-\Gamma$ " " downstairs index.

$$V^{\mu} U^{\nu} g_{\mu\nu}$$

$$\Gamma_{\mu\nu}^{\lambda} A_{\lambda} U^{\mu} U^{\nu}$$

general rule

$$\nabla_\mu (V_\nu^{\alpha\beta}) = \partial_\mu V_\nu^{\alpha\beta} + \Gamma_{\mu\alpha}^\alpha V_\nu^{\alpha\beta} + \Gamma_{\mu\beta}^\beta V_\nu^{\alpha\beta} + \Gamma_{\mu\nu}^\gamma V_\gamma^{\alpha\beta} - \Gamma_{\nu\mu}^\gamma V_\gamma^{\alpha\beta}$$

i.e. $+\Gamma$ for each upstairs index
 $-\Gamma$ " " downstairs index.

$$V^\mu U^\nu g_{\mu\nu}$$

$$\Gamma_{\mu\nu}^\lambda A_\lambda U^\mu U^\nu$$

general rule

$$\nabla_{\mu}(V_{\nu}^{\alpha\beta}) = \partial_{\mu} V_{\nu}^{\alpha\beta} + \Gamma_{\mu}^{\alpha\gamma} V_{\nu}^{\gamma\beta} + \Gamma_{\mu}^{\beta\gamma} V_{\nu}^{\alpha\gamma} - \Gamma_{\mu}^{\gamma\nu} V_{\gamma}^{\alpha\beta}$$

i.e. $+\Gamma$ for this index
 $-\Gamma$ for this index.

$$V^{\mu} U^{\nu} g_{\mu\nu}$$

general rule

$$\nabla_{\mu}(V_{\nu}^{\alpha\beta}) = \partial_{\mu} V_{\nu}^{\alpha\beta} + \Gamma_{\mu}^{\alpha} V_{\nu}^{\delta\beta} + \Gamma_{\mu}^{\beta} V_{\nu}^{\alpha\delta} - \Gamma_{\nu\mu}^{\gamma} V_{\gamma}^{\alpha\beta}$$

i.e. $+\Gamma$ for each upstairs index
 $-\Gamma$ " " downstairs index.

$$V^{\mu} U^{\nu} g_{\mu\nu}$$

$$\Gamma_{\mu\nu}^{\lambda} A_{\lambda} U^{\mu} U^{\nu}$$

$$= \dots (\dots) (\dots) (\partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha)$$

Fid

general rule

$$\nabla_\mu (V_\nu^{\alpha\beta}) = \partial_\mu V_\nu^{\alpha\beta} + \Gamma_{\alpha\mu}^\alpha V_\nu^\beta + \Gamma_{\beta\mu}^\beta V_\nu^\alpha$$

i.e. $+\Gamma$ for each upstairs index
 $-\Gamma$ " " downstairs

$$V^\mu U^\nu g_{\mu\nu}$$

$$\Gamma_{\mu\nu}^\lambda A_\lambda U^\mu U^\nu$$

$$= \left(\right) \left(\right) \left(\right) \left(\partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha \right)$$

general rule

$$\nabla_\mu (V_\nu^{\alpha\beta}) = \partial_\mu V_\nu^{\alpha\beta} + \Gamma_{\alpha\mu}^\alpha V_\nu^\beta + \Gamma_{\beta\mu}^\beta V_\nu^\alpha$$

i.e. $+\Gamma$ for each upstairs index
 $-\Gamma$ " " downstairs

$$V^\mu U^\nu g_{\mu\nu}$$

$$\Gamma_{\mu\nu}^\lambda A_\lambda U^\mu U^\nu$$

$$= \left(\right) \left(\right) \left(\right) \left(\partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha \right)$$

$$\Gamma_{\alpha\mu}^\nu V^\alpha$$

tensor.

$$\Gamma_{\alpha\beta}^\mu$$

$$T^M_{\nu\rho}$$

Fid $\downarrow$

general rule

$$\nabla_\mu (V_\nu^{\alpha\beta}) = \partial_\mu V_\nu^{\alpha\beta} + \Gamma_{\alpha\mu}^\alpha V_\nu^\beta + \Gamma_{\beta\mu}^\beta V_\nu^\alpha$$

i.e. $+\Gamma$ for each upstairs index
 $-\Gamma$ " " downstairs

$$V^\mu U^\nu g_{\mu\nu}$$

$$\Gamma_{\mu\nu}^\lambda A_\lambda U^\mu U^\nu$$

$$= \left(\right) \left(\right) \left(\right) \left(\partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha \right)$$

$$+ \Gamma_{\alpha\mu}^\nu V^\alpha$$

tensor.

$$\Gamma_{\alpha\mu}^\nu$$

$T_{\mu\nu\lambda}$

general rule

$$\nabla_\mu (V_\nu^{\alpha\beta}) = \partial_\mu V_\nu^{\alpha\beta} + \Gamma_{\alpha\mu}^\alpha V_\nu^{\alpha\beta} + \Gamma_{\beta\mu}^\beta V_\nu^{\alpha\beta}$$

i.e. $+\Gamma$ for each upstairs
 $-\Gamma$ " " downstairs

$$V^\mu U^\nu g_{\mu\nu}$$

$$\Gamma_{\mu\nu}^\lambda A_\lambda U^\mu U^\nu$$

$$= (\gamma) (\gamma) (\gamma) (\partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha)$$

$$+ \Gamma_{\alpha\mu}^\nu V^\alpha$$

tensor.

$$U_\nu - \Gamma_{\nu\mu}^\alpha U_\alpha$$

tensor.

$$\Gamma_{\alpha\beta}^\mu$$

$\bar{\psi} i \not{D} \psi$

$$T_{\mu\nu\rho\lambda}$$

$$= \frac{1}{2} \epsilon_{\mu\nu\rho\lambda} J_5^\lambda$$

general rule

$$\nabla_\mu (V_\nu^{\alpha\beta}) = \partial_\mu V_\nu^{\alpha\beta} + \Gamma_{\alpha\mu}^\alpha V_\nu^\beta + \Gamma_{\beta\mu}^\beta V_\nu^\alpha$$

spin current for fermions

i.e. $+\Gamma$ for each upstairs index
 $-\Gamma$ " " downstairs

$$V^\mu U^\nu g_{\mu\nu}$$

$$\Gamma_{\mu\nu}^\lambda A_\lambda U^\mu U^\nu$$

$$= \left(\begin{matrix} \\ \\ \end{matrix} \right) \left(\partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha \right)$$

$$+ \Gamma_{\alpha\mu}^\nu V^\alpha$$

tensor.

$$U_\nu - \Gamma_{\nu\mu}^\alpha U_\alpha$$

tensor.

$$\Gamma_{\alpha\beta}^\mu$$

Field

$$T_{\mu\nu\rho}$$

$$= \epsilon_{\mu\nu\rho\lambda} J_5^\lambda$$

general rule

$$\nabla_\mu (V_\nu^{\alpha\beta}) = \partial_\mu V_\nu^{\alpha\beta} + \Gamma_{\alpha\mu}^\rho V_\nu^{\rho\beta} + \Gamma_{\beta\mu}^\rho V_\nu^{\alpha\rho}$$

spin current for fermions

i.e. $+\Gamma$ for each upstairs index
 $-\Gamma$ " " downstairs

$$V^\mu U^\nu g_{\mu\nu}$$

$$\Gamma_{\mu\nu}^\lambda A_\lambda U^\mu U^\nu$$

$$\Rightarrow \partial'_\mu V'^\nu + \Gamma_{\alpha\mu}^\nu V'^\alpha = (\dots)(\dots)(\dots)(\partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha)$$

$$\text{i.e. } \nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha$$

is a $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ tensor.

$$\text{Since } \nabla_\mu U_\nu = \partial_\mu U_\nu - \Gamma_{\nu\mu}^\alpha U_\alpha$$

is a $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ tensor.

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

$$\Rightarrow \partial'_\mu V'^\nu + \Gamma_{\alpha\mu}^{\nu} V'^\alpha = (\frac{\partial x^\mu}{\partial x'^\mu}) (\frac{\partial x'^\nu}{\partial x^\alpha}) (\partial_\mu V^\alpha + \Gamma_{\alpha\mu}^{\nu} V^\alpha)$$

$$\text{i.e. } \nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma_{\alpha\mu}^{\nu} V^\alpha$$

is a $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ tensor.

$$\text{Since } \nabla_\mu U_\nu = \partial_\mu U_\nu - \Gamma_{\nu\mu}^{\alpha} U_\alpha$$

is a $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ tensor.

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\nu\lambda}^{\mu} \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

$$\Rightarrow \partial'_\mu V'^\nu + \Gamma_{\alpha\mu}^{\nu} V'^\alpha = () () () (\partial_\mu V^\nu + \Gamma_{\alpha\mu}^{\nu} V^\alpha)$$

$$\text{i.e. } \nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma_{\alpha\mu}^{\nu} V^\alpha$$

is a $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ tensor.

$$\text{Since } \nabla_\mu U_\nu = \partial_\mu U_\nu - \Gamma_{\nu\mu}^{\alpha} U_\alpha$$

is a $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ tensor.

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\nu\lambda}^{\mu} \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

$$\Rightarrow \partial'_\mu V'^\nu + \Gamma_{\alpha\mu}^\nu V'^\alpha = (\dots)(\dots)(\dots)(\partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha)$$

$$g_{\mu\nu} = \eta_{\mu\nu}$$

$$g_{\mu\nu,\rho} \stackrel{?}{=} 0 \quad \text{i.e.} \quad \nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha$$

is a $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ tensor.

$$\text{Since} \quad \nabla_\mu U_\nu = \partial_\mu U_\nu - \Gamma_{\nu\mu}^\alpha U_\alpha$$

is a $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ tensor.

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

$$\Rightarrow \partial'_\mu V'^\nu + \Gamma_{\alpha\mu}^\nu V'^\alpha = (\dots)(\dots)(\dots)(\partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha)$$

$$g_{\mu\nu} = \eta_{\mu\nu}$$

$$g_{\mu\nu,\rho} \stackrel{?}{=} 0 \quad \text{i.e., } \nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha$$

is a $[1]$ tensor.

$$\text{Since } \nabla_\mu U_\nu = \partial_\mu U_\nu - \Gamma_{\nu\mu}^\alpha U_\alpha$$

is a $[2]$ tensor.

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

$$\Rightarrow \partial'_\mu V^\alpha + \Gamma_{\alpha\mu}^\alpha V^\alpha = (\quad)(\quad)(\quad)(\partial_\mu V^\alpha + \Gamma_{\alpha\mu}^\alpha V^\alpha)$$

$$g_{\mu\nu} = \eta_{\mu\nu}$$

$$g_{\mu\nu,\rho} \stackrel{?}{=} 0$$

$$\Gamma_{\nu\rho}^\rho$$

$$\text{i.e., } \nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha$$

is a $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ tensor.

$$\text{Since } \nabla_\mu U_\nu = \partial_\mu U_\nu - \Gamma_{\nu\mu}^\alpha U_\alpha$$

is a $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ tensor.

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

$$\Rightarrow \partial'_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha = (\quad)(\quad)(\quad)(\partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha)$$

$$g_{\mu\nu} = \eta_{\mu\nu}$$

$$g_{\mu\nu,\rho} \stackrel{?}{=} 0$$

$$\underbrace{g_{\mu\nu,\rho}}_{40}$$

$$\text{if } \underbrace{\Gamma_{\nu\rho}^\mu}_{40} \text{ Symm}$$

$$=$$

$$\text{i.e., } \nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha$$

is a $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ tensor.

$$\text{Siny } \nabla_\mu U_\nu = \partial_\mu U_\nu - \Gamma_{\nu\mu}^\alpha U_\alpha$$

is a $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ tensor.

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

$$\Rightarrow \partial'_\mu V'^\nu + \Gamma_{\alpha\mu}^\nu V'^\alpha = (\dots)(\dots)(\dots)(\partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha)$$

$$g_{\mu\nu} = \eta_{\mu\nu}$$

$$\stackrel{v,p}{\approx} 0?$$

$$\stackrel{v,p}{\approx} 0$$

$$\Gamma_{\nu\rho}^\mu$$

$$\Gamma_{\nu\rho}^\mu$$

$$\stackrel{v,p}{\approx} 0$$

$$\stackrel{v,p}{\approx} 0$$

$$\stackrel{v,p}{\approx} 0$$

$$\stackrel{v,p}{\approx} 0$$

$$\stackrel{v,p}{\approx} 0$$

$$\text{i.e. } \nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha$$

is a $[1]$ tensor.

$$\text{Since } \nabla_\mu U_\nu = \partial_\mu U_\nu - \Gamma_{\nu\mu}^\alpha U_\alpha$$

is a $[2]$ tensor.

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

$$\Rightarrow \partial'_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha = (\quad)(\quad)(\quad)(\partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha)$$

$$g_{\mu\nu} = \eta_{\mu\nu}$$

$$g_{\mu\nu,\rho} \stackrel{?}{=} 0$$

$$\underbrace{g_{\mu\nu,\rho}}_{40}$$

$$\downarrow \Gamma_{\nu\rho}^\mu$$

$$\underbrace{\Gamma_{\nu\rho}^\mu}_{40}$$

$$=$$

$$=$$

$$=$$

$$=$$

$$=$$

$$=$$

$$=$$

$$\text{i.e., } \nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha$$

is a $[1]$ tensor.

$$\Gamma_{\alpha\beta}^\mu$$

$$\text{Since } \nabla_\mu U_\nu = \partial_\mu U_\nu - \Gamma_{\nu\mu}^\alpha U_\alpha$$

is a $[2]$ tensor.

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

$$V^\alpha = \left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right) \left(\partial_\mu V^\mu + \Gamma_{\alpha\mu}^\mu V^\alpha \right)$$

$$\partial_\mu V^\mu + \Gamma_{\alpha\mu}^\mu V^\alpha$$

[1] tensor.

$$\Gamma_{\alpha\beta}^\mu$$

$$\partial_\mu U_\nu - \Gamma_{\nu\mu}^\alpha U_\alpha$$

tensor.

$$\Sigma^\mu$$

$$\frac{d\Sigma^\mu}{d\tau} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\tau} \Sigma^\beta = 0$$

general rule

$$\nabla_\mu (V_\nu^{\alpha\beta}) = \partial_\mu V_\nu^{\alpha\beta} + \Gamma_{\mu\nu}^\lambda V_\lambda^{\alpha\beta} + \Gamma_{\mu\alpha}^\lambda V_\nu^{\lambda\beta} + \Gamma_{\mu\beta}^\lambda V_\nu^{\alpha\lambda}$$

spin current for fermions

$$J_5^\lambda$$

i.e. +Γ for each up spin
-Γ " " down

$$V^\mu U^\nu g_{\mu\nu}$$

$$\Gamma_{\mu\nu}^\lambda A_\lambda$$

$$V^\alpha = \left(\begin{pmatrix} \end{pmatrix} \right) \left(\begin{pmatrix} \end{pmatrix} \right) \left(\partial_\mu V^\mu + \Gamma_{\alpha\mu}^\mu V^\alpha \right)$$

general rule

$$\nabla_\mu (V_\nu^{\alpha\beta}) = \partial_\mu V_\nu^\alpha + \Gamma_{\mu\nu}^\alpha V^\beta + \Gamma_{\mu\nu}^\beta V^\alpha$$

$$\partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha$$

[1] tensor.

Fib

$$T_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\lambda} J_5^\lambda$$

spin current for fermions

i.e. +Γ for each up spin

-Γ " " down

$$\Gamma_{\alpha\beta}^\mu$$

$$= \partial_\mu U_\nu - \Gamma_{\nu\mu}^\alpha U_\alpha$$

[2] tensor.

$$V^\mu U^\nu g_{\mu\nu}$$

$$\Gamma_{\mu\nu}^\lambda A_\lambda$$

$$\Sigma^\mu$$

Parallel transport

$$\frac{d\Sigma^\mu}{d\tau} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\tau} \Sigma^\beta = 0$$

$$U^\mu \frac{dx^\mu}{d\tau} = 0$$

$$V^\alpha = \left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right) \left(\partial_\mu V^\mu + \Gamma_{\alpha\mu}^\mu V^\alpha \right)$$

general rule

$$\nabla_\mu (V_\nu^{\alpha\beta}) = \partial_\mu V_\nu^{\alpha\beta} + \Gamma_{\mu\nu}^\lambda V_\lambda^{\alpha\beta} + \Gamma_{\mu\alpha}^\lambda V_\nu^{\lambda\beta} + \Gamma_{\mu\beta}^\lambda V_\nu^{\alpha\lambda}$$

$$V^\mu + \Gamma_{\alpha\mu}^\mu V^\alpha$$

tensor.

$$\Gamma_{\alpha\beta}^\mu$$

$$\partial_\mu U_\nu - \Gamma_{\nu\mu}^\alpha U_\alpha$$

tensor.

$$\frac{dS^\mu}{d\tau} = 0$$

$$T_{\mu\nu\rho} = \frac{1}{6} \epsilon_{\mu\nu\rho\lambda} J_5^\lambda$$

spin current for fermions

i.e. $+\Gamma$ for each up spin
 $-\Gamma$ " " " down

$$x^\mu(\tau)$$

Parallel transport

$$V^\mu U^\nu g_{\mu\nu}$$

$$\Gamma_{\mu\nu}^\lambda A_\lambda$$

$$\frac{dS^\mu}{d\tau} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0$$

$$\Rightarrow \partial'_\mu V'^\nu + \Gamma_{\alpha\mu}^\nu V'^\alpha = (\dots)(\dots)(\dots)(\partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha)$$

$$= \eta_{\mu\nu}$$

$$\stackrel{?}{=} 0$$

$$\text{i.e., } \nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma_{\alpha\mu}^\nu V^\alpha$$

is a $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ tensor.

$$\text{Since } \nabla_\mu U_\nu = \partial_\mu U_\nu - \Gamma_{\nu\mu}^\alpha U_\alpha$$

is a $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ tensor.

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

$$\Gamma_{\alpha\beta}^\mu$$

$$\Sigma^\mu$$

$$\frac{dS^\mu}{d\tau}$$

$$\Rightarrow \partial'_\mu V'^\nu + \Gamma'^\nu_{\alpha\mu} V'^\alpha = (\dots)(\dots)(\dots)(\partial_\mu V^\nu + \Gamma^\nu_{\alpha\mu} V^\alpha)$$

$$g_{\mu\nu} = \eta_{\mu\nu}$$

$$\stackrel{?}{=} 0? \quad \text{i.e.} \quad \nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\alpha\mu} V^\alpha$$

is a $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ tensor.

$$\text{Since} \quad \nabla_\mu U_\nu = \partial_\mu U_\nu - \Gamma^\alpha_{\nu\mu} U_\alpha$$

is a $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ tensor.

$$+ \frac{1}{2} (\Gamma^\mu_{\nu\lambda} - \Gamma^\mu_{\lambda\nu}) \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

$$\Gamma^\mu_{\alpha\beta}$$

$$\Sigma^\mu$$

$$\frac{dS^\mu}{d\tau}$$

general rule

$$g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$$

$$\nabla_\mu (V_\nu^{\alpha\beta}) = \partial_\mu$$

$$\bar{\psi} i \gamma^\mu \psi$$

$$\bar{\psi} i \not{D} \psi$$

$$T_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\lambda} J_5^\lambda$$

spin current
for fermions

i.e. $+\Gamma$ for each upstr
 $-\Gamma$ " " " down index.

$$\Gamma_{\mu\nu}^\lambda A_\lambda U^\mu U^\nu$$

Parallel transport

$$\Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0$$

general rule

$$g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$$

$$\nabla_\mu (V_\nu^{\alpha\beta}) = \partial_\mu$$

$$\bar{\psi} i \gamma^a e_\mu^a \partial_\mu \psi$$

$$\bar{\psi} i \not{D} \psi$$

$$T_{\mu\nu\rho}$$

$$= \epsilon_{\mu\nu\rho\lambda}$$

spin current
for fermions

$$J_5^\lambda$$

i.e. +1 for each upstr

-1 " " down index.

$$x^\mu(\tau)$$

$$V^\mu U^\nu g_{\mu\nu}$$

$$\Gamma_{\mu\nu}^\lambda A_\lambda U^\mu U^\nu$$

Parallel transport

$$\Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\tau} \sum^\beta = 0$$