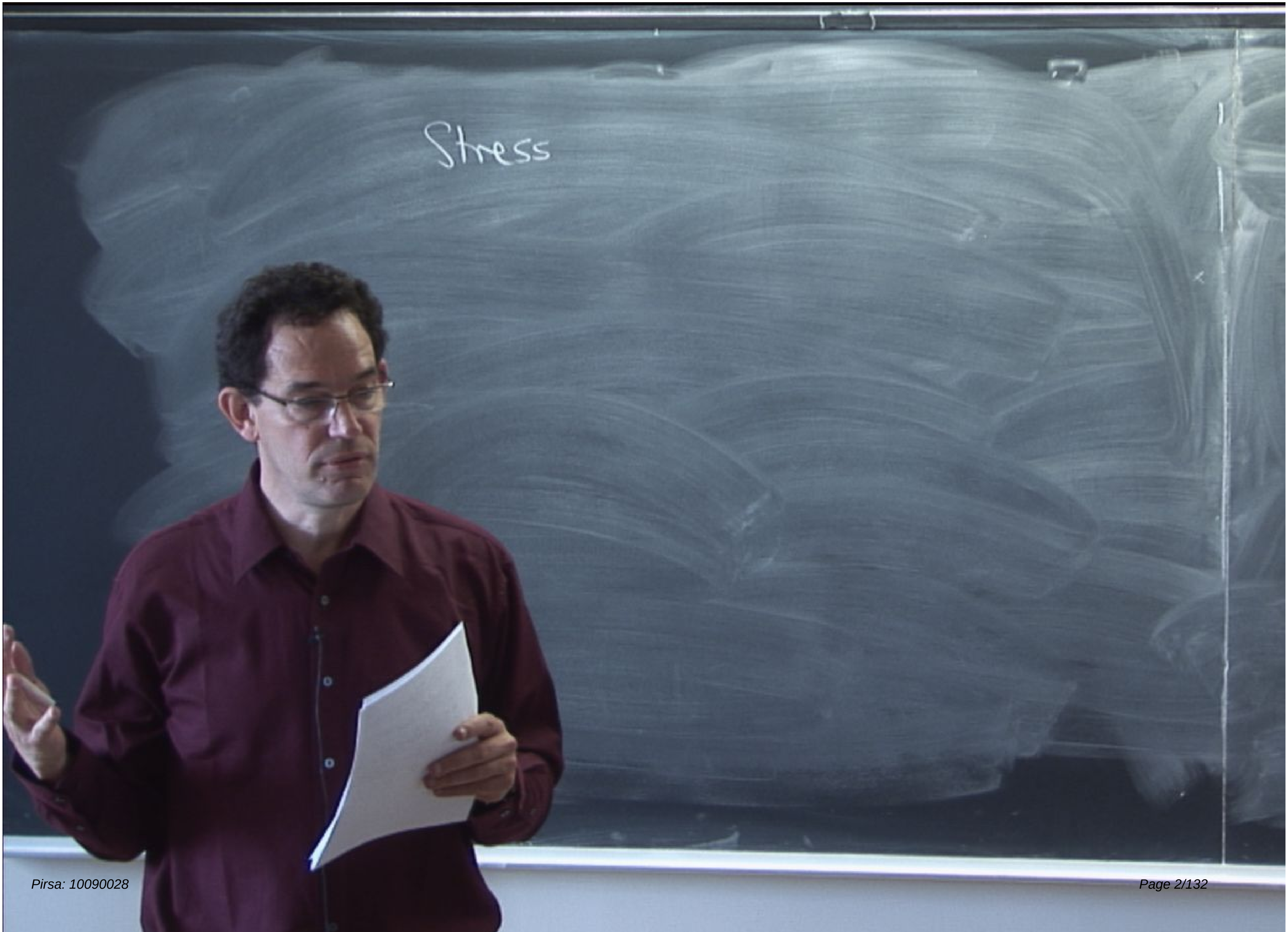
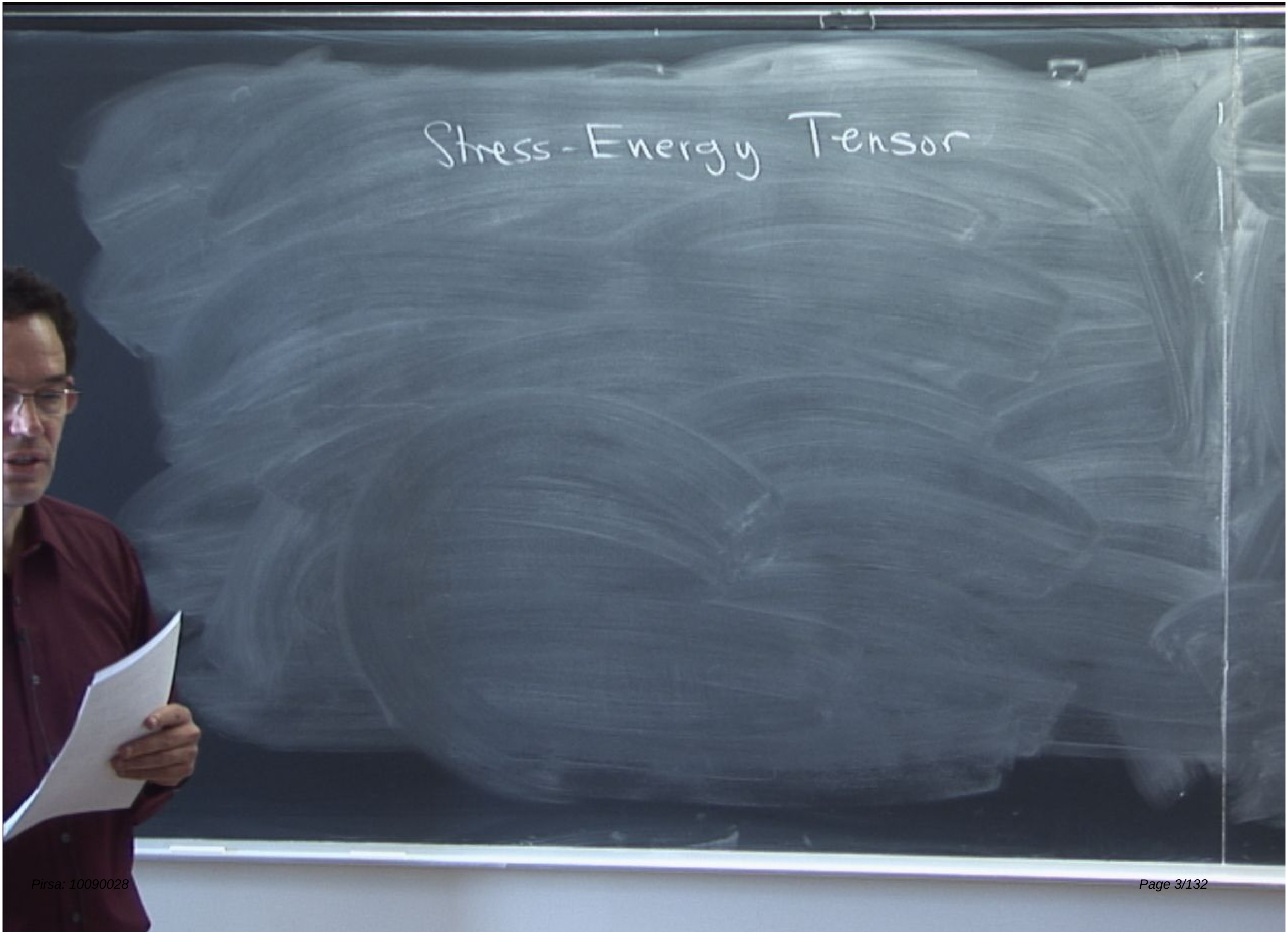
Title: Relativity (PHYS 604) - Lecture 3

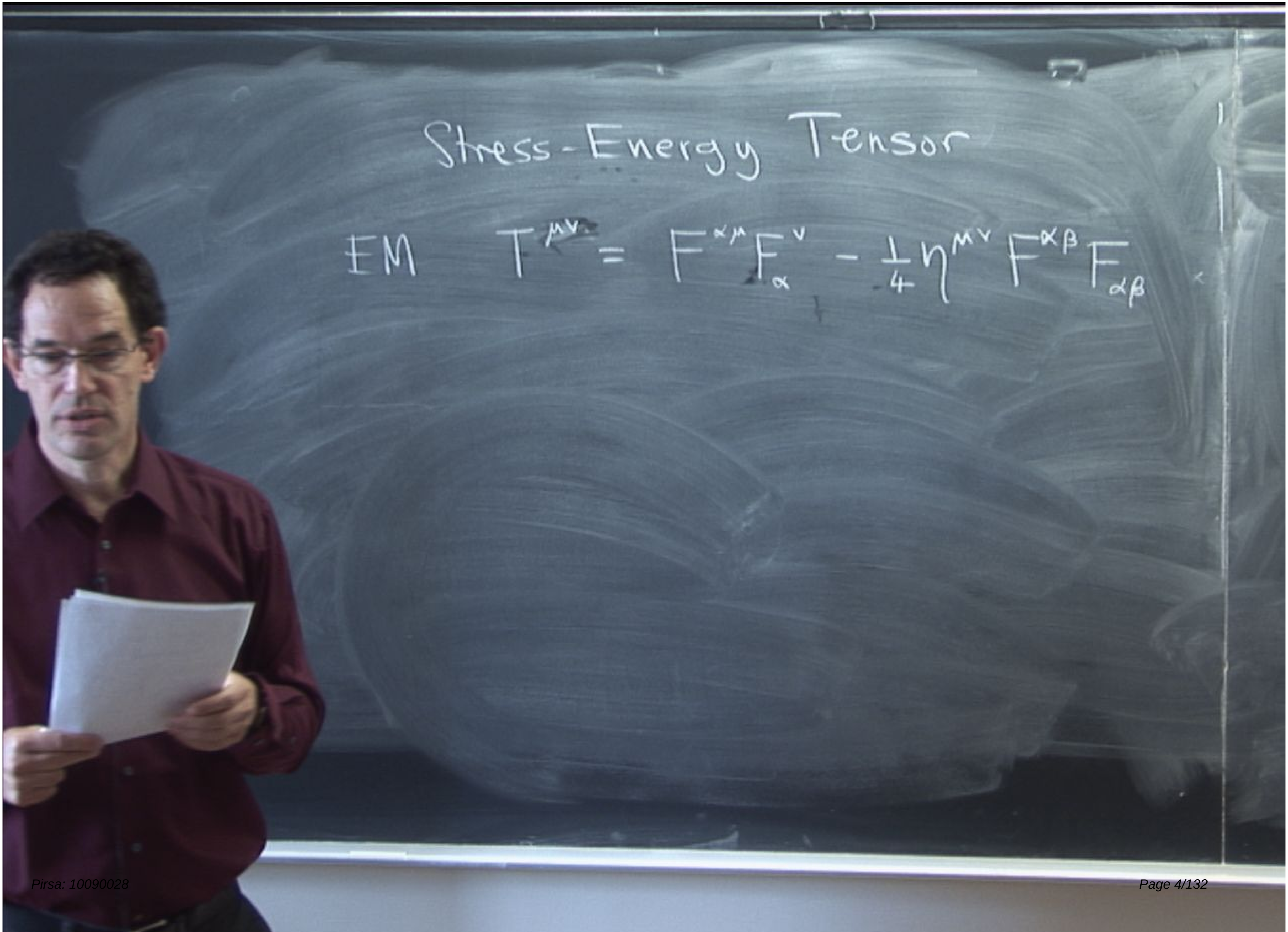
Date: Sep 15, 2010 10:30 AM

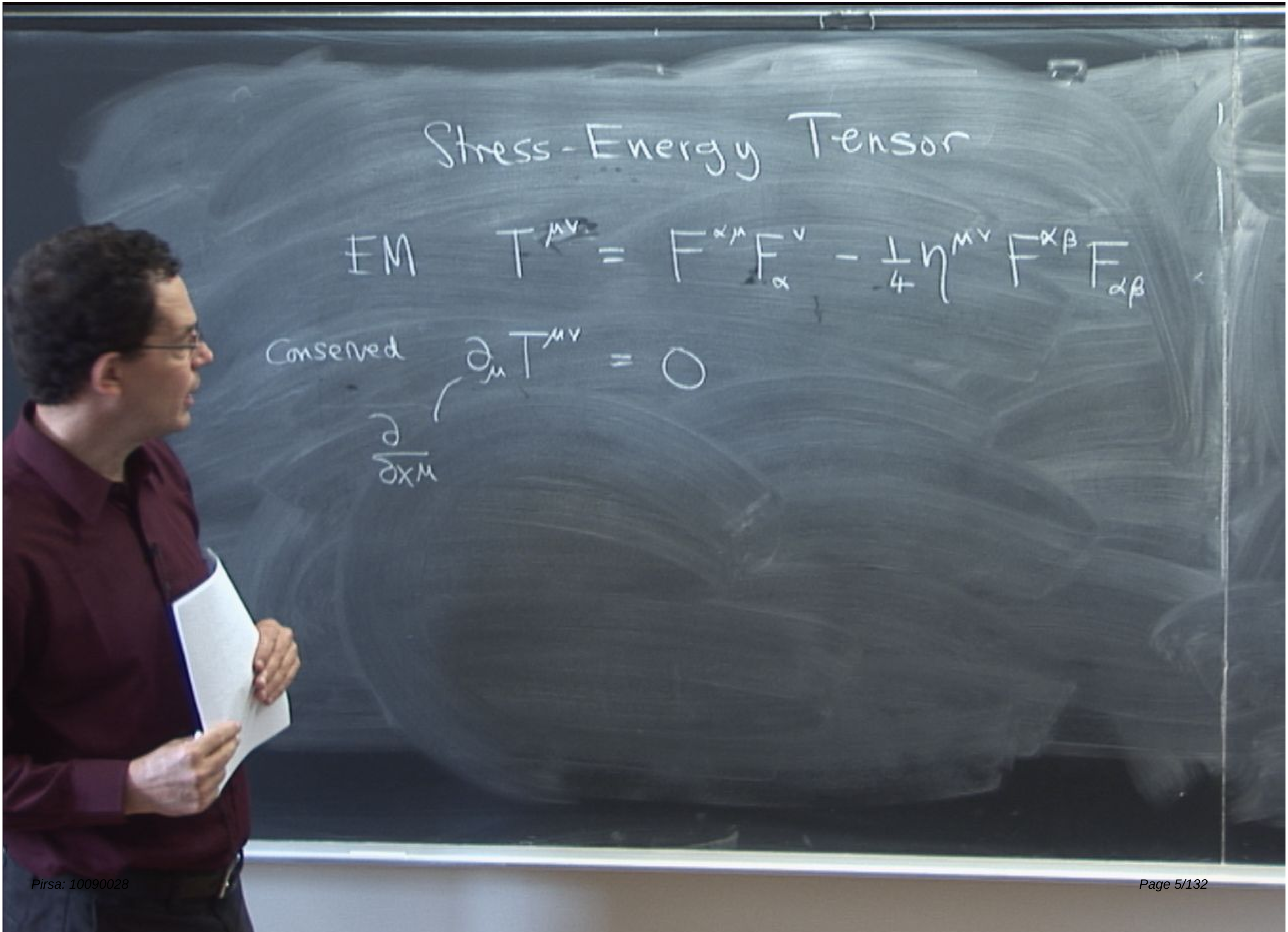
URL: http://pirsa.org/10090028

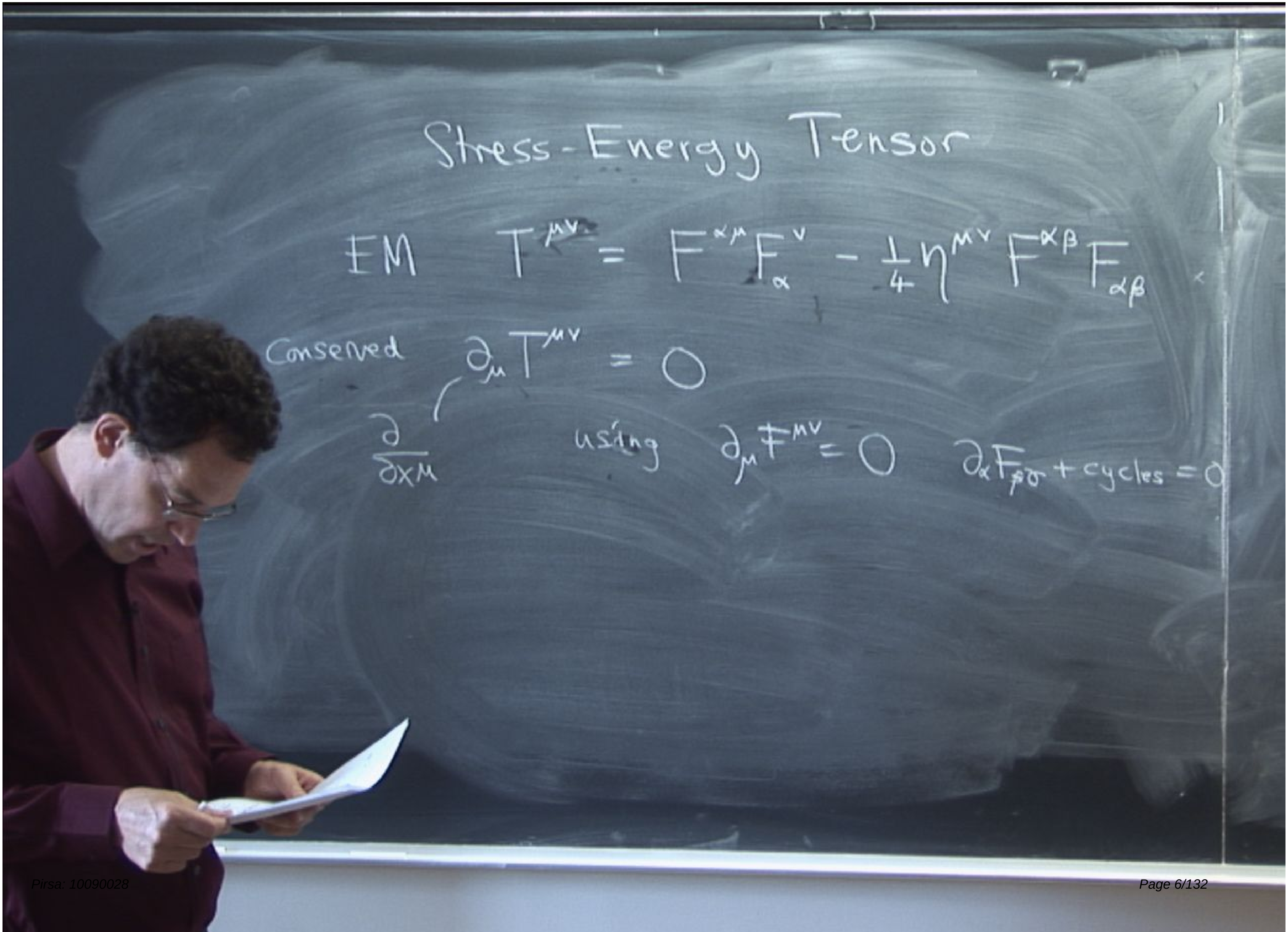
Abstract:

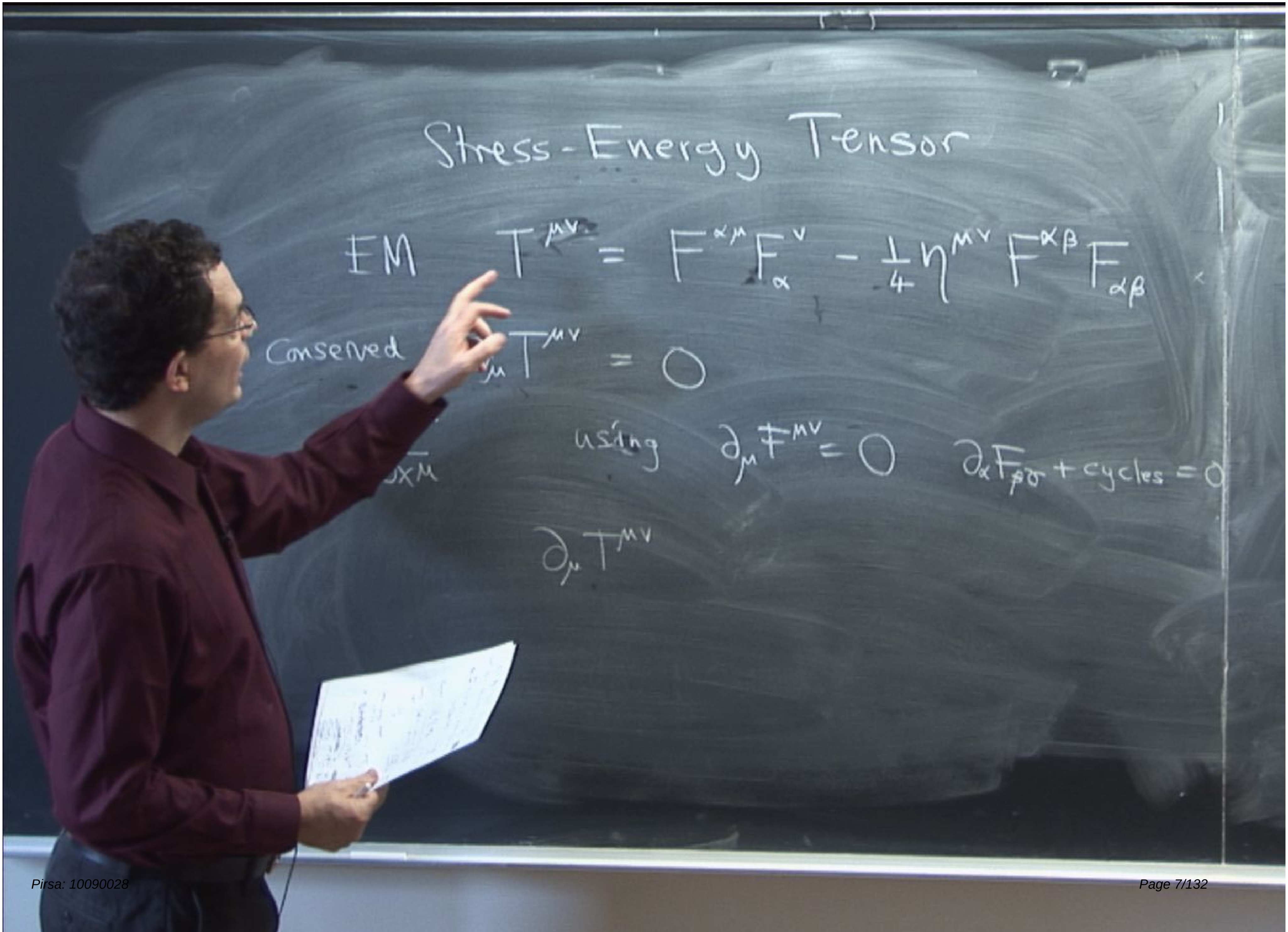

Stress


Stress-Energy Tensor


Stress-Energy Tensor
EM $T^{\mu\nu} = F^{\alpha\mu} F_{\alpha}{}^{\nu} - \frac{1}{4}\eta^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}$


Stress-Energy Tensor
EM $T^{\mu\nu} = F^{\alpha\mu}F_{\alpha}{}^{\nu} - \frac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$
Conserved $\partial_\mu T^{\mu\nu} = 0$
$\frac{\partial}{\partial x^\mu}$


Stress-Energy Tensor
EM $T^{\mu\nu} = F^{\alpha\mu}F_\alpha{}^\nu - \frac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$
Conserved $\partial_\mu T^{\mu\nu} = 0$
$\frac{\partial}{\partial x^\mu}$
using $\partial_\mu F^{\mu\nu} = 0$ $\partial_\alpha F_{\beta\sigma}$ + cycles = 0


Stress-Energy Tensor
EM $T^{\mu\nu} = F^{\alpha\mu}F_{\alpha}^{\ \nu} - \frac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$
Conserved $\partial_\mu T^{\mu\nu} = 0$
using $\partial_\mu F^{\mu\nu} = 0$ $\partial_\alpha F_{\beta\sigma}$ + cycles = 0
$\partial_\mu T^{\mu\nu}$

Stress-Energy Tensor

EM $\quad T^{\mu\nu} = F^{\alpha\mu} F_{\alpha}{}^{\nu} - \frac{1}{4}\eta^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}$

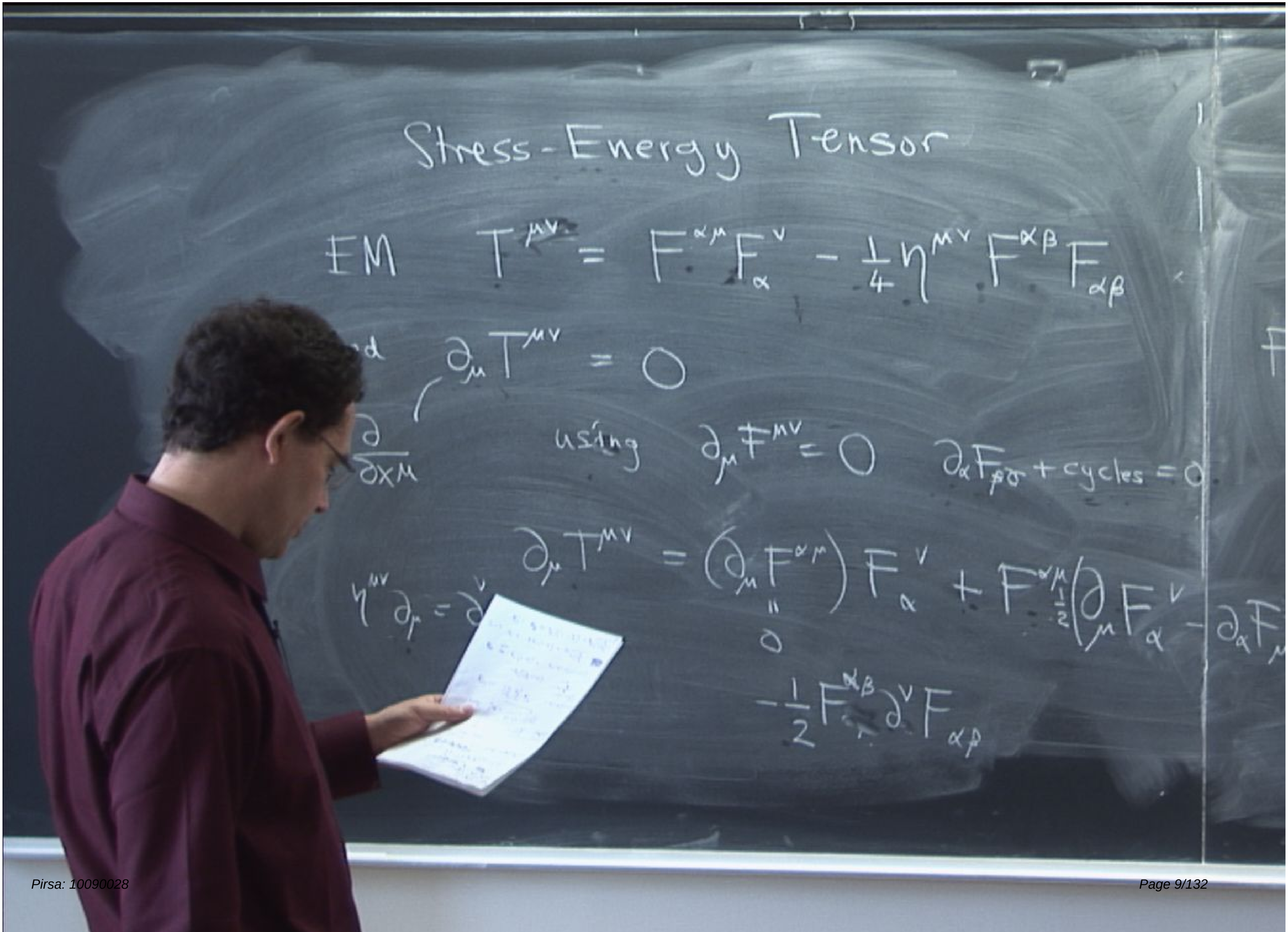
Conserved $\quad \partial_\mu T^{\mu\nu} = 0$

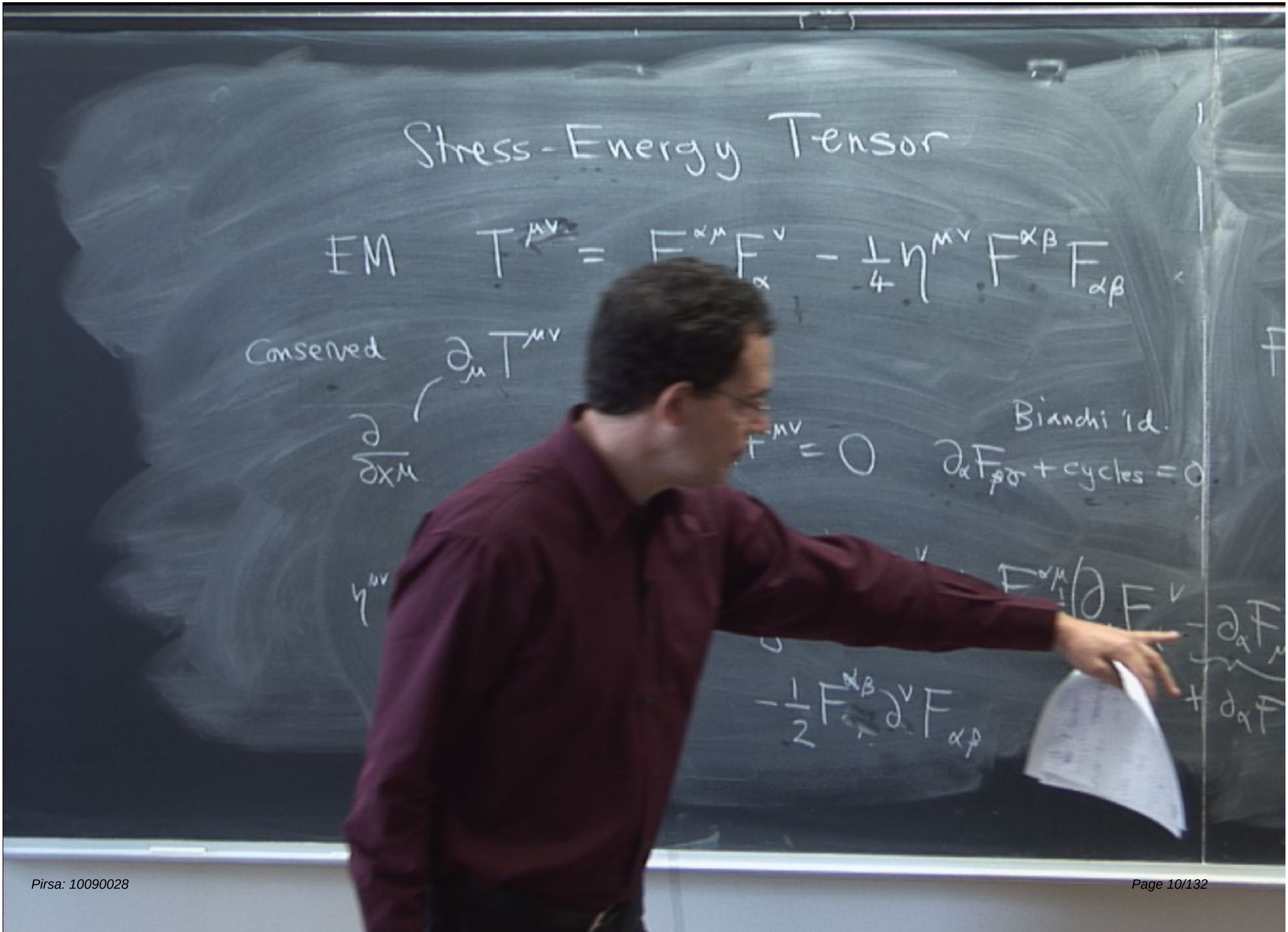
$\frac{\partial}{\partial x^\mu}$

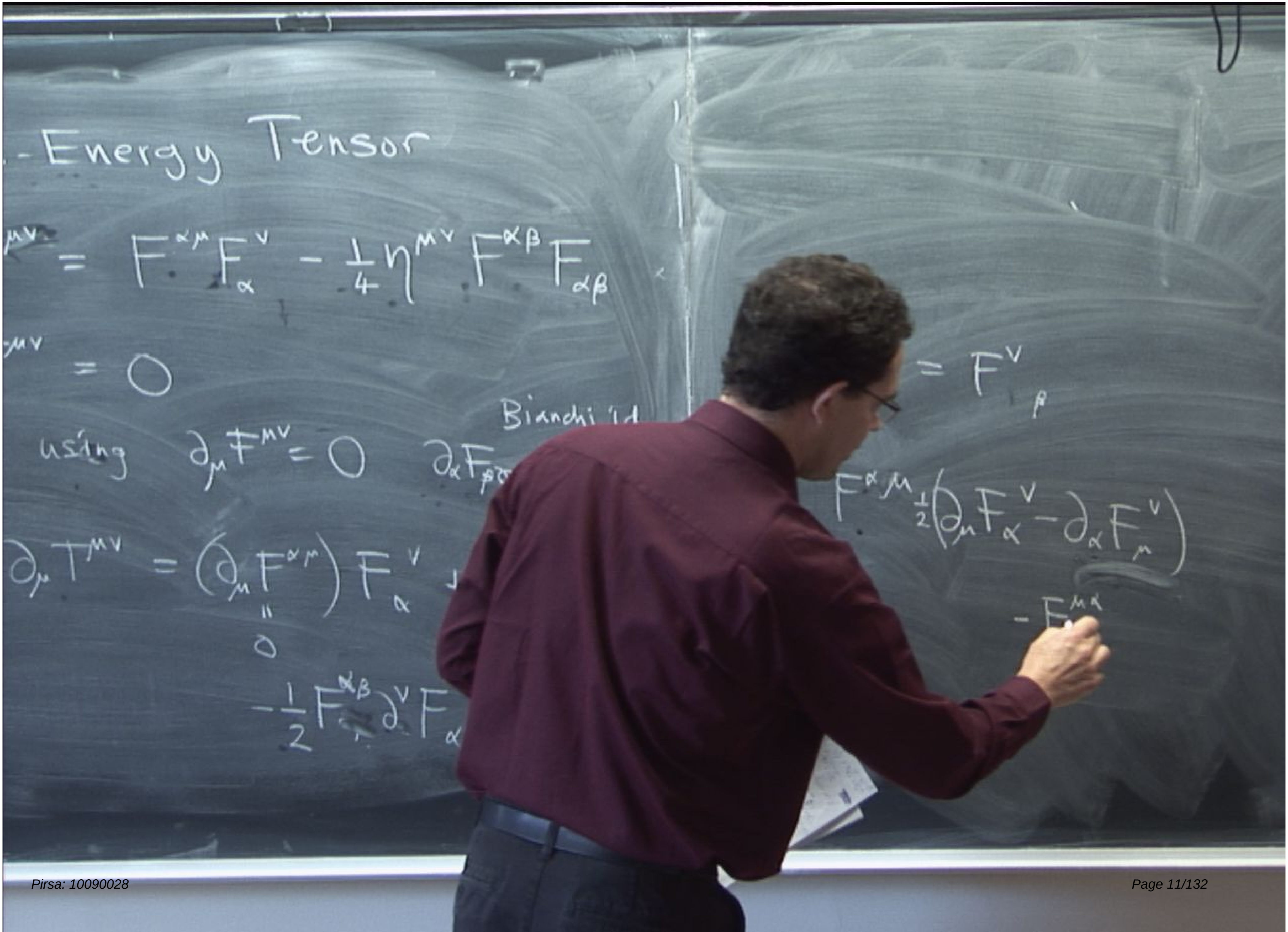
using $\quad \partial_\mu F^{\mu\nu} = 0 \qquad \partial_\alpha F_{\beta\sigma} + \text{cycles} = 0$

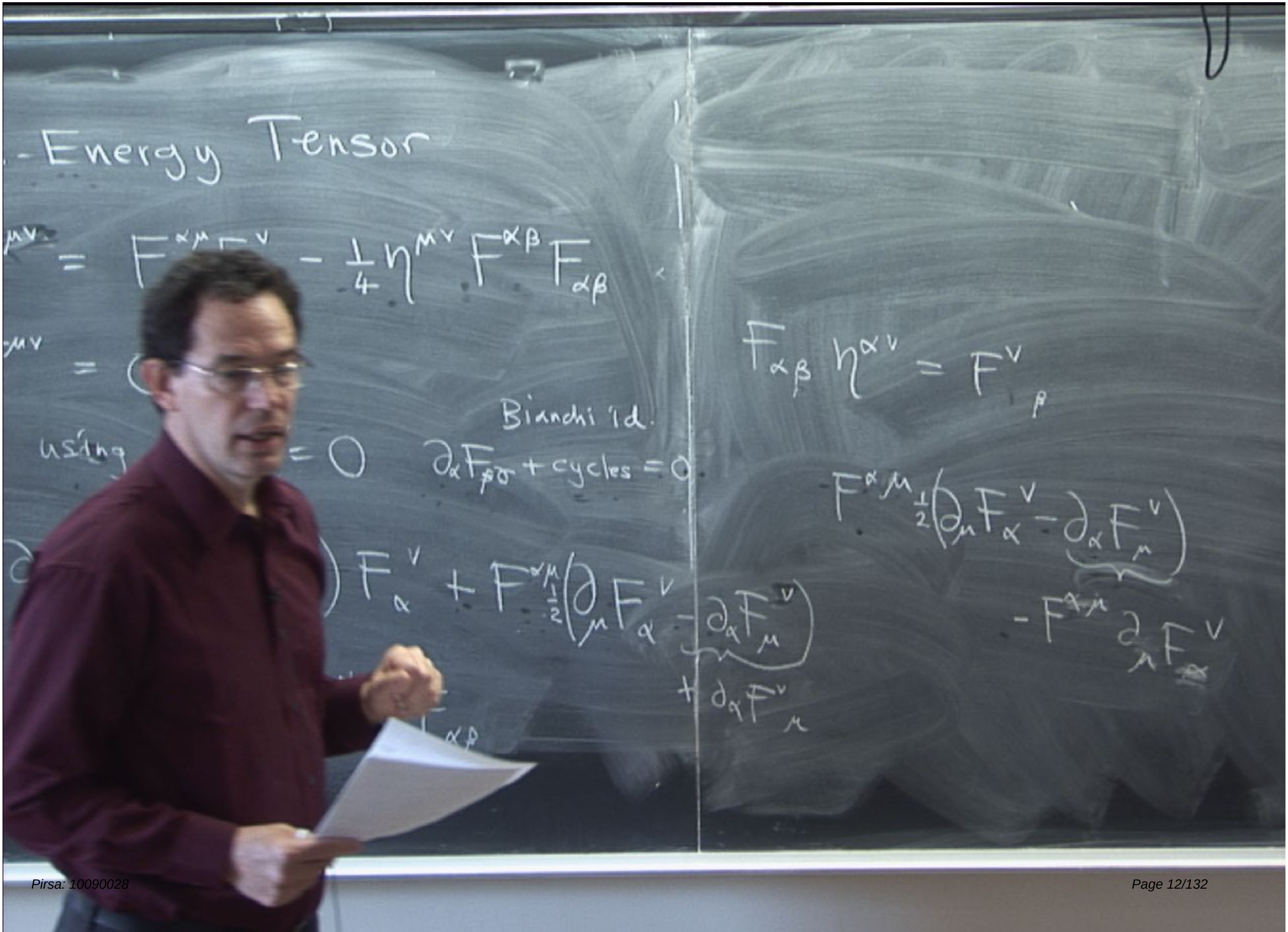
$\eta^{\mu\nu}\partial_\mu = \partial^\nu$

$$\partial_\mu T^{\mu\nu} = \underbrace{(\partial_\mu F^{\alpha\mu})}_{= 0} F_{\alpha}{}^{\nu} + F^{\alpha\mu}\partial_\mu F_{\alpha}{}^{\nu} - \frac{1}{2} F^{\alpha\beta}\partial^\nu F_{\alpha\beta}$$


Stress-Energy Tensor
EM $T^{\mu\nu} = F^{\alpha\mu} F_{\alpha}{}^{\nu} - \frac{1}{4}\eta^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}$
$\partial_\mu T^{\mu\nu} = 0$
$\frac{\partial}{\partial x^\mu}$
using $\partial_\mu F^{\mu\nu} = 0$ $\partial_\alpha F_{\rho\sigma}$ + cycles = 0
$\partial_\mu T^{\mu\nu} = (\partial_\mu F^{\alpha\mu}) F_\alpha{}^\nu + F^{\alpha\mu} \frac{1}{2}(\partial_\mu F_\alpha{}^\nu - \partial_\alpha F_\mu$
$\eta^{\mu\nu}\partial_\mu = \partial^\nu$
$-\frac{1}{2} F^{\alpha\beta} \partial^\nu F_{\alpha\beta}$


Stress-Energy Tensor
EM $T^{\mu\nu} = F^{\alpha\mu}F_{\alpha}{}^{\nu} - \frac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$
Conserved $\partial_\mu T^{\mu\nu}$
$\frac{\partial}{\partial x^\mu}$
$F^{\mu\nu} = 0$
Bianchi id.
$\partial_\alpha F_{\beta\gamma} + \text{cycles} = 0$
$-\frac{1}{2}F^{\alpha\beta}\partial^\nu F_{\alpha\beta}$


-Energy Tensor
$= F^{\alpha\mu} F_\alpha{}^\nu - \frac{1}{4}\eta^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}$
$= 0$
Bianchi id
using $\partial_\mu F^{\mu\nu} = 0$
$\partial_\mu T^{\mu\nu} = (\partial_\mu F^{\alpha\mu}) F_\alpha{}^\nu$
0
$-\frac{1}{2} F^{\alpha\beta} \partial^\nu F_\alpha$
$= F^\nu{}_\beta$
$F^{\alpha\mu} \frac{1}{2}(\partial_\mu F_\alpha{}^\nu - \partial_\alpha F_\mu{}^\nu)$
$-F^{\mu\alpha}$


-Energy Tensor
Bianchi id.
using
$\partial_\alpha F_{\beta\gamma}$ + cycles = 0
$F_{\alpha\beta}\,\eta^{\alpha\nu} = F^{\nu}{}_{\beta}$
$F^{\alpha\mu}\frac{1}{2}(\partial_\mu F_\alpha{}^\nu - \partial_\alpha F_\mu{}^\nu)$
$-F^{\alpha\mu}\partial_\mu F_\alpha{}^\nu$
$+\partial_\alpha F^\nu{}_\mu$

...-Energy Tensor

$$^{\mu\nu} = F^{\alpha\mu} F_{\alpha}{}^{\nu} - \frac{1}{4}\eta^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}$$

$$^{\mu\nu} = 0$$

Bianchi id.

using $\partial_\mu F^{\mu\nu} = 0$ $\quad \partial_\alpha F_{\beta\sigma} + \text{cycles} = 0$

$$^{\mu\nu} = \underbrace{(\partial_\mu F^{\alpha\mu})}_{=0} F_{\alpha}{}^{\nu} + F^{\alpha\mu}\frac{1}{2}\left(\partial_\mu F_{\alpha}{}^{\nu} - \partial_\alpha F_{\mu}{}^{\nu}\right)$$

$$+ \partial_\alpha F^{\nu}{}_{\mu}$$

$$-\frac{1}{2} F^{\alpha\beta} \partial^\nu F_{\alpha\beta}$$

$$F_{\alpha\beta}\,\eta^{\alpha\nu} = F^{\nu}{}_{\beta}$$

$$F^{\alpha\mu}\frac{1}{2}\left(\partial_\mu F_{\alpha}{}^{\nu} - \partial_\alpha F_{\mu}{}^{\nu}\right) = F^{\alpha\mu}\partial_\mu F_{\alpha}{}^{\nu}$$

...-Energy Tensor

$$T^{\mu\nu} = F^{\alpha\mu} F_{\alpha}^{\ \nu} - \frac{1}{4}\eta^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}$$

$$\partial_\mu T^{\mu\nu} = 0$$

Bianchi id.

using $\partial_\mu F^{\mu\nu} = 0$ $\quad \partial_\alpha F_{\beta\sigma} + \text{cycles} = 0$

$$\partial_\mu T^{\mu\nu} = \underbrace{(\partial_\mu F^{\alpha\mu})}_{=0} F_{\alpha}^{\ \nu} + F^{\alpha\mu}\frac{1}{2}\left(\partial_\mu F_{\alpha}^{\ \nu} - \underbrace{\partial_\alpha F_{\mu}^{\ \nu}}_{} \right) + \partial_\alpha F^{\nu}_{\ \mu}$$

$$-\frac{1}{2}F^{\alpha\beta}\partial^\nu F_{\alpha\beta}$$

$$F_{\alpha\beta}\,\eta^{\alpha\nu} = F^{\nu}_{\ \beta}$$

$$F^{\alpha\mu}\frac{1}{2}\left(\partial_\mu F_{\alpha}^{\ \nu} - \partial_\alpha F_{\mu}^{\ \nu}\right)$$
$$\parallel$$
$$F^{\alpha\mu}\partial_\mu F_{\alpha}^{\ \nu}$$

...-Energy Tensor

$$T^{\mu\nu} = F^{\alpha\mu} F_\alpha{}^\nu - \frac{1}{4}\eta^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}$$

$$\partial_\mu T^{\mu\nu} = 0$$

using $\partial_\mu F^{\mu\nu} = 0$ $\quad \partial_\alpha F_{\beta\sigma} + \text{cycles} = 0$ Bianchi id.

$$\partial_\mu T^{\mu\nu} = (\underbrace{\partial_\mu F^{\alpha\mu}}_{0}) F_\alpha{}^\nu + F^{\alpha\mu}\frac{1}{2}(\partial_\mu F_\alpha{}^\nu - \partial_\alpha F_\mu{}^\nu + \partial_\alpha F^\nu{}_\mu)$$

$$-\frac{1}{2}F^{\alpha\beta}\partial^\nu F_{\alpha\beta}$$

c.f. $\frac{1}{2} F^{\alpha\mu}(\partial_\mu F_\alpha{}^\nu + \partial_\alpha F^\nu{}_\mu$

$$F_{\alpha\beta}\,\eta^{\alpha\nu} = F^\nu{}_\beta$$

$$F^{\alpha\mu}\frac{1}{2}(\partial_\mu F_\alpha{}^\nu - \partial_\alpha F_\mu{}^\nu) = F^{\alpha\mu}\partial_\mu F_\alpha{}^\nu$$

...-Energy Tensor

$$T^{\mu\nu} = F^{\alpha\mu} F_\alpha^{\ \nu} - \frac{1}{4}\eta^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}$$

$$\partial_\mu T^{\mu\nu} = 0$$

Bianchi id.

using $\partial_\mu F^{\mu\nu} = 0$ $\quad \partial_\alpha F_{\beta\sigma} + \text{cycles} = 0$

$$\partial_\mu T^{\mu\nu} = \underset{0}{\underset{\parallel}{(\partial_\mu F^{\alpha\mu})}} F_\alpha^{\ \nu} + F^{\alpha\mu}\frac{1}{2}(\partial_\mu F_\alpha^{\ \nu} - \partial_\alpha F_\mu^{\ \nu}) + \partial_\alpha F^\nu_{\ \mu}$$

$$-\frac{1}{2} F^{\alpha\beta}\partial^\nu F_{\alpha\beta}$$

c.f. $\frac{1}{2} F^{\alpha\mu}(\partial_\mu F_\alpha^{\ \nu} + \partial_\alpha F^\nu_{\ \mu} + \partial^\nu F$

$$F_{\alpha\beta}\,\eta^{\alpha\nu} = F^\nu_{\ \beta}$$

$$F^{\alpha\mu}\frac{1}{2}(\partial_\mu F_\alpha^{\ \nu} - \partial_\alpha F_\mu^{\ \nu})$$
$$\parallel$$
$$F^{\alpha\mu}\partial_\mu F_\alpha^{\ \nu}$$

...-Energy Tensor

$$T^{\mu\nu} = F^{\alpha\mu}F_{\alpha}^{\ \nu} - \frac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$$

$$\partial_\mu T^{\mu\nu} = 0$$

Bianchi id.

using $\partial_\mu F^{\mu\nu} = 0$ $\quad \partial_\alpha F_{\beta\sigma} + \text{cycles} = 0$

$$\partial_\mu T^{\mu\nu} = \underbrace{(\partial_\mu F^{\alpha\mu})}_{=0} F_{\alpha}^{\ \nu} + F^{\alpha\mu}\frac{1}{2}\left(\partial_\mu F_{\alpha}^{\ \nu} - \partial_\alpha F_{\mu}^{\ \nu}\right)$$

$$-\frac{1}{2}F^{\alpha\beta}\partial^\nu F_{\alpha\beta}$$

$$+\partial_\alpha F^{\nu}_{\ \mu}$$

$$\text{c.f. } \frac{1}{2}F^{\alpha\mu}\left(\partial_\mu F_{\alpha}^{\ \nu} + \partial_\alpha F^{\nu}_{\ \mu} + \partial^\nu F_{\mu\alpha}\right)$$

$$F_{\alpha\beta}\,\eta^{\alpha\nu} = F^{\nu}_{\ \beta}$$

$$F^{\alpha\mu}\frac{1}{2}\left(\partial_\mu F_{\alpha}^{\ \nu} - \partial_\alpha F_{\mu}^{\ \nu}\right)$$
$$\parallel$$
$$F^{\alpha\mu}\partial_\mu F_{\alpha}^{\ \nu}$$

...-Energy Tensor

$$T^{\mu\nu} = F^{\alpha\mu}F_{\alpha}{}^{\nu} - \frac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$$

$$\partial_\mu T^{\mu\nu} = 0$$

using $\partial_\mu F^{\mu\nu} = 0$ $\quad$ $\partial_\alpha F_{\beta\sigma} + \text{cycles} = 0$ Bianchi id.

$$\partial_\mu T^{\mu\nu} = \underset{0}{\underset{\shortparallel}{(\partial_\mu F^{\alpha\mu})}} F_{\alpha}{}^{\nu} + F^{\alpha\mu}\frac{1}{2}\Big(\overset{①}{\partial_\mu F_{\alpha}{}^{\nu}} - \overset{②}{\partial_\alpha F^{\nu}{}_{\mu}}\Big) \overset{③}{- \frac{1}{2}F^{\alpha\beta}\partial^\nu F_{\alpha\beta}}$$

$$+\partial_\alpha F^{\nu}{}_{\mu}$$

$$F_{\alpha\beta}\,\eta^{\alpha\nu} = F^{\nu}{}_{\beta}$$

$$F^{\alpha\mu}\frac{1}{2}\left(\partial_\mu F_{\alpha}{}^{\nu} - \partial_\alpha F^{\nu}{}_{\mu}\right) = F^{\alpha\mu}\partial_\mu F_{\alpha}{}^{\nu}$$

c.f. $\frac{1}{2}F^{\alpha\mu}\Big(\overset{①}{\partial_\mu F_{\alpha}{}^{\nu}} + \overset{②}{\partial_\alpha F^{\nu}{}_{\mu}} + \overset{③}{\partial^\nu F_{\mu\alpha}}\Big)$

$\alpha\beta$

hi 'id.

ycles = 0

$$F^{\alpha\mu}\,\tfrac{1}{2}\left(\partial_\mu F_\alpha{}^\nu - \partial_\alpha F_\mu{}^\nu\right)$$

$$\parallel$$

$$F^{\alpha\mu}\,\partial_\mu F_\alpha{}^\nu$$

$$\left(\partial_\mu F_\alpha{}^\nu - \partial_\alpha F_\mu{}^\nu\right)$$

$$+\,\partial_\alpha F^\nu{}_\mu$$

c.f. $\frac{1}{2} F^{\alpha\mu}\left(\partial_\mu F_\alpha{}^\nu + \partial_\alpha F^\nu{}_\mu + \partial^\nu F_{\mu\alpha}\right)$

$$\partial_\mu T^{\mu\nu} = 0 \Rightarrow \partial_0 T^{00} + \partial_i T^{i0} = 0$$

define $\rho = T^{00}$ $\qquad \pi^i = T^{i0}$

spatial Volume

V

$$\frac{\partial}{\partial t}\int d \ldots (x)$$

$$F^{\alpha\mu}\frac{1}{2}\left(\partial_\mu F_\alpha{}^\nu - \partial_\alpha F_\mu{}^\nu\right)$$

$$\|$$

$$F^{\alpha\mu}\partial_\mu F_\alpha{}^\nu$$

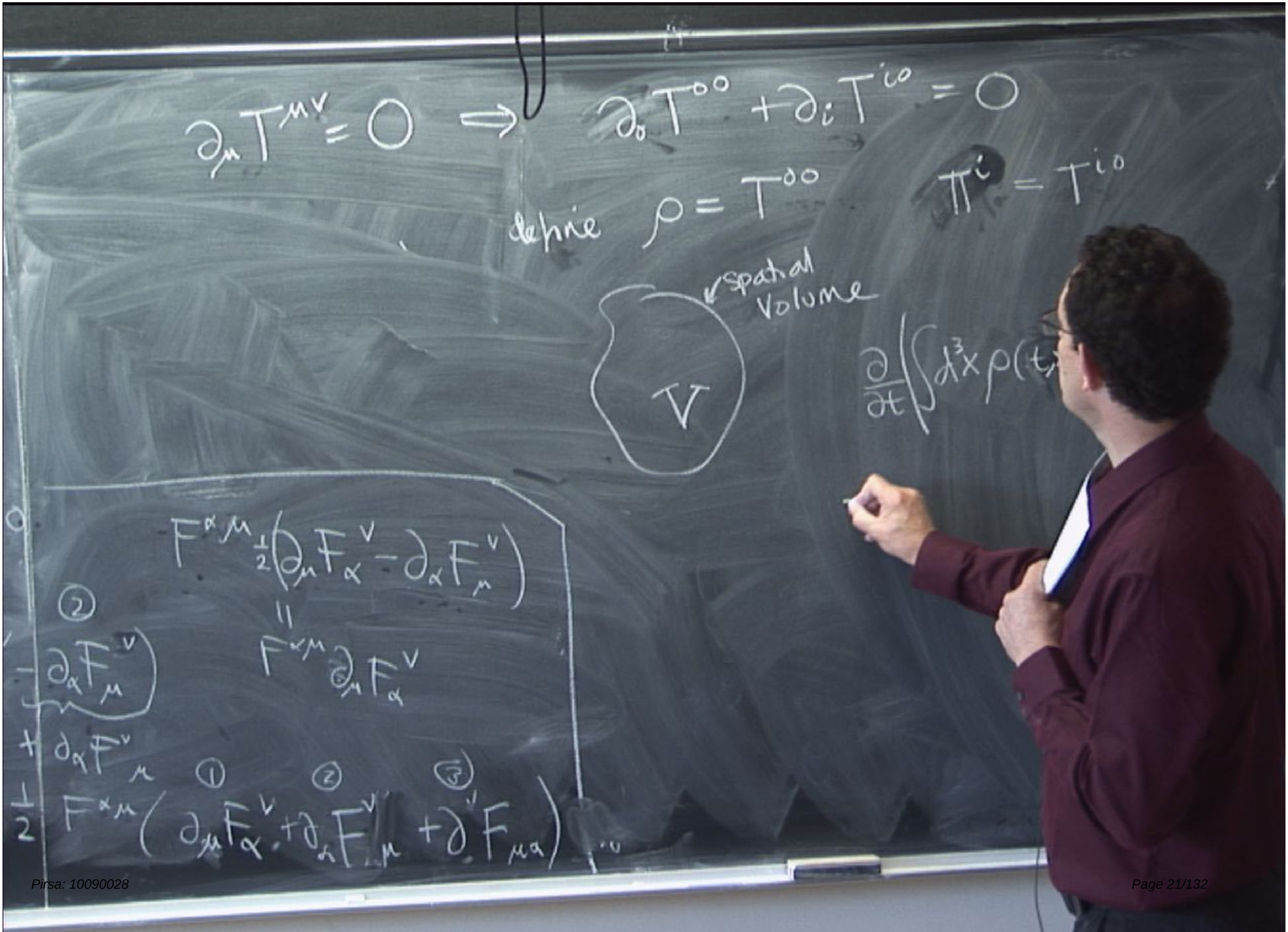
2

$$-\partial_\alpha F_\mu{}^\nu$$

$$+\partial_\alpha F^\nu{}_\mu$$

1 2 3

$$\frac{1}{2} F^{\alpha\mu}\left(\partial_\mu F_\alpha{}^\nu + \partial_\alpha F^\nu{}_\mu + \partial^\nu F_{\mu\alpha}\right)$$


$\partial_\mu T^{\mu\nu} = 0 \Rightarrow \partial_0 T^{00} + \partial_i T^{i0} = 0$
define $\rho = T^{00}$
$\pi^i = T^{i0}$
spatial Volume
V
$\frac{\partial}{\partial t}\left(\int d^3x\, \rho(t,$
$F^{\alpha\mu}\frac{1}{2}(\partial_\mu F_\alpha{}^\nu - \partial_\alpha F_\mu{}^\nu)$
$F^{\alpha\mu}\partial_\mu F_\alpha{}^\nu$
$-\partial_\alpha F_\mu{}^\nu$
$+\partial_\alpha F^\nu{}_\mu$
$\frac{1}{2}F^{\alpha\mu}(\partial_\mu F_\alpha{}^\nu + \partial_\alpha F^\nu{}_\mu + \partial^\nu F_{\mu\alpha})$

$$\partial_\mu T^{\mu\nu} = 0 \Rightarrow \partial_0 T^{00} + \partial_i T^{i0} = 0$$

define $\rho = T^{00}$ $\quad \pi^i = T^{i0}$

$$\frac{\partial \rho}{\partial t} = -\partial_i \pi^i$$

spatial Volume

V

S

$$\frac{\partial}{\partial t}\left(\int d^3x\, \rho(t,\underline{x})\right) = -\int d^3x\, \partial_i \pi^i$$

Gauss divergence thm

$$= -\int_S \pi^i dS^i$$

$$F^{\alpha\mu}\frac{1}{2}\left(\partial_\mu F_\alpha{}^\nu - \partial_\alpha F_\mu{}^\nu\right)$$

$$= F^{\alpha\mu}\partial_\mu F_\alpha{}^\nu$$

2

$$-\partial_\alpha F_\mu{}^\nu$$

$$+\partial_\alpha F^\nu{}_\mu$$

1 2 3

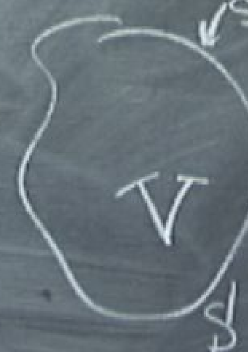
$$\frac{1}{2} F^{\alpha\mu}\left(\partial_\mu F_\alpha{}^\nu + \partial_\alpha F^\nu{}_\mu + \partial^\nu F_{\mu\alpha}\right)$$

$$\partial_\mu T^{\mu\nu} = 0 \Rightarrow \partial_0 T^{00} + \partial_i T^{i0} = 0$$

define $\rho = T^{00}$ $\qquad \pi^i = T^{i0}$

$$\frac{\partial \rho}{\partial t} = -\partial_i \pi^i$$

spatial Volume



$$\frac{\partial}{\partial t}\left(\int d^3x\, \rho(t,x)\right)$$

$$= -\int d^3x\, \partial_i \pi^i$$

Gauss divergence thm $\quad = -\int_S \pi^i\, dS^i$

If S chosen so π^i is zero

$\rightarrow E = \int d^3x\, \rho$ is conserved.

$$F^{\alpha\mu}\tfrac{1}{2}\left(\partial_\mu F_\alpha{}^\nu - \partial_\alpha F_\mu{}^\nu\right)$$

$$\parallel$$

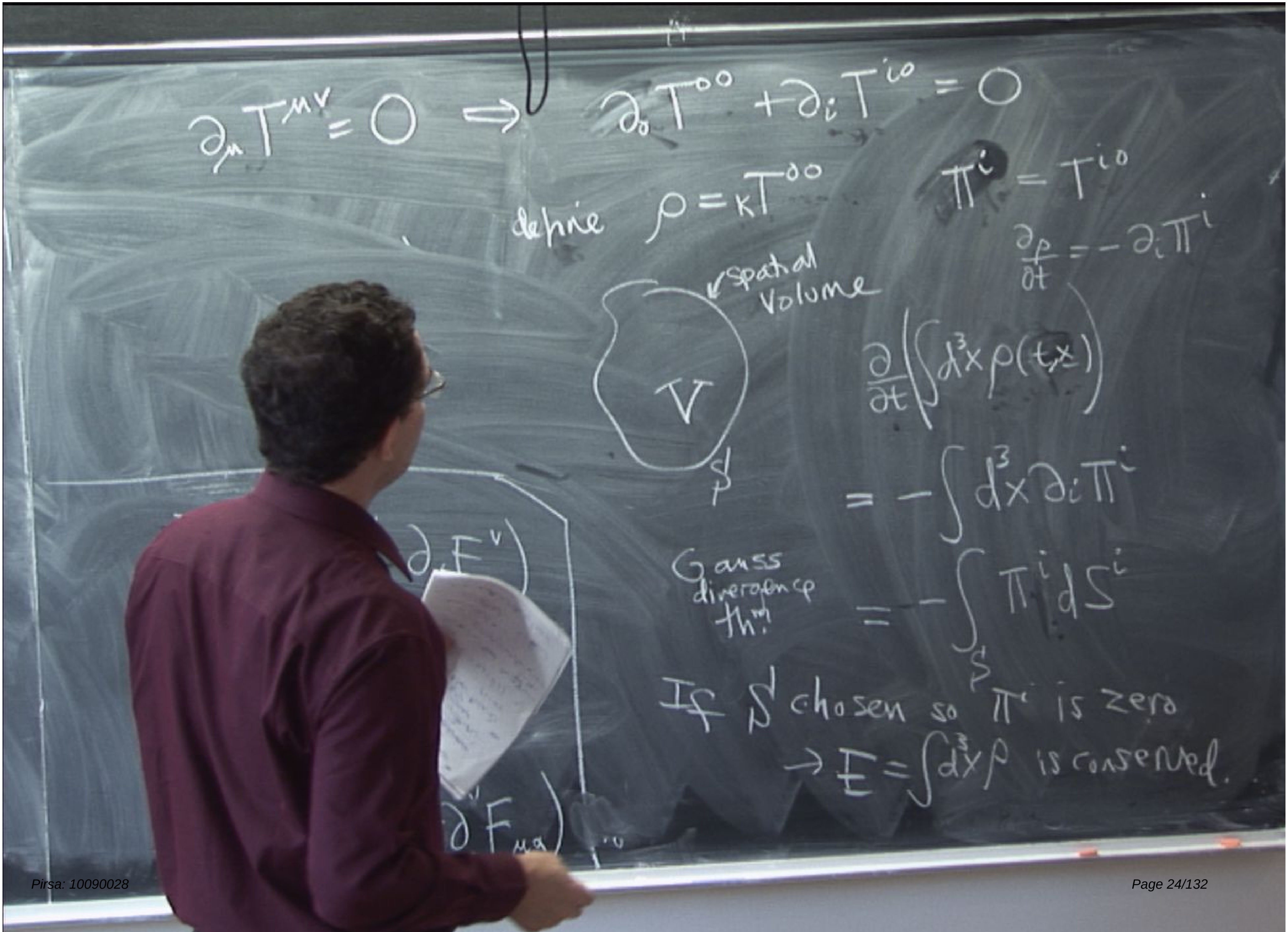
$$F^{\alpha\mu}\partial_\mu F_\alpha{}^\nu$$

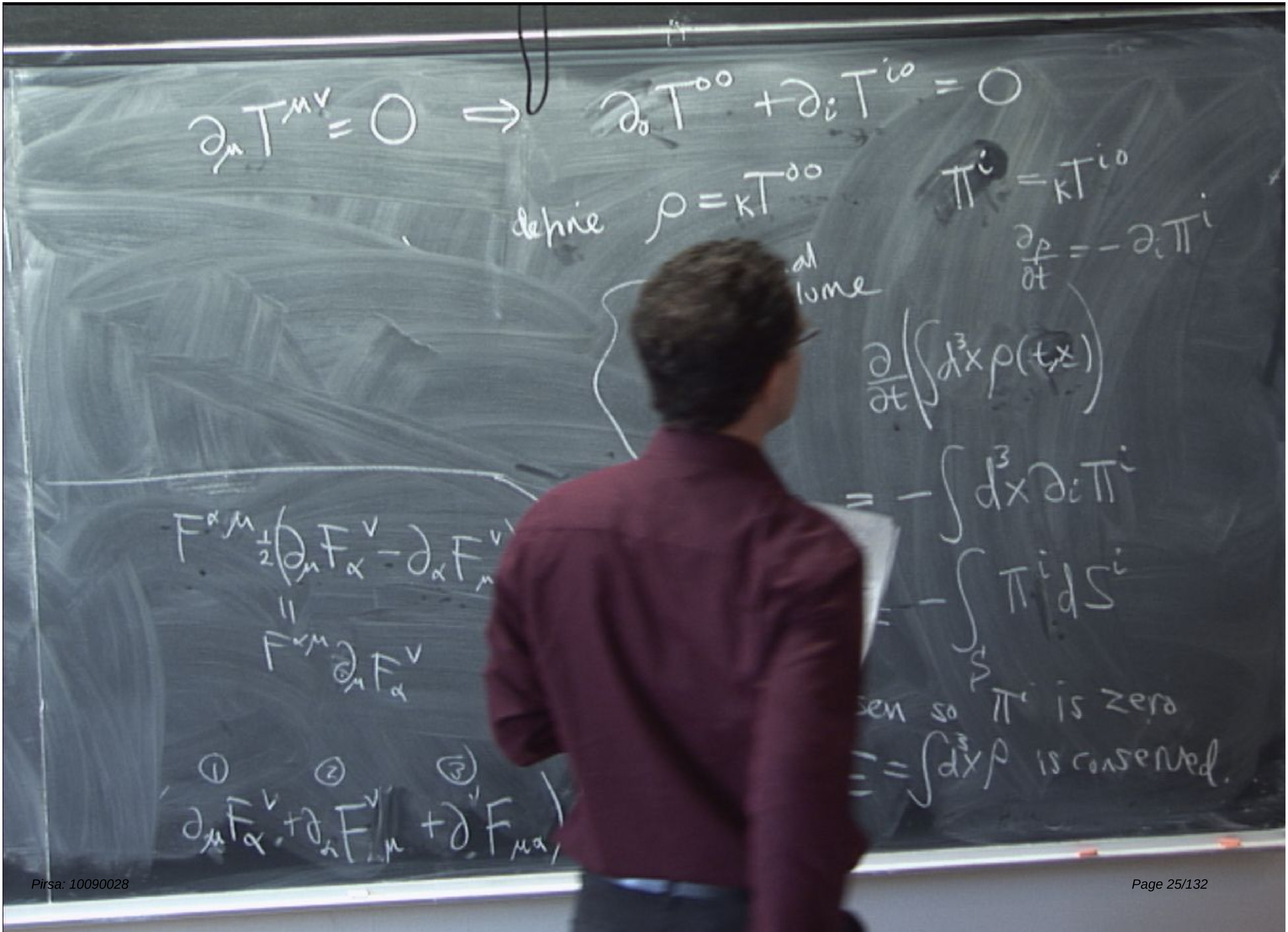
② $\quad -\partial_\alpha F_\mu{}^\nu$

$+\partial_\alpha F^\nu{}_\mu$

① ② ③

$$\frac{1}{2} F^{\alpha\mu}\left(\partial_\mu F_\alpha{}^\nu + \partial_\alpha F^\nu{}_\mu + \partial^\nu F_{\mu\alpha}\right)$$


$\partial_\mu T^{\mu\nu} = 0 \Rightarrow \partial_0 T^{00} + \partial_i T^{i0} = 0$
define $\rho = \kappa T^{00}$
$\pi^i = T^{i0}$
$\frac{\partial \rho}{\partial t} = -\partial_i \pi^i$
spatial Volume
V
S
$\frac{\partial}{\partial t}\left(\int d^3x\, \rho(t,x)\right)$
$= -\int d^3x\, \partial_i \pi^i$
Gauss divergence th^m!
$= -\int_S \pi^i dS^i$
If S chosen so π^i is zero
$\to E = \int d^3x\, \rho$ is conserved.


$\partial_\mu T^{\mu\nu} = 0 \Rightarrow \partial_0 T^{00} + \partial_i T^{i0} = 0$
define $\rho = \kappa T^{00}$
$\pi^i = \kappa T^{i0}$
$\frac{\partial \rho}{\partial t} = -\partial_i \pi^i$
lume
$\frac{\partial}{\partial t}\left(\int d^3x\, \rho(t,x)\right)$
$= -\int d^3x\, \partial_i \pi^i$
$= -\int_S \pi^i dS^i$
sen so π^i is zero
$E = \int d^3x\, \rho$ is conserved.
$F^{\alpha\mu} \frac{1}{2}(\partial_\mu F_\alpha{}^\nu - \partial_\alpha F_\mu{}^\nu$
$F^{\alpha\mu} \partial_\mu F_\alpha{}^\nu$
① ② ③
$\partial_\mu F_\alpha{}^\nu + \partial_\alpha F^\nu{}_\mu + \partial^\nu F_{\mu\alpha}$

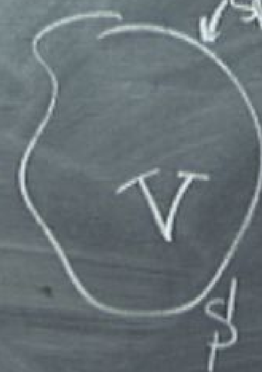
$$\partial_\mu T^{\mu\nu} = 0 \Rightarrow \partial_0 T^{00} + \partial_i T^{i0} = 0$$

$$\partial_0 T^{0i} + \partial_j T^{ji} = 0$$

define $\rho = {}_K T^{00}$ $\quad \Pi^i = {}_K T^{i0}$

$$\frac{\partial \rho}{\partial t} = -\partial_i \Pi^i$$

spatial Volume



$$\frac{\partial}{\partial t}\left(\int d^3x\, \rho(t, x)\right)$$

$$= -\int d^3x\, \partial_i \Pi^i$$

Gauss divergence thm!

$$= -\int_S \Pi^i dS^i$$

If S chosen so Π^i is zero

$\rightarrow E = \int d^3x\, \rho$ is conserved.

$$F^{\alpha\mu} \tfrac{1}{2}\left(\partial_\mu F_\alpha{}^\nu - \partial_\alpha F_\mu{}^\nu\right)$$

$$\shortparallel$$

$$F^{\alpha\mu} \partial_\mu F_\alpha{}^\nu$$

① ② ③

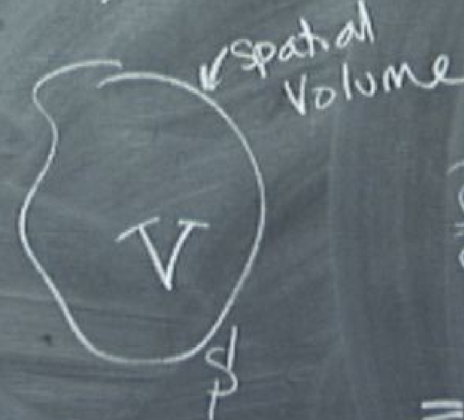
$$\partial_\mu F_\alpha{}^\nu + \partial_\alpha F^\nu{}_\mu + \partial^\nu F_{\mu\alpha}\Big)$$

$$\partial_\mu T^{\mu\nu} = 0 \Rightarrow \partial_0 T^{00} + \partial_i T^{i0} = 0$$

define $\rho = \kappa T^{00}$ $\quad \pi^i = \kappa T^{i0}$

$$\partial_0 T^{0i} + \partial_j T^{ji} = 0$$

$$\frac{\partial \rho}{\partial t} = -\partial_i \pi^i$$



$$p^i = \int d^3x\, T^{0i}$$

$$\frac{\partial}{\partial t}\left(\int d^3x\, \rho(t,x)\right)$$

$$= -\int d^3x\, \partial_i \pi^i$$

Gauss divergence thm:

$$= -\int_S \pi^i dS^i$$

If S chosen so π^i is zero

$\rightarrow E = \int d^3x\, \rho$ is conserved.

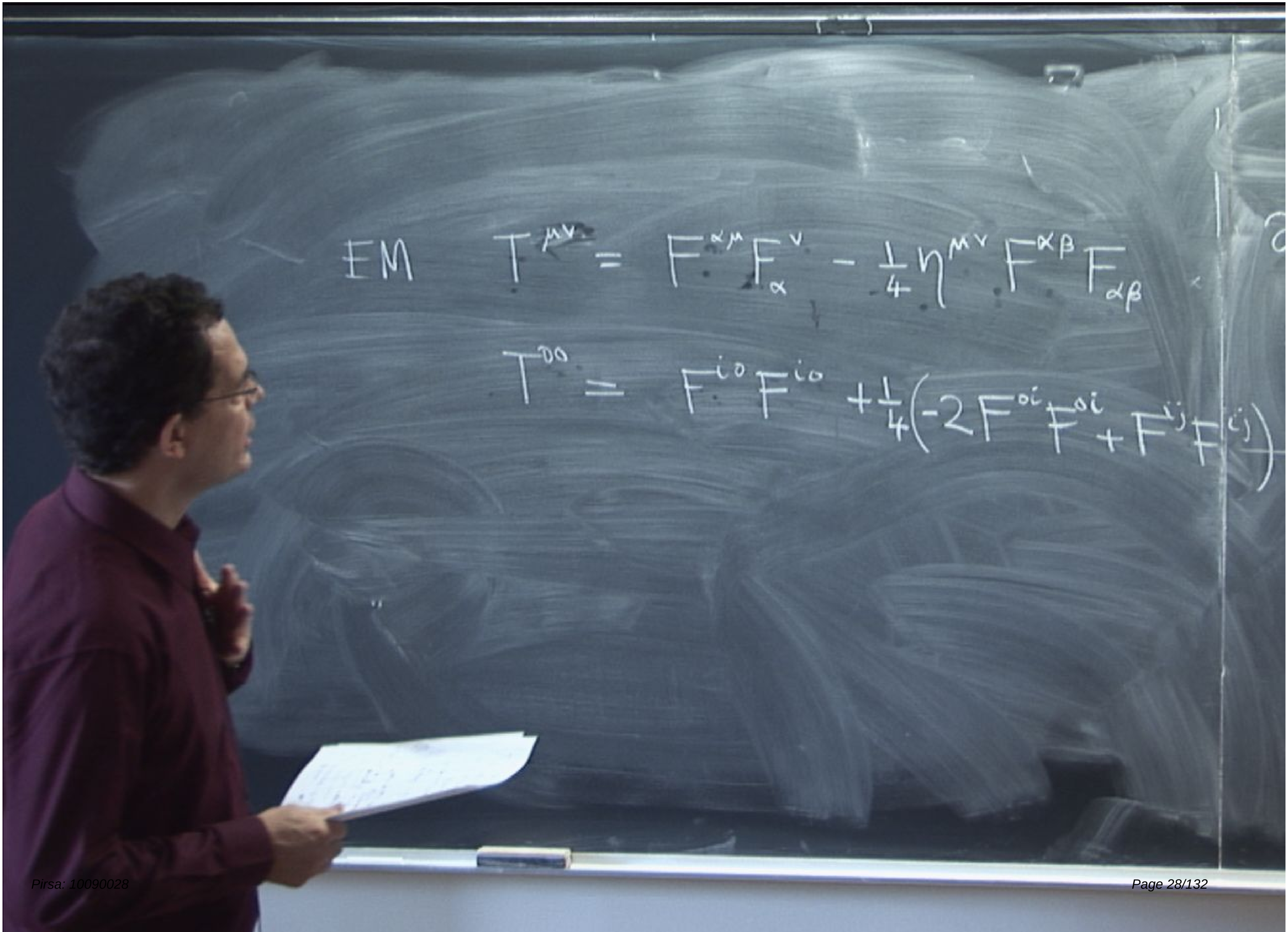
$$F^{\alpha\mu}\tfrac{1}{2}\left(\partial_\mu F_\alpha{}^\nu - \partial_\alpha F_\mu{}^\nu\right)$$

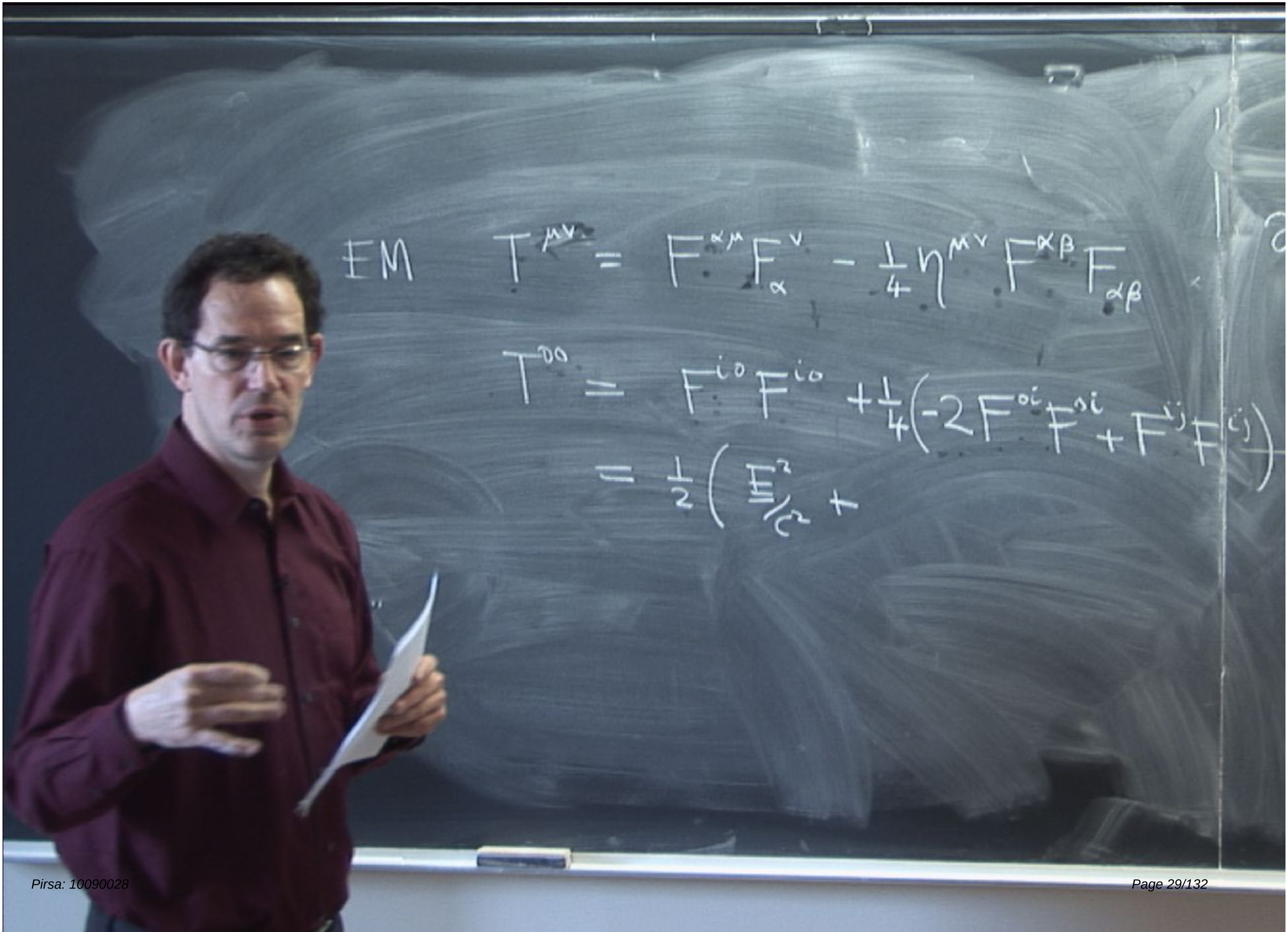
$$\parallel$$

$$F^{\alpha\mu}\partial_\mu F_\alpha{}^\nu$$

① ② ③

$$\partial_\mu F_\alpha{}^\nu + \partial_\alpha F^\nu{}_\mu + \partial^\nu F_{\mu\alpha})$$


EM $T^{\mu\nu} = F^{\alpha\mu}F_{\alpha}{}^{\nu} - \frac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$
$T^{00} = F^{i0}F^{i0} + \frac{1}{4}\left(-2F^{0i}F^{0i} + F^{ij}F^{ij}\right)$

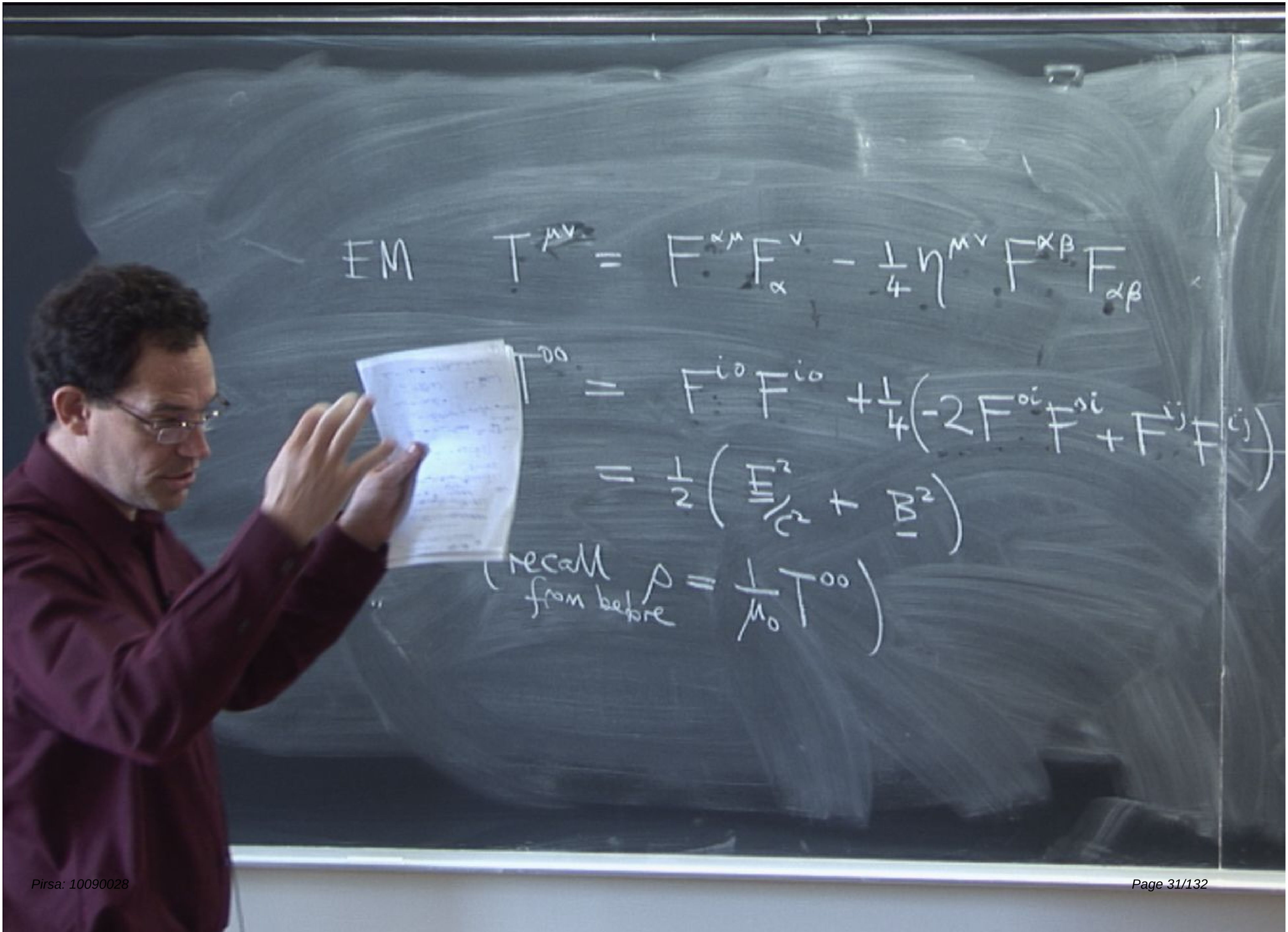

EM $T^{\mu\nu} = F^{\alpha\mu}F_{\alpha}{}^{\nu} - \frac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$
$T^{00} = F^{i0}F^{i0} + \frac{1}{4}\left(-2F^{0i}F^{0i} + F^{ij}F^{ij}\right)$
$= \frac{1}{2}\left(E^2/c^2 + \right.$

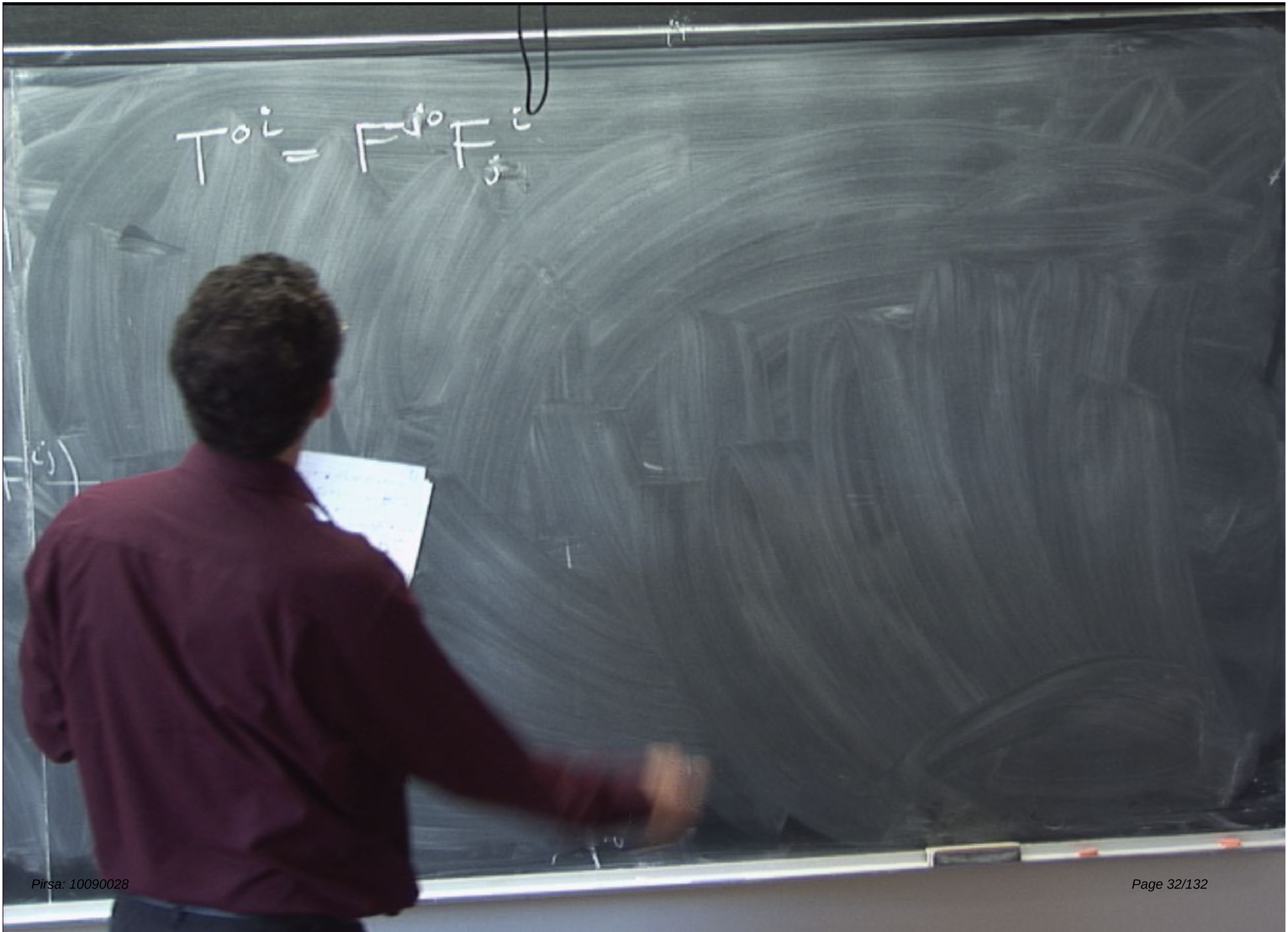
EM $T^{\mu\nu} = F^{\alpha\mu} F_{\alpha}{}^{\nu} - \frac{1}{4}\eta^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}$

$$T^{00} = F^{i0} F^{i0} + \frac{1}{4}\left(-2F^{0i}F^{0i} + F^{ij}F^{ij}\right)$$

$$= \frac{1}{2}\left(\underline{E}^2/c^2 + \underline{B}^2\right)$$

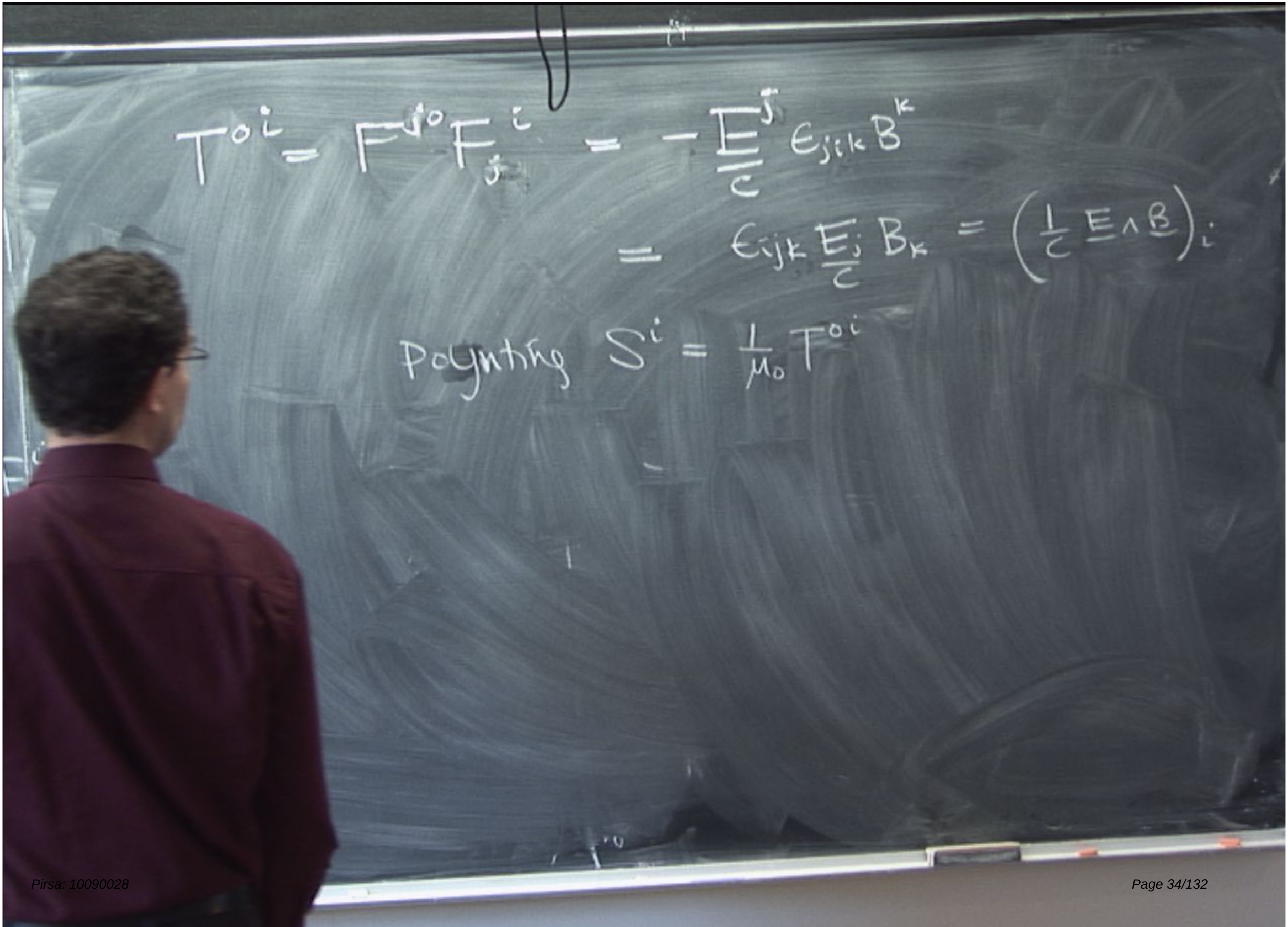
(recall from before $\rho = \frac{1}{\mu_0} T^{00}$)

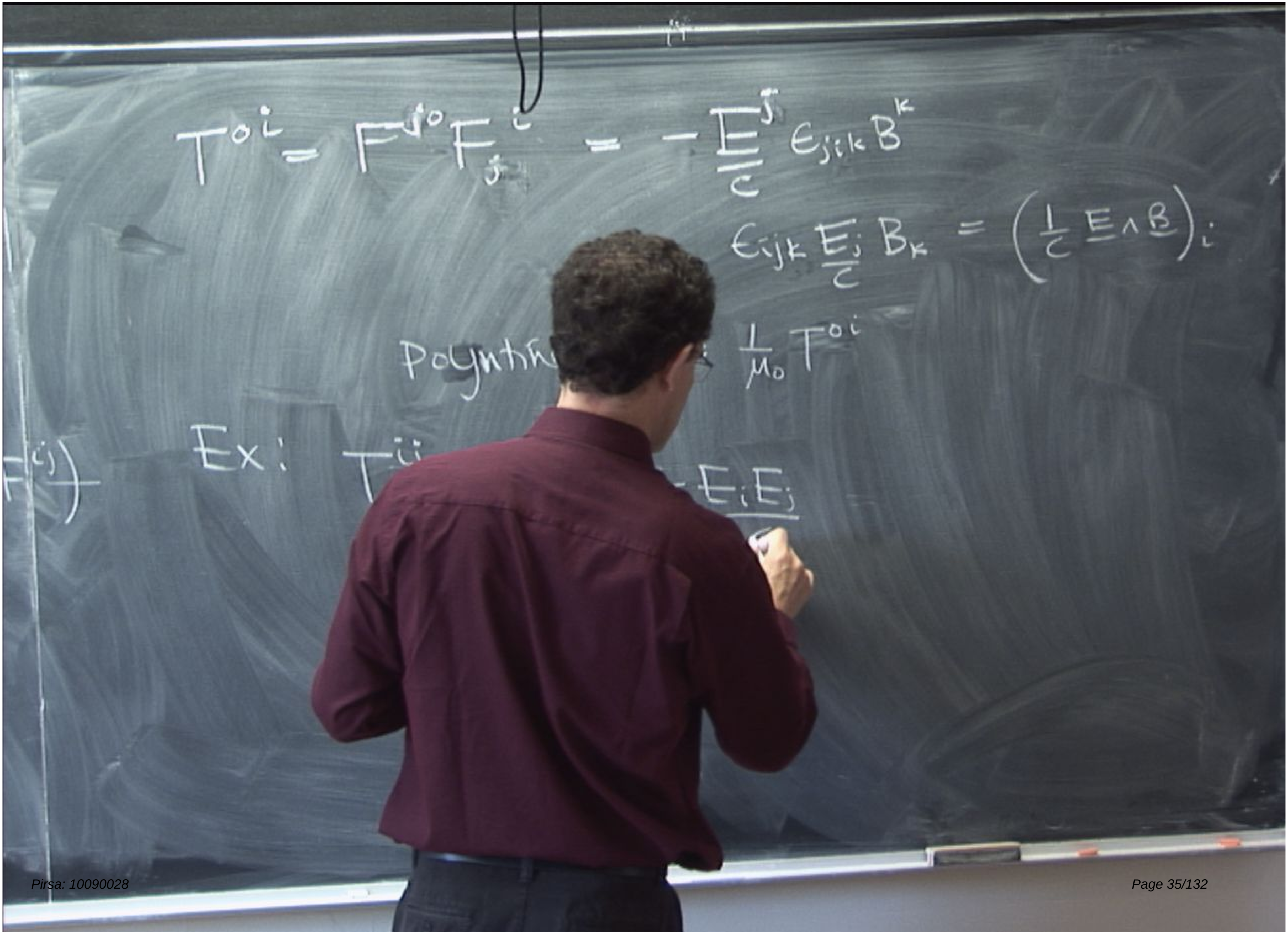

EM $T^{\mu\nu} = F^{\alpha\mu}F_{\alpha}{}^{\nu} - \frac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$
$T^{00} = F^{i0}F^{i0} + \frac{1}{4}\left(-2F^{0i}F^{0i} + F^{ij}F^{ij}\right)$
$= \frac{1}{2}\left(\underline{E}^2/c^2 + \underline{B}^2\right)$
(recall from before $\rho = \frac{1}{\mu_0}T^{00}$)

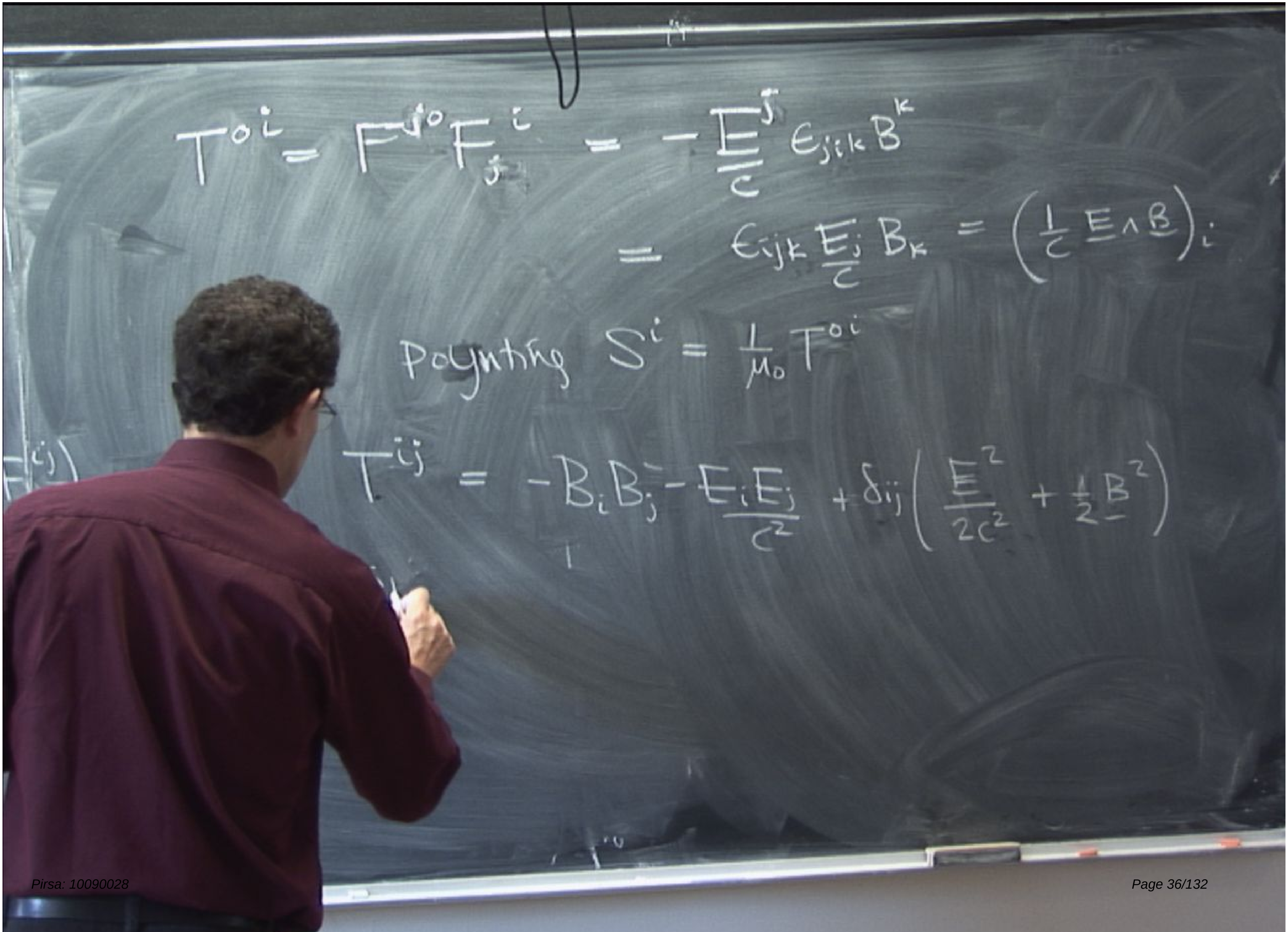

$T^{0i} = F^{j0} F_j{}^i$

$$T^{0i} = F^{j0}F_{j}{}^{i} = -\frac{\underline{E}^{j}}{c}\epsilon_{jik}B^{k}$$

$$= \epsilon_{ijk}\frac{\underline{E}_{j}}{c}B_{k} = \left(\frac{1}{c}\underline{E}\wedge\underline{B}\right)_{i}$$


$T^{0i} = F^{j0} F_j{}^i = -\frac{E^j}{c} \epsilon_{jik} B^k$
$= \epsilon_{ijk} \frac{E_j}{c} B_k = \left(\frac{1}{c} \underline{E} \wedge \underline{B}\right)_i$
Poynting $S^i = \frac{1}{\mu_0} T^{0i}$


$T^{0i} = F^{j0} F_j{}^i = -\frac{E^j}{c} \epsilon_{jik} B^k$
$\epsilon_{ijk} \frac{E_j}{c} B_k = \left(\frac{1}{c} \underline{E} \wedge \underline{B}\right)_i$
Poynting
$\frac{1}{\mu_0} T^{0i}$
Ex:
T^{ij}
$E_i E_j$


$T^{0i} = F^{j0} F_j{}^i = -\frac{E^j}{c} \epsilon_{jik} B^k$
$= \epsilon_{ijk} \frac{E_j}{c} B_k = \left(\frac{1}{c} \underline{E} \wedge \underline{B} \right)_i$
poynting $S^i = \frac{1}{\mu_0} T^{0i}$
$T^{ij} = -B_i B_j - \frac{E_i E_j}{c^2} + \delta_{ij} \left(\frac{E^2}{2c^2} + \frac{1}{2} B^2 \right)$

$$T^{0i} = F^{j0} F_j^{\ i} = -\frac{E^j}{c} \epsilon_{jik} B^k$$

$$= \epsilon_{ijk} \frac{E_j}{c} B_k = \left(\frac{1}{c} \underline{E} \wedge \underline{B} \right)_i$$

Poynting $S^i = \frac{1}{\mu_0} T^{0i}$

Ex: $T^{ij} = -B_i B_j - \frac{E_i E_j}{c^2} + \delta_{ij} \left(\frac{\underline{E}^2}{2c^2} + \frac{1}{2} \underline{B}^2 \right)$

spatial stress in EM field.

$F^{(ij)}$

$$T^{0i} = F^{j0} F_j^{\ i} = -\frac{E^j}{c}\,\epsilon_{jik} B^k$$

$$= \epsilon_{ijk} \frac{E_j}{c} B_k = \left(\frac{1}{c}\,\underline{E} \wedge \underline{B}\right)_i$$

Poynting $S^i = \frac{1}{\mu_0} T^{0i}$

Ex: $$T^{ij} = -B_i B_j - \frac{E_i E_j}{c^2} + \delta_{ij}\left(\frac{\underline{E}^2}{2c^2} + \frac{1}{2}\underline{B}^2\right)$$

spatial stress in EM field.

$$\partial_0 \int T^{0i} = -\int \partial_j T^{ji}$$

$$T^{0i} = F^{j0} F_j{}^i = -\frac{E^j}{c} \epsilon_{jik} B^k$$

$$= \epsilon_{ijk} \frac{E_j}{c} B_k = \left(\frac{1}{c} \underline{E} \wedge \underline{B}\right)_i$$

Poynting $S^i = \frac{1}{\mu_0} T^{0i}$

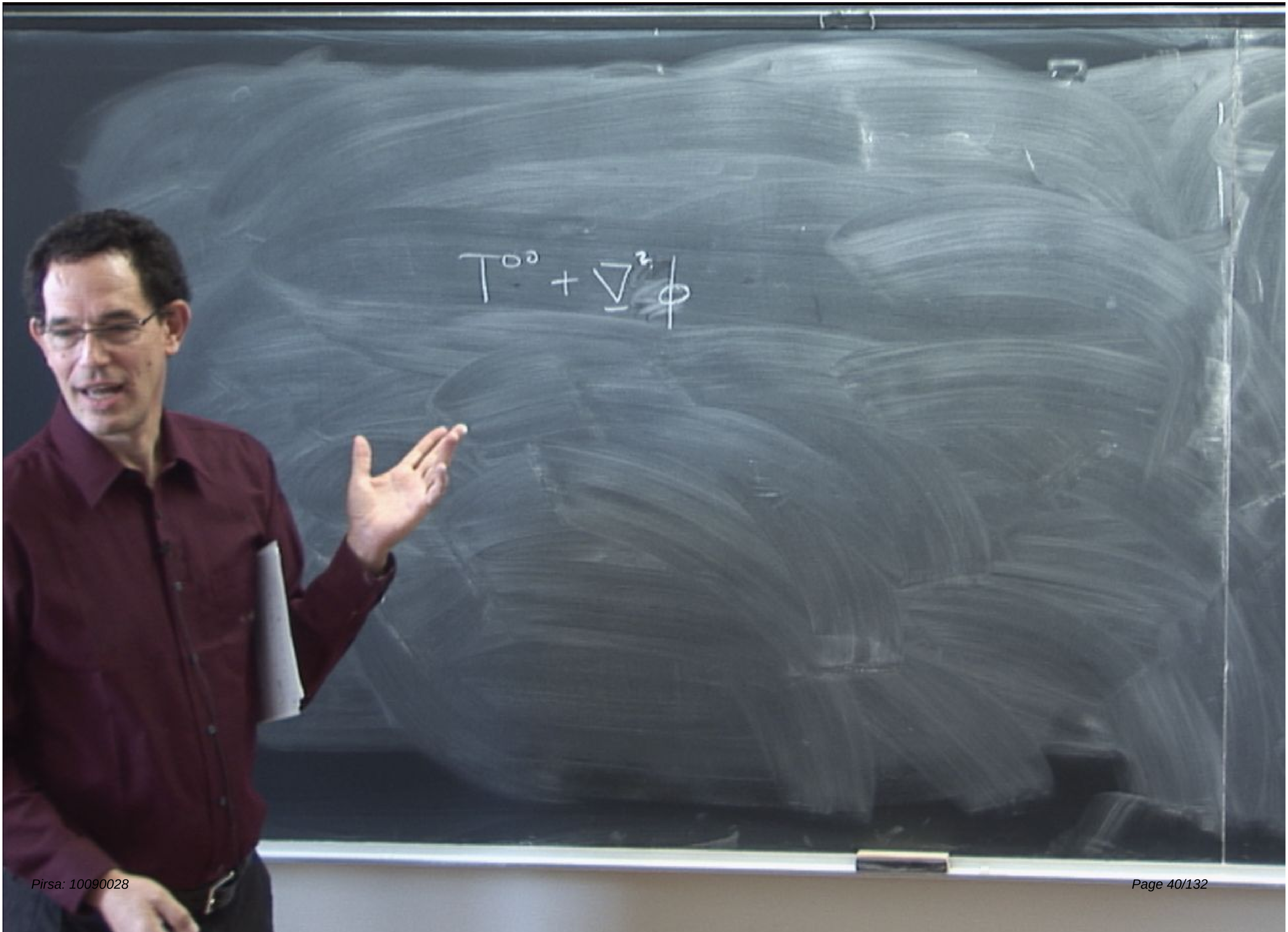
Ex: $T^{ij} = -B_i B_j - \frac{E_i E_j}{c^2} + \delta_{ij}\left(\frac{E^2}{2c^2} + \frac{1}{2} B^2\right)$

spatial stress in EM field.

$\partial_0 \int T^{0i}$

$= -\int \partial_j T^{ji}$

$F^{(ij)}$

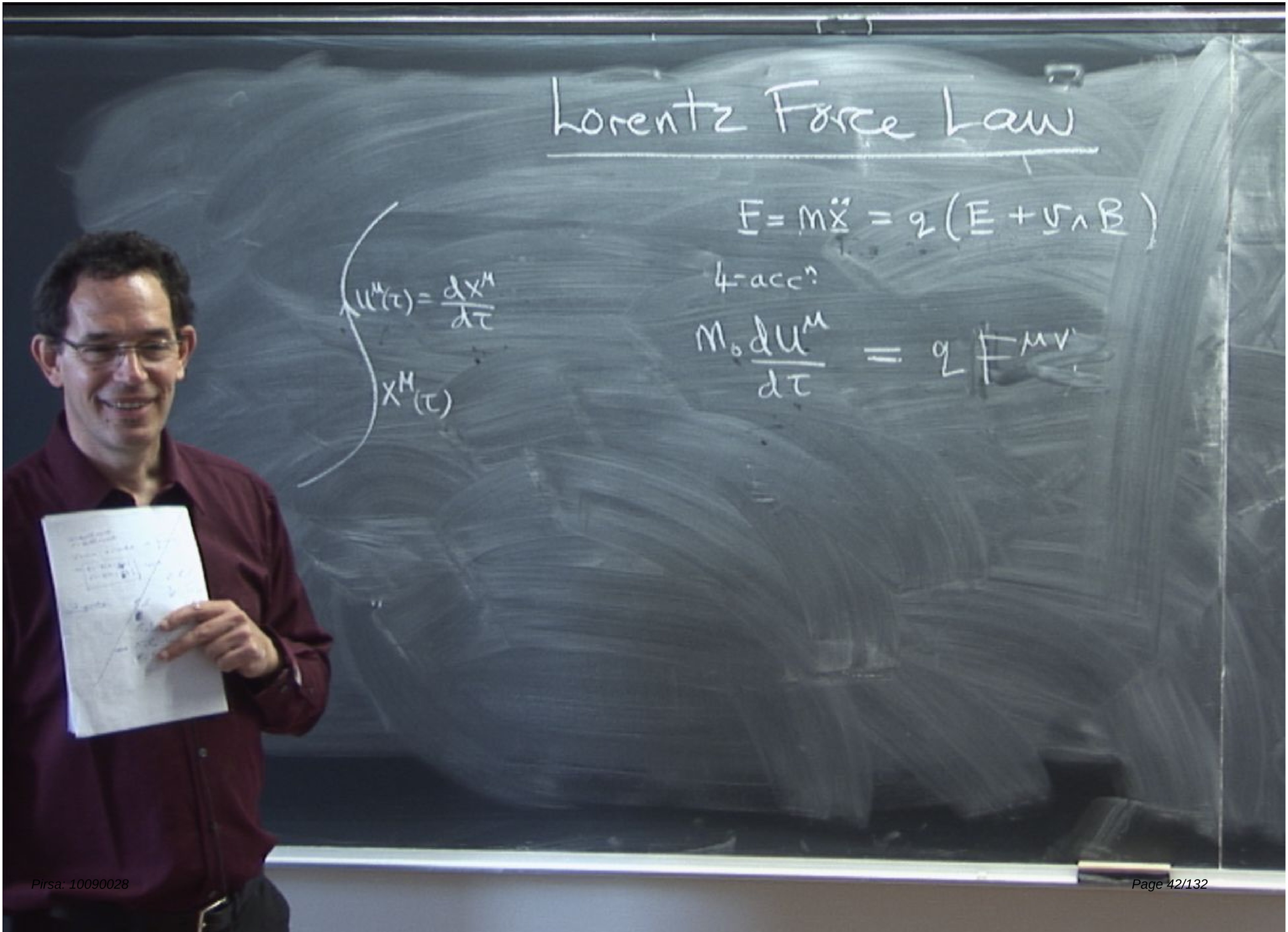

$T^{00} + \underline{\nabla}^2 \phi$

$$T^{00} + \underline{\nabla}^2 \phi$$

$$\partial_\alpha T^{\alpha\mu} = 0$$

$$x'^\mu = x^\mu + c^\mu$$

Emmy Noether


Lorentz Force Law
$\underline{F} = m\ddot{\underline{x}} = q(\underline{E} + \underline{v} \wedge \underline{B})$
4-acc^n?
$u^\mu(\tau) = \frac{dx^\mu}{d\tau}$
$x^\mu(\tau)$
$m_0 \frac{du^\mu}{d\tau} = q F^{\mu\nu}$

Lorentz Force Law

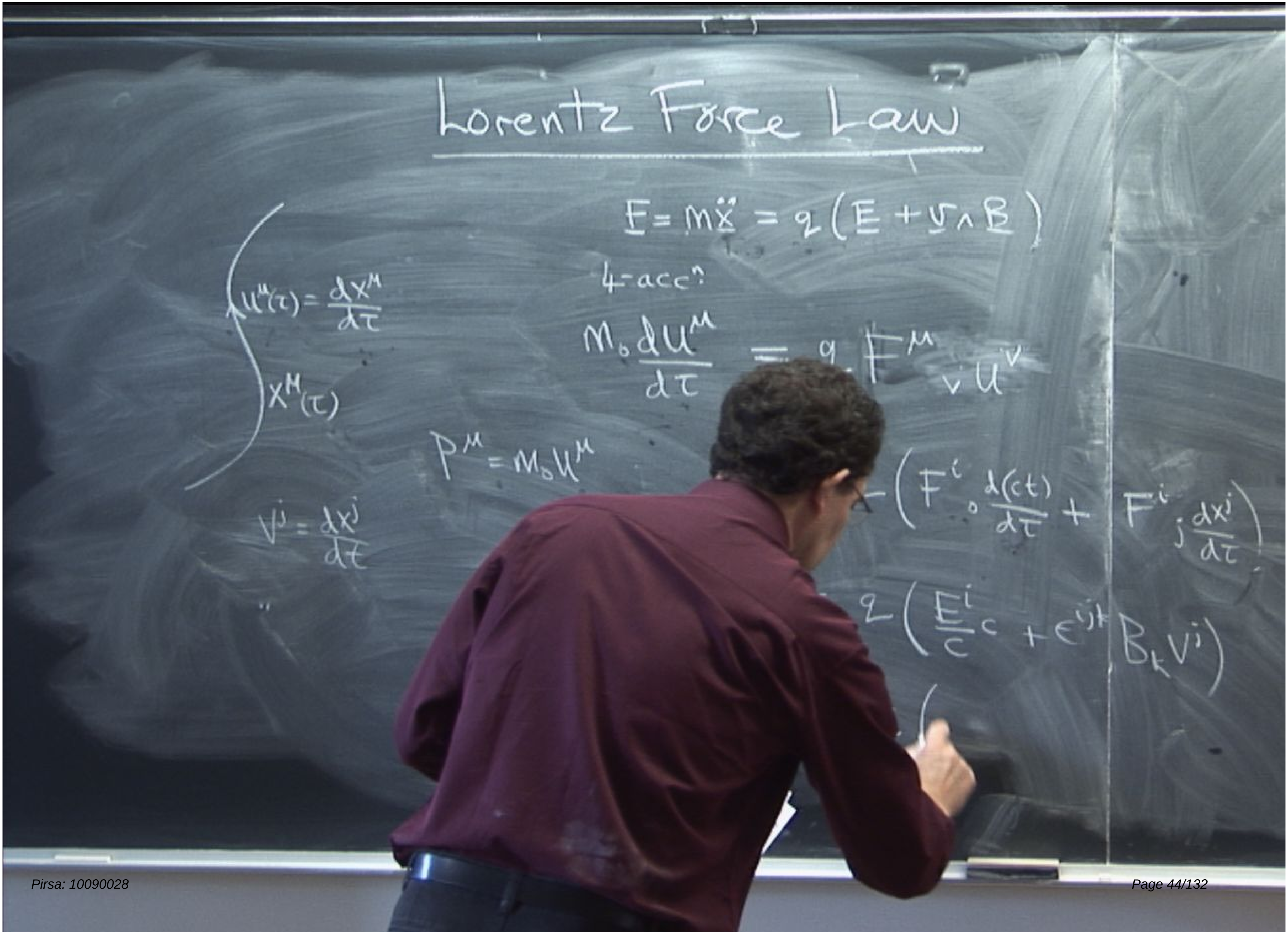
$$\underline{F} = m\ddot{\underline{x}} = q(\underline{E} + \underline{v} \wedge \underline{B})$$

4-accⁿ?

$$m_0 \frac{dU^\mu}{d\tau} = q F^\mu{}_\nu U^\nu$$

$$p^\mu = m_0 U^\mu \qquad \frac{dp^i}{d\tau} = q\left(F^i{}_0 \frac{d(ct)}{d\tau} + F^i{}_j \frac{dx^j}{d\tau}\right)$$

$$\times \frac{d\tau}{dt} \qquad \frac{dp^i}{dt} = q\left(\frac{E^i}{c} c + \epsilon^{ijk} B_k V^j\right)$$


Lorentz Force Law
$\underline{F} = m\ddot{\underline{x}} = q(\underline{E} + \underline{v} \wedge \underline{B})$
4-accⁿ
$u^\mu(\tau) = \frac{dx^\mu}{d\tau}$
$x^\mu(\tau)$
$m_0 \frac{du^\mu}{d\tau} = q F^\mu{}_\nu u^\nu$
$p^\mu = m_0 u^\mu$
$v^j = \frac{dx^j}{dt}$
$\left(F^i{}_0 \frac{d(ct)}{d\tau} + F^i{}_j \frac{dx^j}{d\tau}\right)$
$q\left(\frac{E^i}{c} c + \epsilon^{ijk} B_k v^j\right)$

Lorentz Force Law

$$\underline{F} = m\ddot{\underline{x}} = q\left(\underline{E} + \underline{v} \wedge \underline{B}\right)$$

$$u^{\mu}(\tau) = \frac{dx^{\mu}}{d\tau}$$

$$x^{\mu}(\tau)$$

4-acc^n?

$$m_0 \frac{du^{\mu}}{d\tau} = q F^{\mu}{}_{\nu} u^{\nu}$$

$$p^{\mu} = m_0 u^{\mu}$$

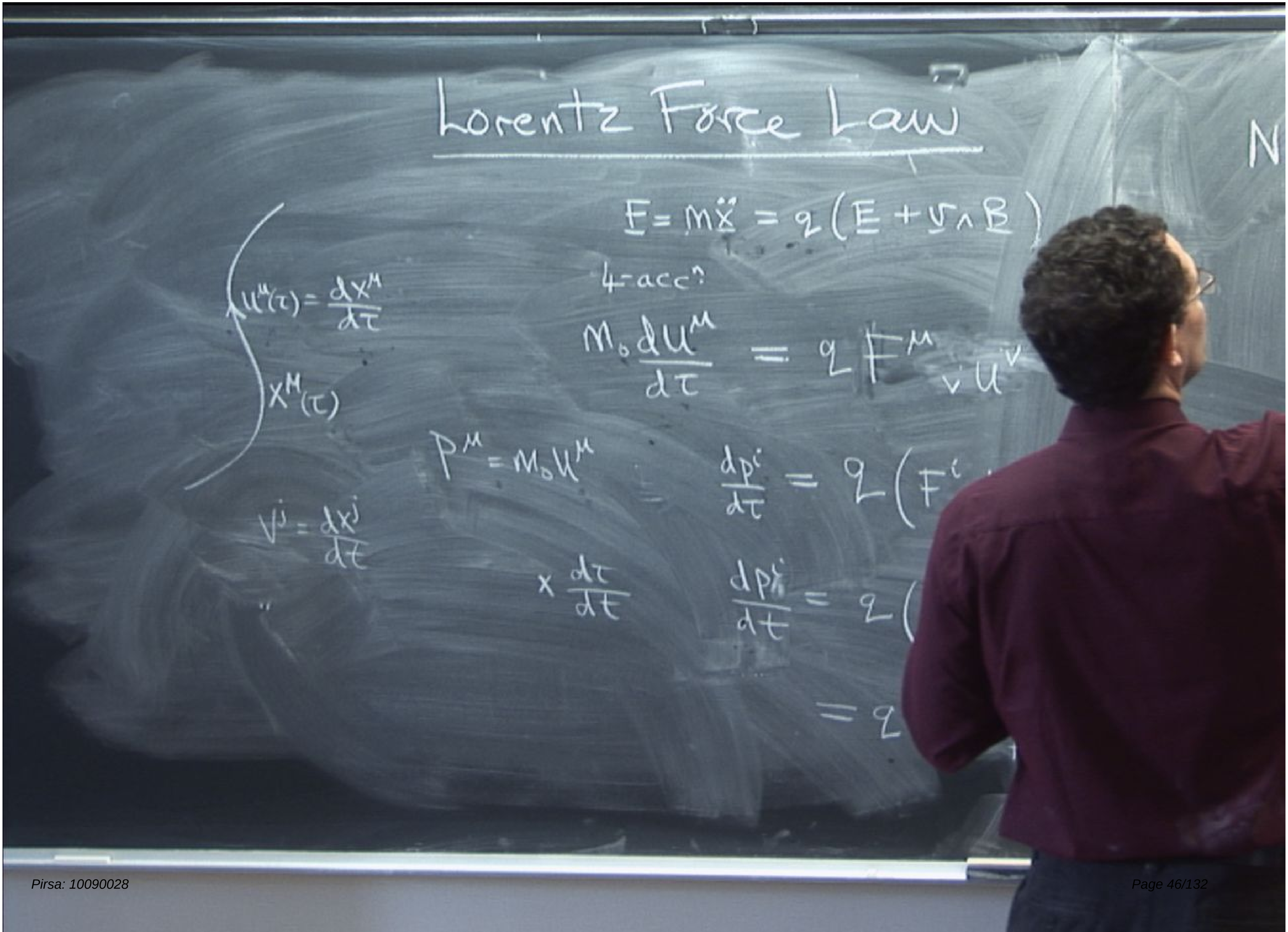
$$\frac{dp^i}{d\tau} = q\left(F^i{}_0 \frac{d(ct)}{d\tau} + F^i{}_j \frac{dx^j}{d\tau}\right)$$

$$v^j = \frac{dx^j}{dt}$$

$$\times \frac{d\tau}{dt}$$

$$\frac{dp^i}{dt} = q\left(\frac{E^i}{c} c + \epsilon^{ijk} B_k v^j\right)$$

$$= q\left(E^i + (v \wedge B)^i\right)$$


Lorentz Force Law
$\underline{F} = m\ddot{\underline{x}} = q(\underline{E} + \underline{v} \wedge \underline{B})$
4-acc^n
$u^\mu(\tau) = \frac{dx^\mu}{d\tau}$
$x^\mu(\tau)$
$m_0 \frac{du^\mu}{d\tau} = q F^\mu{}_\nu u^\nu$
$p^\mu = m_0 u^\mu$
$\frac{dp^i}{d\tau} = q(F^i$
$v^j = \frac{dx^j}{dt}$
$\times \frac{d\tau}{dt}$
$\frac{dp^i}{dt} = q($
$= q$
N

Lorentz Force Law

$$\underline{F} = m\ddot{\underline{x}} = q(\underline{E} + \underline{v} \wedge \underline{B})$$

4-acc^n?

$$u^\mu(\tau) = \frac{dx^\mu}{d\tau}$$

$$m_0 \frac{du^\mu}{d\tau} = q F^\mu{}_\nu u^\nu$$

$$x^\mu(\tau)$$

$$p^\mu = m_0 u^\mu$$

$$\frac{dp^i}{d\tau} = q\left(F^i{}_0 \frac{d(ct)}{d\tau} + F^i{}_j \frac{dx^j}{d\tau}\right)$$

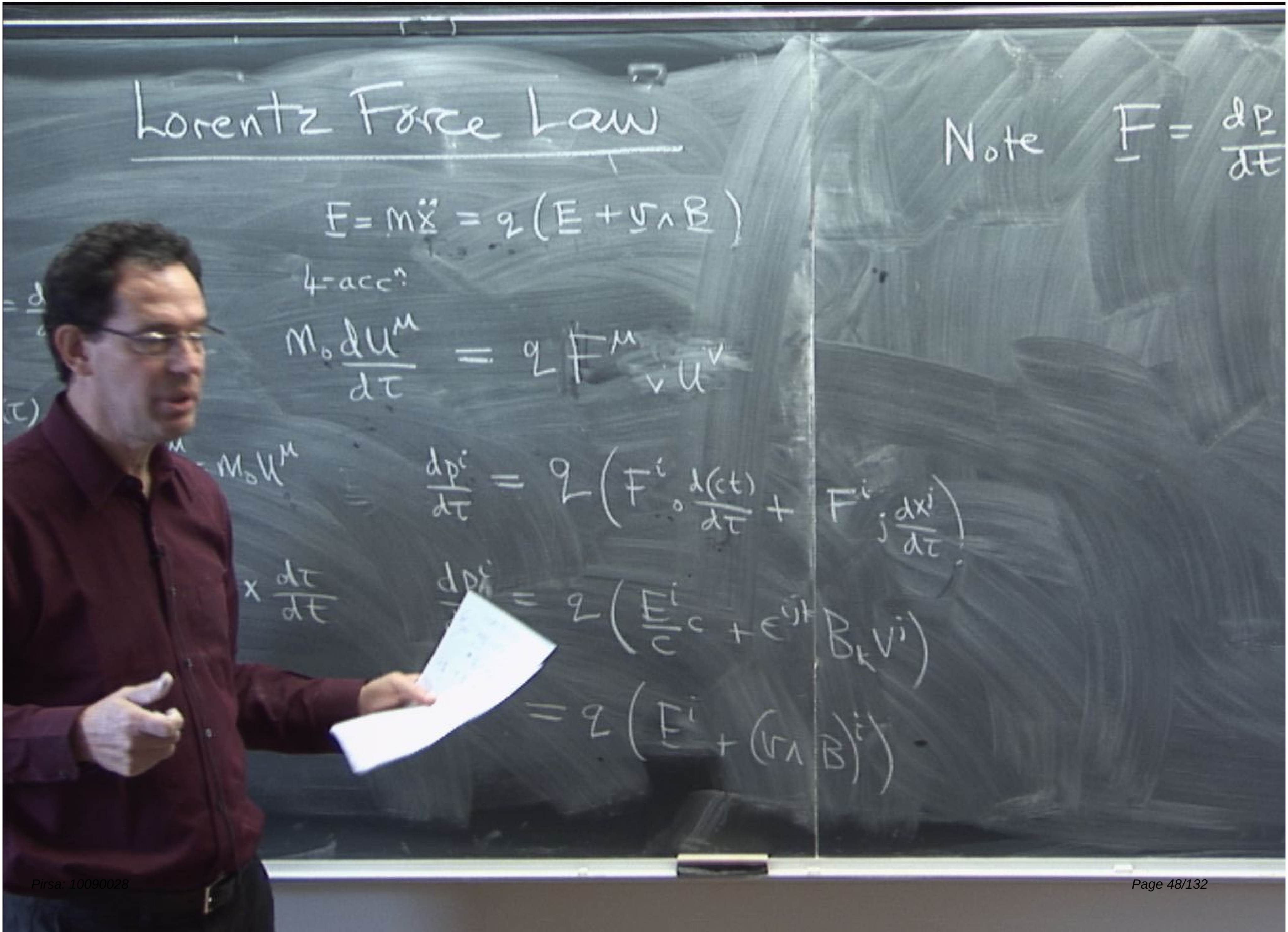
$$v^j = \frac{dx^j}{dt}$$

$$\times \frac{d\tau}{dt}$$

$$\frac{dp^i}{dt} = q\left(\frac{E^i}{c}c + \epsilon^{ijk} B_k v^j\right)$$

$$= q\left(E^i + (v \wedge B)^i\right)$$

N


Lorentz Force Law
$\underline{F} = m\ddot{\underline{x}} = q(\underline{E} + \underline{v} \wedge \underline{B})$
4-acc^n?
$m_0 \frac{dU^\mu}{d\tau} = q F^\mu{}_\nu U^\nu$
$\frac{dp^i}{d\tau} = q\left(F^i{}_0 \frac{d(ct)}{d\tau} + F^i{}_j \frac{dx^j}{d\tau}\right)$
$= q\left(\frac{E^i}{c}c + \epsilon^{ijk} B_k V^j\right)$
$= q\left(E^i + (v \wedge B)^i\right)$
$\times \frac{d\tau}{dt}$
$= M_0 U^\mu$
Note $\underline{F} = \frac{d\underline{p}}{dt}$

Force Law

$$= m\ddot{\underline{x}} = q(\underline{E} + \underline{v} \wedge \underline{B})$$

cc?

$$\frac{dU^\mu}{d\tau} = q F^\mu{}_\nu U^\nu$$

$$\frac{dp^i}{d\tau} = q\left(F^i{}_0 \frac{d(ct)}{d\tau} + F^i{}_j \frac{dx^j}{d\tau}\right)$$

$$\frac{dp^i}{dt} = q\left(\frac{E^i}{c}c + \epsilon^{ijk} B_k V^j\right)$$

$$= q\left(E^i + (\underline{v} \wedge \underline{B})^i\right)$$

Note $\underline{F} = \frac{d\underline{p}}{dt}$ not $\frac{d\underline{p}}{d\tau}$

$$\frac{dp^0}{d\tau} = q F^0{}_i U^i$$

Force Law

$$= m\ddot{\underline{x}} = q(\underline{E} + \underline{v} \wedge \underline{B})$$

cc^n

$$\frac{dU^\mu}{d\tau} = q F^\mu{}_\nu U^\nu$$

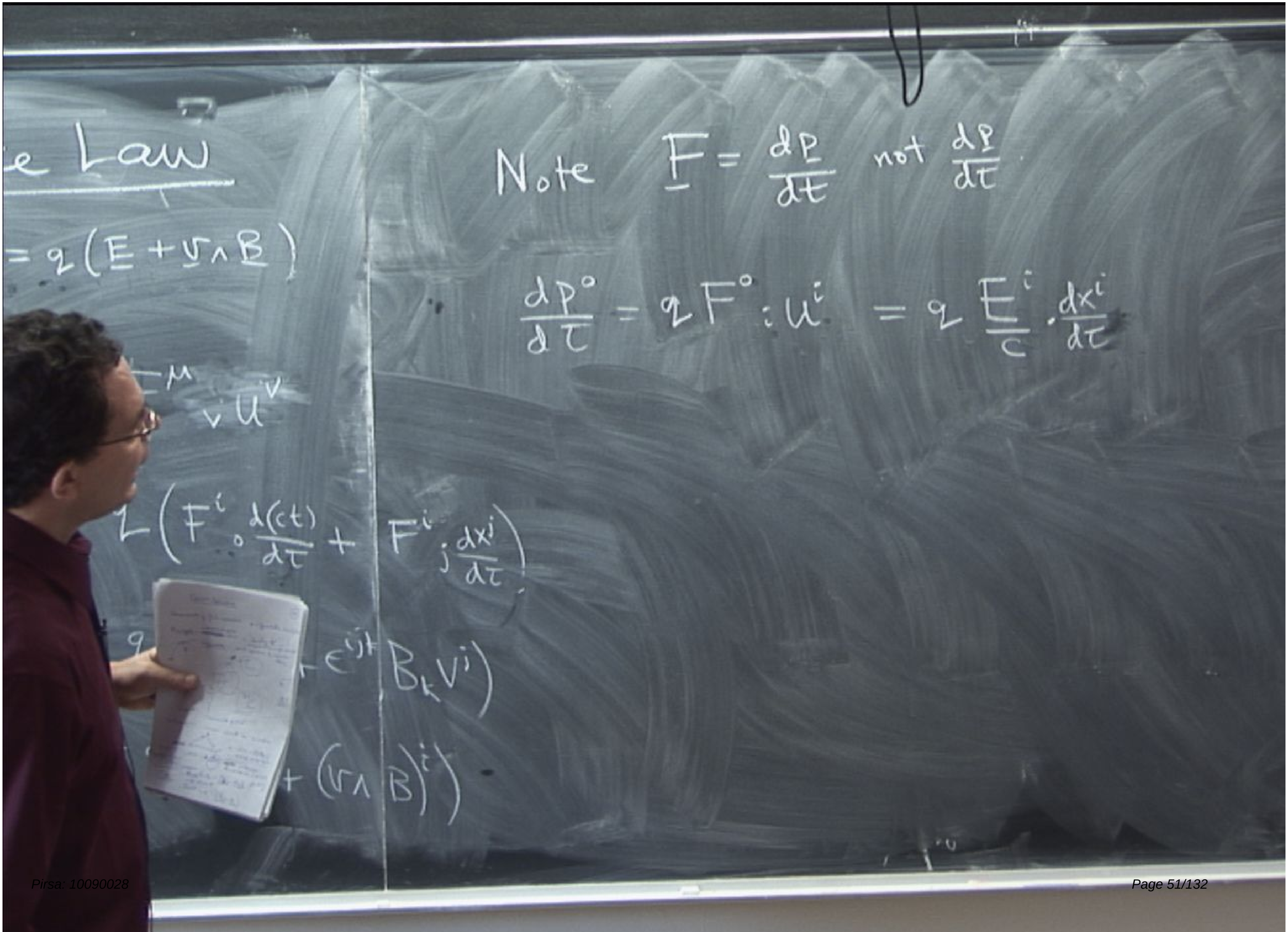
$$\frac{dp^i}{d\tau} = q\left(F^i{}_0 \frac{d(ct)}{d\tau} + F^i{}_j \frac{dx^j}{d\tau}\right)$$

$$\frac{dp^i}{dt} = q\left(\frac{E^i}{c}c + \epsilon^{ijk} B_k v^j\right)$$

$$= q\left(E^i + (v \wedge B)^i\right)$$

Note $\underline{F} = \frac{d\underline{p}}{dt}$ not $\frac{d\underline{p}}{d\tau}$

$$\frac{dp^0}{d\tau} = q F^0{}_i u^i = q \frac{E^i}{c}$$


Note $\underline{F} = \frac{dp}{dt}$ not $\frac{dp}{d\tau}$
$\frac{dp^0}{d\tau} = q F^0{}_i u^i = q \frac{\underline{E}^i}{c} \cdot \frac{dx^i}{d\tau}$
$= q(\underline{E} + \underline{v} \wedge \underline{B})$

e Law

$= q(\underline{E} + \underline{v} \wedge \underline{B})$

$= q F^{\mu}{}_{\nu} u^{\nu}$

$\frac{dp^i}{d\tau} = q\left(F^i{}_0 \frac{d(ct)}{d\tau} + F^i{}_j \frac{dx^j}{d\tau}\right)$

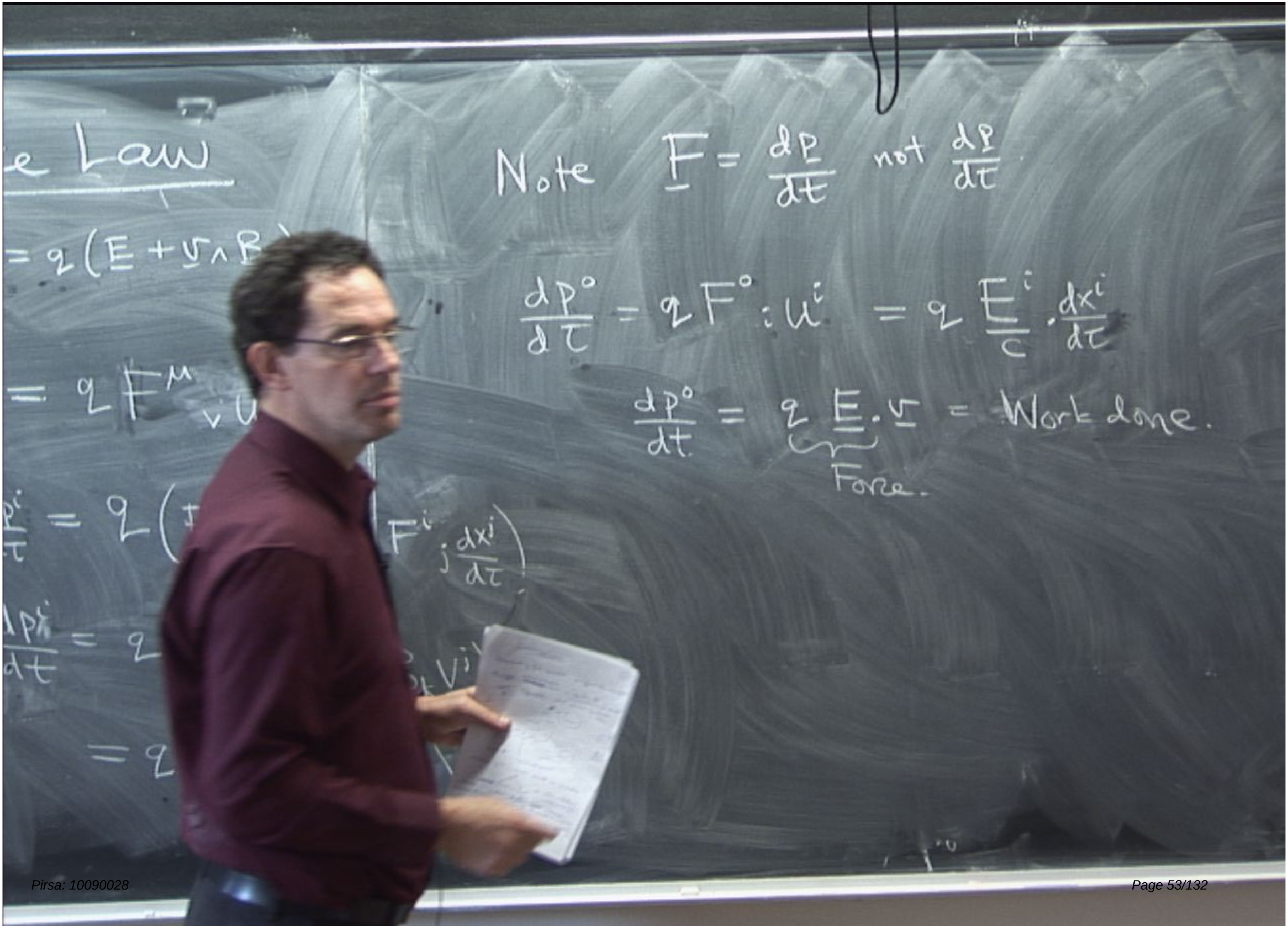
$\frac{dp^i}{dt} = q\left(\frac{E^i}{c} c + \epsilon^{ijk} B_k V^j\right)$

$= q\left(E^i + (v \wedge B)^i\right)$

Note $\underline{F} = \frac{d\underline{p}}{dt}$ not $\frac{d\underline{p}}{d\tau}$

$\frac{dp^0}{d\tau} = q F^0{}_i u^i = q \frac{\underline{E}^i}{c} \cdot \frac{dx^i}{d\tau}$

$\frac{dp^0}{dt} = q\, \underline{E} \cdot \underline{v}$

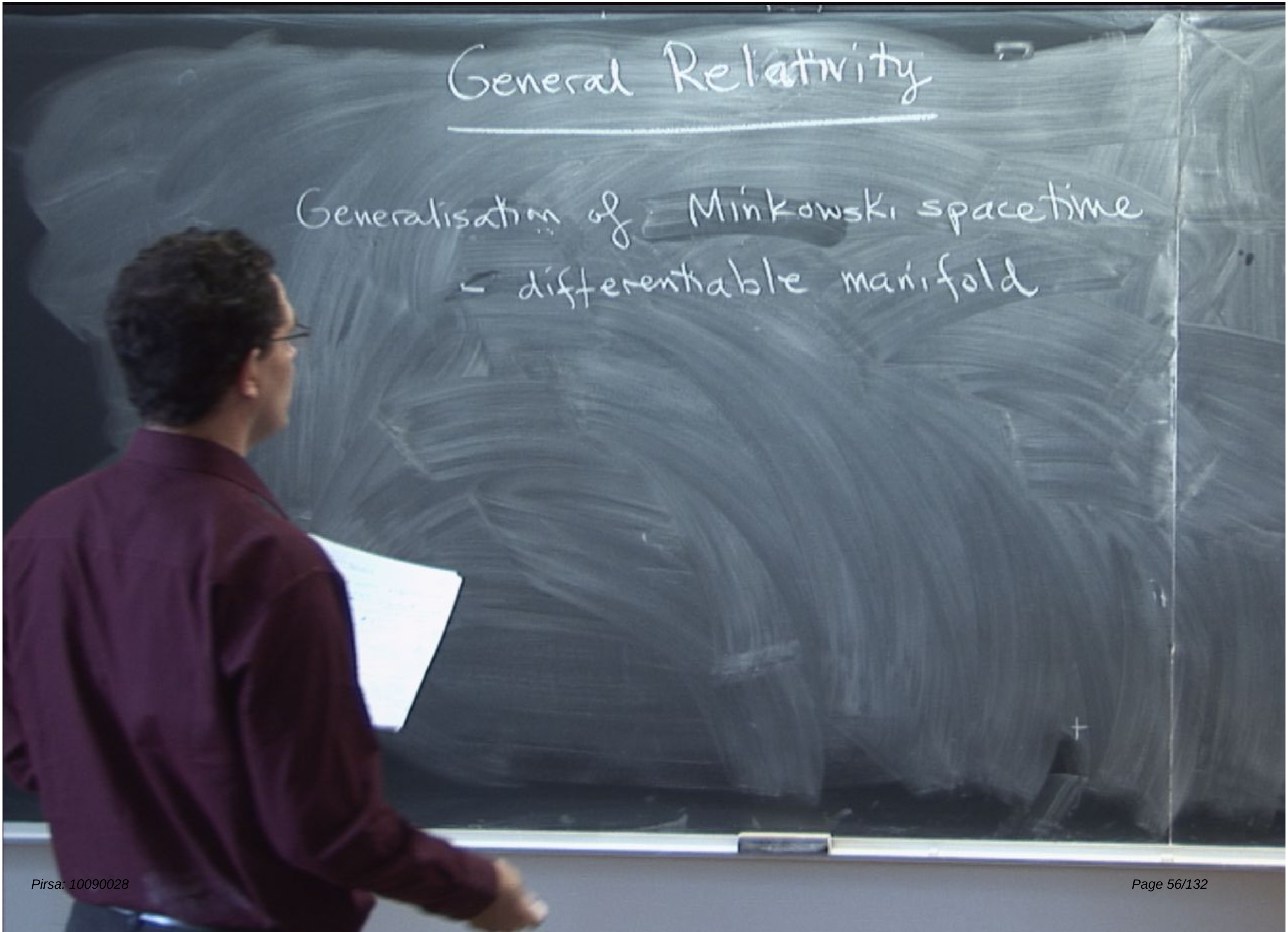


Law
$= q(E + v \wedge B)$
Note $\underline{F} = \frac{dp}{dt}$ not $\frac{dp}{d\tau}$
$\frac{dp^0}{d\tau} = qF^0{}_i u^i = q \frac{E^i}{c} \cdot \frac{dx^i}{d\tau}$
$\frac{dp^0}{dt} = q\,\underbrace{\underline{E}}_{\text{Force.}}\cdot\underline{v}$ = Work done.


General Relativity

General Relativity

Generalisation of Minkowski spacetime

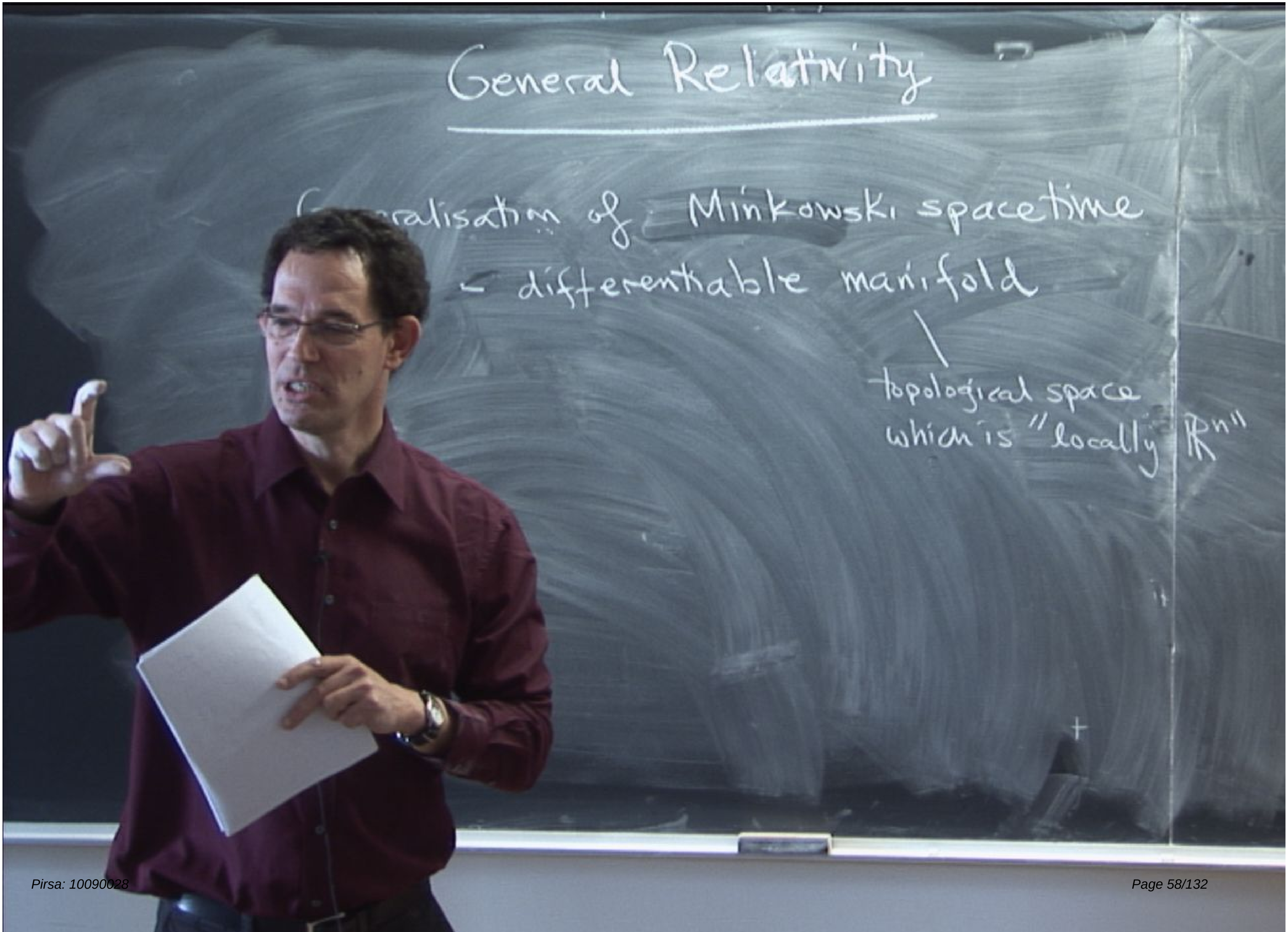

General Relativity
Generalisation of Minkowski spacetime
- differentiable manifold

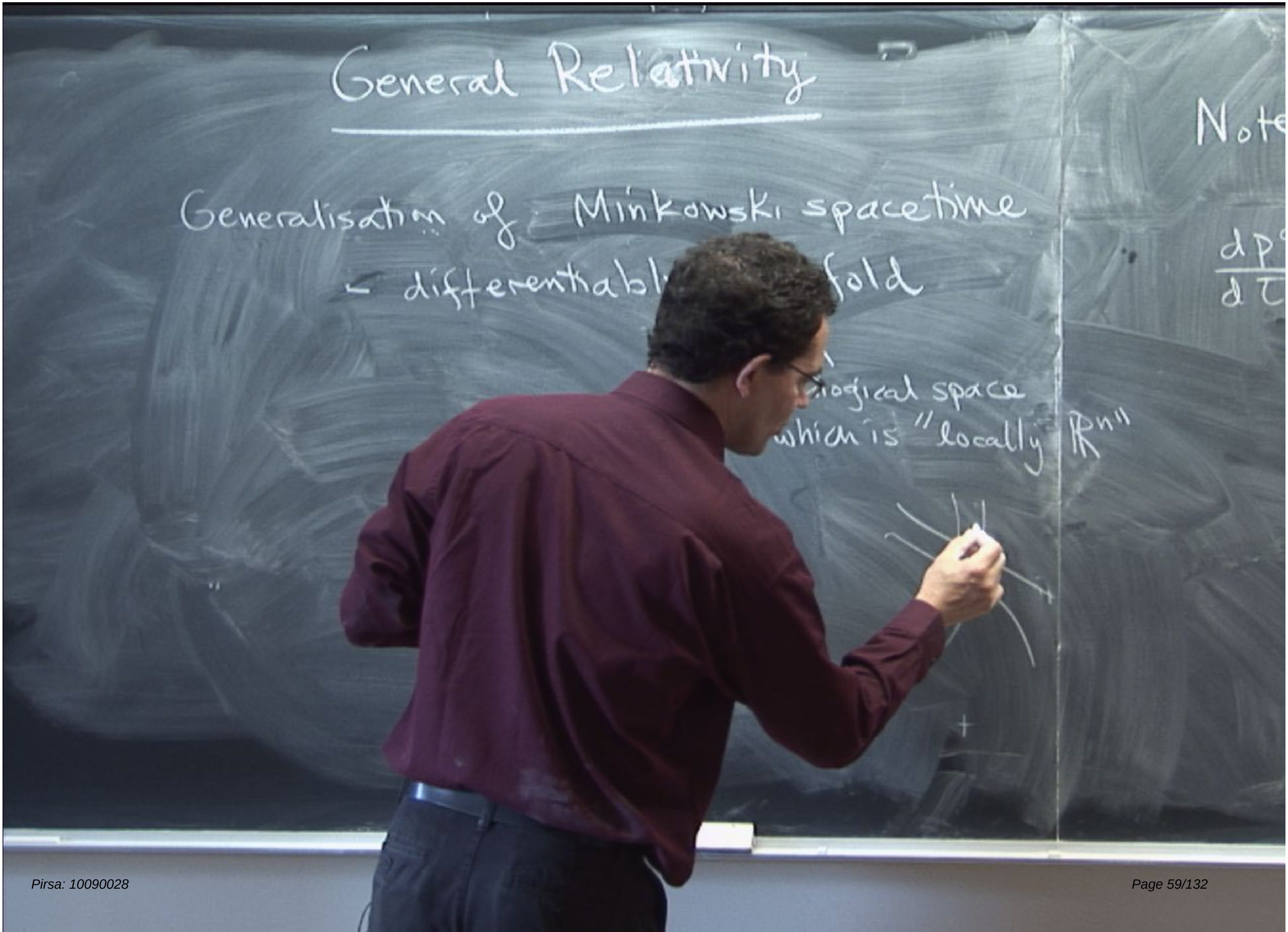
General Relativity

Generalisation of Minkowski spacetime

– differentiable manifold

topological space


General Relativity
ralisation of Minkowski spacetime
– differentiable manifold
topological space
which is "locally $\mathbb{R}^n$"


General Relativity
Generalisation of Minkowski spacetime
– differentiabl
fold
ogical space
which is "locally $\mathbb{R}^n$"
Note
$\frac{dp}{d\tau}$

General Relativity

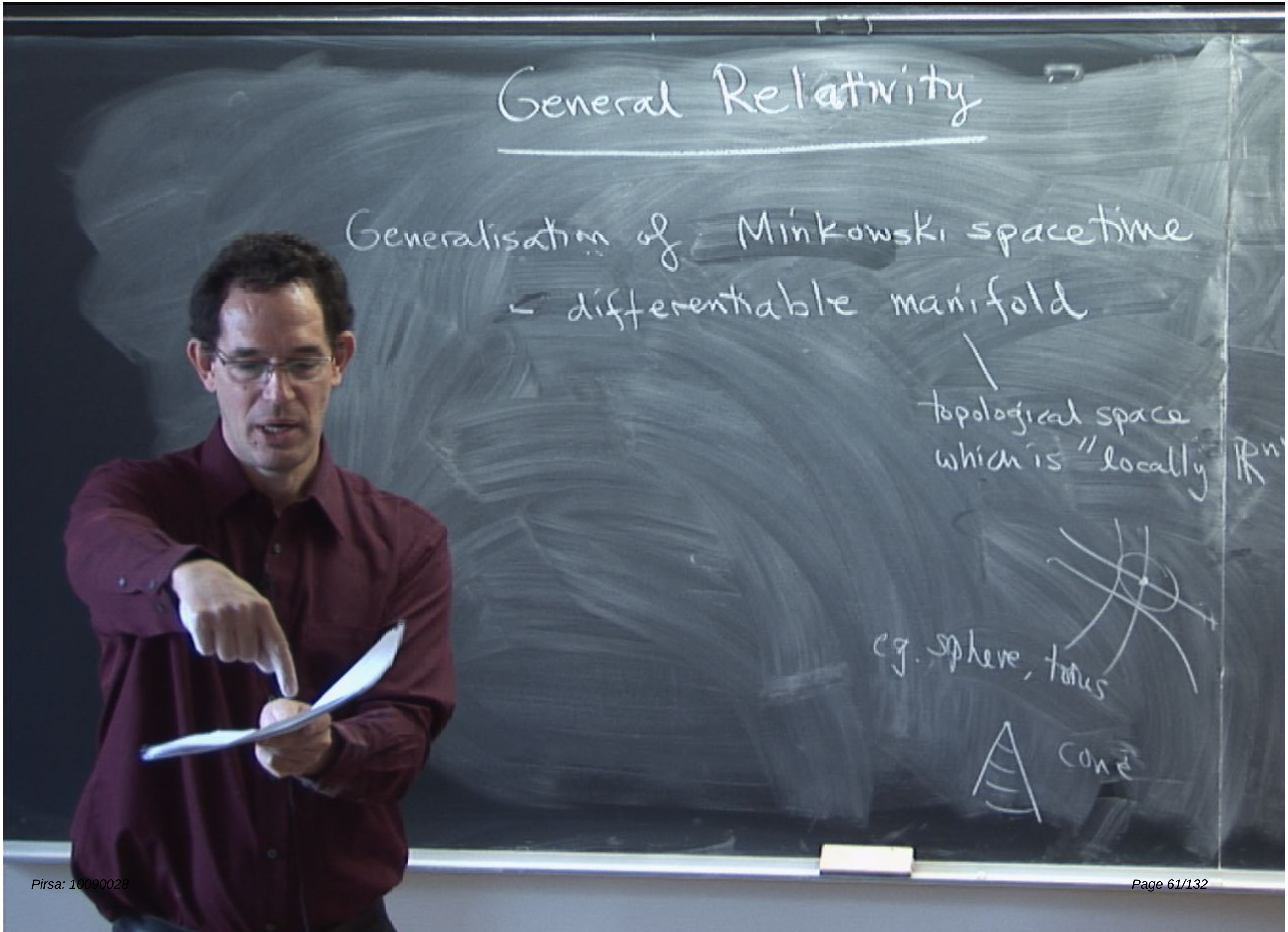
Generalisation of Minkowski spacetime

– differentiable manifold

topological space
which is "locally $\mathbb{R}^n$"

Note

$\frac{dp}{d\tau}$


General Relativity
Generalisation of Minkowski spacetime
- differentiable manifold
topological space
which is "locally" $\mathbb{R}^n$
e.g. sphere, torus
cone

General Relativity

Generalisation of Minkowski spacetime

← differentiable manifold

usual operations of calculus should apply

topological space which is "locally" $\mathbb{R}^n$

eg. sphere, torus

cone

General Relativity

Generalisation of Minkowski spacetime

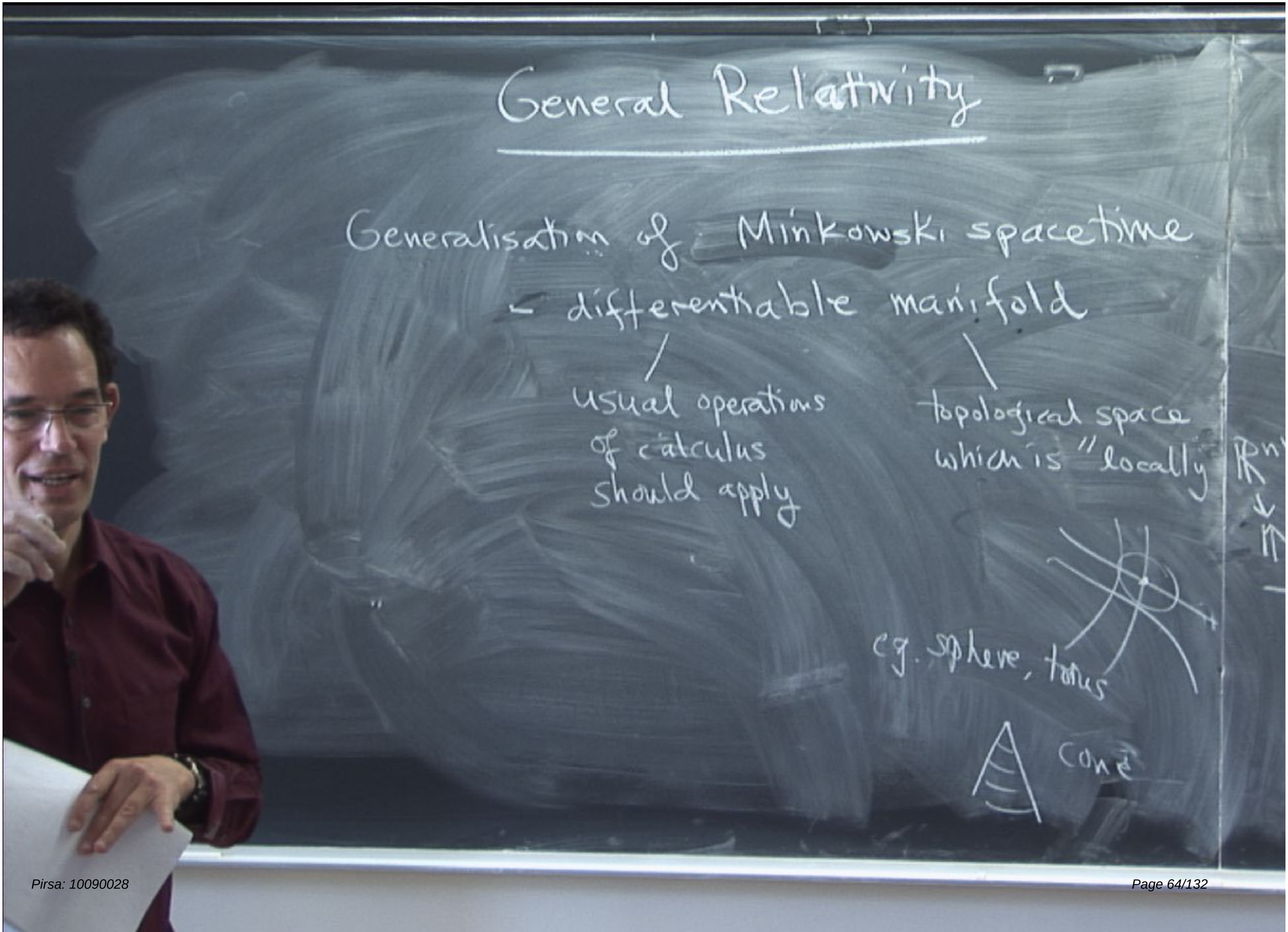
– differentiable manifold

usual operations
of calculus
should apply

topological space
which is "locally $\mathbb{R}^n$"

e.g. sphere, torus

cone

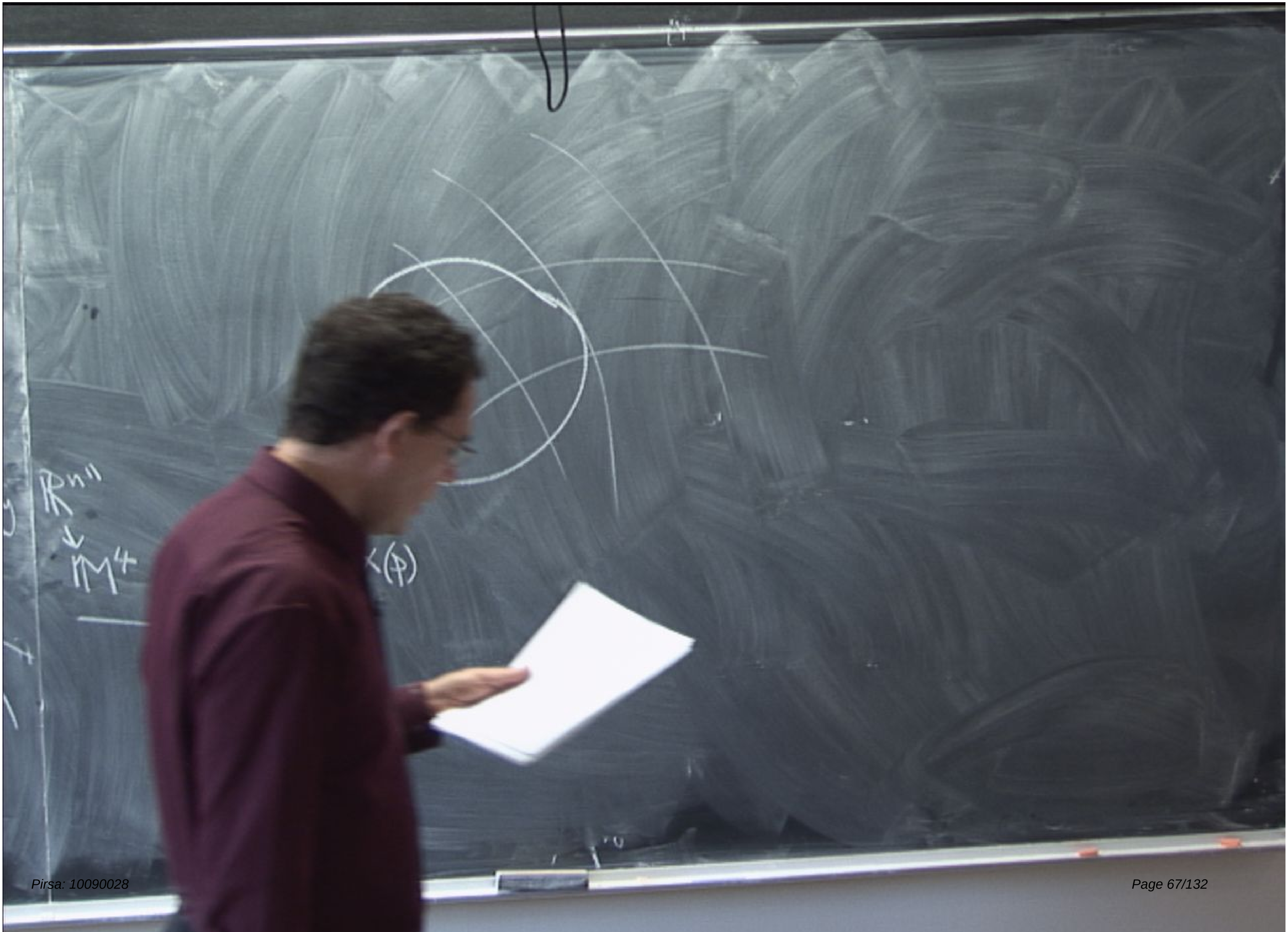

General Relativity
Generalisation of Minkowski spacetime
– differentiable manifold
usual operations of calculus should apply
topological space which is "locally
e.g. sphere, torus
cone

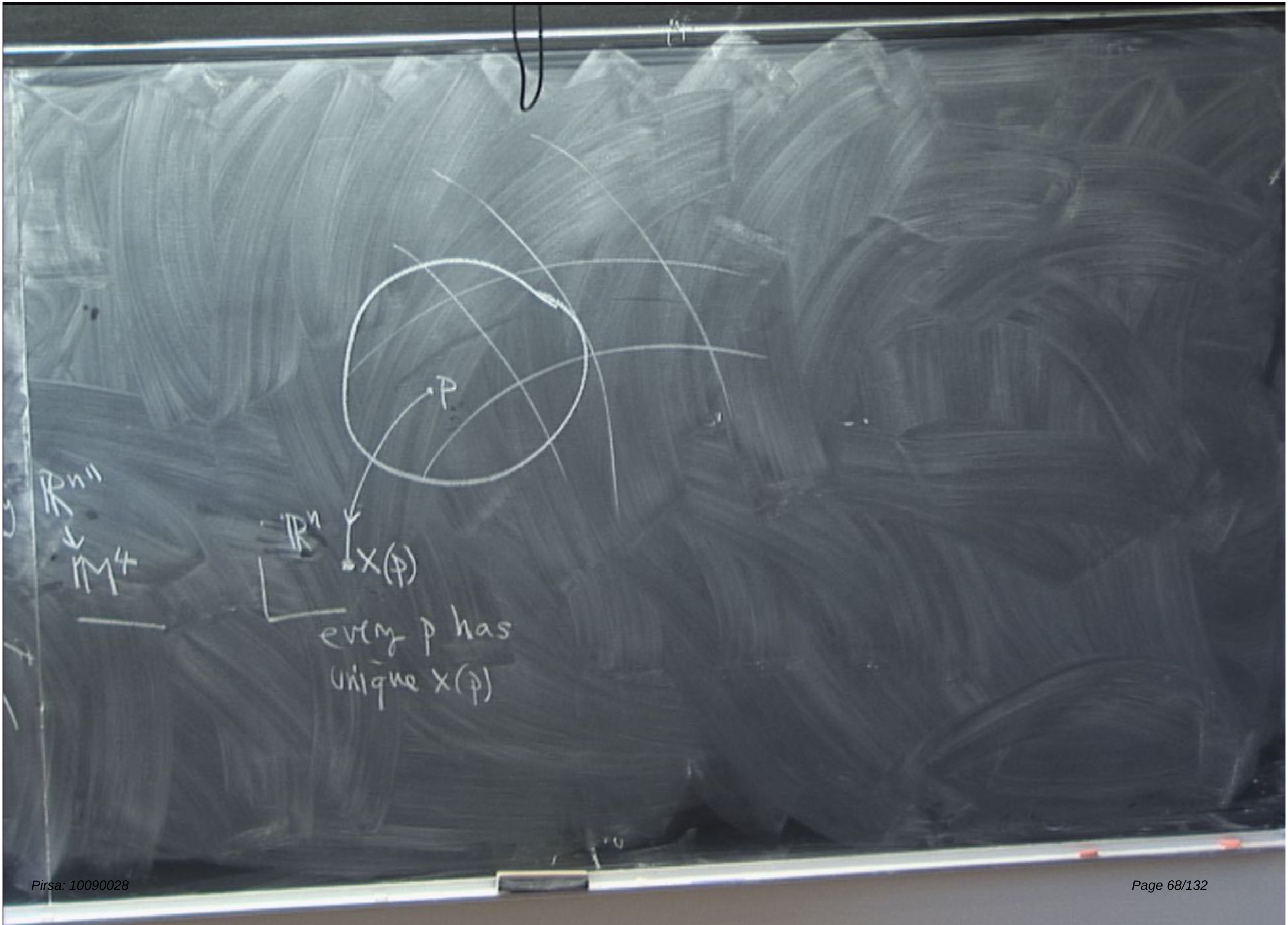
$\mathbb{R}^{``n"}$

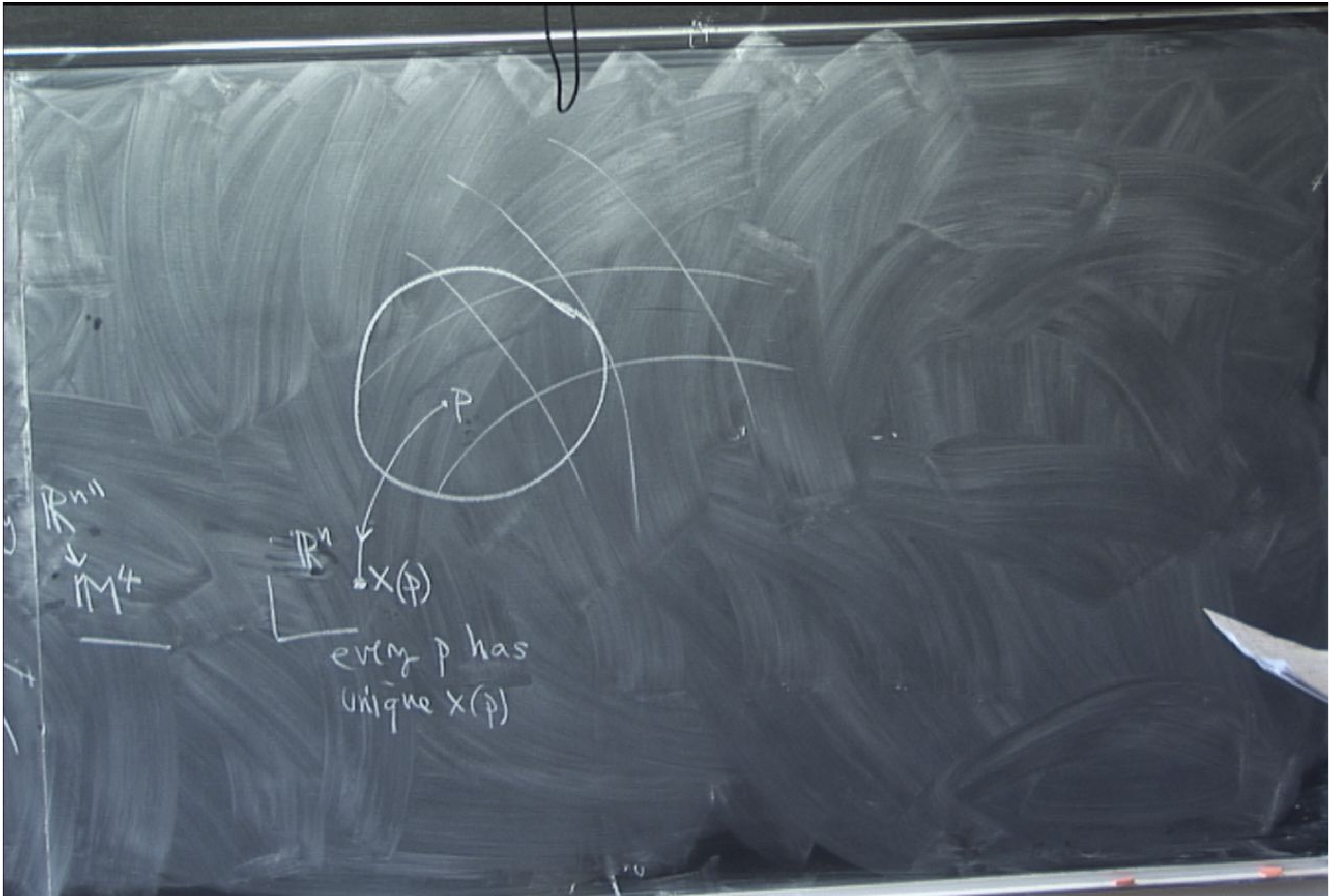
$\downarrow$

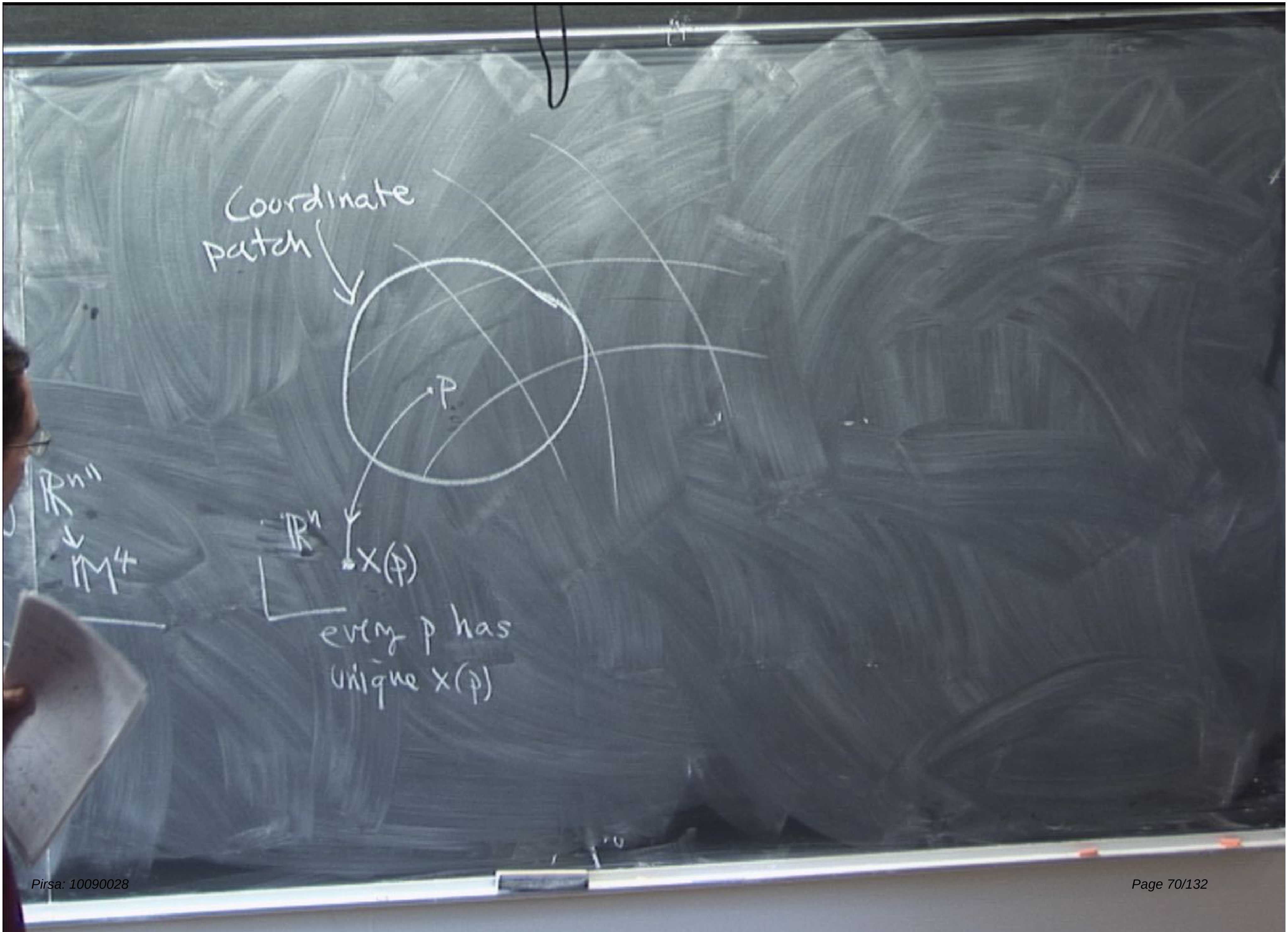
$\mathbb{M}^4$

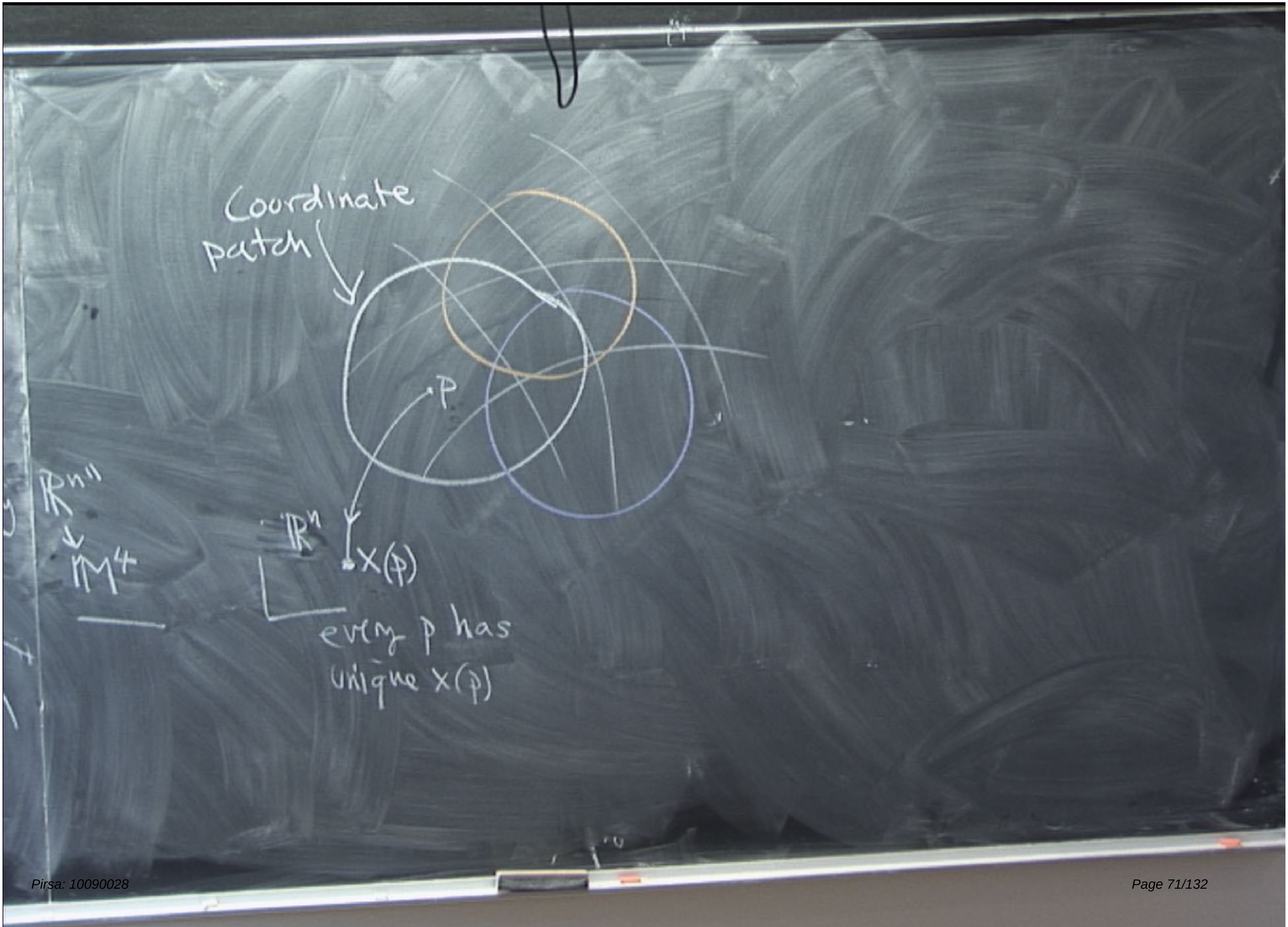

$\mathbb{R}^{n''}$
$\mathbb{M}^4$
$\mathbb{R}^n$

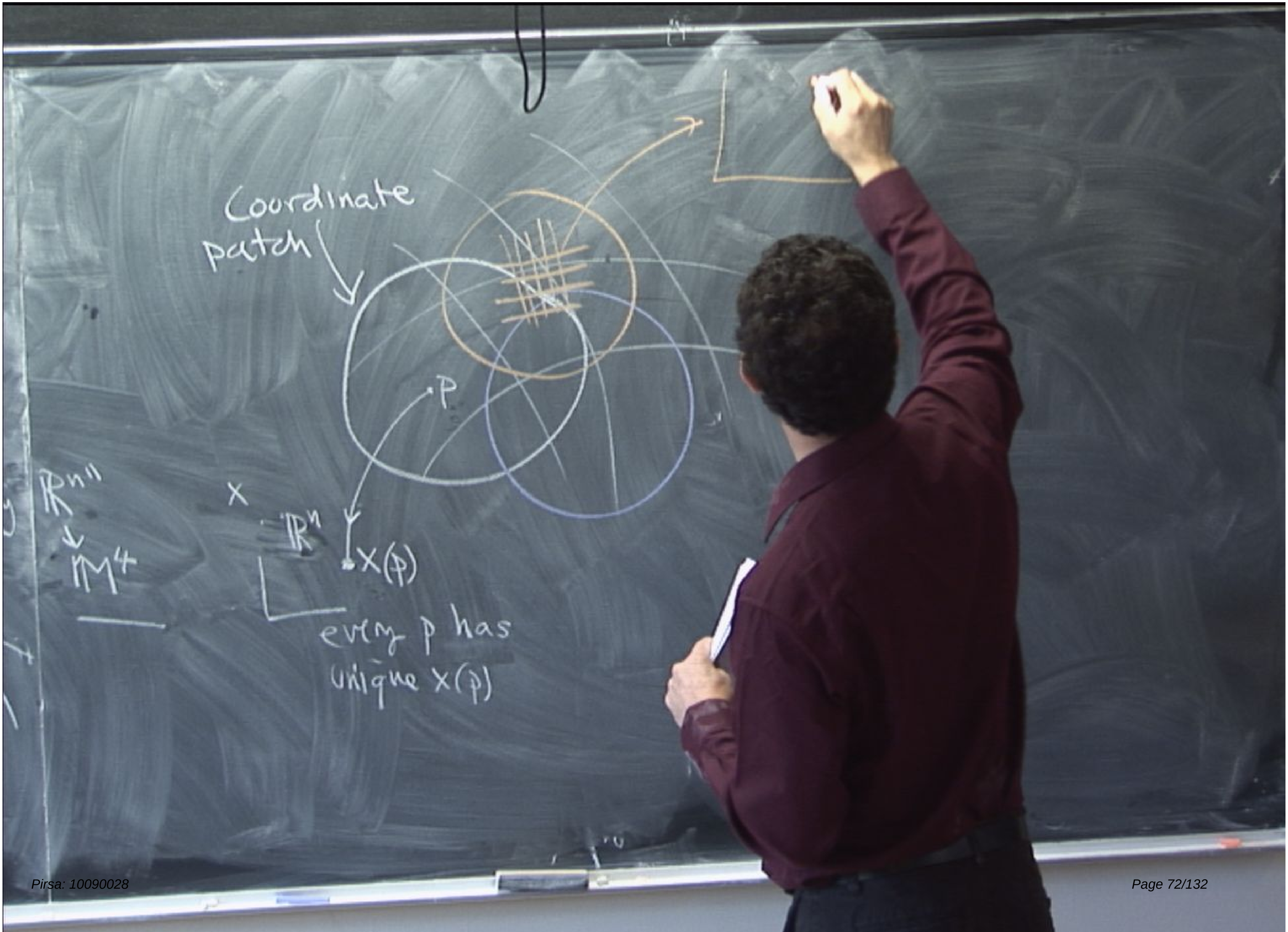



ℝ^n"
M^4
ℝ^n
p
x(p)
every p has
unique x(p)

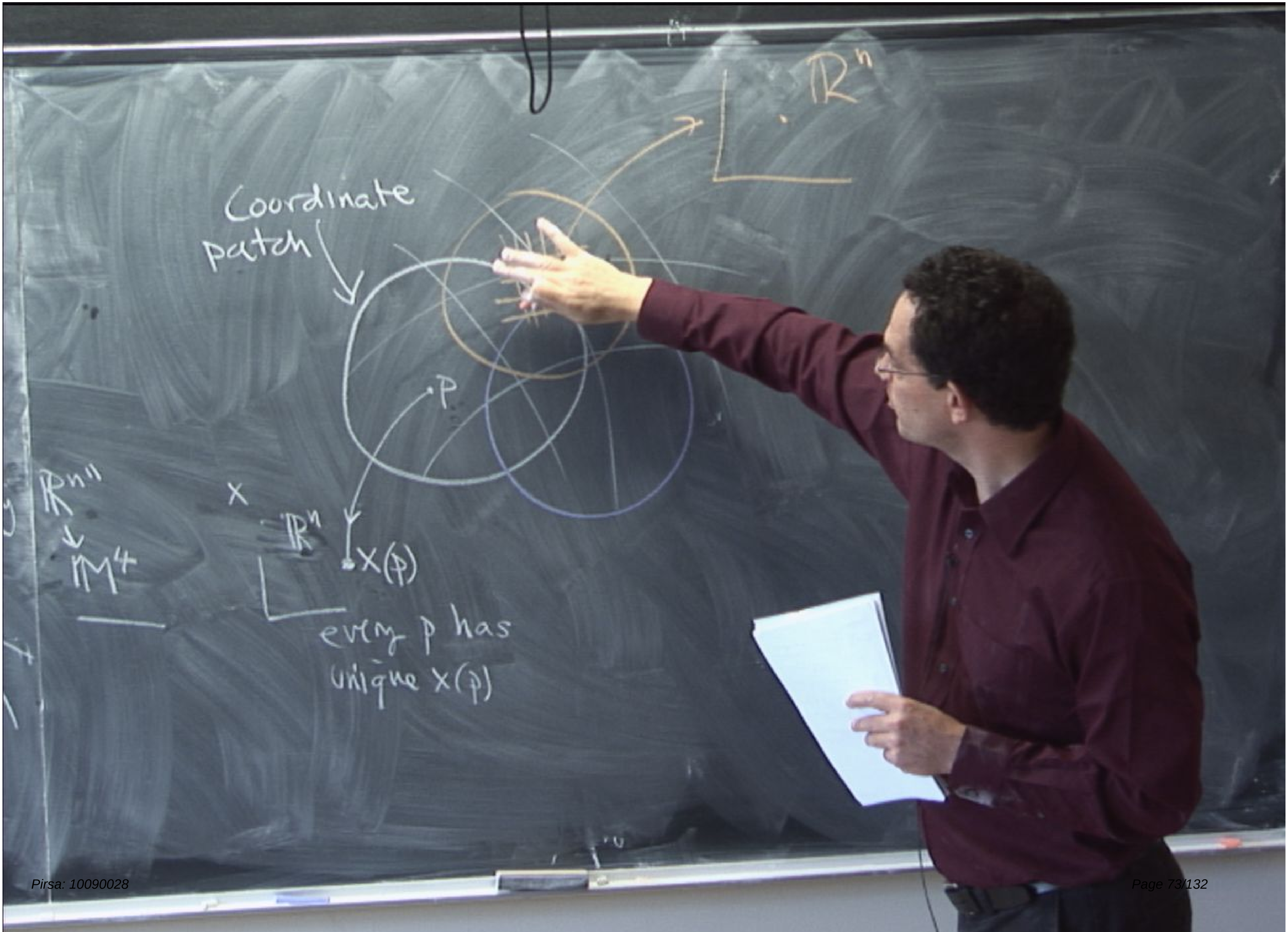

p
ℝⁿ
x(p)
every p has
unique x(p)
"ℝⁿ"
M⁴

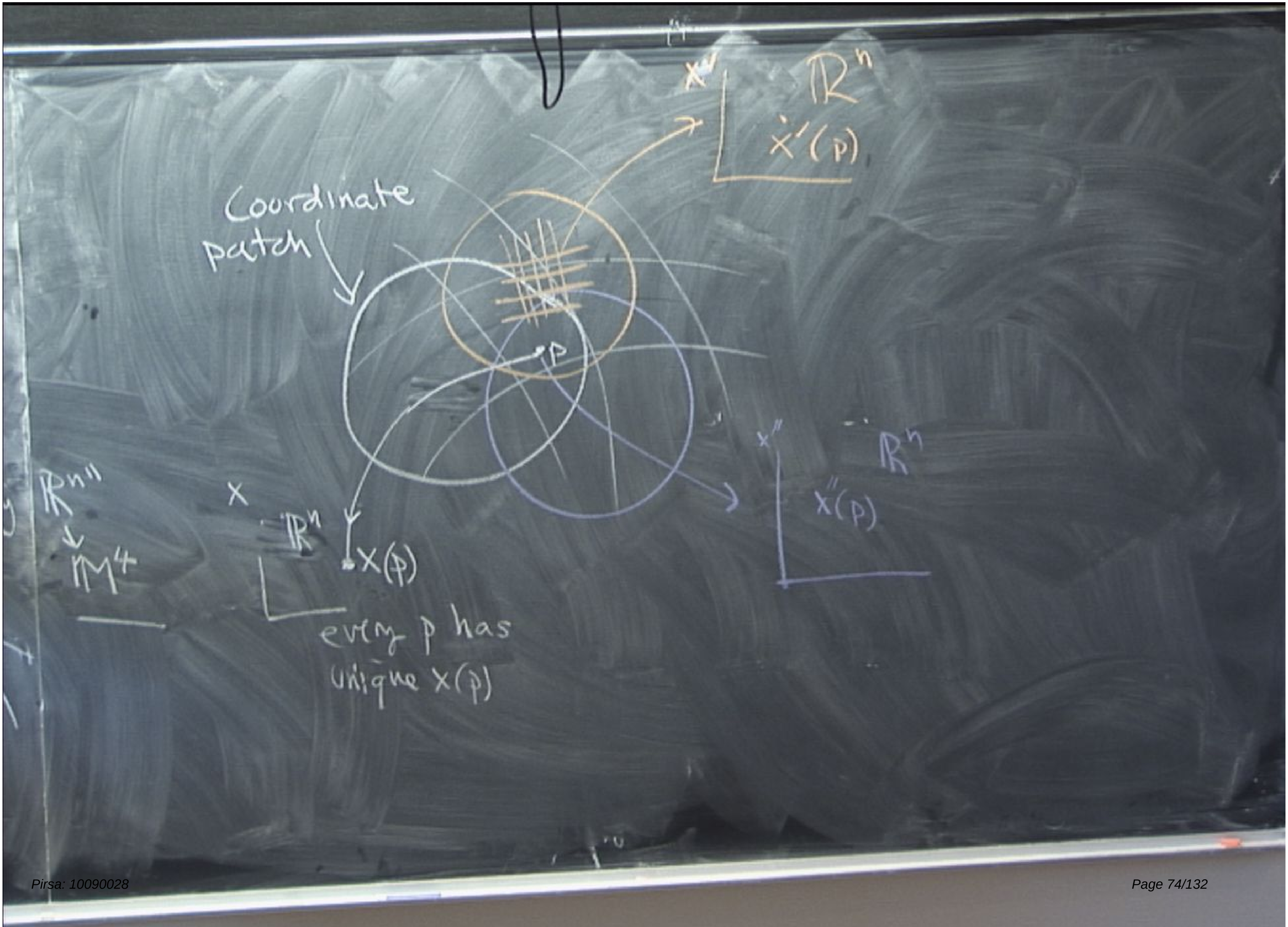

Coordinate
patch
p
$\mathbb{R}^n$
x(p)
every p has
unique x(p)

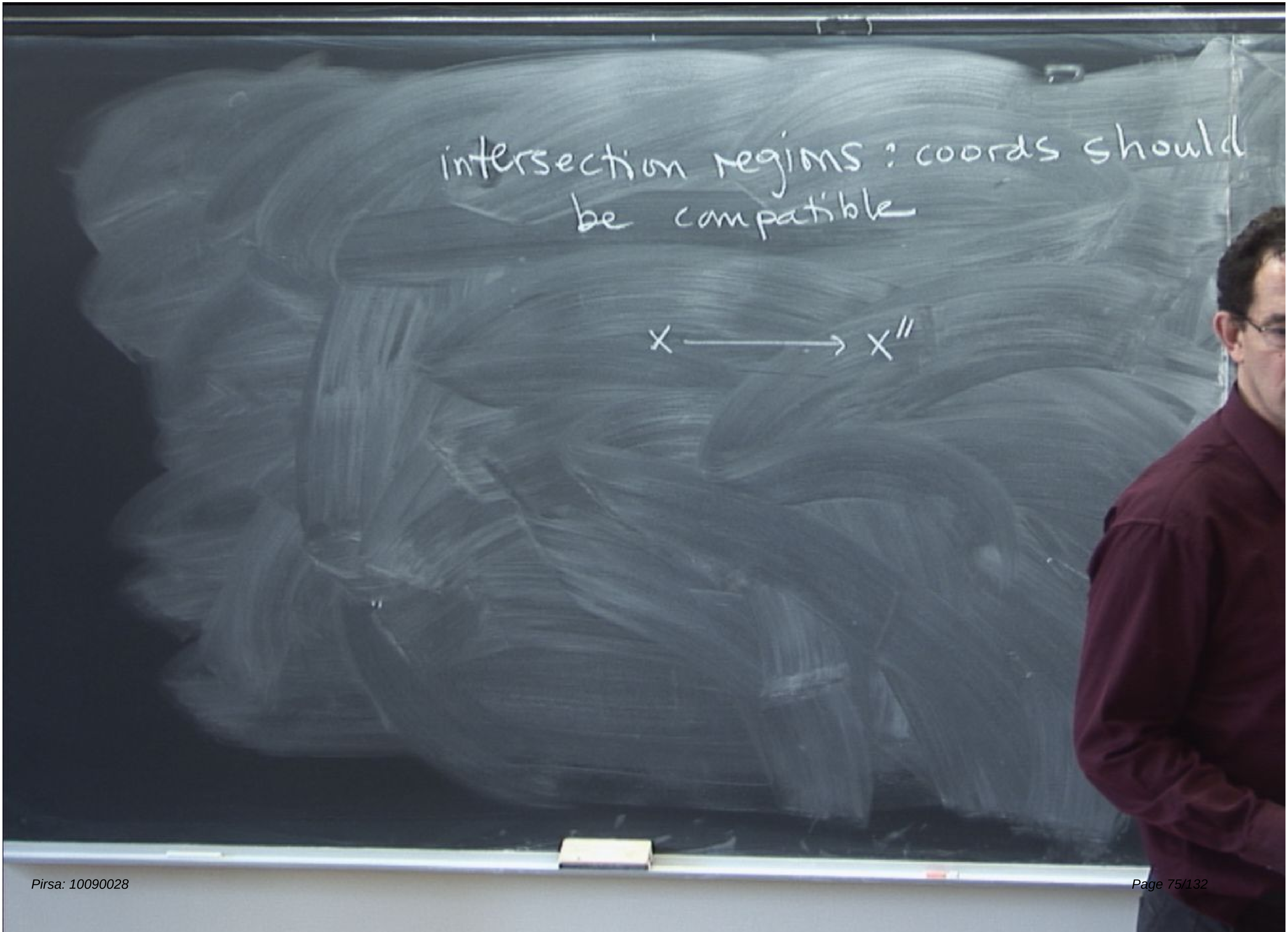

Coordinate
patch
p
ℝⁿ"
M⁴
ℝⁿ
x(p)
every p has
unique x(p)



Coordinate
patch
p
x
$\mathbb{R}^n$
x(p)
"$\mathbb{R}^n$"
$\mathbb{M}^4$
every p has
unique x(p)

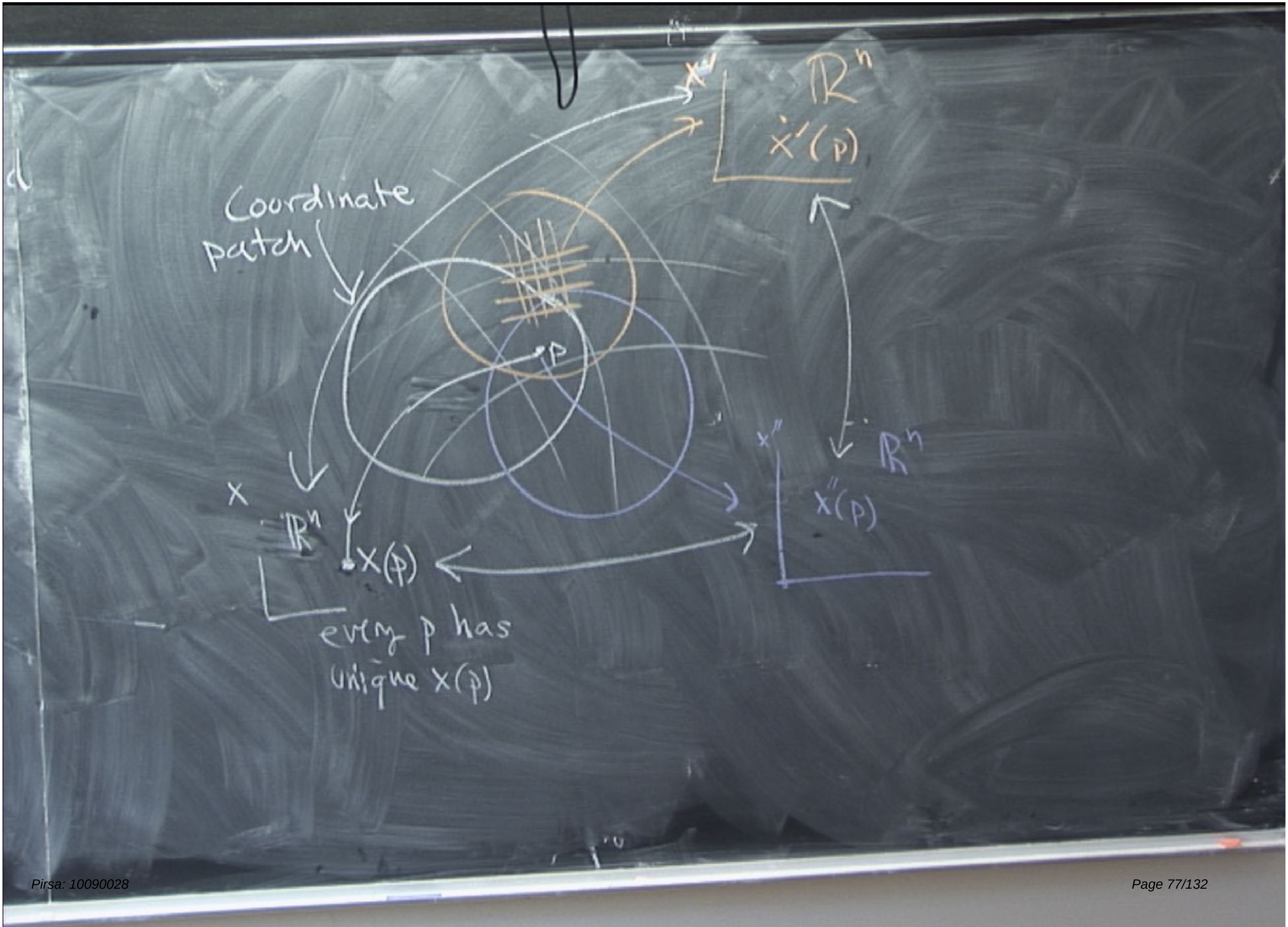

Coordinate
patch
p
x
ℝⁿ
x(p)
every p has
unique x(p)
ℝⁿ
ℳ⁴

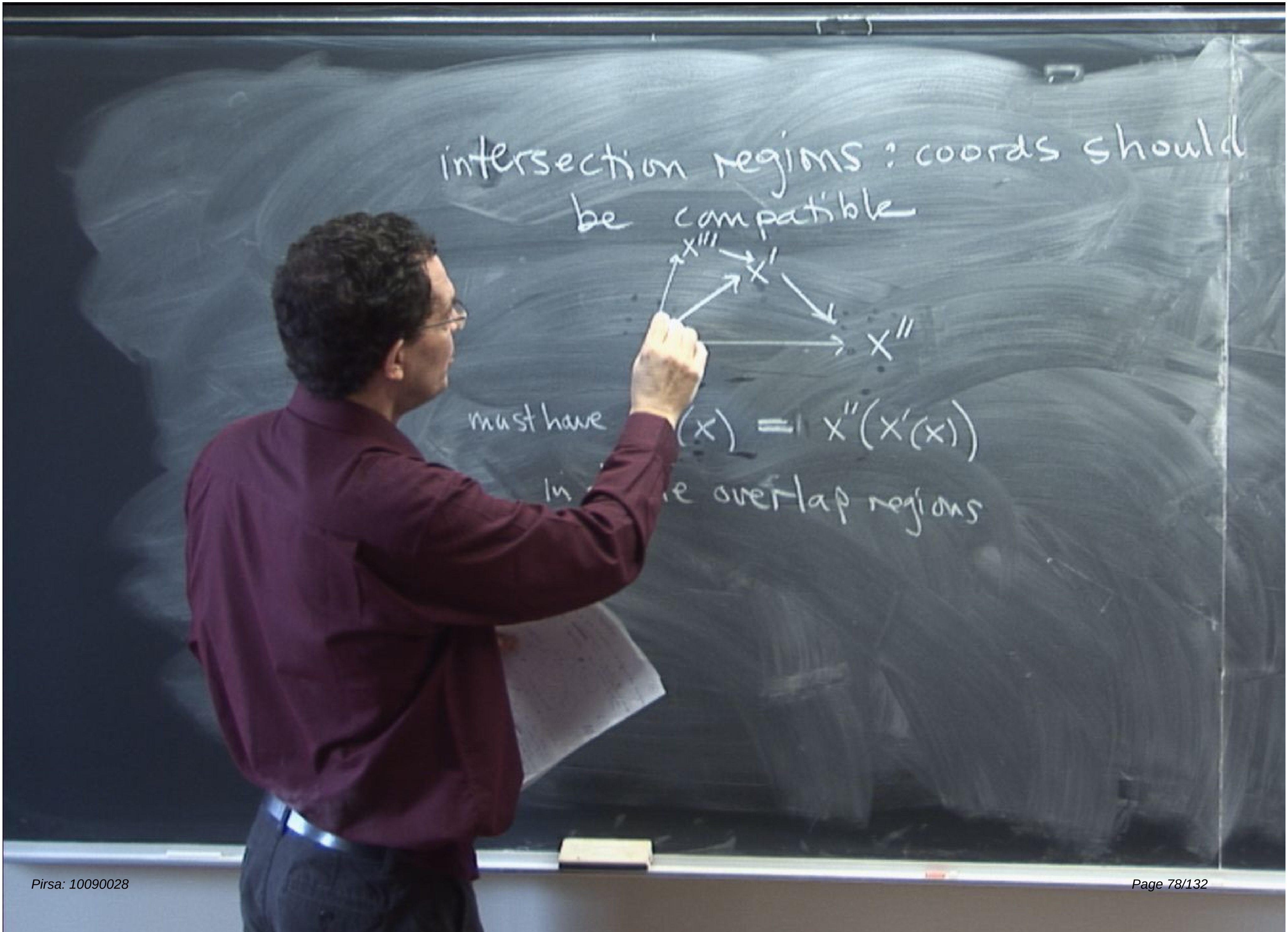

Coordinate
patch
p
x'
$\mathbb{R}^n$
x'(p)
x''
$\mathbb{R}^n$
x''(p)
x
$\mathbb{R}^n$
x(p)
every p has
unique x(p)
M^4


intersection regions : coords should
be compatible
X ⟶ X''

intersection regions : coords should be compatible

$$x \longrightarrow x''$$


Coordinate patch
x'
$\mathbb{R}^n$
$x'(p)$
p
x
$\mathbb{R}^n$
$x(p)$
x''
$\mathbb{R}^n$
$x''(p)$
every p has unique x(p)

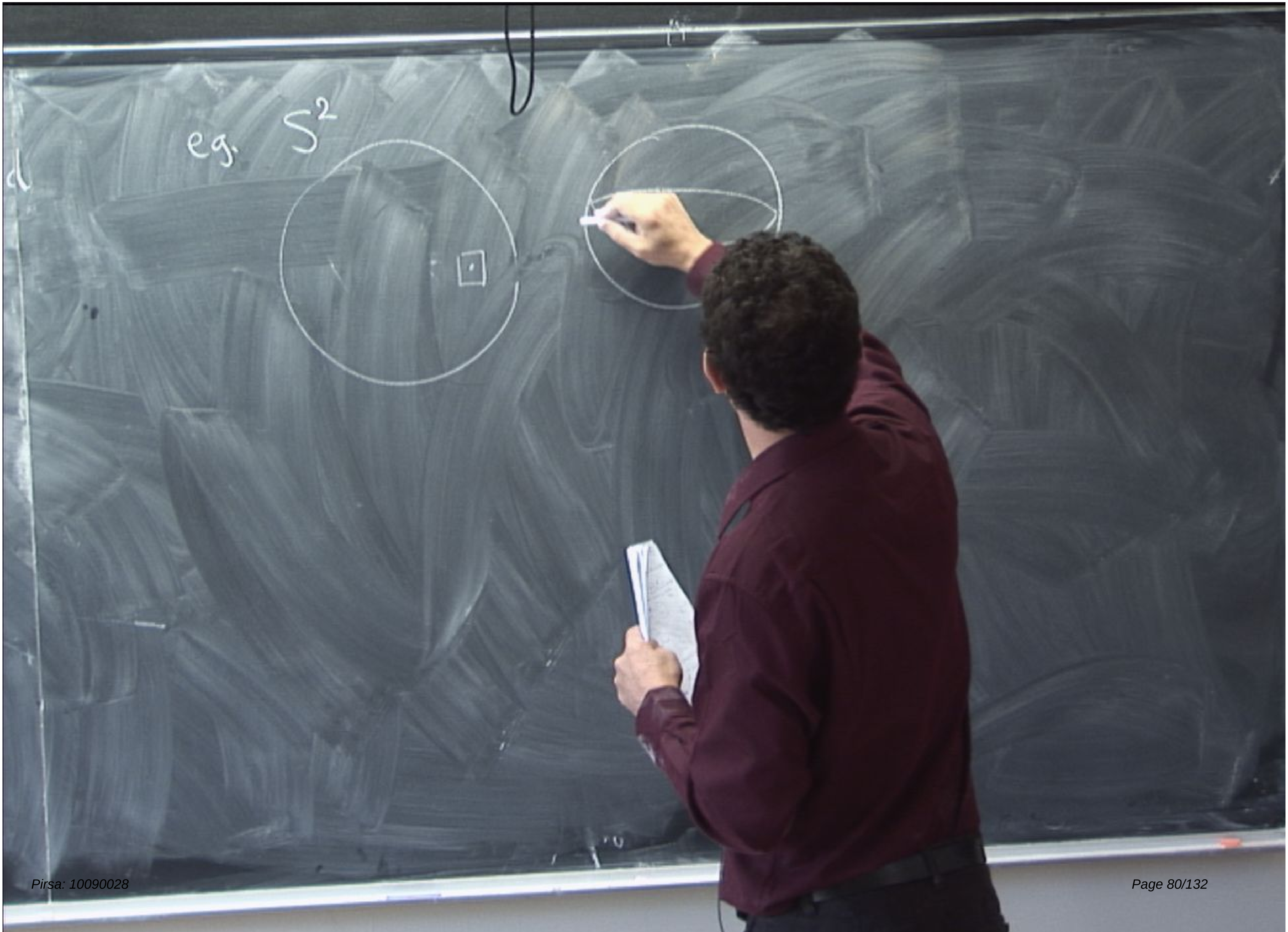

intersection regions : coords should
be compatible
x'''
x'
x''
must have
(x) = x''(x'(x))
in
e overlap regions

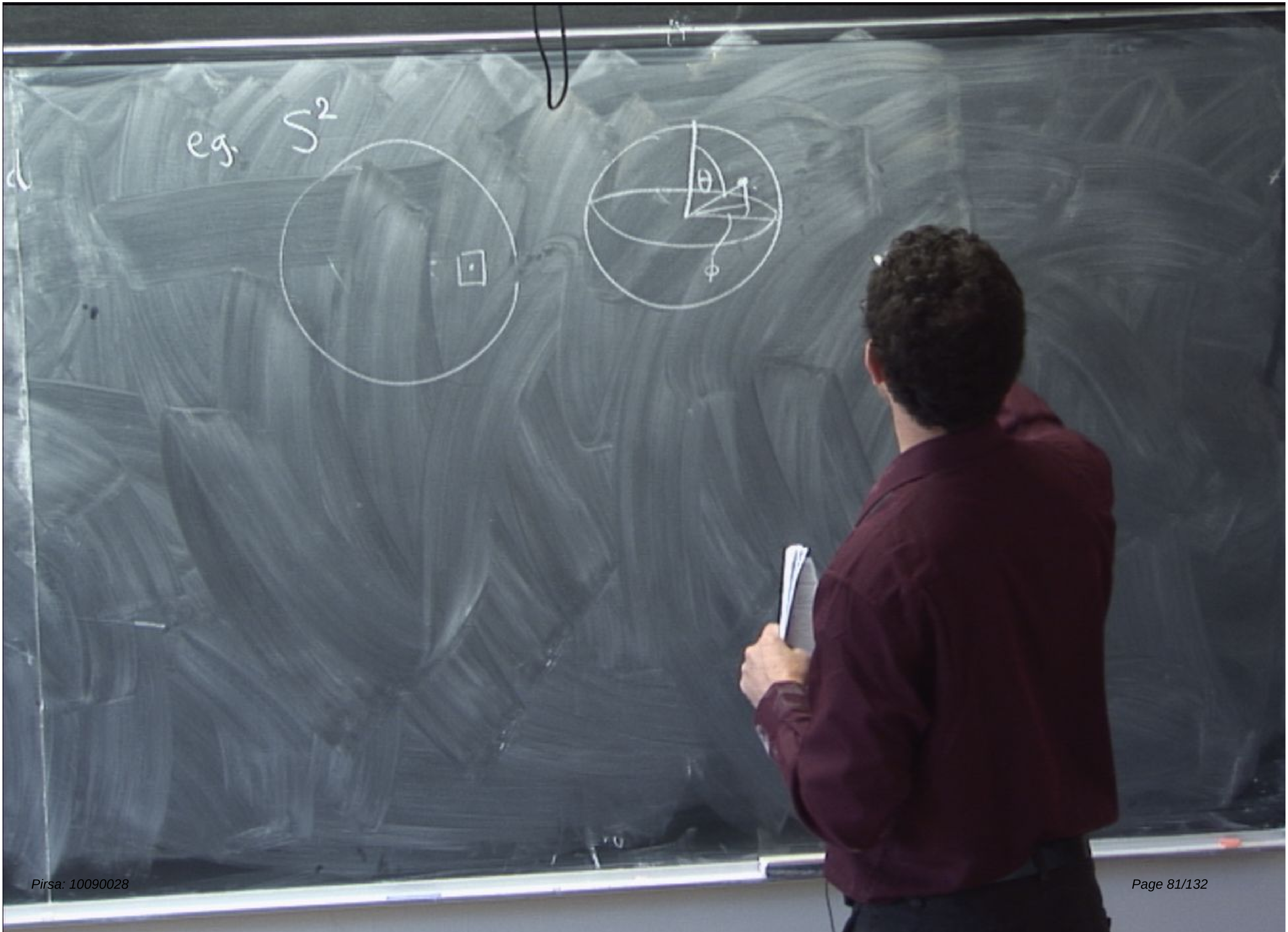
intersection regions : coords should be compatible

$$
\begin{tikzcd}
 & x' \arrow[dr] & \\
x \arrow[ur] \arrow[rr] & & x''
\end{tikzcd}
$$

must have $x''(x) = x''(x'(x))$

in triple overlap regions

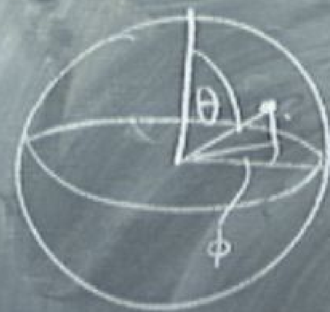

d
eg. S²


d
e.g. S²
θ
φ

eg. S^2

$x^2+y^2+z^2=1$

eg. S^2

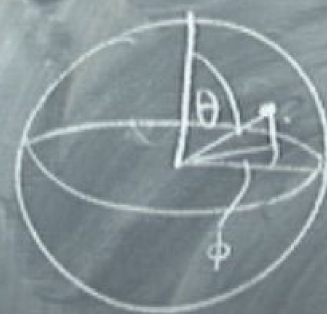


$$x^2+y^2+z^2=1$$

$$x=\sin\theta\cos\phi$$
$$y=\sin\theta\sin\phi$$
$$z=\cos\theta$$

d

eg. S^2



$x^2+y^2+z^2=1$

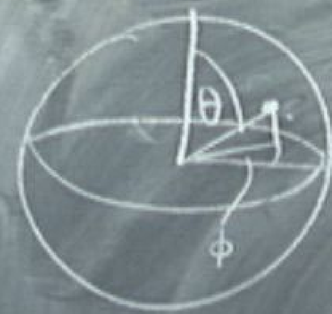
$x = \sin\theta\cos\phi$

$y = \sin\theta\sin\phi$

$z = \cos\theta$

d

eg. S^2



:

$$x^2+y^2+z^2=1$$

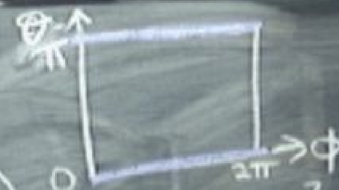
$$x=\sin\theta\cos\phi$$

$$y=\sin\theta\sin\phi$$

$$z=\cos\theta$$

- singular at NP, SP because same pt represented by any $0<\phi\leq 2\pi$

eg. S^2

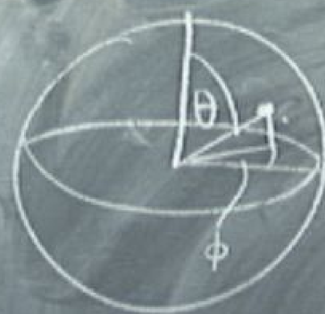


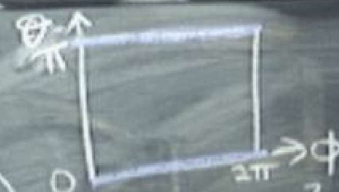
$$x^2+y^2+z^2=1$$

$$x = \sin\theta\cos\phi$$
$$y = \sin\theta\sin\phi$$
$$z = \cos\theta$$

– singular at NP, SP because same pt represented by any $0<\phi\leq 2\pi$

eg. S^2

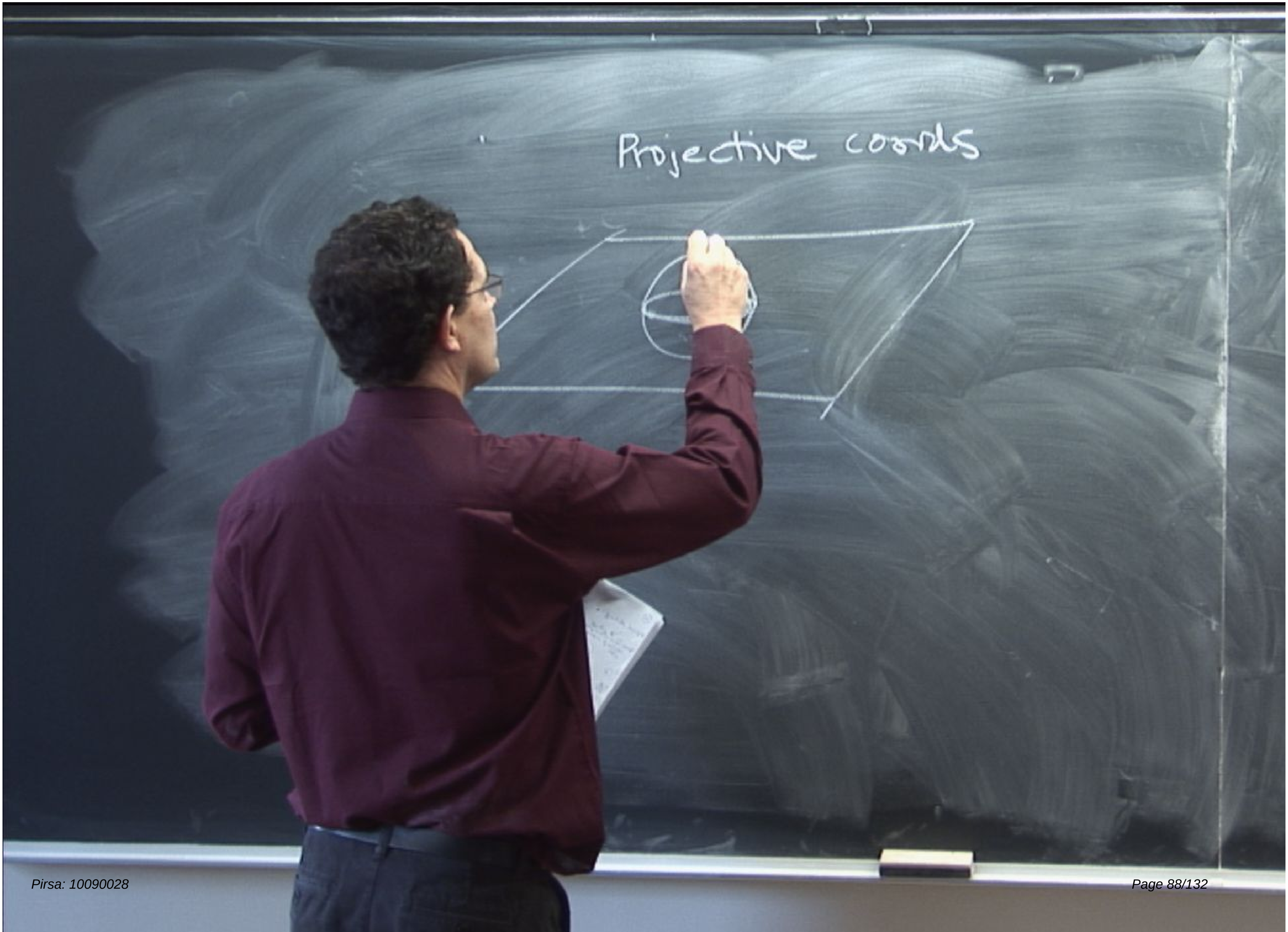


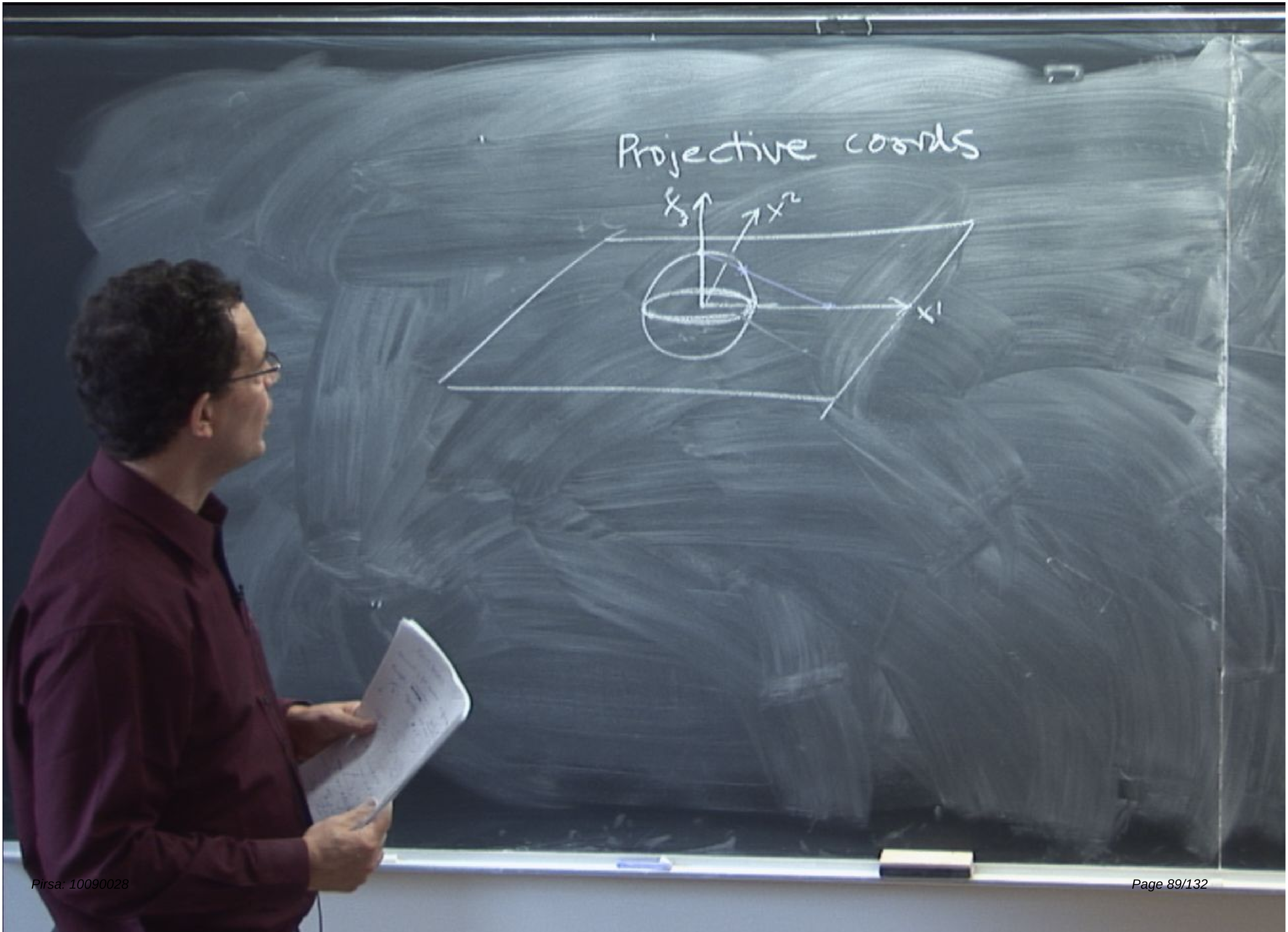


$$x^2+y^2+z^2=1$$

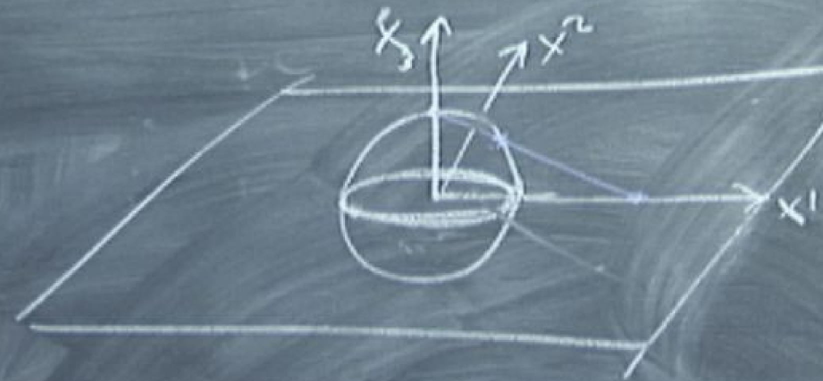
$$x = \sin\theta\cos\phi$$
$$y = \sin\theta\sin\phi$$
$$z = \cos\theta$$

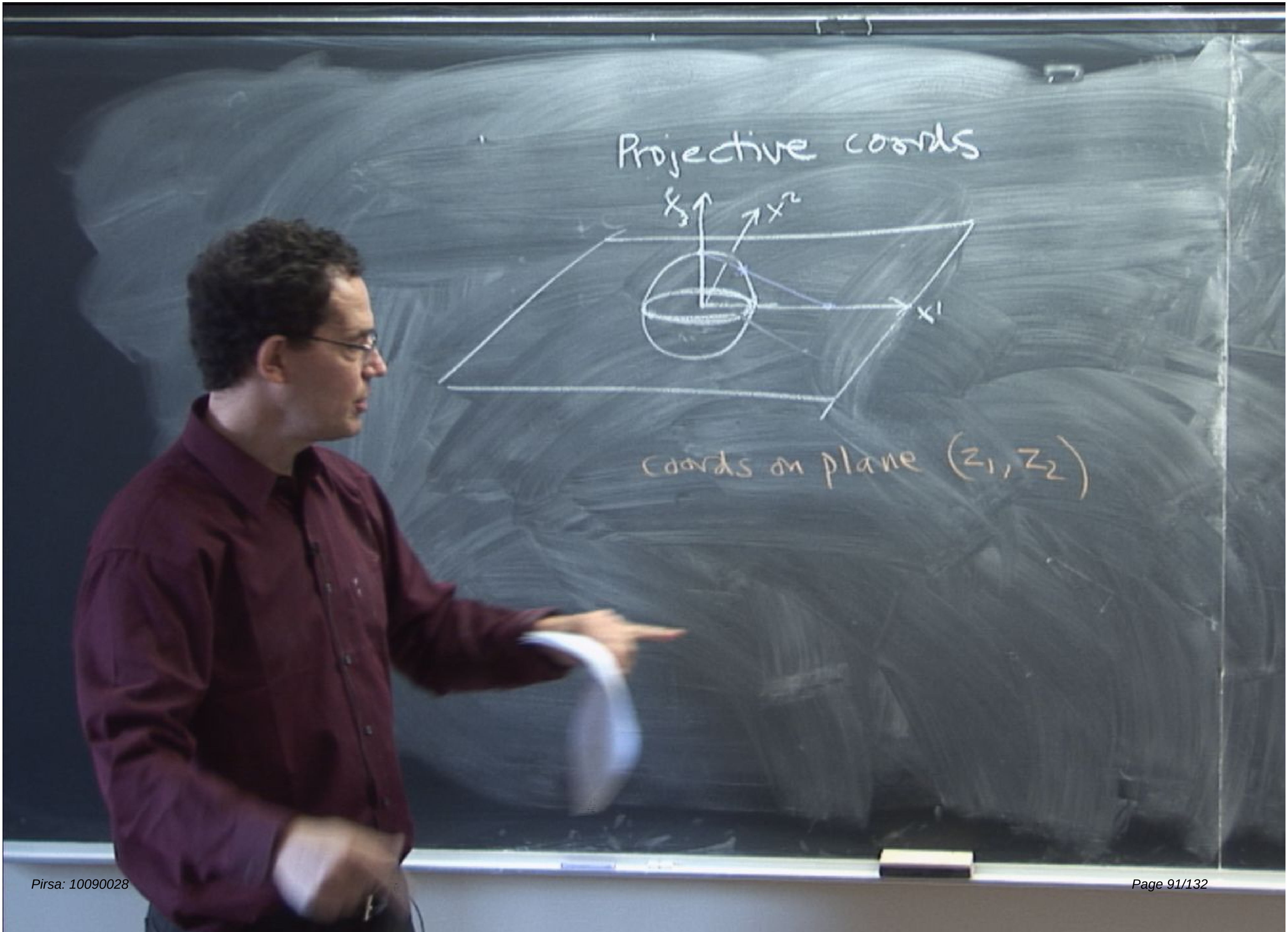
– singular at NP, SP because
same pt represented by any $0<\phi\leq 2\pi$

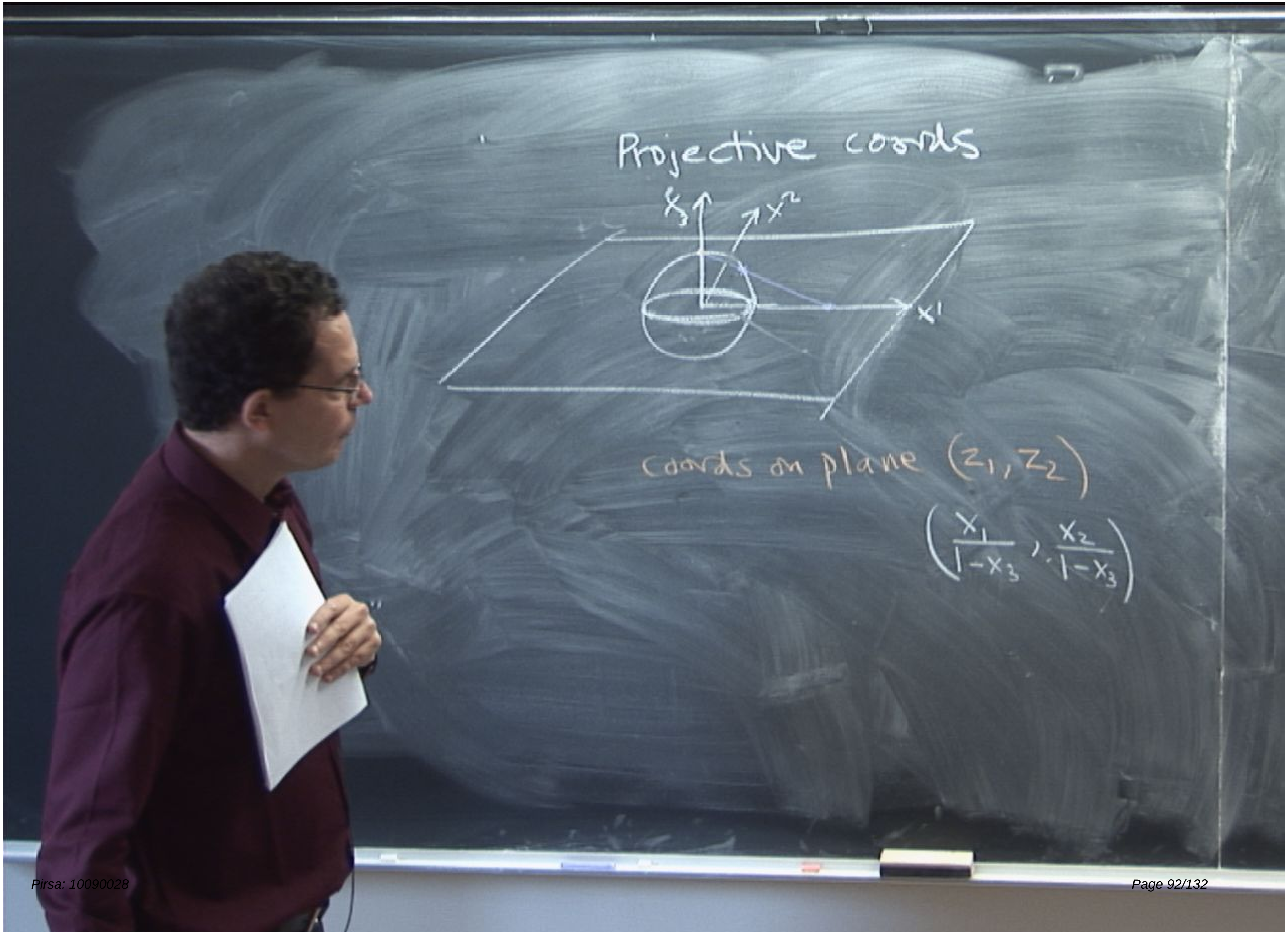

Projective coords


Projective coords
ξ
x^2
x^1

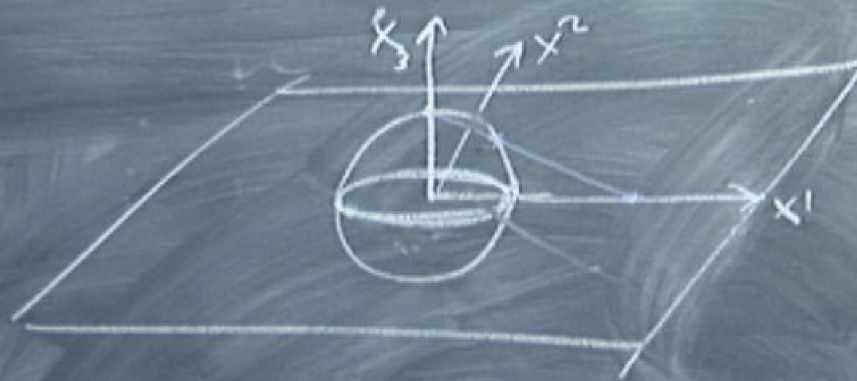
Projective coords




Projective coords
x²
x¹
coords on plane (z₁, z₂)


Projective coords
x_3
x^2
x^1
coords on plane (z_1, z_2)
$\left(\frac{x_1}{1-x_3}, \frac{x_2}{1-x_3}\right)$

Projective coords

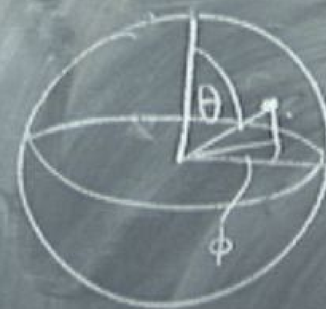


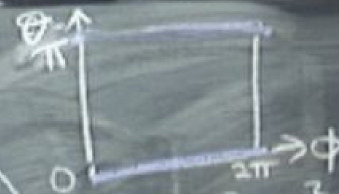
coords on plane (z_1, z_2)

NP proj coords $\left(\frac{x_1}{1-x_3}, \frac{x_2}{1-x_3}\right)$

SP " " $\left(\frac{x_1}{1+x_3}, \frac{x_2}{1+x_3}\right)$

e.g. S^2



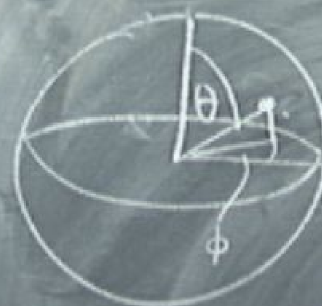


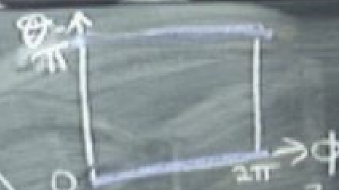
$$x^2+y^2+z^2=1$$

$$x=\sin\theta\cos\phi$$
$$y=\sin\theta\sin\phi$$
$$z=\cos\theta$$

– singular at NP, SP because same pt represented by any $0<\phi\leq 2\pi$

e.g. S^2



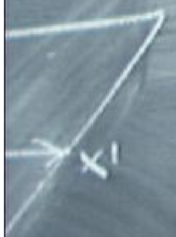


$$x^2+y^2+z^2=1$$

$$x=\sin\theta\cos\phi$$
$$y=\sin\theta\sin\phi$$
$$z=\cos\theta$$

– singular at NP, SP because same pt represented by any $0<\phi\leq 2\pi$

...ords

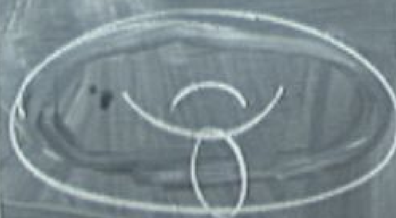


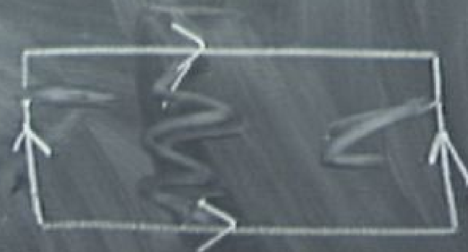
...ane (z_1, z_2)

...oords $\left(\frac{x_1}{1-x_3}, \frac{x_2}{1-x_3}\right)$

'' $\left(\frac{x_1}{1+x_3}, \frac{x_2}{1+x_3}\right)$

eg. S^2





x^2+y...

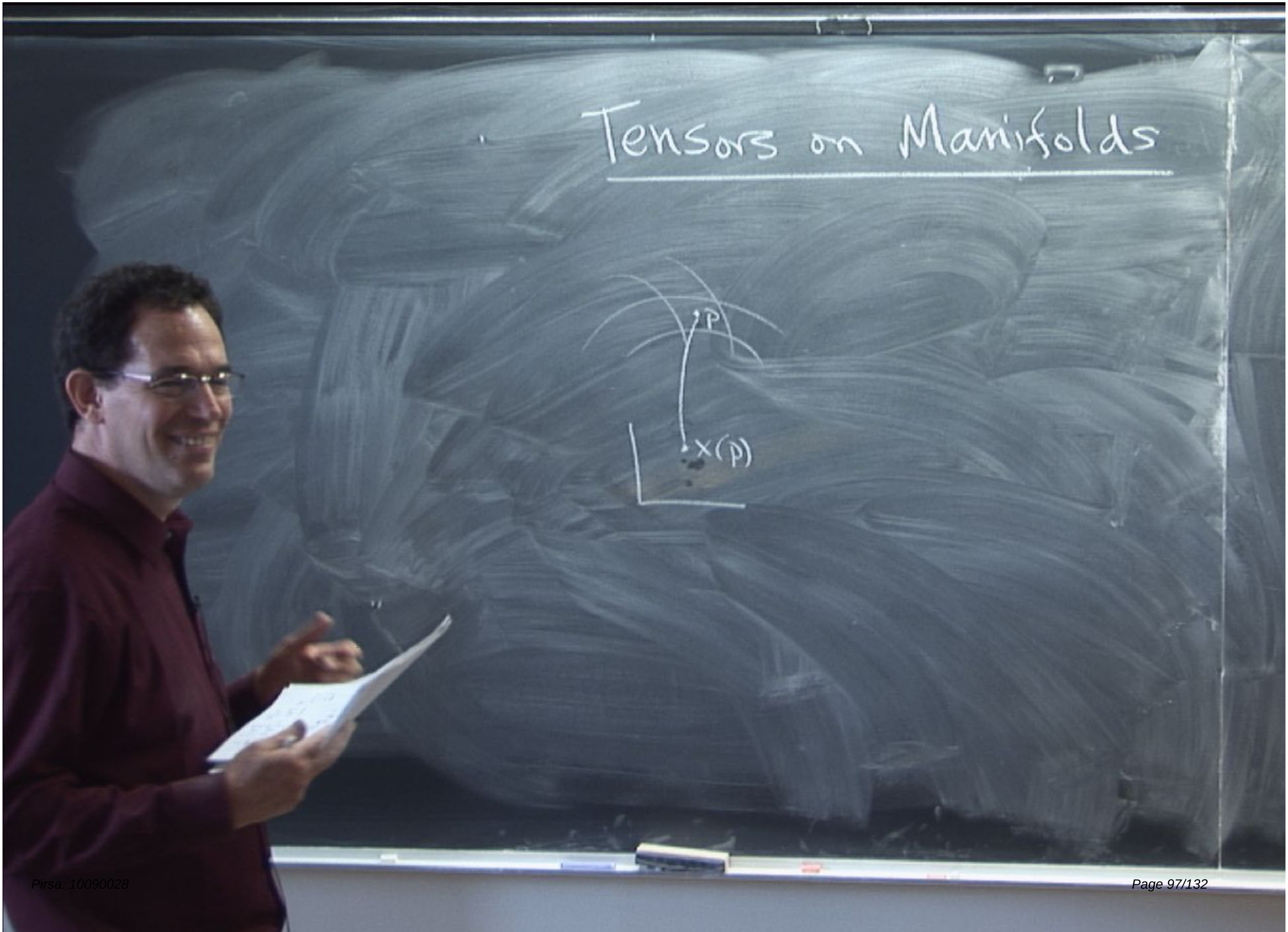
$x =$

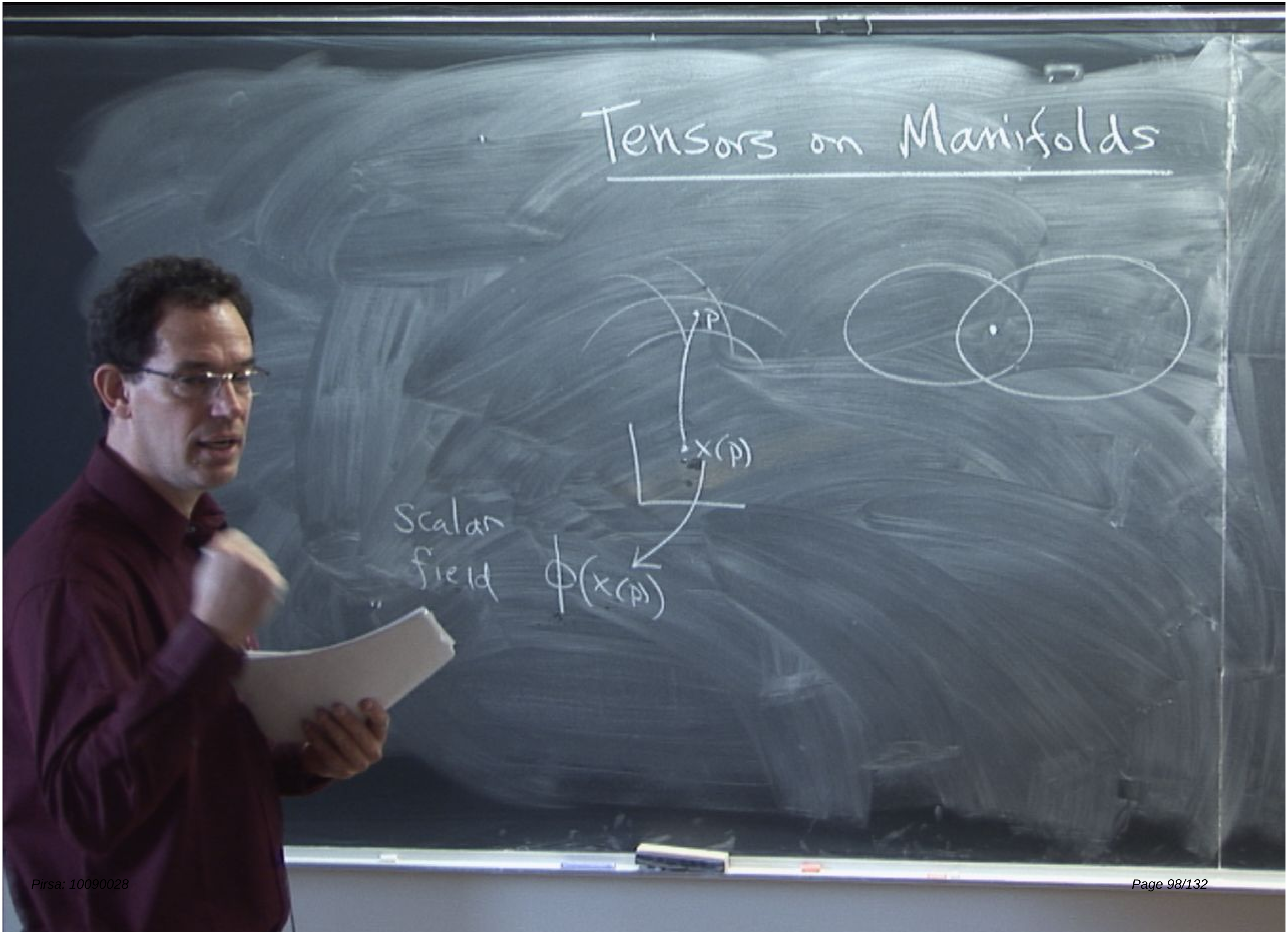
y

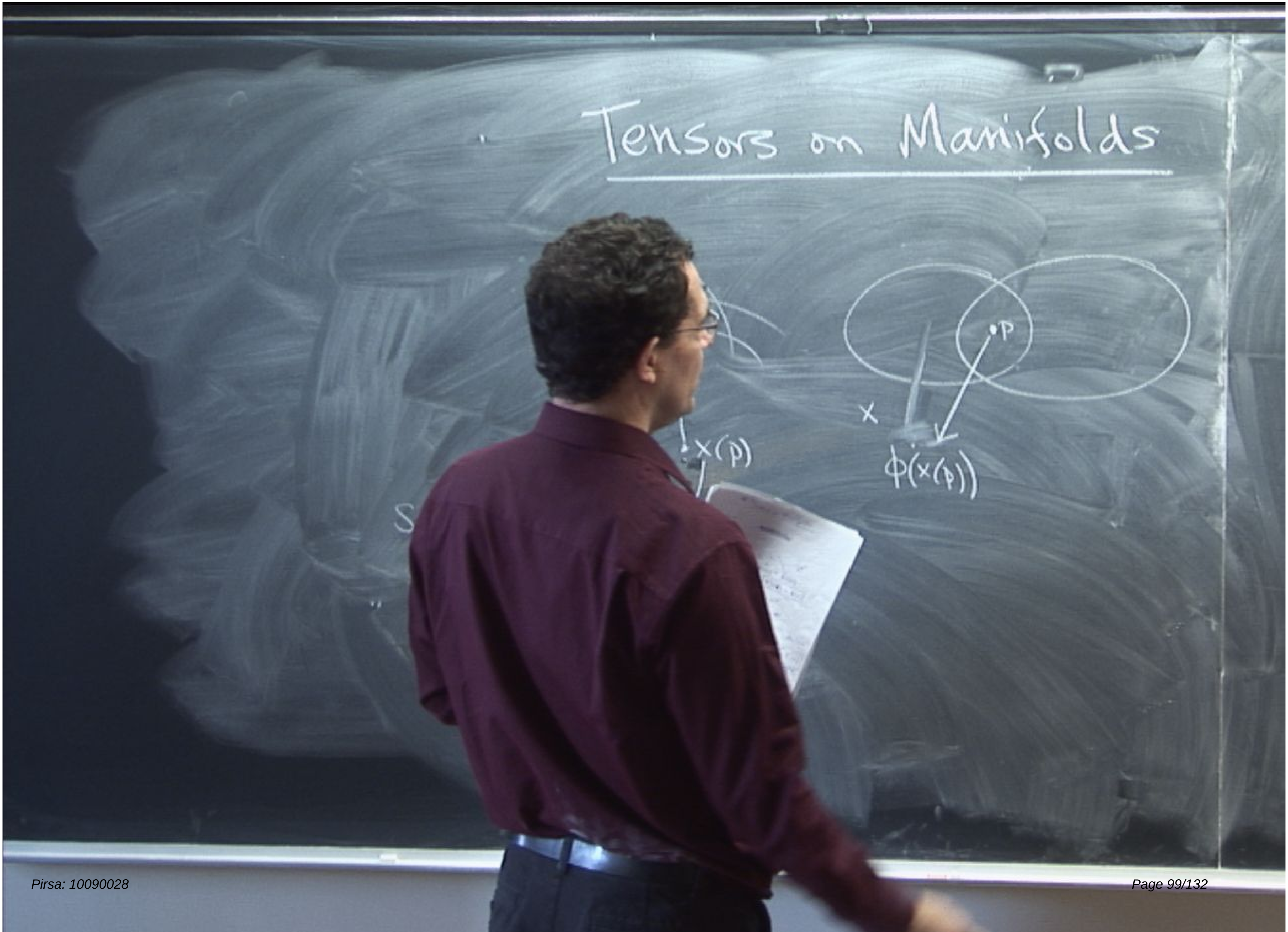
z

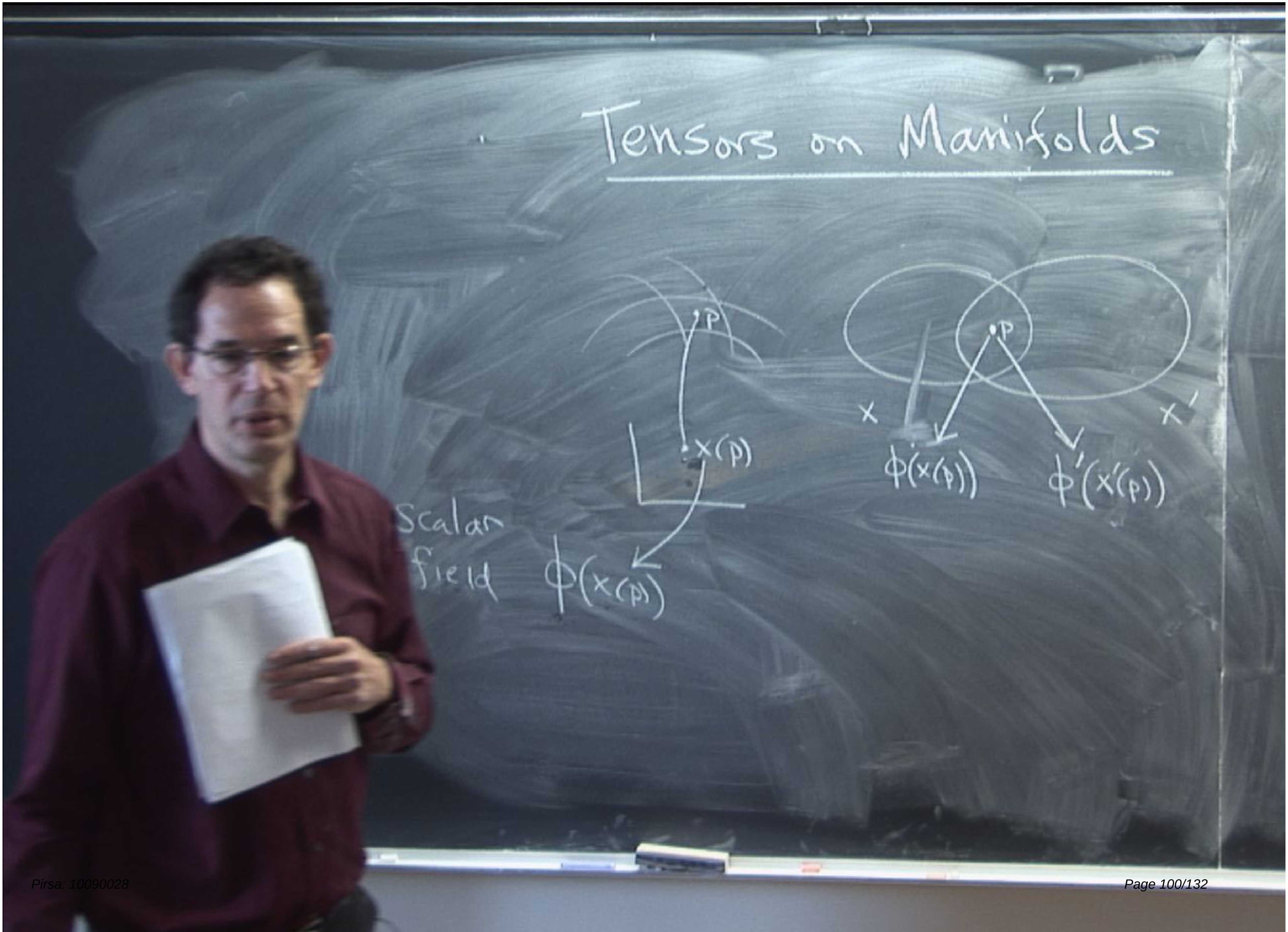
– singular at NP, SP ...

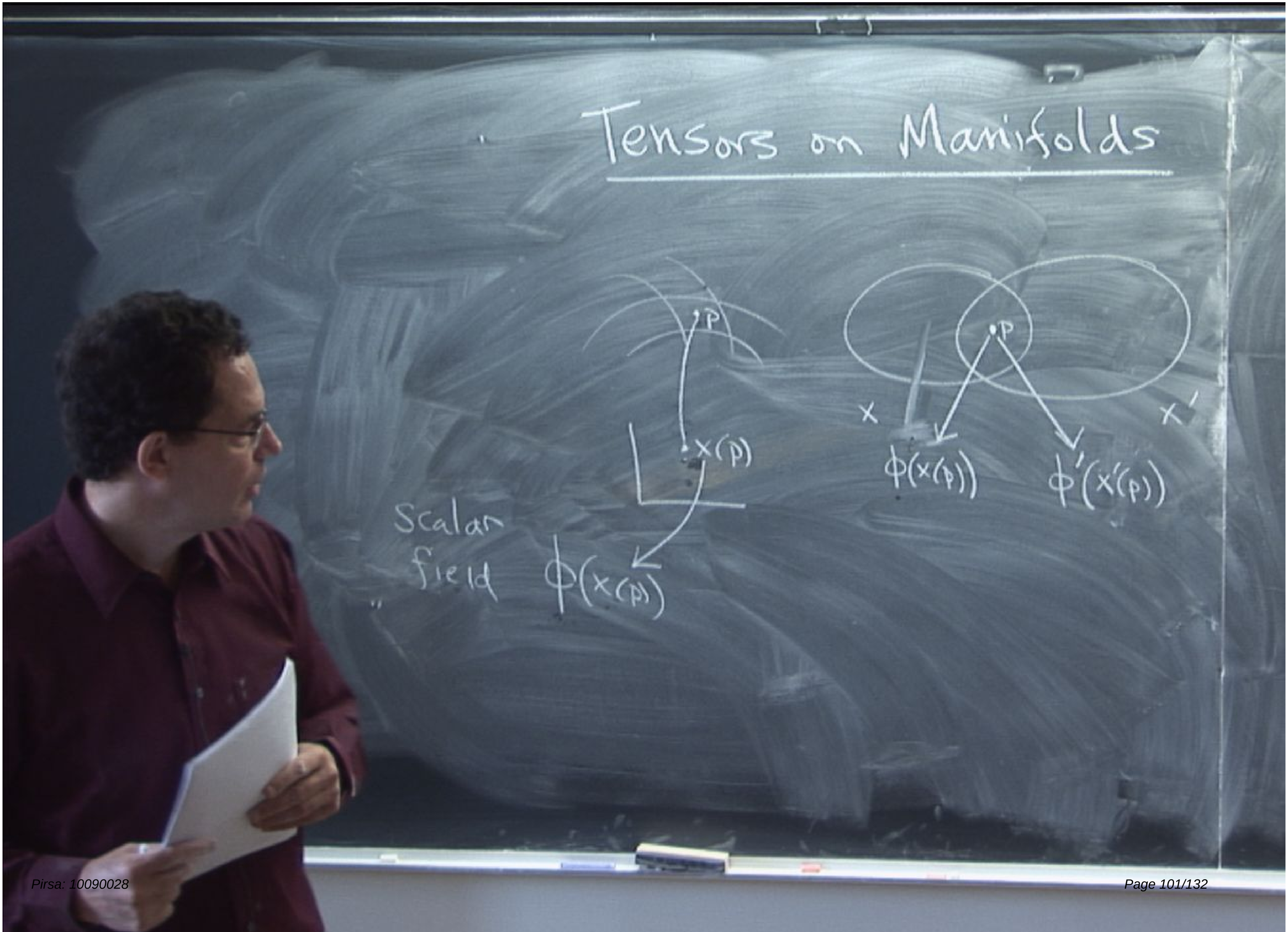
same pt represented ...


Tensors on Manifolds
p
x(p)

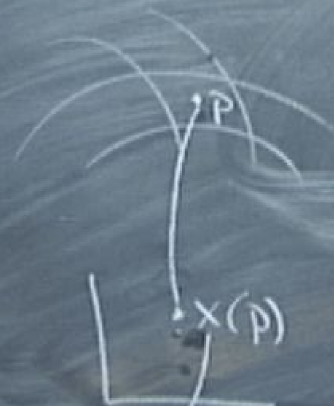

Tensors on Manifolds
p
x(p)
Scalar field
φ(x(p))


Tensors on Manifolds
p
x
x(p)
ϕ(x(p))
S


Tensors on Manifolds
p
x(p)
Scalar
field
φ(x(p))
p
x
x'
φ(x(p))
φ'(x'(p))


Tensors on Manifolds
p
x(p)
Scalar field
φ(x(p))
p
x
x'
φ(x(p))
φ'(x'(p))

Tensors on Manifolds



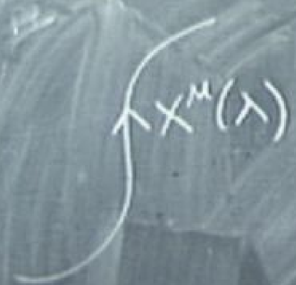
Scalar field $\phi(x(p))$

$\phi(x(p)) = \phi'(x'(p))$

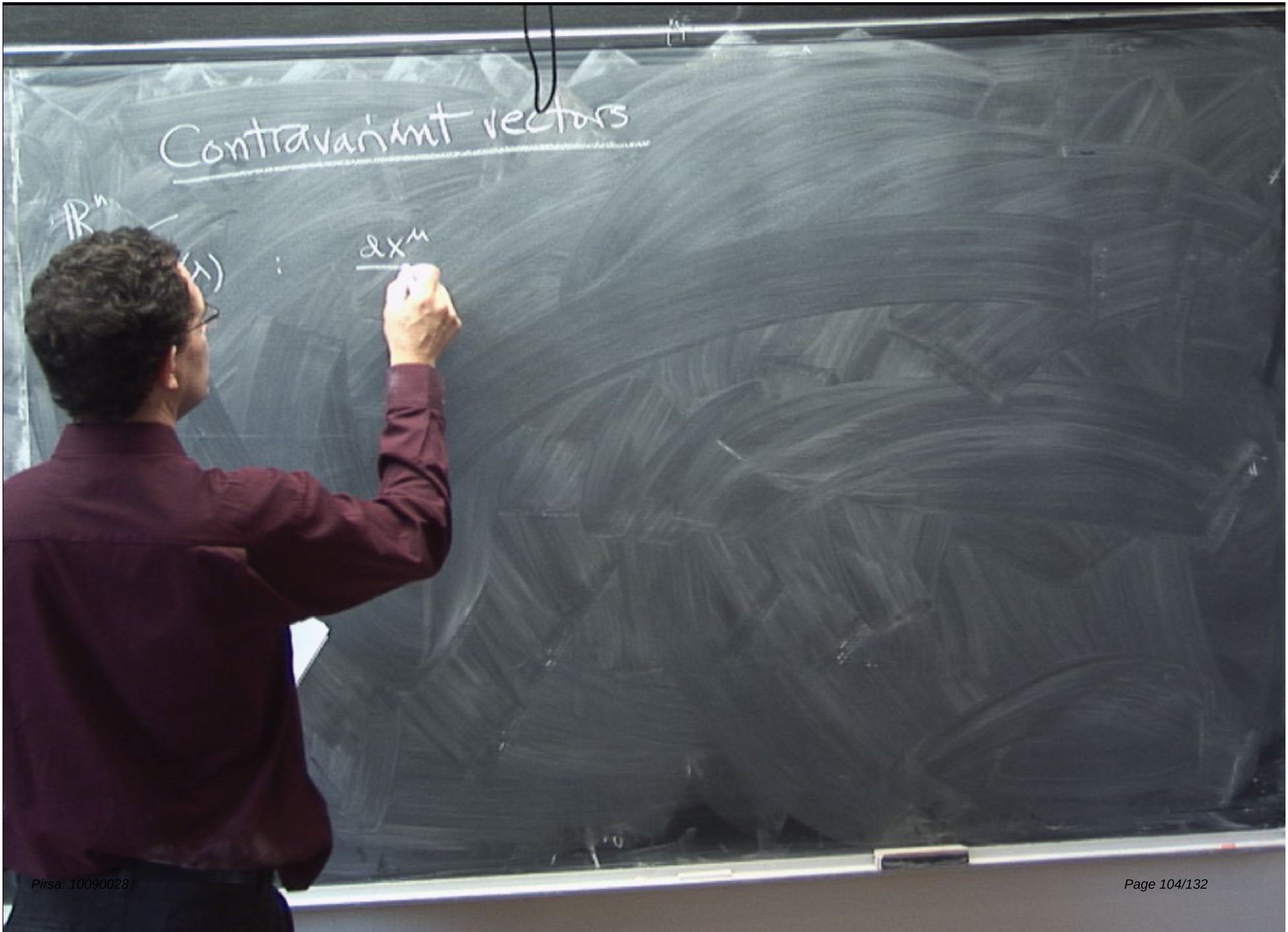
$\Rightarrow$ "scalar" transformation law

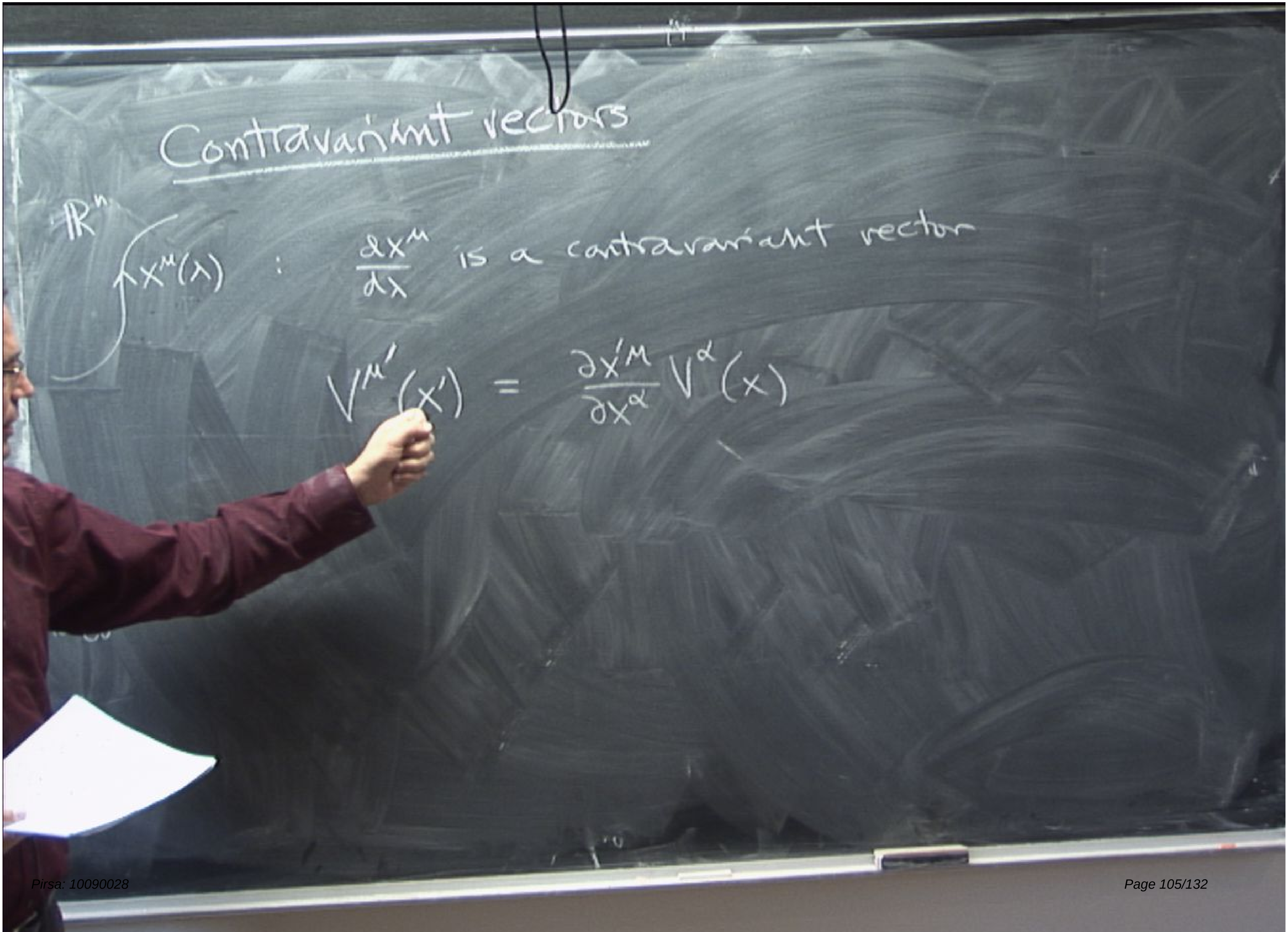
$$\boxed{\phi'(x') = \phi(x)}$$

Contravariant vectors



m law


Contravariant vectors
$\mathbb{R}^n$
dx^μ


Contravariant vectors
$\mathbb{R}^n$
$x^\mu(\lambda)$: $\frac{dx^\mu}{d\lambda}$ is a contravariant vector
$V^{\mu'}(x') = \frac{\partial x'^\mu}{\partial x^\alpha} V^\alpha(x)$

Contravariant vectors

$\mathbb{R}^n$

$x^\mu(\lambda)$: $\dfrac{dx^\mu}{d\lambda}$ is a contravariant vector

$$V^{\mu'}(x') = \frac{\partial x'^\mu}{\partial x^\alpha} V^\alpha(x)$$

m law

Contravariant vectors

$\mathbb{R}^n$

$x^\mu(\lambda)$: $\dfrac{dx^\mu}{d\lambda}$ is a contravariant vector

$$\boxed{V^{\mu'}(x') = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} V^{\alpha}(x)}$$

Covariant vectors

Contravariant vectors

$\mathbb{R}^n$

$x^\mu(\lambda)$: $\frac{dx^\mu}{d\lambda}$ is a contravariant vector

$$\boxed{V^{\mu'}(x') = \frac{\partial x'^\mu}{\partial x^\alpha} V^\alpha(x)}$$

Covariant vectors

e.g. $\frac{\partial}{\partial x'^\mu} = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial}{\partial x^\alpha}$

Contravariant vectors

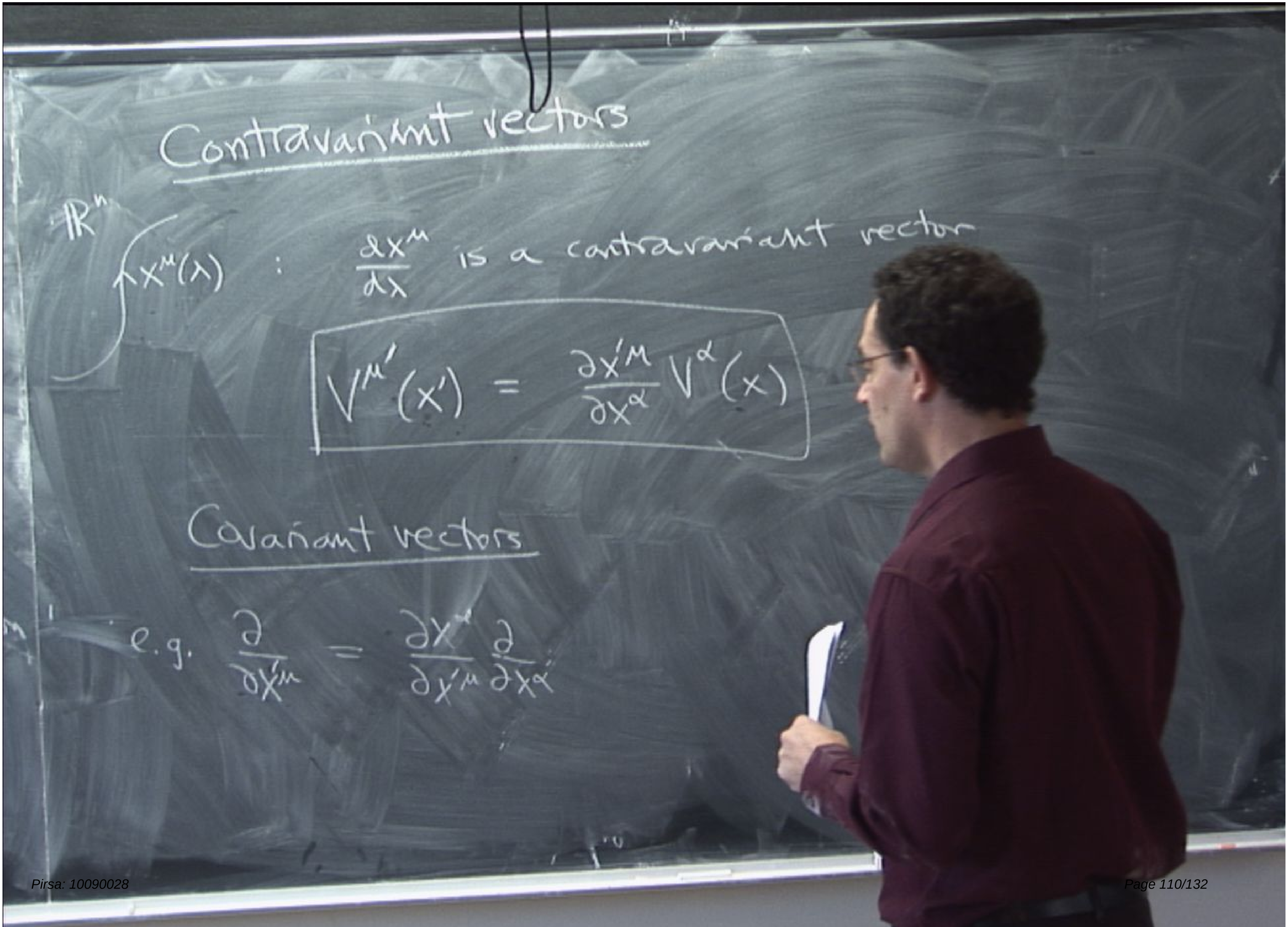
$\mathbb{R}^n$

$x^\mu(\lambda)$: $\frac{dx^\mu}{d\lambda}$ is a contravariant vector

$$\boxed{V^{\mu'}(x') = \frac{\partial x'^\mu}{\partial x^\alpha} V^\alpha(x)}$$

Covariant vectors

e.g. $\frac{\partial}{\partial x'^\mu} = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial}{\partial x^\alpha}$


Contravariant vectors
$\mathbb{R}^n$
$x^\mu(\lambda)$: $\frac{dx^\mu}{d\lambda}$ is a contravariant vector
$\boxed{V^{\mu'}(x') = \frac{\partial x'^\mu}{\partial x^\alpha} V^\alpha(x)}$
Covariant vectors
e.g. $\frac{\partial}{\partial x'^\mu} = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial}{\partial x^\alpha}$

Contravariant vectors

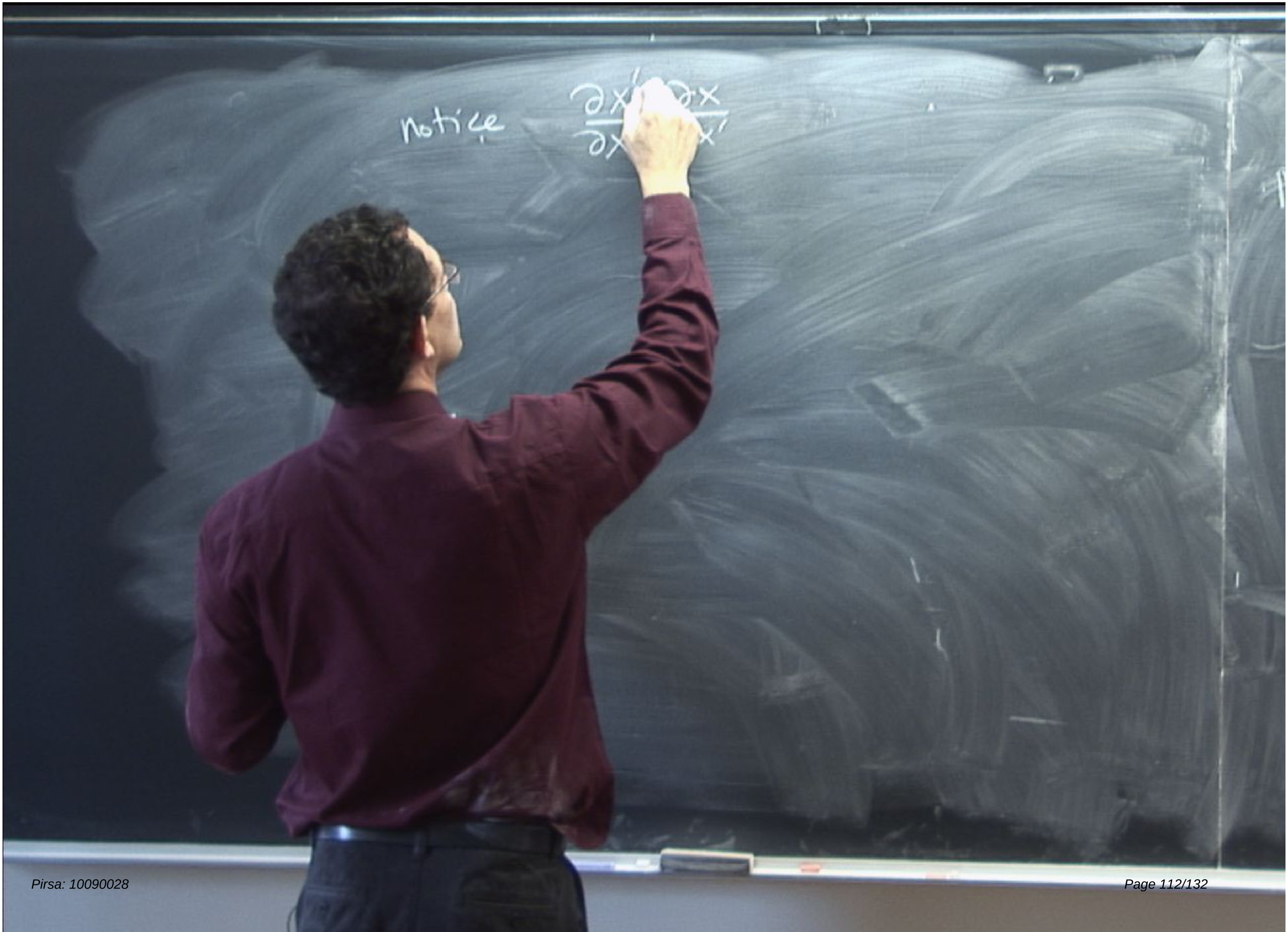
$\mathbb{R}^n$

$x^\mu(\lambda)$: $\dfrac{dx^\mu}{d\lambda}$ is a contravariant vector

$$\boxed{V^{\mu'}(x') = \frac{\partial x'^\mu}{\partial x^\alpha} V^\alpha(x)}$$

Covariant vectors

e.g. $\dfrac{\partial}{\partial x'^\mu} = \dfrac{\partial x^\alpha}{\partial x'^\mu}\dfrac{\partial}{\partial x^\alpha}$ in general $\boxed{W'_\mu(x') = \dfrac{\partial x^\alpha}{\partial x'^\mu} W_\alpha(x)}$


notice

notice $\dfrac{\partial x'^{\mu}}{\partial x^{\alpha}} \dfrac{\partial x^{\beta}}{\partial x'^{\mu}} = \dfrac{\partial x^{\beta}}{\partial x^{\alpha}} = \delta_{\alpha}^{\beta}$

inverses

Contravariant vectors

$\mathbb{R}^n$

$x^\mu(\lambda)$: $\frac{dx^\mu}{d\lambda}$ is a contravariant vector

$$V'^\mu = \Lambda^\mu{}_\alpha V^\alpha$$

$$\boxed{V^{\mu\prime}(x') = \frac{\partial x'^\mu}{\partial x^\alpha} V^\alpha(x)}$$

Covariant vectors

e.g. $\frac{\partial}{\partial x'^\mu} = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial}{\partial x^\alpha}$ in general

$$\boxed{W'_\mu(x') = \frac{\partial x^\alpha}{\partial x'^\mu} W_\alpha(x)}$$

Contravariant vectors

$\mathbb{R}^n$

$x^\mu(\lambda)$: $\frac{dx^\mu}{d\lambda}$ is a contravariant vector

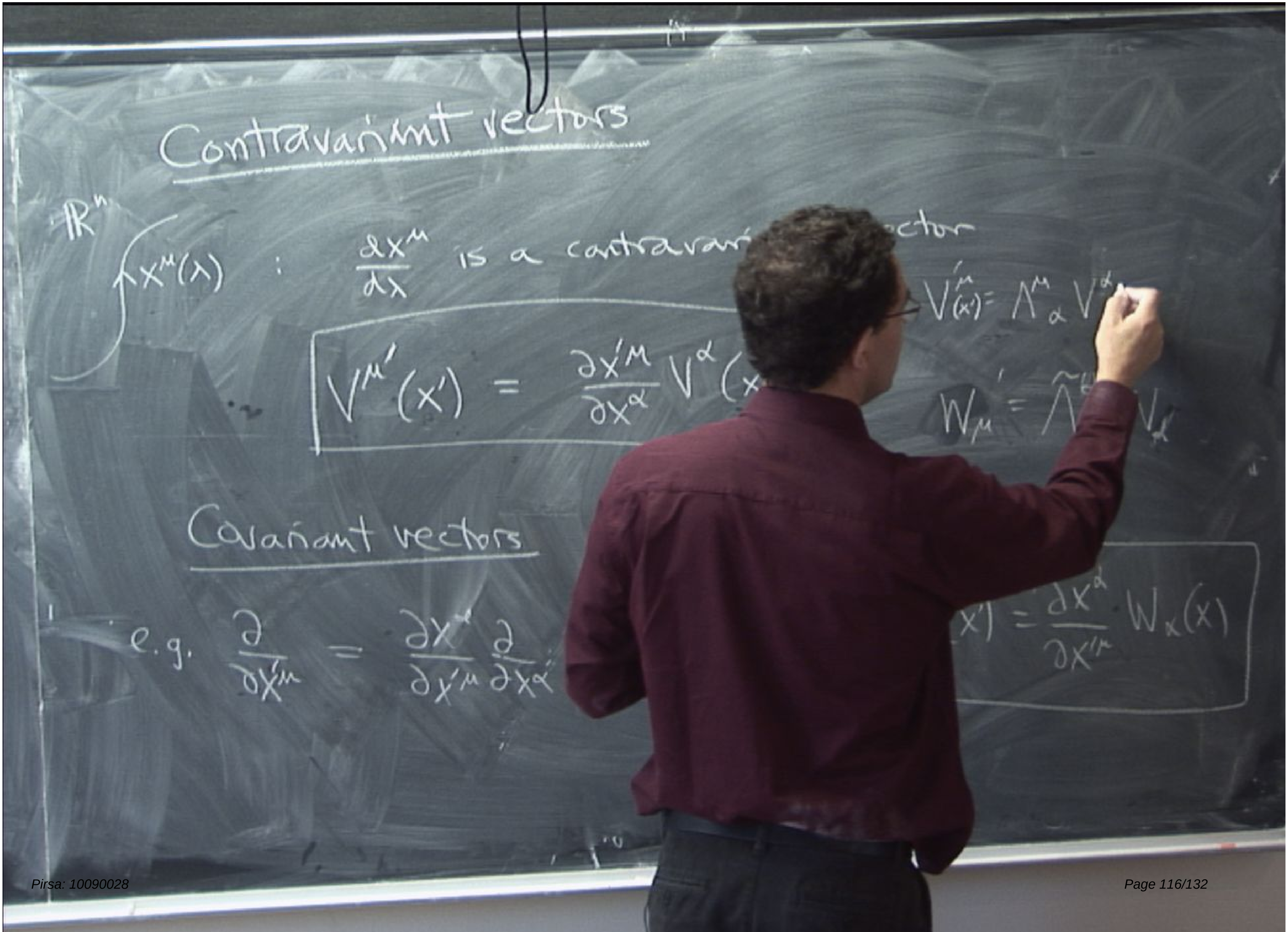
$V'^\mu = \Lambda^\mu{}_\alpha V^\alpha$

$$\boxed{V^{\mu'}(x') = \frac{\partial x'^\mu}{\partial x^\alpha} V^\alpha(x)}$$

$W_\mu' = \tilde{\Lambda}^\alpha{}_\mu W_\alpha$

Covariant vectors

e.g. $\frac{\partial}{\partial x'^\mu} = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial}{\partial x^\alpha}$ in general $\boxed{W_\mu'(x') = \frac{\partial x^\alpha}{\partial x'^\mu} W_\alpha(x)}$


Contravariant vectors
$\mathbb{R}^n$
$x^\mu(\lambda)$: $\frac{dx^\mu}{d\lambda}$ is a contravari... ...ector
$V'^\mu(x') = \Lambda^\mu{}_\alpha V^\alpha$
$V^{\mu'}(x') = \frac{\partial x'^\mu}{\partial x^\alpha} V^\alpha(x$
$W_\mu' = \tilde{\Lambda} \ldots V_\alpha$
Covariant vectors
e.g. $\frac{\partial}{\partial x'^\mu} = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial}{\partial x^\alpha}$
$x') = \frac{\partial x^\alpha}{\partial x'^\mu} W_\alpha(x)$

Contravariant vectors

$$F'^{\alpha\beta}(x') = \Lambda^{\alpha}{}_{\mu}\Lambda^{\beta}{}_{\nu}F^{\mu\nu}(x)$$

$\mathbb{R}^n$

$x^\mu(\lambda)$: $\frac{dx^\mu}{d\lambda}$ is a contravariant vector

$$V'^{\mu}(x') = \Lambda^{\mu}{}_{\alpha}V^{\alpha}(x)$$

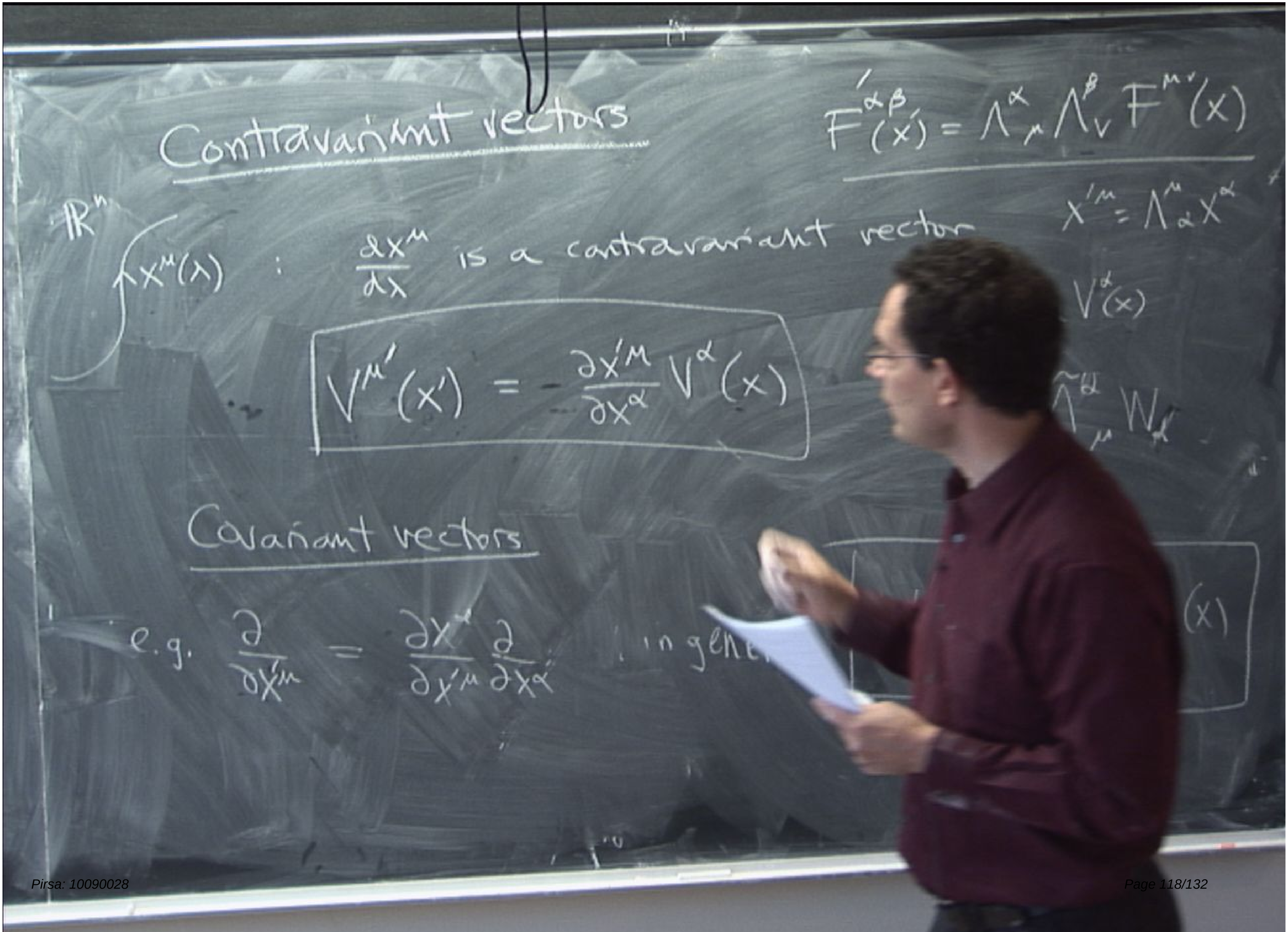
$$\boxed{V^{\mu'}(x') = \frac{\partial x'^{\mu}}{\partial x^{\alpha}}V^{\alpha}(x)}$$

$$W_\mu' = \tilde{\Lambda}^{\alpha}{}_{\mu}W_\alpha$$

Covariant vectors

e.g. $\frac{\partial}{\partial x'^{\mu}} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}}\frac{\partial}{\partial x^{\alpha}}$ in general

$$\boxed{W_\mu'(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}}W_\alpha(x)}$$


Contravariant vectors
$F'^{\alpha\beta}(x') = \Lambda^{\alpha}{}_{\mu} \Lambda^{\beta}{}_{\nu} F^{\mu\nu}(x)$
$x'^{\mu} = \Lambda^{\mu}{}_{\alpha} x^{\alpha}$
$\mathbb{R}^n$
$x^{\mu}(\lambda)$: $\frac{dx^{\mu}}{d\lambda}$ is a contravariant vector
$V^{\alpha}(x)$
$\boxed{V'^{\mu}(x') = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} V^{\alpha}(x)}$
$\tilde{\Lambda}^{\alpha}{}_{\mu} W_{\alpha}$
Covariant vectors
e.g. $\frac{\partial}{\partial x'^{\mu}} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial}{\partial x^{\alpha}}$, in gener
(x)

Contravariant vectors

$$F'^{\alpha\beta}(x') = \Lambda^{\alpha}{}_{\mu}\Lambda^{\beta}{}_{\nu}F^{\mu\nu}(x)$$

$\mathbb{R}^n$

$x'^{\mu} = \Lambda^{\mu}{}_{\alpha}x^{\alpha}$

$x^{\mu}(\lambda)$: $\frac{dx^{\mu}}{d\lambda}$ is a contravariant vector

$V'^{\mu}(x') = \Lambda^{\mu}{}_{\alpha}V^{\alpha}(x)$

$$\boxed{V^{\mu'}(x') = \frac{\partial x'^{\mu}}{\partial x^{\alpha}}V^{\alpha}(x)}$$

$W'_{\mu} = \tilde{\Lambda}^{\alpha}{}_{\mu}W_{\alpha}$

Covariant vectors

e.g. $\frac{\partial}{\partial x'^{\mu}} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}}\frac{\partial}{\partial x^{\alpha}}$ in general

$$\boxed{W'_{\mu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}}W_{\alpha}(x)}$$

notice $\dfrac{\partial x'^{\mu}}{\partial x^{\alpha}} \dfrac{\partial x^{\beta}}{\partial x'^{\mu}} = \dfrac{\partial x^{\beta}}{\partial x^{\alpha}} = \delta_{\alpha}^{\beta}$

inverses

$x'(x)$ well-behaved provided $\dfrac{\partial x'^{\alpha}}{\partial x^{\beta}}$ invertible.

notice $\frac{\partial x'^\mu}{\partial x^\alpha} \frac{\partial x^\beta}{\partial x'^\mu} = \frac{\partial x^\beta}{\partial x^\alpha} = \delta_\alpha^\beta$

inverses

$x'(x)$ well-behaved provided $\frac{\partial x'^\alpha}{\partial x^\beta}$ invertible.

trees

Contravariant

$\mathbb{R}^n$

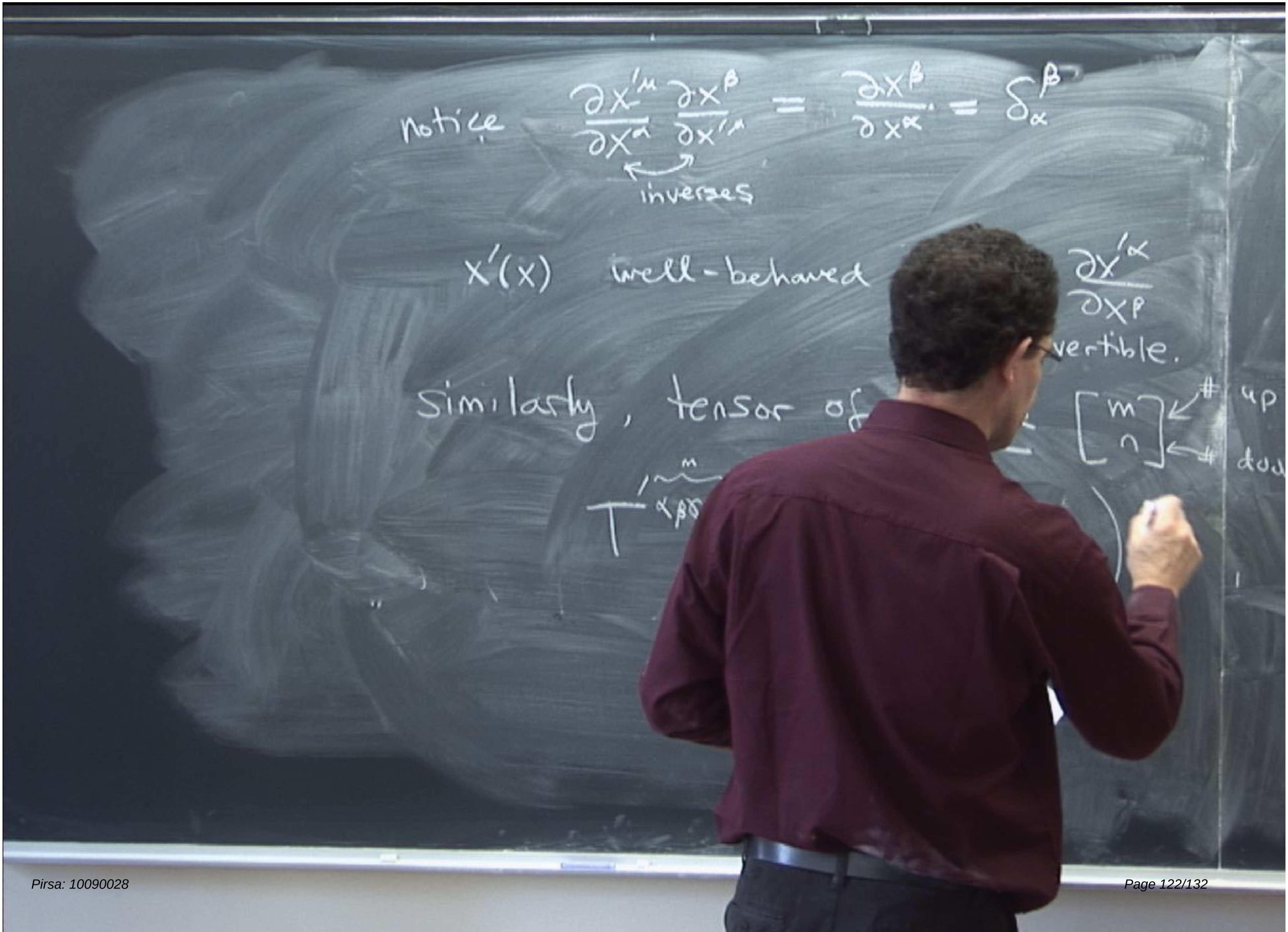
$x^\mu(\lambda)$: $\frac{dx^\mu}{d\lambda}$

$V^{\mu'}(x'$

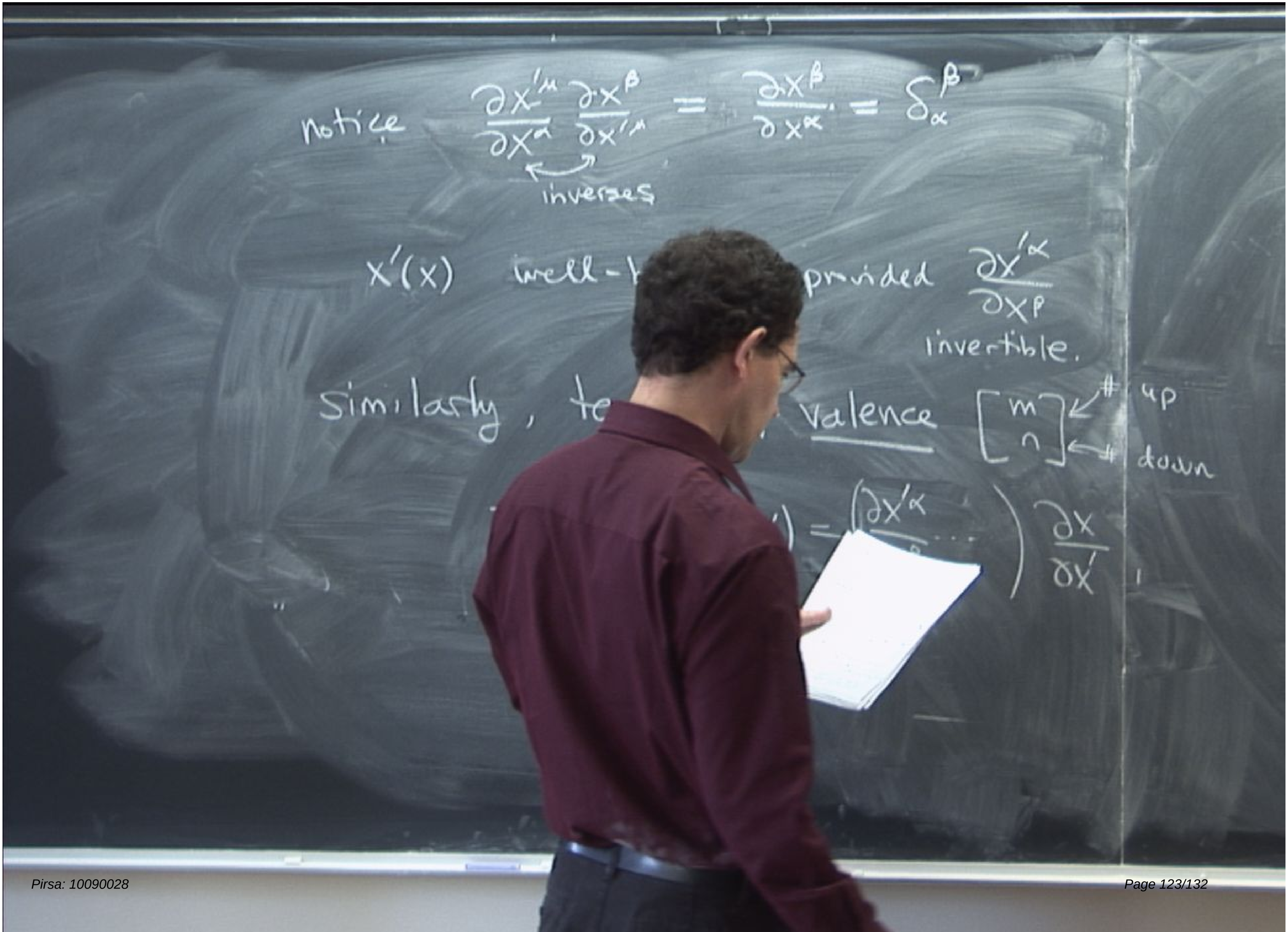
valleys

Covariant vec

e.g. $\frac{\partial}{\partial x'^\mu} = \frac{\partial x}{\partial x'}$



notice $\frac{\partial x'^\mu}{\partial x^\alpha}\frac{\partial x^\beta}{\partial x'^\mu} = \frac{\partial x^\beta}{\partial x^\alpha} = \delta_\alpha^\beta$
inverses
$x'(x)$ well-behaved
$\frac{\partial x'^\alpha}{\partial x^\beta}$
vertible.
similarly, tensor of
$\begin{bmatrix} m \\ n \end{bmatrix}$
up
dow
$T'^{\mu\ldots}_{\alpha\beta\gamma}$


notice $\frac{\partial x'^{\mu}}{\partial x^{\alpha}} \frac{\partial x^{\beta}}{\partial x'^{\mu}} = \frac{\partial x^{\beta}}{\partial x^{\alpha}} = \delta_{\alpha}^{\beta}$
inverses
$x'(x)$ well-
provided $\frac{\partial x'^{\alpha}}{\partial x^{\beta}}$
invertible.
similarly, te
valence $\begin{bmatrix} m \\ n \end{bmatrix}$
up
down
$\frac{\partial x}{\partial x'}$

notice $\frac{\partial x'^{\mu}}{\partial x^{\alpha}} \frac{\partial x^{\beta}}{\partial x'^{\mu}} = \frac{\partial x^{\beta}}{\partial x^{\alpha}} = \delta_{\alpha}^{\beta}$

inverses

$x'(x)$ well-behaved provided $\frac{\partial x'^{\alpha}}{\partial x^{\beta}}$ invertible.

similarly, tensor of valence $\begin{bmatrix} m \\ n \end{bmatrix}$ # up # down

$$T'^{\overbrace{\alpha\beta\gamma\ldots}^{m}}{}_{\underbrace{\mu\nu\ldots}_{n}}(x') = \left(\frac{\partial x'^{\alpha}}{\partial x^{\beta}} \cdots\right)\left(\frac{\partial x^{\varepsilon}}{\partial x'^{\mu}} \cdots\right) T$$

notice $\dfrac{\partial x'^{\mu}}{\partial x^{\alpha}} \dfrac{\partial x^{\beta}}{\partial x'^{\mu}} = \dfrac{\partial x^{\beta}}{\partial x^{\alpha}} = \delta_{\alpha}^{\beta}$

inverses

$x'(x)$ well-behaved provided $\dfrac{\partial x'^{\alpha}}{\partial x^{\beta}}$ invertible.

similarly, tensor of valence $\begin{bmatrix} m \\ n \end{bmatrix}$ # up, # down

$$T'^{\overbrace{\alpha\beta\gamma\ldots}^{m}}{}_{\underbrace{\mu\nu\ldots}_{n}}(x') = \left(\frac{\partial x'^{\alpha}}{\partial x^{\rho}}\frac{\partial x'^{\beta}}{\partial x^{\delta}}\right)\left(\frac{\partial x^{\varepsilon}}{\partial x'^{\mu}}\ldots\right) T^{\rho\delta\ldots}{}_{\varepsilon\ldots}$$

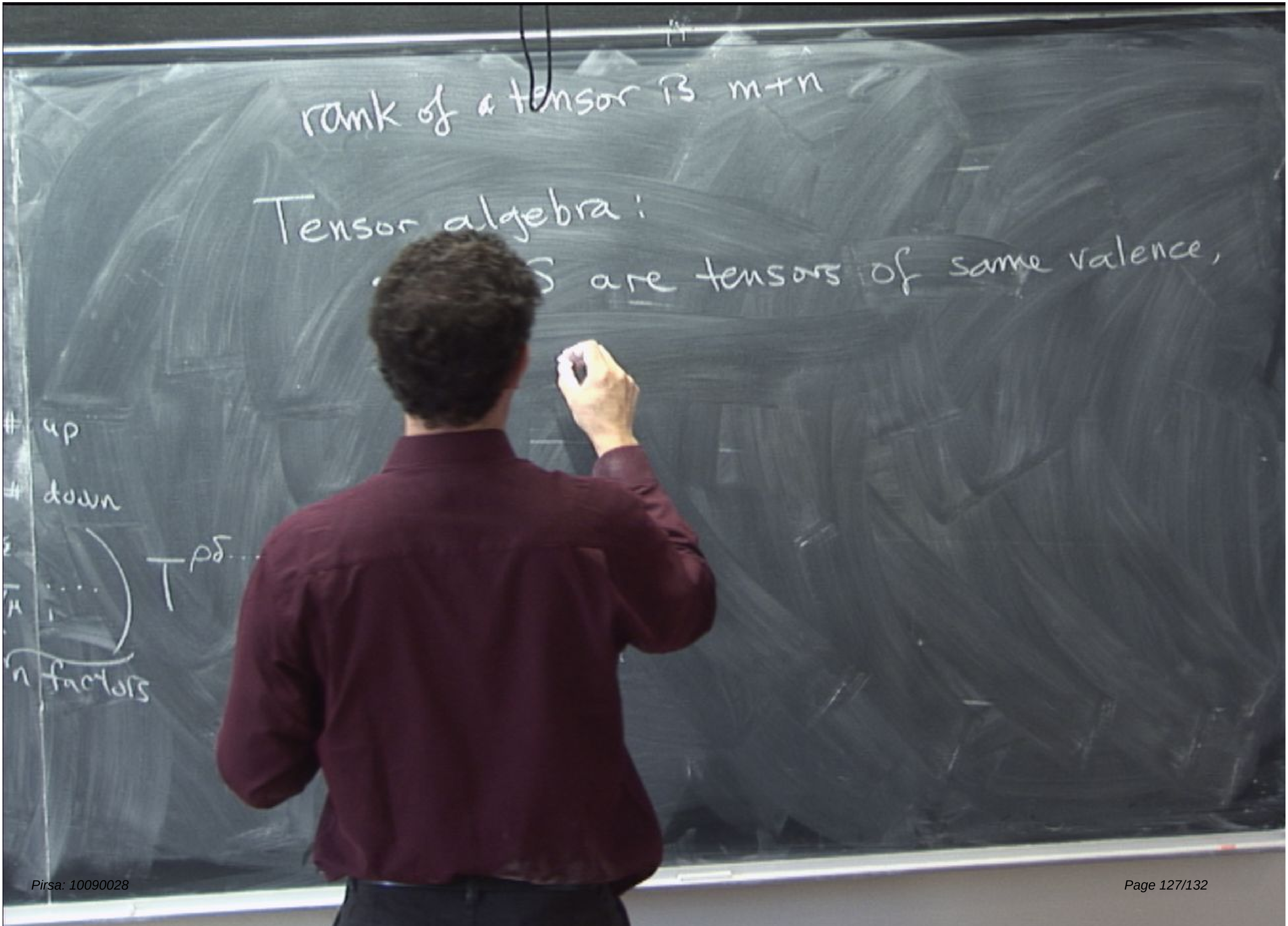
notice $\dfrac{\partial x'^\mu}{\partial x^\alpha}\dfrac{\partial x^\beta}{\partial x'^\mu} = \dfrac{\partial x^\beta}{\partial x^\alpha} = \delta_\alpha^\beta$

inverses

$x'(x)$ well-behaved provided $\dfrac{\partial x'^\alpha}{\partial x^\beta}$ invertible.

Similarly, tensor of valence $\begin{bmatrix} m \\ n \end{bmatrix}$ ← # up, ← # down

$$T'^{\overbrace{\alpha\beta\gamma\ldots}^{m}}{}_{\underbrace{\mu\nu\ldots}_{n}}(x') = \underbrace{\left(\frac{\partial x'^\alpha}{\partial x^\rho}\frac{\partial x'^\beta}{\partial x^\delta}\cdots\right)}_{m\text{ factors}}\underbrace{\left(\frac{\partial x^\epsilon}{\partial x'^\mu}\cdots\right)}_{n\text{ factors}} T^{\rho\delta\ldots}{}_{\mu\ldots}$$


rank of a tensor is m+n
Tensor algebra:
S are tensors of same valence,
up
down
n factors

rank of a tensor is $m+n$

Tensor algebra:

if T, S are tensors of same valence,

$T+S$ is also.

up

down

$T^{\rho\delta\cdots}{}_{\mu\cdots}(x)$

n factors

rank of a tensor is $m+n$

Tensor algebra:

1) if T, S are tensors of same valence,

$T+S$ is also.

2) outer multiplication $T^{\overbrace{\alpha\beta\cdots}^{m}} S_{\underbrace{\mu\nu\cdots}_{n}}$

up

down

$\Big(\cdots\Big) T^{\rho\delta\cdots}{}_{\mu\cdots}(x)$

n factors

rank of a tensor is $m+n$

Tensor algebra:

1) if T, S are tensors of same valence,

$T+S$ is also.

valence

2) outer multiplication $T^{\overbrace{\alpha\beta\cdots}^{m}} S_{\underbrace{\mu\nu\cdots}_{n}} \rightarrow \begin{bmatrix} m \\ n \end{bmatrix}$

3) contractions $T^{\overbrace{\alpha\beta\delta\cdots}^{m}} S_{\underbrace{\alpha\beta\delta\cdots}_{n}} \rightarrow \begin{bmatrix} m-2 \\ n-2 \end{bmatrix}$

\# up

\# down

$T^{\rho\delta\cdots}{}_{\mu\cdots}(x)$

n factors

rank of a tensor is m+n

Tensor algebra:

1) if T, S are tensors of same valence,

T+S is also.

valence

2) outer multiplication $T^{\overbrace{\alpha\beta\dots}^{m}} S_{\underbrace{\mu\nu\dots}_{n}} \to \begin{bmatrix} m \\ n \end{bmatrix}$

3) contractions $T^{\overbrace{\alpha\beta\delta\dots}^{m}} S_{\underbrace{\alpha\beta\delta\dots}_{n}} \to \begin{bmatrix} m-2 \\ n-2 \end{bmatrix}$

up

down

$\Big) T^{\rho\delta\dots}{}_{\mu\dots}(x)$

n factors

If a tensor vanishes in one coordinate system, then it vanishes in any coordinate system.

rank of a tensor is $m+n$

Tensor algebra:

1) if T, S are tensors of same valence, $T+S$ is also.

2) outer multiplication $T^{\overbrace{\alpha\beta\cdots}^{m}} S_{\underbrace{\mu\nu\cdots}_{n}} \rightarrow$ valence $\begin{bmatrix} m \\ n \end{bmatrix}$

3) contractions $T^{\overbrace{\alpha\beta\delta\cdots}^{m}} S_{\underbrace{\alpha\beta\delta\cdots}_{n}} \longrightarrow \begin{bmatrix} m-2 \\ n-2 \end{bmatrix}$

up

down

$\underbrace{\cdots\cdots)}_{n \text{ factors}} T^{\rho\delta\cdots}{}_{\mu\cdots}(x)$

If a tensor vanishes in one coordinate system, then it vanishes in any coordinate system.