

Title: Relativity (PHYS 604) - Lecture 2

Date: Sep 14, 2010 10:30 AM

URL: <http://pirsa.org/10090027>

Abstract:

Summary so far

$$\eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\epsilon^{\alpha\beta\gamma\delta} = \begin{matrix} +1 \\ -1 \end{matrix}$$

$\alpha\beta\gamma\delta =$ even perm
of 0123
" odd "

$$\delta_{\alpha}^{\gamma} = \begin{matrix} 1 & \alpha = \gamma \\ 0 & \text{otherwise} \end{matrix}$$

Maxwell: $\partial_{\alpha} F^{\alpha\beta} = -\mu_0 J^{\beta}$
 $\epsilon^{\alpha\beta\gamma\delta} \partial_{\beta} F_{\gamma\delta} = 0$

Invariant under LTS

$$V^{\alpha} \rightarrow V'^{\alpha} = \Lambda^{\alpha}_{\beta} V^{\beta}$$

$$U_{\alpha} \rightarrow U'_{\alpha} = \Lambda_{\alpha}^{\beta} U_{\beta}$$

Summary so far

$$\eta_{20} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\epsilon^{\alpha\beta\gamma\delta} = +1$$

$\alpha\beta\gamma\delta = \text{even perm of } 0123$
 11 odd "

$$\delta_{\alpha}^{\gamma} = \begin{cases} 1 & \alpha = \gamma \\ 0 & \text{otherwise} \end{cases}$$

Maxwell: $\partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta$
 $\epsilon^{\alpha\beta\gamma\delta} \partial_\beta F_{\gamma\delta} = 0$

Invariant under LTS

$$V^\alpha \rightarrow V'^\alpha = \Lambda^\alpha_\beta V^\beta$$

$$U_\alpha \rightarrow U'_\alpha = U_\beta \hat{\Lambda}_\alpha^\beta$$

$$\epsilon^{\alpha\beta\gamma\delta} = +1$$

$$-1$$

$\alpha\beta\gamma\delta =$ even perm
of 0123

" " odd "

$$\beta = -11 \quad \gamma \beta$$

$$\epsilon_{\alpha\beta\gamma\delta} = +1$$

$$-1$$

$\alpha\beta\gamma\delta =$ even perm
of 0123

" " odd "

0	2	3	1
1	X	X	
0	1	2	3

$$\delta_{\alpha}^{\gamma} = 1 \quad \alpha = \gamma$$

$$0 \quad \text{otherwise}$$

Maxwell:

$$\partial_{\alpha} F^{\alpha\beta} = -\mu_0 J^{\beta}$$

$$\epsilon^{\alpha\beta\gamma\delta} \partial_{\beta} F_{\gamma\delta} = 0$$

... under LTS V^{α}

0 otherwise

Maxwell:

$$\partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta$$

$$\epsilon^{\alpha\beta\gamma\delta} \partial_\beta F_{\gamma\delta} = 0$$

Invariant under LTS

$$V^\alpha \rightarrow V^\alpha$$

$$U_\alpha \rightarrow U_\alpha$$

$$\hat{\Lambda}^\beta_\alpha \hat{\Lambda}^\alpha_\beta$$

Maxwell:

$$\partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta$$

$$\partial_\beta F_{\alpha\beta} = 0$$

invariant under LTS

$$V^\alpha \rightarrow V'^\alpha = \Lambda^\alpha_\beta V^\beta$$

$$U_\alpha \rightarrow U'_\alpha = U_\beta \hat{\Lambda}^\beta_\alpha$$

$$\hat{\Lambda}^\beta_\alpha \Lambda^\alpha_\gamma = \delta^\beta_\gamma$$

Maxwell:

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$$\epsilon^{\alpha\beta\gamma\delta} \partial_\beta F_{\gamma\delta} = 0$$

Invariant under LTS

$$\eta_{\alpha\beta} = \eta_{\mu\nu} \Lambda^\mu_\alpha \Lambda^\nu_\beta$$

$$V^\alpha \rightarrow V'^\alpha = \Lambda^\alpha_\beta V^\beta$$

$$U_\alpha \rightarrow U'_\alpha = U_\beta \hat{\Lambda}^\beta_\alpha$$

$$\hat{\Lambda}^\beta_\alpha \Lambda^\alpha_\gamma = \delta^\beta_\gamma$$

$$\partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta$$

$$\epsilon^{\alpha\beta\gamma\delta} \partial_\beta F_{\gamma\delta} = 0$$

$$V^\alpha \rightarrow V'^\alpha = \Lambda^\alpha_\beta V^\beta$$

$$W'_\mu = \Lambda'_\mu^\nu \Lambda_\nu^\alpha W_\alpha$$

$$\Lambda_\alpha / \Lambda_\gamma = \int_\alpha^\beta$$

$$\partial'_\alpha F'^{\alpha\beta} + j'^\beta = \partial_\gamma F^{\mu\nu} \tilde{\Lambda}^\gamma_\alpha \Lambda^\alpha_\beta$$

$$\partial_\alpha F'^{\alpha\beta} + \mu_0 j'^\beta = \partial_\gamma F^{\mu\nu} \tilde{\Lambda}^\gamma_\alpha \Lambda^\alpha_\mu \Lambda^\beta_\nu$$

$$\partial_\alpha F'^{\alpha\beta} + \mu_0 j'^\beta = \partial_\gamma F^{\mu\nu} \tilde{\Lambda}^\gamma_\alpha \Lambda^\alpha_\mu \Lambda^\beta_\nu + \mu_0 \Lambda^\beta_\nu j^\nu$$

$$\begin{aligned}
 \partial_\alpha F'^{\alpha\beta} + \mu_0 J'^\beta &= \partial_\gamma F^{\mu\nu} \boxed{\tilde{\Lambda}^\gamma_\alpha \Lambda^\alpha_\mu \Lambda^\beta_\nu} \\
 &\quad + \mu_0 \Lambda^\beta_\nu J^\nu \\
 &= (\partial_\mu F^{\mu\nu} + \mu_0 J^\nu) \Lambda^\beta_\nu
 \end{aligned}$$

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 \partial_\alpha F'^{\alpha\beta} + \mu_0 j'^\beta &= \partial_\gamma F^{\mu\nu} \boxed{\tilde{\Lambda}^\gamma_\alpha \Lambda^\alpha_\mu \Lambda^\beta_\nu} \\
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 &= 0.
 \end{aligned}$$

Summary so far

$$\eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$\delta_{\alpha}^{\gamma} = \begin{cases} 1 & \alpha = \gamma \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{matrix} 0 & 2 & 3 & 1 \\ 1 & \times & \times & \times \\ 0 & 1 & 2 & 3 \end{matrix}$$

Maxwell:

$$\partial_{\alpha} F^{\alpha\beta} = -\mu_0 J^{\beta}$$

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Invariant under LTS

$$\eta_{\mu\nu} \Lambda^{\mu}_{\alpha} \Lambda^{\nu}_{\beta}$$

$$V^{\alpha} \rightarrow V'^{\alpha} = \Lambda^{\alpha}_{\beta} V^{\beta}$$

$$U_{\alpha} \rightarrow U'_{\alpha} = U_{\beta} \hat{\Lambda}^{\beta}_{\alpha}$$

$$\hat{\Lambda}^{\beta}_{\alpha} \Lambda^{\alpha}_{\gamma} = \delta^{\beta}_{\gamma}$$

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$$\begin{matrix} 0 & 2 & 3 & 1 \\ 1 & \times & \times & \times \\ 0 & 1 & 2 & 3 \end{matrix}$$

SO(3,1)

Maxwell:

$$\partial_{\alpha} F^{\alpha\beta} = -\mu_0 J^{\beta}$$

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Invariant under LTS

$$\eta_{\alpha\beta} = \eta_{\mu\nu} \Lambda^{\mu}_{\alpha} \Lambda^{\nu}_{\beta}$$

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$$U_{\alpha} \rightarrow U'_{\alpha} = U_{\beta} \hat{\Lambda}^{\beta}_{\alpha}$$

$$\hat{\Lambda}^{\beta}_{\alpha} \Lambda^{\alpha}_{\gamma} = \delta^{\beta}_{\gamma}$$

$SO(3)$



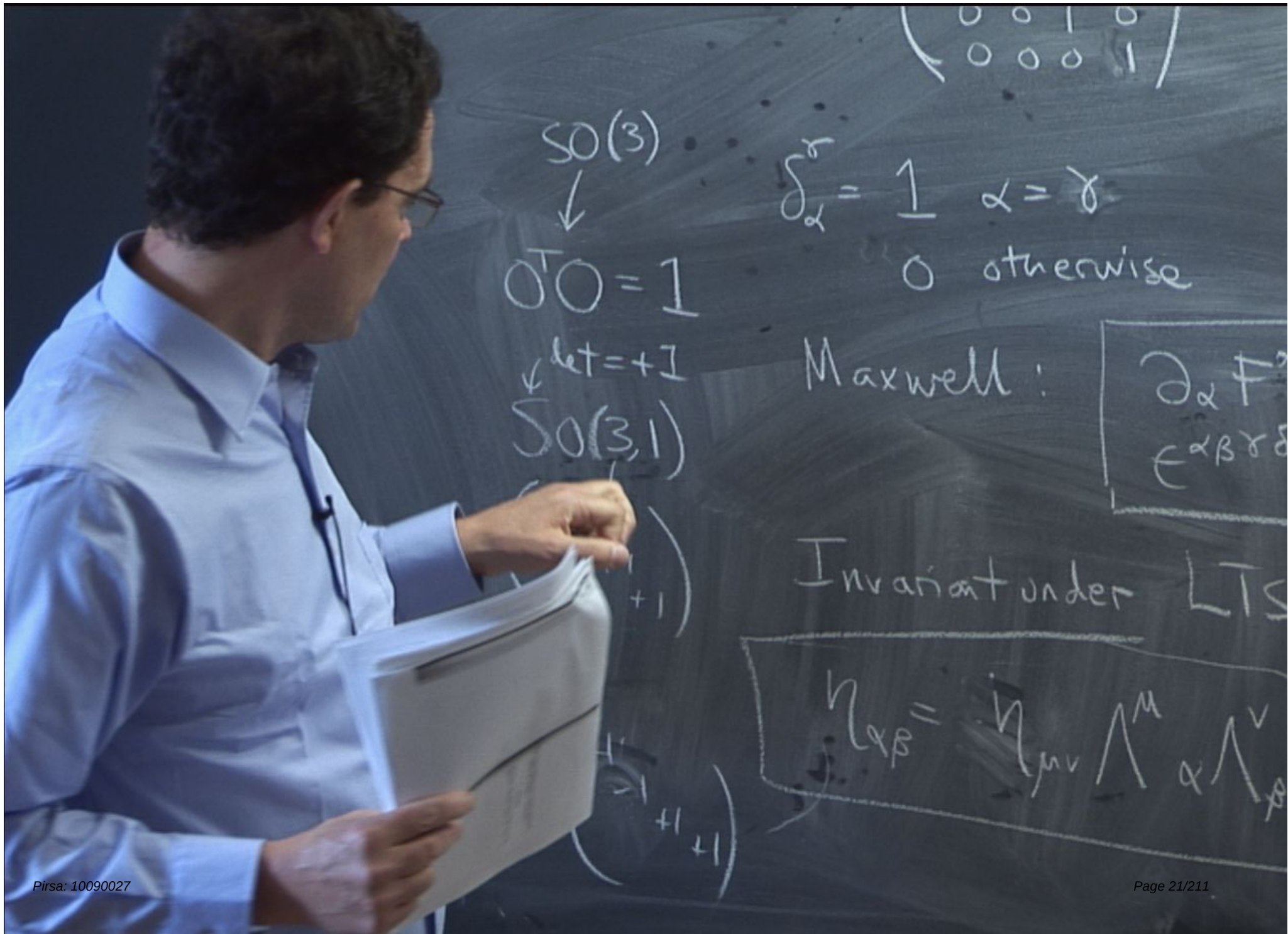
$$O^T O = 1$$

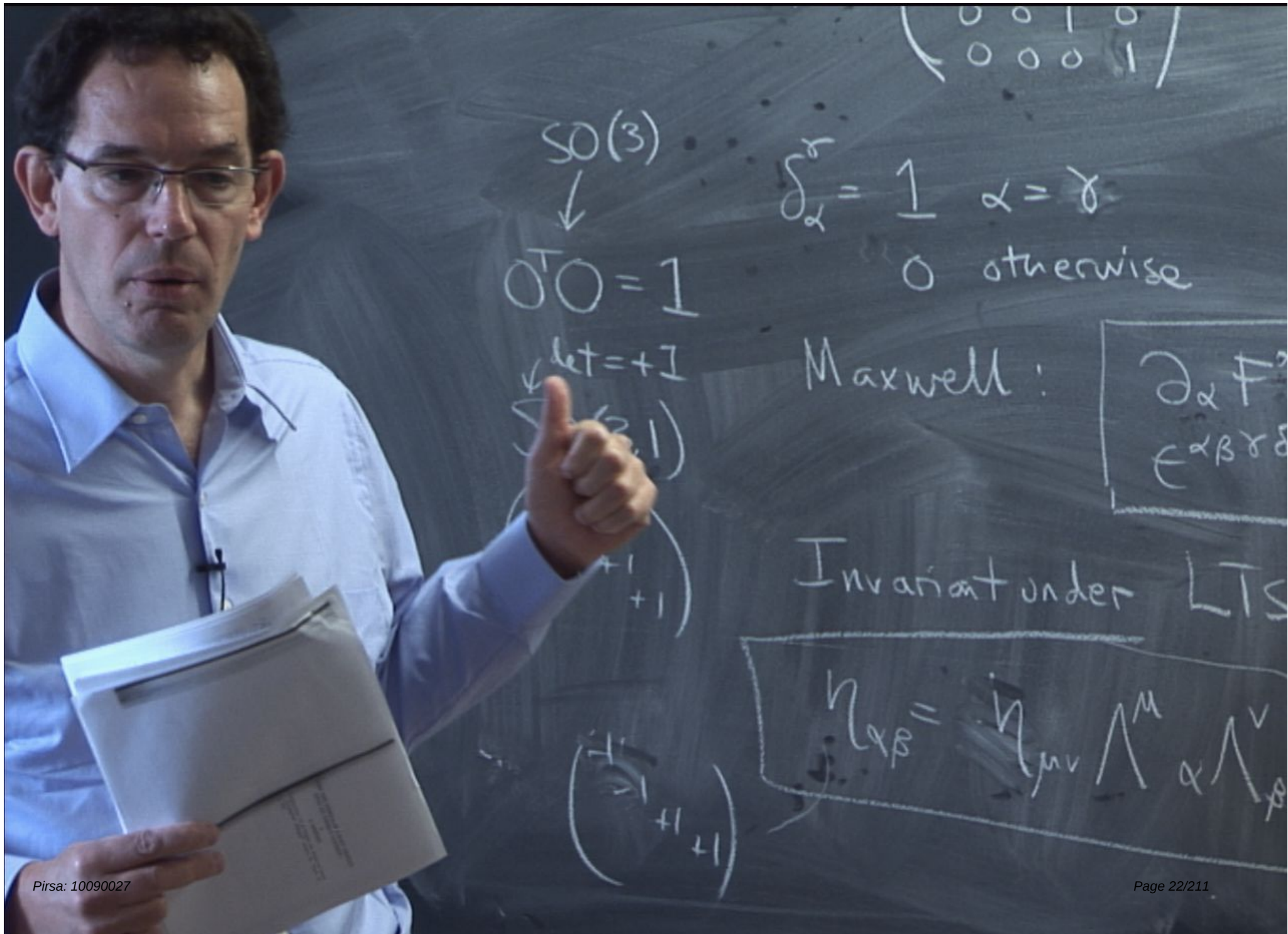
$\downarrow \det = +1$

$SO(3, 1)$

$$\delta_{\alpha}^{\alpha} = 1$$

Maxw





$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$SO(3)$$



$$O^T O = 1$$

$$\det = +1$$

$$S(2,1)$$

$$\begin{pmatrix} +1 & & \\ & +1 & \\ & & +1 \end{pmatrix}$$

$$\begin{pmatrix} +1 & & \\ & -1 & \\ & & +1 & +1 \end{pmatrix}$$

$$\delta_{\alpha}^{\gamma} = 1 \quad \alpha = \gamma$$
$$0 \quad \text{otherwise}$$

Maxwell:

$$\partial_{\alpha} F_{\beta\gamma}$$
$$\epsilon^{\alpha\beta\gamma\delta}$$

Invariant under LTS

$$\eta_{\alpha\beta} = \eta_{\mu\nu} \Lambda^{\mu}_{\alpha} \Lambda^{\nu}_{\beta}$$

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$SO(3)$$



$$O^T O = 1$$

$$\downarrow \det = +1$$

$$SO(3,1)$$



$$\begin{pmatrix} -1 & & & \\ & +1 & & \\ & & +1 & \\ & & & +1 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} -1 & & & \\ & +1 & & \\ & & +1 & \\ & & & +1 \end{pmatrix}$$

$$\delta_{\alpha}^{\gamma} = 1 \quad \alpha = \gamma$$

0 otherwise

Maxwell:

$$\partial_{\alpha} F_{\beta\gamma}$$

Invariant under LTS

$$\eta_{\alpha\beta} = \eta_{\mu\nu} \Lambda^{\mu}_{\alpha} \Lambda^{\nu}_{\beta}$$

$$\partial_\alpha T^{\alpha\beta} + M_0 J^\beta$$

perm
23

Lorentz $\swarrow$ $SO(3,1)$

$$x'^\mu = \Lambda^\mu_\nu x^\nu$$

$$\partial_\alpha T^{\alpha\beta} + M_0 J^\beta$$

perm

23

"

Lorentz $\swarrow$ $SO(3,1)$

$$X'^\mu = \Lambda^\mu_\nu X^\nu$$

Translations $\swarrow$ const

$$X'^\mu = X^\mu + C^\mu$$

$$\partial_\alpha T^{\alpha\beta} + M_0 J^\beta = 0$$

Lorentz $\hookrightarrow SO(3,1)$

$$x^\mu = \Lambda^\mu_\nu x^\nu$$

$$= \left(\partial_\mu F \right)$$

Translations $\hookrightarrow$ const

$$= 0$$

$$x'^\mu = x^\mu + c^\mu \rightarrow x^\mu = x'^\mu - c^\mu$$

$$\frac{\partial}{\partial x'^\mu} = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial}{\partial x^\alpha}$$

$$\partial_\alpha T^{\alpha\mu} + M_0 J^\mu = 0$$

Lorentz $\swarrow$ $SO(3,1)$

$$x'^\mu = \Lambda^\mu_\nu x^\nu$$

$$= (\partial_\mu F$$

Translations $\swarrow$ const

$$x'^\mu = x^\mu + c^\mu$$

$$\rightarrow x^\mu = x'^\mu - c^\mu$$

$$= 0$$

$$\frac{\partial}{\partial x'^\mu} =$$

$$\frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial}{\partial x^\alpha}$$

$$\frac{\partial x^\alpha}{\partial x'^\mu} = \delta^\alpha_\mu$$

$$\partial'_\alpha F'^{\alpha\beta} + \mu_0 j'^\beta = \partial_\gamma F^{\mu\nu} + \mu_0 j^\beta$$

perm
23

Lorentz $\leftarrow SO(3,1)$
 $x'^\mu = \Lambda^\mu_\nu x^\nu$
 Translations $\leftarrow \text{const}$

$$x'^\mu = x^\mu + c^\mu \rightarrow x^\mu = x'^\mu - c^\mu$$

$$\frac{\partial}{\partial x'^\mu} = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial}{\partial x^\alpha} \quad \frac{\partial x^\alpha}{\partial x'^\mu} = \delta^\alpha_\mu$$

$$= \frac{\partial}{\partial x^\alpha}$$

Poincare group

$$\partial'_\alpha F'^{\alpha\beta} + \mu_0 j'^\beta = \partial_\gamma F^{\mu\nu} \Lambda^\gamma_\alpha + \mu_0 j^\beta$$

per m
23

Lorentz $\swarrow$ $SO(3,1)$
 $x'^\mu = \Lambda^\mu_\nu x^\nu$
 Translations $\swarrow$ const

$$x'^\mu = x^\mu + c^\mu \rightarrow x^\mu = x'^\mu - c^\mu$$

$$\frac{\partial}{\partial x'^\mu} = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial}{\partial x^\alpha} \quad \frac{\partial x^\alpha}{\partial x'^\mu} = \delta^\alpha_\mu$$

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Poincare group

$$\partial'_\alpha F'^{\alpha\beta} + \mu_0 j'^\beta = \partial_\gamma F^{\mu\nu} + \mu_0 j^\beta$$

per m
23

$\swarrow$ Lorentz $\swarrow$ $SO(3,1)$
 $\swarrow$ Translations $\swarrow$ const

$$x'^\mu = \Lambda^\mu_\nu x^\nu$$

$$x'^\mu = x^\mu + c^\mu \rightarrow x^\mu = x'^\mu - c^\mu$$

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$$= \frac{\partial}{\partial x^\alpha}$$

Poincare group

perm
23

+ μ_0 /

Lorentz $\leftarrow SO(3,1)$

$$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu}$$

$$= \left(\partial_{\mu} F^{\mu\nu} + \right.$$

Translations $\leftarrow$ const

$$x'^{\mu} = x^{\mu} + c^{\mu} \rightarrow x^{\mu} = x'^{\mu} - c^{\mu}$$

$$= 0.$$

$$\frac{\partial}{\partial x'^{\mu}} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial}{\partial x^{\alpha}}$$

$$= \frac{\partial}{\partial x^{\alpha}}$$

$$\frac{\partial x^{\alpha}}{\partial x'^{\mu}} = \delta^{\alpha}_{\mu}$$

Poincare group

10-dimensional
ctr gp

$$= \mu v + \mu_0 J^v) \wedge v$$

Rule for building
Lorentz-invariant theories:
write equations tensorially!

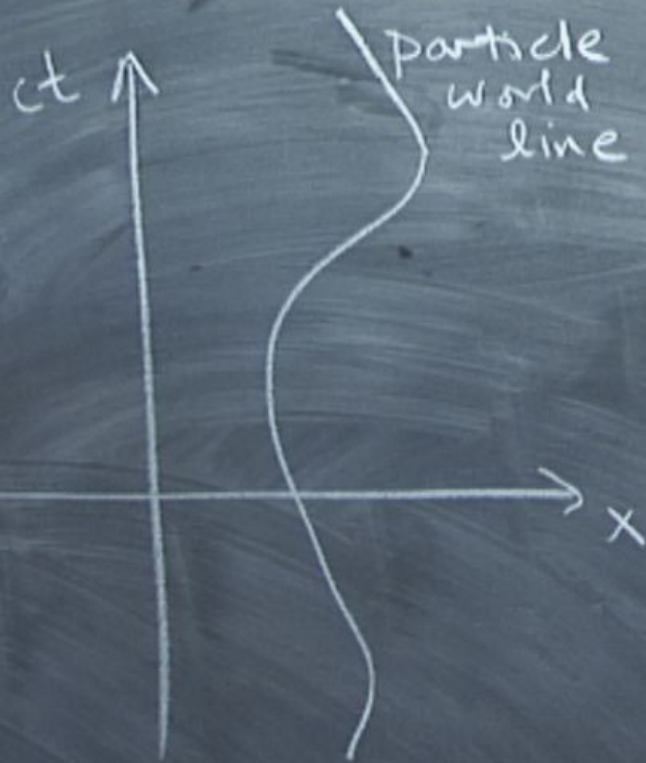
$$x'^{\mu} = \Lambda^{\mu}_{\alpha} x^{\alpha}$$

$$\frac{\partial}{\partial x'^{\alpha}} = \frac{\partial}{\partial x^{\nu}} \tilde{\Lambda}^{\nu}_{\alpha}$$

$$x = (ct, x)$$

$$x'^{\mu} = \Lambda^{\mu}_{\alpha} x^{\alpha}$$

$$\frac{\partial}{\partial x^{\alpha}} = \frac{\partial}{\partial x^{\nu}} \tilde{\Lambda}^{\nu}_{\alpha}$$



$$x^\alpha = (ct, \underline{x})$$

$$x'^\mu = \Lambda^\mu_\alpha x^\alpha$$

$$\frac{\partial}{\partial x'^\alpha} =$$

$$\frac{\partial}{\partial x'^\nu} \tilde{\Lambda}^\nu_\alpha$$

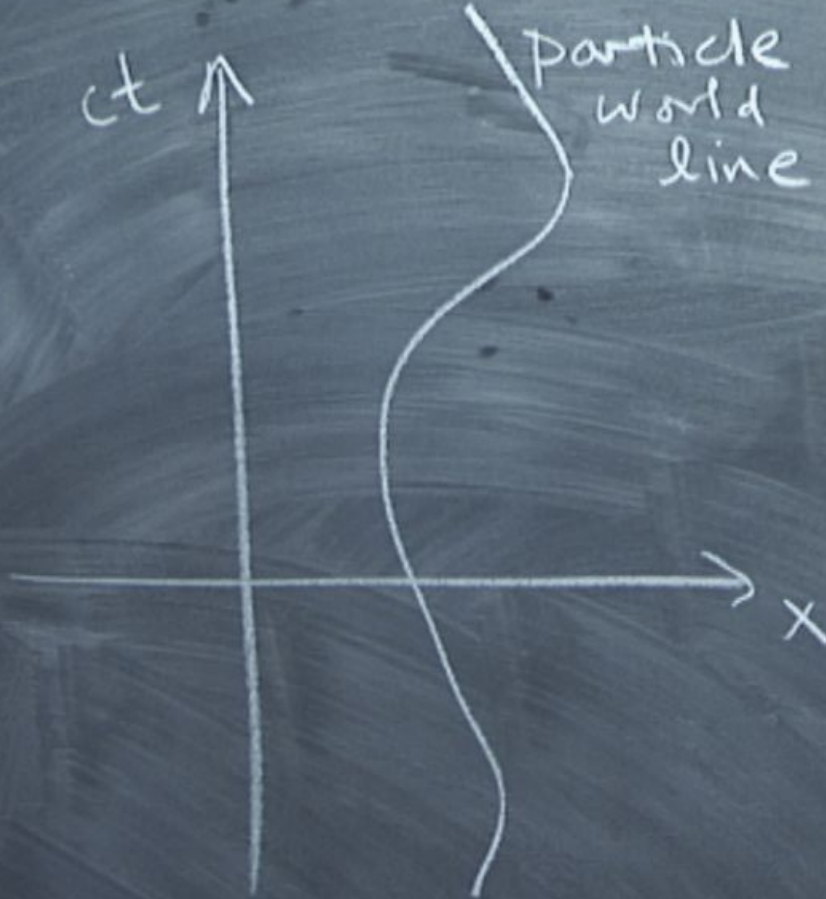


$$x^\alpha = (ct, \underline{x})$$

$$x'^\mu = \Lambda^\mu{}_\alpha x^\alpha$$

$$\frac{\partial}{\partial x'^\alpha} =$$

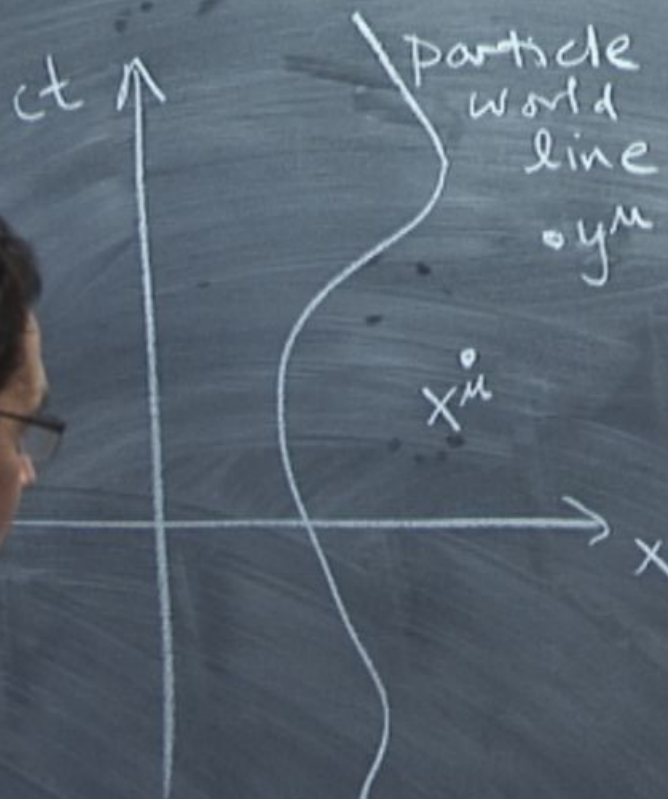
$$\frac{\partial}{\partial x^\nu} \tilde{\Lambda}^\nu{}_\alpha$$



$$x^\alpha = (ct, \underline{x})$$

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$$\frac{\partial}{\partial x'^\alpha} = \frac{\partial}{\partial x^\nu} \tilde{\Lambda}^\nu{}_\alpha$$



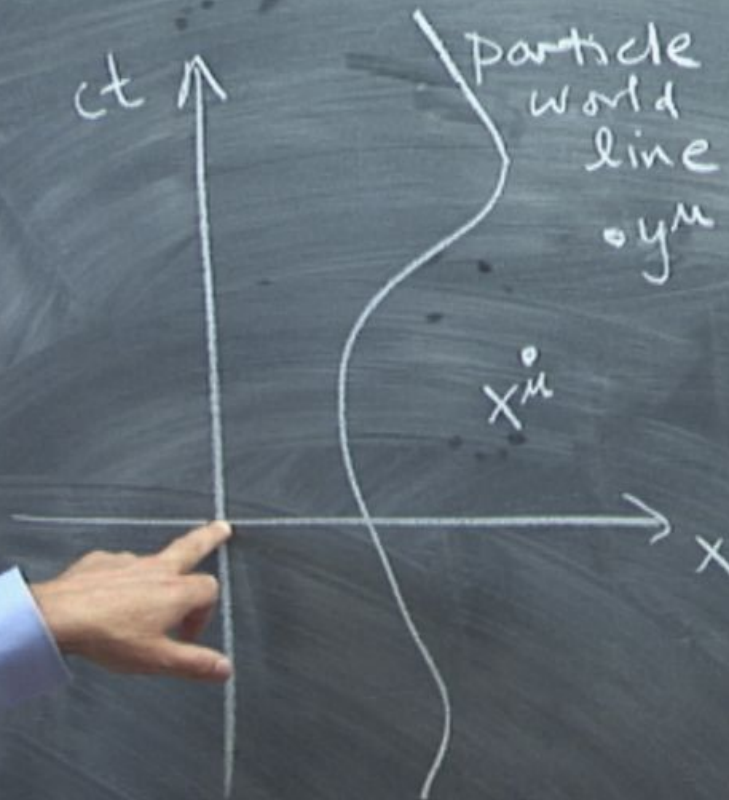
invariant distance
from x to y is

$$\eta_{\alpha\beta} (x^\alpha - y^\alpha) (x^\beta - y^\beta)$$

$$x^\alpha = (ct, \underline{x})$$

$$x'^\mu = \Lambda^\mu{}_\alpha x^\alpha$$

$$\frac{\partial}{\partial x'^\alpha} = \frac{\partial}{\partial x^\nu} \tilde{\Lambda}^\nu{}_\alpha$$



invariant distance
from x to y is

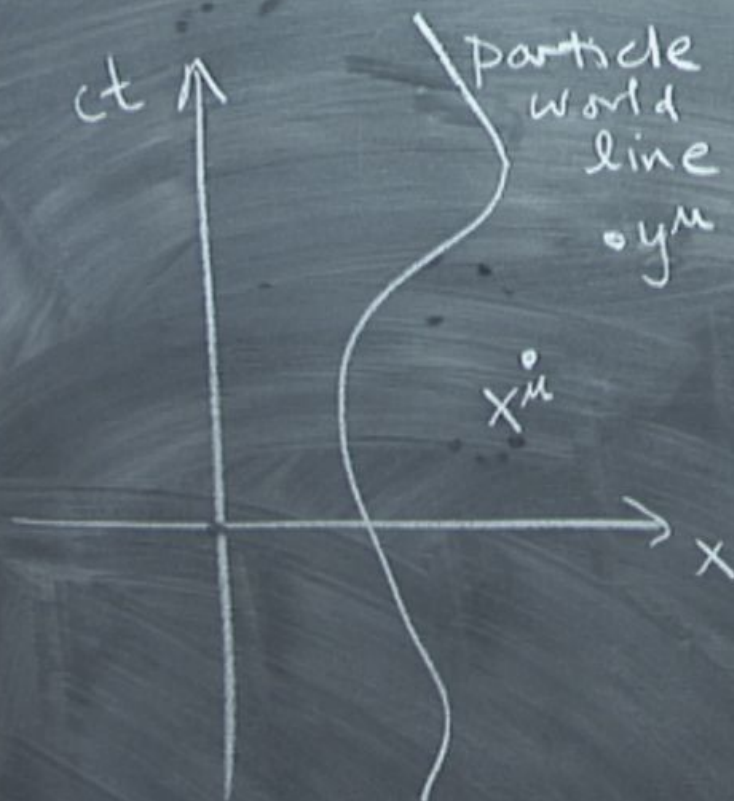
$$\eta_{\alpha\beta} (x^\alpha - y^\alpha) (x^\beta - y^\beta)$$

in particular, if $y^\alpha = 0$

$$x^\alpha = (ct, \underline{x})$$

$$x'^\mu = \Lambda^\mu{}_\alpha x^\alpha$$

$$\frac{\partial}{\partial x^\alpha} = \frac{\partial}{\partial x^\nu} \tilde{\Lambda}^\nu{}_\alpha$$



invariant distance
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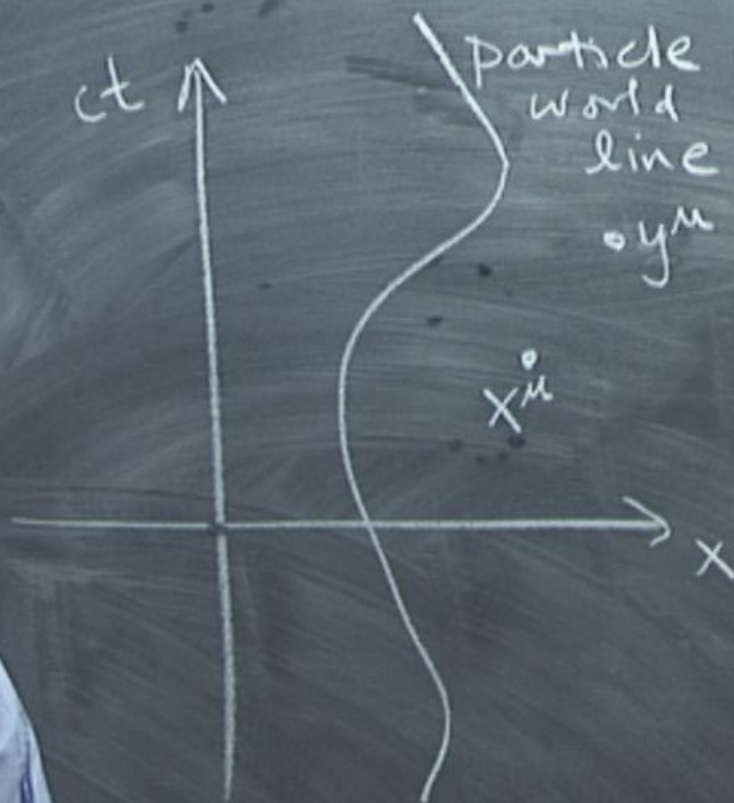
$$\eta_{\alpha\beta} x^\alpha x^\beta = -c^2 t^2 + x^2$$

$$= -c^2 t'^2 + x'^2$$

$$x^\alpha = (ct, \underline{x})$$

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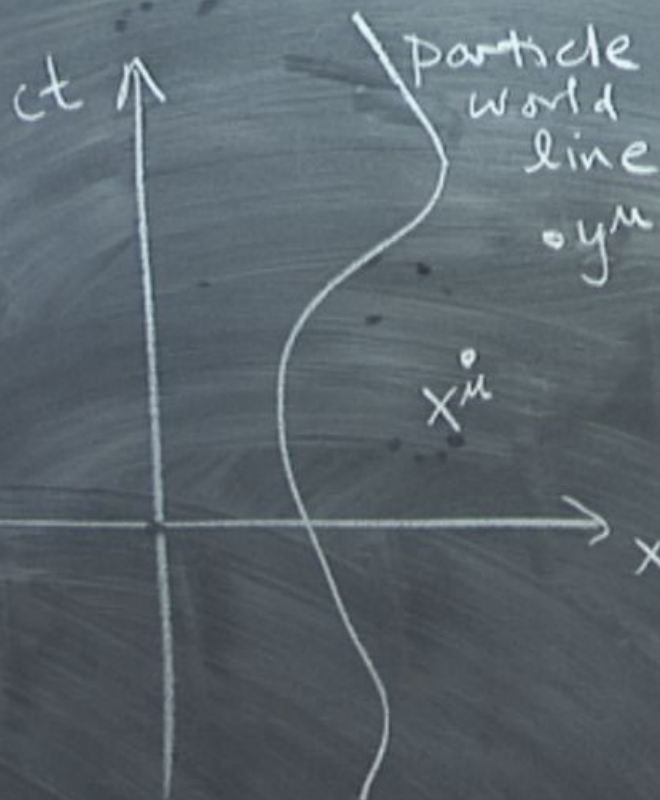
$$x^2 - c^2 t^2 = 0$$

$$= -c^2 t'^2 + x'^2$$

$$x^\alpha = (ct, \underline{x})$$

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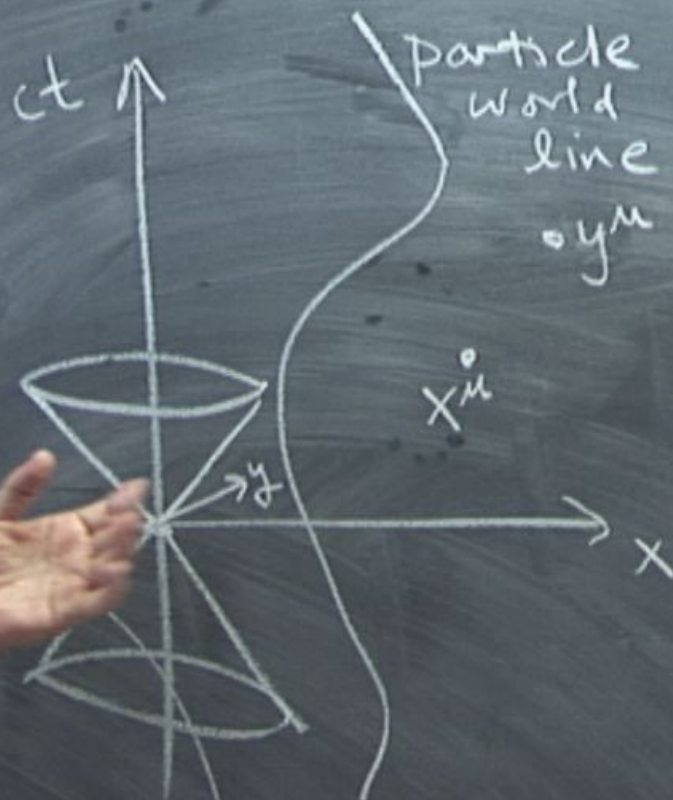
$$x^2 + y^2 + z^2 - c^2 t^2 = 0$$

$$= -c^2 t'^2 + \underline{x}'^2$$

$$x^\alpha = (ct, \underline{x})$$

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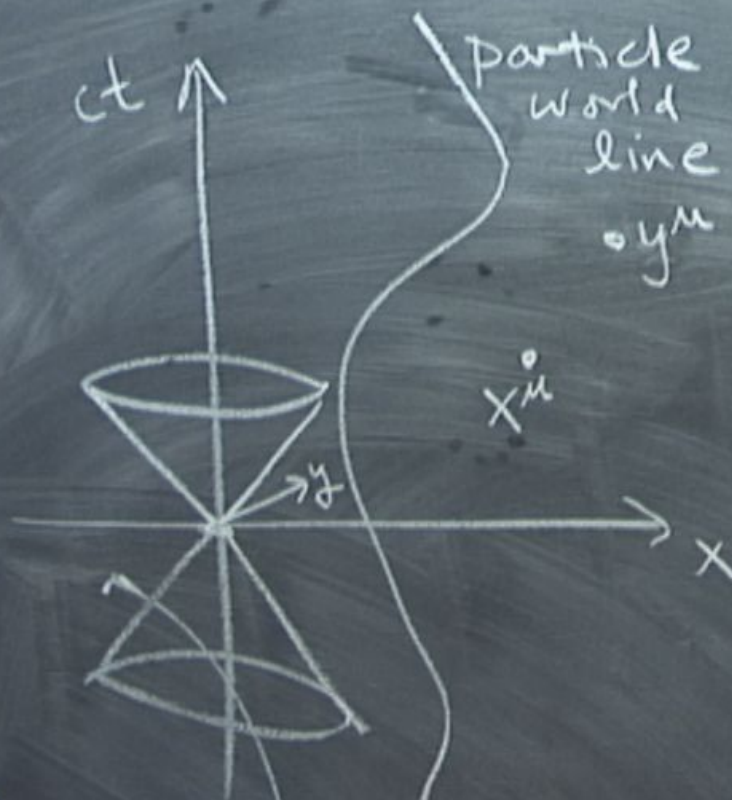
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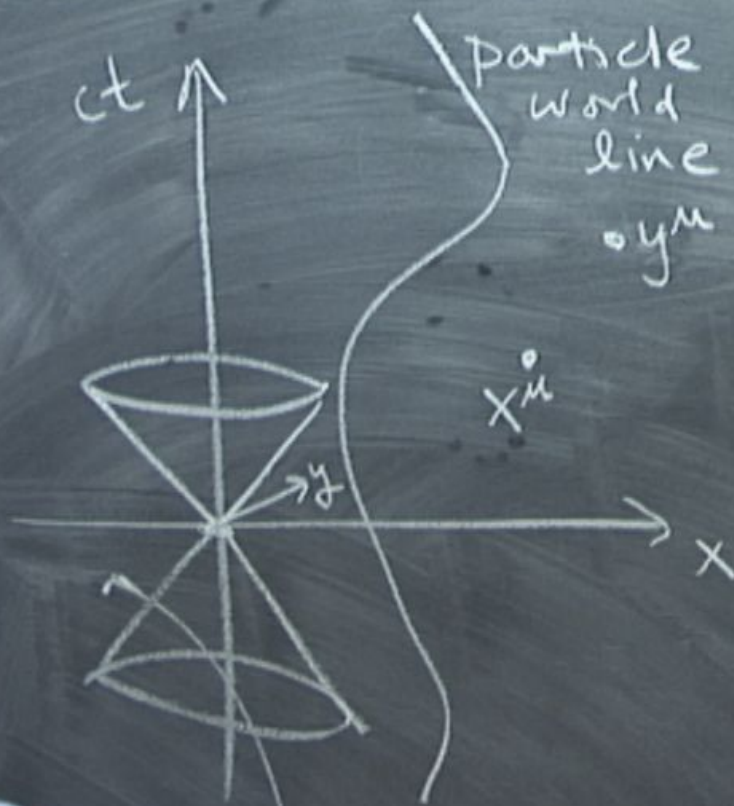
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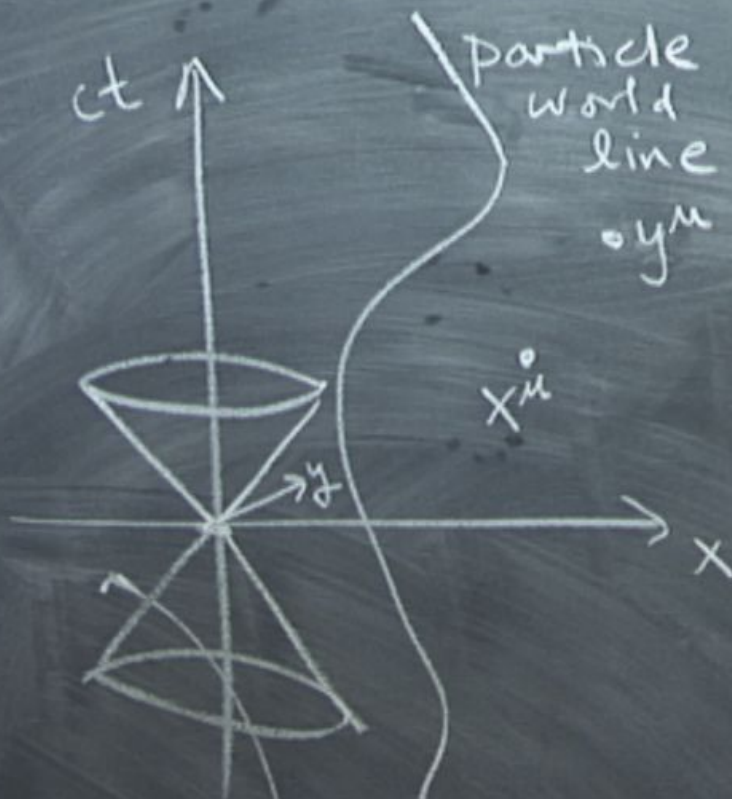
$$= -c^2 t'^2 + \underline{x}'^2$$

$$x'^2 - c^2 t'^2 = 0$$

$$x^\alpha = (ct, \underline{x})$$

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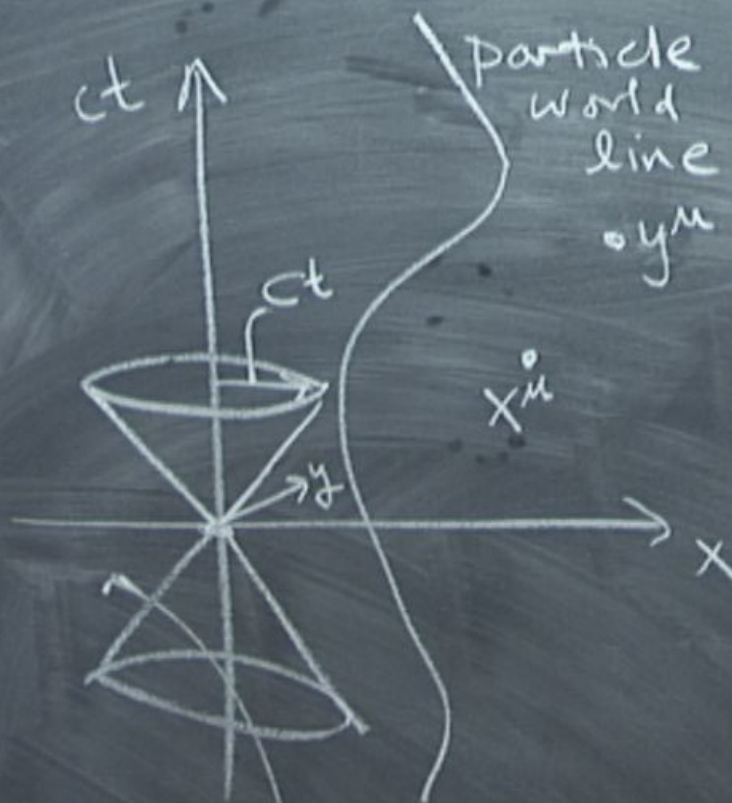
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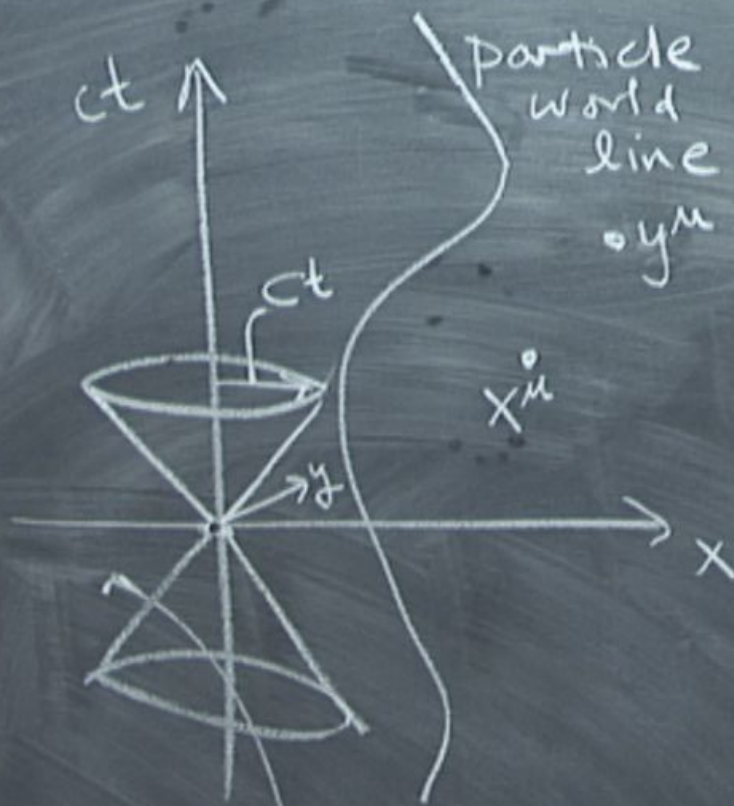
$$x^2 + y^2 + z^2 - c^2 t^2 = 0$$

light cone.

$$x^\alpha = (ct, \underline{x})$$

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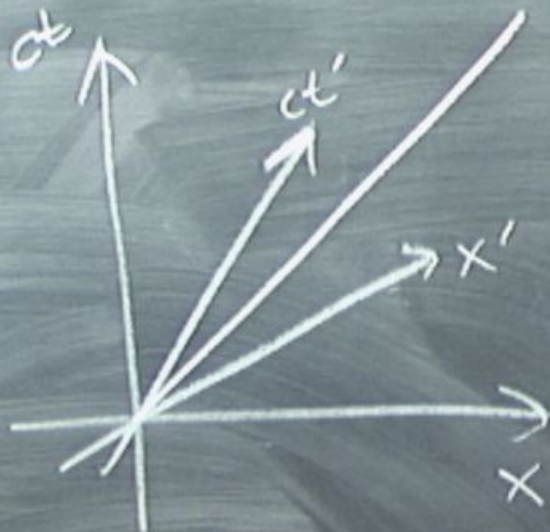
$$\eta_{\alpha\beta} x^\alpha x^\beta = -c^2 t^2 + x^2$$

$$= -c^2 t'^2 + x'^2$$

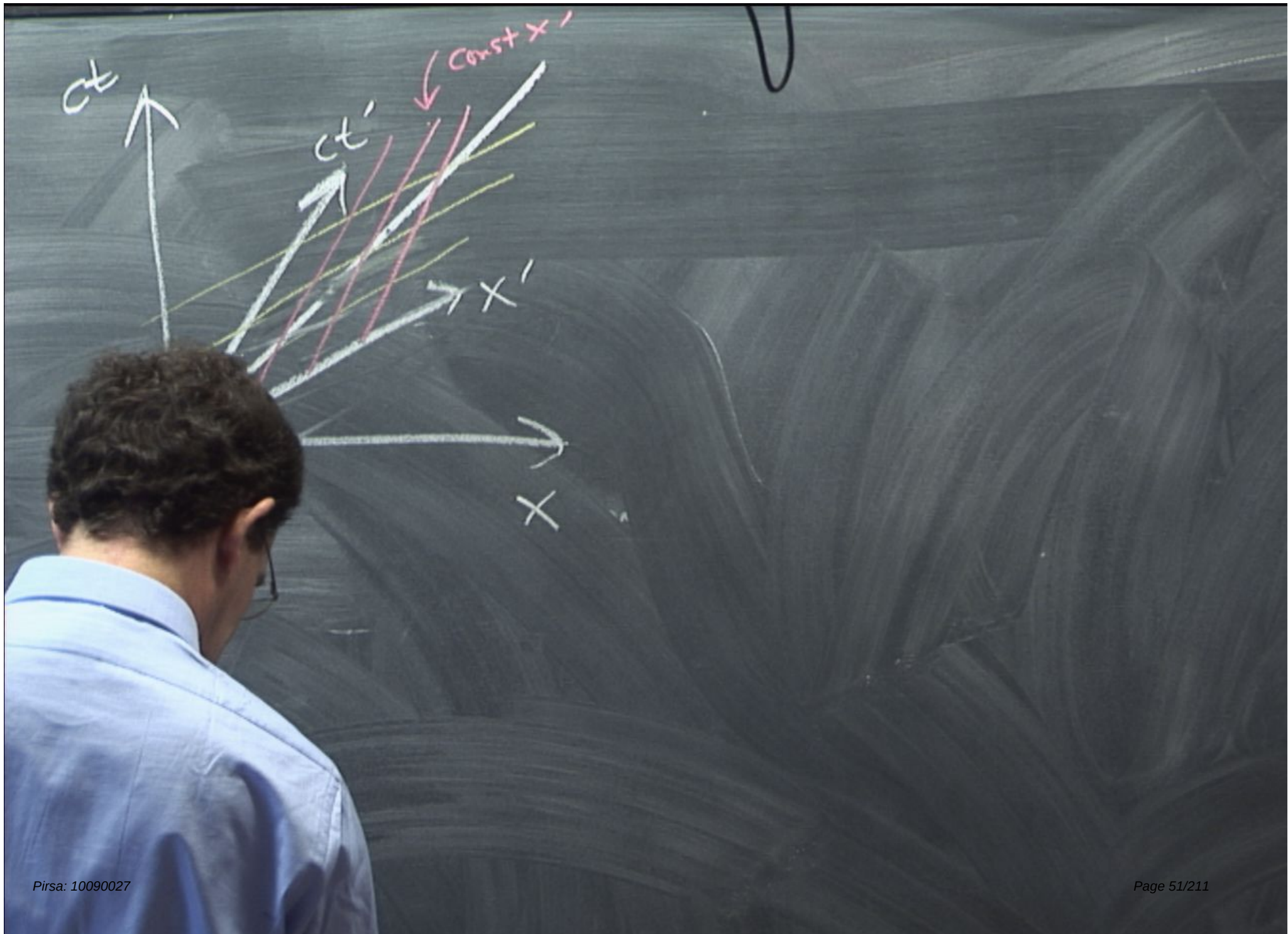
$$x^2 + y^2 + z^2 - c^2 t^2 = 0$$

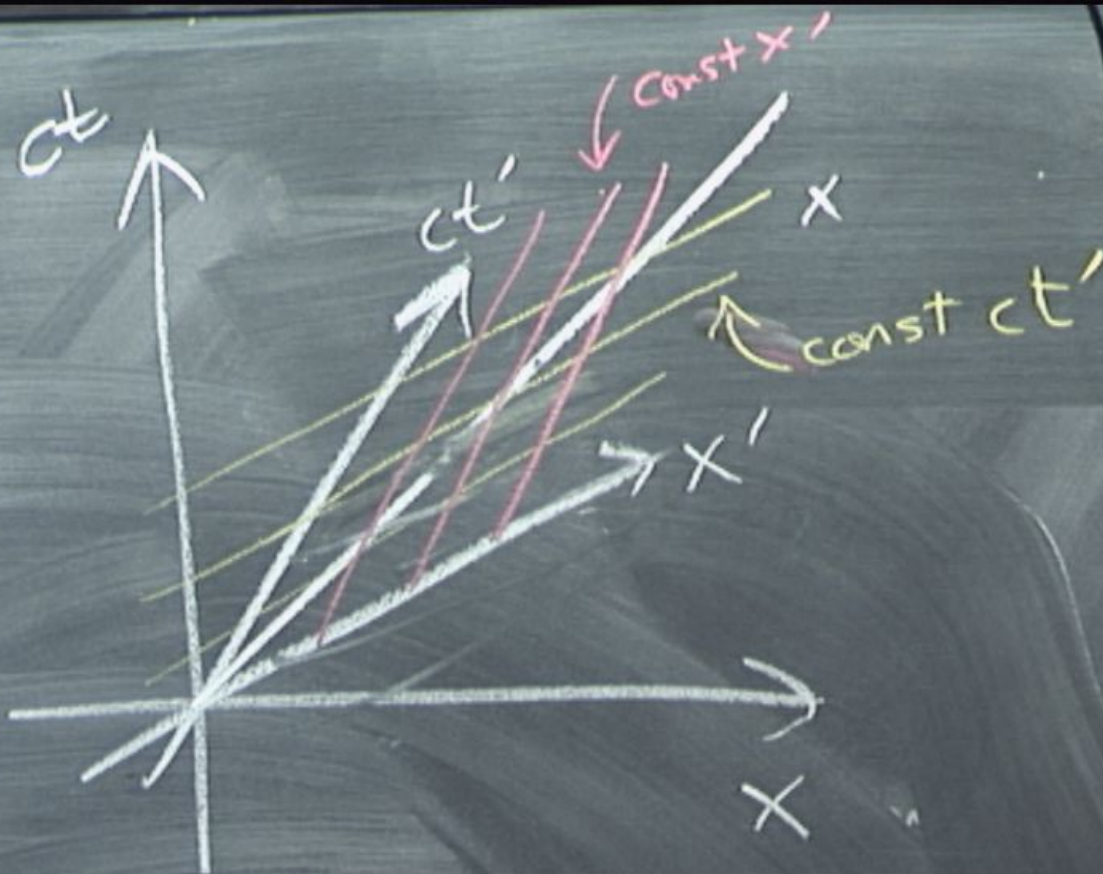
light cone.

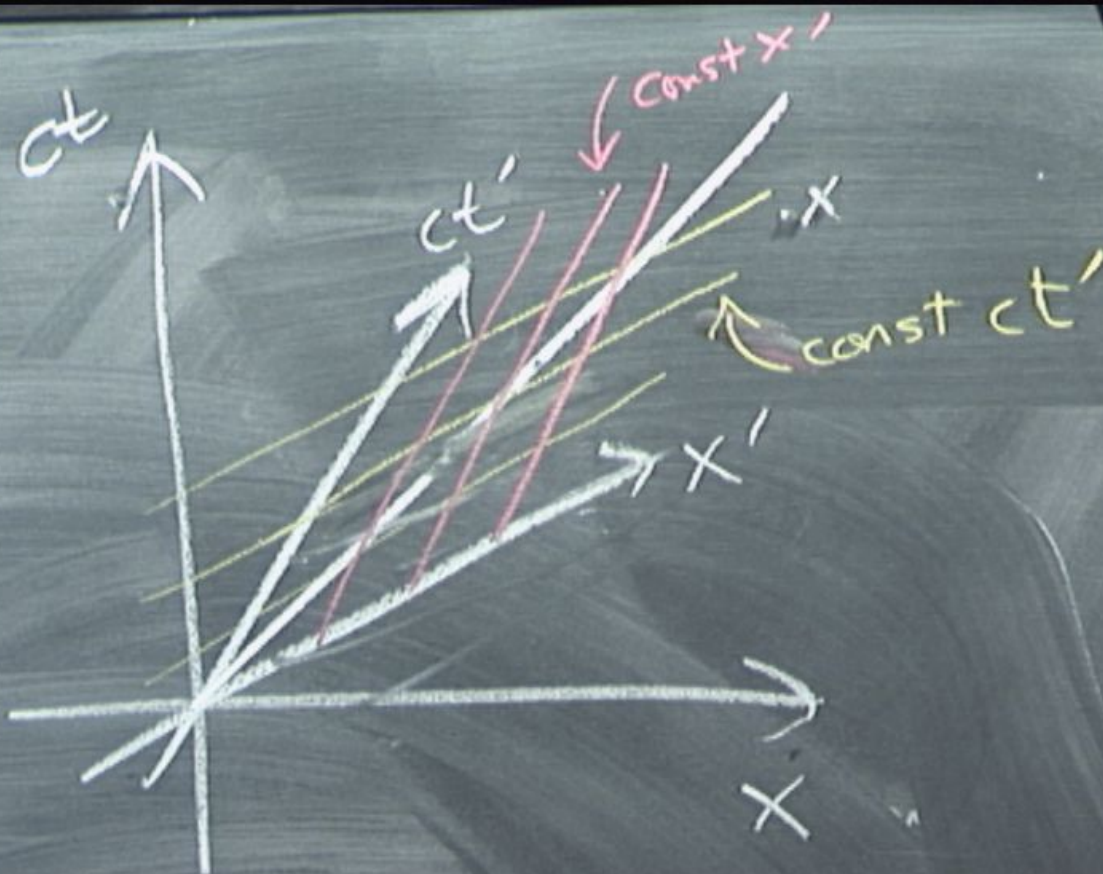




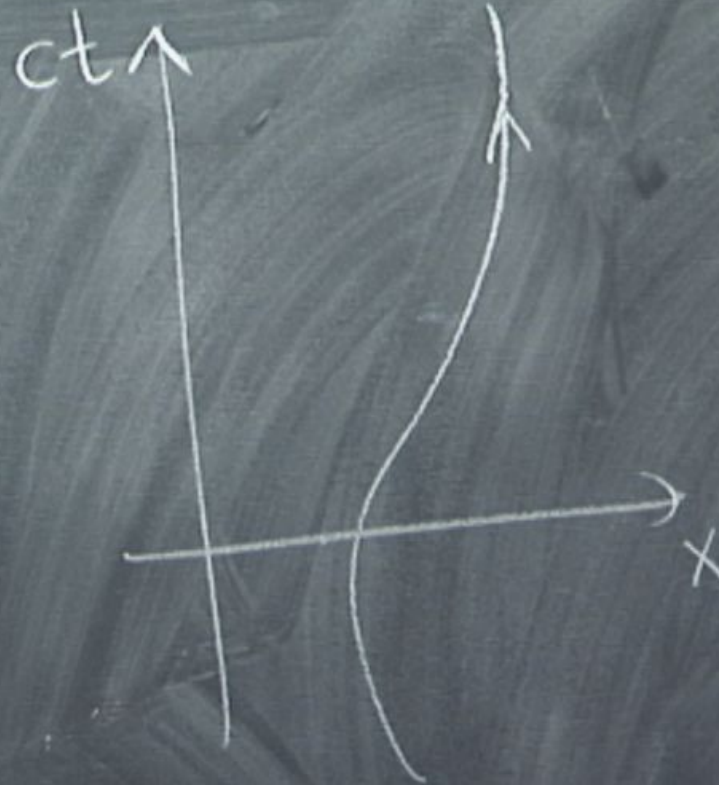
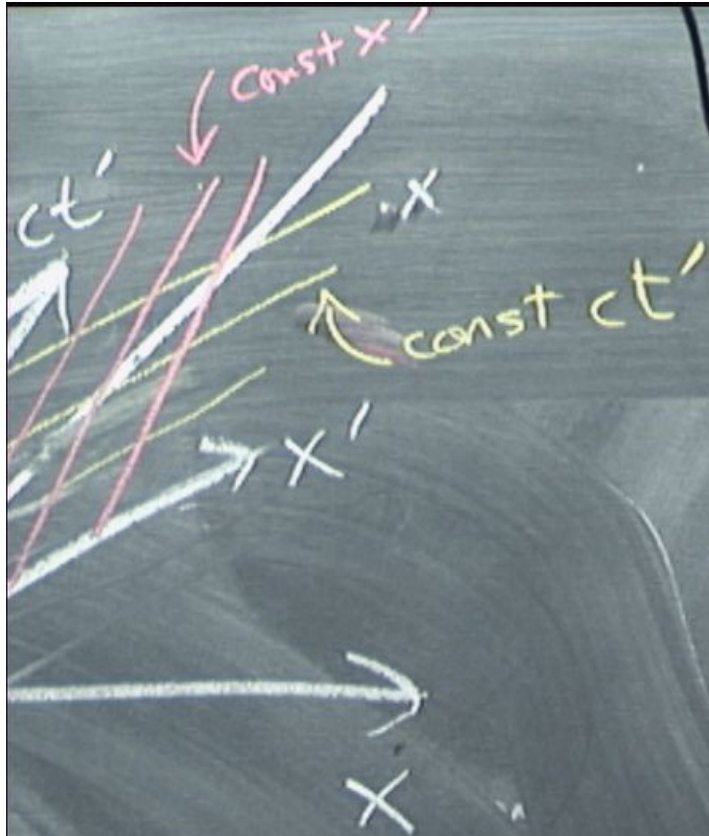








Particles



Particles

const ct'

ct

$x^M(\lambda)$

parameterised curve



Particles

const ct'

ct

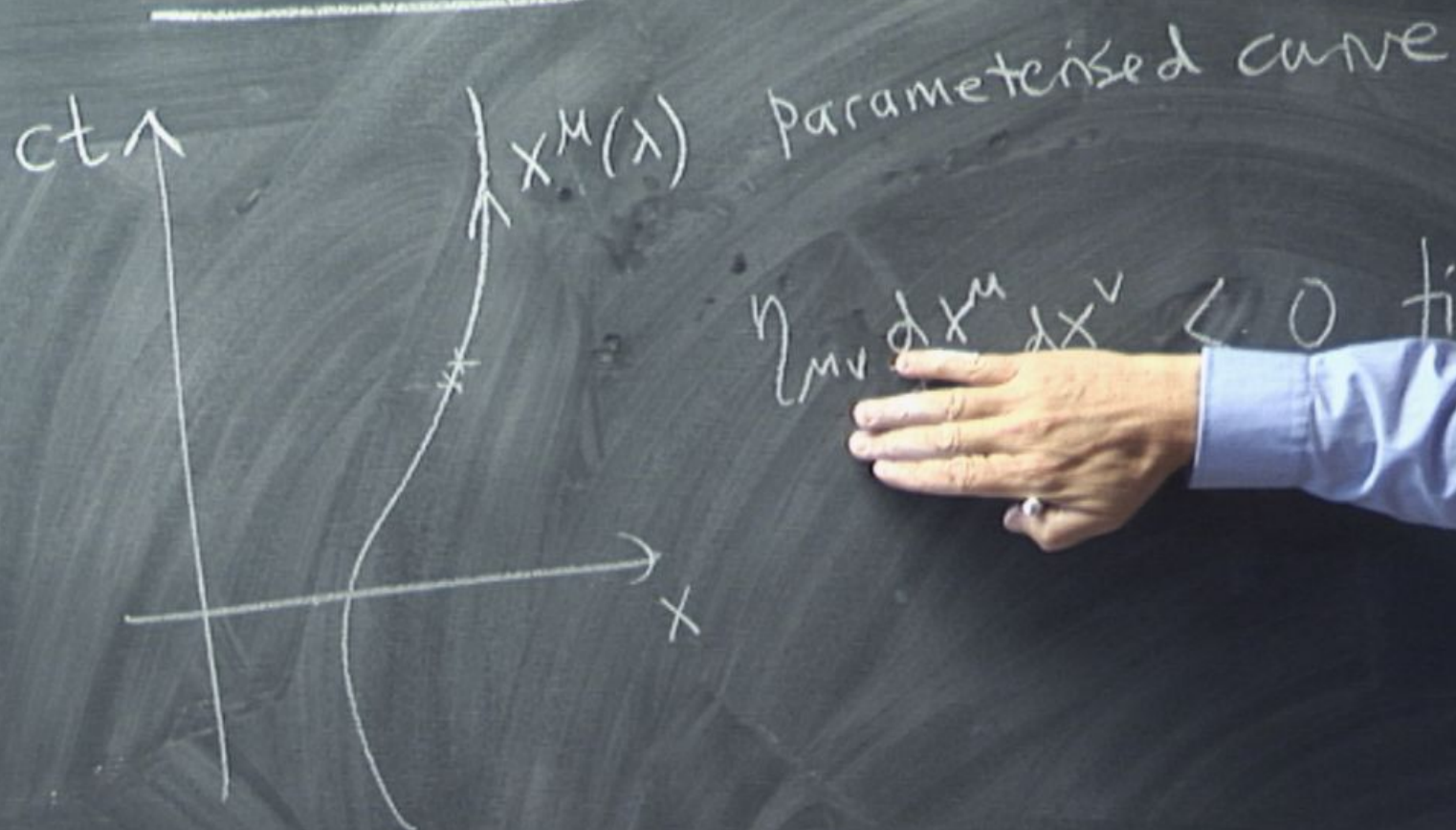
$x^M(\lambda)$

parametrised curve

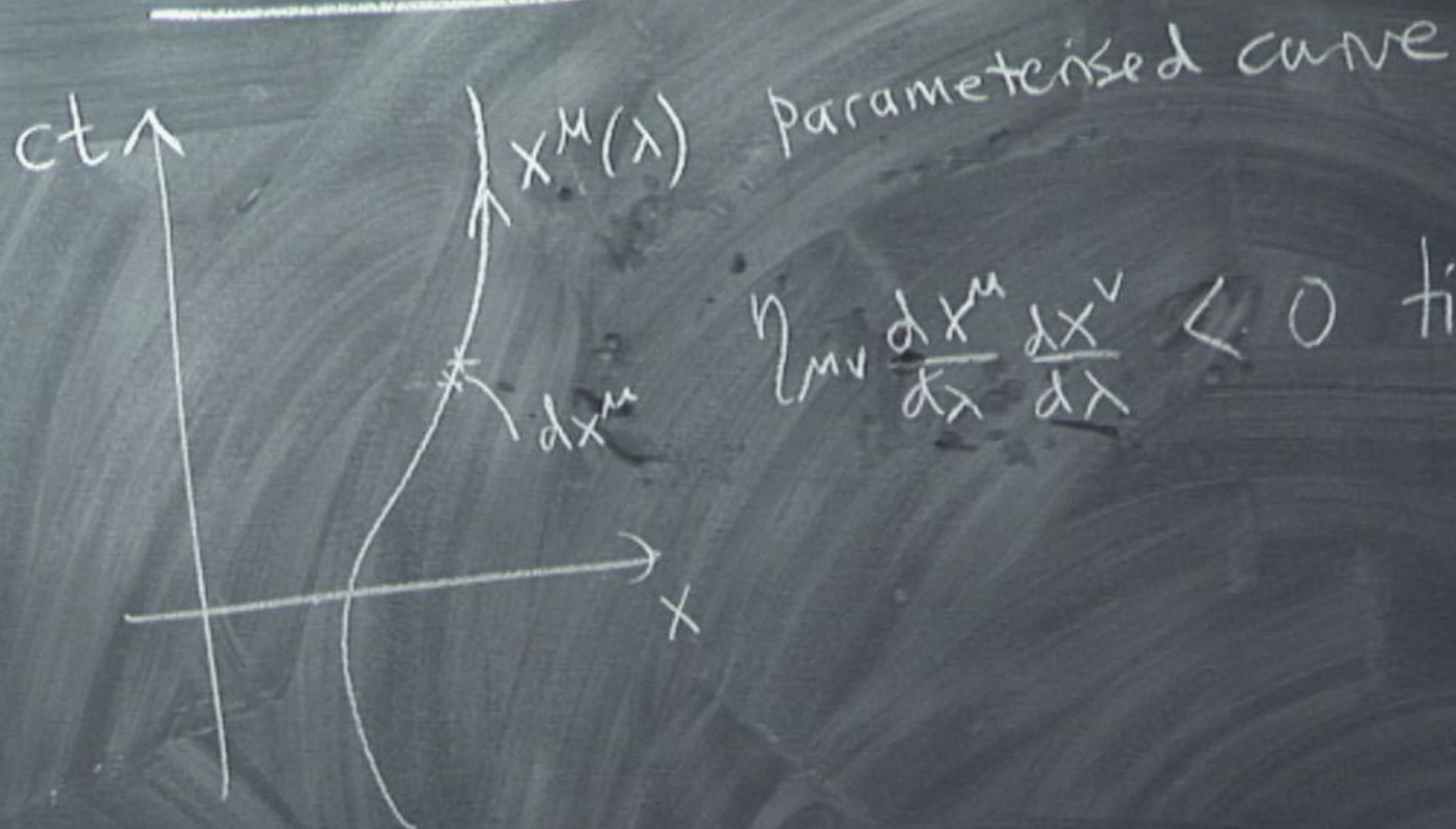
$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} < 0 \text{ time}$$



Particles

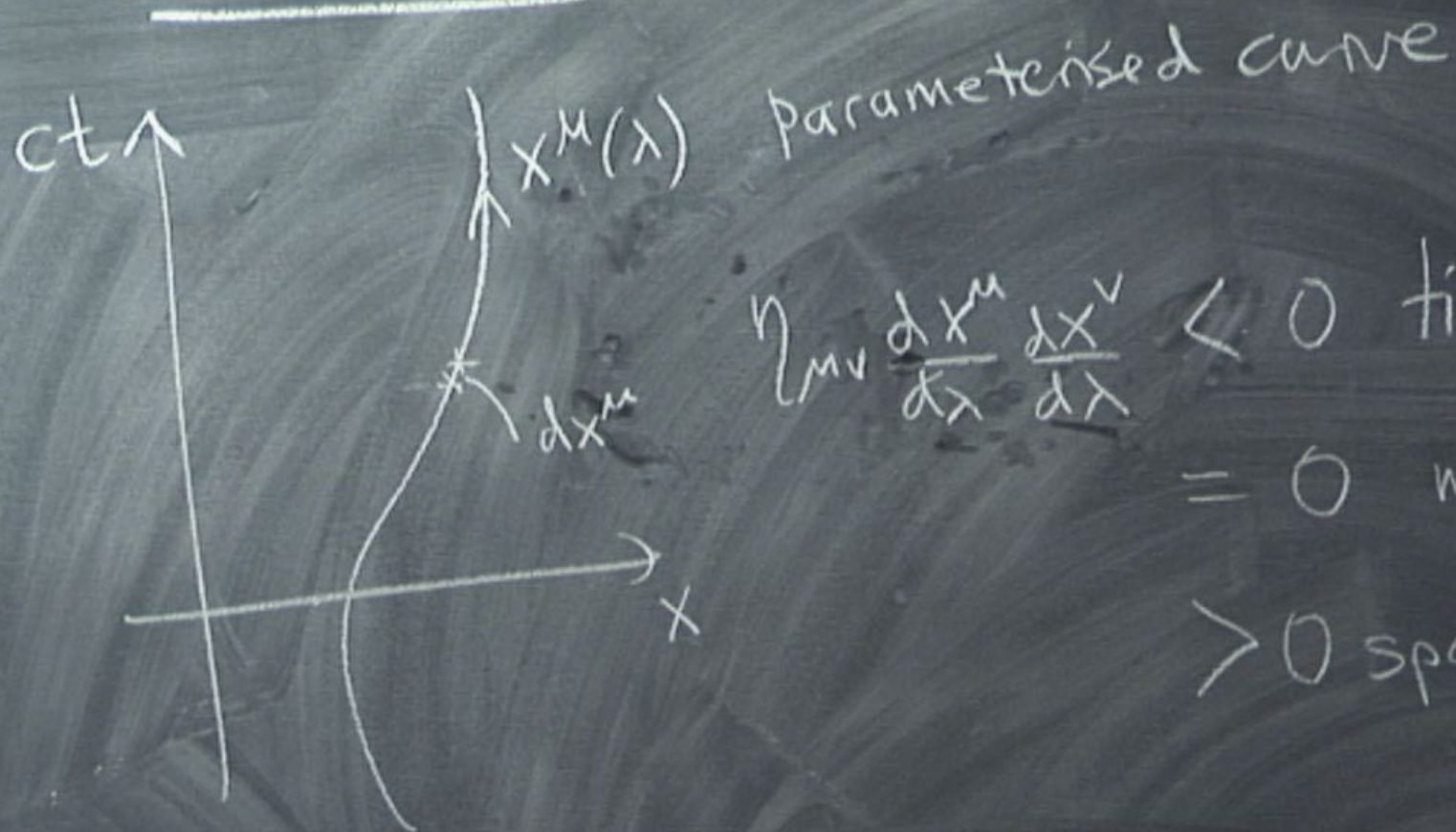


Particles



$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} < 0 \text{ timelike}$$

Particles



$$\eta_{\mu\nu} \frac{dx^{\mu}}{d\lambda} \frac{dx^{\nu}}{d\lambda} < 0 \text{ timelike}$$

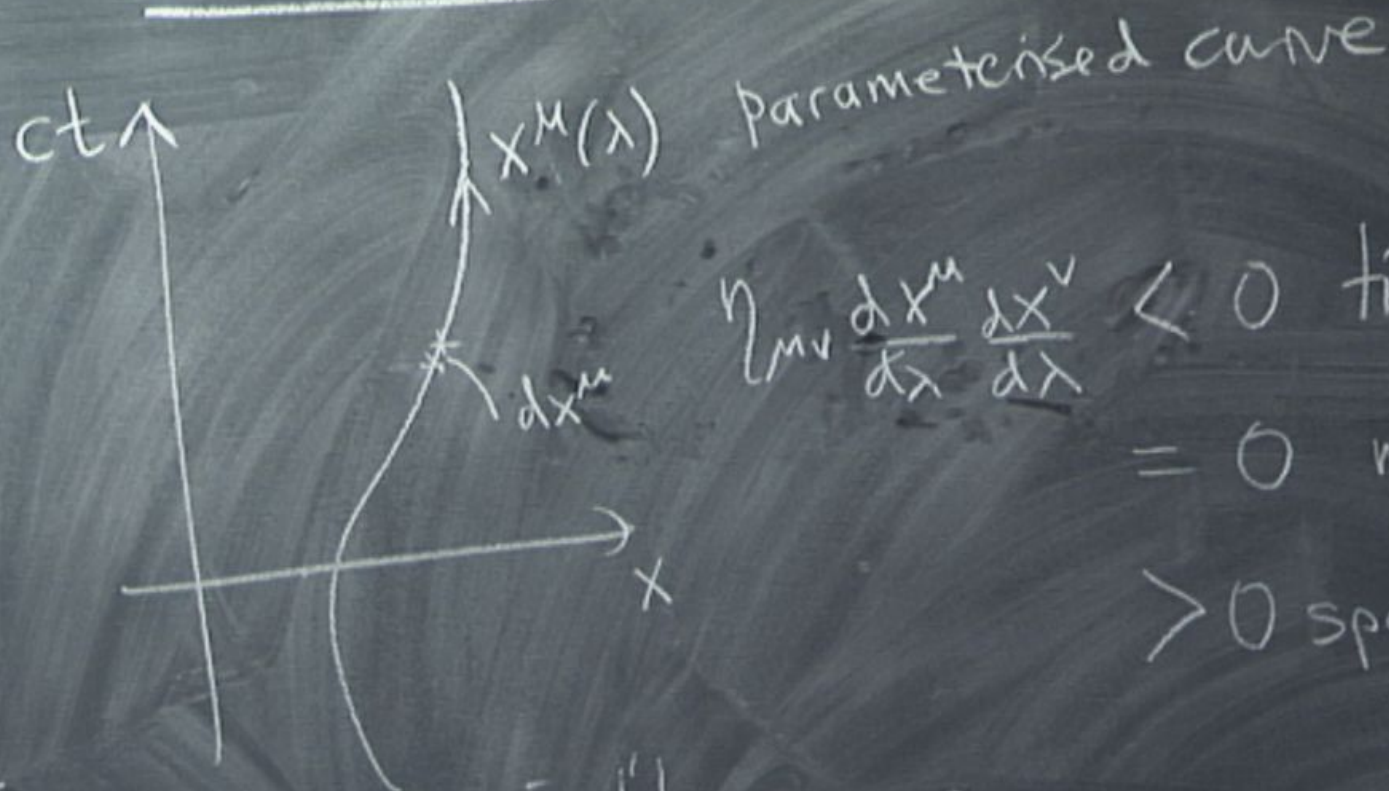
$$= 0 \text{ null}$$

$$> 0 \text{ spacelike}$$

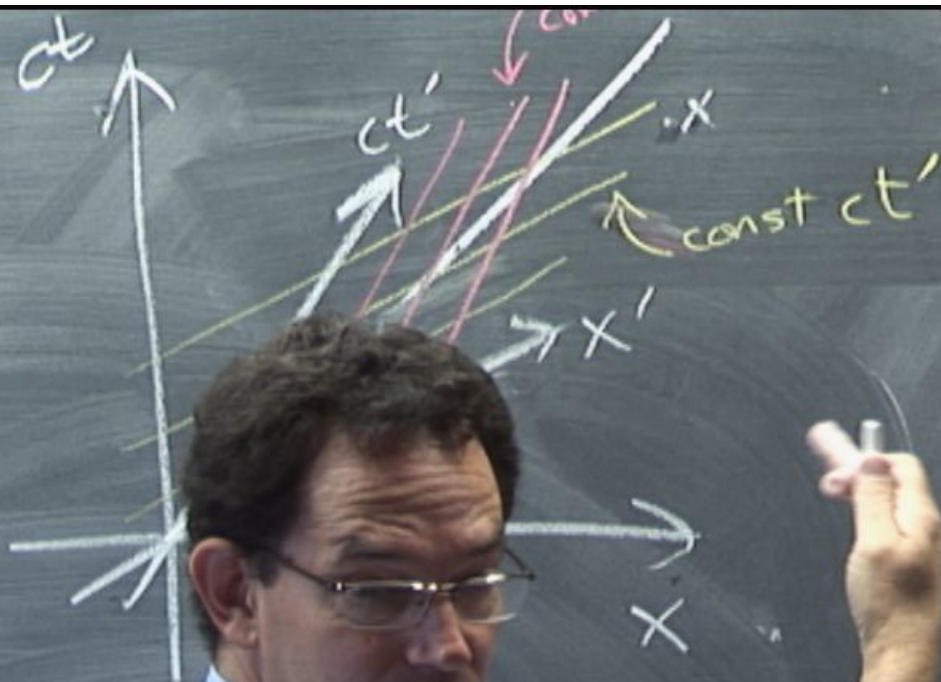


physical massive particles must follow timelike curves.

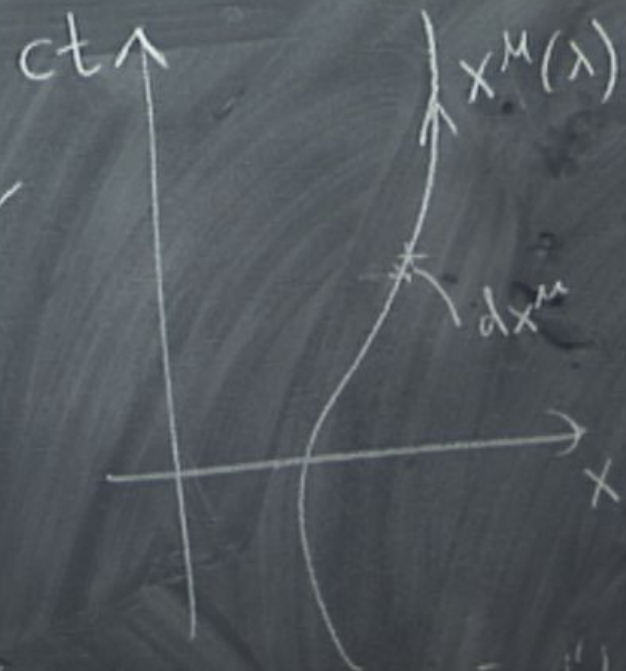
Particles



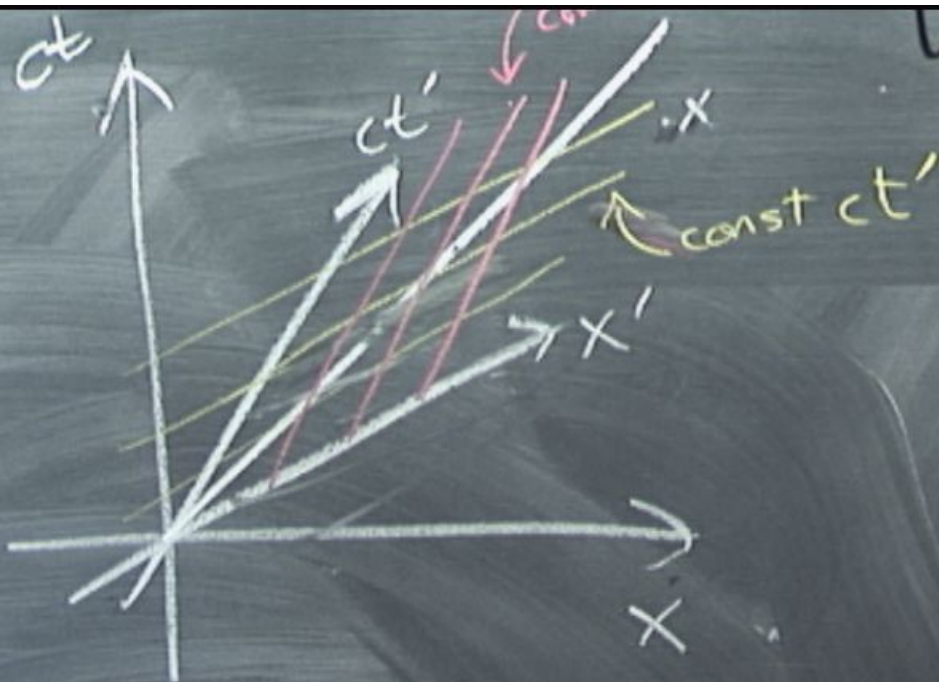
es must follow timelike curves.



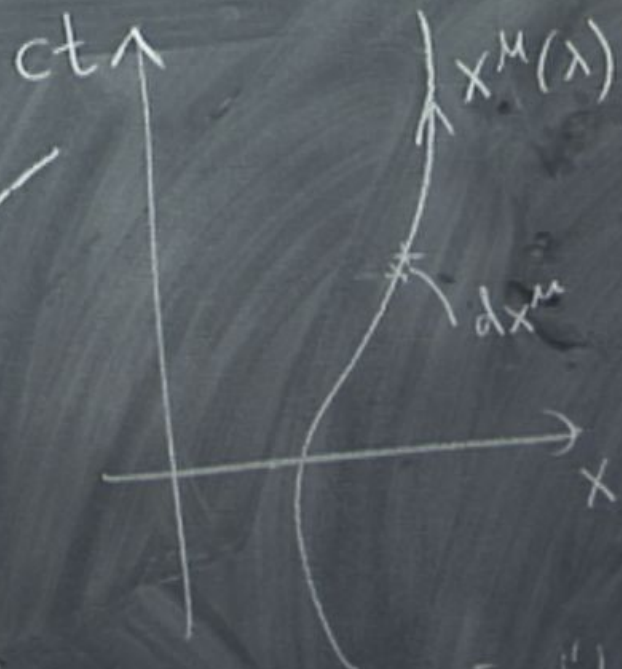
Particles



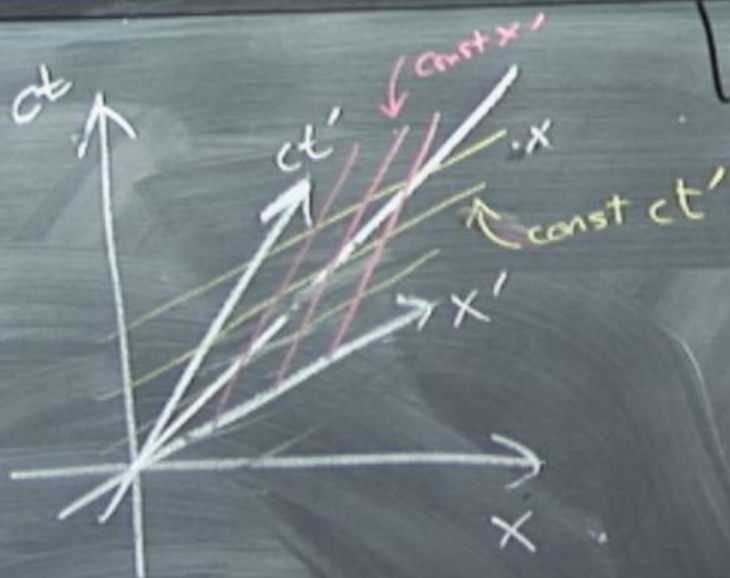
es must follow timelike



Particles



Physikal massive particles must follow timelike



Particles

ct



parameterised curve

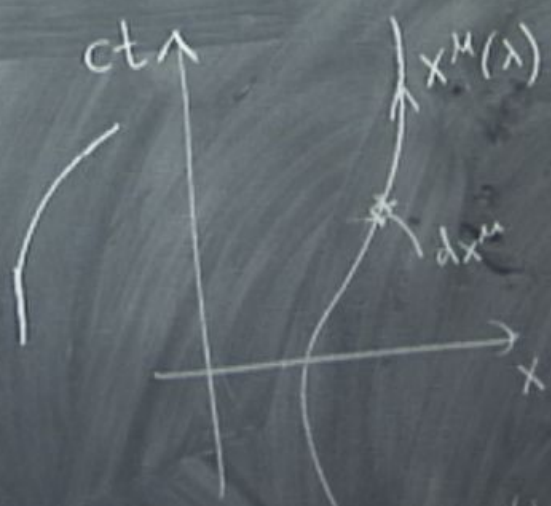
$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} \begin{cases} < 0 & \text{timelike} \\ = 0 & \text{null} \\ > 0 & \text{spacelike} \end{cases}$$

physical massive particles must follow timelike curves.

→ makes sense to parameterise curve using proper time



Particles



parameterised curve

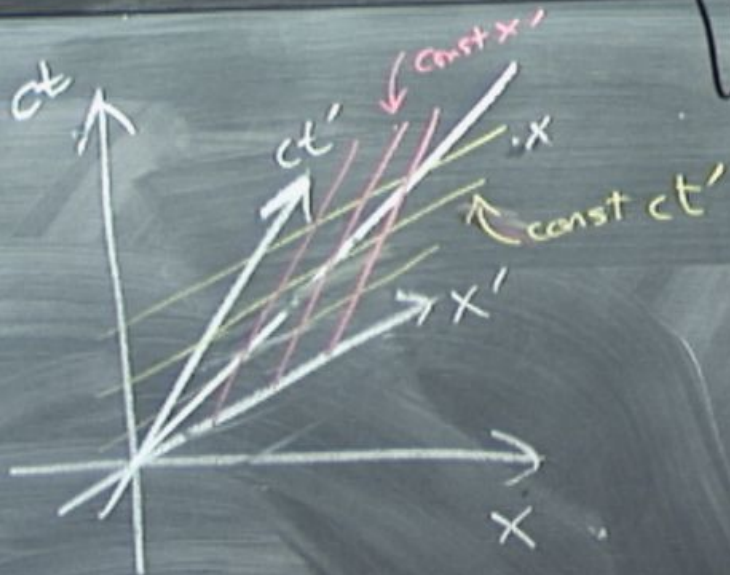
$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} < 0 \text{ timelike}$$

$$= 0 \text{ null}$$

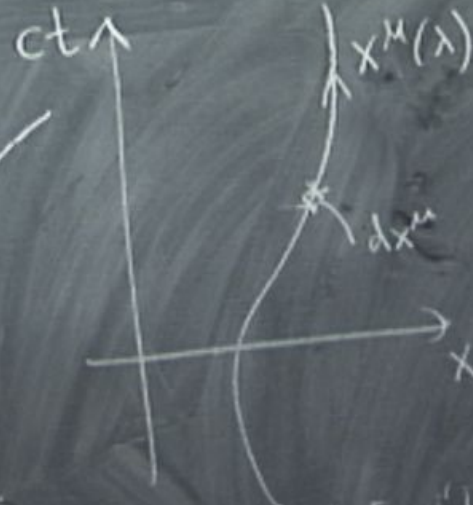
$$> 0 \text{ spacelike}$$

particles must follow timelike curves.

use to parameterise curve using proper time



Particles



parameterised curve

$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} < 0 \text{ timelike}$$

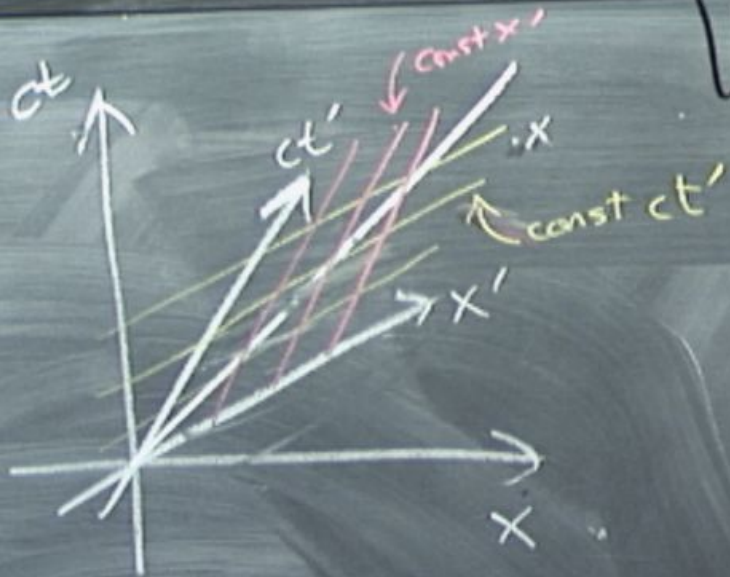
$$= 0 \text{ null}$$

$$> 0 \text{ spacelike}$$

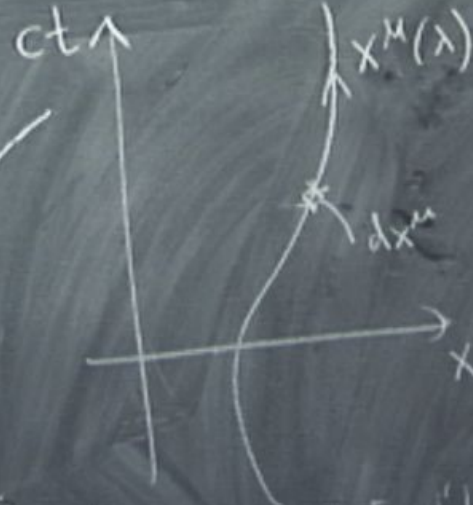
physical massive particles must follow timelike curves.

→ makes sense to parameterise curve using proper time

$$d\tau^2 = (cdt)^2 - dx^2$$



Particles



parameterised curve

$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} < 0 \text{ timelike}$$

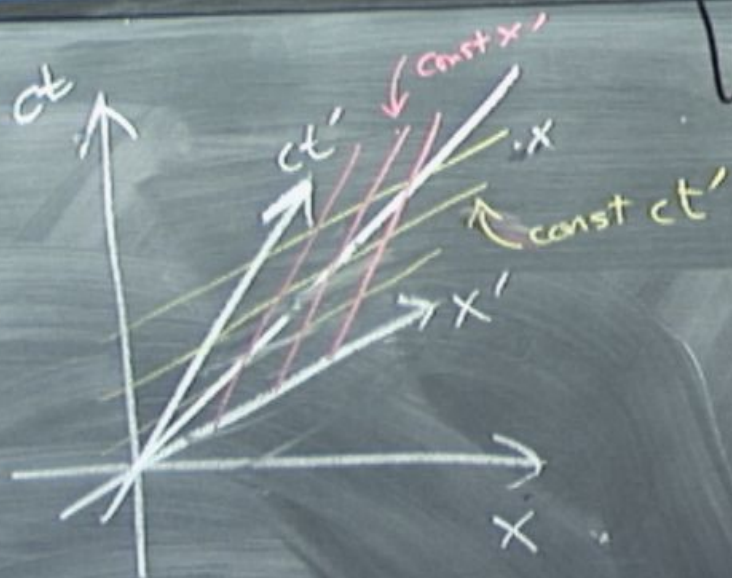
$$= 0 \text{ null}$$

$$> 0 \text{ spacelike}$$

physical massive particles must follow timelike curves.

→ makes sense to parameterise curve using proper time

$$d\tau^2 = (cdt)^2 - dx^2 \rightarrow d\tau = cdt \sqrt{1 - v^2/c^2}$$



Particles

ct



parameterised curve

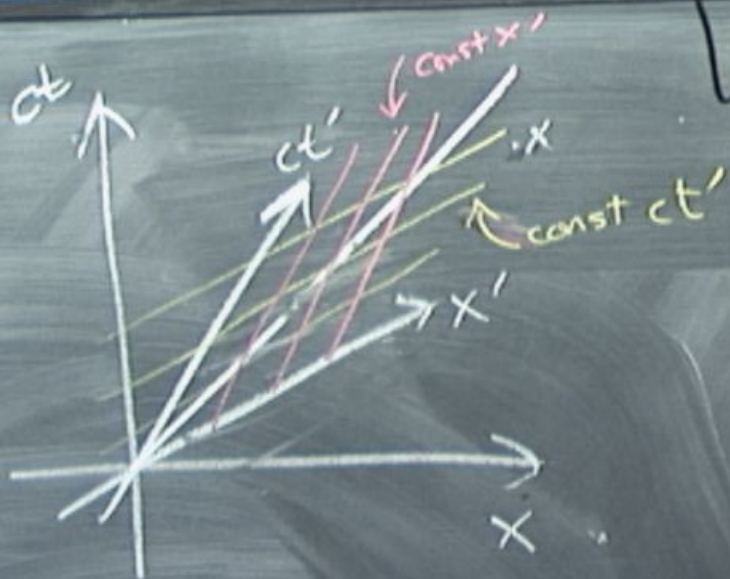
$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} < 0 \text{ timelike}$$

= 0 null

physical massive particles must follow timelike curves.

→ makes sense to parameterise curve using proper time

$$d\tau^2 = (cdt)^2 - dx^2 \rightarrow d\tau = cdt \sqrt{1 - v^2/c^2} \quad v = \frac{dx}{dt}$$



Particles

ct



parameterised curve

$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} < 0 \text{ timelike}$$

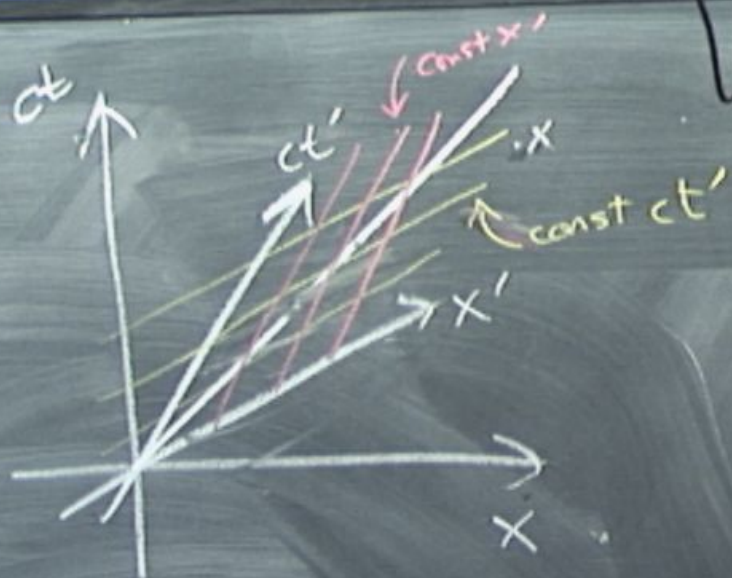
$$= 0 \text{ null}$$

$$> 0 \text{ spacelike}$$

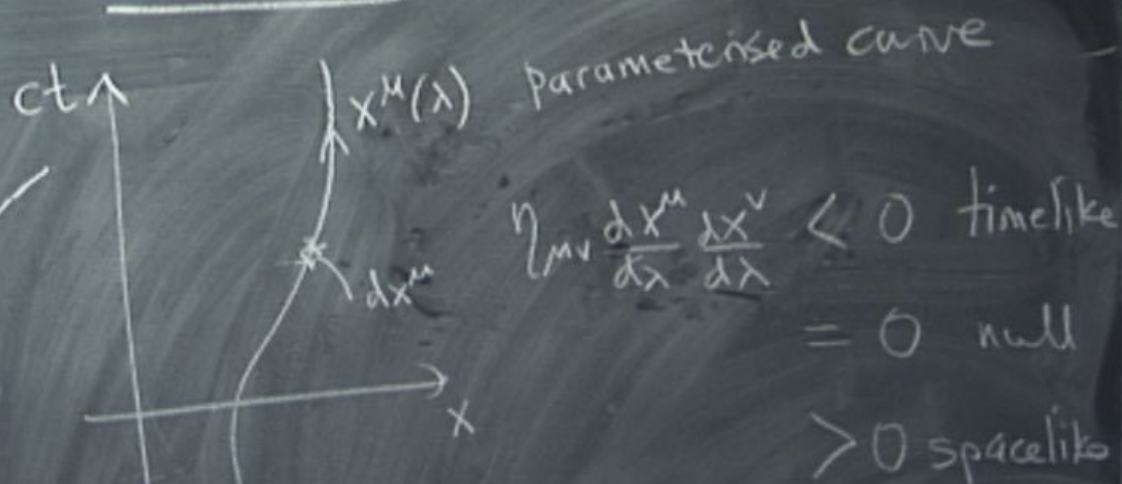
physical massive particles must follow timelike curves.

→ makes sense to parameterise curve using proper time

$$d\tau^2 = (cdt)^2 - dx^2 \rightarrow d\tau = cdt \sqrt{1 - v^2/c^2} \quad v = \frac{dx}{dt}$$



Particles



physical massive particles must follow timelike curves.

→ makes sense to parameterise curve using proper time

$$c^2 d\tau^2 = (cdt)^2 - dx^2 \rightarrow cd\tau = cdt \sqrt{1 - v^2/c^2} \quad v = \frac{dx}{dt}$$

PARTICLES

const ct'



parameterised curve

$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} < 0 \text{ timelike}$$

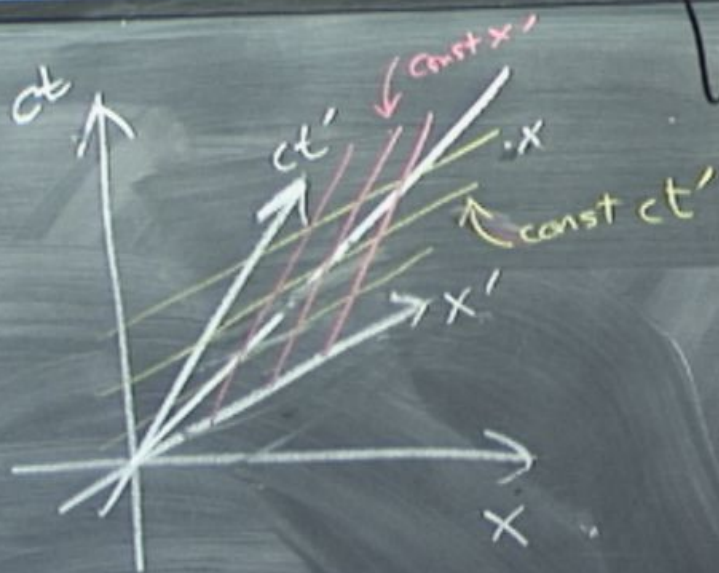
$$= 0 \text{ null}$$

$$> 0 \text{ spacelike}$$

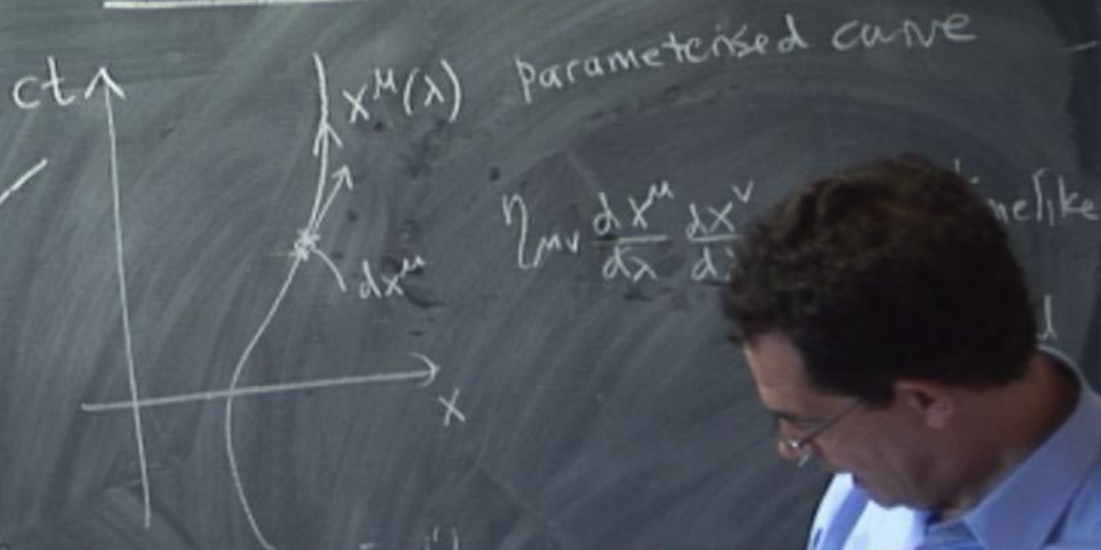
free particles must follow timelike curves.

is sense to parameterise curve using proper time

$$c^2 d\tau^2 = (cdt)^2 - dx^2 \rightarrow cd\tau = cdt \sqrt{1 - v^2/c^2} \quad \underline{v} = \frac{dx}{dt}$$



Particles

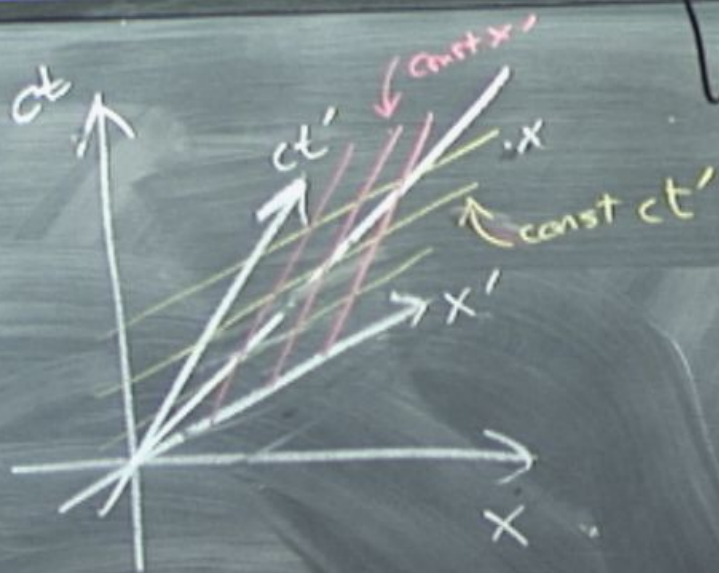


physical massive particles must follow timelike curves.

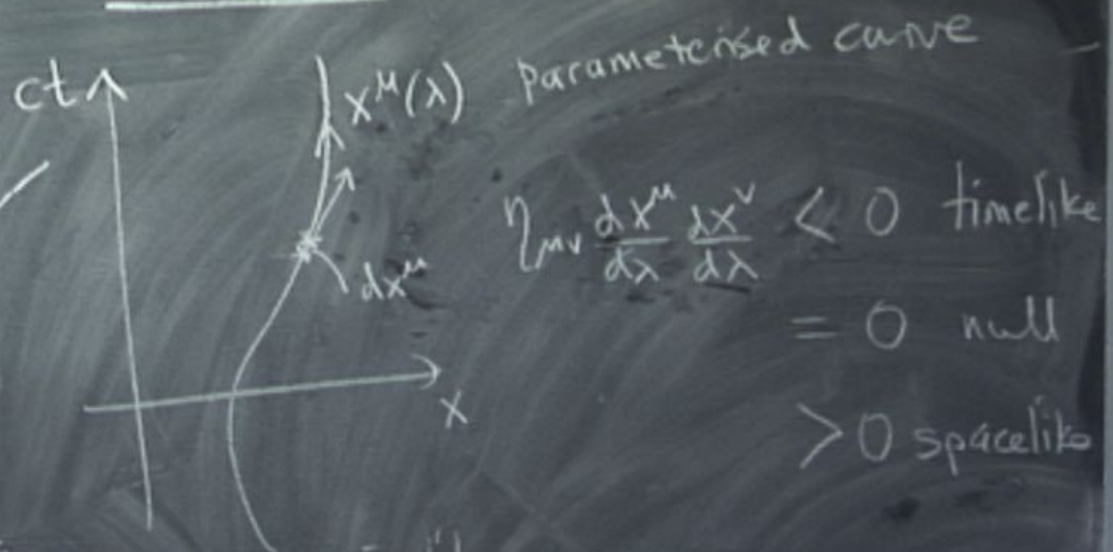
→ makes sense to parameterise curve using proper time

$$c^2 d\tau^2 = (cdt)^2 - dx^2 \rightarrow cd\tau = cdt \sqrt{1 - \frac{v^2}{c^2}}$$

$$\tau = \int dt \sqrt{1 - \frac{v^2}{c^2}}$$



Particles

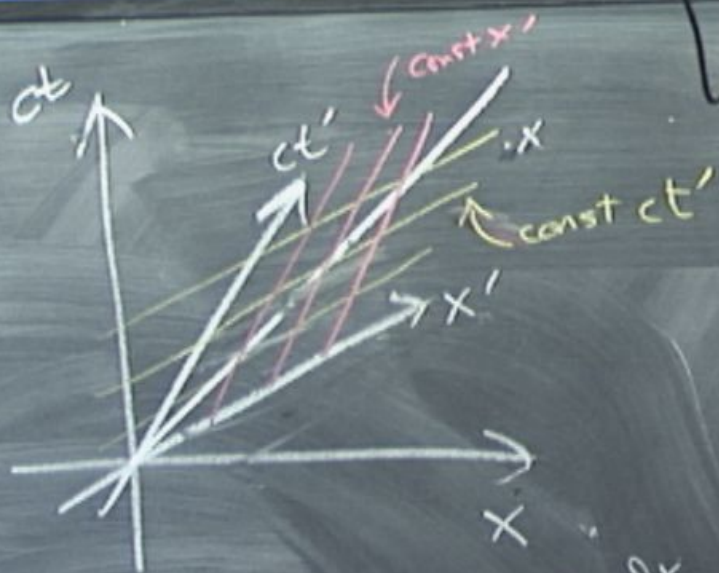


physical massive particles must follow timelike curves.

→ makes sense to parameterise curve using proper time

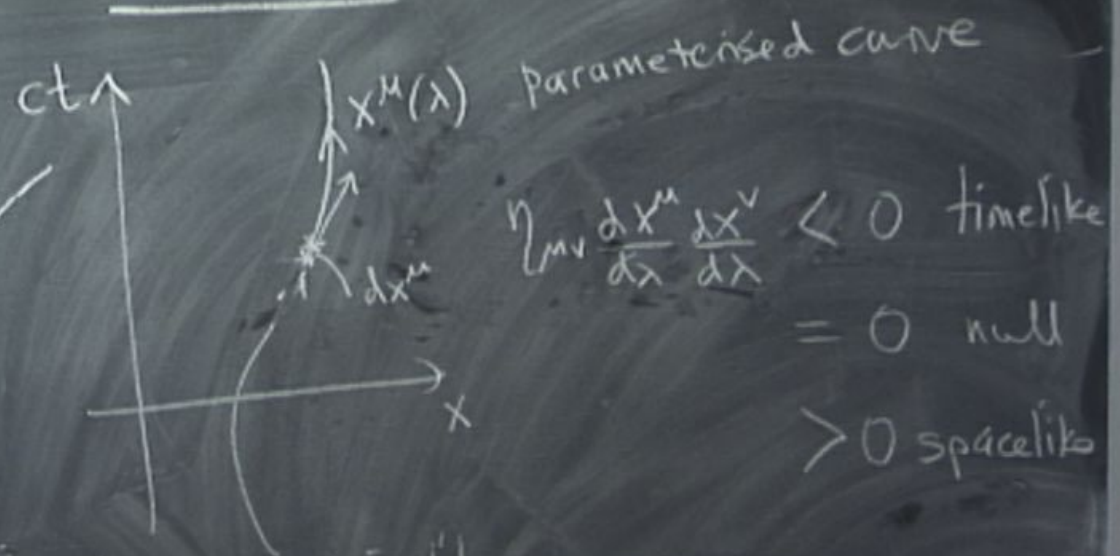
$$c^2 d\tau^2 = (cdt)^2 - dx^2 \rightarrow cd\tau = cdt \sqrt{1 - v^2/c^2} \quad v = \frac{dx}{dt}$$

$$\tau = \int dt \sqrt{1 - v^2/c^2}$$



$$v = \frac{dx}{dt}$$

Particles



$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} < 0 \text{ timelike}$$

$$= 0 \text{ null}$$

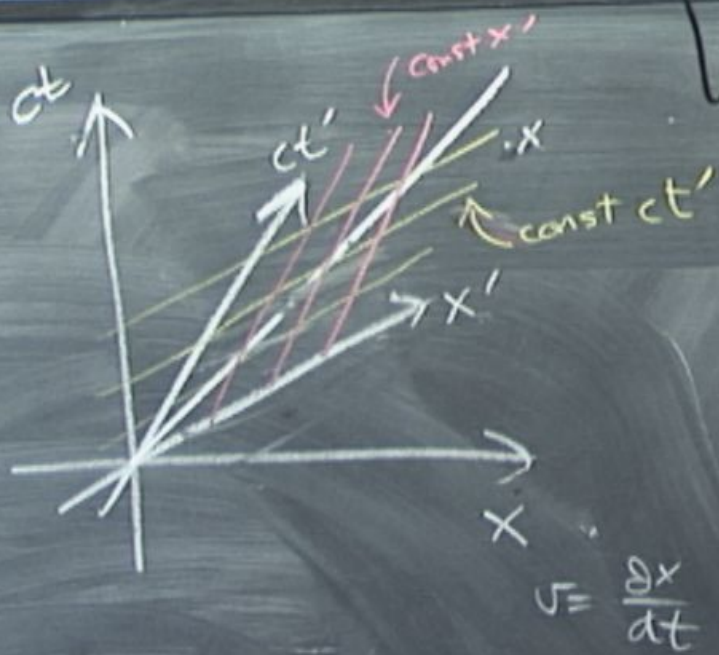
$$> 0 \text{ spacelike}$$

physical particles must follow timelike curves.

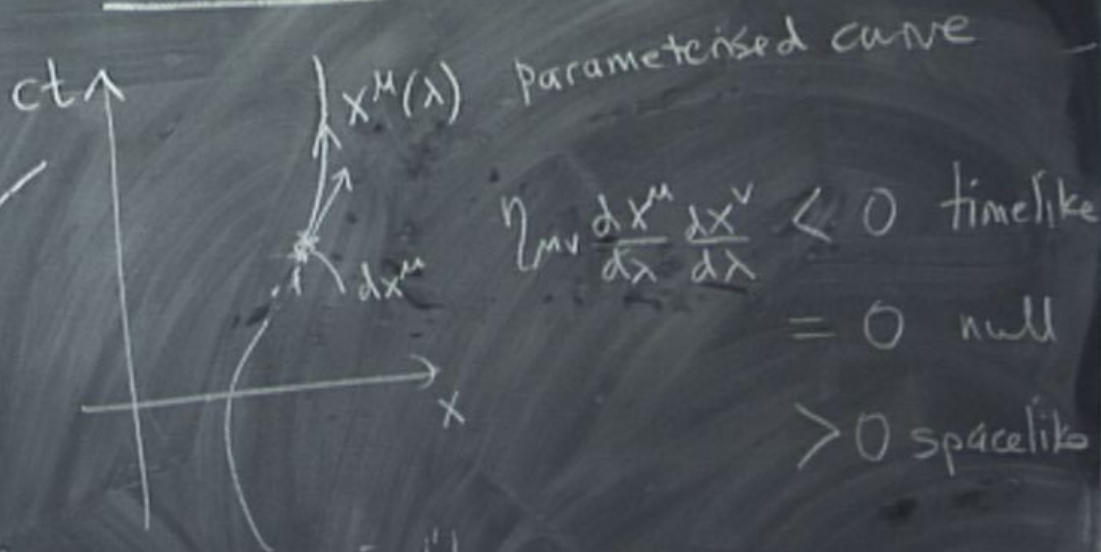
makes sense to parameterise curve using proper time

$$c^2 d\tau^2 = (cdt)^2 - dx^2 \rightarrow cd\tau = cdt \sqrt{1 - v^2/c^2} \quad v = \frac{dx}{dt}$$

$$\tau = \int dt \sqrt{1 - v^2/c^2}$$



Particles

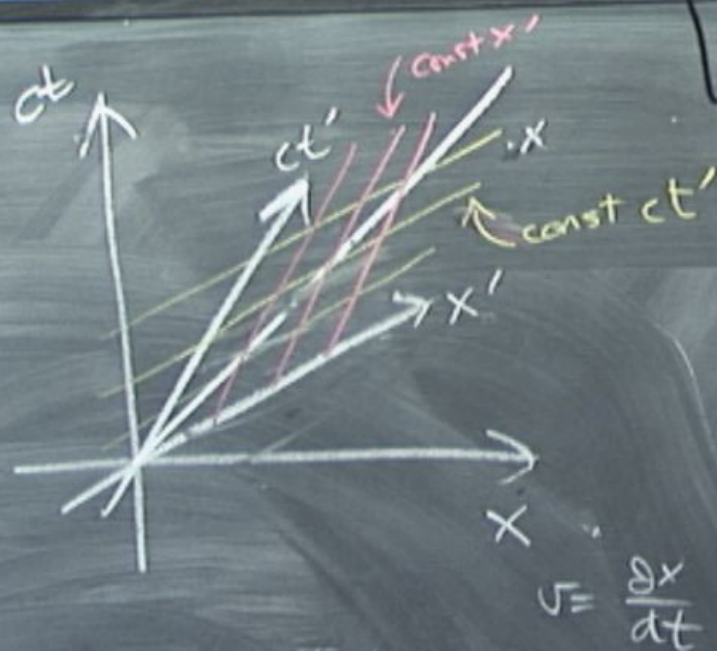


physical massive particles must follow timelike curves.

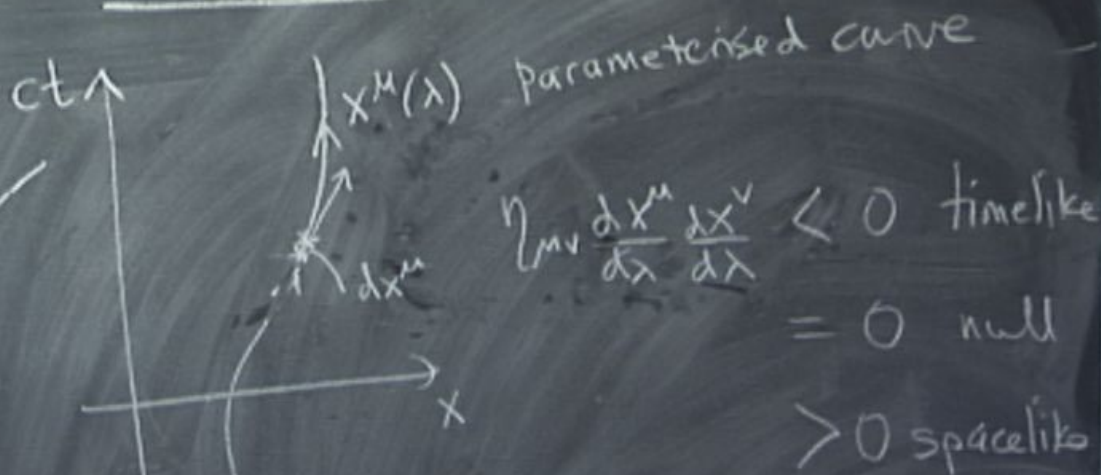
→ makes sense to parameterise curve using proper time

$$c^2 d\tau^2 = (cdt)^2 - dx^2 \rightarrow cd\tau = cdt \sqrt{1 - \frac{v^2}{c^2}} \quad v = \frac{dx}{dt}$$

$$\tau = \int dt \sqrt{1 - \frac{v^2}{c^2}}$$



Particles

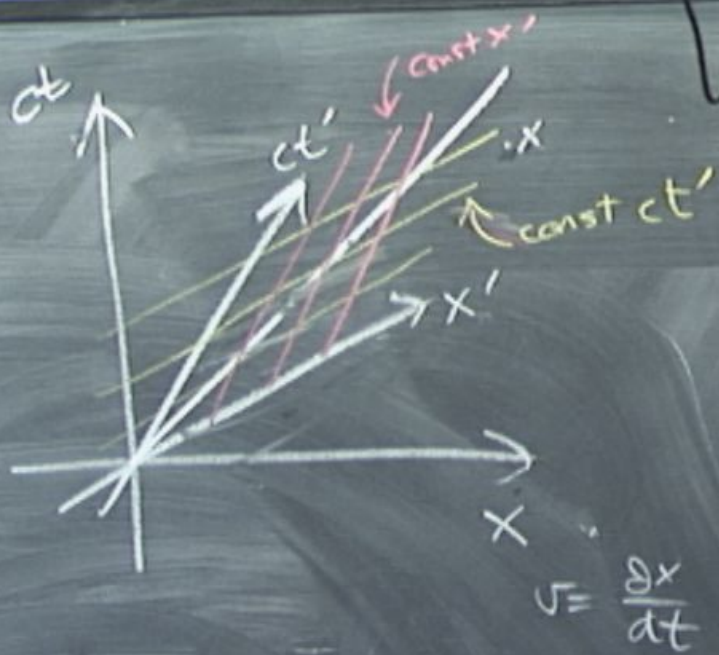


physical massive particles must follow timelike curves.

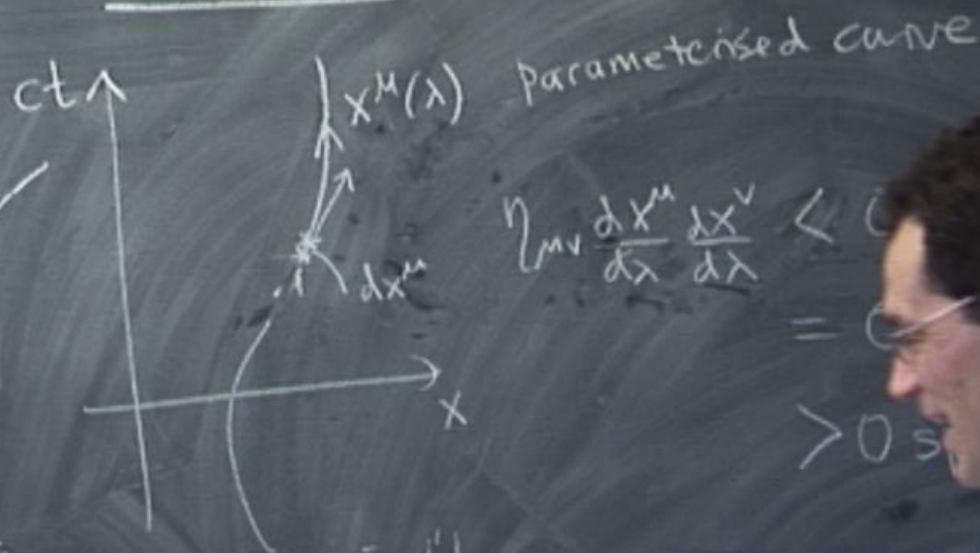
→ makes sense to parameterise curve using proper time

$$c^2 d\tau^2 = (cdt)^2 - dx^2 \rightarrow cd\tau = cdt \sqrt{1 - \frac{v^2}{c^2}} \quad v = \frac{dx}{dt}$$

$$\tau = \int dt \sqrt{1 - \frac{v^2}{c^2}} \Rightarrow \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$$



Particles



$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} < 0$$

$$= 0$$

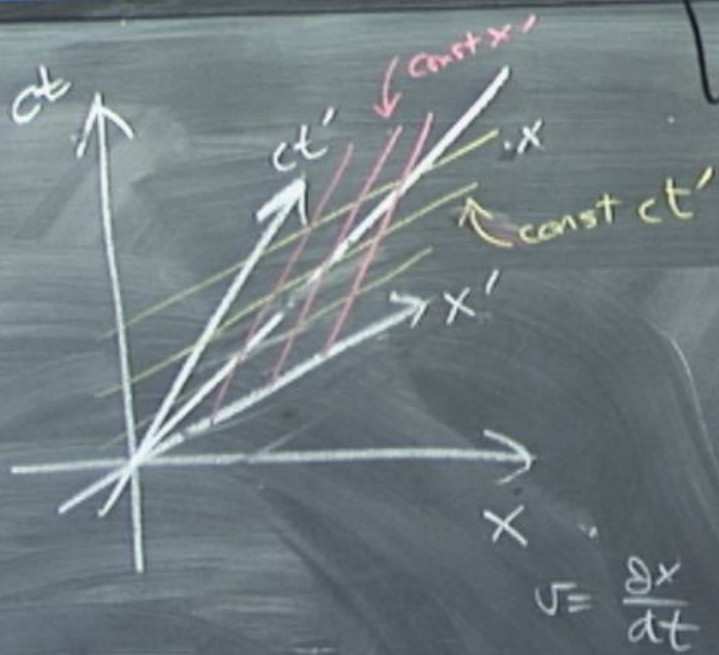
$$> 0$$

physical massive particles must follow timelike curves.

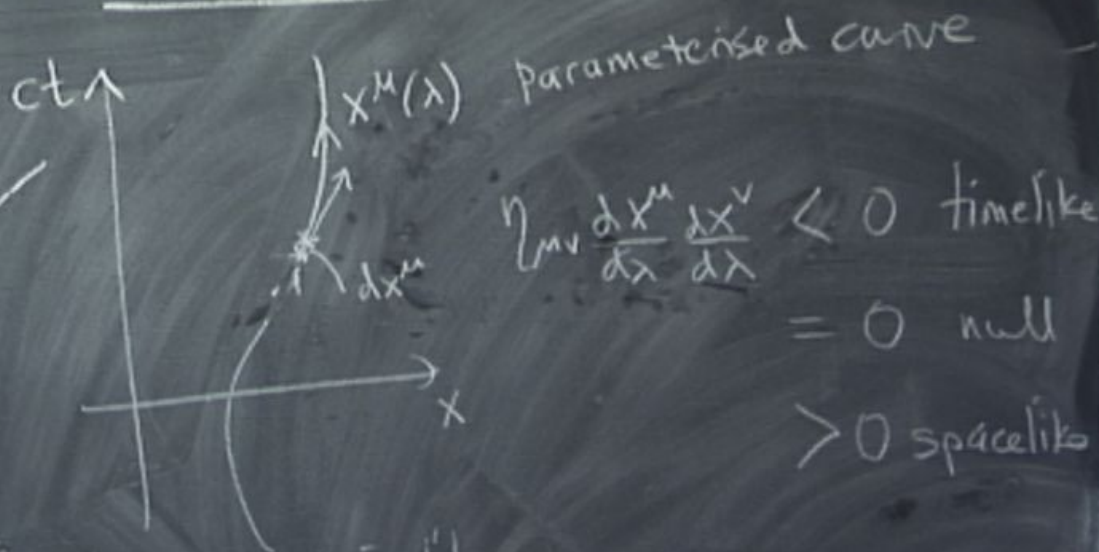
→ makes sense to parametrise curve using proper time

$$d\tau^2 = (cdt)^2 - dx^2 \rightarrow cd\tau = cdt \sqrt{1 - v^2/c^2}$$

$$\tau = \int dt \sqrt{1 - v^2/c^2} \Rightarrow \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$$



Particles



$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} < 0 \text{ timelike}$$

$$= 0 \text{ null}$$

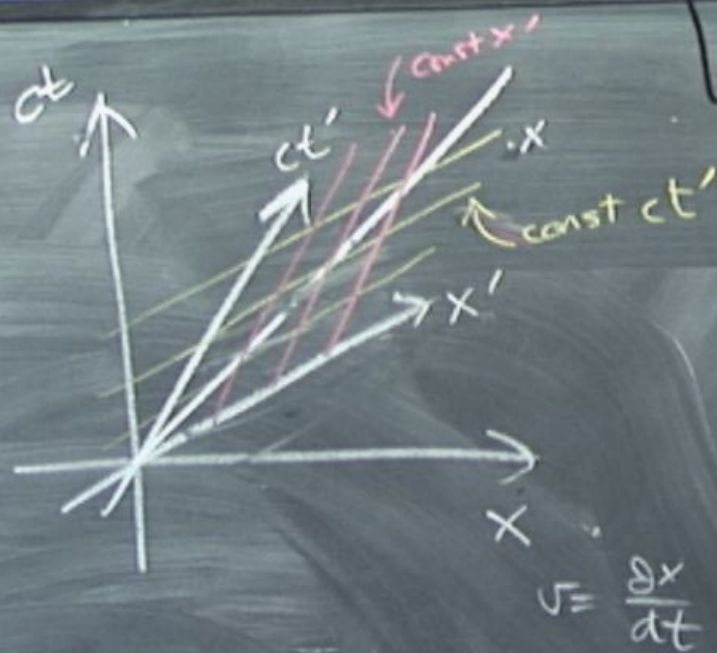
$$> 0 \text{ spacelike}$$

physical massive particles must follow timelike curves.

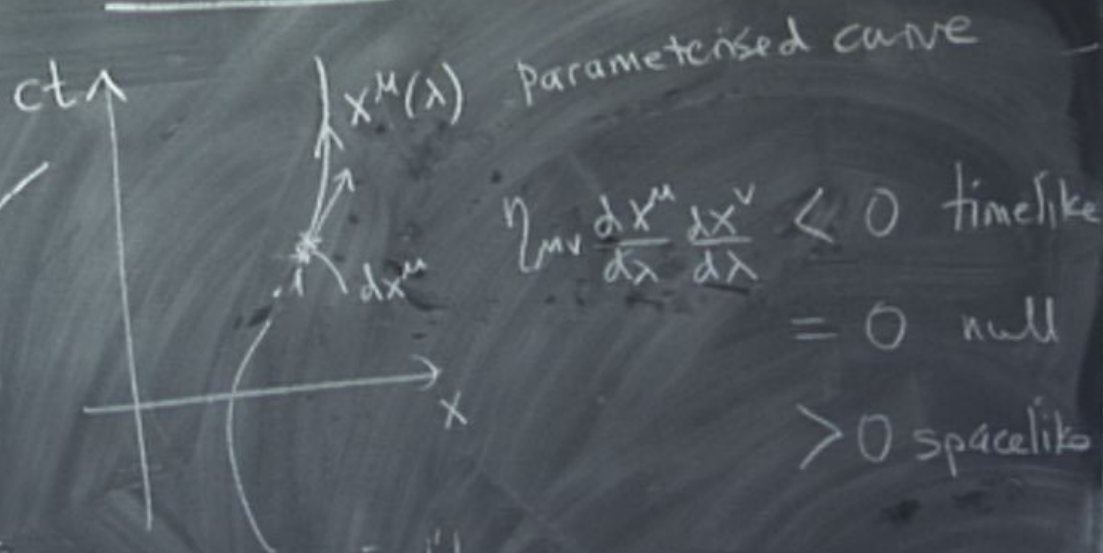
→ makes sense to parameterise curve using proper time

$$d\tau^2 = (cdt)^2 - dx^2 \rightarrow cd\tau = cdt \sqrt{1 - \frac{v^2}{c^2}} \quad v = \frac{dx}{dt}$$

$$\tau = \int dt \sqrt{1 - \frac{v^2}{c^2}} \Rightarrow \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$$



Particles



physical massive particles must follow timelike curves.

→ makes sense to parametrise curve using proper time

$$d\tau^2 = (cdt)^2 - dx^2 \rightarrow d\tau = cdt \sqrt{1 - v^2/c^2} \quad v = \frac{dx}{dt}$$

$$\tau = \int^t dt \sqrt{1 - v^2/c^2} \Rightarrow \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$$

$$v = \frac{dx}{dt}$$

physical massive particles

→ makes sense to p

$$d\tau^2 = (cdt)^2 - dx^2$$

$$\left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dx}{d\tau}\right)^2 = 1$$

$$\tau = \int dt$$

$$v = \frac{dx}{dt}$$

physical massive particles

→ makes sense to p

$$d\tau^2 = (cdt)^2$$

$$-\left(\frac{dt}{d\tau}\right)^2 + \left(\frac{dx}{d\tau}\right)^2 = -1$$

$$\tau = \int dt$$

$$v = \frac{dx}{dt}$$

physical massive particles

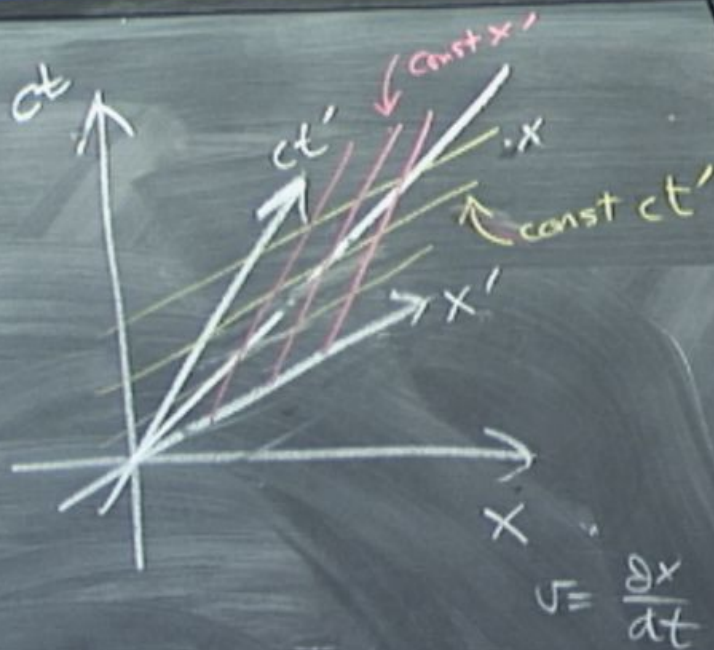
→ makes sense to p

$$d\tau^2 = (cdt)^2 - dx^2$$

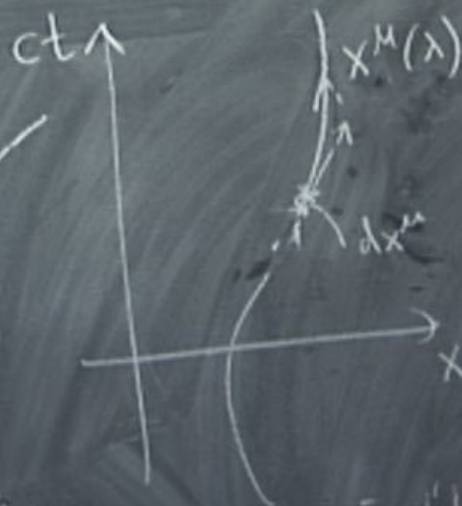
$$-\left(\frac{dt}{d\tau}\right)^2 + \left(\frac{dx}{d\tau}\right)^2 = -1$$

$$\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}$$

$$\tau = \int dt$$



Particles



parameterised curve

$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} < 0 \text{ timelike}$$

$$= 0 \text{ null}$$

$$> 0 \text{ spacelike}$$

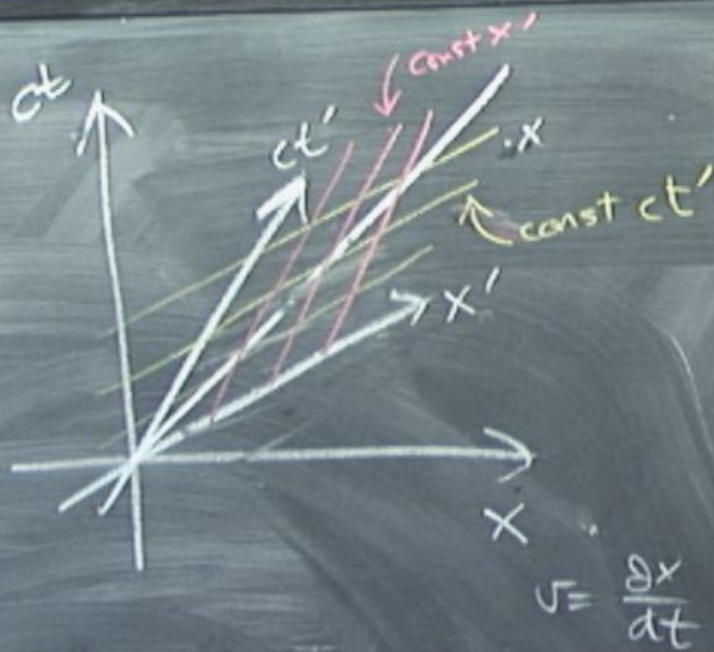
physical massive particles must follow timelike curves.

→ makes sense to parameterise curve using proper time

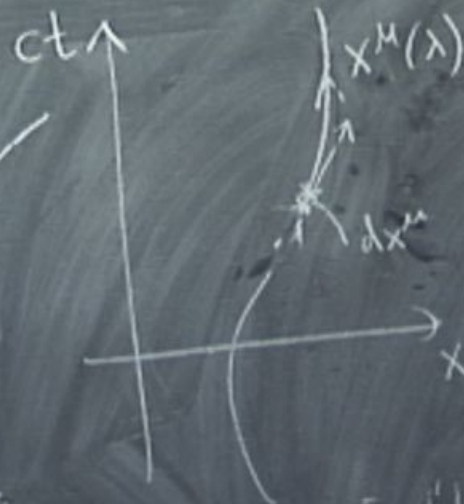
$$d\tau^2 = (cdt)^2 - dx^2 \rightarrow d\tau = cdt \sqrt{1 - v^2/c^2} \quad v = \frac{dx}{dt}$$

$$\underbrace{-\left(\frac{dt}{d\tau}\right)^2 + \left(\frac{dx}{d\tau}\right)^2}_{\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} = -1$$

$$\tau = \int dt \sqrt{1 - v^2/c^2} \Rightarrow \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$$



Particles



parameterised curve

$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} \begin{cases} < 0 & \text{timelike} \\ = 0 & \text{null} \\ > 0 & \text{spacelike} \end{cases}$$

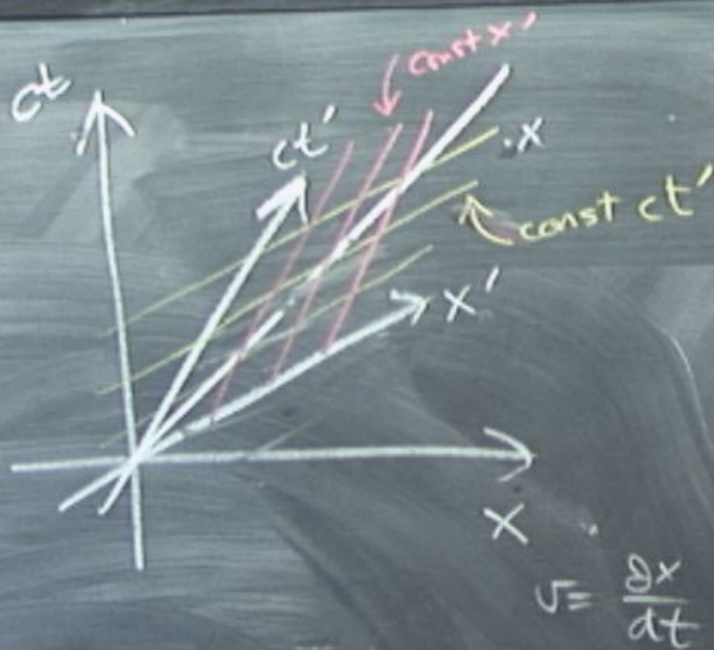
physical massive particles must follow timelike curves.

→ makes sense to parameterise curve using proper time

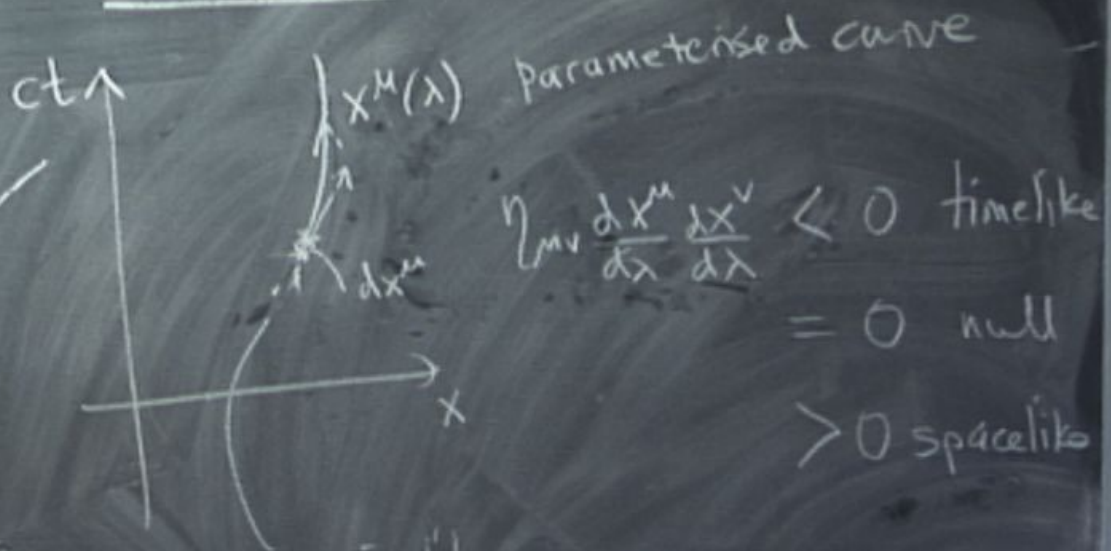
$$d\tau^2 = (cdt)^2 - dx^2 \rightarrow d\tau = \frac{1}{c} \sqrt{c^2 dt^2 - dx^2} \quad \underline{v = \frac{dx}{dt}}$$

$$\underbrace{-\left(\frac{dt}{d\tau}\right)^2 + \left(\frac{dx}{d\tau}\right)^2}_{\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} = -1$$

$$\tau = \int dt \sqrt{1 - \frac{v^2}{c^2}} \Rightarrow \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$$



Particles



parameterised curve

$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} < 0 \text{ timelike}$$

$$= 0 \text{ null}$$

$$> 0 \text{ spacelike}$$

physical massive particles must follow timelike curves.

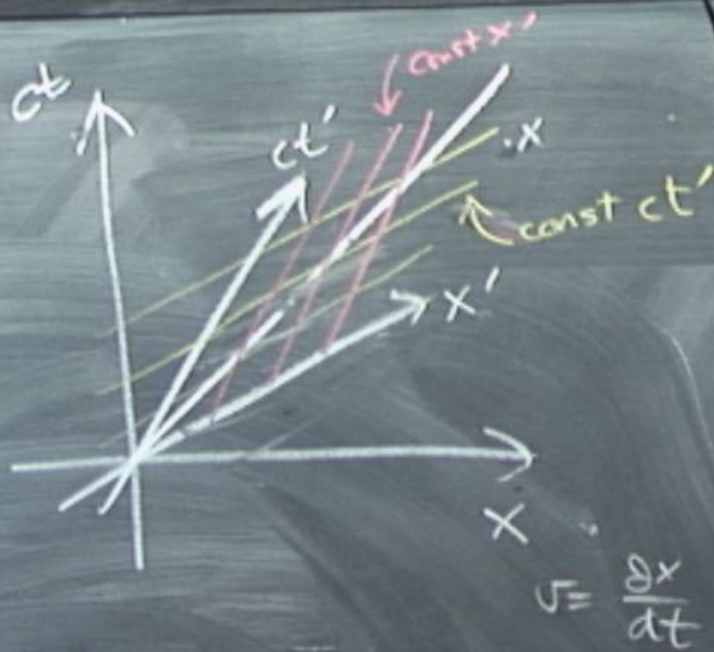
→ makes sense to parameterise curve using proper time

$$d\tau^2 = (cdt)^2 - dx^2 \rightarrow d\tau = cdt \sqrt{1 - v^2/c^2} \quad v = \frac{dx}{dt}$$

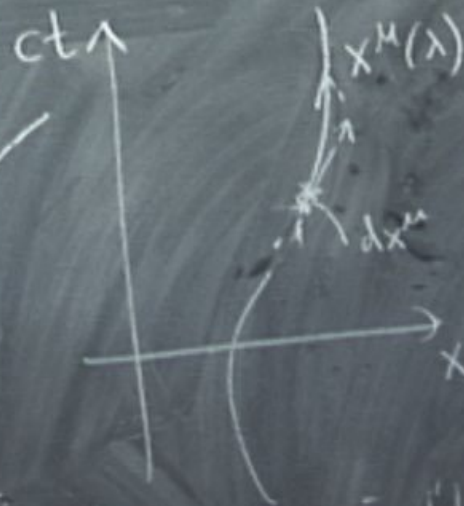
$$\left(\frac{dt}{d\tau} \right)^2 + \left(\frac{dx}{d\tau} \right)^2 = 1$$

$$\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$$

$$\tau = \int dt \sqrt{1 - v^2/c^2} \Rightarrow \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$$



Particles



parameterised curve

$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} < 0 \text{ timelike}$$

$$= 0 \text{ null}$$

$$> 0 \text{ spacelike}$$

physical massive particles must follow timelike curves.

→ makes sense to parameterise curve using proper time

$$d\tau^2 = (cdt)^2 - dx^2 \rightarrow d\tau = \frac{1}{c} \sqrt{c^2 dt^2 - dx^2} \quad \underline{v} = \frac{dx}{dt}$$

$$-\left(\frac{dt}{d\tau}\right)^2 + \left(\frac{dx}{d\tau}\right)^2 = -1$$

$$\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$$

$$\tau = \int dt \sqrt{1 - v^2/c^2} \Rightarrow \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$$

massive particles

$$u^M = \frac{dx^M}{d\tau}$$

4-velocity

massive particles

$$u^M = \frac{dx^M}{d\tau}$$

4-velocity
(dimensionless)

massive particles

$$u^{\mu} = \frac{dx^{\mu}}{d\tau}$$

4-velocity
(dimensionless)

$$\eta_{\mu\nu} u^{\mu} u^{\nu} = -1$$

massive particles

$$u^\mu = \frac{dx^\mu}{d\tau}$$

4-velocity
(dimensionless)

$$\eta_{\mu\nu} u^\mu u^\nu = -1$$

4-momentum

massive particles

$$u^{\mu} = \frac{dx^{\mu}}{d\tau}$$

4-velocity
(dimensionless)

$$\eta_{\mu\nu} u^{\mu} u^{\nu} = -1$$

4-momentum

$$p^{\mu} = m_0 c$$

↓ rest mass

massive particles

$$u^{\mu} = \frac{dx^{\mu}}{d\tau}$$

4-velocity
(dimensionless)

$$\eta_{\mu\nu} u^{\mu} u^{\nu} = -1$$

4-momentum

$$p^{\mu} = \overset{\downarrow \text{rest mass}}{m_0} c u^{\mu}$$

massive particles

$$u^\mu = \frac{dx^\mu}{d\tau}$$

4-velocity
(dimensionless)

$$\eta_{\mu\nu} u^\mu u^\nu = -1$$

rest mass

$$p^\mu = m_0 c u^\mu$$

$\Rightarrow$

$$\eta_{\mu\nu} p^\mu p^\nu$$

$$= -p_0^2 + \mathbf{p}^2 =$$

massive particles

$$u^\mu = \frac{dx^\mu}{d\tau}$$

4-velocity
(dimensionless)

$$\eta_{\mu\nu} u^\mu u^\nu = -1$$

4-momentum

$$p^\mu = \overset{\substack{\downarrow \text{rest mass}}}{m_0} c u^\mu$$

$$\begin{aligned} \Rightarrow p^2 &= \eta_{\mu\nu} p^\mu p^\nu \\ &= -\overset{2}{p_0^2} + \underline{p^2} = \end{aligned}$$

massive particles

$$u^{\mu} = \frac{dx^{\mu}}{d\tau}$$

4-velocity
(dimensionless)

$$\eta_{\mu\nu} u^{\mu} u^{\nu} = -1$$

momentum

$$p^{\mu} = \underset{\substack{\downarrow \text{rest mass}}}{m_0} c u^{\mu}$$

$$\begin{aligned} \Rightarrow p^2 &= \eta_{\mu\nu} p^{\mu} p^{\nu} \\ &= -\underset{^2}{p^0} + \underline{p^2} = m_0^2 c^2 \end{aligned}$$

$$\Rightarrow p_0 = \sqrt{m_0^2 c^2 + \underline{p^2}}$$

massive particles

$$u^\mu = \frac{dx^\mu}{d\tau}$$

4-velocity
(dimensionless)

$$\eta_{\mu\nu} u^\mu u^\nu = -1$$

rest mass

4-momentum

$$p^\mu = m_0 c u^\mu$$

$$\eta_{\mu\nu} p^\mu p^\nu$$

$$= -p_0^2 + \underline{p^2} = m_0^2 c^2$$

$$\Rightarrow p_0 = \sqrt{m_0^2 c^2 + \underline{p^2}}$$

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$$\underline{v} = \frac{dx}{dt}$$

$$\underline{v} = \frac{dx}{dt} = \frac{P}{P_0}$$

massive particles

$$u^\mu = \frac{dx^\mu}{d\tau}$$

4-velocity
(dimensionless)

$$\underline{v} = \frac{dx}{dt}$$

4-moment

$$\eta_{\mu\nu} u^\mu u^\nu = -1$$

rest mass

$$= m_0 c u^\mu = m_0 c \left(\frac{dt}{d\tau}, \frac{dx}{d\tau} \right)$$

p^2

$$\eta_{\mu\nu} p^\mu p^\nu$$

$$-p^0 + p^2 = -m_0^2 c^2$$

$$\boxed{m_0^2 c^2 + p^2}$$

re-pointing u^μ

massive particles

$$u^\mu = \frac{dx^\mu}{d\tau}$$

4-velocity
(dimensionless)

$$\underline{v} = \frac{dx}{dt}$$

$$\eta_{\mu\nu} u^\mu u^\nu = -1$$

4-momentum

$$p^\mu = \overset{\substack{\downarrow \text{rest mass}}}{m_0} c u^\mu = m_0 c \left(\frac{dt}{d\tau}, \frac{dx}{d\tau} \right)$$

$$\begin{aligned} \Rightarrow p^2 &= \eta_{\mu\nu} p^\mu p^\nu \\ &= -p_0^2 + \underline{p}^2 = -M_0^2 c^2 \end{aligned}$$

$$\Rightarrow p_0 = \sqrt{M_0^2 c^2 + \underline{p}^2}$$

(for future-pointing u^μ)

velocity
(dimensionless)

$$\underline{v} = \frac{dx}{dt} = \frac{P}{p_0}$$

$$= m_0 c \left(c \frac{dt}{d\tau} \frac{dx}{d\tau} \right)$$

$$M_0^2 c^2$$

$$u^v = -1$$

rest mass

$$m_0 c u^\mu = m_0 c \left(c \frac{dt}{d\tau}, \frac{dx}{d\tau} \right)$$

$$g_{\mu\nu} p^\mu p^\nu$$

$$\underline{v} = \frac{dx}{dt} = \frac{Pc}{P^0}$$

$$\boxed{\frac{v}{c} = \frac{P}{\sqrt{m_0^2 c^2 + P^2}}}$$

massive particles

$$x^{\mu} = (ct, \underline{x})$$

$$u^{\mu} = \frac{dx^{\mu}}{d\tau}$$

4-velocity
(dimensionless)

$$\eta_{\mu\nu} u^{\mu} u^{\nu} = -1$$

4-momentum

$$p^{\mu} = \overset{\substack{\downarrow \text{rest mass}}}{m_0} c u^{\mu}$$

$$= m_0 c \left(c \frac{dt}{d\tau}, \frac{d\underline{x}}{d\tau} \right)$$

$$\Rightarrow p^2 = \eta_{\mu\nu} p^{\mu} p^{\nu}$$

$$= -p_0^2 + \underline{p}^2 = -m_0^2 c^2$$

$$\Rightarrow p_0 = \sqrt{m_0^2 c^2 + \underline{p}^2}$$

(for future-pointing u^{μ})

$$\underline{v} = \frac{dx}{dt} = \frac{\underline{P}c}{p_0}$$

$$\boxed{\frac{\underline{v}}{c} = \frac{\underline{P}}{\sqrt{m_0^2 c^2 + \underline{P}^2}}}$$

$$\frac{|\underline{v}|}{c} < 1$$

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$|\underline{u}| \rightarrow c$ requires $|\underline{P}| \rightarrow \infty$

$$\underline{v} = \frac{d\underline{x}}{dt} = \frac{\underline{P}c}{p^0}$$

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$$\underline{F} = \frac{d\underline{P}}{dt}$$

$$\underline{v} = \frac{d\underline{x}}{dt} = \frac{\underline{P}c}{p^0}$$

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def of Force $\underline{F} = \frac{d\underline{P}}{dt}$

$$\underline{v} = \frac{d\underline{x}}{dt} = \frac{\underline{P}c}{p^0}$$

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$$\frac{|\underline{v}|}{c} < 1$$

$|\underline{v}| \rightarrow c$ requires $|\underline{P}| \rightarrow \infty$

def of Force $\underline{F} = \frac{d\underline{P}}{dt}$

$$\text{Work} = \underline{F} \cdot \underline{v}$$

$$= \frac{d}{dt} (\gamma m_0 c^2)$$

$$\underline{v} = \frac{d\underline{x}}{dt} = \frac{\underline{P}c}{P^0}$$

$$\boxed{\frac{\underline{v}}{c} = \frac{\underline{P}}{\sqrt{m_0^2 c^2 + \underline{P}^2}}}$$

$$\frac{|\underline{v}|}{c} < 1$$

$|\underline{v}| \rightarrow c$ requires $|\underline{P}| \rightarrow \infty$

def of Force $\underline{F} = \frac{d\underline{P}}{dt}$

$$\begin{aligned} \frac{d(\text{Work})}{dt} &= \underline{F} \cdot \underline{v} \\ &= \frac{d}{dt} (\gamma m_0 c^2) \end{aligned}$$

$$\underline{v} = \frac{d\underline{x}}{dt} = \frac{\underline{P}c}{P^0}$$

$$\boxed{\frac{\underline{v}}{c} = \frac{\underline{P}}{\sqrt{m_0^2 c^2 + \underline{P}^2}}}$$

$$\frac{|\underline{v}|}{c} < 1$$

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def of Force

$$\underline{F} = \frac{d\underline{P}}{dt}$$

$$\begin{aligned} \frac{d(\text{Work})}{dt} &= \underline{F} \cdot \underline{v} \\ &= \frac{d}{dt} (\underbrace{\gamma}_{\text{}} m_0 c^2) \end{aligned}$$

$$\underline{v} = \frac{d\underline{x}}{dt} = \frac{\underline{p}c}{p^0}$$

$$\frac{\underline{v}}{c} = \frac{\underline{p}}{\sqrt{m_0^2 c^2 + \underline{p}^2}}$$

$$\frac{|\underline{v}|}{c} < 1$$

$$|\underline{v}| \rightarrow c \text{ requires } |\underline{p}| \rightarrow \infty$$

def of Force

$$\underline{F} = \frac{d\underline{p}}{dt}$$

$$\frac{d(\text{Work})}{dt} = \underline{F} \cdot \underline{v}$$

$$= \frac{d}{dt} (\underbrace{\gamma m_0 c^2})$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\underline{p} = \gamma m_0 \underline{v}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\underline{v} = \frac{d\underline{x}}{dt} = \frac{\underline{P}c}{p_0}$$

$$\boxed{\frac{\underline{v}}{c} = \frac{\underline{P}}{\sqrt{m_0^2 c^2 + \underline{P}^2}}}$$

$$\frac{|\underline{v}|}{c} < 1$$

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def of Force $\underline{F} = \frac{d\underline{P}}{dt}$

$$\frac{d(\text{Work})}{dt} = \underline{F} \cdot \underline{v}$$

$$= \frac{d}{dt} (\underbrace{\gamma m_0 c^2})$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\underline{P} = \gamma m_0 \underline{v}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\underline{v} = \frac{d\underline{x}}{dt} = \frac{\underline{p}c}{p^0}$$

$$\frac{\underline{v}}{c} = \frac{\underline{p}}{\sqrt{m_0^2 c^2 + \underline{p}^2}}$$

$$\frac{|\underline{v}|}{c} < 1$$

$$|\underline{v}| \rightarrow c \text{ requires } |\underline{p}| \rightarrow \infty$$

def of Force

$$\underline{F} = \frac{d\underline{p}}{dt}$$

$$\frac{d(\text{Work})}{dt} = \underline{F} \cdot \underline{v}$$

$$= \frac{d}{dt} (\underbrace{\gamma m_0 c^2})$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\underline{p} = \gamma m_0 \underline{v}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\underline{v} = \frac{d\underline{x}}{dt} = \frac{\underline{P}}{P_0} c$$

$$\frac{\underline{v}}{c} = \frac{\underline{P}}{\sqrt{m_0^2 c^2 + \underline{P}^2}}$$

$$\frac{|\underline{v}|}{c} < 1$$

$|\underline{v}| \rightarrow c$ requires $|\underline{P}| \rightarrow \infty$

def of Force

$$\underline{F} = \frac{d\underline{P}}{dt}$$

$$\frac{d(\text{Work})}{dt} = \underline{F} \cdot \underline{v} = \frac{d}{dt} (\gamma m_0 c^2)$$

$$\underline{P} = \gamma m_0 \underline{v}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \sqrt{1 + \frac{\underline{P}^2}{m_0^2 c^2}}$$

$$\Rightarrow p^0 = \gamma m_0 c$$

What is p^0 ?

p = momentum

$$\Rightarrow p^0 = \gamma m_0 c$$

What is p^0 ?

p = momentum

$p^0 c$ = energy.

$$\Rightarrow p^0 = \gamma m_0 c$$

What is p^0 ?

p = momentum

$p^0 c$ = energy.

$$E = mc^2$$

$$m = \gamma m_0$$

Einstein 1905

Einstein 1905

before: E_0

Einstein 1905

before: E_0

after

E_1

Einstein 1905

before: E_0

after

E_1 ↗

Einstein 1905

before: E_0

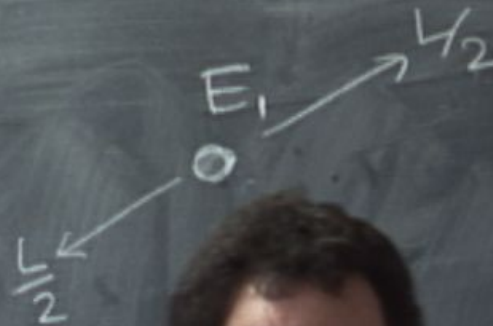
after



Einstein 1905

before: E_0

after



Einstein 1905

before: E_0

after



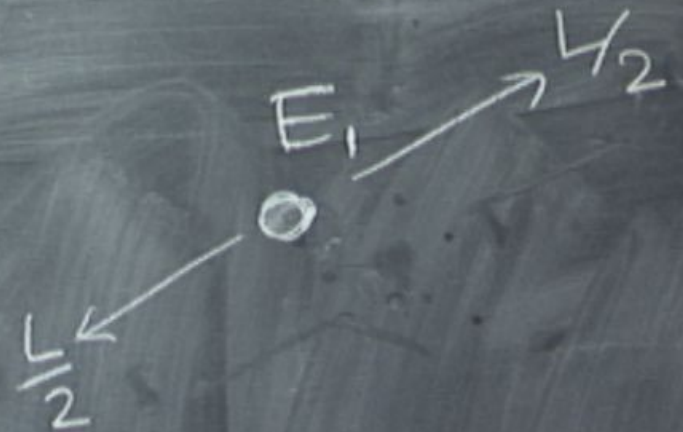
energy
conservation

$$E_0 = E_1 + \frac{L}{2} + \frac{L}{2}$$

Einstein 1905

before: E_0

after



energy
conservation

$$E_0 = E_1 + \frac{L}{2} + \frac{L}{2}$$

under a moving frame.

Einstein 1905

before: E_0

after



energy
conservation

$$E_0 = E_1 + \frac{L}{2} + \frac{L}{2}$$

now consider a moving frame.

Einstein 1905

before: E_0

after



energy
conservation

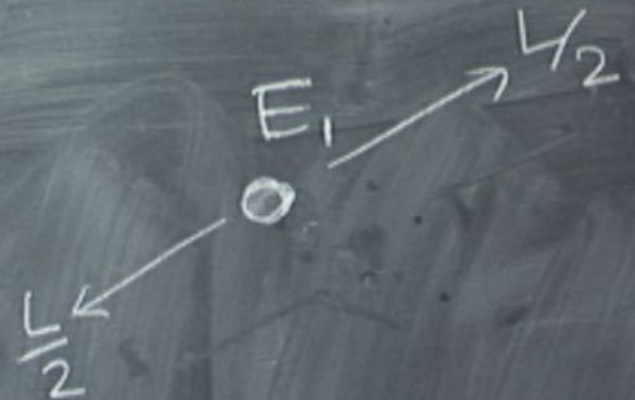
$$E_0 = E_1 + \frac{L}{2} + \frac{L}{2}$$

now consider a moving frame, velocity v along x axis.

Einstein 1905

before: E_0

after



energy
conservation

$$E_0 = E_1 + \frac{L}{2} + \frac{L}{2}$$

now consider a moving frame, velocity v along x axis.

before

H_0

Einstein 1905

before: E_0

after



energy
conservation

$$E_0 = E_1 + \frac{L}{2} + \frac{L}{2}$$

now consider a moving frame, velocity v along x axis.

before

after

$$H_0 = H_1 + \frac{\frac{1}{2}L(1 - \frac{v}{c} \cos \phi)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Einstein 1905

before: E_0

after



energy
conservation

$$E_0 = E_1 + \frac{L}{2} + \frac{L}{2}$$

now consider a moving frame, velocity v along x axis.

before

after

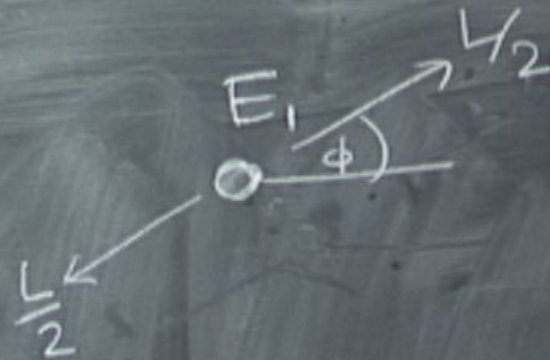
$$H_0 = H_1 + \frac{\frac{1}{2}L(1 - \frac{v}{c}\cos\phi)}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{\frac{1}{2}L(1 + \frac{v}{c}\cos\phi)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Maxwell

Einstein 1905

before: E_0

after



energy
conservation

$$E_0 = E_1 + \frac{L}{2} + \frac{L}{2}$$

now consider a moving frame, velocity v along x axis.

before

after

$$H_0 = H_1 + \frac{\frac{1}{2}L(1 - \frac{v}{c} \cos \phi)}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{\frac{1}{2}L(1 + \frac{v}{c} \cos \phi)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= H_1 + \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Maxwell

$$\Rightarrow H_0 - E_0$$



Kinetic
energy

$\Rightarrow$

$$H_0 - E_0$$

Kinetic energy

$$\Rightarrow \underbrace{H_0 - E_0}_{K_0} = \underbrace{H_1 - E_1}_{K_1} + L \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

Kinetic
energy

$$\Rightarrow \underbrace{H_0 - E_0}_{K_0} = \underbrace{H_1 - E_1}_{K_1} + L \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

$$\Rightarrow K_0 - K_1 = L \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) \approx \quad v \ll c$$

Kinetic
energy

$$\Rightarrow \underbrace{H_0 - E_0}_{K_0} = \underbrace{H_1 - E_1}_{K_1} + L \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

$$\Rightarrow K_0 - K_1 = L \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) \quad v \ll c$$
$$\approx \frac{L v^2}{2 c^2}$$

Kinetic energy

$$\Rightarrow \underbrace{H_0 - E_0}_{K_0} = \underbrace{H_1 - E_1}_{K_1} + L \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

$$v \ll c$$

$$K_0 - K_1 = L \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) \approx \frac{L v^2}{2 c^2}$$

$$= \frac{1}{2} \left(\frac{L}{c^2} \right) v^2$$

Kinetic energy

$$\Rightarrow \underbrace{H_0 - E_0}_{K_0} = \underbrace{H_1 - E_1}_{K_1} + L \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

$$\Rightarrow K_0 - K_1 = L \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) \quad v \ll 1 \quad \approx \frac{L v^2}{2 c^2}$$

$$= \frac{1}{2} \underbrace{\left(\frac{L}{c^2} \right)}_m v^2$$

$\Rightarrow$ mass of particle changes

by $\frac{L}{c^2}$ due to the increase in

Kinetic energy

$$\Rightarrow \underbrace{H_0 - E_0}_{K_0} = \underbrace{H_1 - E_1}_{K_1} + L \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

$$\Rightarrow K_0 - K_1 = L \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) \quad v \ll c$$

$$\approx \frac{L v^2}{2 c^2}$$

$$= \frac{1}{2} \left(\underbrace{\frac{L}{c^2}}_m \right) v^2$$

$\Rightarrow$ mass of particle changes by $\frac{L}{c^2}$ due to the emission of energy

Maxwell eqⁿs have a conserved energy
energy density ρ

Maxwell eqⁿs have a conserved energy
energy density $\rho = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \frac{B^2}{\mu_0}$

Maxwell eqⁿs have a conserved energy

energy density $\rho = \frac{1}{2} \epsilon_0 \underline{E}^2 + \frac{1}{2} \frac{B^2}{\mu_0}$

flux
(Poynting) $\underline{S} = \frac{1}{\mu_0} (\underline{E} \wedge \underline{B})$

Maxwell eqⁿs have a conserved energy

energy density $\rho = \frac{1}{2} \epsilon_0 \underline{E}^2 + \frac{1}{2} \frac{\underline{B}^2}{\mu_0}$

flux
(Poynting) $\underline{S} = \frac{1}{\mu_0} (\underline{E} \wedge \underline{B})$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \underline{S} = 0$$

Maxwell eqⁿs have a conserved energy

energy density $\rho = \frac{1}{2} \epsilon_0 \underline{E}^2 + \frac{1}{2} \frac{B^2}{\mu_0}$



flux
(Poynting)

$$\underline{S} = \frac{1}{\mu_0} (\underline{E} \wedge \underline{B})$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \underline{S} = 0$$

Maxwell eqⁿs have a conserved energy

energy density $\rho = \frac{1}{2} \epsilon_0 \underline{E}^2 + \frac{1}{2} \frac{\underline{B}^2}{\mu_0}$



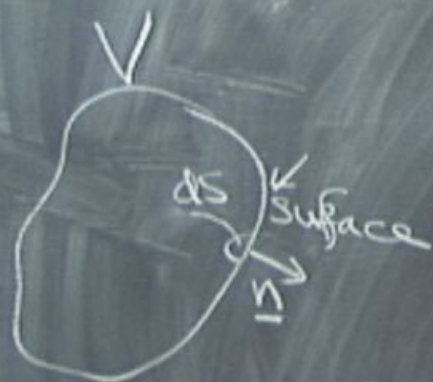
flux
(Poynting) $\underline{S} = \frac{1}{\mu_0} (\underline{E} \wedge \underline{B})$

$$\underline{E} = \int_V \underline{\partial} \times \underline{A}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \underline{S} = 0$$

$$\frac{d\underline{E}}{dt} = - \int_V \underline{\partial} \times \underline{S}$$

Maxwell eqⁿs have a conserved energy
energy density $\rho = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \frac{B^2}{\mu_0}$



flux
(Poynting)

$$\underline{S} = \frac{1}{\mu_0} (\underline{E} \wedge \underline{B})$$

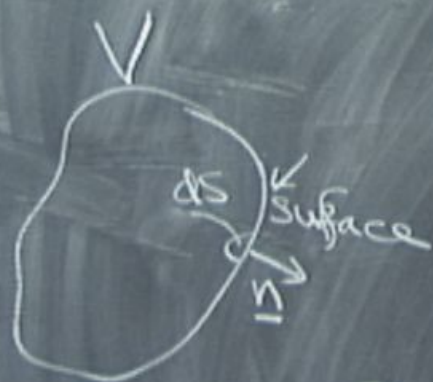
$$\underline{E} = \int_V \underline{\partial} \times \underline{\rho}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \underline{S} = 0$$

$$\frac{d\underline{E}}{dt} = - \int_V \underline{\partial} \times \underline{\nabla} \underline{S}$$

$$= - \int_{\text{area}} \underline{S} \cdot \underline{n} dS$$

Maxwell eqⁿs have a conserved energy
energy density $\rho = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \frac{B^2}{\mu_0}$



flux
(Poynting)

$$\underline{S} = \frac{1}{\mu_0} (\underline{E} \wedge \underline{B})$$

$$\underline{E} = \int_V \underline{\partial} \times \underline{\rho}$$

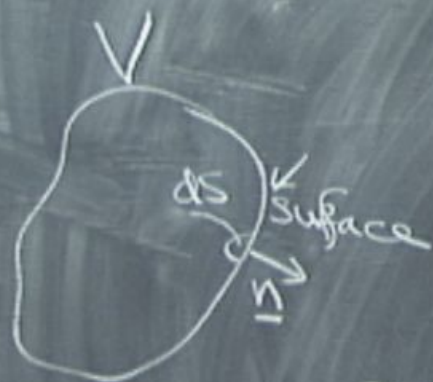
$$\frac{\partial \rho}{\partial t} + \nabla \cdot \underline{S} = 0$$

$$\frac{d\underline{E}}{dt} = - \int_V \underline{\partial} \times \underline{\nabla} \underline{S}$$

$$= - \int_{\text{area}} \underline{S} \cdot \underline{n} ds$$

Maxwell eqⁿs have a conserved energy
energy density

$$\rho = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \frac{B^2}{\mu_0}$$



flux
(Poynting)

$$\underline{S} = \frac{1}{\mu_0} (\underline{E} \wedge \underline{B})$$

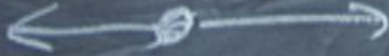
$$\underline{E} = \int_V \underline{\partial} \times \underline{\rho}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \underline{S} = 0$$

$$\frac{d\underline{E}}{dt} = - \int_V \underline{\partial} \times \underline{\nabla} \underline{S}$$

$$= - \int_{\text{area}} \underline{S} \cdot \underline{n} ds$$

Wave packet



pick $\phi=0$

Wave packet



pick $\phi=0$

Wave packet



$$\mathbf{E} = (0, 0, E_z)$$

$$E_z(x-ct)$$

pick $\phi=0$

Wave packet



$$\underline{E} = (0, 0, E_z)$$

$$E_z = E_z(x-ct)$$

$$\nabla \wedge \underline{E} = -\frac{1}{c} \frac{\partial \underline{B}}{\partial t}$$

$$\underline{B} = (0, B_y, 0)$$

pick $\phi=0$

Wave packet



$$\underline{E} = (0, 0, E_z)$$

$$E_z = E_z(x-ct)$$

$$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$(0, -E_z, 0)$$

$$\underline{B} = (0, B_y, 0)$$

pick $\phi=0$

Wave packet



$$\underline{E} = (0, 0, E_z)$$

$$E_z = E_z(x-ct)$$

$$\nabla \wedge \underline{E} = - \frac{\partial \underline{B}}{\partial t}$$

$$\underline{B} = (0, B_y, 0)$$

$$(E_z, 0) = (0, cB'_y, 0)$$

pick $\phi=0$

Wave packet



$$\underline{E} = (0, 0, E_z)$$

$$E_z = E_z(x-ct)$$

$$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$\underline{B} = (0, B_y, 0)$$

$$B_y = B_y(x-ct)$$

$$(0, -E'_z, 0) = (0, cB'_y, 0)$$

$$\Rightarrow B_y = -E_z/c \quad \text{up to a const.}$$

pick $\phi=0$

Wave packet



$$\underline{E} = (0, 0, E_z)$$

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$$\Rightarrow \boxed{B_y = -E_z/c} \text{ up to a const.}$$

$$\underline{B} = (0, B_y, 0)$$

$$B_y = B_y(x-ct)$$

What is light wave in moving frame?

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$$F' \propto \beta$$

What is light wave in moving frame?

L.T. $F'_{\alpha\beta} = \Lambda_{\alpha}^{\gamma} \Lambda_{\beta}^{\delta} F_{\gamma\delta}$

What is light wave in moving frame?

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What is light wave in moving frame?

L.T. $F'^{\alpha\beta} = \Lambda^\alpha_\mu \Lambda^\beta_\nu F^{\mu\nu}$

$$\begin{pmatrix} \gamma - \gamma v/c & & \\ -\gamma v/c & \gamma & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

What is light wave in moving frame?

L.T. $F^{\alpha\beta} = \Lambda^\alpha_\mu \Lambda^\beta_\nu F^{\mu\nu} = \Lambda F \Lambda^T$

$$\begin{pmatrix} \gamma & -\gamma v/c \\ -\gamma v/c & \gamma \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

What is light wave in moving frame?

L.T. $F'^{\alpha\beta} = \Lambda^\alpha_\mu \Lambda^\beta_\nu F^{\mu\nu} = \Lambda F \Lambda^T$

$$\begin{pmatrix} \gamma & -\gamma v/c \\ -\gamma v/c & \gamma \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} \cdot & \cdot & \cdot & E_z/c \\ \cdot & \cdot & \cdot & -B_y \\ \cdot & \cdot & \cdot & \cdot \\ -E_z/c & B_y & \cdot & \cdot \end{pmatrix} \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

What is light wave in moving frame?

L.T. $F'^{\alpha\beta} = \Lambda^\alpha_\mu \Lambda^\beta_\nu F^{\mu\nu} = \Lambda F \Lambda^T$

$$\begin{pmatrix} \gamma & -\gamma v/c \\ -\gamma v/c & \gamma \\ & & 1 \\ & & & 1 \end{pmatrix} \begin{pmatrix} \cdot & \cdot & \cdot & E_z/c \\ \cdot & \cdot & \cdot & -B_y \\ -E_z/c & B_y & \cdot & \cdot \end{pmatrix} \begin{pmatrix} \gamma & -\gamma v/c \\ -\gamma v/c & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$\Rightarrow E'_z = \gamma(E_z + v B_y)$$

$$B'_y = \gamma(B_y + \frac{v}{c} \frac{E_z}{c})$$

What is light wave in moving frame?

L.T. $F'^{\alpha\beta} = \Lambda^\alpha_\mu \Lambda^\beta_\nu F^{\mu\nu} = \Lambda F \Lambda^T$

$$\begin{pmatrix} \gamma & -\gamma v/c \\ -\gamma v/c & \gamma \\ & & 1 \\ & & & 1 \end{pmatrix} \begin{pmatrix} & & & E_z/c \\ & & & -B_y \\ & & & -B_y \\ -E_z/c & B_y & & \end{pmatrix} \begin{pmatrix} \gamma & -\gamma v/c \\ -\gamma v/c & \gamma \\ & & 1 \\ & & & 1 \end{pmatrix}$$

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L.T. $F'^{\alpha\beta} = \Lambda^\alpha_\mu \Lambda^\beta_\nu F^{\mu\nu} = \Lambda F \Lambda^T$

$$\begin{pmatrix} \gamma & -\gamma v/c & 0 & 0 \\ -\gamma v/c & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cdot & \cdot & \cdot & E_z/c \\ \cdot & \cdot & \cdot & -B_y \\ \cdot & \cdot & \cdot & \cdot \\ -E_z/c & B_y & \cdot & \cdot \end{pmatrix} \begin{pmatrix} \gamma & -\gamma v/c & 0 & 0 \\ -\gamma v/c & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow E'_z = \gamma(E_z + v B_y) = \sqrt{\frac{1-v/c}{1+v/c}} E_z$$

$$B'_y = \gamma(B_y + \frac{v}{c} \frac{E_z}{c}) = \sqrt{\frac{1-v/c}{1+v/c}} B_y$$

$$B_y = -E_z/c$$

but $E'_z(x'-ct')$

but $E'_z(x'-ct') = \gamma(E_z(x-ct) + vB_y(x-ct))$

but $E'_z(x'-ct') = \gamma(E_z(x-ct) + vB_y(x-ct))$

$x-ct = \gamma(x'+vt')$

but

$$E'_z(x'-ct') = \gamma(E_z(x-ct) + vB_y(x-ct))$$

but

$$x-ct = \gamma(x' + vt')$$

$$-c\gamma(t' + v\frac{x'}{c^2})$$

$$= \gamma(1 - \frac{v}{c})(x' - ct')$$

but

$$E'_z(x'-ct') = \gamma(E_z(x-ct) + vB_y(x-ct))$$

but

$$x-ct = \gamma(x' + vt')$$

$$-c\gamma(t' + v\frac{x'}{c^2})$$

$$= \gamma(1 - \frac{v}{c})(x' - ct')$$

=

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$= \frac{1}{\sqrt{(1 - \frac{v}{c})(1 + \frac{v}{c})}}$$

but $E'_z(x'-ct') = \gamma(E_z(x-ct) + v B_y(x-ct))$

but $x-ct = \gamma(x' + vt')$

$-c\gamma(t' + vx'/c^2)$

$= \gamma(1 - v/c)(x' - ct')$

$= \sqrt{\frac{1 - v/c}{1 + v/c}} (x' - ct')$

$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$
 $= \frac{1}{\sqrt{(1 - v/c)(1 + v/c)}}$

but

$$E'_z(x'-ct') = \gamma(E_z(x-ct) + v B_y(x-ct))$$

but

$$x-ct = \gamma(x' + vt')$$

$$-c\gamma(t' + vx'/c^2)$$

$$= \gamma(1 - v/c)(x' - ct')$$

$$= \sqrt{\frac{1 - v/c}{1 + v/c}} (x' - ct')$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{1}{\sqrt{(1 - v/c)(1 + v/c)}}$$

(ct))

Poynting

In (t', x') coords

$$\text{Doynting} = \frac{1 - v/c}{1 + v/c}$$

In ct', x' coords

$$p_{\text{ynting}} = \frac{1 - v/c}{1 + v/c} \frac{1}{M_0} \frac{E_z^2}{c} \left(\sqrt{\frac{1 - v/c}{1 + v/c}} \right) (x)$$

In ct', x' coords

$$\text{Poynting} = \frac{1 - v/c}{1 + v/c} \cdot \frac{1}{\mu_0} \frac{E_z^2}{c} \left(\frac{1 - v/c}{1 + v/c} (x' - ct') \right)$$

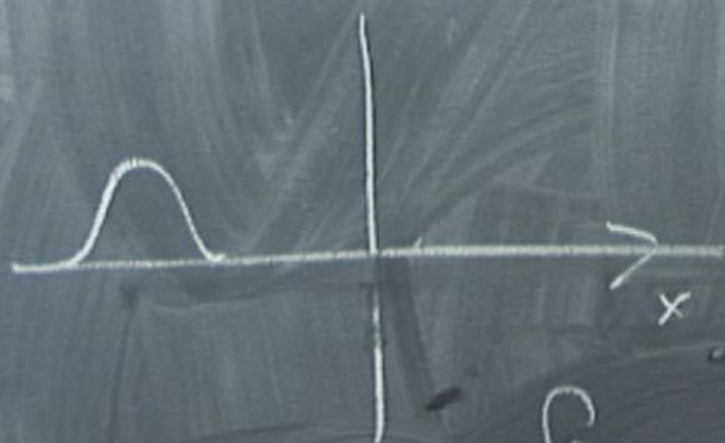
In ct', x' coords

$$\text{Poynting} = \frac{1 - v/c}{1 + v/c} \cdot \frac{1}{\mu_0} \frac{E_z^2}{c} \left(\sqrt{\frac{1 - v/c}{1 + v/c}} (x' - ct') \right)$$

In (t', x') coords

b)

$$\text{Poynting} = \frac{1 - v/c}{1 + v/c} \frac{1}{\mu_0} \frac{E_z^2}{c} \left(\sqrt{\frac{1 - v/c}{1 + v/c}} \right)$$



$$\text{total energy} = \int dt (\text{Poynting flux})$$

In ct', x' coords

$$\text{Poynting} = \frac{1 - v/c}{1 + v/c} \frac{1}{\mu_0} \frac{E_z^2}{c} \left(\sqrt{\frac{1 - v/c}{1 + v/c}} (x' - ct') \right)$$



$$\int dt' \Rightarrow \sqrt{\frac{1 + v/c}{1 - v/c}}$$

$$W = \int dt (\text{Poynting flux})$$

In ct', x' coords

$$\text{Poynting} = \frac{1 - v/c}{1 + v/c} \frac{1}{\mu_0} \frac{E_z^2}{c} \left(\sqrt{\frac{1 - v/c}{1 + v/c}} (x' - ct') \right)$$



$$\int dt' \Rightarrow \sqrt{\frac{1 - v/c}{1 + v/c}}$$

$$\text{Energy} = \int dt (\text{Poynting flux}) \Rightarrow \text{Energy of wpkt} = \text{Energy in origin}$$

In ct', x' coords

$$\text{Poynting} = \frac{1 - v/c}{1 + v/c} \frac{1}{\mu_0} \frac{E_z^2}{c} \left(\sqrt{\frac{1 - v/c}{1 + v/c}} (x' - ct') \right)$$



$$\int dt' \Rightarrow \sqrt{\frac{1 + v/c}{1 - v/c}}$$

$$\text{Energy} = \int dt (\text{Poynting flux}) \Rightarrow \text{Energy of wpkt} \\ = \text{Energy in original frame}$$

$$\times \sqrt{\frac{1 - v/c}{1 + v/c}}$$

In ct', x' coords

$$\text{Poynting} = \frac{1 - v/c}{1 + v/c} \cdot \frac{1}{\mu_0} \frac{E_z^2}{c} \left(\sqrt{\frac{1 - v/c}{1 + v/c}} (x' - ct') \right)$$



$$\int dt' \Rightarrow \sqrt{\frac{1 + v/c}{1 - v/c}}$$

$$\text{Energy} = \int dt (\text{Poynting flux}) \Rightarrow \text{Energy of wpkt} \\ = \text{Energy in original frame} \\ \times \sqrt{\frac{1 - v/c}{1 + v/c}}$$

In ct', x' coords

$$\text{Poynting} = \frac{1 - v/c}{1 + v/c} \cdot \frac{1}{\mu_0} \frac{E_z^2}{c} \left(\sqrt{\frac{1 - v/c}{1 + v/c}} (x' - ct') \right)$$

$$\int dt' \Rightarrow \sqrt{\frac{1 + v/c}{1 - v/c}}$$

$\int dt$ (Poynting flux) $\Rightarrow$ Energy of wpkt
 $=$ Energy in original frame

$$\frac{v \cos \phi}{\sqrt{1 - v^2/c^2}}$$

$$\times \sqrt{\frac{1 - v/c}{1 + v/c}}$$

In ct', x' coords

$$\text{Poynting} = \frac{1 - v/c}{1 + v/c} \frac{1}{\mu_0} \frac{E_z^2}{c} \left(\sqrt{\frac{1 - v/c}{1 + v/c}} (x' - ct') \right)$$



$$\int dt' \Rightarrow \sqrt{\frac{1 + v/c}{1 - v/c}}$$

$$\text{Energy} = \int dt (\text{Poynting flux})$$

$\Rightarrow$ Energy of ...
= Energy in ...

$$\frac{1 - v/c}{\sqrt{1 - v^2/c^2}}$$

$$\times \sqrt{\frac{1 - (v/c)^2}{1 + v/c}}$$

In ct', x' coords

$$\text{Poynting} = \frac{1 - v/c}{1 + v/c} \cdot \frac{1}{\mu_0} \frac{E_z^2}{c} \left(\sqrt{\frac{1 - v/c}{1 + v/c}} (x' - ct') \right)$$



$$\int dt' \Rightarrow \sqrt{\frac{1 + v/c}{1 - v/c}}$$

$$\text{Energy} = \int dt (\text{Poynting flux}) \Rightarrow \text{Energy of wpkt} = \text{Energy in original frame}$$

$$\frac{1 - v/c}{\sqrt{1 - v^2/c^2}}$$

$$\times \sqrt{\frac{1 - v/c}{1 + v/c}}$$

$$E = h\nu$$

$$E = h\nu$$

$$\frac{L}{2} \rightarrow \frac{L}{2} (1 - v_e \cos \phi)$$

$$E = h\nu$$

$$\frac{L}{2} \rightarrow \frac{L}{2} \frac{(1 - v_c \cos \phi)}{\sqrt{1 - v^2/c^2}}$$

$$E = h\nu$$

moving observer

$$\frac{L}{2} \rightarrow \frac{L}{2} \frac{(1 - v/c \cos \phi)}{\sqrt{1 - v^2/c^2}}$$

$$E = h\nu$$

moving observer



$$\frac{\nu}{2} \rightarrow \frac{\nu}{2} \frac{(1 - v/c \cos \phi)}{\sqrt{1 - v^2/c^2}}$$

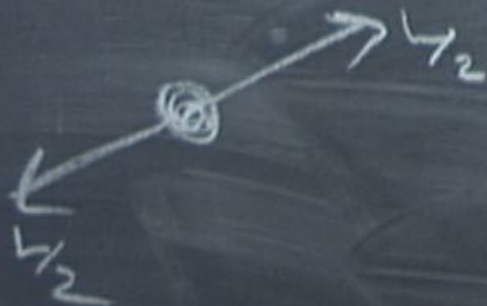
$$c \frac{dt}{d\tau} =$$

$$E = h\nu$$

$$\frac{L}{2} \rightarrow \frac{L}{2} \frac{(1 - v/c \cos \phi)}{\sqrt{1 - v^2/c^2}}$$

moving observer

$$u^M = \gamma$$



$$\frac{cdt}{d\tau} = \frac{cdt}{cdt \sqrt{1 - v^2/c^2}} = \gamma$$

$$E = h\nu$$

$$\frac{L}{2} \rightarrow \frac{L}{2} \frac{(1 - v/c \cos \phi)}{\sqrt{1 - v^2/c^2}}$$

moving observer

$$\left(\gamma, \gamma \frac{v}{c} \right)$$



$$\frac{cdt}{d\tau} = \frac{cdt}{cdt \sqrt{1 - v^2/c^2}} = \gamma$$

$$E = h\nu$$

$$\frac{L}{2} \rightarrow \frac{L}{2} \frac{(1 - v/c \cos \phi)}{\sqrt{1 - v^2/c^2}}$$

moving observer

$$u^M = \left(\gamma, \gamma \frac{v}{c} \right)$$

$\rightarrow \nu_2$

$$\frac{cdt}{d\tau} = \frac{cdt}{cdt \sqrt{1 - v^2/c^2}} = \gamma$$

4-momⁿ of photon

$$p^M =$$

$$E = h\nu$$

$$\frac{L}{2} \rightarrow \frac{L}{2} \frac{(1 - v/c \cos \phi)}{\sqrt{1 - v^2/c^2}}$$

moving

observer

$$u^\mu = \left(\gamma, \gamma \frac{v}{c} \right)$$

$$\frac{cdt}{d\tau} = \frac{cdt}{cdt \sqrt{1 - v^2/c^2}} = \gamma$$

of photon

$$p^\mu = \frac{L}{2} (1, \cos \phi, \sin \phi)$$

$$E = h\nu$$

$$\frac{L}{2} \rightarrow \frac{L}{2} \frac{(1 - v/c \cos \phi)}{\sqrt{1 - v^2/c^2}}$$

moving observer

$$u^M = \left(\gamma, \gamma \frac{v}{c} \right)$$



$$\frac{cdt}{d\tau} = \frac{cdt}{cdt \sqrt{1 - v^2/c^2}} = \gamma$$

4-momⁿ of photon

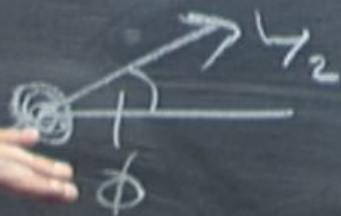
$$p^M = \frac{L}{2a} (1, \cos \phi, \sin \phi)$$

$$E = h\nu$$

$$\frac{L}{2} \rightarrow \frac{L}{2} \frac{(1 - v/c \cos \phi)}{\sqrt{1 - v^2/c^2}}$$

moving observer

$$u^M = \left(\gamma, \gamma \frac{v}{c} \right)$$



$$\frac{cdt}{d\tau} = \frac{cdt}{cdt \sqrt{1 - v^2/c^2}} = \gamma$$

4-momⁿ of photon

$$p^M = \frac{L}{2c} (1, \cos \phi, \sin \phi)$$

$$E = h\nu$$

$$\frac{L}{2} \rightarrow \frac{L}{2} \frac{(1 - v/c \cos \phi)}{\sqrt{1 - v^2/c^2}}$$

moving observer

$$u^\mu = \left(\gamma, \gamma \frac{v}{c} \right)$$



$$\frac{cdt}{d\tau} = \frac{cdt}{cdt \sqrt{1 - v^2/c^2}} = \gamma$$

4-momⁿ of photon

$$p^\mu = \frac{L}{2a} (1, \cos \phi, \sin \phi)$$

$$E = h\nu$$

$$\frac{L}{2} \rightarrow \frac{L}{2} \frac{(1 - v/c \cos \phi)}{\sqrt{1 - v^2/c^2}}$$

moving observer



$$\left(\gamma, \gamma \frac{v}{c} \right)$$

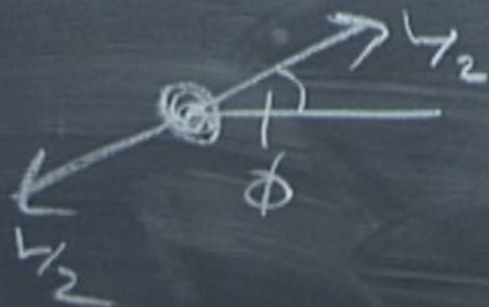
$$\frac{cdt}{dx} = \frac{c}{v} \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$E = h\nu$$

$$\frac{L}{2} \rightarrow \frac{L}{2} \frac{(1 - v/c \cos \phi)}{\sqrt{1 - v^2/c^2}}$$

moving observer

$$u^M = \left(\gamma, \gamma \frac{v}{c}, 0, 0 \right)$$



$$\frac{cdt}{d\tau} = \frac{cdt}{cdt \sqrt{1 - v^2/c^2}} = \gamma$$

4-momⁿ of photon

$$p^M = \frac{L}{2a} (1, \cos \phi, \sin \phi, 0)$$

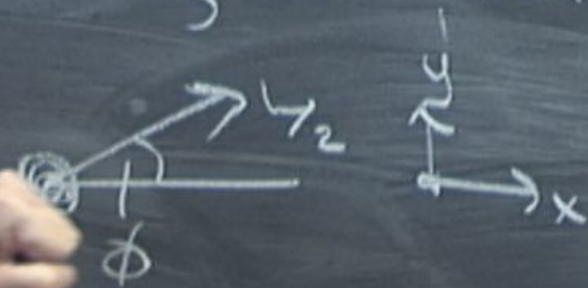
energy of photon

$$E = h\nu$$

$$\frac{L}{2} \rightarrow \frac{L}{2} \frac{(1 - v/c \cos \phi)}{\sqrt{1 - v^2/c^2}}$$

moving observer

$$u^M = \left(\gamma, \gamma \frac{v}{c}, 0, 0 \right)$$



$$\frac{cdt}{d\tau} = \frac{cdt}{cdt \sqrt{1 - v^2/c^2}} = \gamma$$

4-momⁿ of photon

$$p^M = \frac{L}{2a} (1, \cos \phi, \sin \phi, 0)$$

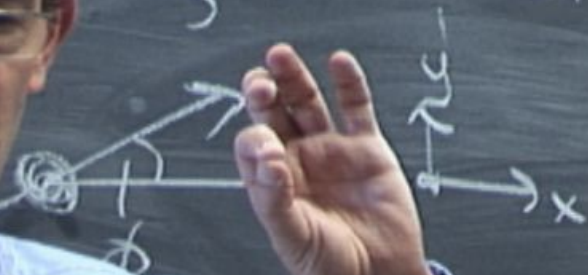
Energy of photon

$$E = h\nu$$

$$\frac{L}{2} \rightarrow \frac{L}{2} \frac{(1 - v/c \cos \phi)}{\sqrt{1 - v^2/c^2}}$$

moving observer

$$u^\mu = \left(\gamma, \gamma \frac{v}{c}, 0, 0 \right)$$



$$\frac{cdt}{d\tau} = \frac{cdt}{cdt \sqrt{1 - v^2/c^2}} = \gamma$$

of photon

$$p^\mu = \frac{L}{2c} (1, \cos \phi, \sin \phi, 0)$$

of photon in rest frame of observer

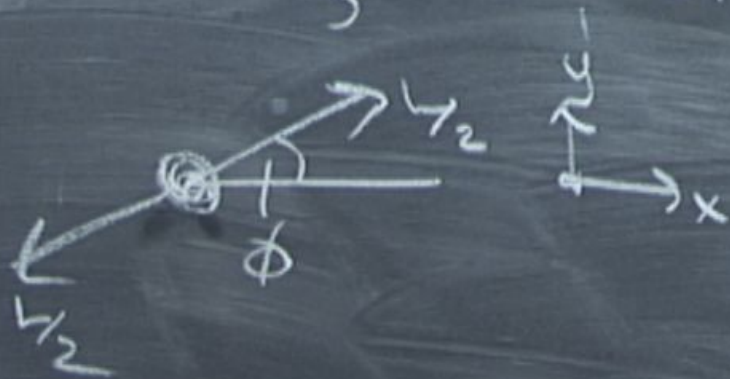
$$= -p^\mu u_\mu$$

$$E = h\nu$$

$$\frac{L}{2} \rightarrow \frac{L}{2} \frac{(1 - v/c \cos \phi)}{\sqrt{1 - v^2/c^2}}$$

moving observer

$$u^M = \left(\gamma, \gamma \frac{v}{c}, 0, 0 \right)$$



$$\frac{cdt}{d\tau} = \frac{cdt}{cdt \sqrt{1 - v^2/c^2}} = \gamma$$

4-momⁿ of photon

$$p^M = \frac{L}{2c} (1, \cos \phi, \sin \phi, 0)$$

Energy of photon in rest frame of observer

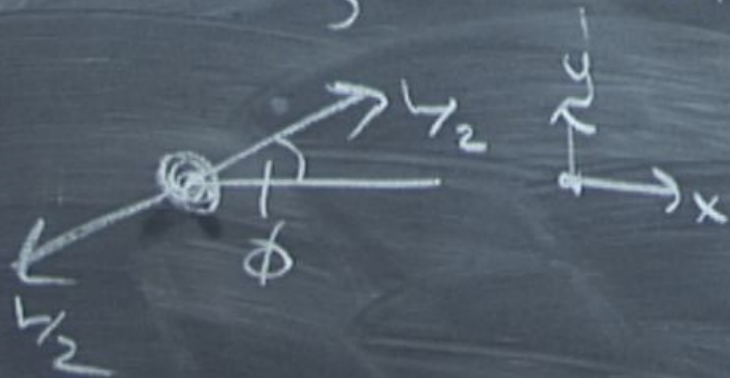
$$= -p^M u_M c = p_c^0$$

$$E = h\nu$$

$$\frac{L}{2} \rightarrow \frac{L}{2} \frac{(1 - v/c \cos \phi)}{\sqrt{1 - v^2/c^2}}$$

moving observer

$$u^\mu = \left(\gamma, \gamma \frac{v}{c}, 0, 0 \right)$$



$$\frac{cdt}{d\tau} = \frac{cdt}{cdt \sqrt{1 - v^2/c^2}} = \gamma$$

4-momⁿ of photon

$$p^\mu = \frac{L}{2c} (1, \cos \phi, \sin \phi, 0)$$

Energy of photon in rest frame of observer

$$= -p^\mu u_\mu c = p^0 c$$

$$= (\gamma, \gamma \frac{v}{c}, 0, 0)$$

$$\frac{cdt}{d\tau} = \frac{cdt}{cdt\sqrt{1-\frac{v^2}{c^2}}} \\ = \gamma$$

$$p^\mu = \frac{L}{2a} (1, \cos\phi, \sin\phi, 0)$$

$$= -p^\mu u_\mu = p^0_c = \frac{\gamma L}{2} (1 - \frac{v}{c} \cos\phi)$$



$$\text{total energy} = \int dt$$