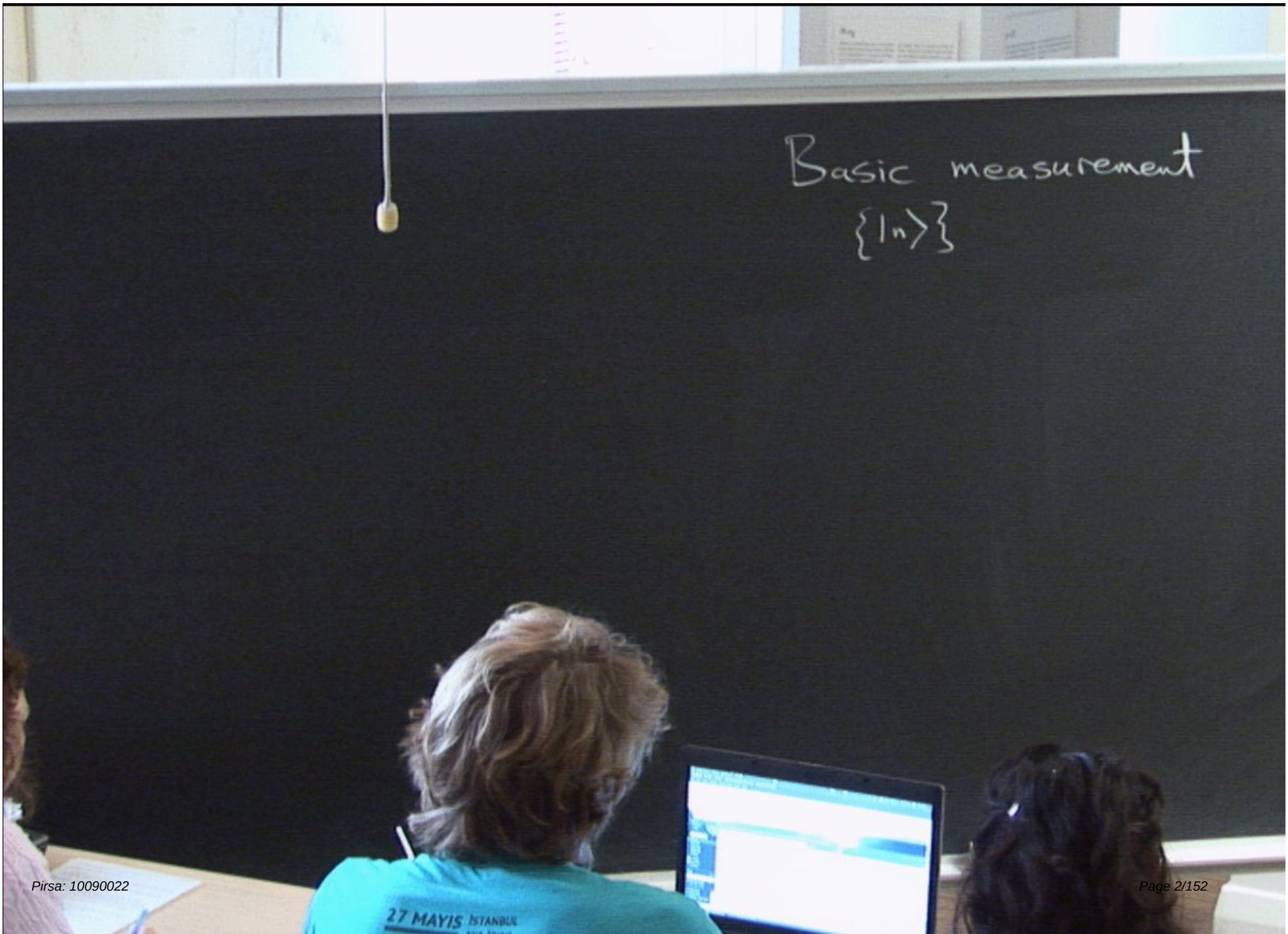
Title: Quantum Theory (PHYS 605) - Lecture 11

Date: Sep 27, 2010 09:00 AM

URL: http://pirsa.org/10090022

Abstract:


Basic measurement
{|n⟩}
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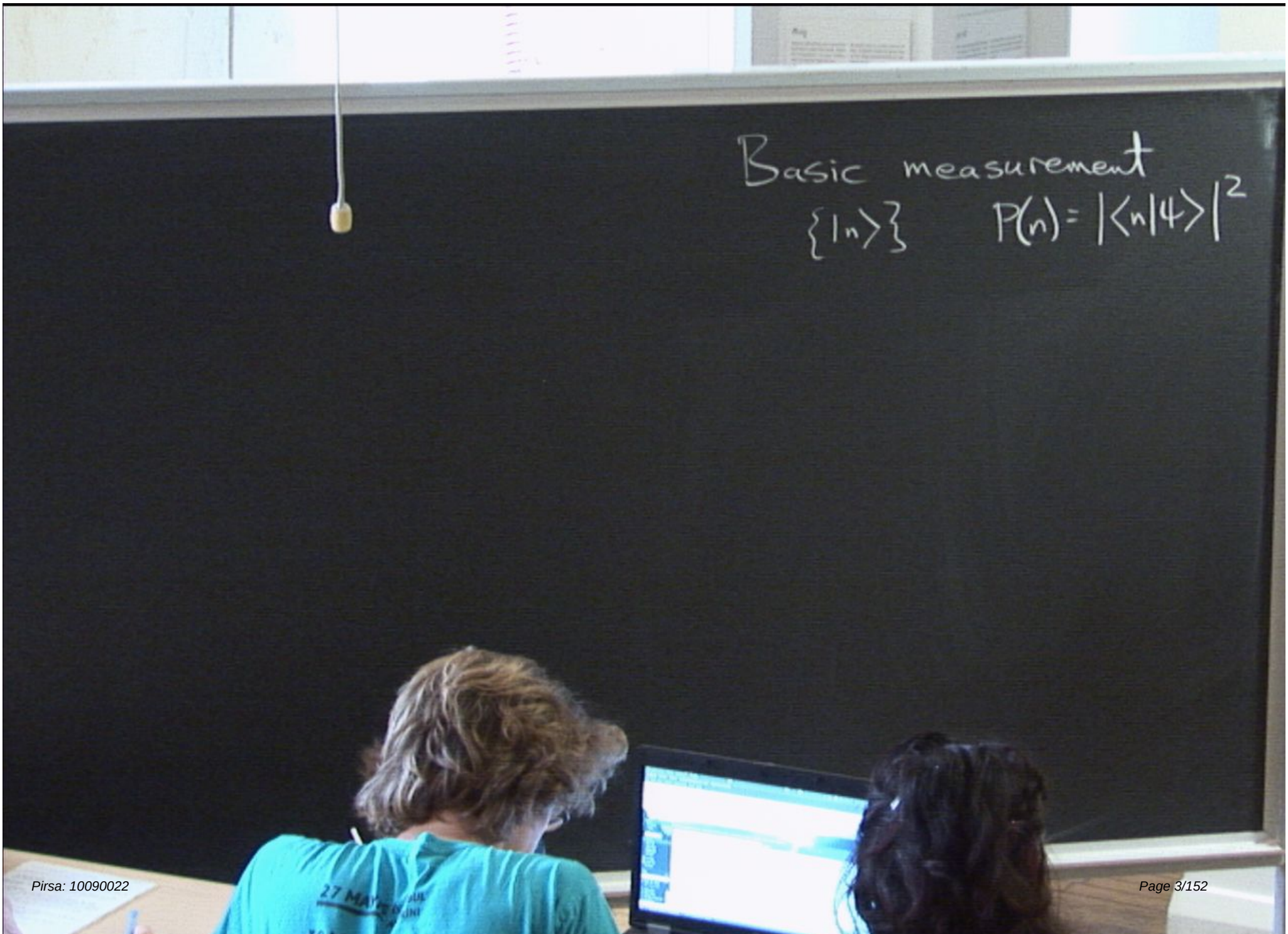
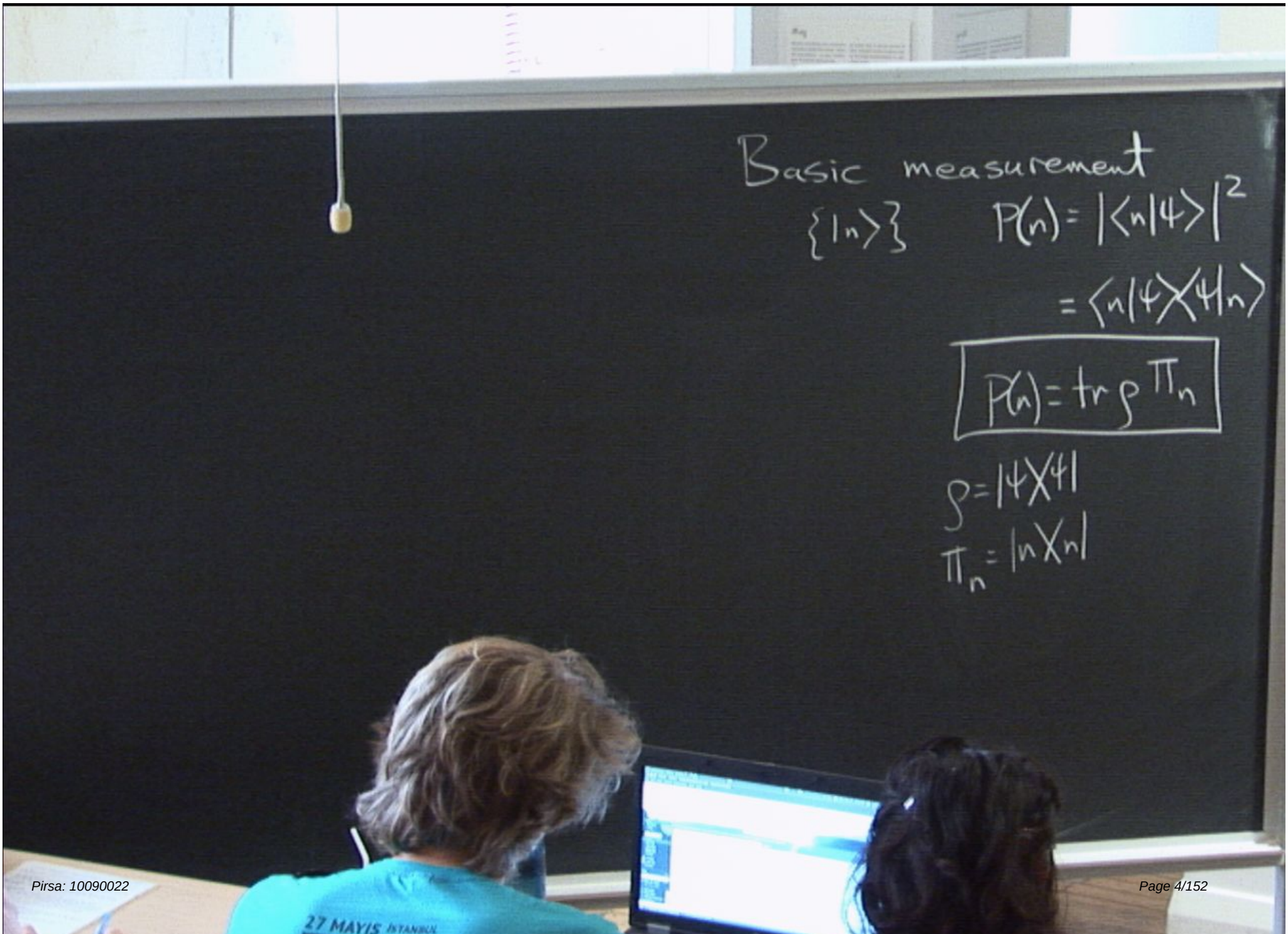
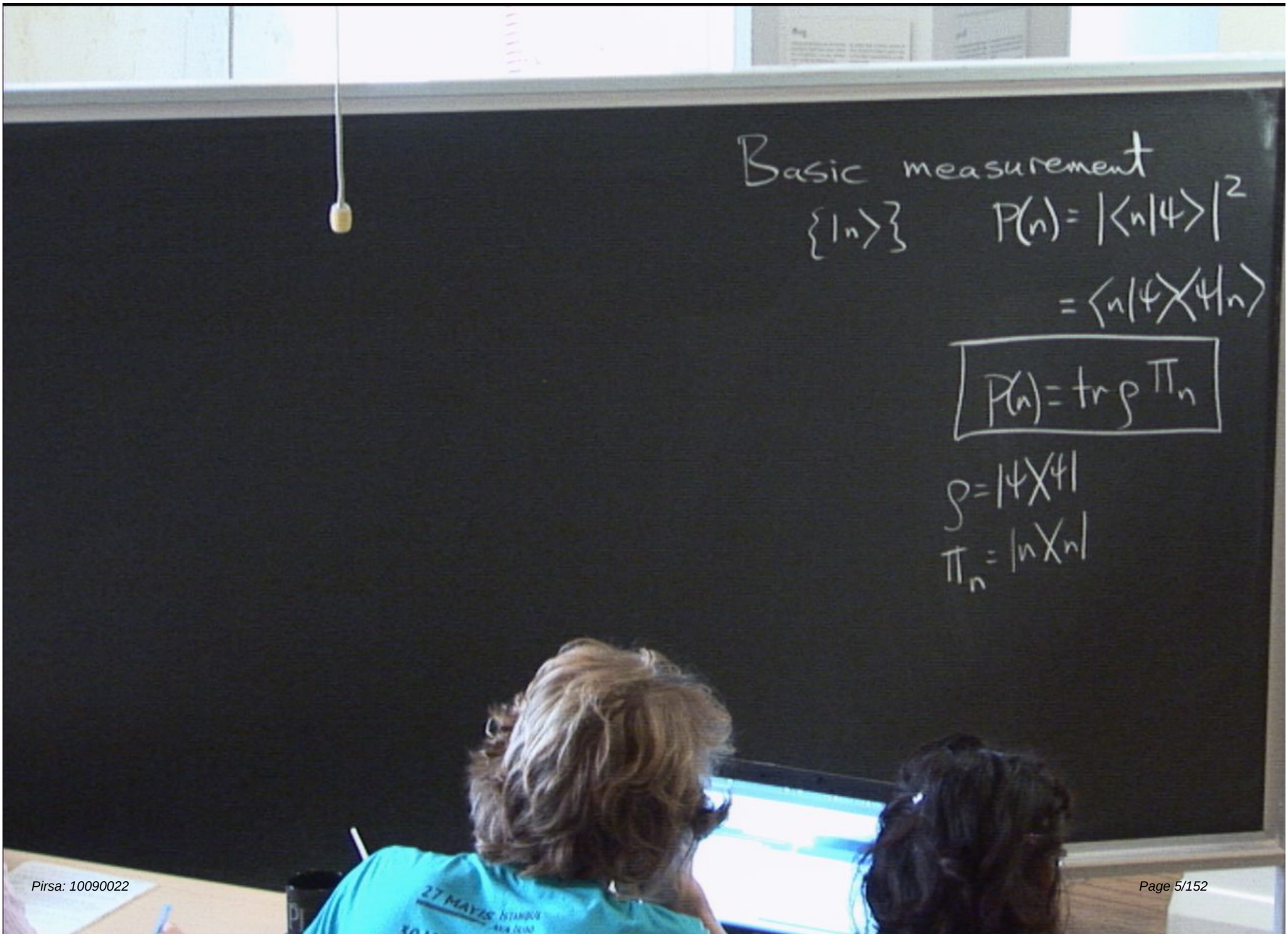

Basic measurement
$\{|n\rangle\}$ $P(n) = |\langle n|\psi\rangle|^2$


Basic measurement
$\{|n\rangle\}$ $P(n) = |\langle n|\psi\rangle|^2$
$= \langle n|\psi\rangle\langle\psi|n\rangle$
$P(n) = \mathrm{tr}\,\rho\,\Pi_n$
$\rho = |\psi\rangle\langle\psi|$
$\Pi_n = |n\rangle\langle n|$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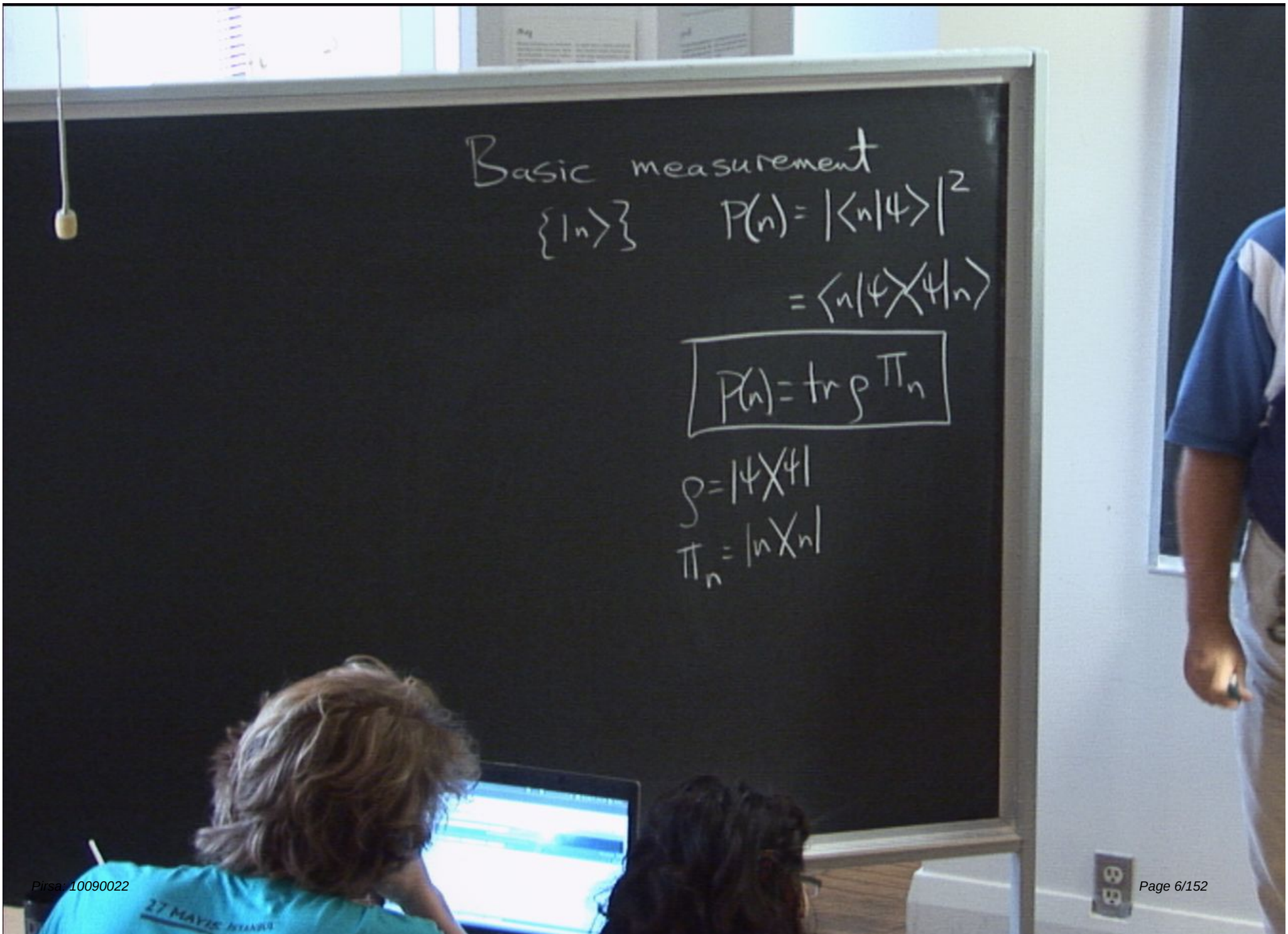
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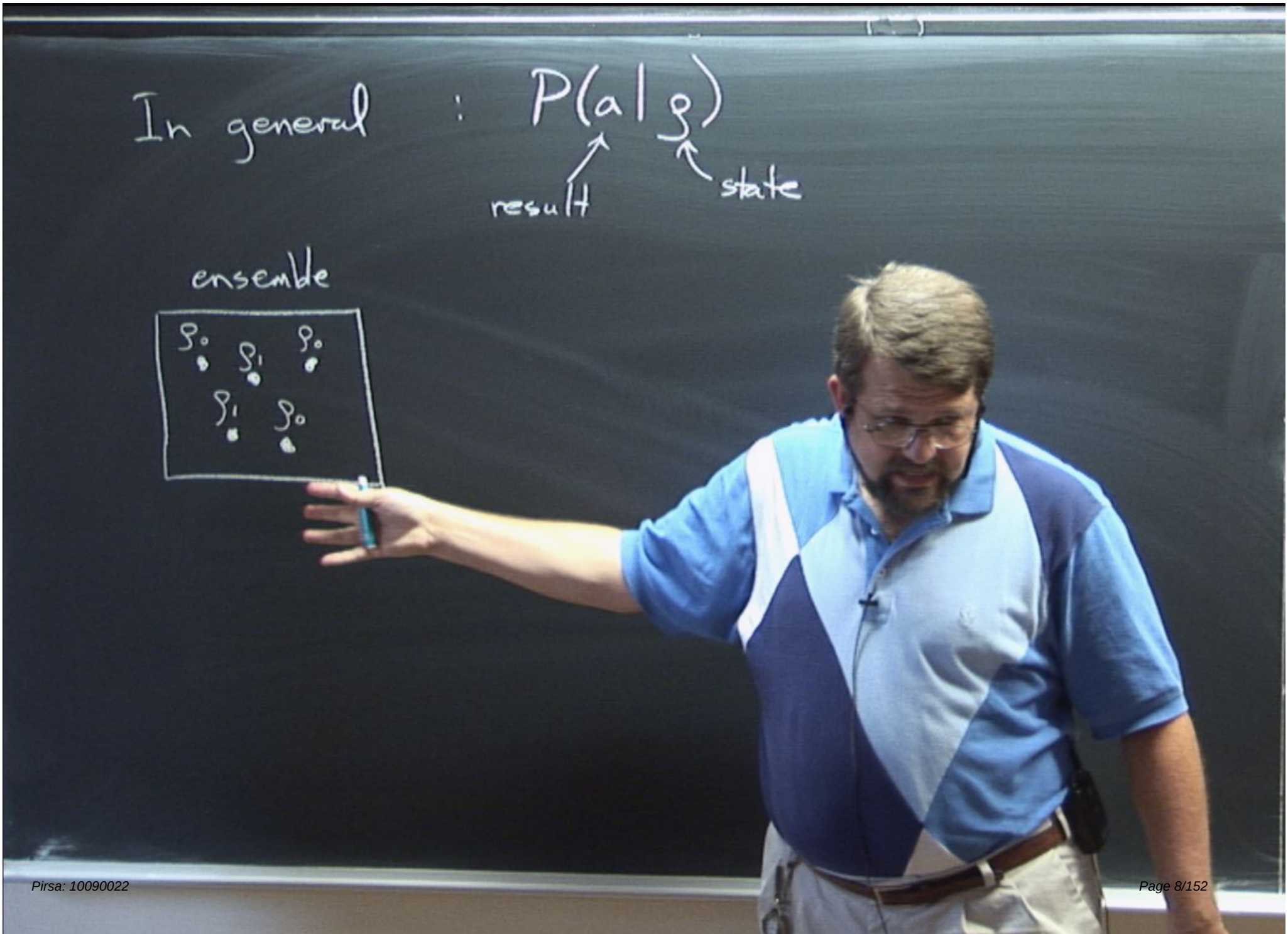
In general : $P(a|\rho)$

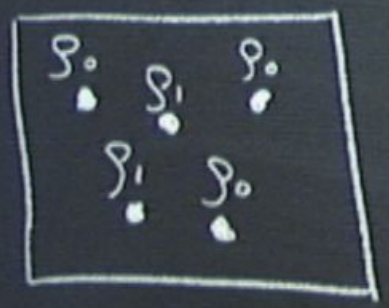
result state


In general : P(a|ρ)
result
state
ensemble
ρ0 ρ1 ρ0
ρ1 ρ0

In general : $P(a \mid \rho)$

result state

ensemble

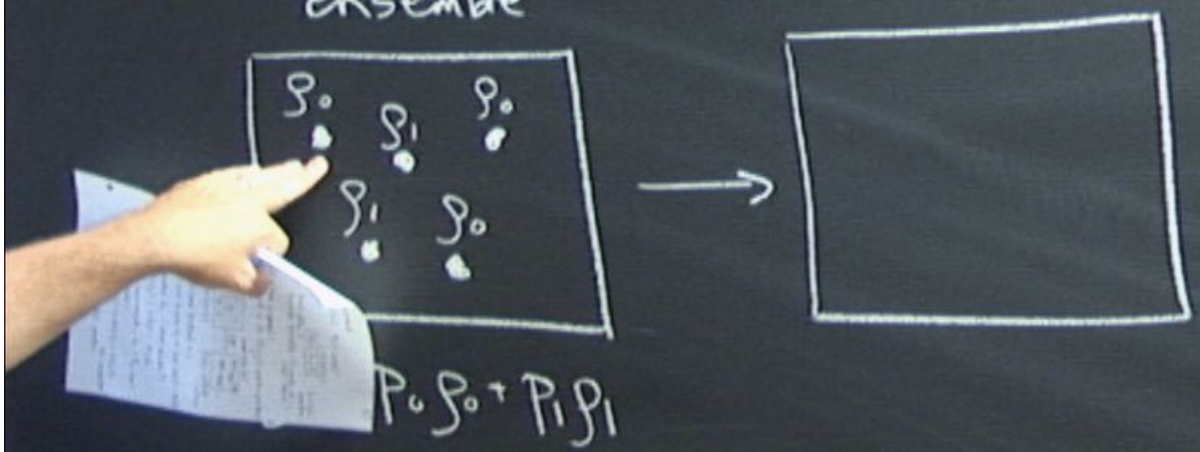


$$\rho = p_0 \rho_0 + p_1 \rho_1$$

In general : $P(a|\rho)$

result state

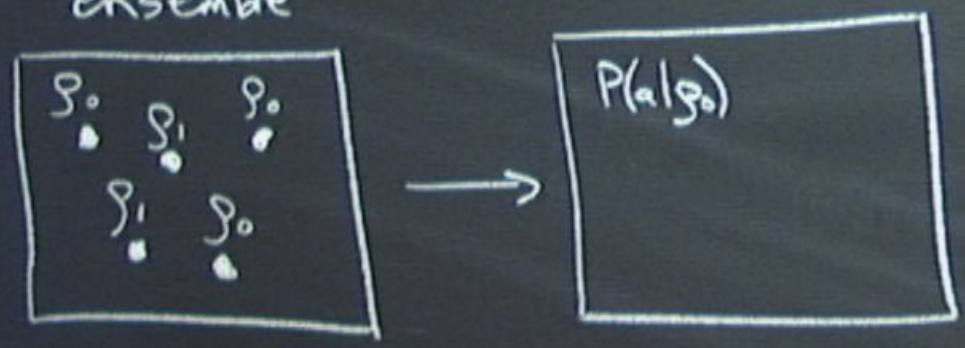
ensemble



In general : $P(a|\rho)$

result state

ensemble



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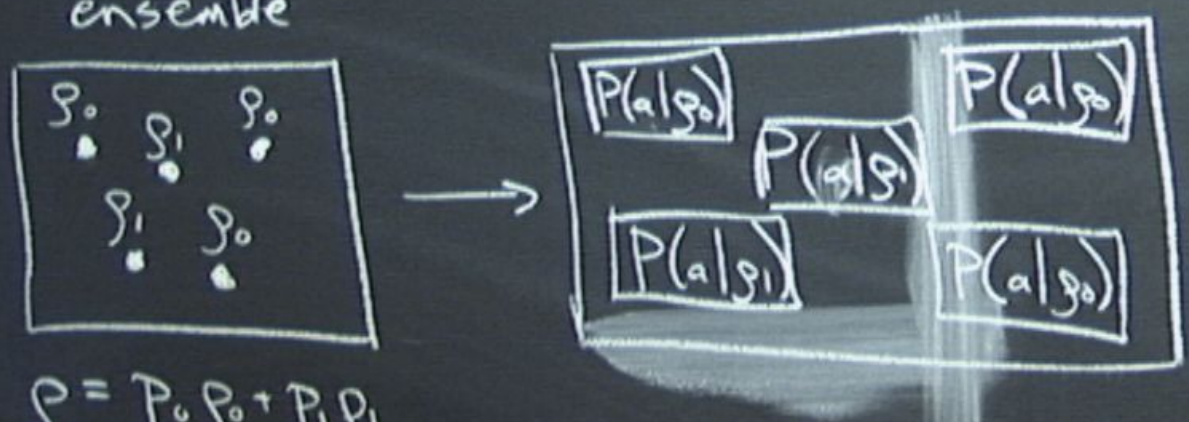
In general : $P(a|\rho)$

result state

ensemble



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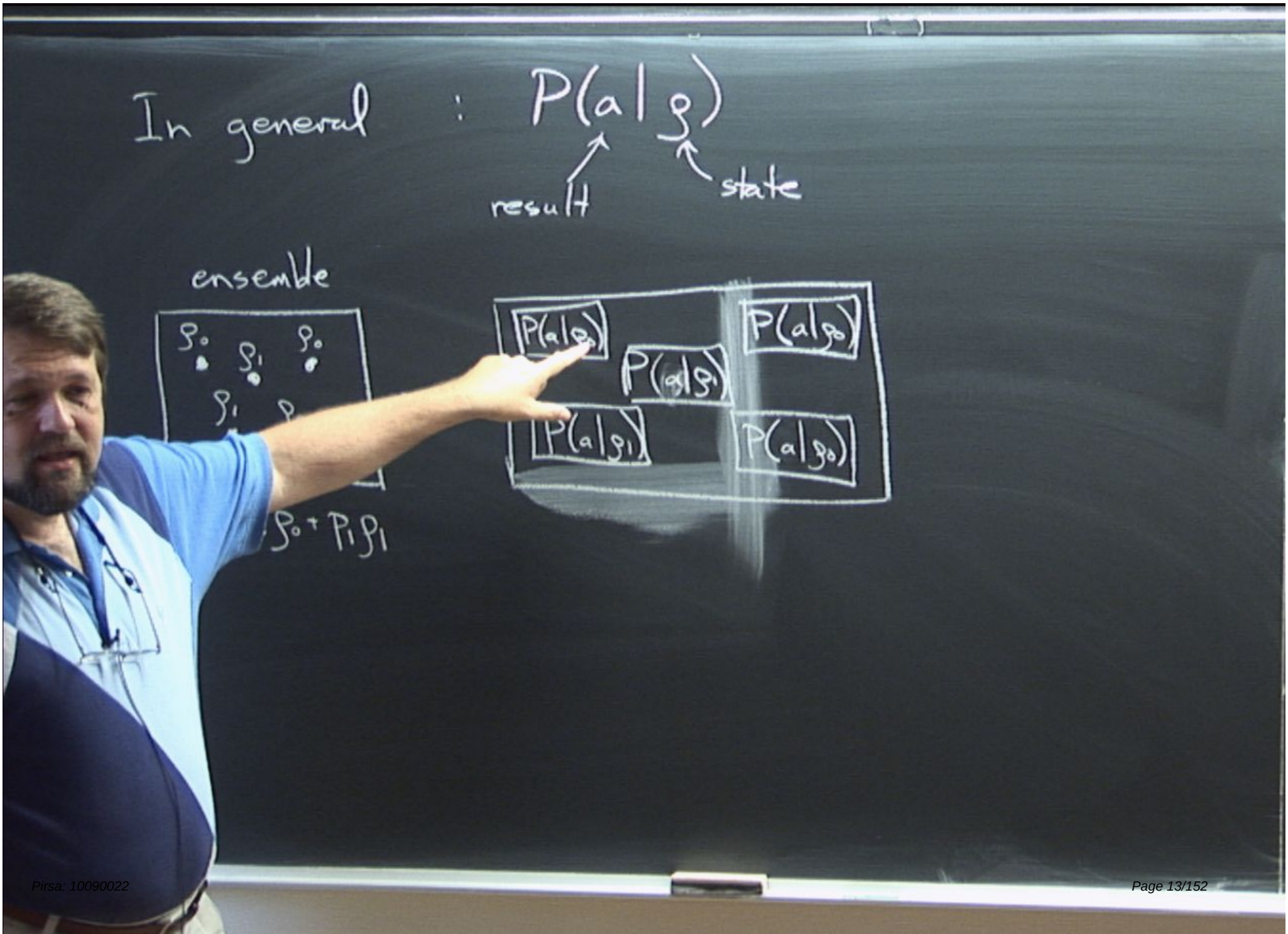
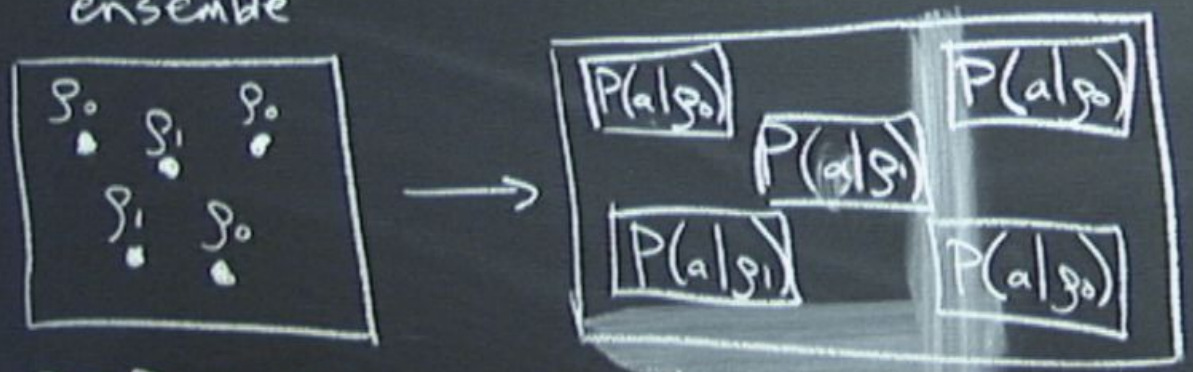

In general : P(a|ρ)
result
state
ensemble
ρ0
ρ1
ρ0 + P1ρ1
P(a|ρ0)
P(a|ρ1)
P(a|ρ0)
P(a|ρ1)
P(a|ρ0)

In general : $P(a|\rho)$

result state

ensemble



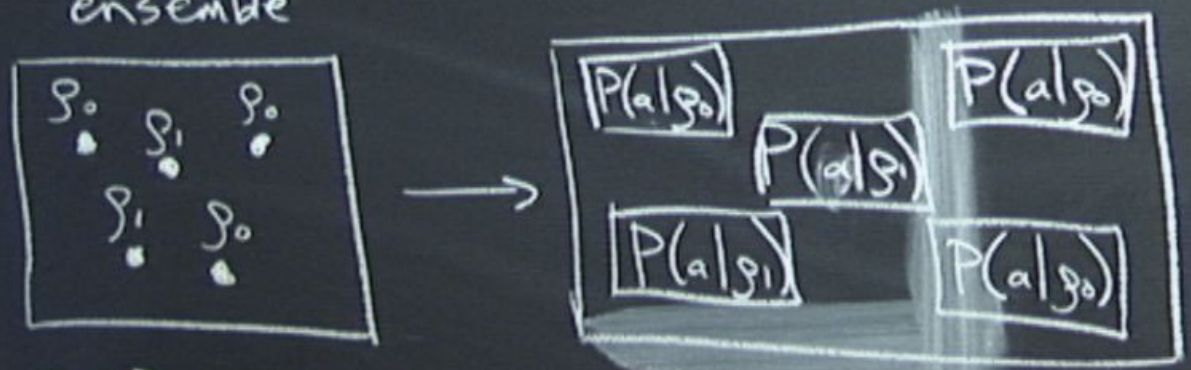
$\rho = p_0\rho_0 + p_1\rho_1$

$P(a|\rho) = p_1 P(a|\rho_1) + p_2 P(a|\rho_2)$

In general : $P(a|\rho)$

result state

ensemble



$\rho = p_0\rho_0 + p_1\rho_1$

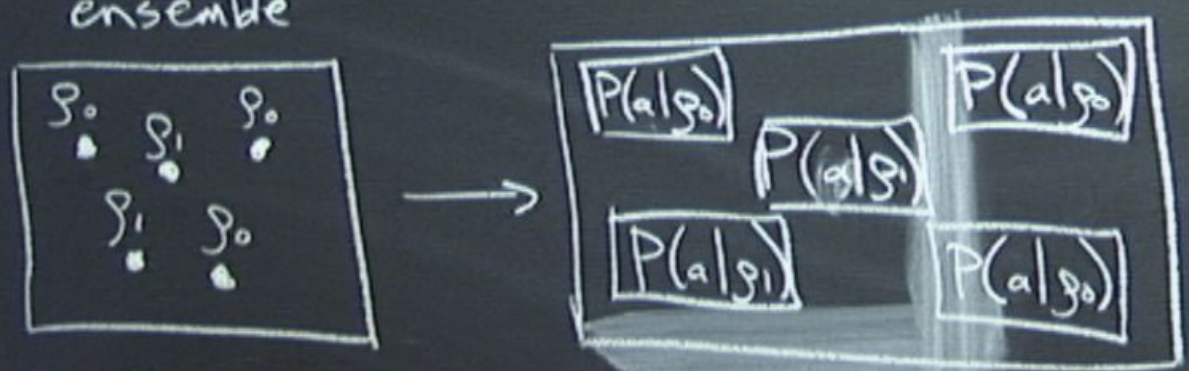
$P(a|\rho) = p_1 P(a|\rho_1) + p_2 P(a|\rho_2)$

$\therefore P(a|\rho)$ is a linear functional on ρ

In general : $P(a|\rho)$ $\qquad \rho \in \mathcal{B}(\mathcal{H})$

result $\qquad$ state

ensemble



$\rho = p_0\rho_0 + p_1\rho_1$ $\qquad P(a|\rho) = p_1 P(a|\rho_1) + p_2 P(a|\rho_2)$

$\therefore P(a|\rho)$ is a linear functional on ρ

$\rho \in \mathcal{B}(\mathcal{H})$

inner product $\langle A, B \rangle = \mathrm{tr}\, A^{+} B$

$\rho \in \mathcal{B}(\mathcal{H})$ inner product $\langle A, B \rangle = \mathrm{tr}\, A^{+} B$

There must exist E_a such that

$$P(a|\rho) = \mathrm{tr}\, \rho E_a$$

$|\rho_0\rangle$

$|\rho_0\rangle$

$) + p_2 P(a|\rho_2)$

on ρ

inner product $\langle A,B\rangle = \mathrm{tr}\, A^{+}B$

$$\boxed{P(a) = \mathrm{tr}\, \rho E_a}$$

must exist E_a such that

$$P(a|\rho) = \mathrm{tr}\, \rho E_a$$

can we say about E_a?

$\rho \in \mathcal{B}(\mathcal{H})$ inner product $\langle A, B \rangle = \mathrm{tr}\, A^{\dagger} B$

There must exist E_a such that

$$P(a|\rho) = \mathrm{tr}\, \rho E_a$$

What can we say about E_a?

Suppose $\rho = |\psi\rangle\langle\psi|$. Then

$$P(a|\rho) = \langle\psi|E_a|\psi\rangle$$

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What can we say about E_a?

Suppose $\rho = |\psi\rangle\langle\psi|$. Then

$$P(a|\rho) = \langle\psi|E_a|\psi\rangle \leftarrow \text{real}, \geq 0$$

$P(a|\rho_2)$

ρ

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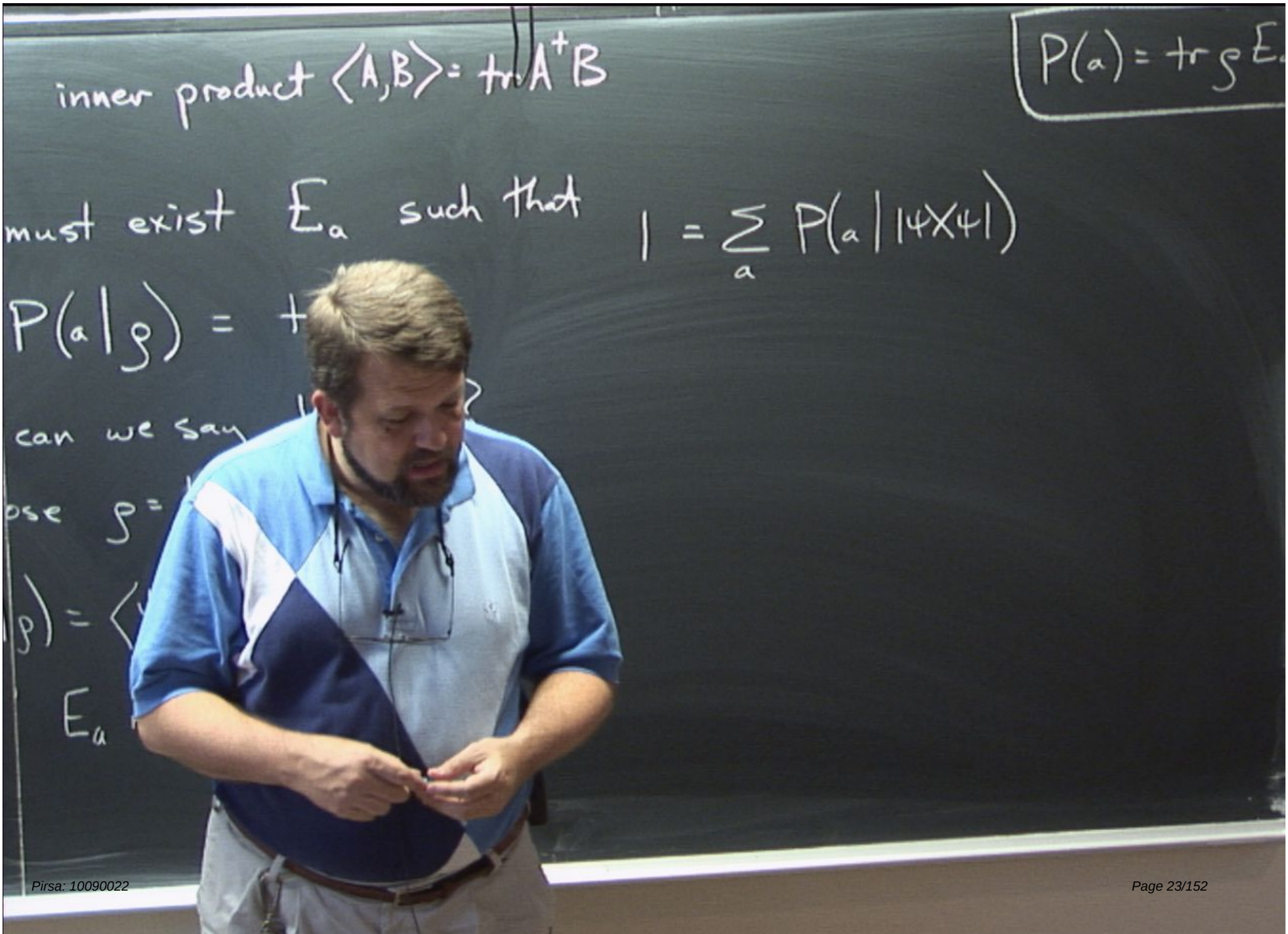
E_a must be a <u>positive op.</u>

$P(a|\rho_2)$

ρ


inner product ⟨A,B⟩ = tr A†B
P(a) = tr ρ E
must exist E_a such that
1 = Σ_a P(a | |ψ⟩⟨ψ|)
P(a|ρ) = t
can we say
ose ρ =
ρ) = ⟨
E_a

inner product $\langle A,B\rangle = \mathrm{tr}\, A^{+}B$

$$\boxed{P(a) = \mathrm{tr}\,\rho E_a}$$

must exist E_a such that

$$P(a|\rho) = \mathrm{tr}\,\rho E_a$$

$$1 = \sum_a P(a\,|\,|\psi\rangle\langle\psi|)$$

$$= \sum_a \langle\psi|E_a|\psi\rangle$$

$$= \langle\psi|\left(\sum_a E_a\right)|\psi\rangle$$

$\langle\psi|\psi\rangle = 1$

can we say about E_a?

pose $\rho = |\psi\rangle\langle\psi|$. Then

$$P(a|\rho) = \langle\psi|E_a|\psi\rangle \leftarrow \text{real}, \geq 0$$

E_a must be a <u>positive op.</u>

inner product $\langle A,B\rangle = \mathrm{tr}\, A^\dagger B$

$$\boxed{P(a) = \mathrm{tr}\, \rho E_a}$$

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inner product $\langle A,B\rangle = \mathrm{tr}\, A^\dagger B$

$$\boxed{P(a) = \mathrm{tr}\, \rho E_a}$$

"effect operators"

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$$\boxed{\sum_a E_a = 1}$$

Open sys. Q

Environment E (initially $|0\rangle$)

interaction U

E-basis $\{|k\rangle\}$

$$U|\psi,0\rangle = \sum_k |\psi_k\rangle \otimes |k\rangle$$

$$|\psi_k\rangle = \langle k|U|\psi,0\rangle$$

$\rho \in \mathcal{B}(\mathcal{H})$

There

What

Supp

$P(a|$

Open sys. Q

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Measure E using $\{|k\rangle\}$ basis

$\rho \in \mathcal{B}(\mathcal{H})$

There

What

Supp

$P(a|$

Open sys. Q

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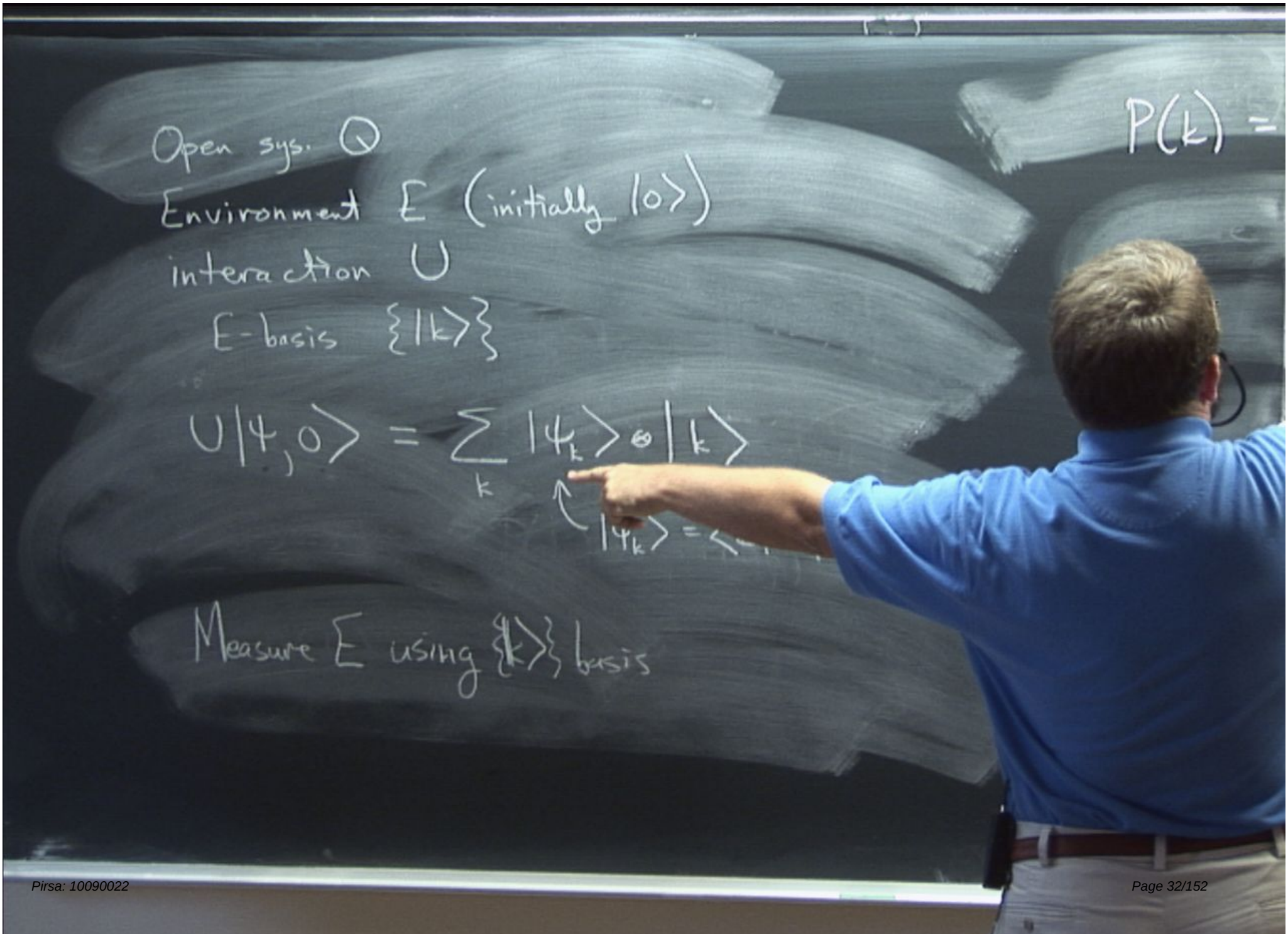
What

Measure E using $\{|k\rangle\}$ basis


Open sys. Q
Environment E (initially |0>)
interaction U
E-basis {|k>}
U|ψ,0> = Σ_k |ψ_k> ⊗ |k>
|ψ_k> =
Measure E using {|k>} basis
P(k) =

$$P(k) = \langle \psi_k | \psi_k \rangle$$

$\rangle)$

$|k\rangle$

$\rangle = \langle k | U | \psi, 0 \rangle = A_k | \psi \rangle$

$$P(k) = \langle \psi_k | \psi_k \rangle$$

$$= \langle \psi | A_k^\dagger A_k | \psi \rangle$$

$$= \mathrm{tr}\, \rho E_k \quad \text{where } E_k = A_k^\dagger A_k$$

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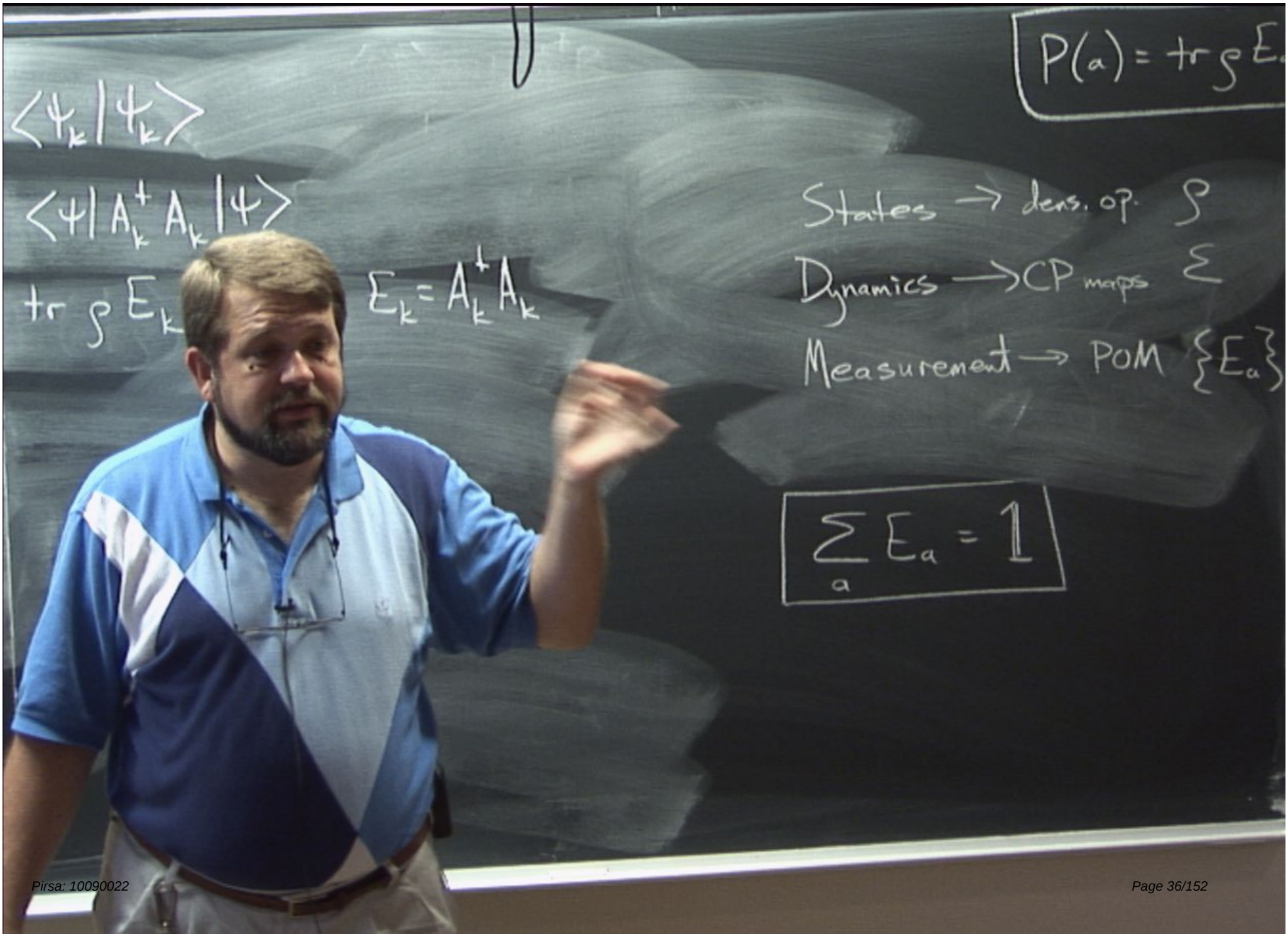
$P(a) = \mathrm{tr}\, \rho E_a$
$\langle \psi_k | \psi_k \rangle$
$\langle \psi | A_k^\dagger A_k | \psi \rangle$
$\mathrm{tr}\, \rho E_k$
$E_k = A_k^\dagger A_k$
States → dens. op. ρ
Dynamics → CP maps $\mathcal{E}$
Measurement → POM $\{E_a\}$
$\sum_a E_a = 1$

$$\langle \psi_k | \psi_k \rangle$$

$$\langle \psi | A_k^\dagger A_k | \psi \rangle$$

$$\mathrm{tr}\, \rho E_k \quad \text{where } E_k = A_k^\dagger A_k$$

$$\boxed{P(a) = \mathrm{tr}\, \rho E_a}$$

States $\rightarrow$ dens. op. ρ

Dynamics $\rightarrow$ CP maps $\mathcal{E}$

Measurement $\rightarrow$ POM $\{E_a\}$

$$\boxed{\sum_a E_a = \mathbb{1}}$$

$$\langle \psi_k | \psi_k \rangle$$

$$\langle \psi | A_k^\dagger A_k | \psi \rangle$$

$$\mathrm{tr}\, \rho E_k \quad \text{where } E_k = A_k^\dagger A_k$$

$$\boxed{P(a) = \mathrm{tr}\, \rho E_a}$$

States $\rightarrow$ dens. op. ρ

Dynamics $\rightarrow$ CP maps $\mathcal{E}$

Measurement $\rightarrow$ POM $\{E_a\}$

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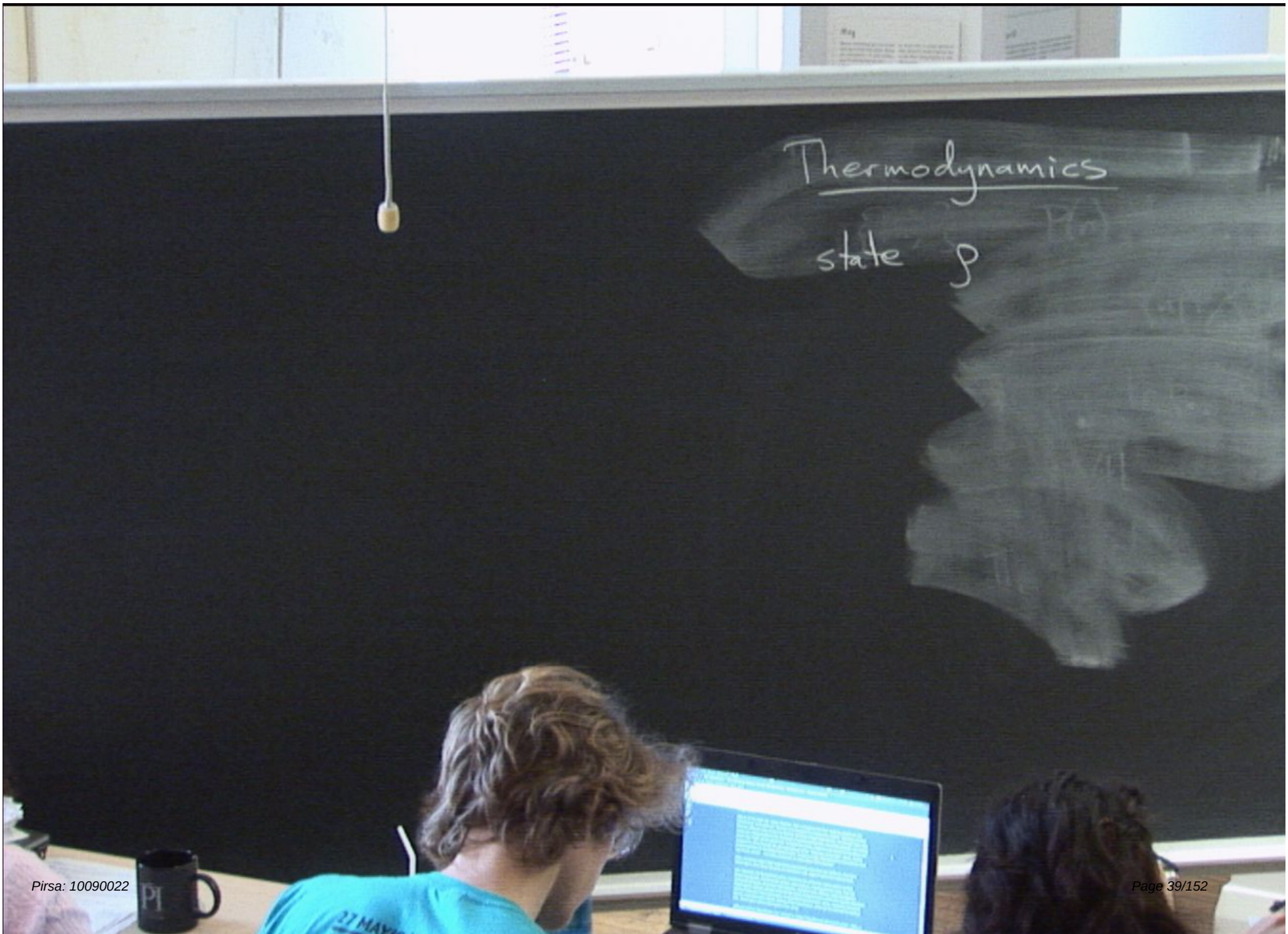
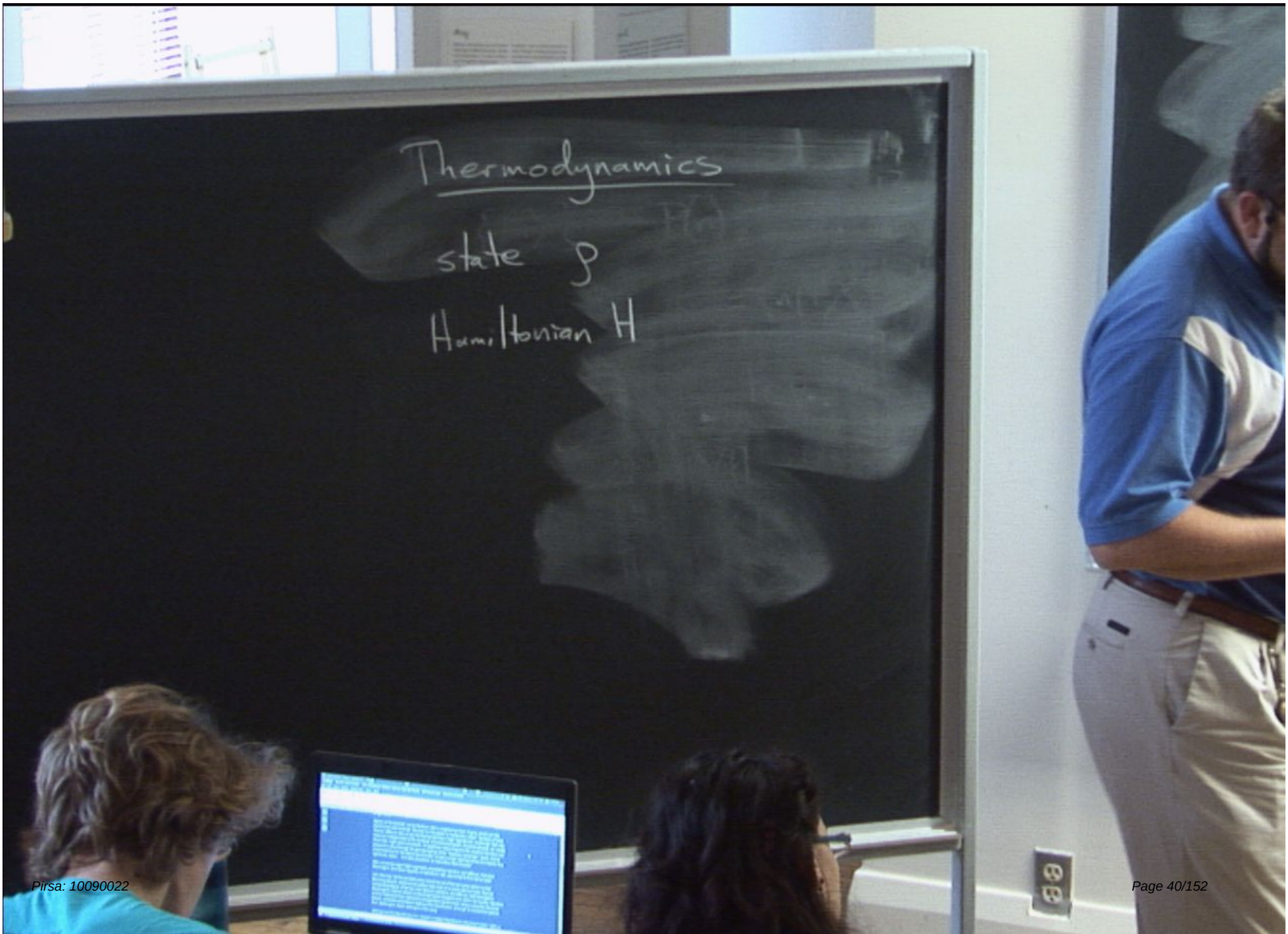
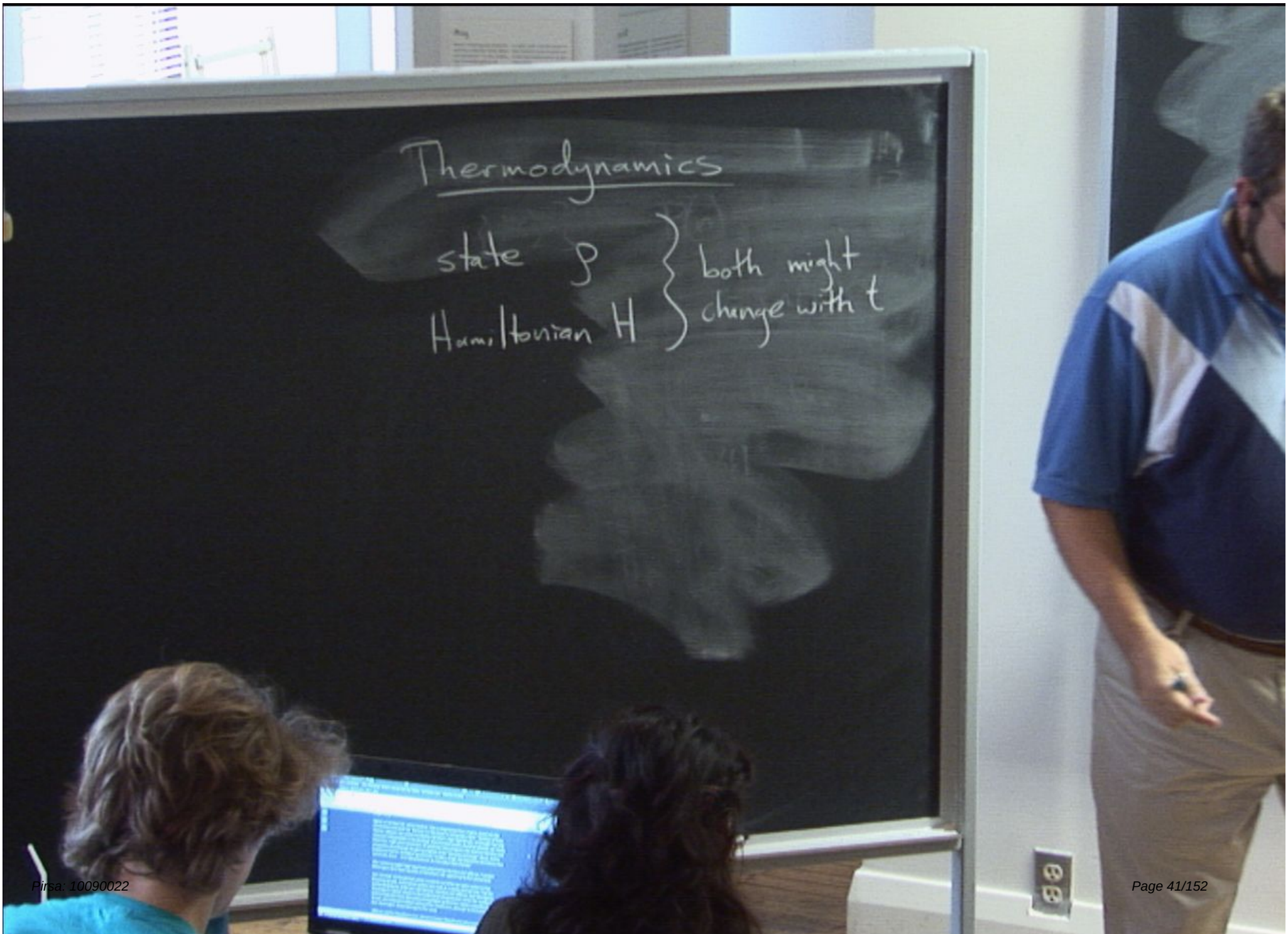
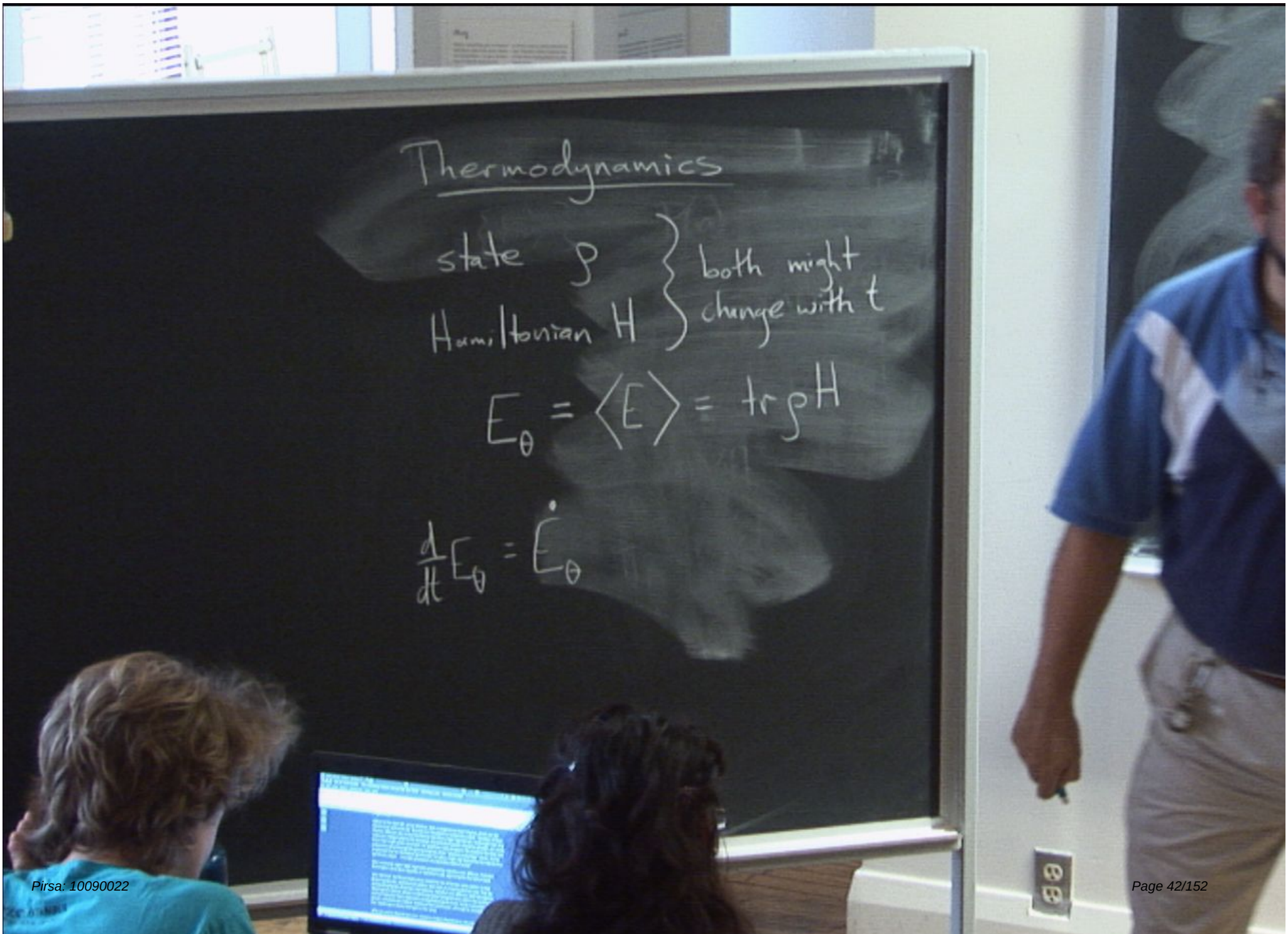
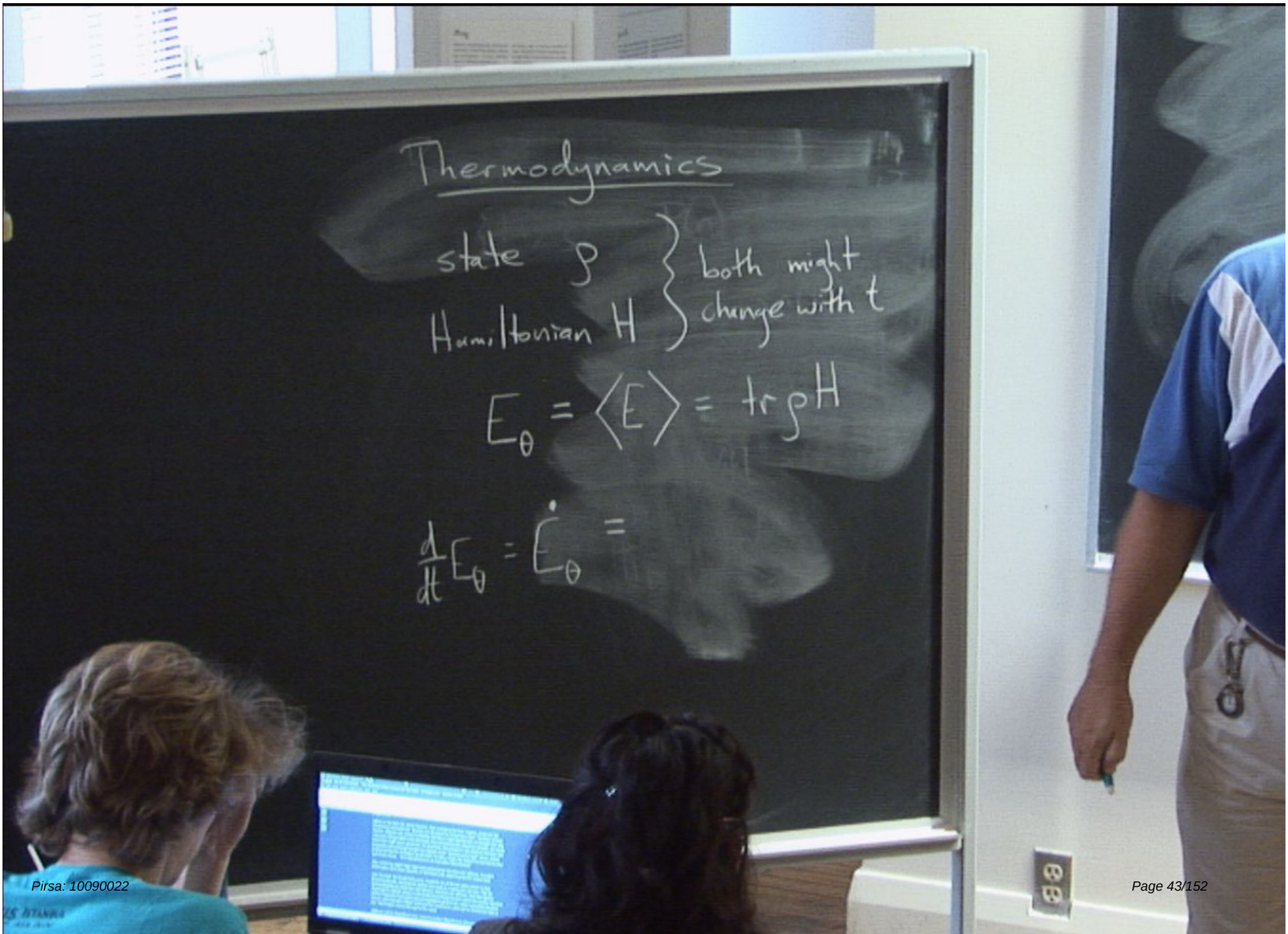
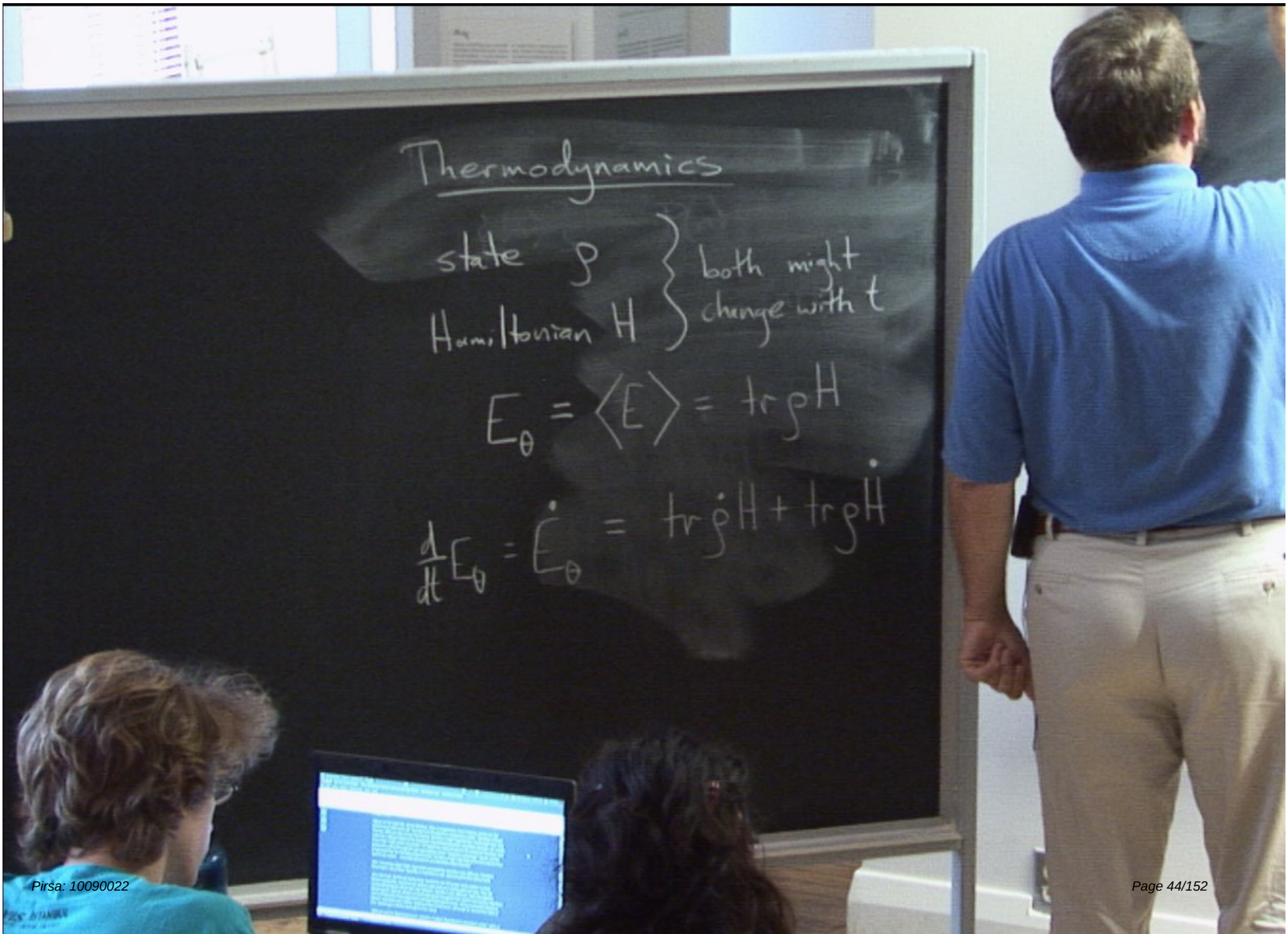

Thermodynamics
state ρ


Thermodynamics
state ρ
Hamiltonian H


Thermodynamics
state ρ
Hamiltonian H
both might
change with t


Thermodynamics
state ρ
Hamiltonian H
both might change with t
$E_0 = \langle E \rangle = \mathrm{tr}\,\rho H$
$\frac{d}{dt} E_0 = \dot{E}_0$


Thermodynamics
state ρ
Hamiltonian H
both might change with t
$E_\theta = \langle E \rangle = \mathrm{tr}\,\rho H$
$\frac{d}{dt} E_\theta = \dot{E}_\theta =$


Thermodynamics
state ρ
Hamiltonian H
both might change with t
$E_0 = \langle E \rangle = \mathrm{tr}\,\rho H$
$\frac{d}{dt} E_0 = \dot{E}_0 = \mathrm{tr}\,\dot{\rho} H + \mathrm{tr}\,\rho \dot{H}$

Over dt

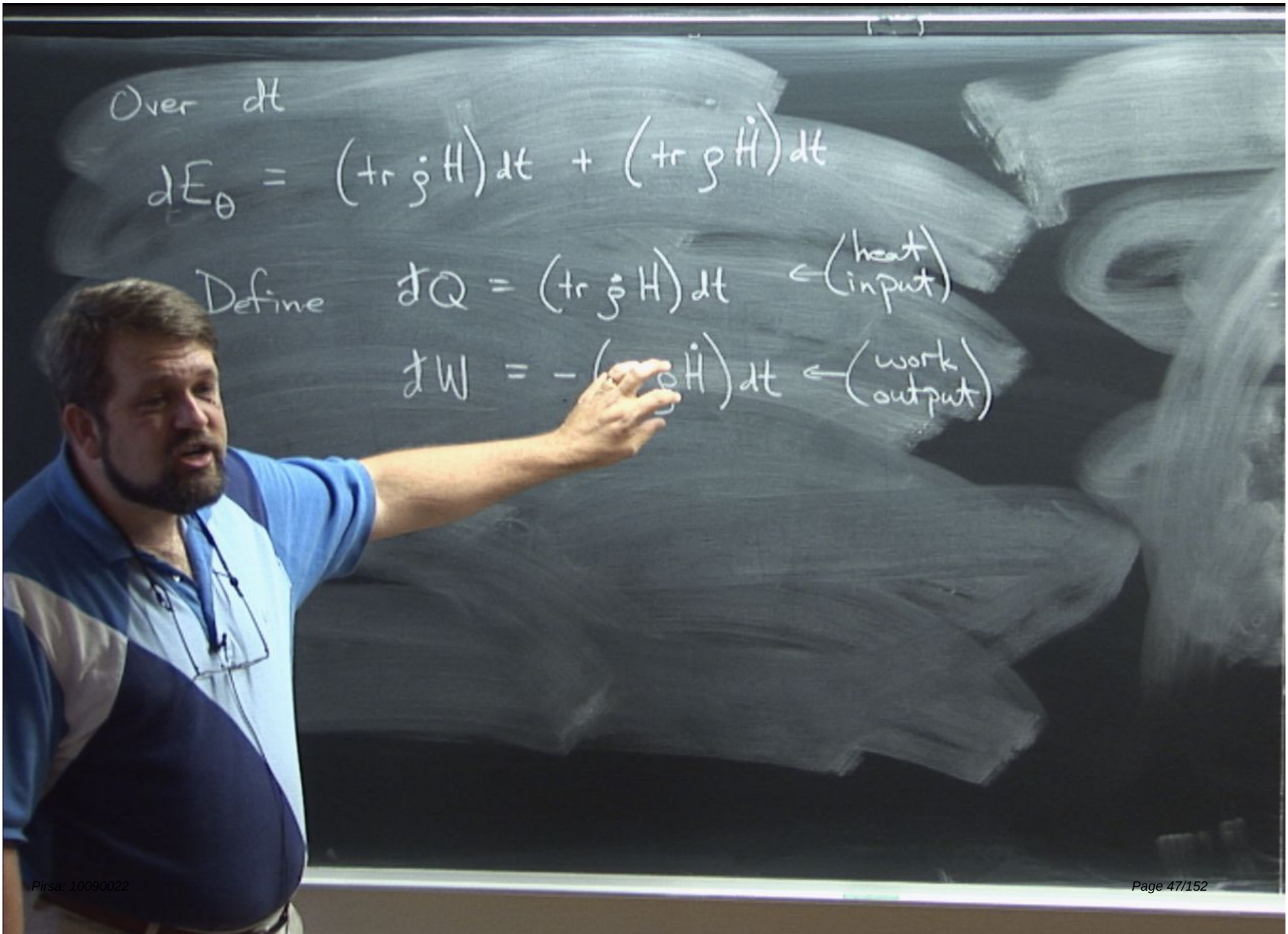
$$dE_\theta = (\mathrm{tr}\,\dot{\rho} H)\,dt + (\mathrm{tr}\,\rho \dot{H})\,dt$$

Over dt

$$dE_\theta = (\mathrm{tr}\,\dot{\rho} H)\,dt + (\mathrm{tr}\,\rho \dot{H})\,dt$$

Define $đQ = (\mathrm{tr}\,\dot{\rho} H)\,dt \leftarrow \begin{pmatrix}\text{heat}\\ \text{input}\end{pmatrix}$

$đW = -(\mathrm{tr}\,\rho \dot{H})\,dt \leftarrow \begin{pmatrix}\text{work}\\ \text{output}\end{pmatrix}$


Over dt
$dE_\theta = (\mathrm{tr}\,\dot{\rho} H)\,dt + (\mathrm{tr}\,\rho \dot{H})\,dt$
Define $đQ = (\mathrm{tr}\,\dot{\rho} H)\,dt$ ← (heat input)
$đW = -(\rho \dot{H})\,dt$ ← (work output)

Over dt

$$dE_\theta = (\mathrm{tr}\,\dot{\rho} H)\,dt + (\mathrm{tr}\,\rho \dot{H})\,dt$$

Define $đQ = (\mathrm{tr}\,\dot{\rho} H)\,dt \leftarrow \begin{pmatrix}\text{heat}\\ \text{input}\end{pmatrix}$

$đW = -(\mathrm{tr}\,\rho \dot{H})\,dt \leftarrow \begin{pmatrix}\text{work}\\ \text{output}\end{pmatrix}$

$$\boxed{dE_\theta = đQ - đW}$$

Over dt

$$dE_\theta = (\mathrm{tr}\,\dot{\rho} H)\,dt + (\mathrm{tr}\,\rho \dot{H})\,dt$$

Define $đQ = (\mathrm{tr}\,\dot{\rho} H)\,dt \leftarrow \begin{pmatrix}\text{heat}\\ \text{input}\end{pmatrix}$

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$$\boxed{dE_\theta = đQ - đW}$$

Suppose we go $(\rho_a, H_a) \to (\rho_b, H_b)$

Over dt

$$dE_\theta = (\mathrm{tr}\,\dot{\rho} H)\,dt + (\mathrm{tr}\,\rho \dot{H})\,dt$$

Define $đQ = (\mathrm{tr}\,\dot{\rho} H)\,dt \leftarrow \begin{pmatrix}\text{heat}\\ \text{input}\end{pmatrix}$

$đW = -(\mathrm{tr}\,\rho \dot{H})\,dt \leftarrow \begin{pmatrix}\text{work}\\ \text{output}\end{pmatrix}$

$$\boxed{dE_\theta = đQ - đW}$$

Suppose we go $(\rho_a, H_a) \to (\rho_b, H_b)$

$$\Delta E_\theta = \mathrm{tr}\,\rho_b H_b - \mathrm{tr}\,\rho_a H_a$$

Over dt

$$dE_\theta = (\mathrm{tr}\,\dot{\rho} H)\,dt + (\mathrm{tr}\,\rho \dot{H})\,dt$$

Define $đQ = (\mathrm{tr}\,\dot{\rho} H)\,dt \leftarrow \begin{pmatrix}\text{heat}\\ \text{input}\end{pmatrix}$

$đW = -(\mathrm{tr}\,\rho \dot{H})\,dt \leftarrow \begin{pmatrix}\text{work}\\ \text{output}\end{pmatrix}$

$$\boxed{dE_\theta = đQ - đW}$$

Suppose we go $(\rho_a, H_a) \to (\rho_b, H_b)$

$$\Delta E_\theta = \mathrm{tr}\,\rho_b H_b - \mathrm{tr}\,\rho_a H_a$$

path independent.

ΔE

$\rho \dot{H}) dt$

$dt \leftarrow \begin{pmatrix} \text{heat} \\ \text{input} \end{pmatrix}$

$\dot{H}) dt \leftarrow \begin{pmatrix} \text{work} \\ \text{output} \end{pmatrix}$

$) \rightarrow (\rho_b, H_b)$

independent.

$$\Delta E|_0 = Q - W$$

$$Q = \int_a^b đQ$$

$$W = \int_a^b đW$$

$\rho \dot{H}) dt$

$dt \leftarrow$ (heat input)

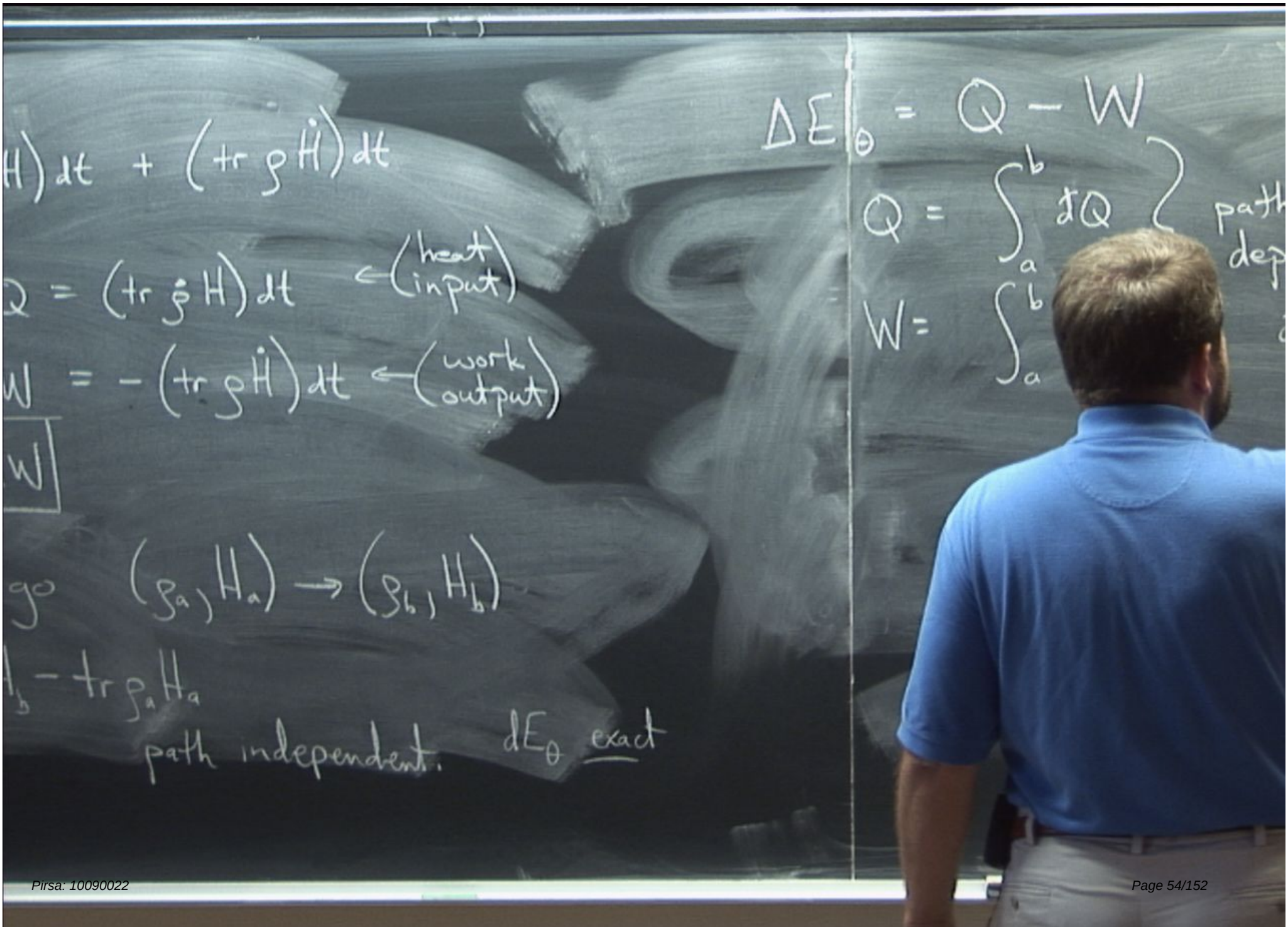
$\dot{H}) dt \leftarrow$ (work output)

$) \rightarrow (\rho_b, H_b)$

independent.

$$\Delta E|_\theta = Q - W$$

$$\left.\begin{aligned} Q &= \int_a^b đQ \\ W &= \int_a^b đW \end{aligned}\right\} \text{path-dependent!}$$


H) dt + (tr ρ Ḣ) dt
Q = (tr ρ̇ H) dt ← (heat input)
W = − (tr ρ Ḣ) dt ← (work output)
W
go (ρa, Ha) → (ρb, Hb)
Hb − tr ρa Ha
path independent. dEθ exact
ΔEθ = Q − W
Q = ∫a b đQ
W = ∫a b
path
dep

$$\Delta E\big|_\theta = Q - W$$

$$\left.\begin{aligned} Q &= \int_a^b đQ \\ W &= \int_a^b đW \end{aligned}\right\} \text{path-dependent!}$$

$đQ,\ đW$ <u>inexact</u>

heat
input)

(work
output)

$H_b)$

dE_θ <u>exact</u>

$$= Q - W$$

$$\left.\begin{aligned} Q &= \int_a^b đQ \\ W &= \int_a^b đW \end{aligned}\right\} \text{path-dependent!}$$

$đQ$, $đW$ <u>inexact</u>

$$đQ = (\mathrm{tr}\,\rho\dot{H})\,dt$$

$$đW = -(\mathrm{tr}\,\rho\dot{H})\,dt$$

$Q - W$

$$\left.\begin{aligned}\int_a^b đQ \\ \int_a^b đW\end{aligned}\right\}$$ path-dependent!

$đQ$, $đW$ <u>inexact</u>

$$đQ = (\mathrm{tr}\,\dot{\rho} H)\,dt$$
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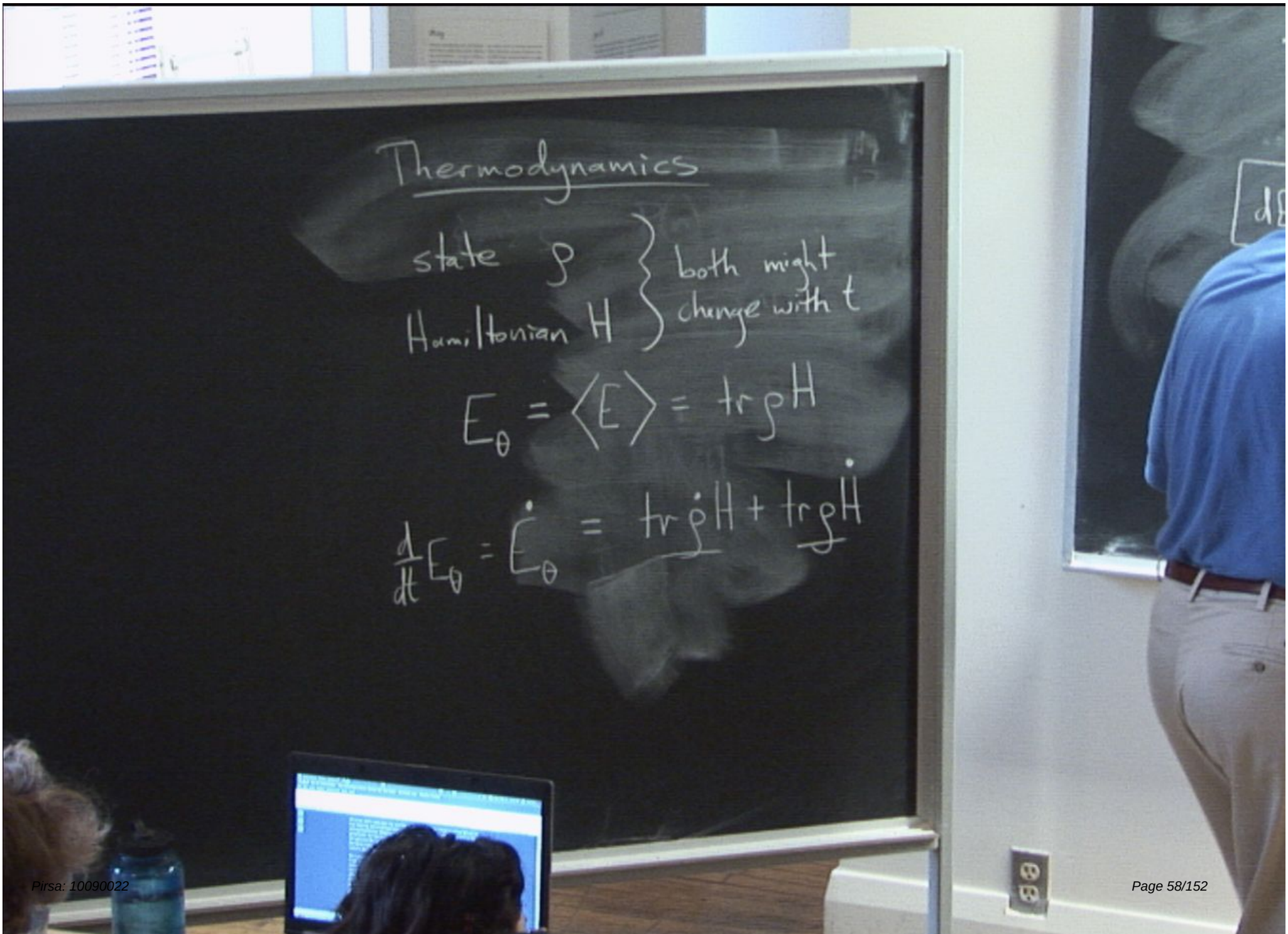
Thermodynamics
state ρ
Hamiltonian H
both might change with t
$E_\theta = \langle E \rangle = \mathrm{tr}\,\rho H$
$\frac{d}{dt} E_\theta = \dot{E}_\theta = \mathrm{tr}\,\dot{\rho} H + \mathrm{tr}\,\rho \dot{H}$

① Hamiltonian evolution $\dot{\rho} = \frac{1}{i\hbar}[H, \rho]$

① Hamiltonian evolution $\dot{\rho} = \frac{1}{i\hbar}[H,\rho]$

$$đQ = \left(\mathrm{tr}\, \dot{\rho} H\right) dt$$

$$= \frac{1}{i\hbar}\left(\mathrm{tr}\, [H,\rho] H\right) dt$$

$$= \frac{1}{i\hbar}\left(\mathrm{tr}\, H\rho H - \mathrm{tr}\, \rho H^2\right) dt$$

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$$đW = -dE_0$$

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$$= 0$$

$$đW = -dE_\theta \quad (\text{exact})$$

E_θ = therm. potential

$$\boxed{W = -\Delta E_\theta}$$

① Hamiltonian evolution $\dot{\rho} = \frac{1}{i\hbar}[H,\rho]$

② Sudden d

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$= \frac{1}{i\hbar}[H,\rho]$

② Sudden change in H: $H_a \longrightarrow H_b$ (fast)

(i.e., ρ has "no time to evolve")

$)dt$

potential

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potential

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$$Q = 0$$

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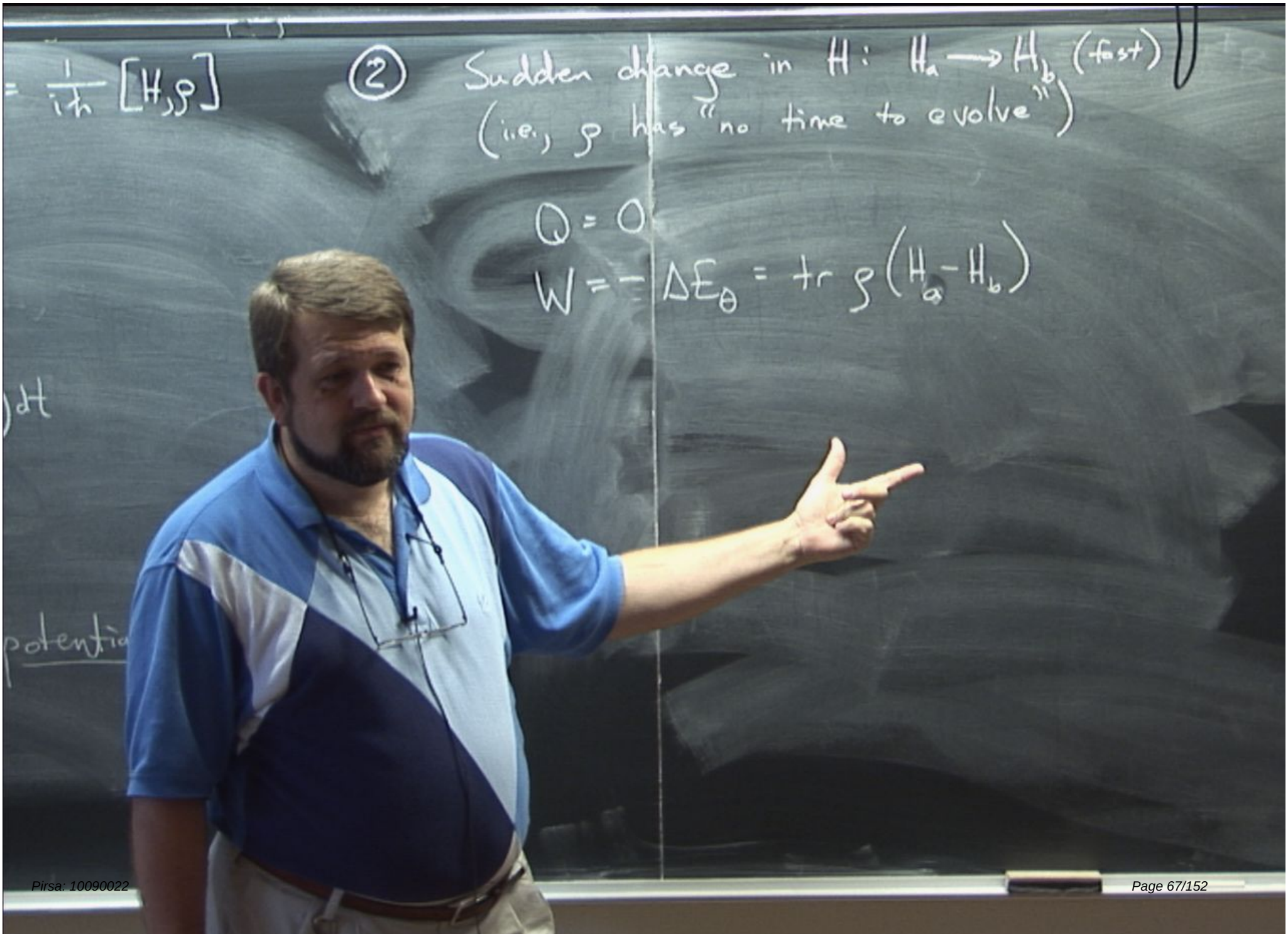
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$Q = 0$
$W = -\Delta E_\theta = \mathrm{tr}\,\rho(H_a - H_b)$
$)dt$
potentia

$= \frac{1}{i\hbar}[H,\rho]$

$)dt$

potential

(2) Sudden change in H: $H_a \longrightarrow H_b$ (fast)

(i.e., ρ has "no time to evolve")

$$Q = 0$$

$$W = -\Delta E_0 = \mathrm{tr}\,\rho\,(H_a - H_b)$$

(3)

$= \frac{1}{i\hbar}[H,\rho]$

$)dt$

potential

② Sudden change in H: $H_a \longrightarrow H_b$ (fast)

(i.e., ρ has "no time to evolve")

$$Q = 0$$

$$W = -\Delta E_\theta = \mathrm{tr}\,\rho\,(H_a - H_b)$$

③ Lindblad evolution

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$$\dot{\rho} = \frac{1}{i\hbar}[H, \rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right)$$

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Imagine $[H, L_k] = 0$

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$$H|n\rangle = E_n|n\rangle$$

$$H(L_k|n\rangle)$$

evolution

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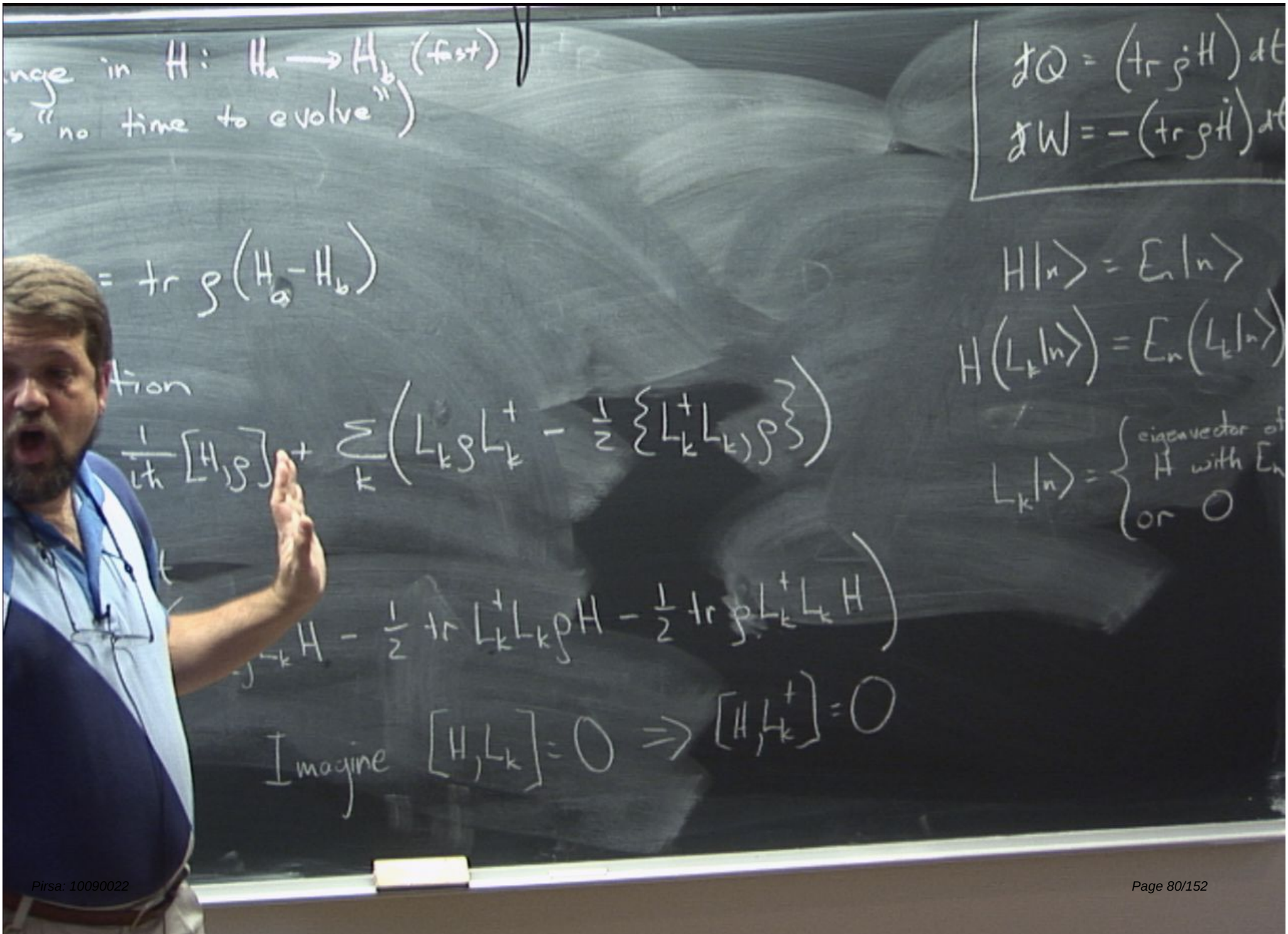
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tion
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Imagine $[H, L_k] = 0 \Rightarrow [H, L_k^\dagger] = 0 \Rightarrow \mathrm{tr}\,\dot{\rho} H =$

$\Rightarrow$

$$\delta Q = (\mathrm{tr}\,\dot{\rho} H)\,dt$$

$$\delta W = -(\mathrm{tr}\,\rho \dot{H})\,dt$$

$$H|n\rangle = E_n|n\rangle$$

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Imagine $[H, L_k] = 0 \Rightarrow [H, L_k^\dagger] = 0 \Rightarrow \mathrm{tr}\,\dot{\rho} H = 0$

$\Rightarrow đQ = 0$

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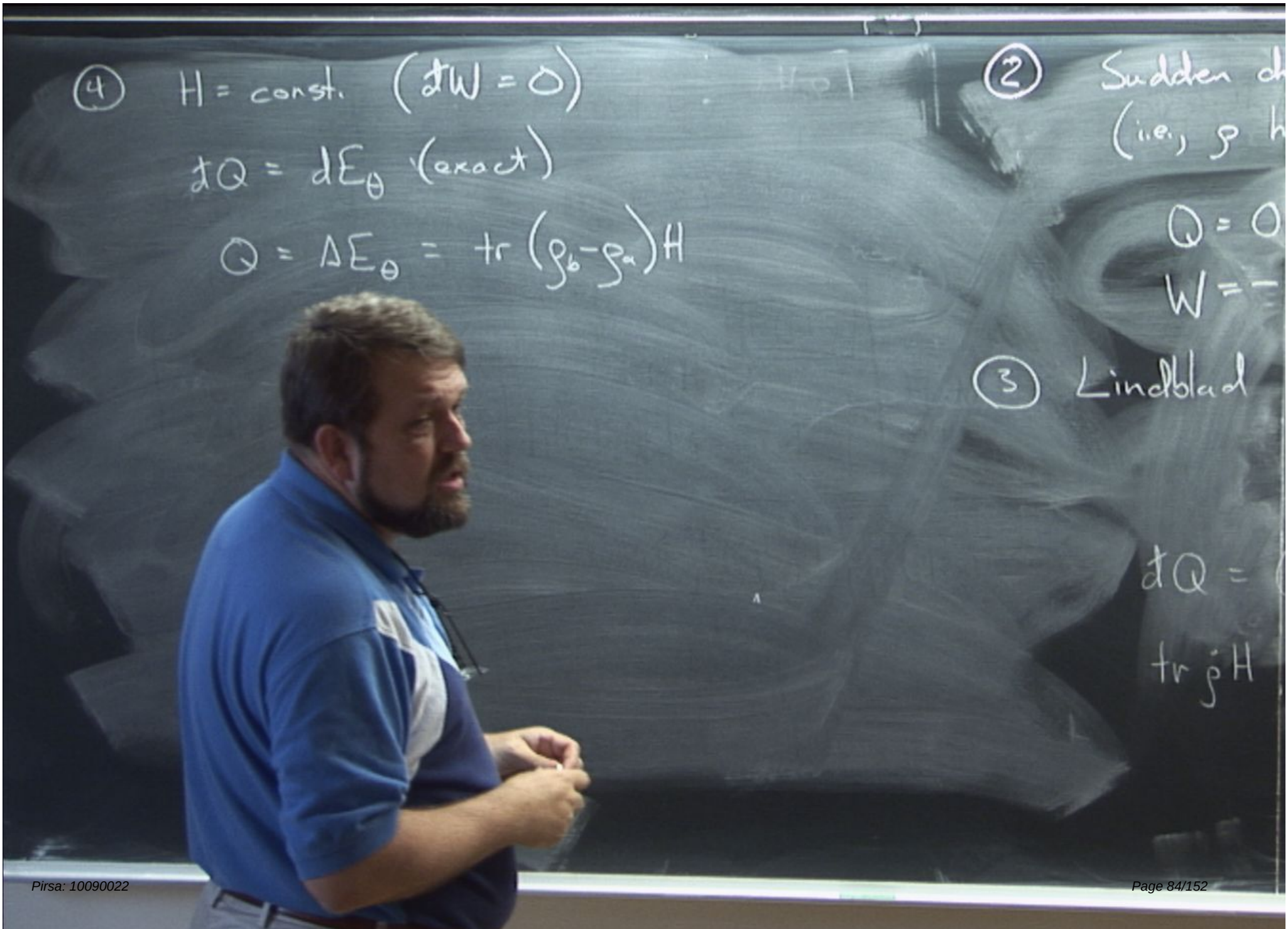
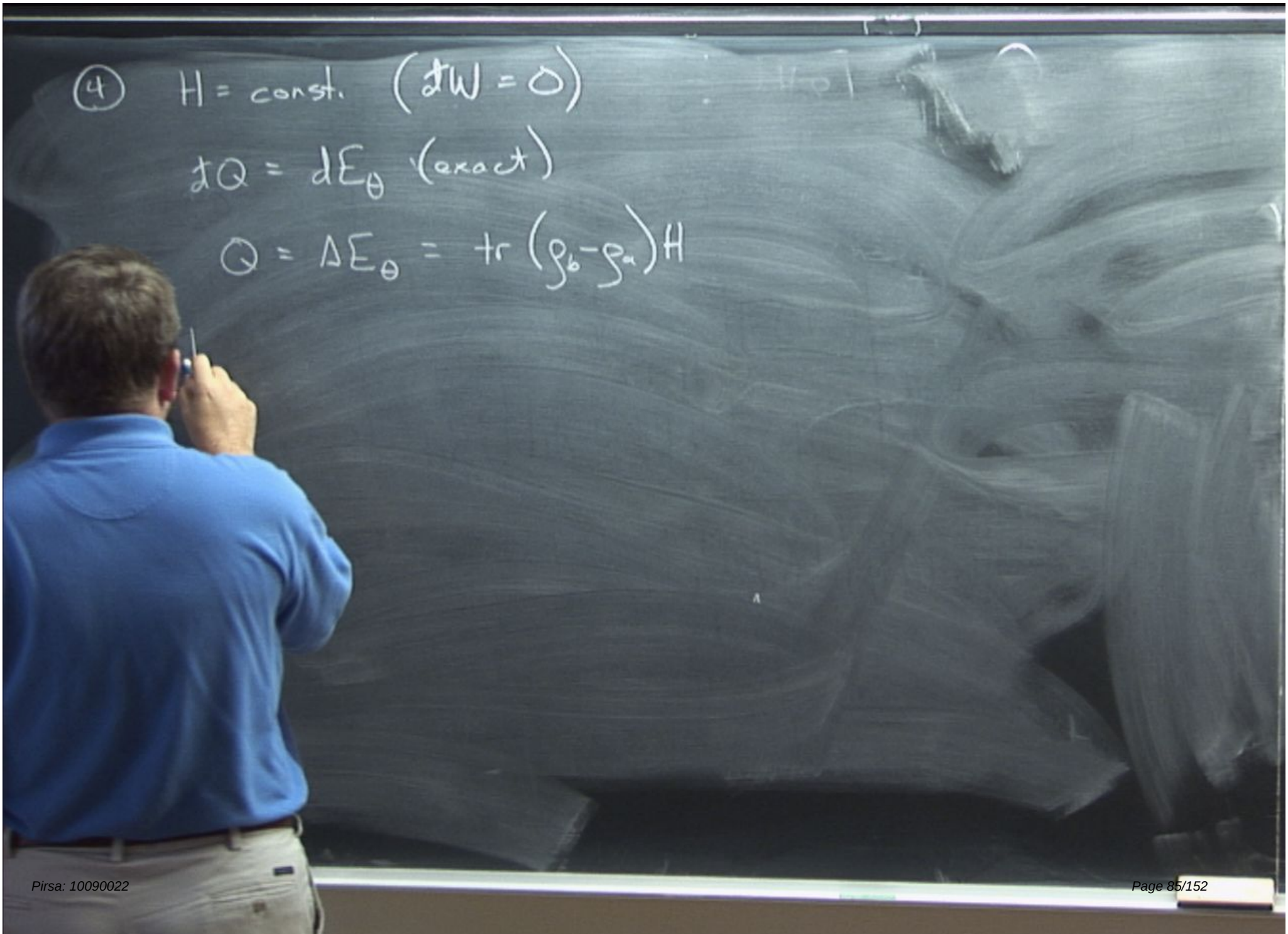

(4) H = const. (đW = 0)
đQ = dE_θ (exact)
Q = ΔE_θ = tr (ρ_b − ρ_a) H
(2) Sudden
(i.e., ρ
Q = 0
W = −
(3) Lindblad
đQ =
tr ρ̇ H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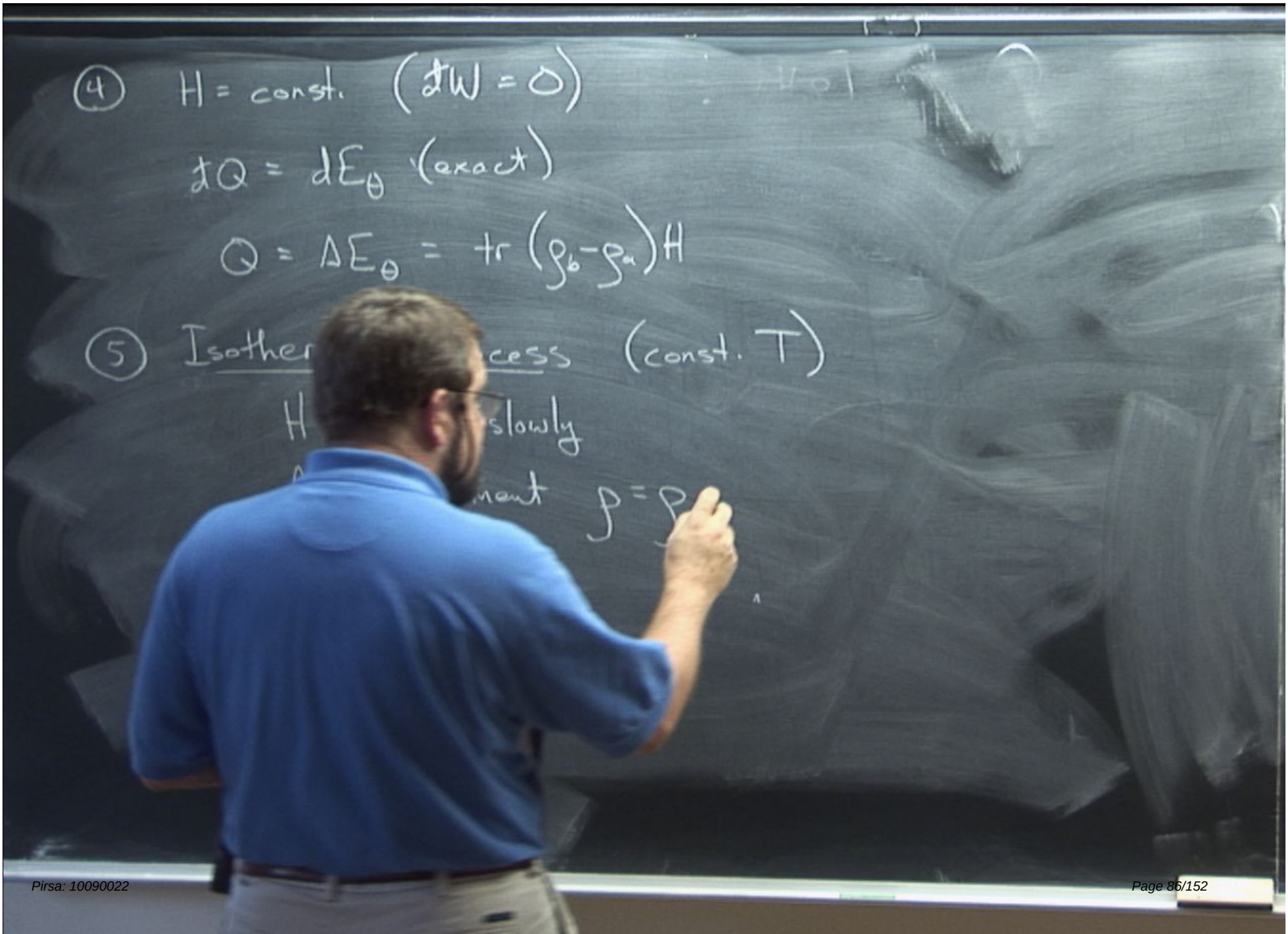
(4) H = const. (đW = 0)
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(5) Isother cess (const. T)
H slowly
nent ρ = ρ

(4) $H = \text{const.}$ $(đW = 0)$

$$đQ = dE_\theta \quad (\text{exact})$$

$$Q = \Delta E_\theta = \mathrm{tr}\,(\rho_b - \rho_a) H$$

<u>Isothermal</u> <u>process</u> (const. T)

H changes slowly

At any moment $\rho = \rho_\theta = \frac{1}{Z} e^{-\beta H}$

$$\beta = \frac{1}{kT} = \text{const.}$$

④ $H = \text{const.}$ $(đW = 0)$

$$đQ = dE_\theta \quad (\text{exact})$$

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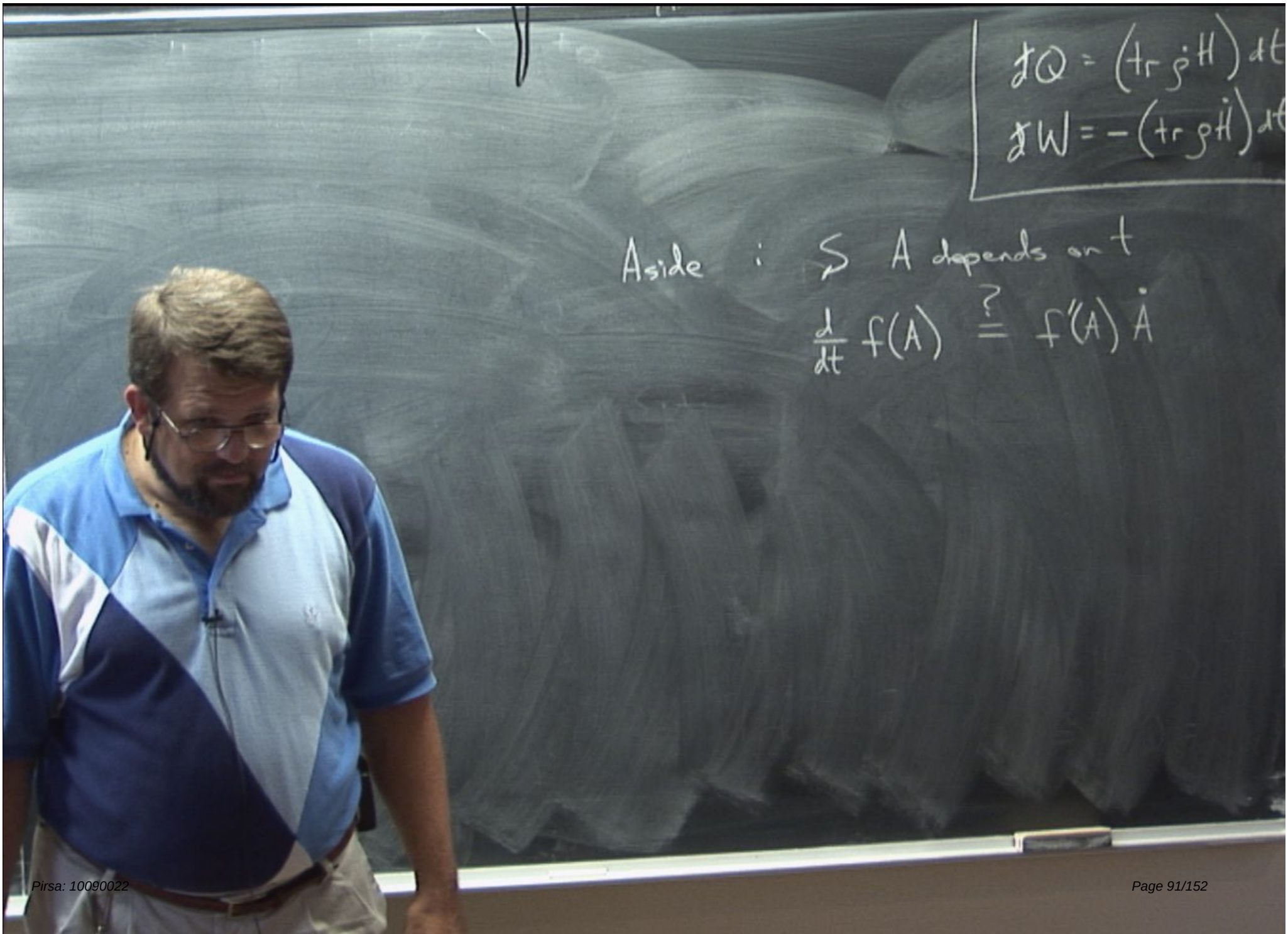
đQ = (tr ρ̇H) dt
đW = −(tr ρḢ) dt
Aside : S A depends on t
d/dt f(A) ≟ f′(A) Ȧ

$$đQ = (\mathrm{tr}\,\dot{\rho} H)\,dt$$
$$đW = -(\mathrm{tr}\,\rho \dot{H})\,dt$$

Aside : $\hookrightarrow$ A depends on t

$$\frac{d}{dt} f(A) \overset{?}{=} f'(A)\,\dot{A}$$

No!

$$f(A) = A^3 = AAA$$

$$\frac{d}{dt} f(A) = \dot{A}AA + A\dot{A}A + AA\dot{A}$$

$$\neq 3A^2\dot{A} \quad \text{in general}$$

$$đQ = (\mathrm{tr}\,\dot{\rho} H)\,dt$$
$$đW = -(\mathrm{tr}\,\rho \dot{H})\,dt$$

Aside : A depends on t

$$\frac{d}{dt} f(A) \overset{?}{=} f'(A)\,\dot{A}$$

No!

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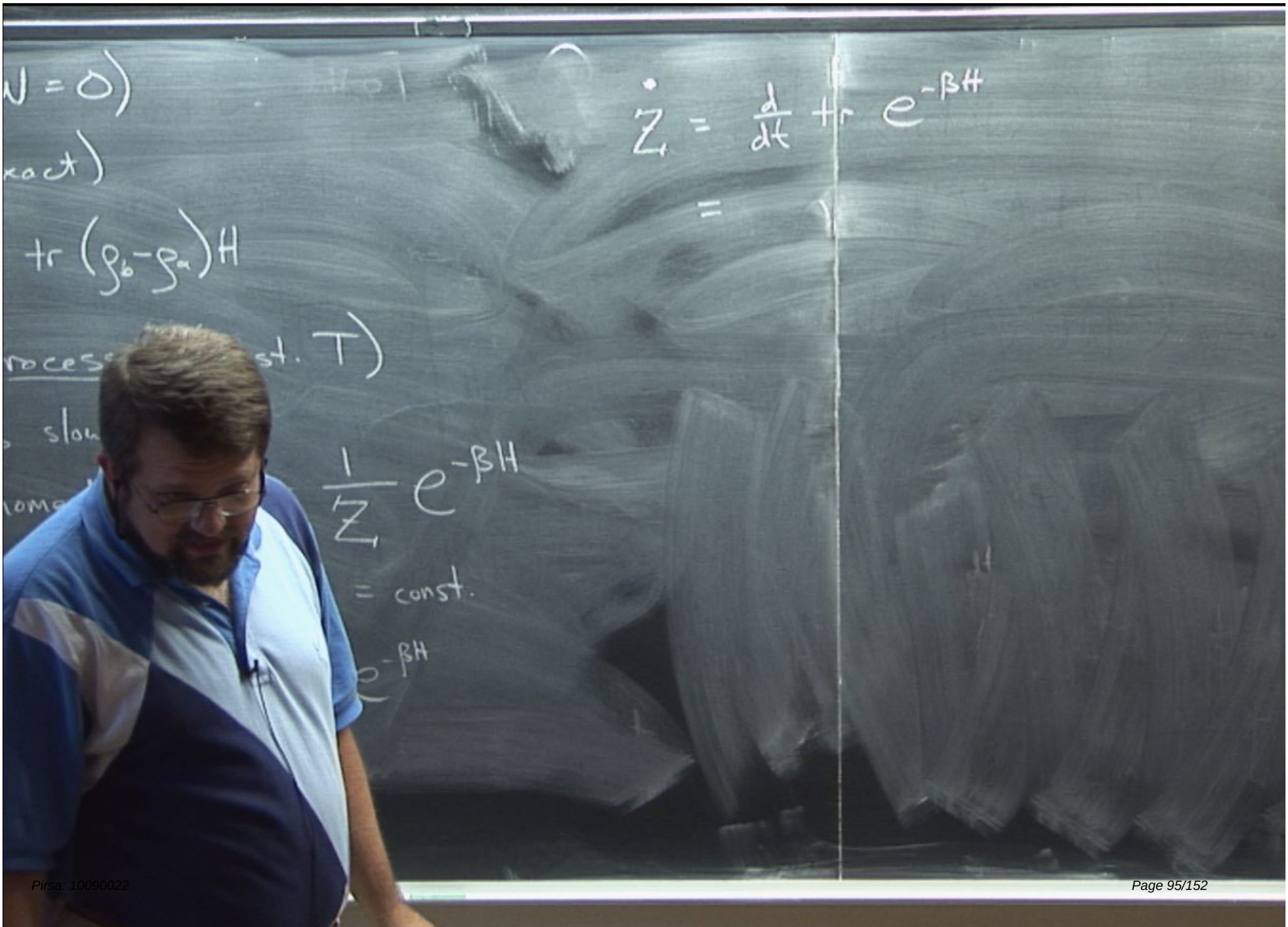
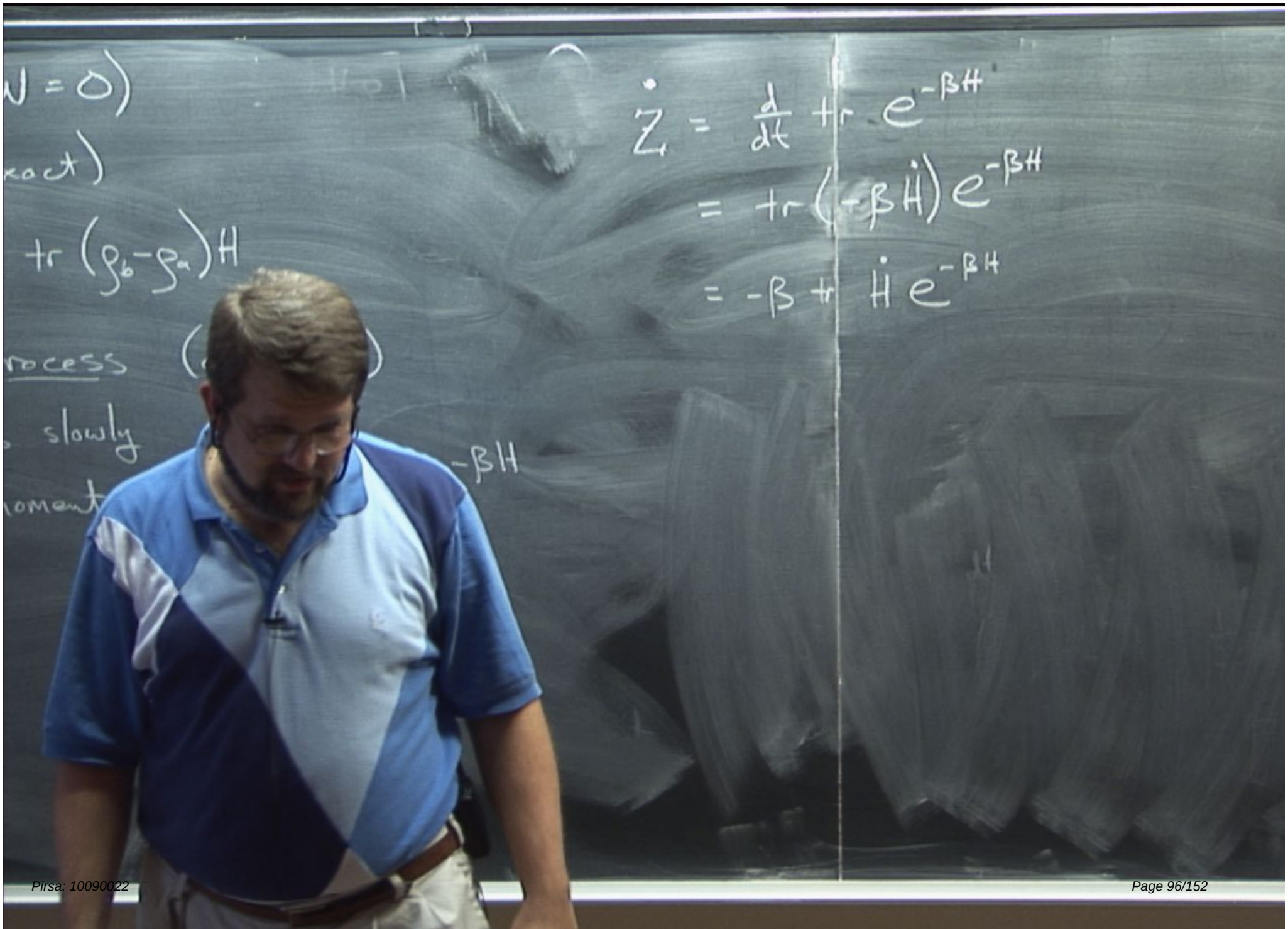
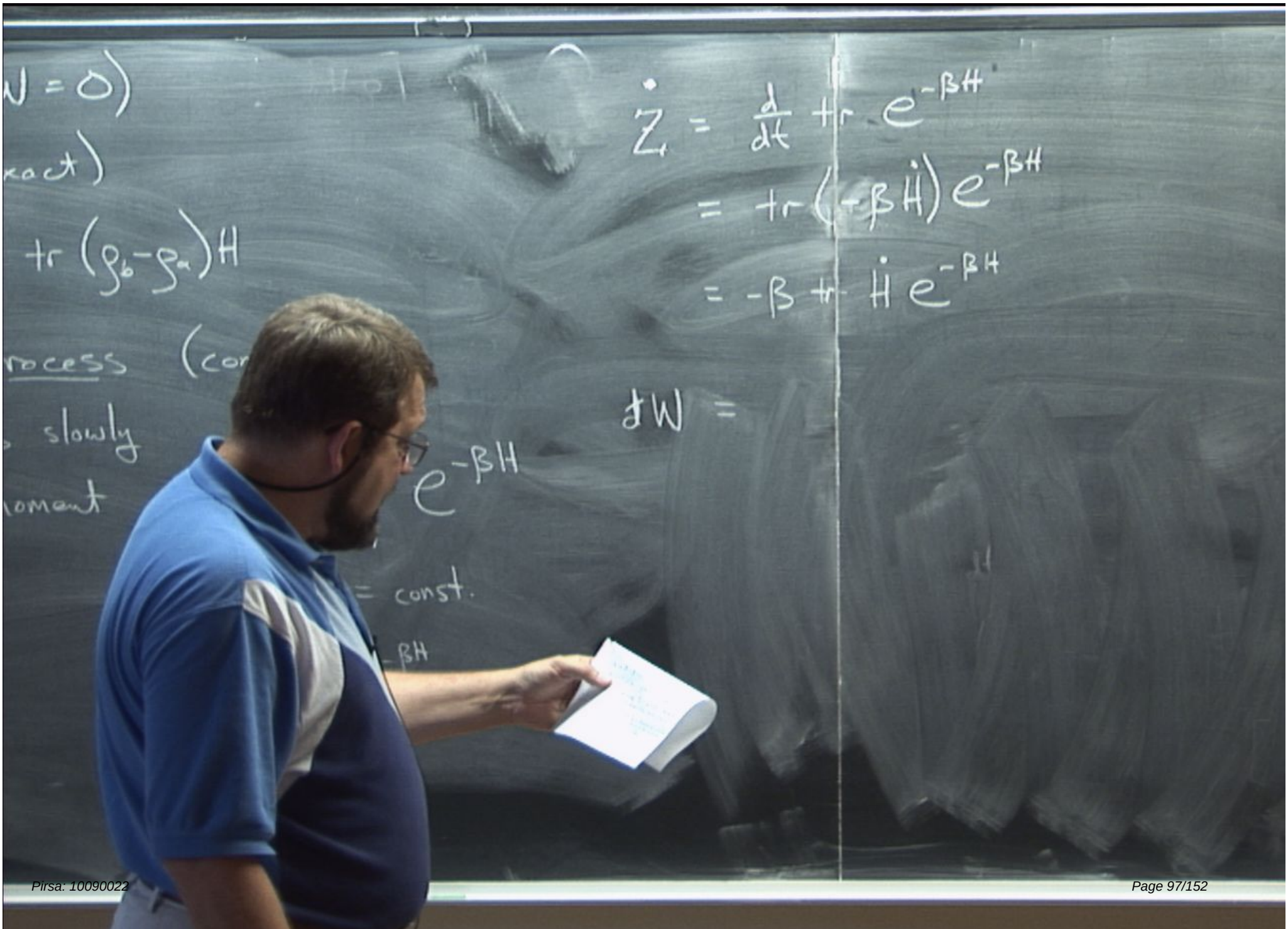
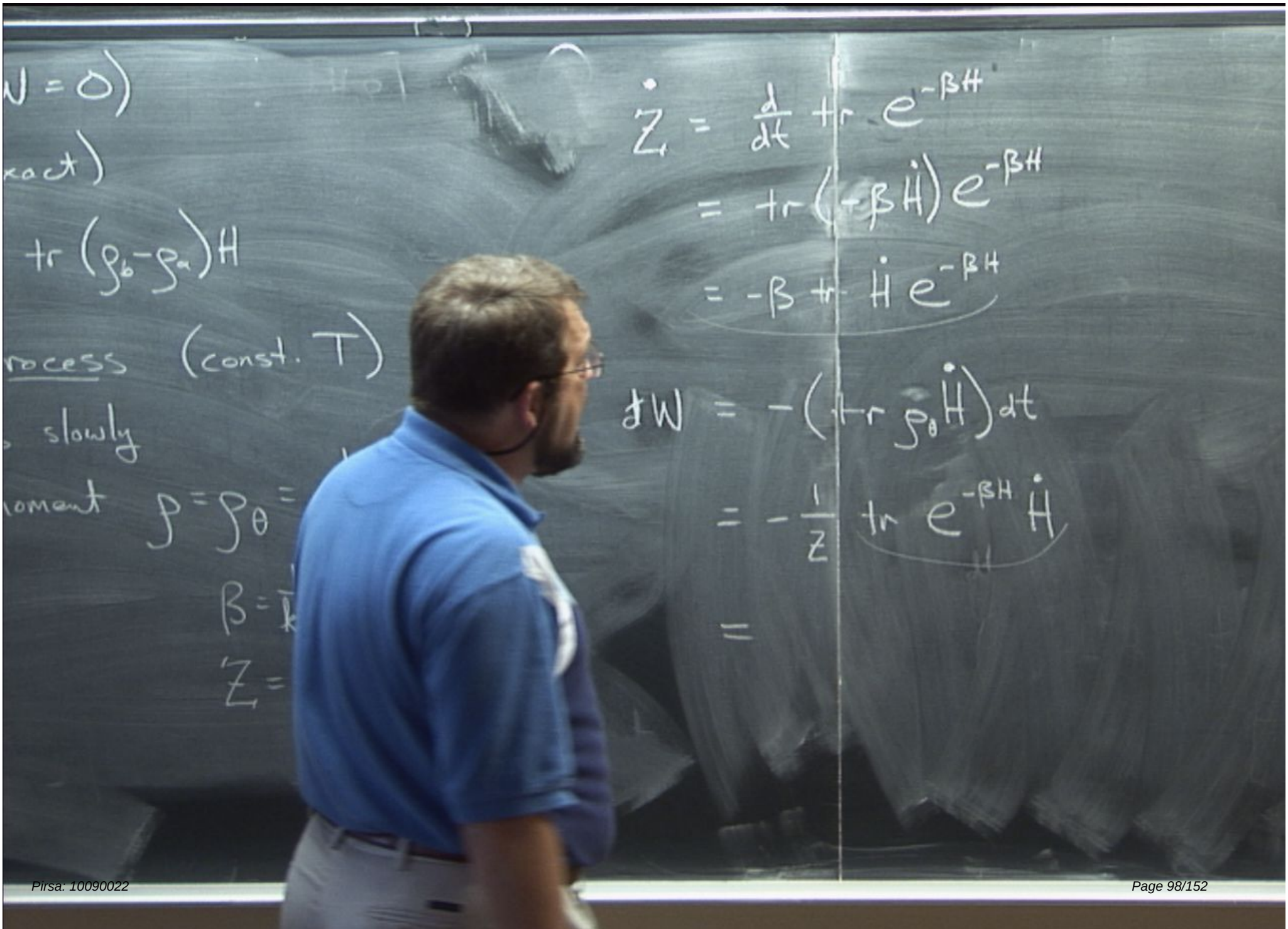
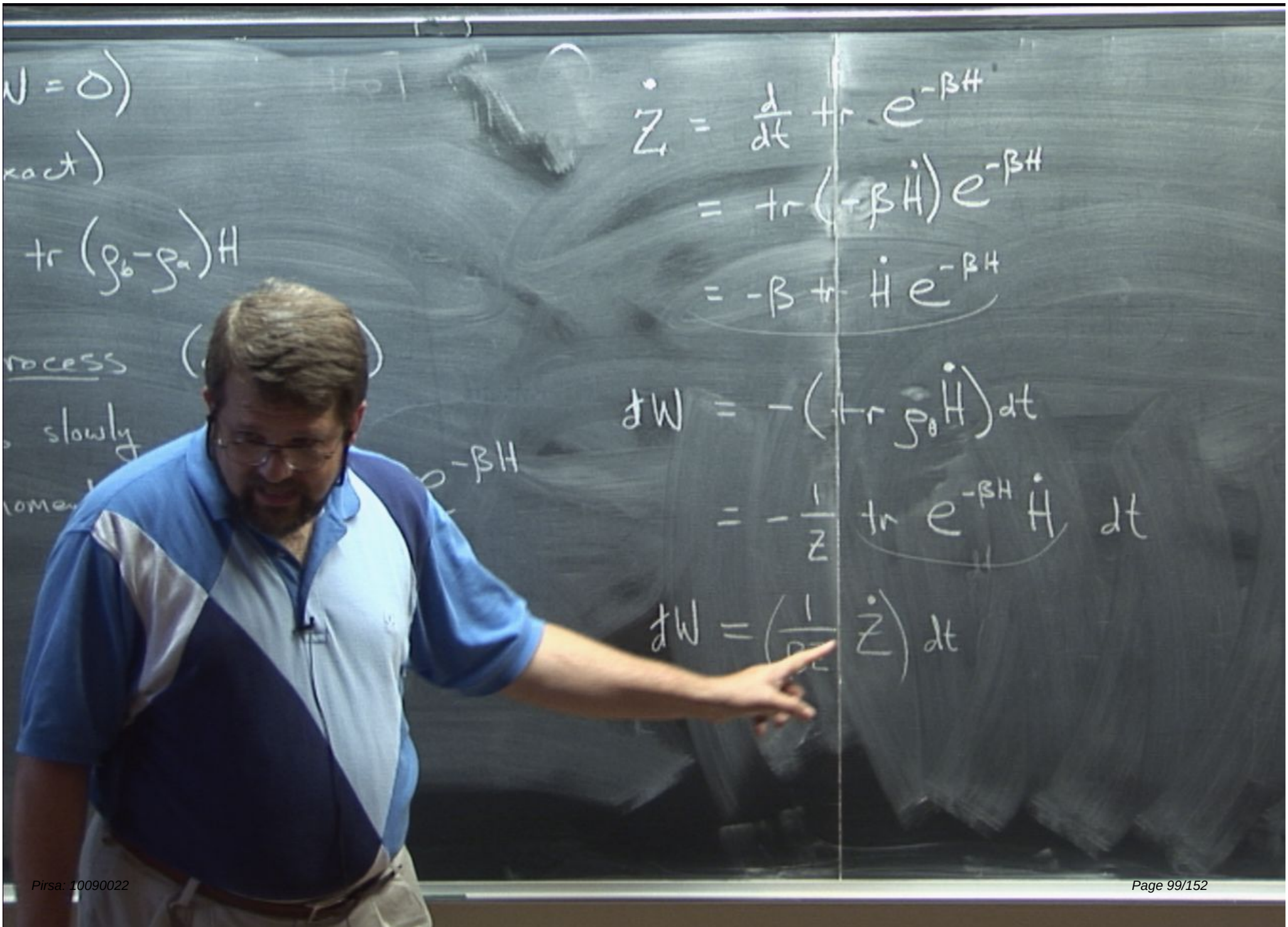

N = 0)
xact)
tr (ρb − ρa) H
rocess st. T)
slow
$\frac{1}{Z} e^{-\beta H}$
= const.
$e^{-\beta H}$
$\dot{Z} = \frac{d}{dt} \mathrm{tr}\, e^{-\beta H}$
=


N = 0)
xact)
tr (ρb − ρa) H
rocess
slowly
−βH
$\dot{Z} = \frac{d}{dt} \mathrm{tr}\, e^{-\beta H}$
$= \mathrm{tr}\,(-\beta \dot{H})\, e^{-\beta H}$
$= -\beta\, \mathrm{tr}\, \dot{H} e^{-\beta H}$


N = 0)
xact)
tr (ρb − ρa) H
rocess (con
slowly
oment
e^{−βH}
= const.
βH
Ż = d/dt tr e^{−βH}
= tr (−β Ḣ) e^{−βH}
= −β tr Ḣ e^{−βH}
đW =


N = 0)
xact)
tr (ρb − ρa) H
rocess (const. T)
slowly
moment ρ = ρθ =
β =
Z =
Ż = d/dt tr e^{−βH}
= tr (−β Ḣ) e^{−βH}
= −β tr Ḣ e^{−βH}
đW = −(tr ρθ Ḣ) dt
= −1/Z tr e^{−βH} Ḣ
=


N = 0)
xact)
tr (ρb − ρa) H
rocess
slowly
$\dot{Z} = \frac{d}{dt} \mathrm{tr}\, e^{-\beta H}$
$= \mathrm{tr}\left(-\beta \dot{H}\right) e^{-\beta H}$
$= -\beta\, \mathrm{tr}\, \dot{H} e^{-\beta H}$
$đW = -\left(\mathrm{tr}\, \rho_\theta \dot{H}\right) dt$
$= -\frac{1}{Z}\, \mathrm{tr}\, e^{-\beta H} \dot{H}\, dt$
$đW = \left(\frac{1}{\beta Z} \dot{Z}\right) dt$

N = 0)

xact)

$\mathrm{tr}\,(\rho_b - \rho_a) H$

rocess (const. T)

slowly

oment $\rho = \rho_\theta = \frac{1}{Z} e^{-\beta H}$

$\beta = \frac{1}{kT} = \text{const.}$

$Z = \mathrm{tr}\, e^{-\beta H}$

$$\dot{Z} = \frac{d}{dt}\,\mathrm{tr}\, e^{-\beta H}$$

$$= \mathrm{tr}\,(-\beta \dot{H})\, e^{-\beta H}$$

$$= -\beta\, \mathrm{tr}\, \dot{H} e^{-\beta H}$$

Defin

$$dW = -(\mathrm{tr}\, \rho_\theta \dot{H})\, dt$$

$$= -\frac{1}{Z}\, \mathrm{tr}\, e^{-\beta H} \dot{H}\, dt$$

$$dW = \left(\frac{1}{\beta Z} \dot{Z}\right) dt$$

$$= \frac{1}{\beta}\left(\frac{d}{dt} \ln Z\right) dt$$

$$\frac{d}{dt}\,\mathrm{tr}\, e^{-\beta H}$$

$$= \mathrm{tr}\left(-\beta \dot{H}\right) e^{-\beta H}$$

$$= -\beta\, \mathrm{tr}\, \dot{H} e^{-\beta H}$$

$$= -\left(\mathrm{tr}\, \rho_\theta \dot{H}\right) dt$$

$$= -\frac{1}{Z}\, \mathrm{tr}\, e^{-\beta H} \dot{H}\, dt$$

$$= \left(\frac{1}{\beta Z} \dot{Z}\right) dt$$

$$= \frac{1}{\beta}\left(\frac{d}{dt} \ln Z\right) dt$$

Define free energy

$$F_\theta = -\frac{1}{\beta} \ln Z_1 = -kT \ln$$

$$đW = -dF_\theta$$

$$đQ = ($$

$$đW = -$$

$$\boxed{\begin{aligned} đQ &= (\mathrm{tr}\,\rho\dot{H})\,dt \\ đW &= -(\mathrm{tr}\,\rho\dot{H})\,dt \end{aligned}}$$

$$e^{-\beta H}$$

$$-\beta\dot{H})\,e^{-\beta H}$$

$$\dot{H}\,e^{-\beta H}$$

$$\mathrm{tr}\,\rho_\theta\dot{H})\,dt$$

$$\mathrm{tr}\,e^{-\beta H}\,\dot{H}\,dt$$

$$\dot{Z})\,dt$$

$$\left(\frac{d}{dt}\ln Z\right)dt$$

Define free energy

$$F_\theta = -\frac{1}{\beta}\ln Z = -kT\ln Z \qquad \left(\text{only for } \rho_\theta\right)$$

$$đW = -dF_\theta$$

$$e^{-\beta H}$$

$$-\beta \dot{H}) e^{-\beta H}$$

$$\dot{H} e^{-\beta H}$$

$$\mathrm{tr}\, \rho_\theta \dot{H}) dt$$

$$\mathrm{tr}\, e^{-\beta H} \dot{H}\, dt$$

$$\dot{Z}) dt$$

$$\left(\frac{d}{dt} \ln Z\right) dt$$

$$đQ = (\mathrm{tr}\, \rho \dot{H})\, dt$$

$$đW = -(\mathrm{tr}\, \rho \dot{H})\, dt$$

Define free energy

$$F_\theta = -\frac{1}{\beta} \ln Z_1 = -kT \ln Z_1 \quad \left(\text{only for } \rho_\theta\right)$$

$$đW = -dF_\theta \leftarrow F_\theta \text{ is a } \underline{\text{potential}}$$

$$\boxed{W = -\Delta F_\theta \quad (\text{isothermal})}$$

$$đQ = (\mathrm{tr}\,\dot{\rho} H)\,dt$$
$$đW = -(\mathrm{tr}\,\rho \dot{H})\,dt$$

Define free energy

$$F_\theta = -\frac{1}{\beta}\ln Z_1 = -kT\ln Z_1 \quad \left(\begin{matrix}\text{only for}\\ \rho_\theta\end{matrix}\right)$$

$$đW = -dF_\theta$$ ← F_θ is a <u>potential</u>

$$\boxed{W = -\Delta F_\theta \quad \text{(isothermal)}}$$

Entropy

Classical

- W accessible states (equally likely)

Quantum

Entropy S_θ

Classical

- W accessible states (equally likely)

$$S_\theta = k \ln W$$

Quantum

Entropy S_θ

Classical

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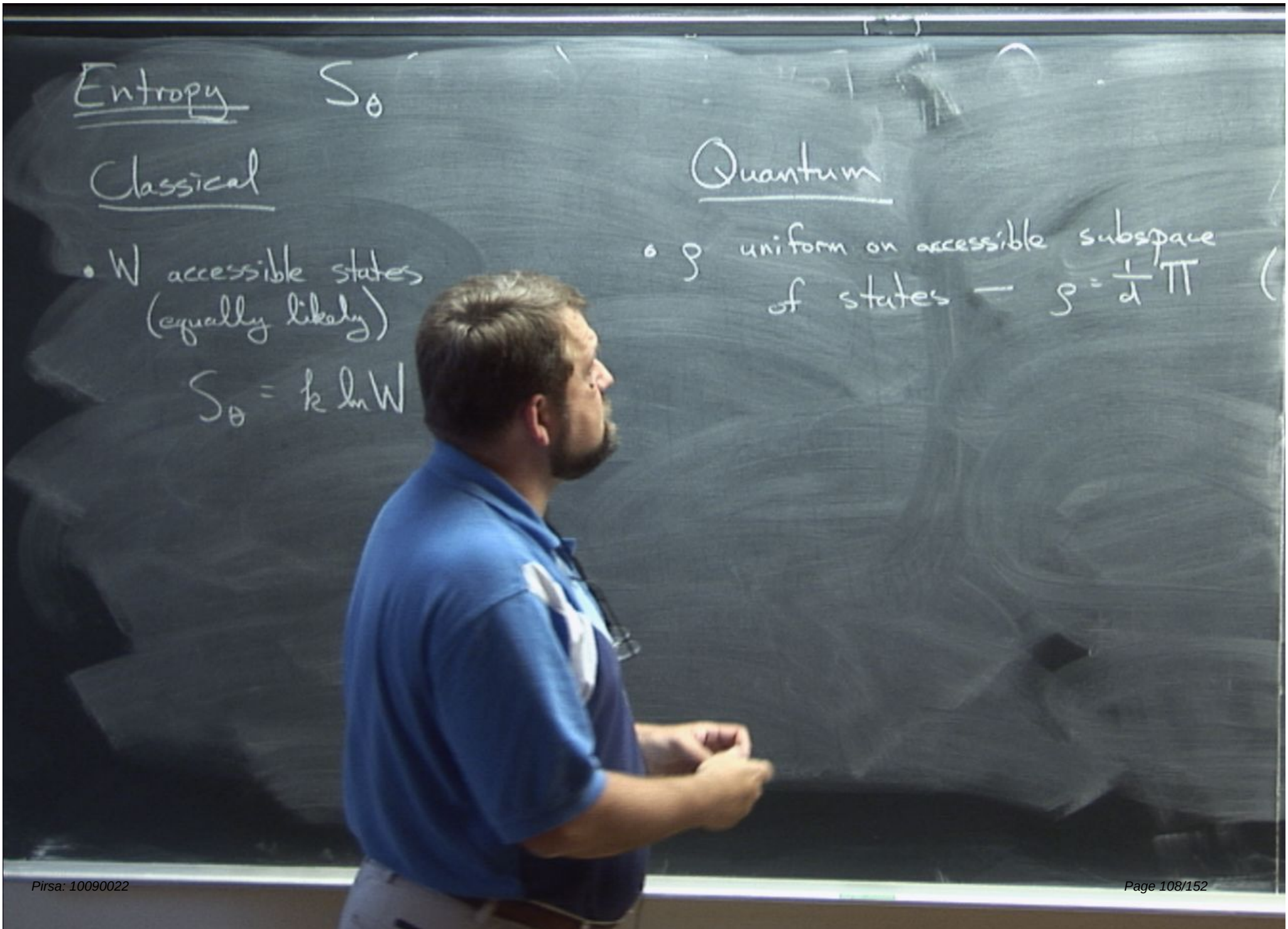
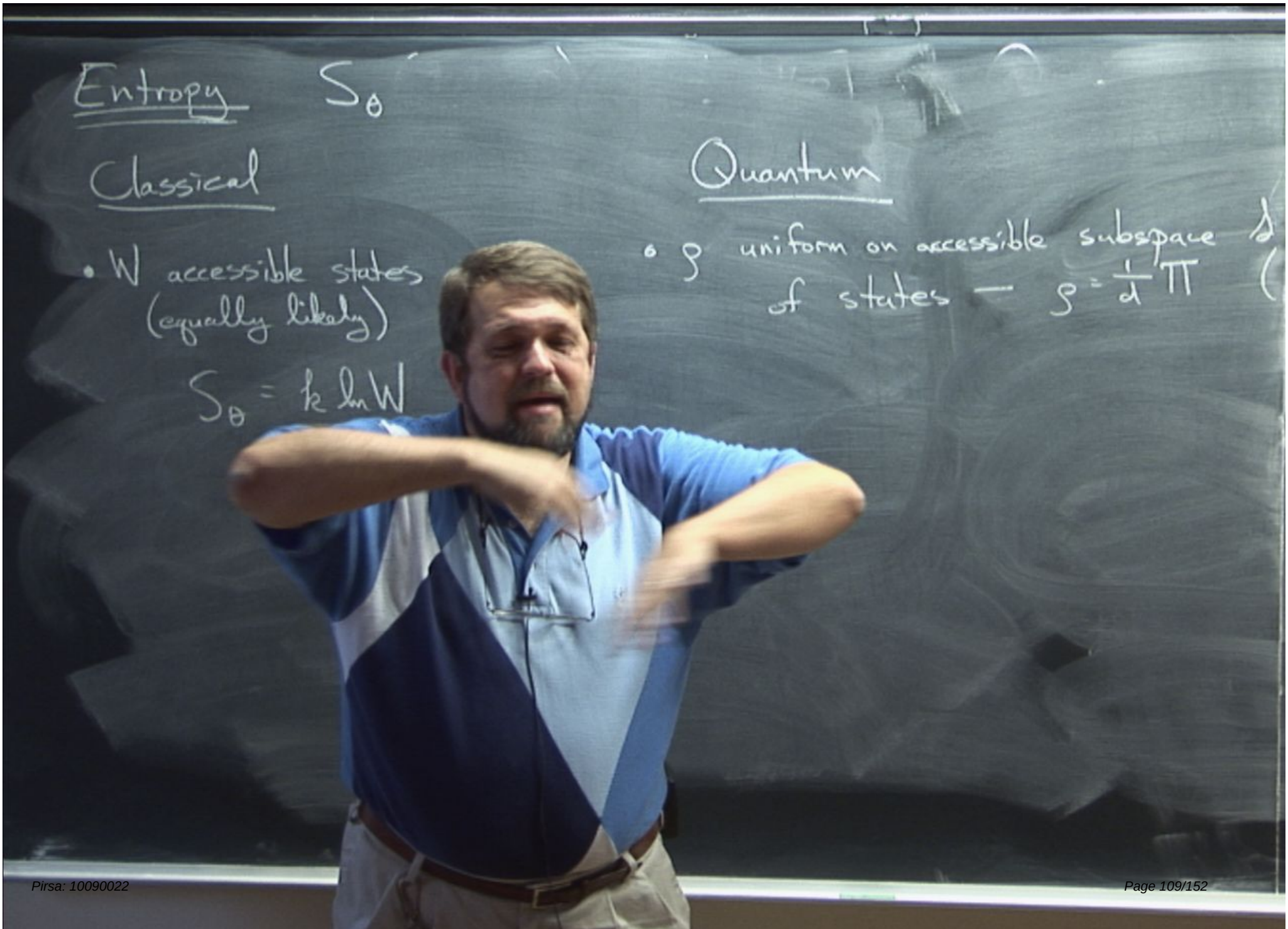
Quantum


Entropy S_θ
Classical
• W accessible states (equally likely)
$S_\theta = k \ln W$
Quantum
• ρ uniform on accessible subspace of states — $\rho = \frac{1}{d}\Pi$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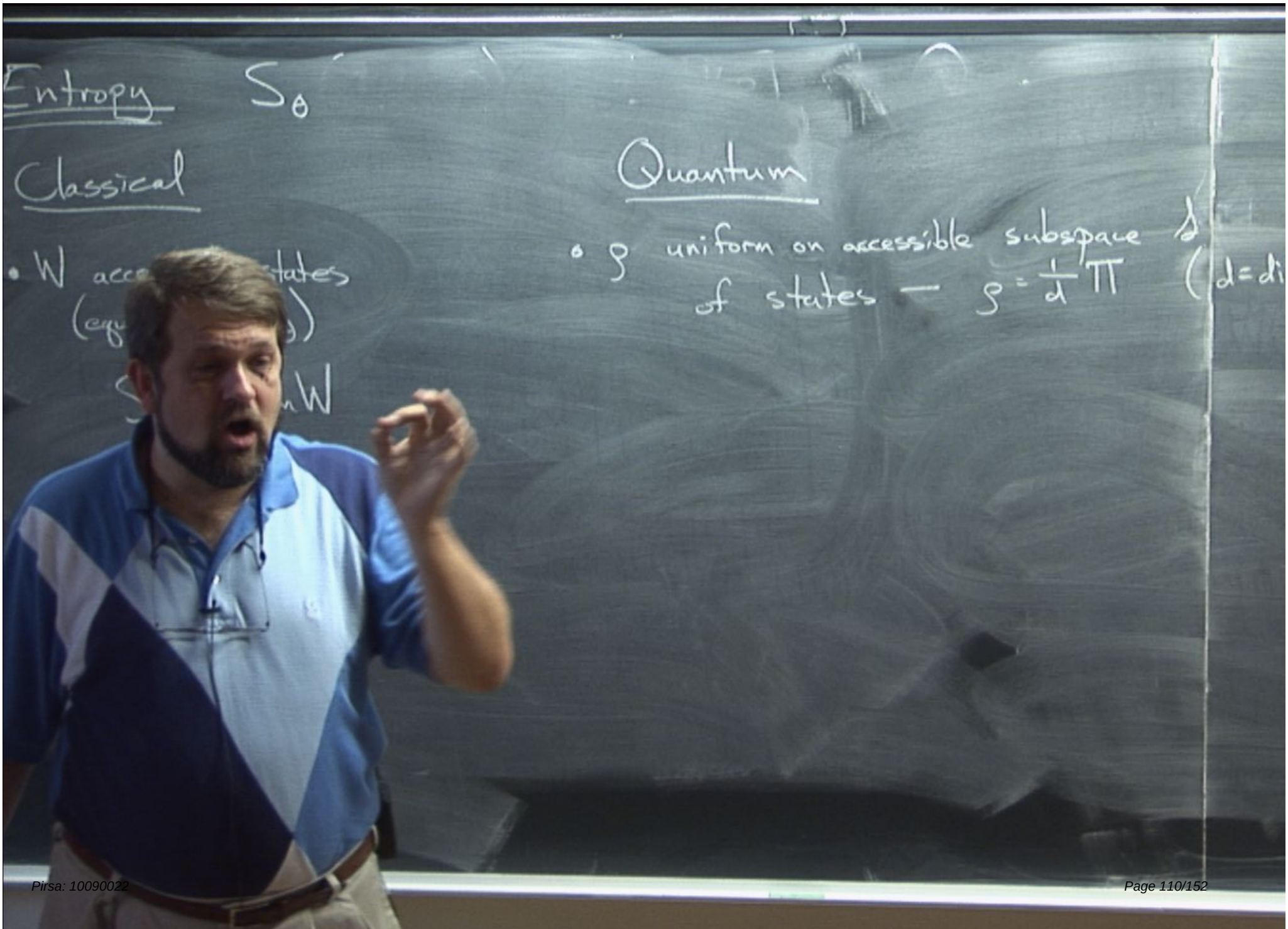
Entropy S_θ
Classical
• W acce states
(equ)
S W
Quantum
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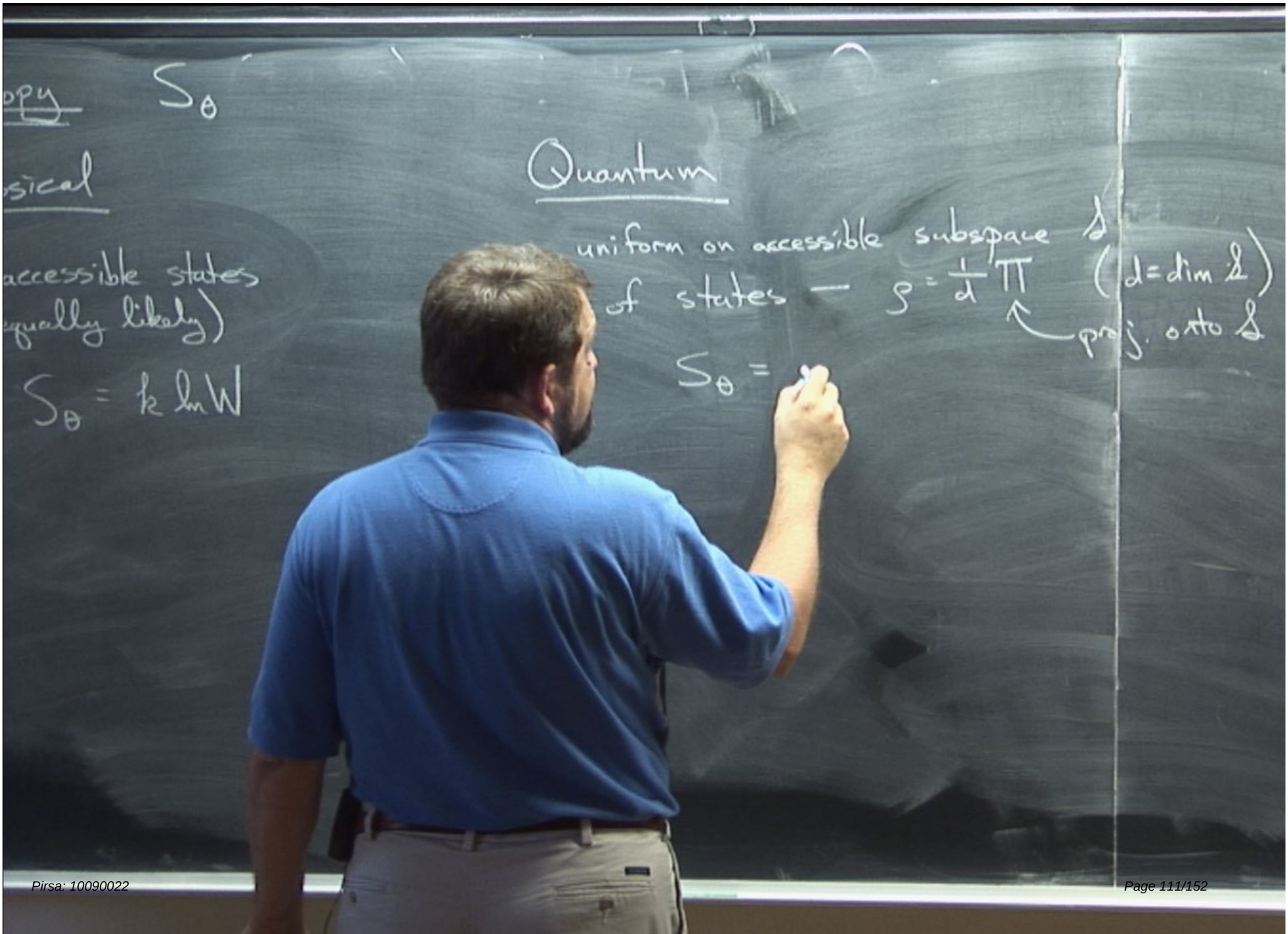
of states — $\rho = \frac{1}{d}\Pi$ (d=di


opy
S_θ
sical
accessible states
equally likely)
$S_\theta = k \ln W$
Quantum
uniform on accessible subspace $\mathcal{S}$
of states — $\rho = \frac{1}{d}\Pi$ ($d = \dim \mathcal{S}$)
proj. onto $\mathcal{S}$
$S_\theta =$

opy S_θ

sical

accessible states
equally likely)

$$\boxed{S_\theta = k \ln W}$$

Quantum

- ρ uniform on accessible subspace $\mathcal{S}$ of states — $\rho = \frac{1}{d}\Pi$ $(d = \dim \mathcal{S})$

proj. onto $\mathcal{S}$

$$\boxed{S_\theta = k \ln d}$$

Entropy S_θ

Classical

- W accessible states (equally likely)

$$\boxed{S_\theta = k \ln W}$$

- State α with prob. p_α

$$\boxed{S_\theta = -k \sum_\alpha p_\alpha \ln p_\alpha}$$

Quantum

- ρ uniform on accessible subspace $\mathcal{S}$ of states — $\rho = \frac{1}{d} \Pi$

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Quantum

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pro

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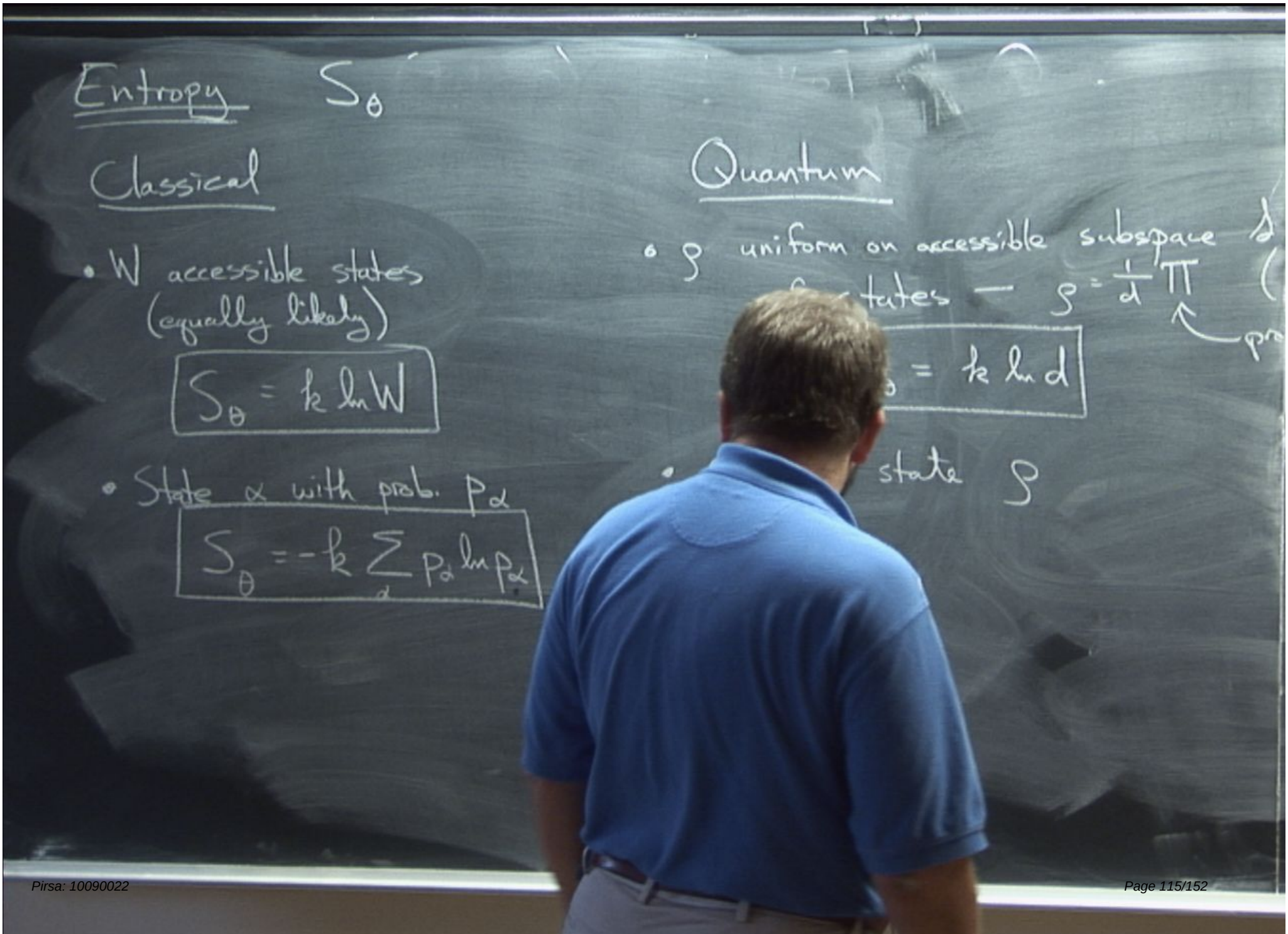
Entropy S_θ
Classical
• W accessible states
(equally likely)
$S_\theta = k \ln W$
• State α with prob. p_α
$S_\theta = -k \sum_\alpha p_\alpha \ln p_\alpha$
Quantum
• ρ uniform on accessible subspace
$\rho = \frac{1}{d}\Pi$
$= k \ln d$
state ρ

Entropy S_θ

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- State α with prob. p_α

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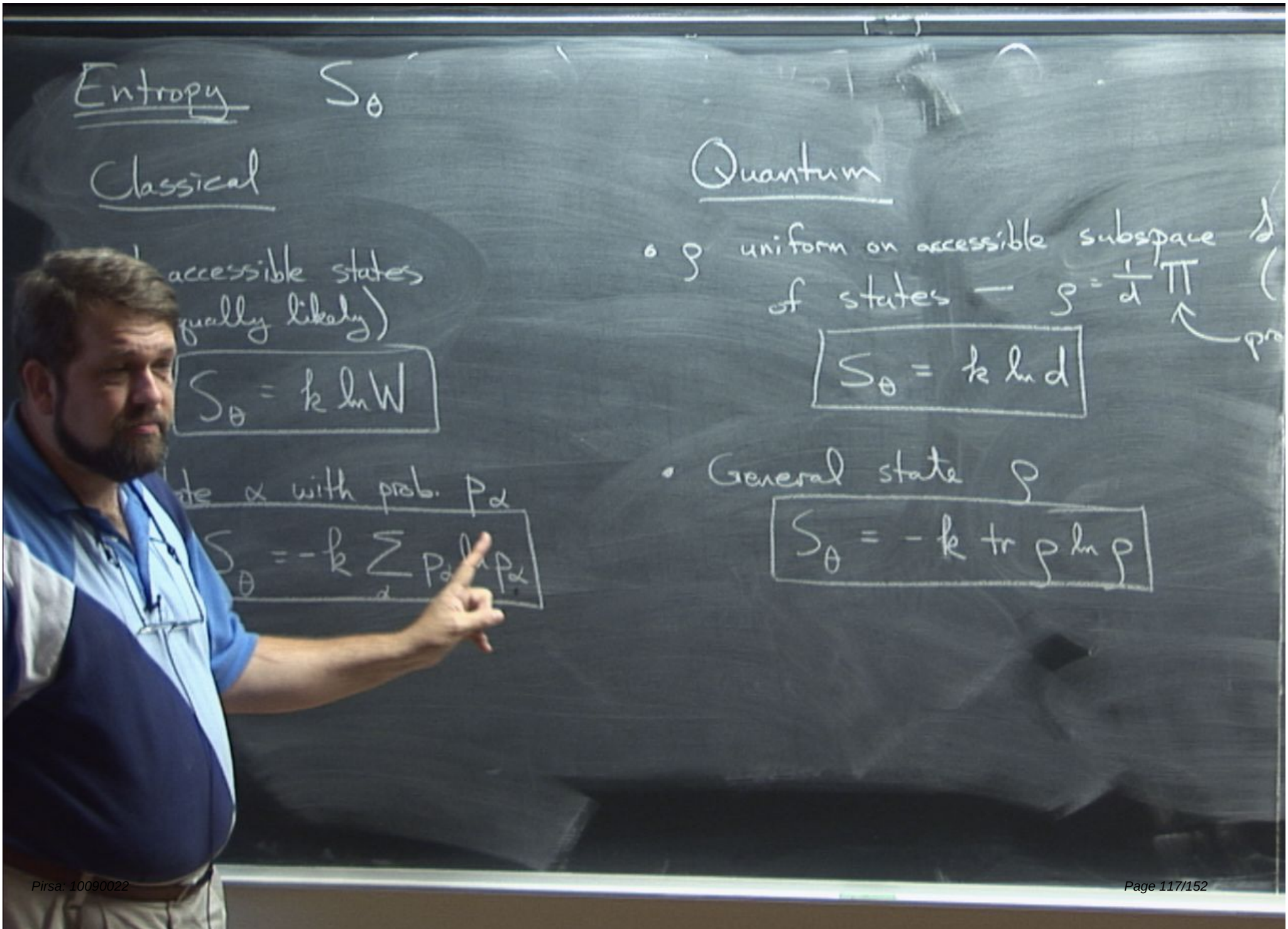
Quantum

- ρ uniform on accessible subspace of states — $\rho = \frac{1}{d}\Pi$ ← pro

$$\boxed{S_\theta = k \ln d}$$

- General state ρ

$$\boxed{S_\theta = -k \operatorname{tr} \rho \ln \rho}$$


Entropy S_θ
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- General state ρ

$$\boxed{S_\theta = -k \operatorname{tr} \rho \ln \rho}$$

Quantum

- ρ uniform on accessible subspace $\mathcal{S}$ of states — $\rho = \frac{1}{d}\Pi$ $(d = \dim \mathcal{S})$

  proj. onto $\mathcal{S}$

$$\boxed{S_\theta = k \ln d}$$

- General state ρ

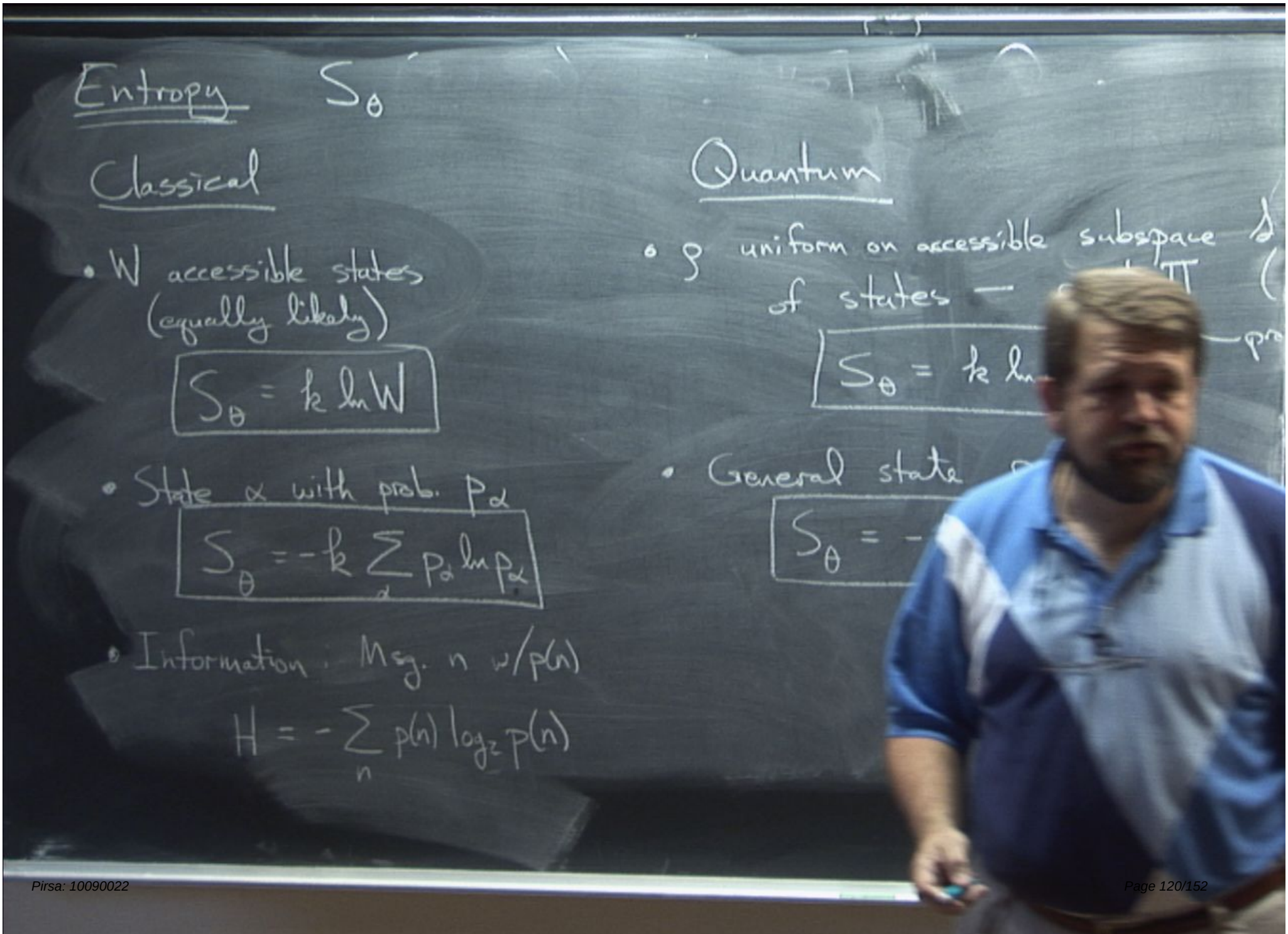
$$\boxed{S_\theta = -k \operatorname{tr} \rho \ln \rho}$$

$S_\theta = 0$ for $\rho = |\psi\rangle\langle\psi|$

p_α

$\ln p_\alpha$

Defin


Entropy S_θ
Classical
• W accessible states (equally likely)
$S_\theta = k \ln W$
• State α with prob. p_α
$S_\theta = -k \sum_\alpha p_\alpha \ln p_\alpha$
• Information: Msg. n w/ p(n)
$H = -\sum_n p(n) \log_2 p(n)$
Quantum
• ρ uniform on accessible subspace
of states
• General state

Quantum

- ρ uniform on accessible subspace $\mathcal{S}$ of states — $\rho = \frac{1}{d}\Pi$ $(d = \dim \mathcal{S})$

  proj. onto $\mathcal{S}$

$$\boxed{S_\theta = k \ln d}$$

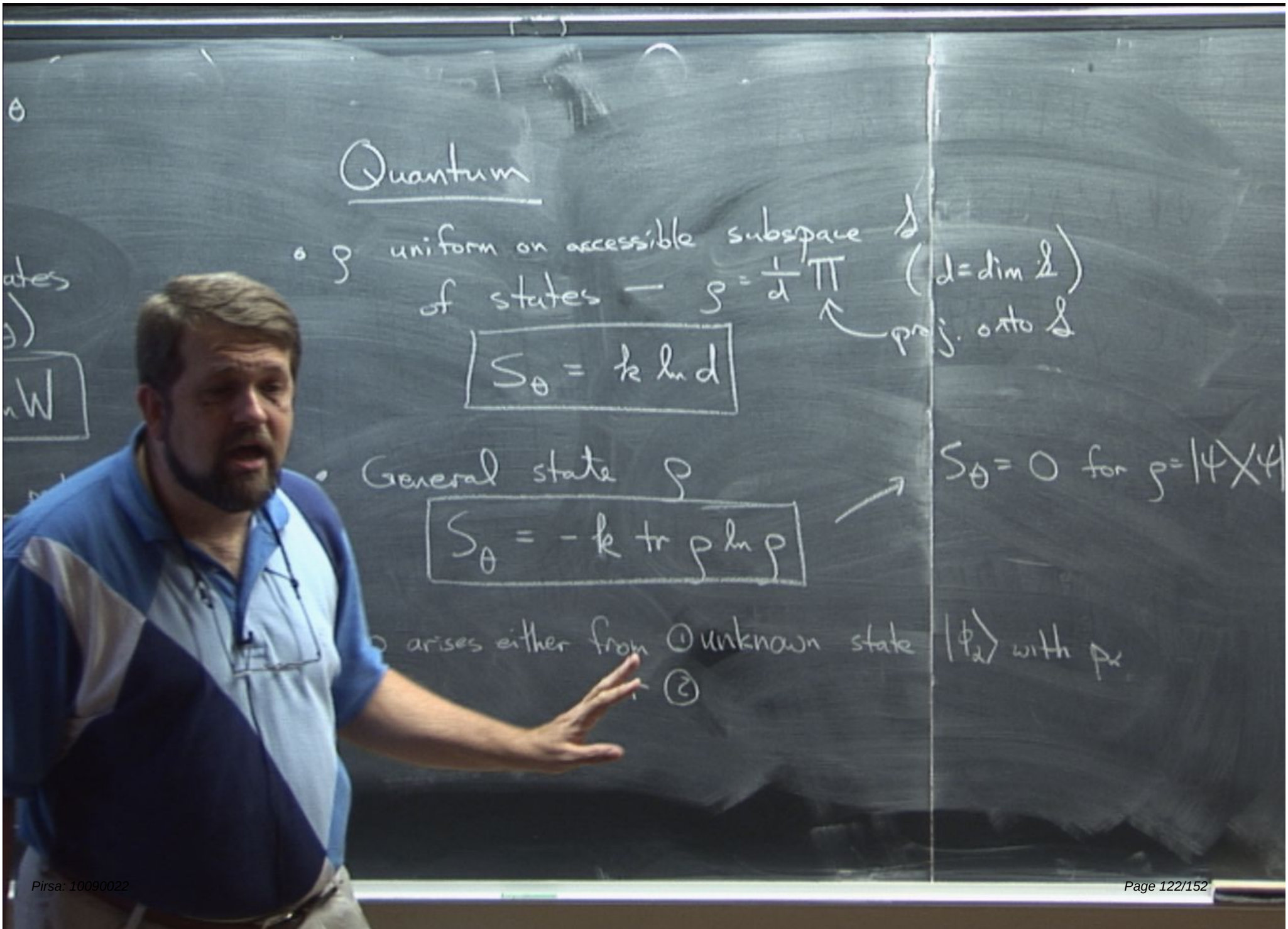
- General state ρ

$$\boxed{S_\theta = -k \operatorname{tr} \rho \ln \rho}$$

$S_\theta = 0$ for $\rho = |\psi\rangle\langle\psi|$

[illegible] form ① unknown state $|\phi_\alpha\rangle$ with p_α

[illegible] or ②


Quantum
• ρ uniform on accessible subspace S
of states — ρ = 1/d Π (d = dim S)
proj. onto S
S_θ = k ln d
• General state ρ
S_θ = − k tr ρ ln ρ
S_θ = 0 for ρ = |ψ⟩⟨ψ|
arises either from ① unknown state |φ_α⟩ with p_α
②

Quantum

- ρ uniform on accessible subspace $\mathcal{S}$ of states — $\rho = \frac{1}{d}\Pi$ $(d = \dim \mathcal{S})$

  proj. onto $\mathcal{S}$

$$\boxed{S_\theta = k \ln d}$$

- General state ρ

$$\boxed{S_\theta = -k \operatorname{tr} \rho \ln \rho}$$

$S_\theta = 0$ for $\rho = |\psi\rangle\langle\psi|$

- ρ arises either from ① unknown state $|\phi_\alpha\rangle$ with p_α or ②

θ

ates

)

$\boxed{\ln W}$

prob. p_α

$\boxed{\sum_\alpha p_\alpha \ln p_\alpha}$

Msg. n w/ p(n)

$\boxed{\sum_n p(n) \log_2 p(n)}$

Quantum

- ρ uniform on accessible subspace $\mathcal{S}$ of states — $\rho = \frac{1}{d}\Pi$ $(d = \dim \mathcal{S})$

  proj. onto $\mathcal{S}$

$$\boxed{S_\theta = k \ln d}$$

- General state ρ

$$\boxed{S_\theta = -k \operatorname{tr} \rho \ln \rho}$$

$S_\theta = 0$ for $\rho = |\psi\rangle\langle\psi|$

- ρ arises either from ① unknown state $|\phi_\alpha\rangle$ with p_α or ② entangled

θ

ates

)

$\boxed{\ln W}$

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$\boxed{\sum_\alpha p_\alpha \ln p_\alpha}$

Msg. n w/ p(n)

$\boxed{\sum_n p(n) \log_2 p(n)}$

Quantum

- ρ uniform on accessible subspace $\mathcal{S}$ of states — $\rho = \frac{1}{d}\Pi$ $(d = \dim \mathcal{S})$

  proj. onto $\mathcal{S}$

$$\boxed{S_\theta = k \ln d}$$

- General state ρ

$$\boxed{S_\theta = -k \operatorname{tr} \rho \ln \rho}$$

$S_\theta = 0$ for $\rho = |\psi\rangle\langle\psi|$

- ρ arises either from ① unknown state $|\phi_\alpha\rangle$ with p_α or ② entangled

$$\boxed{H = -\operatorname{tr} \rho \log_2 \rho}$$

Entropy S_θ

Classical

- W accessible states (equally likely)

$$\boxed{S_\theta = k \ln W}$$

- State α with prob. p_α

$$\boxed{S_\theta = -k \sum_\alpha p_\alpha \ln p_\alpha}$$

- Information: Msg. n w/ $p(n)$

$$\boxed{H = -\sum_n p(n) \log_2 p(n)}$$

"bits"

Quantum

- ρ uniform on accessible subspace of states — $\rho = \frac{1}{d}\Pi$

$$\boxed{S_\theta = k \ln d}$$

- General state ρ

$$\boxed{S_\theta = -k \operatorname{tr} \rho \ln \rho}$$

- ρ arises either from ① unknow or ② extana

$$\boxed{H = -\operatorname{tr} \rho \log_2 \rho}$$

es

)

N

prob. p_α

$$\boxed{\sum_\alpha p_\alpha \ln p_\alpha}$$

sg. n w/ $p(n)$

$$\boxed{-\sum p(n) \log_2 p(n)}$$

"bits"

Quantum

- ρ uniform on accessible subspace $\mathcal{S}$ of states — $\rho = \frac{1}{d}\Pi$ $(d = \dim \mathcal{S})$

  proj. onto $\mathcal{S}$

$$\boxed{S_\theta = k \ln d}$$

- General state ρ

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$S_\theta = 0$ for $\rho = |\psi\rangle\langle\psi|$

- ρ arises either from ① unknown state $|\phi_\alpha\rangle$ with p_α or ② entangled

$$\boxed{H = -\operatorname{tr} \rho \log_2 \rho}$$

"qubits"

Entropy S_θ

Classical

- W accessible states (equally likely)

$$\boxed{S_\theta = k \ln W}$$

- State α with prob. p_α

$$\boxed{S_\theta = -k \sum_\alpha p_\alpha \ln p_\alpha}$$

- Information: Msg. n w/ $p(n)$

$$\boxed{H = -\sum_n p(n) \log_2 p(n)}$$

"bits"

Quantum

- ρ uniform on accessible subspace of states — $\rho = \frac{1}{d}\Pi$ (pro

$$\boxed{S_\theta = k \ln d}$$

- General state ρ

$$\boxed{S_\theta = -k \operatorname{tr} \rho \ln \rho}$$

- ρ arises either from ① unknown state or ② entangled

$$\boxed{H = -\operatorname{tr} \rho \log_2 \rho}$$

"qubits"

Entropy S_θ

Classical

- W accessible states (equally likely)

$$\boxed{S_\theta = k \ln W}$$

- State α with prob. p_α

$$\boxed{S_\theta = -k \sum_\alpha p_\alpha \ln p_\alpha}$$

- Information: Msg. n w/ $p(n)$

$$\boxed{H = -\sum_n p(n) \log_2 p(n)}$$

"bits"

Quantum

- ρ uniform on accessible subspace of states — $\rho = \frac{1}{d}\Pi$ (pr

$$\boxed{S_\theta = k \ln d}$$

- General state ρ

$$\boxed{S_\theta = -k \operatorname{tr} \rho \ln \rho}$$

- ρ arises either from ① unknown state or ② entangled

$$\boxed{H = -\operatorname{tr} \rho \log_2 \rho}$$

"qubits"

Entropy S_θ

Classical

- W accessible states (equally likely)

$$\boxed{S_\theta = k \ln W}$$

- State α with prob. p_α

$$\boxed{S_\theta = -k \sum_\alpha p_\alpha \ln p_\alpha}$$

- Information: Msg. n w/ $p(n)$

$$\boxed{H = -\sum_n p(n) \log_2 p(n)}$$

"bits"

Quantum

- ρ uniform on accessible subspace of states — $\rho = \frac{1}{d}\Pi$ ← pr

$$\boxed{S_\theta = k \ln d}$$

- General state ρ

$$\boxed{S_\theta = -k \operatorname{tr} \rho \ln \rho}$$

- ρ arises either from ① unknown state or ② entangled

$$\boxed{H = -\operatorname{tr} \rho \log_2 \rho}$$

"qubits"

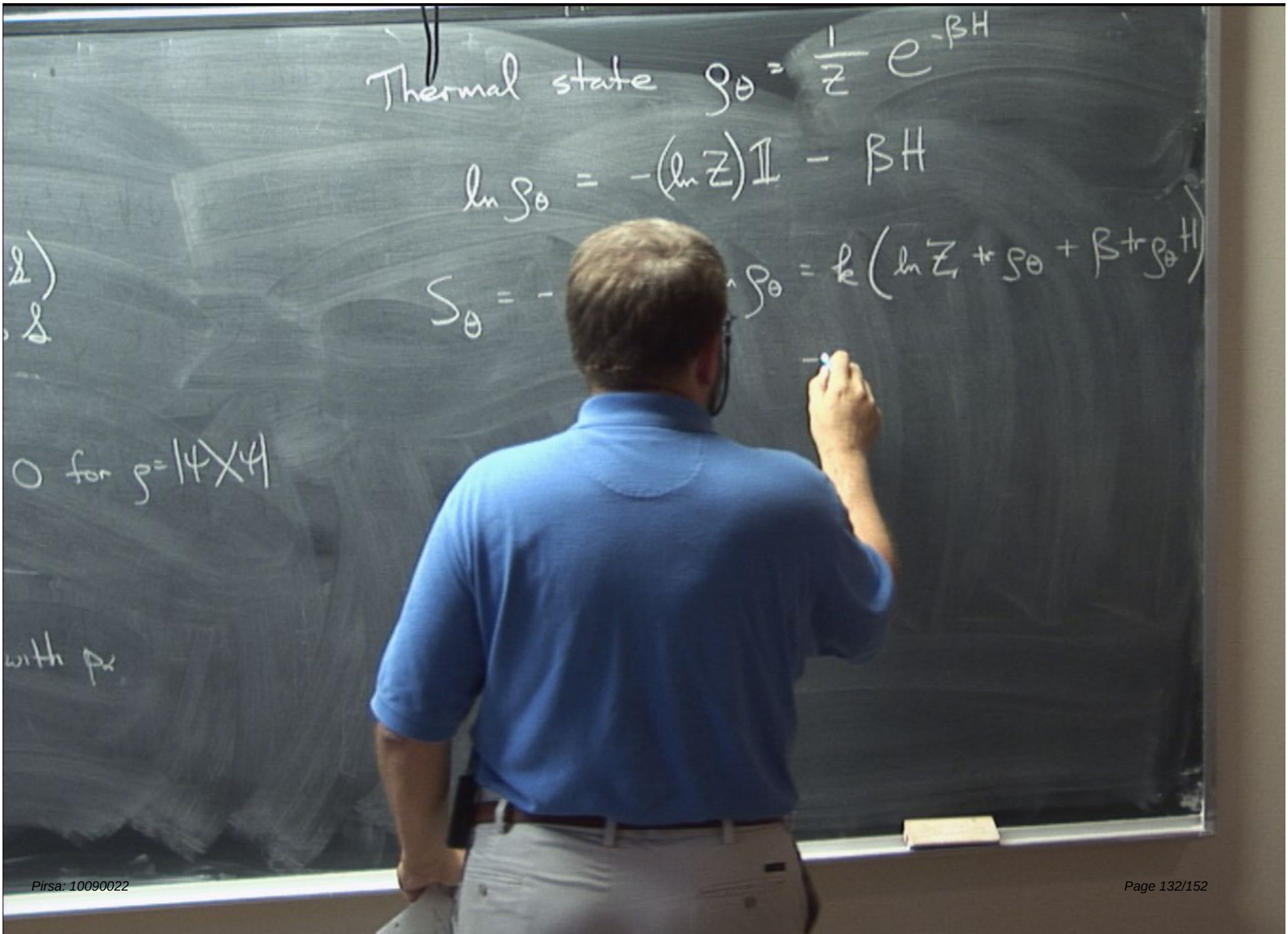
Thermal state $\rho_\theta = \frac{1}{Z} e^{-\beta H}$

$$\ln \rho_\theta = -(\ln Z)\mathbb{1} - \beta H$$

$= \dim \mathcal{L}$)

$\mathcal{L}$

0 for $\rho = |\psi\rangle\langle\psi|$


Thermal state $\rho_\theta = \frac{1}{Z} e^{-\beta H}$
$\ln \rho_\theta = -(\ln Z)\mathbb{1} - \beta H$
$S_\theta = - \ln \rho_\theta = k\left(\ln Z + \mathrm{tr}\,\rho_\theta + \beta\,\mathrm{tr}\,\rho_\theta H\right)$
0 for $\rho = |\psi\rangle\langle\psi|$
with ρ_α

Thermal state $\rho_\theta = \frac{1}{Z} e^{-\beta H}$

$$\ln \rho_\theta = -(\ln Z)\mathbb{1} - \beta H$$

$$S_\theta = -k \operatorname{tr} \rho_\theta \ln \rho_\theta = k\left(\ln Z \operatorname{tr} \rho_\theta + \beta \operatorname{tr} \rho_\theta H\right)$$

$$= k \ln Z + \beta E_\theta$$

Thermal state $\rho_\theta = \frac{1}{Z} e^{-\beta H}$

$$\ln \rho_\theta = -(\ln Z)\mathbb{1} - \beta H$$

$$S_\theta = -k \operatorname{tr} \rho_\theta \ln \rho_\theta = k\left(\ln Z \operatorname{tr} \rho_\theta + \beta \operatorname{tr} \rho_\theta H\right)$$

$$= k \ln Z + k\beta E_\theta$$

$k\beta = \frac{1}{T}$

0 for $\rho = |\psi\rangle\langle\psi|$

with ρ_α

Thermal state $\rho_\theta = \frac{1}{Z} e^{-\beta H}$

$$\ln \rho_\theta = -(\ln Z)\mathbb{1} - \beta H$$

$$S_\theta = -k \operatorname{tr} \rho_\theta \ln \rho_\theta = k\left(\ln Z \operatorname{tr} \rho_\theta + \beta \operatorname{tr} \rho_\theta H\right)$$

$$= k \ln Z + k\beta E_\theta$$

$\frac{F_\theta}{T}$ $\quad k\beta = \frac{1}{T}$

0 for $\rho = |\psi\rangle\langle\psi|$

with ρ_α

Thermal state $\rho_\theta = \frac{1}{Z} e^{-\beta H}$

$$\ln \rho_\theta = -(\ln Z)\mathbb{1} - \beta H$$

$$S_\theta = -k \operatorname{tr} \rho_\theta \ln \rho_\theta = k\left(\ln Z \operatorname{tr} \rho_\theta + \beta \operatorname{tr} \rho_\theta H\right)$$

$$= k \ln Z + k\beta E_\theta$$

$-\frac{F_\theta}{T}$ $\quad k\beta = \frac{1}{T}$

$$\boxed{F_\theta = E_\theta - TS_\theta}$$

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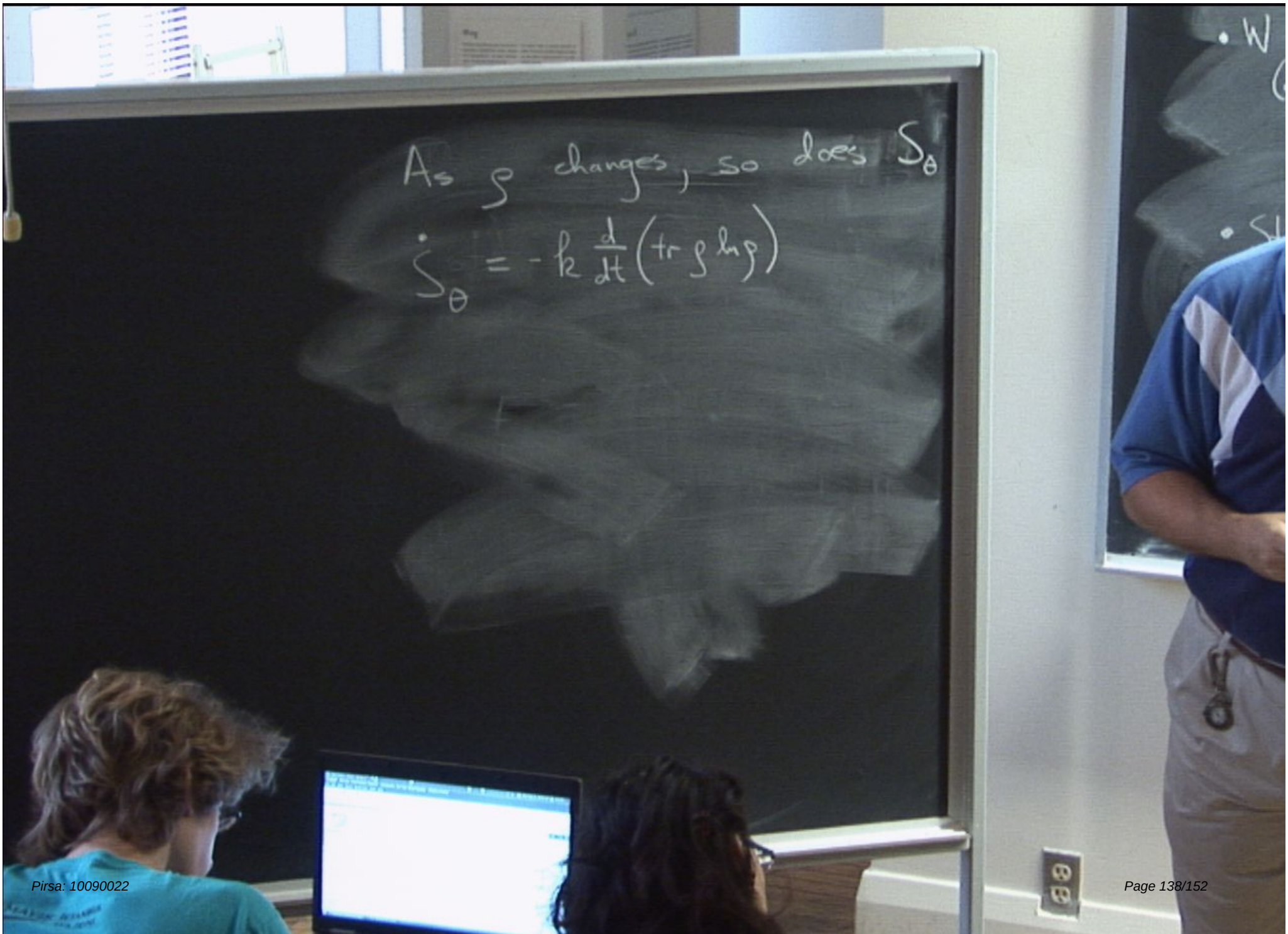
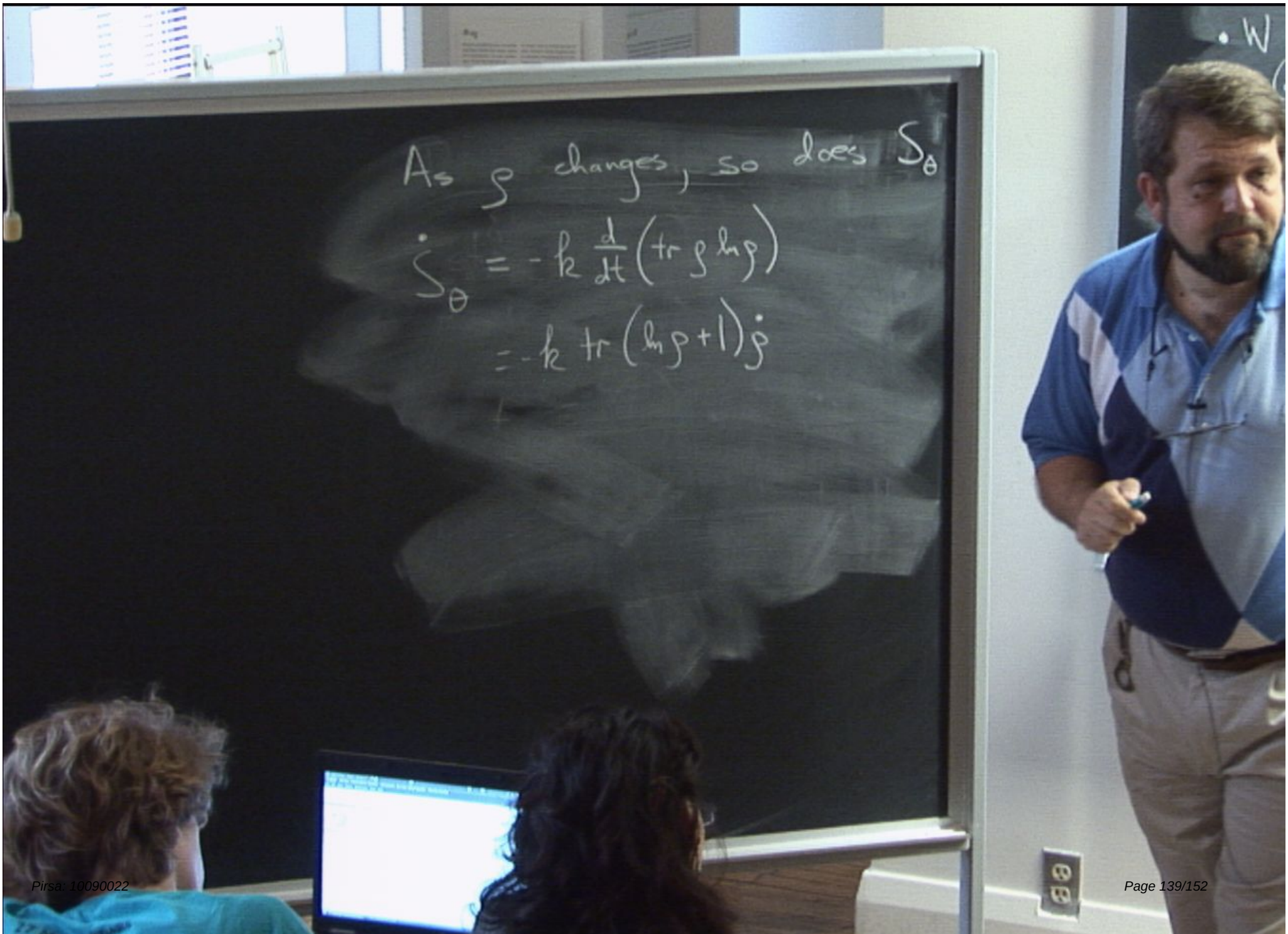
← define for any state

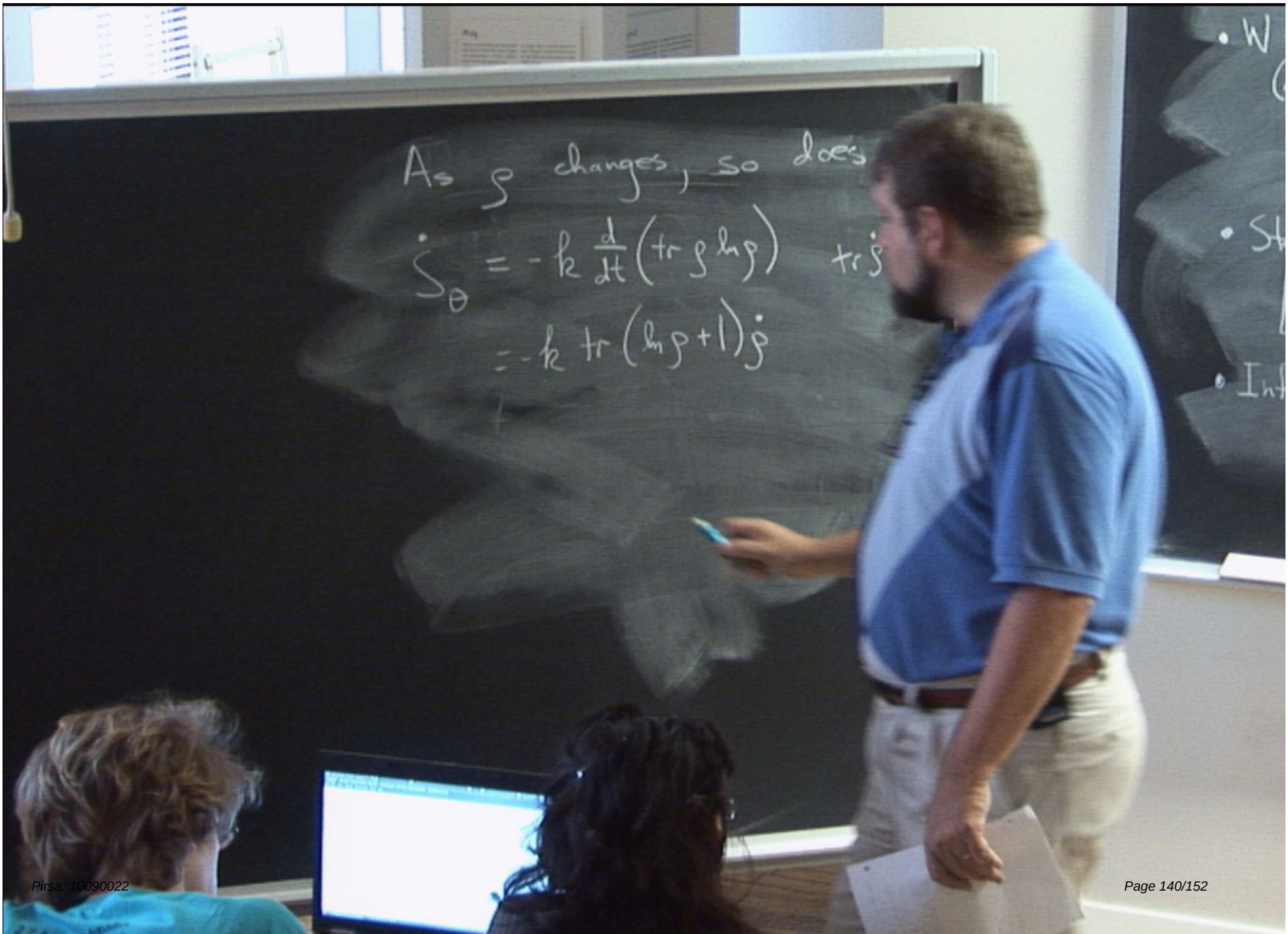
$$F_\theta = (\rho, H, T)$$

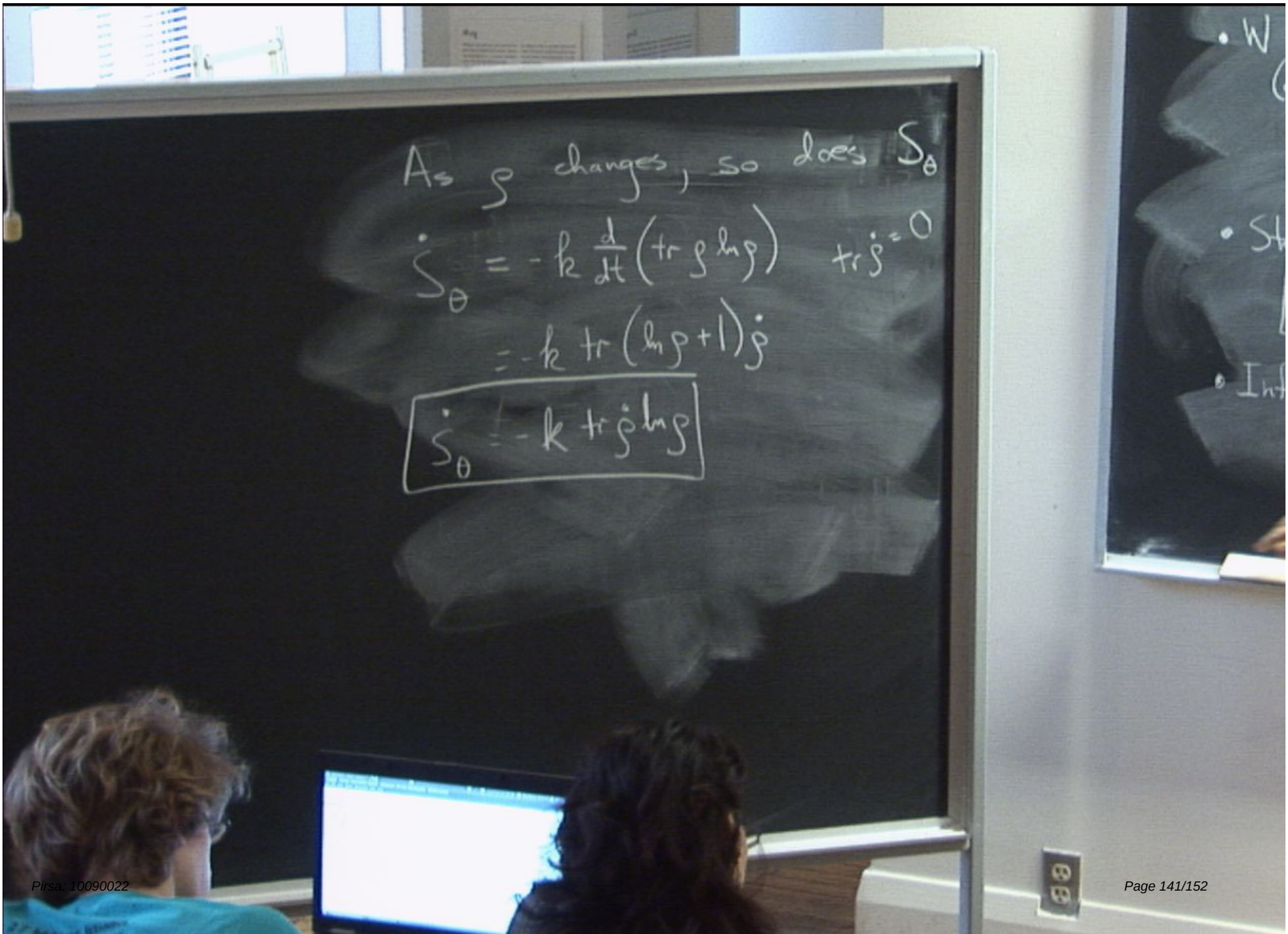
0 for $\rho = |\psi\rangle\langle\psi|$

with ρ_α


As ρ changes, so does S_θ
Ṡ_θ = - k d/dt (tr ρ ln ρ)


As ρ changes, so does S_θ
Ṡ_θ = -k d/dt (tr ρ ln ρ)
= -k tr (ln ρ + 1) ρ̇


As ρ changes, so does
$\dot{S}_\theta = -k \frac{d}{dt}(\mathrm{tr}\, \rho \ln \rho)$ tr $\dot{\rho}$
$= -k\, \mathrm{tr}\, (\ln \rho + 1)\dot{\rho}$


As ρ changes, so does S_θ
Ṡ_θ = -k d/dt (tr ρ ln ρ)
tr ρ̇ = 0
= -k tr (ln ρ + 1) ρ̇
Ṡ_θ = -k tr ρ̇ ln ρ

Quasi-static process $\rho = \rho_\theta$ (equilibrium) at all times

but both H and T may change!

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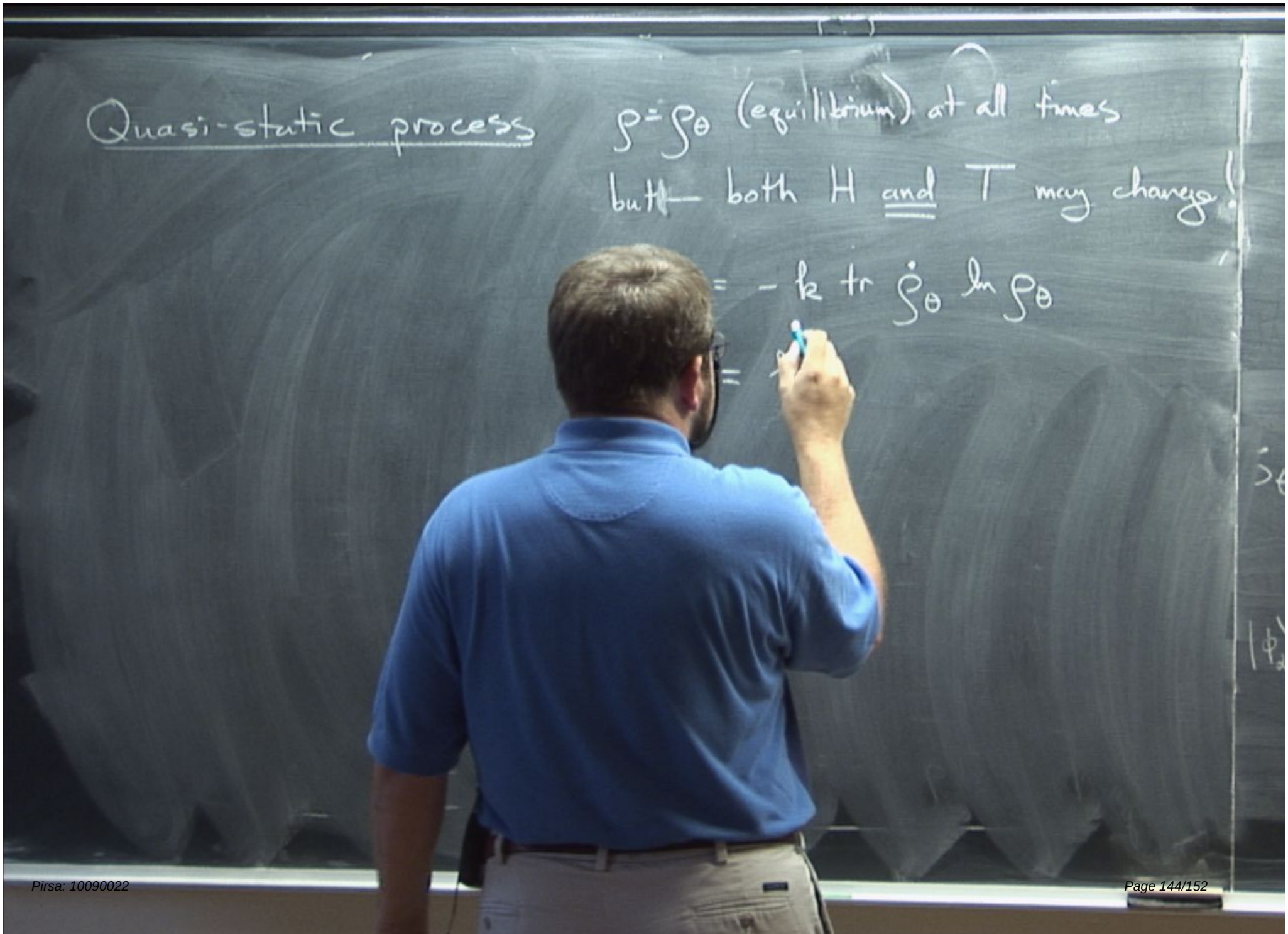
Quasi-static process
$\rho = \rho_\theta$ (equilibrium) at all times
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$= -k \, \mathrm{tr} \, \dot{\rho}_\theta \ln \rho_\theta$

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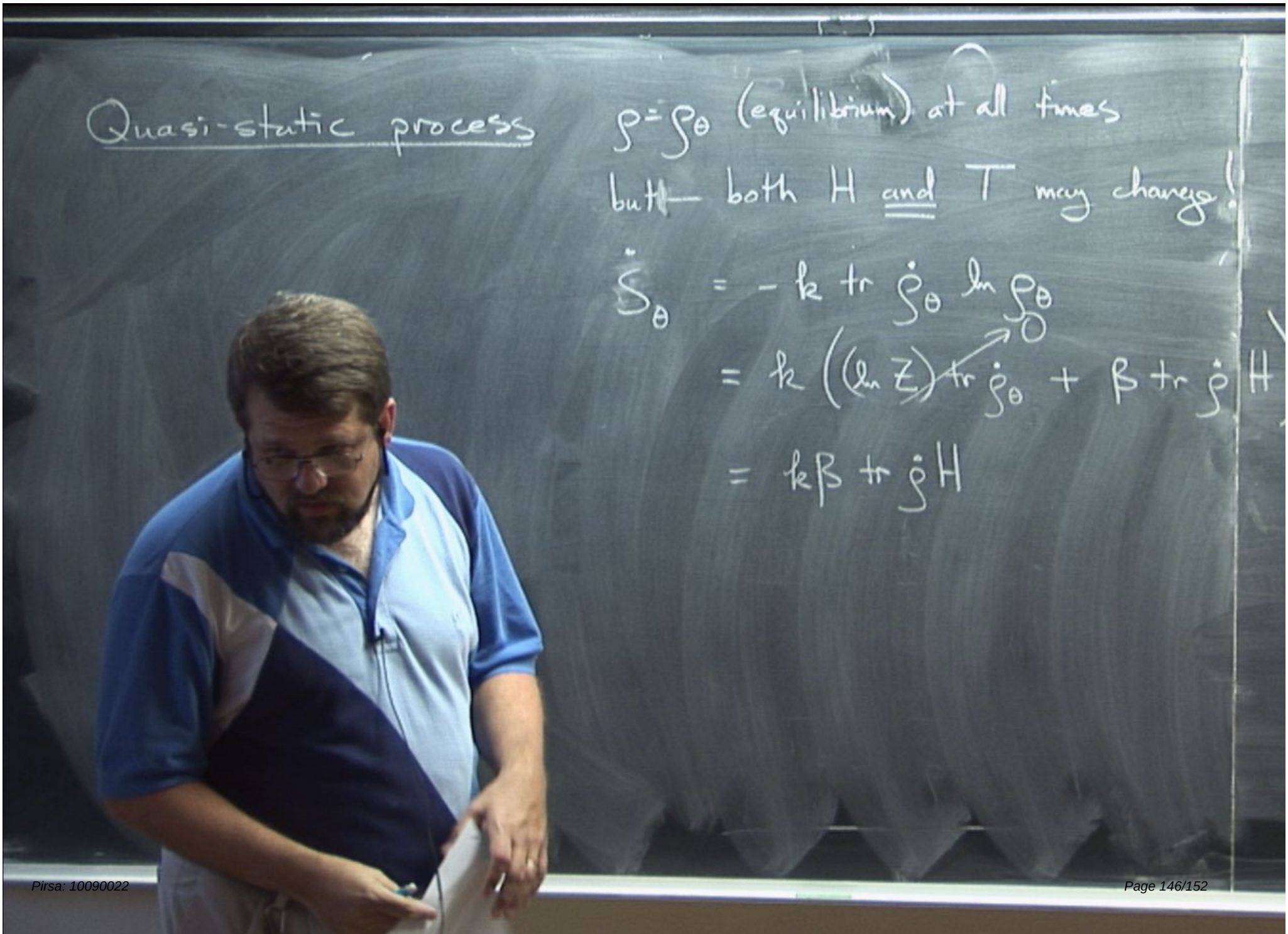
$$= k\left((\ln Z) \operatorname{tr} \dot{\rho}_\theta + \beta \operatorname{tr} \dot{\rho} H\right.$$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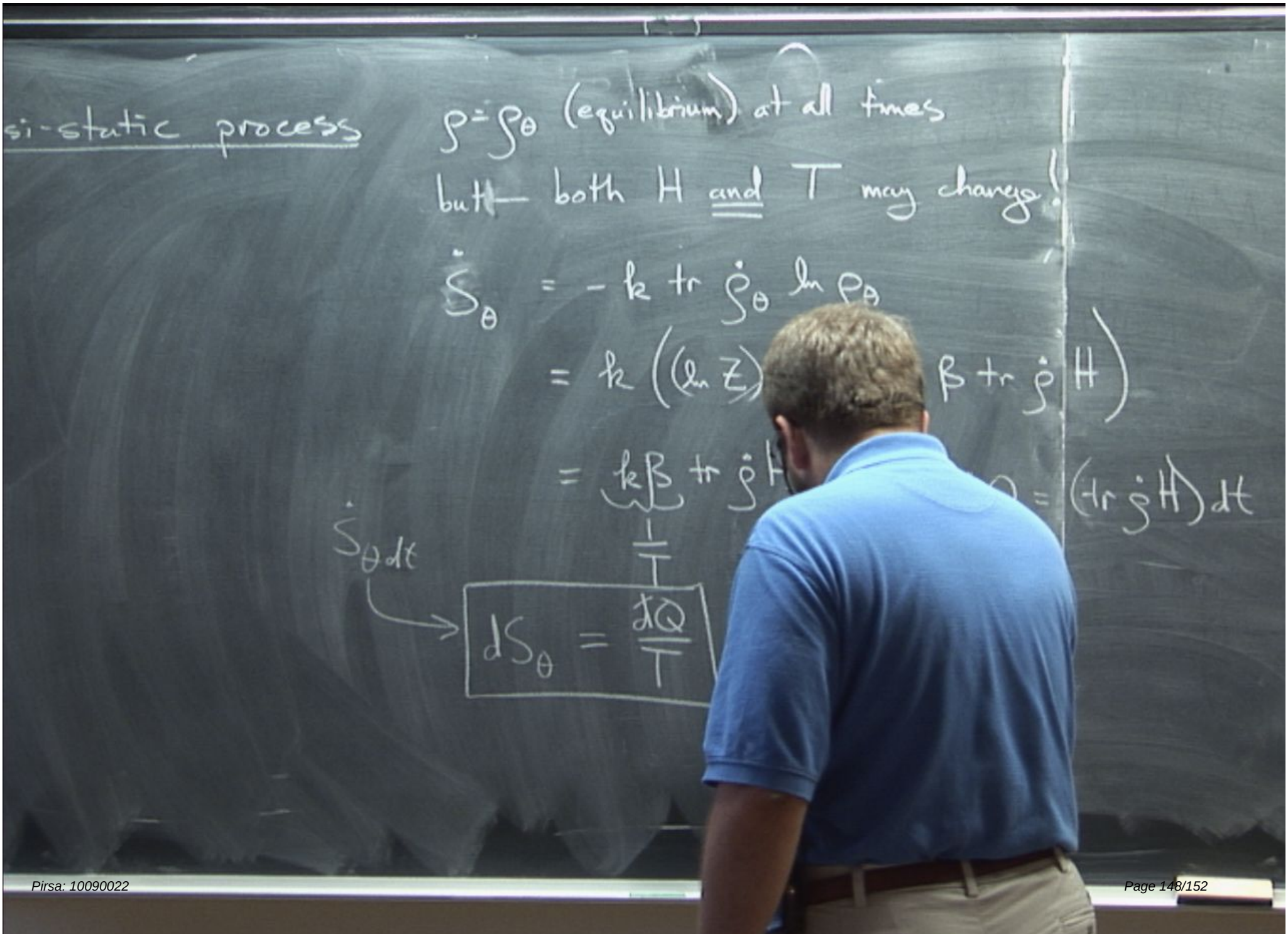
$đQ = (\operatorname{tr}$


si-static process
ρ = ρ_θ (equilibrium) at all times
but both H and T may change!
Ṡ_θ = − k tr ρ̇_θ ln ρ_θ
= k ((ln Z) β tr ρ̇ H)
= kβ tr ρ̇ H
= (tr ρ̇ H) dt
Ṡ_θ dt
dS_θ = đQ/T

si-static process

$\rho = \rho_\theta$ (equilibrium) at all times

but — both H <u>and</u> T may change!

$$\dot{S}_\theta = -k \operatorname{tr} \dot{\rho}_\theta \ln \rho_\theta$$

$$= k\left((\ln Z) \cancelto{0}{\operatorname{tr} \dot{\rho}_\theta} + \beta \operatorname{tr} \dot{\rho} H \right)$$

$$= \underbrace{k\beta}_{\frac{1}{T}} \operatorname{tr} \dot{\rho} H \qquad đQ = (\operatorname{tr} \dot{\rho} H)\, dt$$

$\dot{S}_\theta dt$

$$\boxed{dS_\theta = \frac{đQ}{T}} \quad \text{(quasistatic)}$$

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but — both H and T may change!

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$$= \underbrace{k\beta}_{\frac{1}{T}} \operatorname{tr} \dot{\rho} H$$

$$đQ = (\operatorname{tr} \dot{\rho} H)\, dt$$

$\dot{S}_\theta dt$

$$\Rightarrow \boxed{dS_\theta = \frac{đQ}{T}} \quad \text{(quasistatic)}$$

si-static process

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