Title: Quantum Theory (PHYS 605) - Lecture 10

Date: Sep 24, 2010 09:00 AM

URL: http://pirsa.org/10090021

Abstract:

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Homework :

Lindblad eqn.

$$\frac{d}{dt}\rho = \frac{1}{i\hbar}[H,\rho] + \Lambda a \rho a^{\dagger} - \frac{\Lambda}{2}\left(a^{\dagger}a\rho + \rho a^{\dagger}a\right)$$

initial states $Q : \rho$

$E : |0\rangle\langle 0|$

interaction : U

E-basis : $|k\rangle$

$$A_k|\psi\rangle = \langle k|U|\psi, 0\rangle$$

$$\rho \rightarrow \mathcal{E}(\rho) = \sum_k A_k \rho A_k^{\dagger}$$

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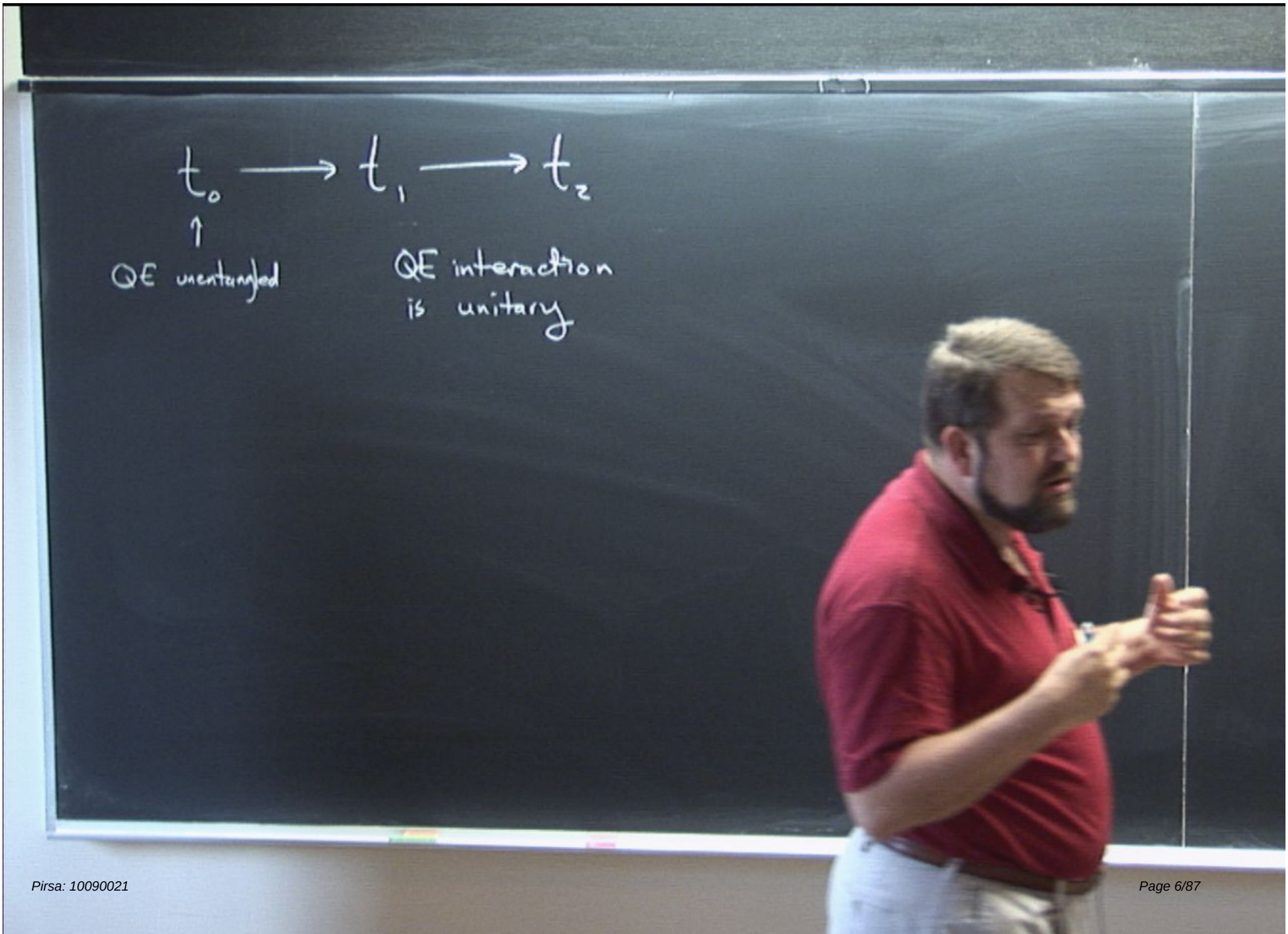
$t_0 \longrightarrow t_1 \longrightarrow t_2$
QE unentangled
QE interaction is unitary

$$t_0 \longrightarrow t_1 \longrightarrow t_2$$

$\uparrow$

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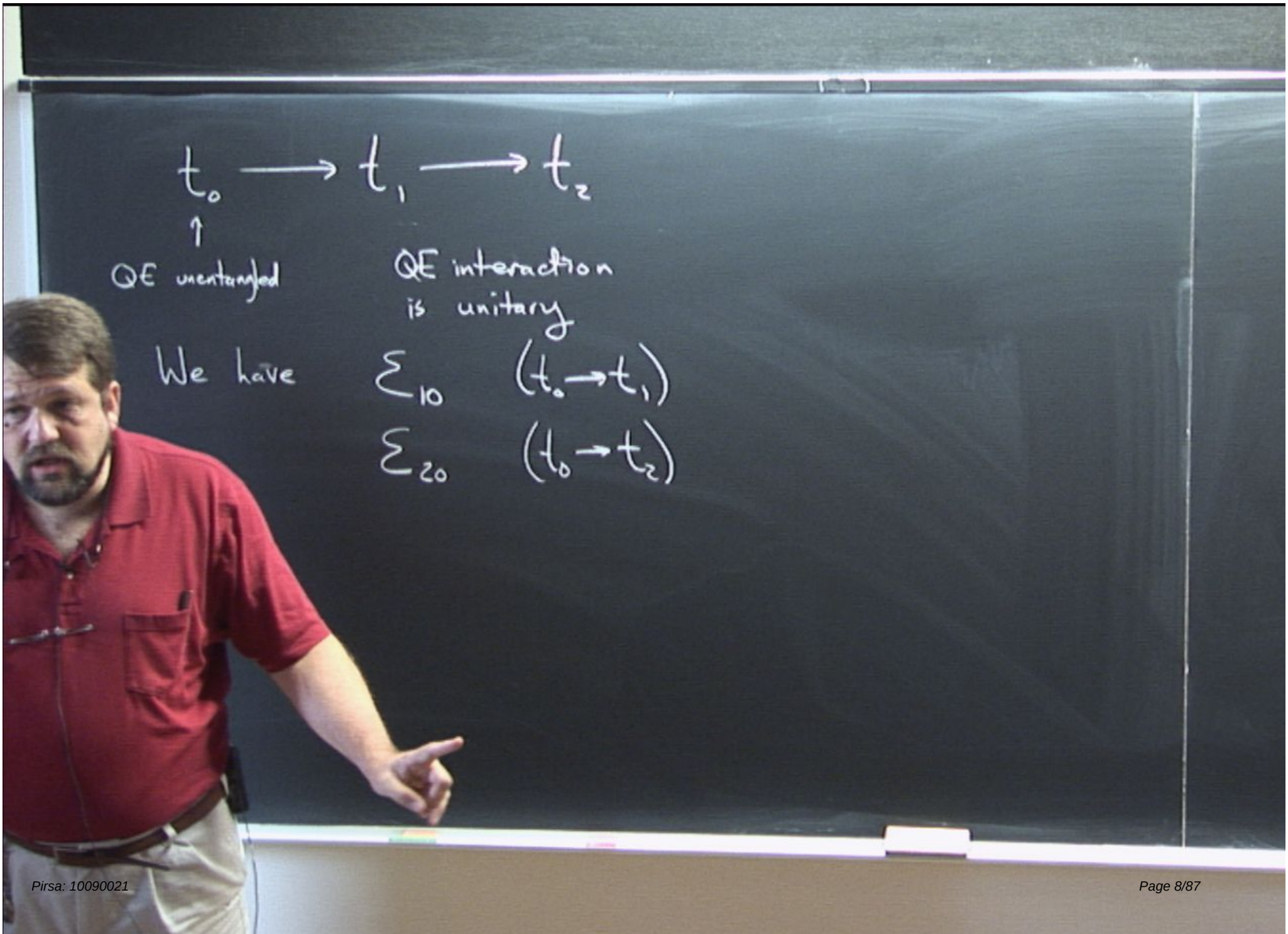
We have $\mathcal{E}_{10} \quad (t_0 \to t_1)$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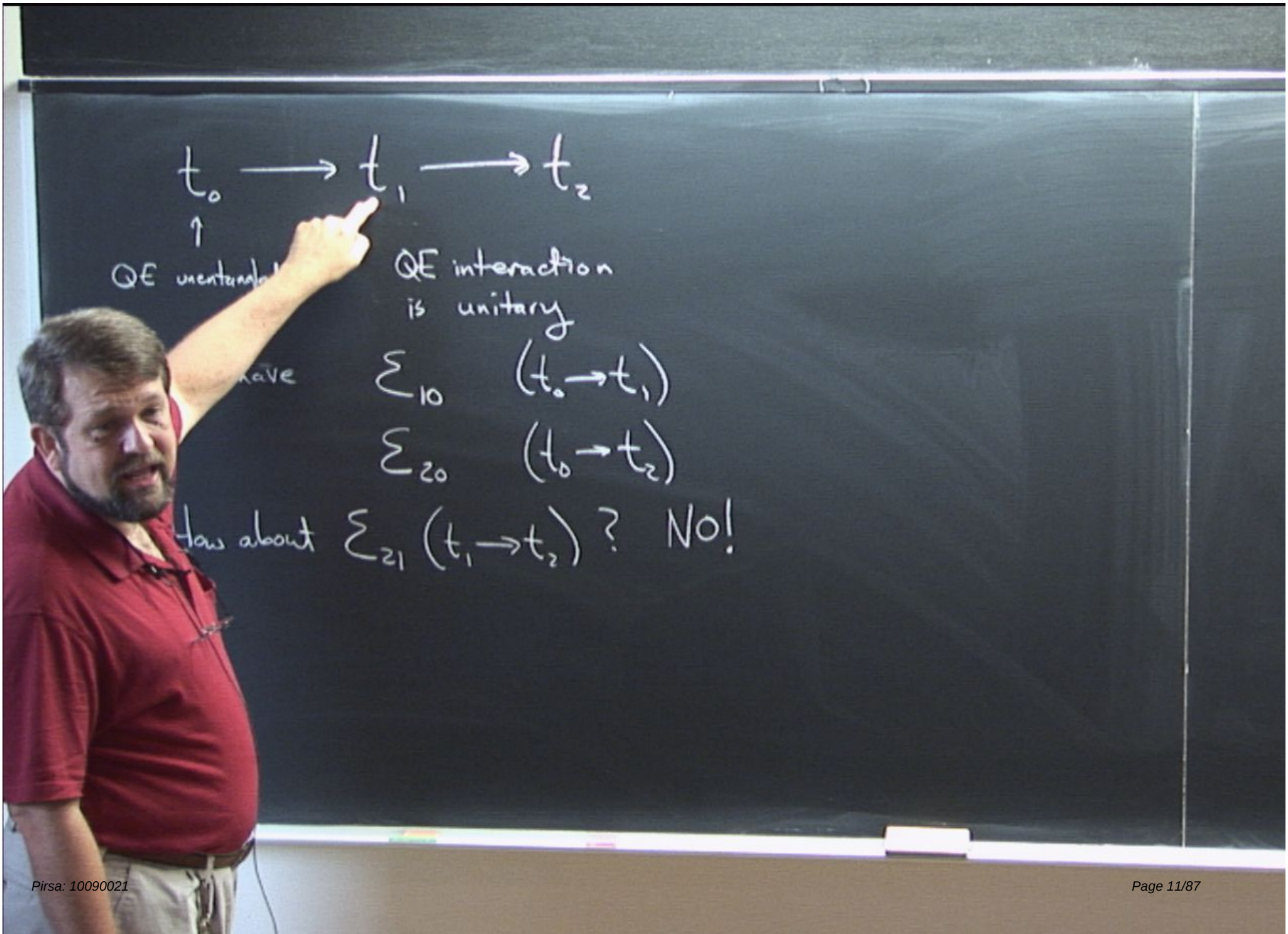
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We don't have $\mathcal{E}_{02} = \mathcal{E}_{21} \circ \mathcal{E}_{01}$

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State at t_{n+1} depends <u>only</u> on state at t_n

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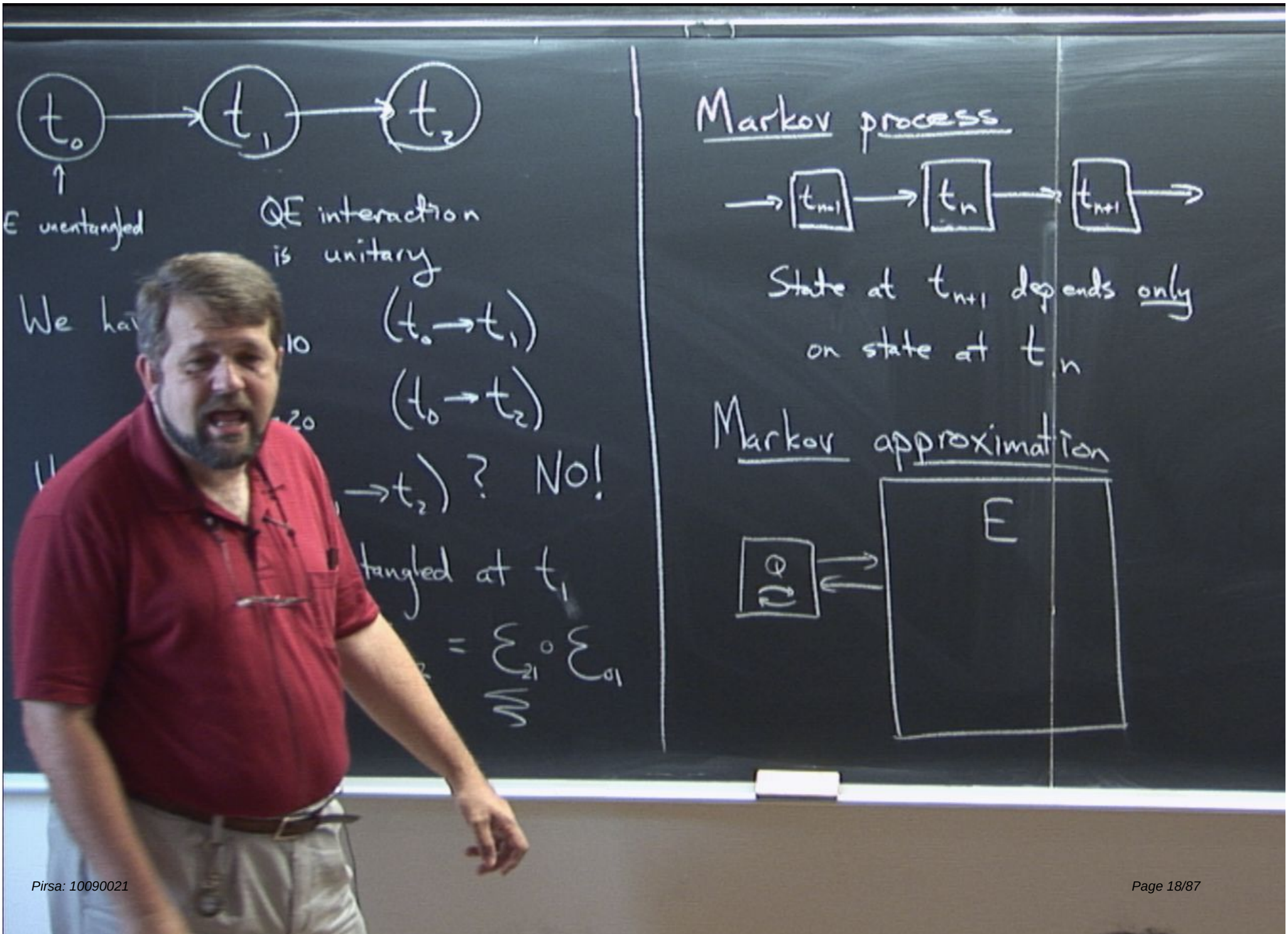
Markov approximation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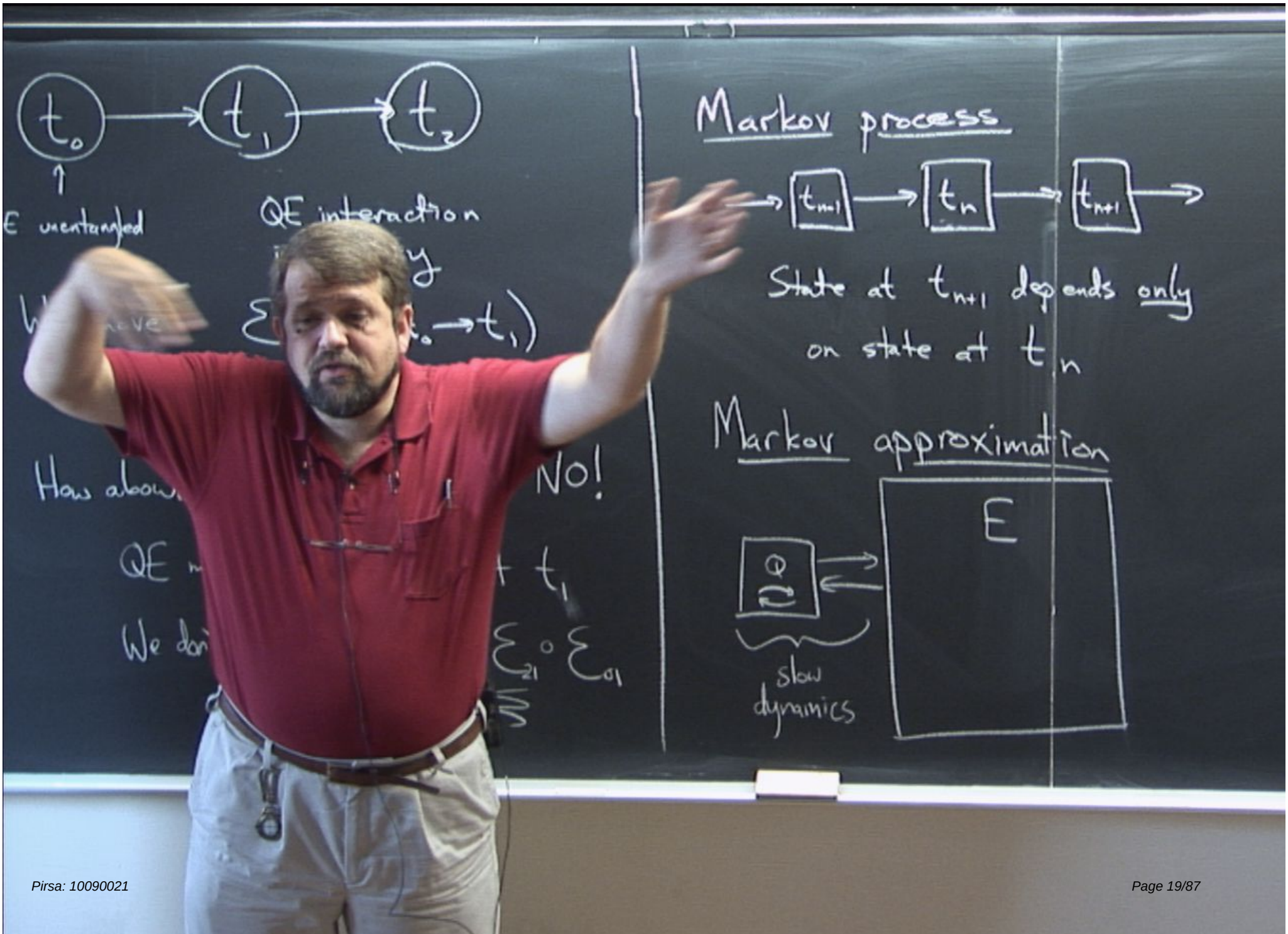
Markov process
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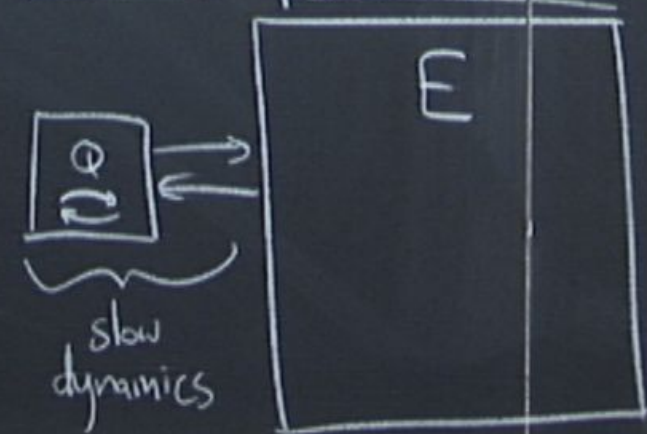

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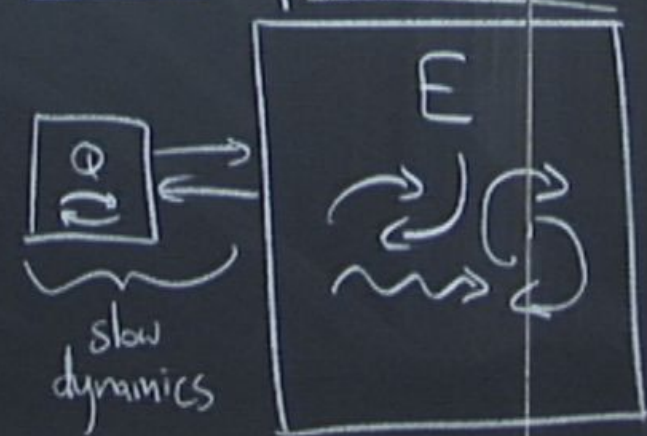

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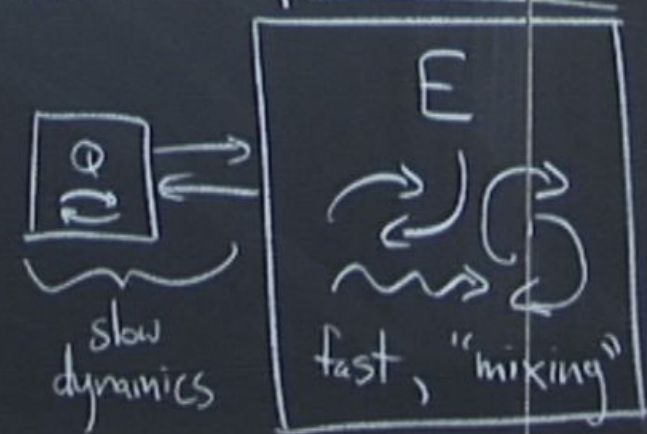

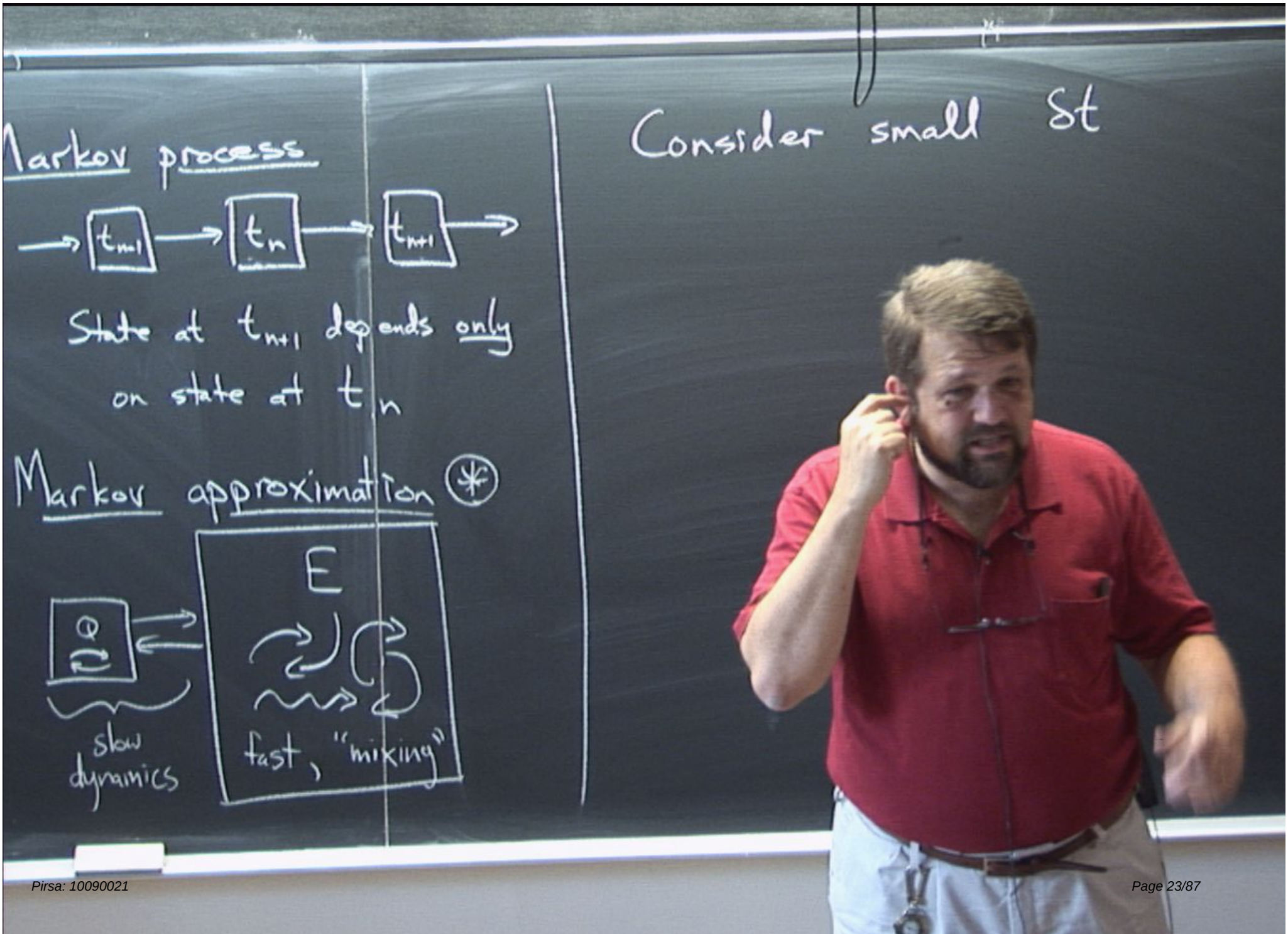
Markov process
t_{n-1}
t_n
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Markov approximation (*)
Q
E
slow dynamics
fast, "mixing"
Consider small δt


Consider small δt
(Q: δt short
E: δt long)
ds only

Consider small δt $\left(\begin{array}{l} Q: \ \delta t \text{ short} \\ E: \ \delta t \text{ long} \end{array}\right)$

Q-state $\rho \longrightarrow \rho + \delta\rho$ $\quad \delta\rho \propto \delta t$ (ignore $O(\delta t^2)$)

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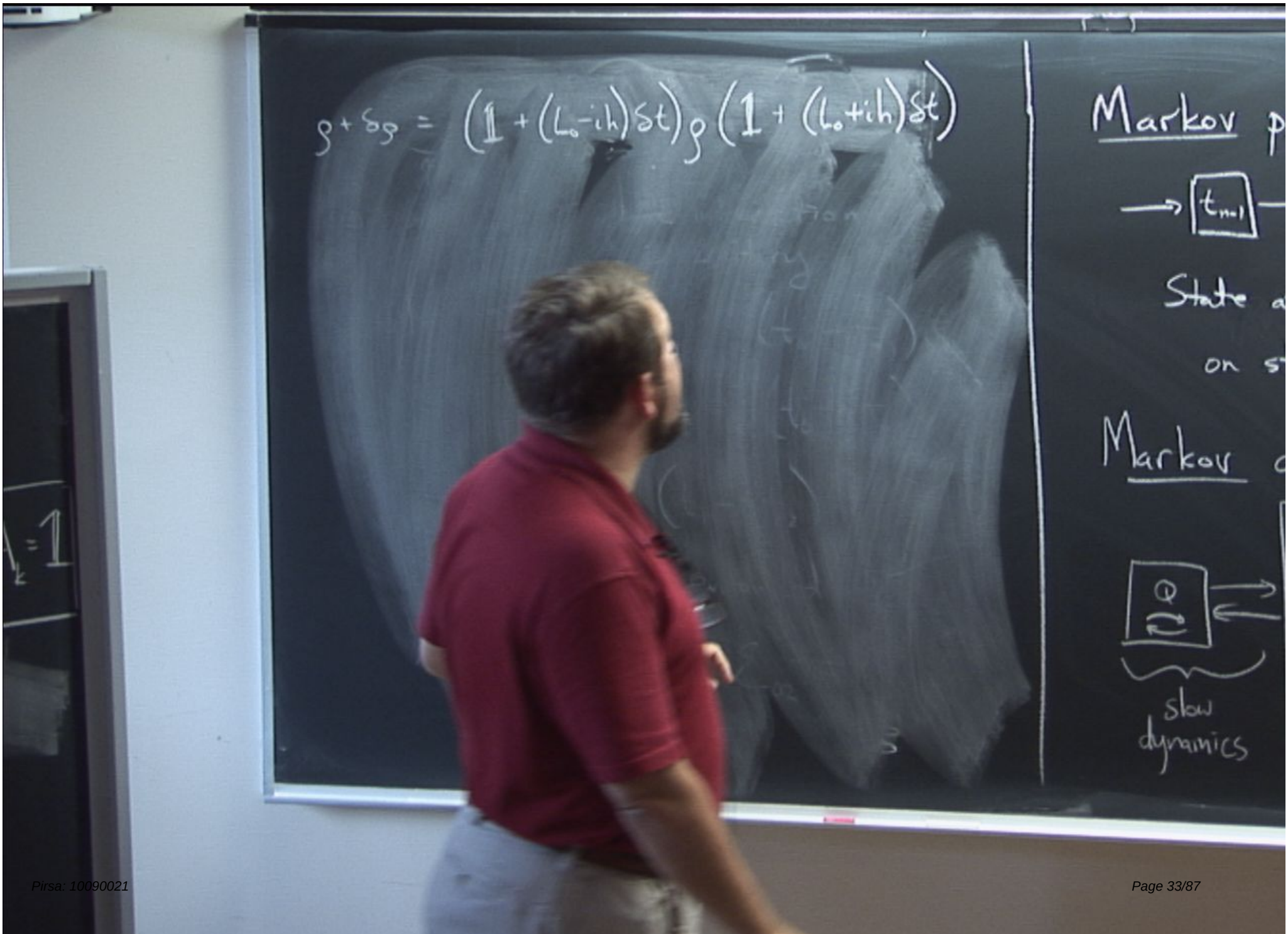
$\rho + \delta\rho = (\mathbb{1} + (L_0 - ih)\delta t)\rho(\mathbb{1} + (L_0 + ih)\delta t)$
Markov
t_{n-1}
State
on
Markov
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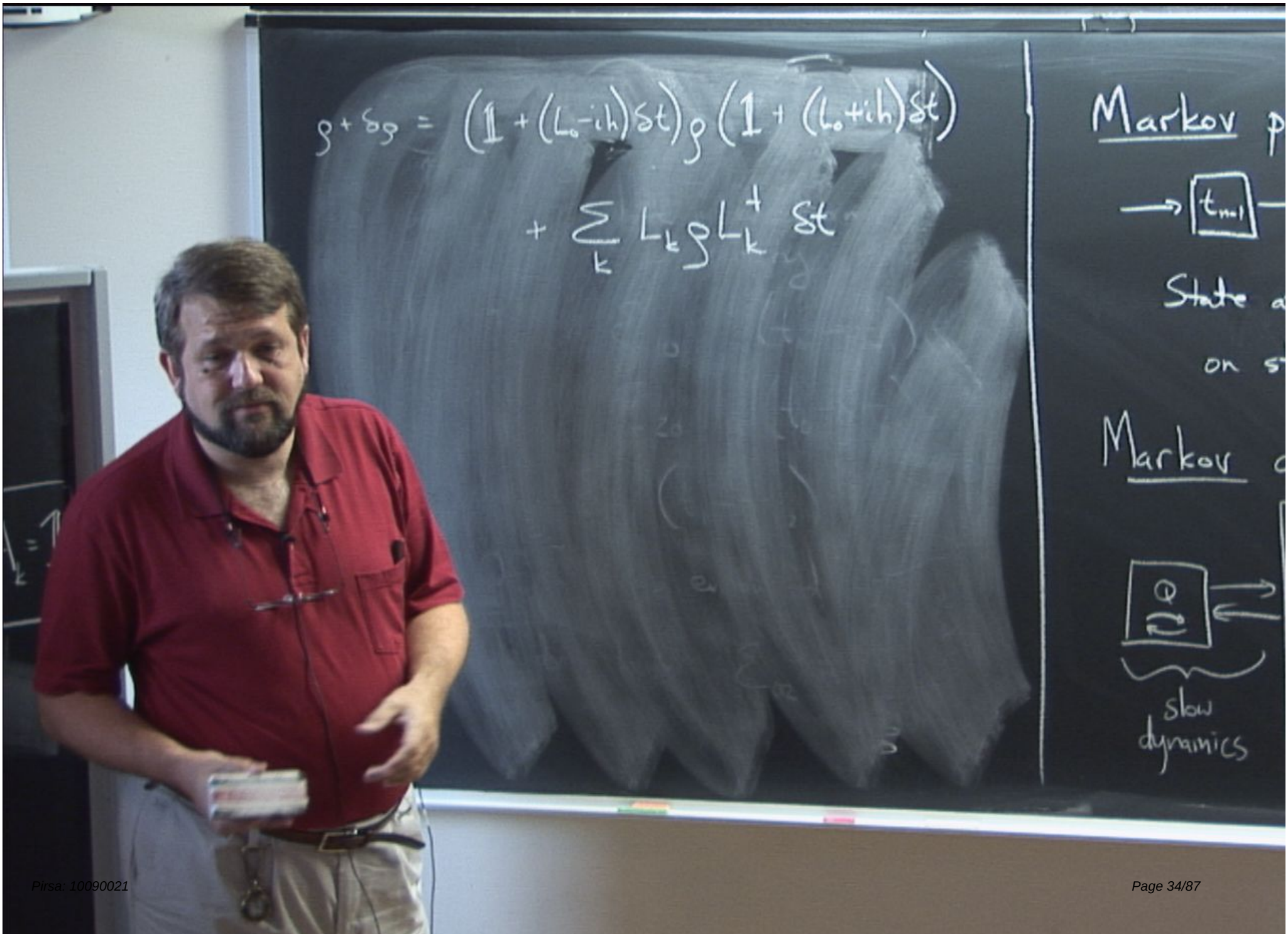
dynamics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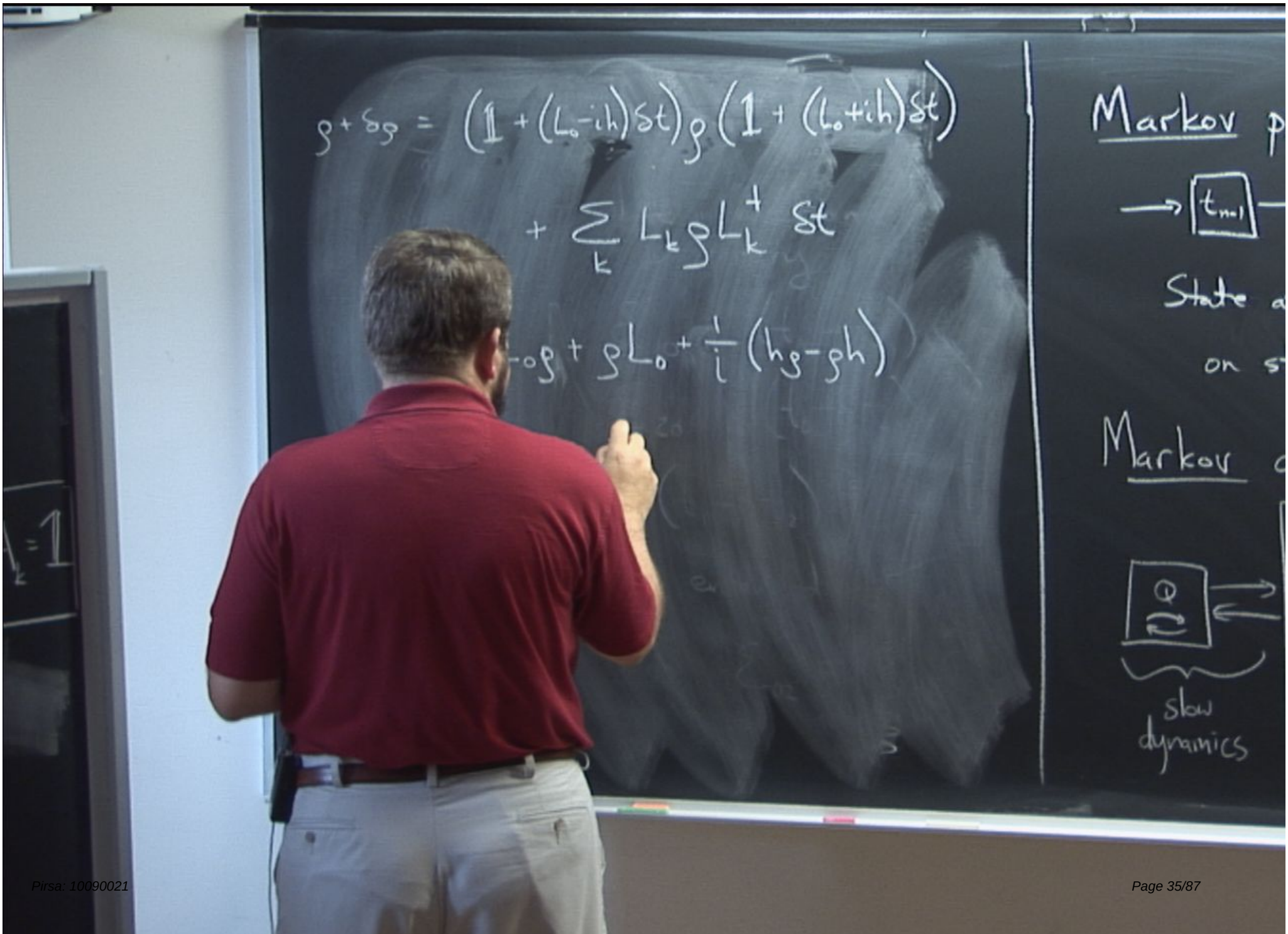
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$\rho L_0 + \frac{1}{i}(h\rho - \rho h)$
Markov
t_{n-1}
State
on
Markov
slow dynamics

$$\rho + \delta\rho = \left(\mathbb{1} + (L_0 - ih)\delta t\right)\rho\left(\mathbb{1} + (L_0 + ih)\delta t\right) + \sum_k L_k \rho L_k^\dagger \, \delta t$$

$$\delta\rho = \left(L_0\rho + \rho L_0 + \frac{1}{i}(h\rho - \rho h) + \sum_k L_k \rho L_k^\dagger\right)\delta t$$

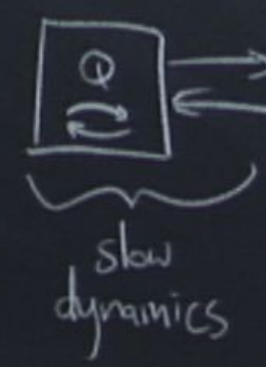
Markov p

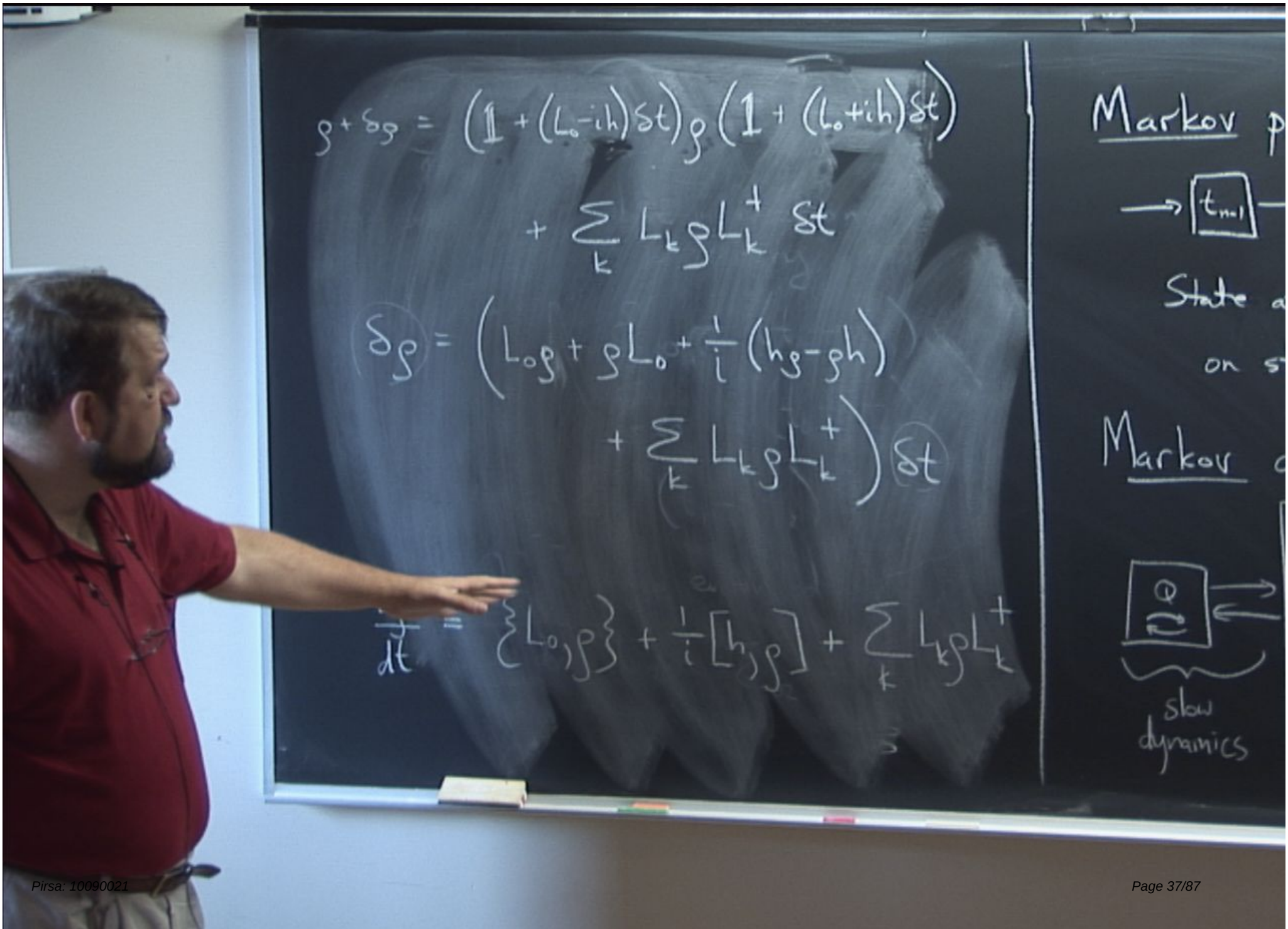
$\rightarrow \boxed{t_{n-1}}$ —

State a

on s

Markov c

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State
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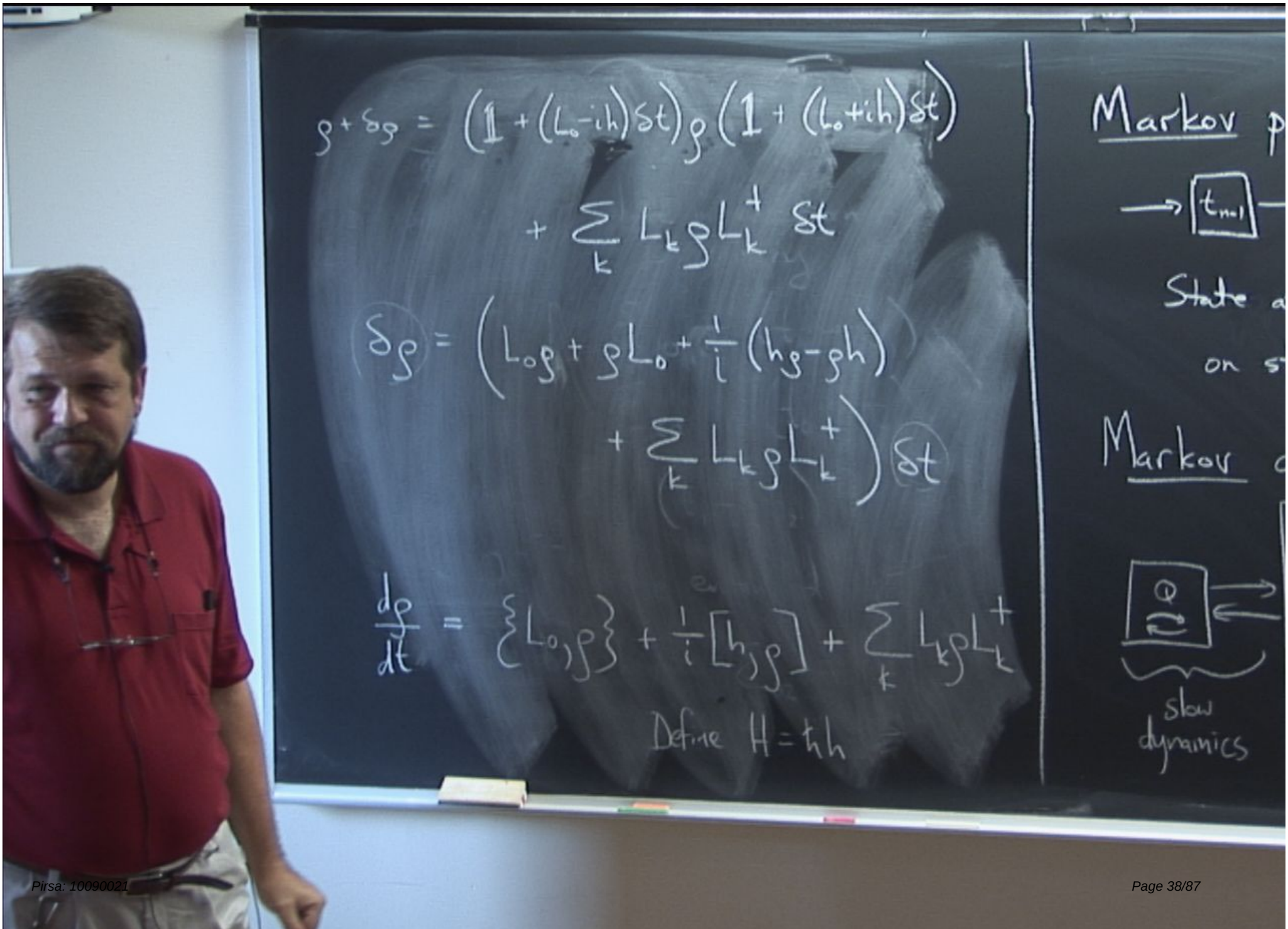
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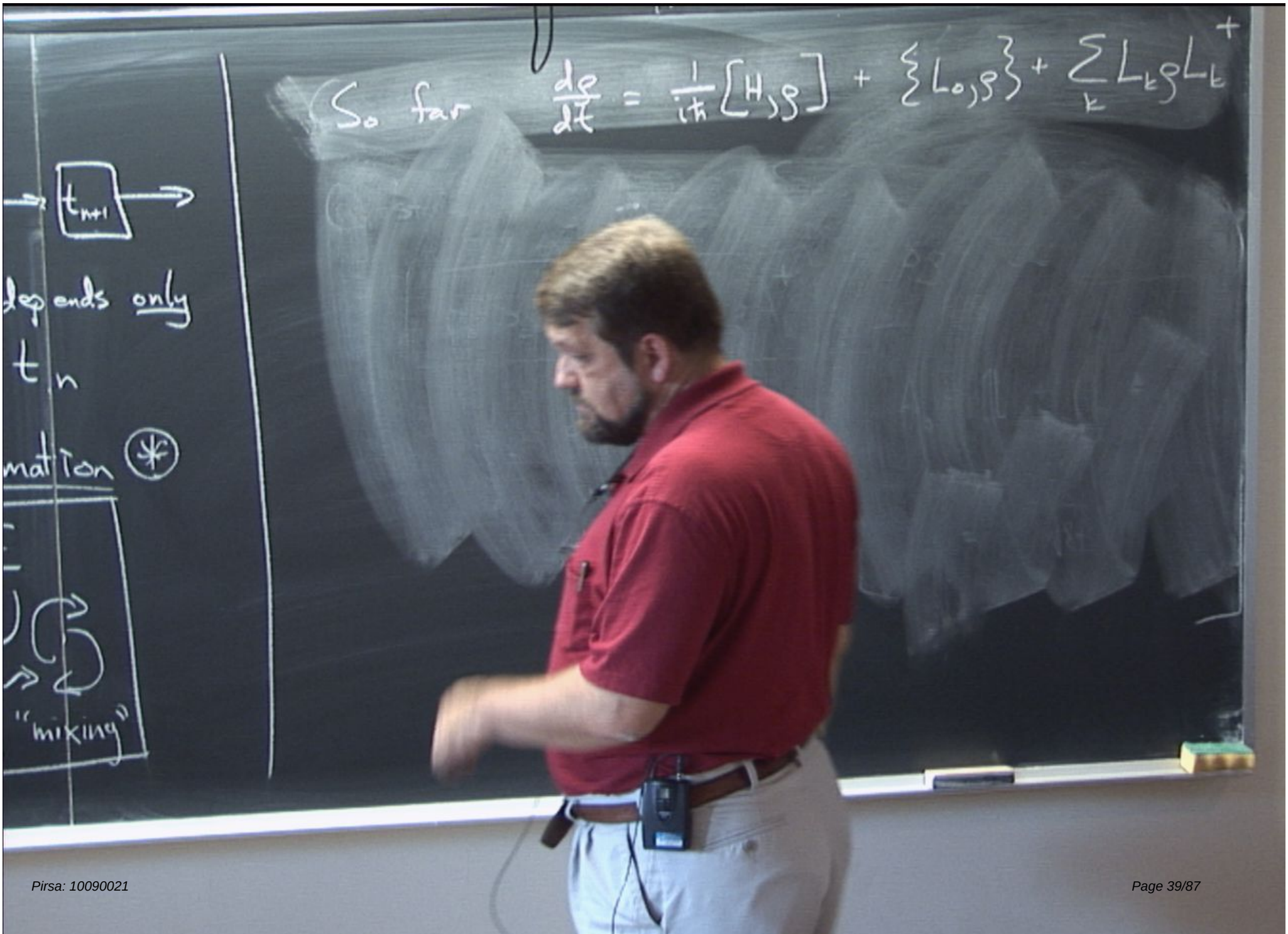
$\frac{d\rho}{dt} = \{L_0, \rho\} + \frac{1}{i}[h, \rho] + \sum_k L_k \rho L_k^\dagger$
Define $H = \hbar h$
Markov
t_{n+1}
State
on
Markov
slow dynamics


So far $\frac{d\rho}{dt} = \frac{1}{i\hbar}[H,\rho] + \{L_0,\rho\} + \sum_k L_k \rho L_k^+$
t_{n+1}
depends only
t_n
mation
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So far $\frac{d\rho}{dt} = \frac{1}{i\hbar}[H,\rho] + \{L_0,\rho\} + \sum_k L_k \rho L_k^\dagger$

$0 = \mathrm{tr}\left(\frac{d\rho}{dt}\right) = 0 \quad 2\,\mathrm{tr}\,\rho L_0$

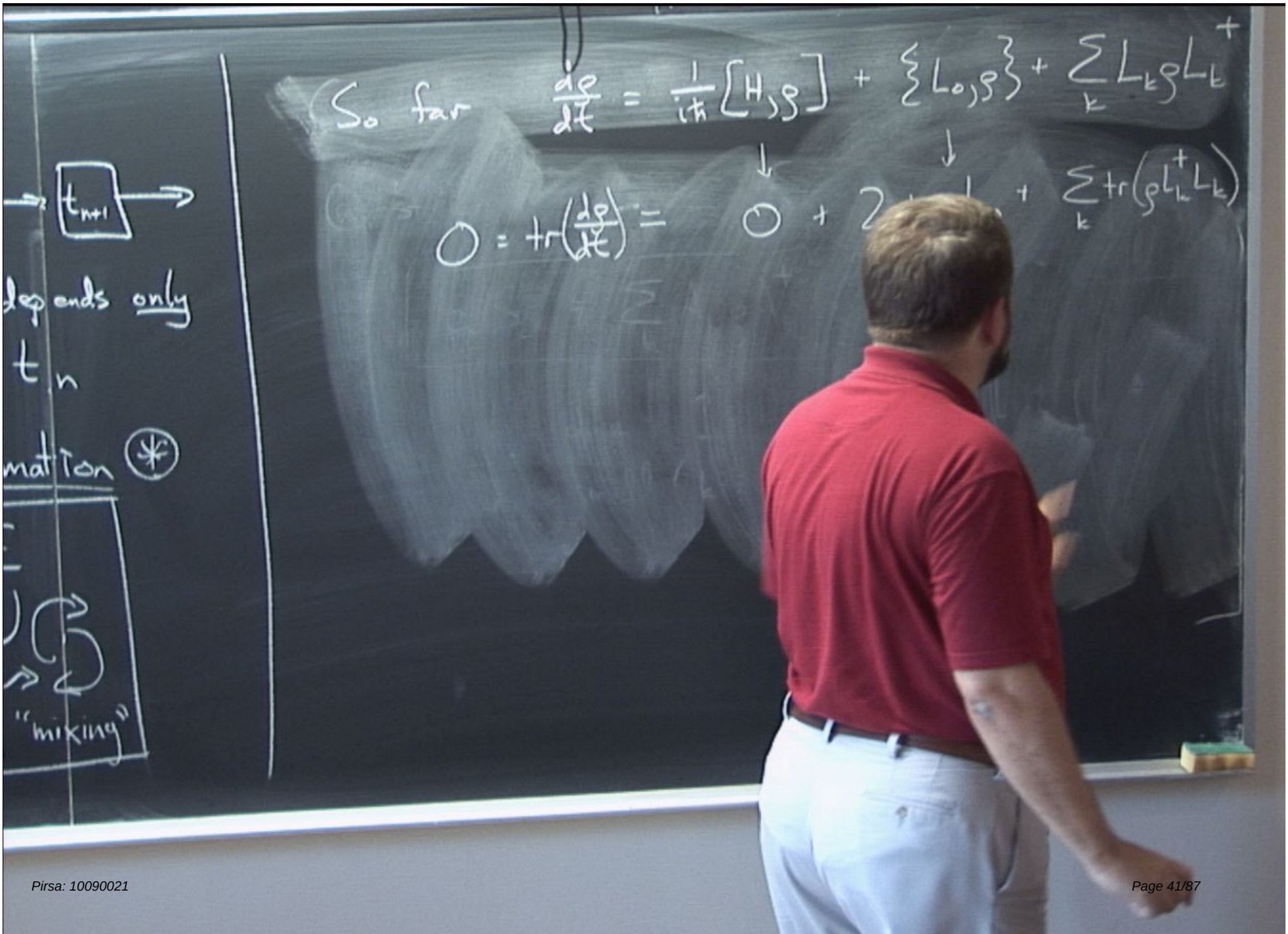
t_{n+1}

depends only

t_n

mation (*)

"mixing"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$0 = tr\left(\frac{d\rho}{dt}\right) = 0 + 2 + \sum_k tr(\rho L_k^+ L_k)$
t_{n+1}
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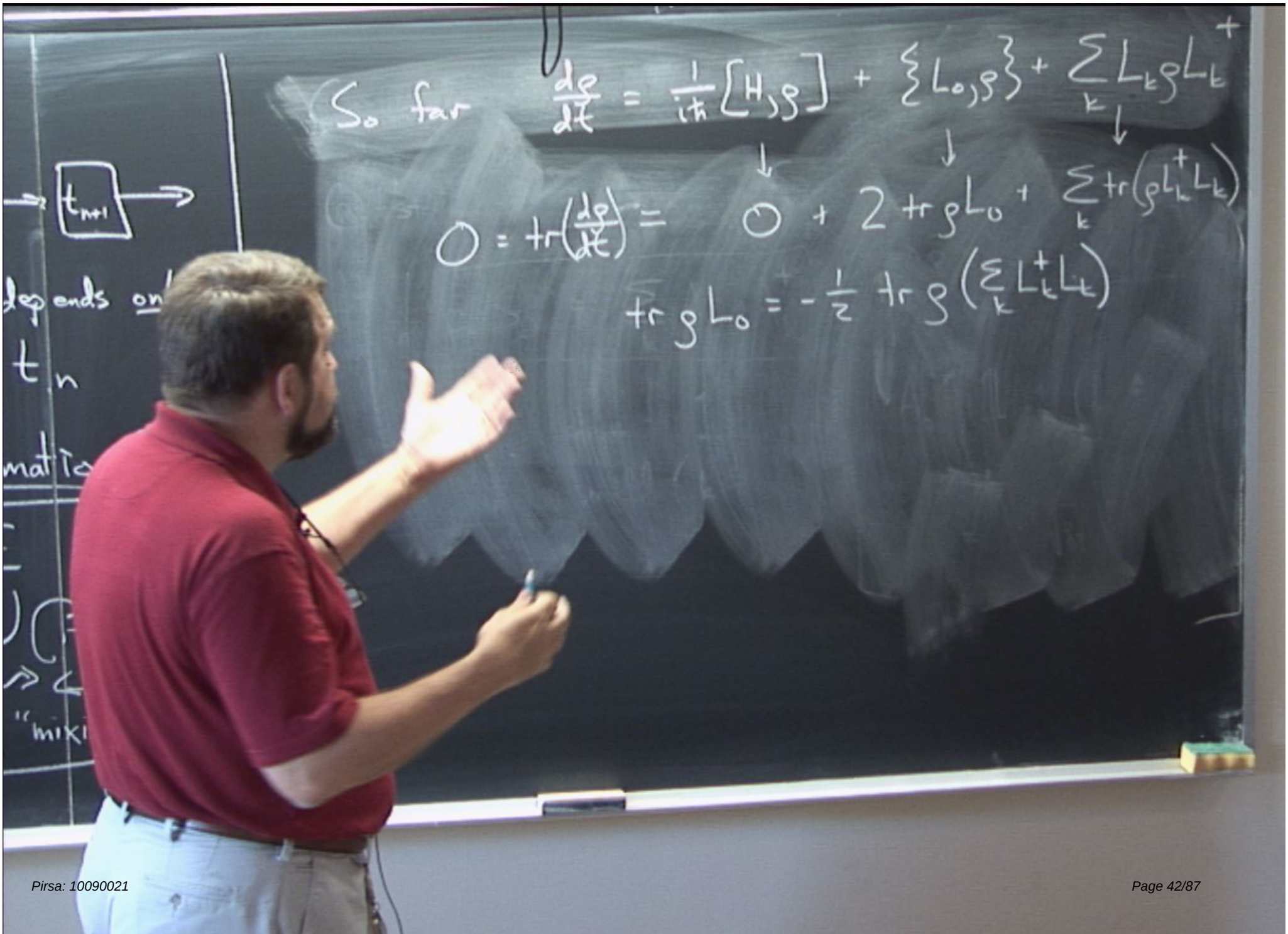
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t_{n+1}
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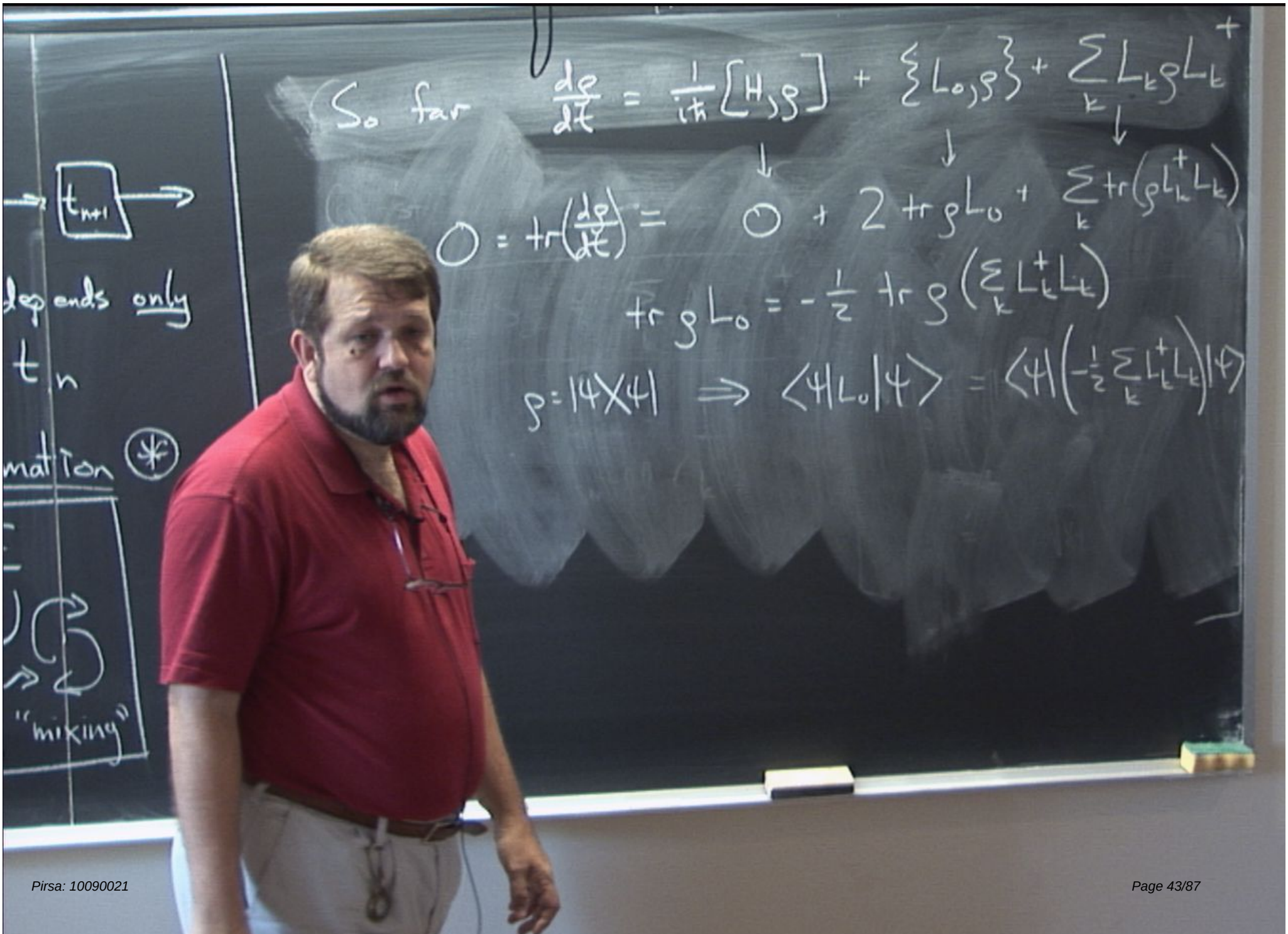
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$\rho = |\psi\rangle\langle\psi| \Rightarrow \langle\psi|L_0|\psi\rangle = \langle\psi|\left(-\frac{1}{2}\sum_k L_k^+ L_k\right)|\psi\rangle$
t_{n+1}
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mation
"mixing"

So far $\frac{d\rho}{dt} = \frac{1}{i\hbar}[H,\rho] + \{L_0,\rho\} + \sum_k L_k \rho L_k^\dagger$

$$0 = \mathrm{tr}\left(\frac{d\rho}{dt}\right) = 0 + 2\,\mathrm{tr}\,\rho L_0 + \sum_k \mathrm{tr}\left(\rho L_k^\dagger L_k\right)$$

$$\mathrm{tr}\,\rho L_0 = -\frac{1}{2}\,\mathrm{tr}\,\rho\left(\sum_k L_k^\dagger L_k\right)$$

$$\rho = |\psi\rangle\langle\psi| \Rightarrow \langle\psi|L_0|\psi\rangle = \langle\psi|\left(-\frac{1}{2}\sum_k L_k^\dagger L_k\right)|\psi\rangle$$

$$\boxed{L_0 = -\frac{1}{2}\sum_k L_k^\dagger L_k}$$

t_{n+1}

depends only

t_n

matrix etc

"mixing"

$L + (L_0 + ih)\delta t)$

$L_k^+ \delta t$

$(h\rho - \rho h)$ $+ \mathcal{O}(\delta t^2)$

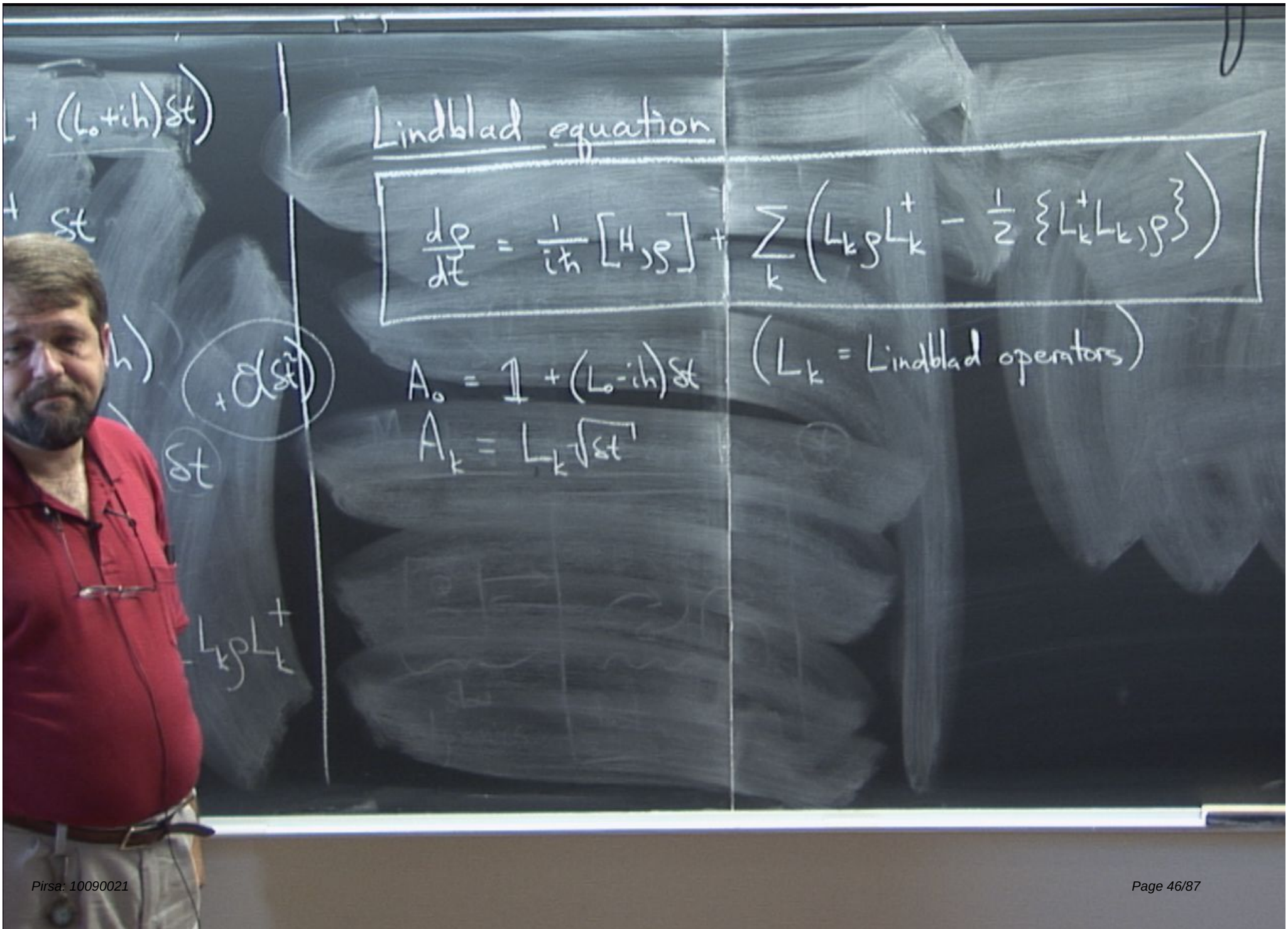
$\rho L_k^+) \delta t$

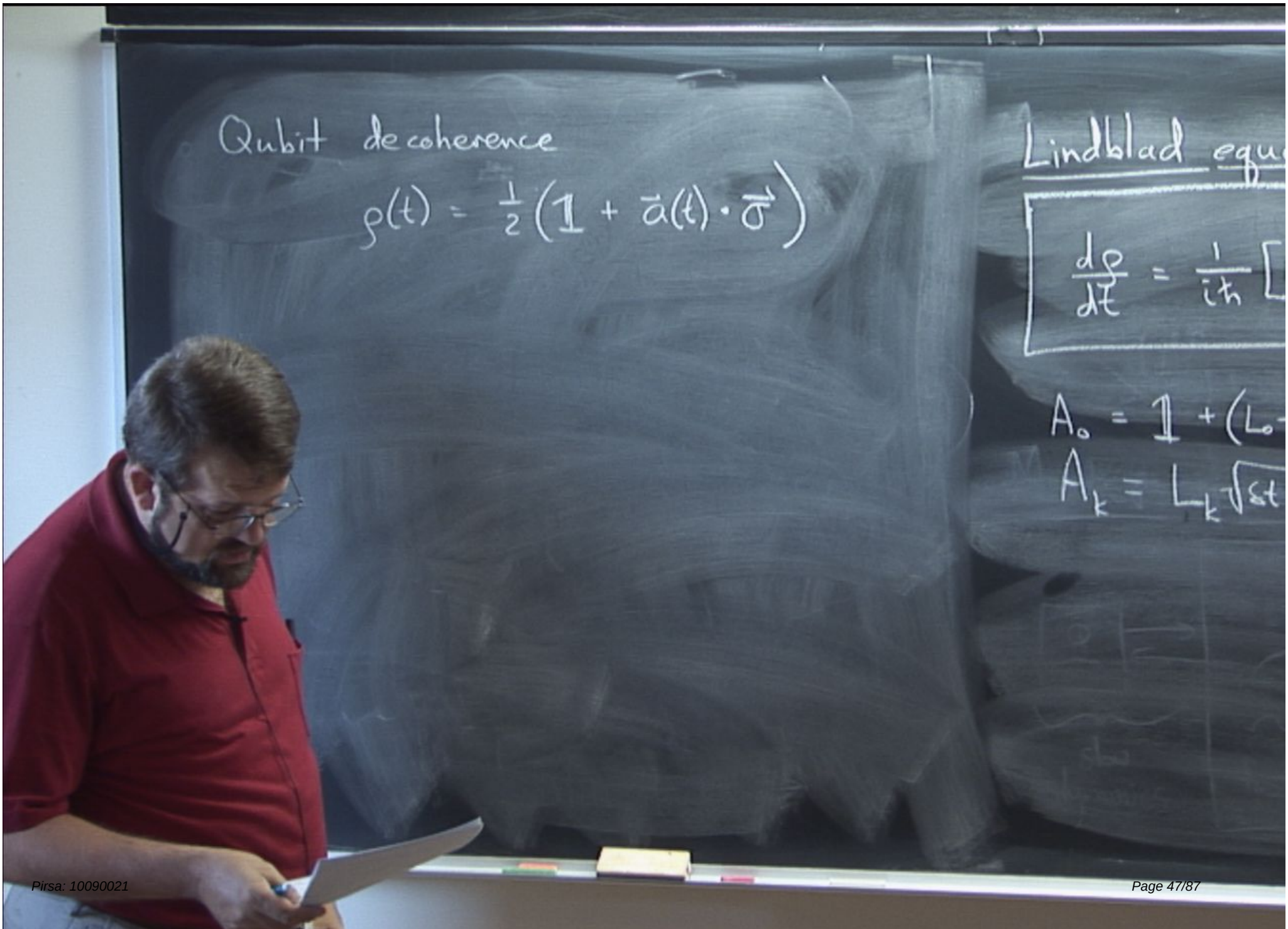
$\sum_k L_k \rho L_k^+$

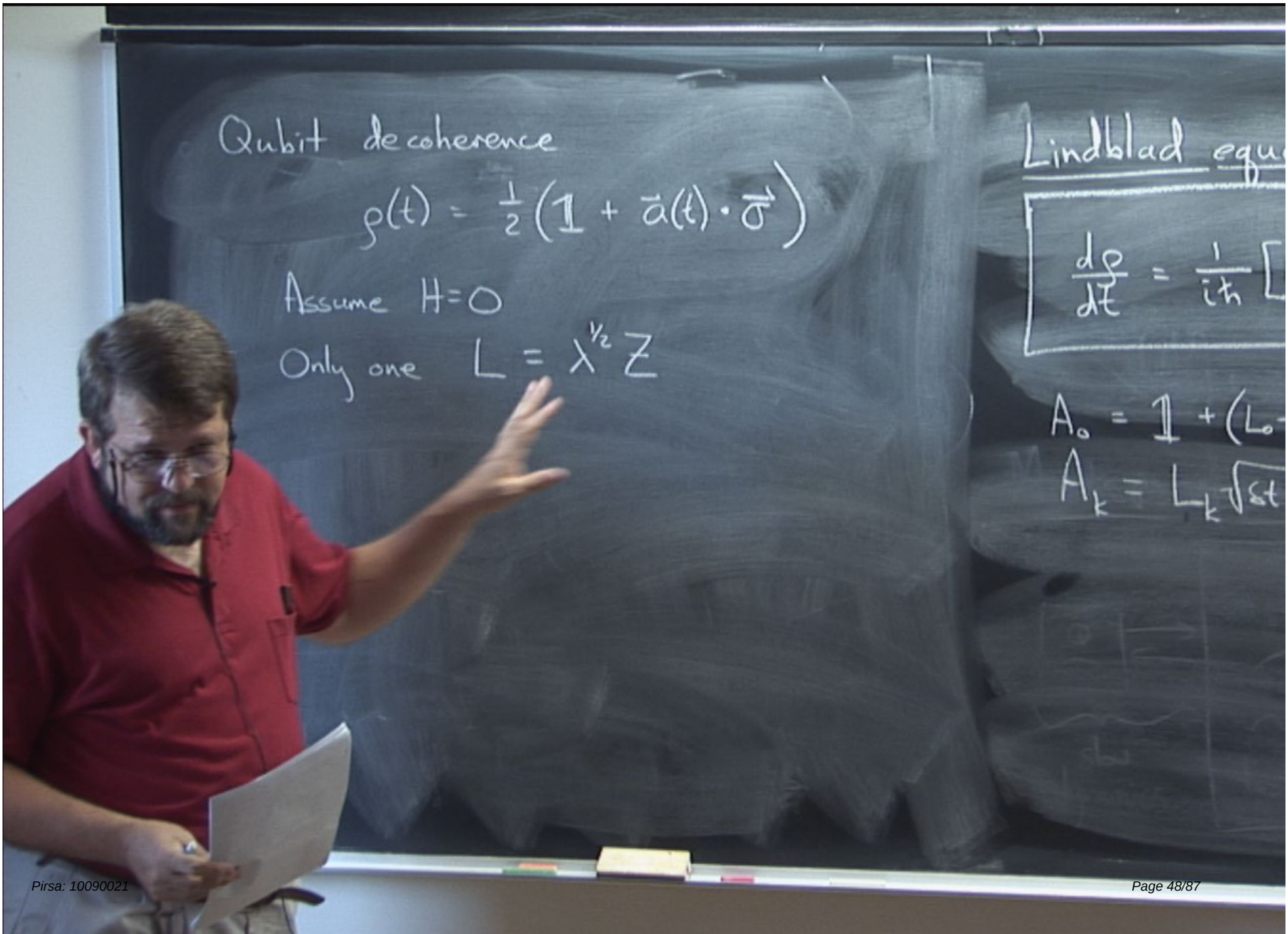
Lindblad equation

$$\boxed{\frac{d\rho}{dt} = \frac{1}{i\hbar}[H, \rho] + \sum_k \left(L_k \rho L_k^+ - \frac{1}{2}\{L_k^+ L_k, \rho\} \right)}$$

$\rho = |\psi$


Lindblad equation
$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H,\rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right)$
(L_k = Lindblad operators)
$A_0 = \mathbb{1} + (L_0 - i\hbar)\delta t$
$A_k = L_k \sqrt{\delta t}$


Qubit decoherence
$\rho(t) = \frac{1}{2}\left(\mathbb{1} + \vec{a}(t)\cdot\vec{\sigma}\right)$
Lindblad equ
$\frac{d\rho}{dt} = \frac{1}{i\hbar}$
$A_0 = \mathbb{1} + (L_0$
$A_k = L_k\sqrt{\delta t}$


Qubit decoherence
$\rho(t) = \frac{1}{2}(\mathbb{1} + \vec{a}(t)\cdot\vec{\sigma})$
Assume $H=0$
Only one $L = \lambda^{1/2} Z$
Lindblad equ
$\frac{d\rho}{dt} = \frac{1}{i\hbar}$
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Lindblad equation

$$\boxed{\frac{d\rho}{dt} = \frac{1}{i\hbar}[H,\rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right)}$$

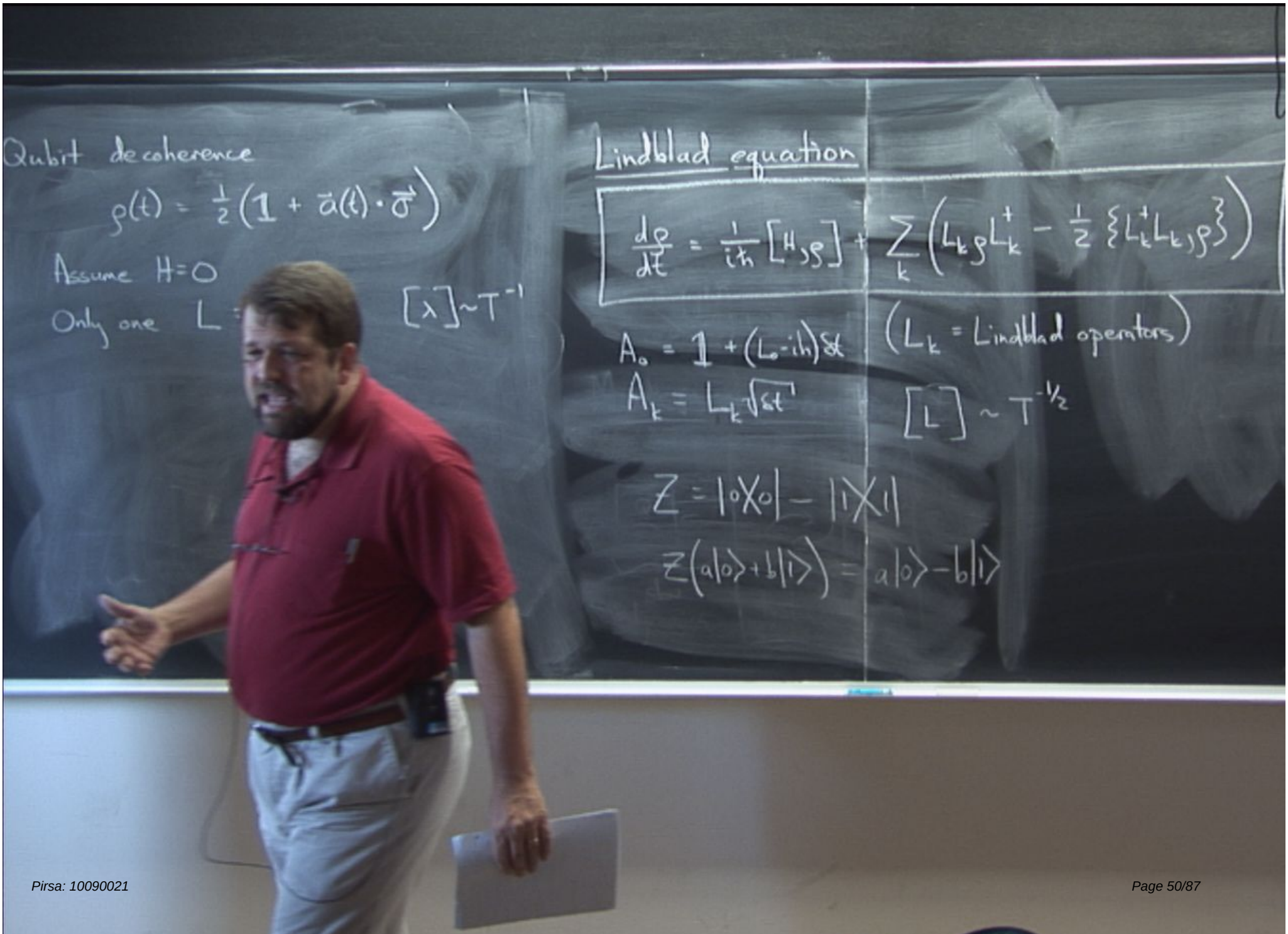
$(L_k = \text{Lindblad operators})$

$A_0 = \mathbb{1} + (L_0 - i\hbar)\delta t$

$A_k = L_k \sqrt{\delta t}$

$[L] \sim T^{-1/2}$

$\mathrm{tr}\,\rho$


Qubit decoherence
$\rho(t) = \frac{1}{2}(\mathbb{1} + \vec{a}(t)\cdot\vec{\sigma})$
Assume $H=0$
Only one L
$[\lambda] \sim T^{-1}$
Lindblad equation
$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H,\rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}\right)$
$A_0 = \mathbb{1} + (L_0 - i\hbar)\delta t$
$A_k = L_k\sqrt{\delta t}$
(L_k = Lindblad operators)
$[L] \sim T^{-1/2}$
$Z = |0\rangle\langle 0| - |1\rangle\langle 1|$
$Z(a|0\rangle + b|1\rangle) = a|0\rangle - b|1\rangle$

$$A_0 = \mathbb{1} + (L_0 - ih)\delta t$$

$$A_k = L_k \sqrt{\delta t}$$

$(L_k$ - Lindblad op...

$$[L] \sim T^{-1/2}$$

$$Z = |0\rangle\langle 0| - |1\rangle\langle 1|$$

$$Z(a|0\rangle + b|1\rangle) = a|0\rangle - b|1\rangle$$

Qubit decoherence

$$\rho(t) = \frac{1}{2}\left(\mathbb{1} + \vec{a}(t)\cdot\vec{\sigma}\right)$$

Assume $H=0$

Only one $L = \lambda^{1/2} Z \qquad [\lambda] \sim T^{-1}$

rep. a random phase flip

Lindblad equatio

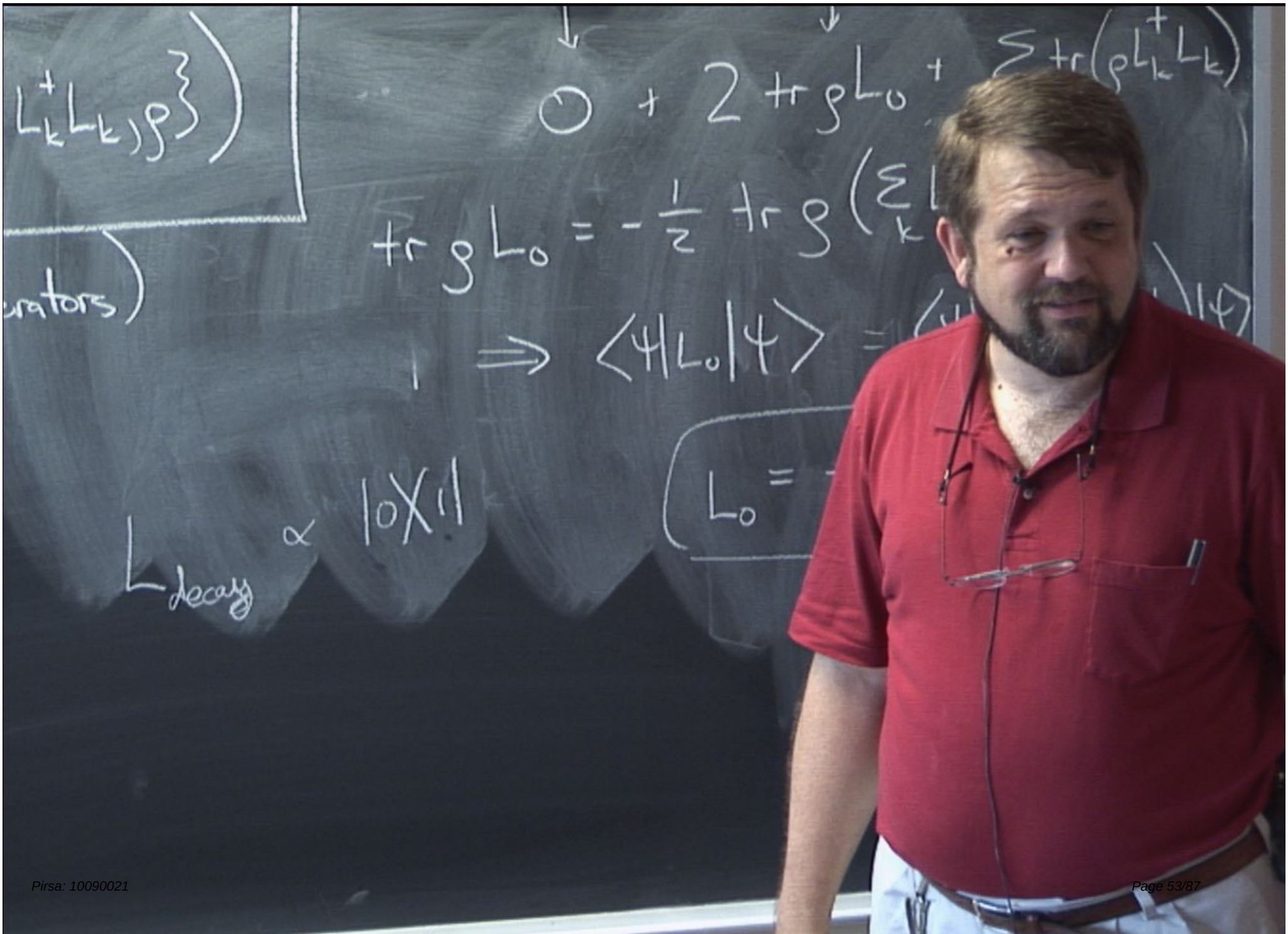
$$\boxed{\frac{d\rho}{dt} = \frac{1}{i\hbar}[H,\rho]}$$

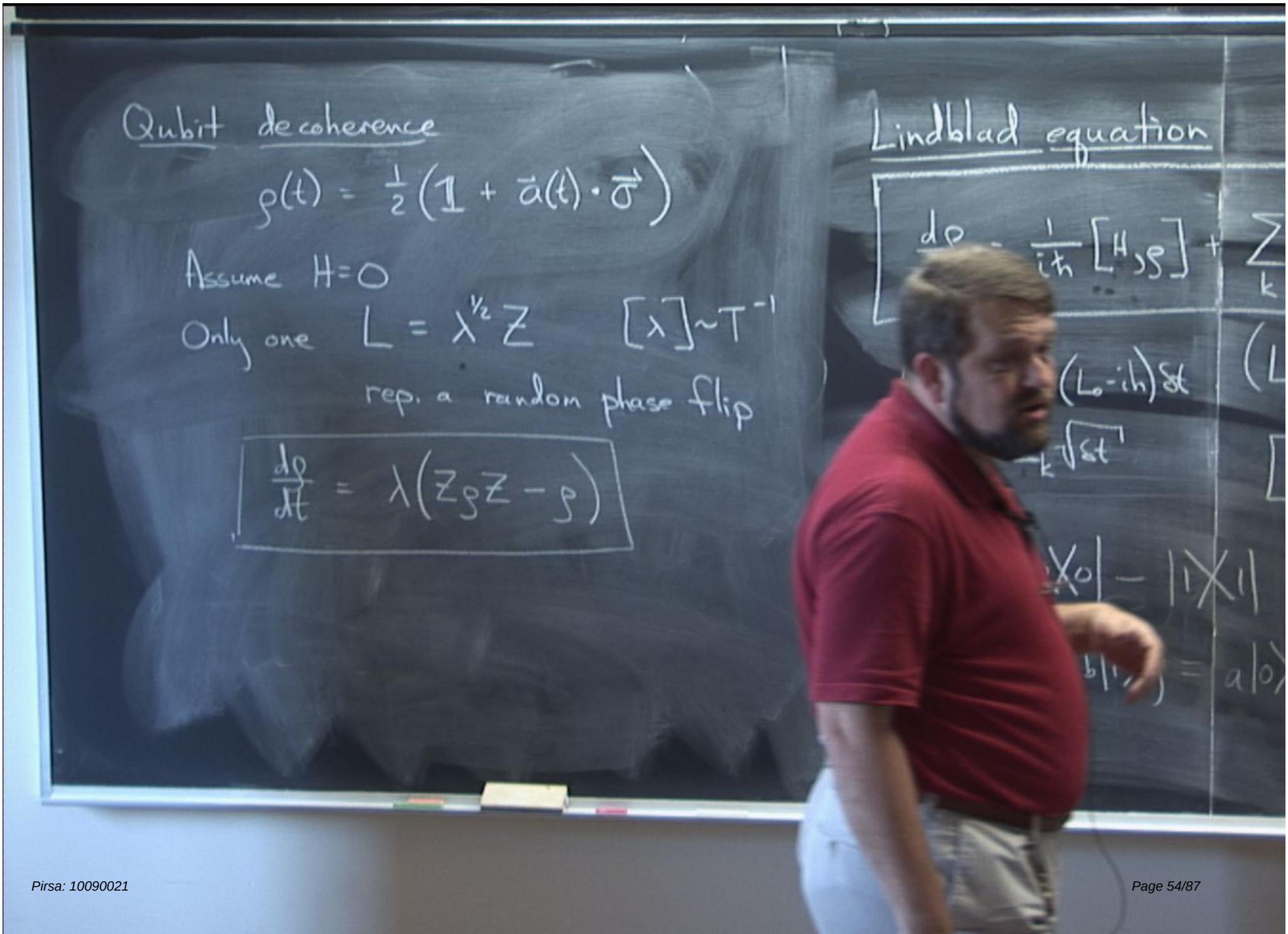
$$A_0 = \mathbb{1} + (L_0 - i\hbar)\delta$$

$$A_k = L_k\sqrt{\delta t}$$

$$Z = |0\rangle\langle 0| - |$$

$$Z\left(a|0\rangle + b|1\rangle\right)$$


$L_k^\dagger L_k)\rho\}$
$0 + 2\,\mathrm{tr}\,\rho L_0 + \sum \mathrm{tr}(\rho L_k^\dagger L_k)$
$\mathrm{tr}\,\rho L_0 = -\frac{1}{2}\,\mathrm{tr}\,\rho\left(\sum_k L\right.$
$\Rightarrow \langle\psi|L_0|\psi\rangle =$
$L_0 =$
$L_{decay} \propto |0X|$


Qubit decoherence
$\rho(t) = \frac{1}{2}(\mathbb{1} + \vec{a}(t)\cdot\vec{\sigma})$
Assume $H=0$
Only one $L = \lambda^{1/2} Z$ $[\lambda] \sim T^{-1}$
rep. a random phase flip
$\frac{d\rho}{dt} = \lambda(Z\rho Z - \rho)$
Lindblad equation

Qubit decoherence

$$\rho(t) = \frac{1}{2}\left(\mathbb{1} + \vec{a}(t)\cdot\vec{\sigma}\right)$$

Assume $H=0$

Only one $L = \lambda^{1/2} Z \qquad [\lambda] \sim T^{-1}$

rep. a random phase flip

$$\boxed{\frac{d\rho}{dt} = \lambda\left(Z\rho Z - \rho\right)}$$

$ZXZ - X =$

Lindblad equation

$$\boxed{\frac{d\rho}{dt} = \frac{1}{i\hbar}[H,\rho] + \sum_k}$$

$A_0 = \mathbb{1} + (L_0 - i\hbar)\delta t$

$A_k = L_k\sqrt{\delta t}$

$Z = |0\rangle\langle 0| - |1\rangle\langle 1|$

$Z(a|0\rangle + b|1\rangle) = a|0\rangle$

Qubit decoherence

$$\rho(t) = \frac{1}{2}\left(\mathbb{1} + \vec{a}(t)\cdot\vec{\sigma}\right)$$

Assume $H=0$

Only one $L = \lambda^{1/2} Z \qquad [\lambda] \sim T^{-1}$

rep. a random phase flip

$$\boxed{\frac{d\rho}{dt} = \lambda\left(Z\rho Z - \rho\right)}$$

$$ZXZ - X = -2X$$

$$ZYZ - Y = -2Y$$

$$ZZZ - Z =$$

Lindblad equation

$$\boxed{\frac{d\rho}{dt} = \frac{1}{i\hbar}[H,\rho] + \sum_k}$$

$$A_0 = \mathbb{1} + (L_0 - i\hbar)\delta t$$

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Qubit decoherence

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rep. a random phase flip

$$\boxed{\frac{d\rho}{dt} = \lambda\left(Z\rho Z - \rho\right)}$$

$$ZXZ - X = -2X$$

$$ZYZ - Y = -2Y$$

$$ZZZ - Z = 0$$

Lindblad equation

$$\boxed{\frac{d\rho}{dt} = \frac{1}{i\hbar}[H,\rho] + \sum_k}$$

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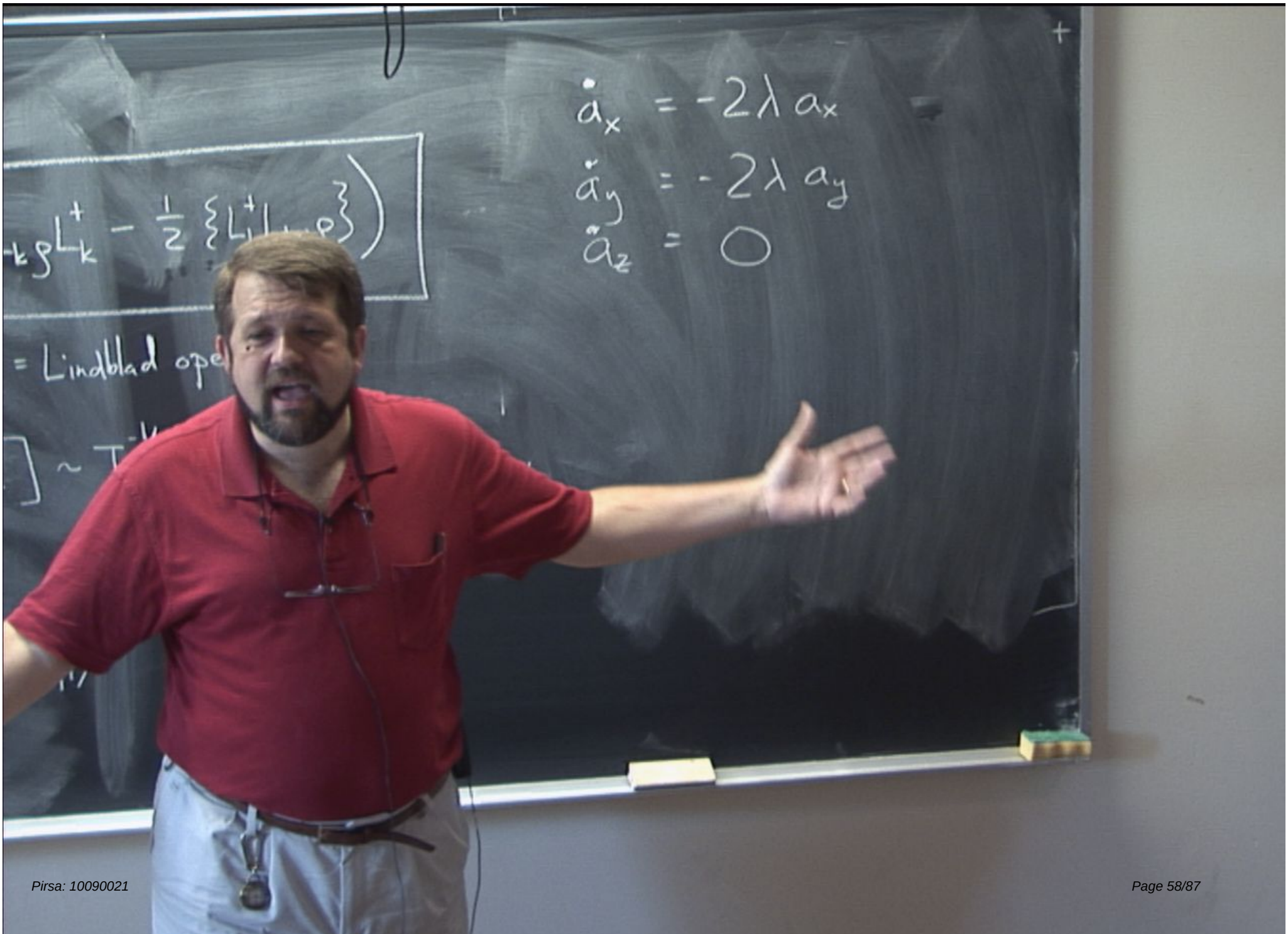

$\dot{a}_x = -2\lambda a_x$
$\dot{a}_y = -2\lambda a_y$
$\dot{a}_z = 0$
= Lindblad ope

$$\boxed{\left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}\right)}$$

L_k = Lindblad operators)

$[L] \sim T^{-1/2}$

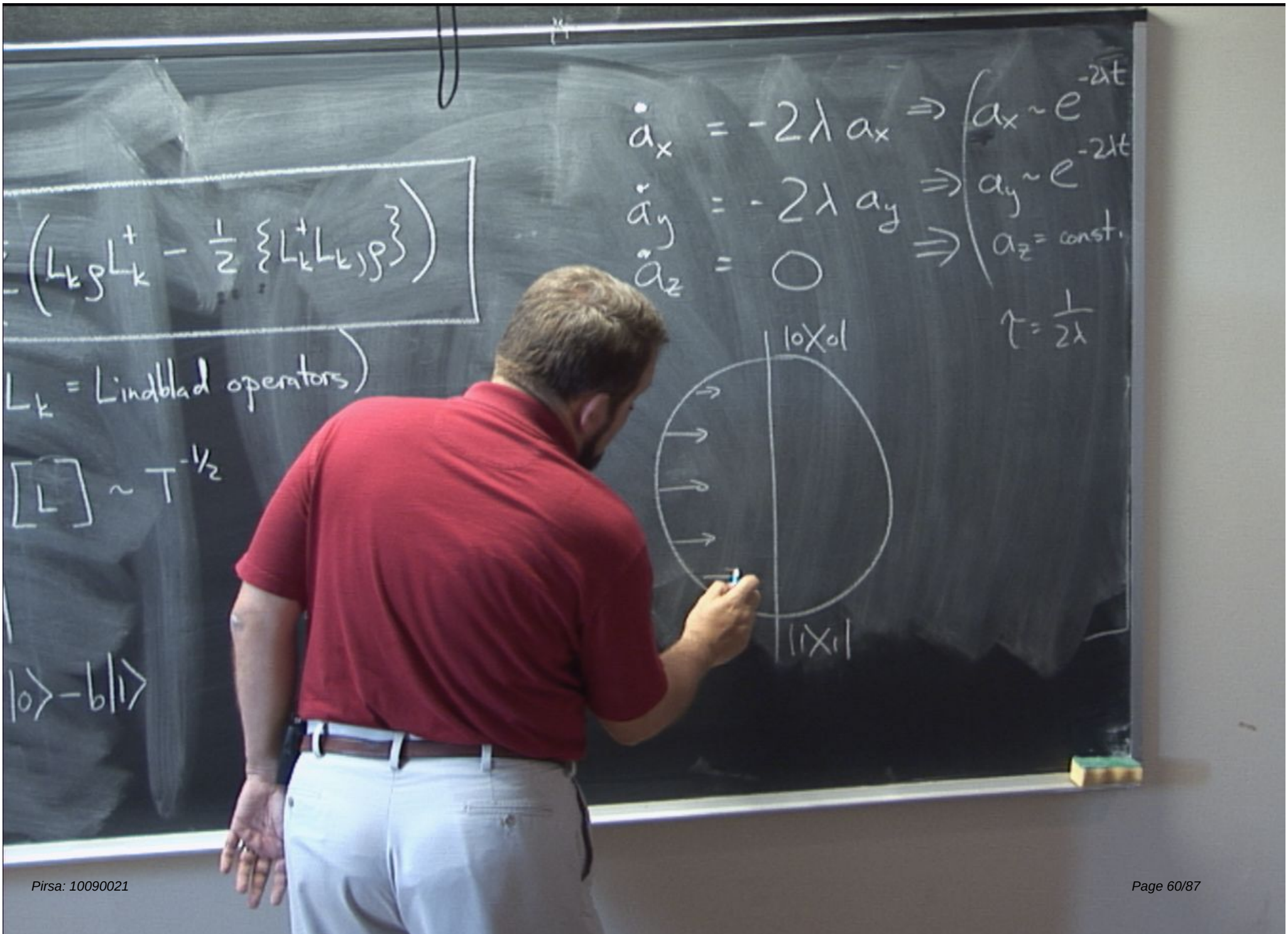
$|0\rangle - b|1\rangle$

$$\dot{a}_x = -2\lambda a_x \Rightarrow a_x \sim e^{-2\lambda t}$$

$$\dot{a}_y = -2\lambda a_y \Rightarrow a_y \sim e^{-2\lambda t}$$

$$\dot{a}_z = 0 \Rightarrow a_z = \text{const.}$$

$$\tau = \frac{1}{2\lambda}$$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$\left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}\right)$
L_k = Lindblad operators
$[L] \sim T^{-1/2}$
$|0\rangle\langle 0|$
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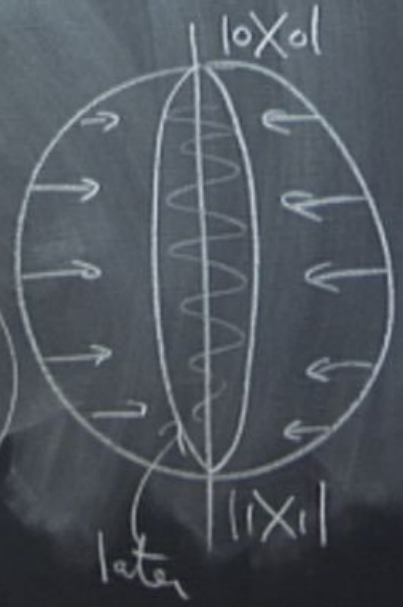
$$\tau = \frac{1}{2\lambda}$$

$$[L] \sim T^{-1/2}$$

superposition

↓

mixture $\{|0\rangle\langle 0|, |1\rangle\langle 1|\}$



$$|0\rangle - b|1\rangle$$

d equation

$$= \frac{1}{i\hbar}[H,\rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right)$$

n $\{|0\rangle, |1\rangle\}$ basis

populations = const.

coherences →

superposition
↓
mixture $\{|0\rangle\langle 0|, |1\rangle\langle 1|\}$

$$\dot{a}_x = -2\lambda a_x \Rightarrow$$
$$\dot{a}_y = -2\lambda a_y \Rightarrow$$
$$\dot{a}_z = 0 \Rightarrow$$

$$- \frac{1}{2}\{L_k^\dagger L_k, \rho\}\Big)$$

$$\dot{a}_y = -2\lambda a_y \Rightarrow a$$

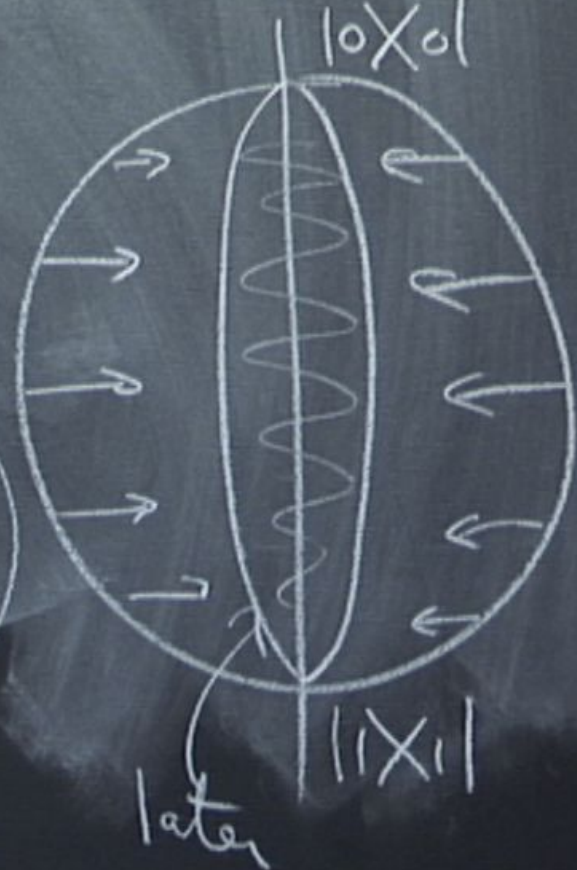
$$\dot{a}_z = 0 \Rightarrow a$$

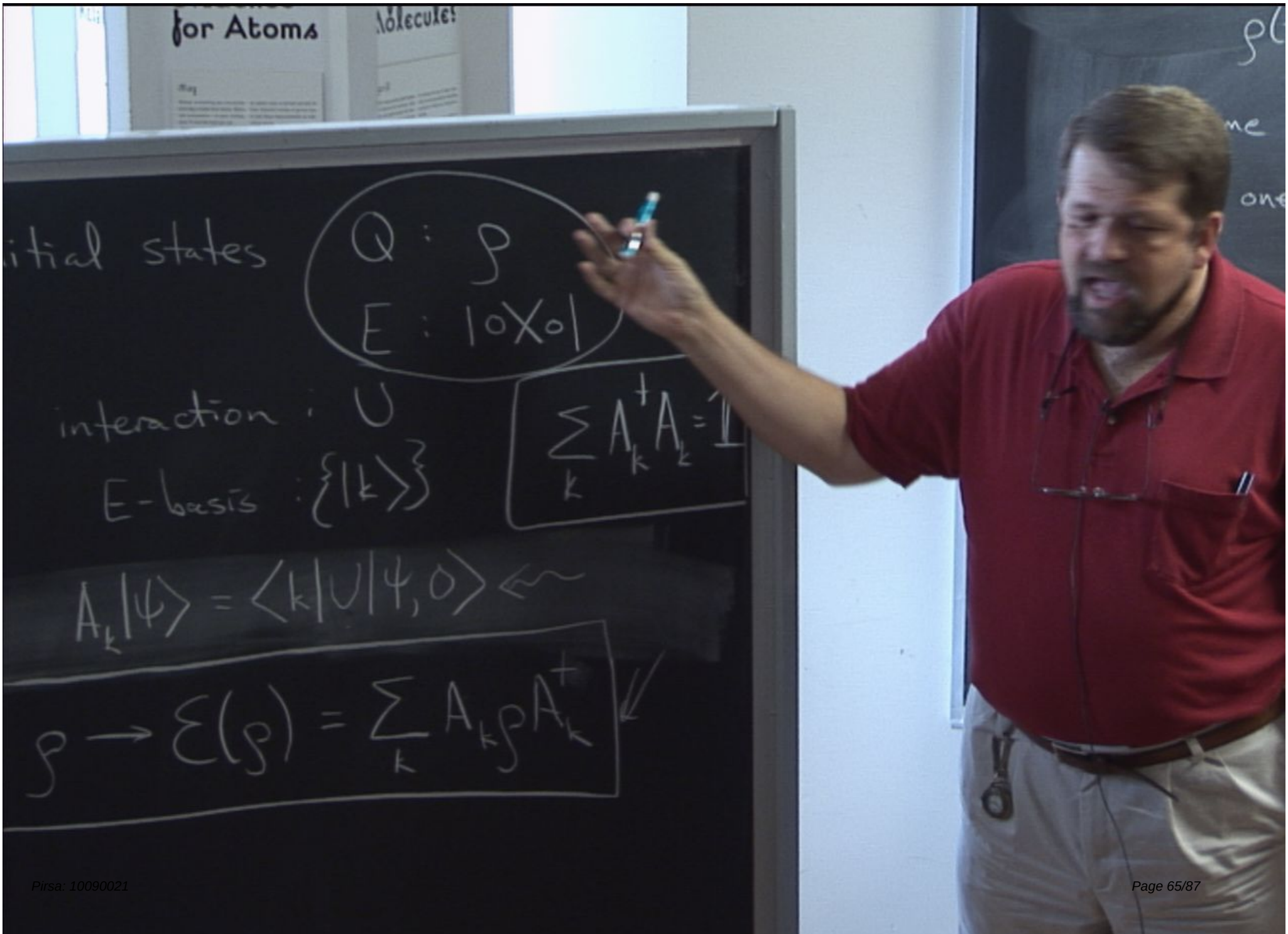
$$L = \Lambda^{1/2} a$$

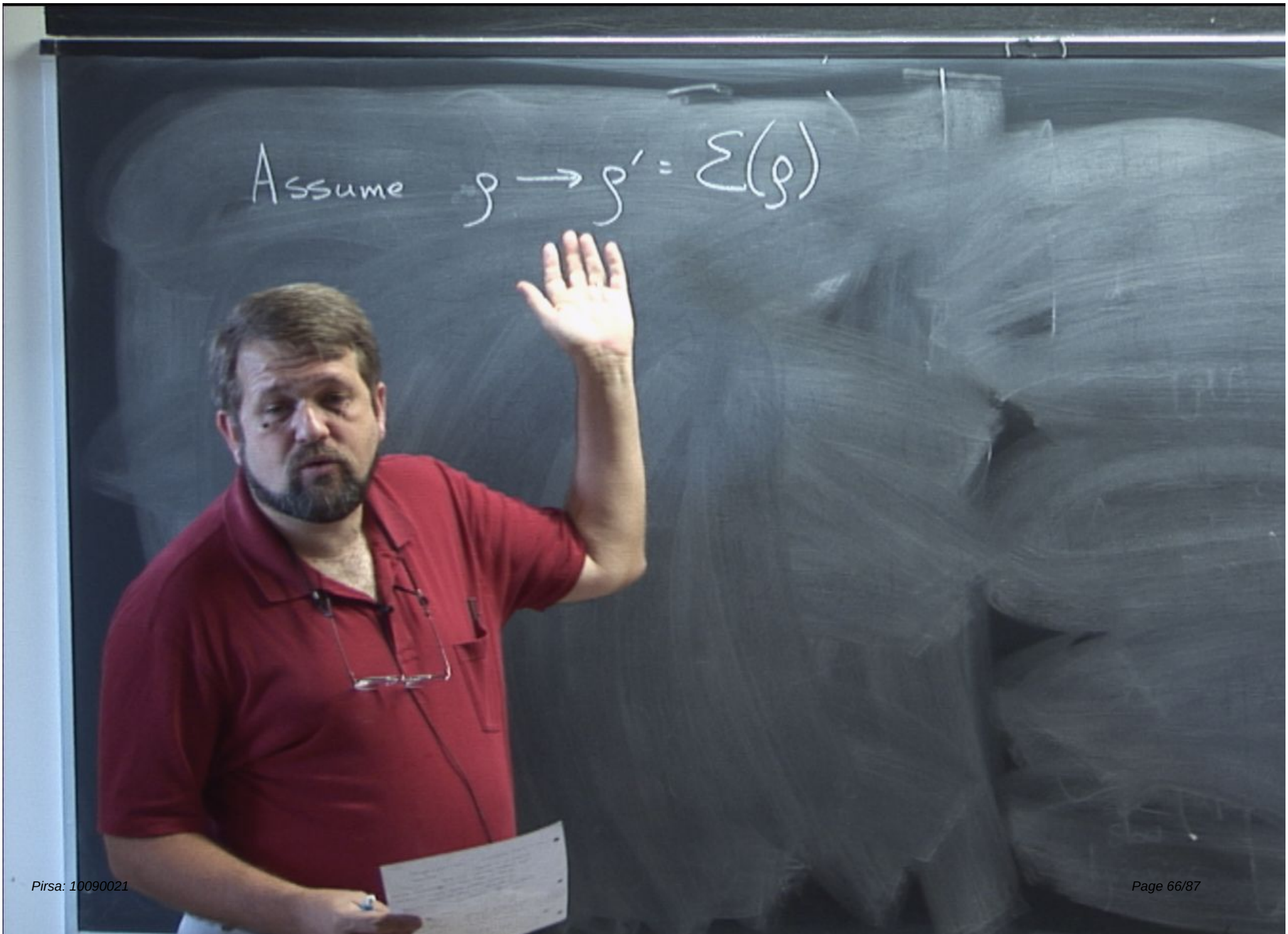
superposition

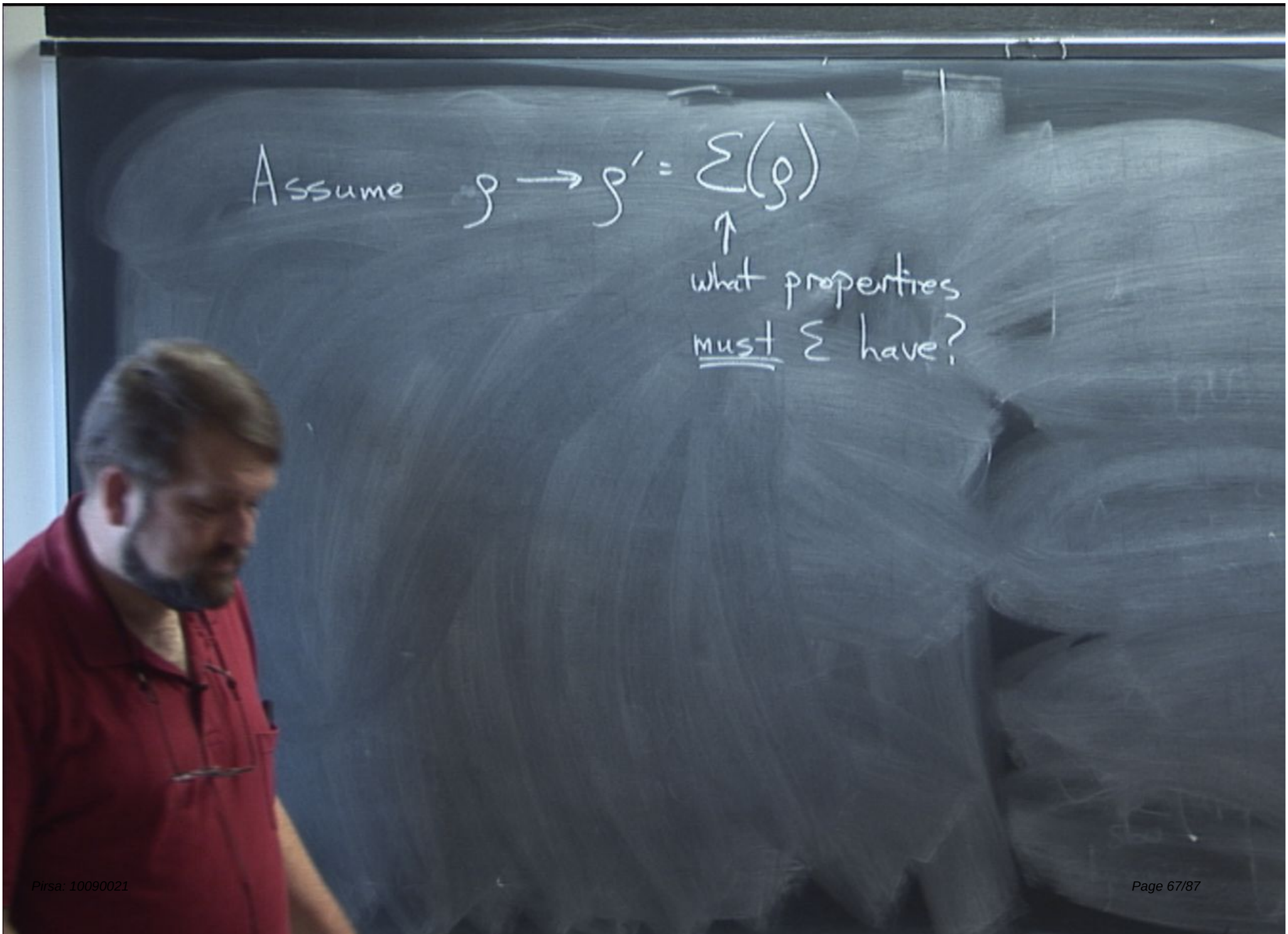
↓

mixture $\{|0\rangle\langle 0|, |1\rangle\langle 1|\}$




for Atoms
itial states
Q : ρ
E : |0X0|
interaction : U
E-basis : {|k⟩}
$\sum_k A_k^\dagger A_k = \mathbb{1}$
$A_k|\psi\rangle = \langle k|U|\psi,0\rangle$
$\rho \to \mathcal{E}(\rho) = \sum_k A_k \rho A_k^\dagger$


Assume $\rho \to \rho' = \mathcal{E}(\rho)$


Assume $\rho \longrightarrow \rho' = \mathcal{E}(\rho)$
what properties
must $\mathcal{E}$ have?

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what properties
<u>must</u> $\mathcal{E}$ have?

ensemble

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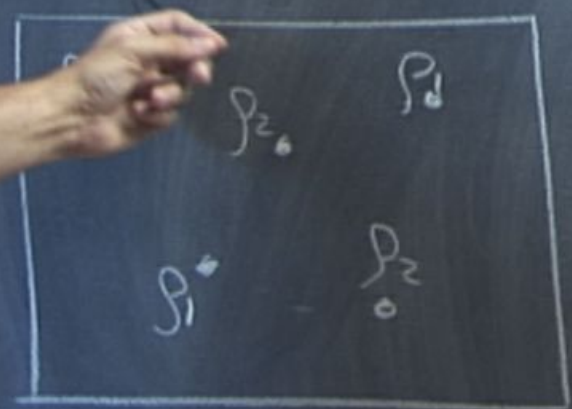
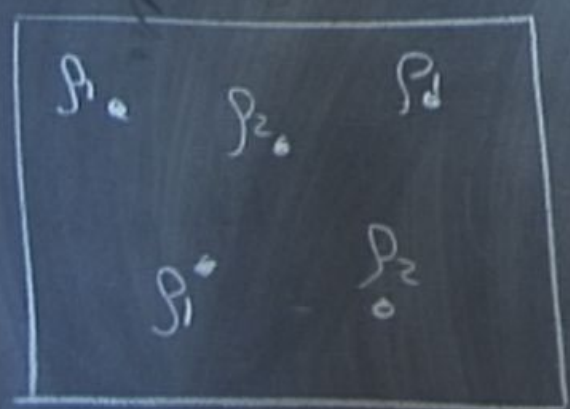


$\rho =$

Assume $\rho \longrightarrow \rho' = \mathcal{E}(\rho)$

what properties
<u>must</u> $\mathcal{E}$ have?

<u>ensemble</u>

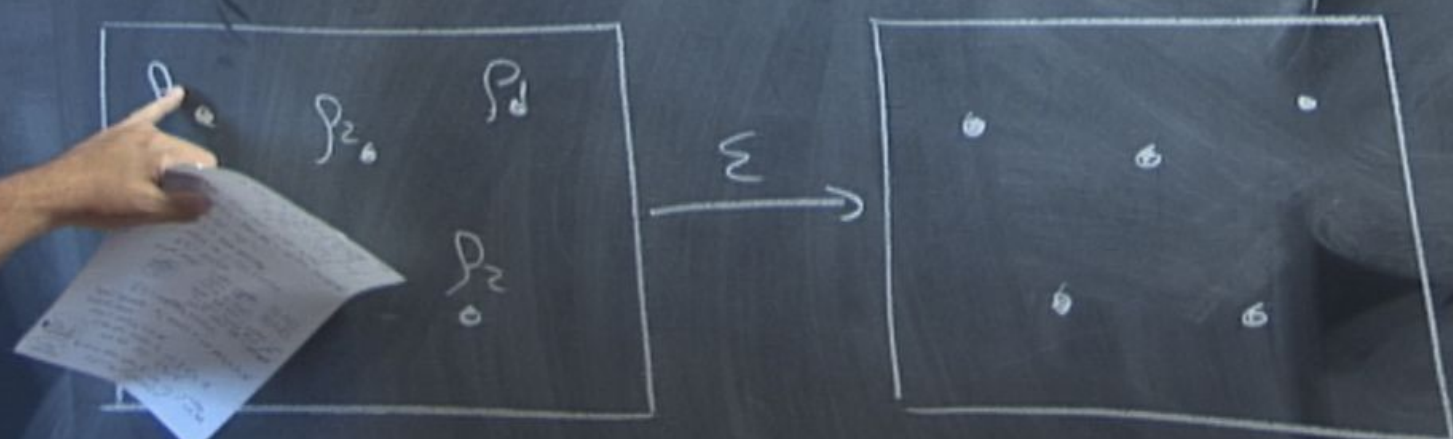


$$\rho = P_1\rho_1 + P_2\rho_2$$

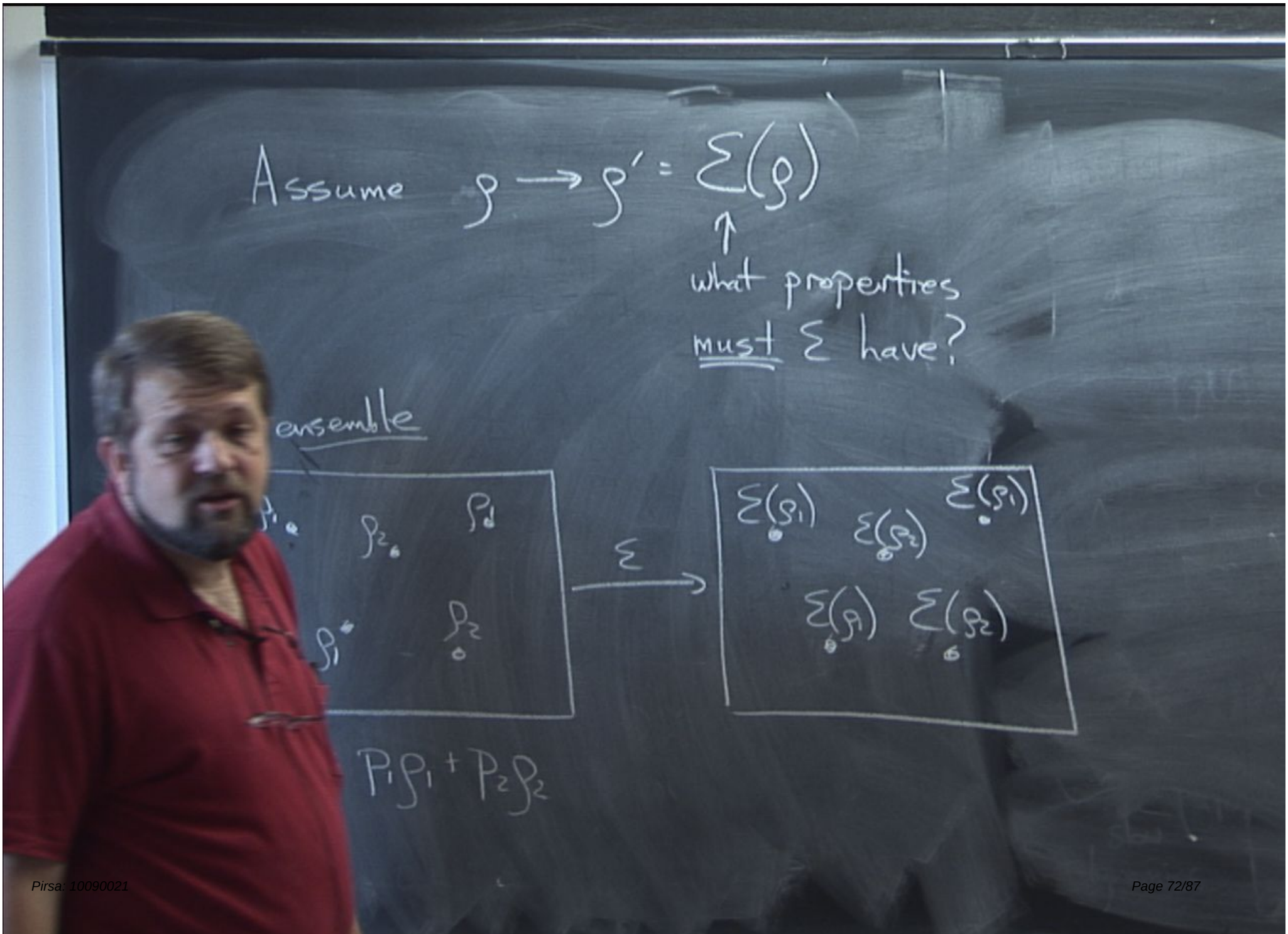
Assume $\rho \longrightarrow \rho' = \mathcal{E}(\rho)$

what properties
must $\mathcal{E}$ have?

ensemble



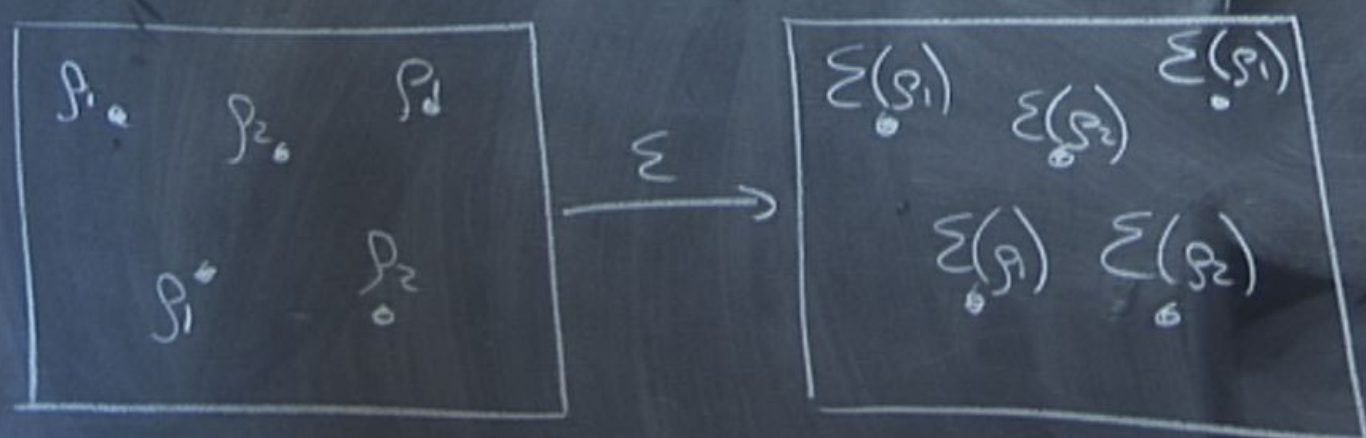
$$\rho = p_1\rho_1 + p_2\rho_2$$


Assume $\rho \to \rho' = \mathcal{E}(\rho)$
what properties must $\mathcal{E}$ have?
ensemble
ρ_1 ρ_2 ρ_1 ρ_2
$\mathcal{E}$
$\mathcal{E}(\rho_1)$ $\mathcal{E}(\rho_2)$ $\mathcal{E}(\rho_1)$ $\mathcal{E}(\rho_1)$ $\mathcal{E}(\rho_2)$
$p_1\rho_1 + p_2\rho_2$

Assume $\rho \longrightarrow \rho' = \mathcal{E}(\rho)$

what properties
<u>must</u> $\mathcal{E}$ have?

<u>ensemble</u>



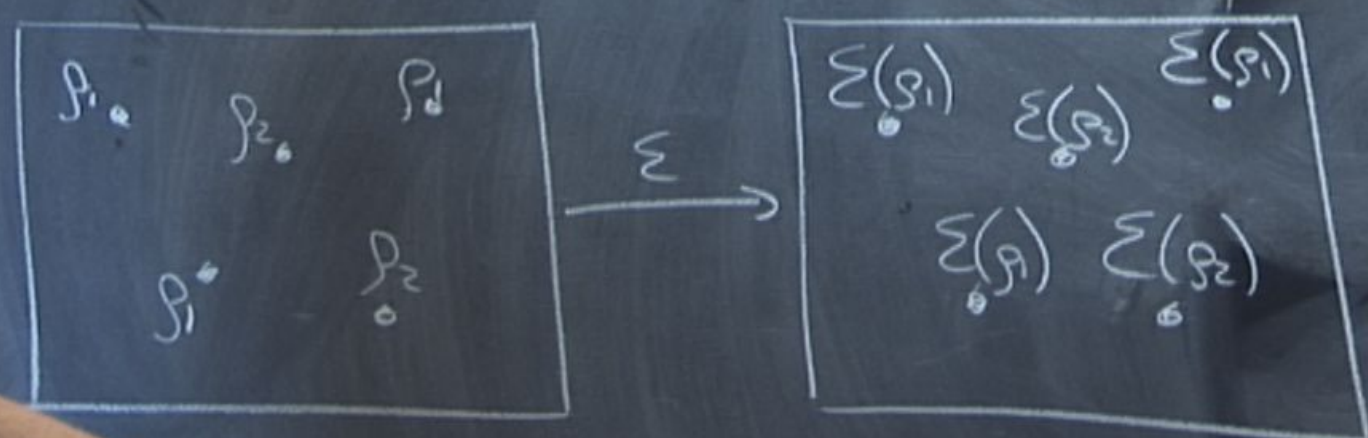
$\rho = p_1\rho_1 + p_2\rho_2$

$p_1\mathcal{E}(\rho_1) + p_2\mathcal{E}(\rho_2)$

Assume $\rho \longrightarrow \rho' = \mathcal{E}(\rho)$

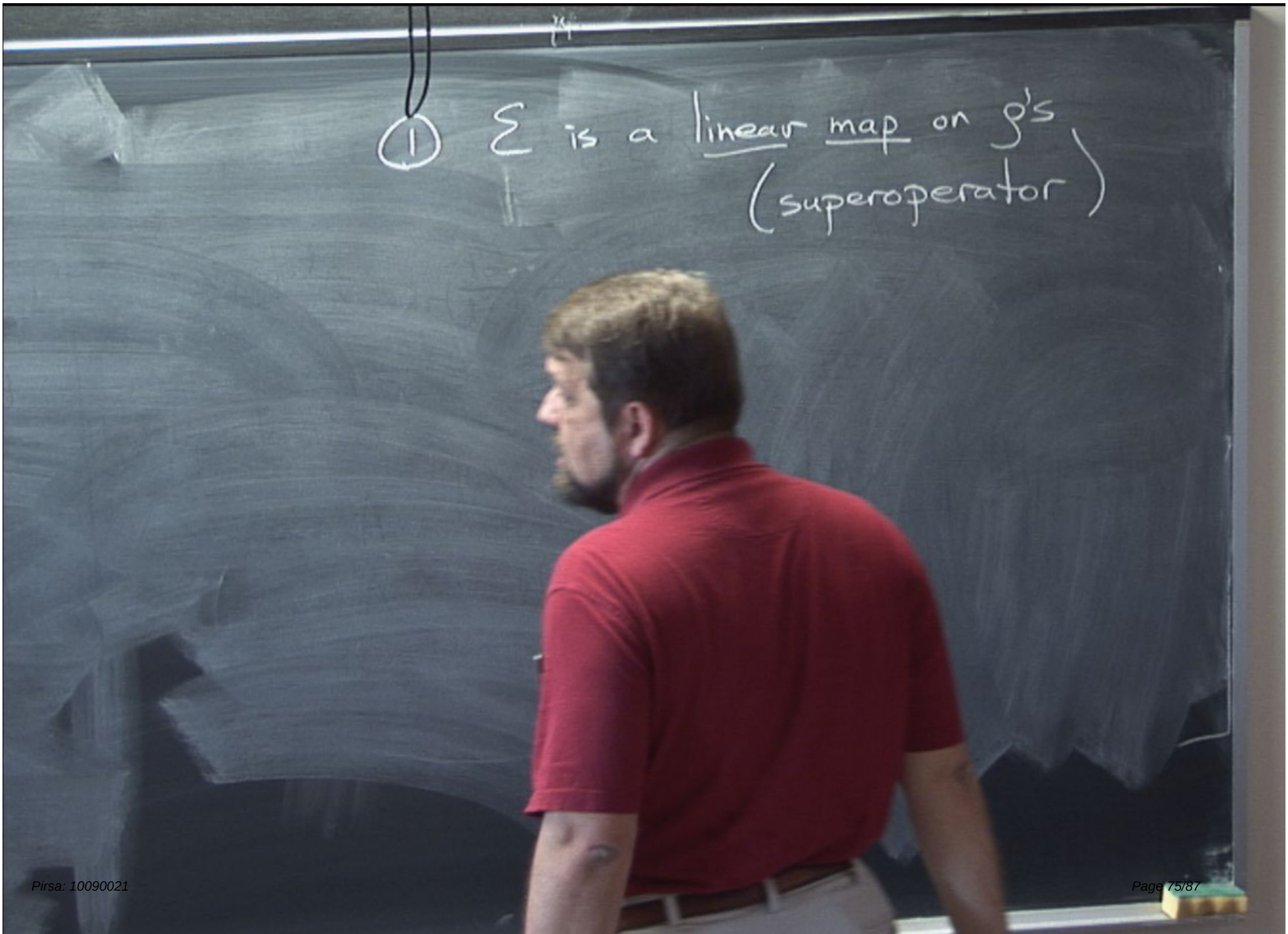
what properties
must $\mathcal{E}$ have?

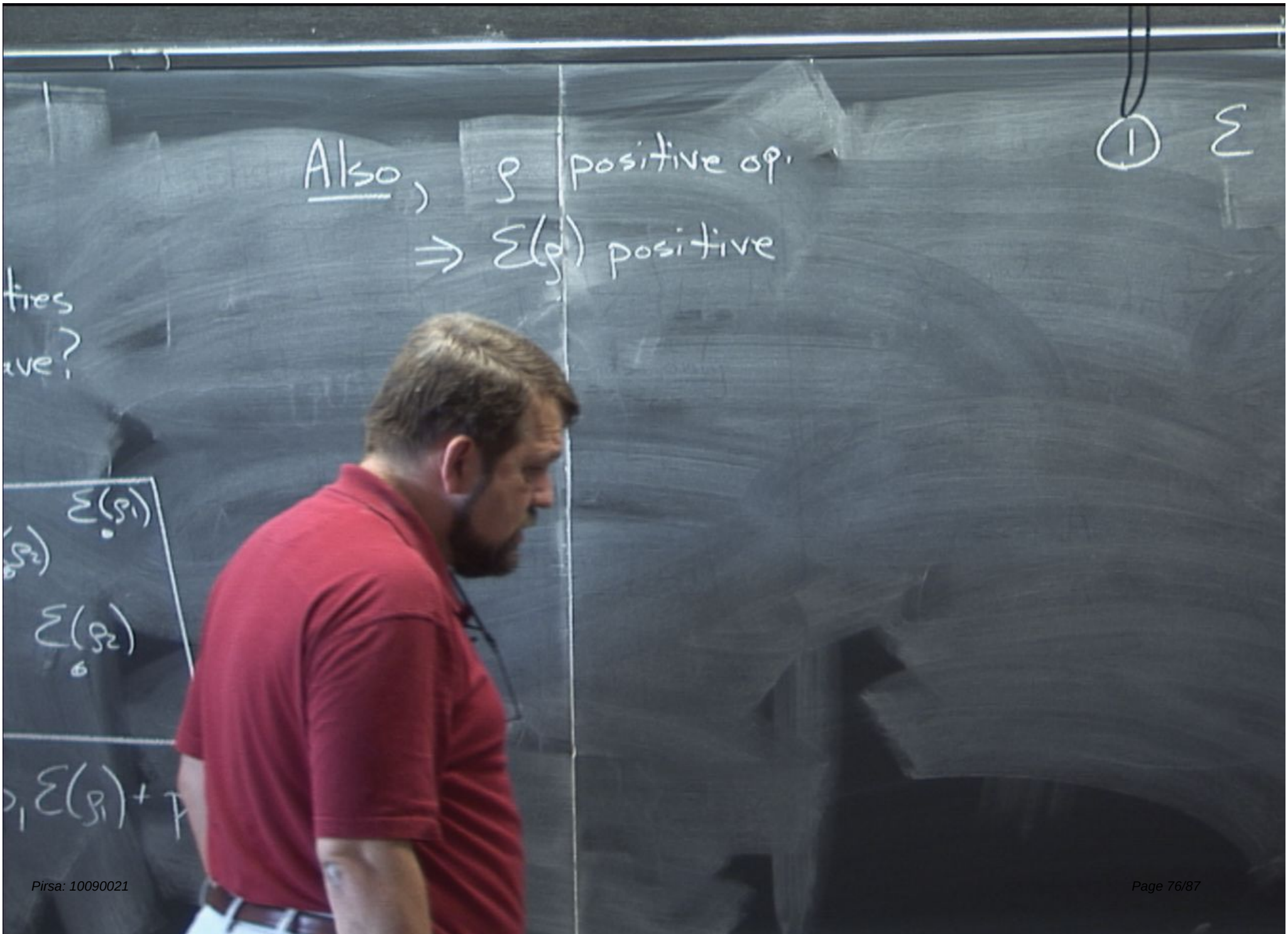
ensemble

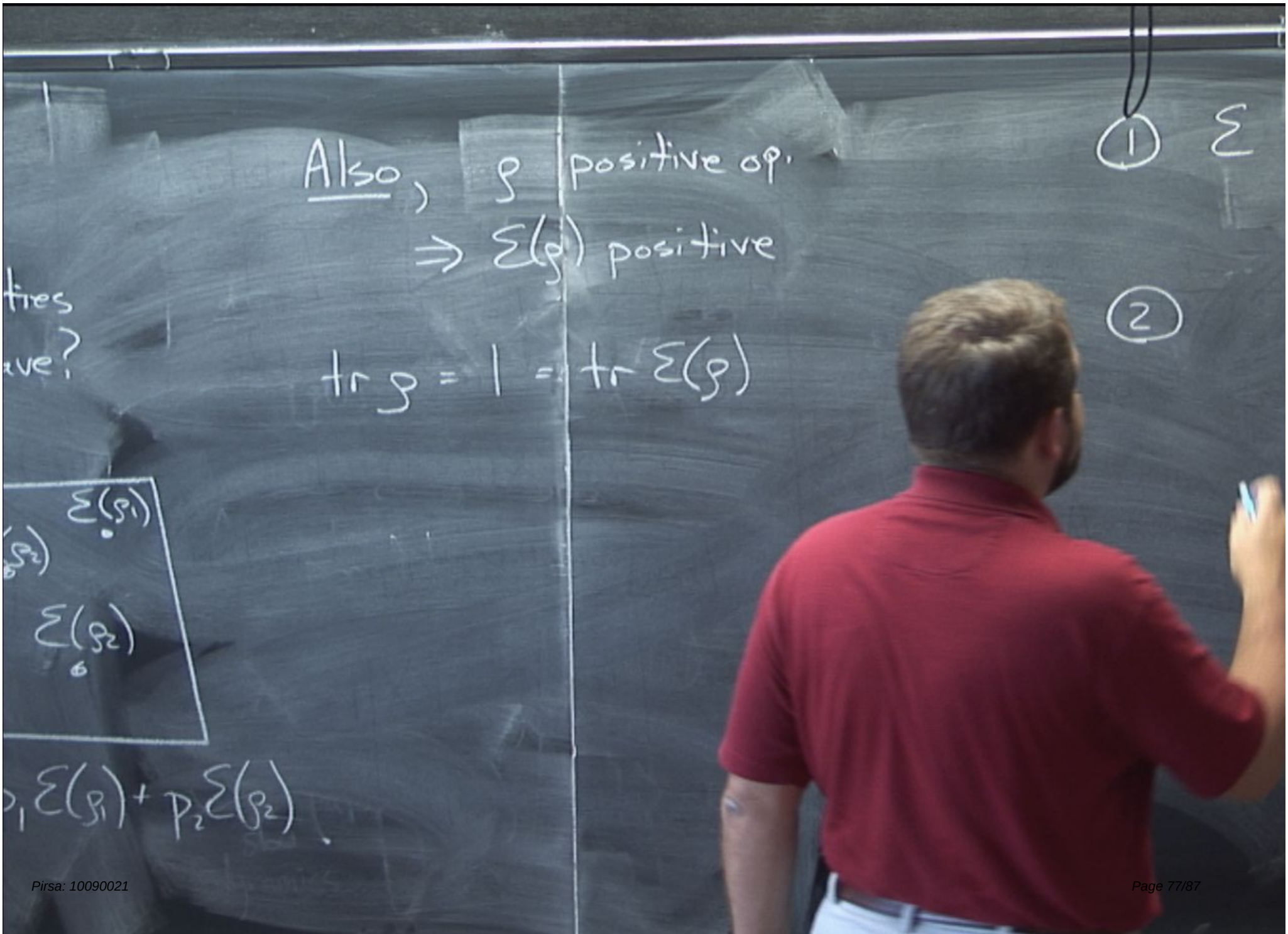


$p_1\rho_1 + p_2\rho_2$

$\mathcal{E}(\rho) = p_1\mathcal{E}(\rho_1) + p_2\mathcal{E}(\rho_2)$


① $\mathcal{E}$ is a linear map on ρ's
(superoperator)


Also, ρ positive op
⇒ Ε(ρ) positive
① Ε
ties
ave?
Ε(ρ1)
Ε(ρ2)
Ε(ρ2)
Ε(ρ1)


Also, ρ positive op.
⇒ Ε(ρ) positive
tr ρ = 1 = tr Ε(ρ)
① Ε
②
tries
ave?
Ε(ρ1)
Ε(ρ2)
p1 Ε(ρ1) + p2 Ε(ρ2)

ve op.

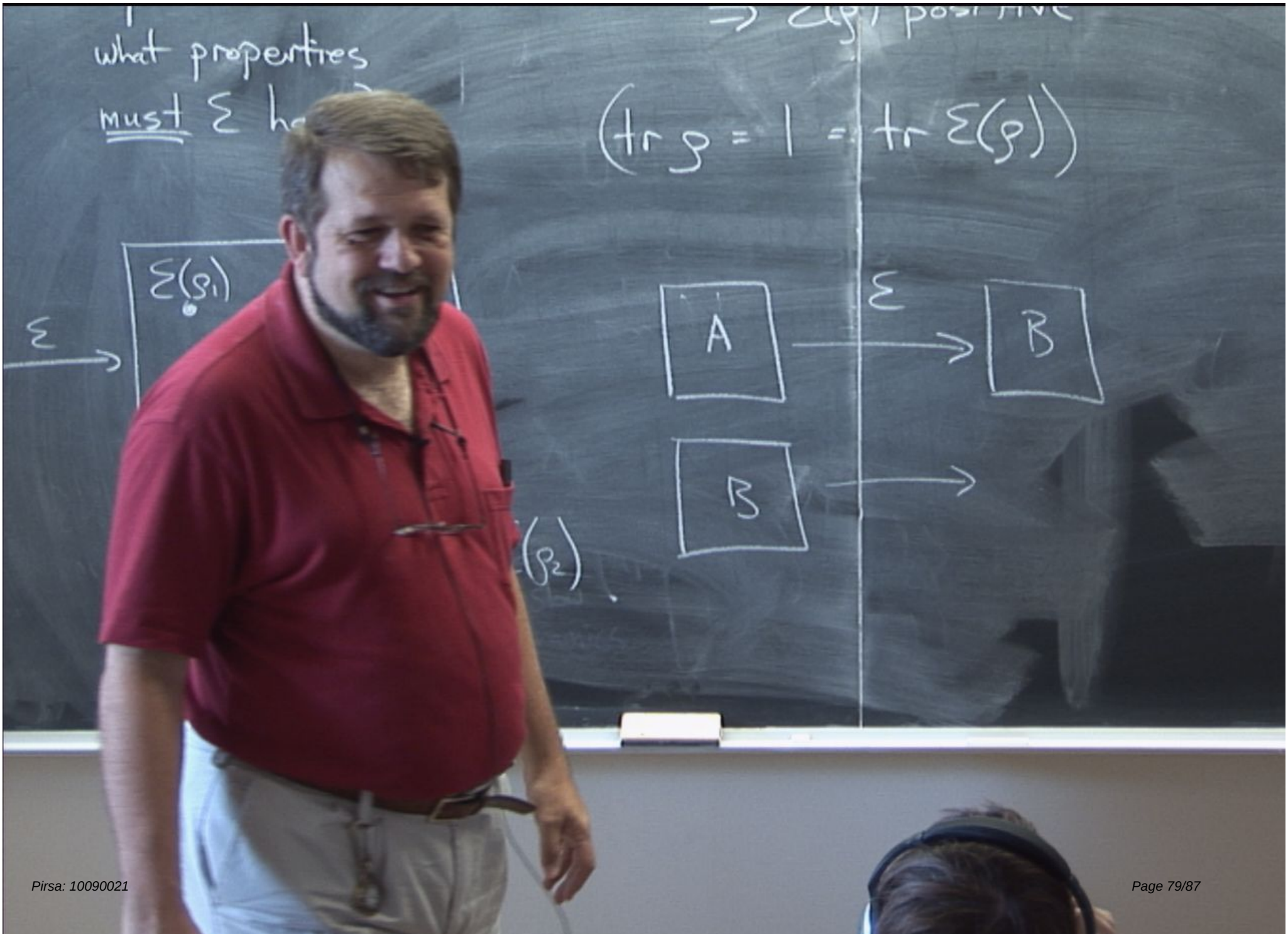
tive

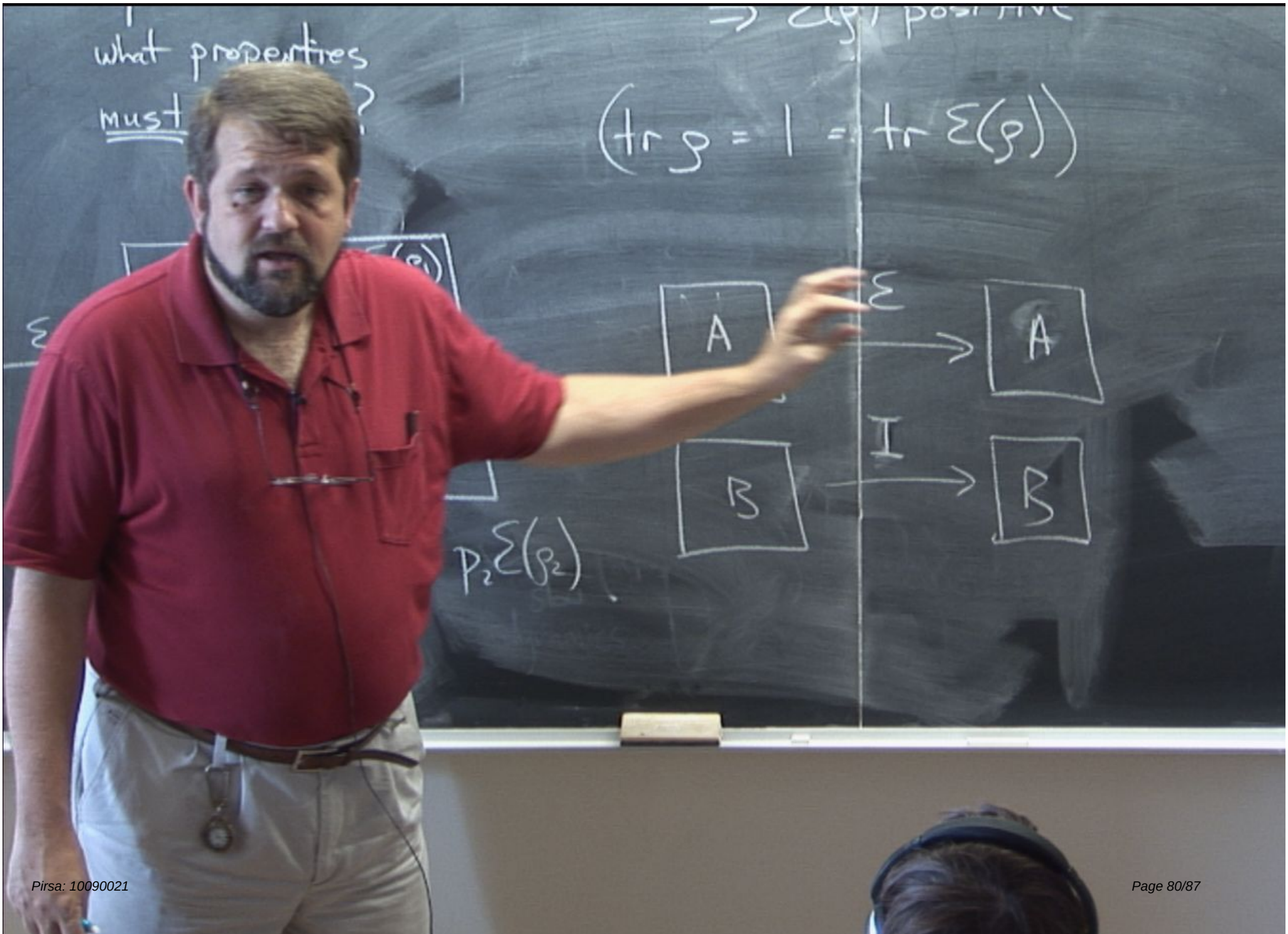
(ρ))

① $\mathcal{E}$ is a <u>linear</u> <u>map</u> on ρ's
(superoperator)

② $\mathcal{E}$ is <u>trace</u>-<u>preserving</u>

③ $\mathcal{E}$ is a <u>positive</u> map
(positive op. $\longrightarrow$ positive op.)


what properties
must Σ h
(tr ρ = 1 = tr Σ(ρ))
Σ(ρ₁)
Σ
A
Σ
B
B

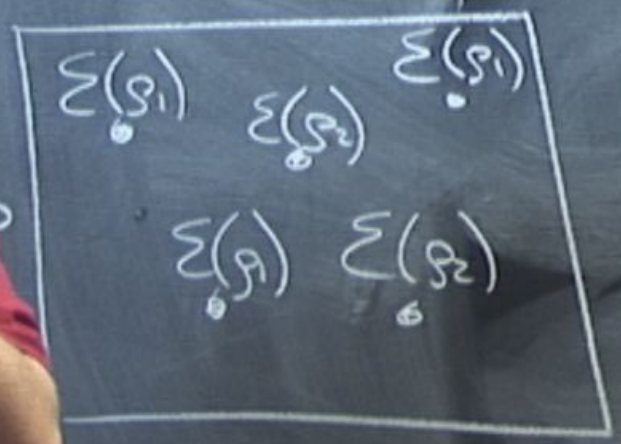

what properties
must
(tr ρ = 1 = tr Ε(ρ))
A
B
Ε
I
A
B
p₂Ε(ρ₂)

what properties

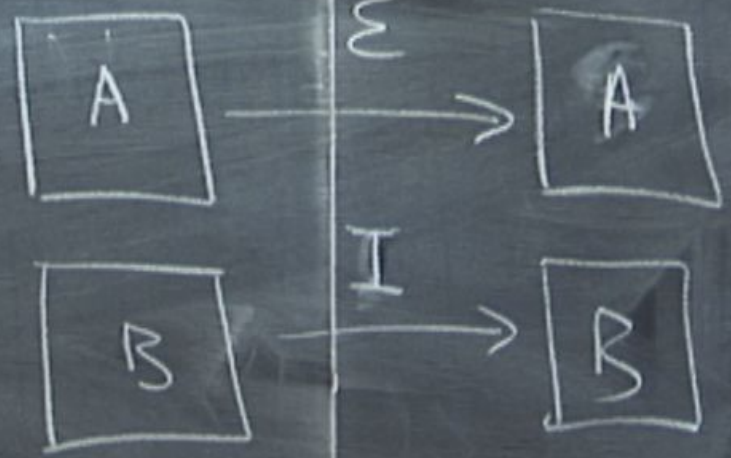
must $\mathcal{E}$ have?

$\Rightarrow \mathcal{E}(\rho)$ positive

$(\mathrm{tr}\,\rho = 1 = \mathrm{tr}\,\mathcal{E}(\rho))$



$\mathcal{E}(\rho) = p_1\mathcal{E}(\rho_1) + p_2\mathcal{E}(\rho_2)$



$\mathcal{E} \otimes$

① $\mathcal{E}$ is a <u>linear</u> <u>map</u> on ρ's
(superoperator)

② $\mathcal{E}$ is <u>trace-preserving</u>

③ $\mathcal{E}$ is a <u>positive</u> map
(positive op. $\longrightarrow$ positive op.)

I ③+ $\mathcal{E}$ is <u>completely</u> <u>positive</u>
($\mathcal{E} \otimes I$ is a positive map)

Assume $\rho \longrightarrow \rho' = \mathcal{E}(\rho)$

what properties

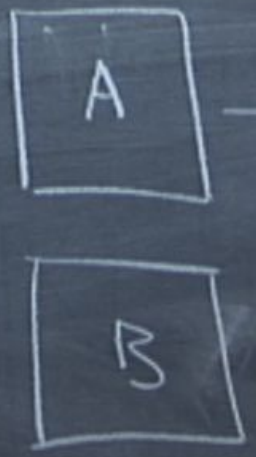
<u>must</u> $\mathcal{E}$ have?

Ans: ① + ② + ③+

<u>Also</u>, ρ

$\Rightarrow \mathcal{E}($

$(\mathrm{tr}\,\rho = 1 =$



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<u>Wonderful</u> <u>thm.</u>

Suppose $\mathcal{E}$ satisfies ① + ② + ③+

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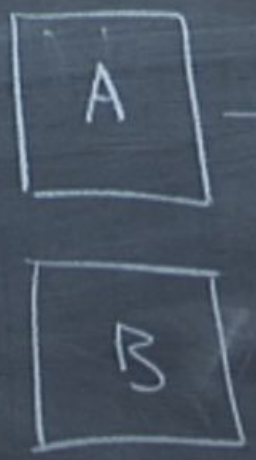

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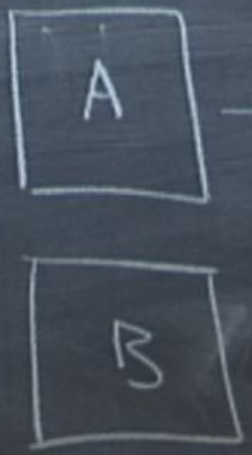

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Homework :

Lindblad eqn.

$$\frac{d}{dt}\rho = \frac{1}{i\hbar}[H,\rho] + \Lambda a \rho a^{\dagger} - \frac{\Lambda}{2}\left(a^{\dagger}a\rho + \rho a^{\dagger}a\right)$$

- We can construct $\mathcal{E}, |0\rangle, U$

$$\mathcal{E}(\rho) = \mathrm{tr}_{(\mathcal{E})}\, U\left(\rho \otimes |0\rangle\langle 0|\right)U^{\dagger}$$