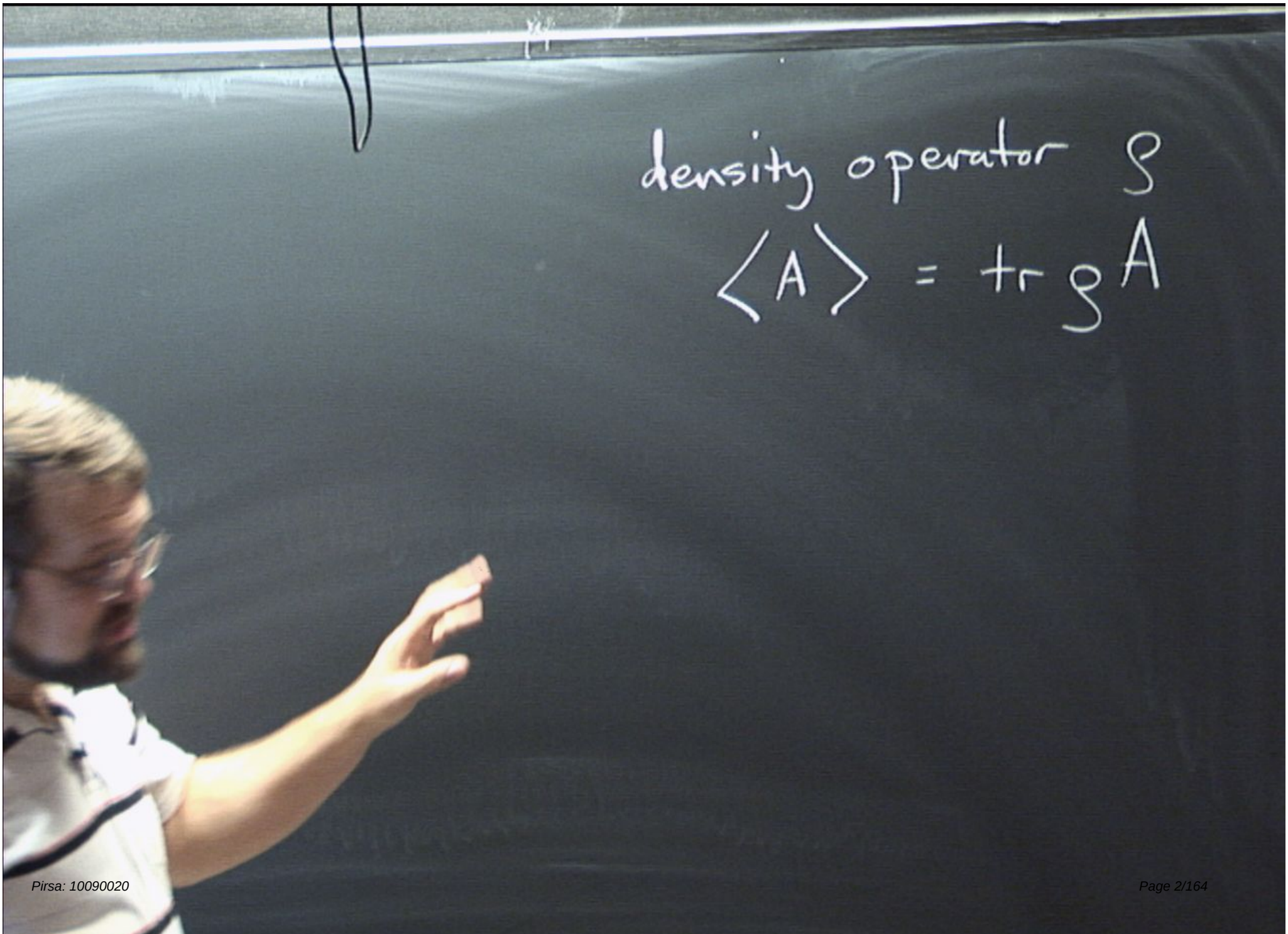
Title: Quantum Theory (PHYS 605) - Lecture 9

Date: Sep 23, 2010 09:00 AM

URL: http://pirsa.org/10090020

Abstract:


density operator ρ
$\langle A \rangle = \mathrm{tr}\, \rho A$

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<u>thermal</u> <u>states</u>

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Hamiltonian H

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Hamiltonian H

Equilibrium with reservoir

at $\beta = \frac{1}{kT}$

density operator ρ

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Equilibrium with reservoir

at $\beta = \frac{1}{kT}$

$$\boxed{\rho_\theta = \frac{1}{Z} e^{-\beta H}}$$

density operator ρ

$$\langle A \rangle = \mathrm{tr}\, \rho A$$

<u>thermal</u> <u>states</u>

Hamiltonian H

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Equilibrium with reservoir

at $\beta = \frac{1}{kT}$

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<u>thermal</u> <u>states</u>

Hamiltonian H

Equilibrium with reservoir

at $\beta = \frac{1}{kT}$

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Z = partition fcn.

$= \mathrm{tr}\, e^{-\beta H}$

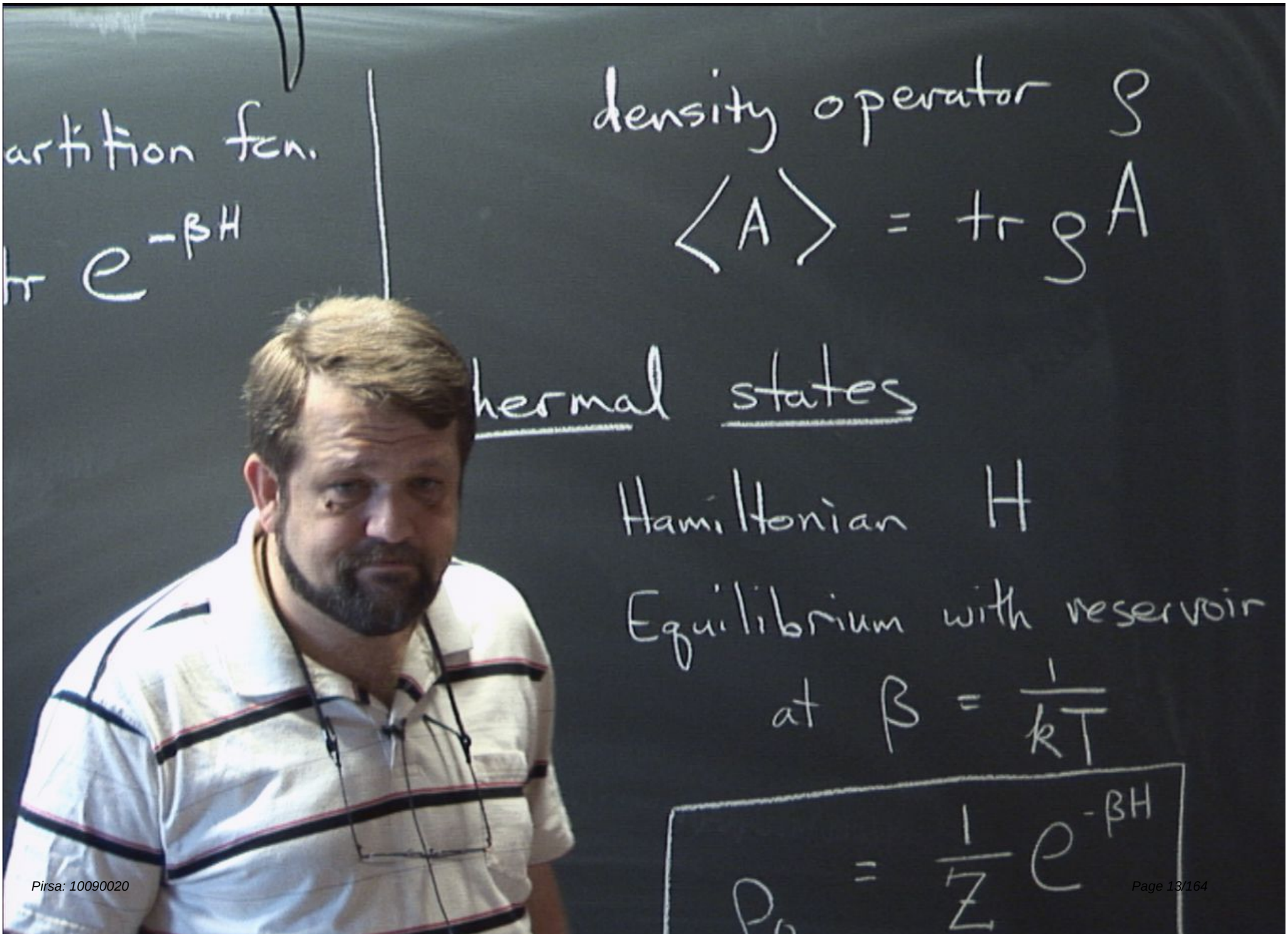
densi

$\langle$

<u>thermal</u>

Hami

Equi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density operator ρ
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hermal states
Hamiltonian H
Equilibrium with reservoir
at β = 1/kT
ρ_0 = 1/Z e^{-βH}

Z = partition fcn.

$= \mathrm{tr}\, e^{-\beta H}$

density operator

$\langle A \rangle = \mathrm{tr}\, \rho$

<u>thermal</u> <u>states</u>

Hamiltonian H

Equilibrium with

at $\beta = \frac{1}{k}$

$\rho_\beta = \frac{1}{Z} e$

Z = partition fcn.

$= \mathrm{tr}\, e^{-\beta H}$

$H|n\rangle = E_n|n\rangle$

ρ_θ is diagonal!

density o

$\langle A \rangle$

<u>thermal</u> <u>stat</u>

Hamiltoni

Equilibri

at

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$$\boxed{\rho_\theta = \sum_n \frac{e^{-\beta E_n}}{Z} |n\rangle\langle n|}$$

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E_n

Entangled states

Entangled states

$$|\Psi^{(RQ)}\rangle$$

Entangled states

$|\Psi^{(RQ)}\rangle$ entangled

Entangled states

$|\Psi^{(RQ)}\rangle$ entangled

How do I describe Q?

Entangled states

$|\Psi^{(RQ)}\rangle$ entangled

How do I describe Q?

Entangled states

$|\Psi^{(RQ)}\rangle$ entangled

How do I describe Q?

Suppose we measure $A^{(Q)}$

Entangled states

$|\Psi^{(RQ)}\rangle$ entangled

How do I describe Q?

Suppose we measure $A^{(Q)}$

$$\langle A^{(Q)} \rangle = \langle \Psi^{(RQ)} | \Pi^{(Q)} \otimes A^{(Q)} | \Psi^{(RQ)} \rangle$$

Entangled states

$|\Psi^{(RQ)}\rangle$ entangled

How do I describe Q?

Suppose we measure $A^{(Q)}$

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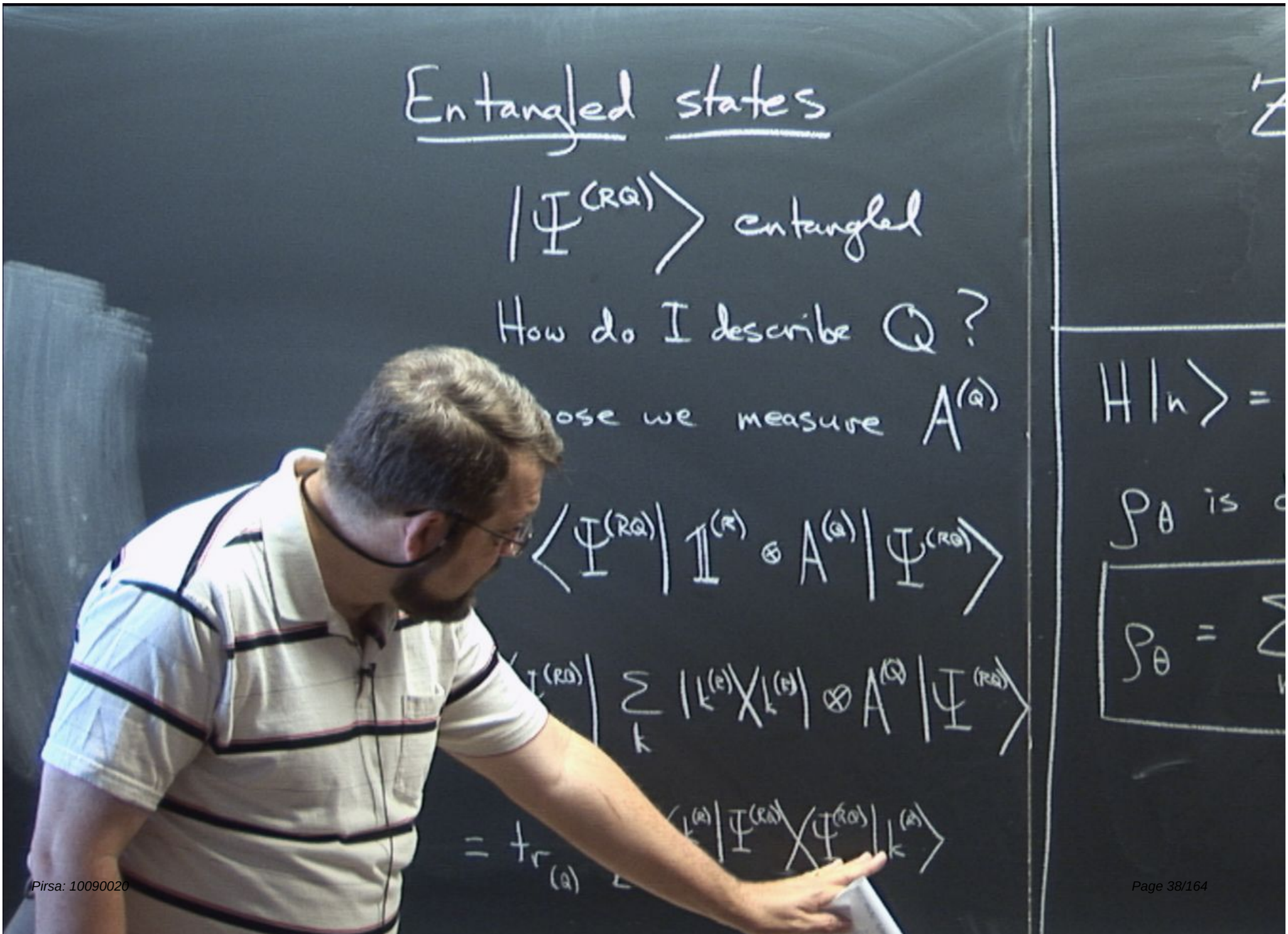
Entangled states
$|\Psi^{(RQ)}\rangle$ entangled
How do I describe Q ?
ose we measure $A^{(Q)}$
$\langle\Psi^{(RQ)}|\,\mathbb{1}^{(R)}\otimes A^{(Q)}\,|\Psi^{(RQ)}\rangle$
$\sum_k |k^{(R)}\rangle\langle k^{(R)}| \otimes A^{(Q)}$
$= tr_{(Q)}$
$H|n\rangle =$
ρ_θ is
$\rho_\theta =$

Entangled states

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$$= \mathrm{tr}_{(Q)} \sum_k \langle k^{(R)} | \Psi^{(RQ)}\rangle\langle \Psi^{(RQ)} | k^{(R)}\rangle \, A^{(Q)}$$

Z

$H|n\rangle =$

ρ_θ is

$$\boxed{\rho_\theta = \sum}$$

Entangled states

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$$= \mathrm{tr}_{(Q)}\left(\sum_k \langle k^{(R)} | \Psi^{(RQ)}\rangle\langle \Psi^{(RQ)} | k^{(R)}\rangle \right) A^{(Q)}$$

$$H|n\rangle =$$

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$$\rho_\theta = \sum$$

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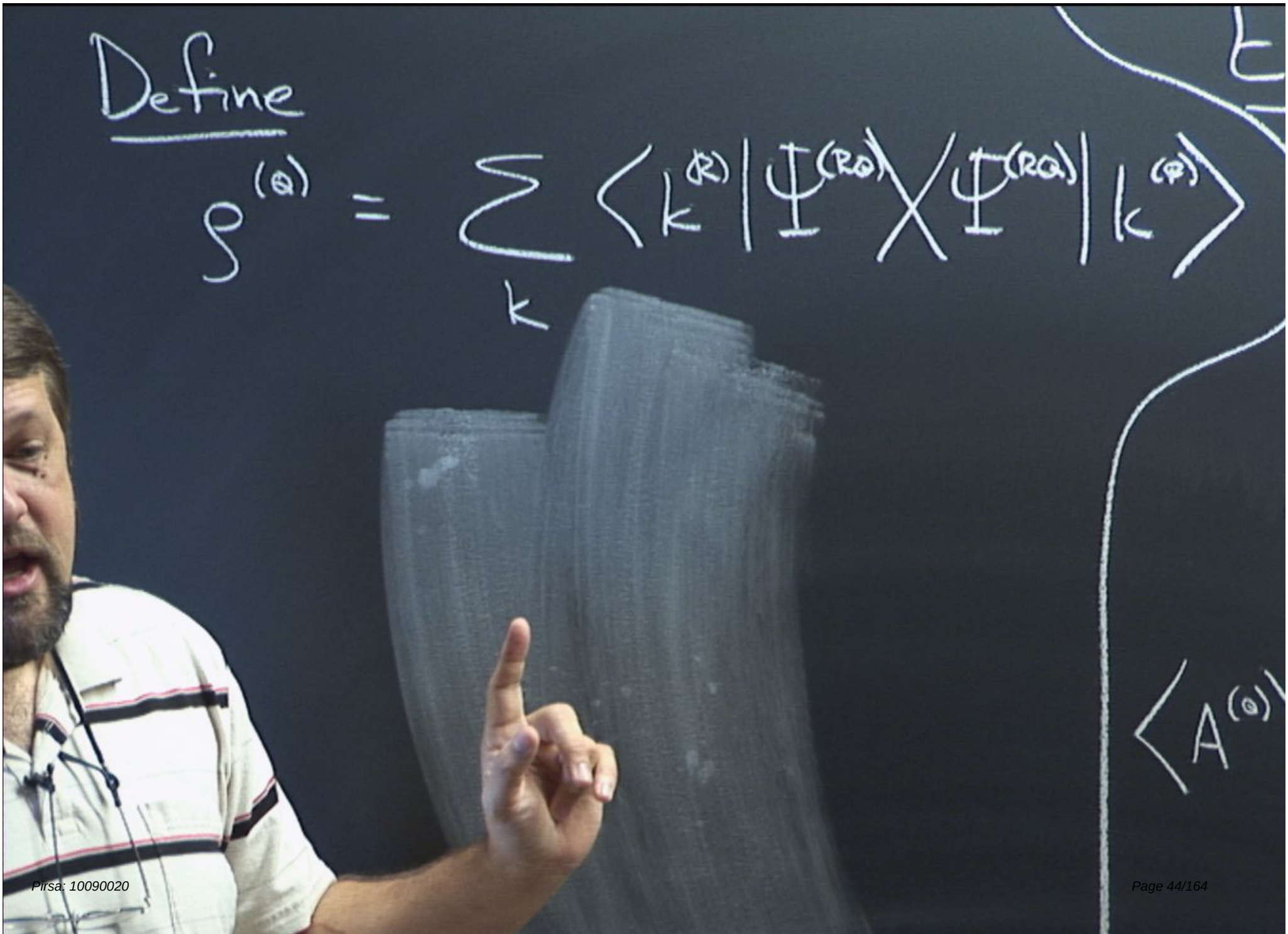
Define
$\rho^{(Q)} = \sum_k \langle k^{(R)} | \Psi^{(RQ)} \rangle \langle \Psi^{(RQ)} | k^{(R)} \rangle$
$\langle A^{(Q)}$

Define

$$\rho^{(Q)} = \sum_k \langle k^{(R)} | \Psi^{(RQ)} \rangle \langle \Psi^{(RQ)} | k^{(R)} \rangle$$

partial trace

$$\boxed{\rho^{(Q)} = \mathrm{tr}_{(R)} | \Psi^{(RQ)} \rangle \langle \Psi^{(RQ)} |}$$

$\langle A^{(Q)} \rangle$

Define

$$\rho^{(Q)} = \sum_k \langle k^{(R)} | \Psi^{(RQ)} \rangle \langle \Psi^{(RQ)} | k^{(R)} \rangle$$

partial trace

$$\boxed{\rho^{(Q)} = \mathrm{tr}_{(R)} | \Psi^{(RQ)} \rangle \langle \Psi^{(RQ)} |}$$

$$\boxed{\langle A^{(Q)} \rangle = \mathrm{tr}_{(Q)} \rho^{(Q)} A^{(Q)}}$$

$\langle A^{(Q)}$

$$|\Psi\rangle = \alpha|00\rangle + \beta|11\rangle$$

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$$|\Psi\rangle\langle\Psi| = \alpha\alpha^*|00\rangle\langle 00| + \alpha\beta^*|00\rangle\langle 11|$$
$$\beta\alpha^*|11\rangle\langle 00| + \beta\beta^*|11\rangle\langle 11|$$

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$$\rho^{(2)} = \mathrm{tr}_{(1)}|\Psi\rangle\langle\Psi| =$$

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$$\rho^{(2)} = \mathrm{tr}_{(1)}|\Psi\rangle\langle\Psi| = \alpha\alpha^*\langle 0|0\rangle|0\rangle\langle 0| + \alpha\beta^*$$

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$$+\beta\alpha^*\cancel{\langle 0|1\rangle}|1\rangle\langle 0| + \beta\beta^*\langle 1|1\rangle|1\rangle\langle 1|$$

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$$+\beta\alpha^*\cancel{\langle 0|1\rangle}|1\rangle\langle 0| + \beta\beta^*\underline{\langle 1|1\rangle}|1\rangle\langle 1|$$
$$= |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|$$

$$|\Psi\rangle = \alpha|00\rangle + \beta|11\rangle$$

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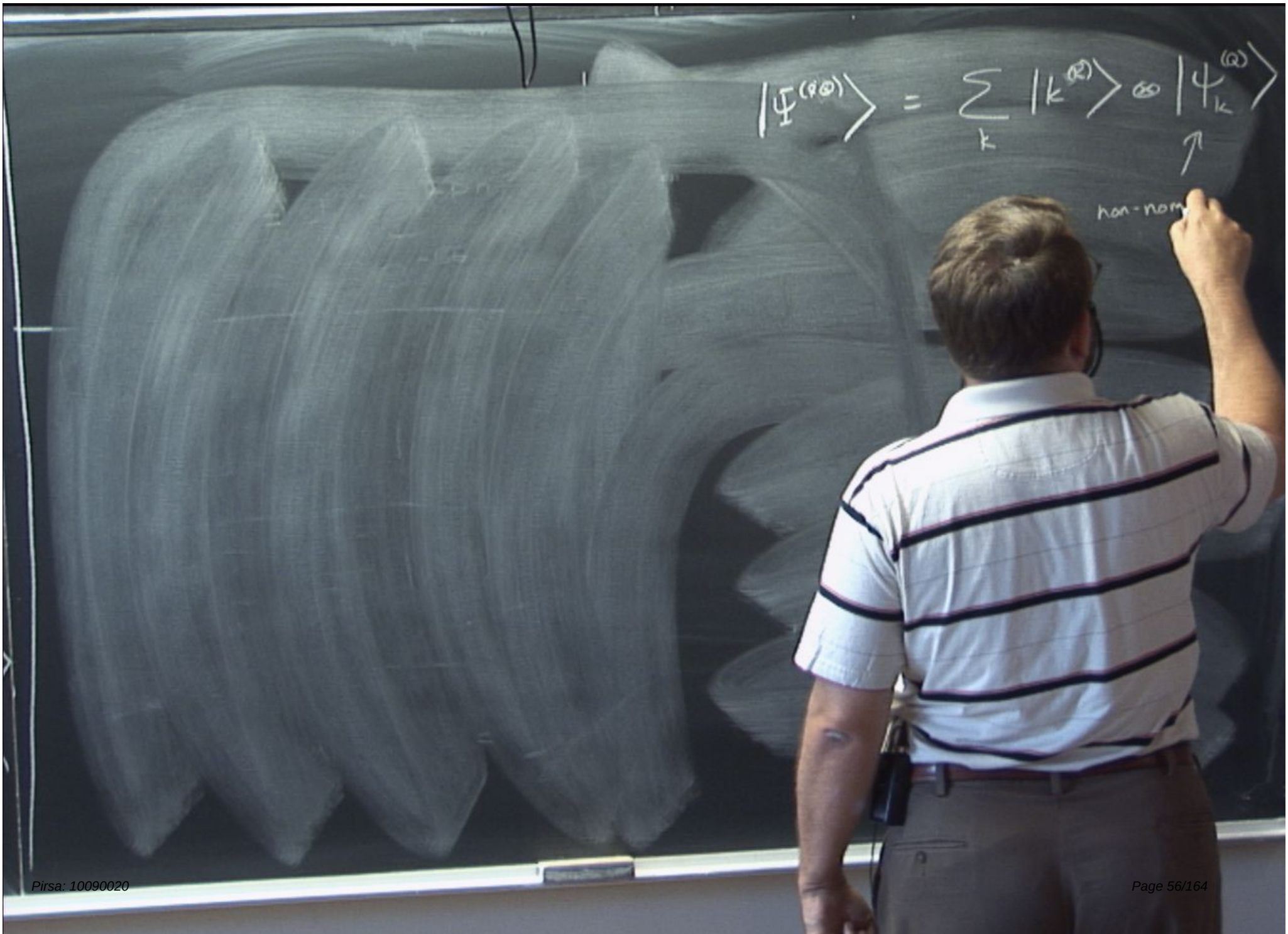
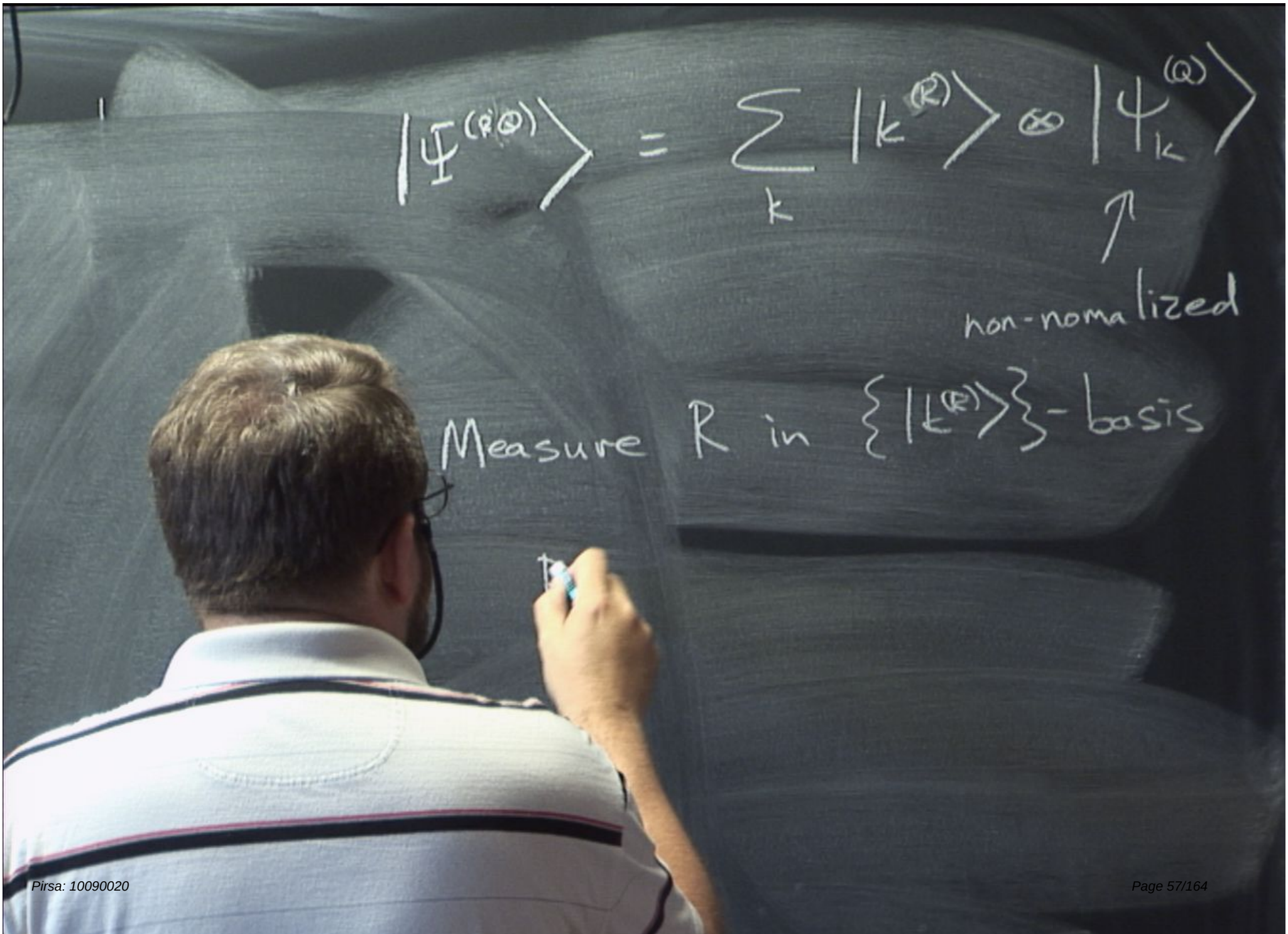
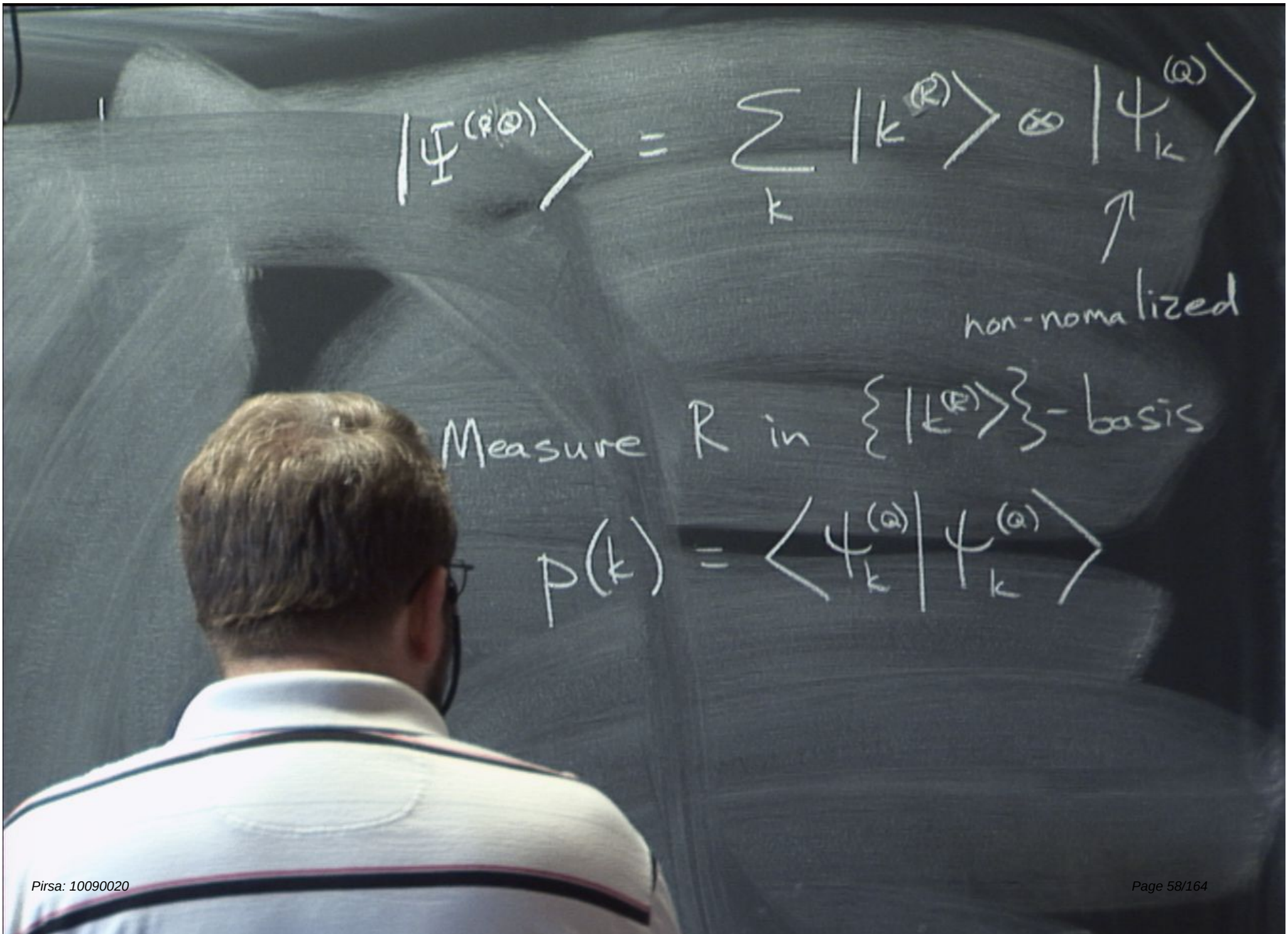

$|\Psi^{(QQ)}\rangle = \sum_k |k^{(R)}\rangle \otimes |\psi_k^{(Q)}\rangle$
non-norm


$|\Psi^{(R,Q)}\rangle = \sum_k |k^{(R)}\rangle \otimes |\psi_k^{(Q)}\rangle$
non-normalized
Measure R in $\{|k^{(R)}\rangle\}$-basis


$|\Psi^{(R,Q)}\rangle = \sum_k |k^{(R)}\rangle \otimes |\psi_k^{(Q)}\rangle$
non-normalized
Measure R in $\{|k^{(R)}\rangle\}$-basis
$p(k) = \langle \psi_k^{(Q)} | \psi_k^{(Q)} \rangle$

$$|\Psi^{(R,Q)}\rangle = \sum_k |k^{(R)}\rangle \otimes |\psi_k^{(Q)}\rangle$$

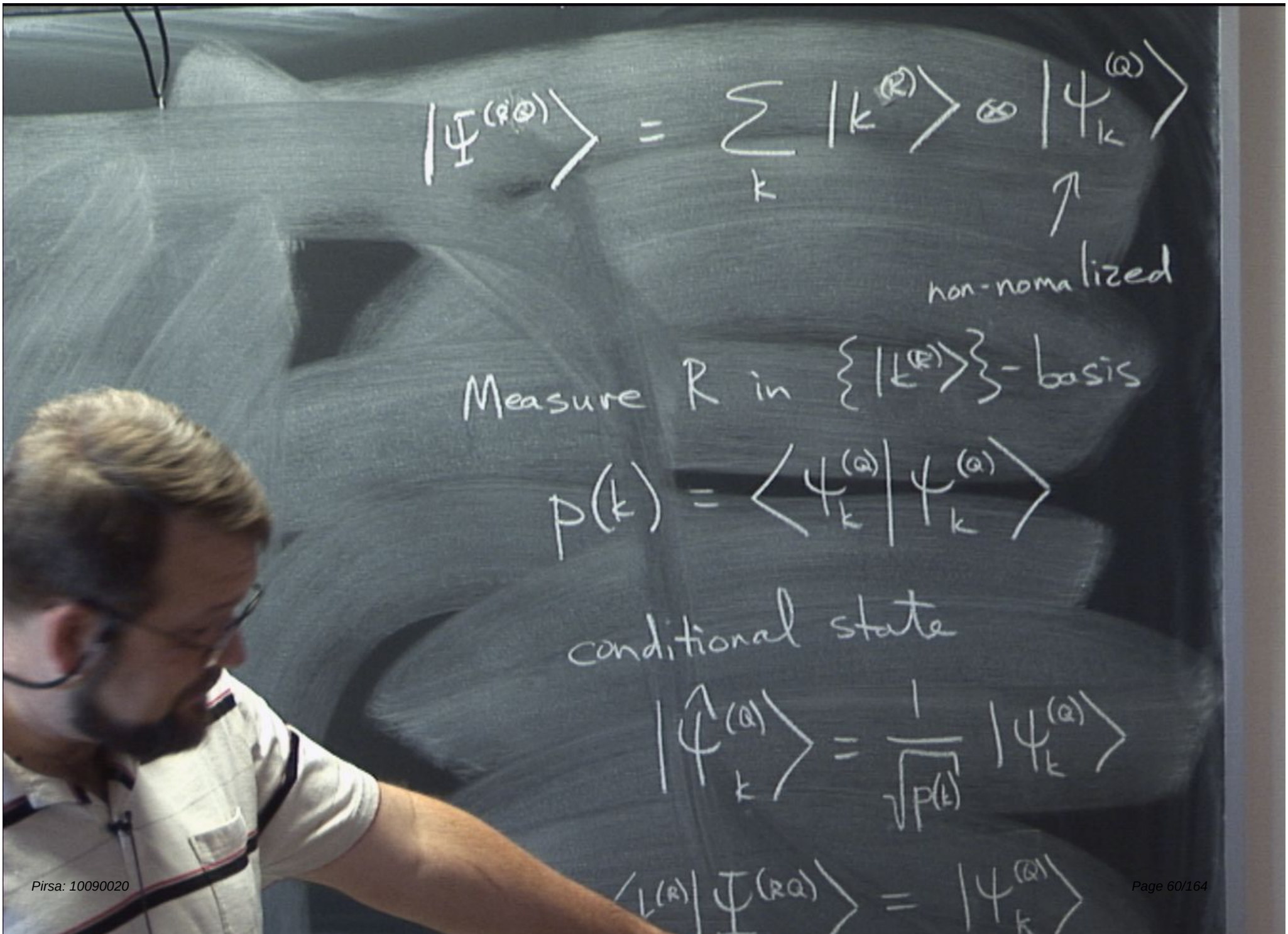
non-normalized

Measure R in $\{|k^{(R)}\rangle\}$-basis

$$p(k) = \langle \psi_k^{(Q)} | \psi_k^{(Q)} \rangle$$

conditional state

$$|\hat{\psi}_k^{(Q)}\rangle = \frac{1}{\sqrt{p(k)}} |\psi_k^{(Q)}\rangle$$


$|\Psi^{(RQ)}\rangle = \sum_k |k^{(R)}\rangle \otimes |\psi_k^{(Q)}\rangle$
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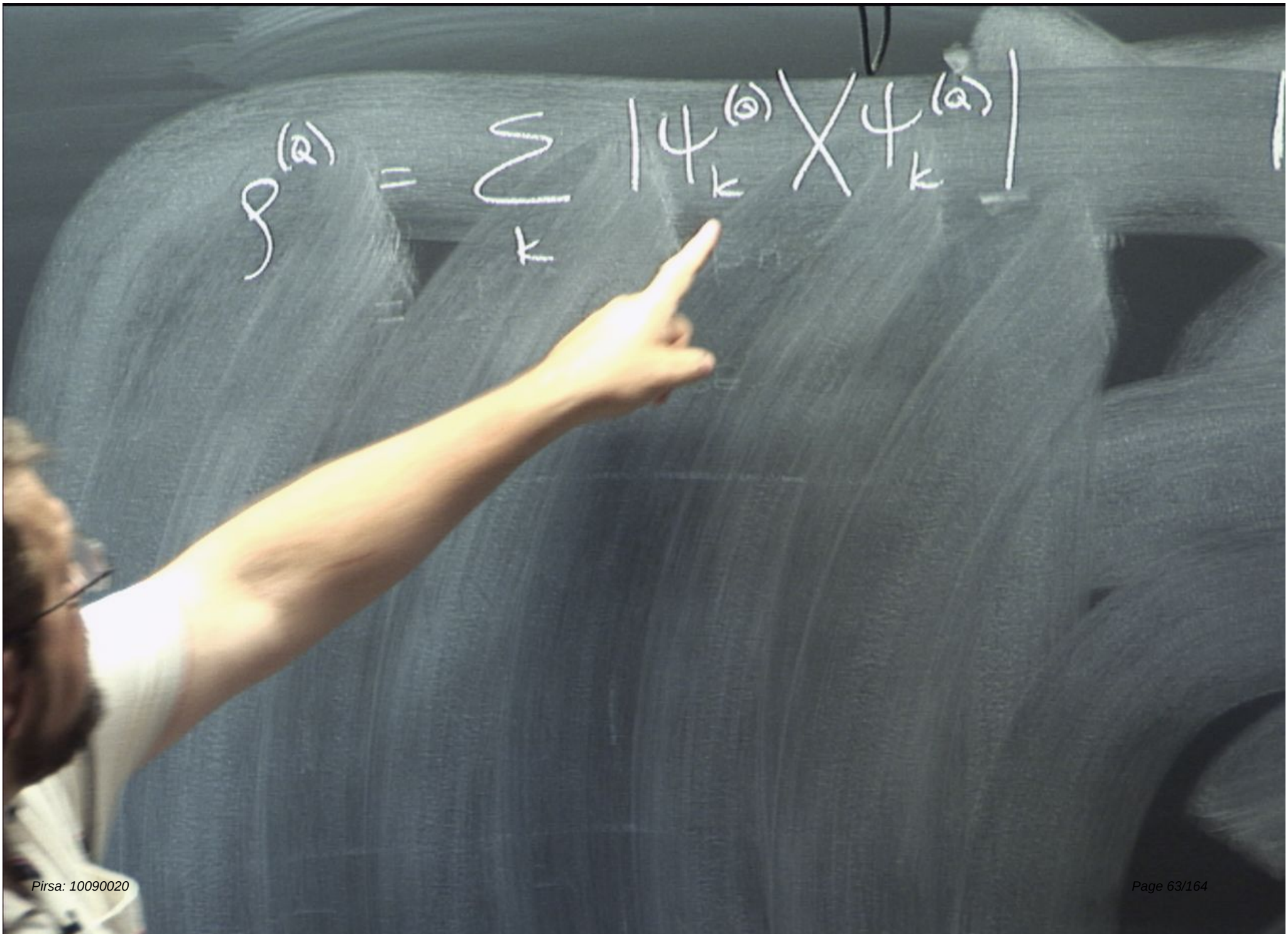
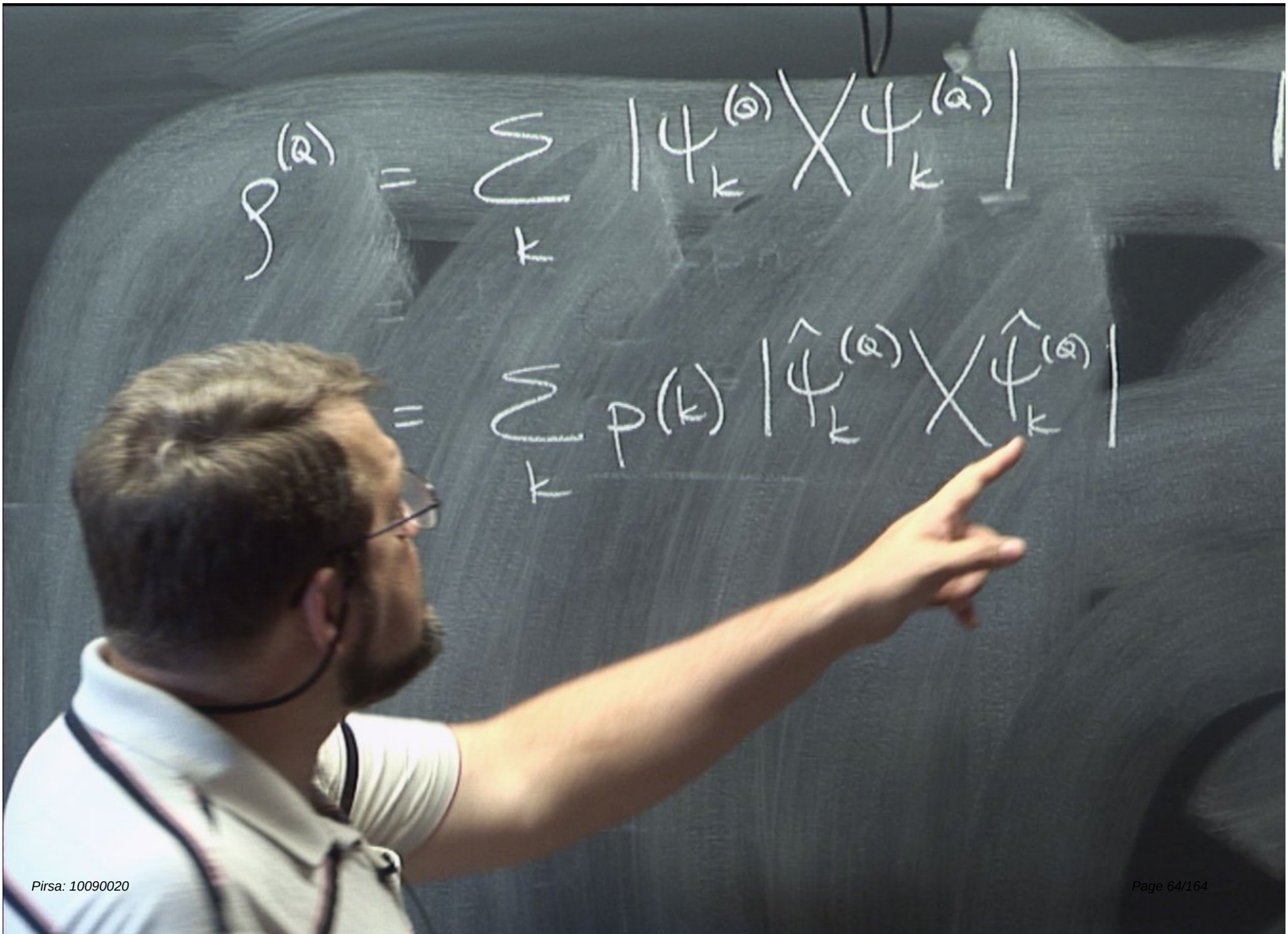
$$\langle k^{(R)} | \Psi^{(RQ)} \rangle = |\psi_k^{(Q)}\rangle$$

$$\rho^{(a)} = \sum_k |\psi_k^{(a)}\rangle\langle\psi_k^{(a)}|$$


$\rho^{(Q)} = \sum_k |\psi_k^{(Q)}\rangle\langle\psi_k^{(Q)}|$


$\rho^{(Q)} = \sum_k |\psi_k^{(Q)}\rangle\langle\psi_k^{(Q)}|$
$= \sum_k p(k)\, |\hat{\psi}_k^{(Q)}\rangle\langle\hat{\psi}_k^{(Q)}|$

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obability $p(k)$

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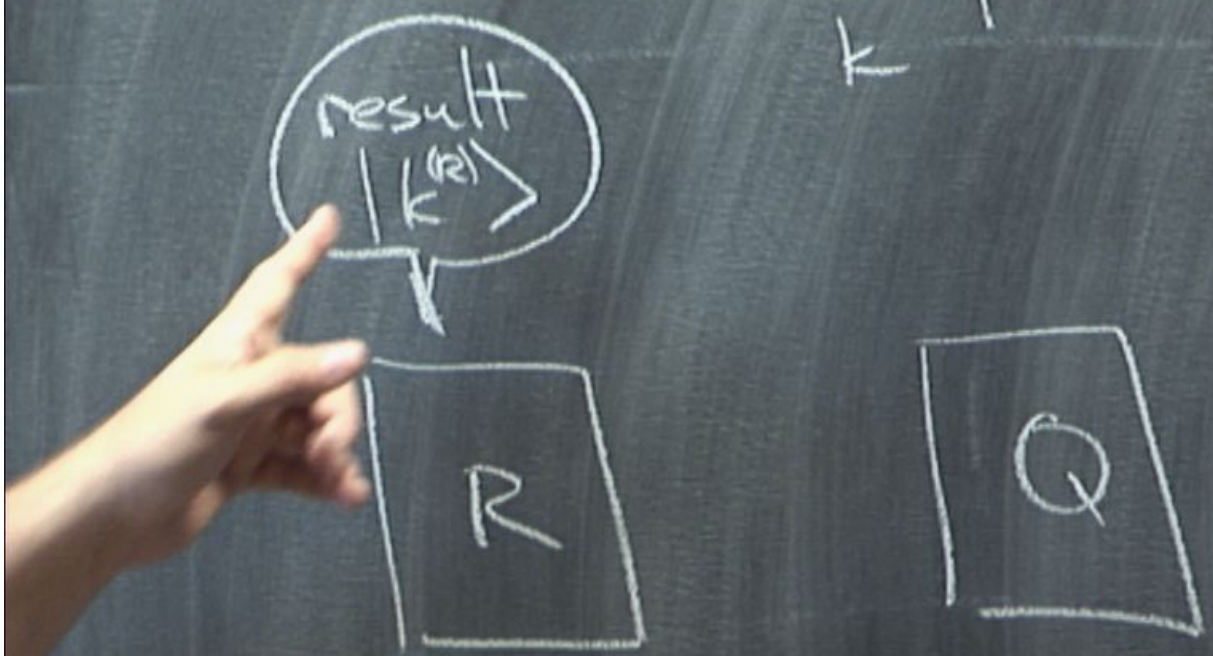

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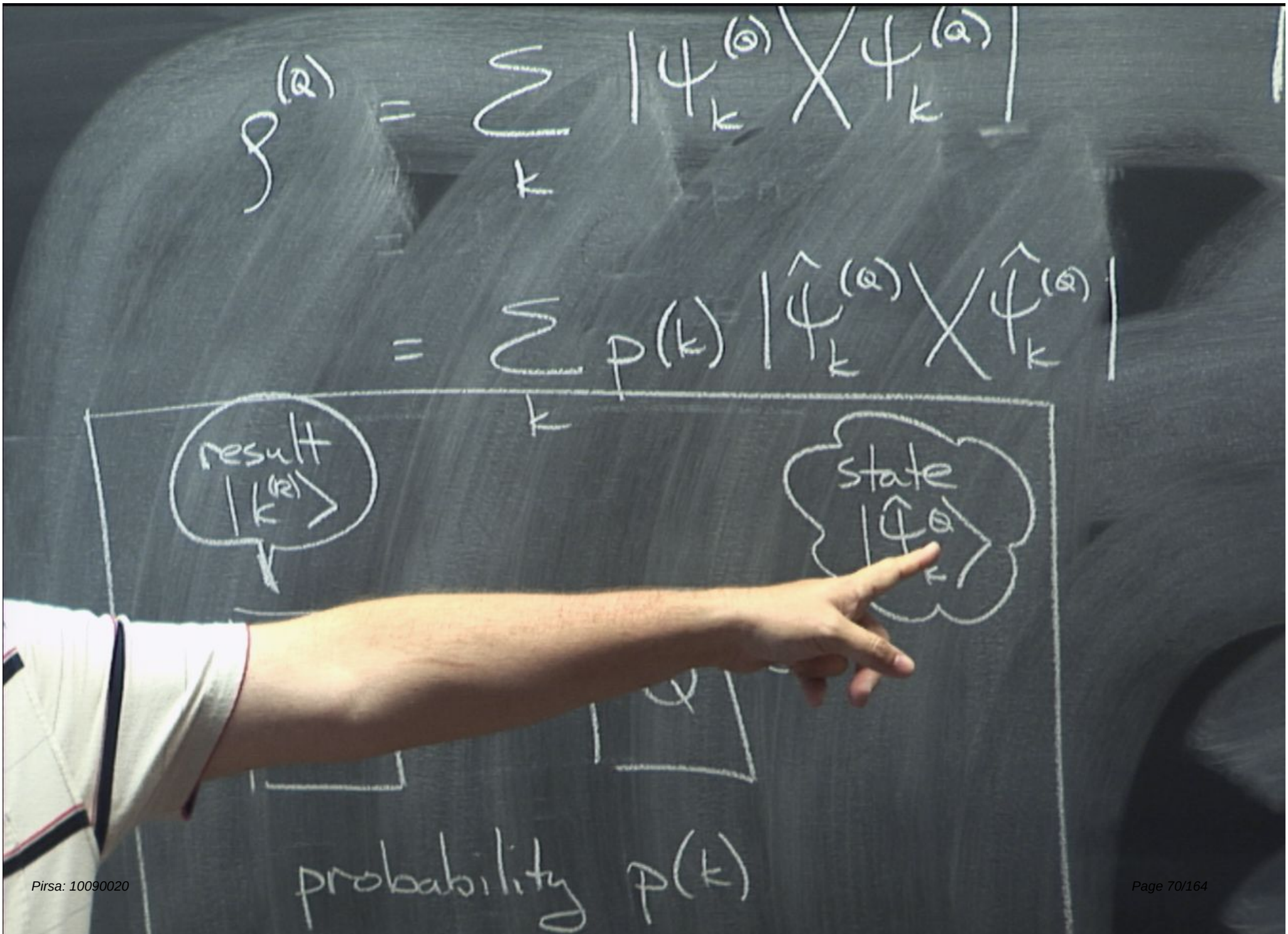
result $|k^{(R)}\rangle$

probability $p(k)$


$\rho^{(Q)} = \sum_k |\psi_k^{(Q)}\rangle\langle\psi_k^{(Q)}|$
$= \sum_k p(k)\, |\hat{\psi}_k^{(Q)}\rangle\langle\hat{\psi}_k^{(Q)}|$
result
$|k^{(R)}\rangle$
state
$|\hat{\psi}_k^{(Q)}\rangle$
probability $p(k)$

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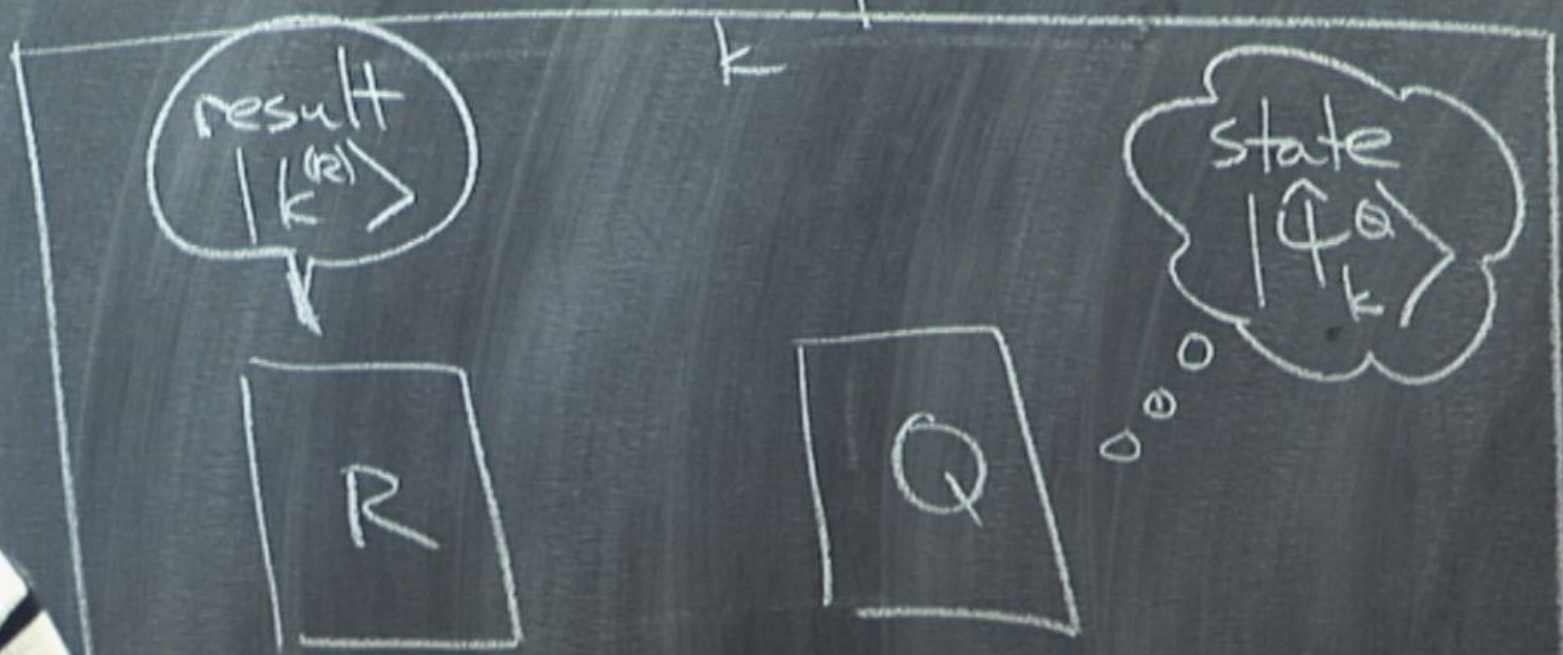

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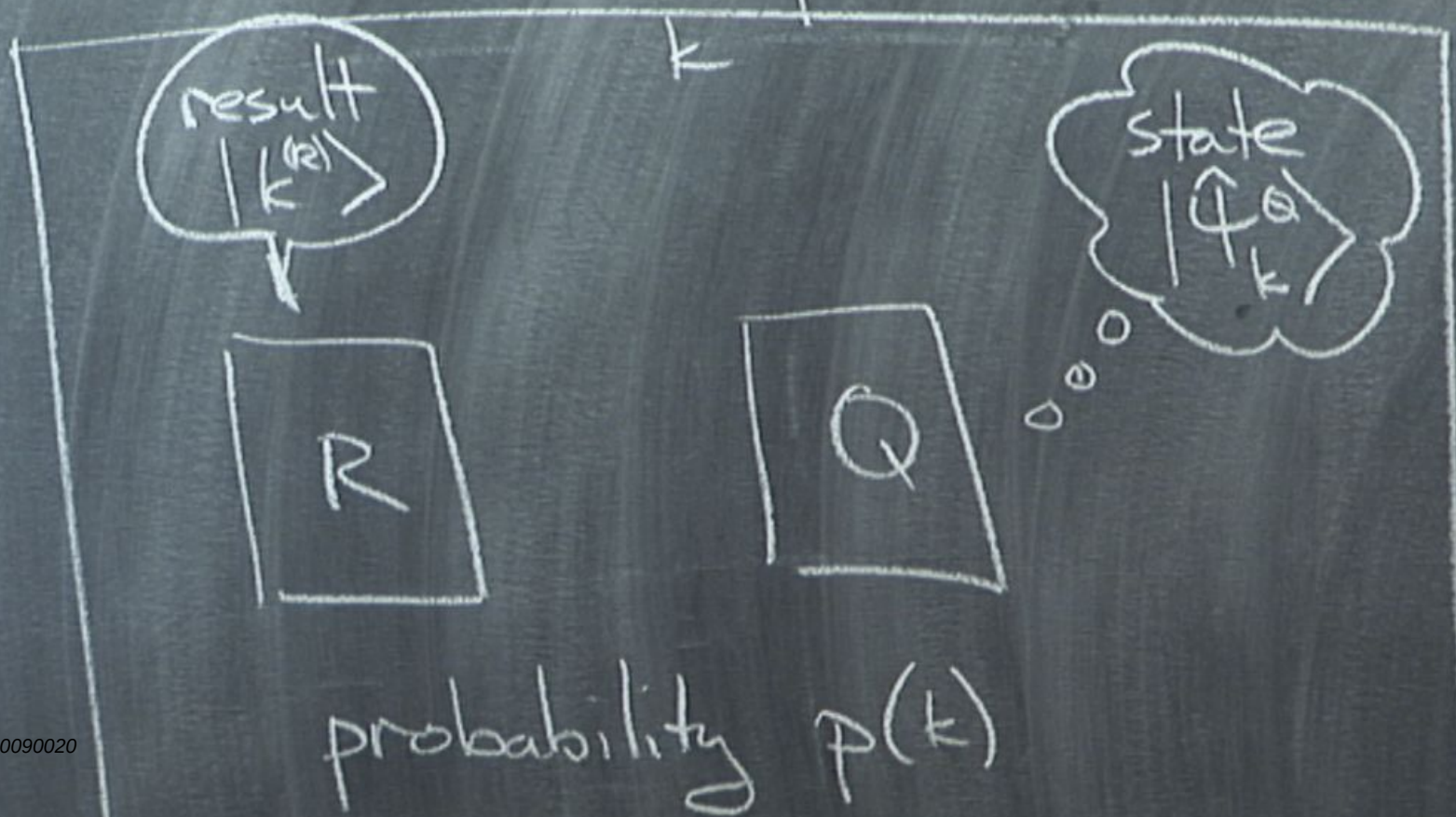
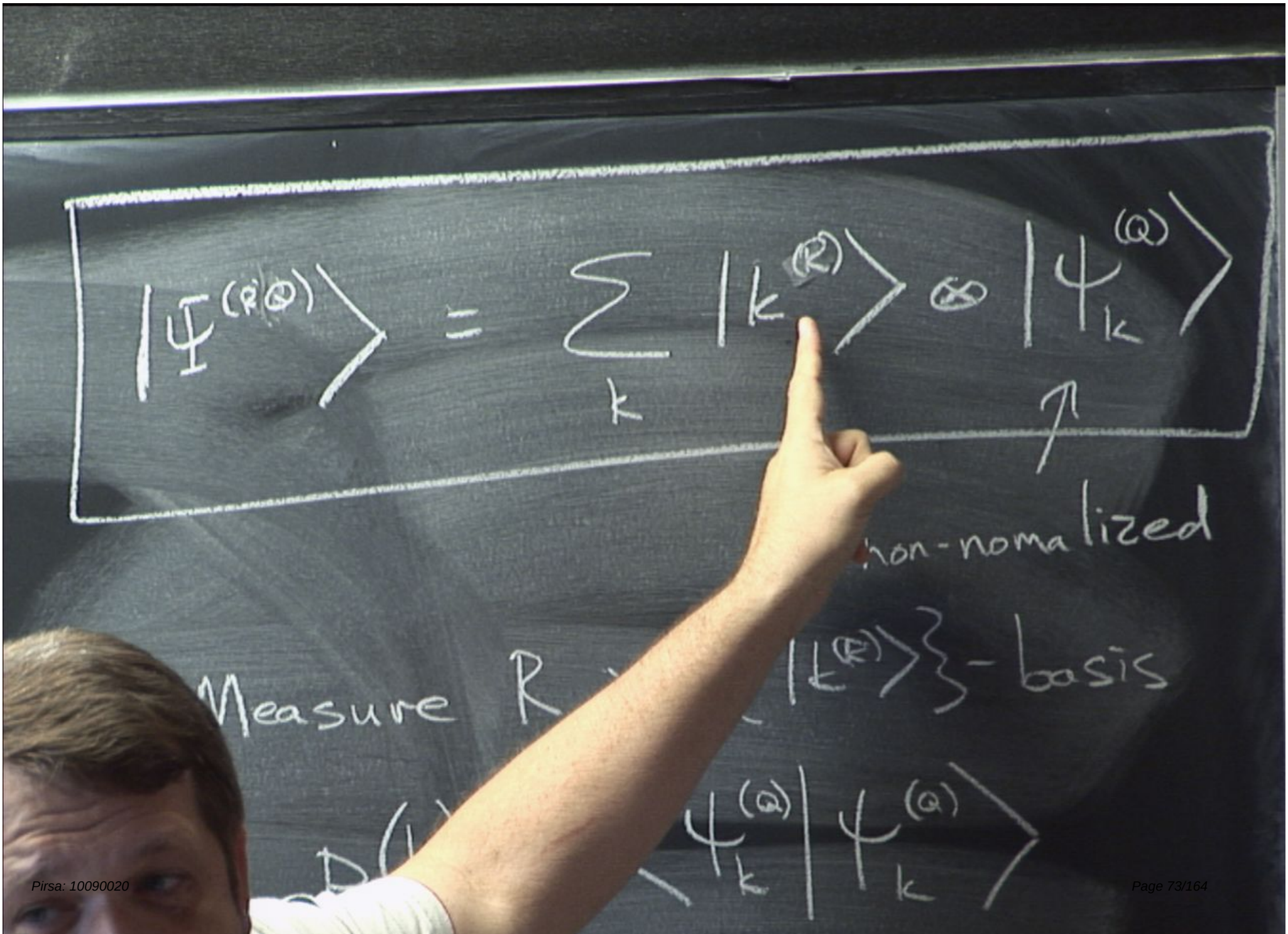



$|\Psi^{(R,Q)}\rangle = \sum_k |k^{(R)}\rangle \otimes |\psi_k^{(Q)}\rangle$
non-normalized
Measure R
$|k^{(R)}\rangle\}$-basis
$\langle\psi_k^{(Q)}|\psi_k^{(Q)}\rangle$

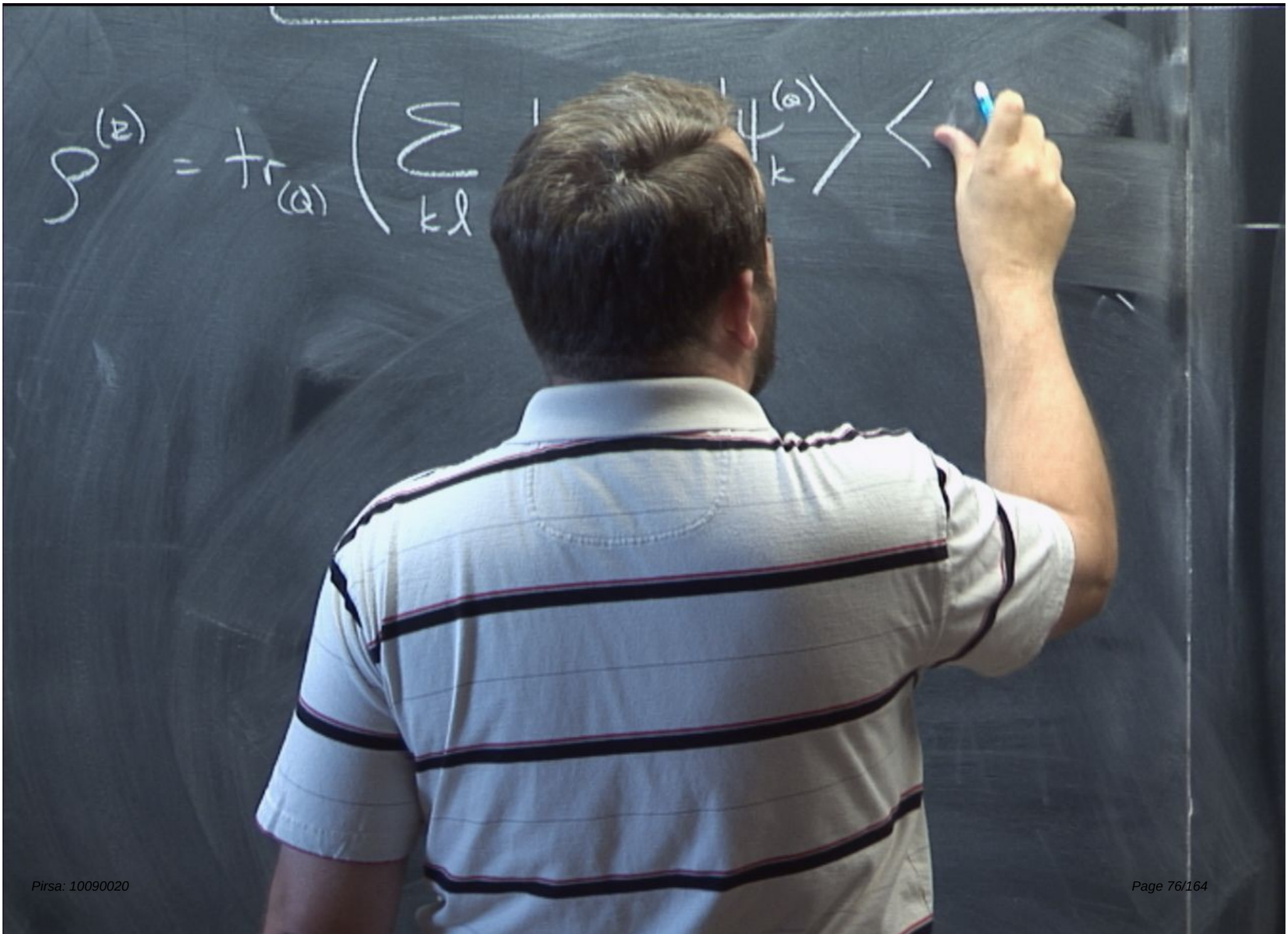
$$\boxed{|\Psi^{(R,Q)}\rangle = \sum_k |k^{(R)}\rangle \otimes |\psi_k^{(Q)}\rangle}$$

non-normalized

Measure R in $\{|k^{(R)}\rangle\}$-basis

$$p(k) = \langle \psi_k^{(Q)} | \psi_k^{(Q)} \rangle$$

What is $\rho^{(R)} = \mathrm{tr}_{(Q)} \, |\Psi^{(RQ)}\rangle\langle\Psi^{(RQ)}|$?


$\rho^{(2)} = tr_{(Q)} \left(\sum_{kl} \right. \ldots \psi_k^{(Q)} \rangle \langle$

$$\rho^{(R)} = \mathrm{tr}_{(Q)}\left(\sum_{kl}\left(|k^{(R)}\rangle \otimes |\psi_k^{(Q)}\rangle\right)\left(\langle l^{(R)}| \otimes \langle \psi_l^{(Q)}|\right)\right)$$

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$$= \sum_{kl} \langle \psi_l^{(Q)}|$$

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$$= \sum_{kl} \langle \psi_l^{(Q)} | \psi_k^{(Q)} \rangle \, |k^{(R)}\rangle\langle l^{(R)}|$$

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populations $\rho_{kk} =$

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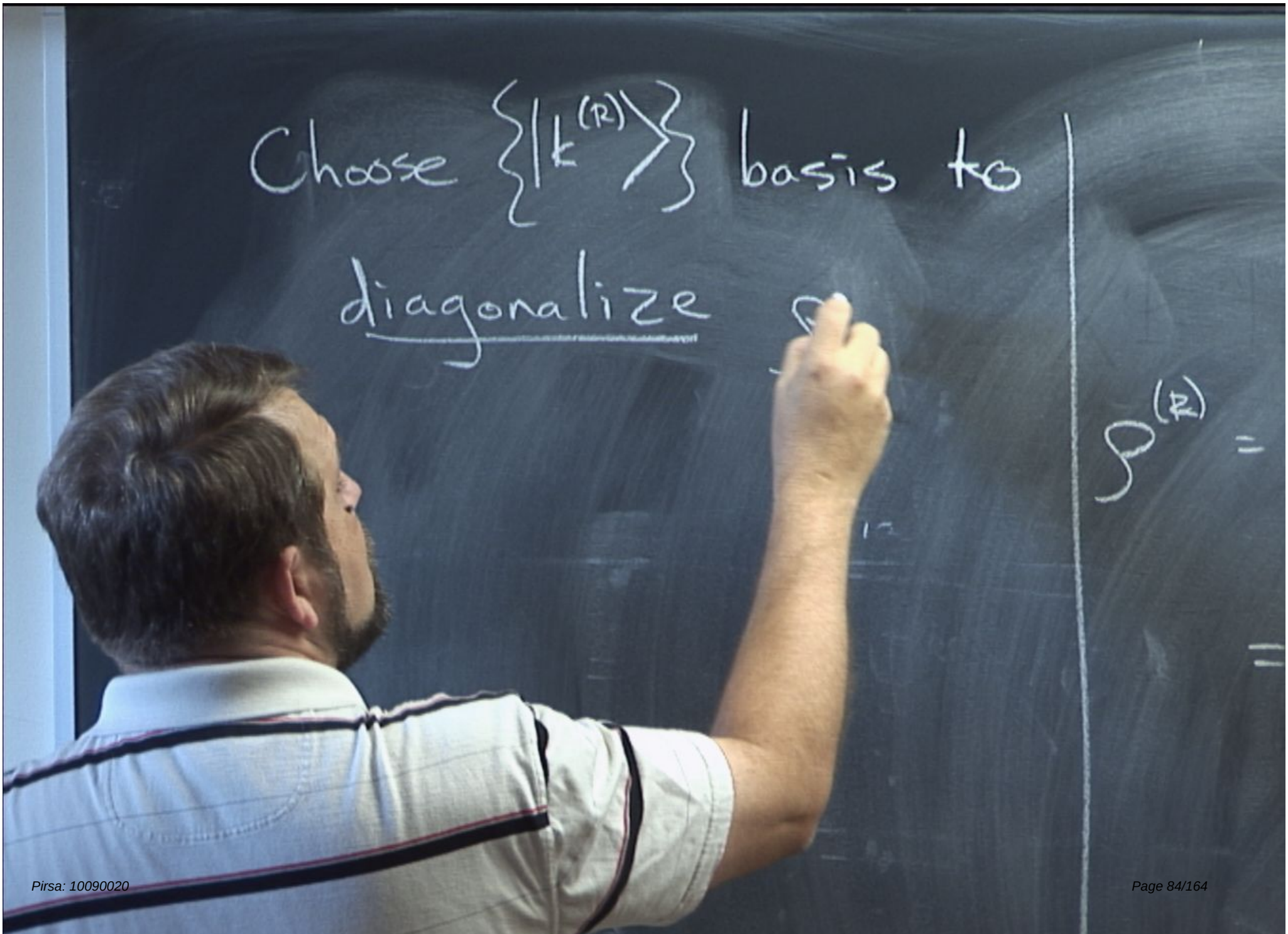
coherences $\neq 0$ in general


Choose $\{|k^{(R)}\rangle\}$ basis to
diagonalize
$\rho^{(R)} =$

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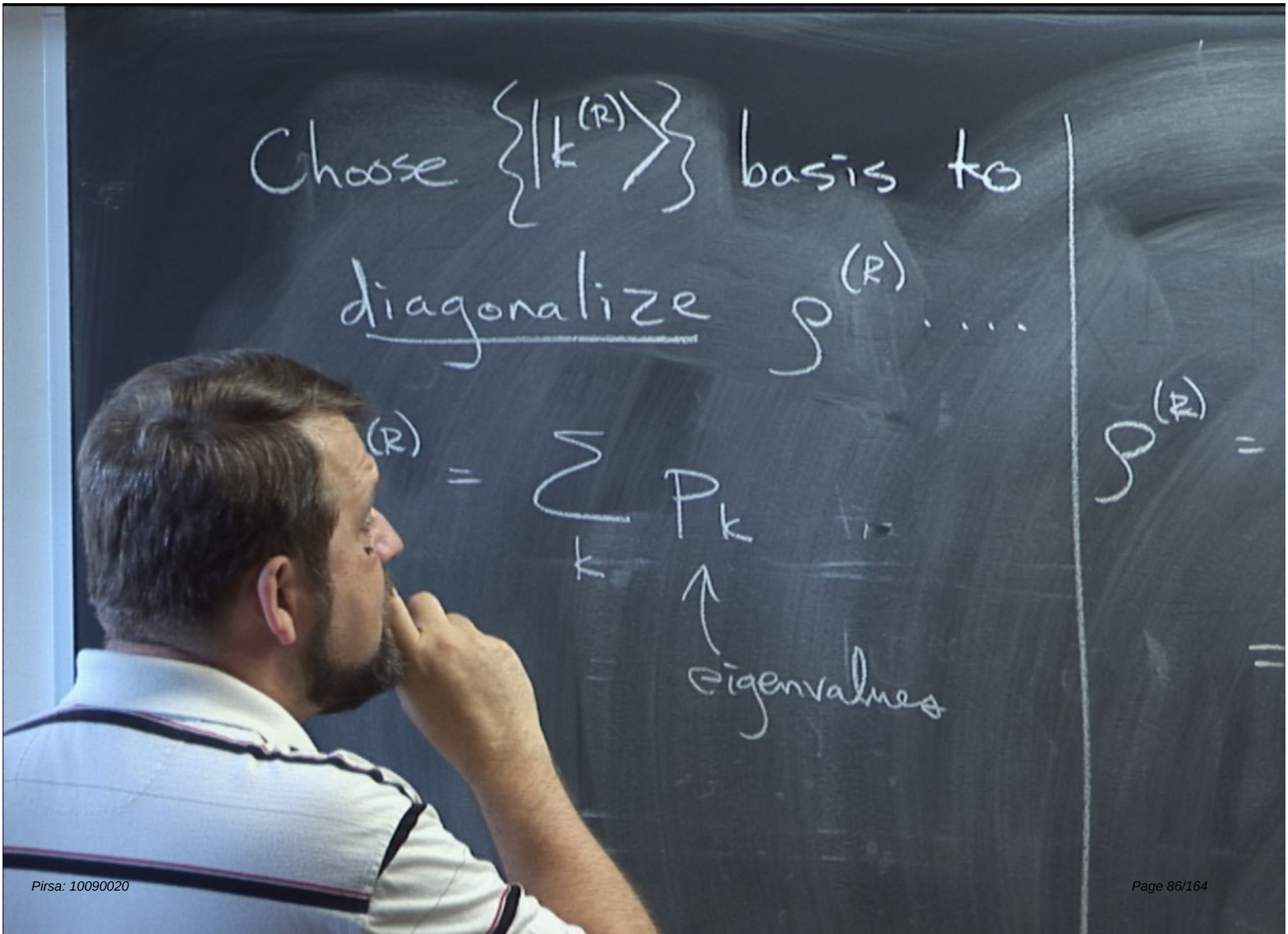
<u>diagonalize</u> $\rho^{(R)}$ ....

$\rho^{(R)} =$

$=$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$\rho^{(R)} = \sum_k P_k$
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eigenvalues

$$\therefore \langle \psi_l^{(Q)} | \psi_k^{(Q)} \rangle = 0 \text{ for } k \neq l$$

$\rho^{(R)} =$

$=$

Choose $\{|k^{(R)}\rangle\}$ basis to

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eigenvalues

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$|\psi_k^{(Q)}\rangle$'s orthogonal set...

$|\hat{\psi}_k^{(Q)}\rangle$'s orthonormal set

(part of a Q-basis

What

$$\rho^{(R)} = tr_{(Q)} \left(\sum_{kl} \right.$$

$$= \sum_{kl} \langle$$

populations

coherence

Q-basis $\quad |k^{(Q)}\rangle = |\hat{\psi}_k^{(Q)}\rangle = \frac{1}{\sqrt{p_k}}$

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$$|k^{(Q)}\rangle = |\hat{\psi}_k^{(Q)}\rangle = \frac{1}{\sqrt{p_k}}|\psi_k^{(Q)}\rangle$$

$$\boxed{|\Psi^{(RQ)}\rangle = \sum_k \sqrt{p_k}\,|k^{(R)}\rangle \otimes |k^{(Q)}\rangle}$$

is

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Schmidt decomposition

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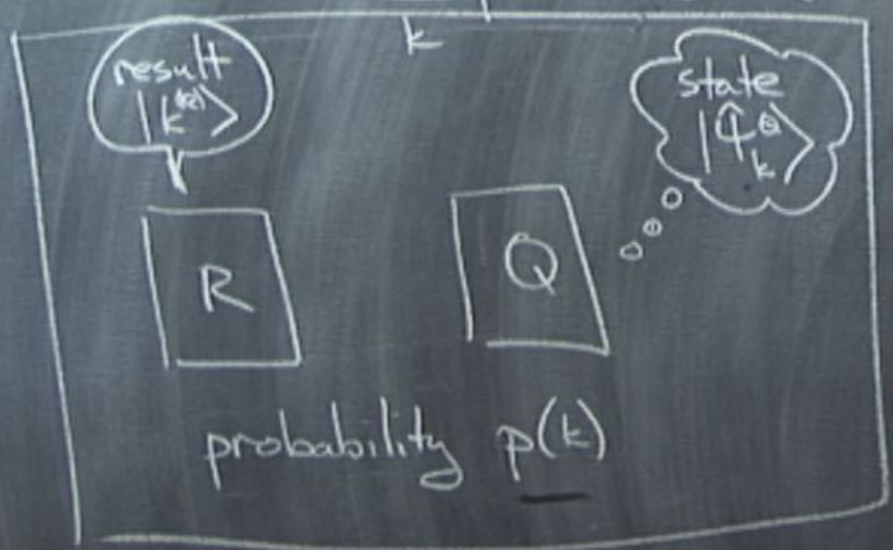
Schmidt decomposition

$$\boxed{\Psi^{(RQ)} \;?}$$

$$\rho^{(Q)} = \sum_k |\psi_k^{(Q)}\rangle\langle\psi_k^{(Q)}|$$

$$= \sum_k p(k)\, |\hat{\psi}_k^{(Q)}\rangle\langle\hat{\psi}_k^{(Q)}|$$



$$\boxed{|\Psi^{(RQ)}\rangle = \sum_k |k^{(R)}\rangle \otimes |\psi_k^{(Q)}\rangle}$$

non-normalized

Measure R in $\{|k^{(R)}\rangle\}$-basis

$$p(k) = \langle\psi_k^{(Q)}|\psi_k^{(Q)}\rangle$$

conditional state

$$|\hat{\psi}_k^{(Q)}\rangle = \frac{1}{\sqrt{p(k)}}\, |\psi_k^{(Q)}\rangle$$

$$\boxed{\langle k^{(R)}|\Psi^{(RQ)}\rangle = |\psi_k^{(Q)}\rangle}$$

Q-basis

$$|k^{(Q)}\rangle = |\hat{\psi}_k^{(Q)}\rangle = \frac{1}{\sqrt{p_k}}|\psi_k^{(Q)}\rangle$$

$$\boxed{|\Psi^{(RQ)}\rangle = \sum_k \sqrt{p_k}\,|k^{(R)}\rangle \otimes |k^{(Q)}\rangle}$$

Schmidt decomposition

Open system dynamics

System Q

Environment E

interact

Open system dynamics

System Q

Environment E

interact

Open system dynamics

System Q

Environment E

interact

$$\begin{pmatrix}\text{pure}\\ \text{Q-state}\end{pmatrix} \longrightarrow \begin{pmatrix}\text{mixed}\\ \text{Q-state}\end{pmatrix}$$ maybe

Environment E

$$\begin{pmatrix}\text{pure}\\ \text{Q-state}\end{pmatrix} \longrightarrow \begin{pmatrix}\text{mixed}\\ \text{Q-state}\end{pmatrix}$$ ma

① Q and E are initially unentangled

Environment E

$$\begin{pmatrix}\text{pure}\\ \text{Q-state}\end{pmatrix} \longrightarrow \begin{pmatrix}\text{mixed}\\ \text{Q-state}\end{pmatrix}$$

ma

1. Q and E are initially unentangled
2. Initial environment state $|0^{(E)}\rangle$

Environment E

$$\begin{pmatrix}\text{pure}\\ \text{Q-state}\end{pmatrix} \longrightarrow \begin{pmatrix}\text{mixed}\\ \text{Q-state}\end{pmatrix}$$ may be

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1. Q and E are initially unentangled
2. Initial environment state $|0^{(E)}\rangle$
3. Unitary evolution $U^{(QE)}$
4. Interested in care about final Q-state

Basis $\{|k^{(\varepsilon)}\rangle\}$ for $\mathcal{H}^{(\varepsilon)}$

Op

Basis $\{|k^{(\varepsilon)}\rangle\}$ for $\mathcal{H}^{(\varepsilon)}$

Op

Basis $\{|k^{(E)}\rangle\}$ for $\mathcal{H}^{(E)}$

initial Q-state $|\psi^{(Q)}, 0^{(E)}\rangle$

$$|\psi^{(Q)}, 0^{(E)}\rangle \rightarrow U^{(QE)} |\psi^{(Q)}, 0^{(E)}\rangle$$

Basis $\{|k^{(E)}\rangle\}$ for $\mathcal{H}^{(E)}$

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$\longrightarrow U^{(QE)} |\psi^{(Q)}, 0^{(E)}\rangle$

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Open su

Syst

Envi

① Q a

② Init

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Open su

Syst

Envi

① Q a

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$$\langle k^{(R)} | U^{(QE)} | \psi^{(Q)}, O^{(E)} \rangle$$

$$|\psi^{(Q)}, O$$

$$\rho^{Q}(\text{after}$$

$$\langle k^{(R)} | U^{(QE)} | \psi^{(Q)}, O^{(E)} \rangle$$

— a Q-vector

—

$|\psi^{(Q)}, O$

ρ^{Q}(after

$$\langle k^{(R)} | U^{(QE)} | \psi^{(Q)}, 0^{(E)} \rangle$$

— a Q-vector

— depends <u>linearly</u> on $|\psi^{(Q)}\rangle$

$|\psi^{(Q)}, 0$

ρ^{Q}(after

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<u>Define</u>

ρ^{Q}(after

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- a Q-vector
- depends <u>linearly</u> on $|\psi^{(Q)}\rangle$

<u>Define</u>

$$A_k^{(Q)} |\psi^{(Q)}\rangle$$

$$\langle k^{(E)} | U^{(QE)} | \psi^{(Q)}, O^{(E)} \rangle$$

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Basi

initia

$|\psi^{(Q)}, O^{(E)}\rangle$

$\rho^{Q}(\text{after}) =$

$=$

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$\rho^{Q}(a$

$A_k^{(Q)}$ depends on $|0^{(E)}\rangle$

U^{QE}

$|k^{(E)}\rangle$

Define Kraus operator

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ρ^{Q}(after)

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ρ^{Q}(after)

$A_k^{(Q)}$ depends on $|0^{(E)}\rangle$

U^{QE}

$|k^{(E)}\rangle$

$$\rho^{Q}(\text{after}) = \sum_k A_k |\psi^{Q}\rangle\langle\psi^{Q}| A_k^{\dagger}$$

$$\rho^{Q}(\text{after}) = \sum_k A_k \underbrace{|\psi^{Q}\rangle\langle\psi^{Q}|}_{\rho^{Q}(\text{before})} A_k^{\dagger}$$

p on density operators

$$\rho_0 \longrightarrow \rho = \mathcal{E}(\rho_0)$$

$$\rho^{Q}(\text{after}) = \sum_k A_k \underbrace{|\psi^{Q}\rangle\langle\psi^{Q}|}_{\rho^{Q}(\text{before})} A_k^{+}$$

Map on density operators

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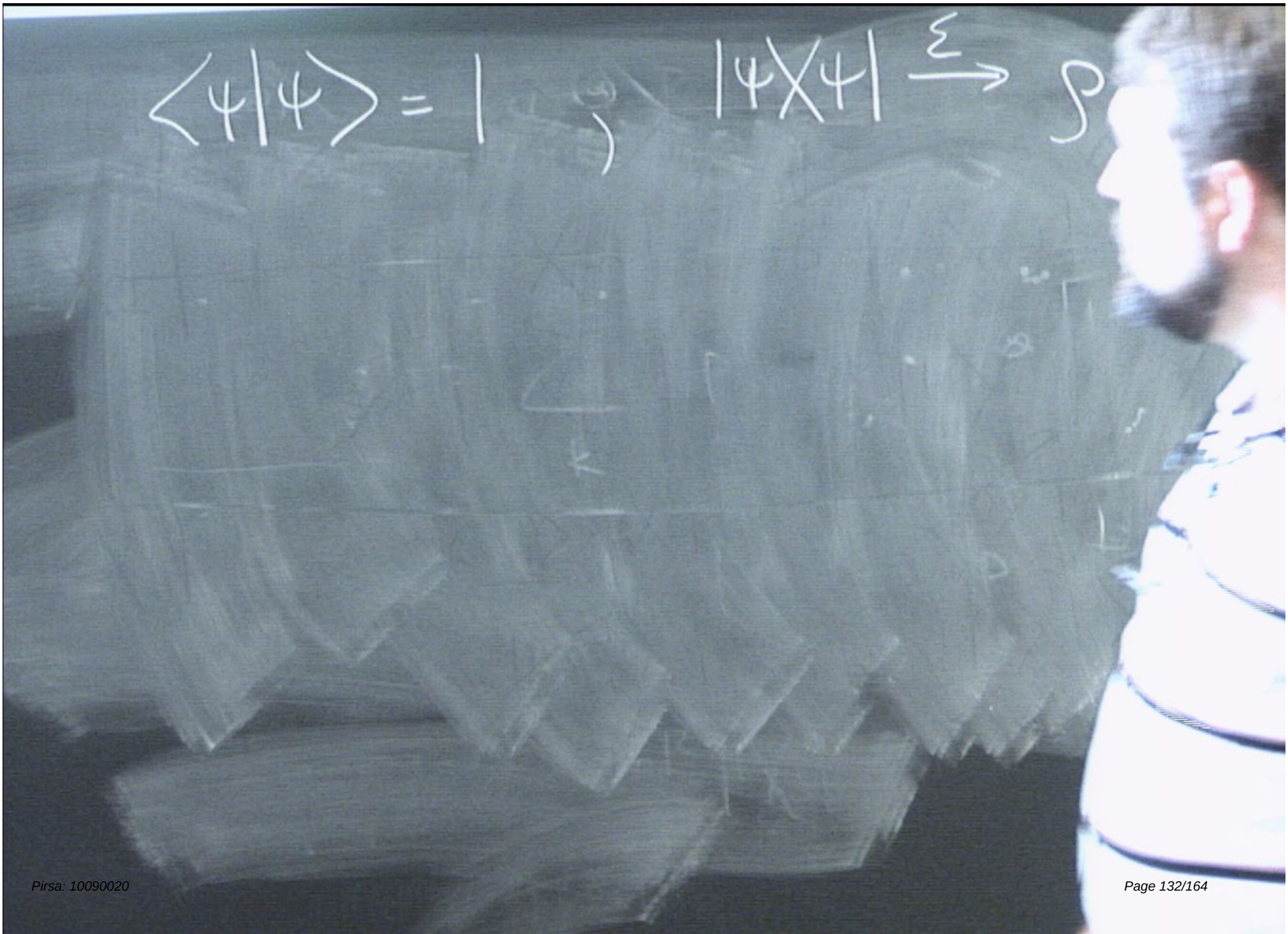
$$= \sum_k A_k \rho_0 A_k^\dagger$$

$\mathcal{E}(\cdot)$ is linear in input

(superoperator)


$\langle\psi|\psi\rangle = 1$; $|\psi\rangle\langle\psi| \xrightarrow{\mathcal{E}} \rho$

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$$=$$

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$$= 1 \quad \text{for any norm } |\psi\rangle$$

$$\boxed{\sum_k A_k^\dagger A_k = \mathbb{1}}$$

(Q), $0^{(E)}\rangle$

ter

linearly on $|\psi^{(Q)}\rangle$

us operator

$\rangle^{(QE)}|\psi^{(Q)}, 0^{(E)}\rangle$

$\rangle^{(E)}$

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Open syste

System

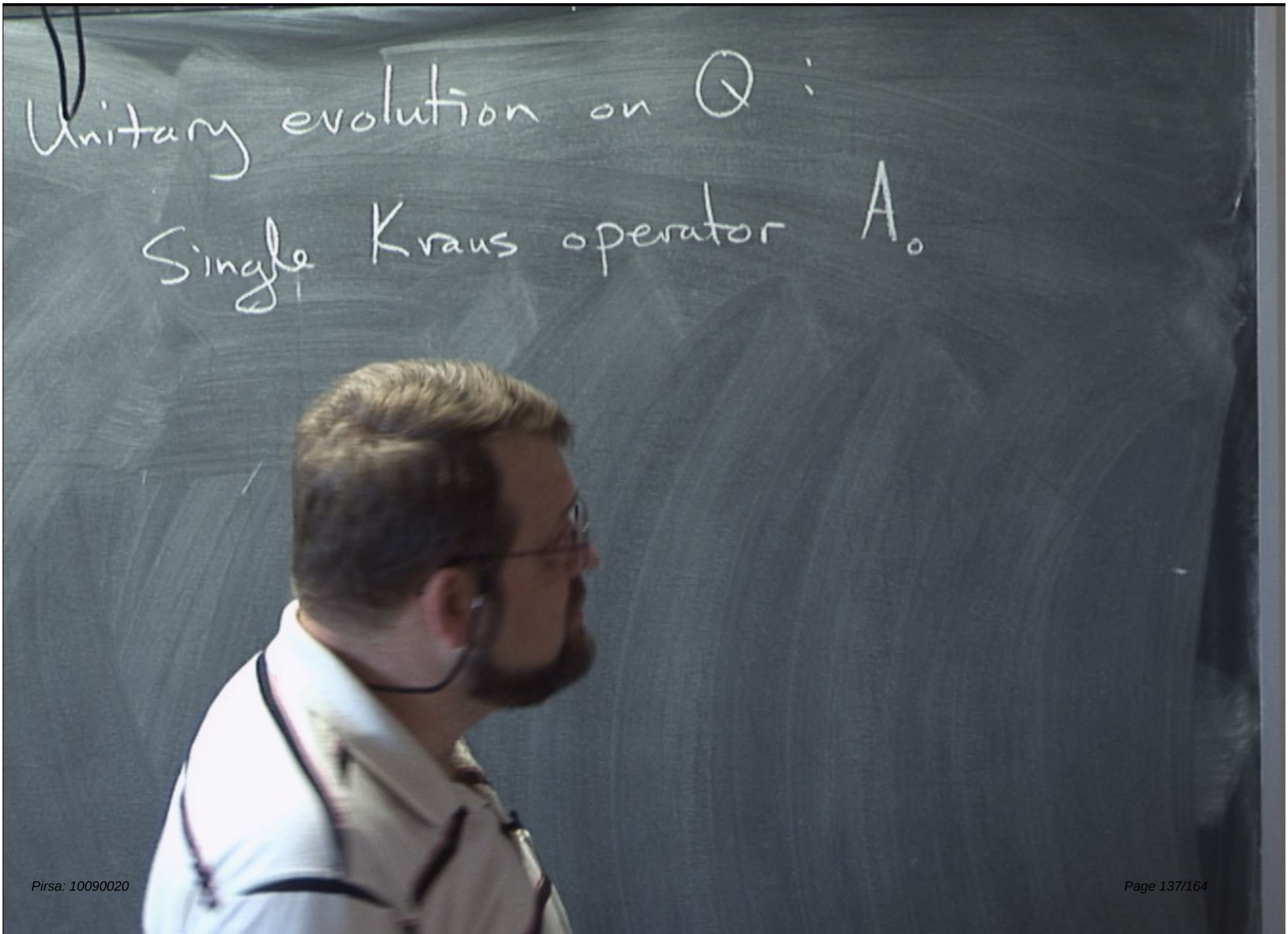
Environ

(1) Q a

(2) In

(3)

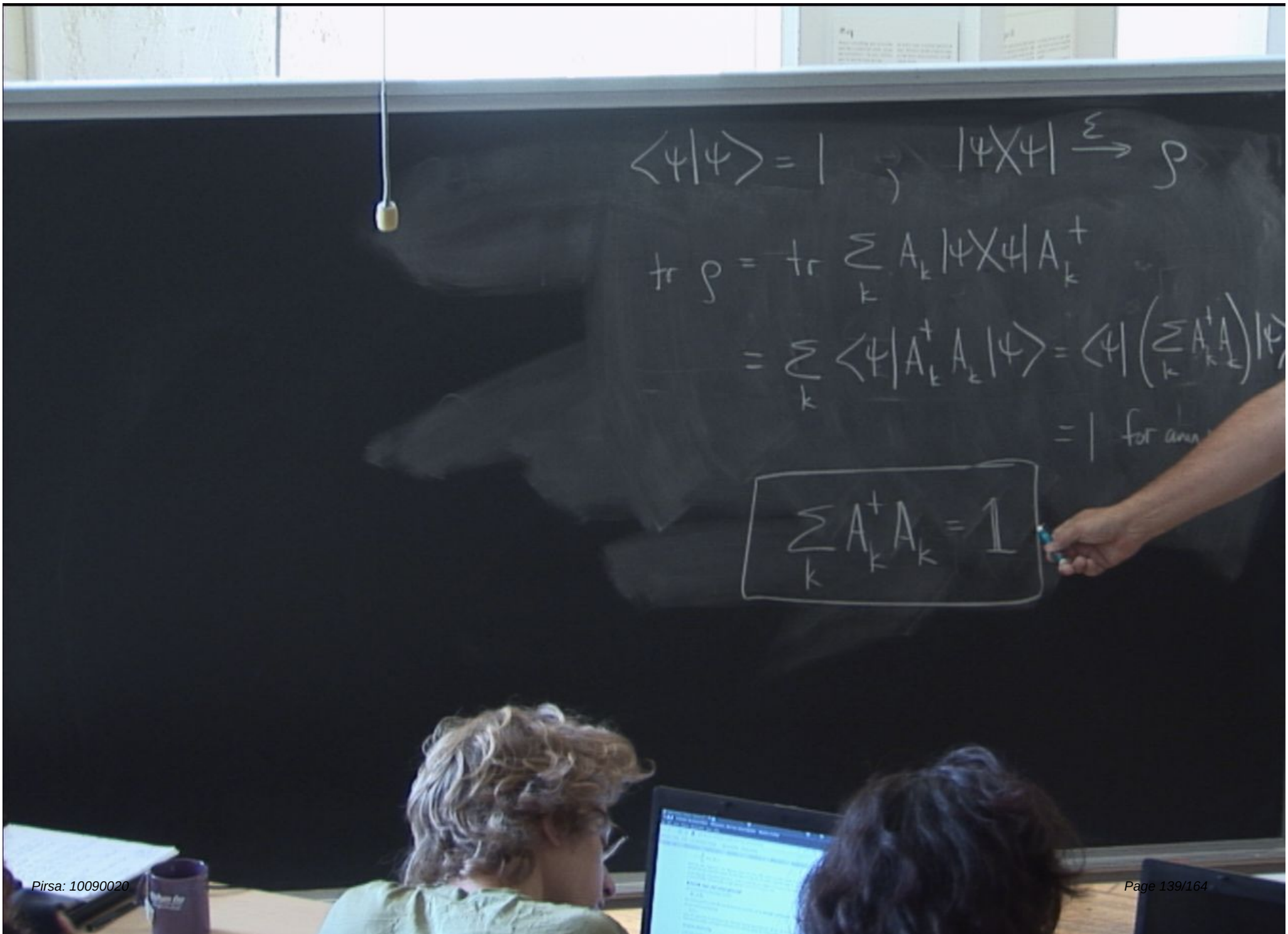
(4)


Unitary evolution on Q :
Single Kraus operator A_0

Unitary evolution on Q:

Single Kraus operator $A = U$

$$\rho_0 \longrightarrow \rho = A \rho_0 A^\dagger = U \rho_0 U^\dagger$$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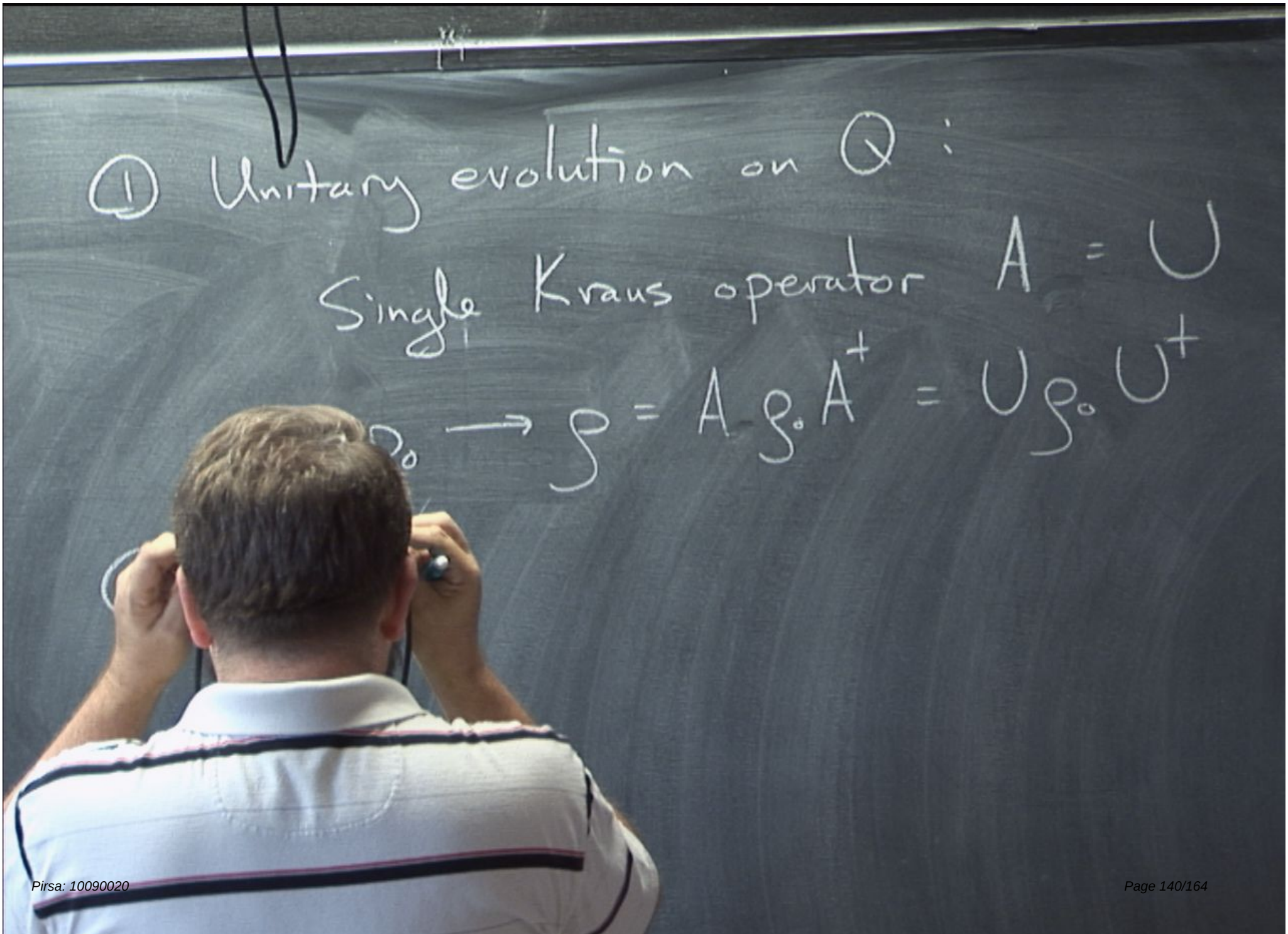
$= 1$ for
$\sum_k A_k^\dagger A_k = 1$


① Unitary evolution on Q :
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$\rho_0 \longrightarrow \rho = A \rho_0 A^+ = U \rho_0 U^+$

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② Random unitary

U_k applies with probability P_k

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① Unitary evolution on Q:

Single Kraus operator $A = U$

$$\rho_0 \longrightarrow \rho = A \rho_0 A^\dagger = U \rho_0 U^\dagger$$

② Random unitary

U_k applies with probability p_k

$$A_k = \sqrt{p_k}\, U_k$$

$$\rho_0 \longrightarrow \rho = \sum_k A_k \rho_0 A_k^\dagger$$

$$= \sum_k p_k \left(U_k \rho U_k^\dagger \right)$$

③ Make a measurement
(projection rule)

$|\psi\rangle \longrightarrow$ with prob. $p_k = |\langle k|\psi\rangle|^2$
wind up in $|k\rangle$

① Un

② Ro

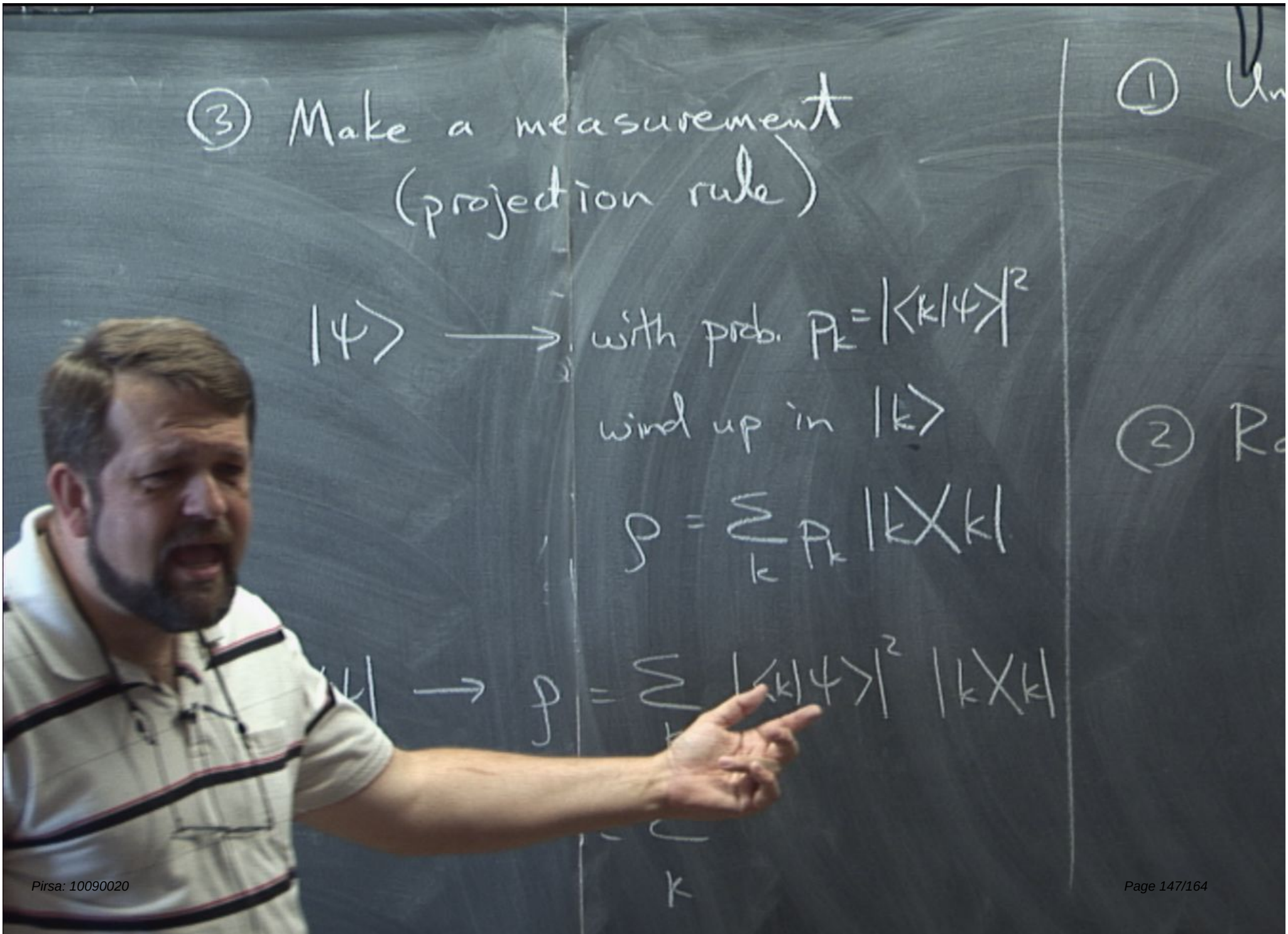
③ Make a measurement
(projection rule)

$|\psi\rangle \longrightarrow$ with prob. $p_k = |\langle k|\psi\rangle|^2$
wind up in $|k\rangle$

$$\rho = \sum_k p_k |k\rangle\langle k|$$

① Un

② Ra


③ Make a measurement
(projection rule)
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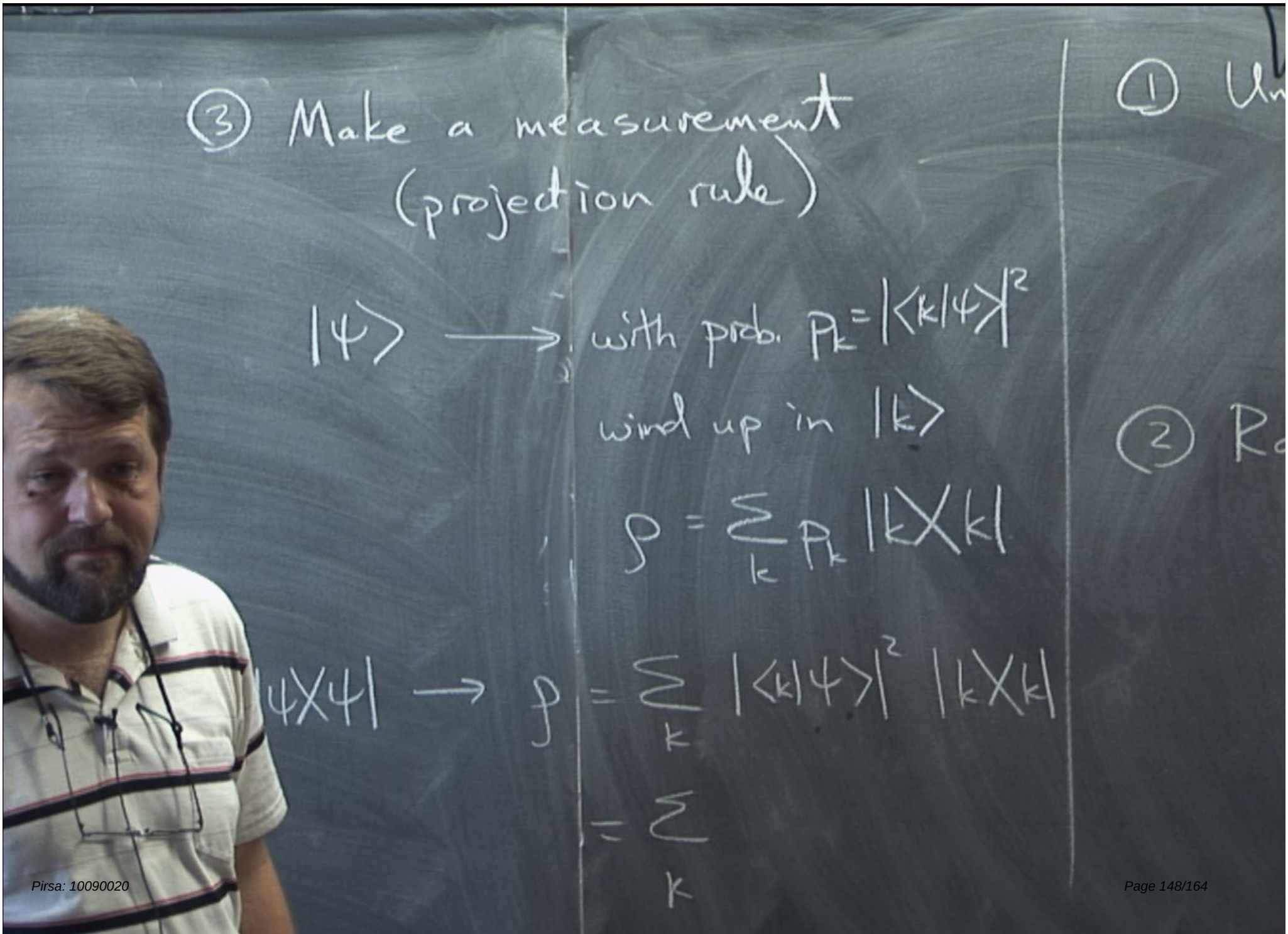
wind up in |k⟩
$\rho = \sum_k p_k |k\rangle\langle k|$
$\rho = \sum_k |\langle k|\psi\rangle|^2 |k\rangle\langle k|$
① Un
② Ra


③ Make a measurement
(projection rule)
$|\psi\rangle \longrightarrow$ with prob. $p_k = |\langle k|\psi\rangle|^2$
wind up in $|k\rangle$
$\rho = \sum_k p_k |k\rangle\langle k|$
$|\psi\rangle\langle\psi| \longrightarrow \rho = \sum_k |\langle k|\psi\rangle|^2 |k\rangle\langle k|$
$= \sum_k$
①
②

③ Make a measurement
(projection rule)

$|\psi\rangle \longrightarrow$ with prob. $p_k = |\langle k|\psi\rangle|^2$
wind up in $|k\rangle$

$$\rho = \sum_k p_k |k\rangle\langle k|$$

$$|\psi\rangle\langle\psi| \longrightarrow \rho = \sum_k |\langle k|\psi\rangle|^2 |k\rangle\langle k|$$

$$= \sum_k \langle k|\psi\rangle\langle\psi|k\rangle |k\rangle\langle k|$$

① Un

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③ Make a measurement
(projection rule)

$|\psi\rangle \longrightarrow$ with prob. $p_k = |\langle k|\psi\rangle|^2$
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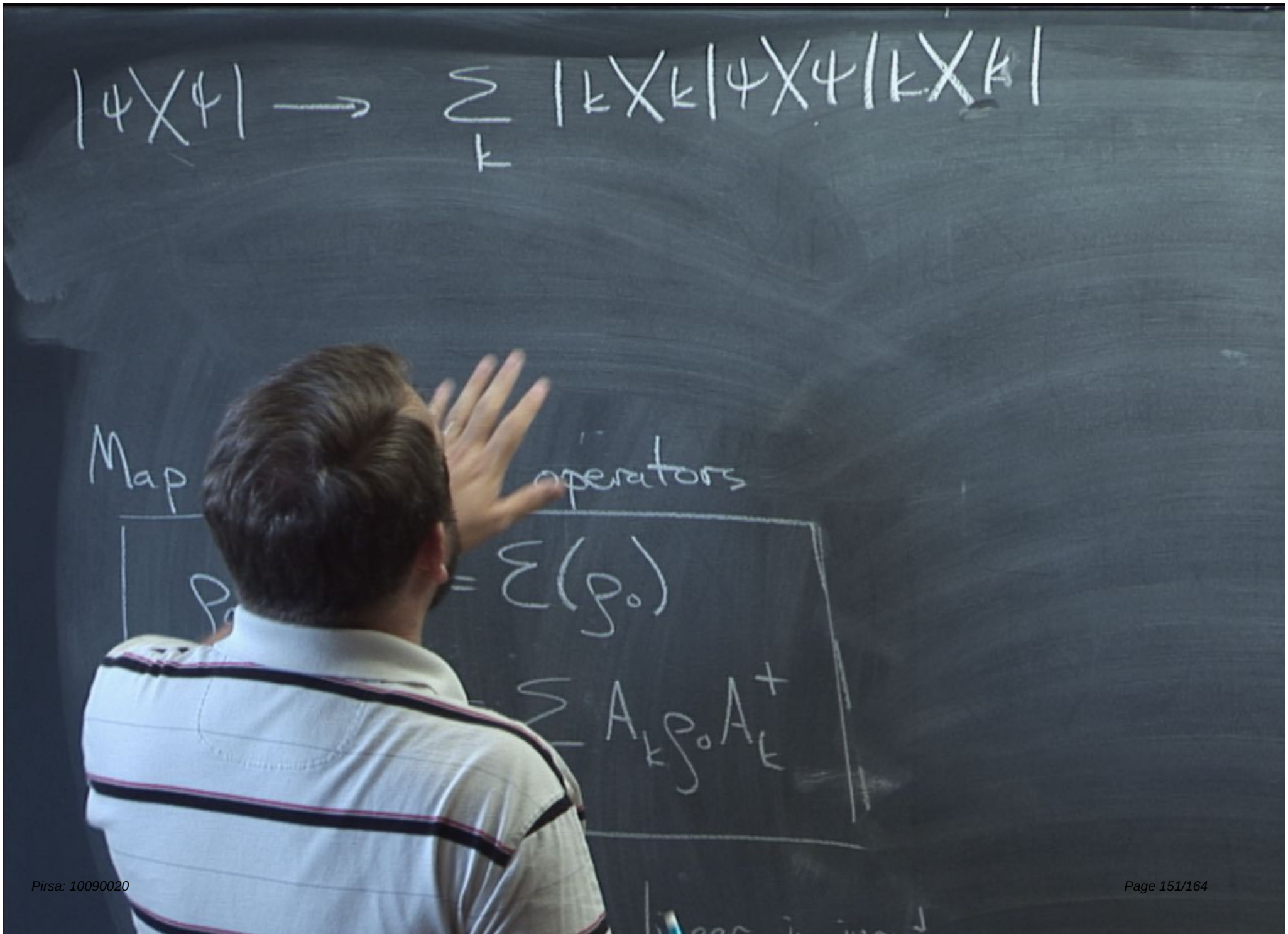
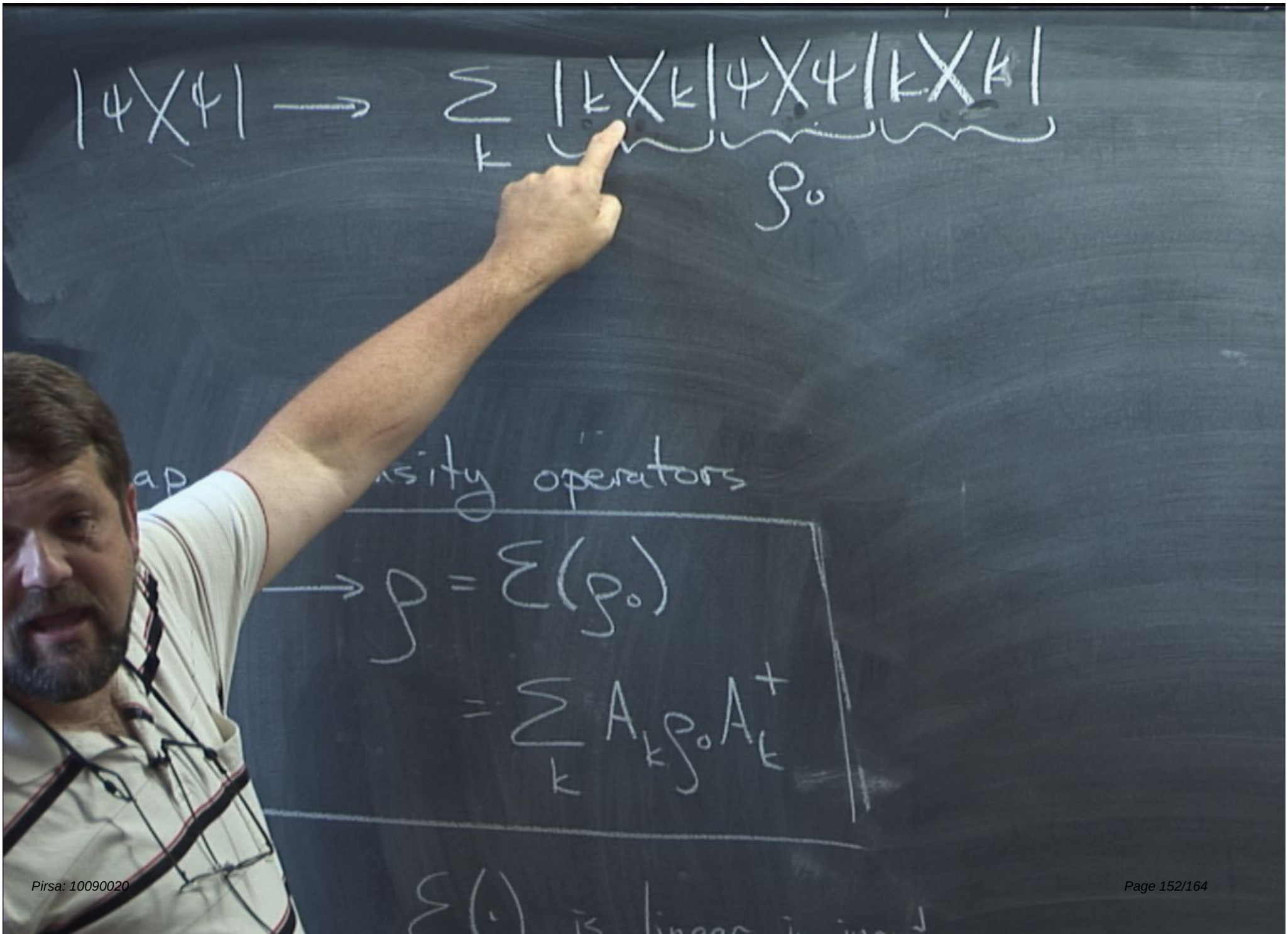

$|\psi\rangle\langle\psi| \longrightarrow \sum_k |k\rangle\langle k|\psi\rangle\langle\psi|k\rangle\langle k|$
Map
operators
$= \mathcal{E}(\rho_0)$
$\sum A_k \rho_0 A_k^\dagger$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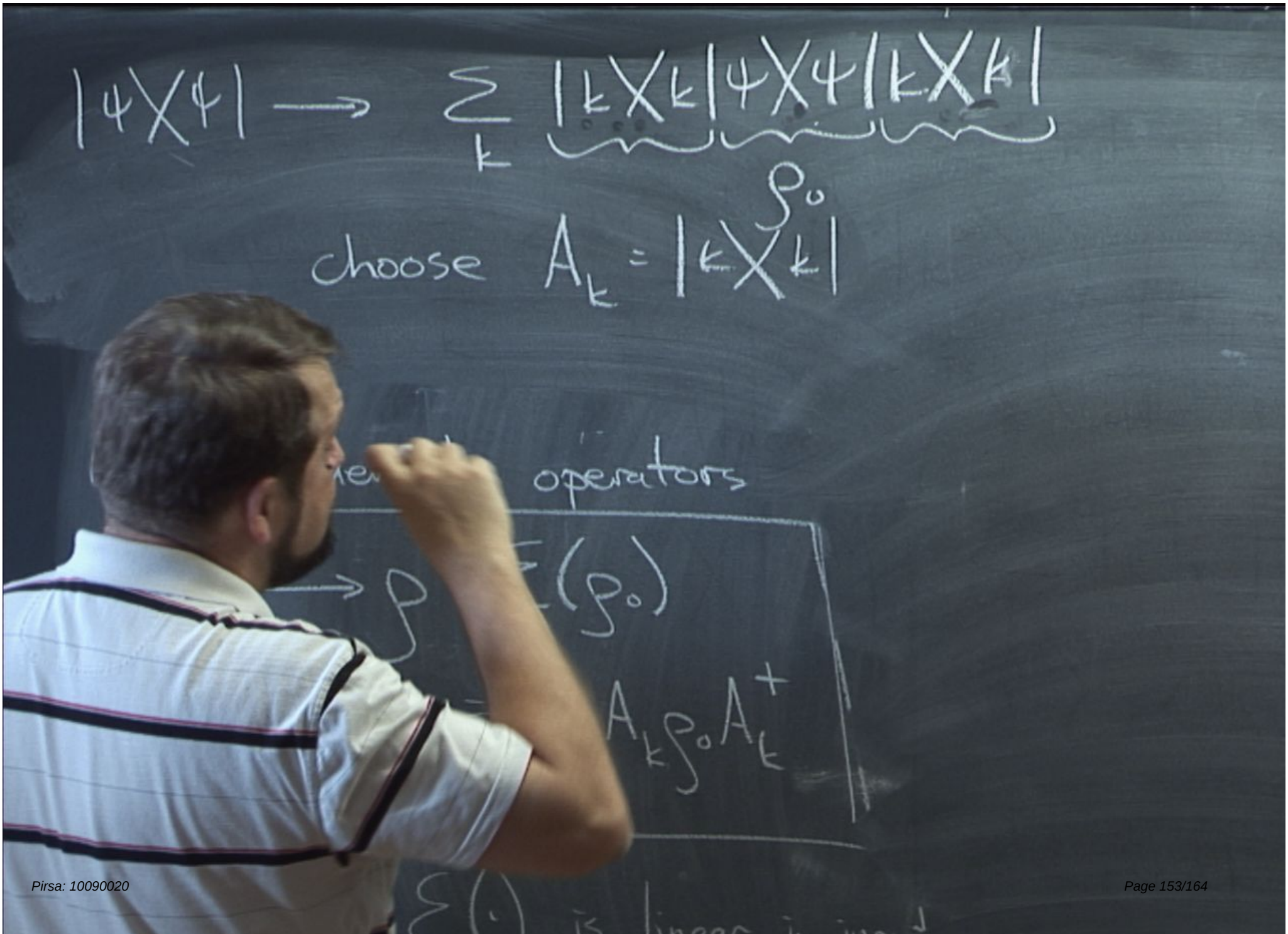
ρ_0
ap
sity operators
$\longrightarrow \rho = \mathcal{E}(\rho_0)$
$= \sum_k A_k \rho_0 A_k^\dagger$
$\mathcal{E}(\cdot)$ is linear


$|\psi\rangle\langle\psi| \longrightarrow \sum_k \underbrace{|k\rangle\langle k|\psi\rangle\langle\psi|k\rangle\langle k|}_{\rho_0}$
choose $A_k = |k\rangle\langle k|$
operators
$\rightarrow \rho \quad \mathcal{E}(\rho_0)$
$A_k \rho_0 A_k^\dagger$

$$|\psi\rangle\langle\psi| \longrightarrow \sum_k \underbrace{|k\rangle\langle k|\psi\rangle\langle\psi|k\rangle\langle k|}_{\rho_0}$$

choose $A_k = |k\rangle\langle k|$

$$\Longrightarrow |\psi\rangle\langle\psi| \to \sum_k A_k$$

Map on density operators

$$\boxed{\begin{aligned}\rho_0 \longrightarrow \rho &= \mathcal{E}(\rho_0) \\ &= \sum_k A_k \rho_0 A_k^{+}\end{aligned}}$$

$\mathcal{E}(\cdot)$ is linear, i.e.

choose $A_k = |k\rangle\langle k|$

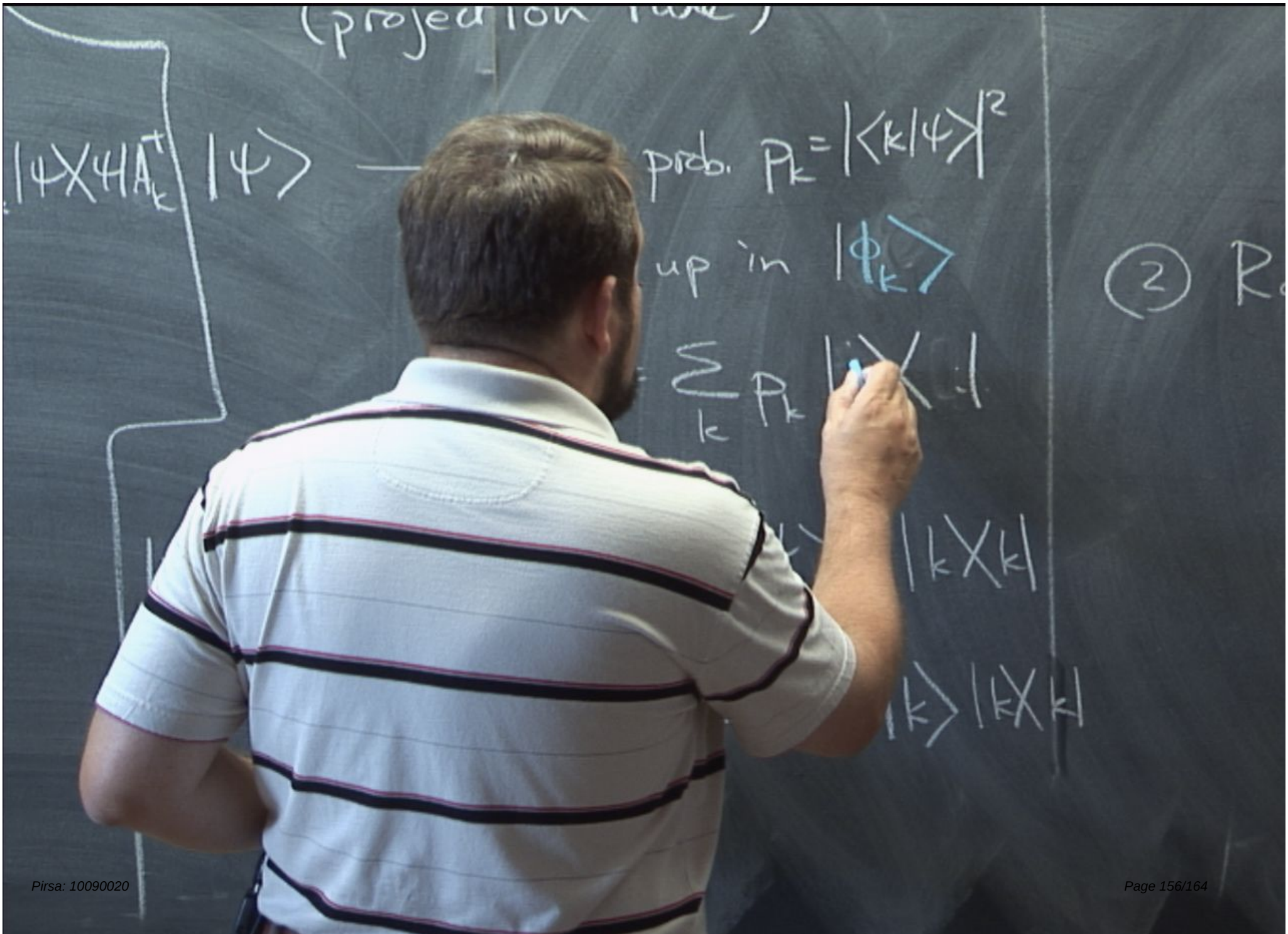
$$\Longrightarrow |\psi\rangle\langle\psi| \to \sum_k A_k$$

Map on density operators

$$\boxed{\begin{aligned} \rho_0 \longrightarrow \rho &= \mathcal{E}(\rho_0) \\ &= \sum_k A_k \rho_0 A_k^\dagger \end{aligned}}$$

$\mathcal{E}(\cdot)$ is linear in inp^t

(superoperator)


prob. $P_k = |\langle k|\psi\rangle|^2$
up in $|\phi_k\rangle$
$\sum_k P_k$
$|k\rangle\langle k|$
$|k\rangle\langle k|$
(2)

(projection rule)

$|\psi\rangle\langle\psi| A_k^\dagger \, |\psi\rangle \longrightarrow$ with prob. $p_k = |\langle k|\psi\rangle|^2$

wind up in $|\phi_k\rangle$

$$\rho = \sum_k p_k \, |\phi_k\rangle\langle\phi_k|$$

$$|\psi\rangle\langle\psi| \longrightarrow \rho = \sum_k |\langle k|\psi\rangle|^2 \, |\phi_k\rangle\langle\phi_k|$$

$$= \sum_k \langle k|\psi\rangle\langle\psi|k\rangle \, |\phi_k\rangle\langle\phi_k|$$

(2) R

(projection rule)

$$|\psi\rangle\langle\psi| A_k^\dagger \, |\psi\rangle \longrightarrow$$

with prob. $p_k = |\langle k|\psi\rangle|^2$

wind up in $|\phi_k\rangle$

$$\rho = \sum_k p_k \, |\phi_k\rangle\langle\phi_k|$$

$$|\psi\rangle\langle\psi| \longrightarrow \rho = \sum_k |\langle k|\psi\rangle|^2 \, |\phi_k\rangle\langle\phi_k|$$

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(2) R

③ Make a measurement

(~~projection rule~~)

$|\psi\rangle\langle\psi| A_k^\dagger |\psi\rangle \longrightarrow$ with prob. $p_k = |\langle k|\psi\rangle|^2$

wind up in $|\phi_k\rangle$

$$\rho = \sum_k p_k |\phi_k\rangle\langle\phi_k|$$

$$|\psi\rangle\langle\psi| \longrightarrow \rho = \sum_k |\langle k|\psi\rangle|^2 |\phi_k\rangle\langle\phi_k|$$

$$= \sum \langle k|\psi\rangle\langle\psi|k\rangle |\phi_k\rangle\langle\phi_k|$$

① U

② R

$$|\psi\rangle\langle\psi| \longrightarrow \sum_k \underbrace{|\phi_k\rangle\langle k|}\,\underbrace{|\psi\rangle\langle\psi|}\,\underbrace{|k\rangle\langle\phi_k|}$$

ρ_0

choose $A_k = |\phi_k\rangle\langle k|$

$$\Longrightarrow \boxed{|\psi\rangle\langle\psi| \longrightarrow \sum_k A_k |\psi\rangle\langle\psi|}$$

ap on density operators

$$\rho_0 \longrightarrow \rho = \mathcal{E}(\rho_0)$$

$$= \sum_k A_k \rho_0 A_k^{+}$$

$\mathcal{E}(\cdot)$ is linear in

$$|\psi\rangle\langle\psi| \longrightarrow \sum_k \underbrace{|\phi_k\rangle\langle k|}\,\underbrace{|\psi\rangle\langle\psi|}_{\rho_0}\,\underbrace{|k\rangle\langle\phi_k|}$$

choose $A_k = |\phi_k\rangle\langle k|$

$$\Longrightarrow \boxed{|\psi\rangle\langle\psi| \to \sum_k A_k |\psi\rangle\langle\psi| A_k^\dagger}$$

Map on density operators

$$\boxed{\begin{aligned}\rho_0 \longrightarrow \rho &= \mathcal{E}(\rho_0)\\ &= \sum_k A_k \rho_0 A_k^\dagger\end{aligned}}$$

$\mathcal{E}(\cdot)$ is linear in inp

(superoperator)

③ M

$|\psi\rangle\langle\psi$

$$\Longrightarrow \boxed{|\psi\rangle\langle\psi| \longrightarrow \sum_k A_k |\psi\rangle\langle\psi| A_k^\dagger} \; |\psi\rangle \longrightarrow$$

$$\langle k|U|\psi, 0\rangle$$

$$= A_k|\psi\rangle$$

$$|\psi\rangle\langle\psi| \longrightarrow$$

$$\Longrightarrow \boxed{|\psi\rangle\langle\psi| \longrightarrow \sum_k A_k |\psi\rangle\langle\psi| A_k^\dagger} \quad |\psi\rangle \longrightarrow$$

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