Title: Quantum Theory (PHYS 605) - Lecture 7

Date: Sep 21, 2010 09:00 AM

URL: http://pirsa.org/10090018

Abstract:

$$U^{(AB)}(t) = e^{-iH^{(AB)}t/\hbar}$$
$$= e^{-i(H^{(A)}+H^{(B)})t/\hbar}$$
$$= e^{-iH^{(A)}t/\hbar}\, e^{-iH^{(B)}t/\hbar}$$
$$= U^{(A)}(t)\; U^{(B)}(t)$$

dynamically <u>independent</u>
or <u>isolated</u>

$$H^{(AB)} = H^{(A)} + H^{(B)}$$

$$\left.\begin{aligned} H^{(A)}|a\rangle &= E_a|a\rangle \\ H^{(B)}|b\rangle &= E_b|b\rangle \end{aligned}\right\} \Rightarrow \begin{aligned} &H^{(AB)}|a,b\rangle \\ &= (E_a+E_b)|a,b\rangle \end{aligned}$$

Assume H's are const. ....

$$U^{(AB)}(t) = e^{-iH^{(AB)}t/\hbar}$$
$$= e^{-i(H^{(A)}+H^{(B)})t/\hbar}$$
$$= e^{-iH^{(A)}t/\hbar}\, e^{-iH^{(B)}t/\hbar}$$
$$= U^{(A)}(t) \otimes U^{(B)}(t)$$

dynamically <u>independent</u>
or <u>isolated</u>

$$H^{(AB)} = H^{(A)} + H^{(B)}$$

$$\left.\begin{aligned} H^{(A)}|a\rangle &= E_a|a\rangle \\ H^{(B)}|b\rangle &= E_b|b\rangle \end{aligned}\right\} \Rightarrow \begin{aligned} &H^{(AB)}|a,b\rangle \\ &= (E_a+E_b)|a,b\rangle \end{aligned}$$

$$|\psi^{(A)}(0), \phi^{(B)}(0)\rangle \longrightarrow \left(U^{(A)}|\psi(0)\rangle\right) \otimes \left(U^{(B)}|\phi(0)\rangle\right)$$
$$= |\psi^{(A)}(t), \phi^{(B)}(t)\rangle$$

Assume H's are const. ....

$$U^{(AB)}(t) = e^{-iH^{(AB)}t/\hbar}$$
$$= e^{-i(H^{(A)}+H^{(B)})t/\hbar}$$
$$= e^{-iH^{(A)}t/\hbar}\, e^{-iH^{(B)}t/\hbar}$$
$$= U^{(A)}(t) \otimes U^{(B)}(t)$$

dynamically <u>independent</u>
or <u>isolated</u>

$$H^{(AB)} = H^{(A)} + H^{(B)}$$

$$\left.\begin{array}{l} H^{(A)}|a\rangle = E_a|a\rangle \\ H^{(B)}|b\rangle = E_b|b\rangle \end{array}\right\} \Rightarrow \begin{array}{l} H^{(AB)}|a,b\rangle \\ = (E_a+E_b)|a,b\rangle \end{array}$$

$$|\psi^{(A)}(0), \phi^{(B)}(0)\rangle \longrightarrow \left(U^{(A)}|\psi(0)\rangle\right) \otimes \left(U^{(B)}|\phi(0)\rangle\right)$$
$$= |\psi^{(A)}(t), \phi^{(B)}(t)\rangle$$

Assume H's are const. ....

"singlet"
$(S=0)$

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

"singlet"
$(S=0)$

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

"singlet"
$(s=0)$

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

"singlet"
$(s=0)$

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

last time: $\hbar = 1$

this time: $\frac{1}{2} = 1$ (sort of)

"singlet" $(s=0)$

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

last time: $\hbar = 1$

this time: $\frac{1}{2} = 1$ (sort of)

Recall: $S_x = \frac{1}{2}\hbar X$ ← etc.

Pauli

"singlet" $(s=0)$

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

last time: $\hbar = 1$

this time: $\frac{1}{2} = 1$ (sort of)

Recall: $S_x = \frac{1}{2}\hbar X$ ← etc.

Pauli

"singlet" $(s=0)$

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

last time: $\hbar = 1$

this time: $\frac{1}{2} = 1$ (sort of)

Recall: $S_x = \frac{1}{2}\hbar X$ ← etc.

Pauli

$$S_x = |\uparrow X \downarrow| + |\downarrow X \uparrow|$$

"singlet" $(s=0)$

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

last time: $\hbar = 1$

this time: $\frac{1}{2} = 1$ (sort of)

Recall: $S_x = \frac{1}{2}\hbar X$ ← etc.

Pauli

$$X = |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|$$

$$Y = -i|\uparrow\rangle\langle\downarrow| + i|\downarrow\rangle\langle\uparrow|$$

$$Z = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|$$

"singlet" $(s=0)$

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

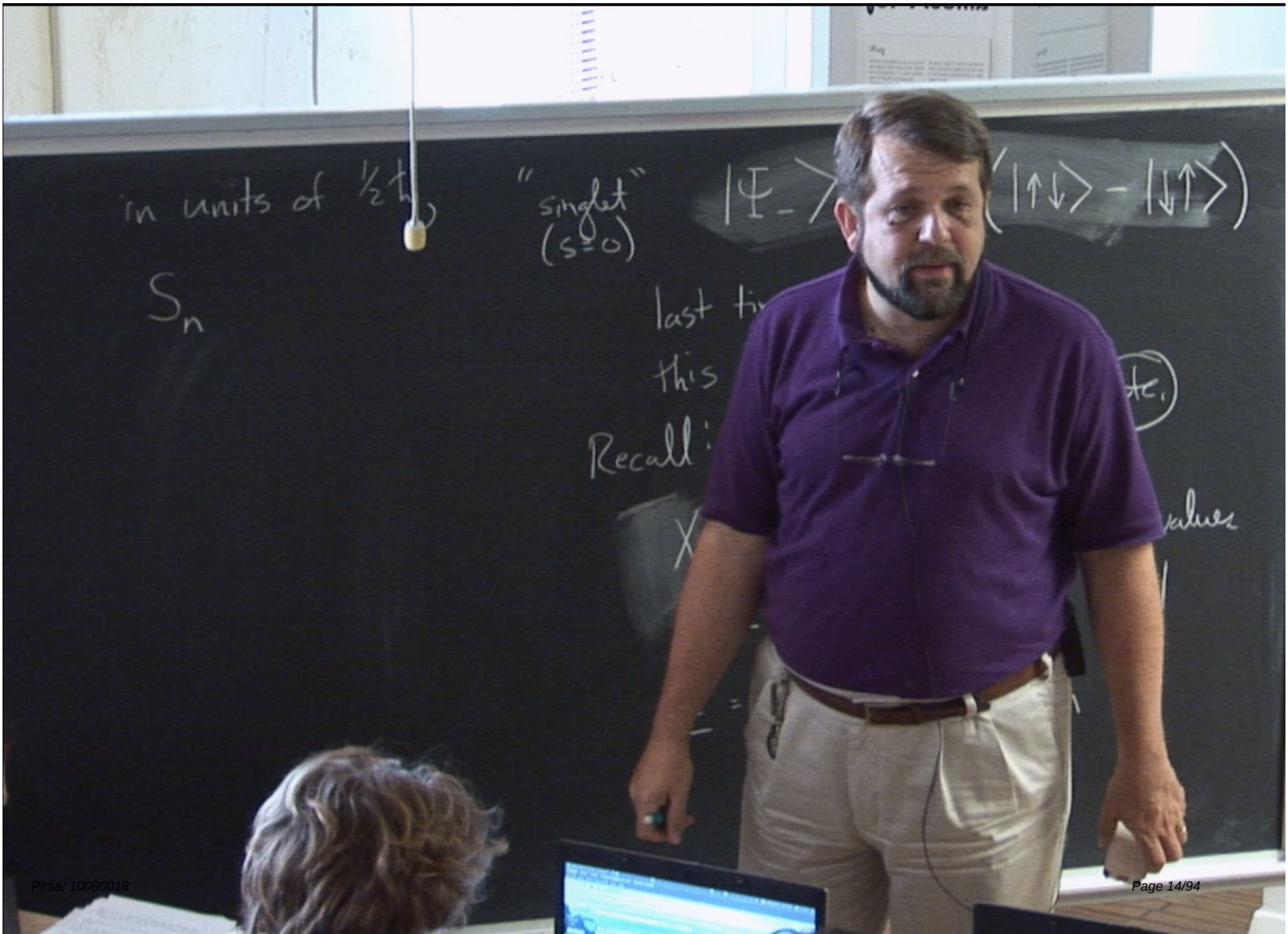
last time: $\hbar = 1$

this time: $\frac{1}{2} = 1$ (sort of)

Recall: $S_x = \frac{1}{2}\hbar X$ ← etc.

Pauli

$$\left.\begin{aligned} X &= |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow| \\ Y &= -i|\uparrow\rangle\langle\downarrow| + i|\downarrow\rangle\langle\uparrow| \\ Z &= |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow| \end{aligned}\right\} \text{eigenvalues } \pm 1$$


in units of ½ħ
"singlet"
(s=0)
$|\Psi_-\rangle$
$(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$
S_n
last ti
this
Recall:
X
values

in units of $\frac{1}{2}\hbar$,

$$\boxed{S_n = \hat{n}\cdot\vec{\sigma}}$$

(X, Y, Z)

"singlet" $(s=0)$

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

last time: $\hbar = 1$

this time: $\frac{1}{2} = 1$ (sort of)

Recall: $S_x = \frac{1}{2}\hbar X$ Pauli

etc.

$$\left.\begin{aligned} X &= |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow| \\ Y &= -i|\uparrow\rangle\langle\downarrow| + i|\downarrow\rangle\langle\uparrow| \\ Z &= |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow| \end{aligned}\right\} \text{eigenvalues } \pm 1$$


in units of ½ħ
"singlet"
(s=0)
$|\Psi_-\rangle$
$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$
$S_n = \hat{n}\cdot\vec{\sigma}$
(X,Y,Z)
Recall:
X =
Y =
Z =

in units of $\frac{1}{2}\hbar$,

$$\boxed{S_n = \hat{n}\cdot\vec{\sigma}}$$

(X, Y, Z)

"singlet" $(s=0)$

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

last time: $\hbar = 1$

this time: $\frac{1}{2} = 1$ (sort of)

Recall: $S_x = \frac{1}{2}\hbar X$ Pauli

etc.

$$\left.\begin{aligned} X &= |\uparrow X \downarrow| + |\downarrow X \uparrow| \\ Y &= -i|\uparrow X \downarrow| + i|\downarrow X \uparrow| \\ Z &= |\uparrow X \uparrow| - |\downarrow X \downarrow| \end{aligned}\right\} \text{eigenvalues } \pm 1$$

in units of $\frac{1}{2}\hbar$

$$\boxed{S_n = \hat{n}\cdot\vec{\sigma}}$$

(X, Y, Z)

"singlet" $(s=0)$

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

last time: $\hbar = 1$

this time: $\frac{1}{2} = 1$ (sort of)

Recall: $S_x = \frac{1}{2}\hbar X$ Pauli

etc.

$$\left.\begin{aligned} X &= |\uparrow X \downarrow| + |\downarrow X \uparrow| \\ Y &= -i|\uparrow X \downarrow| + i|\downarrow X \uparrow| \\ Z &= |\uparrow X \uparrow| - |\downarrow X \downarrow| \end{aligned}\right\} \text{eigenvalues } \pm 1$$

in units of $\frac{1}{2}\hbar$

$$\boxed{S_n = \hat{n}\cdot\vec{\sigma}}$$

(X, Y, Z)

"singlet" $(s=0)$

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

last time: $\hbar = 1$

this time: $\frac{1}{2} = 1$ (sort of)

Recall: $S_x = \frac{1}{2}\hbar X$ ← etc.

Pauli

$$\left.\begin{aligned} X &= |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow| \\ Y &= -i|\uparrow\rangle\langle\downarrow| + i|\downarrow\rangle\langle\uparrow| \\ Z &= |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow| \end{aligned}\right\} \text{eigenvalues } \pm 1$$

For singlet:

$$S_n |\Psi_-\rangle = 0$$

$$\parallel$$

$$\left(S_n^{(1)} + S_n^{(2)}\right) |\Psi_-\rangle$$

For singlet:

$$S_n |\Psi_-\rangle = 0$$

$$\parallel$$

$$\left(S_n^{(1)} + S_n^{(2)}\right) |\Psi_-\rangle$$

Parallel measurements $\Longrightarrow$ opposite results

$|\Psi_-\rangle$ is <u>entangled</u>

For singlet:

$$S_n |\Psi_-\rangle = 0$$

$$\parallel$$

$$\left(S_n^{(1)} + S_n^{(2)}\right) |\Psi_-\rangle$$

Parallel measurements $\Longrightarrow$ opposite results

$|\Psi_-\rangle$ is <u>entangled</u>

For singlet:

$$S_n |\Psi_-\rangle = 0$$

$$\parallel$$

$$(S_n^{(1)} + S_n^{(2)}) |\Psi_-\rangle$$

Parallel measurements $\Rightarrow$ opposite results

$|\Psi_-\rangle$ is <u>entangled</u>

has <u>nice</u> <u>properties</u>

arises <u>naturally</u>

For singlet:

$$S_n |\Psi_-\rangle = 0$$

$$\shortparallel$$

$$(S_n^{(1)} + S_n^{(2)}) |\Psi_-\rangle$$

Parallel measurements $\Rightarrow$ opposite results

$|\Psi_-\rangle$ is <u>entangled</u>

has <u>nice properties</u>

arises <u>naturally</u>

is <u>nothing special</u>

EPR (1935)

For singlet:

$$S_n |\Psi_-\rangle = 0$$

$$\shortparallel$$

$$(S_n^{(1)} + S_n^{(2)}) |\Psi_-\rangle$$

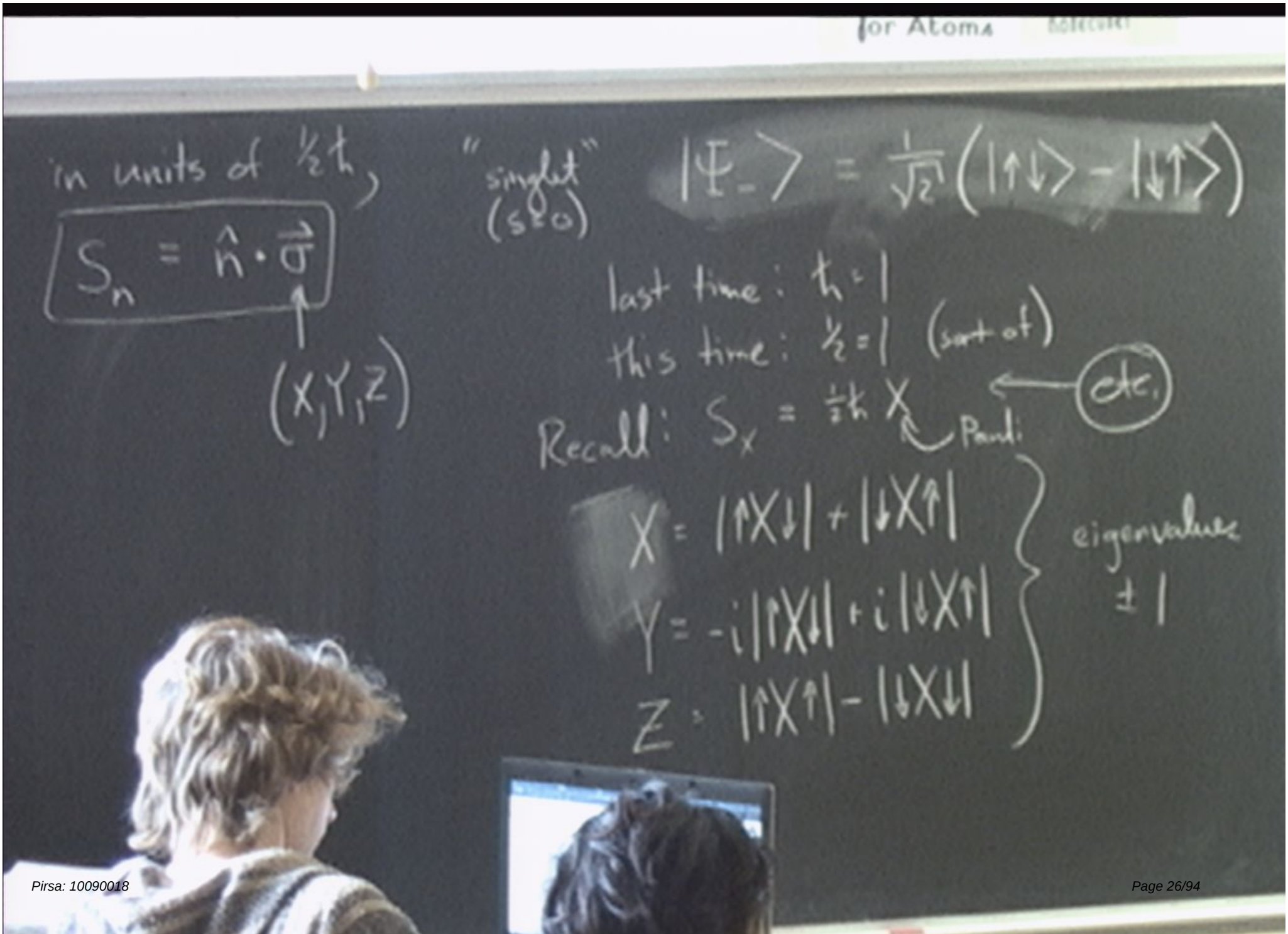
Parallel measurements $\Rightarrow$ opposite results

$|\Psi_-\rangle$ is <u>entangled</u>

has <u>nice properties</u>

arises <u>naturally</u>

is <u>nothing special</u>!


for Atoms
in units of ½ħ,
$S_n = \hat{n} \cdot \vec{\sigma}$
(X, Y, Z)
"singlet" (S=0)
$|\Psi_-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$
last time: ħ = 1
this time: ½ = 1 (sort of)
Recall: $S_x = \frac{1}{2}\hbar X$
Pauli
etc.
$X = |\uparrow X \downarrow| + |\downarrow X \uparrow|$
$Y = -i|\uparrow X \downarrow| + i|\downarrow X \uparrow|$
$Z = |\uparrow X \uparrow| - |\downarrow X \downarrow|$
eigenvalues ±1

EPR (1935)

criterion of reality

For singlet:

$$S_n |\Psi_-\rangle = 0$$

$$\parallel$$

$$(S_n^{(1)} + S_n^{(2)}) |\Psi_-\rangle$$

Parallel measurements $\Rightarrow$ opposite results

$|\Psi_-\rangle$ is <u>entangled</u>

has <u>nice properties</u>

arises <u>naturally</u>

is <u>nothing special</u>!

iterion of reality

$\boxed{|\Psi_-\rangle} \rightsquigarrow$ #2

For singlet:

$$S_n |\Psi_-\rangle = 0$$

$$\shortparallel$$

$$(S_n^{(1)} + S_n^{(2)}) |\Psi_-\rangle$$

Parallel measurements $\Longrightarrow$ opposite results

$|\Psi_-\rangle$ is <u>entangled</u>

has <u>nice</u> <u>properties</u>

arises <u>naturally</u>

is <u>nothing</u> <u>special</u>!

EPR (1935)

criterion of reality

#1 ⟵ $|\Psi_-\rangle$ ⟶ #2

Measure $X^{(1)}$ ⟹

For singlet:

$$S_n|\Psi_-\rangle = 0$$

$$\| $$

$$(S_n^{(1)} + S_n^{(2)})|\Psi_-\rangle$$

Parallel measurements ⟹ opposite results

$|\Psi_-\rangle$ is entangled

has nice properties

arises naturally

is nothing special!

EPR (1935)

criterion of reality

#1 ⟵ $\boxed{|\Psi_-\rangle}$ ⟶ #2

Measure $X^{(1)}$ ⟹ Predict $X^{(2)}$ result with P=1

Measure $Z^{(1)}$ ⟹ Predict $Z^{(2)}$ result with P=1

For singlet:

$$S_n|\Psi_-\rangle = 0$$

$$\| $$

$$(S_n^{(1)} + S_n^{(2)})|\Psi_-\rangle$$

Parallel measurements ⟹ opposite results

$|\Psi_-\rangle$ is entangled

has nice properties

arises naturally

is nothing special!

EPR (1935)

criterion of reality

#1 $\leftsquigarrow \boxed{|\Psi_-\rangle} \rightsquigarrow$ #2

Measure $X^{(1)} \Longrightarrow$ Predict $X^{(2)}$ result with $P=1$

Measure $Z^{(1)} \Longrightarrow$ Predict $Z^{(2)}$ result with $P=1$

For singlet:

$$S_n |\Psi_-\rangle = 0$$

$$\shortparallel$$

$$(S_n^{(1)} + S_n^{(2)}) |\Psi_-\rangle$$

Parallel measurements $\Longrightarrow$ opposite results

$|\Psi_-\rangle$ is entangled

has nice properties

arises naturally

is nothing special!

in units of $\frac{1}{2}\hbar$

$$\boxed{S_n = \hat{n}\cdot\vec{\sigma}}$$

(X, Y, Z)

"singlet" $(s=0)$

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

last time: $\hbar = 1$

this time: $\frac{1}{2} = 1$ (sort of)

Recall: $S_x = \frac{1}{2}\hbar X$ ← etc.

Pauli

$$\left.\begin{aligned} X &= |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow| \\ Y &= -i|\uparrow\rangle\langle\downarrow| + i|\downarrow\rangle\langle\uparrow| \\ Z &= |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow| \end{aligned}\right\} \text{eigenvalues } \pm 1$$

EPR (1935)

criterion of reality

#1 ⟵ $\boxed{|\Psi_-\rangle}$ ⟶ #2

Measure $X^{(1)}$ ⟹ Predict $X^{(2)}$ result with P=1

Measure $Z^{(1)}$ ⟹ Predict $Z^{(2)}$ result with P=1

For singlet:

$$S_n|\Psi_-\rangle = 0$$

$$\| $$

$$(S_n^{(1)} + S_n^{(2)})|\Psi_-\rangle$$

Parallel measurements ⟹ opposite results

$|\Psi_-\rangle$ is entangled

has nice properties

arises naturally

is nothing special!

Assumptions

L (locality) = choice of #1 measurement does not affect #2 outcomes

QM (quantum mech.) = QM correctly predicts correlations in $|\Psi\rangle$

Assumptions

L (locality) = choice of #1 measurement does not affect #2 outcomes

QM (quantum mech.) = QM correctly predicts correlations in $|\Psi_-\rangle$

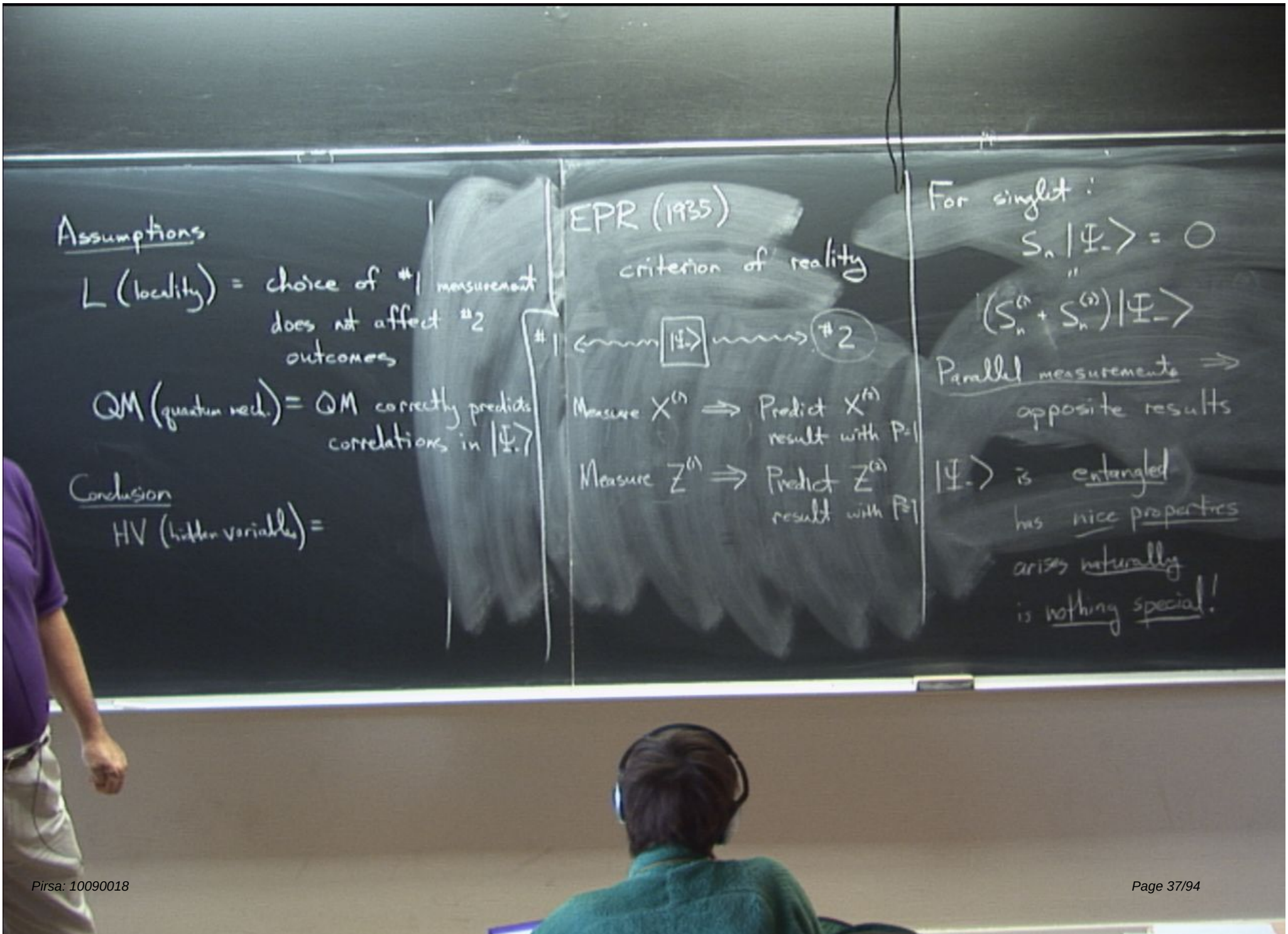
Assumptions

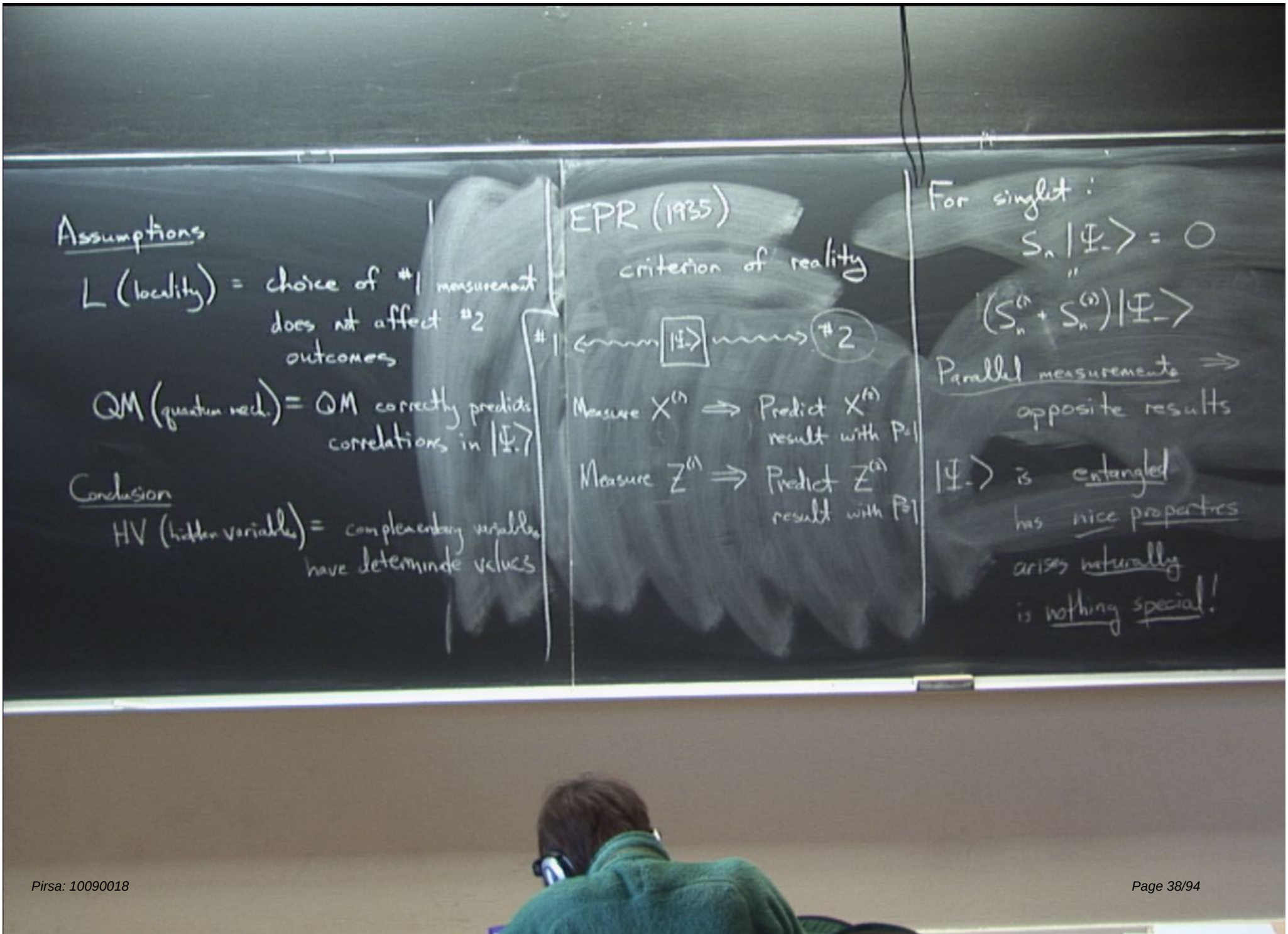
L (locality) = choice of #1 measurement does not affect #2 outcomes

QM (quantum mech.) = QM correctly predicts correlations in $|\Psi_-\rangle$

Conclusion

HV (hidden variables) =


Assumptions
L (locality) = choice of #1 measurement does not affect #2 outcomes
QM (quantum mech.) = QM correctly predicts correlations in |Ψ-⟩
Conclusion
HV (hidden variables) =
EPR (1935)
criterion of reality
#1 |Ψ-⟩ #2
Measure X(1) ⇒ Predict X(2) result with P=1
Measure Z(1) ⇒ Predict Z(2) result with P=1
For singlet:
Sn |Ψ-⟩ = 0
(Sn(1) + Sn(2)) |Ψ-⟩
Parallel measurements ⇒ opposite results
|Ψ-⟩ is entangled
has nice properties
arises naturally
is nothing special!


Assumptions
L (locality) = choice of #1 measurement does not affect #2 outcomes
QM (quantum mech.) = QM correctly predicts correlations in |Ψ-⟩
Conclusion
HV (hidden variables) = complementary variables have determinate values
EPR (1935)
criterion of reality
#1 |Ψ-⟩ #2
Measure X(1) ⇒ Predict X(2) result with P=1
Measure Z(1) ⇒ Predict Z(2) result with P=1
For singlet:
Sn |Ψ-⟩ = 0
(Sn(1) + Sn(2)) |Ψ-⟩
Parallel measurements ⇒ opposite results
|Ψ-⟩ is entangled
has nice properties
arises naturally
is nothing special!

$L + QM$

$\Downarrow$ EPR

HV

Assumptions

L (locality) = choice of #1 measurement does not affect #2 outcomes

QM (quantum mech.) = QM correctly predicts correlations in $|\Psi_-\rangle$

Conclusion

HV (hidden variables) = complementary variables have determinate values

#1

$$L + QM \xRightarrow{\text{EPR}} HV$$

Assumptions

L (locality) = choice of #1 measurement does not affect #2 outcomes

QM (quantum mech.) = QM correctly predicts correlations in $|\Psi_-\rangle$

Conclusion

HV (hidden variables) = complementary variables have determinate values

surement

#2

predicts

in $|\Psi_-\rangle$

variables

values

EPR (1935)

criterion of reality

#1 $\leftarrow\!\sim\!\sim\!\sim$ $\boxed{|\Psi_-\rangle}$ $\sim\!\sim\!\sim\!\rightarrow$ #2

Measure $X^{(1)} \Rightarrow$ Predict $X^{(2)}$ result with $P=1$

Measure $Z^{(1)} \Rightarrow$ Predict $Z^{(2)}$ result with $P=1$

For singlet:

$$S_n |\Psi_-\rangle = 0$$

$$\parallel$$

$$(S_n^{(1)} + S_n^{(2)}) |\Psi_-\rangle$$

Parallel measurements $\Rightarrow$ opposite results

$|\Psi_-\rangle$ is entangled

has nice properties

arises naturally

is nothing special!

L + QM

$\Downarrow$ EPR

HV

Assumptions

L (locality) = choice of #1 measurem
does not affect #2
outcomes

QM (quantum mech.) = QM correctly pred
correlations in 1!

Conclusion

HV (hidden variables) = complementary varial
have determinate valu

L + QM

$\Downarrow$ EPR

HV

Assumptions

L (locality) = choice of #1 measureme
does not affect #2
outcomes

QM (quantum mech.) = QM correctly pred
correlations in 1!

Conclusion

HV (hidden variables) = complementary varial
have determinate valu

$$L + QM \xRightarrow{\text{EPR}} HV$$

Assumptions

L (locality) = choice of #1 measurem
does not affect #2
outcomes

QM (quantum mech.) = QM correctly pred
correlations in 1

Conclusion

HV (hidden variables) = complementary varial
have determinate valu

L + QM

$\Downarrow$ EPR

HV

Assumptions

L (locality) = choice of #1 measurem
does not affect #2
outcomes

QM (quantum mech.) = QM correctly pred
correlations in |ψ⟩

Conclusion

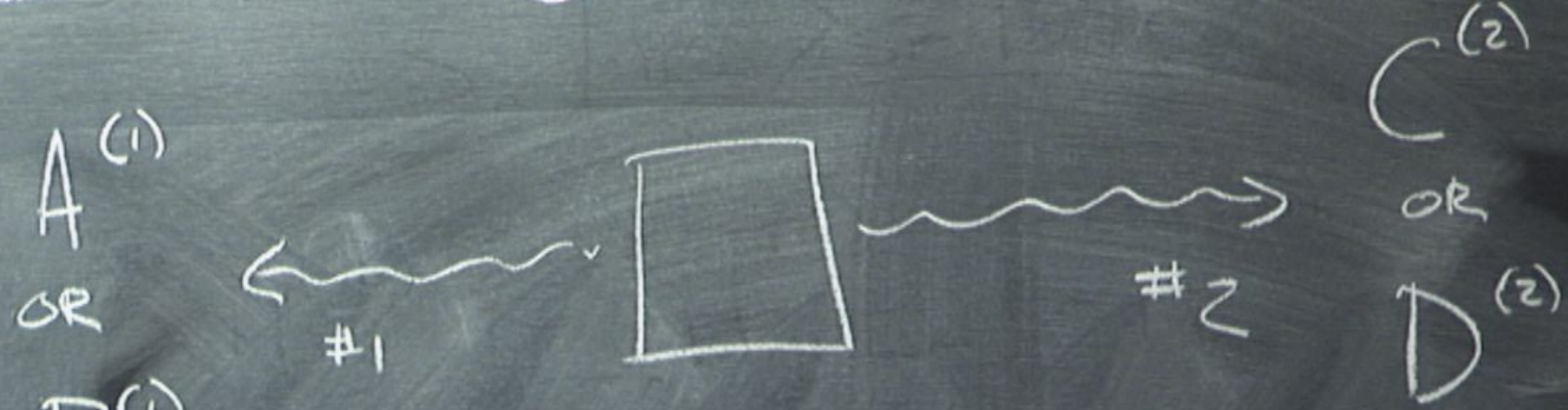
HV (hidden variables) = complementary variables
have determinate value

Bell Imagine a L+HV world

(all results are ± 1)

(CHSH)

Bell Imagine a L+HV world



(all results are ± 1)

(CHSH)

Bell Imagine a L+HV world

(all results are ± 1)

$$AC, AD, BC, BD = \pm$$

(CHSH)

Bell Imagine a L+HV world

$A^{(1)}$ OR $B^{(1)}$ ← #1 [box] #2 → $C^{(2)}$ OR $D^{(2)}$

(all results are ± 1)

Measure $AC, AD, BC, BD = \pm 1$

(CHSH)

Bell Imagine a L+HV world

(all results are ± 1)

Measure $AC, AD, BC, BD = \pm 1$

(CHSH)

Bell Imagine a L+HV world

(all results are ± 1)

Measure $AC, AD, BC, BD = \pm 1$

LHV: All of those have determinate values

$$M = A(C+D) + B(C-D)$$
$$= AC + AD + BC - BD$$

Bell (CHSH) Imagine a L+HV world

(all results are ± 1)

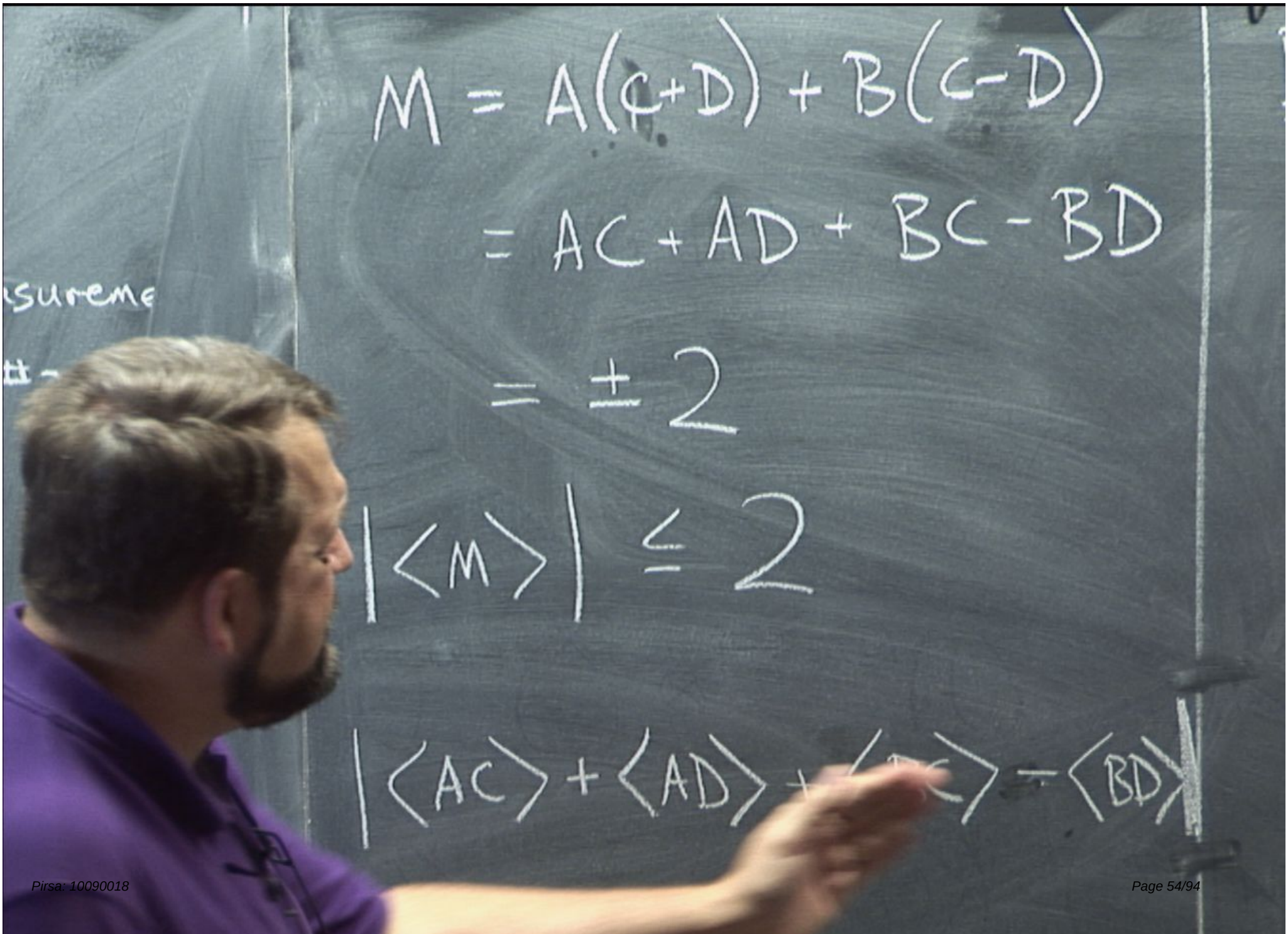
Measure $AC, AD, BC, BD = \pm 1$

LHV: All of those have determinate values

$$M = A(C+D) + B(C-D)$$

$$= AC + AD + BC - BD$$

$$= \pm 2$$


M = A(C+D) + B(C-D)
= AC + AD + BC - BD
= ± 2
|⟨M⟩| ≤ 2
|⟨AC⟩ + ⟨AD⟩ + ⟨BC⟩ - ⟨BD⟩|

$$M = A(C+D) + B(C-D)$$

$$= AC + AD + BC - BD$$

$$= \pm 2$$

$$|\langle M \rangle| \leq 2$$

$$|\langle AC \rangle + \langle AD \rangle + \langle BC \rangle - \langle BD \rangle|$$

#1 measureme

ffect #2

rrectly pred

tions in |ψ

lementary variables

erminate value

$$M = A(C+D) + B(C-D)$$

$$= AC + AD + BC - BD$$

$$= \pm 2$$

$$|\langle M \rangle| \leq 2$$

$$\boxed{|\langle AC \rangle + \langle AD \rangle + \langle BC \rangle - \langle BD \rangle| \leq 2}$$

CHSH ≠

(a type of Bell ≠)

(CHSH

Bell Ima

$A^{(1)}$

OR

$B^{(1)}$

#1

(all results

Measure

LHV: All of

$$M = A(C+D) + B(C-D)$$
$$= AC + AD + BC - BD$$
$$= \pm 2$$
$$|\langle M \rangle| \leq 2$$

$$\boxed{|\langle AC \rangle + \langle AD \rangle + \langle BC \rangle - \langle BD \rangle| \leq 2}$$

CHSH ≠
(a type of Bell ≠)

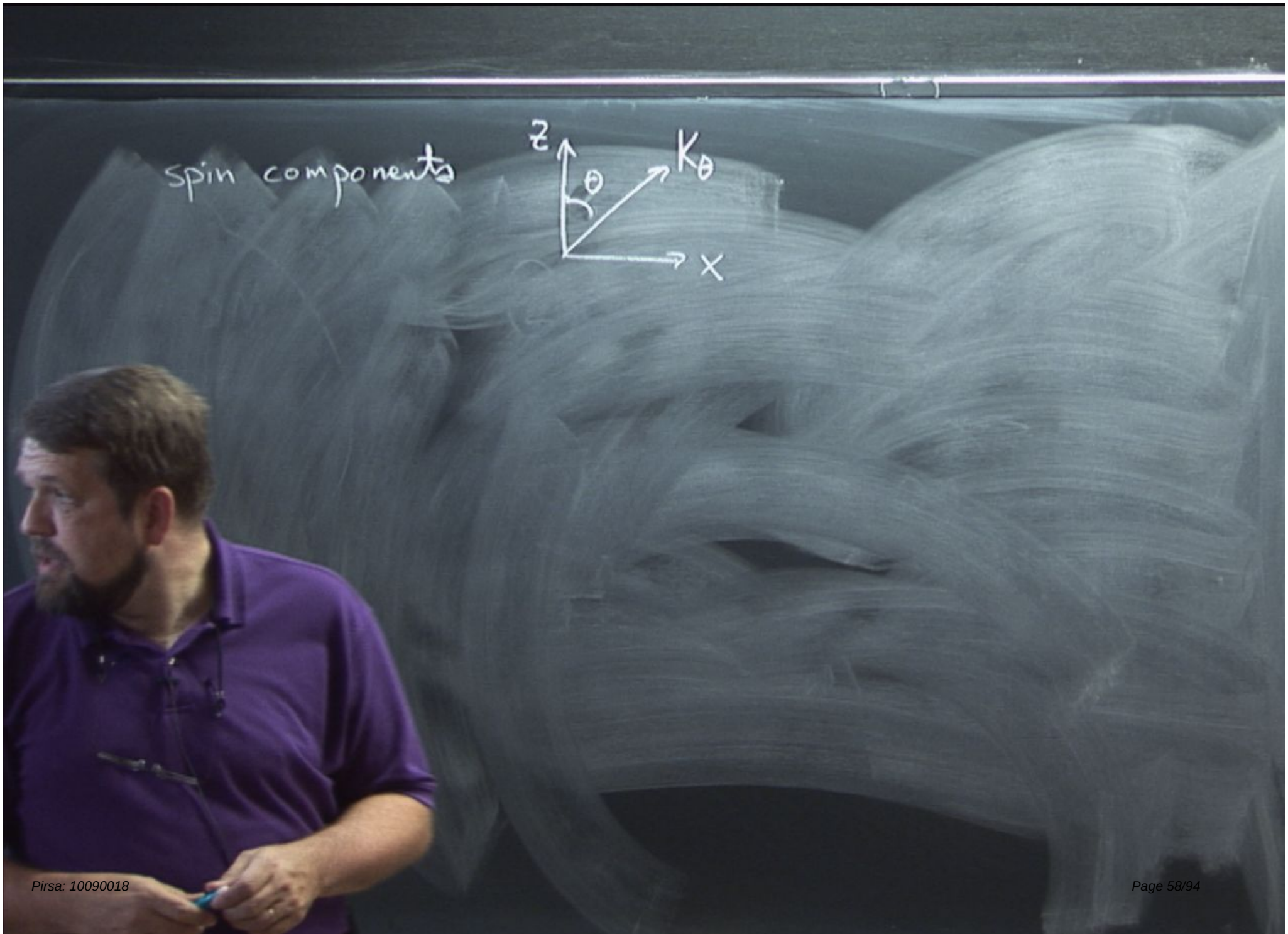
(CHSH)

Bell Imagine a L+HV world

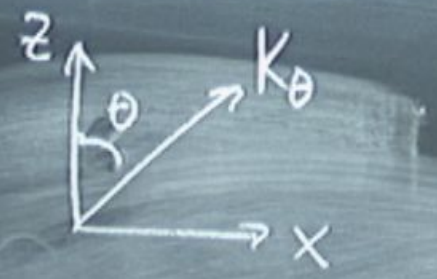
(all results are ± 1)

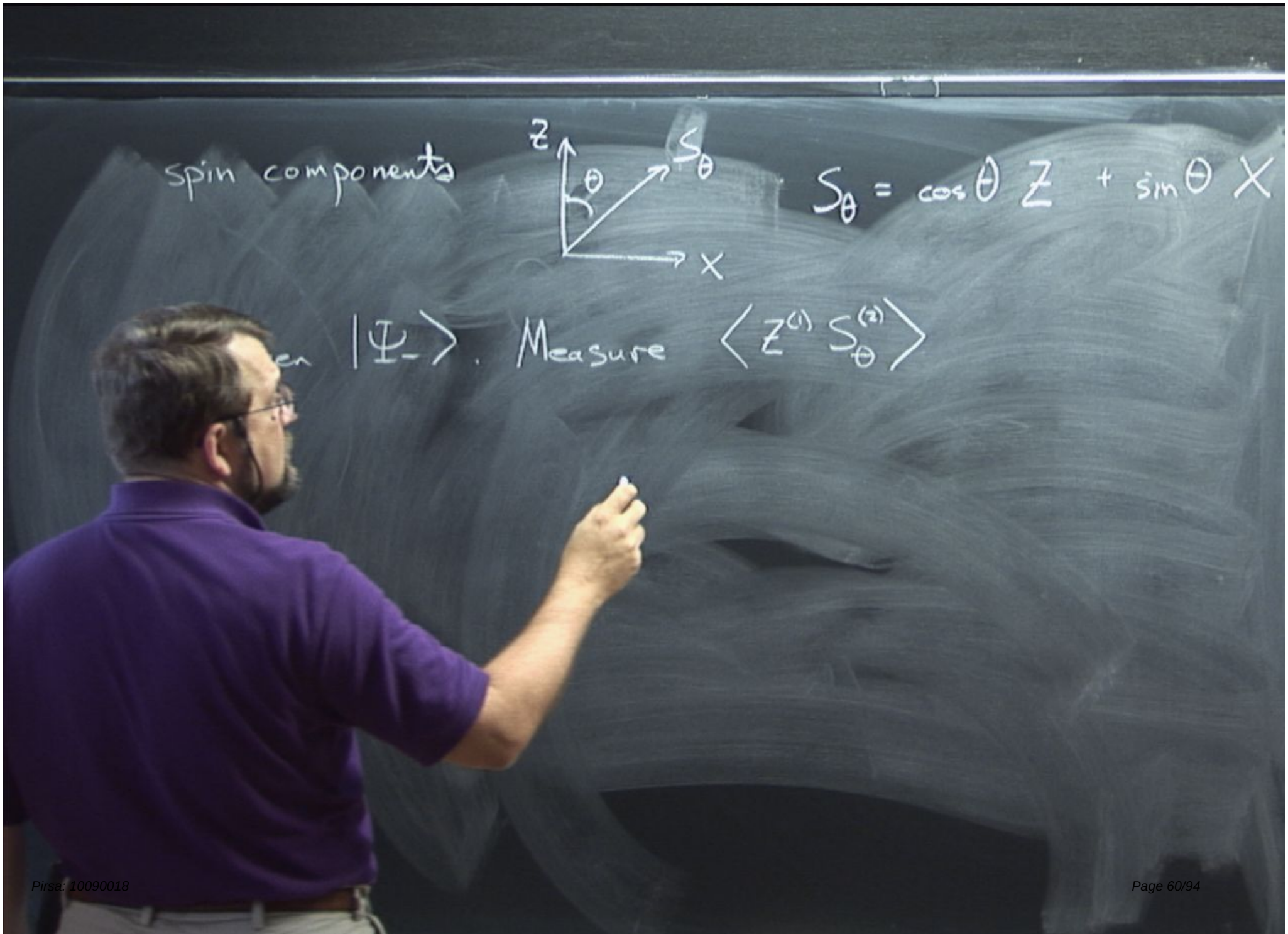
Measure $AC, AD, BC, BD = \pm 1$

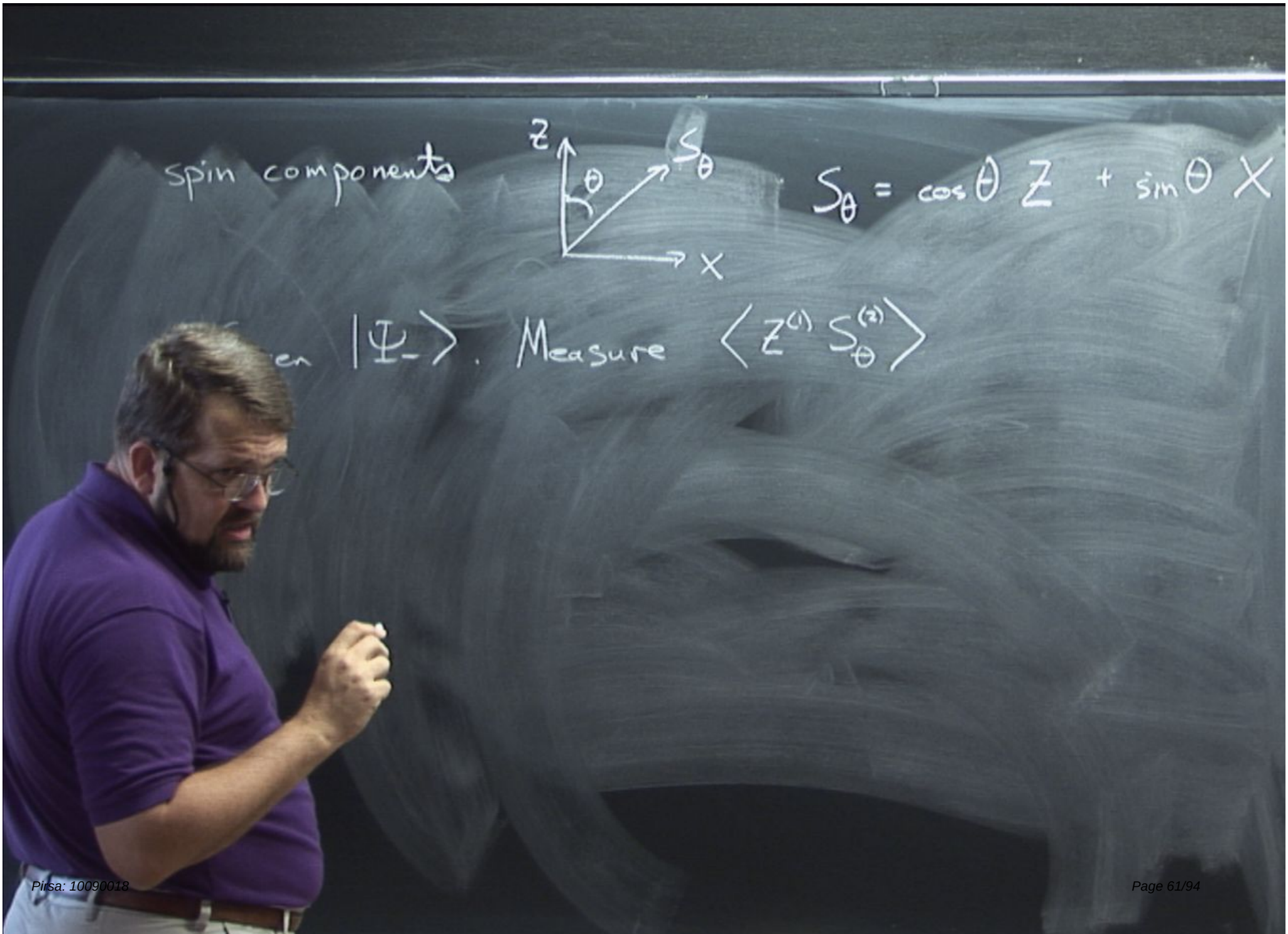
LHV: All of those have determinate values

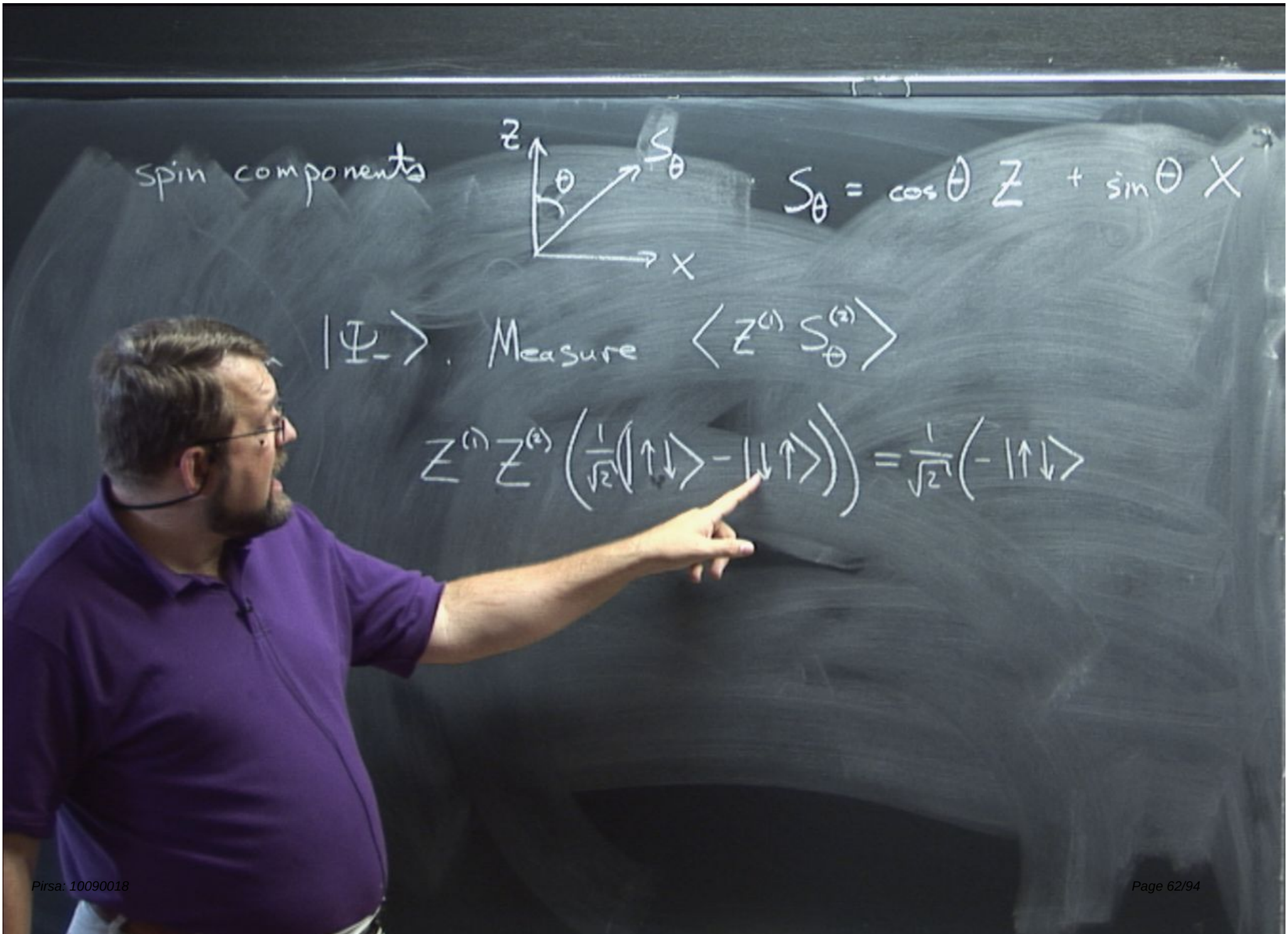

spin components
z
θ
K_θ
x

spin components

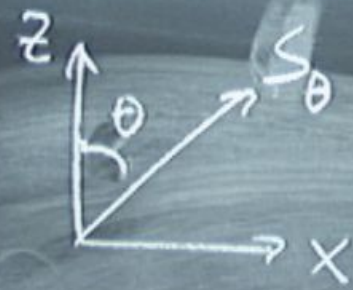



spin components
z
θ
S_θ
x
$S_\theta = \cos\theta\, Z + \sin\theta\, X$
en $|\Psi_-\rangle$. Measure $\langle Z^{(1)} S_\theta^{(2)} \rangle$


spin components
z
θ
S_θ
x
$S_\theta = \cos\theta\, Z + \sin\theta\, X$
en $|\Psi_-\rangle$. Measure $\langle Z^{(1)} S_\theta^{(2)} \rangle$


spin components
z
θ
S_θ
x
$S_\theta = \cos\theta\, Z + \sin\theta\, X$
$|\Psi_-\rangle$. Measure $\langle Z^{(1)} S_\theta^{(2)} \rangle$
$Z^{(1)} Z^{(2)} \left(\frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \right) = \frac{1}{\sqrt{2}} \left(- |\uparrow\downarrow\rangle \right.$

spin components



$$S_\theta = \cos\theta\, Z + \sin\theta\, X$$

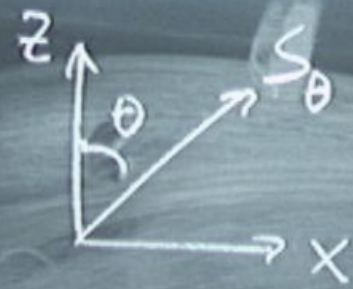
Given $|\Psi_-\rangle$. Measure $\langle Z^{(1)} S_\theta^{(2)} \rangle$

$$Z^{(1)} Z^{(2)} \left(\frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \right) = \frac{1}{\sqrt{2}} \left(-|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right)$$

$$= -|\Psi_-\rangle$$

$$Z^{(1)} X^{(2)} \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) =$$

spin components



$$S_\theta = \cos\theta\, Z + \sin\theta\, X$$

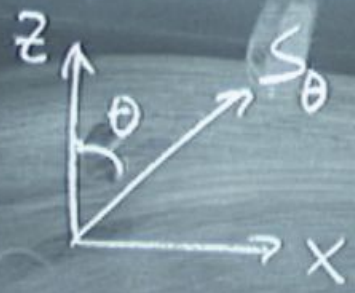
Given $|\Psi_-\rangle$. Measure $\langle Z^{(1)} S_\theta^{(2)} \rangle$

$$Z^{(1)} Z^{(2)} \left(\frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \right) = \frac{1}{\sqrt{2}} \left(-|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right)$$

$$= -|\Psi_-\rangle$$

$$Z^{(1)} X^{(2)} \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) =$$

spin components



$$S_\theta = \cos\theta\, Z + \sin\theta\, X$$

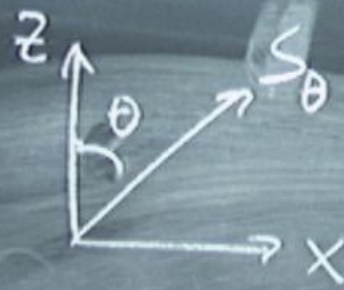
Given $|\Psi_-\rangle$. Measure $\langle Z^{(1)} S_\theta^{(2)} \rangle$

$$Z^{(1)} Z^{(2)} \left(\frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \right) = \frac{1}{\sqrt{2}} \left(- |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right)$$

$$= - |\Psi_-\rangle$$

$$Z^{(1)} X^{(2)} \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) =$$

spin components



$$S_\theta = \cos\theta\, Z + \sin\theta\, X$$

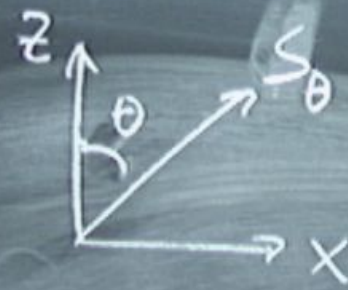
$|\Psi_-\rangle$. Measure $\langle Z^{(1)} S_\theta^{(2)} \rangle$

$$Z^{(1)} Z^{(2)} \left(\frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \right) = \frac{1}{\sqrt{2}} \left(-|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right)$$

$$= -|\Psi_-\rangle$$

$$Z^{(1)} X^{(2)} \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) = \frac{1}{\sqrt{2}} \left(|\uparrow\uparrow\rangle \right.$$

spin components



$$S_\theta = \cos\theta\, Z + \sin\theta\, X$$

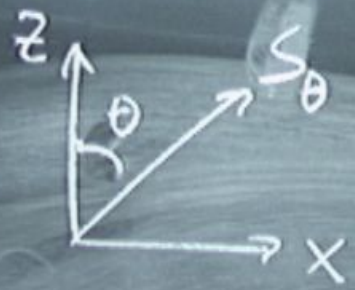
$|\Psi_-\rangle$. Measure $\langle Z^{(1)} S_\theta^{(2)} \rangle$

$$Z^{(1)} Z^{(2)} \left(\frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \right) = \frac{1}{\sqrt{2}} \left(-|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right)$$

$$= -|\Psi_-\rangle$$

$$Z^{(1)} X^{(2)} \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) = \frac{1}{\sqrt{2}} \left(|\uparrow\uparrow\rangle \right.$$

spin components



$$S_\theta = \cos\theta\, Z + \sin\theta\, X$$

Given $|\Psi_-\rangle$. Measure $\langle Z^{(1)} S_\theta^{(2)} \rangle$

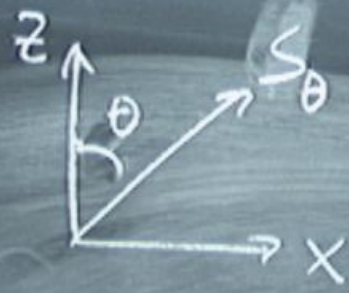
$$Z^{(1)} Z^{(2)} \left(\frac{1}{\sqrt{2}} \; \rangle \right) = \frac{1}{\sqrt{2}} \left(- |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right)$$

$$= - |\Psi_-\rangle$$

$$\downarrow\uparrow\rangle) = \underbrace{\frac{1}{\sqrt{2}} \left(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle \right)}_{|\Phi_+\rangle}$$

$$\langle \Psi_- | \Phi_+ \rangle = 0$$

spin components



$$S_\theta = \cos\theta\, Z + \sin\theta\, X$$

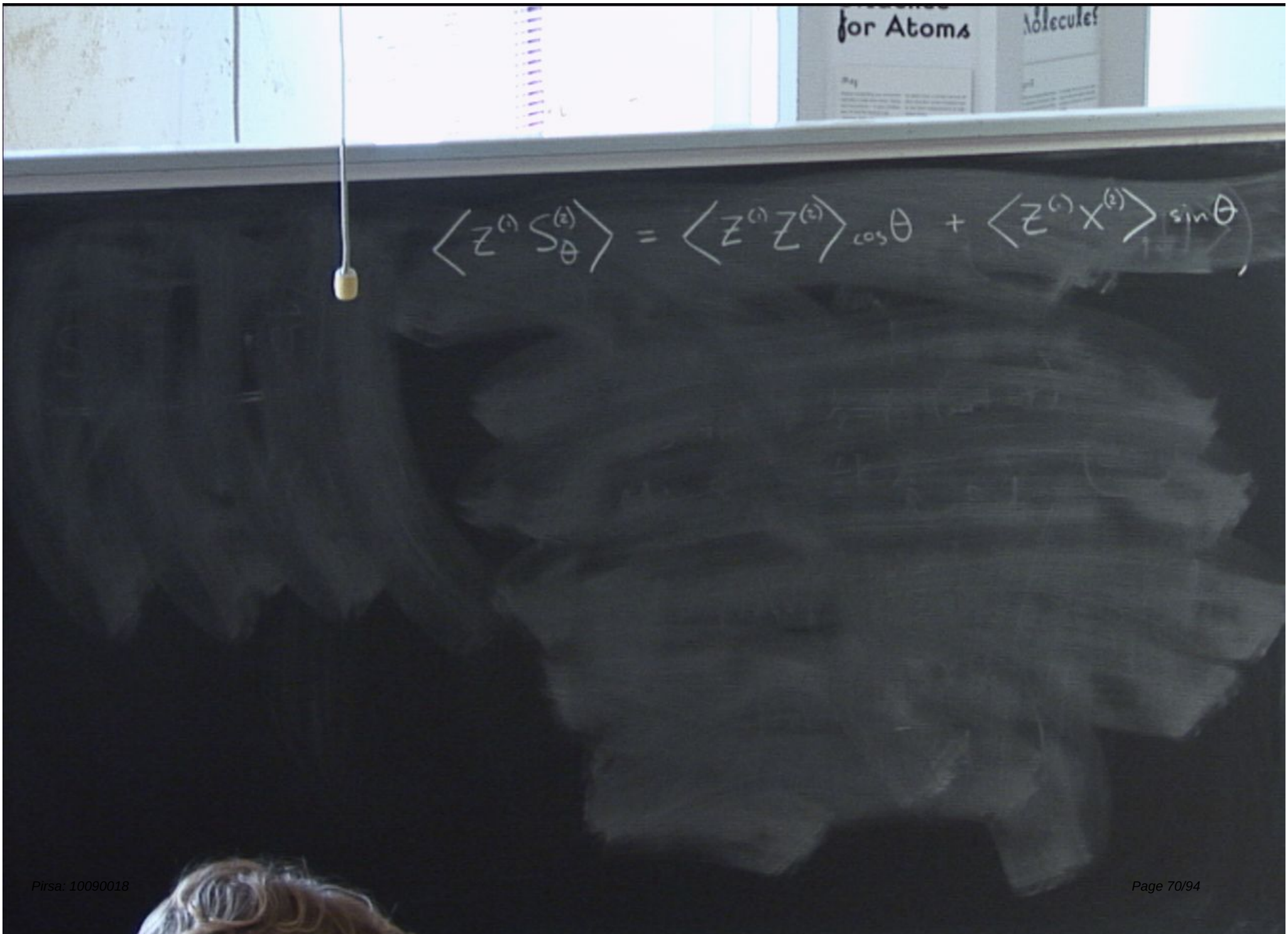
Given $|\Psi_-\rangle$. Measure $\langle Z^{(1)} S_\theta^{(2)} \rangle$

$$Z^{(1)} Z^{(2)} \left(\frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \right) = \frac{1}{\sqrt{2}} \left(-|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right)$$

$$= -|\Psi_-\rangle$$

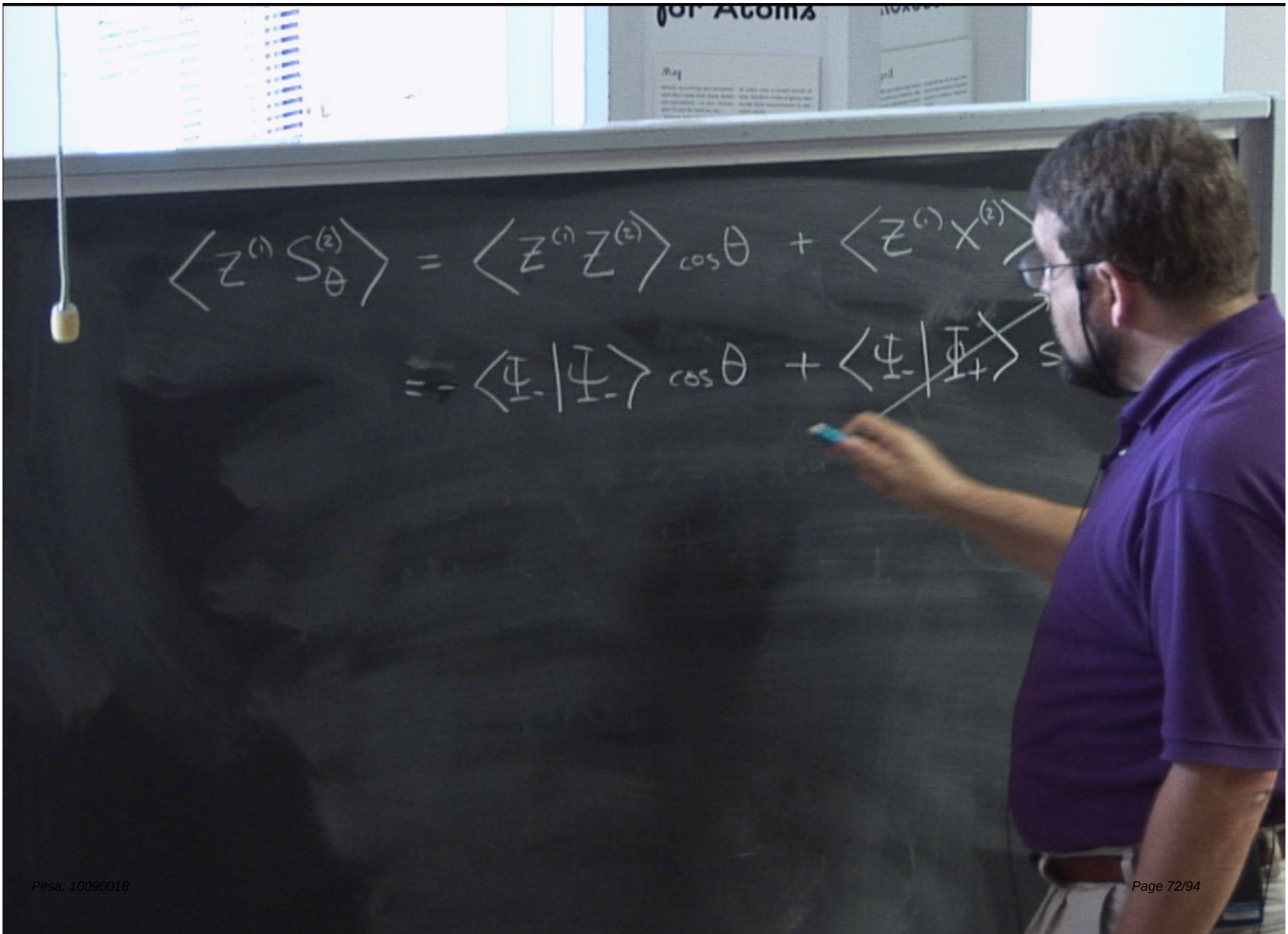
$$Z^{(1)} X^{(2)} \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) = \underbrace{\frac{1}{\sqrt{2}} \left(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle \right)}_{|\Phi_+\rangle}$$

$$\langle \Psi_- | \Phi_+ \rangle = 0$$


for Atoms
$\langle Z^{(1)} S_\theta^{(2)} \rangle = \langle Z^{(1)} Z^{(2)} \rangle \cos\theta + \langle Z^{(1)} X^{(2)} \rangle \sin\theta$

$$\langle Z^{(1)} S_\theta^{(2)} \rangle = \langle Z^{(1)} Z^{(2)} \rangle \cos\theta + \langle Z^{(1)} X^{(2)} \rangle \sin\theta$$

$$= -\langle \Psi_- | \Psi_- \rangle \cos\theta + \langle \Psi_- |$$


$\langle Z^{(1)} S_\theta^{(2)} \rangle = \langle Z^{(1)} Z^{(2)} \rangle \cos\theta + \langle Z^{(1)} X^{(2)} \rangle$
$= -\langle \Psi_- | \Psi_- \rangle \cos\theta + \langle \Psi_- | \Psi_+ \rangle$

$$\langle Z^{(1)} S_\theta^{(2)} \rangle = \langle Z^{(1)} Z^{(2)} \rangle \cos\theta + \langle Z^{(1)} X^{(2)} \rangle \sin\theta$$

$$= -\langle \Psi_- | \Psi_- \rangle \cos\theta + \cancel{\langle \Psi_- | \Psi_+ \rangle} \sin\theta$$

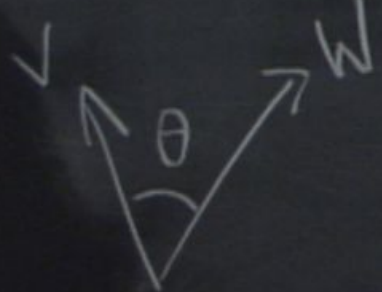
$$\boxed{\langle Z^{(1)} S_\theta^{(2)} \rangle = -\cos\theta}$$

$$\langle Z^{(1)} S_\theta^{(2)} \rangle = \langle Z^{(1)} Z^{(2)} \rangle \cos\theta + \langle Z^{(1)} X^{(2)} \rangle \sin\theta$$

$$= -\langle \Psi_- | \Psi_- \rangle \cos\theta + \cancel{\langle \Psi_- | \Phi_+ \rangle} \sin\theta$$

$$\boxed{\langle Z^{(1)} S_\theta^{(2)} \rangle = -\cos\theta}$$

$|\Psi_-\rangle$ is rotationally invariant!



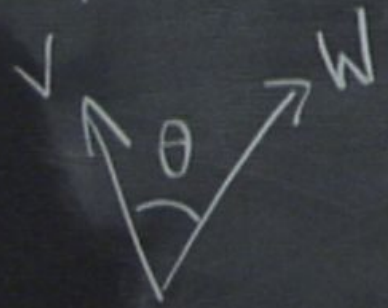
$$\Rightarrow \boxed{\langle V^{(1)} W^{(2)} \rangle = -\cos\theta}$$

$$\langle Z^{(1)} S_\theta^{(2)} \rangle = \langle Z^{(1)} Z^{(2)} \rangle \cos\theta + \langle Z^{(1)} X^{(2)} \rangle \sin\theta$$

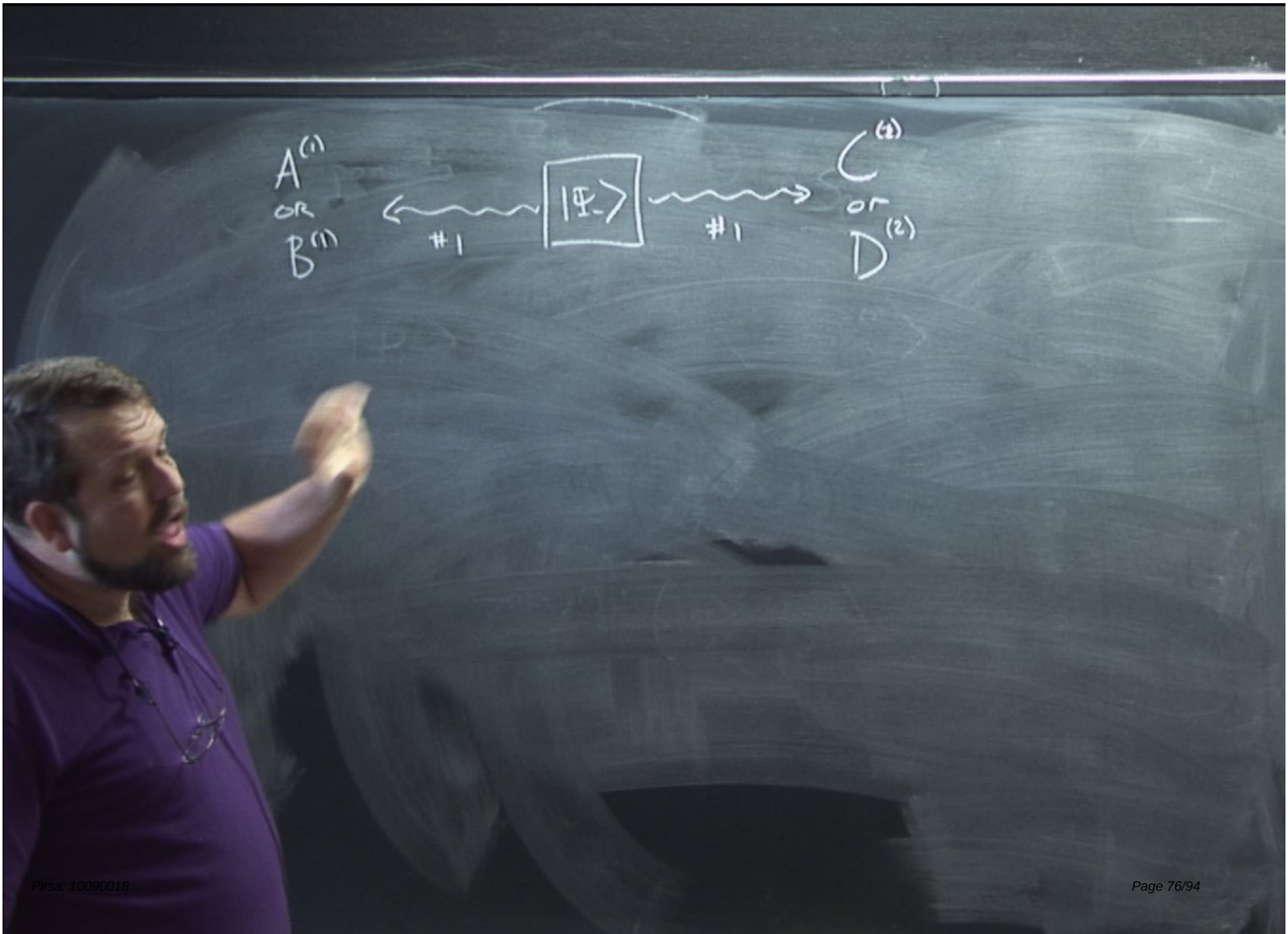
$$= -\langle \Psi_- | \Psi_- \rangle \cos\theta + \cancel{\langle \Psi_- | \Phi_+ \rangle}^{\,0} \sin\theta$$

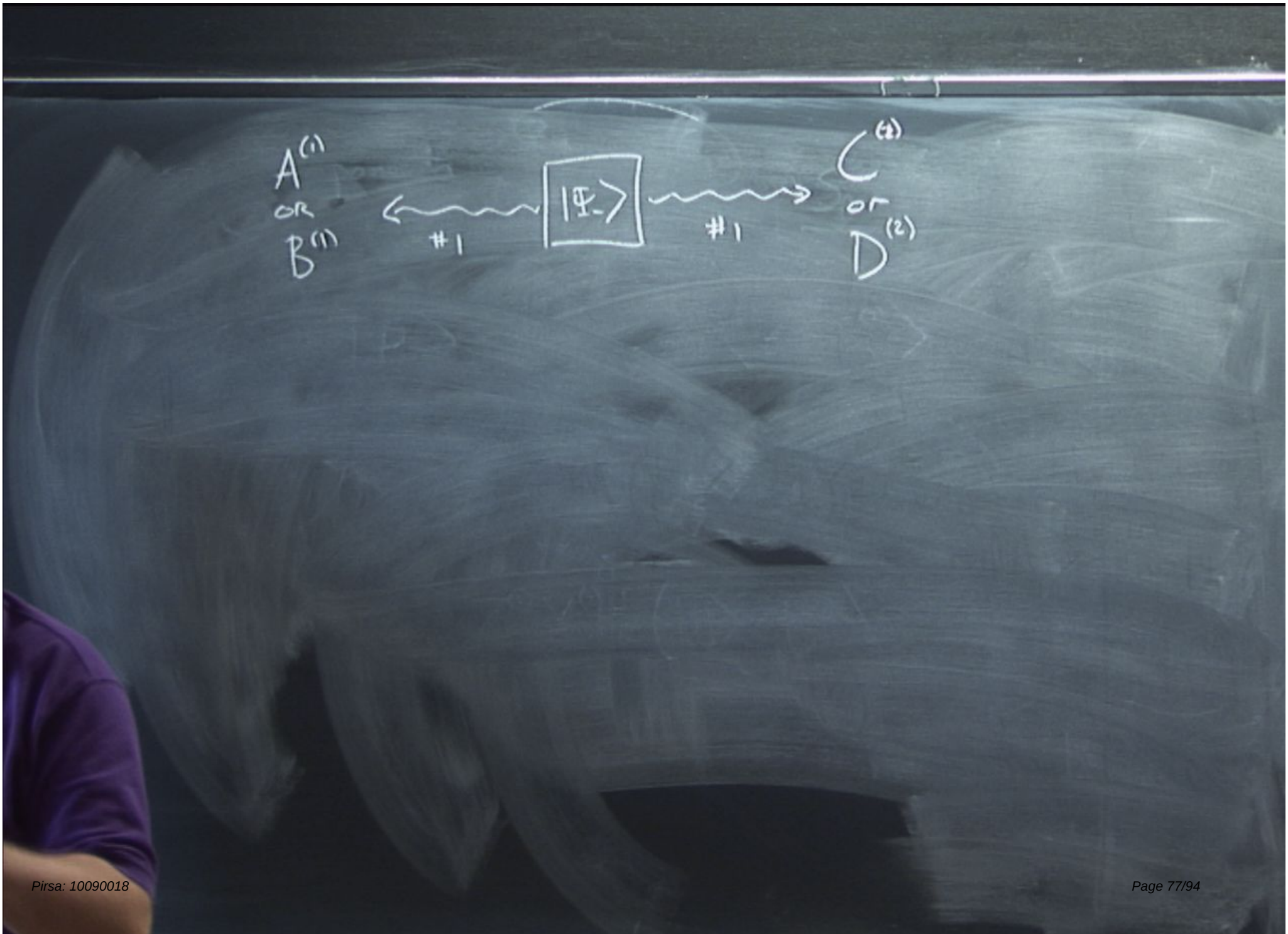
$$\boxed{\langle Z^{(1)} S_\theta^{(2)} \rangle = -\cos\theta}$$

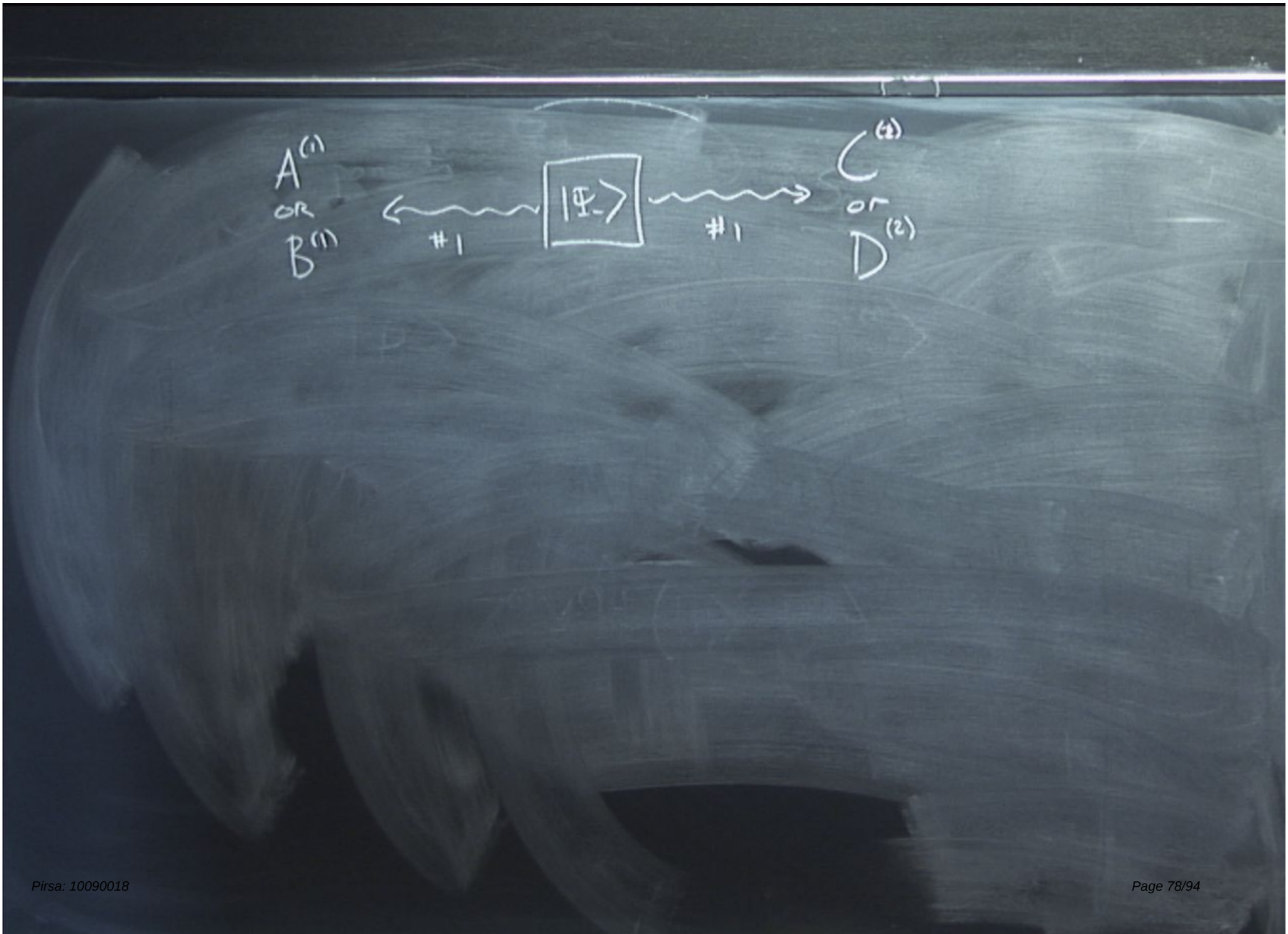
$|\Psi_-\rangle$ is rotationally invariant!

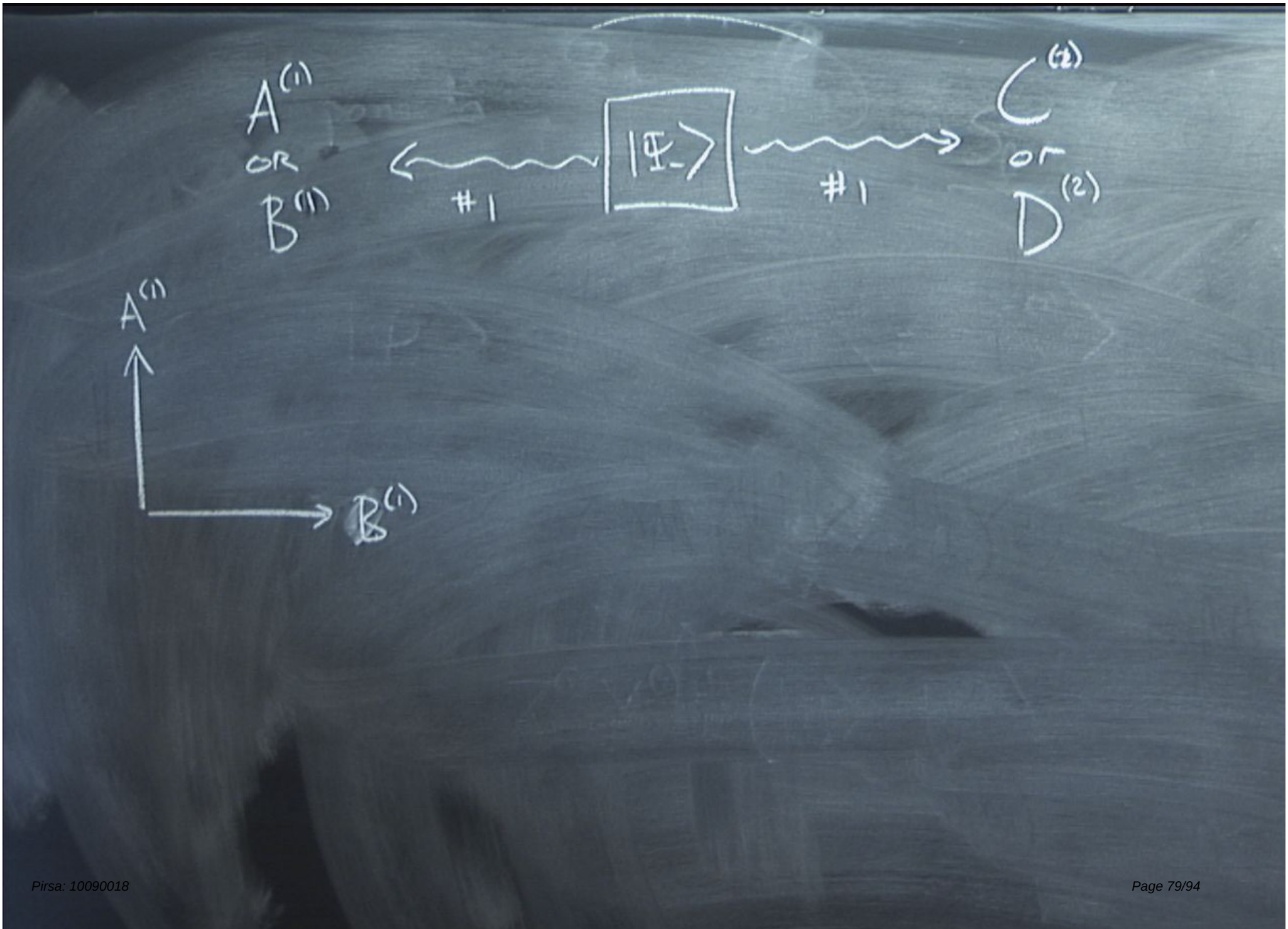


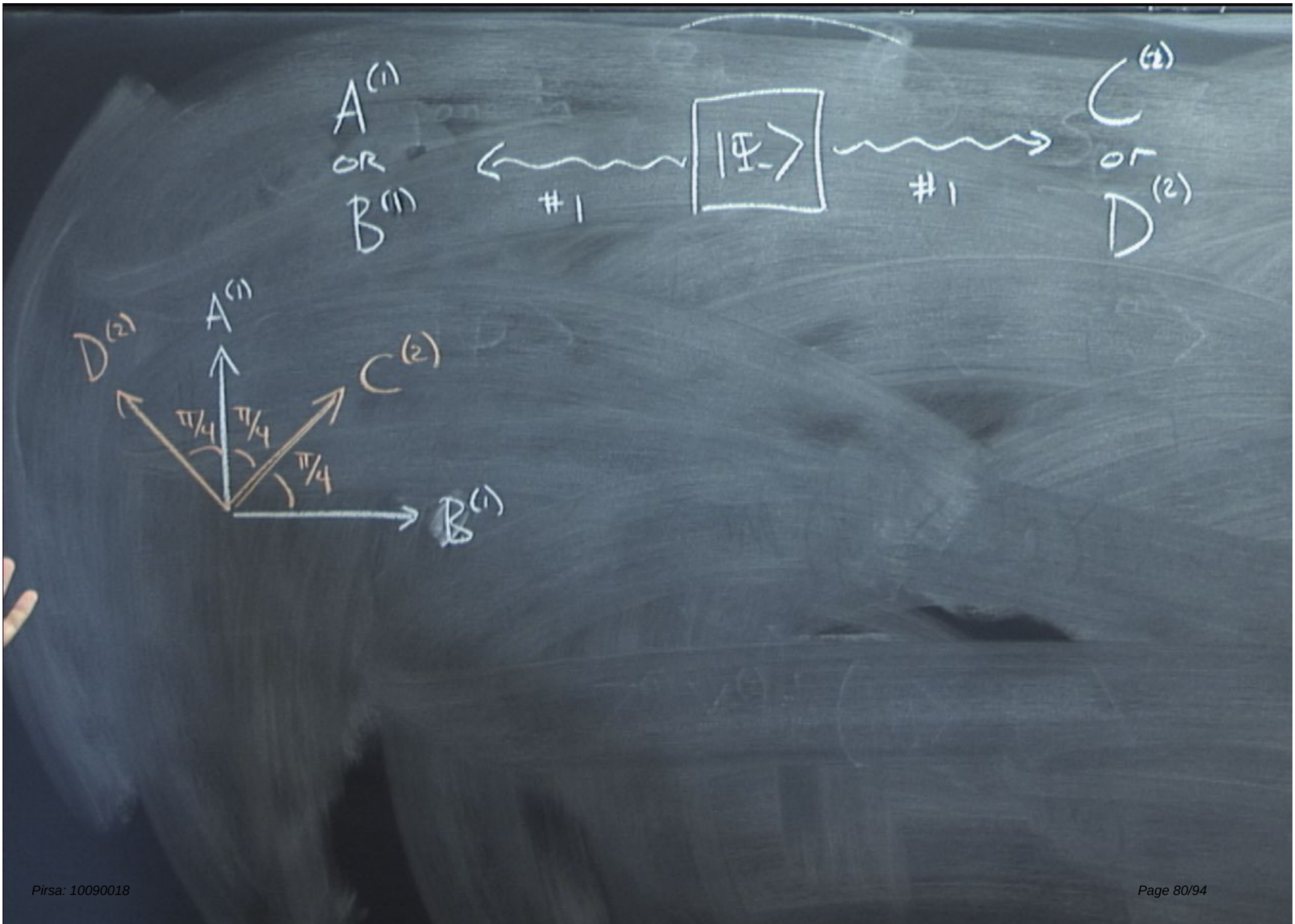
$$\Rightarrow \boxed{\langle V^{(1)} W^{(2)} \rangle = -\cos\theta}$$

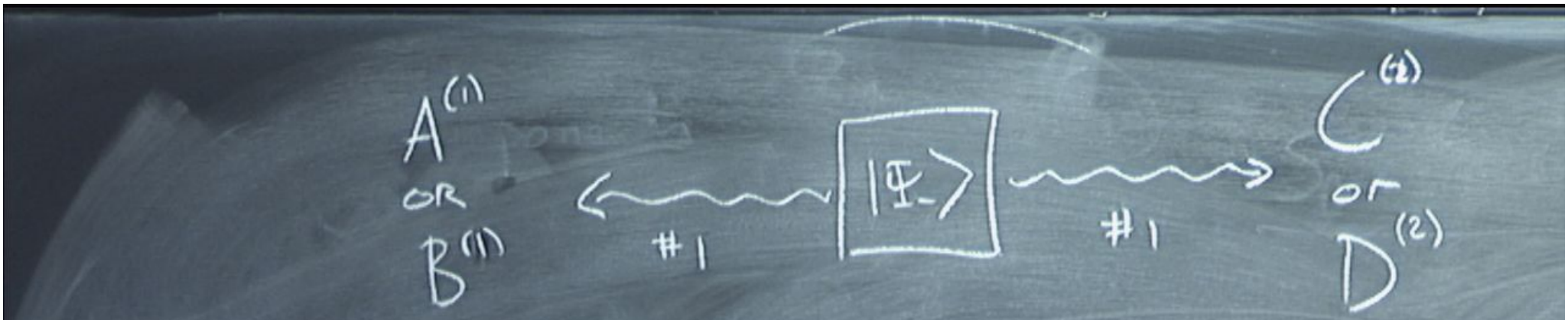

$A^{(1)}$
OR
$B^{(1)}$
#1
$|\Phi_-\rangle$
#1
$C^{(2)}$
OR
$D^{(2)}$

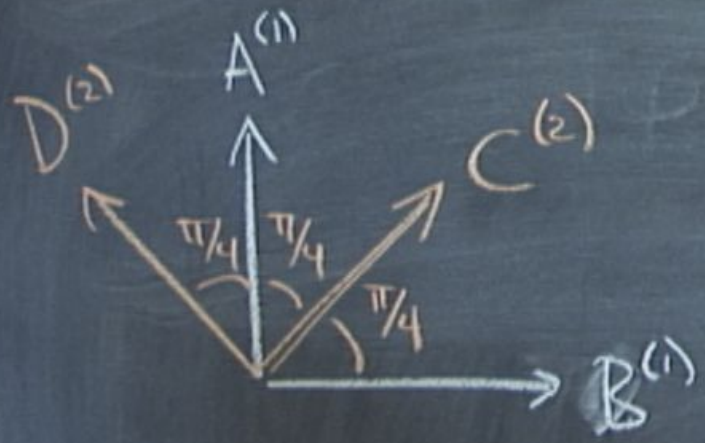

$A^{(1)}$
OR
$B^{(1)}$
#1
$|\Phi_-\rangle$
#1
$C^{(2)}$
OR
$D^{(2)}$

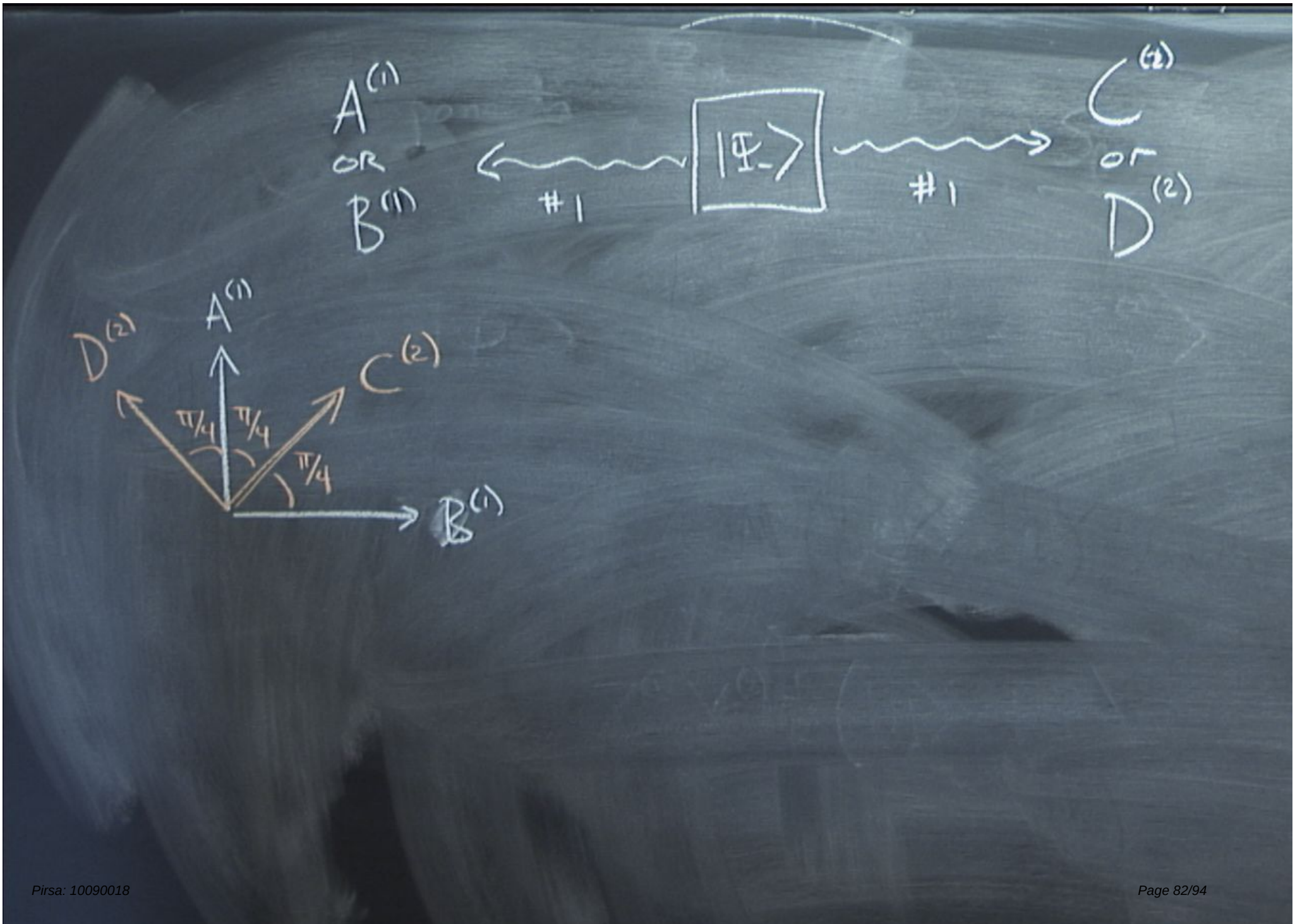

$A^{(1)}$
OR
$B^{(1)}$
#1
$|\Phi_-\rangle$
#1
$C^{(2)}$
OR
$D^{(2)}$

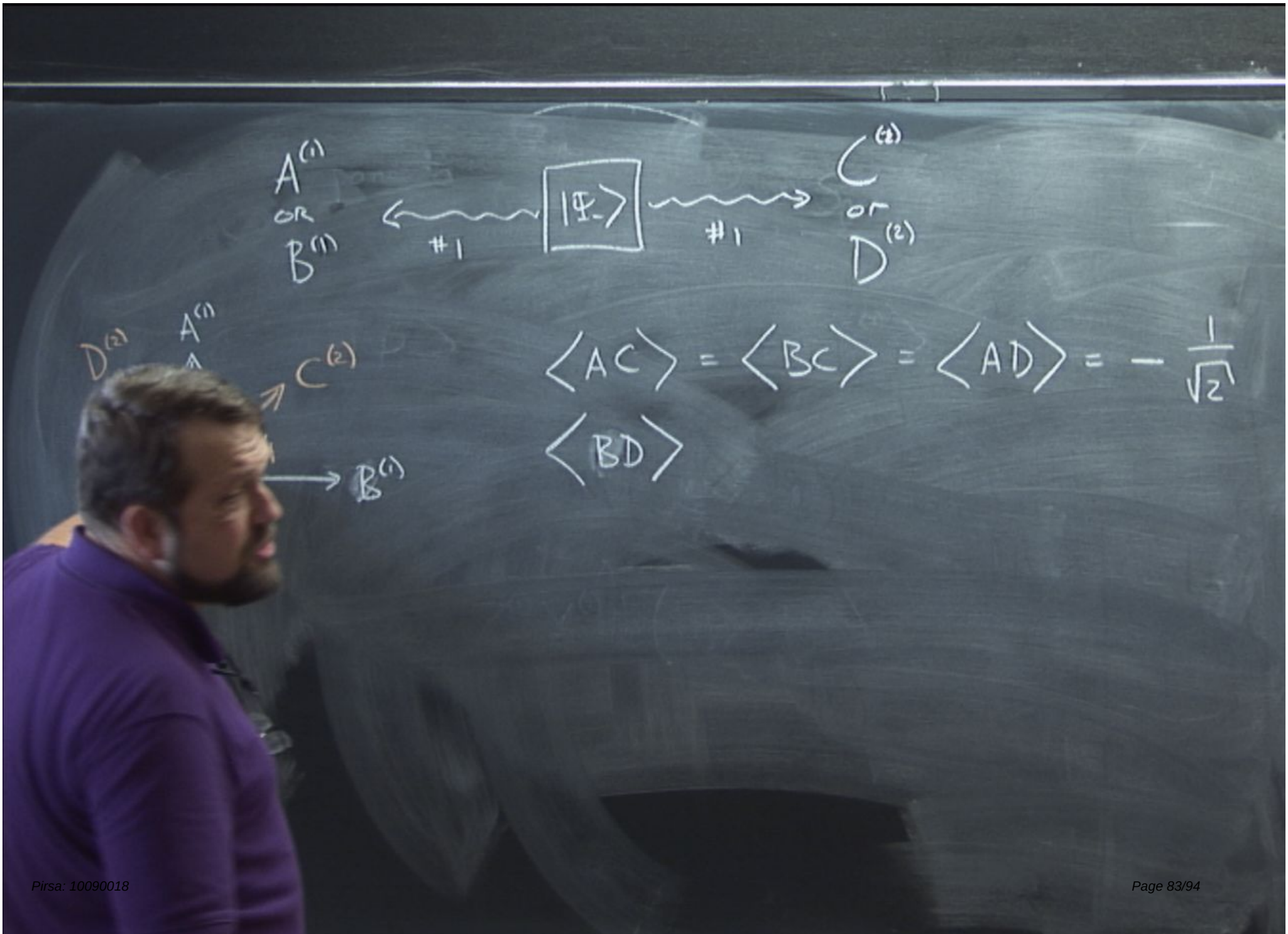

$A^{(1)}$
OR
$B^{(1)}$
#1
$|\Psi_-\rangle$
#1
$C^{(2)}$
OR
$D^{(2)}$
$A^{(1)}$
$B^{(1)}$


A(1)
OR
B(1)
#1
|Ψ-⟩
#1
C(2)
OR
D(2)
D(2)
A(1)
C(2)
π/4
π/4
π/4
B(1)


$A^{(1)}$
OR
$B^{(1)}$
#1
$|\Psi_-\rangle$
#1
$C^{(2)}$
OR
$D^{(2)}$


$D^{(2)}$
$A^{(1)}$
$C^{(2)}$
$\pi/4$
$\pi/4$
$\pi/4$
$B^{(1)}$


A(1)
OR
B(1)
#1
|Ψ-⟩
#1
C(2)
OR
D(2)
D(2)
A(1)
C(2)
π/4
π/4
π/4
B(1)

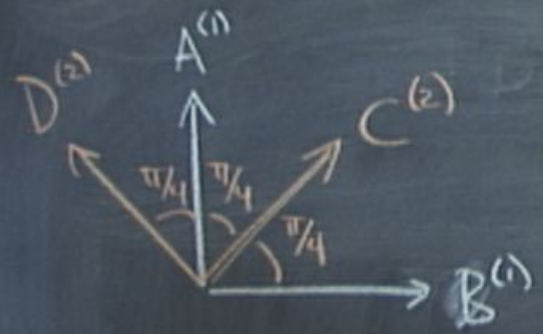

A(1)
OR
B(1)
#1
|Ψ-⟩
#1
C(2)
OR
D(2)
D(2)
A(1)
C(2)
B(1)
⟨AC⟩ = ⟨BC⟩ = ⟨AD⟩ = − 1/√2
⟨BD⟩

$A^{(1)}$ or $B^{(1)}$ $\xleftarrow{\#1}$ $\boxed{|\Psi_-\rangle}$ $\xrightarrow{\#1}$ $C^{(2)}$ or $D^{(2)}$

$$\langle AC \rangle = \langle BC \rangle = \langle AD \rangle = -\frac{1}{\sqrt{2}}$$

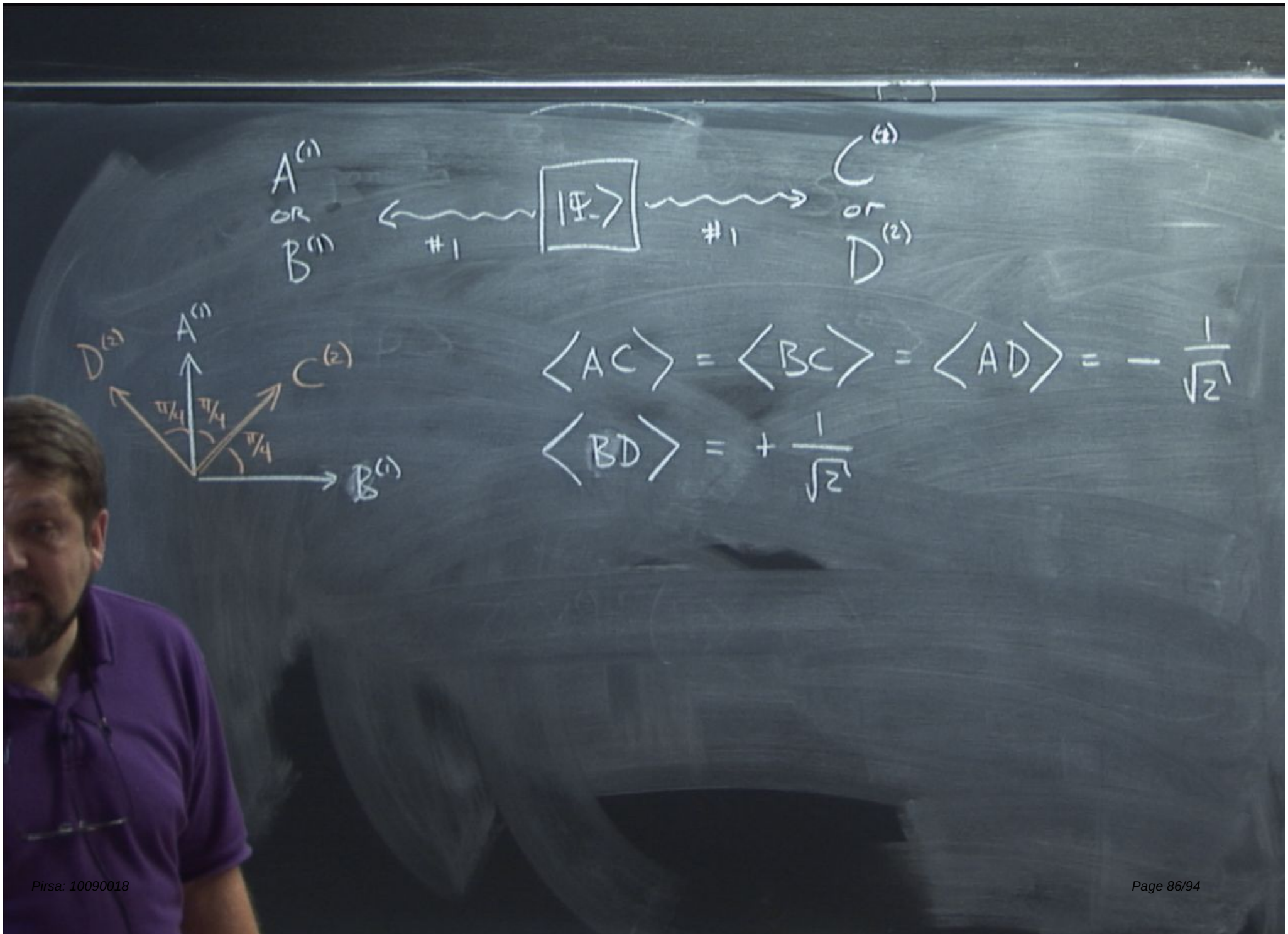
$$\langle BD \rangle$$

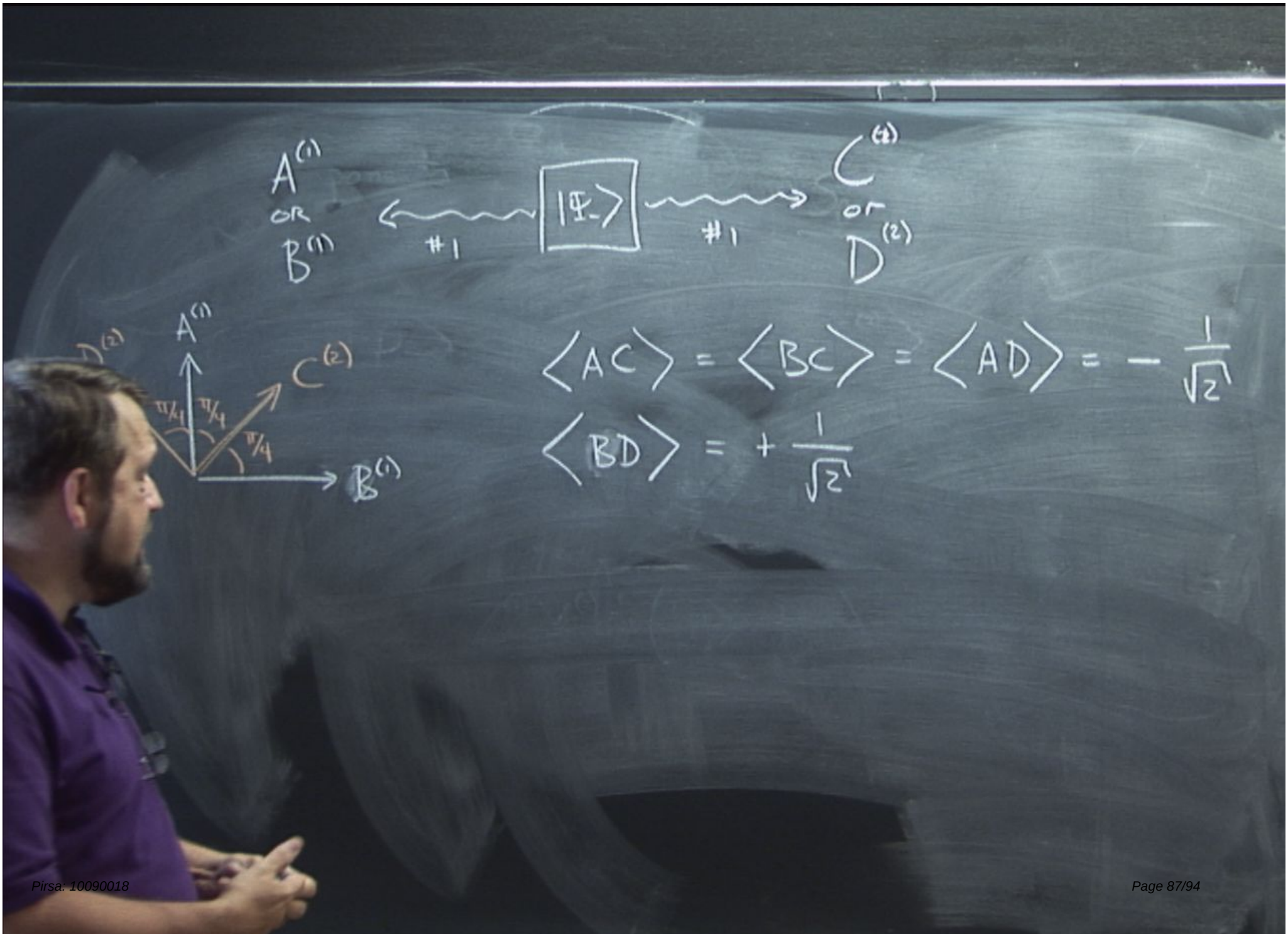
$A^{(1)}$ or $B^{(1)}$ $\xleftarrow{\#1}$ $\boxed{|\Psi_-\rangle}$ $\xrightarrow{\#1}$ $C^{(2)}$ or $D^{(2)}$



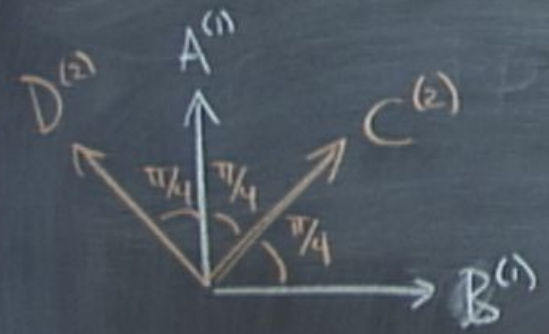
$$\langle AC \rangle = \langle BC \rangle = \langle AD \rangle = -\frac{1}{\sqrt{2}}$$

$$\langle BD \rangle$$


A(1) OR B(1)
#1
|Ψ-⟩
#1
C(2) OR D(2)
D(2)
A(1)
C(2)
π/4
π/4
π/4
B(1)
⟨AC⟩ = ⟨BC⟩ = ⟨AD⟩ = − 1/√2
⟨BD⟩ = + 1/√2


$A^{(1)}$ OR $B^{(1)}$
#1
$|\Psi_-\rangle$
#1
$C^{(2)}$ OR $D^{(2)}$
$A^{(1)}$
$C^{(2)}$
$D^{(2)}$
$B^{(1)}$
$\pi/4$
$\pi/4$
$\pi/4$
$\langle AC \rangle = \langle BC \rangle = \langle AD \rangle = -\frac{1}{\sqrt{2}}$
$\langle BD \rangle = +\frac{1}{\sqrt{2}}$

$$A^{(1)} \text{ OR } B^{(1)} \xleftarrow{\#1} \boxed{|\Psi_-\rangle} \xrightarrow{\#1} C^{(2)} \text{ OR } D^{(2)}$$



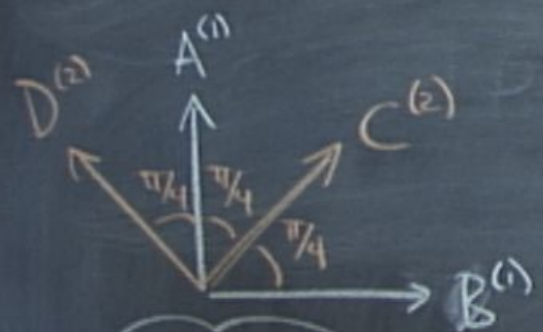
$$\langle AC \rangle = \langle BC \rangle = \langle AD \rangle = -\frac{1}{\sqrt{2}}$$

$$\langle BD \rangle = +\frac{1}{\sqrt{2}}$$

$$\boxed{\langle AC \rangle + \langle AD \rangle + \langle BC \rangle - \langle BD \rangle = -2\sqrt{2}}$$

QM!

$A^{(1)}$ or $B^{(1)}$ $\xleftarrow{\#1}$ $\boxed{|\Psi_-\rangle}$ $\xrightarrow{\#1}$ $C^{(2)}$ or $D^{(2)}$



$$\langle AC\rangle = \langle BC\rangle = \langle AD\rangle = -\frac{1}{\sqrt{2}}$$

$$\langle BD\rangle = +\frac{1}{\sqrt{2}}$$

L + HV

$\Downarrow$ Bell

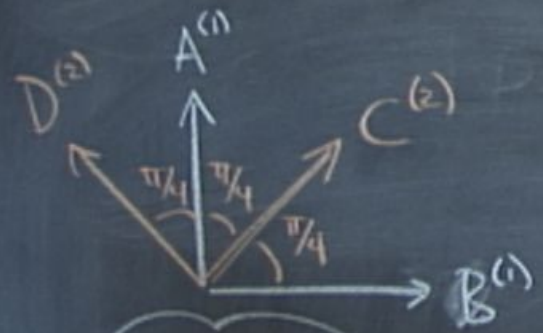
~~QM~~

$$\boxed{\langle AC\rangle + \langle AD\rangle + \langle BC\rangle - \langle BD\rangle = -2\sqrt{2}}$$

QM can <u>violate</u> CHSH $\neq$!!

QM!

$A^{(1)}$ or $B^{(1)}$ $\xleftarrow{\#1}$ $\boxed{|\Psi_-\rangle}$ $\xrightarrow{\#1}$ $C^{(2)}$ or $D^{(2)}$



$$\langle AC\rangle = \langle BC\rangle = \langle AD\rangle = -\frac{1}{\sqrt{2}}$$

$$\langle BD\rangle = +\frac{1}{\sqrt{2}}$$

L + HV

$\Downarrow$ Bell

$\cancel{\text{QM}}$

$$\boxed{\langle AC\rangle + \langle AD\rangle + \langle BC\rangle - \langle BD\rangle = -2\sqrt{2}}$$

QM can <u>violate</u> CHSH $\neq$!!

QM!

$\#1 \leftarrow \boxed{|\psi\rangle} \rightarrow \#1$ $\quad C^{(1)}$ or $D^{(1)}$

$B^{(1)}$

$$\langle AC\rangle = \langle BC\rangle = \langle AD\rangle = -\frac{1}{\sqrt{2}}$$

$$\langle BD\rangle = +\frac{1}{\sqrt{2}}$$

$$\boxed{\langle AC\rangle + \langle AD\rangle + \langle BC\rangle - \langle BD\rangle = -2\sqrt{2}}$$

QM can violate CHSH ≠ !!

QM!

$$M = A(C+D) + B(C-D)$$
$$= AC + AD + BC - BD$$
$$= \pm 2$$
$$|\langle M\rangle| \leq 2$$

$$\boxed{|\langle AC\rangle + \langle AD\rangle + \langle BC\rangle - \langle BD\rangle| \leq 2}$$

CHSH ≠
(a type of Bell ≠)

Bell (CHSH) Imagine a L+H

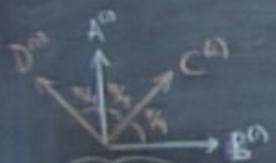
$A^{(1)}$ or $B^{(1)}$ $\leftarrow \#1 \boxed{} \rightarrow$

(all results are ± 1)

Measure AC, AD, BC

LHV: All of these have d

$A^{(1)}$ or $B^{(1)}$ ← #1 $|\psi\rangle$ #1 → $C^{(1)}$ or $D^{(1)}$



L + HV

⇓ Bell

~~QM~~

$$\langle AC\rangle = \langle BC\rangle = \langle AD\rangle = -\frac{1}{\sqrt{2}}$$

$$\langle BD\rangle = +\frac{1}{\sqrt{2}}$$

$$\boxed{\langle AC\rangle + \langle AD\rangle + \langle BC\rangle - \langle BD\rangle = -2\sqrt{2}}$$

QM can violate CHSH ≠ !!

QM!

Bell's thm.

$$M = A(C+D) + B(C-D)$$
$$= AC + AD + BC - BD$$
$$= \pm 2$$
$$|\langle M\rangle| \leq 2$$

$$\boxed{|\langle AC\rangle + \langle AD\rangle + \langle BC\rangle - \langle BD\rangle| \leq 2}$$

CHSH ≠
(a type of Bell ≠)

Bell (CHSH) Imagine a L+HV world

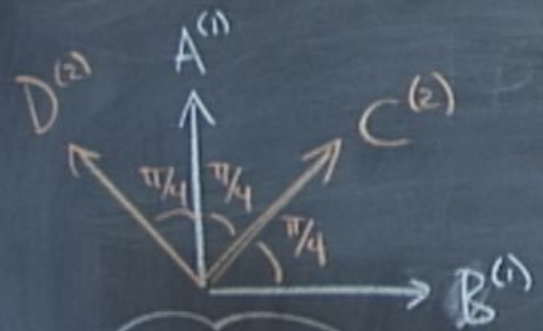
$A^{(1)}$ or $B^{(1)}$ ← #1 □ #2 → $C^{(1)}$ or $D^{(1)}$

(all results are ± 1)

Measure AC, AD, BC, BD = ± 1

LHV: All of these have determinate values

$A^{(1)}$ or $B^{(1)}$ $\xleftarrow{\#1}$ $\boxed{|\Psi_-\rangle}$ $\xrightarrow{\#1}$ $C^{(2)}$ or $D^{(2)}$



$$\langle AC\rangle = \langle BC\rangle = \langle AD\rangle = -\frac{1}{\sqrt{2}}$$

$$\langle BD\rangle = +\frac{1}{\sqrt{2}}$$

L + HV

$\Downarrow$ Bell

~~QM~~

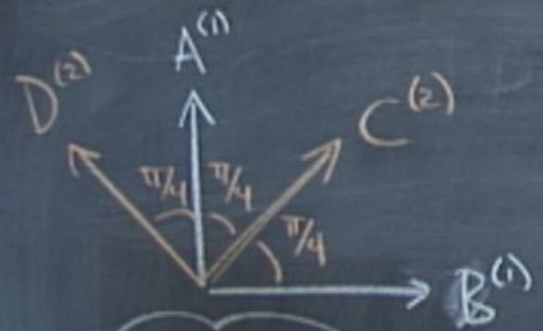
$$\boxed{\langle AC\rangle + \langle AD\rangle + \langle BC\rangle - \langle BD\rangle = -2\sqrt{2}}$$

QM can violate CHSH $\neq$!!

QM!

Bell's thm.

$A^{(1)}$ or $B^{(1)}$ $\xleftarrow{\#1}$ $\boxed{|\Psi_-\rangle}$ $\xrightarrow{\#1}$ $C^{(2)}$ or $D^{(2)}$



$$\langle AC\rangle = \langle BC\rangle = \langle AD\rangle = -\frac{1}{\sqrt{2}}$$

$$\langle BD\rangle = +\frac{1}{\sqrt{2}}$$

L + HV

$\Downarrow$ Bell

~~QM~~

$$\boxed{\langle AC\rangle + \langle AD\rangle + \langle BC\rangle - \langle BD\rangle = -2\sqrt{2}}$$

QM can violate CHSH $\neq$!!

QM!

Bell's thm.