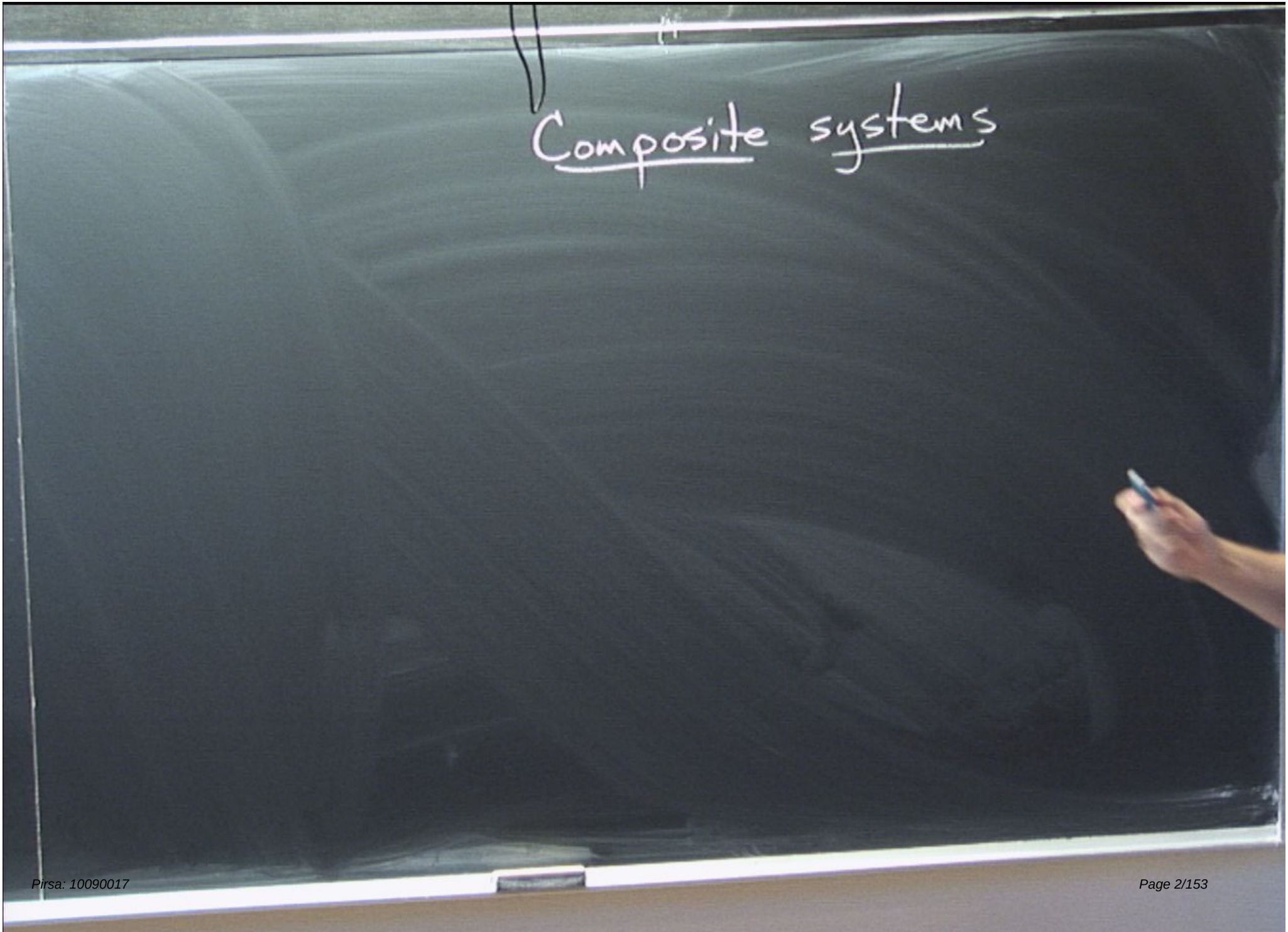
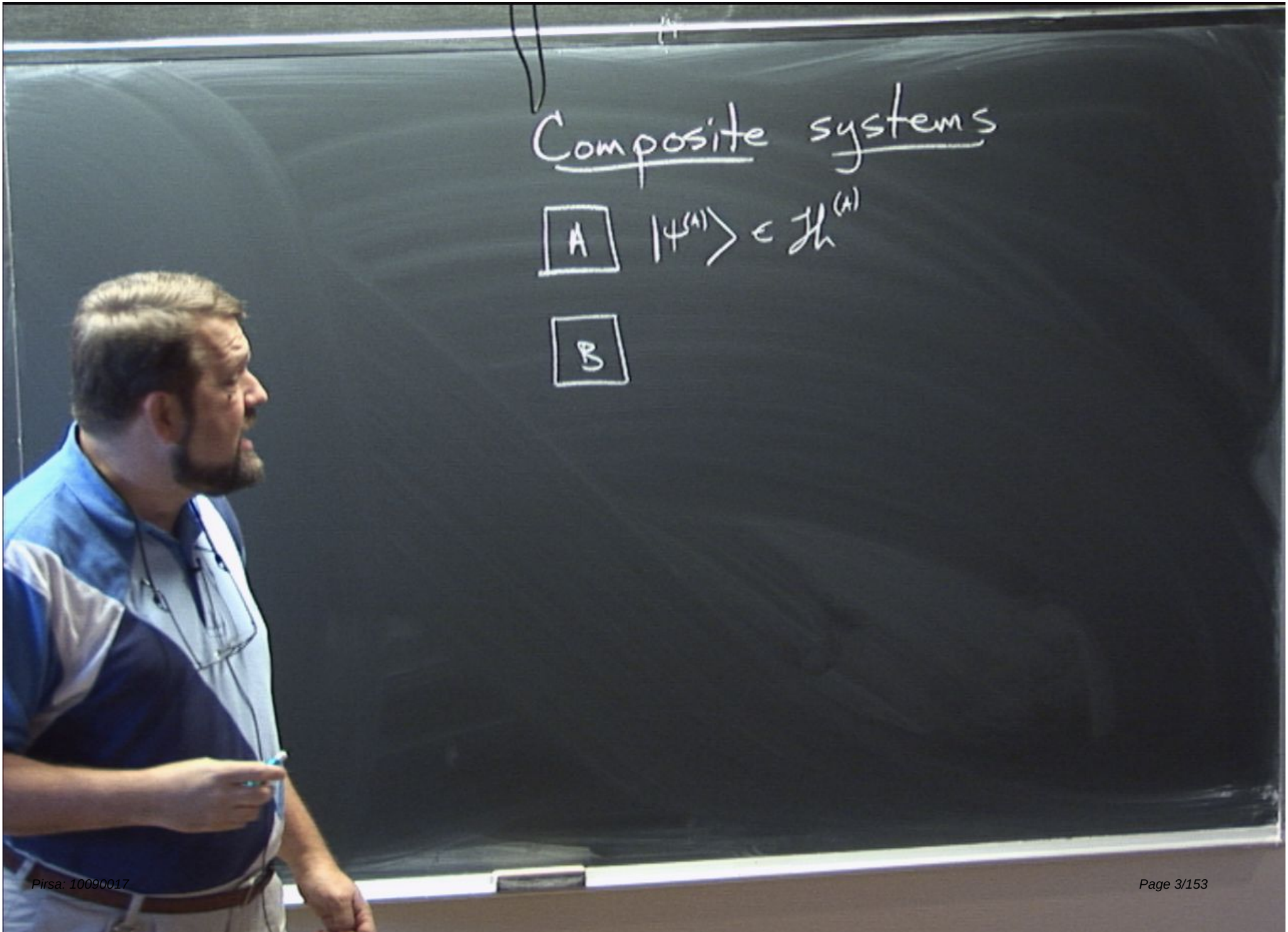
Title: Quantum Theory (PHYS 605) - Lecture 6

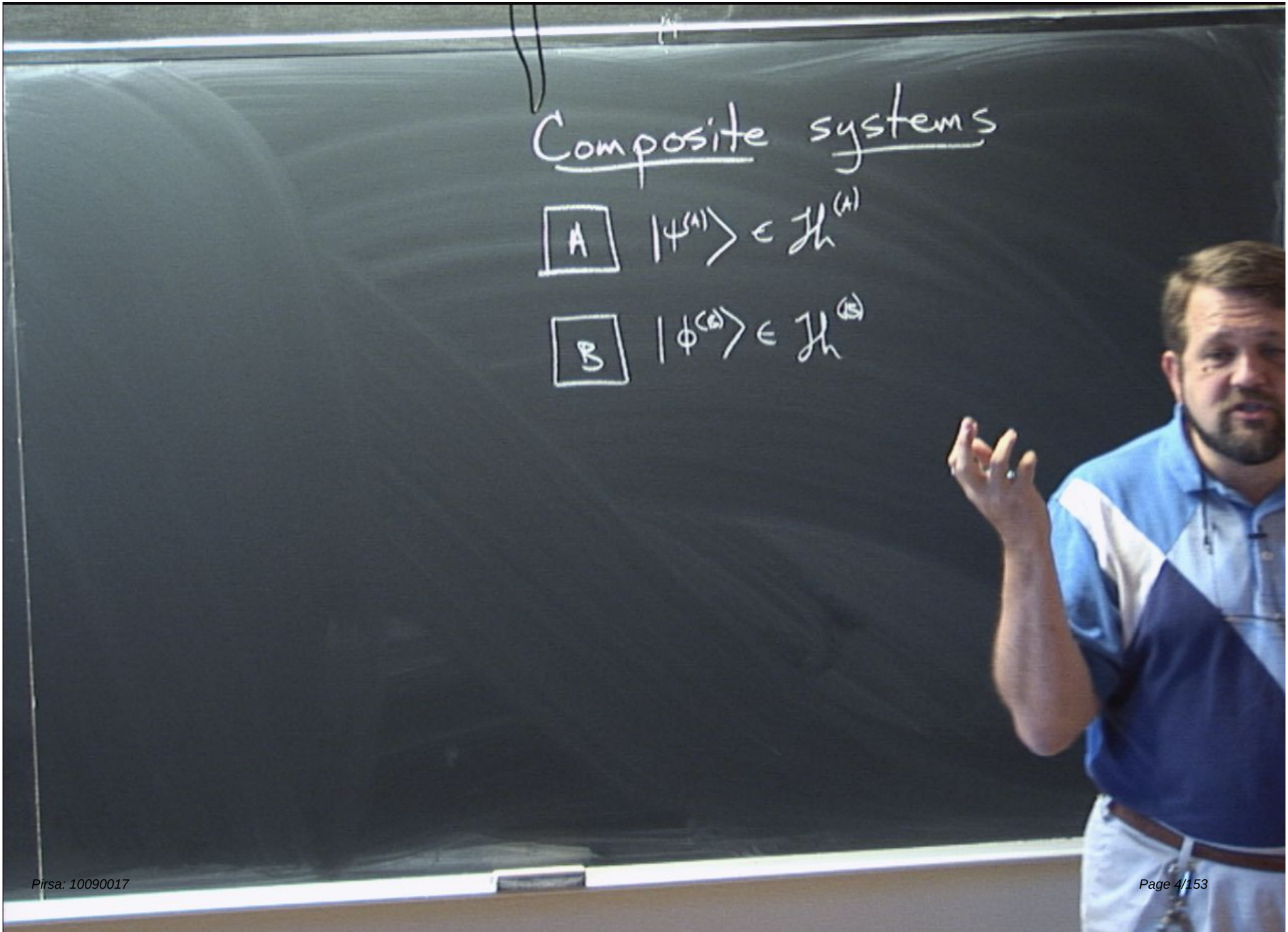
Date: Sep 20, 2010 09:00 AM

URL: http://pirsa.org/10090017

Abstract:


Composite systems


Composite systems
A
$|\psi^{(A)}\rangle \in \mathcal{H}^{(A)}$
B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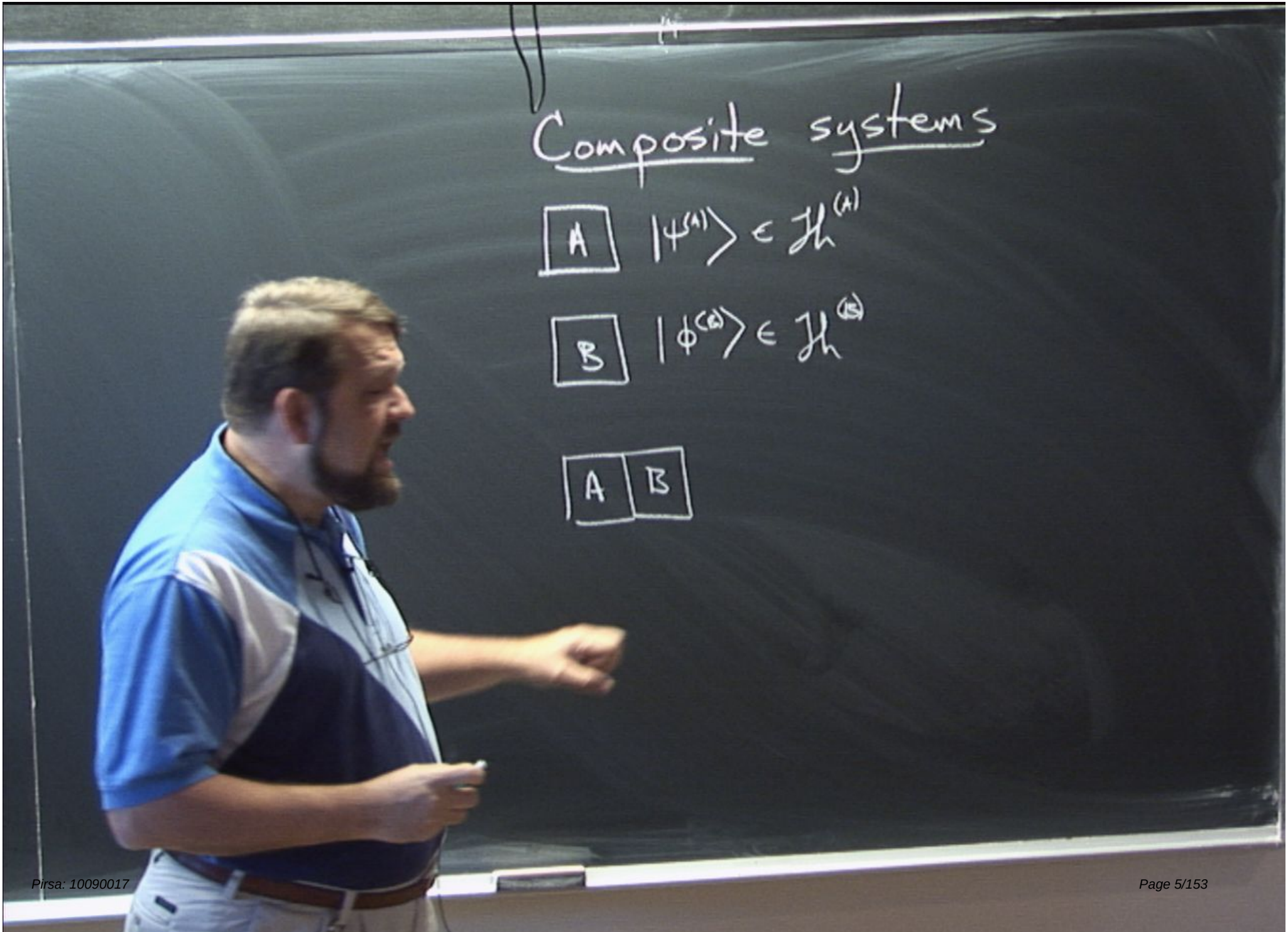
$|\psi^{(A)}\rangle \in \mathcal{H}^{(A)}$
B
$|\phi^{(B)}\rangle \in \mathcal{H}^{(B)}$


Composite systems
A
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A B

Composite systems

A $|\psi^{(A)}\rangle \in \mathcal{H}^{(A)}$

B $|\phi^{(B)}\rangle \in \mathcal{H}^{(B)}$

A B $|\Psi^{(AB)}\rangle \in \mathcal{H}^{(AB)}$

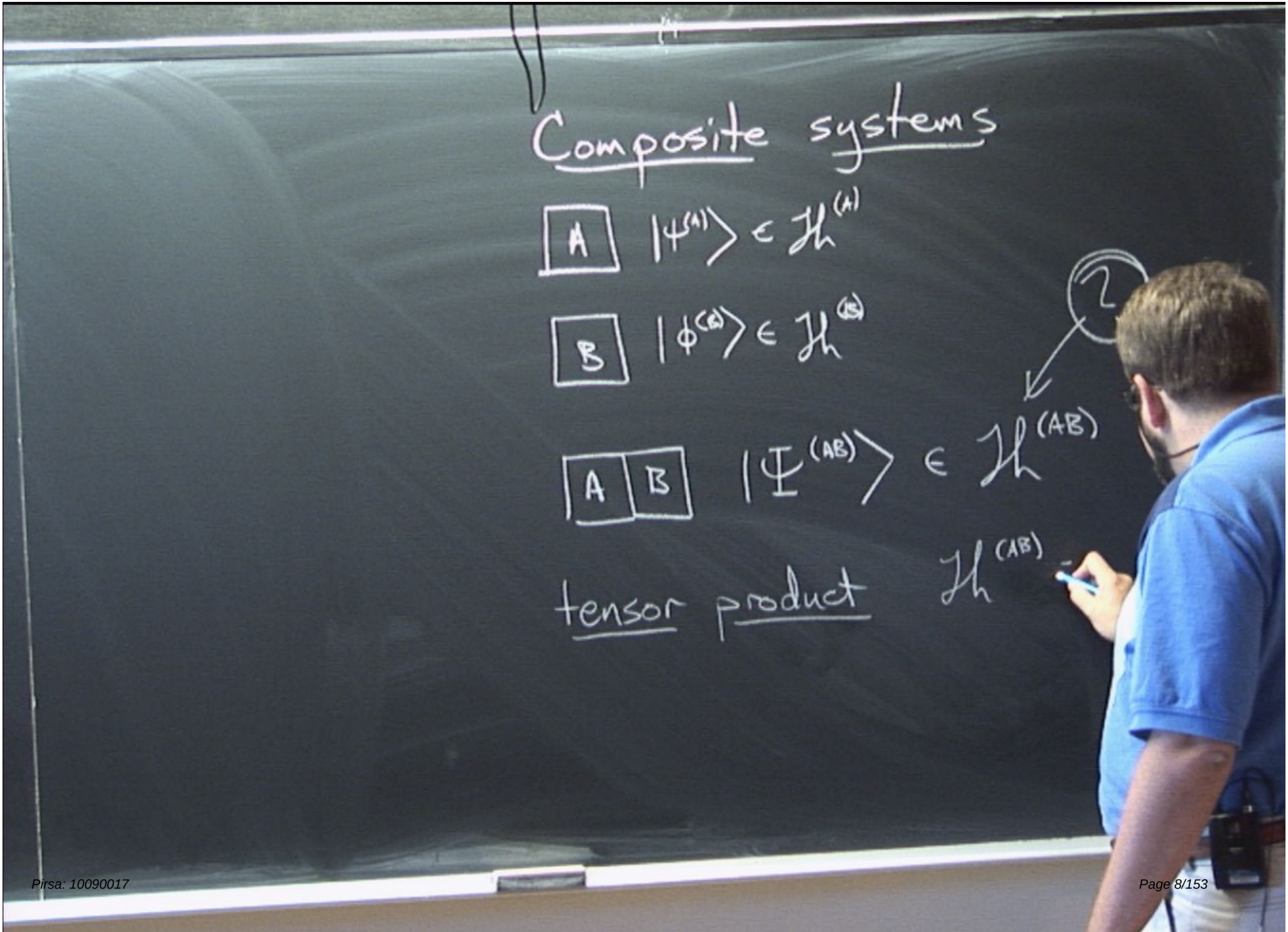
Composite systems

A $|\psi^{(A)}\rangle \in \mathcal{H}^{(A)}$

B $|\phi^{(B)}\rangle \in \mathcal{H}^{(B)}$

A B $|\Psi^{(AB)}\rangle \in \mathcal{H}^{(AB)}$?

tensor product


Composite systems
A
B
A B
tensor product

Composite systems

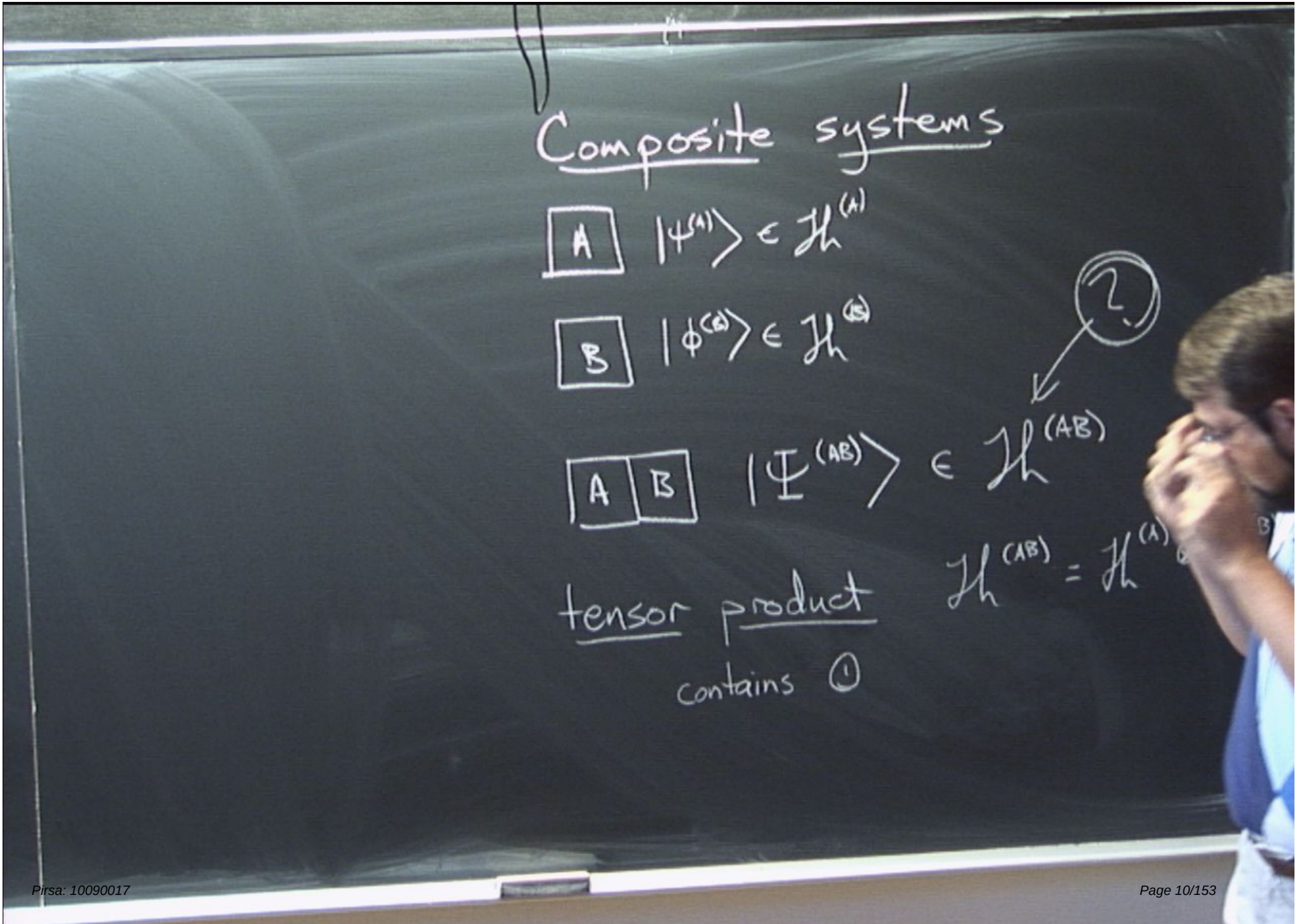
A $|\psi^{(A)}\rangle \in \mathcal{H}^{(A)}$

B $|\phi^{(B)}\rangle \in \mathcal{H}^{(B)}$

?

A B $|\Psi^{(AB)}\rangle \in \mathcal{H}^{(AB)}$

tensor product $\mathcal{H}^{(AB)} = \mathcal{H}^{(A)} \otimes \mathcal{H}^{(B)}$


Composite systems
A
B
A B
tensor product
contains ①
?

Composite systems

A $|\psi^{(A)}\rangle \in \mathcal{H}^{(A)}$

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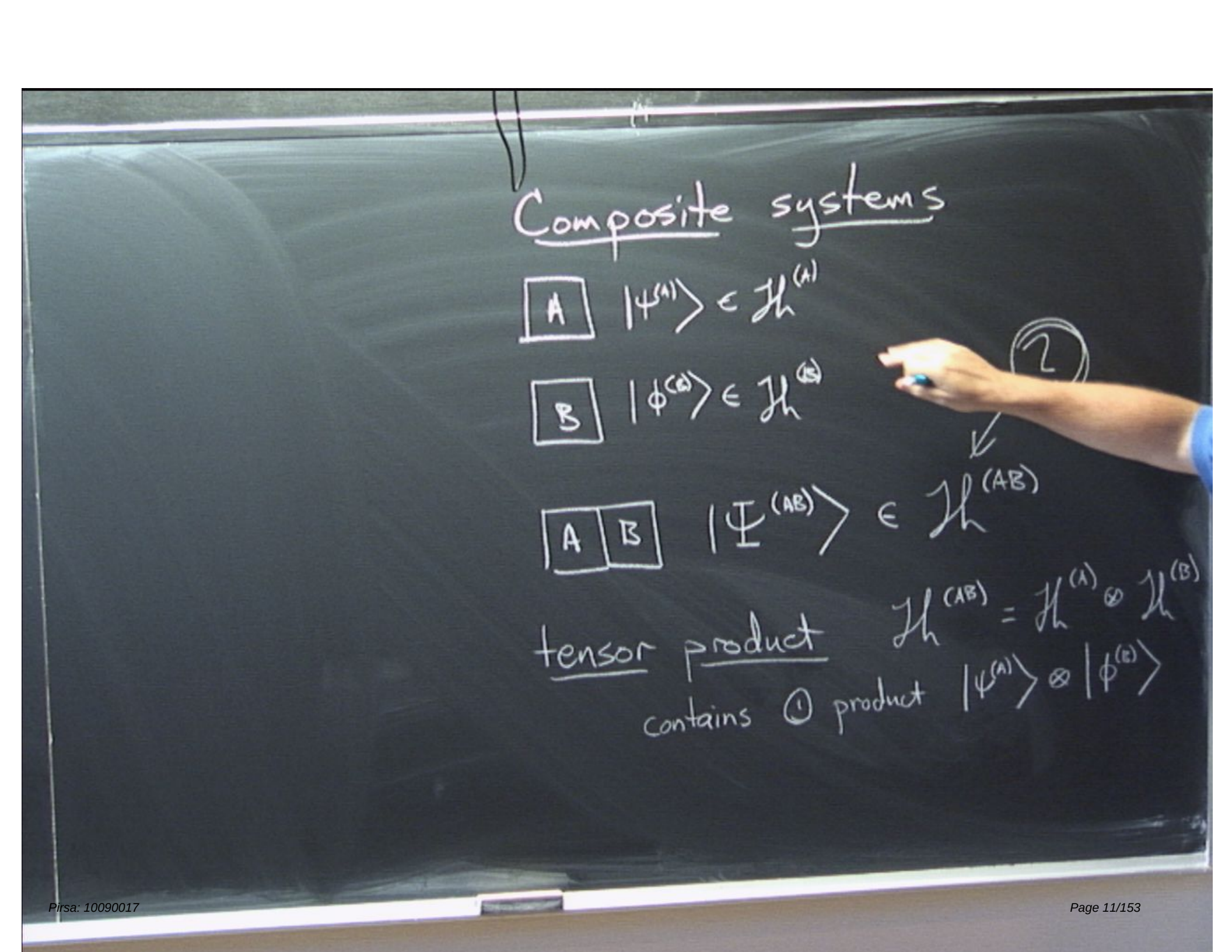
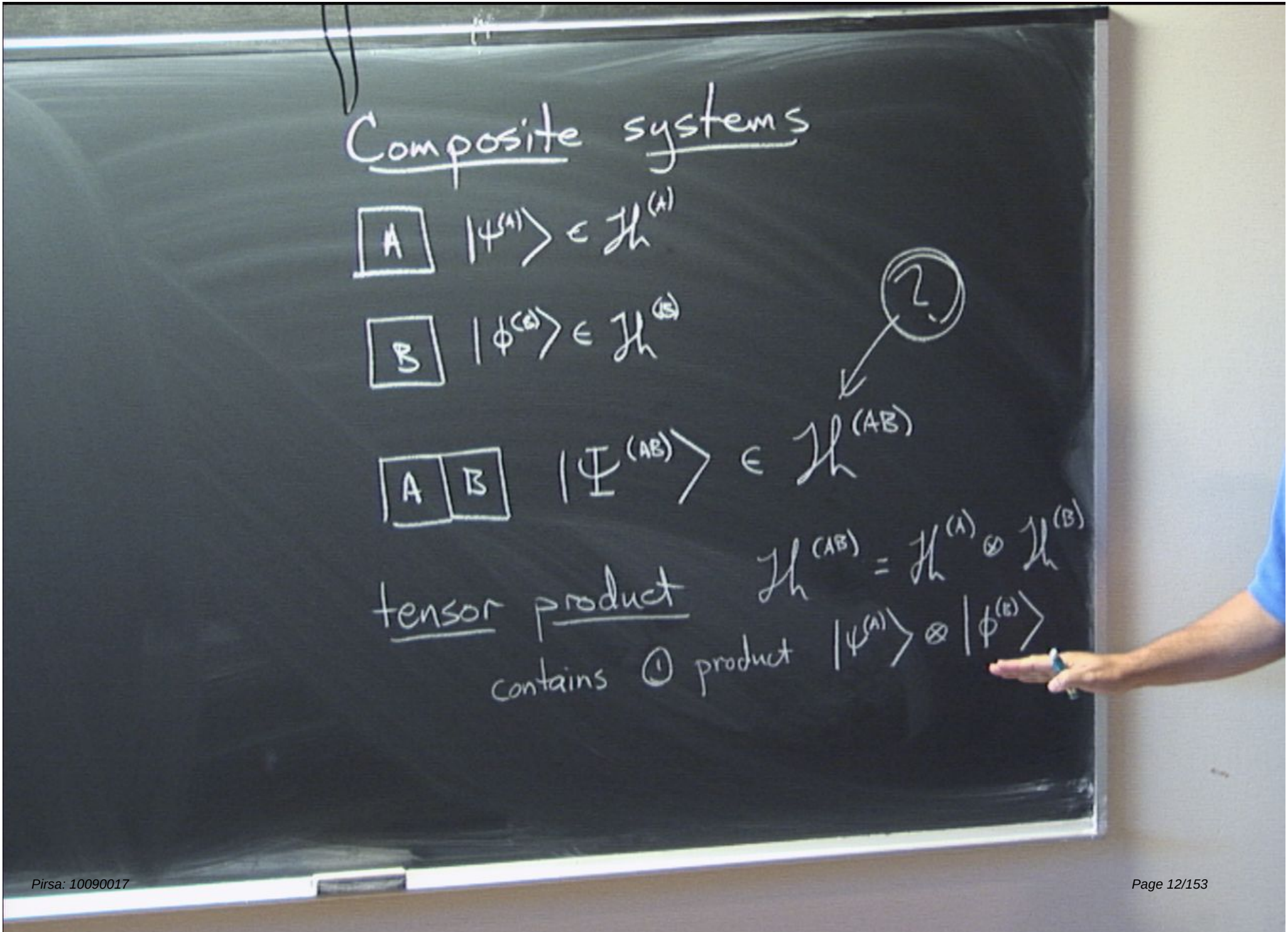


A B $|\Psi^{(AB)}\rangle \in \mathcal{H}^{(AB)}$

tensor product $\mathcal{H}^{(AB)} = \mathcal{H}^{(A)} \otimes \mathcal{H}^{(B)}$

contains ① product $|\psi^{(A)}\rangle \otimes |\phi^{(B)}\rangle$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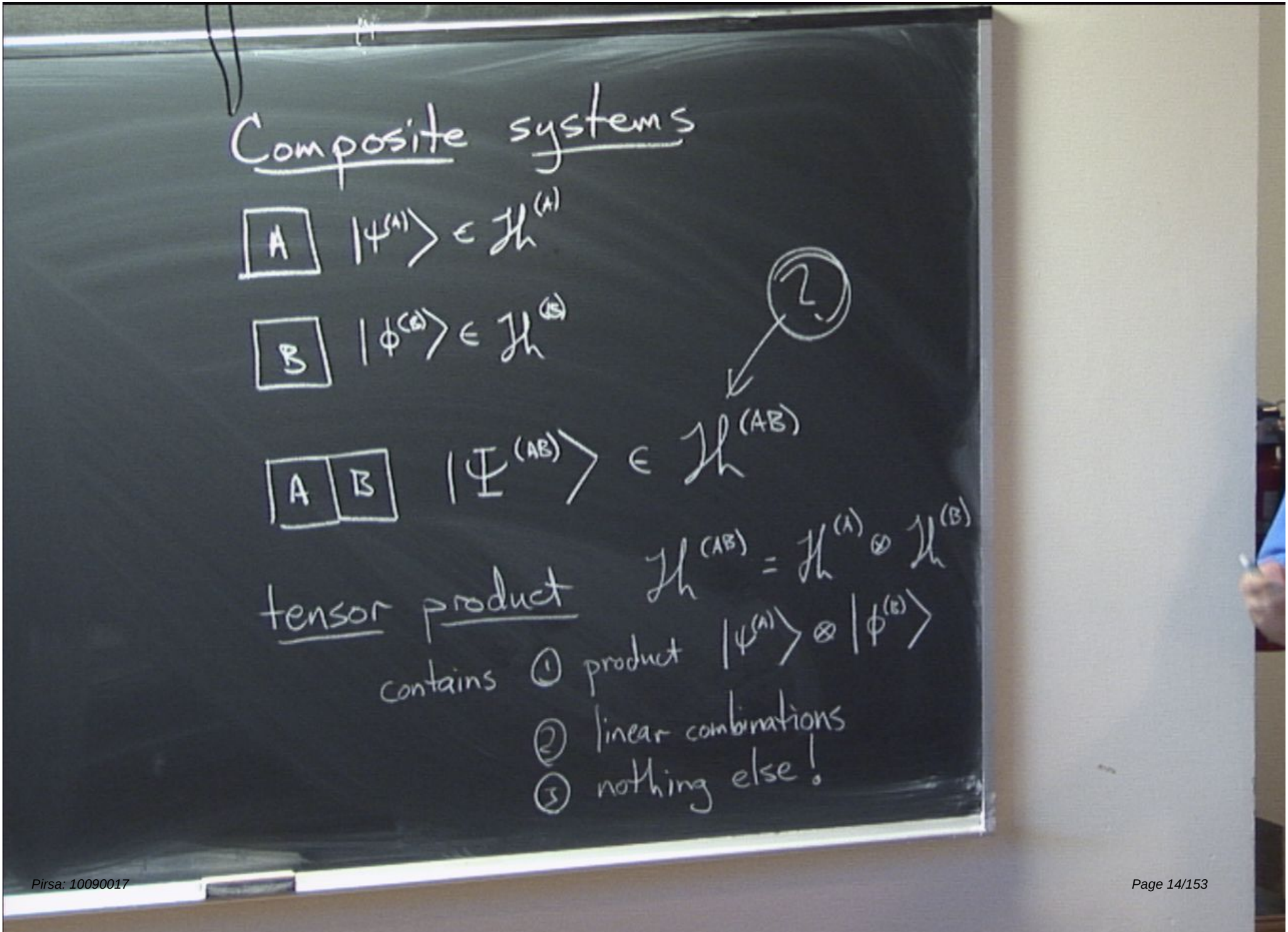
② linear combinations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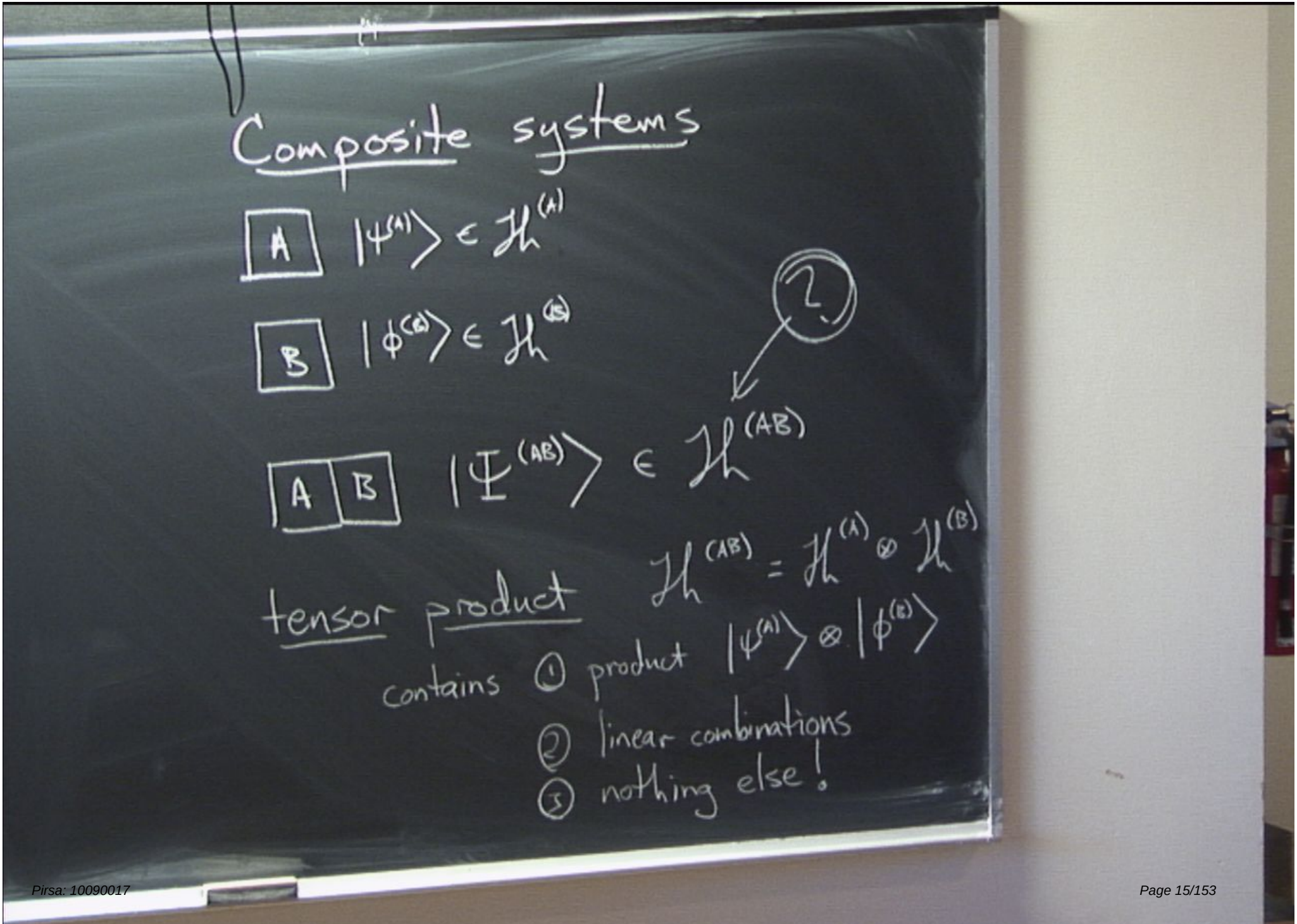
② linear combinations
③ nothing else!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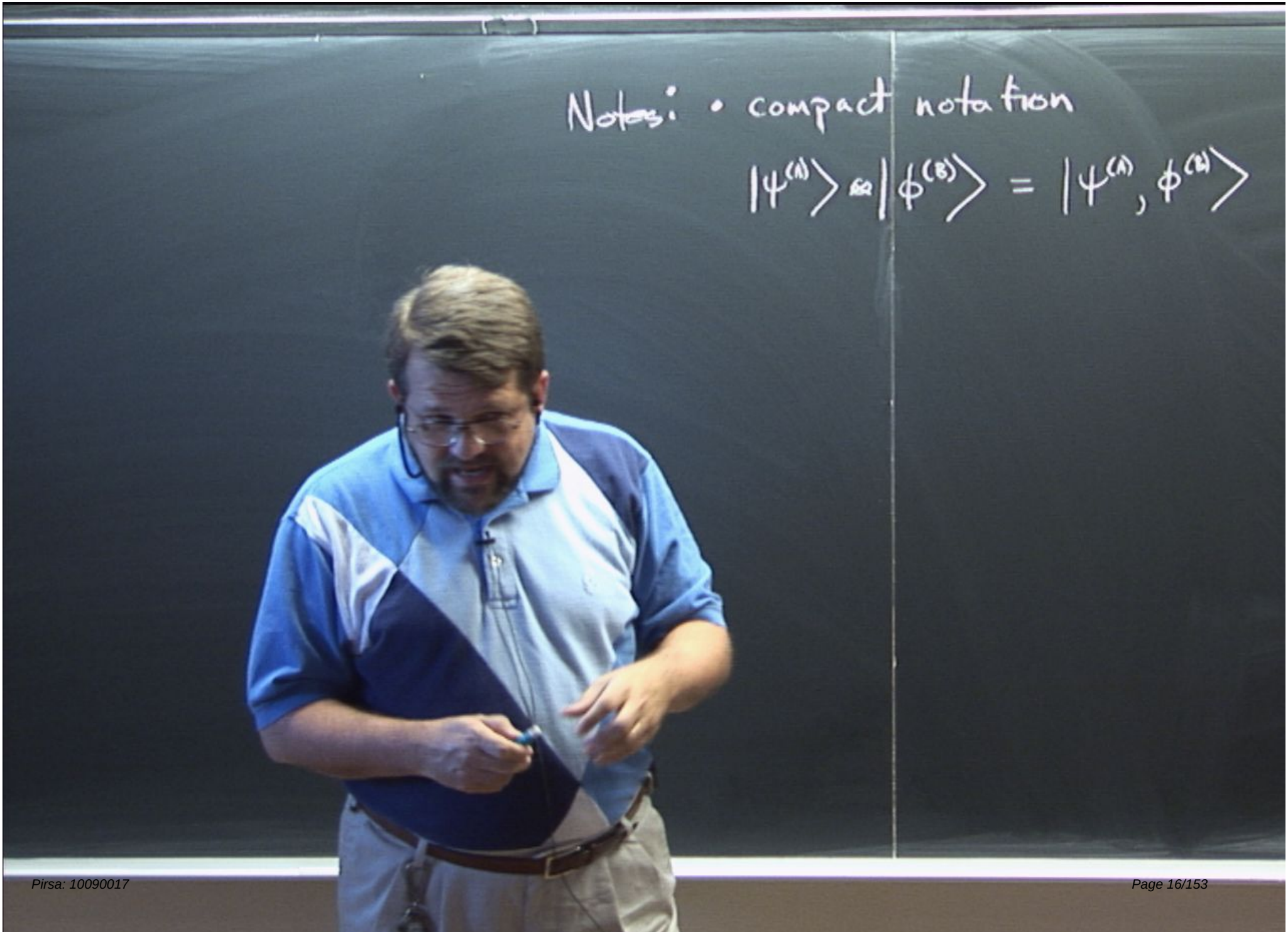
Notes: • compact notation
$|\psi^{(A)}\rangle \otimes |\phi^{(B)}\rangle = |\psi^{(A)}, \phi^{(B)}\rangle$

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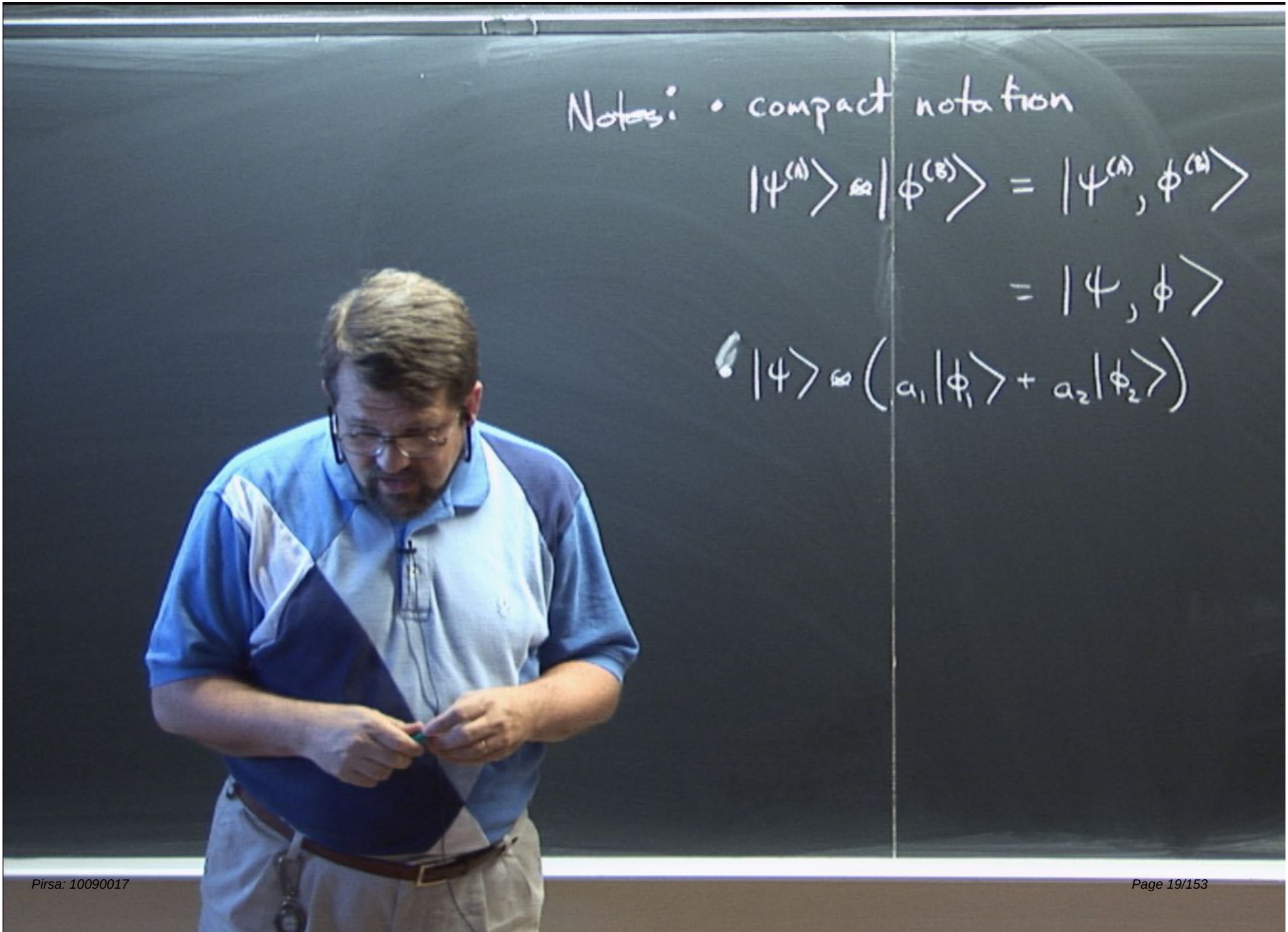
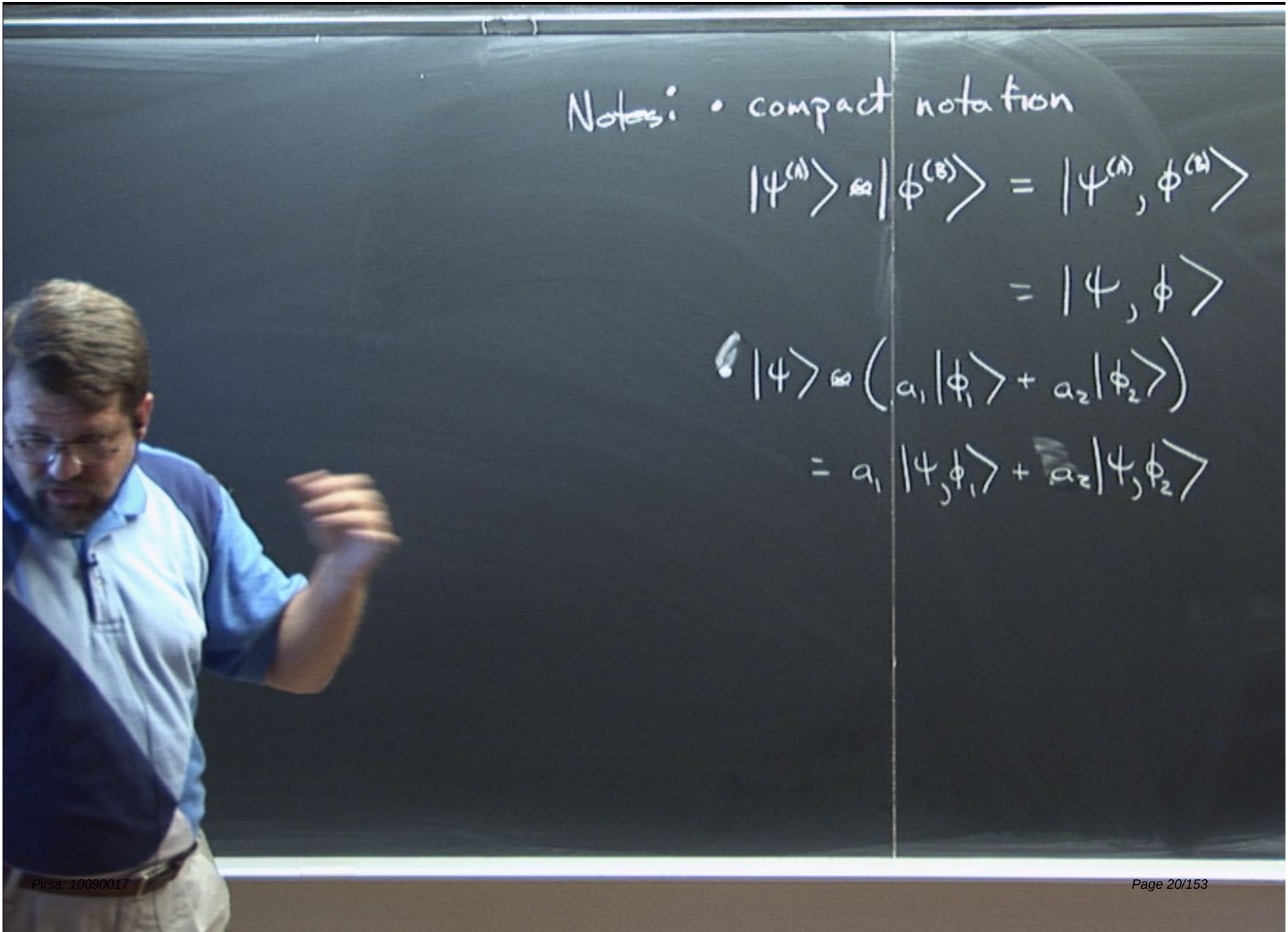

Notes: • compact notation
$|\psi^{(A)}\rangle \otimes |\phi^{(B)}\rangle = |\psi^{(A)}, \phi^{(B)}\rangle$
$= |\psi, \phi\rangle$
• $|\psi\rangle \propto (a_1|\phi_1\rangle + a_2|\phi_2\rangle)$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$= a_1|\psi,\phi_1\rangle + a_2|\psi,\phi_2\rangle$

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- inner product

$$\underbrace{\langle \psi, \phi | \psi', \phi' \rangle}_{\mathcal{H}^{(AB)}} = \underbrace{\langle \psi | \psi' \rangle}_{\mathcal{H}^{(A)}} \underbrace{\langle \phi | \phi' \rangle}_{\mathcal{H}^{(B)}}$$

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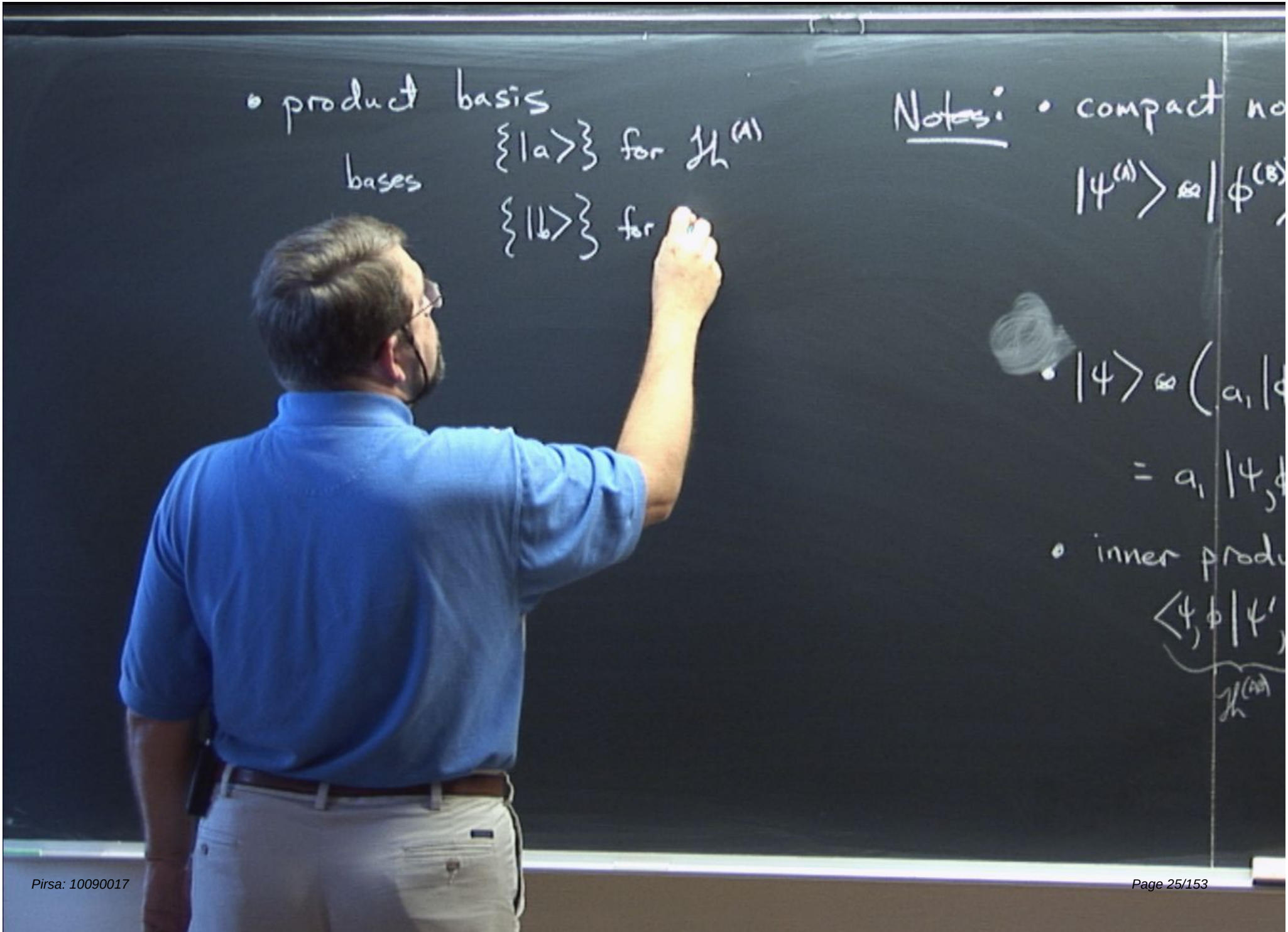
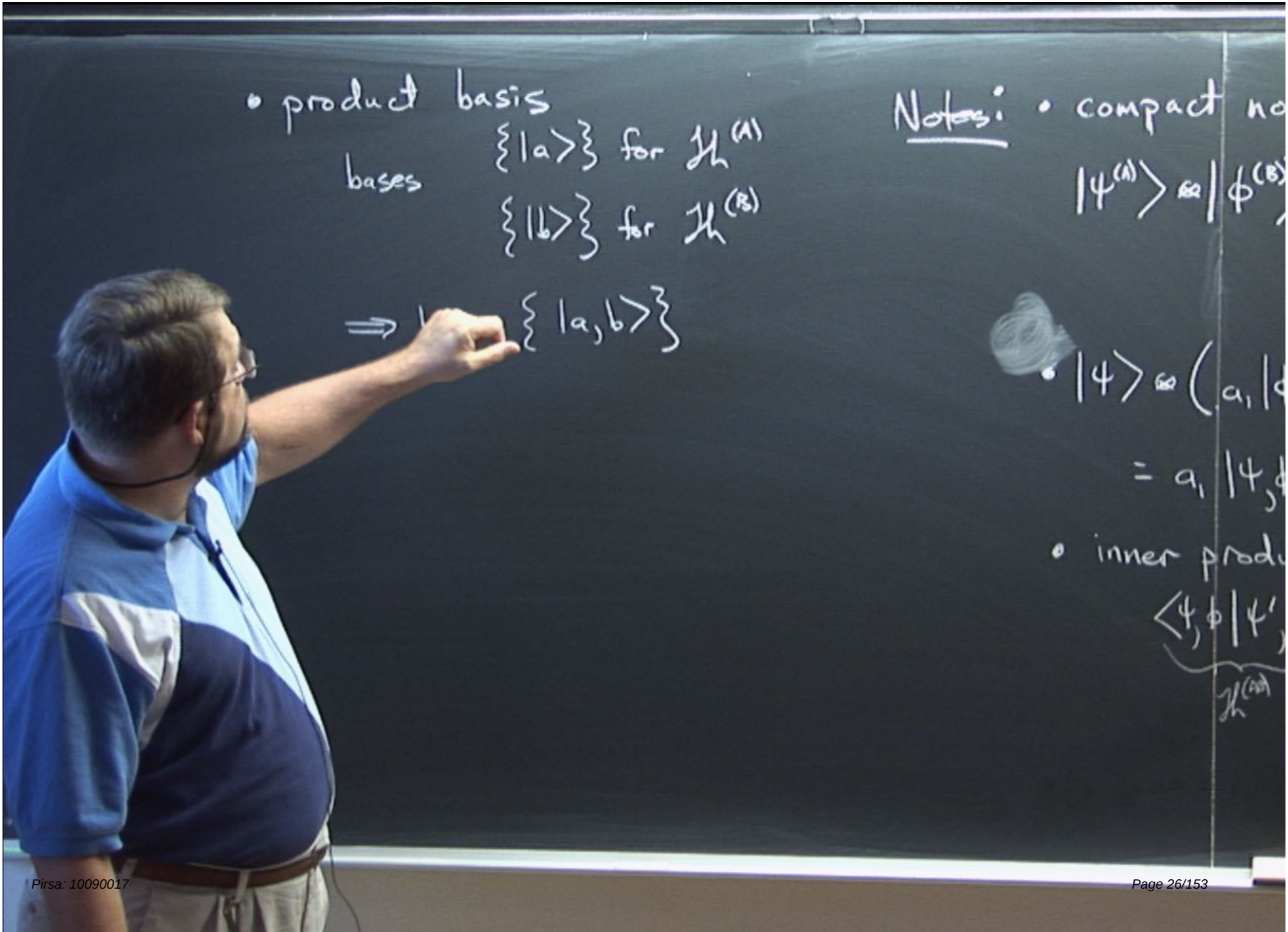
extend by linearity....


• product basis
bases
{|a⟩} for $\mathcal{H}^{(A)}$
{|b⟩} for
Notes:
• compact no
$|\psi^{(A)}\rangle \otimes |\phi^{(B)}$
• $|\psi\rangle \otimes (a_i|$
$= a_i |\psi_j$
• inner produ
$\langle \psi_j \phi | \psi'$
$\mathcal{H}^{(AB)}$


• product basis
bases
$\{|a\rangle\}$ for $\mathcal{H}^{(A)}$
$\{|b\rangle\}$ for $\mathcal{H}^{(B)}$
$\Rightarrow$ $\{|a,b\rangle\}$
Notes:
• compact no
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• $|\psi\rangle \in (a_i|\phi$
$= a_i|\psi_j\phi$
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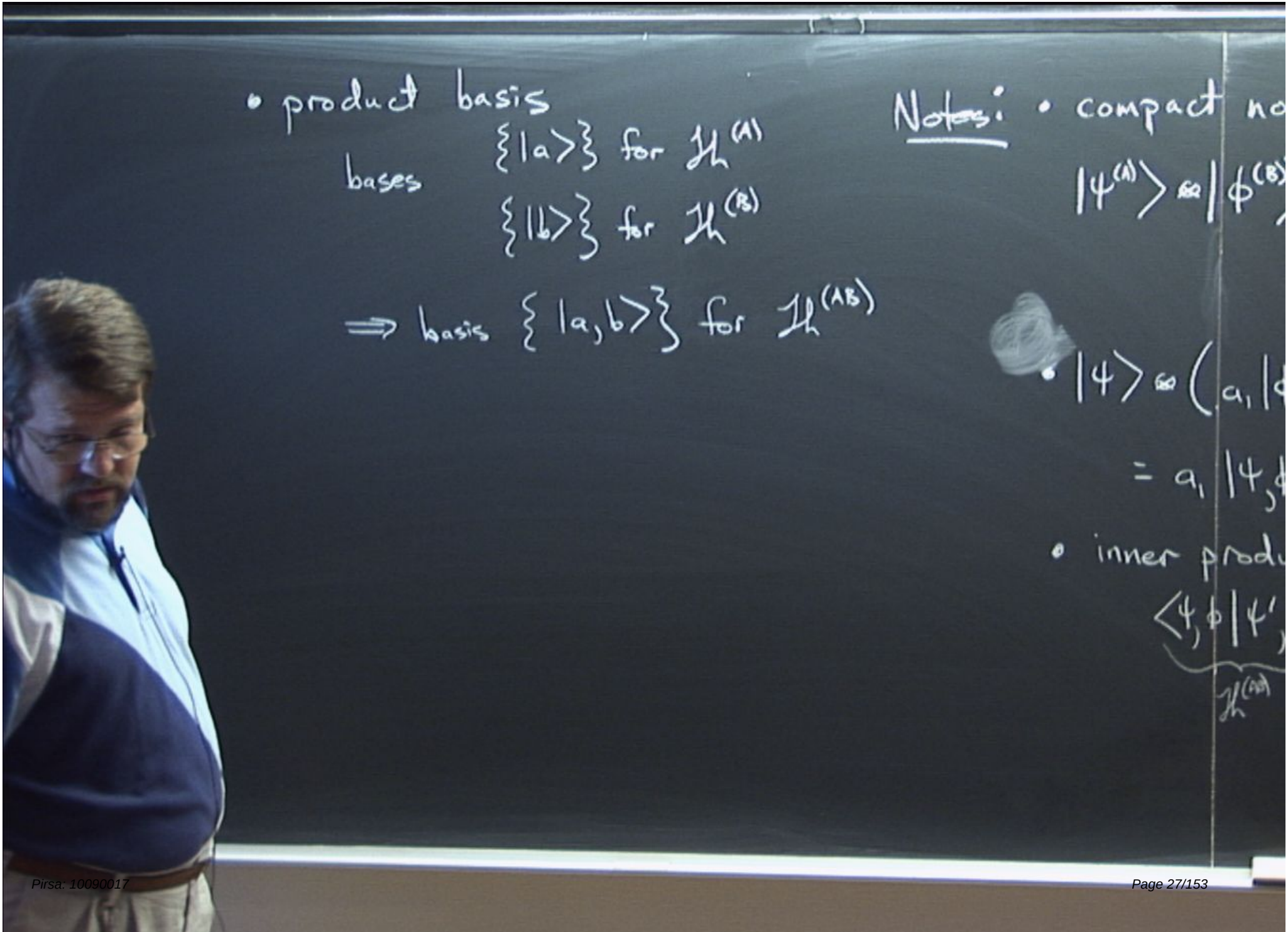
$\langle\psi,\phi|\psi'$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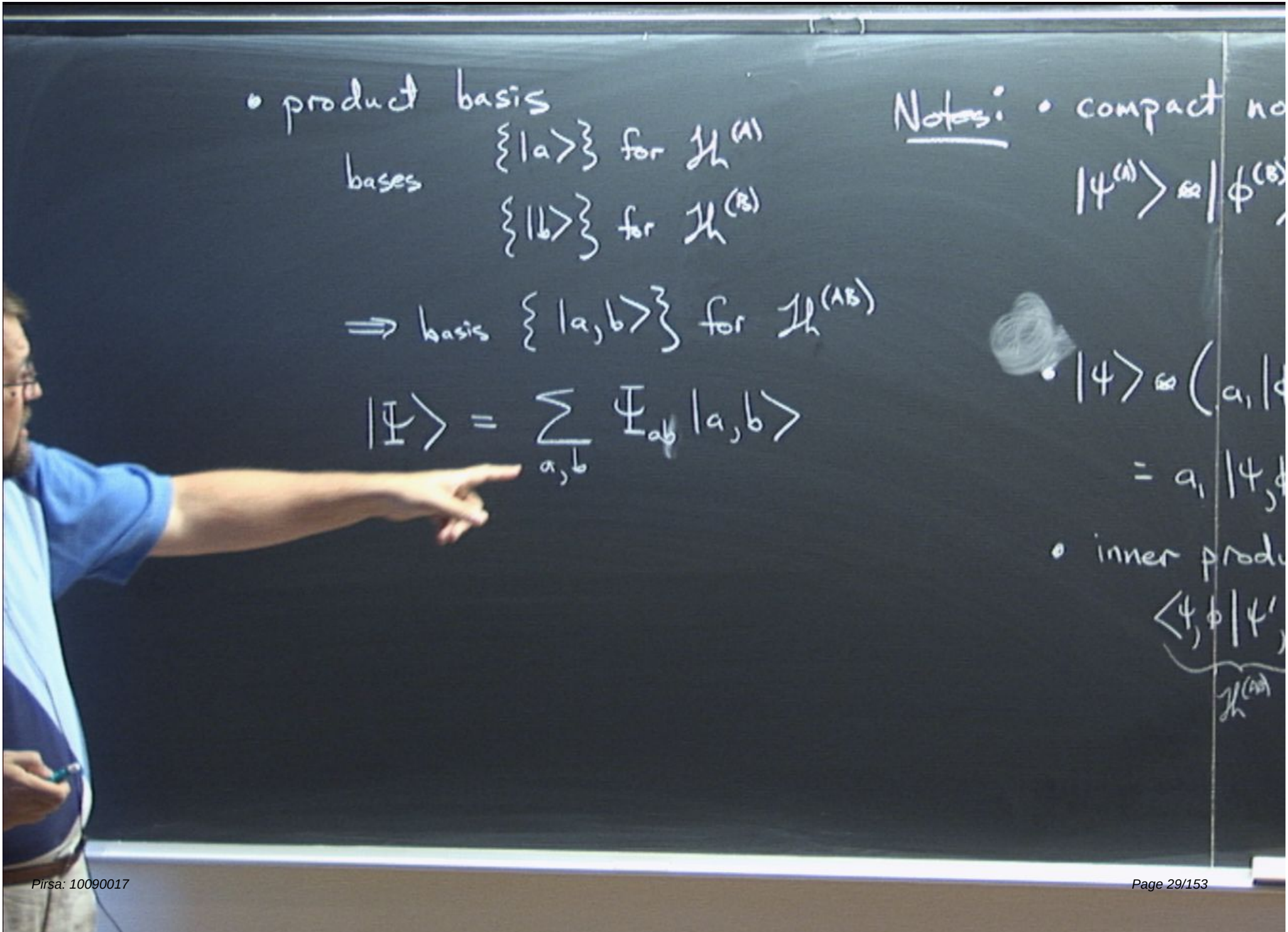
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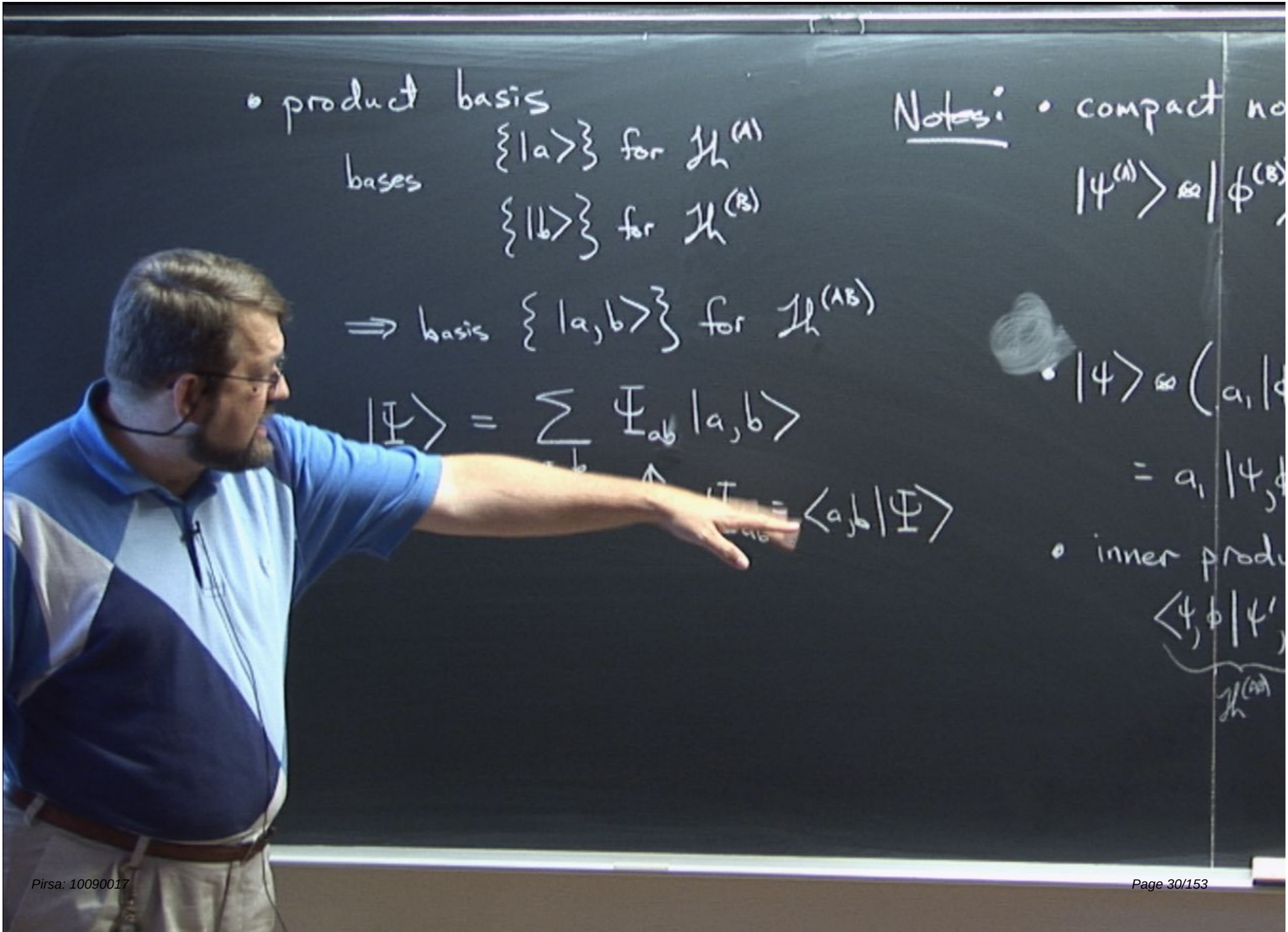
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Q. states: A is in $|\psi^{(A)}\rangle$

B is in $|\phi^{(B)}\rangle$ $\Rightarrow$ AB is in $|\Psi^{(AB)}\rangle = |\psi^{(A)}, \phi^{(B)}\rangle$

"product state"

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$\mathcal{H}^{(AB)}$

Composite systems

notation

$|\phi^{(B)}\rangle = |\psi^{(A)}, \phi^{(B)}\rangle$

$= |\psi, \phi\rangle$

$(a_1|\phi_1\rangle + a_2|\phi_2\rangle)$

$|\psi, \phi_1\rangle + a_2|\psi, \phi_2\rangle$

product

$\langle \psi, \phi | \psi', \phi' \rangle = \underbrace{\langle \psi | \psi' \rangle}_{\mathcal{H}^{(A)}} \underbrace{\langle \phi | \phi' \rangle}_{\mathcal{H}^{(B)}}$

$\mathcal{H}^{(AB)}$

extend by linearity....

A $|\psi^{(A)}\rangle \in \mathcal{H}^{(A)}$

B $|\phi^{(B)}\rangle \in \mathcal{H}^{(B)}$

A B $|\Psi^{(AB)}\rangle \in \mathcal{H}^{(AB)}$

2.

tensor product $\mathcal{H}^{(AB)} = \mathcal{H}^{(A)} \otimes \mathcal{H}^{(B)}$

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② linear combinations

③ nothing else!

- product basis

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"product state"

otherwise, $|\Psi^{(AB)}\rangle$ is entangled

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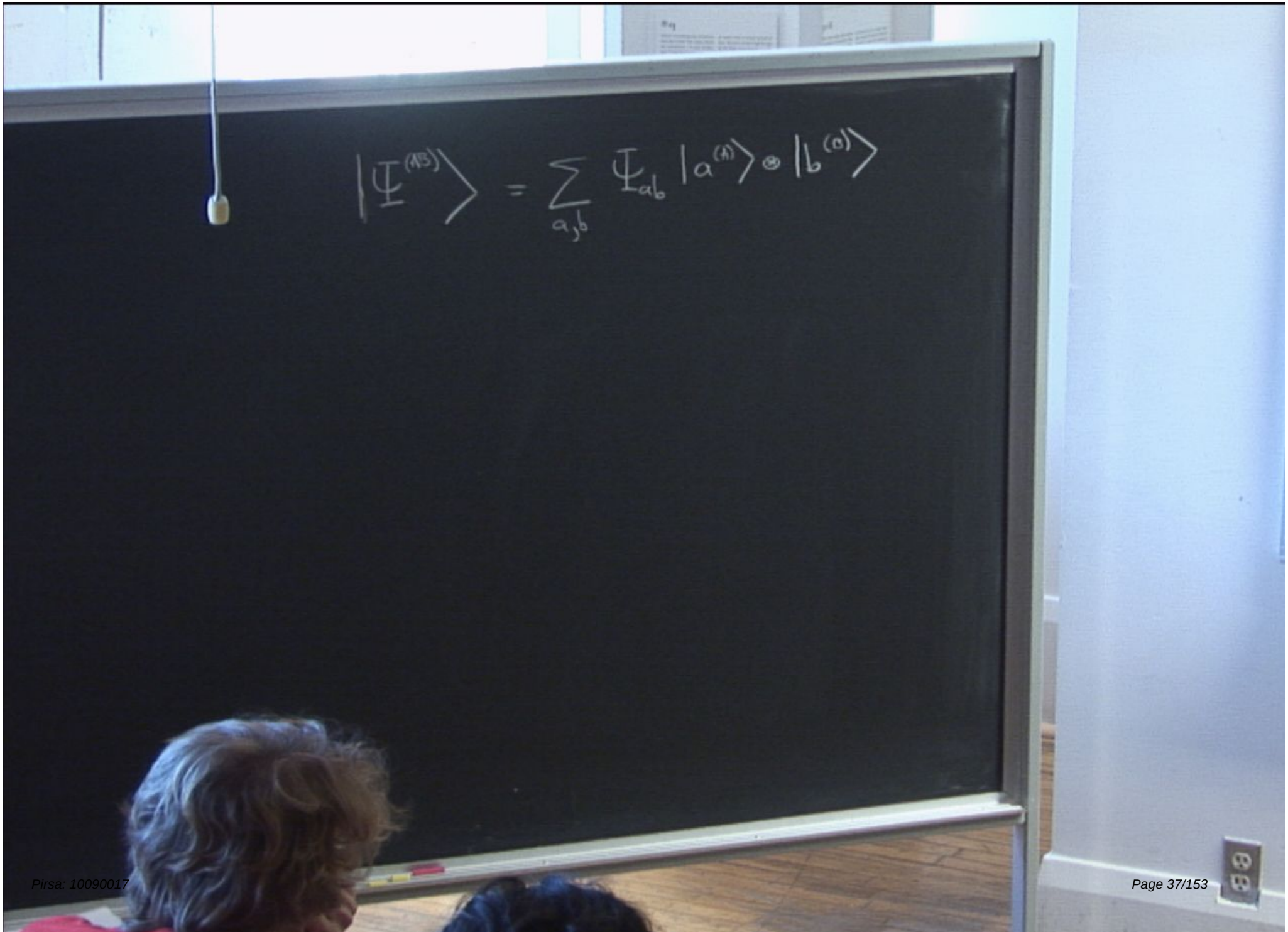
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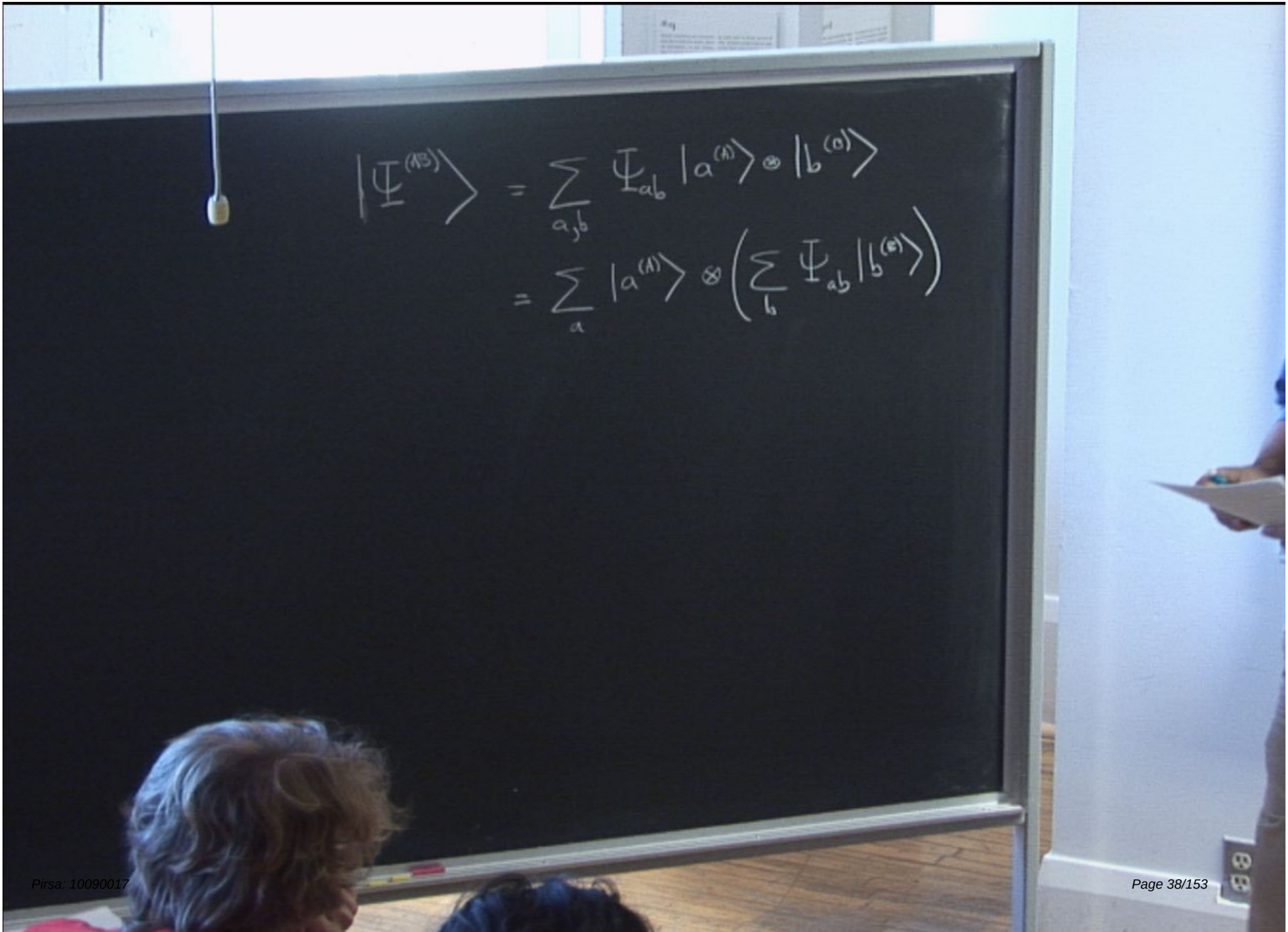
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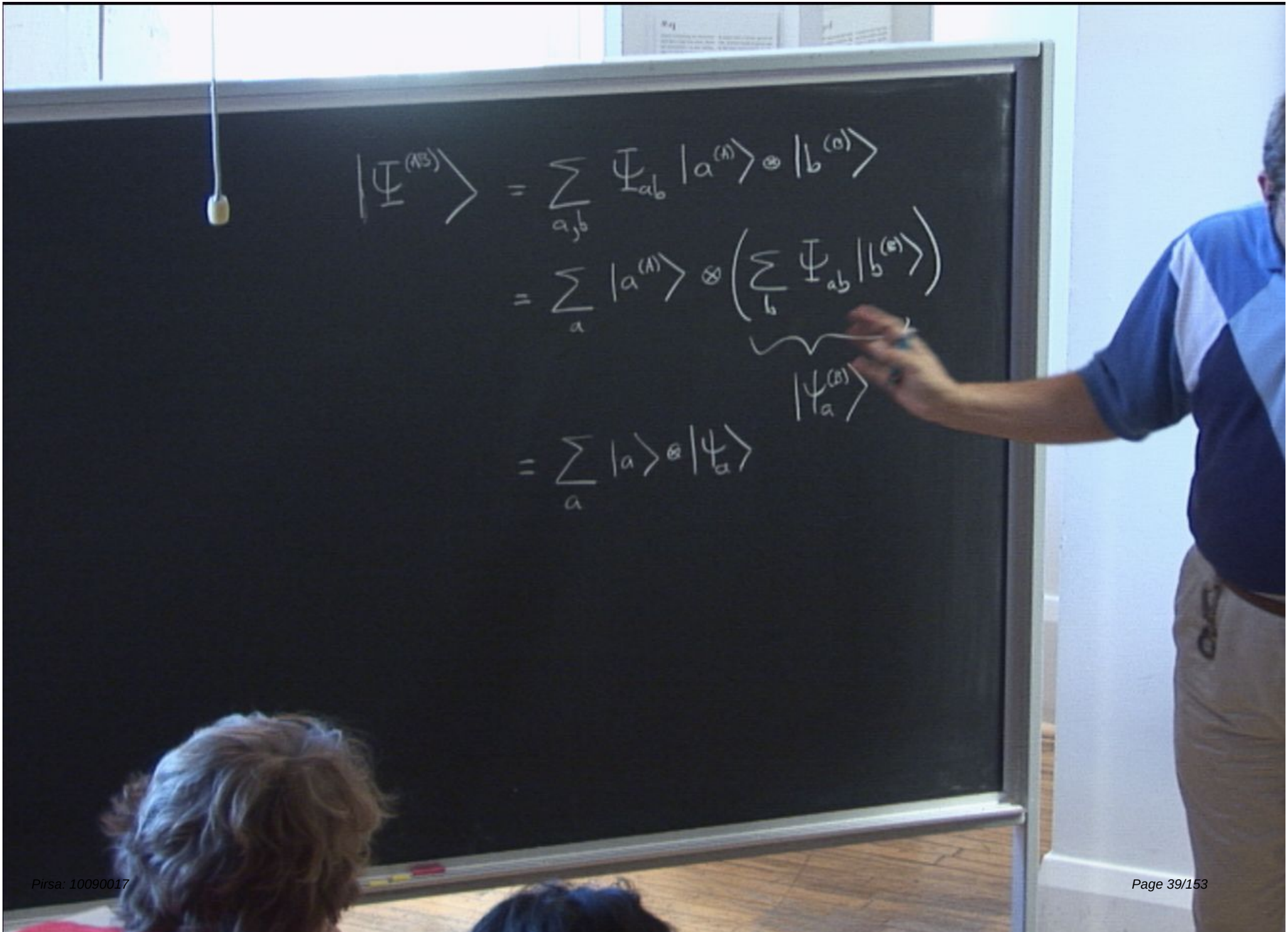
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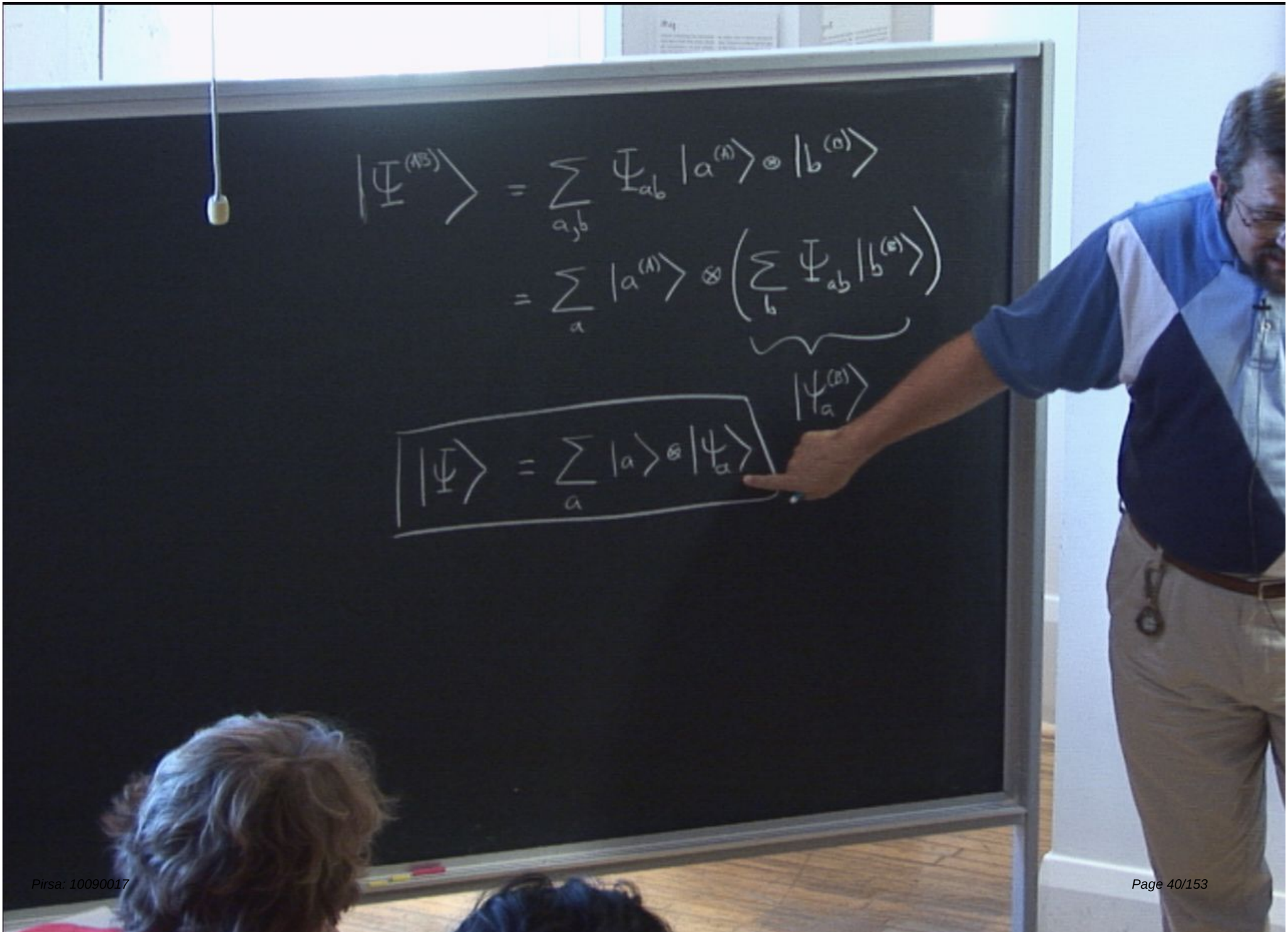
$|\psi_a^{(B)}\rangle$
$= \sum_{a} |a\rangle \otimes |\psi_a\rangle$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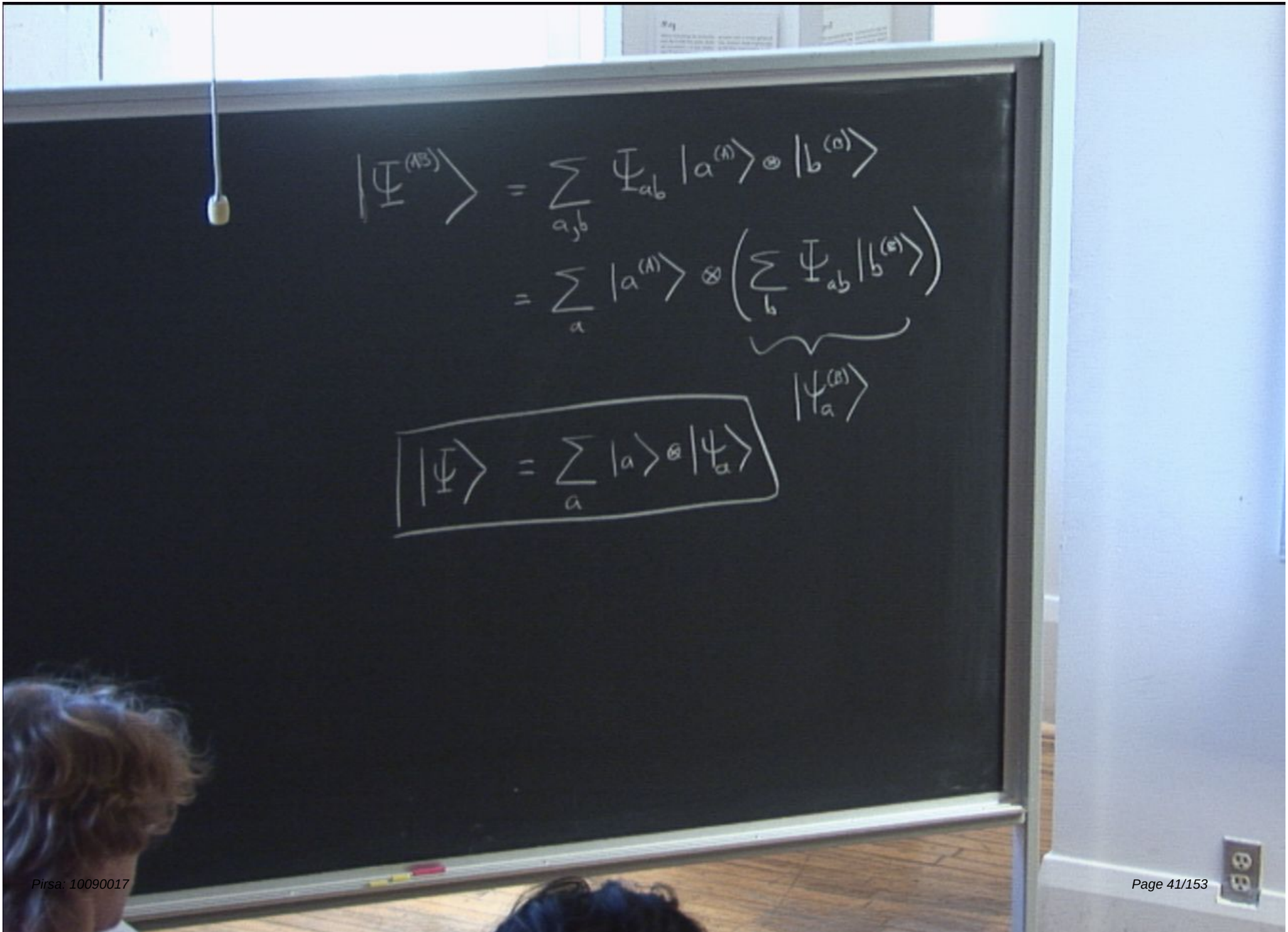
$|\Psi\rangle = \sum_{a} |a\rangle \otimes |\psi_a\rangle$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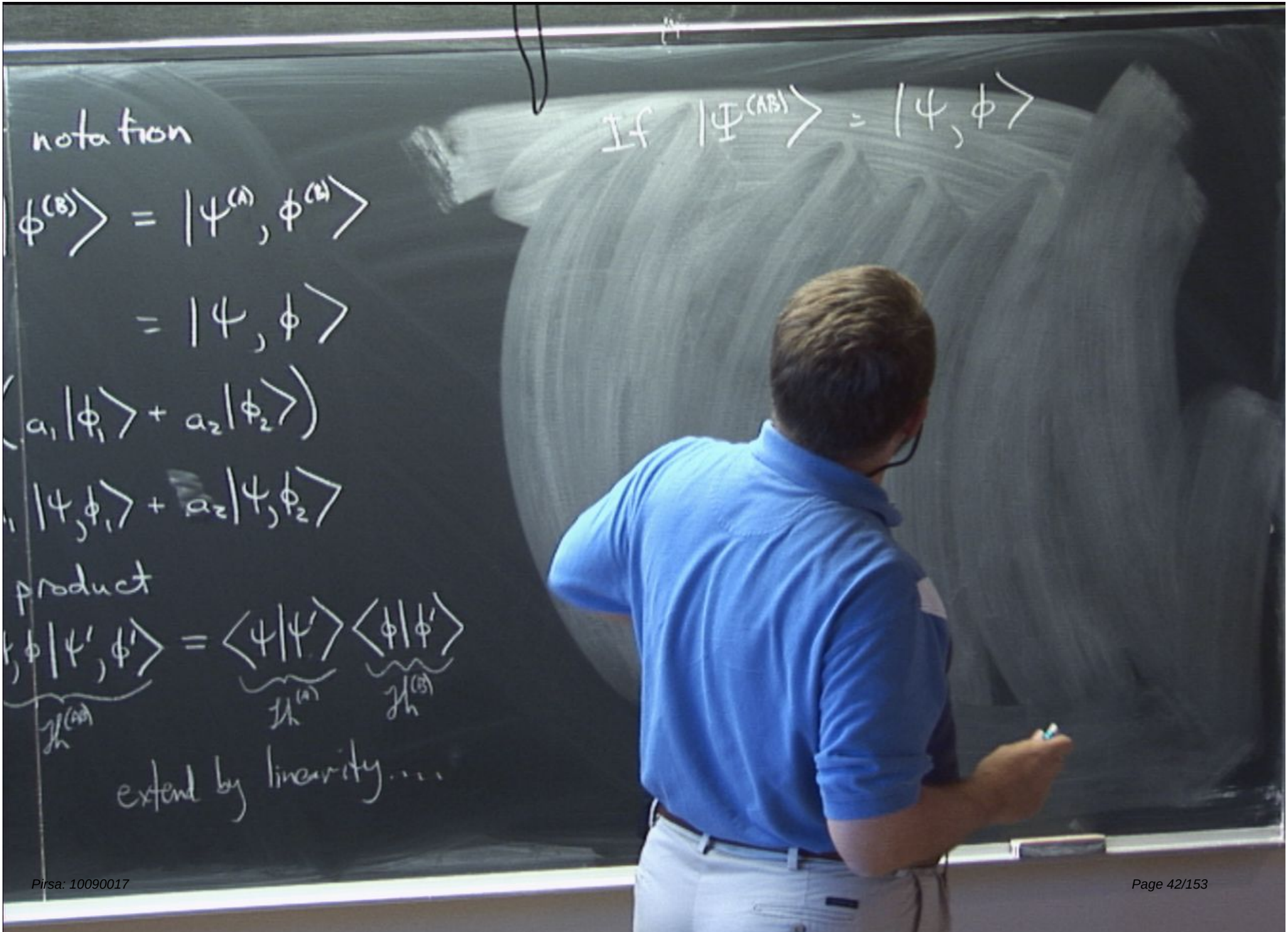
$|\Psi\rangle = \sum_{a} |a\rangle \otimes |\psi_a\rangle$


notation
$|\phi^{(B)}\rangle = |\psi^{(A)}, \phi^{(B)}\rangle$
$= |\psi, \phi\rangle$
$(a_1|\phi_1\rangle + a_2|\phi_2\rangle)$
$|\psi,\phi_1\rangle + a_2|\psi,\phi_2\rangle$
product
$\langle\psi,\phi|\psi',\phi'\rangle = \langle\psi|\psi'\rangle\langle\phi|\phi'\rangle$
$\mathcal{H}^{(AB)}$
$\mathcal{H}^{(A)}$
$\mathcal{H}^{(B)}$
extend by linearity....
If $|\Psi^{(AB)}\rangle = |\psi,\phi\rangle$

notation

$|\phi^{(B)}\rangle = |\psi^{(A)}, \phi^{(B)}\rangle$

$= |\psi, \phi\rangle$

$(a_1|\phi_1\rangle + a_2|\phi_2\rangle)$

$|\psi,\phi_1\rangle + a_2|\psi,\phi_2\rangle$

product

$\langle\psi,\phi|\psi',\phi'\rangle = \underbrace{\langle\psi|\psi'\rangle}_{\mathcal{H}^{(A)}} \underbrace{\langle\phi|\phi'\rangle}_{\mathcal{H}^{(B)}}$

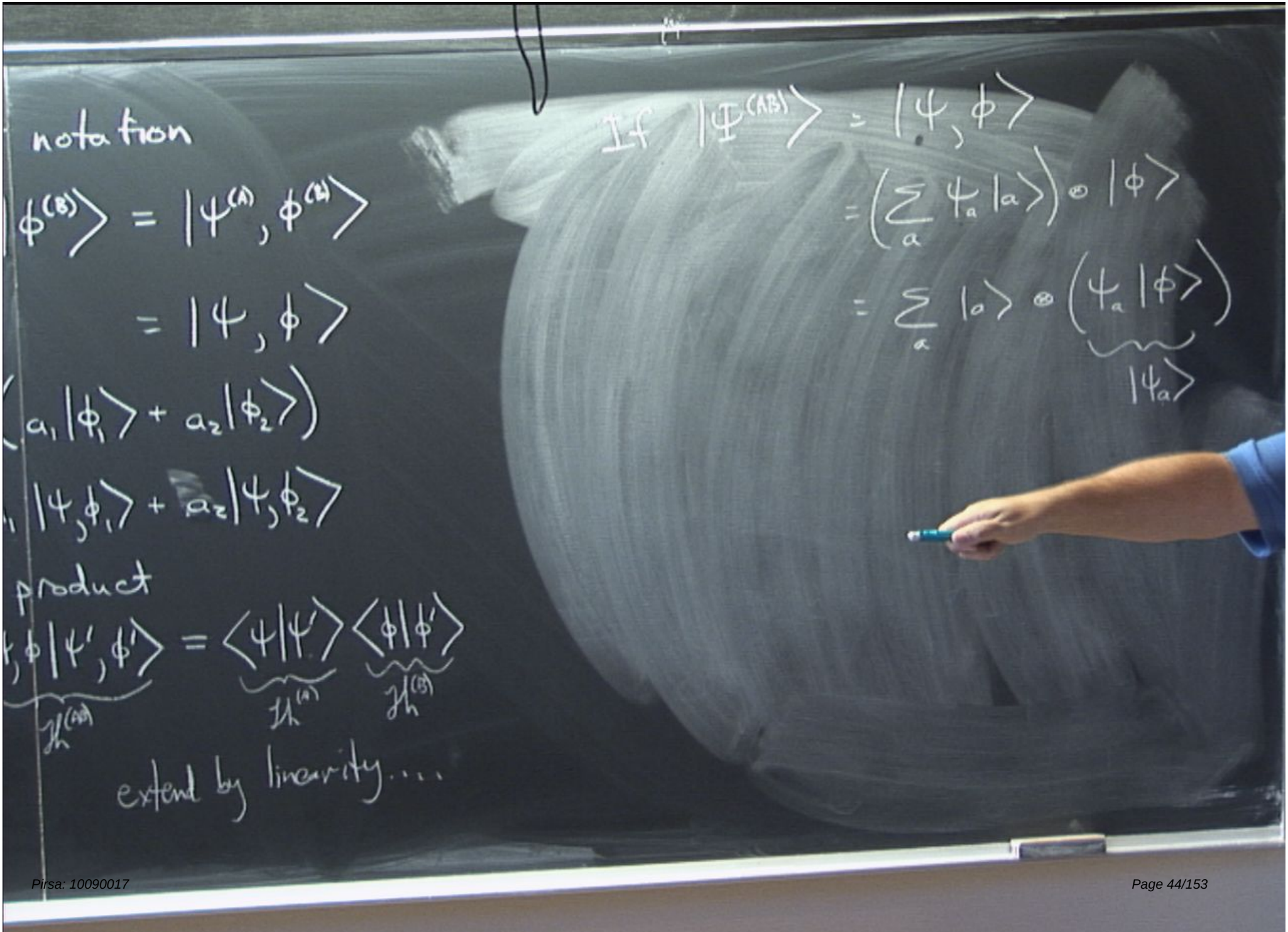
$\mathcal{H}^{(AB)}$

extend by linearity....

If $|\Psi^{(AB)}\rangle = |\psi,\phi\rangle$

$= \left(\sum_a \psi_a |a\rangle\right) \otimes |\phi\rangle$

$= \sum_a |a\rangle \otimes (\psi_a|\phi\rangle)$


notation
$|\phi^{(B)}\rangle = |\psi^{(A)}, \phi^{(B)}\rangle$
$= |\psi, \phi\rangle$
$(a_1|\phi_1\rangle + a_2|\phi_2\rangle)$
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$\mathcal{H}^{(AB)}$ $\mathcal{H}^{(A)}$ $\mathcal{H}^{(B)}$
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$\mathcal{H}^{(AB)}$

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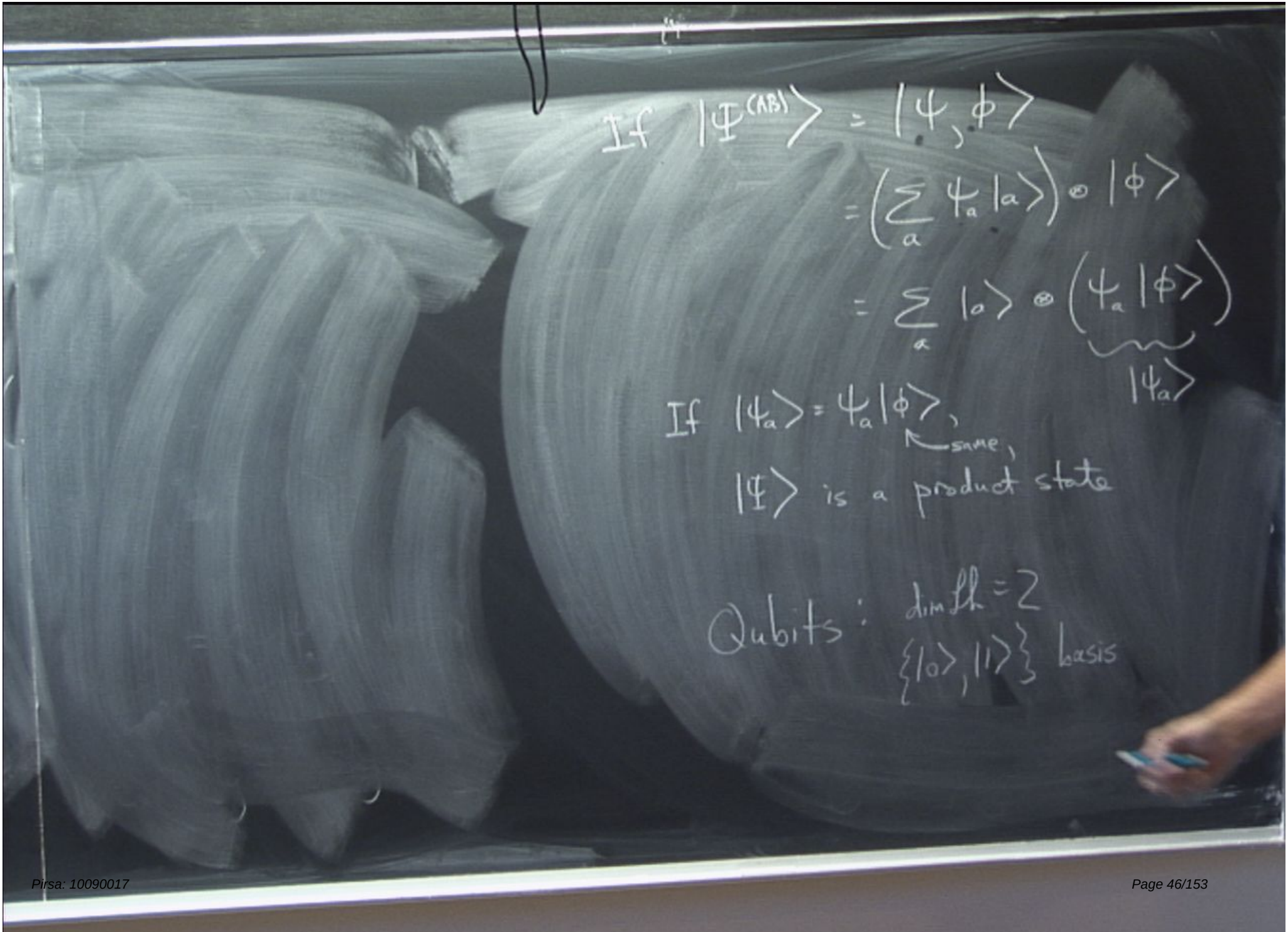
$= \sum_a |a\rangle \otimes \underbrace{(\psi_a|\phi\rangle)}_{|\psi_a\rangle}$

If $|\psi_a\rangle = \psi_a|\phi\rangle$, ← same,

$|\psi\rangle$ is a product state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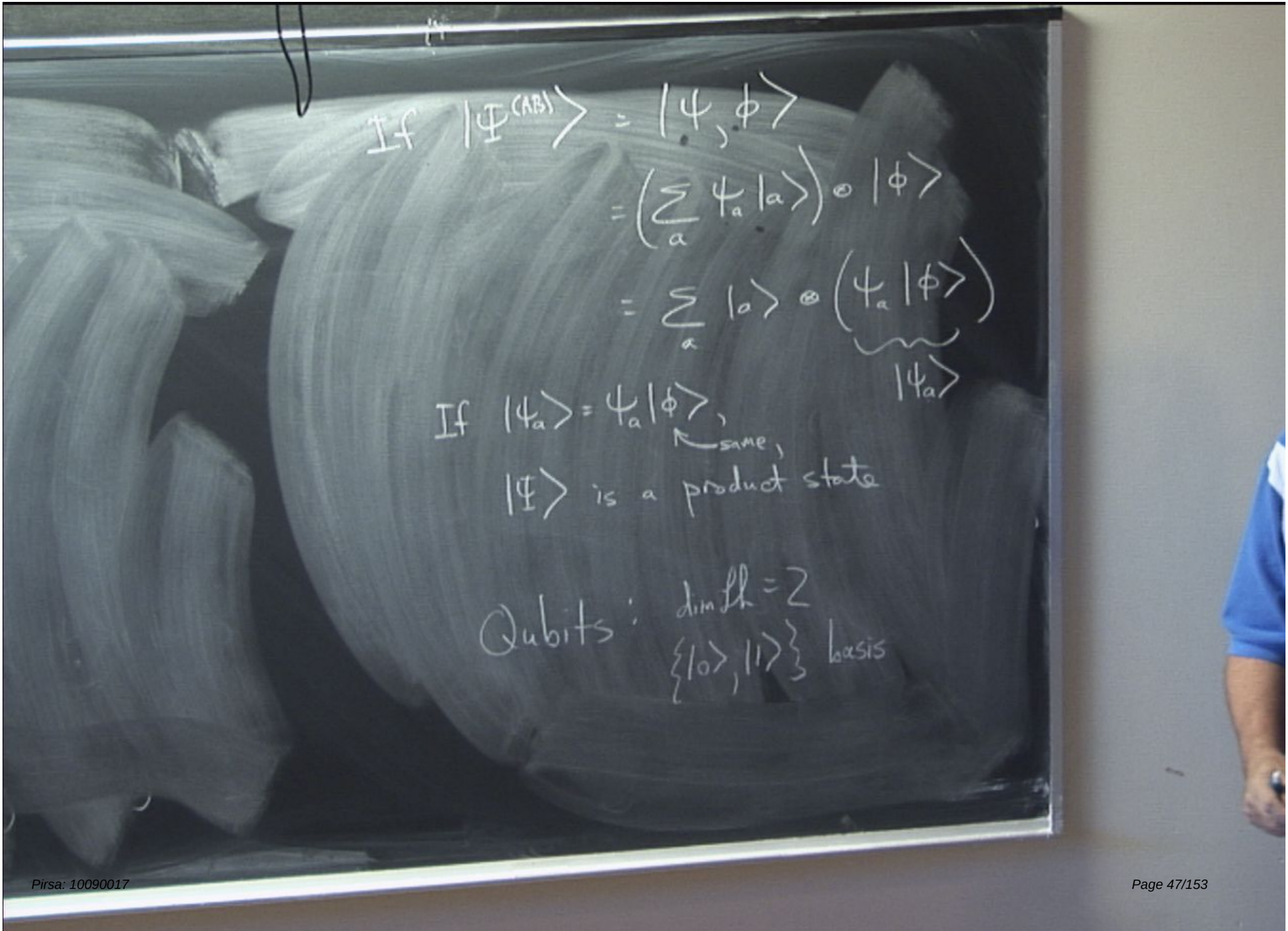
Qubits: $\dim \mathcal{H} = 2$
$\{|0\rangle, |1\rangle\}$ basis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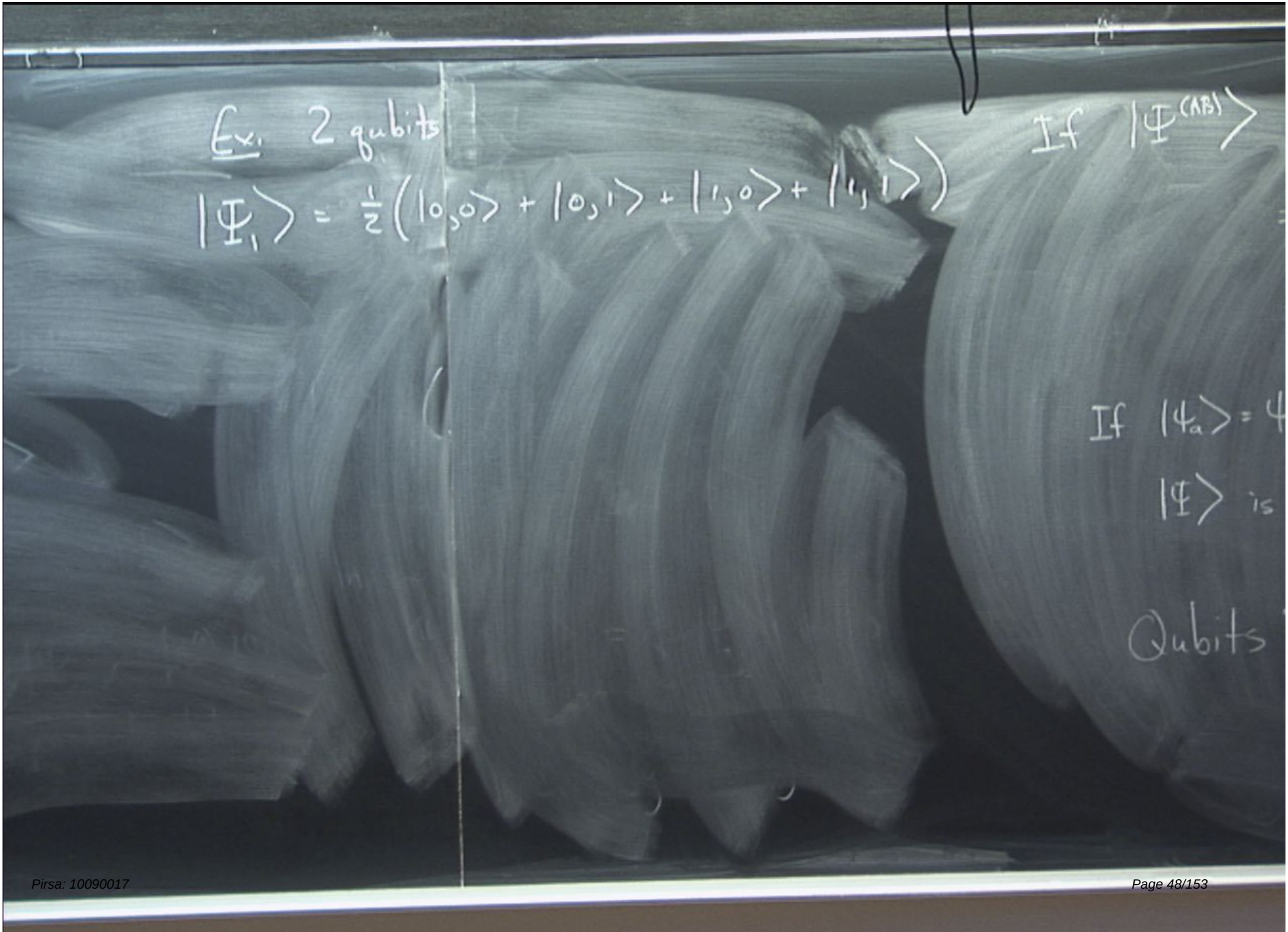
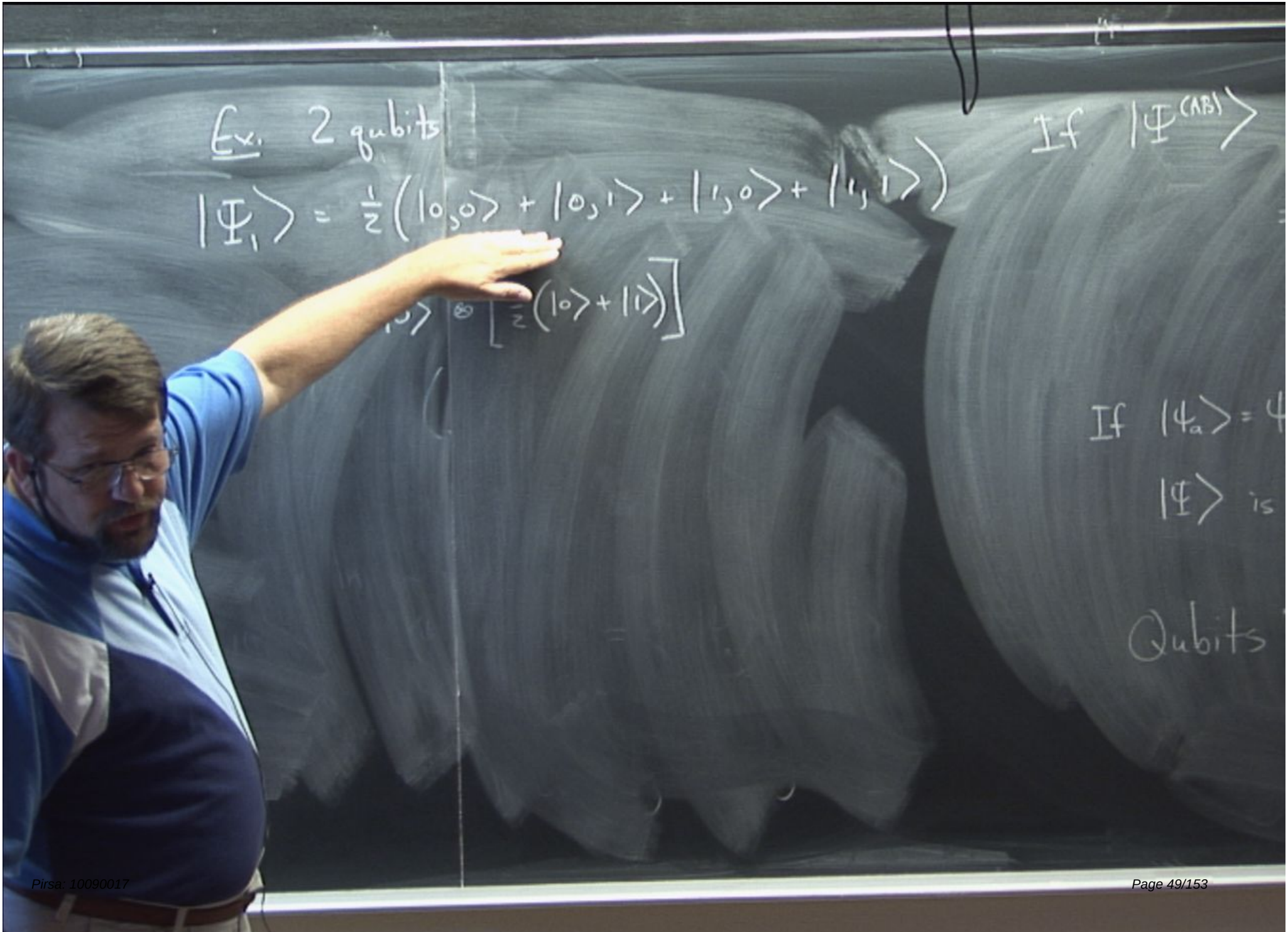

Ex. 2 qubits
$|\Phi_1\rangle = \frac{1}{2}(|0,0\rangle + |0,1\rangle + |1,0\rangle + |1,1\rangle)$
If $|\Psi^{(AB)}\rangle$
If $|\psi_a\rangle =$
$|\Psi\rangle$ is
Qubits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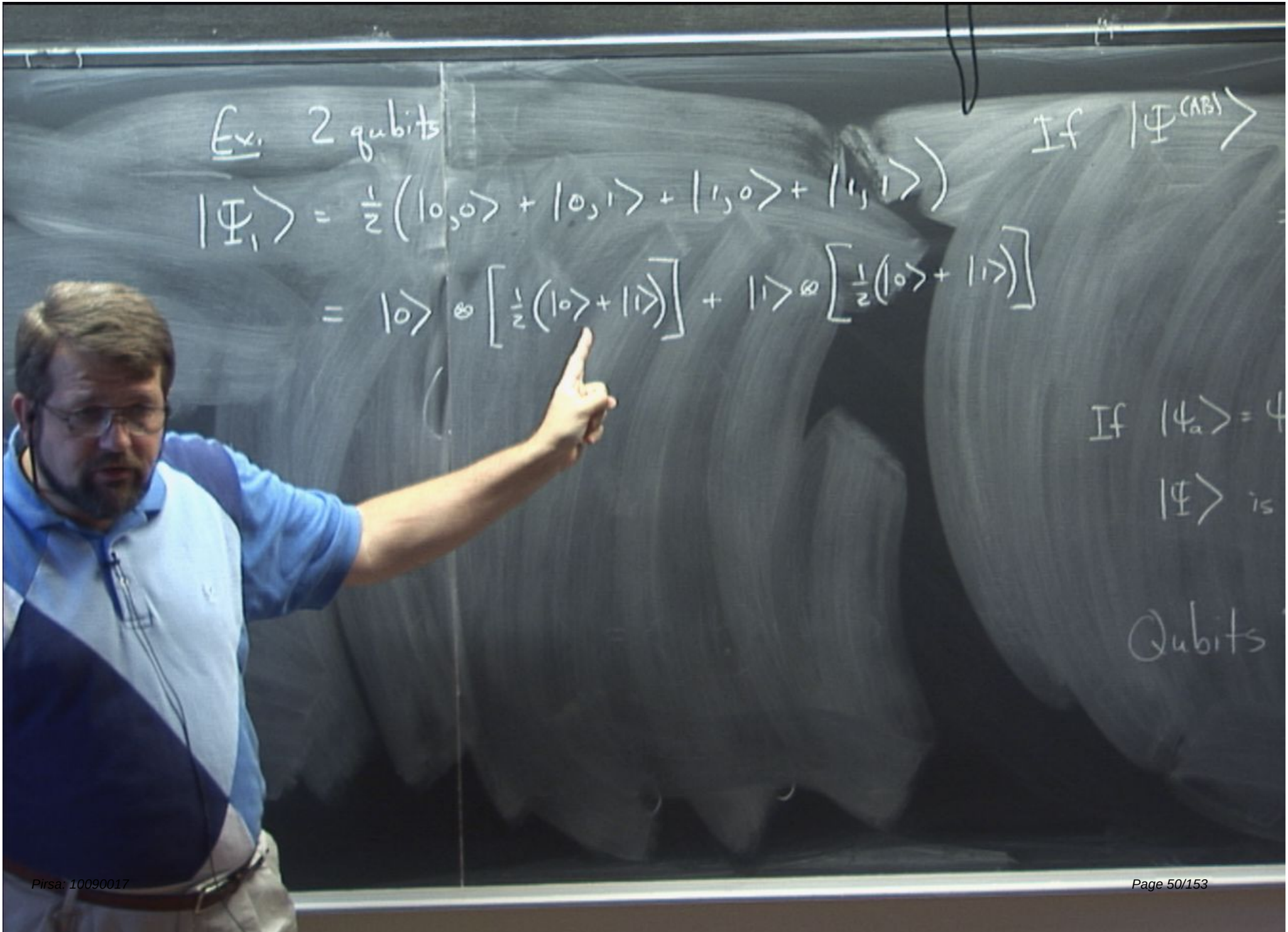
Ex. 2 qubits
$|\Psi_1\rangle = \frac{1}{2}(|0,0\rangle + |0,1\rangle + |1,0\rangle + |1,1\rangle)$
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If $|\Psi^{(AB)}\rangle$
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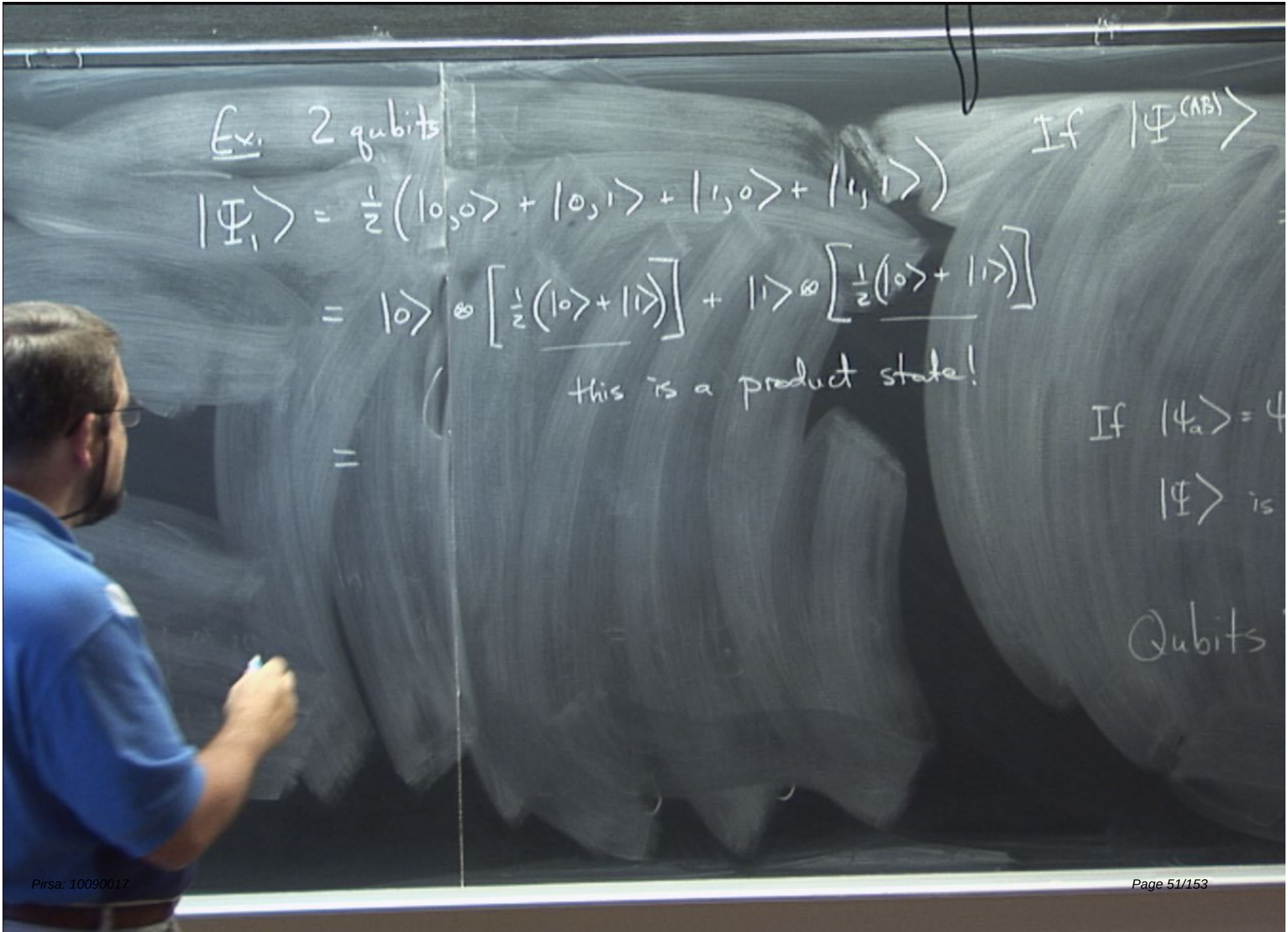
Qubits


Ex. 2 qubits
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$= |0\rangle \otimes \left[\frac{1}{2}(|0\rangle + |1\rangle)\right] + |1\rangle \otimes \left[\frac{1}{2}(|0\rangle + |1\rangle)\right]$
this is a product state!
=
If $|\Psi^{(AB)}\rangle$
If $|\psi_a\rangle$ = 4
$|\Psi\rangle$ is
Qubits

Ex. 2 qubits

$$|\Psi_1\rangle = \frac{1}{2}\left(|0,0\rangle + |0,1\rangle + |1,0\rangle + |1,1\rangle\right)$$

$$= |0\rangle \otimes \left[\frac{1}{2}\left(|0\rangle + |1\rangle\right)\right] + |1\rangle \otimes \left[\frac{1}{2}\left(|0\rangle + |1\rangle\right)\right]$$

this is a product state!

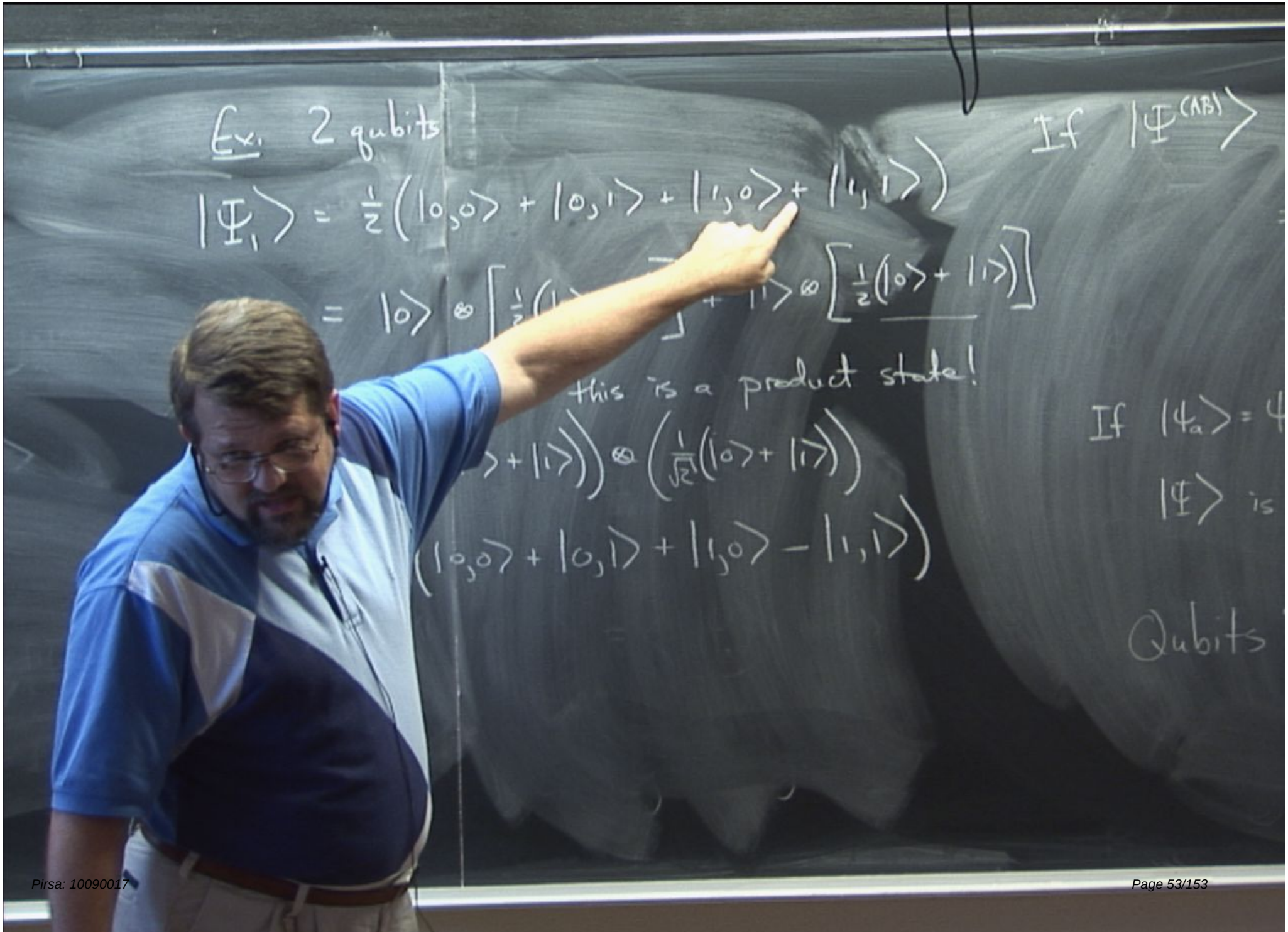
$$= \left(\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\right) \otimes \left(\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\right)$$

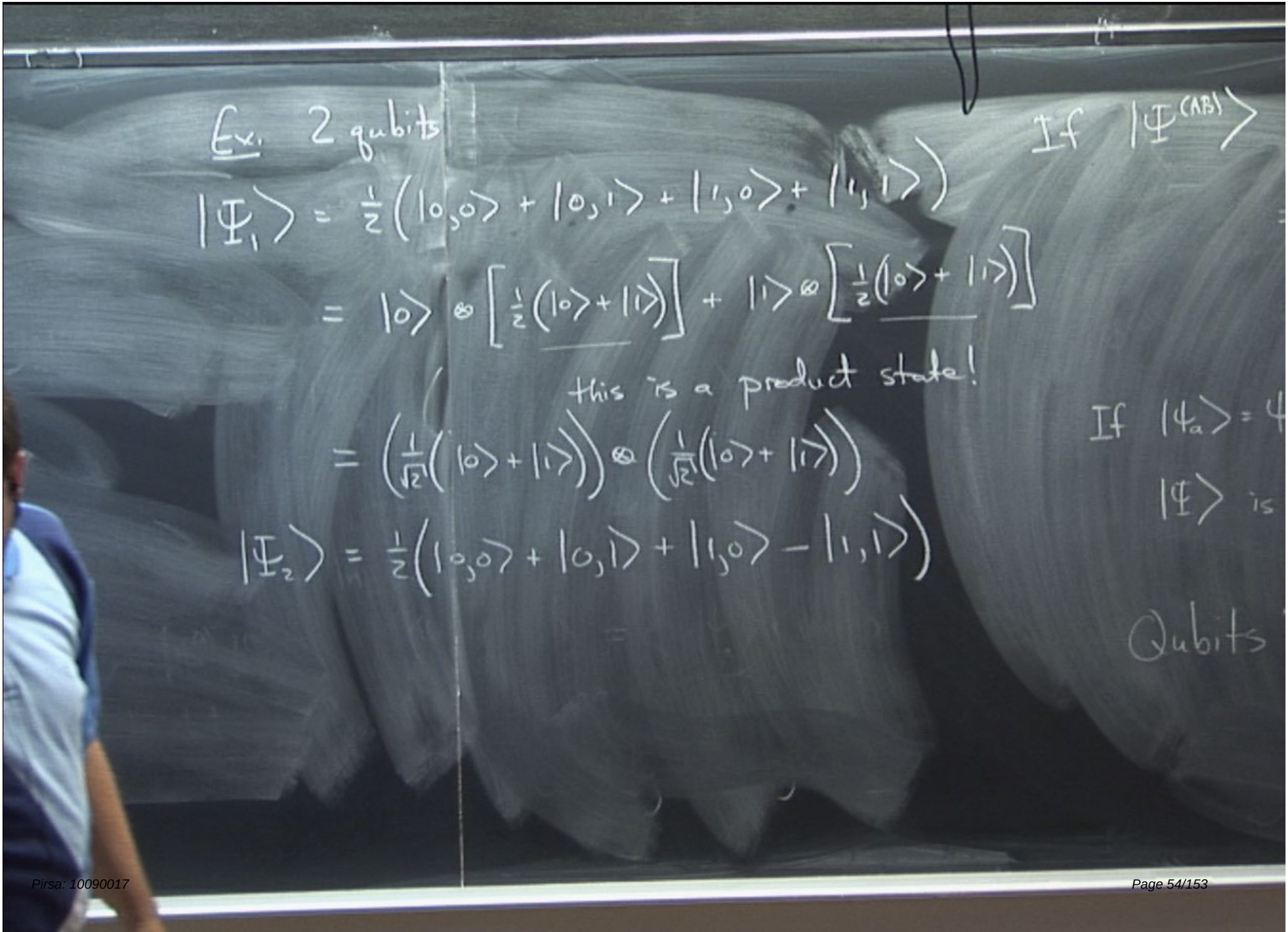
If $|\Psi^{(AB)}\rangle$

If $|\psi_a\rangle = \psi$

$|\Psi\rangle$ is

Qubits


Ex. 2 qubits
$|\Psi_1\rangle = \frac{1}{2}(|0,0\rangle + |0,1\rangle + |1,0\rangle + |1,1\rangle)$
$= |0\rangle \otimes [\quad] + |1\rangle \otimes \left[\frac{1}{2}(|0\rangle + |1\rangle)\right]$
this is a product state!
$\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right) \otimes \left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right)$
$(|0,0\rangle + |0,1\rangle + |1,0\rangle - |1,1\rangle)$
If $|\Psi^{(AB)}\rangle$
If $|\psi_a\rangle$ = 4
$|\Psi\rangle$ is
Qubits


Ex. 2 qubits
$|\Psi_1\rangle = \frac{1}{2}(|0,0\rangle + |0,1\rangle + |1,0\rangle + |1,1\rangle)$
$= |0\rangle \otimes \left[\frac{1}{2}(|0\rangle + |1\rangle)\right] + |1\rangle \otimes \left[\frac{1}{2}(|0\rangle + |1\rangle)\right]$
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$= \left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right) \otimes \left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right)$
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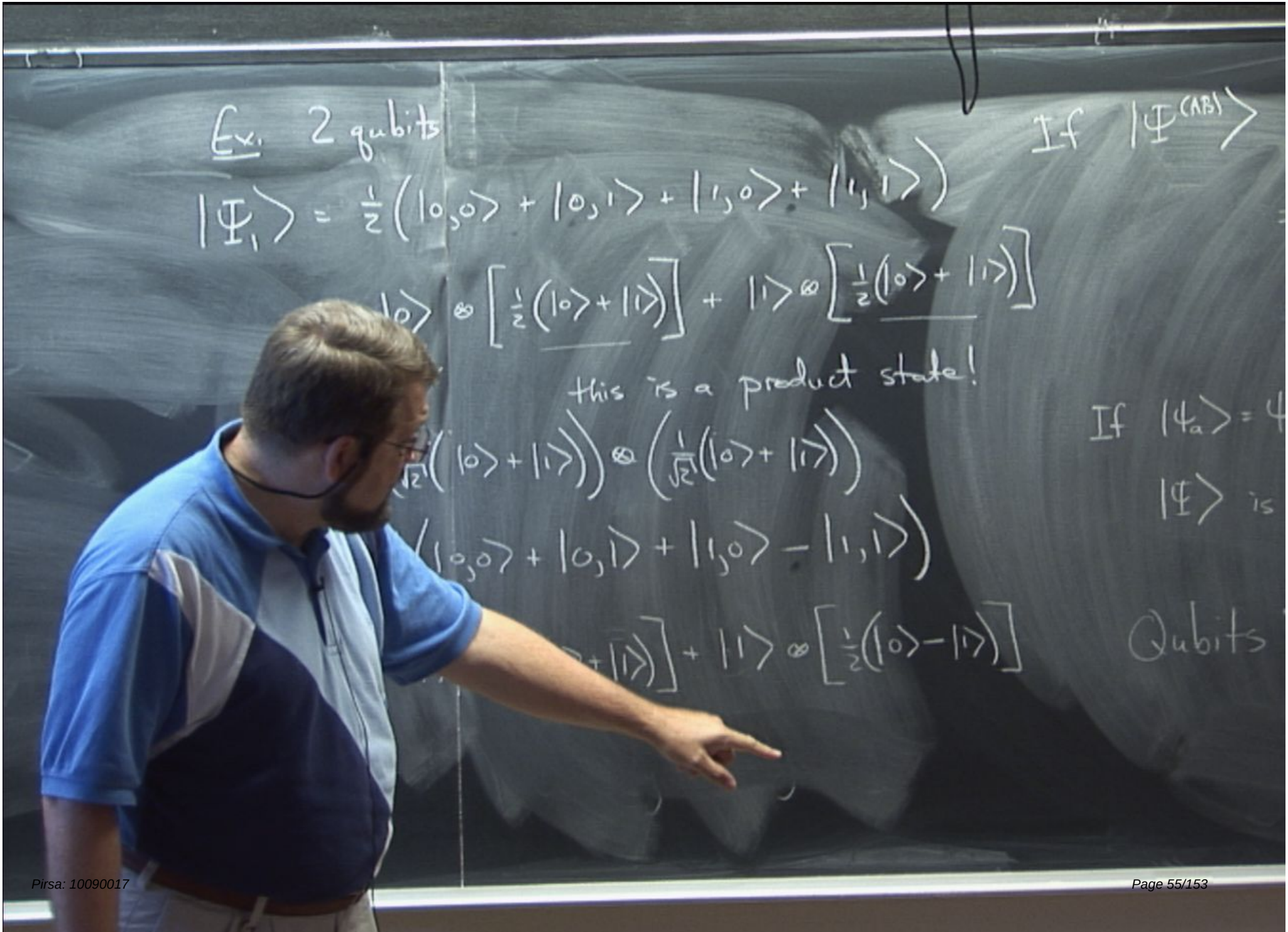
If $|\Psi^{(AB)}\rangle$
If $|\psi_a\rangle$ = ψ
$|\Psi\rangle$ is
Qubits


Ex. 2 qubits
$|\Psi_1\rangle = \frac{1}{2}(|0,0\rangle + |0,1\rangle + |1,0\rangle + |1,1\rangle)$
$|0\rangle \otimes \left[\frac{1}{2}(|0\rangle + |1\rangle)\right] + |1\rangle \otimes \left[\frac{1}{2}(|0\rangle + |1\rangle)\right]$
this is a product state!
$\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right) \otimes \left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right)$
$(|0,0\rangle + |0,1\rangle + |1,0\rangle - |1,1\rangle)$
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Qubits

Ex. 2 qubits

$$|\Psi_1\rangle = \frac{1}{2}\left(|0,0\rangle + |0,1\rangle + |1,0\rangle + |1,1\rangle\right)$$

$$= |0\rangle \otimes \left[\frac{1}{2}\left(|0\rangle + |1\rangle\right)\right] + |1\rangle \otimes \left[\frac{1}{2}\left(|0\rangle + |1\rangle\right)\right]$$

this is a product state!

$$= \left(\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\right) \otimes \left(\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\right)$$

$$|\Psi\rangle = \frac{1}{2}\left(|0,0\rangle + |0,1\rangle + |1,0\rangle - |1,1\rangle\right)$$

$$= |0\rangle \otimes \left[\frac{1}{2}\left(|0\rangle + |1\rangle\right)\right] + |1\rangle \otimes \left[\frac{1}{2}\left(|0\rangle - |1\rangle\right)\right]$$

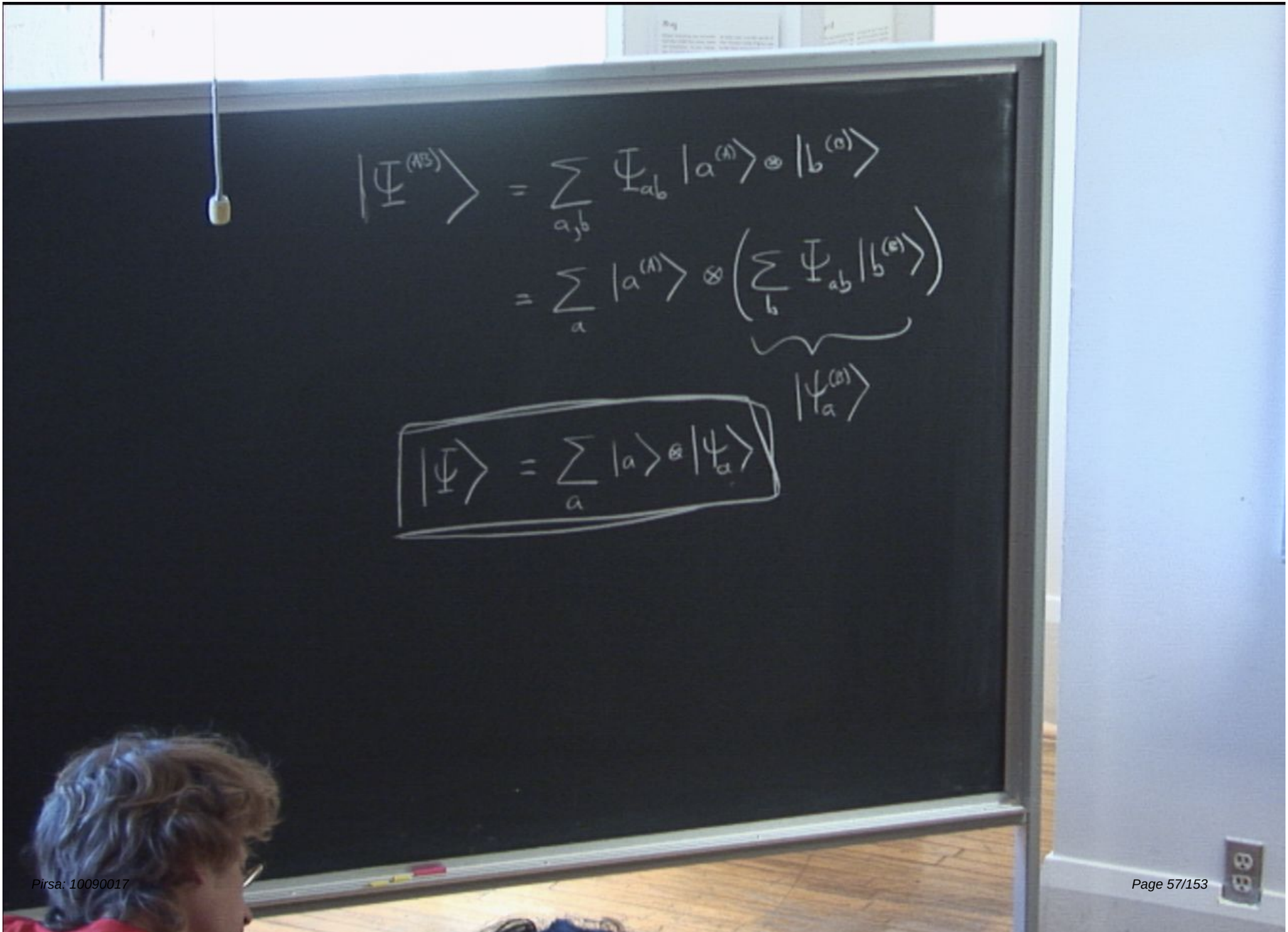
entangled state!

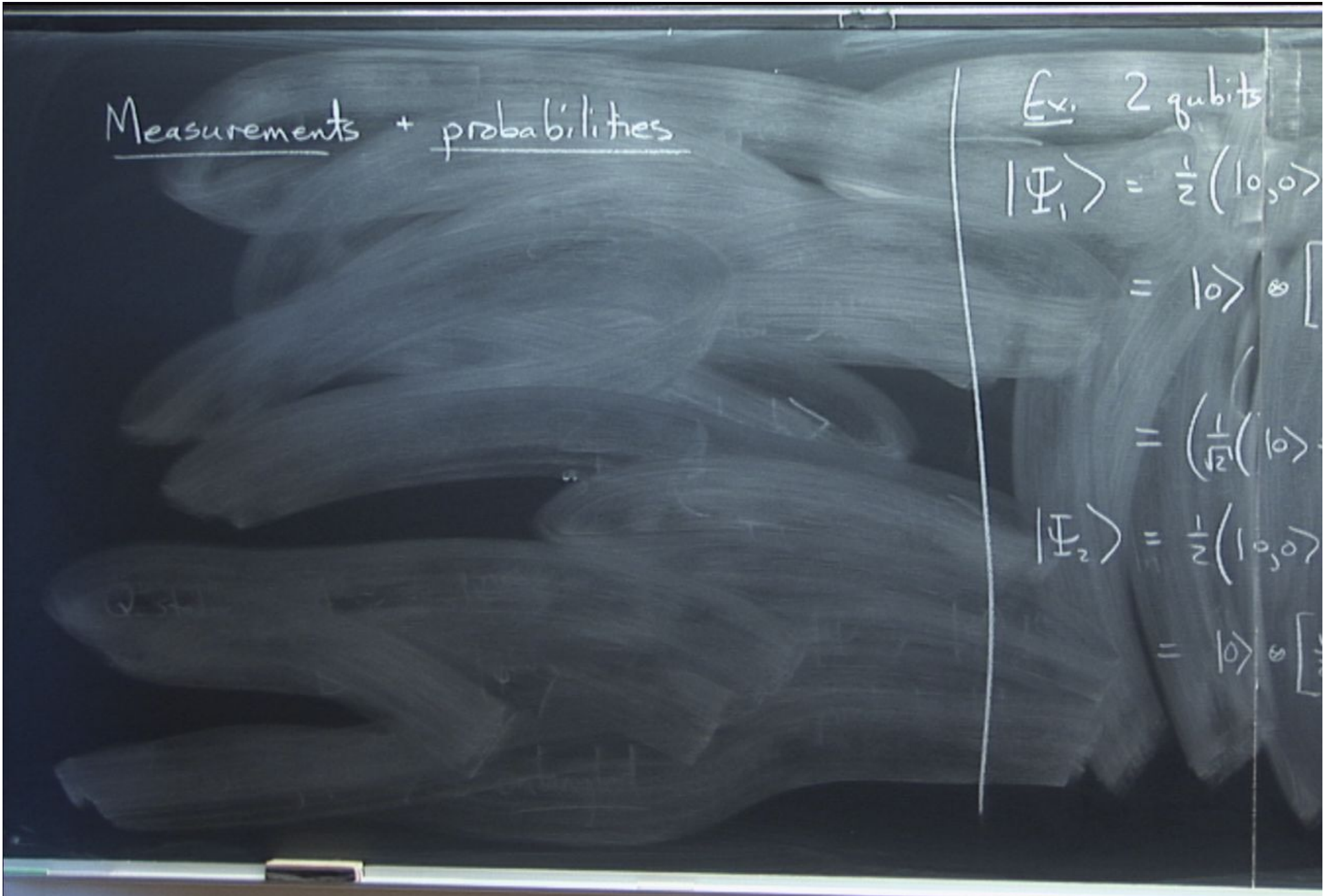
If $|\Psi^{(AB)}\rangle$

If $|\psi_a\rangle = \psi$

$|\Psi\rangle$ is

Qubits


$|\Psi^{(AB)}\rangle = \sum_{a,b} \Psi_{ab} |a^{(A)}\rangle \otimes |b^{(B)}\rangle$
$= \sum_{a} |a^{(A)}\rangle \otimes \left(\sum_{b} \Psi_{ab} |b^{(B)}\rangle \right)$
$|\psi_a^{(B)}\rangle$
$|\Psi\rangle = \sum_{a} |a\rangle \otimes |\psi_a\rangle$


Measurements + probabilities
Ex. 2 qubits
$|\Psi_1\rangle = \frac{1}{2}(|0,0\rangle$
$= |0\rangle \otimes [$
$= \left(\frac{1}{\sqrt{2}}(|0\rangle$
$|\Psi_2\rangle = \frac{1}{2}(|0,0\rangle$
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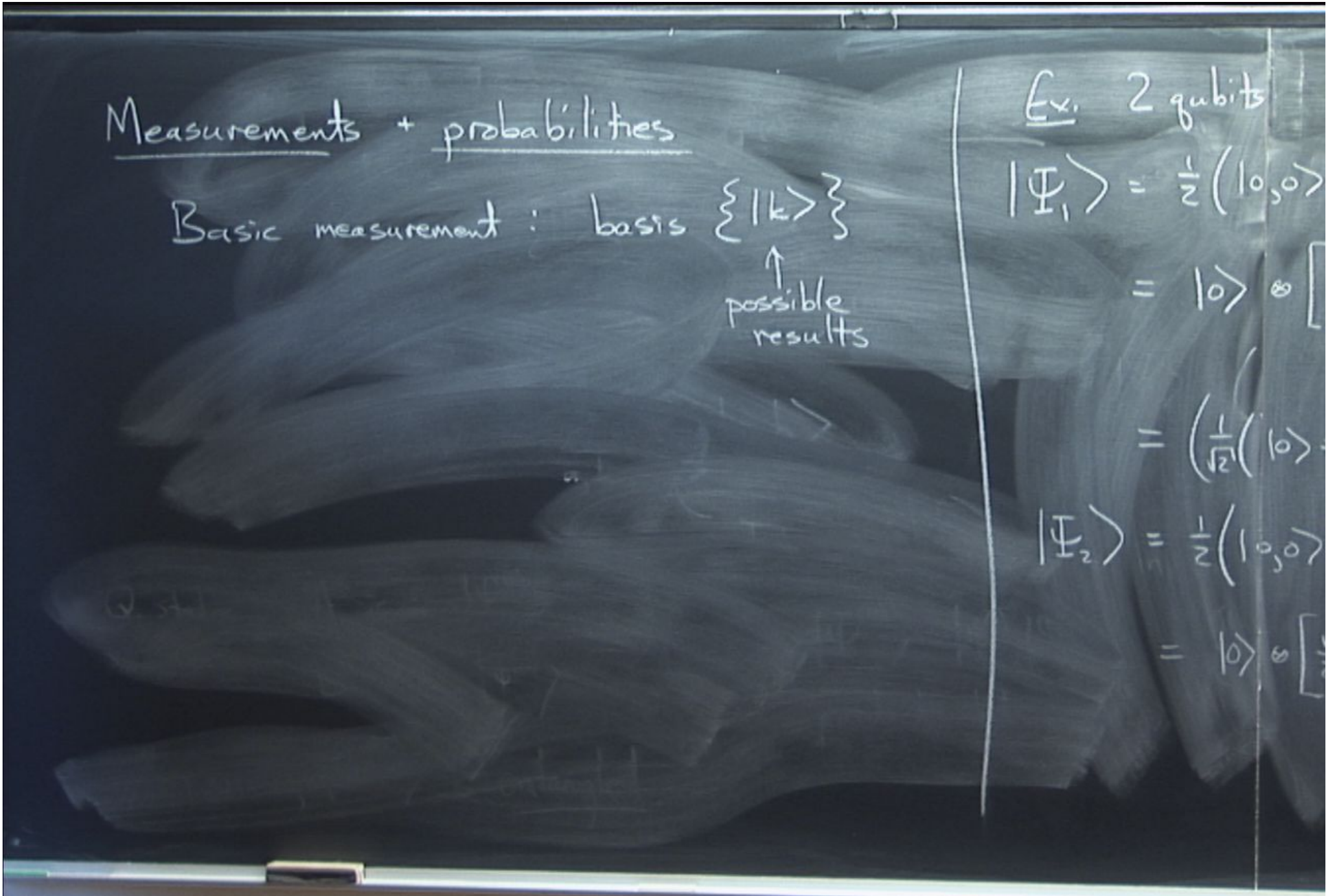
Measurements + probabilities
Basic measurement : basis $\{|k\rangle\}$
possible results
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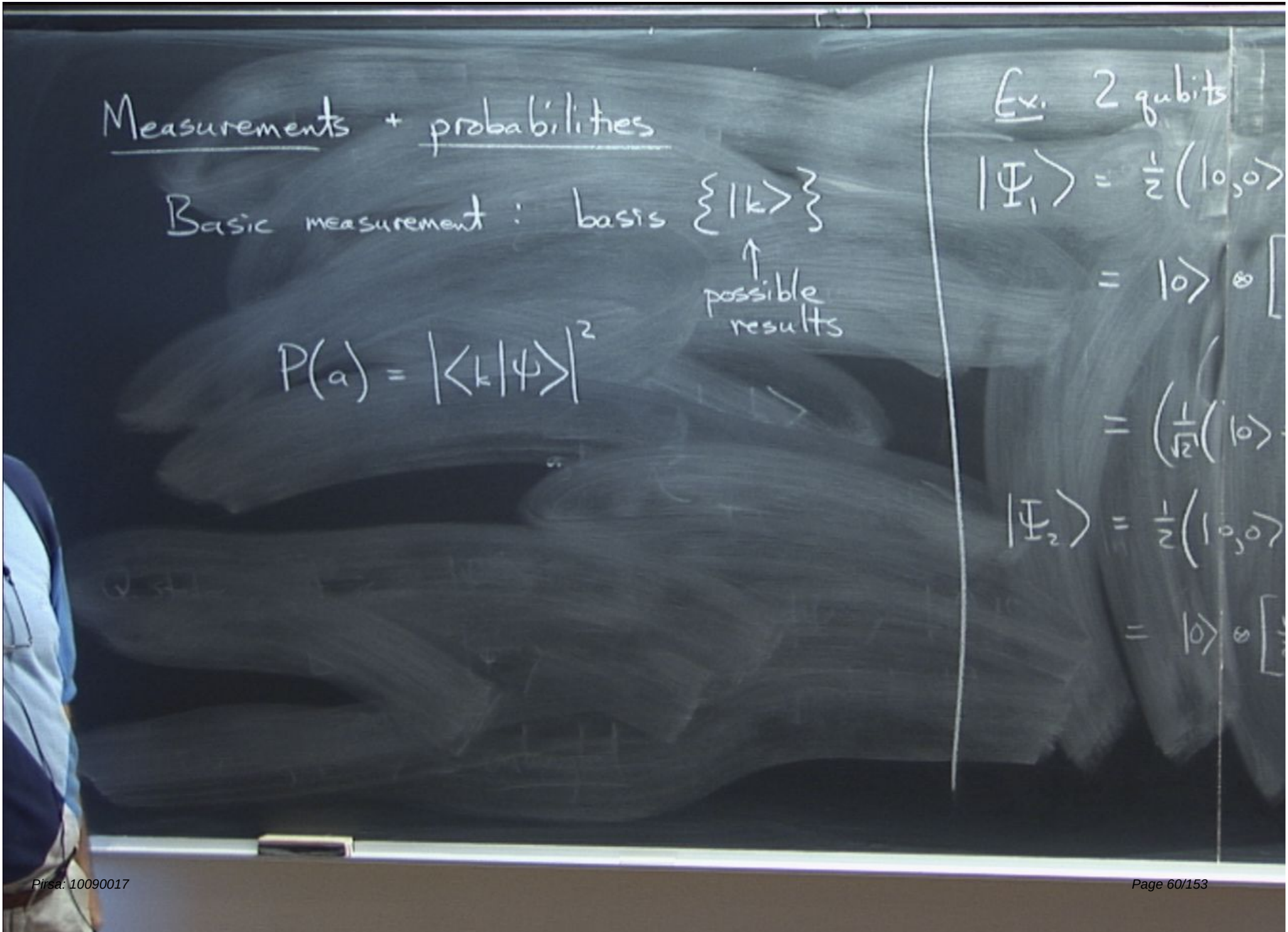
Measurements + probabilities
Basic measurement : basis $\{|k\rangle\}$
possible results
$P(a) = |\langle k|\psi\rangle|^2$
Ex. 2 qubits
$|\Psi_1\rangle = \frac{1}{2}(|0,0\rangle$
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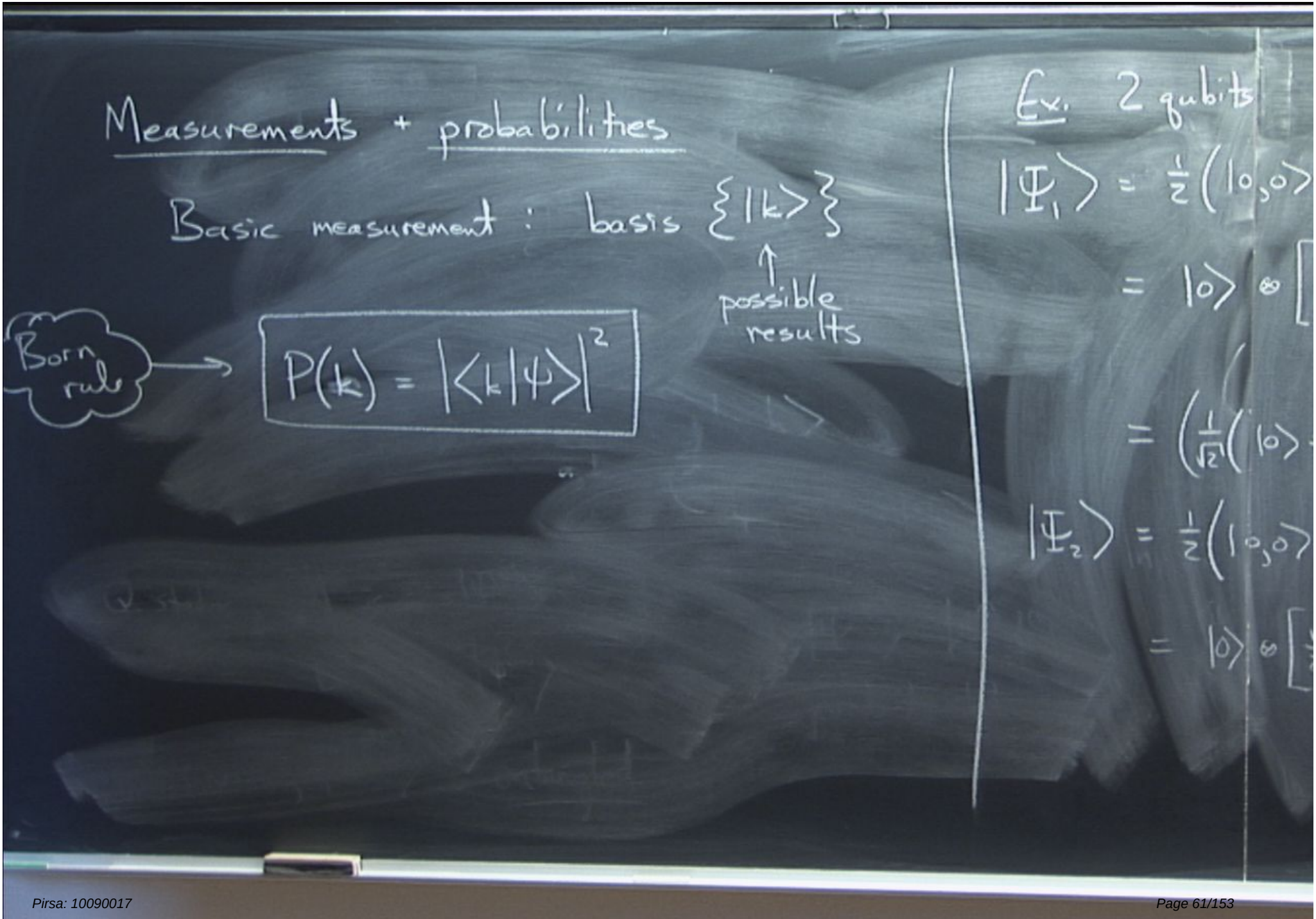
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Measurements + probabilities

Basic measurement : basis $\{|k\rangle\}$

↑ possible results

Born rule → $P(k) = |\langle k|\psi\rangle|^2$

Joint measurements : basis $\{|a\rangle\}$ on A

$\{|b\rangle\}$ on B

Ex. 2 qubits

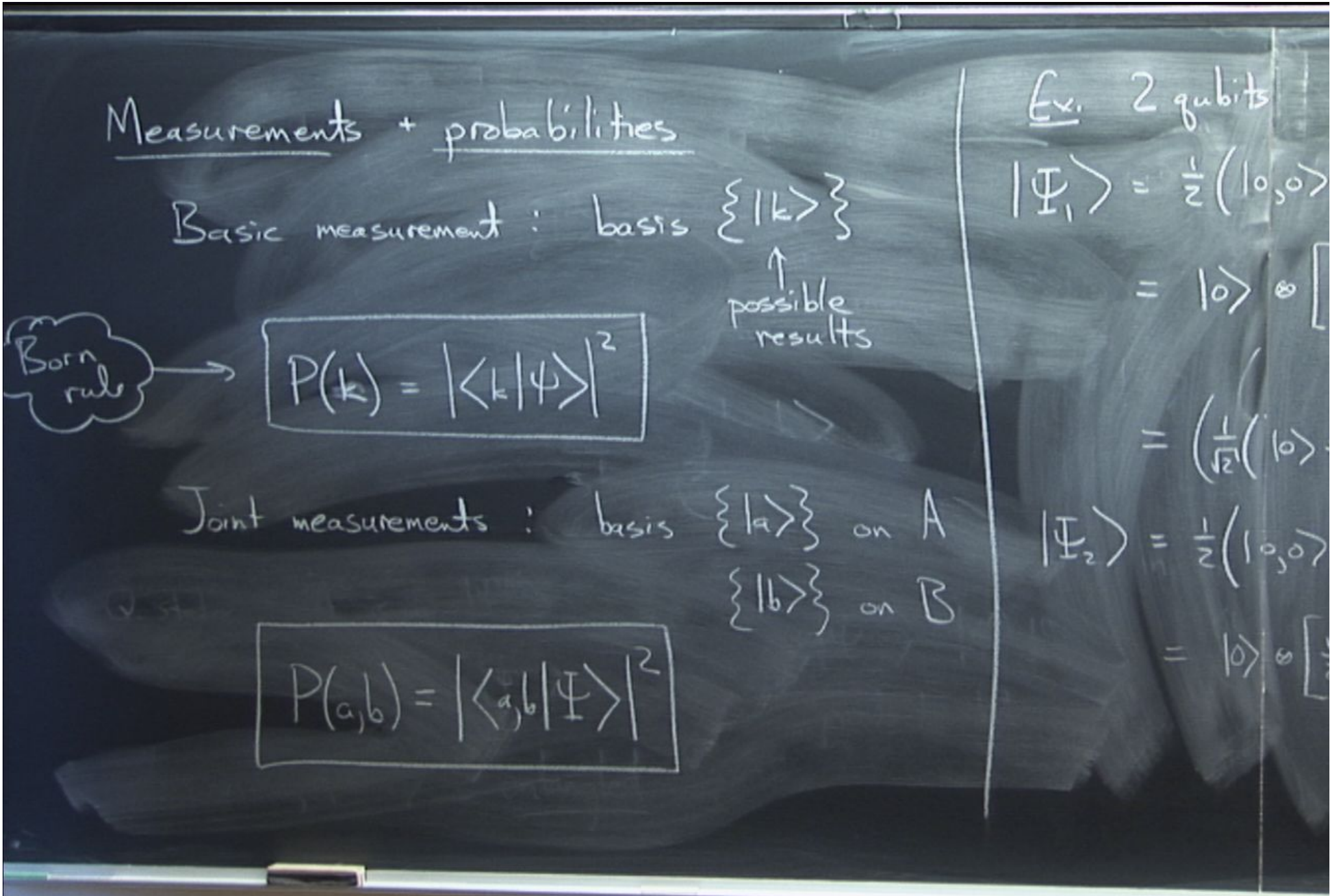
$|\Psi_1\rangle = \frac{1}{2}(|0,0\rangle$

$= |0\rangle \otimes [$

$= \left(\frac{1}{\sqrt{2}}(|0\rangle$

$|\Psi_2\rangle = \frac{1}{2}(|0,0\rangle$

$= |0\rangle \otimes [$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$P(a,b) = |\langle a,b|\Psi\rangle|^2$
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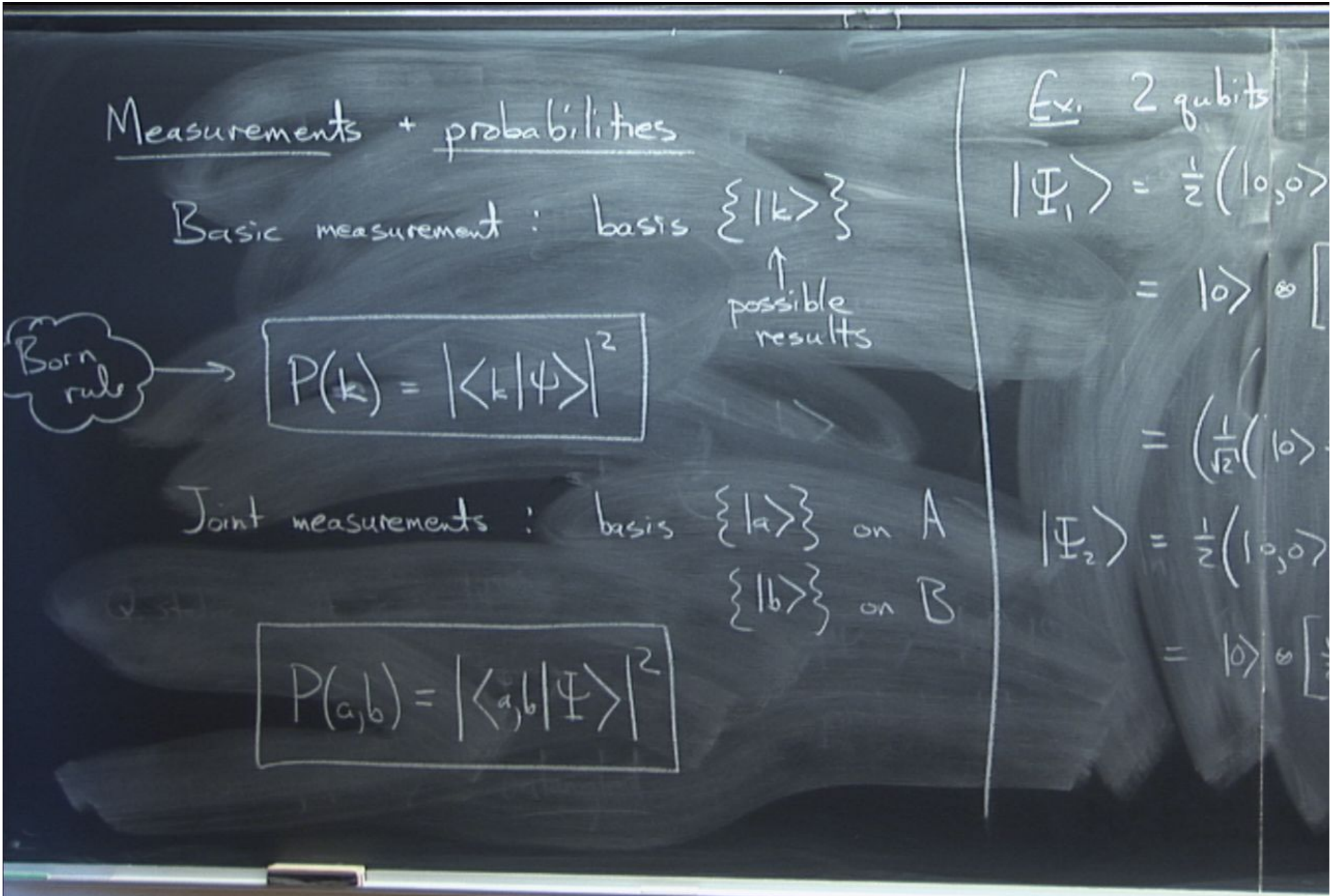
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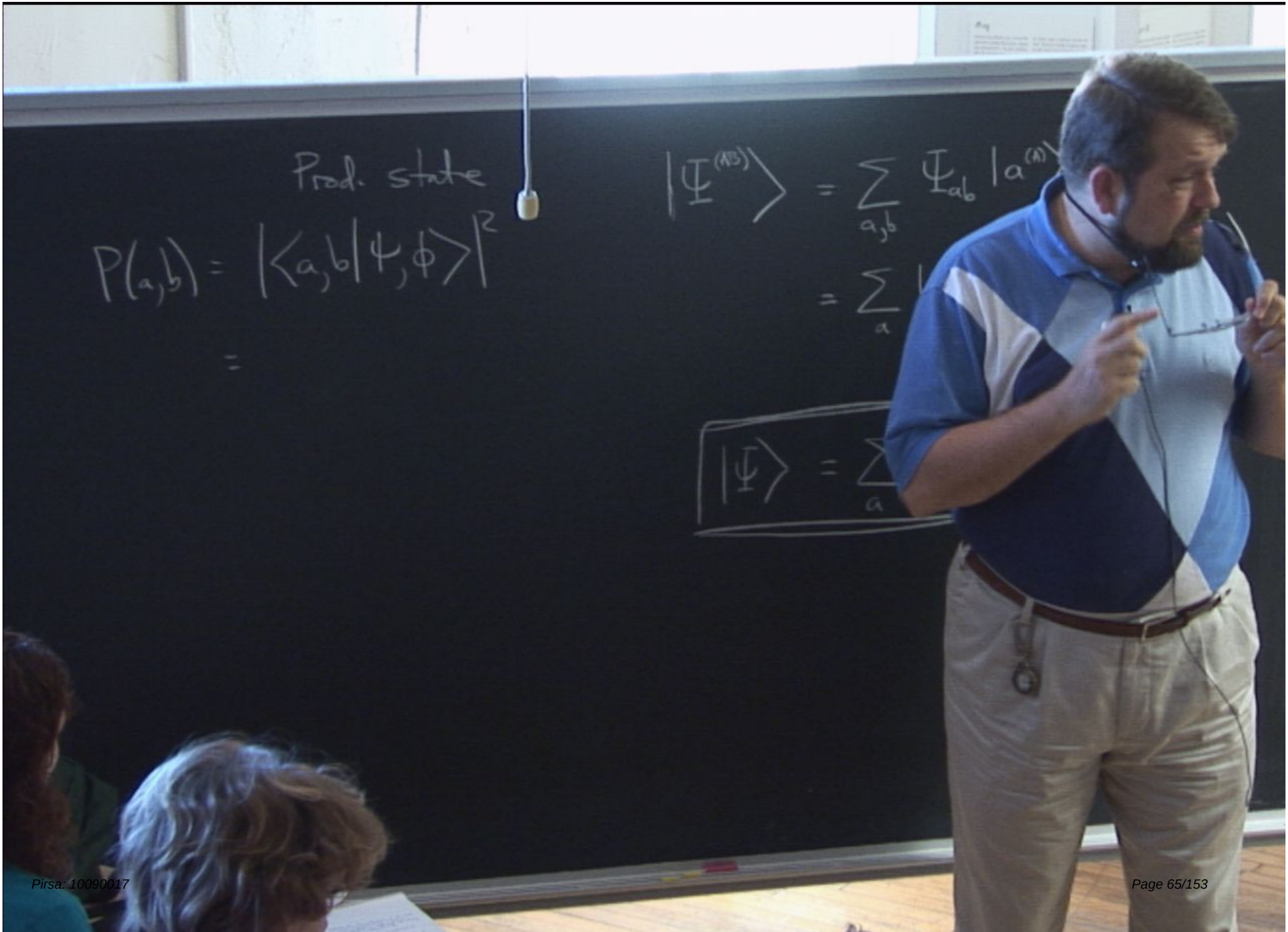
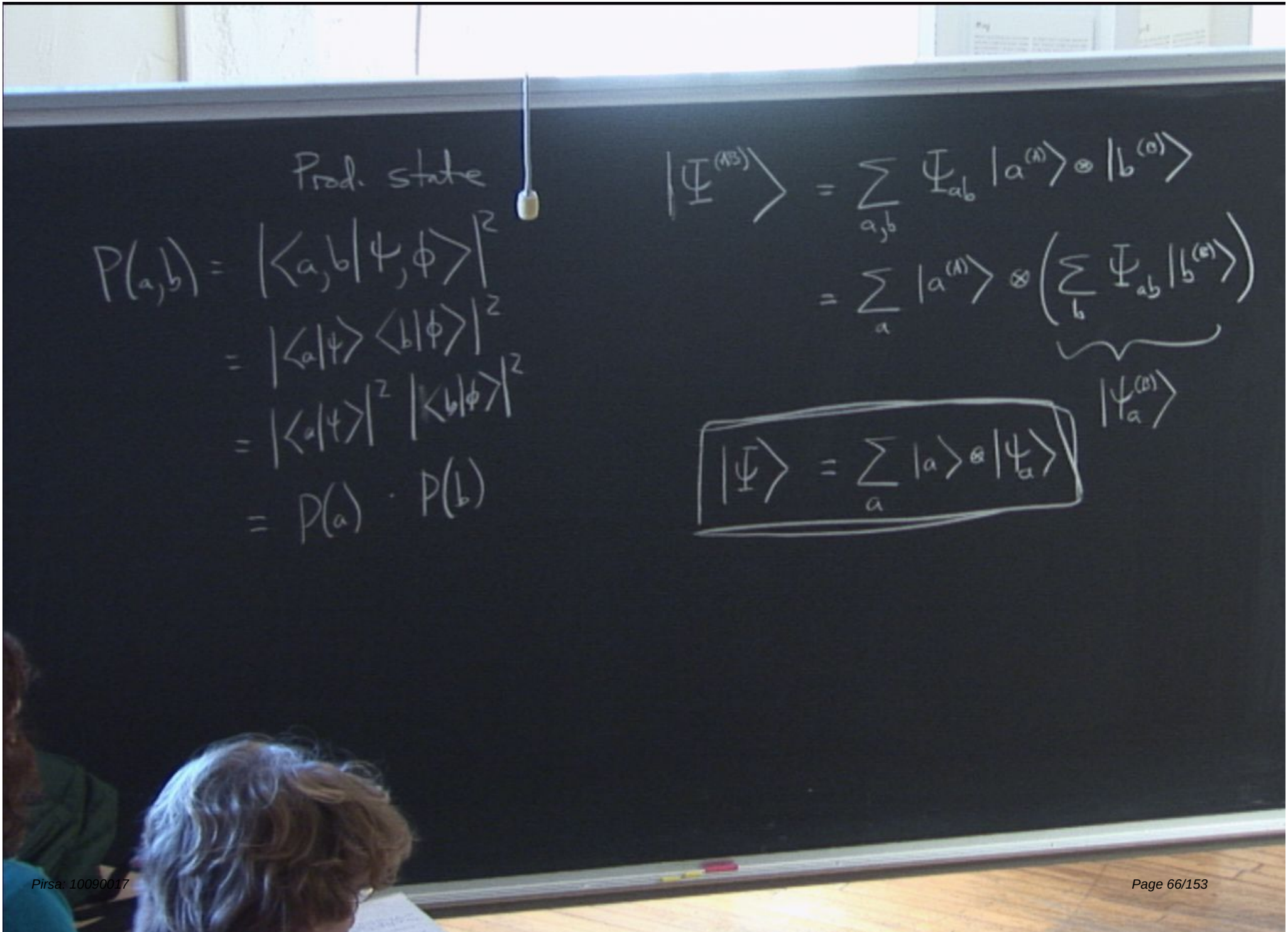
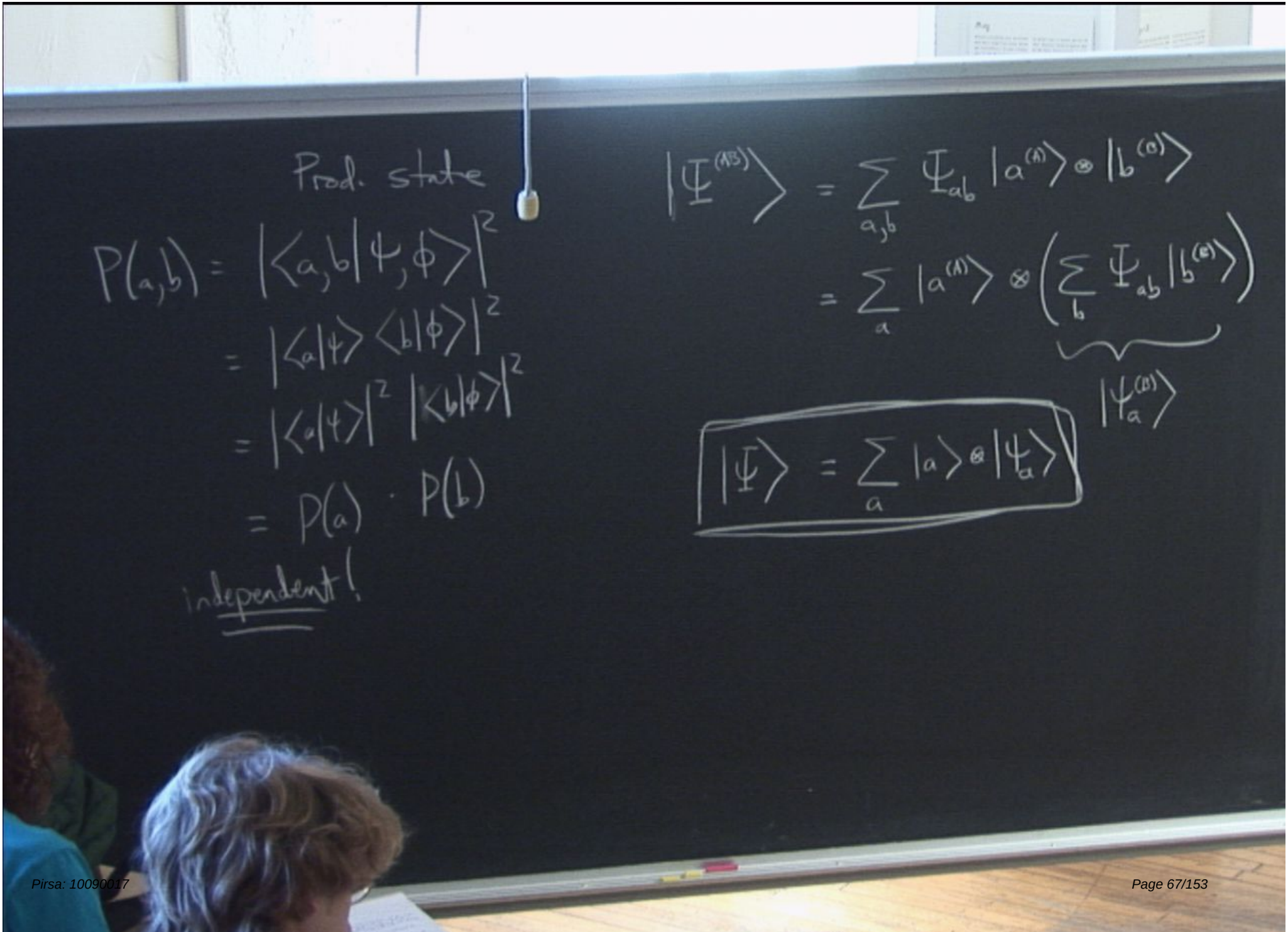
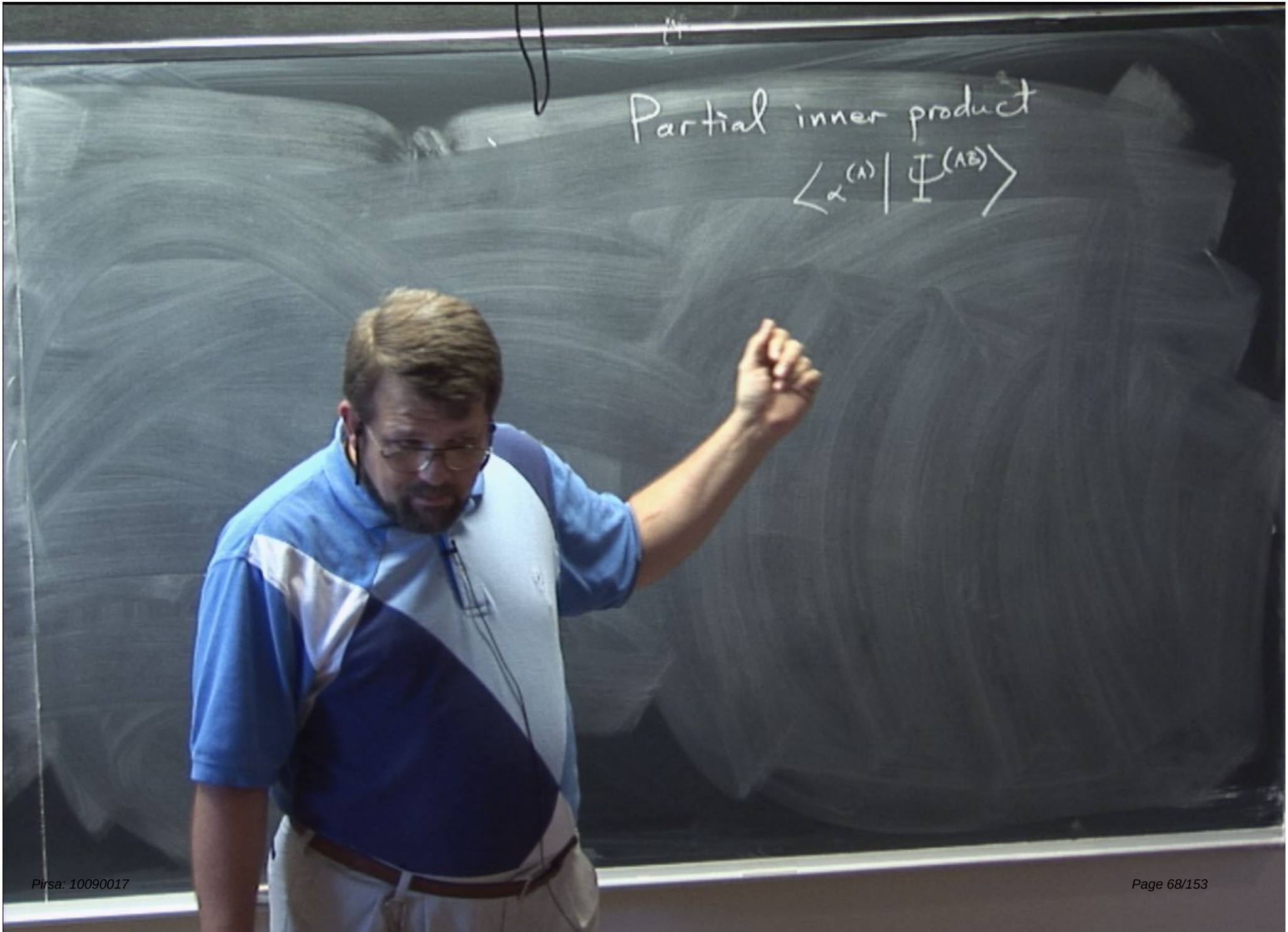
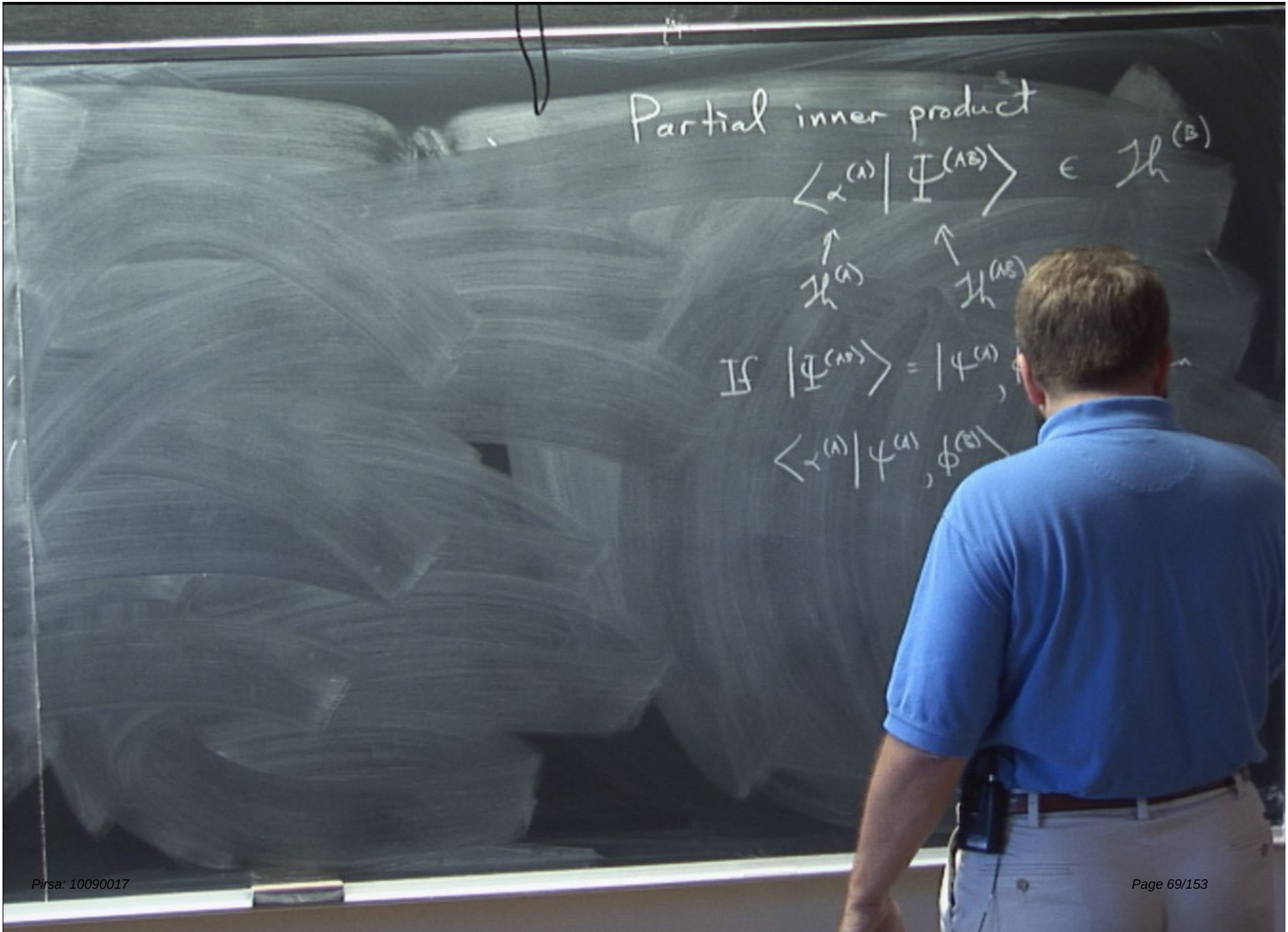
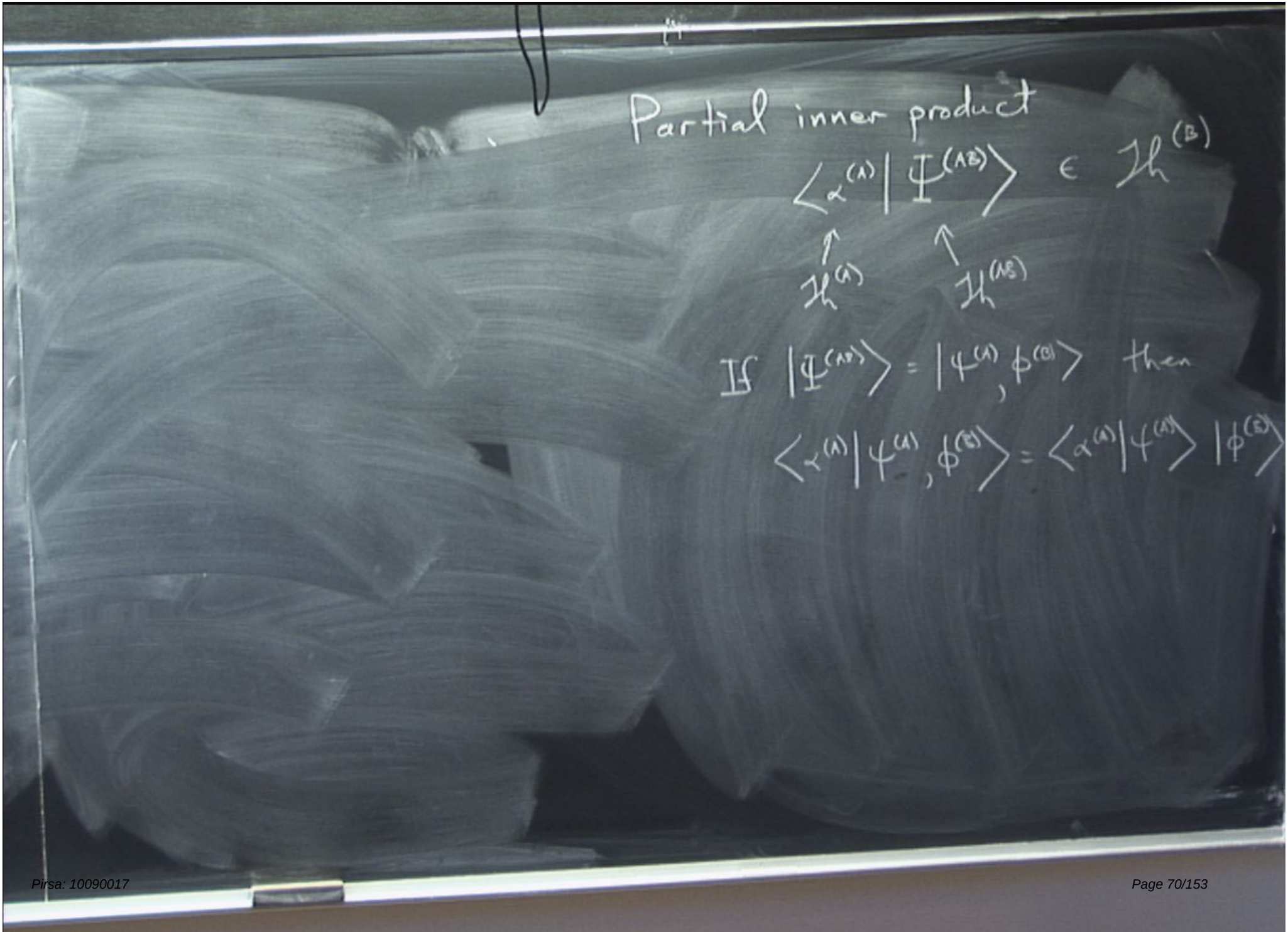

Prod. state
$P(a,b) = |\langle a,b|\psi,\phi\rangle|^2$
$= \sum_{a,b} \Psi_{ab} |a^{(A)}$
$= \sum_{a}$
$|\Psi\rangle = \sum_{a}$


Prod. state
$P(a,b) = |\langle a,b|\psi,\phi\rangle|^2$
$= |\langle a|\psi\rangle\langle b|\phi\rangle|^2$
$= |\langle a|\psi\rangle|^2 |\langle b|\phi\rangle|^2$
$= P(a) \cdot P(b)$
$|\Psi^{(AB)}\rangle = \sum_{a,b} \Psi_{ab} |a^{(A)}\rangle \otimes |b^{(B)}\rangle$
$= \sum_a |a^{(A)}\rangle \otimes \left(\sum_b \Psi_{ab} |b^{(B)}\rangle\right)$
$|\psi_a^{(B)}\rangle$
$|\Psi\rangle = \sum_a |a\rangle \otimes |\psi_a\rangle$


Prod. state
$P(a,b) = |\langle a,b|\psi,\phi\rangle|^2$
$= |\langle a|\psi\rangle\langle b|\phi\rangle|^2$
$= |\langle a|\psi\rangle|^2 |\langle b|\phi\rangle|^2$
$= P(a) \cdot P(b)$
independent!
$|\Psi^{(AB)}\rangle = \sum_{a,b} \Psi_{ab} |a^{(A)}\rangle \otimes |b^{(B)}\rangle$
$= \sum_a |a^{(A)}\rangle \otimes \left(\sum_b \Psi_{ab} |b^{(B)}\rangle\right)$
$|\psi_a^{(B)}\rangle$
$|\Psi\rangle = \sum_a |a\rangle \otimes |\psi_a\rangle$


Partial inner product
$\langle \alpha^{(1)} | \Psi^{(AB)} \rangle$


Partial inner product


Partial inner product
$\langle \alpha^{(A)} | \Psi^{(AB)} \rangle \in \mathcal{H}^{(B)}$
$\mathcal{H}^{(A)}$
$\mathcal{H}^{(AB)}$
If $|\Psi^{(AB)}\rangle = |\varphi^{(A)}, \phi^{(B)}\rangle$ then
$\langle \alpha^{(A)} | \varphi^{(A)}, \phi^{(B)} \rangle = \langle \alpha^{(A)} | \varphi^{(A)} \rangle |\phi^{(B)}\rangle$

Partial inner product

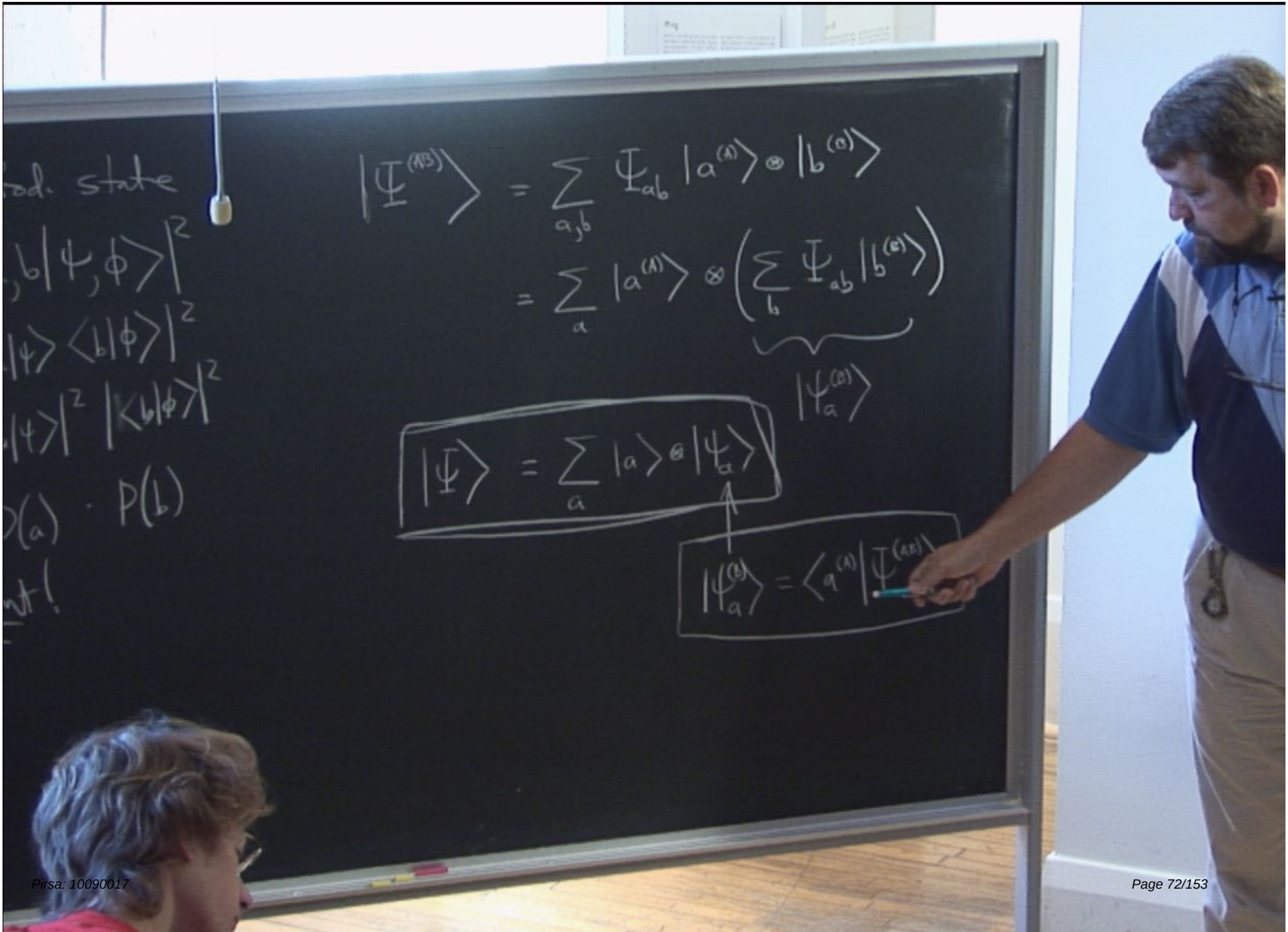
$$\langle \alpha^{(A)} | \Psi^{(AB)} \rangle \in \mathcal{H}^{(B)}$$

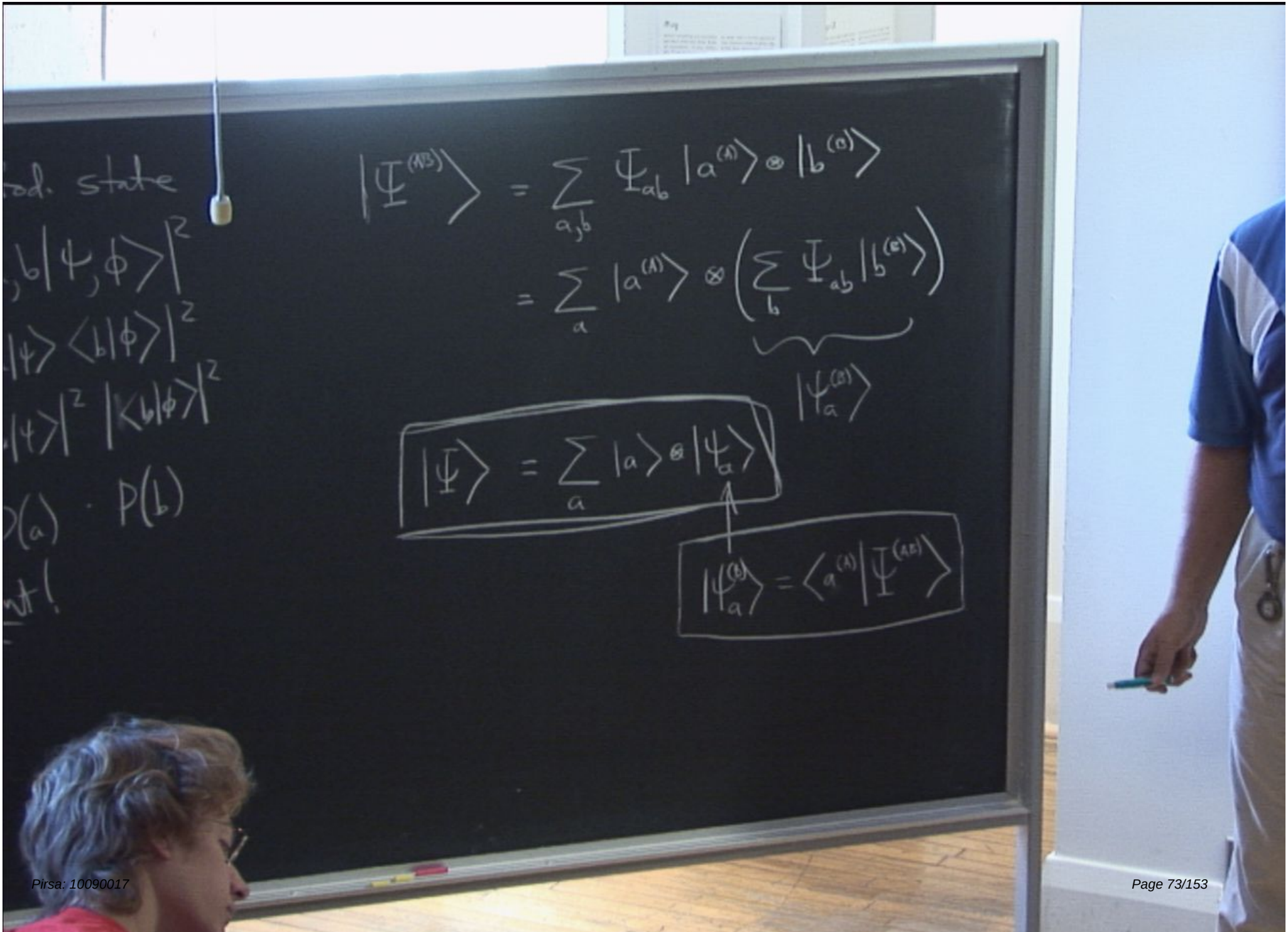
$\mathcal{H}^{(A)}$ $\mathcal{H}^{(AB)}$

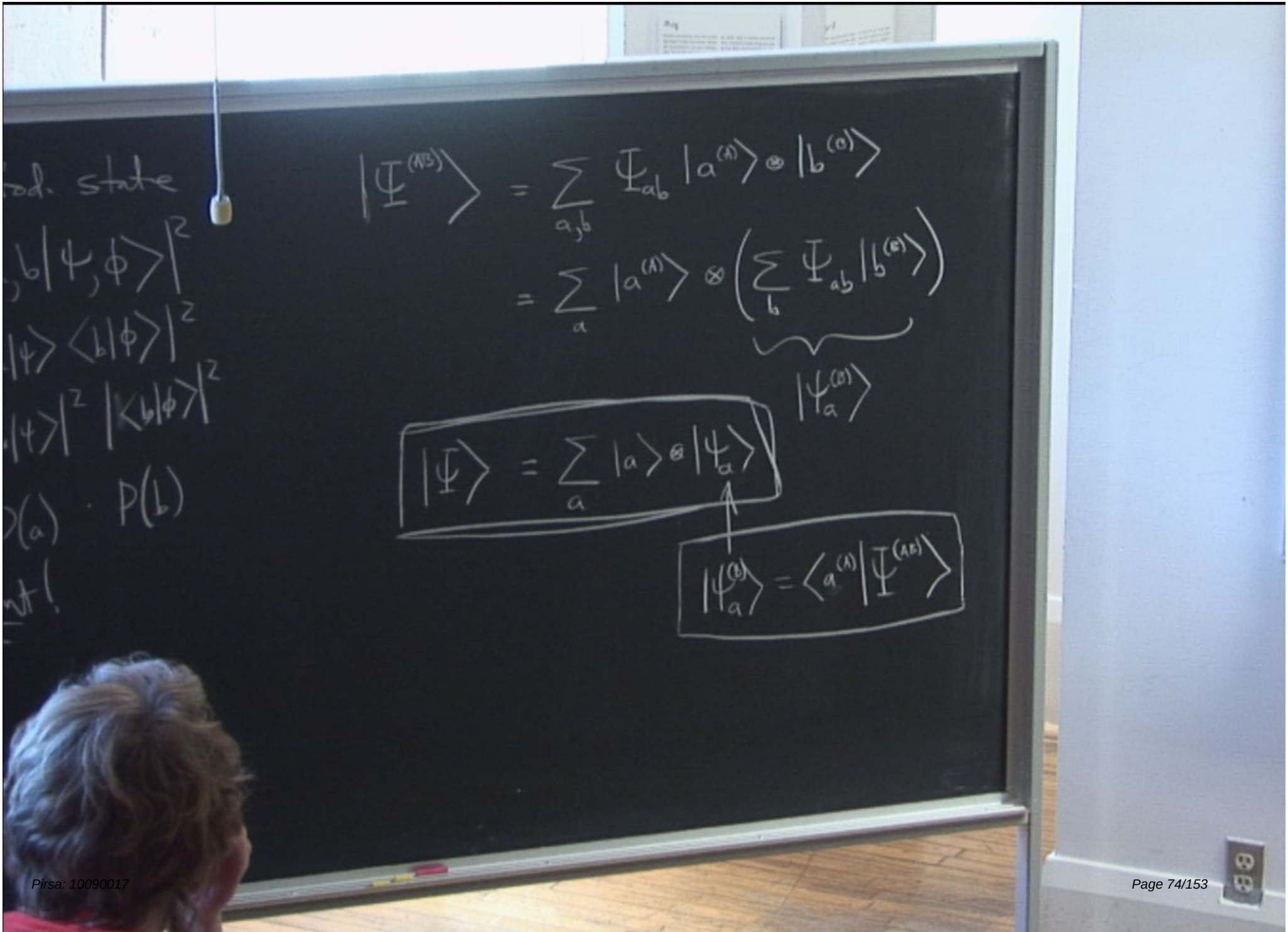
If $|\Psi^{(AB)}\rangle = |\varphi^{(A)}, \phi^{(B)}\rangle$ then

$$\langle \alpha^{(A)} | \varphi^{(A)}, \phi^{(B)} \rangle = \langle \alpha^{(A)} | \varphi^{(A)} \rangle \, |\phi^{(B)}\rangle$$

(+ linearity)


$|\Psi^{(AB)}\rangle = \sum_{a,b} \Psi_{ab} |a^{(A)}\rangle \otimes |b^{(B)}\rangle$
$= \sum_{a} |a^{(A)}\rangle \otimes \left(\sum_{b} \Psi_{ab} |b^{(B)}\rangle \right)$
$|\psi_a^{(B)}\rangle$
$|\Psi\rangle = \sum_a |a\rangle \otimes |\psi_a\rangle$
$|\psi_a^{(B)}\rangle = \langle a^{(A)}| \Psi^{(AB)}$
P(b)


$|\Psi^{(AB)}\rangle = \sum_{a,b} \Psi_{ab} |a^{(A)}\rangle \otimes |b^{(B)}\rangle$
$= \sum_{a} |a^{(A)}\rangle \otimes \left(\sum_{b} \Psi_{ab} |b^{(B)}\rangle \right)$
$|\psi_a^{(B)}\rangle$
$|\Psi\rangle = \sum_{a} |a\rangle \otimes |\psi_a\rangle$
$|\psi_a^{(B)}\rangle = \langle a^{(A)} | \Psi^{(AB)} \rangle$
$P(b)$


$|\Psi^{(AB)}\rangle = \sum_{a,b} \Psi_{ab} |a^{(A)}\rangle \otimes |b^{(B)}\rangle$
$= \sum_a |a^{(A)}\rangle \otimes \left(\sum_b \Psi_{ab} |b^{(B)}\rangle \right)$
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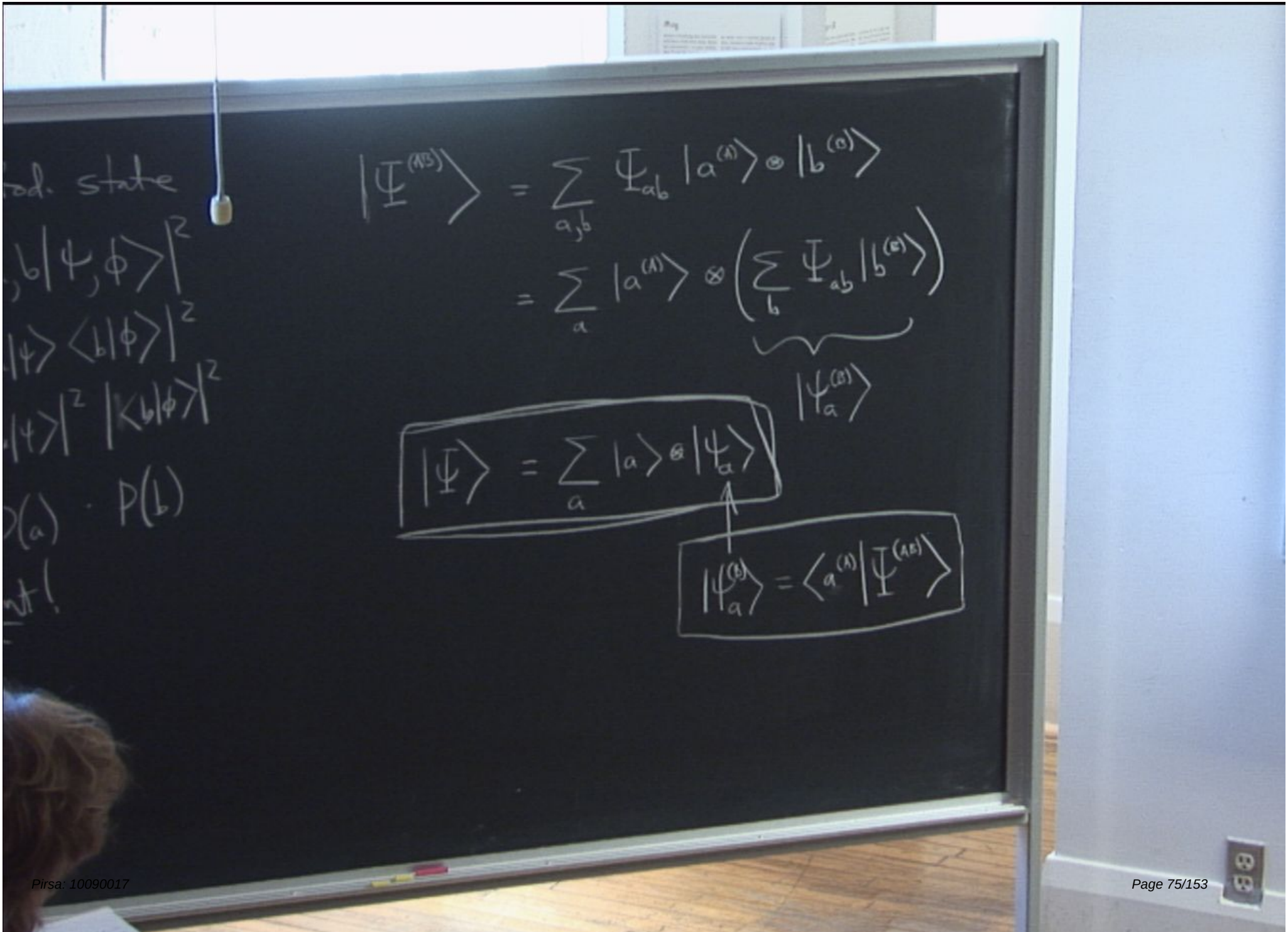
$|\psi_a^{(B)}\rangle = \langle a^{(A)} | \Psi^{(AB)} \rangle$


od. state
$|\Psi^{(AB)}\rangle = \sum_{a,b} \Psi_{ab} |a^{(A)}\rangle \otimes |b^{(B)}\rangle$
$= \sum_{a} |a^{(A)}\rangle \otimes \left(\sum_{b} \Psi_{ab} |b^{(B)}\rangle \right)$
$|\psi_a^{(B)}\rangle$
$|\Psi\rangle = \sum_a |a\rangle \otimes |\psi_a\rangle$
$|\psi_a^{(B)}\rangle = \langle a^{(A)} | \Psi^{(AB)} \rangle$
$P(b)$
$P(a)$

$P(a,b) = |\langle a,b|\Psi\rangle|^2 = |\langle b|\psi_b\rangle|^2$

$P(a) =$

Partial inner

$\langle \alpha^{(A)}|$

$\mathcal{H}^{(A)}$

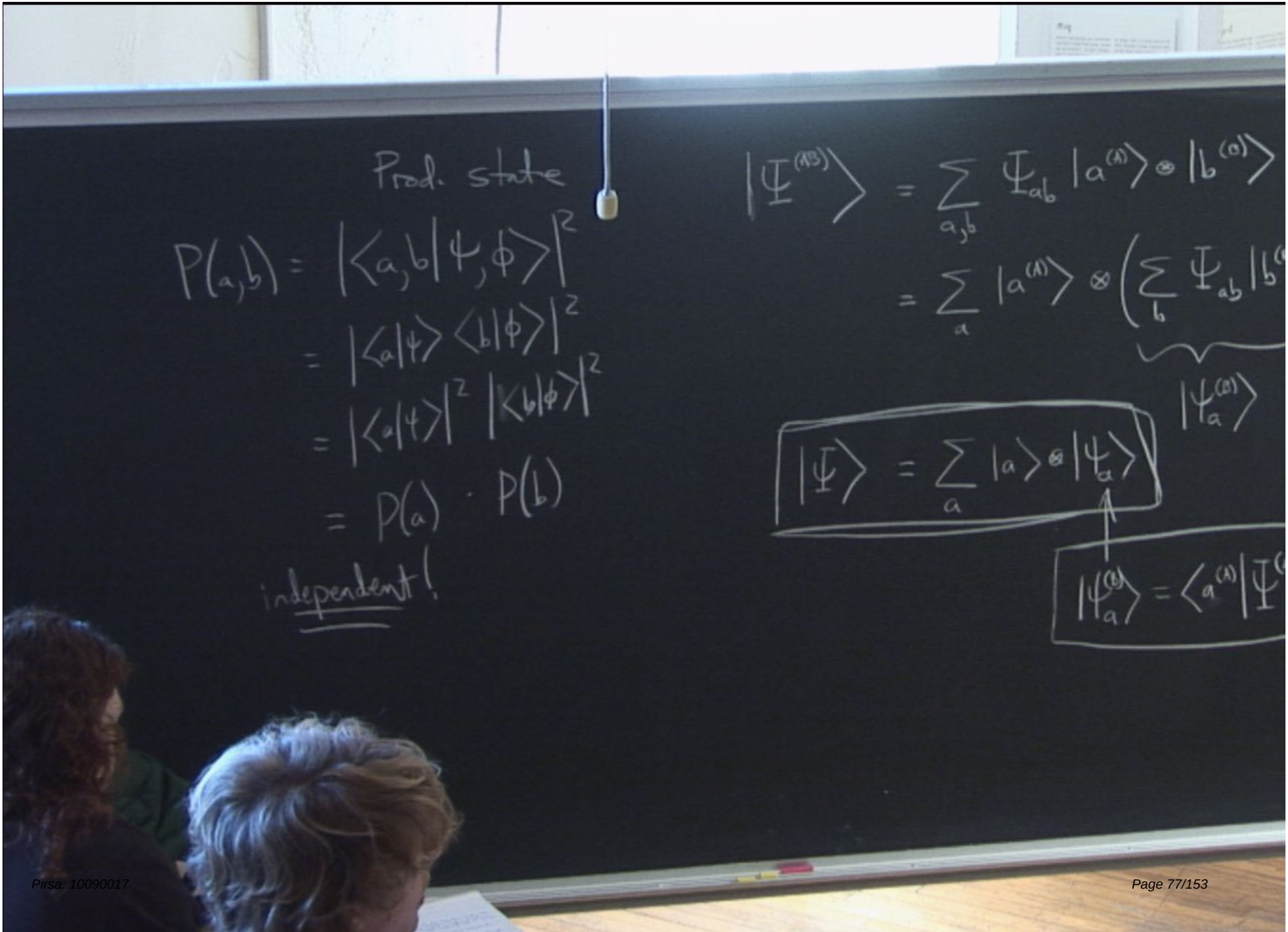
If $|\Psi^{(AB)}\rangle$ =

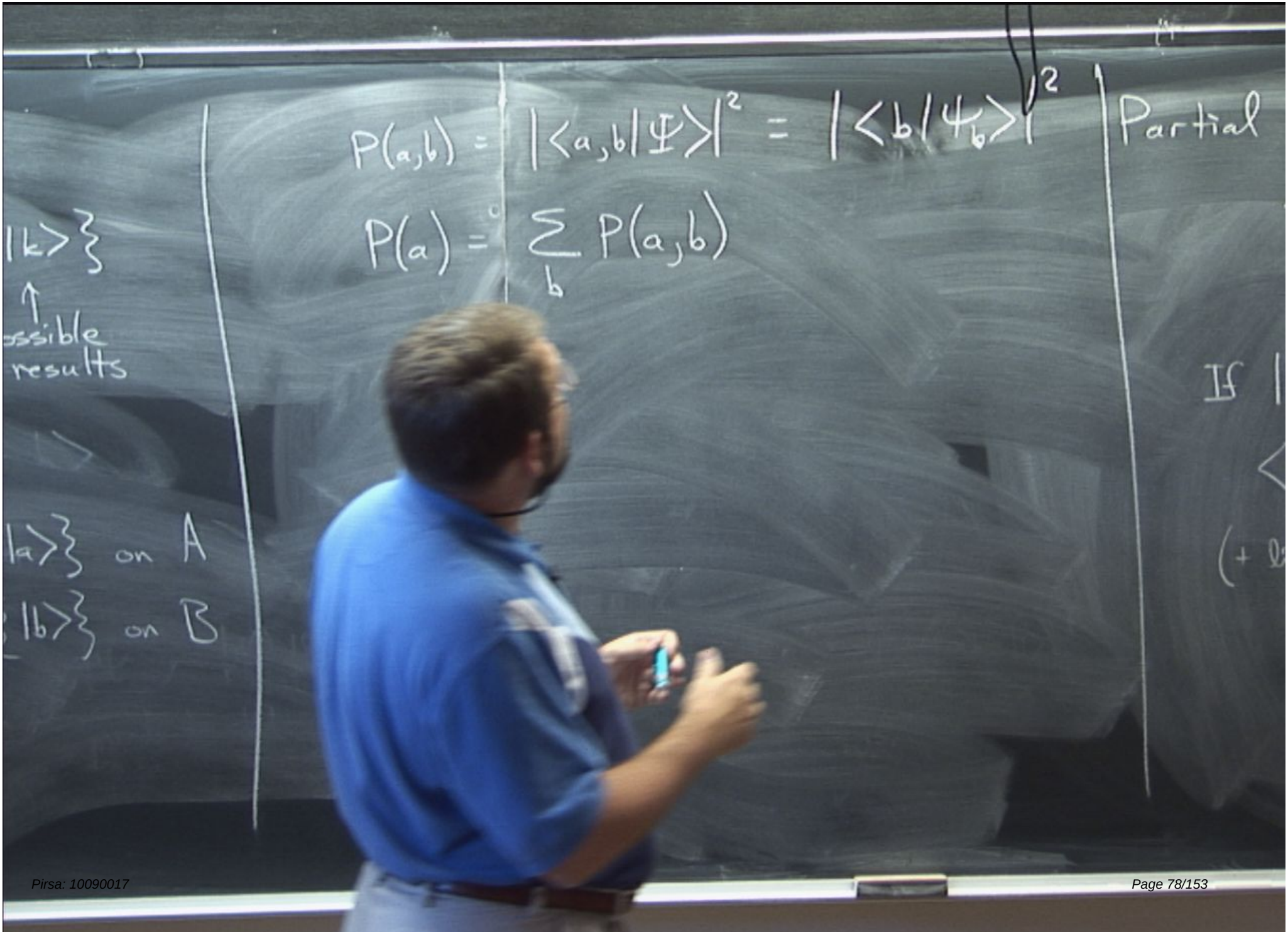
$\langle \alpha^{(A)}|\psi^{(A)}$

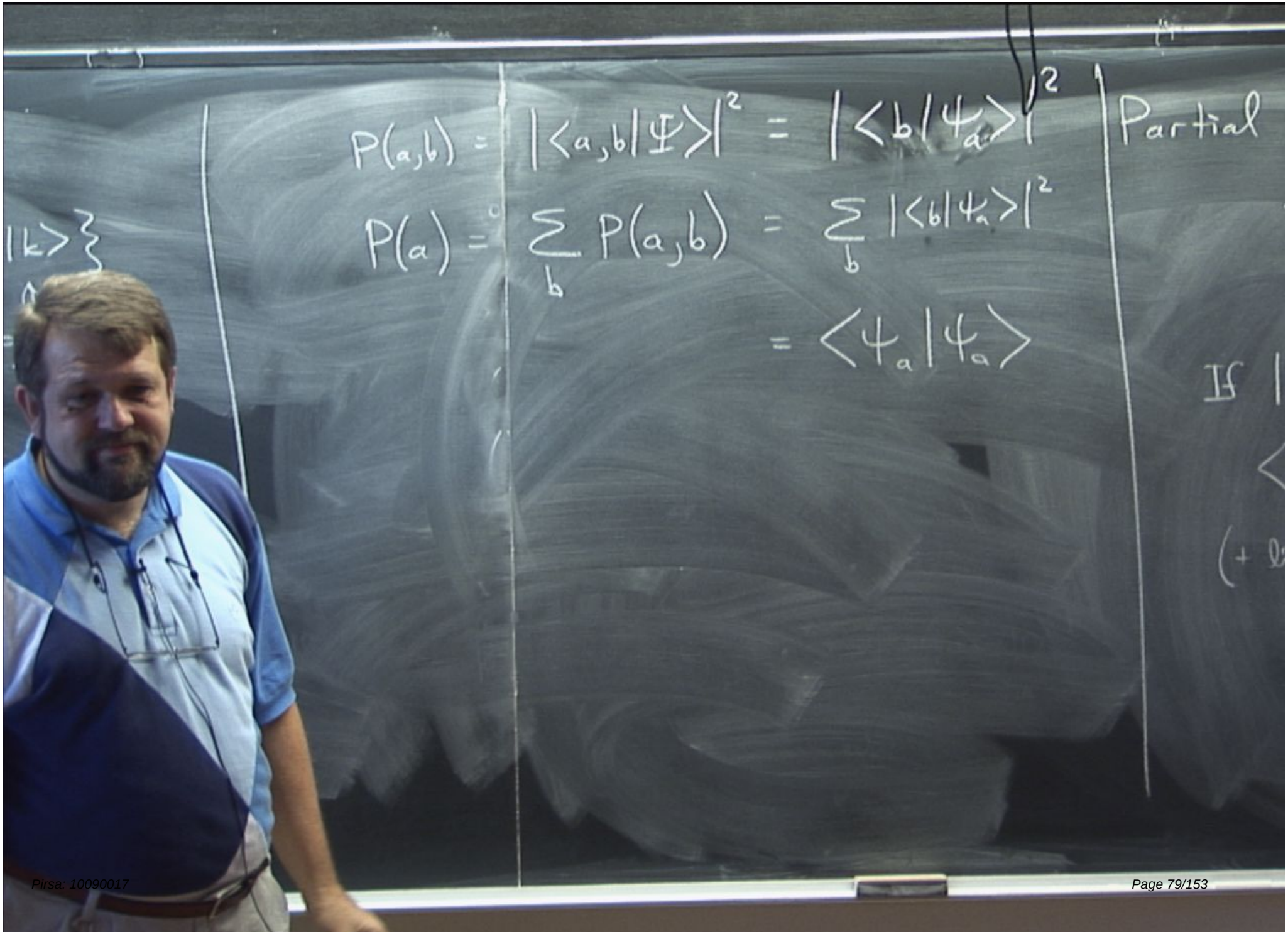
(+ linearity)

on A

on B


Prod. state
$P(a,b) = |\langle a,b|\psi,\phi\rangle|^2$
$= |\langle a|\psi\rangle\langle b|\phi\rangle|^2$
$= |\langle a|\psi\rangle|^2 |\langle b|\phi\rangle|^2$
$= P(a) \cdot P(b)$
independent!
$|\Psi^{(AB)}\rangle = \sum_{a,b} \Psi_{ab} |a^{(A)}\rangle \otimes |b^{(B)}\rangle$
$= \sum_a |a^{(A)}\rangle \otimes \left(\sum_b \Psi_{ab} |b^{(B)}\rangle \right)$
$|\psi_a^{(B)}\rangle$
$|\Psi\rangle = \sum_a |a\rangle \otimes |\psi_a\rangle$
$|\psi_a^{(B)}\rangle = \langle a^{(A)}|\Psi$


$P(a,b) = |\langle a,b|\Psi\rangle|^2 = |\langle b|\psi_b\rangle|^2$
$P(a) = \sum_b P(a,b)$
Partial
$\{|k\rangle\}$
ossible
results
$\{|a\rangle\}$ on A
$\{|b\rangle\}$ on B
If


$P(a,b) = |\langle a,b|\Psi\rangle|^2 = |\langle b|\psi_a\rangle|^2$
$P(a) = \sum_b P(a,b) = \sum_b |\langle b|\psi_a\rangle|^2$
$= \langle\psi_a|\psi_a\rangle$
Partial
$|k\rangle\}$

$|k\rangle\}$

↑

ossible
results

$|a\rangle\}$ on A

$|b\rangle\}$ on B

$P(a,b) = |\langle a,b|\Psi\rangle|^2 = |\langle b|\psi_a\rangle|^2$

$P(a) = \sum_b P(a,b) = \sum_b |\langle b|\psi_a\rangle|^2$

$= \langle\psi_a|\psi_a\rangle$

$P(b|a) = \frac{P(a,b)}{P(a)}$

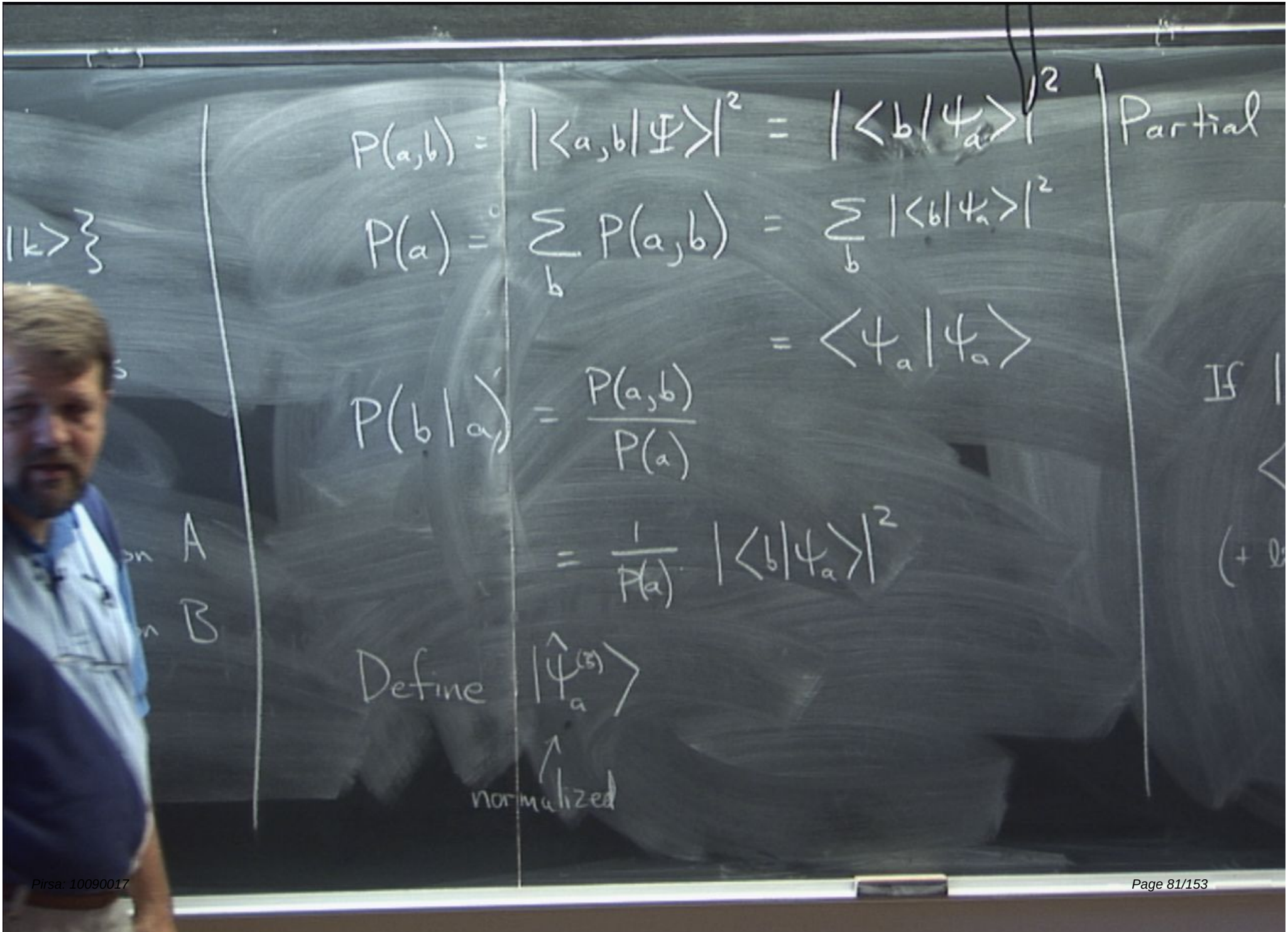
$= \frac{1}{P(a)} |\langle b|\psi_a\rangle|^2$

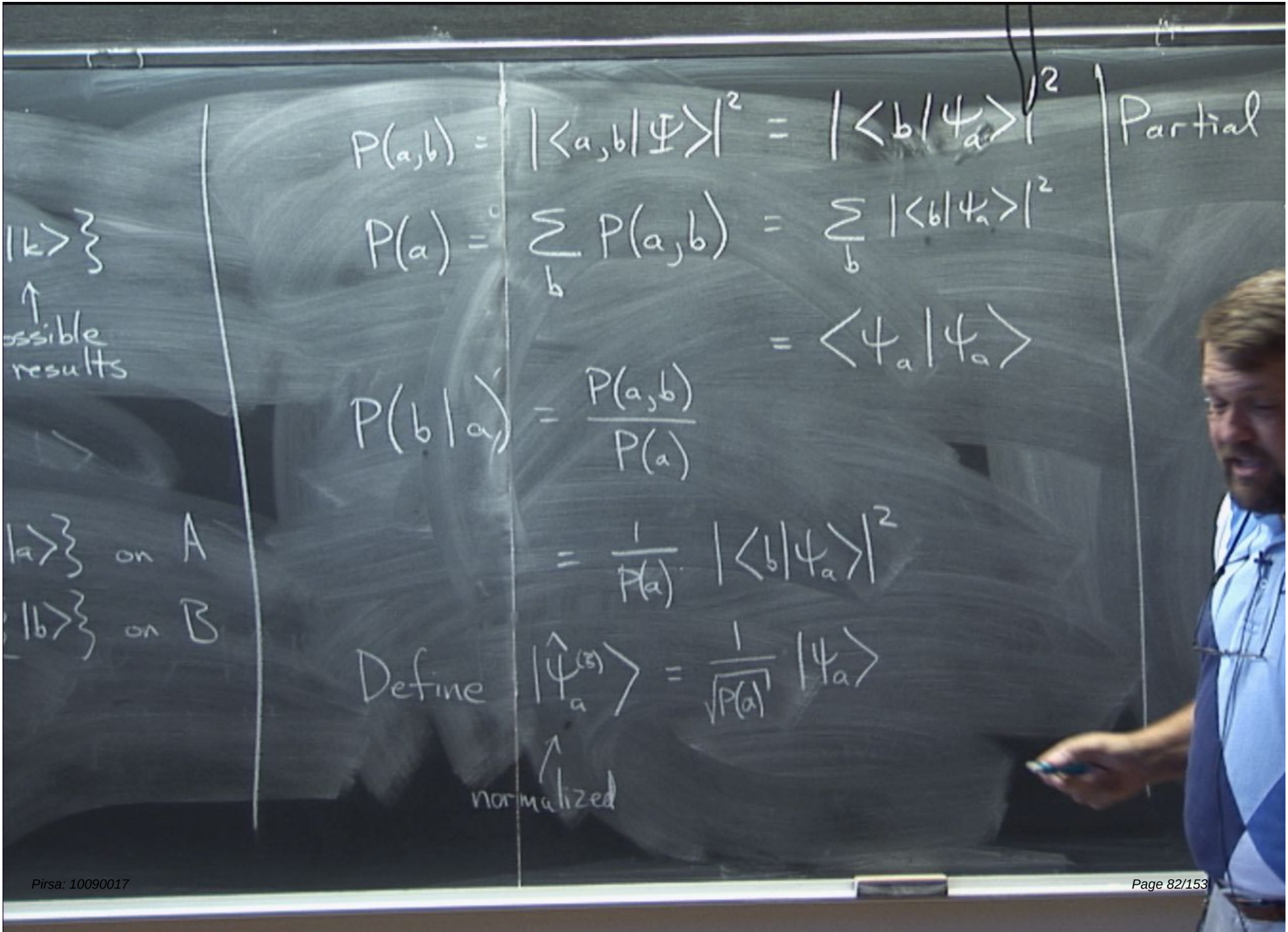
Partial

If |

<

(+ b


$P(a,b) = |\langle a,b|\Psi\rangle|^2 = |\langle b|\psi_a\rangle|^2$
$P(a) = \sum_b P(a,b) = \sum_b |\langle b|\psi_a\rangle|^2$
$= \langle \psi_a|\psi_a\rangle$
$P(b|a) = \frac{P(a,b)}{P(a)}$
$= \frac{1}{P(a)} |\langle b|\psi_a\rangle|^2$
Define $|\hat{\psi}_a^{(b)}\rangle$
normalized
Partial
If
A
B


$P(a,b) = |\langle a,b|\Psi\rangle|^2 = |\langle b|\psi_a\rangle|^2$
$P(a) = \sum_b P(a,b) = \sum_b |\langle b|\psi_a\rangle|^2$
$= \langle\psi_a|\psi_a\rangle$
$P(b|a) = \frac{P(a,b)}{P(a)}$
$= \frac{1}{P(a)} |\langle b|\psi_a\rangle|^2$
Define $|\hat{\psi}_a^{(s)}\rangle = \frac{1}{\sqrt{P(a)}} |\psi_a\rangle$
normalized
Partial
$|k\rangle\}$
ossible results
$|a\rangle\}$ on A
$|b\rangle\}$ on B

$\{|k\rangle\}$

↑ ssible results

$\{|a\rangle\}$ on A

$\{|b\rangle\}$ on B

$P(a,b) = |\langle a,b|\Psi\rangle|^2 = |\langle b|\psi_a\rangle|^2$

$P(a) = \sum_b P(a,b) = \sum_b |\langle b|\psi_a\rangle|^2$

$= \langle\psi_a|\psi_a\rangle$

$P(b|a) = \frac{P(a,b)}{P(a)}$

$= \frac{1}{P(a)} |\langle b|\psi_a\rangle|^2$

Define $|\hat{\psi}_a^{(b)}\rangle = \frac{1}{\sqrt{P(a)}} |\psi_a\rangle$

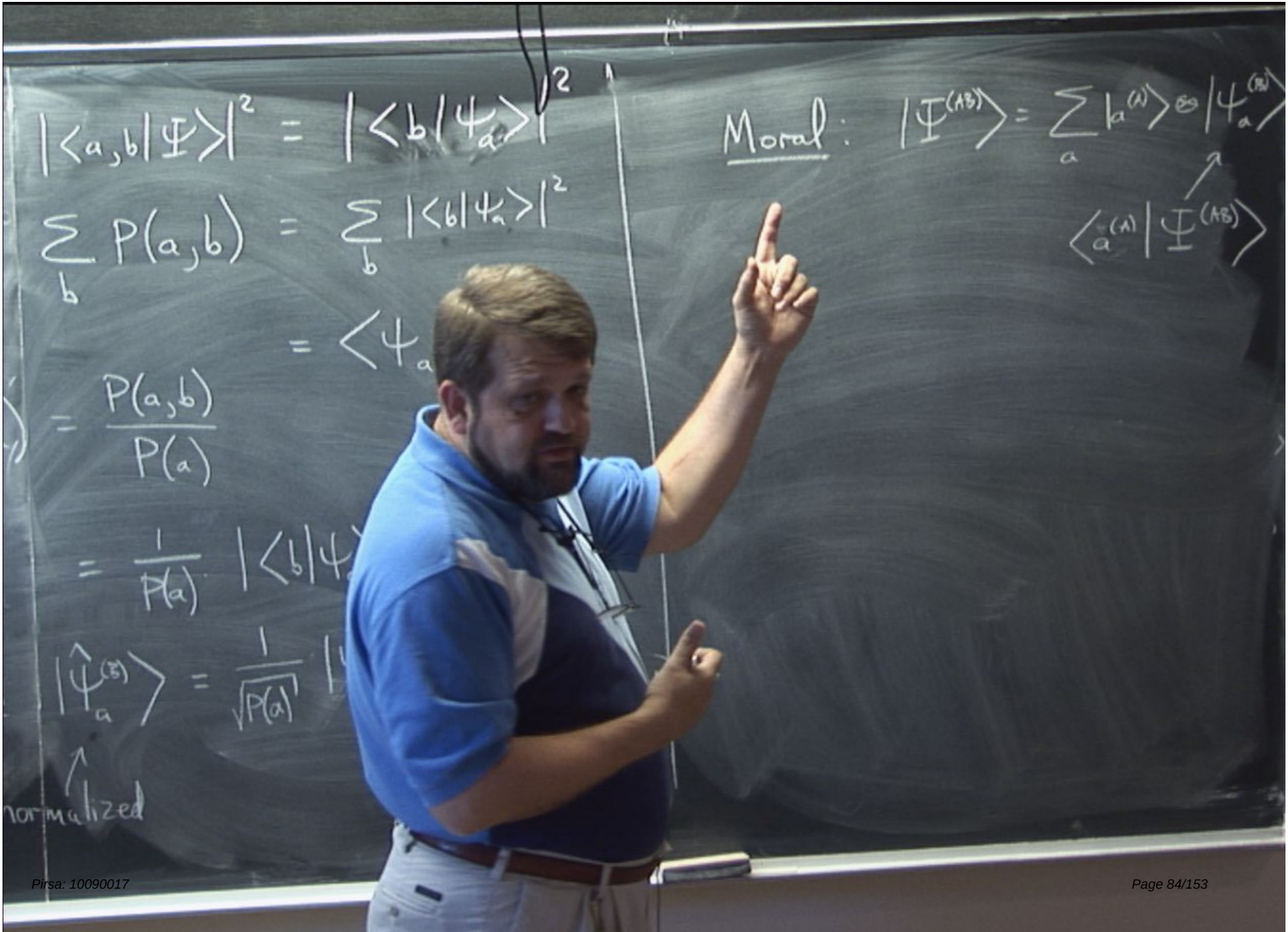
↑ normalized

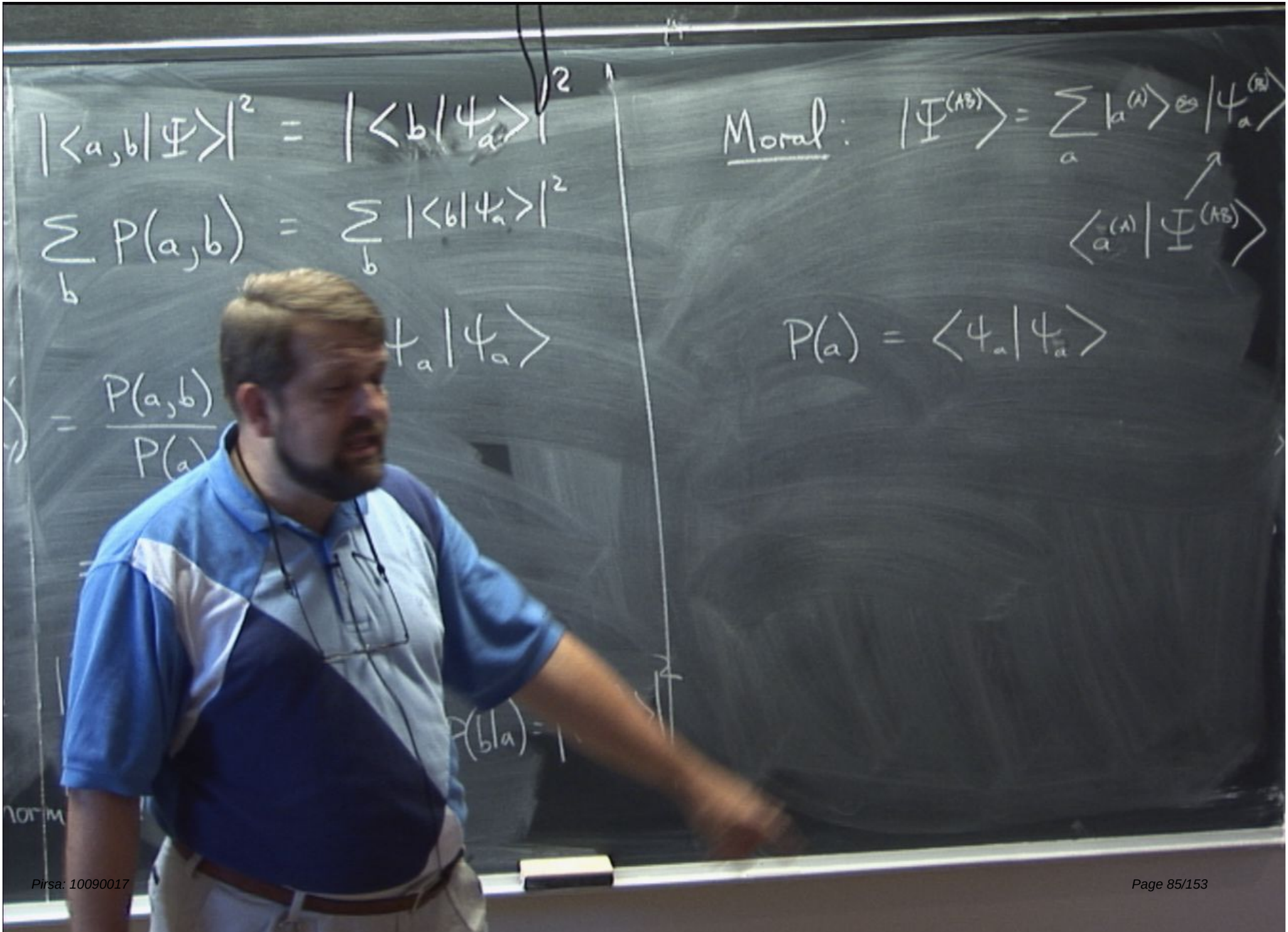
$\Rightarrow P(b|a) = |\langle b|\hat{\psi}_a\rangle|^2$

Partial

If |

(+ b


$|\langle a,b|\Psi\rangle|^2 = |\langle b|\psi_a\rangle|^2$
$\sum_b P(a,b) = \sum_b |\langle b|\psi_a\rangle|^2$
$= \frac{P(a,b)}{P(a)}$
$= \frac{1}{P(a)} |\langle b|\psi$
$|\hat{\psi}_a^{(B)}\rangle = \frac{1}{\sqrt{P(a)}}$
normalized
Moral: $|\Psi^{(AB)}\rangle = \sum_a |a^{(A)}\rangle \otimes |\psi_a^{(B)}\rangle$
$\langle a^{(A)}|\Psi^{(AB)}\rangle$


$|\langle a,b|\Psi\rangle|^2 = |\langle b|\psi_a\rangle|^2$
$\sum_b P(a,b) = \sum_b |\langle b|\psi_a\rangle|^2$
$= \frac{P(a,b)}{P(a)}$
Moral: $|\Psi^{(AB)}\rangle = \sum_a |a^{(A)}\rangle \otimes |\psi_a^{(B)}\rangle$
$\langle a^{(A)}|\Psi^{(AB)}\rangle$
$P(a) = \langle \psi_a|\psi_a\rangle$

$$|\langle a,b|\Psi\rangle|^2 = |\langle b|\psi_a\rangle|^2$$

$$\sum_b P(a,b) = \sum_b |\langle b|\psi_a\rangle|^2$$

$$= \langle\psi_a|\psi_a\rangle$$

$$= \frac{P(a,b)}{P(a)}$$

$$= \frac{1}{P(a)} |\langle b|\psi_a\rangle|^2$$

$$|\hat{\psi}_a^{(B)}\rangle = \frac{1}{\sqrt{P(a)}} |\psi_a\rangle$$

$$\Rightarrow P(b|a) = |\langle b|\hat{\psi}_a\rangle|^2$$

normalized

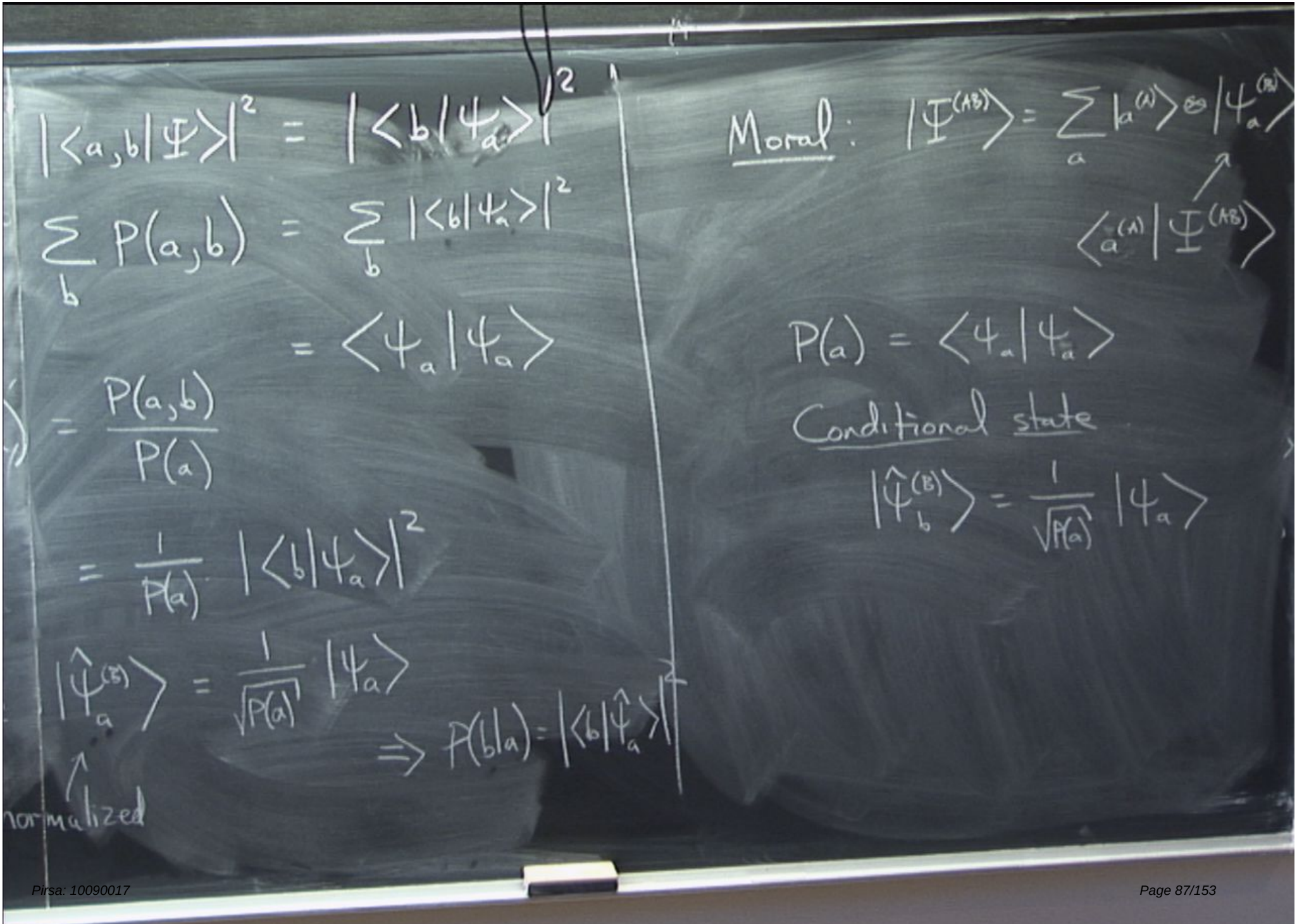
Moral: $|\Psi^{(AB)}\rangle = \sum_a |a^{(A)}\rangle \otimes |\psi_a^{(B)}\rangle$

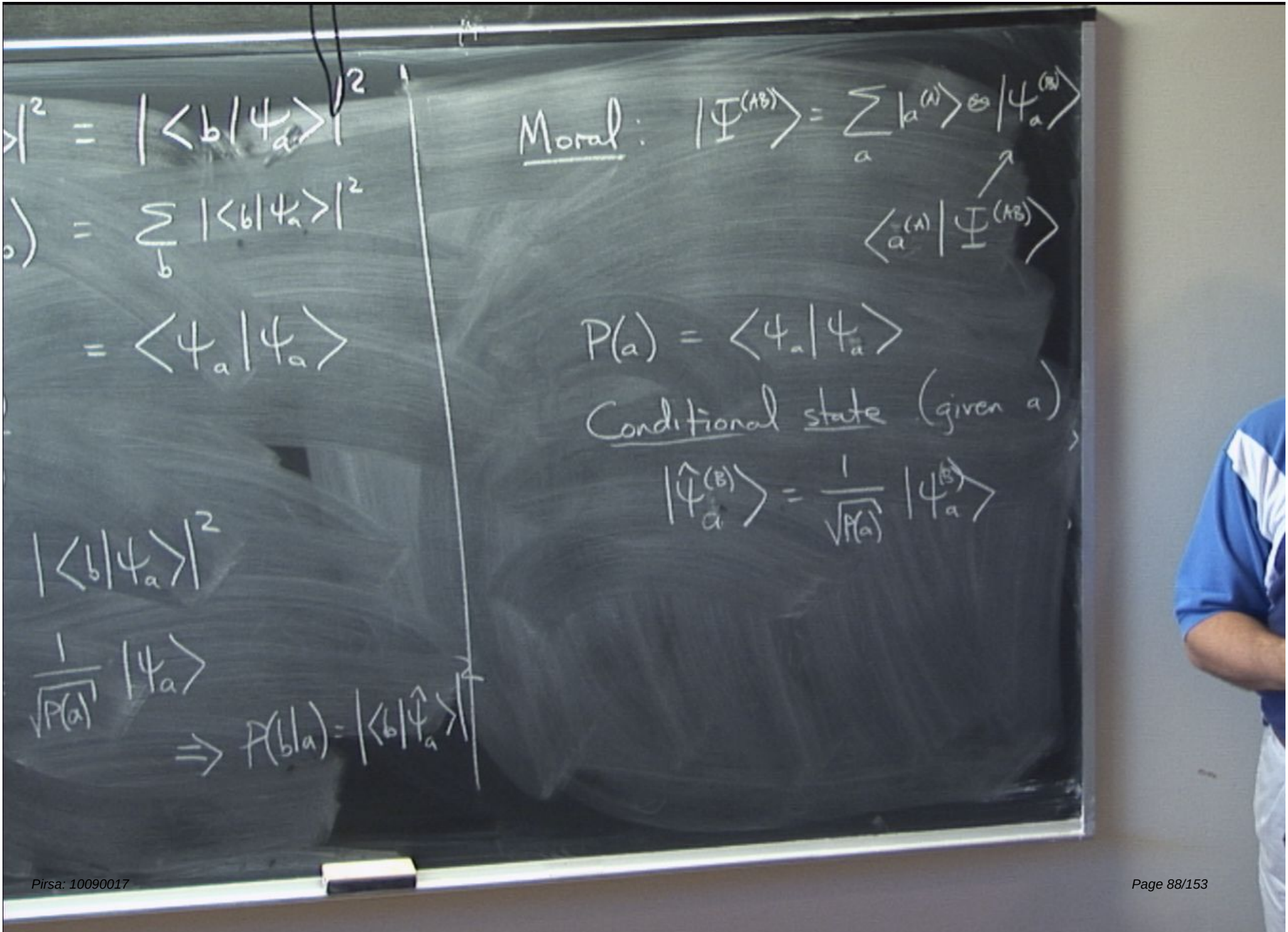
$\langle a^{(A)}|\Psi^{(AB)}\rangle$

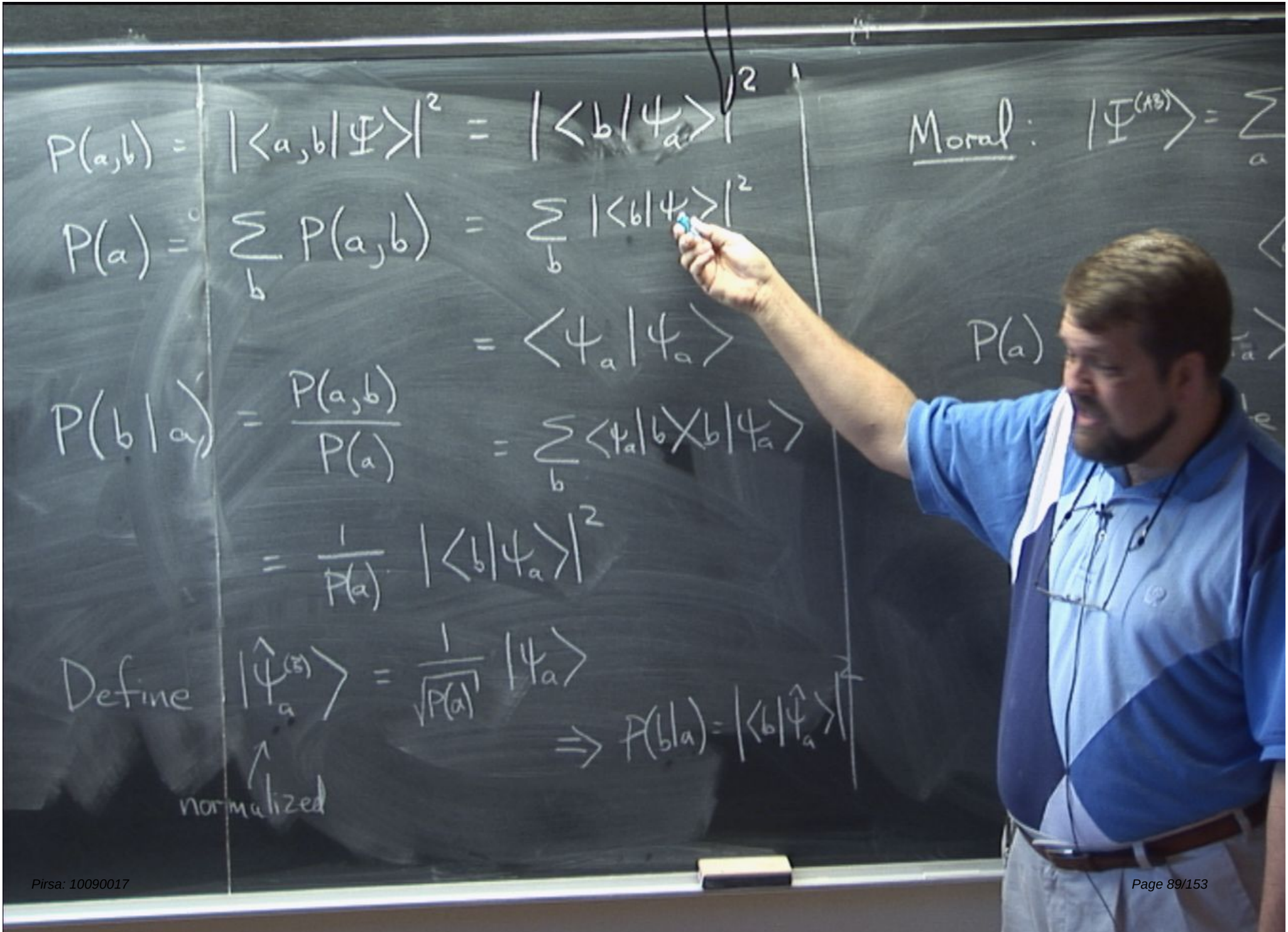
$$P(a) = \langle\psi_a|\psi_a\rangle$$

Conditional state

$$|\hat{\psi}_b^{(B)}\rangle = \frac{1}{\sqrt{P(a)}} |\psi_a\rangle$$


$|\langle a,b|\Psi\rangle|^2 = |\langle b|\psi_a\rangle|^2$
$\sum_b P(a,b) = \sum_b |\langle b|\psi_a\rangle|^2$
$= \langle\psi_a|\psi_a\rangle$
$= \frac{P(a,b)}{P(a)}$
$= \frac{1}{P(a)} |\langle b|\psi_a\rangle|^2$
$|\hat{\psi}_a^{(B)}\rangle = \frac{1}{\sqrt{P(a)}} |\psi_a\rangle$
$\Rightarrow P(b|a) = |\langle b|\hat{\psi}_a\rangle|^2$
normalized
Moral: $|\Psi^{(AB)}\rangle = \sum_a |a^{(A)}\rangle \otimes |\psi_a^{(B)}\rangle$
$\langle a^{(A)}|\Psi^{(AB)}\rangle$
$P(a) = \langle\psi_a|\psi_a\rangle$
Conditional state
$|\hat{\psi}_b^{(B)}\rangle = \frac{1}{\sqrt{P(a)}} |\psi_a\rangle$


$|\langle b|\psi_a\rangle|^2 = |\langle b|\psi_a\rangle|^2$
$= \sum_b |\langle b|\psi_a\rangle|^2$
$= \langle\psi_a|\psi_a\rangle$
$|\langle b|\psi_a\rangle|^2$
$\frac{1}{\sqrt{P(a)}}|\psi_a\rangle$
$\Rightarrow P(b|a) = |\langle b|\hat{\psi}_a\rangle|^2$
Moral: $|\Psi^{(AB)}\rangle = \sum_a |a^{(A)}\rangle \otimes |\psi_a^{(B)}\rangle$
$\langle a^{(A)}|\Psi^{(AB)}\rangle$
$P(a) = \langle\psi_a|\psi_a\rangle$
Conditional state (given a)
$|\hat{\psi}_a^{(B)}\rangle = \frac{1}{\sqrt{P(a)}}|\psi_a^{(B)}\rangle$


$P(a,b) = |\langle a,b|\Psi\rangle|^2 = |\langle b|\psi_a\rangle|^2$
$P(a) = \sum_b P(a,b) = \sum_b |\langle b|\psi_a\rangle|^2$
$= \langle\psi_a|\psi_a\rangle$
$P(b|a) = \frac{P(a,b)}{P(a)} = \sum_b \langle\psi_a|b\rangle\langle b|\psi_a\rangle$
$= \frac{1}{P(a)} |\langle b|\psi_a\rangle|^2$
Define $|\hat{\psi}_a^{(b)}\rangle = \frac{1}{\sqrt{P(a)}} |\psi_a\rangle$
$\Rightarrow P(b|a) = |\langle b|\hat{\psi}_a\rangle|^2$
normalized
Moral: $|\Psi^{(AB)}\rangle = \sum$
$P(a)$

$$P(a,b) = |\langle a,b|\Psi\rangle|^2 = |\langle b|\psi_a\rangle|^2$$

$$P(a) = \sum_b P(a,b) = \sum_b |\langle b|\psi_a\rangle|^2$$

$$= \langle \psi_a|\psi_a\rangle$$

$$P(b|a) = \frac{P(a,b)}{P(a)} = \sum_b \langle \psi_a|b\rangle\langle b|\psi_a\rangle$$

$$= \frac{1}{P(a)} |\langle b|\psi_a\rangle|^2$$

Define $|\hat{\psi}_a^{(b)}\rangle = \frac{1}{\sqrt{P(a)}} |\psi_a\rangle$

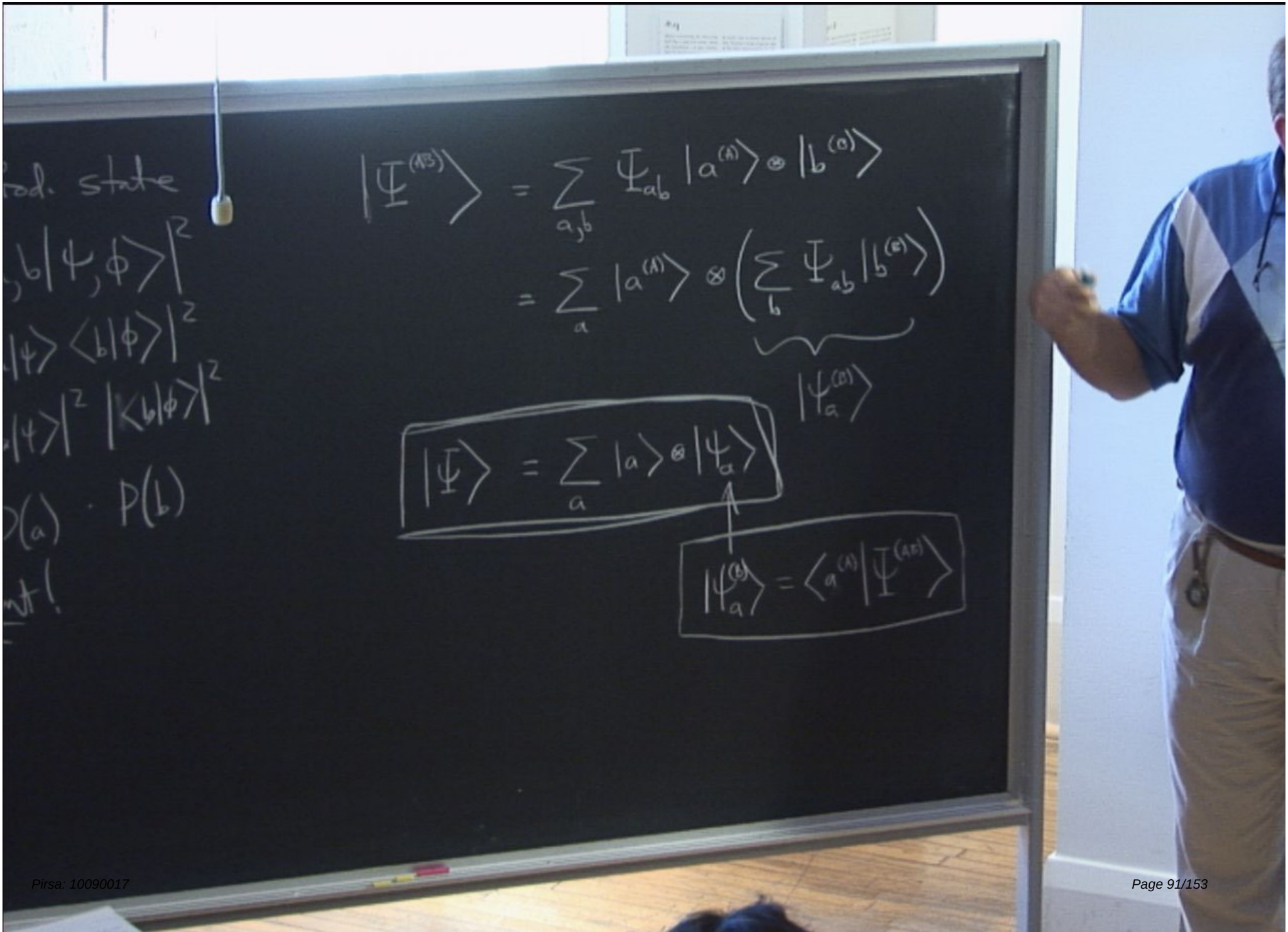
$\Rightarrow P(b|a) = |\langle b|\hat{\psi}_a\rangle|^2$

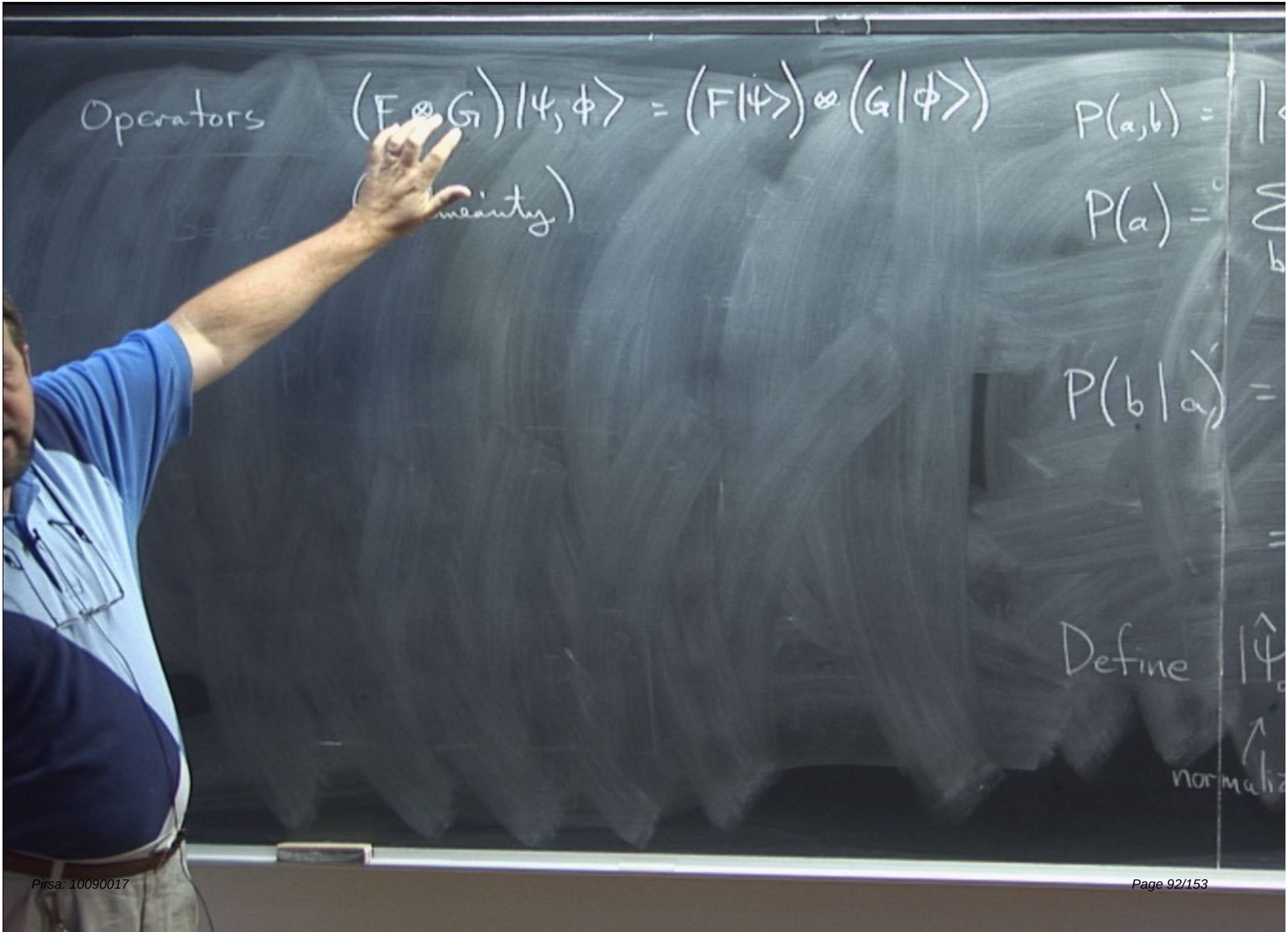
normalized

Moral: $|\Psi^{(AB)}\rangle = \sum_a$

$P(a) =$

Conditio


od. state
$|\Psi^{(AB)}\rangle = \sum_{a,b} \Psi_{ab} |a^{(A)}\rangle \otimes |b^{(B)}\rangle$
$= \sum_a |a^{(A)}\rangle \otimes \left(\sum_b \Psi_{ab} |b^{(B)}\rangle \right)$
$|\psi_a^{(B)}\rangle$
$|\Psi\rangle = \sum_a |a\rangle \otimes |\psi_a\rangle$
$|\psi_a^{(B)}\rangle = \langle a^{(A)}| \Psi^{(AB)} \rangle$
$P(b)$


Operators
$(F \otimes G)|\psi, \phi\rangle = (F|\psi\rangle) \otimes (G|\phi\rangle)$
$P(a,b) =$
$P(a) =$
$P(b|a) =$
Define

Operators $(F \otimes G)|\psi, \phi\rangle = (F|\psi\rangle) \otimes (G|\phi\rangle)$

(+ linearity)

$P(a,b) =$

$P(a) =$

$P(b|a) =$

Define

Operators $(F \otimes G)|\psi, \phi\rangle = (F|\psi\rangle) \otimes (G|\phi\rangle)$

(+ linearity)

Adding spins $(\hbar = 1)$

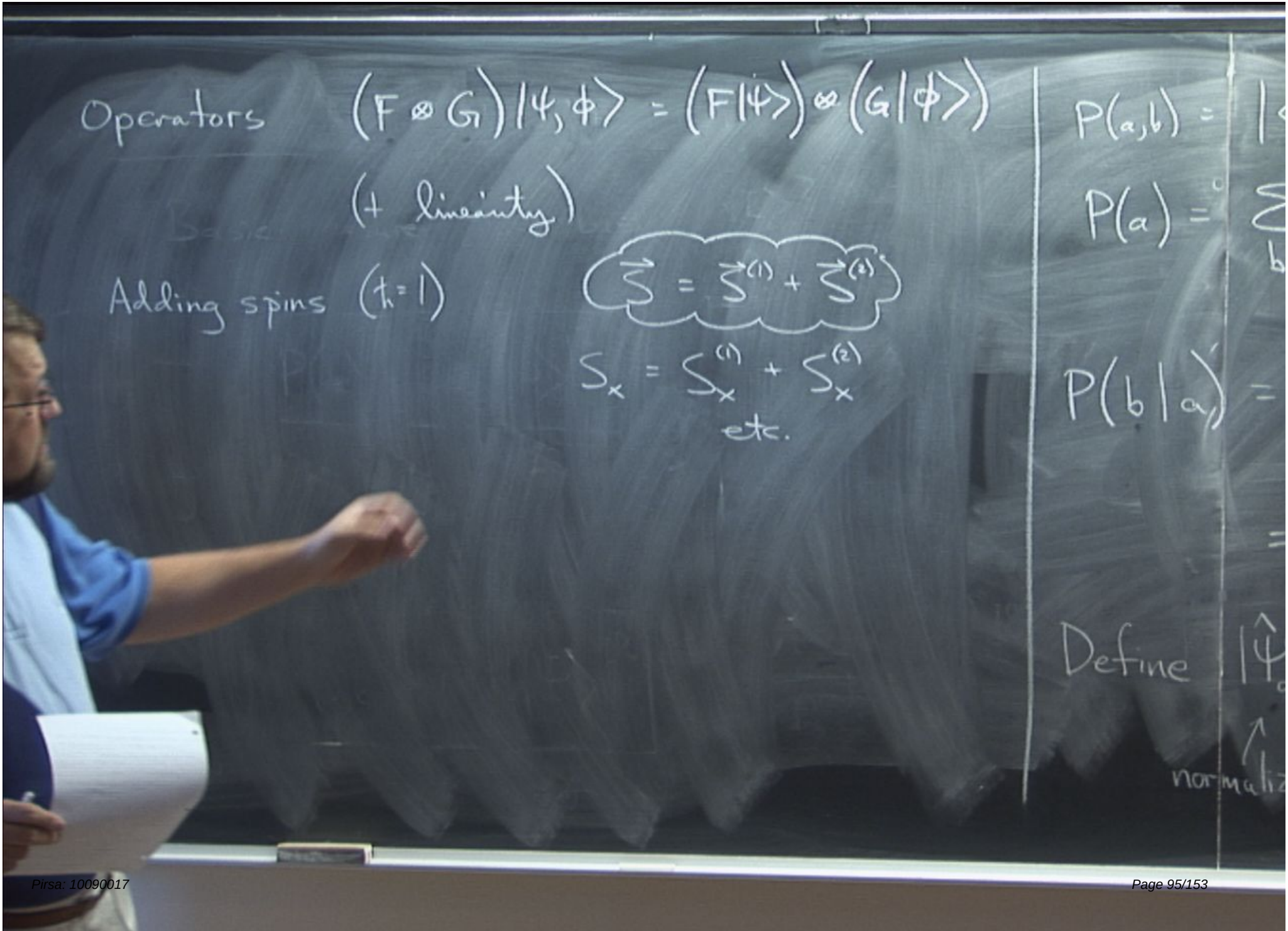
$P(a,b) =$

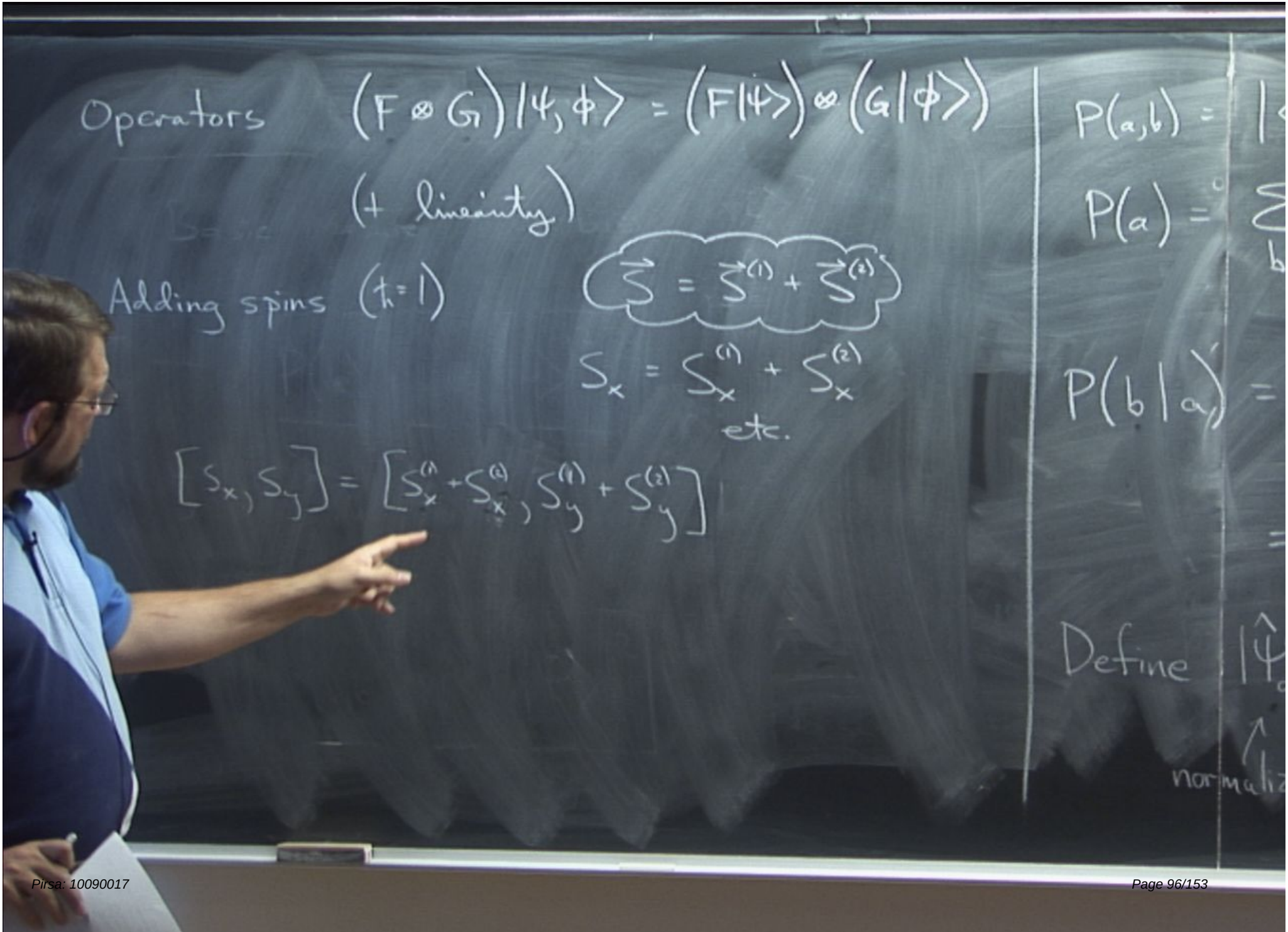
$P(a) =$

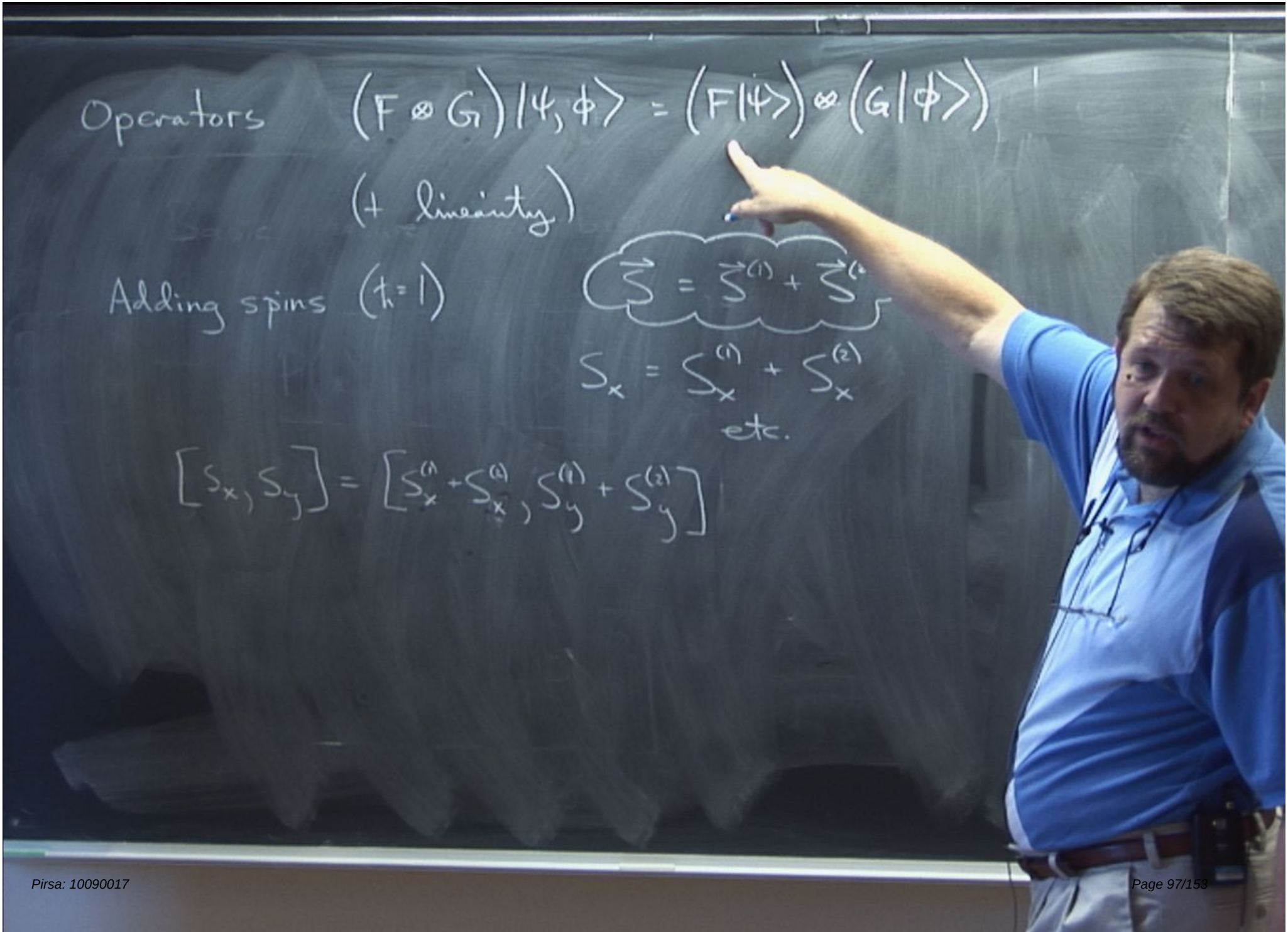
$P(b|a) =$

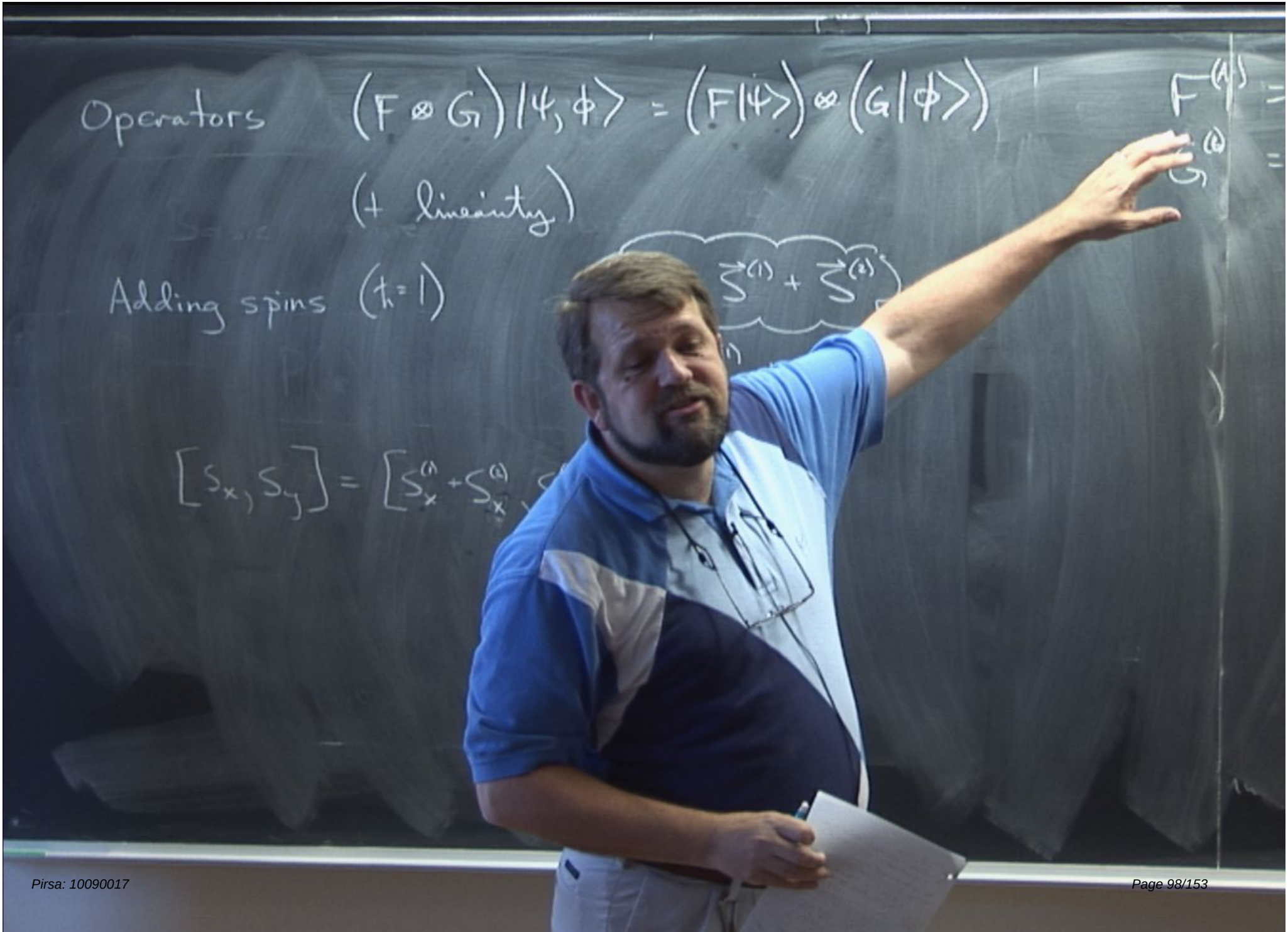
Define $|\hat{\psi}$

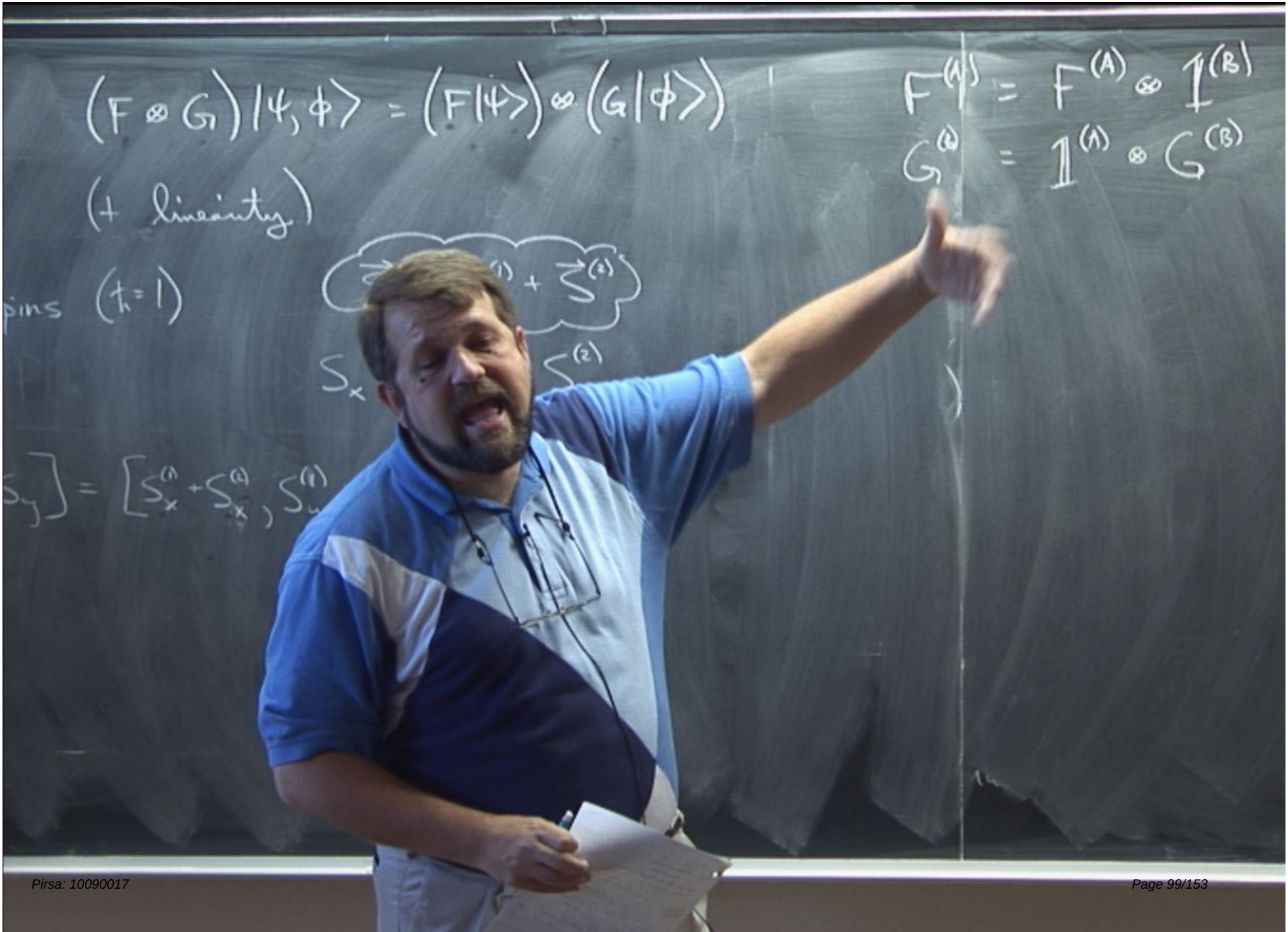
normaliz


Operators
$(F \otimes G)|\psi, \phi\rangle = (F|\psi\rangle) \otimes (G|\phi\rangle)$
(+ linearity)
Adding spins ($\hbar = 1$)
$\vec{S} = \vec{S}^{(1)} + \vec{S}^{(2)}$
$S_x = S_x^{(1)} + S_x^{(2)}$
etc.
$P(a,b) =$
$P(a) =$
$P(b|a) =$
Define
normaliz


Operators
$(F \otimes G)|\psi, \phi\rangle = (F|\psi\rangle) \otimes (G|\phi\rangle)$
(+ linearity)
Adding spins ($\hbar = 1$)
$\vec{S} = \vec{S}^{(1)} + \vec{S}^{(2)}$
$S_x = S_x^{(1)} + S_x^{(2)}$
etc.
$[S_x, S_y] = [S_x^{(1)} + S_x^{(2)}, S_y^{(1)} + S_y^{(2)}]$
$P(a,b) =$
$P(a) =$
$P(b|a) =$
Define
normaliz


Operators
$(F \otimes G)|\psi, \phi\rangle = (F|\psi\rangle) \otimes (G|\phi\rangle)$
(+ linearity)
Adding spins $(\hbar = 1)$
$\vec{S} = \vec{S}^{(1)} + \vec{S}^{(2)}$
$S_x = S_x^{(1)} + S_x^{(2)}$
etc.
$[S_x, S_y] = [S_x^{(1)} + S_x^{(2)}, S_y^{(1)} + S_y^{(2)}]$


Operators
$(F \otimes G)|\psi,\phi\rangle = (F|\psi\rangle) \otimes (G|\phi\rangle)$
(+ linearity)
Adding spins ($\hbar = 1$)
$\vec{S}^{(1)} + \vec{S}^{(2)}$
$[S_x, S_y] = [S_x^{(1)} + S_x^{(2)}$


$(F \otimes G)|\psi,\phi\rangle = (F|\psi\rangle) \otimes (G|\phi\rangle)$
(+ linearity)
$F^{(A)} = F^{(A)} \otimes \mathbb{1}^{(B)}$
$G^{(B)} = \mathbb{1}^{(A)} \otimes G^{(B)}$
spins $(h=1)$
$S^{(1)} + S^{(2)}$
S_x
$S^{(2)}$
$S_y] = [S_x^{(1)} + S_x^{(2)}, S_y^{(1)}$

$$(F \otimes G)|\psi, \phi\rangle = (F|\psi\rangle) \otimes (G|\phi\rangle)$$

(+ linearity)

$$F^{(A)} = F^{(A)} \otimes \mathbb{1}^{(B)}$$

$$G^{(B)} = \mathbb{1}^{(A)} \otimes G^{(B)}$$

spins $(h = 1)$

$$\vec{S} = \vec{S}^{(1)} + \vec{S}^{(2)}$$

$$S_x = S_x^{(1)} + S_x^{(2)}$$

etc.

$$[S_x, S_y] = [S_x^{(1)} + S_x^{(2)}, S_y^{(1)} + S_y^{(2)}]$$

$$=$$

$$(F \otimes G)|\psi, \phi\rangle = (F|\psi\rangle) \otimes (G|\phi\rangle)$$

$$F^{(A)} = F^{(A)} \otimes \mathbb{1}^{(B)}$$

$$G^{(B)} = \mathbb{1}^{(A)} \otimes G^{(B)}$$

(+ linearity)

spins $(\hbar = 1)$

$$\vec{S} = \vec{S}^{(1)} + \vec{S}^{(2)}$$

$$S_x = S_x^{(1)} + S_x^{(2)}$$

etc.

$$[S_x, S_y] = [S_x^{(1)} + S_x^{(2)}, S_y^{(1)} + S_y^{(2)}]$$

$$=$$

erators $(F \otimes G)|\psi, \phi\rangle = (F|\psi\rangle) \otimes (G|\phi\rangle)$

$F^{(A)} = F^{(A)} \otimes$

$G^{(B)} = \mathbb{1}^{(A)} \otimes$

(+ linearity)

lding spins $(\hbar = 1)$

$\vec{S} = \vec{S}^{(1)} + \vec{S}^{(2)}$

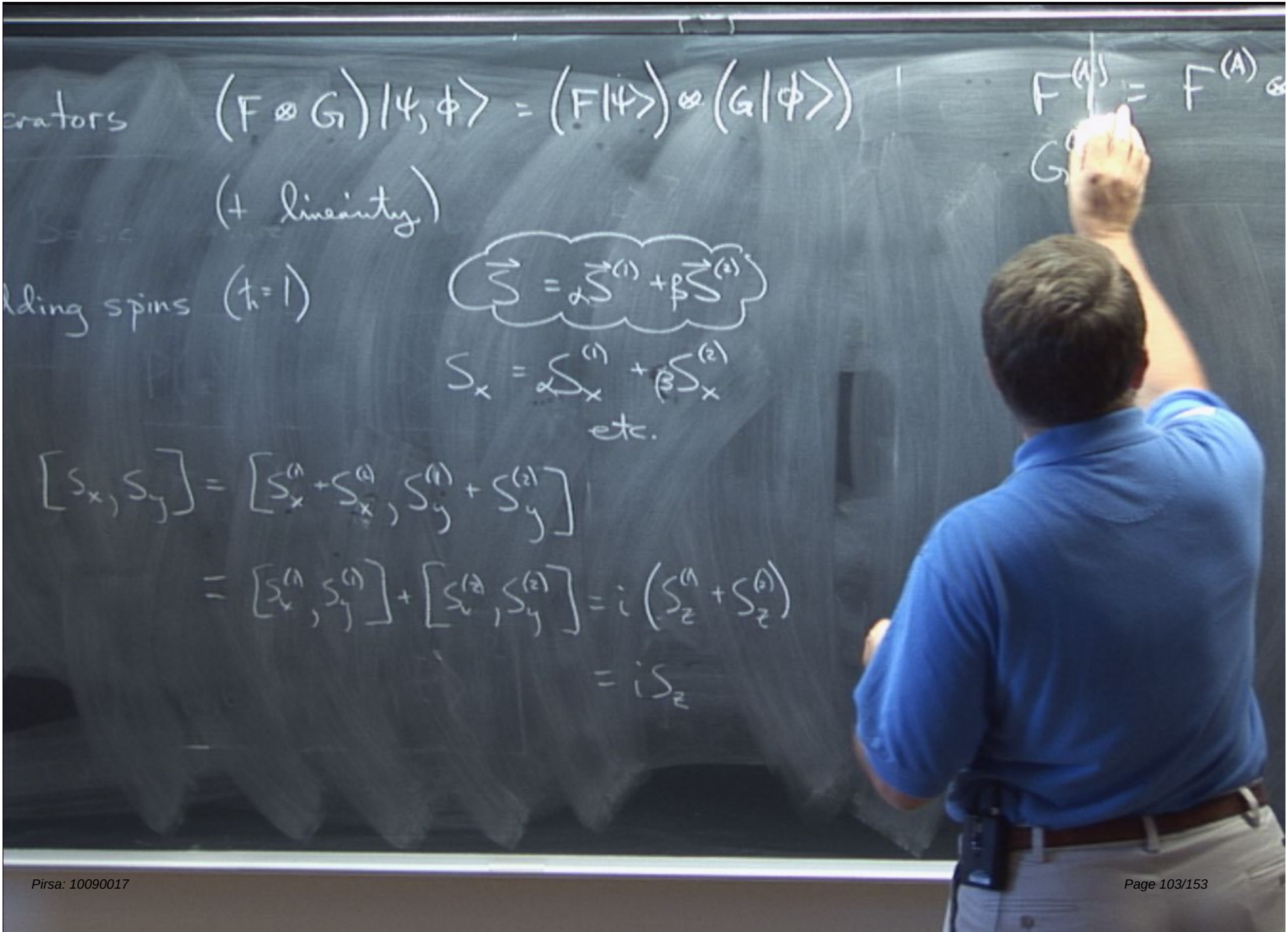
$S_x = S_x^{(1)} + S_x^{(2)}$

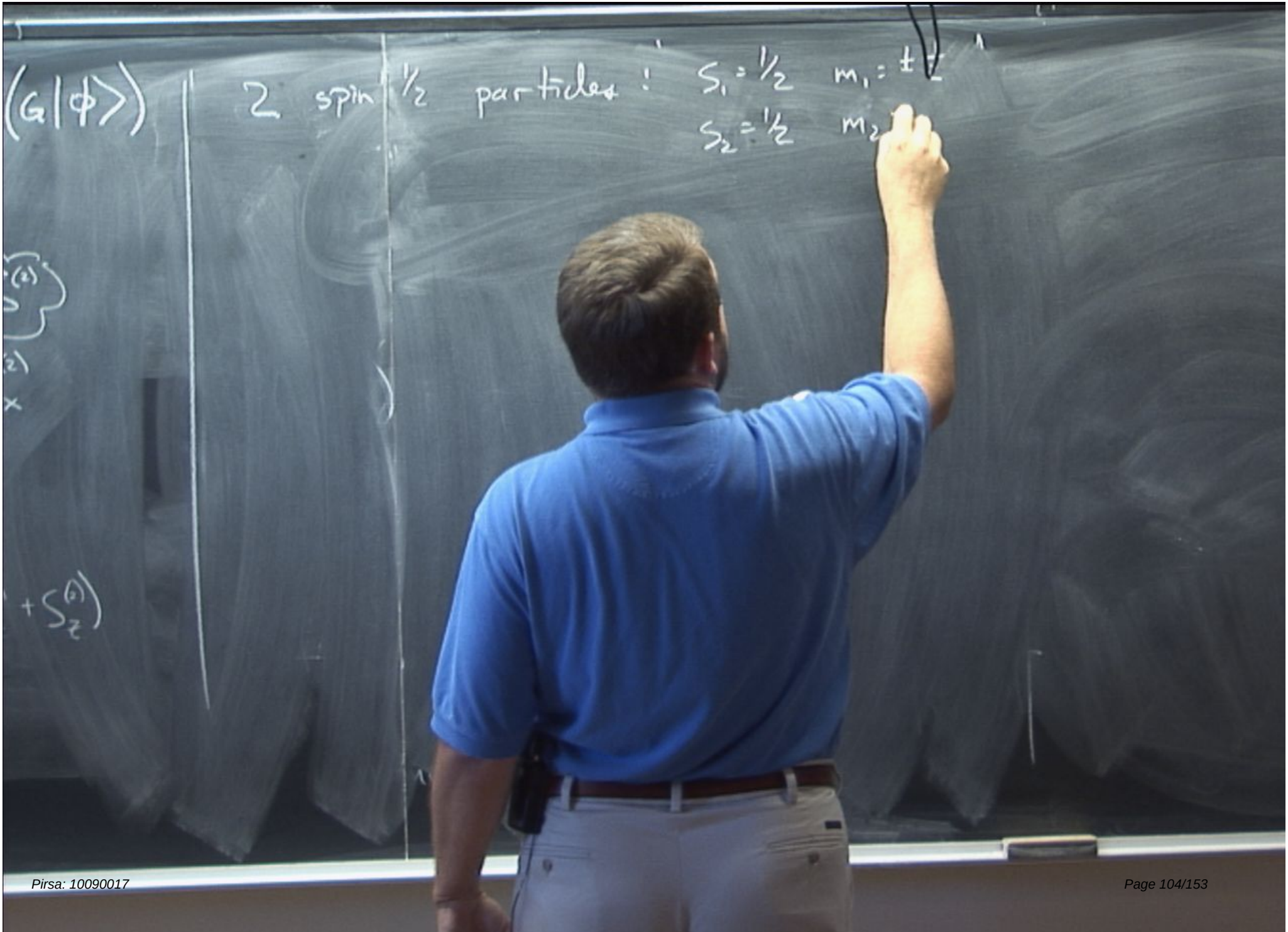
etc.

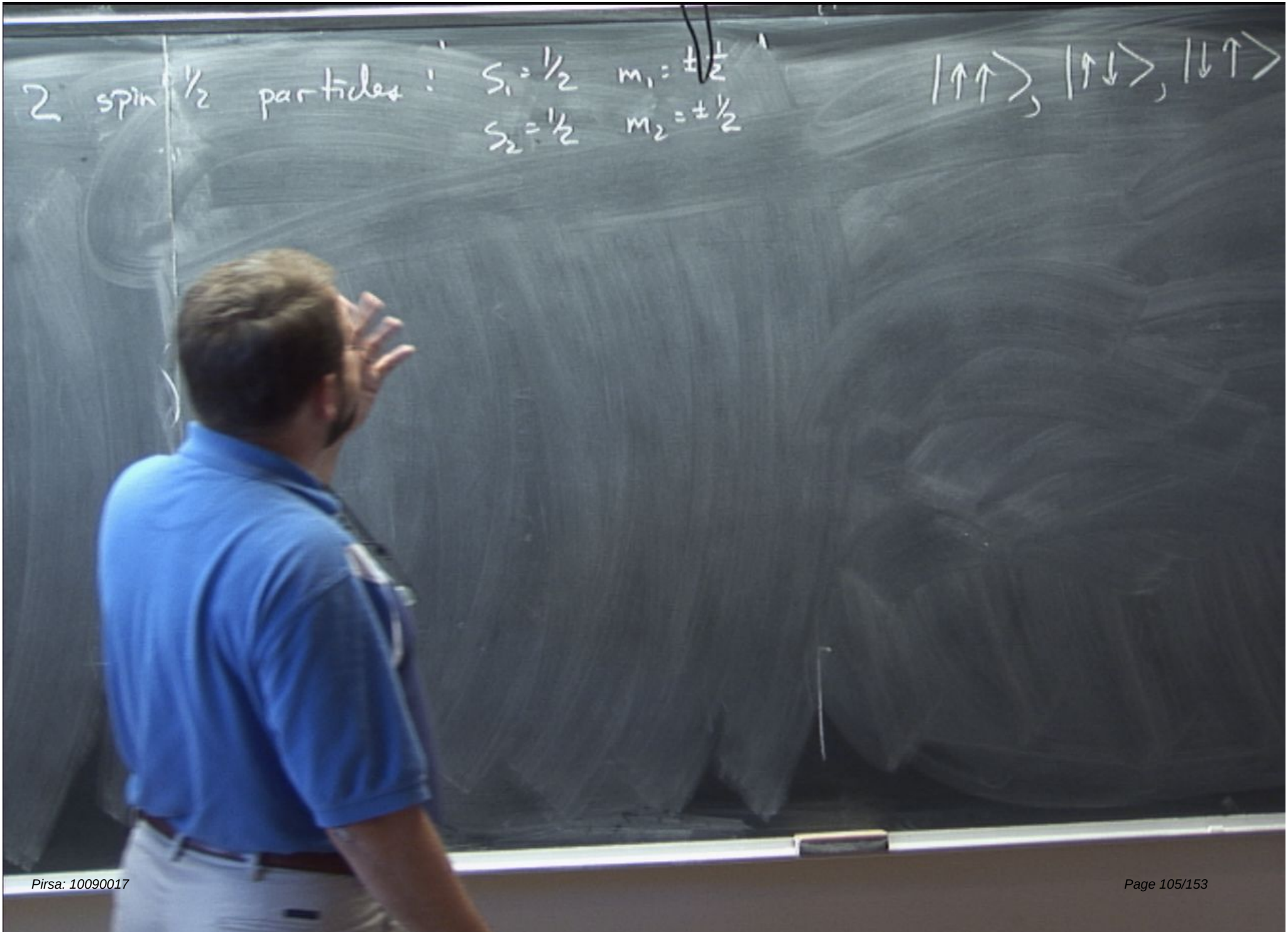
$$[S_x, S_y] = [S_x^{(1)} + S_x^{(2)}, S_y^{(1)} + S_y^{(2)}]$$

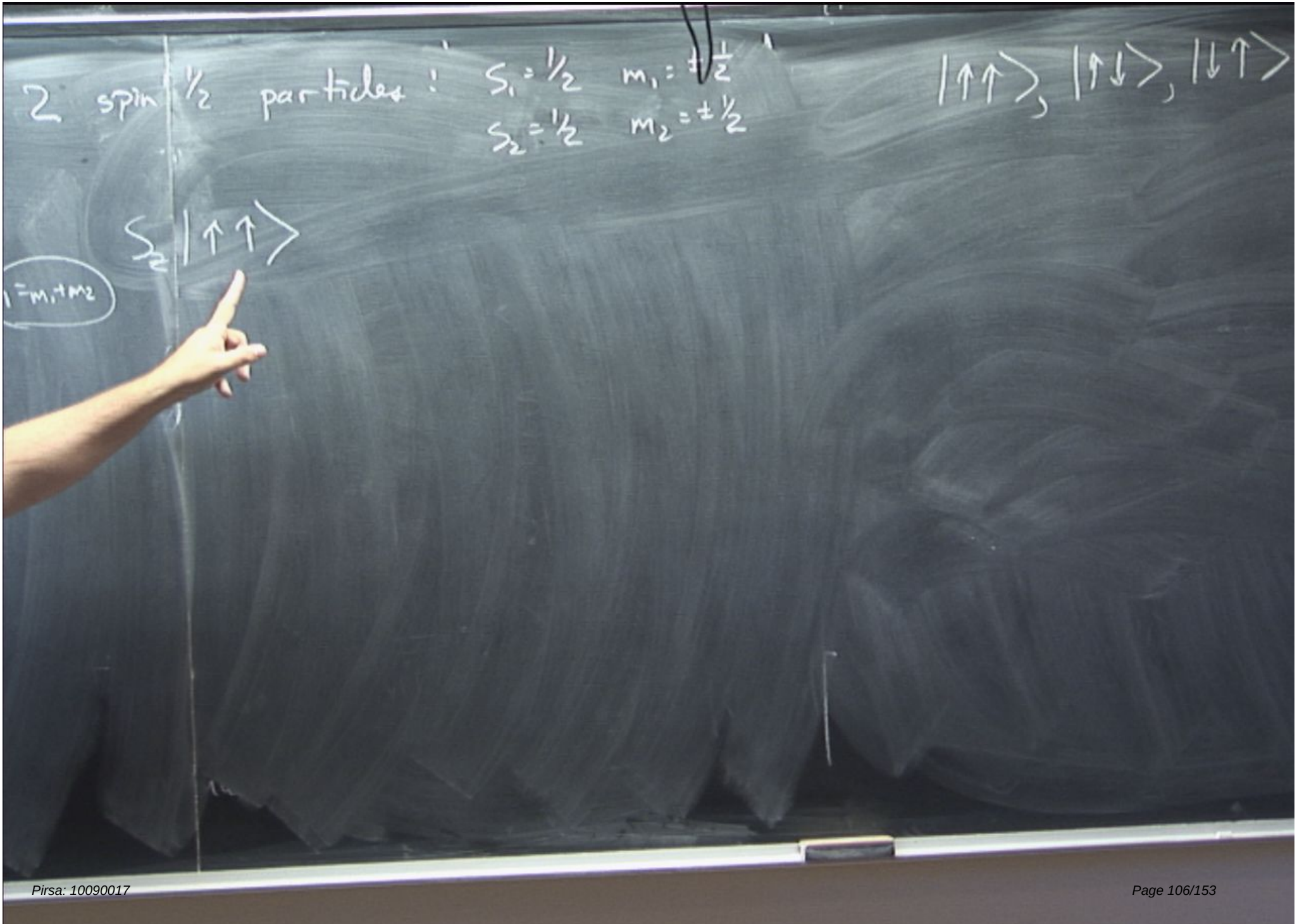
$$= [S_x^{(1)}, S_y^{(1)}] + [S_x^{(2)}, S_y^{(2)}] = i\left(S_z^{(1)} + S_z^{(2)}\right)$$

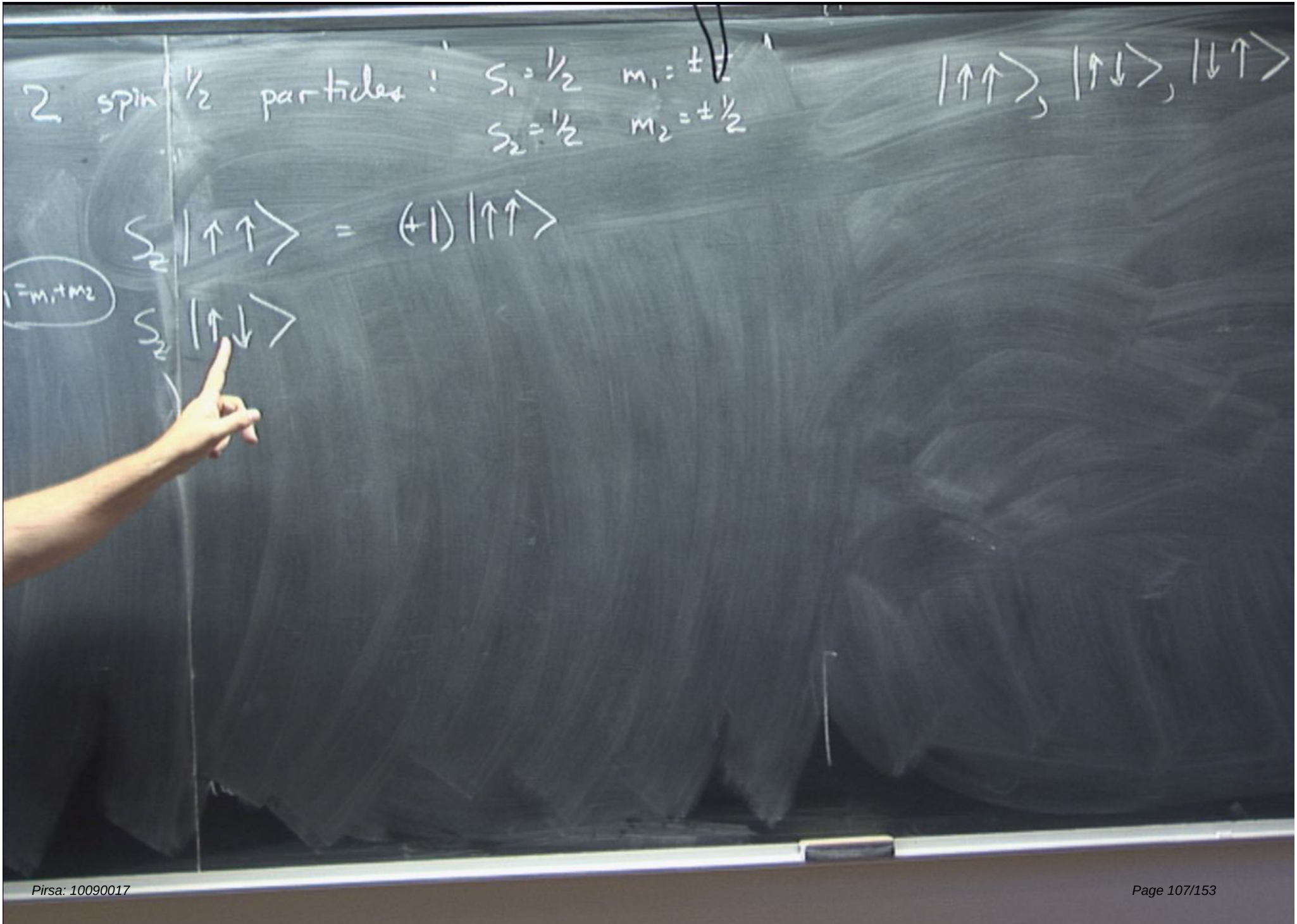
$$= iS_z$$


erators
$(F \otimes G)|\psi, \phi\rangle = (F|\psi\rangle) \otimes (G|\phi\rangle)$
(+ linearity)
lding spins
$(\hbar = 1)$
$\vec{S} = \alpha \vec{S}^{(1)} + \beta \vec{S}^{(2)}$
$S_x = \alpha S_x^{(1)} + \beta S_x^{(2)}$
etc.
$[S_x, S_y] = [S_x^{(1)} + S_x^{(2)}, S_y^{(1)} + S_y^{(2)}]$
$= [S_x^{(1)}, S_y^{(1)}] + [S_x^{(2)}, S_y^{(2)}] = i(S_z^{(1)} + S_z^{(2)})$
$= iS_z$
$F^{(A)} \otimes$
G


(G|ϕ⟩)
2 spin ½ particles : $S_1 = ½$ $m_1 = \pm ½$
$S_2 = ½$ m_2


2 spin 1/2 particles : $s_1 = 1/2$ $m_1 = \pm 1/2$
$s_2 = 1/2$ $m_2 = \pm 1/2$
$|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$


2 spin 1/2 particles : $s_1 = 1/2$ $m_1 = \pm 1/2$
$s_2 = 1/2$ $m_2 = \pm 1/2$
$|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$
$S_z|\uparrow\uparrow\rangle$
$= m_1 + m_2$


2 spin ½ particles : $s_1 = ½$ $m_1 = \pm ½$
$s_2 = ½$ $m_2 = \pm ½$
$|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$
$S_z|\uparrow\uparrow\rangle = (+1)|\uparrow\uparrow\rangle$
$S_z|\uparrow\downarrow\rangle$
$= m_1 + m_2$

2 spin ½ particles : $S_1 = ½$ $m_1 = \pm ½$
$S_2 = ½$ $m_2 = \pm ½$

$|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$

$S_z|\uparrow\uparrow\rangle = (+1)|\uparrow\uparrow\rangle$

$m = m_1 + m_2$

$S_z|\uparrow\downarrow\rangle = 0$

$S_z|\downarrow\uparrow\rangle = 0$

$S_z|\downarrow\downarrow\rangle =$

2 spin ½ particles : $s_1 = \frac{1}{2}$ $m_1 = \pm\frac{1}{2}$

$s_2 = \frac{1}{2}$ $m_2 = \pm\frac{1}{2}$

$|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle$

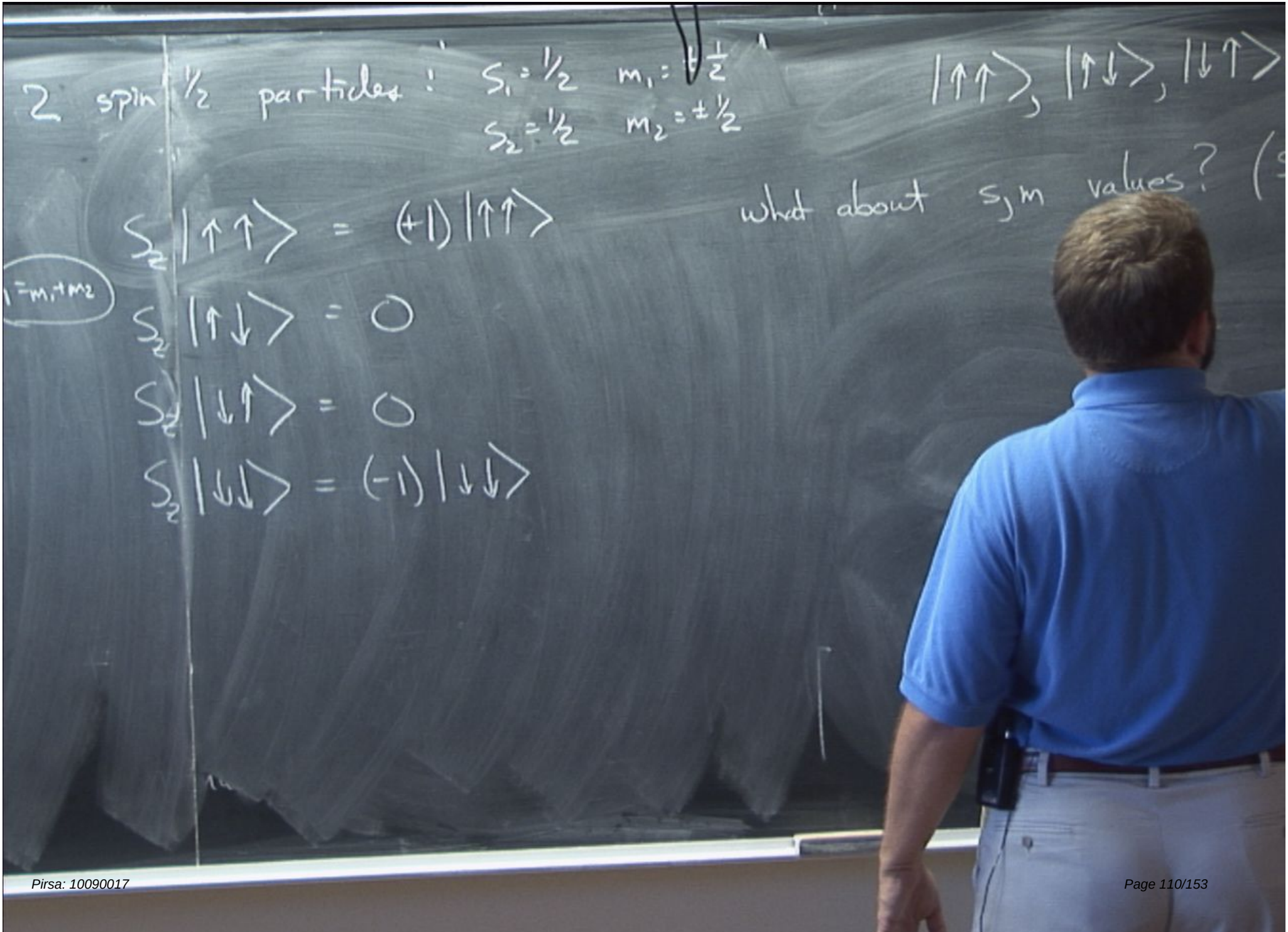
$m = m_1 + m_2$

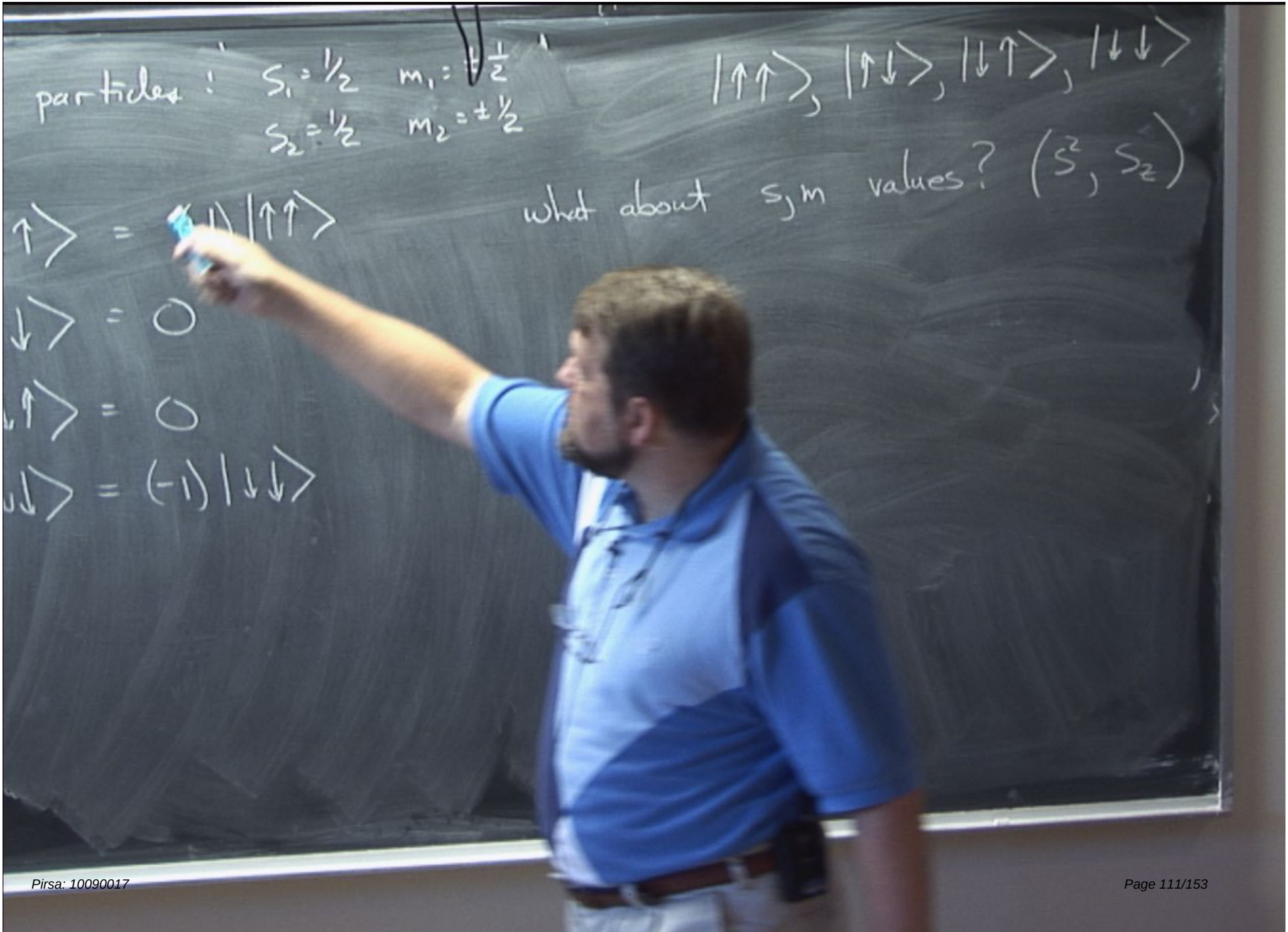
$S_z|\uparrow\uparrow\rangle = (+1)|\uparrow\uparrow\rangle$

$S_z|\uparrow\downarrow\rangle = 0$

$S_z|\downarrow\uparrow\rangle = 0$

$S_z|\downarrow\downarrow\rangle = (-1)|\downarrow\downarrow\rangle$


2 spin ½ particles : $s_1 = ½$ $m_1 = \pm ½$
$s_2 = ½$ $m_2 = \pm ½$
$|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$
what about s, m values?
$S_z|\uparrow\uparrow\rangle = (+1)|\uparrow\uparrow\rangle$
$S_z|\uparrow\downarrow\rangle = 0$
$S_z|\downarrow\uparrow\rangle = 0$
$S_z|\downarrow\downarrow\rangle = (-1)|\downarrow\downarrow\rangle$
$=m_1+m_2$


particles : $s_1 = 1/2$ $m_1 = \pm\frac{1}{2}$
$s_2 = 1/2$ $m_2 = \pm 1/2$
$|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$
what about s, m values? (S^2, S_z)
$|\uparrow\rangle = |\uparrow\uparrow\rangle$
$|\downarrow\rangle = 0$
$|\uparrow\rangle = 0$
$|\downarrow\rangle = (-1)|\downarrow\downarrow\rangle$

particles: $s_1 = 1/2$ $m_1 = \pm 1/2$

$s_2 = 1/2$ $m_2 = \pm 1/2$

$|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$

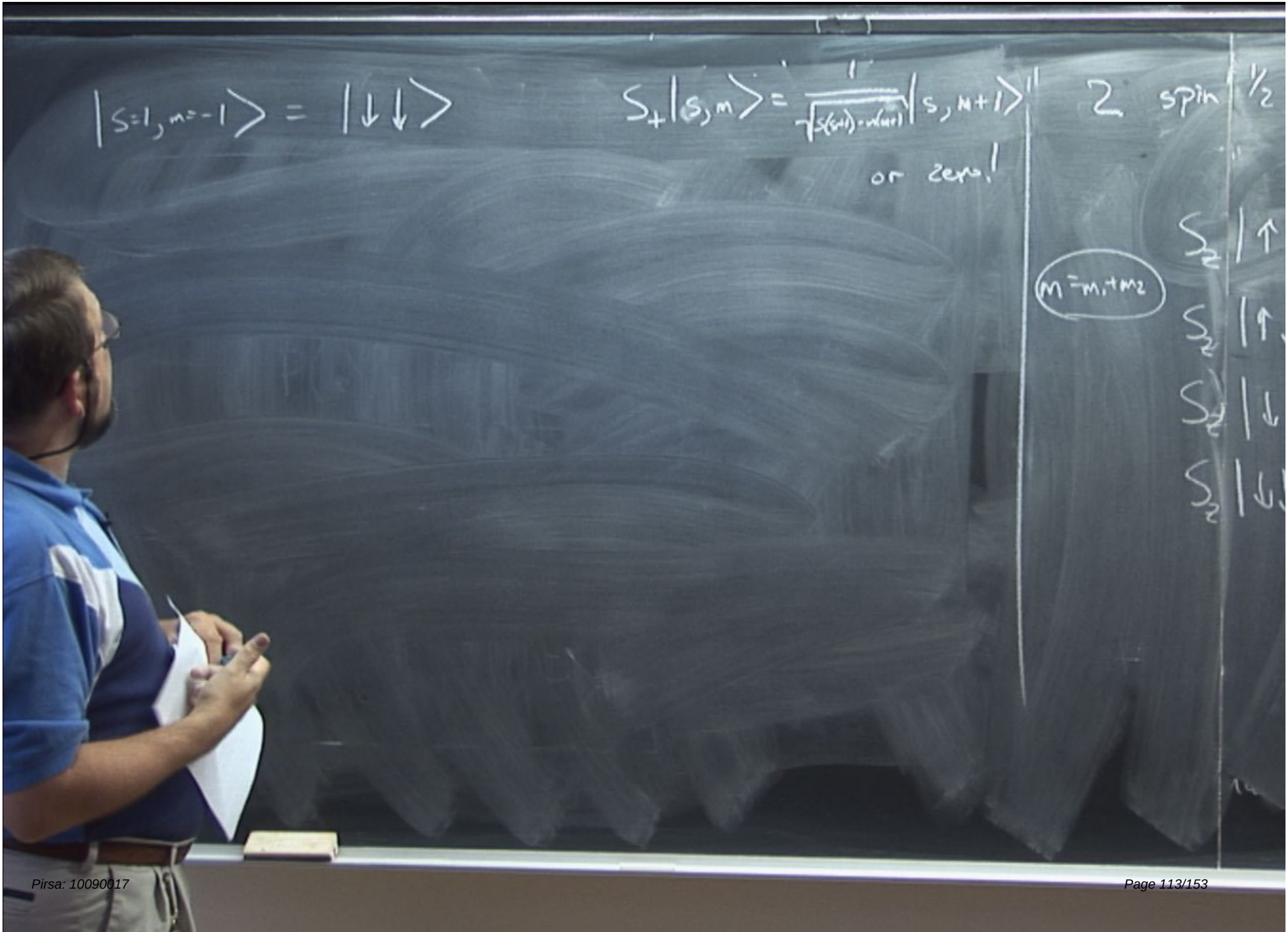
what about s, m values? (S^2, S_z)

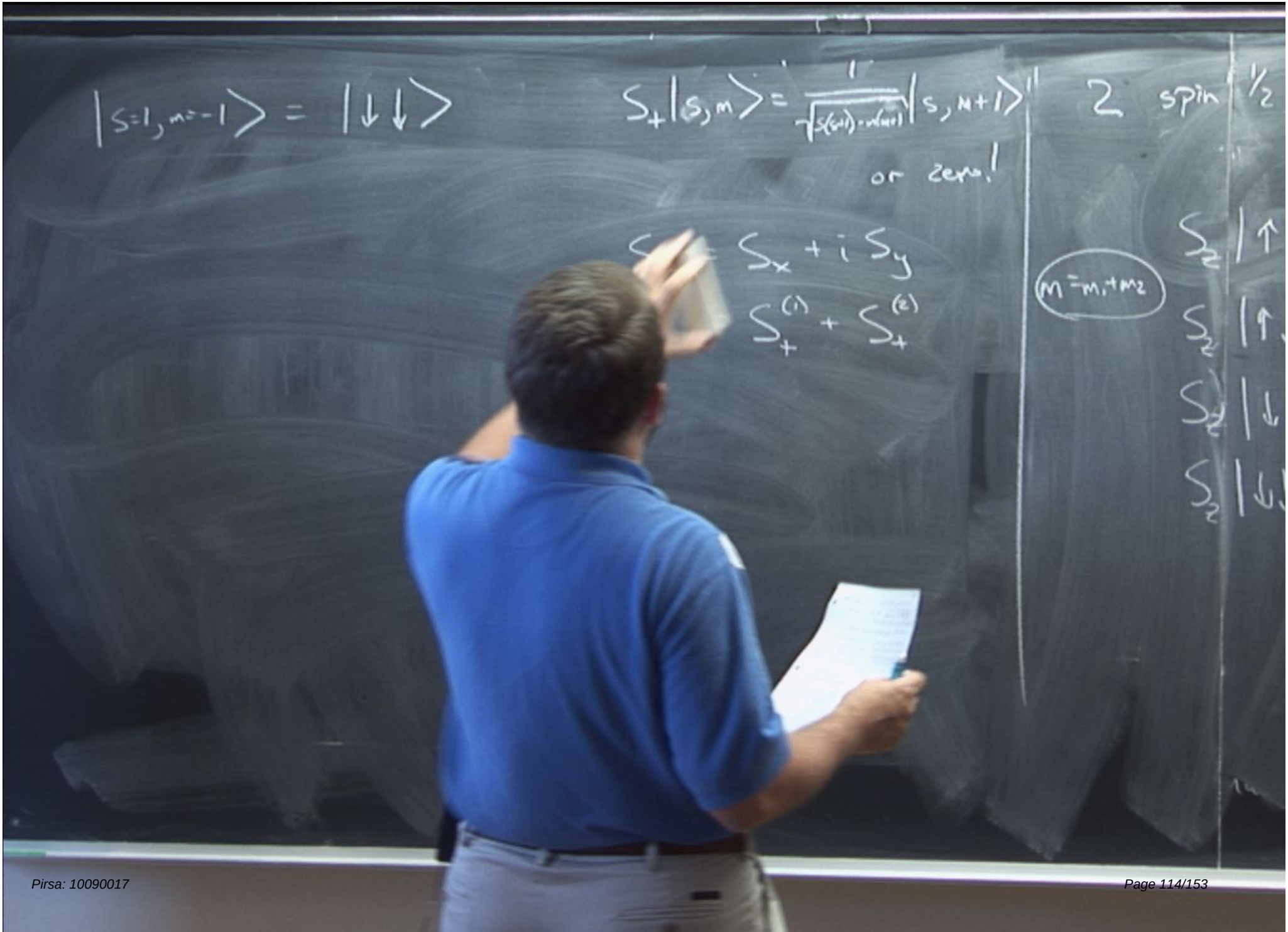
$|\uparrow\rangle = (+1)\,|\uparrow\uparrow\rangle$

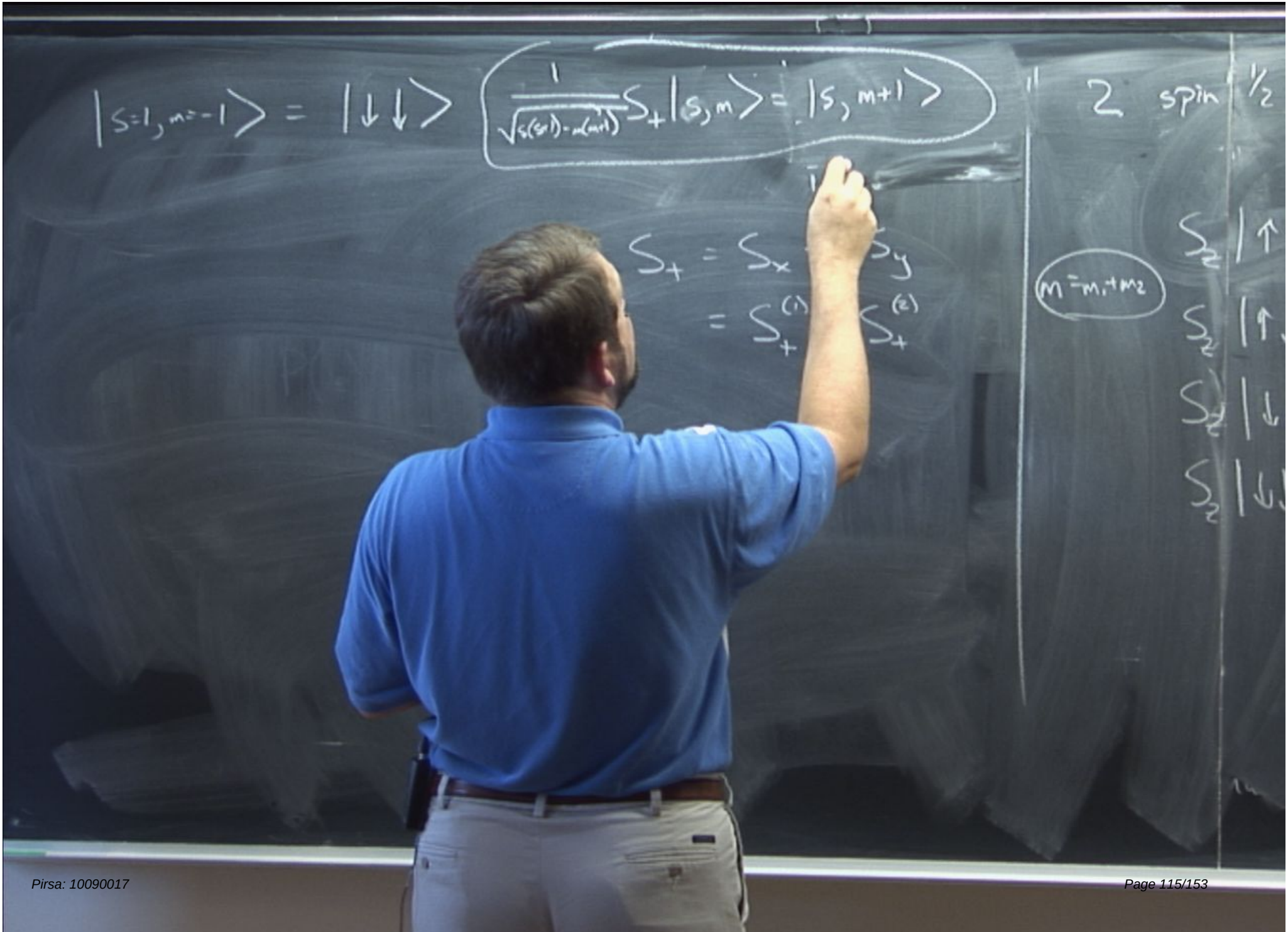
$|\downarrow\rangle = 0$

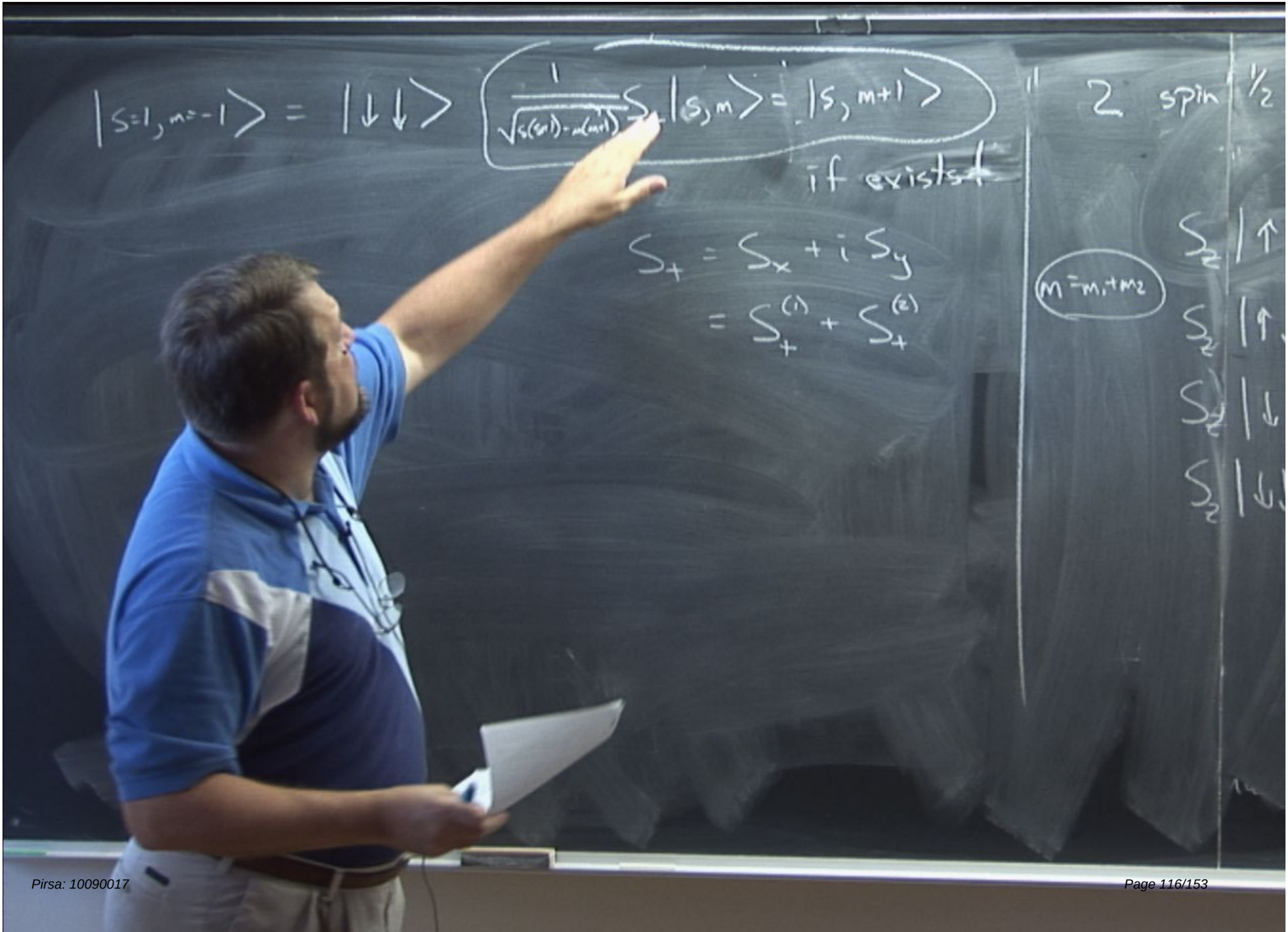
$|\uparrow\rangle = 0$

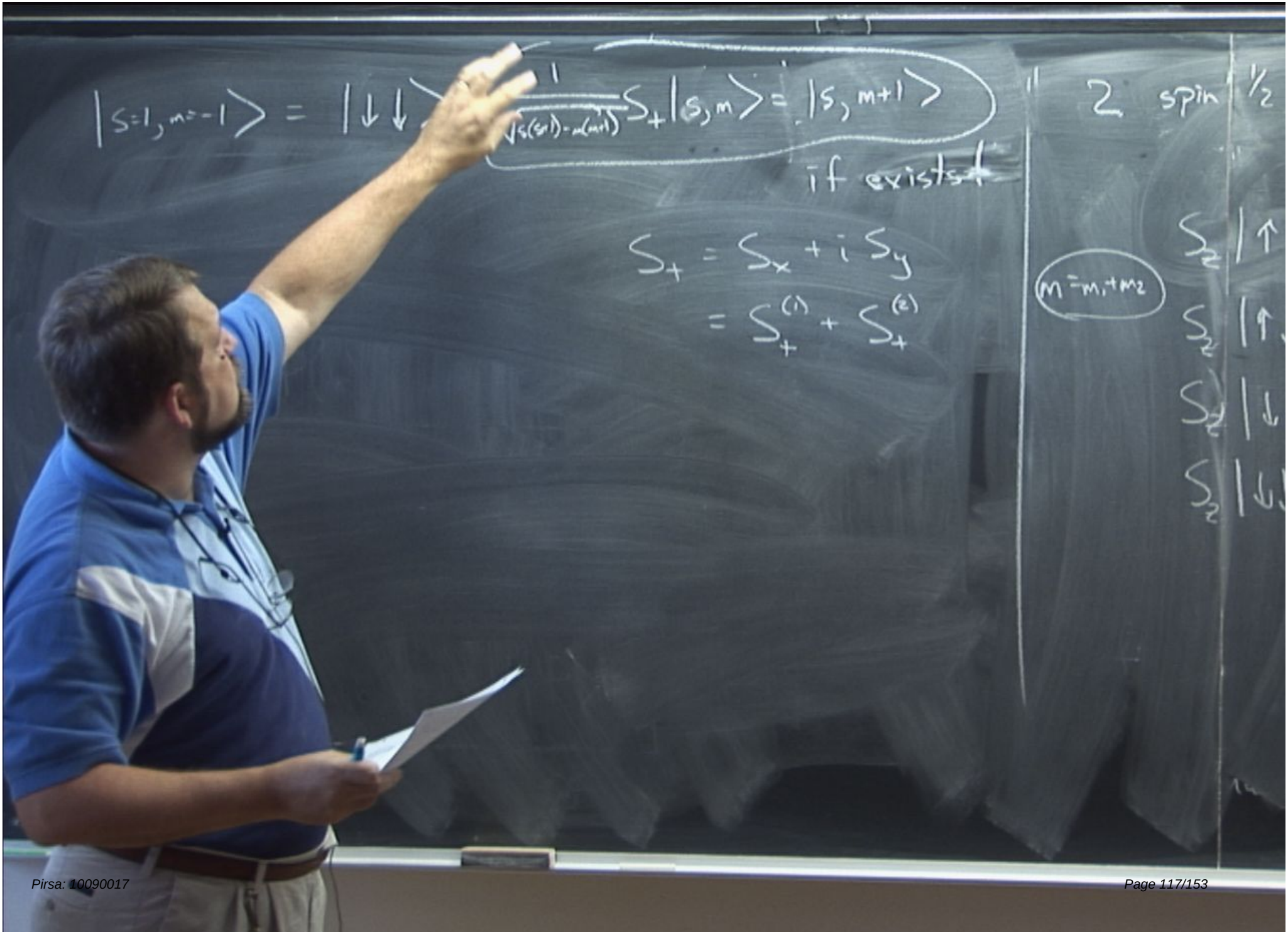
$|\downarrow\rangle = (-1)\,|\downarrow\downarrow\rangle$ ← $m = -1$ ← bottom rung of $s = 1$ ladder

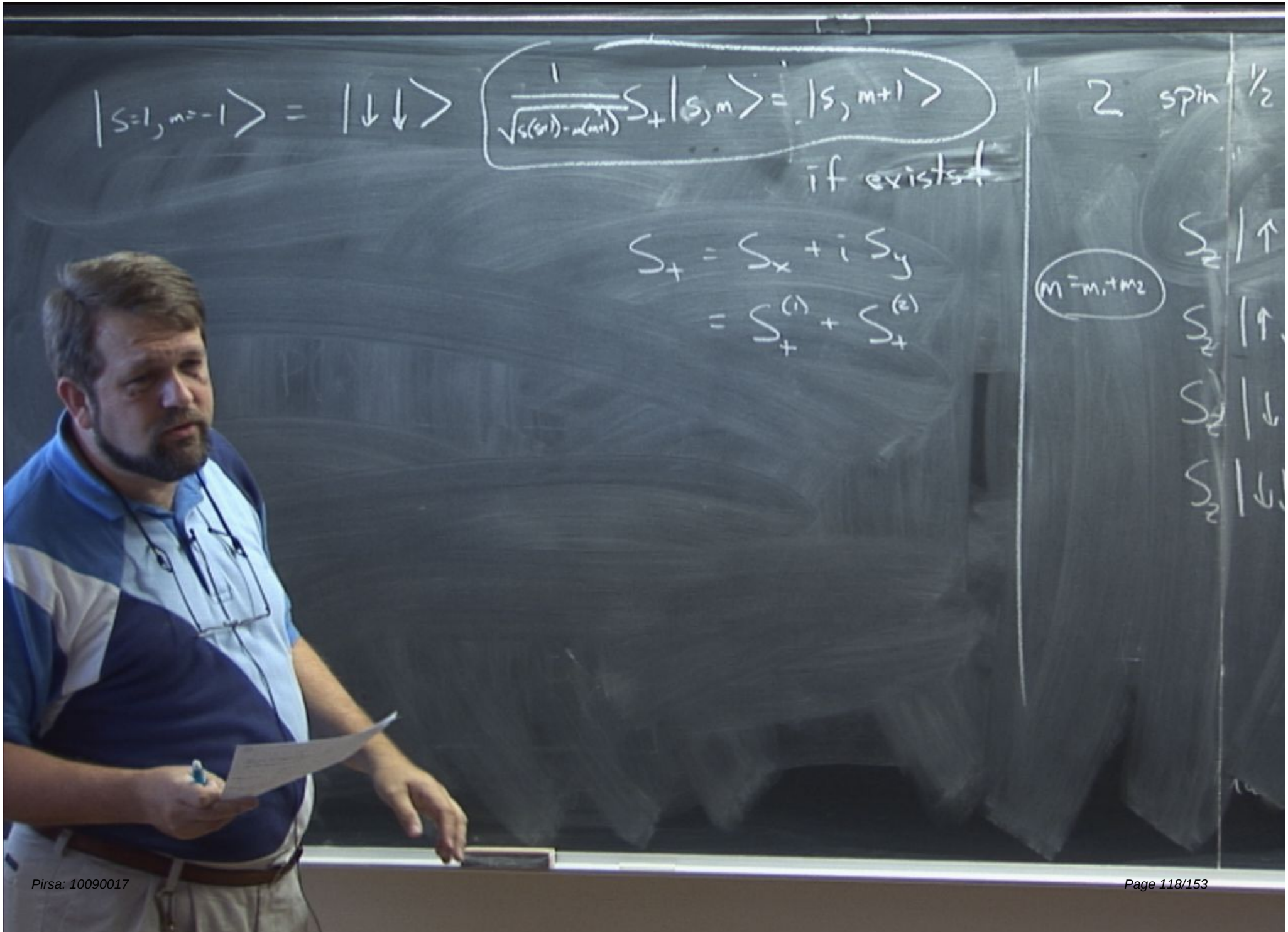

$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$
$S_+|s,m\rangle = \frac{1}{\sqrt{s(s+1)-m(m+1)}}|s,m+1\rangle$
or zero!
2 spin ½
$m = m_1 + m_2$
$S_z|\uparrow$
$S_z|\uparrow$
$S_z|\downarrow$
$S_z|\downarrow$

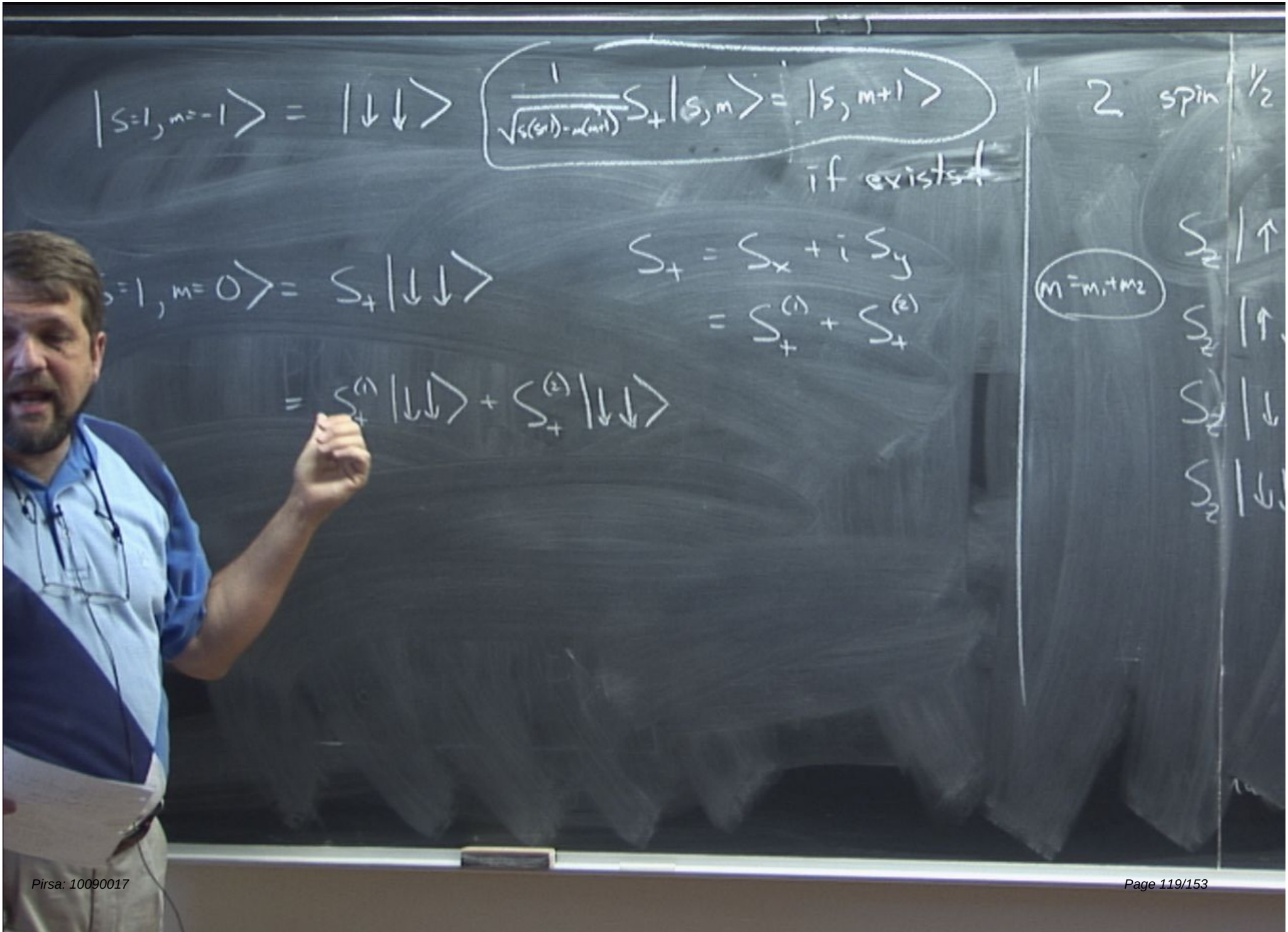

$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$
$S_+|s,m\rangle = \frac{1}{\sqrt{s(s+1)-m(m+1)}}|s,m+1\rangle$
or zero!
$S_+ = S_x + iS_y$
$S_+^{(1)} + S_+^{(2)}$
2 spin ½
$M = m_1 + m_2$
$S_z|\uparrow$
$S_z|\uparrow$
$S_z|\downarrow$
$S_z|\downarrow$


$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$
$\frac{1}{\sqrt{s(s+1)-m(m+1)}} S_+ |s,m\rangle = |s, m+1\rangle$
$S_+ = S_x + S_y$
$= S_+^{(1)} + S_+^{(2)}$
2 spin ½
$m = m_1 + m_2$
$S_z |\uparrow$
$S_z |\uparrow$
$S_z |\downarrow$
$S_z |\downarrow$


$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$
$\frac{1}{\sqrt{s(s+1)-m(m+1)}} S_+ |s,m\rangle = |s, m+1\rangle$
if exists
$S_+ = S_x + iS_y$
$= S_+^{(1)} + S_+^{(2)}$
2 spin ½
$m = m_1 + m_2$
$S_z |\uparrow$
$S_z |\uparrow$
$S_z |\downarrow$
$S_z |\downarrow$


$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$
$\frac{1}{\sqrt{s(s+1)-m(m+1)}} S_+ |s,m\rangle = |s, m+1\rangle$
if exists
$S_+ = S_x + iS_y$
$= S_+^{(1)} + S_+^{(2)}$
2 spin ½
$m = m_1 + m_2$
$S_z |\uparrow$
$S_z |\uparrow$
$S_z |\downarrow$
$S_z |\downarrow$


$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$
$\frac{1}{\sqrt{s(s+1)-m(m+1)}} S_+ |s, m\rangle = |s, m+1\rangle$
if exists
$S_+ = S_x + iS_y$
$= S_+^{(1)} + S_+^{(2)}$
2 spin ½
$m = m_1 + m_2$
$S_2 |\uparrow$
$S_2 |\uparrow$
$S_2 |\downarrow$
$S_2 |\downarrow$


$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$
$\frac{1}{\sqrt{s(s+1)-m(m+1)}} S_+ |s,m\rangle = |s, m+1\rangle$
if exists
$|S=1, m=0\rangle = S_+ |\downarrow\downarrow\rangle$
$= S_+^{(1)} |\downarrow\downarrow\rangle + S_+^{(2)} |\downarrow\downarrow\rangle$
$S_+ = S_x + i S_y$
$= S_+^{(1)} + S_+^{(2)}$
2 spin ½
$m = m_1 + m_2$
$S_z |\uparrow$
$S_z |\uparrow$
$S_z |\downarrow$
$S_z |\downarrow$

$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$

$\frac{1}{\sqrt{s(s+1)-m(m+1)}} S_+ |s, m\rangle = |s, m+1\rangle$ if exists

$|S=1, m=0\rangle = S_+ |\downarrow\downarrow\rangle$

$S_+ = S_x + i S_y$

$= S_+^{(1)} + S_+^{(2)}$

$= S_+^{(1)} |\downarrow\downarrow\rangle + S_+^{(2)} |\downarrow\downarrow\rangle$

$=$

2 spin 1/2

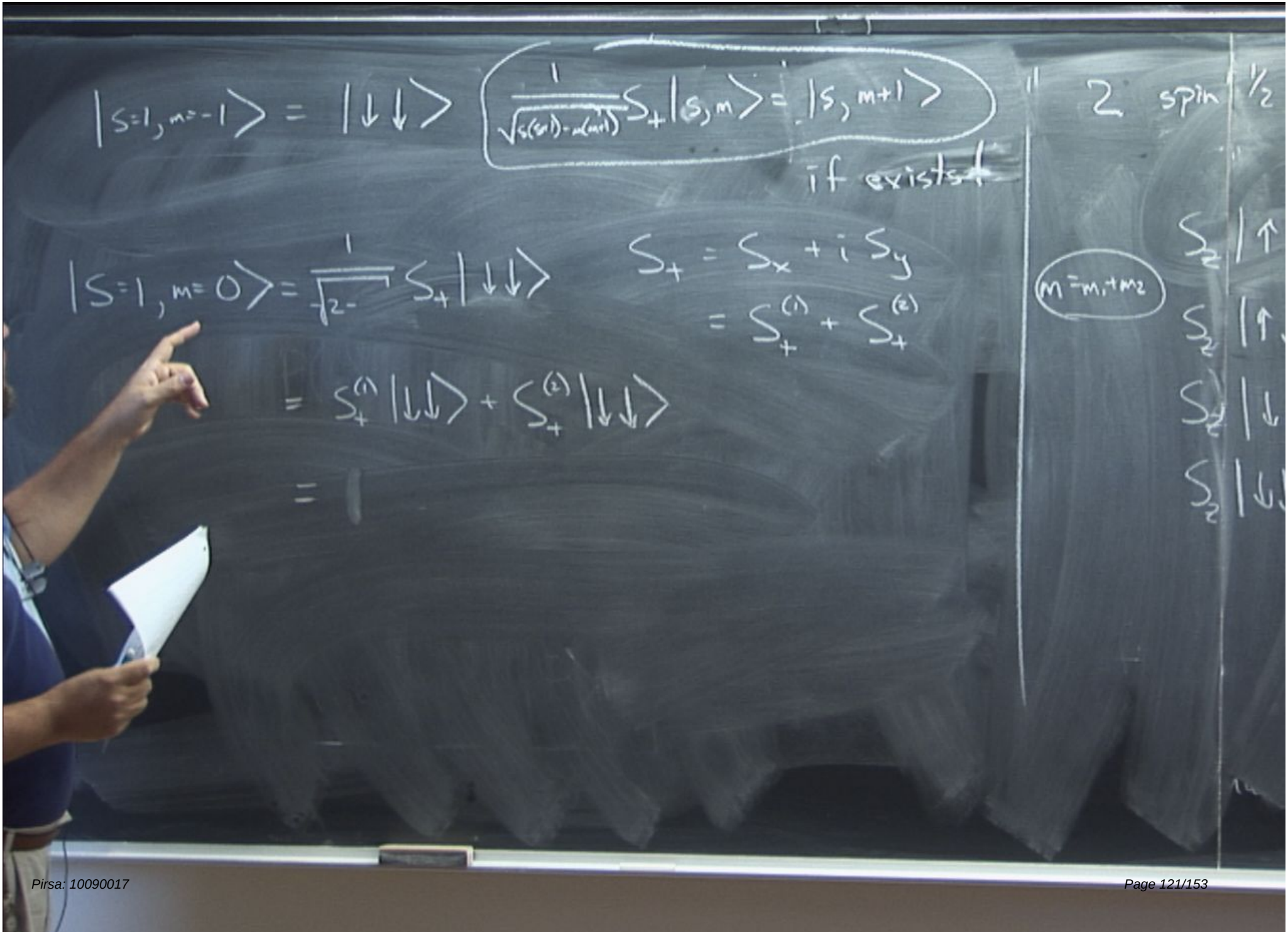
$m = m_1 + m_2$

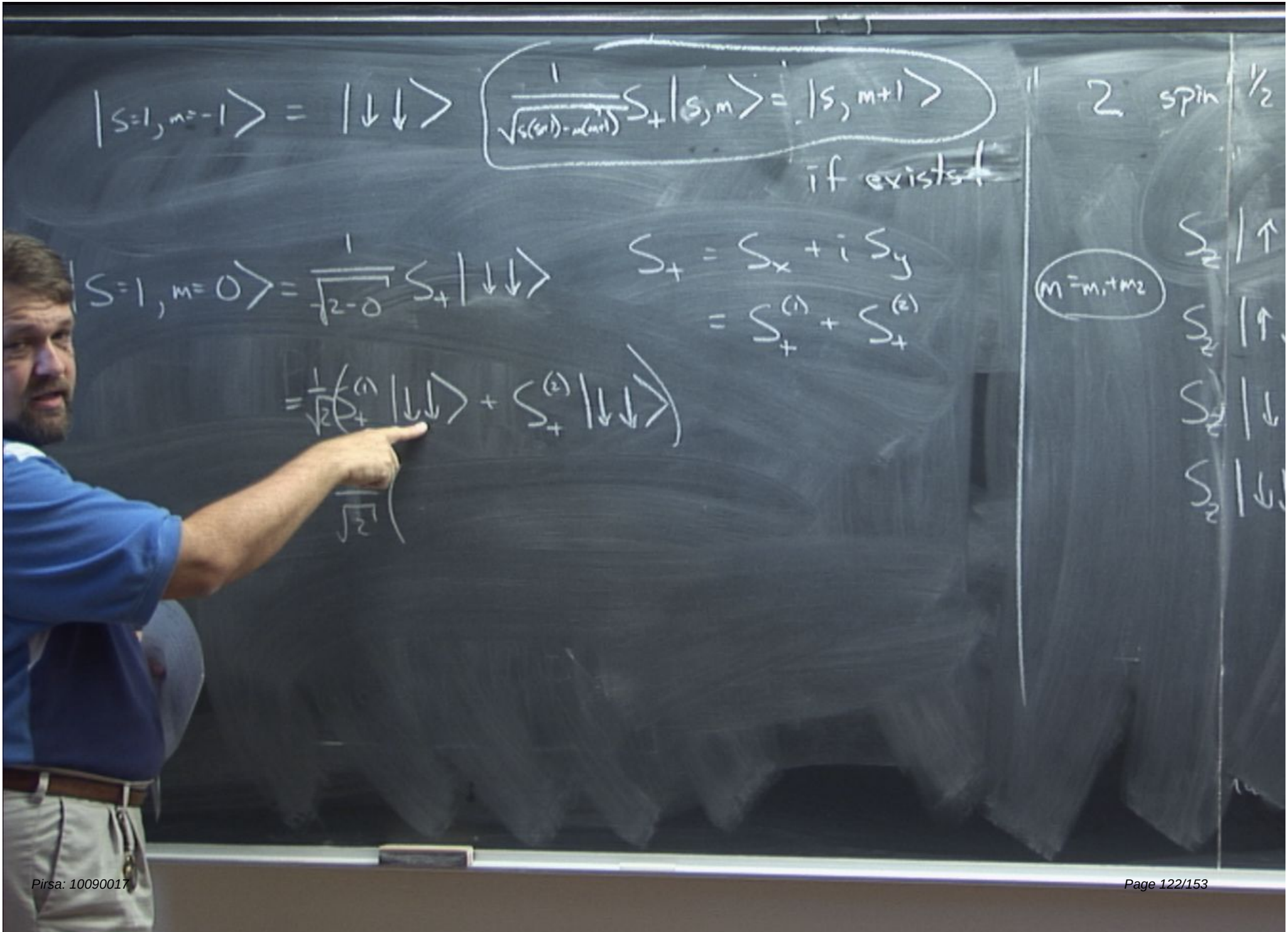
$S_z |\uparrow$

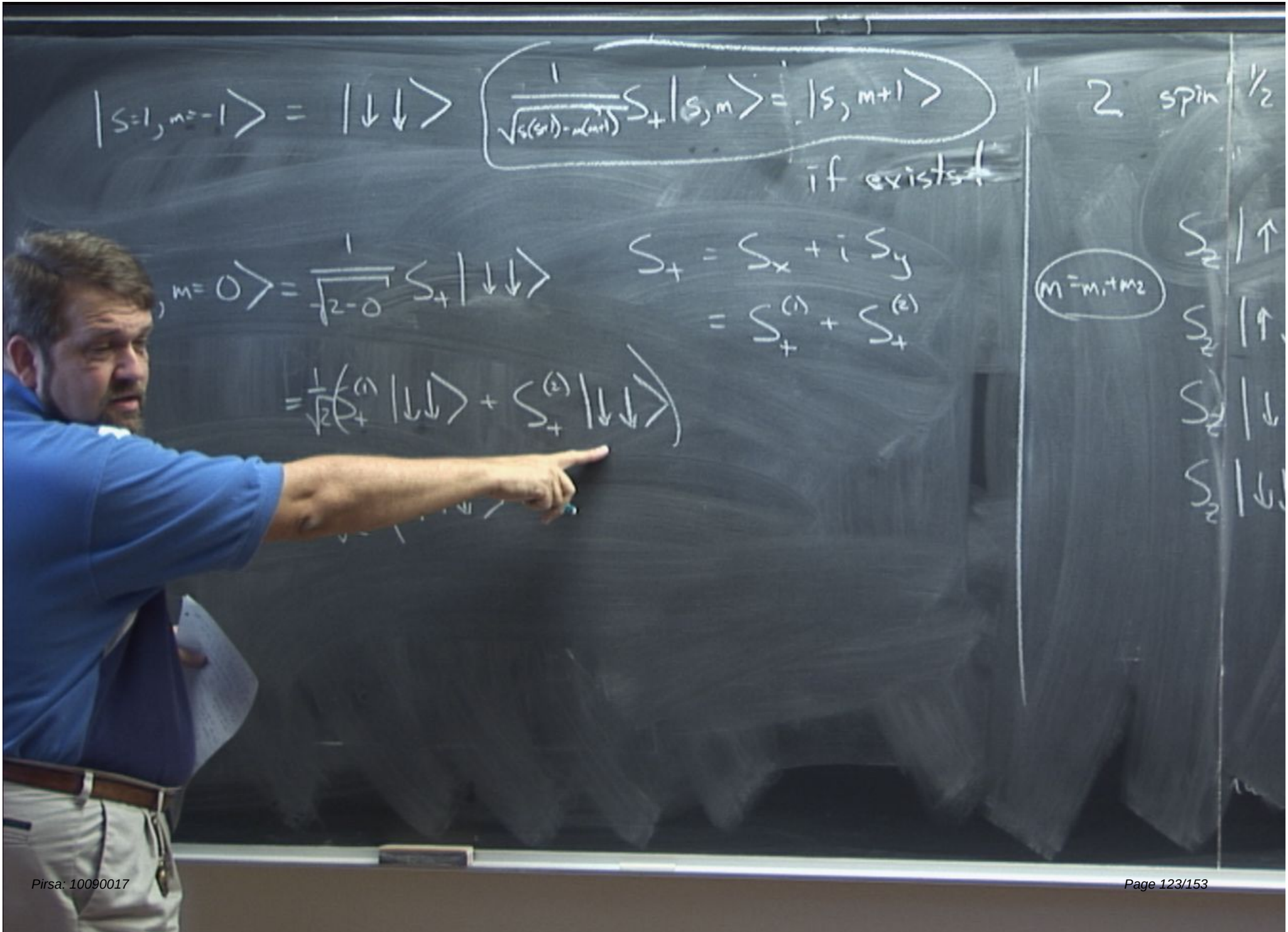
$S_z |\uparrow$

$S_z |\downarrow$

$S_z |\downarrow$


$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$
$\frac{1}{\sqrt{s(s+1)-m(m+1)}} S_+ |s,m\rangle = |s, m+1\rangle$
if exists
$|S=1, m=0\rangle = \frac{1}{\sqrt{2}} S_+ |\downarrow\downarrow\rangle$
$S_+ = S_x + i S_y$
$= S_+^{(1)} + S_+^{(2)}$
$= S_+^{(1)} |\downarrow\downarrow\rangle + S_+^{(2)} |\downarrow\downarrow\rangle$
$=$
2 spin ½
$m = m_1 + m_2$
$S_z |\uparrow$
$S_z |\uparrow$
$S_z |\downarrow$
$S_z |\downarrow$


$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$
$\frac{1}{\sqrt{s(s+1)-m(m+1)}} S_+ |s,m\rangle = |s, m+1\rangle$
if exists
$|S=1, m=0\rangle = \frac{1}{\sqrt{2-0}} S_+ |\downarrow\downarrow\rangle$
$S_+ = S_x + i S_y$
$= S_+^{(1)} + S_+^{(2)}$
$= \frac{1}{\sqrt{2}} \left(S_+^{(1)} |\downarrow\downarrow\rangle + S_+^{(2)} |\downarrow\downarrow\rangle \right)$
$\frac{1}{\sqrt{2}}$(
2 spin ½
$m = m_1 + m_2$
$S_z |\uparrow$
$S_z |\uparrow$
$S_z |\downarrow$
$S_z |\downarrow$


$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$
$\frac{1}{\sqrt{s(s+1)-m(m+1)}} S_+ |s,m\rangle = |s, m+1\rangle$
if exists
$m=0\rangle = \frac{1}{\sqrt{2-0}} S_+ |\downarrow\downarrow\rangle$
$S_+ = S_x + iS_y$
$= S_+^{(1)} + S_+^{(2)}$
$= \frac{1}{\sqrt{2}} \left(S_+^{(1)} |\downarrow\downarrow\rangle + S_+^{(2)} |\downarrow\downarrow\rangle \right)$
2 spin ½
$m = m_1 + m_2$
$S_z |\uparrow$
$S_z |\uparrow$
$S_z |\downarrow$
$S_z |\downarrow$

$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$

$\frac{1}{\sqrt{s(s+1)-m(m-1)}} S_+ |s, m\rangle = |s, m+1\rangle$ if exists

$|S=1, m=0\rangle = \frac{1}{\sqrt{2-0}} S_+ |\downarrow\downarrow\rangle$

$S_+ = S_x + i S_y$

$= S_+^{(1)} + S_+^{(2)}$

$= \frac{1}{\sqrt{2}} \left(S_+^{(1)} |\downarrow\downarrow\rangle + S_+^{(2)} |\downarrow\downarrow\rangle \right)$

$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right)$

2 spin ½

$m = m_1 + m_2$

$S_z |\uparrow$

$S_z |\uparrow$

$S_z |\downarrow$

$S_z |\downarrow$

$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$

$\frac{1}{\sqrt{s(s+1)-m(m-1)}} S_+ |s, m\rangle = |s, m+1\rangle$

if exists

$|S=1, m=0\rangle = \frac{1}{\sqrt{2-0}} S_+ |\downarrow\downarrow\rangle$

$S_+ = S_x + i S_y$

$= S_+^{(1)} + S_+^{(2)}$

$= \frac{1}{\sqrt{2}} \left(S_+^{(1)} |\downarrow\downarrow\rangle + S_+^{(2)} |\downarrow\downarrow\rangle \right)$

$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right)$

2 spin 1/2

$m = m_1 + m_2$

$S_z |\uparrow$

$S_z |\uparrow$

$S_z |\downarrow$

$S_z |\downarrow$

$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$

$\frac{1}{\sqrt{s(s+1)-m(m-1)}} S_+ |s, m\rangle = |s, m+1\rangle$

if exists

$|S=1, m=0\rangle = \frac{1}{\sqrt{2-0}} S_+ |\downarrow\downarrow\rangle$

$S_+ = S_x + i S_y$

$= S_+^{(1)} + S_+^{(2)}$

$= \frac{1}{\sqrt{2}} \left(S_+^{(1)} |\downarrow\downarrow\rangle + S_+^{(2)} |\downarrow\downarrow\rangle \right)$

$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right)$

$|S=1, m=1\rangle = \frac{1}{\sqrt{2-0}} S_+ \left(\frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right) \right)$

2 spin ½

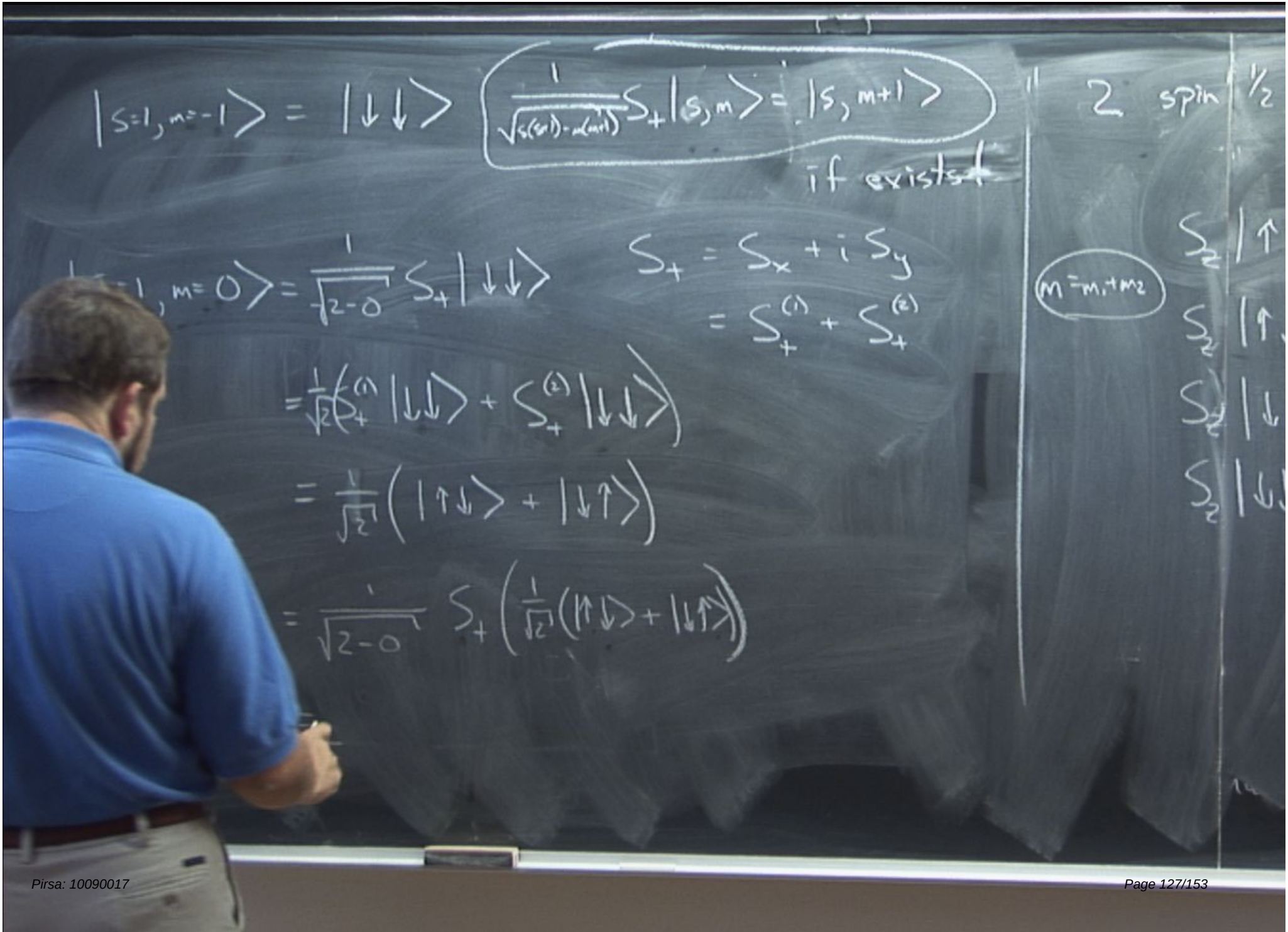
$m = m_1 + m_2$

$S_z |\uparrow$

$S_z |\uparrow$

$S_z |\downarrow$

$S_z |\downarrow$


$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$
$\frac{1}{\sqrt{s(s+1)-m(m-1)}} S_+ |s,m\rangle = |s, m+1\rangle$
if exists
$S_+ = S_x + iS_y$
$= S_+^{(1)} + S_+^{(2)}$
$|1, m=0\rangle = \frac{1}{\sqrt{2-0}} S_+ |\downarrow\downarrow\rangle$
$= \frac{1}{\sqrt{2}}\left(S_+^{(1)}|\downarrow\downarrow\rangle + S_+^{(2)}|\downarrow\downarrow\rangle\right)$
$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$
$= \frac{1}{\sqrt{2-0}} S_+ \left(\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)\right)$
2 spin ½
$m = m_1 + m_2$
$S_z|\uparrow$
$S_z|\uparrow$
$S_z|\downarrow$
$S_z|\downarrow$

$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$

$\frac{1}{\sqrt{s(s+1)-m(m+1)}} S_+ |s,m\rangle = |s, m+1\rangle$ if exists!

$|S=1, m=0\rangle = \frac{1}{\sqrt{2-0}} S_+ |\downarrow\downarrow\rangle$

$S_+ = S_x + iS_y$

$= S_+^{(1)} + S_+^{(2)}$

$= \frac{1}{\sqrt{2}}\left(S_+^{(1)}|\downarrow\downarrow\rangle + S_+^{(2)}|\downarrow\downarrow\rangle\right)$

$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$

$|S=1, m=1\rangle = \frac{1}{\sqrt{2-0}} S_+ \left(\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)\right)$

$= \frac{1}{2}\left(S_+^{(1)}|\uparrow\downarrow\rangle + S_+^{(1)}|\downarrow\uparrow\rangle + S_+^{(2)}|\uparrow\downarrow\rangle + S_+^{(2)}|\downarrow\uparrow\rangle\right)$

0

2 spin ½

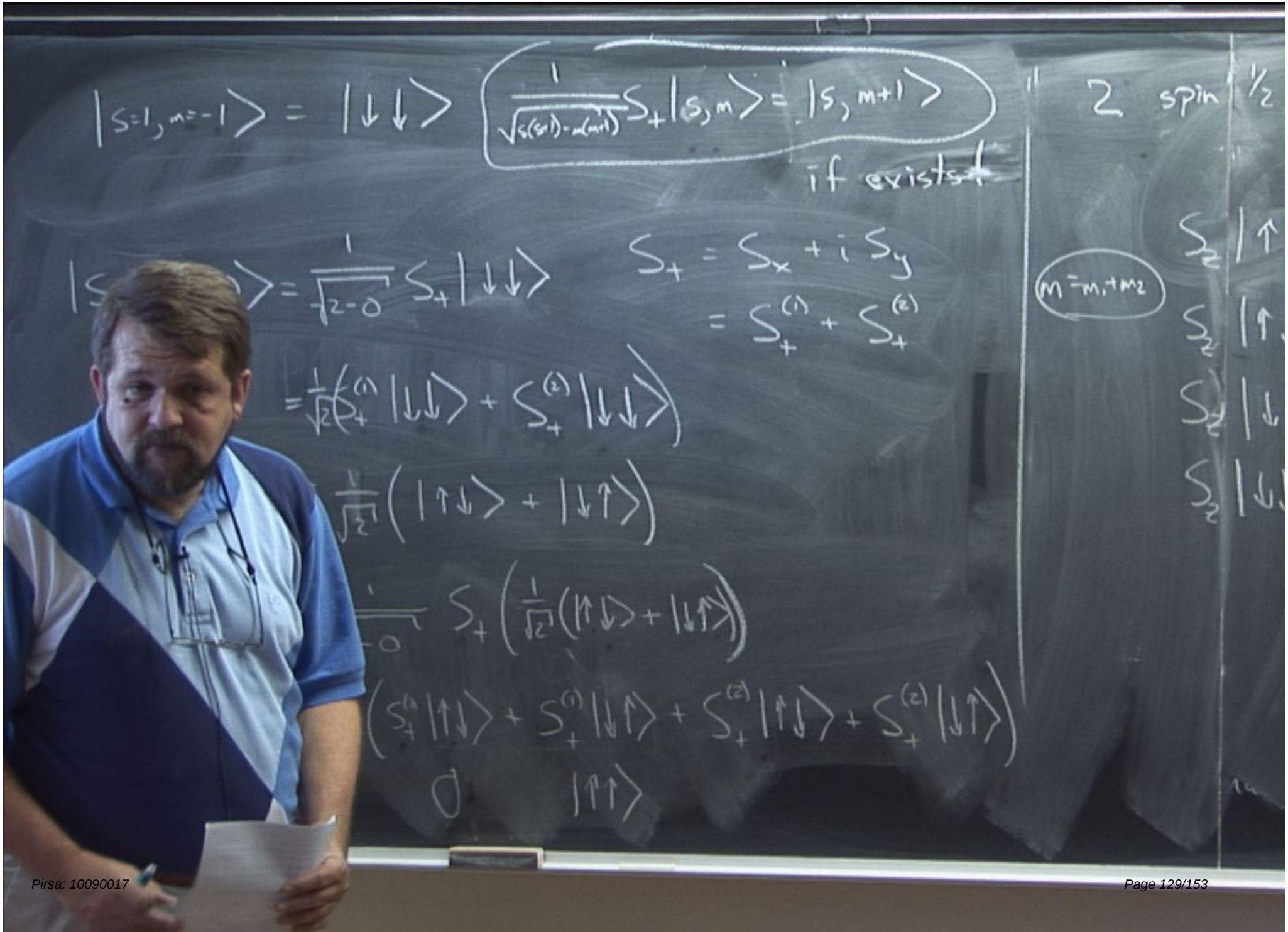
$m = m_1 + m_2$

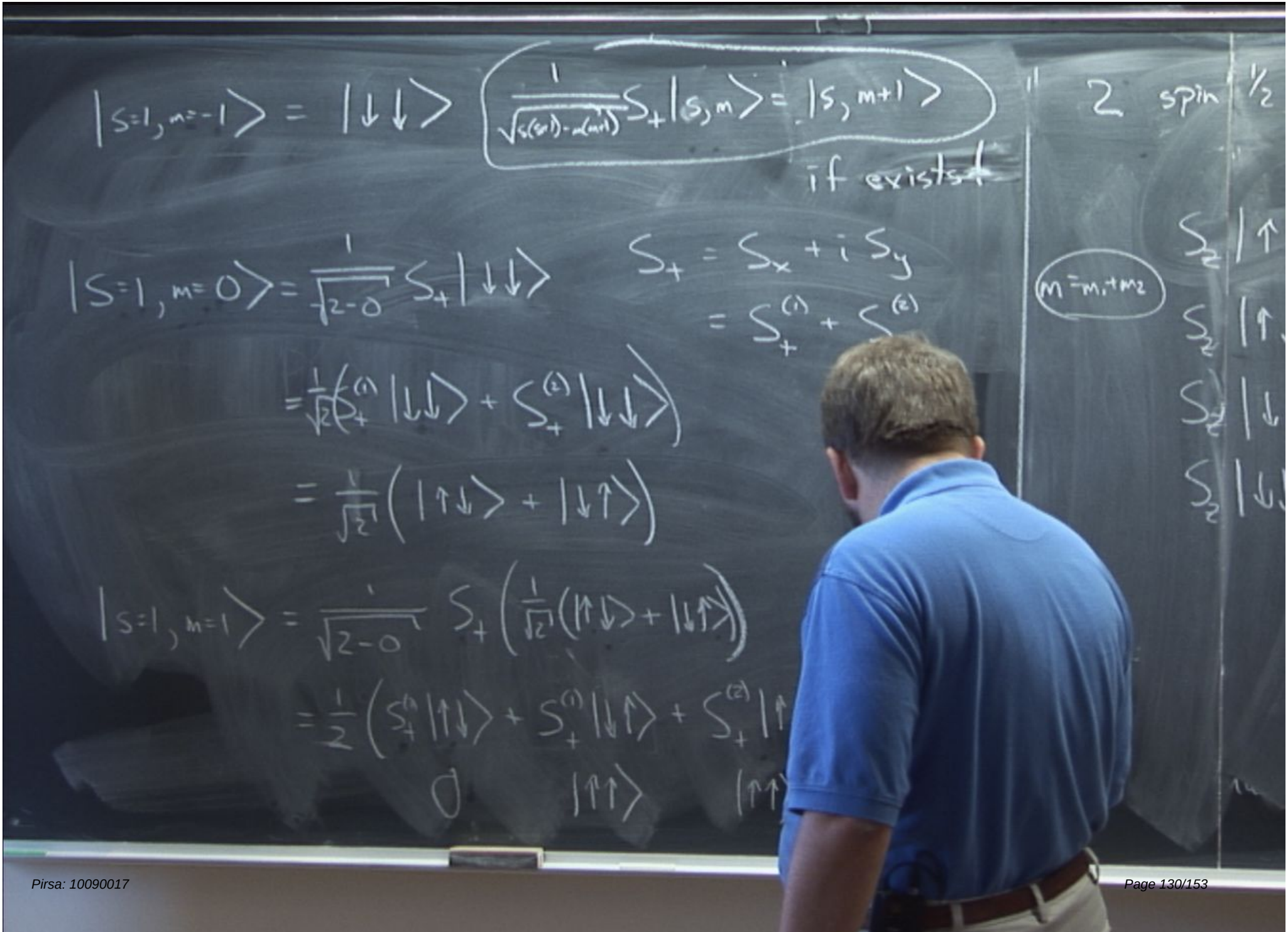
$S_z|\uparrow$

$S_z|\uparrow$

$S_z|\downarrow$

$S_z|\downarrow$


$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$
$\frac{1}{\sqrt{s(s+1)-m(m+1)}} S_+ |s,m\rangle = |s, m+1\rangle$
if exists
$|S \quad \rangle = \frac{1}{\sqrt{2-0}} S_+ |\downarrow\downarrow\rangle$
$S_+ = S_x + i S_y$
$= S_+^{(1)} + S_+^{(2)}$
$= \frac{1}{\sqrt{2}} \left(S_+^{(1)} |\downarrow\downarrow\rangle + S_+^{(2)} |\downarrow\downarrow\rangle \right)$
$\frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right)$
$\frac{1}{\sqrt{2-0}} S_+ \left(\frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right) \right)$
$\left(S_+^{(1)} |\uparrow\downarrow\rangle + S_+^{(1)} |\downarrow\uparrow\rangle + S_+^{(2)} |\uparrow\downarrow\rangle + S_+^{(2)} |\downarrow\uparrow\rangle \right)$
$0 \qquad |\uparrow\uparrow\rangle$
2 spin ½
$m = m_1 + m_2$
$S_z |\uparrow$
$S_z |\uparrow$
$S_z |\downarrow$
$S_z |\downarrow$


$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$
$\frac{1}{\sqrt{s(s+1)-m(m-1)}} S_+ |s,m\rangle = |s, m+1\rangle$
if exists
$|S=1, m=0\rangle = \frac{1}{\sqrt{2-0}} S_+ |\downarrow\downarrow\rangle$
$S_+ = S_x + iS_y$
$= S_+^{(1)} + S_+^{(2)}$
$= \frac{1}{\sqrt{2}}\left(S_+^{(1)}|\downarrow\downarrow\rangle + S_+^{(2)}|\downarrow\downarrow\rangle\right)$
$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$
$|S=1, m=1\rangle = \frac{1}{\sqrt{2-0}} S_+ \left(\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)\right)$
$= \frac{1}{2}\left(S_+^{(1)}|\uparrow\downarrow\rangle + S_+^{(1)}|\downarrow\uparrow\rangle + S_+^{(2)}|\uparrow$
$0 \quad |\uparrow\uparrow\rangle \quad |\uparrow\uparrow$
2 spin ½
$m = m_1 + m_2$
$S_z|\uparrow$
$S_z|\uparrow$
$S_z|\downarrow$
$S_z|\downarrow$

$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$

$\frac{1}{\sqrt{s(s+1)-m(m-1)}} S_+ |s, m\rangle = |s, m+1\rangle$ if exists

$|S=1, m=0\rangle = \frac{1}{\sqrt{2-0}} S_+ |\downarrow\downarrow\rangle$

$S_+ = S_x + i S_y$

$= S_+^{(1)} + S_+^{(2)}$

$= \frac{1}{\sqrt{2}} \left(S_+^{(1)} |\downarrow\downarrow\rangle + S_+^{(2)} |\downarrow\downarrow\rangle \right)$

$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right)$

$|S=1, m=1\rangle = \frac{1}{\sqrt{2-0}} S_+ \left(\frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \right)$

$= \frac{1}{2} \left(S_+^{(1)} |\uparrow\downarrow\rangle + S_+^{(1)} |\downarrow\uparrow\rangle + S_+^{(2)} |\uparrow\downarrow\rangle + S_+^{(2)} |\downarrow\uparrow\rangle \right) = |\uparrow\uparrow\rangle$

$0 \quad |\uparrow\uparrow\rangle \quad |\uparrow\uparrow\rangle \quad 0$

2 spin ½

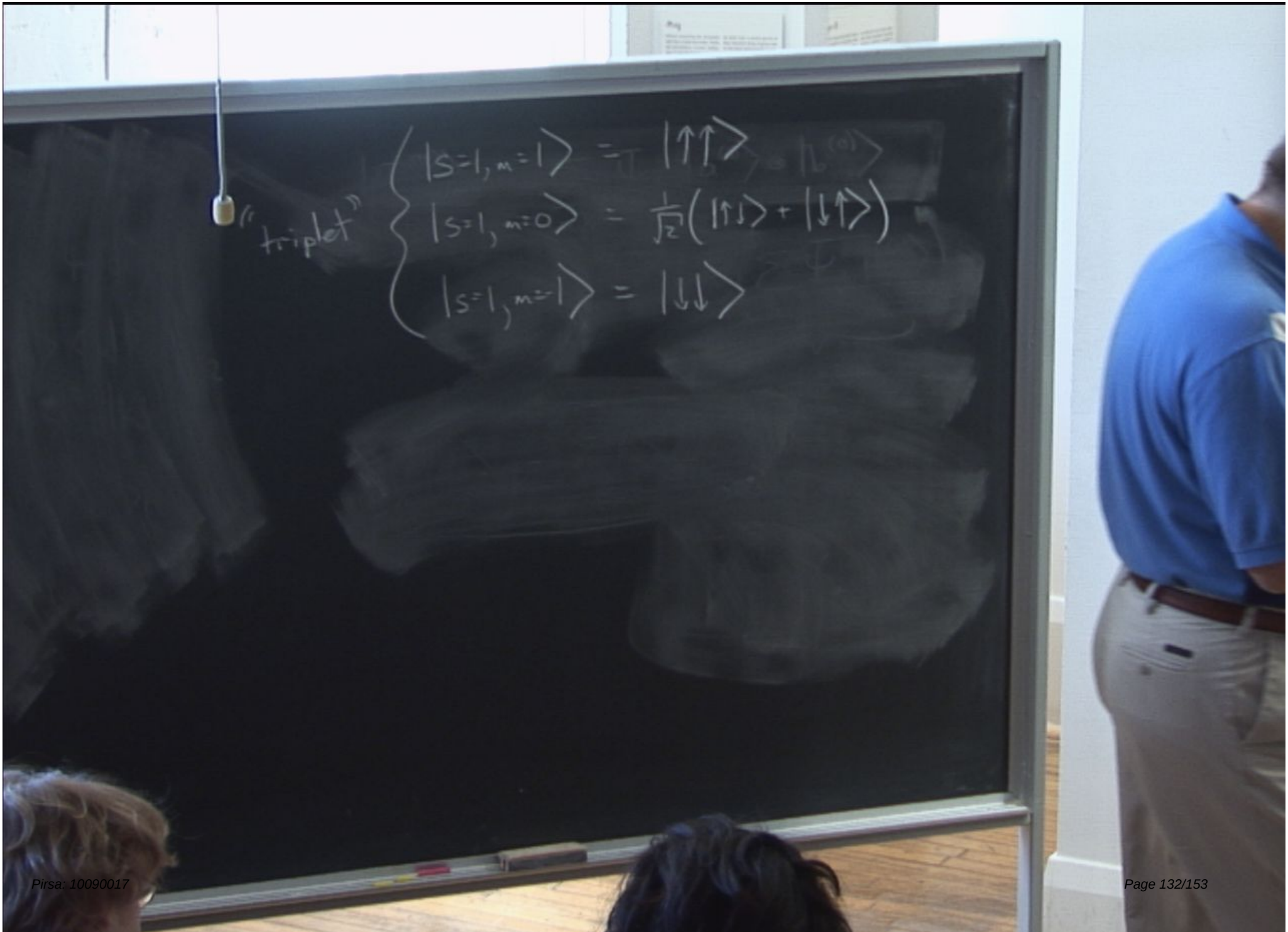
$m = m_1 + m_2$

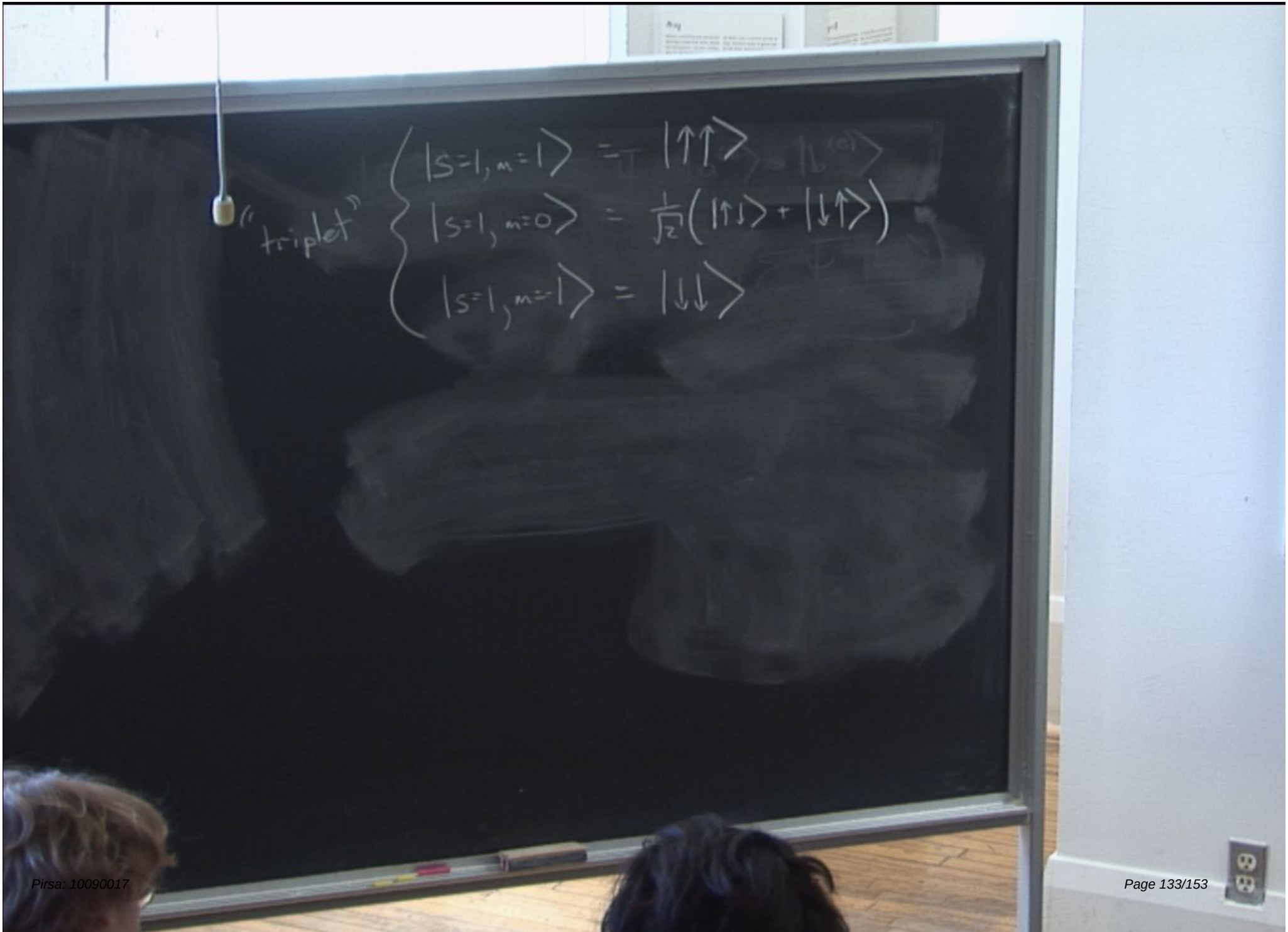
$S_z |\uparrow$

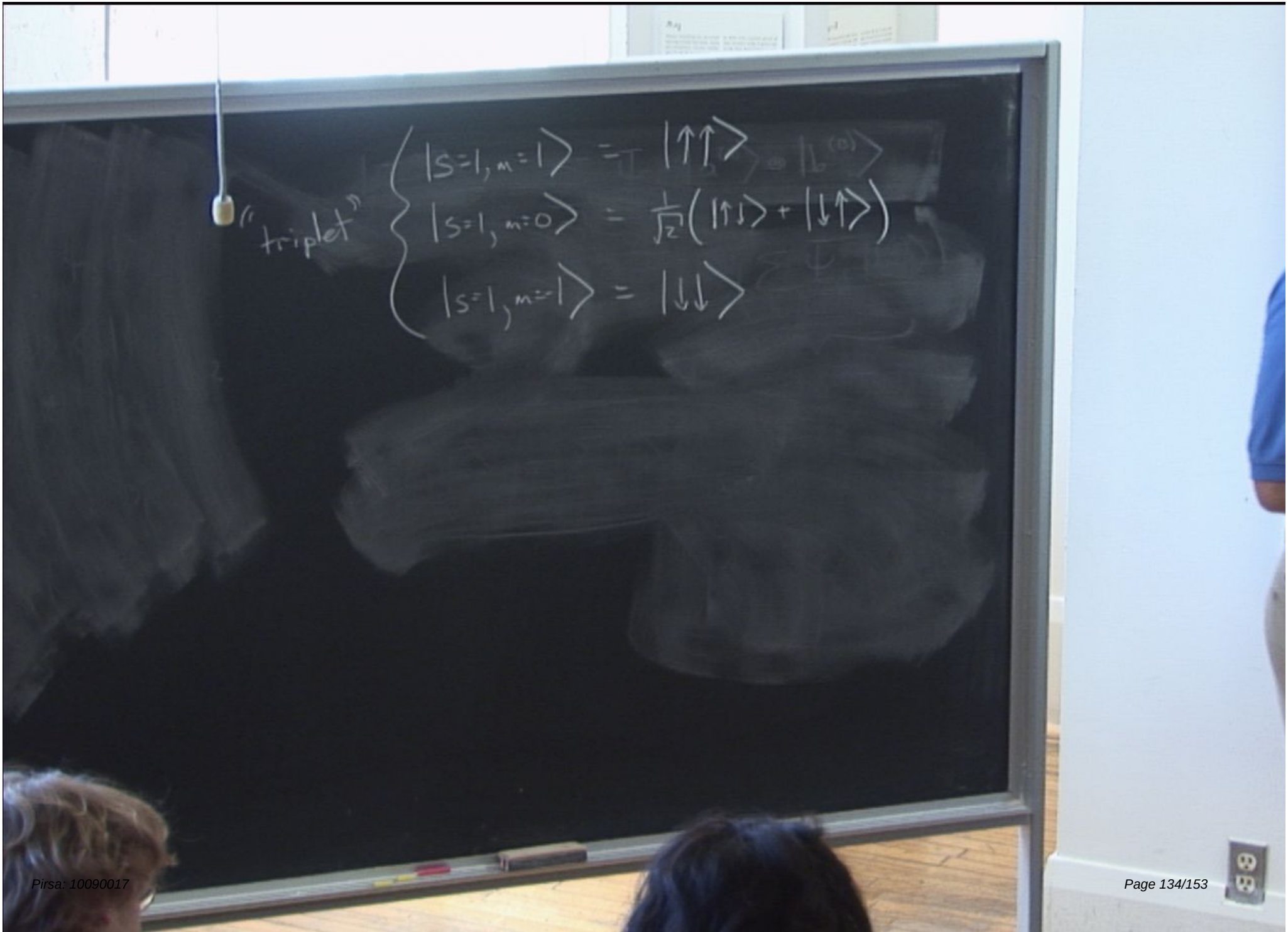
$S_z |\uparrow$

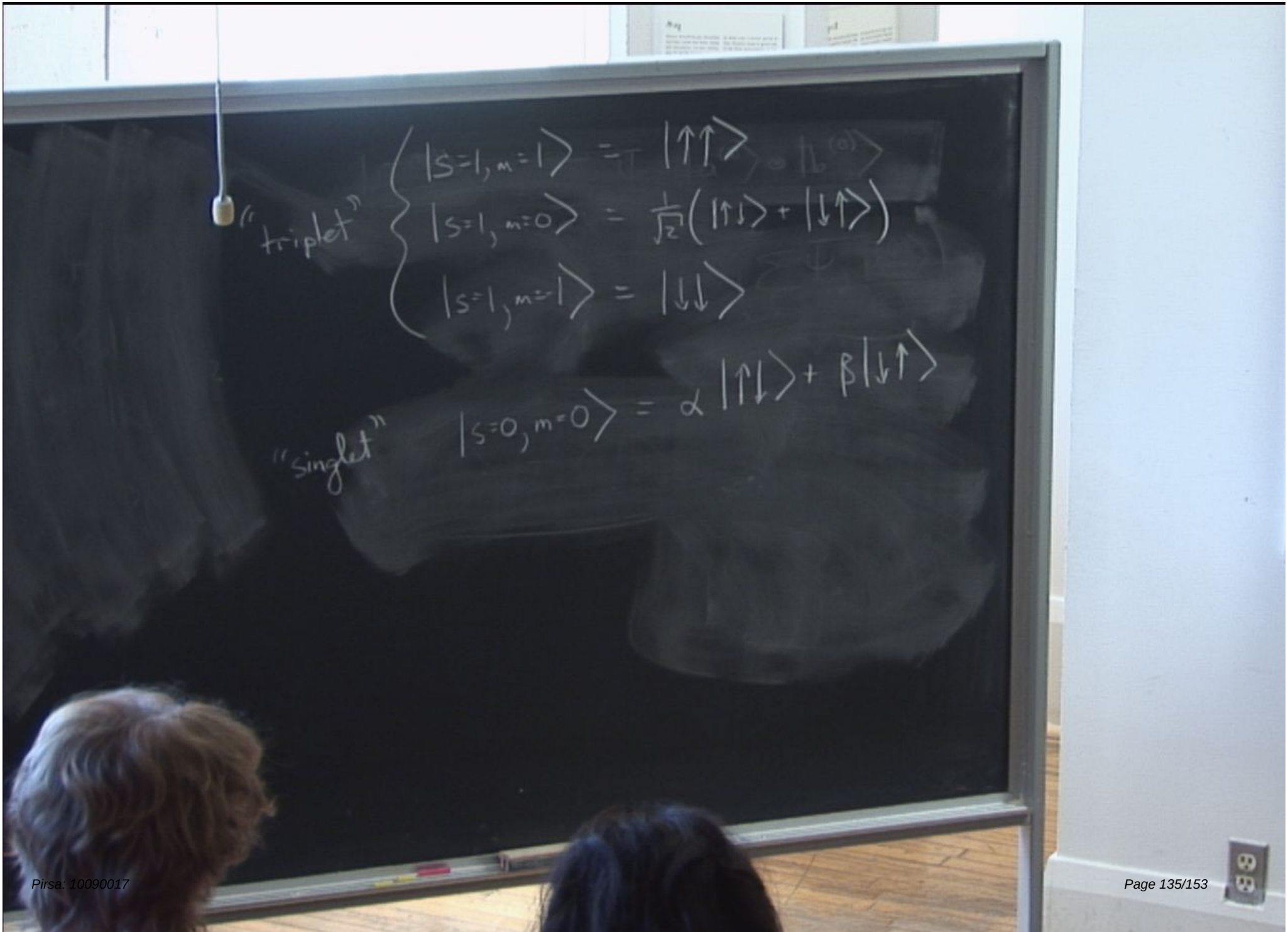
$S_z |\downarrow$

$S_z |\downarrow$


"triplet"
$|s=1, m=1\rangle = |\uparrow\uparrow\rangle$
$|s=1, m=0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$
$|s=1, m=-1\rangle = |\downarrow\downarrow\rangle$


"triplet"
$|s=1, m=1\rangle = |\uparrow\uparrow\rangle$
$|s=1, m=0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$
$|s=1, m=-1\rangle = |\downarrow\downarrow\rangle$


"triplet"
$|s=1, m=1\rangle = |\uparrow\uparrow\rangle$
$|s=1, m=0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$
$|s=1, m=-1\rangle = |\downarrow\downarrow\rangle$


"triplet"
$|S=1, m=1\rangle = |\uparrow\uparrow\rangle$
$|S=1, m=0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$
$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$
"singlet"
$|S=0, m=0\rangle = \alpha|\uparrow\downarrow\rangle + \beta|\downarrow\uparrow\rangle$

2 spin ½ particles : $s_1 = \frac{1}{2}$ $m_1 = \pm\frac{1}{2}$

$s_2 = \frac{1}{2}$ $m_2 = \pm\frac{1}{2}$

$|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$

what about s, m values?

$m = m_1 + m_2$

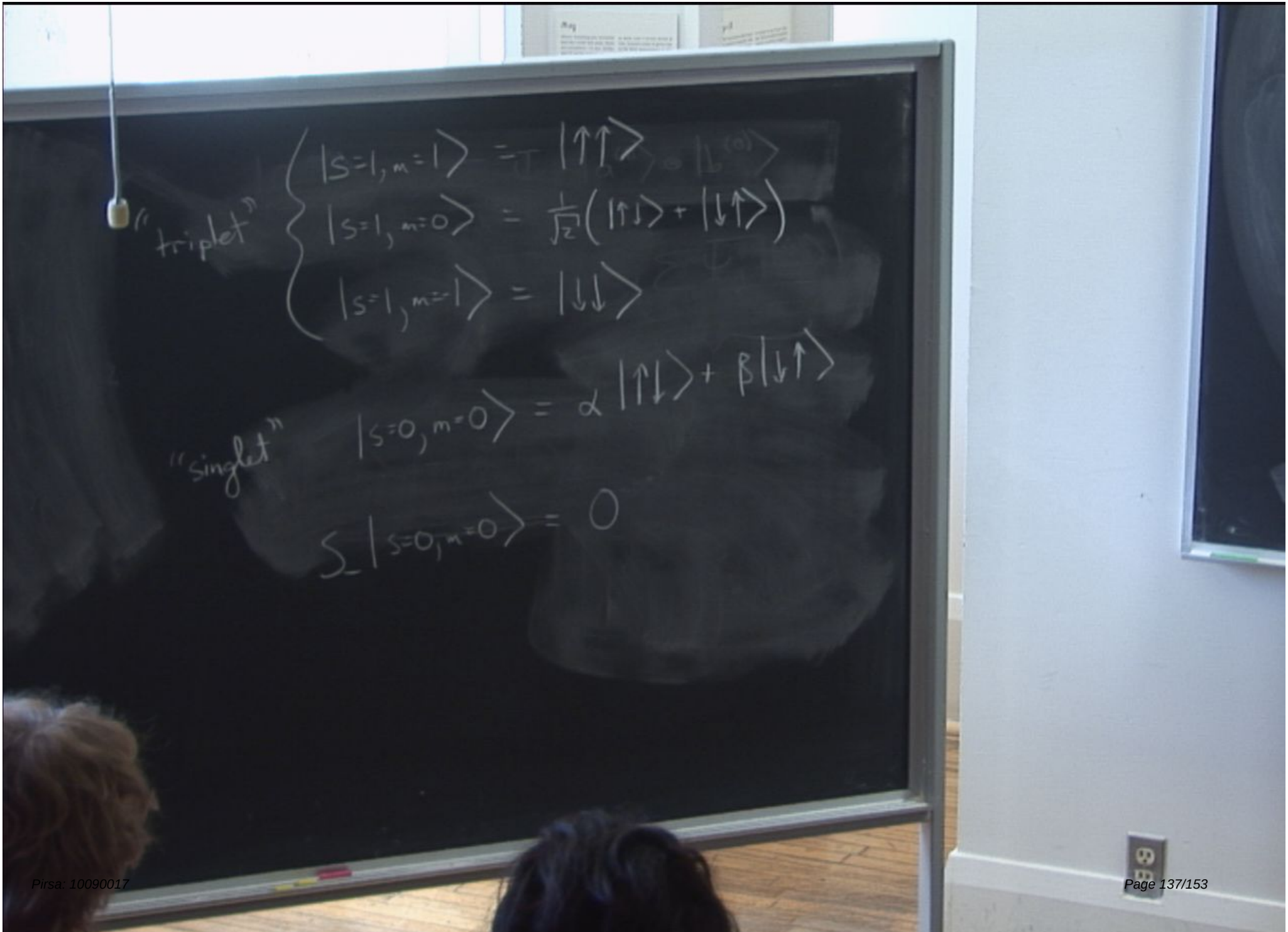
$S_z|\uparrow\uparrow\rangle = (+1)|\uparrow\uparrow\rangle$

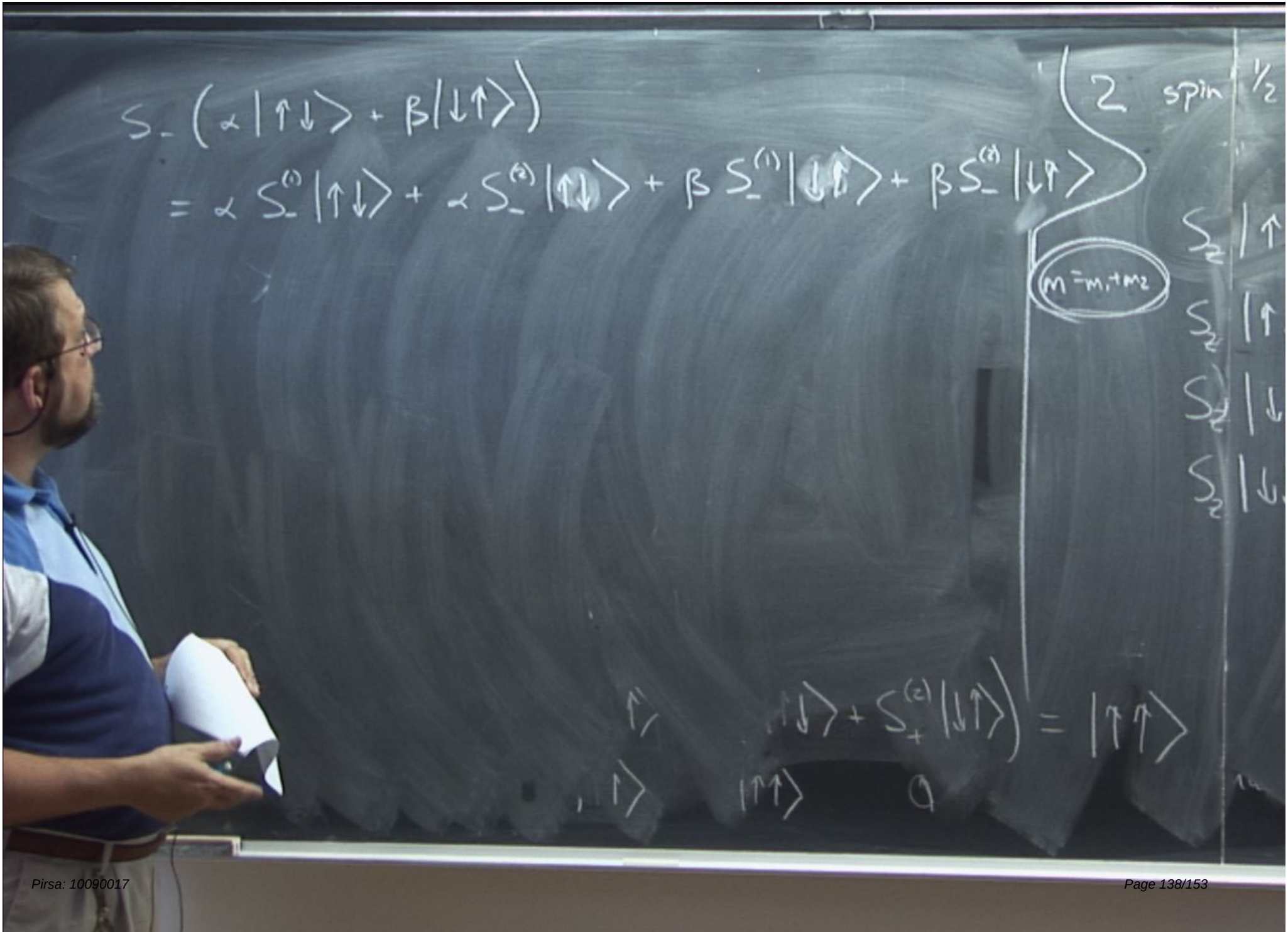
$S_z|\uparrow\downarrow\rangle = 0$

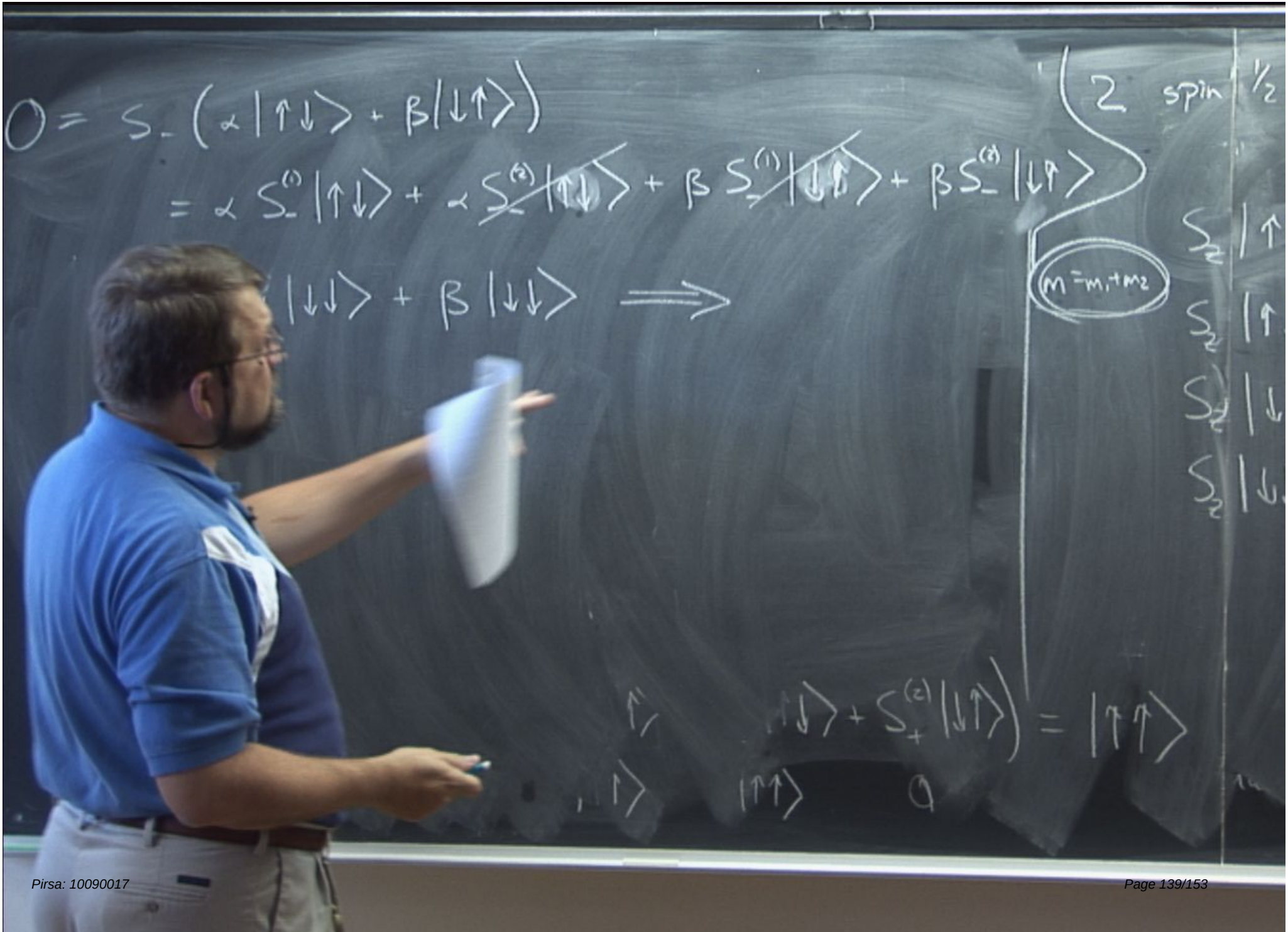
$S_z|\downarrow\uparrow\rangle = 0$

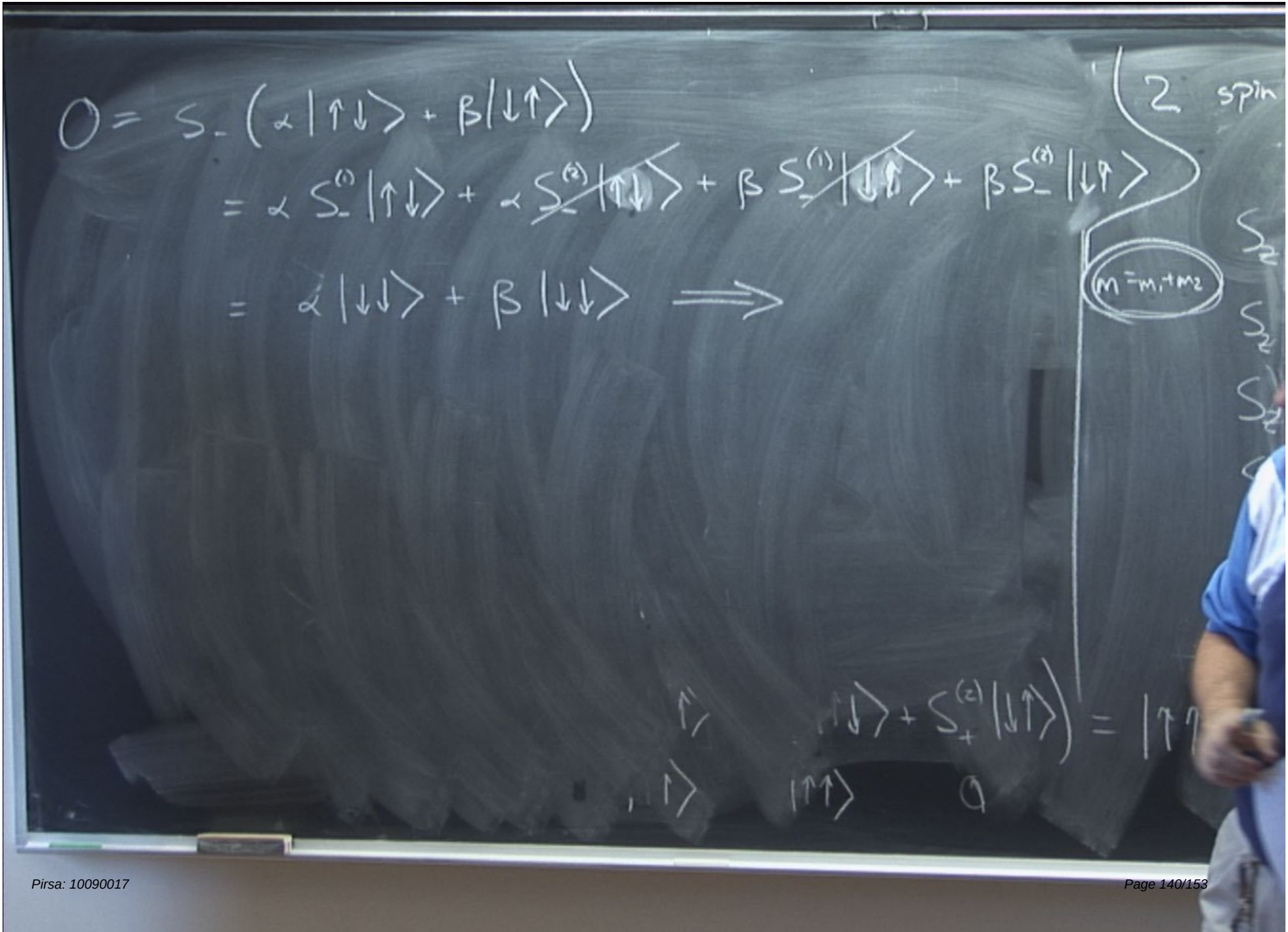
$S_z|\downarrow\downarrow\rangle = (-1)|\downarrow\downarrow\rangle$ ← $m = -1$ ← bottom rung of $s = 1$ ladder

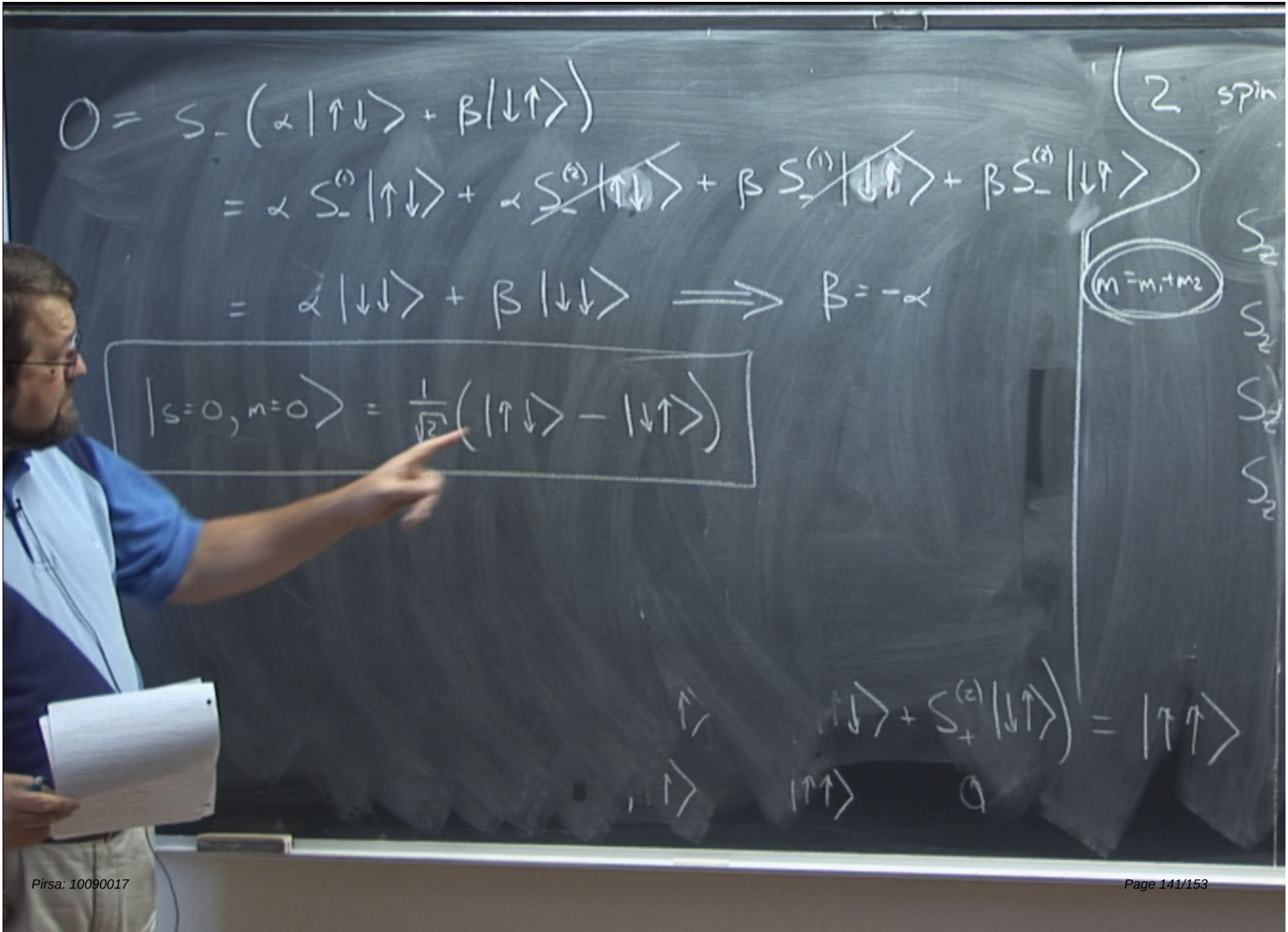
$= |\uparrow\uparrow\rangle$

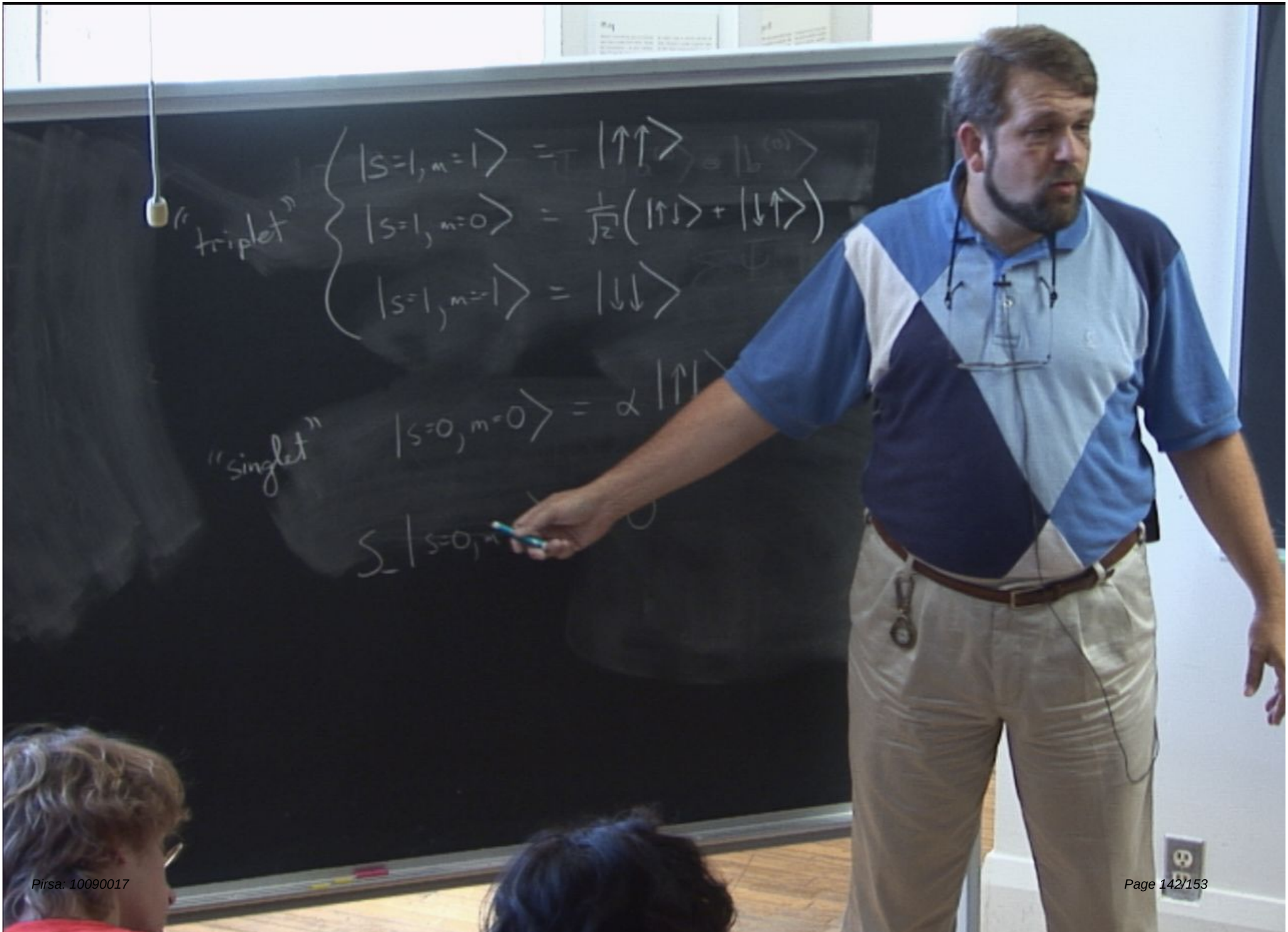

"triplet"
$|S=1, m=1\rangle = |\uparrow\uparrow\rangle$
$|S=1, m=0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$
$|S=1, m=-1\rangle = |\downarrow\downarrow\rangle$
"singlet"
$|S=0, m=0\rangle = \alpha|\uparrow\downarrow\rangle + \beta|\downarrow\uparrow\rangle$
$S_-|S=0, m=0\rangle = 0$

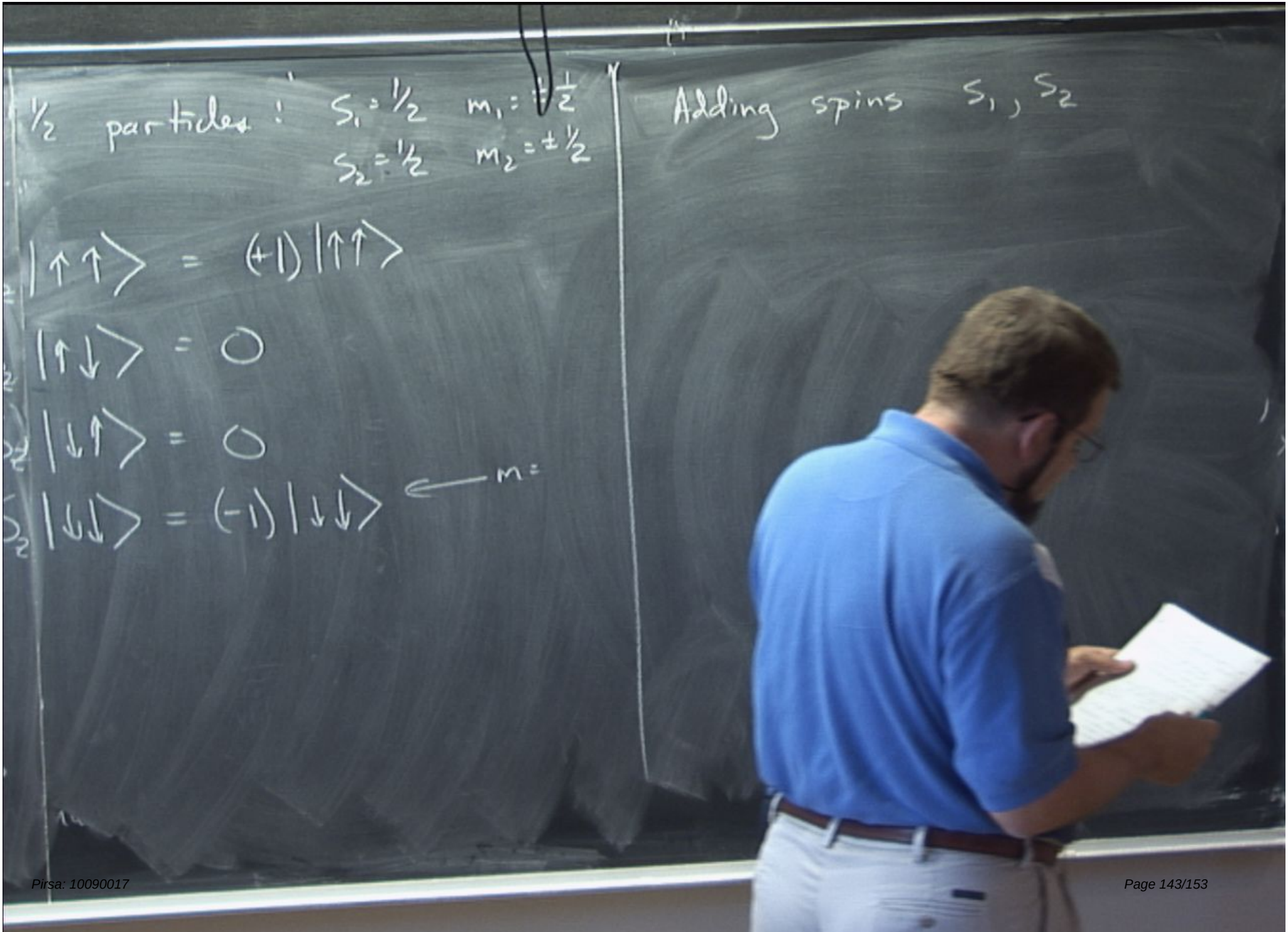

$S_-(\alpha|\uparrow\downarrow\rangle + \beta|\downarrow\uparrow\rangle)$
$= \alpha S_-^{(1)}|\uparrow\downarrow\rangle + \alpha S_-^{(2)}|\uparrow\downarrow\rangle + \beta S_-^{(1)}|\downarrow\uparrow\rangle + \beta S_-^{(2)}|\downarrow\uparrow\rangle$
2 spin 1/2
$m = m_1 + m_2$
$S_z|\uparrow$
$S_z|\uparrow$
$S_z|\downarrow$
$S_z|\downarrow$
$S_+^{(2)}|\downarrow\uparrow\rangle = |\uparrow\uparrow\rangle$
$|\uparrow\uparrow\rangle$

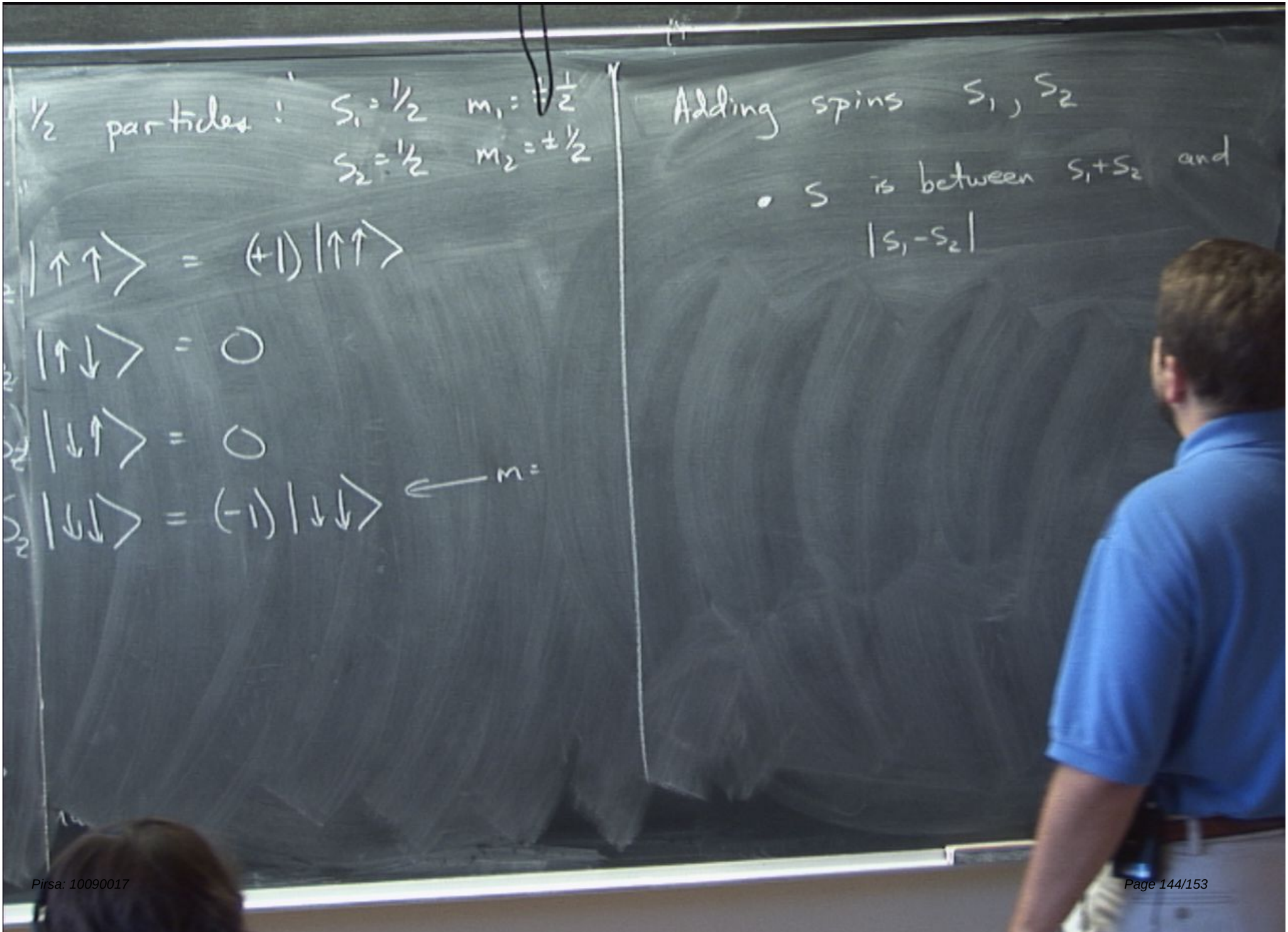

$0 = S_-(\alpha|\uparrow\downarrow\rangle + \beta|\downarrow\uparrow\rangle)$
$= \alpha S_-^{(1)}|\uparrow\downarrow\rangle + \alpha S_-^{(2)}|\uparrow\downarrow\rangle + \beta S_-^{(1)}|\downarrow\uparrow\rangle + \beta S_-^{(2)}|\downarrow\uparrow\rangle$
$|\downarrow\downarrow\rangle + \beta|\downarrow\downarrow\rangle \Longrightarrow$
2 spin 1/2
$m = m_1 + m_2$
$S_z|\uparrow$
$S_z|\uparrow$
$S_z|\downarrow$
$S_z|\downarrow$
$|\downarrow\downarrow\rangle + S_+^{(2)}|\downarrow\uparrow\rangle) = |\uparrow\uparrow\rangle$
$|\uparrow\uparrow\rangle$

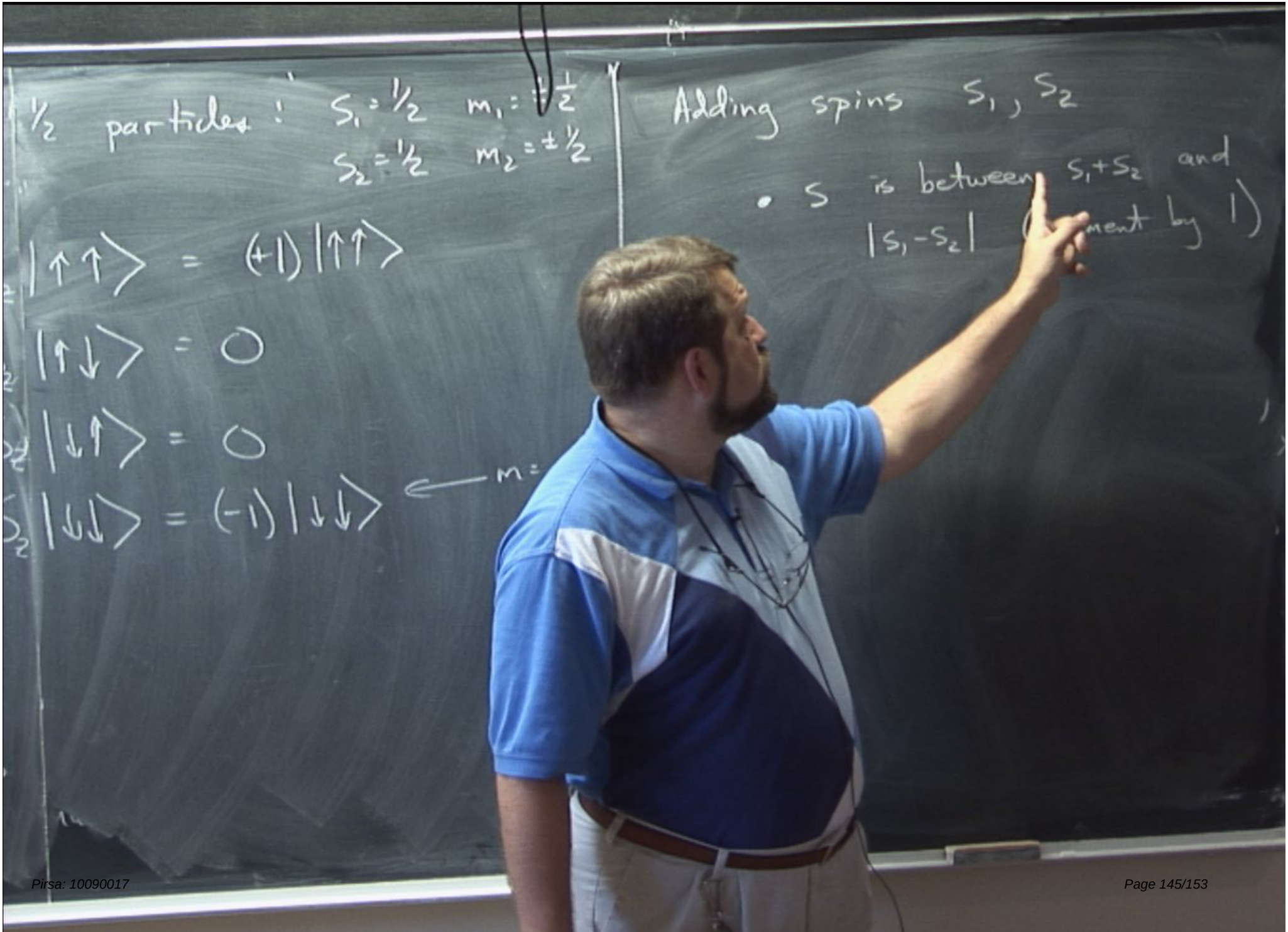

$0 = S_-(\alpha|\uparrow\downarrow\rangle + \beta|\downarrow\uparrow\rangle)$
$= \alpha S_-^{(1)}|\uparrow\downarrow\rangle + \alpha S_-^{(2)}|\uparrow\downarrow\rangle + \beta S_-^{(1)}|\downarrow\uparrow\rangle + \beta S_-^{(2)}|\downarrow\uparrow\rangle$
$= \alpha|\downarrow\downarrow\rangle + \beta|\downarrow\downarrow\rangle \Longrightarrow$
2 spin
$m = m_1 + m_2$
S_z
S_z
S_z
$|\uparrow\downarrow\rangle + S_+^{(2)}|\downarrow\uparrow\rangle) = |\uparrow\uparrow$
$|\uparrow\rangle$
$|\uparrow\uparrow\rangle$
0

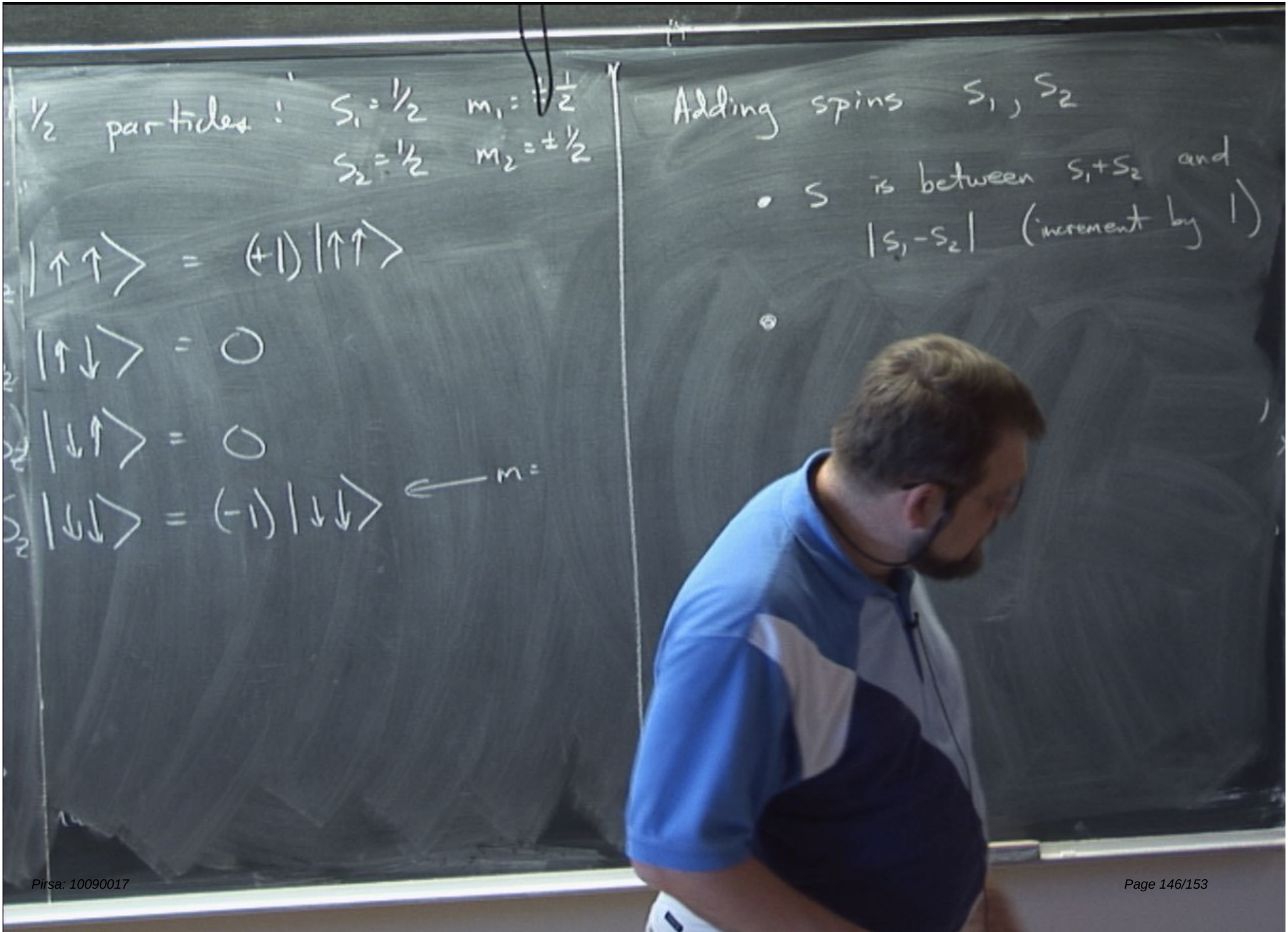

$0 = S_-(\alpha|\uparrow\downarrow\rangle + \beta|\downarrow\uparrow\rangle)$
$= \alpha S_-^{(1)}|\uparrow\downarrow\rangle + \alpha S_-^{(2)}|\uparrow\downarrow\rangle + \beta S_-^{(1)}|\downarrow\uparrow\rangle + \beta S_-^{(2)}|\downarrow\uparrow\rangle$
$= \alpha|\downarrow\downarrow\rangle + \beta|\downarrow\downarrow\rangle \Longrightarrow \beta = -\alpha$
$|s=0, m=0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$
2 spin
$m = m_1 + m_2$
S_z
S_z
S_z
S_z
$+ S_+^{(2)}|\downarrow\uparrow\rangle) = |\uparrow\uparrow\rangle$
$|\uparrow\uparrow\rangle$


"triplet"
$|s=1, m=1\rangle = |\uparrow\uparrow\rangle$
$|s=1, m=0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$
$|s=1, m=-1\rangle = |\downarrow\downarrow\rangle$
"singlet"
$|s=0, m=0\rangle = \alpha|\uparrow\downarrow\rangle$
$S_-|s=0,$


1/2 particles : $s_1 = 1/2$ $m_1 = \pm 1/2$
$s_2 = 1/2$ $m_2 = \pm 1/2$
Adding spins s_1, s_2
$|\uparrow\uparrow\rangle = (+1)|\uparrow\uparrow\rangle$
$|\uparrow\downarrow\rangle = 0$
$|\downarrow\uparrow\rangle = 0$
$|\downarrow\downarrow\rangle = (-1)|\downarrow\downarrow\rangle$ ← m =


½ particles : $s_1 = ½$ $m_1 = \pm ½$
$s_2 = ½$ $m_2 = \pm ½$
$|\uparrow\uparrow\rangle = (+1)|\uparrow\uparrow\rangle$
$|\uparrow\downarrow\rangle = 0$
$|\downarrow\uparrow\rangle = 0$
$|\downarrow\downarrow\rangle = (-1)|\downarrow\downarrow\rangle$ ← m =
Adding spins s_1, s_2
• s is between s_1+s_2 and $|s_1-s_2|$


½ particles : $s_1 = ½$ $m_1 = \pm ½$
$s_2 = ½$ $m_2 = \pm ½$
$|\uparrow\uparrow\rangle = (+1)|\uparrow\uparrow\rangle$
$|\uparrow\downarrow\rangle = 0$
$|\downarrow\uparrow\rangle = 0$
$|\downarrow\downarrow\rangle = (-1)|\downarrow\downarrow\rangle$
Adding spins s_1, s_2
• s is between s_1+s_2 and
$|s_1-s_2|$


½ particles : $s_1 = ½$ $m_1 = \pm ½$
$s_2 = ½$ $m_2 = \pm ½$
$|\uparrow\uparrow\rangle = (+1)|\uparrow\uparrow\rangle$
$|\uparrow\downarrow\rangle = 0$
$|\downarrow\uparrow\rangle = 0$
$|\downarrow\downarrow\rangle = (-1)|\downarrow\downarrow\rangle$ ← m =
Adding spins s_1, s_2
• s is between s_1+s_2 and $|s_1-s_2|$ (increment by 1)

½ particles: $s_1 = 1/2$ $m_1 = \pm\frac{1}{2}$

$s_2 = 1/2$ $m_2 = \pm 1/2$

$|\uparrow\uparrow\rangle = (+1)|\uparrow\uparrow\rangle$

$|\uparrow\downarrow\rangle = 0$

$|\downarrow\uparrow\rangle = 0$

$|\downarrow\downarrow\rangle = (-1)|\downarrow\downarrow\rangle$ ← m =

Adding spins s_1, s_2

- s is between s_1+s_2 and $|s_1-s_2|$ (increment by 1)
- m ranges −s to s for each
- $m = m_1 + m_2$
- $|s,m\rangle$ can be written in terms of $|s_1,m_1;s_2,m_2\rangle$ — coefficients

$\langle s_1,m_1;s_2,m_2|s,m\rangle$

½ particles: $s_1 = 1/2$ $m_1 = \pm\frac{1}{2}$

$s_2 = 1/2$ $m_2 = \pm 1/2$

$|\uparrow\uparrow\rangle = (+1)|\uparrow\uparrow\rangle$

$|\uparrow\downarrow\rangle = 0$

$|\downarrow\uparrow\rangle = 0$

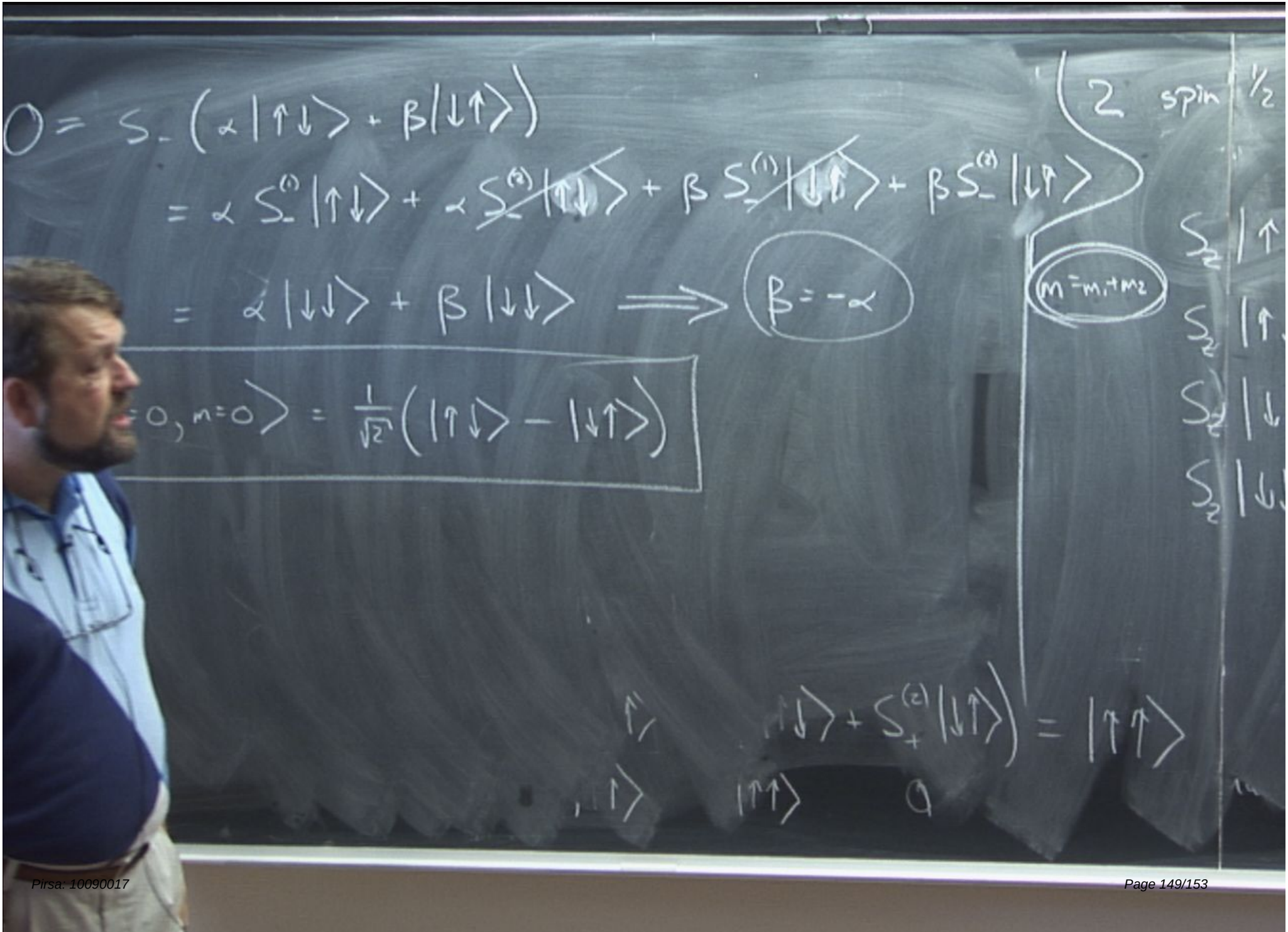
$|\downarrow\downarrow\rangle = (-1)|\downarrow\downarrow\rangle$ ← m =

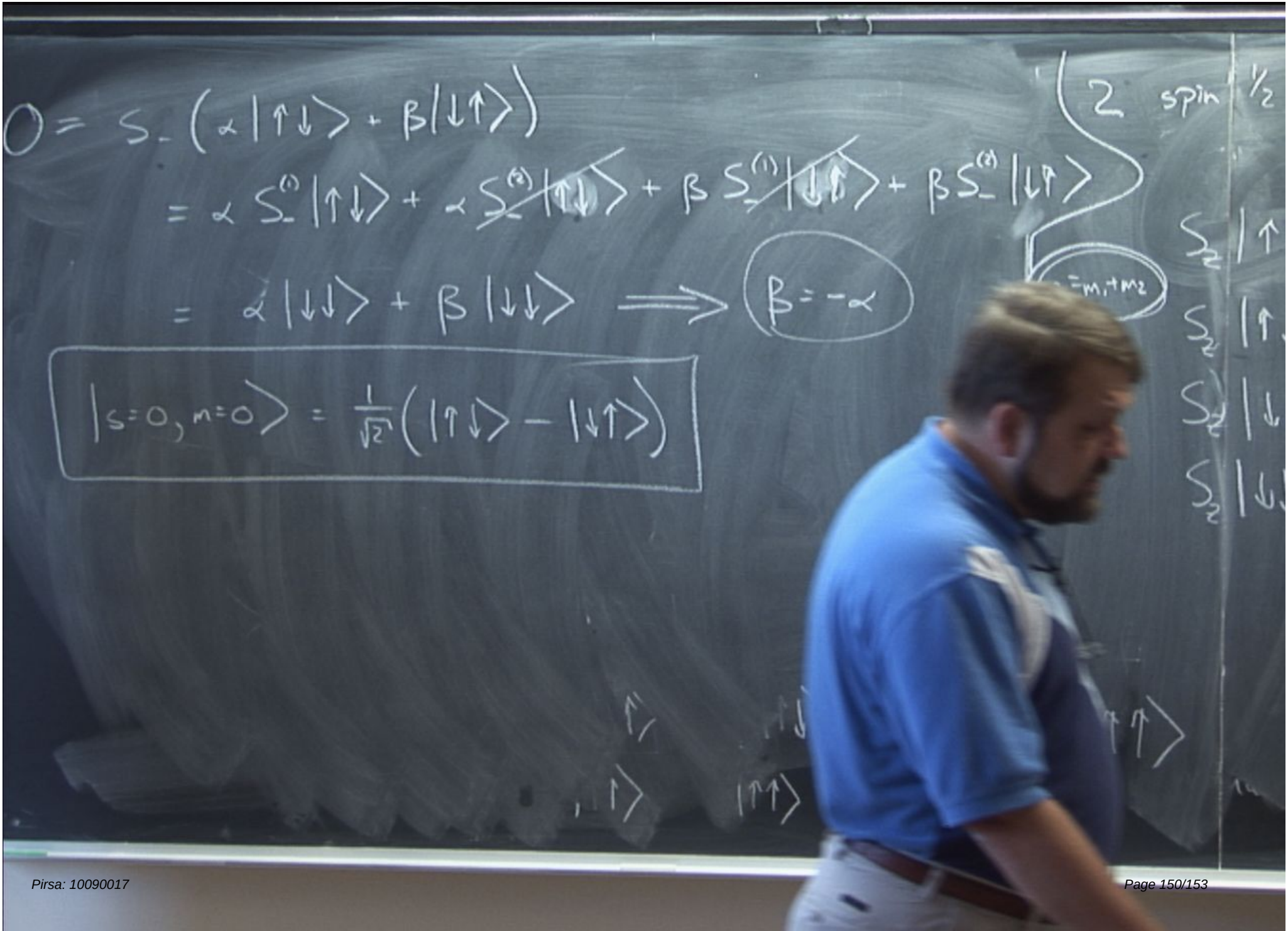
Adding spins s_1, s_2

- s is between s_1+s_2 and $|s_1-s_2|$ (increment by 1)
- m ranges −s to s for each
- $m = m_1 + m_2$
- $|s,m\rangle$ can be written in terms of $|s_1,m_1;s_2,m_2\rangle$ — coefficients

$\langle s_1,m_1;s_2,m_2|s,m\rangle$

↑ Clebsch-Gordon


$0 = S_-(\alpha|\uparrow\downarrow\rangle + \beta|\downarrow\uparrow\rangle)$
$= \alpha S_-^{(1)}|\uparrow\downarrow\rangle + \alpha S_-^{(2)}|\uparrow\downarrow\rangle + \beta S_-^{(1)}|\downarrow\uparrow\rangle + \beta S_-^{(2)}|\downarrow\uparrow\rangle$
$= \alpha|\downarrow\downarrow\rangle + \beta|\downarrow\downarrow\rangle \Longrightarrow \beta = -\alpha$
$=0, m=0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$
2 spin ½
$m = m_1 + m_2$
$S_z|\uparrow$
$S_z|\uparrow$
$S_z|\downarrow$
$S_z|\downarrow$
$\downarrow\rangle + S_+^{(2)}|\downarrow\uparrow\rangle) = |\uparrow\uparrow\rangle$
$,1\rangle$ $|\uparrow\uparrow\rangle$ 0


$0 = S_-(\alpha|\uparrow\downarrow\rangle + \beta|\downarrow\uparrow\rangle)$
$= \alpha S_-^{(1)}|\uparrow\downarrow\rangle + \alpha S_-^{(2)}|\uparrow\downarrow\rangle + \beta S_-^{(1)}|\downarrow\uparrow\rangle + \beta S_-^{(2)}|\downarrow\uparrow\rangle$
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$|s=0, m=0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$
2 spin ½
$m = m_1 + m_2$
$S_z|\uparrow$
$S_z|\uparrow$
$S_z|\downarrow$
$S_z|\downarrow$
$|\uparrow\uparrow\rangle$

$$0 = S_-\left(\alpha|\uparrow\downarrow\rangle + \beta|\downarrow\uparrow\rangle\right)$$

$$= \alpha S_-^{(1)}|\uparrow\downarrow\rangle + \alpha S_-^{(2)}|\uparrow\downarrow\rangle + \beta S_-^{(1)}|\downarrow\uparrow\rangle + \beta S_-^{(2)}|\downarrow\uparrow\rangle$$

$$= \alpha|\downarrow\downarrow\rangle + \beta|\downarrow\downarrow\rangle \Longrightarrow \boxed{\beta = -\alpha}$$

$$\boxed{|s=0, m=0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)}$$

2 spin ½

$m = m_1 + m_2$

$S_z|\uparrow$

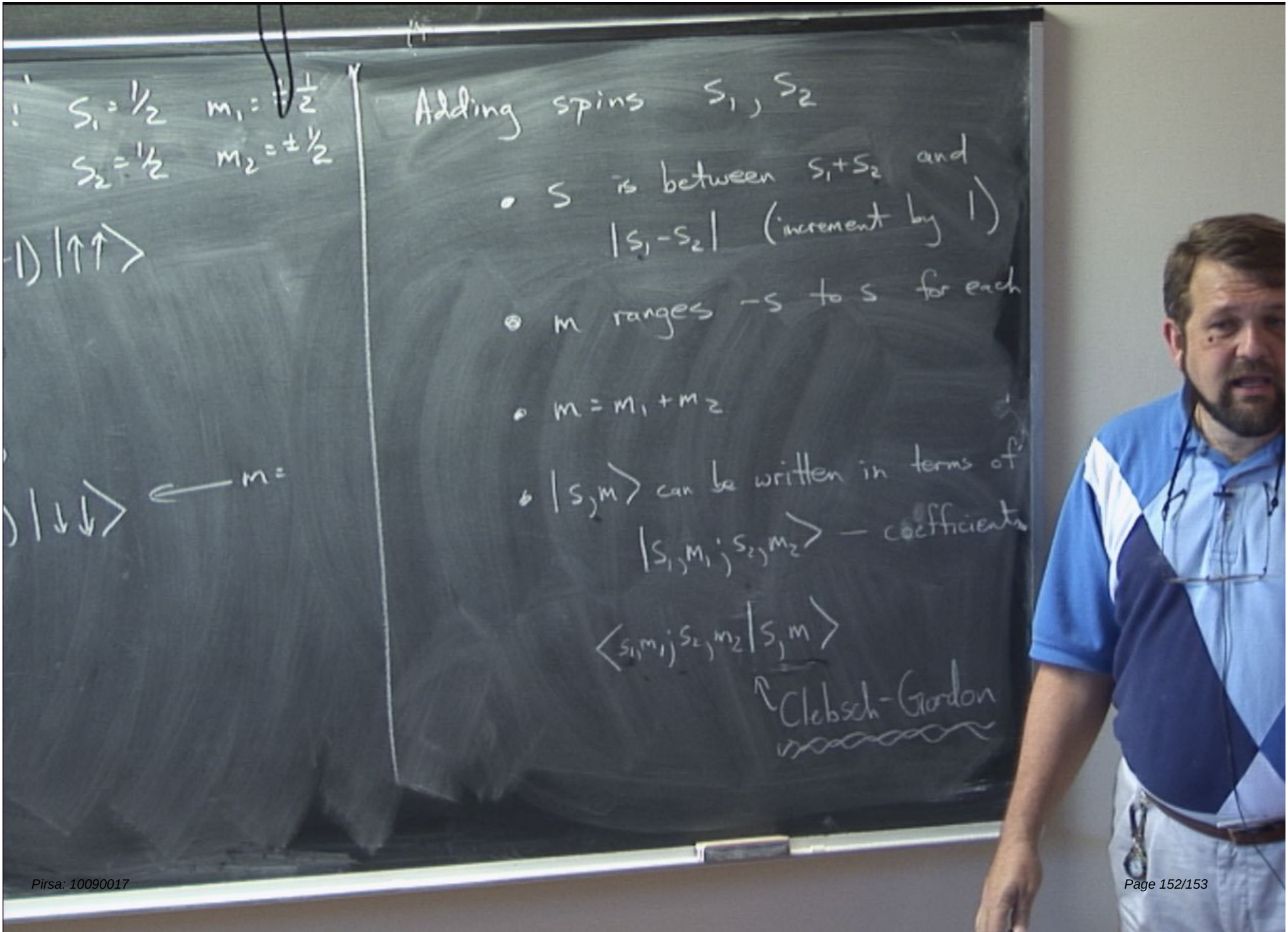
$S_z|\uparrow$

$S_z|\downarrow$

$S_z|\downarrow$

$\left(S_+^{(1)}|\downarrow\uparrow\rangle + S_+^{(2)}|\downarrow\uparrow\rangle\right) = |\uparrow\uparrow\rangle$

$|1\rangle$ $|\uparrow\uparrow\rangle$ 0


$s_1 = 1/2$ $m_1 = \pm 1/2$
$s_2 = 1/2$ $m_2 = \pm 1/2$
$|\uparrow\uparrow\rangle$
$|\downarrow\downarrow\rangle$ ← m =
Adding spins s_1, s_2
• s is between s_1+s_2 and $|s_1-s_2|$ (increment by 1)
• m ranges −s to s for each
• $m = m_1 + m_2$
• $|s,m\rangle$ can be written in terms of $|s_1,m_1;s_2,m_2\rangle$ — coefficients
$\langle s_1,m_1;s_2,m_2|s,m\rangle$
↑ Clebsch-Gordon

$s_1 = 1/2$ $m_1 = \pm 1/2$
$s_2 = 1/2$ $m_2 = \pm 1/2$
$|\uparrow\uparrow\rangle$
$|\downarrow\downarrow\rangle$
$m =$
Adding spins s_1, s_2
• s is between s_1+s_2 and $|s_1-s_2|$ (increment by 1)
• $-s$ to s for each
in terms of
coefficients
$-(s_1+s_2)$
don