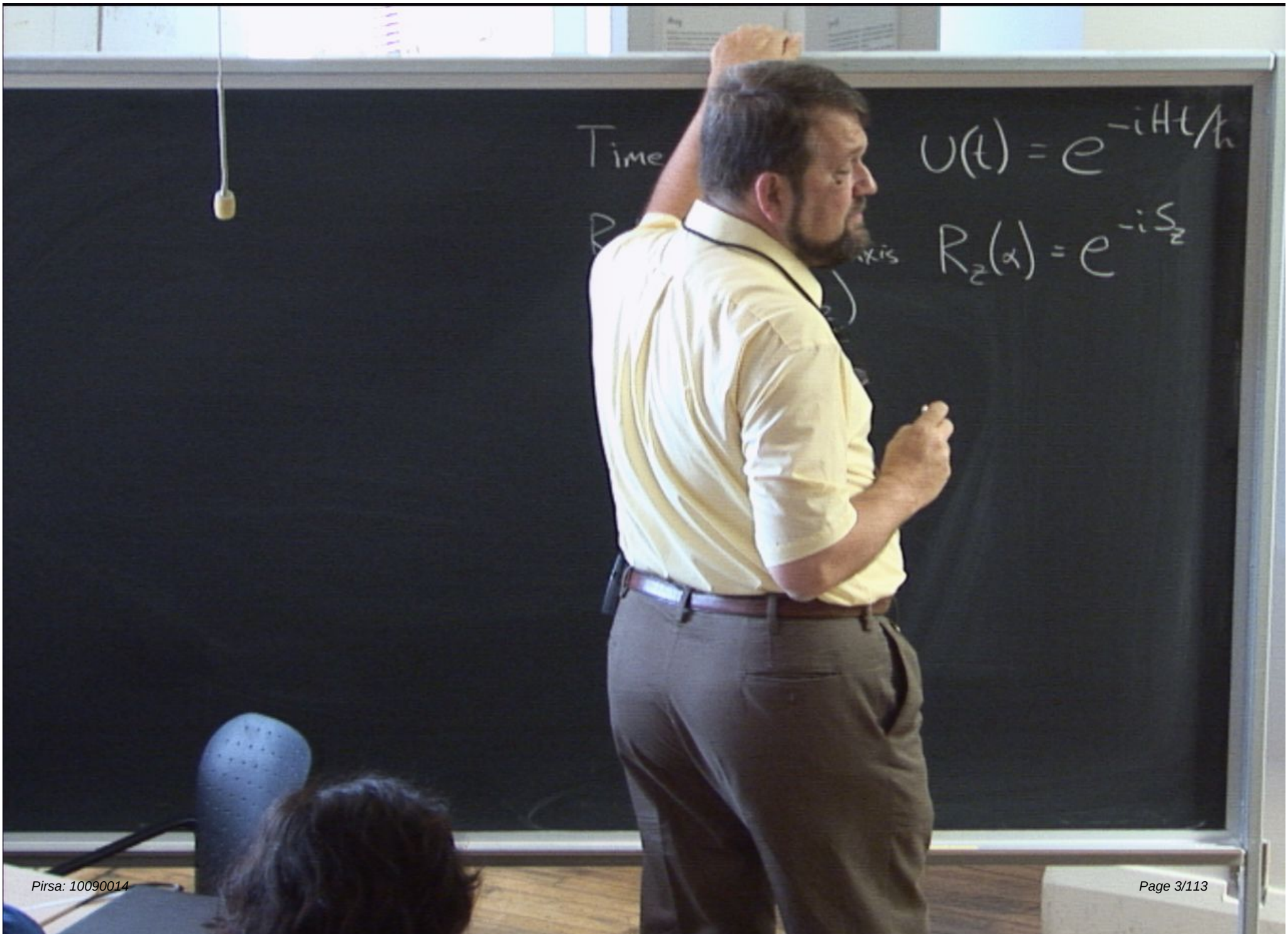
Title: Quantum Theory (PHYS 605) - Lecture 3

Date: Sep 15, 2010 09:00 AM

URL: http://pirsa.org/10090014

Abstract:

Time evolution $U(t) = e^{-iHt/\hbar}$


Time
$U(t) = e^{-iHt/\hbar}$
$R_z(\alpha) = e^{-iS_z}$

Time evolution $U(t) = e^{-iHt/\hbar}$

Rotation about z-axis $R_z(\alpha) = e^{-iS_z\alpha/\hbar}$
(spin-$1/2$)

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(spin-½)

observables : Hermitian

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unitary $= e^{i(\text{Hermitian})}$

dynamical symmetry : unitary V

any

$|\psi\rangle$

dynamical symmetry : unitary V

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$$|\psi_0\rangle \xrightarrow{U(t)} |\psi\rangle = U|\psi_0\rangle$$

dynamical symmetry : unitary V

$$\begin{tikzcd}
\overset{\text{any}}{|\psi_0\rangle} \arrow[r, "U(t)"] \arrow[d, "V"'] & |\psi\rangle = U|\psi_0\rangle \\
V|\psi_0\rangle \arrow[r, "U(t)"] & UV|\psi_0\rangle
\end{tikzcd}$$

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$$[V, U(t)] = 0$$

dynamical symmetry : unitary V

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$U(s) = \mathbb{1} - iHs/\hbar \Rightarrow [V, H] = 0$

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Observable A

$$[A, H] = 0 \Rightarrow A \ \underline{\text{conserved}}$$

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$$V(r+s) = V(r)\,V(s)$$

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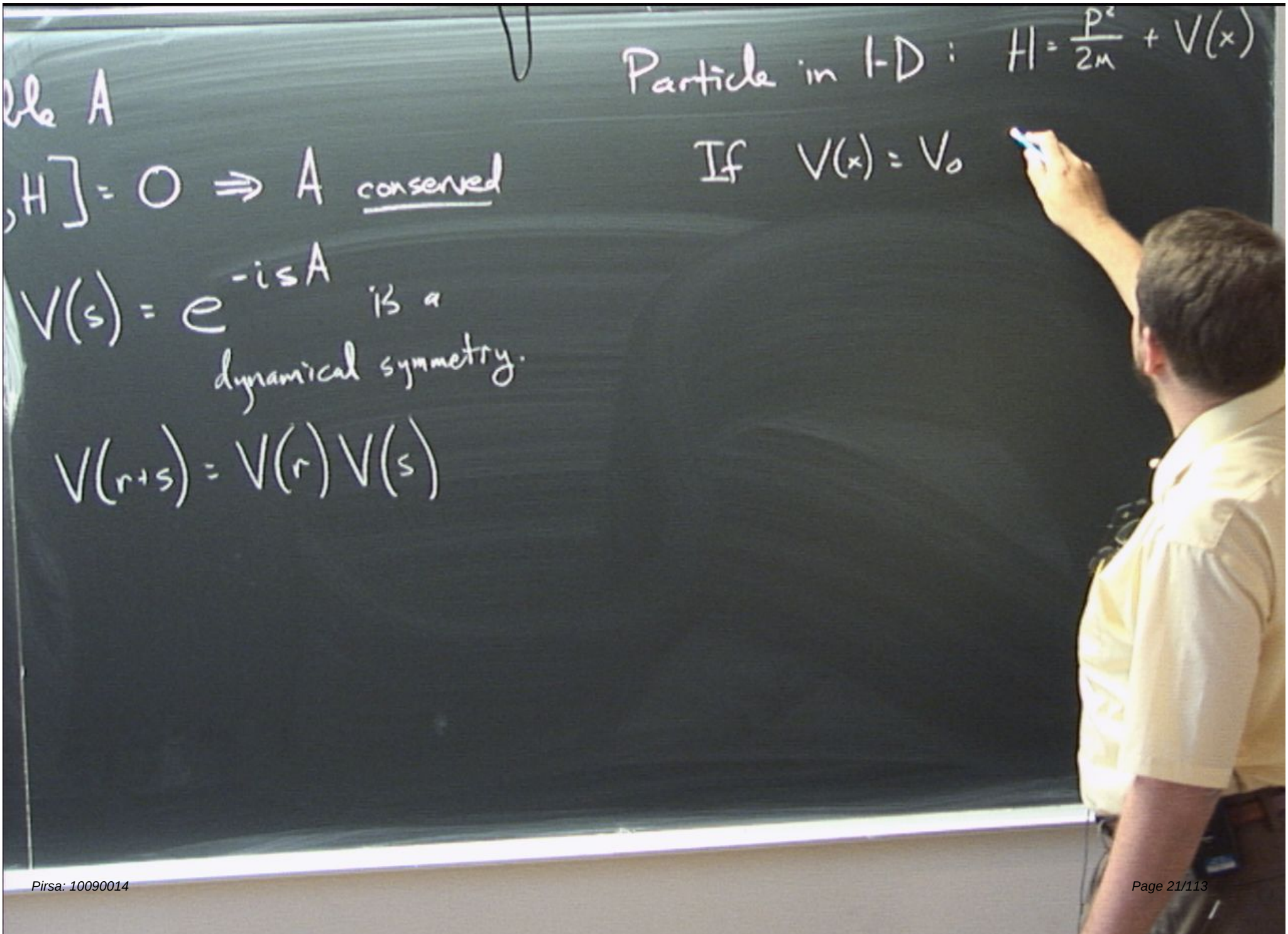
Particle in 1-D : $H = \frac{p^2}{2m} + V(x)$
If $V(x) = V_0$
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Let parameter s have units of L

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$ps/\hbar$

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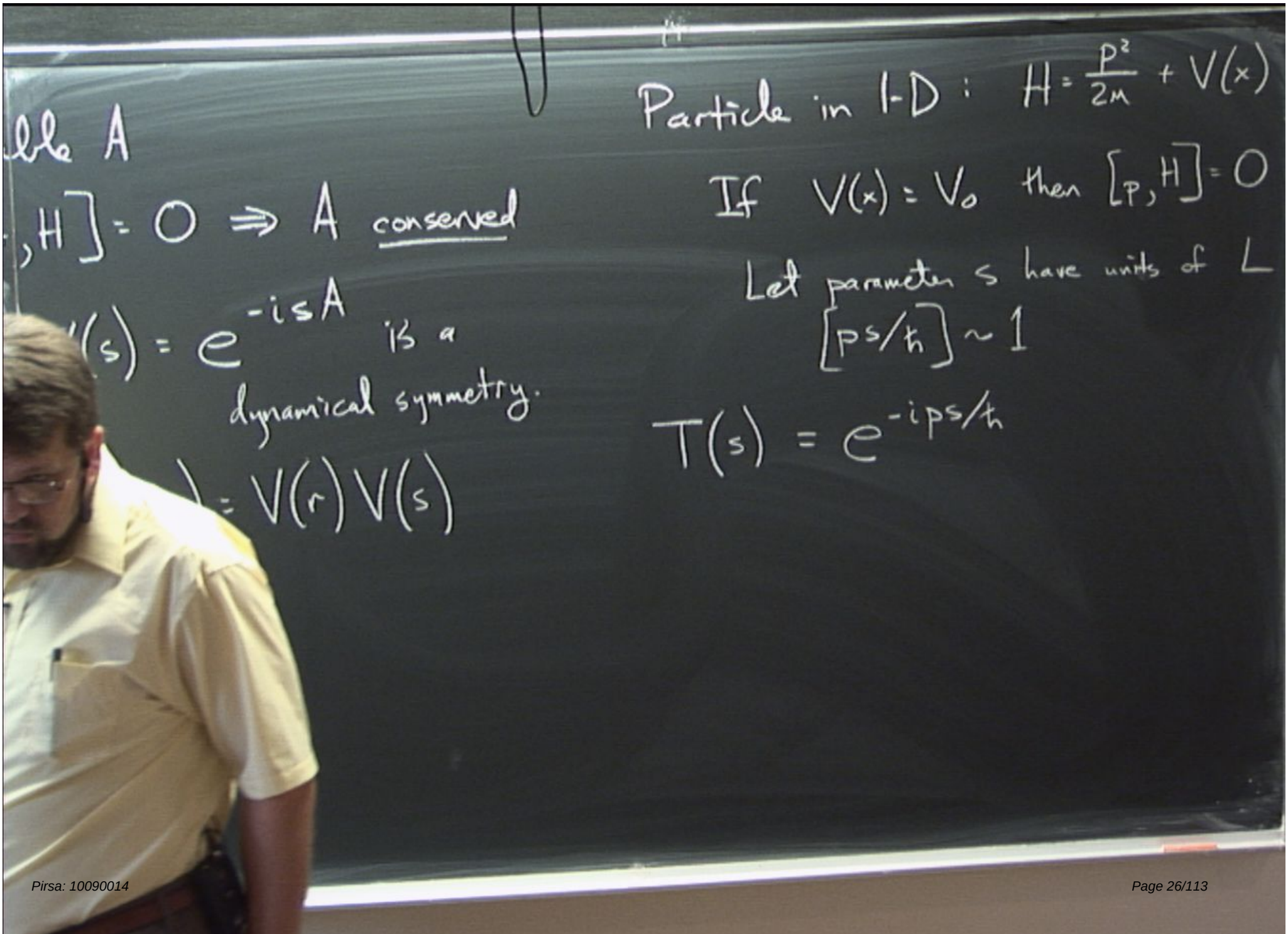
$[ps/\hbar] \sim 1$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$T(s) = e^{-ips/\hbar}$ <u>symmetry</u>

$= \mathbb{1} - ips/\hbar - \frac{1}{2!}\frac{s^2}{\hbar^2}p^2 + \frac{i}{3!}\frac{s^3}{\hbar^3}p^3 + \cdots$

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Position rep.

Particle in 1-D : H

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$$\vdots$$

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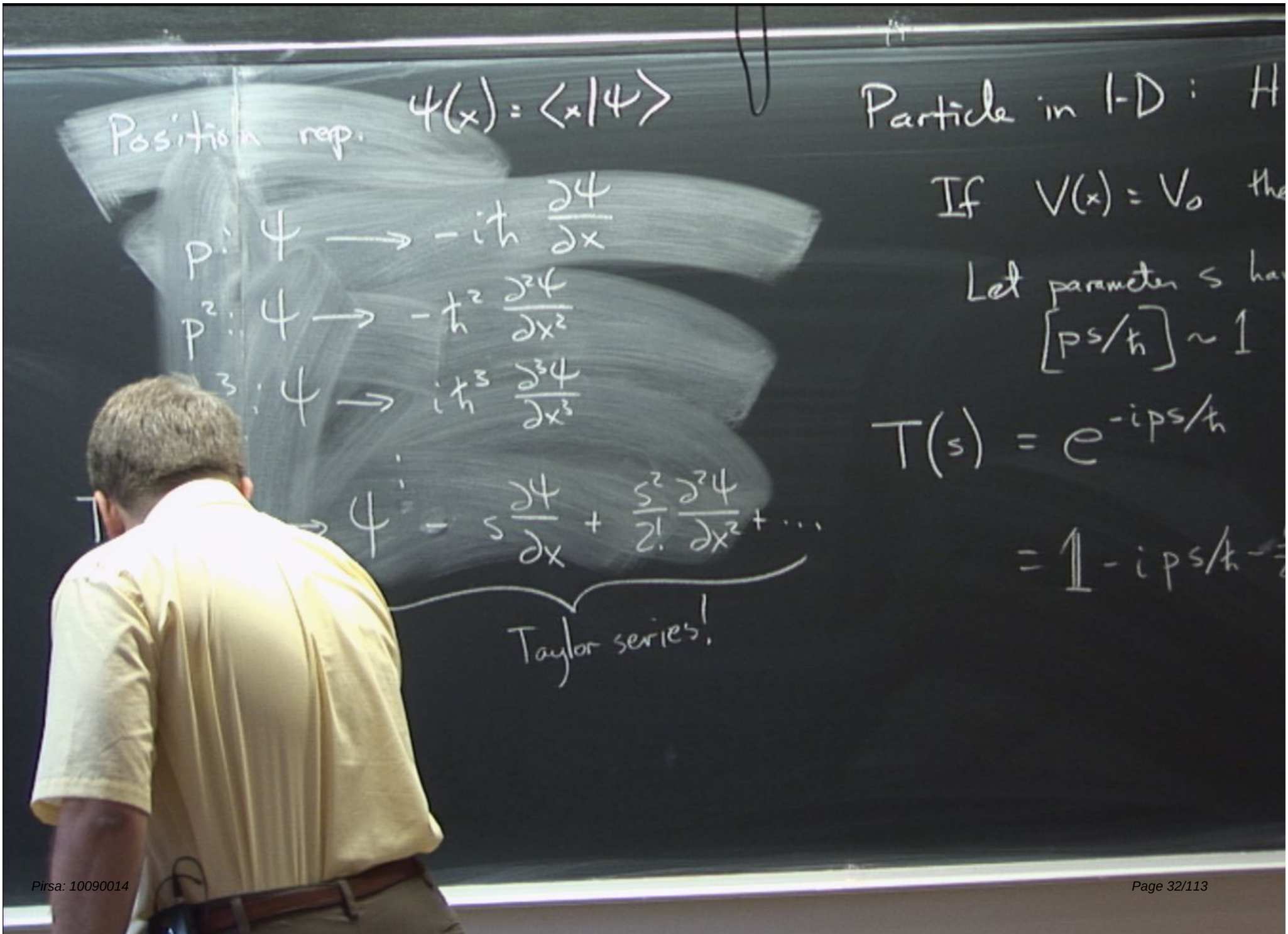
Position rep. $\psi(x) = \langle x|\psi\rangle$
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$\psi - s\frac{\partial\psi}{\partial x} + \frac{s^2}{2!}\frac{\partial^2\psi}{\partial x^2} + \cdots$
Taylor series!
Particle in 1-D :
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Let parameter s
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$$T(s) : \psi(x) \longrightarrow \psi(x-s)$$

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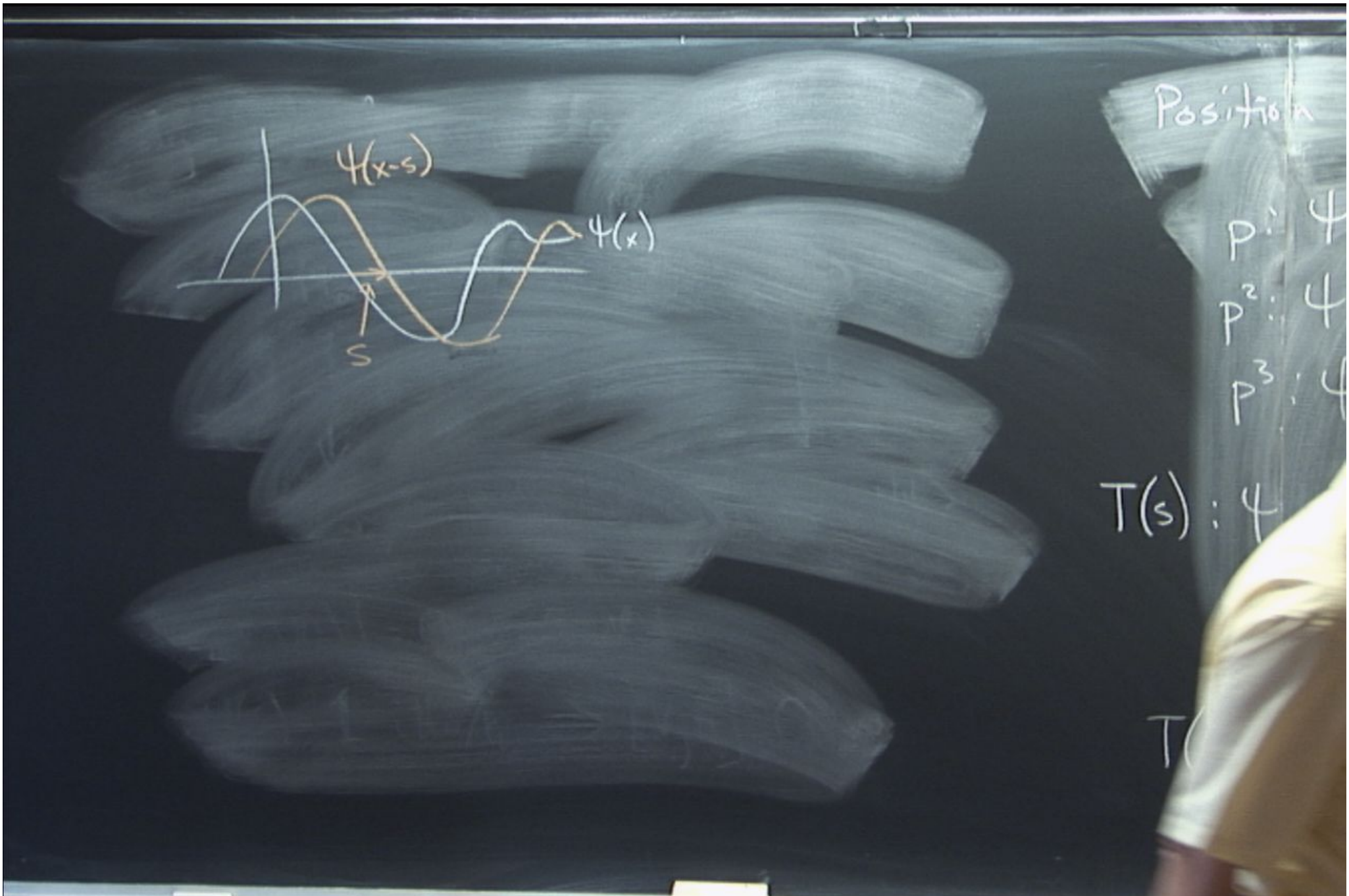
ψ(x-s)
ψ(x)
s
Position
P: ψ
P²: ψ
P³: ψ
T(s): ψ

$\psi(x) = \langle x|\psi\rangle$

rep.

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$$\boxed{T(s) = e^{-ips/\hbar}}$$ symmetry

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Translation: $T(s) = e^{-ips/\hbar}$

Time evolution $U(t) = e^{-iHt/\hbar}$

Rotation about z-axis $R_z(\alpha) = e^{-iS_z\alpha/\hbar}$
(spin-½)

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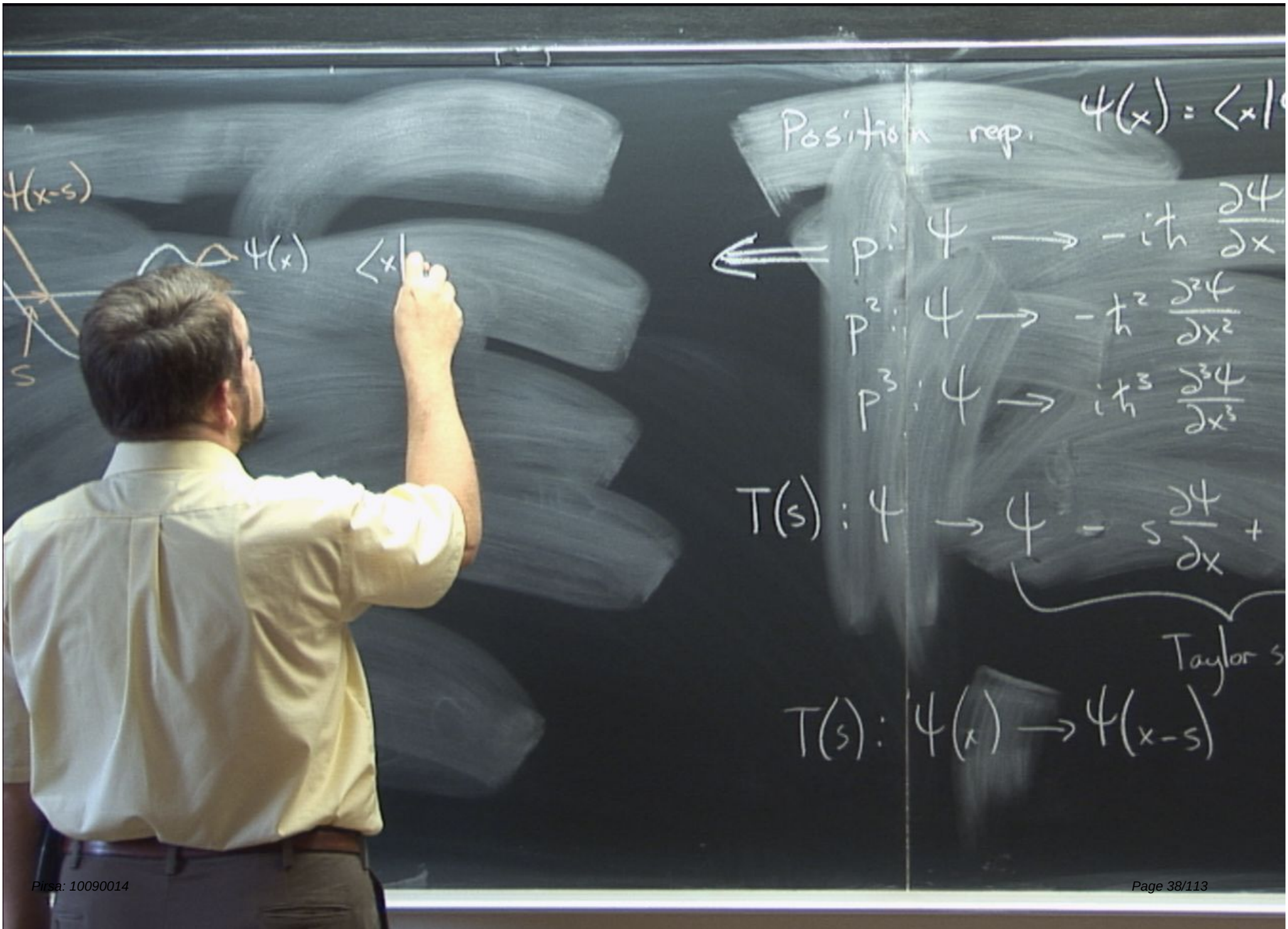
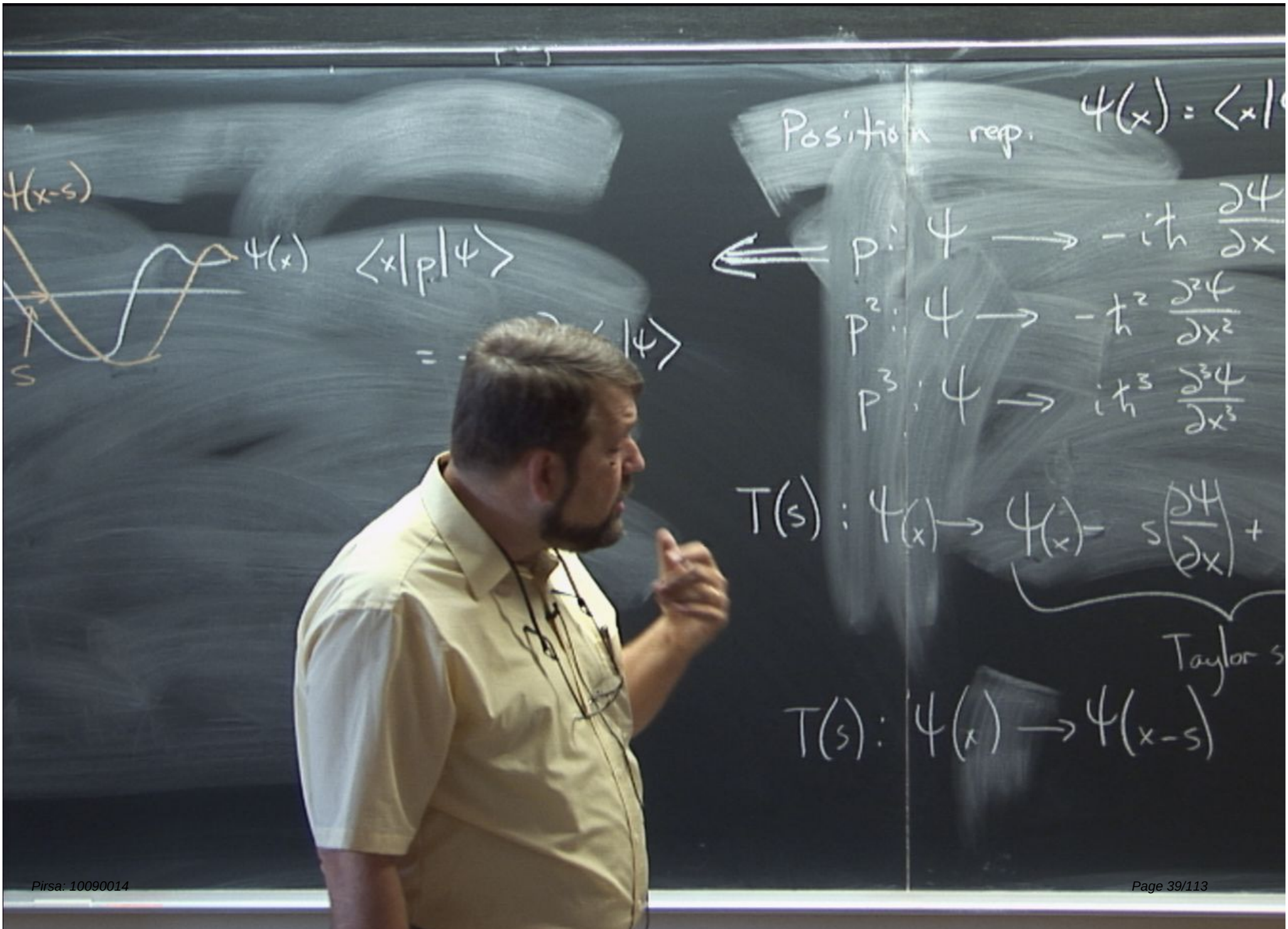

Position rep.
ψ(x) = ⟨x|
ψ(x-s)
ψ(x)
⟨x|
s
p : ψ → -iħ ∂ψ/∂x
p² : ψ → -ħ² ∂²ψ/∂x²
p³ : ψ → iħ³ ∂³ψ/∂x³
T(s) : ψ → ψ - s ∂ψ/∂x +
Taylor s
T(s) : ψ(x) → ψ(x-s)


Position rep.
ψ(x) = ⟨x|
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⟨x|p|ψ⟩
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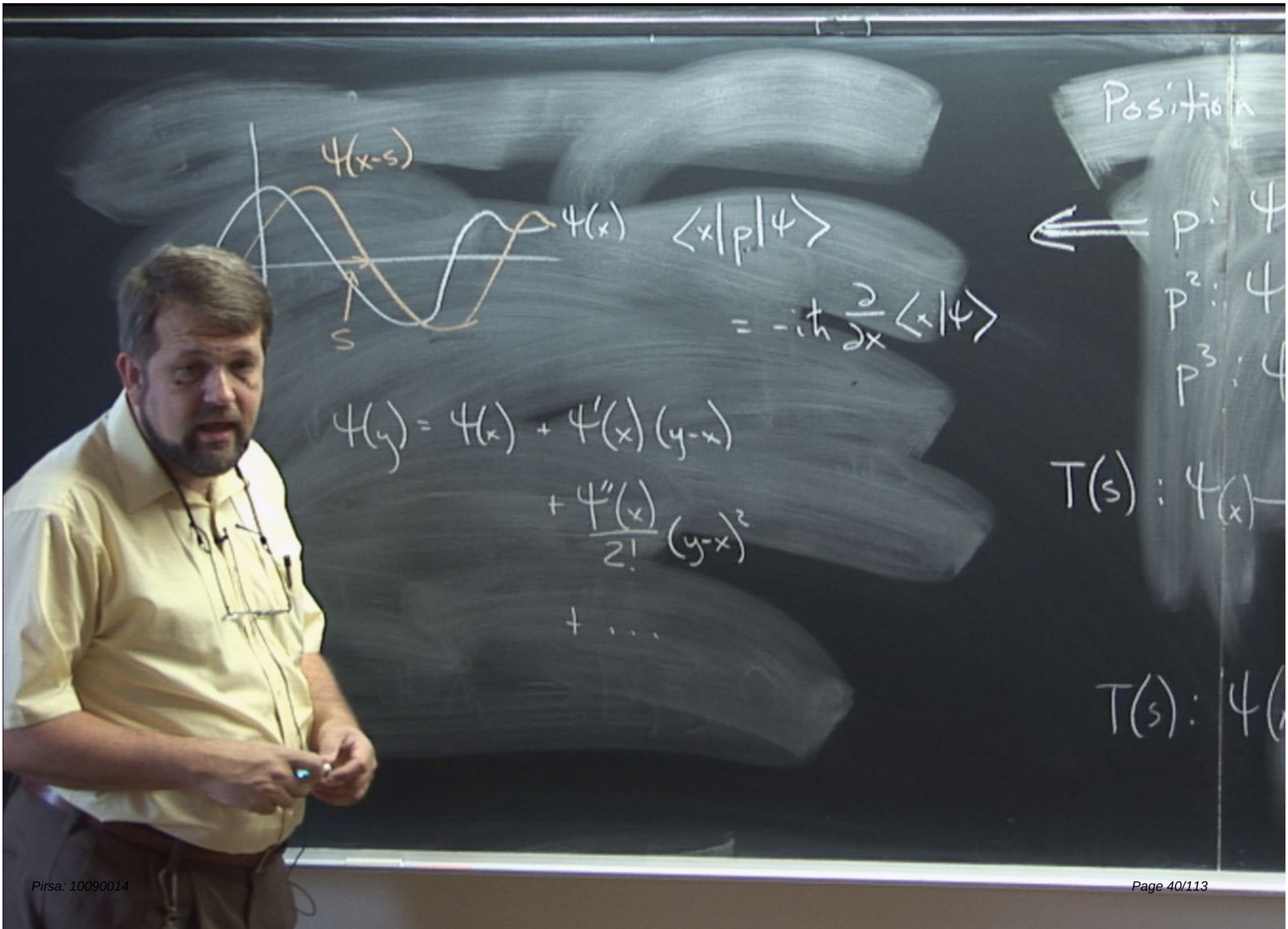
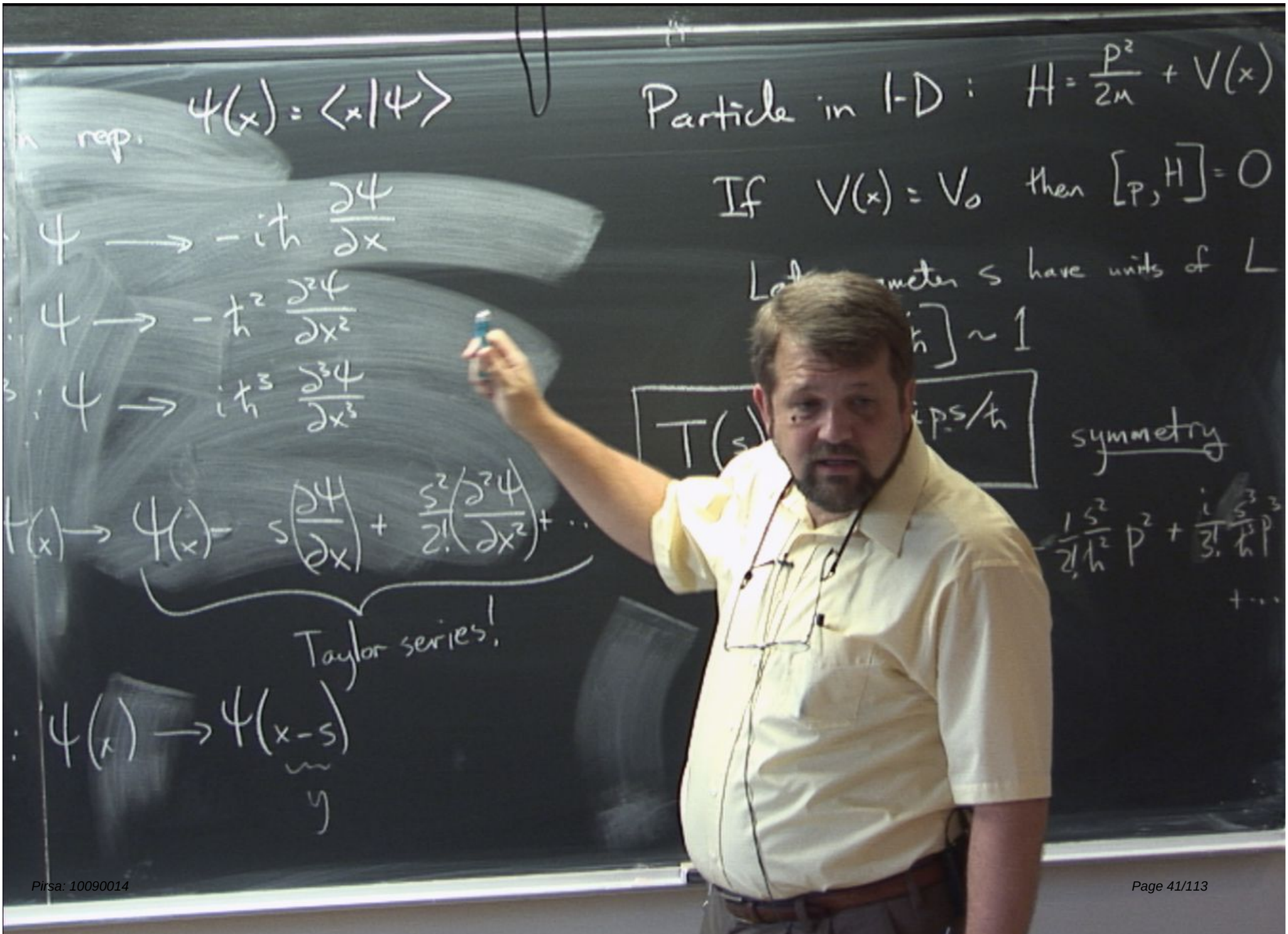

ψ(x-s)
ψ(x)
s
⟨x|p|ψ⟩
= -iħ ∂/∂x ⟨x|ψ⟩
ψ(y) = ψ(x) + ψ'(x)(y-x)
+ ψ''(x)/2! (y-x)²
+ . . .
Position
p : ψ
p² : ψ
p³ : ψ
T(s) : ψ(x)
T(s) : ψ(


$\psi(x) = \langle x|\psi\rangle$
Particle in 1-D : $H = \frac{p^2}{2m} + V(x)$
If $V(x) = V_0$ then $[p, H] = 0$
$\psi \longrightarrow -i\hbar \frac{\partial \psi}{\partial x}$
$\psi \longrightarrow -\hbar^2 \frac{\partial^2 \psi}{\partial x^2}$
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symmetry
Taylor series!
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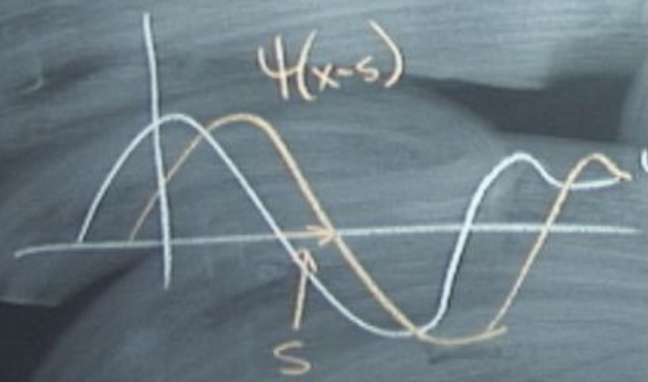


$$\langle x|p|\psi\rangle = -i\hbar \frac{\partial}{\partial x}\langle x|\psi\rangle$$

$$\psi(y) = \psi(x) + \psi'(x)(y-x) + \frac{\psi''(x)}{2!}(y-x)^2 + \dots$$

Position

$p^1 : \psi$

$p^2 : \psi$

$p^3 : \psi$

$T(s) : \psi(x)$

$T(s) : \psi($

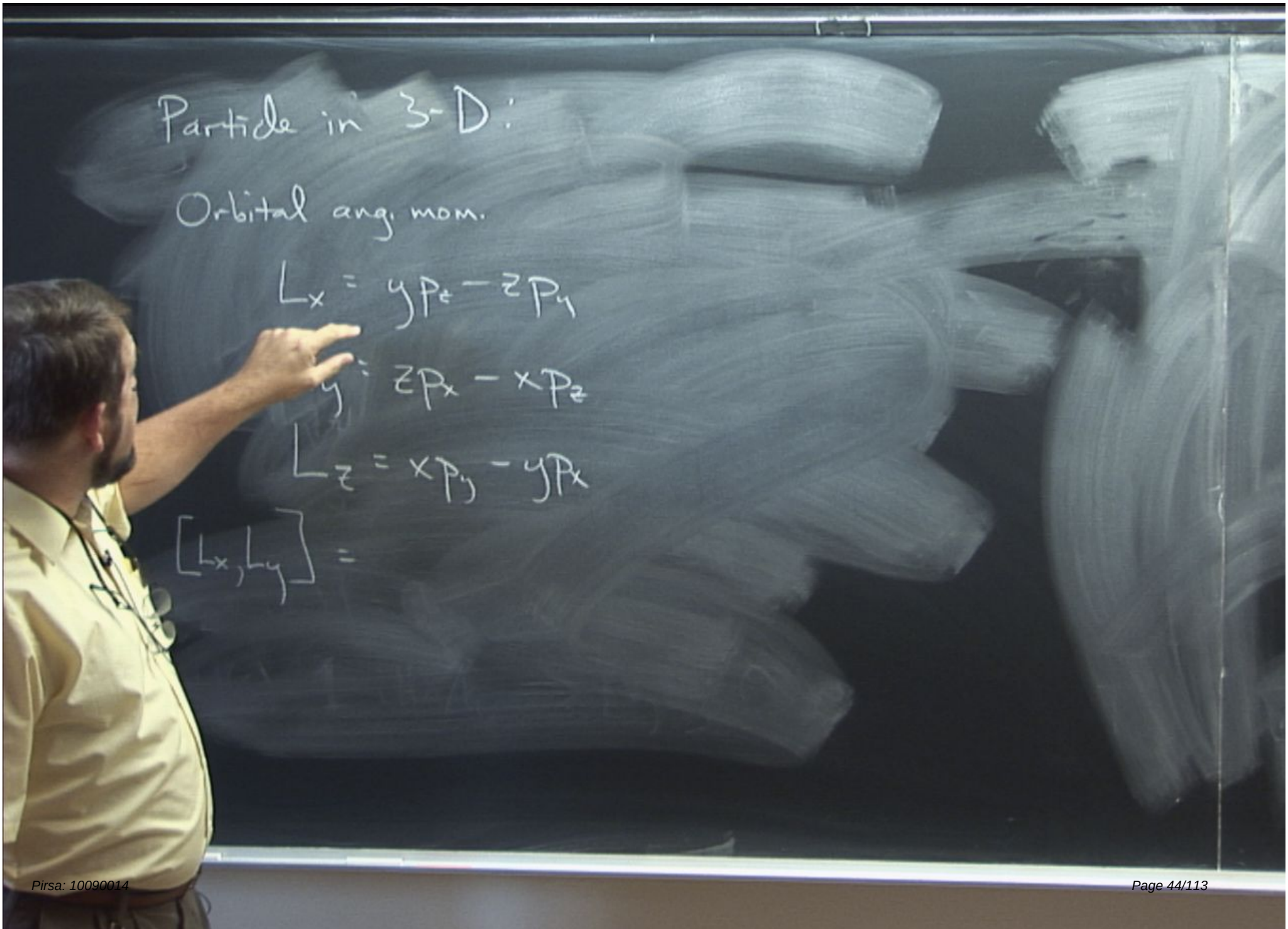
Particle in 3-D:

Orbital ang. mom.

$$L_x = y p_z - z p_y$$

$$L_y = z p_x - x p_z$$

$$L_z = x p_y - y p_x$$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$[L_x, L_y] =$

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$$[L_x, L_y] = y [p_z, z] p_x + x [z, p_z] p_y$$

$$= i\hbar (x p_y - y p_x) = i\hbar L_z$$

Translation: $T(s) = e^{-ips/\hbar}$ Time evolution $U(t) = e^{-iHt/\hbar}$

$$\boxed{[S_x, S_y] = i\hbar S_z}$$

Rotation about z-axis (spin-½) $R_z(\alpha) = e^{-iS_z\alpha/\hbar}$

observables: Hermitian

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Particle in 3-D: $\psi(\vec{r}) = \langle \vec{r} | \psi \rangle$

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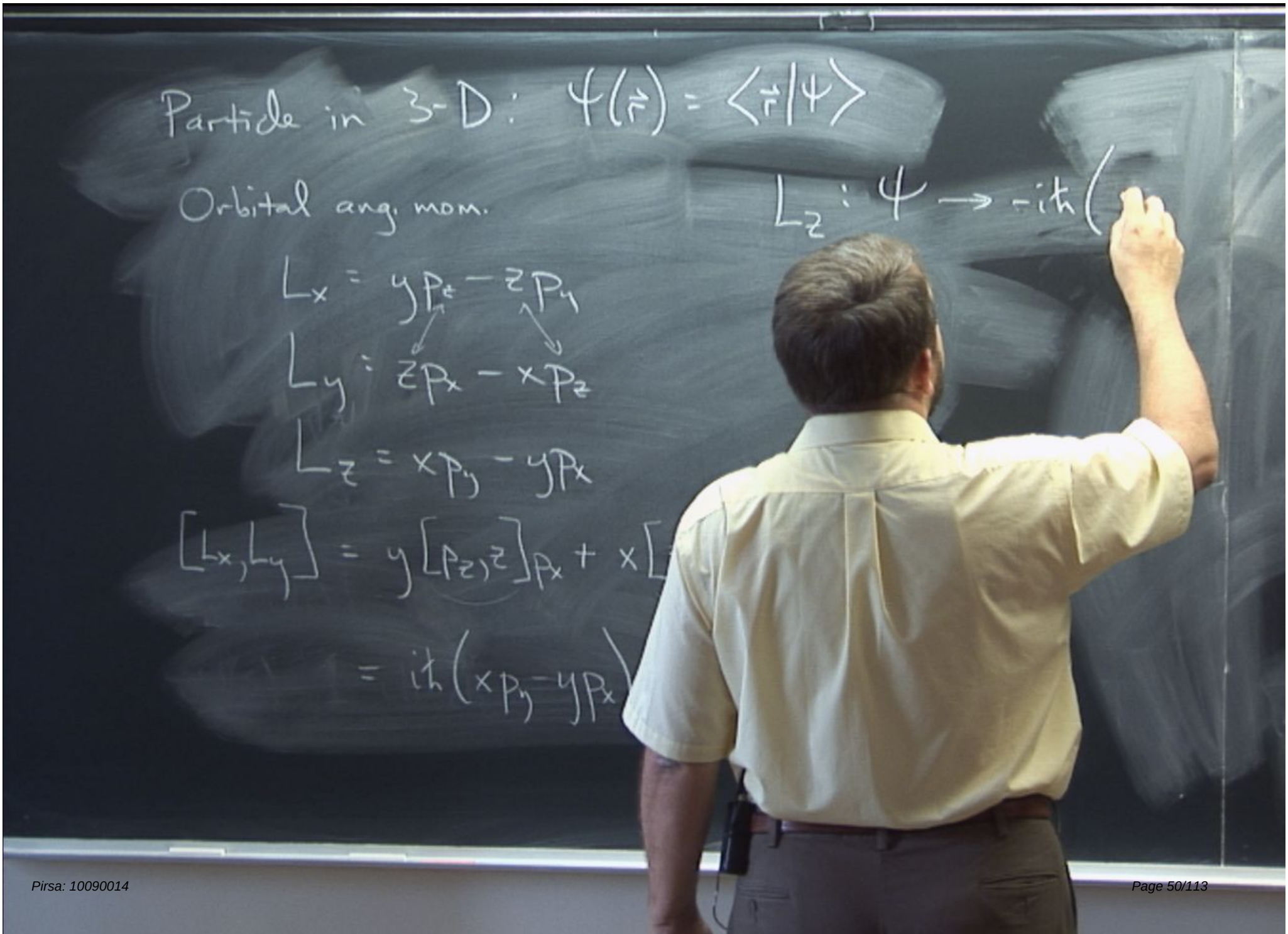
Particle in 3-D: $\psi(\vec{r}) = \langle \vec{r} | \psi \rangle$
Orbital ang. mom.
$L_z : \psi \rightarrow -i\hbar($
$L_x = y p_z - z p_y$
$L_y : z p_x - x p_z$
$L_z = x p_y - y p_x$
$[L_x, L_y] = y [p_z, z] p_x + x [$
$= i\hbar (x p_y - y p_x)$

Particle in 3-D: $\psi(\vec{r}) = \langle \vec{r} | \psi \rangle$

$$\boxed{L_z : \psi \rightarrow -i\hbar\left(x \frac{\partial \psi}{\partial y} - \right.}$$

Orbital ang. mom.

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Particle in 3-D: $\psi(\vec{r}) = \langle \vec{r} | \psi \rangle$

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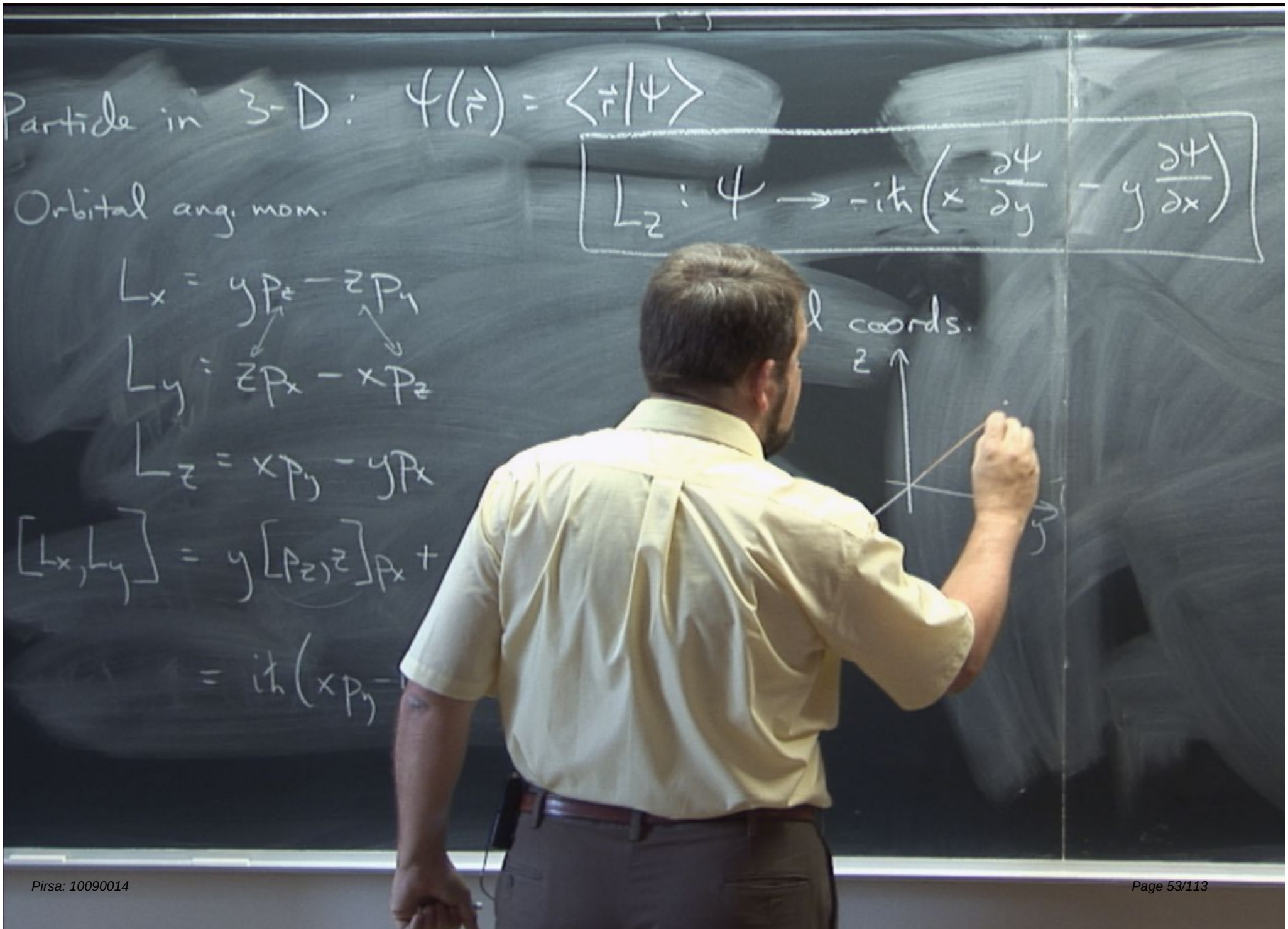
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$L_z = x p_y - y p_x$
$[L_x, L_y] = y [p_z, z] p_x +$
$= i\hbar(x p_y -$
l coords.
z
y

Particle in 3-D: $\psi(\vec{r}) = \langle \vec{r} | \psi \rangle$

Orbital ang. mom.

$$\boxed{L_z : \psi \rightarrow -i\hbar\left(x\frac{\partial\psi}{\partial y} - y\frac{\partial\psi}{\partial x}\right)}$$

$$L_x = y p_z - z p_y$$

$$L_y = z p_x - x p_z$$

$$L_z = x p_y - y p_x$$

$$[L_x, L_y] = y[p_z, z]p_x + x[z, p_z]p_y$$

$$= i\hbar(x p_y - y p_x) = i\hbar L_z$$

Spherical coords.

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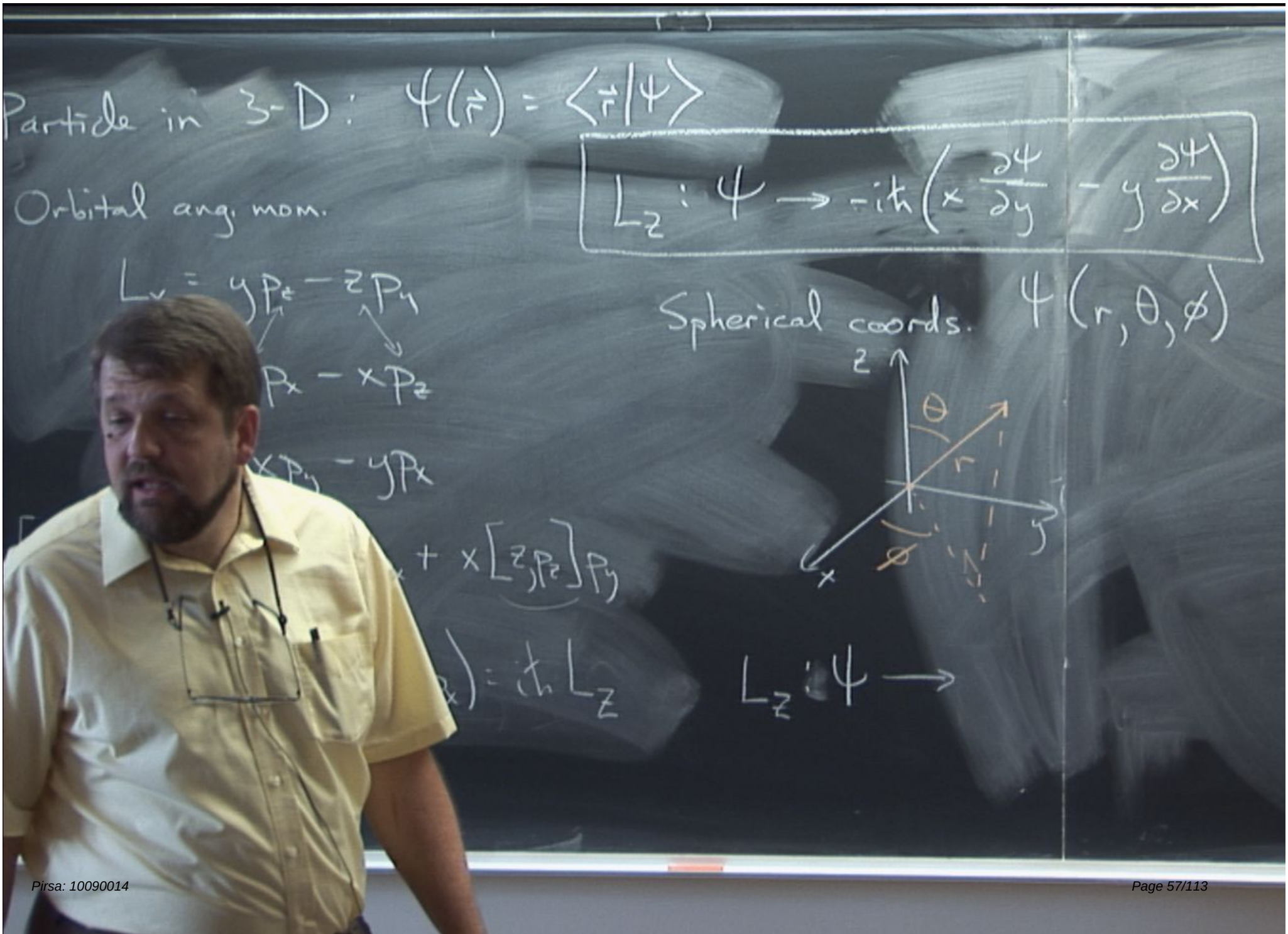
Spherical coords. $\psi(r, \theta, \phi)$


Particle in 3-D: $\psi(\vec{r}) = \langle \vec{r} | \psi \rangle$
Orbital ang. mom.
$L_z : \psi \rightarrow -i\hbar\left(x \frac{\partial \psi}{\partial y} - y \frac{\partial \psi}{\partial x}\right)$
$L_x = y p_z - z p_y$
$p_x - x p_z$
$x p_y - y p_x$
$+ x [z, p_z] p_y$
$= i\hbar L_z$
Spherical coords. $\psi(r, \theta, \phi)$
z
θ
r
y
x
φ
$L_z : \psi \rightarrow$

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Spherical coords. $\psi(r, \theta, \phi)$

$$\boxed{L_z : \psi \longrightarrow -i\hbar \frac{\partial \psi}{\partial \phi}}$$

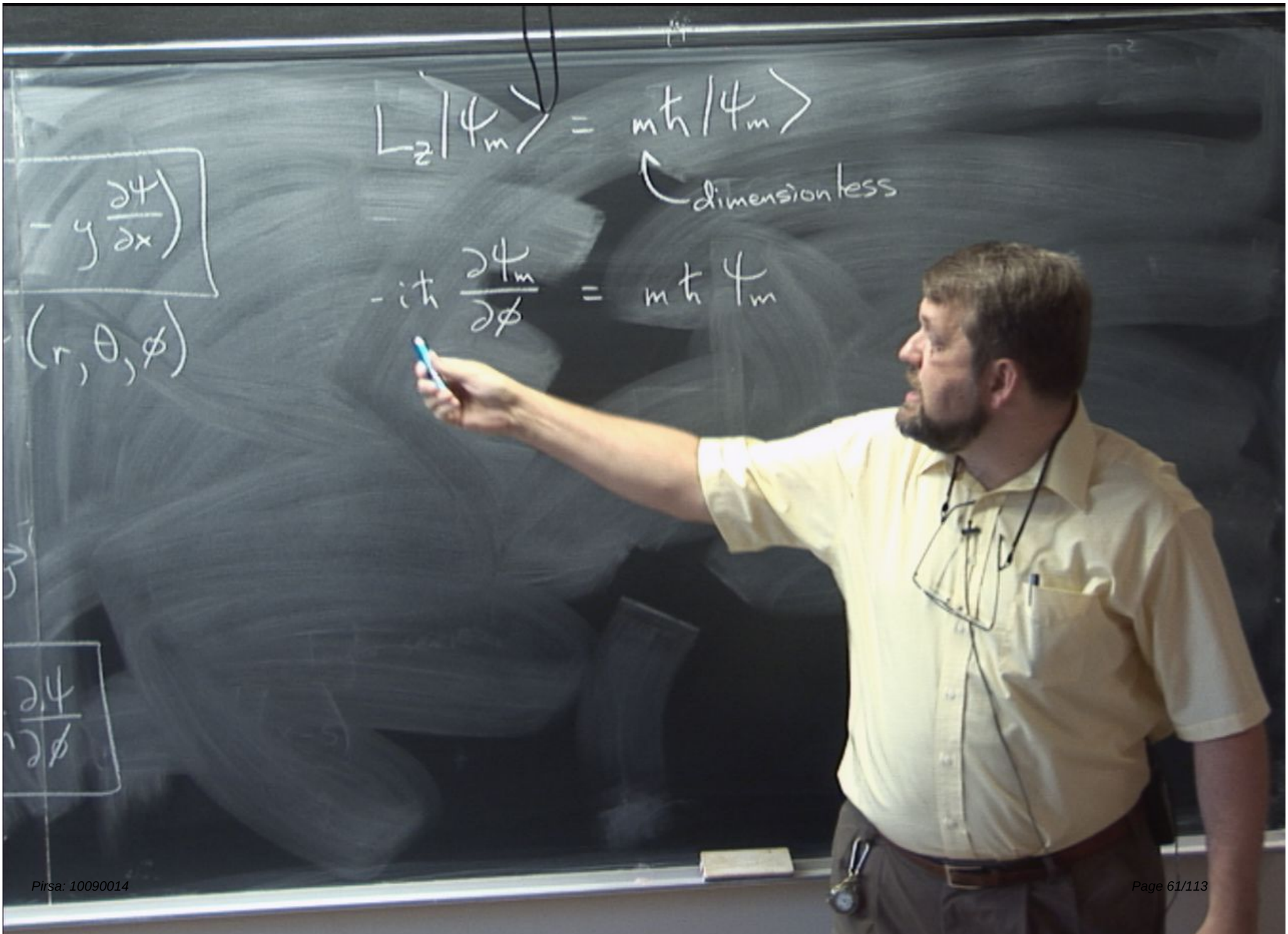
$$L_z|\psi_m\rangle = m\hbar|\psi_m\rangle$$

dimensionless

$$\boxed{-y\frac{\partial\psi}{\partial x}\Big)}$$

$$(r, \theta, \phi)$$

$$\boxed{\frac{\partial\psi}{\partial\phi}}$$


$L_z|\psi_m\rangle = m\hbar|\psi_m\rangle$
dimensionless
$-i\hbar\frac{\partial\psi_m}{\partial\phi} = m\hbar\psi_m$
$-y\frac{\partial\psi}{\partial x}$
(r,θ,ϕ)
$\frac{\partial\psi}{\partial\phi}$

$$L_z|\psi_m\rangle = m\hbar|\psi_m\rangle$$

dimensionless

$$-i\hbar\frac{\partial\psi_m}{\partial\phi} = m\hbar\psi_m$$

$$\Rightarrow \psi_m(r,\theta,\phi) = f(r,\theta)$$

$$\boxed{-y\frac{\partial\psi}{\partial x}}$$

$$(r,\theta,\phi)$$

$$\boxed{\frac{\partial\psi}{\partial\phi}}$$

$$L_z|\psi_m\rangle = m\hbar|\psi_m\rangle$$

dimensionless

$$-i\hbar\frac{\partial\psi_m}{\partial\phi} = m\hbar\psi_m$$

$$\Rightarrow \psi_m(r,\theta,\phi) = c(r,\theta)\,e^{im\phi}$$

$$\boxed{-y\frac{\partial\psi}{\partial x}}$$

$$(r,\theta,\phi)$$

$$\boxed{\frac{\partial\psi}{\partial\phi}}$$

$$L_z|\psi_m\rangle = m\hbar|\psi_m\rangle$$

dimensionless

$$-i\hbar\frac{\partial\psi_m}{\partial\phi} = m\hbar\psi_m$$

$$\Rightarrow \psi_m(r,\theta,\phi) = f(r,\theta)e^{im\phi}$$

$$\boxed{-y\frac{\partial\psi}{\partial x}}$$

$$(r,\theta,\phi)$$

$$\boxed{\frac{\partial\psi}{\partial\phi}}$$

$$L_z|\psi_m\rangle = m\hbar|\psi_m\rangle$$

dimensionless

$$-i\hbar\frac{\partial\psi_m}{\partial\phi} = m\hbar\psi_m$$

$$\Rightarrow \psi_m(r,\theta,\phi) = f(r,\theta)\,e^{im\phi}$$

$$e^{im(\phi+2\pi)} = e^{im\phi}$$

$$\Downarrow$$

$$m = 0, \pm 1, \pm 2, \ldots$$

$$\boxed{-y\frac{\partial\psi}{\partial x}}$$

$$(r,\theta,\phi)$$

$$\boxed{\frac{\partial\psi}{\partial\phi}}$$

$$L_z|\psi_m\rangle = m\hbar|\psi_m\rangle$$

dimensionless

$$-i\hbar\frac{\partial\psi_m}{\partial\phi} = m\hbar\psi_m$$

$$\Rightarrow \psi_m(r,\theta,\phi) = f(r,\theta)\,e^{im\phi}$$

$$R(r)\,Y_{\ell m}(\theta,\phi)$$

$$P_{\ell m}(\theta)\,e^{im\phi}$$

$$e^{im(\phi+2\pi)} = e^{im\phi}$$

$$\Downarrow$$

$$m = 0, \pm 1, \pm 2, \ldots$$

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$$R(r)\,Y_{\ell m}(\theta,\phi)$$

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$$e^{im(\phi+2\pi)} = e^{im\phi}$$

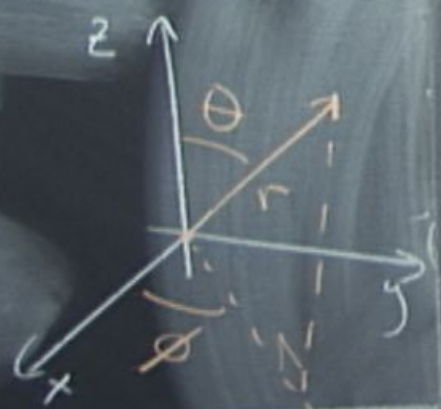
$$m = 0, \pm 1, \pm 2, \ldots$$

Rotate about z-axis:

$$R_z(\alpha) : \psi(r,\theta,\phi) \longrightarrow$$

$$\longrightarrow -i\hbar\left(x\frac{\partial\psi}{\partial y} - y\frac{\partial\psi}{\partial x}\right)$$

ical coords. $\psi(r,\theta,\phi)$



$$-z: \psi \longrightarrow -i\hbar\frac{\partial\psi}{\partial\phi}$$

$$L_z|\psi_m\rangle = m\hbar|\psi_m\rangle$$

dimensionless

$$-i\hbar\frac{\partial\psi_m}{\partial\phi} = m\hbar\psi_m$$

$$\Rightarrow \psi_m(r,\theta,\phi) = f$$

Rotate about z-axis:

$$R_z(\alpha): \psi(r,\theta,\phi) \longrightarrow \psi(r,\theta,\phi-\alpha)$$

$$\psi_m = f(r,\theta)e^{im\phi} \rightarrow$$

$$L_z|\psi_m\rangle = m\hbar|\psi_m\rangle$$

dimensionless

$$-i\hbar\frac{\partial\psi_m}{\partial\phi} = m\hbar\psi_m$$

$$\Rightarrow \boxed{\psi_m(r,\theta,\phi) = f(r,\theta)e^{im\phi}}$$

$$R(r)Y_{\ell m}(\theta,\phi)$$

$$P_{\ell m}(\theta)e^{im\phi}$$

$$e^{im(\phi+2\pi)} = e^{im\phi}$$

$$m = 0, \pm 1, \pm 2, \ldots$$

Rotate about z-axis:

$$R_z(\alpha): \psi(r,\theta,\phi) \longrightarrow \psi(r,\theta,\phi-\alpha)$$

$$\psi_m = f(r,\theta)e^{im\phi} \rightarrow e^{-im\alpha}f(r,\theta)e^{im\phi}$$

$$= e^{-im\alpha}\psi_m$$

$$-y\frac{\partial\psi}{\partial x}$$

$$(r,\theta,\phi)$$

$$\frac{\partial\psi}{\partial\phi}$$

Particle in 3-D: $\psi(\vec{r}) = \langle \vec{r} | \psi \rangle$

$$\boxed{R_z(\alpha) = e^{-iL_z\alpha/\hbar}}$$

$$\boxed{L_z : \psi \longrightarrow -i\hbar\left(x\frac{\partial\psi}{\partial y} - \right.}$$

Spherical coords. $\psi(r,$

$$\boxed{L_z : \psi \longrightarrow -i\hbar\frac{\partial\psi}{\partial\phi}}$$

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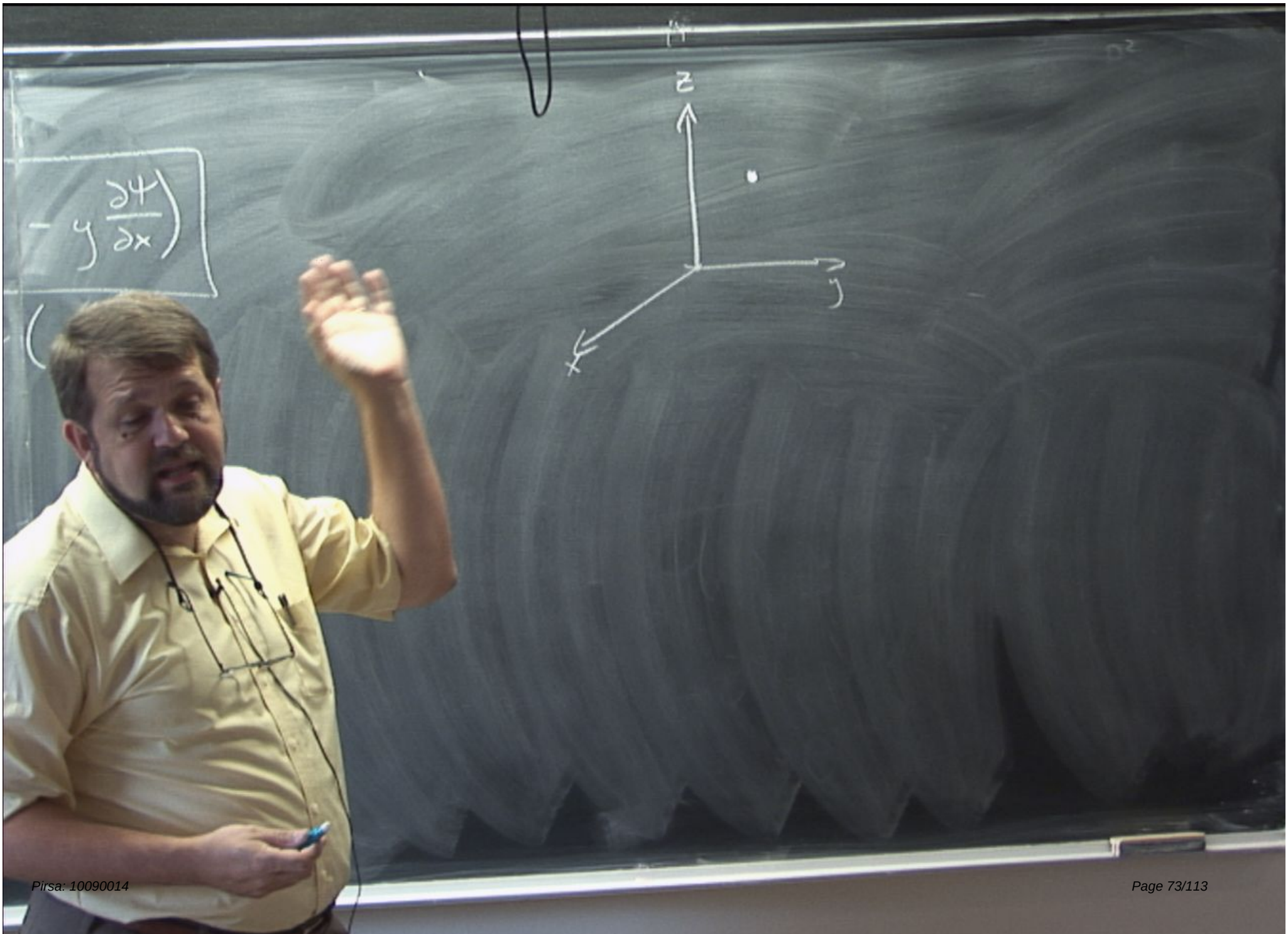
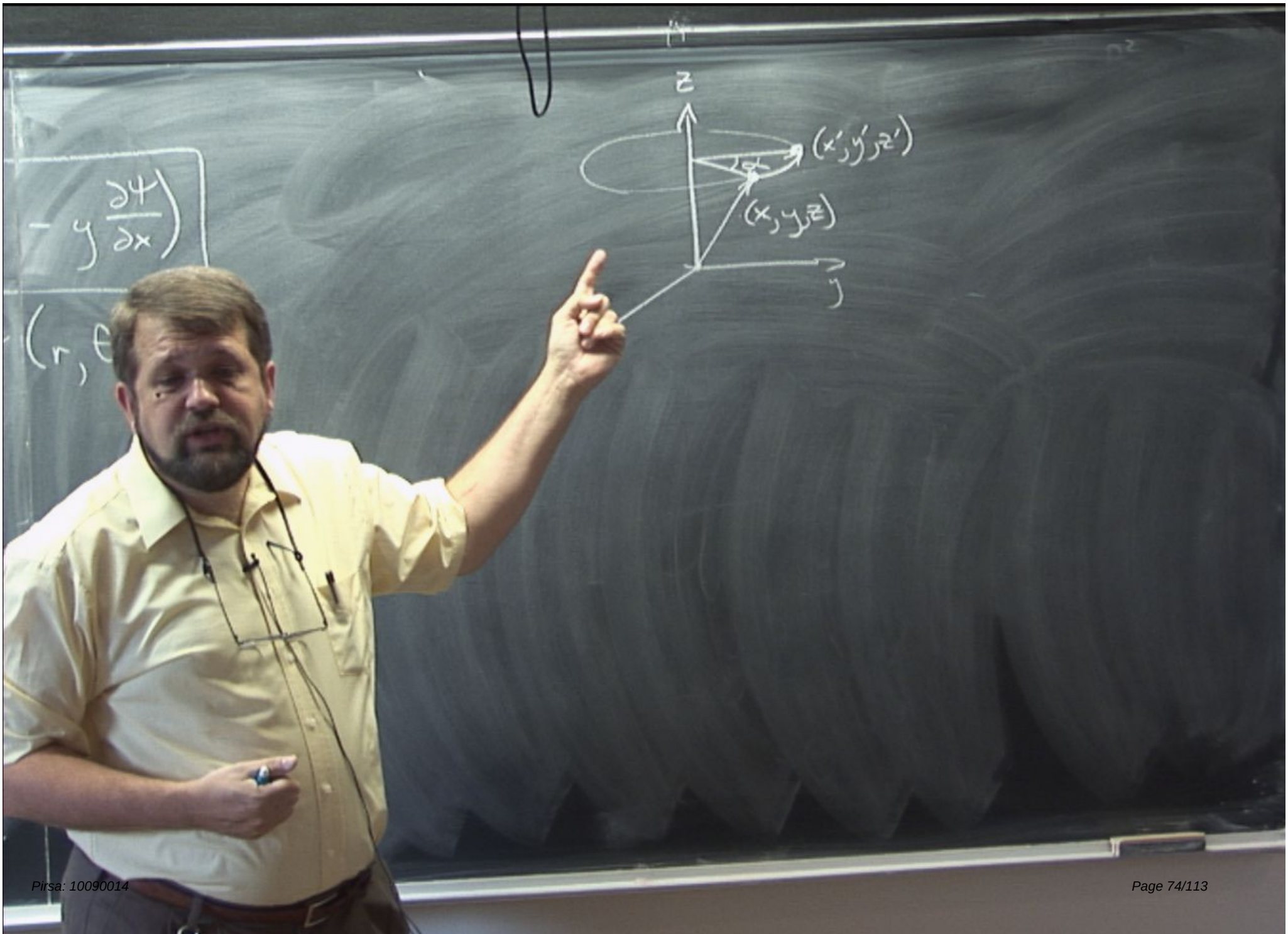
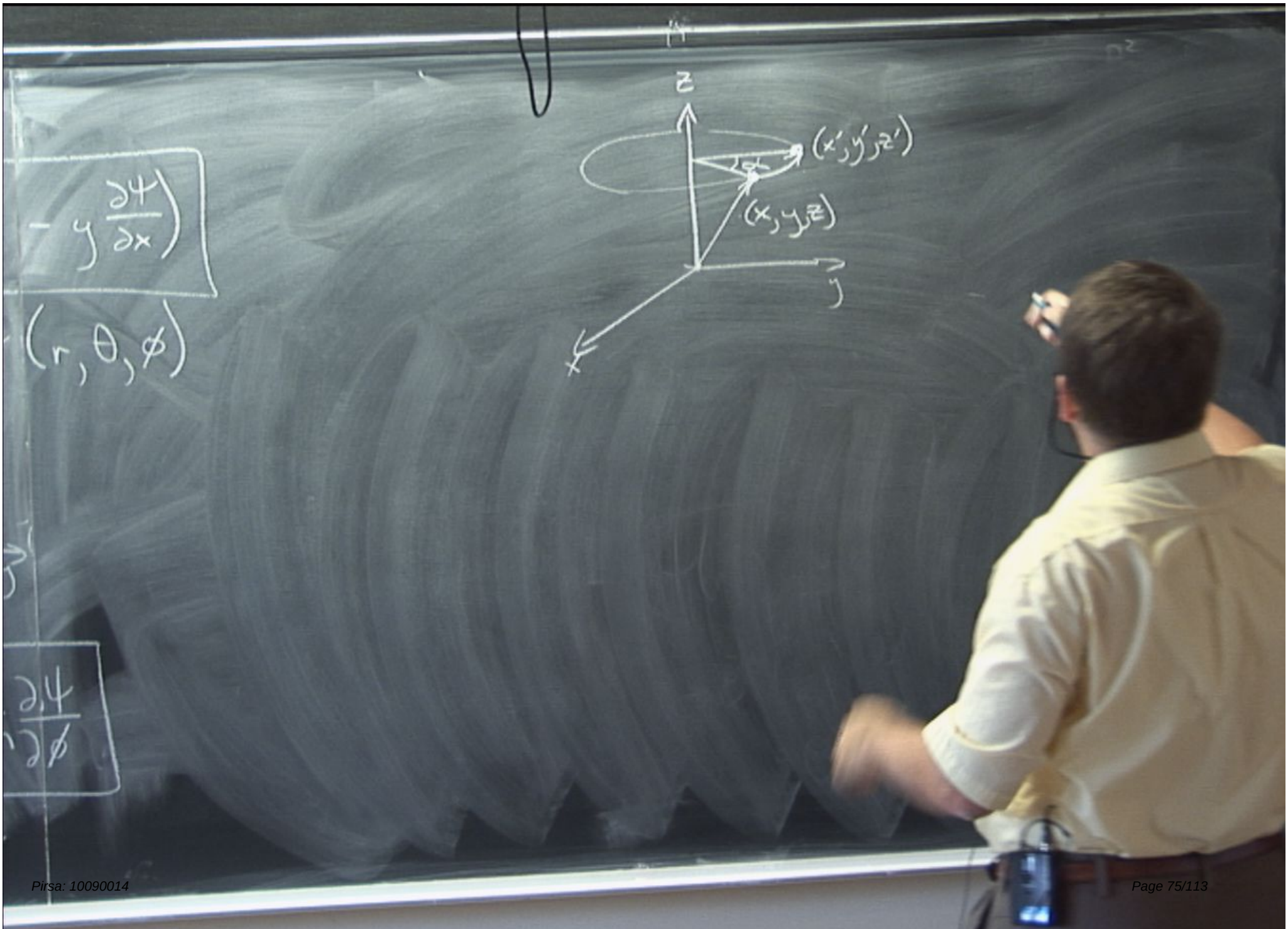

z
y
x

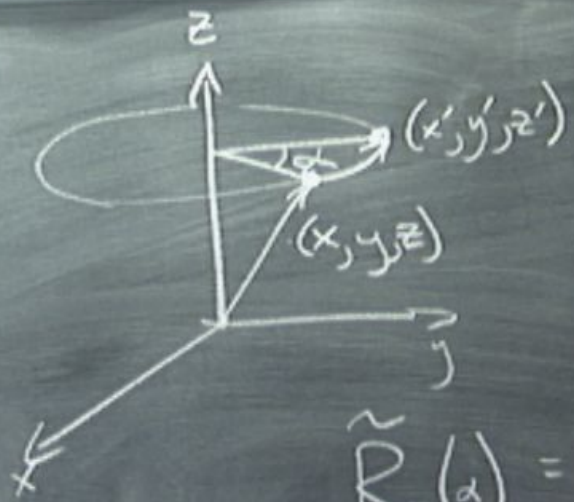

z
(x', y', z')
(x, y, z)
y


z
(x', y', z')
(x, y, z)
y
x
$-y\frac{\partial\psi}{\partial x}$
(r, θ, ϕ)
$\frac{\partial\psi}{\partial\phi}$

$$\boxed{-y\frac{\partial\psi}{\partial x}}$$

$$(r,\theta,\phi)$$

$$\boxed{\frac{\partial\psi}{\partial\phi}}$$



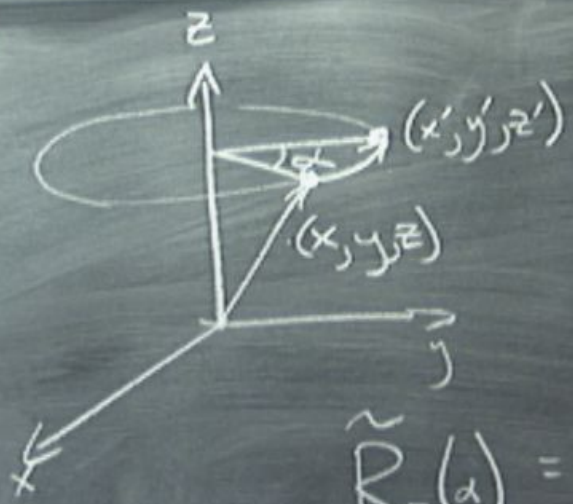
$$\begin{pmatrix}x\\y\\z\end{pmatrix}\to\begin{pmatrix}x'\\y'\\z'\end{pmatrix}=\begin{pmatrix}\text{matrix}\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}$$

$$\tilde{R}_z(\alpha)=\begin{pmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$\boxed{-y\frac{\partial\psi}{\partial x}\Big)}$$

$$(r,\theta,\phi)$$

$$\boxed{\frac{\partial\psi}{\partial\phi}}$$



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \text{matrix} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

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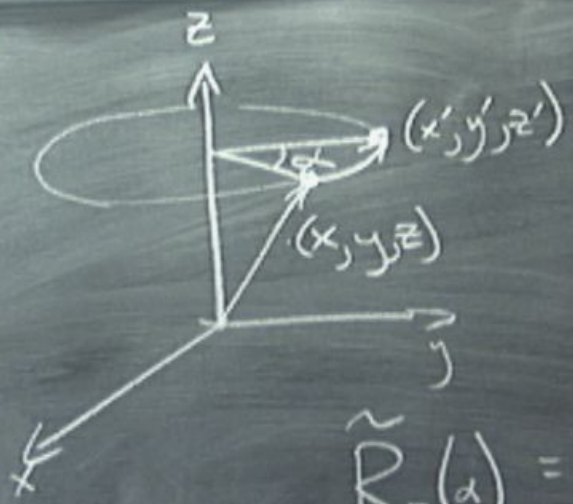
$$(r,\theta,\phi)$$

$$\boxed{\frac{\partial\psi}{\partial\phi}}$$



$$\begin{pmatrix}x\\y\\z\end{pmatrix}\to\begin{pmatrix}x'\\y'\\z'\end{pmatrix}=\begin{pmatrix}\text{matrix}\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}$$

$$\tilde{R}_z(\alpha)=\begin{pmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{pmatrix}$$

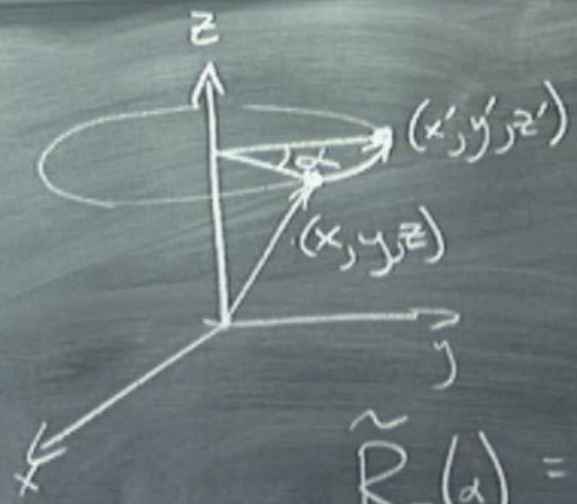
Rotate by ϵ (keep terms incl. $O(\epsilon^2)$, ...)

$$\tilde{R}_z(\epsilon)=\Big($$

$$\boxed{-y\frac{\partial\psi}{\partial x}\Big)}$$

$$(r,\theta,\phi)$$

$$\boxed{\frac{\partial\psi}{\partial\phi}}$$



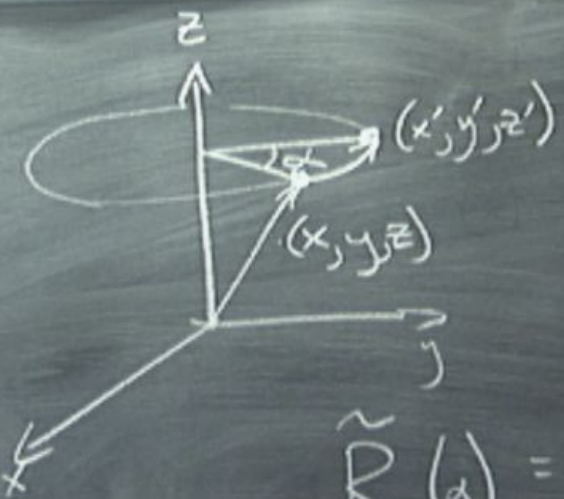
$$\begin{pmatrix}x\\y\\z\end{pmatrix}\to\begin{pmatrix}x'\\y'\\z'\end{pmatrix}=\begin{pmatrix}\text{matrix}\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}$$

$$\tilde{R}_z(\alpha)=\begin{pmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{pmatrix}$$

Rotate by ϵ (keep terms incl. $O(\epsilon^2)$, ...)

$$\tilde{R}_z(\epsilon)=\begin{pmatrix}1-\epsilon^2/2 & -\epsilon & 0\\ \epsilon & 1-\epsilon^2/2 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$\tilde{R}_x(\epsilon) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1-\epsilon^2/2 & -\epsilon \\ 0 & \epsilon & 1-\frac{\epsilon^2}{2} \end{pmatrix}$$



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \text{matrix} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

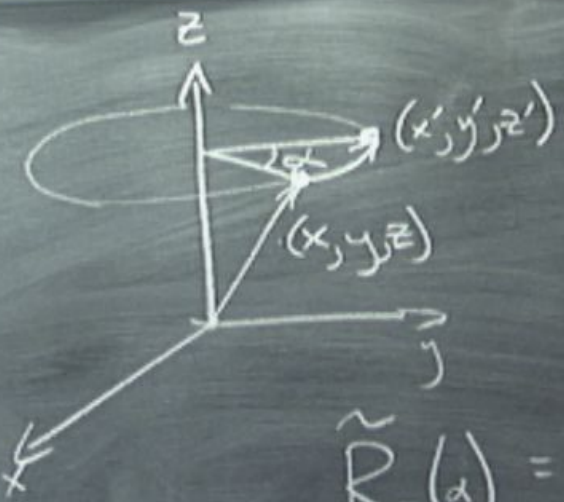
$$\tilde{R}_y(\epsilon) = \begin{pmatrix} 1-\epsilon^2/2 & 0 & \epsilon \\ 0 & 1 & 0 \\ -\epsilon & 0 & 1-\epsilon^2/2 \end{pmatrix}$$

$$\tilde{R}_z(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Rotate by ϵ (keep terms incl. $O(\epsilon^2)$, ...)

$$\tilde{R}_z(\epsilon) = \begin{pmatrix} 1-\epsilon^2/2 & -\epsilon & 0 \\ \epsilon & 1-\epsilon^2/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} + O(\epsilon^3)$$

$$\tilde{R}_x(\epsilon) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1-\epsilon^2/2 & -\epsilon \\ 0 & \epsilon & 1-\frac{\epsilon^2}{2} \end{pmatrix}$$



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \text{matrix} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

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Rotate by ϵ (keep terms incl. $O(\epsilon^2)$, ...)

$$\tilde{R}_z(\epsilon) = \begin{pmatrix} 1-\epsilon^2/2 & -\epsilon & 0 \\ \epsilon & 1-\epsilon^2/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} + O(\epsilon^3)$$

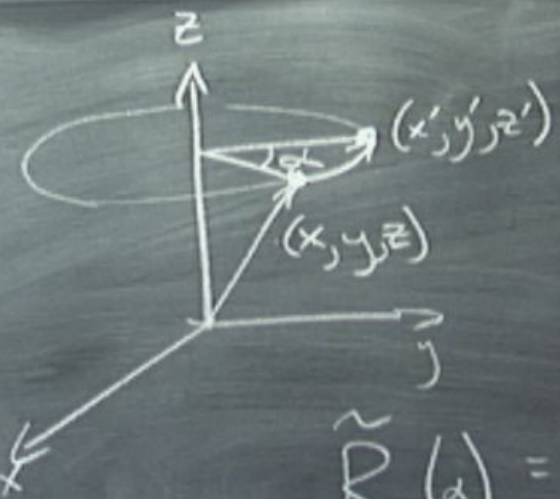
$$\tilde{R}_x(-\epsilon)\tilde{R}_y(-\epsilon)\,\tilde{R}_x(\epsilon)\,\tilde{R}_y(\epsilon) \quad \text{not quite } \tilde{1}$$

$$\tilde{R}_x(-\epsilon)\tilde{R}_y(-\epsilon)\,\tilde{R}_x(\epsilon)\,\tilde{R}_y(\epsilon) \quad \text{not quite } \tilde{1}$$

$$= \cdots \text{dull but easy} \cdots = \begin{pmatrix} 1 & -\epsilon^2 & 0 \\ \epsilon^2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + O(\epsilon^3)$$

$$\tilde{R}_x(\epsilon) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1-\epsilon^2/2 & -\epsilon \\ 0 & \epsilon & 1-\frac{\epsilon^2}{2} \end{pmatrix}$$

$$\tilde{R}_y(\epsilon) = \begin{pmatrix} 1-\epsilon^2/2 & 0 & \epsilon \\ 0 & 1 & 0 \\ -\epsilon & 0 & 1-\epsilon^2/2 \end{pmatrix}$$



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \text{matrix} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\tilde{R}_z(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Rotate by ϵ (keep terms incl. $O(\epsilon^2)$, ...)

$$\tilde{R}_z(\epsilon) = \begin{pmatrix} 1-\epsilon^2/2 & -\epsilon & 0 \\ \epsilon & 1-\epsilon^2/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} + O(\epsilon^3)$$

$$\tilde{R}_x(-\epsilon)\tilde{R}_y(-\epsilon)\,\tilde{R}_x(\epsilon)\,\tilde{R}_y(\epsilon) \quad \text{not quite } \tilde{1}$$

$$= \cdots \text{dull but easy} \cdots = \begin{pmatrix} 1 & -\epsilon^2 & 0 \\ \epsilon^2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + O(\epsilon^3)$$

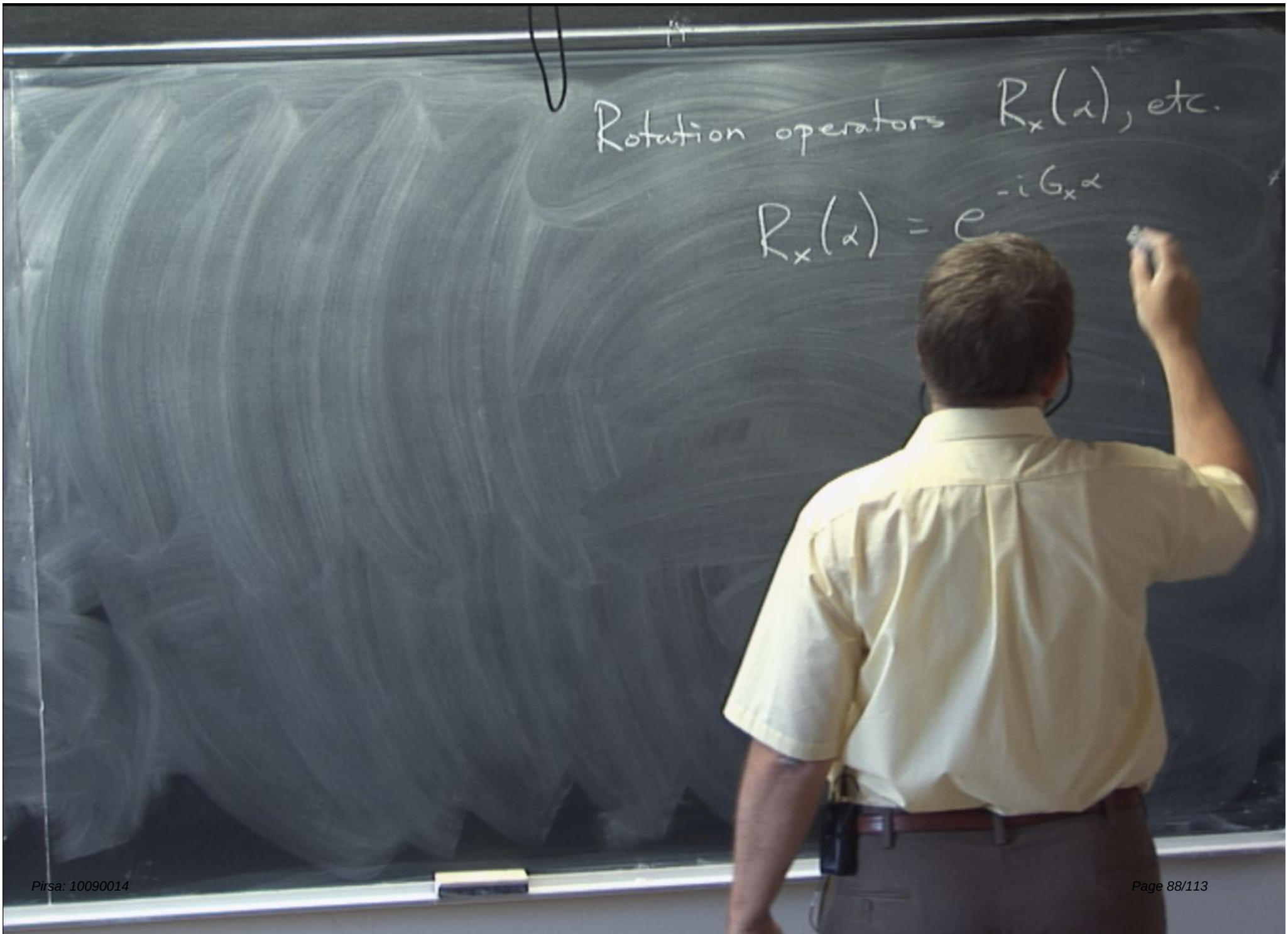
$$= R_z(\epsilon^2)$$

$$\tilde{R}_x(-\epsilon)\tilde{R}_y(-\epsilon)\,\tilde{R}_x(\epsilon)\,\tilde{R}_y(\epsilon) \quad \text{not quite } \tilde{1}$$

$$= \cdots \text{dull but easy} \cdots = \begin{pmatrix} 1 & -\epsilon^2 & 0 \\ \epsilon^2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + O(\epsilon^3)$$

$$= R_z(\epsilon^2)$$

Rotation operators $R_x(\alpha)$, etc.


Rotation operators $R_x(\alpha)$, etc.
$R_x(\alpha) = e^{-iG_x\alpha}$

Rotation operators $R_x(\alpha)$, etc.

$$R_x(\alpha) = e^{-iG_x\alpha}$$

(what is G_x? $G_x =$

Rotation operators $R_x(\alpha)$, etc.

$$R_x(\alpha) = e^{-iG_x\alpha}$$

$$R_y(\alpha) = e^{-iG_y\alpha}$$

$$R_z(\alpha) = e^{-iG_z\alpha}$$

Rotation operators $R_x(\alpha)$, etc.

$$R_x(\alpha) = e^{-iG_x\alpha}$$

$$R_y(\alpha) = e^{-iG_y\alpha}$$

$$R_z(\alpha) = e^{-iG_z\alpha}$$

G_x, G_y, G_z <u>generate</u> rotations...

$$R_x(-\epsilon)\,R_y(-\epsilon)\,R_x(\epsilon)\,R_y(\epsilon)$$

$$= \left(\mathbb{1} + i G_x \epsilon - \frac{\epsilon^2}{2} G_x^2\right) \cdots \left(\right.$$

Rotation operators $R_x(\alpha)$, etc.

$$R_x(\alpha) = e^{-i G_x \alpha}$$

$$R_y(\alpha) = e^{-i G_y \alpha}$$

$$R_z(\alpha) = e^{-i G_z \alpha}$$

G_x, G_y, G_z <u>generate</u> rotations...

$$R_x(-\epsilon)\,R_y(-\epsilon)\,R_x(\epsilon)\,R_y(\epsilon)$$

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$$\cdots \left(\mathbb{1} - i G_y \epsilon - \frac{\epsilon^2}{2} G_y\right)$$

Rotation operators $R_x(\alpha)$, etc.

$$R_x(\alpha) = e^{-i G_x \alpha}$$

$$R_y(\alpha) = e^{-i G_y \alpha}$$

$$R_z(\alpha) = e^{-i G_z \alpha}$$

G_x, G_y, G_z <u>generate</u> rotations...

$$R_x(-\epsilon)\,R_y(-\epsilon)\,R_x(\epsilon)\,R_y(\epsilon)$$

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$$\cdots \left(\mathbb{1} - i G_y \epsilon - \frac{\epsilon^2}{2} G_y^2\right)$$

$$= \mathbb{1} +$$

Rotation operators $R_x(\alpha)$, etc.

$$R_x(\alpha) = e^{-i G_x \alpha}$$

$$R_y(\alpha) = e^{-i G_y \alpha}$$

$$R_z(\alpha) = e^{-i G_z \alpha}$$

G_x, G_y, G_z <u>generate</u> rotations...

Rotation operators $R_x(\alpha)$, etc.

$$R_x(-\epsilon) R_y(-\epsilon) R_x(\epsilon) R_y(\epsilon)$$

$$= \left(\mathbb{1} + i G_x \epsilon - \frac{\epsilon^2}{2} G_x^2\right) \cdots$$

$$\cdots \left(\mathbb{1} - i G_y \epsilon - \frac{\epsilon^2}{2} G_y^2\right)$$

$$= \mathbb{1} +$$

$$R_x(\alpha) = e^{-i G_x \alpha}$$

$$R_y(\alpha) = e^{-i G}$$

$$R_z(\alpha) = e^{-i}$$

$$R_x(-\epsilon)\,R_y(-\epsilon)\,R_x(\epsilon)\,R_y(\epsilon)$$

$$= \left(\mathbb{1} + i G_x \epsilon - \frac{\epsilon^2}{2} G_x^2\right) \cdots$$

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$$R_z(\epsilon^2) = \mathbb{1} - i\epsilon^2 G_z$$

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G_x, G_y, G_z <u>generate</u> rotations...

$$\therefore \left[G_x, G_y\right] = i\, G_z$$

R_x

$R_z($

$$\therefore \; [G_x, G_y] = i\, G_z \quad (+\text{cyclic})$$

<u>Define</u> angular momentum operators: $S_x = \hbar G_x$

etc.

$\vdots$

R_x

$R_z($

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<u>Define</u> angular momentum operators: $S_x = \hbar G_x$

etc.

$$\vdots$$

$$R_x(\alpha) = e^{-i S_x \alpha/\hbar}$$

$$R_y(\alpha) = e^{-i S_y \alpha/\hbar}$$

$$R_z(\alpha) = e^{-i S_z \alpha/\hbar}$$

R_x

$R_z($

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$$R_x(-\epsilon)\,R_y(-\epsilon)\,R_x(\epsilon)\,R_y(\epsilon)$$

$$= \left(\mathbb{1} + i G_x \epsilon - \frac{\epsilon^2}{2} G_x^2\right) \cdots$$

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orientations of a 3-D object

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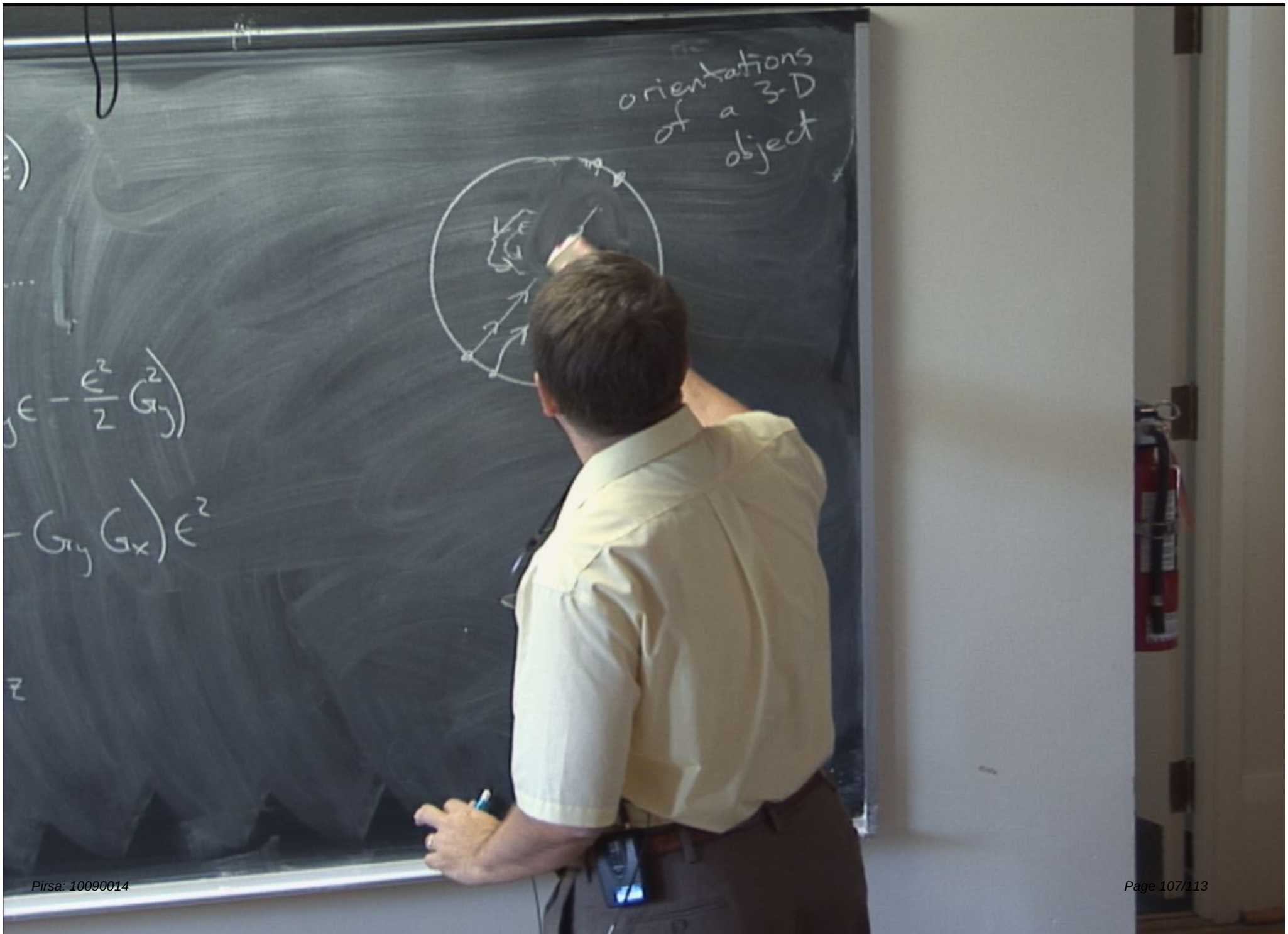
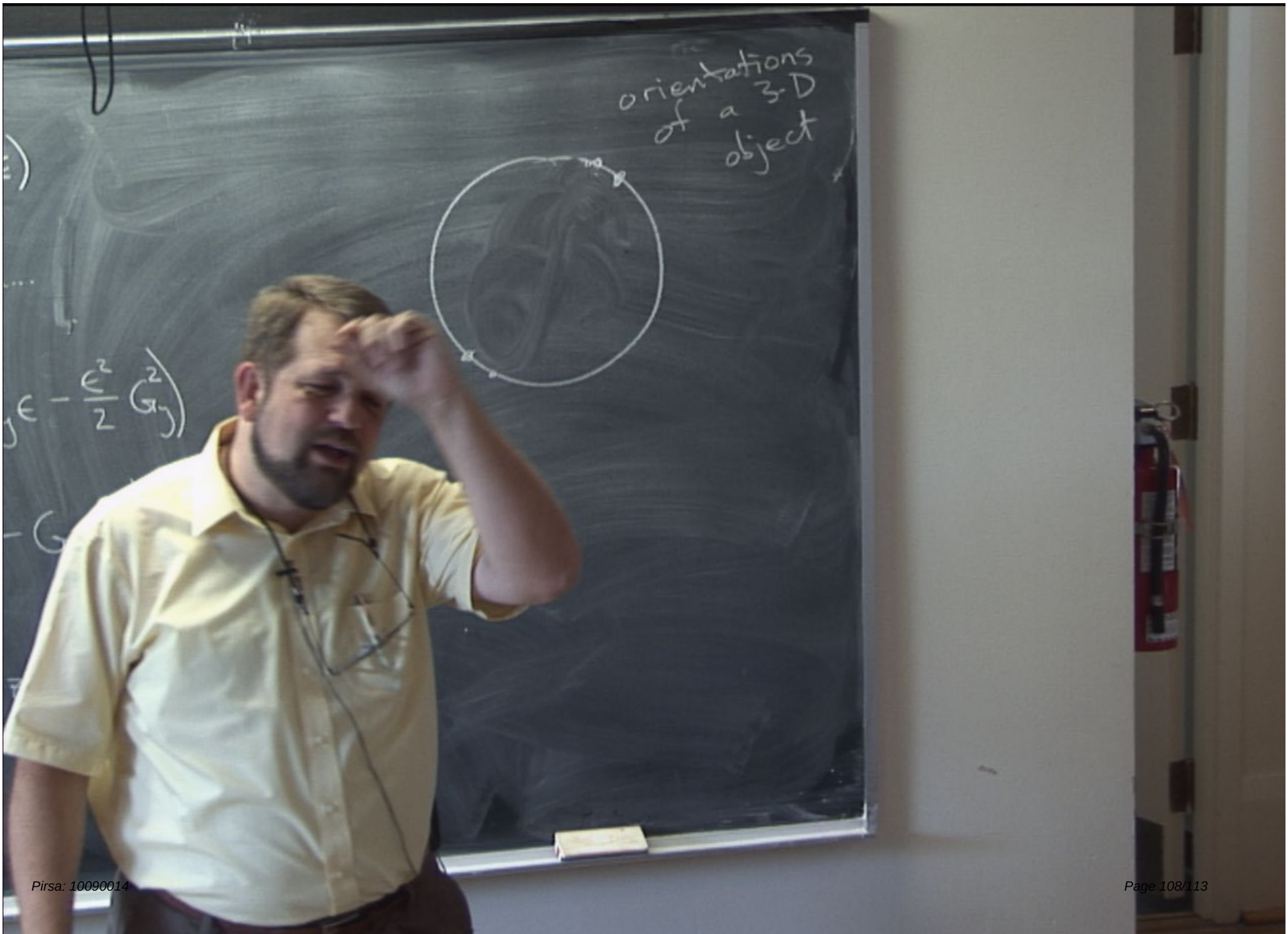
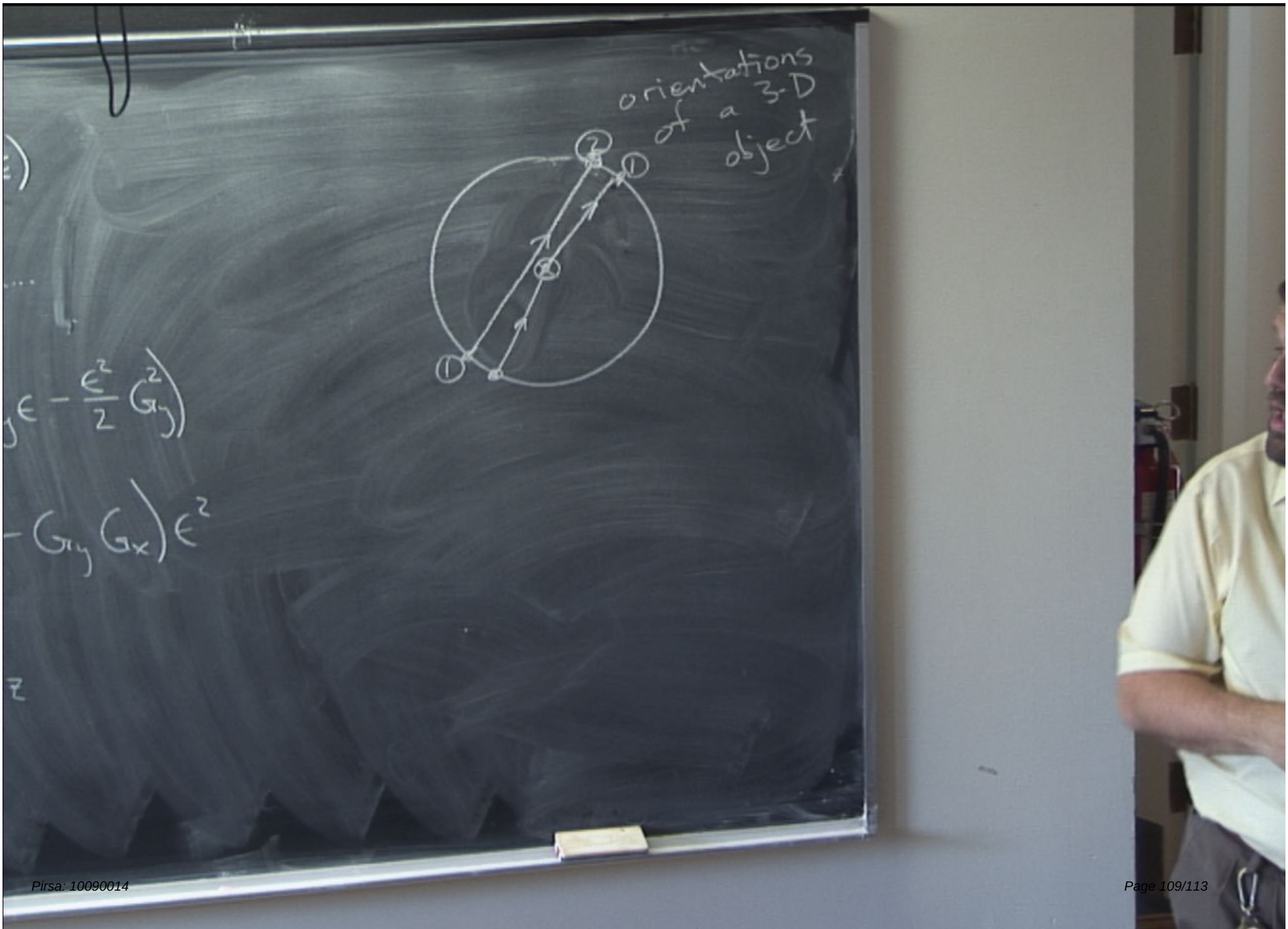
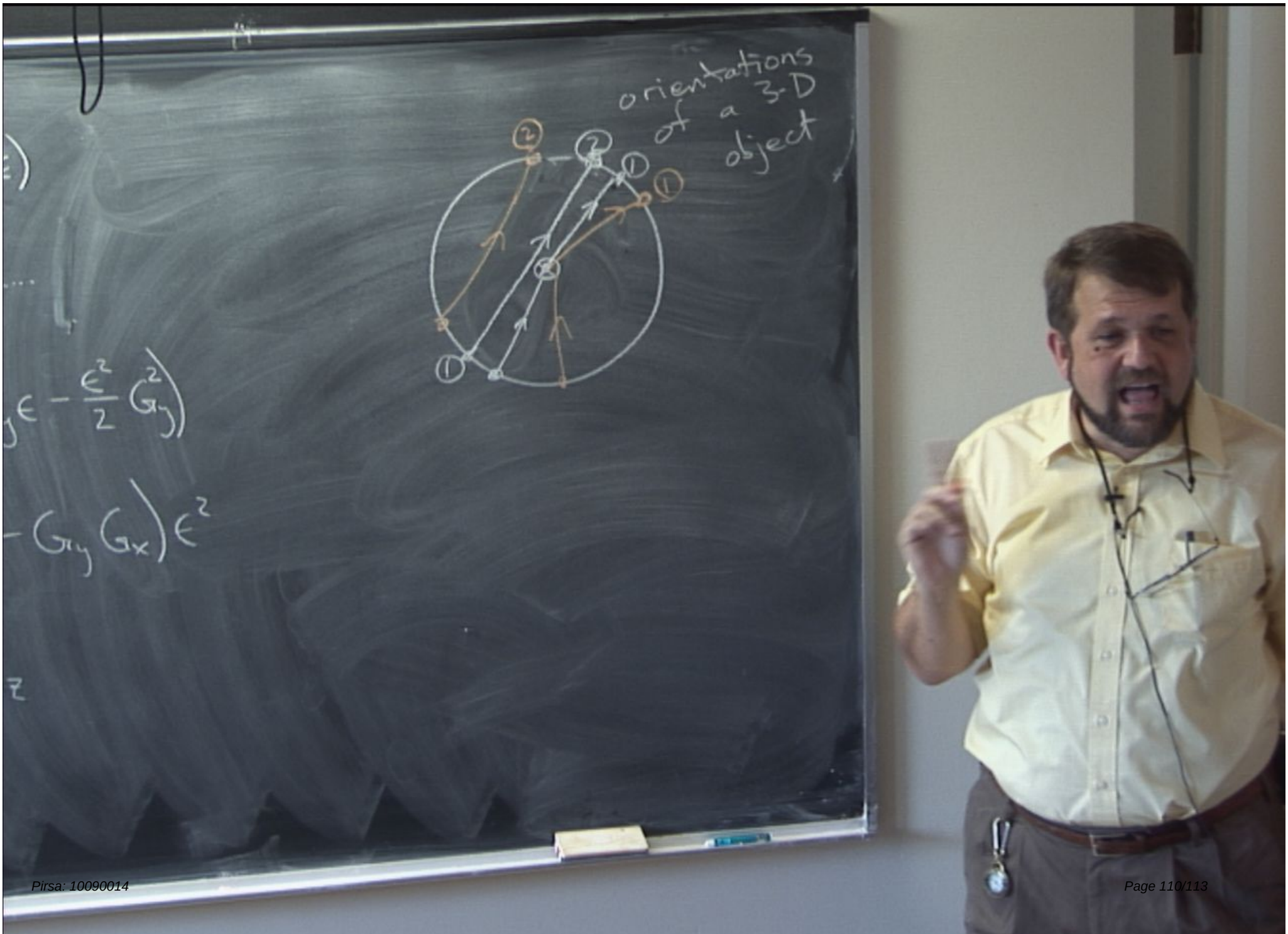
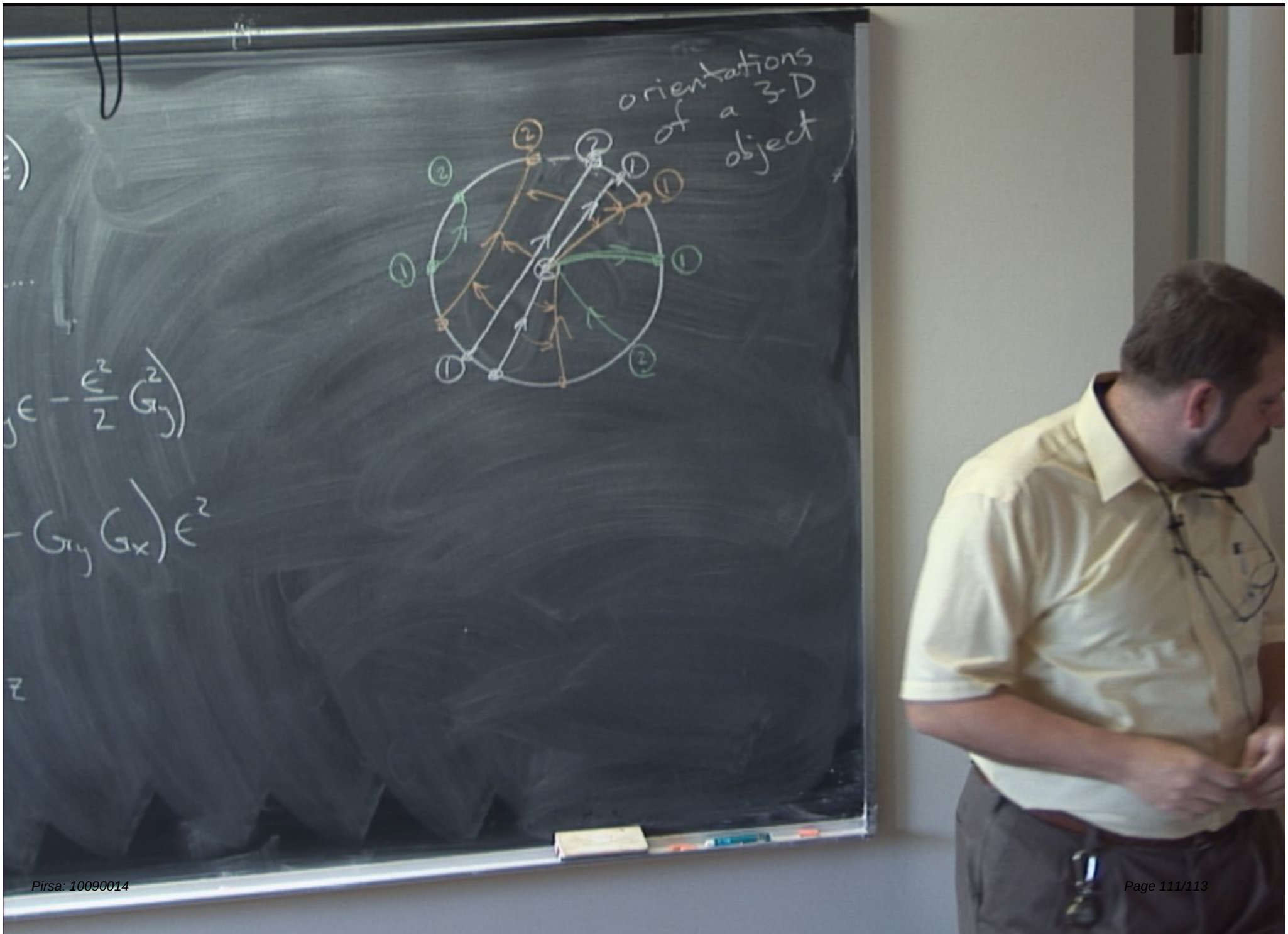
orientations of a 3-D object


orientations of a 3-D object


orientations
of a 3-D
object


orientations of a 3-D object
$\epsilon - \frac{\epsilon^2}{2} G_y^2$
$- G_y G_x) \epsilon^2$


orientations of a 3-D object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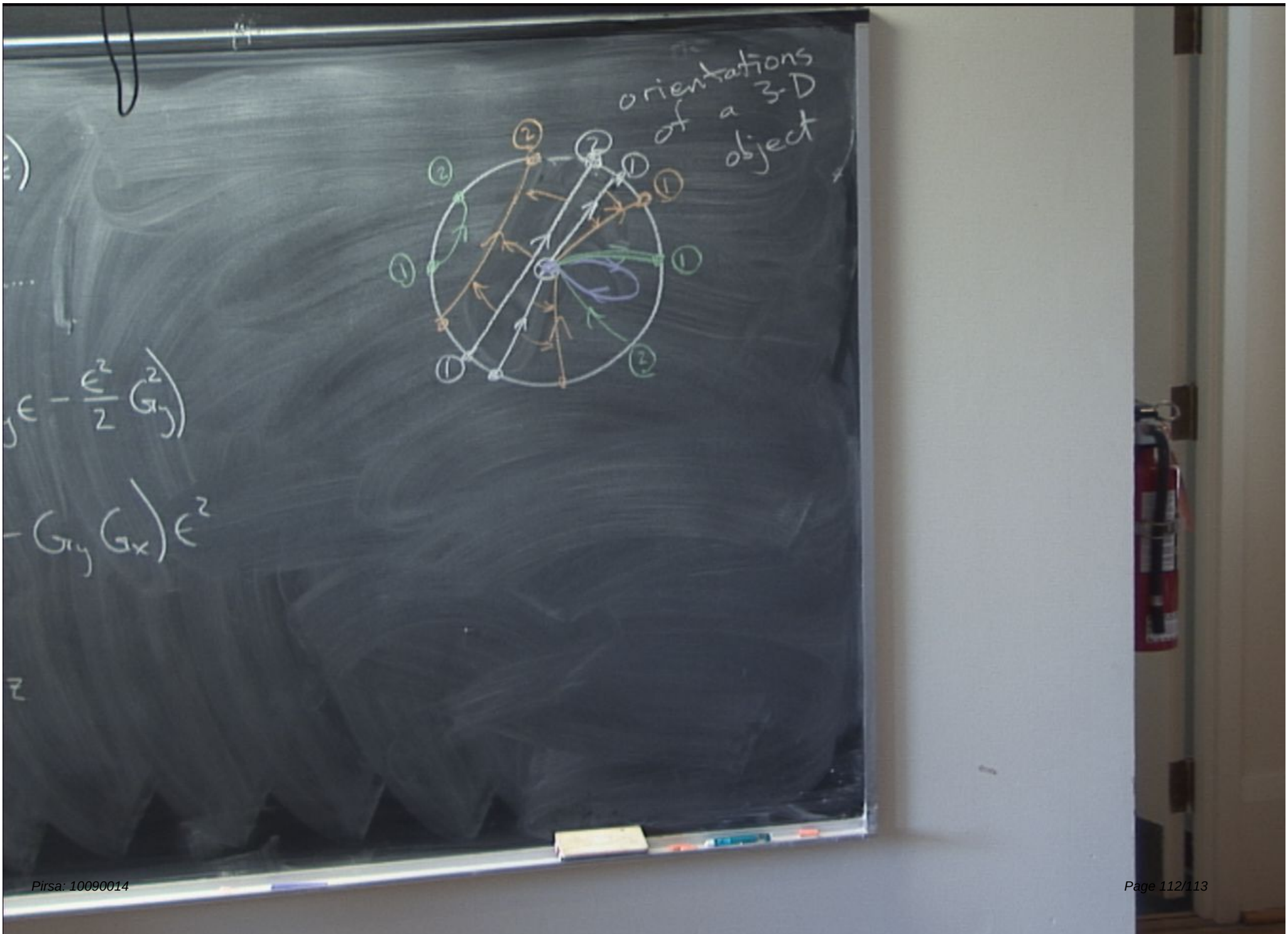
orientations of a 3-D object
$y\epsilon - \frac{\epsilon^2}{2} G_y^2)$
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orientations of a 3-D object