Title: Majorana Fermions in Solid State Systems

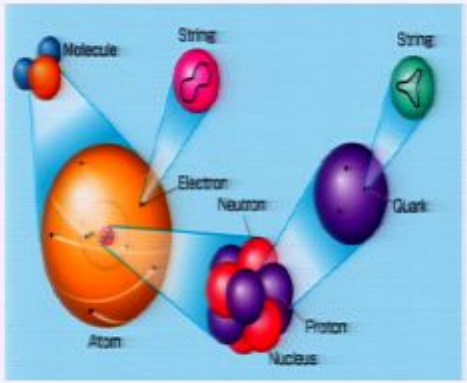
Date: Sep 29, 2010 02:00 PM

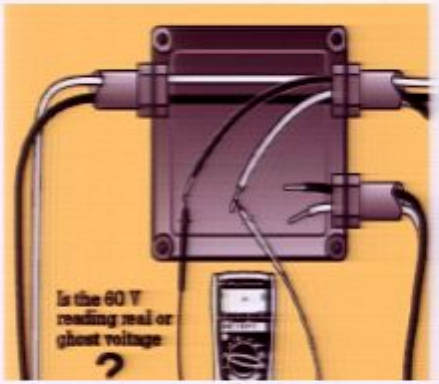
URL: http://pirsa.org/10090001

Abstract: A Majorana fermion is a particle that is its own antiparticle. It has been studied in high energy physics for decades, but has not been definitely observed. In condensed matter physics, Majorana fermions appear as low energy fractionalized quasi-particles with non-Abelian statistics and inherent nonlocality. In this talk I will first discuss recent theoretical proposals of realizing Majorana fermions in solid-state systems, including topological insulators and nanowires. I will next propose experimental setups to detect the existence of Majorana fermions and their striking properties.

Majorana Fermions: Realization and Detection in Solid-State Systems

Liang Fu





Collaboration with Charlie Kane at UPenn

Colloquium at Perimeter Institute on 09/29/2010

A Brief History of Majorana Fermion

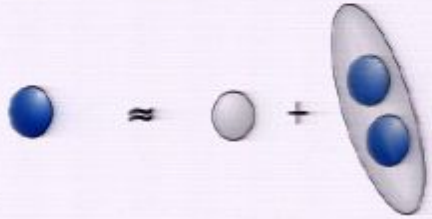
- 1937: Particle = Anti-particle $\gamma = \gamma^\dagger$

  - a candidate for neutrinos

  - no definite evidence

- 2000: Majorana fermion in the solid state

  - may exist as a quasiparticle in superconductors Read & Green, Kitaev



electron hole Cooper pair

$\gamma = uc^\dagger + uc$ "real part of a complex fermion"

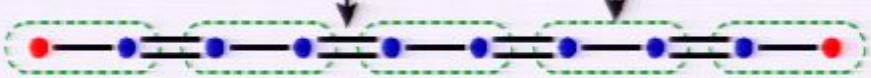
Kitaev model for 1D p-wave superconductor

1D spinless fermion: $H-\mu N=\sum_i t(c_i^\dagger c_{i+1}+c_{i+1}^\dagger c_i)-\mu c_i^\dagger c_i+\Delta(c_i c_{i+1}+c_{i+1}^\dagger c_i^\dagger)$

$c_i \to \gamma_{1i}+i\gamma_{2i}$

Majorana chain:

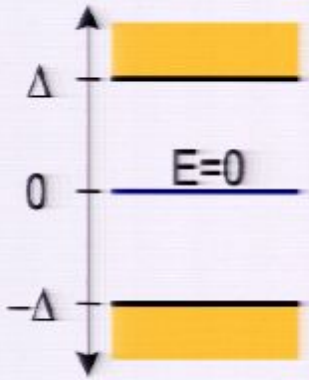
For Δ=t $t_1=\mu$ $t_2=2t$



$(t_1<t_2)$

Unpaired Majorana Fermion at end

BdG spectrum



$$H=(c^\dagger,c)H_{BdG}\begin{pmatrix} c \\ c^\dagger \end{pmatrix}=\sum_{E_m>0} E_m\Gamma_m^\dagger\Gamma_m$$

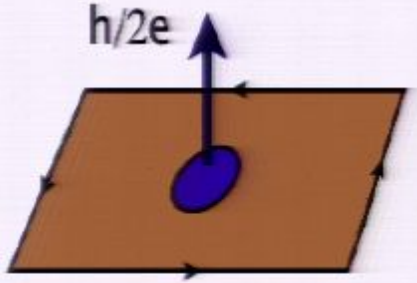
$\Gamma_{E=0}^\dagger=\Gamma_{E=0}\equiv\gamma$: a zero energy Majorana bound state

$$\gamma = \int d^2\mathbf{r}u^*(\mathbf{r})e^{-i\phi/2}c^\dagger(\mathbf{r})+u(\mathbf{r})e^{i\phi/2}c(\mathbf{r})=\gamma^\dagger$$

- ground state degeneracy encoded in $\Psi=\gamma_1+i\gamma_2$

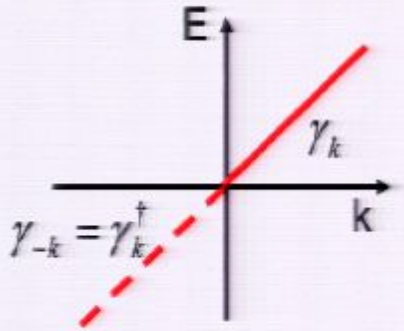
Read-Green model for 2D p-wave superconductor

2D spinless $p_x + ip_y$ superconductor



- BdG Hamiltonian is analogous to quantum Hall state

1. chiral Majorana edge state

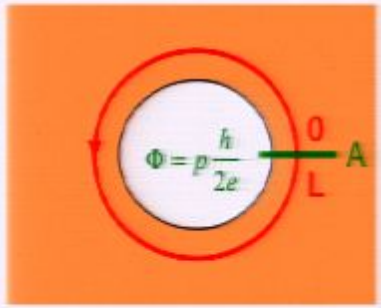


- half of a 1D chiral electron

$$H = -i\hbar v_F \gamma \partial_x \gamma$$

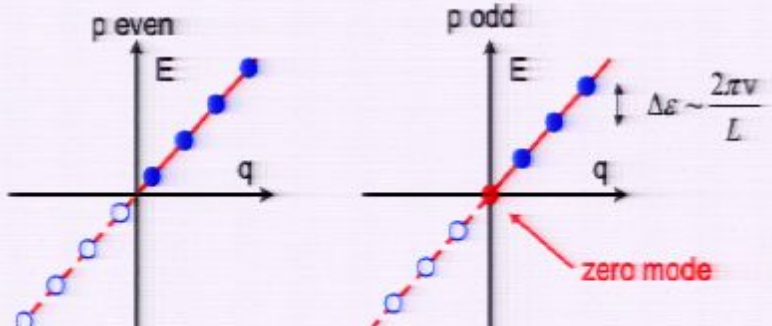
2. Majorana bound state at the vortex

(Ising anyon)



Boundary condition

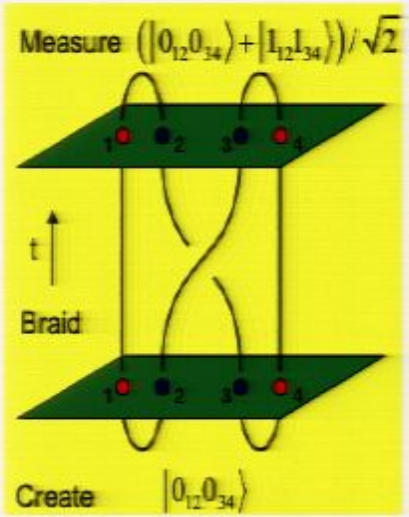
$$\psi(L) = (-1)^{p+1}\psi(0)$$



Non-locality and Non-Abelian Statistics

Degenerate ground states associated with Majorana zero modes

- 2 Majorana separated bound states = 1 fermion $\Psi = \gamma_1 + i\gamma_2$
  - 2 degenerate states (full/empty) = 1 qubit
- 2N separated Majoranas = N qubits
- Quantum Information is stored non locally
  - Immune from local decoherence



Braiding performs unitary operations

Non-Abelian statistics

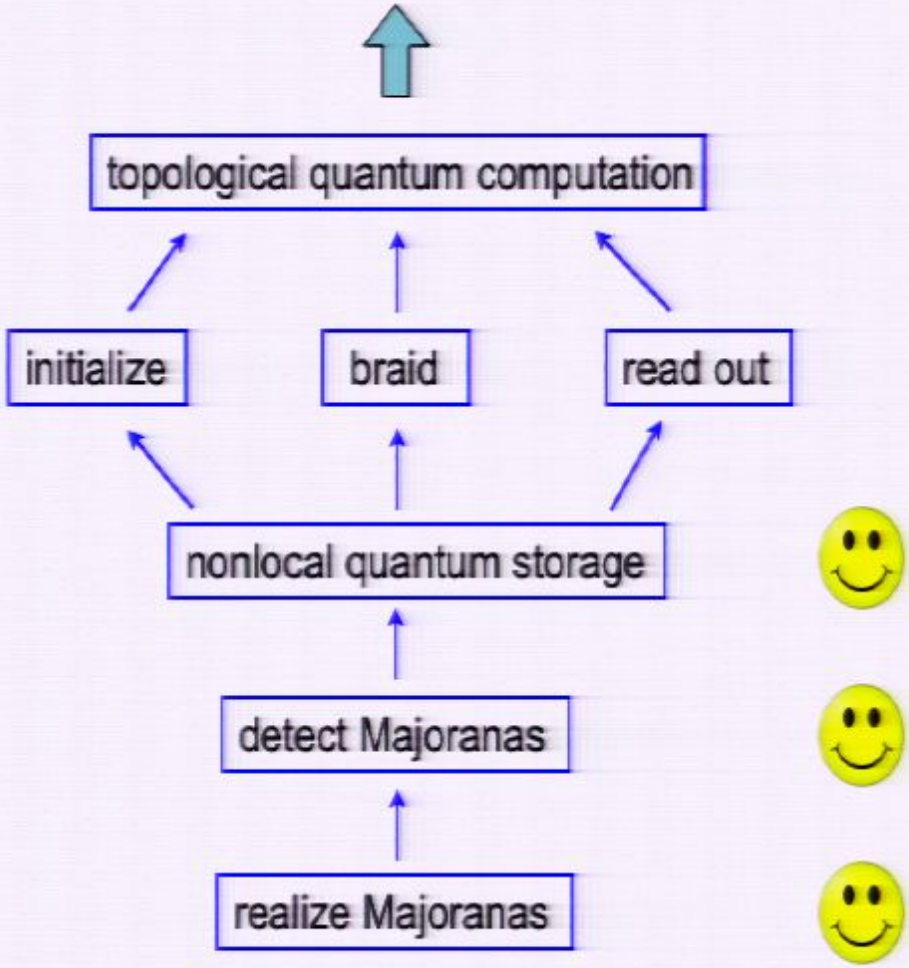
Interchange rule (Ivanov 03)

$$\gamma_i \to \gamma_j$$

$$\gamma_j \to -\gamma_i$$

Potentially useful in fault-tolerant quantum computation (Kitaev, 1997)

Universal quantum computer?



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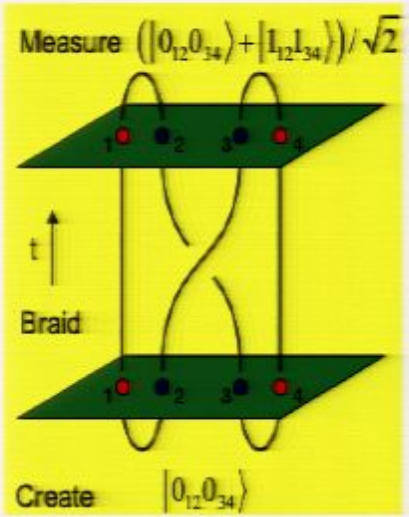

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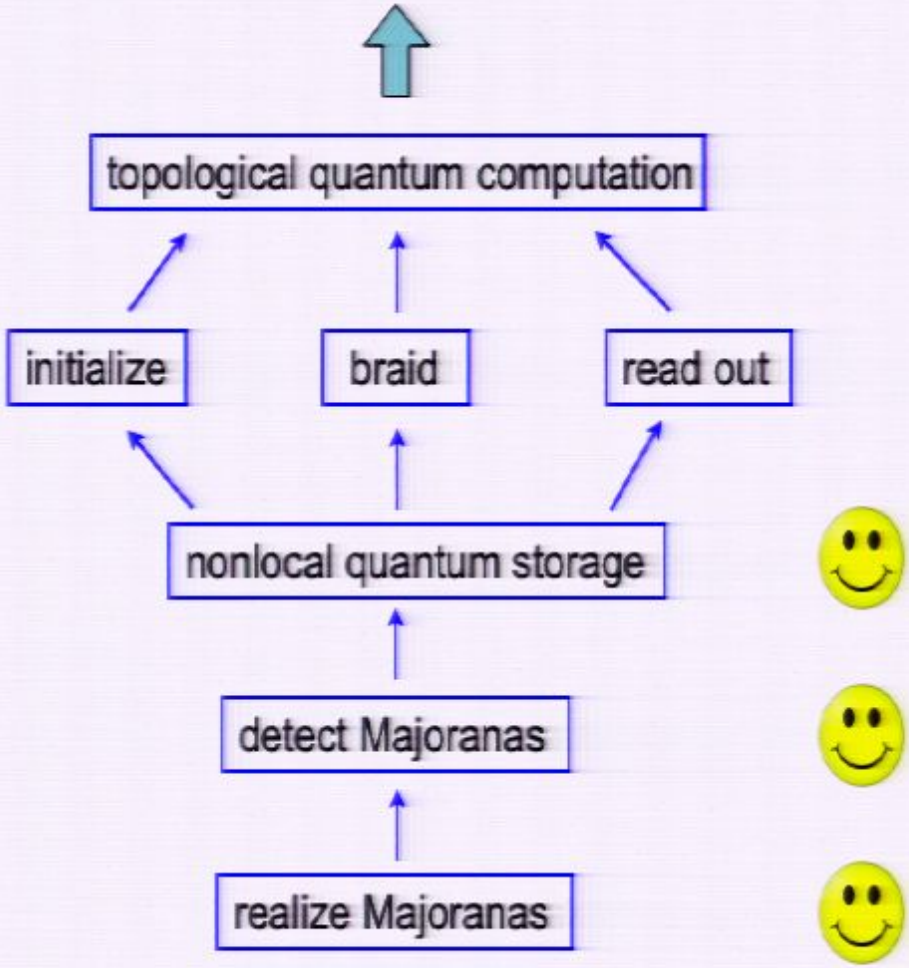

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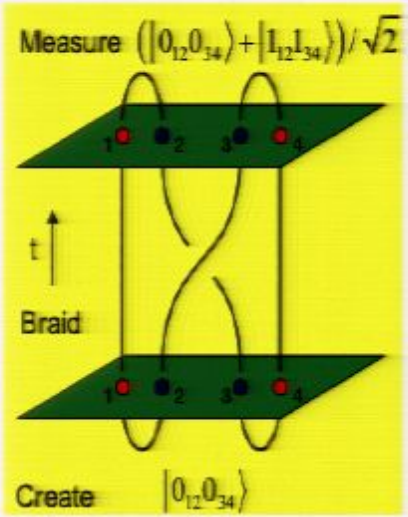

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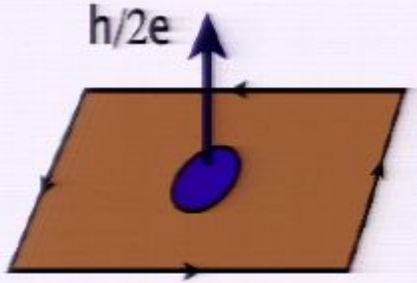

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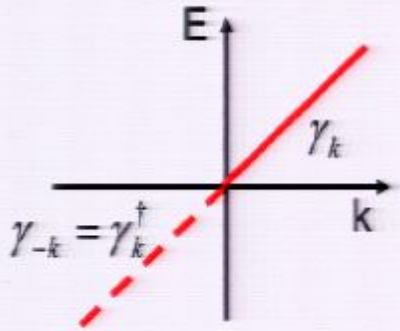

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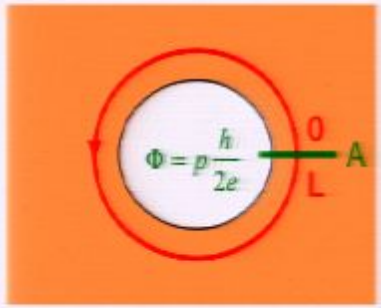

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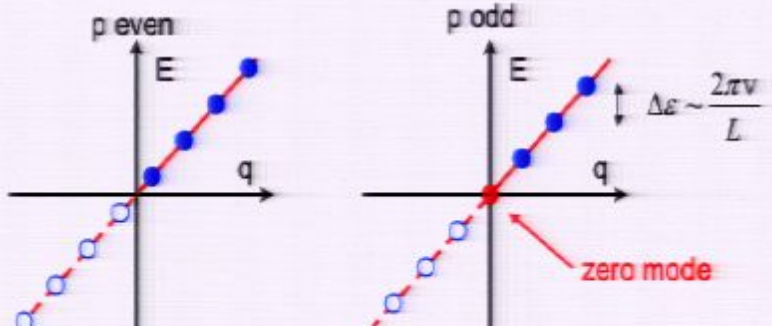

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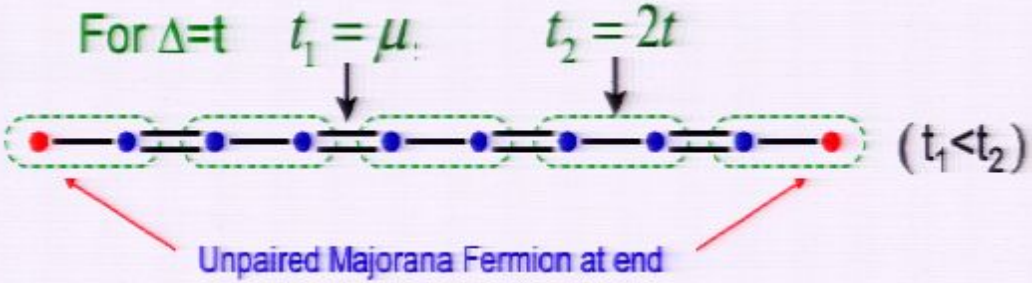

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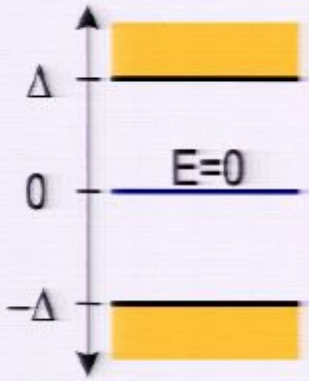

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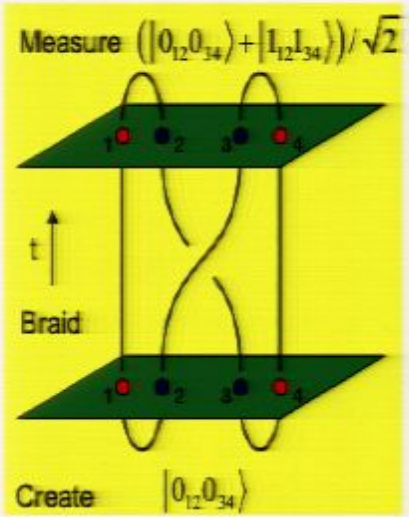

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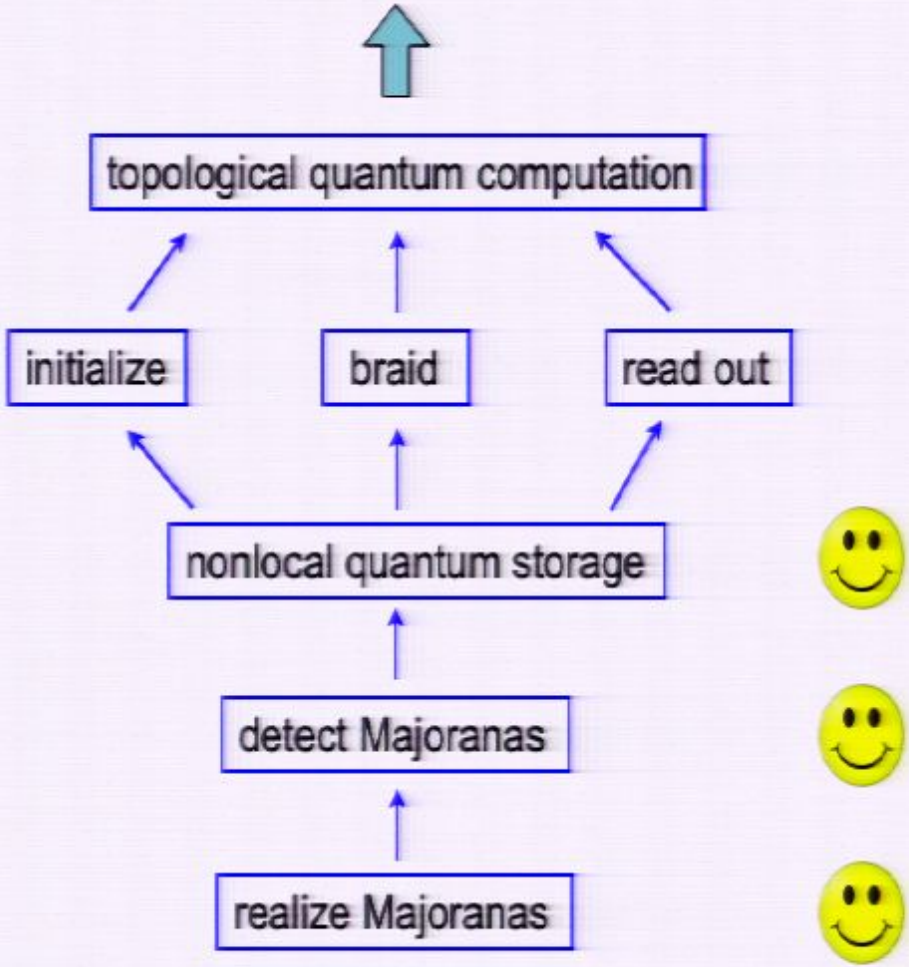


Potential Solid-State Hosts for Majorana Fermions

From models to reality:

Challenge: avoid fermion doubling to achieve pairing with the same species

Counter-example: singlet SC $\gamma_\uparrow^\dagger = uc_\uparrow^\dagger + vc_\downarrow$ 

- 5/2 quantum Hall state: (Moore-Read)
- Unconventional Superconductor/Superfluid:
  1. SrRuO
  2. fermion superfluid in cold atoms
- Proximity Effect Devices using Ordinary s-wave superconductor
  1. Topological insulator (Fu & Kane, PRL 2008)

     a novel metal without fermion doubling
     + externally induce superconductivity

  2. Semiconductor/magnetic heterstructure (Das Sarma et al, 2009, Oreg et al 2010, Lee 2010...)

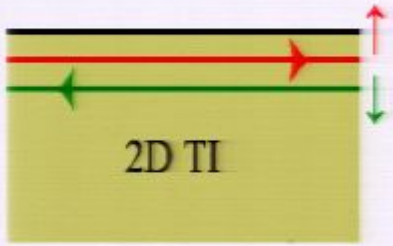
Topological Insulators

A new class of band insulators with protected surface states

Hasan & Kane, arXiv:1002.3895

- an odd number of spin non-degenerate Fermi surface

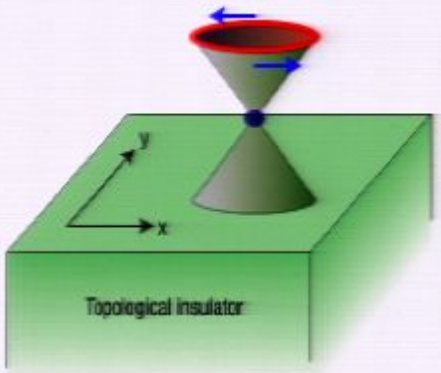
1. helical edge states



- observed in HgTe/CdTe

(Konig et al, 2007)

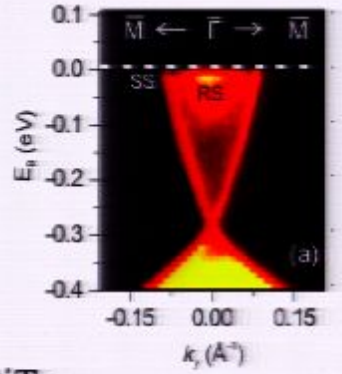
2. Dirac surface states



- observed in Bi-Sb, BiSe & BiTe...

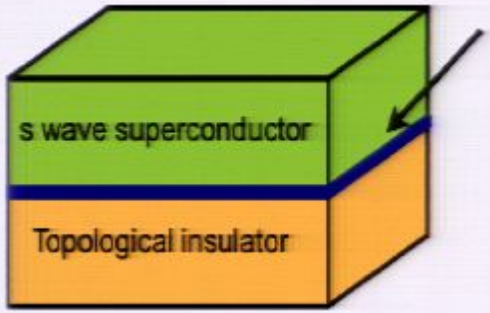
(Hsieh et al, 2008)

Bi_2Se_3



- surface/edge states:
  "half" of an ordinary metal; time-reversal-invariant
  bypass "fermion doubling" theorem

Superconducting Proximity Effect: Route to Majorana



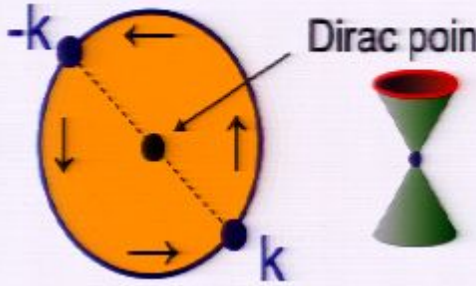
Proximity induced superconductivity at interface

$$H = \psi^\dagger(-iv\vec{\sigma}\cdot\vec{\nabla} - \mu)\psi + \Delta\psi_\uparrow^\dagger\psi_\downarrow^\dagger + \Delta^*\psi_\downarrow\psi_\uparrow$$

Dirac surface state s-wave pairing

projected to FS

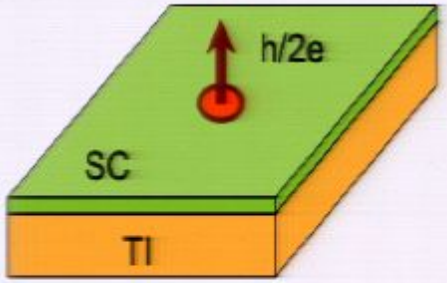
(Fu & Kane, PRL 08)



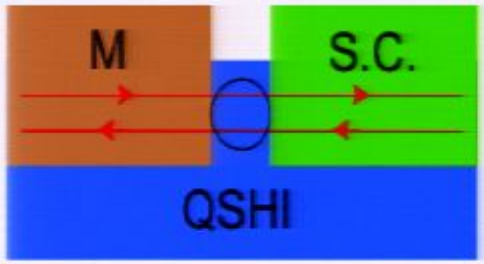
$$e^{i\theta_k}c_k^\dagger c_{-k}^\dagger + e^{-i\theta_k}c_k c_{-k}$$

"p-wave" pairing

- Majorana bound states at vortex and domain wall



Quasiparticle Bound state at E=0



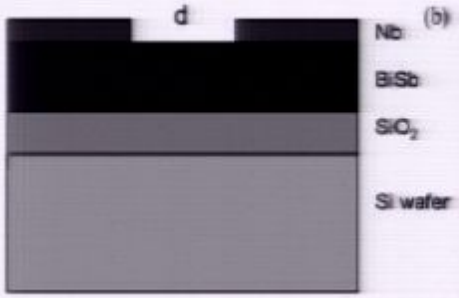
Domain wall bound state

Experimental Status

VOLUME 77, NUMBER 14 PHYSICAL REVIEW LETTERS 30 SEPTEMBER 1996

Anomalous Proximity Effect in the Nb-BiSb-Nb Junctions

A. Yu. Kasumov, O. V. Kononenko, V. N. Matveev, T. B. Borsenko, V. A. Tulin, E. E. Vdovin, and I. I. Khodos



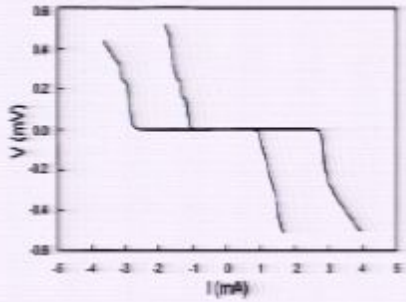


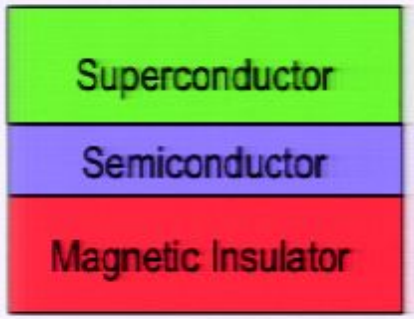
FIG. 2. *I-V* characteristics for the junction at 4.2 K. I_c = 1 mA for 0.6 μm slit junction; I_c = 2.5 mA for 1.2 μm slit junction.

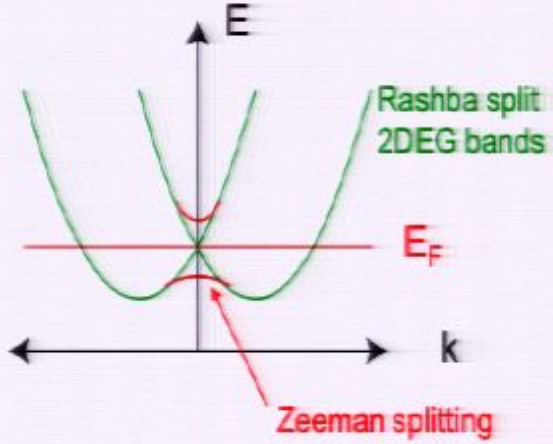
- supercurrent observed in Nb-BiSb junctions: evidence of good proximity effect
- Tc ~ 10K for Nb

Another Candidate System for Majorana

Semiconductor - Magnet - Superconductor structure

Sau, Lutchyn, Tewari, Das Sarma '09





- semiconductor with spin-orbit coupling: InAs

  2D inversion layer or 1D nanowire

- additional Zeeman field to avoid fermion doubling
- requires very low electron density → high purity

Race for Majorana Fermions !

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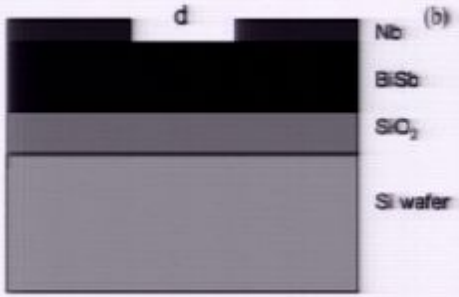

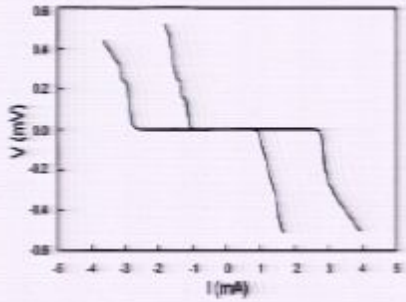

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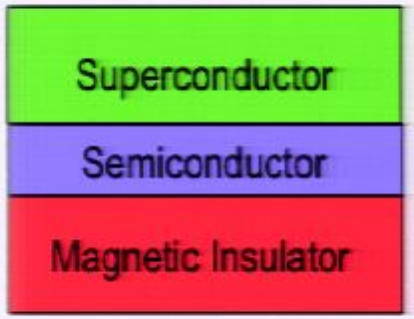

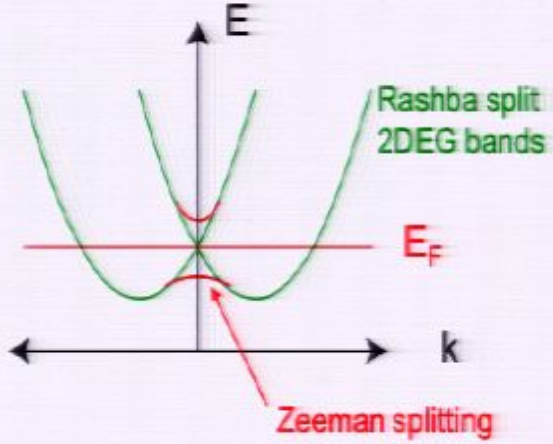

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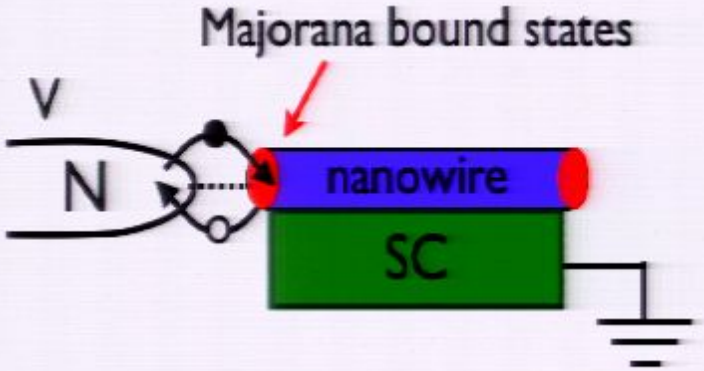
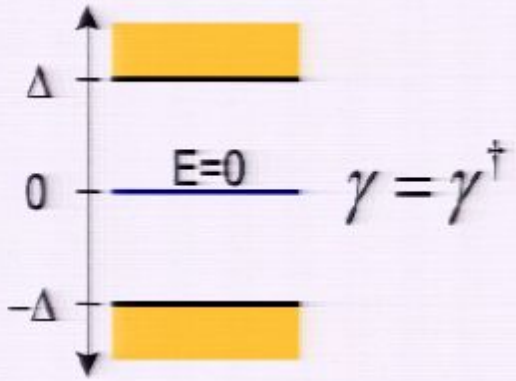
Detecting the *Presence* of Majorana Bound States





Zero-bias resonant Andreev reflection:

- tunneling Hamiltonian: $c^\dagger \sim e^{i\phi/2}\gamma,\ c \sim e^{-i\phi/2}\gamma$

  in a grounded superconductor, phase is a c-number.

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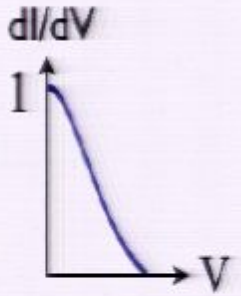

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Experimental Status

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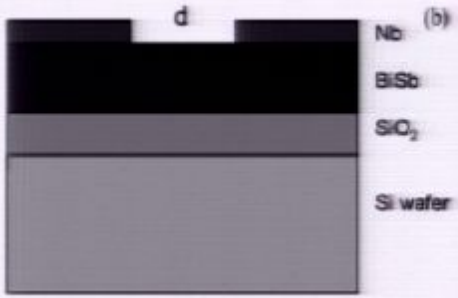

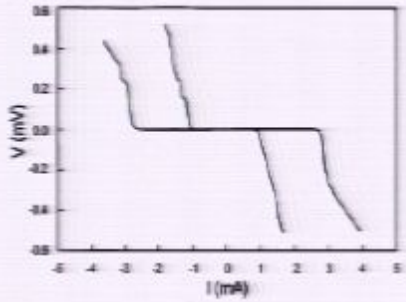

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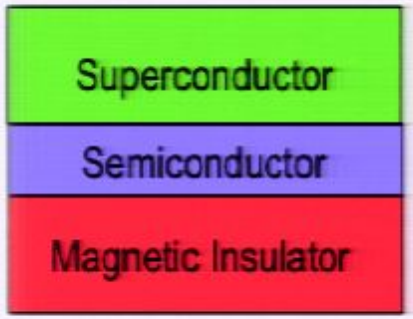

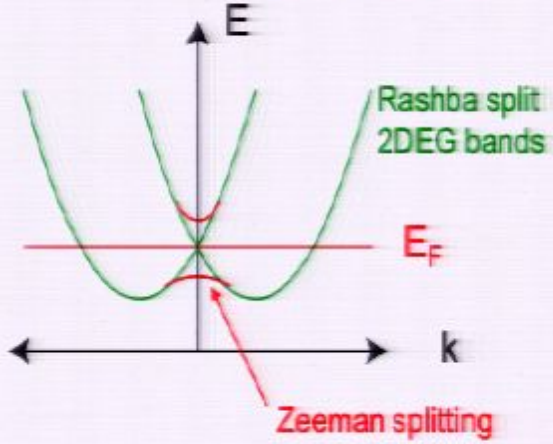

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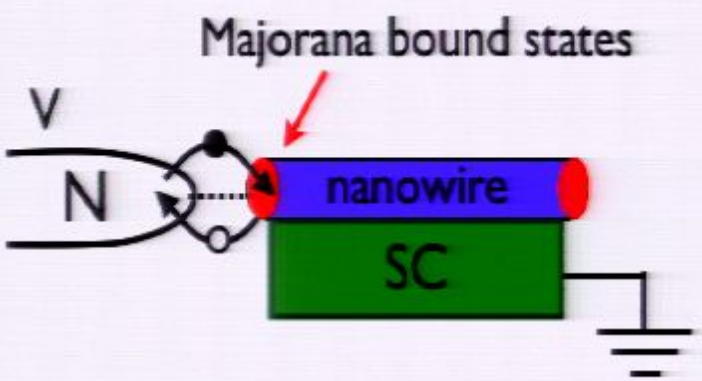

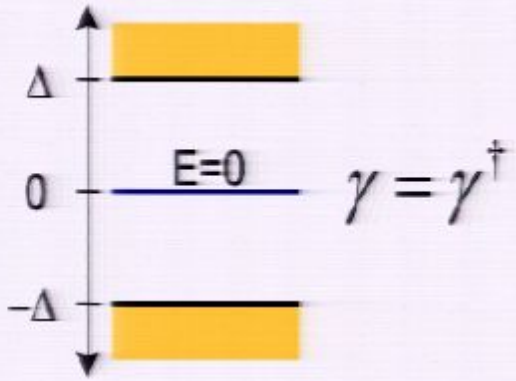

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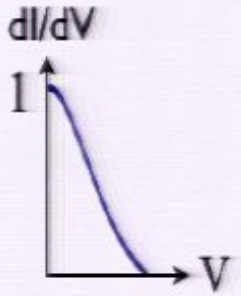

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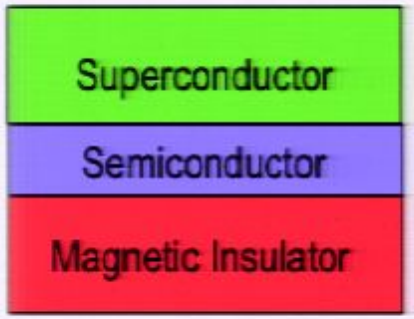

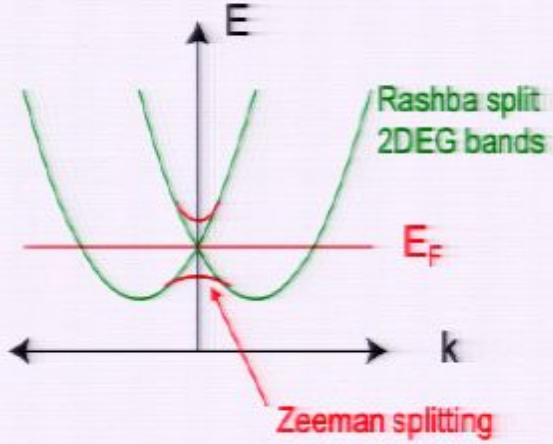

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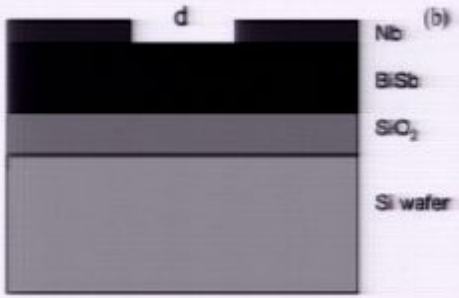

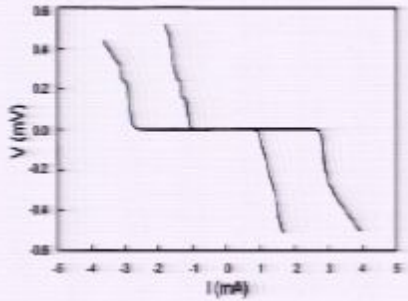

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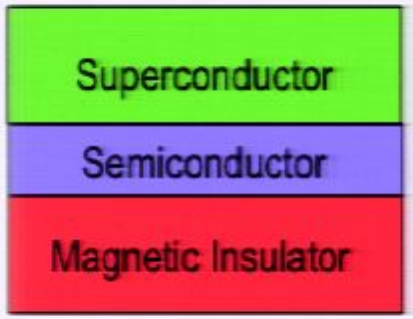

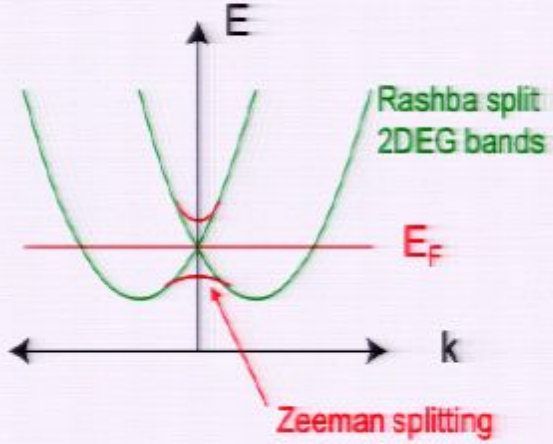

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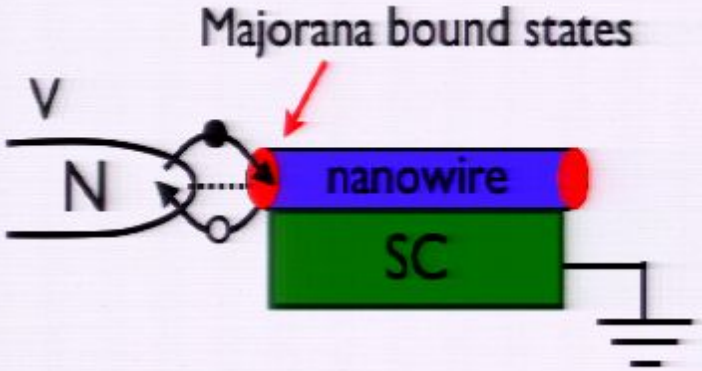

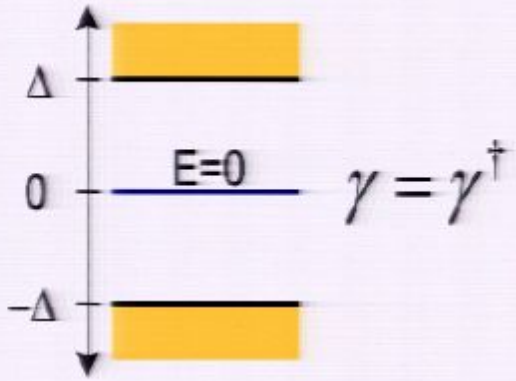

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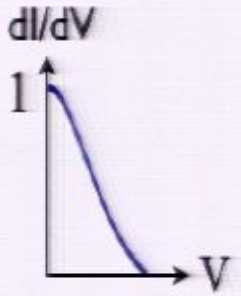

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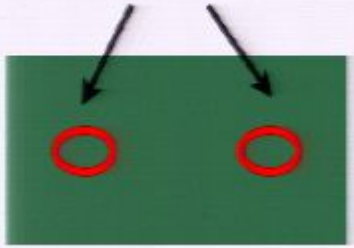

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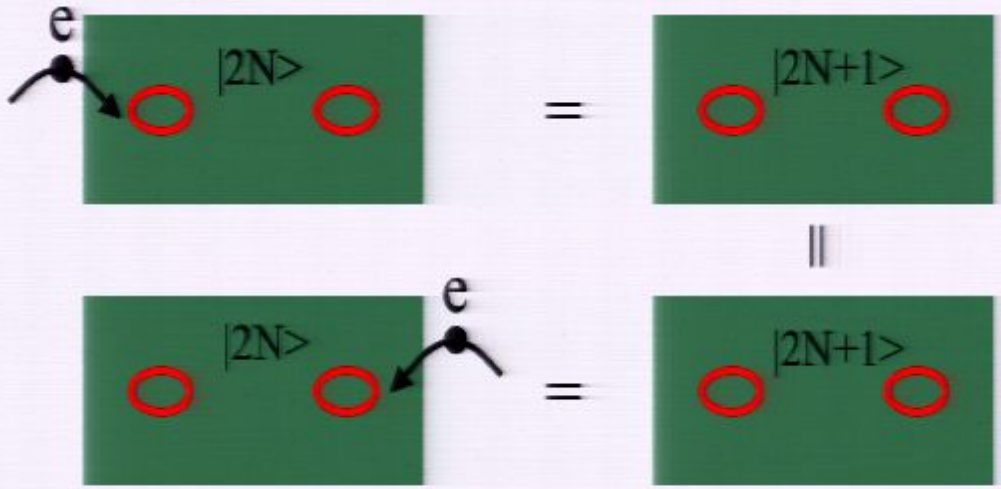

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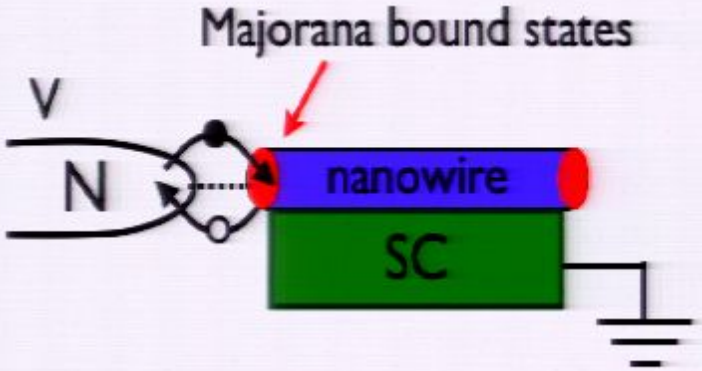

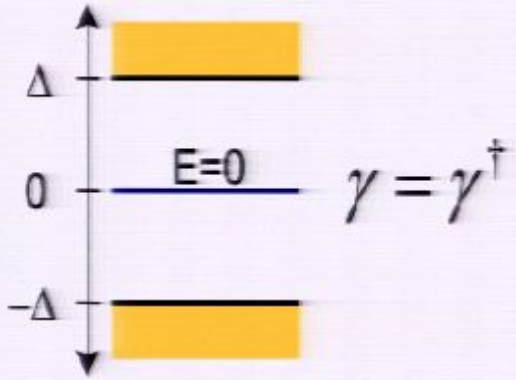

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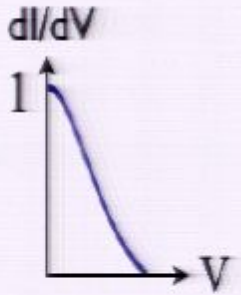

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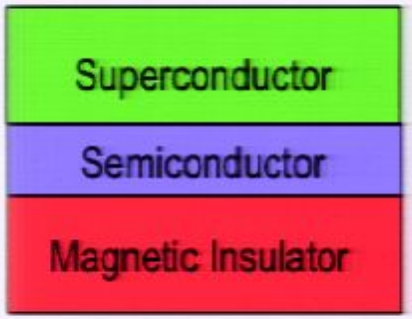

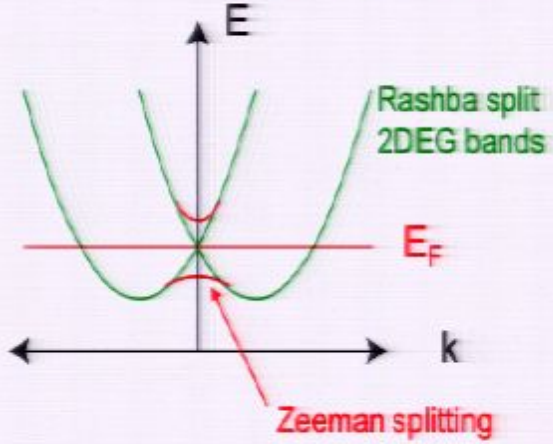

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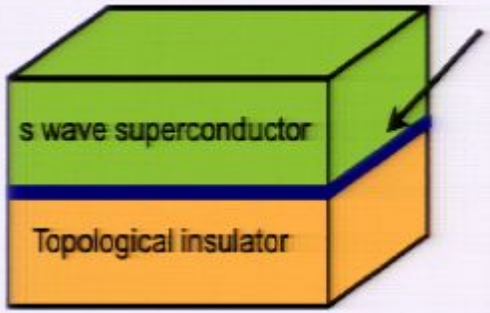
Superconducting Proximity Effect: Route to Majorana



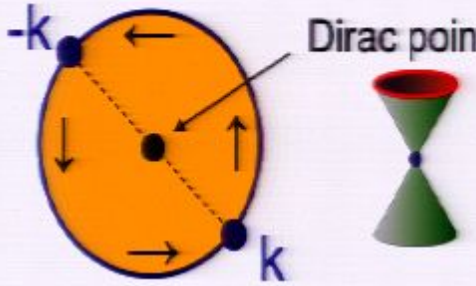
Proximity induced superconductivity at interface

$$H = \psi^\dagger(-iv\vec{\sigma}\cdot\vec{\nabla} - \mu)\psi + \Delta\psi_\uparrow^\dagger\psi_\downarrow^\dagger + \Delta^*\psi_\downarrow\psi_\uparrow$$

Dirac surface state s-wave pairing

projected to FS

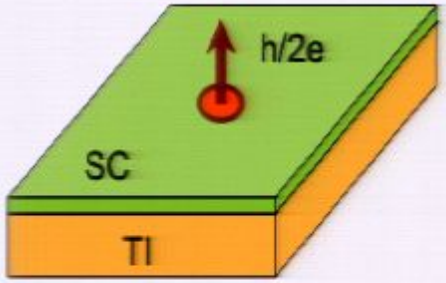
(Fu & Kane, PRL 08)



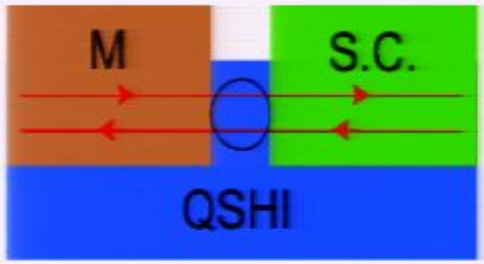
$$e^{i\theta_k}c_k^\dagger c_{-k}^\dagger + e^{-i\theta_k}c_k c_{-k}$$

"p-wave" pairing

- Majorana bound states at vortex and domain wall



Quasiparticle Bound state at E=0



Domain wall bound state

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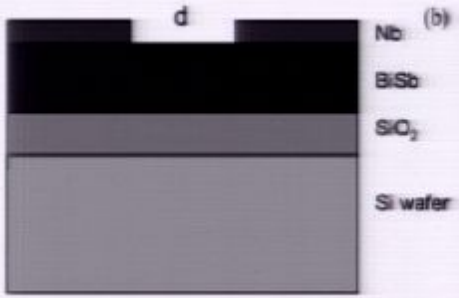

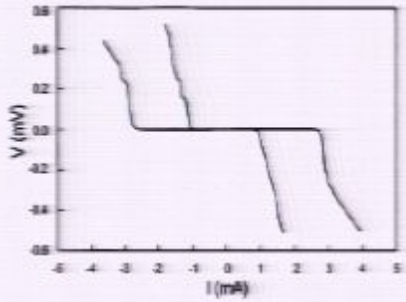

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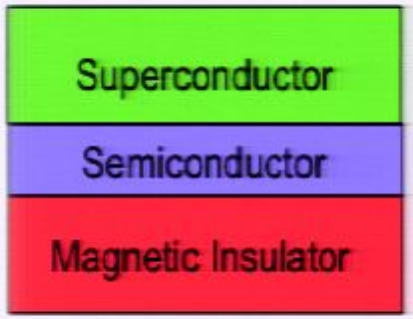

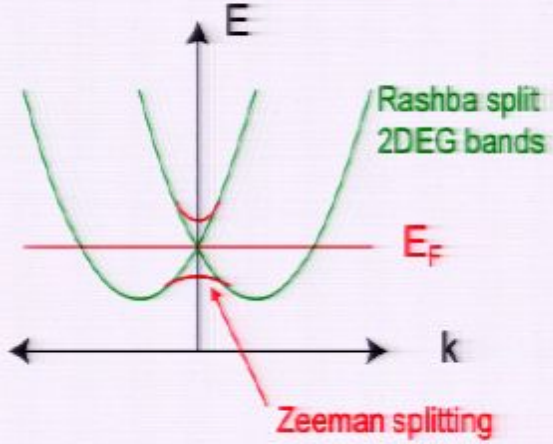

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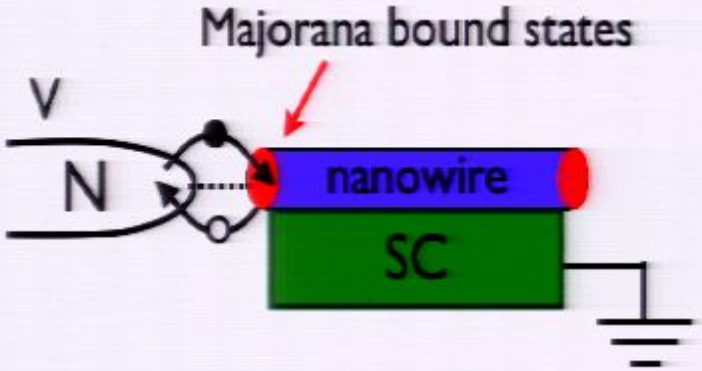

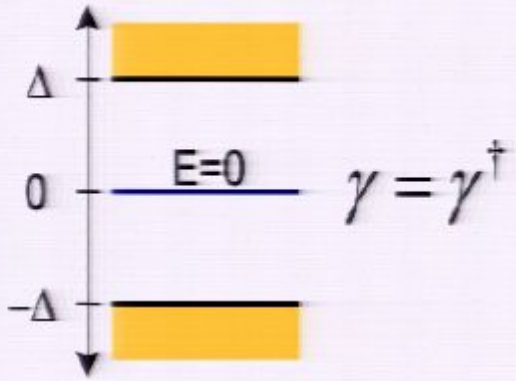

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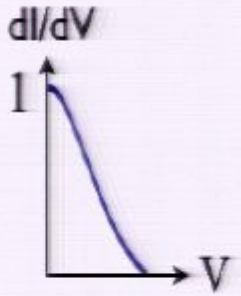

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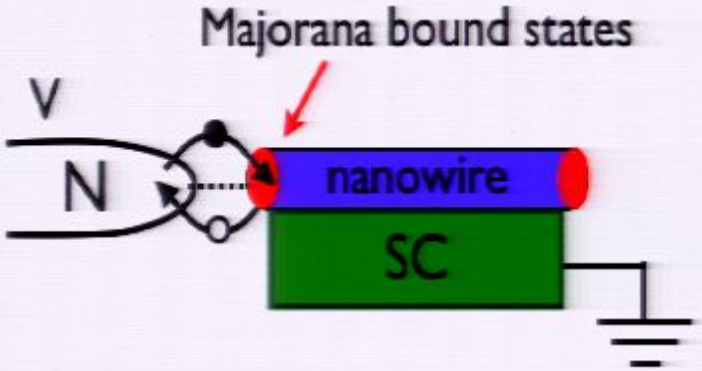

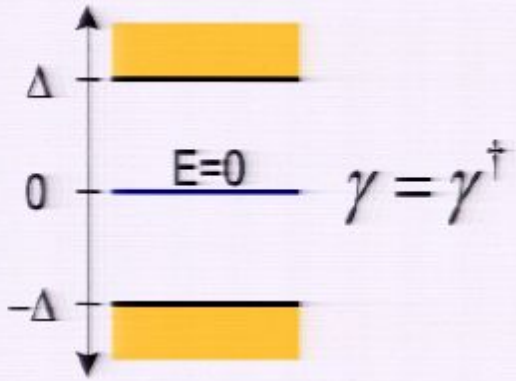

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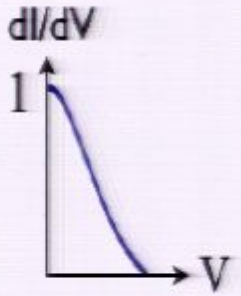

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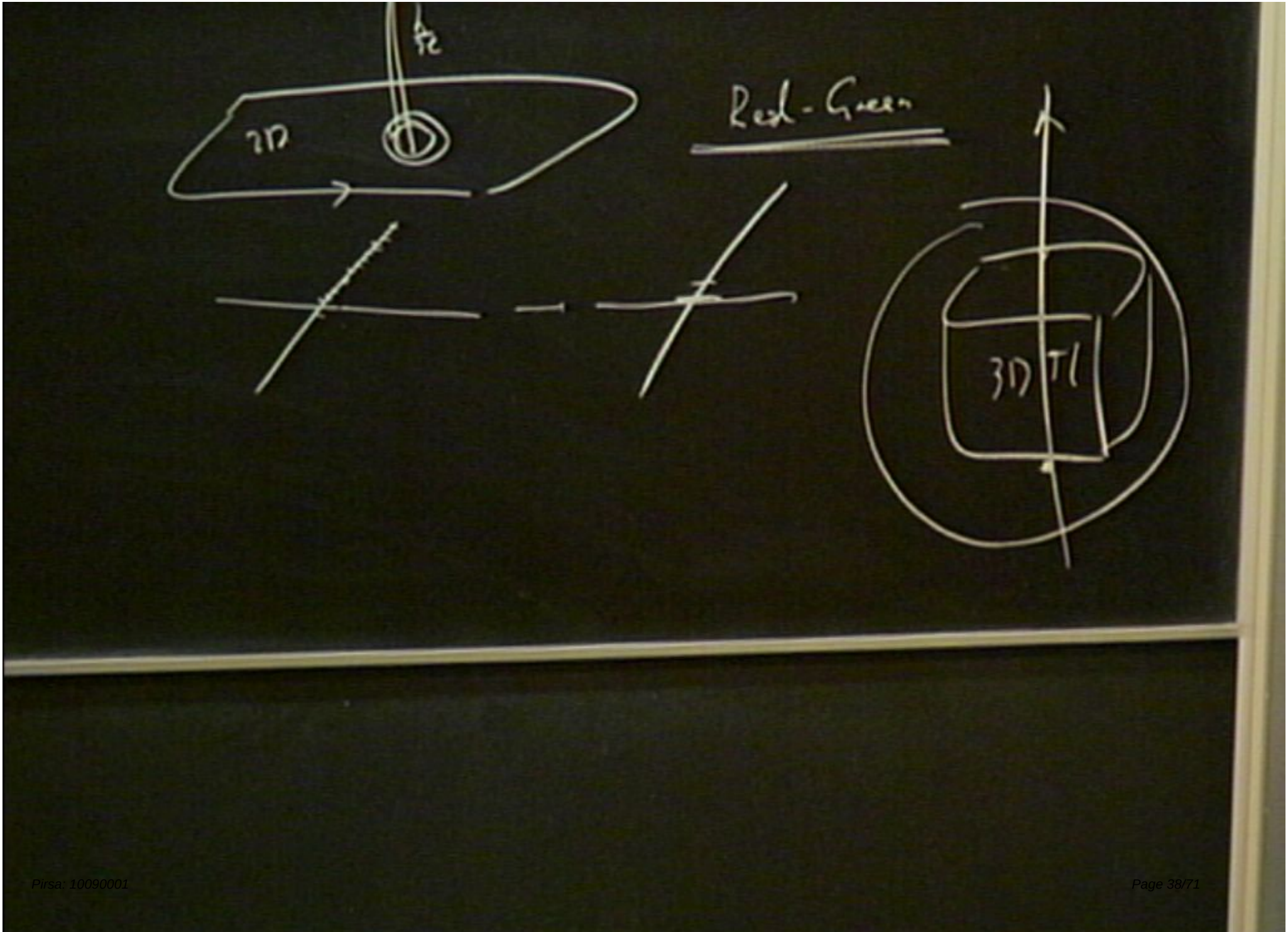
Red - Green

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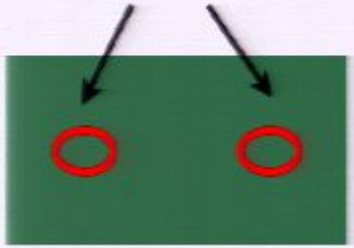

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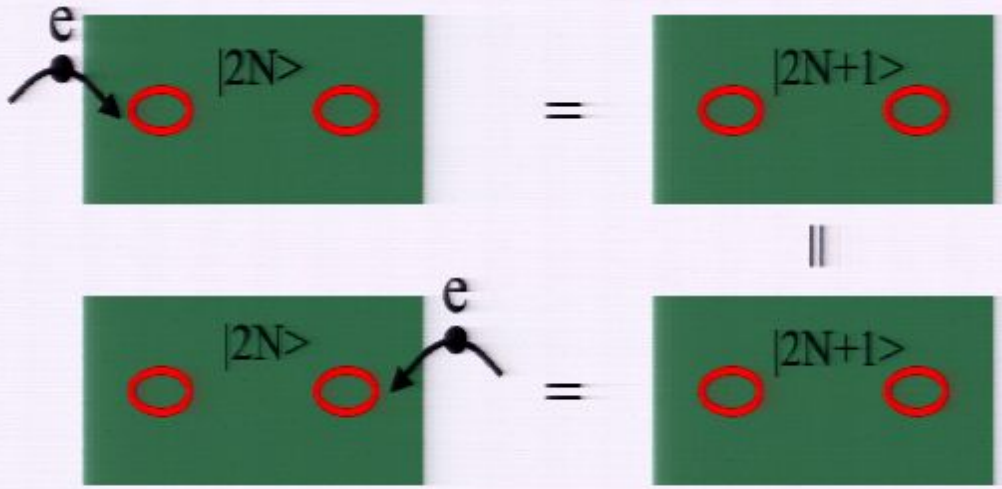
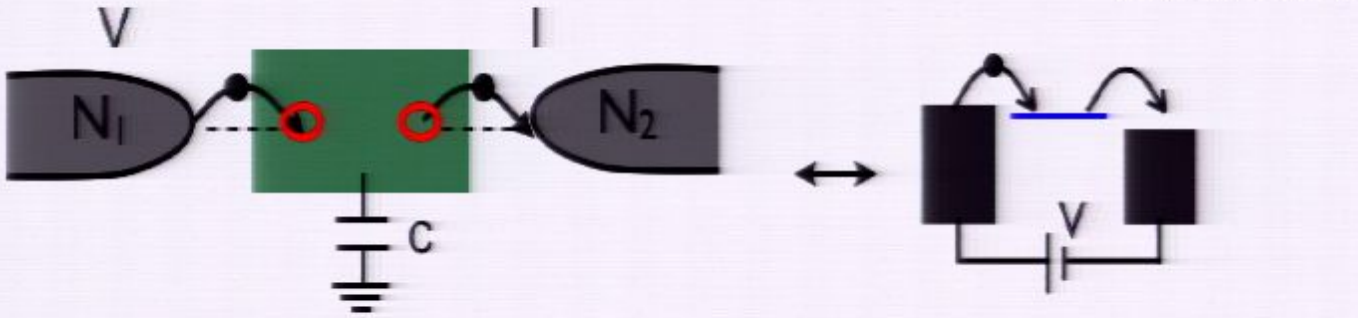


(Fu, PRL 2010)

Detecting the *Nonlocality*

"Teleportation" Experiment: nonlocal conductance through Majorana bound states in a *mesoscopic* superconductor.

(Fu, PRL 2010)



- with finite charging energy:

  - fluctuates between two charge states |2N> or |2N+1>

  - Andreev reflection suppressed; single electron tunneling

- Experimental signatures:

  - conductance peak vs. gate voltage; absence of even-odd effect

  - phase coherent transport

  - transmission phase shift independent of distance: "teleportation"

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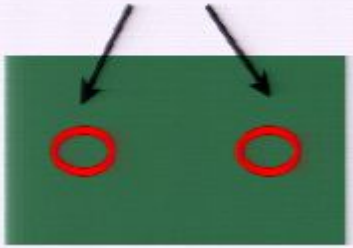
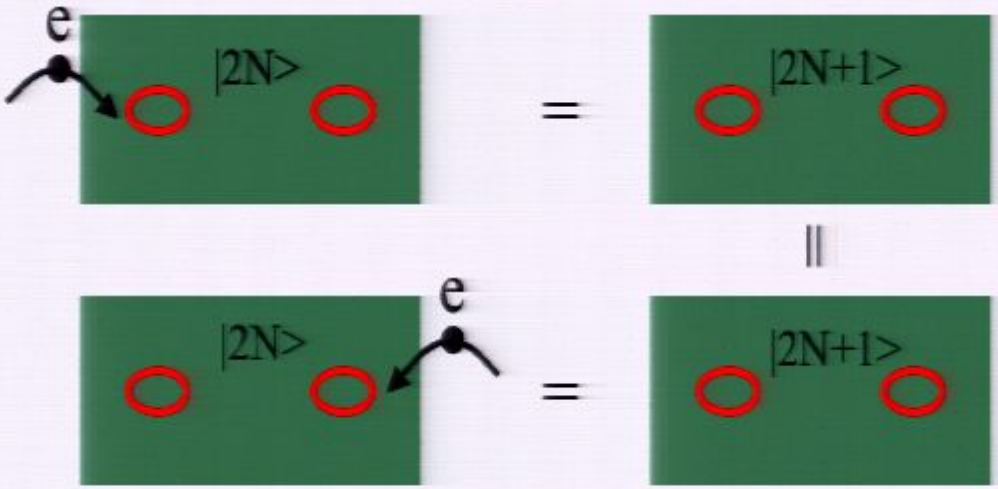
two-fold degeneracy:

$\Psi|0\rangle=0 \quad \Psi|1\rangle=|0\rangle$



- ground state can have either an even or odd # of electrons
- hidden entanglement of two spatially separated Majoranas $i\gamma_1\gamma_2 = (-1)^N$

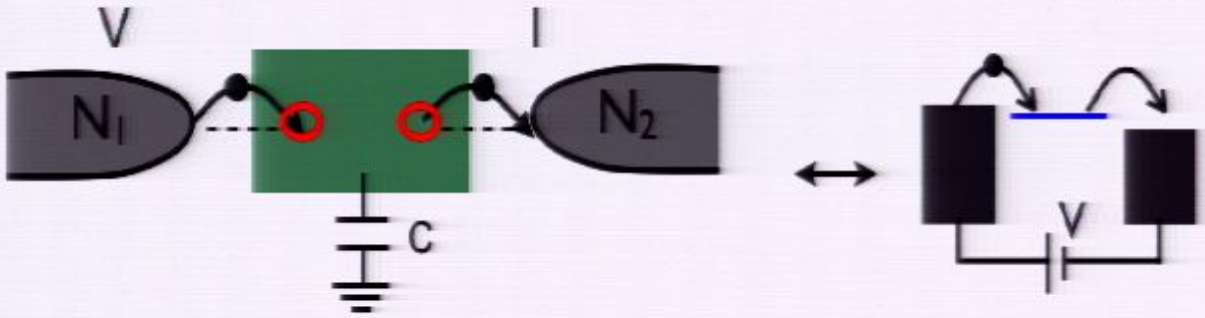


(Fu, PRL 2010)

Detecting the *Nonlocality*

"Teleportation" Experiment: nonlocal conductance through Majorana bound states in a *mesoscopic* superconductor.

(Fu, PRL 2010)



- with finite charging energy:

  - fluctuates between two charge states $|2N\rangle$ or $|2N+1\rangle$

  - Andreev reflection suppressed; single electron tunneling

- Experimental signatures:

  - conductance peak vs. gate voltage; absence of even-odd effect

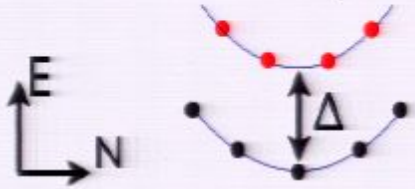
  - phase coherent transport

  - transmission phase shift independent of distance: "teleportation"

Theory of Teleportation via Majorana Fermions

Energy spectrum of SC with charging energy: $\frac{(N-N_0)^2 e^2}{2C}$

1. without Majoranas



2. with a pair of Majoranas

Hilbert space: $|N=2n\rangle \otimes |0\rangle_{12}$ & $|N=2n+1\rangle \otimes |1\rangle_{12}$ $\quad i\gamma_1\gamma_2 = (-1)^N$

Tunneling Hamiltonian: $H_T = \sum_{j=1,2} \left[\lambda_j c_j^\dagger(R_j)\gamma_j e^{-i\phi/2} + \lambda_j^* \gamma_j c_j(R_j) e^{i\phi/2}\right]$

- Two-level approximation: $e^{-i\phi/2} \longrightarrow s^+$
- Exact mapping to resonant tunneling:

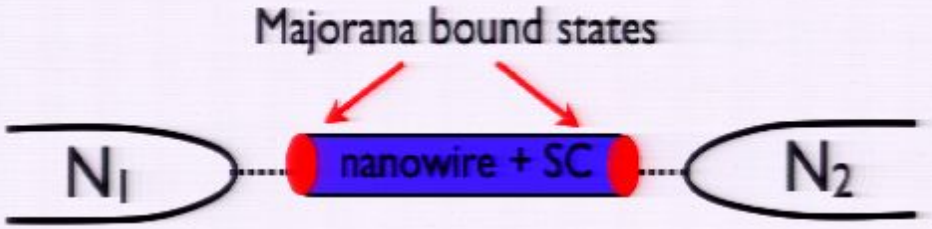
$$\gamma_1 s^+ \to f^+, \quad \gamma_1 s^- \to f$$
$$\gamma_2 s^+ \to i(-1)^{n_0} f^+, \quad \gamma_2 s^- \to i(-1)^{n_0+1} f.$$

$$H_T = \lambda_1(c_1^\dagger f + h.c.) + \lambda_2(c_2^\dagger f + h.c.)$$

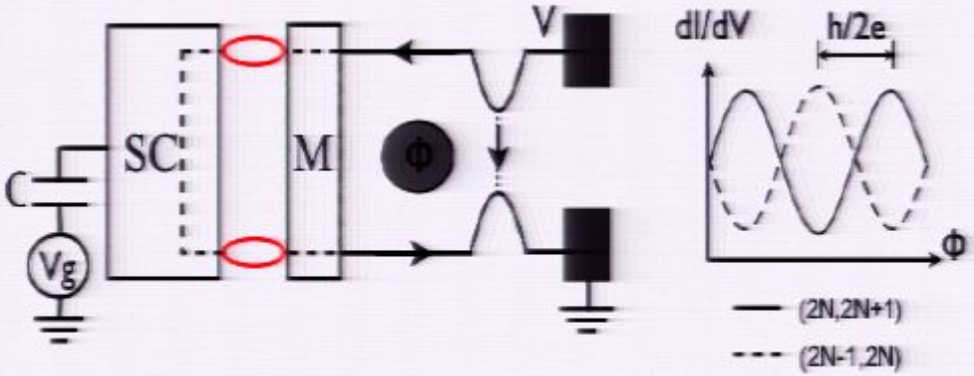
- Phase shift changes by π when N_0 changes by one: detect the qubit

Experimental Setups

1. Conductance measurement:



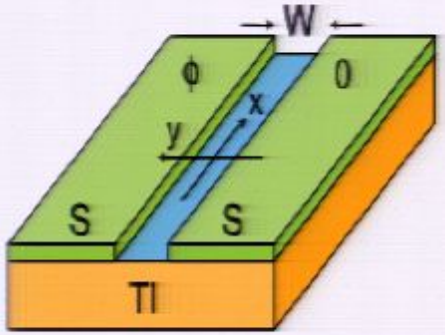
2. Phase measurement:

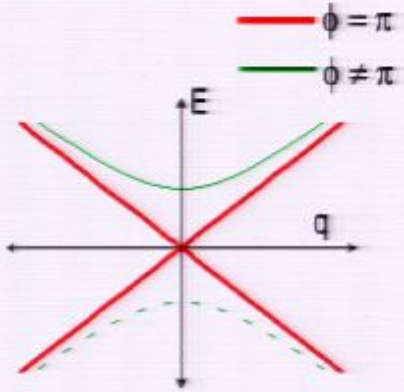


- Majorana bound states in 2D topological insulators
- edge states are used as leads
- interference measures the phase shift

Braiding Majorana Fermions in TI/SC Interface

- S-TI-S line junction

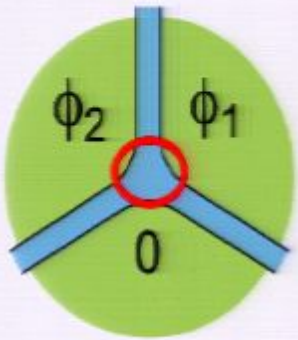


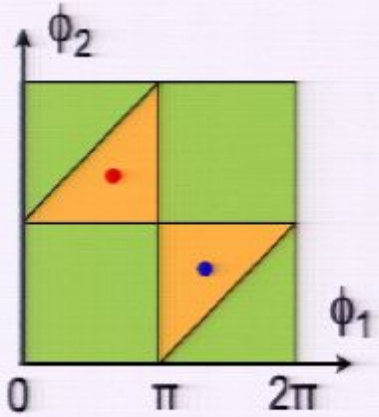


Fu & Kane, PRL 08

"Majorana wire": gapless at phase difference π

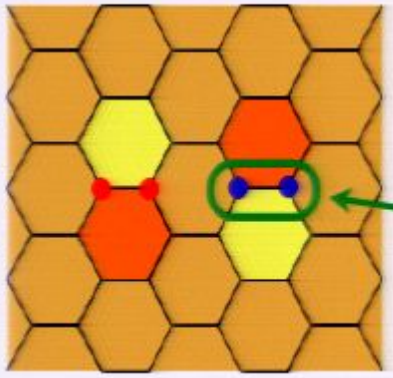
- Tri-junction



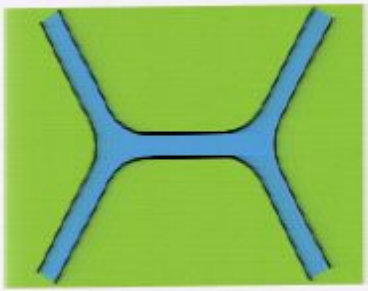


Storage register for Majorana

Superconductor Arrays on Topological Insulators

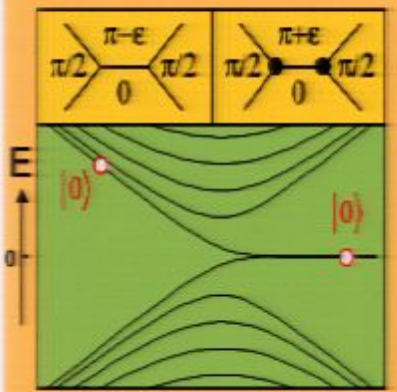


Control phases of Josephson junction



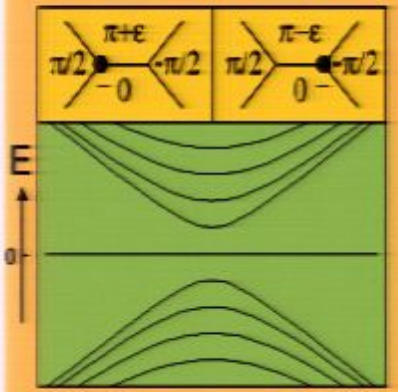
Create

A pair of Majorana bound states can be created from the vacuum in a well defined state |0>.



Braid

A single Majorana can be moved between junctions. Allows braiding of multiple Majoranas

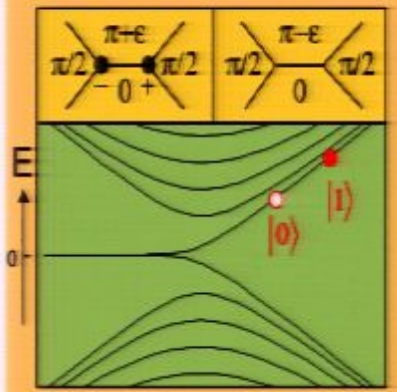


Measure

Fuse a pair of Majoranas.

States |0,1> distinguished by

- presence of quasiparticle.
- supercurrent across line junction



Probing Non-Abelian Statistics

Platform for topological quantum computation?

Probing Non-Abelian Statistics

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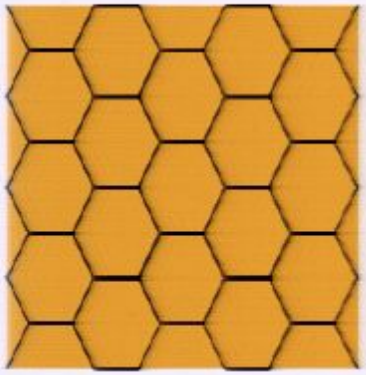
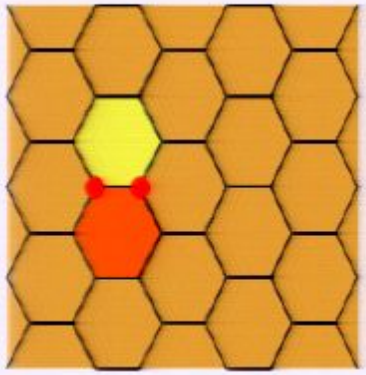
Probing Non-Abelian Statistics

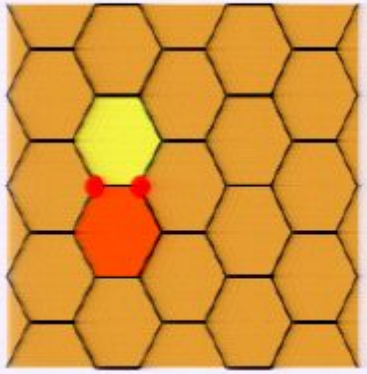


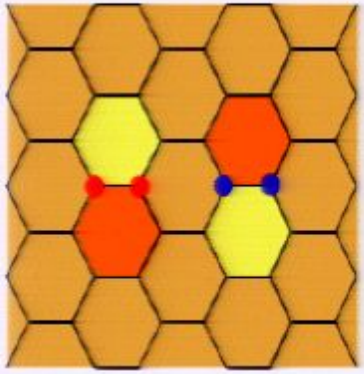


phase ϕ

Platform for topological quantum computation?

Probing Non-Abelian Statistics



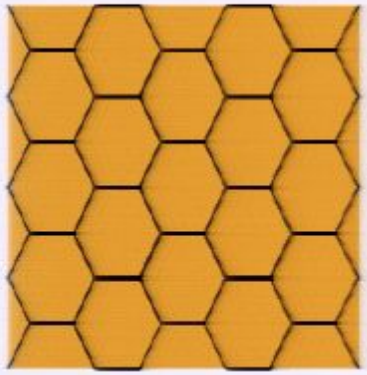


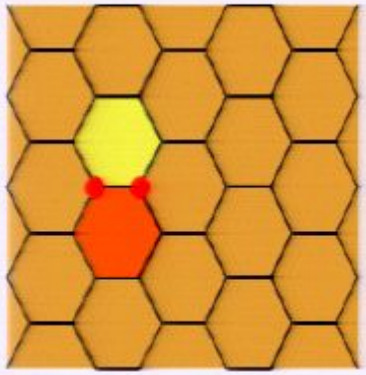
$|0\rangle \otimes |0\rangle$

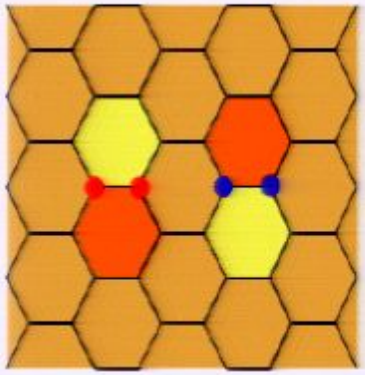
phase ϕ

Platform for topological quantum computation?

Probing Non-Abelian Statistics







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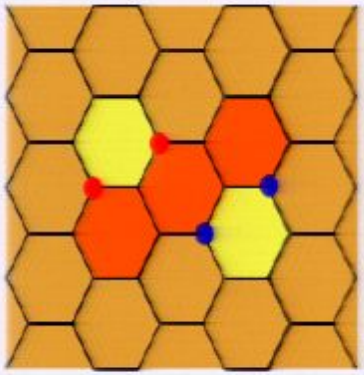
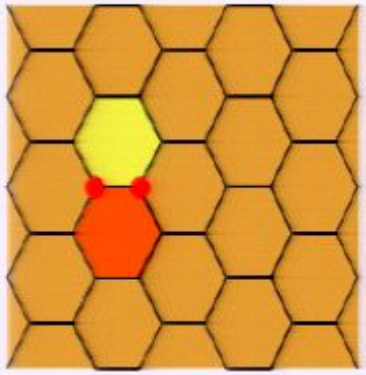
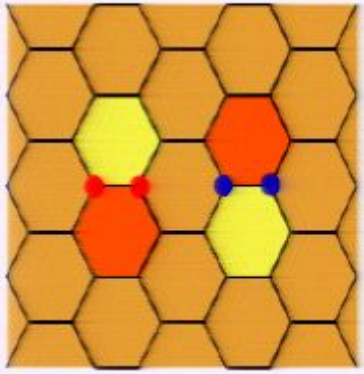


phase ϕ

Platform for topological quantum computation?

Probing Non-Abelian Statistics





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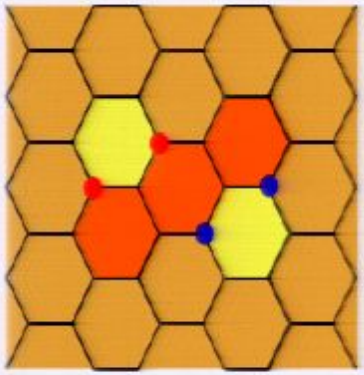
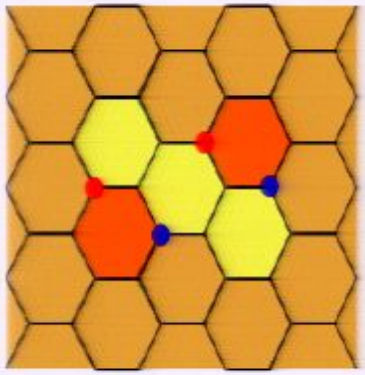
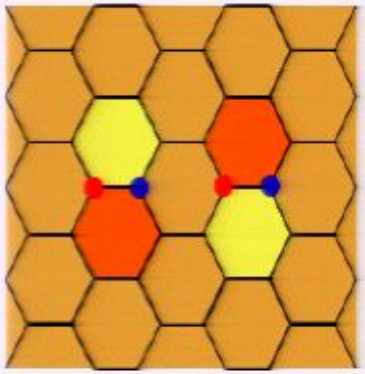




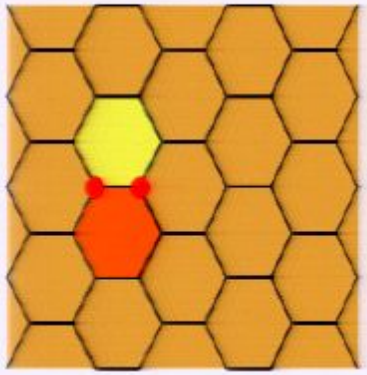


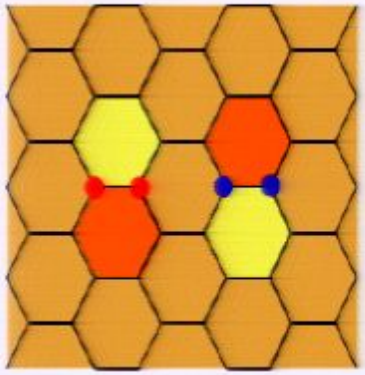
$(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle)/\sqrt{2}$

phase ϕ

Platform for topological quantum computation?

Probing Non-Abelian Statistics





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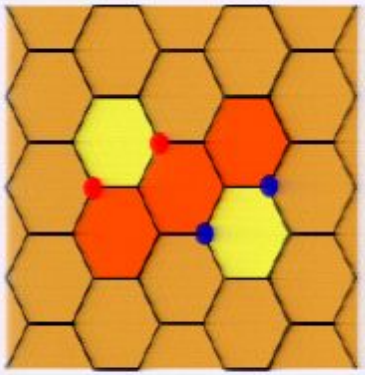

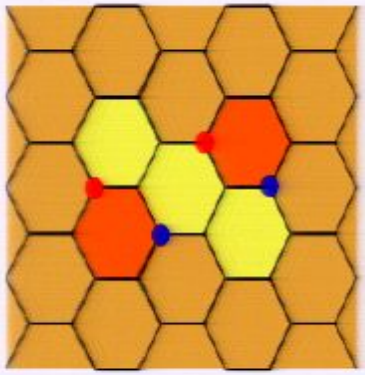

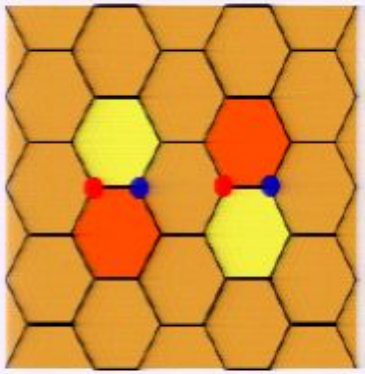

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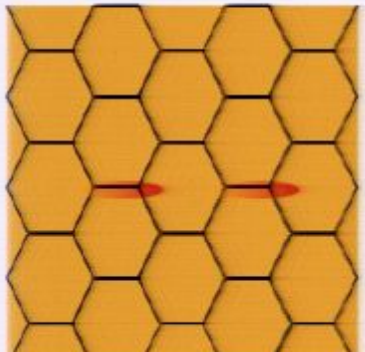


Platform for topological quantum computation?

Conclusion

Take-home message:

1. Majorana fermion is the simplest non-Abelian quasi-particle, and likely the most promising one to be realized and detected in engineered solid state systems in the future.

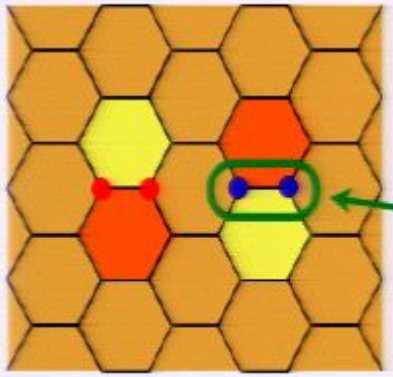
2. By measuring current and voltage, one can learn something fundamental about nature in the search for Majorana fermion.

Thank you !

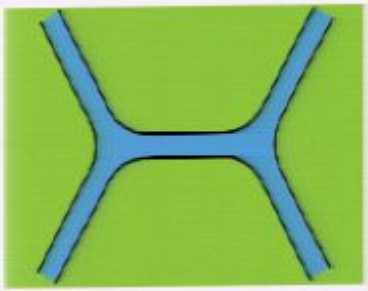
Probing Non-Abelian Statistics

Platform for topological quantum computation?

Superconductor Arrays on Topological Insulators



Control phases of Josephson junction



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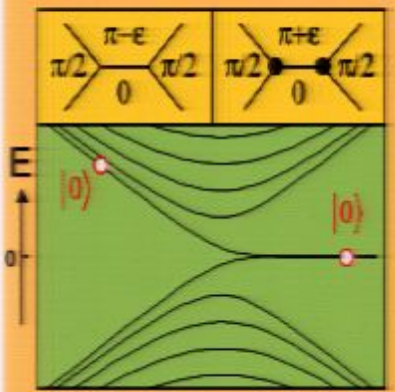

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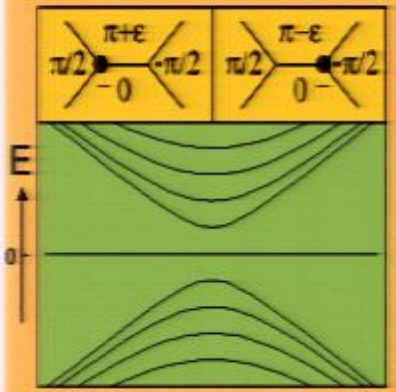

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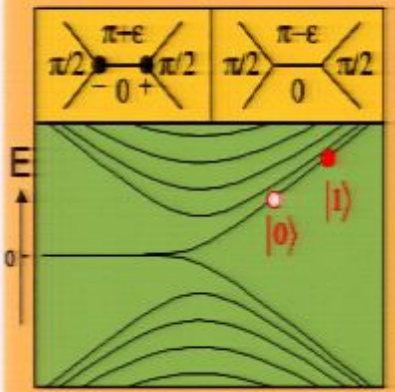
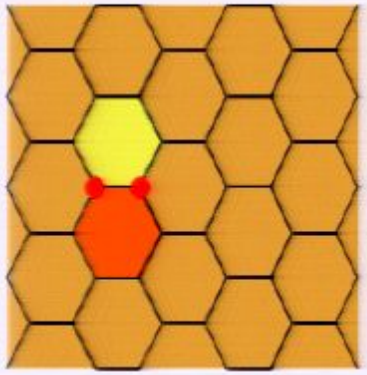
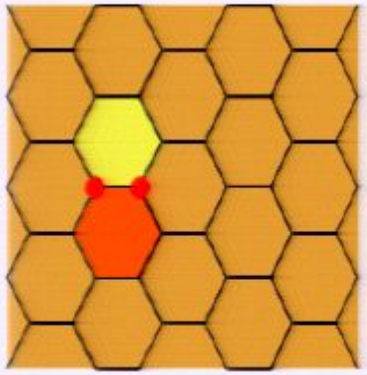


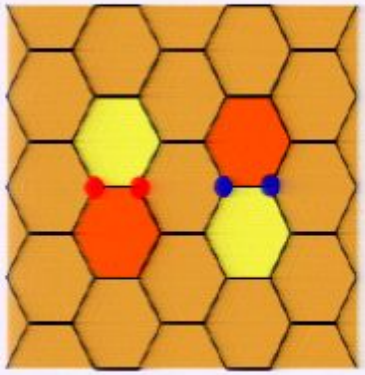
Probing Non-Abelian Statistics



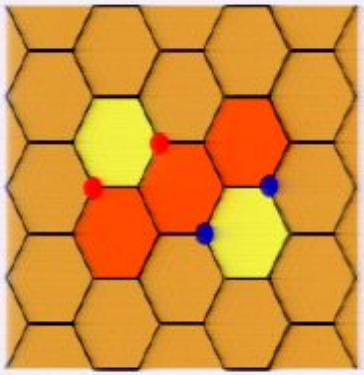
Platform for topological quantum computation?

Probing Non-Abelian Statistics



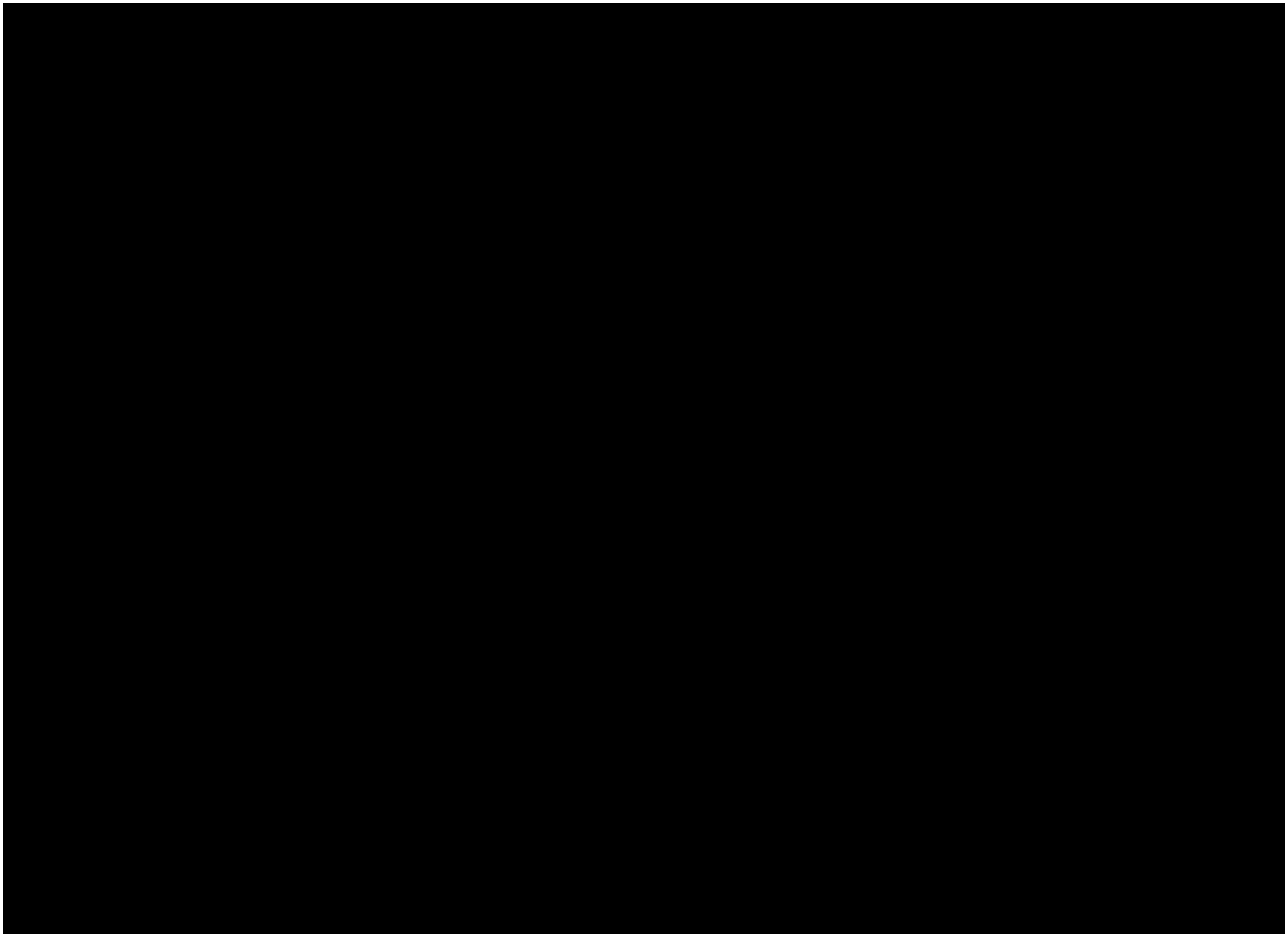


$|0\rangle \otimes |0\rangle$



phase ϕ

Platform for topological quantum computation?



Kitaev model for 1D p-wave superconductor

1D spinless fermion: $H-\mu N=\sum_i t(c_i^\dagger c_{i+1}+c_{i+1}^\dagger c_i)-\mu c_i^\dagger c_i+\Delta(c_i c_{i+1}+c_{i+1}^\dagger c_i^\dagger)$

$c_i \to \gamma_{1i}+i\gamma_{2i}$

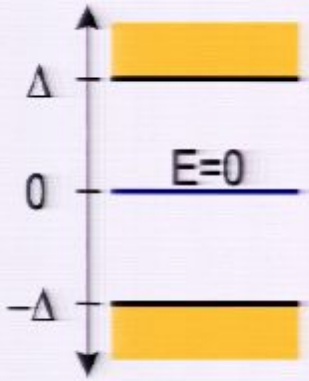
Majorana chain:

For Δ=t $t_1=\mu$ $t_2=2t$

$(t_1<t_2)$

Unpaired Majorana Fermion at end

BdG spectrum



$$H=(c^\dagger,c)H_{BdG}\begin{pmatrix} c \\ c^\dagger \end{pmatrix}=\sum_{E_m>0} E_m\Gamma_m^\dagger\Gamma_m$$

$\Gamma_{E=0}^\dagger=\Gamma_{E=0}\equiv\gamma$: a zero energy Majorana bound state

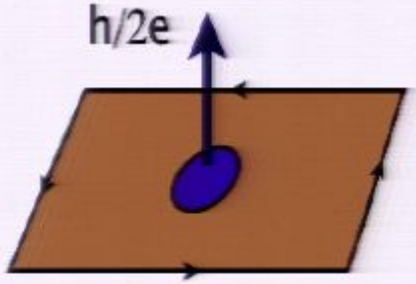
$$\gamma = \int d^2\mathbf{r}u^*(\mathbf{r})e^{-i\phi/2}c^\dagger(\mathbf{r})+u(\mathbf{r})e^{i\phi/2}c(\mathbf{r})=\gamma^\dagger$$

- ground state degeneracy encoded in $\Psi=\gamma_1+i\gamma_2$

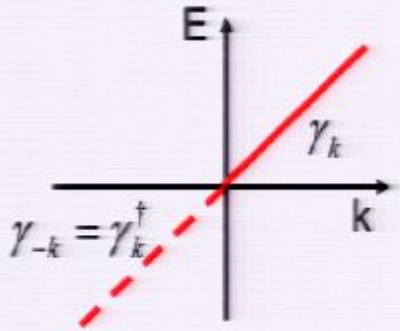
Read-Green model for 2D p-wave superconductor

2D spinless $p_x + ip_y$ superconductor

- BdG Hamiltonian is analogous to quantum Hall state



1. chiral Majorana edge state

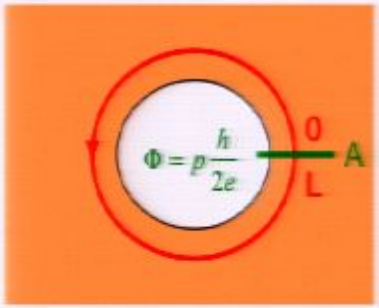


- half of a 1D chiral electron

$$H = -i\hbar v_F \gamma \partial_x \gamma$$

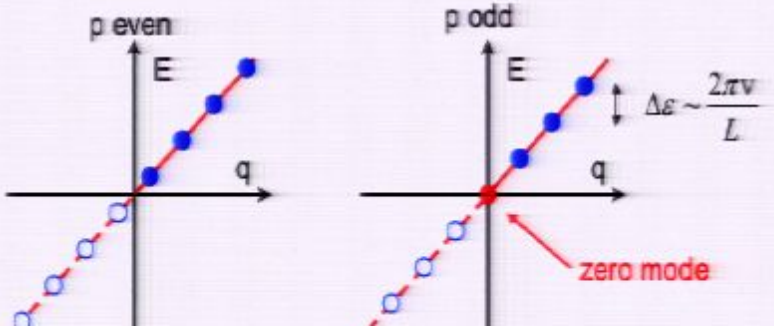
2. Majorana bound state at the vortex

(Ising anyon)



Boundary condition

$$\psi(L) = (-1)^{p+1}\psi(0)$$

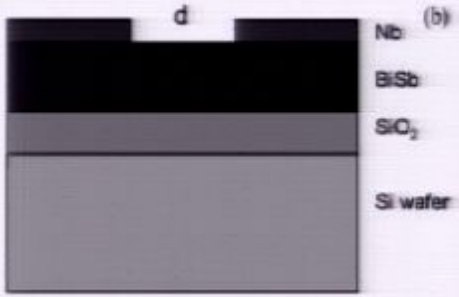


Experimental Status

VOLUME 77, NUMBER 14 PHYSICAL REVIEW LETTERS 30 SEPTEMBER 1996

Anomalous Proximity Effect in the Nb-BiSb-Nb Junctions

A. Yu. Kasumov, O. V. Kononenko, V. N. Matveev, T. B. Borsenko, V. A. Tulin, E. E. Vdovin, and I. I. Khodos



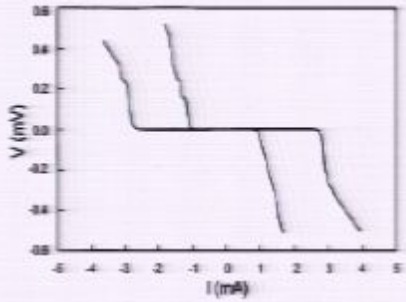


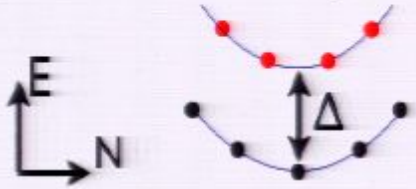
FIG. 2. *I-V* characteristics for the junction at 4.2 K. I_c = 1 mA for 0.6 μm slit junction; I_c = 2.5 mA for 1.2 μm slit junction.

- supercurrent observed in Nb-BiSb junctions: evidence of good proximity effect
- Tc ~ 10K for Nb

Theory of Teleportation via Majorana Fermions

Energy spectrum of SC with charging energy: $\frac{(N-N_0)^2 e^2}{2C}$

1. without Majoranas



2. with a pair of Majoranas

Hilbert space: $|N=2n\rangle \otimes |0\rangle_{12}$ & $|N=2n+1\rangle \otimes |1\rangle_{12}$ $\quad i\gamma_1\gamma_2 = (-1)^N$

Tunneling Hamiltonian: $H_T = \sum_{j=1,2} \left[\lambda_j c_j^\dagger(R_j)\gamma_j e^{-i\phi/2} + \lambda_j^* \gamma_j c_j(R_j) e^{i\phi/2}\right]$

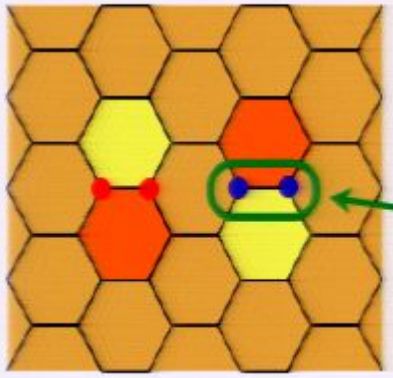
- Two-level approximation: $e^{-i\phi/2} \longrightarrow s^+$
- Exact mapping to resonant tunneling:

$$\gamma_1 s^+ \to f^+, \quad \gamma_1 s^- \to f$$
$$\gamma_2 s^+ \to i(-1)^{n_0} f^+, \quad \gamma_2 s^- \to i(-1)^{n_0+1} f$$

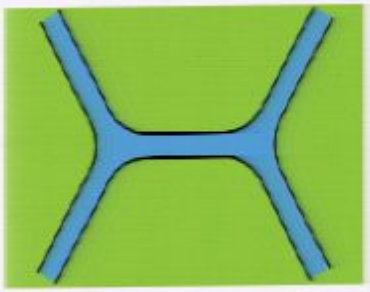
$$H_T = \lambda_1(c_1^\dagger f + h.c.) + \lambda_2(c_2^\dagger f + h.c.)$$

- Phase shift changes by π when N_0 changes by one: detect the qubit

Superconductor Arrays on Topological Insulators

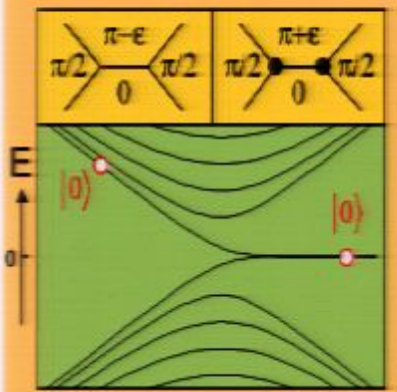


Control phases of Josephson junction



Create

A pair of Majorana bound states can be created from the vacuum in a well defined state |0>.



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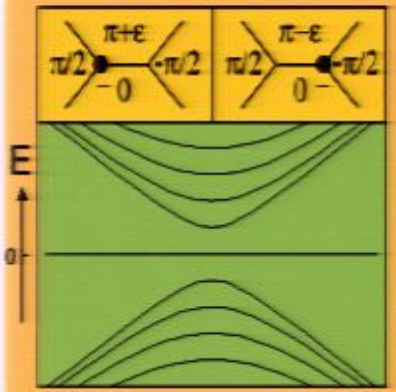

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States |0,1> distinguished by

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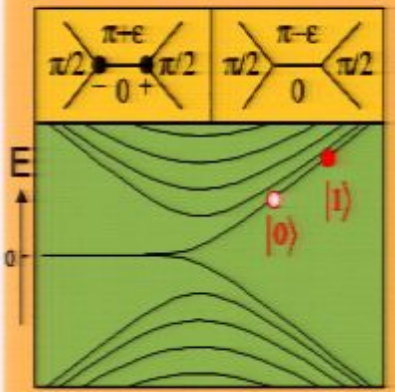
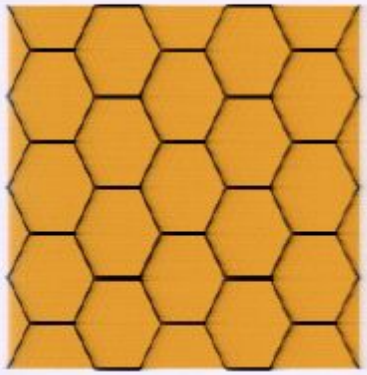
- supercurrent across line junction



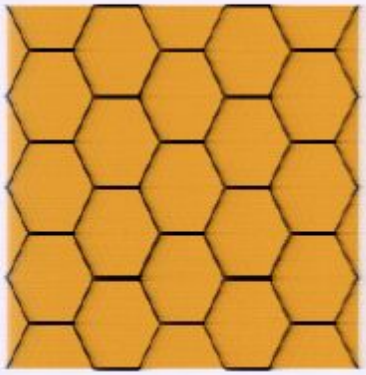
Probing Non-Abelian Statistics

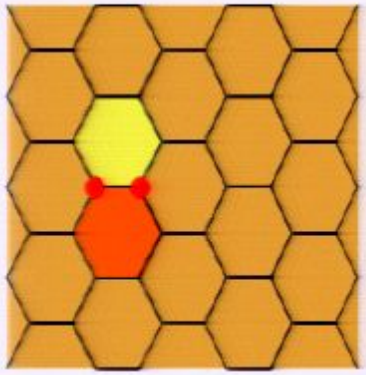


phase ϕ

Platform for topological quantum computation?

Probing Non-Abelian Statistics





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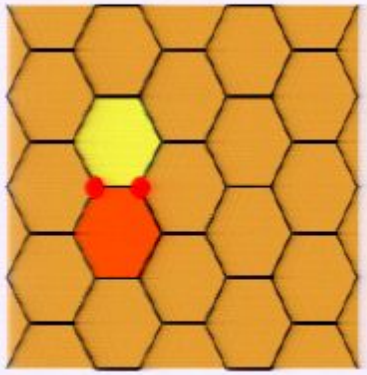
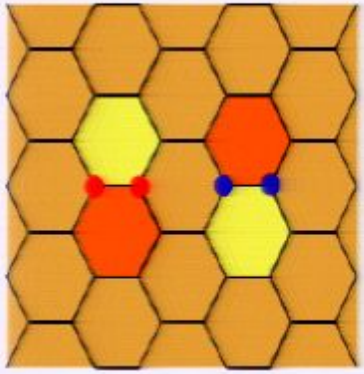
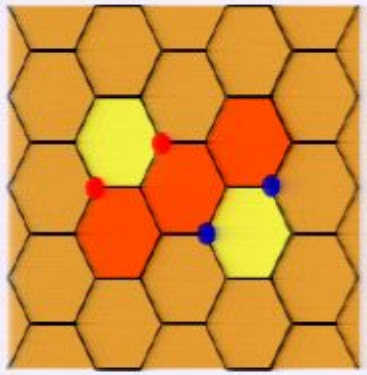




$|0\rangle \otimes |0\rangle$



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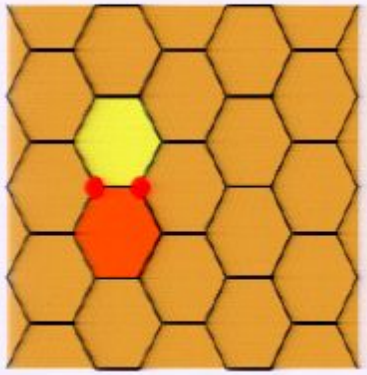

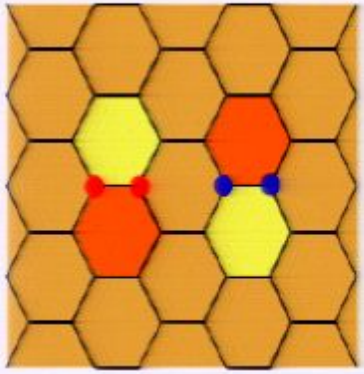

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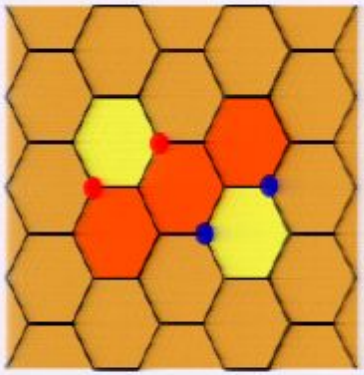

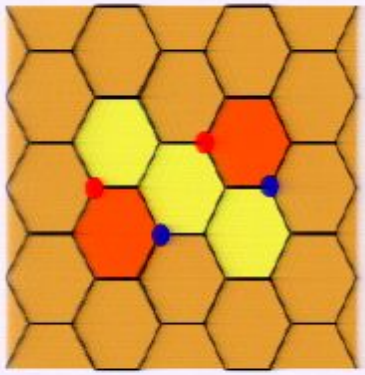

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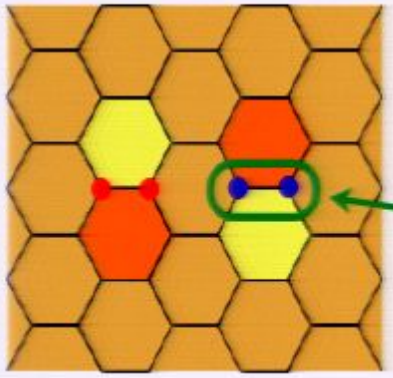

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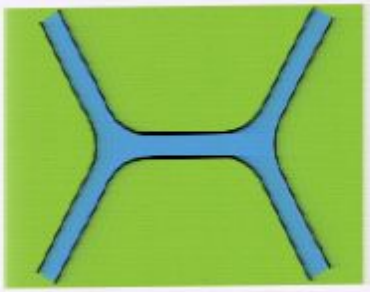

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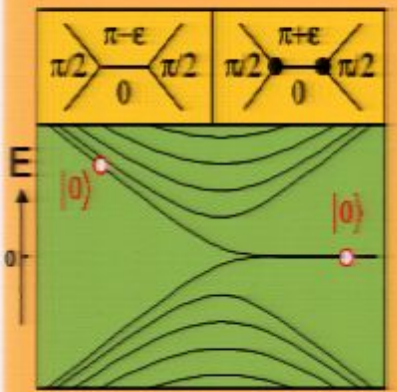

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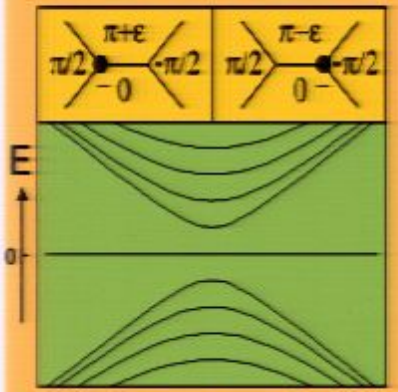

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