Title: Probability of slow roll inflation

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Abstract:

Probability of Slow-roll Inflation

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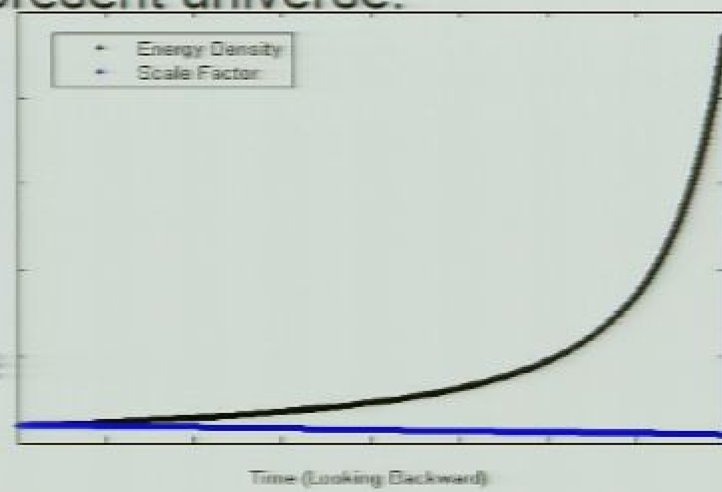
Slow-roll Inflation

- Inflationary cosmology successfully resolves the flatness and horizon problems of standard cosmology.
- Inflation also generates density perturbations in the early universe that could correspond to the origin of the large-scale structure observed in the present universe.

➢ Slow-roll inflationary paradigm does not however resolve the singularity problem of standard cosmology.

➢ Slow-roll inflation may also have a measure problem and thus like standard cosmology, may require a fine-tuning of initial conditions.



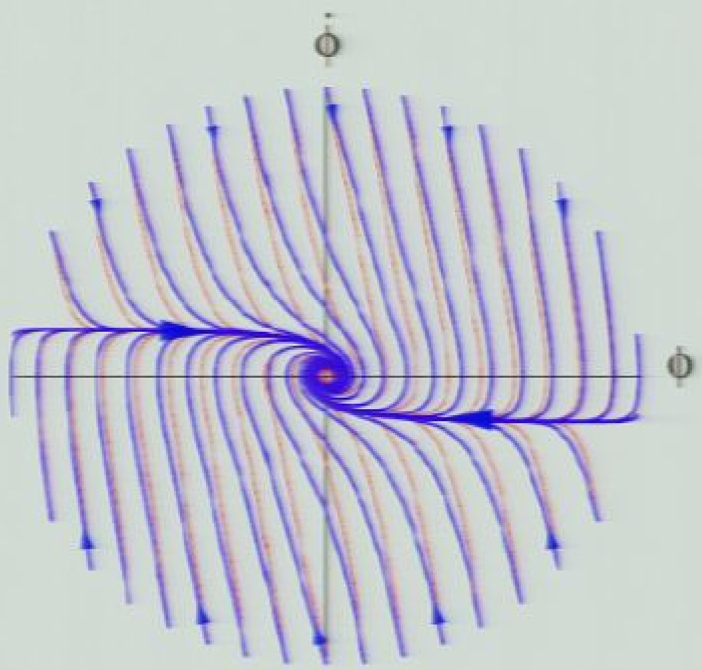
The Problem

What is the probability that our universe experienced a sufficient number of e-foldings of inflation to resolve the fine-tuning problems of standard cosmology?

To answer this question it will be assumed that:

1. Inflation is caused by a minimally-coupled scalar field.
2. The field dynamics is governed by the theory of general relativity.
3. Our universe is a Friedmann-Robertson-Walker universe.

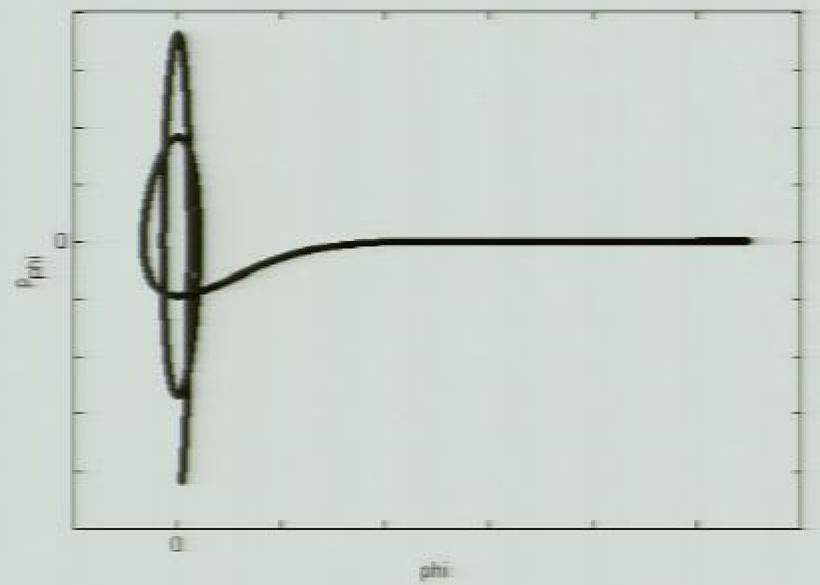
Attractor Behavior



Kofman, Linde and Mukhanov
(arxiv: JHEP 0210:057,2002)

Attractor Behavior

$$\hat{H} = a^3 V(\phi) + \frac{p_\phi^2}{2a^3} - 3ak - \frac{p_a^2}{12a} = 0$$



The system is not an attractor
in the canonical phase space!

Probability Theory

- All classical probabilities are based on some conditional knowledge.
- Classical probabilistic arguments can be used reliably ***only*** when one ***completely*** understands both the nature of the underlying dynamics and the source of its randomness. Hollands and Wald (arxiv: hep-th0210001)

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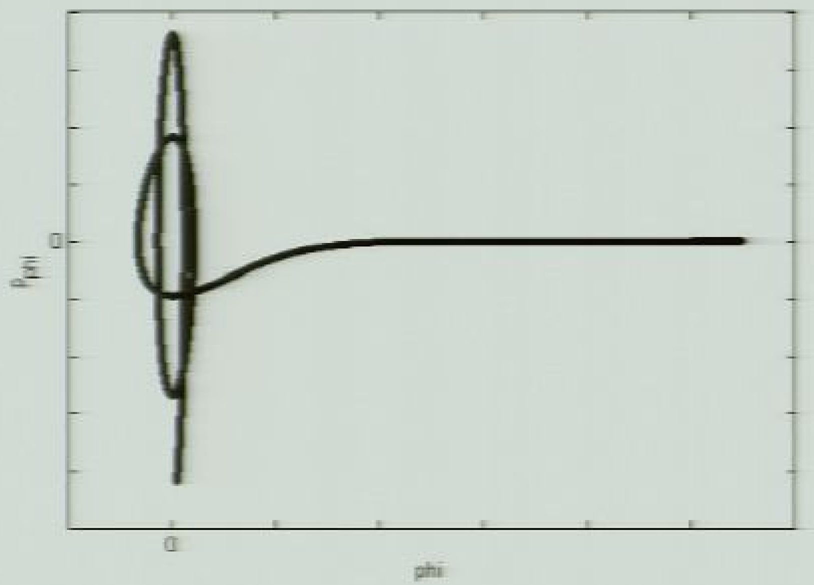

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An example from Hollands and Wald (arxiv: hep-th0210001)

- What is the conditional probability that if cows were to be discovered on a distant planet, that their color would be green?

- A physicist might proceed by taking the bandwidth of the green part of the visible spectrum and dividing this quantity by the bandwidth of the entire visible spectrum.

- A chemist might proceed by taking the number of chemical compounds that are green and then dividing this quantity by the total number of chemical compounds.

An example from Hollands and Wald (arxiv: hep-th0210001)

- A biologist might first estimate the likelihood of the color of the planet itself, by methods similar to that of the chemist and then take camouflage and other survival factors into account to estimate the probability of the cows being green.

- All of the above methods seem to be "*reasonable*" measures to obtain a first order estimate of the probability, yet they could result in drastically different results.

A "*Natural*" Measure

Gibbons, Hawking and Stewart (Nucl. Phys. B281, 736 (1987)) argued that a *natural* measure on the set of solutions to the cosmological equations should satisfy the following three conditions:

1. It should be positive.
2. It should depend only on the intrinsic dynamics and neither on any choice of time slicing nor on the choice of dependent variables.
3. It should respect all the symmetries of the space of solutions without introducing any additional ad hoc structures which do not arise from the field equations themselves.

Canonical Liouville Measure

- GHS showed that the canonical Liouville measure satisfies their three conditions.
- Since the cosmological field equations do not provide initial conditions on the canonical coordinates it is most *natural* to invoke Laplace's principle of indifference.
- This assumption may ultimately be inaccurate in light of a more physically relevant theory of cosmology but it is an **unbiased** estimate in light of our present ignorance.
- Probabilities from such a priori distributions can still be useful if they are extremely low or high because then it is a challenge for the fundamental theory to significantly alter them.

Probability of Slow-roll Inflation?

- Hawking and Page (Nucl. Phys. B298, (1988)) showed that both the set of inflationary and the set of noninflationary solutions have infinite measure when considering:

1. The GHS measure.
2. A flat probability distribution.
3. FRW model with a scalar field.

- They concluded that this scenario gives an **ambiguous probability** for inflation.

- Hawking and Page also pointed out that the divergence in the measure is due to the scale factor.

Another Condition On The Measure

Hollands and Wald (arxiv: hep-th0210001)

- Consider the case where there exists a manifold M representing the possible states of a system, on which there is defined a measure μ such that $\mu(M) = \infty$.

- Let there be a property Q of the system that corresponds to a measurable subset, S, of M.

- There are three possibilities for the probability that Q holds, $P(Q)$;

1. $\mu(S) < \infty,\ \mu(M\text{-}S) = \infty \rightarrow P(Q) \equiv \mu(S)/\ \mu(M) = 0$
2. $\mu(M\text{-}S) < \infty,\ \mu(S) = \infty \rightarrow P(Q) \equiv 1\text{-}\ \mu(M\text{-}S)/\ \mu(M) = 1$
3. $\mu(S) = \mu(M\text{-}S) = \infty \rightarrow P(Q)$ is undefined

- Thus a fourth condition is that if $\mu(M)$ is infinite then $\mu(S)$ and $\mu(M\text{-}S)$ can not both be infinite.

Gibbons and Turok Measure

- Gibbons and Turok (Phys. Rev. D77:063516, 2008) showed that the canonical measure can be visualized as a *magnetic flux* of solutions through phase space.
- They demonstrated that the divergence in the GHS measure is from a divergence in the scale factor, a.
- They argued that in this limit the value of a "*is neither geometrically meaningful nor physically observable*."
- They deal with this divergence by identifying and removing all universes which are indistinguishable. That is a **cutoff** of a is introduced.
- Using Stokes' theorem the canonical measure is

$$\oint p_\phi \, d\phi = 2 \int a^3 |\dot{\phi}| \, d\phi$$

Adiabatic Invariant

- Gibbons and Turok demonstrated that their measure is independent of the cut-off in a, provided that it is evaluated when the Hubble parameter H is small, so that the expansion of the universe is adiabatic as far as the matter fields are concerned.
- In this regime the scalar field oscillates about the potential minimum and a becomes a background function of time.

$$\hat{H}_\phi = \frac{p_\phi^2}{2a(t)^3} + a(t)^3 V \qquad V = \frac{1}{2} m^2 \phi^2 \qquad \oint p_\phi \, d\phi = 2\pi \frac{\hat{H}_\phi}{m}$$

The Strategy

- Since the GT measure loses its cutoff dependence in the regime of low *H* and adiabatic expansion, it is most *natural* to evaluate the measure in this regime.
- Thus the measure will be evaluated at the end of slow-roll inflation.

The approach is to calculate the measure at the end of slow-roll inflation, for those solutions which experience *N* or more e-folds of inflation as the scalar field evolution is traced back in time, and for all the solutions. The ratio of the two measures is then interpreted as the probability of slow-roll inflation.

Assuming a flat universe;

$$H^2=\frac{1}{3}\left(\frac{1}{2}\dot{\phi}+V\right) \qquad \dot{H}=\frac{-\dot{\phi}^2}{2}\Leftrightarrow|\dot{\phi}|=2\left|\frac{dH}{d\phi}\right|$$

$$\ddot{\phi}+3\mathrm{H}\dot{\phi}+\frac{dV}{d\phi}=0 \qquad H^2=\frac{V}{3}+\frac{2}{3}\left(\frac{dH}{d\phi}\right)^2$$

and the measure can be written as $\int 4\mathrm{a}^3\left|\frac{dH}{d\phi}\right|d\phi$

Thus in the regime of low H and adiabatic expansion where a behaves as a background function of time, the probability for N e-folds of inflation is

$$P(N)\approx\frac{\int_N\left|\frac{dH}{d\phi}\right|_S d\phi}{\int\left|\frac{dH}{d\phi}\right|_S d\phi}$$

To first order the slow-roll solution to the Friedmann equation is

$$H^2 = \frac{V}{3} + \frac{2}{3}\left(\frac{dH}{d\phi}\right)^2 \qquad H_{SR} = \sqrt{\frac{V}{3}}\left(1 + \frac{1}{12V^2}\left(\frac{dV}{d\phi}\right)^2\right)$$

Perturbing around the slow-roll solution to first order

$$H \Rightarrow H_{SR} + \delta H \qquad V \Rightarrow V + \delta V$$

and setting $\delta V = 0$ since the derivatives of V are small in the slow-roll approximation

$$\frac{d\delta H}{dN} = 3\delta H \qquad \delta H = \delta H_S e^{3N}$$

The solution departs from slow roll when the perturbation is of the same order as the first order correction term

$$\delta H \approx \frac{m}{\sqrt{N}}$$

Recall that the probability for N e-folds of inflation is

$$P(N) \approx \frac{\int_N \left|\frac{dH}{d\phi}\right|_S d\phi}{\int \left|\frac{dH}{d\phi}\right|_S d\phi}$$

The numerator can be approximated as δH_S, since

$$H \Rightarrow H_{SR} + \delta H \qquad H_{SR} = \sqrt{\frac{V}{3}}\left(1 + \frac{1}{12V^2}\left(\frac{dV}{d\phi}\right)^2\right)$$

and the derivatives of V are small in the slow-roll approximation.

The denominator is approximately m.

$$\int \left|\frac{dH}{d\phi}\right|_S d\phi = \frac{1}{2}\int_S \sqrt{6H^2 - 2V}\, d\phi \approx \frac{m}{3}$$

Thus the probability for N or more e-folds of inflation is

$$P(N) \approx N^{-1/2} e^{-3N}$$

Measure Conservation

- Kofman, Linde and Mukhanov (JHEP 0210:057, 2002) argue that the decay of the scalar field and particle production are irreversible processes.
- They state that "*in inflationary cosmology the total energy and entropy of the scalar field and particles created by its decay is not conserved.*"
- They argue that for these reasons inflationary dynamics is definitely not measure preserving.
- Based on this consideration they argue that the probability of slow-roll inflation should not be evaluated at the end of inflation.

Kofman, Linde and Mukhanov Measure

- They defined a measure to be $\iint d\dot{\phi}\, d\phi$
- They evaluated this measure when the energy density of the scalar field was equal to the Planck density.
- They assumed Laplace's principle of indifference for the initial coordinates in phase space **but** imposed the restriction that the energy density is equal to the Planck density.
- The resulting probability of slow-roll inflation is *1-O(m)* where *m* is the mass of the inflaton.
- We calculated that using these conditions the probability for 60 or more e-folds of inflation is $\approx 1 - O(m)$. For example, if $m = 1e\text{-}6\ m_{pl}$ the probability for 60 or more e-folds is 0.999997.

Kofman, Linde and Mukhanov Measure

- Using the KLM measure, the probability of slow-roll inflation depends **sensitively** on the surface of initial conditions chosen to evaluate the probability.
- The decision to evaluate the measure when the scalar field has Planck density is certainly ad hoc.
- Kofman, Linde and Mukhanov note that even if their measure is evaluated at the end of slow-roll inflation the probability that inflation occurred is $O(m)$.
- We calculated the probability for 60 or more e-folds of inflation does get small as we approach the end of inflation.

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Carroll and Tam Measure

- Carroll and Tam (arxiv: hep-th1007.1417) argued that the Gibbons and Turok approach to resolving the divergence of the canonical measure, "*seems to be throwing away almost all of the solutions and keeping a set of measure zero.*"

- They propose a new regularization scheme that ultimately results in the measure

$$\mu \propto \int \sqrt{1 - \frac{V(\phi)}{3H^2}}\, d\phi$$

- This is effectively the same as the KLM measure and is surface dependent!

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Carroll and Tam Measure

- They chose to evaluate the measure at the Planck density and let the mass of the inflaton be $m = 3e\text{-}3\ m_{pl}$.
- They argued that 60 or more e-folds of inflation would occur for all Φ excluding the region $-24 \leq \Phi \leq 6$ and determined that the normalized measure integrates to 0.99996.
- We calculated this probability to be 0.95950 which is closer to $1\text{-}O(m)$ as expected.

Conclusion

What is the probability that our universe experienced a sufficient number of e-foldings of inflation to resolve the fine-tuning problems of standard cosmology?

Gibbons and Turok measure assuming Laplace's principle of indifference

$$\oint p_\phi d\phi = 2\int a^3 |\dot{\phi}| d\phi$$

Probability for N or more e-folds of inflation

$$P(N) \approx N^{-1/2} e^{-3N}$$

"*The question of why or how inflation started remains a deep mystery and a challenge for the fundamental theory.*" *(Gibbons and Turok, Phys. Rev. D77:063516, 2008)*