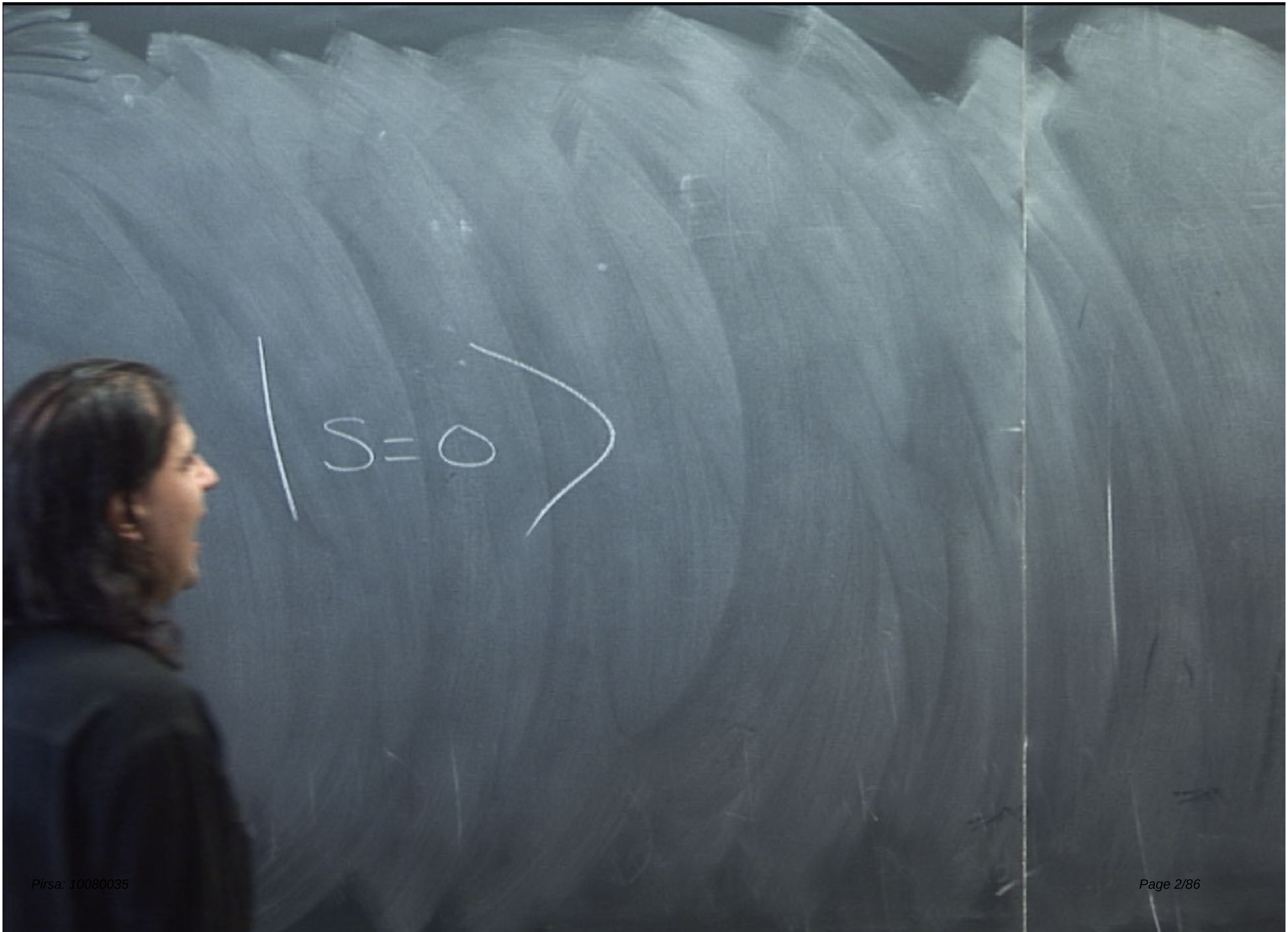
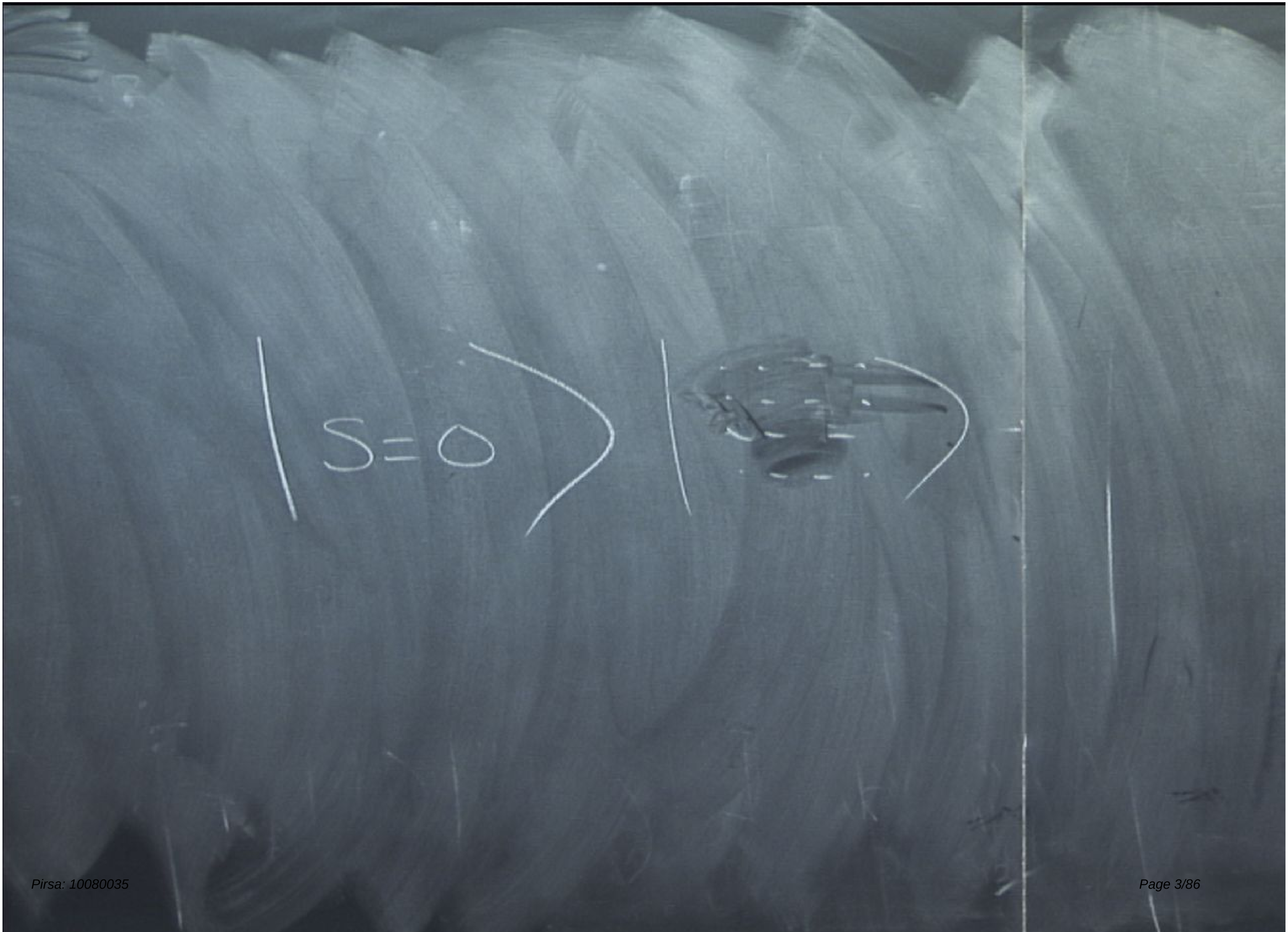
Title: PSI 2010 - Theoretical Physics - Lecture 3B

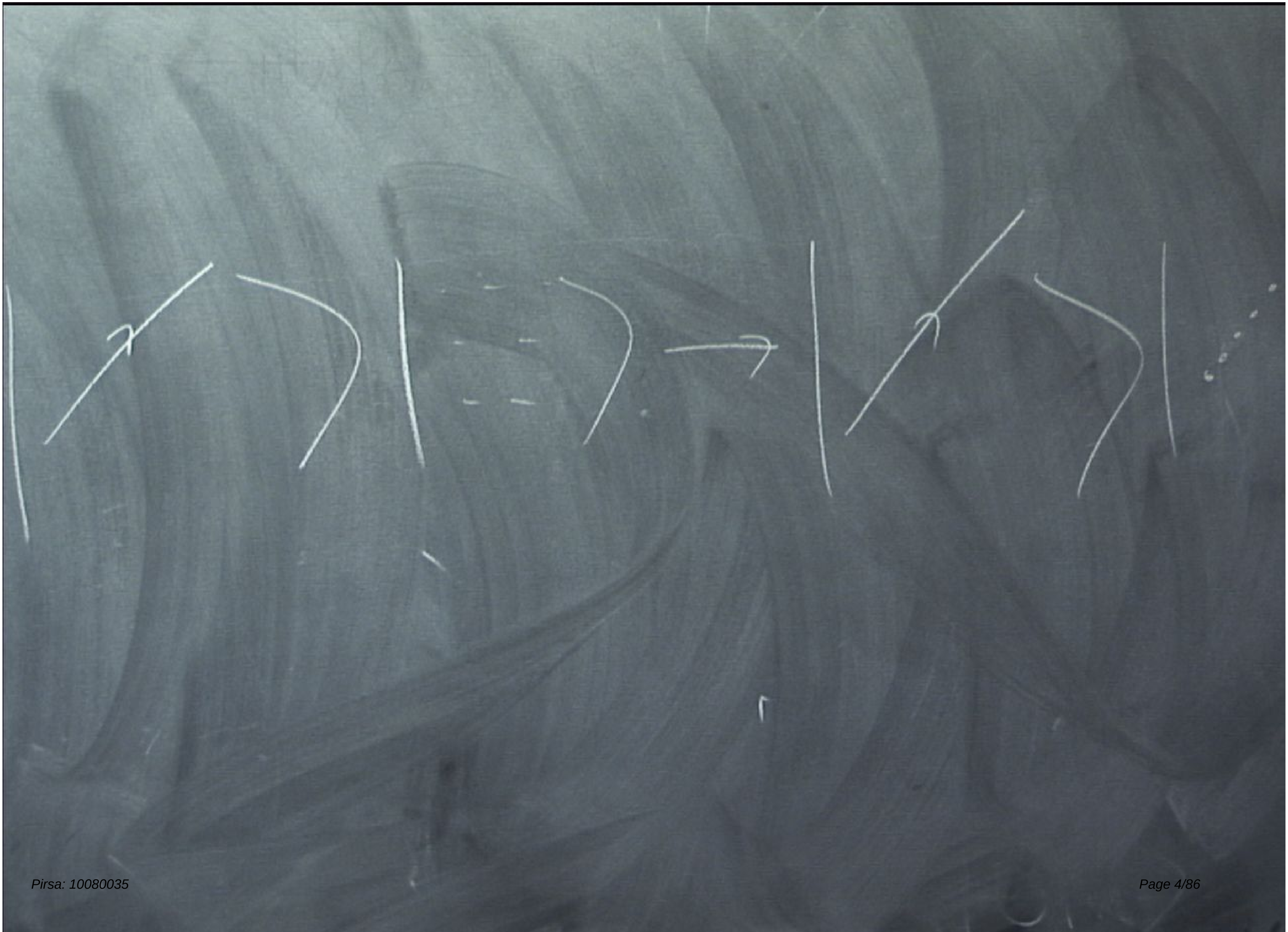
Date: Aug 25, 2010 10:30 AM

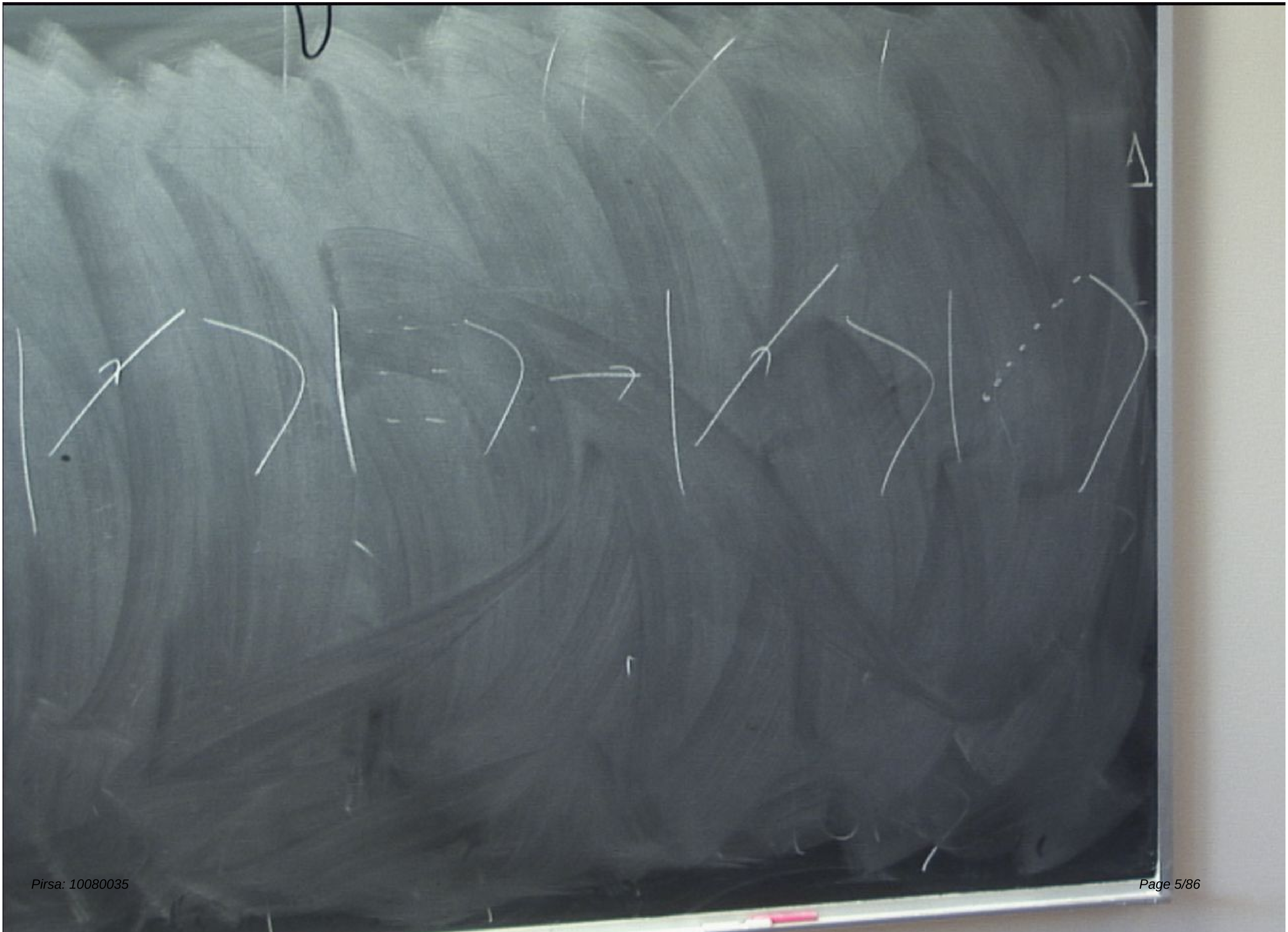
URL: http://pirsa.org/10080035

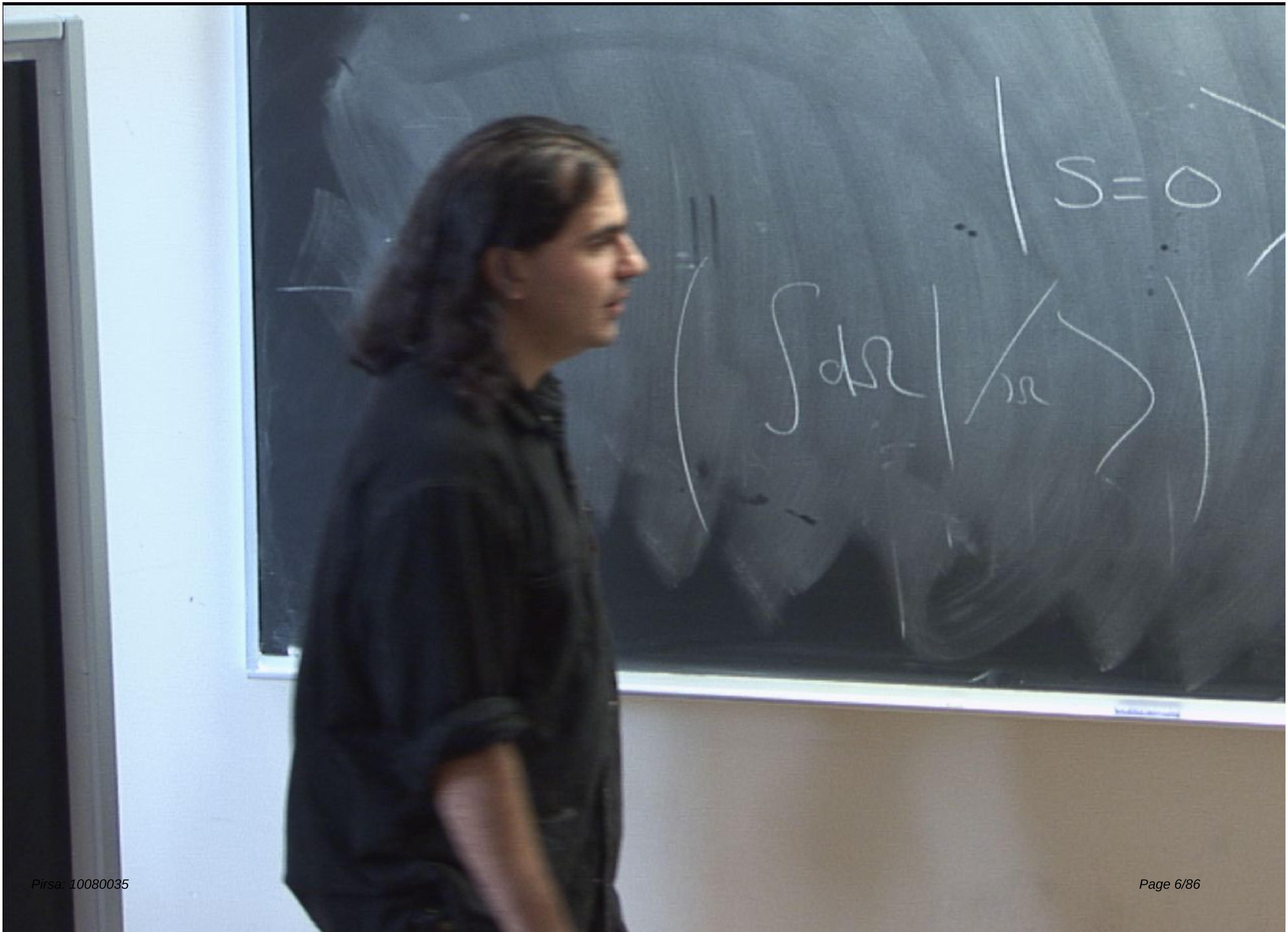
Abstract:

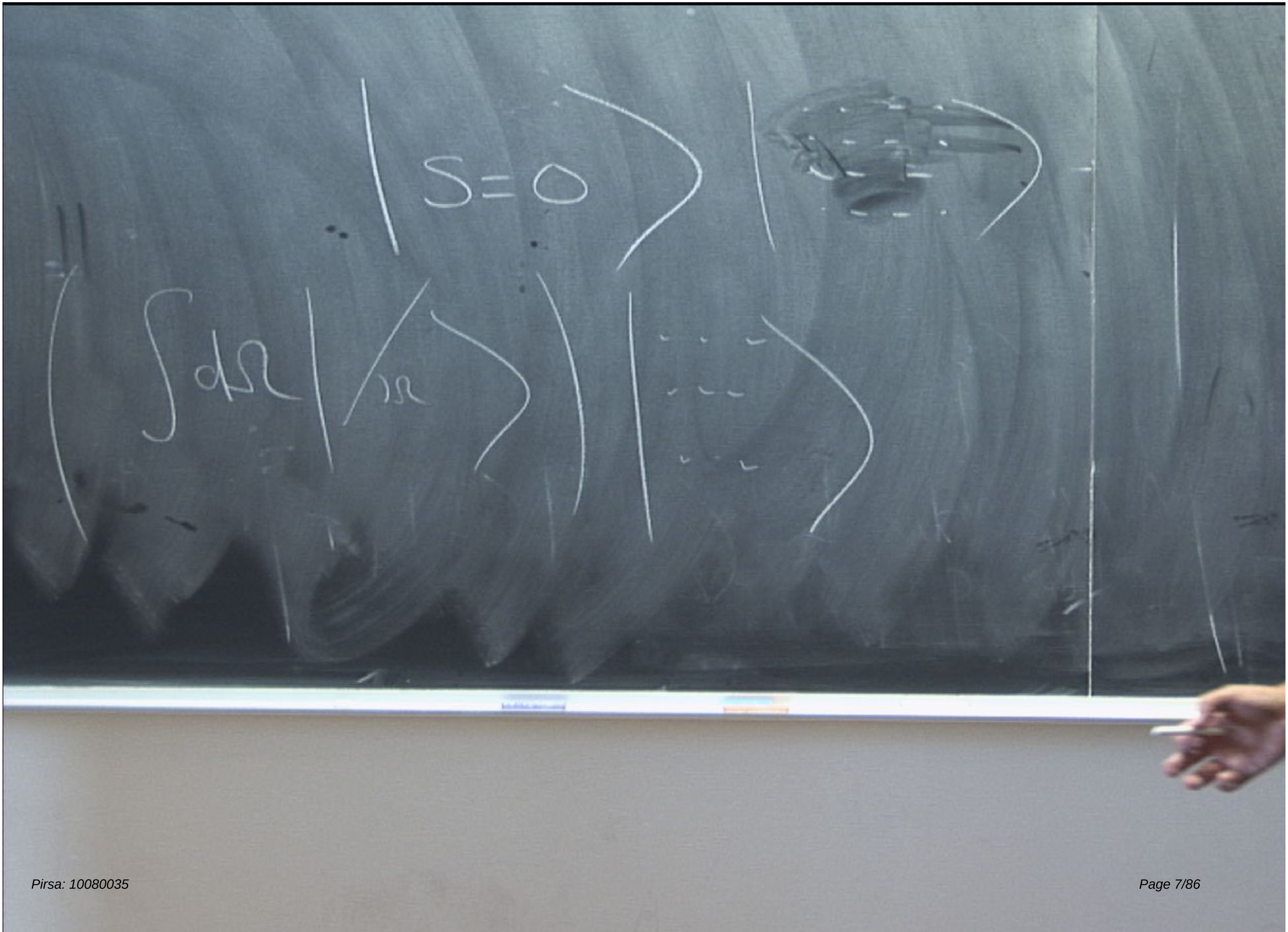

$|s=0\rangle$

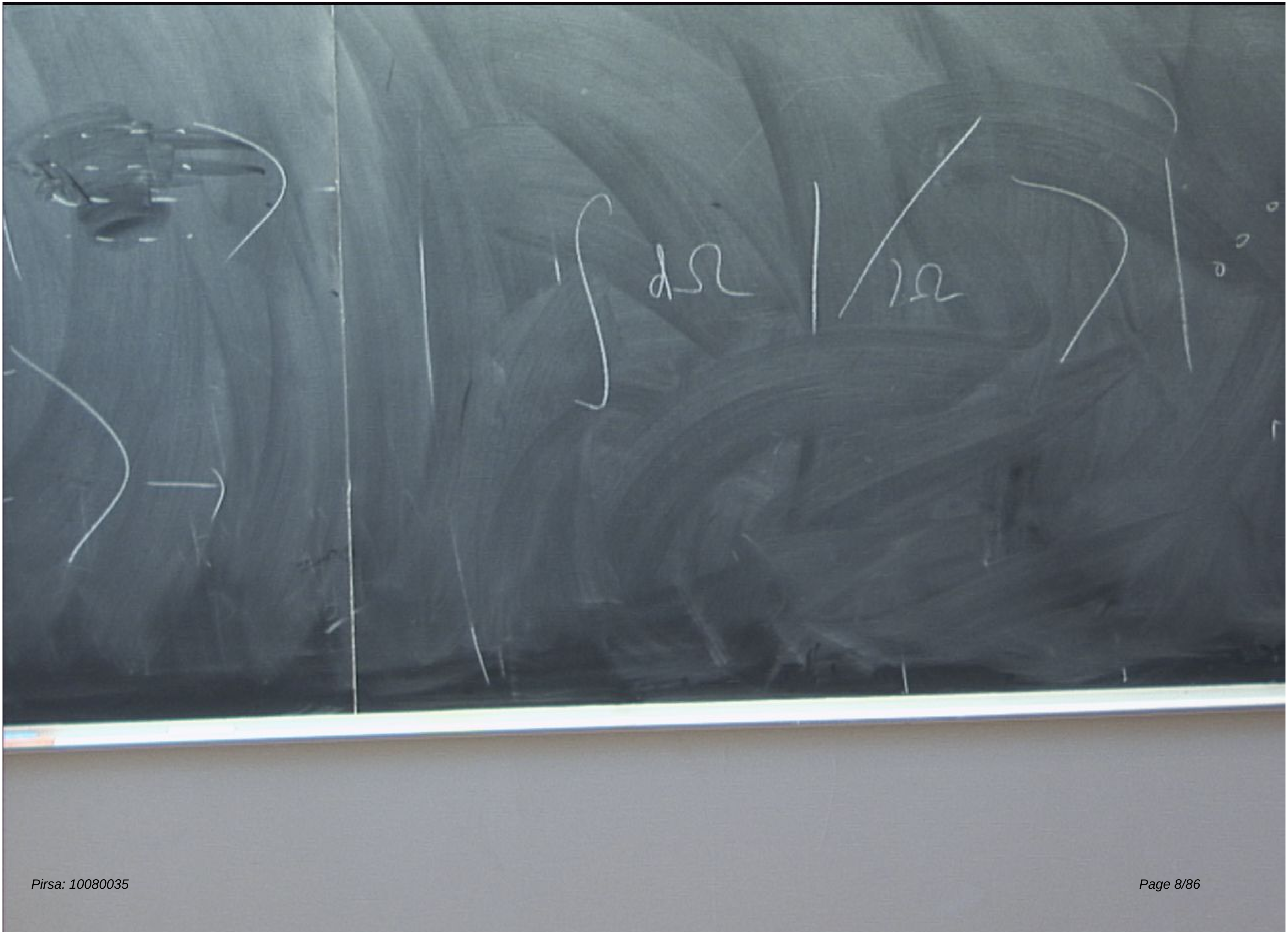

$|S=0\rangle|\ \rangle$

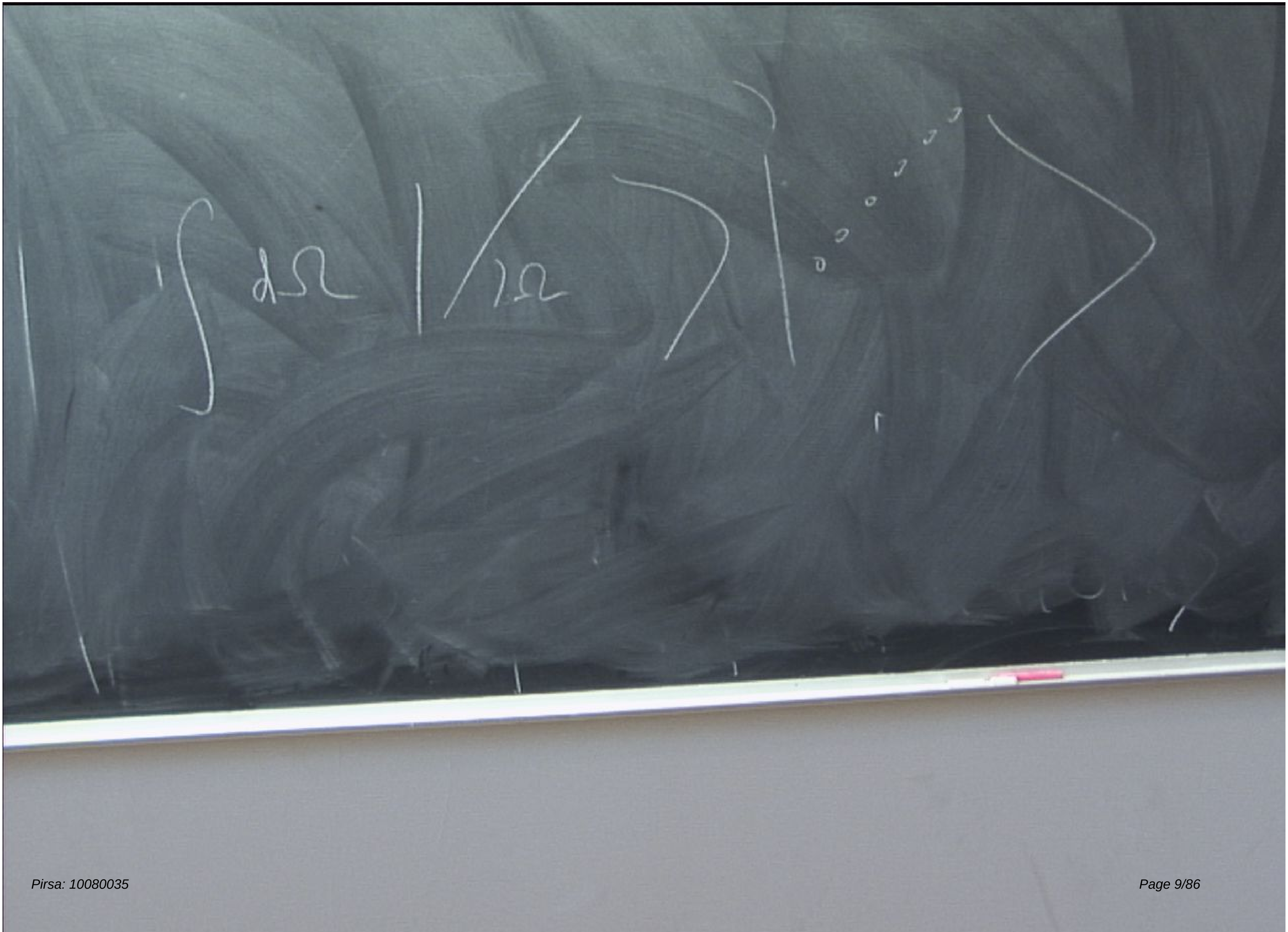


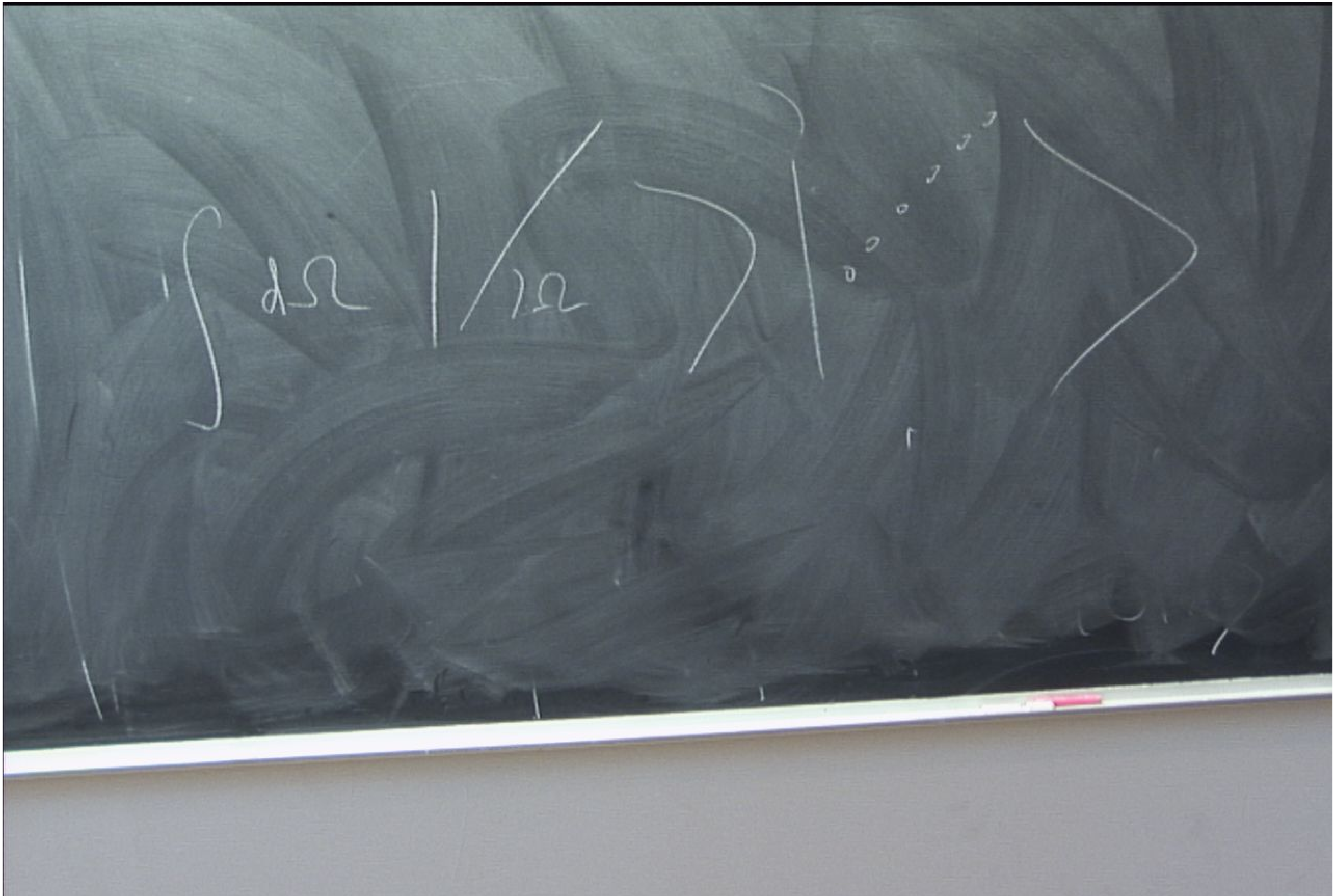



S=0


S=0
∫dΩ



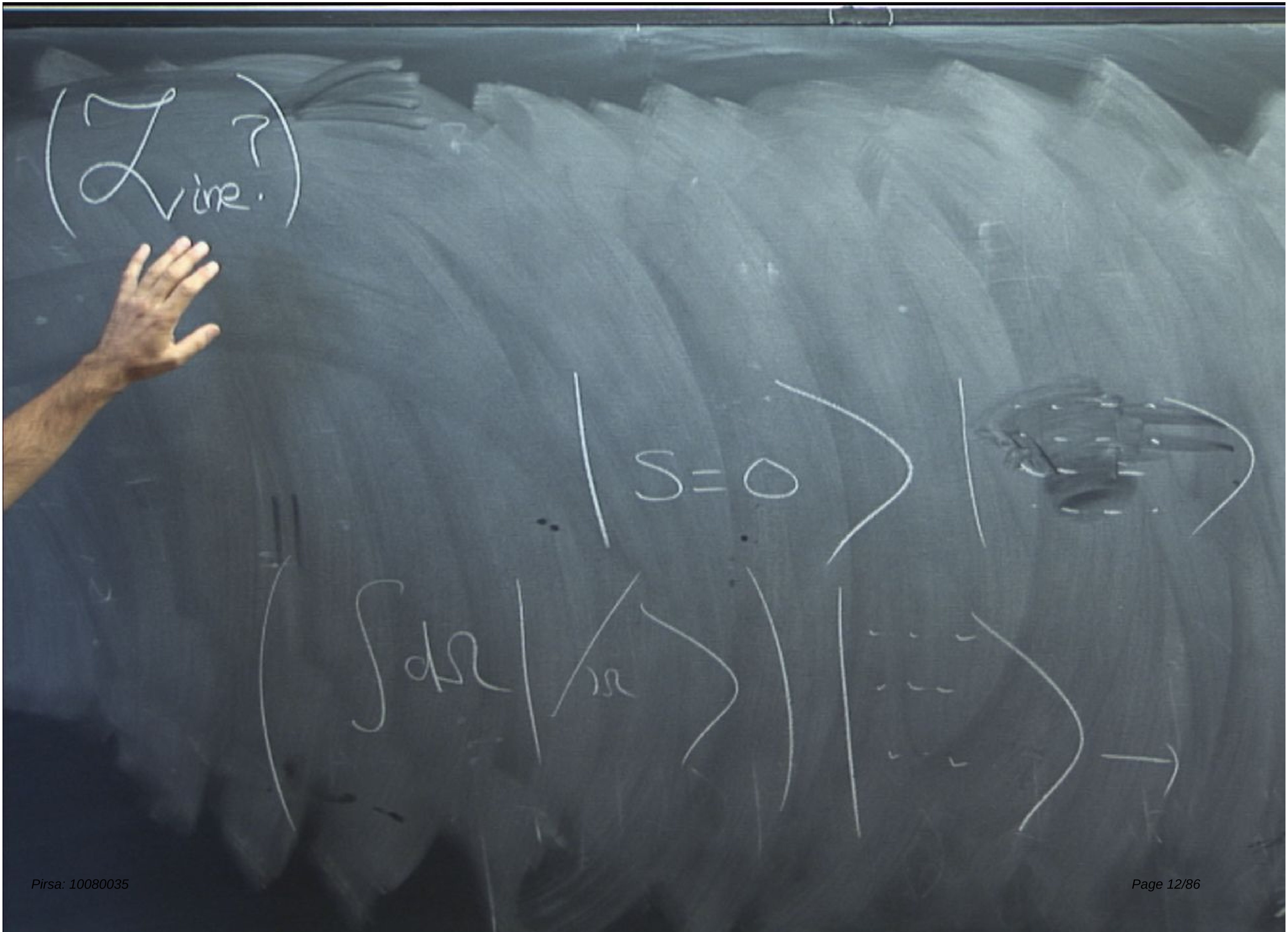


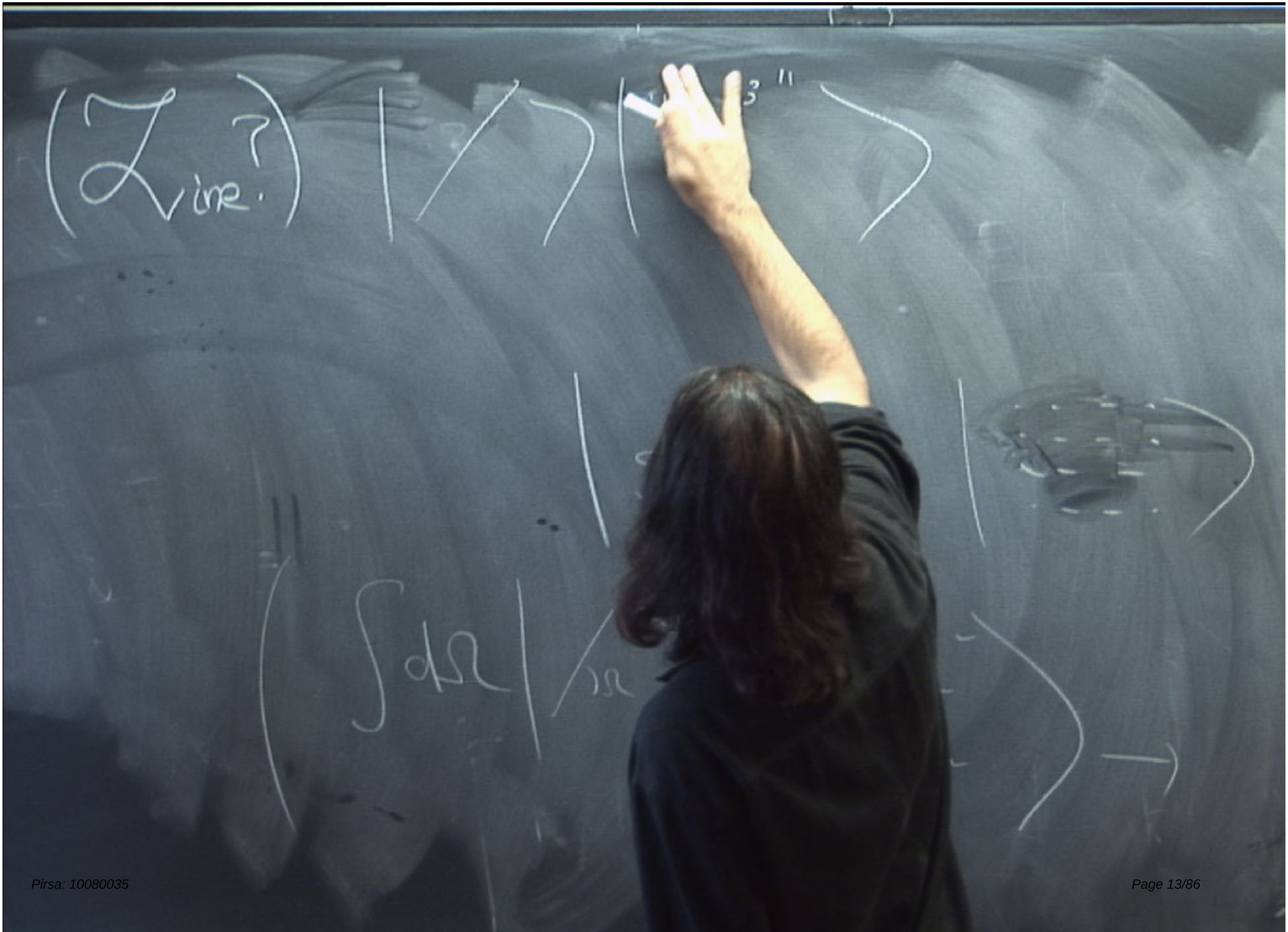


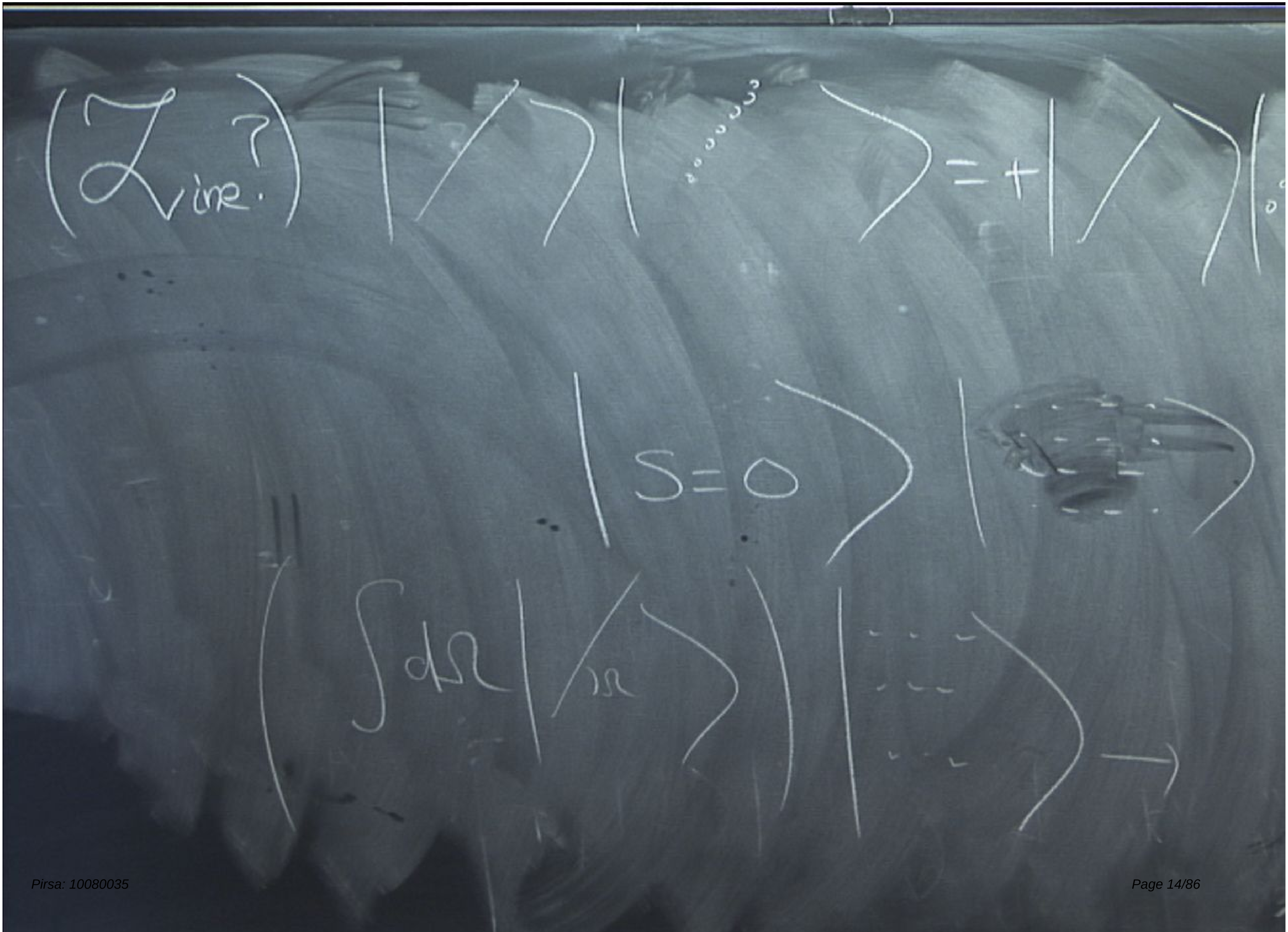
$\left(\mathcal{Z}_{\text{vine}}?\right)$

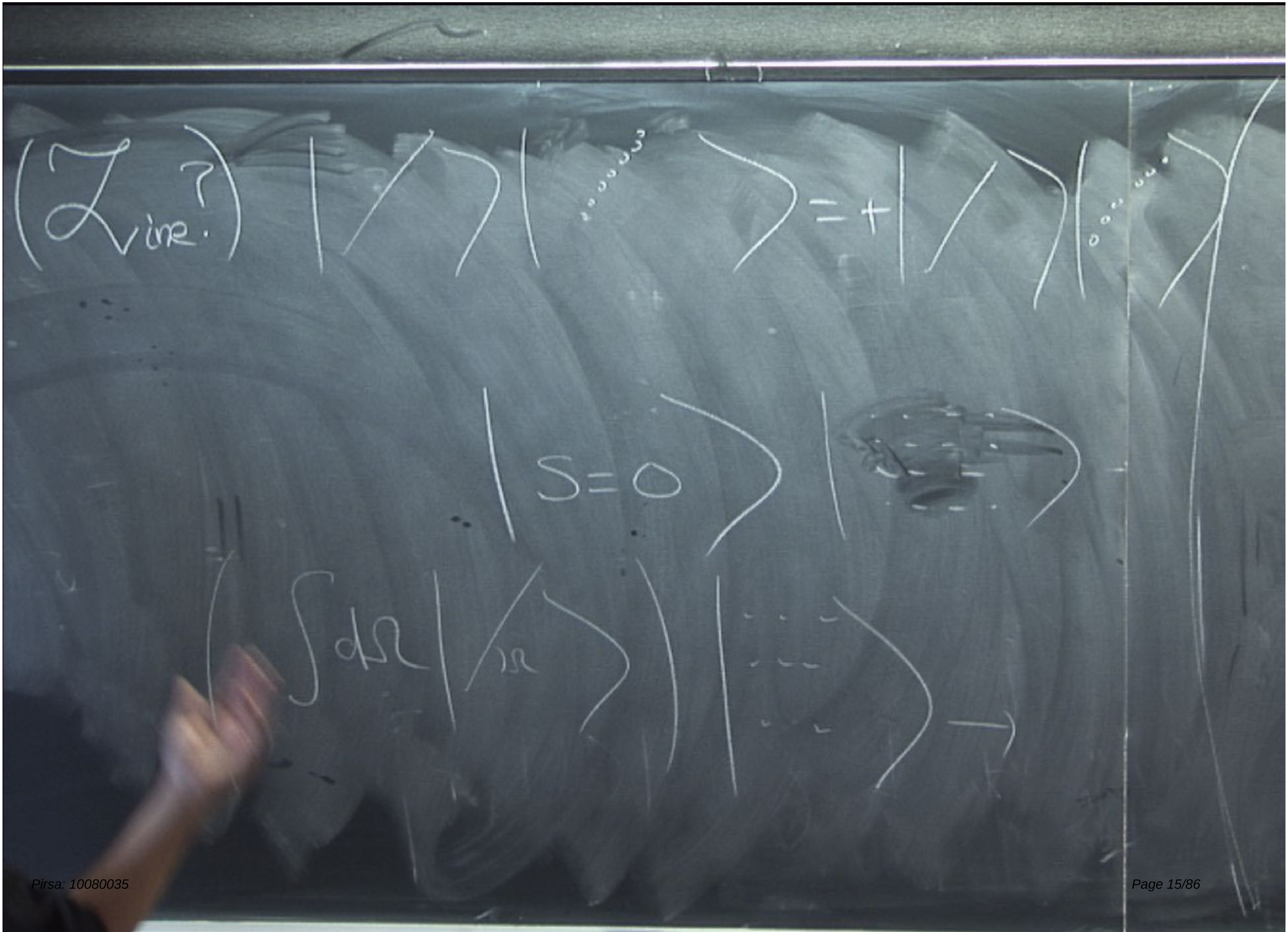
$$\Big|\, s=0 \Big\rangle \Big|\, \cdots\cdots \Big\rangle$$

$$= \left(\int d\Omega \,\Big|\, \diagup_{\Omega} \Big\rangle\right) \Big|\, \cdots \Big\rangle$$


$(Z_{vine}?)$
$|S=0\rangle$
$\int d\Omega$




S=0
∫dΩ



$(Z_{inv}?)$
$= +$
$S=0$
$\int d\Omega$

$(\mathcal{L}_{\mathrm{ine}}?)\; |\diagup\rangle |\,\cdots\rangle = + |\diagup\rangle |\,\cdots\rangle$

$|S=0\rangle \,|\,\cdots\rangle$

$\left(\int d\Omega\, |\diagup_{\Omega}\rangle\right) |\,\cdots\rangle \rightarrow (\mathcal{L}_{\mathrm{ine}}?)$

$$|\psi_N\rangle = \left(\alpha|\uparrow\rangle + \beta|\downarrow\rangle\right)^N$$

$$\left(\frac{N_{\uparrow\downarrow}}{N}\right) = P_{\uparrow\downarrow}.$$

$$|\psi_N\rangle = \left(\alpha|\uparrow\rangle + \beta|\downarrow\rangle\right)^{N}$$

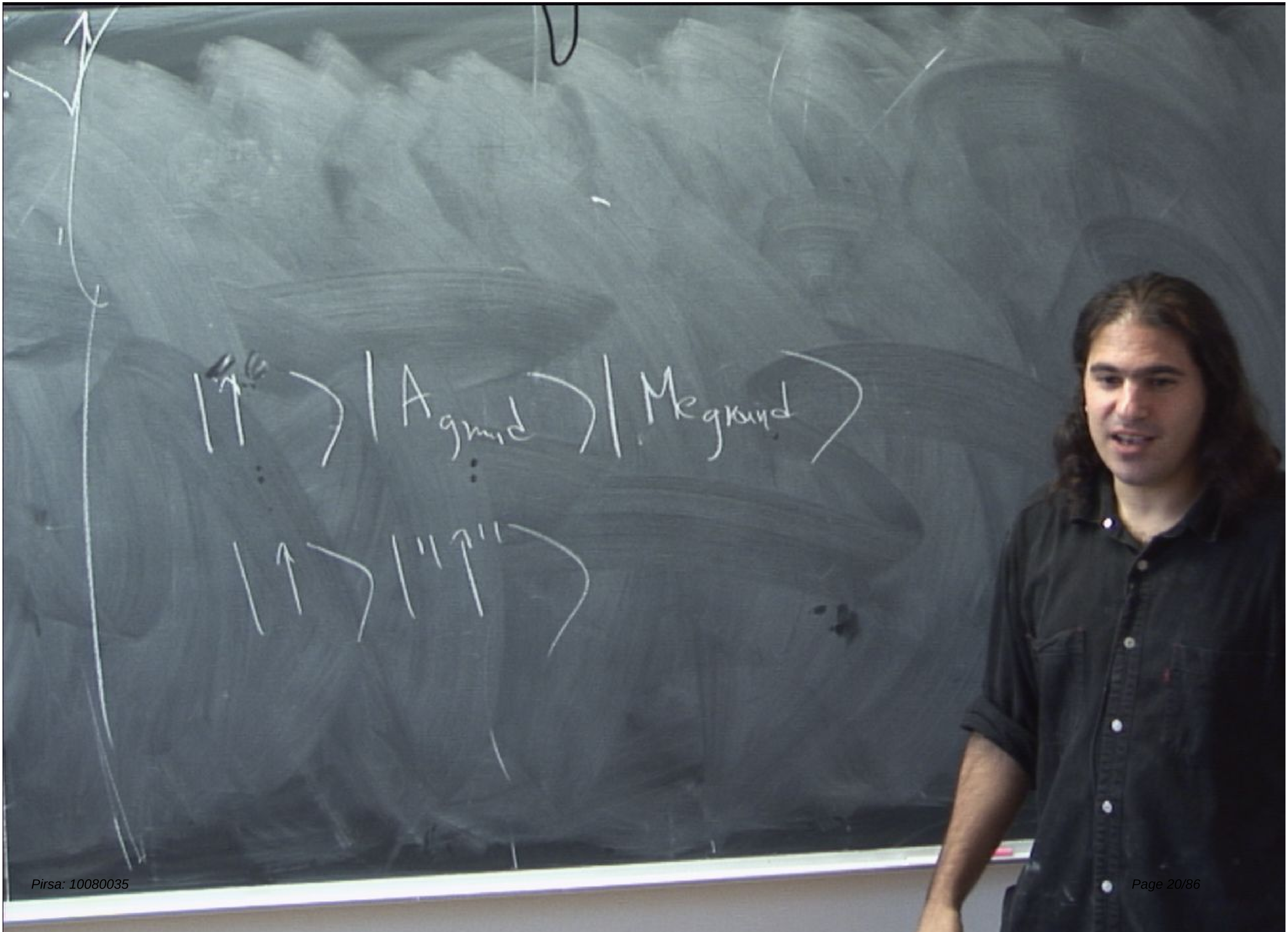
$$\left(\frac{N_{\uparrow\downarrow}}{N}\right) = P_{\uparrow\downarrow}.$$

$$\lim_{N\to\infty} P_{\uparrow\downarrow}|\psi_N\rangle = \begin{matrix}|\alpha|^2 \\ |\beta|^2\end{matrix}\,|\psi_N\rangle_{N\to\infty}$$

$$(\mathcal{Z}_{\text{vine}}?)\quad |\nearrow\rangle|\cdots\rangle = +|\nearrow\rangle|\cdots\rangle \checkmark$$

$$\frac{1}{\sqrt{2}}|\uparrow\rangle|\text{“}\uparrow\text{”}\rangle|\text{“It's up!”}\rangle$$

$$+\frac{1}{\sqrt{2}}|\downarrow\rangle|\text{“}\downarrow\text{”}\rangle|\text{“It's down”}\rangle$$


|"↑"⟩|A_ground⟩|Me_ground⟩
|↑⟩|"↑"⟩

$$|\uparrow\rangle|A_{\text{grnd}}\rangle|\text{Me}_{\text{ground}}\rangle$$

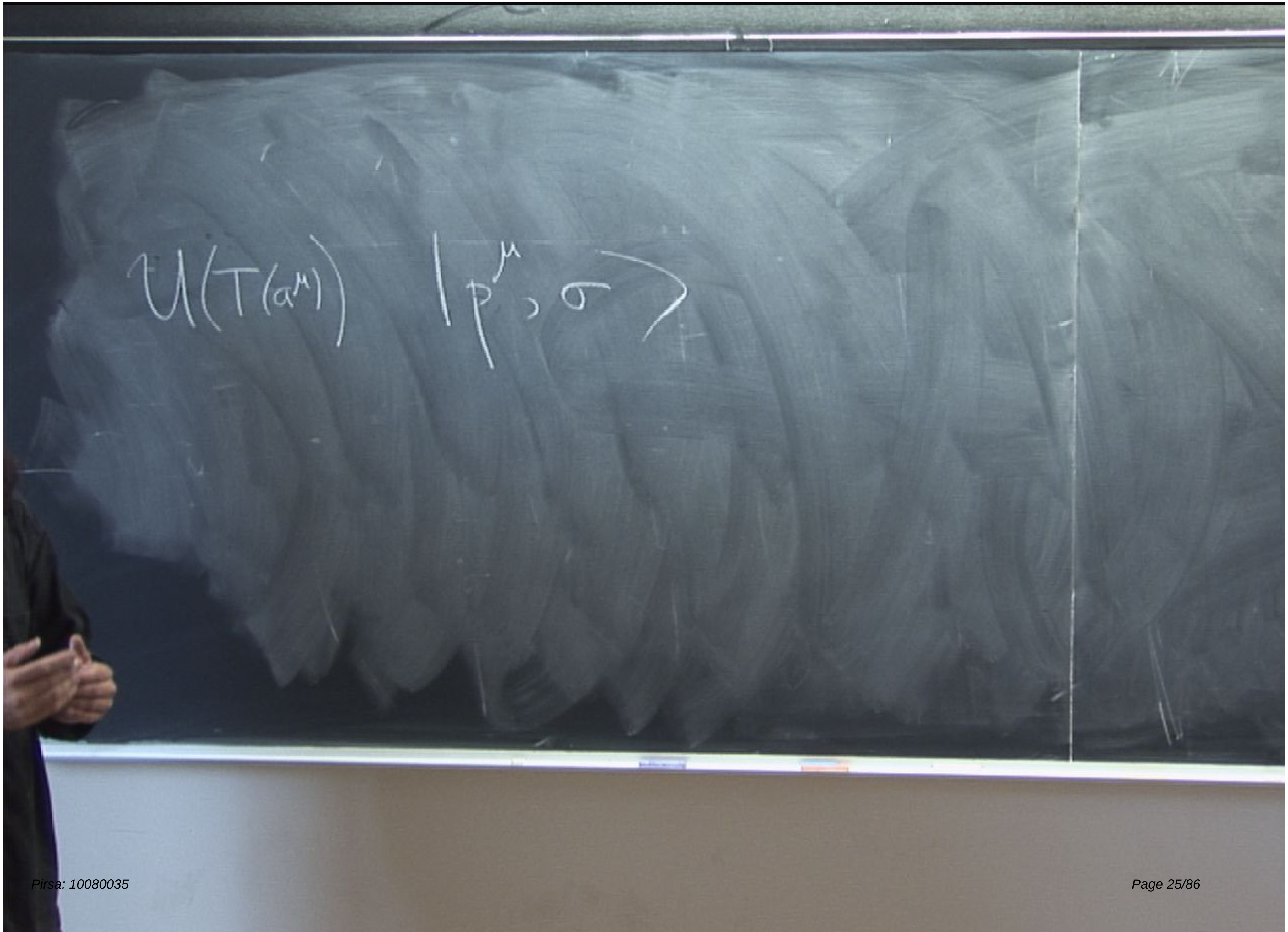
$$|\uparrow\rangle|``\uparrow"\rangle|``\text{I think } \uparrow"\rangle$$

QM + SR

QM + SR

Particles, Particles, Particles

$|p^\mu, \sigma\rangle$


$\mathcal{U}(T(a^\mu))\ |p^\mu, \sigma\rangle$

$$x^\mu \to x^\mu + a^\mu$$

$$\mathcal{U}(T(a^\mu)) \, |p^\mu, \sigma\rangle = e^{ip\cdot a} \, |p^\mu, \sigma\rangle$$

$$|p^\mu, \sigma\rangle = e^{i p^\mu a_\mu} |p^\mu, -\sigma\rangle$$

$\Lambda^{\mu}{}_{\nu}$

$$\Lambda^{\mu}{}_{\nu}$$

$$U[\Lambda]$$

$$x^\mu \to x^\mu + a^\mu .$$

$$U(T(a^\mu)) \, |p^\mu, \sigma\rangle = e^{i p^\mu a_\mu} \, |p^\mu, \sigma\rangle$$

$$U(\Lambda)|p, \sigma\rangle$$

$x^\mu \to x^\mu + a^\mu$.

$$U(T(a^\mu))\,|p^\mu, \sigma\rangle = e^{i p^\mu a_\mu}\,|p^\mu, \sigma\rangle$$

$$U(\Lambda)\,|p, \sigma\rangle = |\Lambda p, \sigma\rangle$$

$x^\mu \to x^\mu + a^\mu$.

$$\mathcal{U}(T(a^\mu)) \, |p^\mu, \sigma\rangle = e^{i p^\mu a_\mu} |p^\mu, \sigma\rangle$$

$$\mathcal{U}(\Lambda) |p, \sigma\rangle = |\Lambda p, \sigma\rangle$$

$x^\mu \to x^\mu + a^\mu.$

$$U(T(a^\mu))\,|p^\mu, \sigma\rangle = e^{i p^\mu a_\mu}\,|p^\mu, \sigma\rangle$$

$$U(\Lambda)|p, \sigma\rangle = \sum_{\sigma'} D_{\sigma\sigma'}(\Lambda)$$

$+a^\mu$.

$(\mu)) \; |p^\mu, \sigma\rangle = e^{i p^\mu a_\mu} |p^\mu, -\sigma\rangle$

$U(\Lambda)|p, \sigma\rangle = \sum_{\sigma'} D_{\sigma\sigma'}(\Lambda) |\Lambda p, \sigma'\rangle$

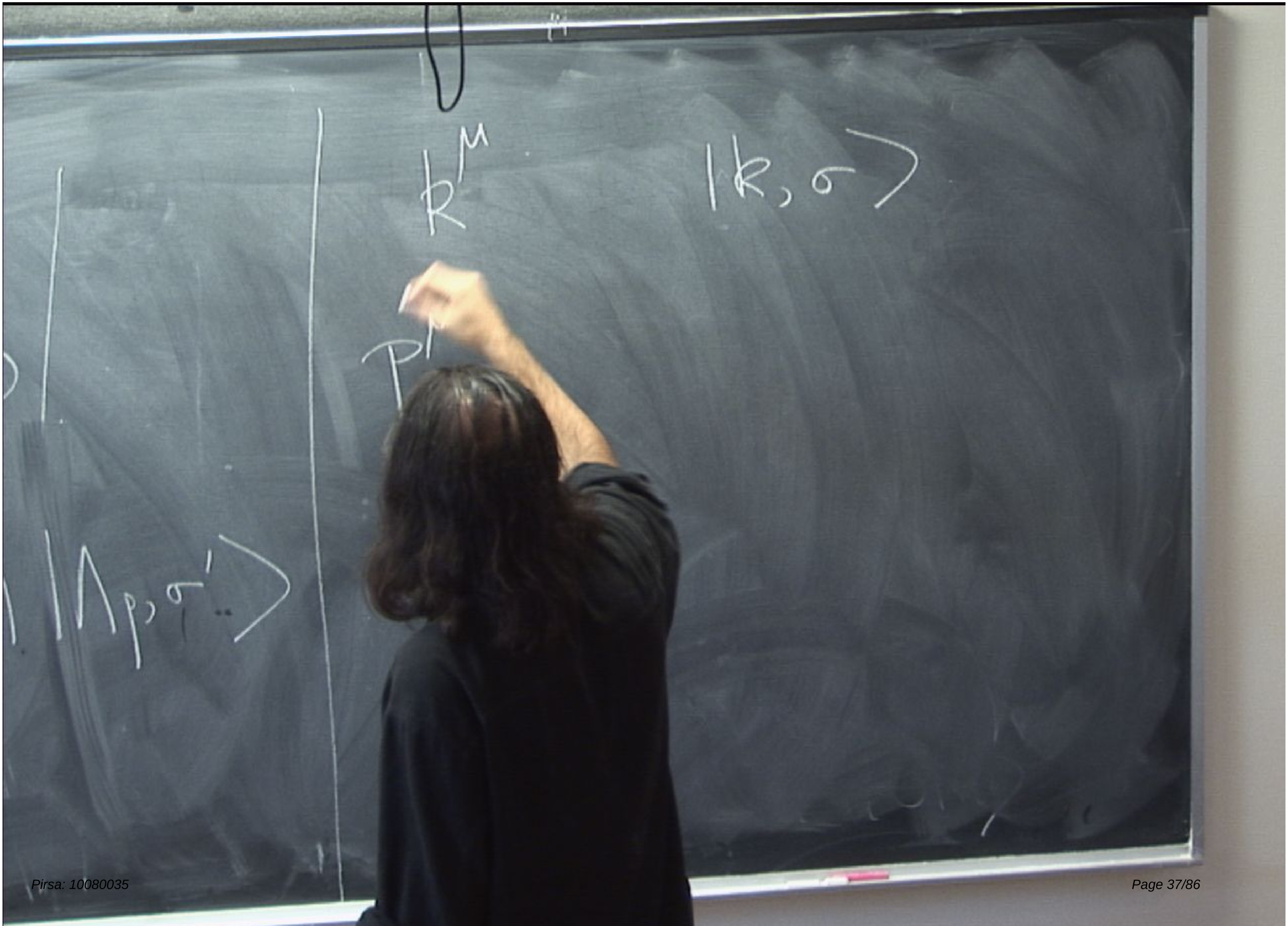
$$x^\mu \to x^\mu + a^\mu$$

$$\mathcal{U}(T(a^\mu))\,|p^\mu, \sigma\rangle = e^{i p^\mu a_\mu}\,|p^\mu, \sigma\rangle$$

$$\mathcal{U}(\Lambda)\,|p, \sigma\rangle = \sum_{\sigma'} D_{\sigma\sigma'}(\Lambda)\,|\Lambda p, \sigma'\rangle$$

k^{μ}

$|\Lambda_{p,\sigma'}\rangle$


k^{μ}
$|k, \sigma\rangle$
P^{μ}
$|\Lambda p, \sigma' \ldots\rangle$

k^μ $|k,\sigma\rangle$

$p^\mu = L^\mu{}_\nu(p)\, k^\nu$

$|\Lambda p, \sigma'\rangle$

k^μ $|k,\sigma\rangle$

$p^\mu = L^\mu{}_\nu(p)\, k^\nu$

$|\Lambda p, \sigma'\rangle$

$$k^\mu \qquad |k, \sigma\rangle$$

$$p^\mu = L^\mu{}_\nu(p)\, k^\nu$$

$$|\Lambda p, \sigma'\rangle \qquad |p, \sigma\rangle \equiv U(L(p))\,|k, \sigma\rangle$$

$$|\Lambda p, \sigma'\rangle$$

$$k^\mu \qquad |k, \sigma\rangle$$

$$p^\mu = L^\mu{}_\nu(p)\, k^\nu$$

$$|p, \sigma\rangle \underset{\text{def}}{\equiv} U(L(p))\,|k, \sigma\rangle$$

$$k^\mu \qquad |k,\sigma\rangle$$

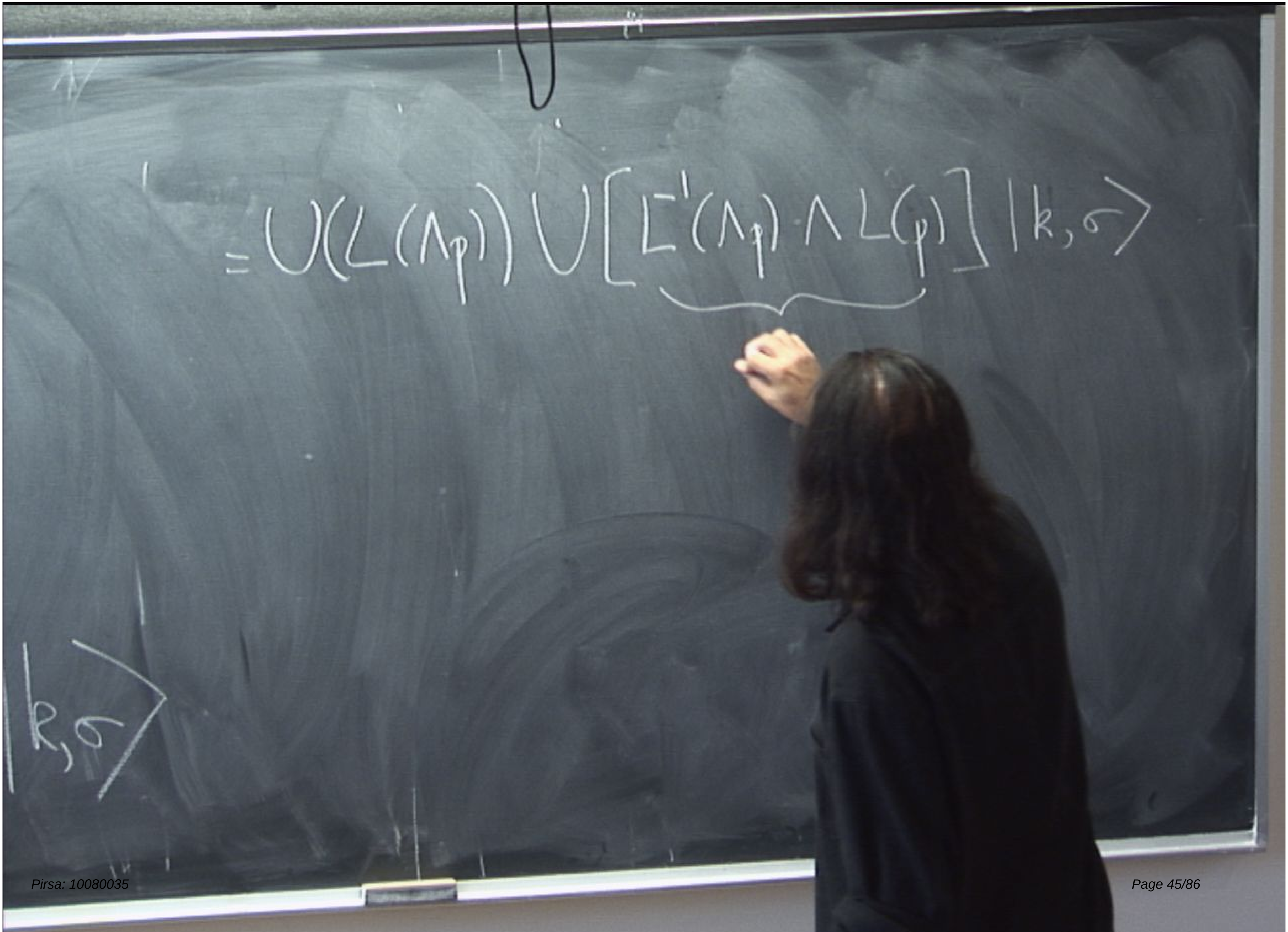
$$p^\mu = L^\mu{}_\nu(p)\, k^\nu$$

$$|\Lambda p, \sigma'\rangle \qquad |p,\sigma\rangle \underset{\text{def}}{\equiv} U(L(p))\,|k,\sigma\rangle$$

$$U(\Lambda)|p,\sigma\rangle$$

$$= U(\Lambda)U(L(p))|k,\sigma\rangle$$

$$U(\Lambda)\left|p,\sigma\right\rangle$$
$$= U(\Lambda)U(L(p))\left|k,\sigma\right\rangle$$
$$= U(L(\Lambda p))U(L^{-1}(\Lambda p))U(\Lambda)U(L(p))\left|k,\sigma\right\rangle$$

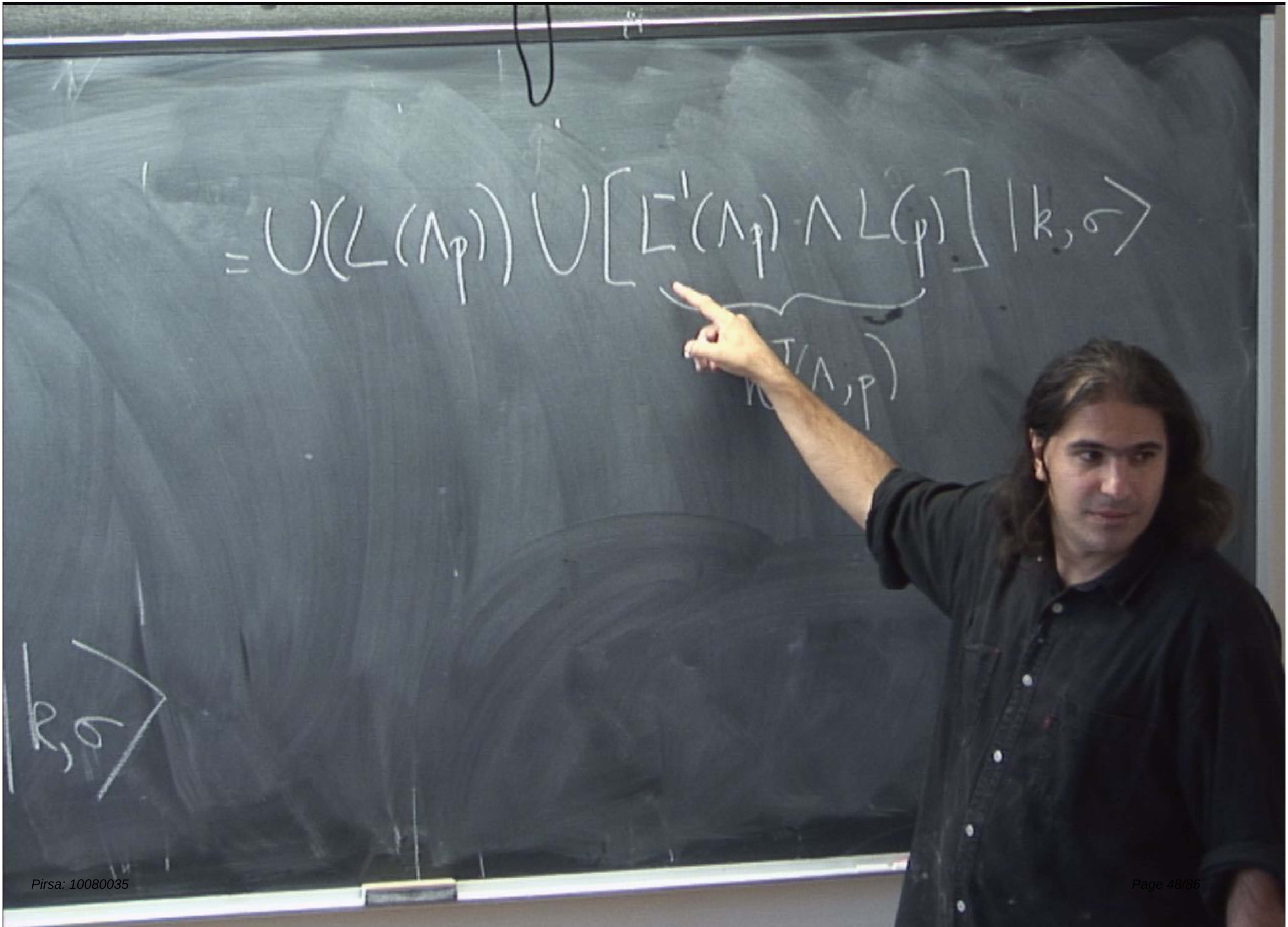

$= U(L(\Lambda p))\, U[\underbrace{L^{-1}(\Lambda p)\, \Lambda\, L(p)}]\, |k, \sigma\rangle$
$|k, \sigma\rangle$

$$= U(L(\Lambda p))\, U\big[\underbrace{L^{-1}(\Lambda p)\,\Lambda\, L(p)}_{W}\big]\,|k,\sigma\rangle$$

$|k,\sigma\rangle$

$$= U(L(\Lambda p))\, U\big[\underbrace{L^{-1}(\Lambda p)\,\Lambda\, L(p)}_{W(\Lambda,p)}\big]\,|k,\sigma\rangle$$

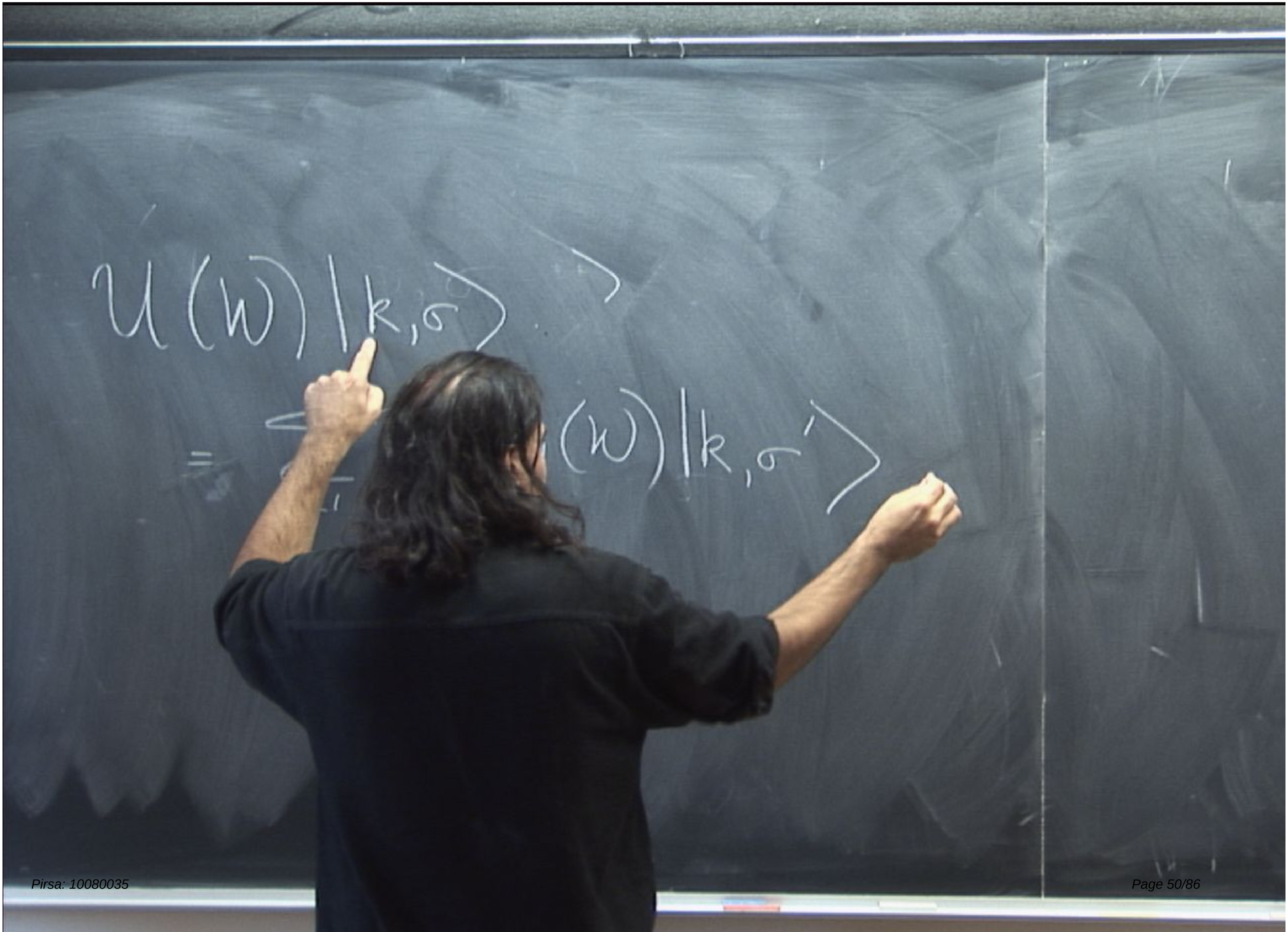
$|k,\sigma\rangle$


= U(L(Λp)) U[L⁻¹(Λp) Λ L(p)] |k, σ>
|k, σ>

$$= U(L(\Lambda p))\, U\big[\underbrace{L^{-1}(\Lambda p)\,\Lambda\, L(p)}_{W(\Lambda, p)}\big]\, |k, \sigma\rangle$$

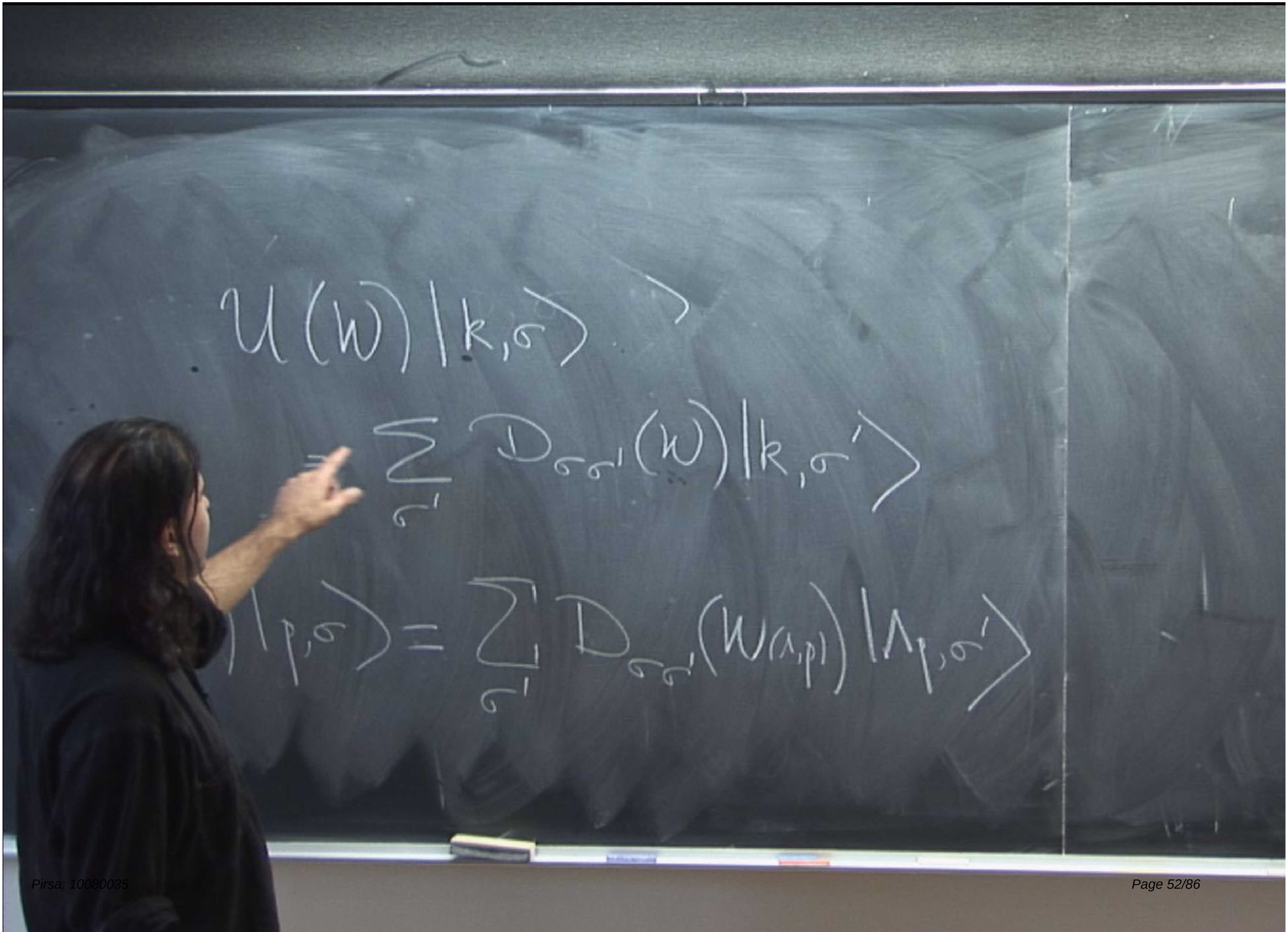
$$\boxed{W^{\mu}{}_{\nu}(\Lambda, p)\, k^{\nu} = k^{\mu}}$$

$$|k, \sigma\rangle$$


U(W)|k,σ>
(W)|k,σ'>

$$\mathcal{U}(W)|k,\sigma\rangle$$

$$= \sum_{\sigma'} D_{\sigma\sigma'}(W)|k,\sigma'\rangle$$


$\mathcal{U}(W)|k,\sigma\rangle$
$= \sum_{\sigma'} D_{\sigma\sigma'}(W)|k,\sigma'\rangle$
$)|p,\sigma\rangle = \sum_{\sigma'} D_{\sigma\sigma'}(W(\Lambda,p))|\Lambda p,\sigma'\rangle$

$$U(W)|k,\sigma\rangle$$

$$= \sum_{\sigma'} D_{\sigma\sigma'}(W)|k,\sigma'\rangle$$

$$(\Lambda)|p,\sigma\rangle = \sum_{\sigma'} D_{\sigma\sigma'}(W(\Lambda,p))|\Lambda p,\sigma'\rangle$$

$\mathcal{U}(L(\Lambda p))$ $\mathcal{U}(W)|k,\sigma\rangle$

$$= \mathcal{U}(L(\Lambda p)) \sum_{\sigma'} D_{\sigma\sigma'}(W)|k,\sigma'\rangle$$

$$(\Lambda)|p,\sigma\rangle = \sum_{\sigma'} D_{\sigma\sigma'}(W(\Lambda,p))|\Lambda p,\sigma'\rangle$$

$\mathcal{U}(L(\Lambda p))$ $\quad \mathcal{U}(W)|k,\sigma\rangle$

$= \mathcal{U}(L(\Lambda p)) \sum_{\sigma'} D_{\sigma\sigma'}(W)|k,\sigma'\rangle$

$$U(\Lambda)|p,\sigma\rangle = \sum_{\sigma'} D_{\sigma\sigma'}(W(\Lambda,p))|\Lambda p,\sigma'\rangle$$

$\mathcal{U}(L(\Lambda p))$ $\quad \mathcal{U}(W)|k,\sigma\rangle$

$$= \mathcal{U}(L(\Lambda p)) \sum_{\sigma'} D_{\sigma\sigma'}(W)|k,\sigma'\rangle$$

$$\boxed{U(\Lambda)|p,\sigma\rangle = \sum_{\sigma'} D_{\sigma\sigma'}(W(\Lambda,p))|\Lambda p,\sigma'\rangle}$$

$$= U(L(\Lambda p))\, U\big[\underbrace{L^{-1}(\Lambda p)\,\Lambda\, L(p)}_{W(\Lambda,p)}\big]\,|k,\sigma\rangle$$

$$\boxed{W^{\mu}{}_{\nu}(\Lambda,p)\, k^{\nu} = k^{\mu}}$$

"Little group"

$$W \cdot k = k$$

$$= U(L(\Lambda p))\, U[\underbrace{L^{-1}(\Lambda p)\,\Lambda\, L(p)}_{W(\Lambda, p)}]\, |k, \sigma\rangle$$

$$\boxed{W^{\mu}{}_{\nu}(\Lambda, p)\, k^{\nu} = k^{\mu}}$$

"Little group"

$$W \cdot k = k$$

$$= U(L(\Lambda p))\, U\Big[\underbrace{L^{-1}(\Lambda p)\,\Lambda\, L(p)}_{W(\Lambda, p)}\Big]|k,\sigma\rangle$$

$$\boxed{W^{\mu}{}_{\nu}(\Lambda, p)\, k^{\nu} = k^{\mu}}$$

"Little group"

$$W \cdot k = k$$

$$W^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} + \epsilon\,\omega^{\mu}{}_{\nu}$$

$$W^{\mu}{}_{\nu}\, k^{\nu} = k^{\mu} \implies \omega^{\mu}{}_{\nu}\, k^{\nu} = 0$$

$$\omega'^{\mu}{}_{\nu} = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & & & & \\ 1 & & & & \\ 2 & & & & \\ 3 & & & & \end{array} \left[\begin{array}{c|c|c|c} 0 & a & b & c \\ \hline a & 0 & & \\ \hline & & 0 & \\ \hline & & & 0 \end{array}\right] \Lambda$$

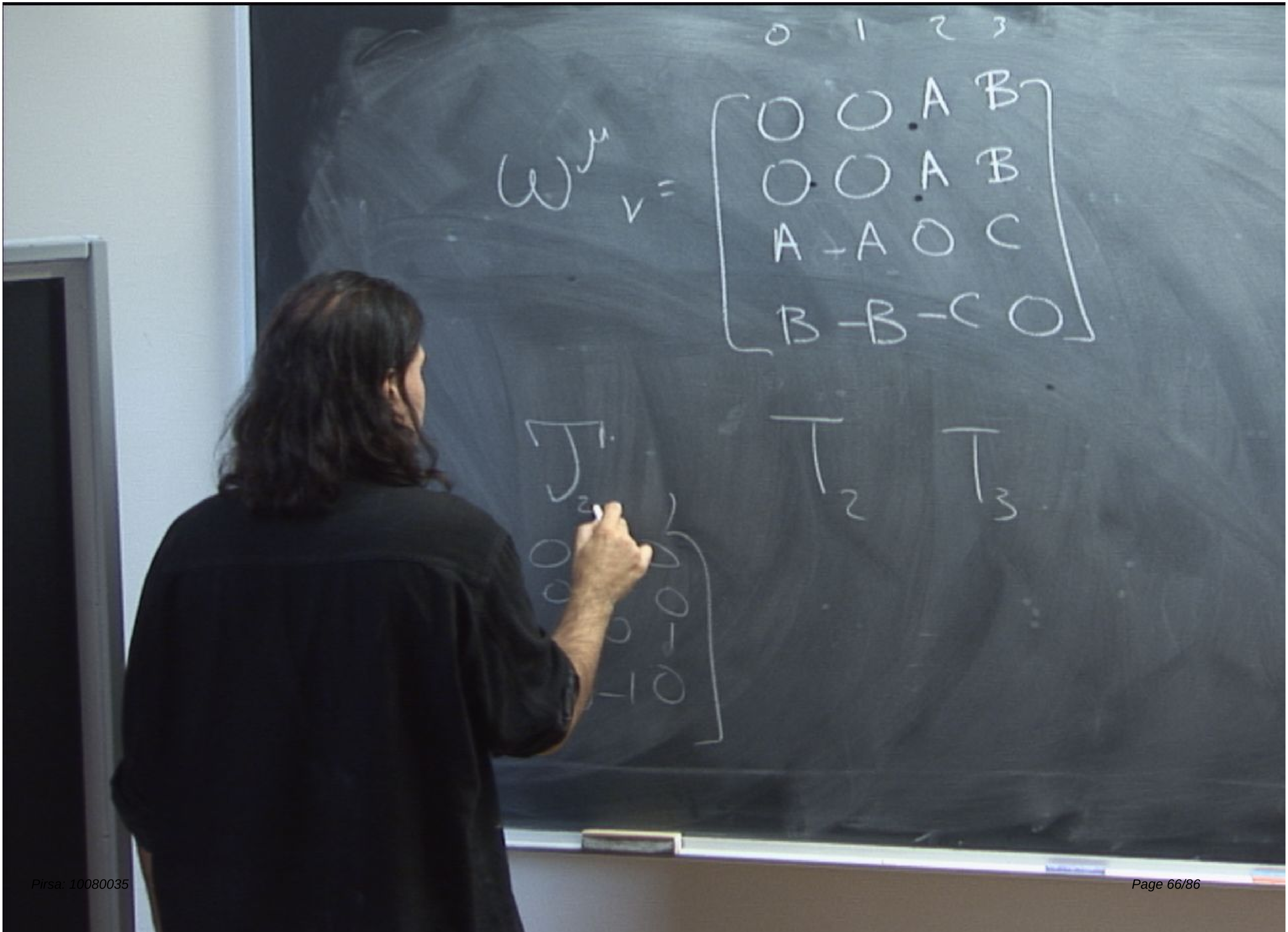
$$
\omega^{\mu}{}_{\nu} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}
\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \left[\begin{array}{c|c|c|c} 0 & a & b & c \\ \hline a & 0 & A & B \\ \hline b & -A & 0 & C \\ \hline c & -B & -C & 0 \end{array}\right] \end{array}
$$

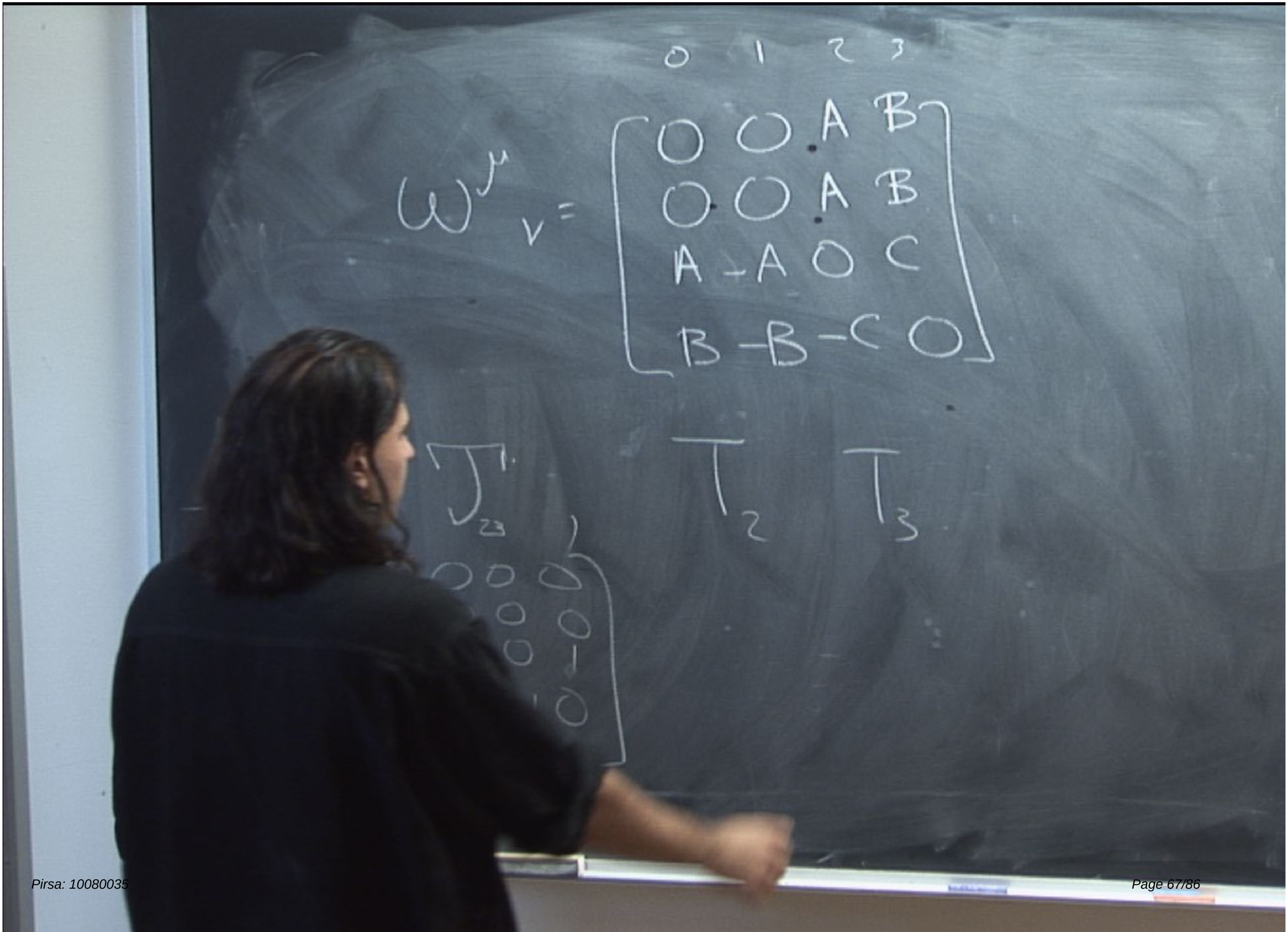
$$\omega^{\mu}{}_{\nu} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \left[\begin{array}{c|c|c|c} 0 & a & b & c \\ \hline a & 0 & A & B \\ \hline b & -A & 0 & C \\ \hline c & -B & -C & 0 \end{array}\right] \end{array} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad k = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$
\omega^{\mu}{}_{\nu} =
\begin{array}{c}
 \\ 0 \\ 1 \\ 2 \\ 3
\end{array}
\begin{array}{c}
\begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\
\begin{bmatrix}
0 & a & b & c \\
a & 0 & A & B \\
b & -A & 0 & C \\
c & -B & -C & 0
\end{bmatrix}
\end{array}
\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}
\qquad
k = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}
$$

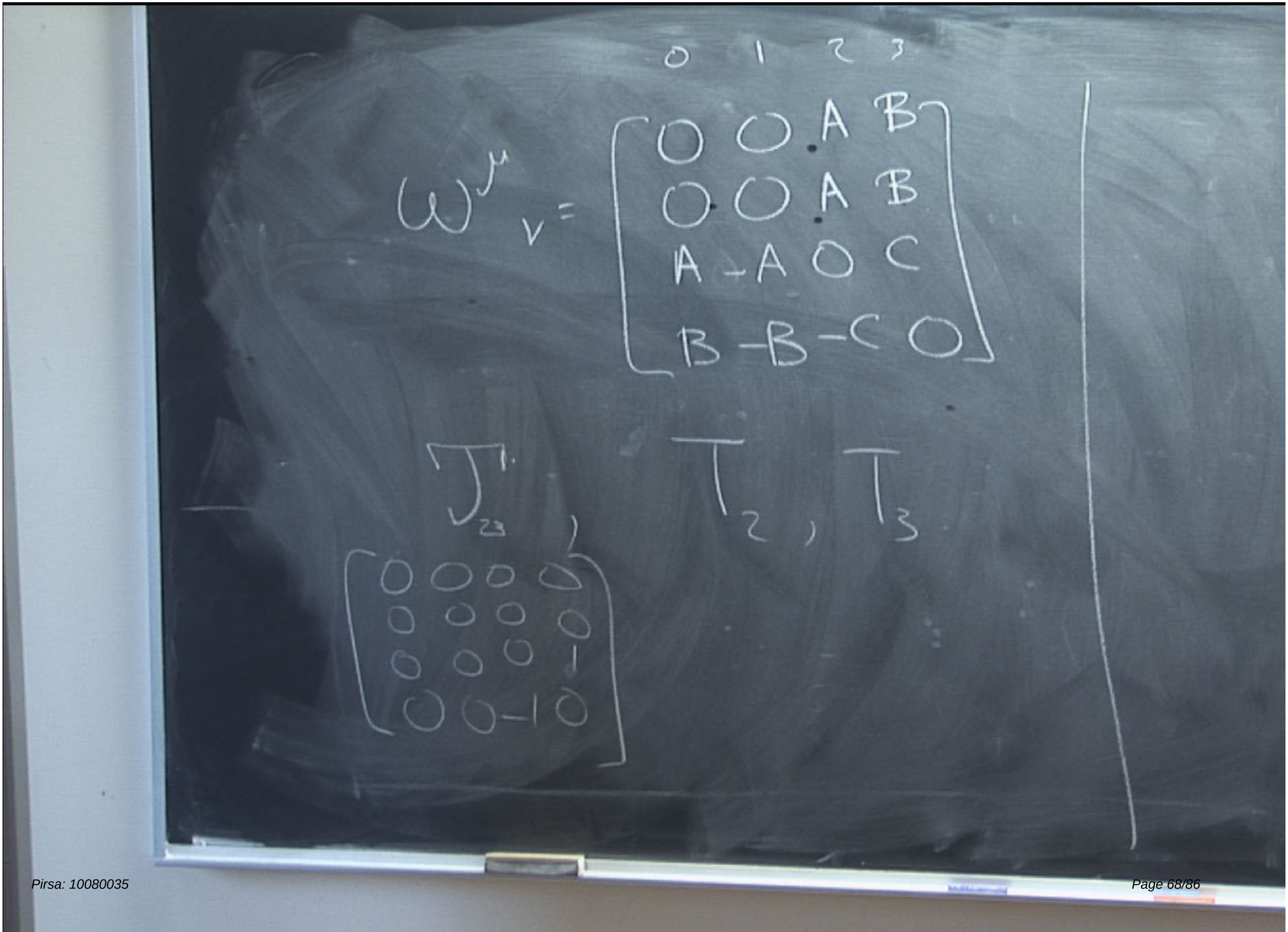
$$
= \begin{pmatrix} a \\ a \\ b - A \\ c - B \end{pmatrix}
$$

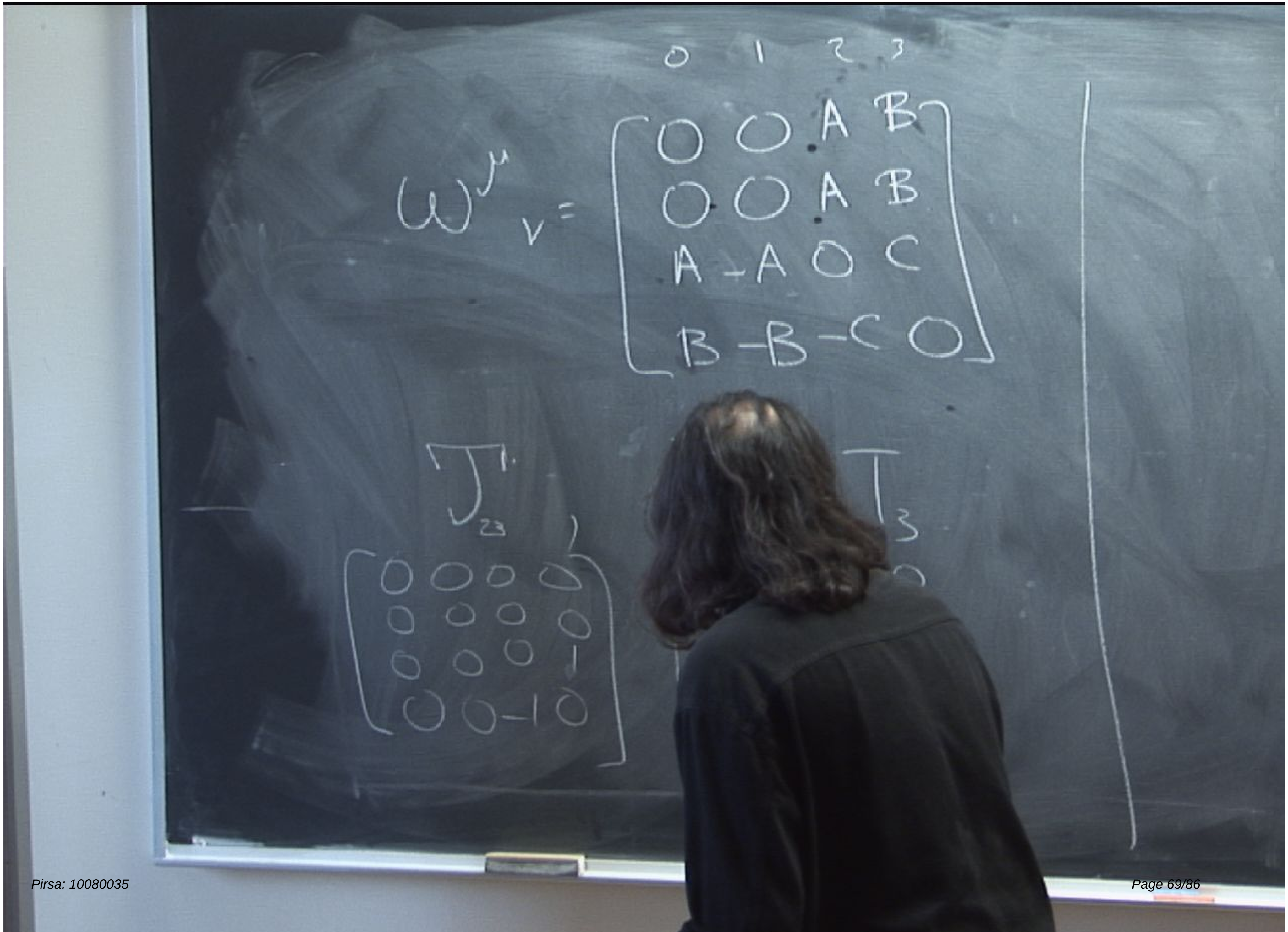
$$\omega^{\mu}{}_{\nu} = \begin{bmatrix} 0 & 0 & A & B \\ 0 & 0 & A & B \\ A & -A & 0 & C \\ B & -B & -C & 0 \end{bmatrix}$$

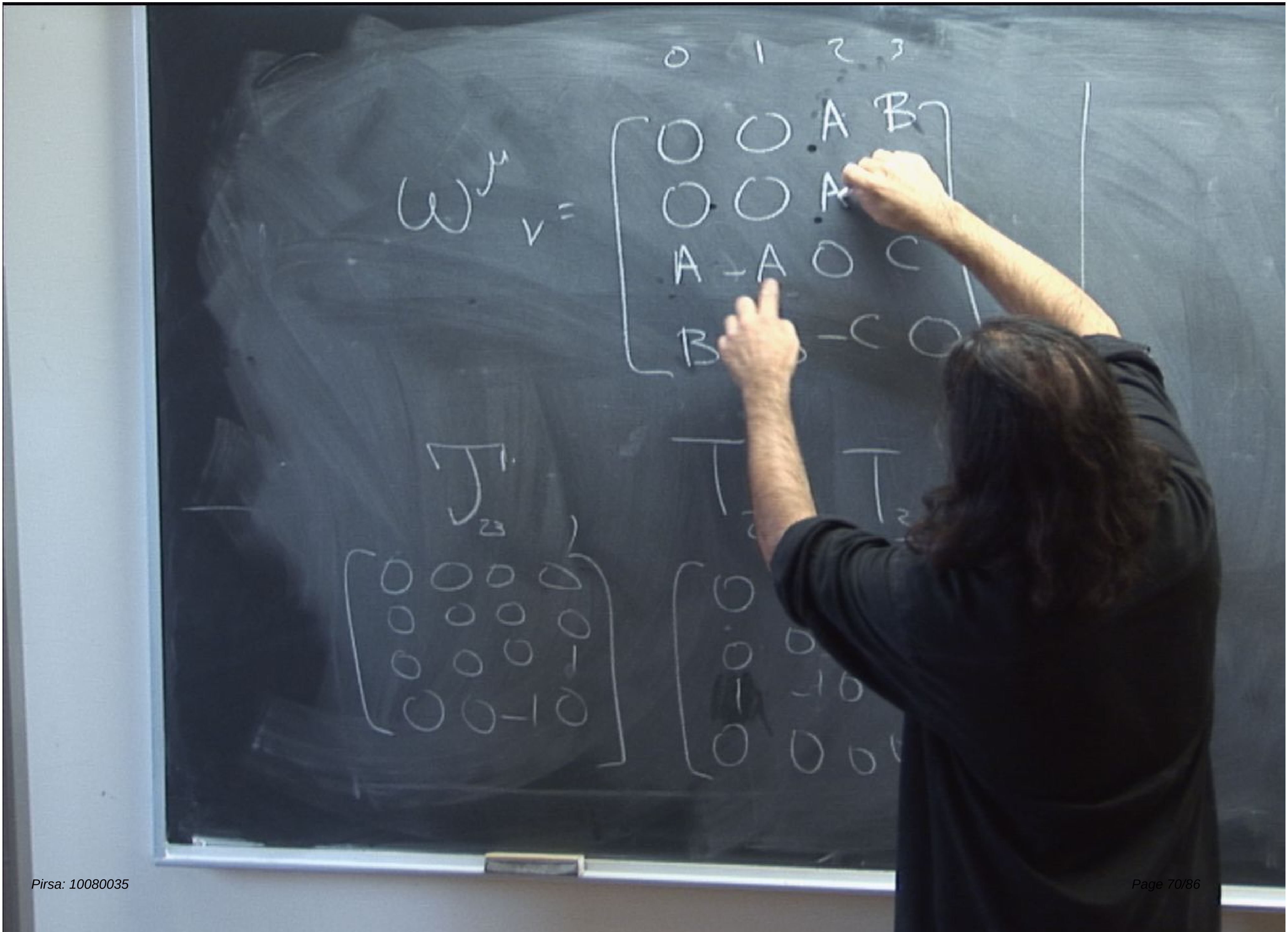

0 1 2 3
ω^μ_ν =
0 0 A B
0 0 A B
A -A 0 C
B -B -C 0
T_1
T_2
T_3



0 1 2 3
$\omega^{\mu}{}_{\nu} = \begin{bmatrix} 0 & 0 & A & B \\ 0 & 0 & A & B \\ A & -A & 0 & C \\ B & -B & -C & 0 \end{bmatrix}$
J_{23}
T_2
T_3


$$\omega^{\mu}{}_{\nu} = \begin{array}{cccc} 0 & 1 & 2 & 3 \\ \end{array}$$
$$\begin{bmatrix} 0 & 0 & A & B \\ 0 & 0 & A & B \\ A & -A & 0 & C \\ B & -B & -C & 0 \end{bmatrix}$$
J_{23} , T_2 , T_3
$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$


0 1 2 3
ω^μ_ν =
0 0 A B
0 0 A B
A -A 0 C
B -B -C 0
J_23
0 0 0 0
0 0 0 0
0 0 0 1
0 0 -1 0
T_3



$$
\omega^{\mu}{}_{\nu} = \begin{array}{c} \begin{matrix} 0 & 1 & 2 & 3 \end{matrix} \\ \begin{bmatrix} 0 & 0 & A & B \\ 0 & 0 & A & B \\ A & -A & 0 & C \\ B & -B & -C & 0 \end{bmatrix} \end{array}
$$

$$
J_{23}, \qquad T_2, \qquad T_3
$$

$$
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$\omega^{\mu}{}_{\nu} = \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & A & B \\ 0 & 0 & A & B \\ A & -A & 0 & C \\ B & -B & -C & 0 \end{bmatrix} \end{array}$$

$$J_{23}, \quad T_2, \quad T_3$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

ex.

$$[T_2, T_3] = 0$$

$$[T_2, J_{23}] = T_3$$

$$[T_3, J_{23}] = -T_2$$

$$
\begin{array}{c}
\begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\
\omega^{\mu}{}_{\nu} = \begin{bmatrix} 0 & 0 & A & B \\ 0 & 0 & A & B \\ A & -A & 0 & C \\ B & -B & -C & 0 \end{bmatrix}
\end{array}
$$

$$
J_{23}, \quad T_2, \quad T_3
$$

$$
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad
\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

ex.

$$[T_2, T_3] = 0$$

$$[T_2, J_{23}] = T_3$$

$$[T_3, J_{23}] = -T_2$$

$$E(2)$$

$$\omega^{\mu}{}_{\nu} = \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & A & B \\ 0 & 0 & A & B \\ A & -A & 0 & C \\ B & -B & -C & 0 \end{bmatrix} \end{array}$$

$$J_{23}, \quad \text{"} T_2 \text{"}, \quad \text{"} T_3 \text{"}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

ex.

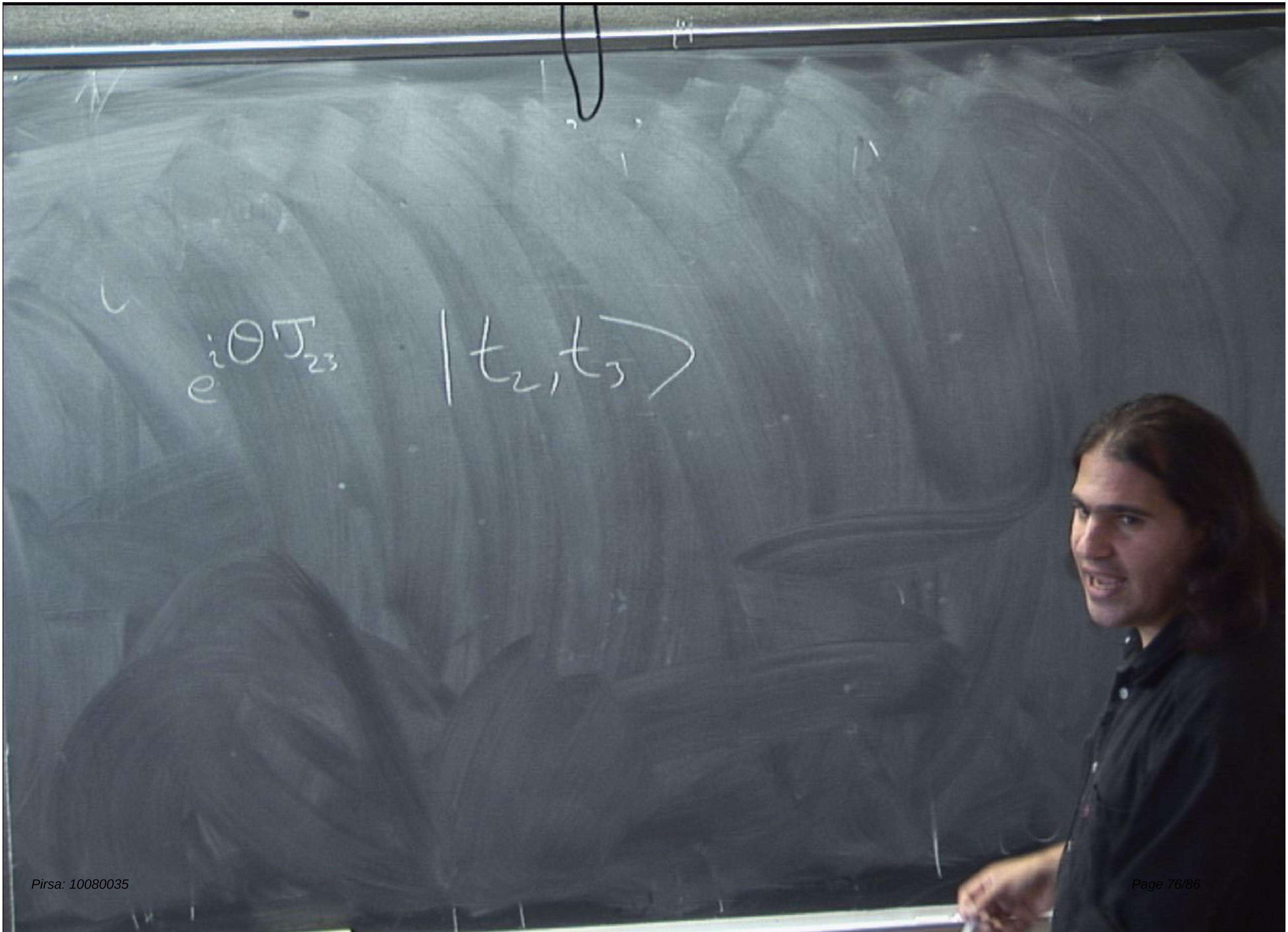
$$[T_2, T_3] = 0$$

$$[T_2, J_{23}] = T_3$$

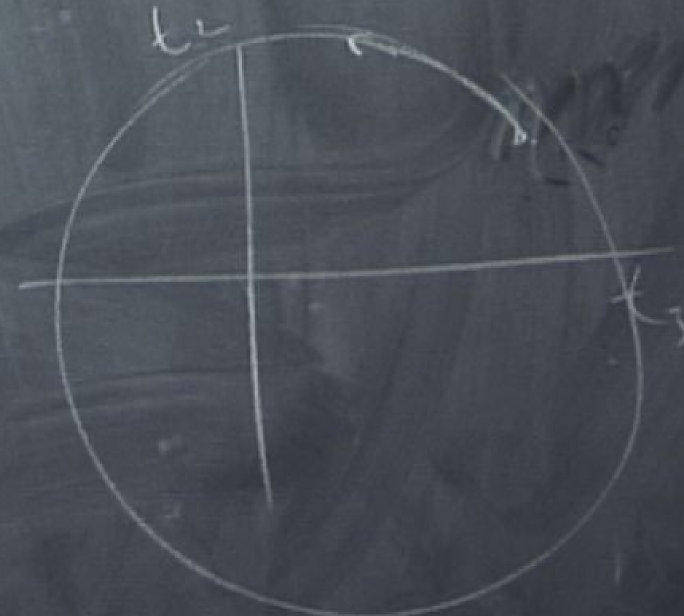
$$[T_3, J_{23}] = -T_2$$

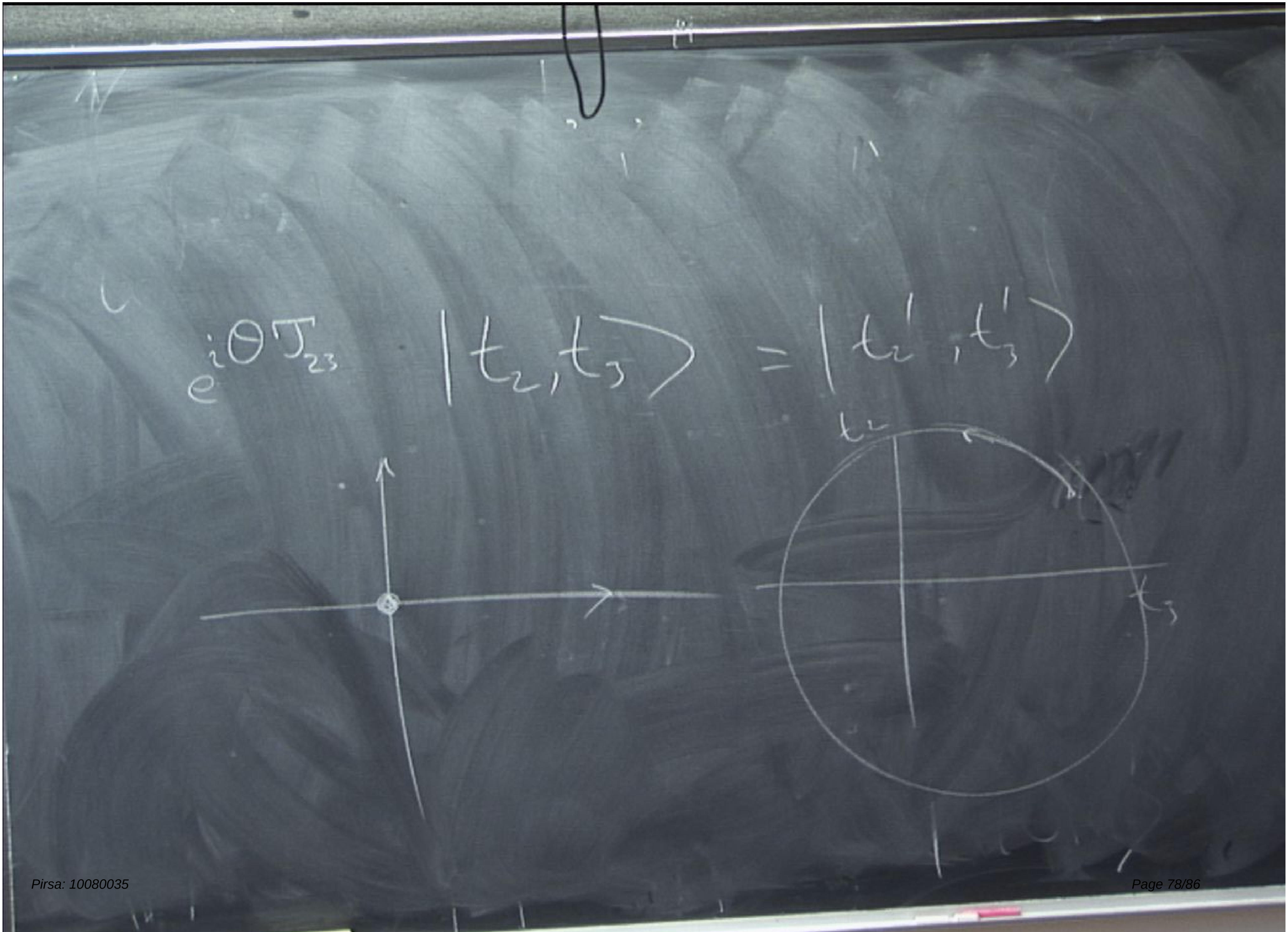
$E(2)$

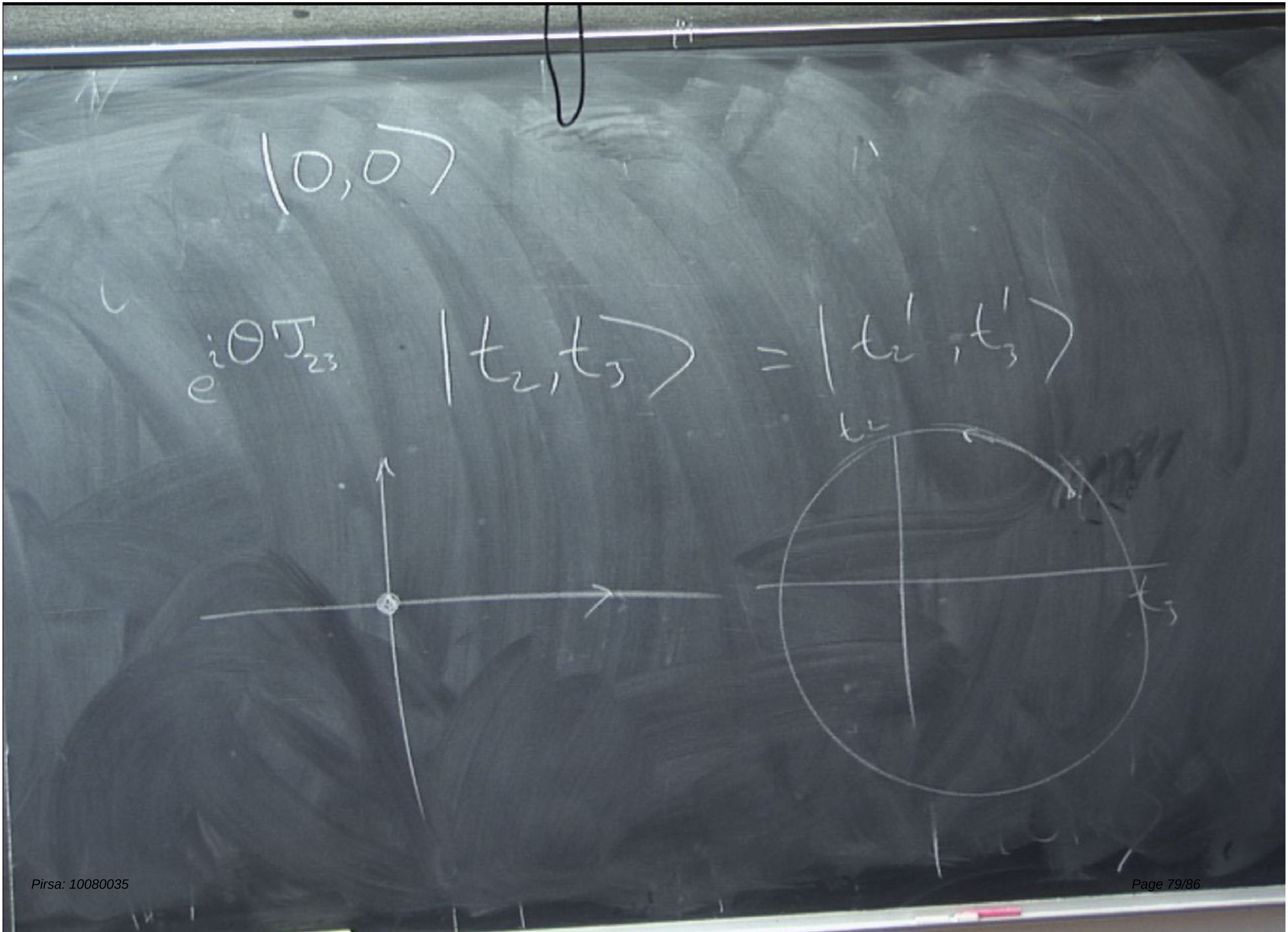
$|t_2, t_3\rangle$


$e^{i\Theta J_{23}} \quad |t_2, t_3\rangle$

$$e^{i\theta J_{23}} \, |t_2, t_3\rangle = |t_2', t_3'\rangle$$

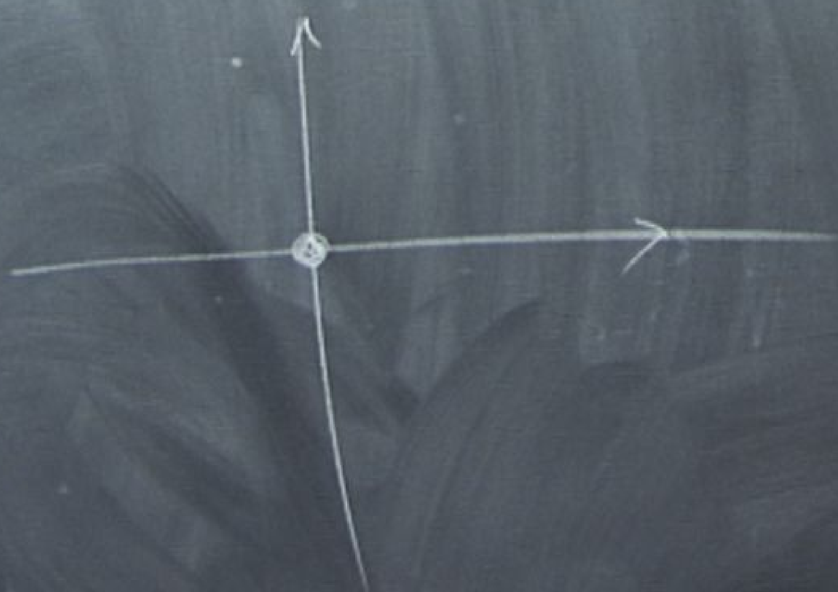


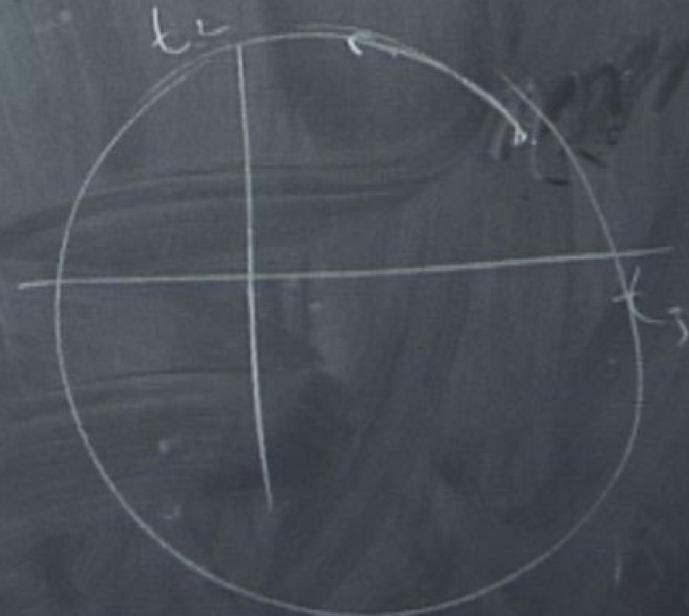

$e^{i\theta J_{23}} |t_2, t_3\rangle = |t_2', t_3'\rangle$
t_2
t_3


|0,0⟩
e^{iθJ_23} |t_2, t_3⟩ = |t_2', t_3'⟩
t_2
t_3

$$|0,0\rangle$$

$$e^{i\theta J_{23}} |t_2, t_3\rangle = |t_2', t_3'\rangle$$





Massive:

$$|p,\sigma\rangle \qquad U(\Lambda)|p,\sigma\rangle = \sum_{\sigma'} R_{\sigma\sigma'}(W)\,|\Lambda p,\sigma'\rangle$$

Massive:

$$|p,\sigma\rangle$$

$$U(\Lambda)|p,\sigma\rangle = \sum_{\sigma'} R_{\sigma\sigma'}(W)\,|\Lambda p,\sigma'\rangle$$

Massless:

$$|p,$$

Massive:

$$|p,\sigma\rangle \qquad U(\Lambda)|p,\sigma\rangle = \sum_{\sigma'} R_{\sigma\sigma'}(W)\,|\Lambda p,\sigma'\rangle$$

Massless:

$$|p,h\rangle \qquad U(\Lambda)|p,\sigma\rangle = e^{ih\Theta(W)}$$

Massive:

$$|p,\sigma\rangle \qquad U(\Lambda)|p,\sigma\rangle$$

$$= \sum_{\sigma'} R_{\sigma\sigma'}(W)\,|\Lambda p,\sigma'\rangle$$

$m=0$

Massless:

$$|p,h\rangle \qquad U(\Lambda)|p,\sigma\rangle = e^{ih\Theta(W)}\,|\Lambda p,h\rangle$$

Massive:

$|p,\sigma\rangle$. $U(\Lambda)|p,\sigma\rangle$

$= \sum_{\sigma'} R_{\sigma\sigma'}(W)\,|\Lambda p,\sigma'\rangle$

Massless:

$|p,h\rangle$, $U(\Lambda)|p,h\rangle = e^{ih\Theta(W)}|p,h\rangle$

MASSIVE

$$|p,\sigma\rangle \qquad U(\Lambda)|p,\sigma\rangle$$

$$= \sum_{\sigma'} R_{\sigma\sigma'}(W)\,|\Lambda p, \sigma'\rangle$$

massless

$$|p,h\rangle \qquad U(\Lambda)|p,h\rangle = e^{ih\Theta(W)}\,|\Lambda p, h\rangle$$