Title: PSI 2010 - Theoretical Physics - Lecture 2B

Date: Aug 24, 2010 10:30 AM

URL: http://pirsa.org/10080034

Abstract:

(Poincaré) Group (Symmetry groups more generally).

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$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x'^2 + y'^2 = x^2 + y^2$$

Poincaré) Group (Symmetry groups more generally).

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \qquad \begin{pmatrix} A & B \\ C & D \end{pmatrix} = ($$

$(x' \quad y$

groups more generally).

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \epsilon \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

$\begin{pmatrix} x \\ y \end{pmatrix}$

ré) Group (Symmetry groups more generally).

$$\begin{pmatrix}x'\\y'\end{pmatrix} = \begin{pmatrix}A & B\\C & D\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$$

$$\begin{pmatrix}x'\\y'\end{pmatrix} = \begin{pmatrix}A & B\\C & D\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

$$y')\begin{pmatrix}x'\\y'\end{pmatrix} = (x\ \ y)\begin{pmatrix}x\\y\end{pmatrix}$$

$$x' = x + (ax + by)\epsilon$$
$$y' = y + (cx + dy)\epsilon$$

$$x'^2 + y'^2 = x$$

groups more generally).

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \epsilon \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

$$x' = x + (ax + by)\epsilon$$
$$y' = y + (cx + dy)\epsilon$$

$$x'^2 + y'^2 = x^2 + y^2 + 2\epsilon\left[ax^2 + bxy + dy^2 + cxy\right]$$

groups more generally)).

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \epsilon \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

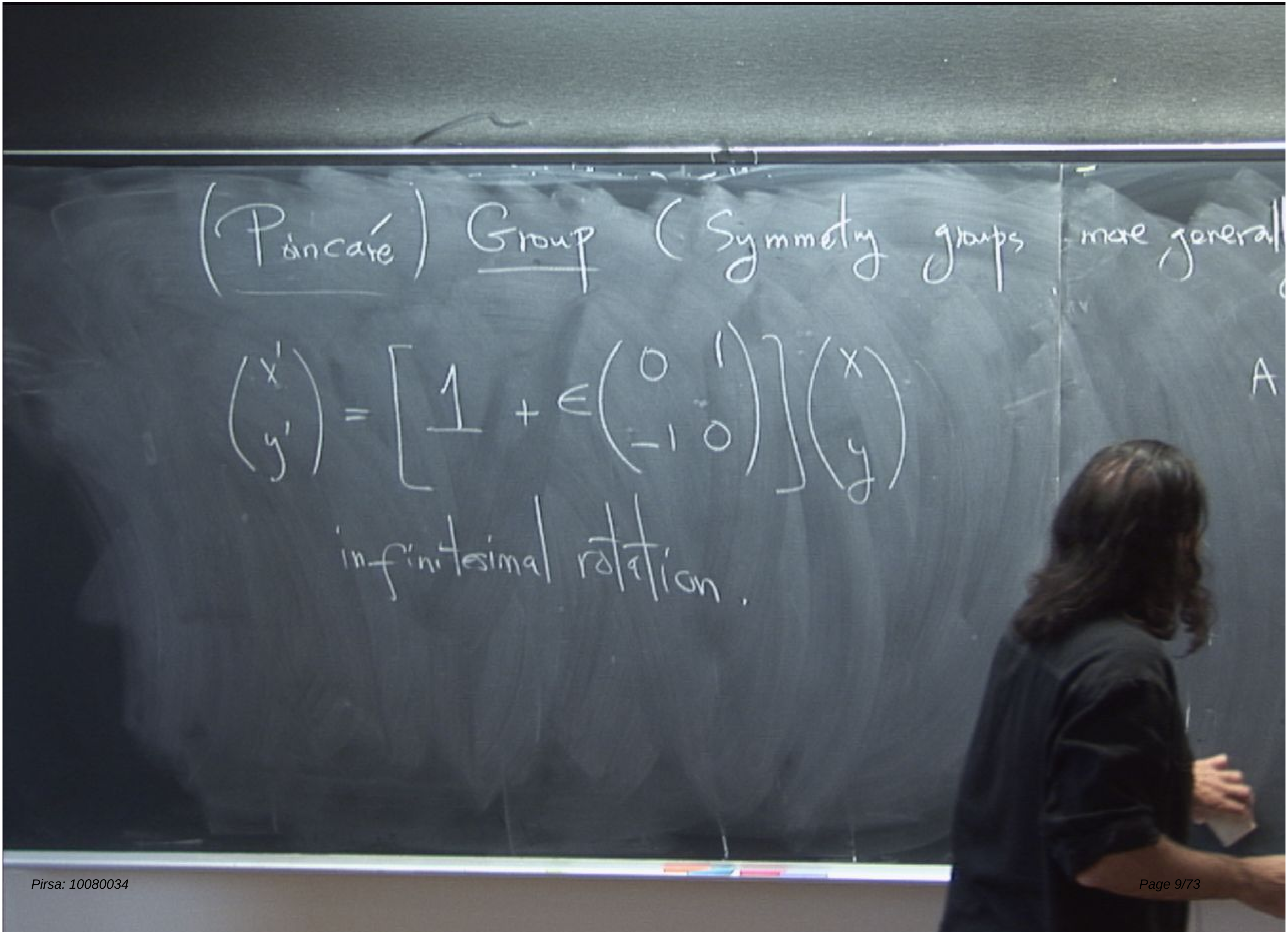
$$x' = x + (ax + by)\epsilon$$

$$y' = y + (cx + dy)\epsilon$$

$$x'^2 + y'^2 = x^2 + y^2 + 2\epsilon \underbrace{\left[ax^2 + bxy + dy^2 + cxy\right]}_{= 0}$$

$$\Rightarrow a = d = 0, \quad b = -c$$

$$ax^2 + (b+c)xy + dy^2 = 0$$


(Poincaré) Group (Symmetry groups more generall
$\begin{pmatrix} x' \\ y' \end{pmatrix} = \left[1 + \epsilon \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right] \begin{pmatrix} x \\ y \end{pmatrix}$
infinitesimal rotation.
A

roups

more generally)

$i = 1,2$

X_i

roups | more generally)

$i = 1, 2$

X_i

$X'_i =$

roups | more generally)

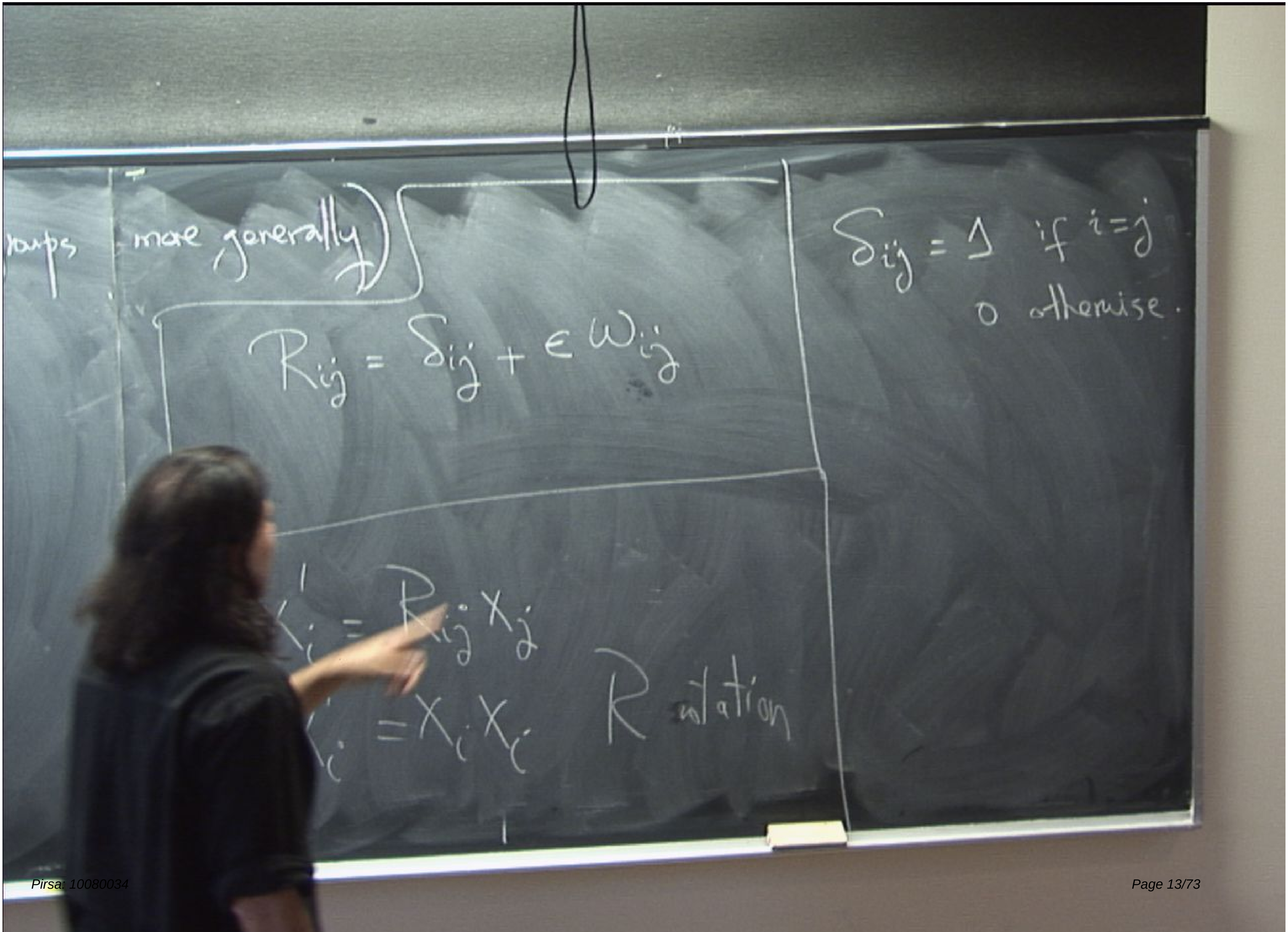
$i = 1, 2$

X_i

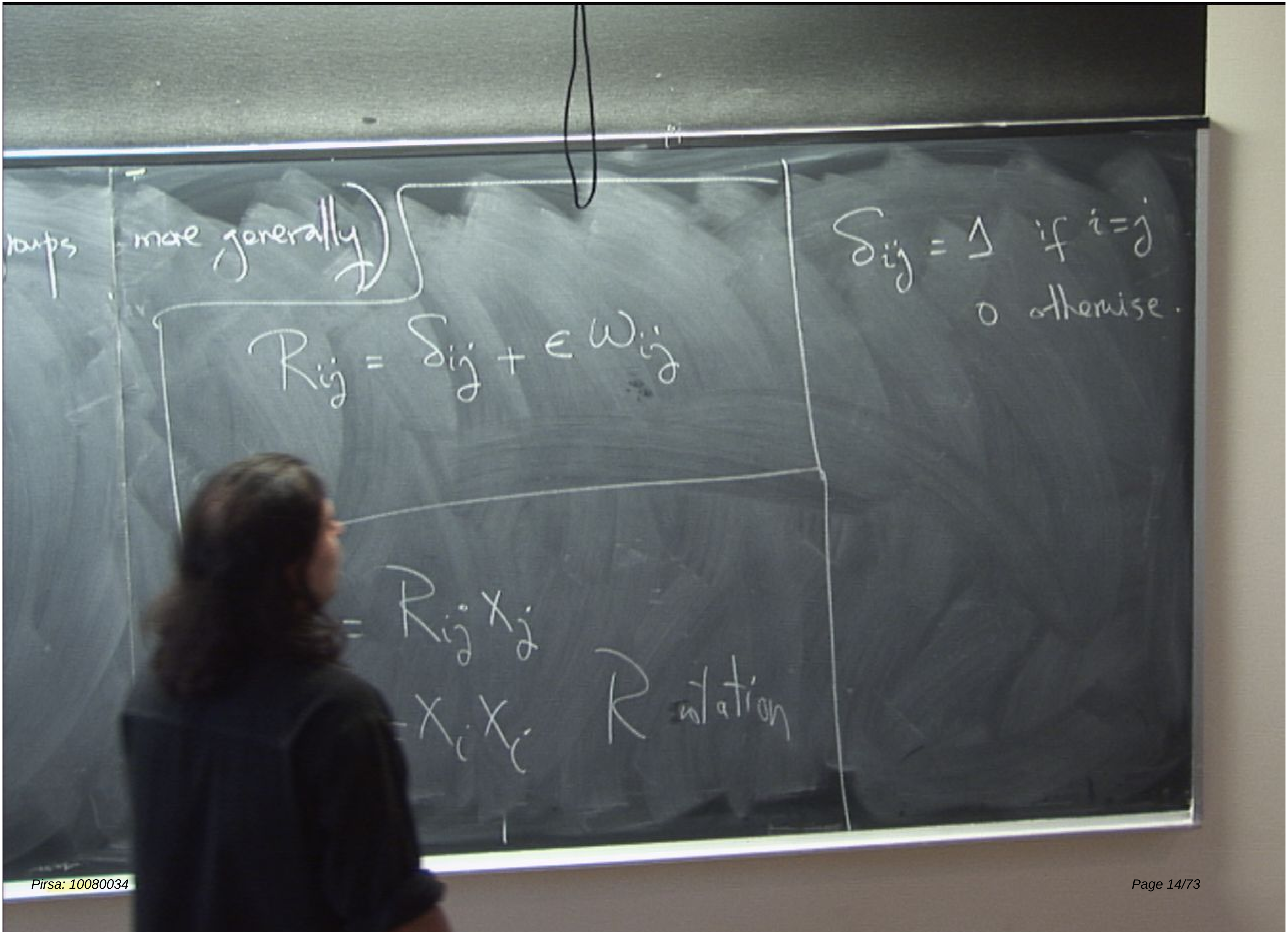
$$X'_i = R_{i1} X_1 + R_{i2} X_2 = \sum_{j=1}^{2} R_{ij} X_j$$

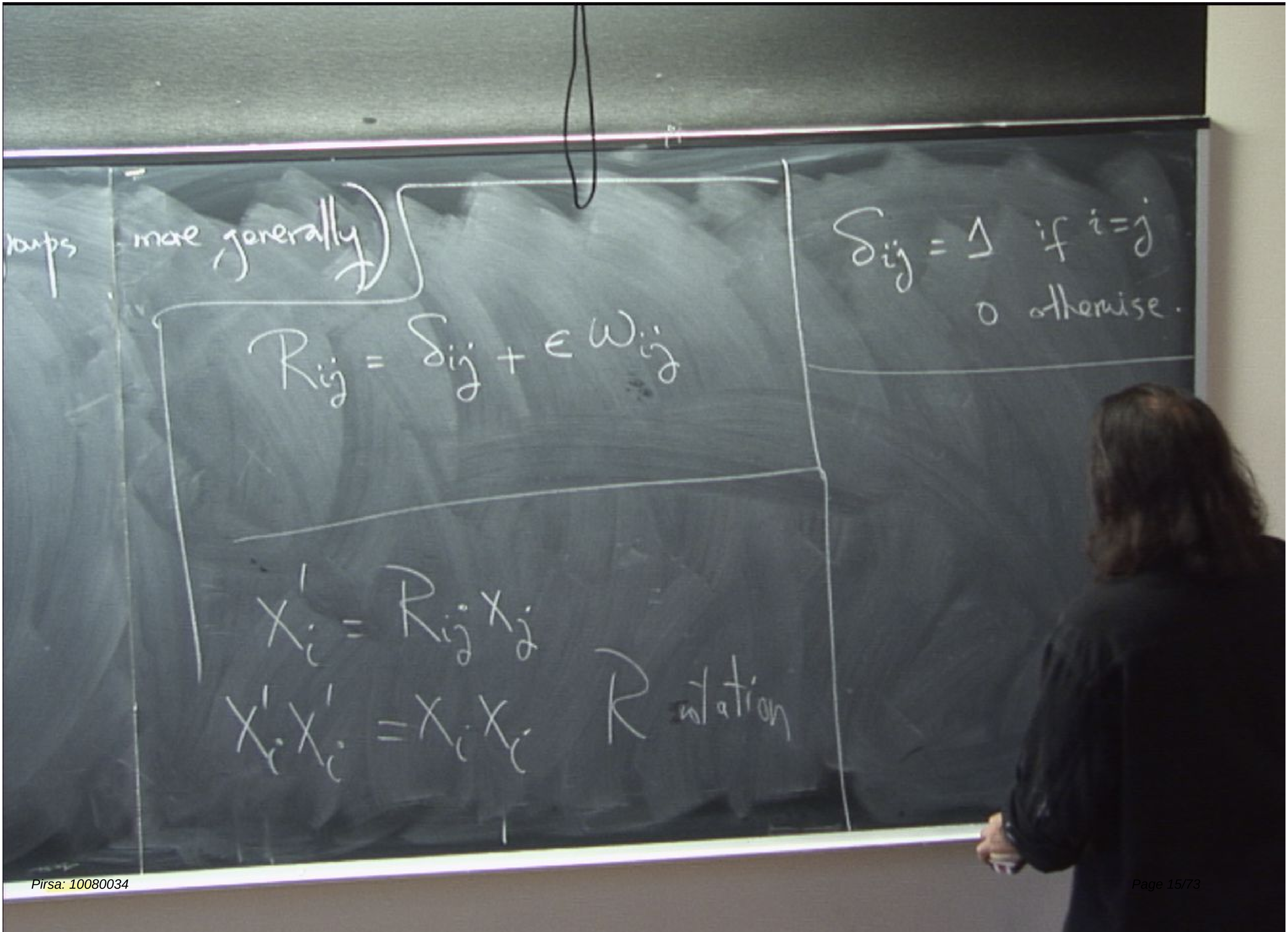
$$= R_{ij} X_j$$

$$X'_i = R_{ij} X_j$$



roups
more generally
$R_{ij} = \delta_{ij} + \epsilon \omega_{ij}$
$x'_i = R_{ij} x_j$
$x_i = x_i x_i$
Rotation
$\delta_{ij} = 1$ if $i=j$
0 otherwise.


roups
more generally
$R_{ij} = \delta_{ij} + \epsilon\, \omega_{ij}$
$\delta_{ij} = 1$ if $i=j$
0 otherwise.
$= R_{ij} x_j$
$= x_i x_i$
Rotation


roups
more generally
$R_{ij} = \delta_{ij} + \epsilon \omega_{ij}$
$X'_i = R_{ij} X_j$
$X'_i X'_i = X_i X_i$
R rotation
$\delta_{ij} = 1$ if $i=j$
0 otherwise.

roups

more generally)

$$R_{ij} = \delta_{ij} + \epsilon\, \omega_{ij}$$

$$X'_i = R_{ij} X_j$$

$$X'_i X'_i = X_i X_i$$

R otation

$\delta_{ij} = 1$ if $i=j$

0 otherwise.

$$X'_i = X_i + \epsilon\, \omega_{ij} X_j$$

$$X'_i X'_i = X_i X_i + 2\epsilon X_i \omega_{ij} X_j$$

$$\equiv 0$$

(Poincaré) Group (Symmetry groups more generall

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \left[\mathbb{1} + \epsilon \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right] \begin{pmatrix} x \\ y \end{pmatrix}$$

infinitesimal rotation.

(Poincaré) Group (Symmetry groups more generall

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \left[\mathbb{1} + \epsilon \overbrace{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}^{I} \right] \begin{pmatrix} x \\ y \end{pmatrix}$$

infinitesimal rotation.

θ

$(1 +$

more generally,

$$\boxed{R(\theta) = e^{I\theta}}$$

$$X'(\theta) = R(\theta) X .$$

$$= e^{I\theta} X$$

more generally,

$$\boxed{R(\theta) = e^{\mathbb{I}\theta}}$$

$$x'(\theta) = R(\theta)\,x.$$

$$= e^{\mathbb{I}\theta}\,x$$

$$e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

-1

groups

more generally

$\omega_{ij} x_j$

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

$$\epsilon\,\omega_{ij} = \epsilon_{12}\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \epsilon_{13}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} + \epsilon_{23}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

(Poincaré) Group (Symmetry groups more generally)

$$e^{\theta_{12} T_{12}} \, e^{\theta_{13} T_{13}} \, e^{\theta_{23} T_{23}}$$

$$= e^{\left(\varphi_{12} T_{12} + \varphi_{13} T_{13} + \varphi_{23} T_{23}\right)}$$

$$\begin{bmatrix} 0 & a & \\ -a & 0 & \\ -b & -c & \end{bmatrix}$$

$$\in \omega_{ij} =$$

(Poincaré) Group (Symmetry groups more generally)

$$e^{\theta_{12} T_{12}} e^{\theta_{13} T_{13}} e^{\theta_{23} T_{23}}$$

$$= e^{(\varphi_{12} T_{12} + \varphi_{13} T_{13} + \varphi_{23} T_{23})}$$

$$e^{A} e^{B} = e^{A + B + [A,B] + [A,[A,B]] + \cdots}$$

$$\begin{bmatrix} 0 & a \\ -a & 0 \\ -b & -c \end{bmatrix}$$

$$\in \omega_{ij} =$$

(Poincaré) Group (Symmetry groups more generally)

$$\begin{bmatrix} 0 & a & \\ -a & 0 & \\ -b & -c & \end{bmatrix}$$

$\epsilon\, \omega_{ij} =$

s   more generally)

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

$$\boxed{\begin{aligned} &\epsilon_{12} T_{12} + \epsilon_{13} T_{13} + \epsilon_{23} T_{23} \\ &= i\epsilon_3 J_3 + i\epsilon_2 J_2 + i\epsilon_1 J_1 \end{aligned}}$$

$$\epsilon\,\omega_{ij} = \epsilon_{12}\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \epsilon_{13}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} + \epsilon_{23}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

(Poincaré) Group (Symmetry groups more

$$J_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

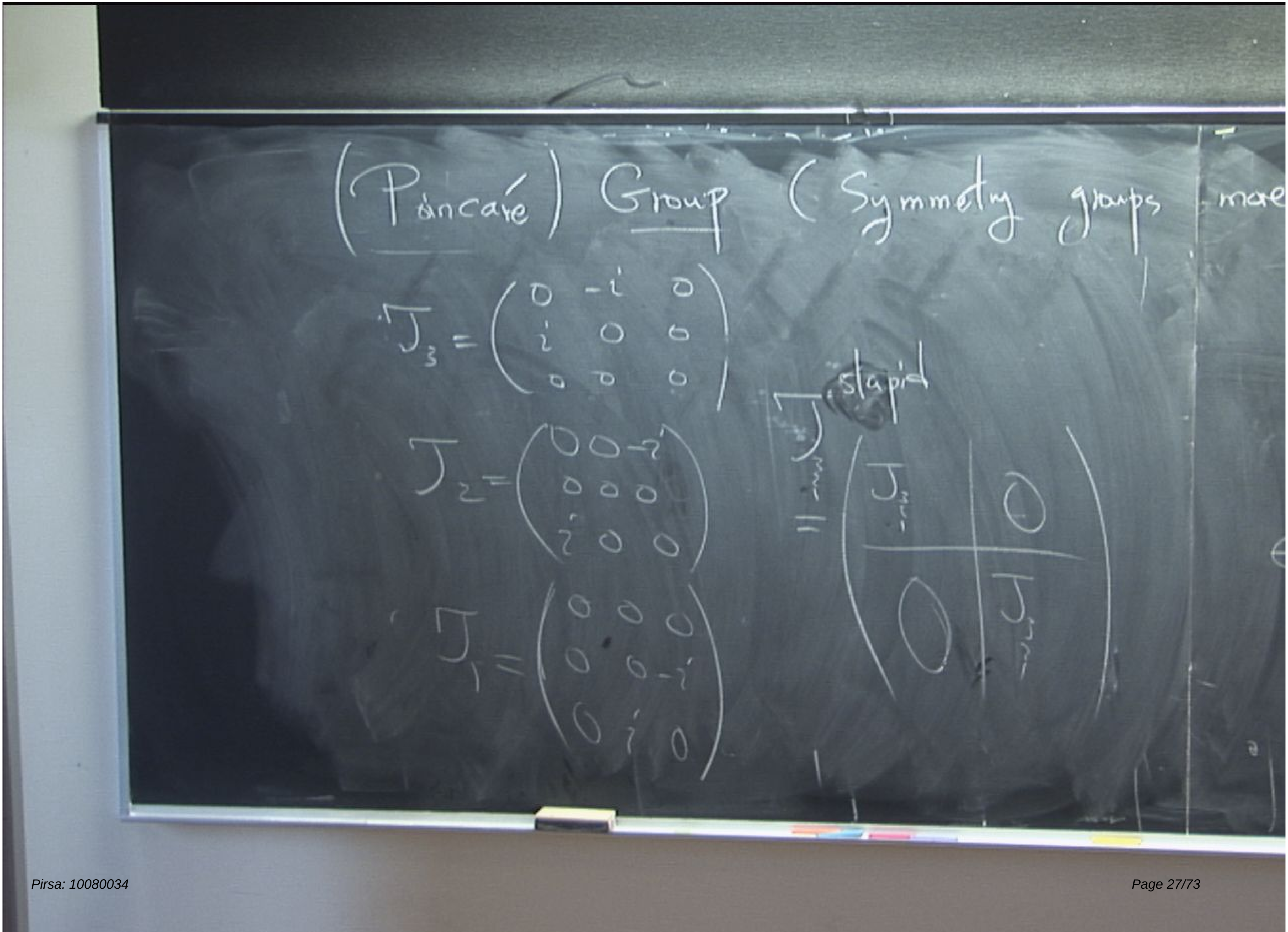
$$e^{i\theta_3 J_3}$$

$$J_2 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$[J_1, J_2] = i J_3 .$$

$+$ cyclic.

$$J_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$


(Poincaré) Group (Symmetry groups more
$J_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$J_2 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$
$J_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$
stupid
$J_i = \left(\begin{array}{c|c} J_i & 0 \\ \hline 0 & J_i \end{array}\right)$

$$\boxed{[J^a, J^b] = i f^{abc} J^c}$$

$J^a \; , \; a=1, \ldots, d$

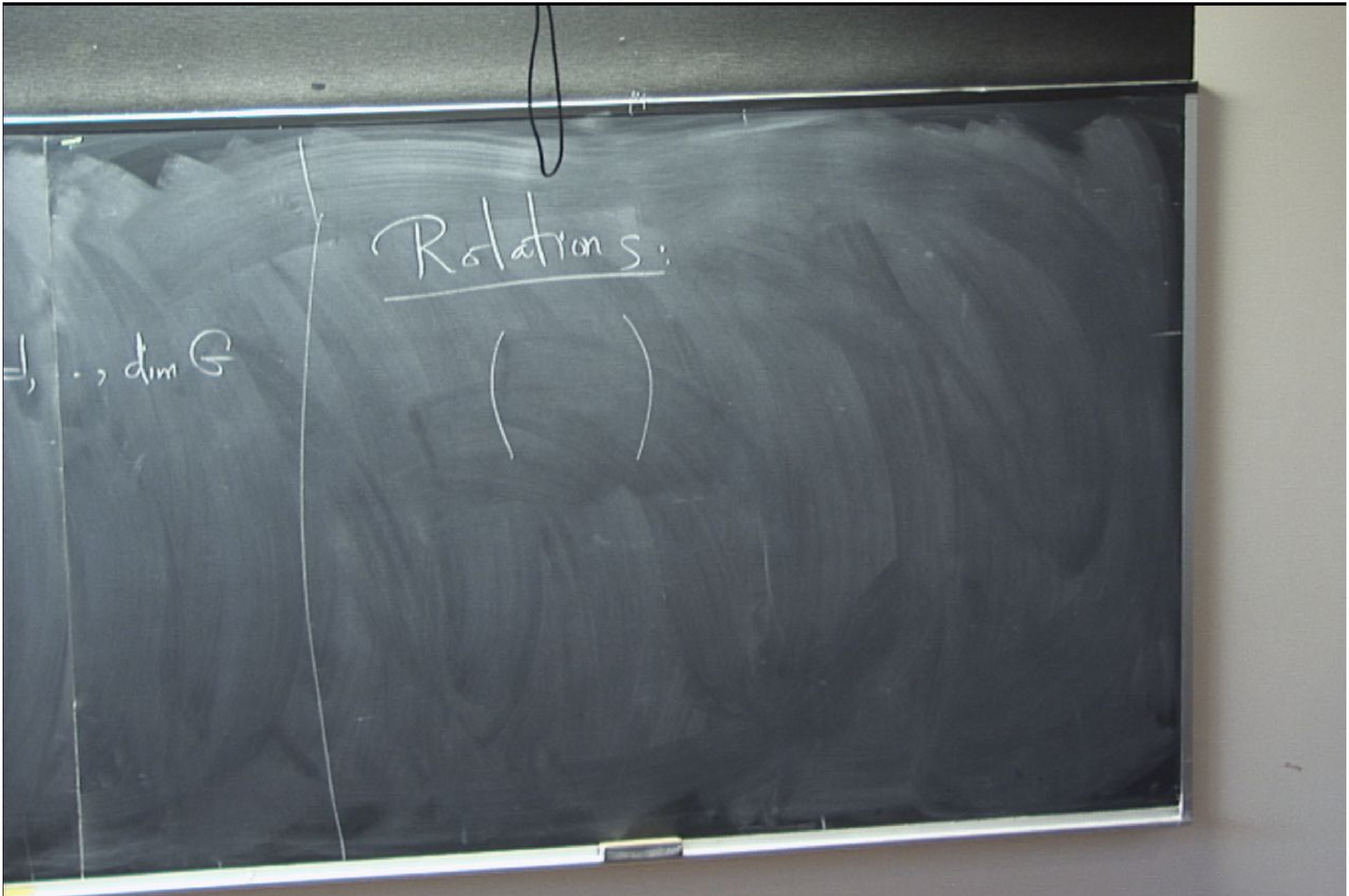
$$\boxed{[J^a, J^b] = i f^{abc} J^c}$$

J^a , $a=1, \dots, \dim G$

J^a's are Hermitian

$$\boxed{[J^a, J^b] = i f^{abc} J^c}$$

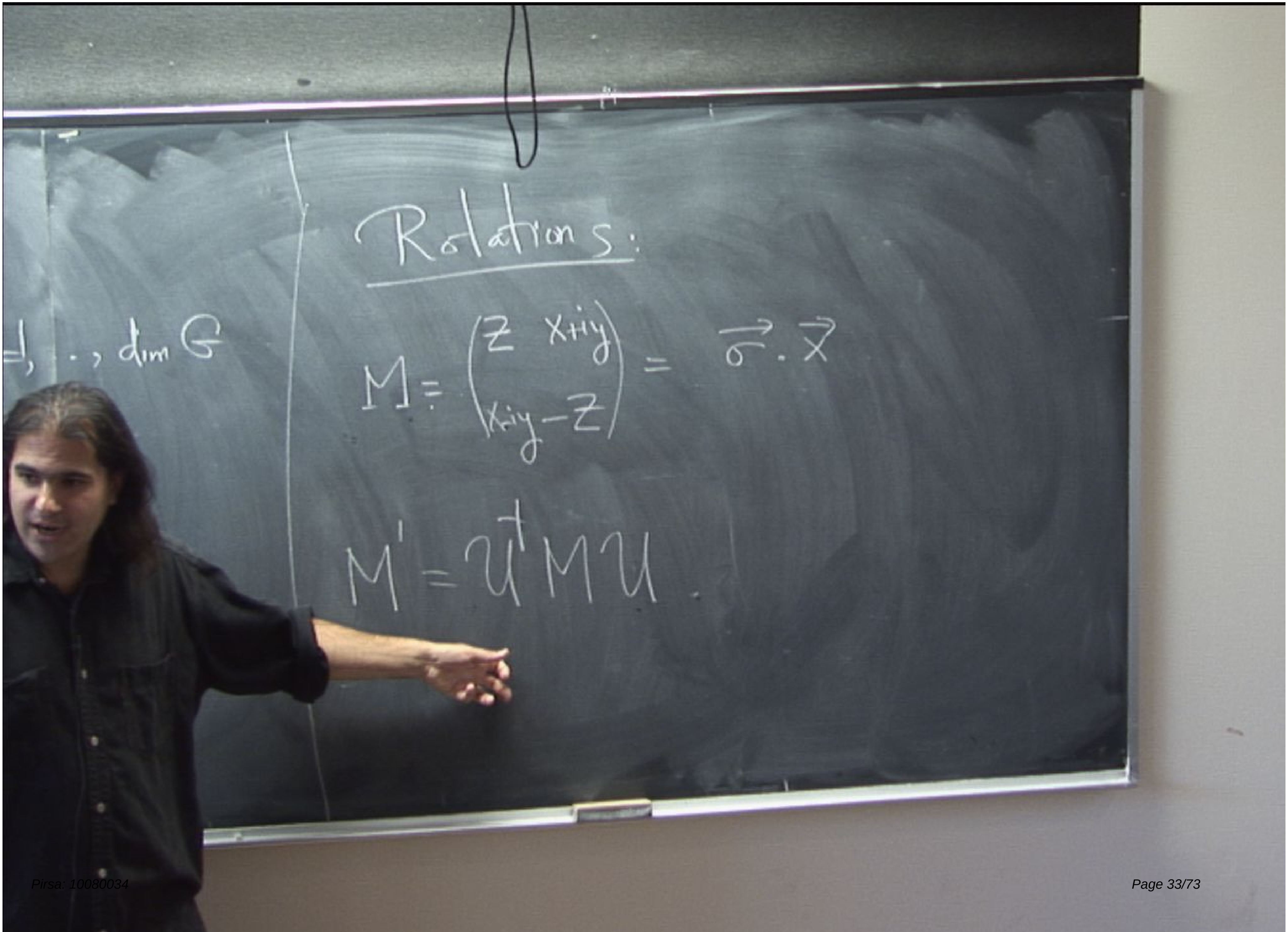
$J^a, \; a=1, \ldots, \dim G$


Relations:
$\ldots, \dim G$

$d, \ldots, \dim G$

Rotations:

$$M = \begin{pmatrix} z & x+iy \\ x-iy & -z \end{pmatrix} = \vec{\sigma} \cdot \vec{x}$$


Rotations:
d, .., dim G
$M = \begin{pmatrix} Z & x+iy \\ x-iy & -Z \end{pmatrix} = \vec{\sigma} . \vec{x}$
$M' = U^{\dagger} M U$

$d, \ldots, \dim G$

Rotations:

$$M \doteq \begin{pmatrix} z & x+iy \\ x-iy & -z \end{pmatrix} = \vec{\sigma} \cdot \vec{x}$$

$$M' = U^{\dagger} M U .$$

U unitary.
$\det U = 1.$

$U \in SU(2)$.

$\ldots, \dim G$

Rotations:

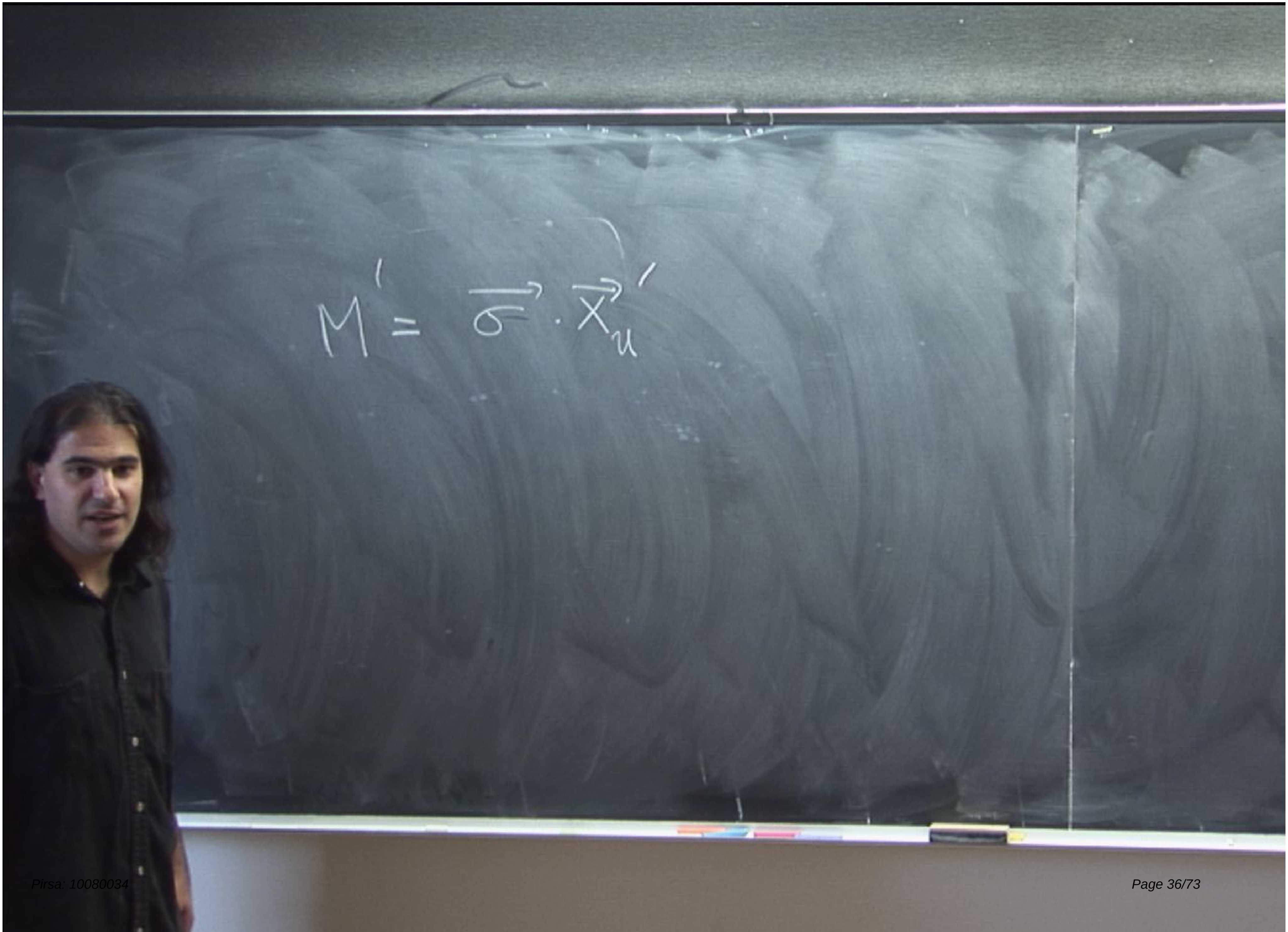
$$M = \begin{pmatrix} z & x+iy \\ x-iy & -z \end{pmatrix} = \vec{\sigma}\cdot\vec{x}$$

$$M' = U^\dagger M U .$$

U unitary. $\det U = 1$.

$U \in SU(2)$.

$$\operatorname{tr} M' = \operatorname{tr} U^\dagger M U = \operatorname{tr} M = 0$$


$M' = \vec{\sigma} \cdot \vec{X}_u'$

$$M' = \vec{\sigma} \cdot \vec{X}_u'$$

$$\vec{\sigma} \cdot \vec{x} = \sigma_x x + \sigma_y y + \sigma_z z$$

$$M' = \vec{\sigma} \cdot \vec{X}'_u \qquad M' = \begin{pmatrix} z' & x'-iy' \\ x'+iy' & -z' \end{pmatrix}$$

$$\vec{\sigma} \cdot \vec{x} = \sigma_x x + \sigma_y y + \sigma_z z$$

$$M' = \vec{\sigma} \cdot \vec{X}'_u \qquad M' = \begin{pmatrix} z' & x'-iy' \\ x'+iy' & -z' \end{pmatrix}$$

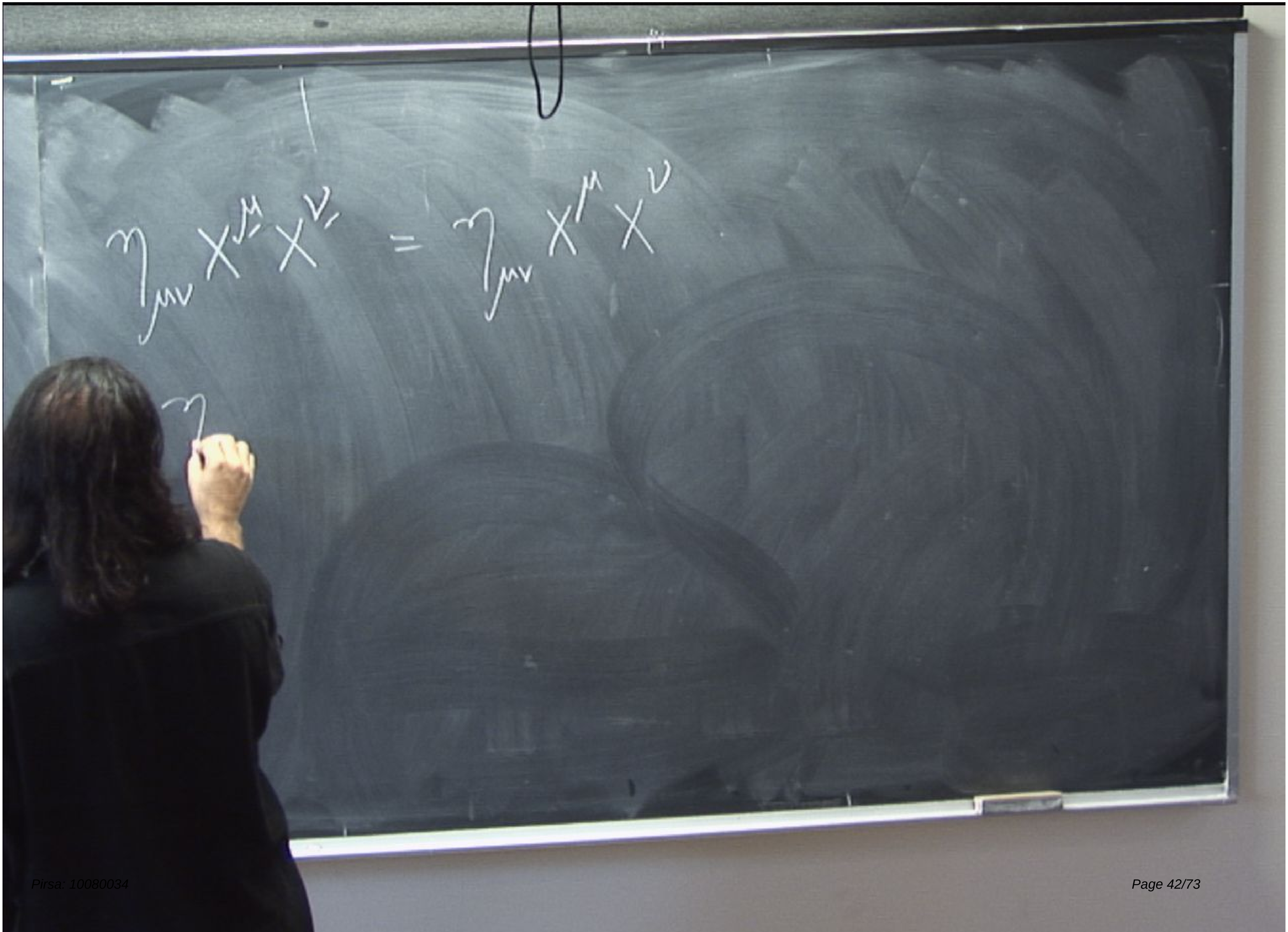
$$\vec{\sigma} \cdot \vec{x} = \sigma_x x + \sigma_y y + \sigma_z z$$

$$\det M = -z^2 - (x^2 + y^2)$$
$$= -\vec{x}^2$$

X^{μ}

$$x^\mu = (t, \vec{x})$$

$$x'^\mu = \underline{\Lambda^\mu{}_\nu}\, x^\nu$$


$\eta_{\mu\nu} X'^{\mu} X'^{\nu} = \eta_{\mu\nu} X^{\mu} X^{\nu}$
η

$$\eta_{\mu\nu} x'^{\mu} x'^{\nu} = \eta_{\mu\nu} x^{\mu} x^{\nu}$$

$$\eta_{\mu\nu} = \begin{cases} 1 & \mu = \nu = 0 \\ -1 & \mu = \nu = 1,2,3 \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$\eta_{\mu\nu} X^{\mu'} X^{\nu'} = \eta_{\mu\nu} X^{\mu} X^{\nu}$$

$$\eta_{\mu\nu} = \begin{cases} 1 & \mu = \nu = 0 \\ -1 & \mu = \nu = 1,2,3 \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$x_\mu = (t, -\vec{x})$$

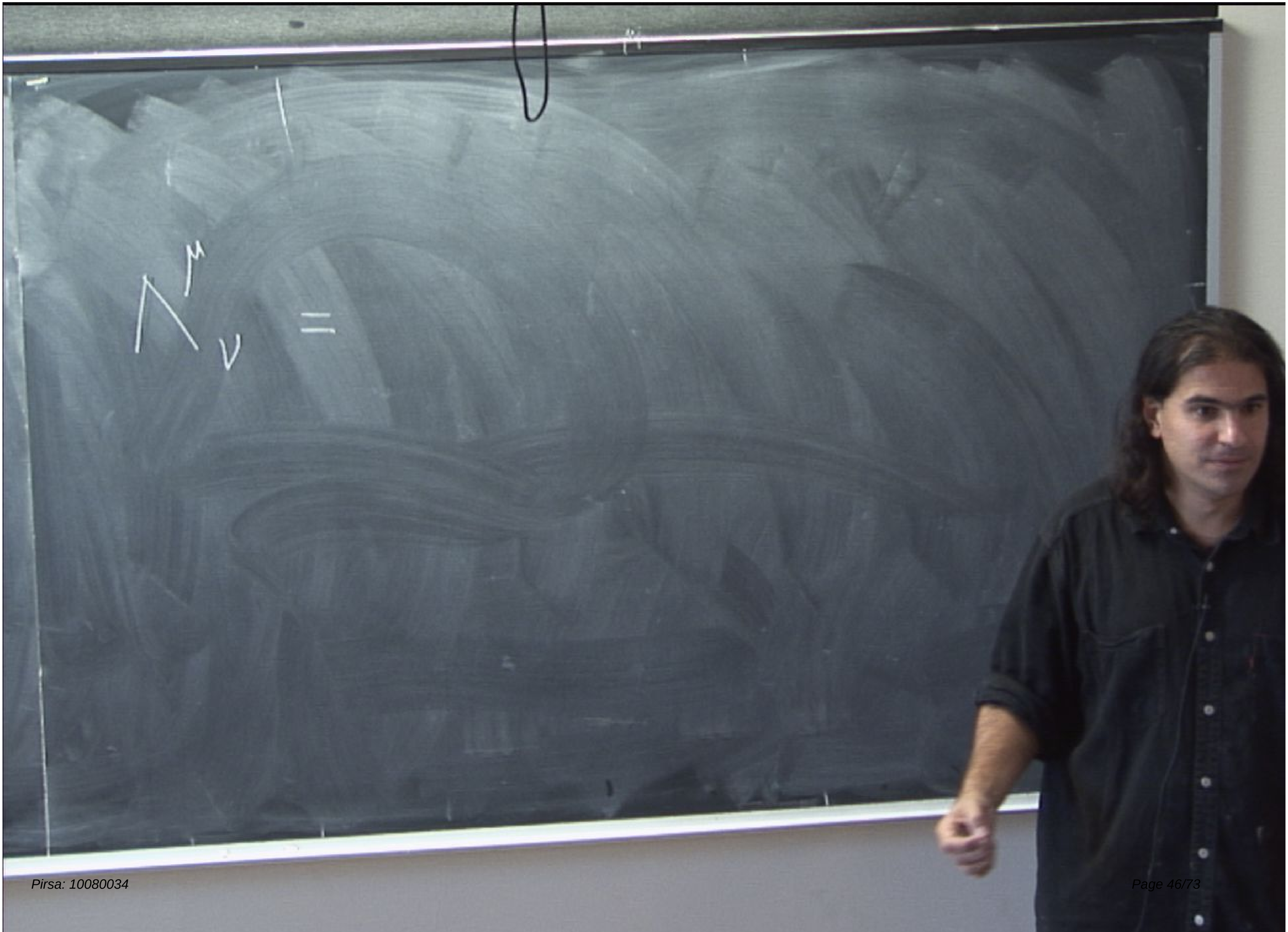
$$x^\mu = (t, \vec{x}) \qquad x_\mu = \eta_{\mu\nu} x^\nu$$

$$x'^\mu = \underline{\Lambda^\mu{}_\nu} \, x^\nu$$

$$\eta_{\mu\nu} x^\mu x^\nu$$

$$= x_\mu x^\mu$$

η


$\Lambda^{\mu}{}_{\nu} =$

$$\Lambda^{\mu}{}_{\nu} = \delta^{\mu}_{\nu} + \epsilon\, \omega^{\mu}{}_{\nu}$$

$$x'^{\mu} = x^{\mu} + \epsilon\, \omega^{\mu}{}_{\nu} x^{\nu}$$

$$\Lambda^\mu{}_\nu = \delta^\mu_\nu + \epsilon\,\omega^\mu{}_\nu$$

$$x'^\mu = x^\mu + \epsilon\,\omega^\mu{}_\rho x^\rho$$

$$x'^\nu x'^\mu = \eta_{\mu\nu} x^\mu x^\nu$$

$$= \eta_{\mu\nu} x^\mu x^\nu + 2\epsilon\,\eta_{\mu\nu} x^\nu \omega^\mu{}_\rho x^\rho$$

$$\Lambda^\mu{}_\nu = \delta^\mu_\nu + \epsilon\,\omega^\mu{}_\nu$$

$$X'^\mu = X^\mu + \epsilon\,\omega^\mu{}_\rho X^\rho$$

$$X'^\nu X'^\mu = \eta_{\mu\nu} X^\mu X^\nu$$

$$\shortparallel$$

$$\eta_{\mu\nu} X^\mu X^\nu + 2\epsilon\,\eta_{\mu\nu} X^\nu\,\omega^\mu{}_\rho X^\rho$$

$$\boxed{\Rightarrow \underbrace{\eta_{\mu\nu}\,\omega^\mu{}_\rho}_{\equiv\ \omega_{\nu\rho}}\, X^\nu X^\rho = 0}$$

$$\Lambda^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} + \epsilon\, \omega^{\mu}{}_{\nu}$$

$$\omega_{\nu\rho} = -\omega_{\rho\nu}$$

antisymm. 4x4 matrices.

$$x'^{\mu} = x^{\mu} + \epsilon\, \omega^{\mu}{}_{\rho}\, x^{\rho}$$

$$\Rightarrow \underbrace{\eta_{\mu\nu}\, \omega^{\mu}{}_{\rho}}_{\equiv}\; x^{\nu} x^{\rho} = 0$$

$$\omega_{\nu\rho}\, x^{\nu} x^{\rho} = 0$$

$$\eta_{\mu\nu}\, x'^{\nu} x'^{\mu} = \eta_{\mu\nu}\, x^{\mu} x^{\nu}$$

$$= \eta_{\mu\nu}\, x^{\mu} x^{\nu} + 2\epsilon\, \eta_{\mu\nu}\, x^{\nu}\, \omega^{\mu}{}_{\rho}\, x^{\rho}$$

$$\Lambda^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} + \epsilon\, \omega^{\mu}{}_{\nu}$$

$$x'^{\mu} = x^{\mu} + \epsilon\, \omega^{\mu}{}_{\rho} x^{\rho}$$

$$x'^{\nu} x'^{\mu} = \eta_{\mu\nu} x^{\mu} x^{\nu}$$

$$\shortparallel$$

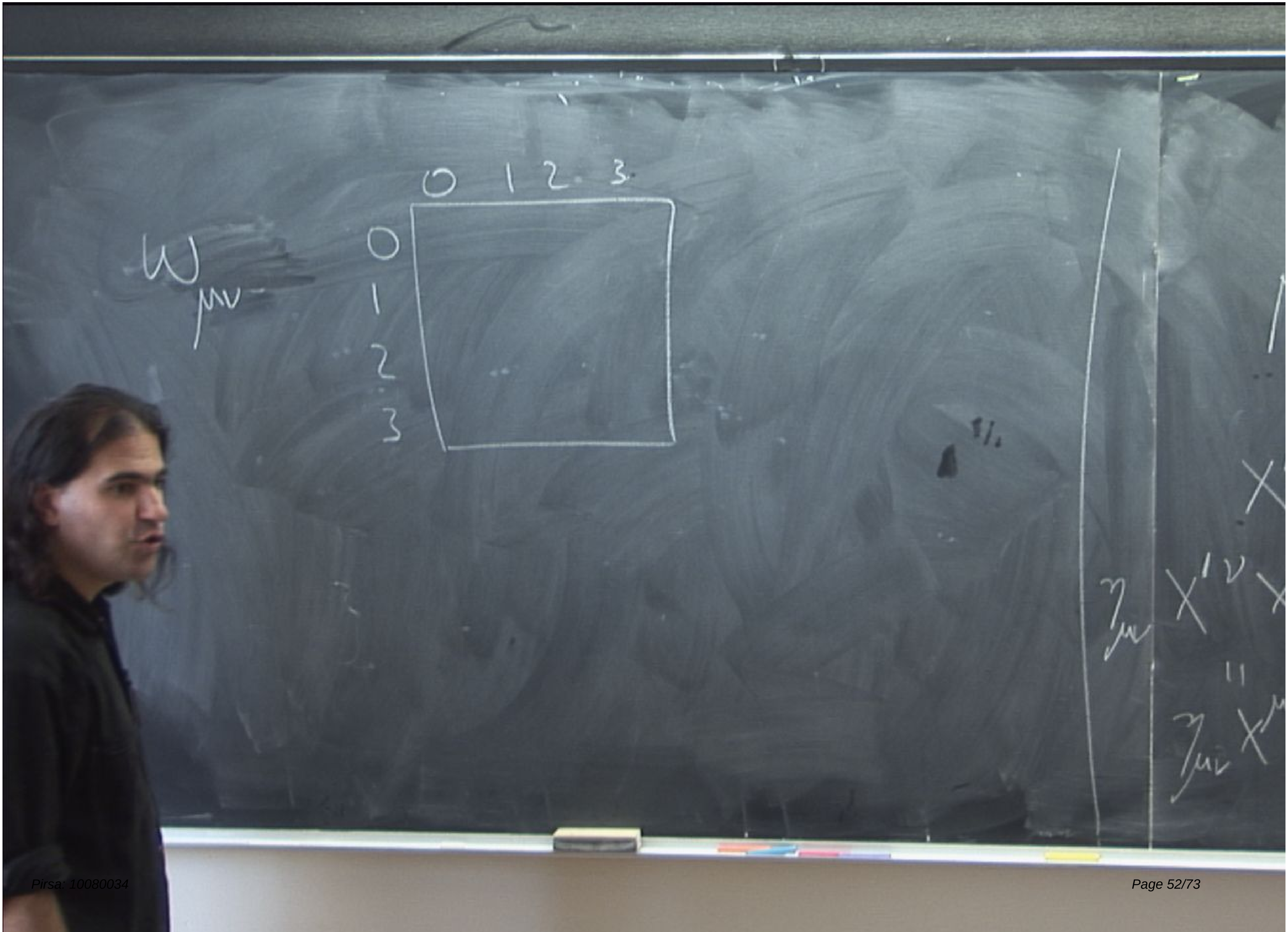
$$\eta_{\mu\nu} x^{\mu} x^{\nu} + 2\epsilon\, \eta_{\mu\nu} x^{\nu} \omega^{\mu}{}_{\rho} x^{\rho}$$

$$\omega_{\nu\rho} = -\omega_{\rho\nu}$$

antisymm. 4x4 matrices.

$$\Rightarrow \underbrace{\eta_{\mu\nu}\, \omega^{\mu}{}_{\rho}}_{\equiv}\, x^{\nu} x^{\rho} = 0$$

$$\omega_{\nu\rho} x^{\nu} x^{\rho} = 0$$


0 1 2 3
$W_{\mu\nu}$
0
1
2
3
$\eta_{\mu\nu}$ X'^{ν}
$\eta_{\mu\nu}$

$$W_{\mu\nu} = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & a & b & c \\ 1 & -a & 0 & A & B \\ 2 & -b & -A & 0 & C \\ 3 & -c & -B & -C & 0 \end{array}$$

$\eta_{\mu\nu} X'^{\nu} X$

$=$

$\eta_{\mu\nu} X^{\mu}$

$$\omega_{\mu\nu} \qquad \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & a & b & c \\ 1 & -a & 0 & A & B \\ 2 & -b & -A & 0 & C \\ 3 & -c & -B & -C & 0 \end{array}$$

$$\omega^{\mu}{}_{\nu}$$

$$\eta_{\mu\nu} X'^{\nu} X' \ldots$$

$$= \eta_{\mu\nu} X^{\mu} \ldots$$

$\omega^{\mu}{}_{\nu}$

	0	1	2	3
0	0	a	b	c
1	±a	0	A	B
2	±b	±A	0	C
3	±c	±B	±C	0

$$\eta_{\rho\mu}\omega^{\mu}{}_{\nu} = \omega_{\rho\nu}$$

$$\eta_{\mu\nu} X'^{\mu} X'^{\nu} = \eta_{\mu\nu} X^{\mu} X$$

$$\omega^{\mu}{}_{\nu}$$

	0	1	2	3
0	0	a	b	c
1	±a	0	A	B
2	±b	±A	0	C
3	±c	±B	±C	0

$$\omega_{0i} = -\omega_{i0}$$

$$\Longrightarrow \omega^{0}{}_{i} = +\omega^{i}{}_{0}$$

$$\eta_{\rho\mu}\,\omega^{\mu}{}_{\nu} = \omega_{\rho\nu}$$

$$\omega_{0i} = \omega^{0}{}_{i}$$

$$\omega_{i0} = -\omega^{i}{}_{0}.$$

$$\eta_{\mu\nu} X'^{\nu} X \ldots = \eta_{\mu\nu} X^{\mu} \ldots$$

$$\omega^{\mu}{}_{\nu} \rightarrow \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & a & b & c \\ 1 & a & 0 & A & B \\ 2 & b & -A & 0 & C \\ 3 & c & -B & -C & 0 \end{array}$$

$$\omega_{0i} = -\omega_{i0}$$

$$\Longrightarrow \omega^{0}{}_{i} = +\omega^{i}{}_{0}$$

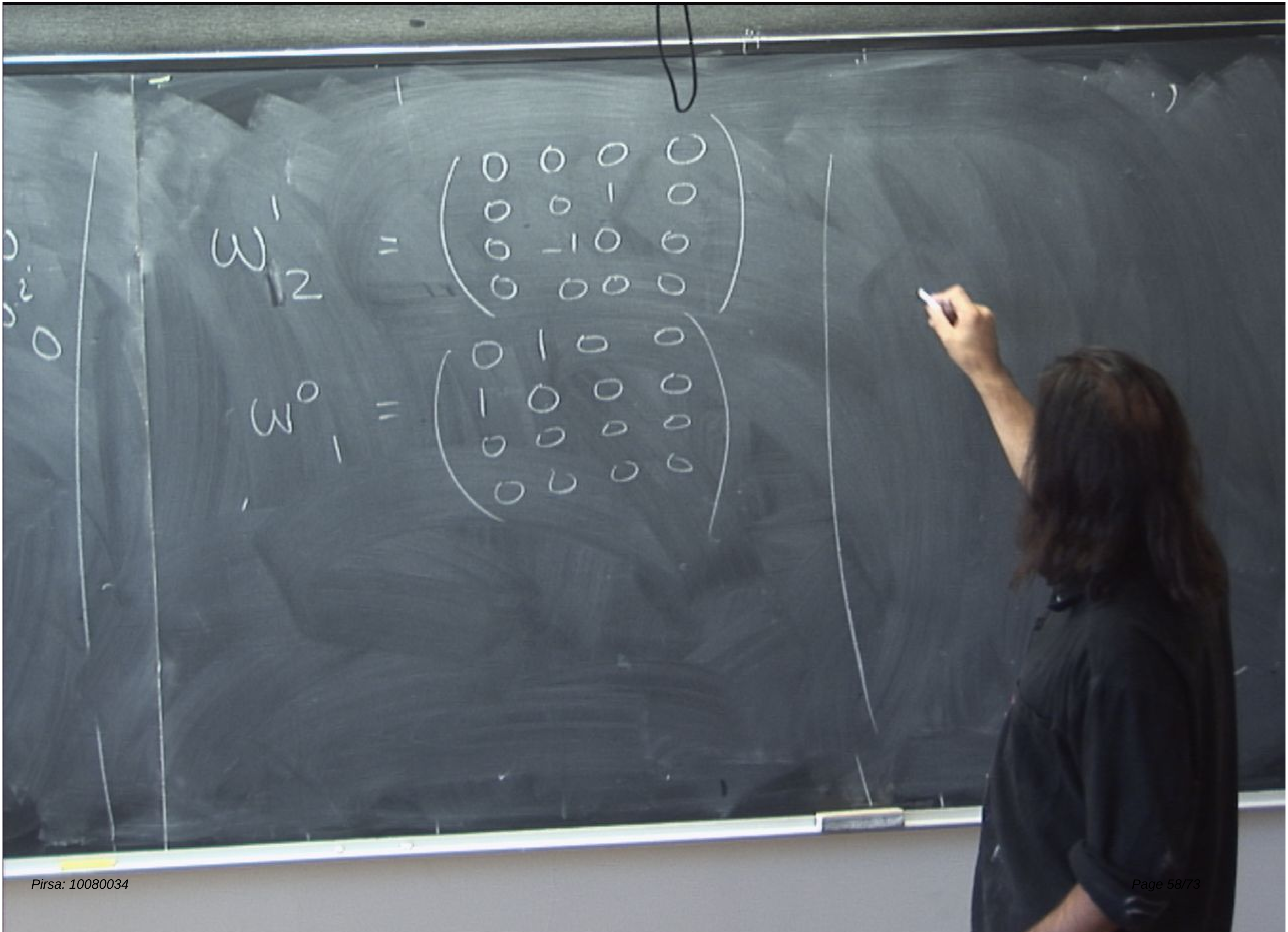
$$\eta_{\rho\mu}\,\omega^{\mu}{}_{\nu} = \omega_{\rho\nu}$$

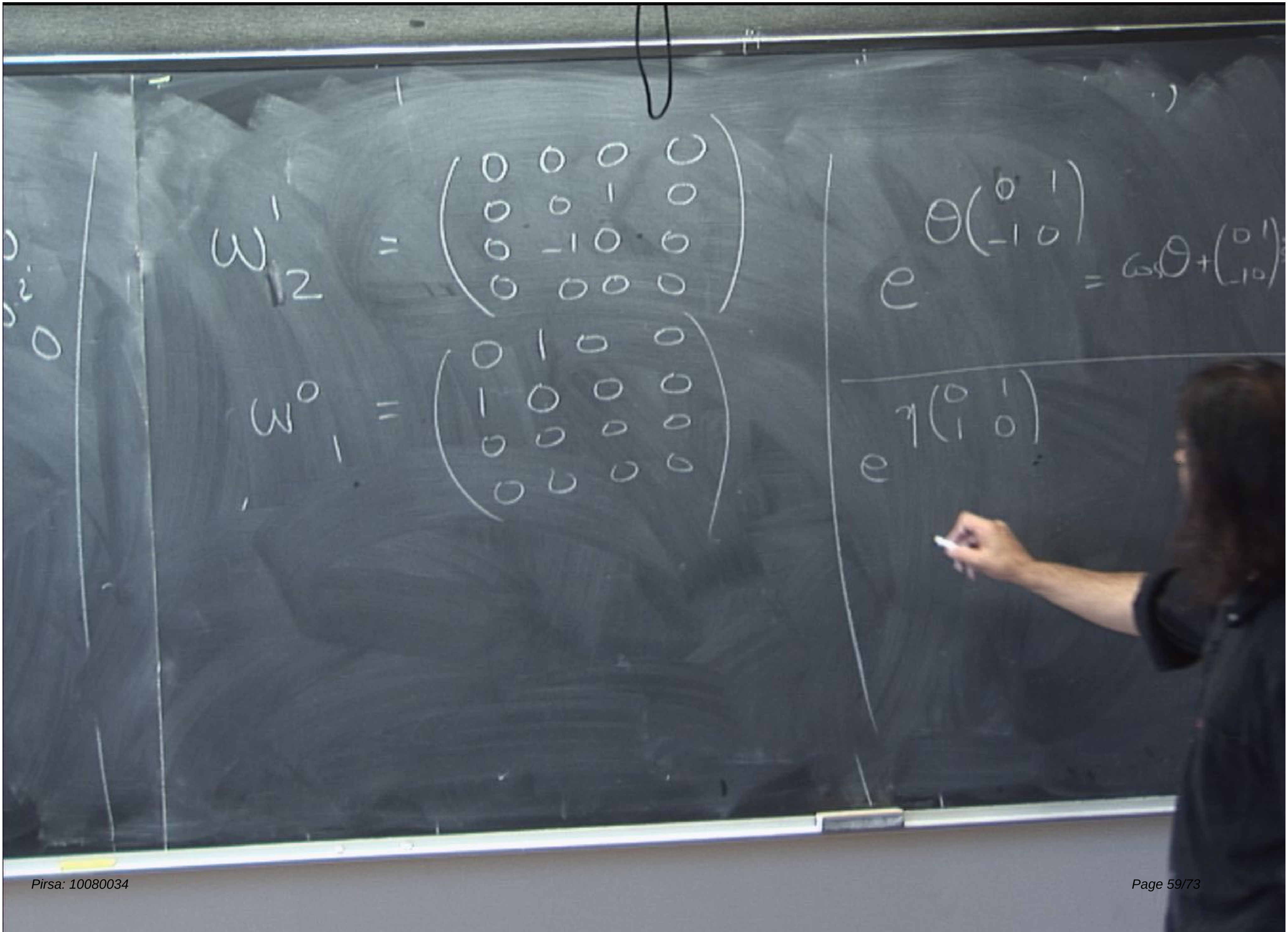
$$\omega_{0i} = \omega^{0}{}_{i}$$

$$\omega_{i0} = -\omega^{i}{}_{0}.$$

ν

$\eta_{\mu\nu}$


$\omega^1{}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
$\omega^0{}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$


$\omega^1{}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
$\omega^0{}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
$e^{\theta\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}} = \cos\theta + \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
$e^{\eta\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}$

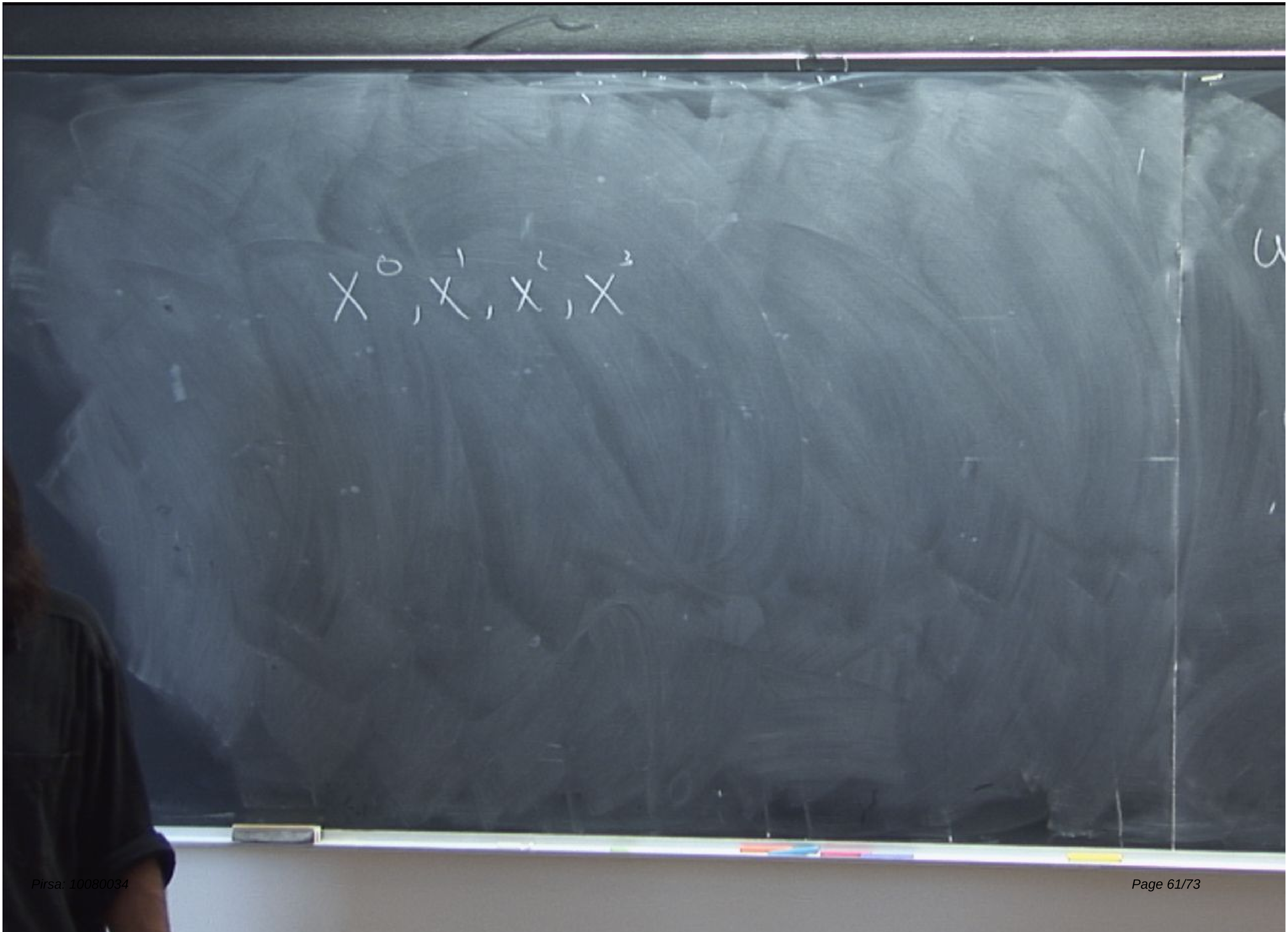
$$\omega^{1}{}_{2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\omega^{0}{}_{1} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$e^{\theta\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}} = \cos\theta + \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\sin\theta$$

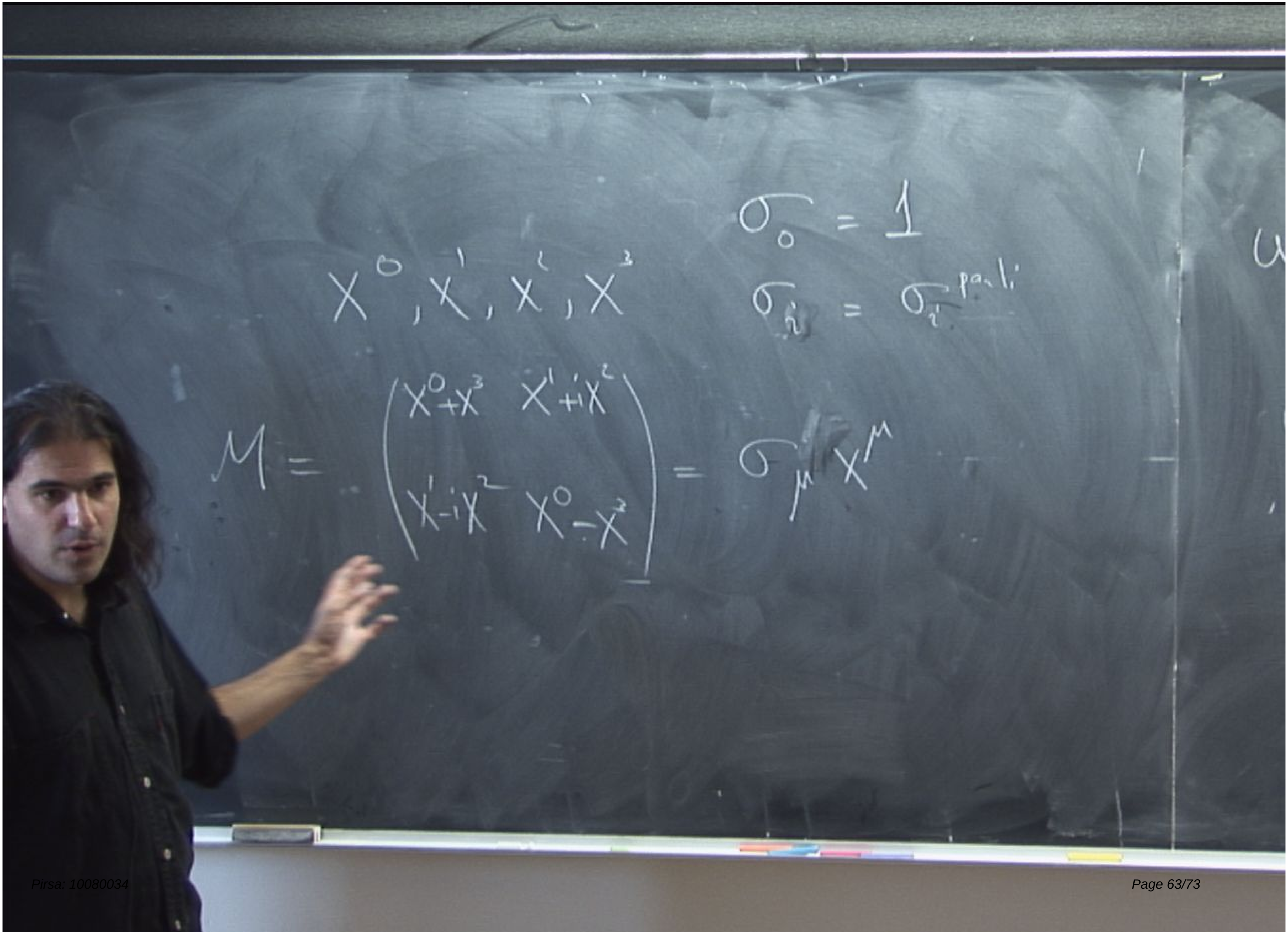
$$e^{\eta\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} = \cosh\eta + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\sinh\eta$$

$$\begin{pmatrix} x_0' \\ x_1' \end{pmatrix} = \begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix}\begin{pmatrix} x_0 \\ x_1 \end{pmatrix}$$


X^0, X^1, X^2, X^3

$$X^0, X^1, X^2, X^3$$

$$\begin{pmatrix} X^0 + X^3 & X^1 + iX^2 \\ X^1 - iX^2 & X^0 - X^3 \end{pmatrix}$$


X^0, X^1, X^2, X^3
$\sigma_0 = 1$
$\sigma_i = \sigma_i^{\text{pauli}}$
$M = \begin{pmatrix} X^0+X^3 & X^1+iX^2 \\ X^1-iX^2 & X^0-X^3 \end{pmatrix} = \sigma_\mu X^\mu$

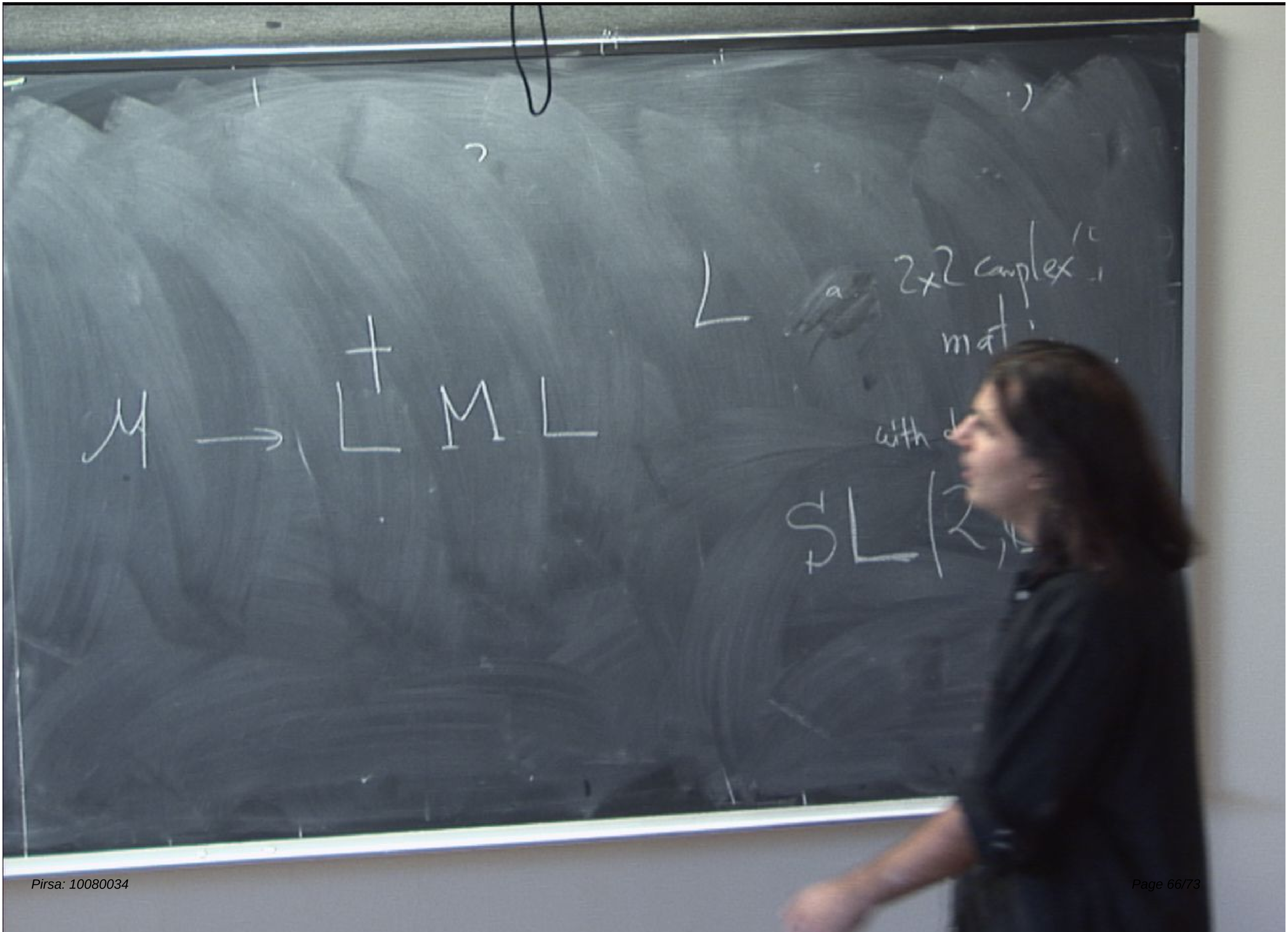
$$\sigma_0 = 1$$

$$X^0, X^1, X^2, X^3 \qquad \sigma_i = \sigma_i^{\text{pauli}}$$

$$M = \begin{pmatrix} X^0+X^3 & X^1+iX^2 \\ X^1-iX^2 & X^0-X^3 \end{pmatrix} = \sigma_\mu X^\mu$$

L any 2×2 complex matrix.

$$M \to L^\dagger M L$$


L
2x2 complex
$\mathcal{M} \to L^\dagger M L$
with
SL(2,

$$M' = L^{\dagger} M L$$

L a 2×2 complex matrix with $\det L = 1$

$SL(2,\mathbb{C})$

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L a 2×2 complex matrix with $\det L = 1$

$SL(2,\mathbb{C})$

$$X^0, X^1, X^2, X^3 \qquad \sigma_0 = \mathbb{1}, \quad \sigma_i = \sigma_i^{\text{pauli}}$$

$$M = \begin{pmatrix} X^0+X^3 & X^1+iX^2 \\ X^1-iX^2 & X^0-X^3 \end{pmatrix} = \sigma_\mu X^\mu$$

$$\det M = X^{0\,2} - X^{3\,2} - \left(X^{1\,2} + X^{2\,2}\right) = X^\mu X^\nu \eta_{\mu\nu} \,!$$

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$$\Rightarrow \boxed{X'^{\mu} = \Lambda_{L}{}^{\mu}{}_{\nu} X^{\nu}}$$

Lorentz Grp $SO(3,1)$

L a 2×2 complex matrix

with $\det L = \underline{1}$

$SL(2,\mathbb{C})$

$$M' = L^{\dagger} M L$$

$$= \sigma_{\mu} X_L'^{\mu}$$

$$\det M' = |\det L|^2 \det M = \det M$$

$$\Rightarrow \boxed{X'^{\mu} = \Lambda_{L}{}^{\mu}{}_{\nu} X^{\nu}}$$

$$\boxed{\begin{array}{l}\text{Lorentz Grp } SO(3,1) \\ \quad = SL(2,\mathbb{C}) \; \ldots\end{array}}$$

L a 2×2 complex matrix.

with $\det L = 1$

$SL(2,\mathbb{C})$

$$M' = L^{\dagger} M L$$

$$= \sigma_{\mu} X'^{\mu}_{L}$$

$$\det M' = |\det L|^{2} \det M = \det M$$

$$\Rightarrow \boxed{x'^{\mu} = \Lambda_{L}{}^{\mu}{}_{\nu}\, x^{\nu}}$$

$$\boxed{\begin{array}{c}\text{Lorentz Grp } SO(3,1) \\ \parallel \\ SL(2,\mathbb{C}) \ldots\end{array}}$$

L a 2×2 complex matrix.

with $\det L = \underline{1}$

$SL(2,\mathbb{C})$

$$\begin{pmatrix} e^{i\frac{\theta}{2}} & \\ & e^{-i\frac{\theta}{2}} \end{pmatrix} \qquad \begin{pmatrix} e^{\eta/2} & \\ & e^{-\eta/2} \end{pmatrix}$$