

Title: Undergraduate Talk

Date: Aug 12, 2010 02:00 PM

URL: <http://pirsa.org/10080025>

Abstract:

# Quantum State Tomography via Compressed Sensing



# Quantum State Tomography via compressed Sensing

$d^2$  measurement settings

$$d = 2^n$$

$$Q(r, d, b, d)$$

↑  $shb$



# Quantum State Tomography via compressed Sensing

$d^2$  measurement settings

$$d = 2^n$$

$$Q(r, d, b, d)$$

$r \text{ int}$        $\swarrow$   $shb$



# Quantum State Tomography via compressed Sensing

$d^2$  ent settings  
 $d = 2$   
 $q/r$

$T$  stable state preparation

# Quantum State Tomography via Compressed Sensing

$d^2$  measurement settings

$$d = 2^n$$

$$O(d \log d)$$

$r \log d$   $\leftarrow$   $s \log d$

$T$  stable state preparation

$$C \frac{\text{number of measurements}}{\text{number of samples}}$$

$r = \text{ink}$        $\text{ink}$        $\text{ink}$        $\text{ink}$

$\text{ink} = \text{ink}$  of  $\text{ink}$



$\exists T, \epsilon$  s.t.  $\forall n > T$  either  $\|x_n - x\| < \epsilon$  or  $\|x_n - x\| > \epsilon$





ink

shd

$$V = \frac{W}{m} = \frac{T - cM}{m}$$

$m$  = # of meetings

$\exists T, c$  st CS offers an advantage to standard method.



$\rho$  in state

$$\beta \sigma_i = \text{tr}(X \sigma_i)$$

$\rho$  in state

$$z = 30.3$$

$$\psi(x)$$

$\rho$  the state

$$\rho = \sum_{i=1}^m p_i |i\rangle\langle i|$$

$$p_i(x)$$

$\rho$  the state

$$\rho = \{\sigma_i\}_{i=1}^m$$

$$f(\rho) \rightarrow \{\text{tr}(\rho \sigma_i)\}_{i=1}^m$$

$$b \rightarrow \{c_i\}_{i=1}^m$$

$\rho$  the state

$$\Omega = \{\sigma_i\}_{i=1}^m$$

$$A_n(x) \rightarrow \{\text{tr}(x\sigma_i)\}_{i=1}^m$$

$$b \rightarrow \{c_i\}_{i=1}^m$$

$$\|A_n(x) \cdot b\|_2 < \epsilon$$

Algorithm

$\rho$  the state  
 $\Omega = \{ \sigma_i \}$

$A_n(x) = \{ \langle x, \sigma_i \rangle \}_{i=1}^n$   
 $b$

## Algorithms

Least squares

$$\min_x \left\{ \|A_n(x) - b\|_2^2 \right\} \quad \text{st } x \geq 0$$
$$\pi(x) = 1$$

$\rho$  the state

$$\Omega = \{\sigma_i\}_{i=1, \dots, m}$$

$$A_n(x) \rightarrow \{\text{tr}(x\sigma_i)\}$$

$$b \rightarrow \{c_i\}$$

$$\|A_n(x) \cdot b\|_2 < \epsilon$$

## Algorithms

Least squares

$$\min_x \left[ \|A_n(x) - b\|_2^2 \right] \quad \text{st } x \geq 0, \text{tr}(x) = 1$$

L1-regularized

$$\min_x \left[ \lambda \text{Tr}(x) + \|A_n(x) \cdot b\|_2^2 \right] \quad \text{st } x \geq 0$$



$\rho$  the state

$$\Omega = \{\sigma_i\}_{i=1, \dots, m}$$

$$A_n(x) \rightarrow \{\text{tr}(x\sigma_i)\}$$

$$b \rightarrow \{c_i\}_{i=1, \dots, m}$$

$$\|A_n(x) \cdot b\|_2 < \epsilon$$

## Algorithms

Least squares

$$\min_x \left[ \|A_n(x) \cdot b\|_2^2 \right] \quad \text{st } x \geq 0 \\ \text{tr}(x) = 1$$

L1-regularized

$$\min_x \left[ \lambda \text{Tr}(x) + \|A_n(x) \cdot b\|_2^2 \right] \quad \text{st } x \geq 0$$

$$A_n(x) \rightarrow \{\text{tr}(x\sigma_i)\}$$

$$b \rightarrow \{\sigma_i\}$$

$$\|A_n(x) \cdot b\|_2 < \epsilon$$

$$\min_x \left\{ \|A_n(x) - b\|_1^2 \right\} \quad \text{st } x \geq 0, \pi(x) = 1$$

$$\min_x \left\{ \mu \text{Tr}(x) + \|A_n(x) \cdot b\|_1^2 \right\} \quad \text{st } x \geq 0$$

$\rho$  in state

$$\Omega = \{\sigma_i\}$$

$$A_n(x) \rightarrow \{\text{tr}(x\sigma_i)\}$$

$$b \rightarrow \{c_i\}$$

$$\|A(x) \cdot b\|_2 <$$

### Algorithms

Least squares

$$\min_x \left[ \|A_n(x) - b\|_2^2 \right] \quad \text{st } x \geq 0, \text{tr}(x) = 1$$

$\ell_1$ -regularized

$$\min_x \left[ \mu \text{Tr}(x) + \|A_n(x) \cdot b\|_2^2 \right] \quad \text{st } x \geq 0$$

Trace heuristic

$$\min_x \left[ \text{Tr}(x) \right] \quad \text{st } x \geq 0$$



$\rho$  in state  
 $\Omega = \{\sigma_i\}$   
... m  
 $A_n(x) \rightarrow \{\text{tr}(x\sigma_i)\}$   
... n  
 $b \rightarrow \{c_i\}$   
... n  
 $\|A_n(x) \cdot b\|_1 < \epsilon$

Algorithm

Least squares

$\min \|A_n(x) \cdot b\|_1$  st  $x \geq 0$   
 $\text{tr}(x) = 1$

$\text{tr}(x) = \|A_n(x) \cdot b\|_1$  st  $x \geq 0$

$\min \|A_n(x) \cdot b\|_1$  st  $x \geq 0$   
 $\|A_n(x) \cdot b\|_1 < \epsilon$

$\rho$  the state

$$\Omega = \{\sigma_i\}$$

$$A_n(x) \rightarrow \{\text{tr}(x\sigma_i)\}$$

$$b \rightarrow \{c_i\}$$

$$\|A_n(x) \cdot b\|_2 < \epsilon$$

### Algorithms

least squares

$$\min_x \left\{ \|A_n(x) - b\|_2^2 \right\} \quad \text{st } x \geq 0, \text{tr}(x) = 1$$

L1-regularized

$$\min_x \left\{ \mu \text{Tr}(x) + \|A_n(x) \cdot b\|_2^2 \right\} \quad \text{st } x \geq 0$$

Trace heuristic

$$\min_x \left\{ \text{Tr}(x) \right\} \quad \text{st } x \geq 0, \|A_n(x) - b\|_2^2 < \epsilon$$

# Maximum Likelihood



# Maximum Likelihood

$$P_{ij} = 1 +$$

$n$

$a_1 = \# \text{ of } 1\text{'s you mean} + 1$

$a_2 = \# \text{ of } 2\text{'s} \dots - 1$

Maximum Likelihood

$$P_{\pm}(i) = \frac{1 \pm t_i(\sigma x)}{2}$$

$n$

$a_+ = \# \text{ of } + \text{ 's you mean } + 1$

$a_- = \# \text{ of } - \text{ 's } - 1$

$$L_{\pm}(x) = (P_{\pm,+})^{a_+} (P_{\pm,-})^{a_-}$$

$$L(x) = \prod_{\pm} L_{\pm}(x)$$

$$\max_x \{L(x)\} \quad \text{if } x \geq 0$$
  
$$\quad \quad \quad \text{if } x < 0$$



# Maximum Likelihood

$$P_{y_i}(i) = \frac{1 \pm t_i(\sigma x)}{2}$$

$n$

$a_+ = \# \text{ of } +1 \text{ s you mean } +1$

$a_- = \# \text{ of } -1 \text{ s}$

$$L_i(x) = (P_{+i})^{a_+} (P_{-i})^{a_-}$$

$$L(x) = \prod_i L_i(x)$$

$$\max_x \{L(x)\} \quad \text{if } x \geq 0$$

$t(x) = 1$

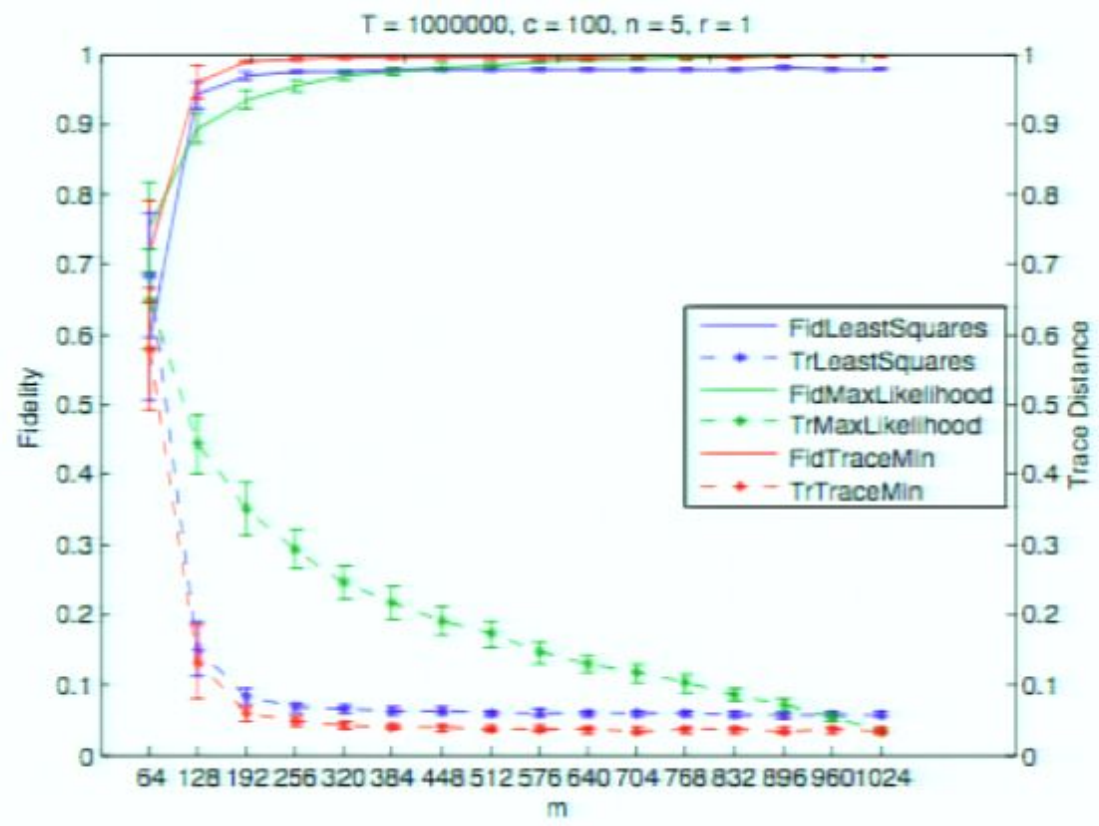
# Variables for Testing

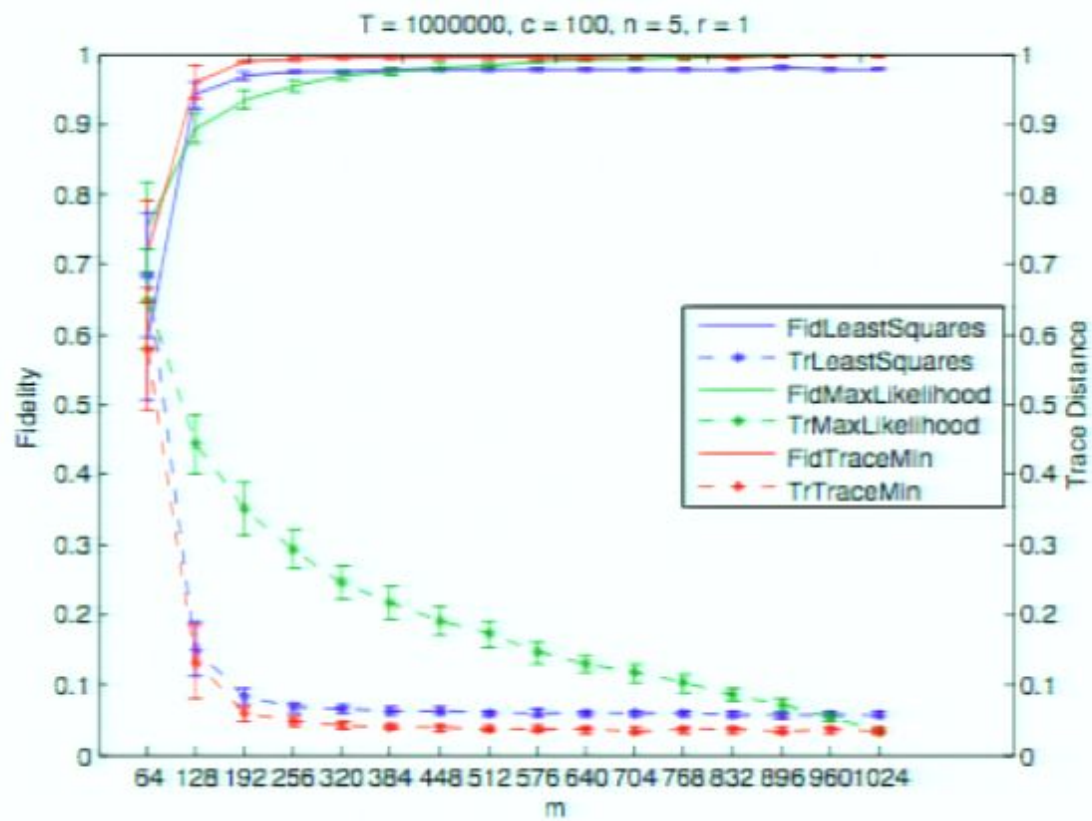
$$c = 1, 25, 100$$

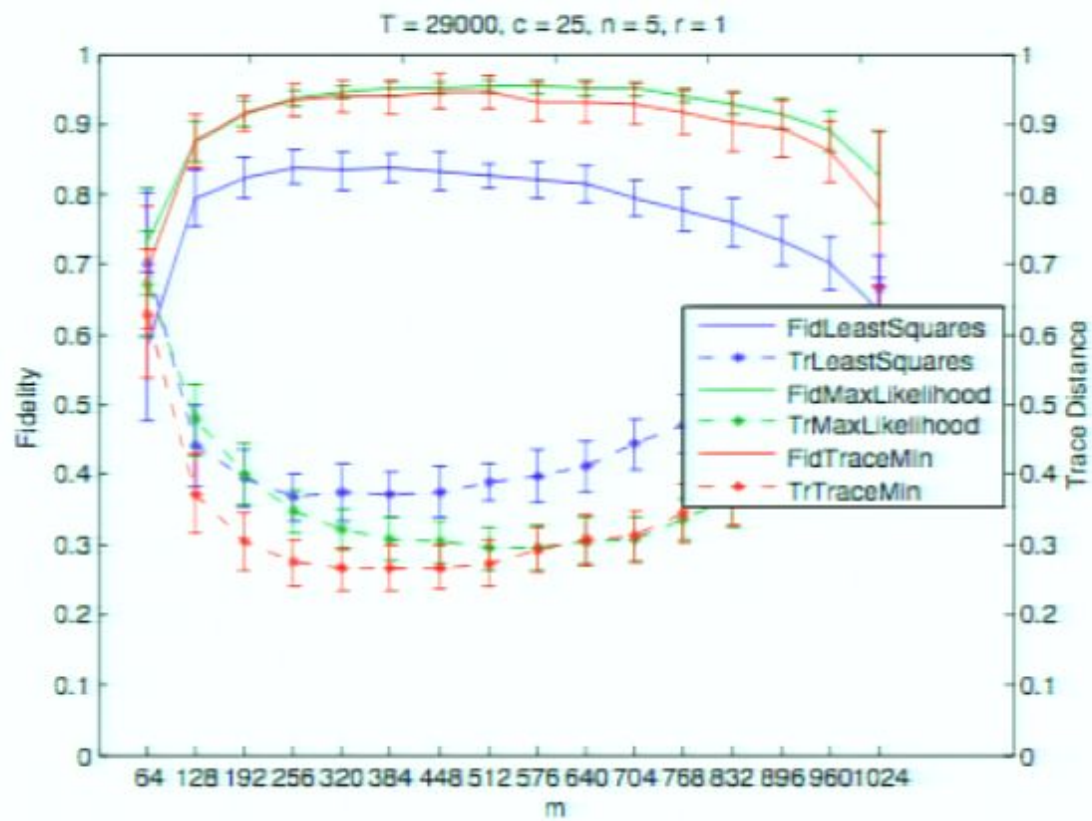
$$m = 2^{\text{rd}} \dots d^2$$

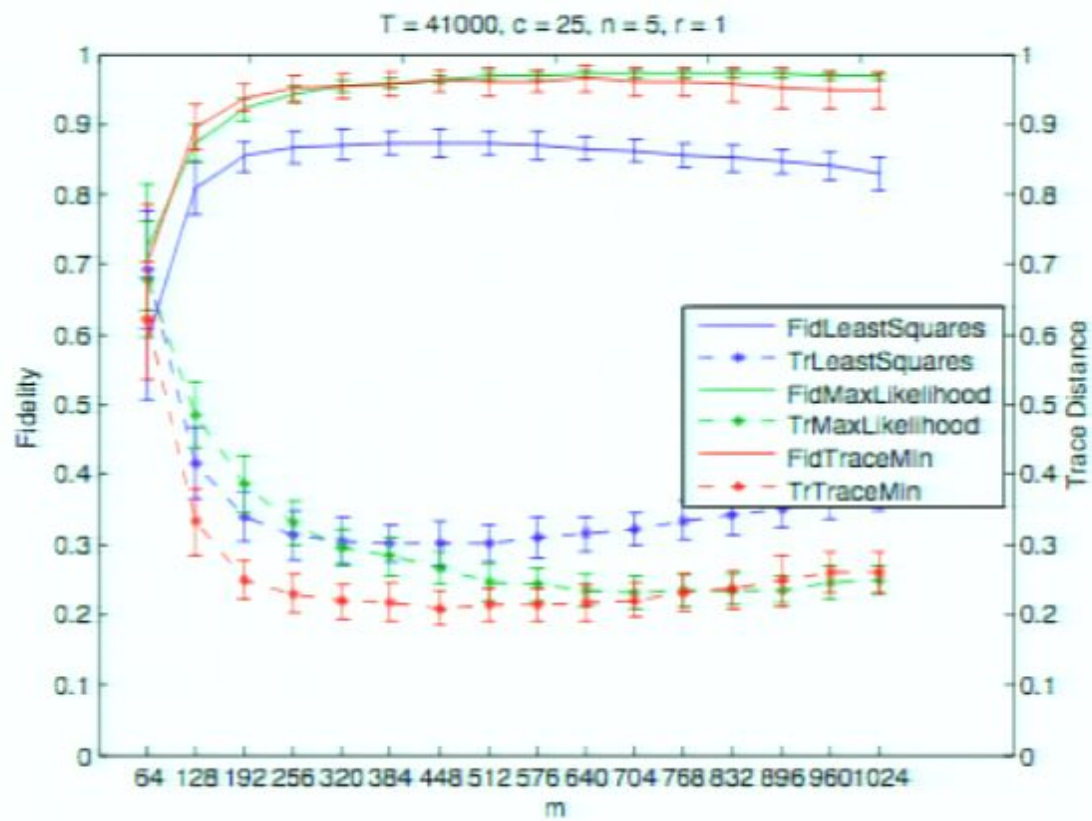
$$n = 5, r = 1, 2$$

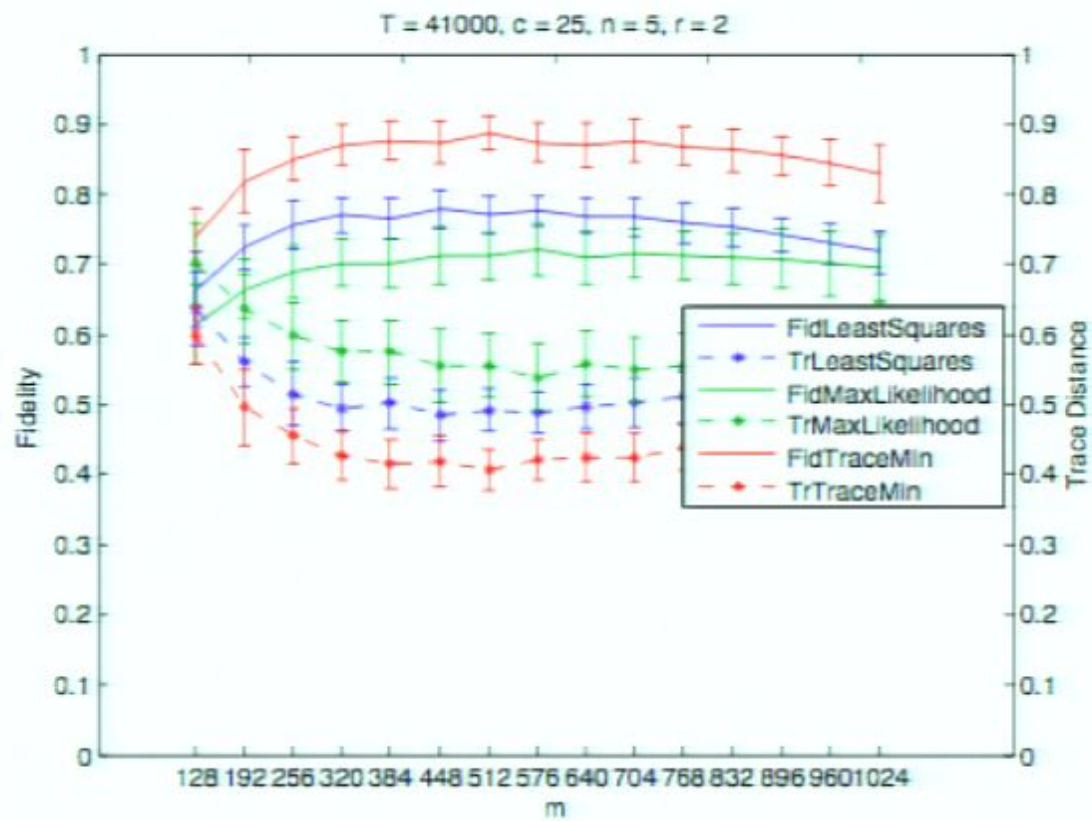
$$k = 60$$











# Variables for Testing

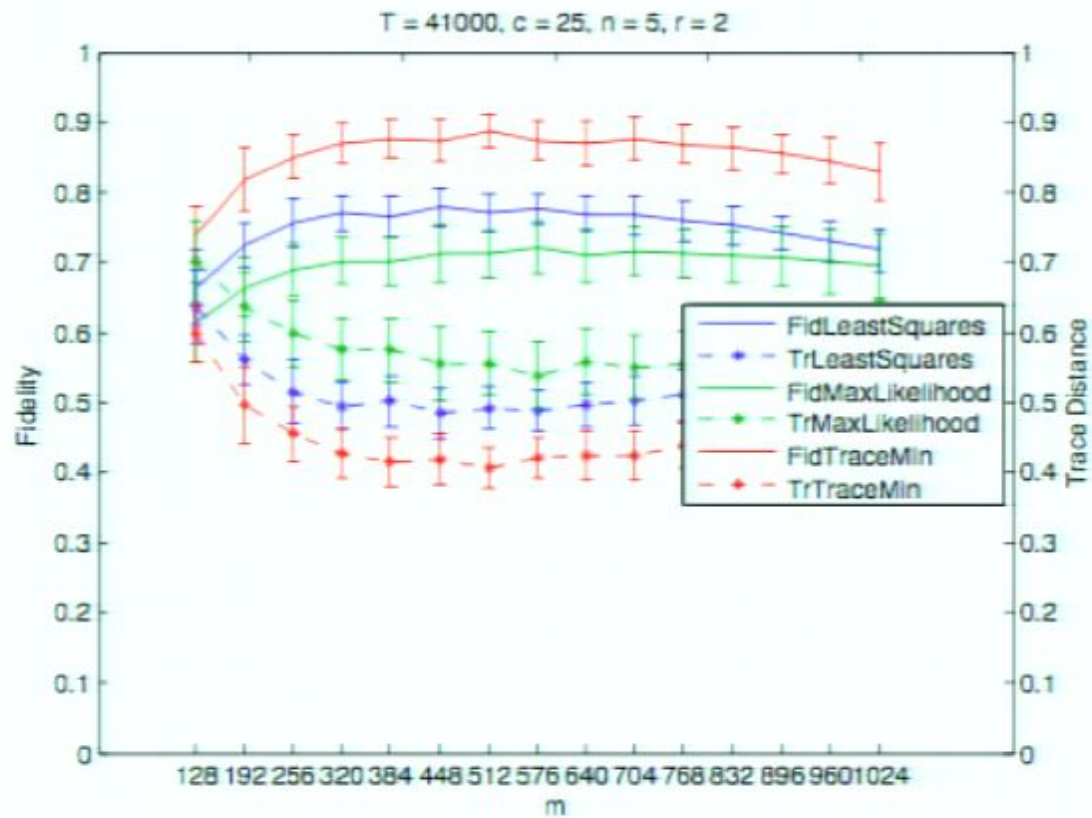
$$c = 1, 25, 100$$

$$m = 2^{\text{rd}} \dots d^2$$

$$n = 5, r = 1, 2$$

$$k = 60$$







Density of states  
Grand state degeneracy

Density of states  
Grand state degeneracy

Density of states

Grand state degeneracy

Computational Complexity

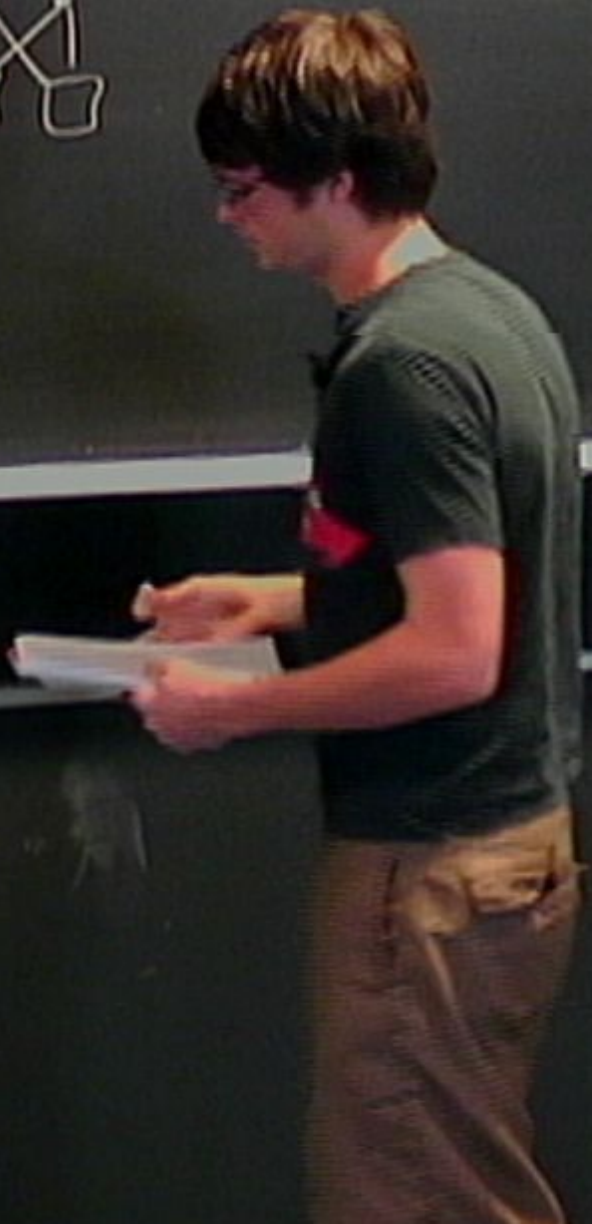
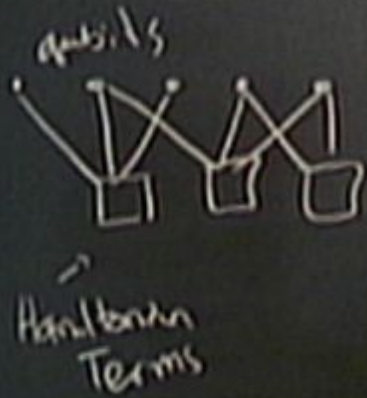
density of states

Grand state degeneracy

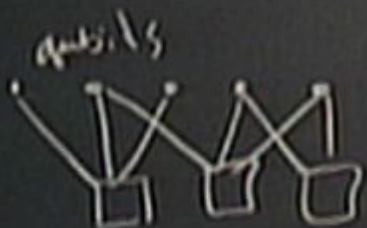
Computational Complexity



# Computational Complexity



# Computational Complexity



Hamiltonian Terms

$$H = \sum_j M_j$$

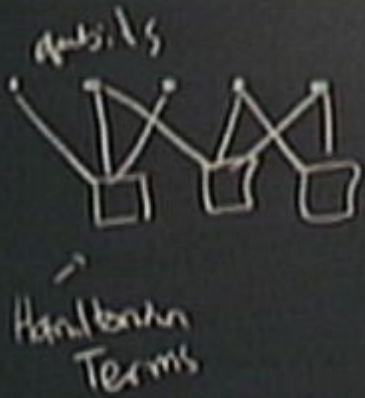
$$\lambda_0 = 0?$$

$$\gg \alpha$$





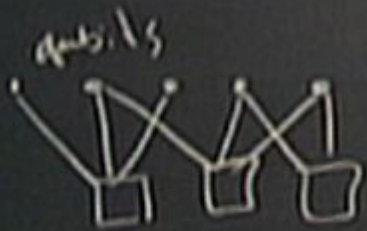
# Computational Complexity



$$H = \sum_i \Pi_i$$

$$\lambda_0 = 0? \\ > \lambda^k$$

# Computational Complexity



Hamiltonian Terms

$$H = \sum_i \Pi_i$$

$$r_b = 0?$$

$$\lambda^k$$

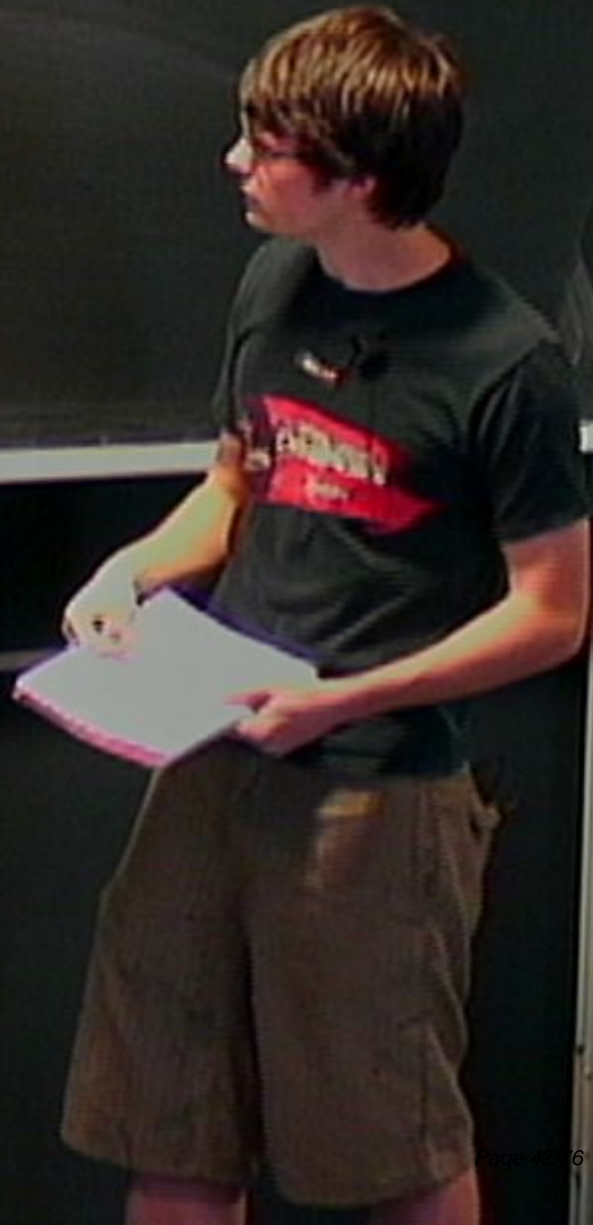
$$H = \sum_i H_i$$

$$\lambda < a$$

$$\lambda > b$$

$$a \geq \frac{1}{p(n)}$$

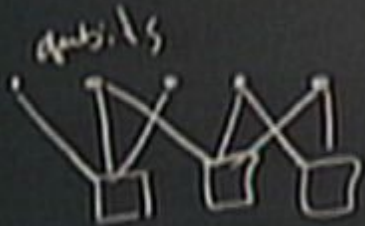
$$a - b \geq \frac{1}{p(n)}$$



Density of states

Grand state degeneracy

Computational Complexity



Hamiltonian Terms

$$H = \sum_i \Pi_i$$

$$\lambda_b = 0? \quad a \geq \frac{1}{\rho_{\text{min}}}$$

$$H = \sum_i H_i \quad a-b \geq \frac{1}{\rho_{\text{min}}}$$

$$\lambda < a$$

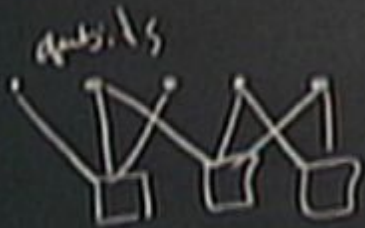
$$\lambda > b$$

density  
of  
states  
for a

LARK

Density of states  
 Grand state degeneracy

Computational Complexity



Hamiltonian Terms

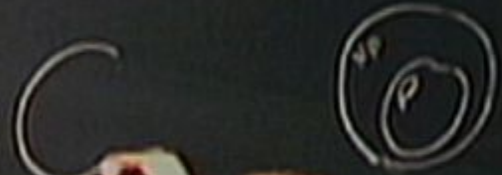
$$H = \sum_i \Pi_i$$

$$\lambda_0 = 0? \quad a \geq \frac{1}{\sqrt{N}}$$

$$H = \sum_i H_i \quad (a-b \geq \frac{1}{\sqrt{N}})$$

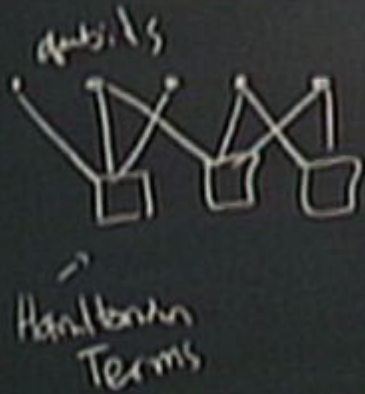
$$\lambda < a$$

$$\lambda > b$$



Open Questions  
 $\lambda_0$   
 + Questions  
 $\rightarrow \lambda_0$

Density of states  
 Grand state degeneracy  
 Computational Complexity



$$H = \sum_T \Pi_i$$

$$\lambda_0 = 0? \quad a \approx \frac{1}{N \ln N}$$

$$H = \sum_i H_i \quad a-b \approx \frac{1}{p \ln p}$$

$$\lambda < a$$

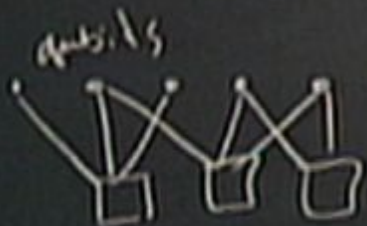
$$\lambda > b$$

density question  
 $\rightarrow \lambda_0$   
 Counting Question  
 how many  $\lambda_0$

Density of states

Grand state degeneracy

Computational Complexity



Hamiltonian Terms

$$H = \sum_i H_i$$

$$\lambda_0 = 0?$$

$$\lambda > \lambda_0$$

$$H = \sum_i H_i$$

$$\lambda < a$$

$$\lambda > b$$

$$a \geq \frac{1}{M} H$$

$$a - b \geq \frac{1}{p} H$$



$$\#BQP = \#P$$

density questions

$\rightarrow \lambda_0$

Counting Questions

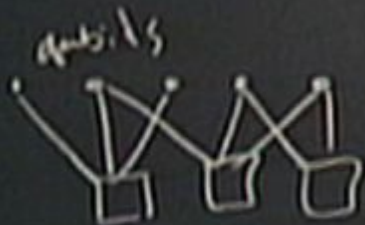
how many  $\lambda_0$

LA 101

Density of states

Grand state degeneracy

Computational Complexity



Hamiltonian Terms

$$H = \sum_i \Pi_i$$

$$\lambda_0 = 0?$$

$\gg \lambda$

$$H = \sum_i H_i$$

$$\lambda < a$$

$$\lambda > b$$

$$a \approx \frac{1}{M^2}$$

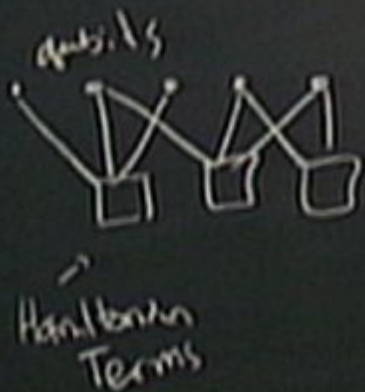
$$a-b \approx \frac{1}{p^2 \ln}$$



$$\#BQP = \#P$$

density questions  
 $\rightarrow \lambda_0$   
 Counting Questions  
 how many  $\lambda_0$

Density of states  
 Grand state degeneracy  
 Computational Complexity



$$H = \sum_i \Pi_i$$

$$\lambda_0 = 0? \quad a \geq \frac{1}{\rho_0} H$$

$$H = \sum_i H_i \quad a-b \geq \frac{1}{\rho_0} H$$

$$\lambda < a$$

$$\lambda > b$$



$$\#BQP = \#P$$

density questions  
 →  $\lambda_0$   
 Coupling Questions  
 how big  $\lambda_0$





Small red sign with illegible text, possibly a notice or warning.



A man with dark hair and glasses, wearing a black t-shirt with a red graphic and khaki pants. He is standing in front of a chalkboard, looking down at a piece of paper he is holding with both hands. He appears to be presenting or reading from the paper.

Small red sign with white text, partially legible. The text includes "FIRE" and "EXIT".



QNA



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$\epsilon_n$

$$x \in U \Rightarrow P\{0 < |x_n - x| < \epsilon_n\} > \alpha$$



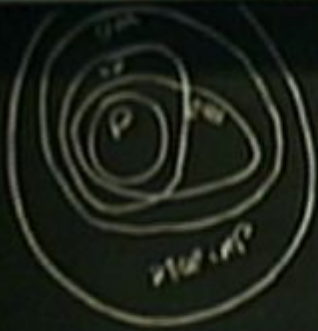
QNA

$$X \in L \Rightarrow \exists \epsilon > 0 \ P\{|X_n - \mu| > \epsilon\} > \alpha$$

$$X \notin L \Rightarrow \forall \epsilon > 0 \ P\{|X_n - \mu| > \epsilon\} < \beta$$

$$a \cdot b \Rightarrow \frac{1}{a} \cdot \frac{1}{b}$$





QMA

$$x \in L \Rightarrow \exists |u\rangle P[Q|x, u] > \alpha \quad \alpha \cdot b \geq \frac{1}{p}$$

$$x \notin L \Rightarrow \forall |u\rangle P[Q|x, u] < \beta$$

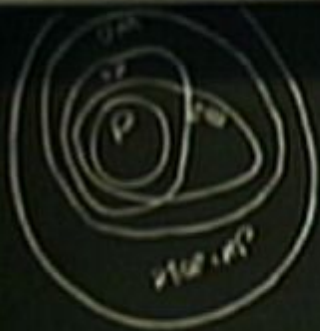
Space QMA  $\mathcal{L} = \text{SPACE}(S)$

$$x \in L \Rightarrow \exists |u\rangle \in S \quad P[Q|x, u] > \alpha'$$

$$\forall |u\rangle \in S \quad P[Q|x, u] < \beta'$$

$$x \notin L \Rightarrow \forall |u\rangle \quad P[Q|x, u] < \epsilon'$$





QMA

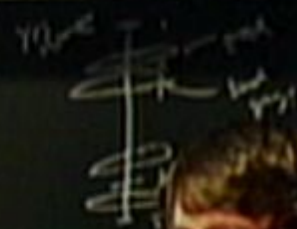
$x \in L \Rightarrow \exists |u\rangle P[|a\rangle, |u\rangle, |u\rangle] > \alpha$   $a, b \Rightarrow \frac{1}{\text{poly}}$

$x \notin L \Rightarrow \forall |u\rangle P[|a\rangle, |u\rangle, |u\rangle] < \beta$

Space QMA  $\mathcal{H} = S_0 \oplus S_1$

$x \in L \Rightarrow \forall |u\rangle \in S_0 P[|a\rangle, |u\rangle, |u\rangle] > \alpha'$   
 $\forall |u\rangle \in S_1 P[|a\rangle, |u\rangle, |u\rangle] < \beta'$

$x \notin L \Rightarrow \forall |u\rangle P[|a\rangle, |u\rangle, |u\rangle] < \epsilon'$



$S_0$  accepts subspace

$S_1$  rejects subspace

$\mathcal{H} = S_0 \oplus S_1$   
 $R_0 \oplus R_1$



$X \subseteq Y \Rightarrow \text{VIDES } P\{x \in X\} \subseteq P\{x \in Y\}$   
 $\text{VIDES } P\{x \in X\} \subseteq P\{x \in Y\}$   
 $X \subseteq Y \Rightarrow P\{x \in X\} \subseteq P\{x \in Y\}$   
 $H$

$S_1$  resulta integrado  
 $H \subseteq S \subseteq S_1$   
 $R \subseteq R_1$



A small red rectangular sign is affixed to the left side of the chalkboard frame. The text on the sign is mostly illegible but appears to contain some safety or warning information in Spanish.



$XCL \rightarrow V \cap C$

$V \cap C \subseteq P \cap Q$  by subgraph

$XCL \rightarrow \forall H, P[0,1,2,3] \subseteq C'$

$H \subseteq S_0 \subseteq S_1$

$R_0 \subseteq R_1$

HCP

Compute  $dm(s)$  where  
 $S$  is a set of vertices  
of a vertex

Complete direct sum  
S exactly opposite of  
A vector

$$H = S \oplus S^\perp \\ = R \oplus R^\perp$$

$$\dim(S) < \dim(R) \\ \dim(S) = \dim(R) \\ \Rightarrow S \cap R^\perp$$

Complete bipartite graph  
 $S$  arbitrary subset of  
 $n$  vertices

$$H = S \circ S^c \\ = R \circ R^c$$

$$\dim(S) < \dim(R)$$

$$\dim(S) + \dim(R^c) > D$$

$$\Rightarrow R \in S \cap R^c$$



Complete lattice  
 $S$  arbitrary subspace of  
a vector

$$H = S \circ S^\perp \\ = R \circ R^\perp$$

$$\dim(S) < \dim(R) \\ \dim(S) + \dim(R^\perp) > D \\ \Rightarrow \in S \cap R^\perp$$

Equality



Complete graph  $K_n$   
 $S$  arbitrary subset of  
 $n$  vertices

$$H = S \circ S^c \\ = R \circ R^c$$

$$\dim(S) < \dim(R) \\ \dim(S) + \dim(R^c) > D \\ \Rightarrow S \cap R^c \neq \emptyset$$

Equality

w/ pr  
 $f, g$

$$f = \tau(g \circ \sigma)$$

Complete graph  
 $S$  strictly subspace of  
a vertex

$$H = S \circ S^{\perp} \\ = R \circ R^{\perp}$$

$$\dim(S) < \dim(R) \\ \dim(S) + \dim(R^{\perp}) > D \\ \Rightarrow S \cap R^{\perp}$$

Equality

$\omega_{pr}$

$$f = \tau(g(\sigma(\omega)))$$



Complete direct sum  
 $S$  exactly opposite of  
a vector

$$H = S \oplus S^\perp \\ = R \oplus R^\perp$$

$$\dim(S) < \dim(R)$$

$$\dim(S) + \dim(R^\perp) > D$$

$$\Rightarrow \in S \cap R^\perp$$

Equality

w/ pr  
 $f, g$

$$f = \tau(g(\sigma(\cdot)))$$

$$H = S \circ S^2 \\ = R \circ R^1$$

$$\dim(S) < \dim(R)$$

$$\dim(S) \cdot \dim(R) > D$$

$$\Rightarrow \in S \cap R_2$$

Equality

w/ pr

f, g

$$f = \tau(g(\sigma(v)))$$

---

$$QMA(a, b) = QMA(1 - \epsilon^{a/b}, \epsilon^{a/b})$$



$$H = S \circ S^T$$

$$= R \circ R^T$$

$$\dim(S) < \dim(R)$$

$$\dim(S) \cdot \dim(R) > D$$

$$h \in S \cap R_2$$

Equality

w/ pr

f, g

$$f = \tau(g(\sigma(v)))$$

$$QMA(a, b) \sim QMA(1 - \epsilon^{a+b}, \epsilon^{a+b})$$

$$(U^a \otimes I \otimes b \otimes I \otimes U) | \psi \rangle \otimes | \phi \rangle \otimes | \chi \rangle$$

↑  
input state  
state

↑  
witness  
state

↑  
state

$$H = SOS^T$$

$$= R \Theta R^T$$

$$\dim(S) < \dim(R)$$

$$\dim(S) + \dim(R) > D$$

$$h \in S \cap R_2$$

Equality

w.p.

f, g

$$f = \tau(g(\sigma(\cdot)))$$

$$QMM(a, b) \sim QMM(1 - z^{p(a)}, z^{p(b)})$$

$$V = V_{r_1}, V_2, V_3$$

$$c(V_i) = \sum c_i$$

$$\text{tr} \left[ \left( \underbrace{U^T \Sigma \Theta \Theta^T U}_{\text{power onto signal}} \right) \underbrace{I \otimes \Theta \Theta^T}_{\text{wired to receiver}} \underbrace{I}_{\text{noise}} \right] = \text{tr}(\Pi_{r_1}) + f_p - f_n$$

$$H = SOS^T$$

$$= R \Theta R^T$$

$$\dim(S) < \dim(R)$$

$$\dim(S) \cdot \dim(R) > D$$

$$h \in S \cap R_2$$

Equality

w.p.  
f, g

$$f = \tau(g(\sigma(v)))$$

$$QMA(a, b) = QMA(1 - \epsilon^{p(n)}, \epsilon^{p(n)})$$

$$V = V_{r_1}, V_2, V_3$$

$$\langle v, v \rangle = \sum \langle v_i, v_i \rangle$$

$$\text{tr} \left[ \left( \underbrace{U^T D \Theta B \Theta^T U}_{\text{input onto output}} \right) \underbrace{10 \times 10 \Theta \mathbb{1}}_{\text{input}} \right] = \text{tr}(\Pi_{r_1}) + f_p - f_n$$

$$= R \circ R' \quad \det(s) \cdot \det(R_{11}) > D$$

$$\Rightarrow \in S \cap R_2$$

$$\omega_{f,g} \quad f = \tau(g(\sigma(\cdot)))$$

$$Q_{M}(a,b) = Q_{M}(1-z^{a}, z^{b})$$

$$V = V_r, V_s, V_t$$

$$C(V) = \sum C(V_i) X_i V_i$$

$$\text{tr} \left[ \left( \underbrace{U^* \Sigma \Sigma^* U}_{\text{input onto output}} \otimes \underbrace{U \otimes U}_{\text{input of input}} \otimes \underbrace{I}_{\text{input}} \right) \otimes \otimes \otimes I \right] = \text{tr}(\Pi_r) + (p - r_n)$$



CAUTION  
 ELECTRICAL  
 EQUIPMENT  
 INSIDE

$$= R \circ R' \quad d_i(s) \cdot d_i(R_i) > D$$

$$\lambda \in S \cap R_\perp$$

$$\omega_{pf} \quad f, g \quad f = \tau(g(\sigma(v)))$$

$$Q_{MM}(a, b) = Q_{MM}(1 - z^{p(a)}, z^{p(b)})$$

$$V = V_{r_1} \cdot V_1 \cdot V_1$$

$$c(v_i) = \sum c_j v_i x_j v_i$$

$$\text{tr} \left[ \left( \underbrace{U^* U a b x d U}_{\substack{\text{point onto} \\ \text{actual}}} \right) \underbrace{10 x d 10 U}_{\substack{\text{using the} \\ \text{idea}}} \right] = \text{tr}(\Pi_{r_1}) + f_p - f_n$$

$$= \sum_x f(x)$$

$$\begin{aligned}
 & \text{define } g(v, w) = \begin{cases} 1 & \text{if } (v, w) \in E \\ 0 & \text{else} \end{cases} \\
 & = \sum_v f(v) \\
 & = \sum_v c(v)
 \end{aligned}$$



CAUTION  
 NO SMOKING  
 NO ALCOHOL  
 NO DRUGS

$$= \sum_x f(x)$$
$$= \sum_x c(x)$$

$$= \sum_{x,y} g(x,y)$$

$$= \sum_x f(x)$$
$$= \sum_x c(x)$$

$$= \sum_{x,y} g(x,y)$$



CAUTION  
DO NOT TOUCH  
EQUIPMENT



$$= \sum_x f(x)$$

$$= c(b)$$

point into  
 equal

using of  
 value

not

define  $g(y, x) = \begin{cases} 1 & \text{if } f(x) = y \\ 0 & \text{else} \end{cases}$



CAUTION  
 NO SMOKING  
 NO ALCOHOL  
 NO DRUGS

$$= \mathbb{R} \otimes \mathbb{R} \quad d_1(s) \cdot d_2(K_2) > 0$$

$$\omega_{pr} \quad f, g \quad f = \tau(g(\sigma(v)))$$

$$Q_{ML}(a, b) = Q_{ML}(1 - \tau^{(a)}, \tau^{(b)})$$

$$V = V_{r_1}, V_2, V_3$$

$$c(v_i) = \sum c(v_i) X_j v_i$$

$$\text{tr} \left[ \left( \begin{array}{c|c|c} U^* & I & b \times d \\ \hline U & & \end{array} \right) \otimes \sigma \otimes \mathbb{1} \right] = \text{tr}(\Pi_{r_1}) + f_p - f_n$$

$$= \sum_r f(r) \\ \sum_r c(r)$$

$$\text{define } g(v, v) = \begin{cases} 1 \\ 0 \end{cases}$$

Complete bipartite graph  
 $\hookrightarrow$  counting number of  
 a vertex

$$= \sum_{x,y} g(x,y) \quad \square$$

$$H = S \otimes S^2$$

$$\dim(S) \in \dim(R)$$

$$d(S) + d(R) \geq D$$

$$W \in S \cap R_2$$

Equality

where  
 $f, g$

$$QMA(a,b) = QMA(1-\gamma^{(a)}, \gamma^{(b)})$$

$$\text{tr} \left[ \left( \underbrace{U^{\otimes a}}_{\text{input}} \otimes \underbrace{V^{\otimes b}}_{\text{output}} \right) \rho \otimes \Omega \right] = \text{tr}(\Pi \rho)$$

$$= \sum_{i=1}^n f(i)$$

$$= \sum_{i=1}^n g(i)$$

define  $g(v,y) = \{$

$$= V_{i_1}, V_{i_2}, \dots, V_{i_b}$$

$$g(v,y) = \sum_{i_1, \dots, i_b} c_{i_1, \dots, i_b} V_{i_1} \dots V_{i_b}$$

Complete graph  
 ↳ every vertex of  
 a vertex

$$= \sum_{x,y} g(x,y) \quad \square$$

$$H = S \circ S^T$$

$$= R \circ R^T$$

$$\dim(S) < \dim(R)$$

$$\dim(S) + \dim(R) > D$$

$$W \in S \cap R^\perp$$

Equality  
 w.r.t  
 f, g

$$QMA(a,b) = QMA(1-\epsilon^{10}, \epsilon^{10})$$

$$\text{tr} \left[ \left( \underbrace{U^\dagger \Pi_0 U}_{\text{input state}} \otimes \underbrace{V^\dagger \Pi_1 V}_{\text{output state}} \right) \otimes \rho \right] = \text{tr}(\Pi_{01}) + \dots$$

$V, V^\dagger$   
 $\sum_{i,j} \Pi_{ij} X_{ij}$