Title: Summer Undergraduate Research Talks - 2010

Date: Aug 03, 2010 10:00 AM

URL: http://pirsa.org/10080023

Abstract:

INFLATION: Highly accelerated expansion of the universe $\rightarrow e^{Ht}$

FRW (flat): $ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2 d\Omega^2\right]$

Plug in the metric into $R^\alpha_\beta - \frac{R}{2}\delta^\alpha_\beta + \Lambda \delta^\alpha_\beta = 8\pi G T^\alpha_\beta$

$\Lambda \to 0$

With $T^\alpha_\beta = \begin{bmatrix} -\rho & & & 0 \\ & P & & \\ & & P & \\ 0 & & & P \end{bmatrix} \Rightarrow$ Friedmann equations

$$\Rightarrow \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} \Rightarrow \boxed{\left(\frac{\dot{a}}{a}\right) = H}$$

$$\Rightarrow \left(\frac{\ddot{a}}{a}\right) = -4\pi G(\rho + 3P), \text{ if } -\rho/3 > P$$

$$\rightarrow \ddot{a} > 0$$

- Considering $H = H_0$ (constant) $\Leftrightarrow a = e^{H_0 t}$

$$\rightarrow ds^2 = -dt^2 + e^{2H_0 t}(dr^2 + r^2 d\Omega^2)$$ [Exact de Sitter geometry]

- Considering $H(t) = H_0 + \varepsilon(t) + \mathcal{O}_2(H) \ldots$ with $\mathcal{O}_2(H) \ll \varepsilon(t)$, $\varepsilon(t) \ll H_0$

$$\rightarrow a(t) = \exp\left(\int^t H(t')dt' \ldots\right.$$

So: $$ds^2 = -dt^2 + \exp\left(2\int^t H(\ldots\right.$$

[Quasi de Sitter geometry]

Inflation generated by a scalar field

Dynamics of a scalar field:

$$S[\phi] = \int d^4x \sqrt{-g} \left[\frac{1}{2} \phi^{,\mu}\phi_{,\mu} - V(\phi) \right]$$

Finding ${}^{(\phi)}T^{\alpha}{}_{\beta}$ from $S[\phi]$:

$$\rho = \frac{1}{2}\phi^{,\mu}\phi_{,\mu} + V(\phi)$$

$$P = \frac{1}{2}\phi^{,\mu}\phi_{,\mu} - V(\phi)$$

Considering $\phi(\bar{x},t) = \underline{\phi(t)} + \underline{\delta\phi(\bar{x},t)}$

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Homogeneous part.

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Motion equations for the field:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

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If $\dot{\phi}^2 \ll V(\phi)$

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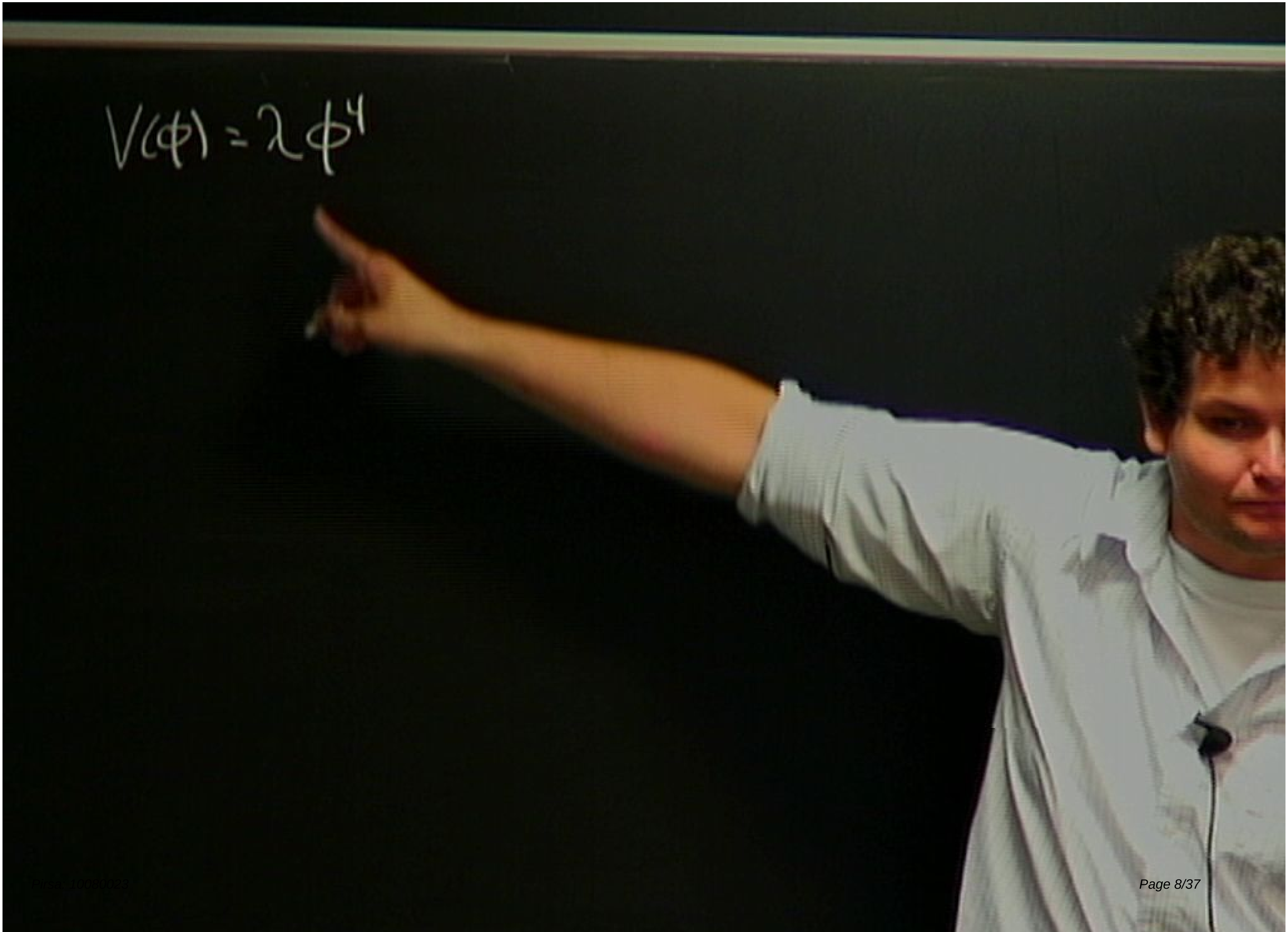
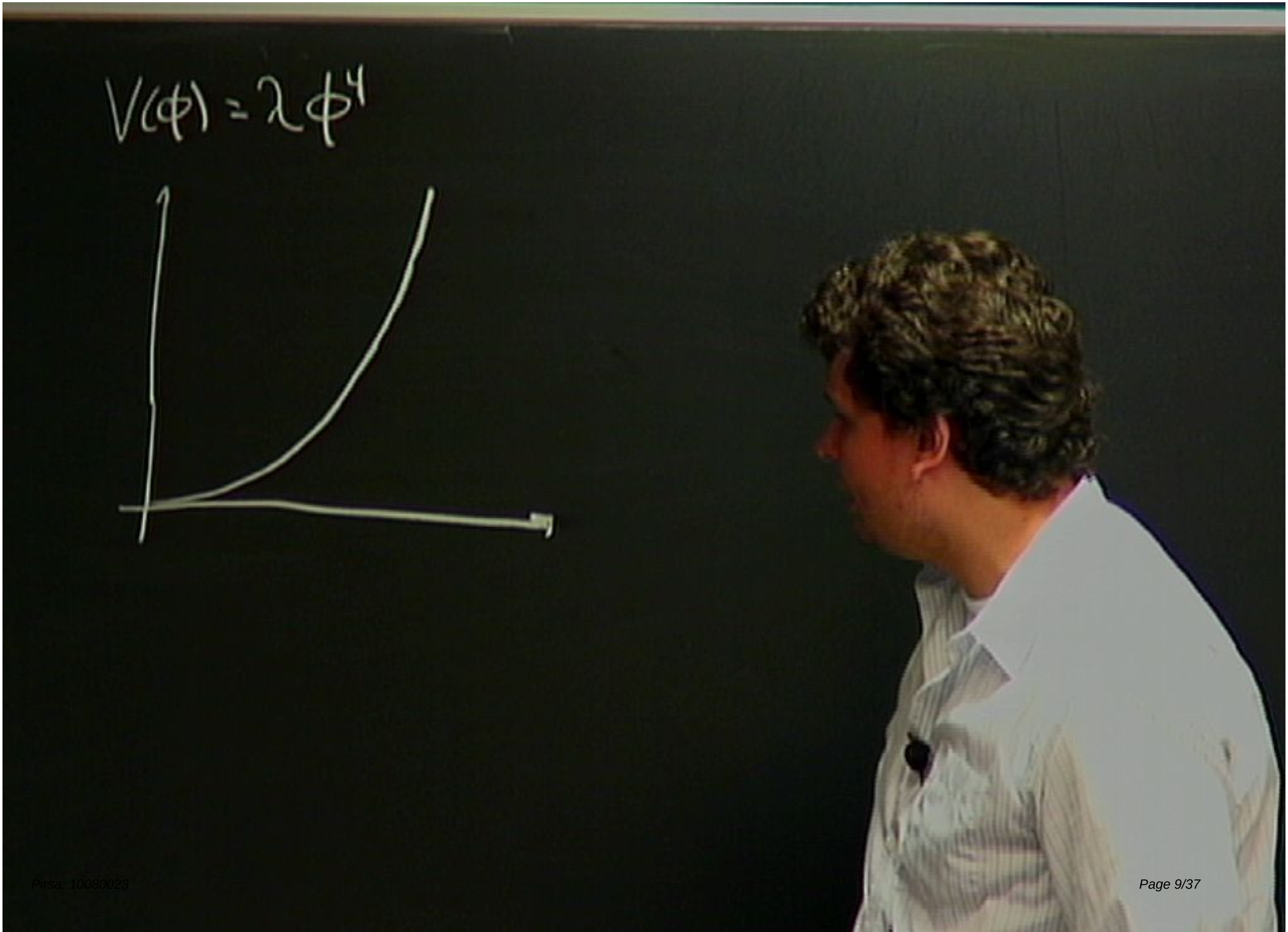

$V(\phi) = \lambda \phi^4$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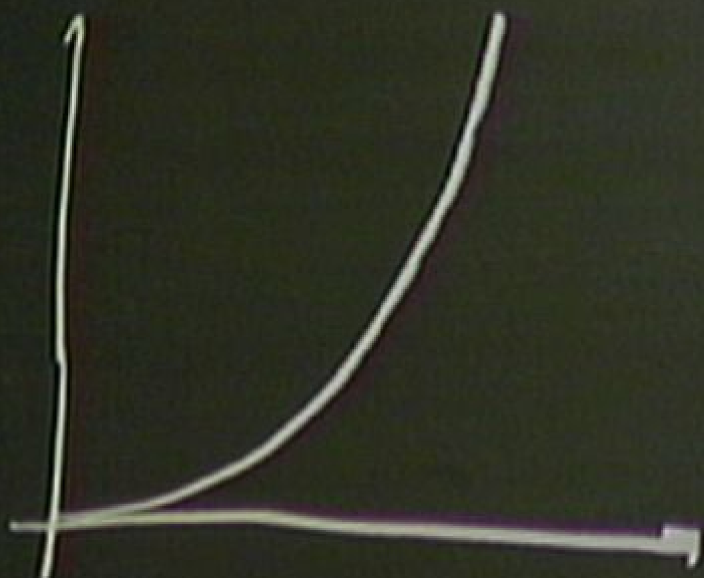
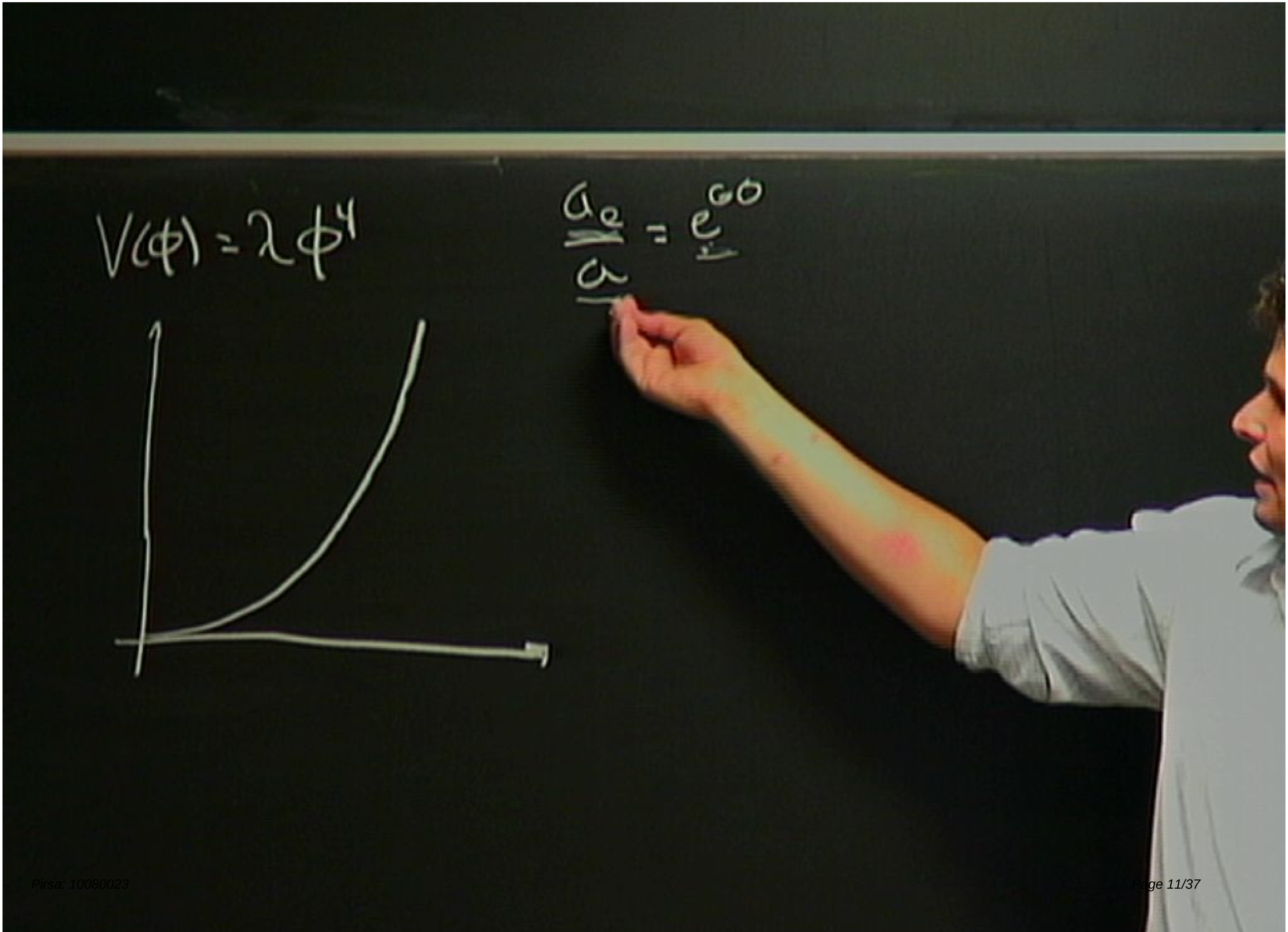
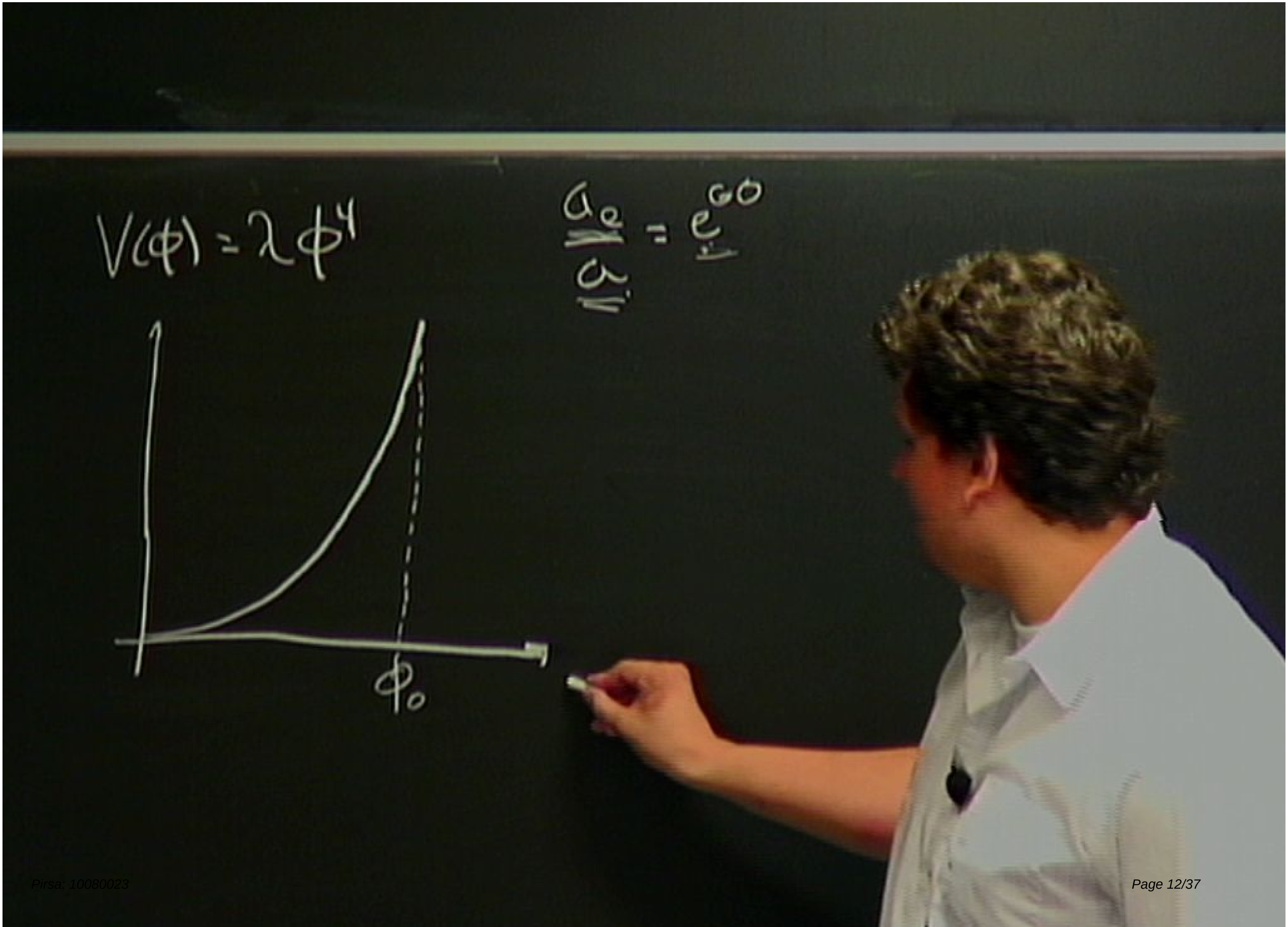



$V(\phi) = \lambda \phi^4$
$\frac{a_e}{a} = e^{60}$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ϕ_0

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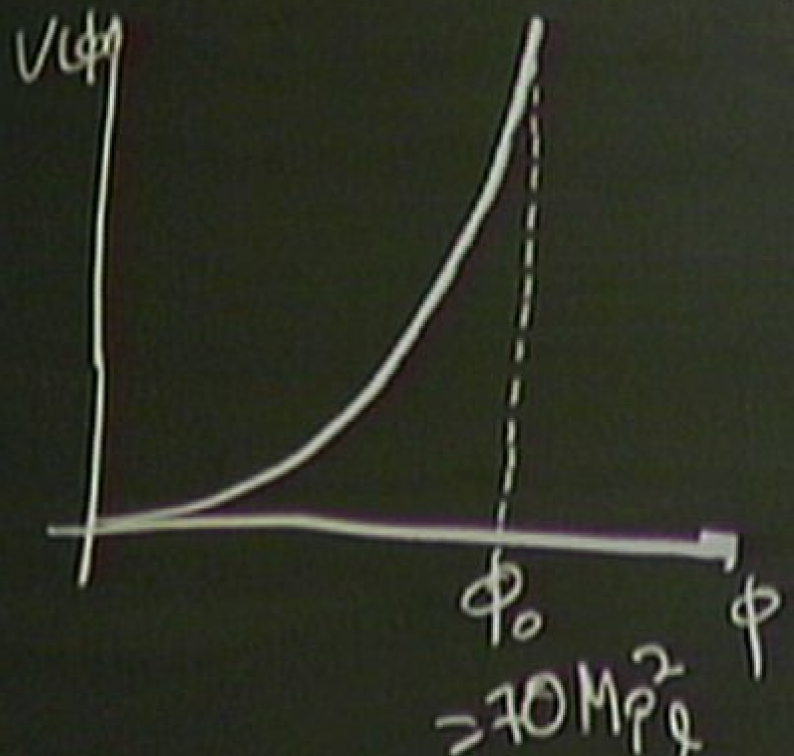

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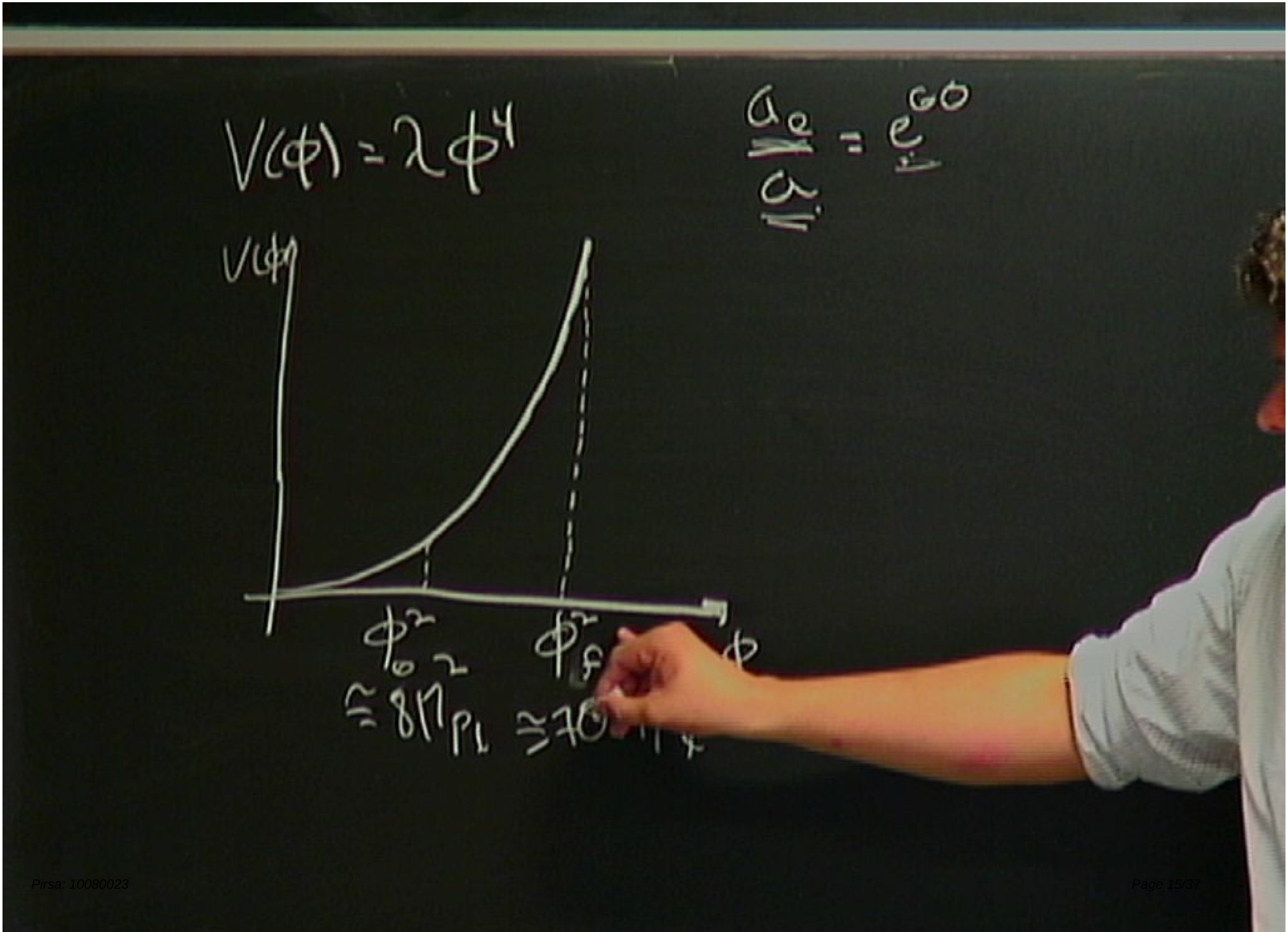
V(φ) = λφ^4
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V(φ)
φ

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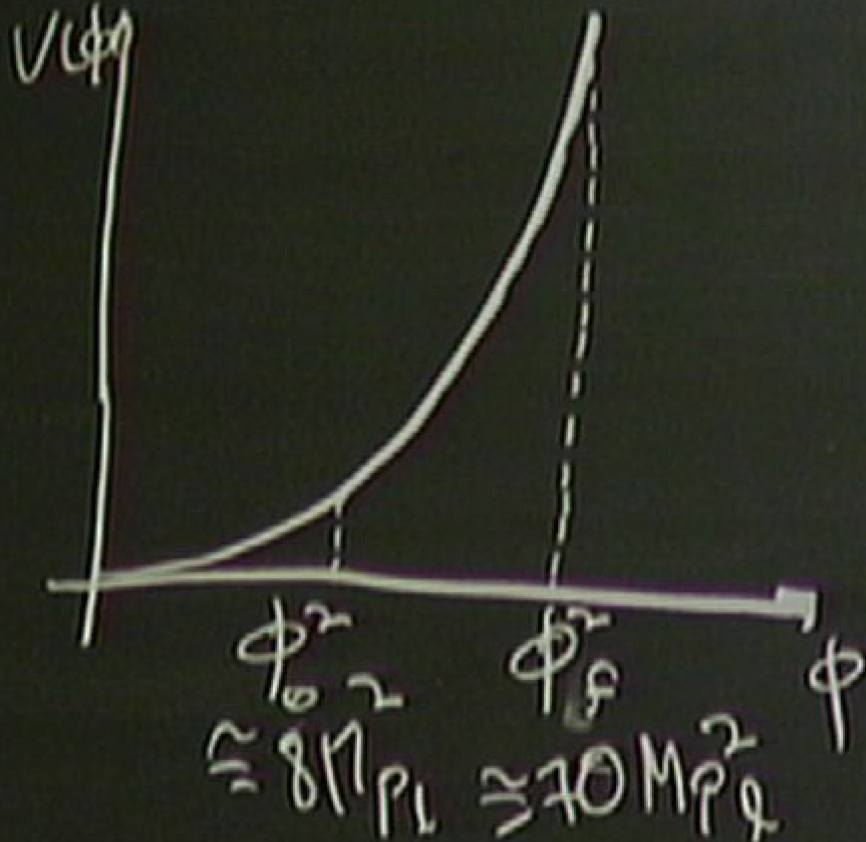

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If $\dot{\phi}^2 \ll V(\phi)$

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e^{KT}

- Considering $H = H_0$ (constant) $\Leftrightarrow a = e^{H_0 t}$

$$\rightarrow ds^2 = -dt^2 + e^{2H_0 t}(dr^2 + r^2 d\Omega^2) \quad \text{[Exact de Sitter geometry]}$$

- Considering $H(t) = H_0 + \varepsilon(t) + \mathcal{O}_2(H) \cdots$ with $\mathcal{O}_2(H) \ll \varepsilon(t)$, $\varepsilon(t) \ll H_0$

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So:

$$ds^2 = -dt^2 + \exp\left(2\int^t H(t')dt'\right)\left[dr^2 + r^2 d\Omega^2\right]$$

[Quasi de Sitter geometry]

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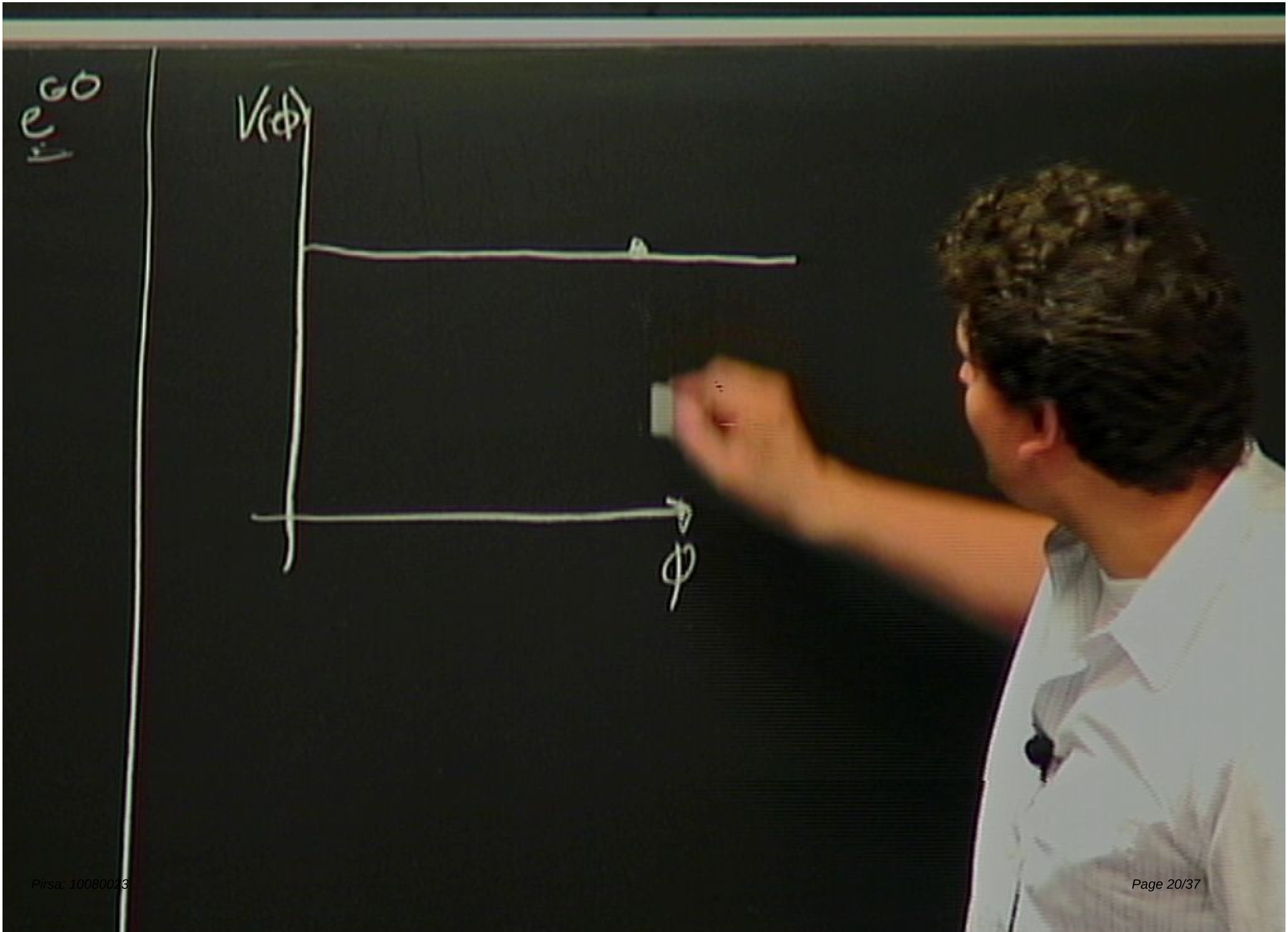
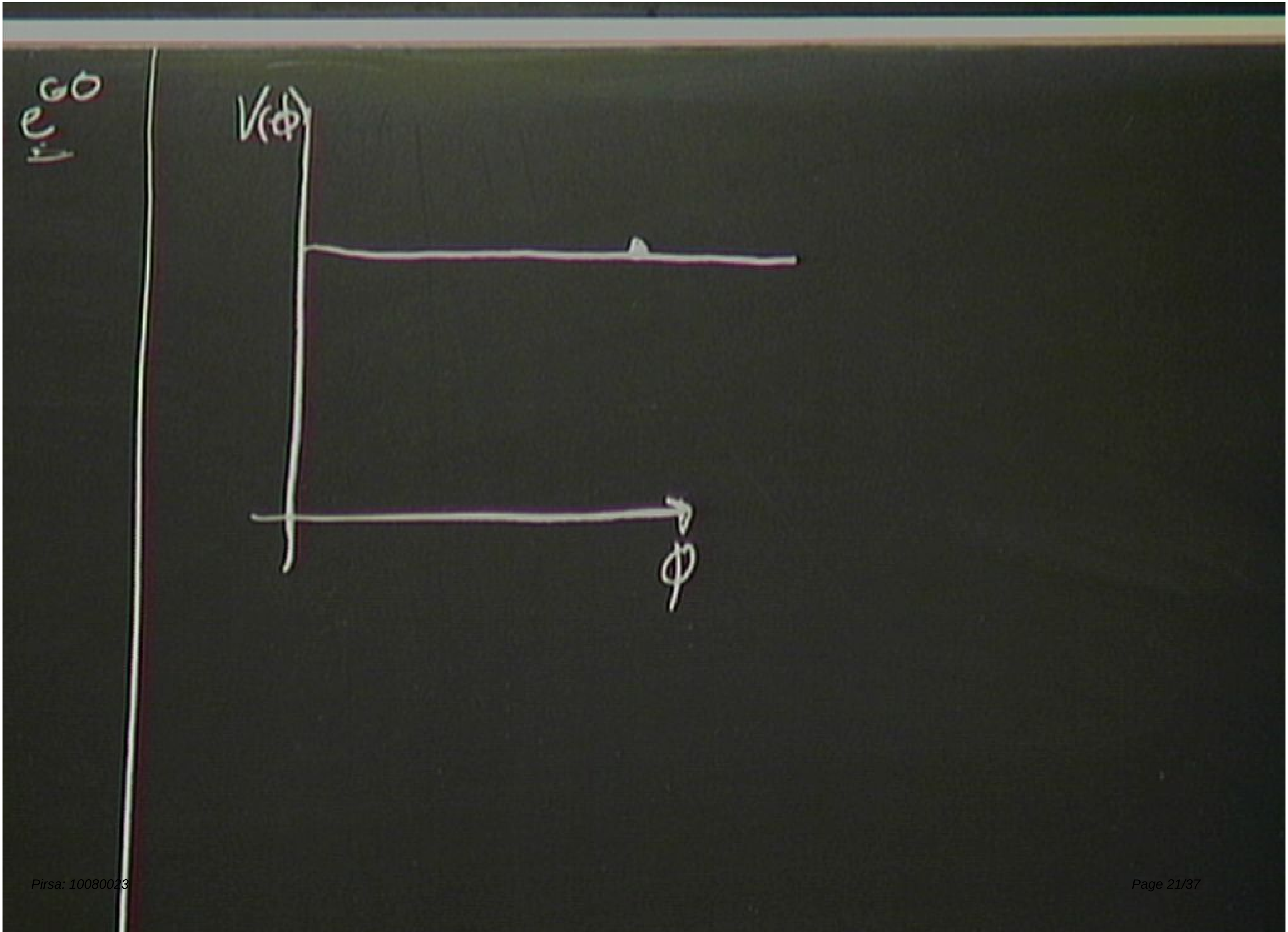
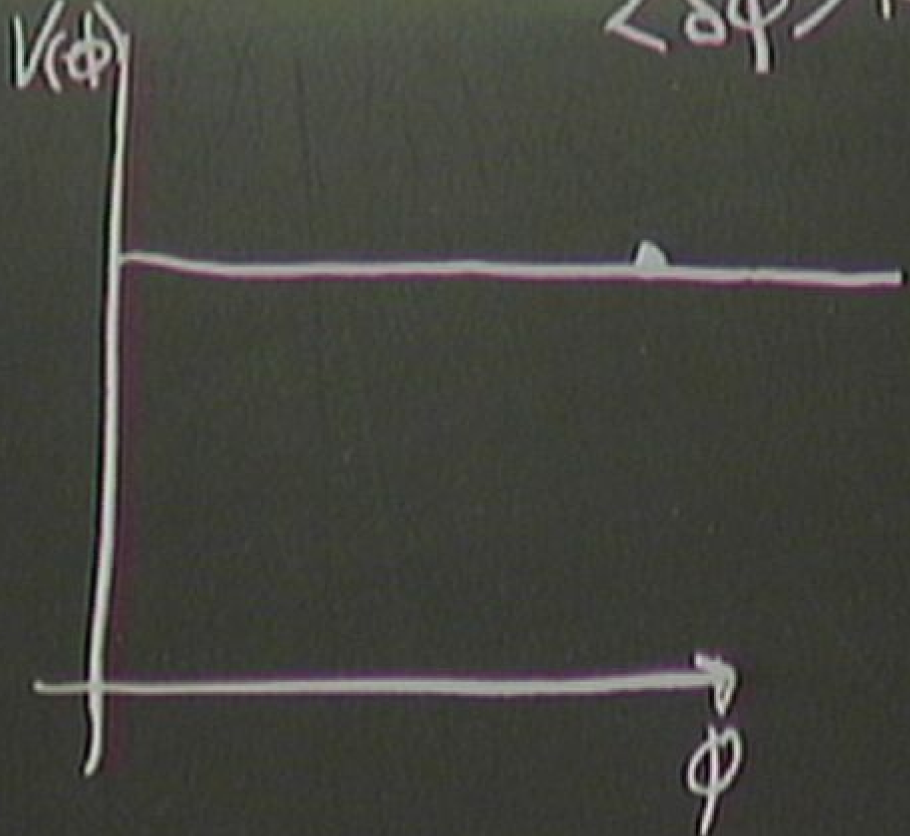
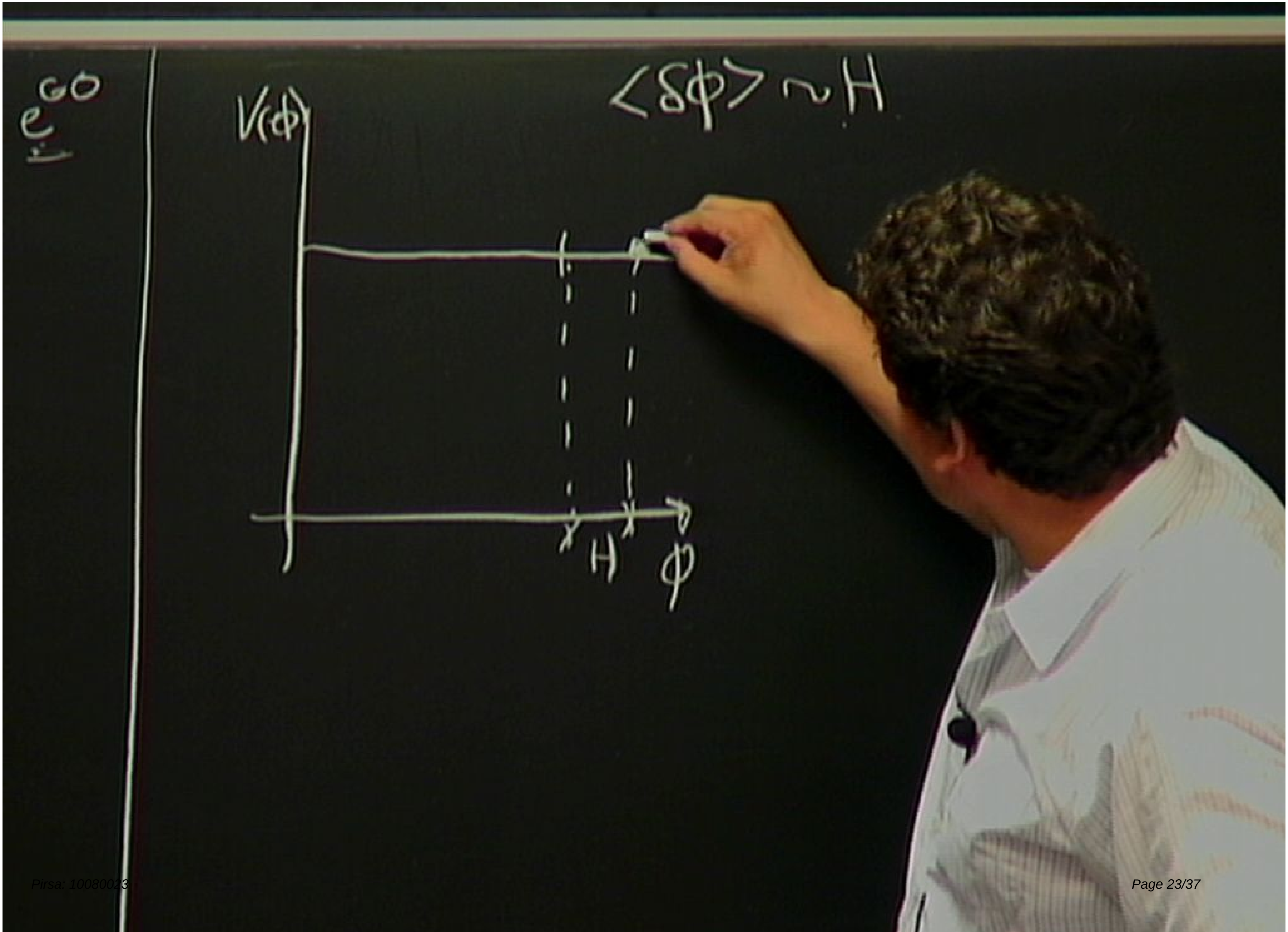

V(φ)
φ

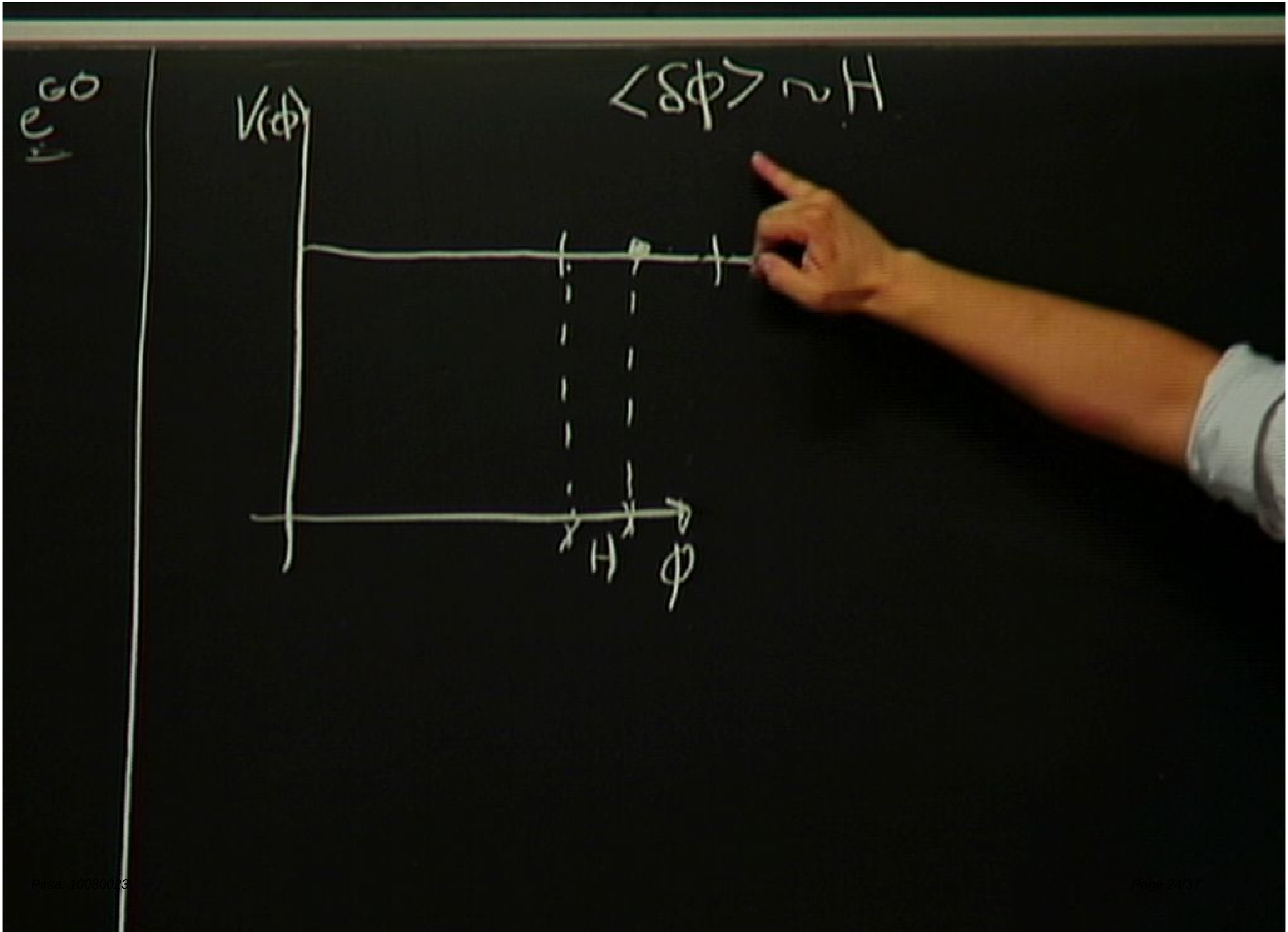

V(φ)
φ

e^{60}

$\langle \delta\phi \rangle \sim H$

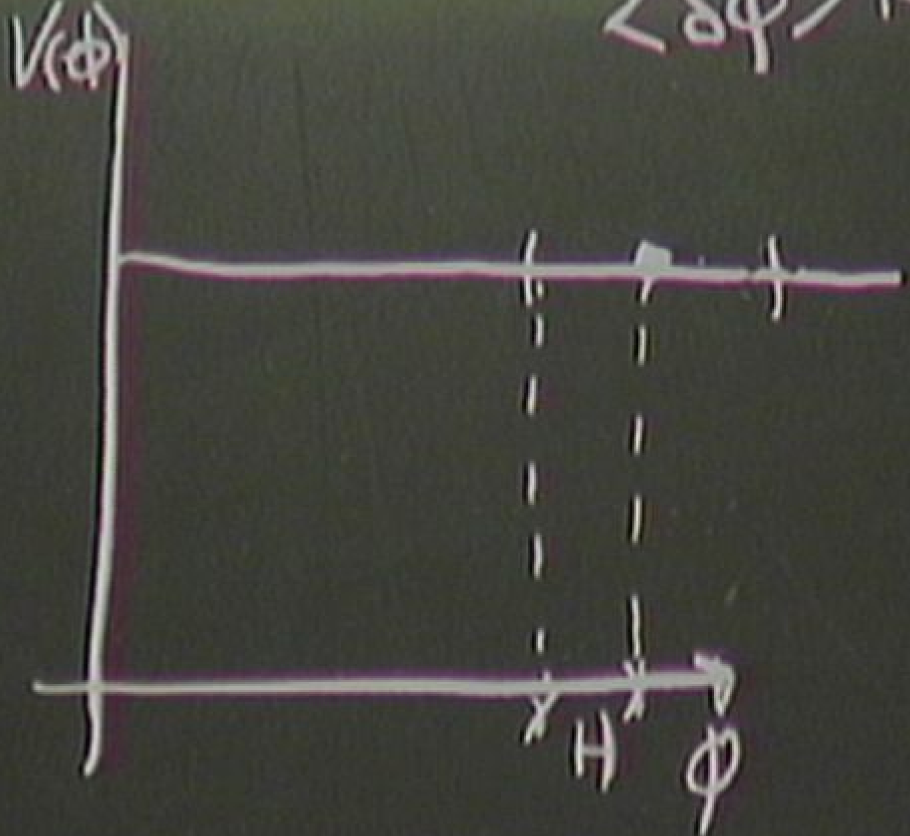



$\langle \delta\phi \rangle \sim H$
$V(\phi)$
H
ϕ


V(φ)
<δφ> ~ H
H
φ

e^{GO}

$$\langle \delta\phi \rangle \sim H$$



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$$\langle 0|\delta\phi^{+}\delta\phi|0\rangle^{1/2}$$

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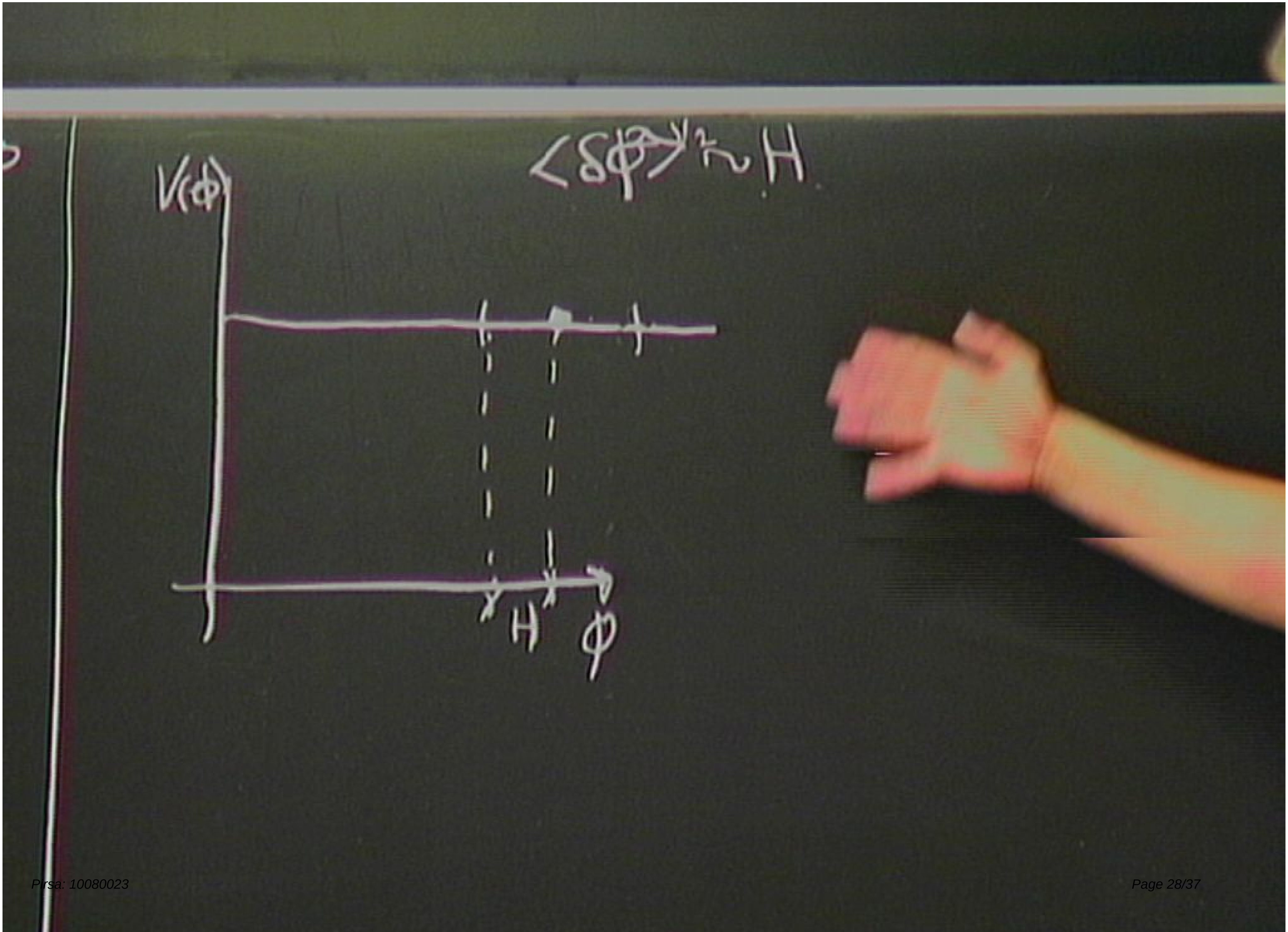

$V(\phi)$
$\langle \delta\phi^2 \rangle^{1/2} \sim H$
H
ϕ

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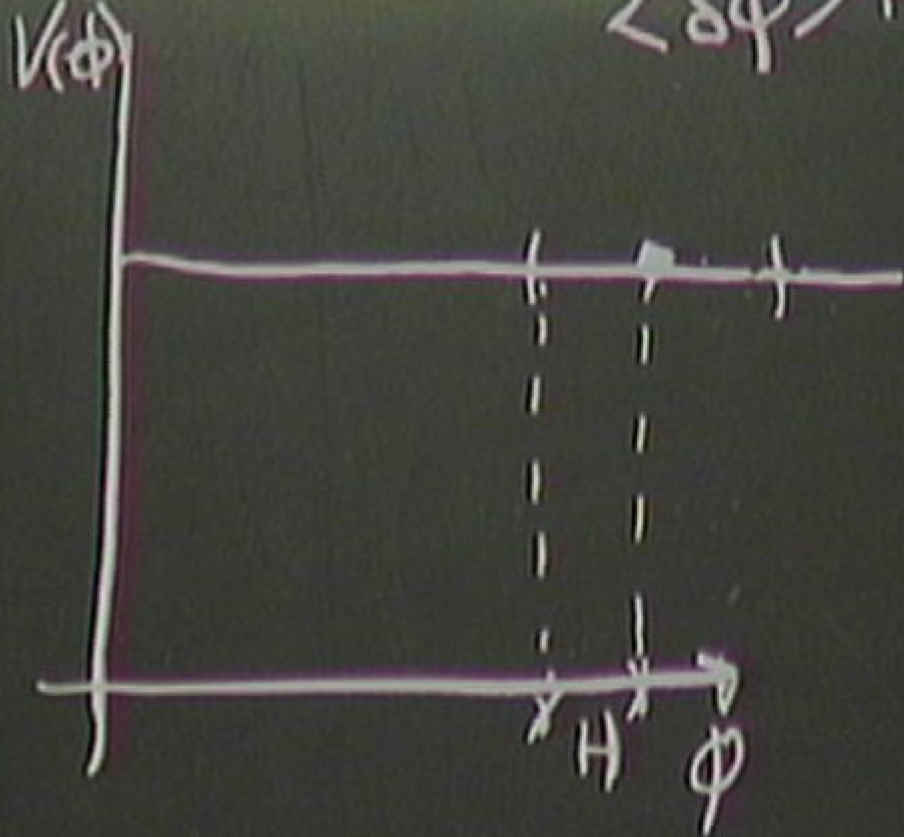
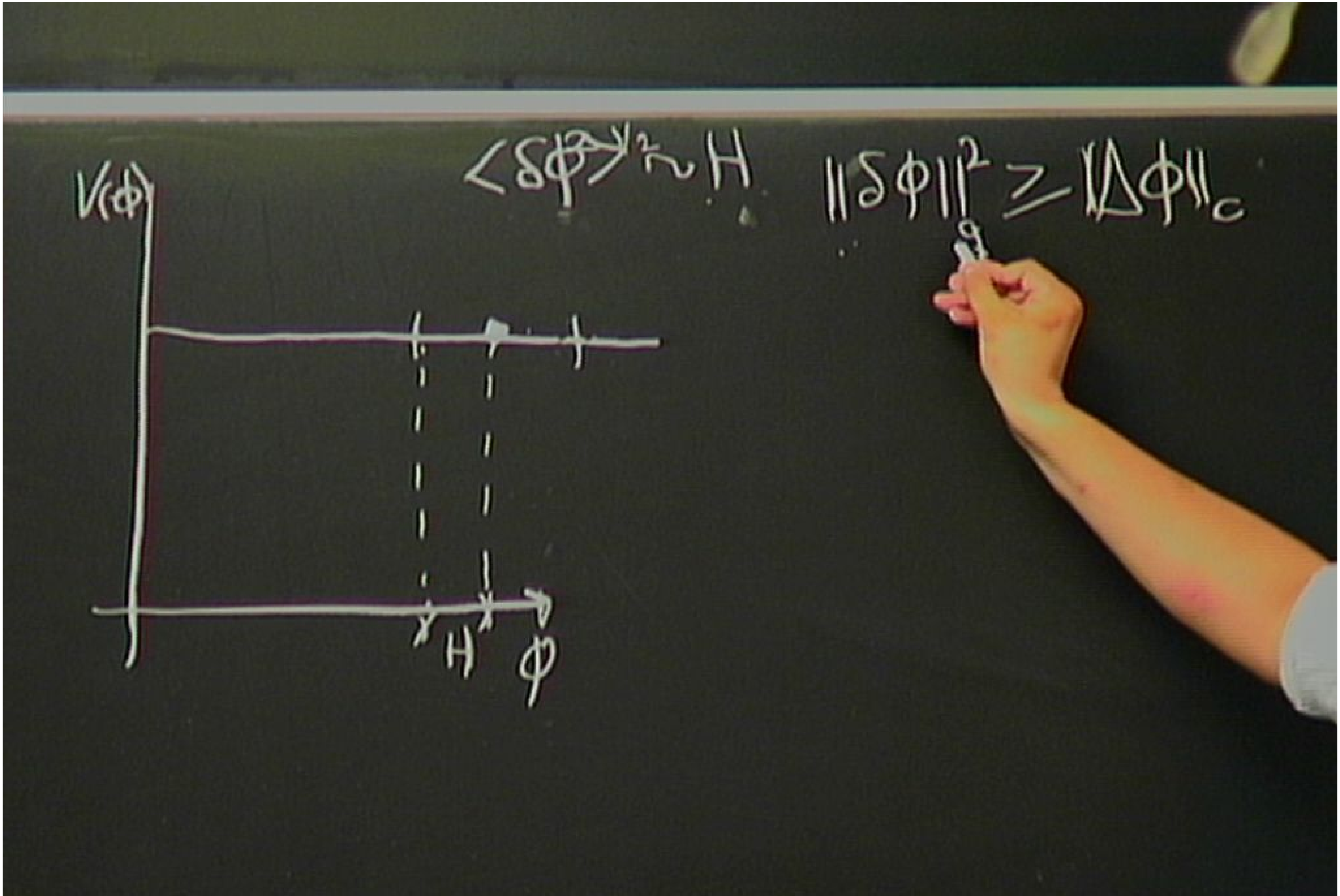
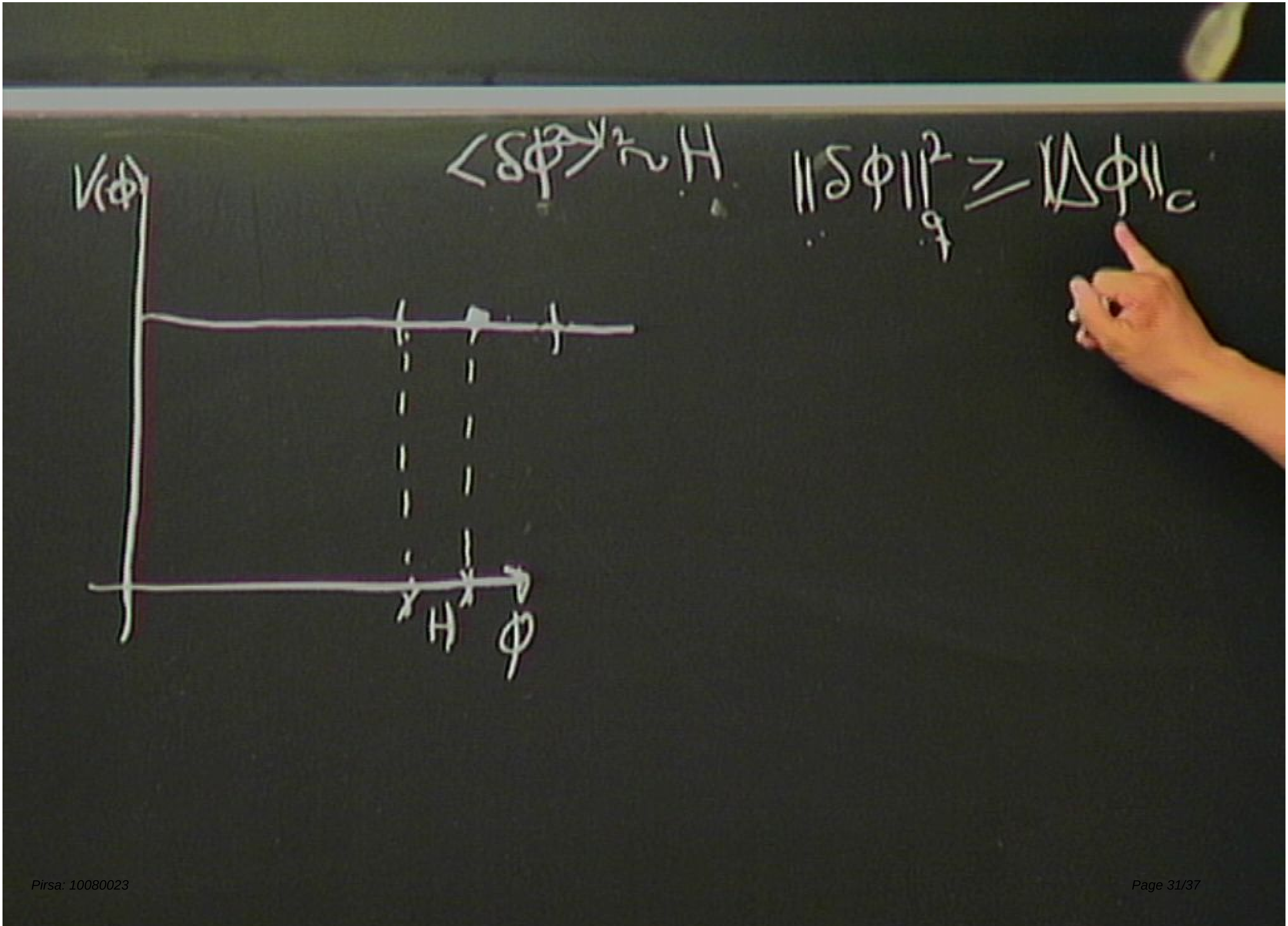
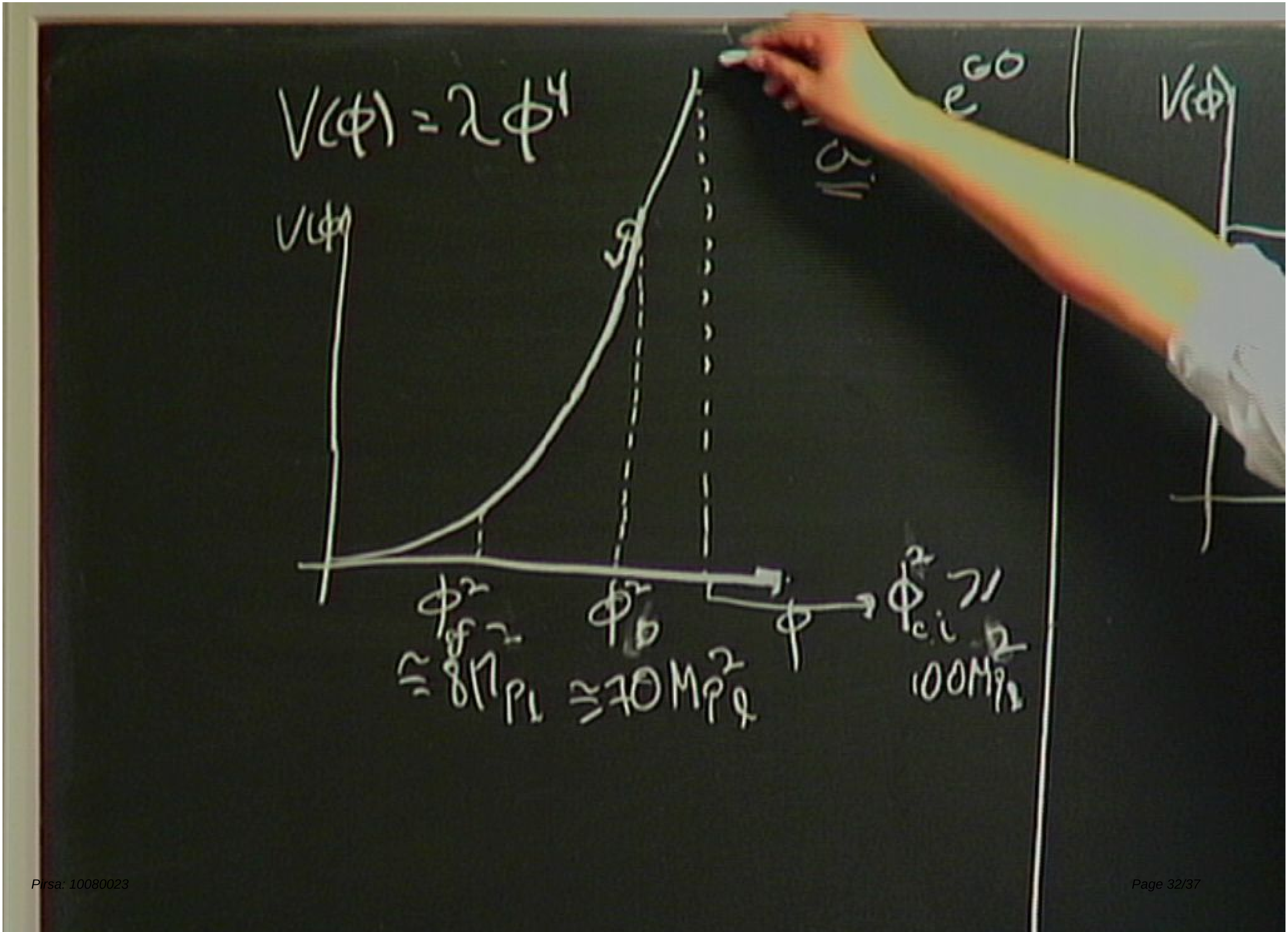
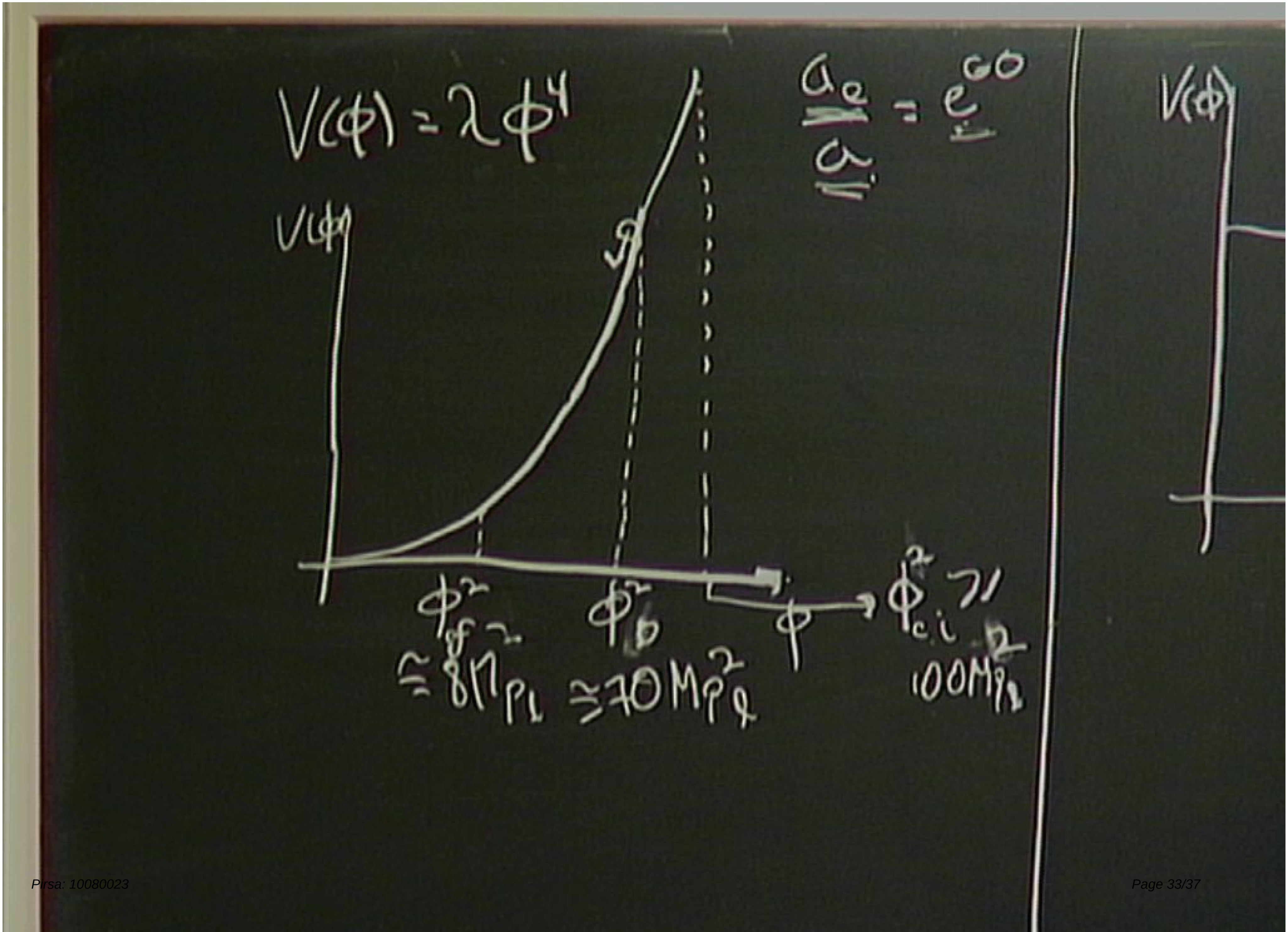




$\langle \delta\phi^2 \rangle^{1/2} \sim H$
$\|\delta\phi\|^2 \geq \|\Delta\phi\|_c$
$V(\phi)$
H
ϕ


$\langle \delta\phi^2 \rangle^{1/2} \sim H$
$\|\delta\phi\|_q^2 \gtrsim \|\Delta\phi\|_c$
$V(\phi)$
H
ϕ


V(φ) = λφ⁴
V(φ)
φ
e^60
V(φ)
≃ 8π M_Pl
≃ 70 M_Pl²
100 M_Pl²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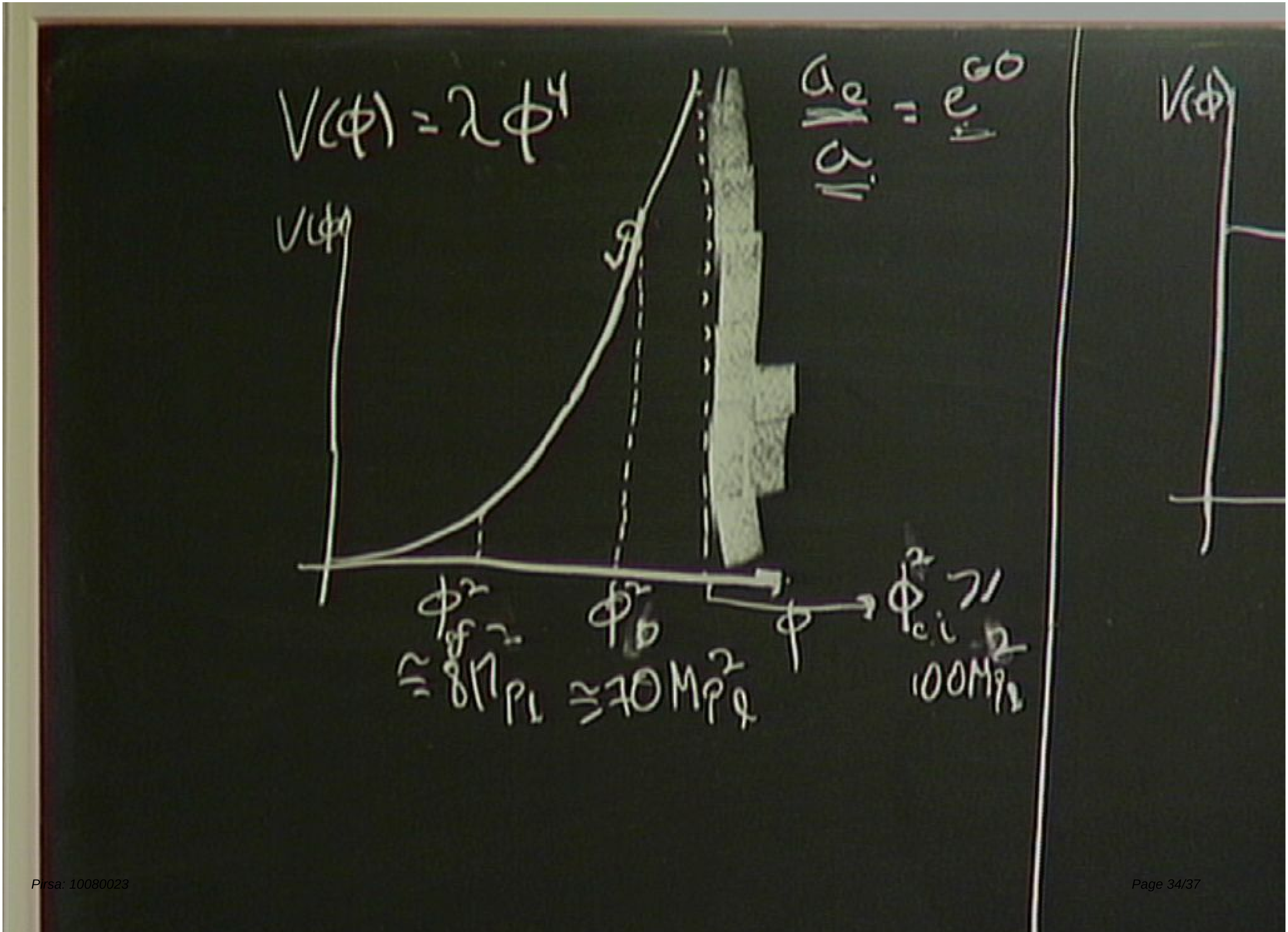
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[Quasi de Sitter geometry]

$$dQ = T\,dS$$

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[Quasi de Sitter geometry]

$\underbrace{dQ}_{\text{energy}} = TdS$

= q. horizon

= a. horizon

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[Quasi de Sitter geometry]

$$\underbrace{dQ}_{\text{energy}} = T\,dS \rightarrow \frac{dA}{4G}$$

$$T \sim \frac{H}{2\pi}$$

— e. horizon

— a. horizon