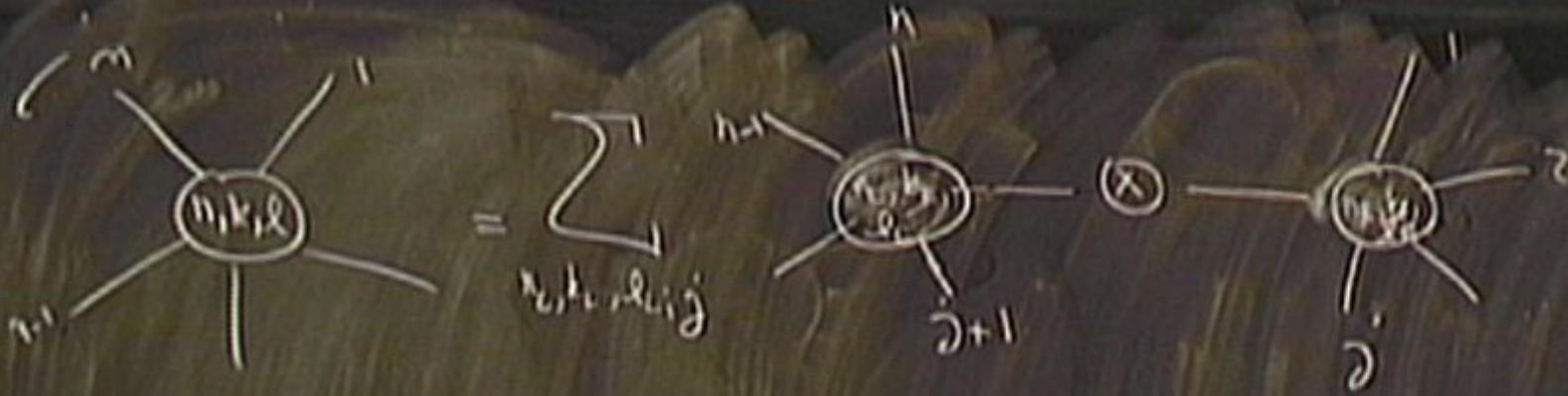


Title: Space-time, Quantum Mechanics and Scattering Amplitudes

Date: Aug 27, 2010 03:00 PM

URL: <http://pirsa.org/10080022>

Abstract:





$$= \sum_{i, k, l, j} \dots$$

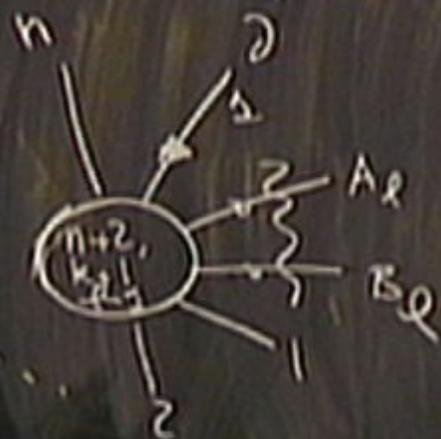


$$\begin{aligned} n_l + n_k - 2 &= n \\ k_l + k_{l+1} &= k \\ l_{l+1} &= l \end{aligned}$$



$$= \sum_{i, k, l} n_{i, k, l} g$$

$$\begin{aligned} n_l + n_k - 2 &= n \\ k_l + k_r - 1 &= k \\ l + l &= l \end{aligned}$$



+



= $\sum_{k_2, k_1, l_1, j} \dots$

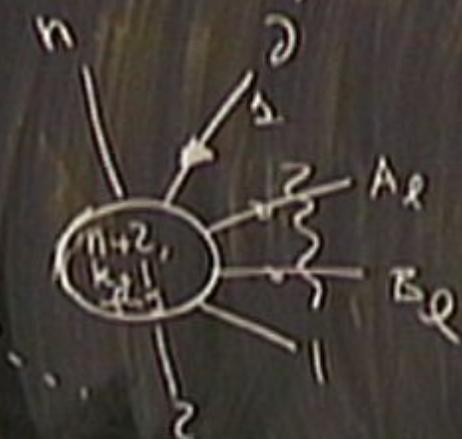
$$n_2 + n_1 - 2 = n$$

$$k_2 + k_1 + l_1 = k$$

$$l_2 + l_1 = l$$



\otimes



Symmetrisch
in A_2, B_2



= $\sum_{k, l, i, j}$
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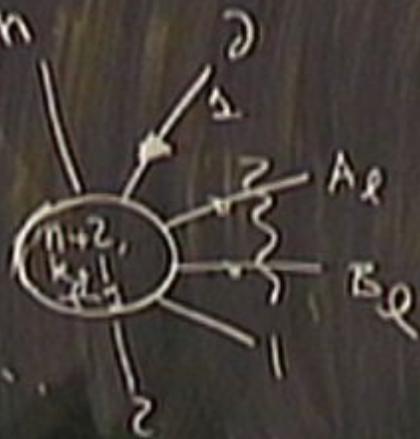
$$n_L + n_R - 2 = n$$

$$k_L + k_R + 1 = k$$

$$l_L + l_R = l$$



\otimes



Symmetrie
 in A_l, B_l

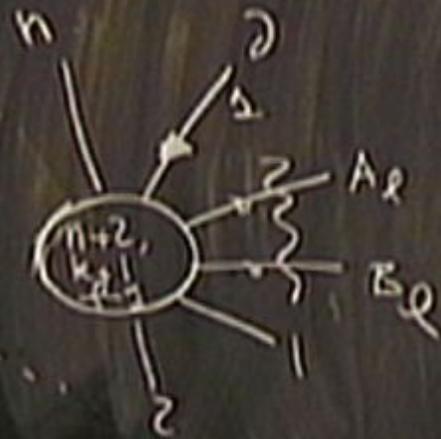
$$\mathbb{Z}_n \rightarrow \mathbb{Z}_{n+1} \otimes \mathbb{Z}_{n-1}$$



$$= \sum_{k_1, k_2, \dots, k_j}^{n-1}$$



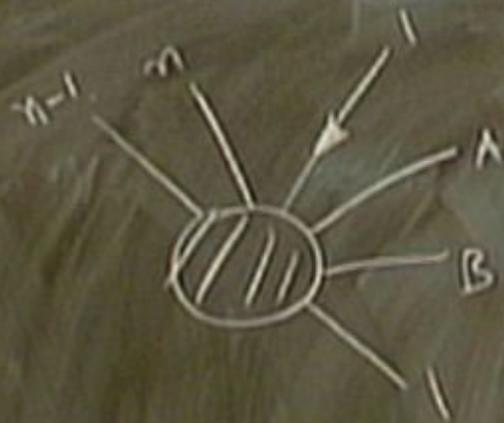
$$\begin{aligned} n_2 + n_3 - 2 &= n \\ k_2 + k_3 - 1 &= k \\ l_2 + l_3 - 1 &= l \end{aligned}$$



Symmetrie
in A_2, B_2

$$\mathbb{Z}_n \rightarrow \mathbb{Z}_{n+2} \wr \mathbb{Z}_{n-1}$$

Ex: 1-loop MHV amplitudes



$$[n-1 \ n | AB]$$

$$\times M_{\text{NMHV tree}} \left(\begin{array}{l} (n-1 \ n | A B) \\ (A B) \cap (n-1 \ n) \end{array} \right)$$

Ex: 1-loop MHV amplitudes

$$\sum_{j \cdot} [AB | j j+1]$$

Ex: 1-loop MHV amplitudes

$$\sum_{\vec{j}} [AB | \vec{j} \vec{j}+1]$$

$$M_n = M_{n-1} + F_n$$

$$= M_{n-2} + F_{n-2} + F_n$$

$$= M_{n-3} + F_{n-3} + F_{n-1} + F_n$$

⋮

Ex: 1-loop MHV amplitudes

$$\sum_{i \cdot} [AB | i i+1]$$

$$\sum_{i < j}$$

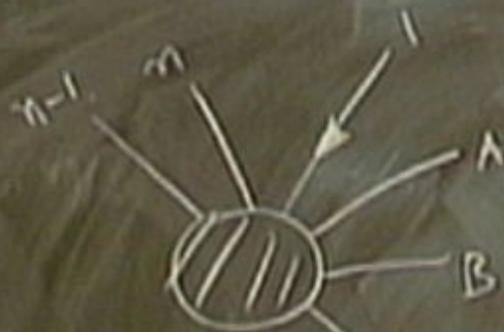
$$M_n = M_{n-1} + F_n$$

$$= M_{n-2} + F_{n-2} + F_n$$

$$= M_{n-3} + F_{n-3} + F_{n-1} + F_n$$

$$\vdots$$

$$= \left(F_n + F_{n-1} + \dots \right)$$



$$[n-1 \ n \ | \ AB]$$

$$\times M_{\text{NMHV tree}} \left(\begin{array}{l} (n-1 \ n \ | \ AB) \\ (AB) \wedge (n-1 \ n) \end{array} \right)$$



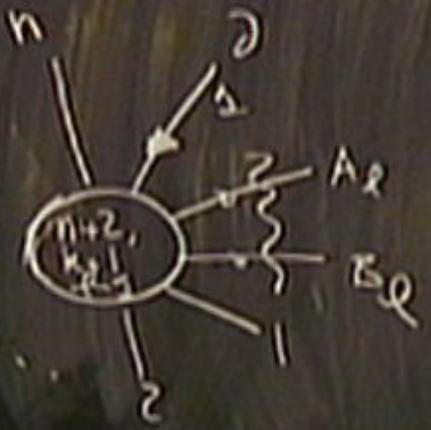


= \sum
 n, k, l \in \mathbb{Z}

$$\begin{aligned} n_2 + n_3 - 2 &= n \\ k_2 + k_3 - 1 &= k \\ l_2 + l_3 - 1 &= l \end{aligned}$$



(X)



Symmetrisch
 in A_2, B_2

$$\mathbb{Z}_n \rightarrow \mathbb{Z}_{n+1} \oplus \mathbb{Z}_{n+1}$$

$$\delta^4 [\eta_A \langle B | \psi \rangle + \eta_B \langle$$

$$\delta^4 \left[\eta_A \langle B | j, j+1 \rangle + \eta_B \langle A | j, j+1 \rangle + \dots \right]$$

$$\langle A | j \rangle \langle B | j, j+1 \rangle \langle j, j+1 | A \rangle \langle j, j+1 | AB \rangle \langle j+1 | AB | \rangle$$

$$\delta^4 \left[\eta_A \langle B | j j+1 \rangle + \dots \right]$$

$$\frac{\langle AB | j \rangle \langle B | j j+1 \rangle \langle j j+1 | A \rangle \langle j j+1 | AB \rangle \langle j+1 | AB | \rangle}{\langle \hat{A} B | i i+1 \rangle \langle B | i i+1 \rangle \langle i i+1 | \hat{A} B \rangle \langle i i+1 | \hat{A} B \rangle \langle \hat{A} B | i \rangle}$$

$$\delta^4 \left[\eta_{\hat{A}} \langle B | i i+1 \rangle + \dots \right]$$

$$\frac{\langle \hat{A} B | i i+1 \rangle \langle B | i i+1 \rangle \langle i i+1 | \hat{A} B \rangle \langle i i+1 | \hat{A} B \rangle \langle \hat{A} B | i \rangle}{\dots}$$

$$\hat{A} = \langle B | j+1 | \hat{A} \rangle - \langle A | j+1 \rangle B$$

$$\int d\eta_A d\eta_B \delta^4 \left[\eta_A \langle B | j+1 \rangle + \dots \right] \\ \frac{\delta^4 \left[\eta_A \langle B | j+1 \rangle + \dots \right]}{\langle A B | j \rangle \langle B | j+1 \rangle \langle | j+1 A \rangle \langle j+1 AB \rangle \langle j+1 AB | \rangle}$$

$$\delta^4 \left[\eta_{\hat{A}} \langle B | i+1 | \rangle + \dots \right] \\ \frac{\delta^4 \left[\eta_{\hat{A}} \langle B | i+1 | \rangle + \dots \right]}{\langle \hat{A} B | i+1 \rangle \langle B | i+1 | \rangle \langle i+1 | \hat{A} B \rangle \langle i+1 | \hat{A} B \rangle \langle \hat{A} B | i \rangle}$$



$$\begin{aligned} \hat{A} &= \langle B | j+1 \rangle \langle A \\ &- \langle A | j+1 \rangle \langle B \end{aligned}$$

$$\int d\eta_A d\eta_B \delta^4 \left[\frac{\eta_A \langle B | j+1 \rangle + \eta_B \langle A | j+1 \rangle + \dots}{\langle AB | j \rangle \langle B | j+1 \rangle \langle j+1 | A \rangle \langle j+1 | AB \rangle \langle j+1 | AB | \rangle} \right]$$

$$\delta^4 \left[\frac{\eta_B \langle A | i+1 \rangle + \dots}{\langle \hat{A} B | i+1 \rangle \langle B | i+1 \rangle \langle i+1 | \hat{A} B \rangle \langle i+1 | \hat{A} B \rangle \langle \hat{A} B | i \rangle} \right]$$

$$\langle B | j, j+1 \rangle \langle \hat{A} | i, i+1 \rangle$$

$$\langle B | j_{j+1} \rangle^4 \langle \hat{A} | i_{i+1} \rangle^4$$

$\langle AB$

$$\langle \hat{A} | i, i+1 \rangle$$

$$= \langle A | i, i+1 \rangle \langle B | j, j+1 \rangle - \langle A | j, j+1 \rangle \langle B | i, i+1 \rangle$$

$$\equiv \langle AB (|i, i+1\rangle \wedge |j, j+1\rangle) \rangle$$

$$\langle B | j, j+1 \rangle \langle \hat{A} | i, i+1 \rangle^4$$

$$\langle AB | j \rangle \langle AB | j+1 \rangle \langle AB | j+1 \rangle \langle AB | i+1 \rangle \langle AB | i+1 \rangle \langle$$

$$\langle B | j, j+1 \rangle \langle \hat{A} | i, i+1 \rangle^4$$

$$\langle AB | j \rangle \langle AB | j+1 \rangle \langle AB | j+1 \rangle \langle AB | i, i+1 \rangle \langle AB | i+1 \rangle \langle AB | i \rangle$$

$$\int d\eta_A d\eta_B \frac{\delta^4 \left[\eta_A \langle B | j j+1 \rangle + \eta_B \langle A | j j+1 \rangle + \dots \right]}{\langle A B | j \rangle \langle B | j j+1 \rangle \langle | j j+1 A \rangle \langle j j+1 A B \rangle \langle j+1 A B | \rangle}$$

$$\left. \begin{aligned} \eta_A &= \langle B | j j+1 | D A \\ &= \langle A | j j+1 | P B \end{aligned} \right\}$$

$$\delta^4 \left[\eta_B \langle \hat{A} | i i+1 \rangle + \dots \right]$$

$$\frac{\langle \hat{A} B | i i+1 \rangle \langle B | i i+1 \rangle \langle i i+1 | \hat{A} \rangle \langle i i+1 | \hat{A} B \rangle \langle \hat{A} B | i \rangle}{|\hat{A} B | i i+1 \rangle}$$

$$\sum_{i < j}$$

$$\langle AB C (i+1) n (i+1) j+1 \rangle$$

$$\langle AB | i \rangle \langle AB | i+1 \rangle \langle AB | i+2 \rangle \dots \langle AB | j \rangle \langle AB | j+1 \rangle \langle AB | j+2 \rangle$$

=



MHV:

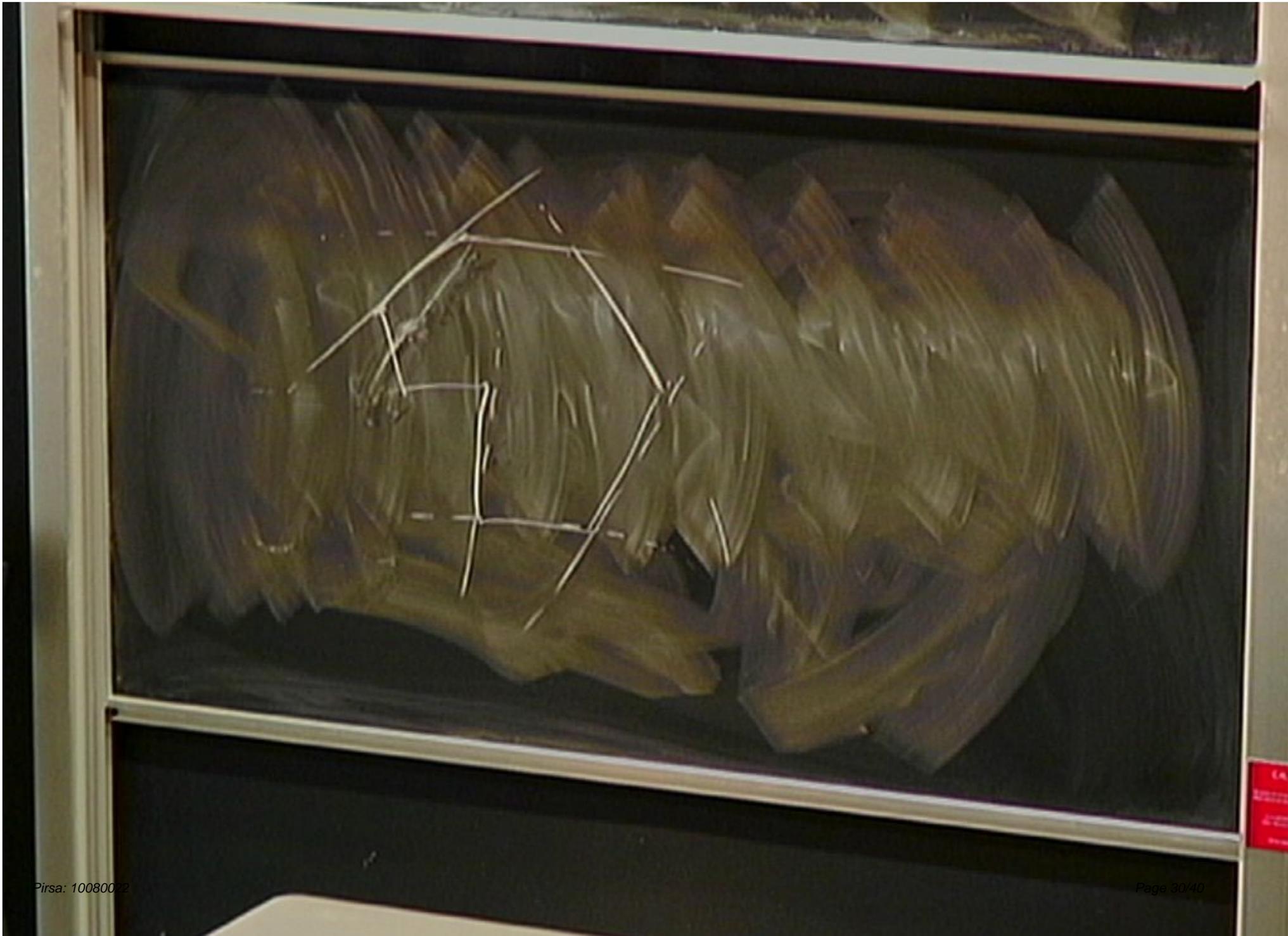
$$\sum_a \eta_a^k \frac{\partial}{\partial z_a^k} M = 0$$

$$\frac{\partial}{\partial z_a^k} M = 0$$

$$\frac{\partial I}{\partial z_A} = \frac{\partial I_A}{\partial z_A} + \frac{\partial I_B}{\partial z_B}$$



$$\frac{\partial I}{\partial z_A} = \frac{\partial I_A}{\partial z_A} + \frac{\partial I_B}{\partial z_A}$$



MHV:

$$\sum_a \eta_a^k \frac{\partial}{\partial z_a} M = 0$$

$$\frac{\partial}{\partial z_a} M = 0$$

$$\begin{aligned} & (\kappa - \kappa_j)^2 \\ & \Downarrow \\ & \langle AB j-1 j \rangle \\ & \langle AB \kappa j \rangle \end{aligned}$$

MHV:

$$\sum_a \eta_a^k \frac{\partial}{\partial z_a} M = 0$$

$$\frac{\partial}{\partial z_a} M = 0$$

$$\begin{aligned} & (x - x_j)^2 \\ & \quad \downarrow \\ & \langle AB j-1 j \rangle \\ & \langle AB k \rangle \\ & (x - x_{k^*})^2 \end{aligned}$$



$$1 \leftrightarrow (5\ 1\ 2)$$

$$\vec{v} \leftrightarrow (j-1\ j\ j+1)$$

$$\langle ABxy \rangle$$

$$\langle ABxy \rangle = \langle AB(\dots i) \cap (y+1 \dots) \rangle$$

$$\int \left(\frac{d^4 z_A d^4 z_B}{\text{vol}(GL(2))} \right)$$

$$\frac{d^4 z_{12}}{\langle AB12 \rangle \langle CAB12 \rangle \cdot \langle AB412 \rangle}$$

$$\int \left(\frac{d^4 z_A d^4 z_B}{\text{vol}(GL(2))} \right)$$

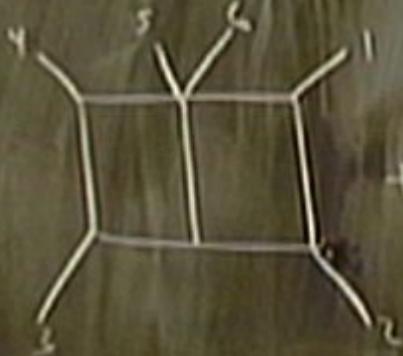
$$\frac{\langle AB13 \rangle \langle 1245 \rangle \langle 3245 \rangle}{\langle AB12 \rangle \cdot \langle AB51 \rangle}$$







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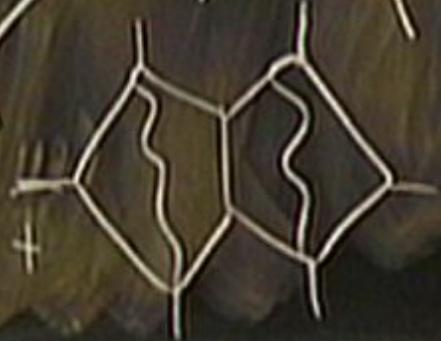
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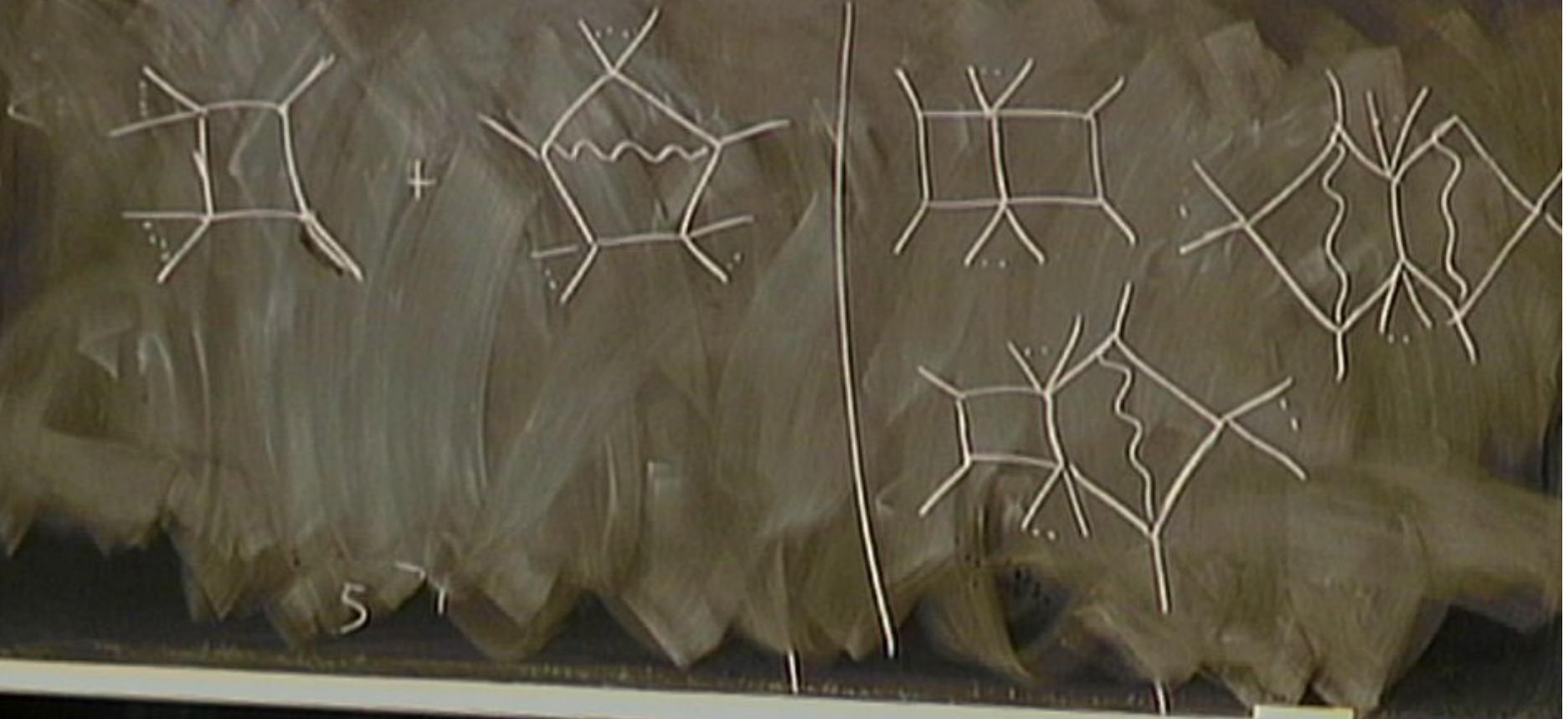
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