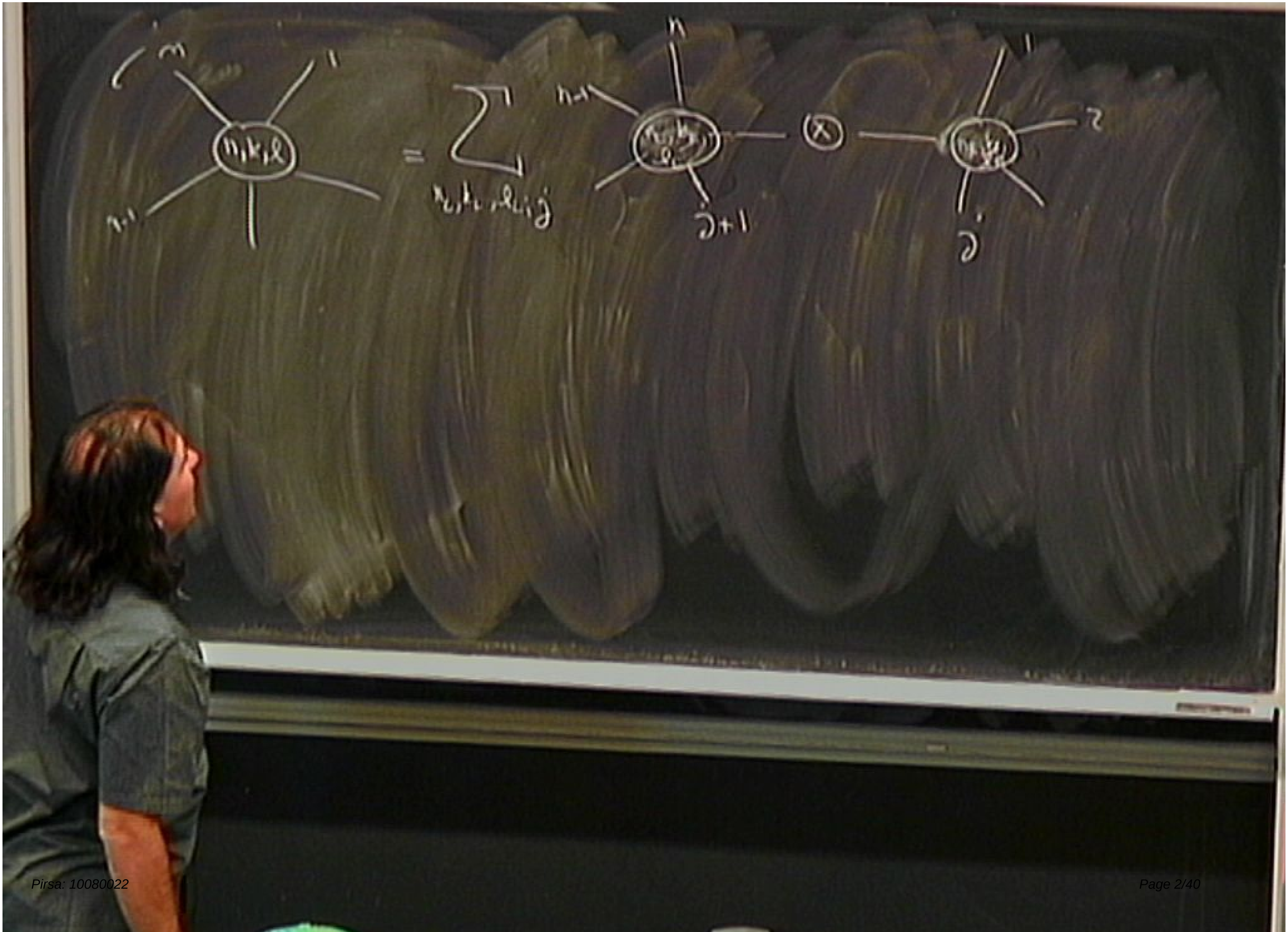
Title: Space-time, Quantum Mechanics and Scattering Amplitudes

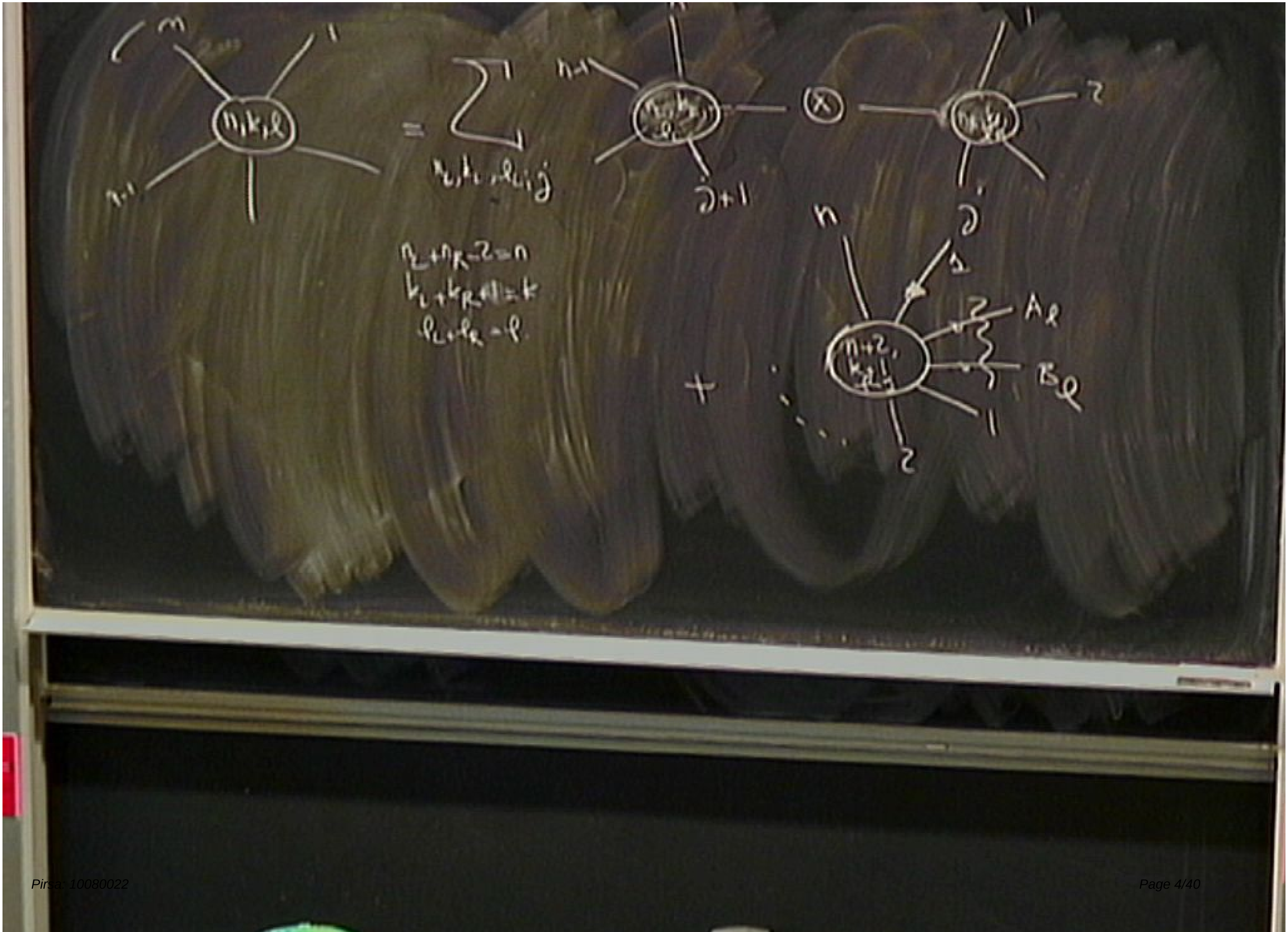
Date: Aug 27, 2010 03:00 PM

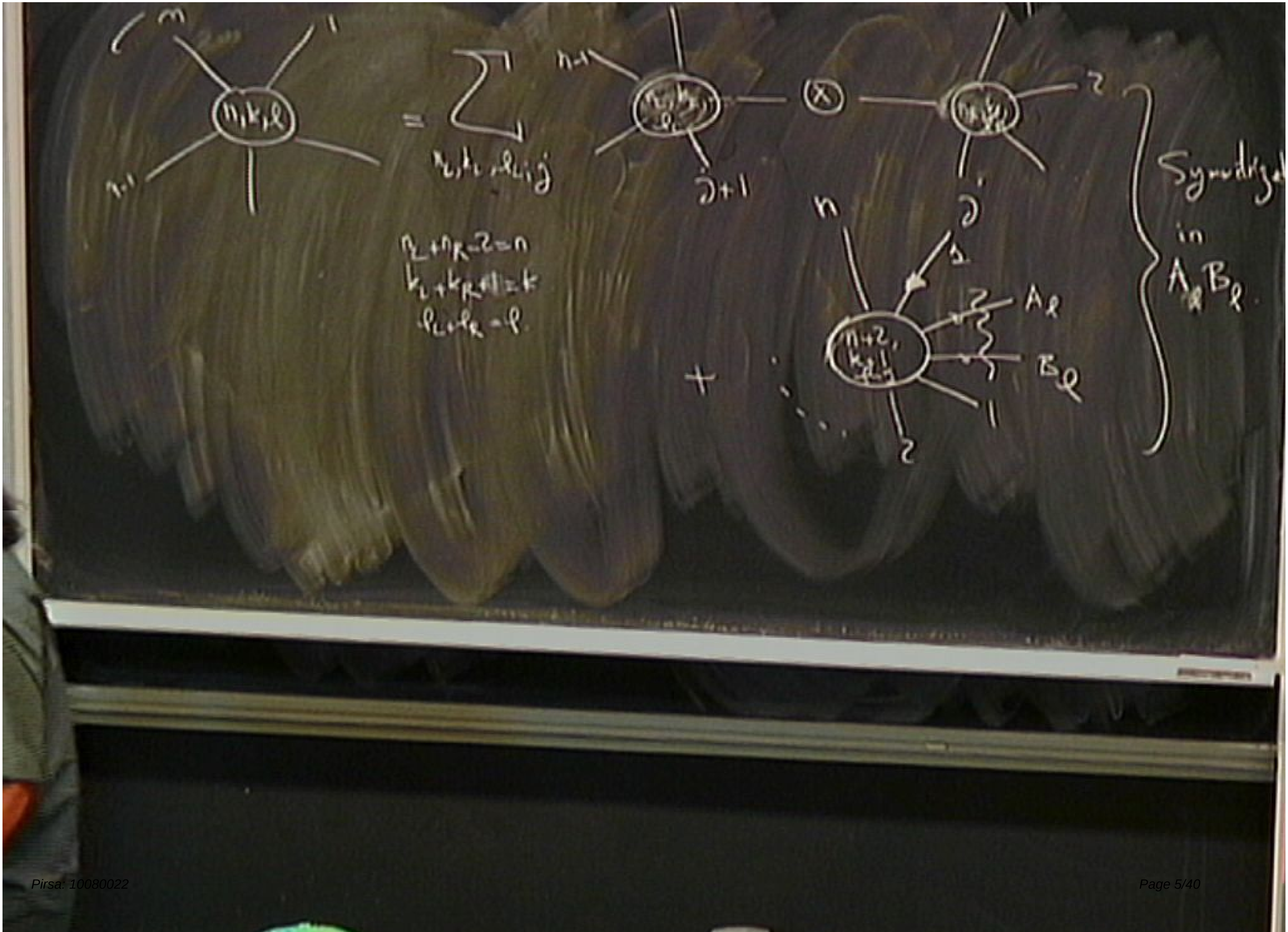
URL: http://pirsa.org/10080022

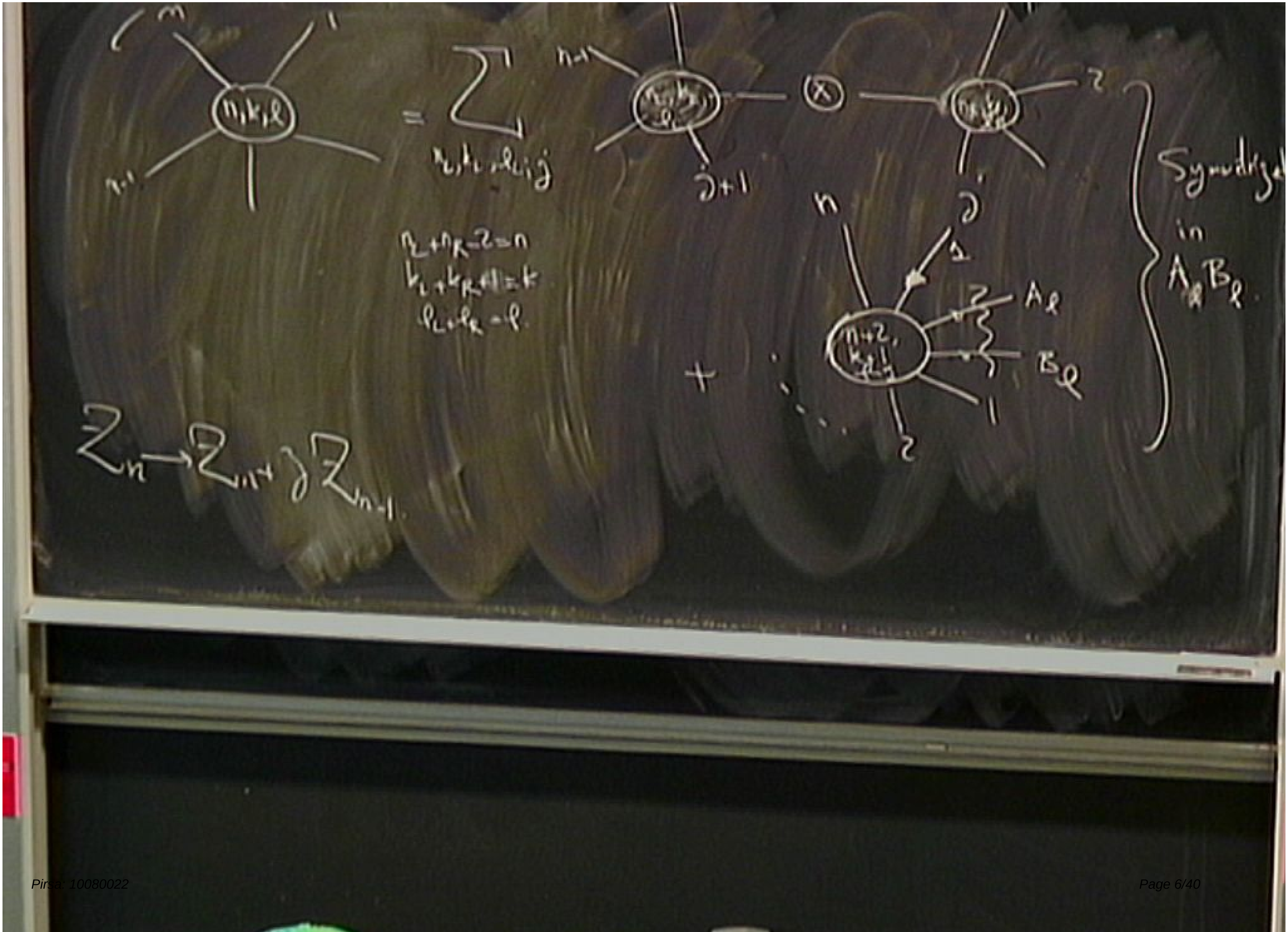
Abstract:

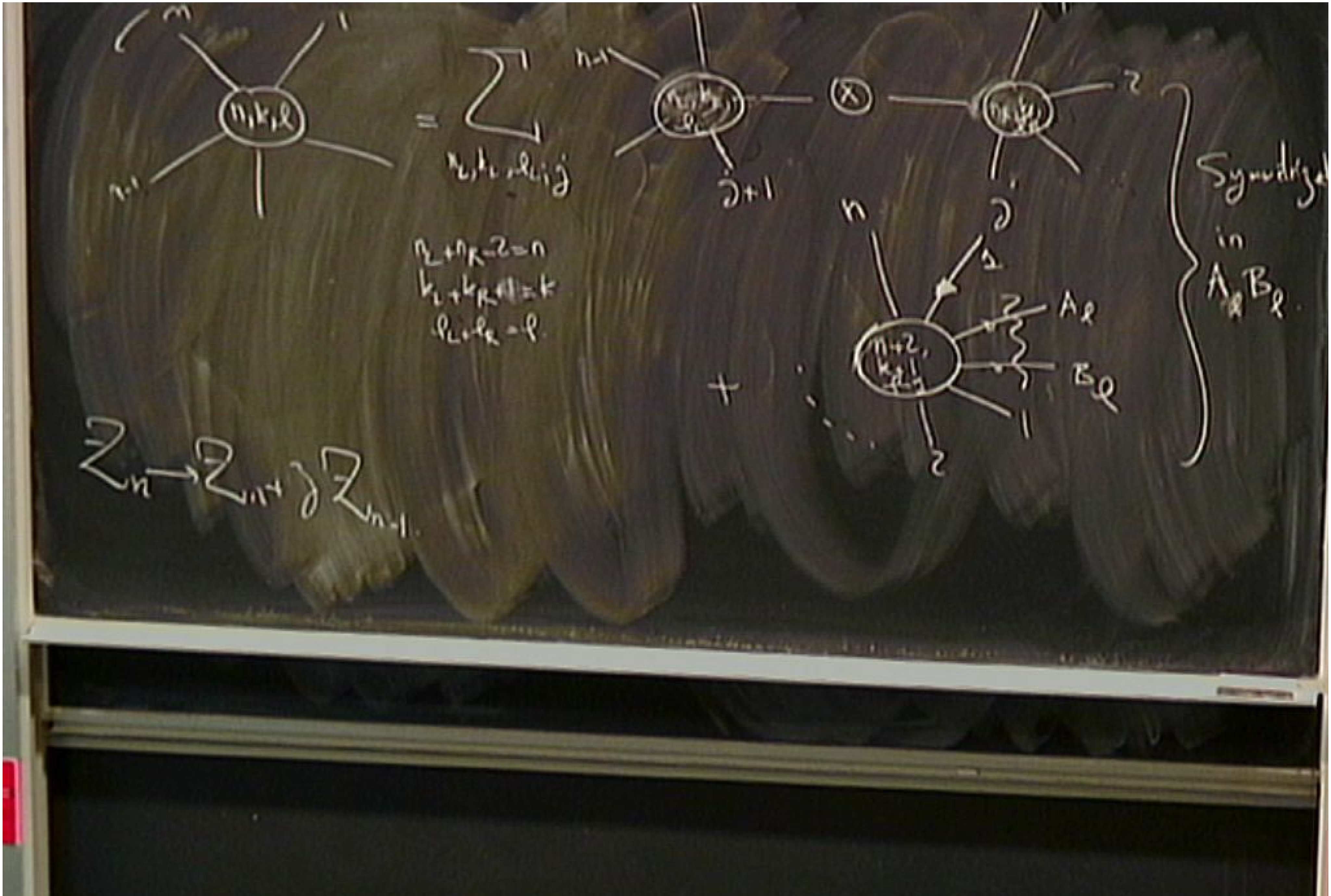


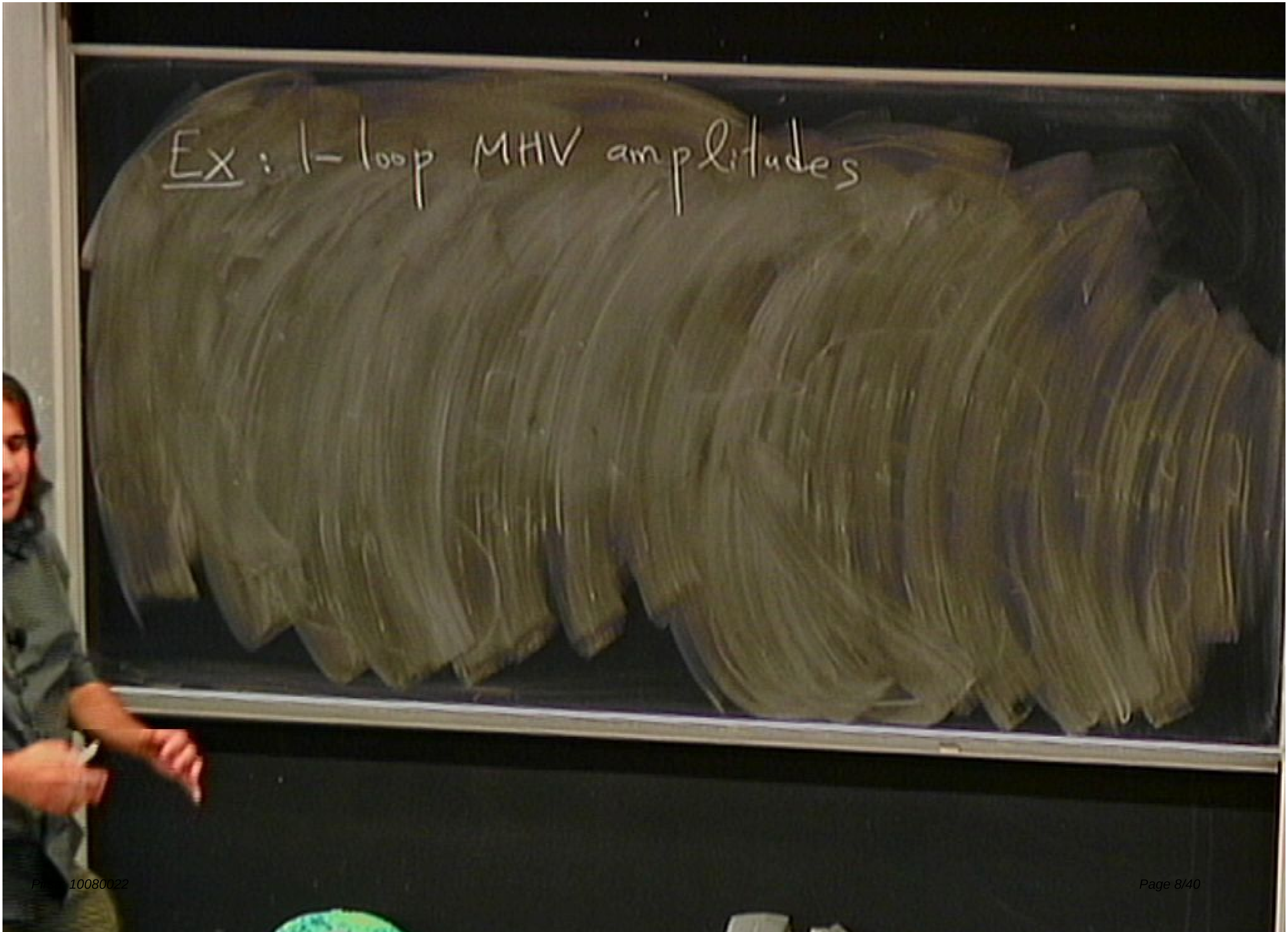

n_L+n_R-2=n
k_L+k_R+1=k
l_L+l_R=l

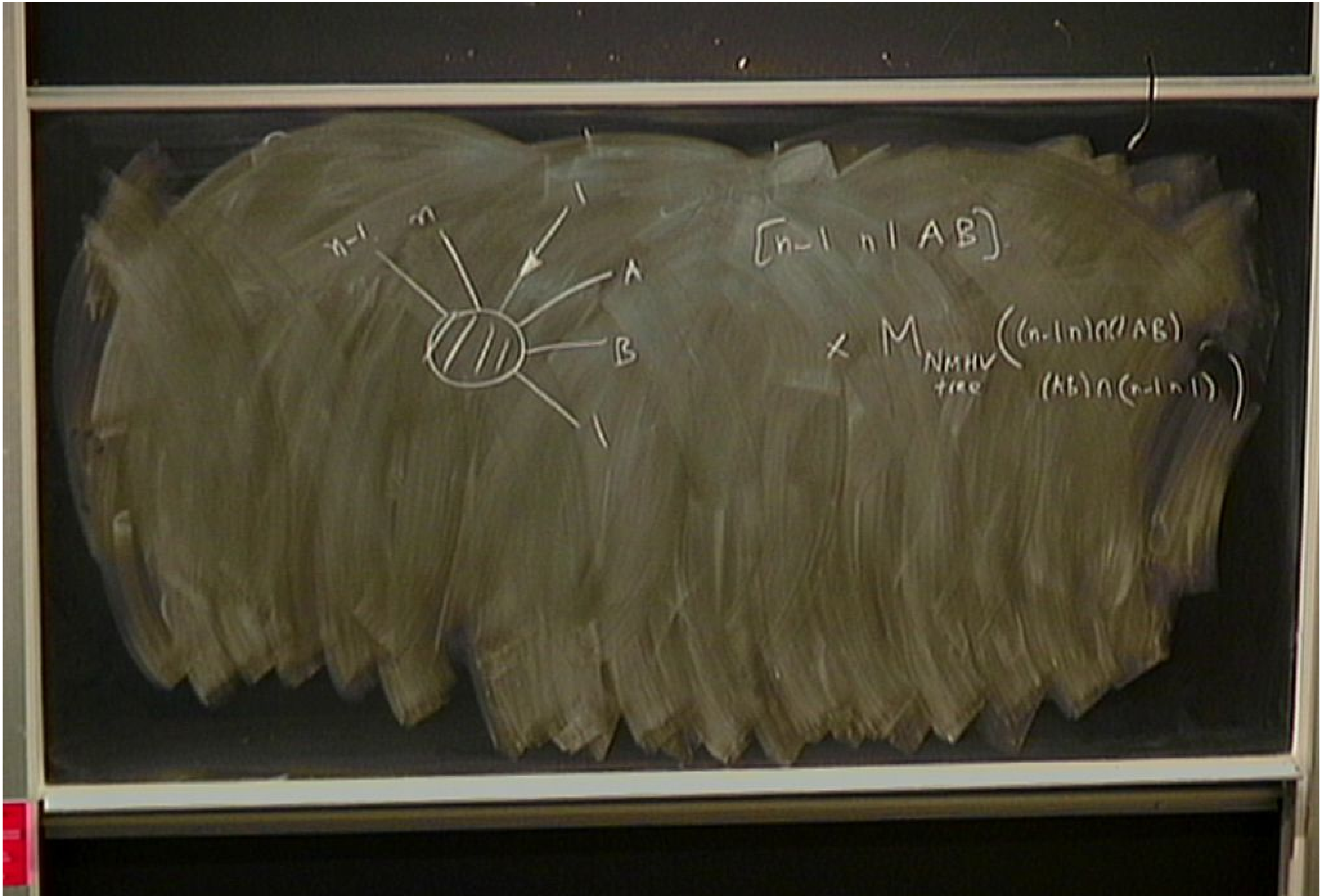

n, k, l
n-1
n
j+1
A_l
B_l
n+2, k-1, l-1
n_L + n_R - 2 = n
k_L + k_R + 1 = k
l_L + l_R = l


n,k,ℓ
nL+nR-2=n
kL+kR+1=k
ℓL+ℓR=ℓ
in
Aℓ
Bℓ


n,k,l
n_L+n_R-2=n
k_L+k_R+1=k
l_L+l_R=l
A_l
B_l
in
A_l B_l

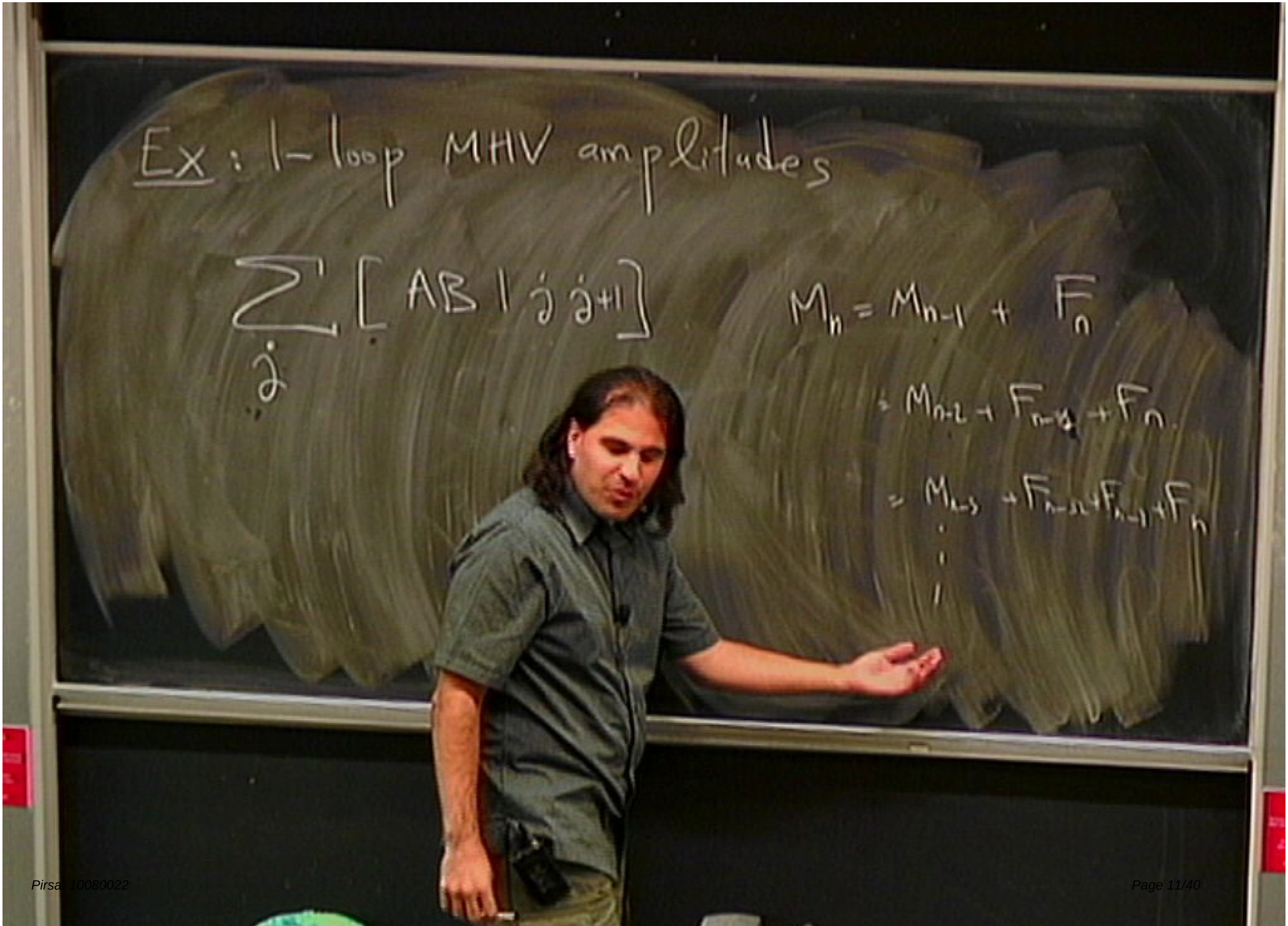

n,k,ℓ
n_L+n_R-2=n
k_L+k_R+1=k
ℓ_L+ℓ_R=ℓ
Syzygies in A_ℓ B_ℓ
Z_n → Z_{n+1} γ Z_{n-1}


Ex: 1-loop MHV amplitudes


n-1
n
1
A
B
1
[n-1 n | A B]
× M
NMHV
tree
((n-1 n)∩(1 A B)
(AB)∩(n-1 n 1))

Ex: 1-loop MHV amplitudes

$$\sum_{j} [AB \mid j \; j{+}1]$$


Ex: 1-loop MHV amplitudes
$\sum_j [AB \mid j\, j{+}1]$
$M_n = M_{n-1} + F_n$
$= M_{n-2} + F_{n-1} + F_n$
$= M_{n-3} + F_{n-2} + F_{n-1} + F_n$

Ex: 1-loop MHV amplitudes

$$\sum_{j} [AB \mid j \; j+1]$$

$$\sum_{i<j}$$

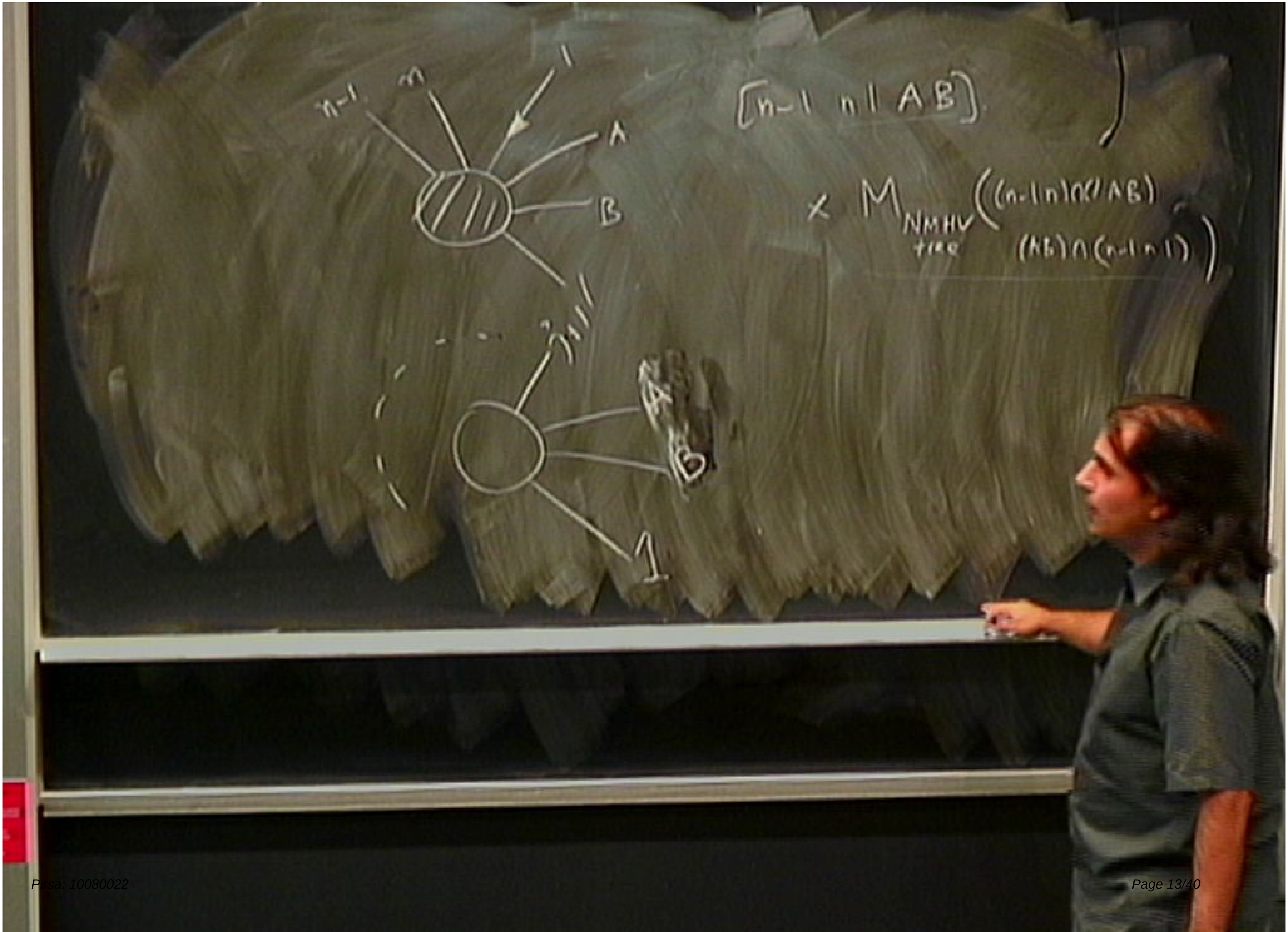
$$M_n = M_{n-1} + F_n$$

$$= M_{n-2} + F_{n-1} + F_n$$

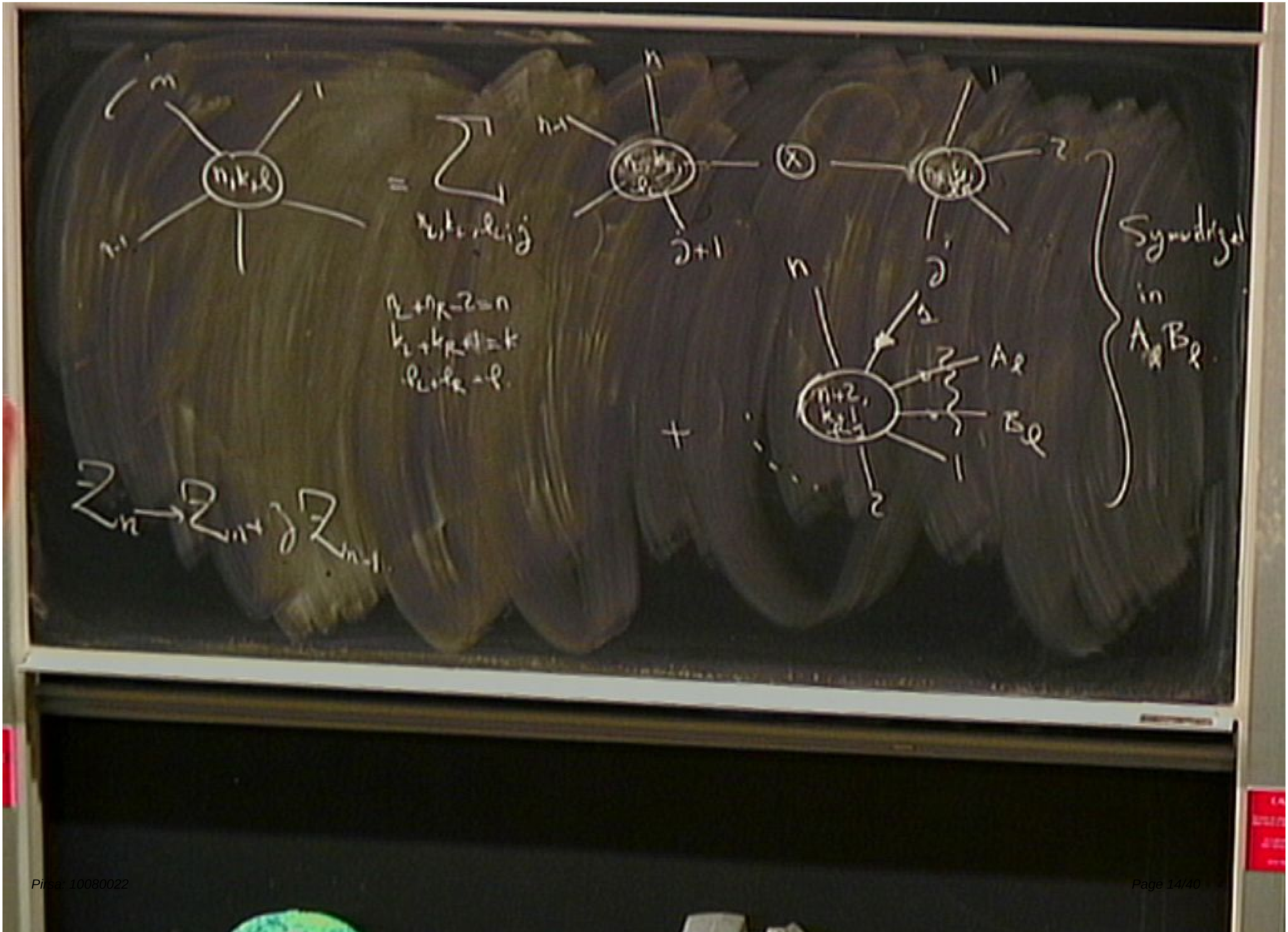
$$= M_{n-3} + F_{n-2} + F_{n-1} + F_n$$

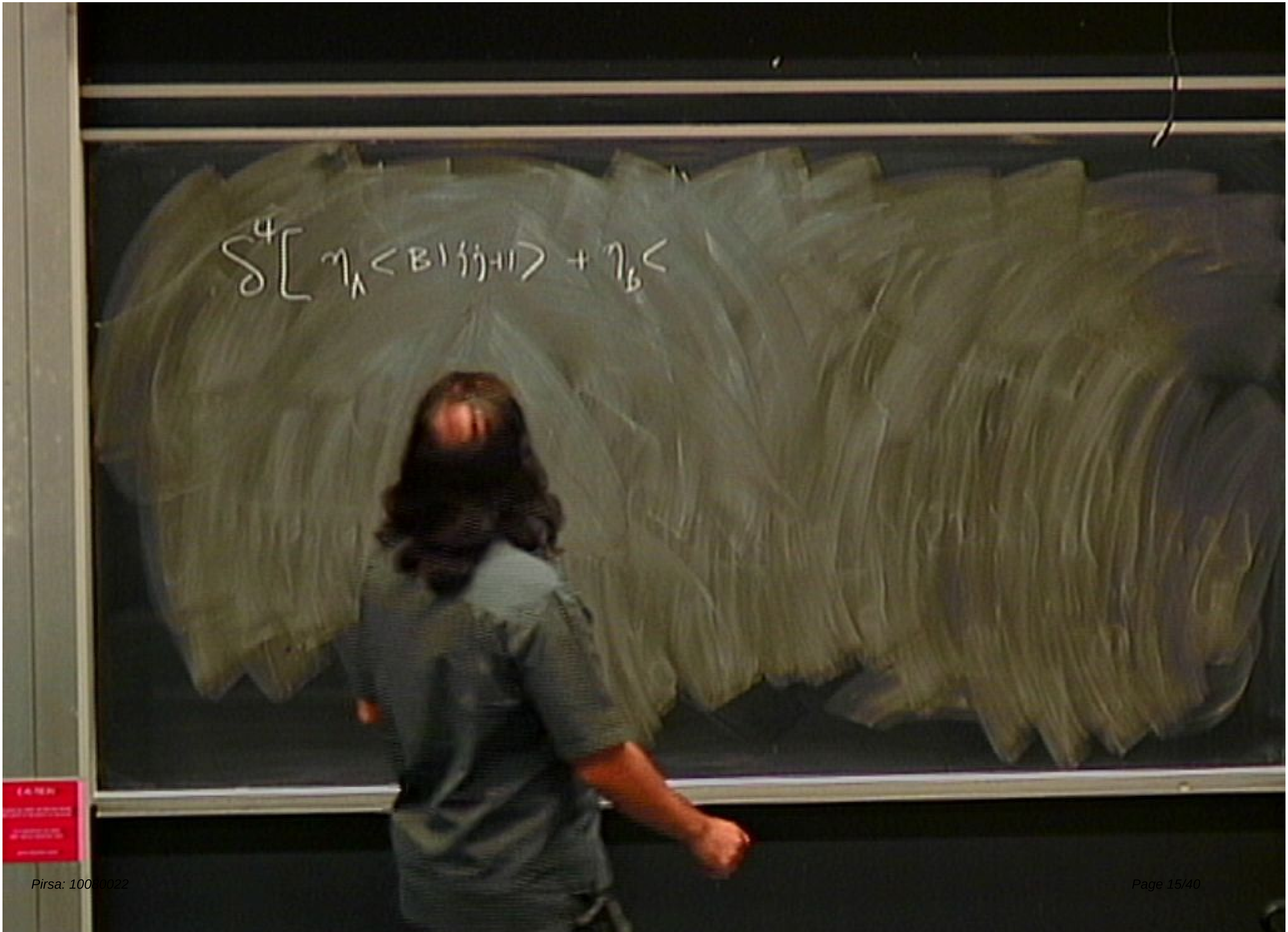
$$\vdots$$

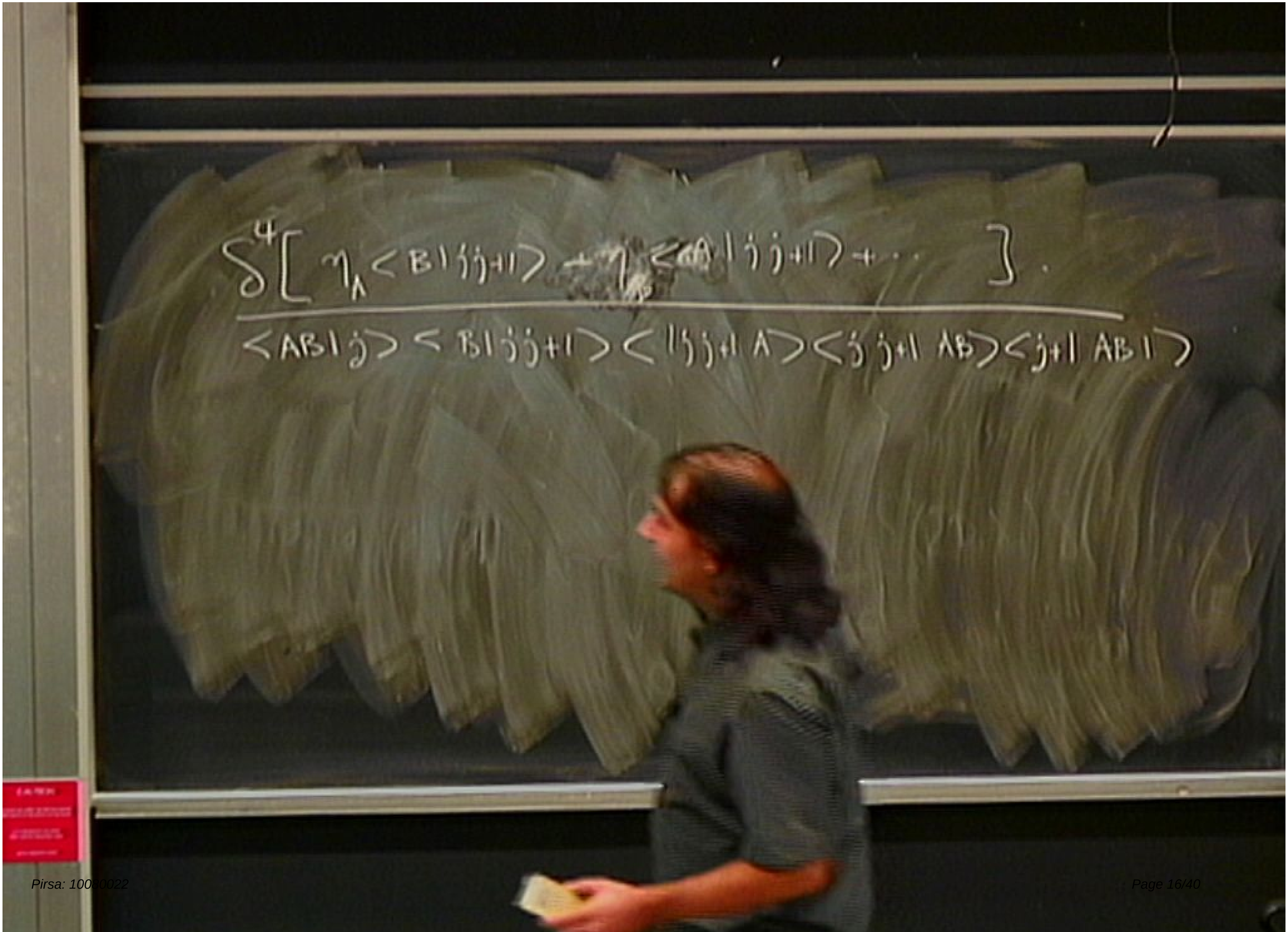
$$= \left(F_n + F_{n-1} + \cdots \right)$$

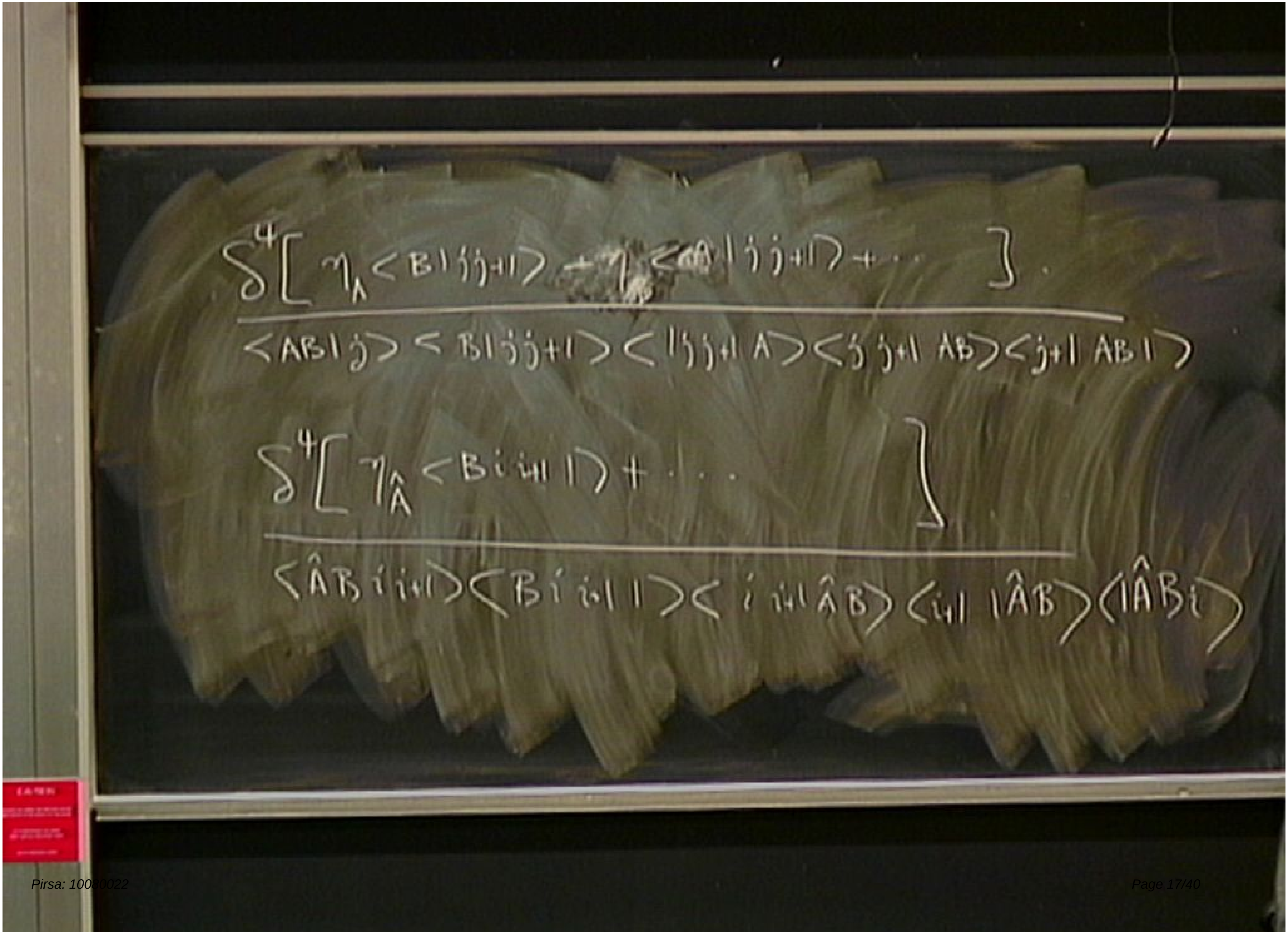


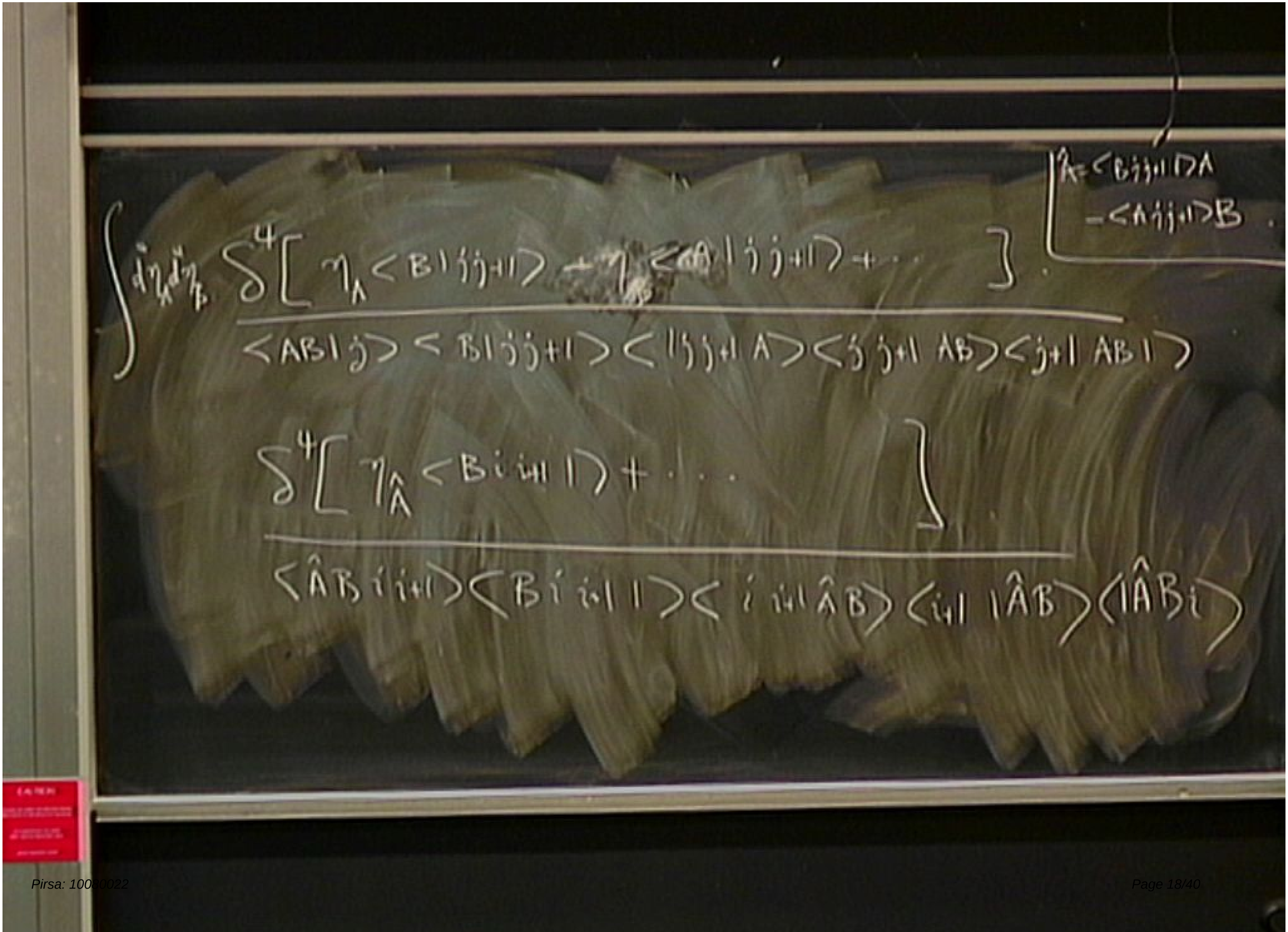
n-1
n
1
A
B
[n-1 n | A B]
× M NMHV tree ((n-1 n) ∩ (1 A B)
(A B) ∩ (n-1 n 1))
A
B
1


Symmetrized
in
$A_\ell B_\ell$
A_ℓ
B_ℓ


$\delta^4 [\eta_A \langle B | \zeta(j+1) \rangle + \eta_b \langle$


<AB | j> < B | j j+1 > < | j j+1 A > < j j+1 AB > < j+1 | AB | >

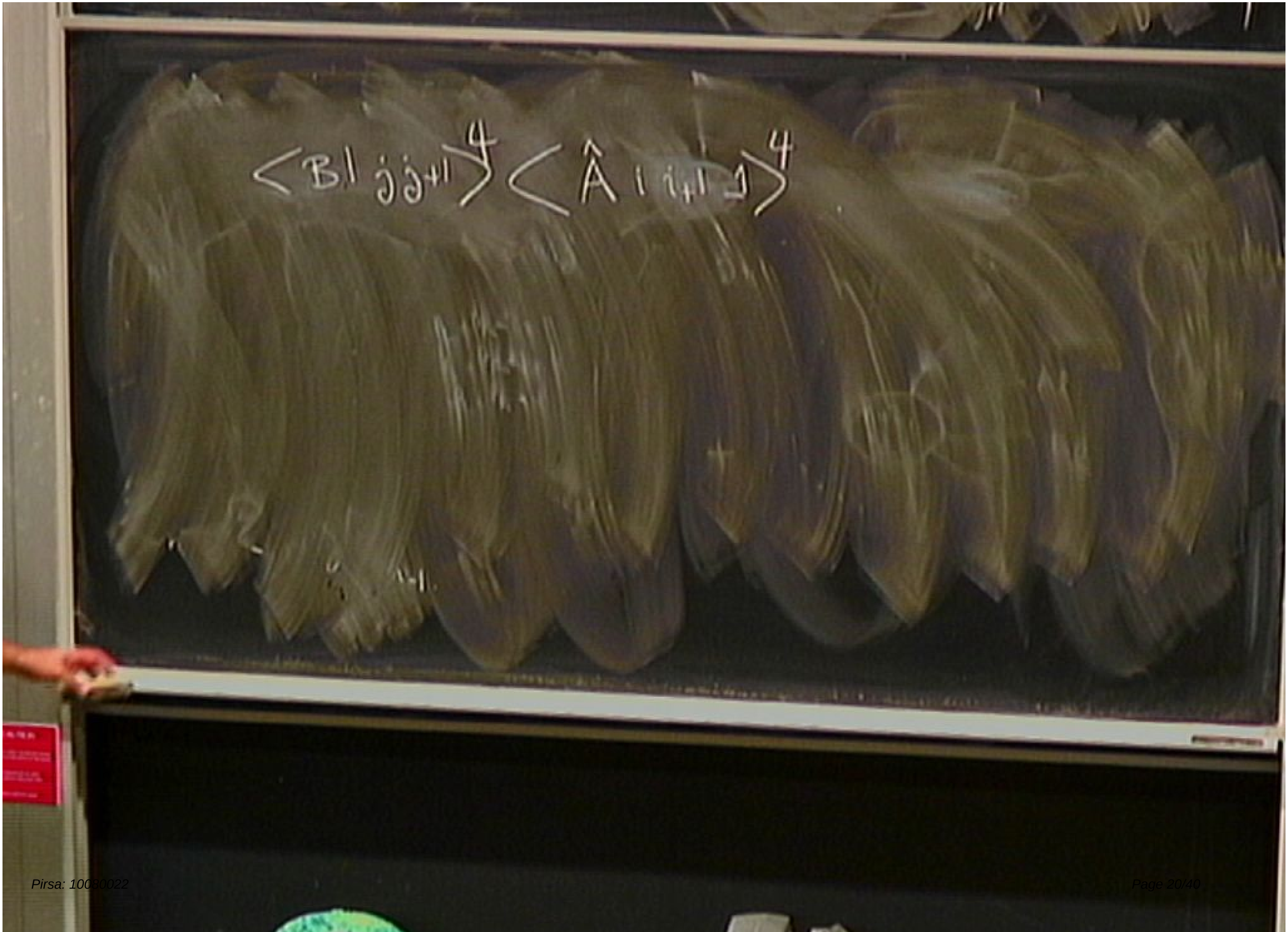

δ4[η_A <B|j j+1> + η <A|j j+1> + ...] .
<AB|j> <B|j j+1> <|j j+1 A> <j j+1 AB> <j+1 AB|>
δ4[η_Â <B i i+1 1> + ...]
<ÂB i i+1> <B i i+1 1> <i i+1 ÂB> <i+1 1ÂB> <1ÂB i>

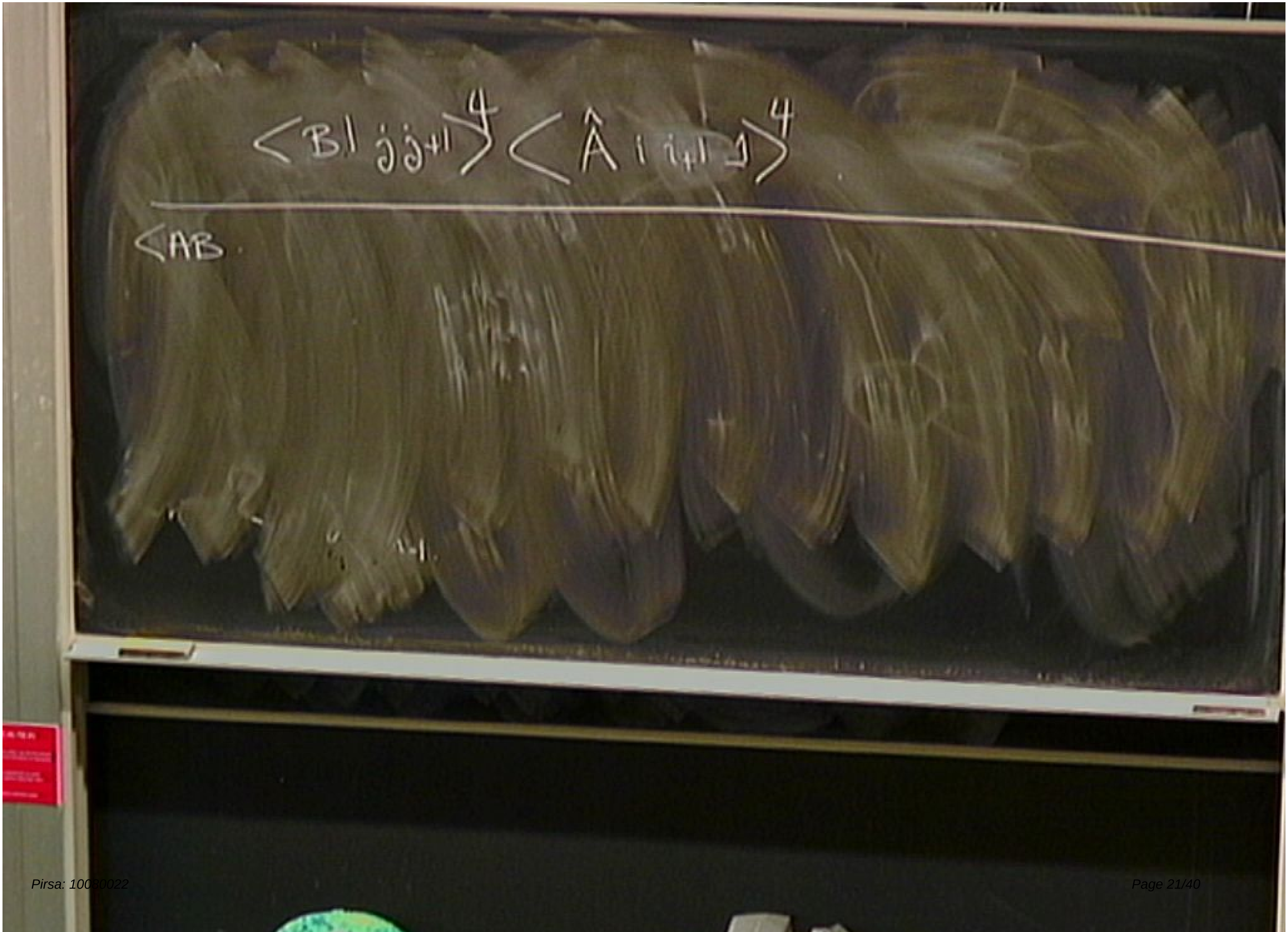

Â = <B j j+1 1>A − <A j j+1 1>B
∫ d⁴η_Â d⁴η_B̂ δ⁴[η_A <B 1 j j+1> + η_B <A 1 j j+1> + . . .]
<AB 1 j> <B 1 j j+1> <1 j j+1 A> <j j+1 AB> <j+1 AB 1>
δ⁴[η_Â <B i i+1 1> + . . .]
<ÂB i i+1> <B i i+1 1> <i i+1 ÂB> <i+1 1 ÂB> <1 ÂB i>

$$\int d^4\eta_A\, d^4\eta_B\ \frac{\delta^4\Big[\eta_A \langle B \mid j\, j{+}1\rangle + \overbrace{\eta_B \langle A \mid j\, j{+}1\rangle + \cdots}^{\eta_{\hat{A}}}\Big]}{\langle AB \mid j\rangle \langle B \mid j\, j{+}1\rangle \langle 1\, j\, j{+}1\, A\rangle \langle j\, j{+}1\, AB\rangle \langle j{+}1\, AB\, 1\rangle}$$

$$\hat{A} = \langle B\, j\, j{+}1\, 1\rangle A - \langle A\, j\, j{+}1\, 1\rangle B$$

$$\frac{\delta^4\big[\eta_B \langle \hat{A}\, i\, i{+}1\, 1\rangle + \cdots\big]}{\langle \hat{A} B\, i\, i{+}1\rangle \langle B\, i\, i{+}1\, 1\rangle \langle i\, i{+}1\, \hat{A} B\rangle \langle i{+}1\, 1\, \hat{A} B\rangle \langle 1\, \hat{A} B\, i\rangle}$$

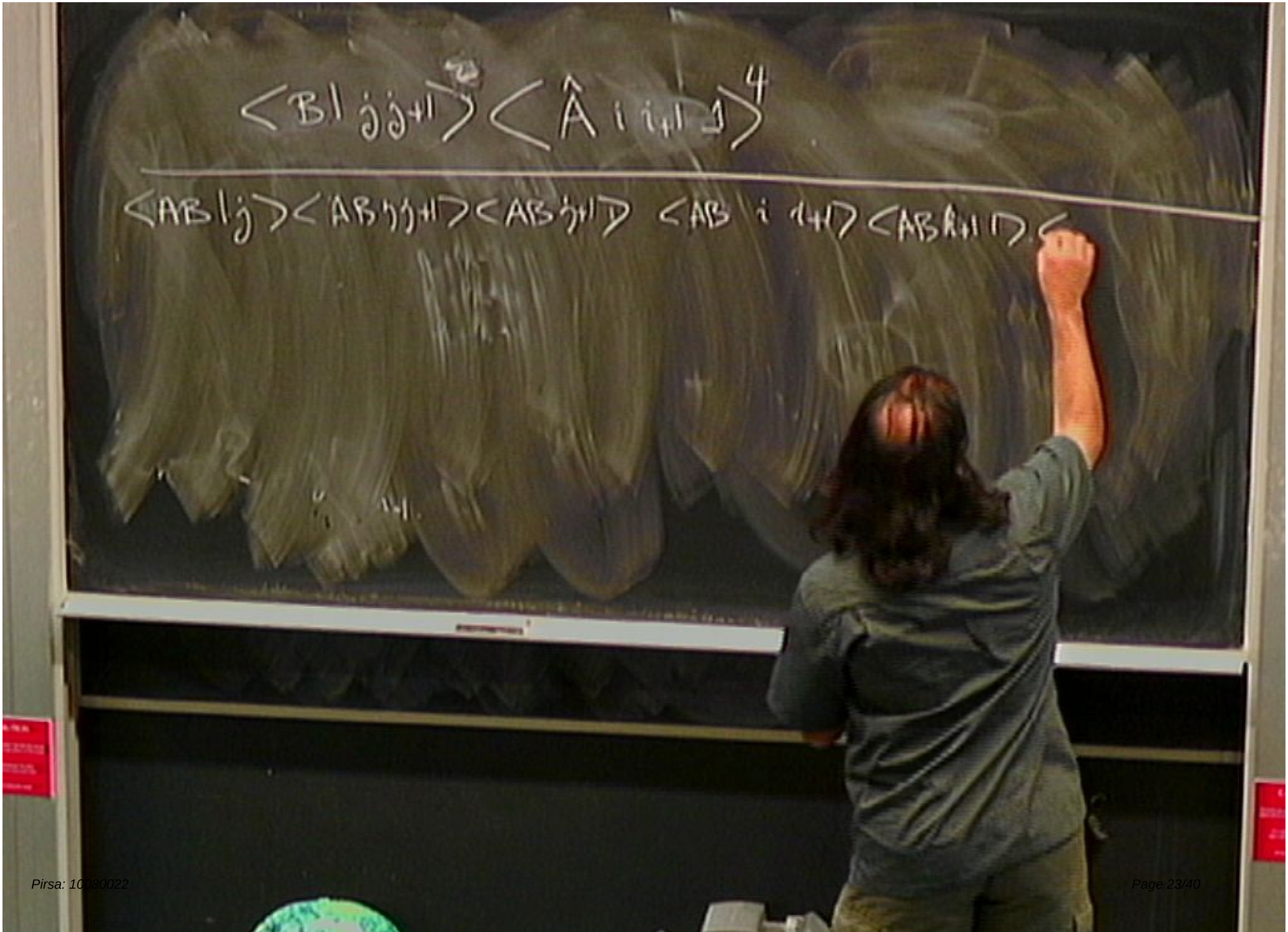

⟨B| j j+1⟩^4 ⟨Â | i i+1⟩^4

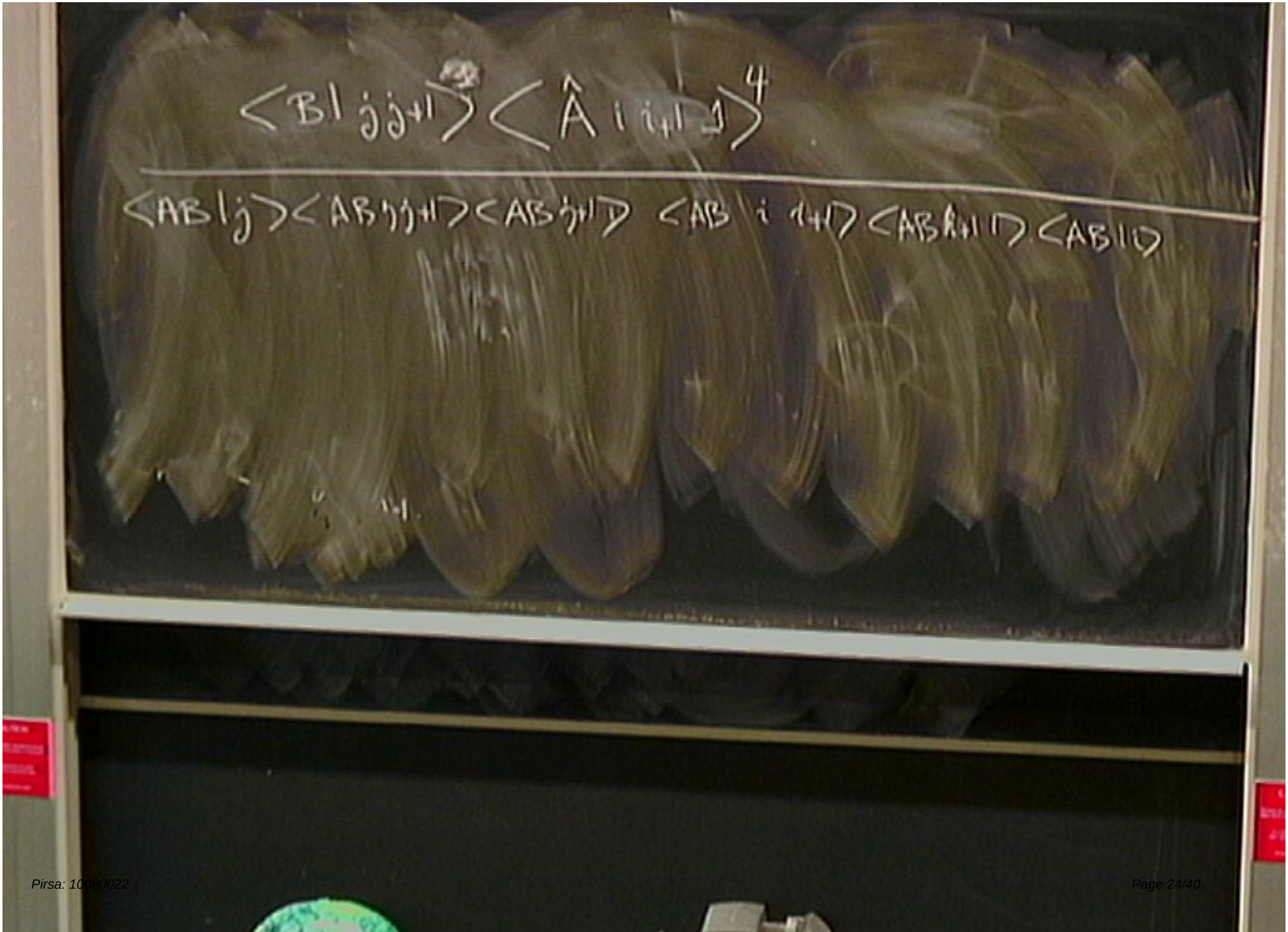

<B| j j+1>4 <Â i i+1 1>4
<AB

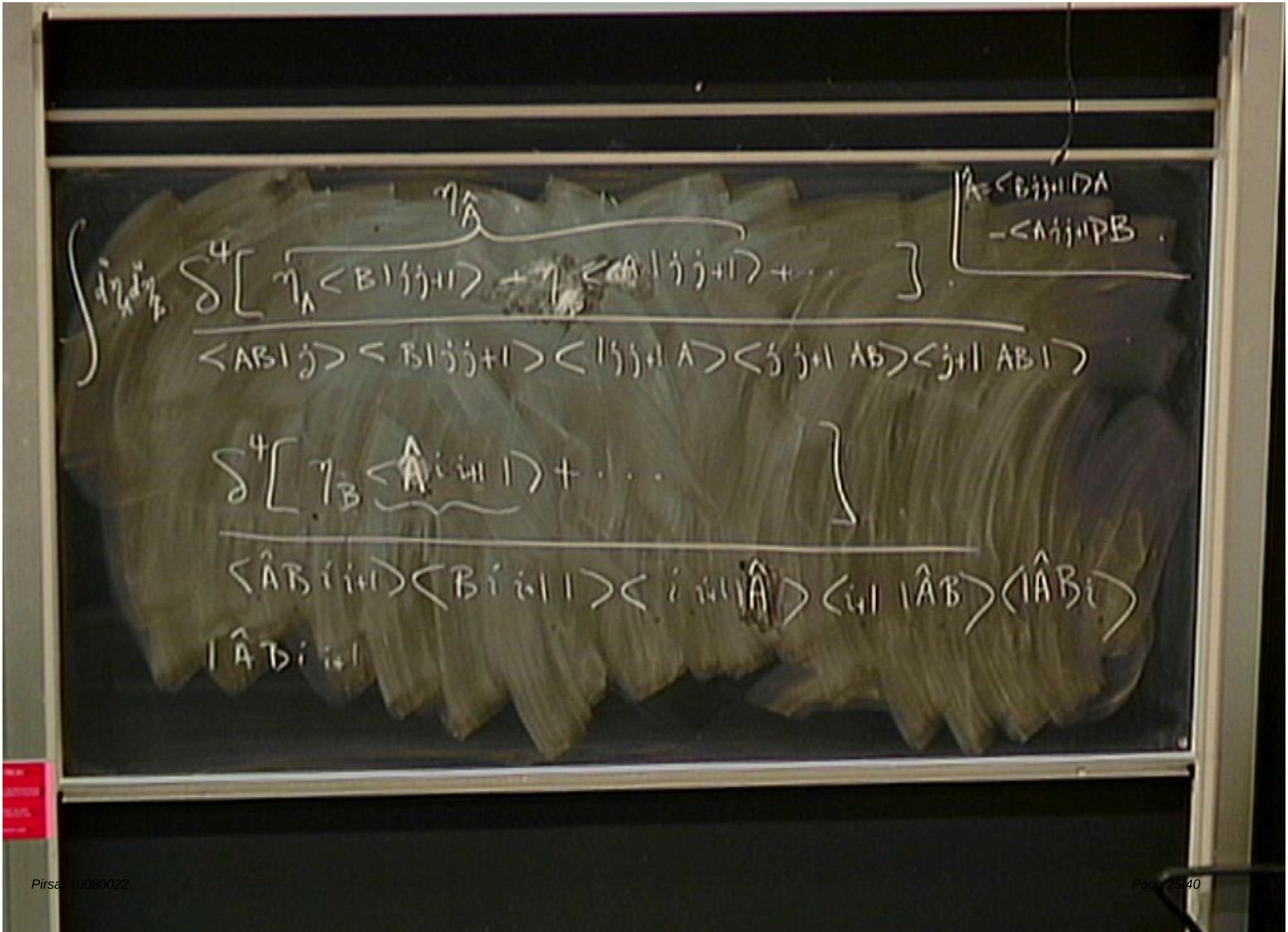
$$\langle \hat{A} \,|\, i\, i{+}1 \rangle .$$

$$= \langle A \,|\, i\, i{+}1\rangle \langle B \,|\, j\, j{+}1\rangle - \langle A \,|\, j\, j{+}1\rangle \langle B \,|\, i\, i{+}1\rangle$$

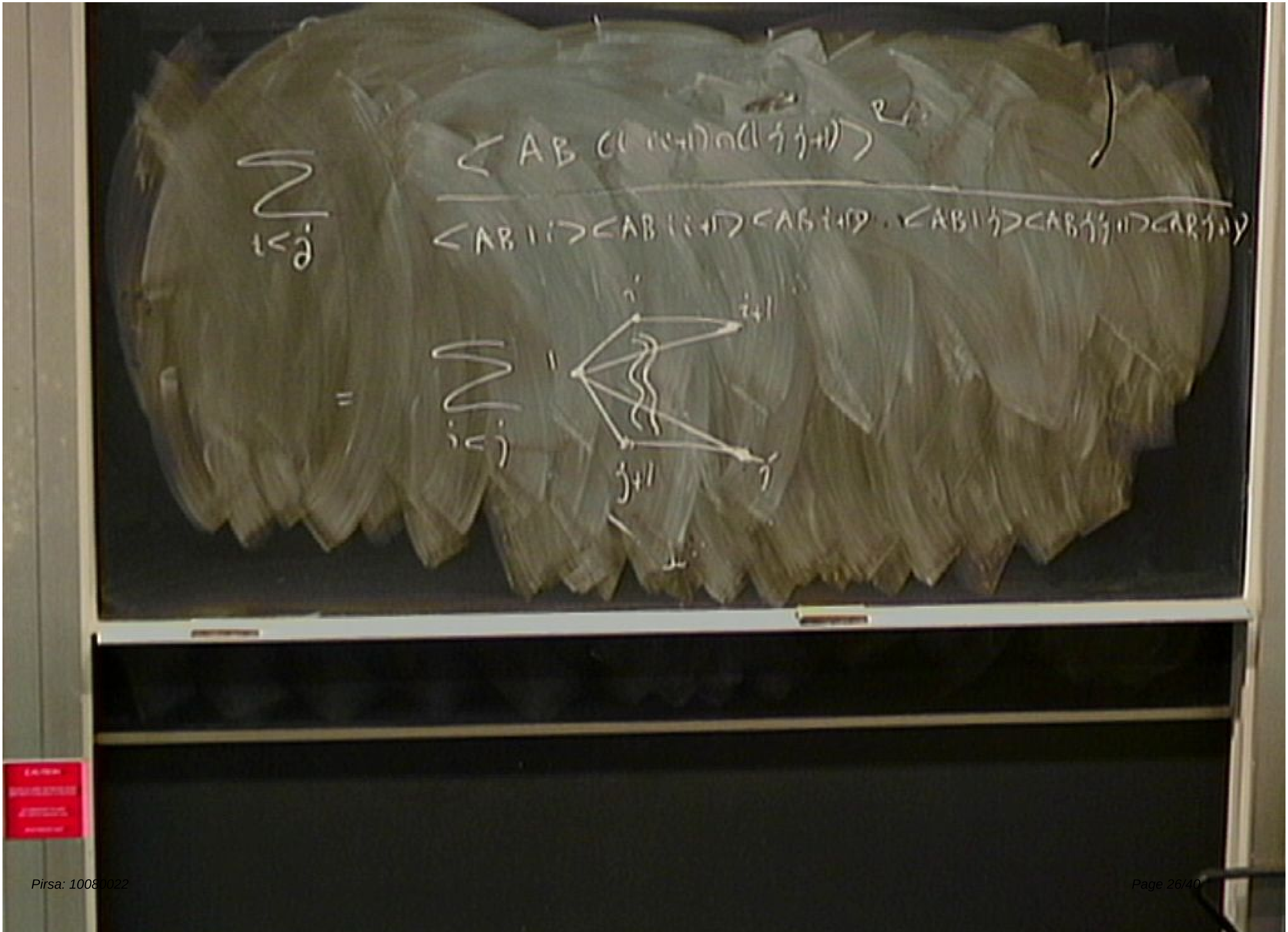
$$\equiv \langle A B \,(\, |\, i\, i{+}1 \wedge |\, j\, j{+}1) \rangle .$$

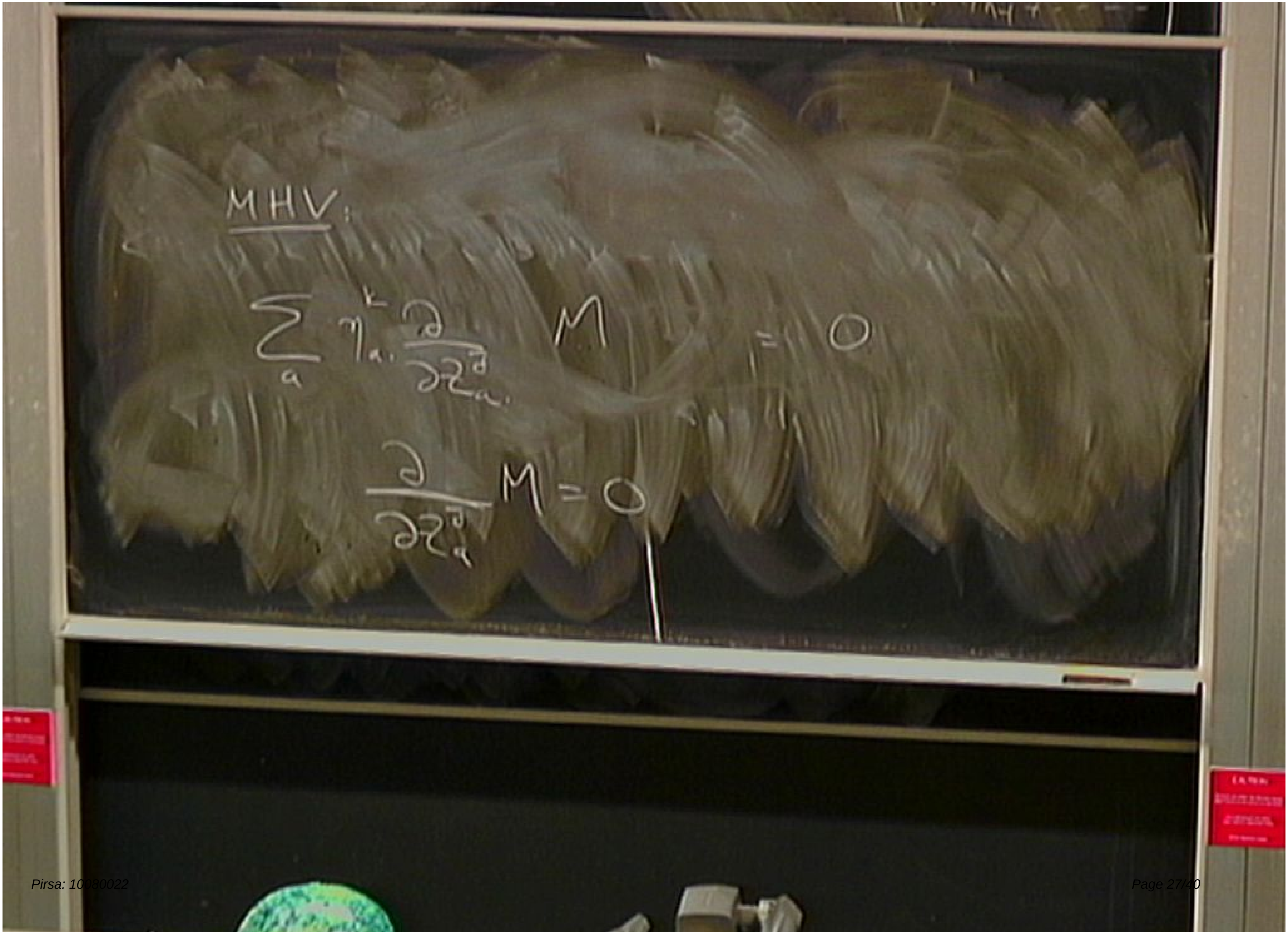


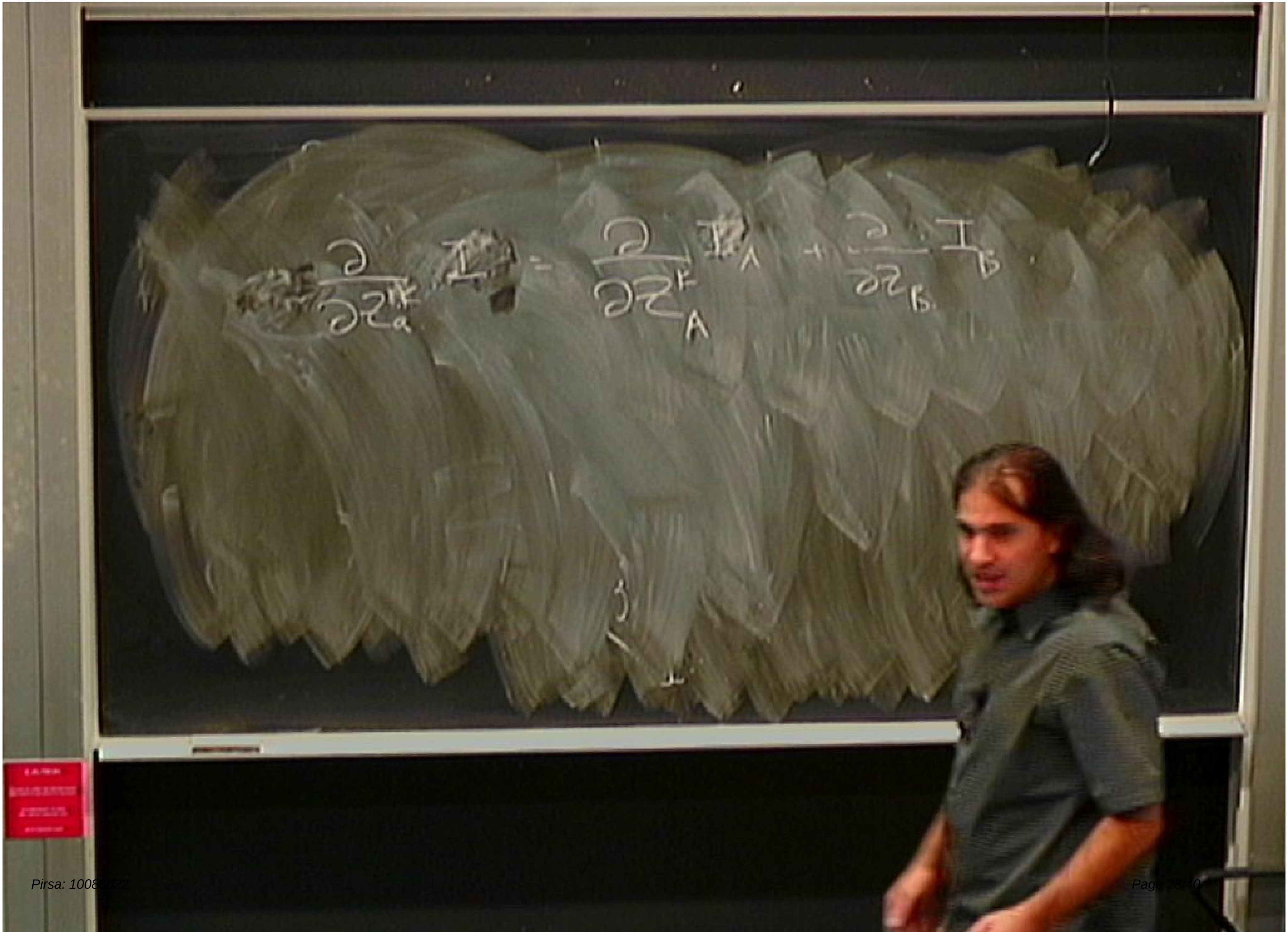

<B| j j+1> <Â i i+1 1>4
<AB|j><AB j j+1><AB j+1> <AB i i+1><AB i+1 1><AB|1>

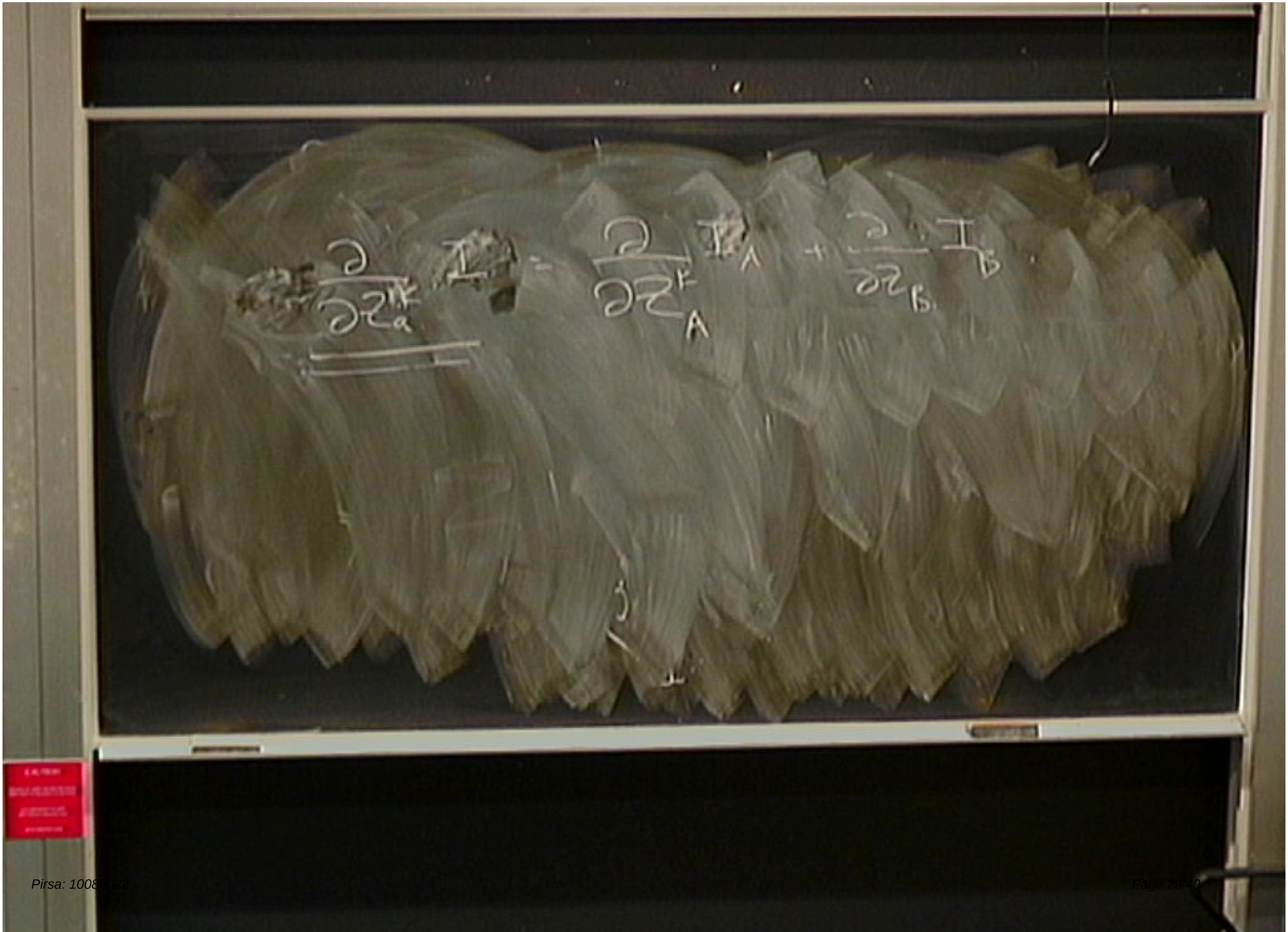


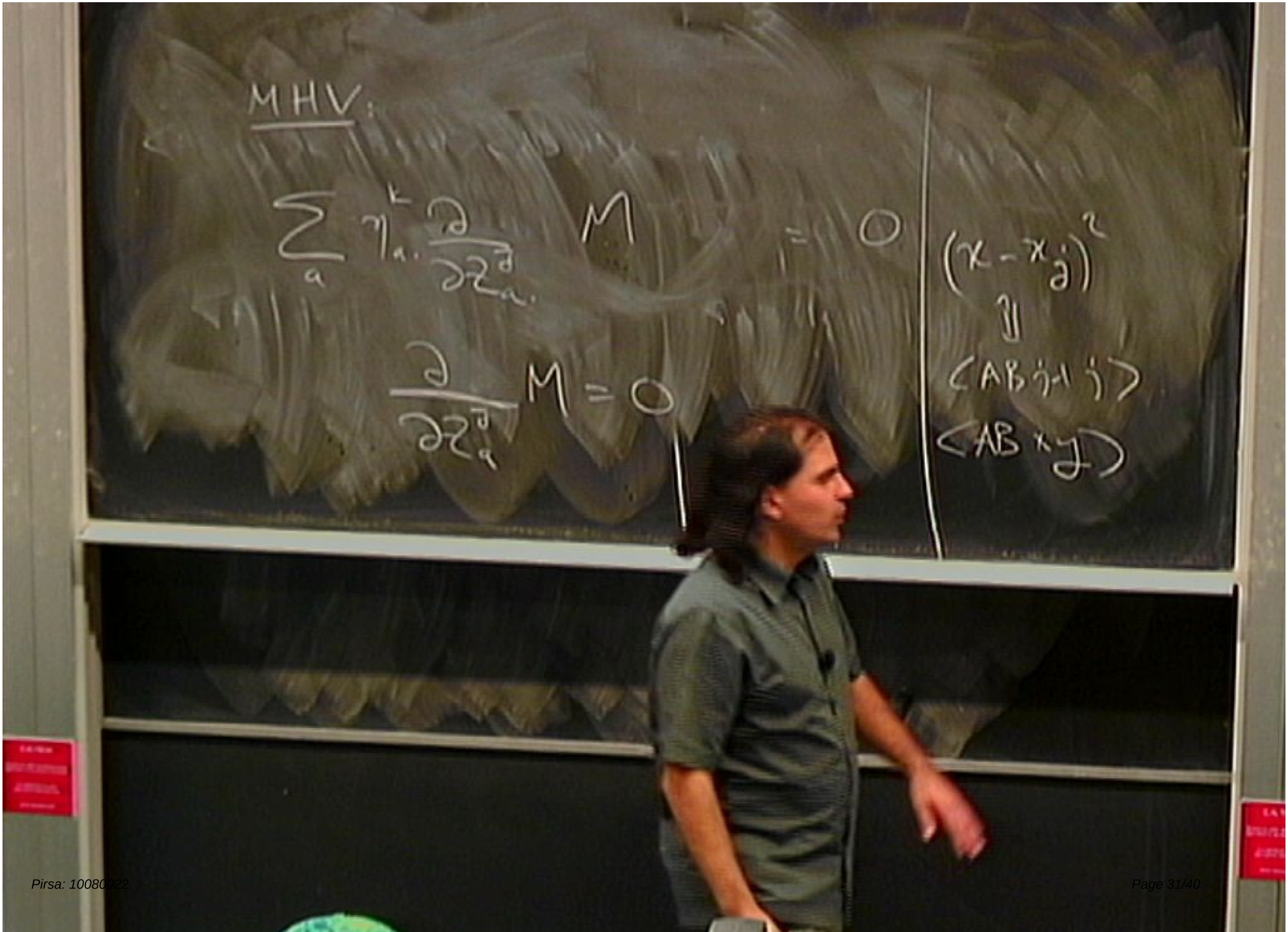
Â = <B j j+1 D A − <A j j+1 D B
<AB | j> <B | j j+1> <| j j+1 A> <j j+1 AB> <j+1 AB |>
δ⁴[η_B <Â i i+1 |> + . . .]
<Â B i i+1> <B i i+1 |> <i i+1 Â> <i+1 | Â B> <| Â B i>
| Â B i i+1

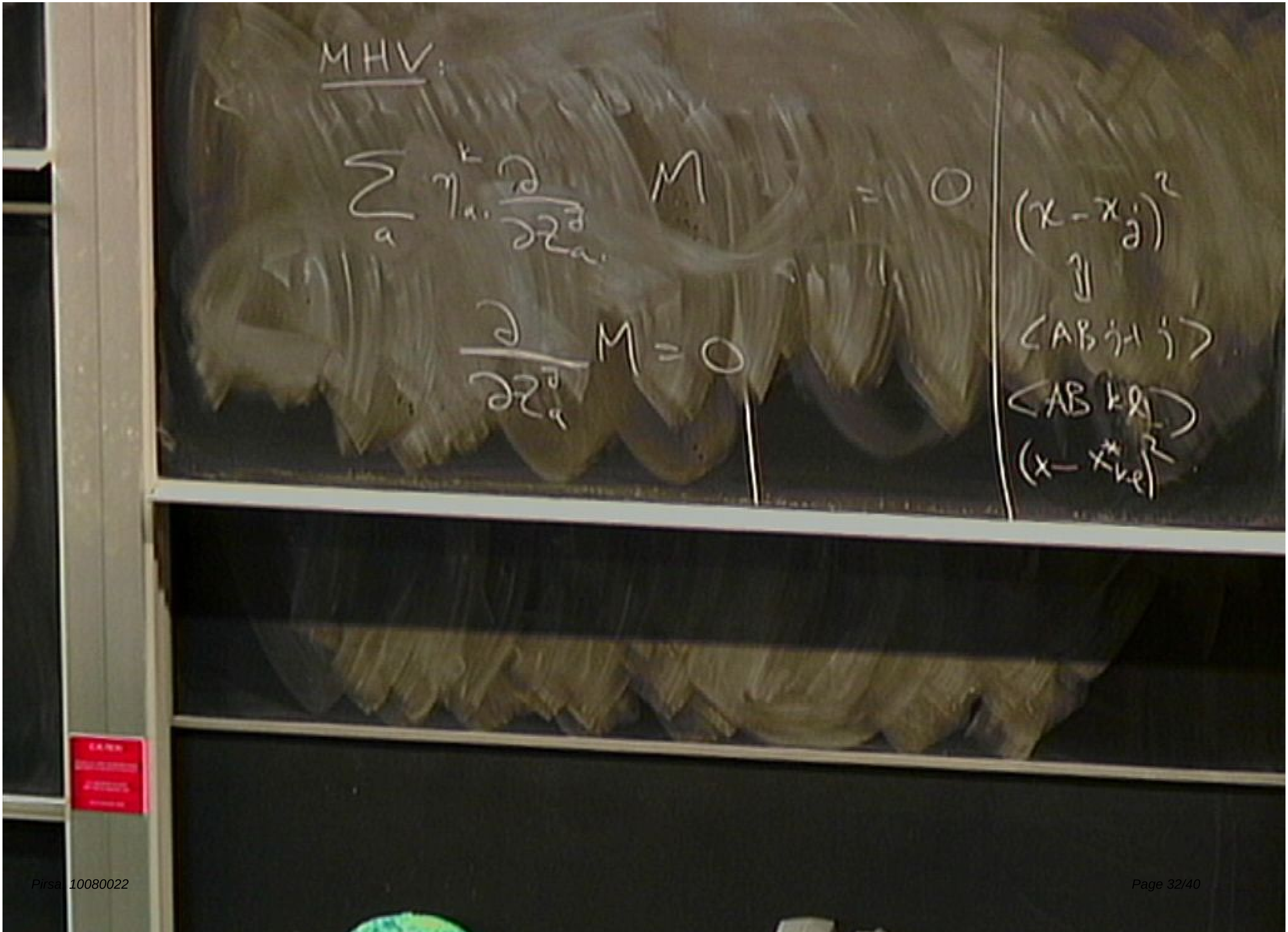

i<j
i
i+1
j
j+1

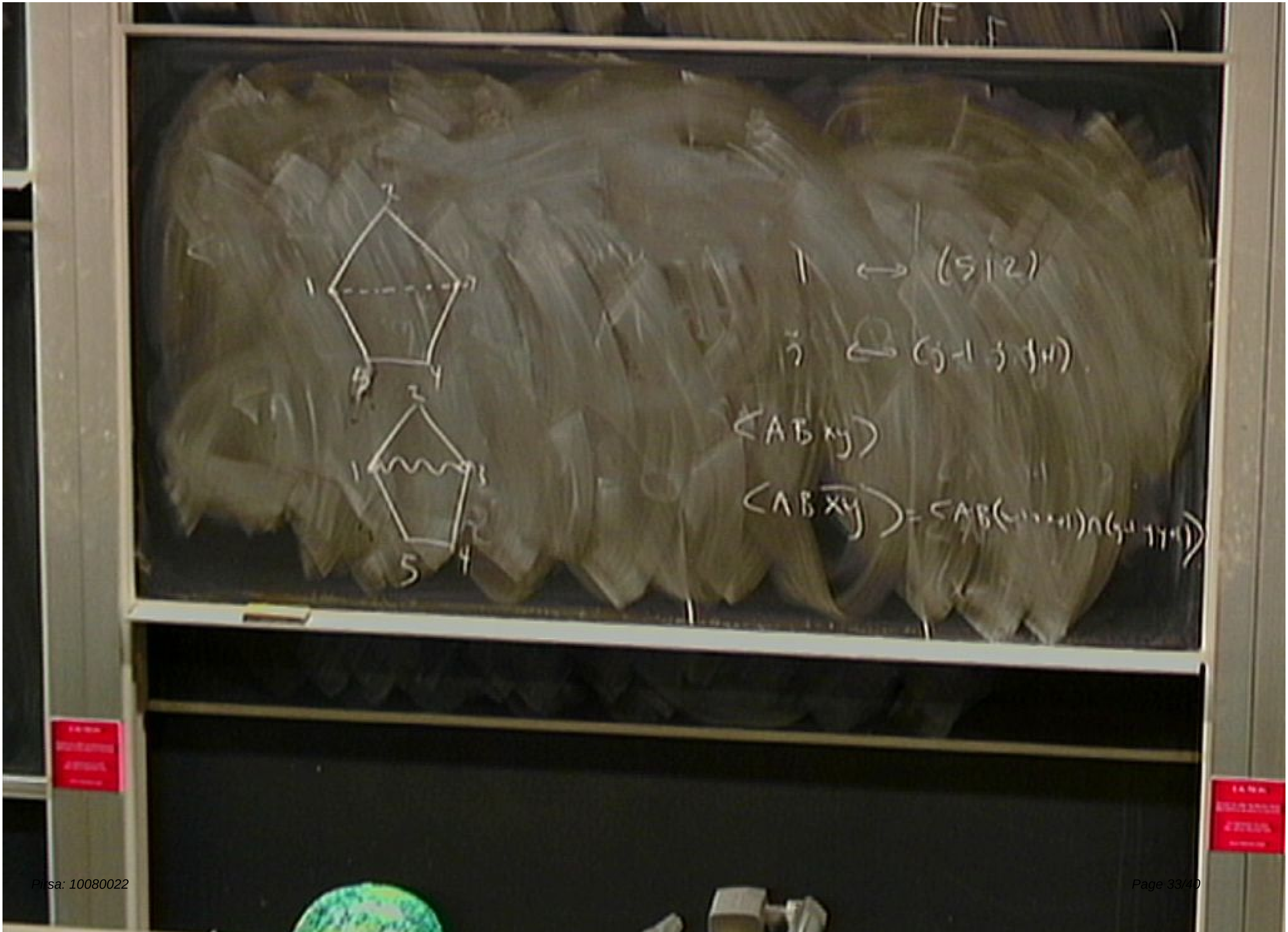

MHV:
$\sum_a^k \eta_a \frac{\partial}{\partial z_a^{\eta}} M = 0$
$\frac{\partial}{\partial z_a^{\eta}} M = 0$

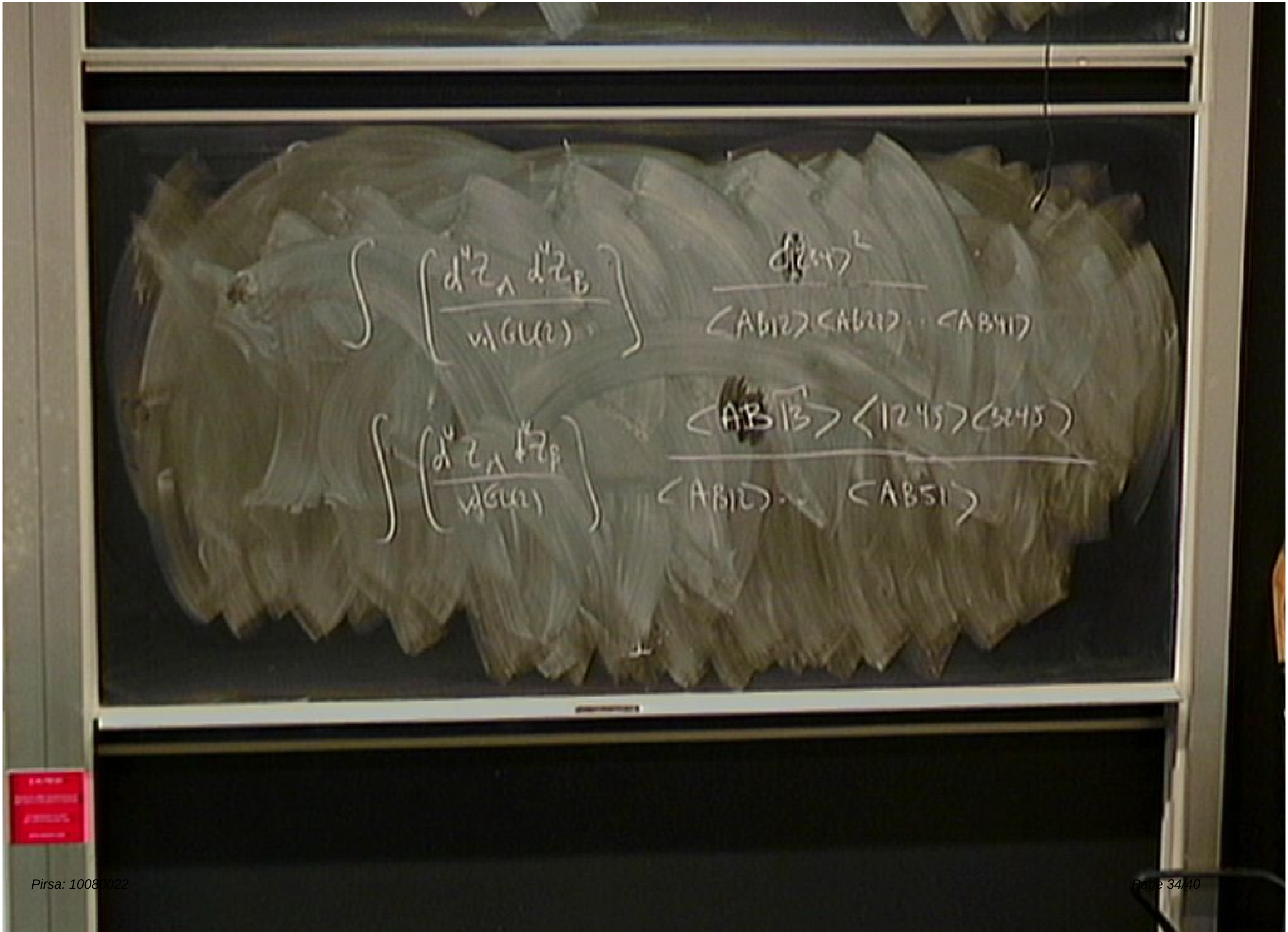


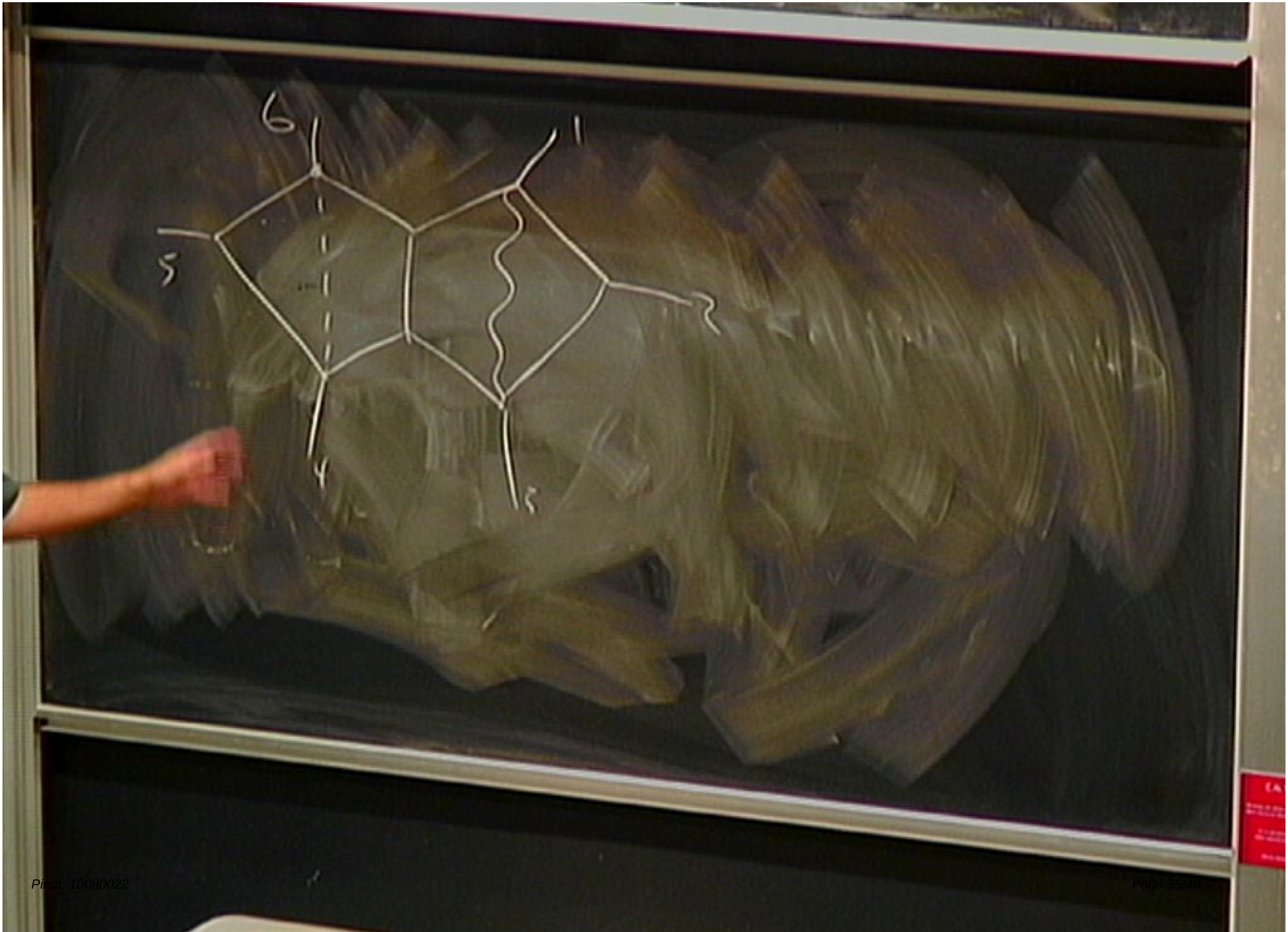

$\frac{\partial}{\partial z^K_a} I = \frac{\partial}{\partial z^K_A} I_A + \frac{\partial}{\partial z_B} I_B$

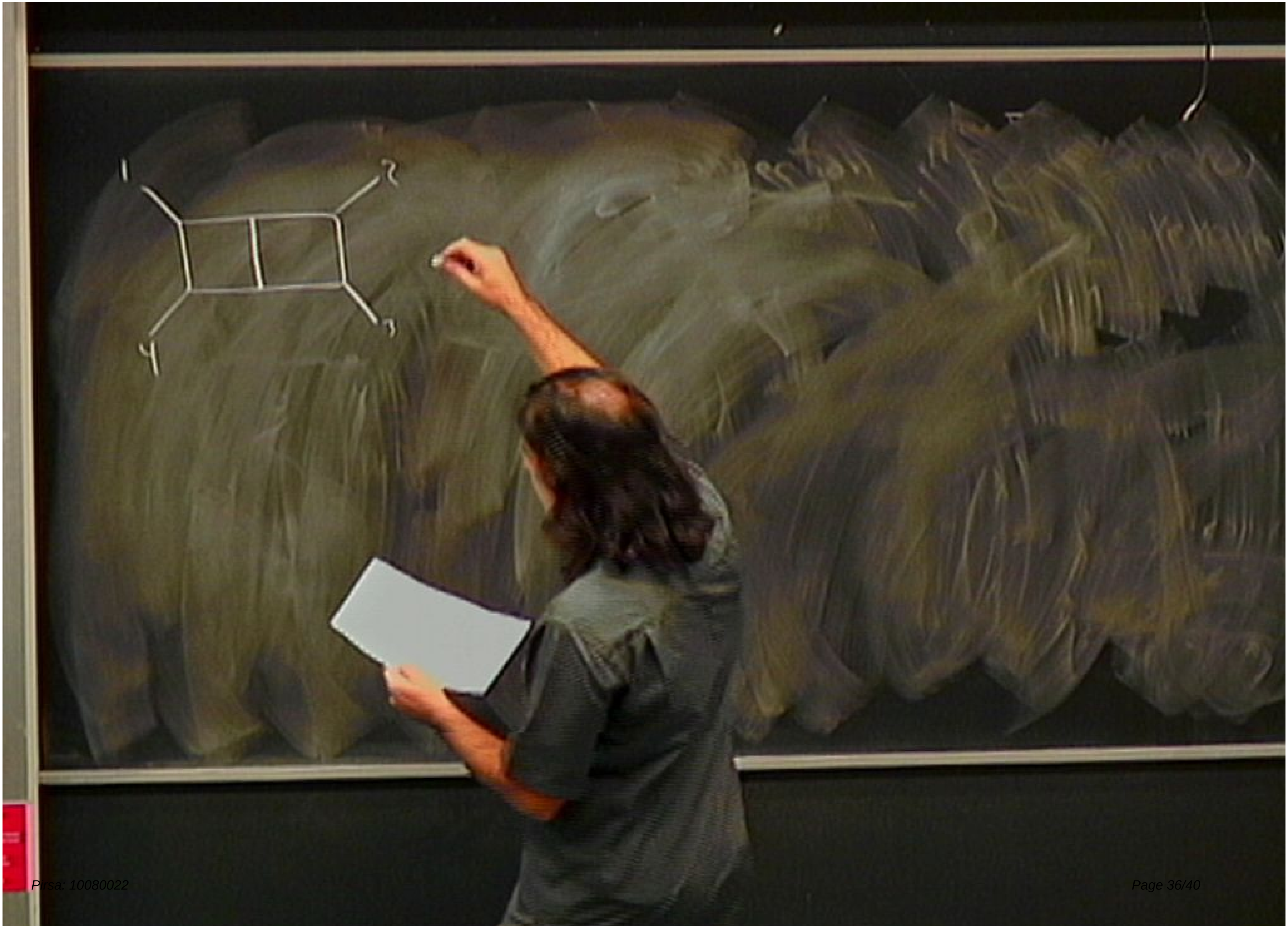

MHV:
Σ_a η_a^k ∂/∂Z_a^J M = 0
∂/∂Z_a^J M = 0
(x − x_j)^2
⟨AB j-1 j⟩
⟨AB x y⟩


MHV:
∑_a η_a^k ∂/∂z_a^J M = 0
∂/∂z_a^J M = 0
(x − x_j)^2
⇕
⟨AB j−1 j⟩
⟨AB k l⟩
(x − x*_{k,l})^2


⟨AB xy⟩


d^4 Z_A d^4 Z_B
vol GL(2)
<AB12><AB23> ... <AB41>
d^4 Z_A d^4 Z_B
vol GL(2)
<1245><3245>
<AB12> ... <AB51>


6
1
5
2
4


1
2
3
4