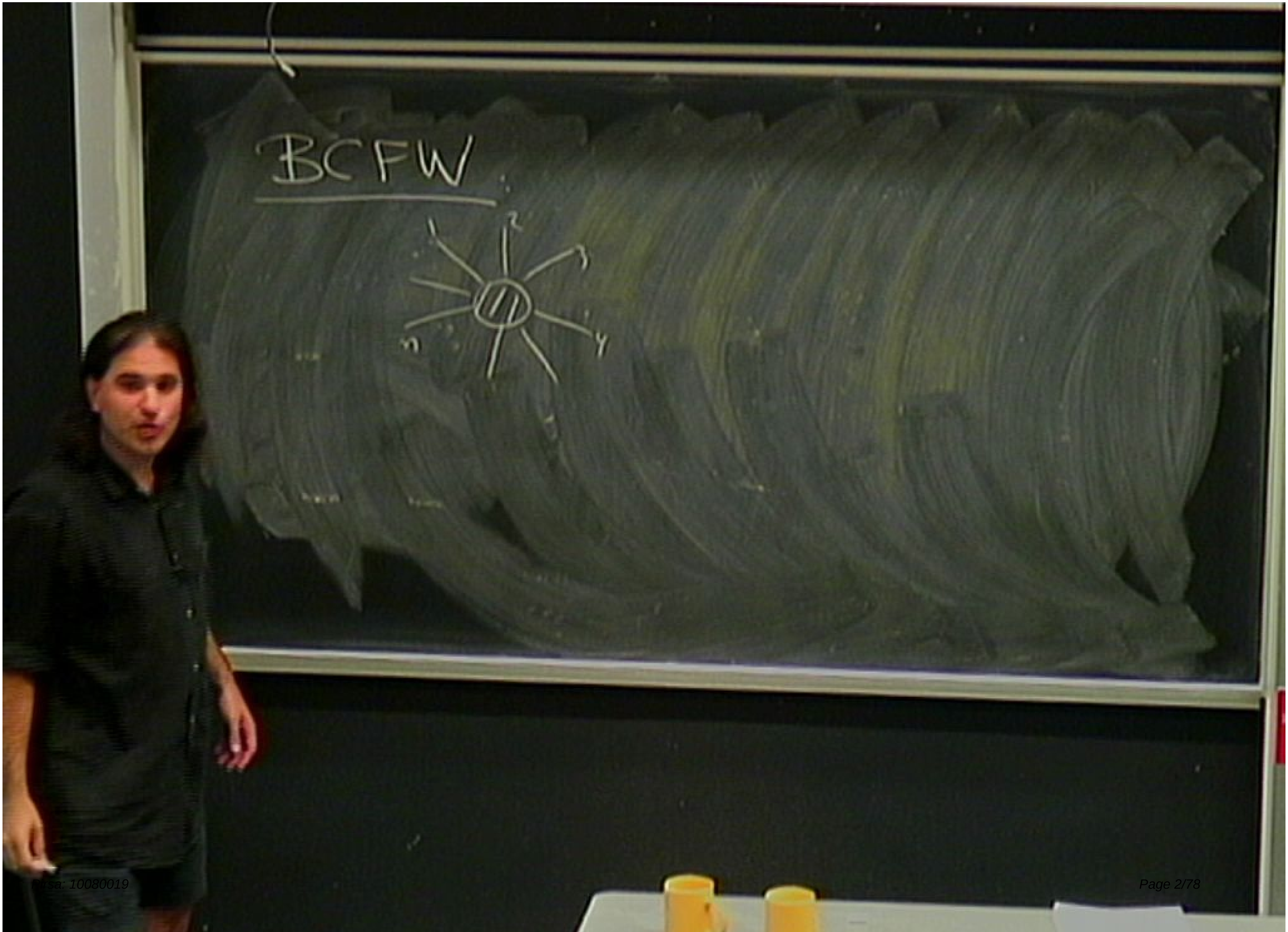
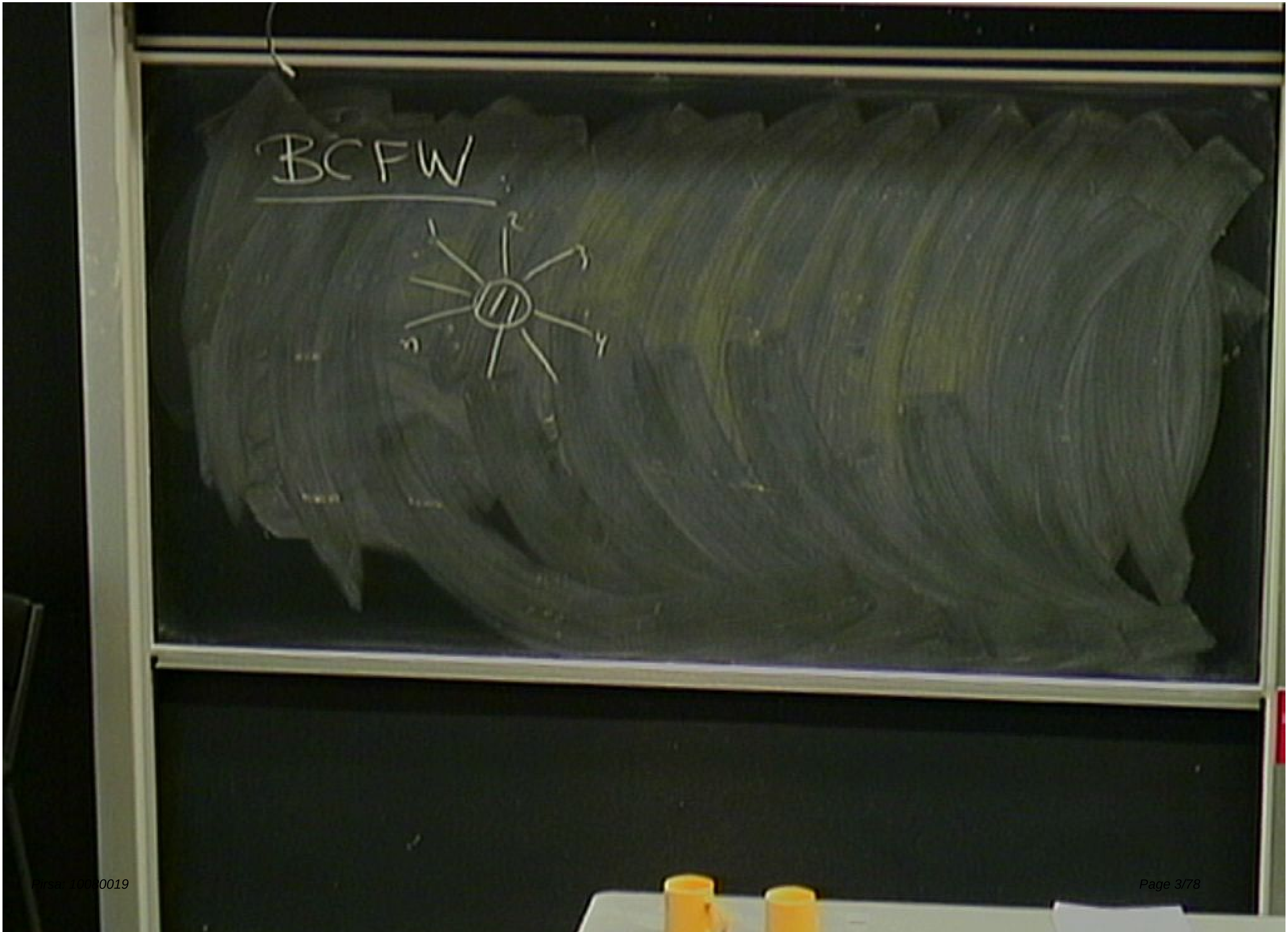
Title: Space-time, Quantum Mechanics and Scattering Amplitudes - Lecture 2B

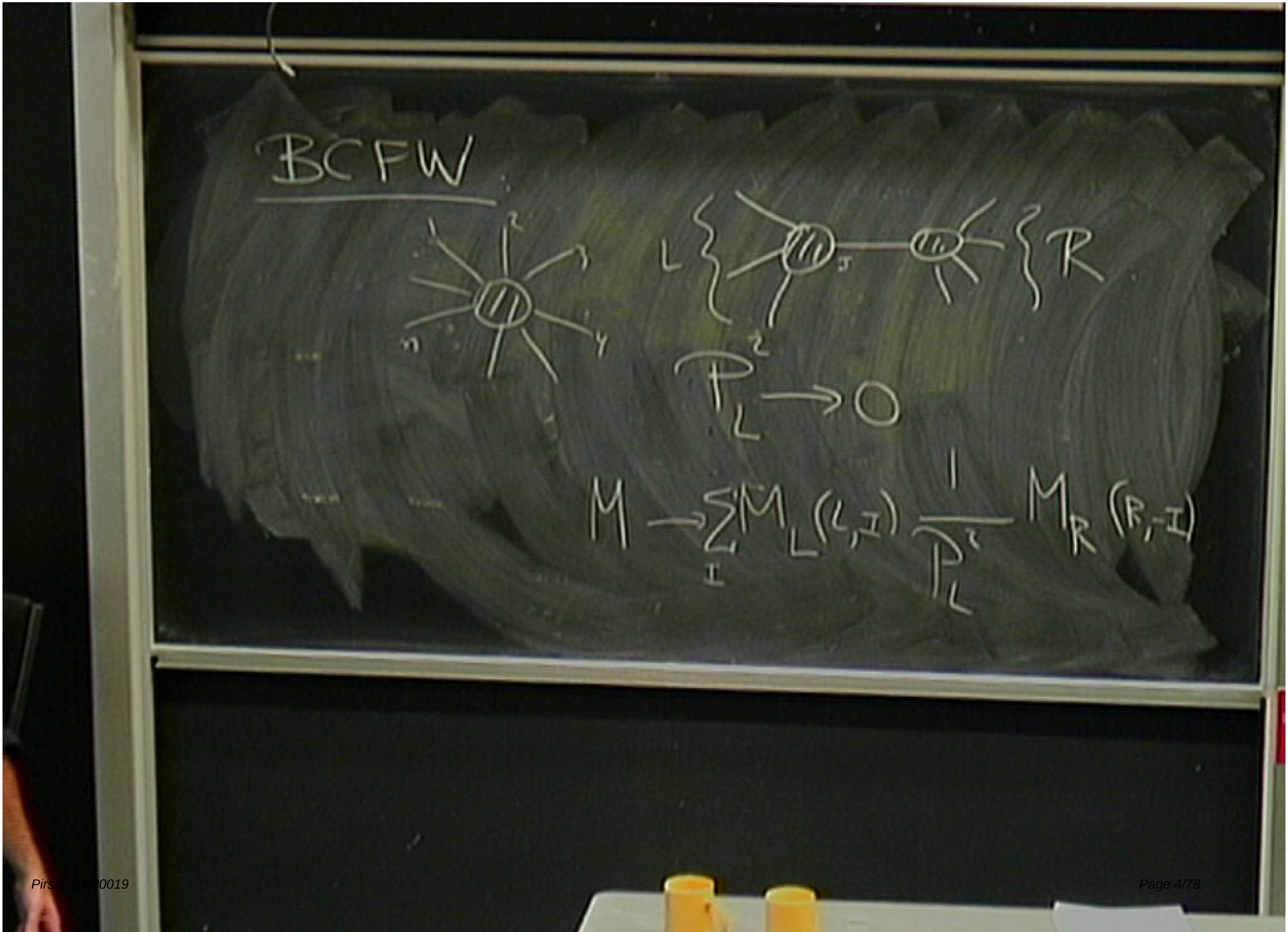
Date: Aug 24, 2010 03:00 PM

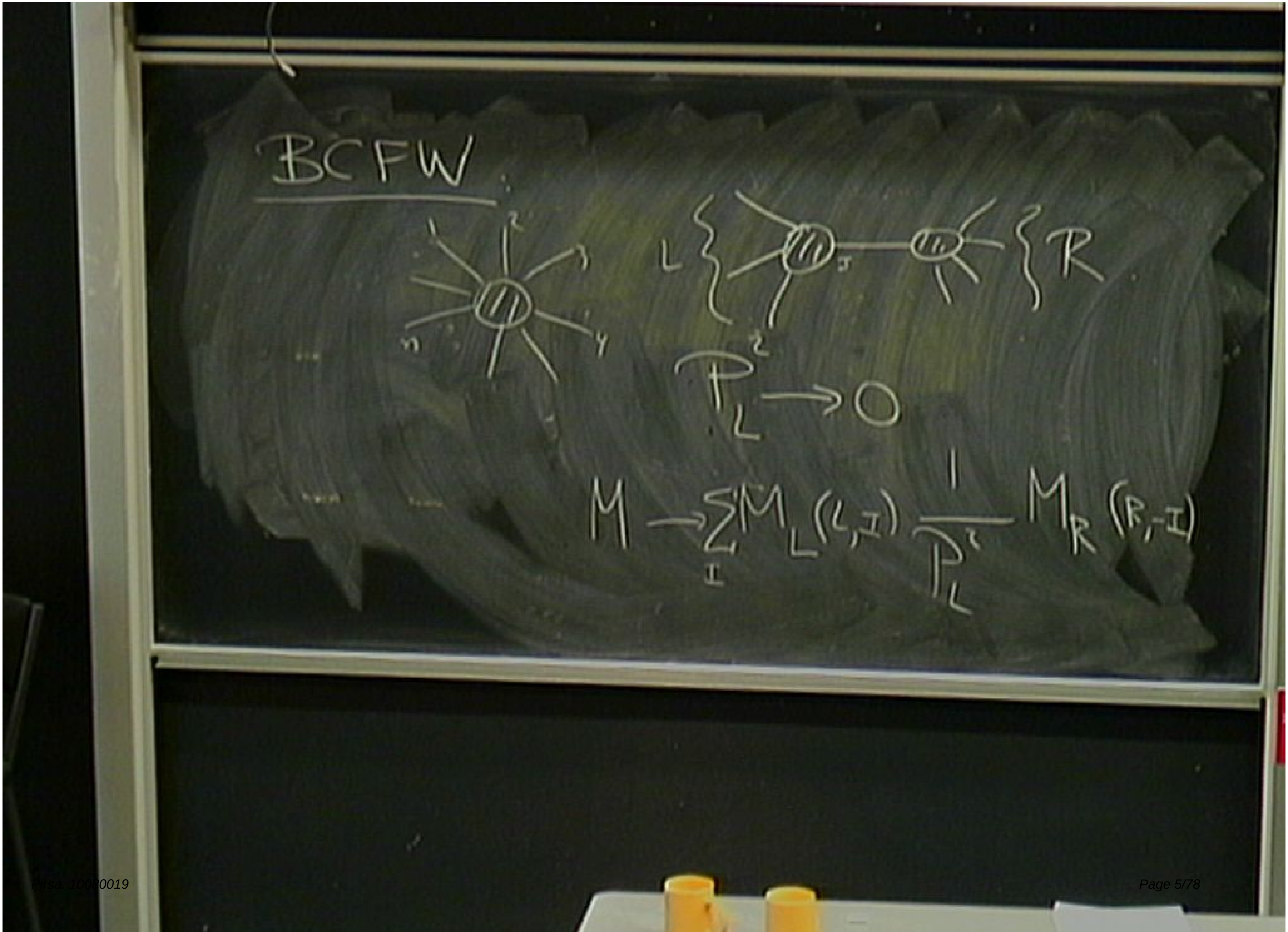
URL: http://pirsa.org/10080019

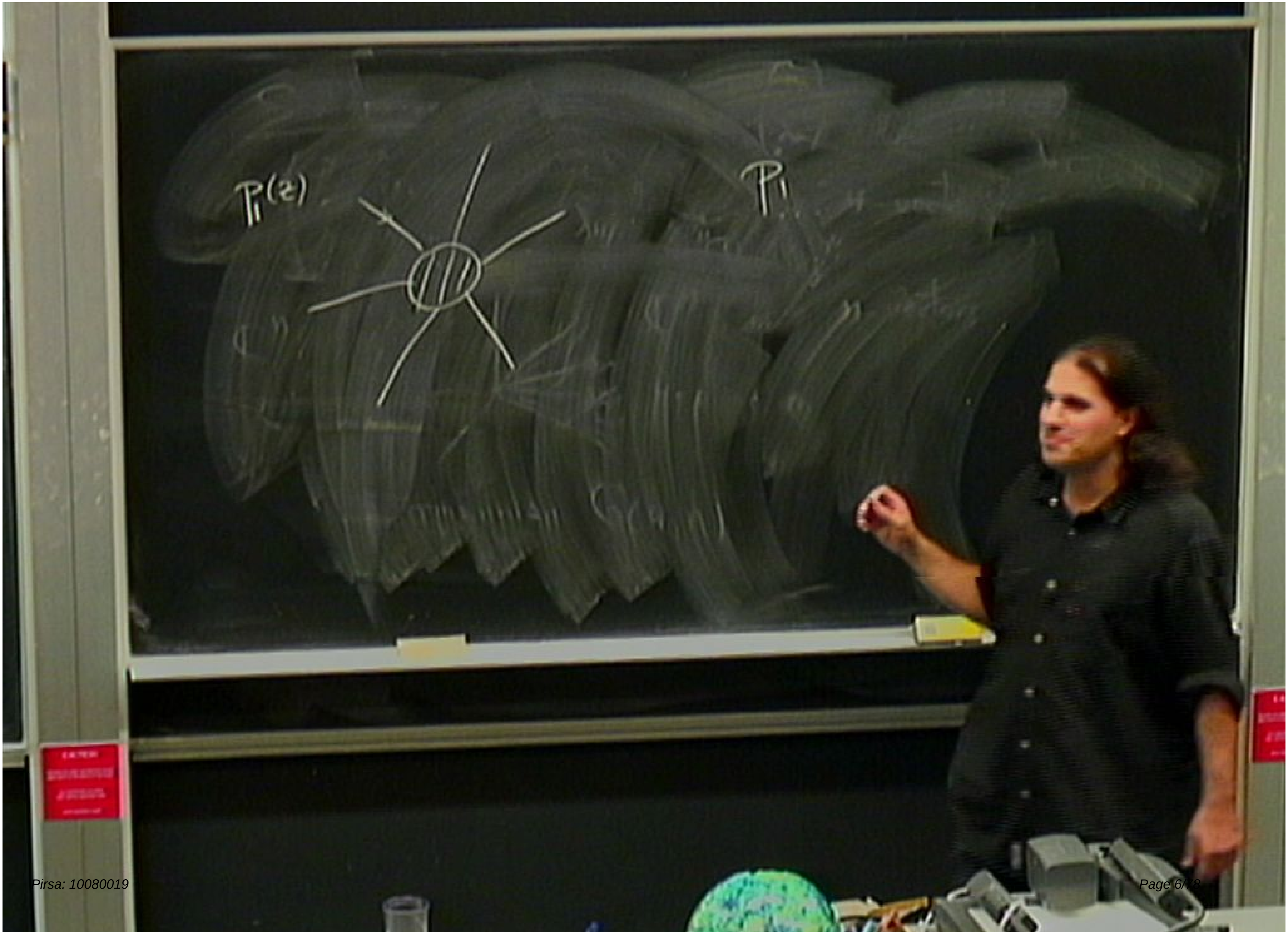
Abstract:


BCFW

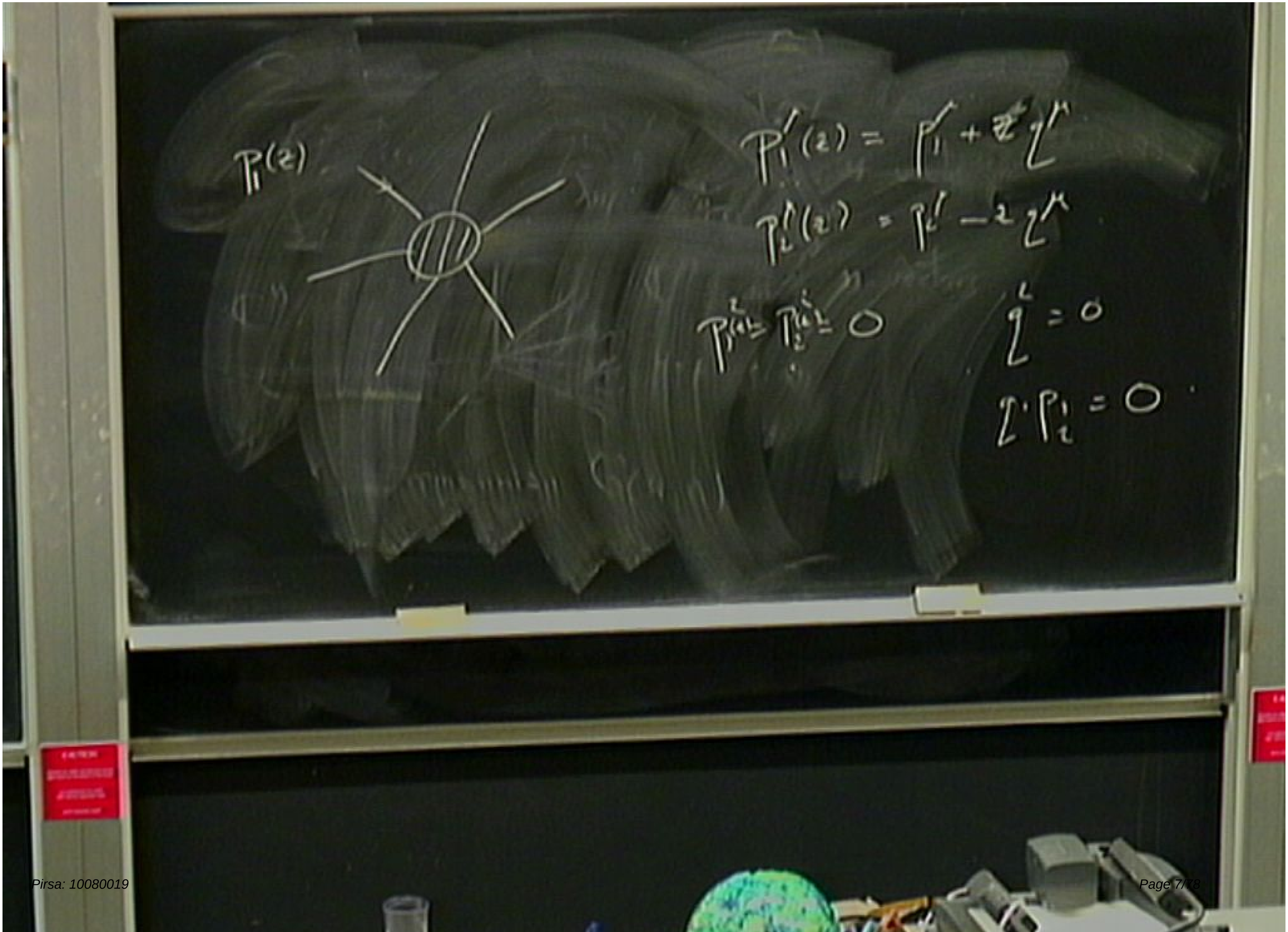

BCFW
1
2
3
4
n


BCFW
L
R
$P_L^2 \to 0$
$M \to \sum_I M_L(L, I) \frac{1}{P_L^2} M_R(R, -I)$


BCFW
L
R
I
$P_L^2 \to 0$
$M \to \sum_I M_L(L,I) \frac{1}{P_L^2} M_R(R,-I)$



P1(z)
P1

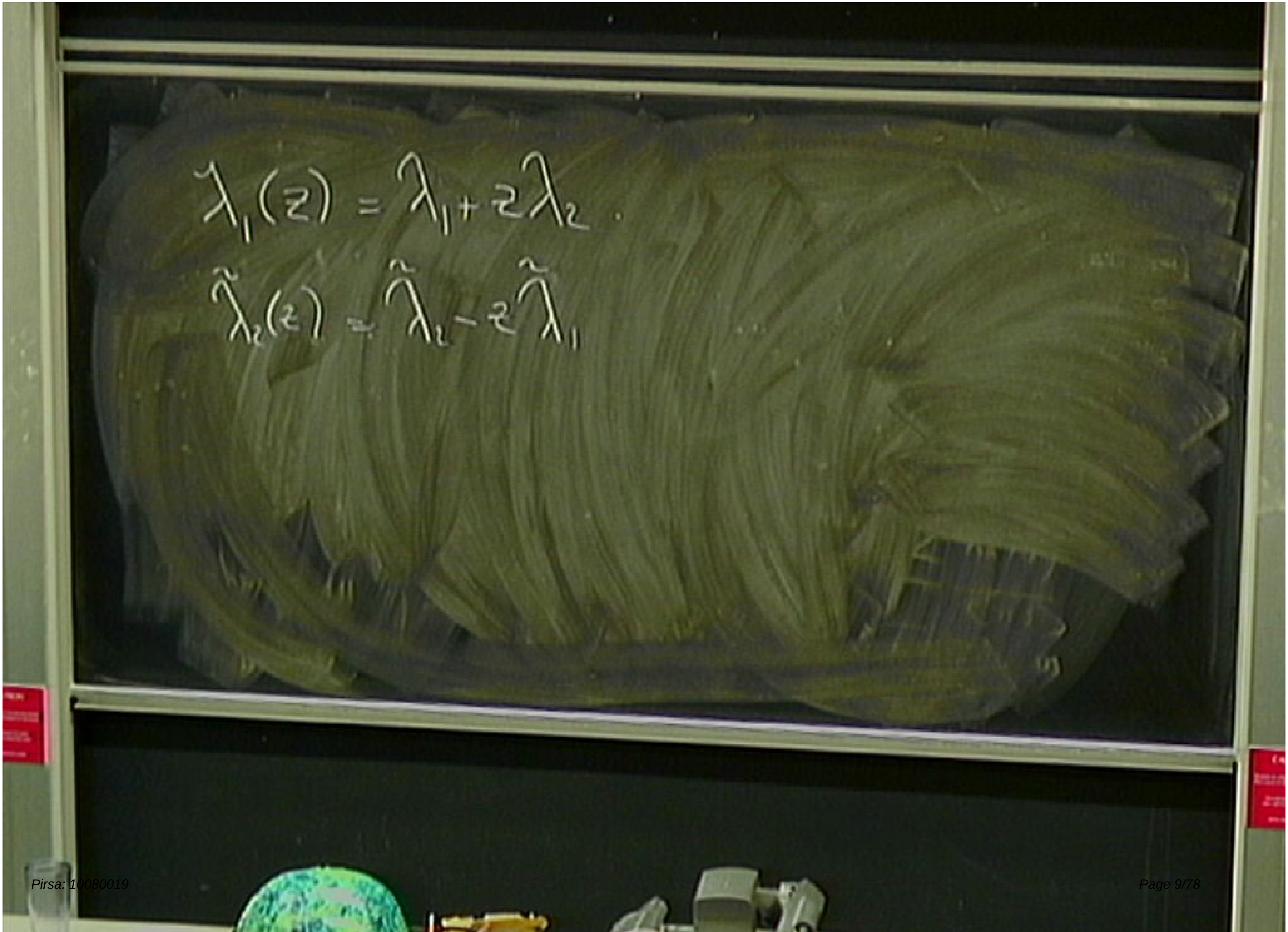


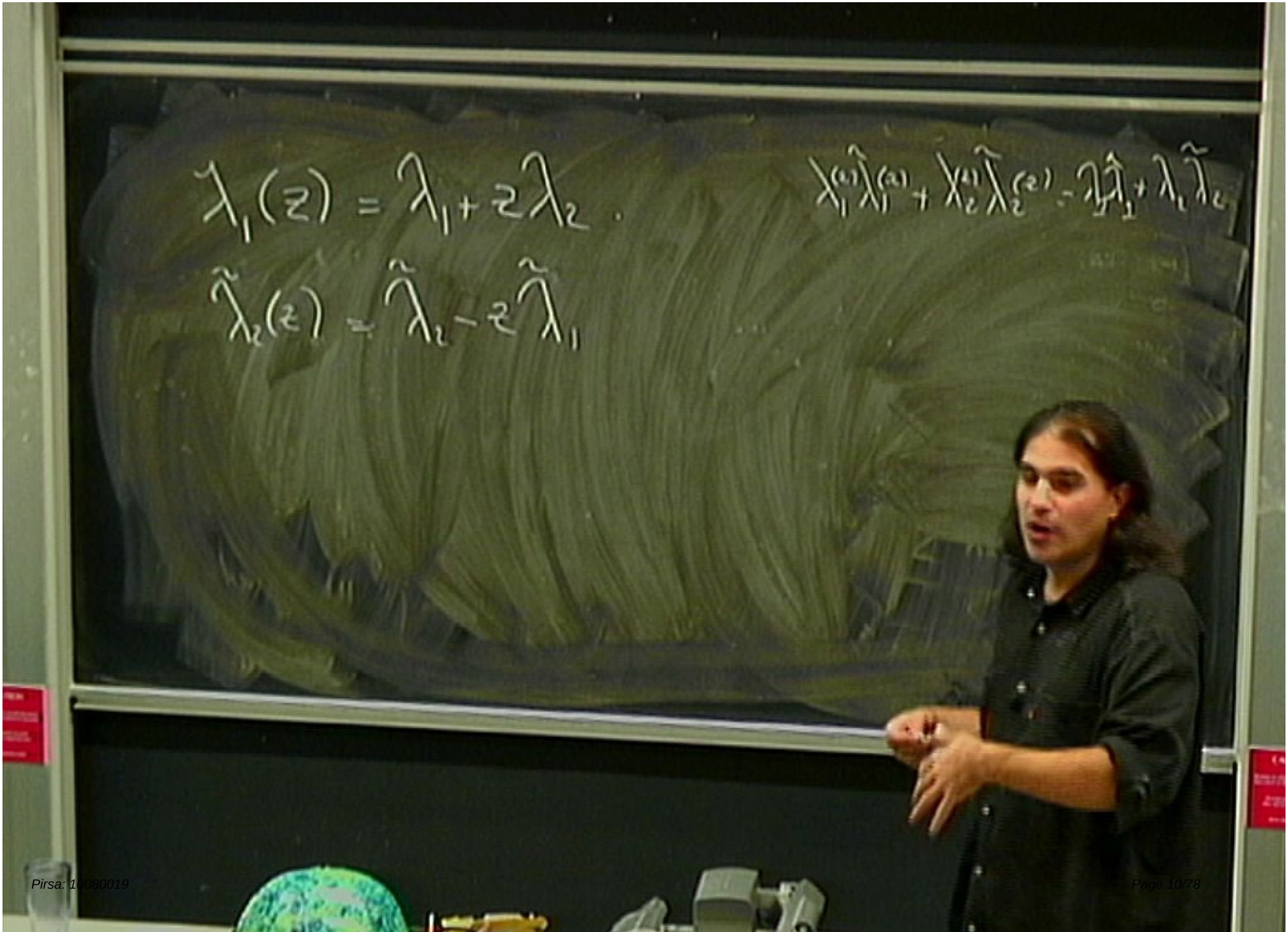
$p_1(z)$
$p_1^\mu(z) = p_1^\mu + z\, q^\mu$
$p_2^\mu(z) = p_2^\mu - z\, q^\mu$
$p_1^2(z) = p_2^2(z) = 0$
$q^2 = 0$
$q \cdot p_{1,2} = 0$

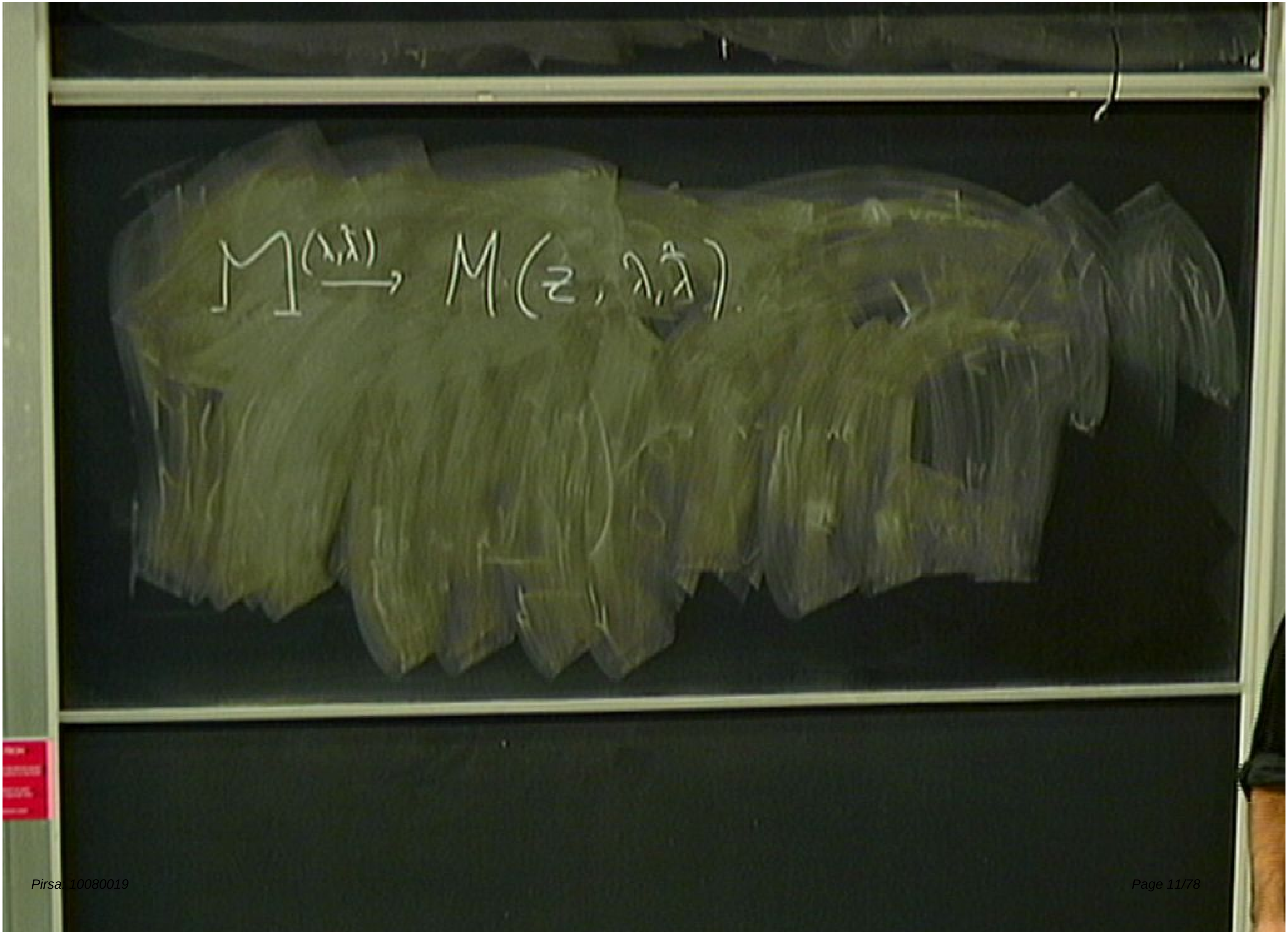
$$P_1 = (1 \; 1 \; 0 \; 0 \ldots 0).$$

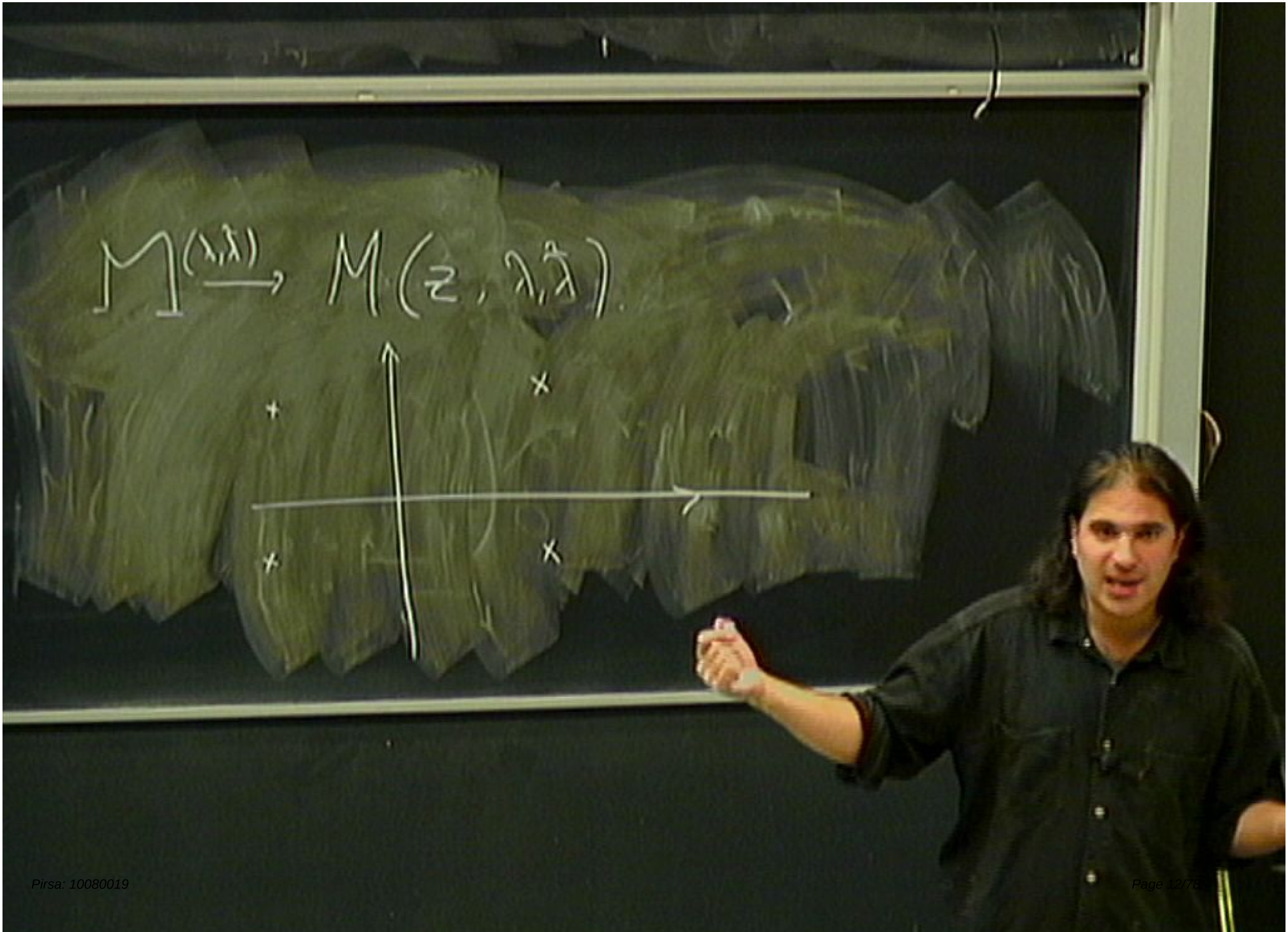
$$P_2 = (1 \; {-1} \; 0 \; 0 \ldots 0)$$

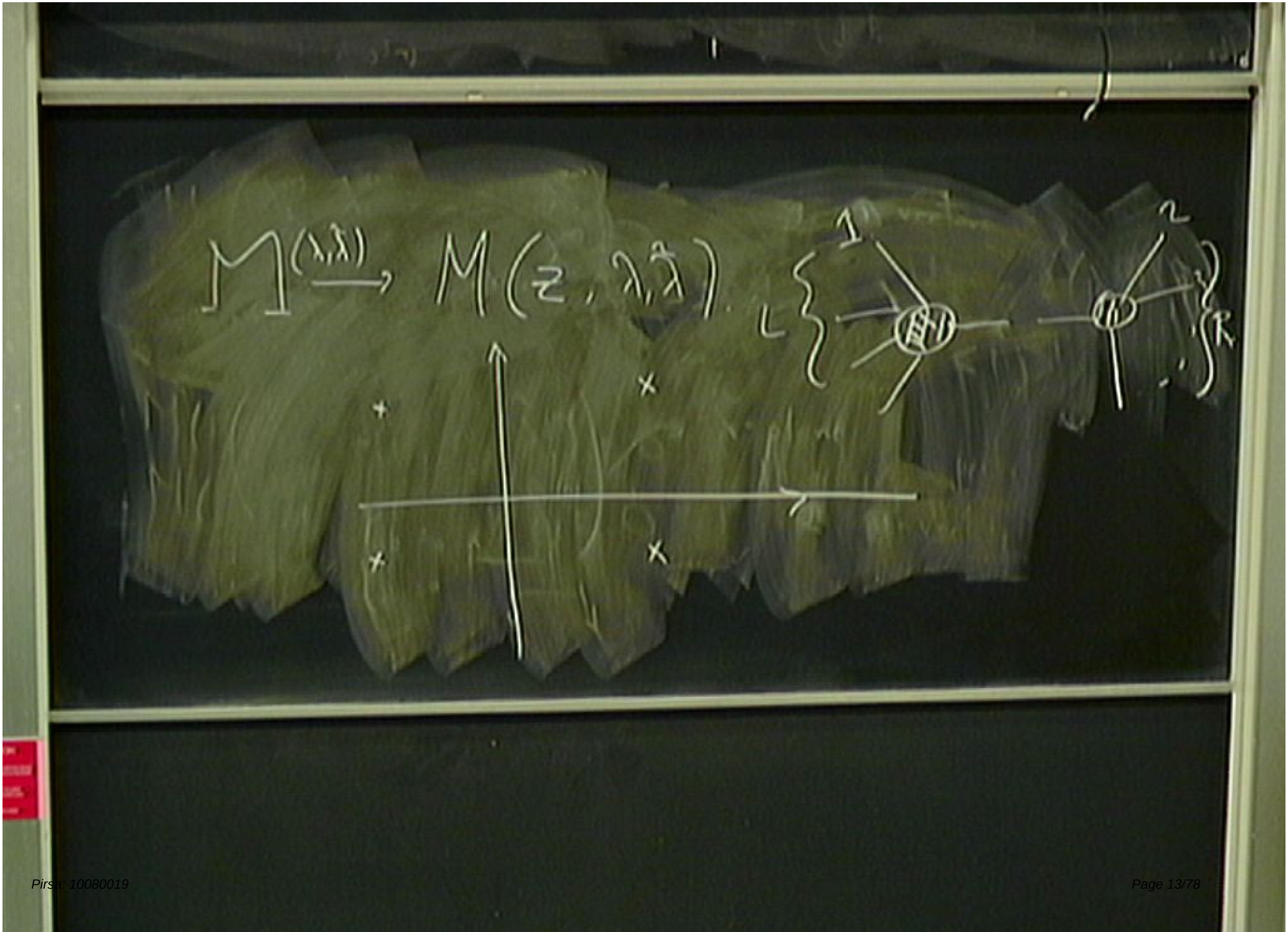
$$q = (0 \; 0 \; 1 \; i \; 0 \ldots 0).$$

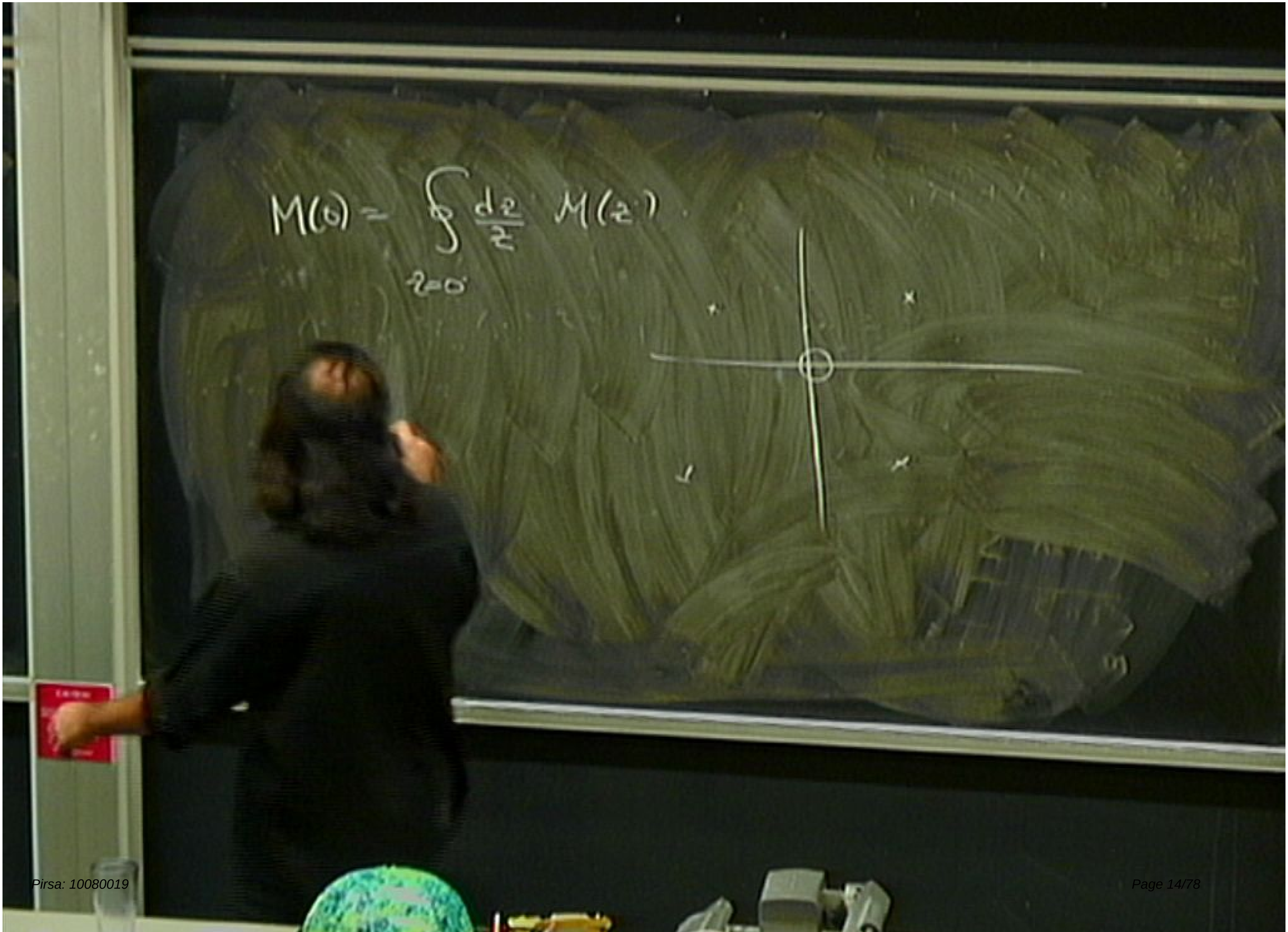

$\lambda_1(z) = \lambda_1 + z\lambda_2$.
$\tilde{\lambda}_2(z) = \tilde{\lambda}_2 - z\tilde{\lambda}_1$

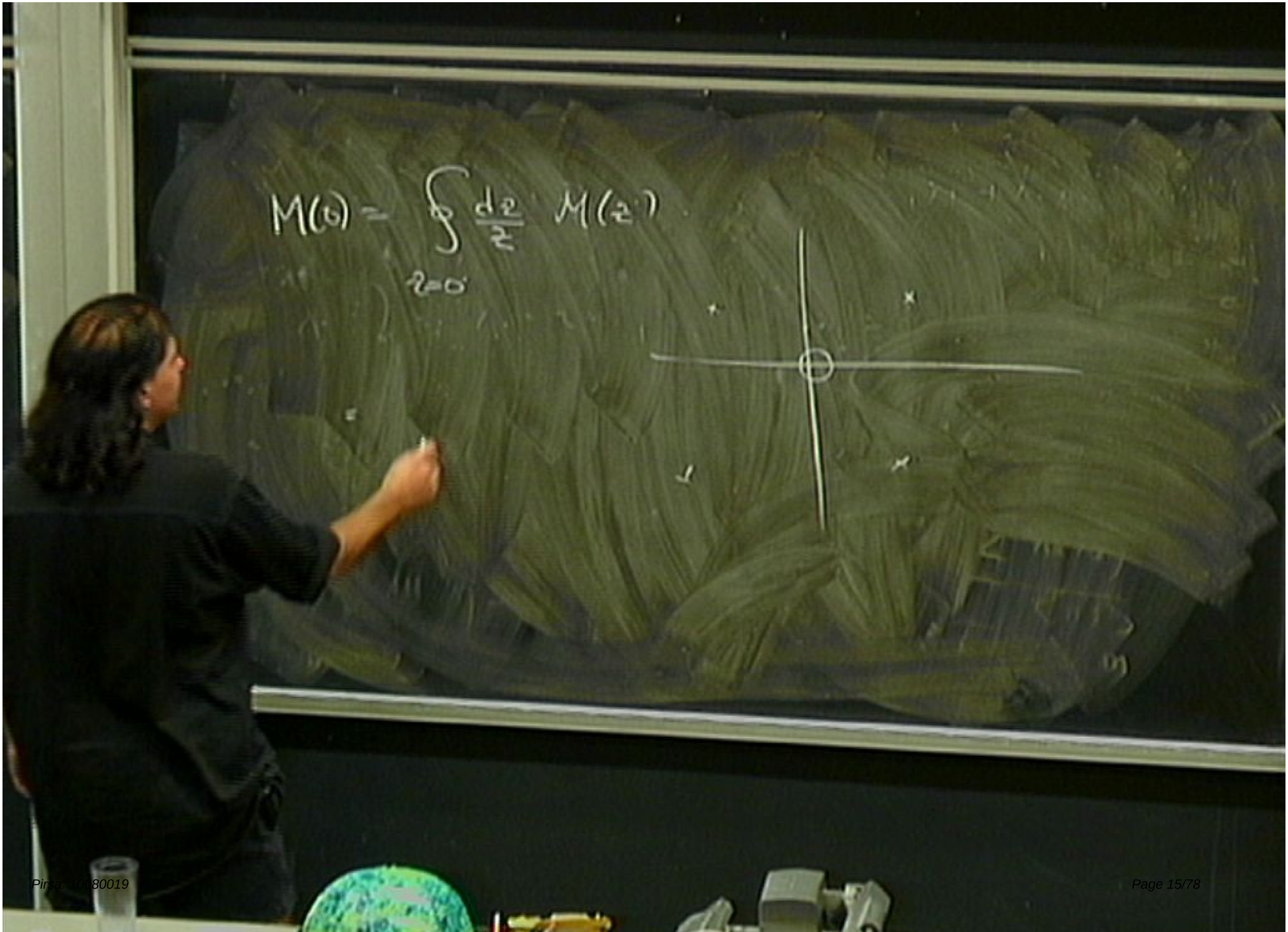

$\lambda_1(z) = \lambda_1 + z\lambda_2$
$\tilde{\lambda}_2(z) = \tilde{\lambda}_2 - z\tilde{\lambda}_1$
$\lambda_1^{(z)}\tilde{\lambda}_1^{(z)} + \lambda_2^{(z)}\tilde{\lambda}_2^{(z)} = \lambda_1\tilde{\lambda}_1 + \lambda_2\tilde{\lambda}_2$

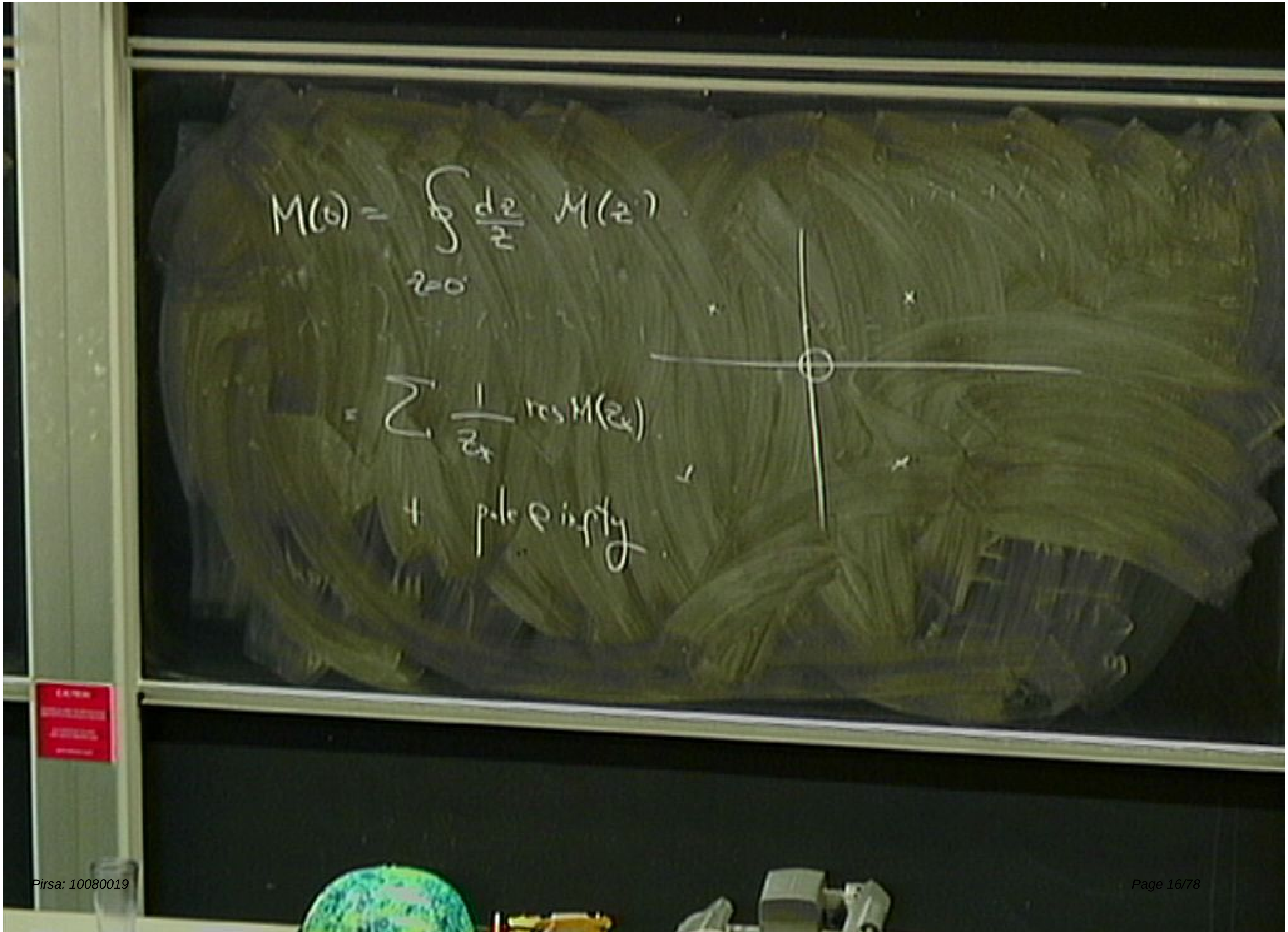

$M^{(\lambda,\tilde{\lambda})} \longrightarrow M(z, \lambda, \tilde{\lambda})$


$M^{(\lambda,\tilde\lambda)} \rightarrow M(z, \lambda, \tilde\lambda)$

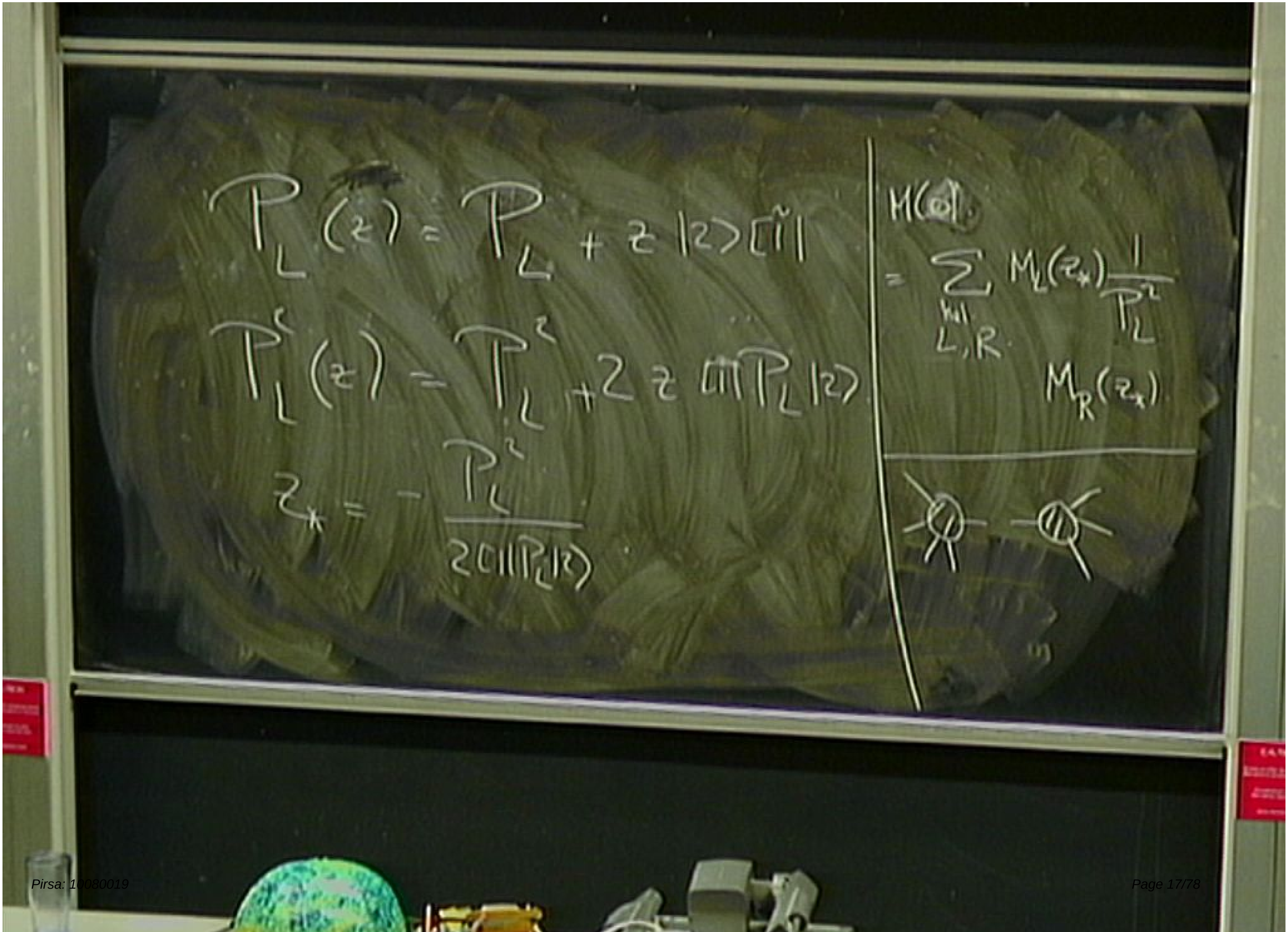

M(λ,λ̃) → M(z, λ, λ̃)
L
R
1
2

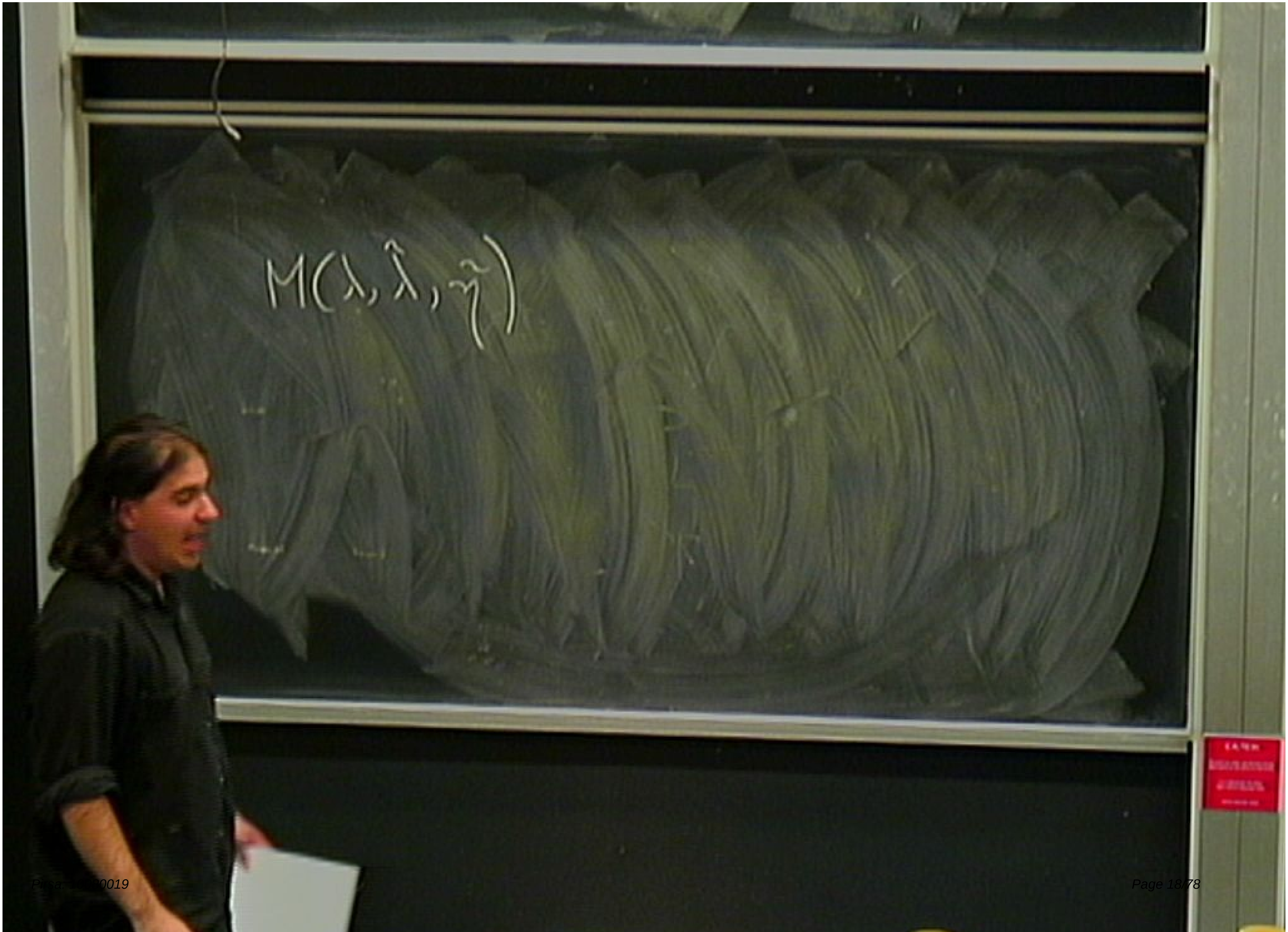

M(0) = ∮ dz/z M(z)
z=0

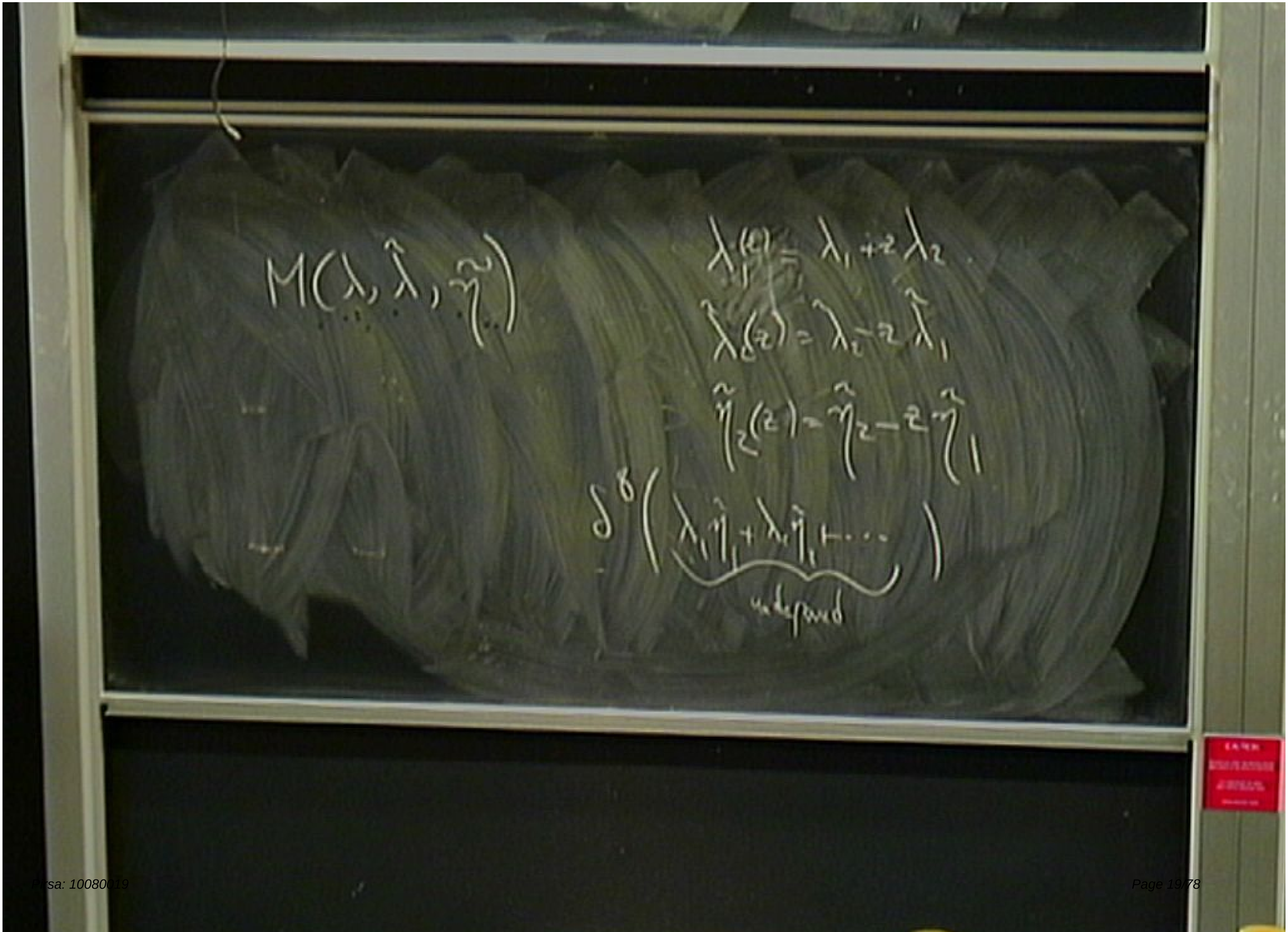

$M(0) = \oint \frac{dz}{z} M(z)$
$z=0$



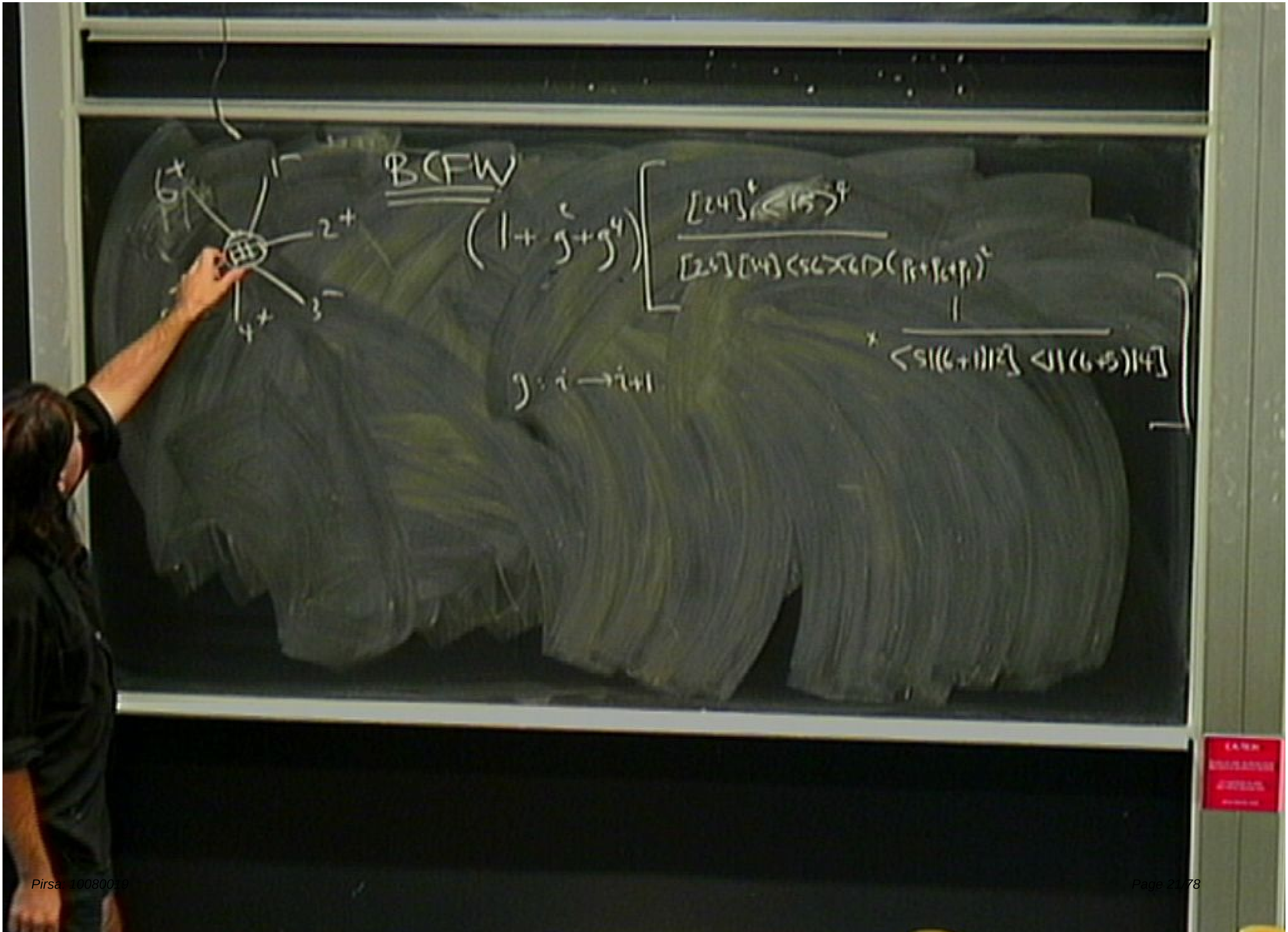
M(0) = ∮ dz/z M(z)
z=0
= Σ 1/z_k res M(z_k)
+ pole @ infinity

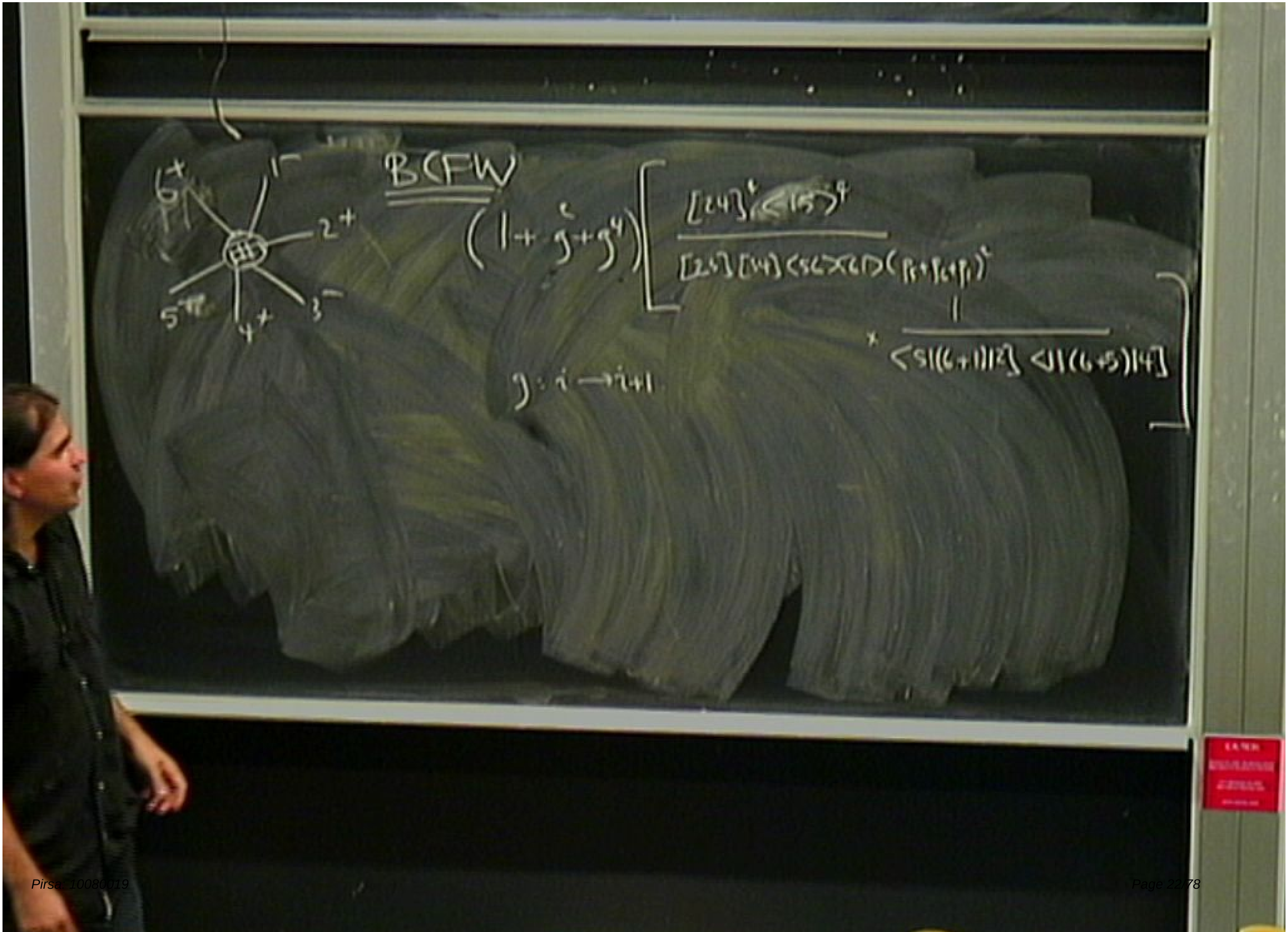

P_L(z) = P_L + z |2⟩[1̃|
P_L^2(z) = P_L^2 + 2 z [1|P_L|2⟩
z_* = − P_L^2 / 2[1|P_L|2⟩
M(0)
= Σ_{L,R} M_L(z_*) 1/P_L^2 M_R(z_*)

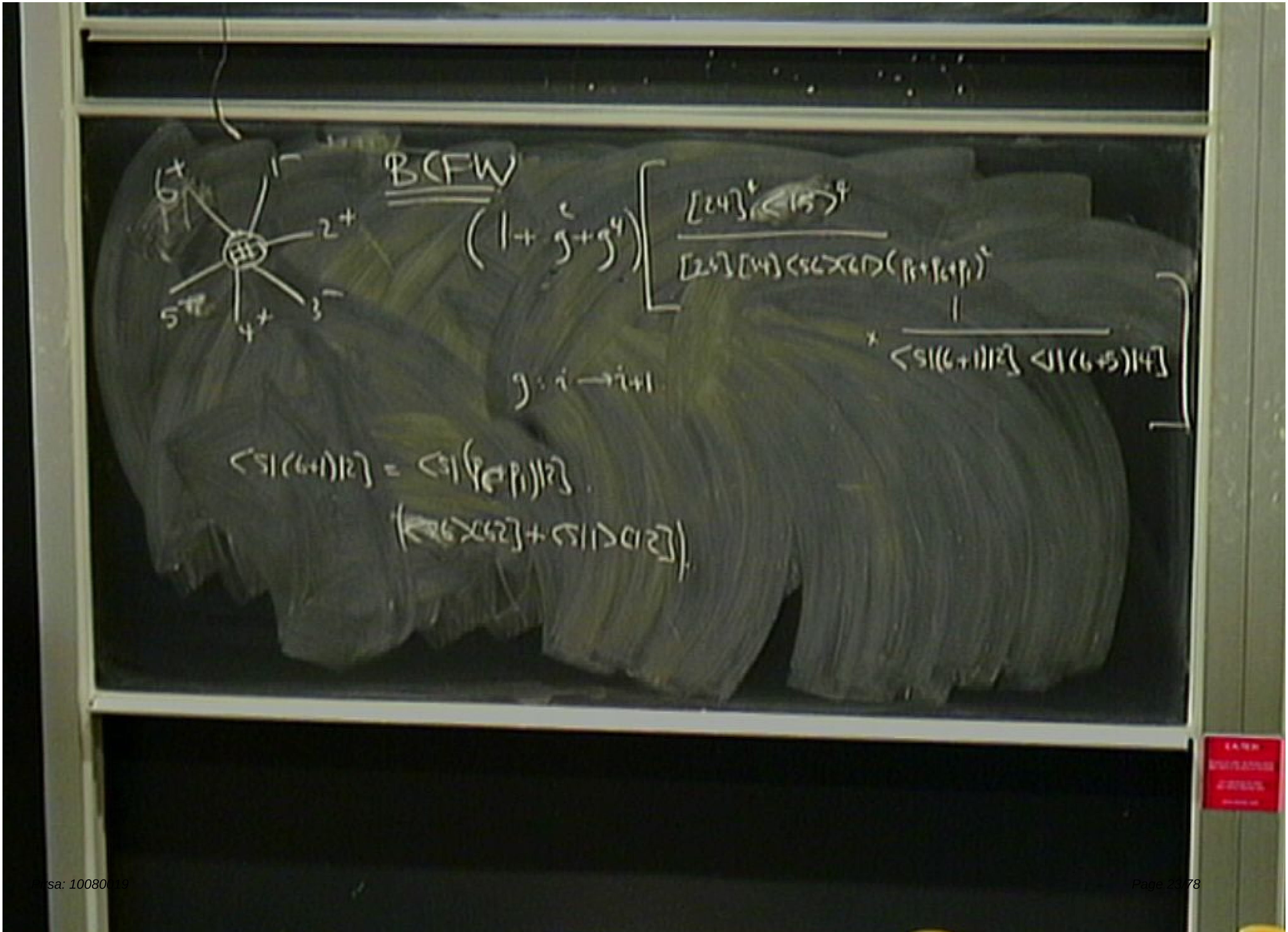

$M(\lambda, \hat{\lambda}, \tilde{\eta})$

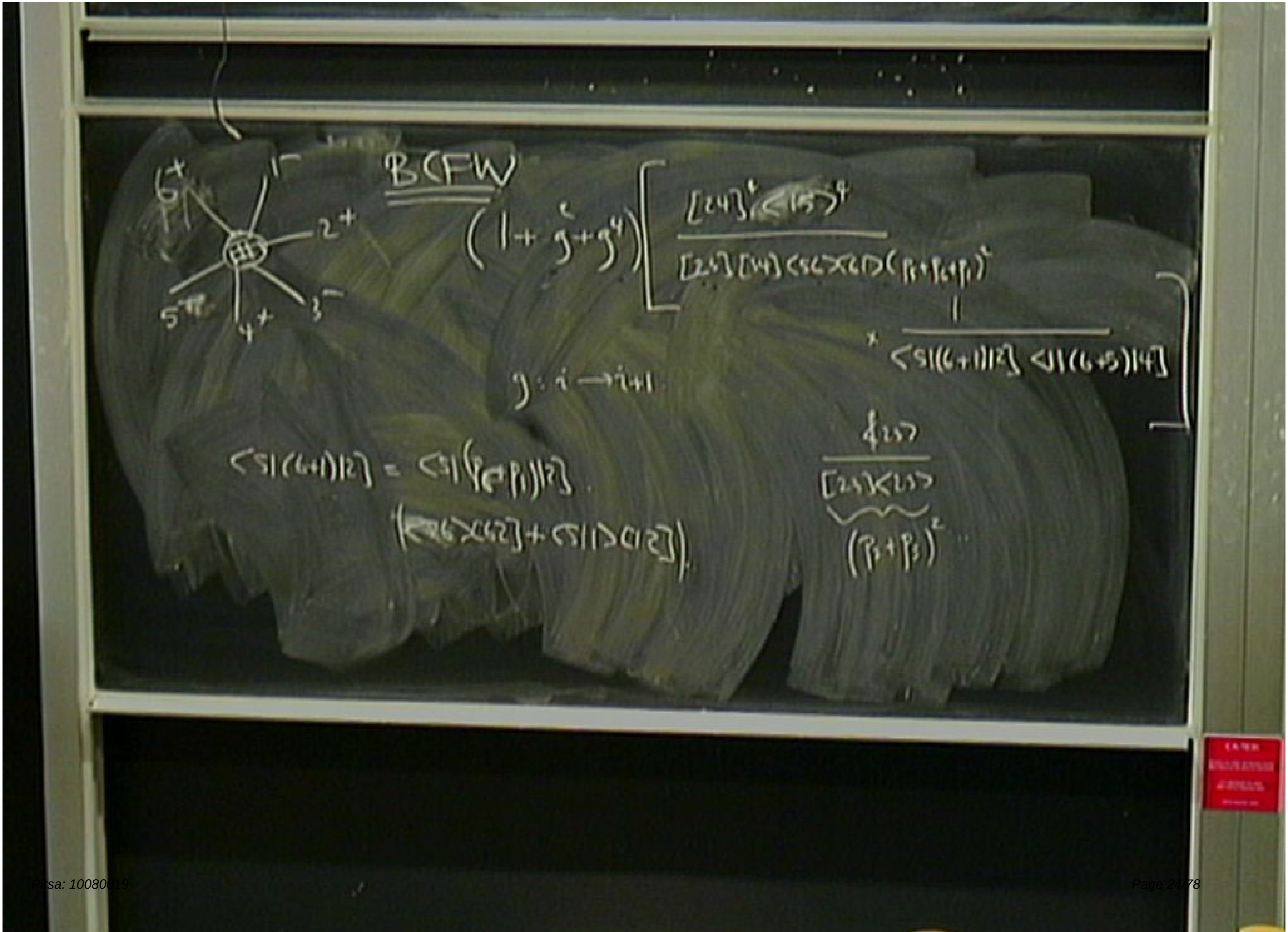

$M(\lambda, \hat{\lambda}, \tilde{\eta})$
$\lambda_1(z) = \lambda_1 + z\lambda_2$
$\tilde{\lambda}_2(z) = \tilde{\lambda}_2 - z\tilde{\lambda}_1$
$\tilde{\eta}_2(z) = \tilde{\eta}_2 - z\tilde{\eta}_1$
$\delta^8(\lambda_1\tilde{\eta}_1 + \lambda_2\tilde{\eta}_2 + \dots)$
independent

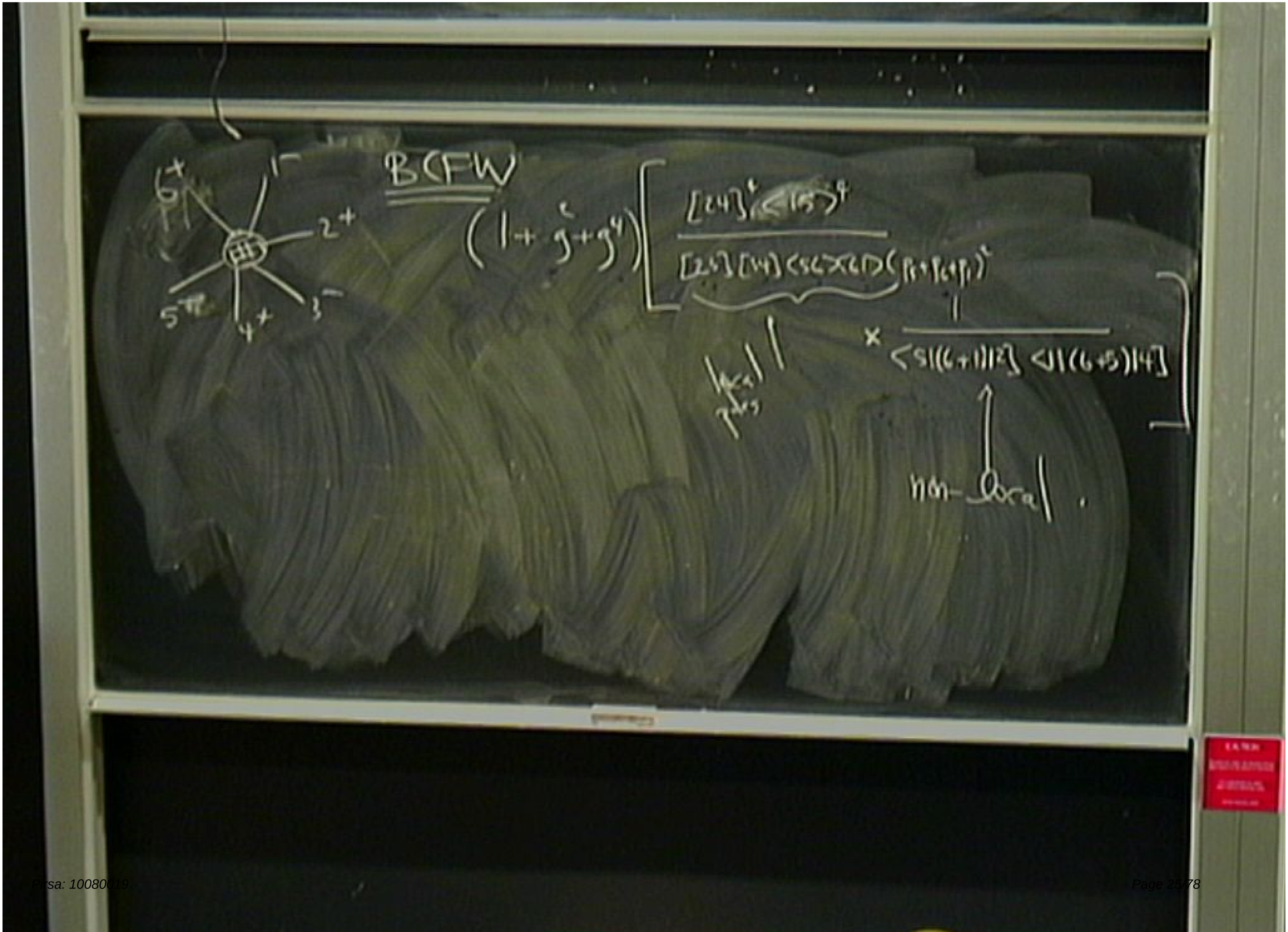

1⁻
2⁺
3⁻
4⁺
6⁺


BCFW
g: i → i+1


BCFW
1−
2+
3−
4+
6+
g : i → i+1


BCFW
g : i → i+1


BCFW
g : i → i+1


BCFW
(1+g²+g⁴)
× 1/⟨5|(6+1)|2] ⟨1|(6+5)|4]
non-local

$$P(BCFW) = (1+g^2+g^4)\left[\frac{[4|(3+5)|1\rangle^4}{[34][45]\langle 61\rangle\langle 12\rangle(p_6+p_1+p_2)^2} \times \frac{1}{\langle 4|1+2|3]\,\langle 2|(1+6)|5]}\right]$$

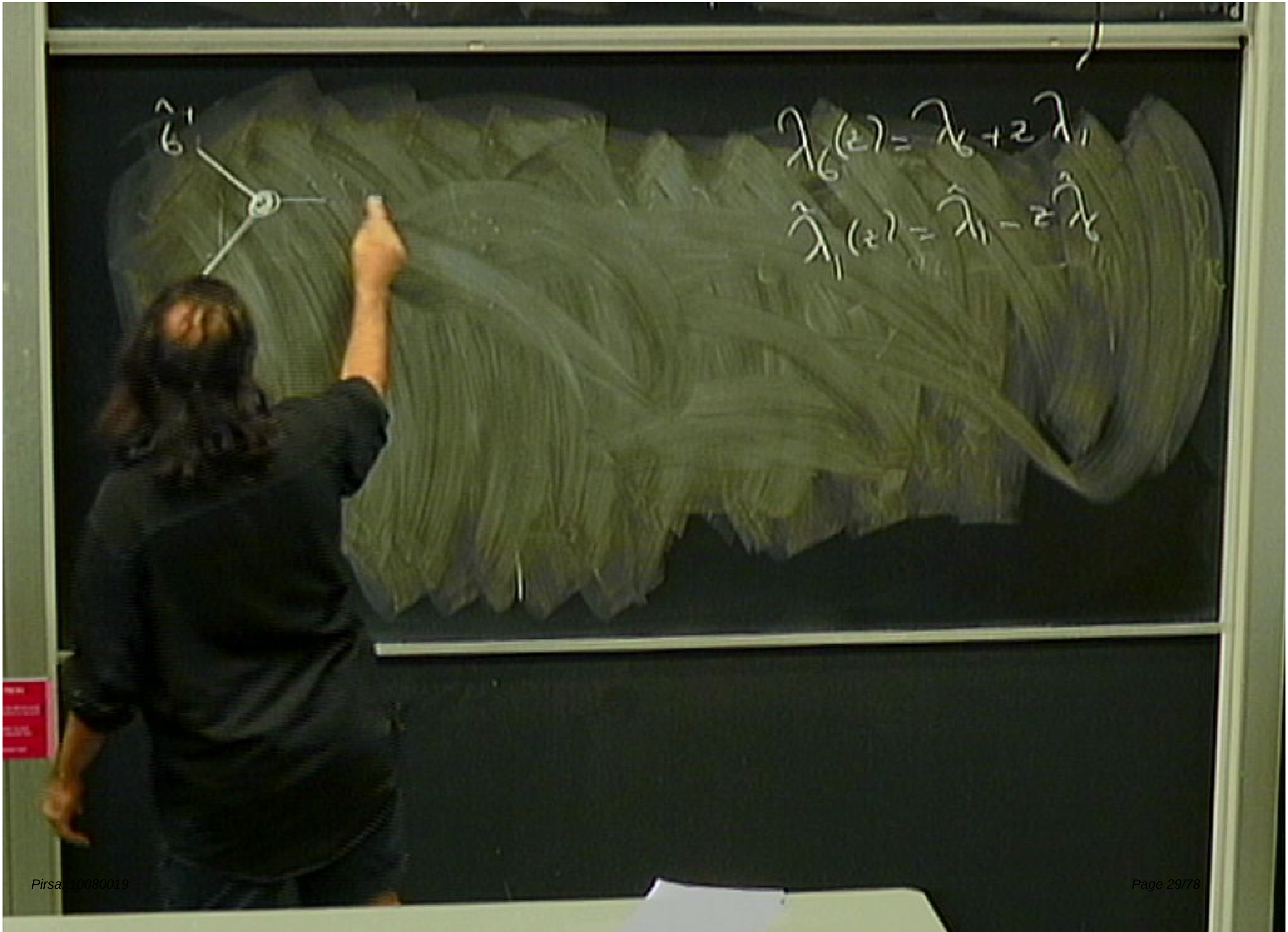
$$P(BCFW) = (1+g^2+g^4)\left[\frac{[4|(3+5)|1\rangle^4}{[54][45]\langle 61\rangle\langle 12\rangle(p_4+p_5+p_6)^2} \times \frac{1}{\langle 4|1+2|3]\,\langle 2|(1+6)|5]}\right]$$

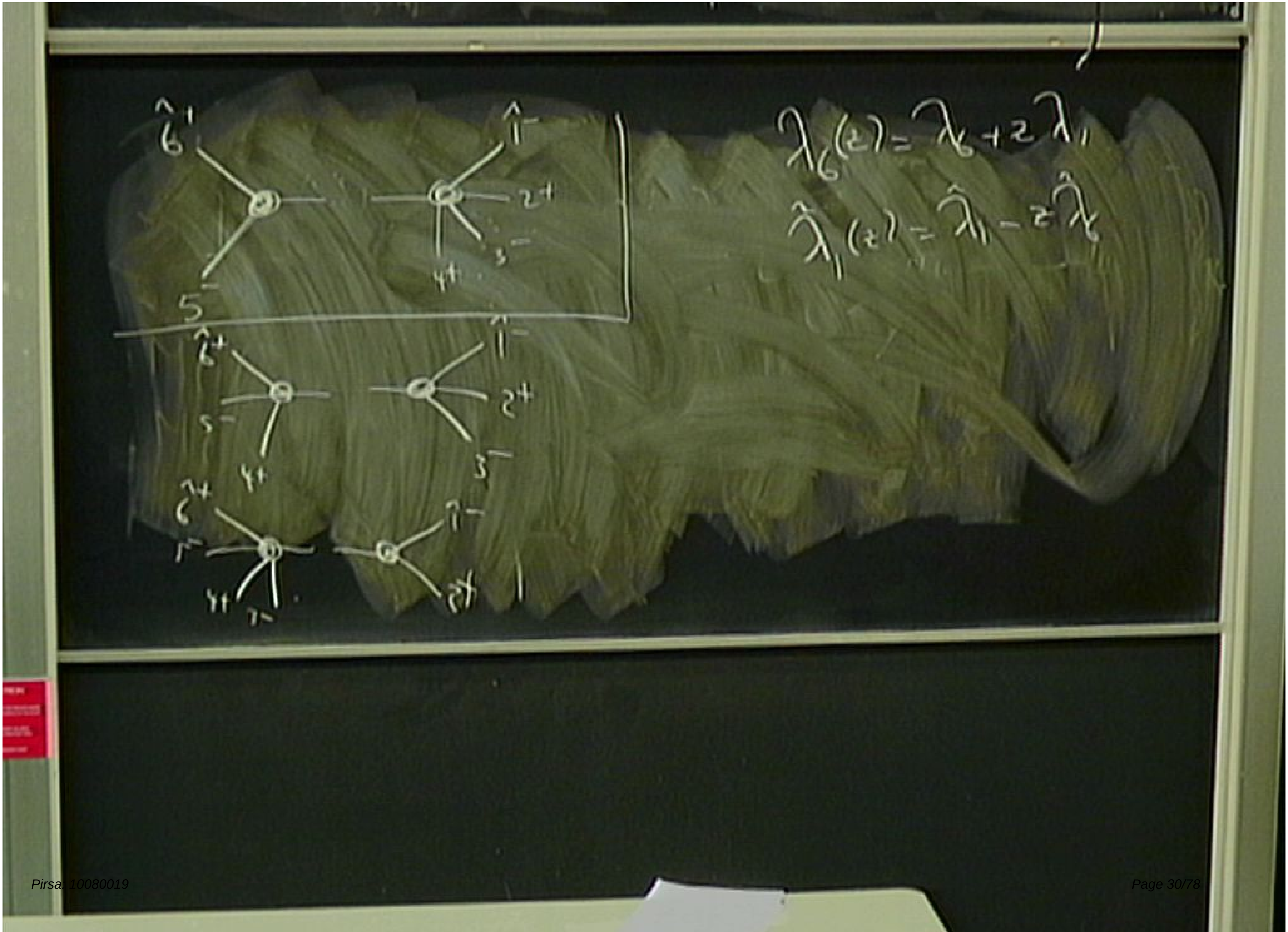
$$M_{BCFW} = M_{P(BCFW)}\Big|$$

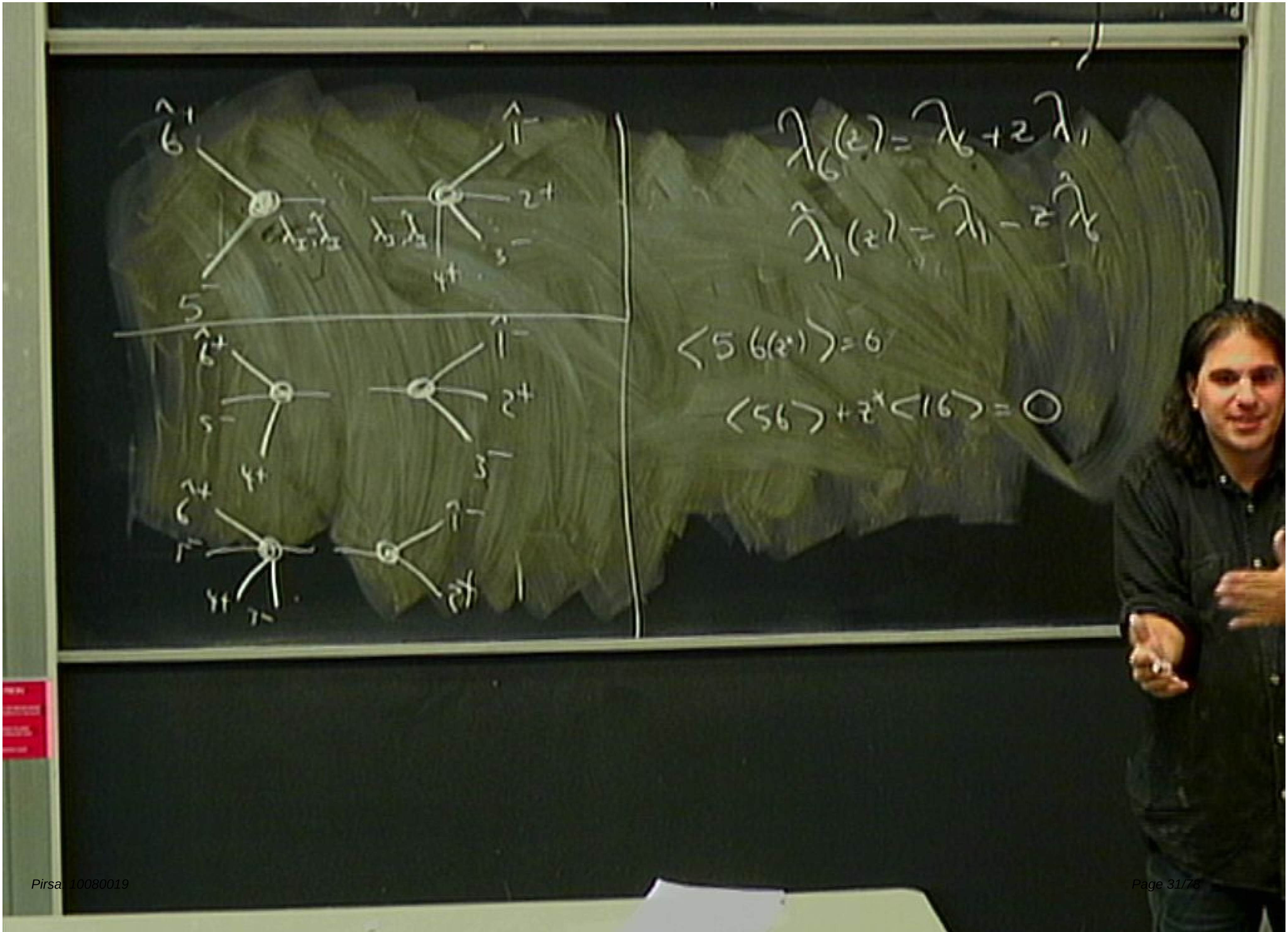
$$P(BCFW) = (1+g^2+g^4)\left[\frac{[4|(3+5)|1\rangle^4}{[54][45]\langle 61\rangle\langle 12\rangle(p_4+p_5+p_6)^2} \times \frac{1}{\langle 4|1+2|5]\,\langle 2|(1+6)|5]}\right]$$

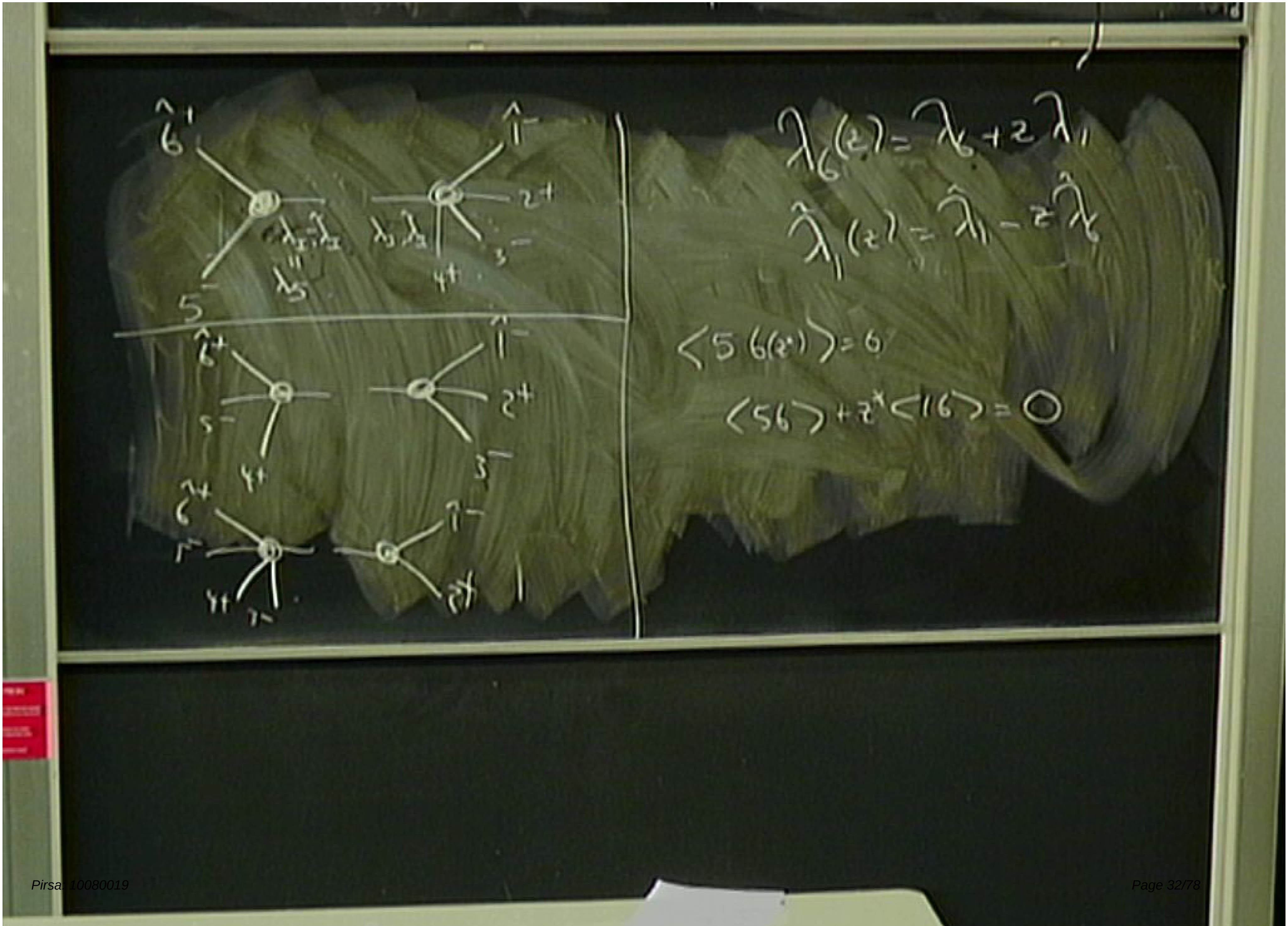
$$M_{BCFW} = M_{P(BCFW)}\,!$$

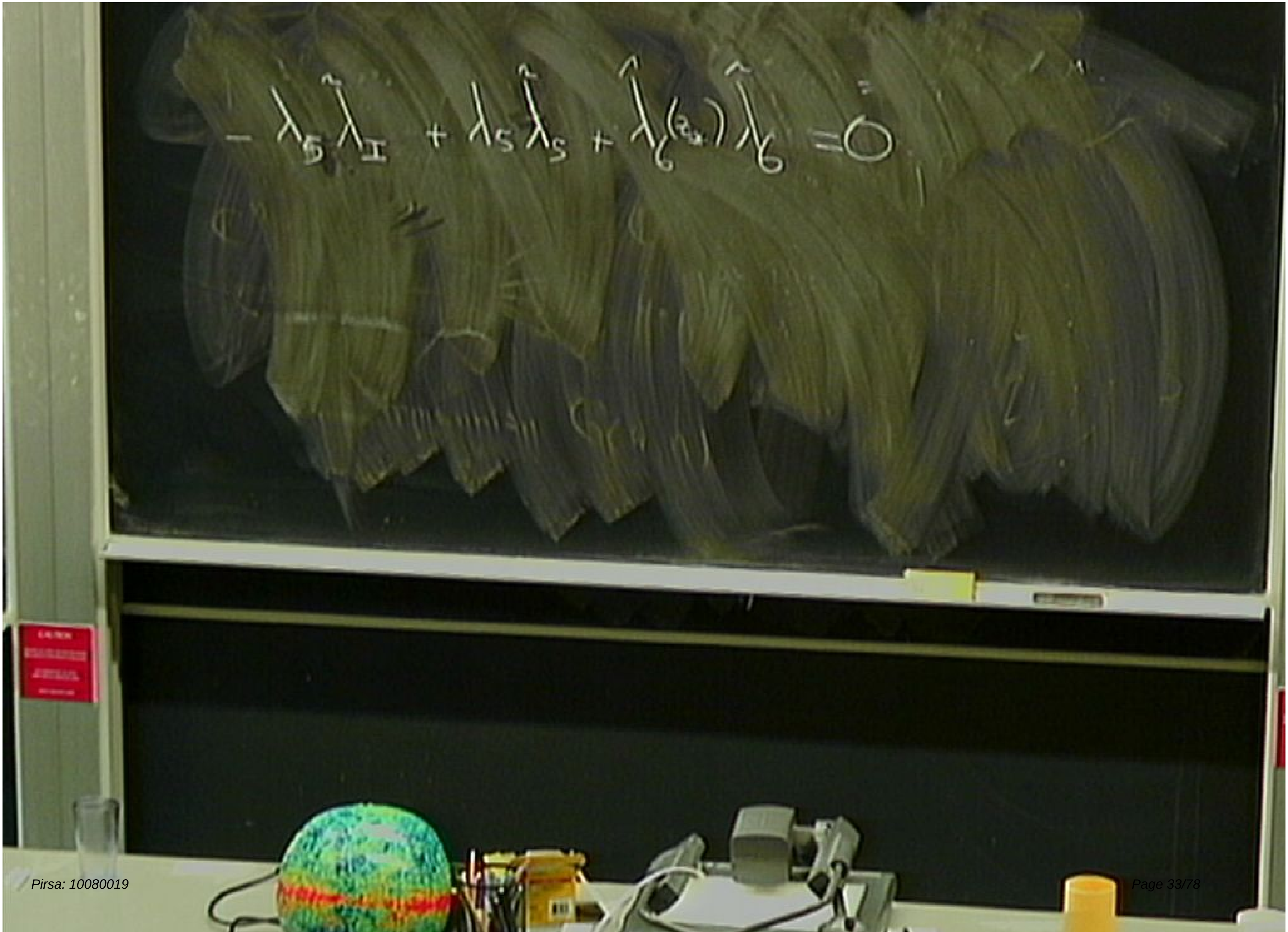
Remarkable 6-term identity.

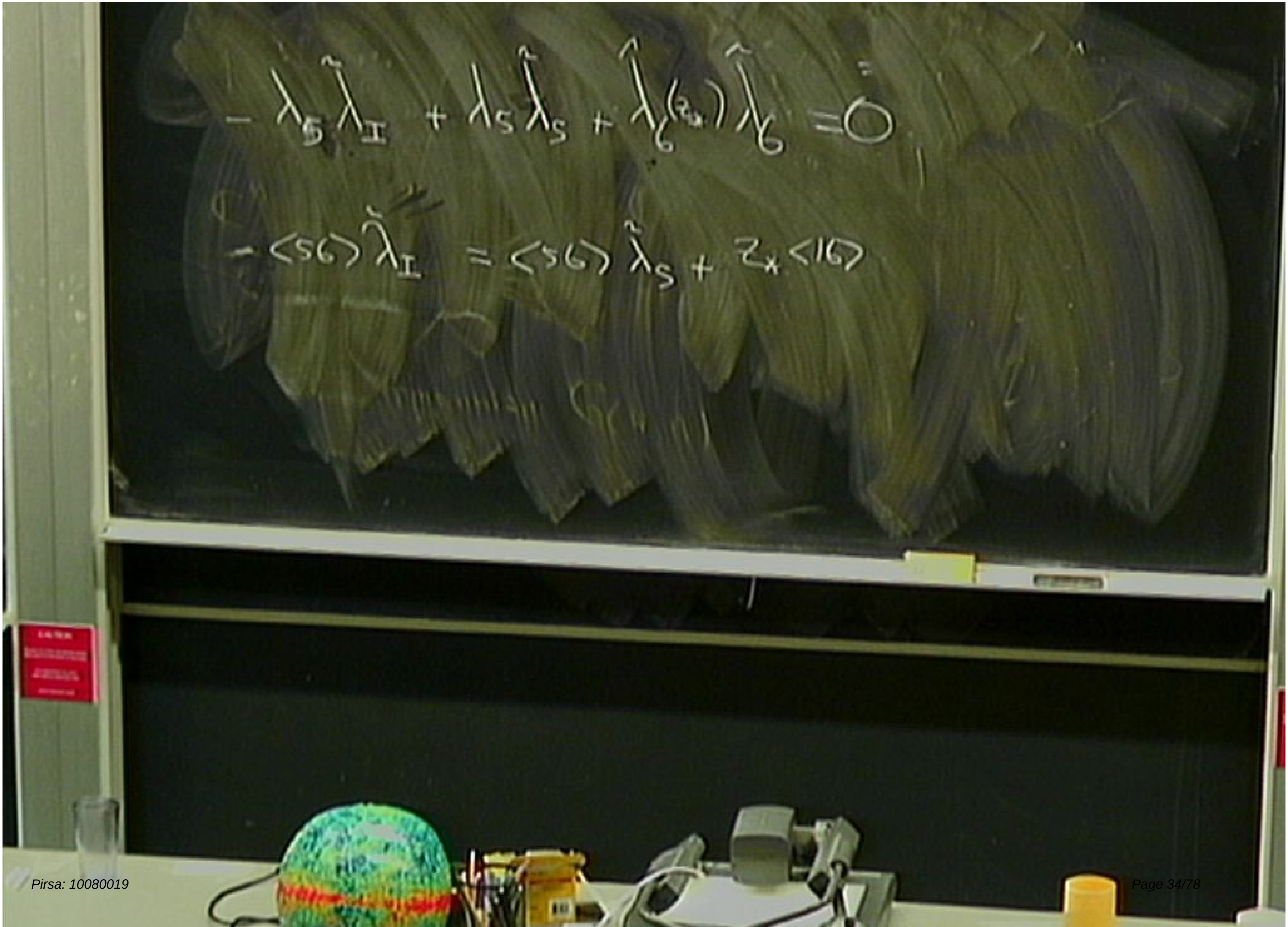

$\lambda_6(z) = \lambda_6 + z\lambda_1$
$\hat{\lambda}_1(z) = \hat{\lambda}_1 - z\hat{\lambda}_6$

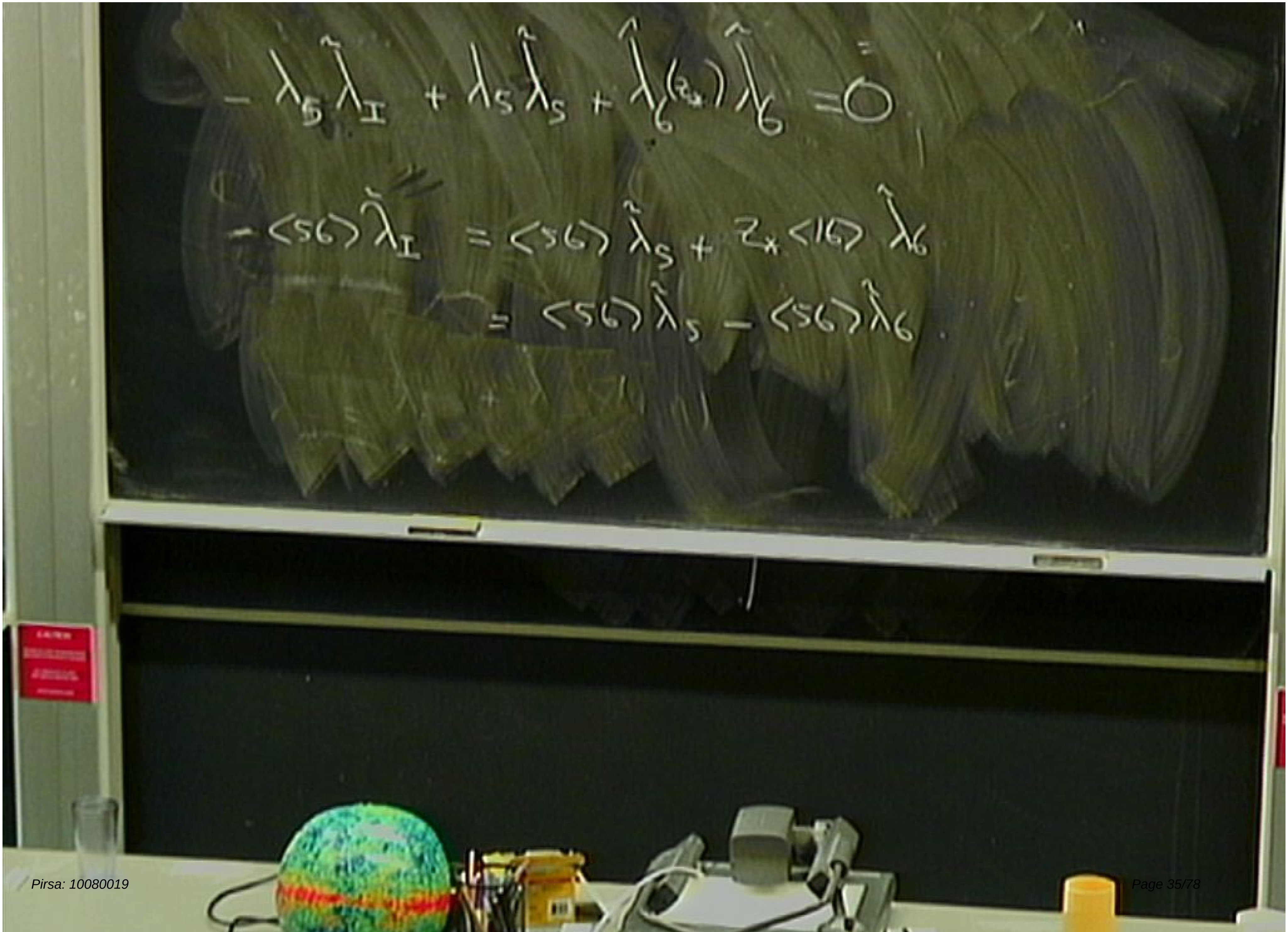

$\lambda_6(z) = \lambda_6 + z\lambda_1$
$\tilde{\lambda}_1(z) = \tilde{\lambda}_1 - z\tilde{\lambda}_6$

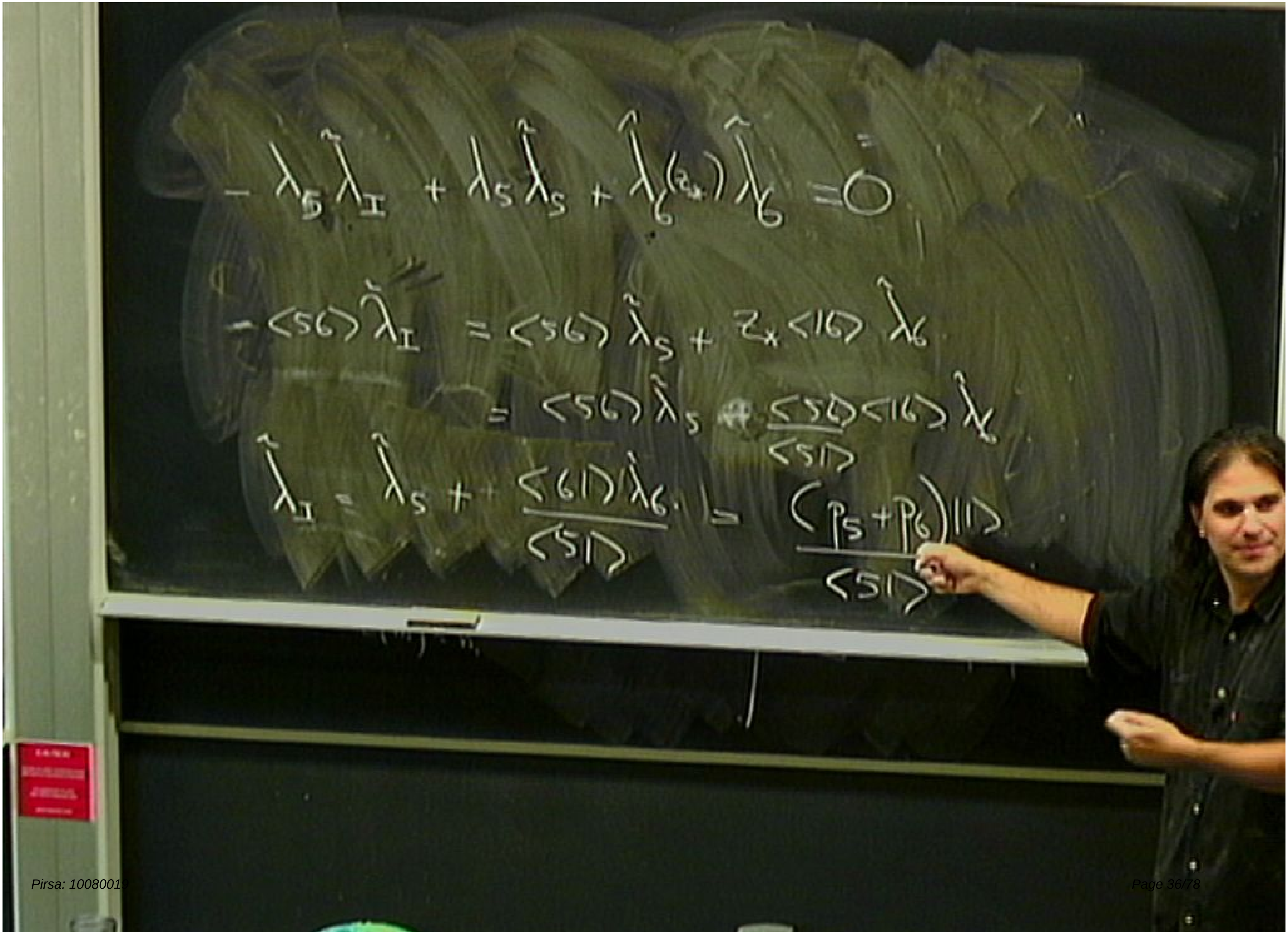

λ6(z) = λ6 + z λ1
λ̃1(z) = λ̃1 − z λ̃6
<5 6(z*)> = 0
<56> + z*<16> = 0

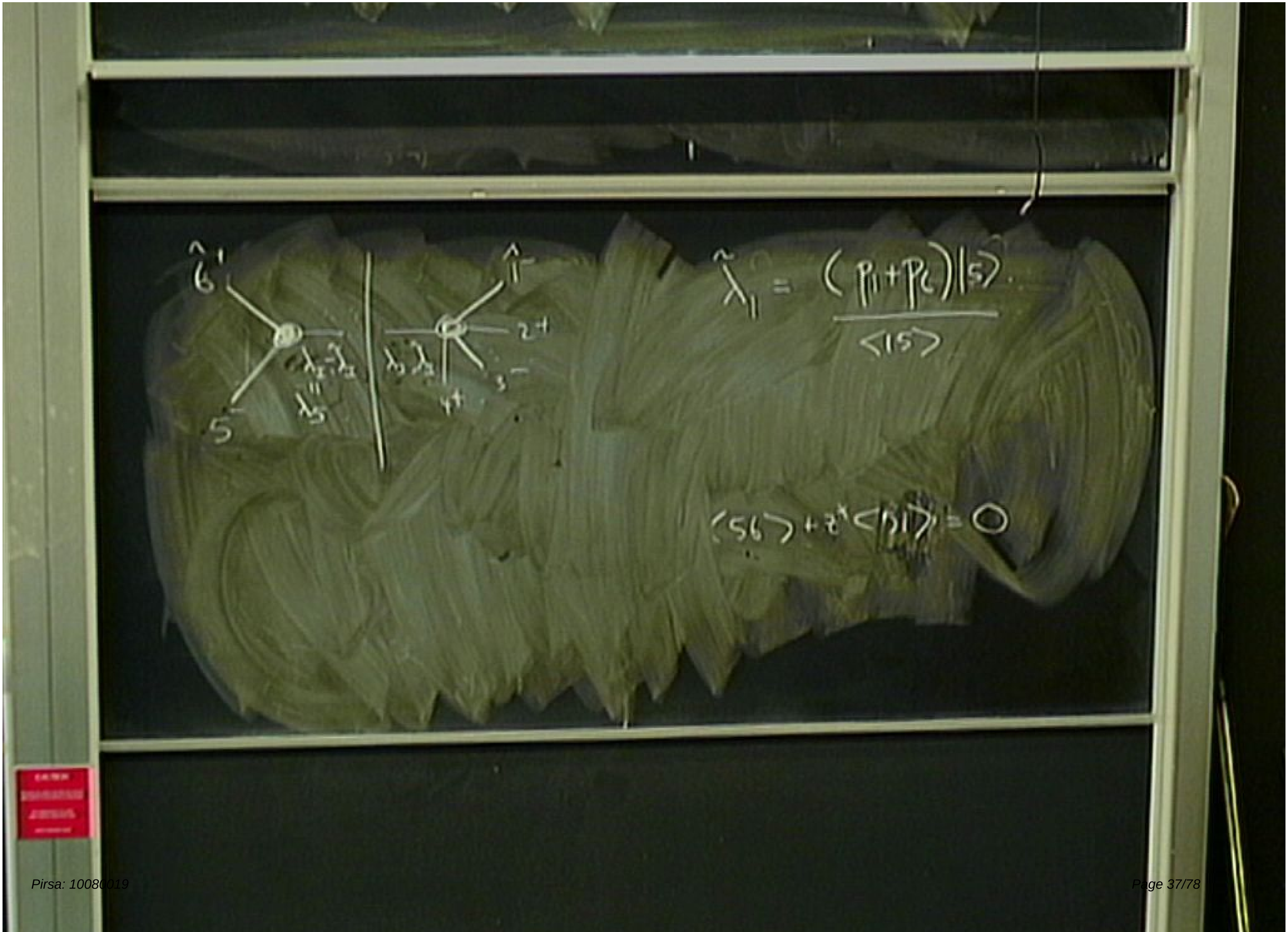

λ6(z) = λ6 + z λ1
λ̃1(z) = λ̃1 − z λ̃6
<5 6(z*)> = 0
<56> + z* <16> = 0


$-\lambda_5 \hat{\lambda}_{\bar{5}} + \lambda_5 \hat{\lambda}_5 + \hat{\lambda}_6(z_*) \hat{\lambda}_6 = 0$

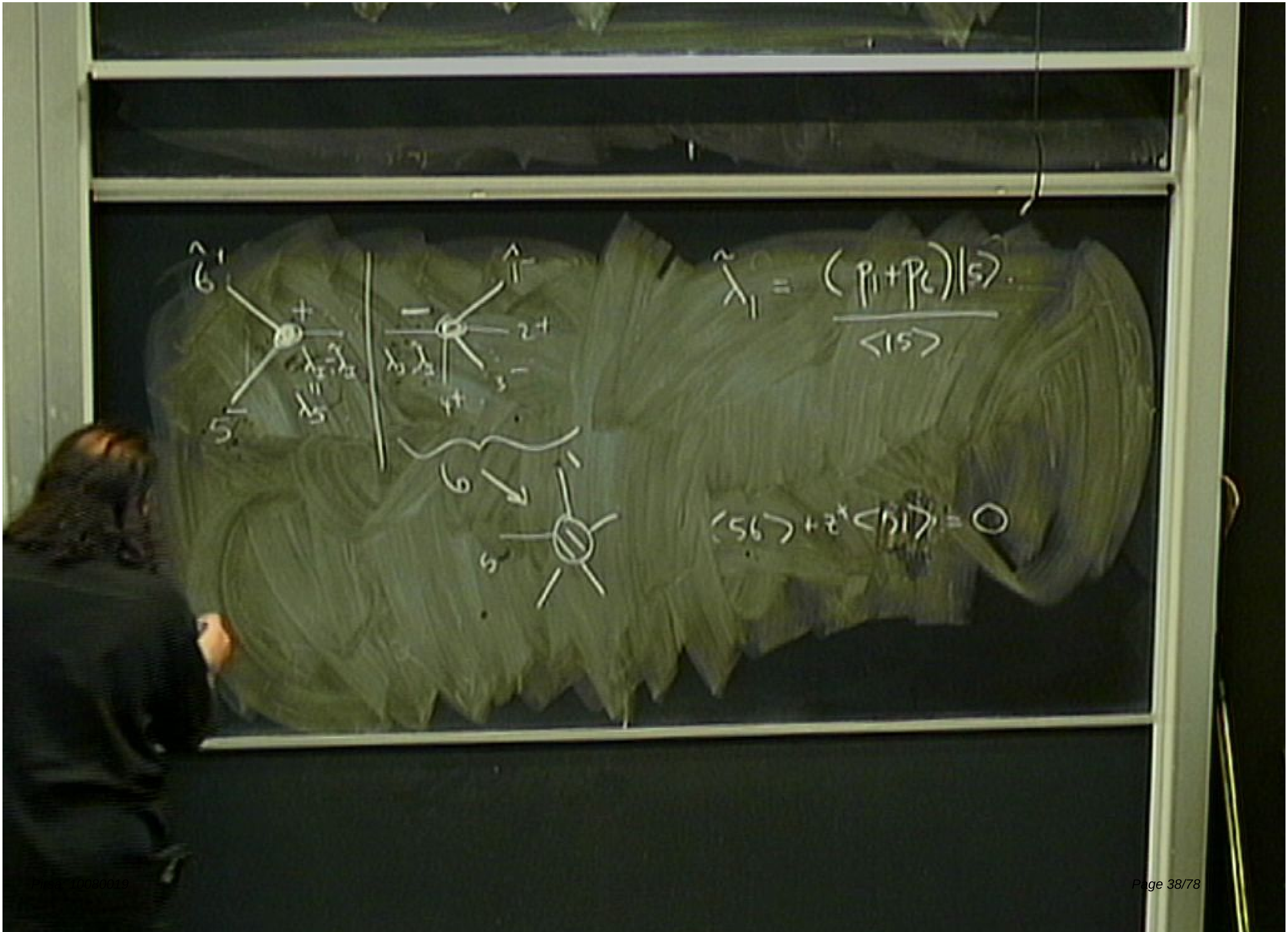

$-\lambda_5\tilde{\lambda}_I + \lambda_5\tilde{\lambda}_5 + \hat{\lambda}_6(z_*)\hat{\tilde{\lambda}}_6 = 0$
$-\langle 56\rangle\tilde{\lambda}_I = \langle 56\rangle\tilde{\lambda}_5 + z_*\langle 16\rangle$

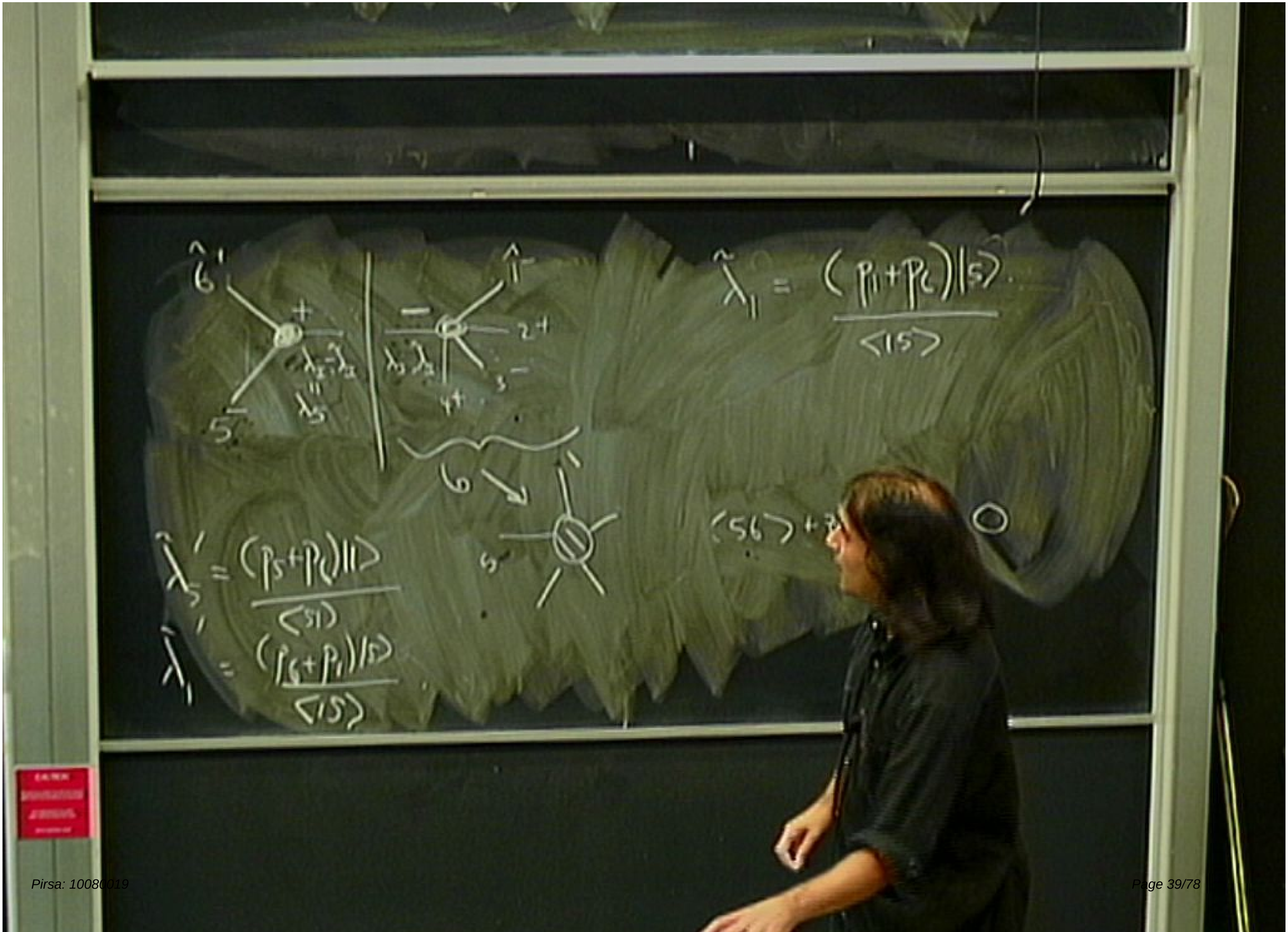

$-\lambda_5 \tilde{\lambda}_I + \lambda_5 \hat{\lambda}_5 + \hat{\lambda}_6(z_*) \hat{\lambda}_6 = 0$
$-\langle 56 \rangle \tilde{\lambda}_I = \langle 56 \rangle \tilde{\lambda}_5 + z_* \langle 16 \rangle \hat{\lambda}_6$
$= \langle 56 \rangle \tilde{\lambda}_5 - \langle 56 \rangle \hat{\lambda}_6$

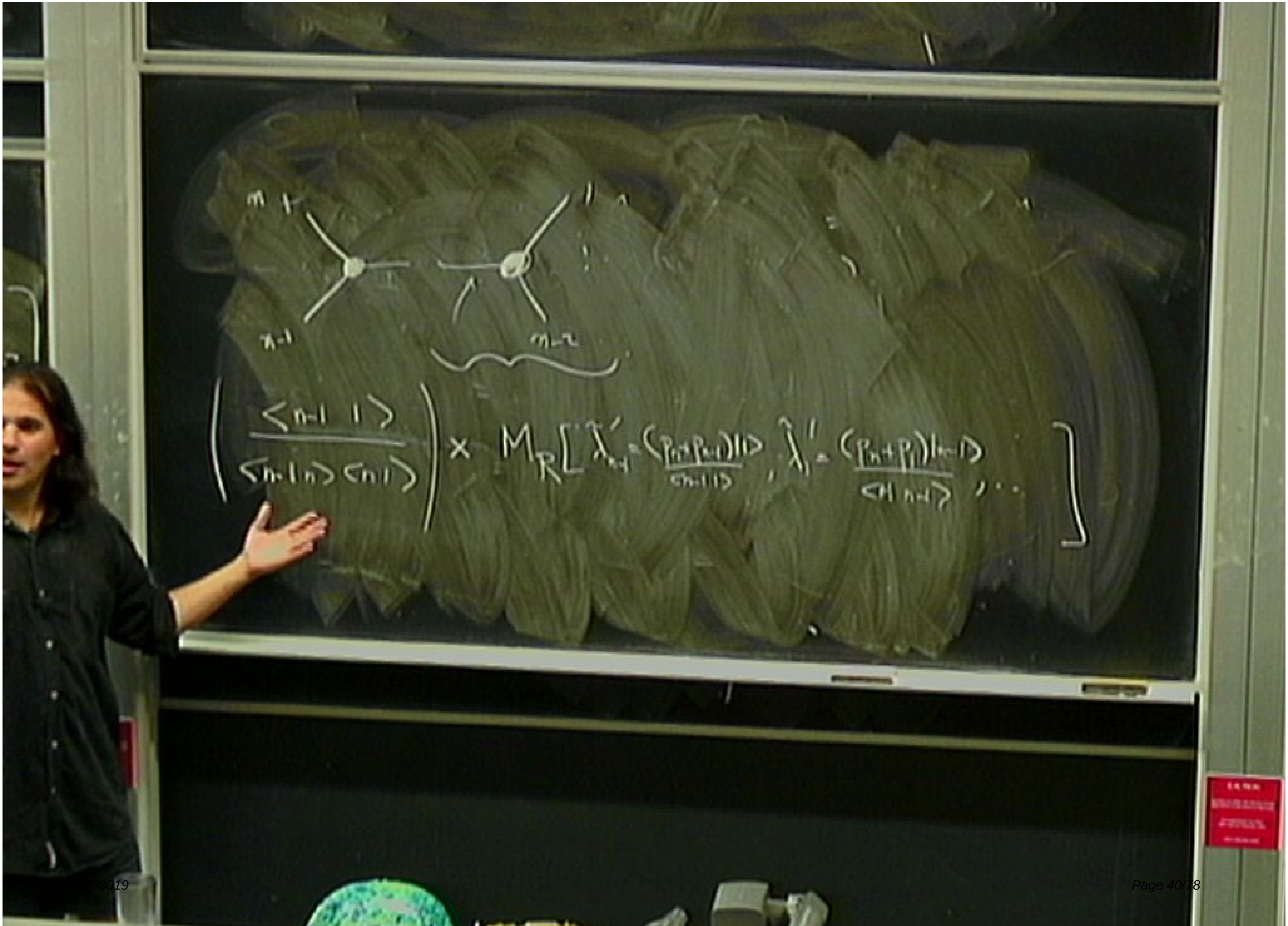

$-\lambda_5 \hat{\tilde\lambda}_I + \lambda_5 \hat{\tilde\lambda}_5 + \hat\lambda_6(z_*) \hat{\tilde\lambda}_6 = 0$
$-\langle 56 \rangle \hat{\tilde\lambda}_I = \langle 56 \rangle \hat{\tilde\lambda}_5 + z_* \langle 16 \rangle \hat{\tilde\lambda}_6$
$= \langle 56 \rangle \hat{\tilde\lambda}_5 + \frac{\langle 56 \rangle \langle 16 \rangle}{\langle 51 \rangle} \hat{\tilde\lambda}_6$
$\hat{\tilde\lambda}_I = \hat{\tilde\lambda}_5 + \frac{\langle 61 \rangle \hat{\tilde\lambda}_6}{\langle 51 \rangle} = \frac{(p_5 + p_6)|1\rangle}{\langle 51 \rangle}$

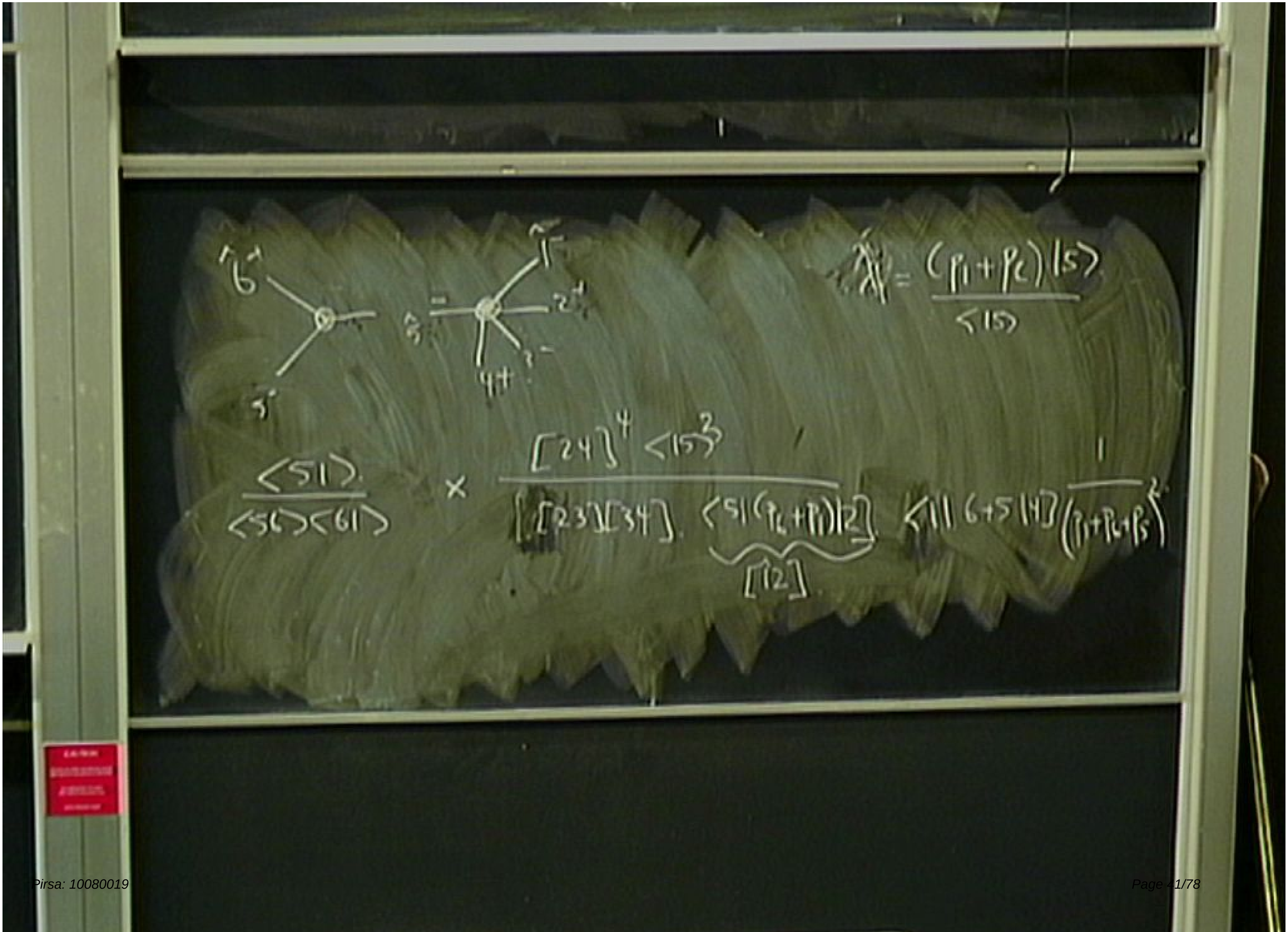


⟨15⟩

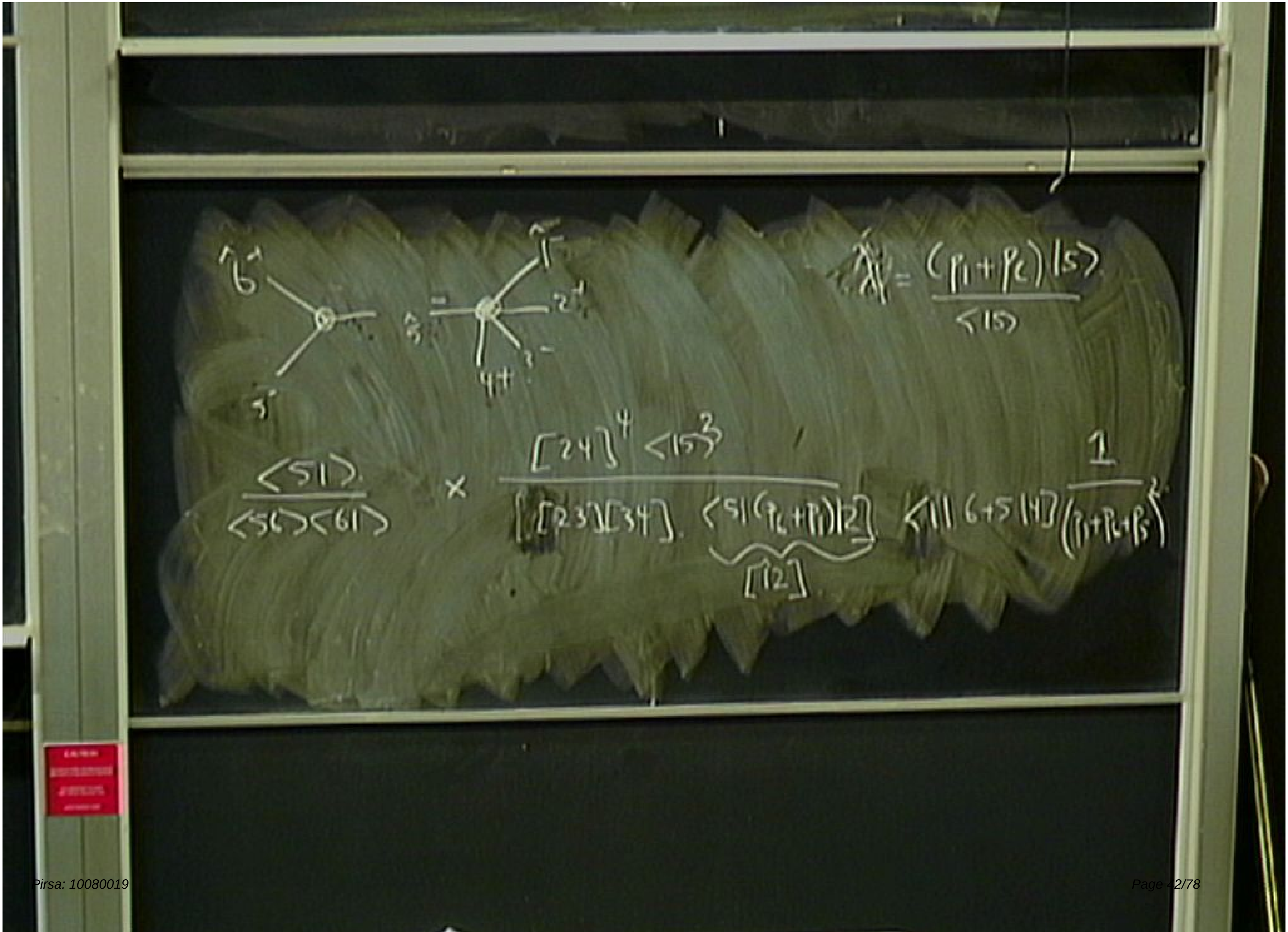


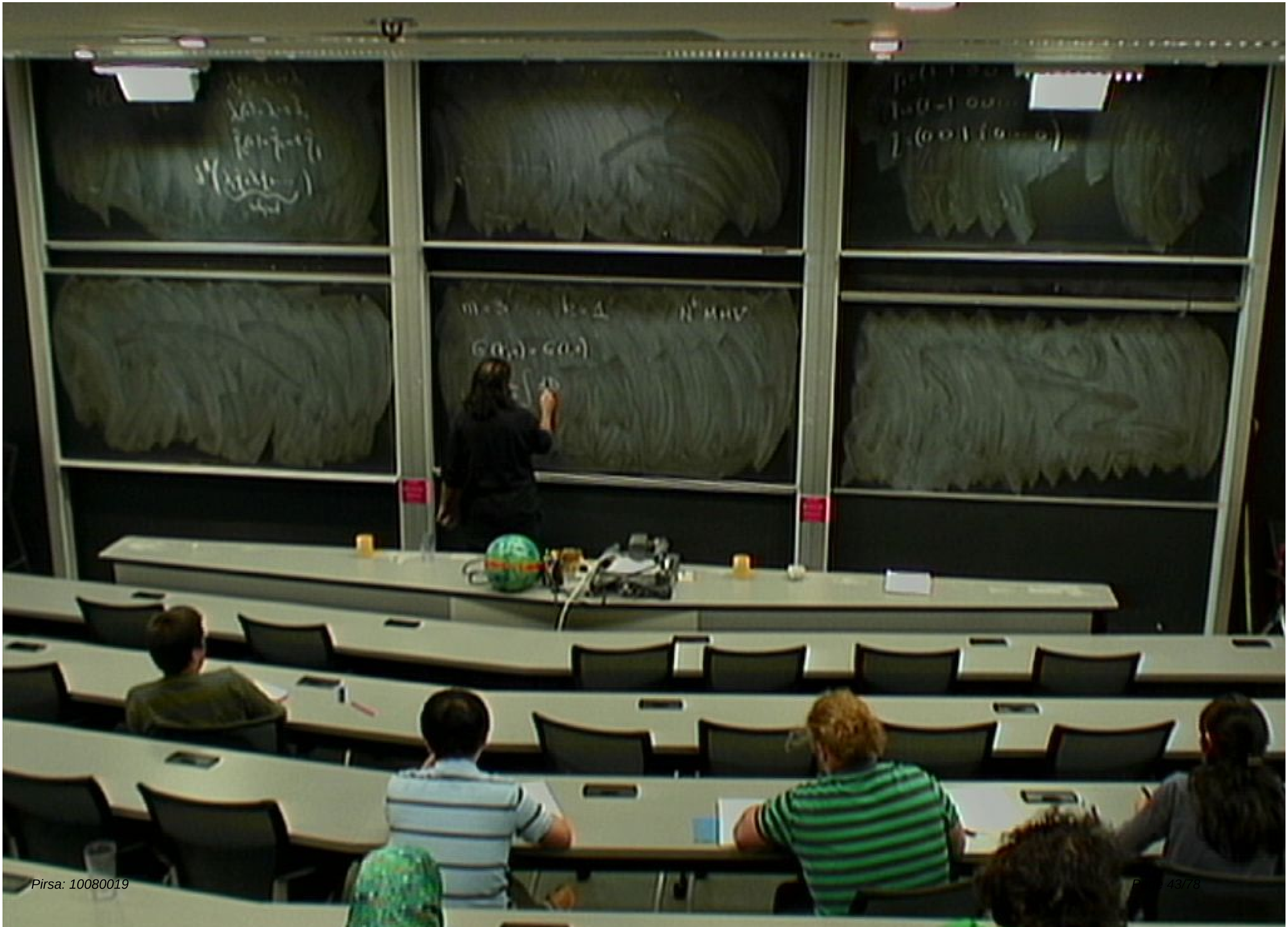

(p1+p6)|5⟩
⟨15⟩
(p5+p6)|1⟩
⟨51⟩
(p6+p1)|5⟩
⟨15⟩

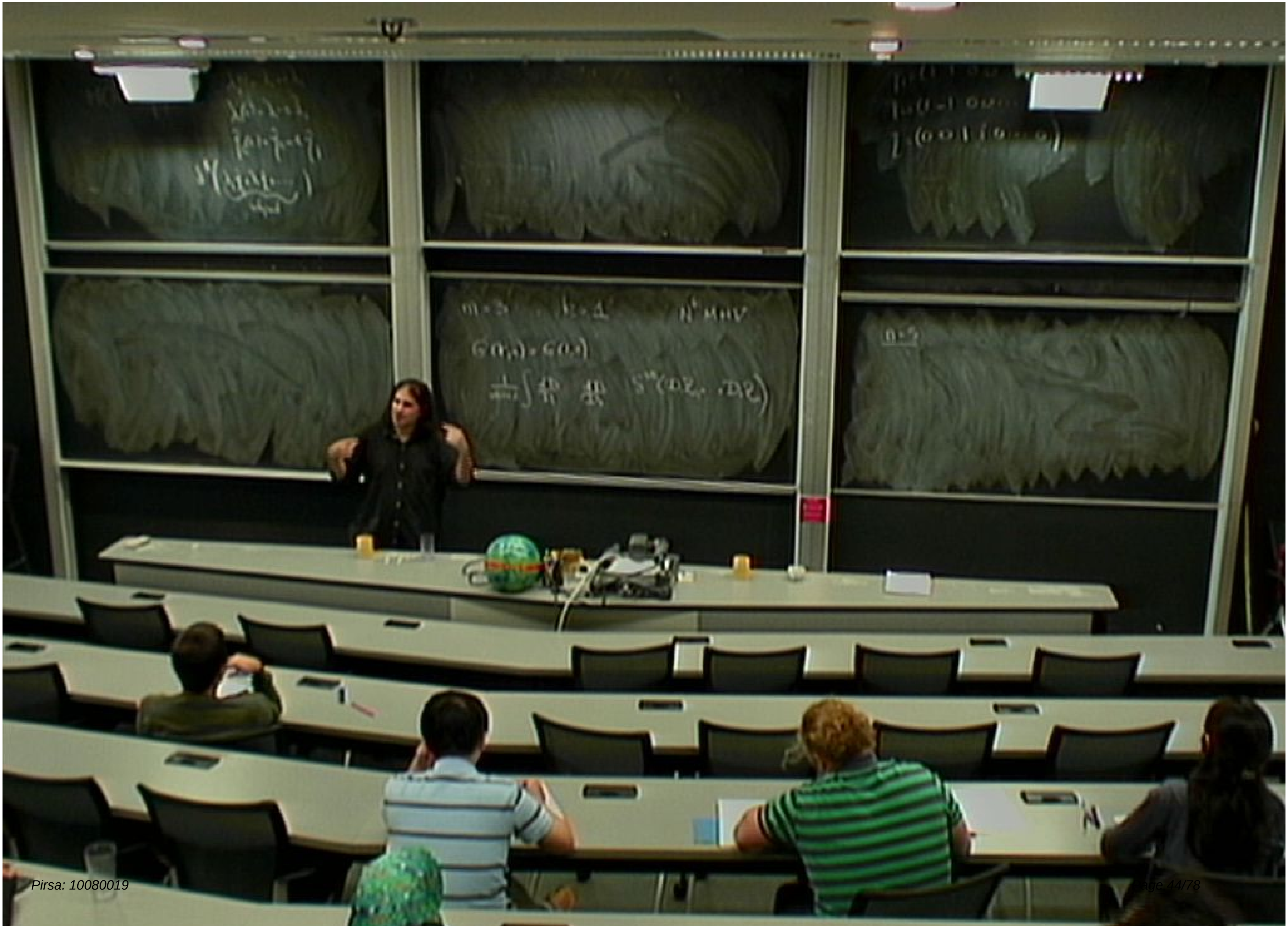


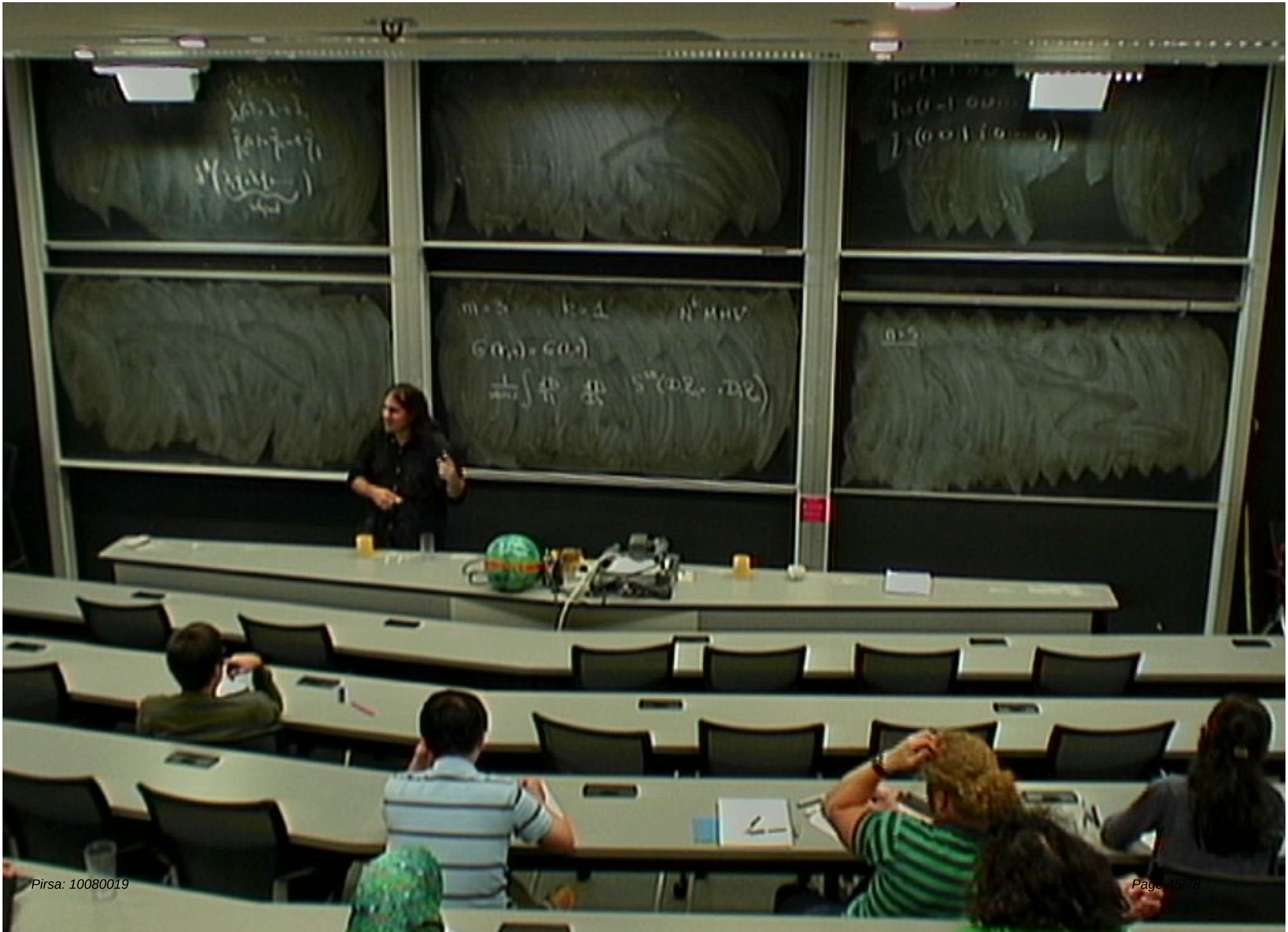


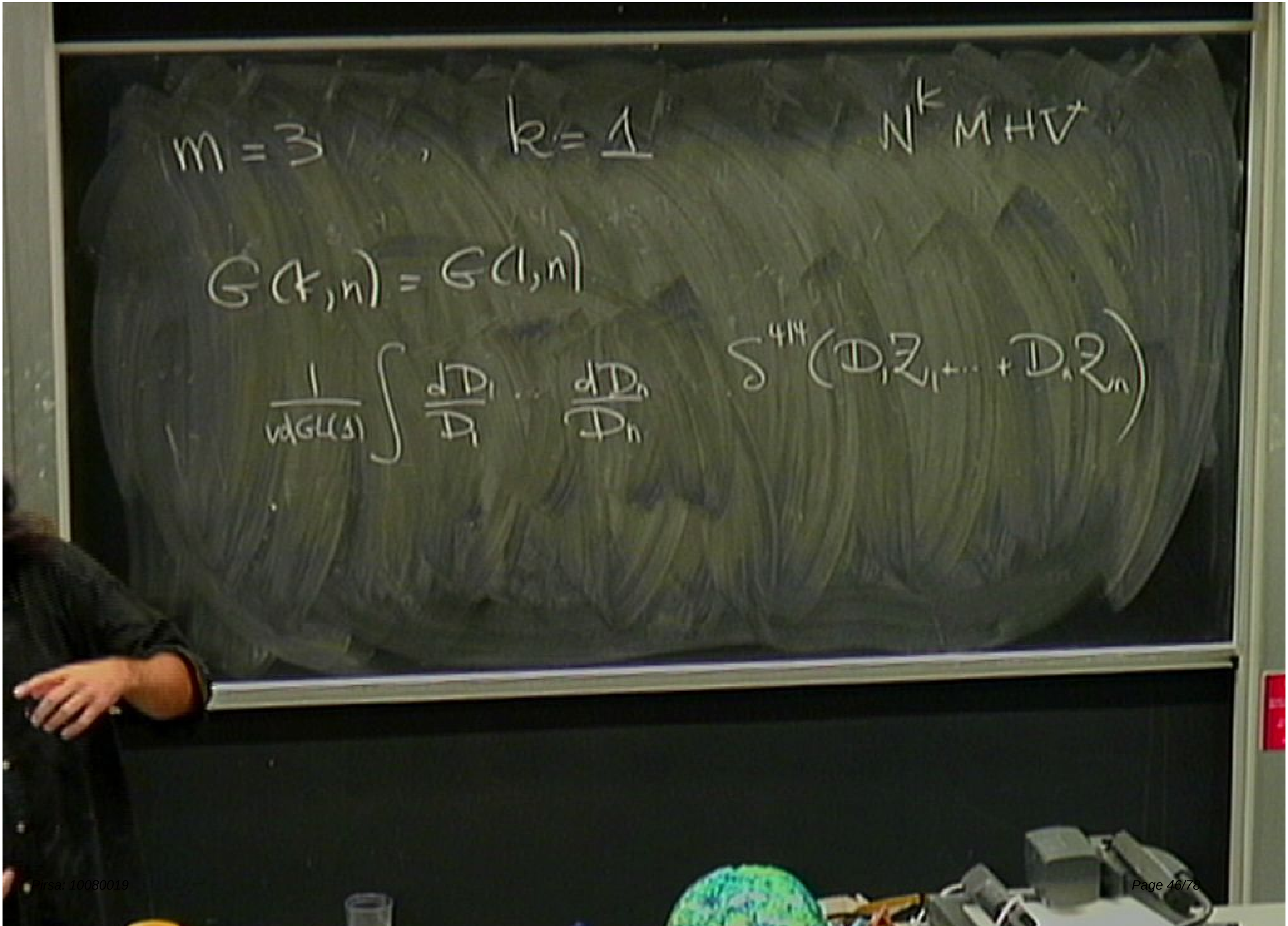
[24]^4 <15>^3
<51> / <56><61> × [24]^4 <15>^3 / [23][34] <5|(p_6+p_1)|2] <1|6+5|4] (p_1+p_6+p_5)^2
[12]

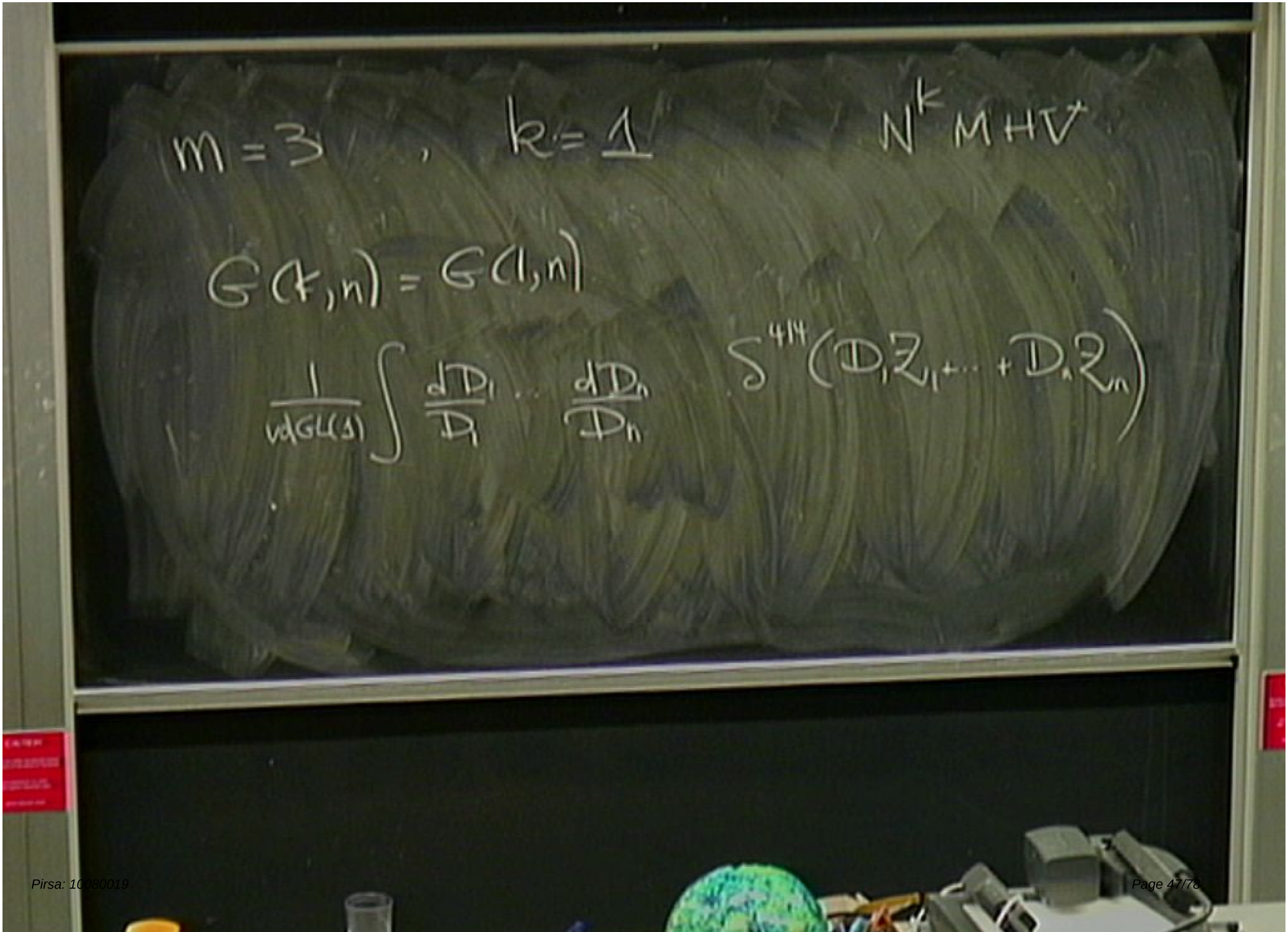


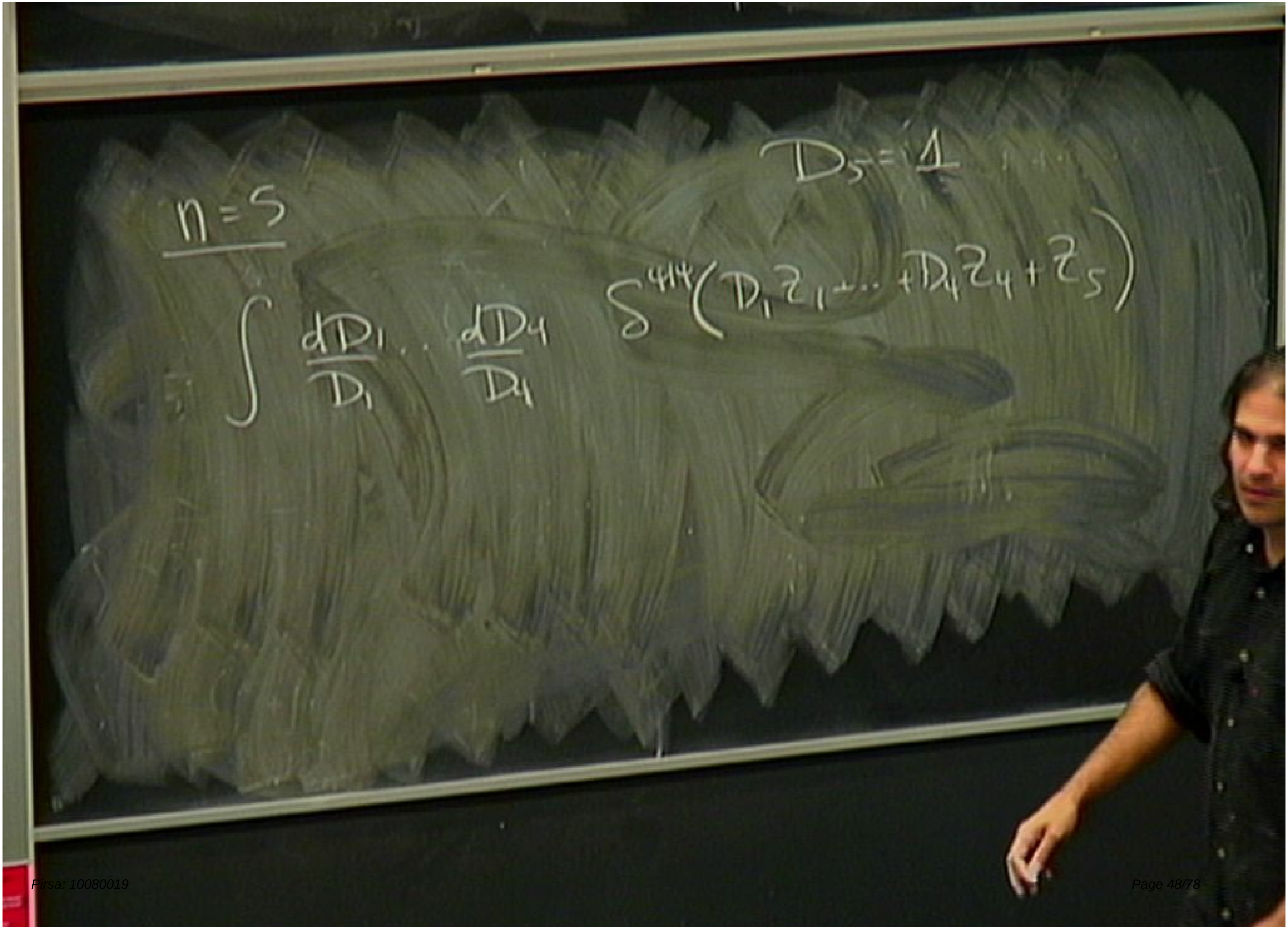







$m=3$, $k=1$
$N^k MHV$
$G(k,n) = G(1,n)$
$\frac{1}{\text{vol}\,GL(1)} \int \frac{dD_1}{D_1} \cdots \frac{dD_n}{D_n} \; \delta^{4|4}(D_1 Z_1 + \cdots + D_n Z_n)$


m = 3 , k = 1
N^k MHV
G(k,n) = G(1,n)
1/vol GL(1) ∫ dD1/D1 ... dDn/Dn δ^4|4(D1 Z1 + ... + Dn Zn)

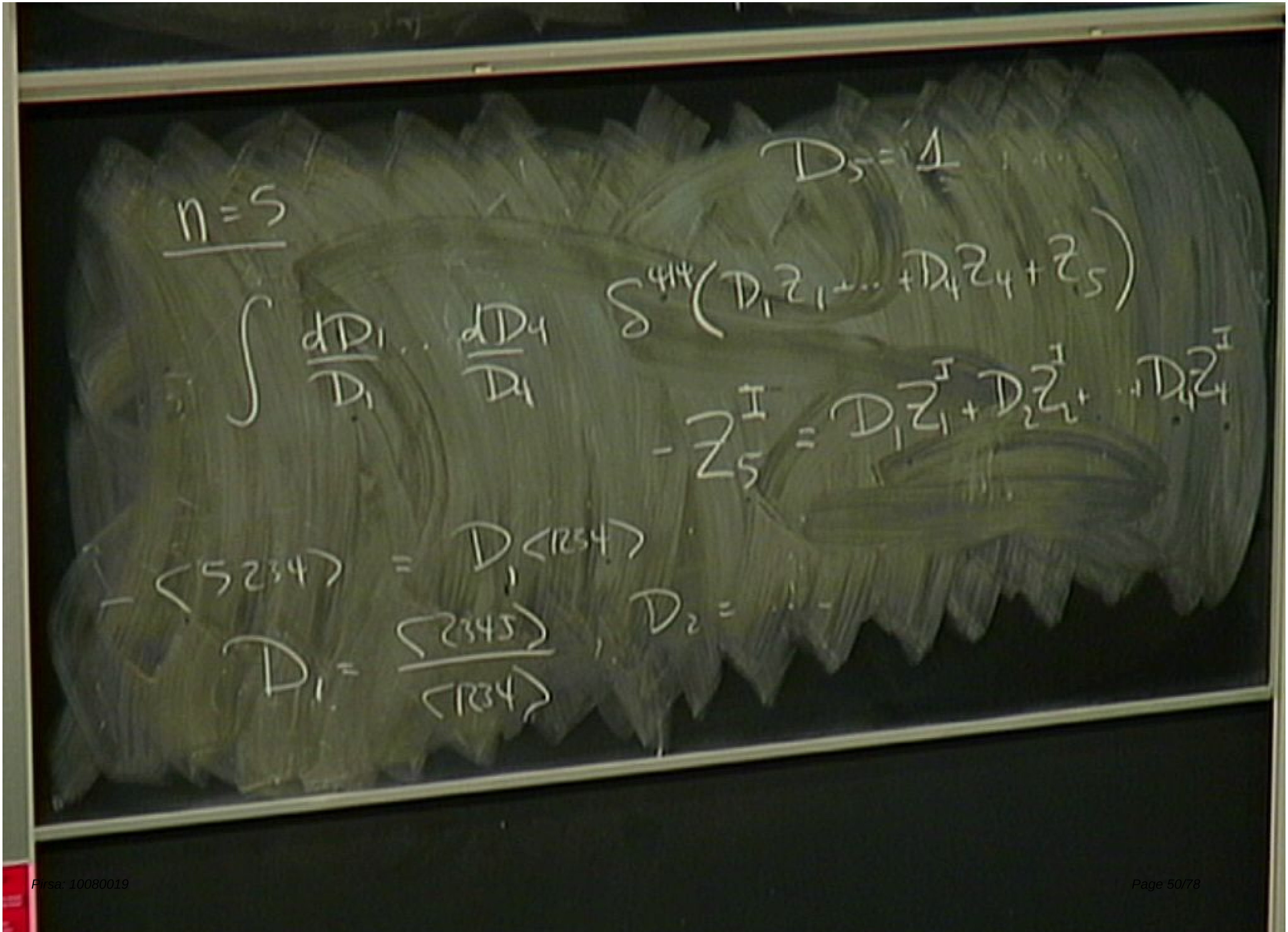

$D_5 = 1$
$n = 5$
$\int \frac{dD_1}{D_1} \cdots \frac{dD_4}{D_4} \, \delta^{4|4}(D_1 Z_1 + \cdots + D_4 Z_4 + Z_5)$

$n = 5$

$D_5 = 1$

$$\int \frac{dD_1}{D_1} \cdots \frac{dD_4}{D_4} \, \delta^{4|4}(D_1 Z_1 + \cdots + D_4 Z_4 + Z_5)$$

$$-Z_5^I = D_1 Z_1^I + D_2 Z_2^I + \cdots + D_4 Z_4^I$$



D5 = 1
n = 5
∫ dD1/D1 … dD4/D4 δ^{4|4}(D1 Z1 + … + D4 Z4 + Z5)
−Z5^I = D1 Z1^I + D2 Z2^I + … + D4 Z4^I
−⟨5234⟩ = D1⟨1234⟩
D1 = ⟨2345⟩/⟨1234⟩ , D2 = …

$D_5 = 1$

$\underline{n = 5}$

$$\int \frac{dD_1}{D_1} \cdots \frac{dD_4}{D_4} \, \delta^{4|4}\left(D_1 Z_1 + \dots + D_4 Z_4 + Z_5\right)$$

$$-Z_5^I = D_1 Z_1^I + D_2 Z_2^I + \dots + D_4 Z_4^I$$

$$-\langle 5234 \rangle = D_1 \langle 1234 \rangle$$

$$D_1 = \frac{\langle 2345 \rangle}{\langle 1234 \rangle}, \quad D_2 = \dots$$

$\underline{n=5}$

$D_5 = 1$

$$\int \frac{dD_1}{D_1} \cdots \frac{dD_4}{D_4} \, \delta^{4|4}\left(D_1 Z_1 + \cdots + D_4 Z_4 + Z_5\right)$$

$$-Z_5^I = D_1 Z_1^I + D_2 Z_2^I + \cdots + D_4 Z_4^I$$

$$-\langle 5234 \rangle = D_1 \langle 1234 \rangle$$

$$D_1 = \frac{\langle 2345 \rangle}{\langle 1234 \rangle}, \quad D_2 = \cdots$$

$$\frac{\langle 1234\rangle^4}{\langle 2345\rangle\langle 1345\rangle\langle 1245\rangle\langle 1235\rangle}\,\frac{1}{\langle 1234\rangle}\,\delta^{0|4}\left[\eta_5+\frac{\langle 2345\rangle}{\langle 1234\rangle}\eta_1+\cdots\right]$$

$$=\frac{\delta^{0|4}\left[\langle 1234\rangle\eta_5+\text{cyclic}\right]}{\langle 1234\rangle\langle 2345\rangle\langle 3451\rangle\langle 4512\rangle\langle 5123\rangle}$$

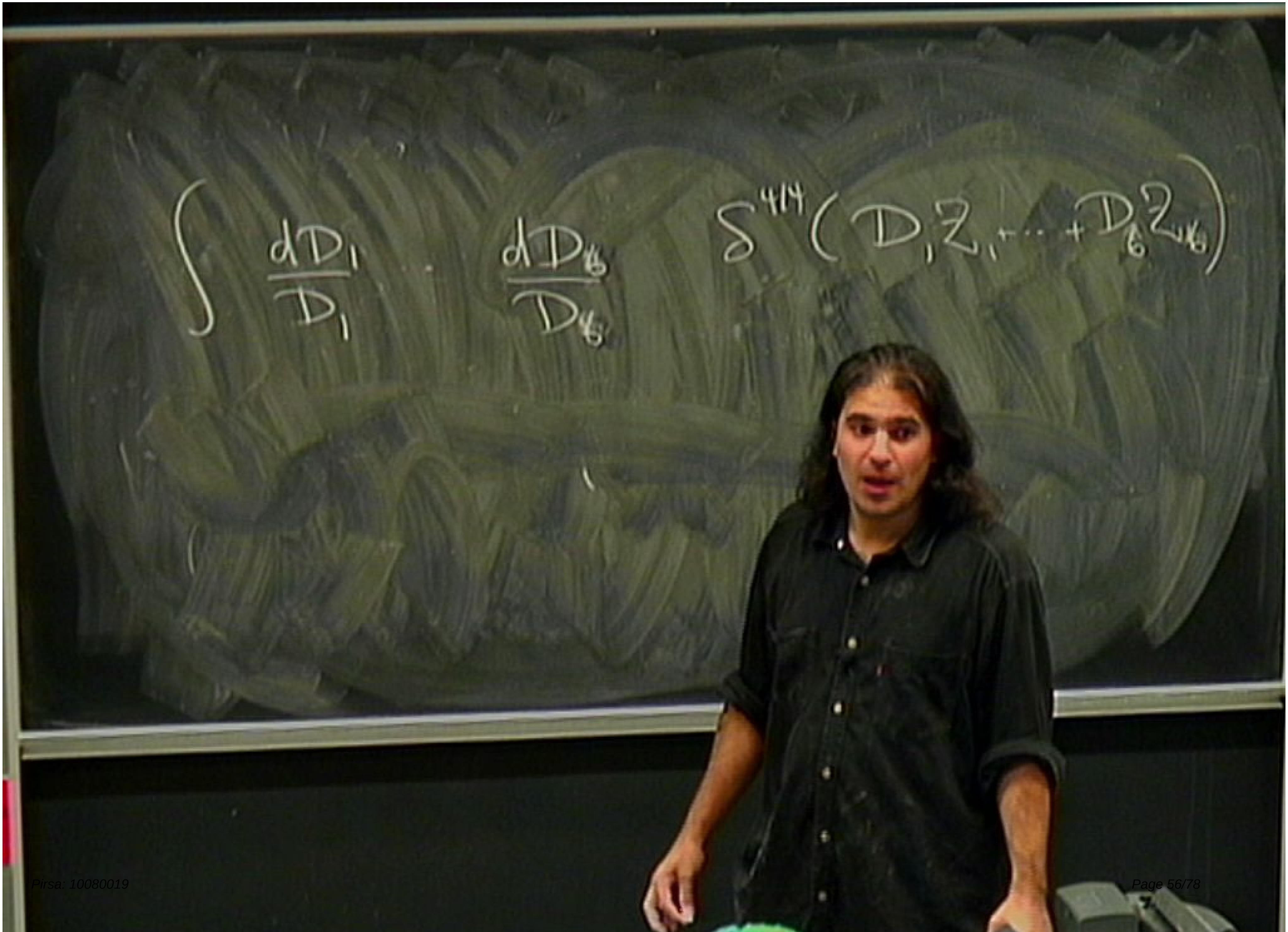
$$\frac{\langle 1234\rangle^4}{\langle 2345\rangle\langle 1345\rangle\langle 1245\rangle\langle 1235\rangle}\,\frac{1}{\langle 1234\rangle}\,\delta^{0|4}\left[\eta_5 + \frac{\langle 2345\rangle}{\langle 1234\rangle}\eta_1 + \cdots\right]$$

$$= \frac{\delta^{0|4}\left[\langle 1234\rangle\eta_5 + \text{cyclic}\right]}{\langle 1234\rangle\langle 2345\rangle\langle 3451\rangle\langle 4512\rangle\langle 5123\rangle}$$

$$\int \frac{dD_1}{D_1} \cdots \frac{dD_n}{D_n} \; \delta^{4|4}\left(D_1 Z_1 + \cdots + D_n Z_n\right)$$

$k(n-k-4)$ $\qquad$ $(n-5)$

$(m-2)(n-m-2)$


$\int \frac{dD_1}{D_1} \cdots \frac{dD_6}{D_6} \; \delta^{4|4}(D_1 Z_1 + \cdots + D_6 Z_6)$

$$\int \frac{dD_1}{D_1} \oint \frac{dD_3}{D_3} \frac{dD_6}{D_6} \; \delta^{4|4}\left(D_1 Z_1 + \cdots + D_6 Z_6\right)$$

$\downarrow \Delta$

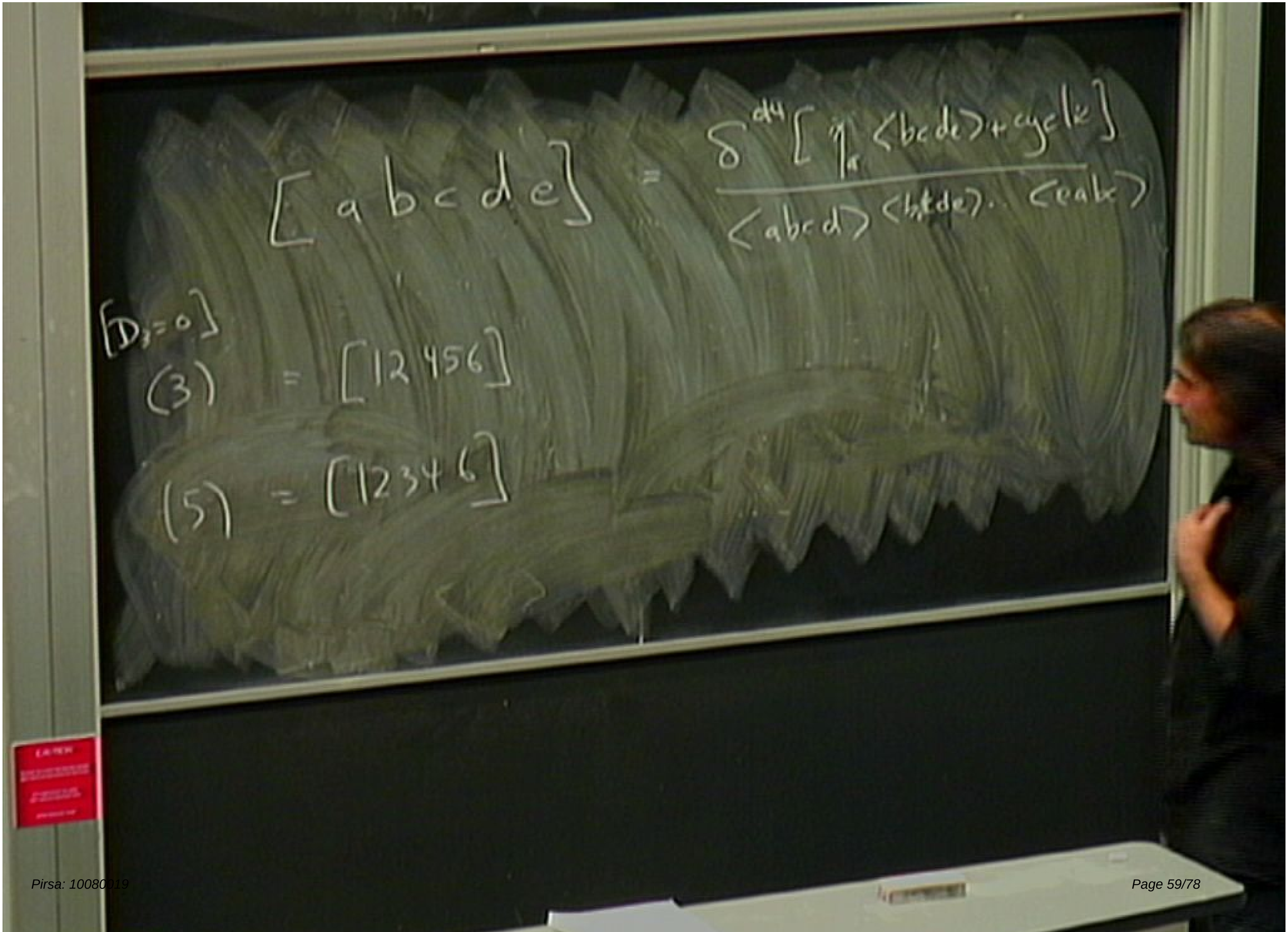
$$D_3 = 0$$

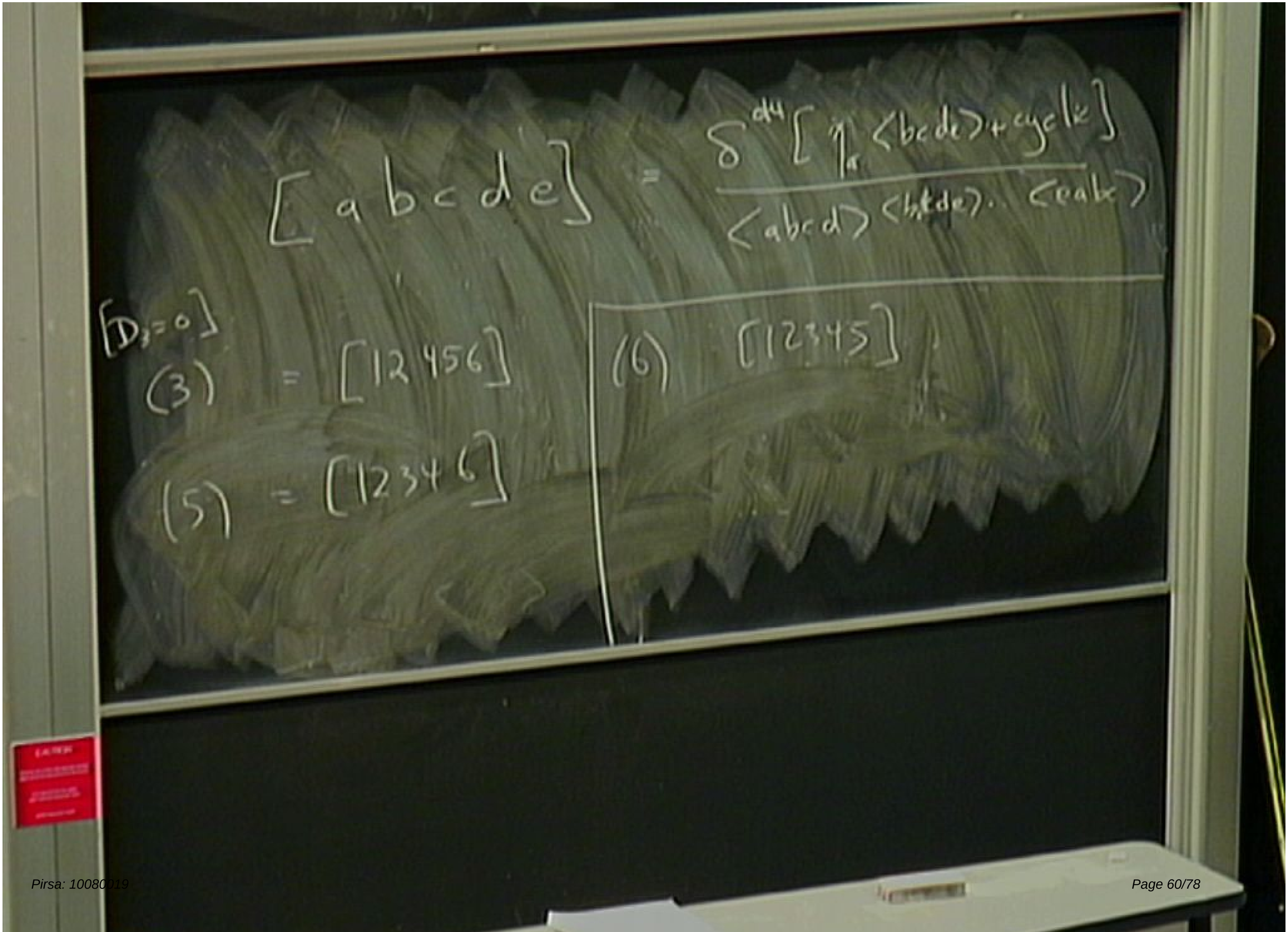
$$\int \frac{dD_1}{D_1} \frac{dD_2}{D_2} \frac{dD_4}{D_4} \frac{dD_5}{D_5} \frac{dD_6}{D_6} \; \delta^{4|4}\left(D_1 Z_1 + \quad D_6 Z_6\right)$$

$$\int \frac{dD_1}{D_1} \underbrace{\oint \frac{dD_3}{D_3}}_{\Delta} \frac{dD_6}{D_6}\, \delta^{4|4}\left(D_1 Z_1 + \cdots + D_6 Z_6\right)$$

$$D_3 = 0$$

$$\int \frac{dD_1}{D_1} \frac{dD_2}{D_2} \frac{dD_4}{D_4} \frac{dD_5}{D_5} \frac{dD_6}{D_6}\, \delta^{4|4}\left(D_1 Z_1 + \quad D_6 Z_6\right)$$


[abcde] =
⟨abcd⟩
[D3=0]
(3) = [12456]
(5) = [12346]


[a b c d e] = δ^4|4 [η <bcde> + cycle] / <abcd> <bcde> .. <eabc>
[D3 = 0]
(3) = [12456]
(5) = [12346]
(6) [12345]

$$[a\,b\,c\,d\,e] = \frac{\delta^{0|4}\left[\eta_a \langle bcde\rangle + \text{cyclic}\right]}{\langle abcd\rangle\langle bcde\rangle \cdots \langle eabc\rangle}$$

$[D_3 = 0]$

$(3) = [12456]$

$(5) = [12346]$

$(6) \leftrightarrow [12345]$

BCFW: $(6) + (4) + (2)$

$P(\text{BCFW})$: $(5) + (3) + (1)$

$$[a\,b\,c\,d\,e] = \frac{\delta^{0|4}\left[\eta_a \langle bcde\rangle + \text{cyclic}\right]}{\langle abcd\rangle \langle bcde\rangle \cdots \langle eabc\rangle}$$

$[D_3 = 0]$

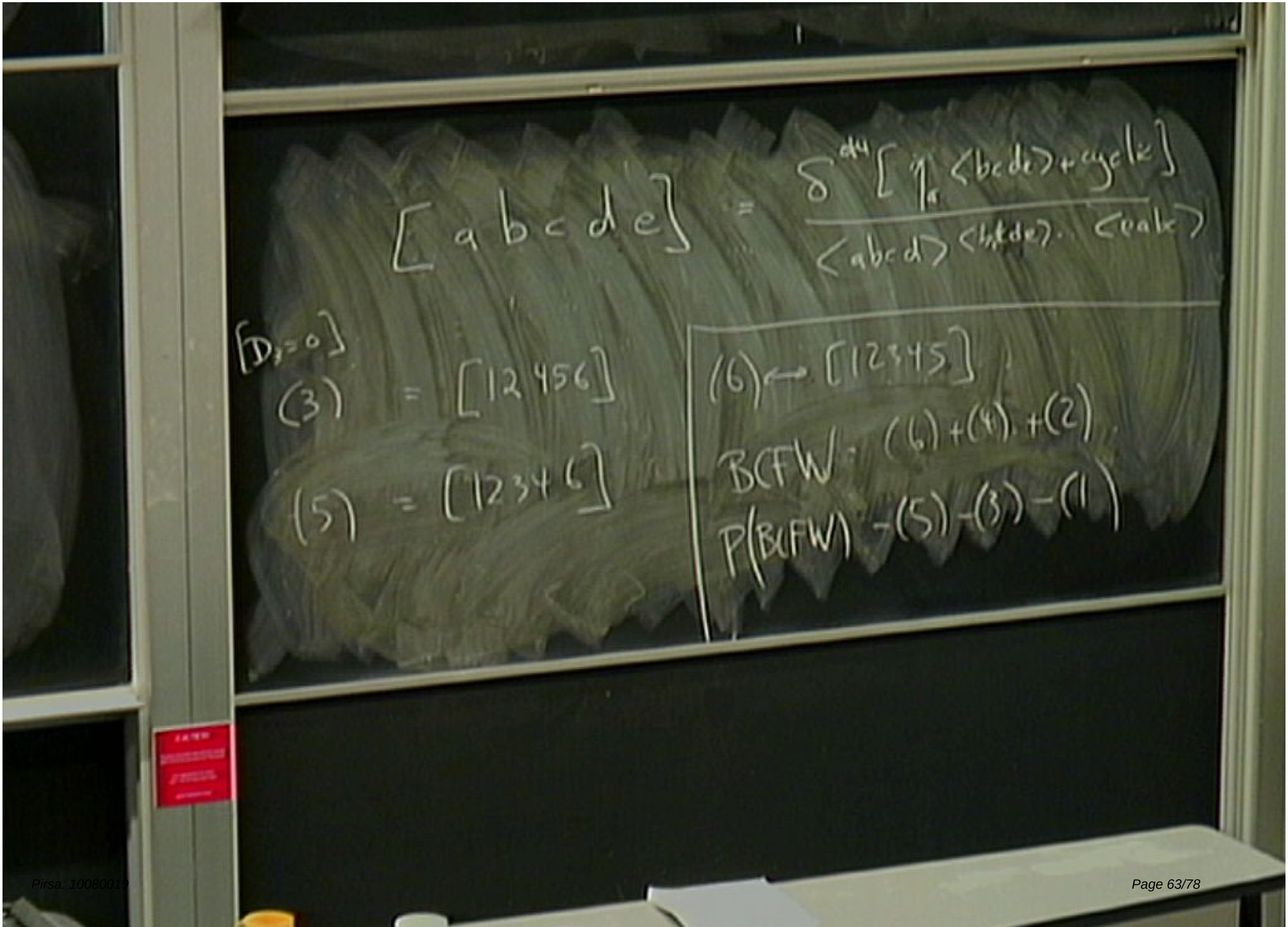
$(3) = [12456]$

$(5) = [12346]$

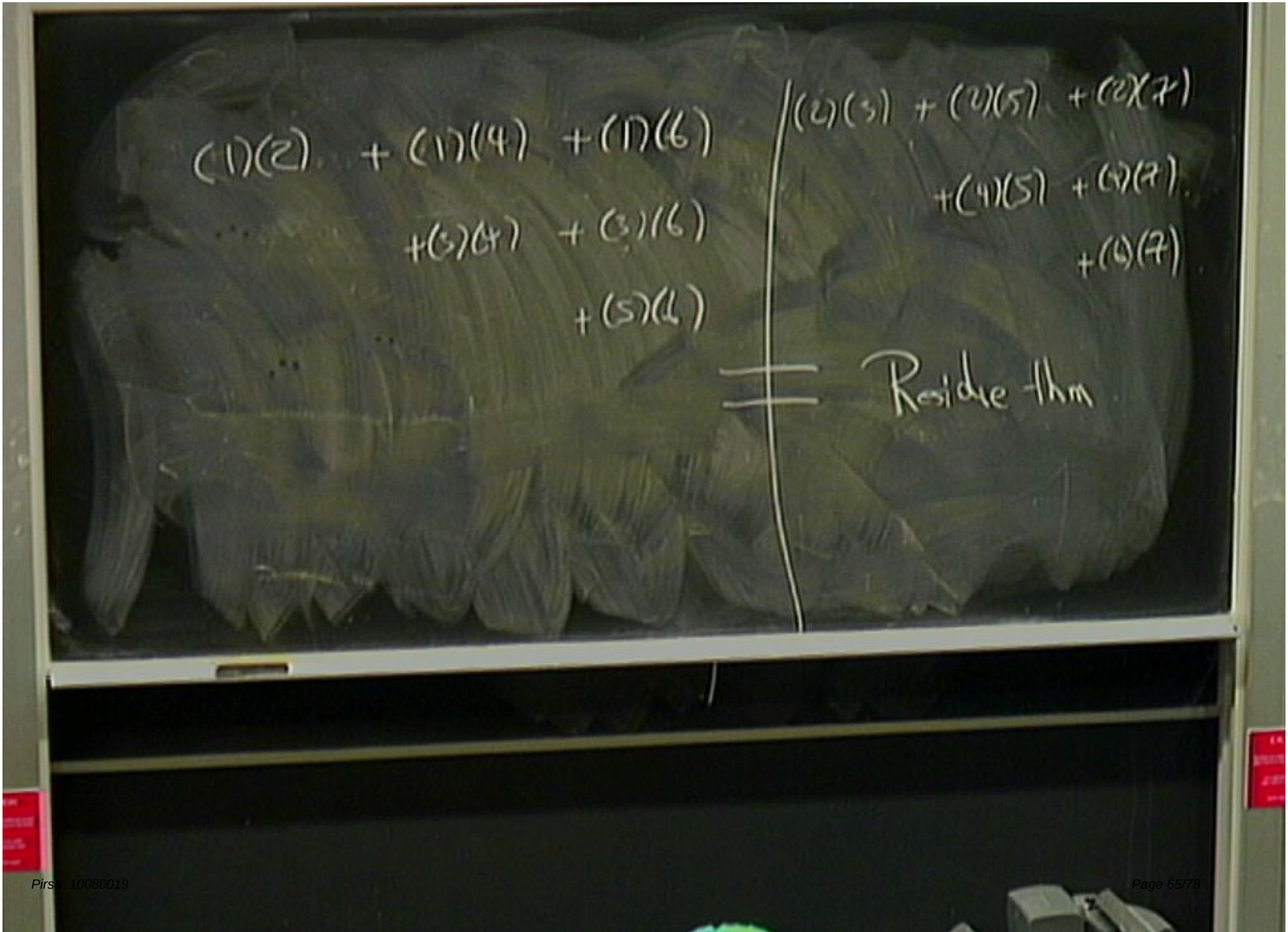
$(6) \leftrightarrow [12345]$

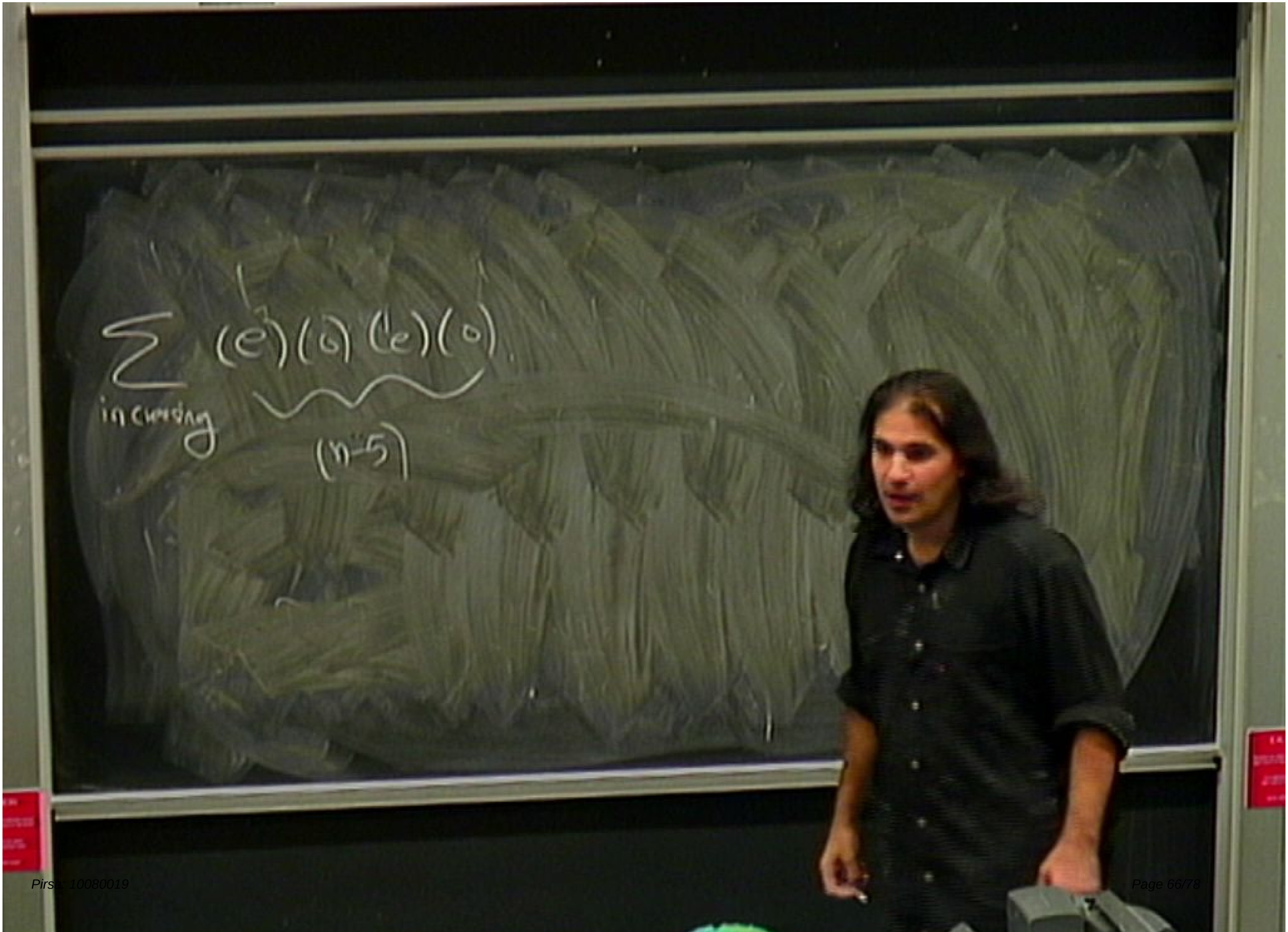
BCFW: $(6) + (4) + (2)$

$P(\text{BCFW}) - (5) - (3) - (1)$

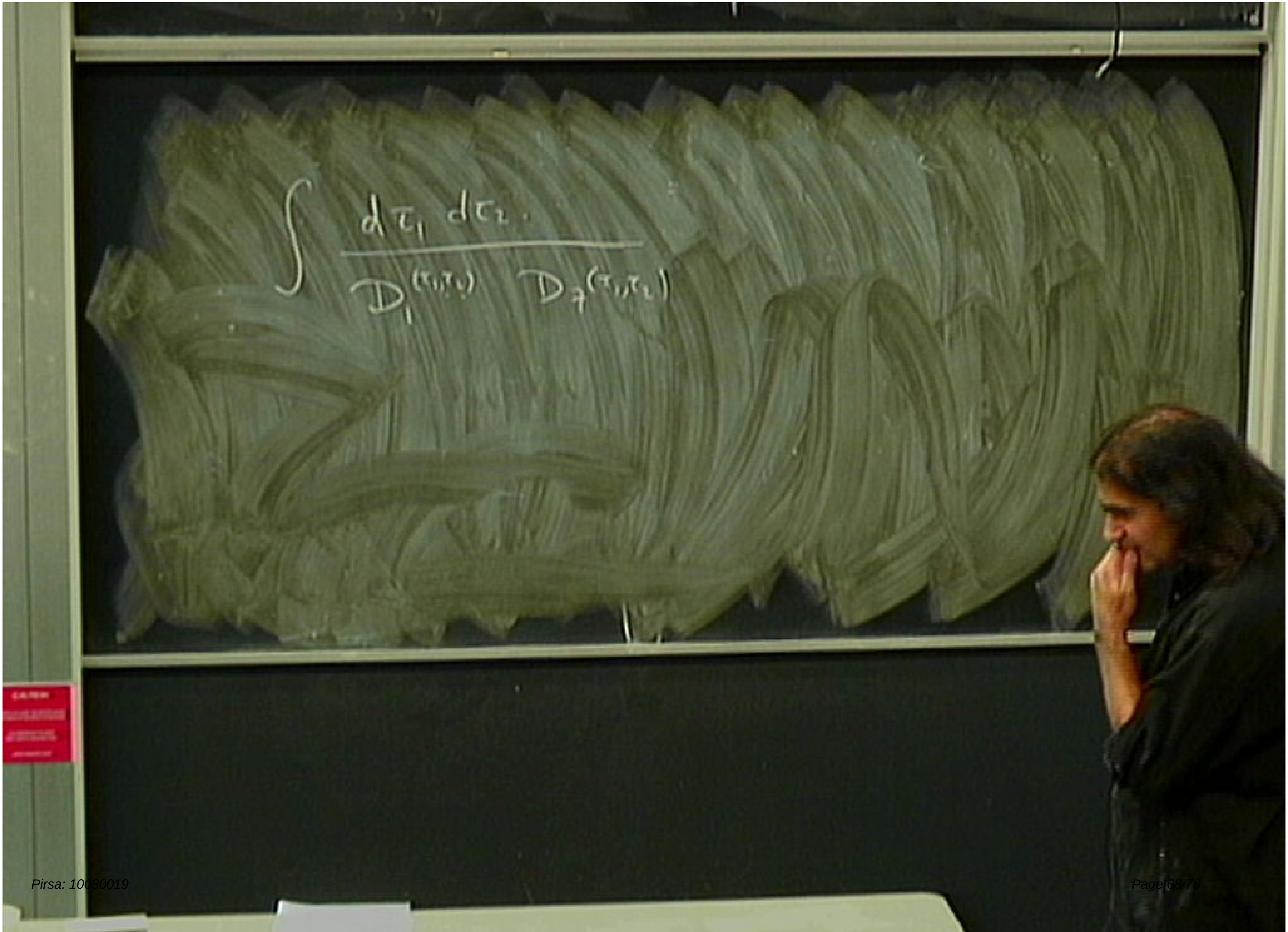

[a b c d e] =
[D3 = 0]
(3) = [12456]
(5) = [12346]
(6) ⟷ [12345]
BCFW : (6) + (4) + (2)
P(BCFW) : (5) - (3) - (1)

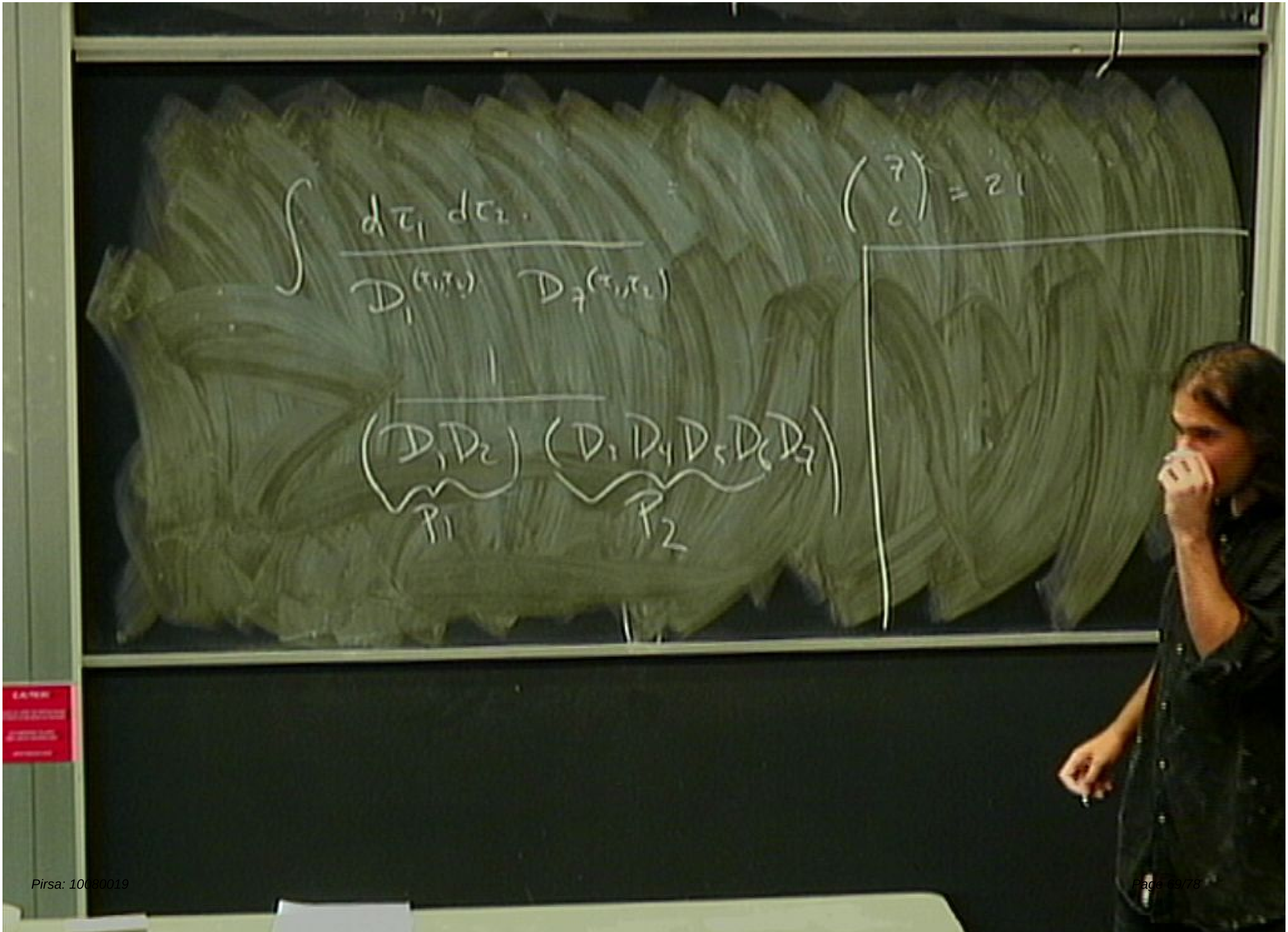

(1)(2) + (

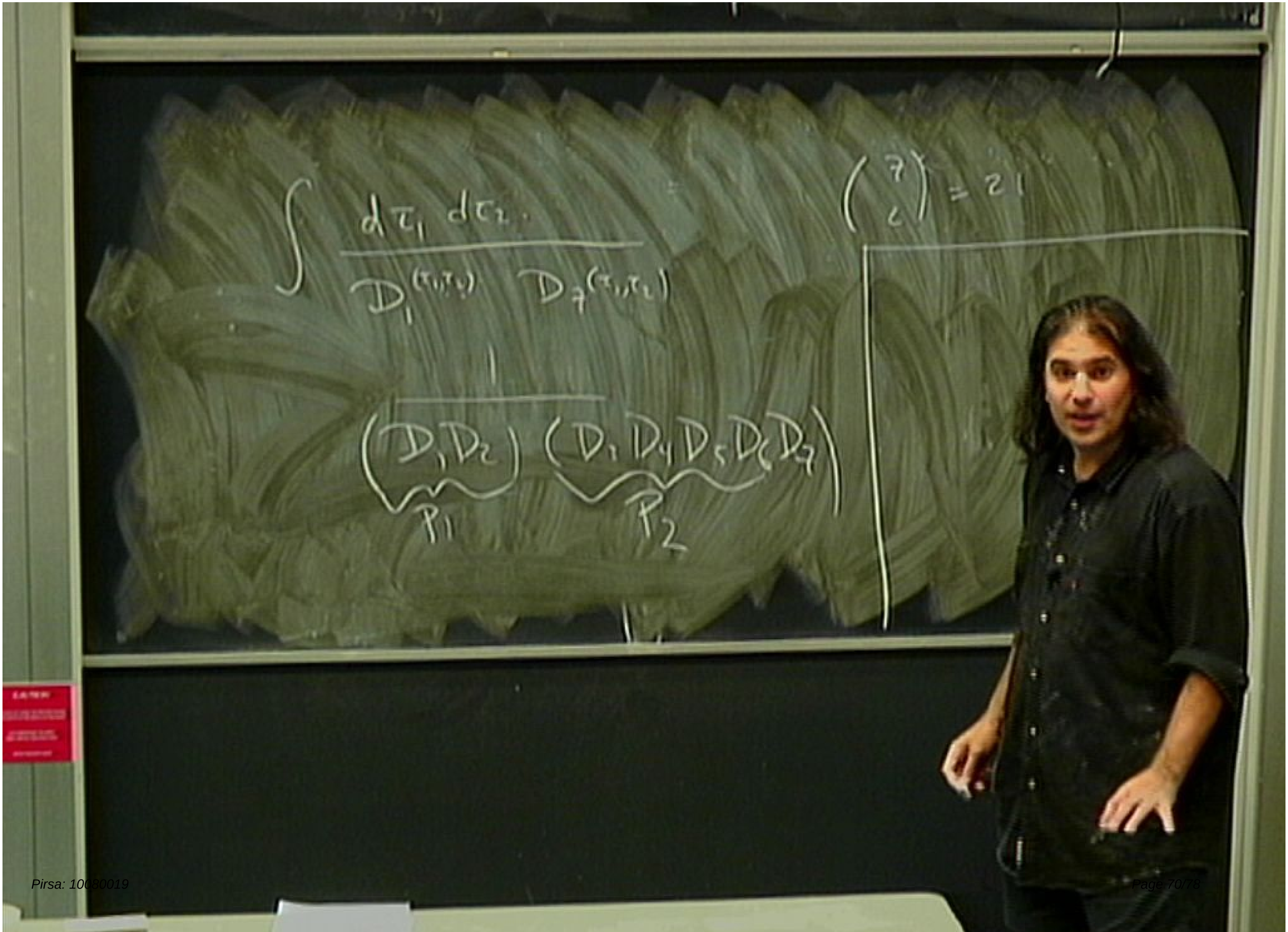

(1)(2) + (1)(4) + (1)(6)
+(3)(4) + (3)(6)
+(5)(6)
(2)(3) + (2)(5) + (2)(7)
+(4)(5) + (4)(7)
+(6)(7)
= Residue thm

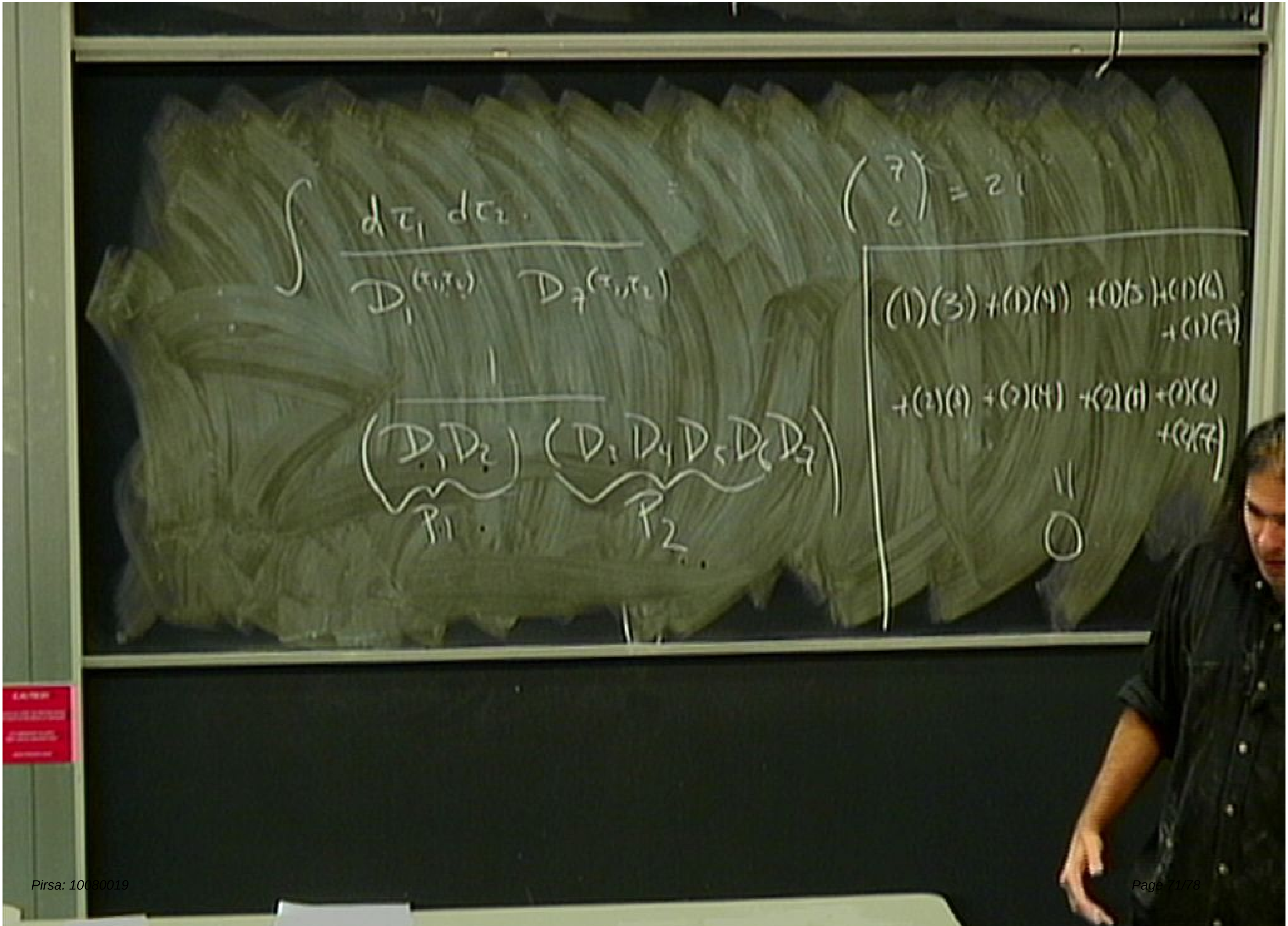

incoming
(n-5)

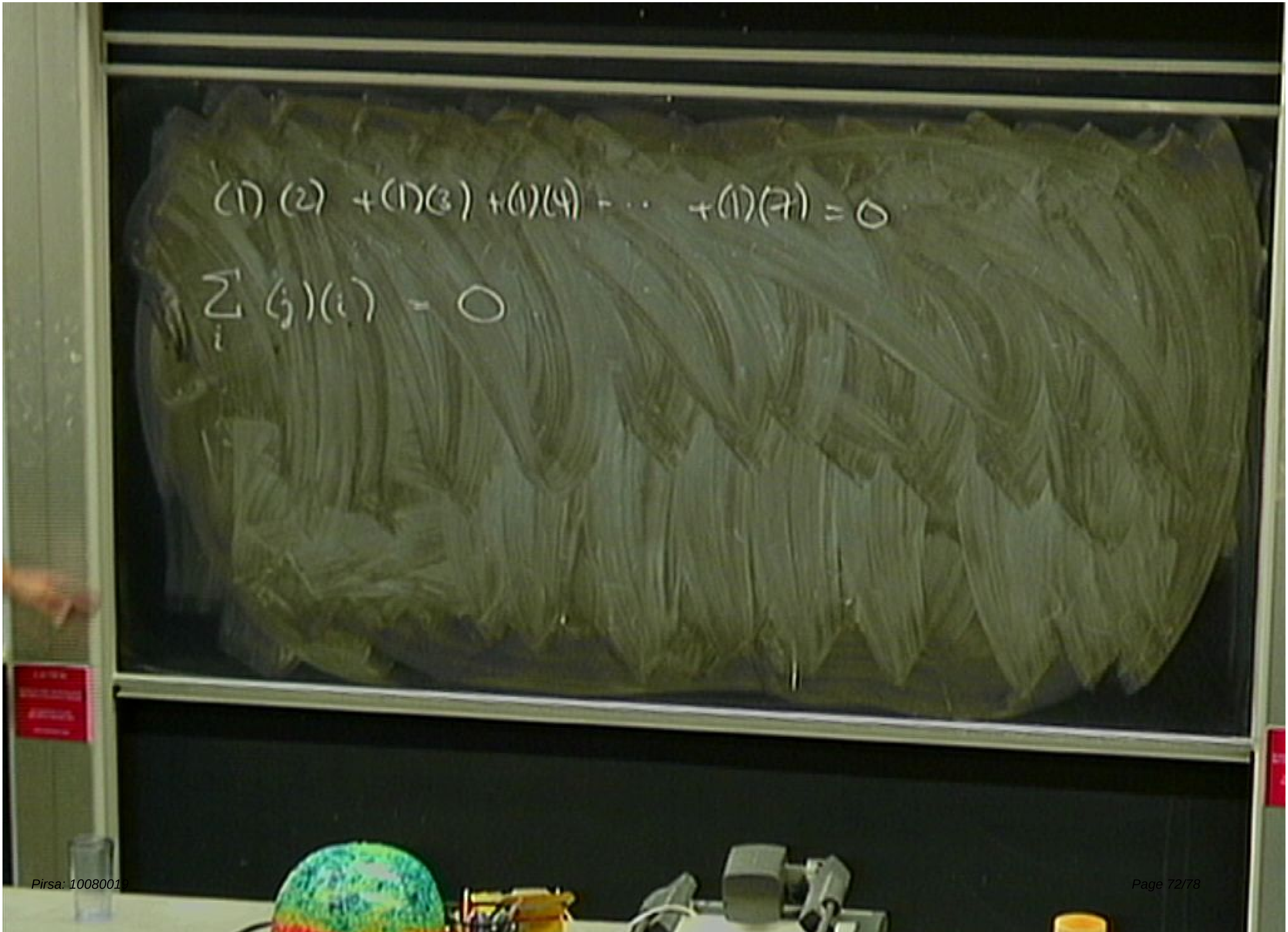
$$\sum_{\text{incoming}} \underbrace{(e)(o)(e)(o)}_{(n-5)} = (-1)^{n-5} \sum \underbrace{(o)(e)(o)\ldots}_{n-5}$$

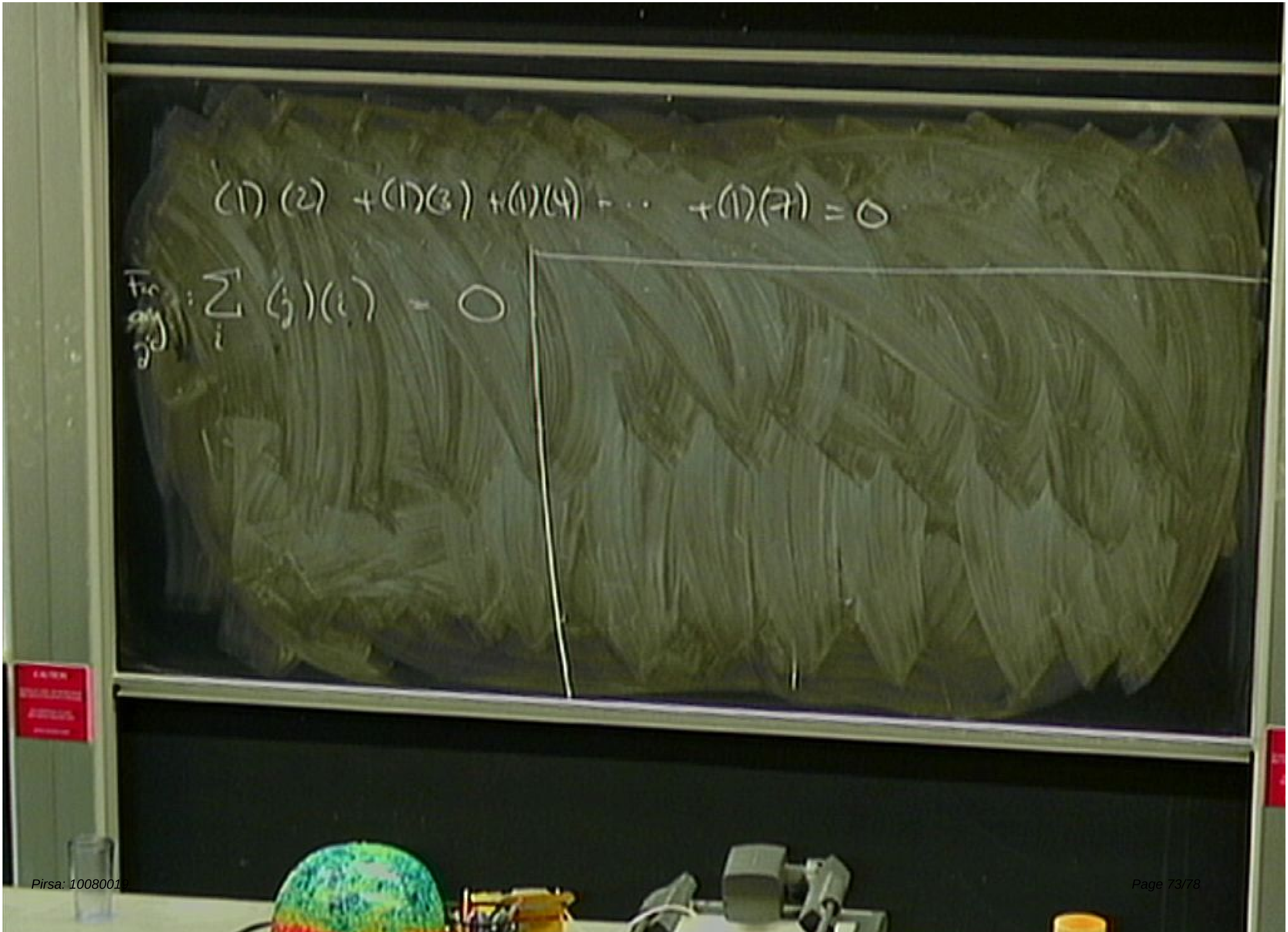

∫ dτ1 dτ2 /
D1(τ1,τ2) D7(τ1,τ2)

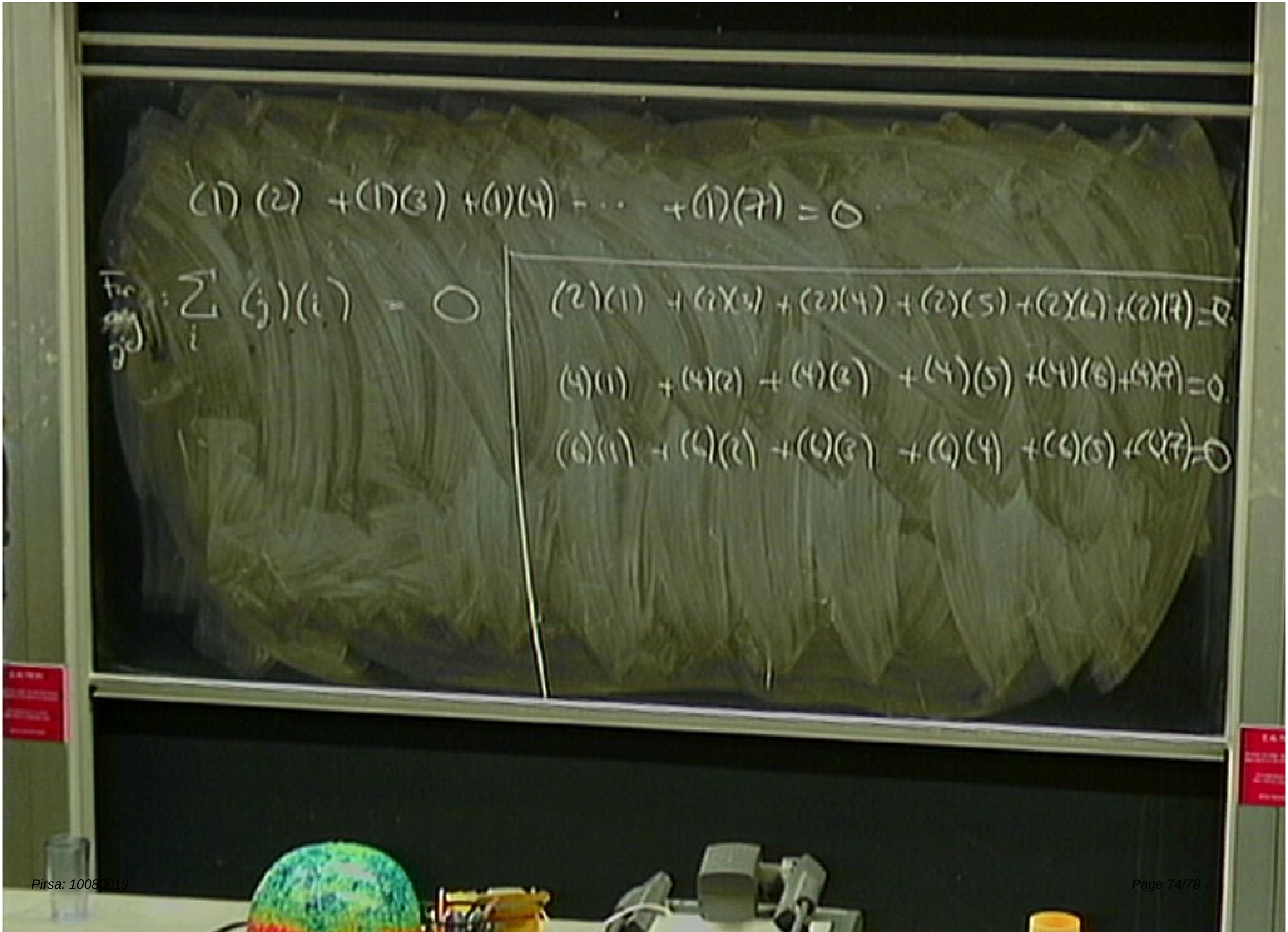

$\int \frac{d\tau_1\, d\tau_2}{D_1(\tau_1,\tau_2) \cdots D_7(\tau_1,\tau_2)}$
$\frac{1}{(D_1 D_2)(D_3 D_4 D_5 D_6 D_7)}$
P_1
P_2
$\binom{7}{2} = 21$

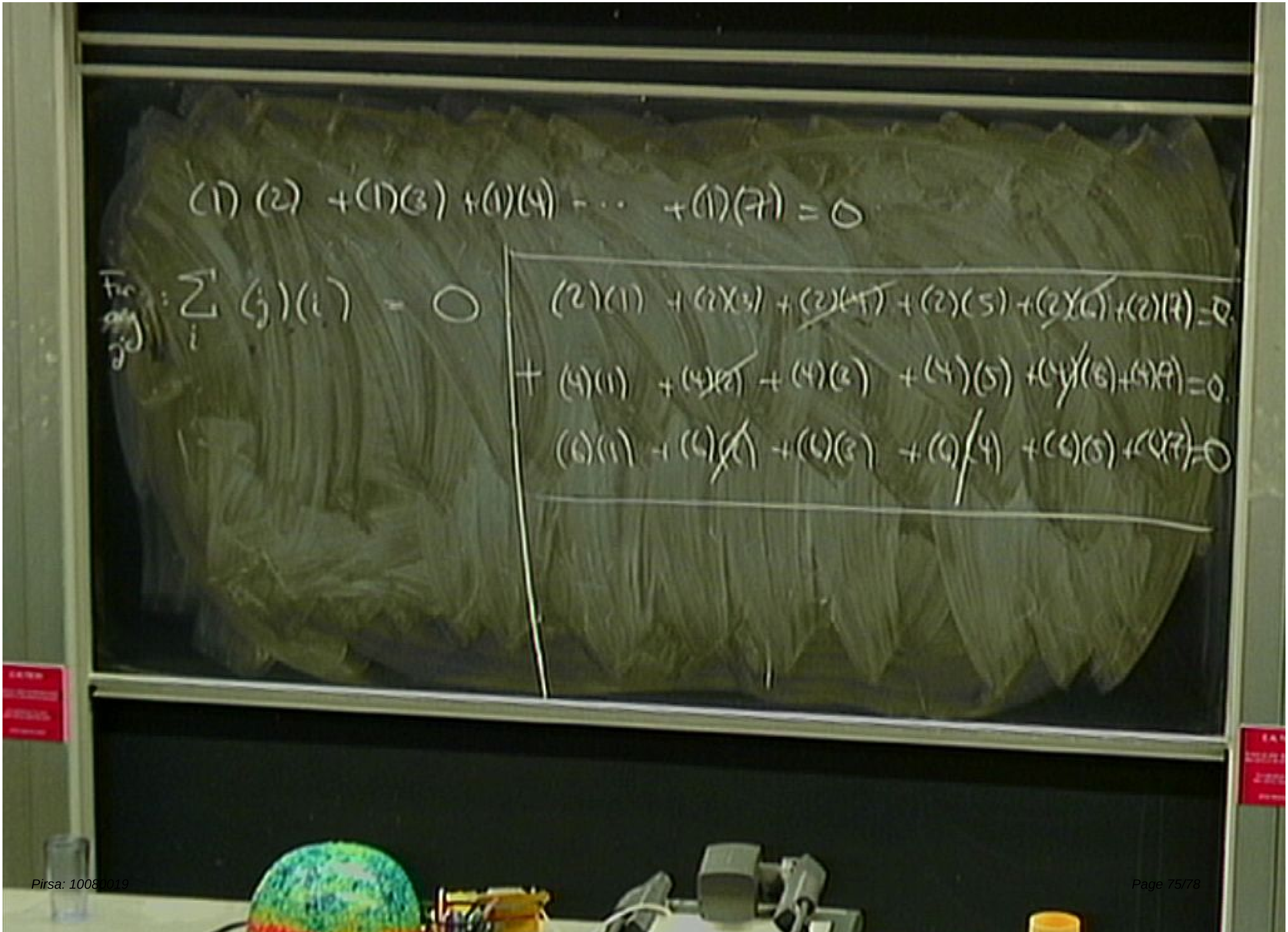

∫ dτ₁ dτ₂ / D₁(τ₁,τ₂) D₇(τ₁,τ₂)
(7 2) = 21
1 / (D₁D₂)(D₃D₄D₅D₆D₇)
P₁
P₂

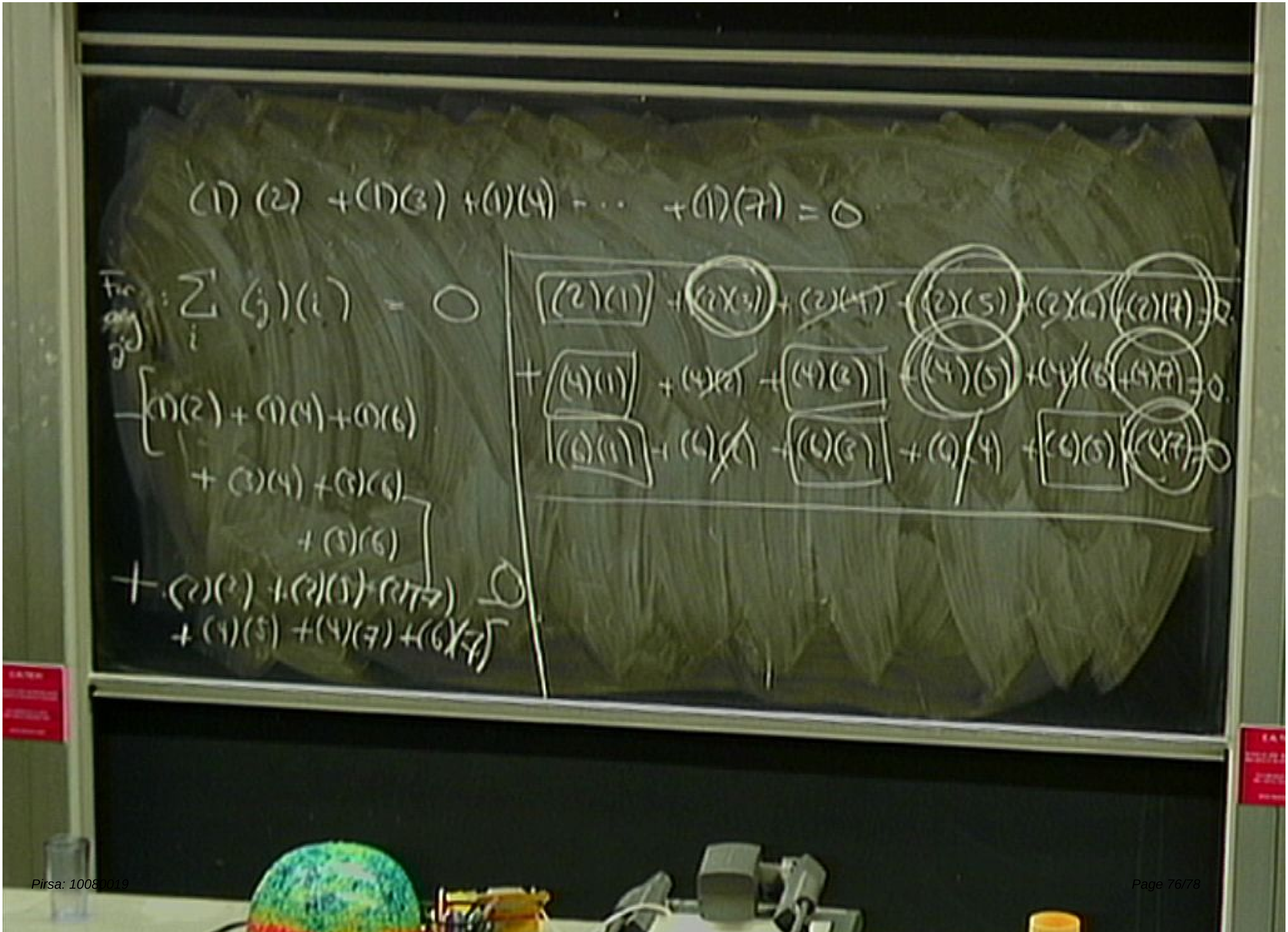

$\int \frac{d\tau_1\, d\tau_2}{D_1(\tau_1,\tau_2)\; D_7(\tau_1,\tau_2)}$
$\frac{1}{(D_1 D_2)(D_3 D_4 D_5 D_6 D_7)}$
P_1
P_2
$\binom{7}{2} = 21$
(1)(3) + (1)(4) + (1)(5) + (1)(6) + (1)(7)
+ (2)(3) + (2)(4) + (2)(5) + (2)(6) + (2)(7)
= 0

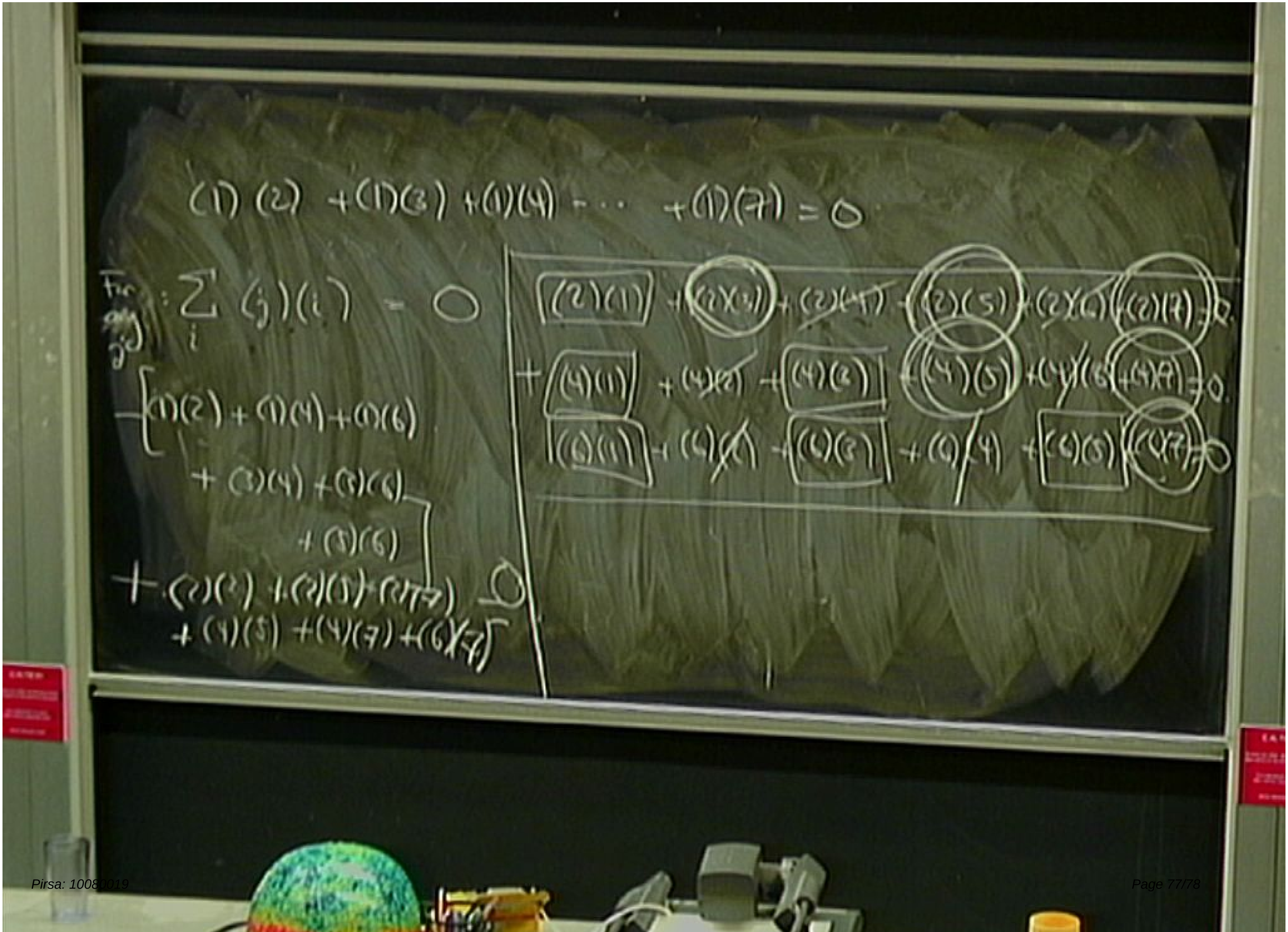

(1)(2) + (1)(3) + (1)(4) + ··· + (1)(7) = 0
Σ_i (j)(i) = 0

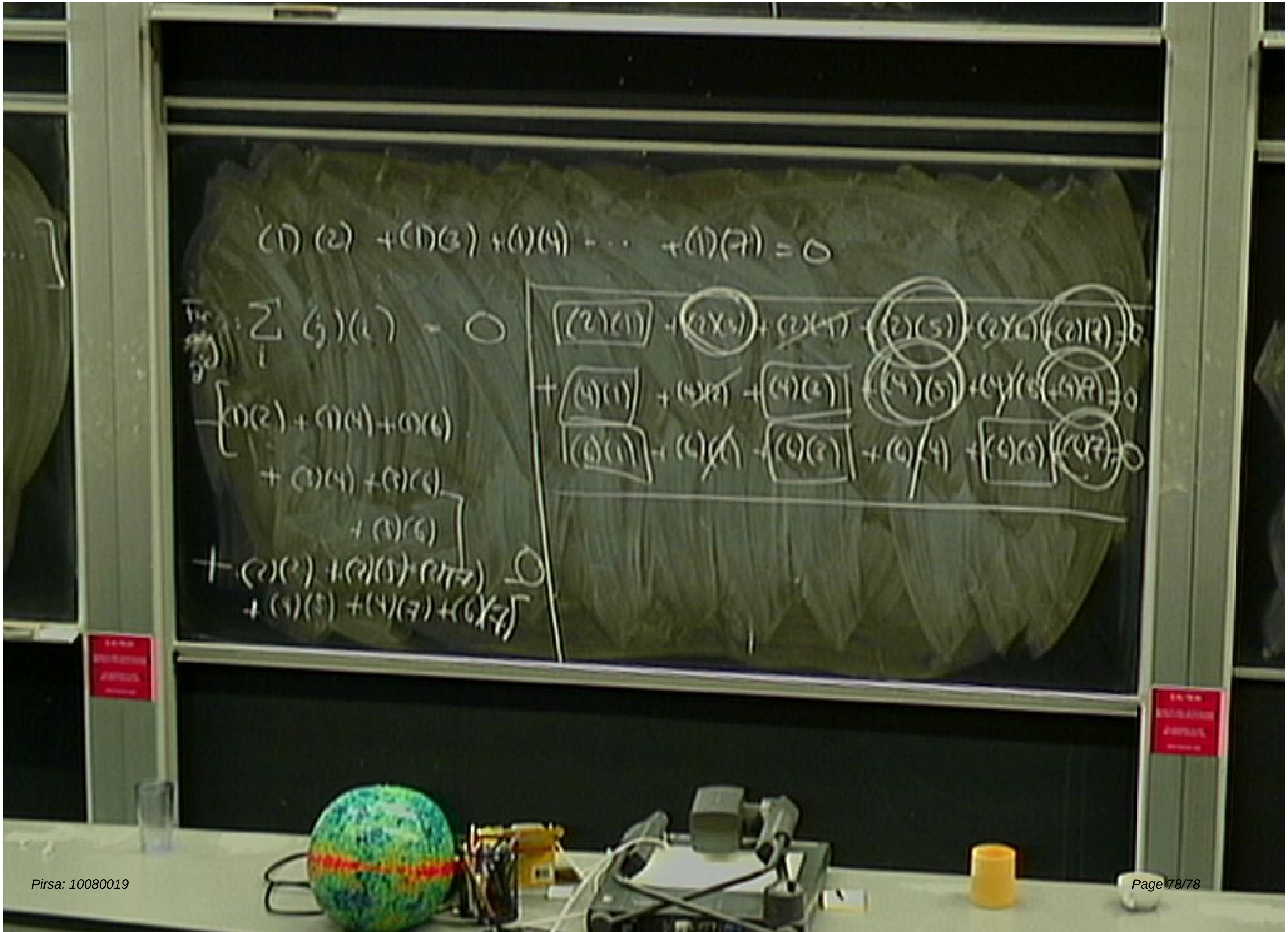

(1)(2) + (1)(3) + (1)(4) + ··· + (1)(7) = 0
Σ (j)(i) = 0


(1)(2) + (1)(3) + (1)(4) + · · · + (1)(7) = 0
For any j: $\sum_i$ (j)(i) = 0
(2)(1) + (2)(3) + (2)(4) + (2)(5) + (2)(6) + (2)(7) = 0
(4)(1) + (4)(2) + (4)(3) + (4)(5) + (4)(6) + (4)(7) = 0
(6)(1) + (6)(2) + (6)(3) + (6)(4) + (6)(5) + (6)(7) = 0


(1)(2) + (1)(3) + (1)(4) + ··· + (1)(7) = 0
For any j: Σ_i (j)(i) = 0
(2)(1) + (2)(3) + (2)(4) + (2)(5) + (2)(6) + (2)(7) = 0
+ (4)(1) + (4)(2) + (4)(3) + (4)(5) + (4)(6) + (4)(7) = 0
(6)(1) + (6)(2) + (6)(3) + (6)(4) + (6)(5) + (6)(7) = 0


(1)(2) + (1)(3) + (1)(4) + · · · + (1)(7) = 0
For any j: Σ (j)(i) = 0
i
(1)(2) + (1)(4) + (1)(6)
+ (3)(4) + (3)(6)
+ (5)(6)
+ (2)(3) + (2)(5) + (2)(7)
+ (4)(5) + (4)(7) + (6)(7)
(2)(1) + (2)(3) + (2)(4) + (2)(5) + (2)(6) + (2)(7) = 0.
+ (4)(1) + (4)(2) + (4)(3) + (4)(5) + (4)(6) + (4)(7) = 0.
(6)(1) + (6)(2) + (6)(3) + (6)(4) + (6)(5) + (6)(7) = 0


(1)(2) + (1)(3) + (1)(4) + ··· + (1)(7) = 0
For any j: Σ (j)(i) = 0
i
(1)(2) + (1)(4) + (1)(6)
+ (3)(4) + (3)(6)
+ (5)(6)
+ (2)(3) + (2)(5) + (2)(7) = 0
+ (4)(5) + (4)(7) + (6)(7)
(2)(1) + (2)(3) + (2)(4) + (2)(5) + (2)(6) + (2)(7) = 0
+ (4)(1) + (4)(2) + (4)(3) + (4)(5) + (4)(6) + (4)(7) = 0
(6)(1) + (6)(2) + (6)(3) + (6)(4) + (6)(5) + (6)(7) = 0


(1)(2) +(1)(3) +(1)(4) + · · · +(1)(7) = 0