Title: Space-time, Quantum Mechanics and Scattering Amplitudes - 5A

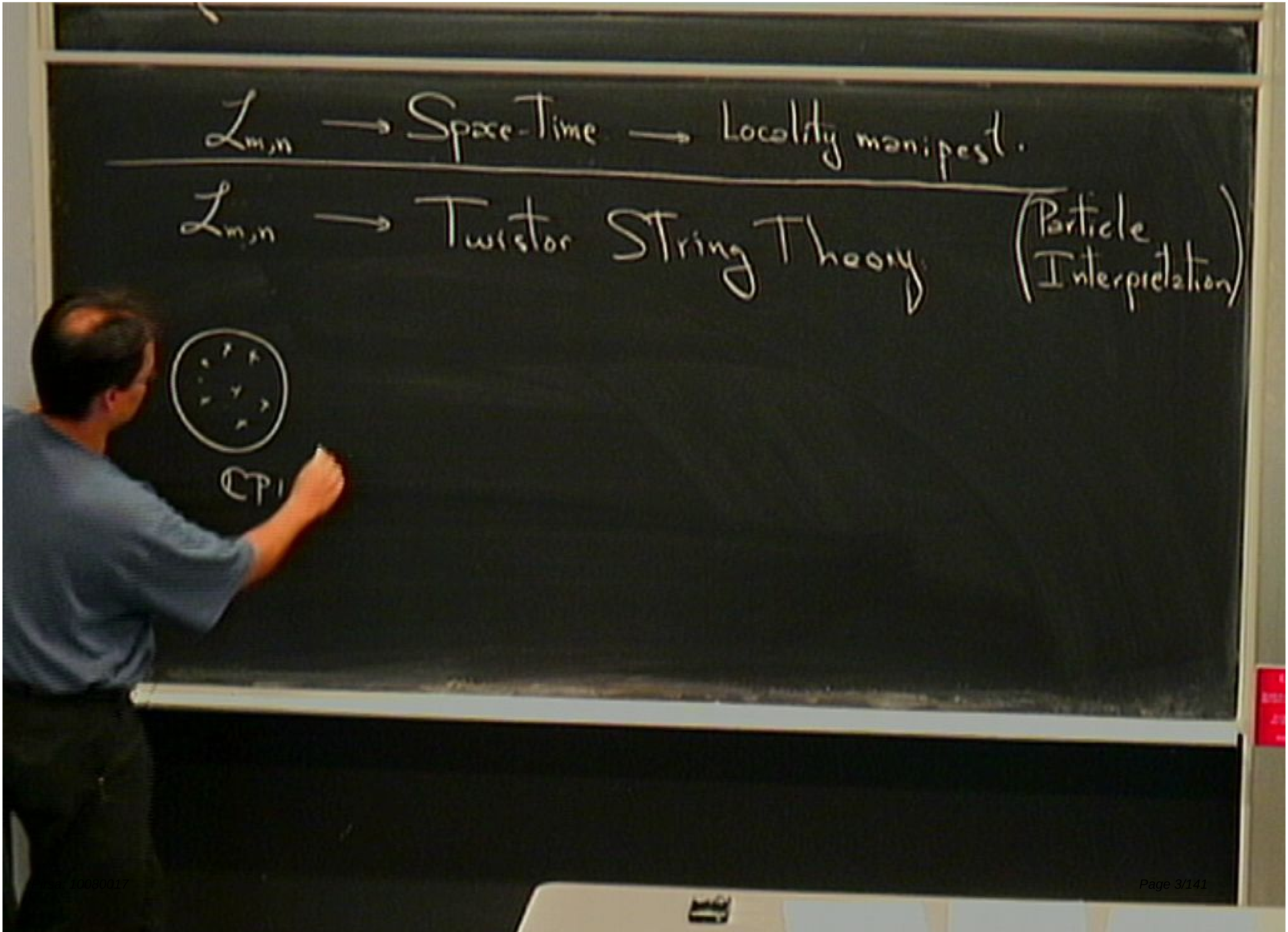
Date: Aug 27, 2010 10:00 AM

URL: http://pirsa.org/10080017

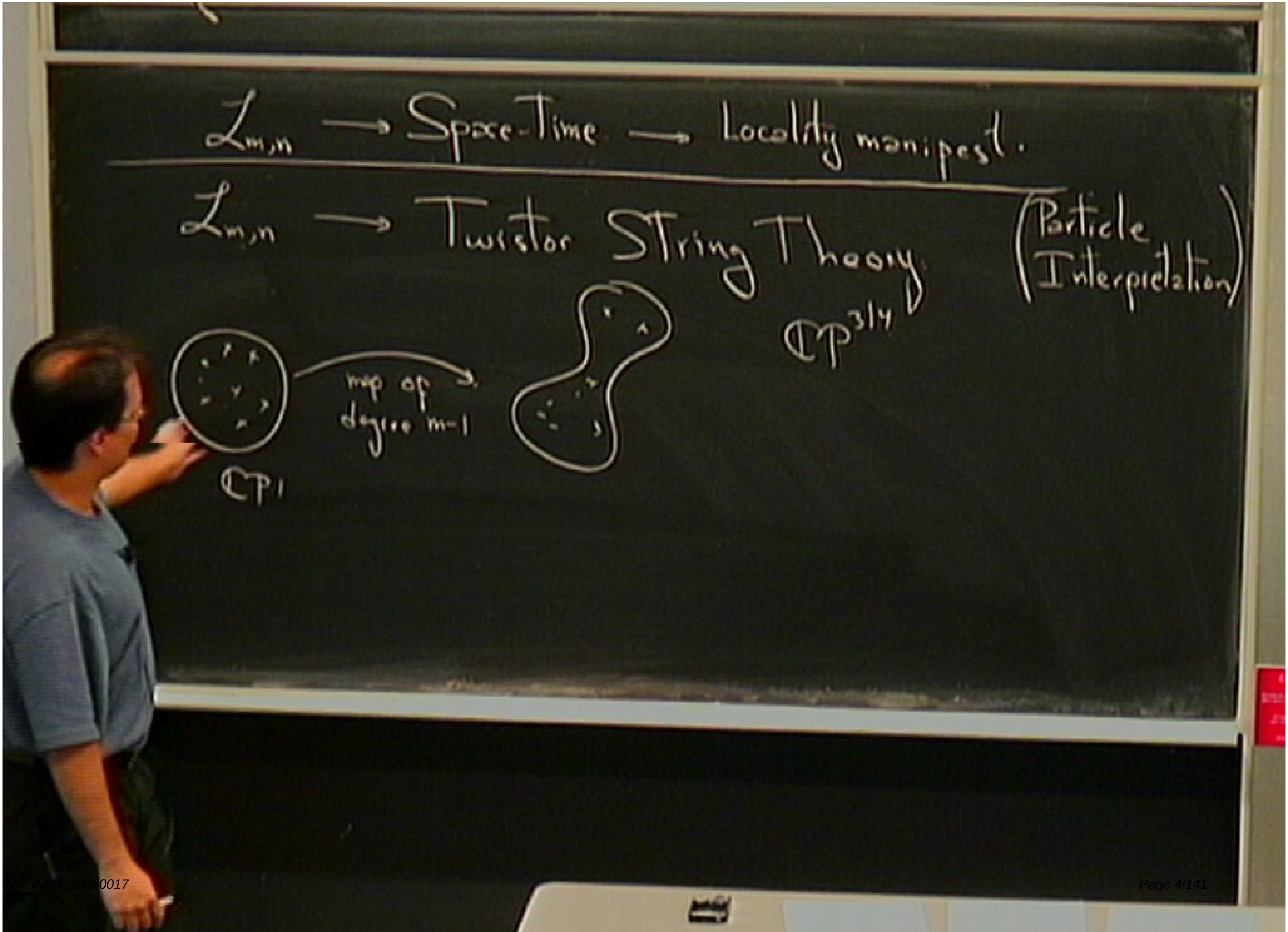
Abstract: This mini-course will review recent developments in our understanding of scattering amplitudes in gauge theories, particularly N=4 SYM in four dimensions. We will discuss a dual theory for these scattering amplitudes associated with contour integrals over Grassmannians, where the remarkable symmetries of the theory are manifest, but space-time and unitarity emerge as secondary concepts.

$\mathcal{L}_{m,n} \longrightarrow$ Space-Time $\longrightarrow$ Locality manifest.

$\mathcal{L}_{m,n} \longrightarrow$ Twistor String Theory



$\mathcal{L}_{m,n} \longrightarrow$ Space-Time $\longrightarrow$ Locality manifest.
$\mathcal{L}_{m,n} \longrightarrow$ Twistor String Theory (Particle Interpretation)
$\mathbb{CP}^1$


$\mathcal{L}_{m,n} \longrightarrow$ Space-Time $\longrightarrow$ Locality manifest.
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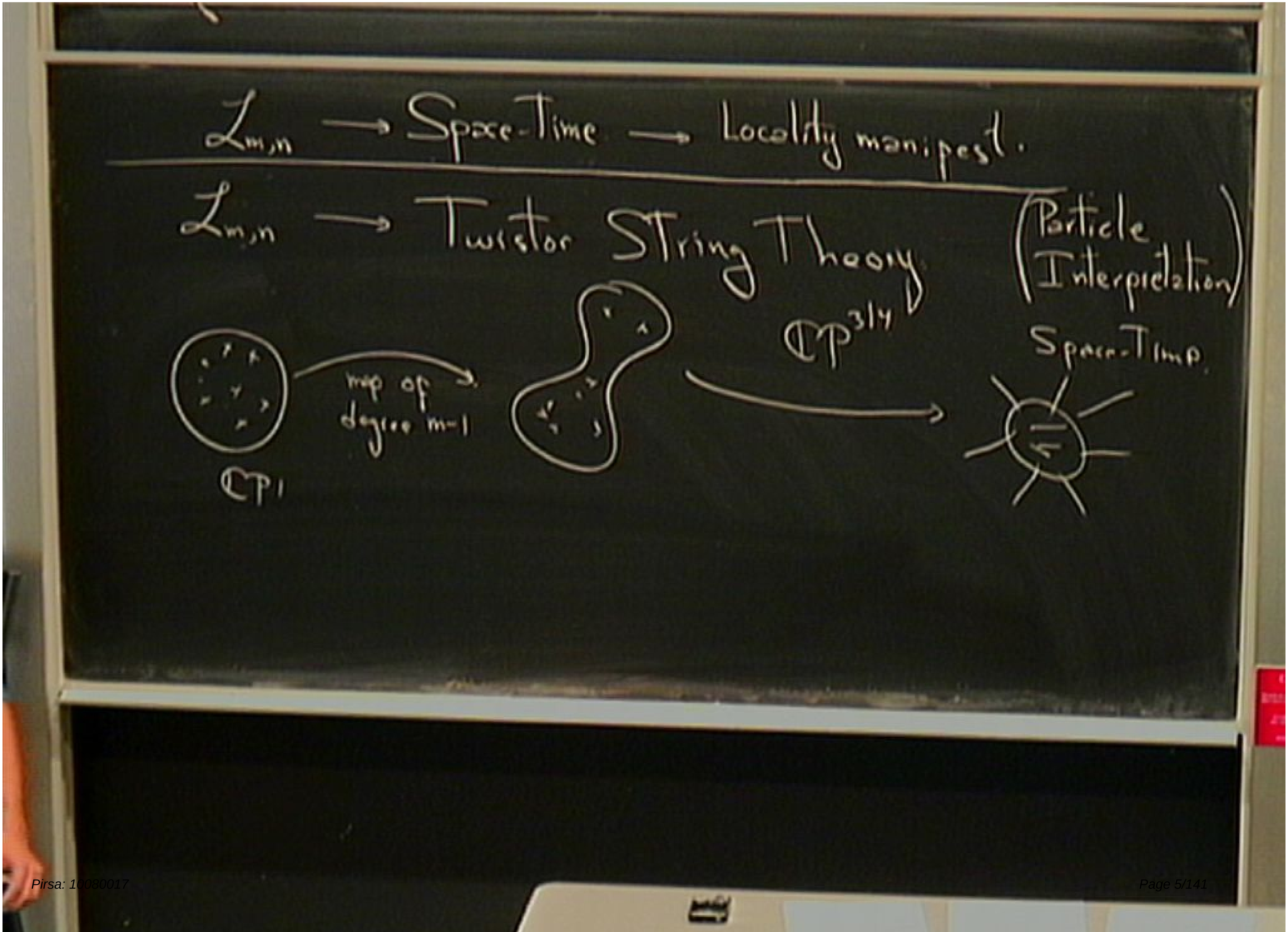
(Particle Interpretation)
$\mathbb{CP}^{3|4}$
map of degree m-1
$\mathbb{CP}^1$


$\mathcal{L}_{m,n}$ → Space-Time → Locality manifest.
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$\mathbb{CP}^{3|4}$
Space-Time
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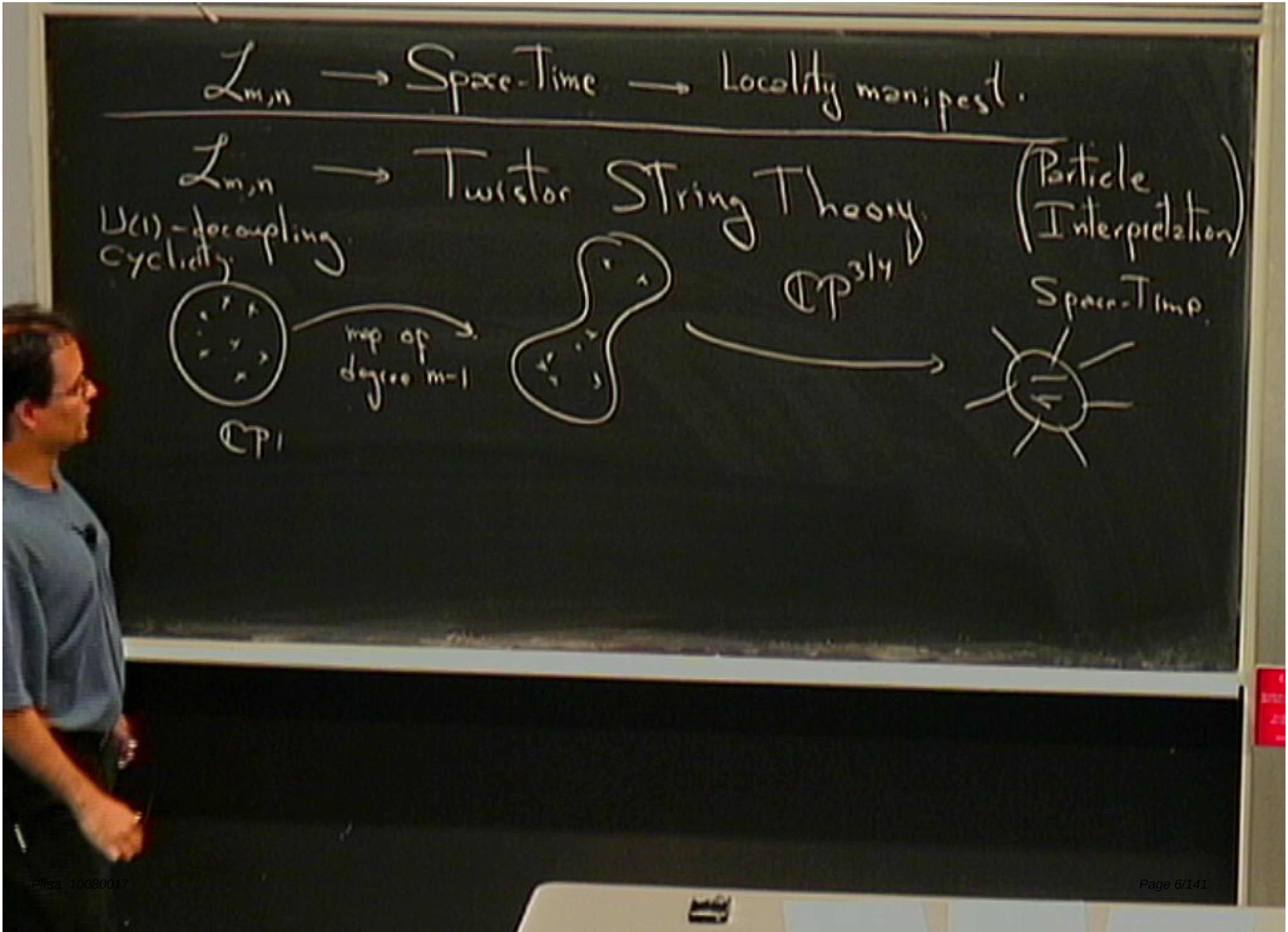
$\mathcal{L}_{m,n}$ → Space-Time → Locality manifest.
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(Particle Interpretation)
U(1)-decoupling
cyclicity
map of degree m-1
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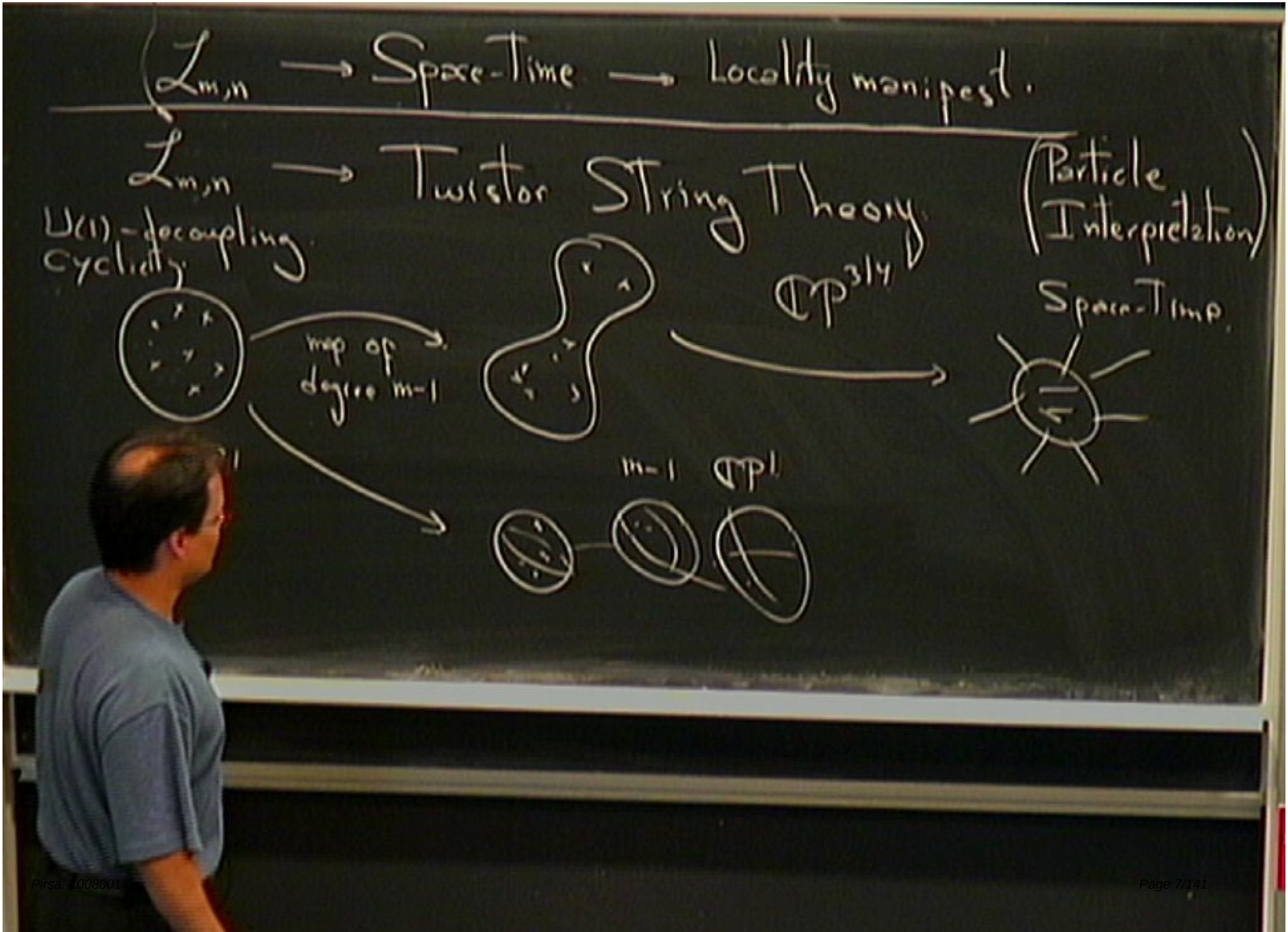
$\mathbb{CP}^1$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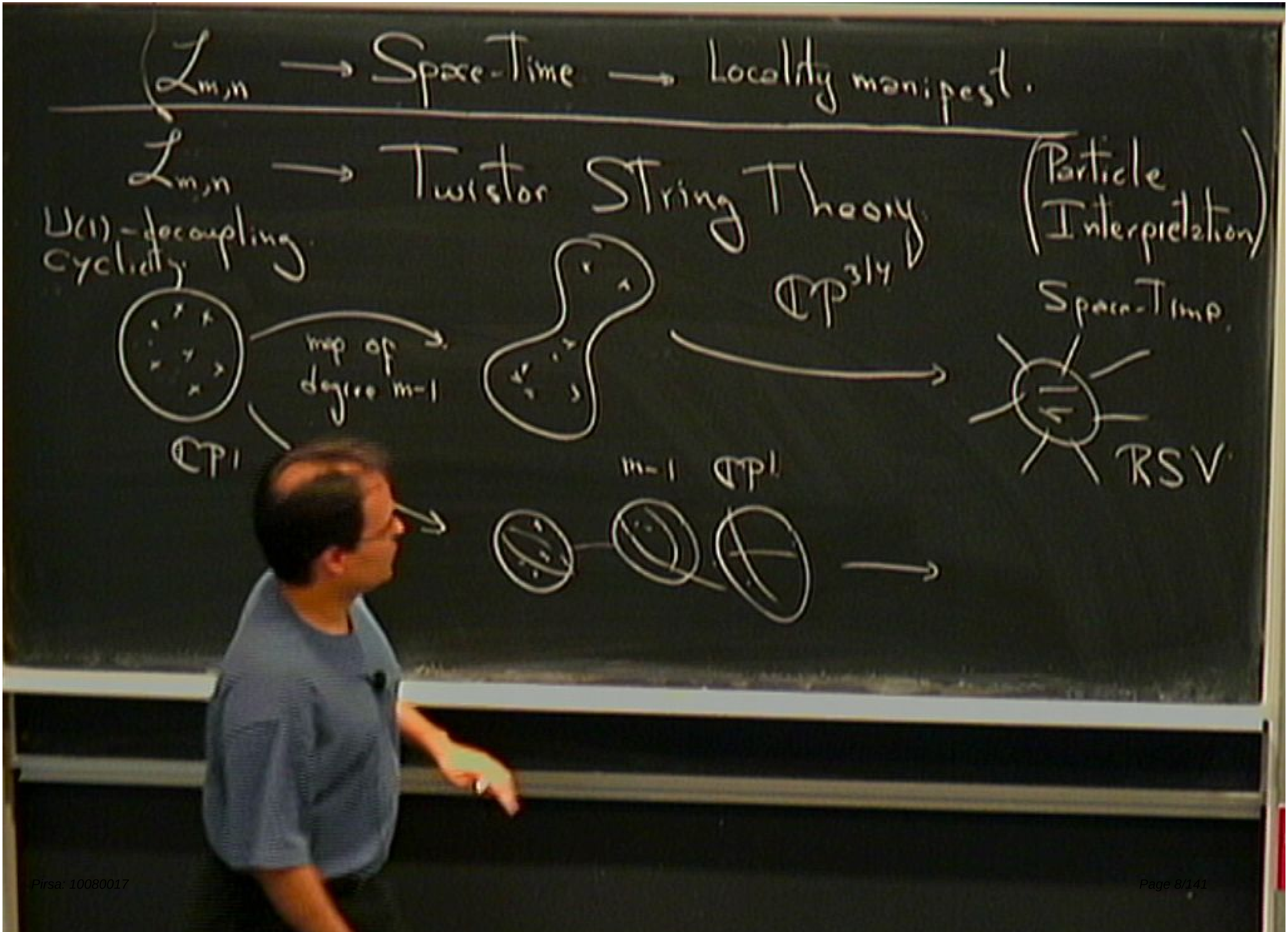

(Lm,n → Space-Time → Locality manifest.
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U(1)-decoupling
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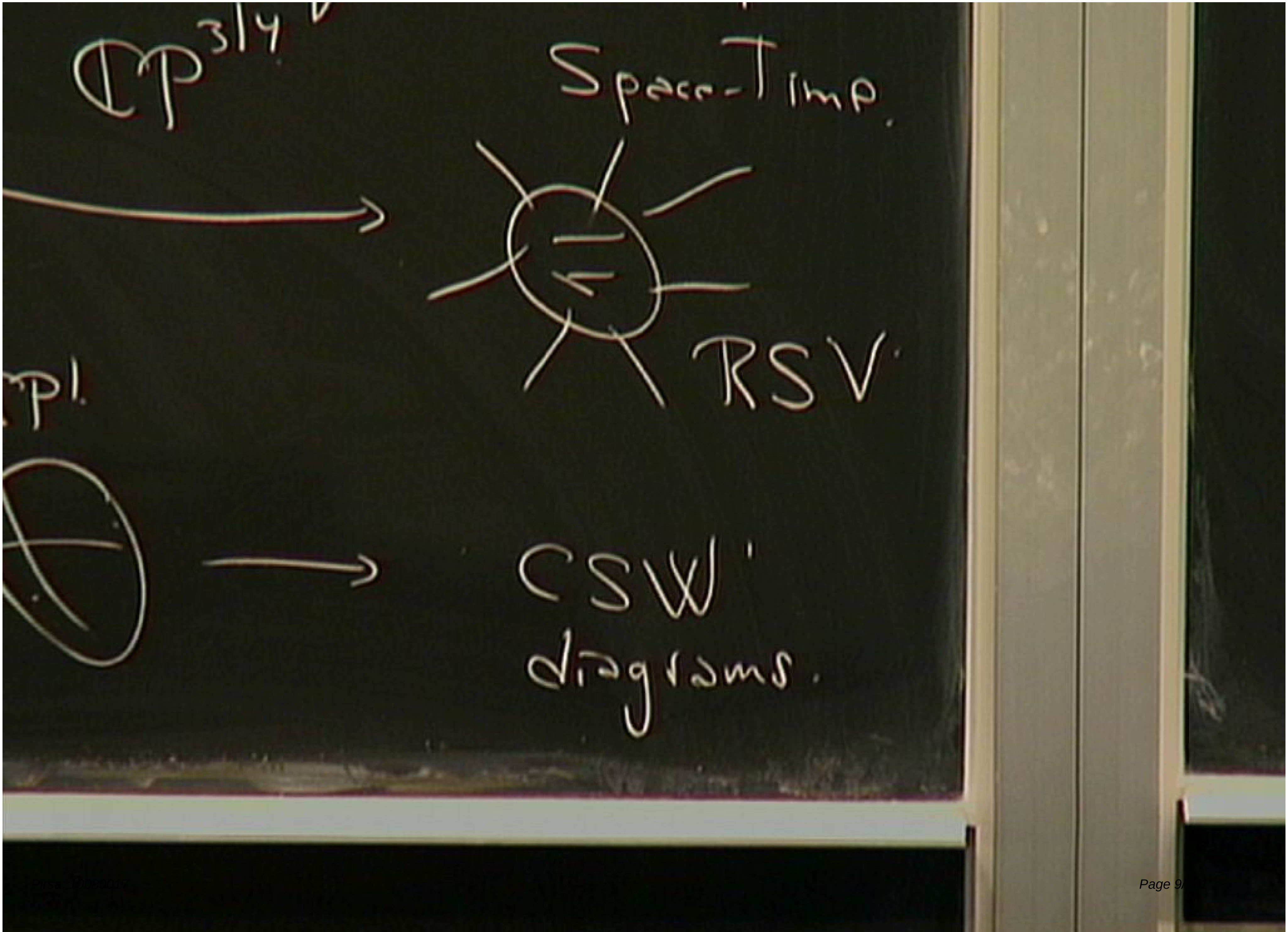
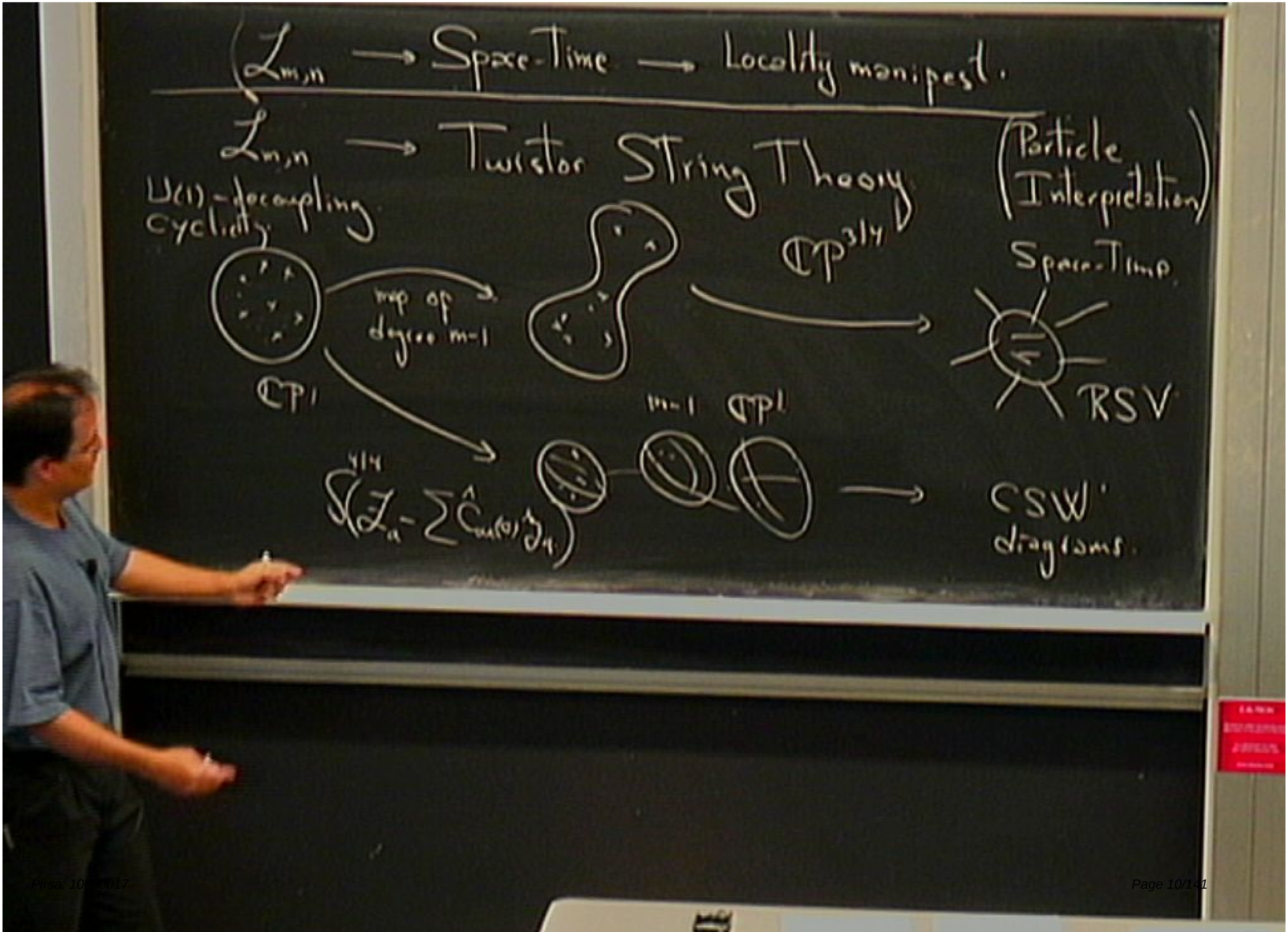
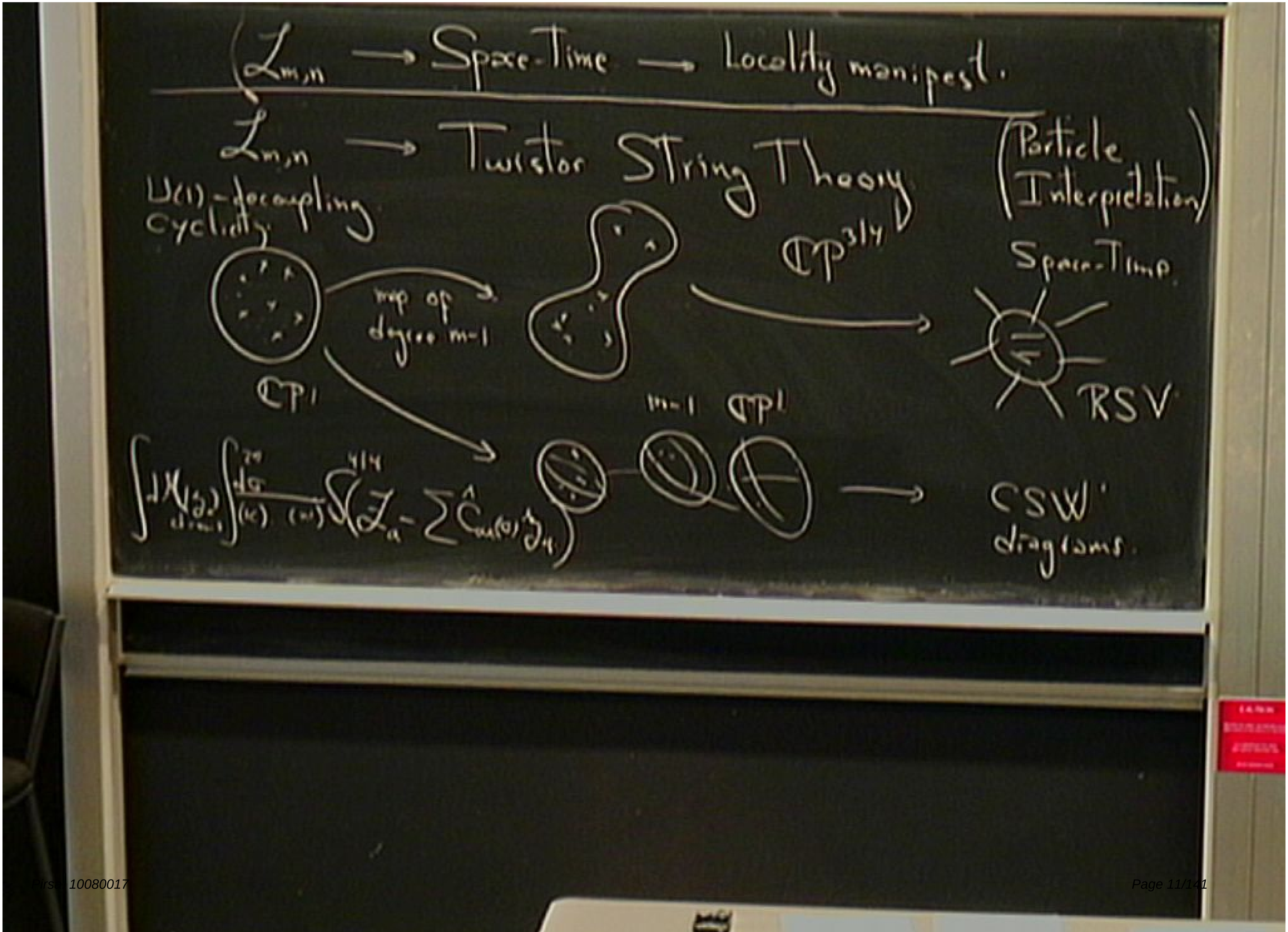
CP3|4
Space-Time
CP1
m-1 CP1
RSV


CP3|4
Space-Time
RSV
CSW
diagrams



ℒm,n → Space-Time → Locality manifest.
ℒm,n → Twistor String Theory
(Particle Interpretation)
U(1)-decoupling
cyclicity
map of degree m-1
CP^{3|4}
CP^1
m-1
CP^1
Space-Time
RSV
CSW'
diagrams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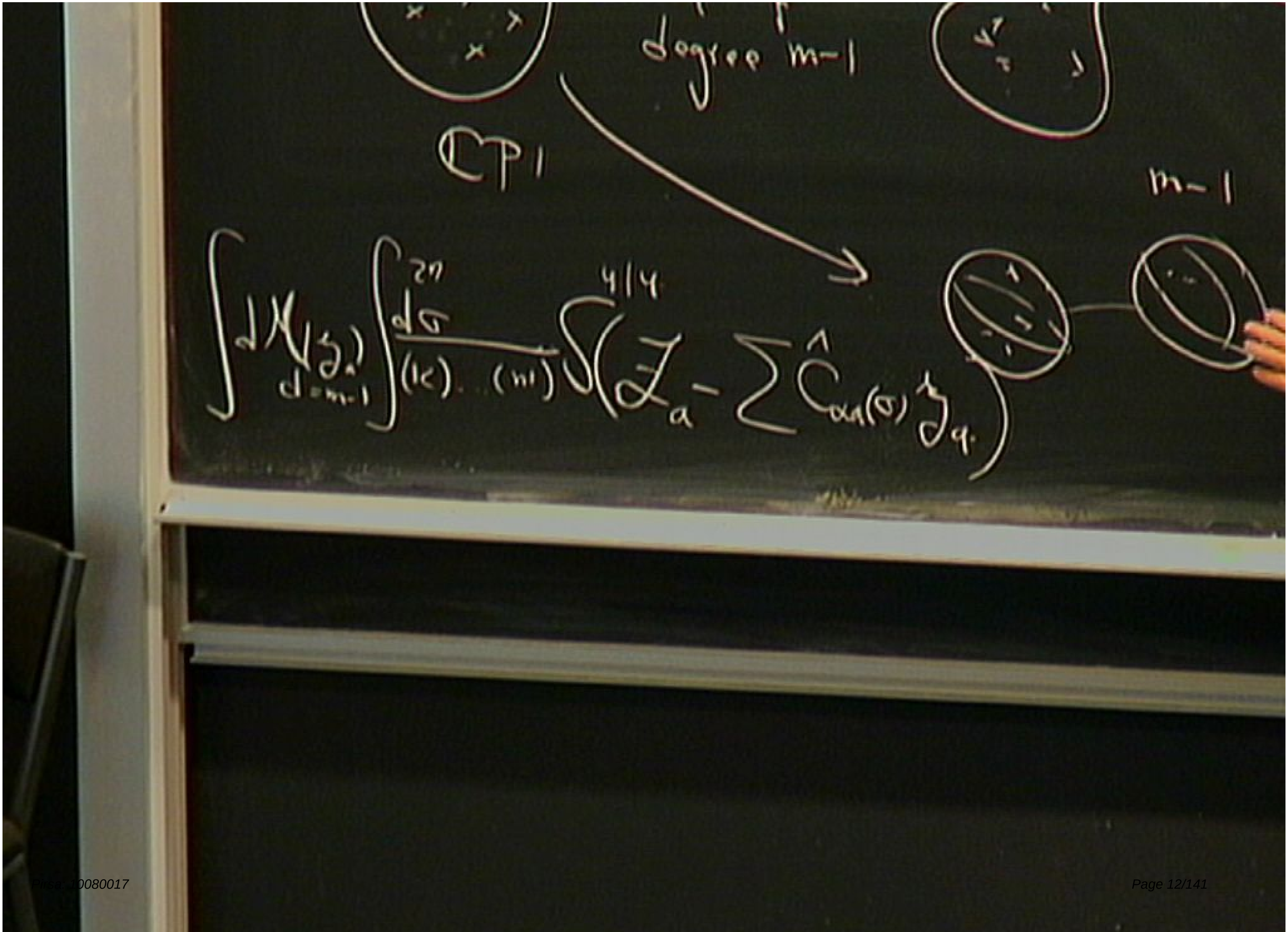
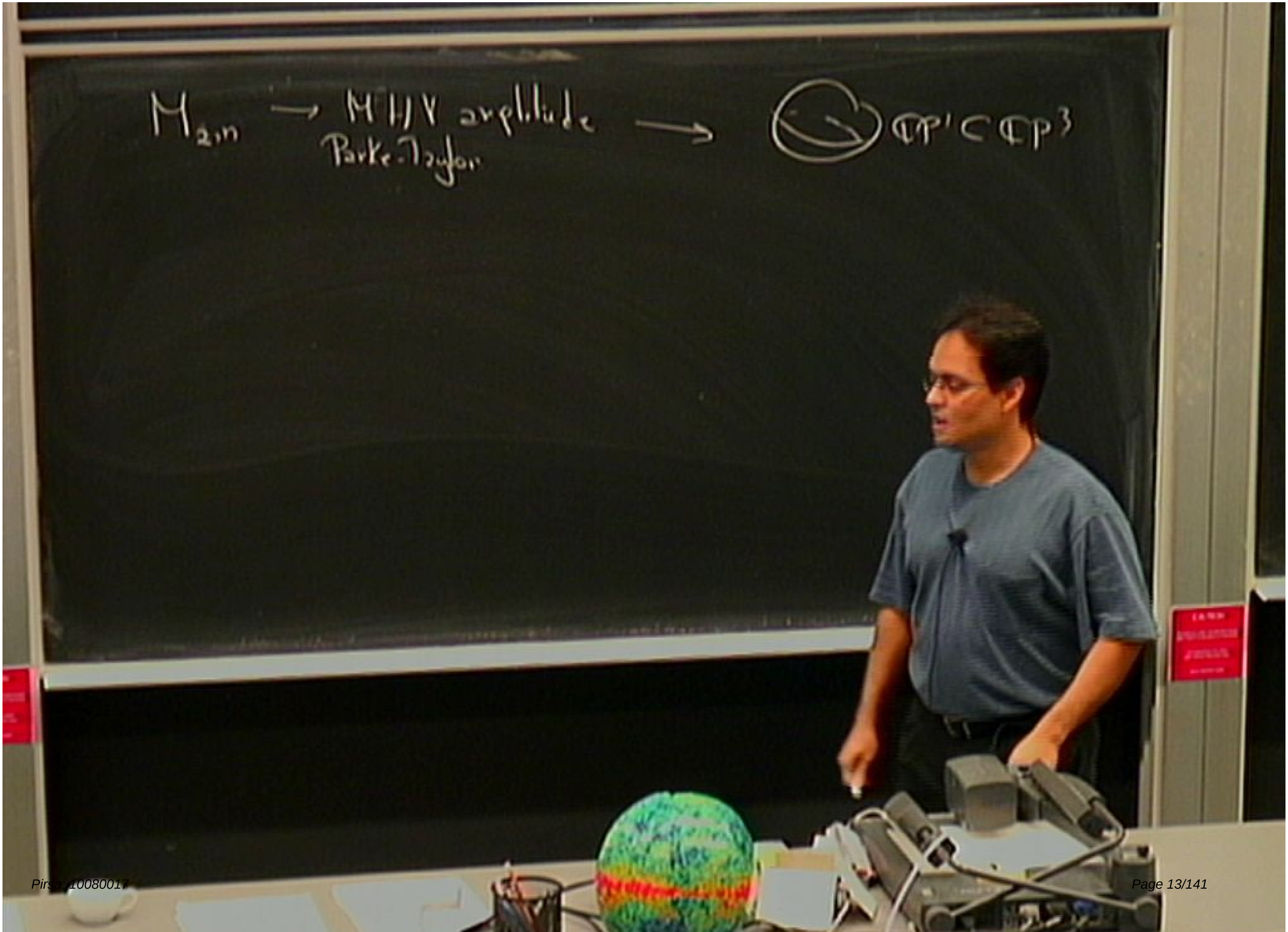
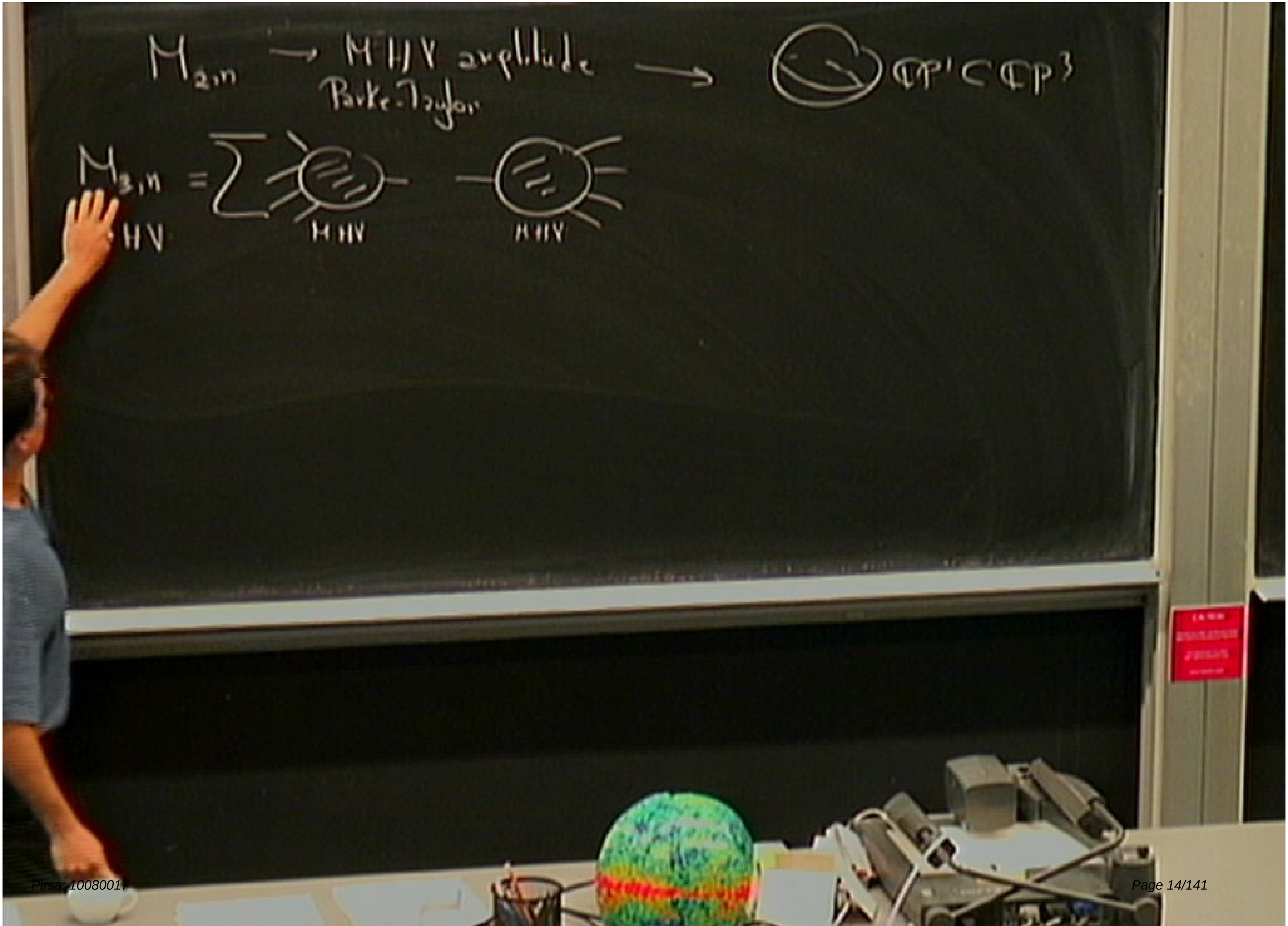
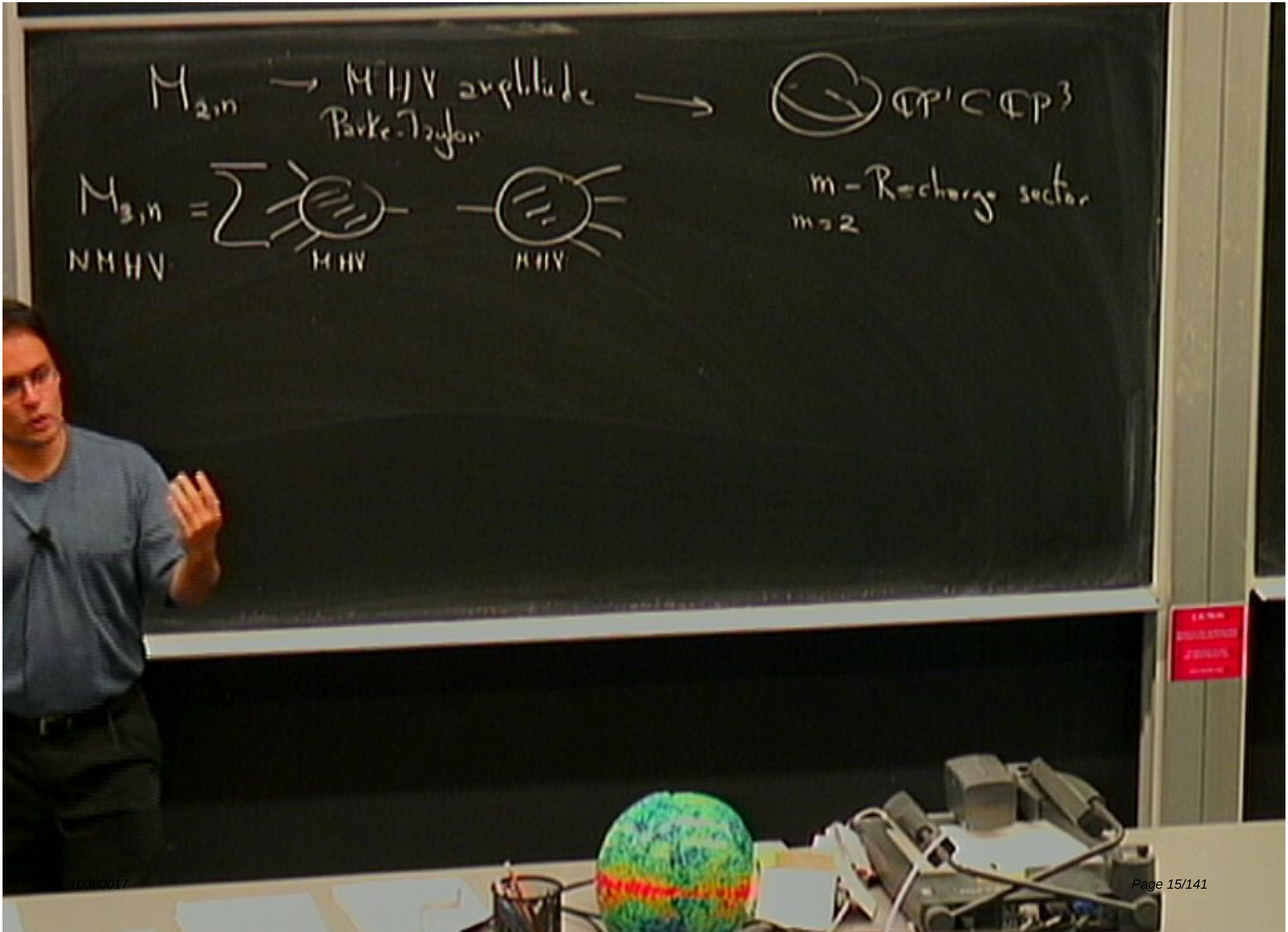
$\mathbb{CP}^{3|4}$
Space-Time
$\mathbb{CP}^1$
m-1 $\mathbb{CP}^1$
RSV
"CSW" diagrams.

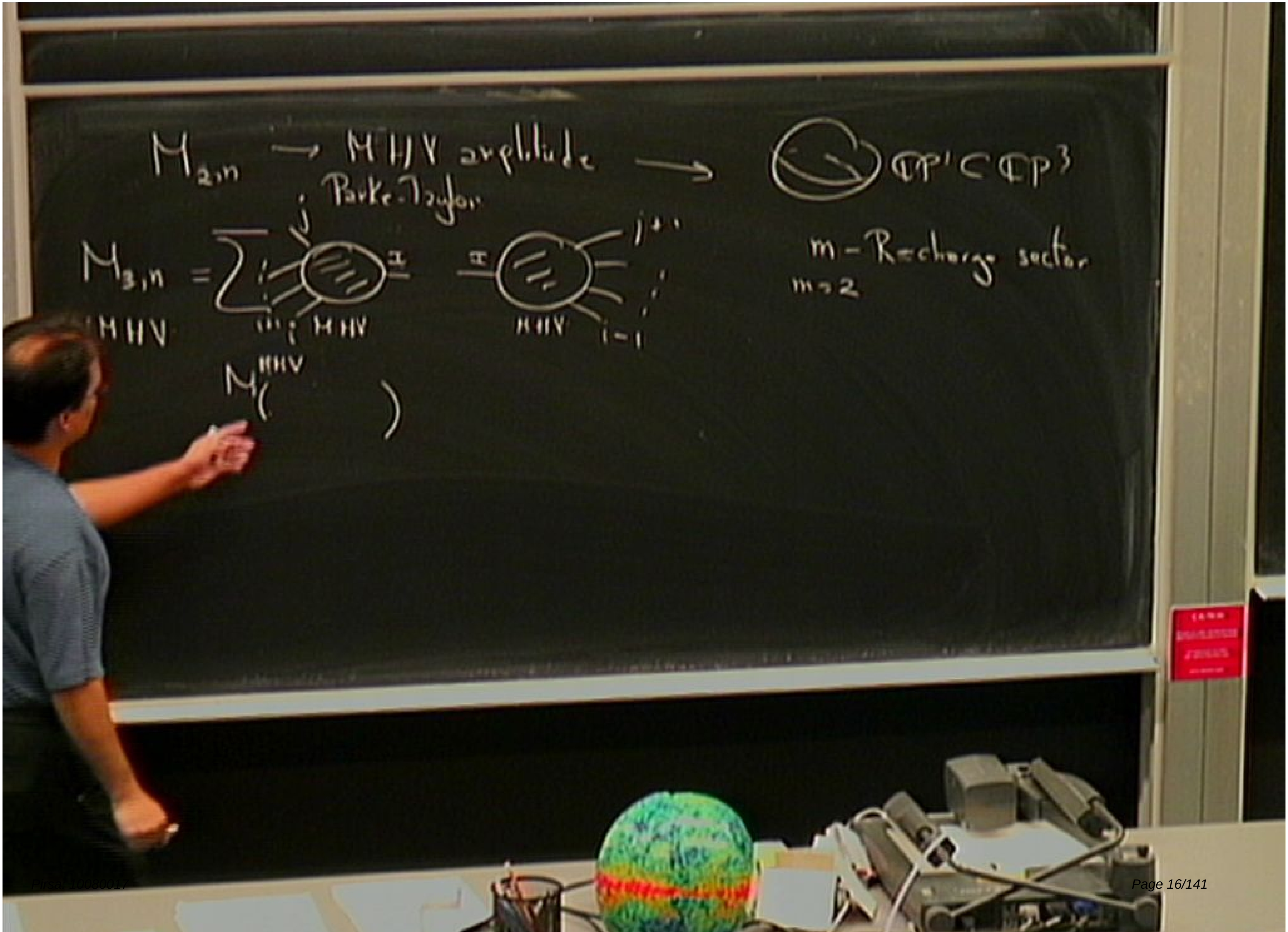

degree m−1
ℂP1
m−1
∫dM(ζa) d=m−1 ∫2π dσ/(12)…(n1) δ4|4(Za − Σ Ĉaa(σ) ζa)


$M_{2,n} \to$ MHV amplitude
Parke-Taylor
$\mathbb{CP}^1 \subset \mathbb{CP}^3$



$M_{2,n} \rightarrow$ MHV amplitude
Parke-Taylor
$\mathbb{CP}^1 \subset \mathbb{CP}^3$
$M_{3,n} = \sum$
MHV
MHV
HV


M2,n → MHV amplitude → CP1 ⊂ CP3
Parke-Taylor
M3,n = Σ
NMHV
MHV
MHV
m-Recharge sector
m=2


M2,n → MHV amplitude →
Parke-Taylor
CP1 ⊂ CP3
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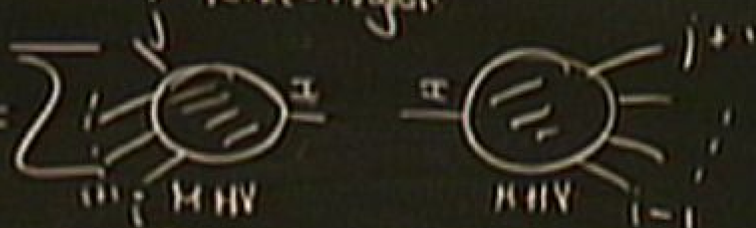
m-Recharge sector
m=2
MHV
MHV
MHV
MMHV

$M_{2,n} \longrightarrow$ MHV amplitude $\longrightarrow$  $\mathbb{CP}^1 \subset \mathbb{CP}^3$

Parke-Taylor

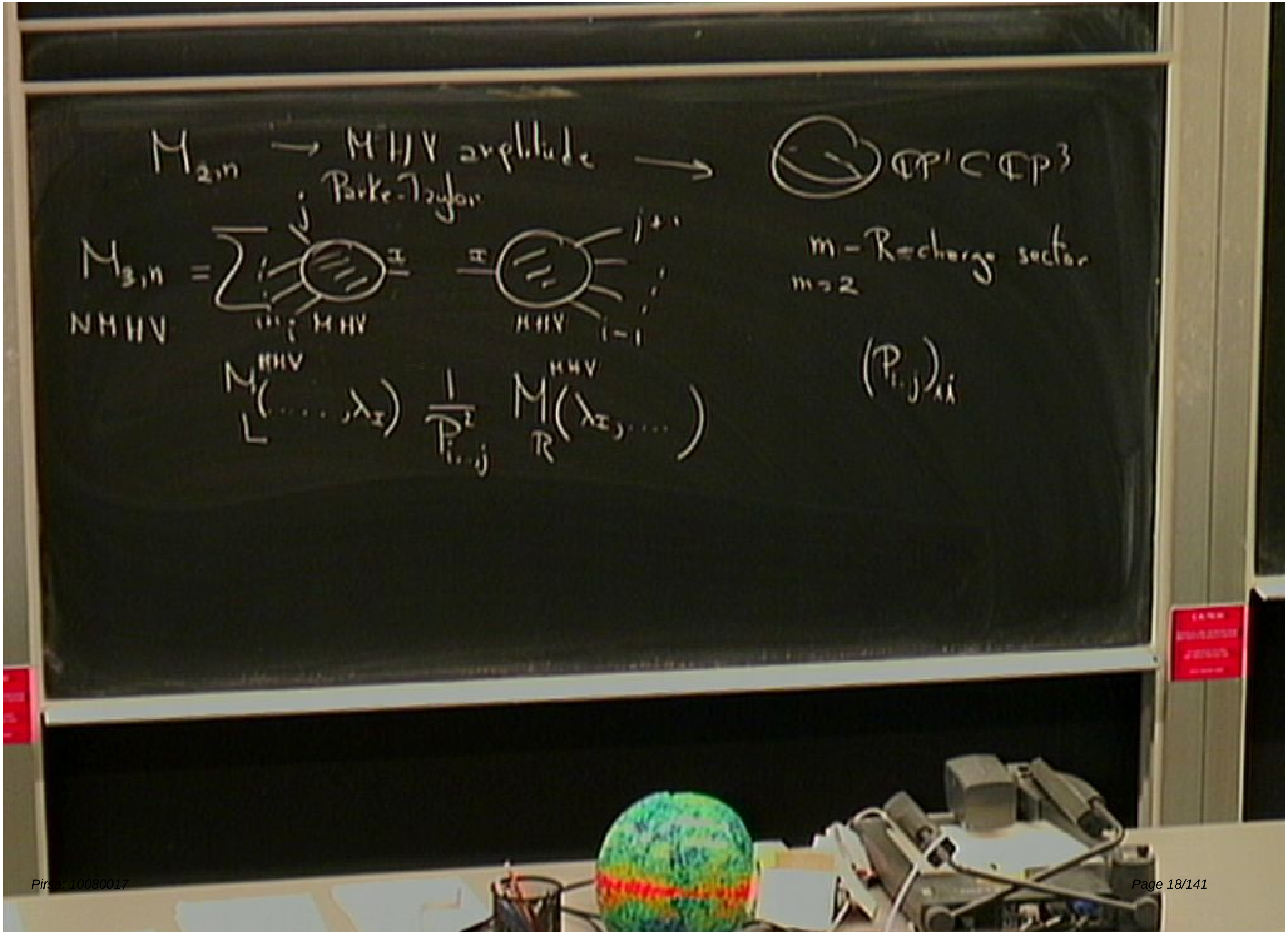
m - R-charge sector

m=2

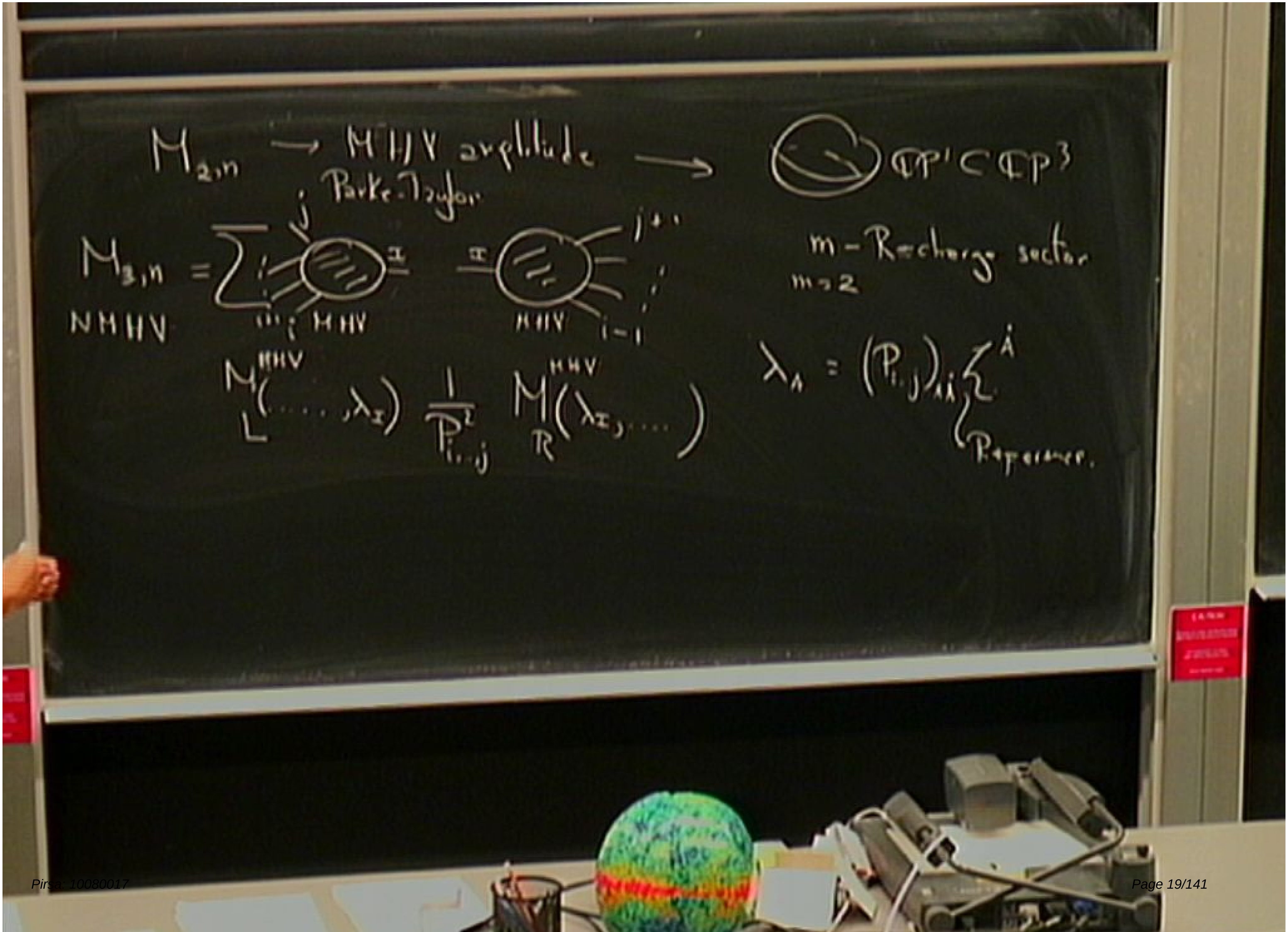
$M_{3,n} = \sum$ 

NMHV

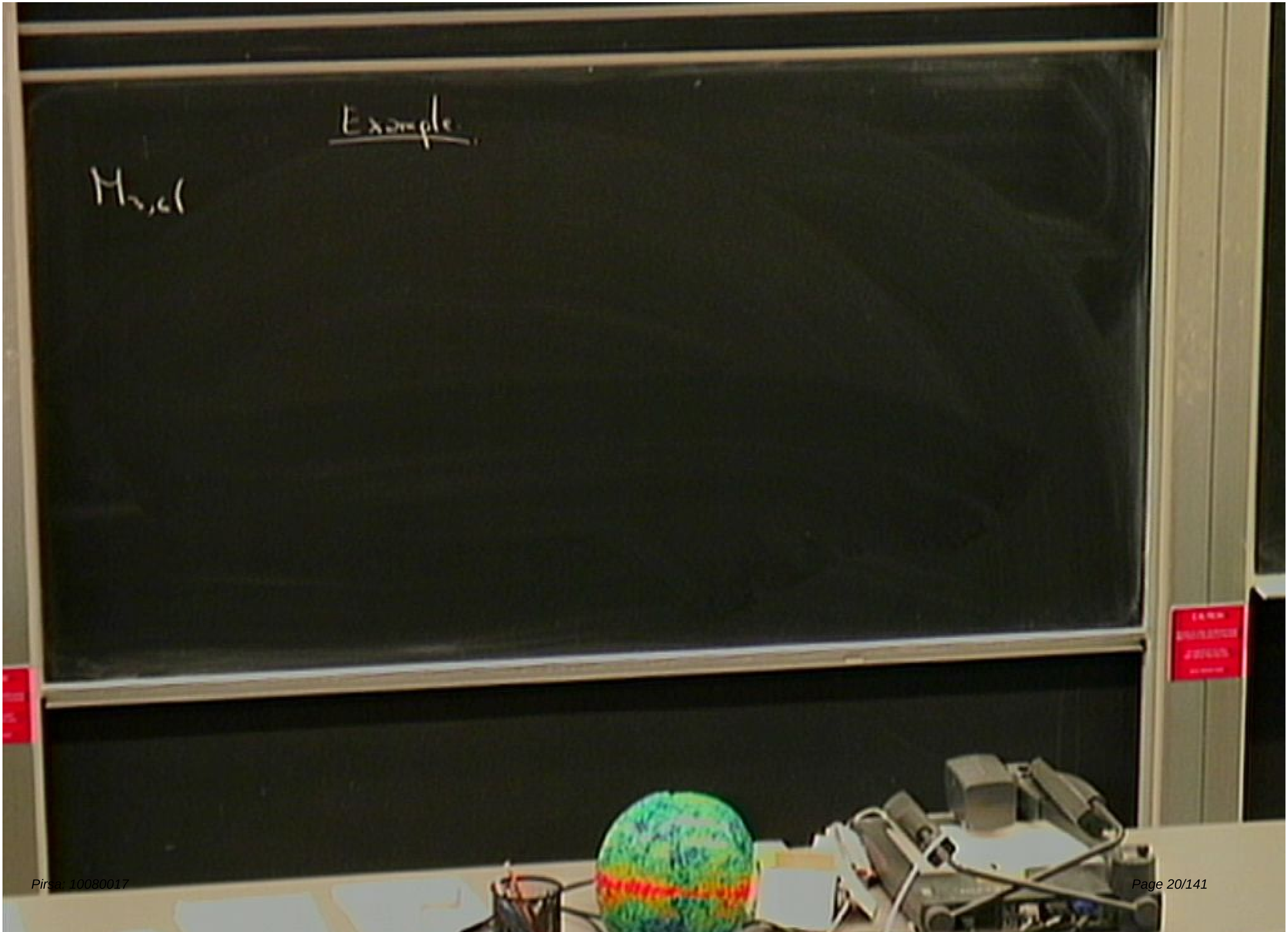
$$M_L^{MHV}(\ldots,\lambda_I)\,\frac{1}{P^2_{i\ldots j}}\,M_R^{MHV}(\lambda_I,\ldots)$$



M2,n → MHV amplitude → CP1 ⊂ CP3
Parke-Taylor
m - R-charge sector
m=2
M3,n = Σ
NMHV
MHV
MHV
(Pi..j)



$M_{2,n} \rightarrow$ MHV amplitude $\rightarrow$ $\mathbb{CP}^1 \subset \mathbb{CP}^3$
Parke-Taylor
$M_{3,n} = \sum$
NMHV
MHV
MHV
m-Recharge sector
m=2
$M^{MHV}_{L}(\ldots, \lambda_I) \frac{1}{P^2_{i..j}} M^{MHV}_{R}(\lambda_I, \ldots)$
$\lambda_A = (P_{i..j})_{A\dot{A}} \zeta^{\dot{A}}$
Reference.

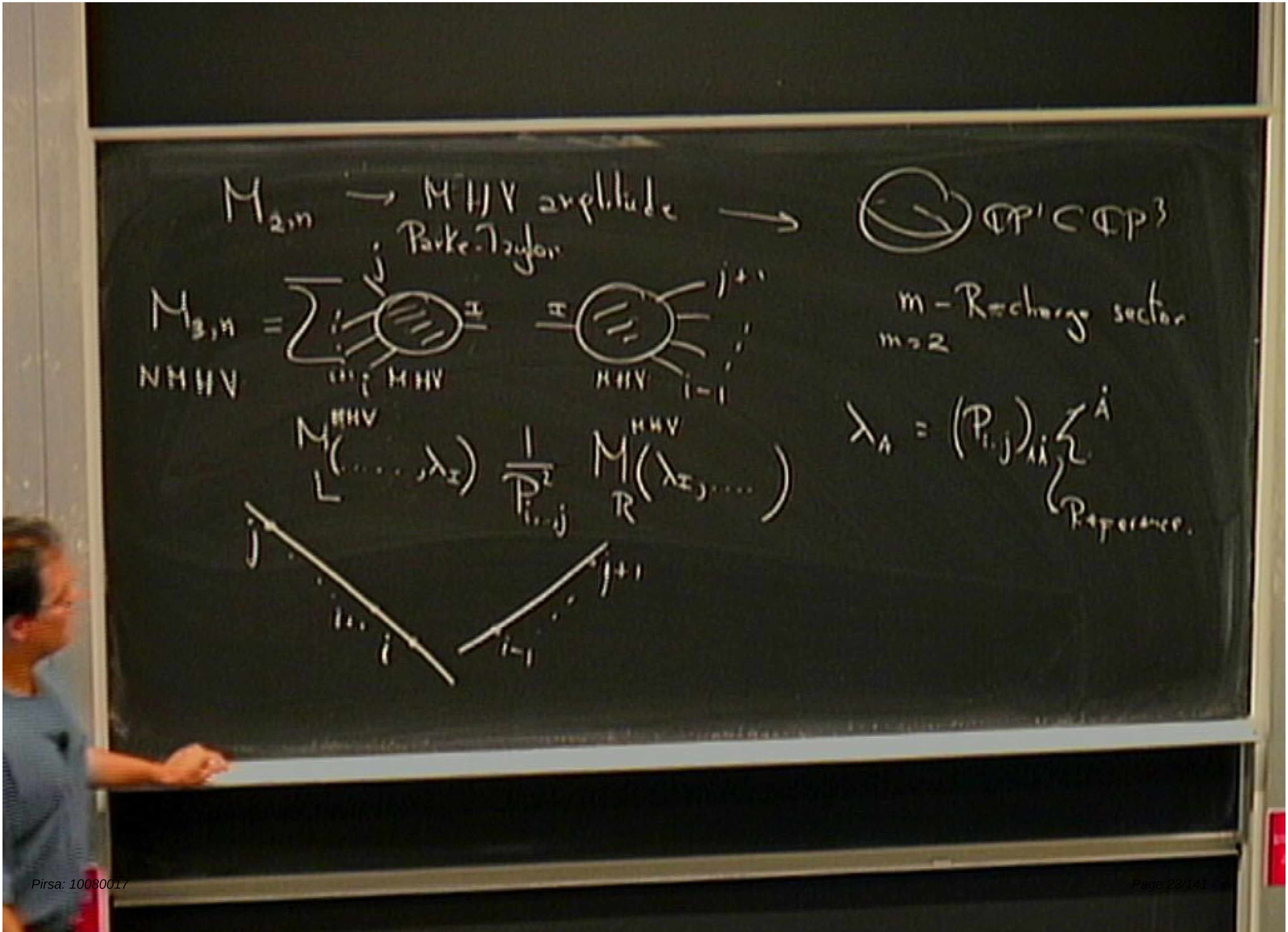

Example
$M_{n,d}($

Example

$$M_{3,6}(1^-,2^+,3^-,4^+,5^-,6^+)$$

Example.

$$M_{3,6}(1^-, 2^+, 3^-, 4^+, 5^-, 6^+) =$$



$M_{2,n}$ → MHV amplitude → $\mathbb{CP}^1 \subset \mathbb{CP}^3$
Parke-Taylor
$M_{3,n}$ =
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MHV
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$M^{MHV}_L(\ldots,\lambda_I)\ \frac{1}{P^2_{i..j}}\ M^{MHV}_R(\lambda_I,\ldots)$
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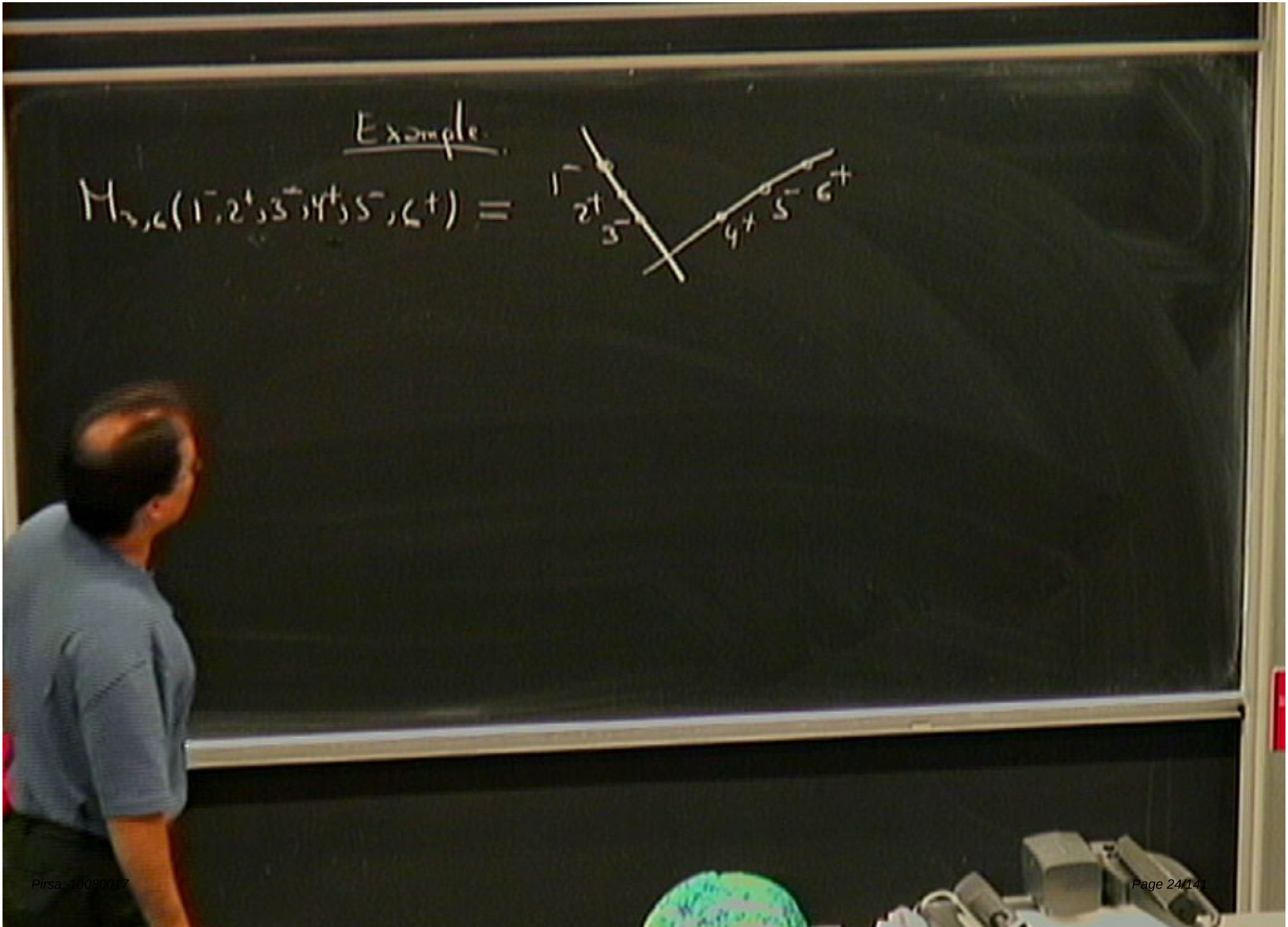
Example
$M_{3,6}(1^-,2^+,3^-,4^+,5^-,6^+) =$
1^- 2^+ 3^-
4^+ 5^- 6^+

Example

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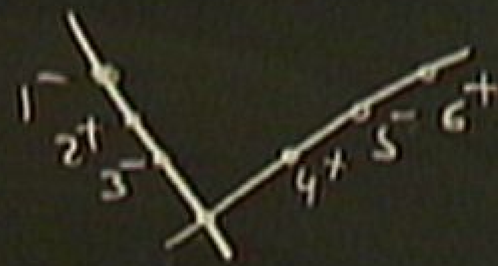

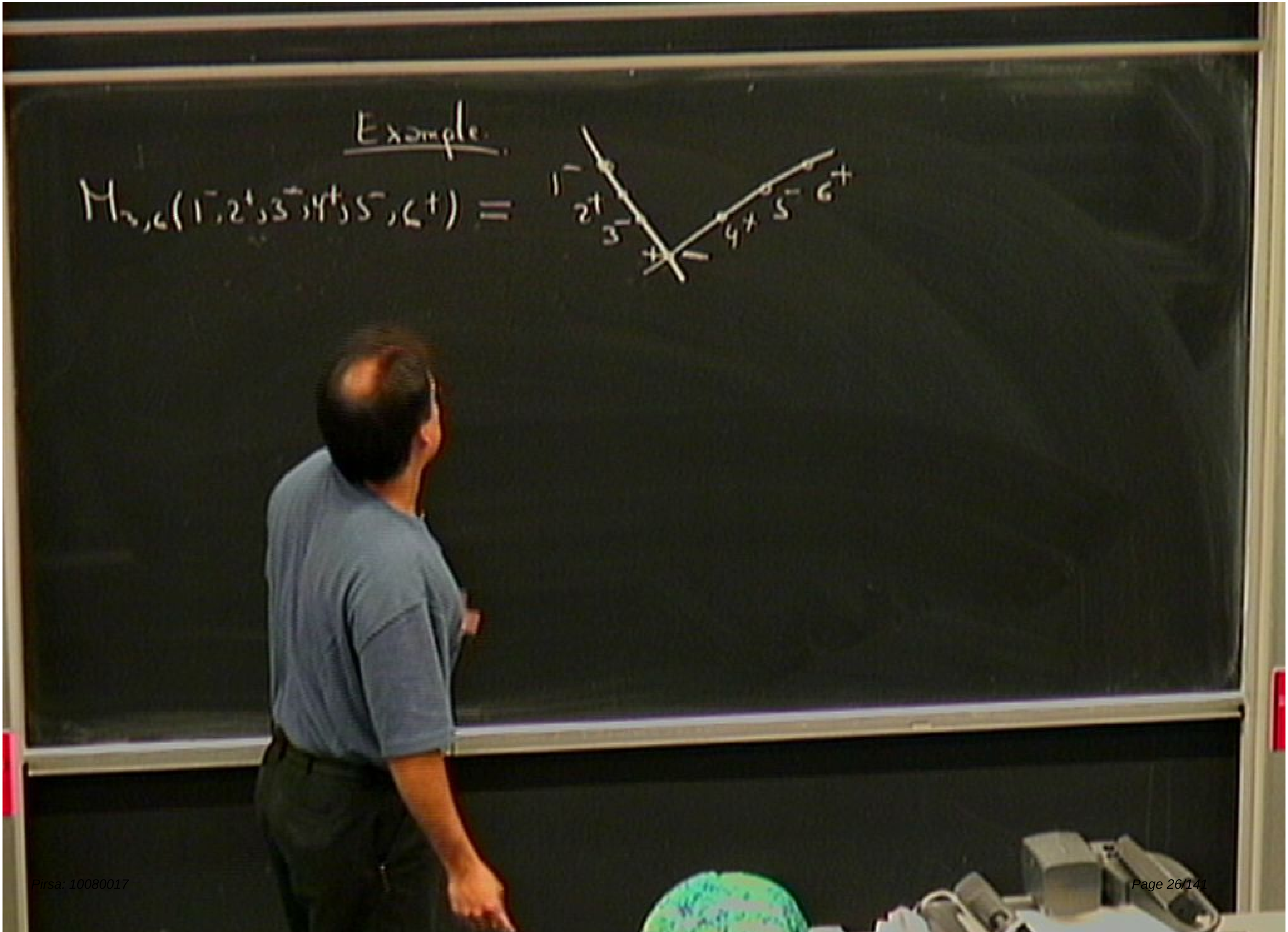
Example
$M_{3,6}(1^-,2^+,3^-,4^+,5^-,6^+) =$
1^- 2^+ 3^- $+$ $-$ 4^+ 5^- 6^+

Example

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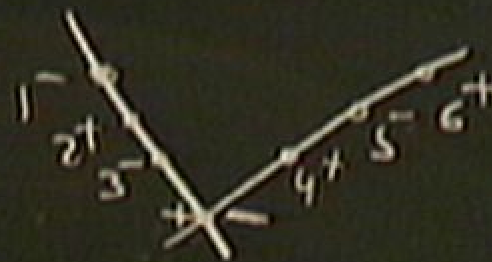
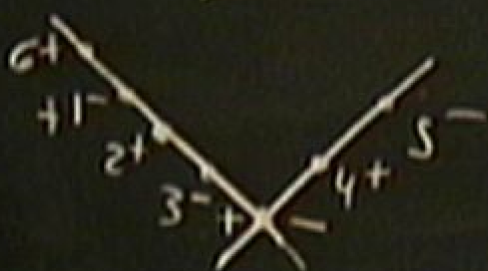
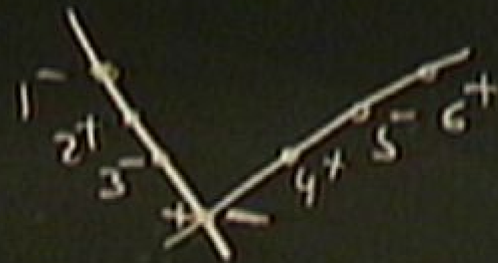


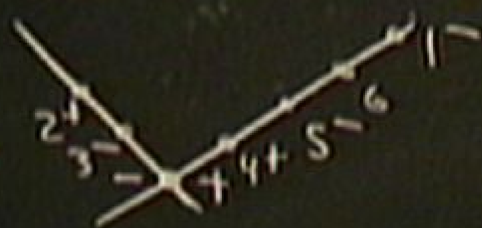
$+$

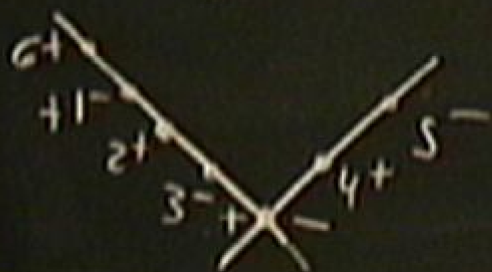


$+$

Example.

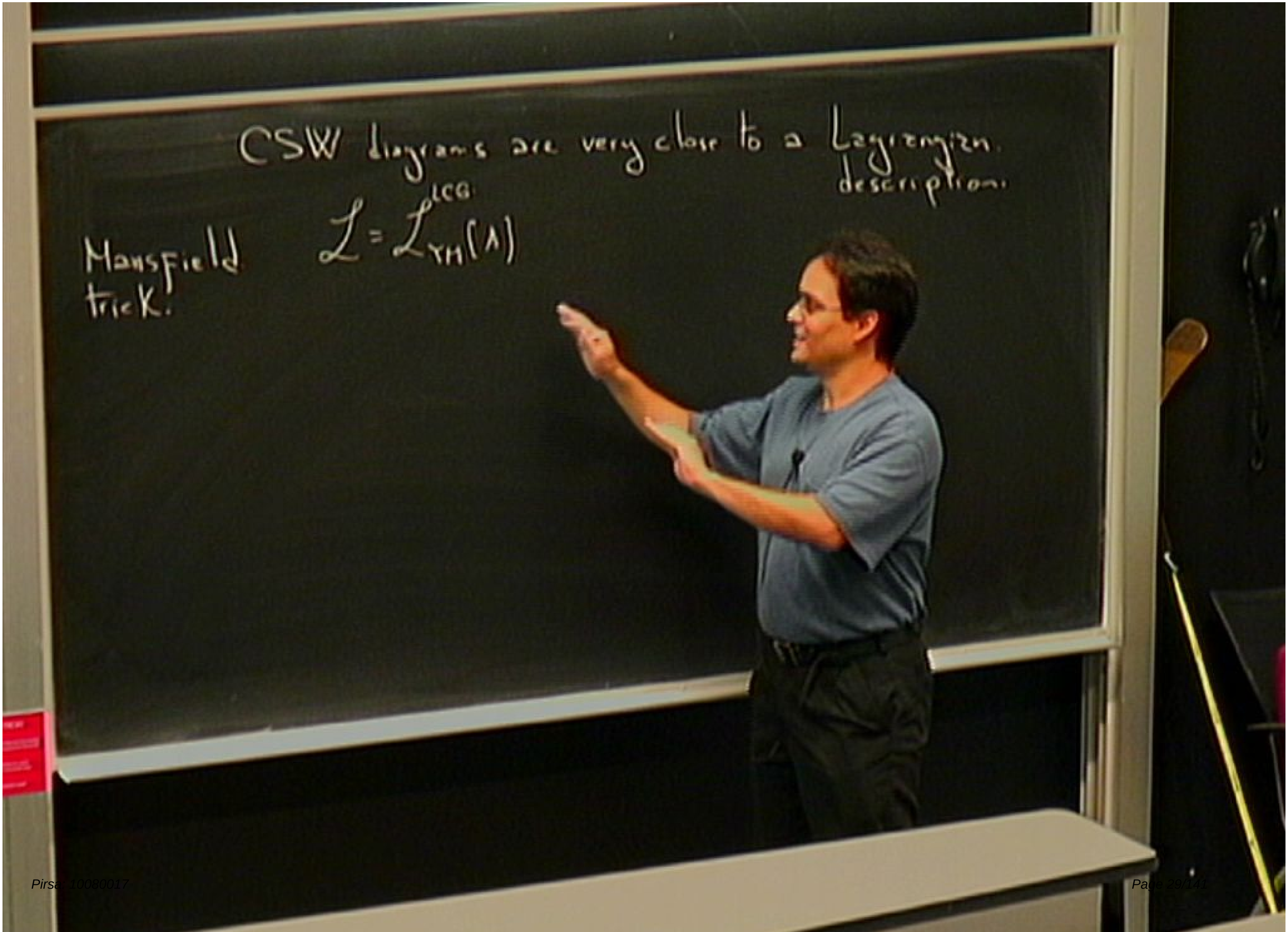
$M_{3,6}(1^-,2^+,3^-,4^+,5^-,6^+) =$ 

$+$ 

$+$ 

$+$ 6 more diagrams.

. . . .


CSW diagrams are very close to a Lagrangian description.
Mansfield trick.
L = L^{LCG}_{YM}[A]

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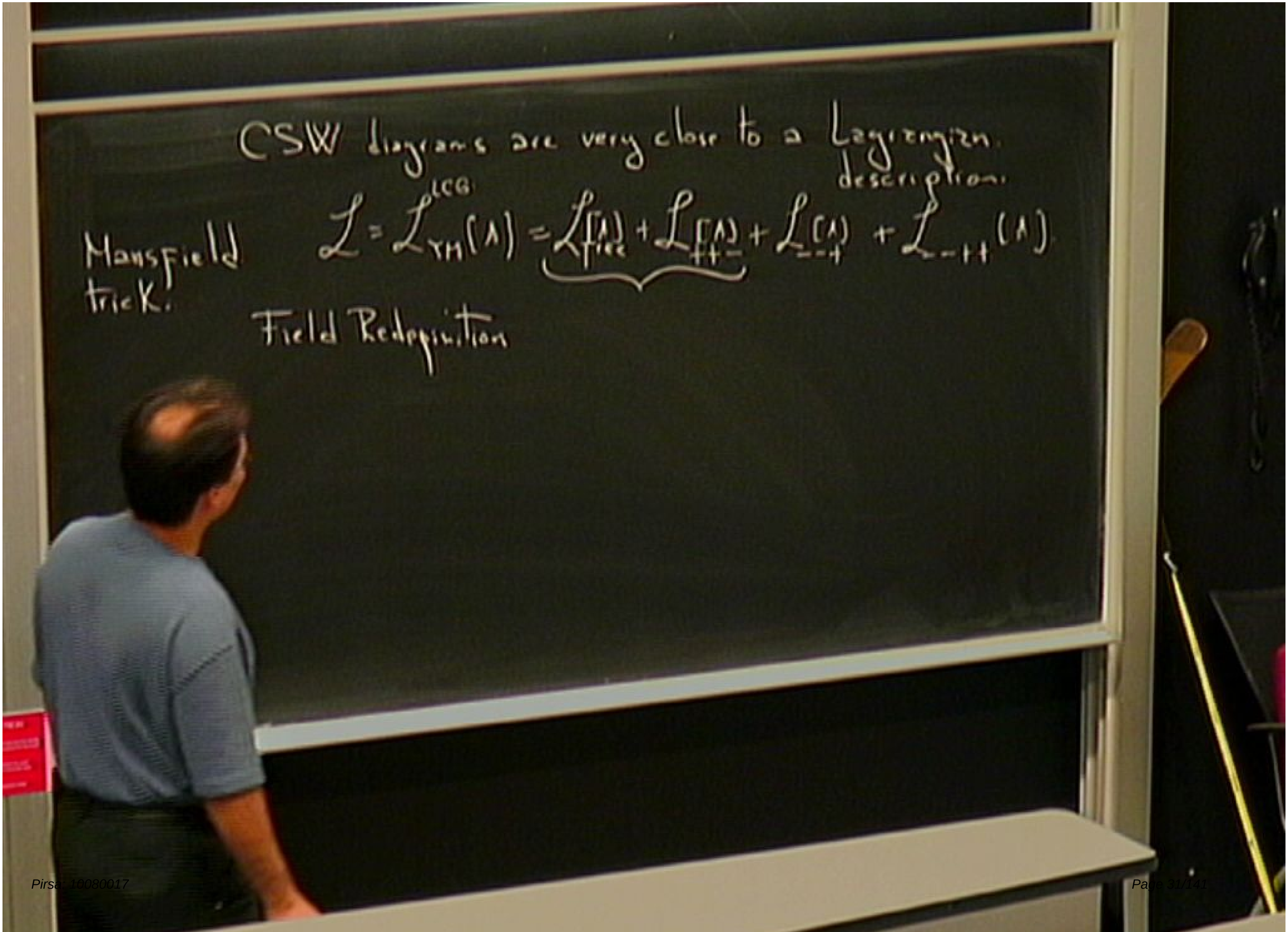
Mansfield trick:

$$\mathcal{L} = \mathcal{L}^{LCG}_{YM}(A) = \mathcal{L}_{free}(A) + \mathcal{L}_{++-}(A) + \mathcal{L}_{--+}(A) + \mathcal{L}_{--++}(A)$$


CSW diagrams are very close to a Lagrangian description.
Mansfield trick.
$\mathcal{L} = \mathcal{L}^{LCG}_{YM}(A) = \underbrace{\mathcal{L}_{free}(A) + \mathcal{L}_{++-}(A)}_{} + \mathcal{L}_{--+}(A) + \mathcal{L}_{--++}(A)$
Field Redefinition

CSW diagrams are very close to a Lagrangian description.

Mansfield trick:

$$\mathcal{L} = \mathcal{L}^{LCG}_{YM}(A) = \underbrace{\mathcal{L}_{free}[A] + \mathcal{L}_{++-}[A]} + \mathcal{L}_{--+}(A) + \mathcal{L}_{--++}(A)$$

Field Redefinition $\quad \mathcal{L}_{free}[B] + \sum_{p=1}^{\infty} \mathcal{L}_{--\underbrace{+\cdots+}_{p}}[B]$

Mansfield trick:

$$\mathcal{L} = \mathcal{L}_{YM}(A) = \underbrace{\mathcal{L}_{free}[A] + \mathcal{L}_{++-}[A]}_{} + \mathcal{L}_{--+}(A) + \mathcal{L}_{--++}(A)$$

description:

Field Redefinition $\quad \mathcal{L}_{free}[B] + \sum_{p=1}^{\infty} \mathcal{L}_{--\underbrace{+\cdots+}_{p}}[B]$

BSM: Twistor Space $\quad CS[\mathcal{A}] \longrightarrow$ Partial g...

$$S^2 \to \mathbb{CP}^3$$
$$\downarrow$$
$$S^4 \longrightarrow \text{Euclidean}$$

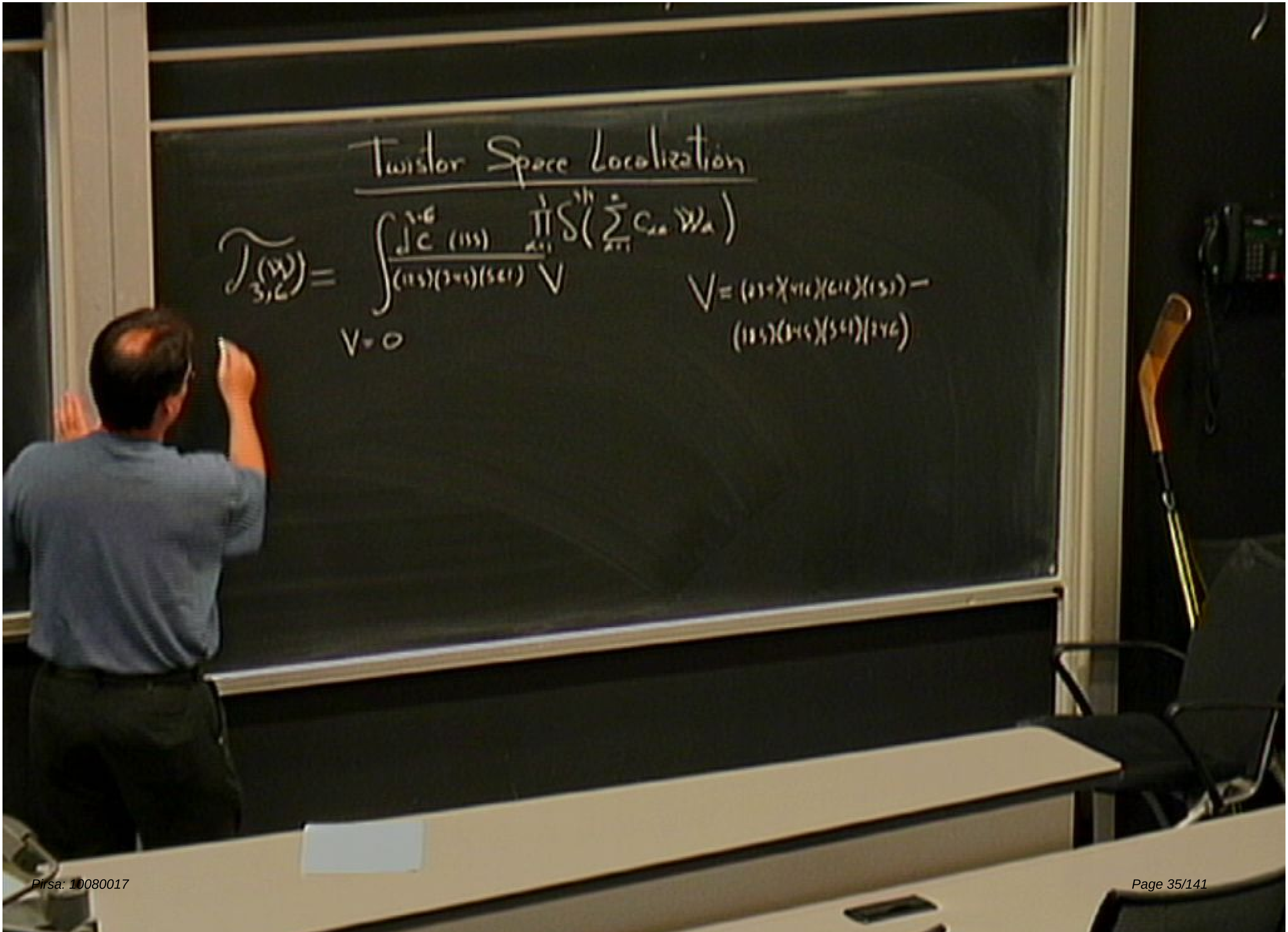
$$\mathcal{L} = \mathcal{L}_{YM}(A) = \underbrace{\mathcal{L}_{free}[A] + \mathcal{L}_{++-}[A]} + \mathcal{L}_{--+}[A] + \mathcal{L}_{--++}(A).$$

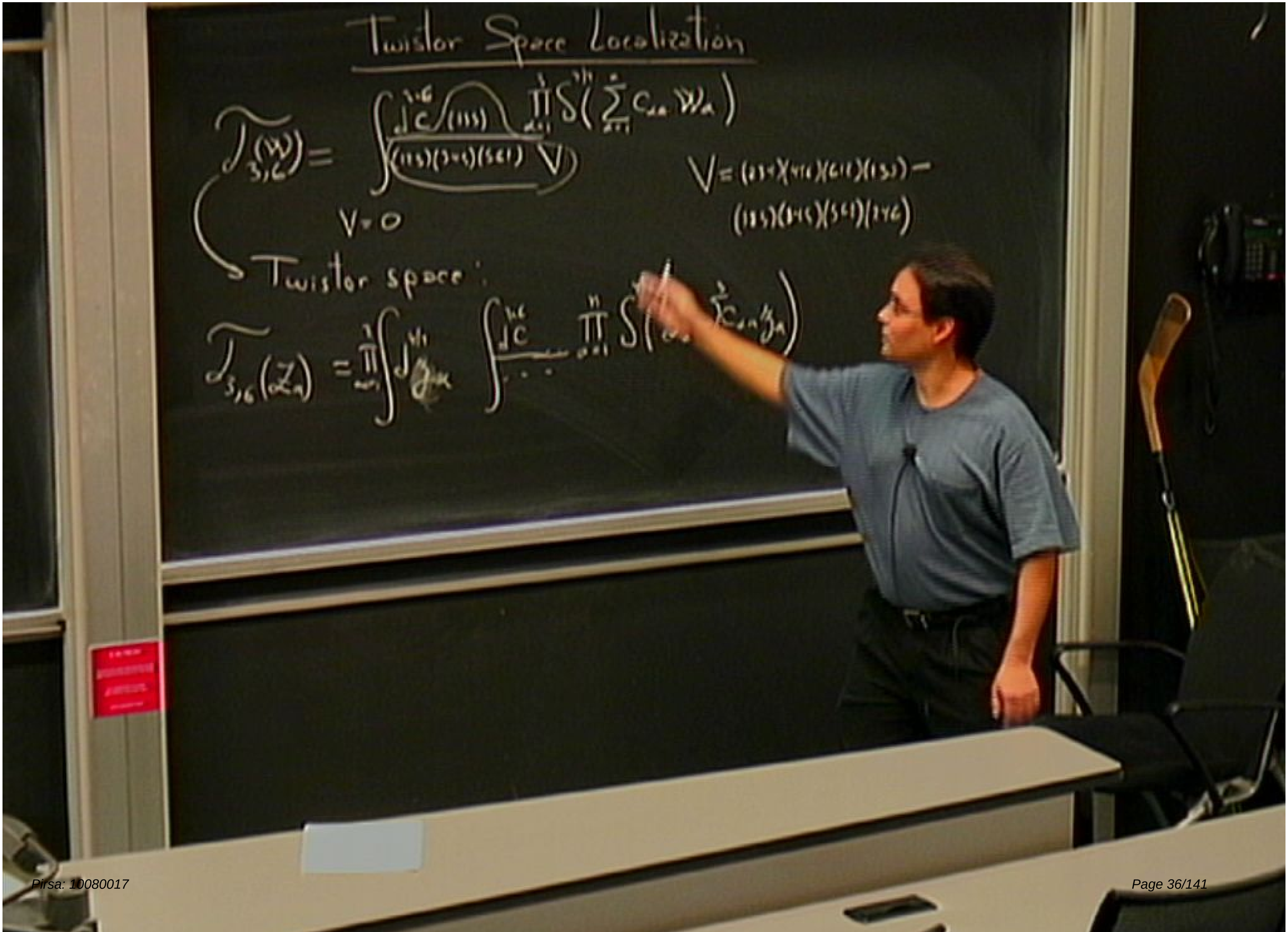
Field Redefinition $\quad \mathcal{L}_{free}[B] + \sum_{p=1}^{\infty} \mathcal{L}_{--\underbrace{+\cdots+}_{p}}[B]$

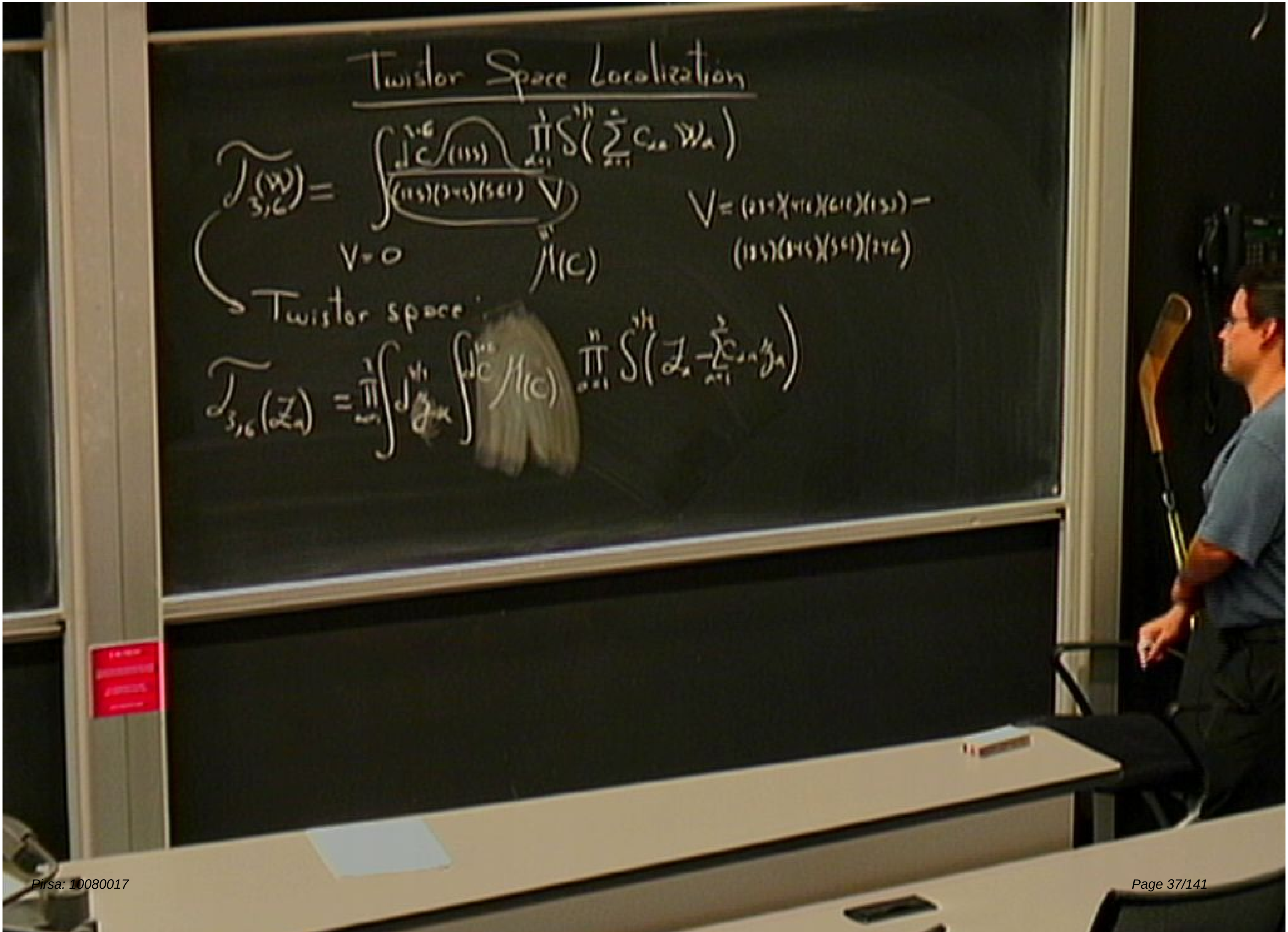
Twistor Space. $\quad C.S.[\mathcal{A}] \longrightarrow$ Partial g.F. Spacetime g.F. $\longrightarrow \mathcal{L}_{YM}[A]$

$S^2 \longrightarrow \mathbb{CP}^3$ $\quad$ Partial g.F.

$\longrightarrow$ Euclidean


Twistor Space Localization
V = 0


Twistor Space Localization
V=0
Twistor space:


Twistor Space Localization
V=0
Twistor space:

Twistor Space Localization

$$\tilde{\mathcal{J}}_{3,6}(\mathcal{W}) = \int \frac{d^{3\times 6}C\,(123)}{(123)(345)(561)\,V} \prod_{a=1}^{3} \delta^{4|4}\left(\sum_{i=1}^{6} C_{ai}\mathcal{W}_i\right)$$

$V=0$ $\qquad \mu(C)$

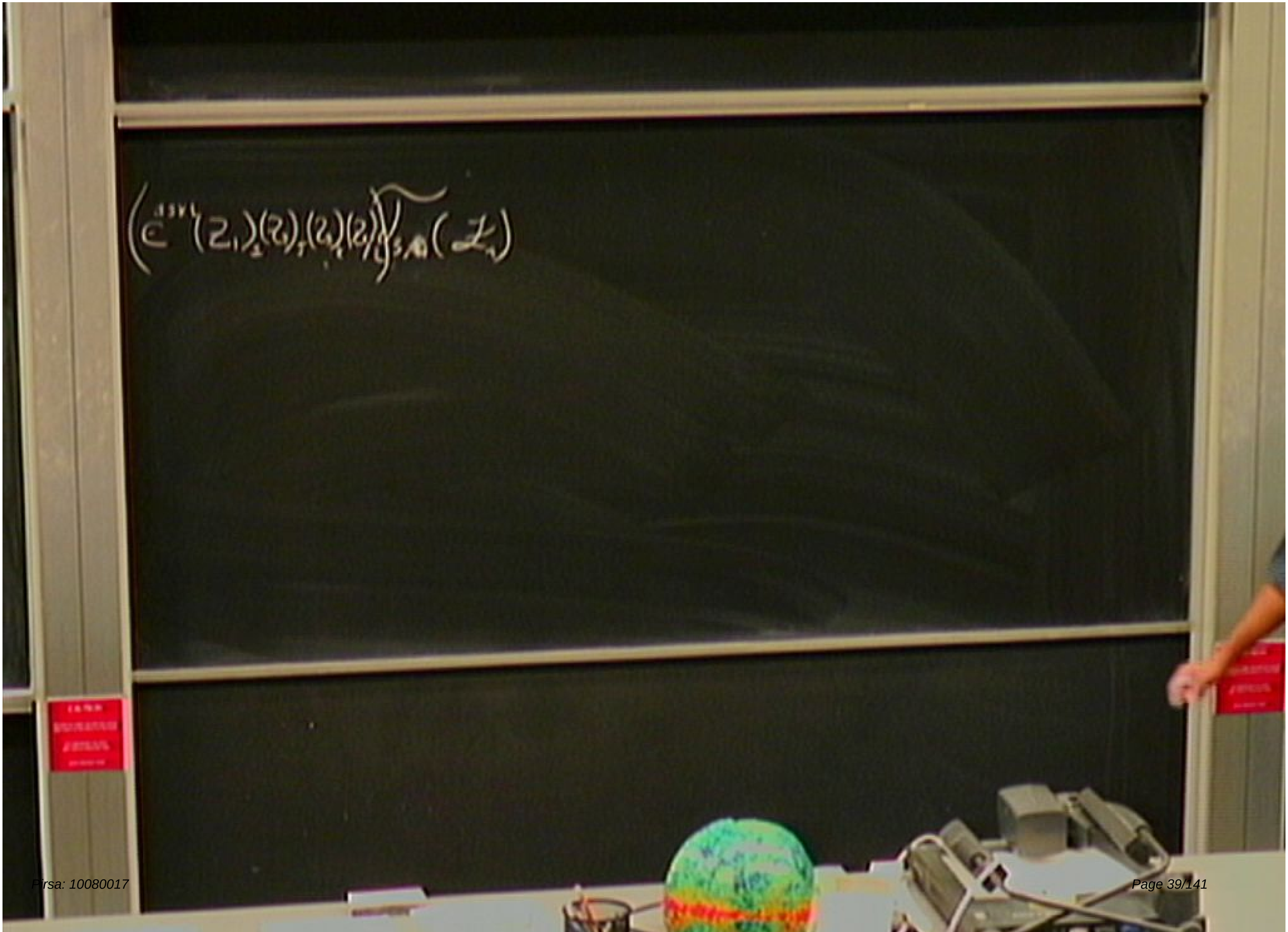
$$V = (234)(456)(612)(135) - (123)(345)(561)(246)$$

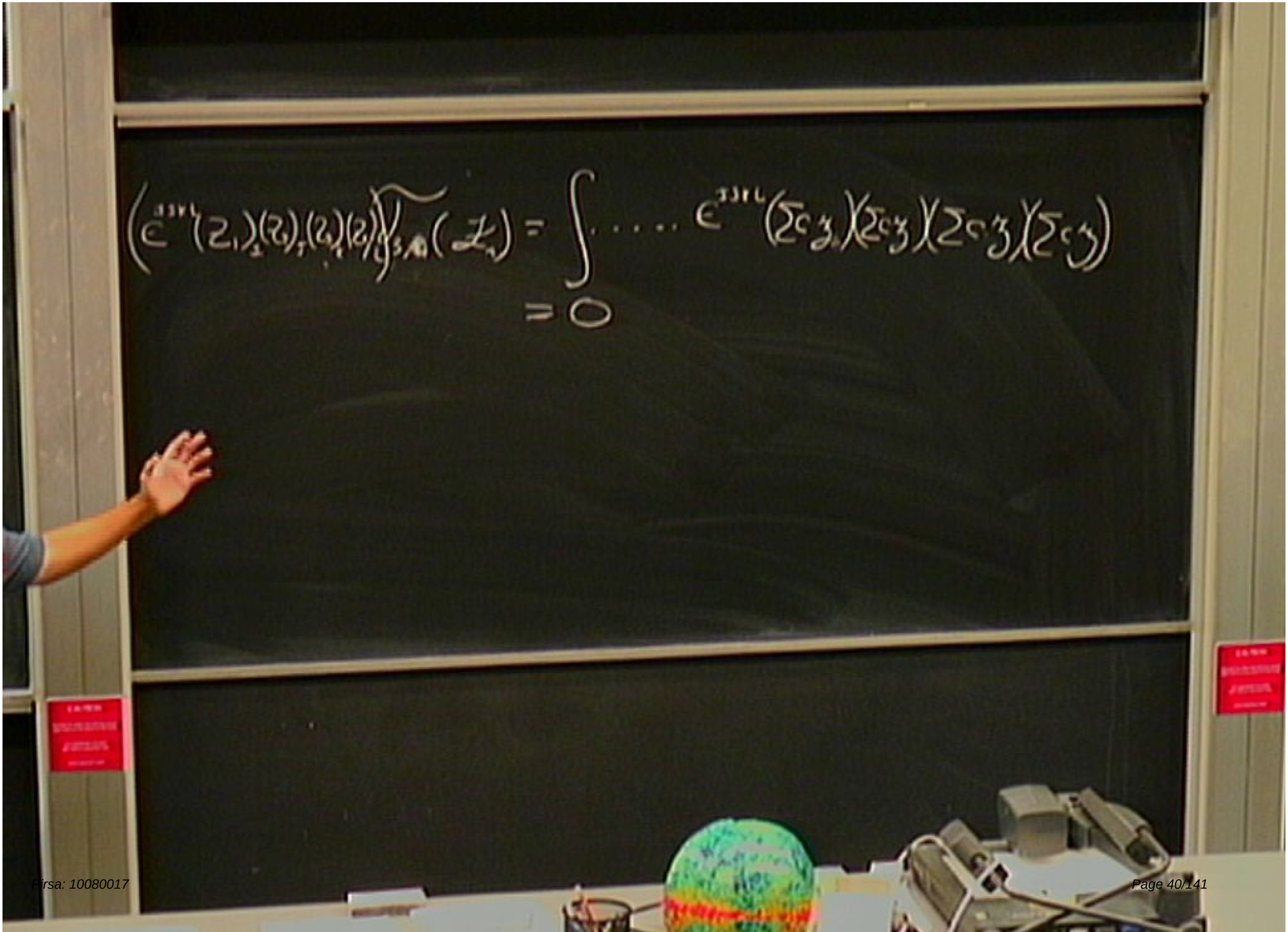
Twistor space:

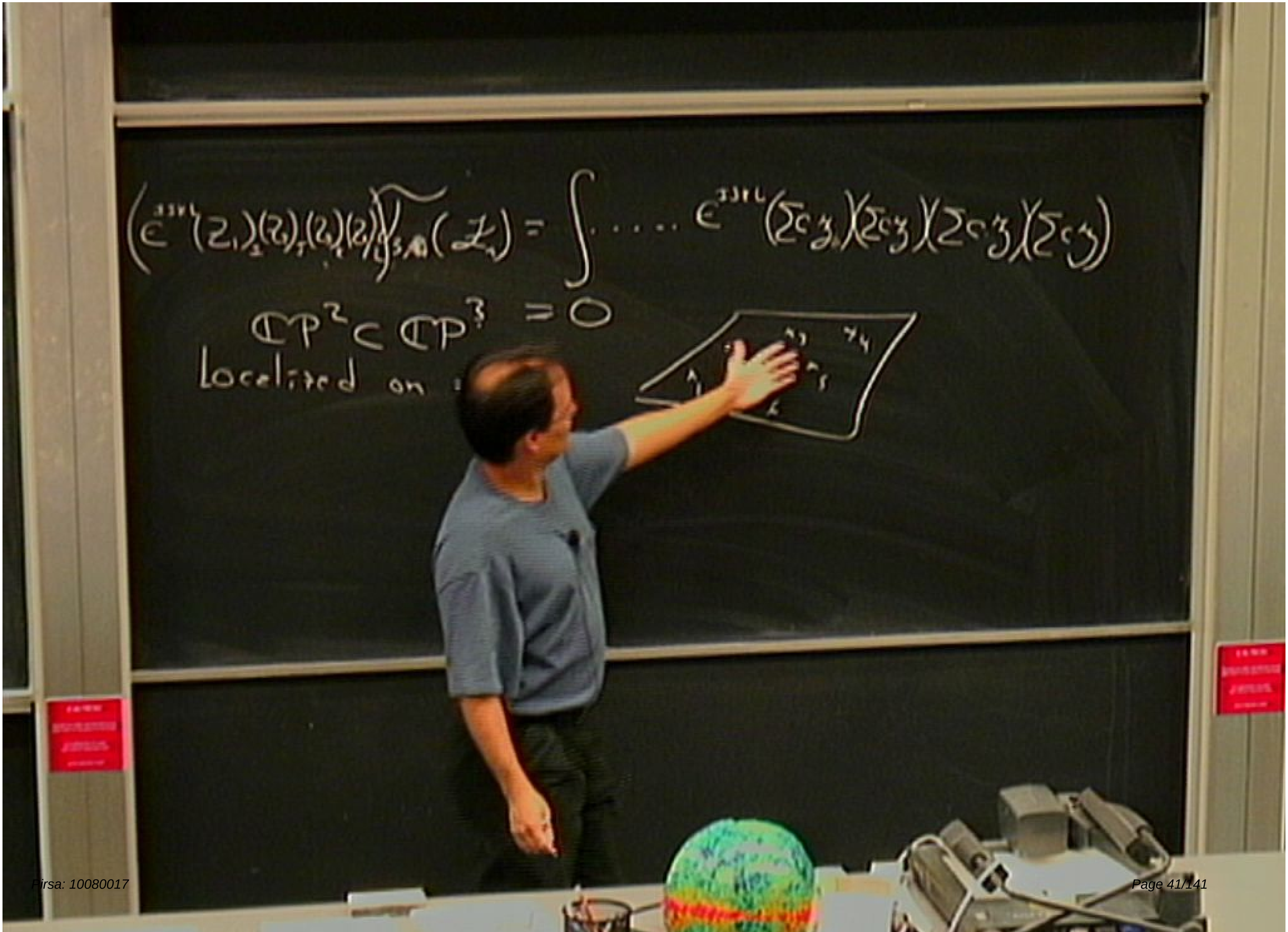
$$\tilde{\mathcal{J}}_{3,6}(\mathcal{Z}_i) = \prod_{a=1}^{3}\int d^{4|4}\mathcal{Z}_a \int d^{3\times 6}C\, \mu(C) \prod_{i=1}^{6} \delta^{4|4}\left(\mathcal{Z}_i - \sum_{a=1}^{3} C_{ai}\mathcal{Z}_a\right)$$

Claim: $\tilde{\mathcal{J}}(\mathcal{Z})$ is not a function.

It is a distribution





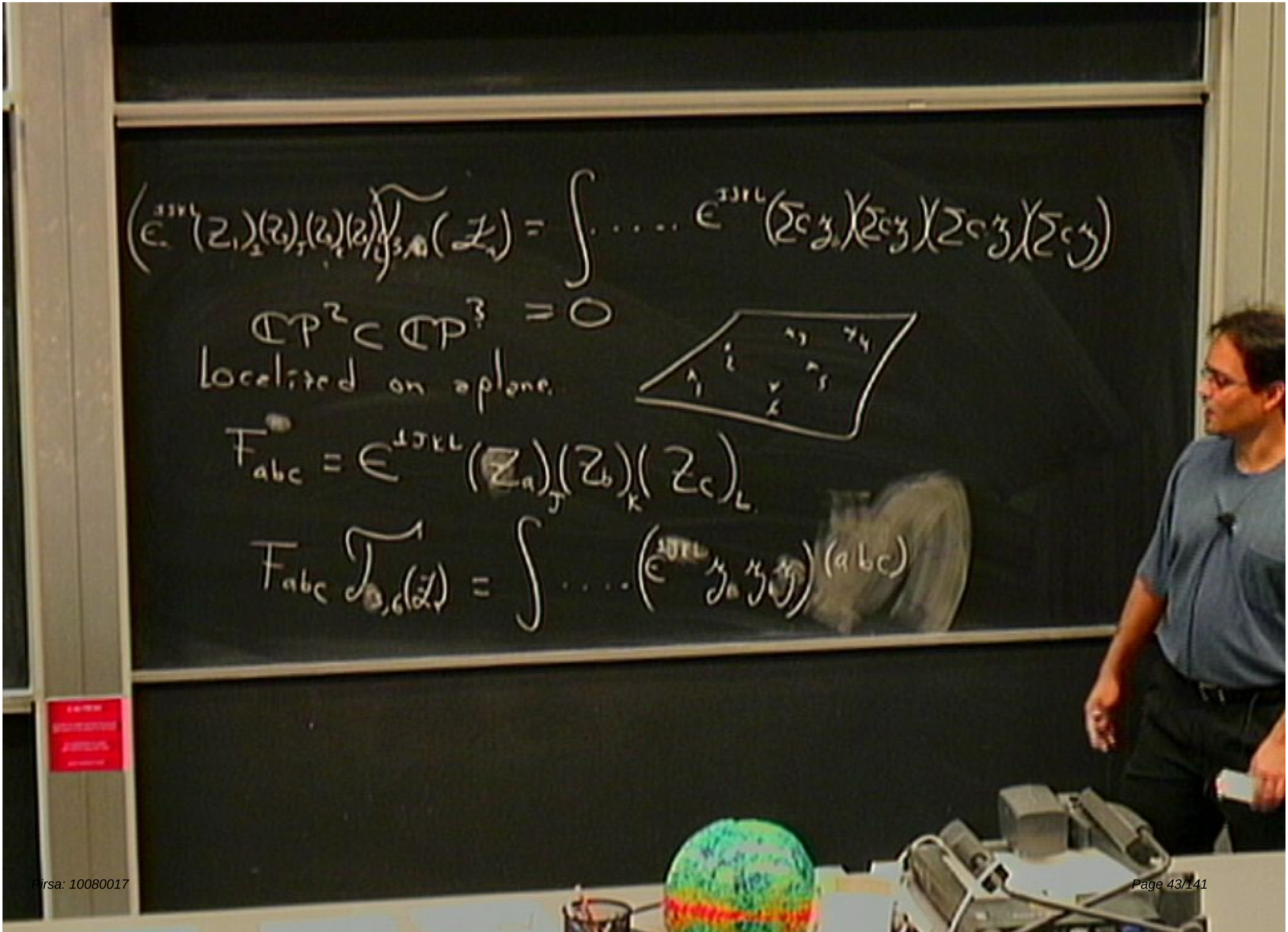

$\mathbb{CP}^2 \subset \mathbb{CP}^3 = 0$
localized on

$$\left(\epsilon^{IJKL}(Z_1)_I(Z_2)_J(Z_3)_K(Z_4)_L\right)\widetilde{f}_{3,4}(\mathcal{Z}_a) = \int \cdots\cdots \epsilon^{IJKL}\left(\sum c\,z\right)\left(\sum c\,z\right)\left(\sum c\,z\right)\left(\sum c\,z\right)$$

$$\mathbb{CP}^2 \subset \mathbb{CP}^3 \Rightarrow 0$$

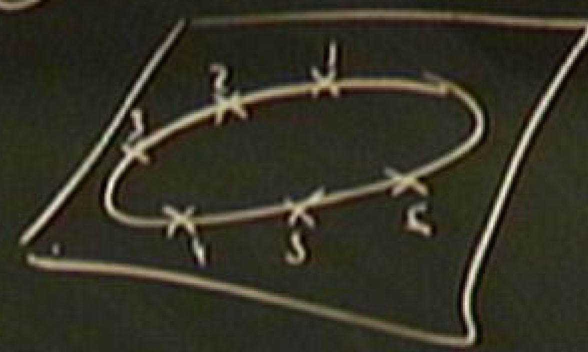
Localized on a plane.

$$F_{abc} = \epsilon^{IJKL}(Z_a)_J(Z_b)_K(Z_c)_L$$



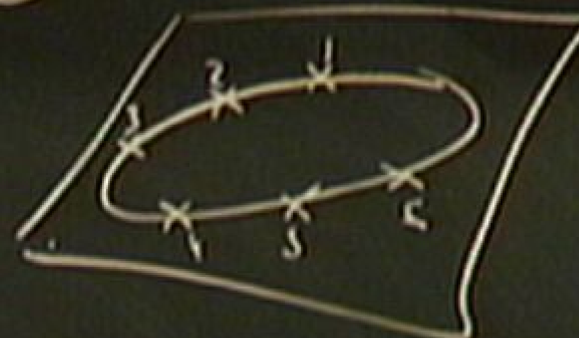
$\left(\epsilon^{IJKL}(Z_1)_I(Z_2)_J(Z_3)_K(Z_4)_L\right)\widetilde{\mathcal{J}}_{3,4}(\mathcal{Z}_4) = \int \cdots \epsilon^{IJKL}(\sum c z)(\sum c z)(\sum c z)(\sum c z)$
$\mathbb{CP}^2 \subset \mathbb{CP}^3 = 0$
localized on a plane.
$F_{abc} = \epsilon^{IJKL}(Z_a)_J(Z_b)_K(Z_c)_L$
$F_{abc}\,\widetilde{\mathcal{J}}_{3,6}(\mathcal{Z}) = \int \cdots \left(\epsilon^{IJKL} z_a z_b z_c\right)^{(abc)}$

$$\left(F_{234}F_{456}F_{612}F_{123} - FFFF\right)\mathcal{T}_{3,6} = 0$$



$$F_{123}F_{345}F_{561}\,\mathcal{T}_{3,6} =$$

$$\left(F_{234}F_{456}F_{612}F_{123} - FFFF\right)\mathcal{J}_{3,6} = 0$$



$$F_{123}F_{345}F_{561}\,\mathcal{J}_{3,6} = 0$$

cyclic

$$F_{i\,i+1\,i+2}\,F_{i+2\,i+3\,i+4}\,F_{i+4\,i+5\,i+6}\,\mathcal{J} = 0$$

$$\left(F_{234}F_{456}F_{612}F_{123} - FFFF\right)\mathcal{J}_{3,6} = 0$$

$$F_{123}F_{345}F_{561}\,\mathcal{J}_{3,6} = 0$$

$\to$ cyclic.

$$F_{i\,i+1\,i+2}F_{i+2\,i+3\,i+4}F_{i+4\,i+5\,i+6}\,\mathcal{J} = 0$$

Claim:

More precise connection :

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$$\mathcal{L}_{3,6} =$$

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More precise connection :

$\mathcal{L}_{3,n} =$

More precise connection:

$$\mathcal{L}_{3,n} = \frac{\delta^8(\sum \lambda\tilde{\eta})\,\delta^4(\sum \lambda\tilde{\lambda})}{\langle 12\rangle\langle 23\rangle \cdots}$$

More precise connection :

$$\mathcal{L}_{3,n} = \frac{\delta^8(\sum \lambda\tilde{\eta})\,\delta^4(\sum \lambda\tilde{\lambda})}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle}\,\mathcal{R}_{3,n}$$

$\mathcal{R}$

$$\mathcal{R}_{3,n} = \int$$

More precise connection :

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$$\mathcal{R}_{m-1,n} \quad \mathcal{R}_{k}$$

$$\mathcal{R}_{3,n} = \int$$

More precise connection:

$$\mathcal{L}_{3,n} = \frac{\delta^8(\sum \lambda\tilde{\eta})\,\delta^4(\sum \lambda\tilde{\lambda})}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle}\,\mathcal{R}_{3,n}$$

$$\mathcal{R}_{m-1,n}$$
$$\searrow k$$

$$\mathcal{R}_{3,n} = \int \frac{d^n D_a}{D_1 D_2 \cdots D_n}\,\delta^{4|4}\left(\sum_{a=1}^{n} D_a \mathcal{Z}_a\right)$$

It is a distribution

More precise connection:

$$\mathcal{L}_{3,n} = \frac{\delta^{8}(\sum\lambda\tilde{\eta})\,\delta^{4}(\sum\lambda\tilde{\lambda})}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}\,\mathcal{R}_{3,n}$$

$$\mathcal{R}_{3,n} = \int \frac{d^{n}D_a}{D_1 D_2 \cdots D_n}\,\delta^{4|4}\Big(\sum_{a=1}^{n} D_a \mathcal{Z}_a\Big)$$

$$\int d$$

$$\mathcal{R}_{m-1,n}$$

$\hookrightarrow k$

$\mathcal{Z}$ = mome... ...wistors

$$\mathcal{Z} = \begin{pmatrix} \lambda \\ \mu \\ \eta \end{pmatrix}$$

It is a distribution

More precise connection :

$$\mathcal{L}_{3,n} = \frac{\delta^{8}(\sum \lambda\tilde{\eta})\,\delta^{4}(\sum \lambda\tilde{\lambda})}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle}\,\mathcal{R}_{n}$$

$$\mathcal{R}_{m-1,n} \to k$$

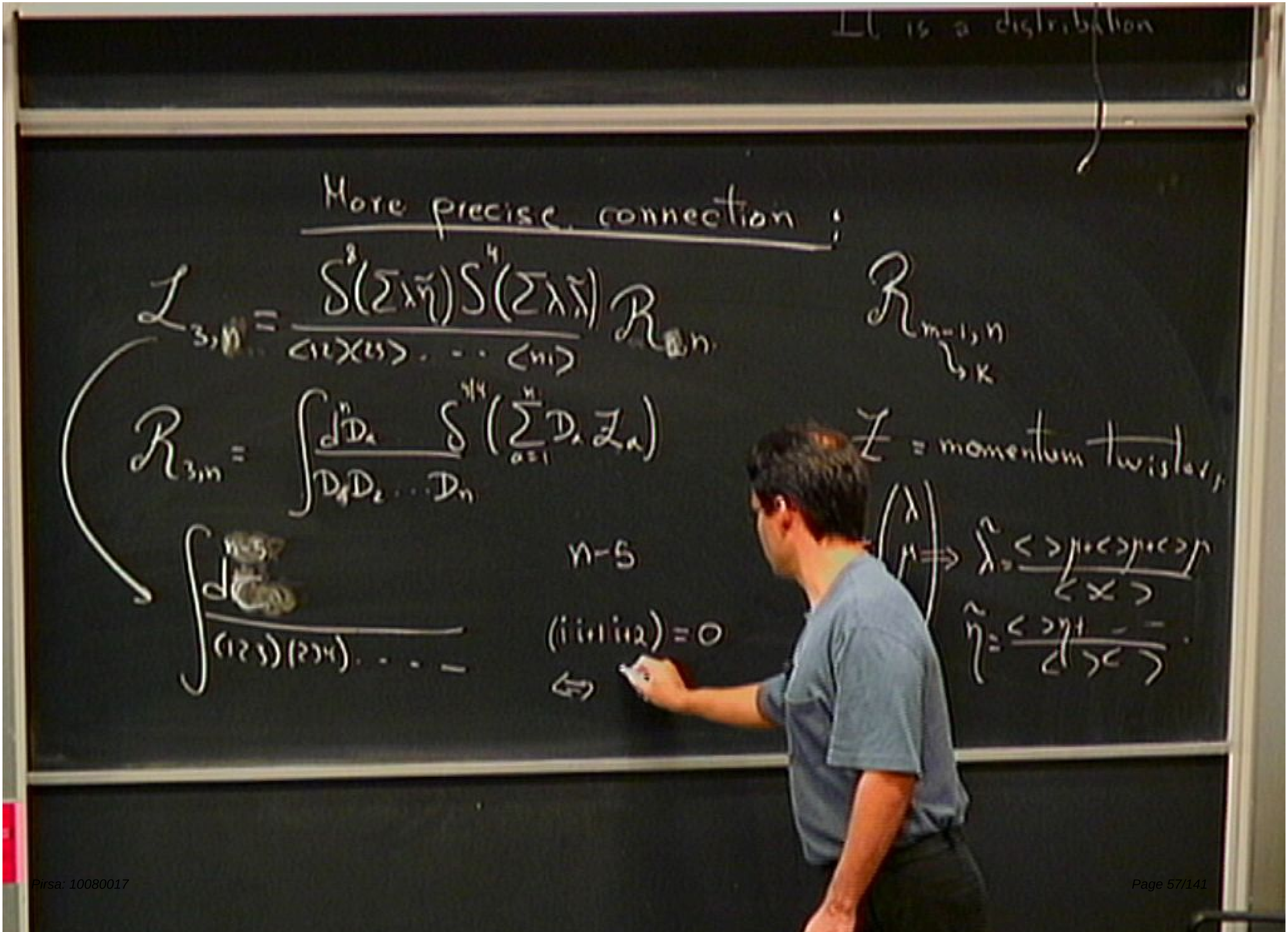
$$\mathcal{R}_{3,n} = \int \frac{d^{n}D_a}{D_1 D_2 \cdots D_n}\,\delta^{4|4}\Big(\sum_{a=1}^{n} D_a \mathcal{Z}_a\Big)$$

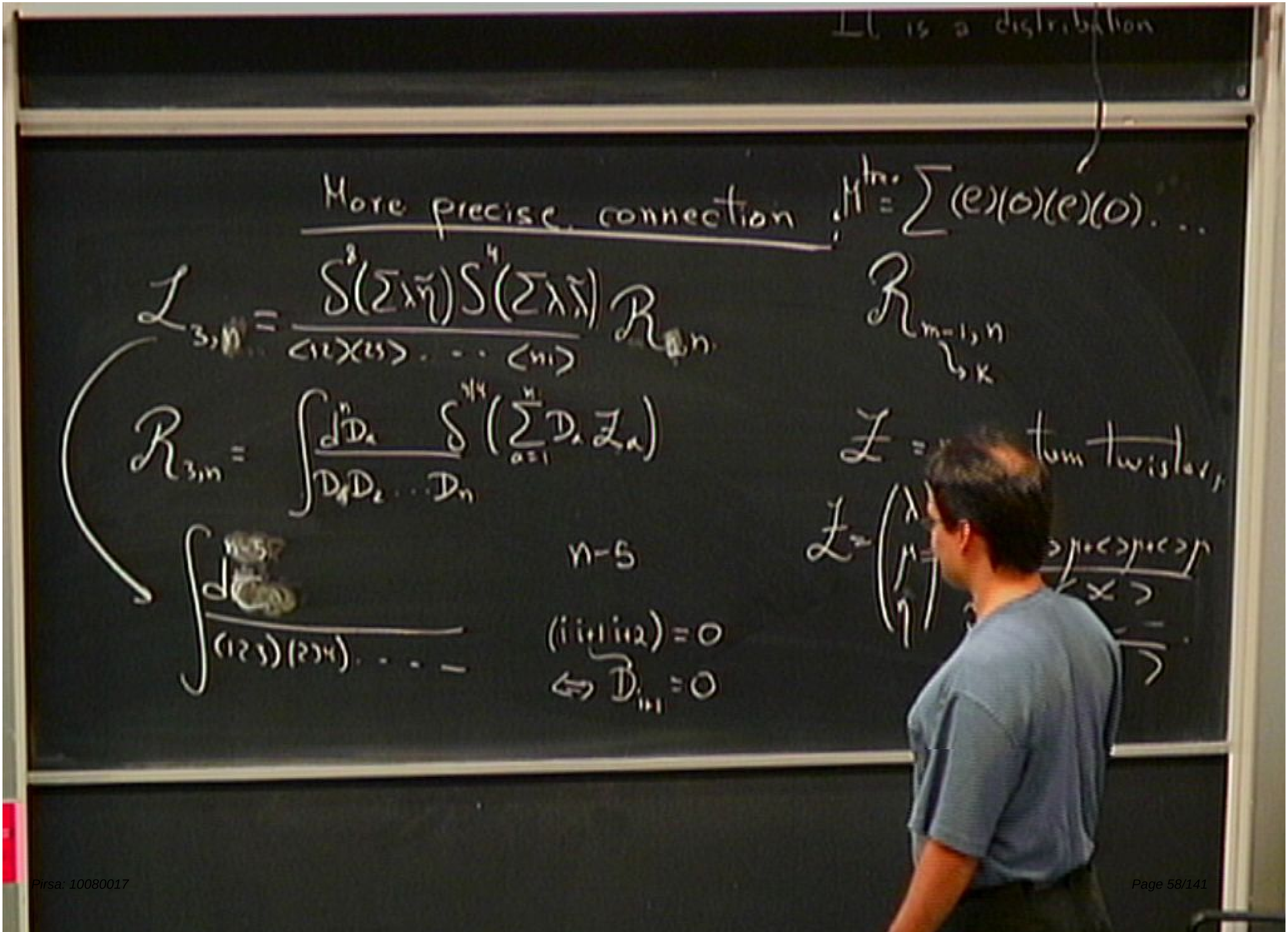
$\mathcal{Z}$ = momentum twistors

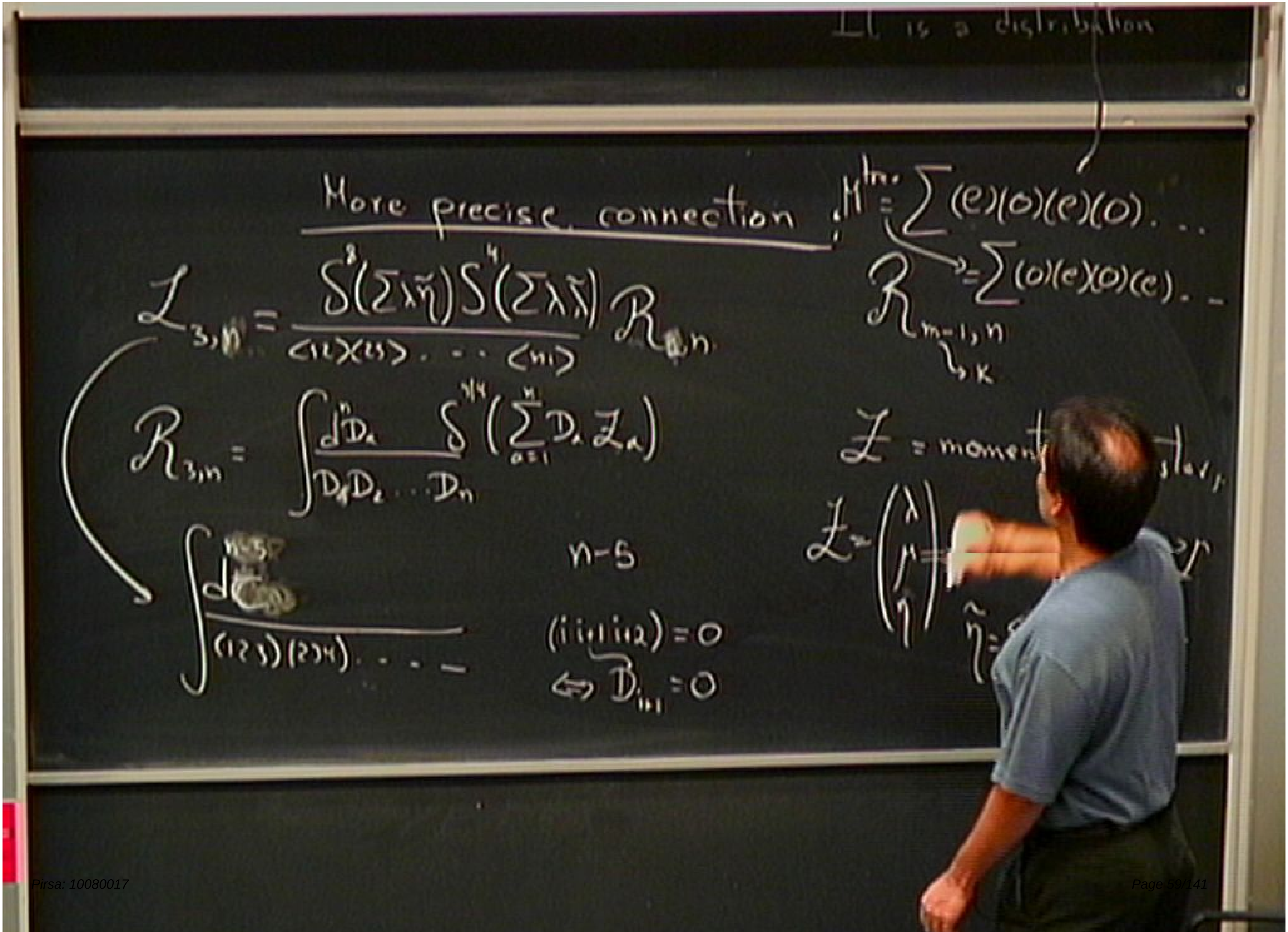
$$\int \frac{d^{3\times n} C_{\kappa a}}{(123)(234)\cdots}$$

$$\mathcal{Z} = \begin{pmatrix} \lambda \\ \mu \\ \eta \end{pmatrix} \Rightarrow \tilde{\lambda} = \frac{\langle\ \rangle\mu + \langle\ \rangle\mu + \langle\ \rangle\mu}{\langle\ \rangle\langle\ \rangle}$$

$$\tilde{\eta} = \frac{\langle\ \rangle\eta + \cdots}{\langle\ \rangle\langle\ \rangle}$$


It is a distribution
More precise connection:
$\mathcal{L}_{3,n} = \frac{\delta^8(\sum \lambda\tilde{\eta})\,\delta^4(\sum \lambda\tilde{\eta})}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle} R_{3,n}$
$R_{n-1,n}$
$\mathcal{R}_{3,n} = \int \frac{d^n D_a}{D_1 D_2 \cdots D_n}\, \delta^{4|4}\left(\sum_{a=1}^{n} D_a \mathcal{Z}_a\right)$
$\mathcal{Z}$ = momentum twistor
$\int \frac{d}{(123)(234)\cdots}$
$n-5$
$(i\; i{+}1\; i{+}2) = 0$
$\Leftrightarrow$


It is a distribution
More precise connection
$M^{\text{tree}} = \sum (e)(o)(e)(o)\ldots$
$\mathcal{L}_{3,n} = \frac{\delta^8(\sum \lambda \tilde{\eta})\,\delta^4(\sum \lambda \tilde{\eta})}{\langle 12\rangle\langle 23\rangle \ldots \langle n1\rangle} \mathcal{R}_{3,n}$
$\mathcal{R}_{m-1,n}$
$\mathcal{R}_{3,n} = \int \frac{d^n D_a}{D_1 D_2 \ldots D_n} \delta^{4|4}\left(\sum_{a=1}^{n} D_a Z_a\right)$
Z =
tum twistor
$\int \frac{d}{(123)(234)\ldots}$
$n-5$
$(i\ i{+}1\ i{+}2) = 0$
$\Leftrightarrow D_{i+1} = 0$


is a distribution
More precise connection
$M^{\text{tree}} = \sum (e)(o)(e)(o) \ldots$
$= \sum (o)(e)(o)(e) \ldots$
$\mathcal{L}_{3,n} = \frac{\delta^8(\sum \lambda\tilde{\eta})\,\delta^4(\sum \lambda\tilde{\lambda})}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle} R_{n}$
$R_{m-1,n}$
$R_{3,n} = \int \frac{d^n D_a}{D_1 D_2 \cdots D_n}\, \delta^{n/4}\left(\sum_{a=1}^{n} D_a Z_a\right)$
Z = momentum twistor
$n-5$
$\int \frac{d \ldots}{(123)(234) \cdots}$
$(i\, i{+}1\, i{+}2) = 0$
$\Leftrightarrow D_{i+1} = 0$

It is a distribution

More precise connection

$M^{\text{tree}} = \sum (e)(o)(e)(o) \ldots$

$\mathcal{R} = \sum (o)(e)(o)(e) \ldots$

$$\mathcal{L}_{3,n} = \frac{\delta^3(\sum \lambda \tilde{\eta})\,\delta^4(\sum \lambda \tilde{\lambda})}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle} \mathcal{R}_{3,n}$$

$\mathcal{R}_{n-1,n}$

$\hookrightarrow K$

$$\mathcal{R}_{3,n} = \int \frac{d^n D_a}{D_1 D_2 \cdots D_n} \delta^{4|4}\left(\sum_{a=1}^{n} D_a \mathcal{Z}_a\right)$$

$\mathcal{Z}$ = momentum twistors

$$\mathcal{Z} = \begin{pmatrix} \lambda \\ \mu \\ \eta \end{pmatrix} \Rightarrow \tilde{\lambda} = \frac{\langle\ \rangle \mu + \langle\ \rangle \mu + \langle\ \rangle \mu}{\langle\ \rangle\langle\ \rangle}$$

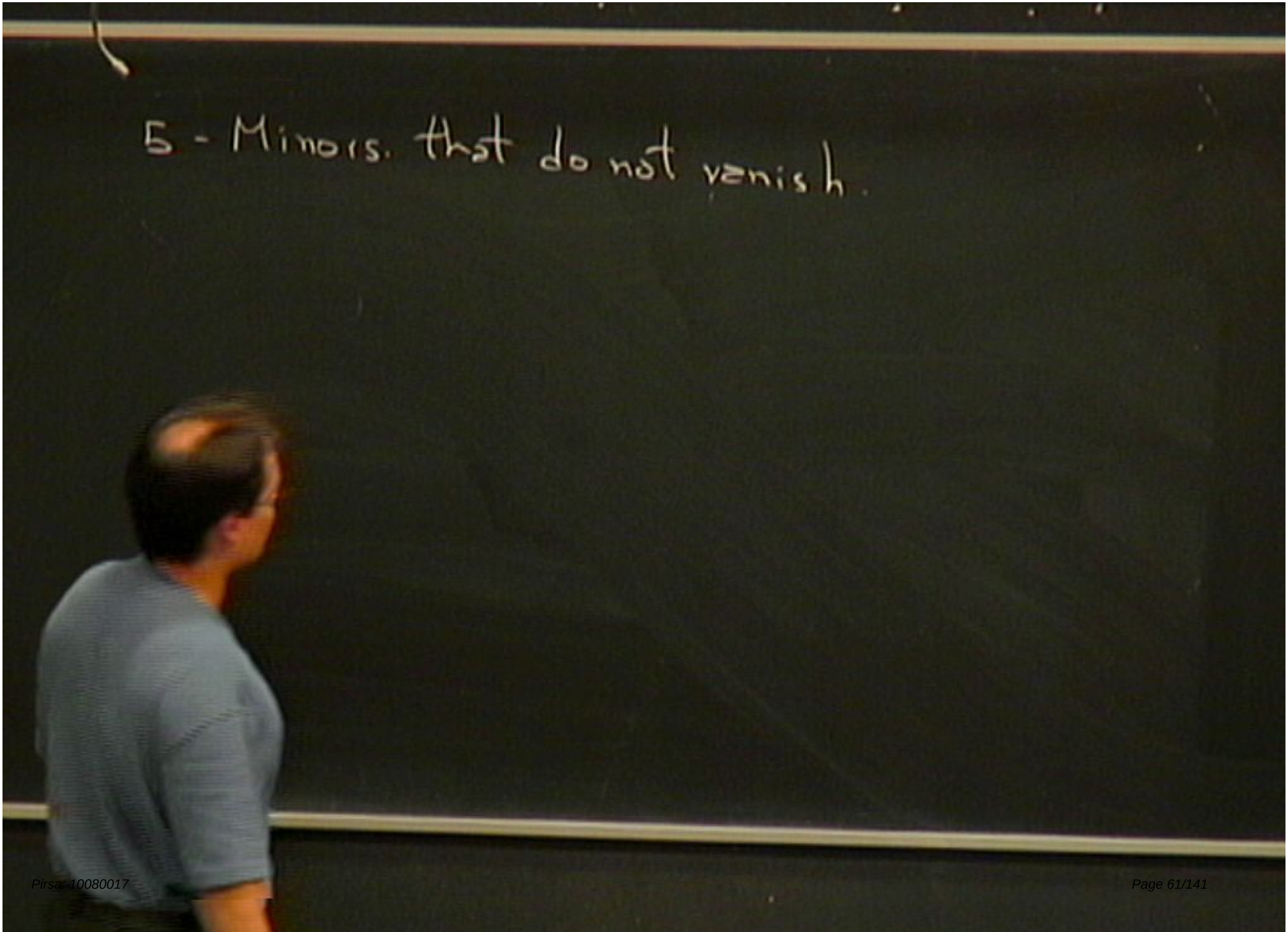
$$\tilde{\eta} = \frac{\langle\ \rangle \eta + \ldots}{\langle\ \rangle\langle\ \rangle}$$

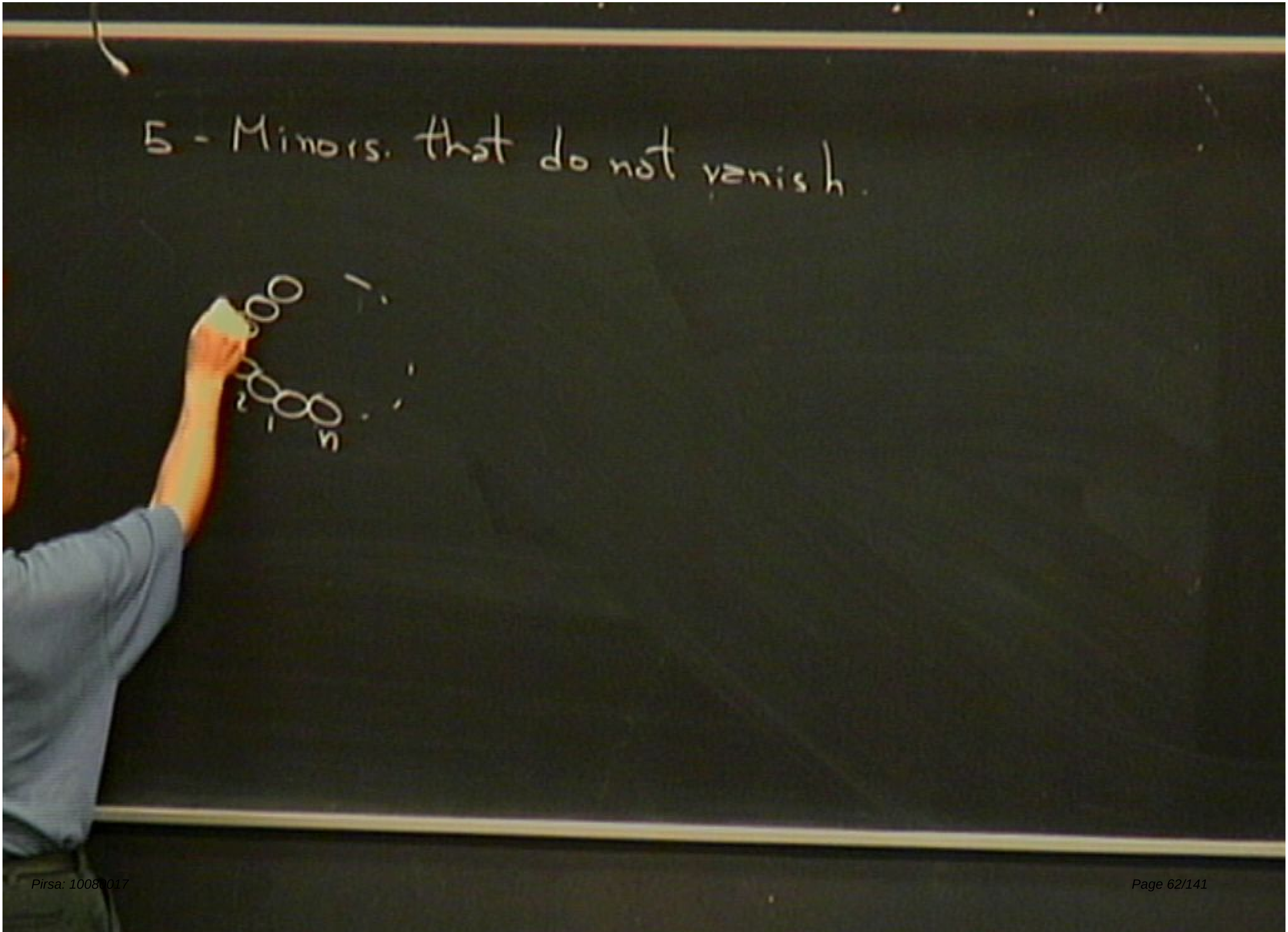
$$\int \frac{d^{n-5} c}{(123)(234) \cdots}$$

$n-5$

$(i\, i{+}1\, i{+}2) = 0$

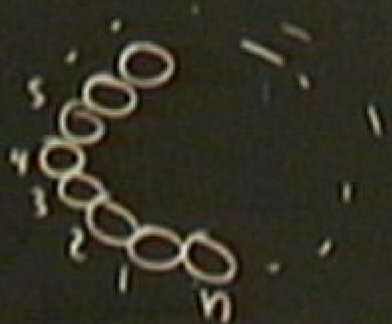
$\Leftrightarrow D_{i+1} = 0$

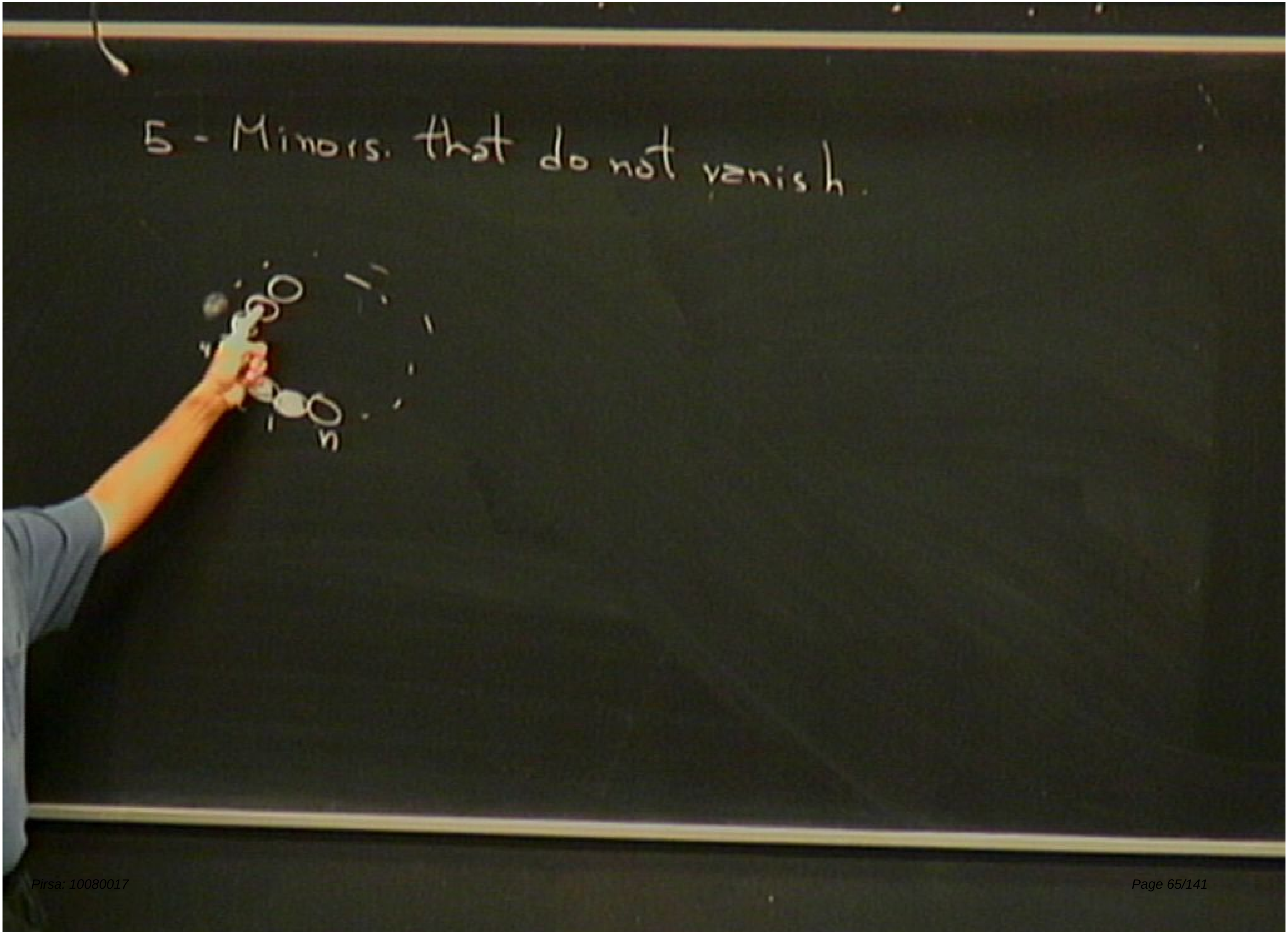

5 - Minors that do not vanish.

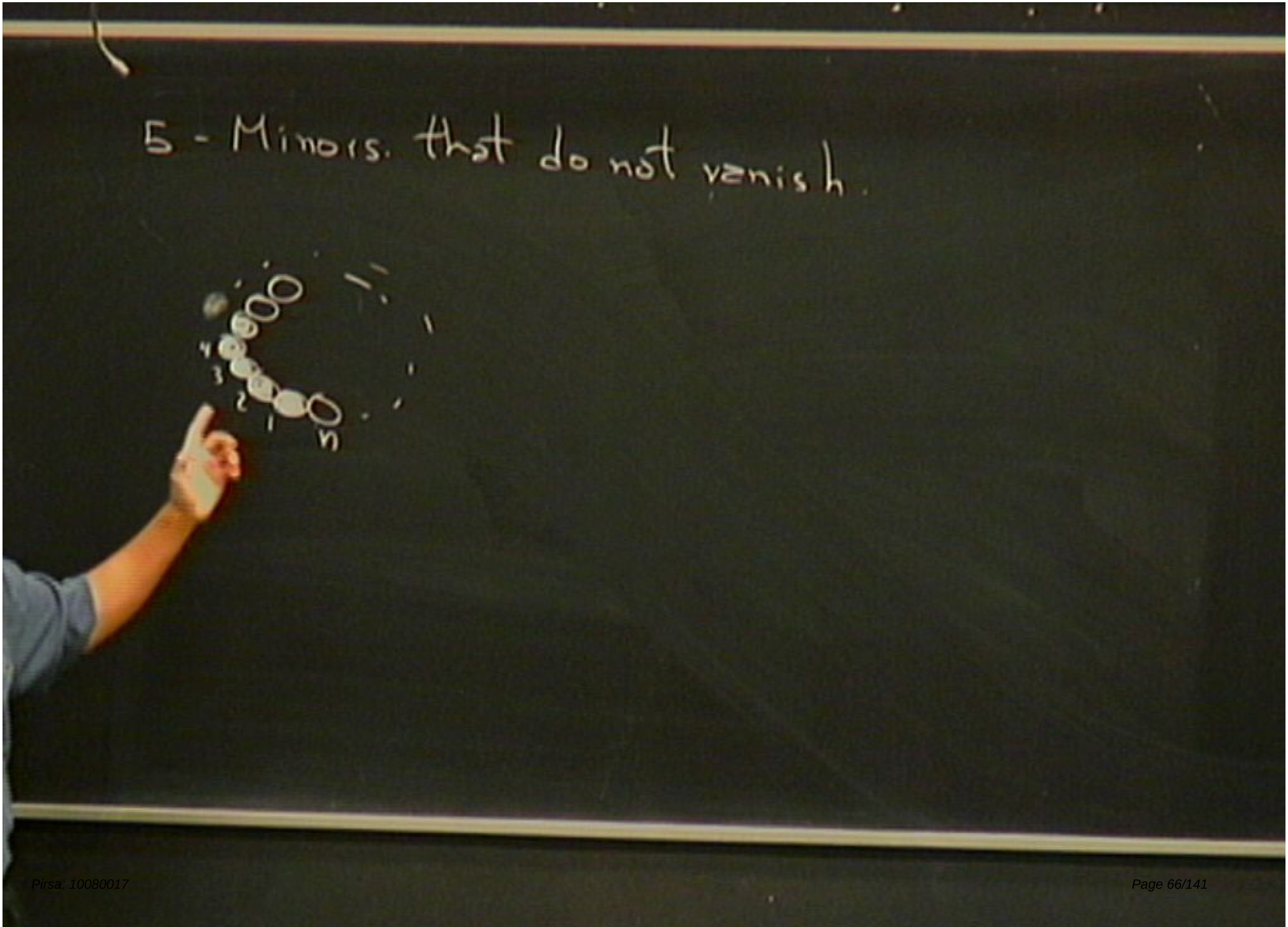

5 - Minors that do not vanish.
2
1
n

5 - Minors that do not vanish.

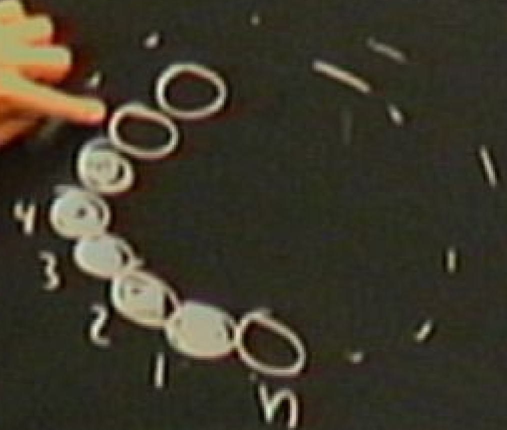
5 - Minors that do not vanish.

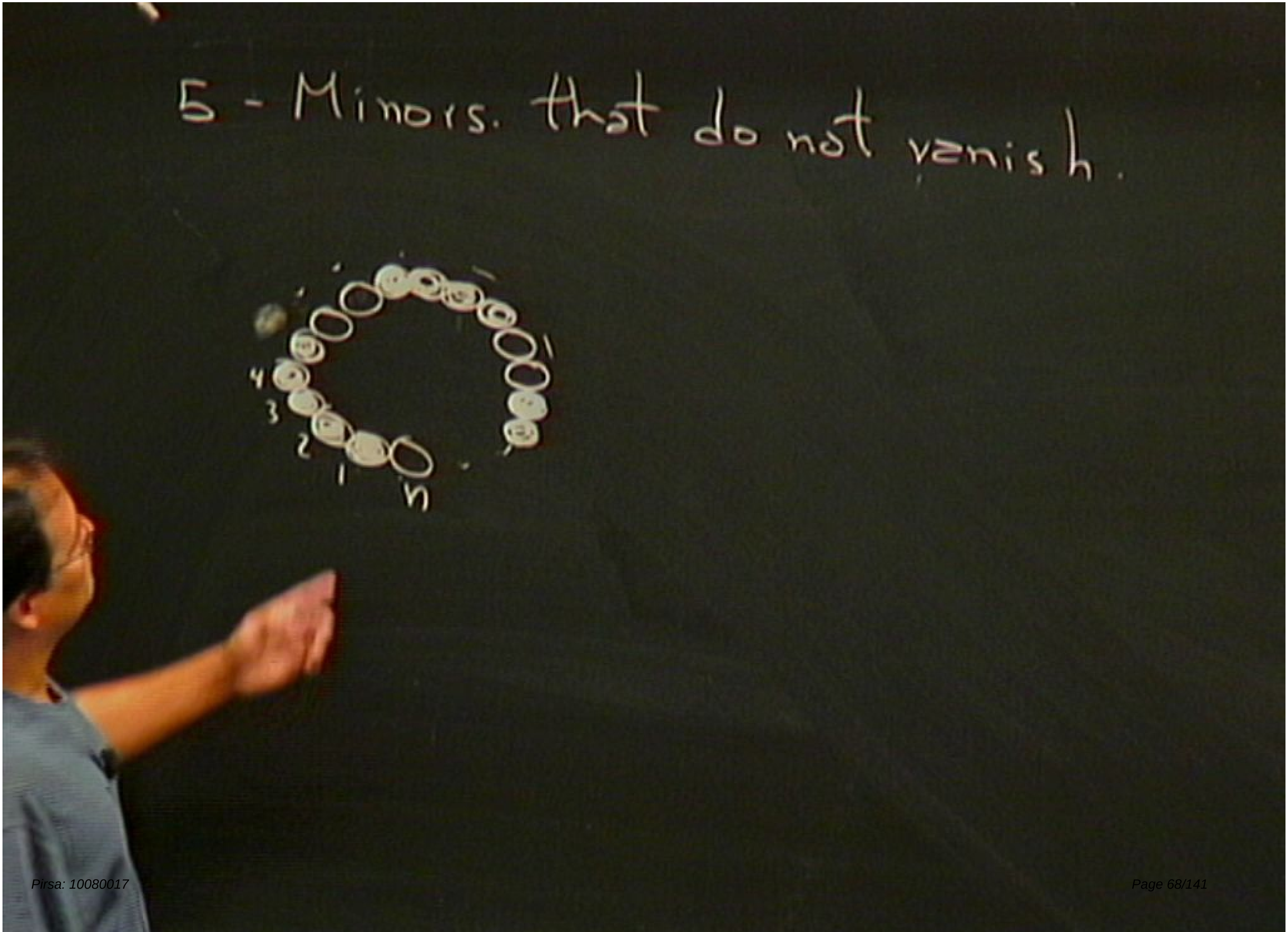



5 - Minors that do not vanish.
1
n


5 - Minors that do not vanish.
4
3
2
1
n

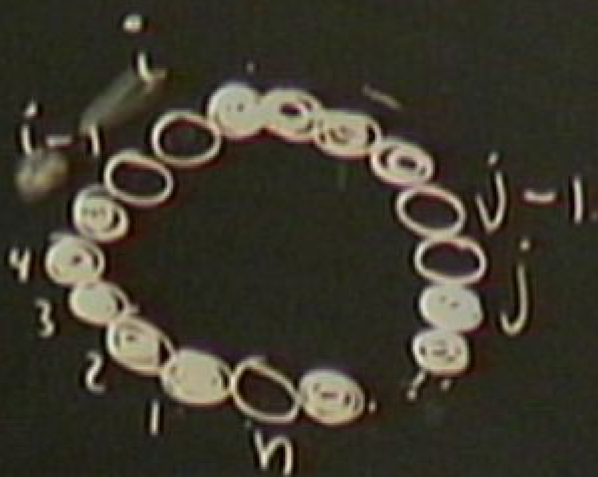
5 - Minors that do not vanish.




5 - Minors that do not vanish.
4
3
2
1
n

5 - Minors. that do not vanish.

$$\sum_{i,j}$$



5 - Minors that do not vanish.

$$M^{\text{tree}} = \sum_{i,j}$$

5 - Minors that do not vanish.

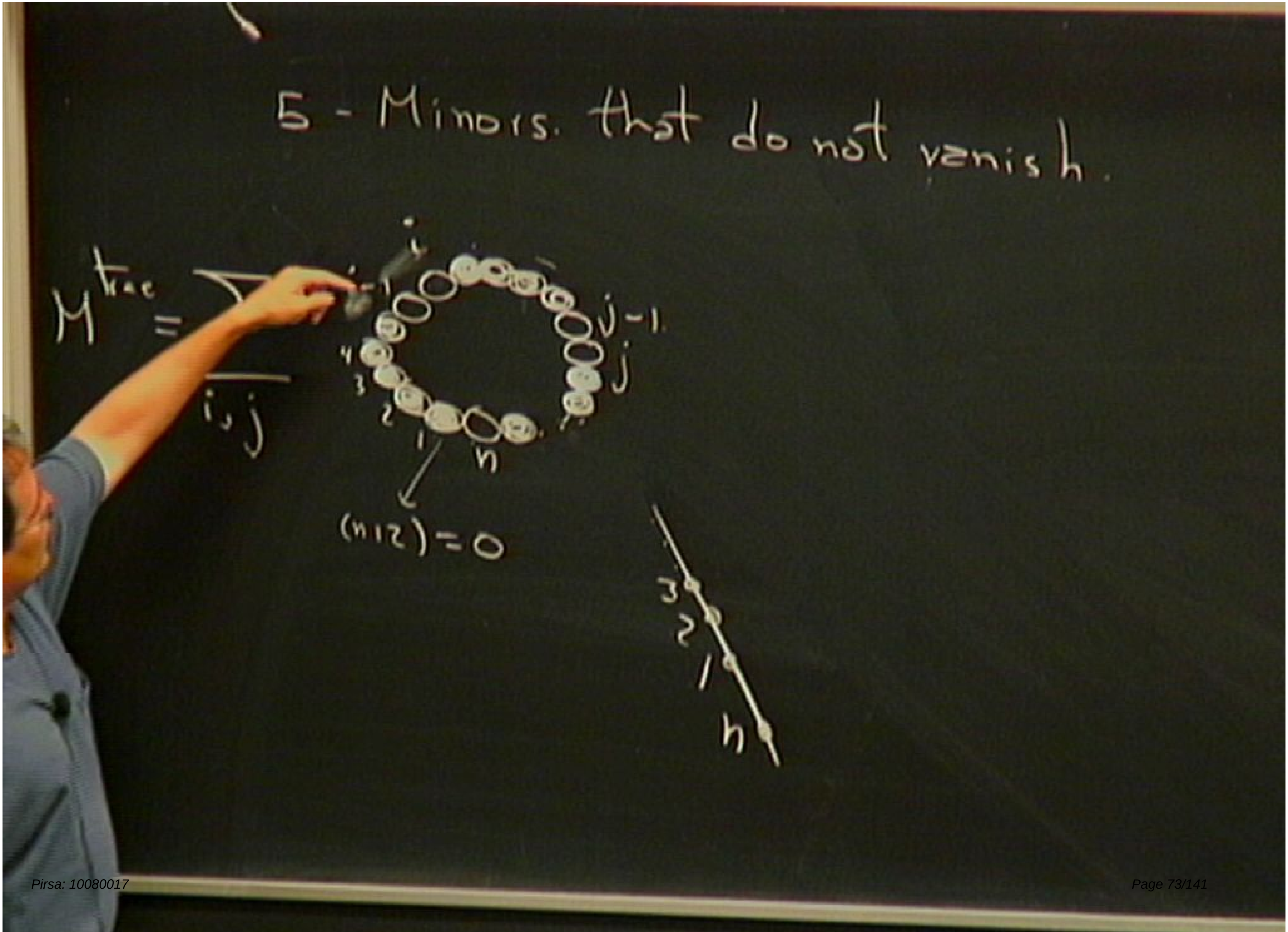
$$M^{\text{trac}} = \sum_{i,j}$$

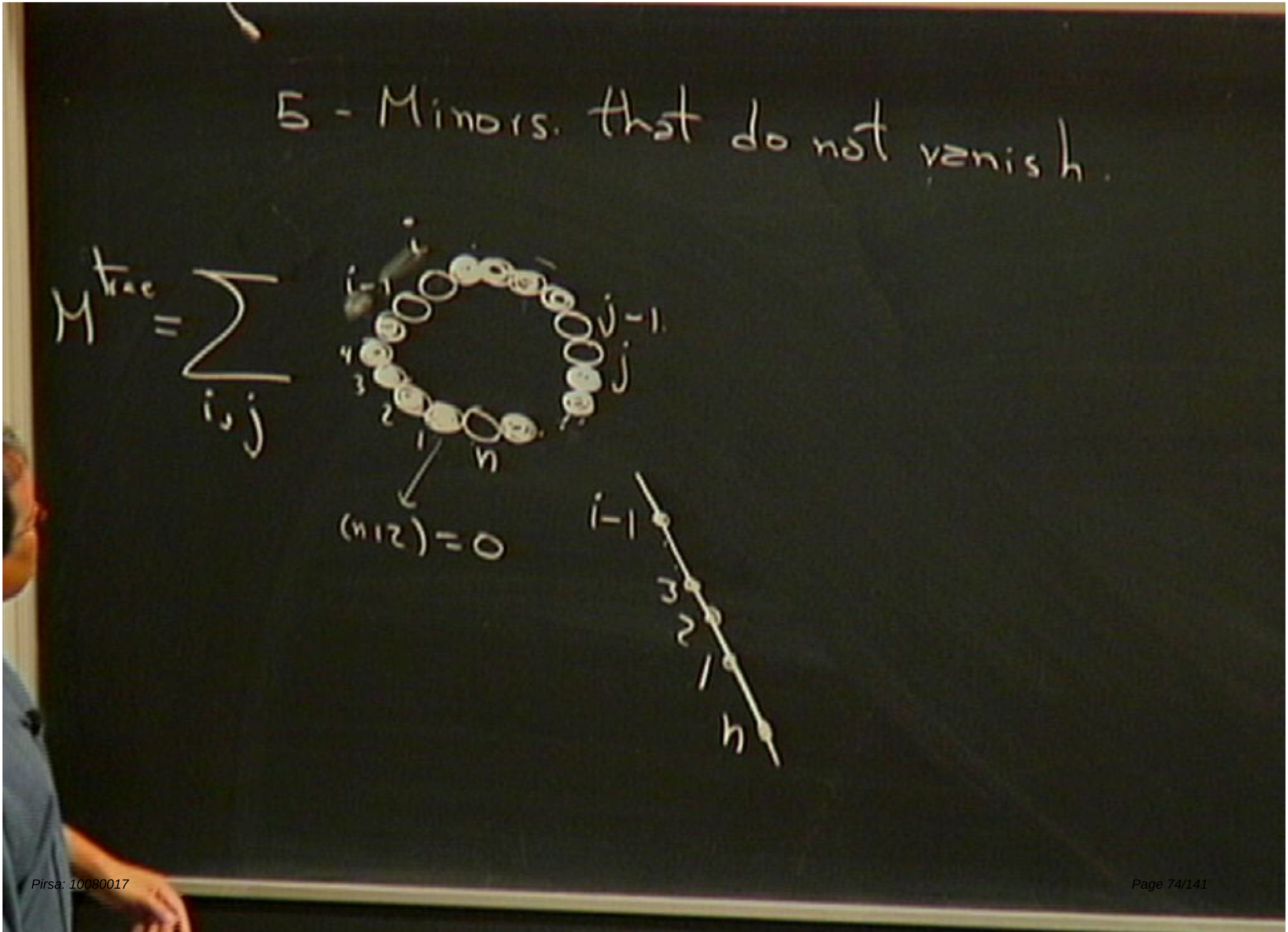
$(n12) = 0$

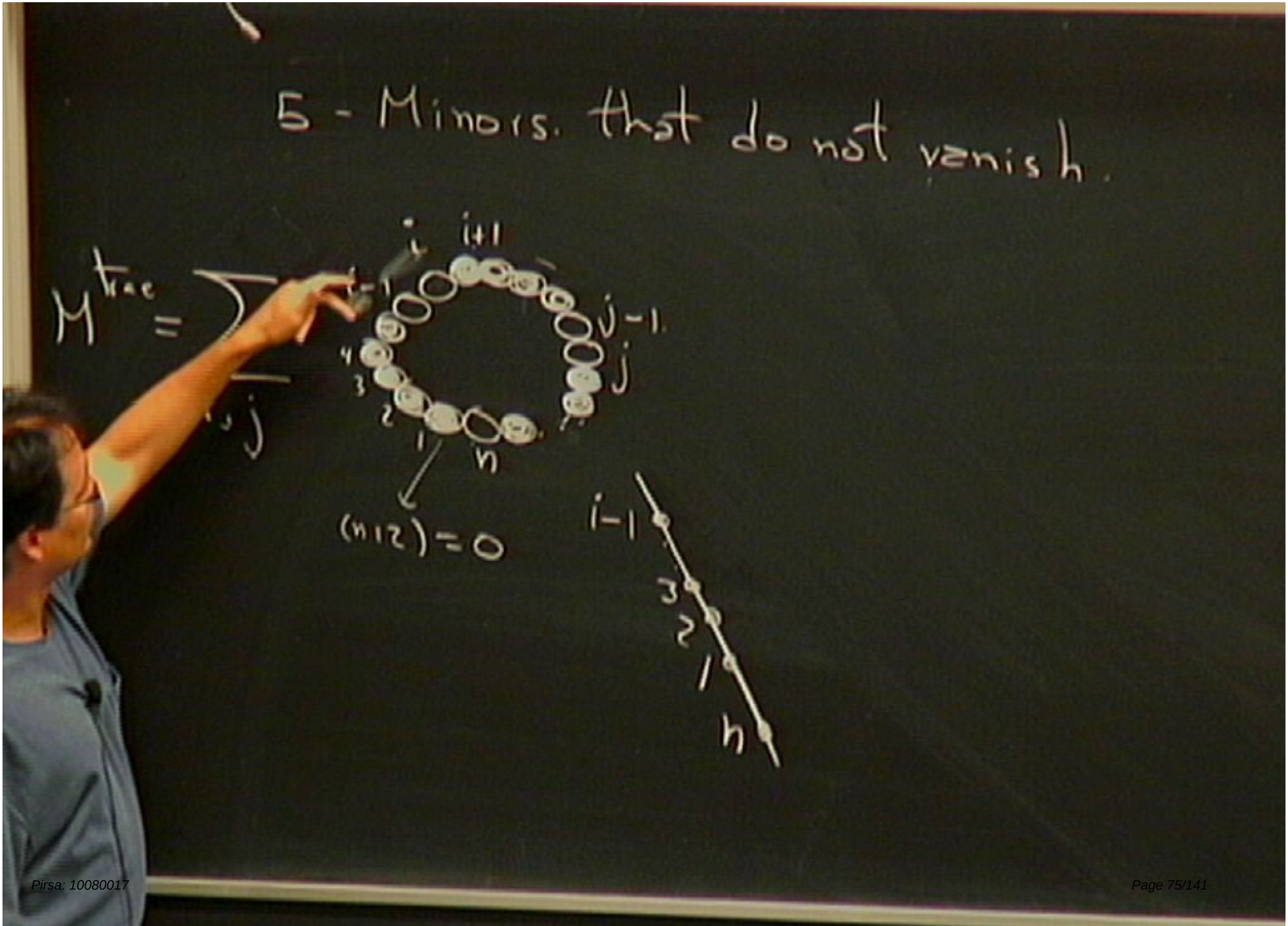
5 - Minors that do not vanish.

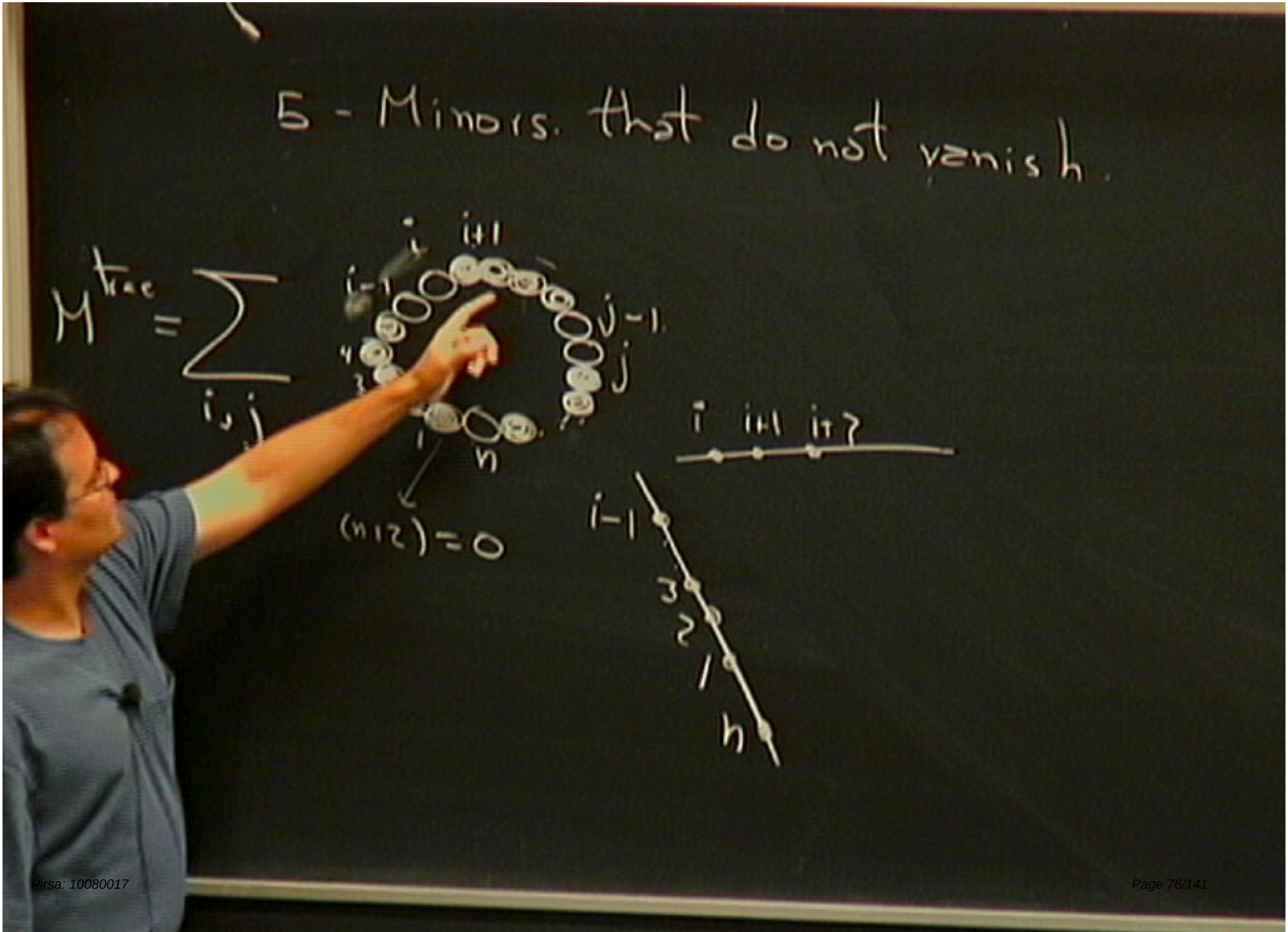
$$M^{tree} = \sum_{i,j}$$

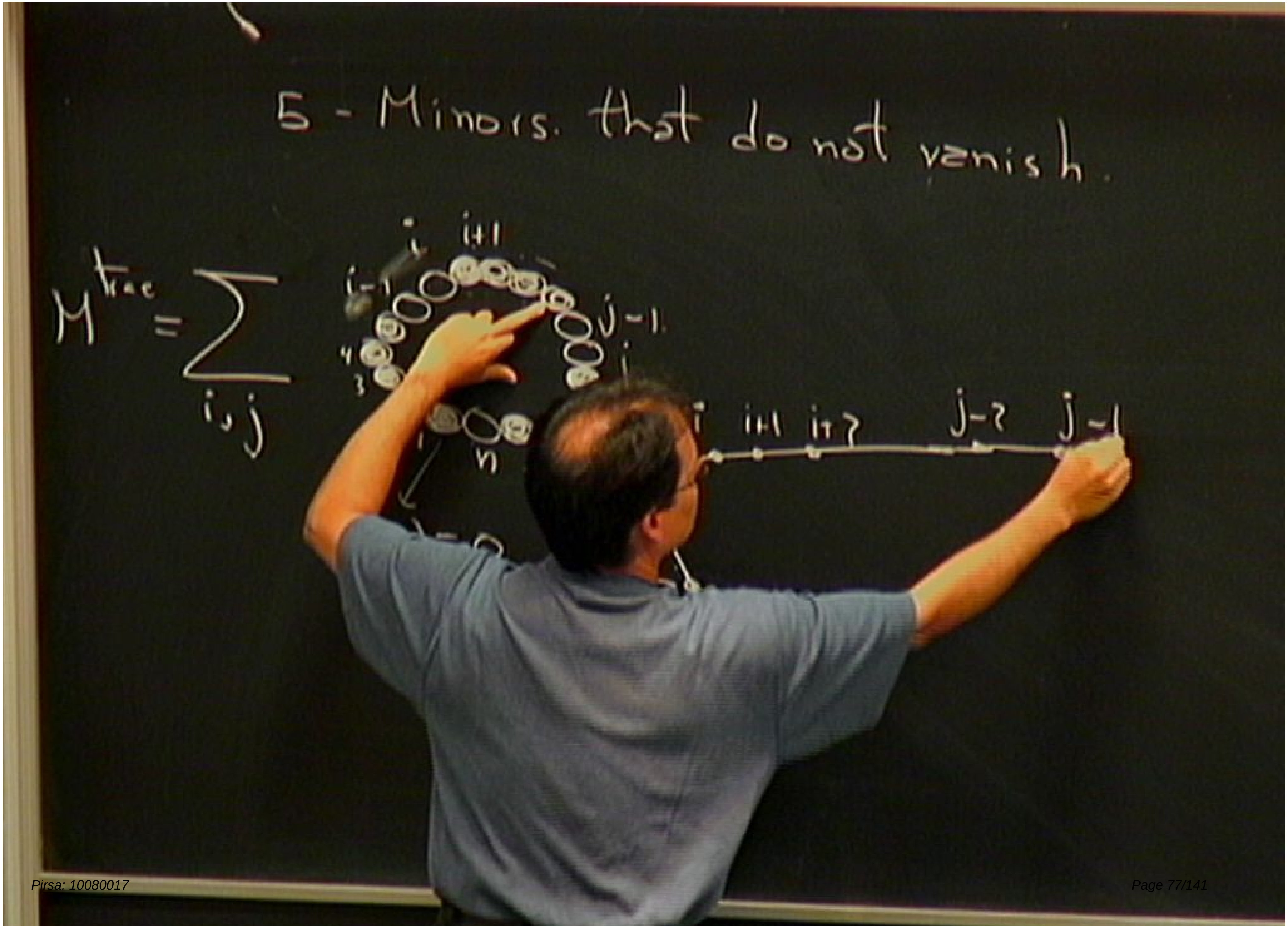
$(n12) = 0$


5 - Minors that do not vanish.
M^trac =
i, j
i
j-1
j
4
3
2
1
n
(n12) = 0
3
2
1
n


5 - Minors that do not vanish.
$M^{tree} = \sum_{i,j}$
i
i-1
j-1
j
4
3
2
1
n
(n12)=0
i-1
3
2
1
n

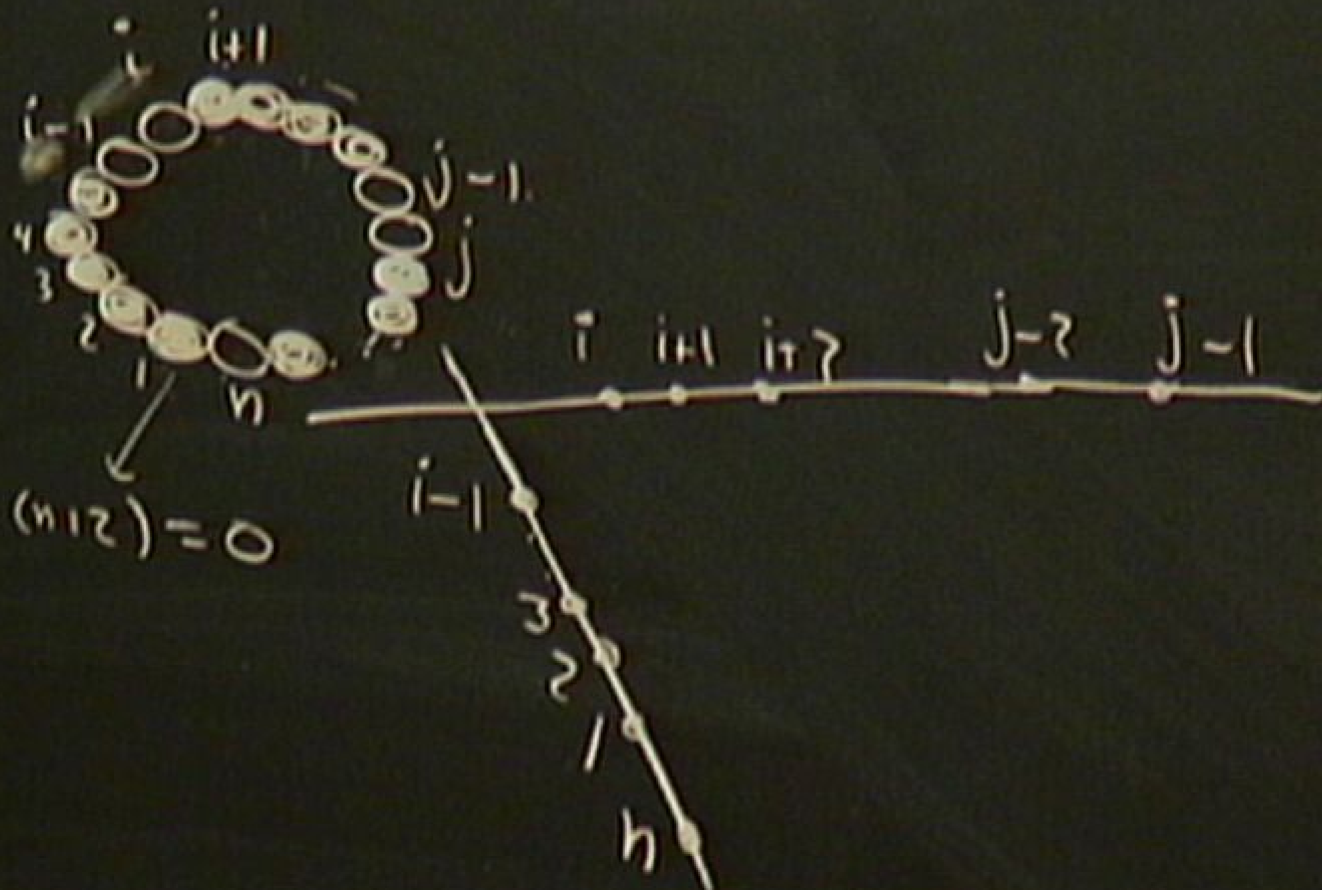

5 - Minors that do not vanish.
M trace = Σ i,j
i
i+1
i-1
j-1
j
4
3
2
1
n
(n12)=0
i-1
3
2
1
n

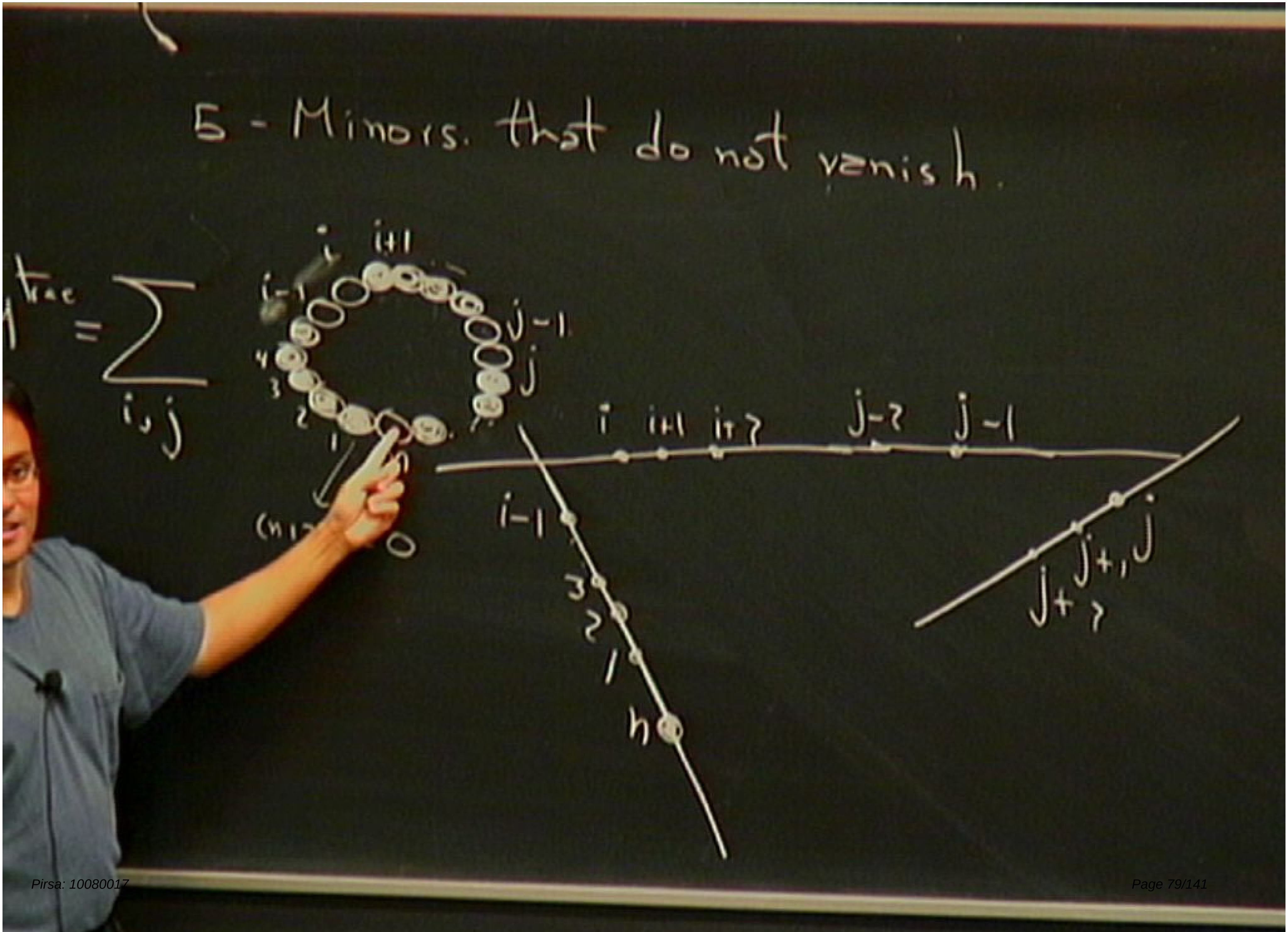

5 - Minors that do not vanish.
M^{tree} = Σ_{i,j}
i i+1
i-1
j-1
j
4
3
1
n
(n12)=0
i i+1 i+2
i-1
3
2
1
n

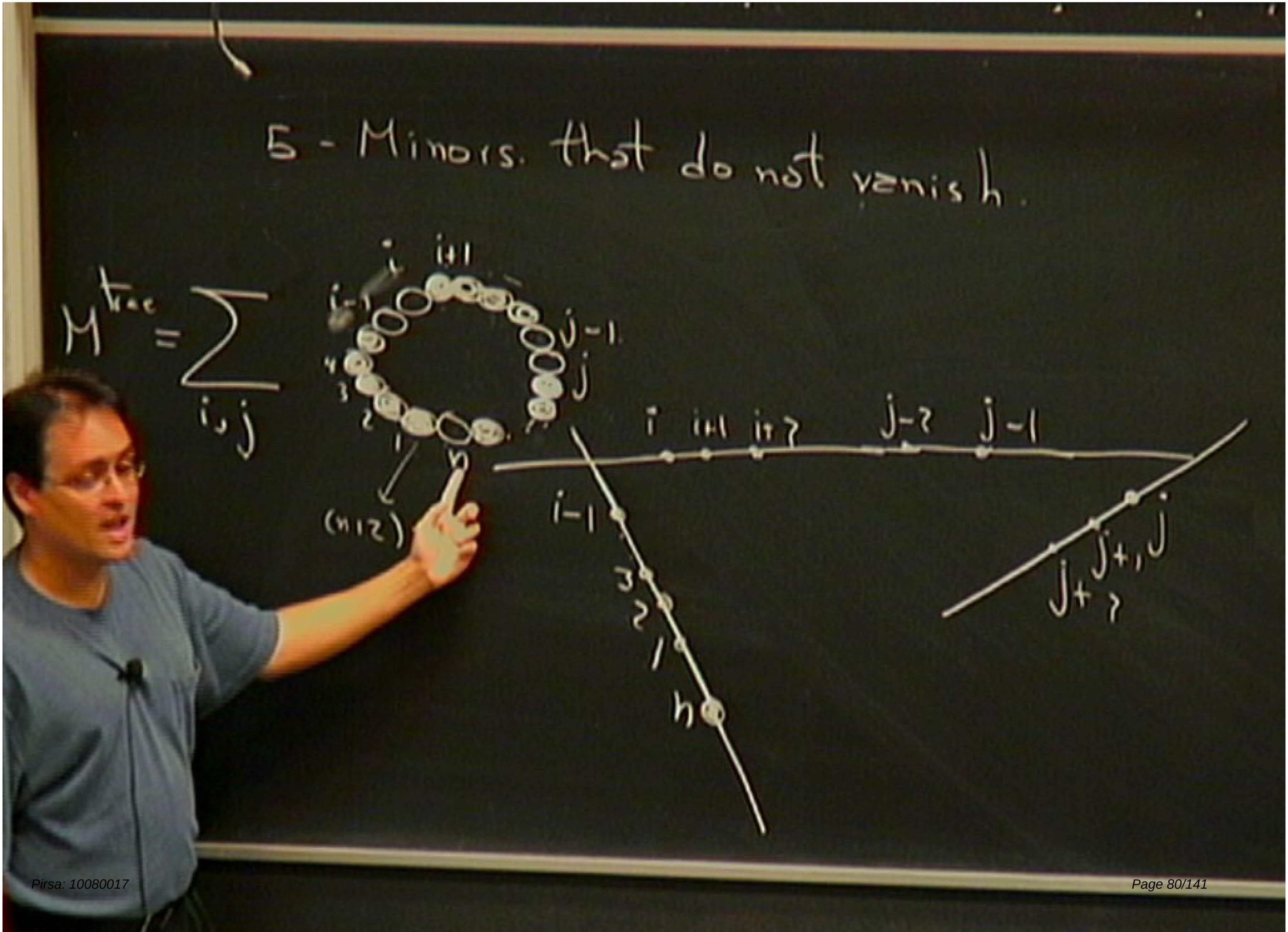

5 - Minors that do not vanish.
M^trac = Σ i, j
i
i+1
i-1
j-1
j
4
3
1
n
i
i+1
i+2
j-2
j-1

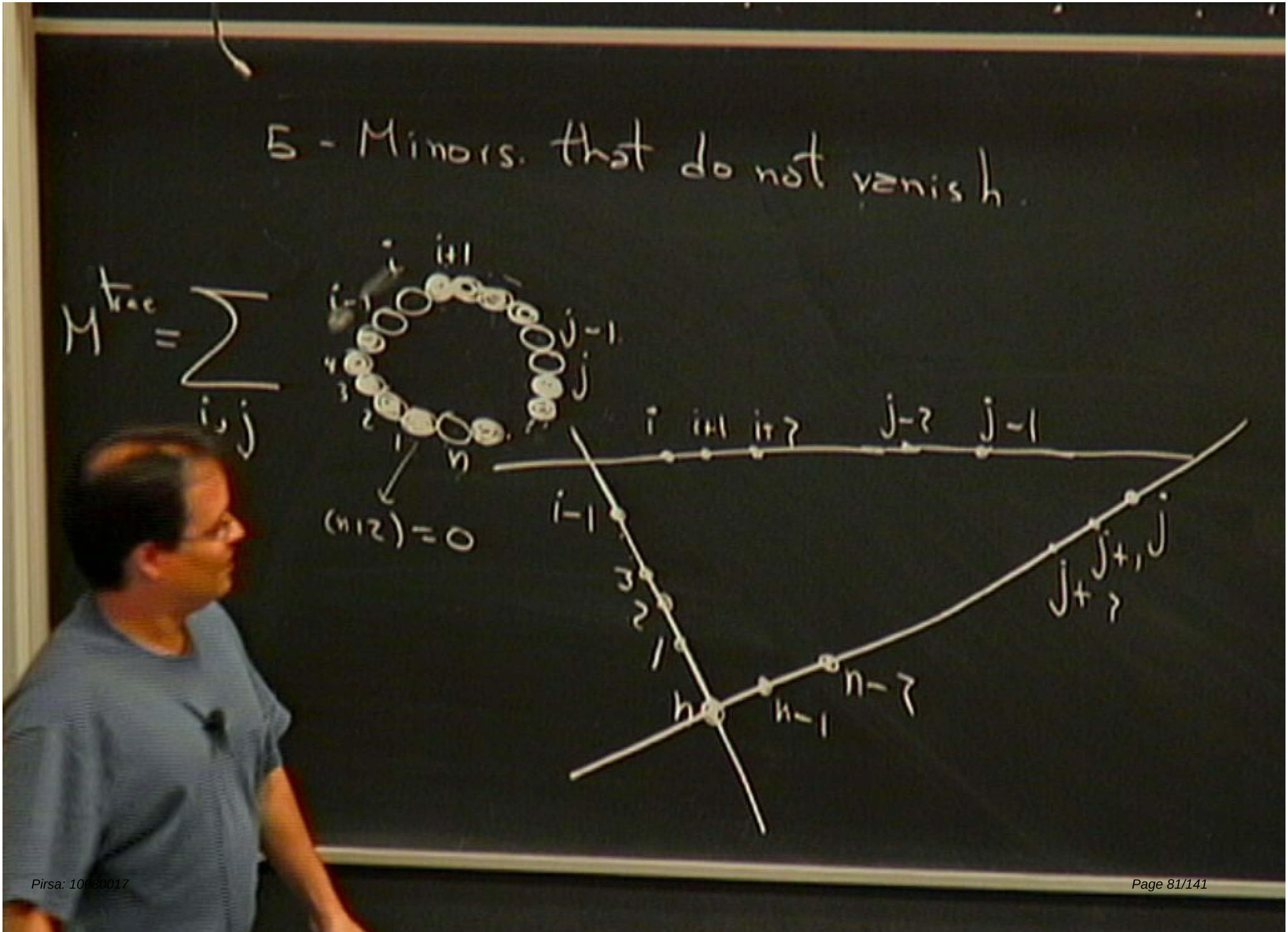
5 - Minors that do not vanish.

$$M^{\text{tree}} = \sum_{i,j}$$




5 - Minors that do not vanish
tree =
i, j
i
i+1
i-1
j-1
j
i+2
j-2
3
2
1
h
j+1
j+2


5 - Minors that do not vanish.
M^tree = Σ_{i,j}
i
i+1
i-1
j-1
j
(n,2)
n
i i+1 i+2 j-2 j-1
i-1
3
2
1
n
j+1 j
j+2


5 - Minors that do not vanish.
M trac = Σ i,j
(n12) = 0
i-1
i
i+1
j-1
j
1
2
3
4
n
i+1
i+2
j-2
j-1
i-1
3
2
1
n
n-1
n-2
j+2
j+1
j

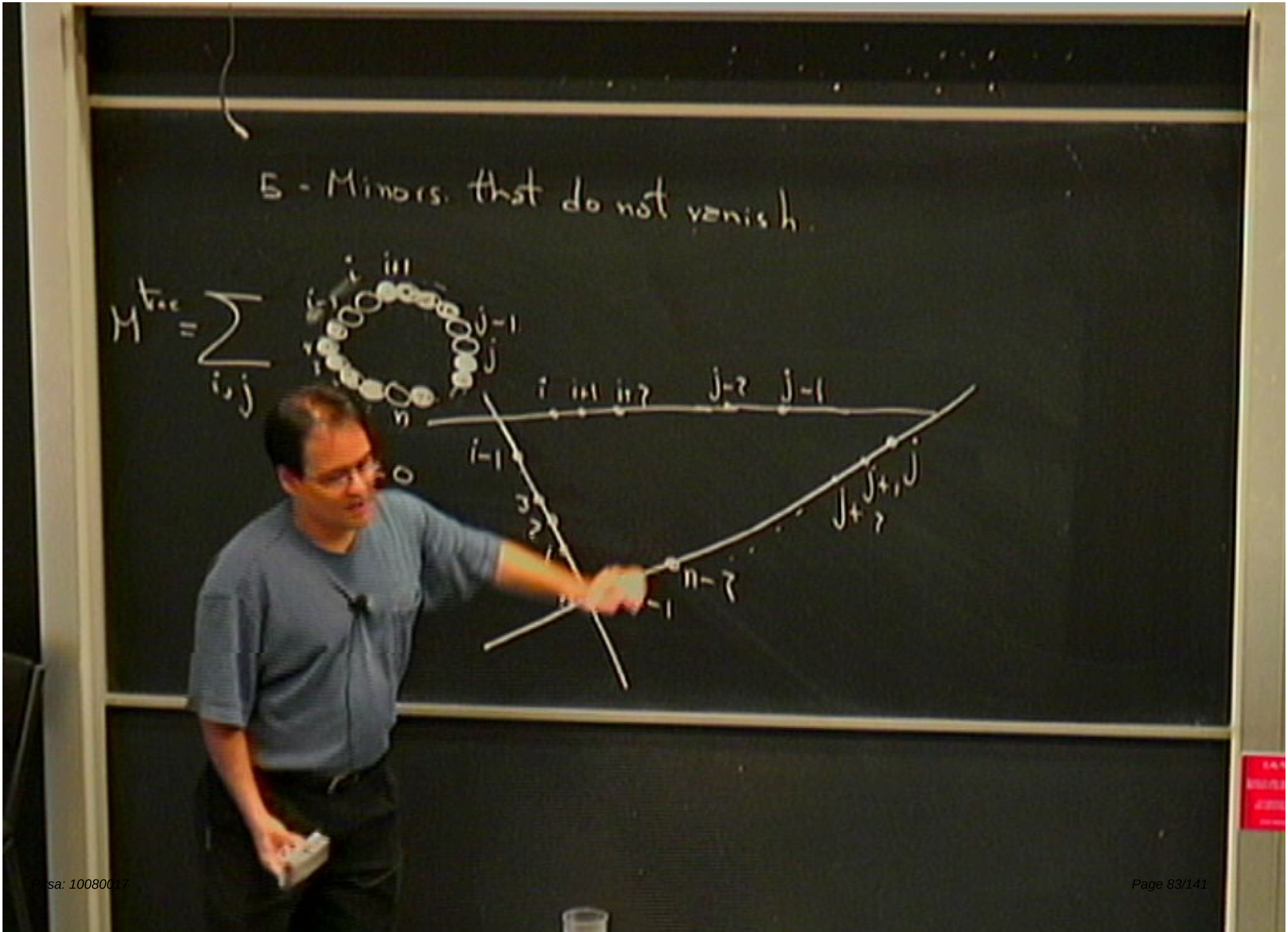
$$\left(\epsilon^{IJKL}(Z_1)_I(Z_2)_J(Z_3)_K(Z_4)_L\right)\widetilde{\mathcal{J}}_{3,4}(\mathcal{Z}_n) = \int \cdots\cdots \epsilon^{IJKL}\left(\sum c\, z\right)\left(\sum c\, z\right)\left(\sum c\, z\right)\left(\sum c\, z\right)$$

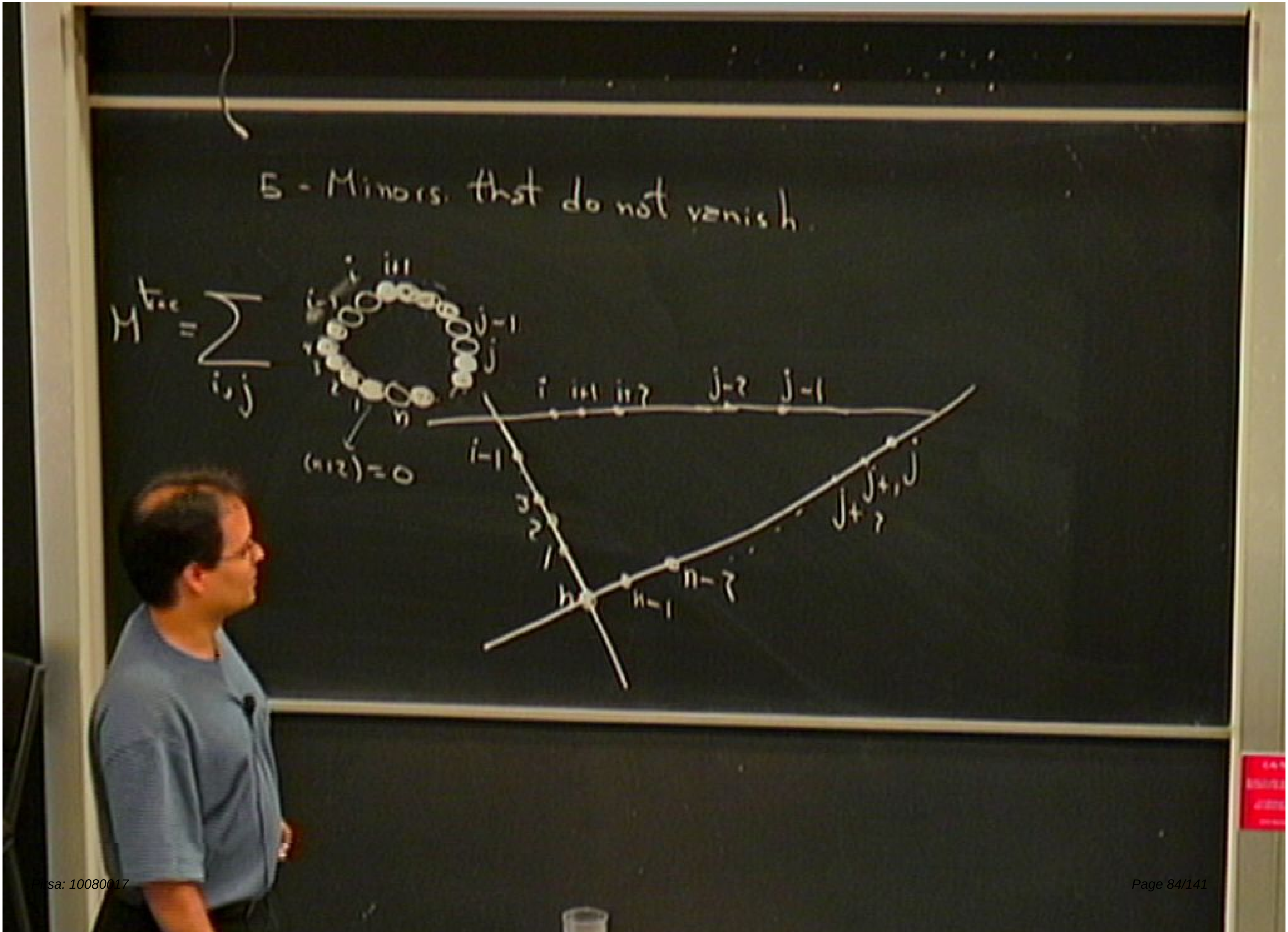
$$\mathbb{CP}^2 \subset \mathbb{CP}^3 = 0$$

Localized on a plane.

$$F_{abc} = \epsilon^{IJKL}(Z_a)_J(Z_b)_K(Z_c)_L$$

$$F_{abc}\,\widetilde{\mathcal{J}}_{3,6}(\mathcal{Z}) = \int \cdots\cdots \left(\epsilon^{IJKL} z\, z\, z\right)^{(abc)}$$


5 - Minors that do not vanish.
M^tree = ∑ i,j
i
i+1
j-1
j
n
i-1
i+2
j-2
n-2
j+1


5 - Minors that do not vanish.
M^tree = ∑ i,j
(n12)=0
i i+1 i+2 j-2 j-1
i-1
n
n-1
n-2
j+2 j+1 j

$\mathcal{L}$ is a distribution

More precise connection

$M^{\text{tree}} = \sum (e)(o)(e)(o) \ldots$

$\mathcal{R} = \sum (o)(e)(o)(e) \ldots$

$\mathcal{R}_{m-1,n} \to k$

$$\mathcal{L}_{3,n} = \frac{\delta^8(\sum \lambda\tilde{\eta})\,\delta^4(\sum \lambda\tilde{\lambda})}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle}\,\mathcal{R}_{3,n}$$

$$\mathcal{R}_{3,n} = \int \frac{d^n D_a}{D_1 D_2 \cdots D_n}\,\delta^{4|4}\Big(\sum_{a=1}^{n} D_a \mathcal{Z}_a\Big)$$

$$\int \frac{d^{n-5}\ldots}{(123)(234)\cdots}$$

$n-5$

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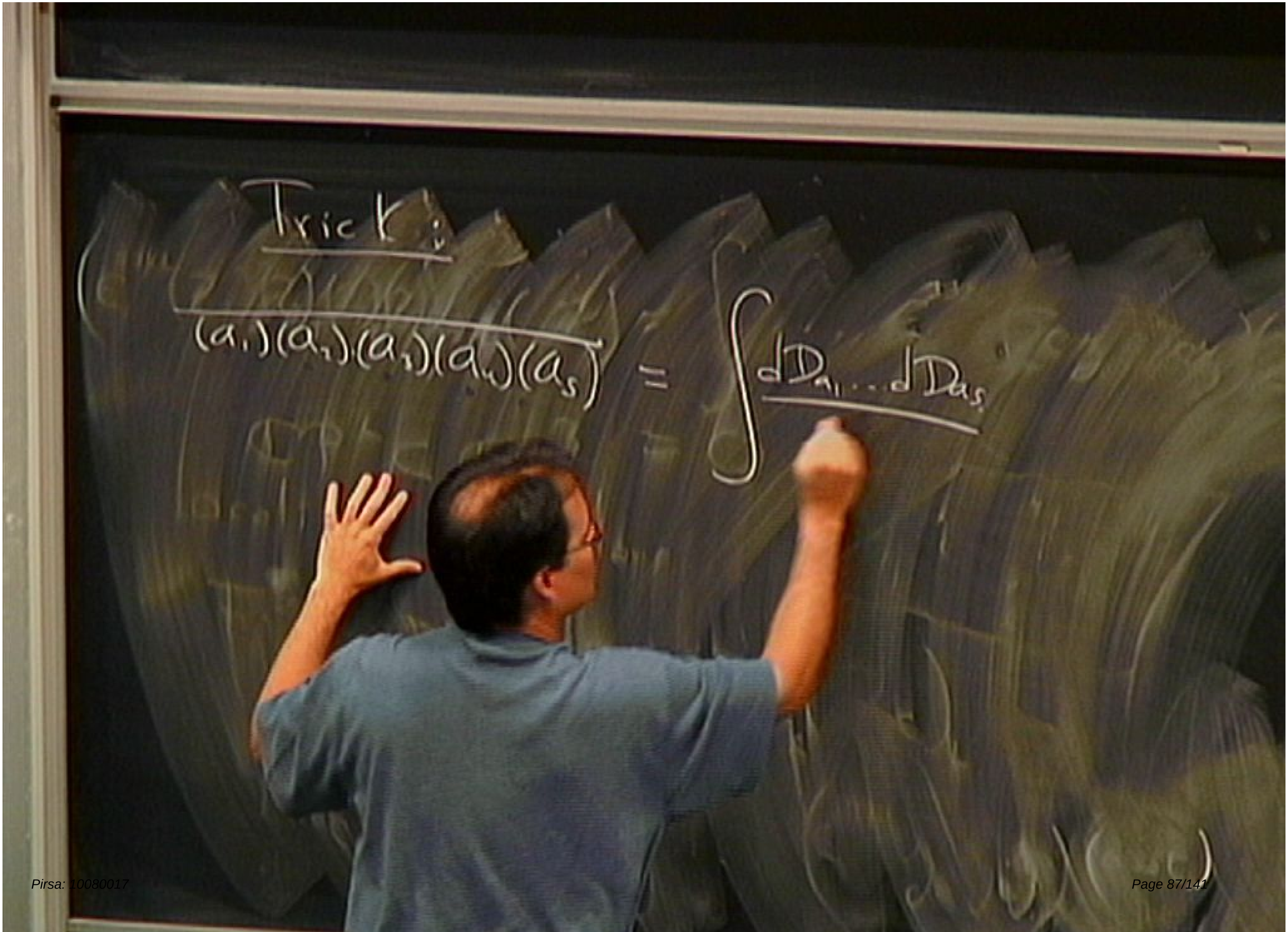
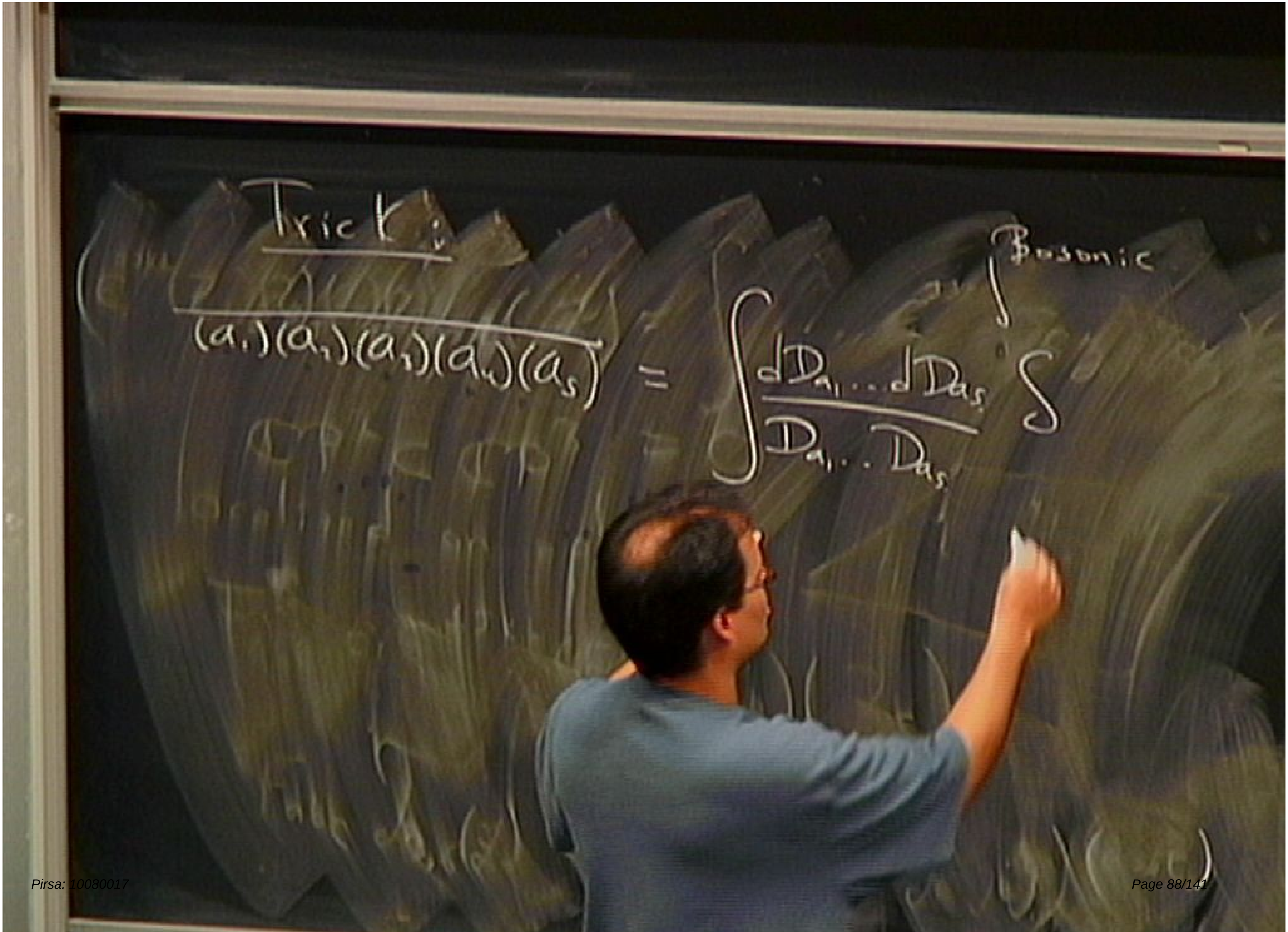
$$\mathcal{Z} = \begin{pmatrix} \lambda \\ \mu \\ \eta \end{pmatrix} \Rightarrow \tilde{\lambda} = \frac{\langle\,\rangle\mu + \langle\,\rangle\mu + \langle\,\rangle\mu}{\langle\,\rangle\langle\,\rangle}$$

$$\tilde{\eta} = \frac{\langle\,\rangle\eta + \ldots}{\langle\,\rangle\langle\,\rangle}$$

Trick:

$$(a_1)(a_2)(a_3)(a_4)(a_5)$$

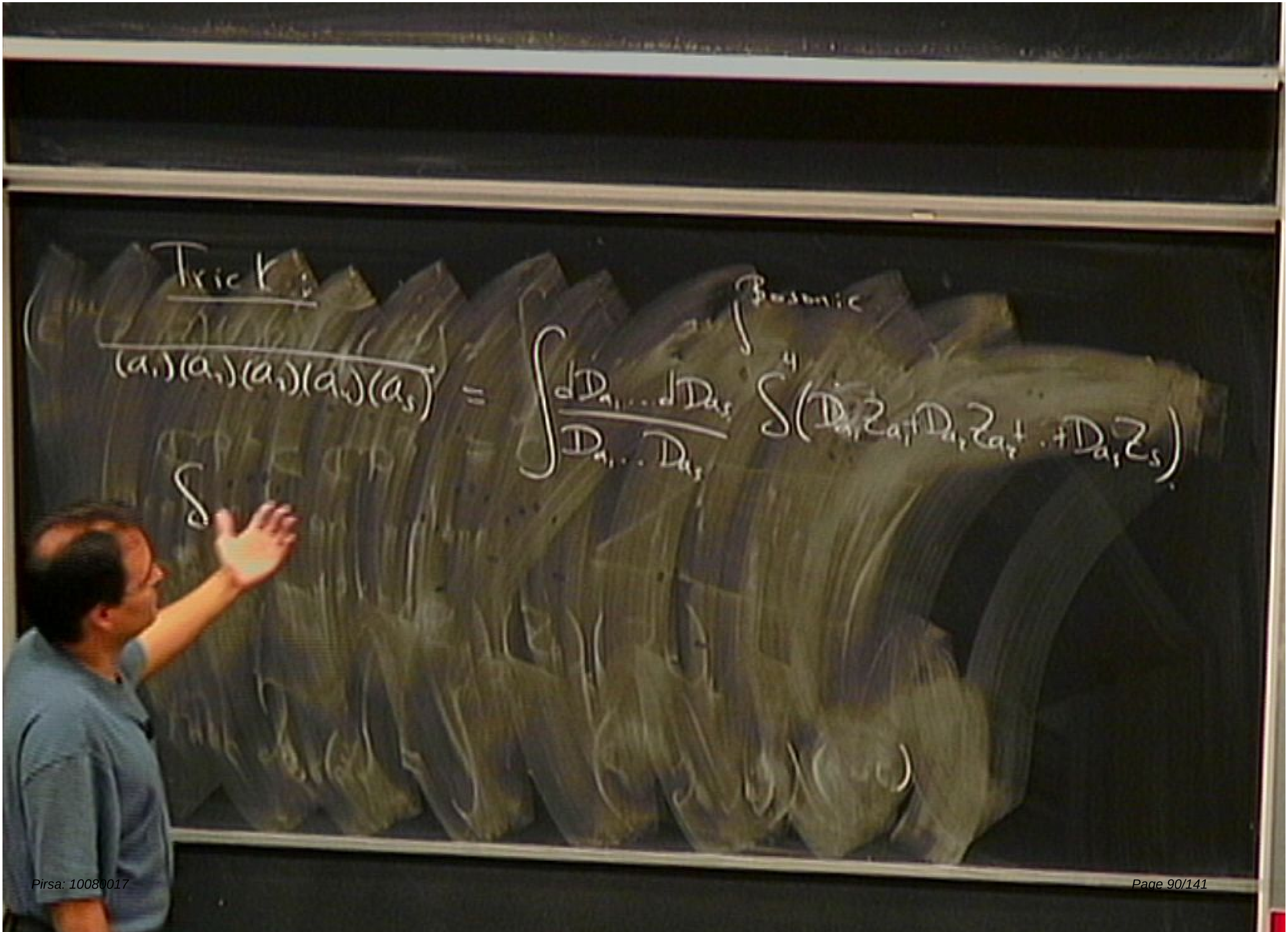

Trick:
$(a_1)(a_2)(a_3)(a_4)(a_5) = \int dD_{a_1} \ldots dD_{a_5}$


Trick:
Bosonic
$\frac{\quad}{(a_1)(a_2)(a_3)(a_4)(a_5)} = \int \frac{dD_{a_1}\dots dD_{a_5}}{D_{a_1}\dots D_{a_5}} \int$

Trick:

Bosonic

$$\frac{1}{(a_1)(a_2)(a_3)(a_4)(a_5)} = \int \frac{dD_{a_1}\ldots dD_{a_5}}{D_{a_1}\ldots D_{a_5}}\, \delta^4\left(D_{a_1} Z_{a_1} + D_{a_2} Z_{a_2} + \ldots + D_{a_5} Z_5\right).$$


Trick:
Bosonic
$\frac{1}{(a_1)(a_2)(a_3)(a_4)(a_5)} = \int \frac{dD_{a_1}\ldots dD_{a_5}}{D_{a_1}\ldots D_{a_5}} \delta^4(D_{a_1}Z_{a_1}+D_{a_2}Z_{a_2}+\ldots+D_{a_5}Z_{a_5})$

Trick:

Bosonic

$$\frac{}{(a_1)(a_2)(a_3)(a_4)(a_5)} = \int \frac{dD_{a_1}\cdots dD_{a_5}}{D_{a_1}\cdots D_{a_5}}\, \delta^4\left(D_{a_1}Z_{a_1}+D_{a_2}Z_{a_2}+\cdots+D_{a_5}Z_{a_5}\right)$$

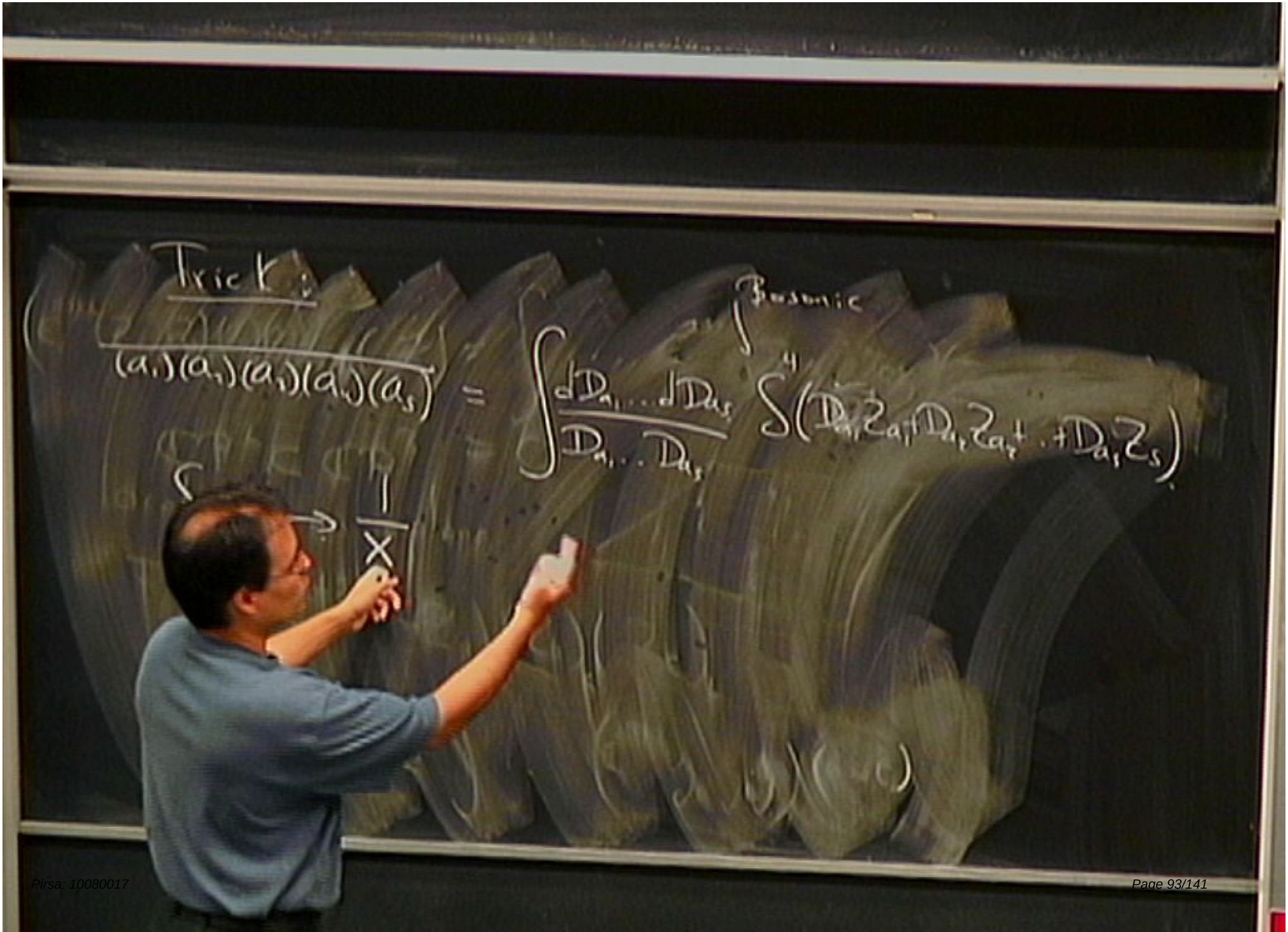
δ

Trick:

Bosonic

$$\frac{\;}{(a_1)(a_2)(a_3)(a_4)(a_5)} = \int \frac{dD_{a_1}\ldots dD_{a_5}}{D_{a_1}\ldots D_{a_5}}\,\delta^4\left(D_{a_1}Z_{a_1}+D_{a_2}Z_{a_2}+\ldots+D_{a_5}Z_{a_5}\right)$$

δ


Trick:
Bosonic
$\frac{}{(a_1)(a_2)(a_3)(a_4)(a_5)} = \int \frac{dD_{a_1} \ldots dD_{a_5}}{D_{a_1} \ldots D_{a_5}} \delta^4(D_{a_1}Z_{a_1} + D_{a_2}Z_{a_2} + \ldots + D_{a_5}Z_{a_5})$
$\frac{1}{X}$

Trick:

Bosonic

$$\frac{\;}{(a_1)(a_2)(a_3)(a_4)(a_5)} = \int \frac{dD_{a_1}\cdots dD_{a_5}}{D_{a_1}\cdots D_{a_5}}\, \delta^4\left(D_{a_1} Z_{a_1} + D_{a_2} Z_{a_2} + \cdots + D_{a_5} Z_{a_5}\right)$$

$$\delta(x) \rightarrow \frac{1}{x}$$

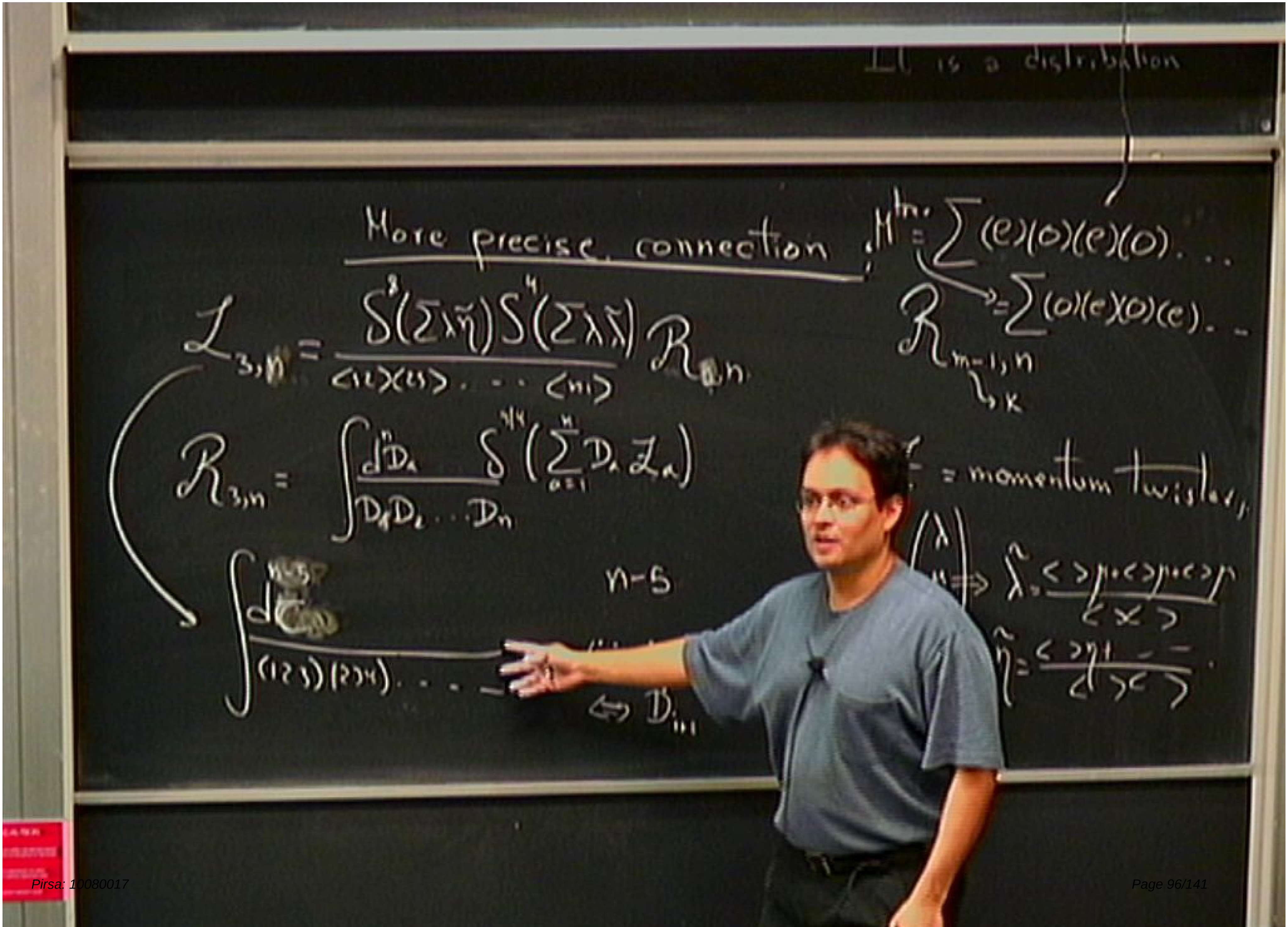
Trick:

Bosonic

Manifest DCI

$$\frac{1}{(a_1)(a_2)(a_3)(a_4)(a_5)} = \int \frac{dD_{a_1}\cdots dD_{a_5}}{D_{a_1}\cdots D_{a_5}}\, \delta^4\left(D_{a_1}Z_{a_1}+D_{a_2}Z_{a_2}+\cdots+D_{a_5}Z_{a_5}\right)$$

$$\delta(x) \to \frac{1}{x}$$


It is a distribution
More precise connection
= momentum twistors
n-5

It is a distribution

More precise connection:

$$M^{tree} = \sum (e)(o)(e)(o)\ldots$$

$$\mathcal{R} = \sum (o)(e)(o)(e)\ldots$$

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$$\mathcal{L}_{3,n} = \frac{\delta^8(\sum\lambda\tilde{\eta})\,\delta^4(\sum\lambda\tilde{\lambda})}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}\,\mathcal{R}_{3,n}$$

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$$\int \frac{d^{n-5}C}{(123)(234)\cdots}$$

$n-5$

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$$\tilde{\eta} = \frac{\langle\ \rangle\eta + \cdots}{\langle\ \rangle\langle\ \rangle}$$

Trick:

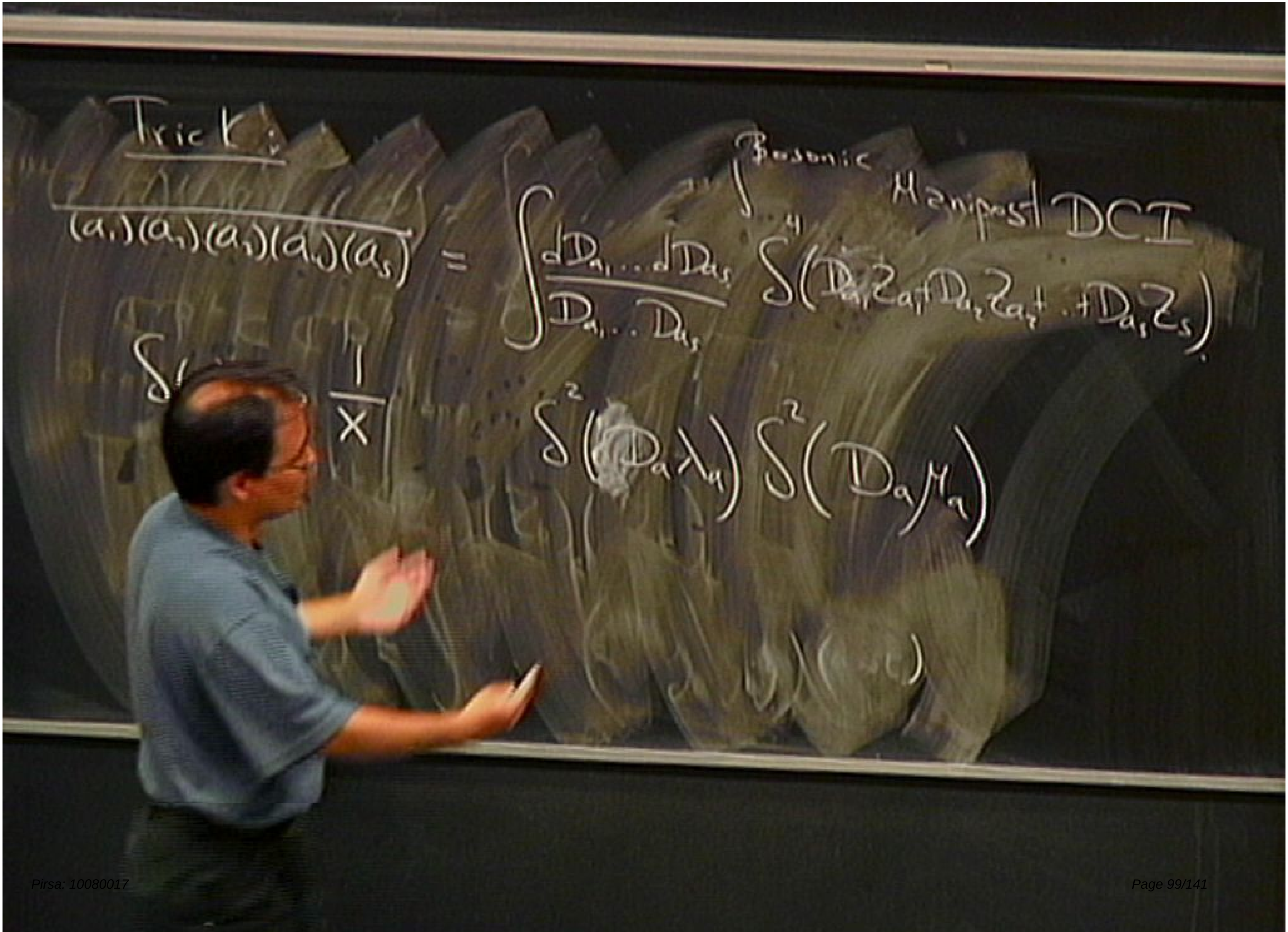
Bosonic

Manifest DCI

$$\frac{\quad}{(a_1)(a_2)(a_3)(a_4)(a_5)} = \int \frac{dD_{a_1}\ldots dD_{a_5}}{D_{a_1}\ldots D_{a_5}}\, \delta^4\left(D_{a_1}Z_{a_1}+D_{a_2}Z_{a_2}+\ldots+D_{a_5}Z_{a_5}\right)$$

$$\delta(x) \rightarrow \frac{1}{x}$$

$$\delta^2(D_a \lambda_a)\, \delta^2(D_a \mu_a)$$


Trick:
(a1)(a2)(a3)(a4)(a5)
Bosonic
Manifest DCI
dDa1...dDa5
Da1...Da5
δ(Da1Za1+Da2Za2+...+Da5Z5)

Trick:

$$\frac{1}{(a_1)(a_2)(a_3)(a_4)(a_5)} = \int \frac{dD_{a_1}\ldots dD_{a_5}}{D_{a_1}\ldots D_{a_5}}\, \delta^4\left(D_{a_1}Z_{a_1} + D_{a_2}Z_{a_2} + \ldots + D_{a_5}Z_{a_5}\right)$$

Bosonic $\int$ Manifest DCI

$$\delta(x) \to \frac{1}{x}$$

$$\delta^2(D_a \lambda_a)\, \delta^2(D_a \mu_a)$$

Trick:

Bosonic

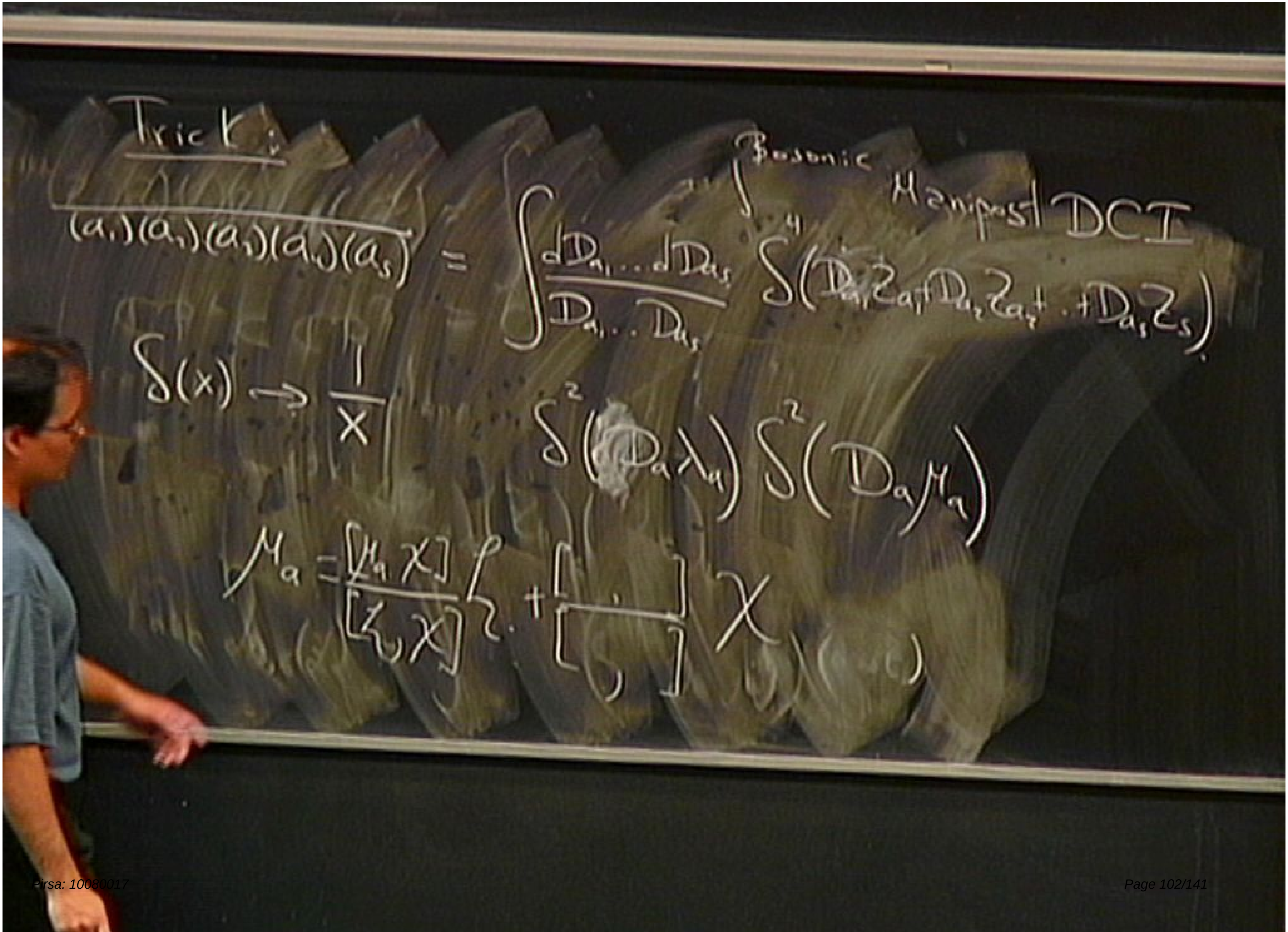
Manifest DCI

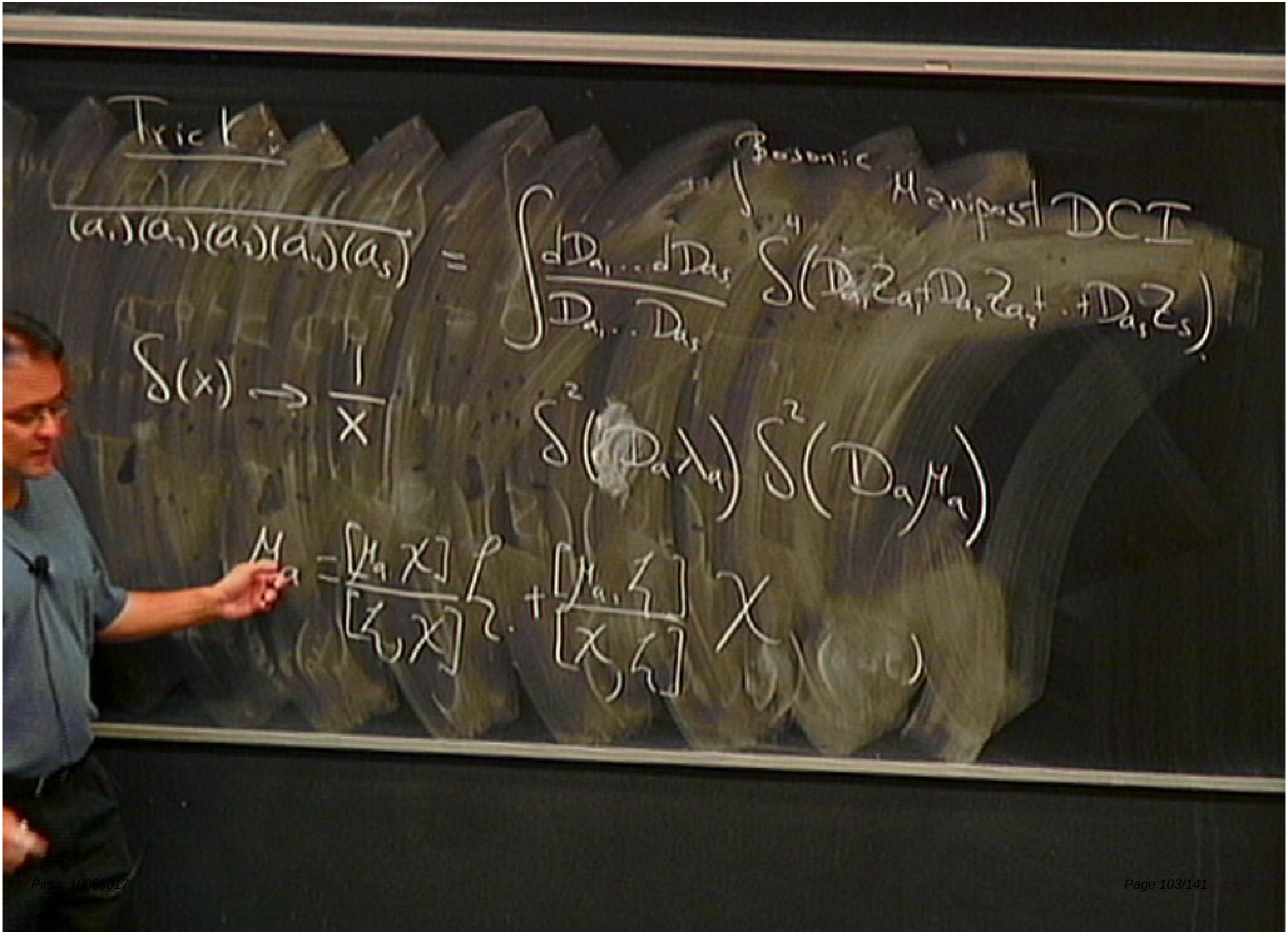
$$\frac{\quad}{(a_1)(a_2)(a_3)(a_4)(a_5)} = \int \frac{dD_{a_1}\ldots dD_{a_5}}{D_{a_1}\ldots D_{a_5}}\, \delta(D_{a_1}Z_{a_1}+D_{a_2}Z_{a_2}+\ldots+D_{a_5}Z_5)$$

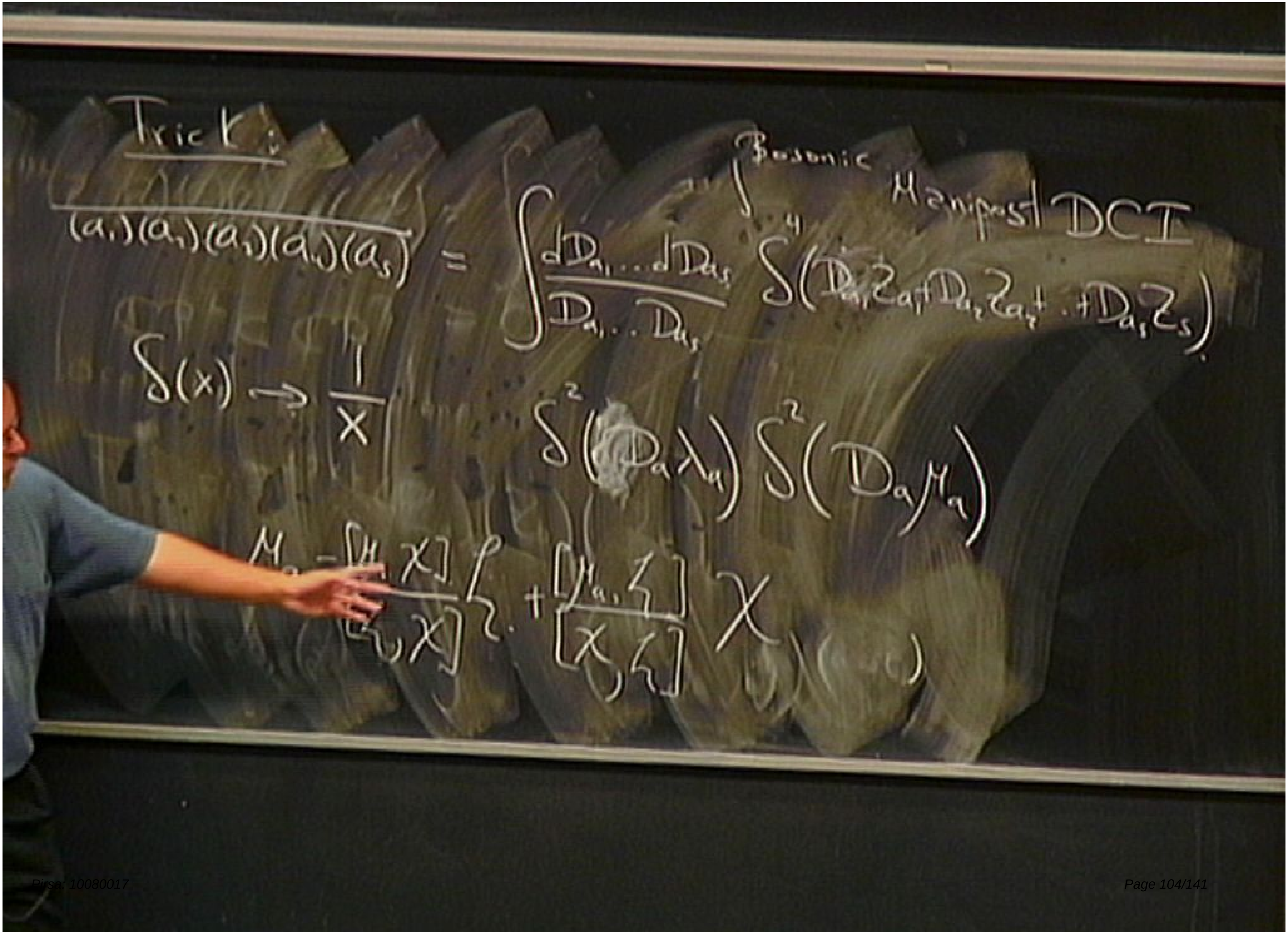
$$\delta(x) \to \frac{1}{x}$$

$$\delta^2(D_a\lambda_a)\,\delta^2(D_a\mu_a)$$

$$\mu_a = \zeta. \qquad X$$


Trick:
Bosonic
Manifest DCI
1/((a1)(a2)(a3)(a4)(a5)) = ∫ dDa1 ... dDa5 / Da1 ... Da5 δ^4(Da1 Za1 + Da2 Za2 + . + Da5 Z5)
δ(x) → 1/x
δ^2(Da λa) δ^2(Da μa)
μa = [μa X]/[λa X] λa + [,]/[,] X


Trick:
Bosonic
Manifest DCI
δ(x) → 1/x



Trick:
$\frac{\langle\ \rangle}{(a_1)(a_2)(a_3)(a_4)(a_5)} = \int \frac{dD_{a_1}\dots dD_{a_5}}{D_{a_1}\dots D_{a_5}} \delta^4(D_{a_1}Z_{a_1}+D_{a_2}Z_{a_2}+\dots+D_{a_5}Z_{a_5})$
Bosonic
Manifest DCI
$\delta(x) \to \frac{1}{x}$
$\delta^2(D_a\lambda_a)\,\delta^2(D_a\mu_a)$
$\mu_a = \frac{[\mu_a, X]}{[\zeta, X]}\zeta + \frac{[\mu_a, \zeta]}{[X, \zeta]}X$

Trick:

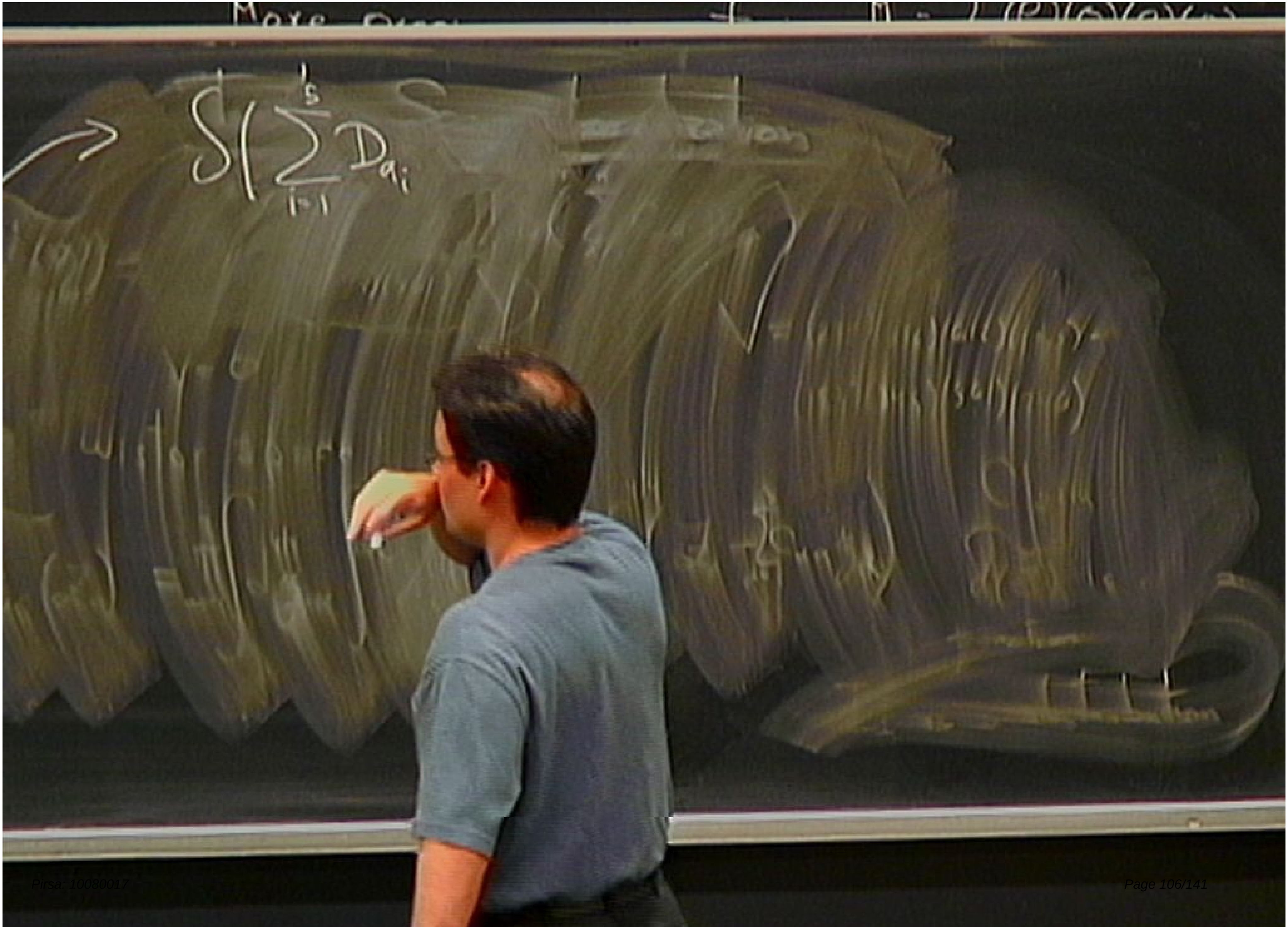
Bosonic Manifest DCI

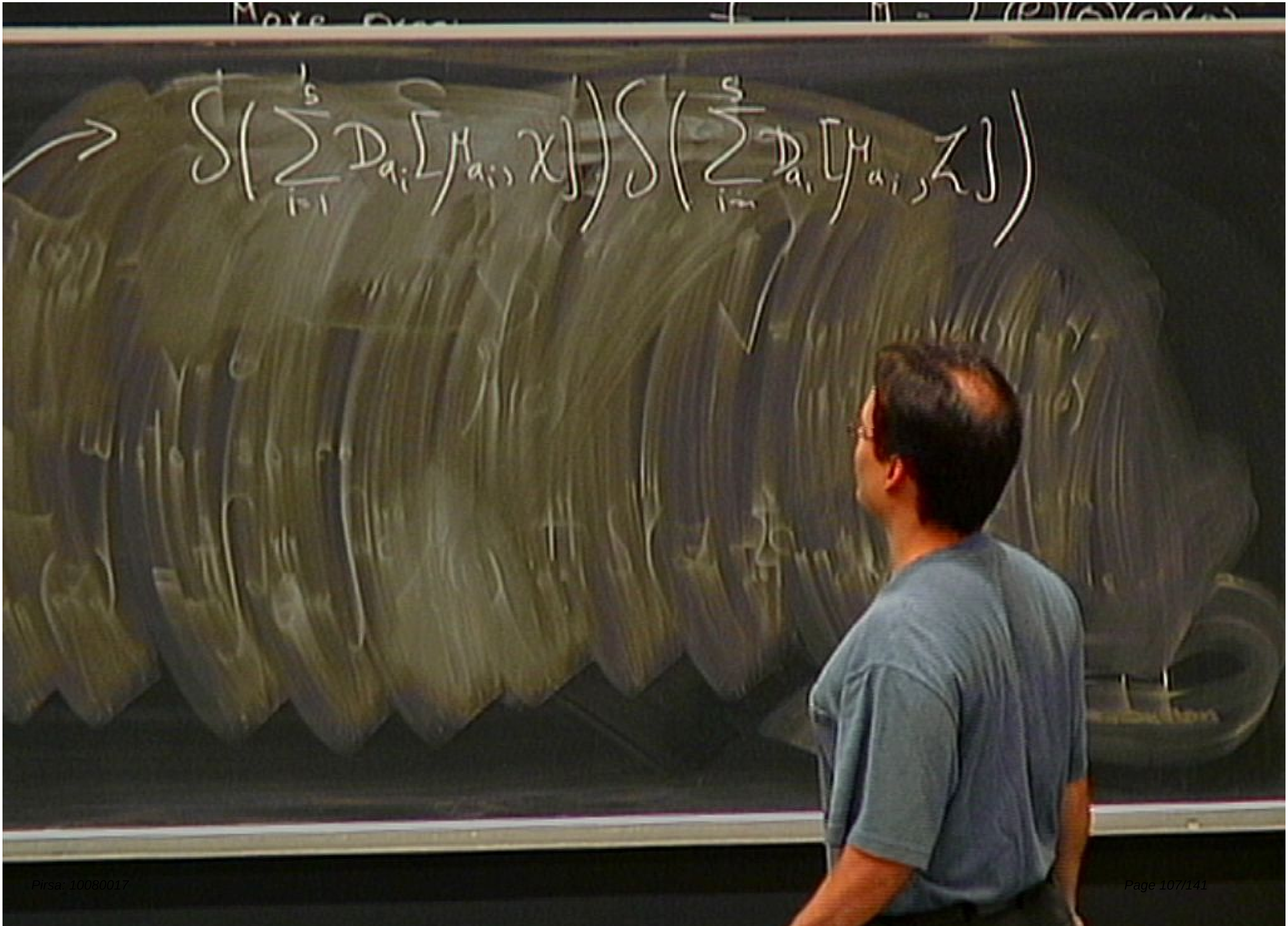
$$\frac{1}{(a_1)(a_2)(a_3)(a_4)(a_5)} = \int \frac{dD_{a_1}\dots dD_{a_5}}{D_{a_1}\dots D_{a_5}} \delta^4(D_{a_1}Z_{a_1}+D_{a_2}Z_{a_2}+\dots+D_{a_5}Z_{a_5})$$

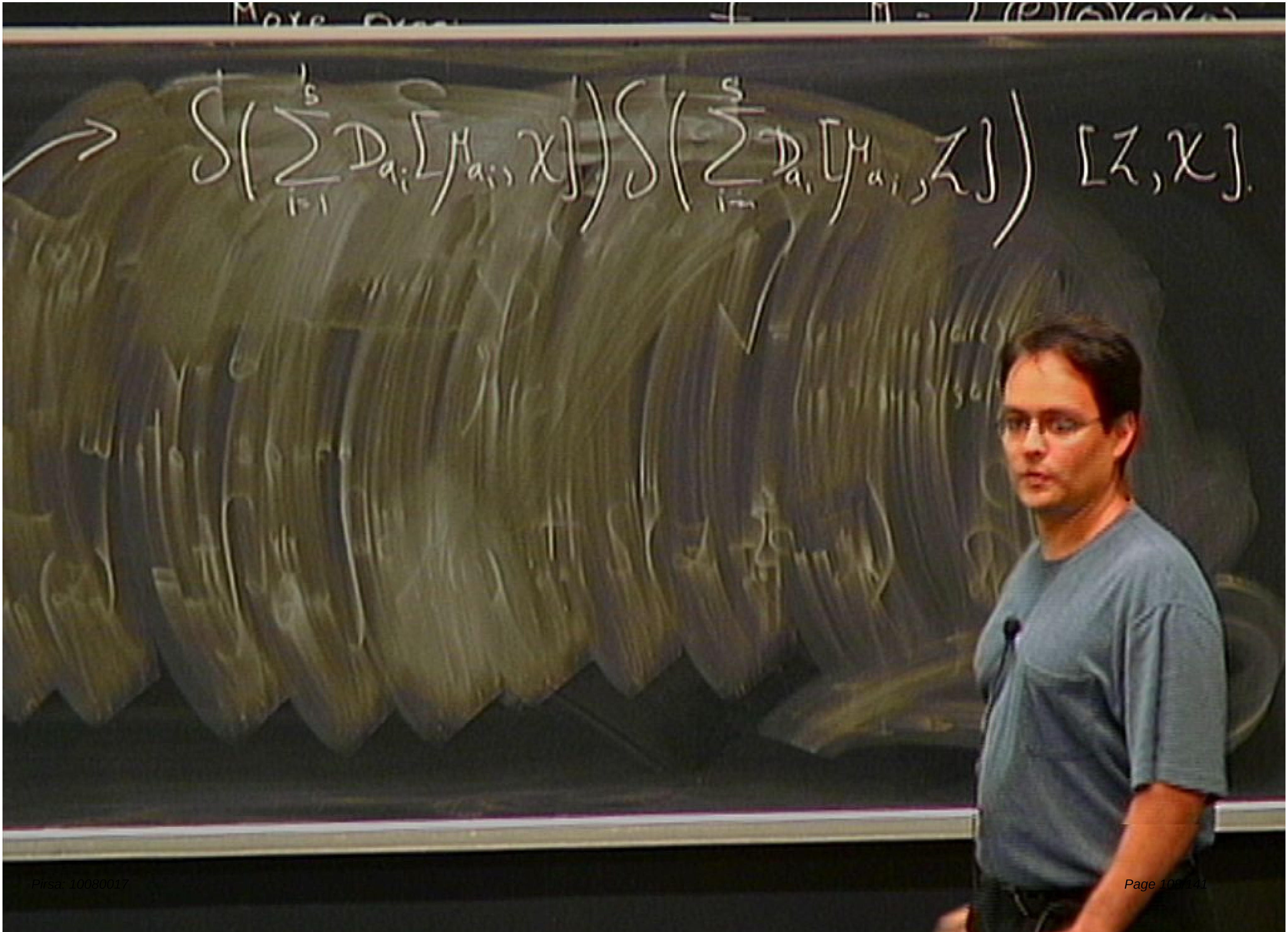
$$\delta(x) \to \frac{1}{x}$$

$$\delta^2(D_a\lambda_a)\,\delta^2(D_a\mu_a)$$

$$\mu_a = \frac{[\mu_a\,X]}{[\zeta,X]}\zeta + \frac{[\mu_a\,\zeta]}{[X,\zeta]}X$$

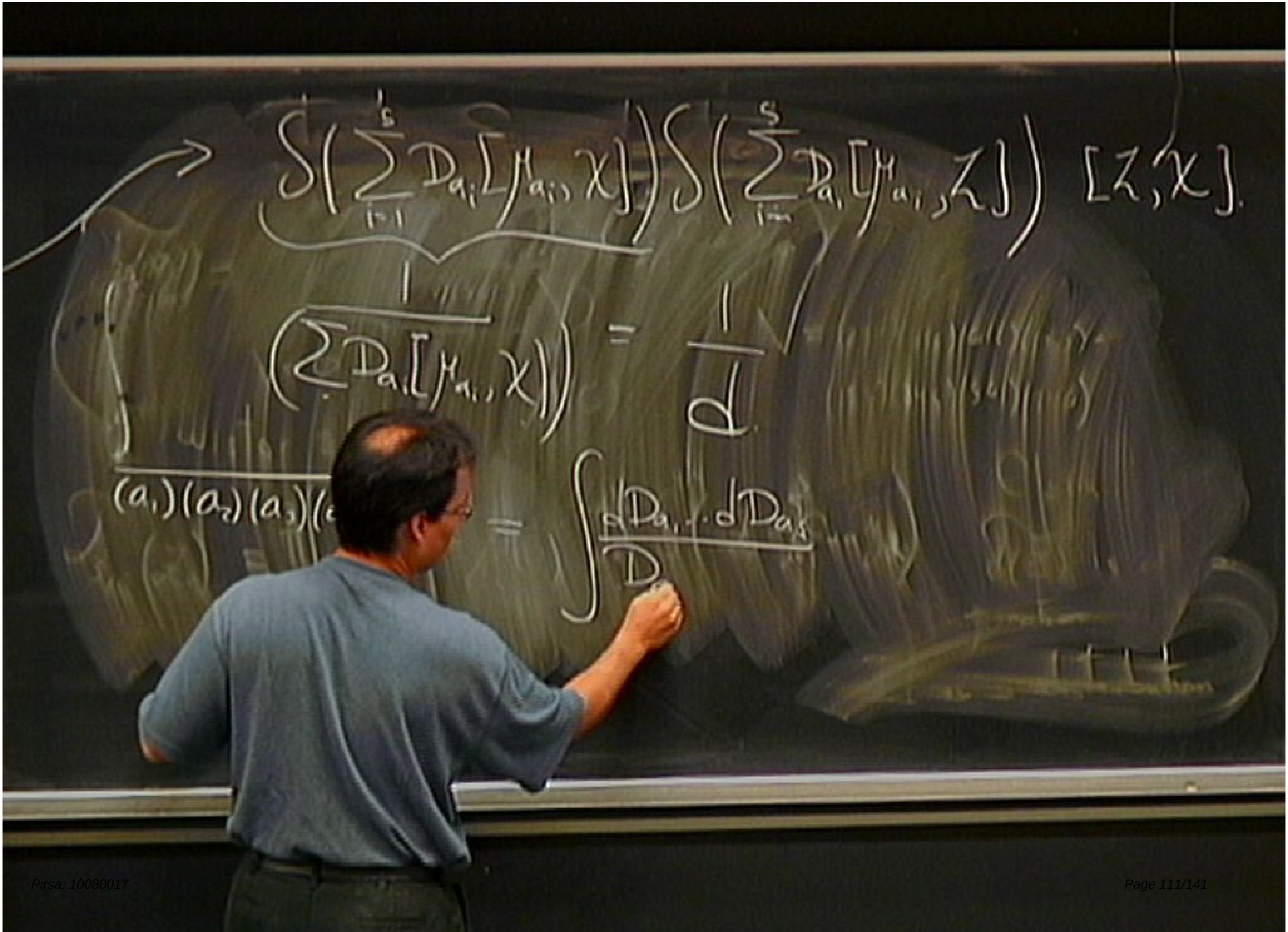

$S(\sum_{i=1}^{s} D_{a_i}$

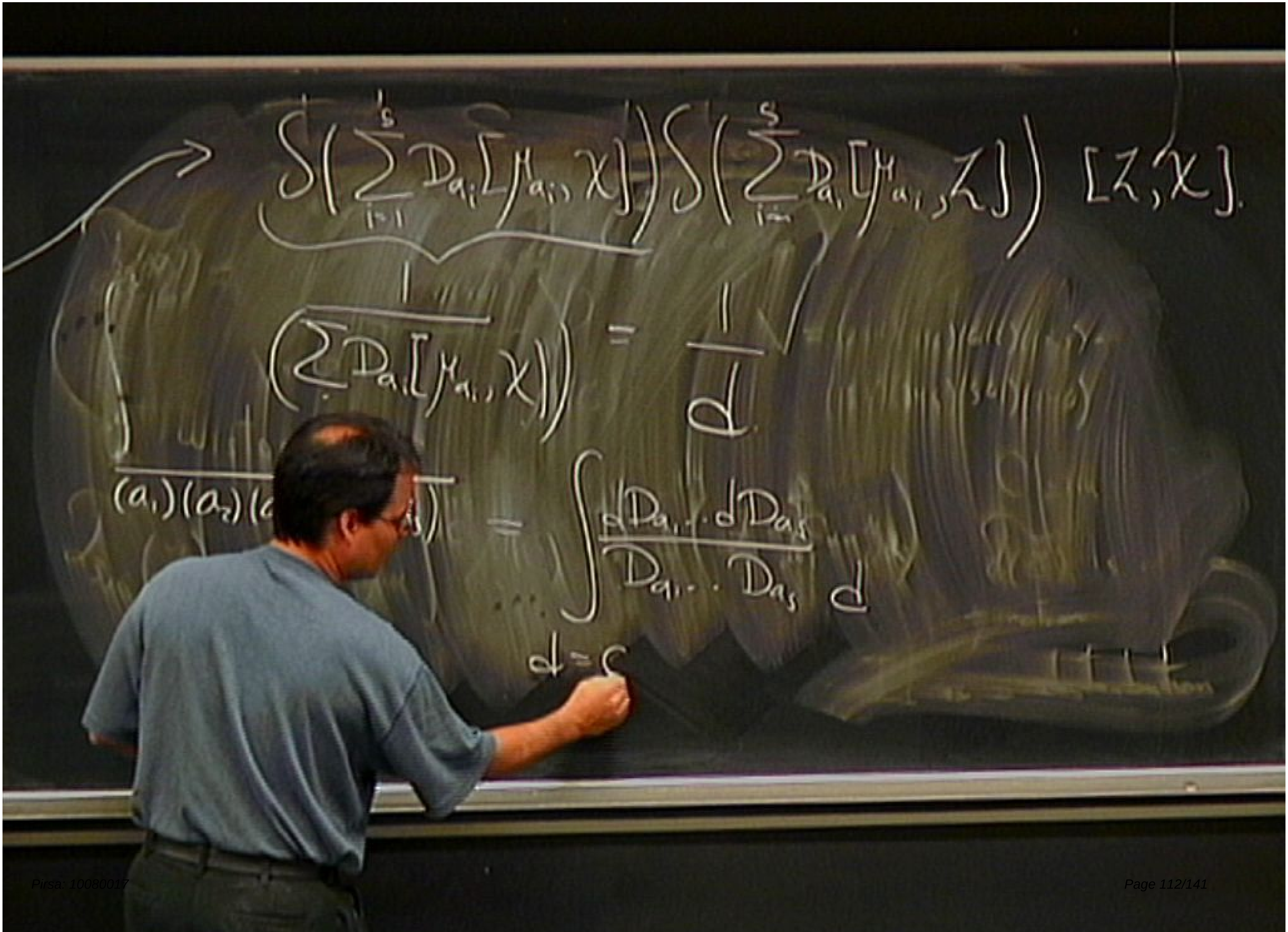

$\delta\left(\sum_{i=1}^{s} D_{a_i}[H_{a_i}, X]\right)\delta\left(\sum_{i=1}^{s} D_{a_i}[H_{a_i}, Z]\right)$


$S\left(\sum_{i=1}^{s} D_{a_i}[H_{a_i}, X]\right) S\left(\sum_{i=1}^{s} D_{a_i}[H_{a_i}, Z]\right) [Z, X].$

$$\Rightarrow \; S\Big(\sum_{i=1}^{s} D_{a_i}[M_{a_i}, X]\Big)\, S\Big(\sum_{i=1}^{s} D_{a_i}[M_{a_i}, Z]\Big)\, [Z, X].$$

$$\underbrace{\delta\left(\sum_{i=1}^{s} D_{a_i}[M_{a_i}, X]\right)}_{\frac{1}{\left(\sum D_{a_i}[M_{a_i}, X]\right)} = \frac{1}{d}}\delta\left(\sum_{i=1}^{s} D_{a_i}[M_{a_i}, Z]\right) [Z, X].$$

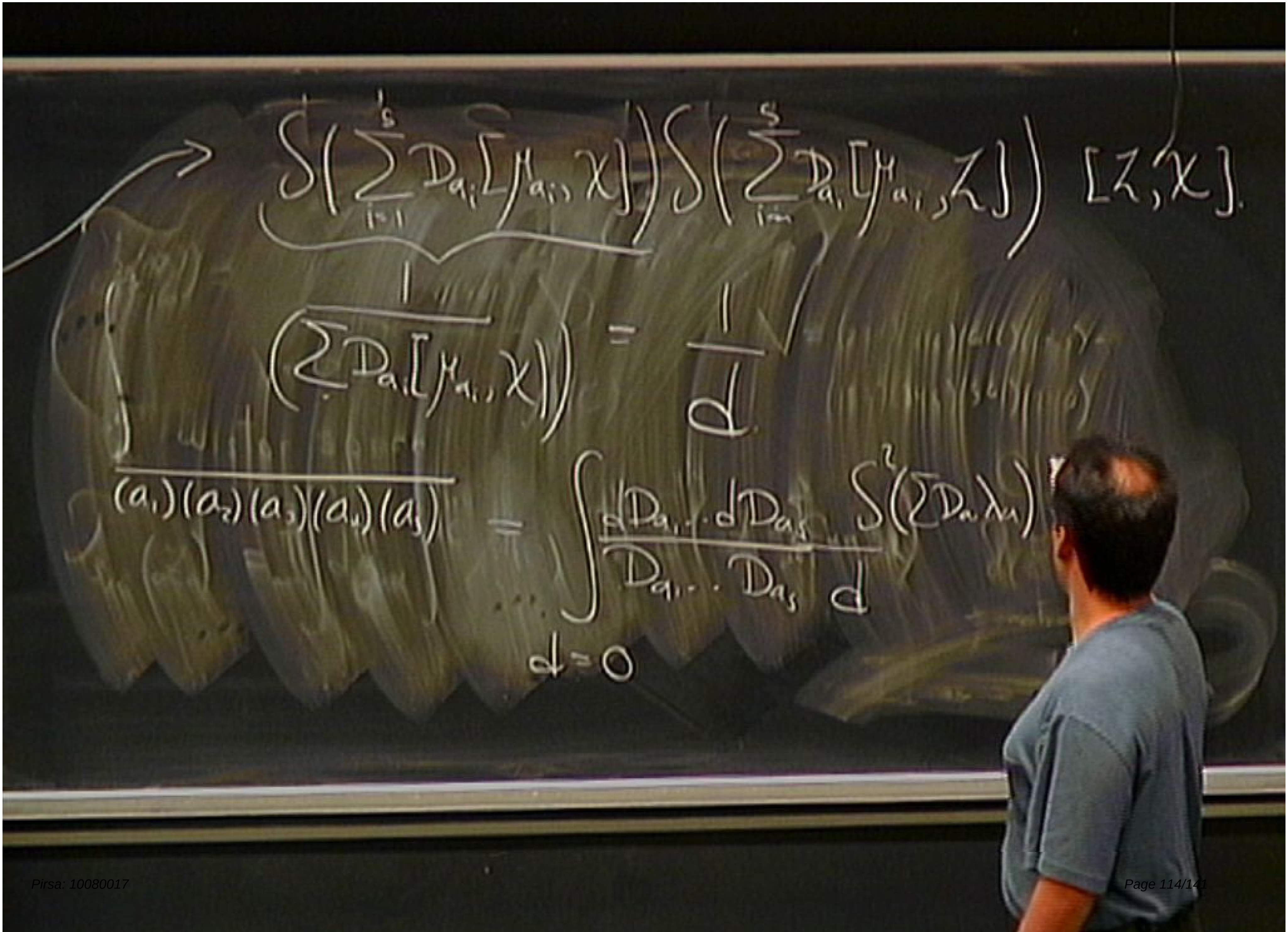


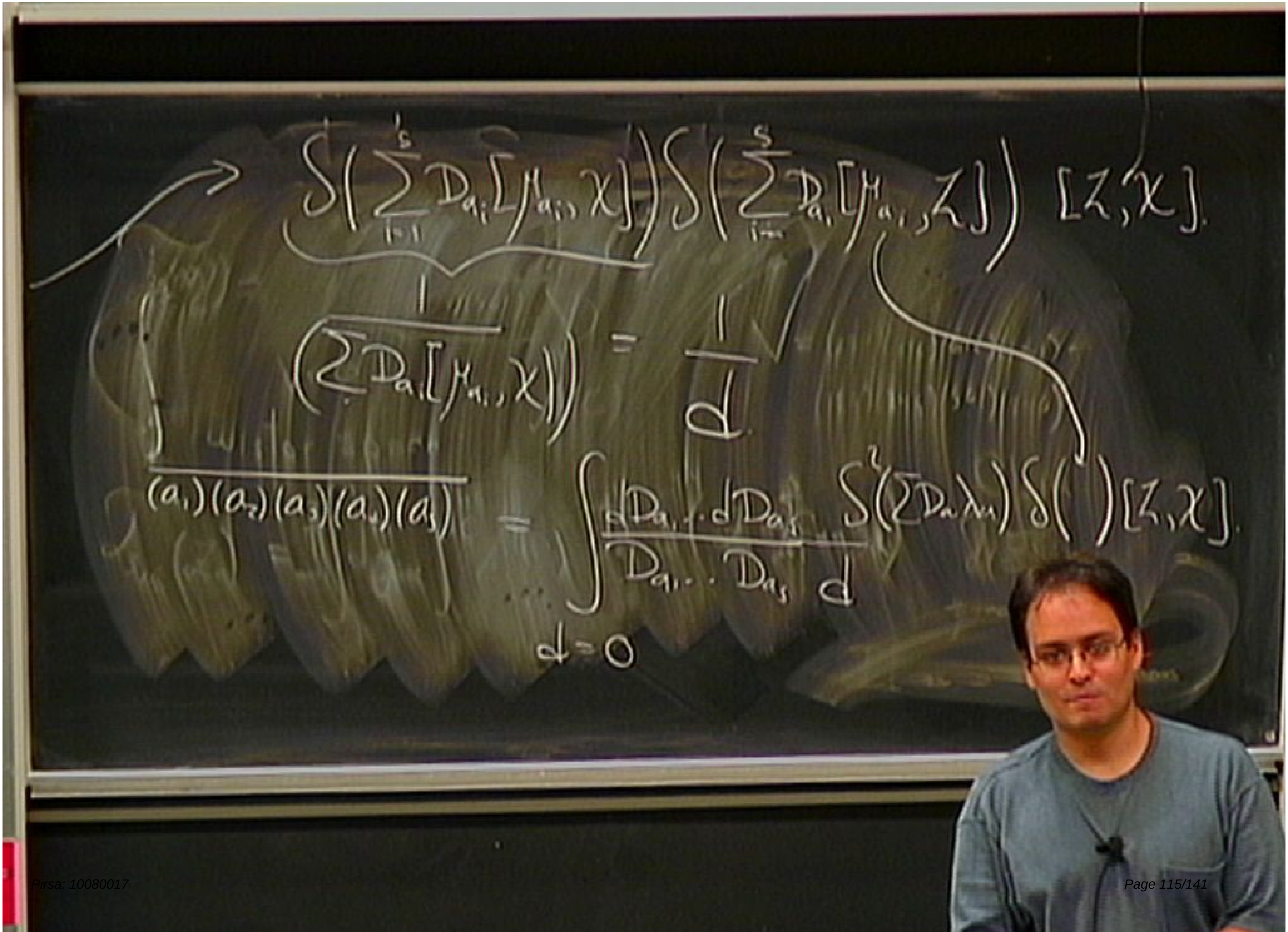

[Z, X].
$\frac{1}{\left(\sum D_{a_i}[\mu_{a_i}, X]\right)} = \frac{1}{d}$
$= \int \frac{dD_{a_1} \dots dD_{a_s}}{D_{a_1} \dots D_{a_s}}\, d$

$$\delta\Big(\underbrace{\sum_{i=1}^{s} D_{a_i}[M_{a_i}, X]}\Big)\,\delta\Big(\sum_{i=1}^{s} D_{a_i}[M_{a_i}, Z]\Big)\ [Z, X].$$

$$\frac{1}{\left(\sum D_{a_i}[M_{a_i}, X]\right)} = \frac{1}{d}$$

$$\frac{\int}{(a_1)(a_2)(a_3)(a_4)(a_5)} = \int_{d=0} \frac{dD_{a_1}\cdots dD_{a_5}}{D_{a_1}\cdots D_{a_5}\ d}$$


$\delta\left(\sum_{i=1}^{s} D_{a_i}[\mu_{a_i}, X]\right)\delta\left(\sum_{i=1}^{s} D_{a_i}[\mu_{a_i}, Z]\right)[Z,X]$
$\frac{1}{\left(\sum D_{a_i}[\mu_{a_i}, X]\right)} = \frac{1}{d}$
$\frac{\ldots}{(a_1)(a_2)(a_3)(a_4)(a_5)} = \int \frac{dD_{a_1}\cdots dD_{a_5}}{D_{a_1}\cdots D_{a_5}}\,\frac{1}{d}\,\delta^s\left(\sum D_a\lambda_a\right)$
$d=0$


$\delta\left(\sum_{i=1}^{s} D_{a_i}[\mu_{a_i}, X]\right)\delta\left(\sum_{i=1}^{s} D_{a_i}[\mu_{a_i}, Z]\right)[Z,X]$
$\frac{1}{\left(\sum D_{a_i}[\mu_{a_i}, X]\right)} = \frac{1}{d}$
$(a_1)(a_2)(a_3)(a_4)(a_5)$
$= \int \frac{dD_{a_1} \cdots dD_{a_5}}{D_{a_1} \cdots D_{a_5}\, d}\, \delta(\sum D_a \lambda_a)\,\delta(\;)[Z,X]$
$d=0$

$$\delta\Big(\sum_{i=1}^{s} D_{a_i}[\mu_{a_i}, X]\Big)\,\delta\Big(\sum_{i=1}^{s} D_{a_i}[\mu_{a_i}, Z]\Big)\,[Z, X].$$

$$\frac{1}{\left(\sum D_{a_i}[\mu_{a_i}, X]\right)} = \frac{1}{d}$$

$$\frac{\cdots}{(a_1)(a_2)(a_3)(a_4)(a_5)} = \int_{d=0} \frac{dD_{a_1}\cdots dD_{a_5}}{D_{a_1}\cdots D_{a_5}\; d}\,\delta'\Big(\sum D_{a_i}\lambda_{a_i}\Big)\,\delta(\ )\,[Z, X]$$

$$\delta\left(\sum_{i=1}^{5} D_{a_i}[\mu_{a_i}, X]\right)\delta\left(\sum_{i=1}^{5} D_{a_i}[\mu_{a_i}, Z]\right)[Z,X].$$

$$\frac{1}{\left(\sum D_a[\mu_a, X]\right)} = \frac{1}{d}$$

$$\frac{\cdots}{(a_1)(a_2)(a_3)(a_4)(a_5)} = \int \frac{dD_{a_1}\cdots dD_{a_5}}{D_{a_1}\cdots D_{a_5}\; d}\,\delta\left(\sum D_a\lambda_a\right)\delta(\;)\,[Z,X].$$

$d=0$

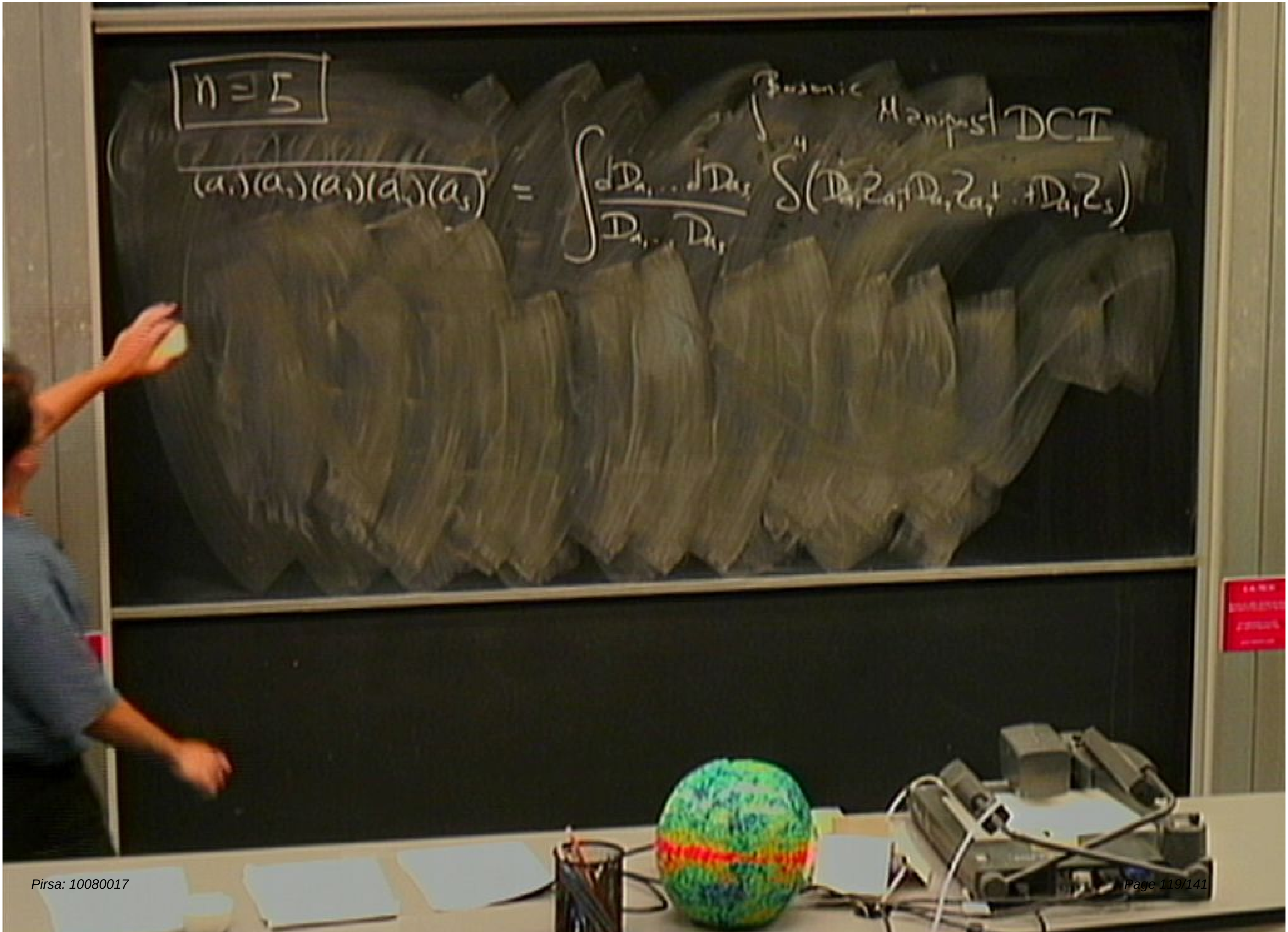
$$\rightarrow \delta\Big(\sum_{i=1}^{5} D_{a_i}[M_{a_i}, X]\Big)\,\delta\Big(\sum_{i=1}^{5} D_{a_i}[M_{a_i}, Z]\Big)\,[Z, X].$$

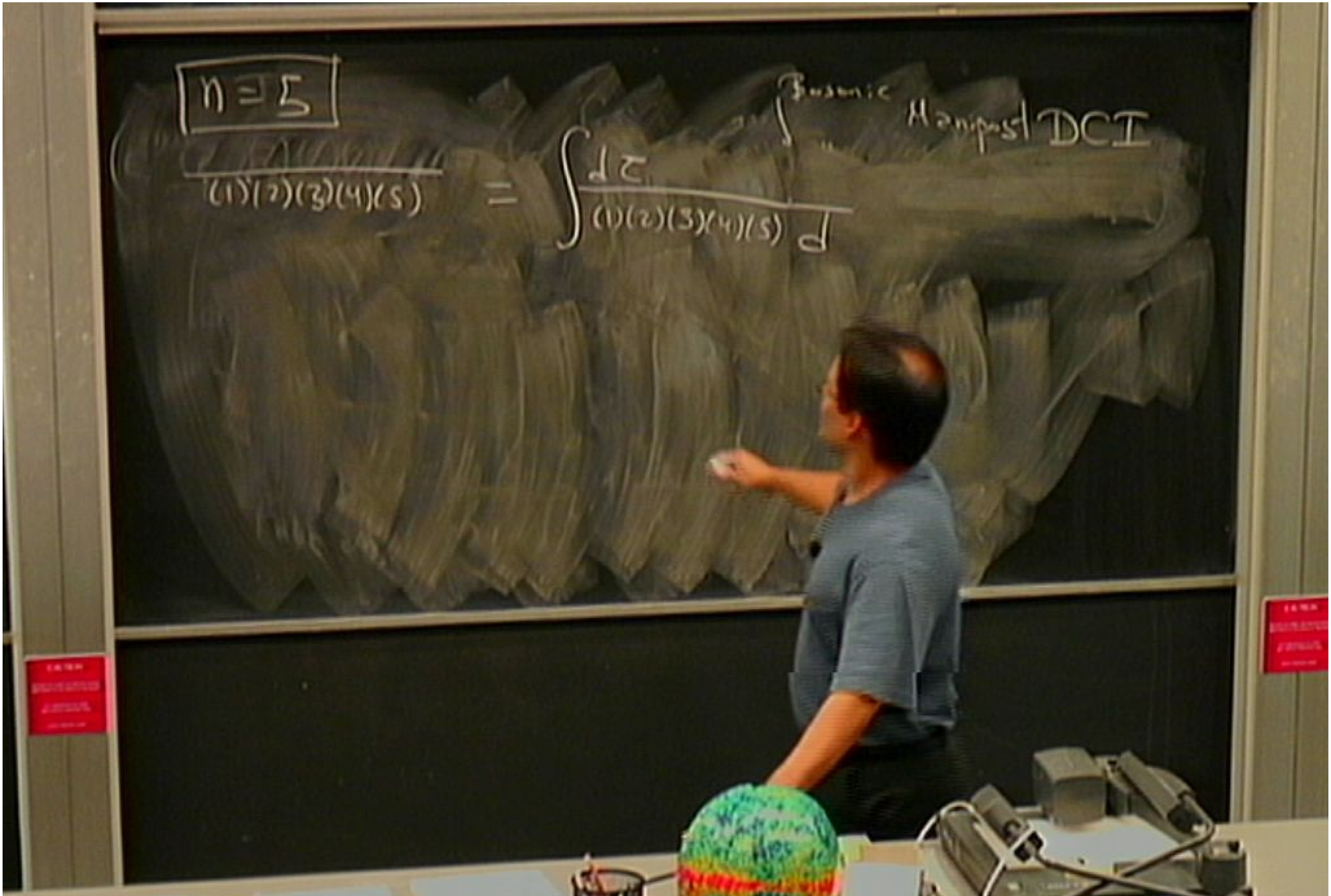
$$\frac{1}{\left(\sum D_{a_i}[M_{a_i}, X]\right)} = \frac{1}{d}$$

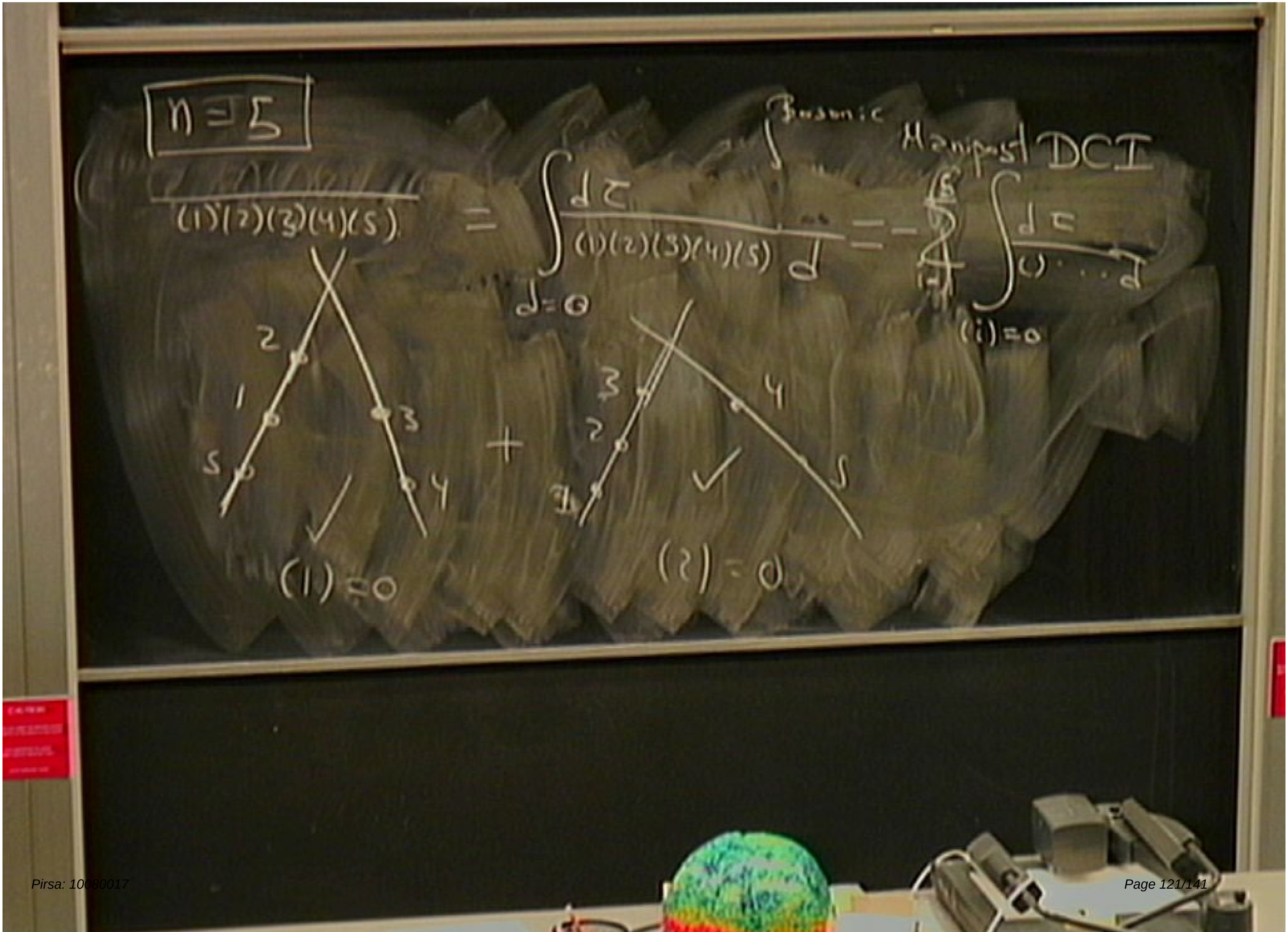
$$\frac{1}{(a_1)(a_2)(a_3)(a_4)(a_5)} = \int \frac{dD_{a_1} \cdots dD_{a_5}}{D_{a_1} \cdots D_{a_5}\; d}\, \delta'\left(\sum D_{a_i} \lambda_{a_i}\right) \delta(\;)\,[Z, X].$$

$$d = 0$$

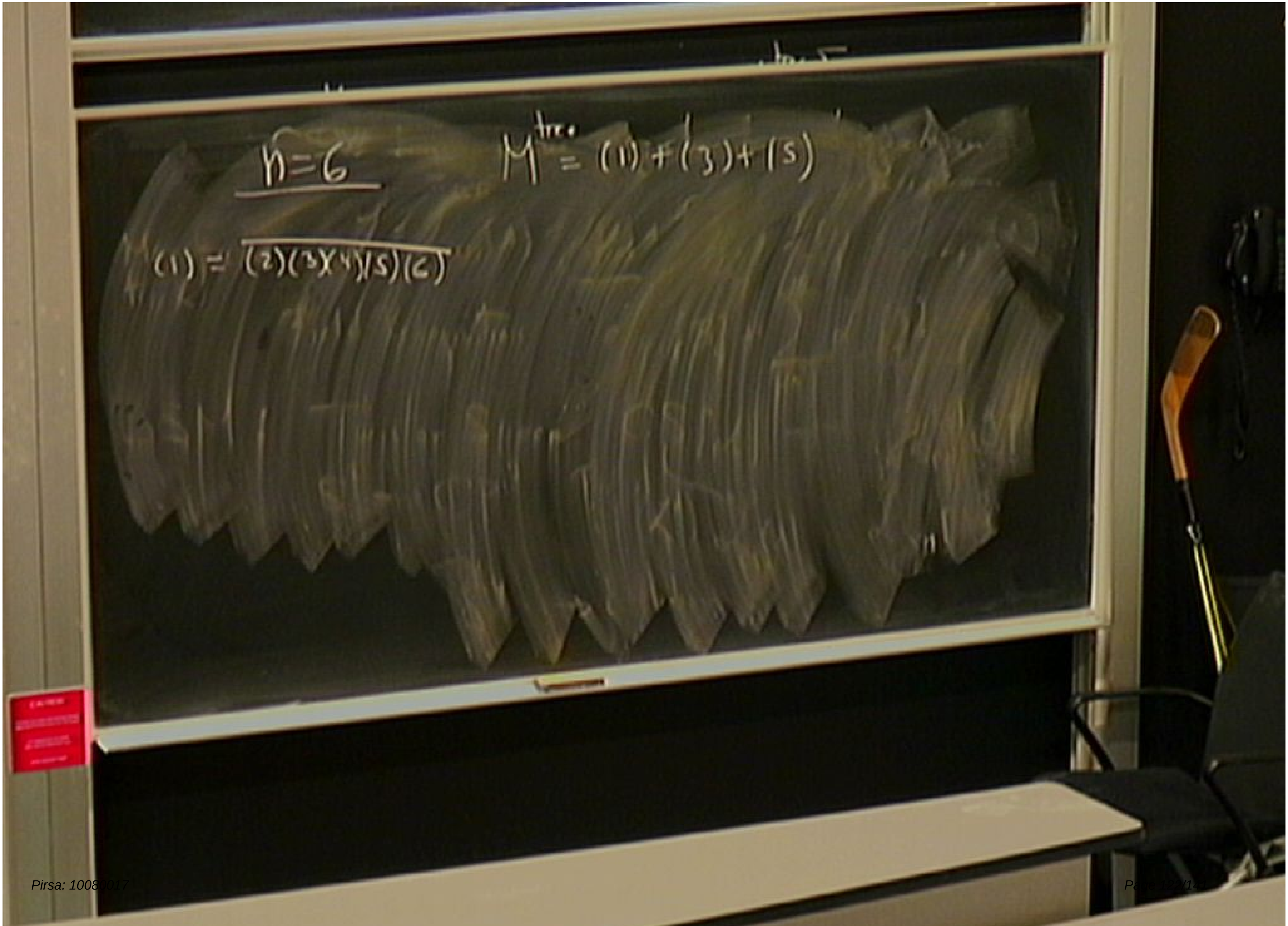
D_{a_5} D_{a_1} D_{a_1} $D_{a_3} = 0$ D_{a_2} D_{a_4}

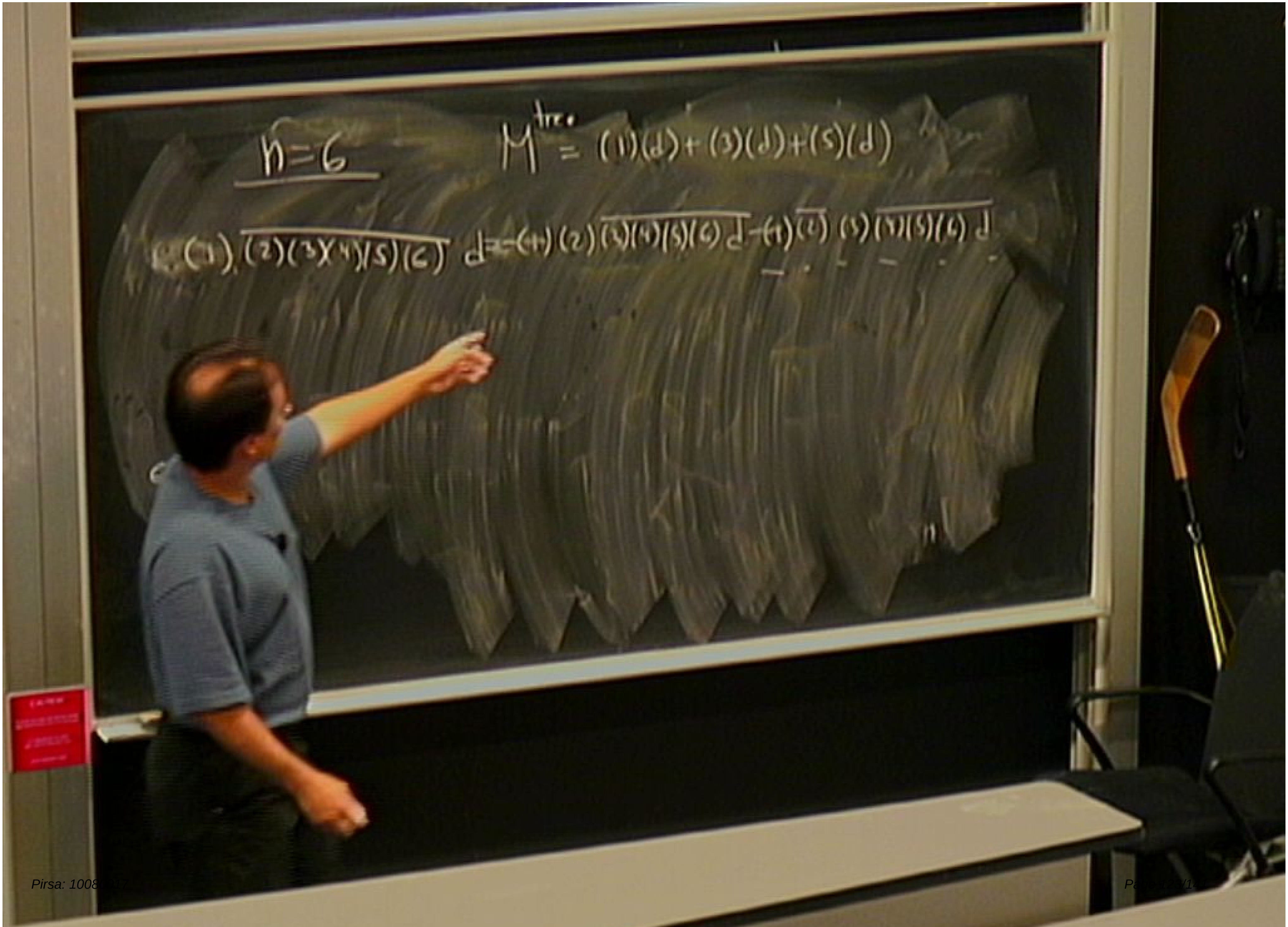

n = 5
Bosonic
Manifest DCI

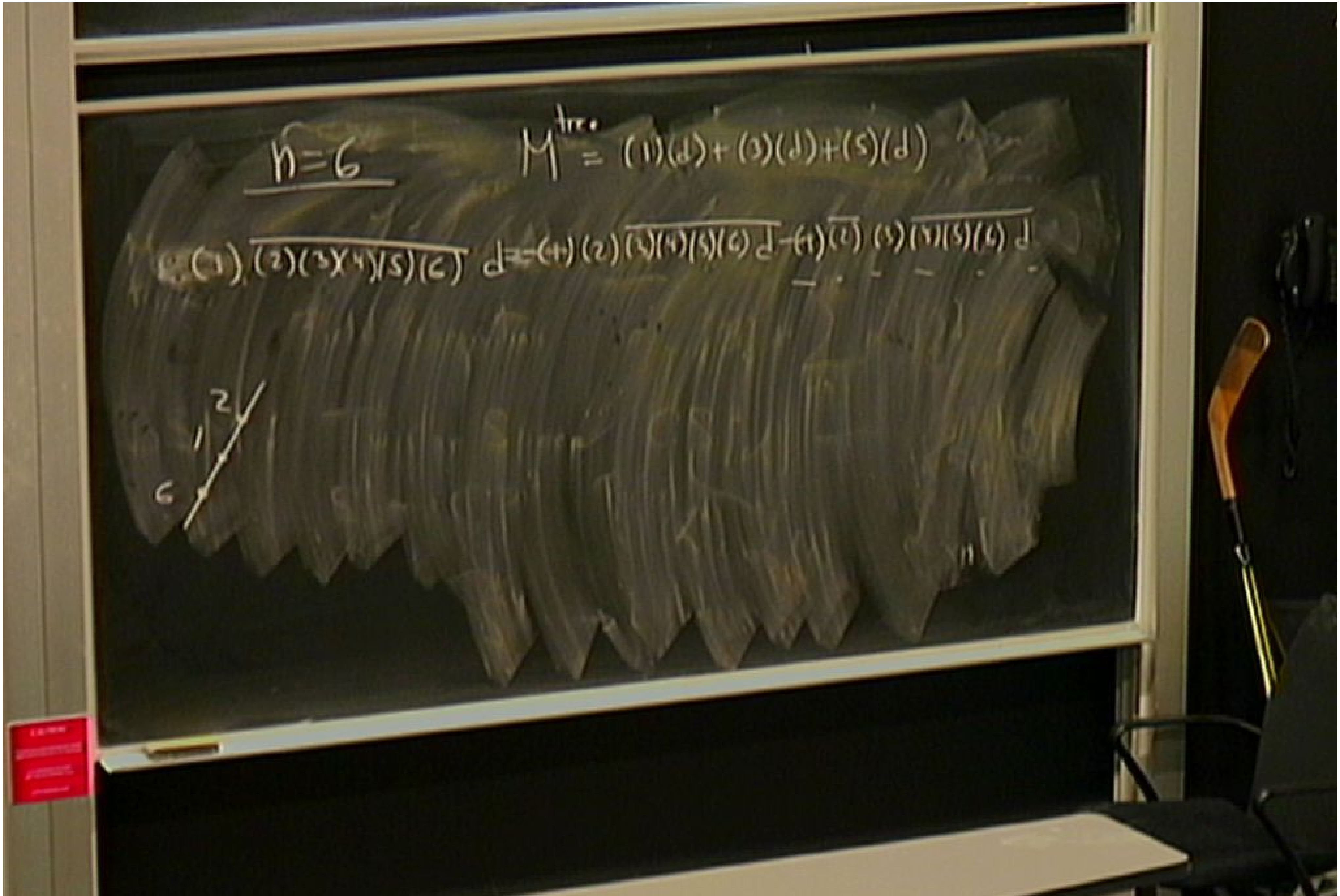

n = 5
(1)(2)(3)(4)(5)
dτ
(1)(2)(3)(4)(5)
Bosonic
Manifest DCI

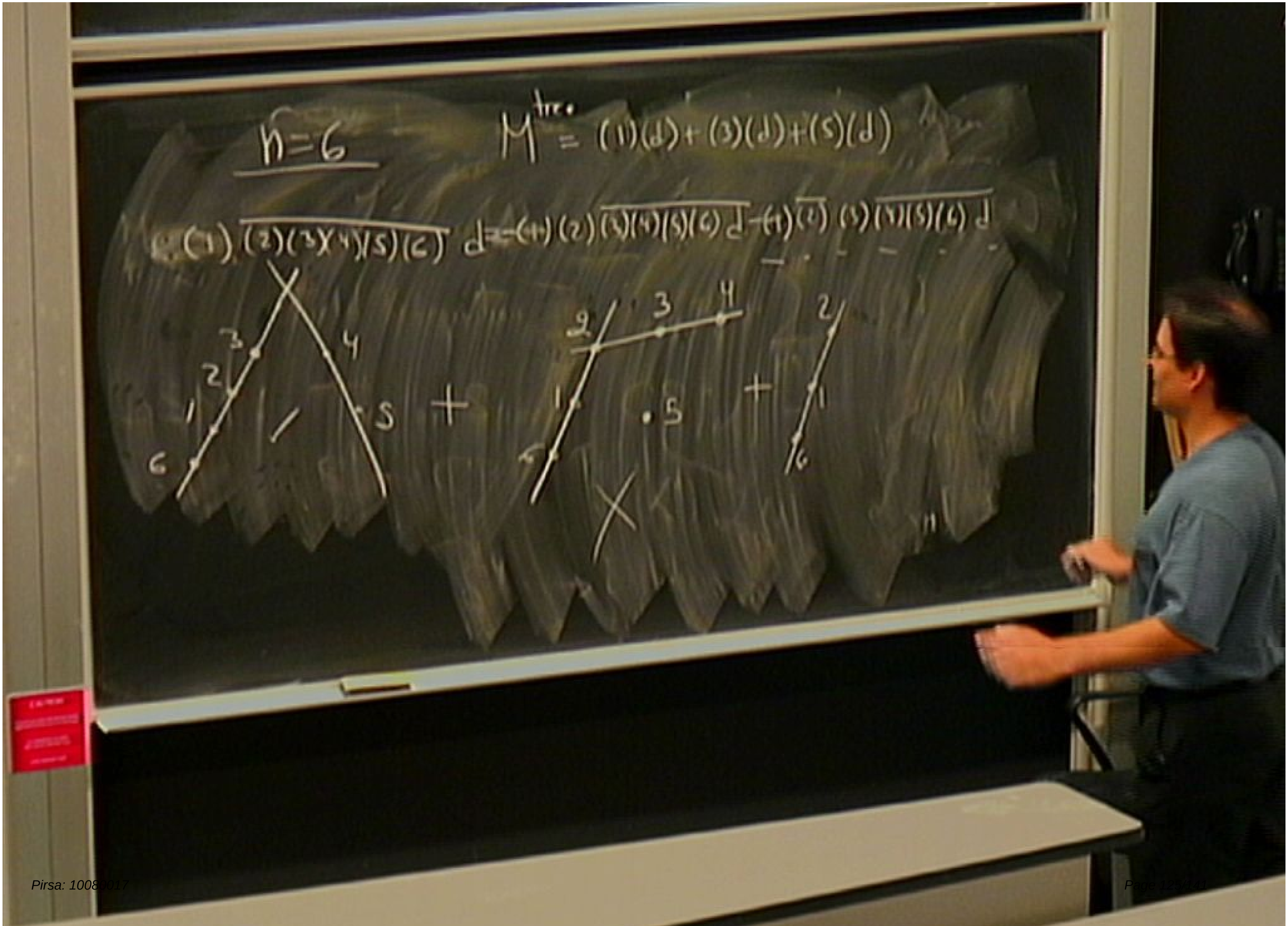


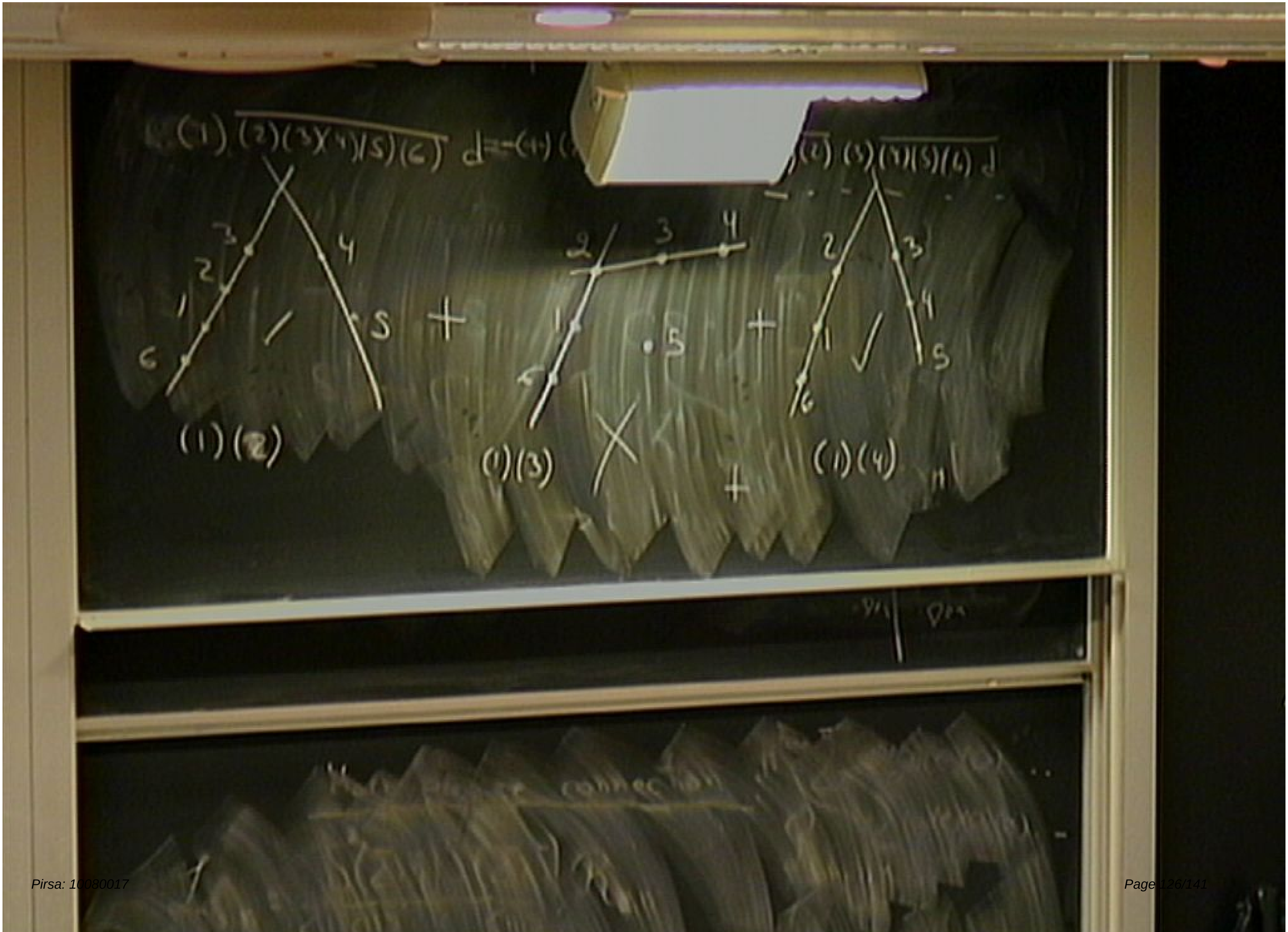
n = 5
(1)(2)(3)(4)(5)
d = 0
Bosonic
Manifest DCI
(i) = 0
(1) = 0
(2) = 0

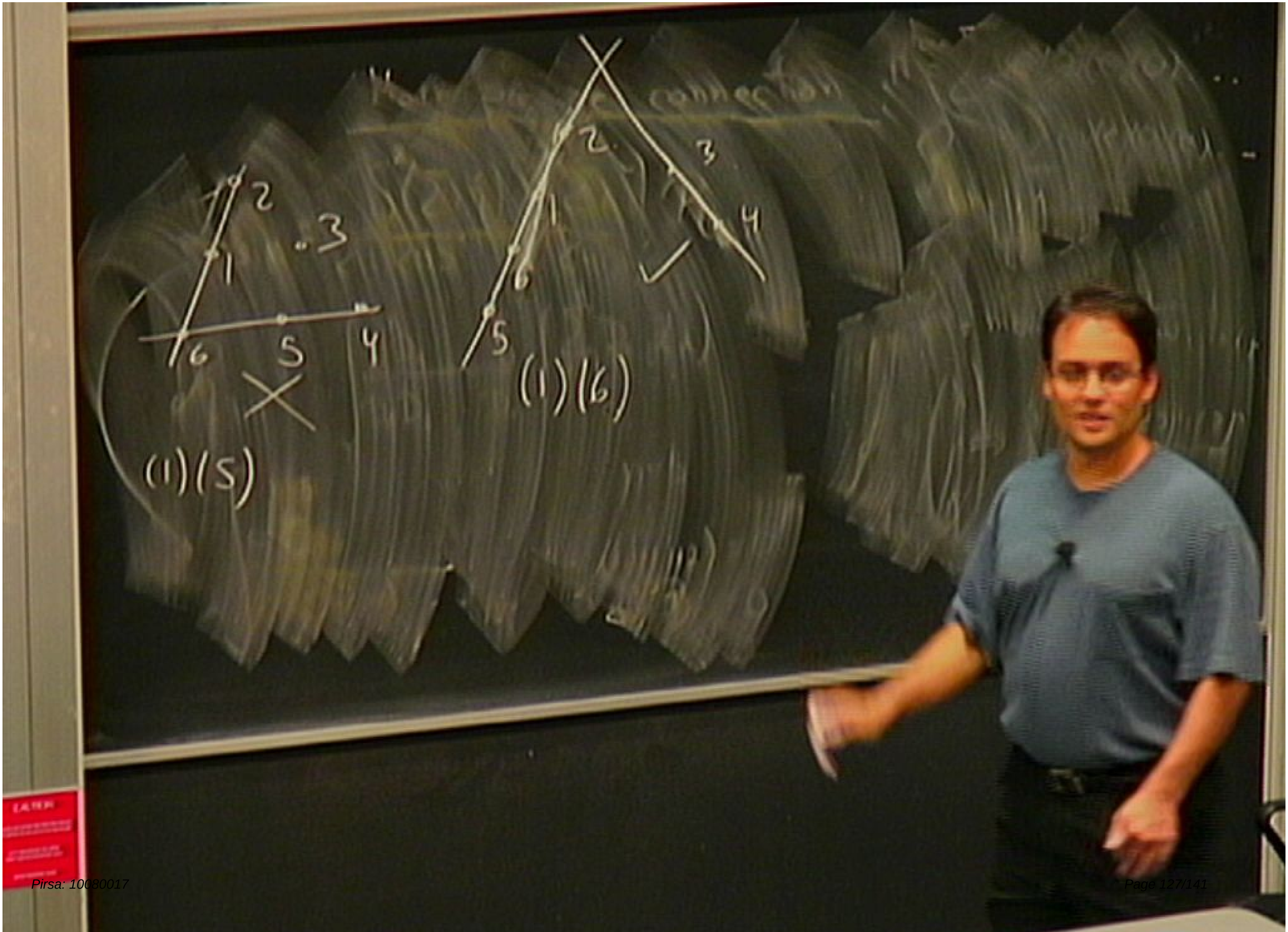

$\hat{n}=6$
$M^{tree} = (1)+(3)+(5)$
$(1) = \overline{(2)(3)(4)(5)(6)}$

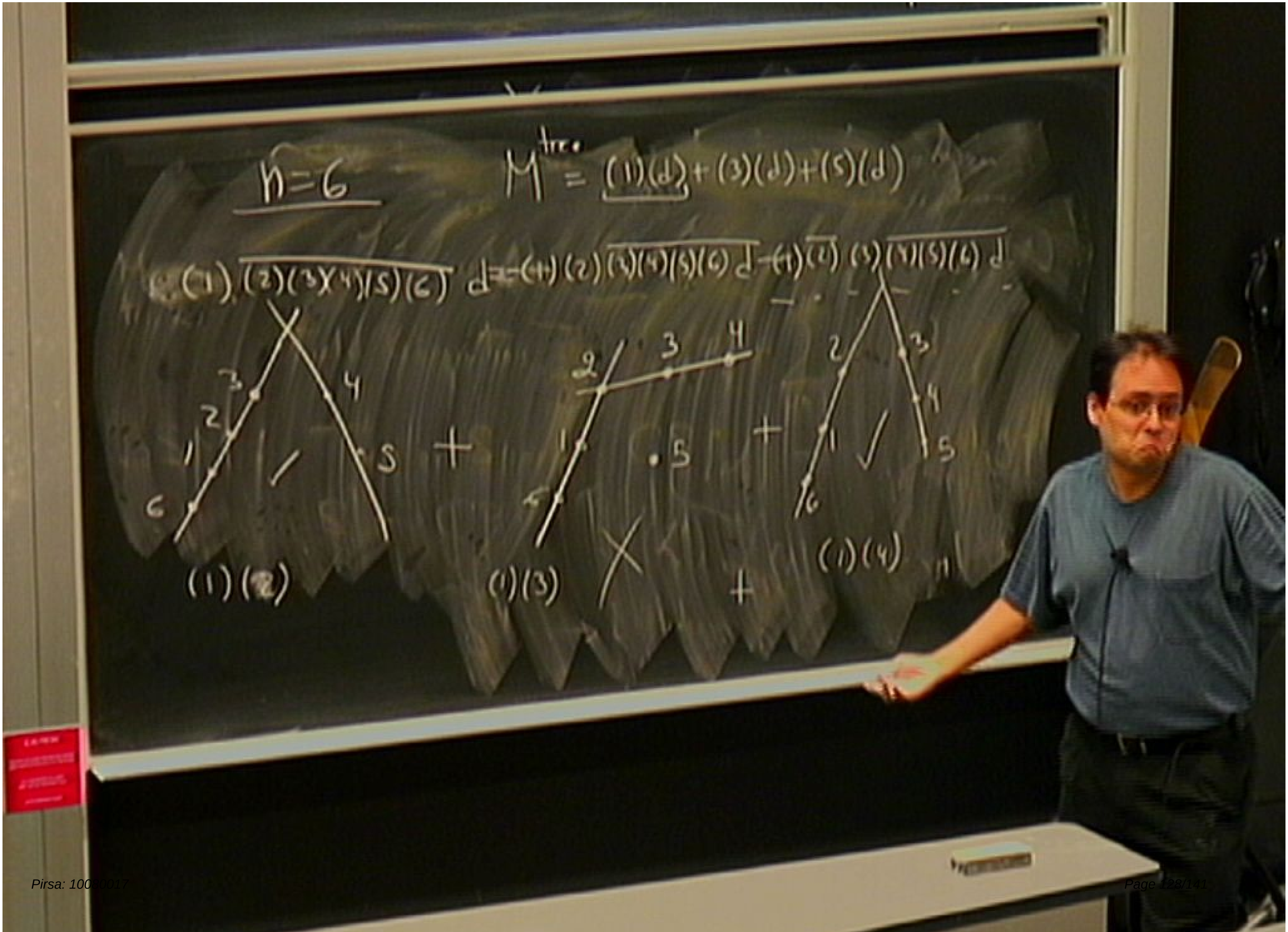

n=6
M tree = (1)(d) + (3)(d) + (5)(d)

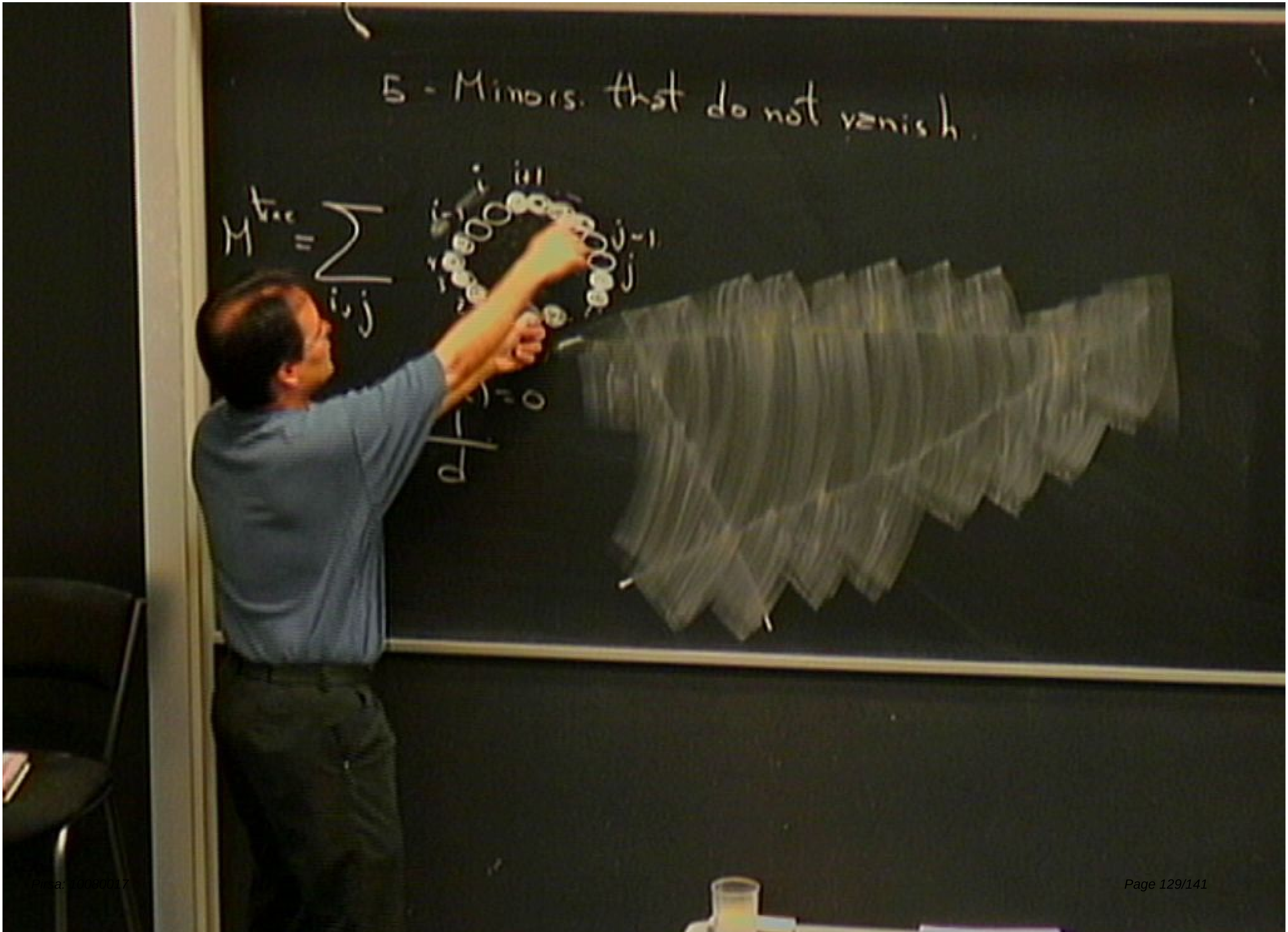

$\hat{n}=6$
$M^{tree} = (1)(d) + (3)(d) + (5)(d)$
$(1)\overline{(2)(3)(4)(5)(6)}\, d = -(1)(2)\overline{(3)(4)(5)(6)}\, d - (1)\overline{(2)}\,(3)\overline{(4)(5)(6)}\, d$
2
1
6


n=6
M tree = (1)(d) + (3)(d) + (5)(d)

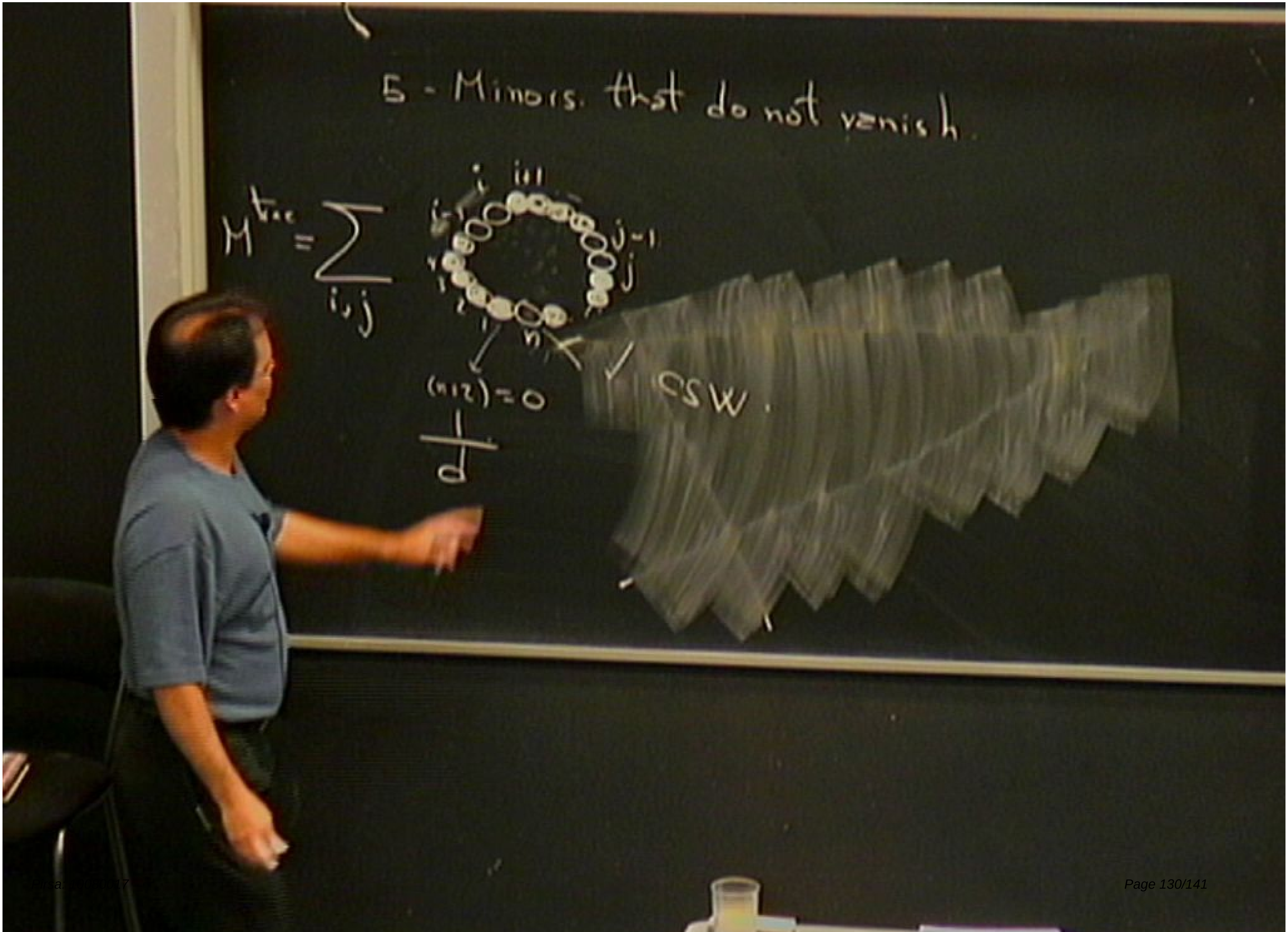

(1)(2)
(1)(3)
(1)(4)


2
3
1
6
5
4
(1)(5)
(1)(6)

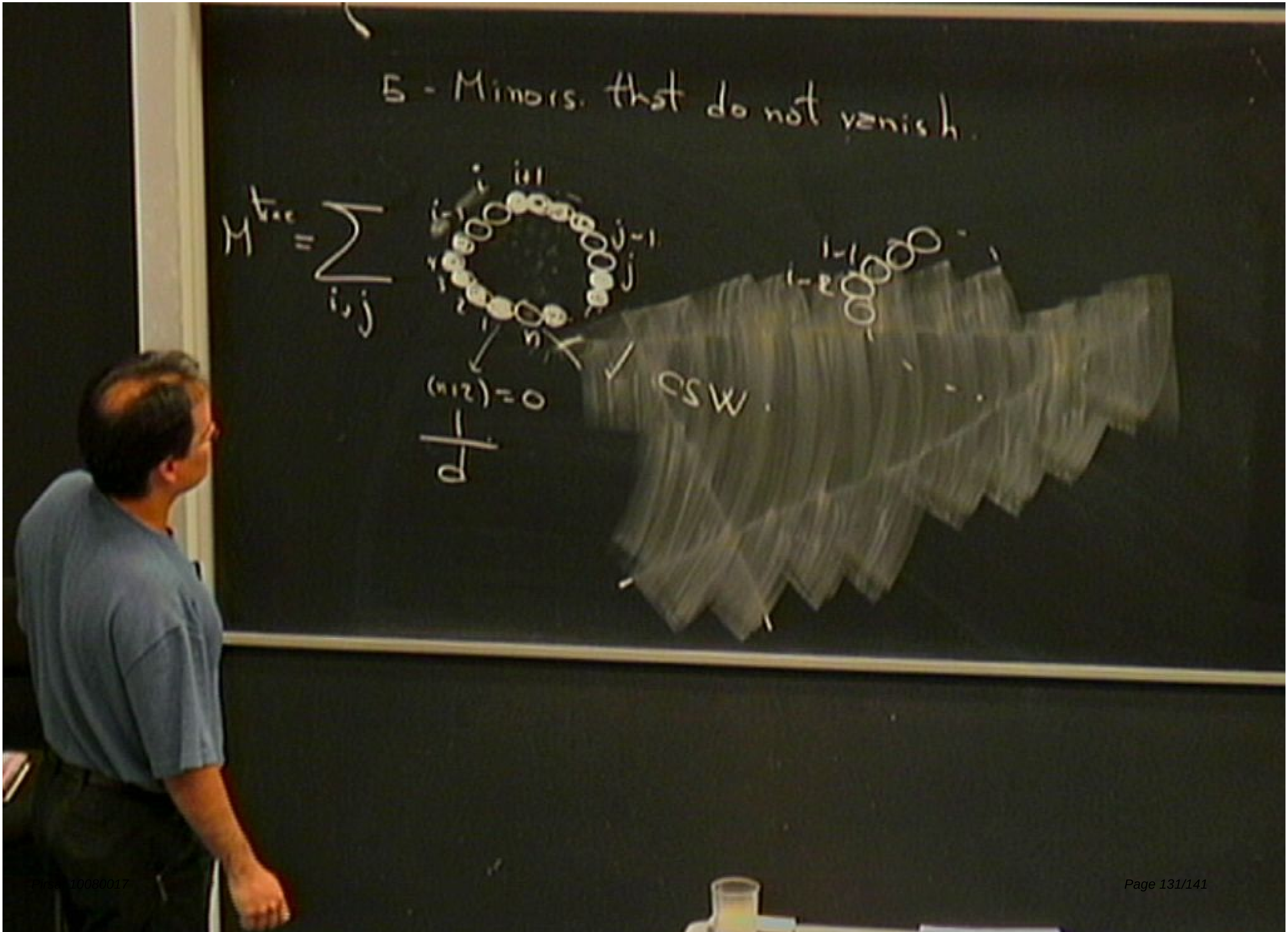

n=6
M tree = (1)(d)+(3)(d)+(5)(d)
(1)(2)
(1)(3)
(1)(4)

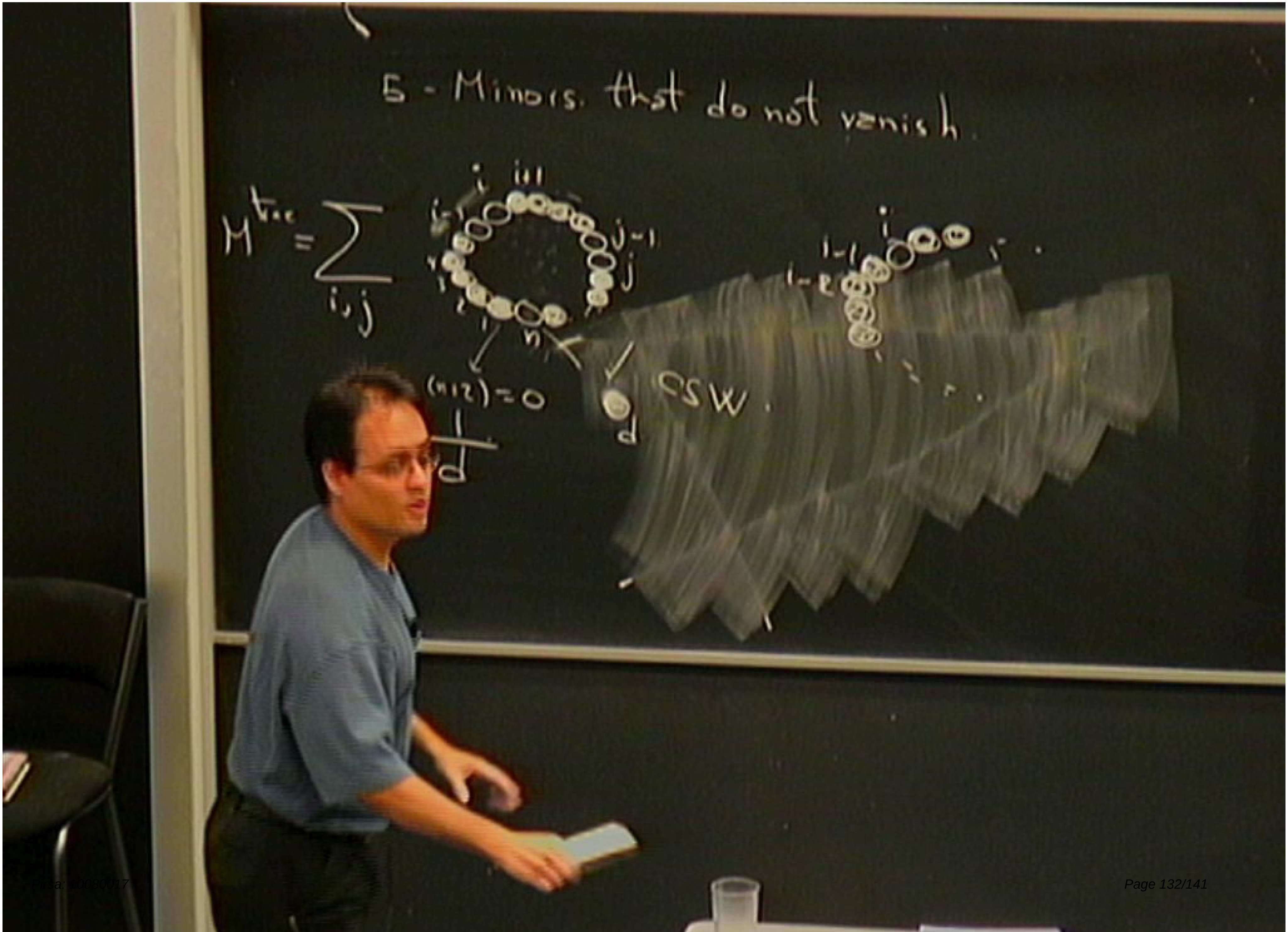


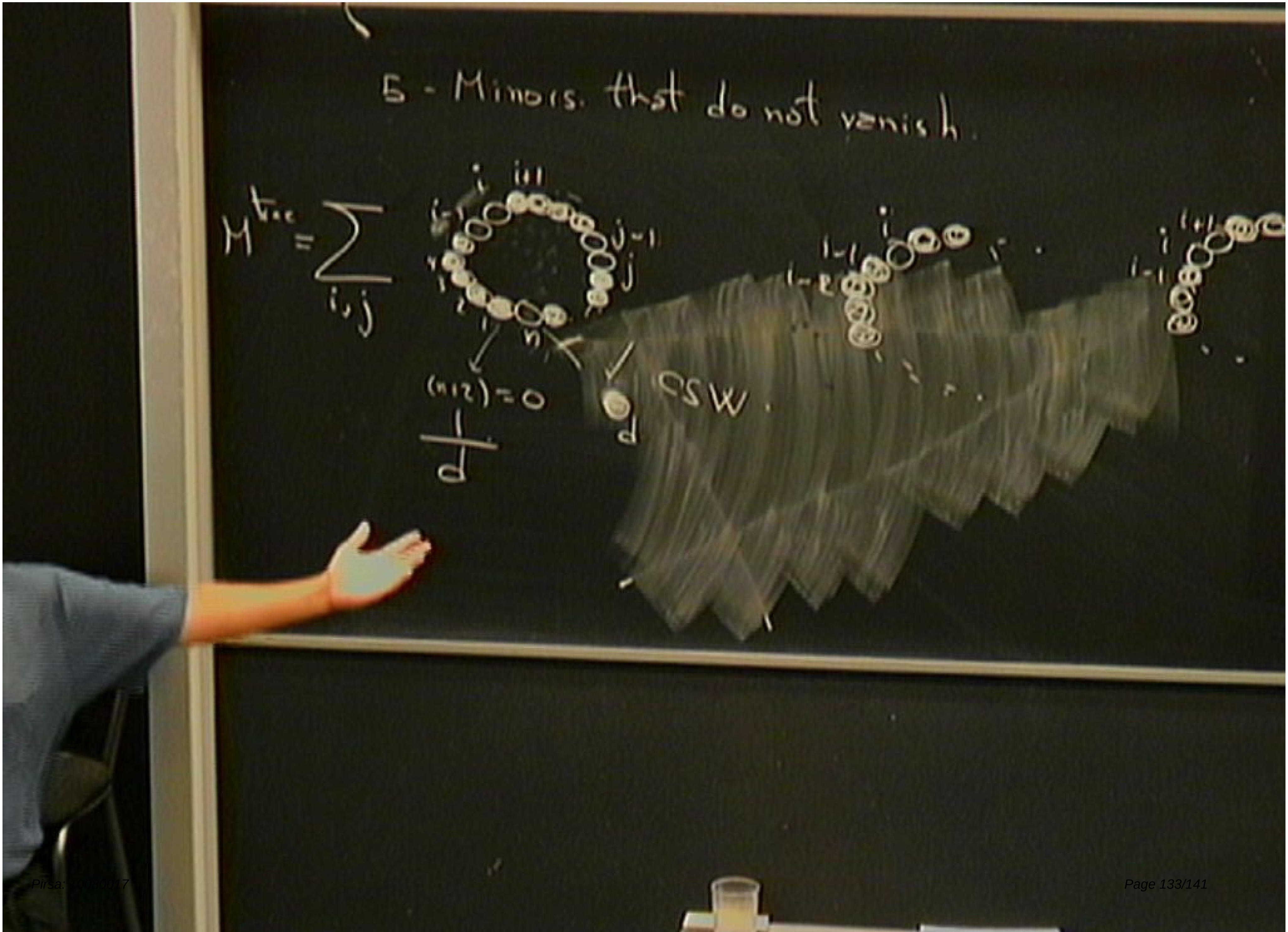
5 - Minors that do not vanish.
$M^{tree} = \sum_{i,j}$
i
i+1
j-1
j

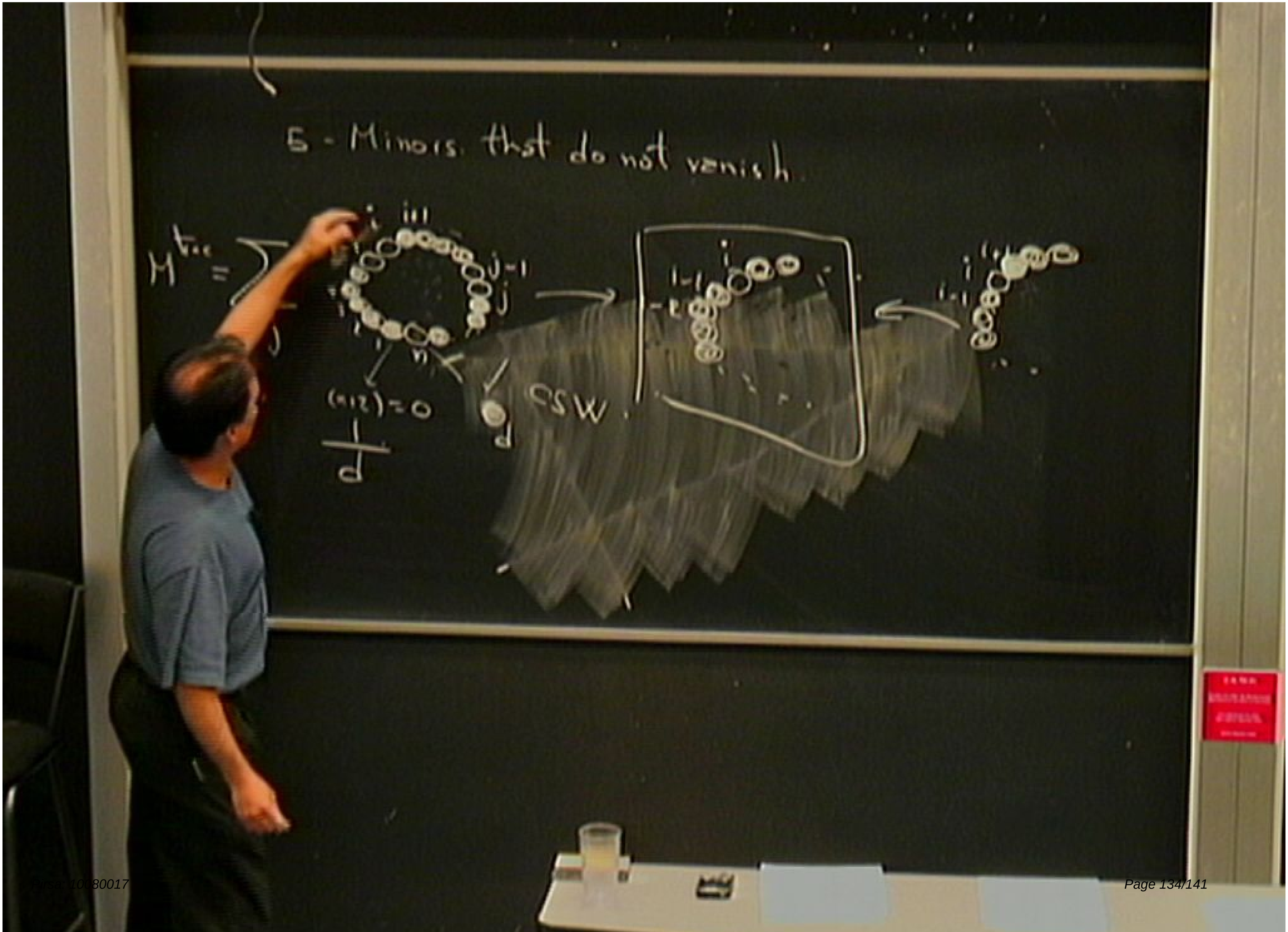


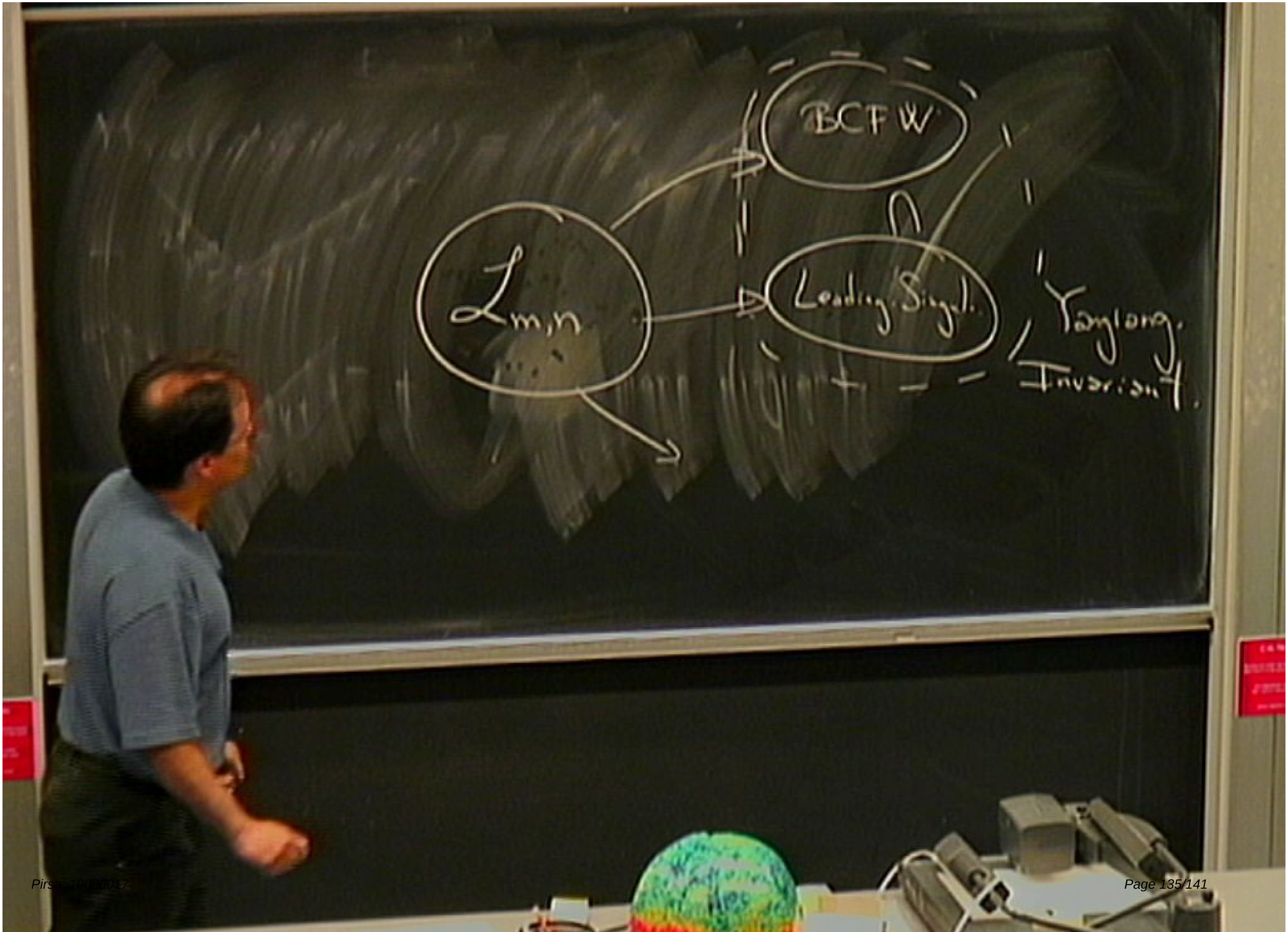
5 - Minors that do not vanish.
$M^{tree} = \sum_{i,j}$
i
i+1
j-1
j
$\langle n\,1\,2\rangle = 0$
CSW.

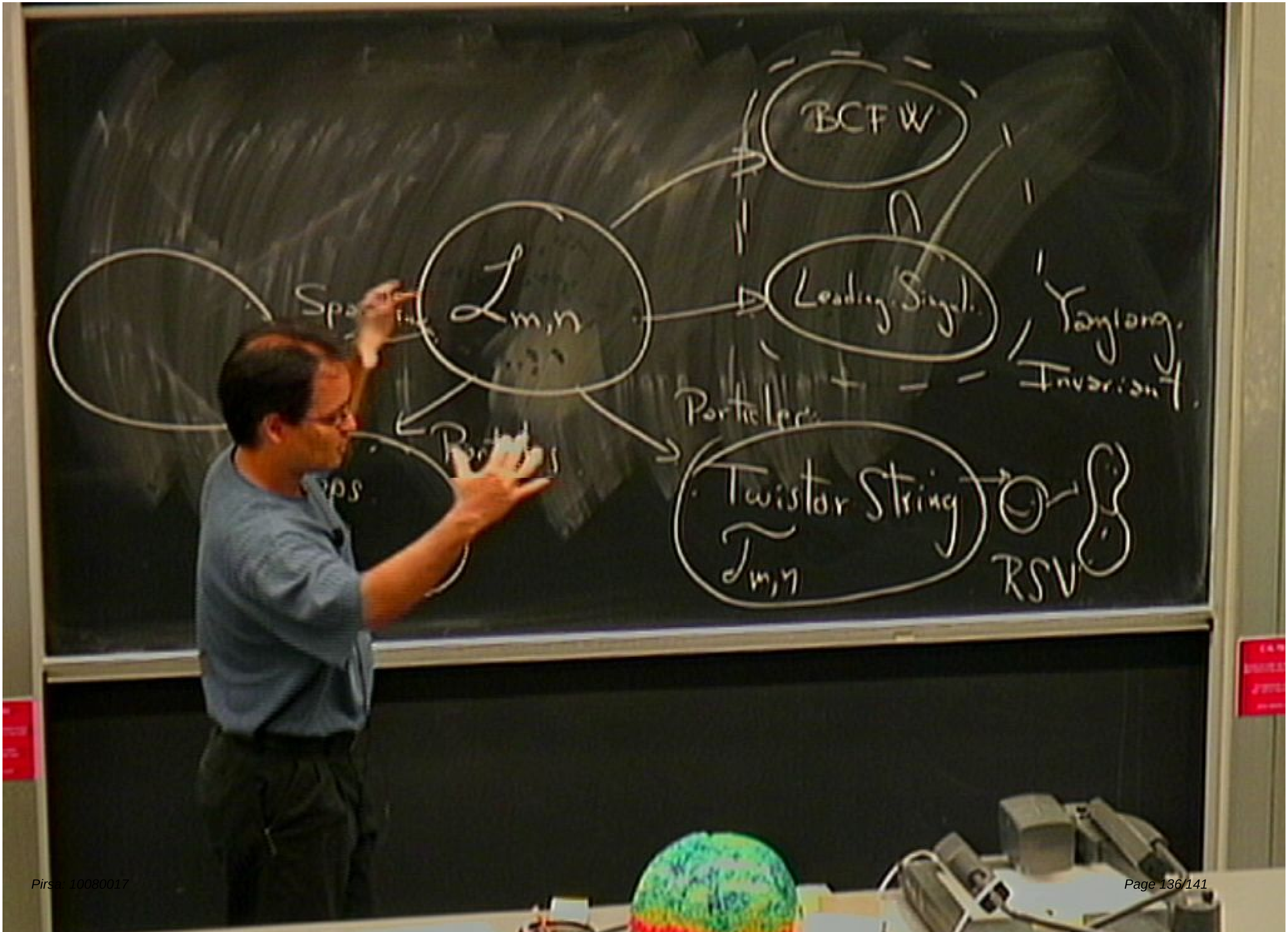

5 - Minors that do not vanish.
$M^{tree} = \sum_{i,j}$
$(n12)=0$
CSW

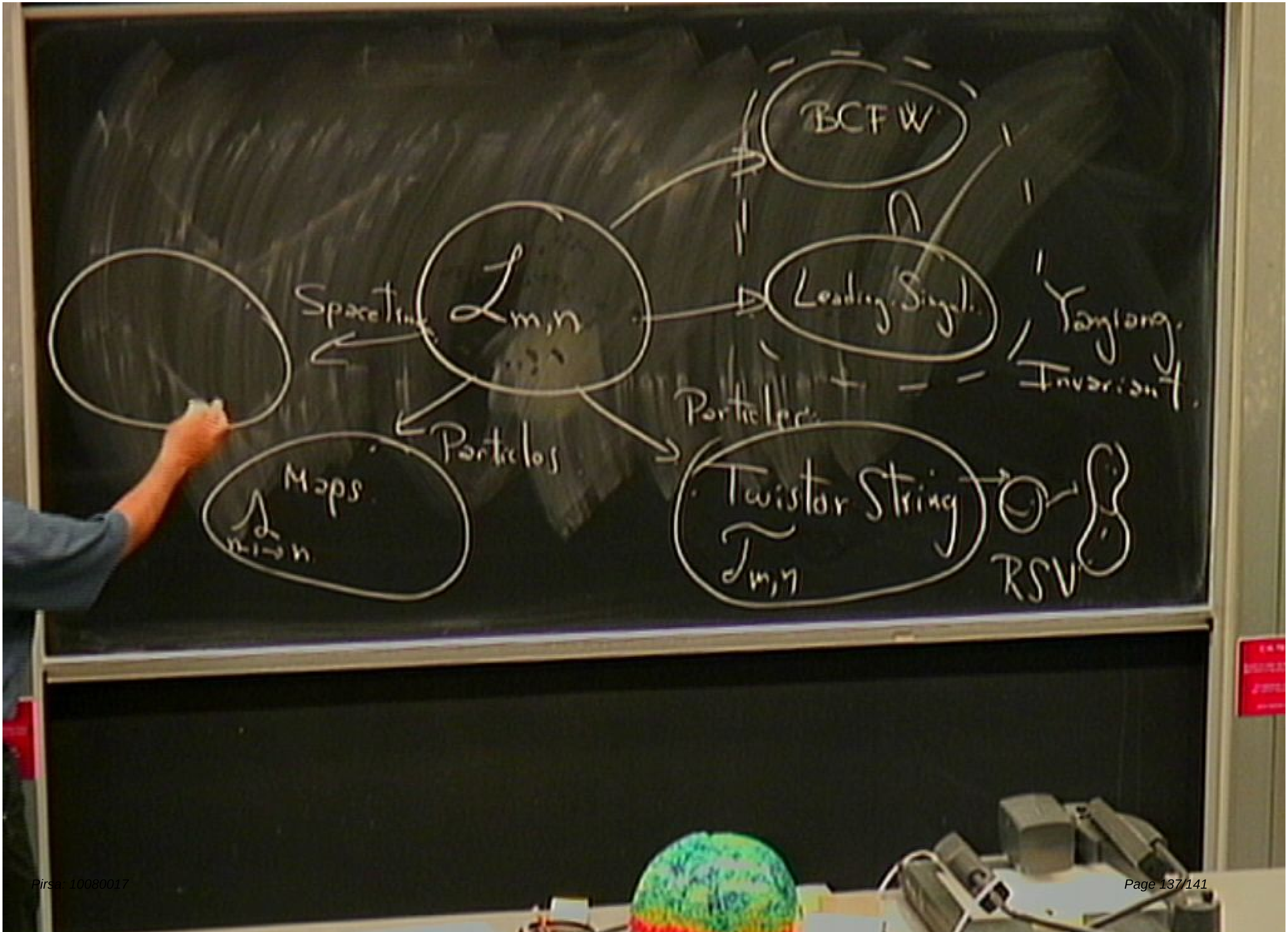

5 - Minors that do not vanish.
CSW

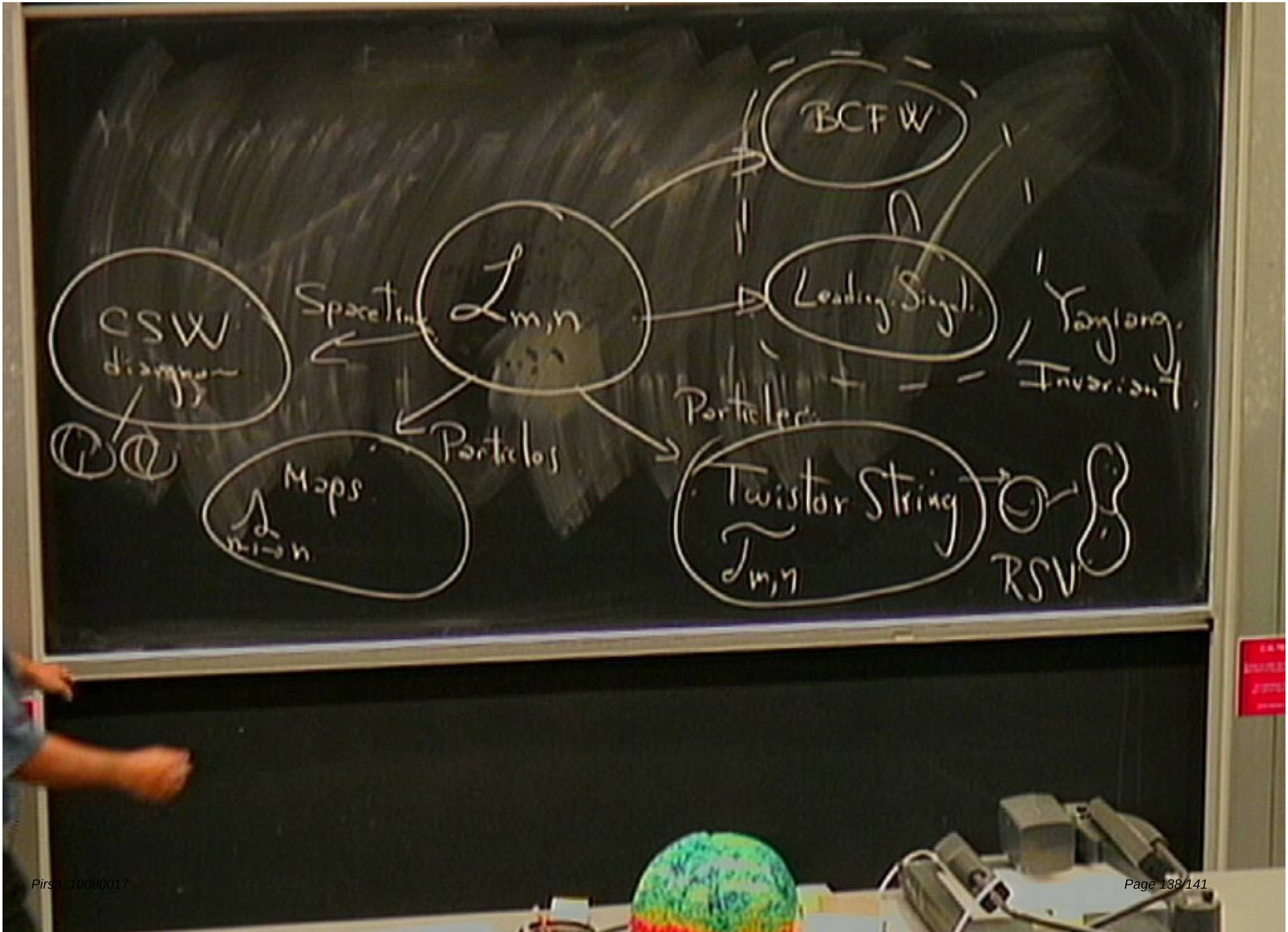

5 - Minors that do not vanish.
M^tree = Σ i,j
(n12) = 0
CSW.

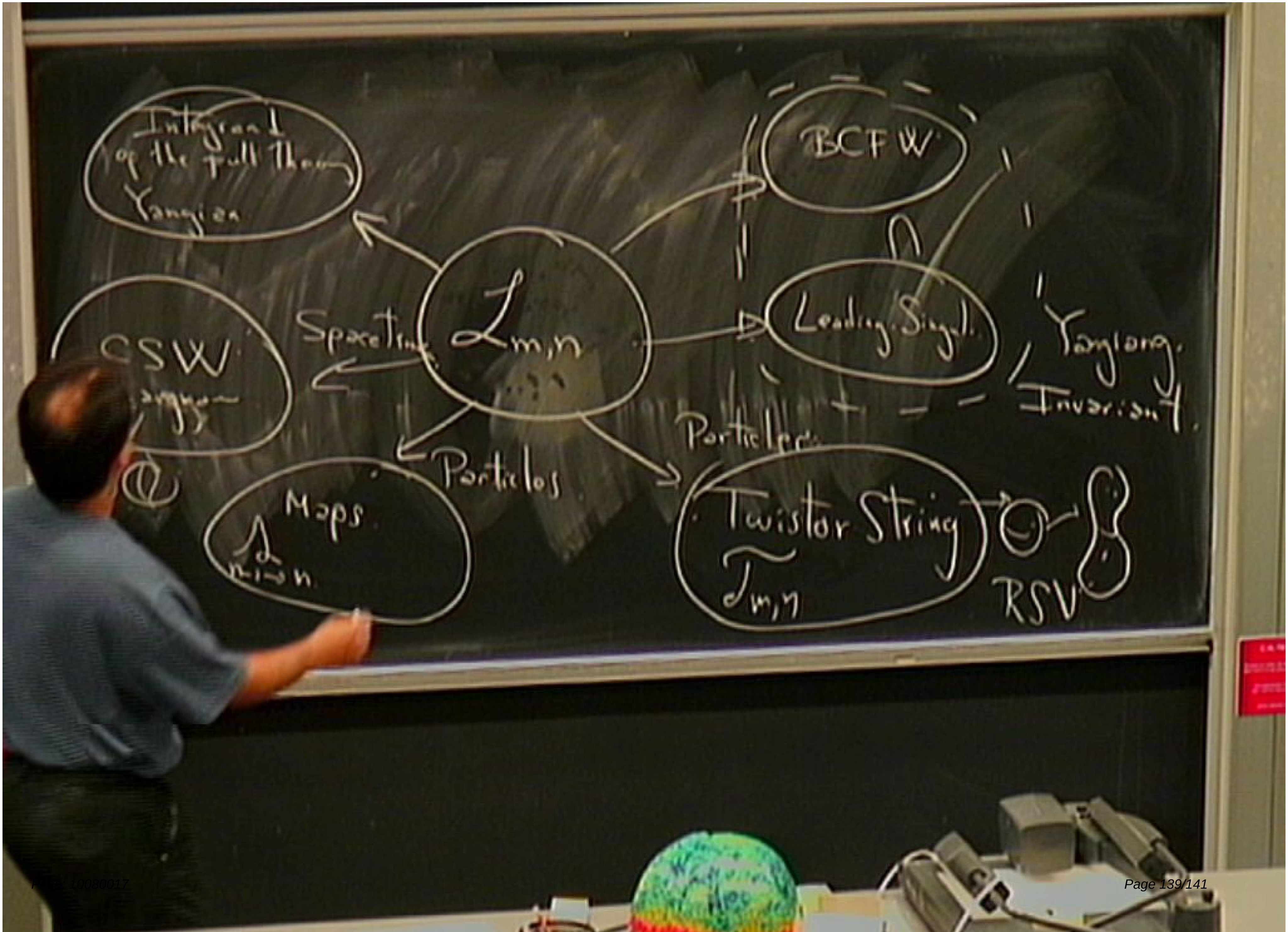

5 - Minors that do not vanish.
CSW

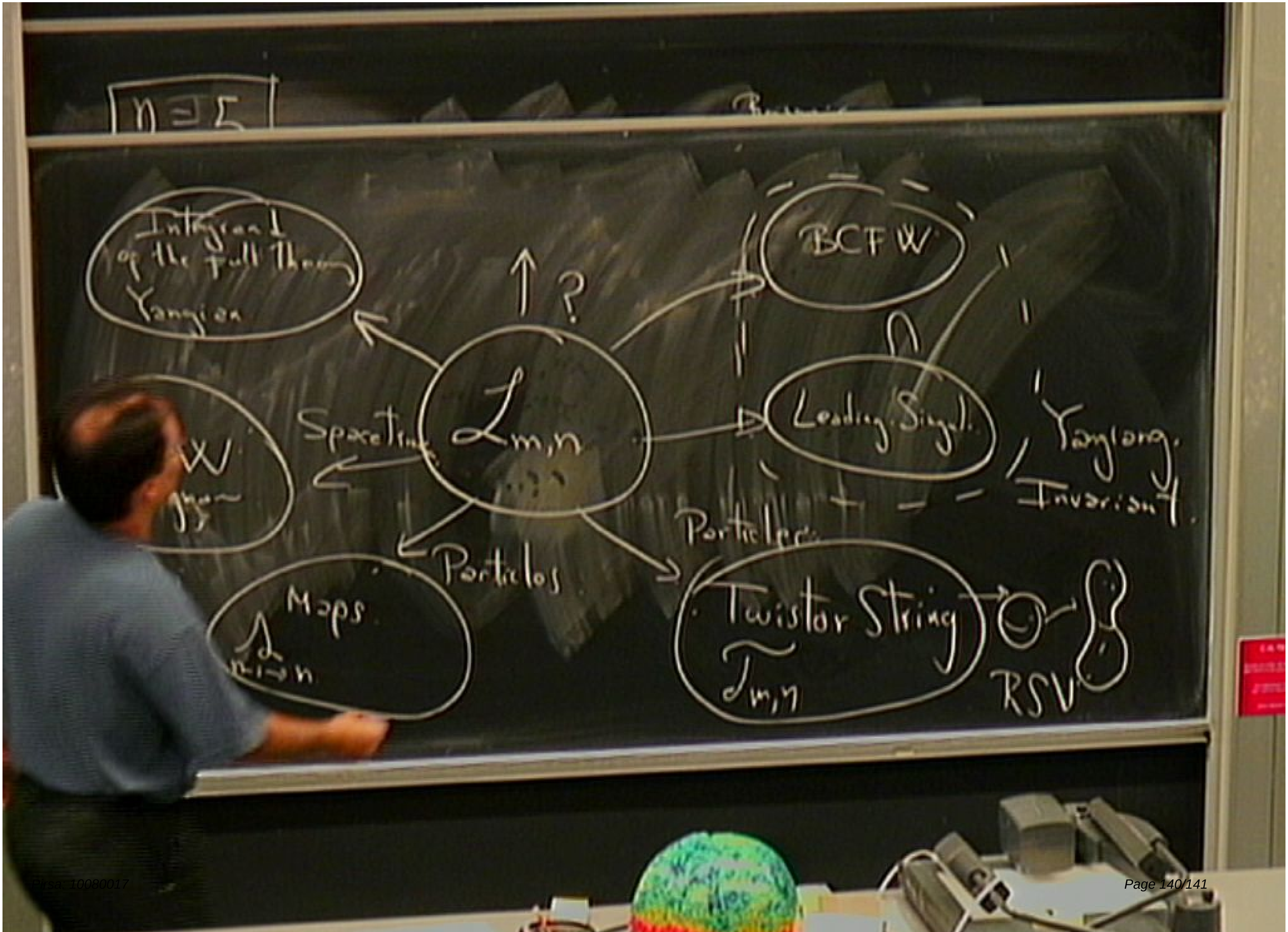

BCFW
$\mathcal{L}_{m,n}$
Leading Singul.
Yangian
Invariant


BCFW
Leading Singl.
Yangiang Invariant
Particles
Twistor String
RSV

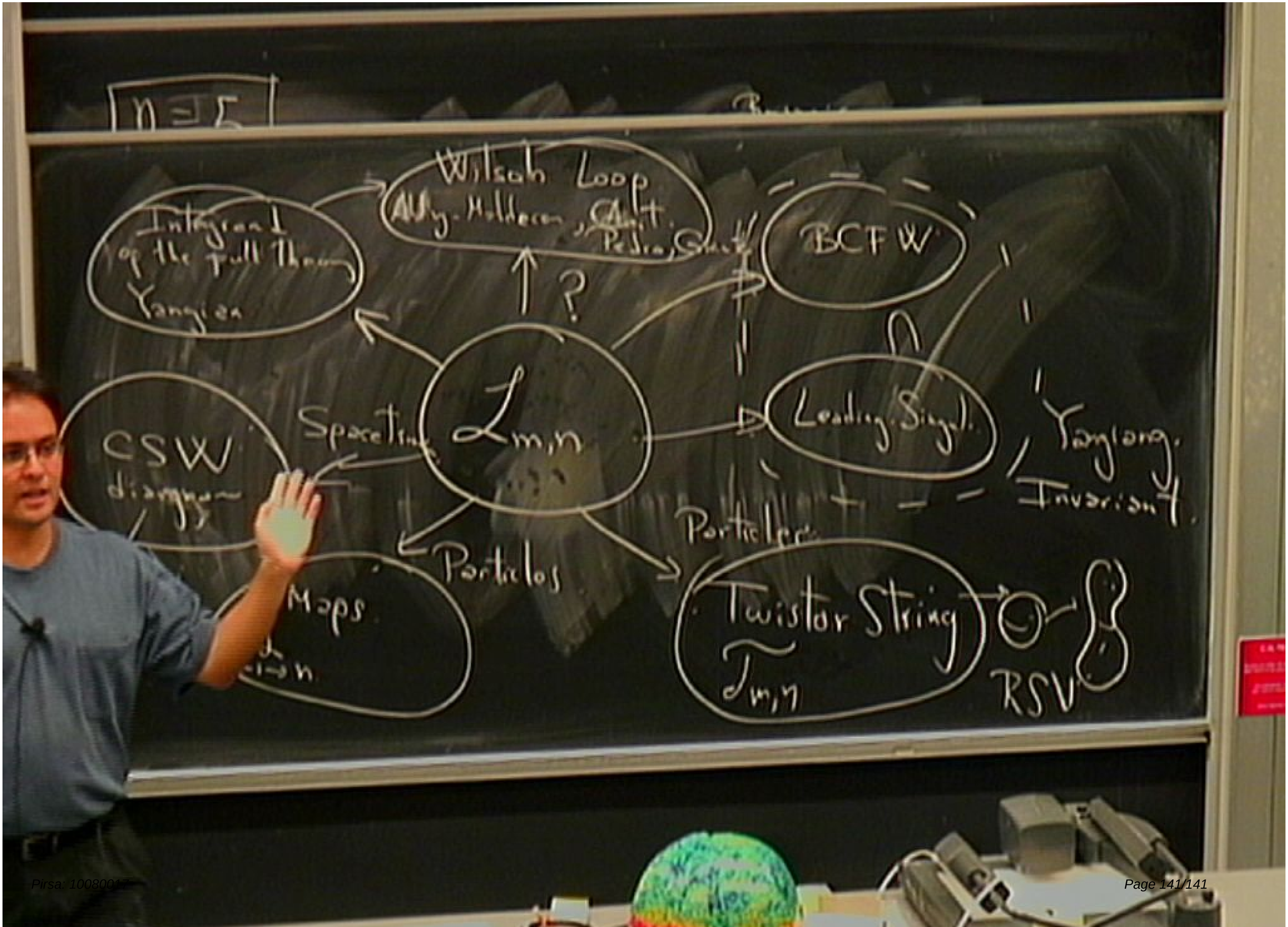

BCFW
Spacetime
$\mathcal{L}_{m,n}$
Leading Singul.
Yangian Invariant
Particles
Particles
Maps
Twistor String
$\tilde{\mathcal{L}}_{m,n}$
RSV


BCFW
Leading Singul.
Yangiang
Invariant
CSW
diagrams
Spacetime
Particles
Particles
Maps
Twistor String
RSV


Integrand of the full theory
Yangian
BCFW
CSW
Spacetime
$\mathcal{L}_{m,n}$
Leading Singul.
Yangiang Invariant
Particles
Particles
Maps
Twistor String
$\tilde{\mathcal{L}}_{m,n}$
RSV



Integrand of the Full Theory
Yangian
?
BCFW
Spacetime
$\mathcal{L}_{m,n}$
Leading Singul.
Yangian Invariant
Particles
Particles
Maps
Twistor String
$\tilde{\mathcal{L}}_{m,n}$
RSV



Wilson Loop
Integrand of the full theory
Yangian
BCFW
CSW diagram
Spacetime
Particles
Leading Singl.
Yangiang. Invariant
Maps
Twistor String
RSV