Title: Topological defects and vacuum decay

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Abstract: False vacua in QFT are liable to undergo spontaneous decay. Slowness of quantum tunneling can however allow a long lifetime to the false vacuum state. In supersymmetric theories this is a crucial criterion for obtaining a long lived universe with spontaneously broken supersymmetry. We have explored false vacua which admit topological defects, including in a supersymmetric model with O'Rafeartaigh type supersymmetry breaking. We show that the presence of topological defects significantly alters the stability of the false vacuum. For some values of parameters, it also results in the putative false vacuum being rendered unstable. Finally I report a formula derived for instanton assisted tunneling applicable to metastable vacua with monopoles.

Topology and vacuum stability

by U. A. Yajnik

Indian Institute of Technology, Bombay; (also U de M and McGill)
collaborators
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August 20, 2010, Perimeter Institute, Canada

Determining supersymmetric vacua

- A persistent problem of supersymmetry breakdown is that such vacua may be only local minima and have the danger of relaxing to a true minimum where supersymmetry is restored.
- This unpleasant possibility is however easy to avoid if the tunneling rate to the true vacuum can be made much longer than the known age of the Universe.

Implications of topological solutions

- It is not sufficient to study the translationally invariant i.e., spatially homogeneous avatars of the vacua.
- Generically, topological defects such as cosmic strings or monopoles are possible, though model dependent.
- A class of topological defects can exist which can nucleate the formation of the true vacuum
  - A local minimum can be rendered unstable against decay to a vacuum of lower energy
  - the presence of the cosmic string entailed the consequence that false vacuum would "roll-over" smoothly to the true vacuum

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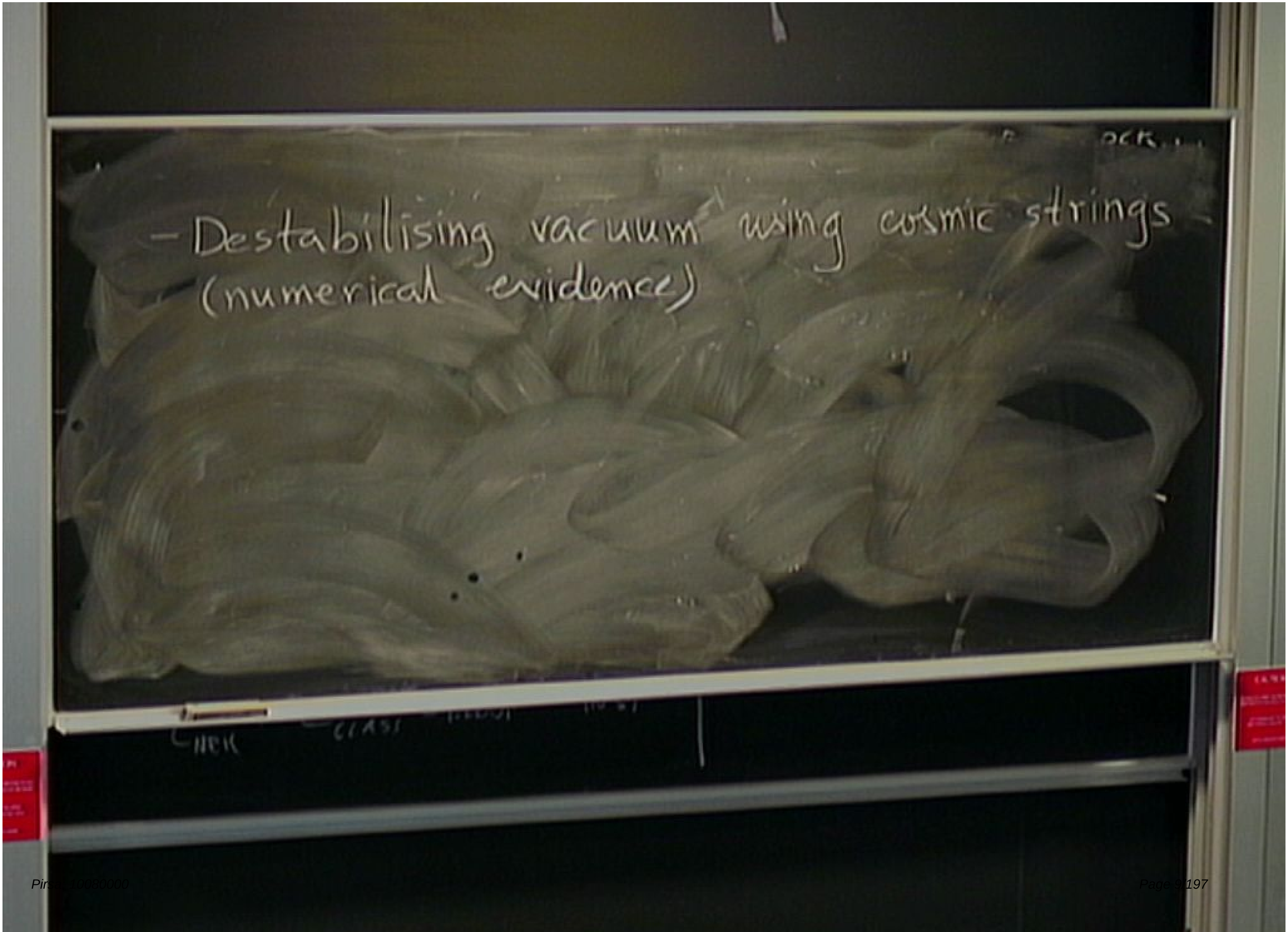
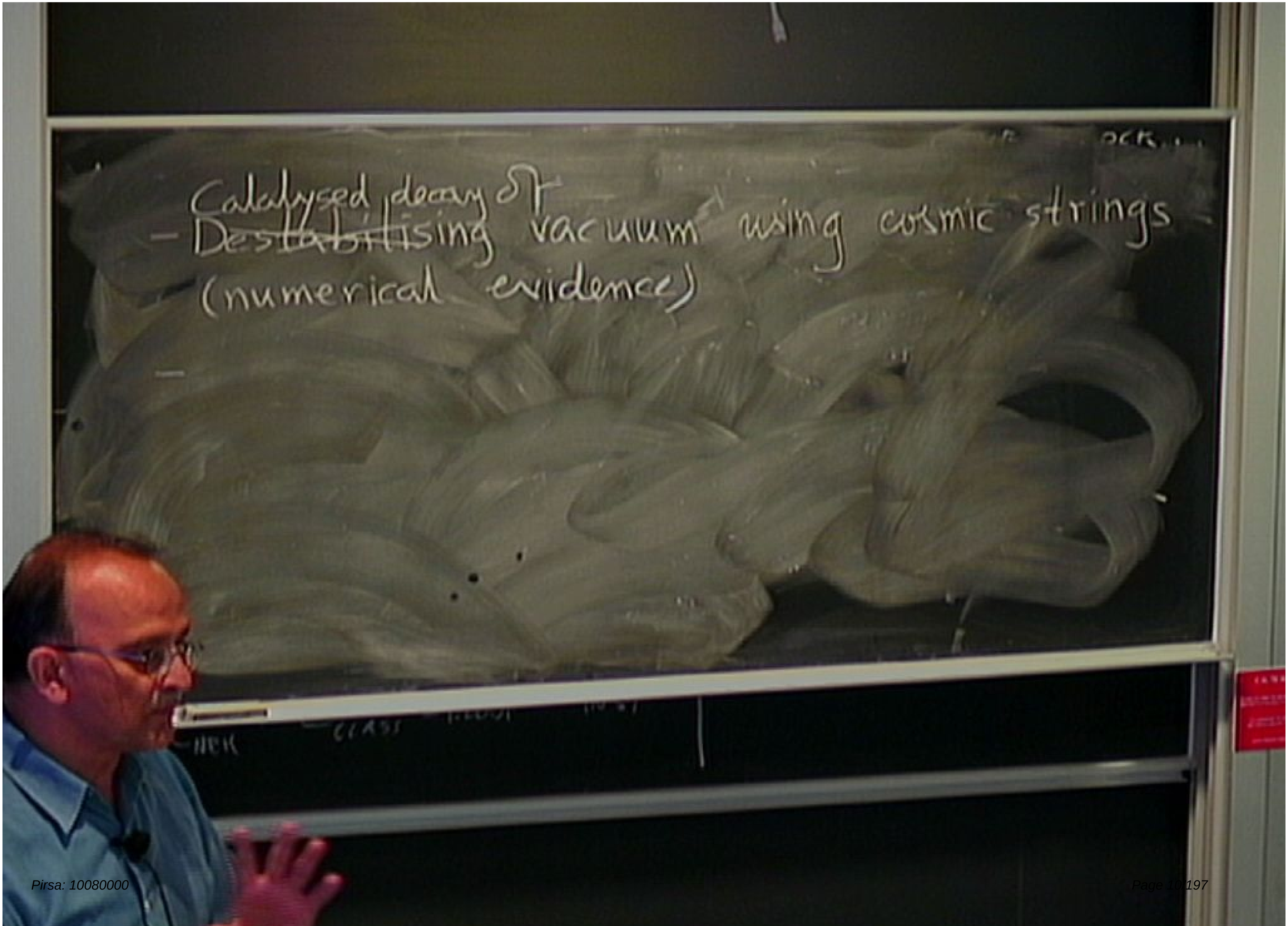

- Destabilising vacuum using cosmic strings
(numerical evidence)


Catalysed decay of
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- ~~Destabilising~~ vacuum using cosmic strings
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- A closed form formula* (compare V-K-O-C)
- Numerical evidence (monopoles)

* Monopoles in SU(2) with a triplet

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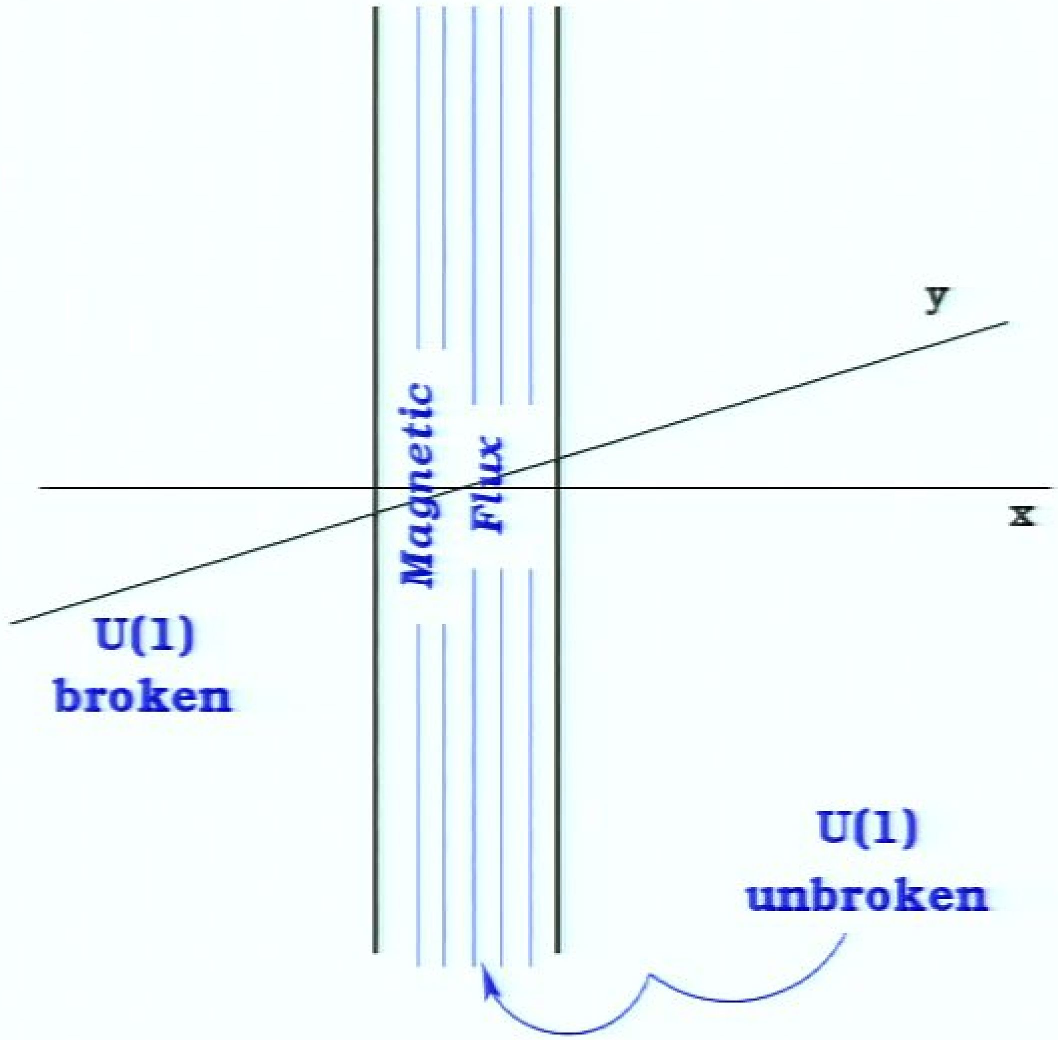
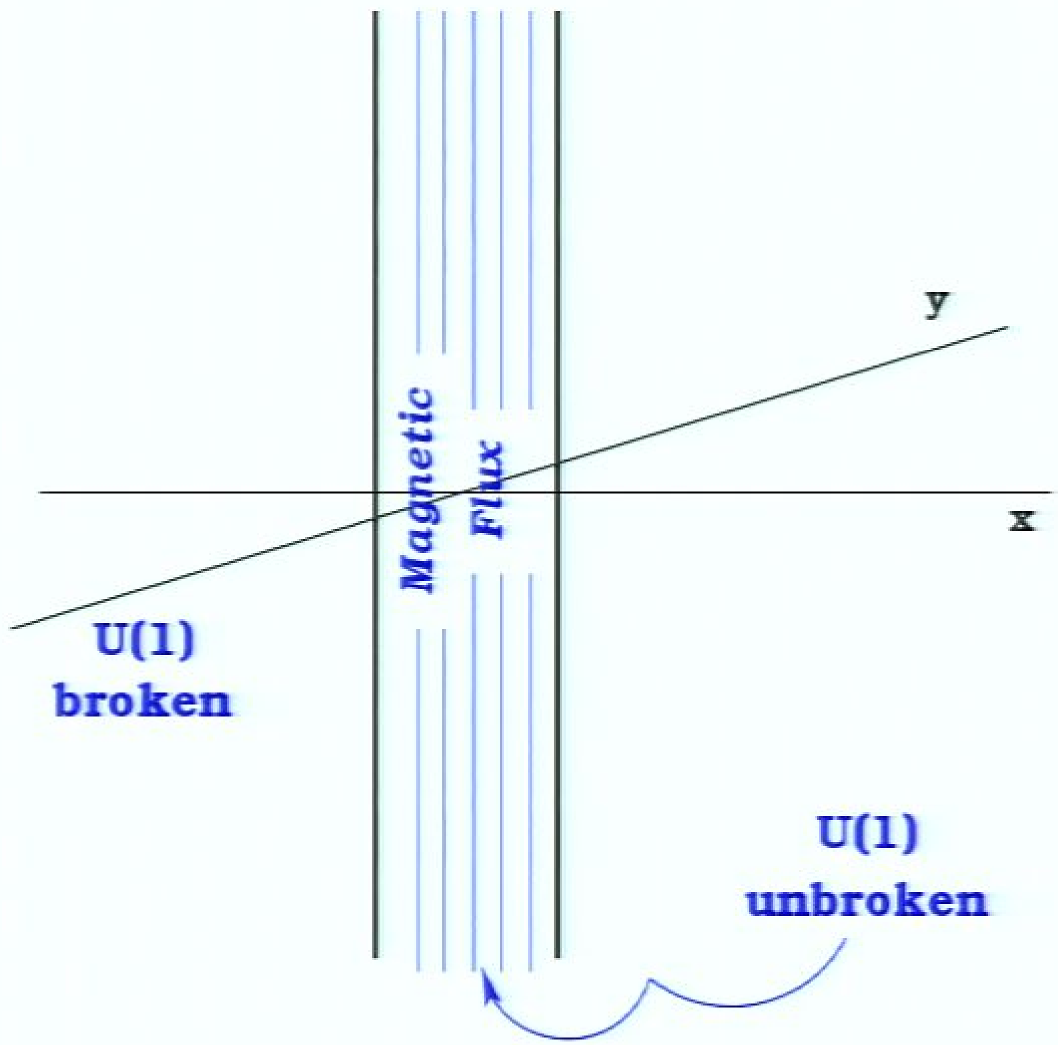
Modification of vacuum

Nielsen and Olesen (1973)



y
x
Magnetic
Flux
U(1)
broken
U(1)
unbroken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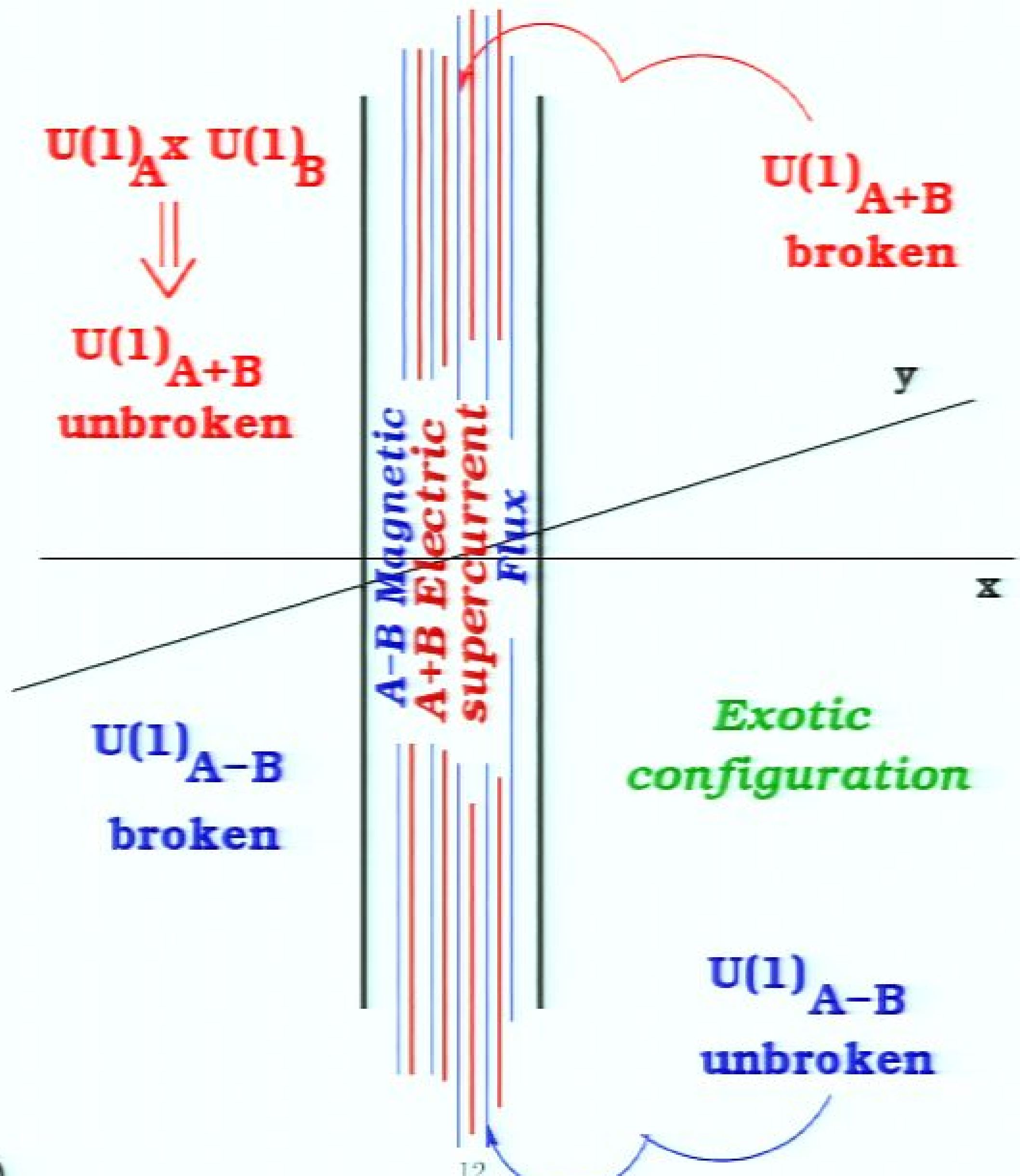
a macroscopic signature of gauge symmetry breakdown

Superconducting string



Witten (1985)

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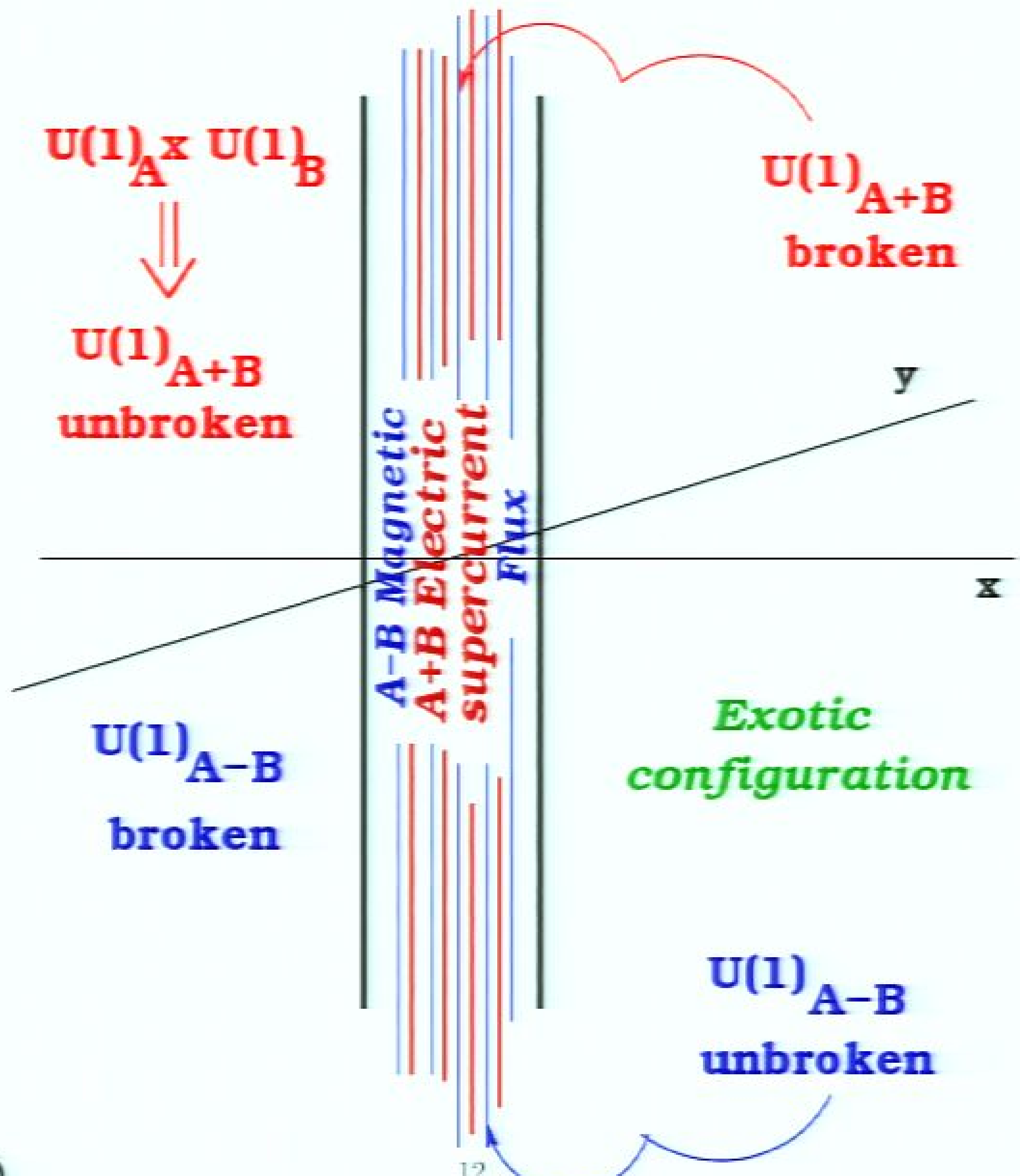

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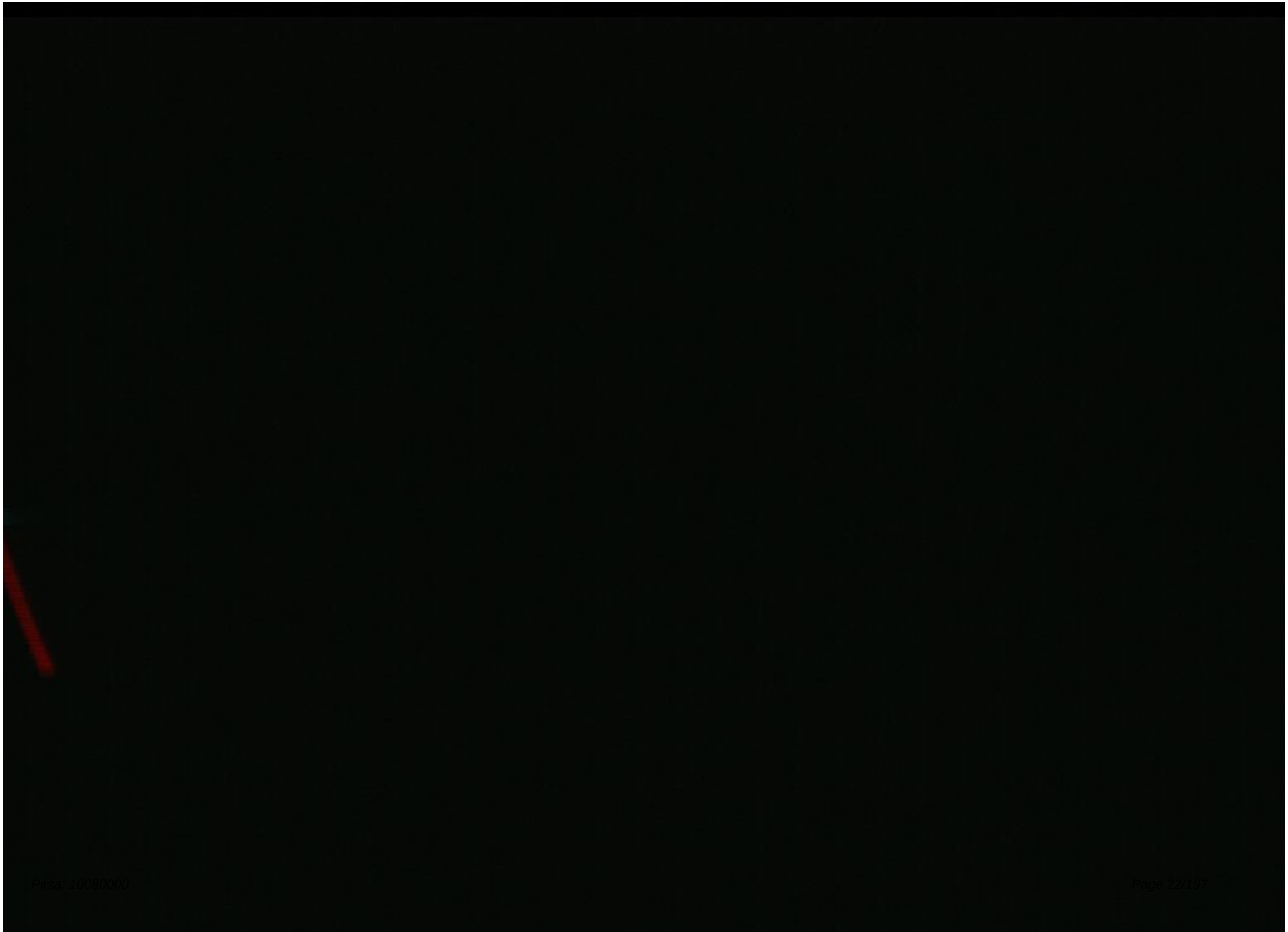

Exotic configurations in general

- $V^{\rm eff}(\Phi)$ has minima at Φ_1, Φ_2 etc.
- The vortex ansatz is generated by $K = \sum \alpha_j T_j$
- Generically a VEV Φ_i decomposes into subspaces Φ_{iS} and Φ_{iB} such that

$$[K, \Phi_{iS}] = 0; \qquad [K, \Phi_{iB}] \neq 0$$

- Resulting vortex ansatz for Φ_i becomes

$$\Phi_i^{\rm vortex}(r, \theta) = s_i(r)\Phi_{iS} + b_i(r)\exp(i\theta)\Phi_{iB}$$

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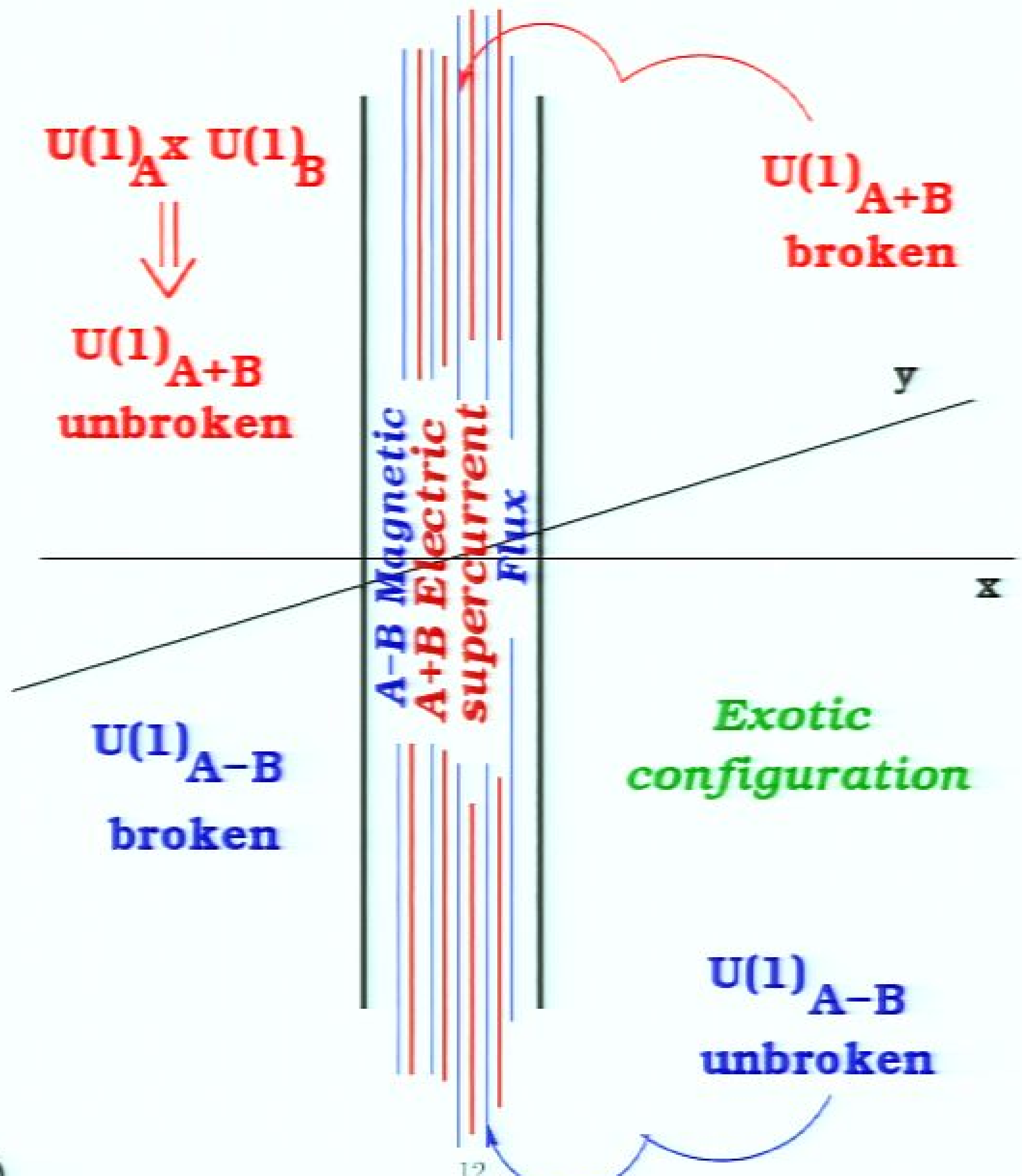

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- Compute the minima of $V^{\text{eff}}(s)$, say $s=0$, $s=s_1$
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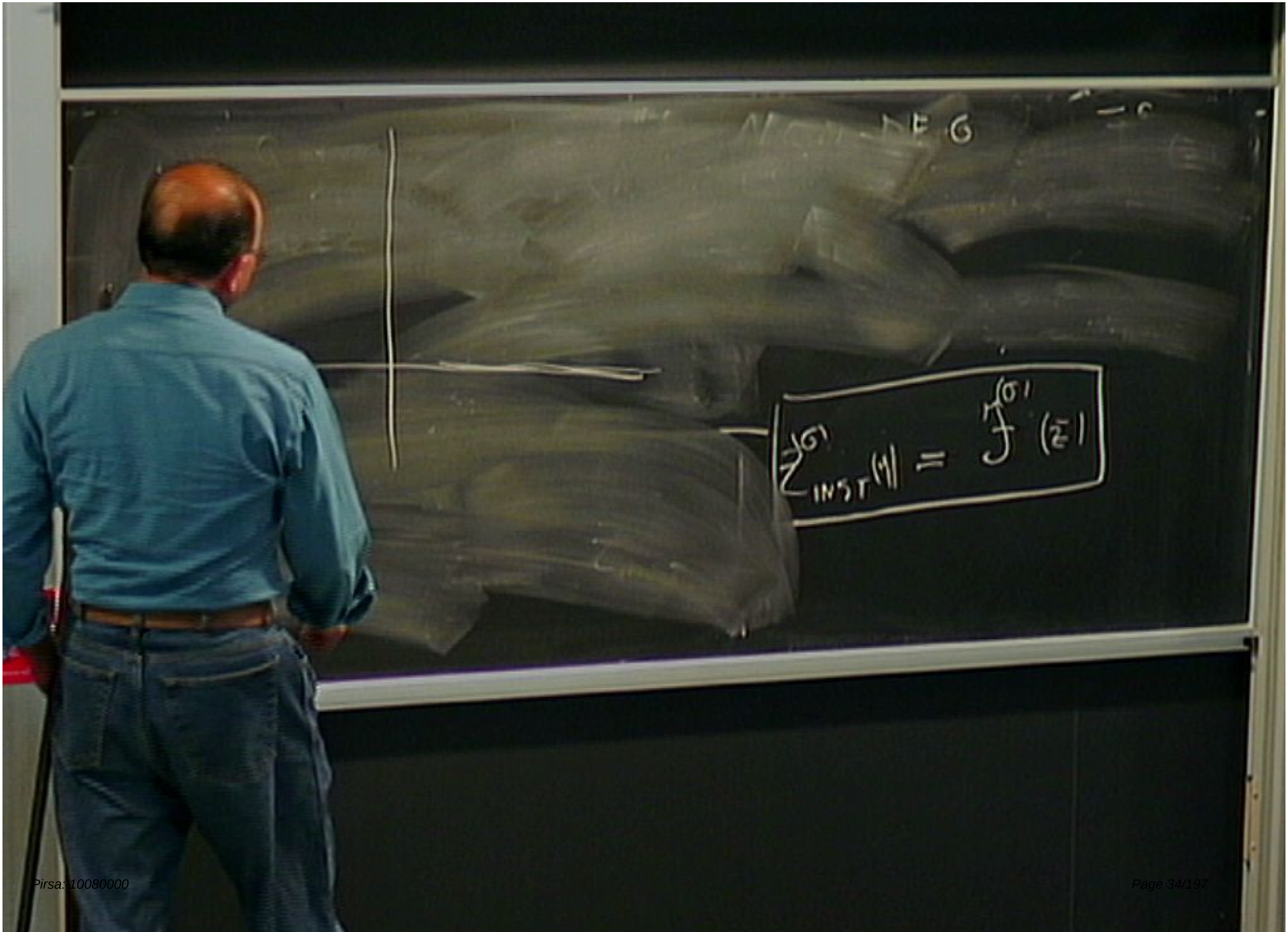
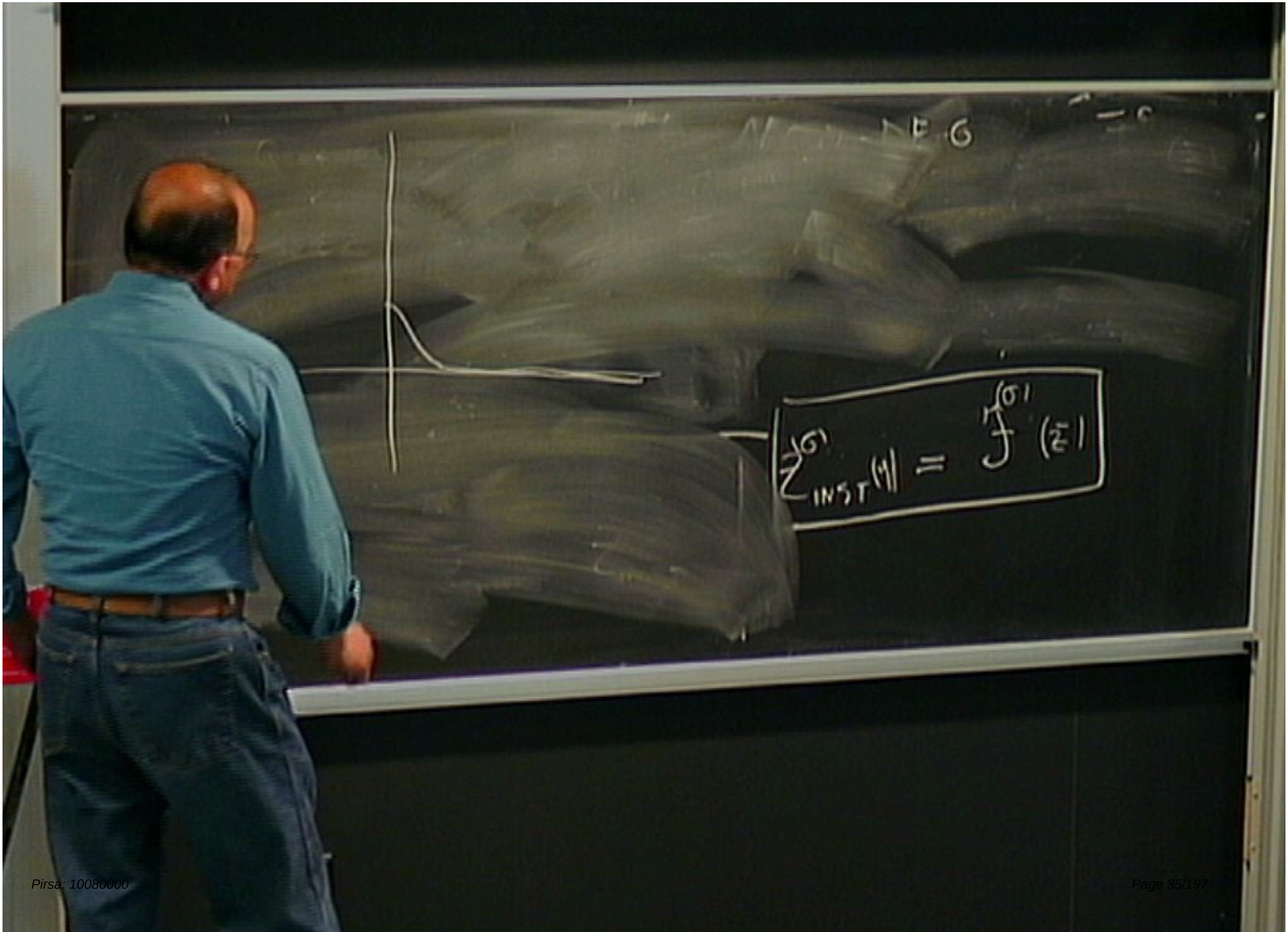
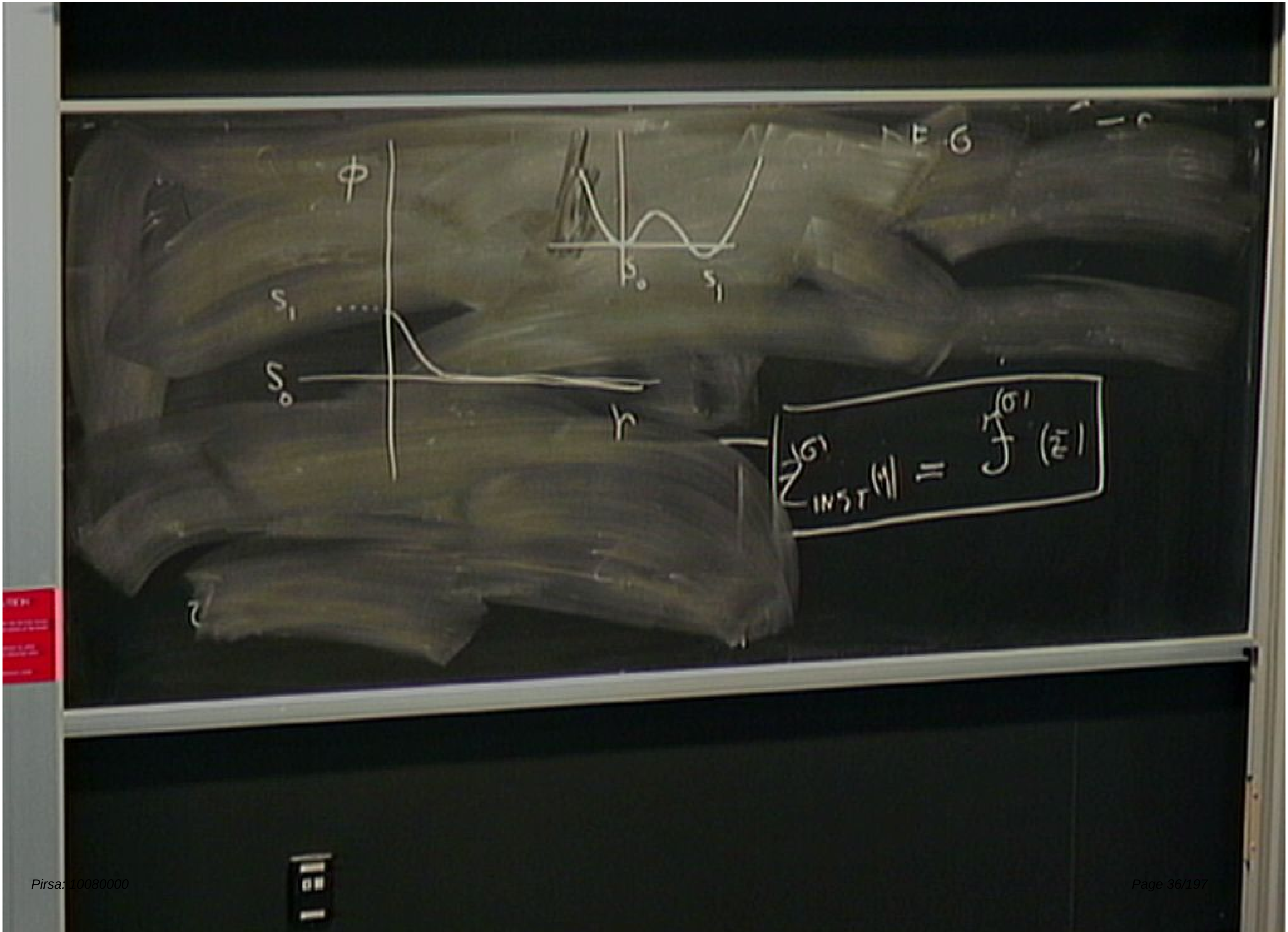
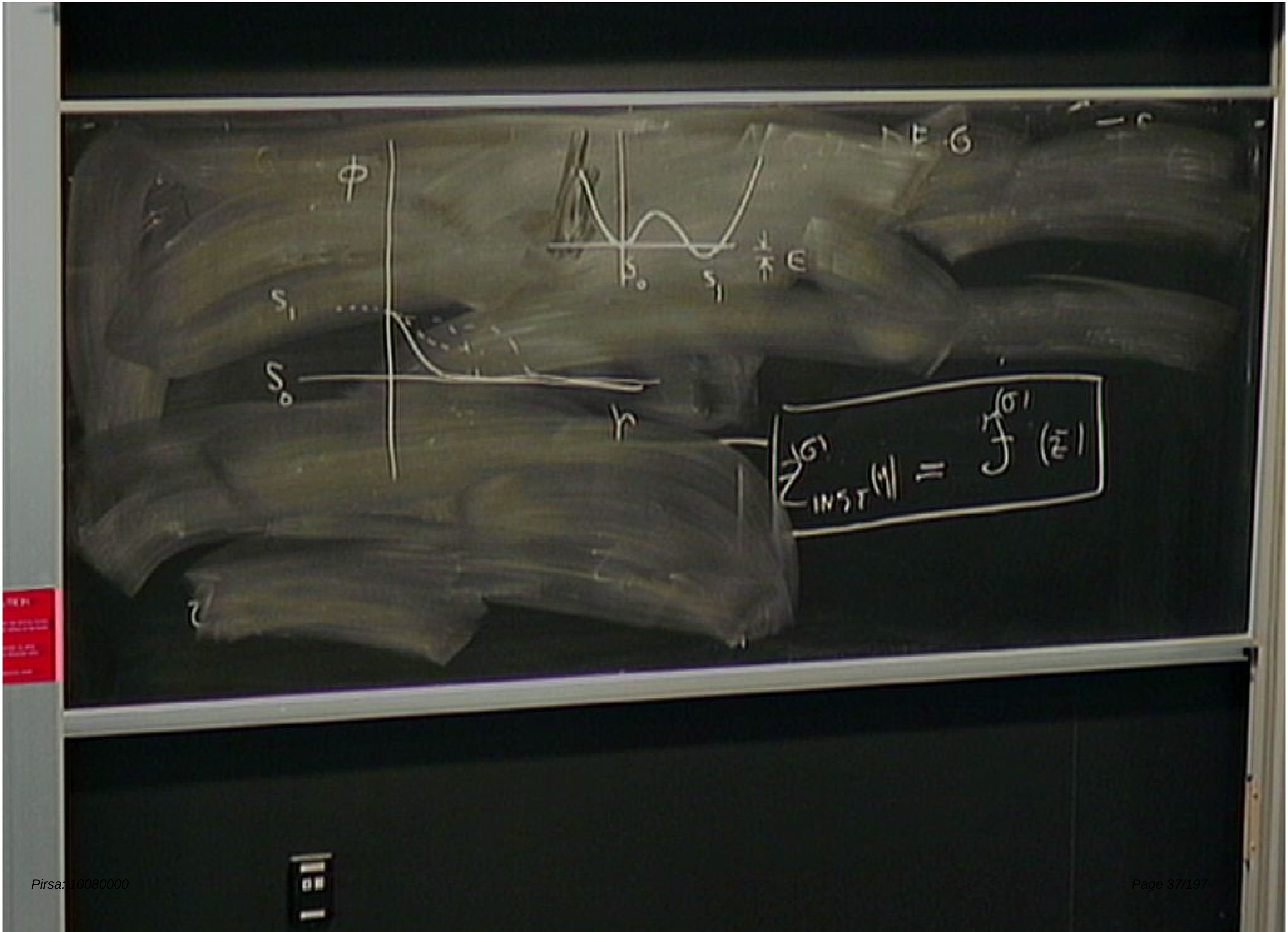
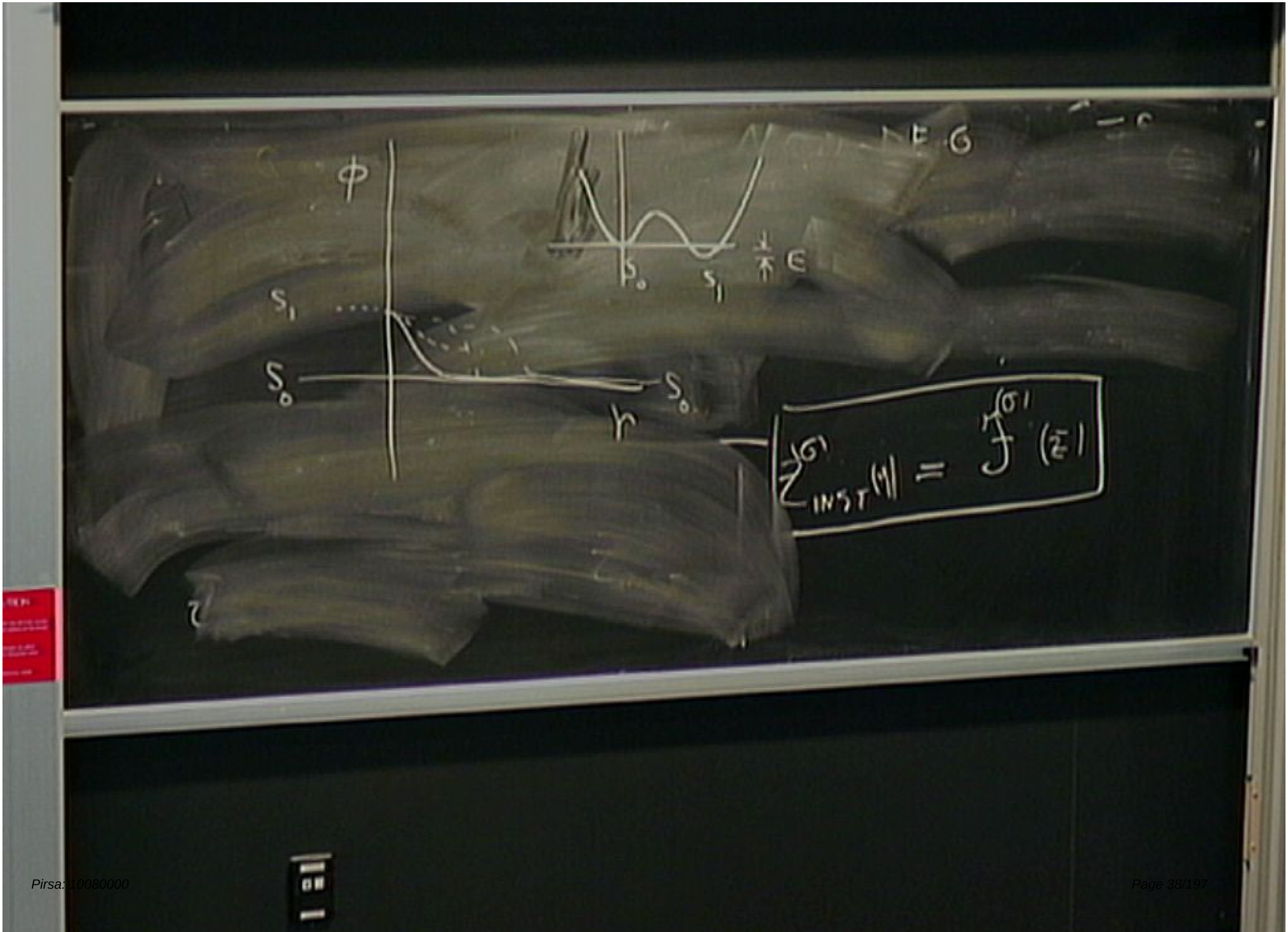
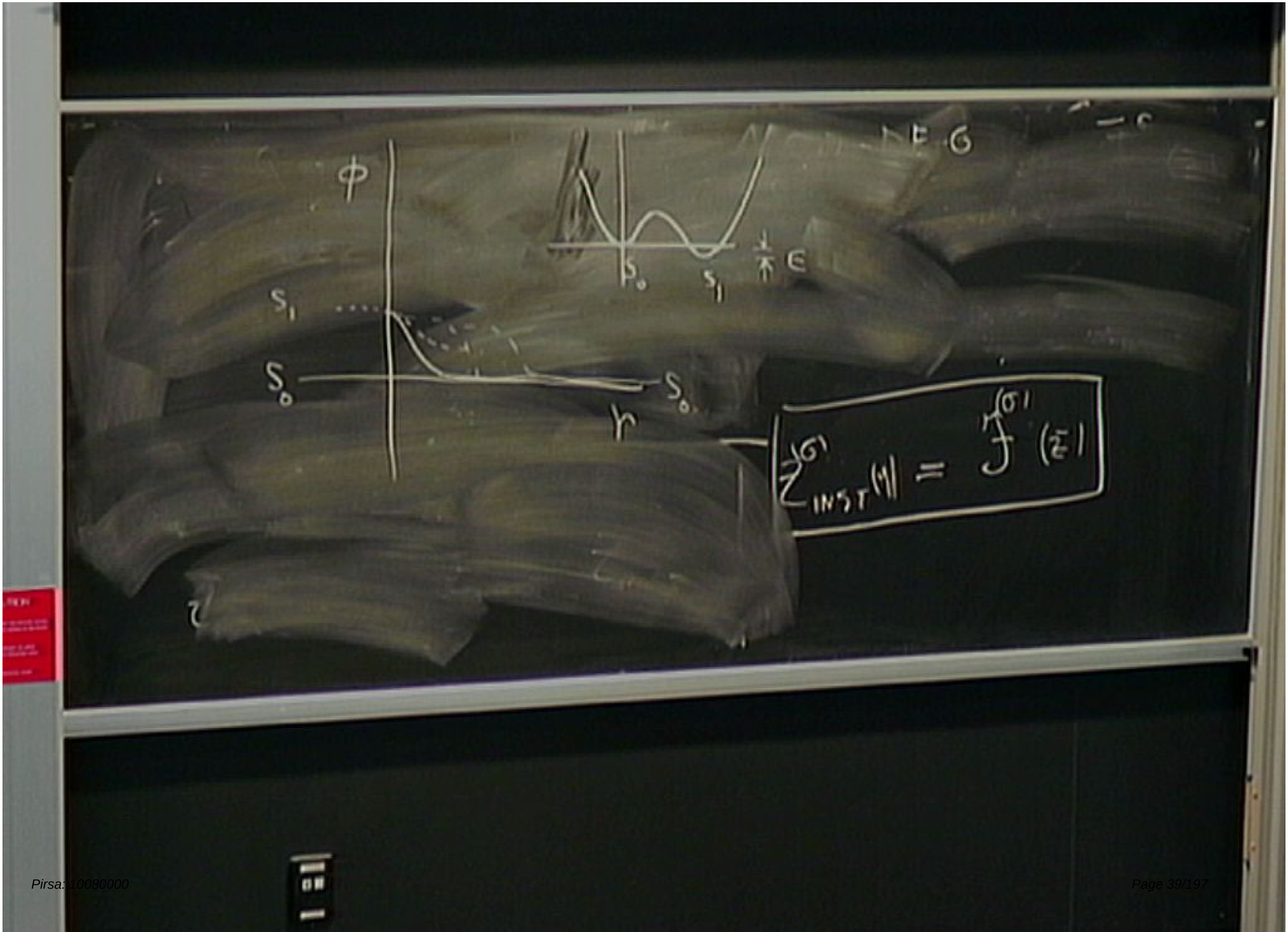


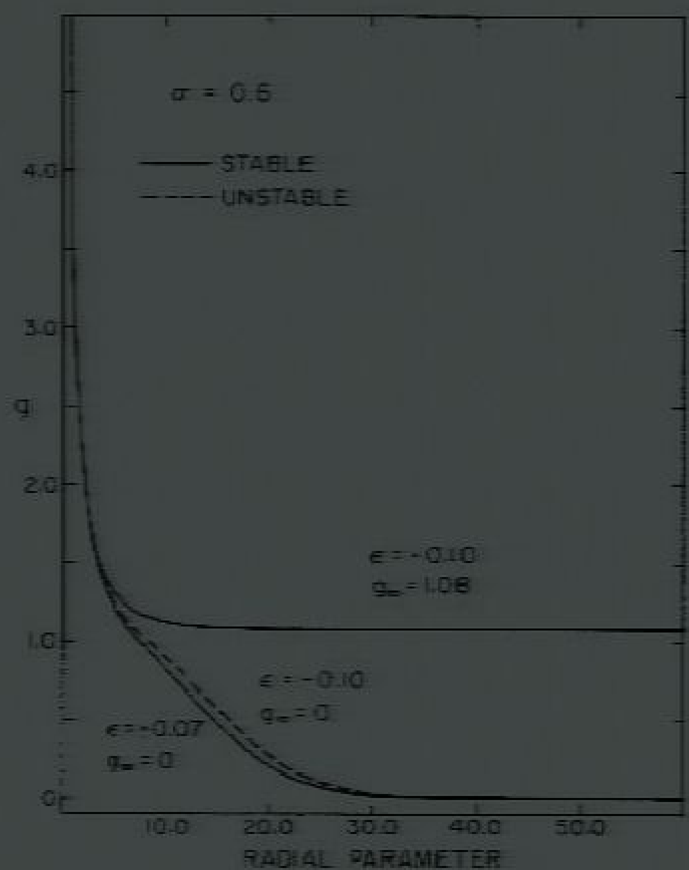



NEG
φ
s0
s1
S1
S0
r
Z INST (q) = f (z)

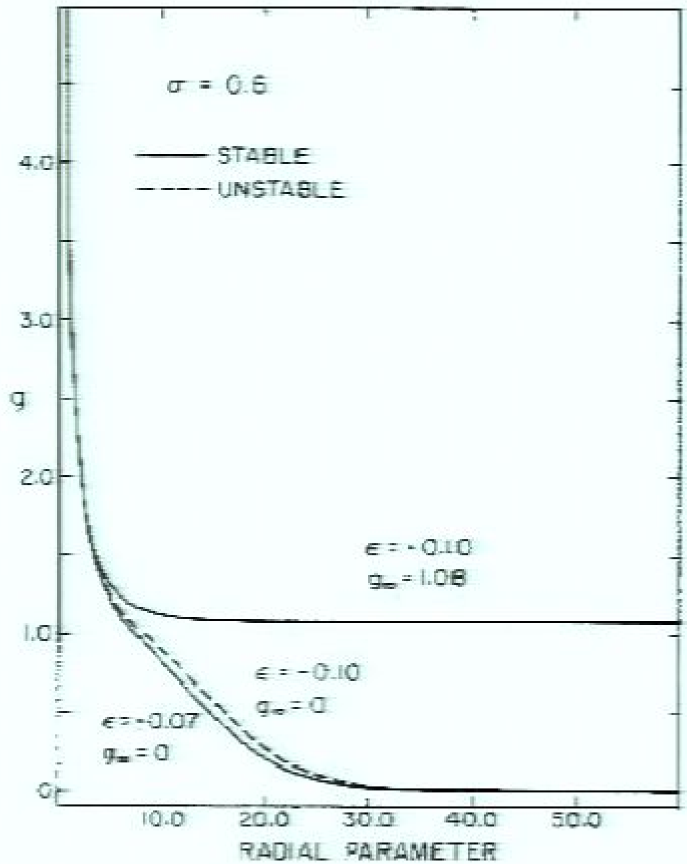

φ
S₁
S₀
r
s₀
s₁


φ
S1
S0
S0
r
E


φ
s₁
s₀
s₀
s₁
s₀
r
E
Z^(σ)_INST(τ) = f^(σ)(τ)



σ = 0.6
STABLE
UNSTABLE
4.0
3.0
2.0
1.0
0
q
ε = −0.10
q∞ = 1.08
ε = −0.10
q∞ = 0
ε = −0.07
q∞ = 0
10.0
20.0
30.0
40.0
50.0
RADIAL PARAMETER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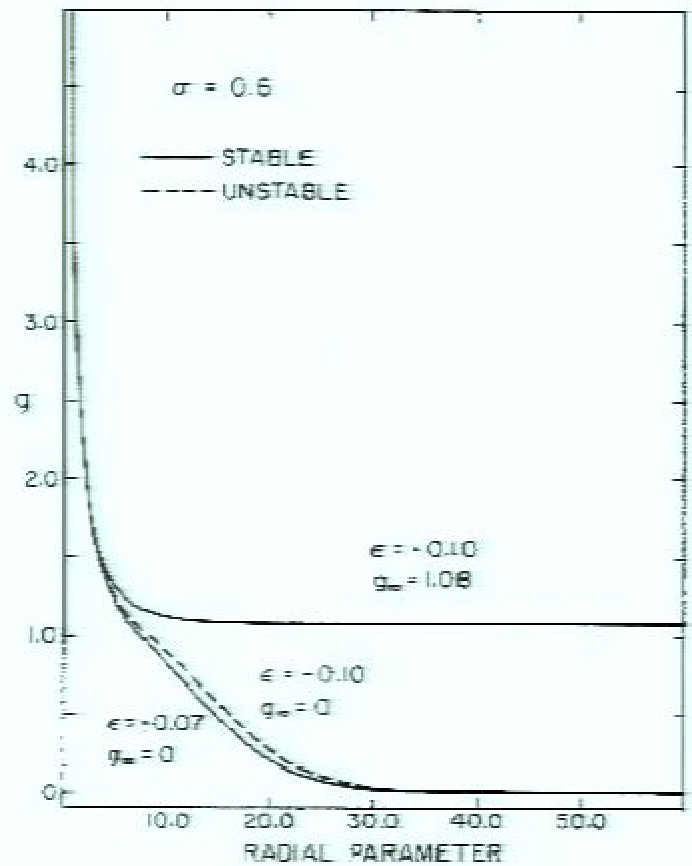
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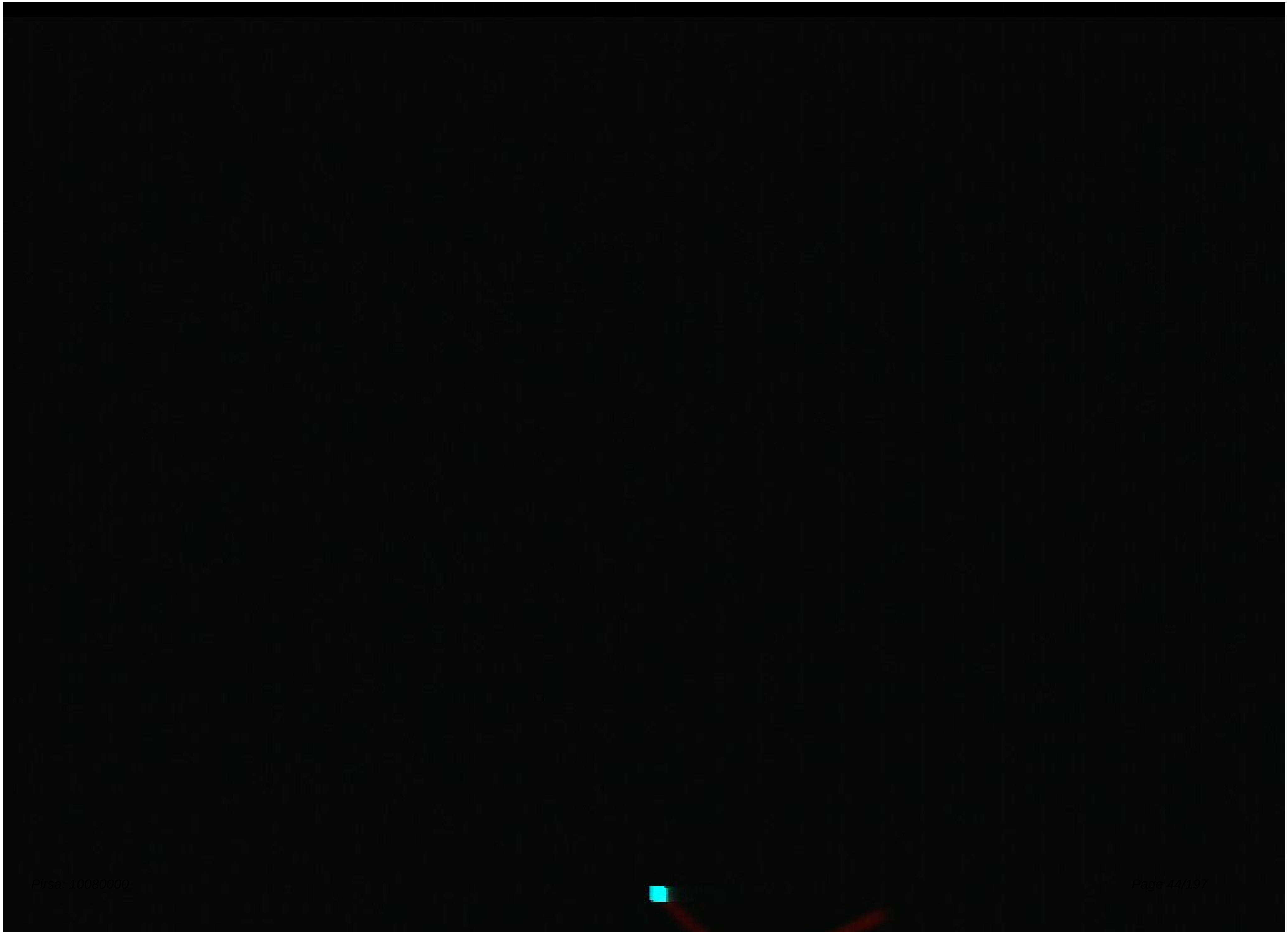

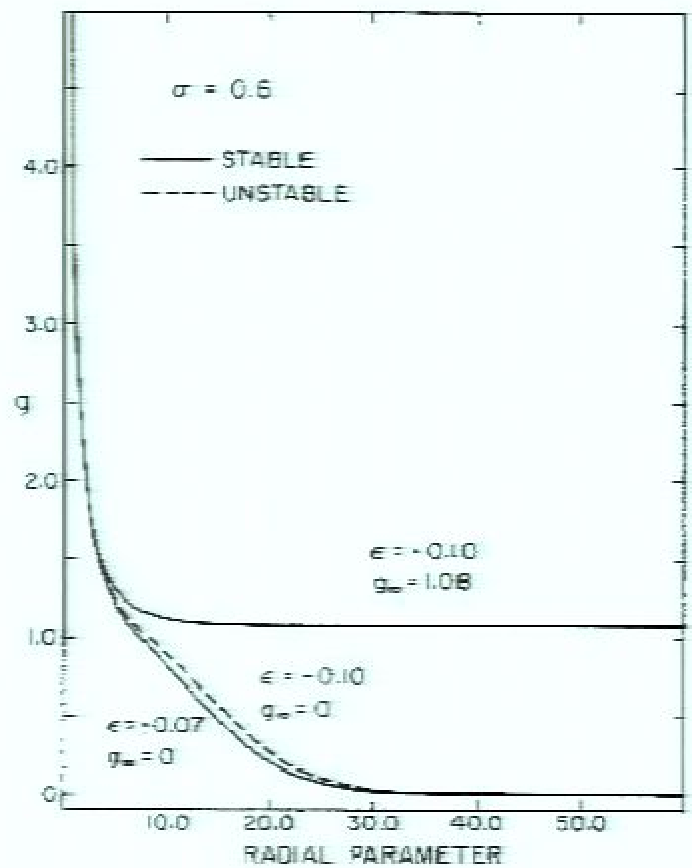
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Onset of time dependence beyond criticality

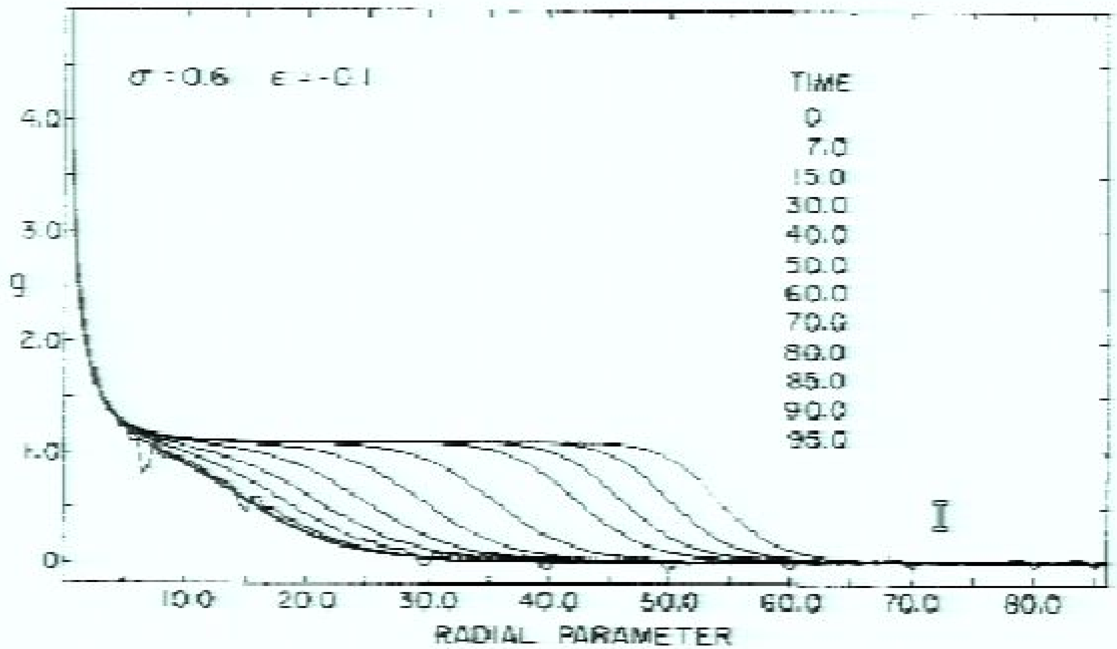

FIG. 2. Real-time rollover.

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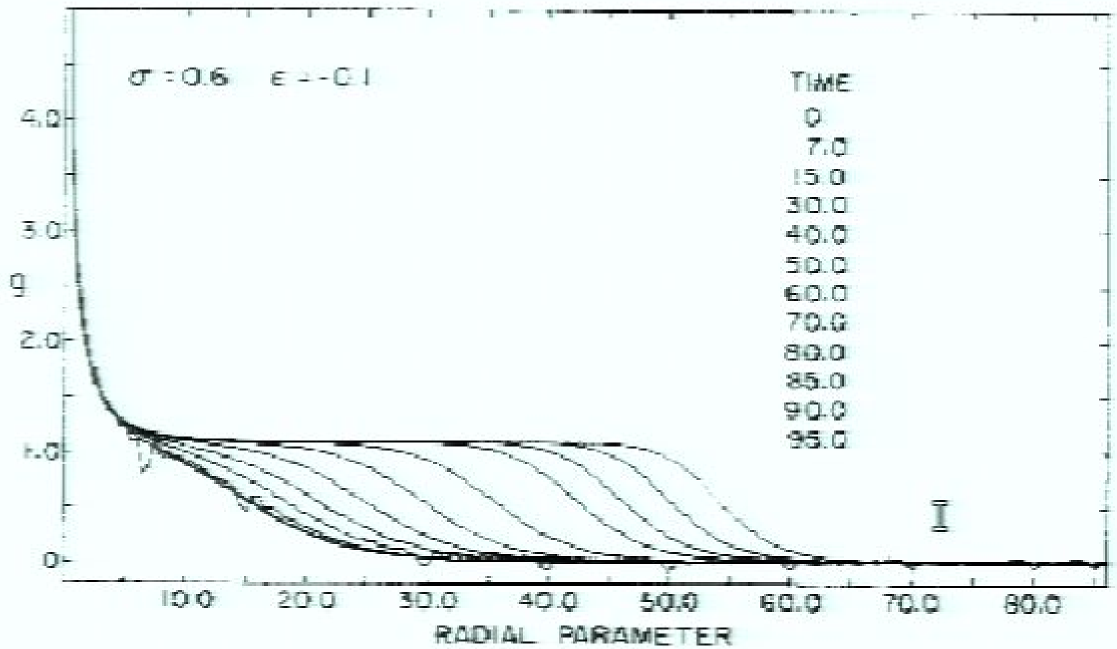

FIG. 2. Real-time rollover.

An example of GMSB

- Hidden sector breaking of SUSY at a scale Λ_s
- A messenger sector with symmetry group G_m :
  a gauge singlet S,
  a pair of messenger quarks q and $\bar{q}$,
  a pair of chiral superfields, N and P, in vector-like representations of G_m

$$W_{mes} = \kappa S\bar{q}q + \frac{\lambda}{3}S^3 + \lambda_1 PNS \tag{1}$$

$$\kappa, \lambda, \lambda_1 > 0$$

Consider G_m to be $U(1)$, and P, N charges $+e$ and $-e$.

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After including the $U(1)$ D-terms, the F-terms, and V_{SB} as above, the scalar potential of the messenger sector becomes

$$\begin{aligned} V_{mes} &= \frac{e^2}{2}(|P|^2 - |N|^2)^2 + (M_2^2 + \lambda_1^2|S|^2)(|P|^2 + |N|^2) \\ &+ \kappa^2|S|^2(|q|^2 + |\overline{q}|^2) + |\kappa\overline{q}q + \lambda S^2 + \lambda_1 PN|^2 \end{aligned} \quad (2)$$

The M_2^2 term arises from integrating out the hidden sector, with $M_2^2 \sim \Lambda_s^2 e^4 < 0$.

This potential is unbounded from below due to sign of M_2^2, but higher-order terms in V_{SB} result in a deep global minimum far away in field space in which the visible sector is supersymmetric.

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For a viable local minimum, one must set $q = \overline{q} = 0$ in the expression for V_{mes} and in order to have $S \neq 0$ simultaneously, we must have

$$\lambda > \lambda_1. \tag{3}$$

The local minima lie at $q = \overline{q} = 0$ and

$$|P|^2 = |N|^2 = -M_2^2 \frac{\lambda}{\lambda_1^3(2 - \lambda_1/\lambda)} \tag{4}$$

$$|S|^2 = -M_2^2 \frac{1 - \lambda_1/\lambda}{\lambda_1^2(2 - \lambda_1/\lambda)} \tag{5}$$

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Along with the condition $\lambda > \lambda_1$, stability of the local minima require the following relations between the couplings :

$$\lambda_1^3 \leq 2\lambda e^2 \tag{7}$$

$$\lambda_1 \leq \frac{\kappa\lambda}{\kappa + \lambda} \tag{8}$$

Two important types of vacua :

$$|V_1\rangle : \langle S\rangle \neq 0 \quad \text{and} \quad \langle q\rangle = \langle \bar{q}\rangle = 0$$

which means that SUSY is broken while the color gauge group is unbroken.

$$|V_2\rangle : \langle S\rangle \neq 0, \text{ but also, } \langle q\rangle \neq 0, \quad \langle \bar{q}\rangle \neq 0$$

SUSY is still broken but the color gauge group is also broken.

The interesting feature is that for all ranges of couplings,

$$\langle V_2|V_{mes}|V_2\rangle < \langle V_1|V_{mes}|V_1\rangle \tag{9}$$

... troublesome from the point of view of phenomenology

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If cosmic strings are supported, these vacua are modified to what will be denoted as $|V_1^{(string)}\rangle$ and $|V_2^{(string)}\rangle$ respectively.

Next, we demonstrate that $|V_1^{(string)}\rangle$ becomes parametrically unstable towards decay into $|V_2^{(string)}\rangle$.

The string ansatz

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The string ansatz

In a local minimum, $|P| = |N|$. The ansatz functions for the

scalar fields are a simple generalization of the Abrikosov-Nielsen-Olesen string,

$$P = \eta f(r) e^{i\theta} \tag{10}$$

$$N = \eta f(r) e^{-i\theta} \tag{11}$$

$$S = \eta g(r) e^{i\pi/2} \tag{12}$$

$$q = \overline{q} = \eta h(r) e^{i\pi/2} \tag{13}$$

The value of η is as required for global vacua.

The gauge field behavior is described by a function $a(r)$ defined through

$$A_\theta(r) = \frac{1}{e\,r} a(r) \tag{14}$$

$$A_0 = A_r = A_z = 0 \tag{15}$$

where for continuity of $A_\theta(r)$, we have $a(0) = 0$, and at infinity, $A_\theta(r)$ is pure gauge and goes as $1/r$ and hence $a(\infty) = 1$.

Choice of boundary conditions

$$|V_1\rangle: \qquad f = a = 1;\ g \neq 0,\ h = 0 \tag{16}$$

$$|V_2\rangle: \qquad f \lesssim 1,\ a = 1;\ g \neq 0,\ h \neq 0 \tag{17}$$

$|V_1\rangle$ is the desired supersymmetry breaking and color preserving local minimum.

Recall that for all ranges of couplings,

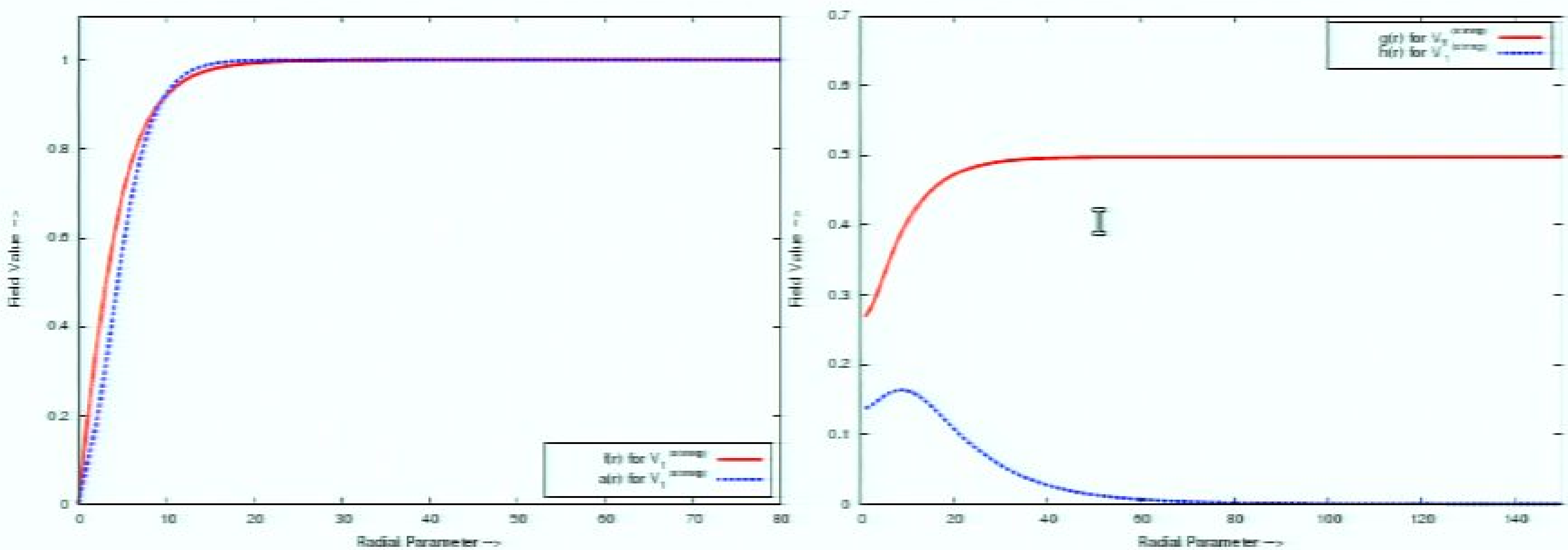
$$\langle V_2|V_{mes}|V_2\rangle < \langle V_1|V_{mes}|V_1\rangle \quad (18)$$

Vacuum with gauge string

We denote by $|V_1^{(string)}\rangle$ the static state of the system in which the cosmic string configuration approaches $|V_1\rangle$ and by

$|V_2^{(string)}\rangle$ a static string solution which asymptotes to $|V_2\rangle$.

An example of the solutions in a case where both $|V_1^{(string)}\rangle$ and $|V_2^{(string)}\rangle$ are stable is shown below



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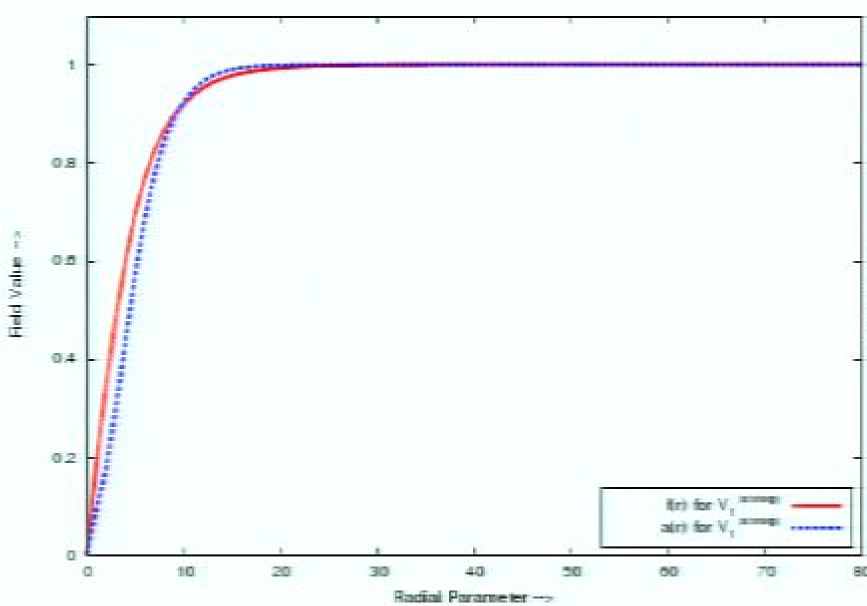
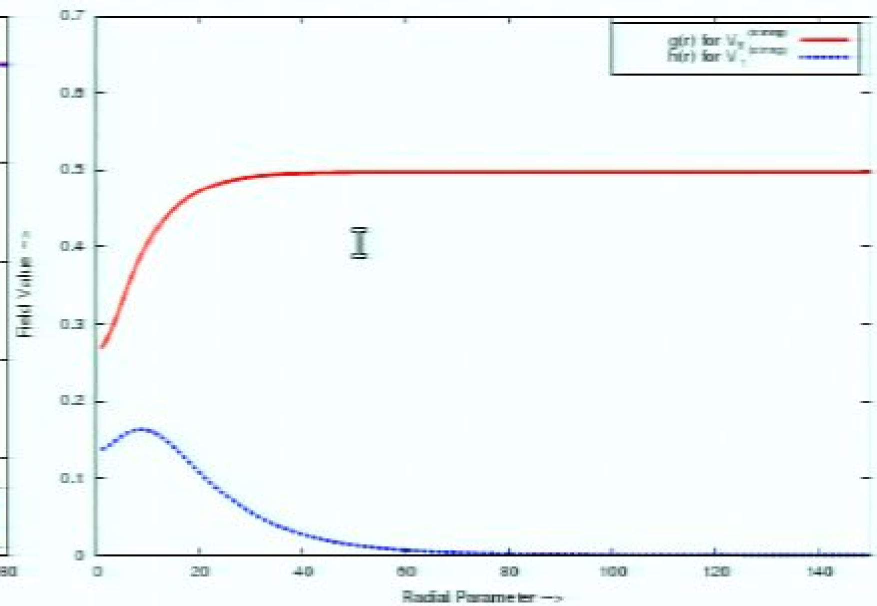
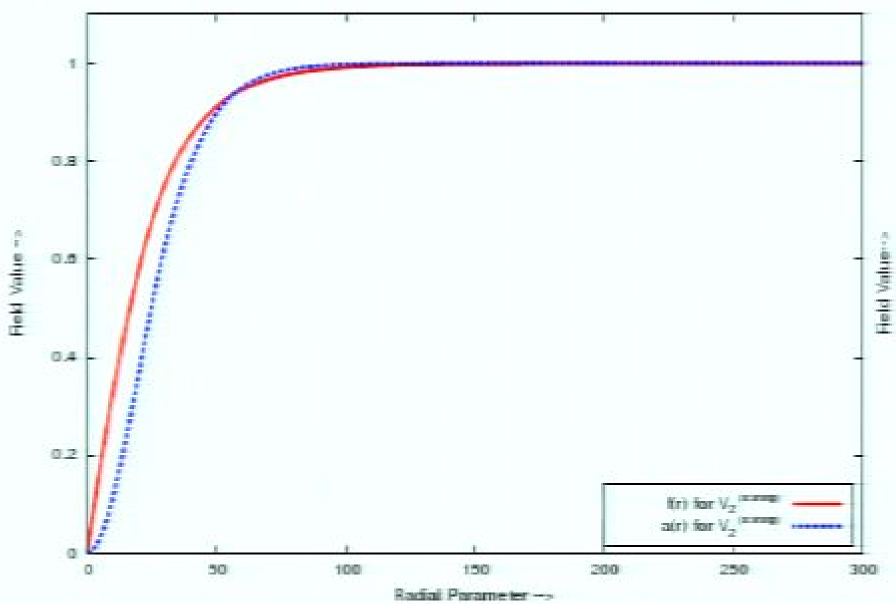
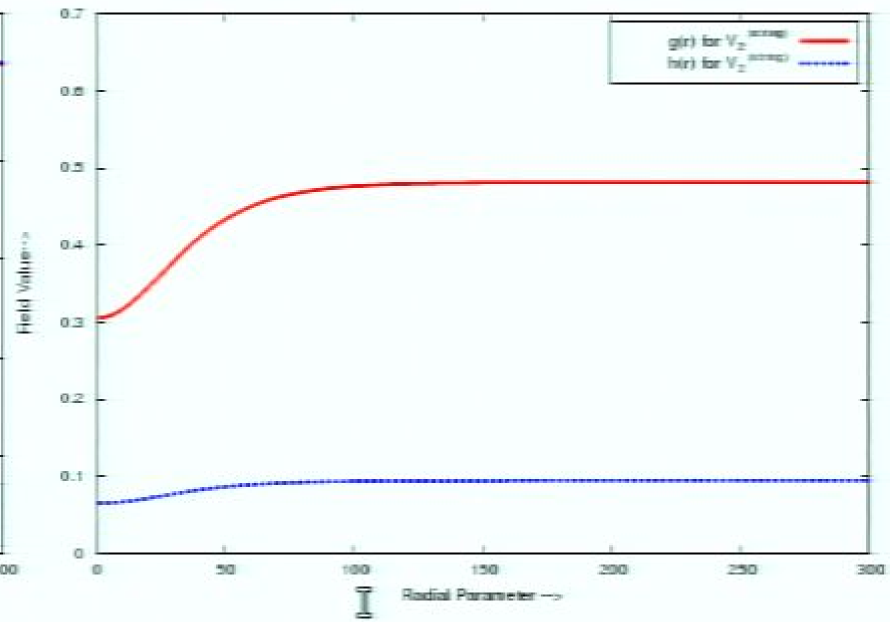




The configurations for $|V_2^{(string)}\rangle$





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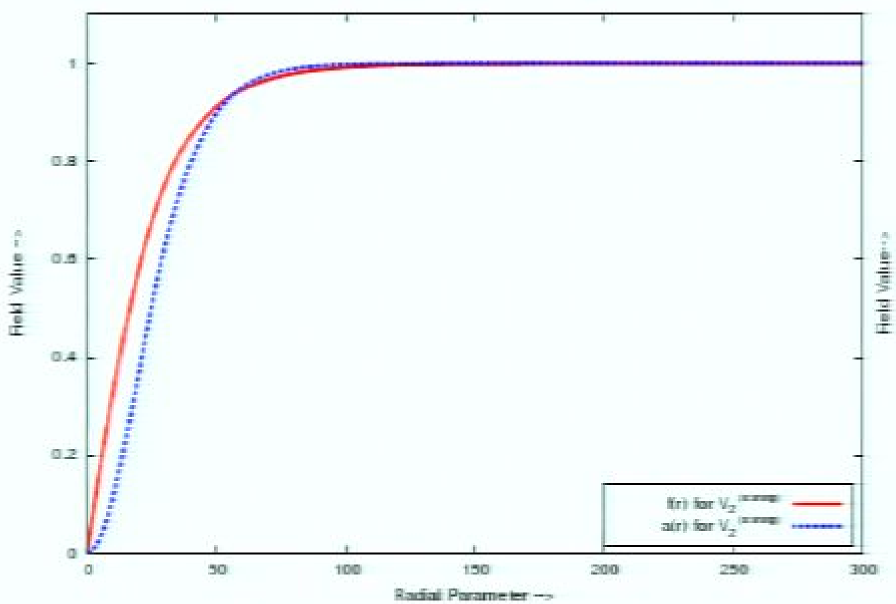

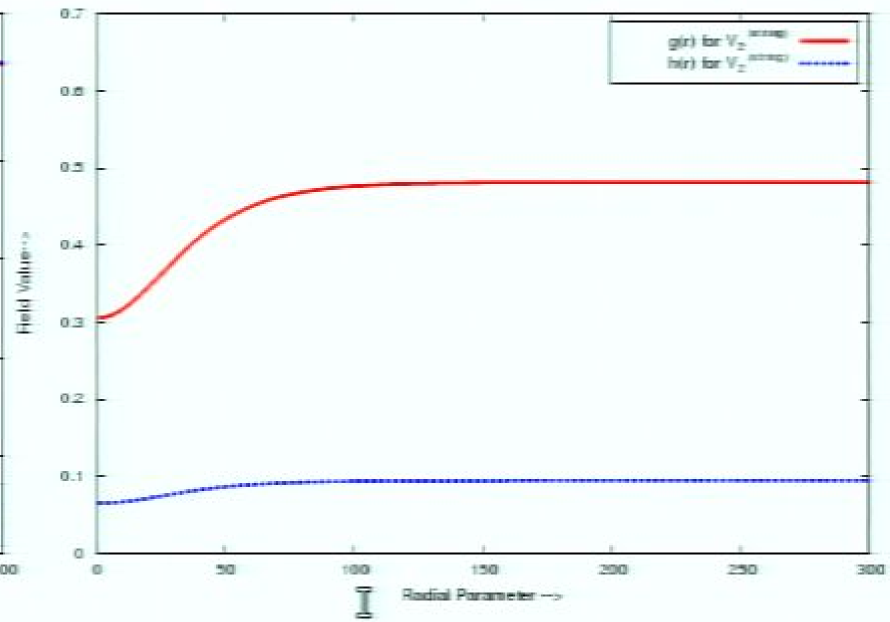

κ	λ_1	λ	$\lvert V_1^{(string)}\rangle$	$\lvert V_2^{(string)}\rangle$
2.4	1.2	≥ 2.41	×	✓
2	1	≥ 2.01	×	✓
2	≤ 0.75	2.5	×	✓
1.7	0.85	≥ 1.71	×	✓
1.3	0.65	≤ 1.42	×	✓
1.3	0.65	≥ 1.43	✓	✓
1.3	≤ 0.65	1.5	✓	✓
1	0.5	≤ 1.07	×	✓
1	0.5	≥ 1.1	✓	✓
1	≥ 0.3	1.1	✓	✓
1	≤ 0.15	1.1	×	✓
0.6	0.3	≤ 0.68	×	✓

Table 1.

κ	λ_1	λ	$\lvert V_1^{(string)}\rangle$	$\lvert V_2^{(string)}\rangle$
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1	≥ 0.3	1.1	✓	✓
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Table 1.

Stability analysis -- semi-analytic

Effective potential for small fluctuations of the squark fields.

Negative energy bound state necessary for instability.

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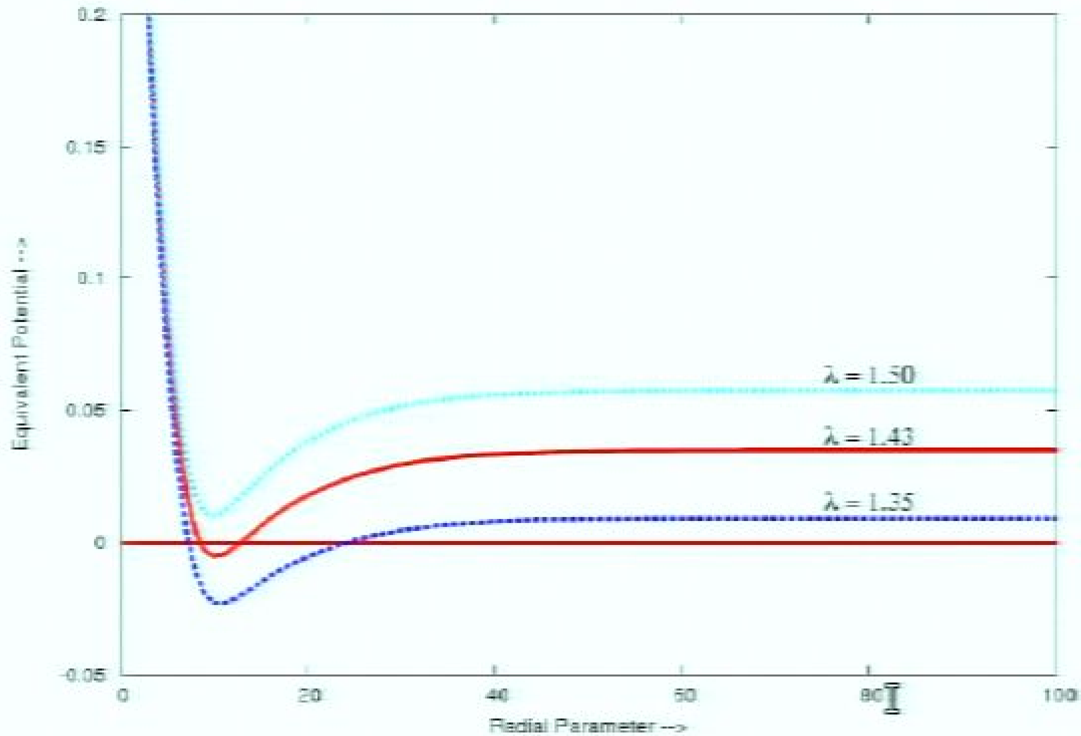
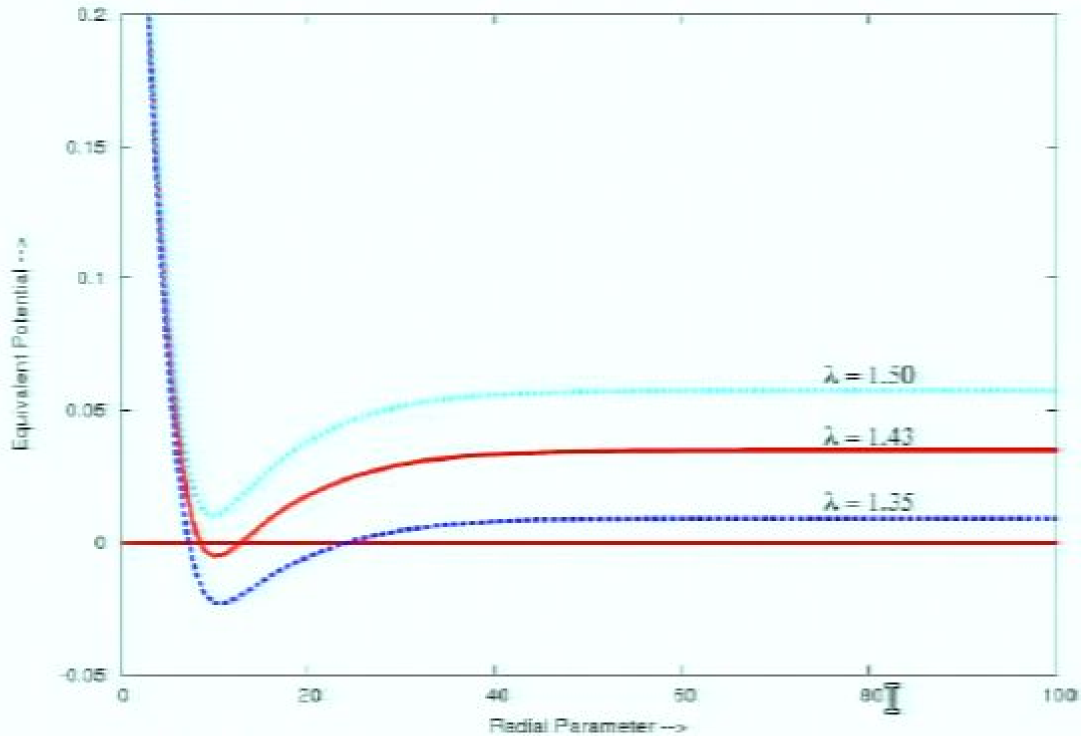


Topological objects can invalidate certain putative ground states



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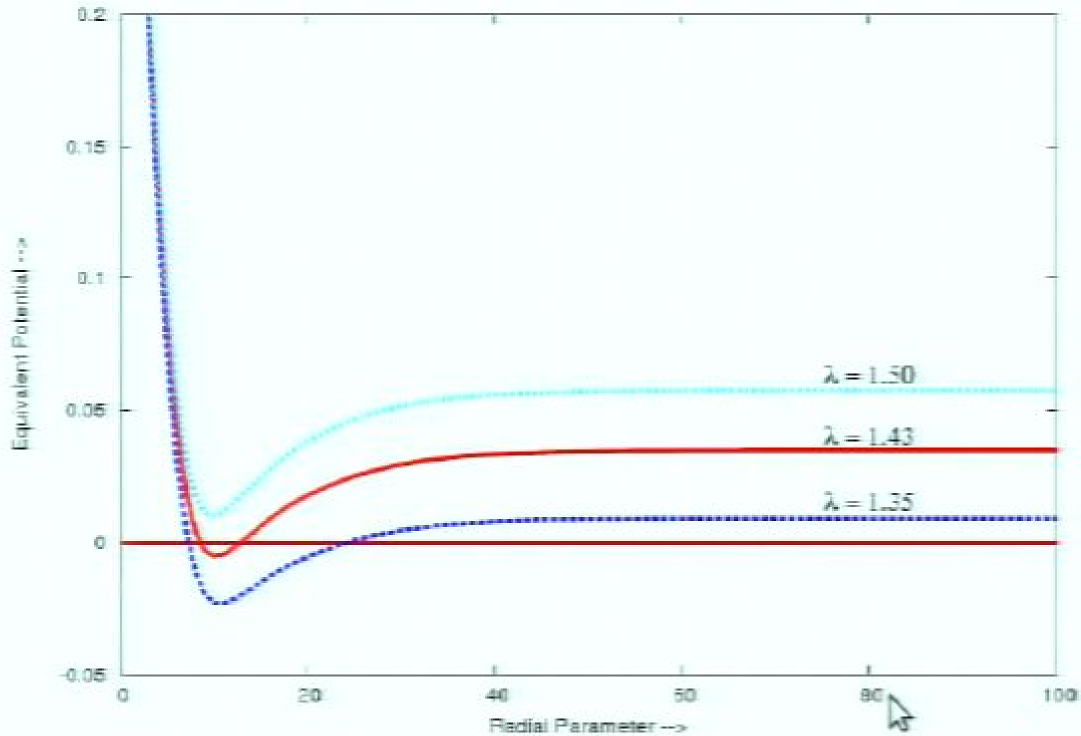

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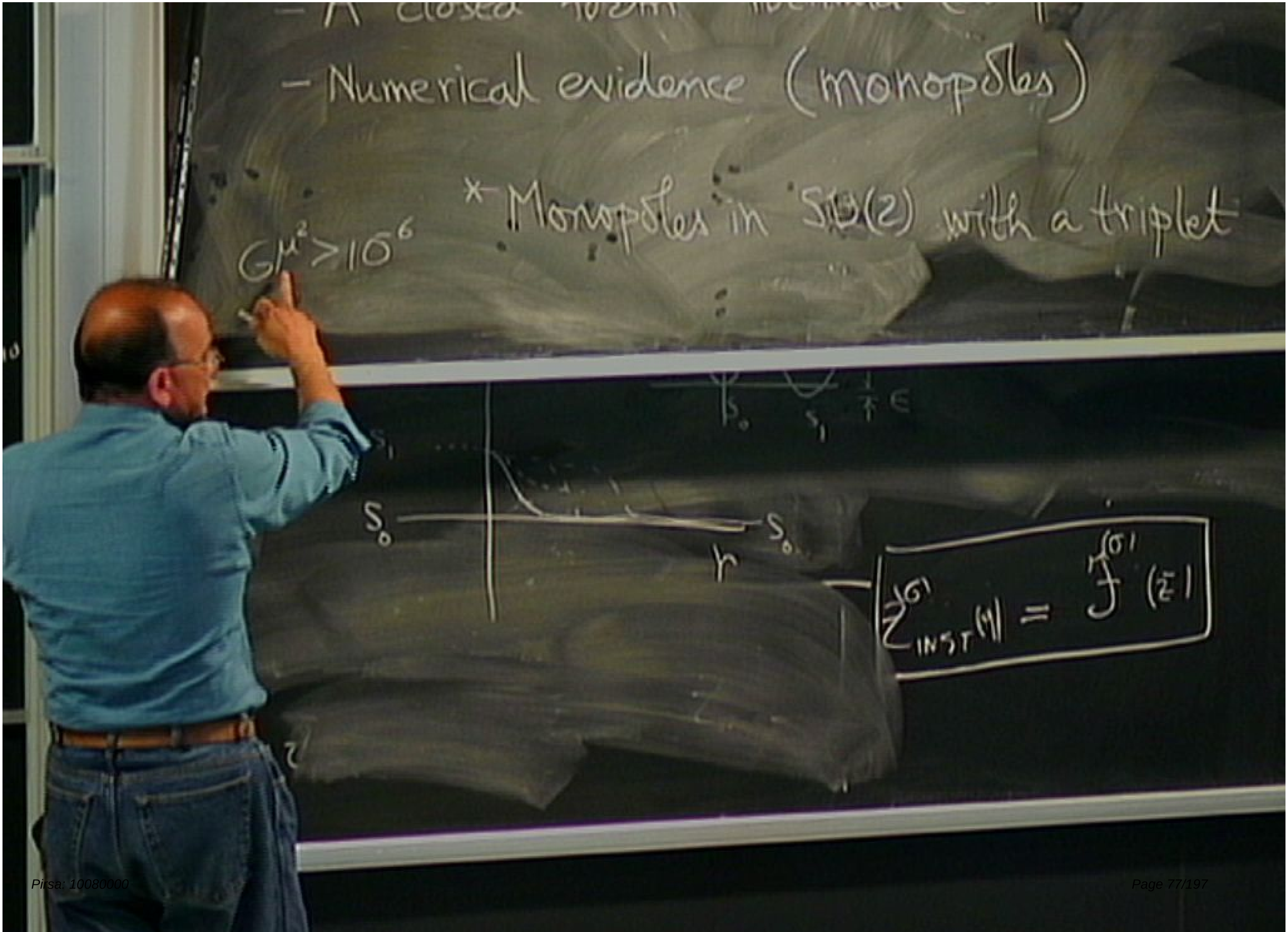
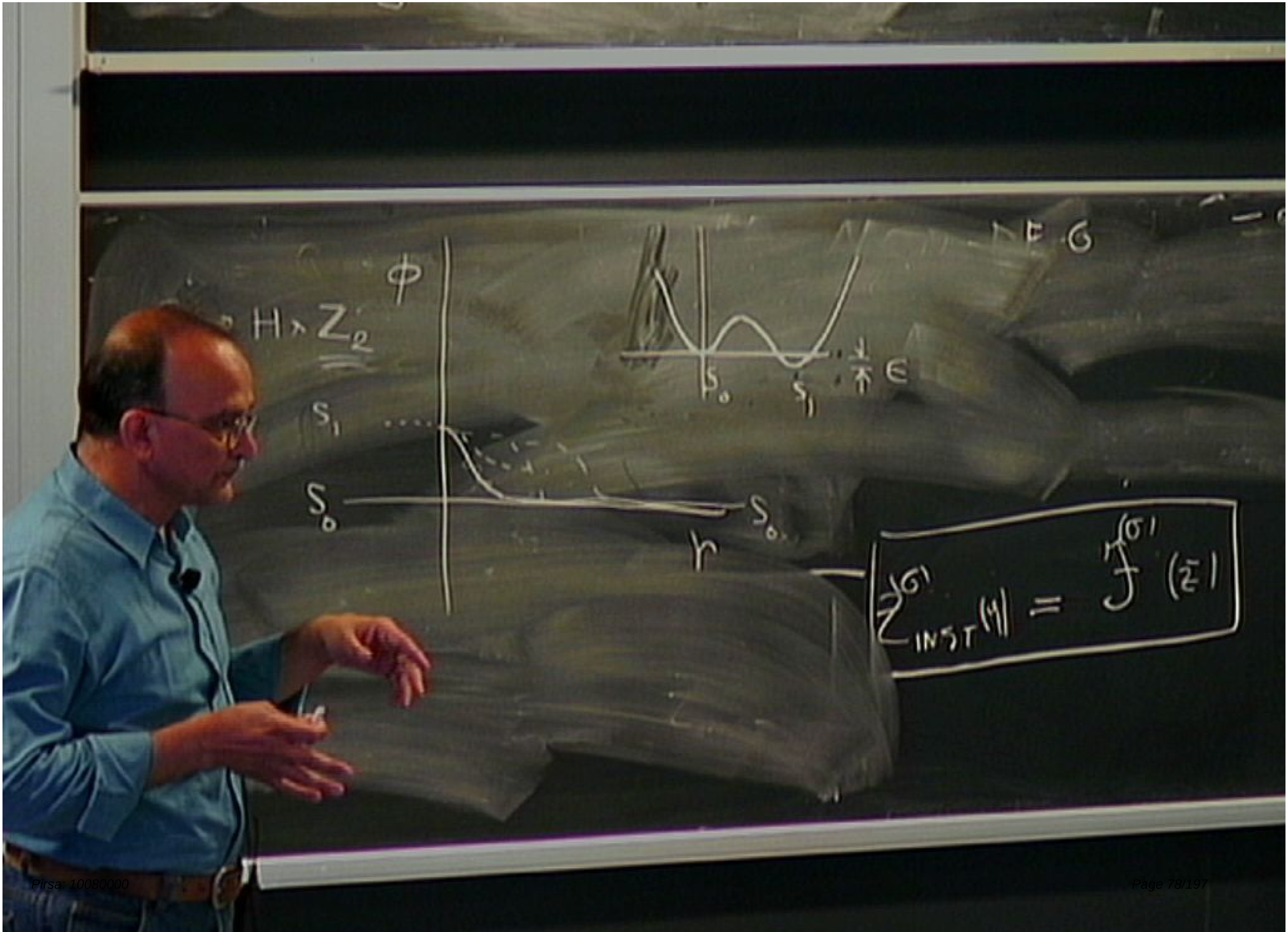
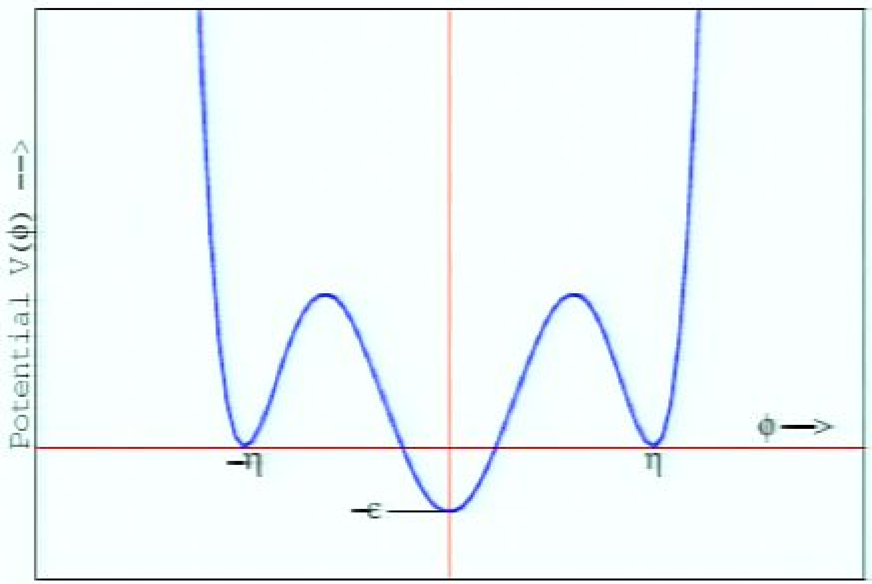

- Numerical evidence (monopoles)
* Monopoles in SU(2) with a triplet
$G\mu^2 > 10^{-6}$

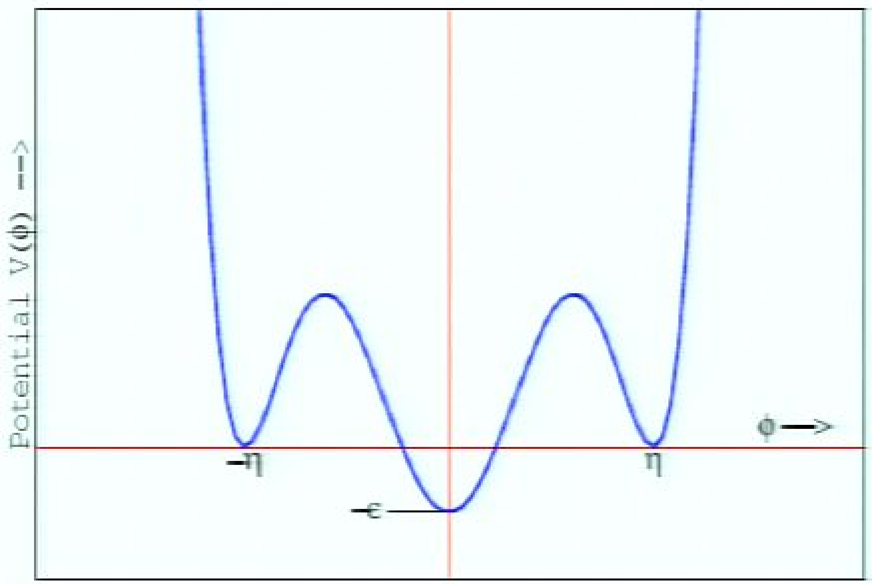

φ
H × Z_2
S_1
S_0
S_0
r
ε
Z^{σ'}_{INST}(y) = f^{σ'}(z)

[redacted]



Time independent spherically symmetric ansatz for the monopole

$$\begin{aligned}\phi_a &= \hat{r}_a h(r) \\ A^a_\mu &= \epsilon_{\mu ab}\hat{r}_b \frac{1-K(r)}{e\,r} \\ A_0 &= 0\end{aligned} \quad (21)$$



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where $\hat{r}$ is a unit vector in spherical polar coordinates.
The energy function becomes

$$E(K,h) = 4\pi \int_0^\infty dr \Big(\frac{(K')^2}{e^2} + \frac{(1-K^2)^2}{2e^2r^2} + \frac{1}{2}r^2(h')^2 + K^2h^2 + r^2V(h) \Big)$$

Thin wall approximation

In the thin wall approximation, the functions h and K can be written as

$$\begin{aligned} h &= h(r-R) \\ K &= K(r-R) \end{aligned} \quad (22)$$

which cna be ensured in our model by tuning

$$\frac{h^2}{\mu^2} << \frac{\tilde{\lambda}\tilde{a}^4 + \tilde{\gamma}^2}{2\tilde{\lambda}\tilde{a}^2} \quad (23)$$

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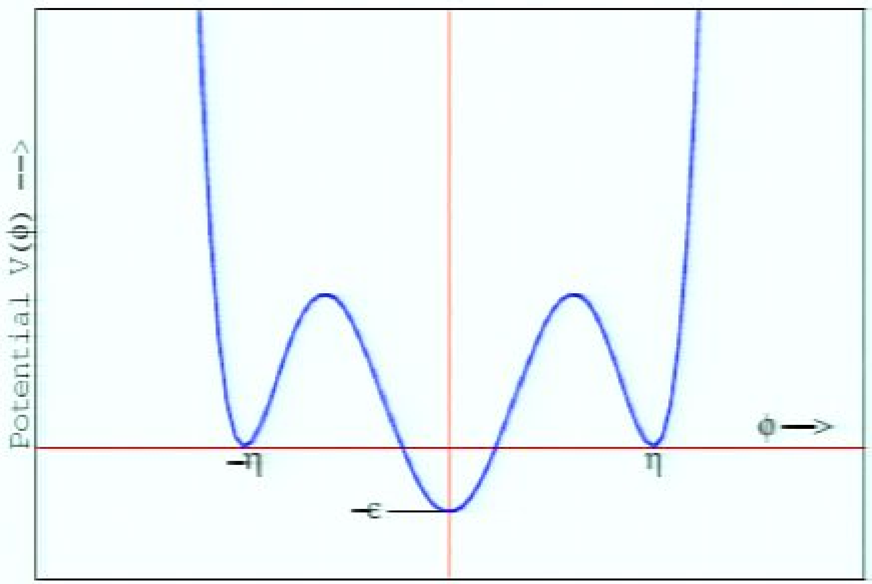

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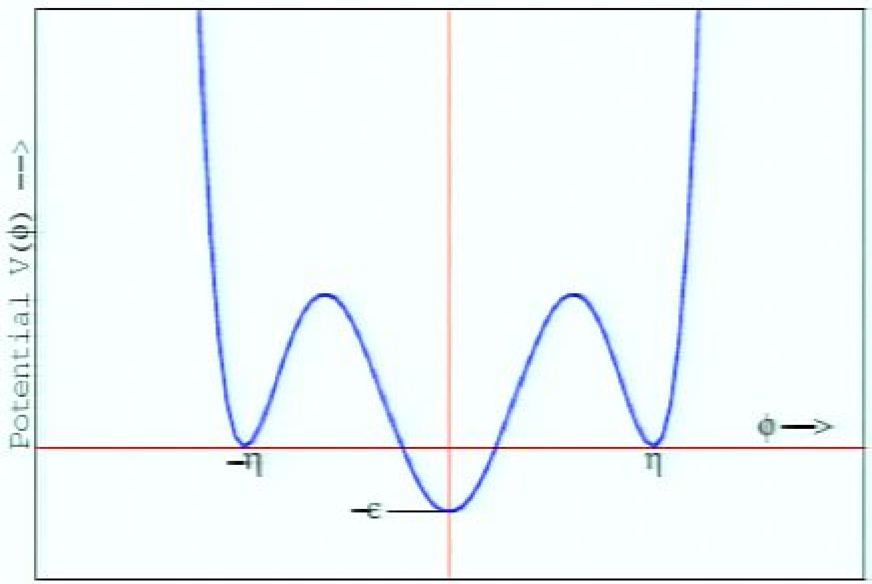

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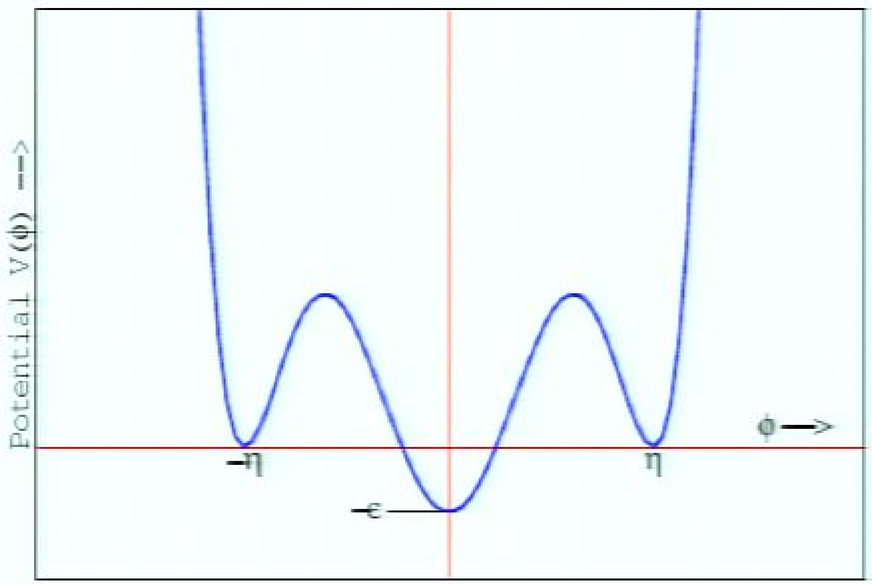

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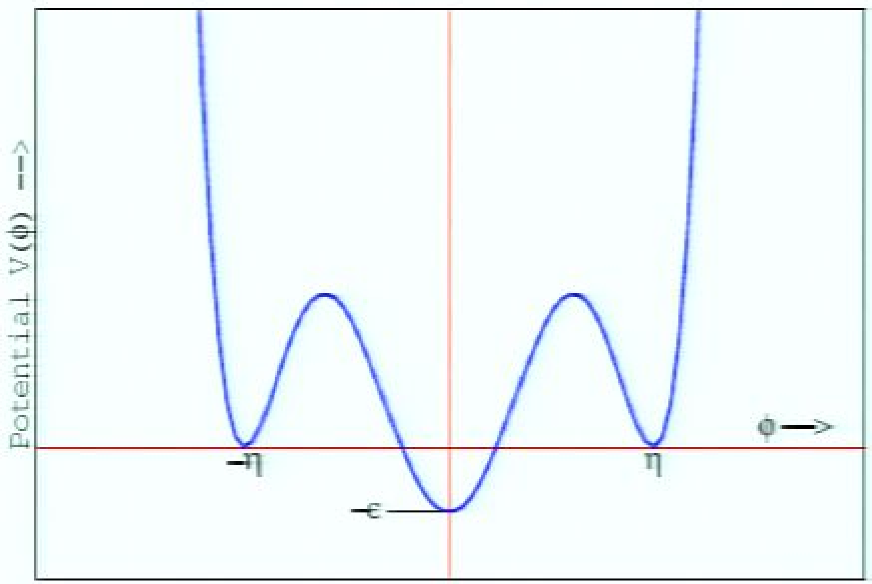

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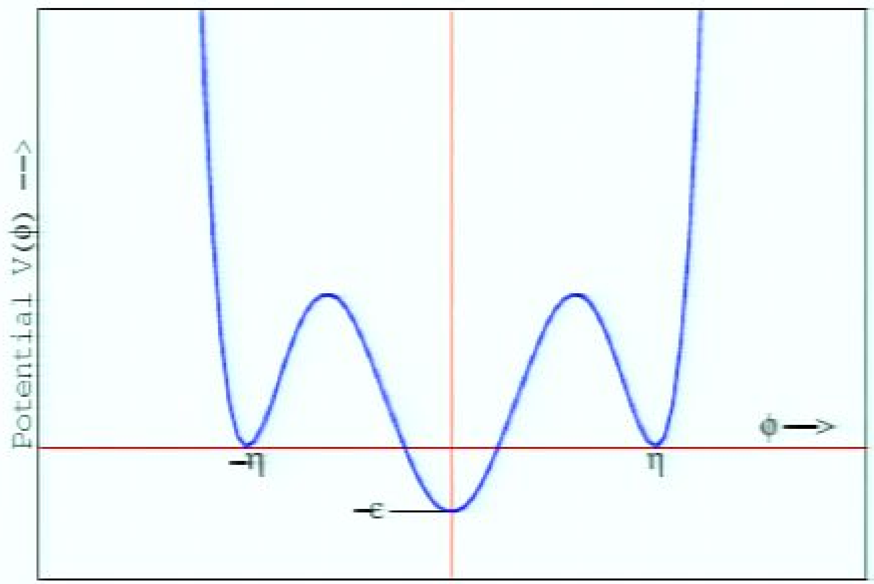

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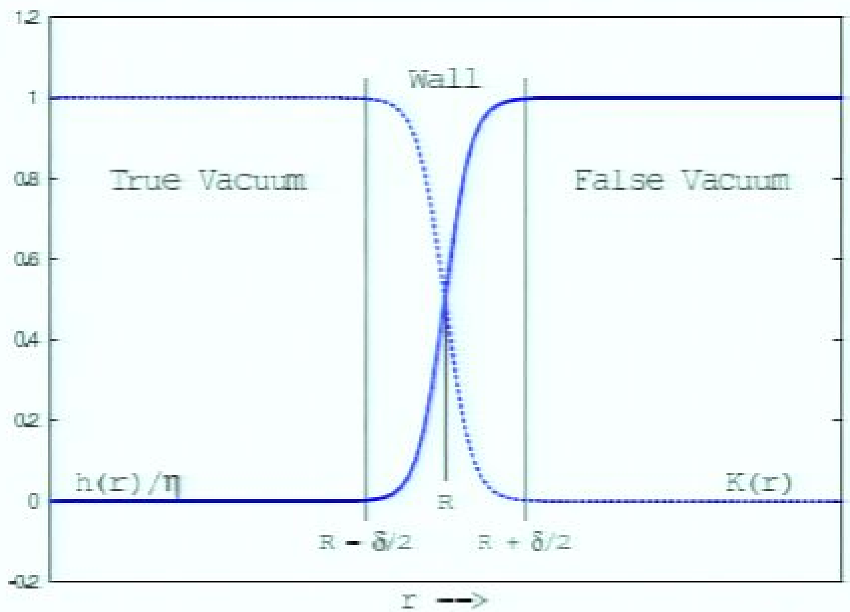
When the above condition is satisfied, $\partial V/\partial h$ is linear in h.



Total energy can be expressed as

$$E = 4\pi \quad [\quad \int_0^{R-\frac{\delta}{2}} d\,r\,r^2 V(h) + \int_{R+\frac{\delta}{2}}^{\infty} d\,r \frac{1}{2e^2 r^2}$$

$$+ \quad \int_{R-\frac{\delta}{2}}^{R+\frac{\delta}{2}} d\,r \left(\frac{(K')^2}{e^2} + \frac{(1-K^2)^2}{2e^2 r^2} \right.$$

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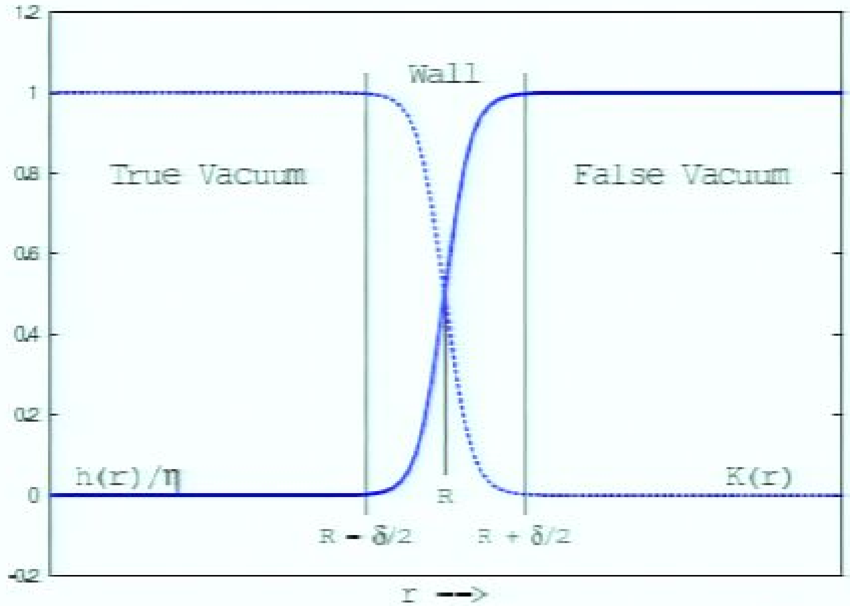

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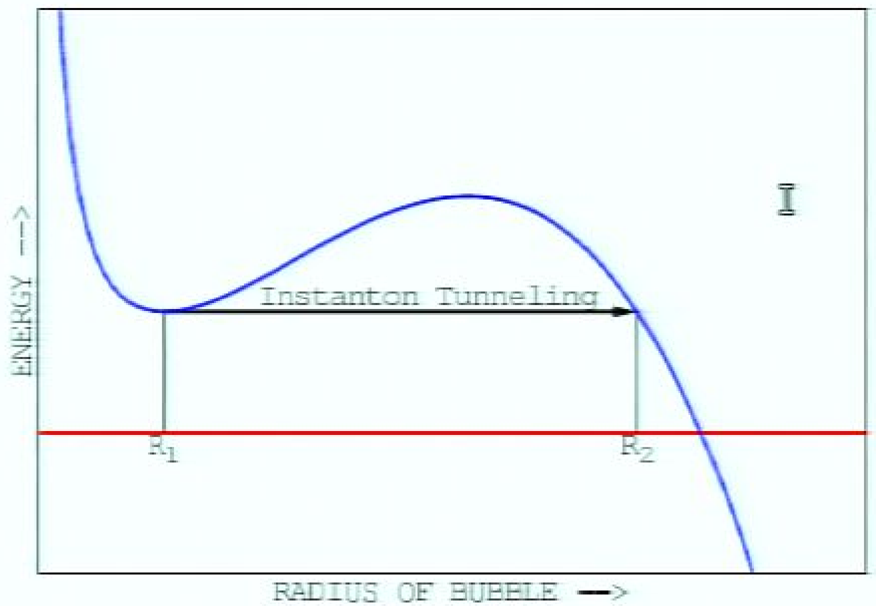
$$+ \quad \frac{1}{2} r^2 (h')^2 + K^2 h^2 + r^2 V(h) \Big) \Big]. \tag{24}$$

This has the form

$$E(R) = -\alpha R^3 + 4\pi\sigma R^2 + \frac{C}{R} \tag{25}$$



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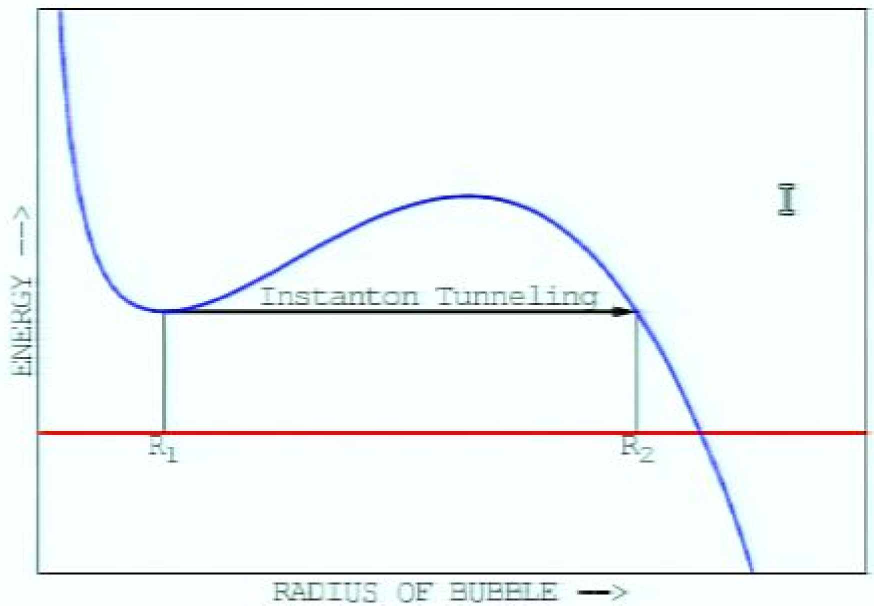

For the Euclidean action we get

$$S_E = \int_{-\infty}^{\infty} d\tau \Big(2\pi \dot{R}^2 (S_1 R^2 + S_2) + E(R) \Big) \tag{26}$$

where $\tau = i\, t$ is the Euclidean time and $\dot{R}$ is the derivative with respect to τ.

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39

The instanton

Boundary conditions

$$R = R_1 \text{ for } \tau = \pm\infty,$$

$$R = R_2 \text{ for } \tau = 0, \text{ and}$$

$$dR/d\tau = 0 \text{ for } \tau = 0.$$

The equation of motion for R can be written

$$(R^2 S_1 + S_2)\ddot{R} + S_1 R \dot{R}^2 - \frac{1}{4\pi}\frac{\partial E}{\partial R} = 0. \quad (27)$$

This has an "integral of motion"

$$E(R) = 2\pi(S_2 + S_1 R^2)\dot{R}^2. \quad (28)$$

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$$\begin{aligned} S_E &= \int_{-\infty}^{\infty} d\tau (\frac{dR}{d\tau}) 4\pi (S_2 + S_1 R^2) \dot{R} \\ &\approx \sqrt{32\pi} \int_{R_1}^{R_2} dR \sqrt{(S_1 R^2) E(R)}. \end{aligned} \quad (29)$$

Neglecting S_2 in comparison to $S_1 R^2$,

Thus the action for the bounce can be written

$$S_E = A \int_{R_1}^{R_2} dR \sqrt{-(\alpha R^5 - 4\pi\sigma R^4 - CR + E_0 R^2)} \quad (30)$$

where $A = \sqrt{32\pi S_1}$.

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The integrand has a double root at $R = R_1$, a positive root at $R = R_2$, and a negative root at $R = R_3$.

$$R_1 \approx (4e^2\sigma)^{-1/3}$$

$$R_3 = -2R_1$$

$$R_2 \text{ is large, } R_2 \approx 4\pi\sigma/\alpha = 3\sigma/\epsilon$$

Thus

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$$\frac{\Gamma}{V} \sim \left(\frac{\kappa}{2}\right) \exp\left\{ \frac{16}{105} \sqrt{\frac{2 S_1 \pi^2 \epsilon}{3}} \mathcal{F}(R_1, R_2, R_3) \right\}$$

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$$+ \quad \frac{1}{2}r^2(h')^2 + K^2h^2 + r^2V(h)\Big)\Big]. \tag{24}$$

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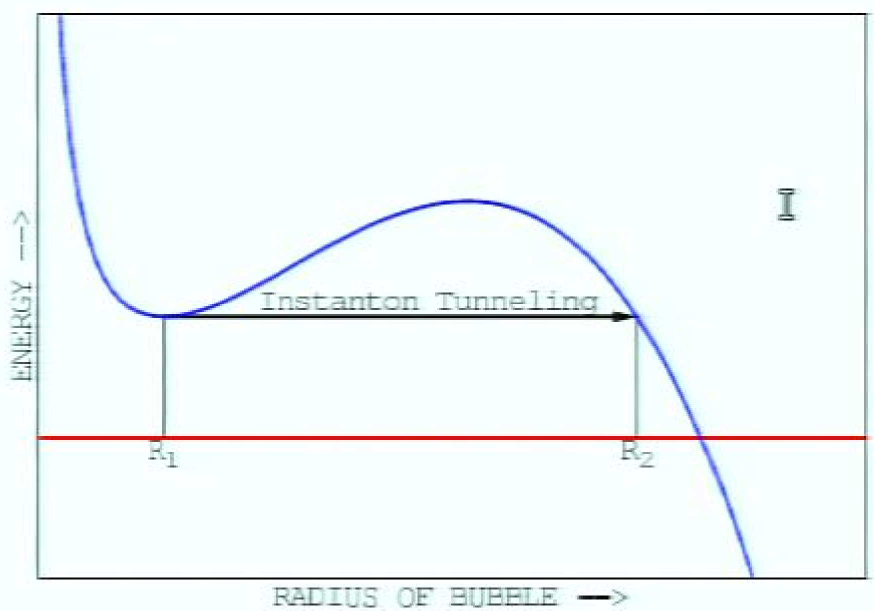

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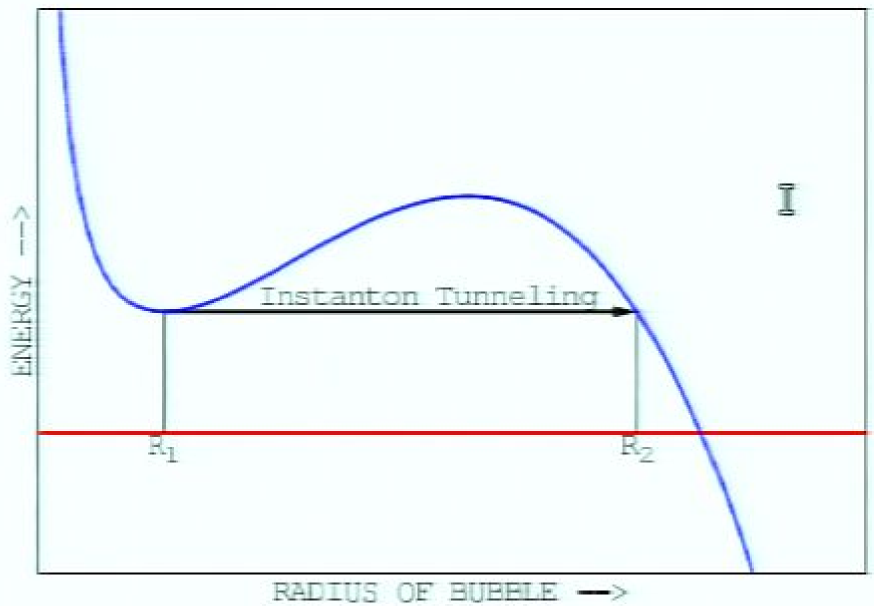

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- The presence of a monopole makes naive minimum of effective potential unphysical
- If the limit $R_1 \to R_2$ is controlled by external conditions like temperature in the early Universe, then the monopoles will become sites where the true vacuum is nucleated without the need for quantum tunneling.

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$$\frac{R_1}{R_2} = \frac{1}{e^{2/3}}\frac{1}{(4\sigma)^{1/3}}\frac{\epsilon}{3\sigma} \tag{32}$$

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The integrand has a double root at $R=R_1$, a positive root at $R=R_2$, and a negative root at $R=R_3$.

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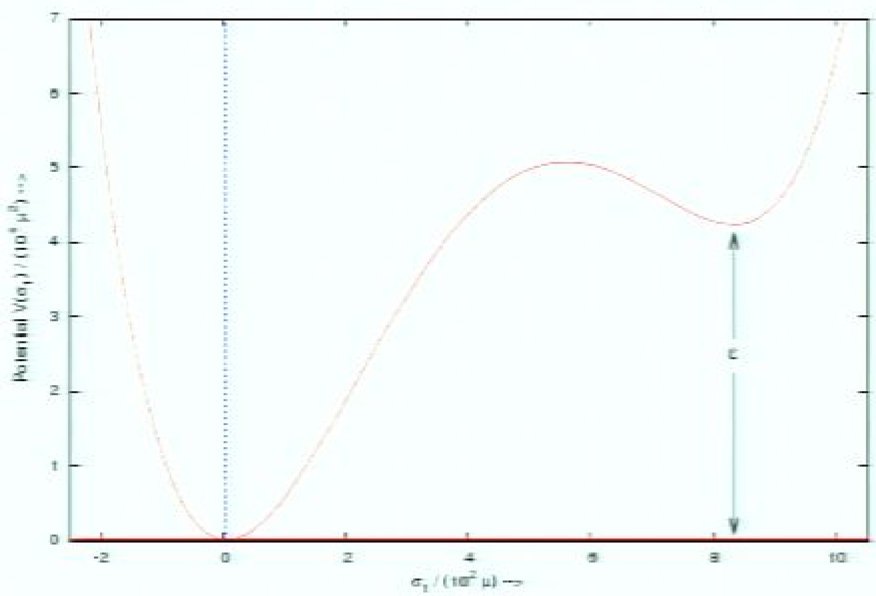
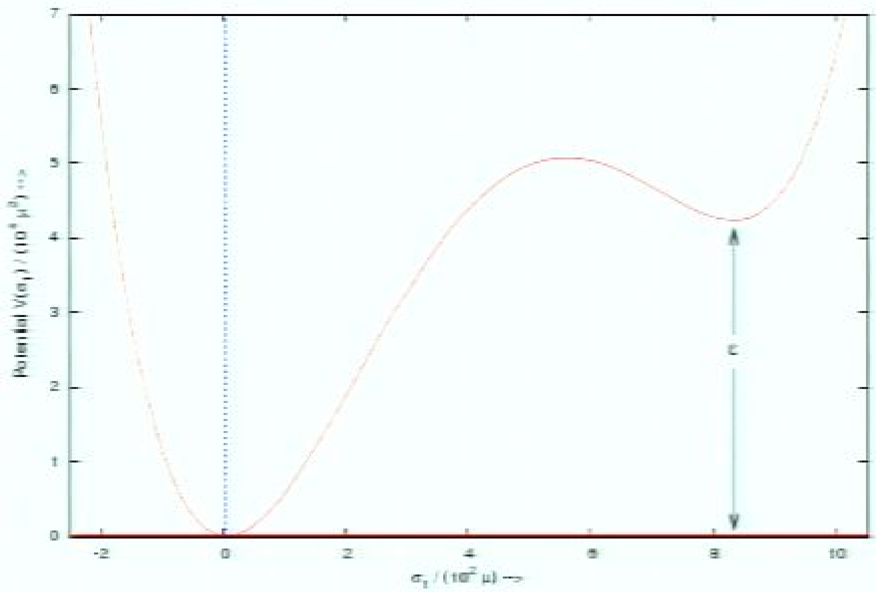


Graph of effective potential for the direct SUSY breaking model



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Stability of the monopole configurations

Study the availability of both $|V_1^{monopole}\rangle$ and $|V_2^{monopole}\rangle$ by first choosing a value of v_2 and then varying $\tilde{M}$.

An example of $|V_1^{monopole}\rangle$ when $\tilde{M} = 1400$ and $\sigma_2 = 500\mu$ is shown in figures.
In this case, the value of v_1 is 685.3μ and this is independent of σ_2. A similar solution exists for $|V_2^{monopole}\rangle$ for which the value of v_1 is 5.5μ.

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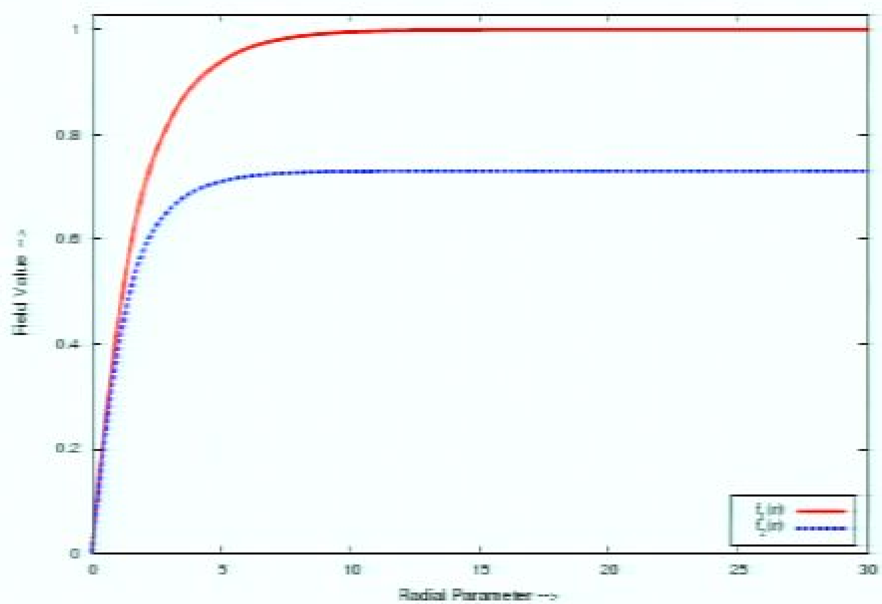
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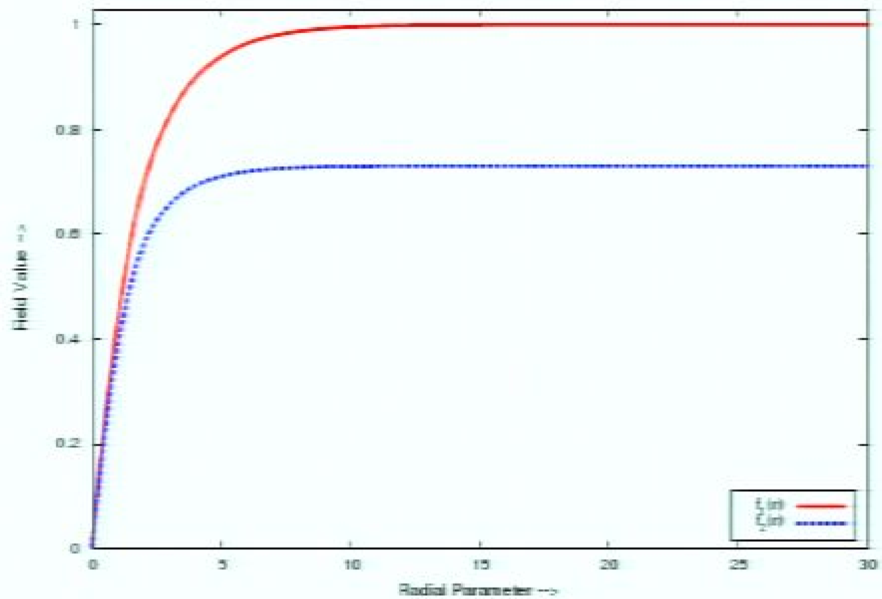
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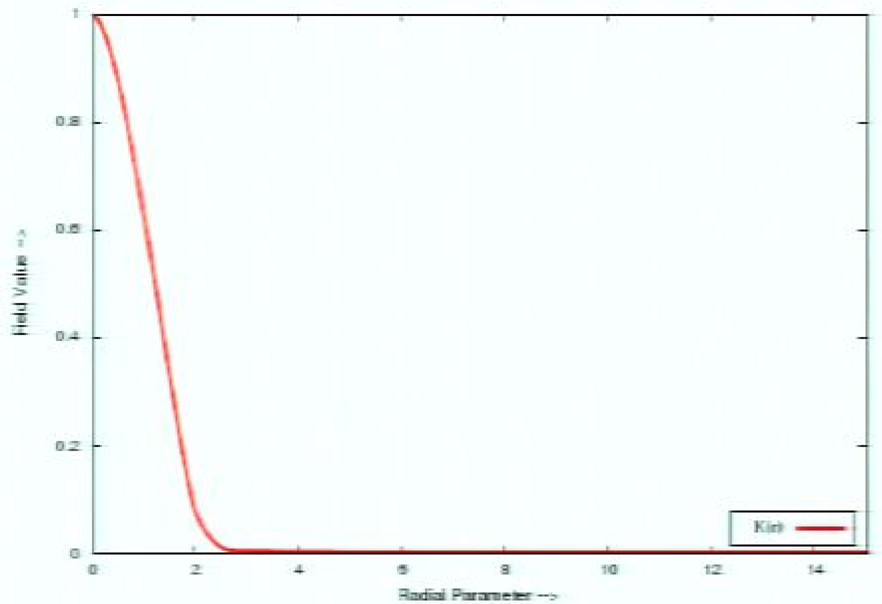

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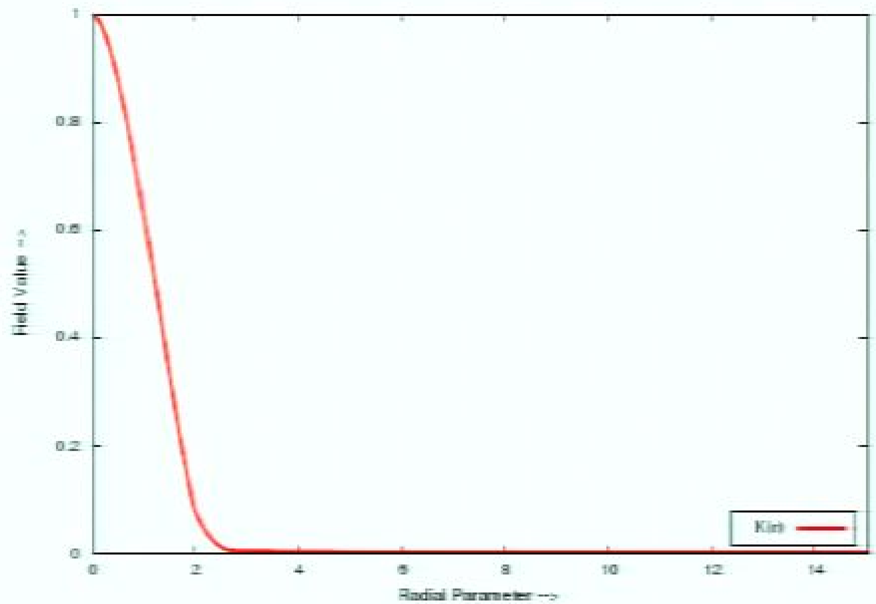

Field Value -->
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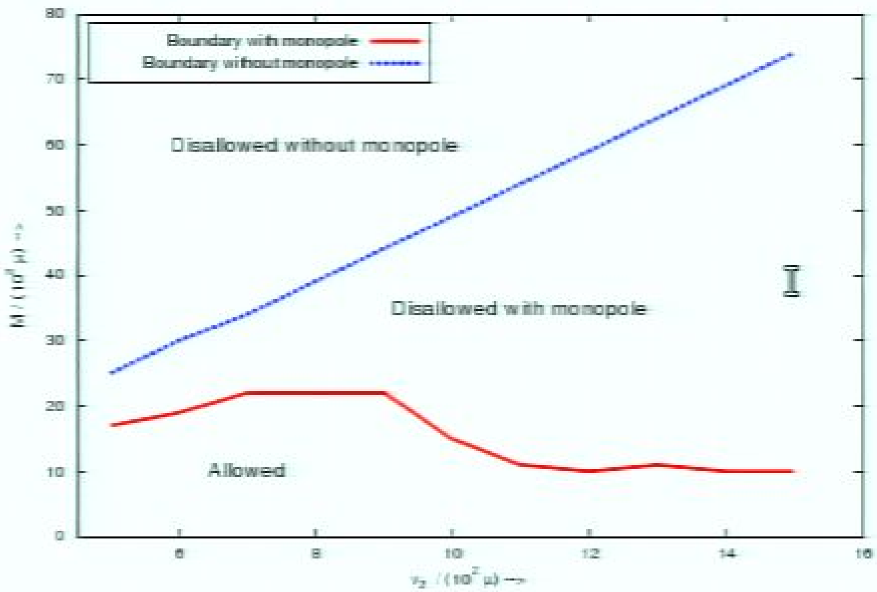
A scatter plot in parameter space

There is a large region in the parameter space of v_2 and $\tilde{M}$ for which a numerical solution for $|V_1^{monopole}\rangle$ cannot be obtained.



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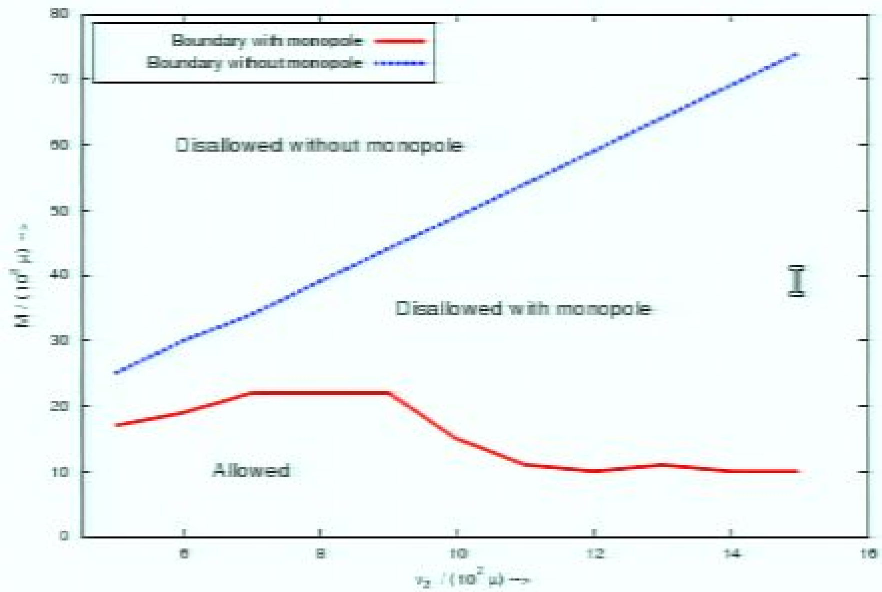

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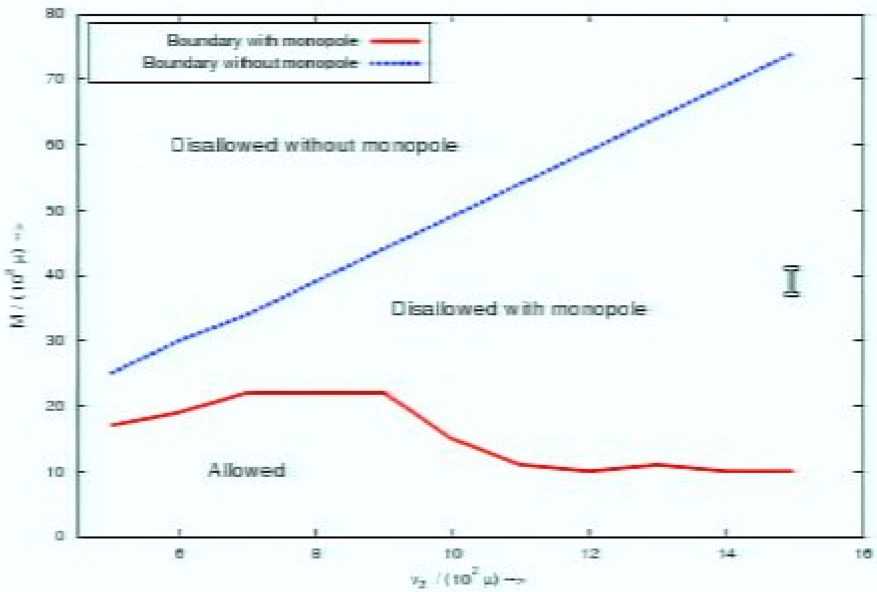

- We also have a closed form formula for false monopoles interpolating between seeded tunneling and roll-over decay.

Thank you!

Typeset using TeXmacs

Summary

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$$\begin{aligned} V &= \left(\mu\sigma_1 - \frac{\lambda\sigma_1^2}{\sqrt{30}} + \frac{7\alpha_1\sigma_1^3}{30M} + \frac{\alpha_2\sigma_1^3}{M} \right)^2 \\ &+ \frac{\sigma_2^2}{900M^2}\left(30M\mu - 2\sqrt{30}M\lambda\sigma_1 + 3(7\alpha_1 + 30\alpha_2)\sigma_1^2\right)^2. \end{aligned} \quad (37)$$

Comparing homogeneous nucleation

Voloshin-Kobzarev-Okun-Coleman answer B_0,

$$B_0 = \frac{27\pi^2}{2}\frac{S_1^4}{\epsilon^3} \tag{34}$$
$$= \frac{27\pi^2}{128}\tilde{\lambda}^2\tilde{a}^{16}\frac{\mu^{12}}{\epsilon^3}. \tag{35}$$

Comparing this expression with our bounce $B \equiv S_E$ for the monopole assisted tunneling given in (32), we see that

$$B = \frac{32\sqrt{2}}{105\pi}B_0\left(1-\frac{R_1}{R_2}\right)^{5/2} I\left(\frac{R_1}{R_2},\frac{R_3}{R_2}\right). \tag{36}$$

- Unlike the homogeneous case, the bounce can parametrically become indefinitely small and vanish in the limit $R_1 \to R_2$

Solving for $\dot{R}$ from (28) and using this in the above equation yields

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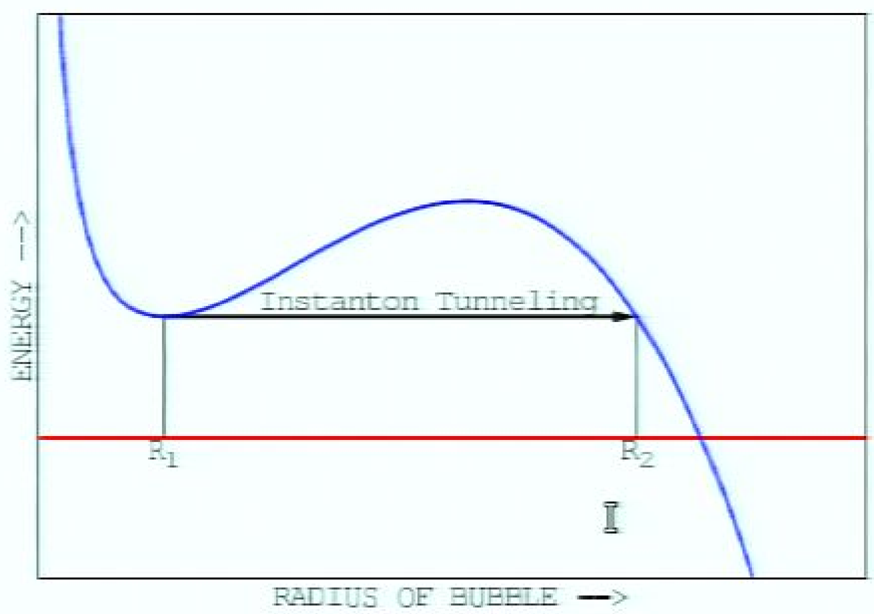

where $\hat{r}$ is a unit vector in spherical polar coordinates. The energy function becomes

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In the thin wall approximation, the functions h and K can be written as

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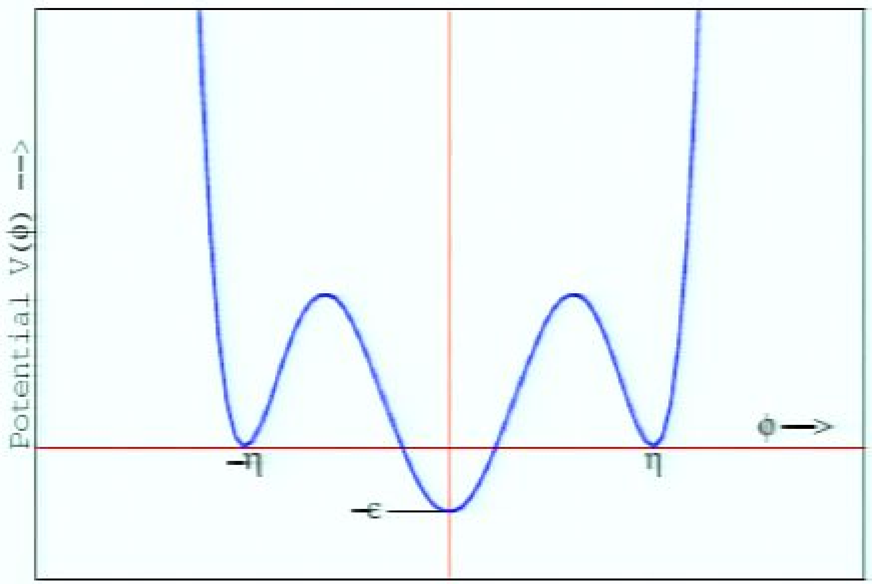

Time independent spherically symmetric ansatz for the monopole

$$\begin{aligned}\phi_a &= \hat{r}_a h(r) \\ A^a_\mu &= \epsilon_{\mu a b}\hat{r}_b \frac{1-K(r)}{er} \\ A_0 &= 0\end{aligned} \qquad (21)$$

Metastable monopoles :

Consider an $SU(2)$gauge theory with a triplet scalar field ϕ with the Lagrangian density given by

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \frac{1}{2}(D_\mu\phi^a)(D^\mu\phi^a) - V(\phi^a\phi^a) \tag{19}$$

The potential we use is a polynomial of order 6 in ϕ and may conveniently be written as

$$V(\phi) = \lambda\phi^2(\phi^2 - a^2)^2 + \gamma^2\phi^2 - \epsilon \tag{20}$$

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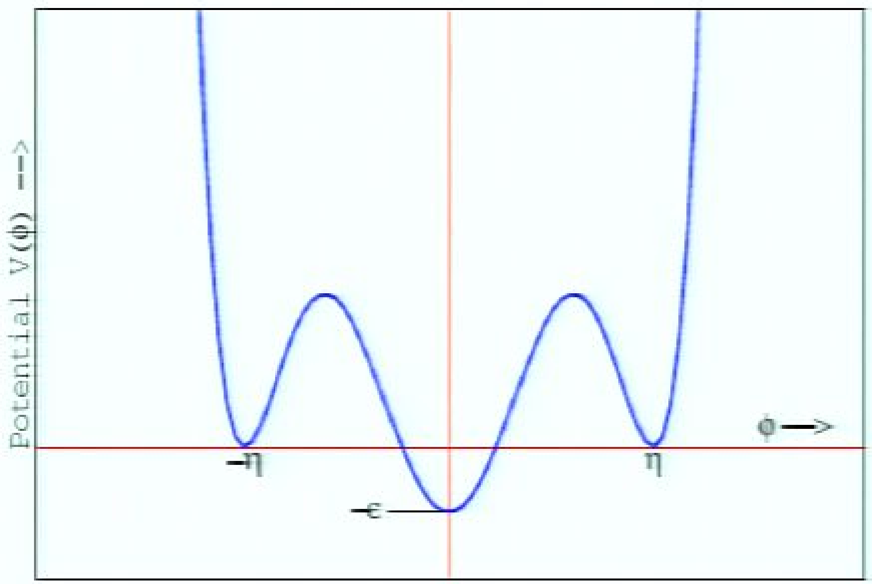

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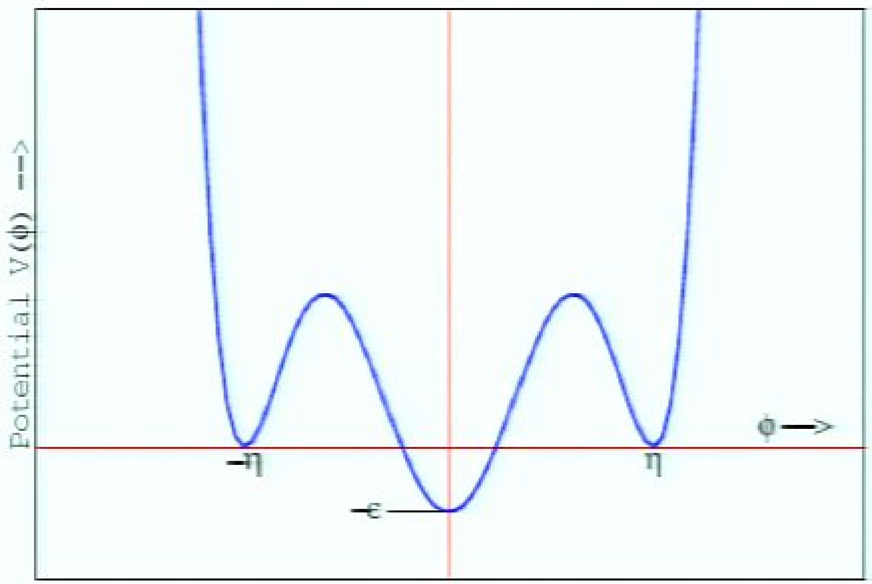

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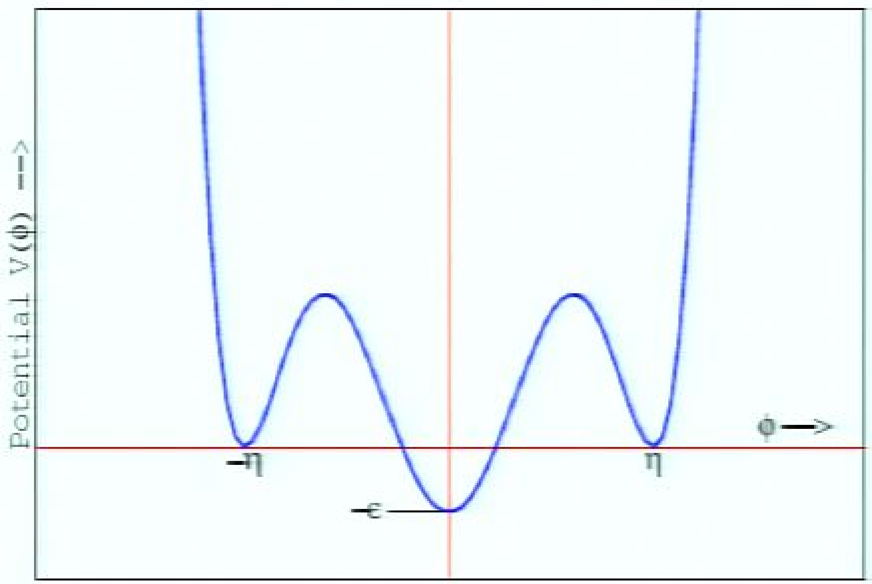

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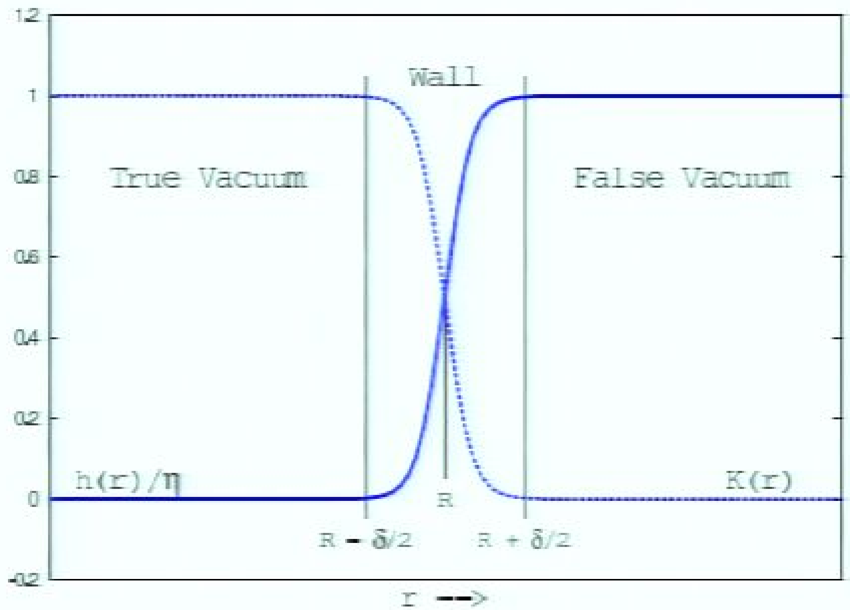

Total energy can be expressed as

$$E = 4\pi \left[\int_0^{R-\frac{\delta}{2}} dr\, r^2 V(h) + \int_{R+\frac{\delta}{2}}^{\infty} dr \frac{1}{2e^2r^2} \right.$$
$$+ \int_{R-\frac{\delta}{2}}^{R+\frac{\delta}{2}} dr \left(\frac{(K')^2}{e^2} + \frac{(1-K^2)^2}{2e^2r^2} \right.$$

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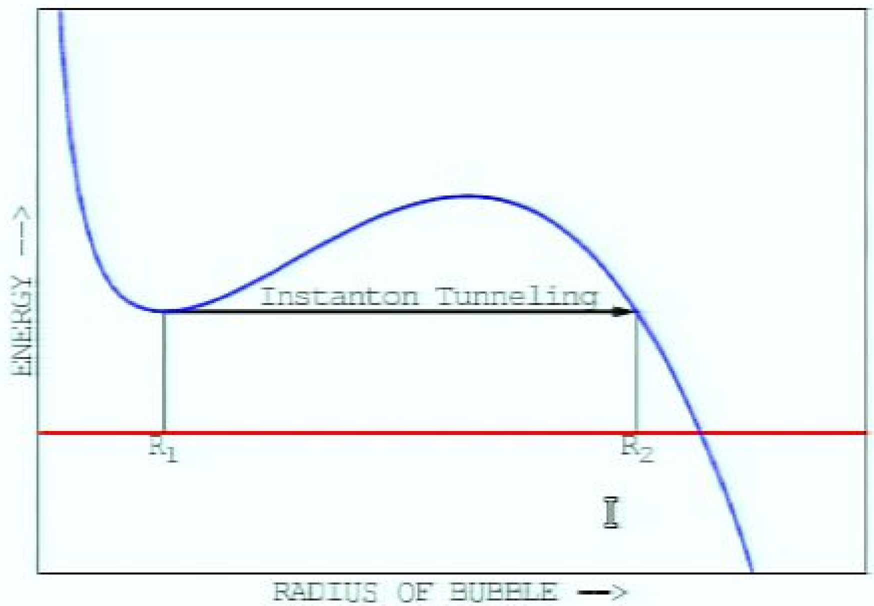

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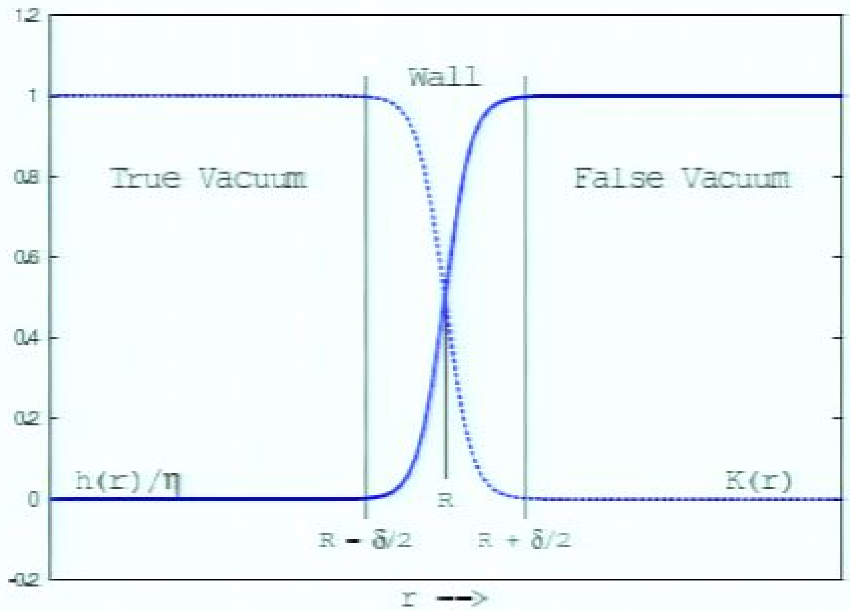

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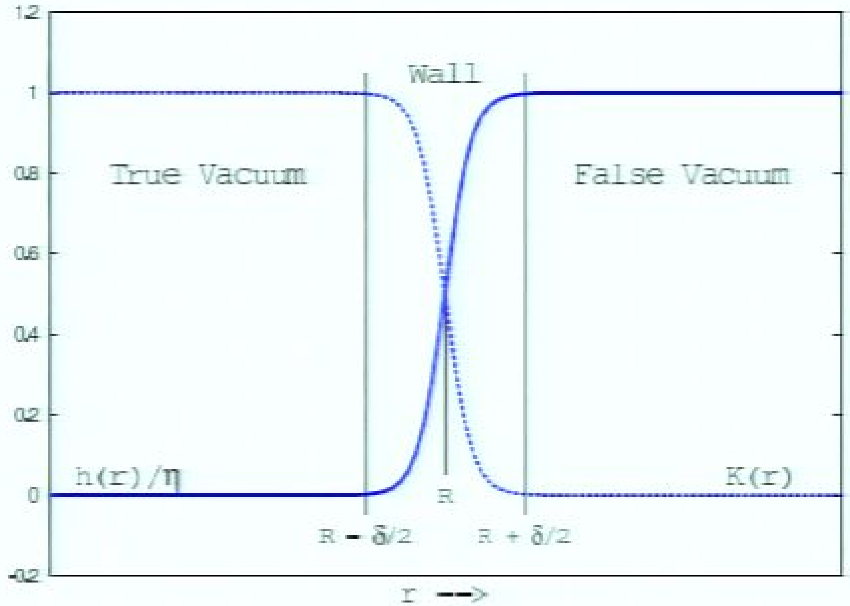

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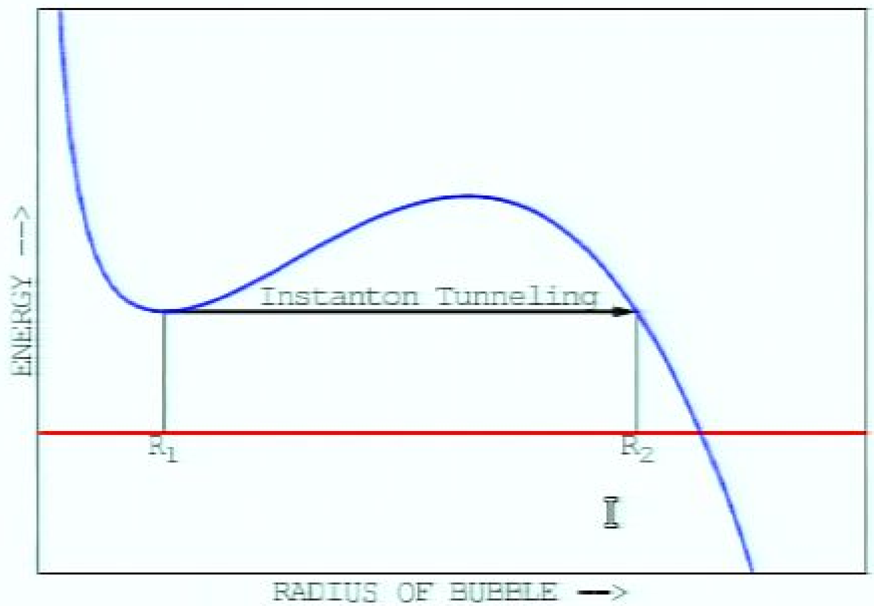

For the Euclidean action we get

$$S_E = \int_{-\infty}^{\infty} d\tau \Big(2\pi \dot{R}^2 (S_1 R^2 + S_2) + E(R) \Big) \tag{26}$$

where $\tau = i\, t$ is the Euclidean time and $\dot{R}$ is the derivative with respect to τ.

The instanton

Boundary conditions

$$R = R_1 \text{ for } \tau = \pm\infty,$$
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From the values of R_1 and R_2, we have

$$\frac{R_1}{R_2} = \frac{1}{e^{2/3}} \frac{1}{(4\sigma)^{1/3}} \frac{\epsilon}{3\sigma} \tag{32}$$

$$= \frac{1}{(\tilde{\lambda} e)^{2/3}} \left(\frac{16}{27}\right)^{1/3} \frac{\epsilon}{\tilde{a}^{16/3} \mu^4} \tag{33}$$

$$\frac{\Gamma}{V} \sim \left(\frac{\kappa}{2}\right) \exp\left\{ \frac{16}{105} \sqrt{\frac{2 S_1 \pi^2 \epsilon}{3}} \mathcal{F}(R_1, R_2, R_3) \right\}$$

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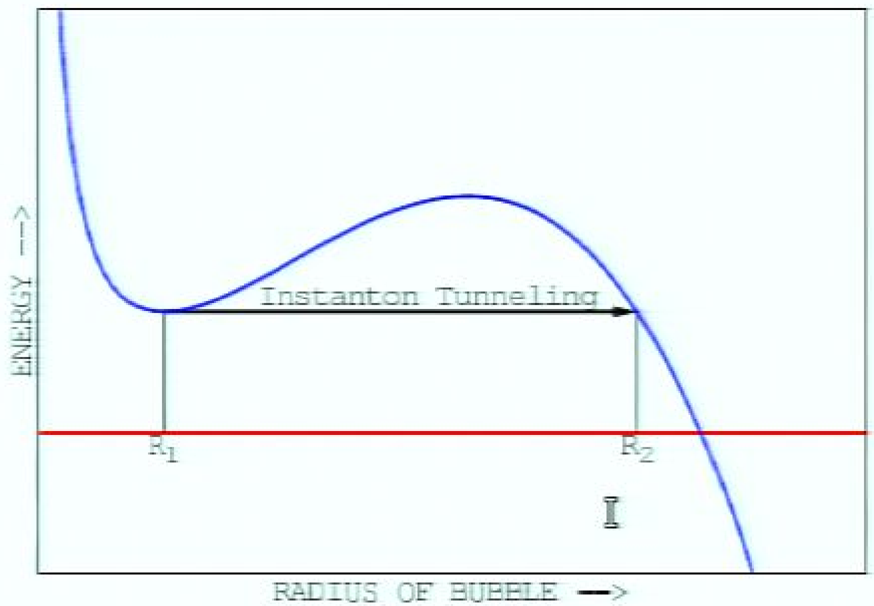

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