Title: Closed time-like curves in measurement-based quantum computation

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URL: http://pirsa.org/10070029

Abstract: Many results have been recently obtained regarding the power of hypothetical closed time-like curves (CTC’s) in quantum computation. Most of them have been derived using Deutsch’s influential model for quantum CTCs [D. Deutsch, Phys. Rev. D 44, 3197 (1991)]. Deutsch’s model demands self-consistency for the time-travelling system, but in the absence of (hypothetical) physical CTCs, it cannot be tested experimentally. In this paper we show how the one-way model of measurement-based quantum computation (MBQC) can be used to test Deutsch’s model for CTCs. Using the stabilizer formalism, we identify predictions that MBQC makes about a specific class of CTCs involving travel in time of quantum systems. Using a simple example we show that Deutsch’s formalism leads to predictions conflicting with those of the one-way model. There exists an alternative, little-discussed model for quantum time-travel due to Bennett and Schumacher (in unpublished work, see http://bit.ly/cjWUT2), which was rediscovered recently by Svetlichny [arXiv:0902.4898v1]. This model uses quantum teleportation to simulate (probabilistically) what would happen if one sends quantum states back in time. We show how the Bennett/ Schumacher/ Svetlichny (BSS) model for CTCs fits in naturally within the formalism of MBQC. We identify a class of CTC’s in this model that can be simulated deterministically using techniques associated with the stabilizer formalism. We also identify the fundamental limitation of Deutsch's model that accounts for its conflict with the predictions of MBQC and the BSS model. This work was done in collaboration with Raphael Dias da Silva and Elham Kashefi, and has appeared in preprint format (see website). Website: http://arxiv.org/abs/1003.4971

# Closed timelike curves in measurement-based quantum computation

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Perimeter Institute – July 16th 2010

arXiv:1003.4971v1 [quant-ph]

# Outline

- Closed time-like curves (CTCs) = time travel
- Deutsch's model for CTCs
- How CTCs show up in measurement-based quantum computation
- CTC model by Bennett/Schumacher/Svetlichny
- Conclusion

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# Time travel

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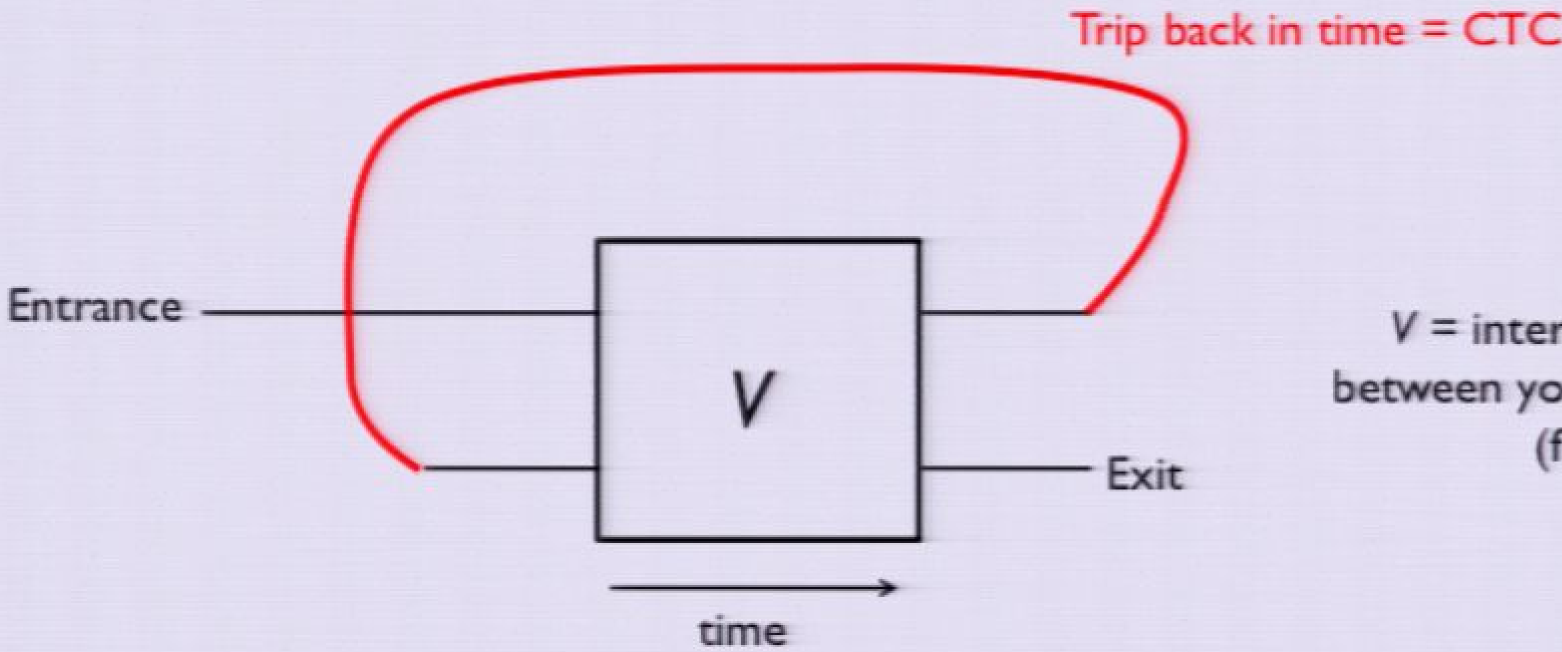
- To the future? Easy, use relativity.

- To the past? More involved:
  - Relativity predicts solutions with *closed timelike curves* (CTCs)= travel to the past
  - It's not known whether CTCs are physically possible, perhaps with clever black-hole engineering.
  - To avoid paradoxes, self-consistency conditions on time-travellers must apply – more on that later.

# Time travel - CTCs

- Time travel scenario:




$V$ = interaction region between you (past) and you (future)

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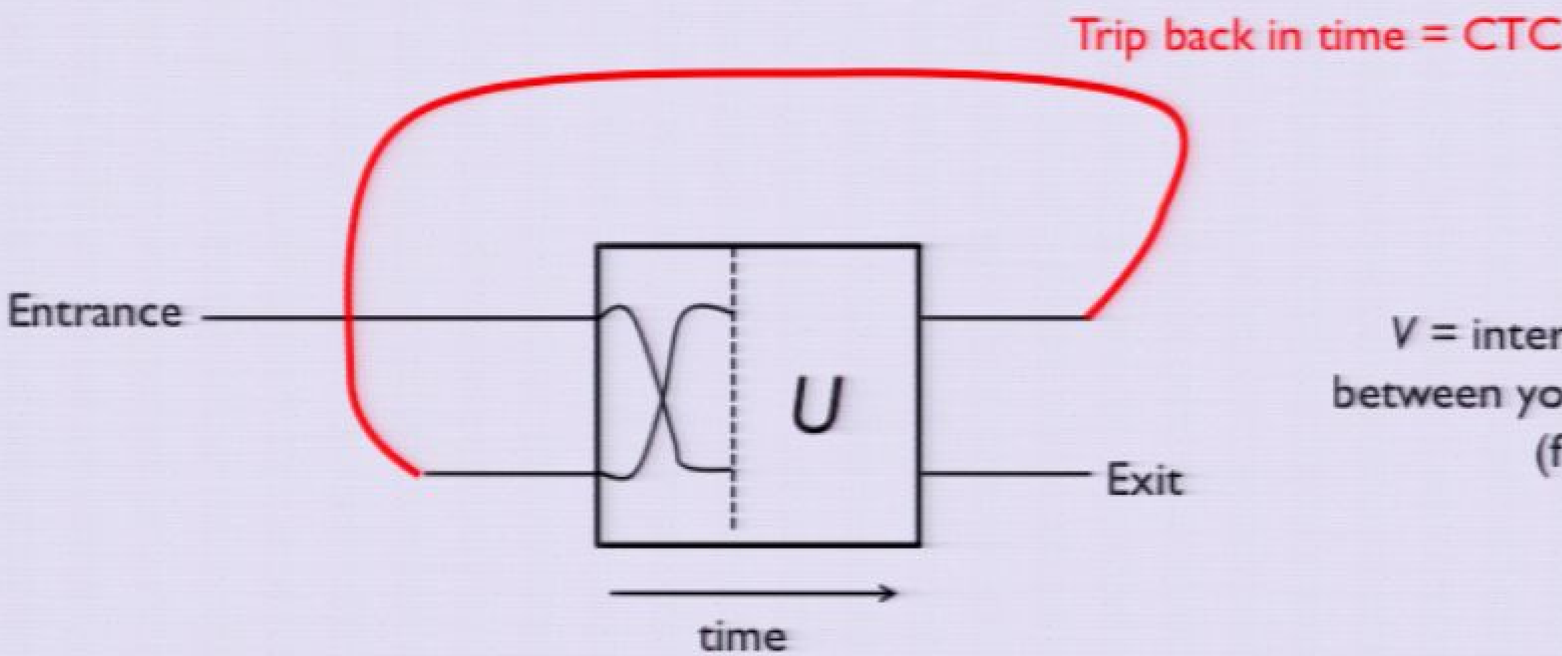


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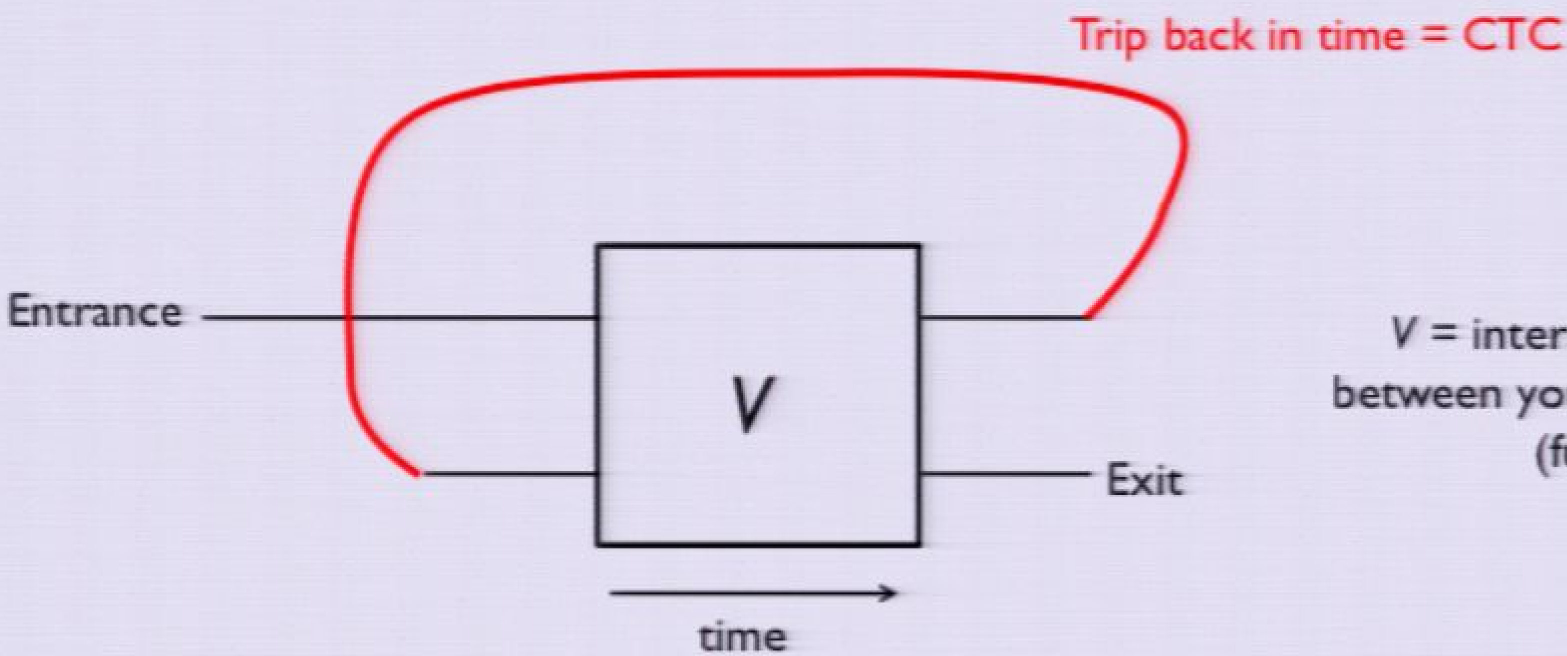


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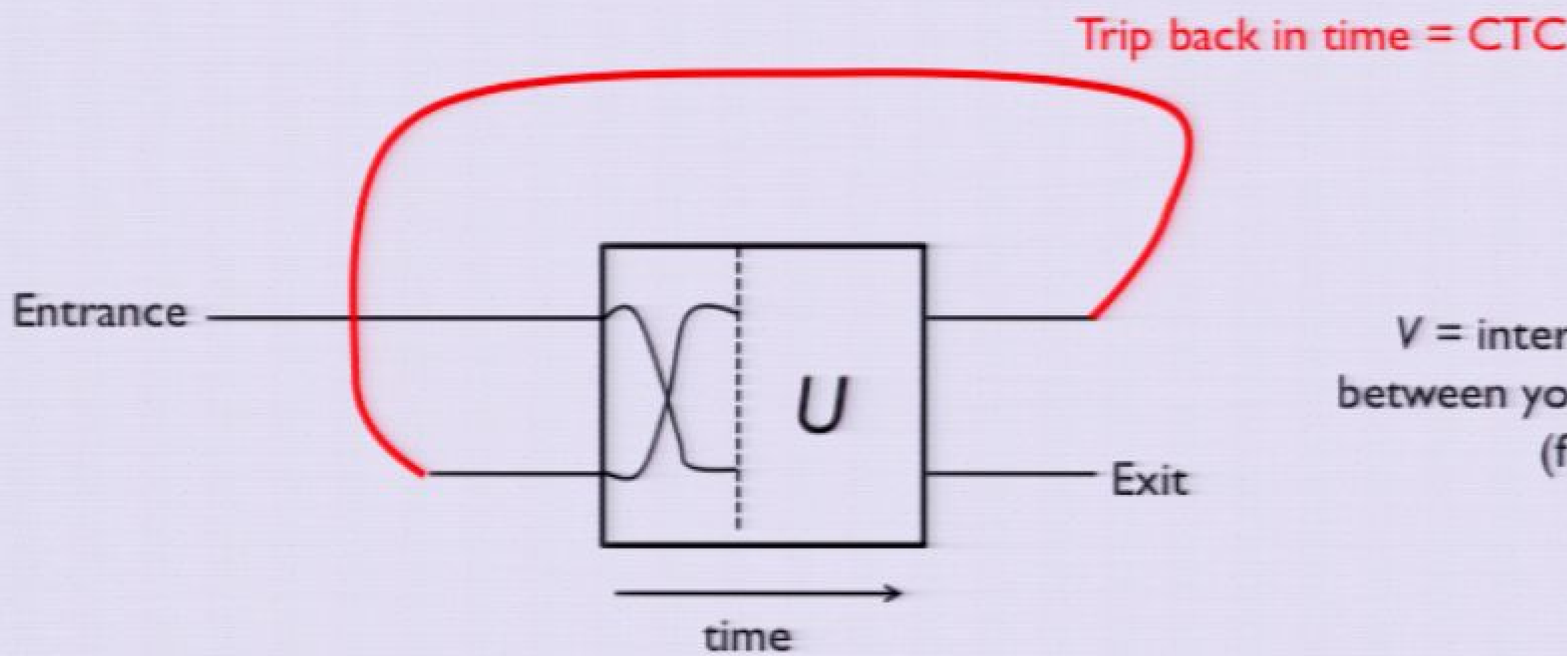


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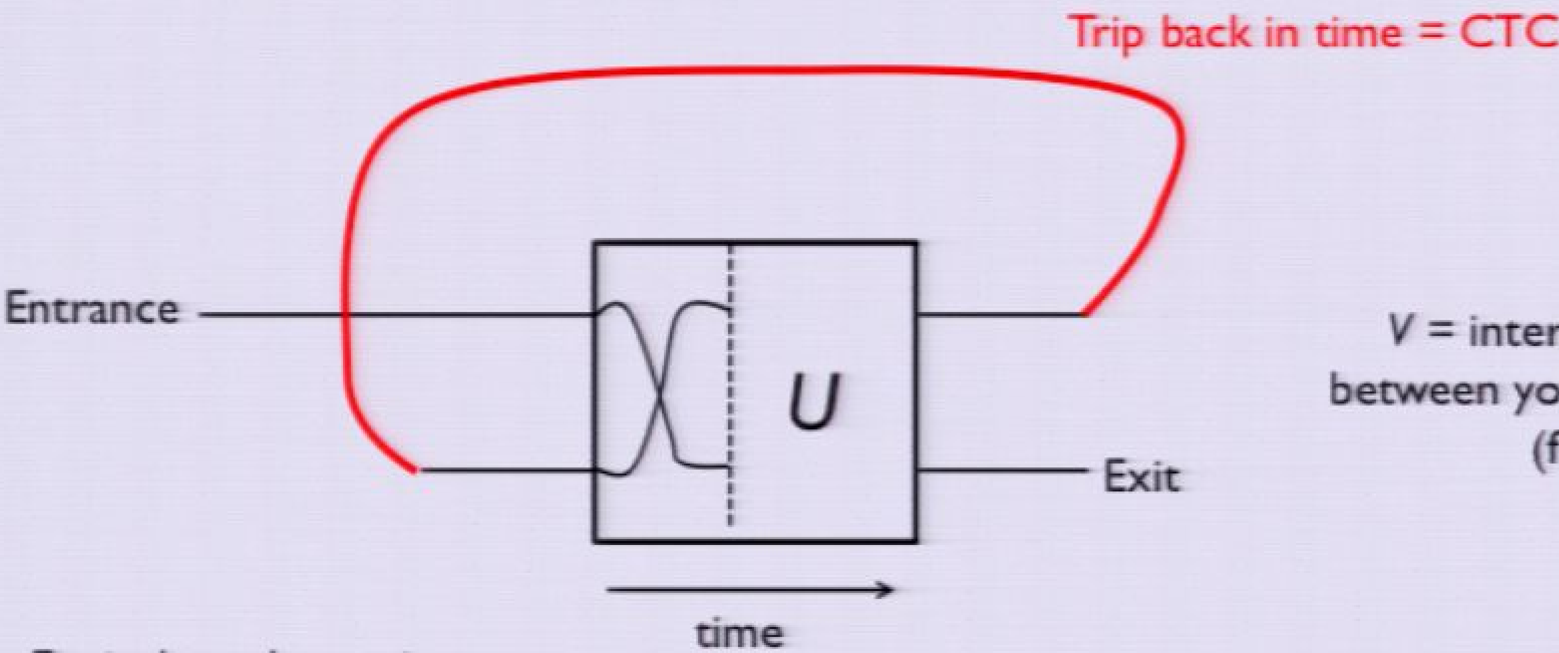


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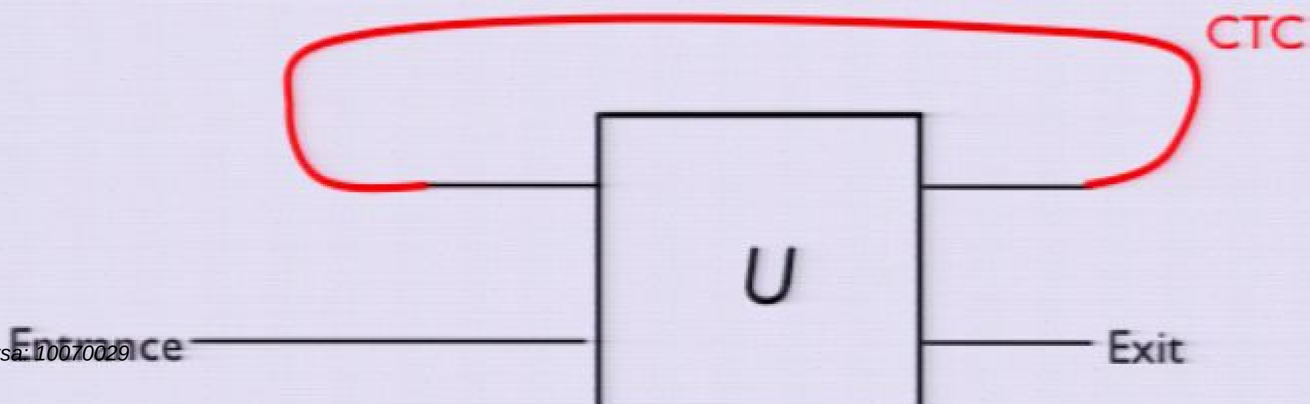
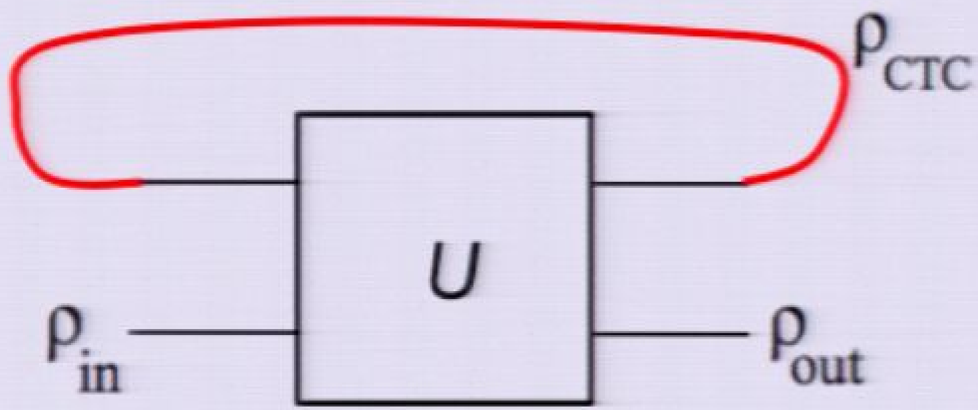
- Equivalent alternative:




*U* = interaction region

# Deutsch's model for CTCs

Deutsch, Phys. Rev. D 44, 3197 (1991)




- *U* = 2-qubit unitary
  - 1 qubit travels back in time
  - 1 qubit doesn't
- Initial state: $\rho_{CTC} \otimes \rho_{in}$
- After *U*: $U(\rho_{CTC} \otimes \rho_{in})U^{+}$
- Self-consistency condition: $\rho_{CTC} = Tr_B\left[U(\rho_{CTC} \otimes \rho_{in})U^{+}\right]$

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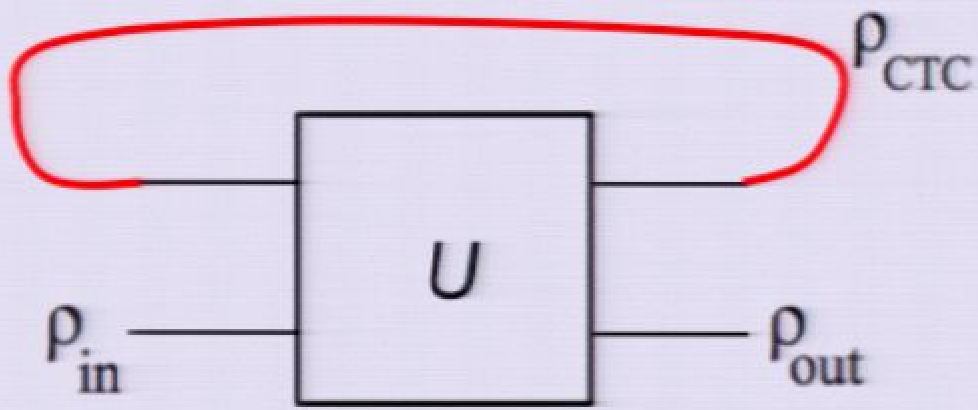


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- Self-consistency condition: $$\rho_{CTC} = Tr_B\left[U(\rho_{CTC} \otimes \rho_{in})U^+\right]$$
- Deutsch showed that:
  - there's always at least 1 self-consistent solution;
  - there can be multiple such solutions;
  - each solution corresponds to an input-output map, in general non-linear.

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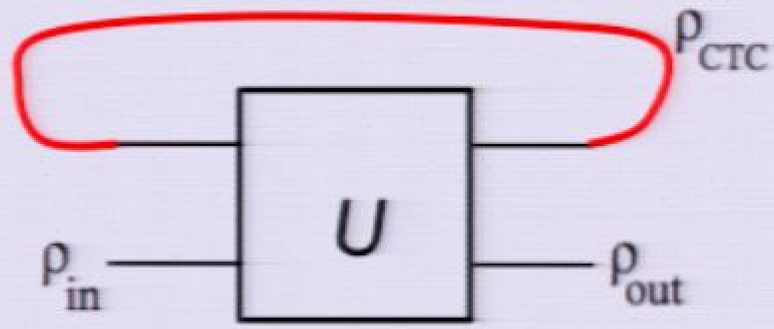


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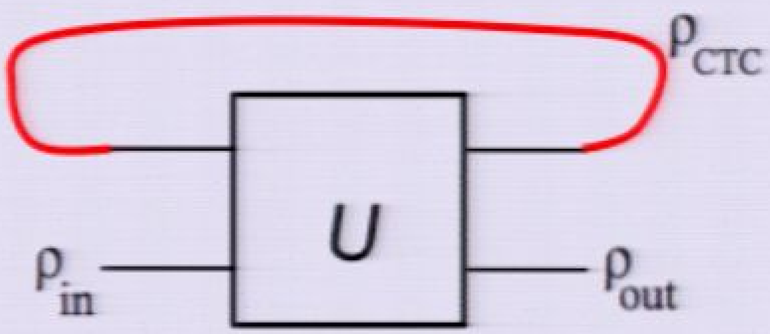


- **Computational power** of Deutsch's CTCs :

  Non-orthogonal state discrimination? Solution to NP-complete problems?

  - Bacon arXiv:quant-ph/0309189v3 (solves NP-complete problems)
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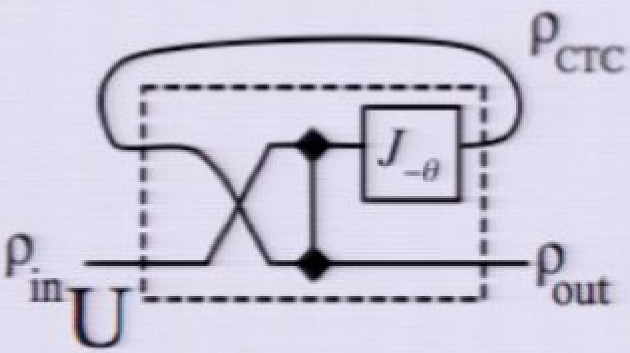
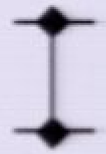
# Deutsch - example



$$= CTR - Z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad J_{-\theta} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{-i\theta} \\ 1 & -e^{-i\theta} \end{pmatrix}$$

$$\rho_{in} = \frac{1}{2}(1 + \vec{n} \cdot \vec{\sigma}) \qquad \rho_{CTC} = \frac{1}{2}(1 + \vec{m} \cdot \vec{\sigma}) \qquad \rho_{out} = \frac{1}{2}(1 + \vec{r} \cdot \vec{\sigma})$$

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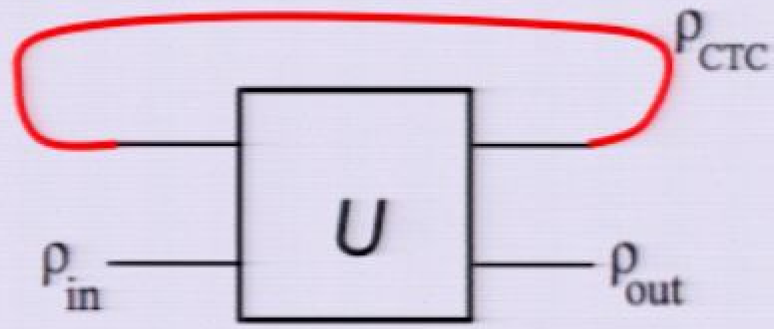


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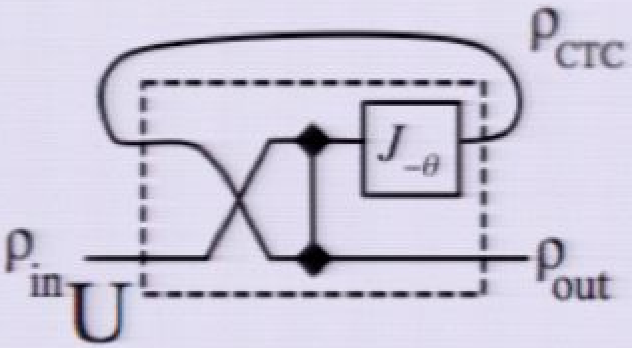


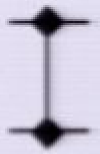

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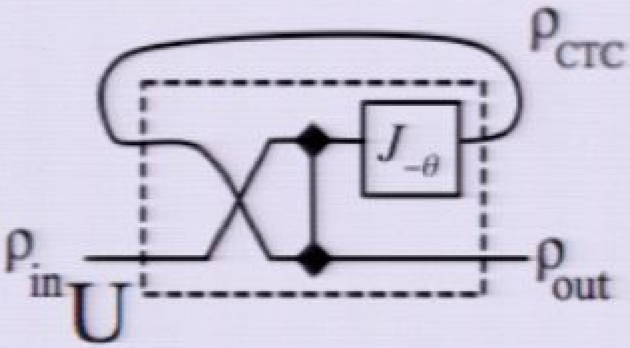


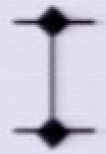

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$$\begin{aligned} m_x &= n_z, \\ m_y &= m_z(n_x \sin\theta - n_y \cos\theta), \\ m_z &= m_z(n_x \cos\theta + n_y \sin\theta). \end{aligned}$$

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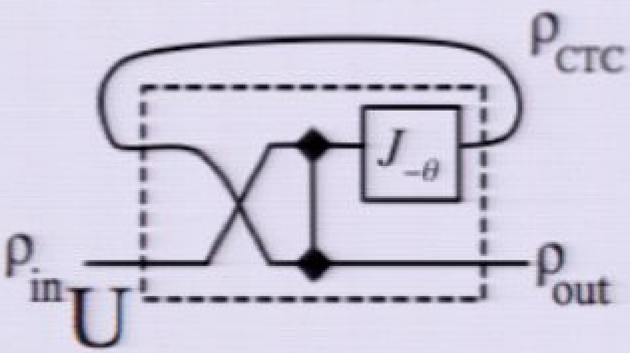


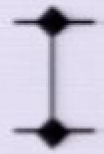

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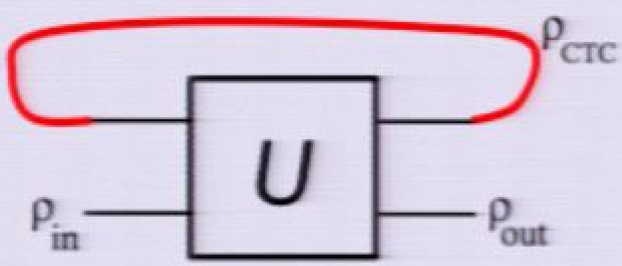
- Solution for generic input: $\begin{cases} \rho_{CTC} : \vec{m} = (n_z, 0, 0) \\ \rho_{out} : \vec{r} = (n_z^2, 0, 0) \end{cases}$ ⇐ Non-linear map!

- Alternative solution for $|\psi_{in}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\theta}|1\rangle) : \rho_{CTC} = \rho_{out} : \vec{m} = (0, 0, m_z)$

# Testing Deutsch's model



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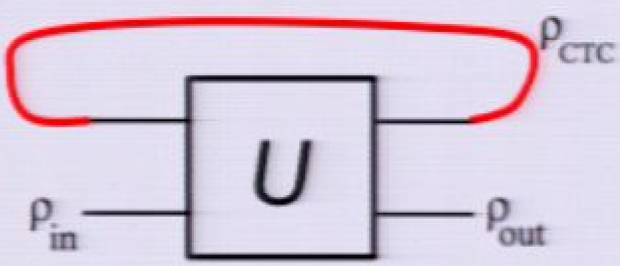


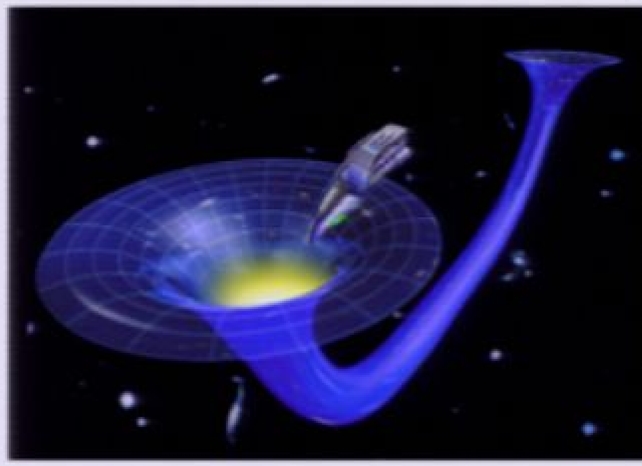

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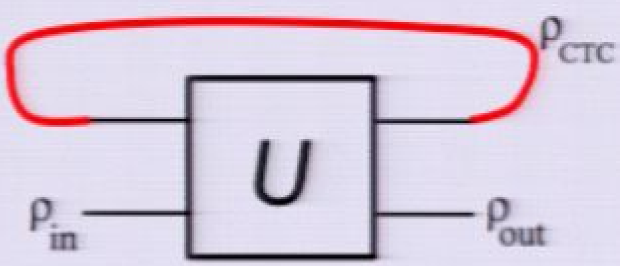


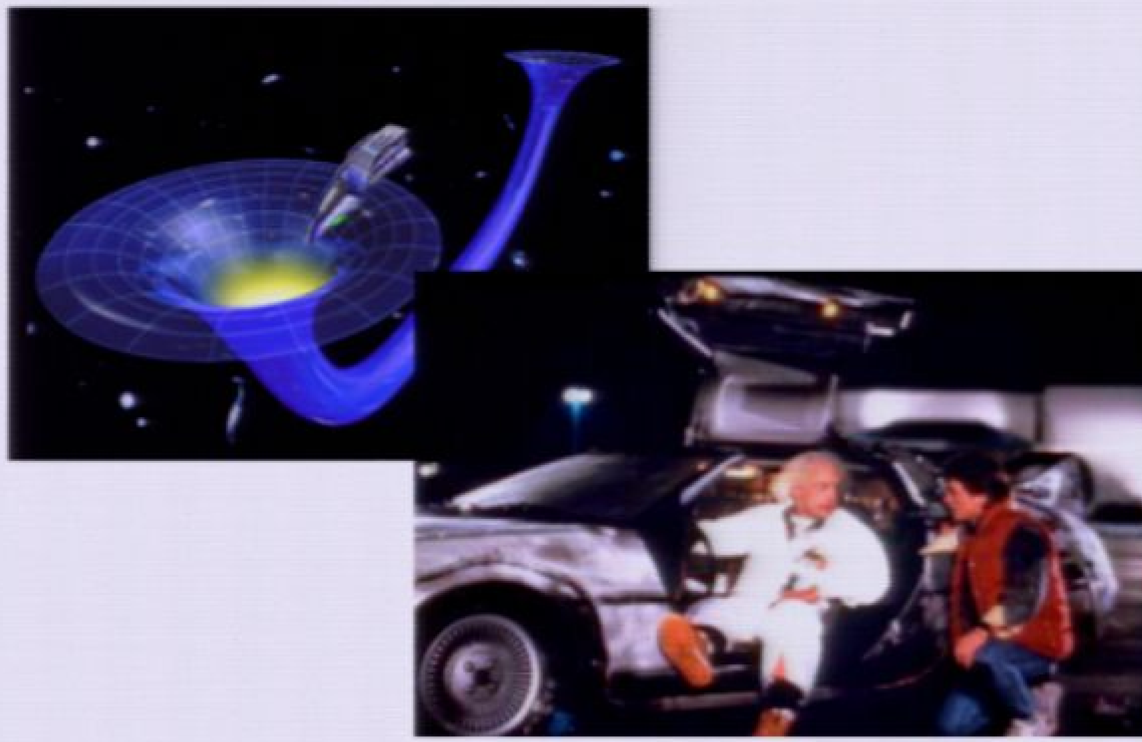

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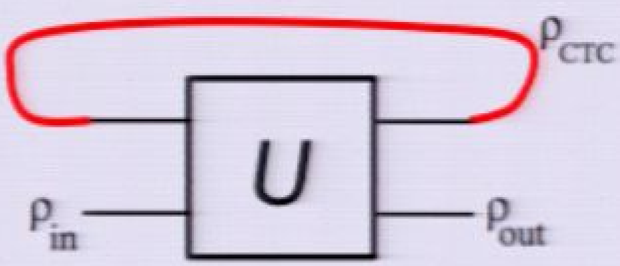
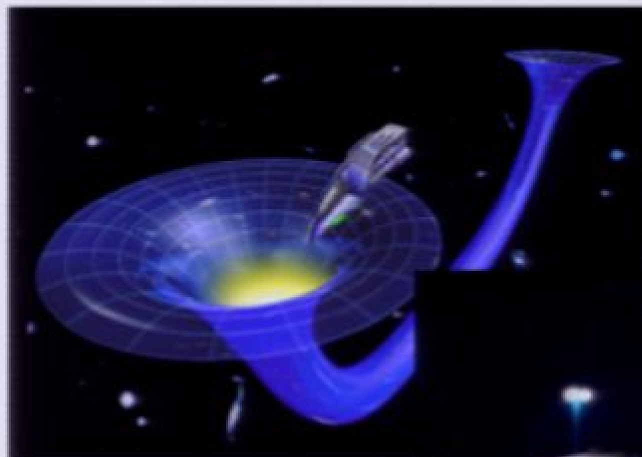


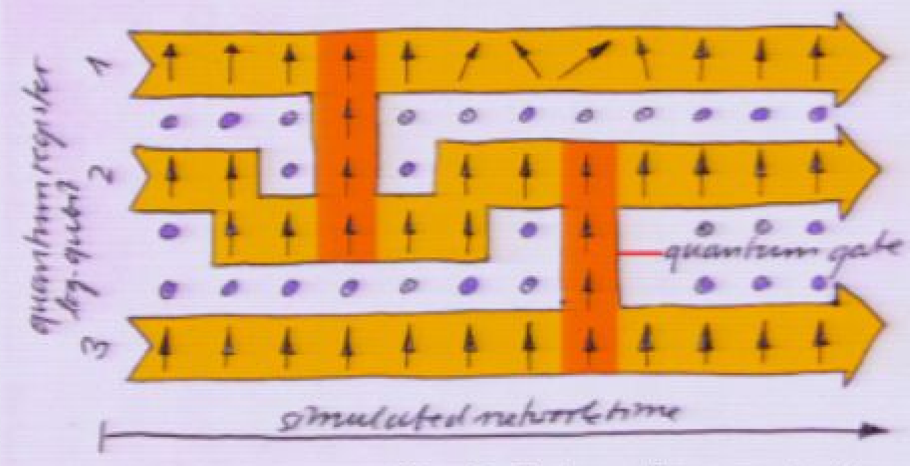



- Discussion of computational power of CTCs uses Deutsch's model. In the absence of experiments, how to check if the model is sound?

- We'll see that **measurement-based quantum computation** offers an answer.

# The one-way model of quantum computing

- Proposed by Raussendorf/Briegel
  [PRL 86, 5188 (2001)]



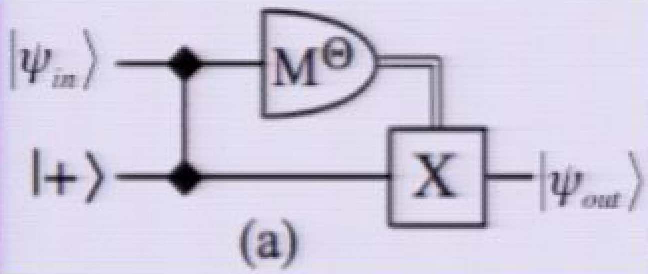

Credit: Robert Raussendorf

- Consists in:
  - Preparation of standard entangled states via Heisenberg interactions – *cluster* states;
  - Adaptive sequence of 1-qubit measurements.

- Computational resource = quantum correlations of initial state

- Algorithm = choice of adaptive sequence of measurements

# Example: *J* gate




$$|\psi_{in}\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |+\rangle \equiv 1/\sqrt{2}(|0\rangle + |1\rangle)$$

$$M^{\Theta} = \text{Meas. on basis} \begin{Bmatrix} |0\rangle + e^{i\theta}|1\rangle \leftrightarrow \text{ outcome } s1 = 0 \\ |0\rangle - e^{i\theta}|1\rangle \leftrightarrow \text{ outcome } s1 = 1 \end{Bmatrix}$$

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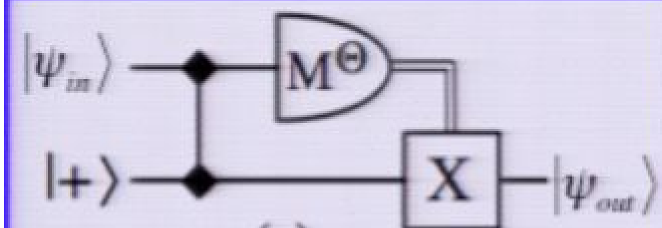

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$$X_2^{s1} M_1^{\theta} CTR_Z |G\rangle, \qquad |G\rangle \equiv |\psi_{in}\rangle_1 \otimes |+\rangle_2$$

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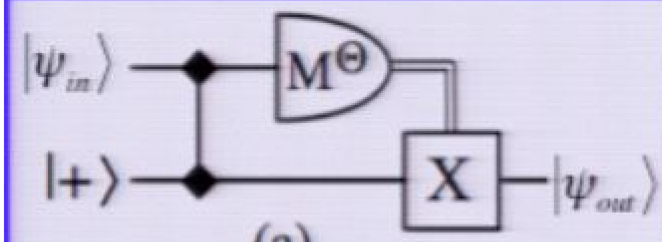

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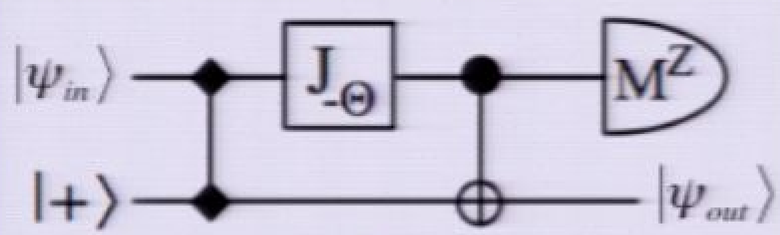
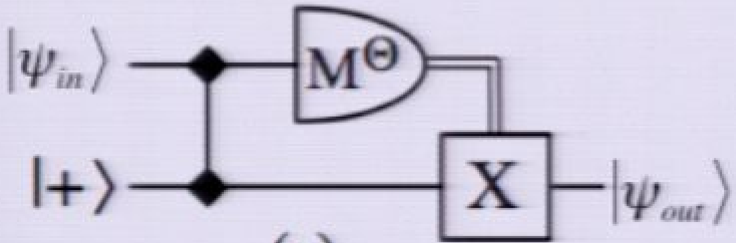
- Equivalent circuit – measure *Z*, implement *CTR-X* (*CNOT*) coherently:



(b)

# $J$ gate in MBQC = CTC



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$$\Leftrightarrow \quad X_2^{s1} M_1^{\theta} |G\rangle,$$
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$$\begin{cases} (Z_1 \otimes X_2)^0 = Z_1^0 \otimes X_2^0 = 1 \text{ (identity)} \\ (Z_1 \otimes X_2)^1 = Z_1^1 \otimes X_2^1 \end{cases}$$

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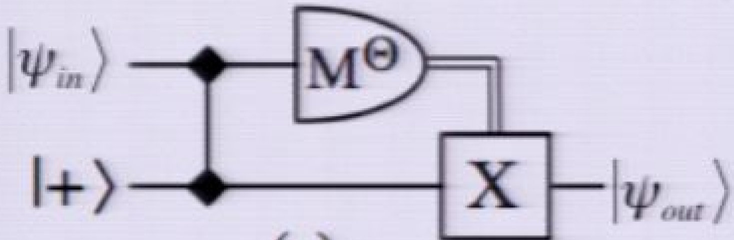


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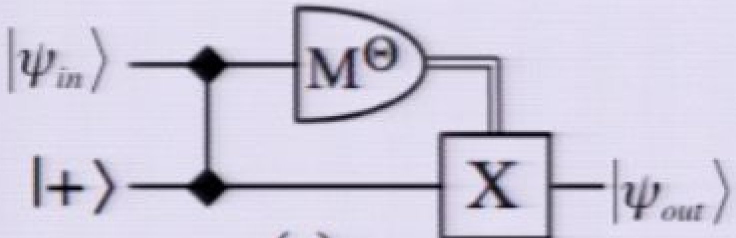

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- We can then perform stabilizer manipulation:

$$X_2^{s1} M_1^{\theta} |G\rangle = X_2^{s1} M_1^{\theta} (Z_1^{s1} X_2^{s1} |G\rangle)$$
$$= X_2^{s1+s1} M_1^{\theta} Z_1^{s1} |G\rangle = M_1^{\theta} Z_1^{s1} |G\rangle$$

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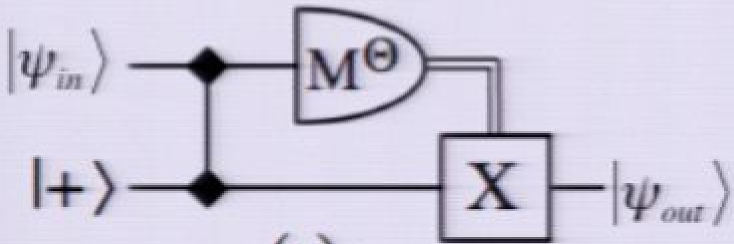



(a)

$$\Leftrightarrow \quad X_2^{s1} M_1^{\theta} |G\rangle, \quad |G\rangle \equiv CTR_Z |\psi_{in}\rangle_1 \otimes |+\rangle_2$$

- Stabilizers of state $|G\rangle$ :

$$\begin{cases} (Z_1 \otimes X_2)^0 = Z_1^0 \otimes X_2^0 = 1 \text{ (identity)} \\ (Z_1 \otimes X_2)^1 = Z_1^1 \otimes X_2^1 \end{cases}$$

That is, $Z_1^{s1} \otimes X_2^{s1}$ is stabilizer independently of the outcome $s_1$ of the measurement on qubit 1:

$$Z_1^{s1} \otimes X_2^{s1} |G\rangle = |G\rangle$$

- We can then perform stabilizer manipulation:

$$X_2^{s1} M_1^{\theta} |G\rangle = X_2^{s1} M_1^{\theta} (Z_1^{s1} X_2^{s1} |G\rangle)$$

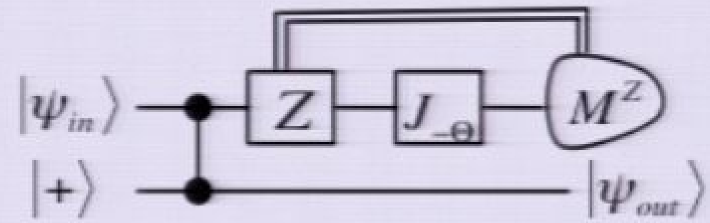
$$= X_2^{s1+s1} M_1^{\theta} Z_1^{s1} |G\rangle = M_1^{\theta} Z_1^{s1} |G\rangle$$

**Time-travel** situation: we need to apply Z depending on outcome of measurement not yet made.

# *J* gate in MBQC = CTC

- Putting this CTC in Deutsch format:

$$M_1^{\theta} Z_1^{s1} |G\rangle \quad \Leftrightarrow$$



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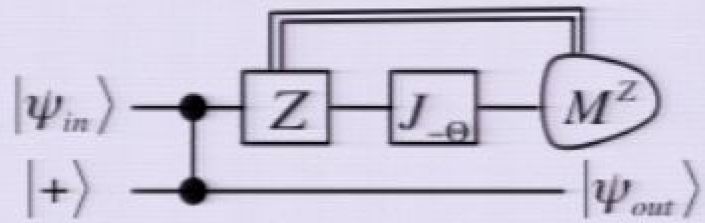
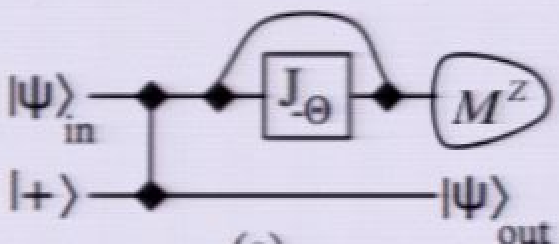



$\Leftrightarrow$

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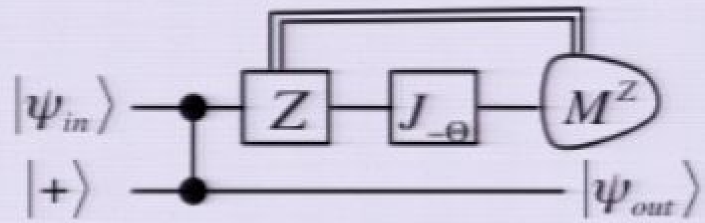

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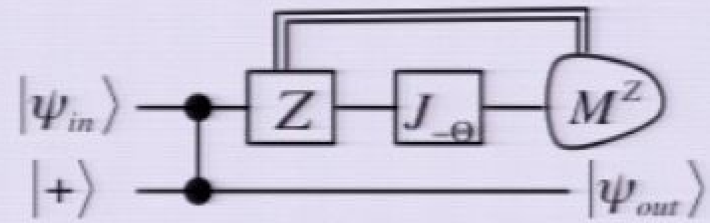


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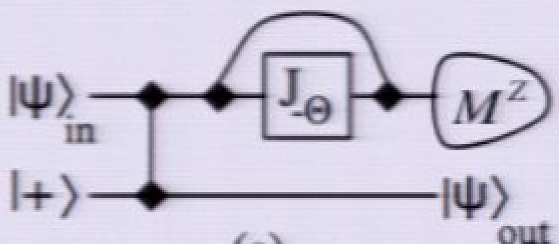


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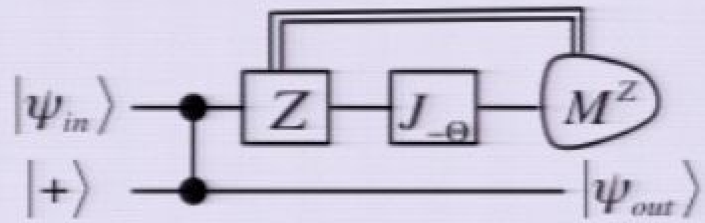


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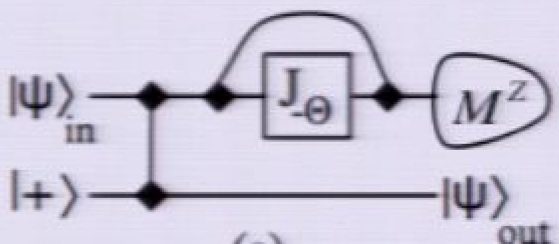


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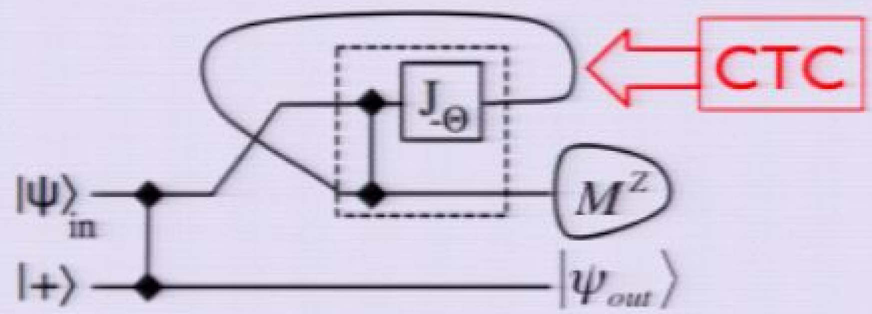

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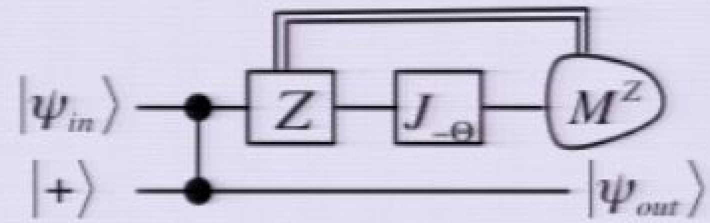


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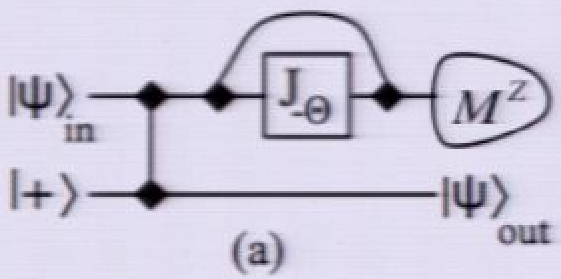


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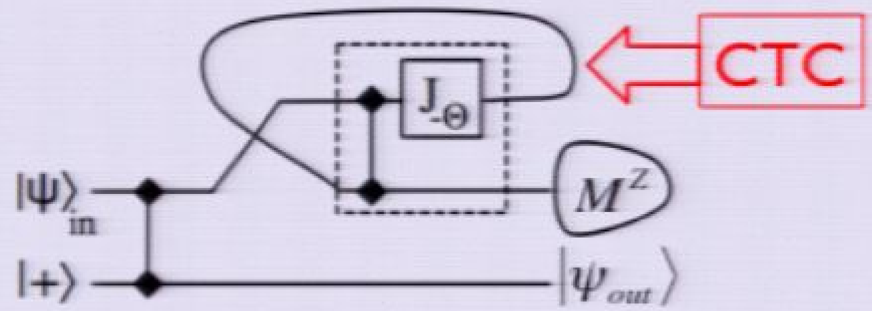


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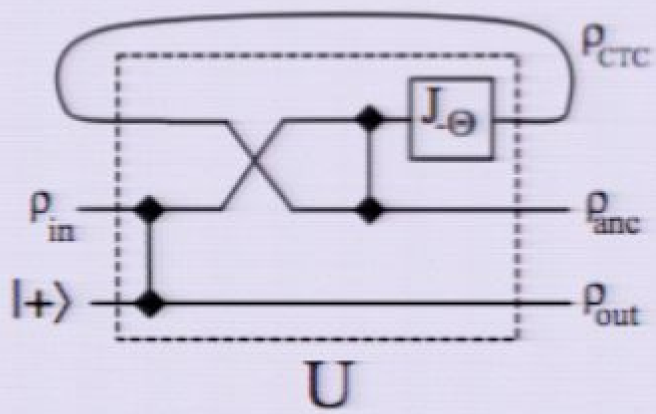

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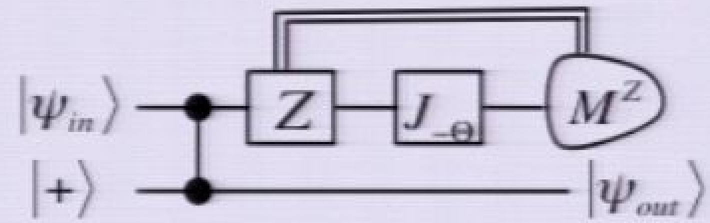


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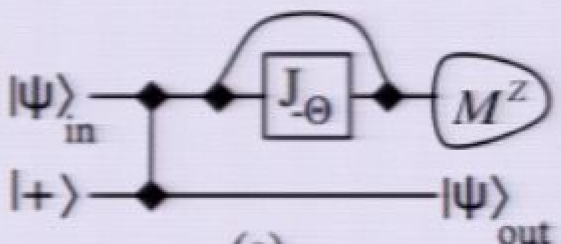


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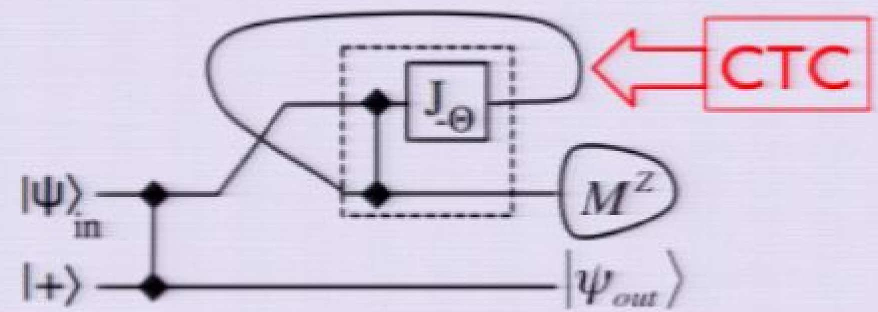


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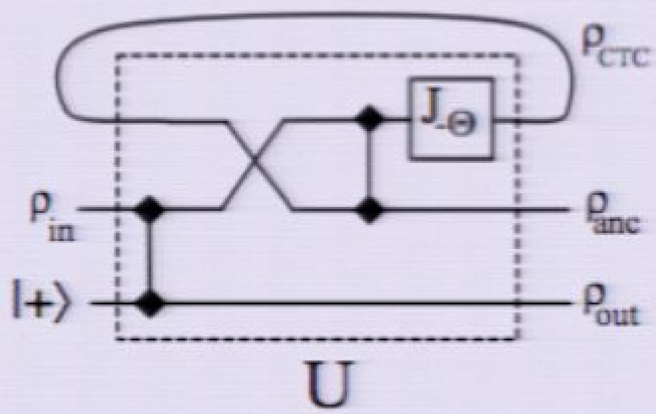


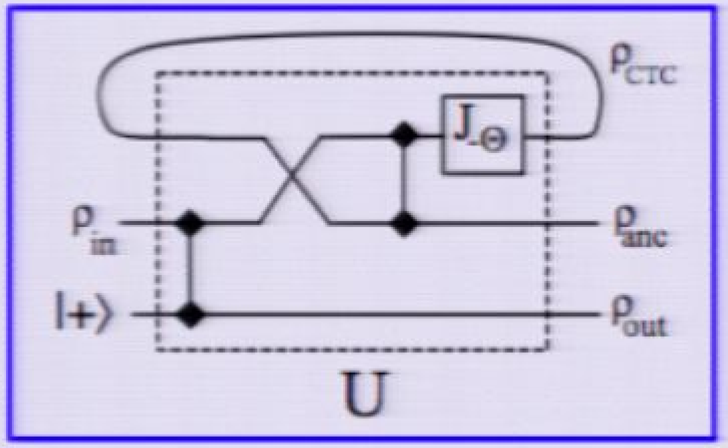

<u>Deutsch consistency conditions:</u>

$$\rho_{in} : \vec{n} = (n_x, n_y, n_z)$$

$$\rho_{anc} : \vec{m} = (n_z^2, 0, 0)$$

$$\rho_{out} = \rho_{CTC} : \vec{r} = (n_z, 0, 0)$$

# Comparing the one-way model with Deutsch:

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$$\rho_{in} = \frac{1}{2}\begin{pmatrix} 1+n_z & n_x - in_y \\ n_x + in_y & 1-n_z \end{pmatrix} \longrightarrow \rho_{out} = \frac{1}{2}\begin{pmatrix} 1 & n_z \\ n_z & 1 \end{pmatrix}$$

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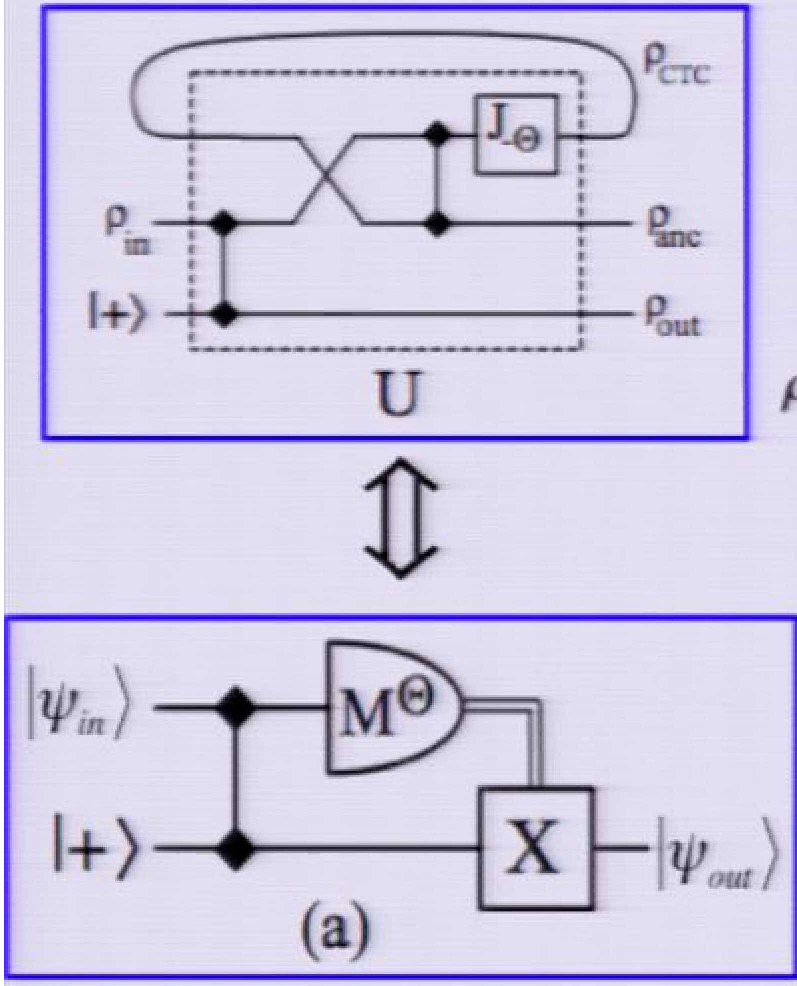


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One-way model:

$$|\psi_{in}\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\psi_{out}\rangle = J_{-\theta}|\psi_{in}\rangle = \alpha|+\rangle + \beta e^{-i\theta}|-\rangle$$

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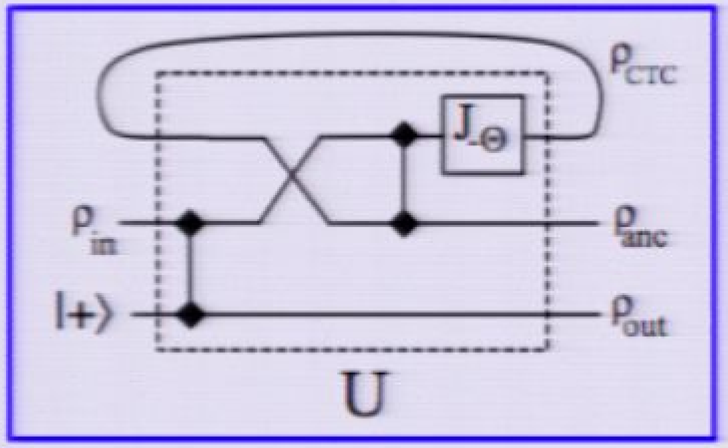


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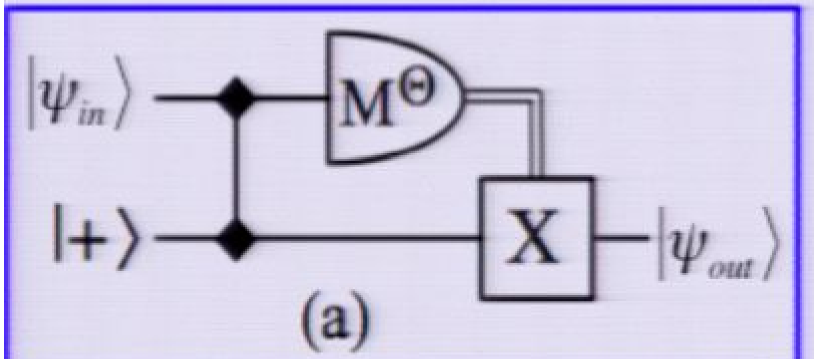
$\Updownarrow$

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Deutsch's prediction is *different* from the one based on the one-way model. What's wrong with Deutsch?

# CTCs: model by Bennett/Schumacher/Svetlichny (BSS)

- Bennett and Schumacher, unpublished (2002) – see seminar http://bit.ly/crs8Lb
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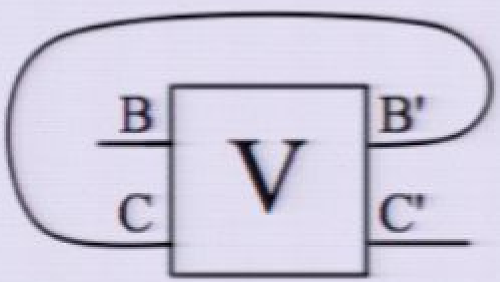
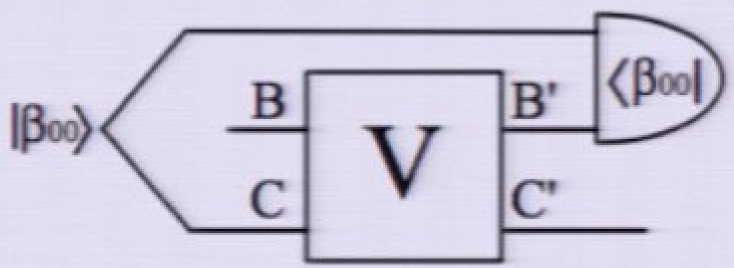



CTC

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Simulation using teleportation and post-selection: B'=C

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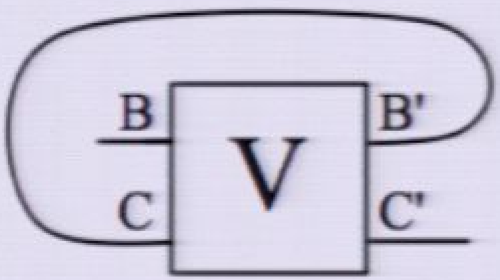


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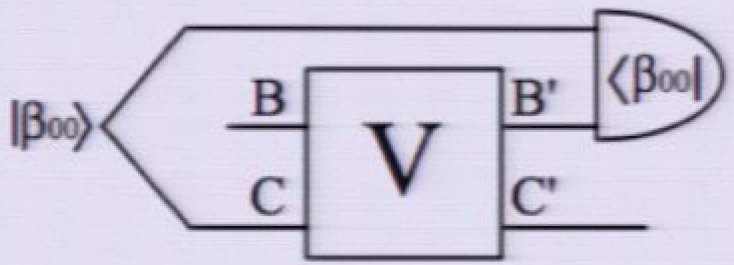


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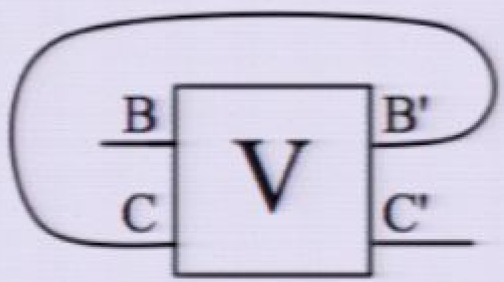


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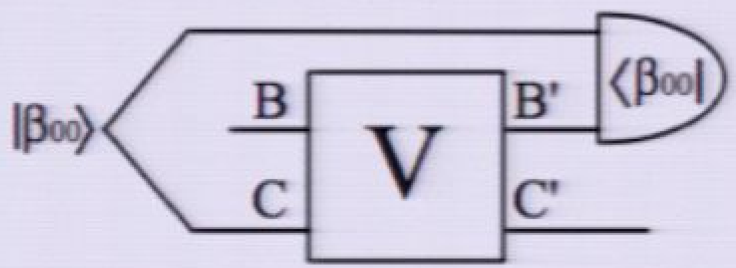


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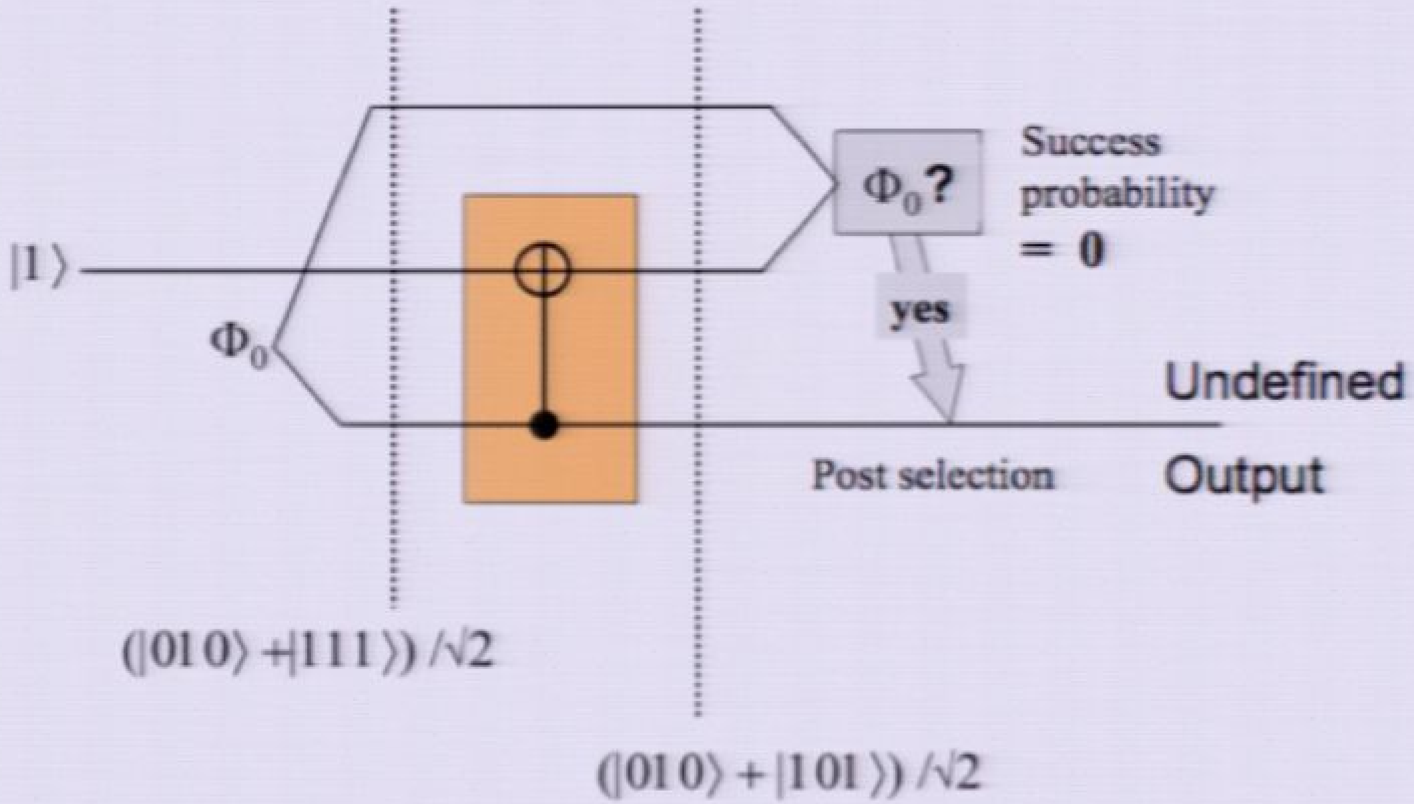
**What are BSS's predictions for our CTC?**

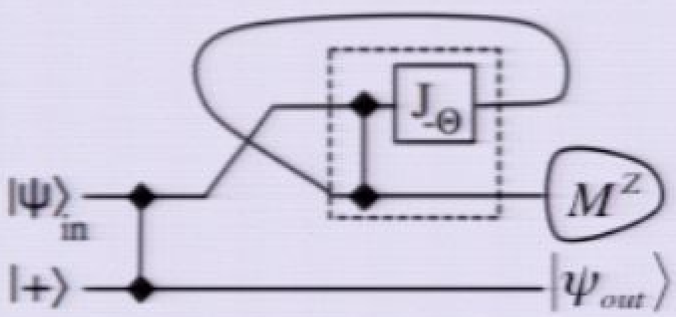
# How BSS deals with the grandfather paradox

- From Bennett's talk slides: http://bit.ly/crs8Lb



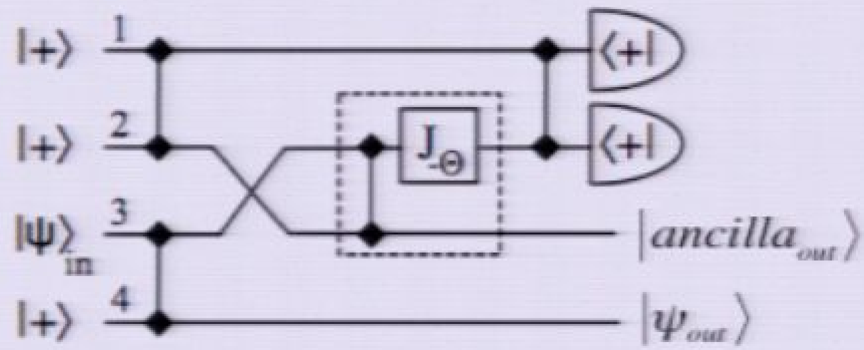

- Paradoxical combinations of input and unitary result in post-selection with success probability $p=0$.

# BSS x MBQC




CTC we studied

$\Longrightarrow$




Its BSS simulation circuit

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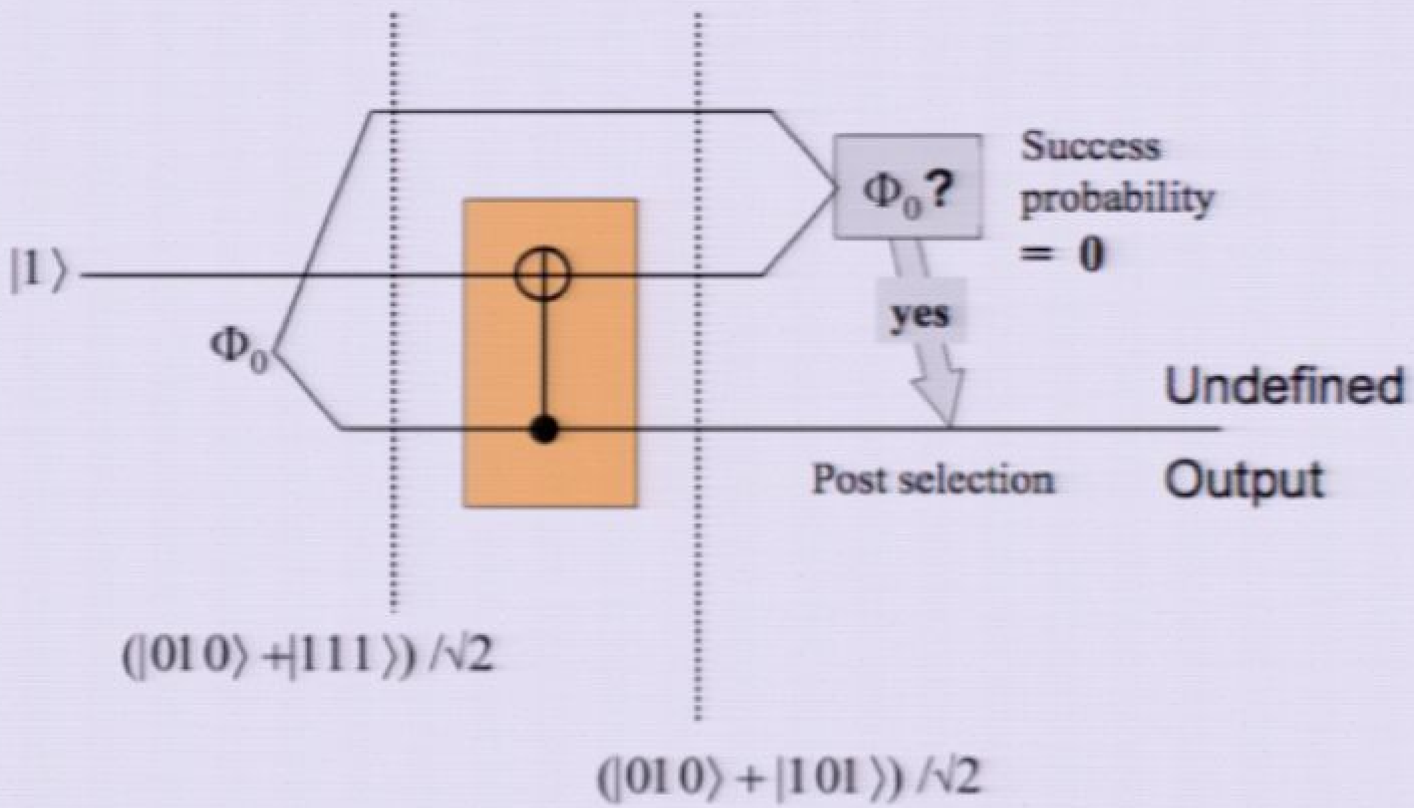


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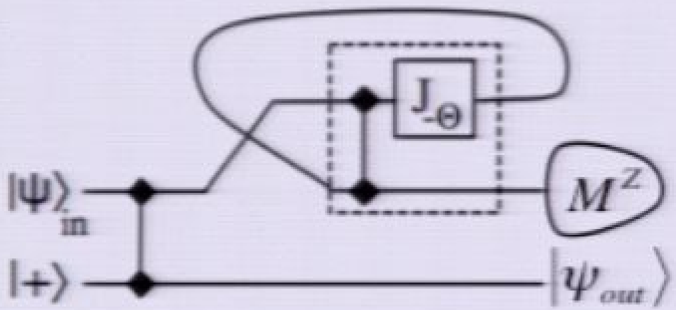


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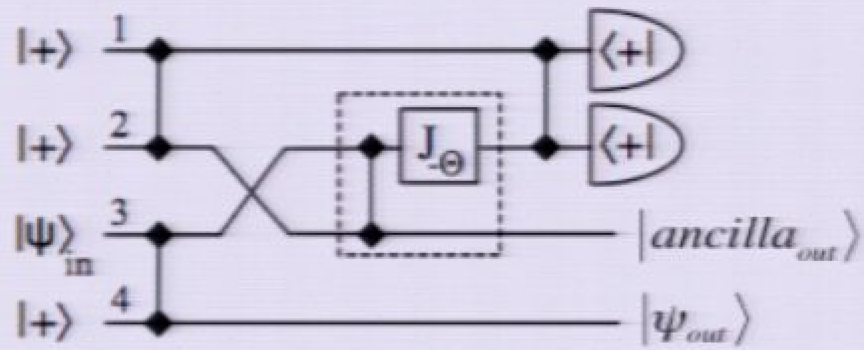
⟹

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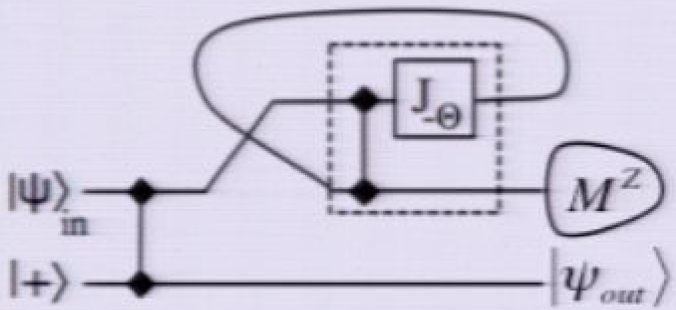


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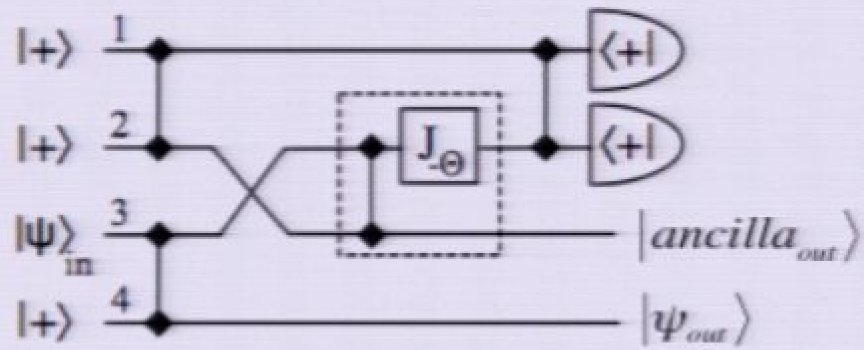



Its BSS simulation circuit

- Simple calculation shows that BSS circuit implements (probabilistically) the map

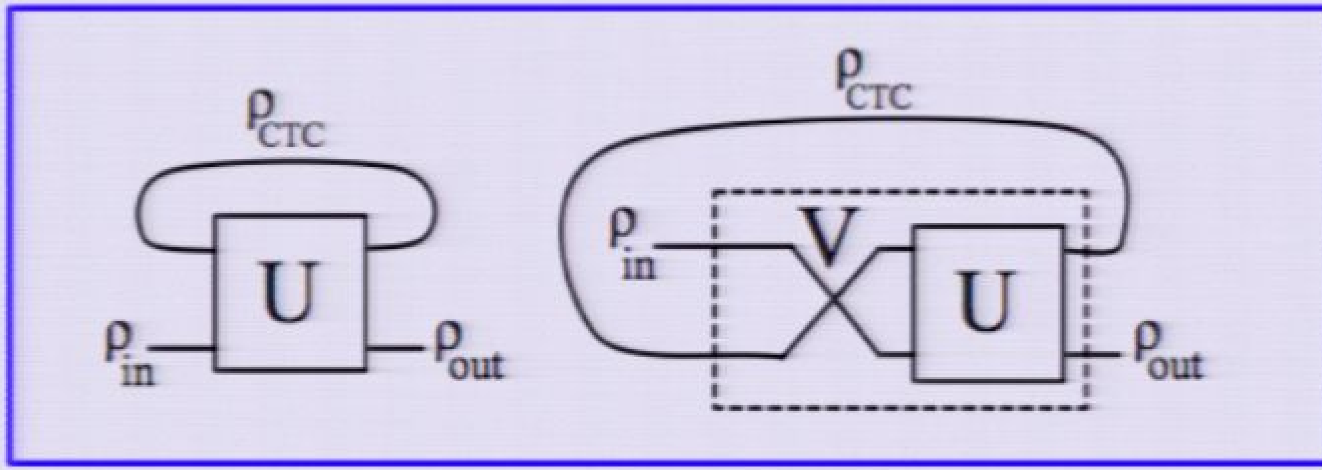
$$|\psi_{in}\rangle \rightarrow |\psi_{out}\rangle = J_{-\theta}|\psi_{in}\rangle$$

… recovering exactly MBQC's prediction!

BSS is the right model to explain CTCs in MBQC, and not Deutsch's…

# Deutsch/BSS comparison

- CTCs in both models can be trivially put in the same format:



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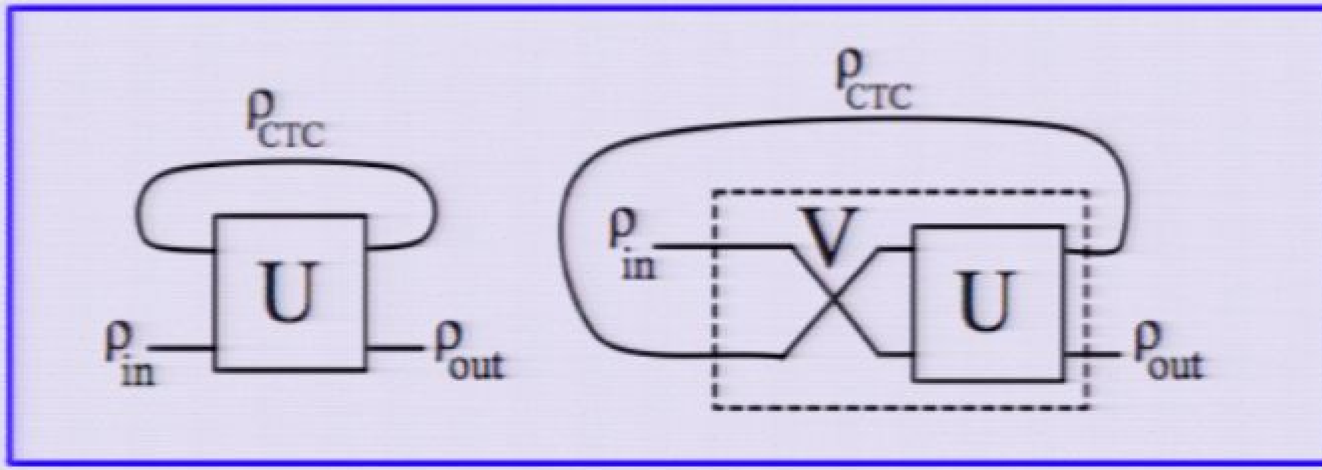
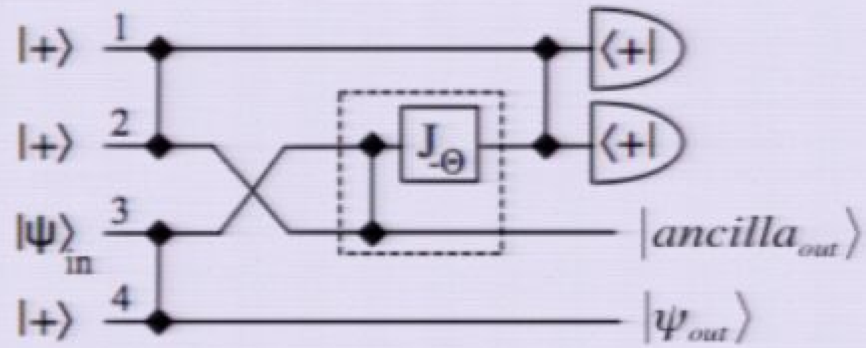



- Deutsch demands self-consistency: that is necessary
- but there appear artificial mixed solutions for CTC qubit, yielding multiple self-consistent solutions.
- Deutsch sends mixed state to past – the correlations of CTC qubit with other systems are lost!
- BSS preserves entanglement and correlations of CTC qubit, due to the teleportation step.
- Deutsch solutions with pure-state CTC qubit coincide with the BSS solution.

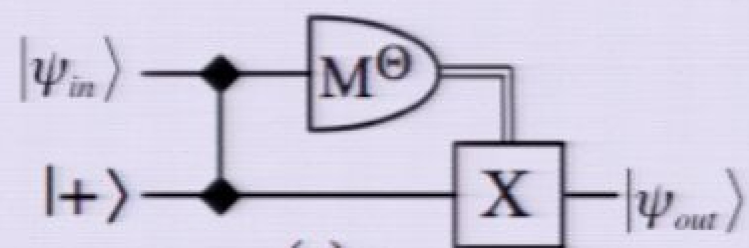
# MBQC as deterministic simulations of CTCs

- Stabilizer techniques enable us to simplify BSS circuits
  - Z deletion;
  - Local complementation.



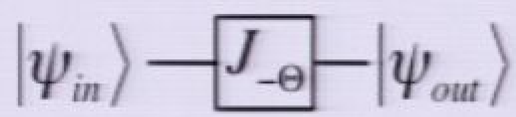

- Some BSS circuits reduce to MBQC patterns that deterministically simulate a unitary.
  - flow, gflow theorems.

- For example, the BSS circuit above is equivalent to:




(a)

- Or more simply:



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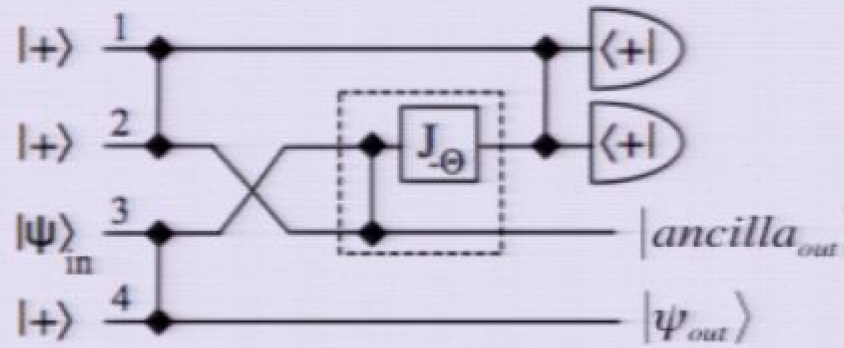


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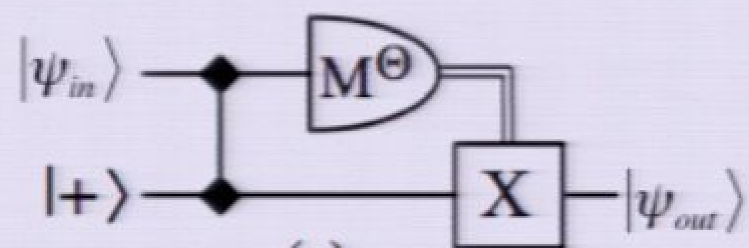


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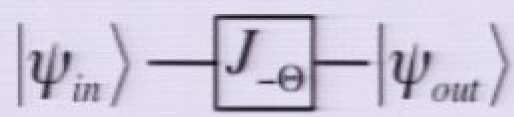



- Or more simply:

# Conclusions

- CTCs show up in MBQC; these can be analyzed using different CTC models.
- BSS model predictions agree with MBQC.
- Deutsch's model predictions are in conflict with MBQC – Deutsch's CTC qubit is sent to past stripped of its entanglement.
- We characterized a class of CTCs that admit deterministic simulation circuits using the BSS model.
- More work is needed to better understand implications of the BSS model:
  - MQ + Deutsch's CTCs = PSPACE (Aaronson/Watrous 2008)
  - BSS is associated with complexity class PostBQP=PP (Aaronson 2004)
  - See recent work (and experiment) by Lloyd *et al.*: arXiv:1005.2219v1

# PP versus PSPACE

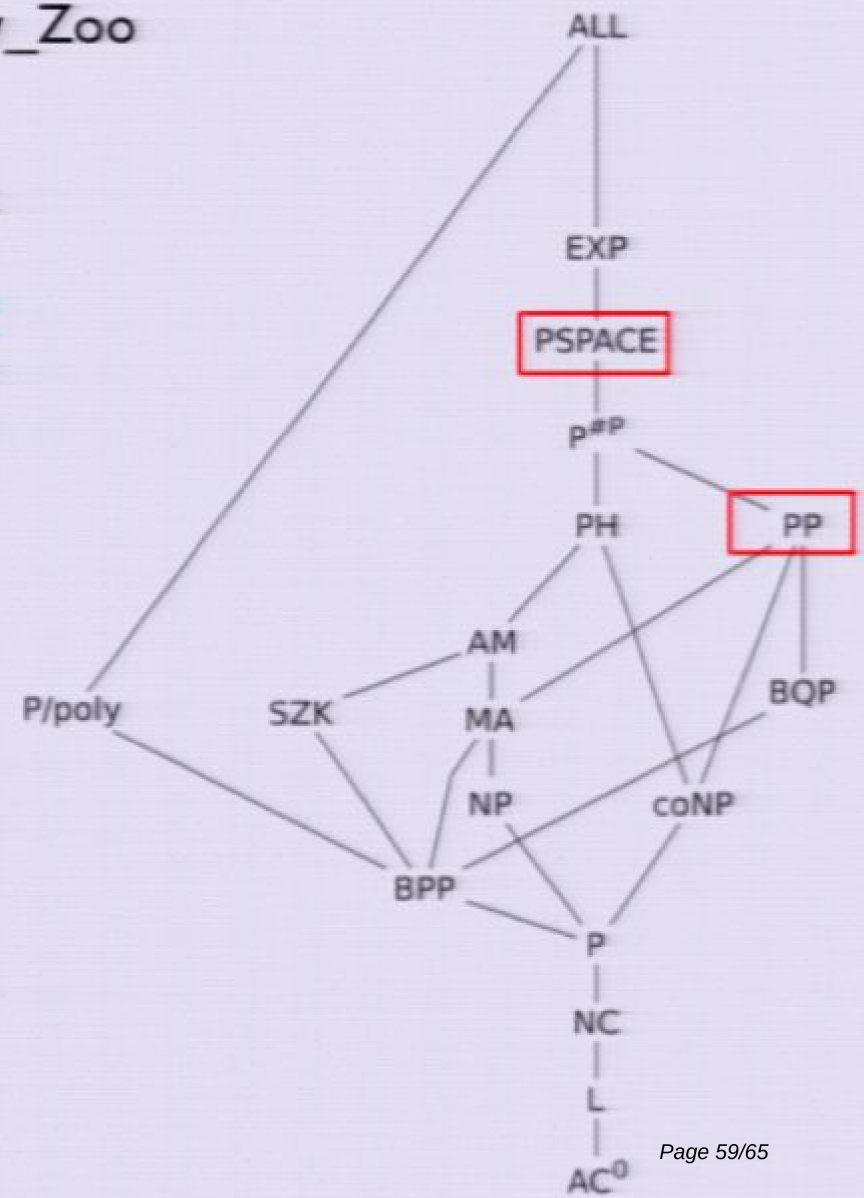
From http://qwiki.stanford.edu/wiki/Complexity_Zoo

## PP

Like BPP, PP is a class defined in an attempt to find out what randomness allows us to do algorithmically. Formally, PP is the class of problems solvable by an NP machine such that, given a "yes" instance, strictly more than 1/2 of the computation paths accept, while given a "no" instance, strictly less than 1/2 of the computation paths accept

## PSPACE

Whereas P is a class of problems that can be solved in a polynomially-bounded amount of time, PSPACE is the class of problems that can be solved by a deterministic Turing machine that uses only a polynomially-bounded amount of space, regardless of how long the computation takes.



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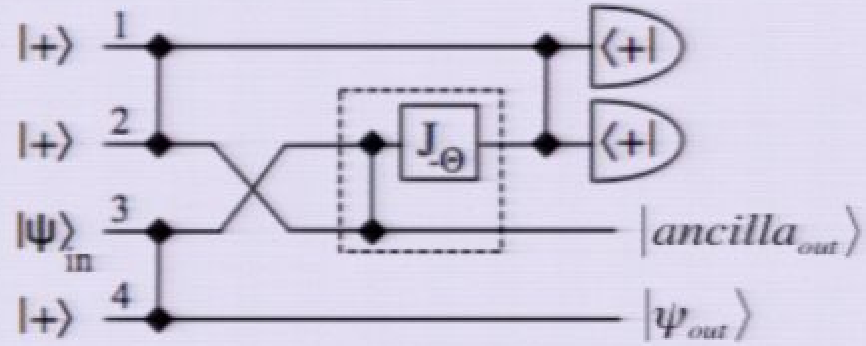


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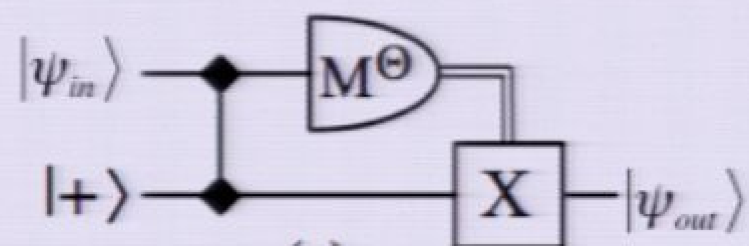
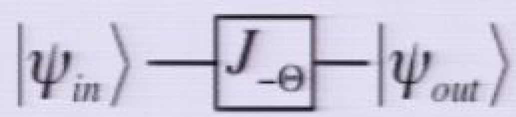
  - flow, gflow theorems.

- For example, the BSS circuit above is equivalent to:




(a)

- Or more simply:



# CTCs: model by Bennett/Schumacher/Svetlichny (BSS)

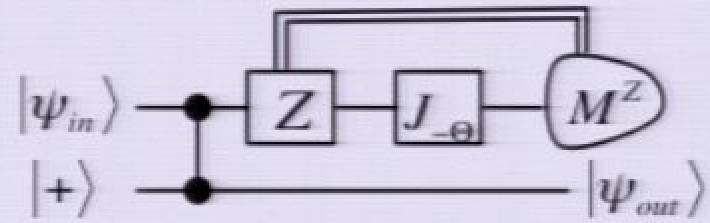
- Bennett and Schumacher, unpublished (2002) – see seminar http://bit.ly/crs8Lb
- Rediscovered independently by Svetlichny (2009) - arXiv:0902.4898v1

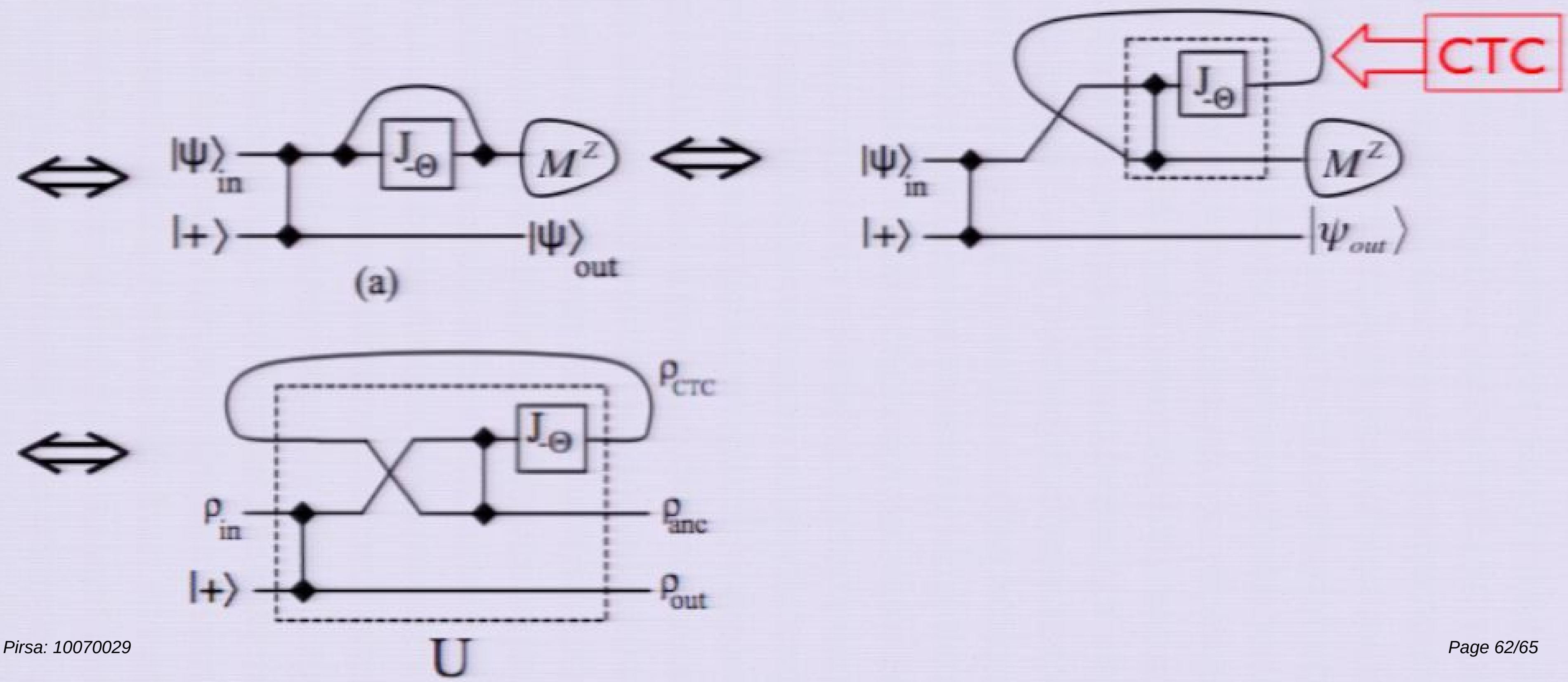
  - Related work on black holes by Horowitz/Maldacena (2004), Preskill/Gottesman (2004)

# *J* gate in MBQC = CTC

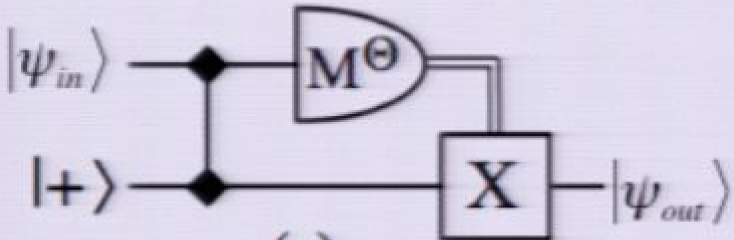
- Putting this CTC in Deutsch format:

$$M_1^{\theta} Z_1^{s1}|G\rangle \quad \Leftrightarrow$$



# $J$ gate in MBQC = CTC



(a)

$$\Leftrightarrow \quad X_2^{s1} M_1^{\theta} |G\rangle,$$
$$|G\rangle \equiv CTR_Z |\psi_{in}\rangle_1 \otimes |+\rangle_2$$

- Stabilizers of state $|G\rangle$ :

$$\begin{cases} (Z_1 \otimes X_2)^0 = Z_1^0 \otimes X_2^0 = 1 \text{ (identity)} \\ (Z_1 \otimes X_2)^1 = Z_1^1 \otimes X_2^1 \end{cases}$$

That is, $Z_1^{s1} \otimes X_2^{s1}$ is stabilizer independently of the outcome $s_1$ of the measurement on qubit 1:

$$Z_1^{s1} \otimes X_2^{s1} |G\rangle = |G\rangle$$

- We can then perform stabilizer manipulation:

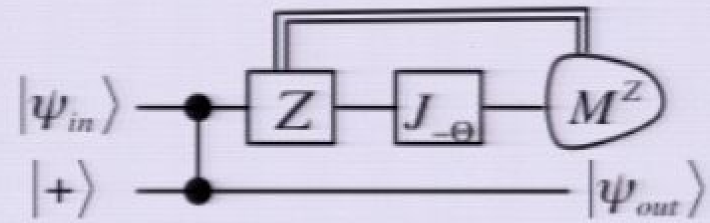
$$X_2^{s1} M_1^{\theta} |G\rangle = X_2^{s1} M_1^{\theta} (Z_1^{s1} X_2^{s1} |G\rangle)$$
$$= X_2^{s1+s1} M_1^{\theta} Z_1^{s1} |G\rangle = M_1^{\theta} Z_1^{s1} |G\rangle$$

**Time-travel** situation: we need to apply $Z$ depending on outcome of measurement not yet made.

# *J* gate in MBQC = CTC

- Putting this CTC in Deutsch format:

$$M_1^{\theta} Z_1^{s1} |G\rangle \quad \Leftrightarrow$$



# $J$ gate in MBQC = CTC

- Putting this CTC in Deutsch format:

$$M_1^{\theta} Z_1^{s1} |G\rangle$$