

Title: Black hole microstate counting and its macroscopic counterpart III

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Abstract: In this talk I shall describe a general formalism based on $\text{AdS}_2/\text{CFT}_1$ correspondence that allows us to systematically calculate the entropy, index and other physical observables of an extremal black hole taking into account higher derivative and quantum corrections to the action. I shall also describe precise microscopic computation of the same quantities for a class of supersymmetric extremal black holes and compare this with the prediction of $\text{AdS}_2/\text{CFT}_1$ correspondence.

For black holes in $D=4$ which break $4n$ supersymmetries the protected index is

$$B_{2n} = \frac{1}{(2n)!} \text{Tr}(-1)^F (2h)^{2n} = \frac{1}{(2n)!} \text{Tr}(-1)^{2h} (2h)^{2n}$$

For 1/4 BPS black holes in $\mathcal{N} = 4$ supersymmetric string theories, $4n=12$ and the relevant index is B_6 .

How can we use d_{hor} to calculate this index on the black hole side?

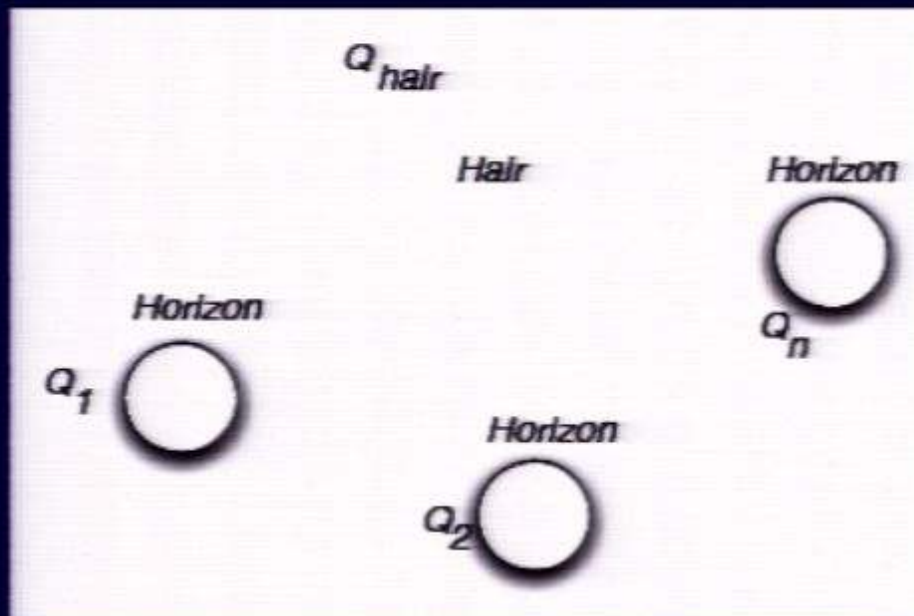
In general the macroscopic degeneracy / index can have two kinds of contributions:

1. From the horizon.

2. From degrees of freedom living outside the horizon (hair).

N. Banerjee, Mandal, A.S.; Jatkar, A.S., Srivastava

Example: The fermion zero modes associated with the broken supersymmetry generators are always part of the hair modes.



Q_i denotes both electric and magnetic charges of the i -th black hole.

We shall denote the degeneracy associated with the hair degrees of freedom by d_{hair} .

d_{hair} can be calculated by explicitly identifying and quantizing the hair modes.

The total degeneracy:

$$\sum_n \sum_{\substack{\{\vec{Q}_i\}, \vec{Q}_{\text{hair}} \\ \sum_{i=1}^n \vec{Q}_i + \vec{Q}_{\text{hair}} = \vec{Q}}} \left\{ \prod_{i=1}^n d_{\text{hor}}(\vec{Q}_i) \right\} d_{\text{hair}}(\vec{Q}_{\text{hair}}; \{\vec{Q}_i\})$$

Twisted index

Suppose we have a \mathbb{Z}_N symmetry generated by g that commutes with all the supersymmetries.

We can define a twisted index:

$$B_6^g = \frac{1}{6!} \text{Tr} \left[(-1)^{2h} (2h)^6 g \right]$$

Following the same logic as in the case of B_6 one can show that on the black hole side

$$B_6^g = -\text{Tr}_{\text{hor}}(g)$$

Microstate counting

We shall now describe the results of microscopic analysis in a class of $\mathcal{N} = 4$ supersymmetric string theories.

The simplest example: Heterotic string theory on T^6 .

This theory has 28 gauge fields.

Thus a generic charged state is characterized by 28 dimensional electric charge vector Q and magnetic charge vector P .

The theory has T-duality symmetry $O(6, 22; \mathbb{Z})$ under which Q and P transform as vectors.

This allows us to define T-duality invariant bilinears in the charges

$$Q^2, P^2, Q \cdot P$$

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$$Q^2 - P^2, \quad Q \cdot P$$

More general class of $\mathcal{N} = 4$ supersymmetric string theories can be constructed by taking orbifolds of heterotic string theory on T^6 .

– **CHL models**

Chaudhuri, Hockney, Lykken

These theories have $(r + 6)$ $U(1)$ gauge fields for different values of r .

Thus Q and P are $(r+6)$ dimensional vectors.

In each of these theories, the index $B_6(Q, P)$ has been computed for a wide class of charge vectors (Q, P) .

The computation is done in a dual frame

type IIB on $K3 \times T^2$

and its orbifolds.

In each of these theories we compute B_6 for a rotating D1-D5-p system in KK monopole background.

In the weak coupling limit the dynamics is given by that of a system of decoupled harmonic oscillators, and exact computation of B_6 is possible.

The result is then expressed in terms of the T-duality invariant bilinears $Q^2, P^2, Q \cdot P$ in the original heterotic frame.

In each case the result is given as Fourier expansion coefficients of some known functions $Z(\rho, \sigma, \nu)$:

$$\mathbf{B}_6 = (-1)^{\mathbf{Q}\cdot\mathbf{P}} \int \mathbf{d}\rho \int \mathbf{d}\sigma \int \mathbf{d}\nu e^{-\pi i(\rho\mathbf{Q}^2 + \sigma\mathbf{P}^2 + 2\nu\mathbf{Q}\cdot\mathbf{P})} Z(\rho, \sigma, \nu)$$

$Z(\rho, \sigma, \nu)$: explicitly known in each of the examples, and transform as modular forms of certain weights under subgroups of $\text{Sp}(2, \mathbb{Z})$.

Dijkgraaf, Verlinde, Verlinde; Shih, Strominger, Yin;

Cardoso, de Wit, Kappeli, Mohaupt

David, Jatkar, A.S.; Dabholkar, Gaiotto, Nampuri;

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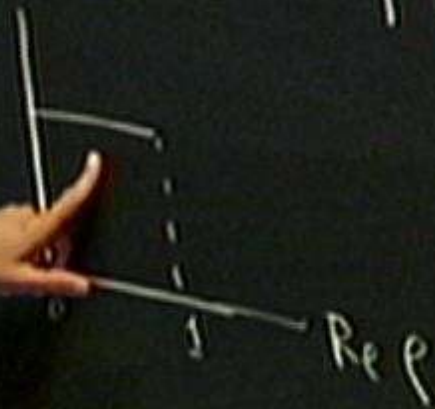
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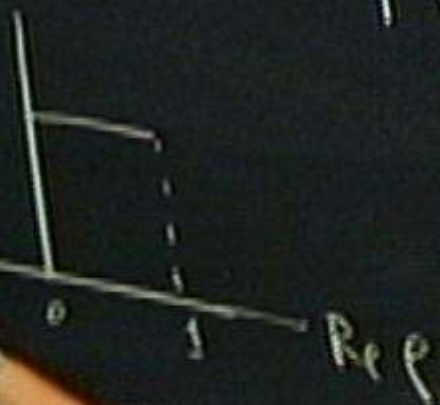
$$T \sim \text{hom}(g)$$

$$\text{Im} p \sim \exp\left(\frac{1}{N} S_{\text{wald}} + \dots\right)$$



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subspaces of the moduli
Space the contour hits
a pole

$\int_{\gamma} \text{tr}(\rho(g))$

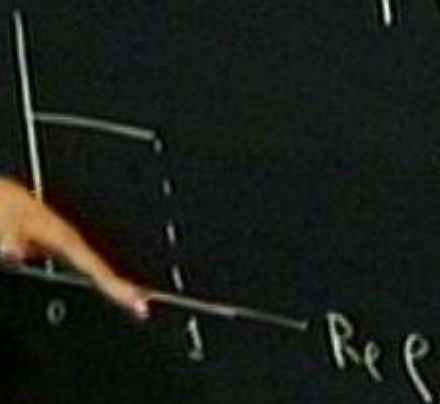
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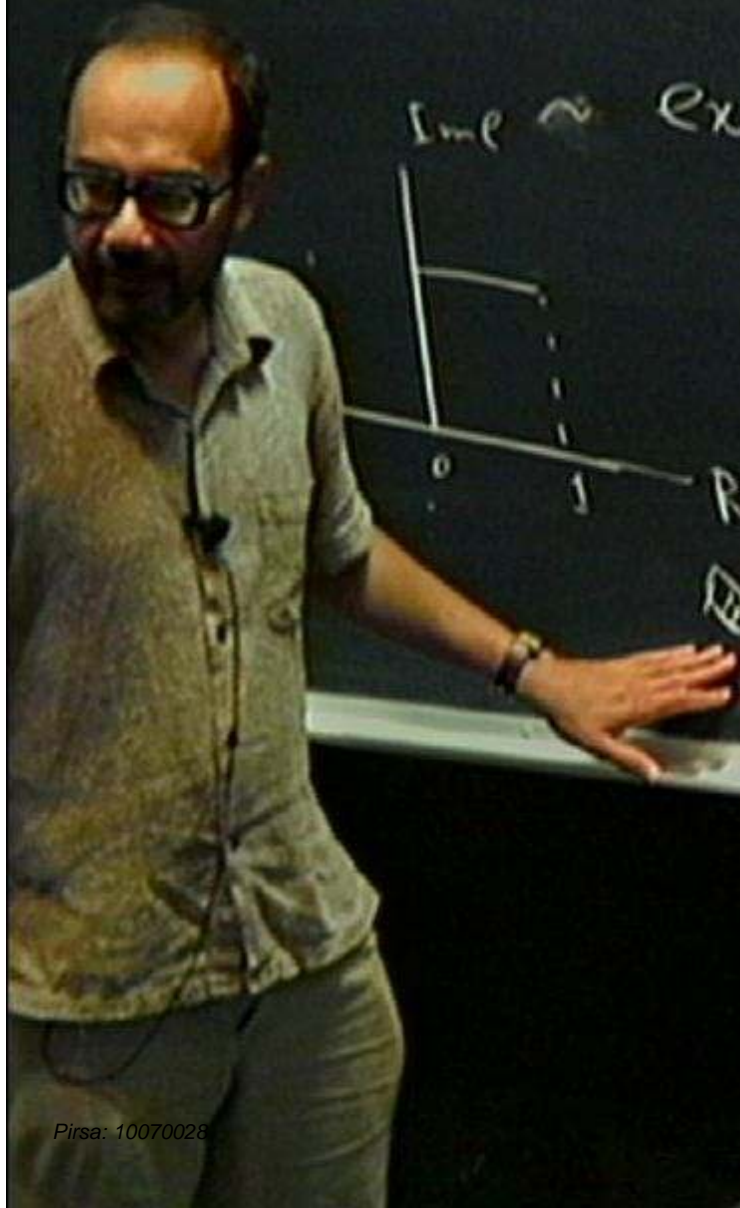


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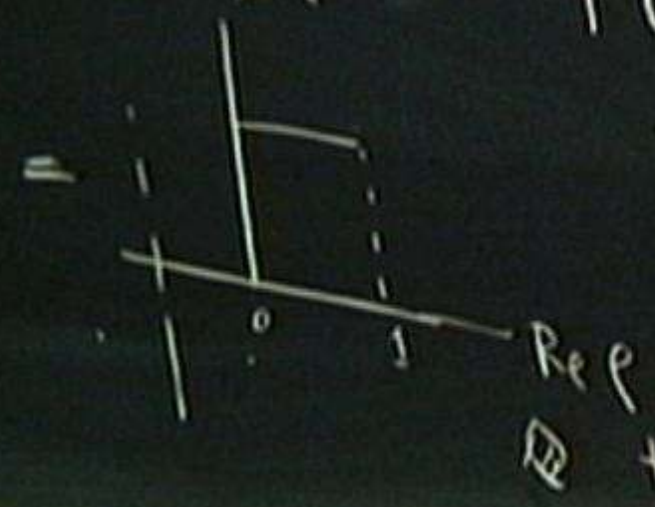
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$\mathbb{R} + \dots$



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General structure of $Z(\rho, \sigma, \mathbf{v})$:

$$Z(\rho, \sigma, \mathbf{v}) = e^{-2\pi i(a\rho + b\sigma + c\mathbf{v})} \prod_{m,n,p} (1 - e^{2\pi i(m\rho + n\sigma + p\mathbf{v})})^{-c(m,n,p)}$$

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It is also possible to find the systematic expansion of B_6 for large charges.

In each case we find $B_6 < 0$, in agreement with the macroscopic prediction.

$$\ln |B_6| = \pi \sqrt{Q^2 P^2 - (Q \cdot P)^2} + f \left(\frac{Q \cdot P}{P^2}, \frac{\sqrt{Q^2 P^2 - (Q \cdot P)^2}}{P^2} \right) + \mathcal{O}(\text{charge}^{-2})$$

f : a known function.

For example, for heterotic string theory compactified on a six dimensional torus,

$$f(\tau_1, \tau_2) = 12 \ln \tau_2 + 24 \ln \eta(\tau_1 + i\tau_2) + 24 \ln \eta(-\tau_1 + i\tau_2) \quad \text{Page 29/44}$$

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Can we explain these results from the black hole side?

For black holes carrying the same charges as the microscopic system we find:

$$S_{\text{wald}} = \pi \sqrt{Q^2 P^2 - (Q \cdot P)^2}$$

– reproduces the leading result for $\ln|B_6|$.

By integrating out the massive modes the 1PI action receives a local one loop correction term.

The correction to the Wald entropy due to these terms:

$$f\left(\frac{Q \cdot P}{P^2}, \frac{\sqrt{Q^2 P^2 - (Q \cdot P)^2}}{P^2}\right)$$

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What about logarithmic corrections?

The microscopic result suggests that they must be absent in all $\mathcal{N} = 4$ supersymmetric string theories.

Can we verify this by evaluating the one loop contribution to Z_{AdS_2} due to massless particles?

So far the computation has been done for the matter multiplet fields.

Result: The logarithmic corrections cancel.

– consistent with the microscopic results.

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\mathbb{Z}_N twisted index

On special subspaces of the parameter space of the $\mathcal{N} = 4$ supersymmetric string theories in (3+1) dimensions, the theory develops \mathbb{Z}_N discrete symmetry generated by an element g which commutes with supersymmetry.

Each theory has a certain set of allowed values of N .

Example: For heterotic on T^6 we can have $N=2,3,4,5,6,7,8$

On these special subspaces we can define the twisted index:

$$B_6^g = \frac{1}{6!} \text{Tr} \left[(-1)^{2h} (2h)^6 g \right]$$

In each case we can calculate the twisted index B_6^g , and find that the result is again given by Fourier integrals of modular forms of subgroups of $Sp(2, \mathbb{Z})$.

$$B_6^g = (-1)^{Q \cdot P} \int d\rho \int d\sigma \int d\nu e^{-\pi i(\rho Q^2 + \sigma P^2 + 2\nu Q \cdot P)} Z_g(\rho, \sigma, \nu)$$

Z_g are known functions.

Furthermore for large charges we find

$$B_6^g = \exp[\pi \sqrt{Q^2 P^2 - (Q \cdot P)^2} / N + \dots]$$

in agreement with the prediction from the black hole side.

All these results provide us with the ‘experimental verification’ of the theory of extremal black holes based on Wald’s formula and $\text{AdS}_2/\text{CFT}_1$ correspondence.

Eventually we hope to reproduce the complete asymptotic expansion of the microscopic result for $\ln|\mathbf{B}_6|$ from the string theory path integral over AdS_2 .

Basic tool: localization of the path integral to a finite dimensional subspace using supersymmetry.

In this process we hope to learn not only about black holes but also about string theory, *e.g.* the rules for carrying out path integral over string fields.

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