Title: Black hole microstate counting and its macroscopic counterpart II

Date: Jul 08, 2010 11:00 AM

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Abstract: In this talk I shall describe a general formalism based on AdS_2/CFT_1 correspondence that allows us to systematically calculate the entropy, index and other physical observables of an extremal black hole taking into account higher derivative and quantum corrections to the action. I shall also describe precise microscopic computation of the same quantities for a class of supersymmetric extremal black holes and compare this with the prediction of AdS_2/CFT_1 correspondence.

Review

AdS_2/CFT_1 correspondence $\Rightarrow$ a prescription for computing the degeneracy d_{hor} associated with the black hole horizon

– given by the finite part of the partition function of string theory on the near horizon $AdS_2 \times K$ geometry.

$$Z_{AdS_2} = d_{hor}\, e^{CL + \mathcal{O}(L^{-1})}$$

L: length of the boundary of regularized AdS_2.

C: some constant

Note: Near the boundary of AdS_2, the θ independent solution to the Maxwell's equation has the form:

$$A_r = 0, \quad A_\theta = C_1 + C_2 r$$

C_1 (chemical potential) represents normalizable mode

C_2 (electric charge) represents non-normalizable mode

$\rightarrow$ the path integral must be carried our keeping C_2 (charge) fixed and integrating over C_1 (chemical potential).

Thus the AdS_2 path integral computes the entropy in the microcanonical ensemble.

This is also the reason why we need to insert the boundary term $\exp[-iq_k \oint_{\partial(AdS_2)} d\theta\, A_\theta^{(k)}]$ in the path integral.

Consistency check:

In the classical limit

$$\mathbf{Z}_{\mathbf{AdS_2}} = \exp[-\mathbf{Action} - \mathbf{i q_k} \oint_{\partial(\mathbf{AdS_2})} \mathbf{d}\theta\, \mathbf{A}_\theta^{(\mathbf{k})}]\Big|_{\mathbf{classical}}$$

$$= \exp\left[-2\pi\left(\mathbf{q_i e_i} - \sqrt{\det \mathbf{g}_{\mathbf{AdS_2}}}\, \mathcal{L}_{\mathbf{AdS_2}}\right)(\cosh\eta_0 - 1)\right]$$

$$= \exp\left[2\pi\left(\mathbf{q_i e_i} - \sqrt{\det \mathbf{g}_{\mathbf{AdS_2}}}\, \mathcal{L}_{\mathbf{AdS_2}}\right) + \mathbf{CL}\right]$$

$$= \exp\left[\mathbf{S}_{\mathbf{wald}} + \mathbf{CL}\right]$$

Note: $\mathbf{L} = \mathbf{a}\sinh\eta_0$

$\Rightarrow \cosh\eta_0 = \mathbf{L}/\mathbf{a} + \mathcal{O}(\mathbf{L}^{-1})$

Thus $\mathbf{d}_{\mathbf{hor}} = \exp[\mathbf{S}_{\mathbf{wald}}]$ in the classical limit.

2. Define Z_{AdS_2}: Path integral over string fields in the euclidean near horizon background geometry weighted by

$$\exp[-\mathbf{Action} - \mathbf{i q_k} \oint_{\partial(\mathbf{AdS_2})} \mathbf{d\theta\, A_\theta^{(k)}}]$$

$\{q_k\}$: electric charges carried by the black hole under the U(1) gauge field $A^{(k)}$.

3. By AdS_2/CFT_1 correspondence:

$$Z_{AdS_2} = Z_{CFT_1}$$

$$Z_{CFT_1} = \mathrm{Tr}(e^{-LH}) = d_0\, e^{-L\, E_0}$$

H: Hamiltonian of dual CFT_1 at the boundary of AdS_2.

(d_0, E_0): (degeneracy, energy) of the states of CFT

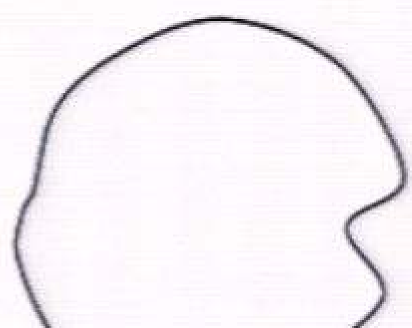
Steps for computing d_{hor}

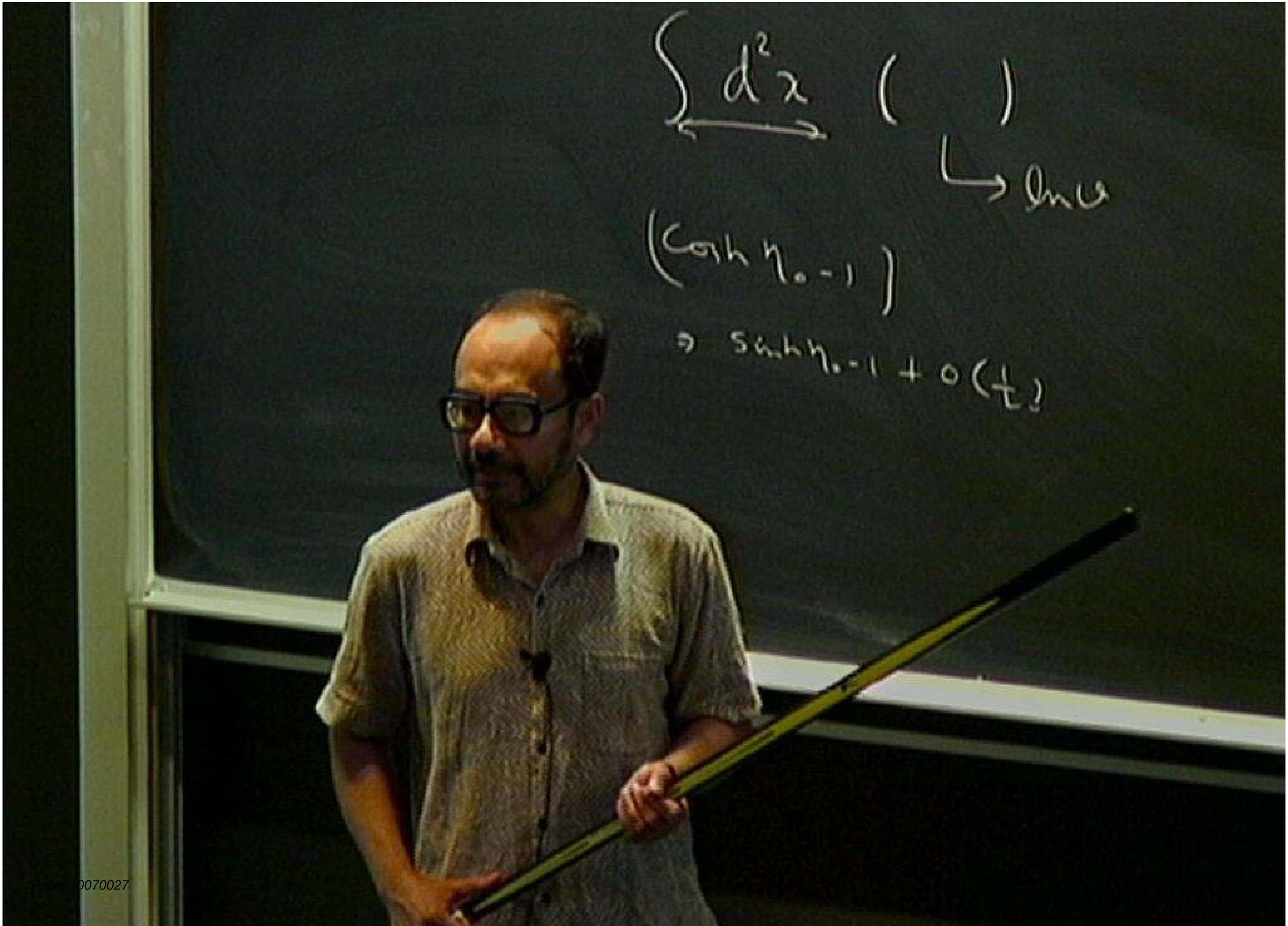
1. Consider the euclidean AdS_2 metric:

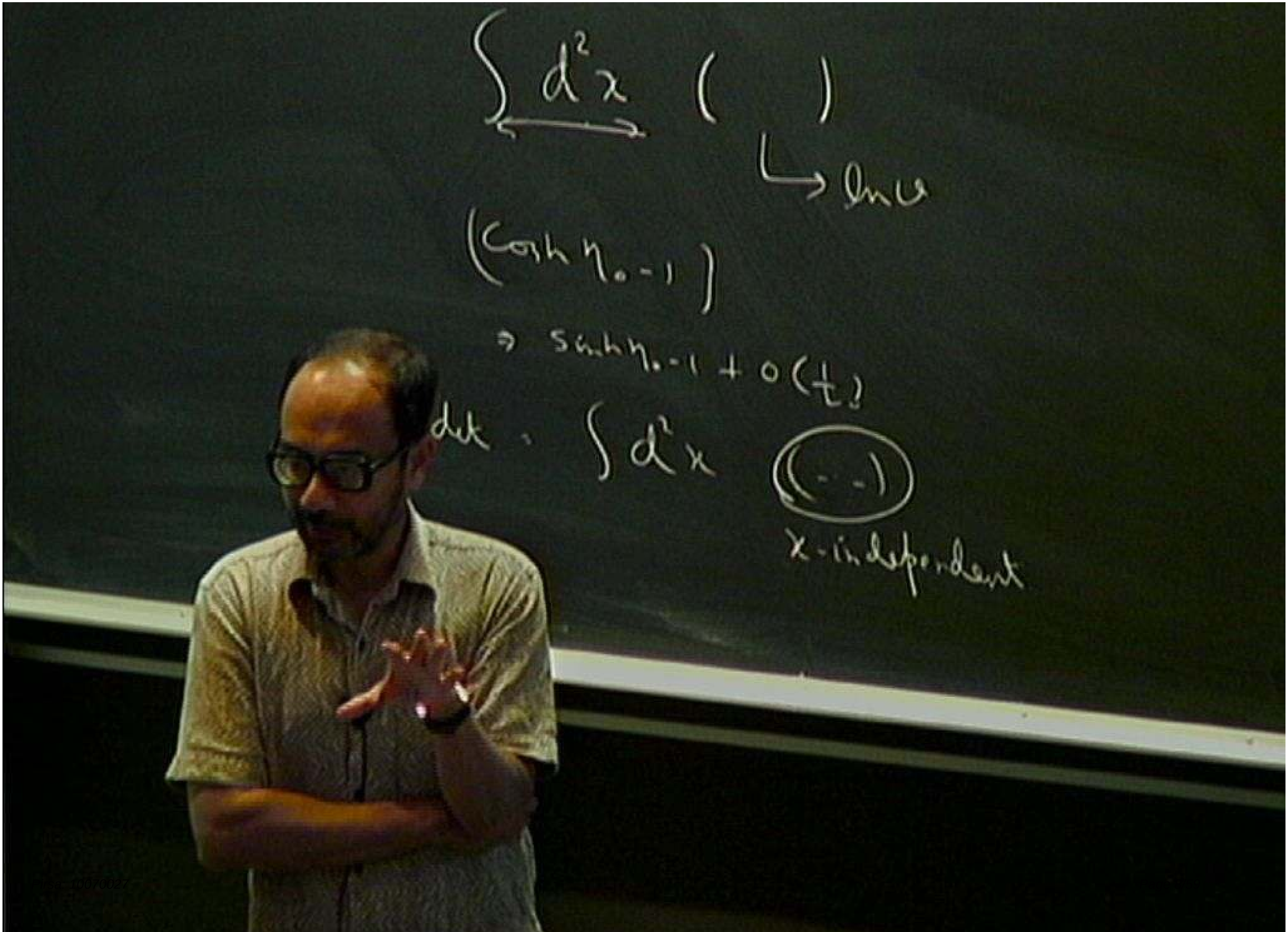
$$\begin{aligned} ds^2 &= v\left((r^2-1)d\theta^2 + \frac{dr^2}{r^2-1}\right), \quad 1 \le r < \infty, \quad \theta \equiv \theta + 2\pi \\ &= v(\sinh^2\eta\, d\theta^2 + d\eta^2), \quad r \equiv \cosh\eta, \quad 0 \le \eta < \infty \end{aligned}$$

Regularize the infinite volume of AdS_2 by putting a cut-off $r \le r_0 f(\theta)$ for some smooth periodic function $f(\theta)$.

This makes the AdS_2 boundary have a finite length L.




∫d²x (   )
(cosh η₀ -1)
⇒ sinh η₀ -1 + O(t)


x-independent

$$\int d^2x \; (\quad)$$

$\hookrightarrow \ln a$

$$(\cosh \eta_0 - 1)$$

$$\Rightarrow \sinh \eta_0 - 1 + O\left(\tfrac{1}{t}\right)$$

$$\ln \det = \int d^2x \; (\ldots)$$

x-independent

Weighted degeneracy

In string theory one often finds vacua with discrete symmetries.

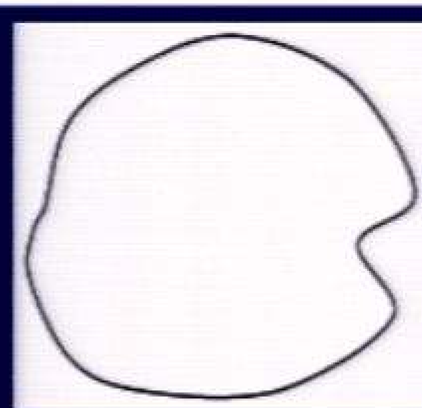
Example: As we move around in the moduli space of heterotic string theory on T^6, on special subspaces the theory develops discrete $\mathbb{Z}_N$ symmetries ($N = 2, 3, 4, 5, 6, 7, 8$) which commute with supersymmetry.

Suppose we want to compute the weighted degeneracy

$$Tr_{hor}(g)$$

g: some $\mathbb{Z}_N$ symmetry generator.

What macroscopic computation should we carry out?



By following the logic of AdS_2/CFT_1 correspondence we find that we need to again compute the partition function on AdS_2, but this time with a g twisted boundary condition on the fields under $\theta \to \theta + 2\pi$.

Other than this the asymptotic boundary condition must be identical to that of the original near horizon geometry since the charges have not changed

The 'finite part' of this partition function gives us $\mathrm{Tr}_{hor}(g)$.

Recall AdS_2 metric:

$$ds^2 = v\left[(r^2-1)d\theta^2 + \frac{dr^2}{r^2-1}\right] = v\left[\sinh^2\eta d\theta^2 + d\eta^2\right]$$

The circle at infinity, parametrized by θ, is contractible at the origin $r = 1$.

Thus a g twist under $\theta \to \theta + 2\pi$ is not admissible.

$\to$ the $AdS_2 \times S^2$ geometry is not a valid saddle point of the path integral.

Question: Are there other saddle points which could contribute to the path integral?

Constraints:

1. It must have the same asymptotic geometry as the $AdS_2 \times S^2$ geometry.

2. It must have a g twist under $\theta \to \theta + 2\pi$.

3. It must preserve sufficient amount of supersymmetries so that integration over the fermion zero modes do not make the integral vanish.

Beasley, Gaiotto, Guica, Huang, Strominger, Yin;
N. Banerjee, S. Banerjee, Gupta, Mandal, A.S.

There are indeed such saddle points in the path integral, constructed as follows.

1. Take the original near horizon geometry of the black hole.

2. Take a $\mathbb{Z}_N$ orbifold of this background with $\mathbb{Z}_N$ generated by simultaneous action of

a) $2\pi/N$ rotation in AdS_2

a) $2\pi/N$ rotation in S^2 (needed for preserving SUSY)

c) g.

N. Banerjee, Jatkar, A.S.; A.S.; Pioline, Murthy

To see that this has the same asymptotic geometry as the attractor geometry we make a rescaling:

$$\theta \to \theta/N, \qquad r \to N r$$

The metric takes the form:

$$v\left((r^2 - N^{-2})d\theta^2 + \frac{dr^2}{r^2 - N^{-2}}\right)$$

Orbifold action: $\theta \to \theta + 2\pi$, $\phi \to \phi + 2\pi/N$, g

The g transformation provides us with the correct boundary condition.

The ϕ shift can be regarded as a Wilson line, and hence is an allowed fluctuation in AdS_2.

The classical action associated with this saddle point, after removing the divergent part proportional to the length of the boundary, is S_{wald}/N.

Thus the contribution to the weighted degeneracy from this saddle point is

$$\mathrm{Tr}_{hor}(g) = Z_g^{finite} = \exp\left[S_{wald}/N\right]$$

(agrees with the microscopic results)

Note: $\exp\left[S_{wald}/N\right] << \exp\left[S_{wald}\right]$

Thus the $\mathbb{Z}_N$ quantum numbers must be delicately distributed among the microstates of the black hole so that a charge of order unity, averaged over $\exp\left[S_{wald}\right]$ number of states, gives a contribution of order $\exp\left[S_{wald}/N\right]$.

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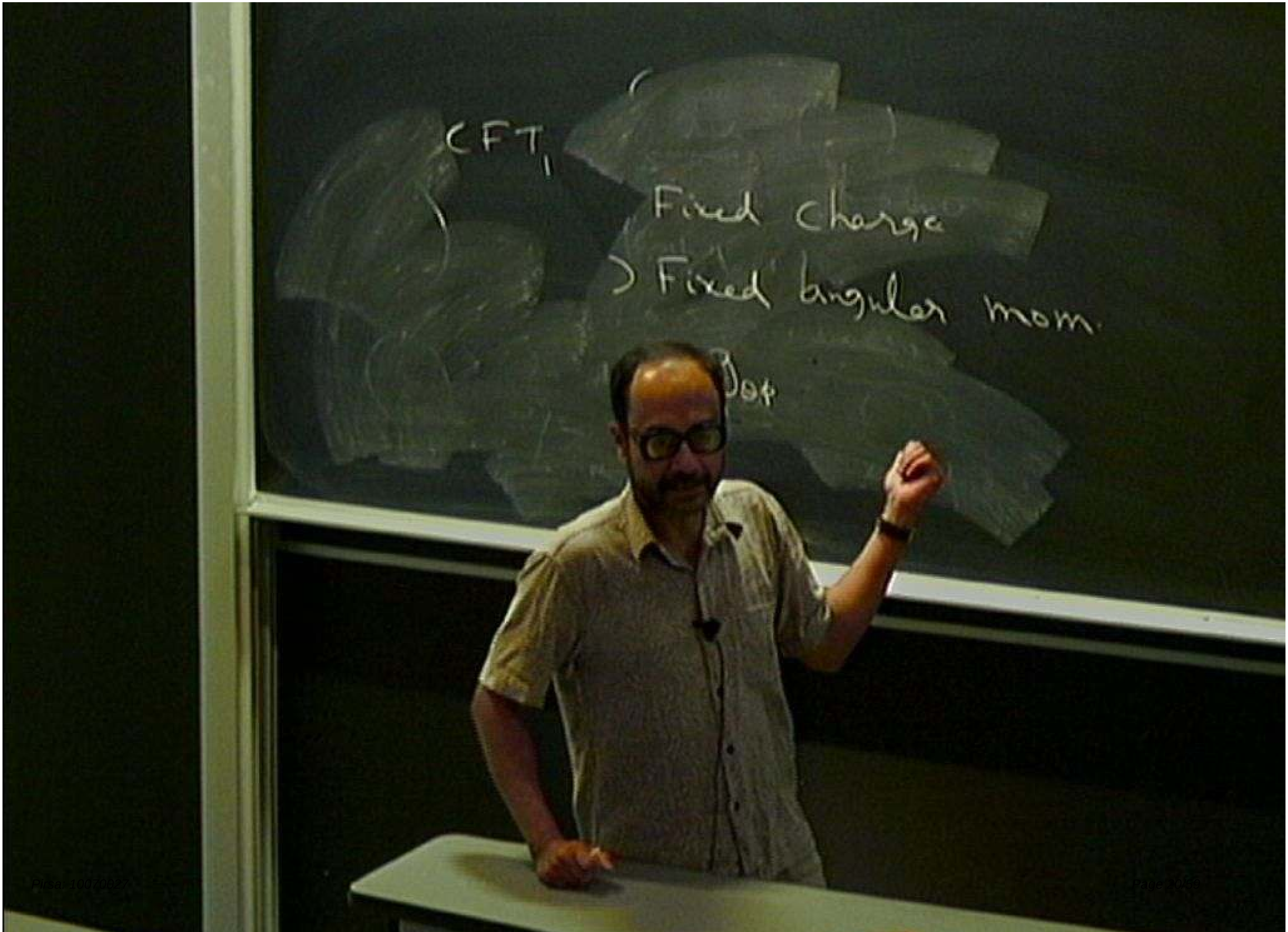
CFT
Fixed charge
Fixed angular mom.

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N. Banerjee, Jatkar, A.S.; A.S.; Pioline, Murthy

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Perturbative corrections to d_{hor}

The effect of integrating out the massive mode contribution to Z_{AdS_2} can be regarded as a modification of the effective Lagrangian density.

– can be accommodated using Wald's formula.

What about the contribution from the massless modes?

At one loop this requires finding the eigenvalues of the kinetic operator in the near horizon geometry and then calculating the determinant.

Example:

Consider a massless scalar field with standard kinetic term in the near horizon $AdS_2 \times S^2$ background for a spherically symmetric extremal black hole in D=4.

All the eigenvalues and eigenfunctions of $\Box$ on $AdS_2 \times S^2$ can be found explicitly.

$\Rightarrow$ can be used to compute det $\Box$ and hence one loop contribution to Z_{AdS_2}.

Result: A contribution to $\ln d_{hor}$ of the form:

$$-\frac{1}{180}\ln A$$

For black holes in supergravity / superstring theory the kinetic operator for fluctuations around the near horizon geometry mixes scalars, vectors and tensors.

One needs to diagonalize the kinetic operator, find the determinant and then compute its contribution to Z_{AdS_2} and hence d_{hor}.

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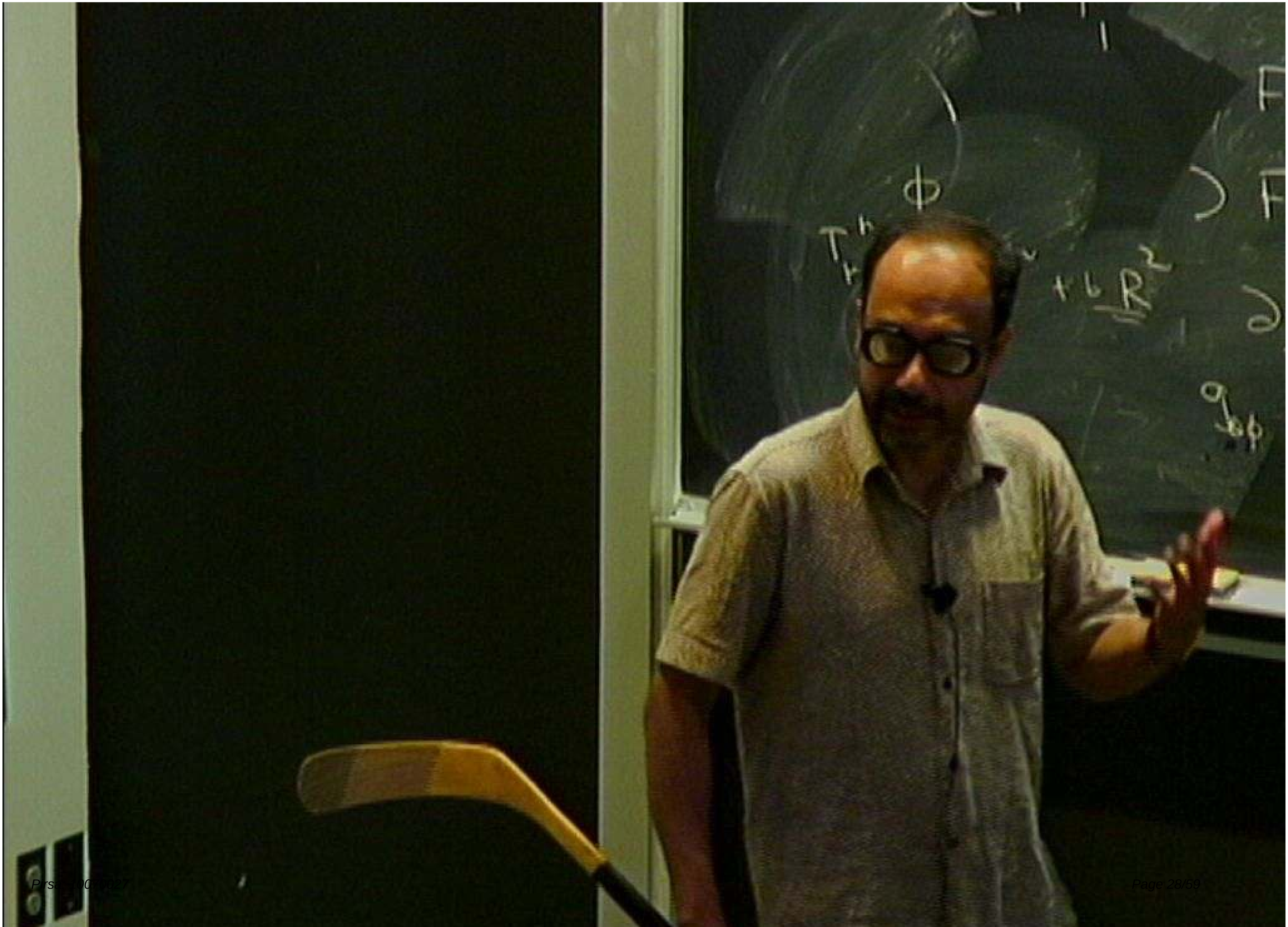

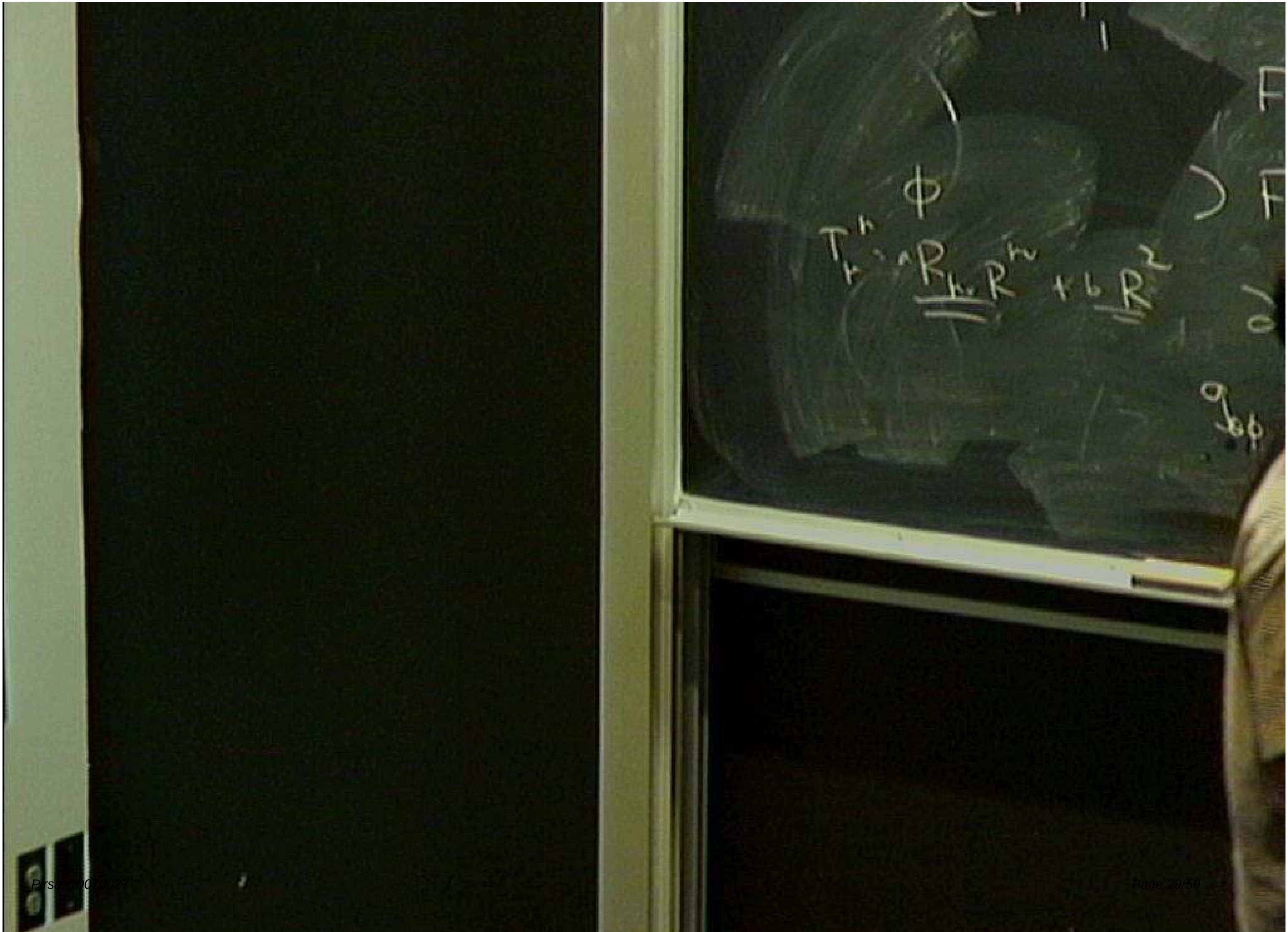

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Degeneracy to index

The microscopic analysis is always done in a region of the moduli space where gravity can be ignored and hence the states do not form a black hole.

In order to be able to compare it with the results from the black hole side we must focus on quantities which do not change as we change the coupling from small to large value.

– needs appropriate supersymmetric index.

The appropriate index in D=4 is the helicity trace index.

Bachas, Kiritsis

Suppose we have a BPS state that breaks 4n supersymmetries.

$\rightarrow$ there will be 4n fermion zero modes (goldstino) on the world-line of the state.

Quantization of these zero modes will produce Bose-Fermi degenerate states.

Thus $\mathrm{Tr}(-1)^F$ vanishes.

Define: $B_{2n} = \frac{1}{(2n)!}\mathrm{Tr}(-1)^F(2h)^{2n} = \frac{1}{(2n)!}\mathrm{Tr}(-1)^{2h}(2h)^{2n}$

h: third component of angular momentum in rest frame.

For every pair of fermion zero modes $\mathrm{Tr}(-1)^F(2h)$ gives a non-vanishing result, leading to a non-zero B_{2n}.

Most of our studies will be on 1/4 BPS black holes in $\mathcal{N} = 4$ supersymmetric string theories in D=4.

Preserves 4 out of 16 supersymmetries

$\Leftarrow$ breaks 12 supersymmetries.

Thus the relevant helicity trace index is B_6.

Since on the microscopic side we compute an index, we must ensure that on the black hole side also we compute an index.

Otherwise we cannot compare the two results.

How can we use d_{hor} to compute the index B_6 on the black hole side?

In general the macroscopic degeneracy / index can have two kinds of contributions:

1. From the horizon.

2. From degrees of freedom living outside the horizon (hair).

N. Banerjee, Mandal, A.S.; Jatkar, A.S., Srivastava

Example: The fermion zero modes associated with the broken supersymmetry generators are always part of the hair modes.

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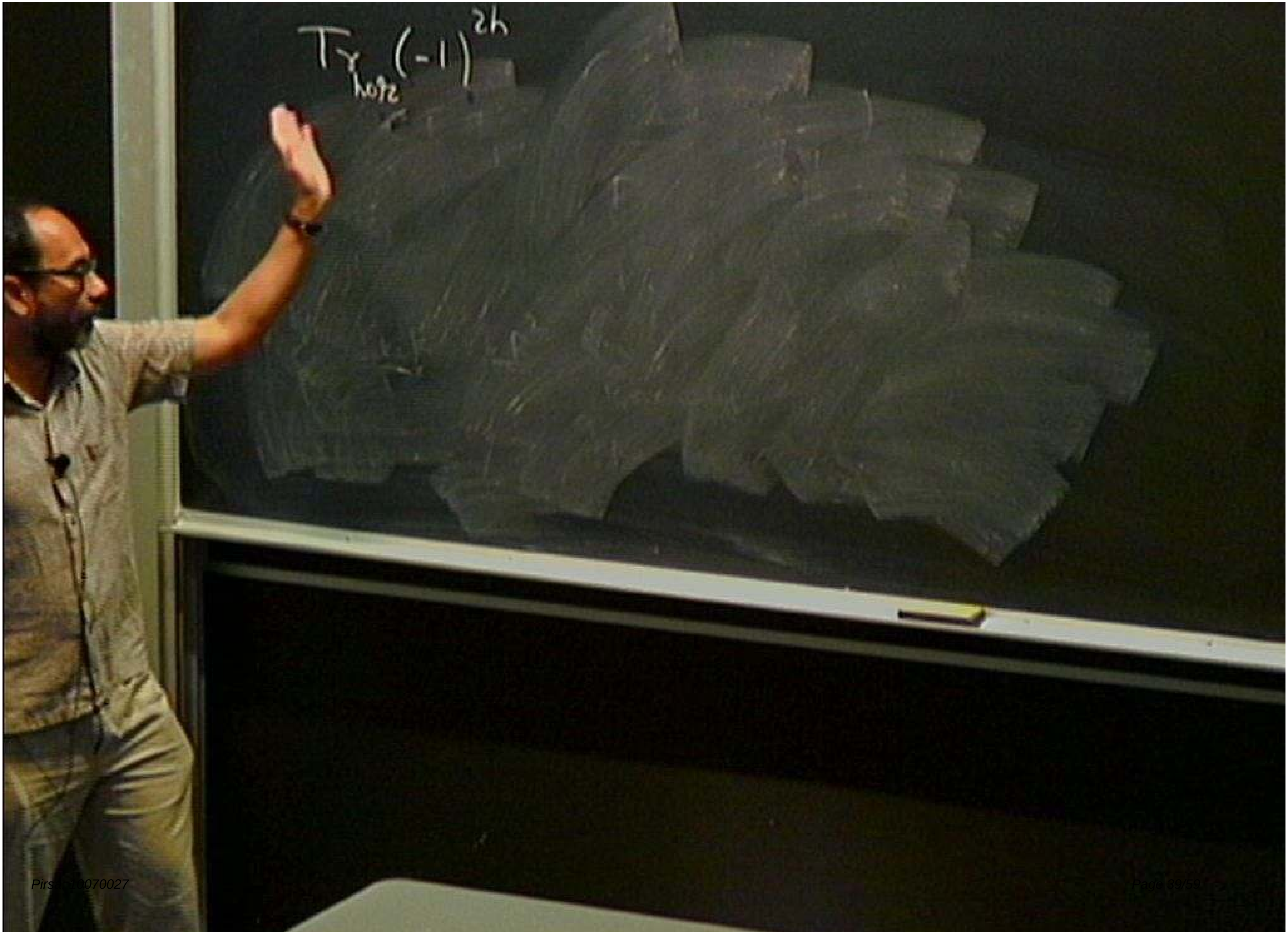
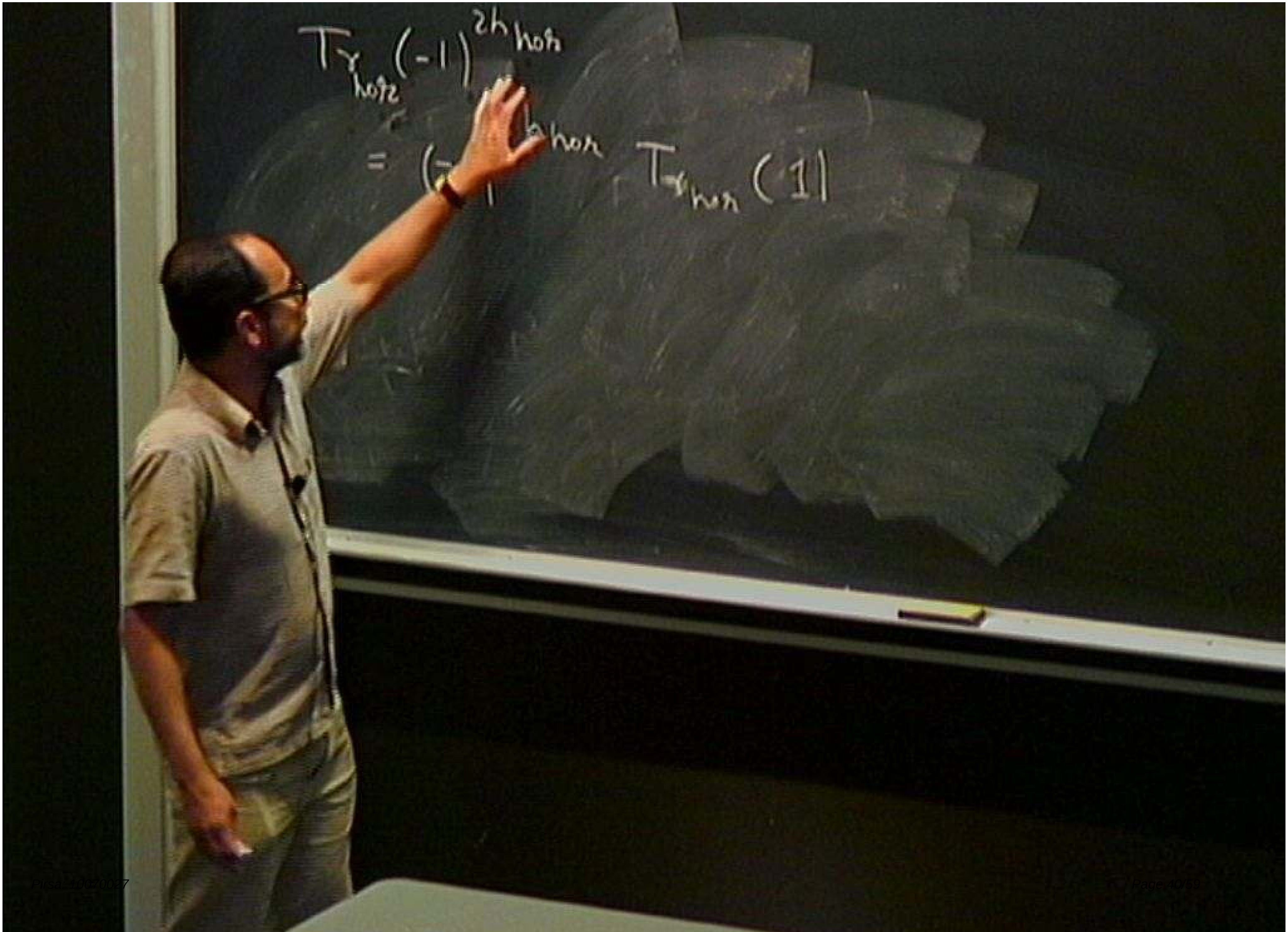
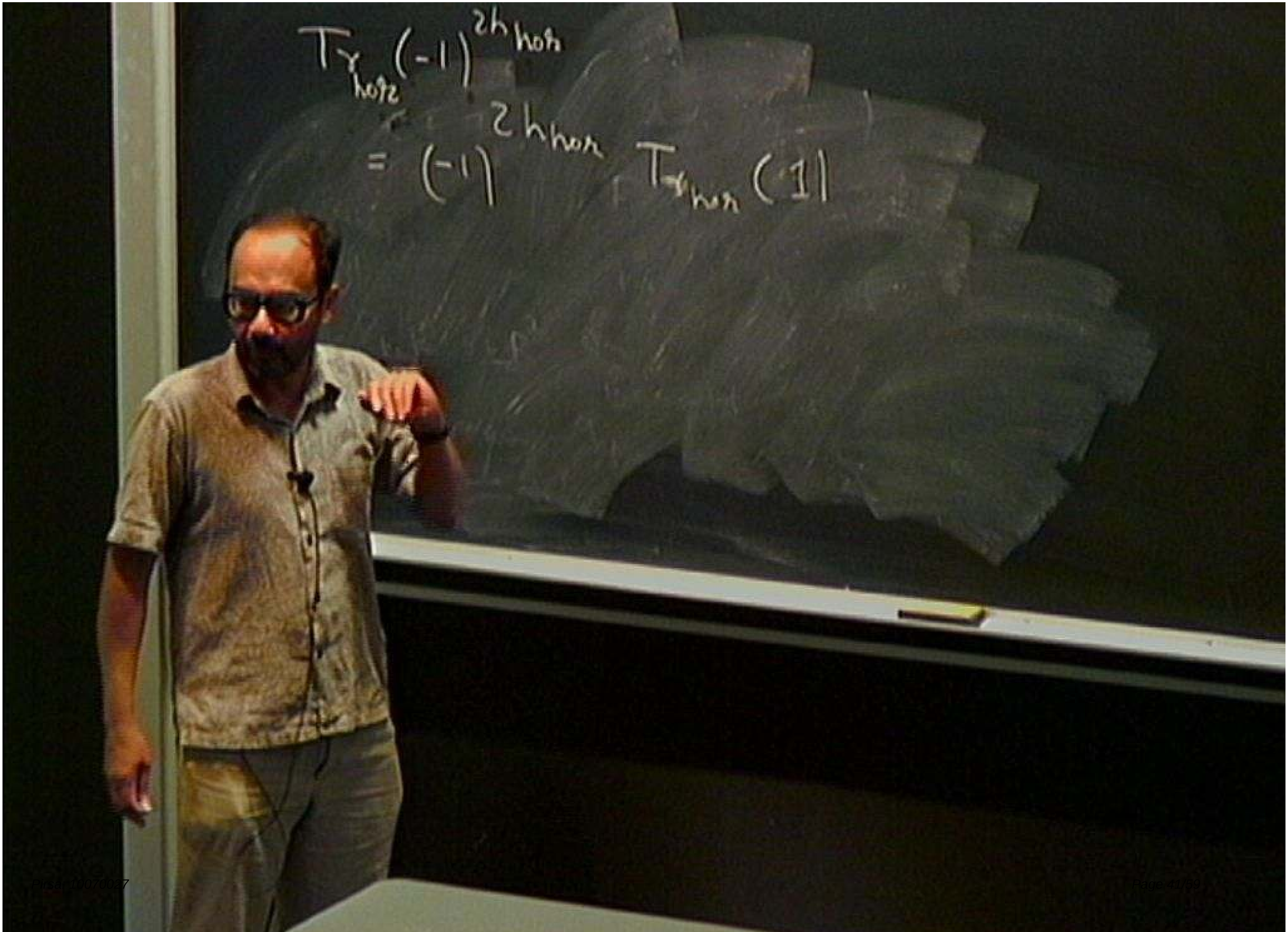
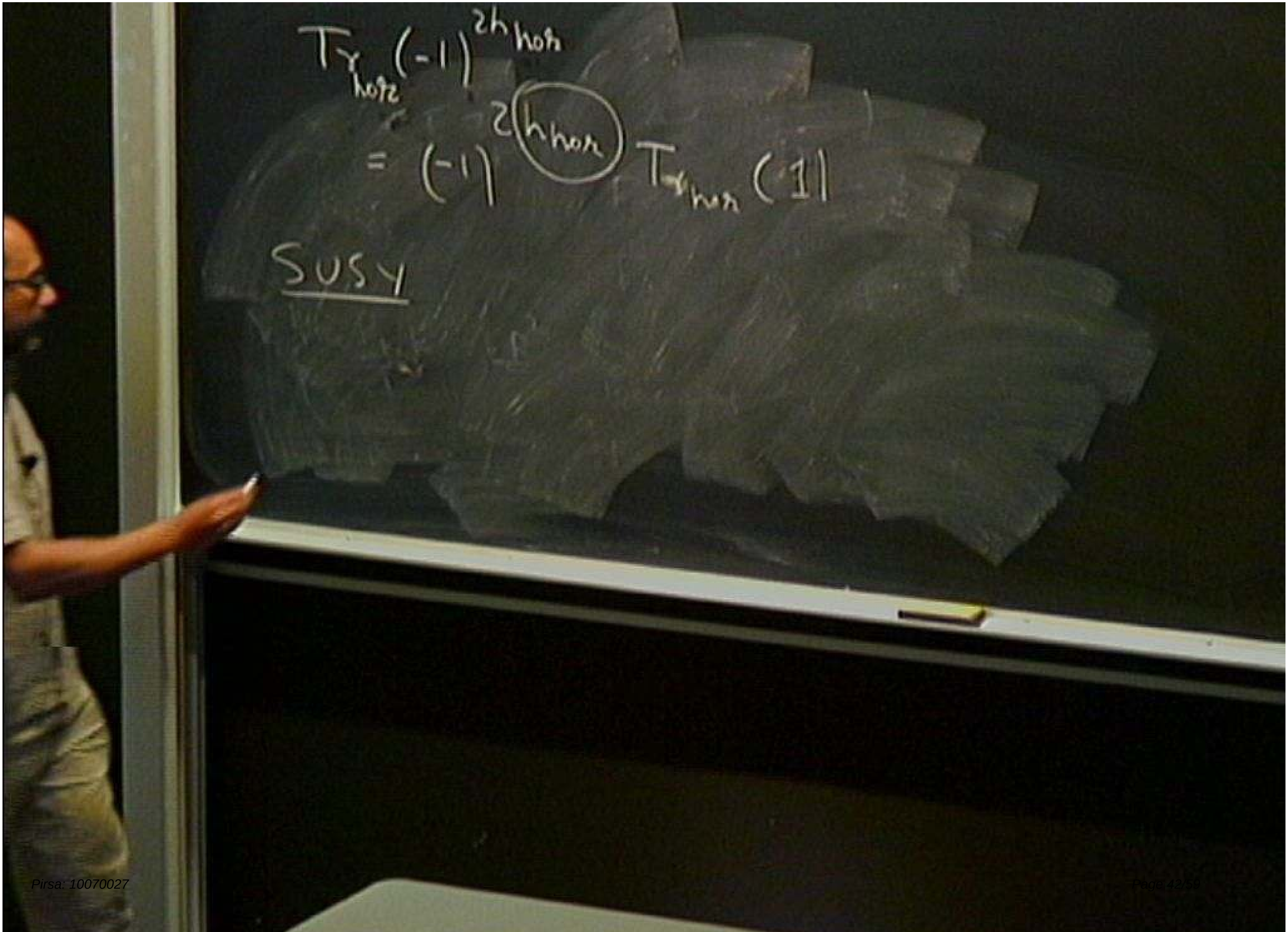
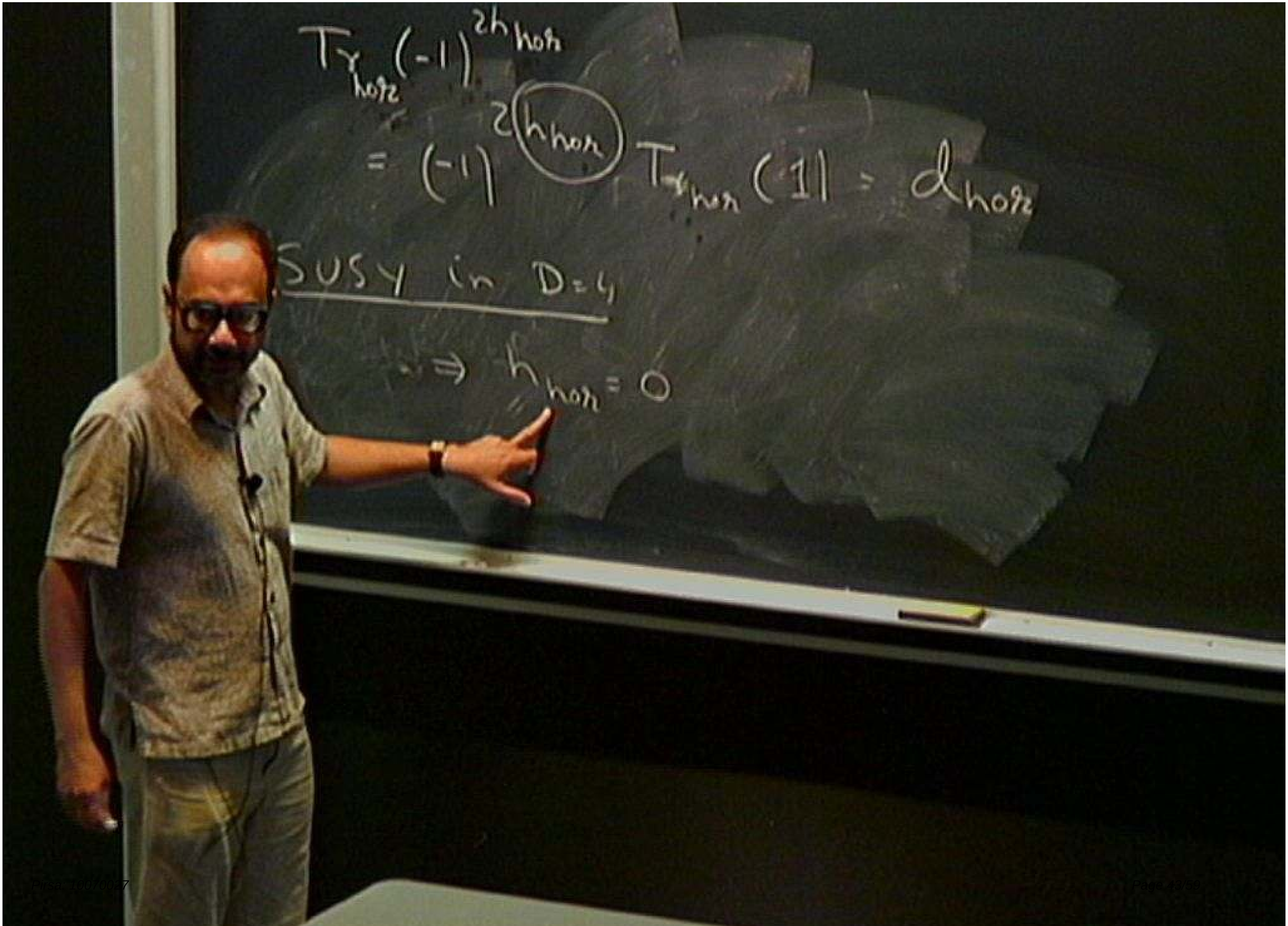

Tr (-1)^2h
hor

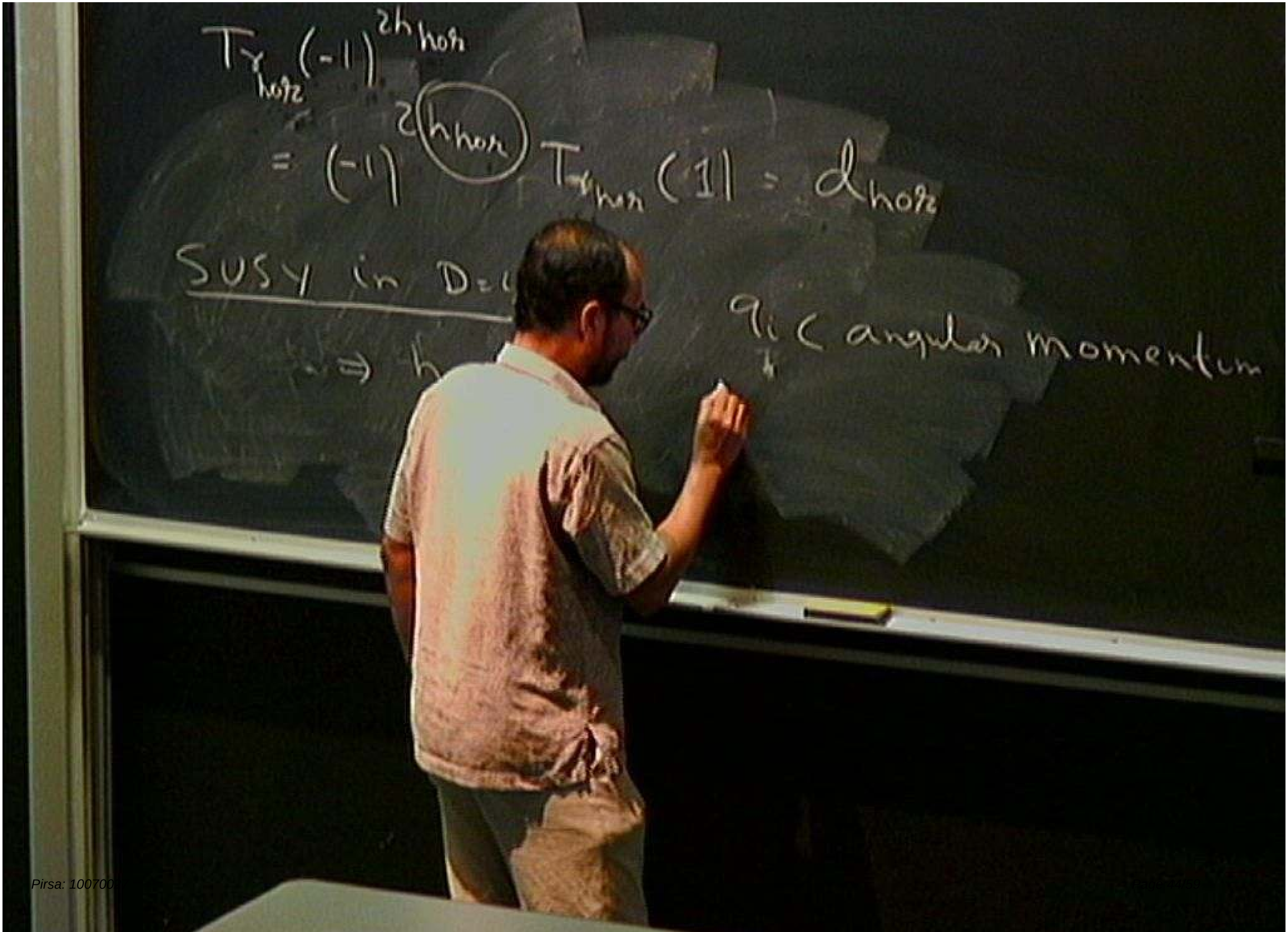

Tr_hor (-1)^2h_hor
= (   )^h_hor Tr_hor (1)



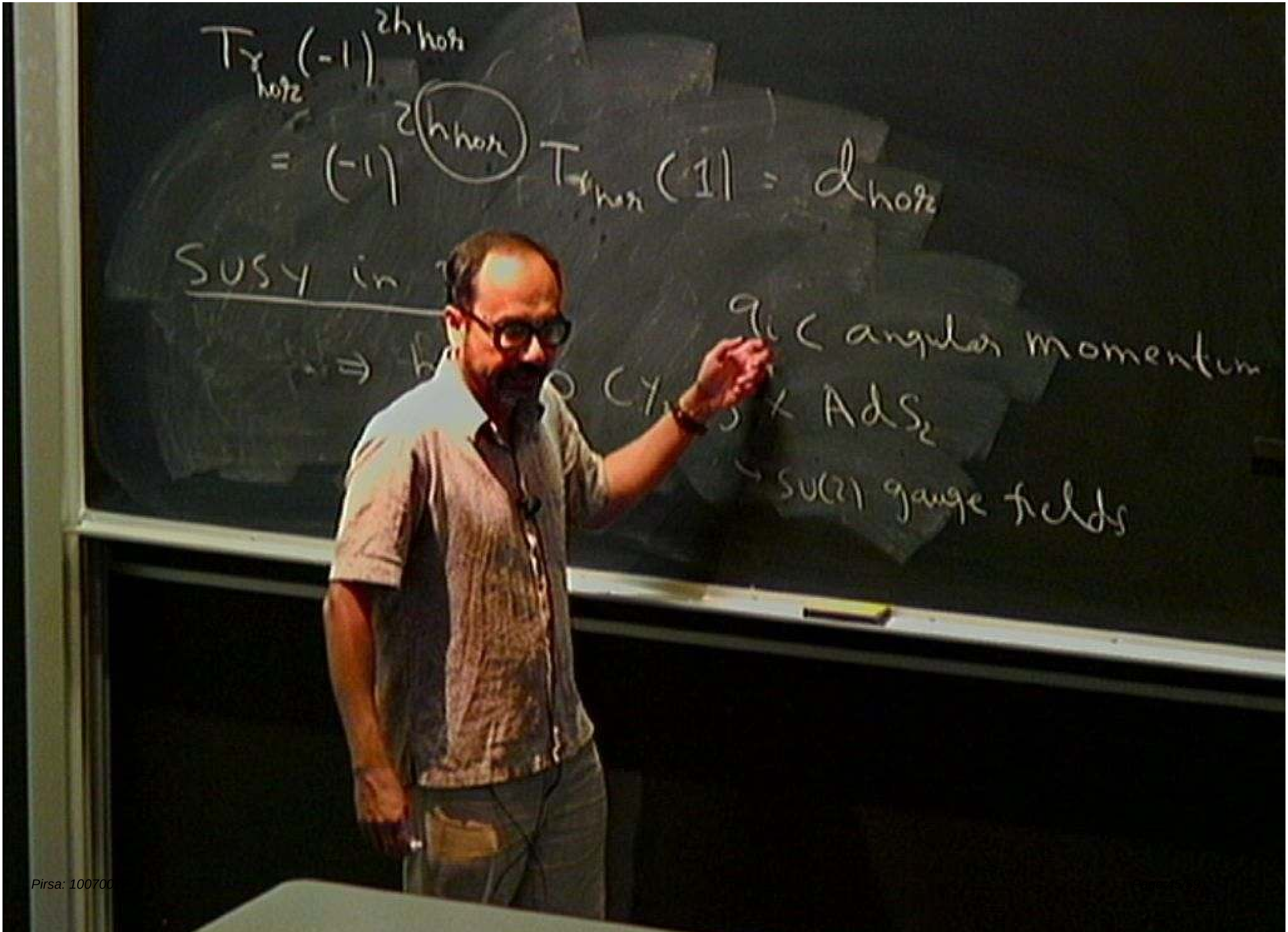
$Tr_{hor}(-1)^{2h_{hor}}$
$= (-1)^{2h_{hor}} \, Tr_{hor}(1)$

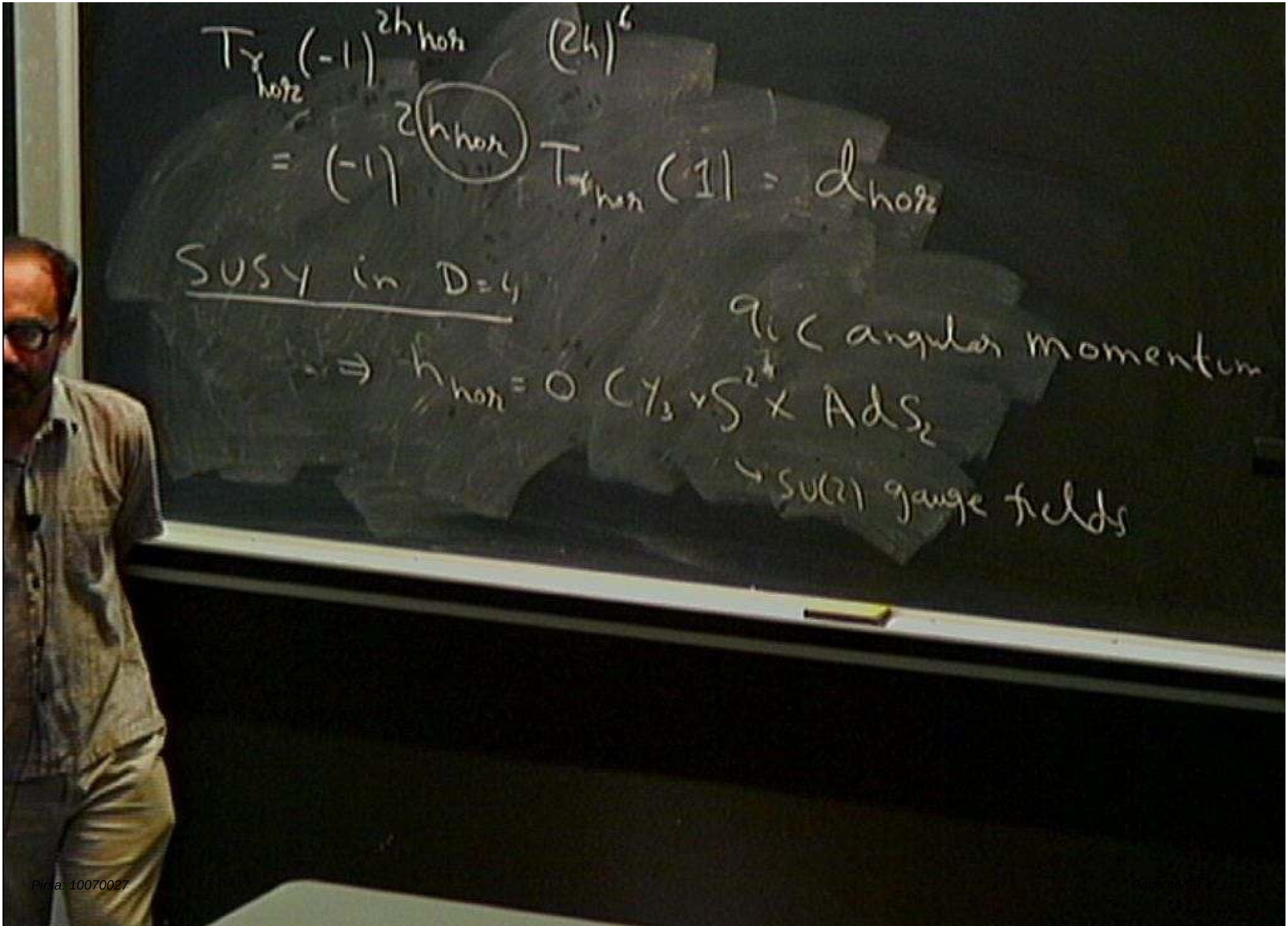

$Tr_{hor} (-1)^{2h_{hor}}$
$= (-1)^{2h_{hor}} Tr_{hor} (1)$
SUSY

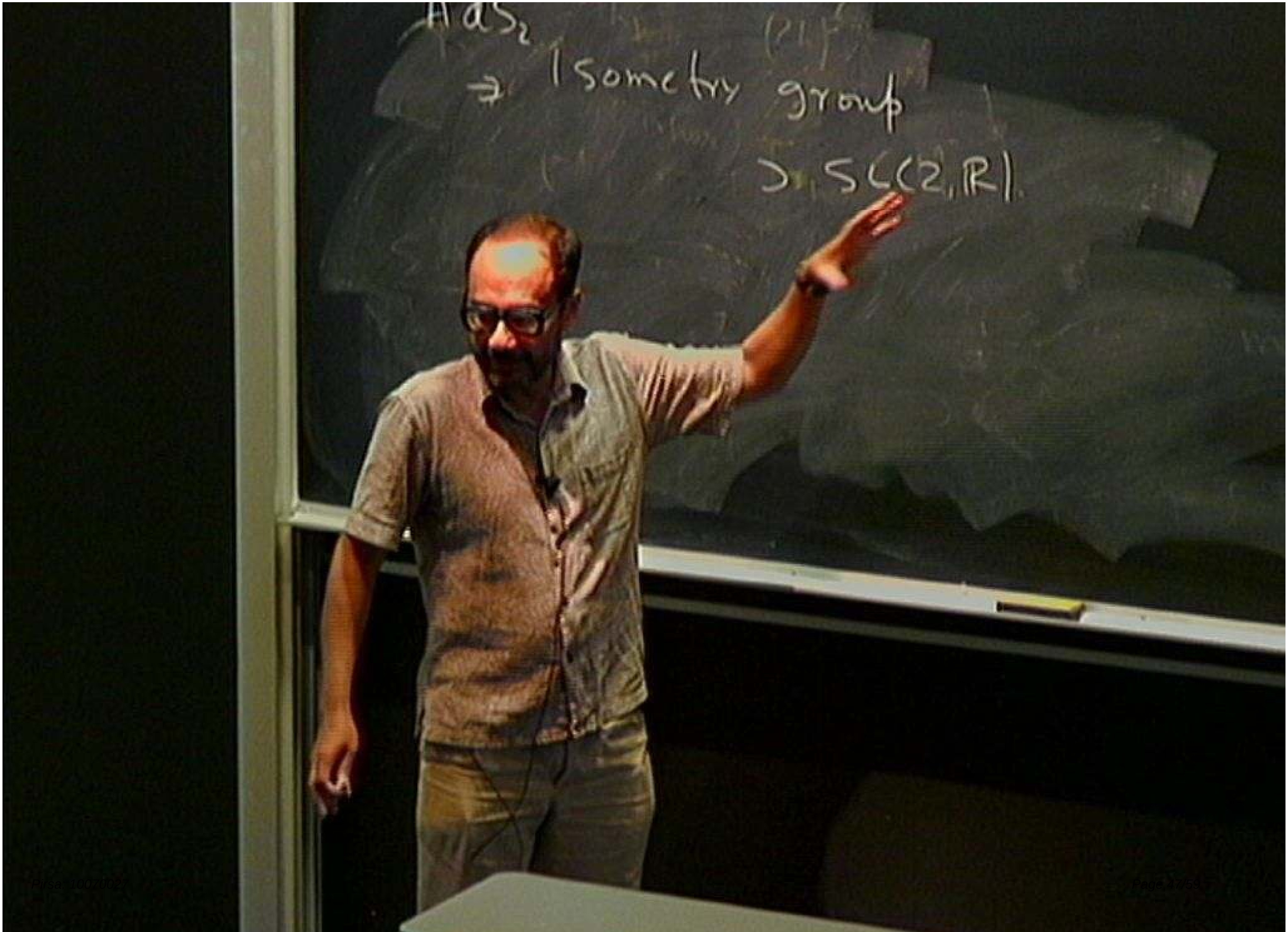

Tr_hor (-1)^2h_hor
= (-1)^2(h_hor) Tr_hor (1) = d_hor
SUSY in D=4
⇒ h_hor = 0

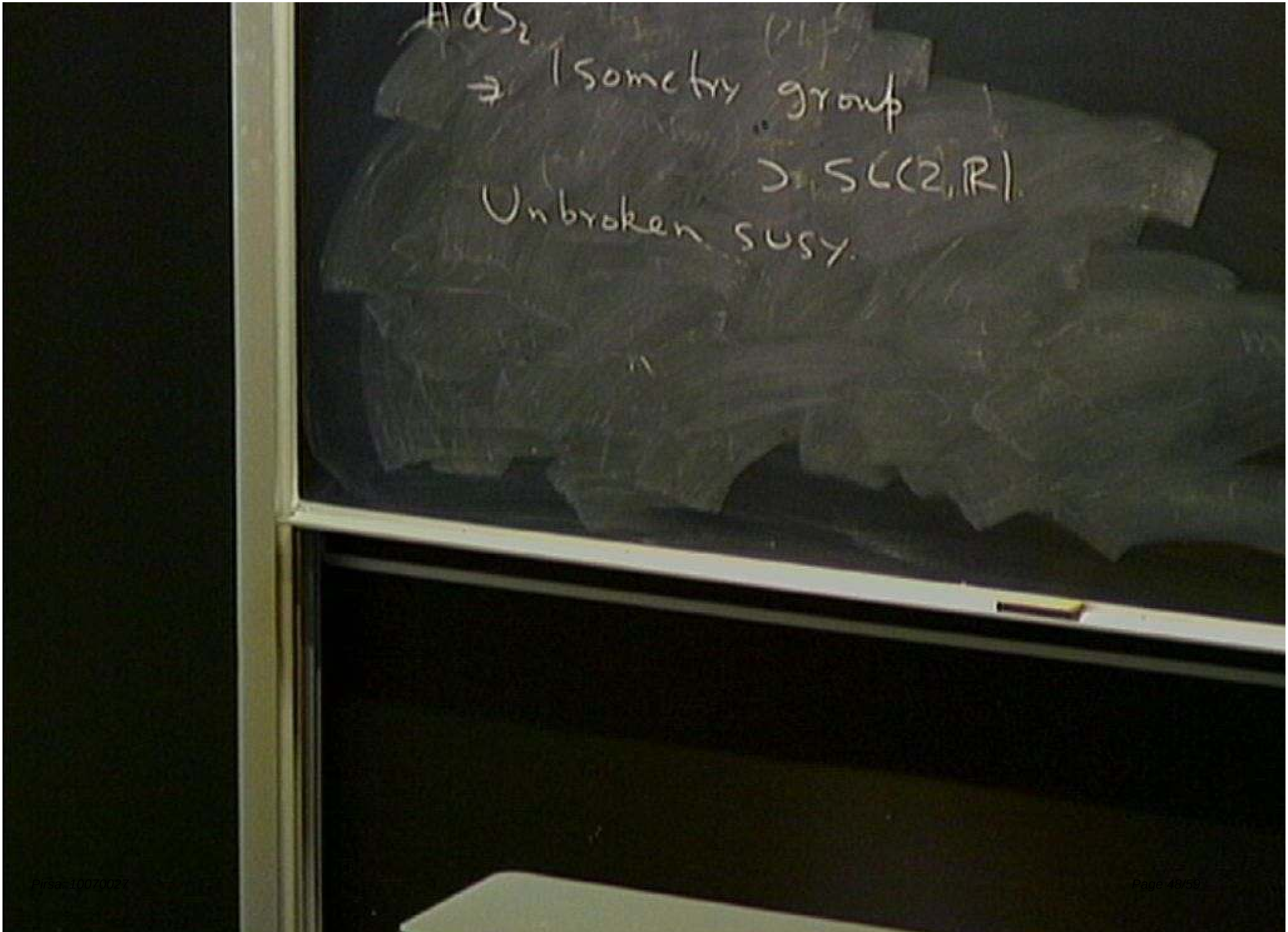


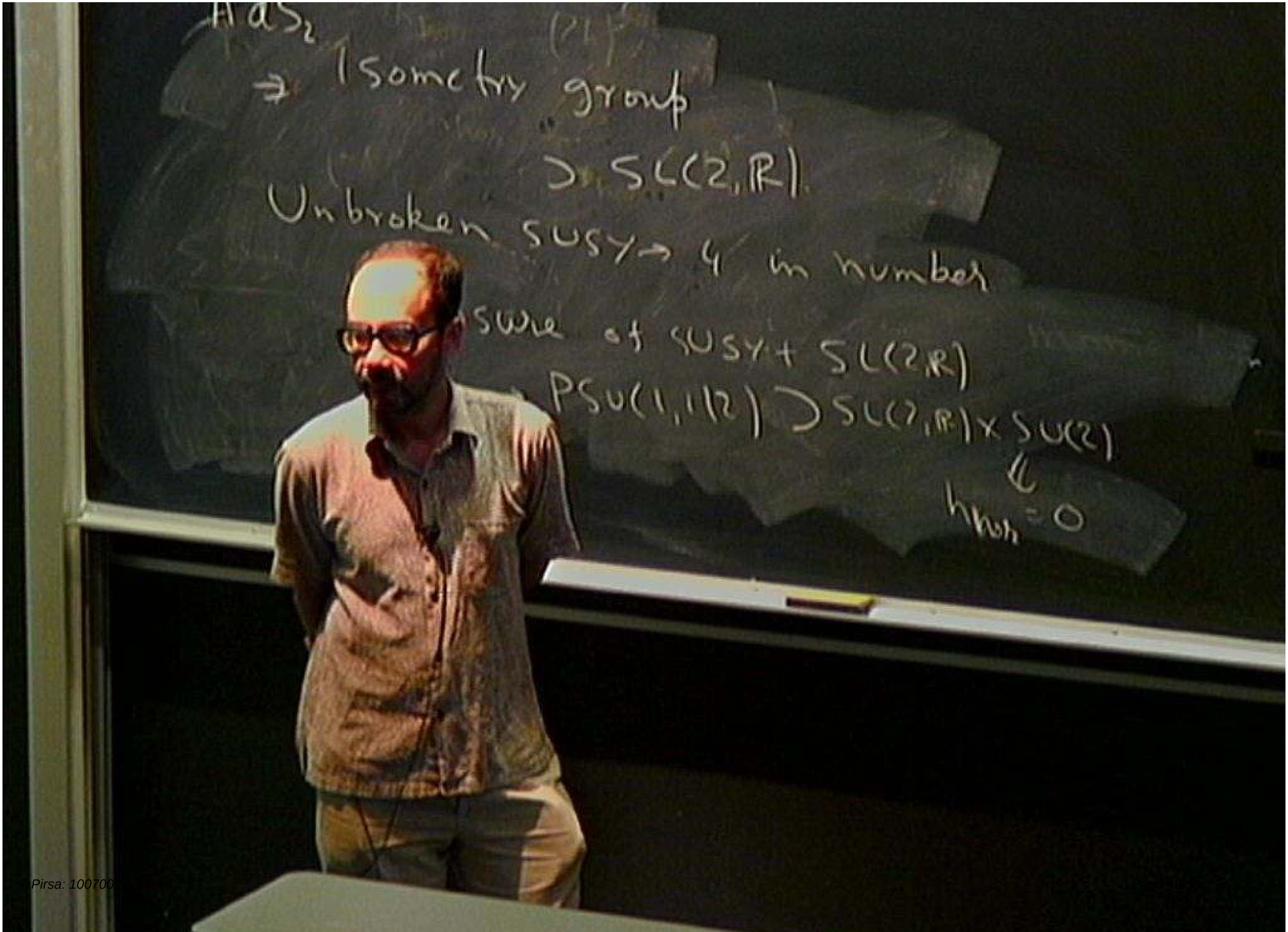
Tr hor (-1) 2h hor
= (-1) 2h hor Tr hor (1) = d hor
SUSY in D=1
q i (angular momentum

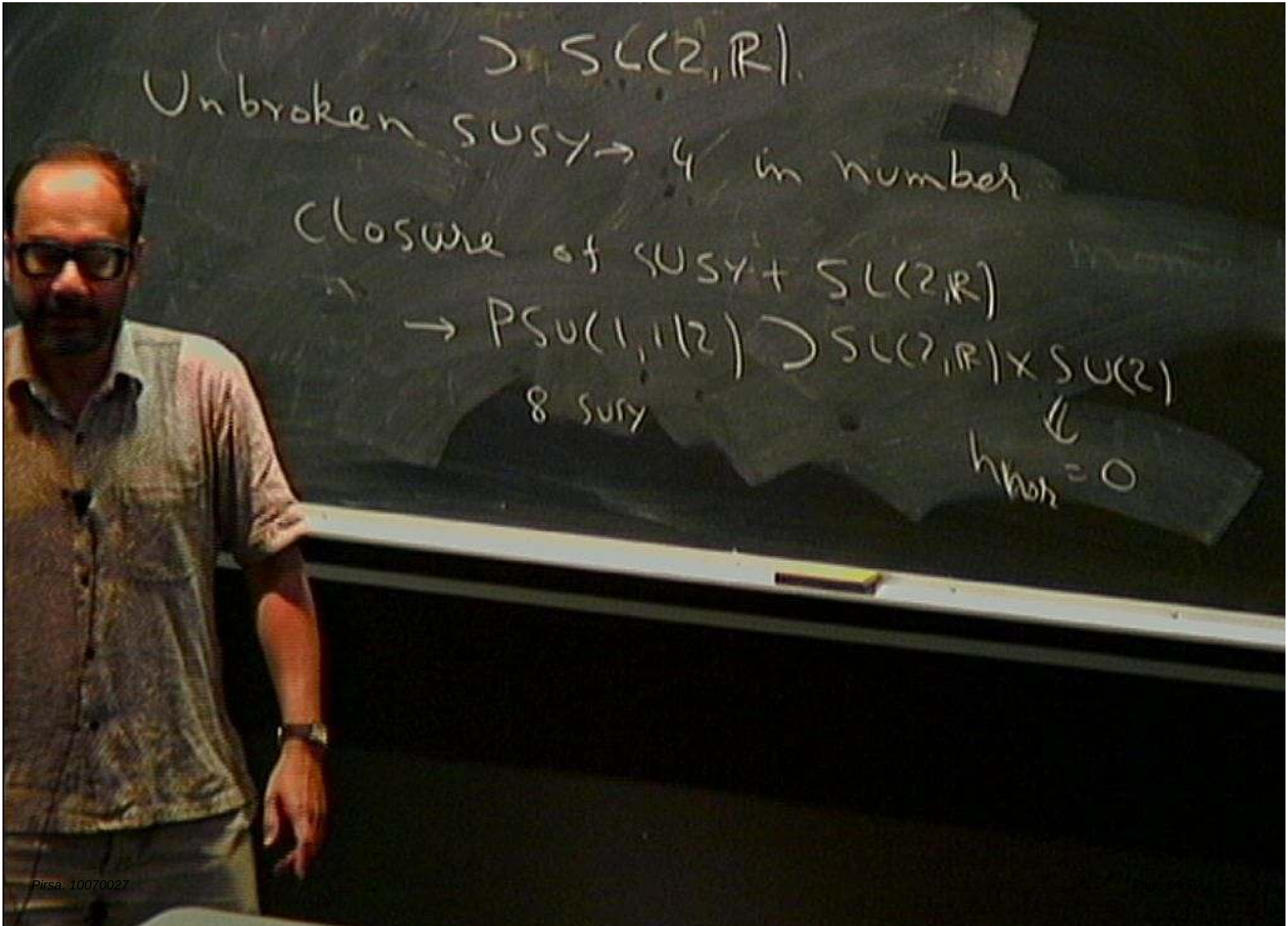

SUSY in
angular momentum
SU(2) gauge fields


Tr_hor (-1)^2h hor (2h)^6
= (-1)^2h hor Tr_hor (1) = d_hor
SUSY in D=4
⇒ h_hor = 0 (Y_3 × S^2 × AdS_2
q_i ⊂ angular momentum
→ SU(2) gauge fields


AdS_2
∃ Isometry group
⊃ SL(2,R)


AdS₂
⇒ Isometry group
⊃ SL(2,R)
Unbroken susy.


AdS2
⇒ Isometry group
⊃ SL(2,R)
Unbroken susy → 4 in number
sure of susy + SL(2,R)
PSU(1,1|2) ⊃ SL(2,R) x SU(2)
hhor = 0


⊃ SL(2,R)
Unbroken susy → 4 in number
Closure of susy + SL(2,R)
→ PSU(1,1|2) ⊃ SL(2,R) × SU(2)
8 susy
h_hor = 0

Isometry group $\quad O \quad$ hair

$\supset SL(2,\mathbb{R}) \quad (-1)^{2h} \quad (2h)^{6}$

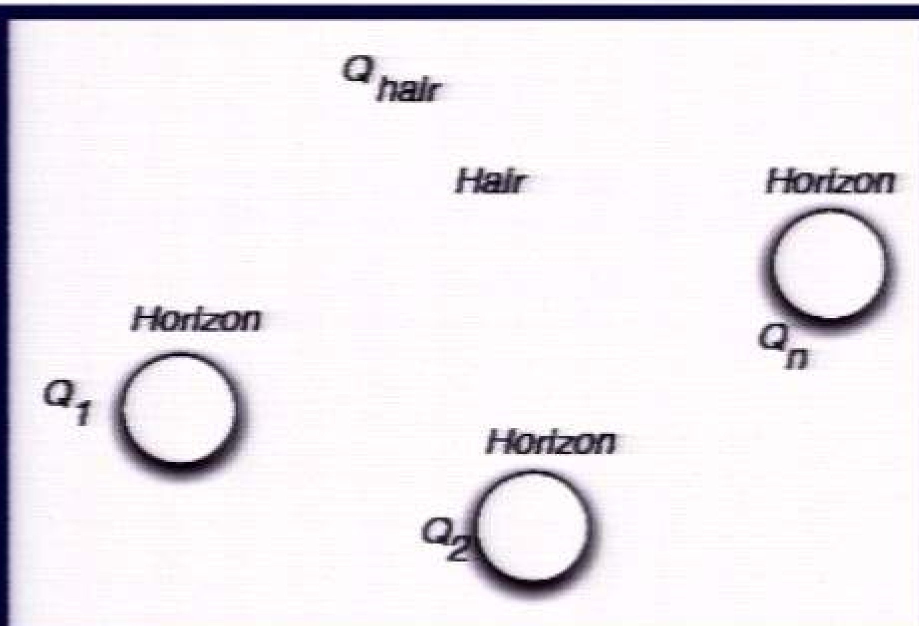
Unbroken susy → 4 in number

Closure of susy + $SL(2,\mathbb{R})$

$\rightarrow PSU(1,1|2) \supset SL(2,\mathbb{R}) \times SU(2)$

8 susy $\qquad \Downarrow$

$h_{hor} = 0$



Q_i denotes both electric and magnetic charges of the i-th black hole.

Now let us compute B_6 for the same configuration.

$$B_6 = \frac{1}{6!}\mathrm{Tr}(-1)^{2h}(2h)^6 = \frac{1}{6!}\mathrm{Tr}(-1)^{h_{hor}+h_{hair}}(2h_{hor}+2h_{hair})^6$$

In four dimensions, supersymmetry $\rightarrow h_{hor} = 0$.

Thus

$$B_6 = \frac{1}{6!}\mathrm{Tr}(-1)^{2h_{hair}}(2h_{hair})^6$$

$$\|$$

$$\sum_n \sum_{\substack{\{\vec{Q}_i\},\vec{Q}_{hair} \\ \sum_{i=1}^n \vec{Q}_i+\vec{Q}_{hair}=\vec{Q}}} \left\{\prod_{i=1}^{n} d_{hor}(\vec{Q}_i)\right\} B_{6;hair}(\vec{Q}_{hair};\{\vec{Q}_i\})$$

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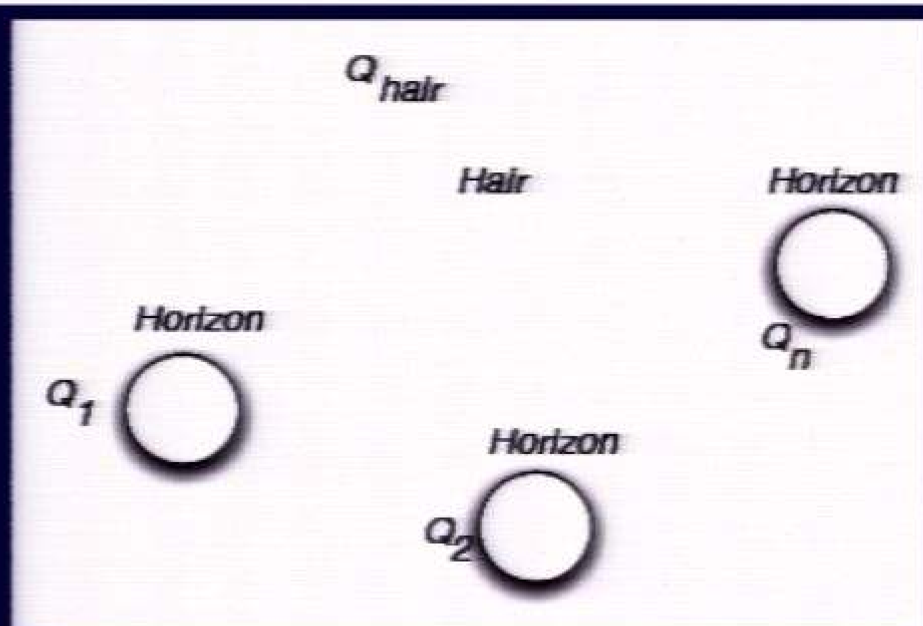
We shall denote the degeneracy associated with the hair degrees of freedom by d_{hair}.

d_{hair} can be calculated by explicitly identifying and quantizing the hair modes.

The total degeneracy:

$$\sum_{n} \sum_{\substack{\{\vec{Q}_i\}, \vec{Q}_{hair} \\ \sum_{i=1}^{n} \vec{Q}_i + \vec{Q}_{hair} = \vec{Q}}} \left\{ \prod_{i=1}^{n} d_{hor}(\vec{Q}_i) \right\} d_{hair}(\vec{Q}_{hair}; \{\vec{Q}_i\})$$



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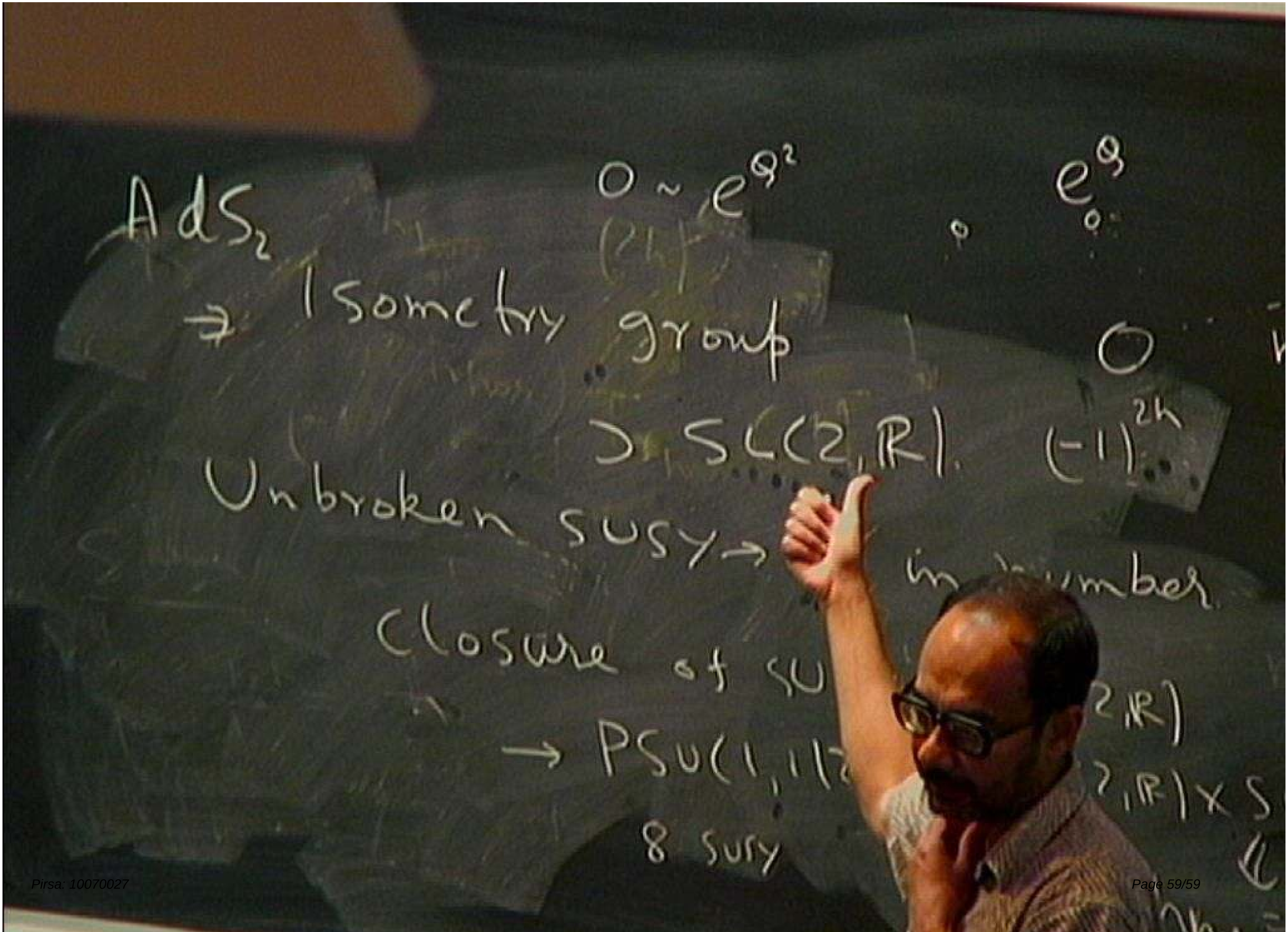
$$B_6 = \frac{1}{6!}\mathrm{Tr}(-1)^{2h_{hair}}(2h_{hair})^6$$

$$\|$$

$$\sum_n \sum_{\substack{\{\vec{Q}_i\},\vec{Q}_{hair} \\ \sum_{i=1}^n \vec{Q}_i + \vec{Q}_{hair} = \vec{Q}}} \left\{\prod_{i=1}^n d_{hor}(\vec{Q}_i)\right\} B_{6;hair}(\vec{Q}_{hair};\{\vec{Q}_i\})$$


AdS2
O ~ e^φ2
e^φ
⇒ Isometry group
⊃ SL(2,R)
(-1)^2h
Unbroken susy →
in number
Closure of SU
→ PSU(1,1|2
8 susy