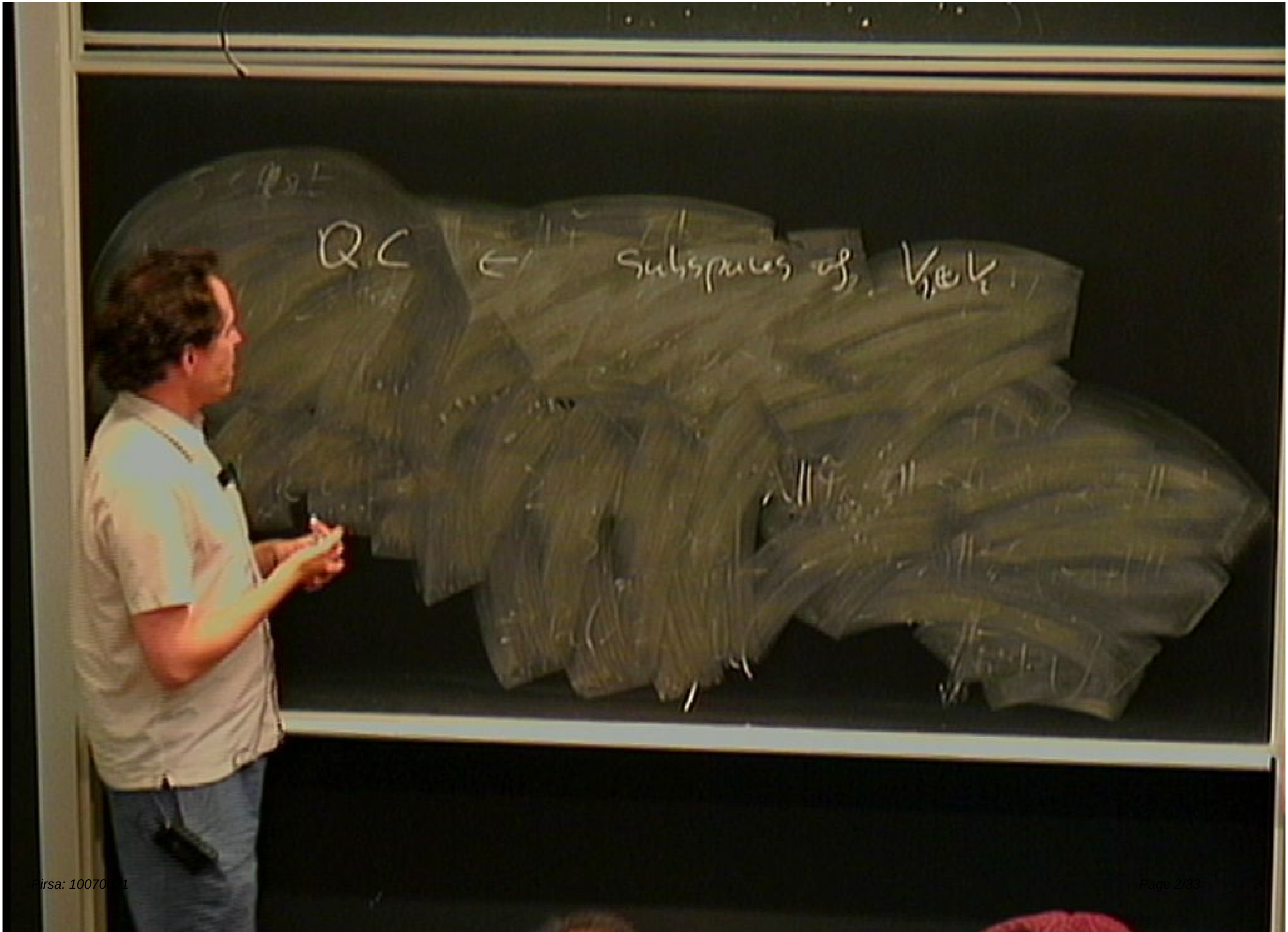
Title: Random matrices and random quantum channels

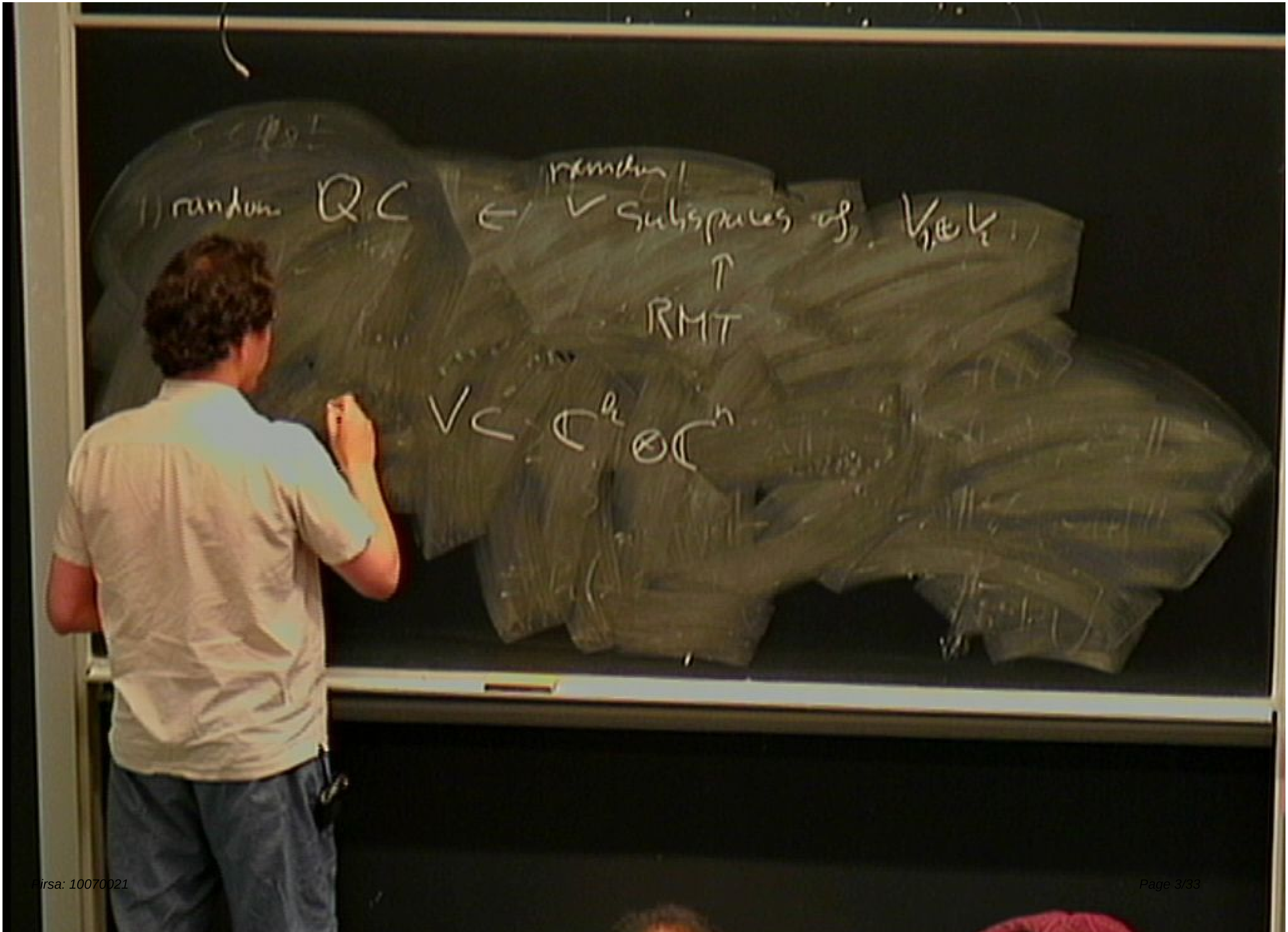
Date: Jul 06, 2010 02:15 PM

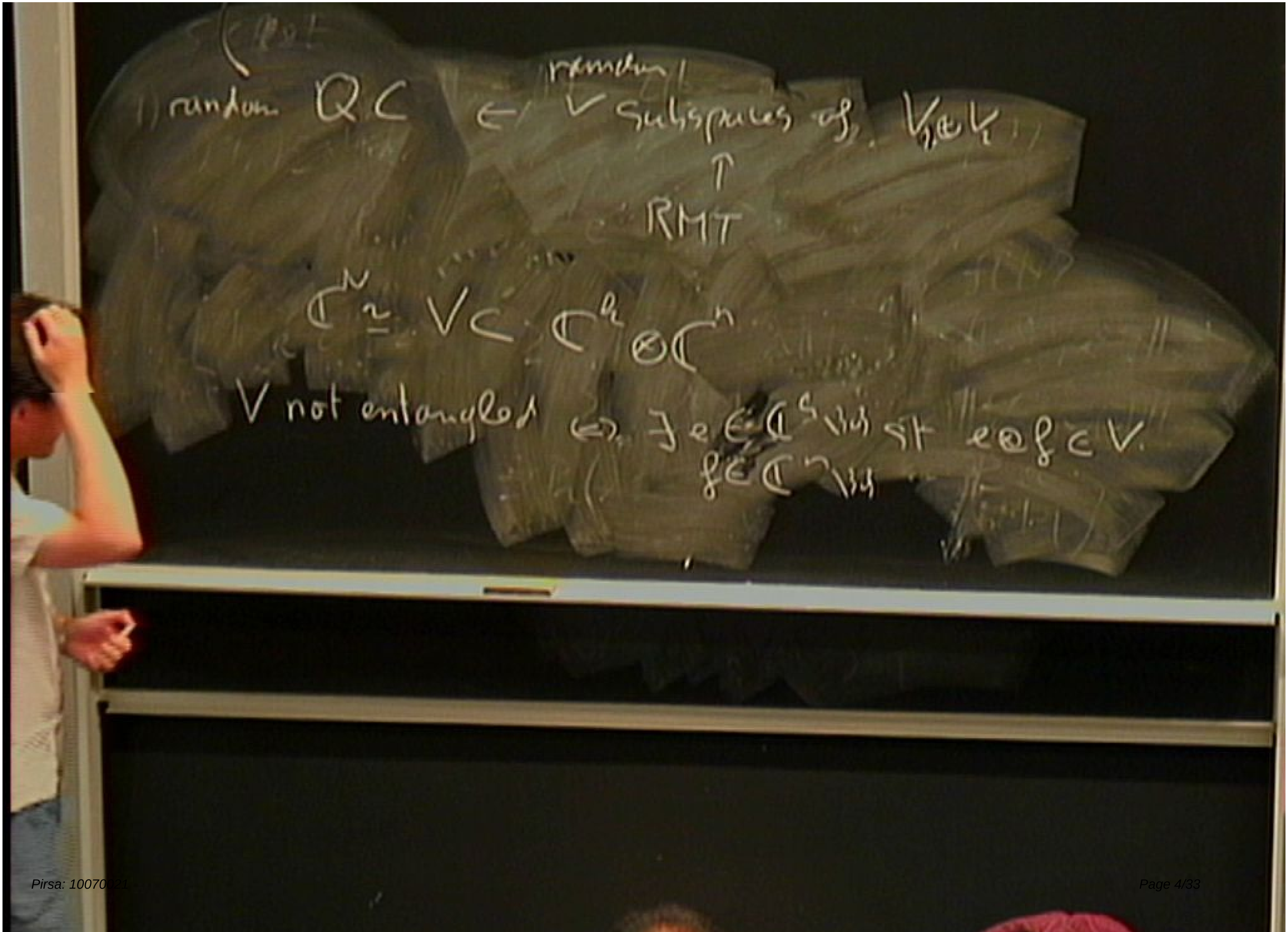
URL: http://pirsa.org/10070021

Abstract: In this talk I will describe how random matrix theory and free probability theory (and in particular, results of Haagerup and Thorbjornsen) can give insight into the problem of understanding all possible eigenvalues of the output of important classes of random quantum channels. I will also describe applications to the minimum output entropy additivity problems.



QC ∈ subspaces of $V_1 \otimes V_2$


1) random QC ∈ random V subspaces of V1⊗V2
RMT
V ⊂ C^k ⊗ C^n


1) random QC ⇐ random V subspaces of V₁⊗V₂
RMT
ℂᴺ ≅ V ⊂ ℂᵏ ⊗ ℂⁿ
V not entangled ⇔ ∃ e ∈ ℂᵏ \ {0} st e⊗f ⊂ V.
f ∈ ℂⁿ \ {0}

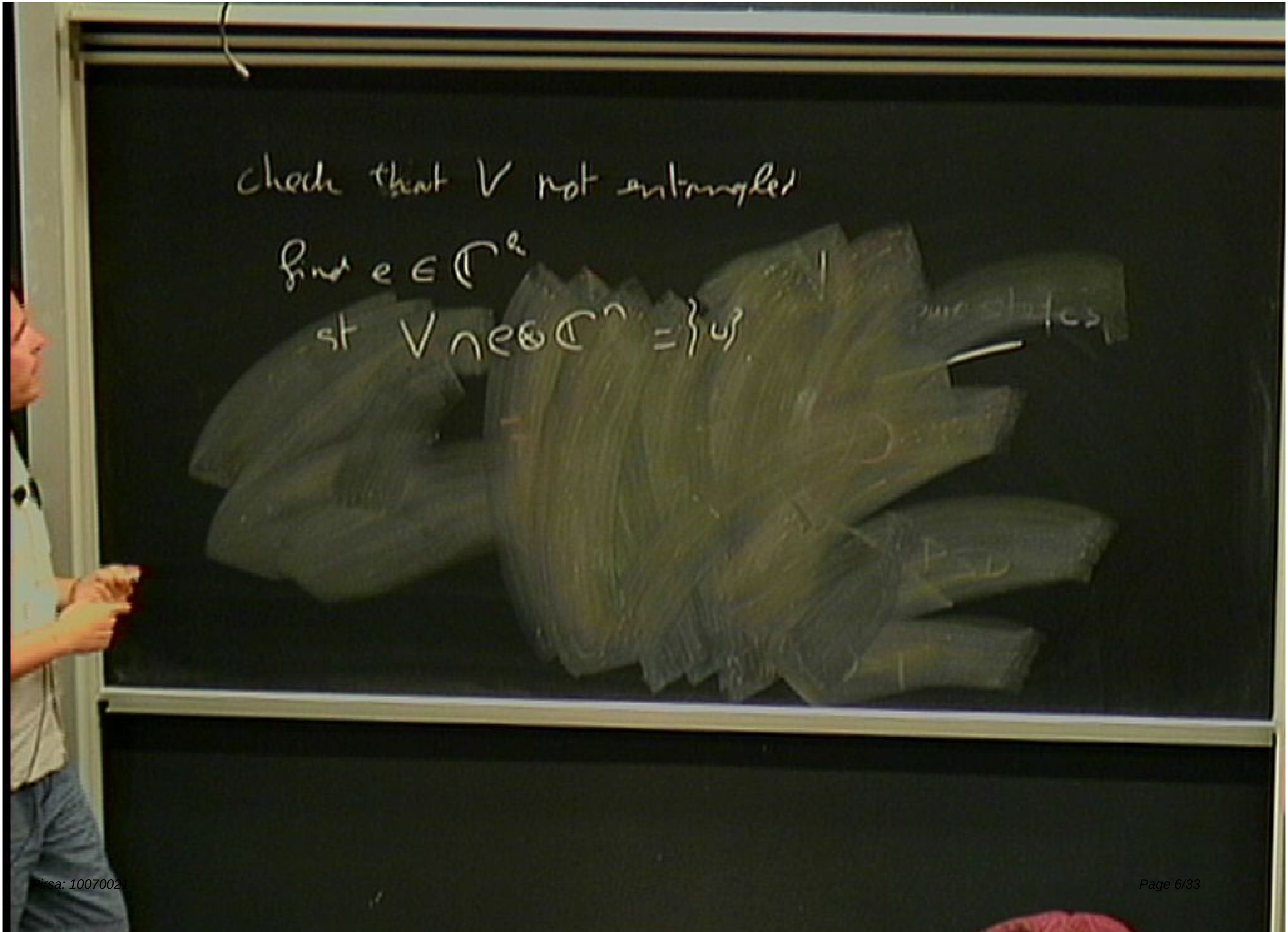
1) random QC $\leftarrow$ random V subspaces of $V_1 \otimes V_2$

$\uparrow$

RMT

$$\mathbb{C}^N \cong V \subset \mathbb{C}^k \otimes \mathbb{C}^n$$

V not entangled $\Leftrightarrow$ $\exists\, e \in \mathbb{C}^k \setminus \{0\}$, $f \in \mathbb{C}^n \setminus \{0\}$ s.t. $e \otimes f \in V$.


check that V not entangled
find $e \in \mathbb{C}^2$
st $V \cap e \otimes \mathbb{C}^n = \{0\}$

check that V not entangled

find $e \in \mathbb{C}^{\ell}$

st $V \cap e \otimes \mathbb{C}^{n} = \{0\}$.

if $N + n > n\ell$, $\exists$ V not entangled.

· check that V not entangled?

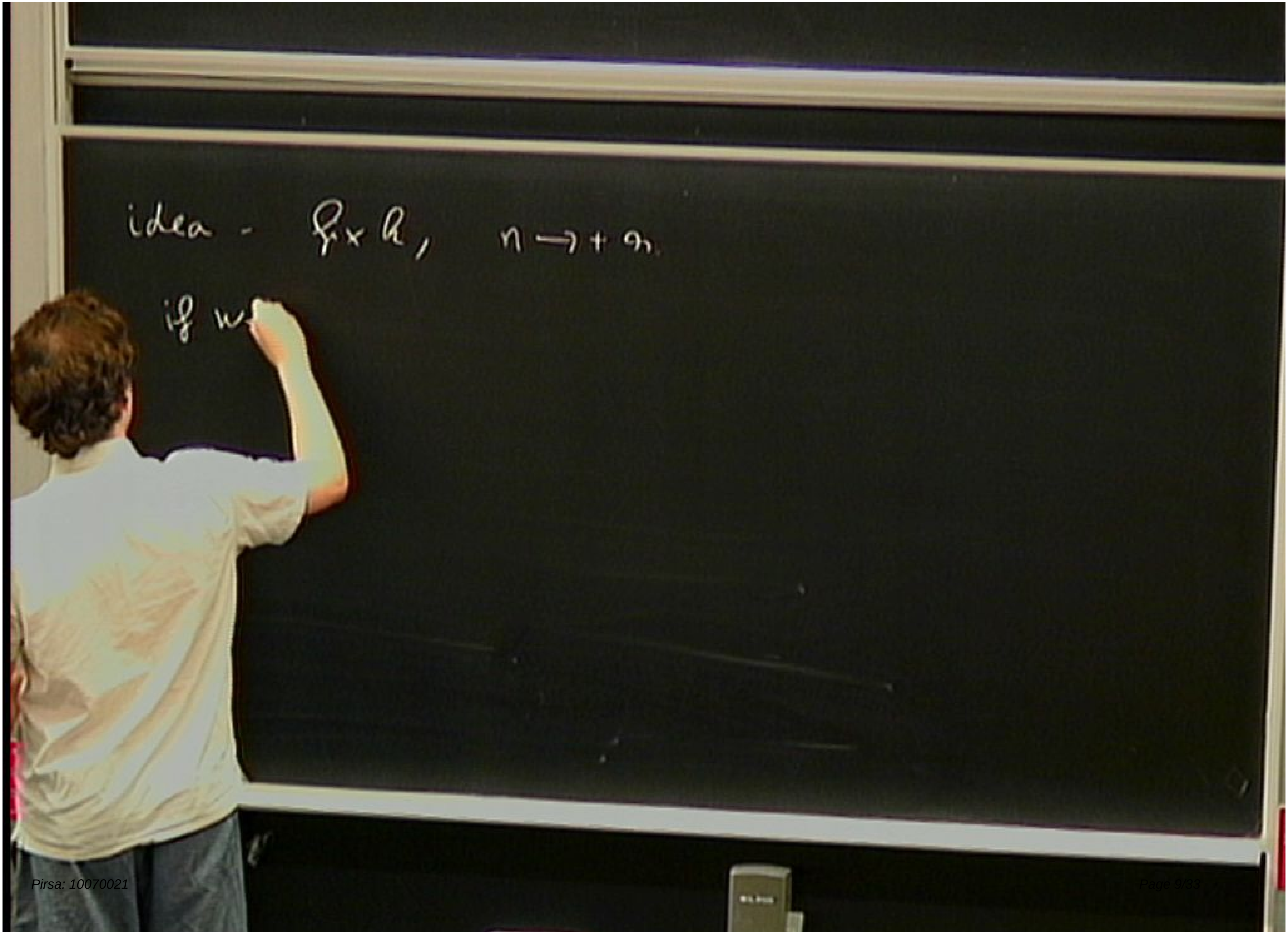
find $\underline{e} \in \mathbb{C}^k$

st $V \cap e \otimes \mathbb{C}^n \neq \{0\}$.

if $N + n > nk$. $\exists$ V not entangled.

· check that V entangled?

$\forall e \in \mathbb{C}^k$ $\quad e \otimes \mathbb{C}^n \cap V = \{0\}$.


idea - fix h, n → +∞
if w

idea - fix k, $n \to +\infty$

if we can find $e_1, \dots, e_k \in B_k(0,1)$

idea - fix R, $n \to +\infty$

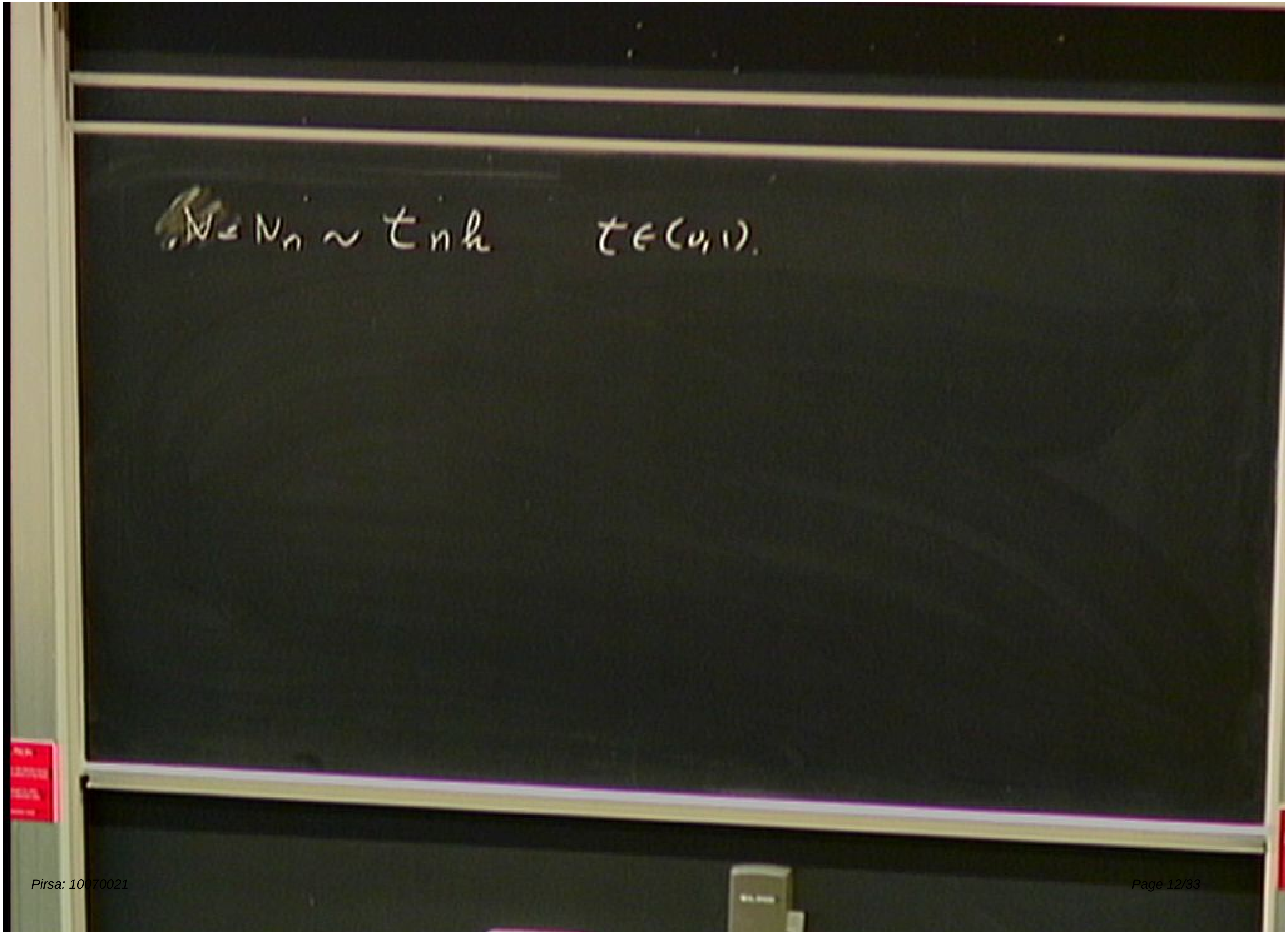
if we can find $e_1, \dots, e_\ell \in B_R(0,1)$

$0 < \alpha, \beta$ $\qquad \alpha + \beta < 1$

$\forall e \in B_R(0,1) \ \bigwedge_{\exists i \in \{1,\dots,\ell\}} \ \|P_e - P_{e_i}\| < \alpha.$

$\forall i \in \{1,\dots,\ell\} \qquad \|P_V \cdot P_{e_i}\| < \beta.$

$\Rightarrow \forall e \quad \|P_V \cdot P_e\| < 1 \quad \Rightarrow$ V entangled.


$N = N_n \sim t n h \qquad t \in (0,1)$

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for each n, choose V at random of dim N.

in $\mathbb{C}^q \otimes \mathbb{C}^N$.

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P_V.

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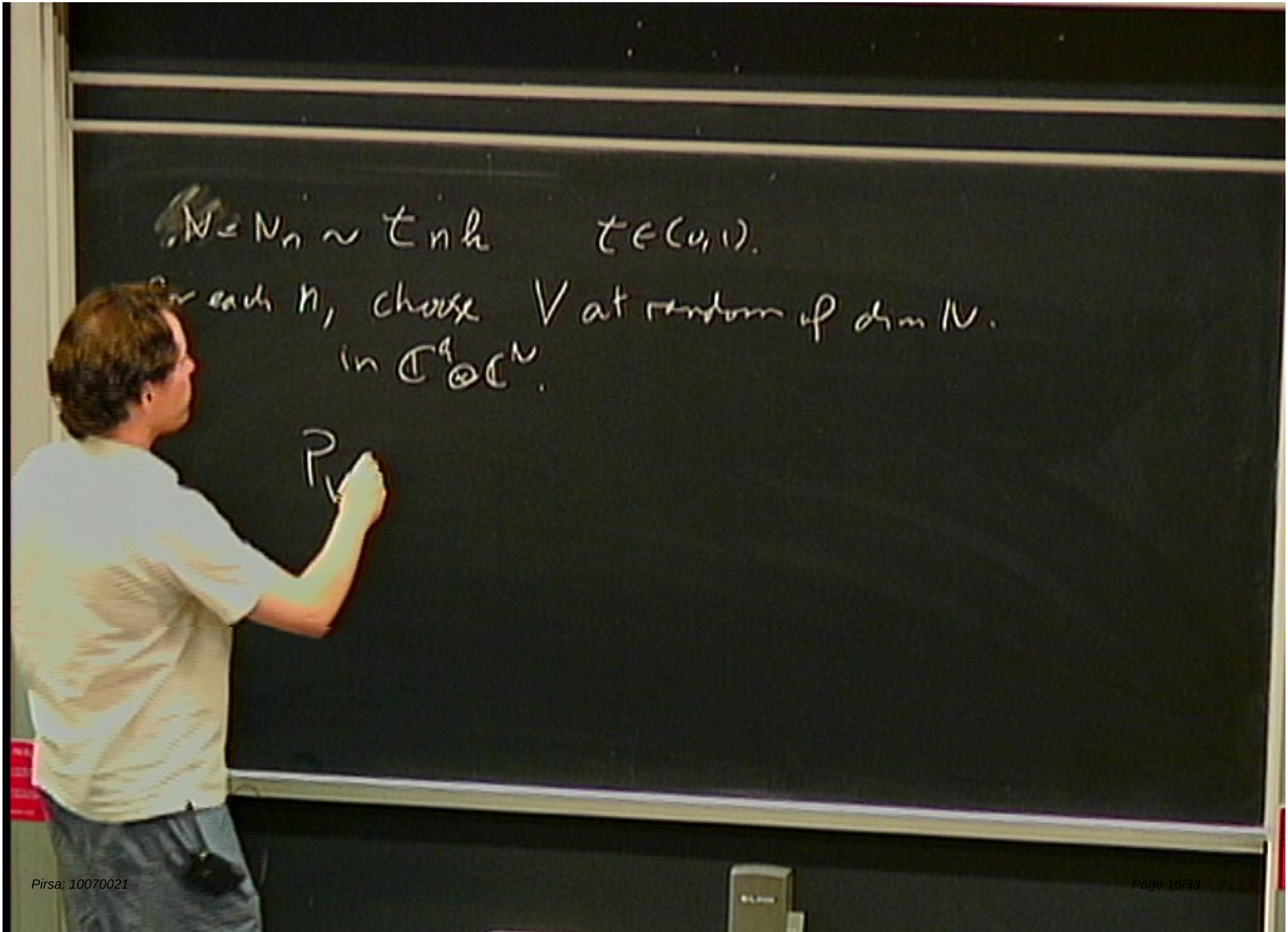
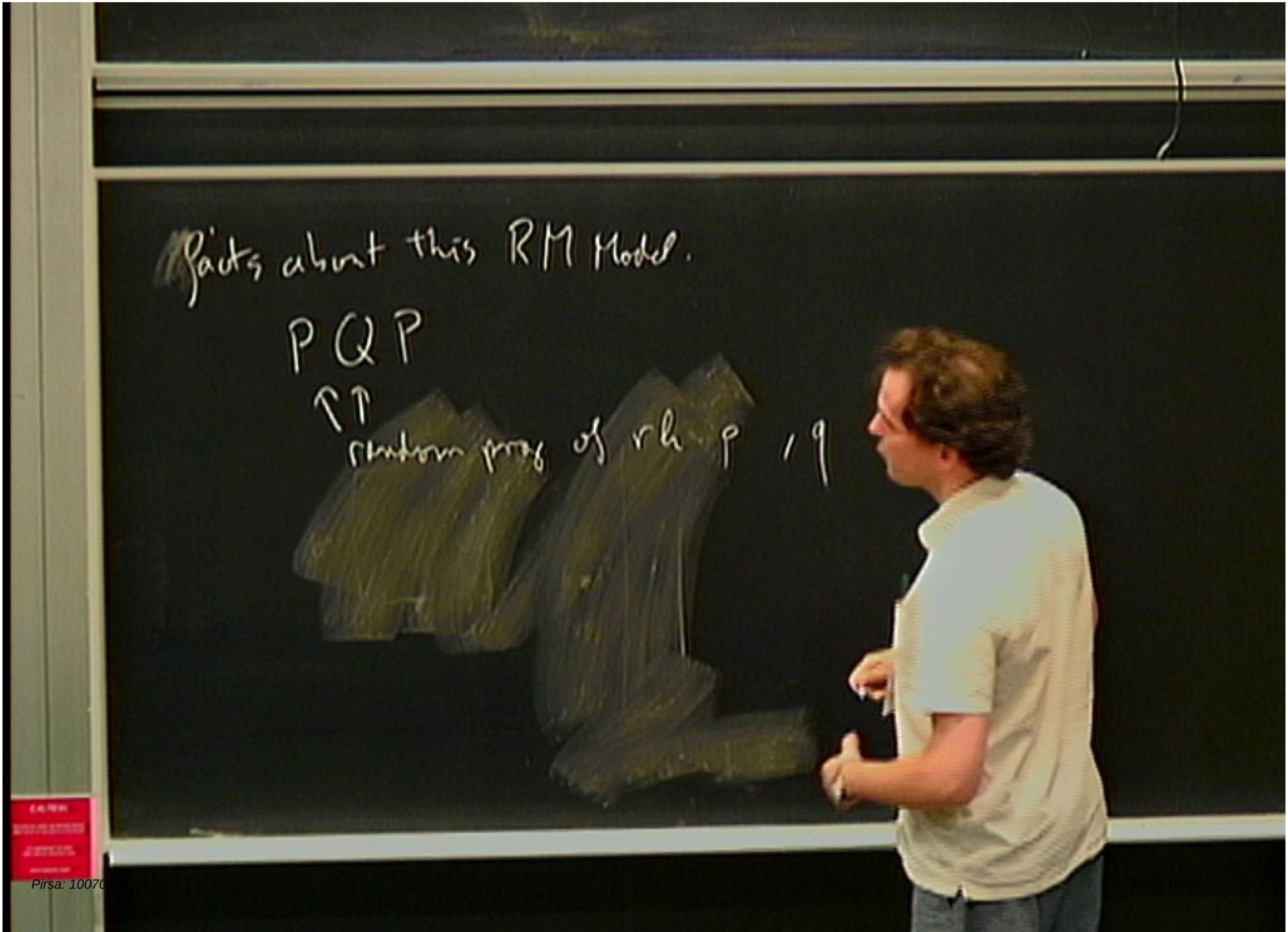

N = N_n ~ t n h   t ∈ (0,1).
For each n, choose V at random of dim N.
in C^q ⊗ C^N.
P_V

$N = N_n \sim t n k \qquad t \in (0,1).$

for each n, choose V at random of dim N.
in $\mathbb{C}^k \otimes \mathbb{C}^N$.

$$P_V \cdot P_{e_i} \otimes I_n \cdot P_V \quad \rightsquigarrow \text{RM model}.$$


Facts about this RM Model.
PQP
random proj of rk p , q

Facts about this RM Model.

$P\,Q\,P$

$\uparrow\uparrow$ random proj of rk p, q $\in M_n(\mathbb{C})$

wlog $q \le p$, $p+q \le n$

joint dist of ev's is $\propto \prod_{i<j} (\lambda_i - \lambda_j)^2 \cdot \prod_{i=1}^{p} (1-\lambda_i)^{n-p-q} \lambda_i$

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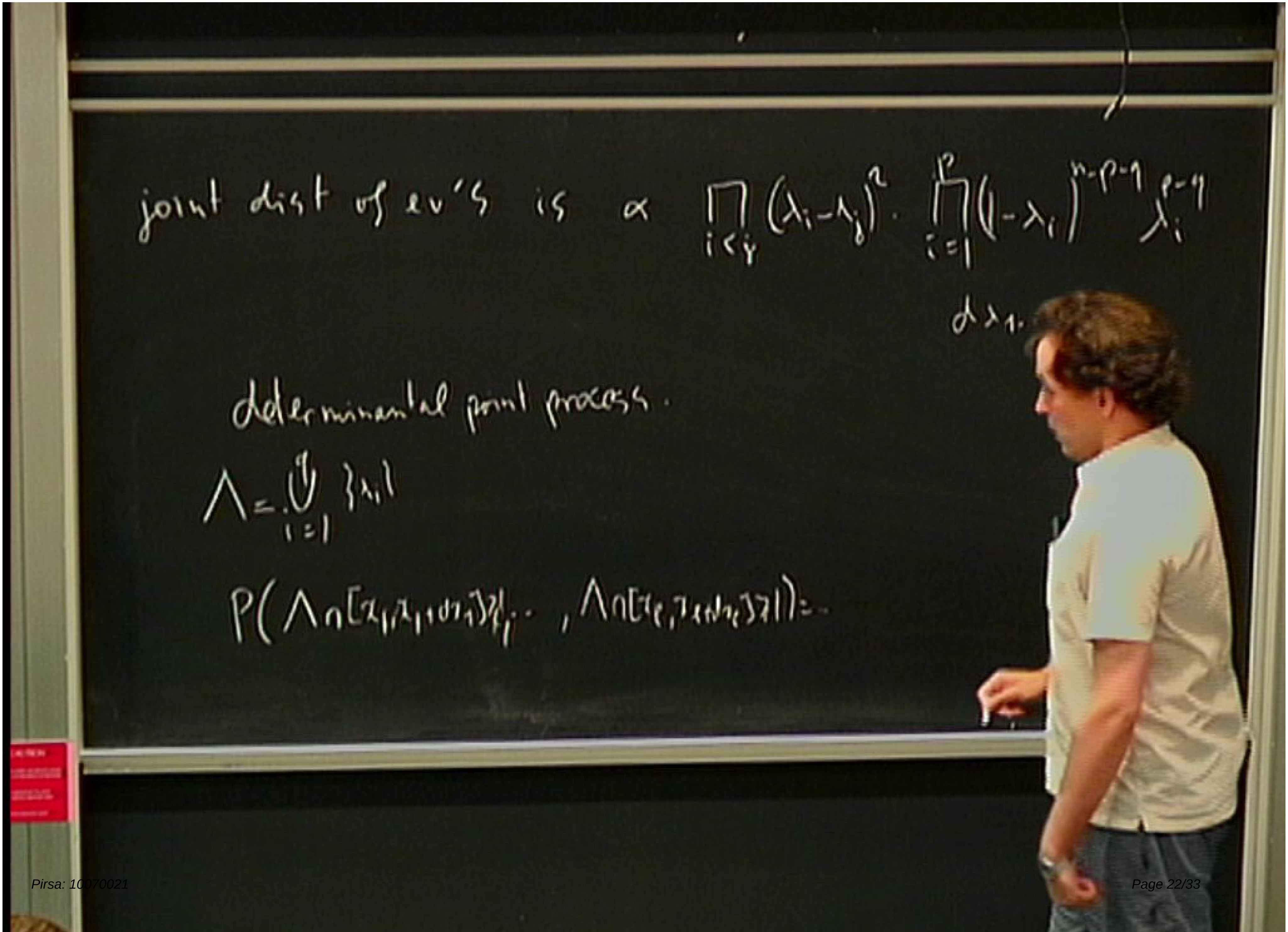
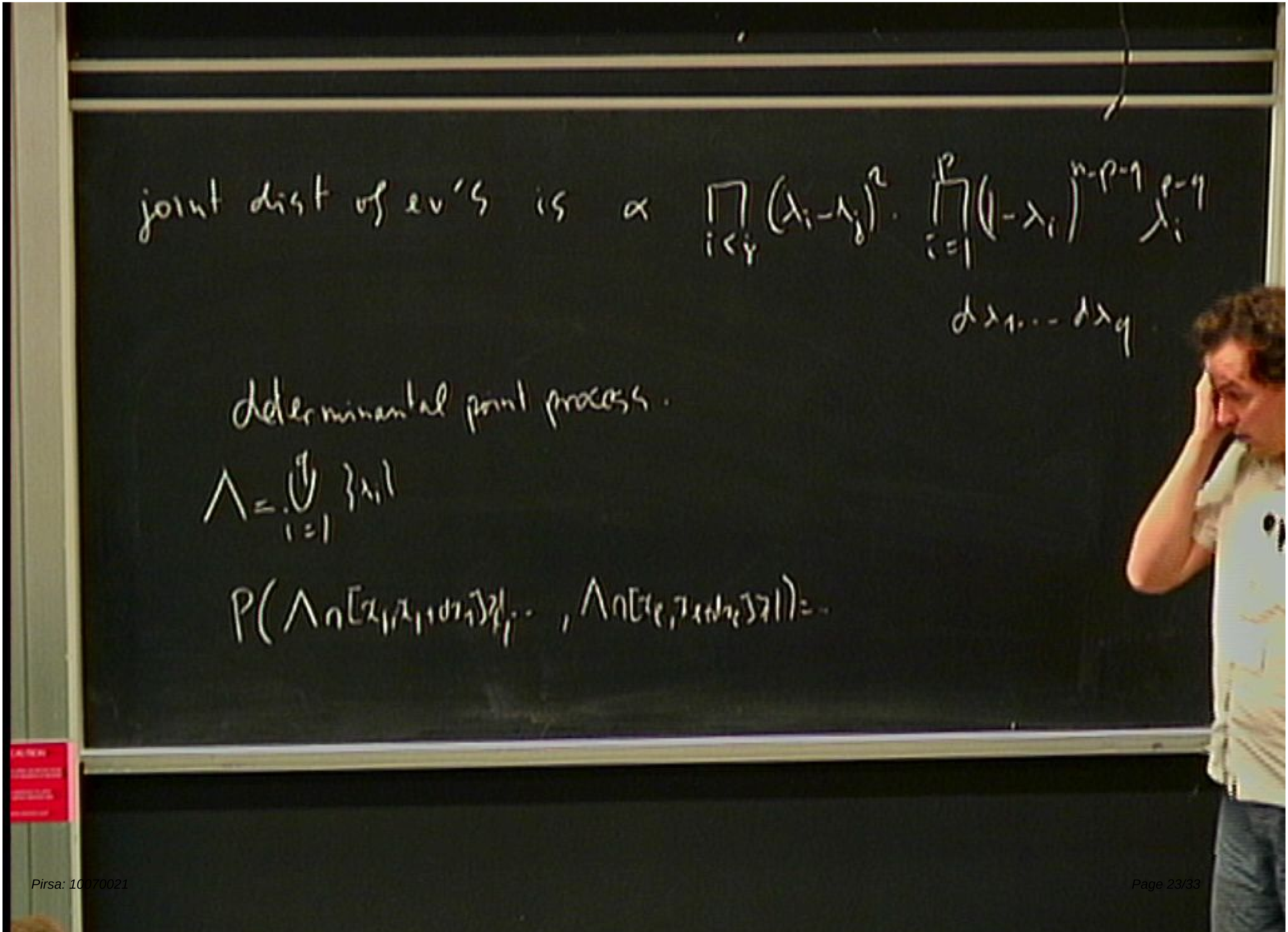
$d\lambda_1 \dots d\lambda_q$


joint dist of ev's is ∝ ∏_{i<j} (λ_i − λ_j)^2 · ∏_{i=1}^p (1 − λ_i)^{n−p−1} λ_i^{p−1}
dλ_1 ...
determinantal point process.
Λ = ⋃_{i=1}^p {λ_i}
P(Λ ∩ [x_1, x_1+dx_1] ≠ ∅, ..., Λ ∩ [x_k, x_k+dx_k] ≠ ∅) = ..


joint dist of ev's is $\propto \prod_{i<j} (\lambda_i - \lambda_j)^2 \cdot \prod_{i=1}^{p} (1-\lambda_i)^{n-p-q} \lambda_i^{p-q}$
$d\lambda_1 \dots d\lambda_q$
determinantal point process.
$\Lambda = \bigcup_{i=1}^{q} \{\lambda_i\}$
$P(\Lambda \cap [x_1, x_1+dx_1] \neq \emptyset, \dots, \Lambda \cap [x_k, x_k+dx_k] \neq \emptyset) = \dots$

joint dist of ev's is $\propto \prod_{i<j} (\lambda_i - \lambda_j)^2 \cdot \prod_{i=1}^{p} (1-\lambda_i)^{n-p-q} \lambda_i^{p-q}$

$d\lambda_1 \dots d\lambda_q$

determinantal point process.

$\Lambda = \bigcup_{i=1}^{q} \{\lambda_i\}$

$P(\Lambda \cap [x_1, x_1 + dx_1] \neq \emptyset, \dots, \Lambda \cap [x_k, x_k + dx_k] \neq \emptyset) = dx_1 \dots dx_k \det(K(x_i, x_j))$

$$K(x,y) = \sqrt{w(x)}\,\sqrt{w(y)}\,\sum_{i=0}^{q-1} p_i(x)\, p_i(y)$$

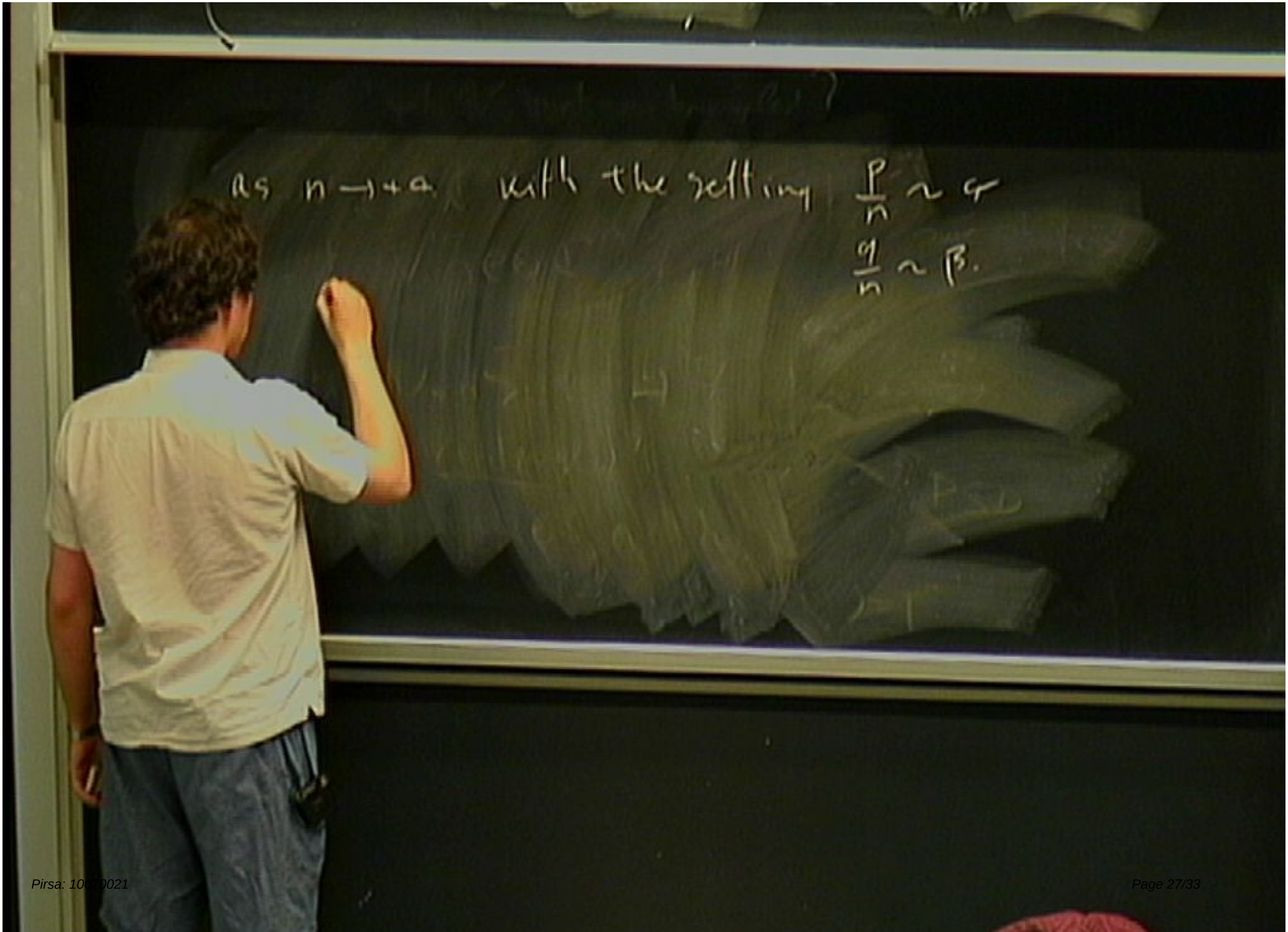
$$w(x) = (1-x)^{n-p-q}\, x^{p-q}$$

p_i onp's

$$K(x,y) = \sqrt{w(x)}\,\sqrt{w(y)} \sum_{i=0}^{q-1} p_i(x)\, p_i(y)$$

$$w(x) = (1-x)^{n-p-q}\, x^{p-q}$$

p_i onp's wrt w.


as n → +∞ with the setting p/n ~ α
q/n ~ β.

as $n \to +\infty$ with the setting $\frac{p}{n} \sim \alpha$

$\frac{q}{n} \sim \beta.$

- $\frac{1}{q} \sum_{i=1}^{q} \delta_{\lambda_i}$ converges.

- largest ev $\to \alpha + \beta - 2\alpha\beta + \sqrt{4\alpha\beta(1-\alpha)(1-\beta)}$

$\; = 1$

<u>Cor</u>: as soon as $t < 1 - k$?

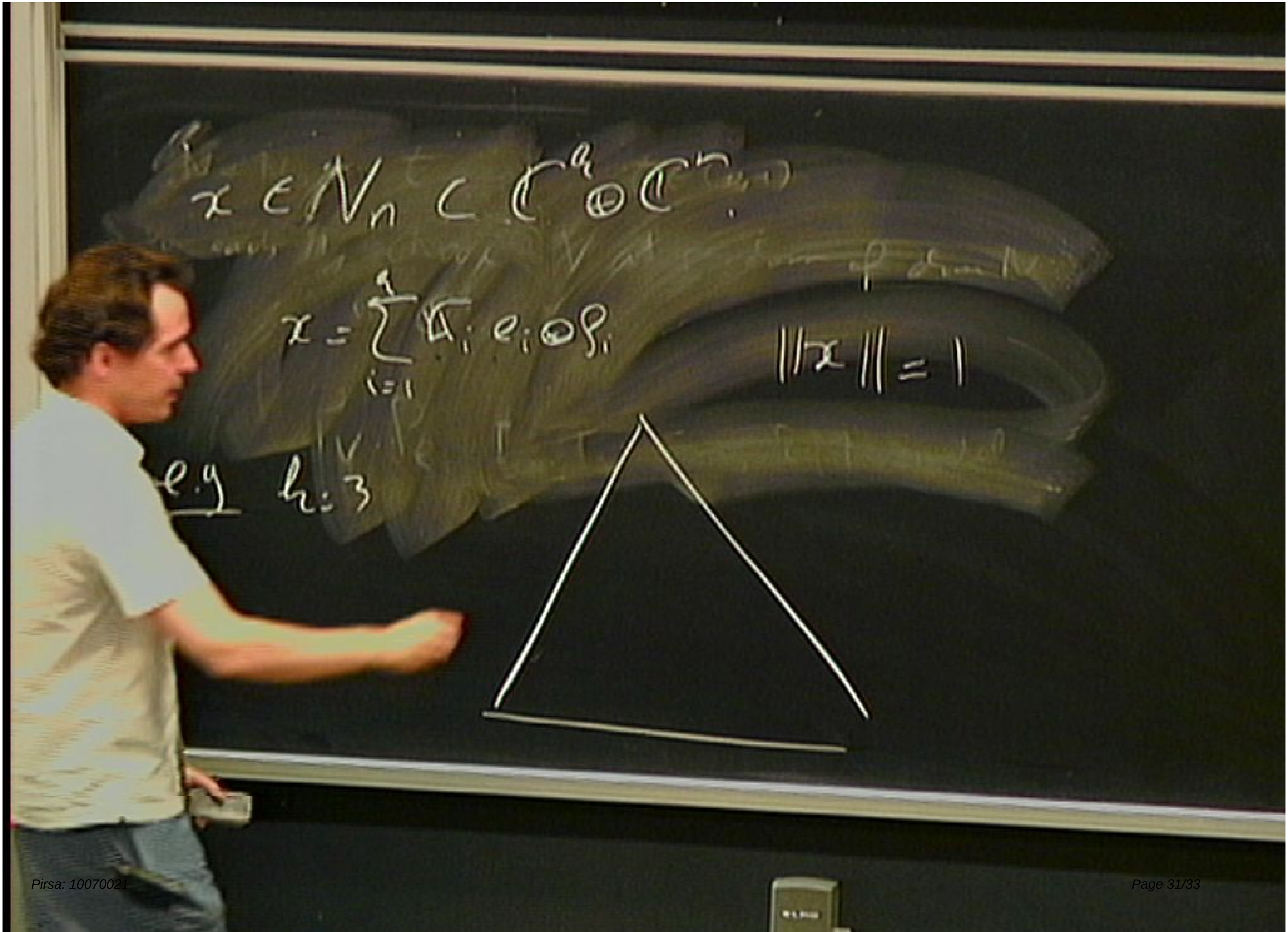
a.s, as $n \to +\infty$, $V_n \subset \mathbb{C}^k \otimes \mathbb{C}^n$

$\forall$ is entangled.

$N \sim t\,n\,k \qquad t \in (0,1).$

n, choose V at random of dim N.
in $\mathbb{C}^q \otimes \mathbb{C}^N$.

$P_V \cdot P_{e_i} \otimes I_n \cdot P_V \rightsquigarrow$ RM model.

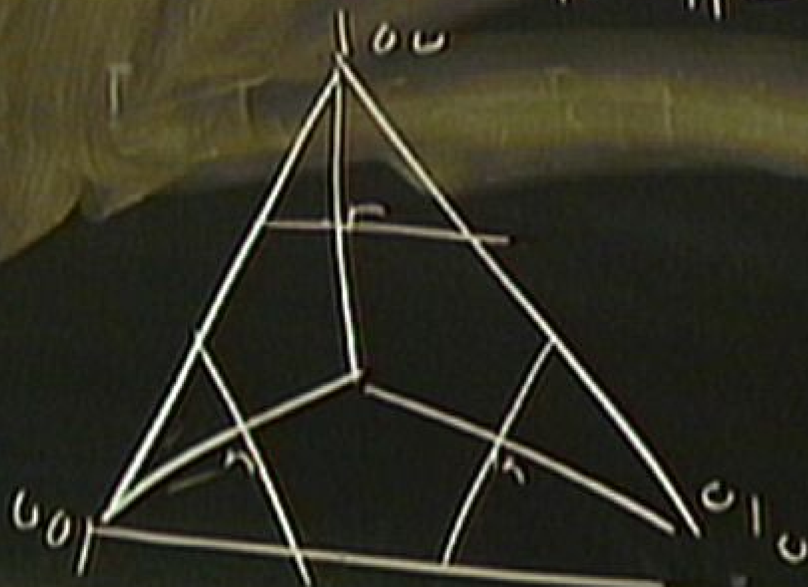


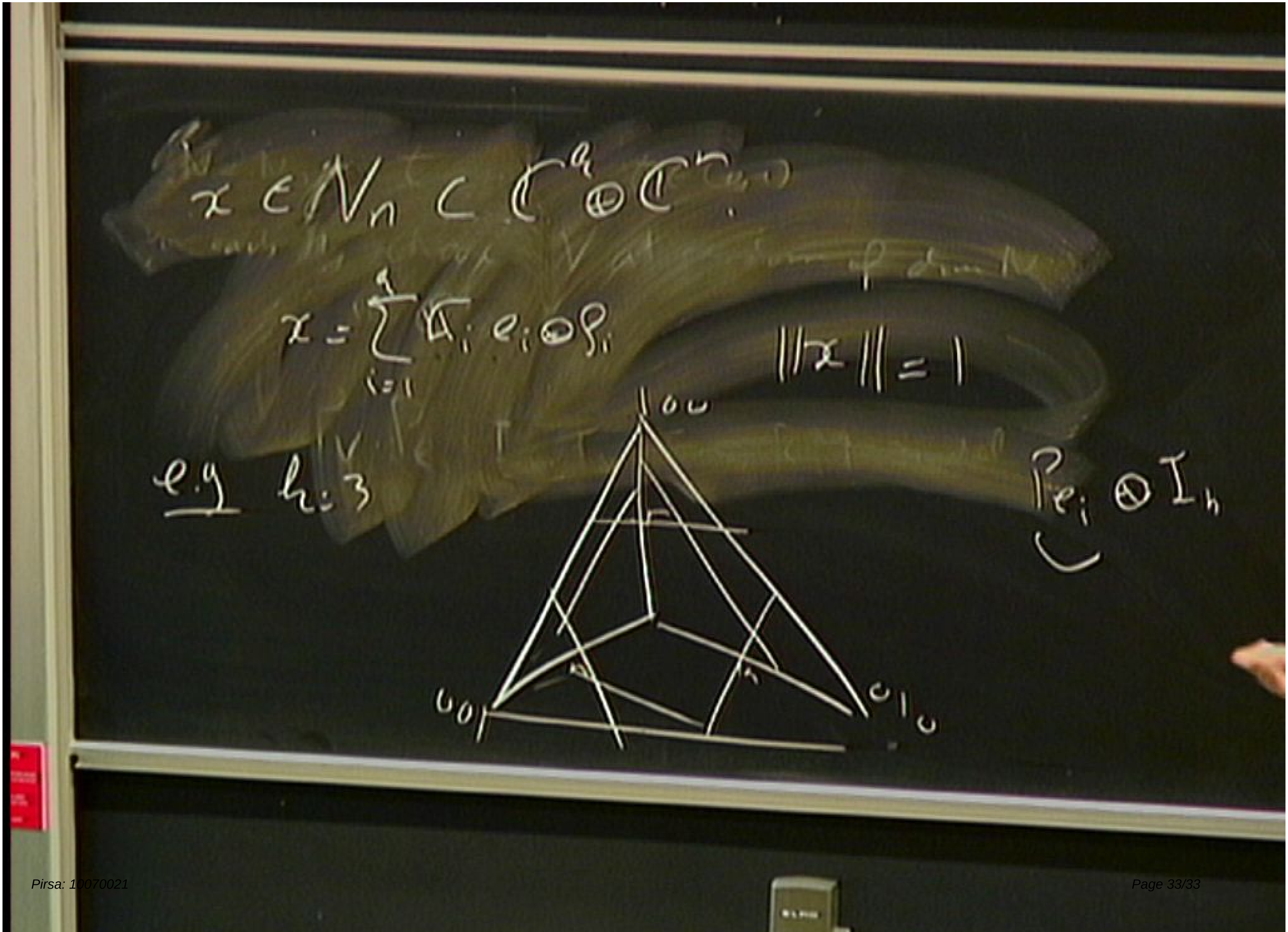
$x \in N_n \subset \mathbb{C}^q \otimes \mathbb{C}^n$
$x = \sum_{i=1}^{q} \sqrt{\lambda_i}\, e_i \otimes \rho_i$
$\|x\| = 1$
e.g. $q = 3$

$x \in N_n \subset \mathbb{C}^q \otimes \mathbb{C}^n$

$x = \sum_{i=1}^{q} \sqrt{\lambda_i}\, e_i \otimes \rho_i$ $\|x\| = 1$

e.g. $n=3$




$x \in N_n \subset \mathbb{C}^d \otimes \mathbb{C}^n$
$x = \sum_{i=1}^{d} \alpha_i e_i \otimes \rho_i$
$\|x\| = 1$
e.g. $n=3$
100
001
010
$P_{e_i} \otimes I_n$