Title: Hausdorff and spectral dimension of random graphs

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Abstract: We introduce a class of probability spaces whose objects are infinite graphs and whose probability distributions are obtained as limits of distributions for finite graphs. The notions of Hausdorff and spectral dimension for such ensembles are defined and some results on their value in koncrete examples, such as random trees, will be described.

HAUSDORFF AND SPECTRAL DIMENSION OF RANDOM GRAPHS

B. DURHUUS

"Random Matrix Techniques in Quantum Information Theory", Perimeter Institute, July 4, 2010.

Some motivation:

1) Random graphs in discretized gravity.

   2 dim: Matrix modes ↔ Topological gravity

   Intrinsic geometry ?

   Ref: J. Ambjørn, B.D., T. Jonsson, Quantum Geometry, Cambridge Univ. Press, 1997.

2) Random media. Anomalous diffusion.

3) Percolation on a fixed graph, e.g. $\mathbb{Z}^d$

   ↪ Random percolation clusters.

4) Structure of networks.

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2. Basic concepts.

- Graph G. $V(G)$ = vertex set
  $E(G)$ = edge set

d_v = degree of $v < \infty$

- Path in G: $(v_0, v_1), (v_1, v_2), \dots, (v_{k-1}, v_k)$
  all v_i different.

  Circuit = closed path $(v_0 = v_k)$.

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  Hausdorff dimension of G connected and infinite:

  $$d_h = \lim_{R \to \infty} \frac{\ln |B_R(G, v_0)|}{\ln R}$$

  Indep. of v_0, but not well-defined for all G.

- A walk in G $\omega : v_0 \to v_k$ is a sequence of edges $(v_0, v_1), (v_1, v_2), \ldots, (v_{k-1}, v_k)$ with v_i's not necess. distinct.

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ρ defines a probability distribution

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$$p(\omega) = \prod_{i=0}^{k-1} \sigma_{\omega(i)}^{-1}$$

where $\omega(i)$ = i'th vertex in ω.

p defines a probability distribution on walks of fixed length and starting point:

Hausdorff dimension of G connected and infinite:

$$d_h = \lim_{R\to\infty} \frac{\ln |B_R(G, v_0)|}{\ln R}$$

Indep. of v_0, but not well-defined for all G.

- A walk in G $\omega : v_0 \to v_k$ is a sequence of edges $(v_0, v_1), (v_1, v_2), \ldots, (v_{k-1}, v_k)$ with v_i's not necess. distinct.

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where $\Pi_n(v_0)$ = set of walks of length n emerging from v_0. These probability distributions define the simple random walk on G.

For G connected: $p_t(G, v_0)$ = return probability to v_0 after time $t \geq 0$

$$p_t(G, v_0) = \sum_{\substack{\omega: v_0 \to v_0 \\ |\omega| = t}} p(\omega).$$

Spectral dimension for G connected and infinite:

$$d_s = -2 \lim_{t \to \infty} \frac{\ln p_t(G, v_0)}{\ln t}$$

Indep. of v_0 but not well-defined for all G.

Note: $\qquad d_h \geq 1, \quad d_s \geq 1 \quad$ all G.

$$d_h = d_s = d \quad \text{for } G = \mathbb{Z}^d.$$

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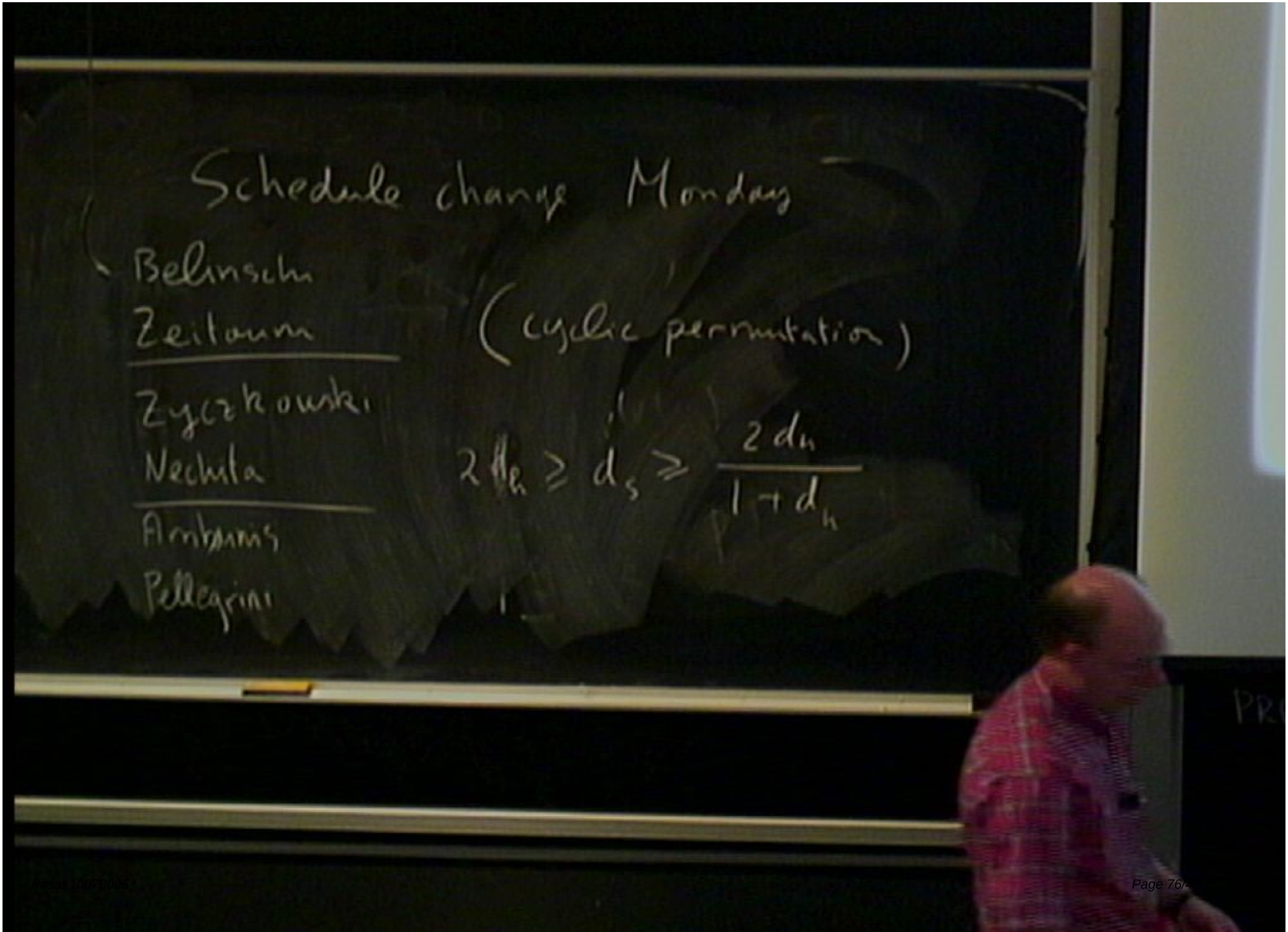
Schedule change Monday
Belinschi
Zeitouni
(cyclic permutation)
Zyczkowski
Nechita
Pellegrini
PR

⑤

- Ensemble of graphs = random graph = set of graphs $\mathcal{G}$ with a probability measure μ.

  $\langle \cdot \rangle_\mu$ = expectation w.r.t. μ.

  Assume graphs are rooted with root vertex r:

  $$B_R(G) = B_R(G, r) \quad , \quad p_t(G) = p_t(G, r).$$

  Hausdorff and spectral dimension of random graph:

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- Generally obtain μ as a limit of probability distributions μ_N defined on finite sets of finite graphs $\mathcal{G}_N \subseteq \mathcal{G}$.

Consider $\mathcal{G}$ as metric space:

$$d(G, G') = \inf\{(R+1)^{-1} \mid B_R(G) = B_R(G')\}$$

(μ_N) converges to μ if

$$\int_{\mathcal{G}} f(G)\, d\mu_N(G) \to \int_{\mathcal{G}} f(G)\, d\mu(G)$$

for bounded continuous functions f.

Equivalent to

$$\mu_N(\{B_R(G) = G_0\}) \to \mu(\{B_R(G) = G_0\})$$

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$0 \le p \le 1$ fixed.

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on configurations $(c_e)_{e \in E(G)}$ in $\{0,1\}^{E(G)}$.

Given a configuration c define the cluster C_r as the maximal connected subgraph of G containing r such that c assumes value 1 on all edges in C_r.

The case $G = \mathbb{Z}^d$, $d \geq 2$: Choose $r = \underline{0}$.

There exists a critical probability $p_{cr} \in]0,1[$ such that

$$\varrho(|C_r| = \infty) \begin{cases} = 0 & \text{if } p < p_{cr} \\ > 0 & \text{if } p > p_{cr}. \end{cases}$$

For $d = 2$ or $d \geq 19$ (but expected to hold for all $d \geq 2$):

$$\varrho_{p_{cr}}(|C_r| = \infty) = 0$$

i.e. percolation cluster C_r is almost surely finite at percolation threshold.

For $p < p_{cr}$

$$\varrho_p(|C_r| = n) \sim e^{-\alpha(p) \cdot n}, \quad n \to \infty,$$

where $\alpha(p) > 0$. Hence mean cluster size

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Let $\mathcal{C}\ell$ denote the set of clusters containing $\underline{0}$, considered as a set of graphs.

The incipient infinite cluster $= (\mathcal{C}\ell, \mu)$ where

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provided the limit exists.

$d = 2$: Existence shown by H. Kesten '86.

d large: Existence shown by R. van der Hofstad & A.A. Járai 2004.

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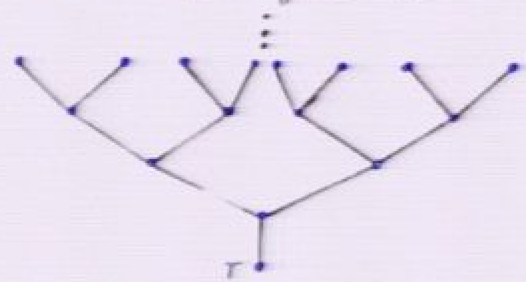
4. The case G = Cayley graph Γ ($d=\infty$)

Assume vertices of Γ have degree n except root of degree 1.



Cayley tree with $n=3$

For a (non-trivial) rooted subtree T of Γ with $1 \le |T| < \infty$:

$$\varrho_p(C_r = T) = p \prod_{v \in T \setminus r} \binom{n-1}{d_v - 1} p^{d_v - 1} (1-p)^{n-d_v}$$

Hence, conditioning on configurations with root edge of value 1 we get the distribution of Galton-Watson type

$$\varrho(T) = \prod_{v \in T \setminus r} p_{d_v - 1} ,$$

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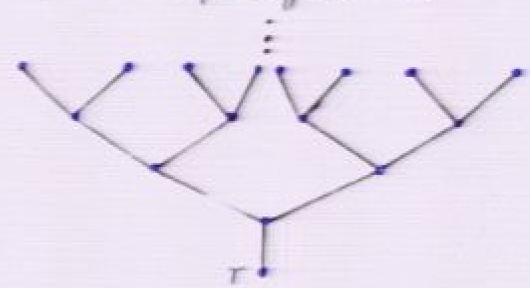

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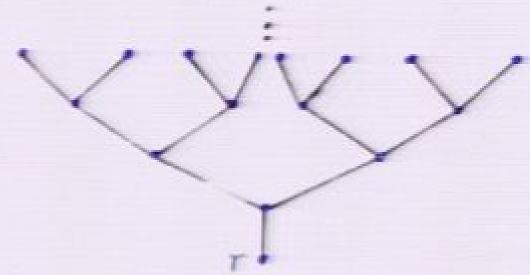

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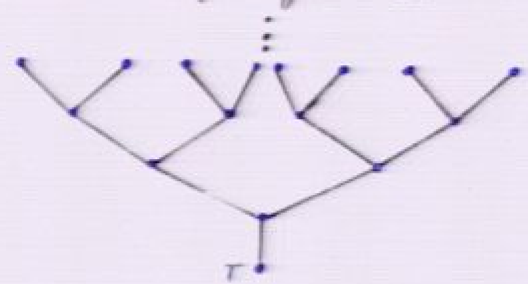

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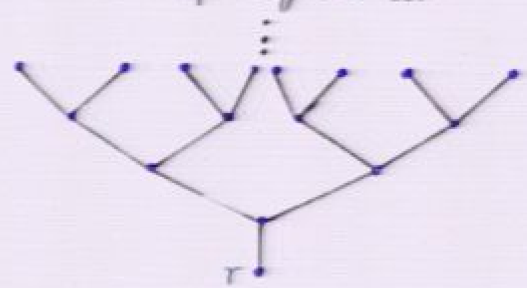

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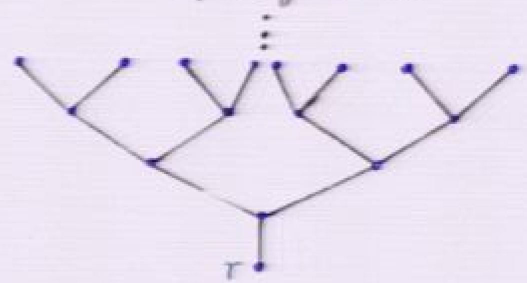

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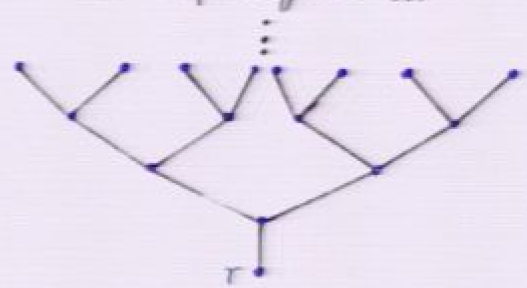

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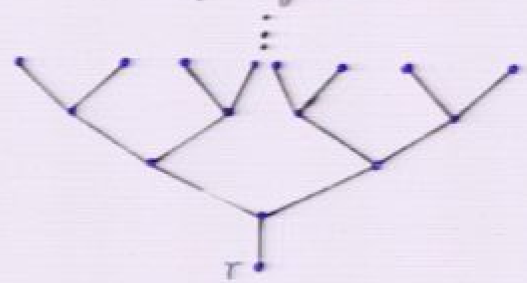

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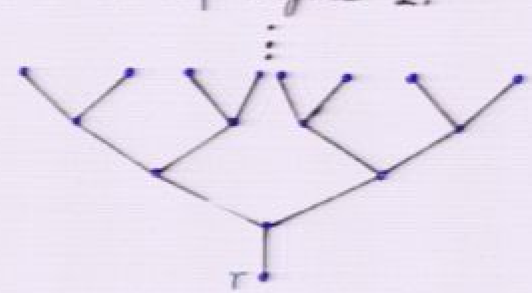

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Hence, conditioning [illegible] configurations with root edge of [illegible] 1 we get the distribution of Ga[illegible]-Watson type

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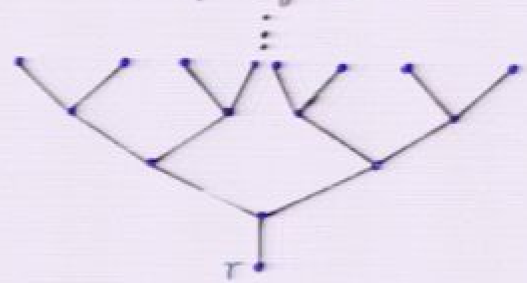

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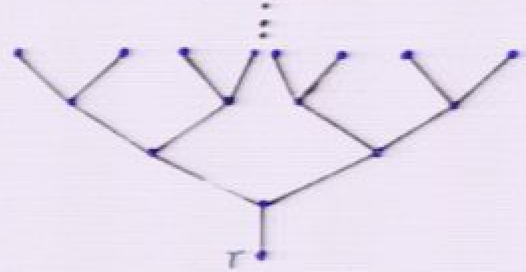

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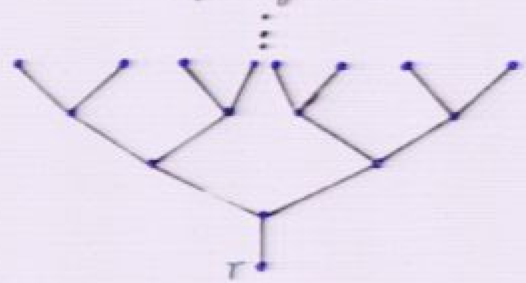

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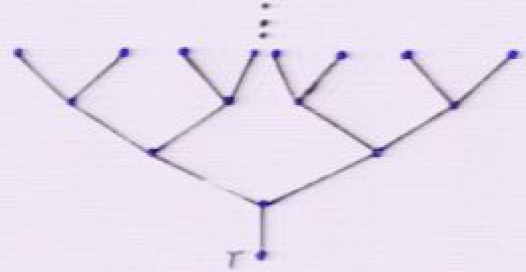

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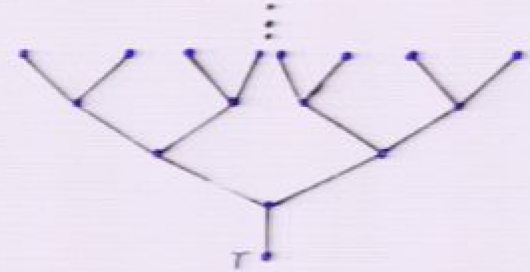

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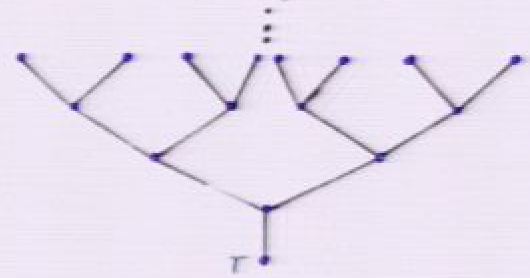

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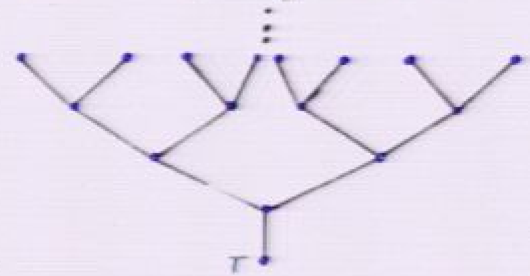

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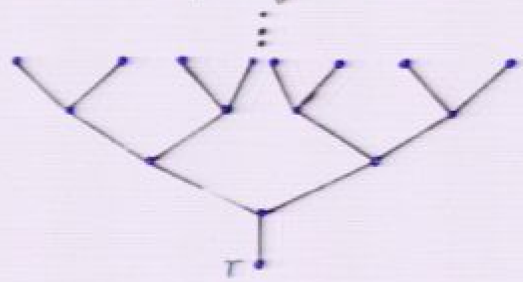

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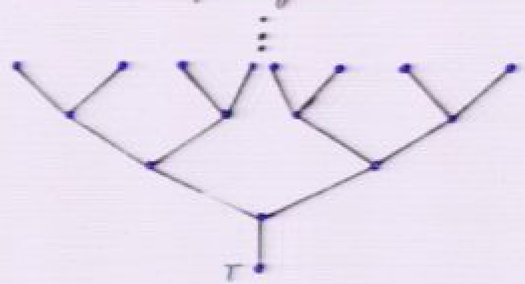

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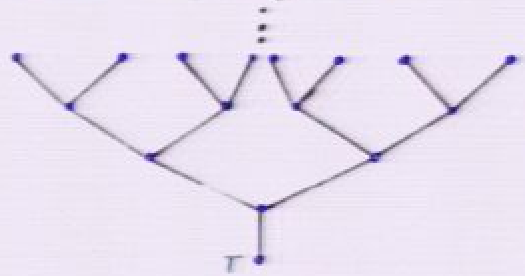

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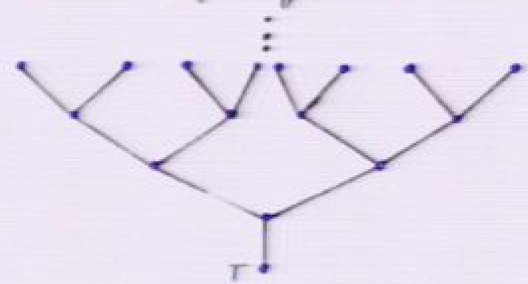

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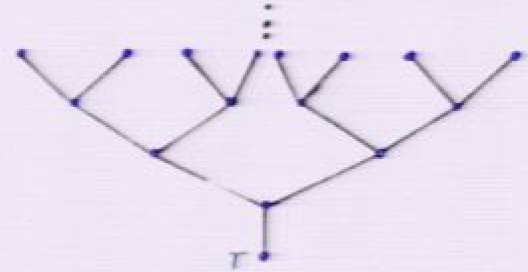

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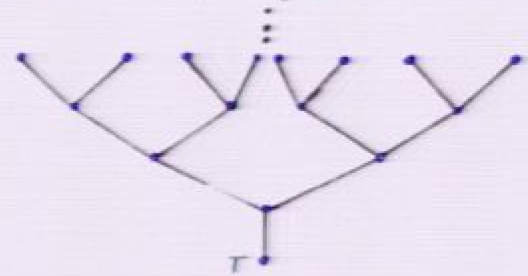

Cayley tree with $n=3$

For a (non-trivial) rooted subtree T of Γ with $1 \le |T| < \infty$:

$$\varrho_p(C_r = T) = p \prod_{v \in T\setminus r} \binom{n-1}{d_v - 1} p^{d_v - 1} (1-p)^{n-d_v}$$

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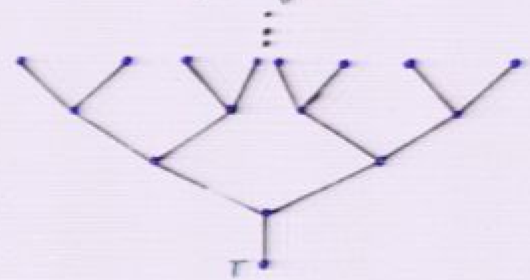

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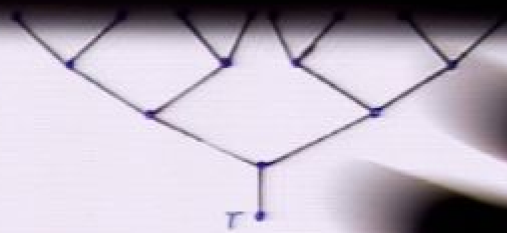

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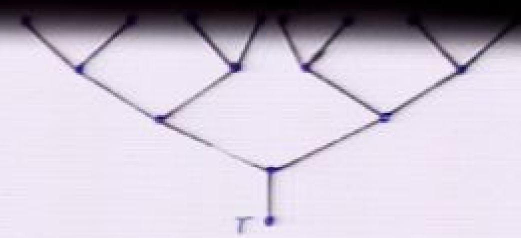

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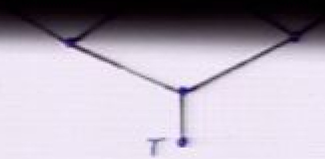

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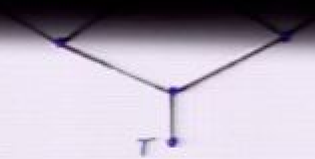

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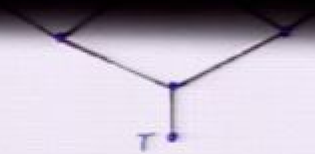

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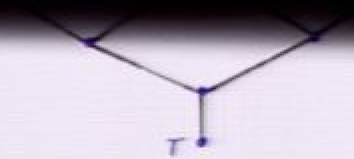

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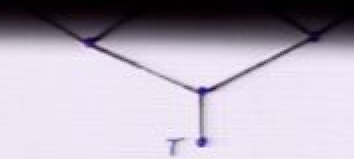

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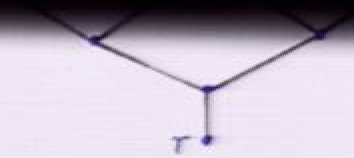

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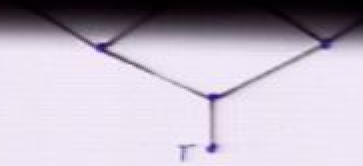

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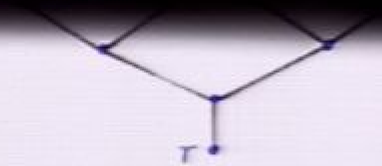

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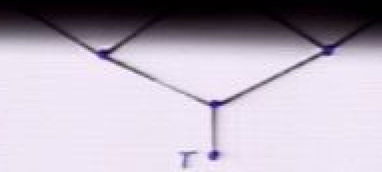

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[illegible]

Hence [illegible] with [illegible] edge of [illegible] the distribution of [illegible]

[illegible]

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[illegible]

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Set

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Generating functions

$$Z(t) = \sum_{N=1}^{\infty} Z_N t^N$$

and

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$$Z(t) = \sum_{N=1}^{\infty} Z_N t^N$$

and

$$f(t) = \sum_{k=0}^{\infty} p_k t^k$$

are related by

$$\boxed{Z(t) = t\, f(Z(t))}$$

For Γ: $\quad f(t) = (1-p+tp)^{n-1} = f_p(t)$

Determine p_{cr}:

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For Γ: $f(t) = (1-p+tp)^{n-1} = f_p(t)$

Determine p_{cr}:

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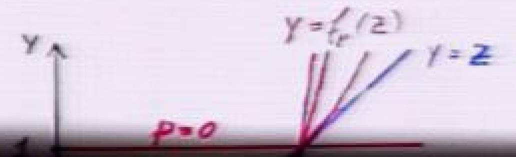

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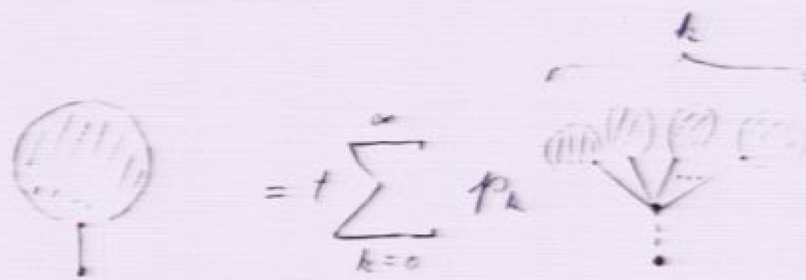
(11')



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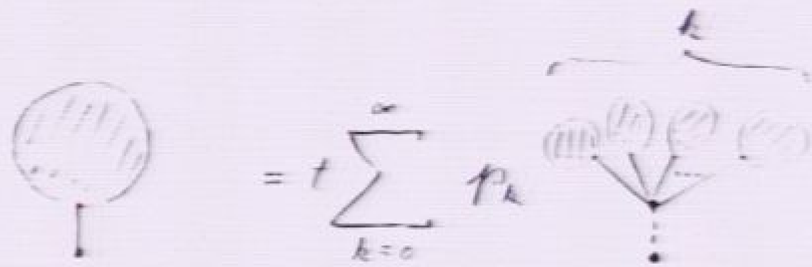
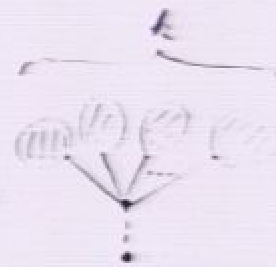


(11')

$$= t \sum_{k=0}^{\infty} p_k$$



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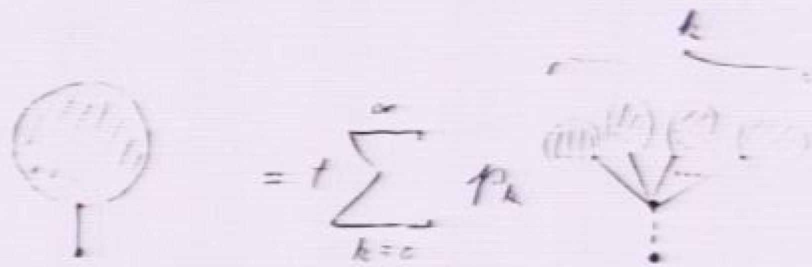

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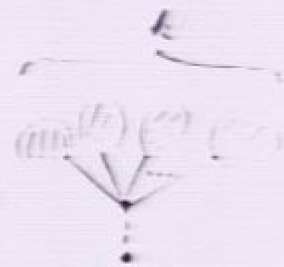

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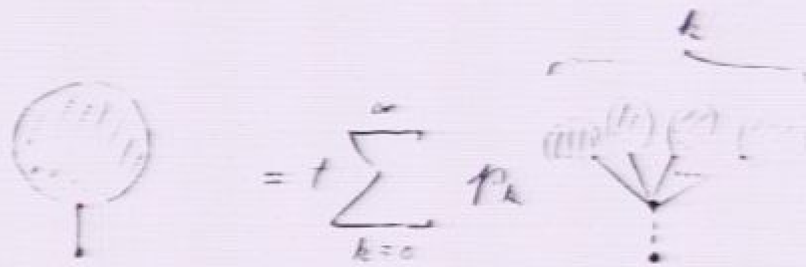

(11')

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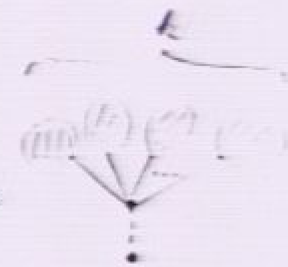
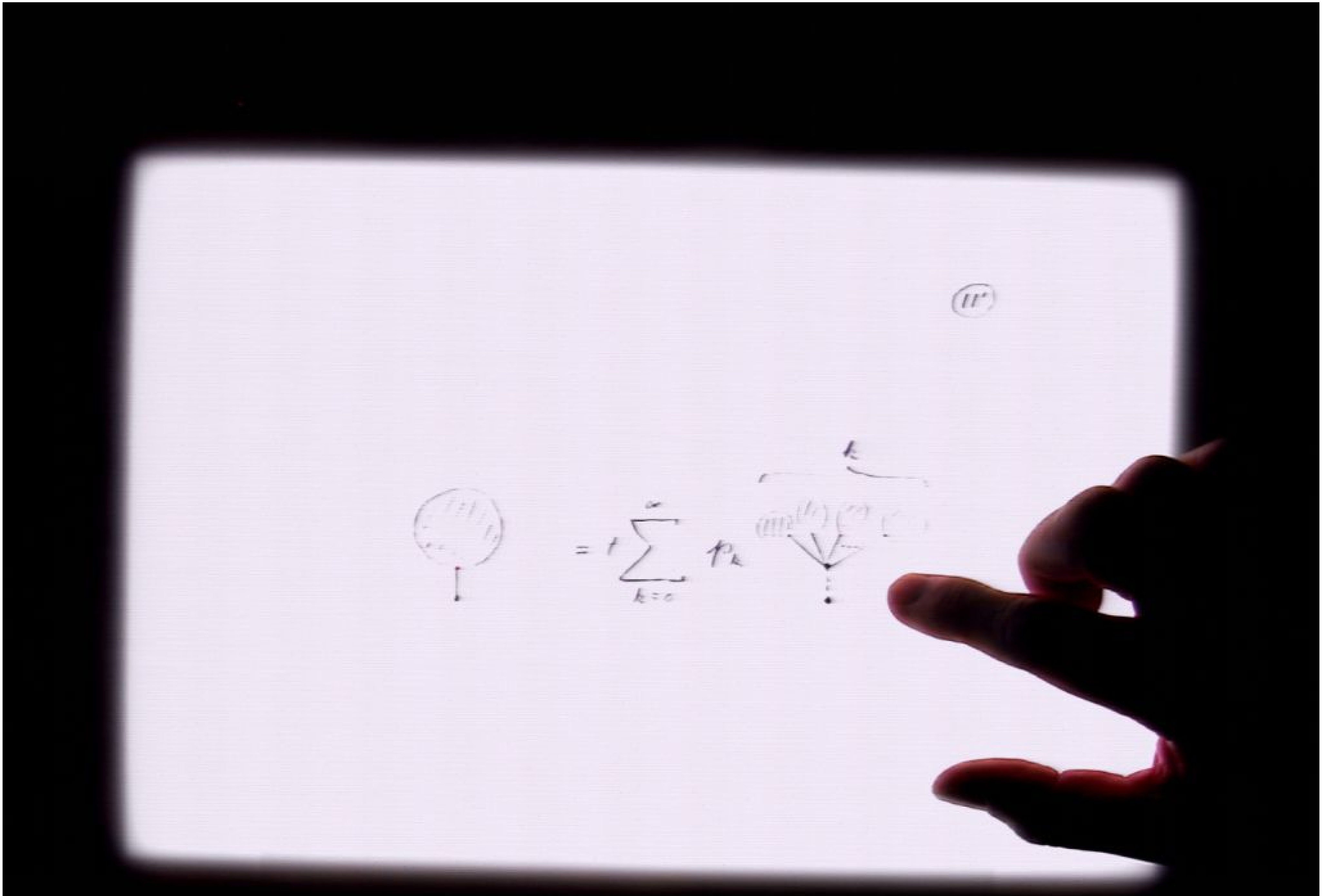
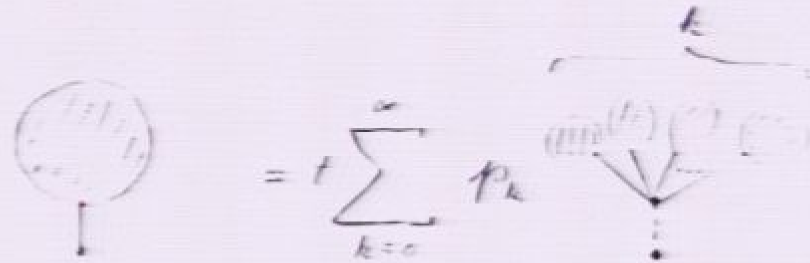
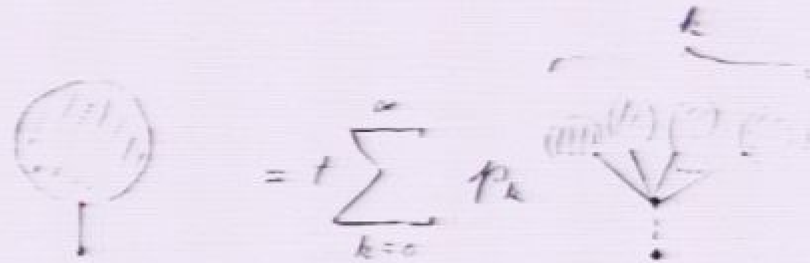



(II')
k
$= t \sum_{k=0}^{\infty} p_k$

(11')





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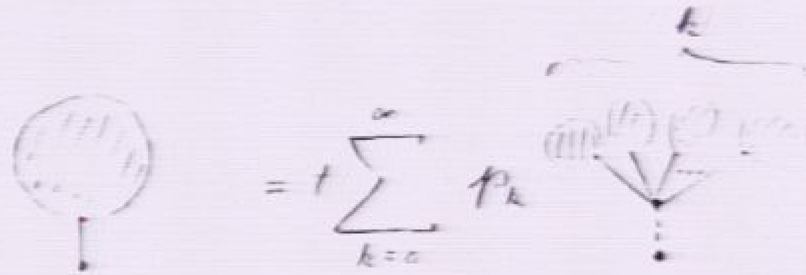
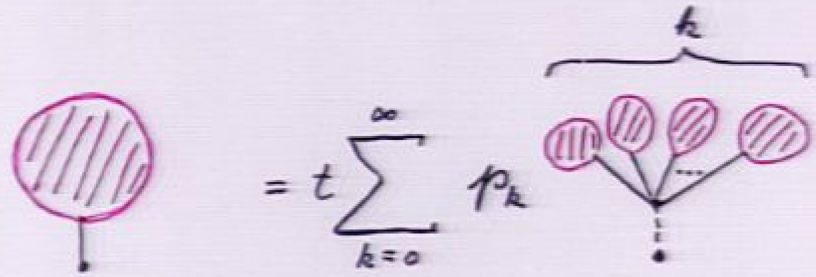


$$= t \sum_{k=0}^{\infty} f_k$$

(11')



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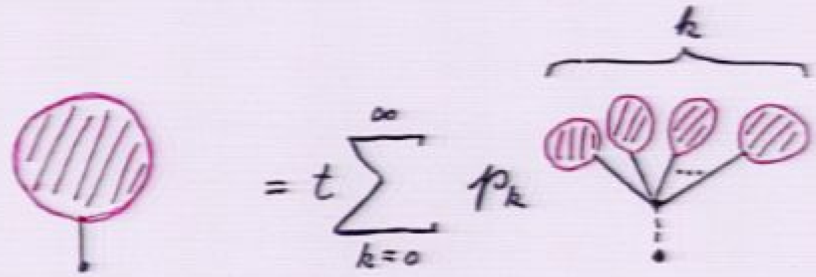

11'

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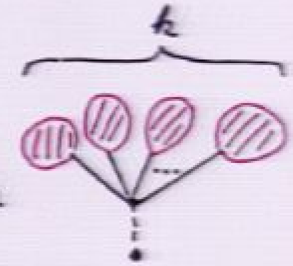

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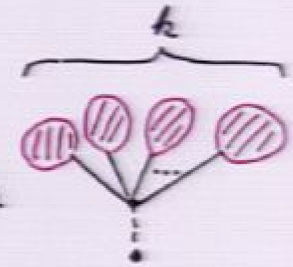

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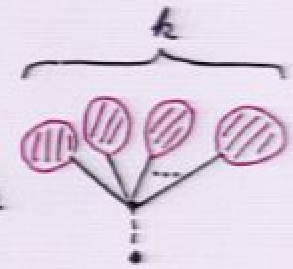
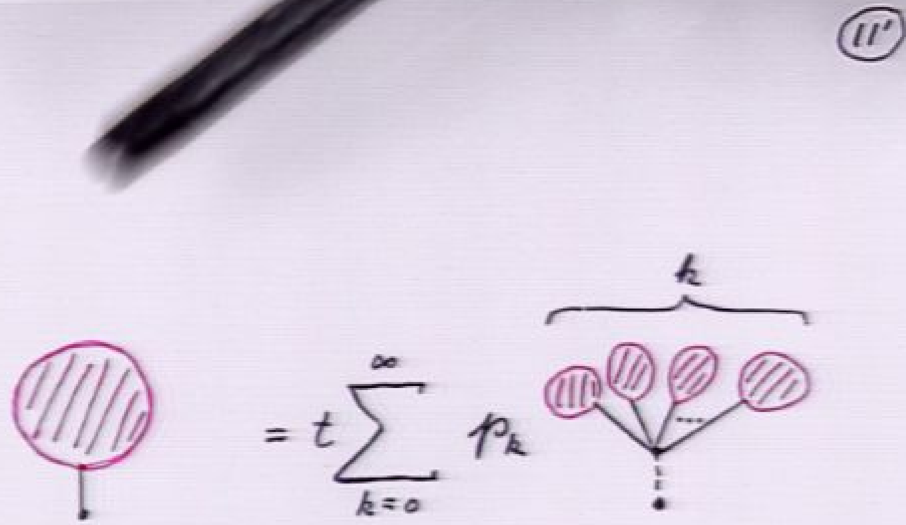
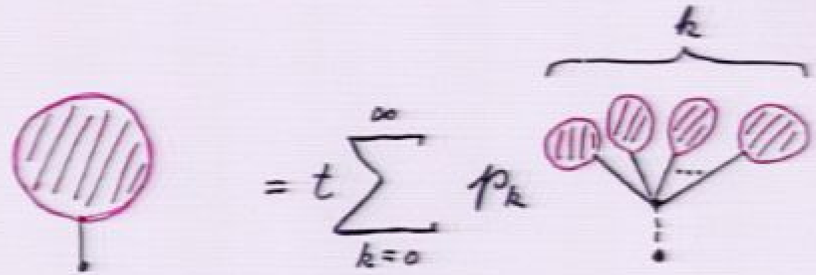
 $= t \sum_{k=0}^{\infty} p_k$ 


11'
k
= t
∞
k=0
p_k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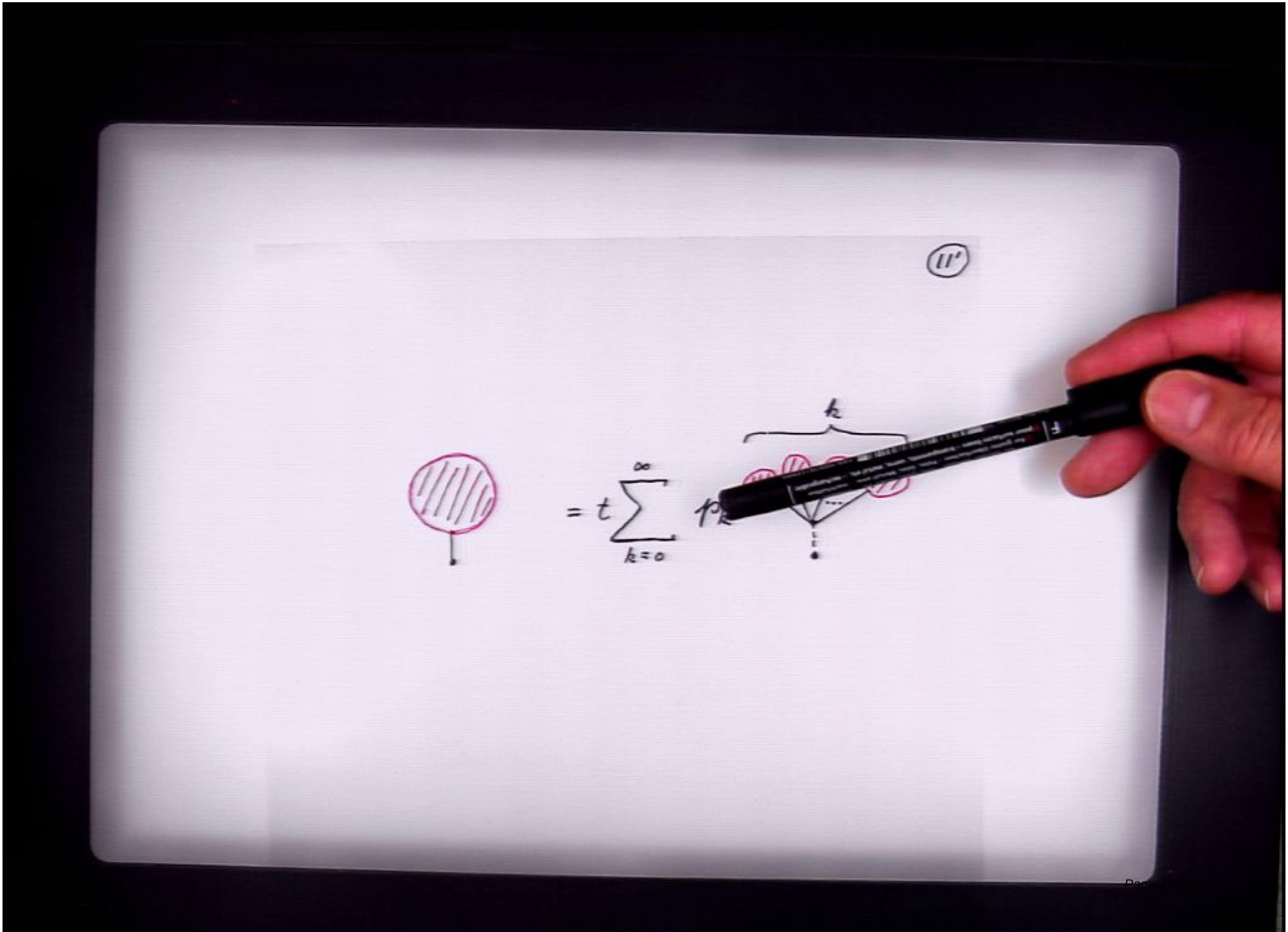
(11')
k
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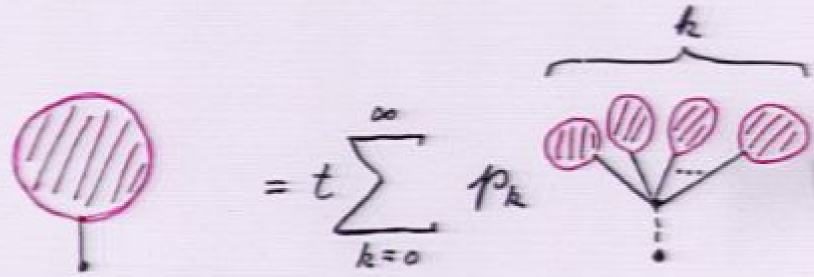
(11')



(11′)

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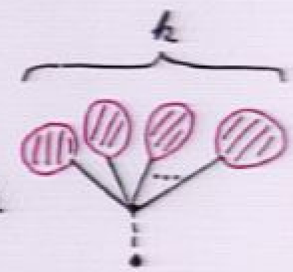

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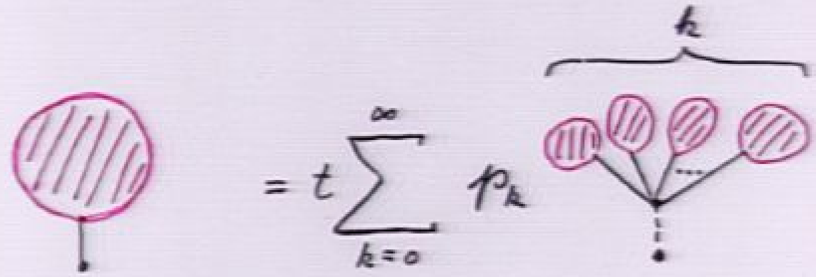

(41)

Generating f[illegible]

$$Z(t) = \sum_{n=1}^{\infty} Z_n t^n$$

and

$$f(t) = \sum_{k=0}^{\infty} f_k t^k$$

are related by

$$\boxed{Z(t) = t\, f(Z(t))}$$

Eg: P: $f(t) = (1-p+tp)^{n-1} = f_p(t)$

$T: |T| = N$

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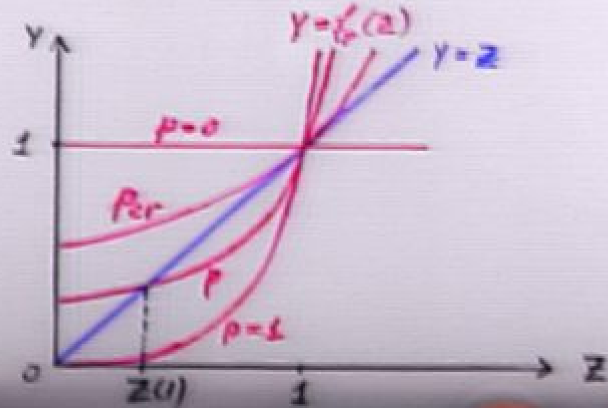

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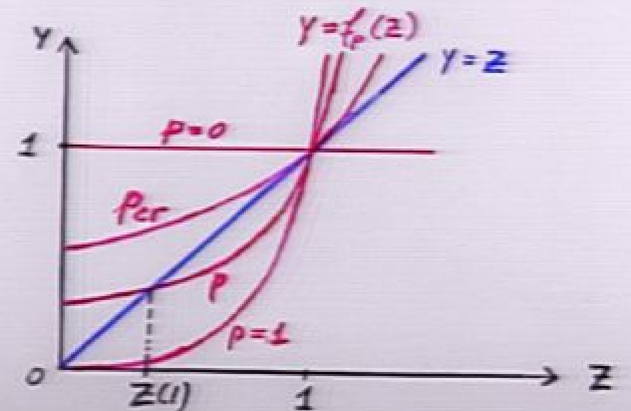

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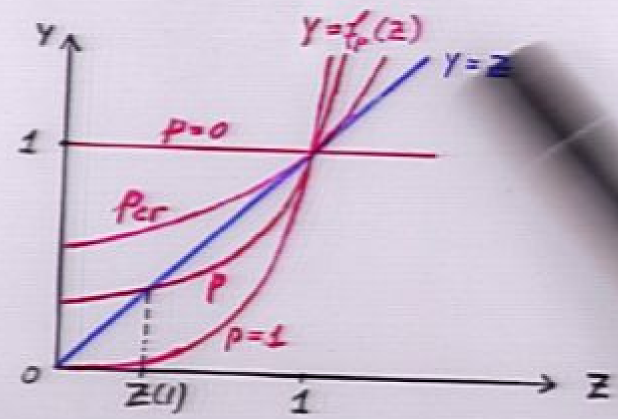

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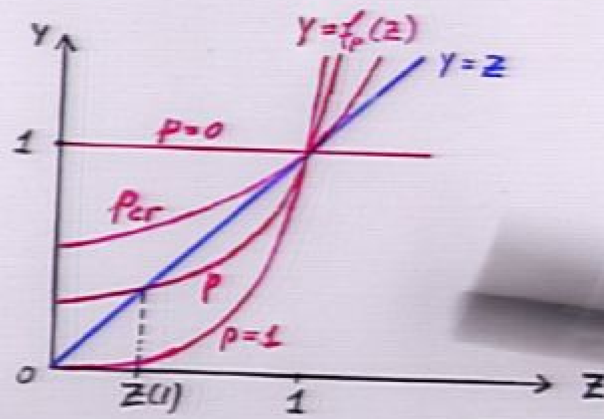

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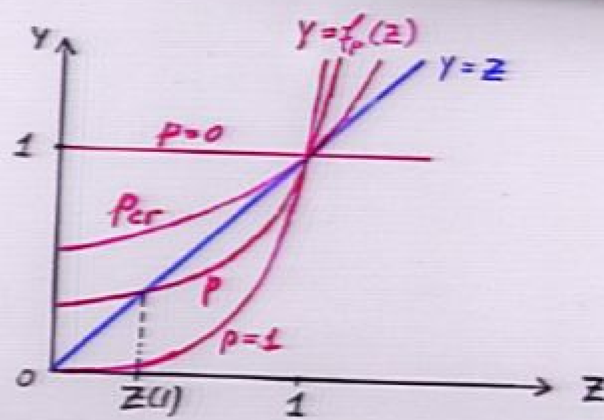

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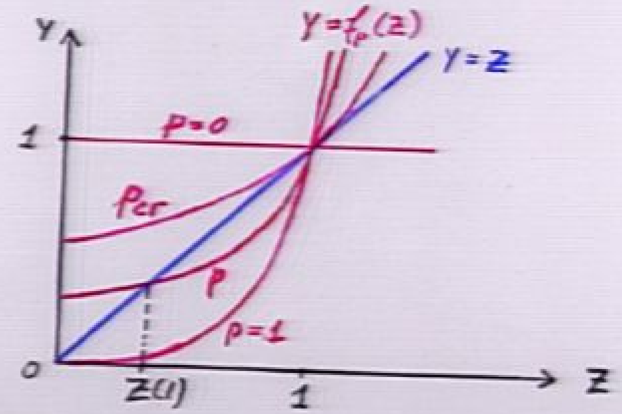

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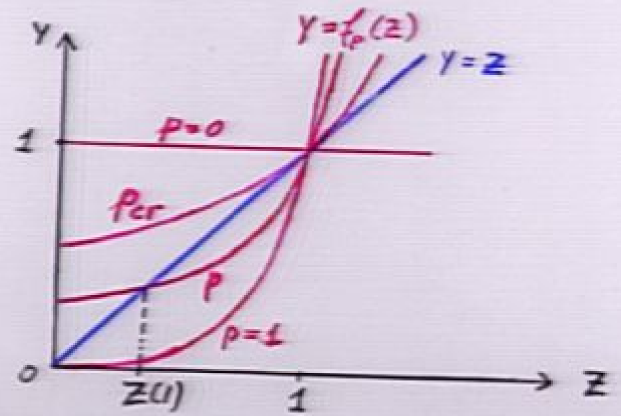

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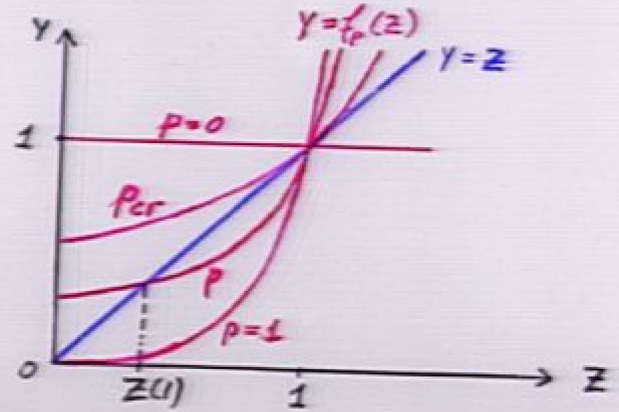

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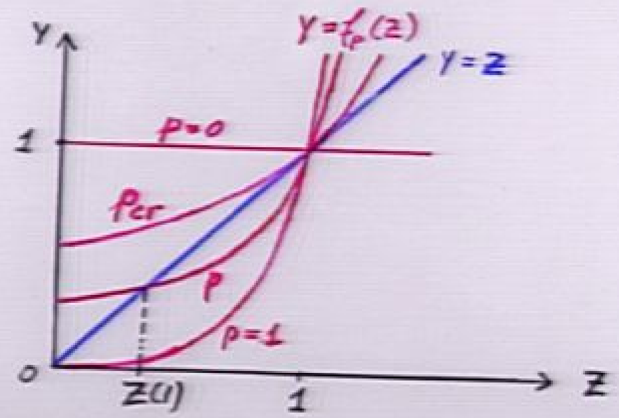

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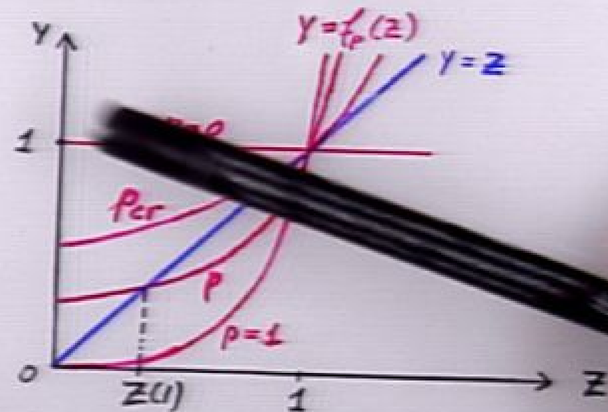

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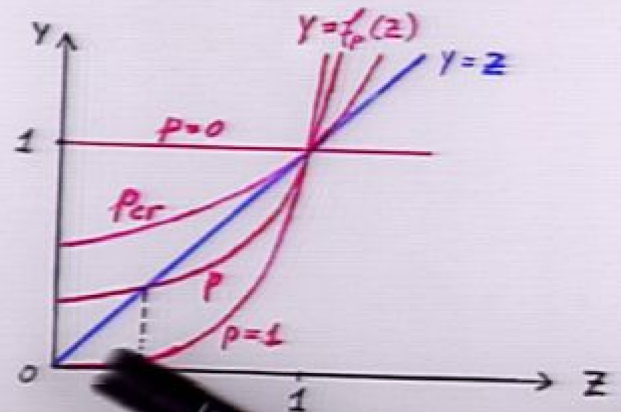

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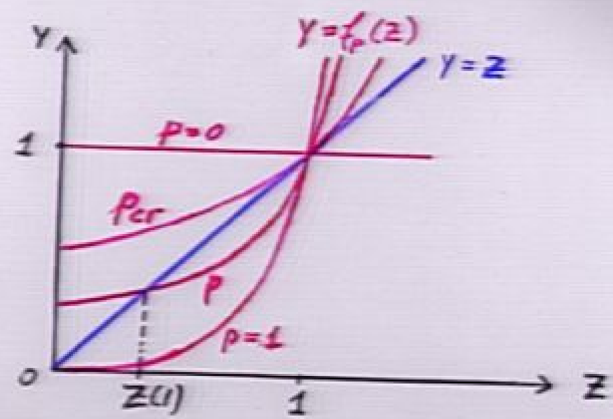

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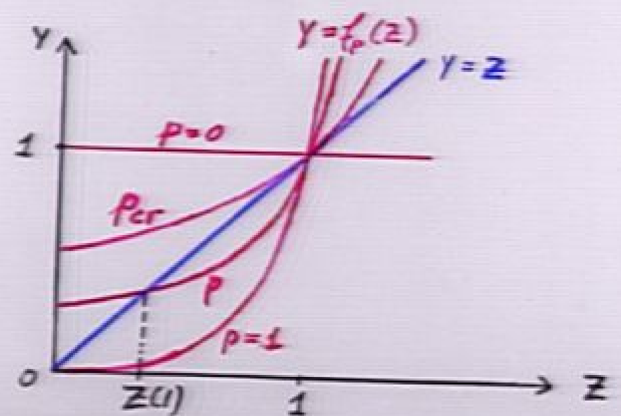

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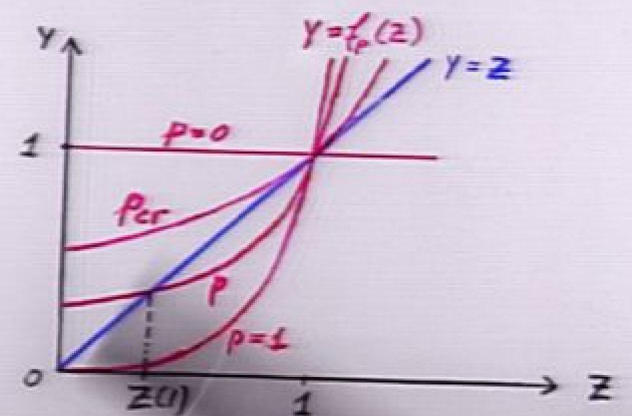

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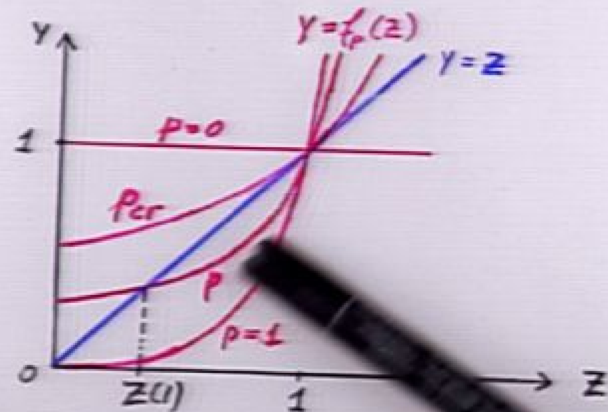

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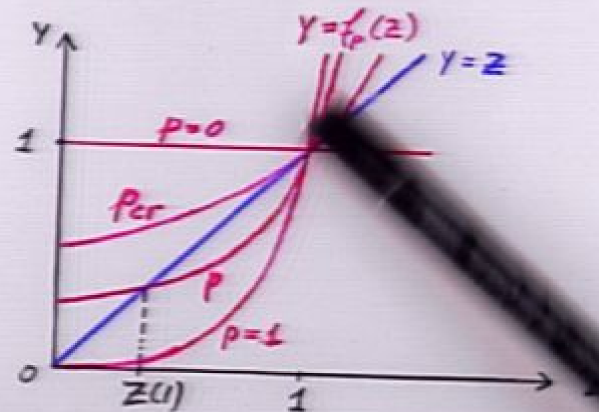

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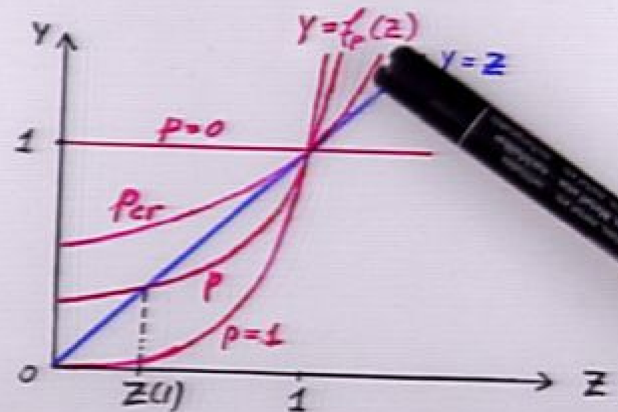

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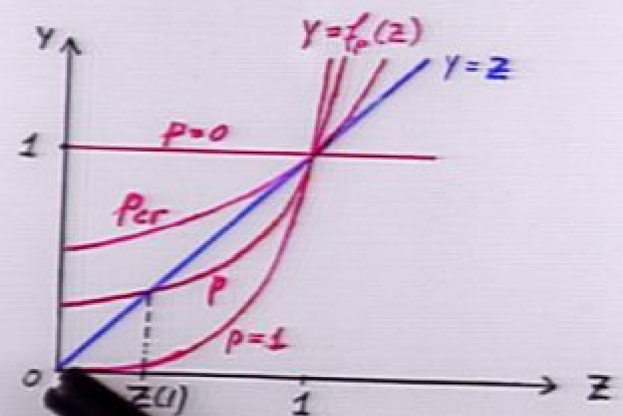

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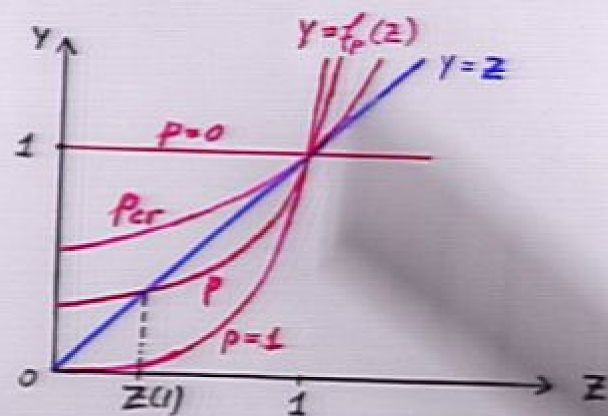

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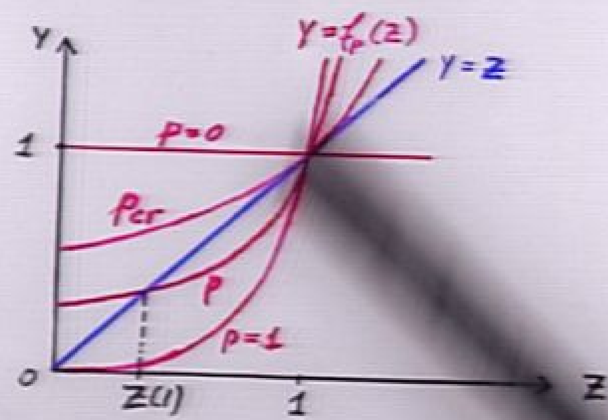

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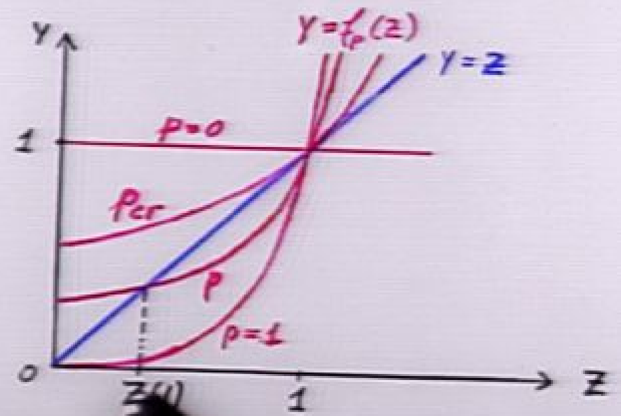

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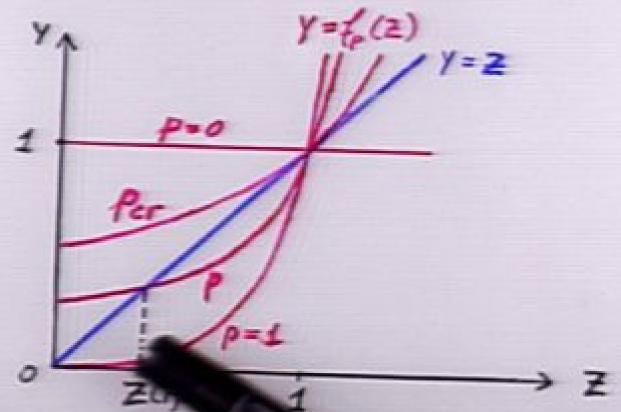

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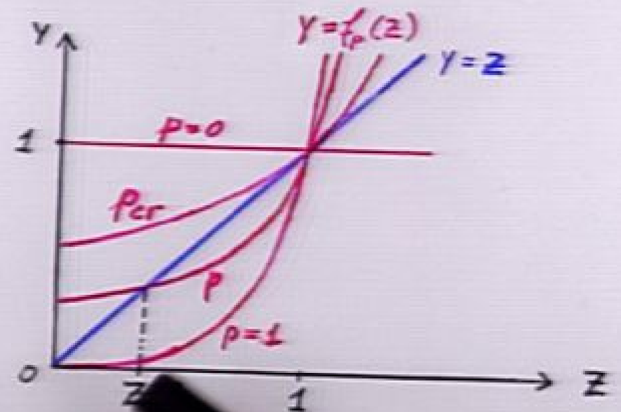

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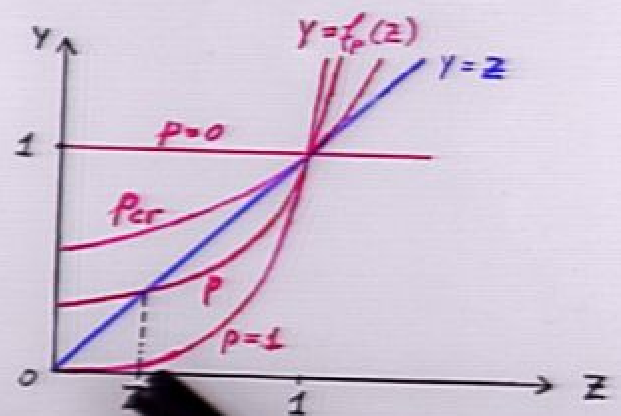

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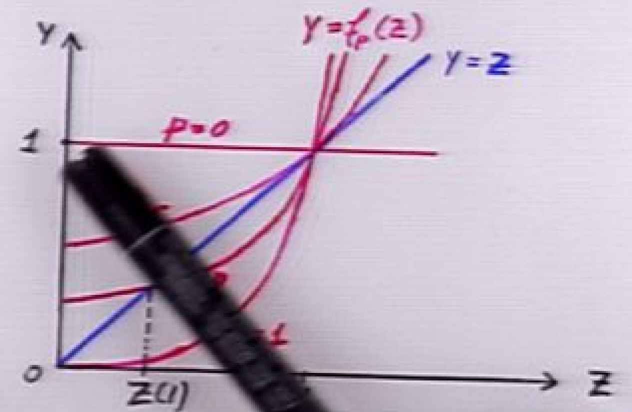

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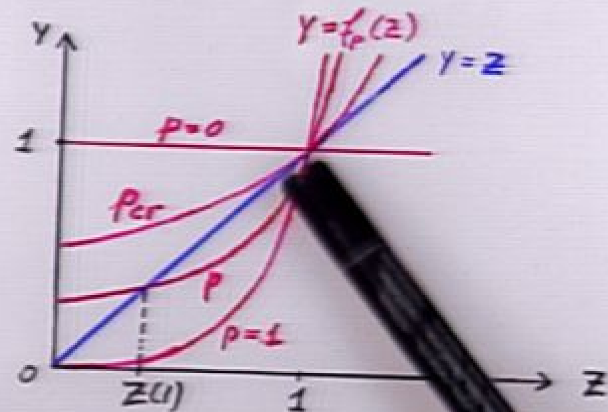

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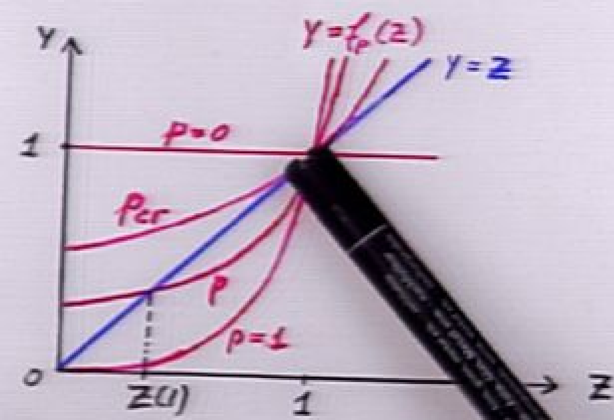

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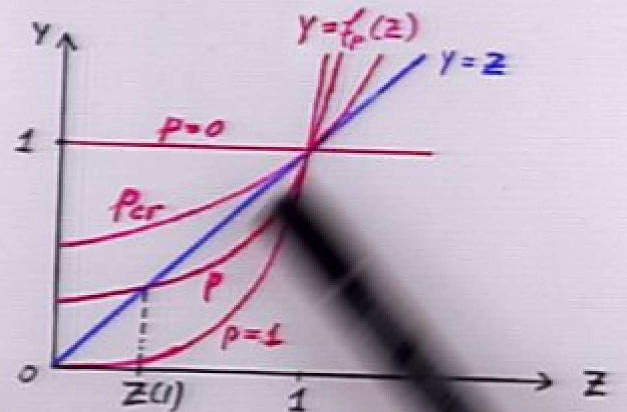

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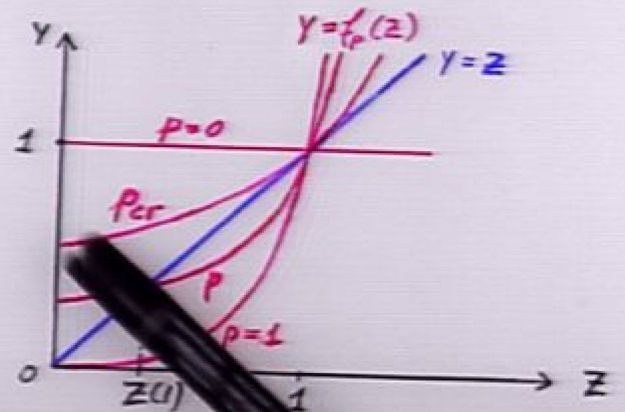

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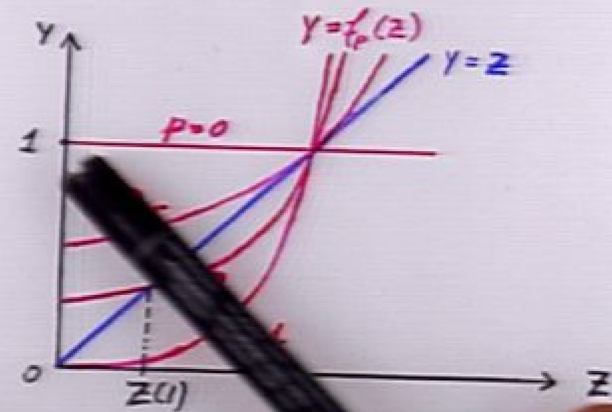

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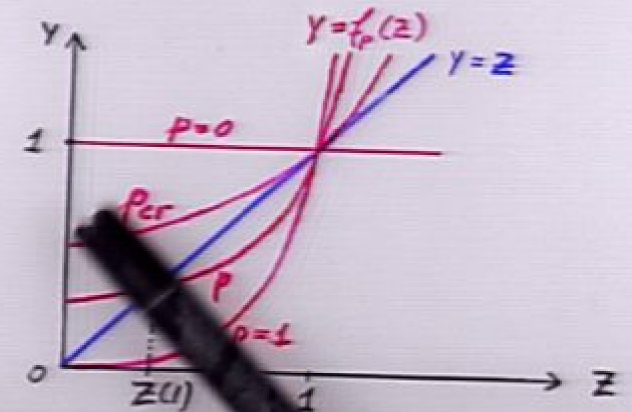

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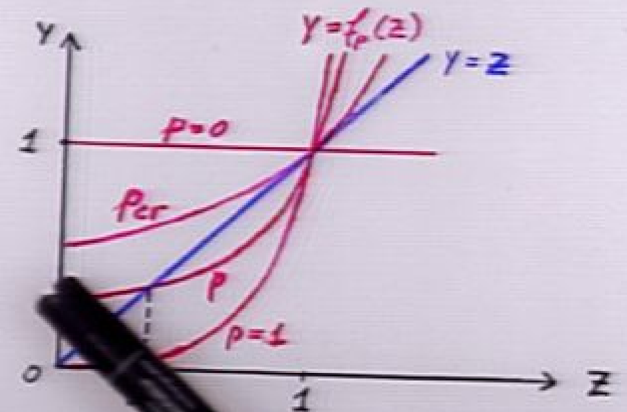

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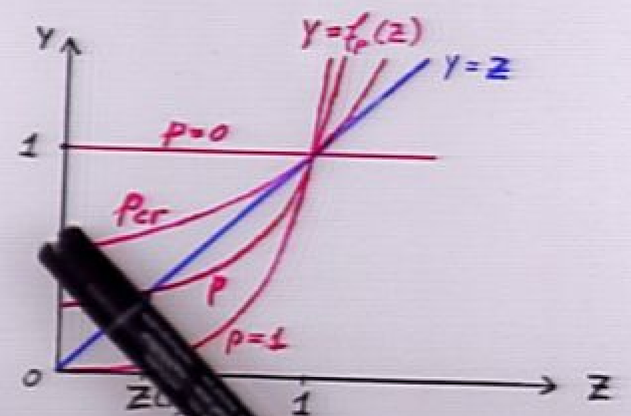

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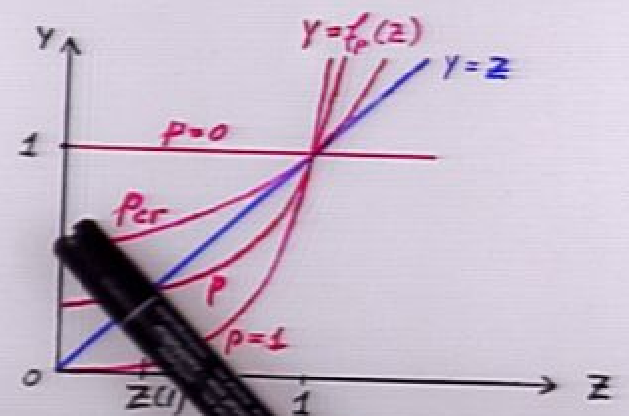

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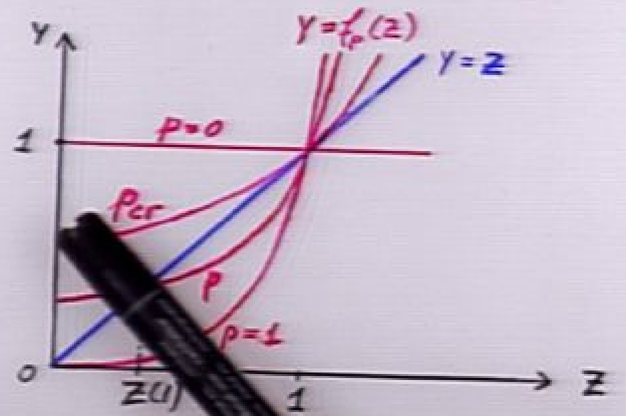

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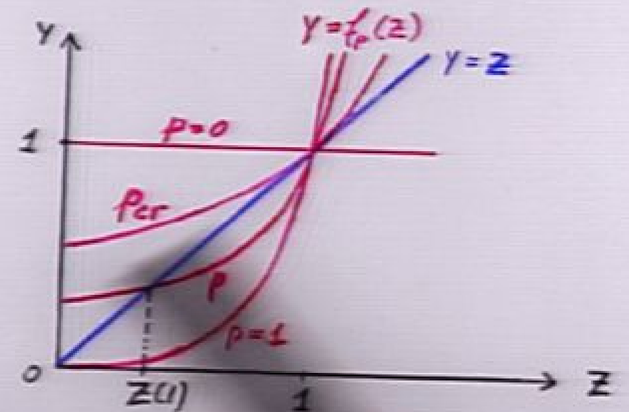

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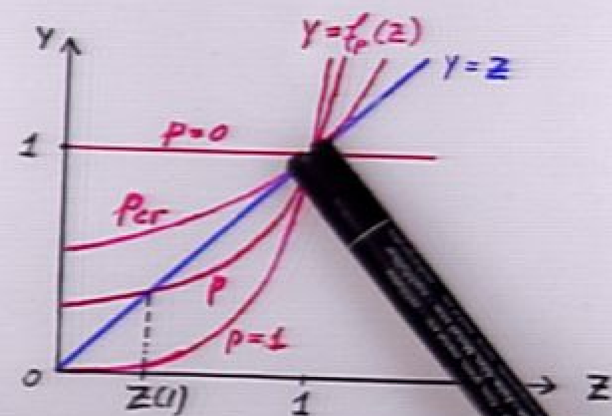

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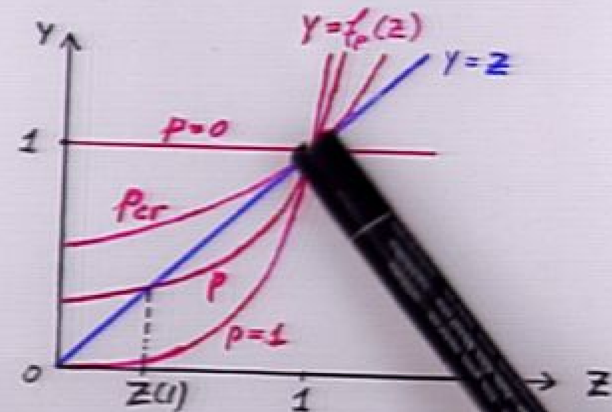

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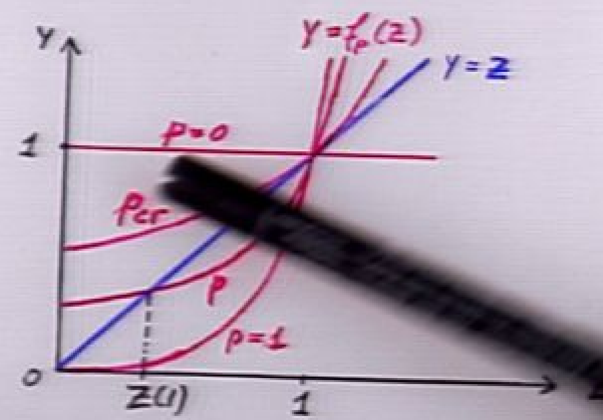

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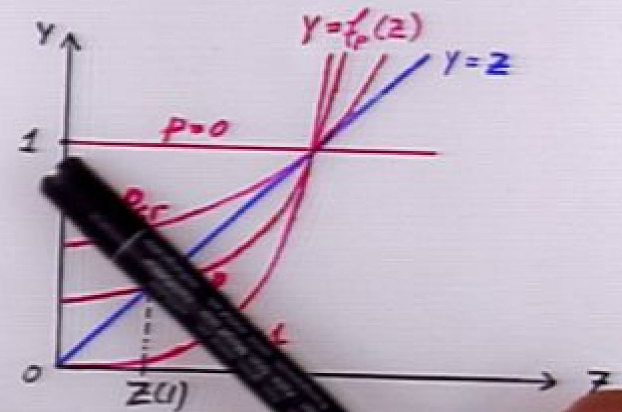

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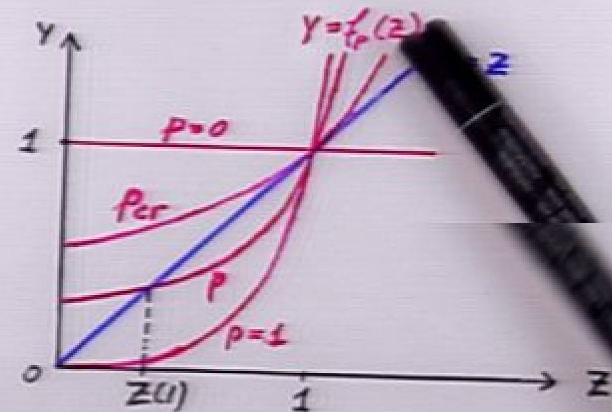

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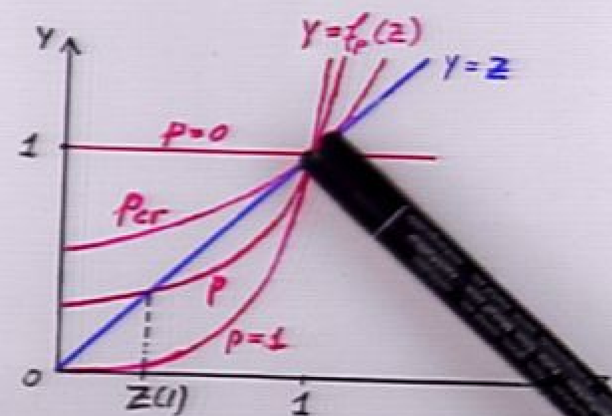

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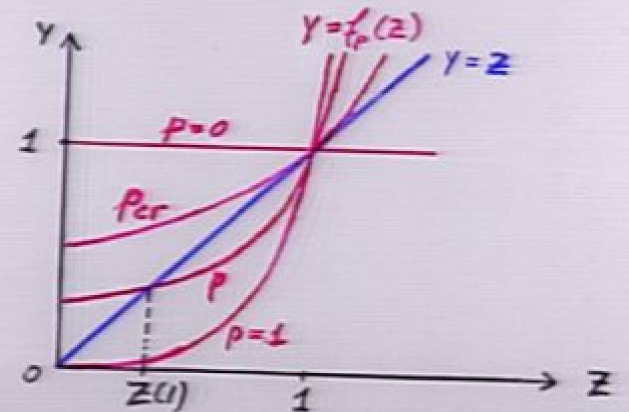

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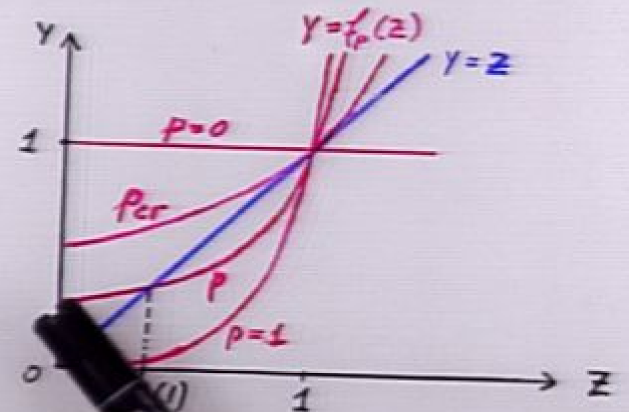

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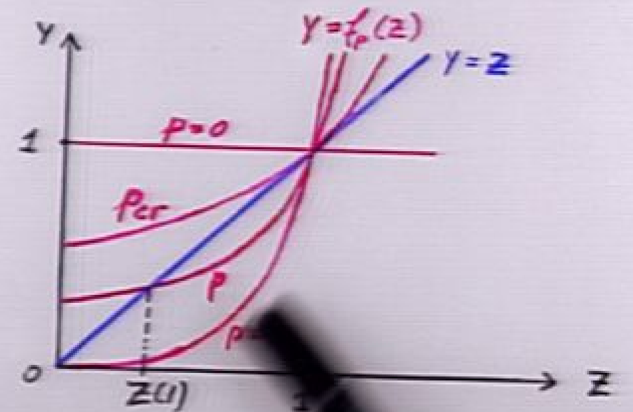

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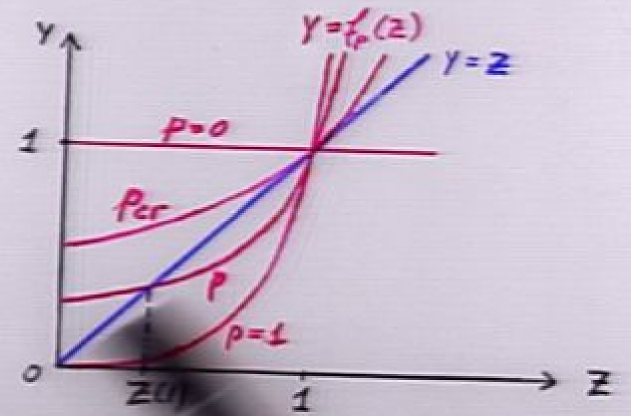

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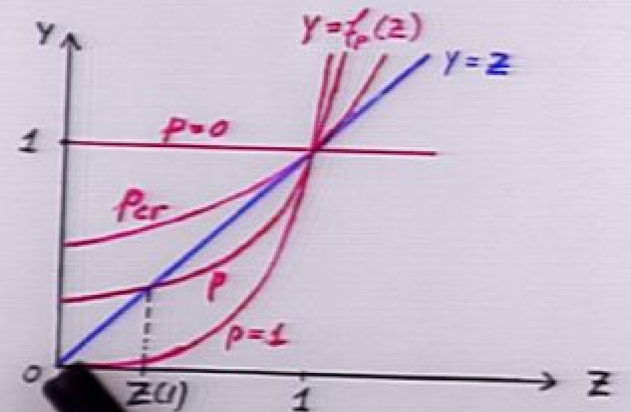

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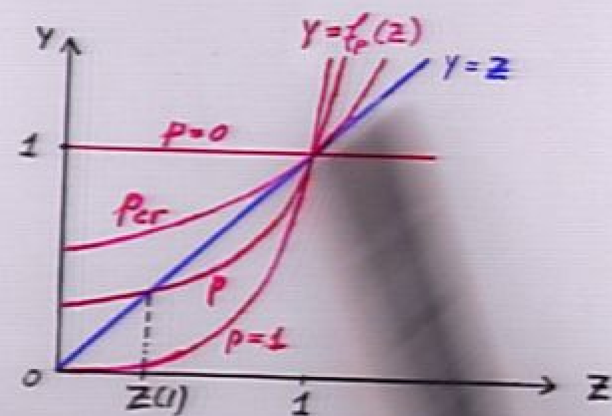

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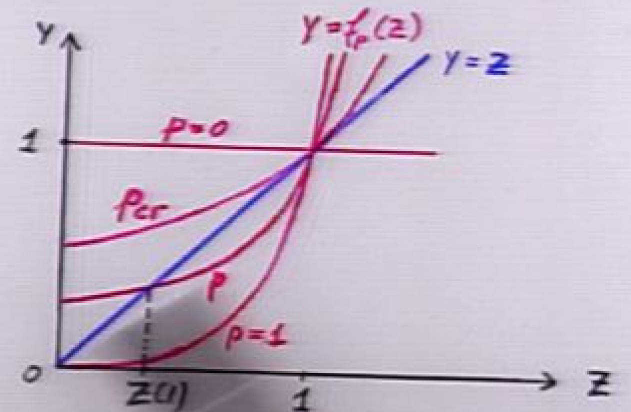

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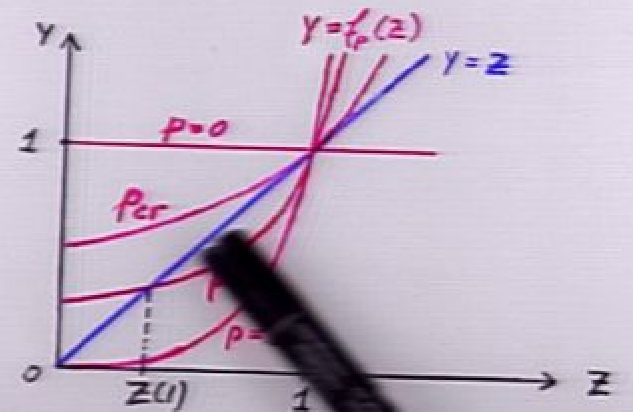

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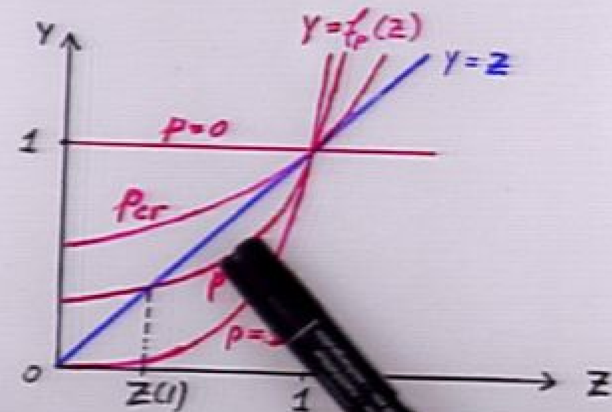

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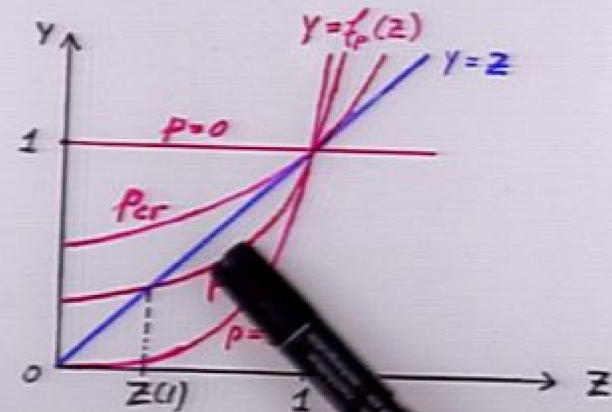

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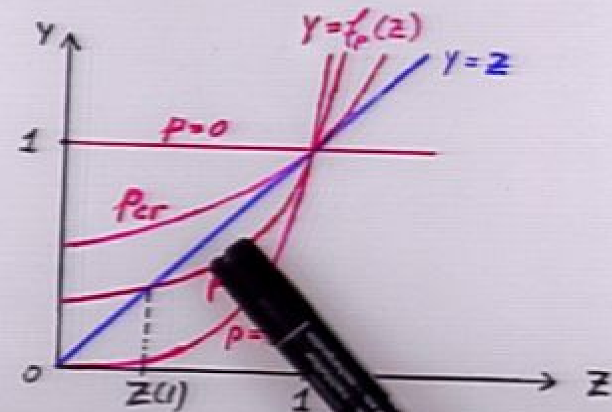

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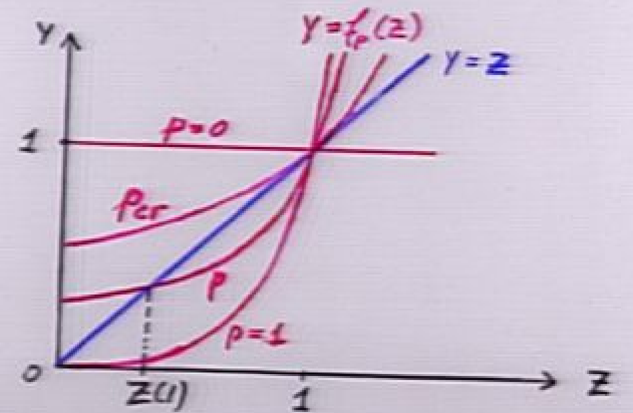

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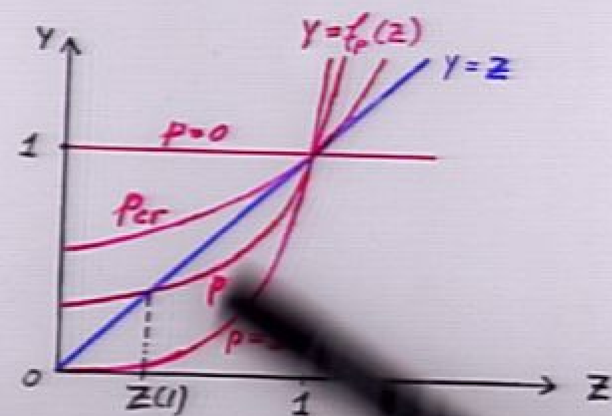

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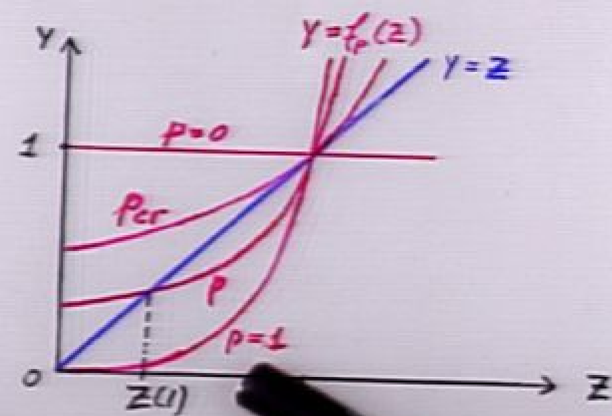

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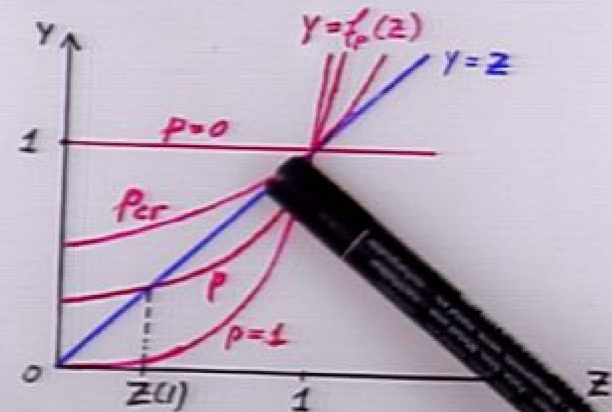

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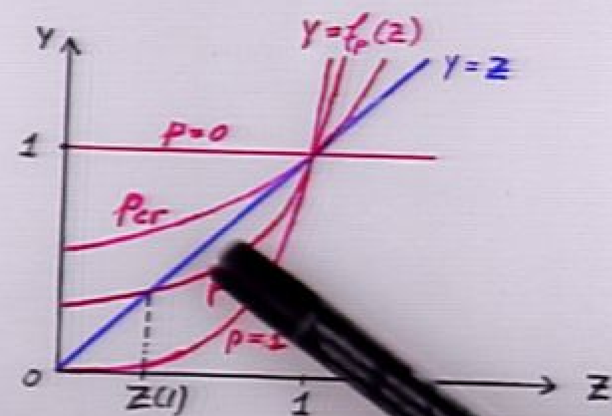

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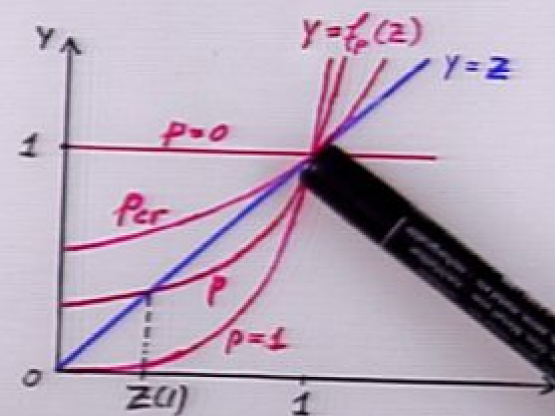

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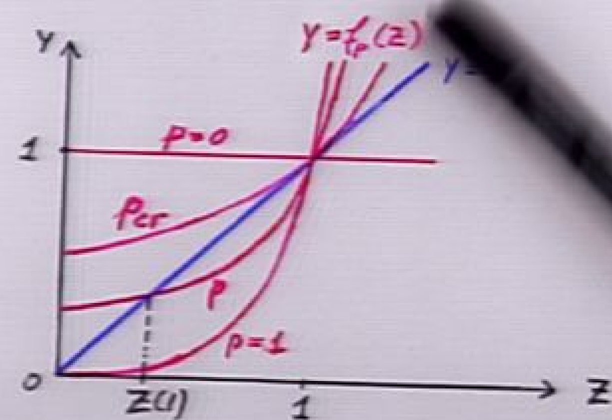

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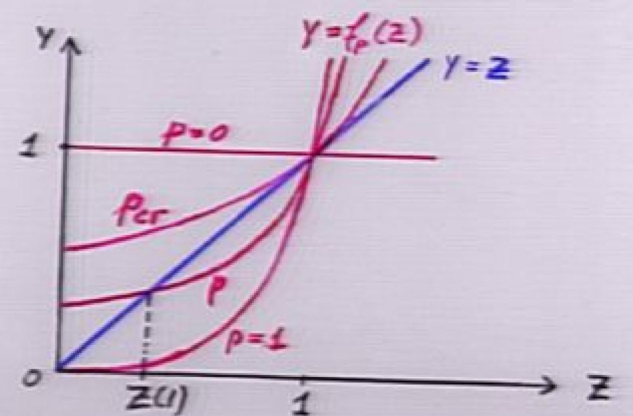

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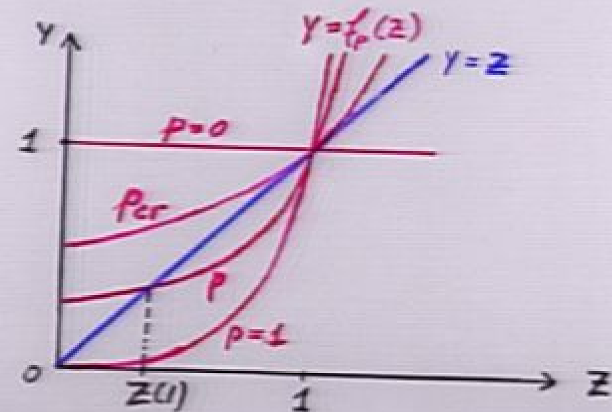

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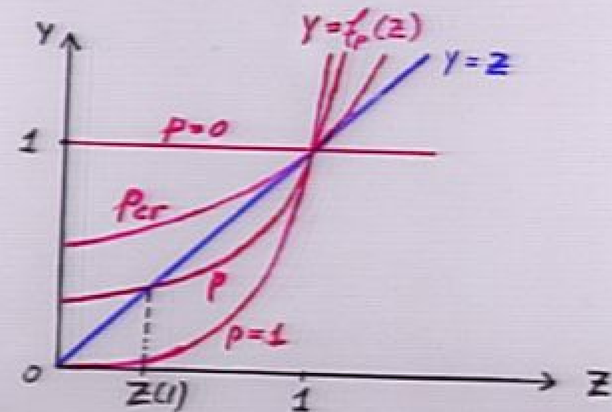

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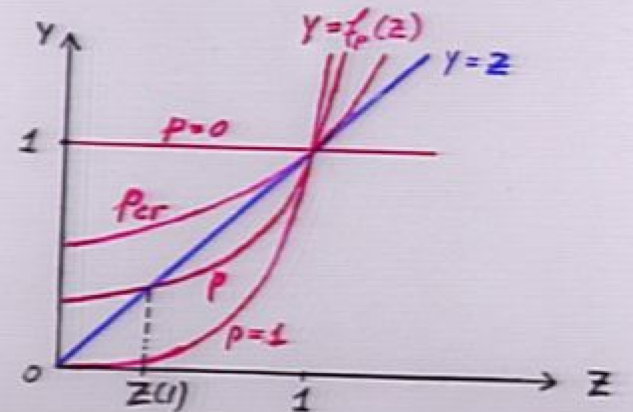

Probability that T is finite:

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From figure: $\Xi(1) = 1$ if $p \le p_{cr}$, where p_{cr} is determined by $f_p'(1) = 1$.

Hence

$$f_p'(1) = (n-1)p \Rightarrow p_{cr} = \frac{1}{n-1}.$$

Alternatively: For $p = p_{cr}$

$$\sum_{k=1}^{\infty} k p_k = 1$$

i.e. average off-spring is 1, Galton-Watson process is critical.

Conditioning on trees of fixed size N gives

(12)

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Consider limit of (μ_N) in more general context below.

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5. Generic tree ensembles.

Assume off-spring probabilities $(p_k)_{k\geq 0}$ satisfy

$$\sum_{k=0}^{\infty} p_k = 1 \quad \text{and} \quad \sum_{k=1}^{\infty} k p_k = 1 .$$

The corresponding critical Galton-Watson process is generic if generating funct

$$f(t) = \sum_{k=0}^{\infty} p_k t^k$$

is analytic in a neighborhood of the unit circle, i.e. has radius of convergence > 1

Note: f is a polynomial for percolation on a Cayley tree, hence generic

Define probability distribution ρ on finite rooted trees by

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$$\rho(T) = \prod_{v \in T \setminus r} p_{d_v - 1} .$$

Conditioning on trees of fixed size gives probability distribution

$$\mu_N(T) = \frac{1}{Z_N} \rho(T)$$

on $\mathcal{T}_N$, where $Z_N = \rho(\mathcal{T}_N)$.

The corresponding critical Galton-Watson process is generic if generating function

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$$Z(t) = t\,f(Z(t))$$

follows

$$Z(t) \sim 1 - cst \cdot \sqrt{1-t}$$

for $t \to 1$. Hence by genericity

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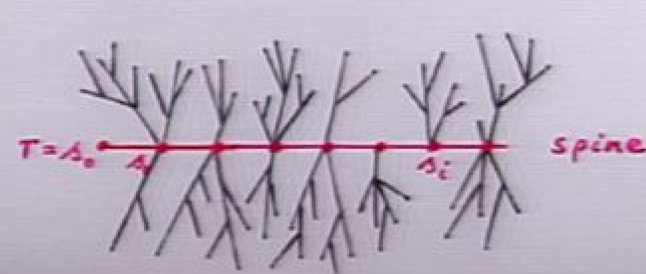

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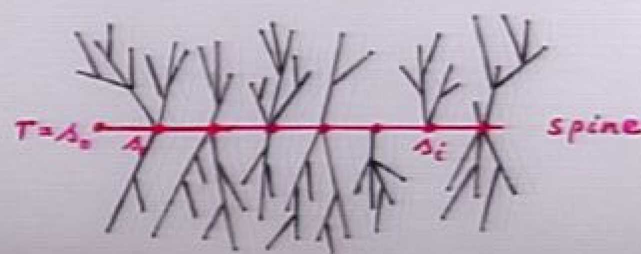

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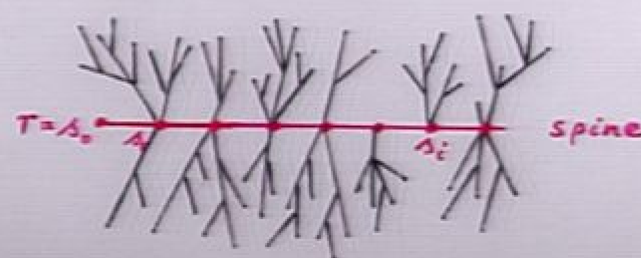

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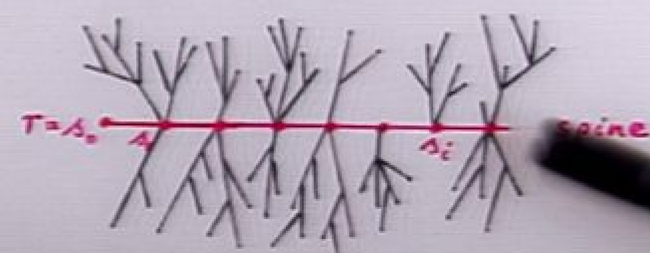

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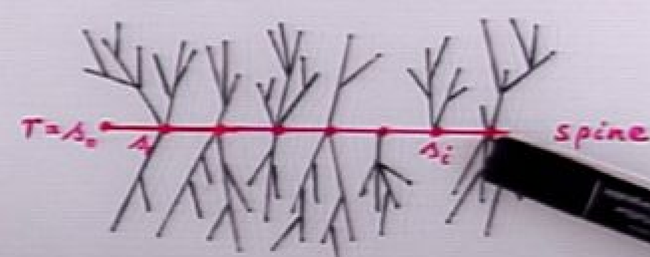

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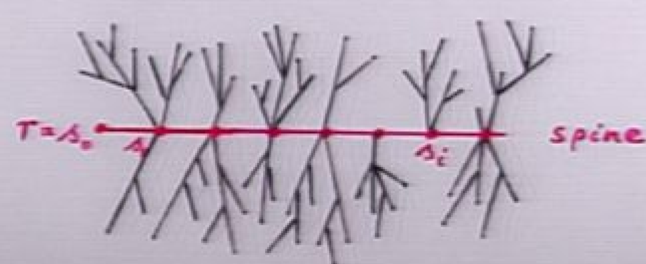

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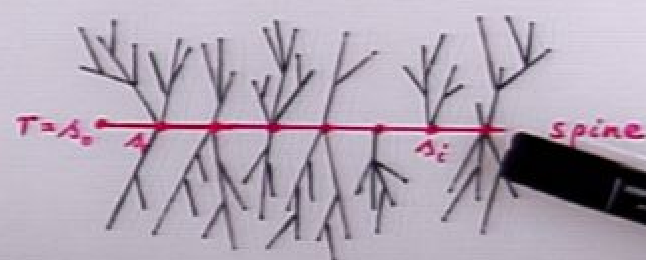

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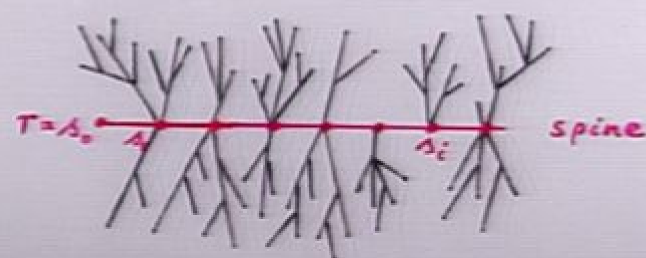

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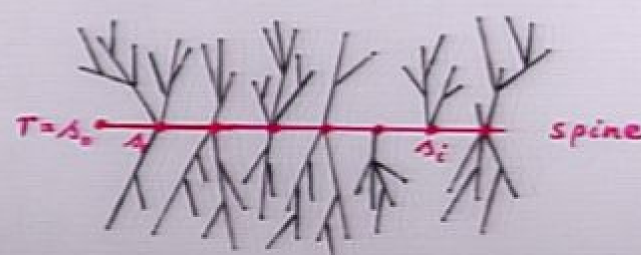

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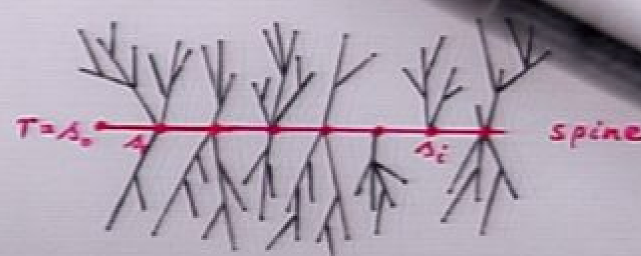

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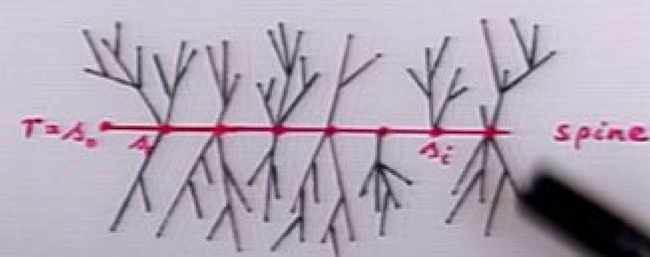

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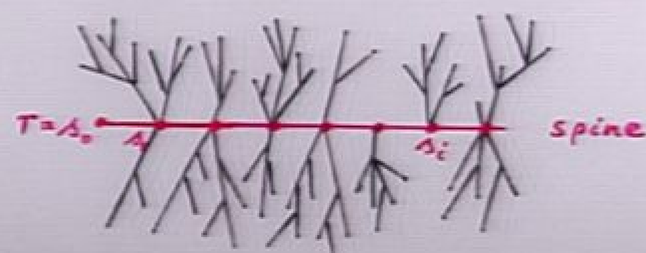

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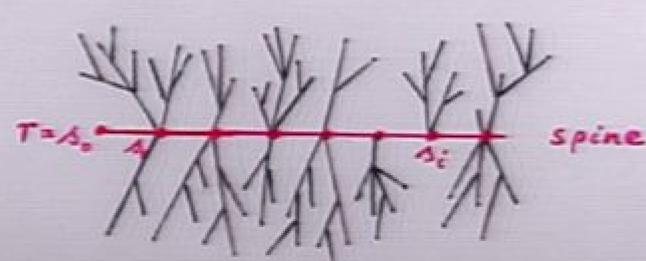

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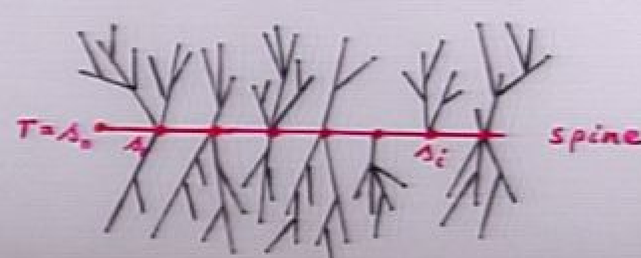

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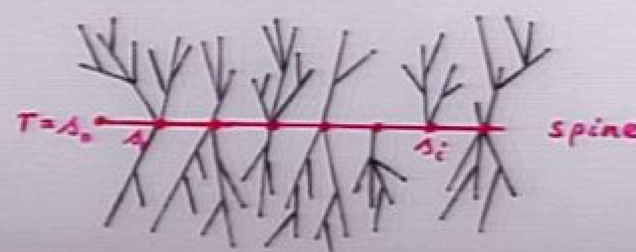

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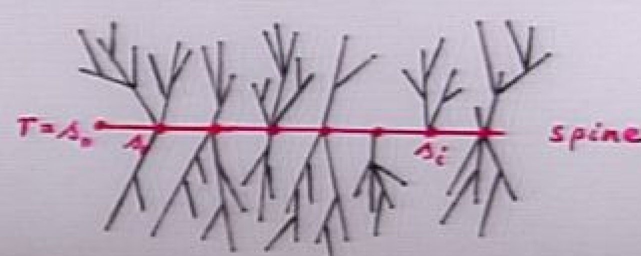

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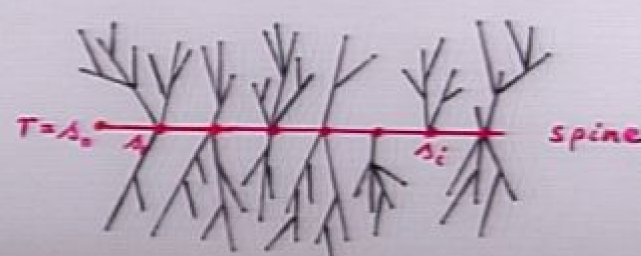

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Theorem 2 For a generic tree ensemble $(\mathcal{S}, \mu)$ we have

$$d_h = 2 \quad \text{and} \quad d_s = \frac{4}{3}.$$

Almost sure statements also hold, in particular

$$c'(\ln R)^{-2} R^2 \leq |B_R| \leq c R^2 \ln R$$

almost surely.

- $d_h = 2$ follows from previous theorem and

  $$\langle |B_R| \rangle_g \sim R$$

  Average number of individuals in a Galton-Watson tree is constant.

  For d_s one uses, among other results, Kolmogorov '32: Probability that more than R generations survive $\sim \frac{cst}{R}$ for $R \to \infty$.

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For d_s one uses, among other results, Kolmogorov '32: Probability that more than R generations survive $\sim \frac{cst}{R}$ for $R \to \infty$.

Theorem 2 For a generic tree ensemble $(\mathcal{T}, \mu)$ we have

$$d_h = 2 \quad \text{and} \quad d_s = \frac{4}{3}.$$

Almost sure statements also hold, in particular

$$c'(\ln R)^{-2} R^2 \le |B_R| \le c R^2 \ln R$$

almost surely.

- $d_h = 2$ follows from previous theorem and

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