

Title: Bayesian Inference on Numerical Injections

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Abstract: We describe a Markov-Chain Monte-Carlo technique to study the source parameters of gravitational-wave signals from the inspirals of stellar-mass compact binaries detected with ground-based detectors such as LIGO and Virgo. We can apply this technique to both spinning and non-spinning waveforms and we use a variety of tools like parallel tempering to improve the sampling efficiency of the algorithm in a multi-dimensional parameter space. We describe new developments in model-selection techniques for distinguishing between alternative signal models. We present preliminary results from the application of these techniques to data sets containing injections of numerical-relativity waveforms into simulated Gaussian detector noise. We study the source parameters of signals from the inspirals of stellar-mass compact binaries detected with ground-based gravitational-wave detectors such as LIGO and Virgo. We use automatic adaptation of the step size and take into account the correlations between parameters to efficiently probe the parameter space while keeping the algorithm suitable for a wide range of signals. We shall discuss the performance of the MCMC algorithm and the typical measurement accuracy of the source parameters as a function of the binary parameters and the number of detectors in the network. We will show that despite the lower positional accuracy compared to other astronomical observations an association of a gravitational-wave event with e.g. an electromagnetic detection is possible with three or even two 4-km-size interferometers.

Bayesian Inference on Numerical Injections

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on behalf of the NINJA Bayesian parameter
estimation group



Participants



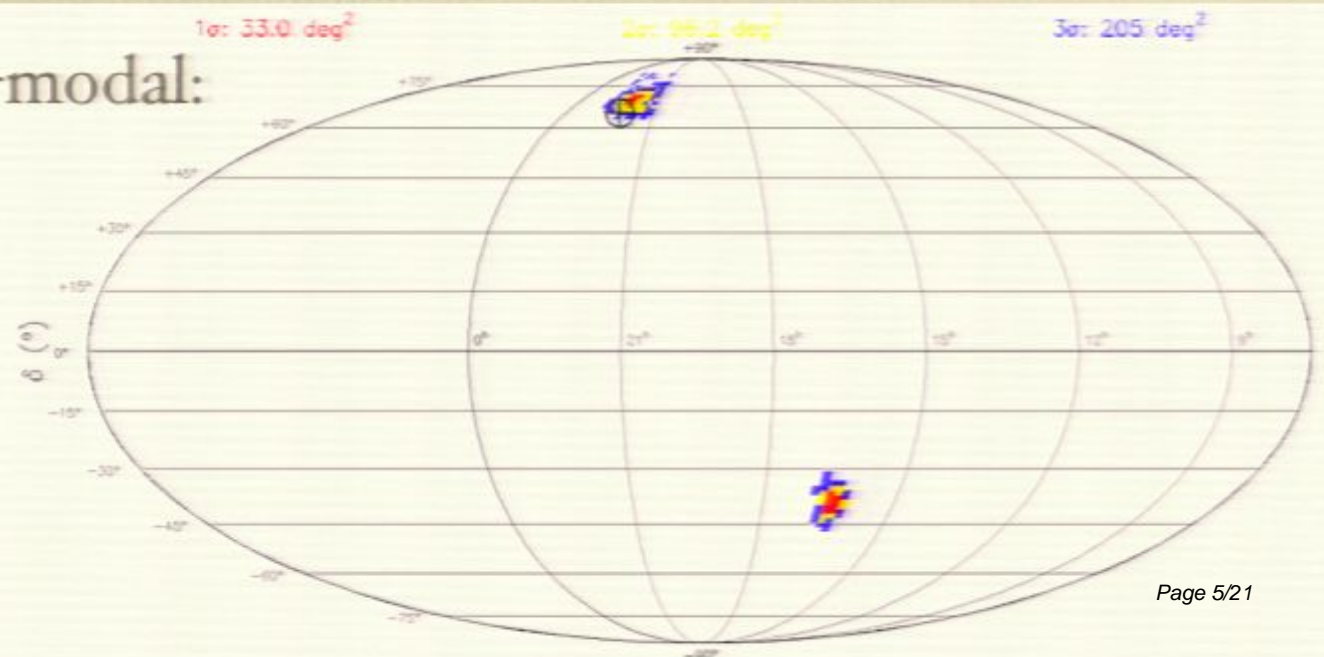
- Several data analysis groups are involved in parameters estimation on NINJA data:
 - FollowupMCMC - C. Röver (AEI Hannover)
 -  SPINspiral MCMC - B. Farr, V. Raymond, I. Mandel, V. Kalogera (Northwestern), M. v. d. Sluys (U Alberta)
 - S. Nissanke, P. Ajith, M. Vallisneri (JPL/Caltech), J. Sievers, D. Holz (LANL), S. Hughes (MIT), N. Dalal (CITA)
 -  Nested Sampling - B. Aylott, R. Smith, A. Vecchio (Birmingham), J. Veitch (Cardiff)
 - MultiNest - F. Feroz, J. Gair, P. Graff, M. Hobson (Cambridge)

NINJA Parameter Estimation

- Show that we can still accurately estimate parameters on more realistic injections using approximate waveforms
- Study systematic errors
- Determine appropriate waveform family to use “on the fly” via model selection
- Develop/test pipeline of passing triggers from detection searches
- Compare and validate parameter-estimation codes

Challenges

- Explore a large physical parameter space: 9 to 15 dimensions
- Analyze data streams coherently
- Make use of a priori information
- Infer posterior distribution on signal parameters
- Could be multi-modal:



Solution: Bayesian Inference

Compute the full posterior probability density function on the parameter space θ of the signal model H , given data $\{d\}$.

$$\begin{array}{c} \text{Posterior} \swarrow \\ p(\vec{\theta}|\{d\}, H) = \frac{\overset{\text{Prior}}{p(\theta|H)} \overset{\text{Likelihood}}{p(\{d\}|\theta, H)}}{\underset{\text{Evidence}}{p(\{d\}|H)}} \end{array}$$

$$\text{Likelihood: } p(\{d\}|\theta, H) \propto e^{-\langle d-h(\theta)|d-h(\theta)\rangle/2}$$

$$\text{Evidence: } p(\{d\}|H) = \int p(\theta|H)p(\{d\}|\theta, H)d\theta$$

Approaches

Markov Chain Monte Carlo

- Stochastic sampling in multi-dimensional parameter space
- Designed for evaluating posterior PDFs
- Can also be used to compute evidence for model selection
- Sample from the posterior accepting/rejecting new samples according to Metropolis-Hastings ratio [new vs. current sample]
- Variations to improve sampling: jump proposal distributions; parallel tempering

Nested Sampling

- Stochastic sampling in multi-dimensional parameter space
- Designed compute evidence for model selection
- Can also be used for evaluating PDFs
- Samples iteratively drawn from shrinking prior volume with new sample replacing lowest-likelihood sample [N live samples]
- Variations in selection of next live point (e.g., InspNest uses MCMC; MultiNest - ellipsoidal clustering)

Markov Chain Monte Carlo

$\mathcal{M} (M_{\odot})$

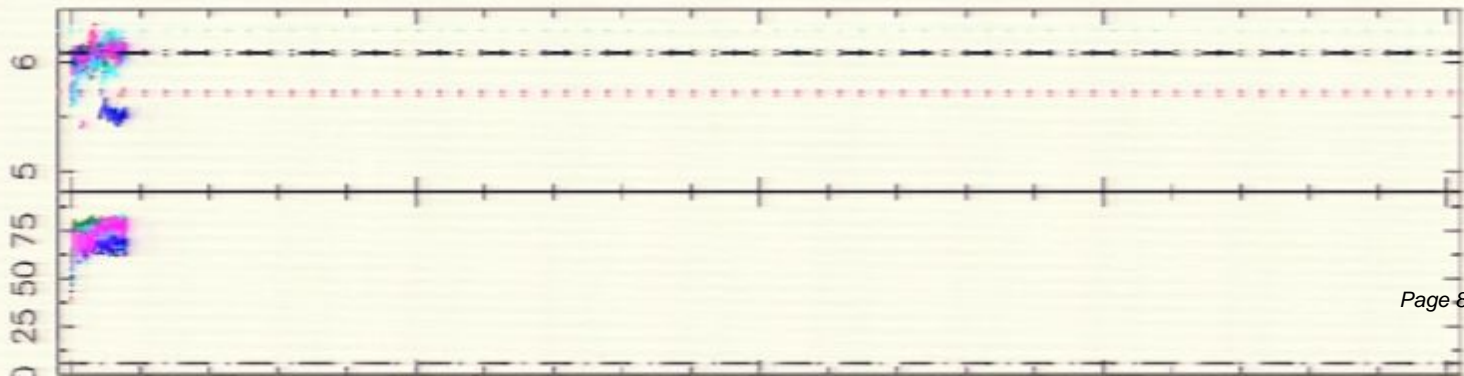
Signal: 6.084

Iteration: 7.97E+04

Data points: 8.00E+02

Chain:

log(L):



Markov Chain Monte Carlo

$\mathcal{M} (M_{\odot})$

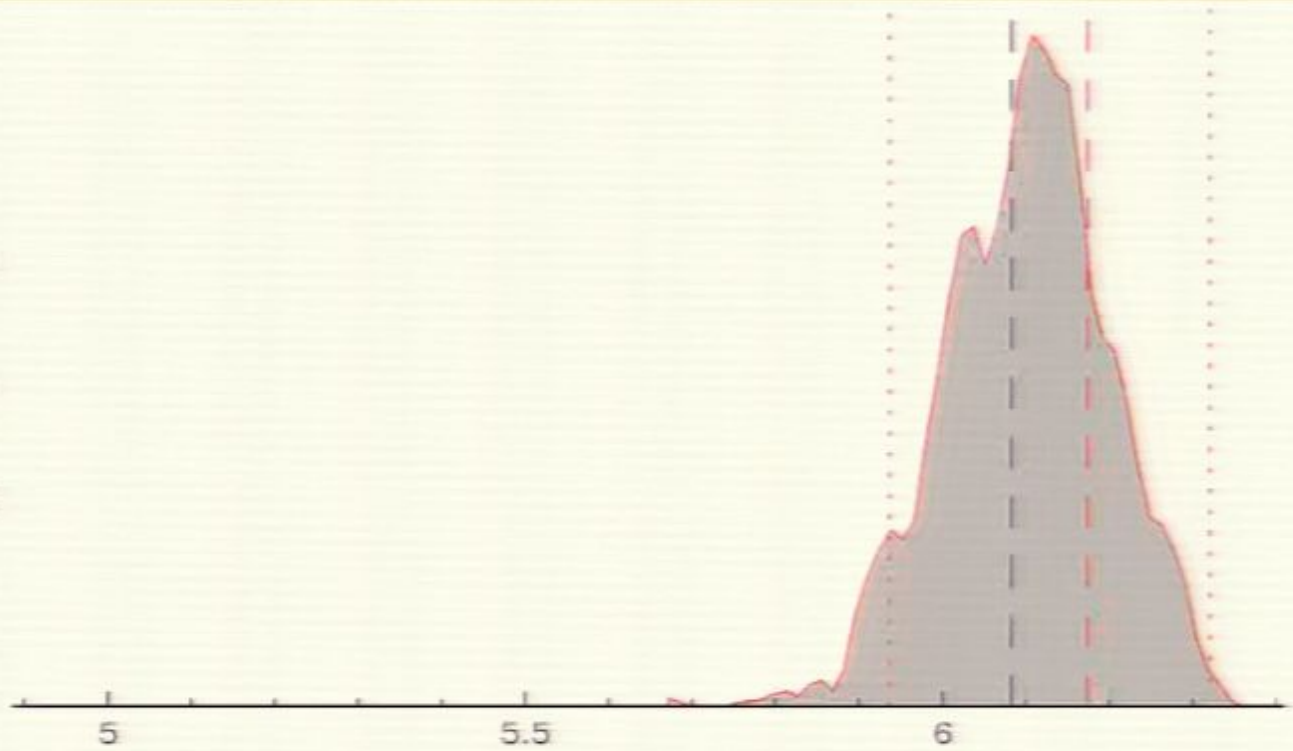
Signal: 6.084

Median: 6.174

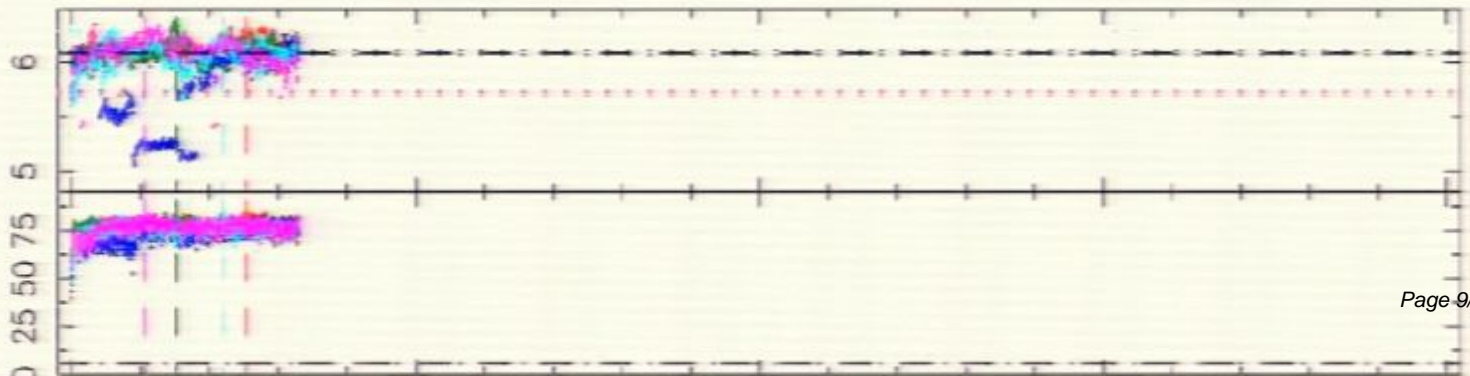
$\Delta_{95\%}$: 6.26%

Iteration: 3.30E+05

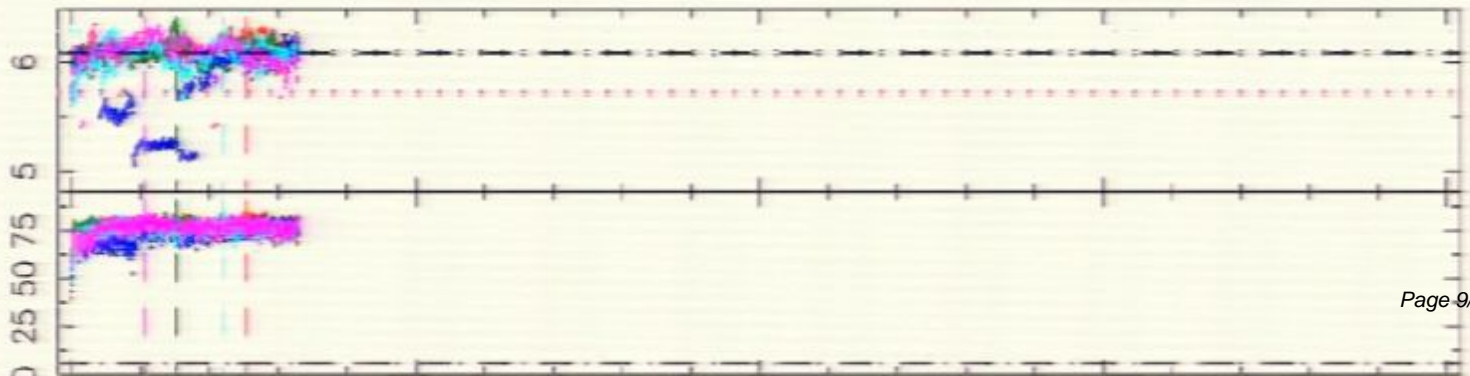
Data points: 5.85E+03



Chain:



log(L):



Markov Chain Monte Carlo

$\mathcal{M} (M_{\odot})$

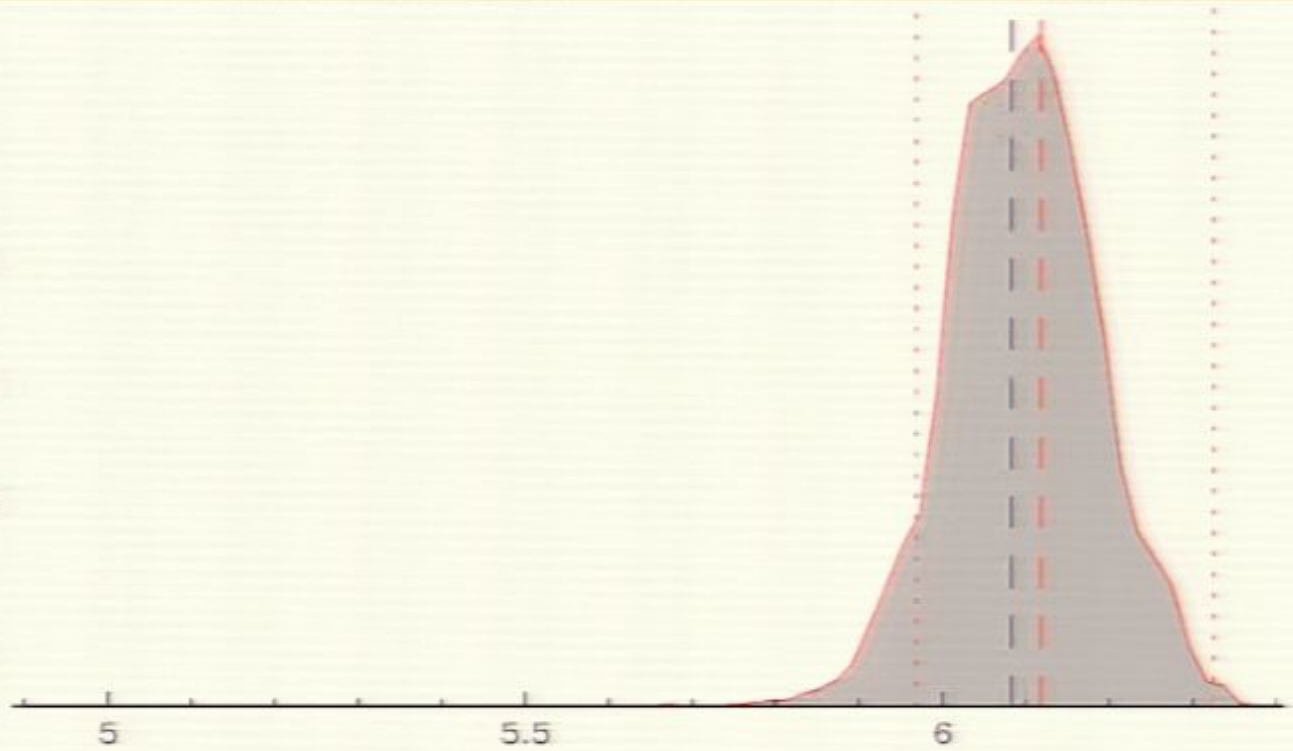
Signal: 6.084

Median: 6.118

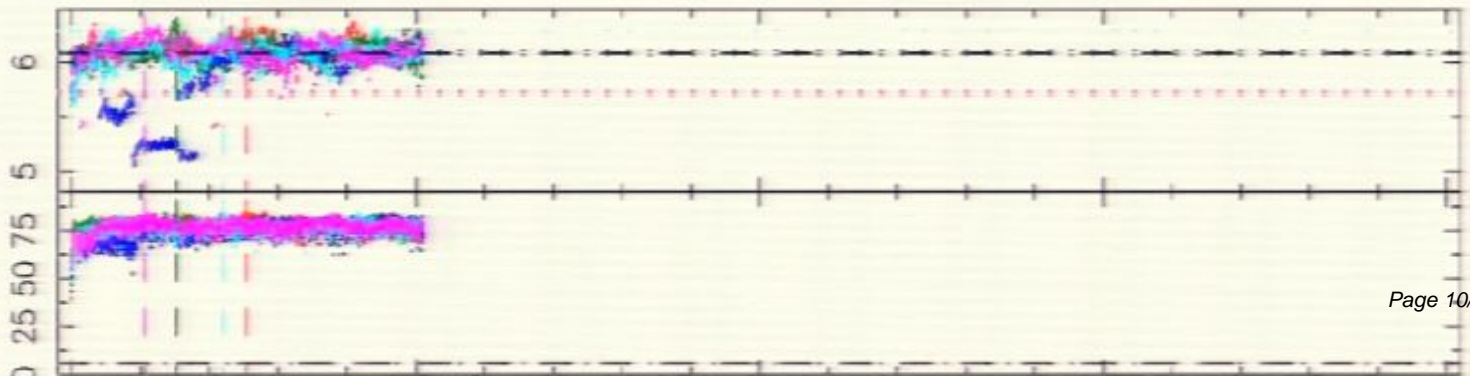
$\Delta_{95\%}$: 5.80%

Iteration: 5.10E+05

Data points: 1.31E+04



Chain:



log(L):

Markov Chain Monte Carlo

$\mathcal{M} (M_{\odot})$

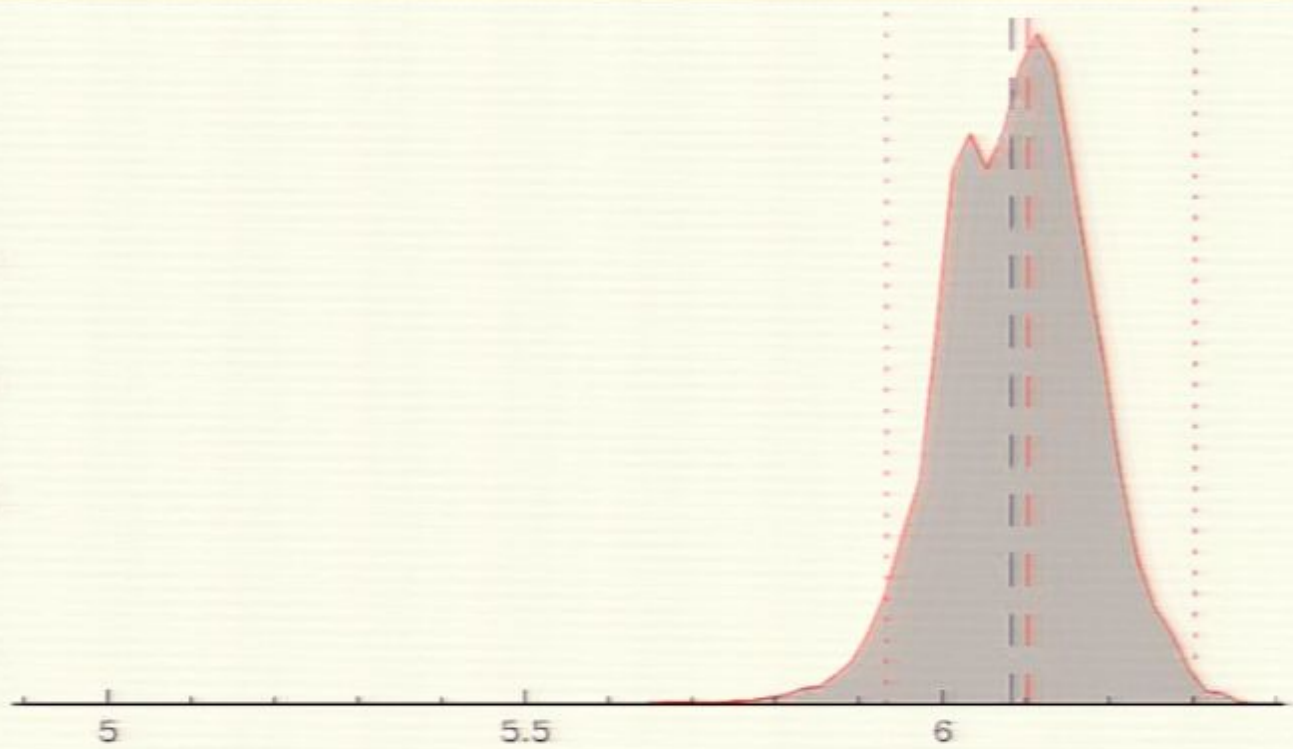
Signal: 6.084

Median: 6.103

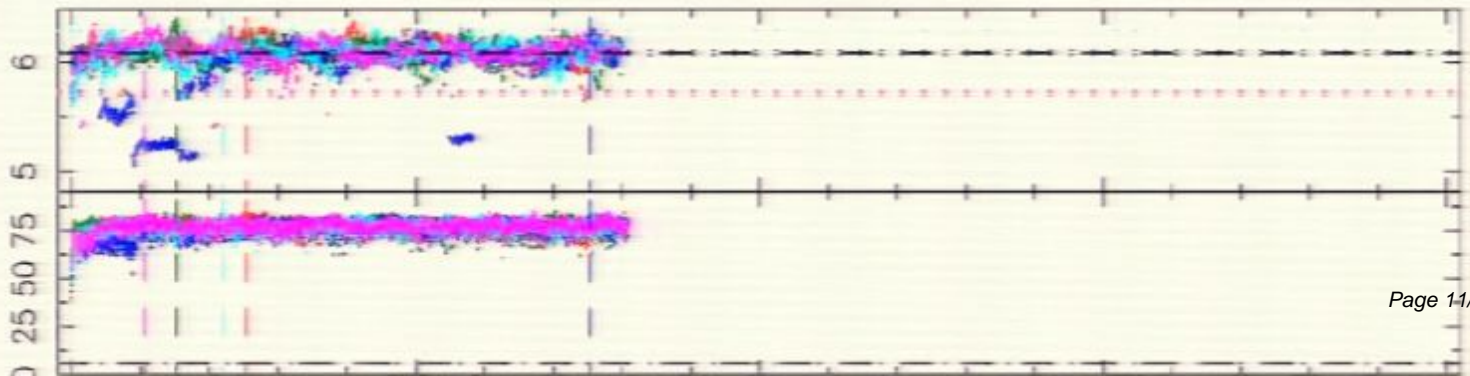
$\Delta_{95\%}$: 6.03%

Iteration: 8.00E+05

Data points: 2.51E+04



Chain:



$\log(L)$:

Markov Chain Monte Carlo

$\mathcal{M} (M_{\odot})$

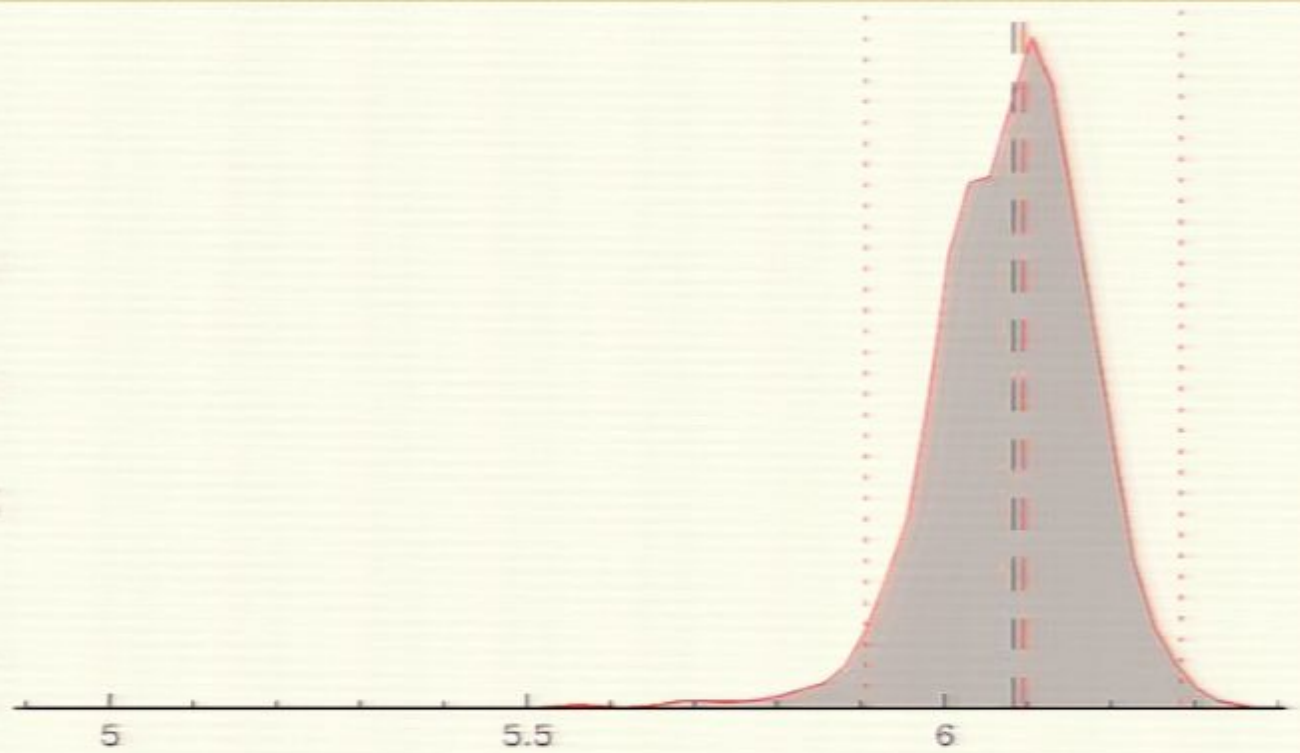
Signal: 6.084

Median: 6.095

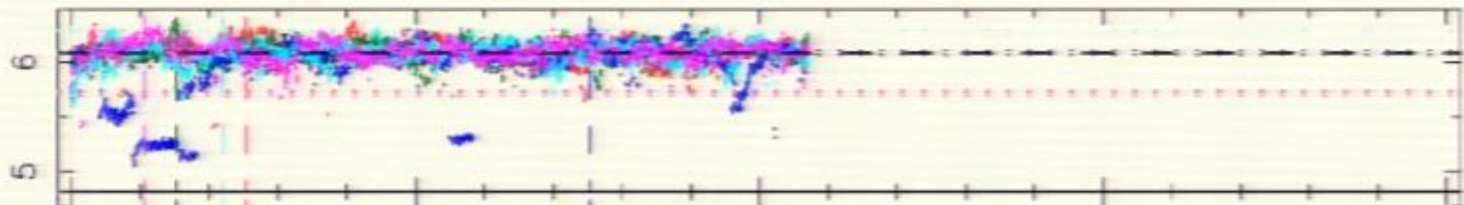
$\Delta_{95\%}$: 6.20%

Iteration: 1.07E+06

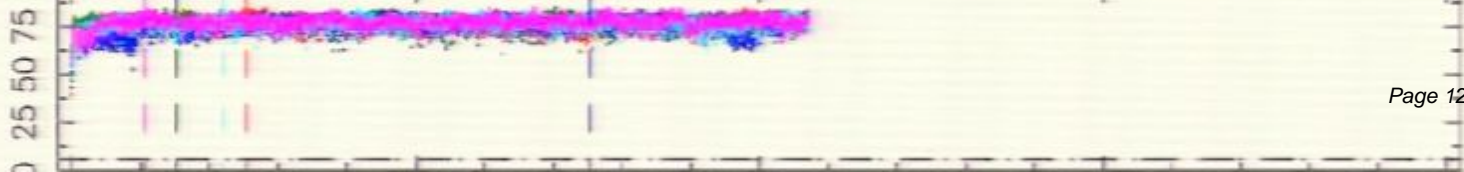
Data points: 3.86E+04



Chain:



log(L):



Markov Chain Monte Carlo

$\mathcal{M} (M_{\odot})$

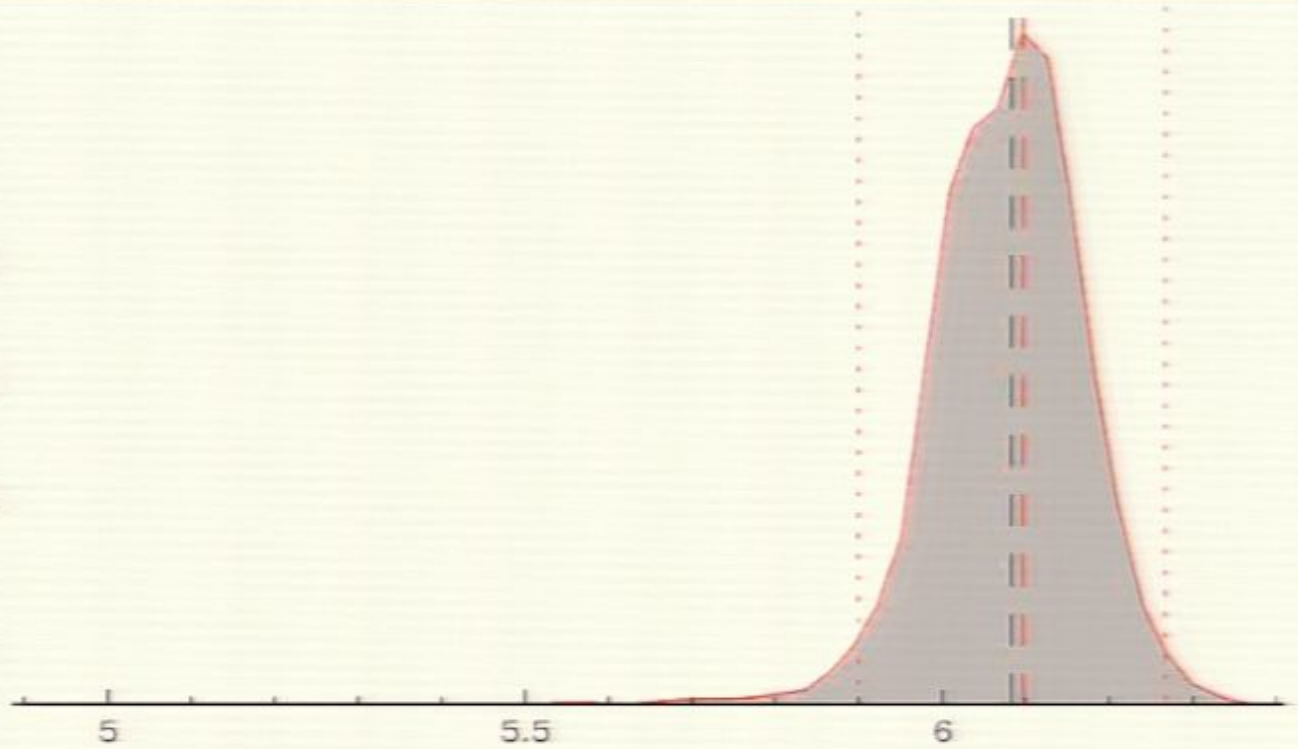
Signal: 6.084

Median: 6.098

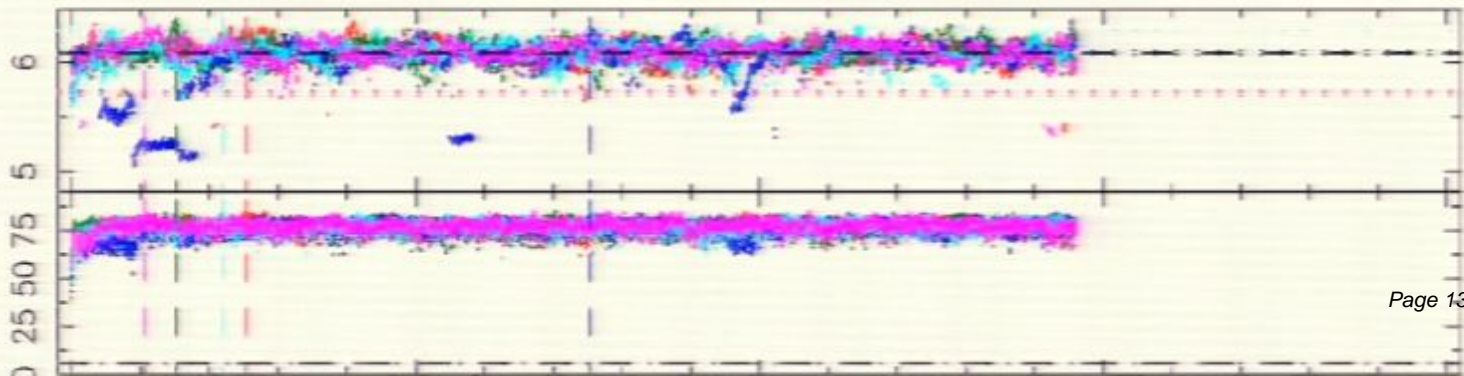
$\Delta_{95\%}$: 6.06%

Iteration: 1.46E+06

Data points: 5.81E+04



Chain:



log(L):

Markov Chain Monte Carlo

$\mathcal{M} (M_{\odot})$

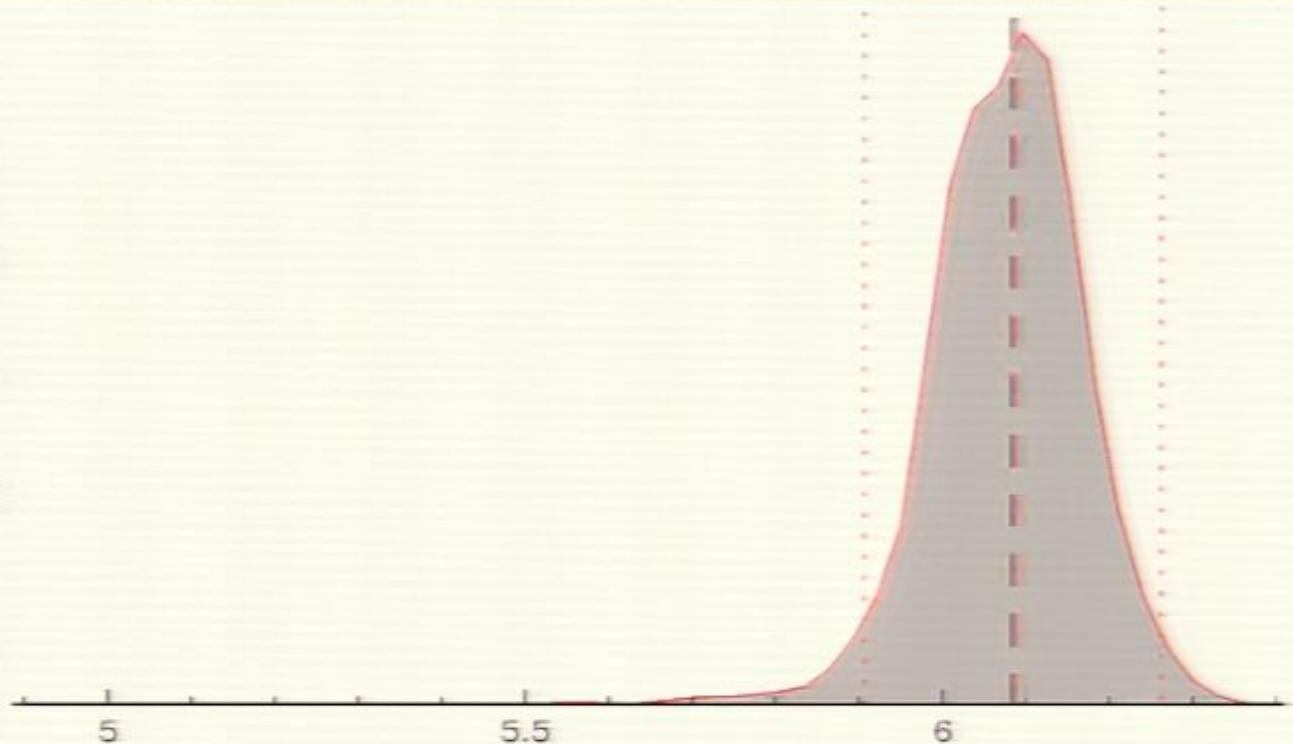
Signal: 6.084

Median: 6.089

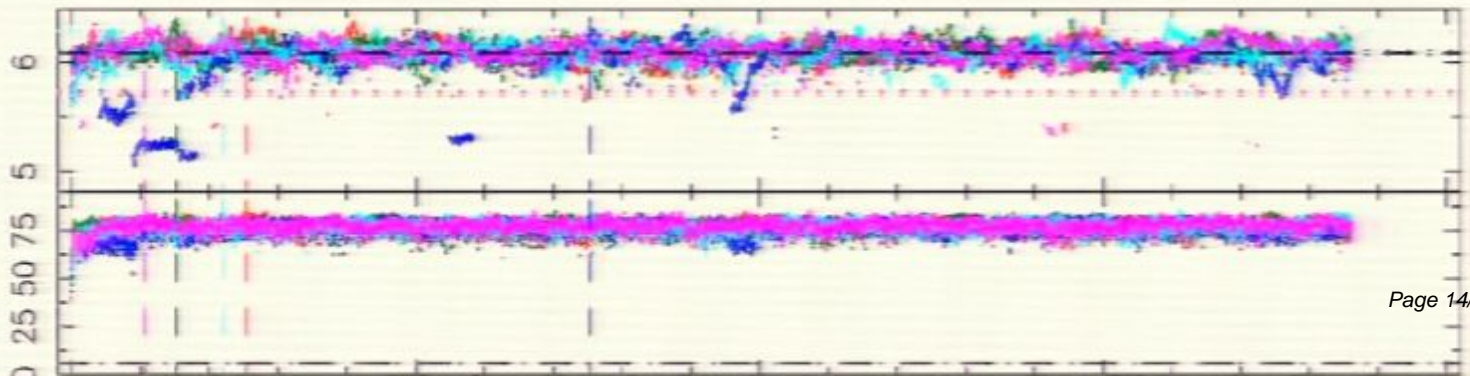
$\Delta_{95\%}$: 5.85%

Iteration: 1.86E+06

Data points: 7.81E+04

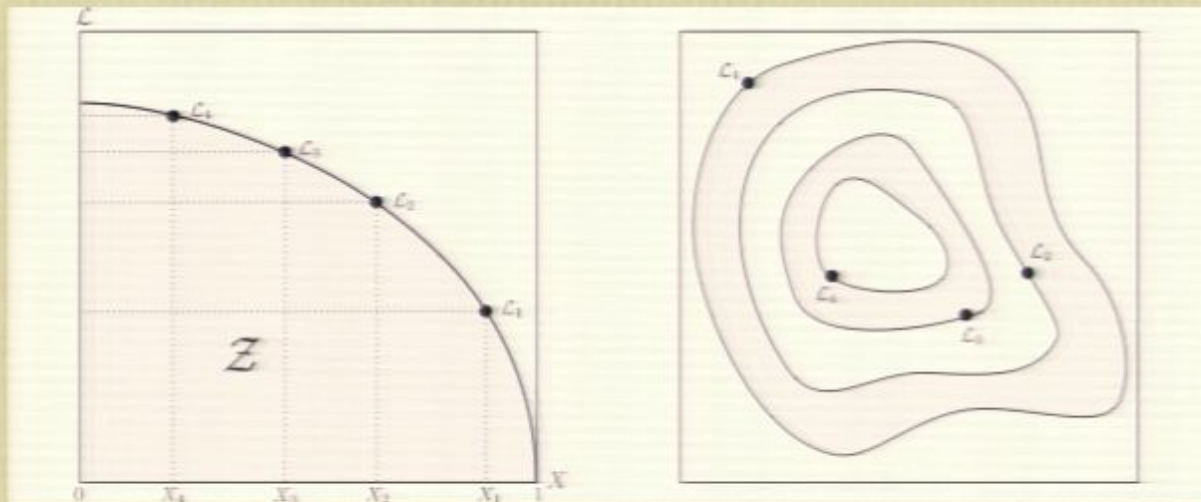


Chain:

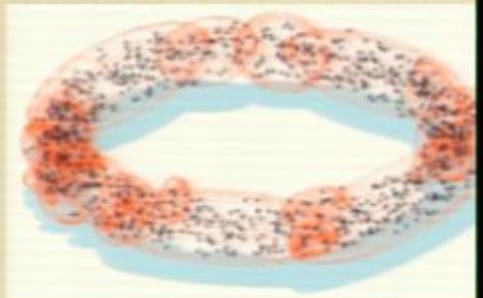


log(L):

Nested Sampling



- Use a collection of N_{Live} samples from prior. At each iteration replace outermost sample with one drawn from within the contour.
- At each iteration the volume enclosed shrinks by factor $\sim e^{1/N_{\text{Live}}}$.
- Computes marginal likelihood: fit of data to a model
- Re-sample to get samples from posterior PDF
- MultiNest: Mode separation achieved via ellipsoidal decomposition of live point set. Live points updated



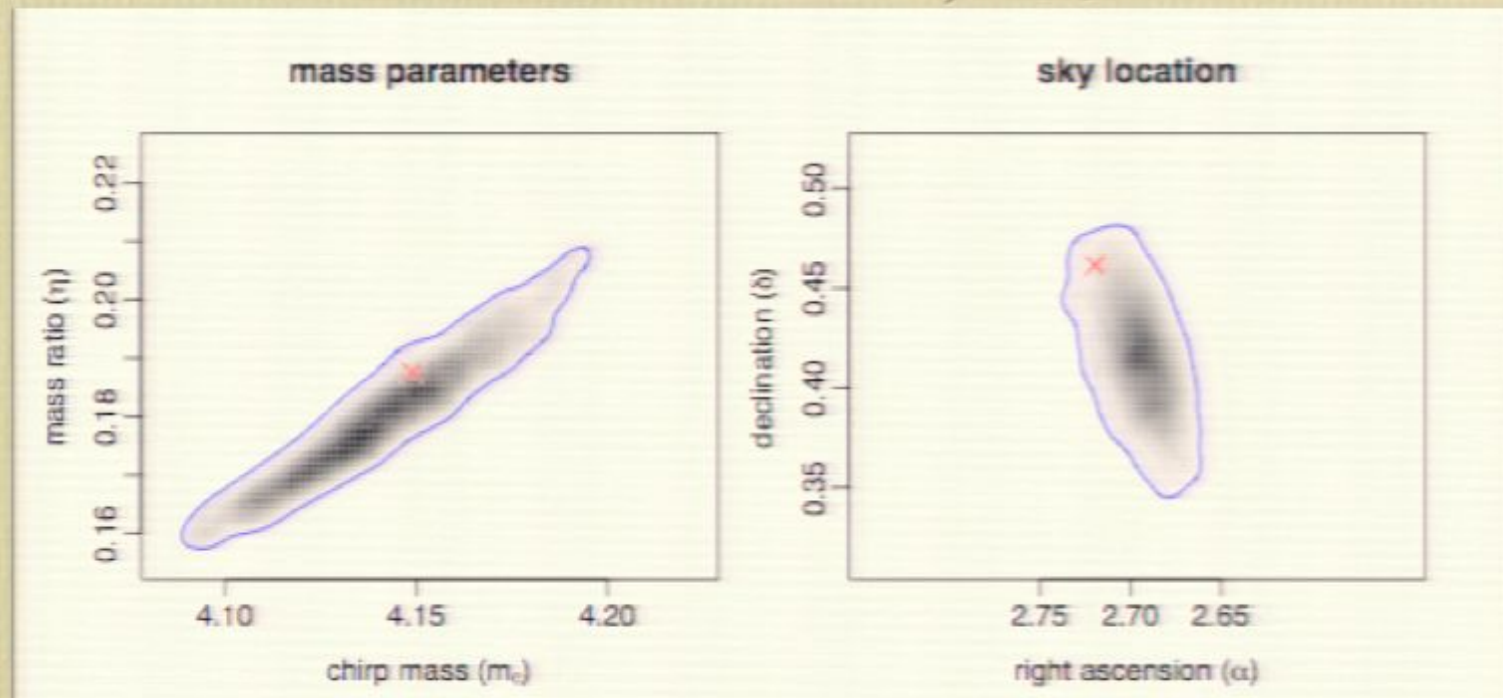
Preliminary NINJA Results

FollowupMCMC

non-spinning binary parameter estimation via MCMC (AEI)

[Röver et al, P.R.D. 75 62004 (2007)]

test2, lowmass, inj. #40



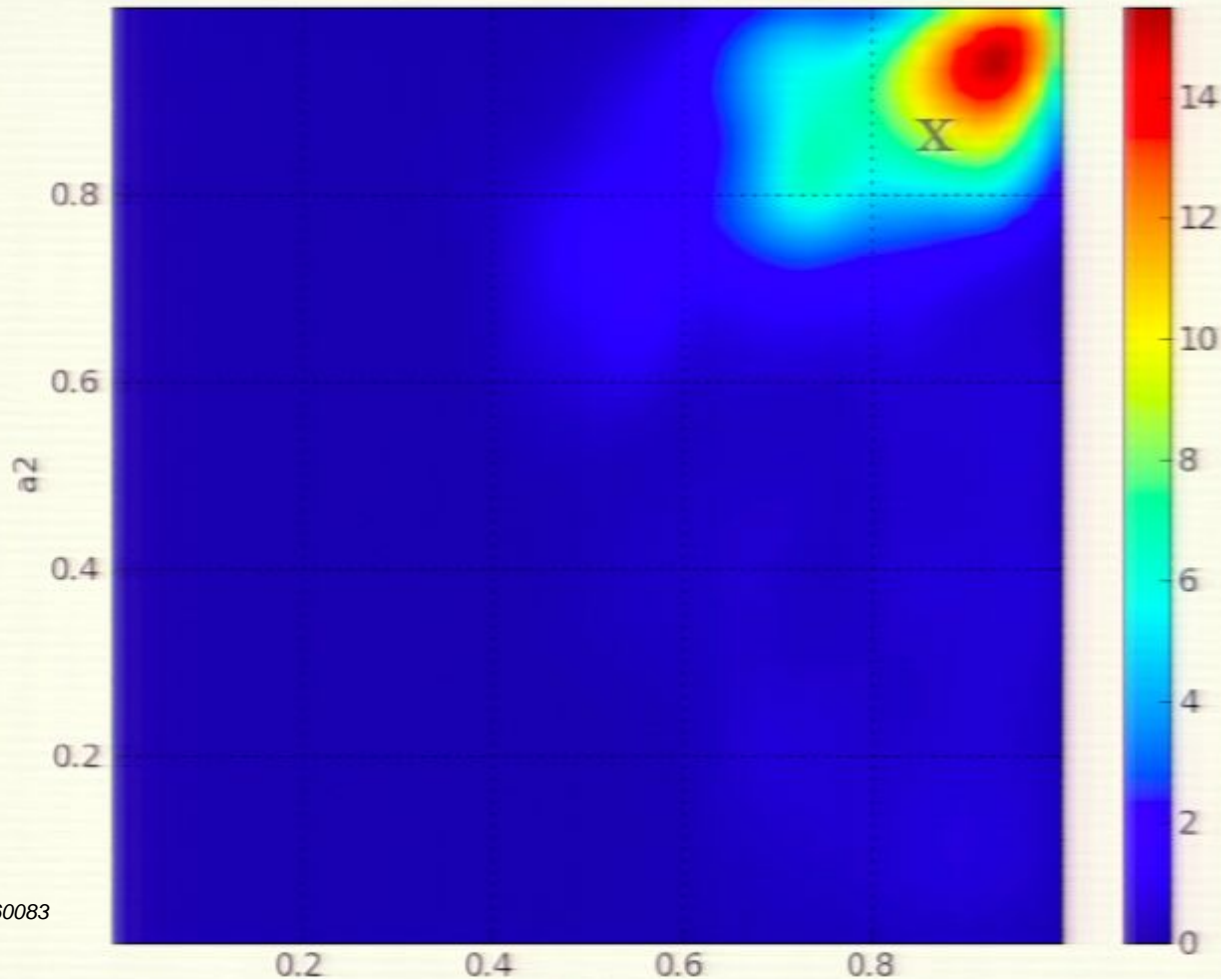
- $M_c=4.148$, $\eta=0.1785$, SNR=14.2

- used IMRPhenomA waveforms

SPINspiral

(spinning binary parameter estimation via MCMC (Northwestern)
[van der Sluys et al., CQG 25 184011 (2008)]

test2, lowmass, inj. #0

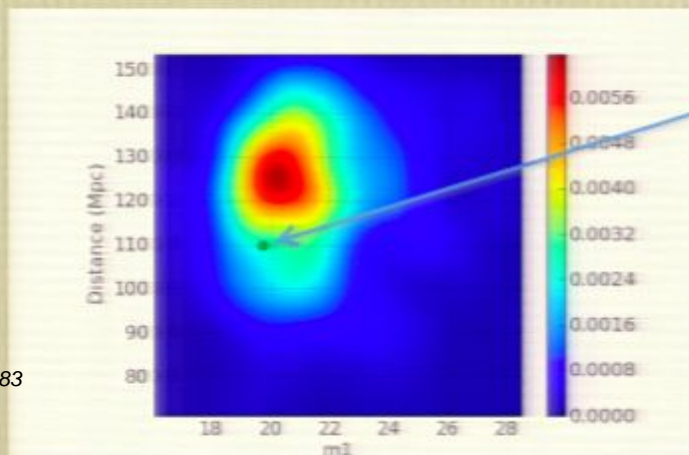
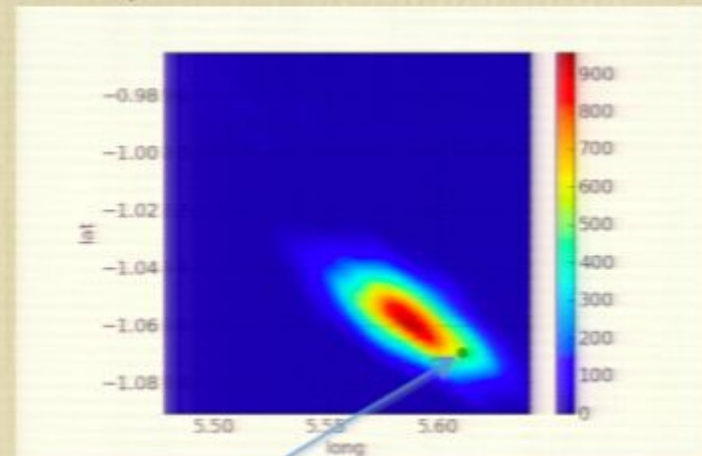
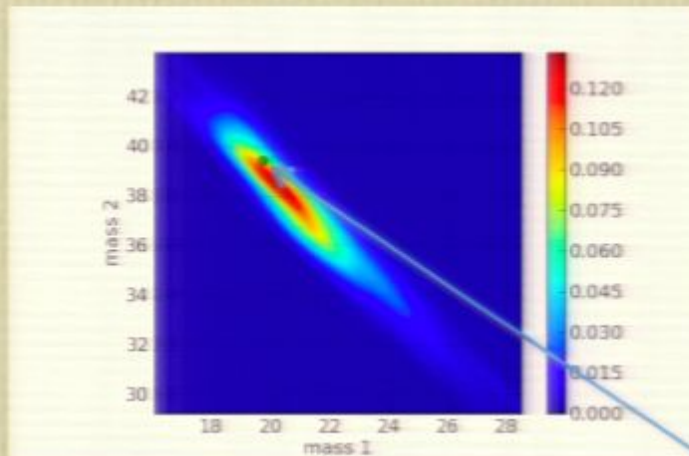


- SNR=17
- $\eta=0.25$
- $M_c=15.4$
- used SpinTaylor waveforms
- based on 1 MCMC chain

InspNest

(non-spinning binary nested sampling code (Birmingham/Cardiff))
[Veitch & Vecchio, P.R.D. 81 62003 (2010)]

test2, highmass, inj. #98



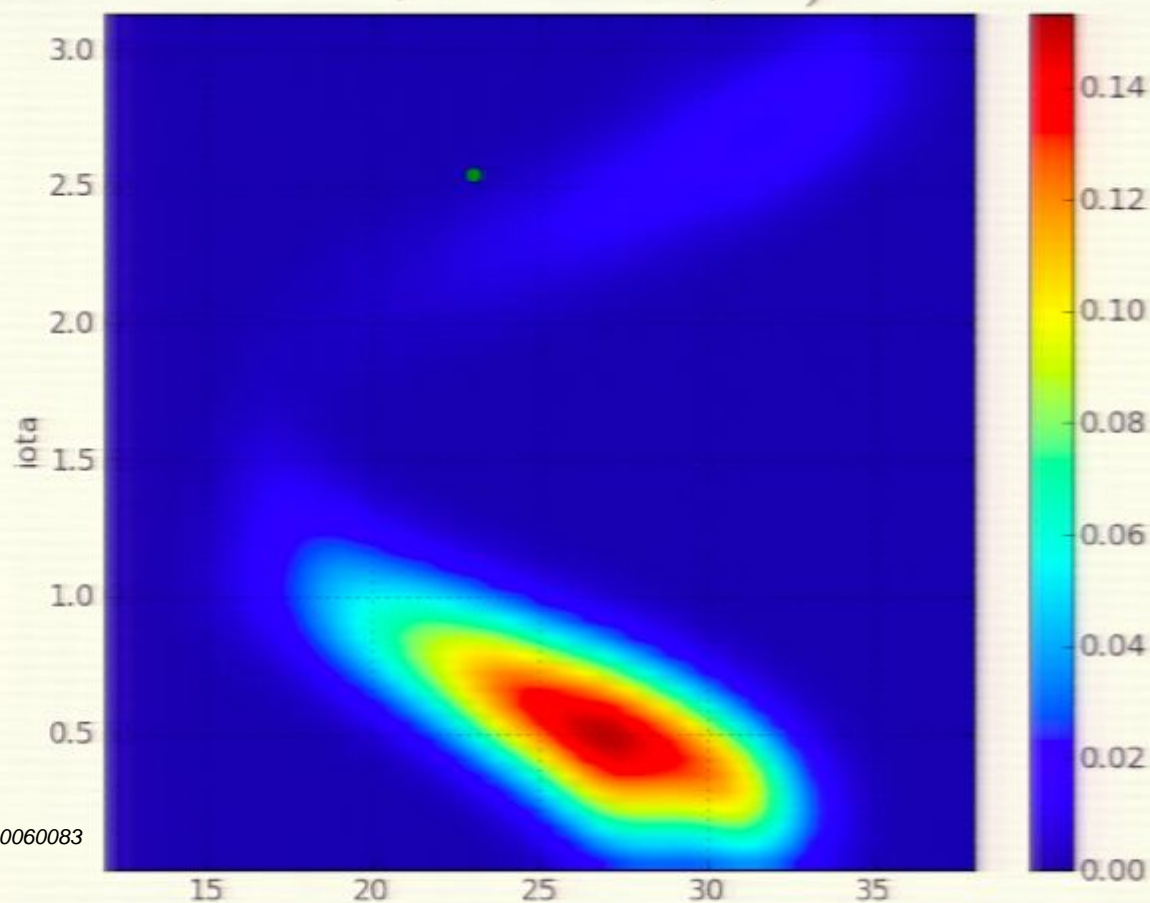
- Injection parameters, SNR-19
- used IMRPhenomA waveforms
- $B_{\text{signal/noise}}=227$

InspNest

(non-spinning binary nested sampling via MultiNest (Cambridge))

[Feroz et al., MNRAS 398 4 1601 (2009)]

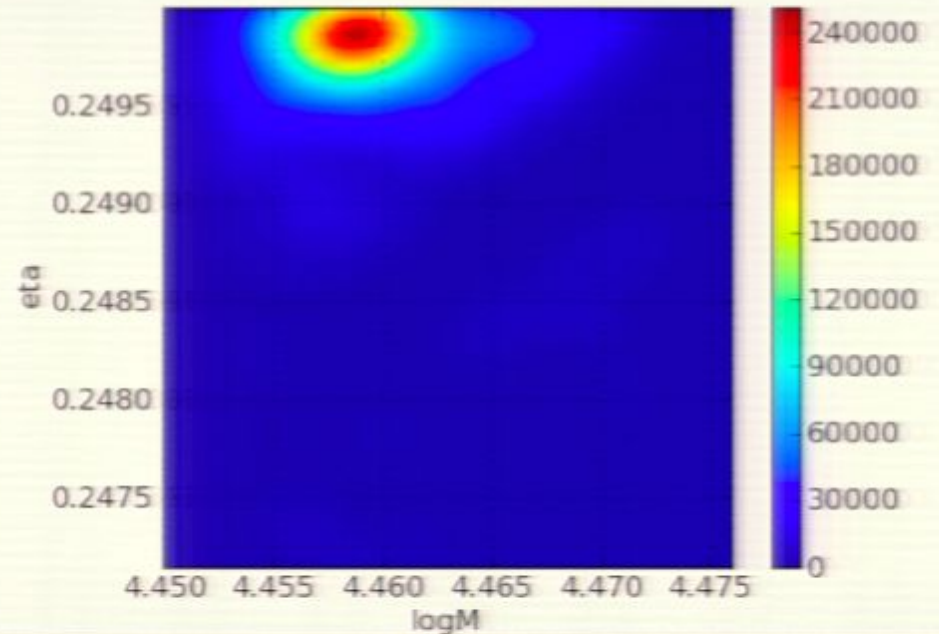
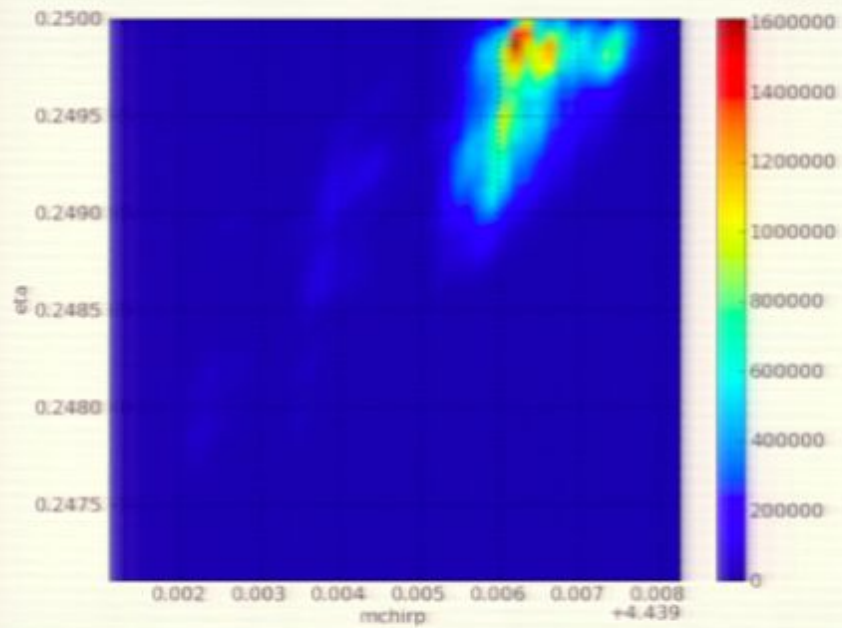
test2, lowmass, inj. #2



- SNR=33
- $\eta=0.25$
- $M_c=9.54$
- used IMRPhenomA waveforms
- Note the inclination/distance

Systematic Errors

test2, lowmass, inj. #23



SPINspiral

-20 MCMC chains converged

InspNest

- used TaylorT2 & Taylor F2 waveforms