Title: Understanding spinning black-hole binaries: a new effective-one-body model

Date: Jun 25, 2010 04:50 PM

URL: http://pirsa.org/10060079

Abstract: The dynamics of black-hole binaries is a very complex problem which has been solved only very recently through time-expensive numerical-relativity calculations. In spite of this mathematical complexity many results of these calculations can be accurately reproduced with phenomenological approaches based on test particles combined with Post-Newtonian theory and black-hole perturbation theory. In this talk I will focus on effective-one-body models, which have proved a useful and fast tool to accurately reproduce numerical-relativity waveforms. In particular I will present a novel, self-consistent effective-one-body model for spinning black-hole binaries, and show that this model does not suffer from the shortcomings of the existing models which have been put forward in the literature.

A new effective-one-body model for spinning black-hole binaries

Enrico Barausse (University of Maryland)

based on
EB, E. Racine and A. Buonanno, PRD 80 104025 (2009),
EB and A. Buonanno, PRD 81 084024 (2010)

25th June 2010

Motivation

Dynamics of binary systems in GR is tough! (No analytic solution)

- Numerical relativity simulations for BH binaries produced only recently (Pretorius '05, Campanelli et al '06, Baker et al '06)
- Simulations are very time consuming (take weeks) $\Rightarrow$ impossible to cover entire parameter space of BH binaries

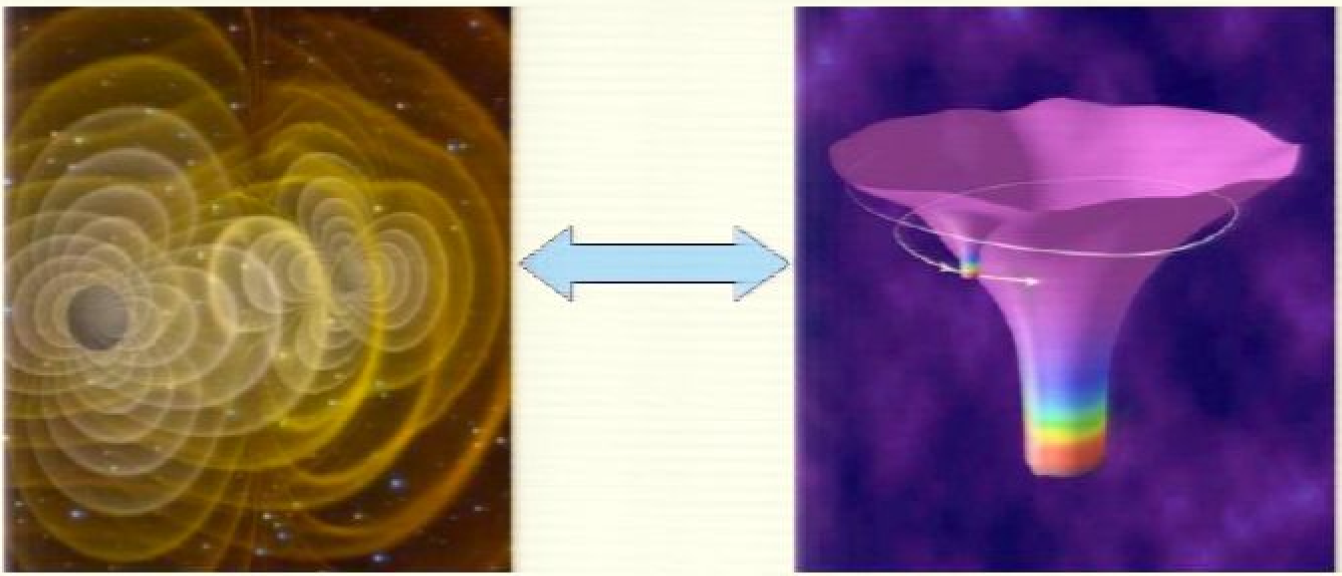
Motivation

Dynamics of binary systems in GR is tough! (No analytic solution)

- Numerical relativity simulations for BH binaries produced only recently (Pretorius '05, Campanelli et al '06, Baker et al '06)
- Simulations are very time consuming (take weeks) $\Rightarrow$ impossible to cover entire parameter space of BH binaries

To do astrophysics or gravitational-wave astronomy, we need faster analytical-relativity approaches: e.g effective-one-body (EOB) model

Main idea: map two-body problem into test-particle problem



$m_1 = m_2$ $\qquad$ $m_1 \ll m_2$

s this mapping possible?

s this mapping possible?

- Newtonian non-spinning binaries can be mapped to non-spinning test-particle with mass $\mu = m_1 m_2/(m_1 + m_2)$ around mass $M = m_1 + m_2$

s this mapping possible?

- Newtonian non-spinning binaries can be mapped to non-spinning test-particle with mass $\mu = m_1 m_2/(m_1 + m_2)$ around mass $M = m_1 + m_2$

- Energy levels of positronium $(e^+\text{-}e^-)$ can be mapped to those of hydrogen through

$$\frac{E_{\rm H}}{\mu c^2} = \frac{E_{\rm pos}^2 - m_1^2 c^4 - m_2^2 c^4}{2 m_1 m_2 c^4}$$

ffective-one-body models for non-spinning black-hole inaries (Buonanno and Damour 2001)

ffective-one-body models for non-spinning black-hole inaries (Buonanno and Damour 2001)

1. Conservative dynamics of non-spinning BH binaries = "Real" PN Hamiltonian $H_{\mathrm{PN,\,real}}$

ffective-one-body models for non-spinning black-hole inaries (Buonanno and Damour 2001)

1. Conservative dynamics of non-spinning BH binaries = "Real" PN Hamiltonian $H_{\mathrm{PN,\,real}}$
2. particle with mass $\mu = m_1 m_2/(m_1 + m_2)$ around a $M = m_1 + m_2$ *deformed* Schwarzschild BH ("effective problem") has Hamiltonian H_{eff}, which satisfies

$$\frac{H_{\mathrm{eff}}}{\mu c^2} = \frac{H^2_{\mathrm{PN,\,real}} - m_1^2 c^4 - m_2^2 c^4}{2 m_1 m_2 c^4} \text{ (up to 3 PN) } (*)$$

ffective-one-body models for non-spinning black-hole inaries (Buonanno and Damour 2001)

1. Conservative dynamics of non-spinning BH binaries = "Real" PN Hamiltonian $H_{\mathrm{PN,\,real}}$
2. particle with mass $\mu = m_1 m_2/(m_1 + m_2)$ around a $M = m_1 + m_2$ *deformed* Schwarzschild BH ("effective problem") has Hamiltonian H_{eff}, which satisfies

$$\frac{H_{\mathrm{eff}}}{\mu c^2} = \frac{H^2_{\mathrm{PN,\,real}} - m_1^2 c^4 - m_2^2 c^4}{2 m_1 m_2 c^4} \text{ (up to 3 PN) } (*)$$

3. H_{eff} can be calculated at all PN orders (derformed Schwarzschild is given at all PN orders) $\Rightarrow$ invert Eq. $(*)$ and get "real" Hamiltonian valid at all PN orders:

$$H_{\mathrm{real}} = M\sqrt{1 + 2\frac{\mu}{M}\left(\frac{H_{\mathrm{eff}}}{\mu} - 1\right)}.$$

ffective-one-body models for non-spinning black-hole inaries (Buonanno and Damour 2001)

1. Conservative dynamics of non-spinning BH binaries = "Real" PN Hamiltonian $H_{\mathrm{PN,\,real}}$
2. particle with mass $\mu = m_1 m_2/(m_1+m_2)$ around a $M = m_1 + m_2$ *deformed* Schwarzschild BH ("effective problem") has Hamiltonian H_{eff}, which satisfies

$$\frac{H_{\mathrm{eff}}}{\mu c^2} = \frac{H^2_{\mathrm{PN,\,real}} - m_1^2 c^4 - m_2^2 c^4}{2 m_1 m_2 c^4} \text{ (up to 3 PN) } (*)$$

3. H_{eff} can be calculated at all PN orders (derformed Schwarzschild is given at all PN orders) $\Rightarrow$ invert Eq. $(*)$ and get "real" Hamiltonian valid at all PN orders:

$$H_{\mathrm{real}} = M\sqrt{1 + 2\frac{\mu}{M}\left(\frac{H_{\mathrm{eff}}}{\mu} - 1\right)}.$$

4. H_{real}, with model for the fluxes (remember Nico Yunes' talk) and recipe to attach QNMs describing the ringdown (see Yi Pan's talk).

How to build an EOB for spinning black-hole binaries?

1. Conservative dynamics of spinning BH binaries = "Real" PN Hamiltonian $H_{\mathrm{PN,\,real}}$

How to build an EOB for spinning black-hole binaries?

1. Conservative dynamics of spinning BH binaries = "Real" PN Hamiltonian $H_{\mathrm{PN,\,real}}$
2. particle with mass $\mu = m_1 m_2/(m_1+m_2)$ and suitable spin σ around a $M = m_1 + m_2$ *deformed* Kerr BH with suitable spin ("effective problem") $\Rightarrow$ Hamiltonian H_{eff}, which satisfies

$$\frac{H_{\mathrm{eff}}}{\mu c^2} = \frac{H^2_{\mathrm{PN,\,real}} - m_1^2 c^4 - m_2^2 c^4}{2 m_1 m_2 c^4} \text{ (up to 3 PN) } (*)$$

How to build an EOB for spinning black-hole binaries?

1. Conservative dynamics of spinning BH binaries = "Real" PN Hamiltonian $H_{\mathrm{PN,\,real}}$
2. particle with mass $\mu = m_1 m_2/(m_1+m_2)$ and suitable spin σ around a $M = m_1 + m_2$ *deformed* Kerr BH with suitable spin ("effective problem") $\Rightarrow$ Hamiltonian H_{eff}, which satisfies

$$\frac{H_{\mathrm{eff}}}{\mu c^2} = \frac{H^2_{\mathrm{PN,\,real}} - m_1^2 c^4 - m_2^2 c^4}{2 m_1 m_2 c^4} \text{ (up to 3 PN) } (*)$$

3. H_{eff} can be calculated at all PN orders (derformed Kerr is given at all PN orders) $\Rightarrow$ invert Eq. $(*)$ and get "real" Hamiltonian valid at all PN orders:

$$H_{\mathrm{real}} = M\sqrt{1 + 2\frac{\mu}{M}\left(\frac{H_{\mathrm{eff}}}{\mu} - 1\right)}.$$

How to build an EOB for spinning black-hole binaries?

1. Conservative dynamics of spinning BH binaries = "Real" PN Hamiltonian $H_{\mathrm{PN,\,real}}$
2. particle with mass $\mu = m_1 m_2/(m_1 + m_2)$ and suitable spin σ around a $M = m_1 + m_2$ *deformed* Kerr BH with suitable spin ("effective problem") $\Rightarrow$ Hamiltonian H_{eff}, which satisfies

$$\frac{H_{\mathrm{eff}}}{\mu c^2} = \frac{H^2_{\mathrm{PN,\,real}} - m_1^2 c^4 - m_2^2 c^4}{2 m_1 m_2 c^4} \text{ (up to 3 PN) } (*)$$

3. H_{eff} can be calculated at all PN orders (derformed Kerr is given at all PN orders) $\Rightarrow$ invert Eq. $(*)$ and get "real" Hamiltonian valid at all PN orders:

$$H_{\mathrm{real}} = M\sqrt{1 + 2\frac{\mu}{M}\left(\frac{H_{\mathrm{eff}}}{\mu} - 1\right)}.$$

Problem: Hamiltonian H_{eff} for spinning test-particle in curved spacetime was not known!

Can one guess H_{eff}?

Damour, Jaranowski and Schafer '08 tried phenomenological coupling between the particle's spin and the geometry, but this model has shortcomings:

Can one guess H_{eff}?

Damour, Jaranowski and Schafer '08 tried phenomenological coupling between the particle's spin and the geometry, but this model has shortcomings:

- Test particle limit recovered only approximately
- At order $\geq 3PN$:
  - No effective ISCO for spin aligned with $\boldsymbol{L}$
  - When ISCO exists, dependence on spins is non-monotonic

Can one guess H_{eff}?

Damour, Jaranowski and Schafer '08 tried phenomenological coupling between the particle's spin and the geometry, but this model has shortcomings:

- Test particle limit recovered only approximately
- At order $\geq 3PN$:
  - No effective ISCO for spin aligned with $\boldsymbol{L}$
  - When ISCO exists, dependence on spins is non-monotonic
- No natural way of attaching to attach quasi-normal modes to describe ringdown (*ad-hoc* matching)

Can we derive H for a spinning particle in curved spacetime?

Can we derive H for a spinning particle in curved spacetime?

Ideally, we would proceed as in the non-spinning particle case

$$S = -m \int d\tau \ \text{(geodetic motion)} \ \Rightarrow$$

Can we derive H for a spinning particle in curved spacetime?

Ideally, we would proceed as in the non-spinning particle case

$$S = -m \int d\tau \text{ (geodetic motion) } \Rightarrow$$

$$L = -m \frac{d\tau}{dt} = -m \sqrt{-\left(g_{tt} + 2 g_{ti} \frac{dx^i}{dt} + g_{ij} \frac{dx^i}{dt} \frac{dx^j}{dt} \right)}$$

$$(L = -m\sqrt{1 - \dot{x}^2} \text{ in Minkowski}) \Rightarrow$$

Can we derive H for a spinning particle in curved spacetime?

Ideally, we would proceed as in the non-spinning particle case

$$S = -m \int d\tau \text{ (geodetic motion) } \Rightarrow$$

$$L = -m\frac{d\tau}{dt} = -m\sqrt{-\left(g_{tt} + 2g_{ti}\frac{dx^i}{dt} + g_{ij}\frac{dx^i}{dt}\frac{dx^j}{dt}\right)}$$

$$(L = -m\sqrt{1-\dot{\boldsymbol{x}}^2} \text{ in Minkowski}) \Rightarrow$$

Legendre transformation gives

$$P_i = \frac{\partial L}{\partial \dot{x}^i} \quad \text{and} \quad H = P_i\dot{x}^i - L = \beta^i P_i + \alpha\sqrt{m^2 + \gamma^{ij}P_iP_j}$$

where $\alpha = \frac{1}{\sqrt{-g^{tt}}}$, $\beta^i = \frac{g^{ti}}{g^{tt}}$, $\gamma^{ij} = g^{ij} - \frac{g^{ti}g^{tj}}{g^{tt}}$

$$(H = \sqrt{m^2 + \boldsymbol{P}^2} \text{ in Minkowski})$$

ut the case of spinning particles is *much* more complicated

But the case of spinning particles is *much* more complicated

Deviations from geodetic motion due to coupling between spin and curvature: described by Mathisson-Papapetrou-Pirani eqs

$$\frac{Dp^{\mu}}{D\tau} = -\frac{1}{2} R^{\mu}{}_{\alpha\beta\gamma}\, u^{\alpha}\, S^{\beta\gamma}\,, \quad \frac{DS^{\mu\nu}}{D\tau} = p^{\mu}\, u^{\nu} - p^{\nu}\, u^{\mu}\,.$$

ut the case of spinning particles is *much* more complicated

Deviations from geodetic motion due to coupling between spin and curvature: described by Mathisson-Papapetrou-Pirani eqs

$$\frac{Dp^\mu}{D\tau} = -\frac{1}{2} R^\mu{}_{\alpha\beta\gamma}\, u^\alpha\, S^{\beta\gamma}\,, \quad \frac{DS^{\mu\nu}}{D\tau} = p^\mu\, u^\nu - p^\nu\, u^\mu\,.$$

- Particle's spin encoded in antisymmetric tensor $S^{\mu\nu}$ (6 degrees of freedom instead of 3) $\Rightarrow$ system underdetermined: 10 eqs vs 13 variables

But the case of spinning particles is *much* more complicated

Deviations from geodetic motion due to coupling between spin and curvature: described by Mathisson-Papapetrou-Pirani eqs

$$\frac{Dp^\mu}{D\tau} = -\frac{1}{2} R^\mu{}_{\alpha\beta\gamma}\, u^\alpha\, S^{\beta\gamma}\,, \quad \frac{DS^{\mu\nu}}{D\tau} = p^\mu\, u^\nu - p^\nu\, u^\mu\,.$$

- Particle's spin encoded in antisymmetric tensor $S^{\mu\nu}$ (6 degrees of freedom instead of 3) $\Rightarrow$ system underdetermined: 10 eqs vs 13 variables
- To close system, need to choose spin supplementary condition to reduce spin's degrees of freedom to 3

$$S^{\mu\nu}\omega_\nu = 0, \text{ with } \omega \text{ timelike}$$

But the case of spinning particles is *much* more complicated

Deviations from geodetic motion due to coupling between spin and curvature: described by Mathisson-Papapetrou-Pirani eqs

$$\frac{Dp^\mu}{D\tau} = -\frac{1}{2} R^\mu{}_{\alpha\beta\gamma}\, u^\alpha\, S^{\beta\gamma}\,, \quad \frac{DS^{\mu\nu}}{D\tau} = p^\mu\, u^\nu - p^\nu\, u^\mu\,.$$

- Particle's spin encoded in antisymmetric tensor $S^{\mu\nu}$ (6 degrees of freedom instead of 3) $\Rightarrow$ system underdetermined: 10 eqs vs 13 variables
- To close system, need to choose spin supplementary condition to reduce spin's degrees of freedom to 3

$$S^{\mu\nu}\omega_\nu = 0, \text{ with } \omega \text{ timelike}$$

- MPP eqs + SSC = motion of body's center of energy with respect to observer ω (spinning particle has finite size to avoid superluminal motion)

A sketch of the derivation (EB, Racine and Buonanno '08)

A sketch of the derivation (EB, Racine and Buonanno '08)

- Write down the action giving the MPP equations: done by Hanson and Regge '74 (flat spacetime) and by Porto '06 (curved spacetime)

A sketch of the derivation (EB, Racine and Buonanno '08)

- Write down the action giving the MPP equations: done by Hanson and Regge '74 (flat spacetime) and by Porto '06 (curved spacetime)
- Introduce a time slicing and write the unconstrained Lagrangian: this has 6 degrees of freedom for the spin

A sketch of the derivation (EB, Racine and Buonanno '08)

- Write down the action giving the MPP equations: done by Hanson and Regge '74 (flat spacetime) and by Porto '06 (curved spacetime)
- Introduce a time slicing and write the unconstrained Lagrangian: this has 6 degrees of freedom for the spin
- Perform a Legendre transformation and derive the unconstrained Hamiltonian: this still has 6+6 degrees of freedom for the spin

A sketch of the derivation (EB, Racine and Buonanno '08)

- Write down the action giving the MPP equations: done by Hanson and Regge '74 (flat spacetime) and by Porto '06 (curved spacetime)
- Introduce a time slicing and write the unconstrained Lagrangian: this has 6 degrees of freedom for the spin
- Perform a Legendre transformation and derive the unconstrained Hamiltonian: this still has 6+6 degrees of freedom for the spin
- Impose a SSC and another set of 3 "conjugate" conditions: use Dirac brackets formalism to compute the constrained Hamiltonian and the constrained phase-space algebra

sketch of the derivation (EB, Racine and Buonanno '08)

- Write down the action giving the MPP equations: done by Hanson and Regge '74 (flat spacetime) and by Porto '06 (curved spacetime)
- Introduce a time slicing and write the unconstrained Lagrangian: this has 6 degrees of freedom for the spin
- Perform a Legendre transformation and derive the unconstrained Hamiltonian: this still has 6+6 degrees of freedom for the spin
- Impose a SSC and another set of 3 "conjugate" conditions: use Dirac brackets formalism to compute the constrained Hamiltonian and the constrained phase-space algebra
- IF the SSC and conjugate constraint are chosen properly, the constrained algebra is canonical $\Rightarrow$ Hamilton eqs take usual form $\dot{x}^i = \frac{\partial H}{\partial P_i}$, $\dot{P}_i = -\frac{\partial H}{\partial x^i}$ $\Rightarrow$ Hamiltonian has the usual meaning!

Hamiltonian for spinning particle with mass m and spin S^I in curved spacetime, at linear order in the particle's spin

$$H = H_{\rm NS} - \left(\beta^i F_i^K + F_t^K + \frac{\alpha \gamma^{ij} P_i F_j^K}{\sqrt{m^2 + \gamma^{ij} P_i P_j}} \right) S_K .$$

where $H_{\rm NS}$ is the Hamiltonian for a non-spinning particle:

$$H_{\rm NS} = \beta^i P_i + \alpha \sqrt{m^2 + \gamma^{ij} P_i P_j} .$$

with $\alpha = \frac{1}{\sqrt{-g^{tt}}}$, $\beta^i = \frac{g^{ti}}{g^{tt}}$, $\gamma^{ij} = g^{ij} - \frac{g^{ti} g^{tj}}{g^{tt}}$ and

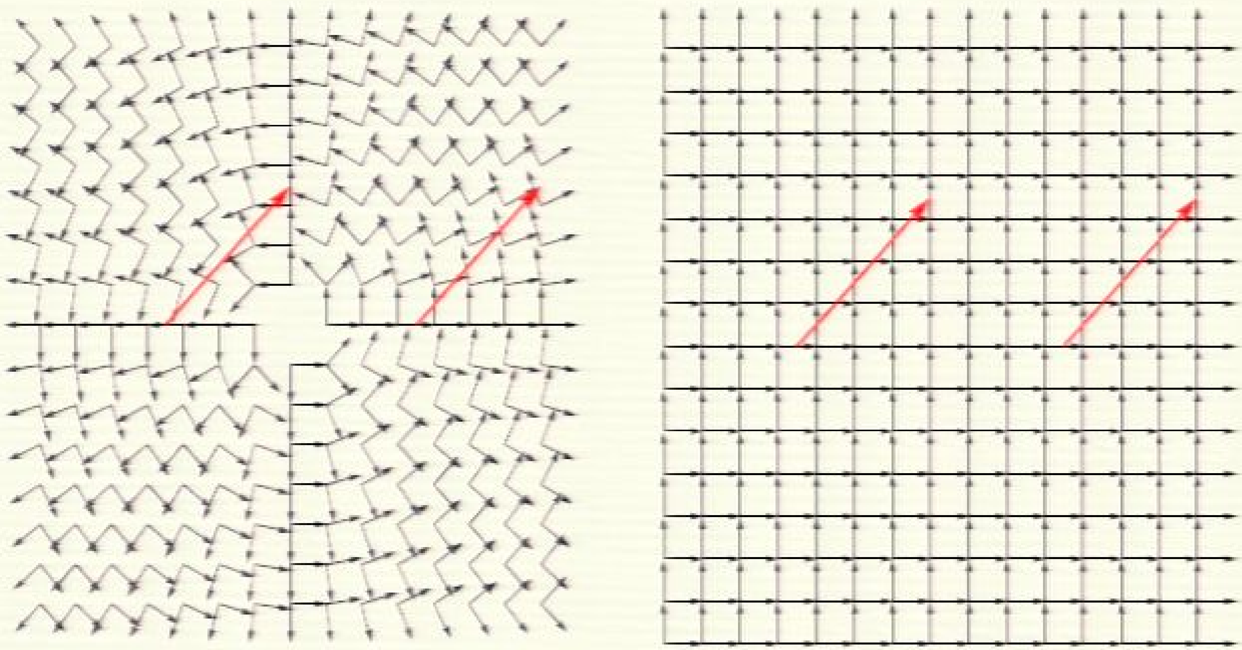
$$F_\mu^K = \left(2 E_{\mu T I} \frac{\bar{\omega}_J}{\bar{\omega}_T} + E_{\mu IJ} \right) \epsilon^{IJK} , \qquad E_{\lambda\mu\nu} \equiv \frac{1}{2} \eta_{AB} \, \tilde{e}^A_\mu \, \tilde{e}^B_{\nu;\lambda} .$$

$$\bar{\omega}_T = \bar{\omega}_\mu \, \tilde{e}^\mu_T = \bar{P}_\mu \, \tilde{e}^\mu_T - m , \qquad \bar{\omega}_I = \bar{\omega}_\mu \, \tilde{e}^\mu_I = \bar{P}_\mu \, \tilde{e}^\mu_I$$

$\tilde{e}_A(x^i, t)$ is an arbitrary reference tetrad field: it's a gauge field because H's built with different $\tilde{e}_A$'s are linked by canonical transformations

What is the meaning of the reference tetrad field?

- It's a "gauge" degree of freedom: different choice of the tetrad produce Hamiltonian related by canonical transformation
- It gives the reference frame with respect to which the spin is measured: e.g. the spin *vector* is constant in flat spacetime, but the *components* in general are not!



To avoid gauge terms, tetrad must become cartesian in the flat limit!

Ve now know the Hamiltonian for a spinning particle!

We now know the Hamiltonian for a spinning particle!

New effective-one-body model works!

- Correct test-particle limit: when $m_1/m_2 \to 0$ it gives particle with mass m_1 and spin S_1 around Kerr BH with mass m_2 and spin S_2

Ve now know the Hamiltonian for a spinning particle!

New effective-one-body model works!

- Correct test-particle limit: when $m_1/m_2 \to 0$ it gives particle with mass m_1 and spin S_1 around Kerr BH with mass m_2 and spin S_2
- There is an ISCO for any values of spin and mass-ratio, and the dependence on the parameters looks sensible

Ve now know the Hamiltonian for a spinning particle!

New effective-one-body model works!

- Correct test-particle limit: when $m_1/m_2 \to 0$ it gives particle with mass m_1 and spin S_1 around Kerr BH with mass m_2 and spin S_2
- There is an ISCO for any values of spin and mass-ratio, and the dependence on the parameters looks sensible
- The frequency during the plunge reaches a peak $\Rightarrow$ because peak indicates that particle's motions is getting locked to the black hole rotation (Hughes '01, EB, Hughes and Rezzolla '07), it is natural to attach quasi-normal modes there $\Rightarrow$ easy to describe the ringdown (Yi Pan's talk)

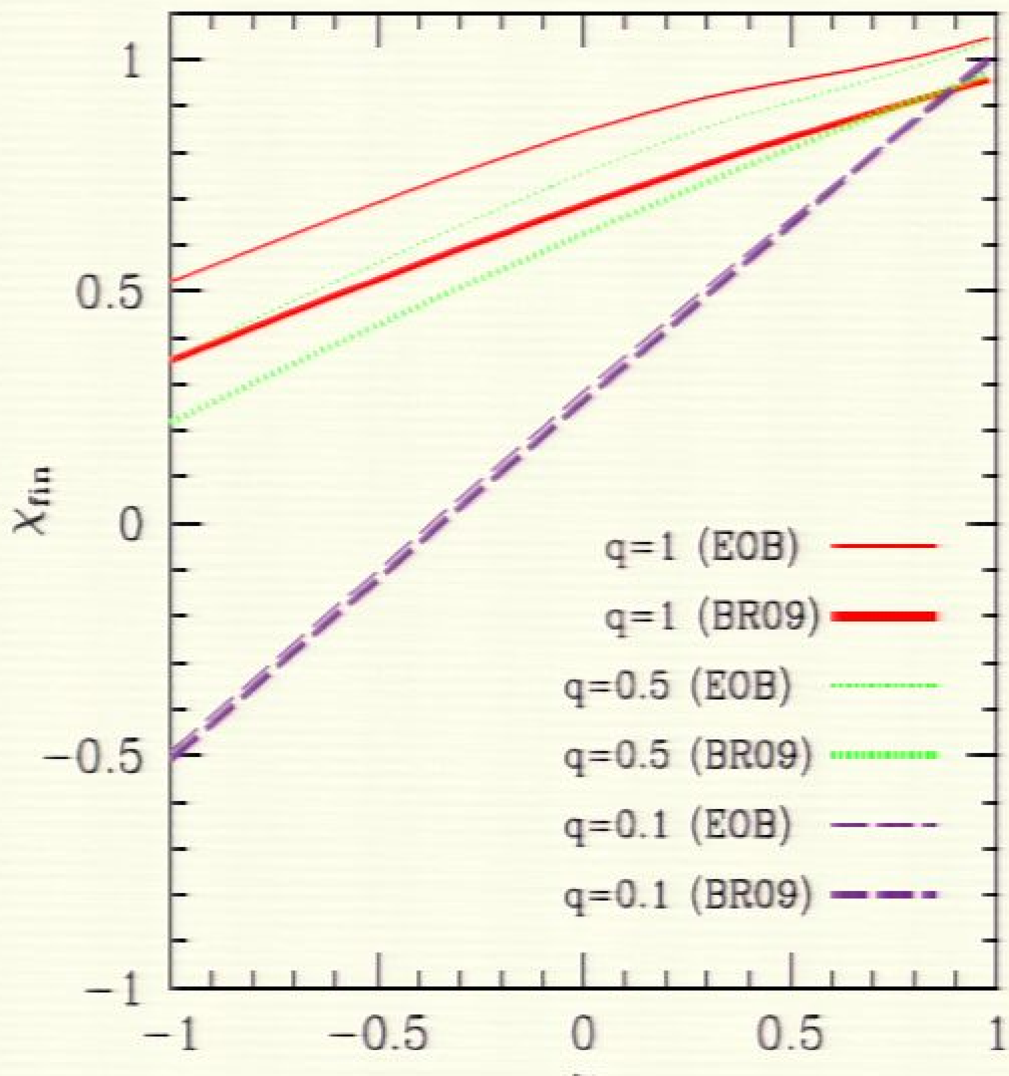
We now know the Hamiltonian for a spinning particle!

New effective-one-body model works!

- Correct test-particle limit: when $m_1/m_2 \to 0$ it gives particle with mass m_1 and spin S_1 around Kerr BH with mass m_2 and spin S_2
- There is an ISCO for any values of spin and mass-ratio, and the dependence on the parameters looks sensible
- The frequency during the plunge reaches a peak $\Rightarrow$ because peak indicates that particle's motions is getting locked to the black hole rotation (Hughes '01, EB, Hughes and Rezzolla '07), it is natural to attach quasi-normal modes there $\Rightarrow$ easy to describe the ringdown (Yi Pan's talk)
- Agrees with the self-force computation of the ISCO shift for a non-spinning particle in a Schwarzschild spacetime after calibration of free parameter describing effect of unknown high PN orders (Barack and Sago 2009)

$\chi_{\rm fin} = J_{\rm fin}/M_{\rm fin}^2$ vs $S_1/m_1^2 = S_2/m_2^2 = \chi$, $q = m_2/m_1$

vs formula for final spin reproducing NR data (EB and Rezzolla 2009)



Conclusions

- We derived the Hamiltonian for a spinning particle in a curved spacetime, 60 years after the Mathisson-Papapetrou-Pirani eqs of motion were written

Conclusions

- We derived the Hamiltonian for a spinning particle in a curved spacetime, 60 years after the Mathisson-Papapetrou-Pirani eqs of motion were written

- New EOB model for spinning black-hole binary which reproduces test particle limit: this model does not suffer from the shortcomings of existing models

Conclusions

- We derived the Hamiltonian for a spinning particle in a curved spacetime, 60 years after the Mathisson-Papapetrou-Pirani eqs of motion were written
- New EOB model for spinning black-hole binary which reproduces test particle limit: this model does not suffer from the shortcomings of existing models
- EOB faster than NR and has applications to astroph and gravitational-wave astronomy, but requires calibration against NR results and self-force calculations

For more comparison with NR data, see Yi Pan's talk!