Title: Achievable Directional Reconstruction for Gravitational waves generated by Binary Systems.

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Abstract: Recently generated asymptotic expansions zanolin et al. arXiv:0912.0065 [gr-qc] showa frequentist approach to go beyond Fisher information assessments of the accuracy for maximum likelihood parameter estimations. In this talk we describe the application of these techniques to directional reconstruction fornumerical relativity waveforms.

LSC LIGO Scientific Collaboration

Asymptotic expansions for the accuracy of parameter estimations in binary systems

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NRDA 2010, June 25th Perimeter Institute

Motivations

- CRLB provides analytical understanding but it does not explain errors accurately for low SNR because it is only sensitive to the main lobe of the likelihood function .
- The first detections might be with an SNR 20 or less.
- Even if other approaches are available (e.g MonteCarlo, Bayesian pipelines on real data) we should apply all the techniques that provide new knowledge on the errors.
- 3 applications discussed here (NO REAL DATA).

Parameter estimation papers

- Inspiral phase only. (no direction errors)
  M. Zanolin, S. Vitale, N. Makris, Phys. Rev. D. 81.124048 (2010) .
- IMR BBH (no spin - source direction reconstruction not included). submitted to PRD. S. Vitale M. Zanolin gr-qc arXiv:1004.4537
- IMR BBH directional reconstruction (no spin) in preparation.
- IMR BBH with spin Marc LeBourdais (SURF Caltech ERAU). Mentors: A. Paramesan. M. Vallisneri.

Disclaimer

- Finn, Flanagan, Vallisneri (PRD 2008): proposed an algorithm to derive : “expansions of the signal” in SNR for Gaussian noise. Final explicit form not available (until last week). Explicit GW examples were not studied yet (they currently are this summer)
- Zanolin, Vitale, Makris: expansions of the likelihood expansions for arbitrary noise.
- LeBourdais Vallisneri Zanolin: the two expansions are equivalent in Gaussian noise.

Asymptotic Expansions

- Given any (known) probability distribution of the data it is possible to derive, in the frequentist framework expansions of the moments of Maximum likelihood estimators in terms of inverse powers of SNR. The first order is the CRLB while higher orders depend on higher order derivatives of the likelihood function (4th for the second order and therefore are dependent on secondary maxima of the likelihood function)

$$\sigma^2_{\vartheta^i} = \frac{S_1^2}{\rho^2} + \frac{S_2^2}{\rho^4} + \cdots = \sigma^2_{\vartheta^i}[1] + \sigma^2_{\vartheta^i}[2] + \cdots$$

$$b_{\vartheta^i} = \frac{B_1}{\rho} + \frac{B_2}{\rho^2} + \cdots = b_{\vartheta^i}[1] + b_{\vartheta^i}[2] + \cdots$$

1. We will work in the frequency space. In this case the expectations become scalar products:

$$\langle a(f), b(f) \rangle \equiv 2 \int_{f_{low}}^{f_{cut}} df \frac{a(f)^* b(f) + a(f) b(f)^*}{S_h(f)}$$

where $S_h(f)$ is the *one sided noise spectral density*.

2. The (i, j) element of the Fisher information is then:

$$\Gamma_{ij} = \langle \frac{\partial h(f)}{\partial \vartheta^i}, \frac{\partial h(f)}{\partial \vartheta^j} \rangle \equiv \langle h_i(f) \, , \, h_j(f) \rangle$$

3. We define the optimal SNR to be:

$$\rho^2 \equiv \langle h(f) \, , \, h(f) \rangle = \int_{f_{low}}^{f_{cut}} df \frac{|h(f)|}{S_h(f)}$$

First and second order variance

1. The CRLB for the i-th parameter is easily calculated as the (i,i) element of the inverse matrix of the Fisher information:

$$\sigma^2_{\vartheta^i} \geq \Gamma^{ii}$$

2. The second order variance can be expressed like a combination of means of the likelihod's derivatives:

$$\begin{aligned} C_2(\vartheta^j) &= \Gamma^{jm}\Gamma^{jn}\Gamma^{pq}(2\upsilon_{nq,m,p}+\upsilon_{nmpq}+3\upsilon_{nq,pm}+2\upsilon_{nmp,q}+\upsilon_{mpq,n} \\ &+ \Gamma^{jm}\Gamma^{jn}\Gamma^{pz}\Gamma^{qt}[(\upsilon_{npm}+\upsilon_{n,mp})(\upsilon_{qzt}+2\upsilon_{t,zq}) \\ &+ \upsilon_{npq}(\frac{5}{2}\upsilon_{mzt}+2\upsilon_{m,tz}+\upsilon_{m,t,z}) \\ &+ \upsilon_{nq,z}(6\upsilon_{mpt}+2\upsilon_{pt,m}+\upsilon_{mpt})] - \Gamma^{jj} \end{aligned}$$

Where

$$\upsilon_{a_1a_2..a_s,\,..\,,b_1b_2..b_s} = E\left[\ell_{a_1a_2..a_s}\,..\,\ell_{b_1b_2..b_s}\right]$$

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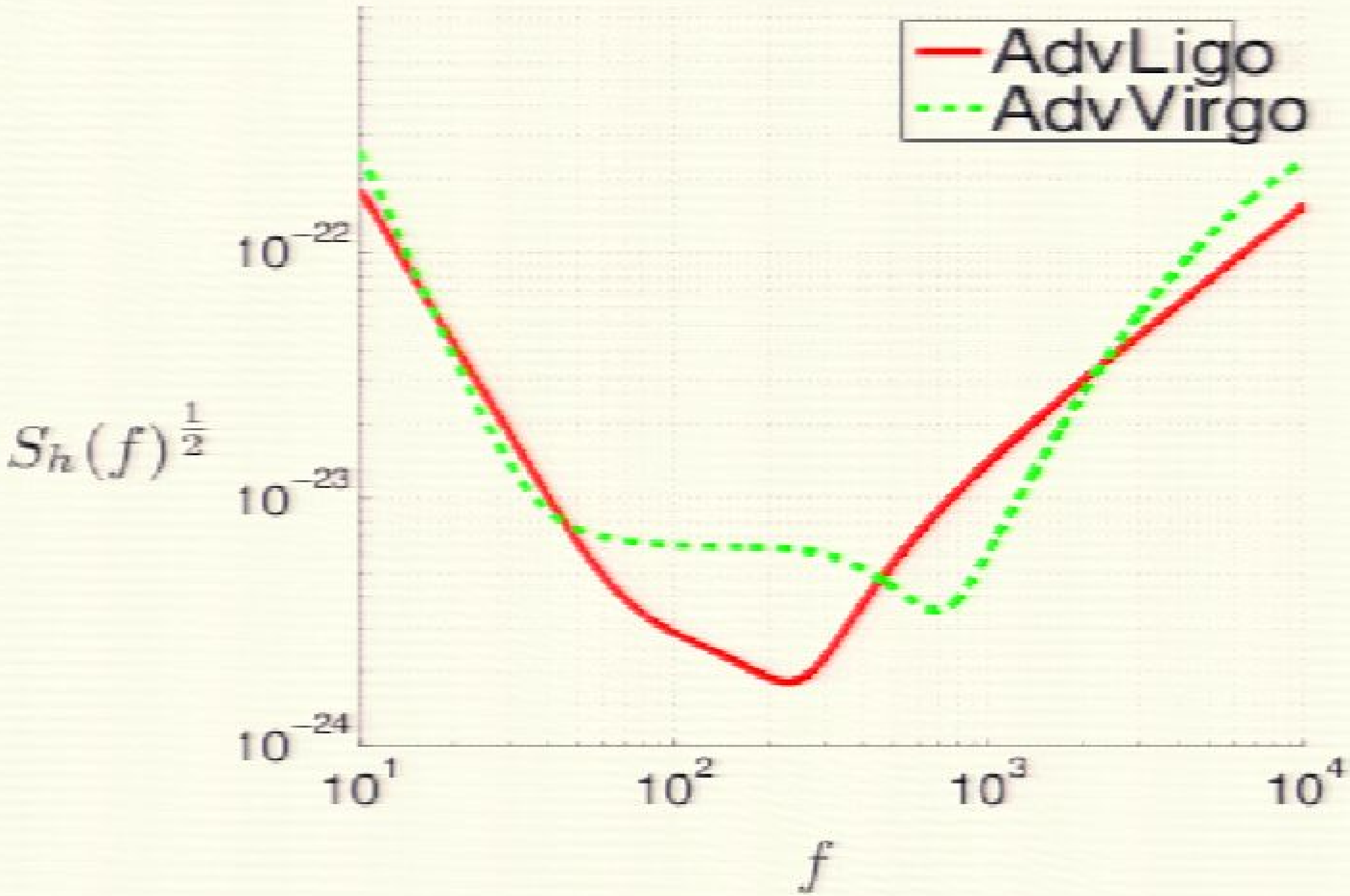
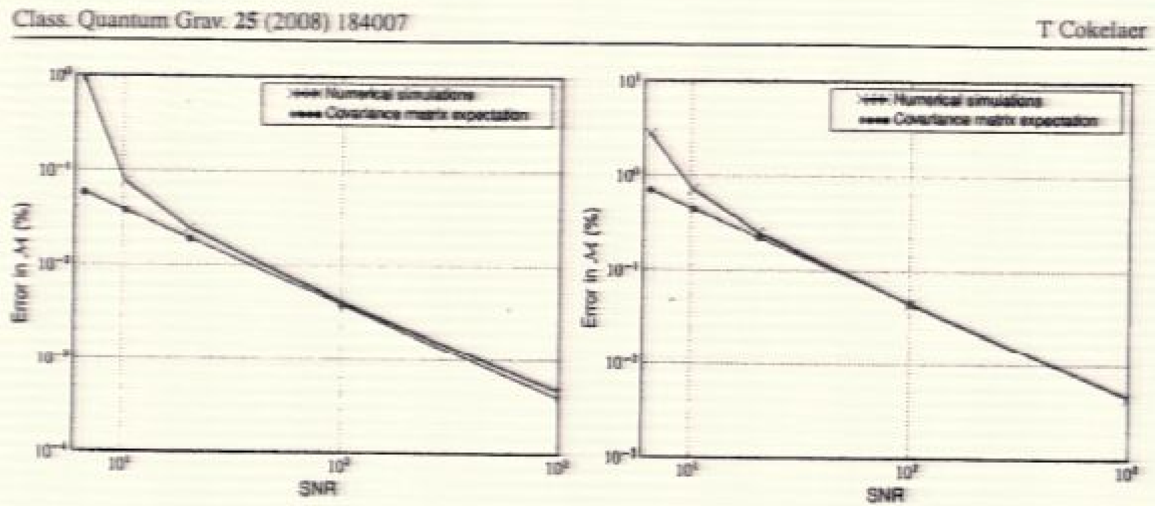
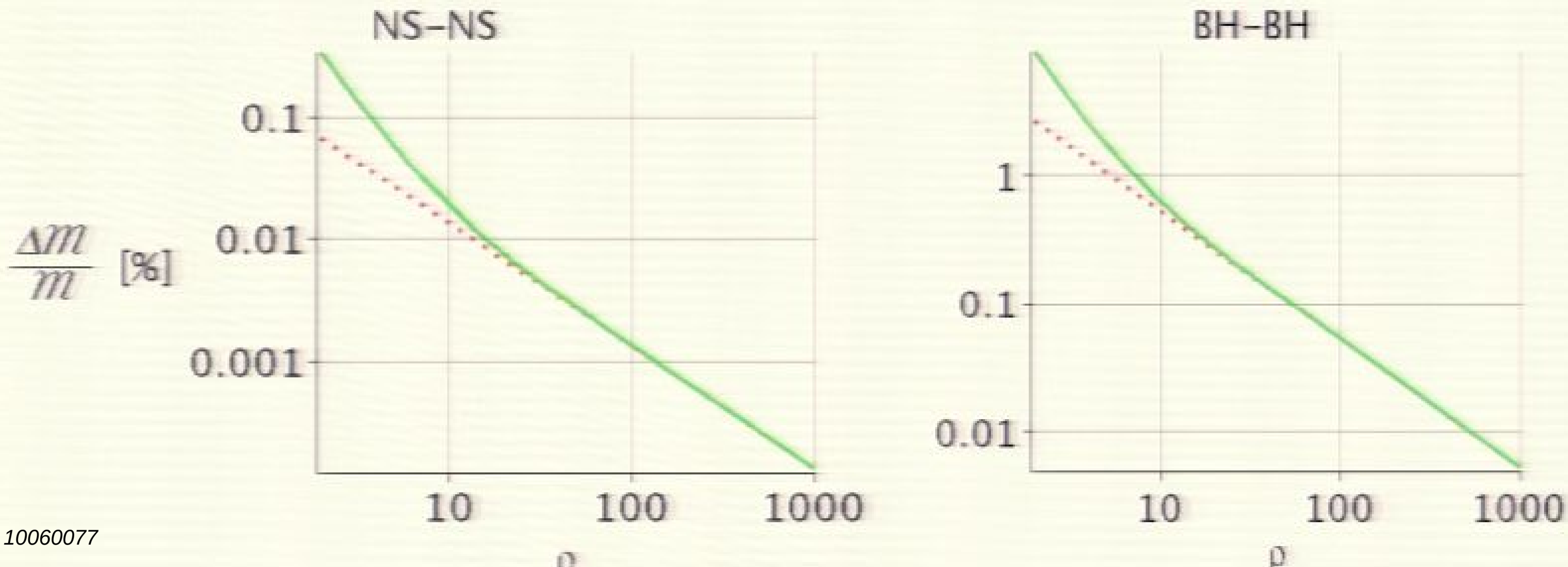

AdvLigo
AdvVirgo
$S_h(f)^{\frac{1}{2}}$
10^{-22}
10^{-23}
10^{-24}
10^{1}
10^{2}
10^{3}
10^{4}
f

Inspiral phase only

NSNS-1.4-1.4-arrival time-full par space

Numerical simulations (T. Cokelaer, Class. Q. Grav. 25,184007, (2008))





1. Let's consider the time t as the only parameter, all the others quantities being known. Working at 1PN order, we obtain:

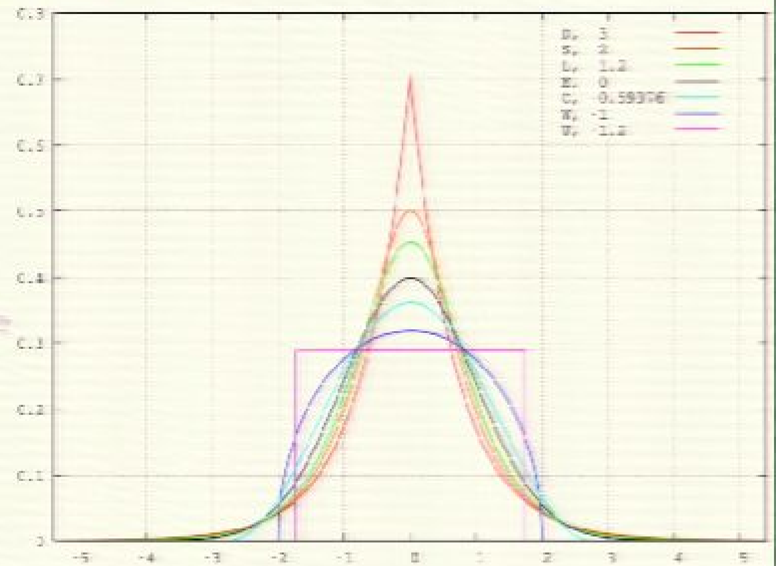
$$\left(\frac{C_{2t}}{\sigma_t}\right)^2 = \frac{1}{4\pi^2\rho^2}\frac{\frac{J_4}{J_1}}{\left(\frac{J_2}{J_1}\right)^2}$$

where

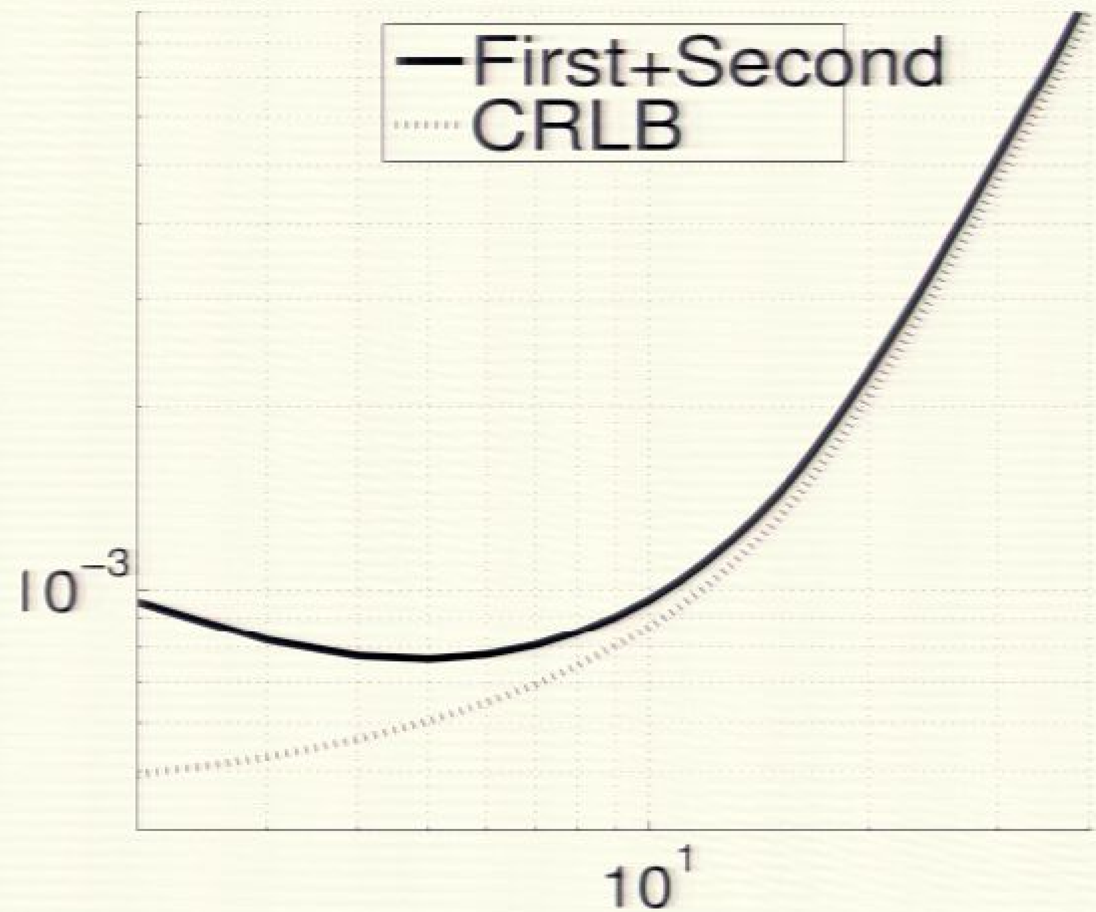
$$J_\beta \equiv \int_{f_{low}}^{f_{cut}} f^\beta |h(f)|^2 df$$

2. The the second order correction is much smaller that the first order one if the SRN ρ^2 satisfies:

$$\rho^2 \gg \frac{\frac{J_4}{J_1}}{\left(\frac{J_2}{J_1}\right)^2}$$



Error minima with only inspiral phase



BBH

BBH Avanced LIGO errors at SNR=10

		$M = 200M_\odot$				$M = 100M_\odot$				$M = 20M_\odot$			
		$\sigma[1]$	$\sigma[2]$	$b[1]$	$b[2]$	$\sigma[1]$	$\sigma[2]$	$b[1]$	$b[2]$	$\sigma[1]$	$\sigma[2]$	$b[1]$	$b[2]$
$\eta = 0.25$	Δt	9.47	12.5	-0.45	-1.04	3.54	2.92	-0.10	0.07	0.22	0.20	$-6.6\,10^{-3}$	$-8.9\,10^{-3}$
	$\Delta\phi$	40.7	59.7	-7.85	-18.0	24.2	21.6	-2.27	-0.98	9.98	11.0	-0.42	-0.57
	$\frac{\Delta M}{M}$	6.02	9.60	0.82	1.87	2.61	2.12	$-2.24\,10^{-4}$	$-2.92\,10^{-2}$	1.38	1.44	$5.34\,10^{-3}$	$1.25\,10^{-2}$
	$\frac{\Delta\eta}{\eta}$	11.3	16.4	0.81	1.26	6.53	5.82	0.37	0.58	2.58	2.88	0.04	$3.64\,10^{-2}$
$\eta = 0.22$	Δt	12.0	16.3	-0.74	-1.35	4.33	3.73	-0.16	$3.79\,10^{-2}$	0.23	0.22	$-6.14\,10^{-3}$	$-8.24\,10^{-3}$
	$\Delta\phi$	63.3	94.7	-9.69	-21.6	36.5	33.9	-2.70	-1.1	12.9738	14.8	-0.36	-0.49
	$\frac{\Delta M}{M}$	5.10	8.21	0.74	1.93	2.62	1.88	$9.63\,10^{-3}$	$-3.57\,10^{-2}$	1.27	1.37	$4.52\,10^{-3}$	$0.70\,10^{-2}$
	$\frac{\Delta\eta}{\eta}$	12.1	18.0	0.84	1.77	6.87	6.35	0.34	0.51	2.36	2.72	$3.38\,10^{-2}$	$4.03\,10^{-2}$
$\eta = 0.16$	Δt	21.8	30.0	-2.12	-3.17	7.04	6.65	-0.31	-1.04	0.25	0.26	$-4.93\,10^{-3}$	$-5.79\,10^{-3}$
	$\Delta\phi$	143	213	-18.0	-33.2	77.0	77.7	-3.75	-2.26	19.2	22.4	-3.76	-0.25
	$\frac{\Delta M}{M}$	3.07	4.21	0.48	1.26	2.75	1.70	$1.27\,10^{-3}$	$-4.84\,10^{-2}$	0.99	1.10	$2.19\,10^{-3}$	$-1.23\,10^{-3}$
	$\frac{\Delta\eta}{\eta}$	14.2	20.8	0.85	2.37	7.53	7.58	0.37	0.52	1.84	2.15	0.02	$3.57\,10^{-2}$

Figure 2. The errors in an Advanced Ligo detector (table above) and Advanced Virgo (below). σ and b have the meanings explained in eqs. 2.6 and 2.7. The time errors are in milliseconds, the phase errors are in radians, while the errors in the mass parameters are in percent

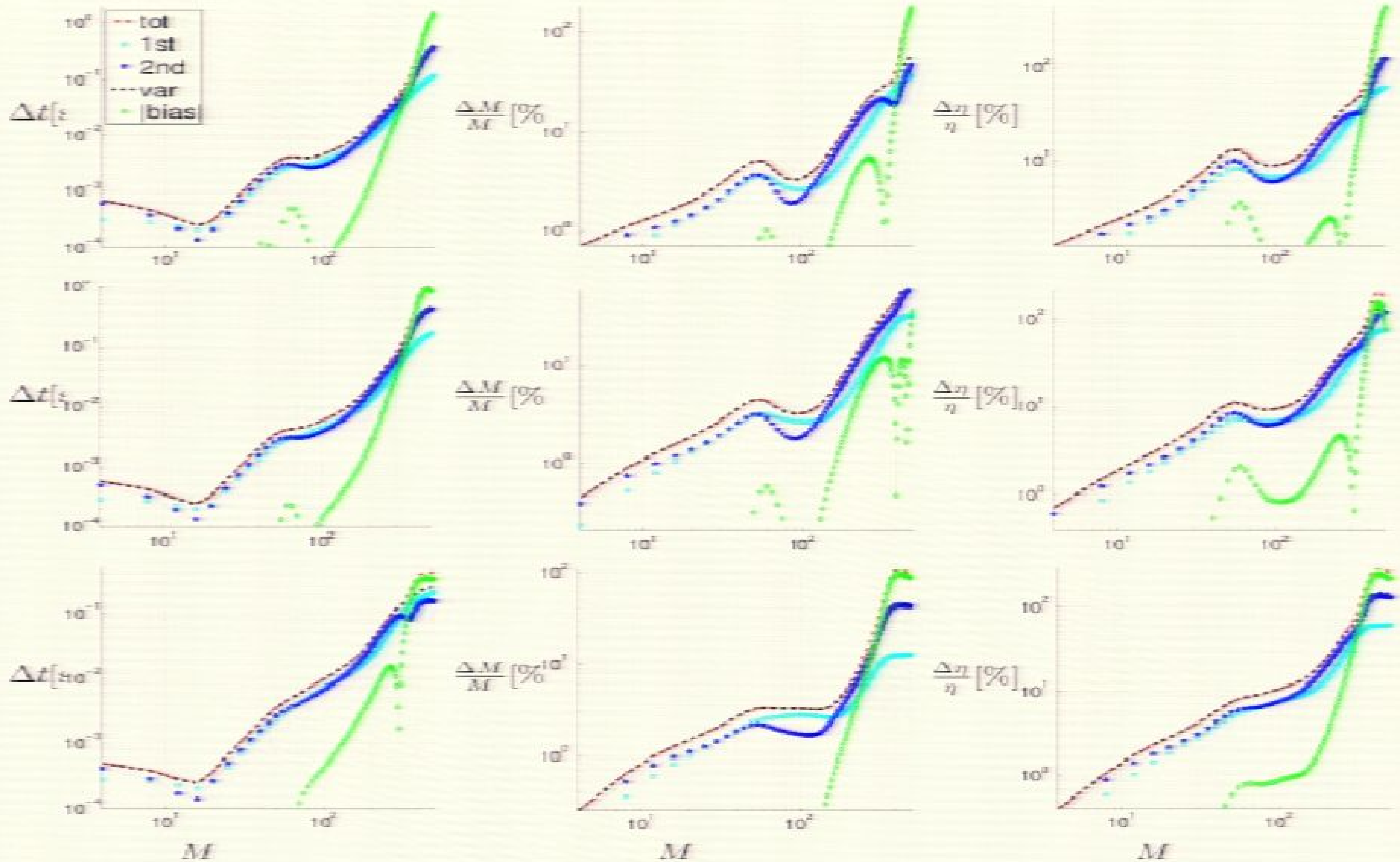


Figure 6. (Color Online) The errors plotted against the total mass, using the AdvLigo noise, for a fixed value of $\rho = 10$, and with $\eta = 0.25$ (top), $\eta = 0.2222$ (middle) and $\eta = 0.16$ (bottom).

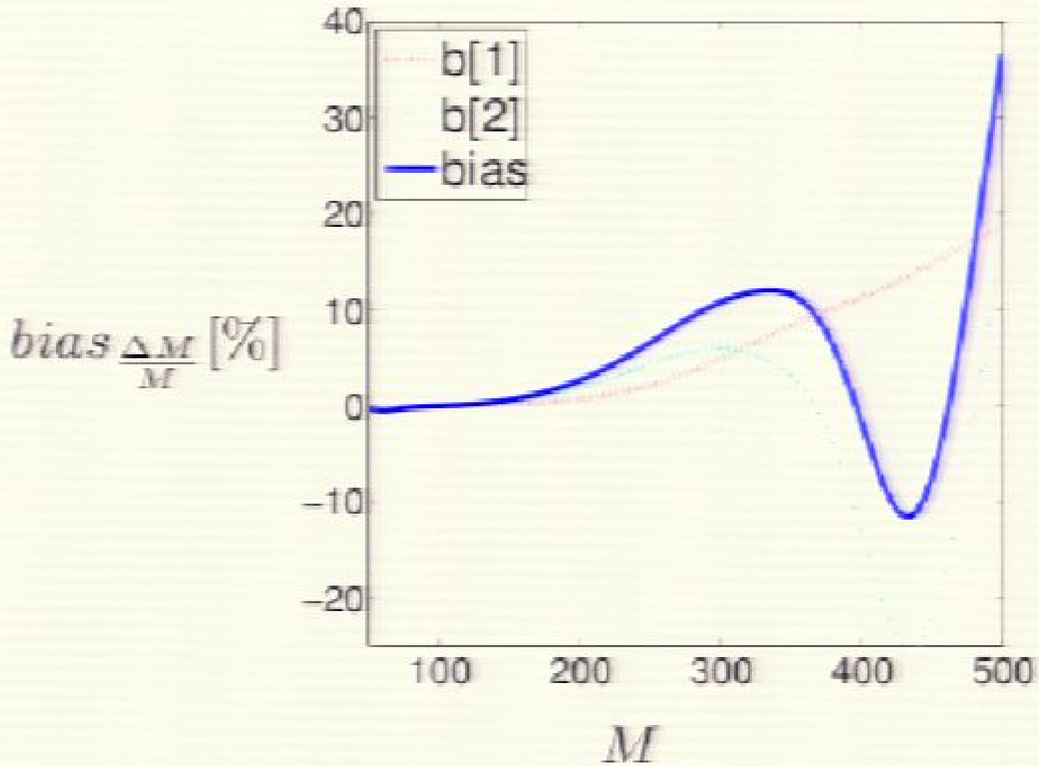
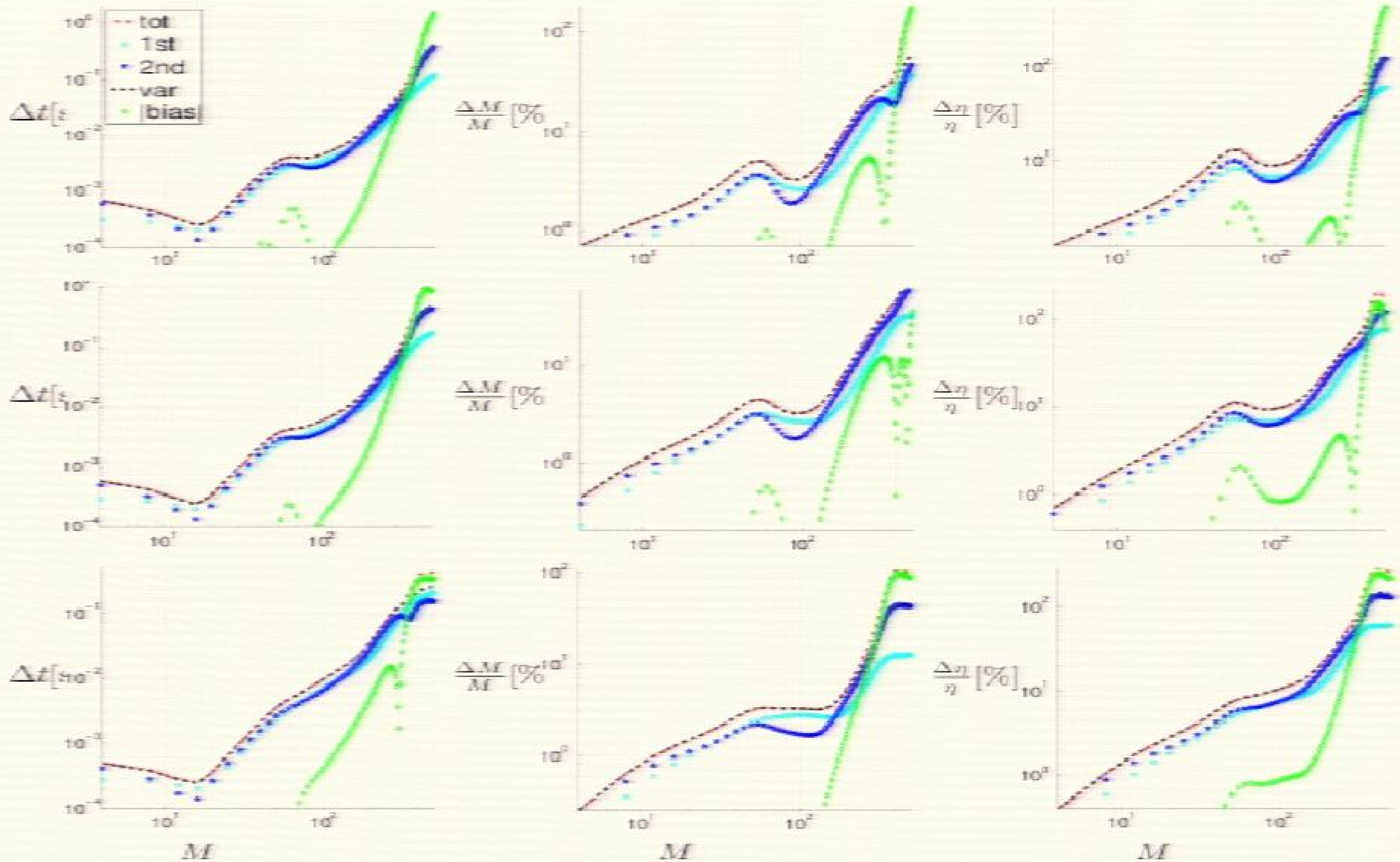


Figure 8. (Color Online) The bias, first order (red dashed), second order (cyan dotted) and total (blue line) using the AdvLigo noise, for a fixed value of $\rho = 10$, and with $\eta = 0.2222$



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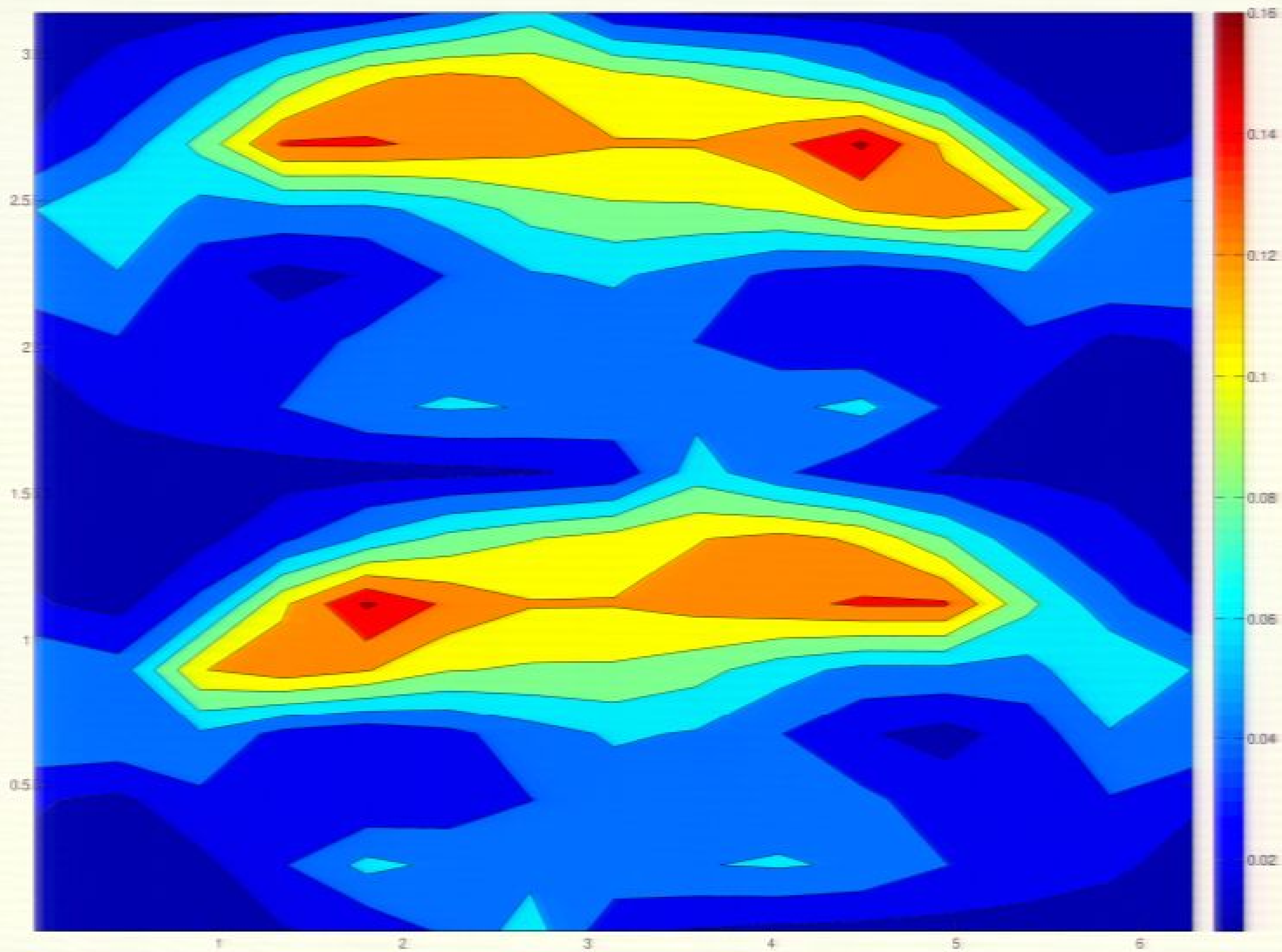
Direction Reconstruction

theta



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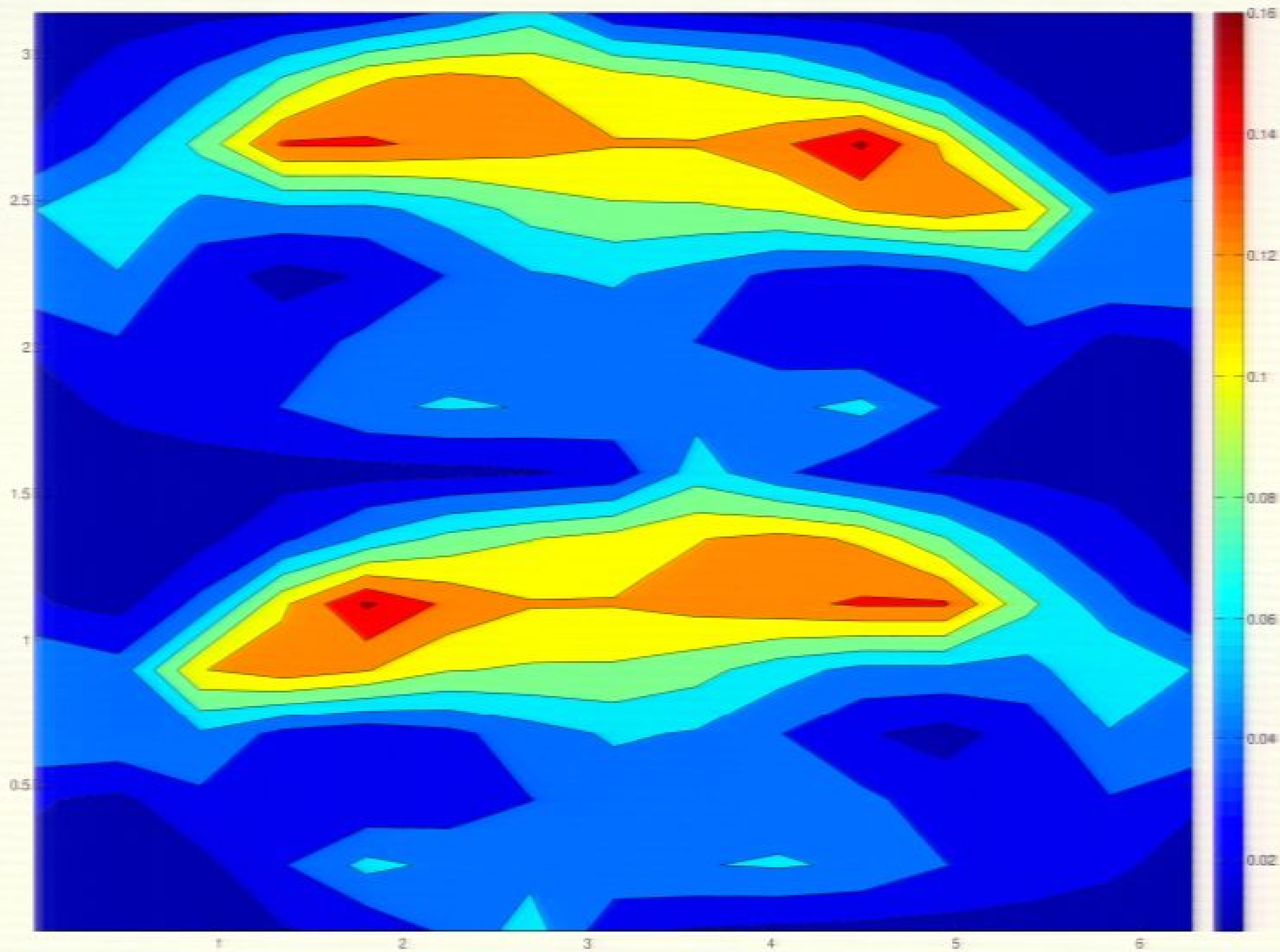
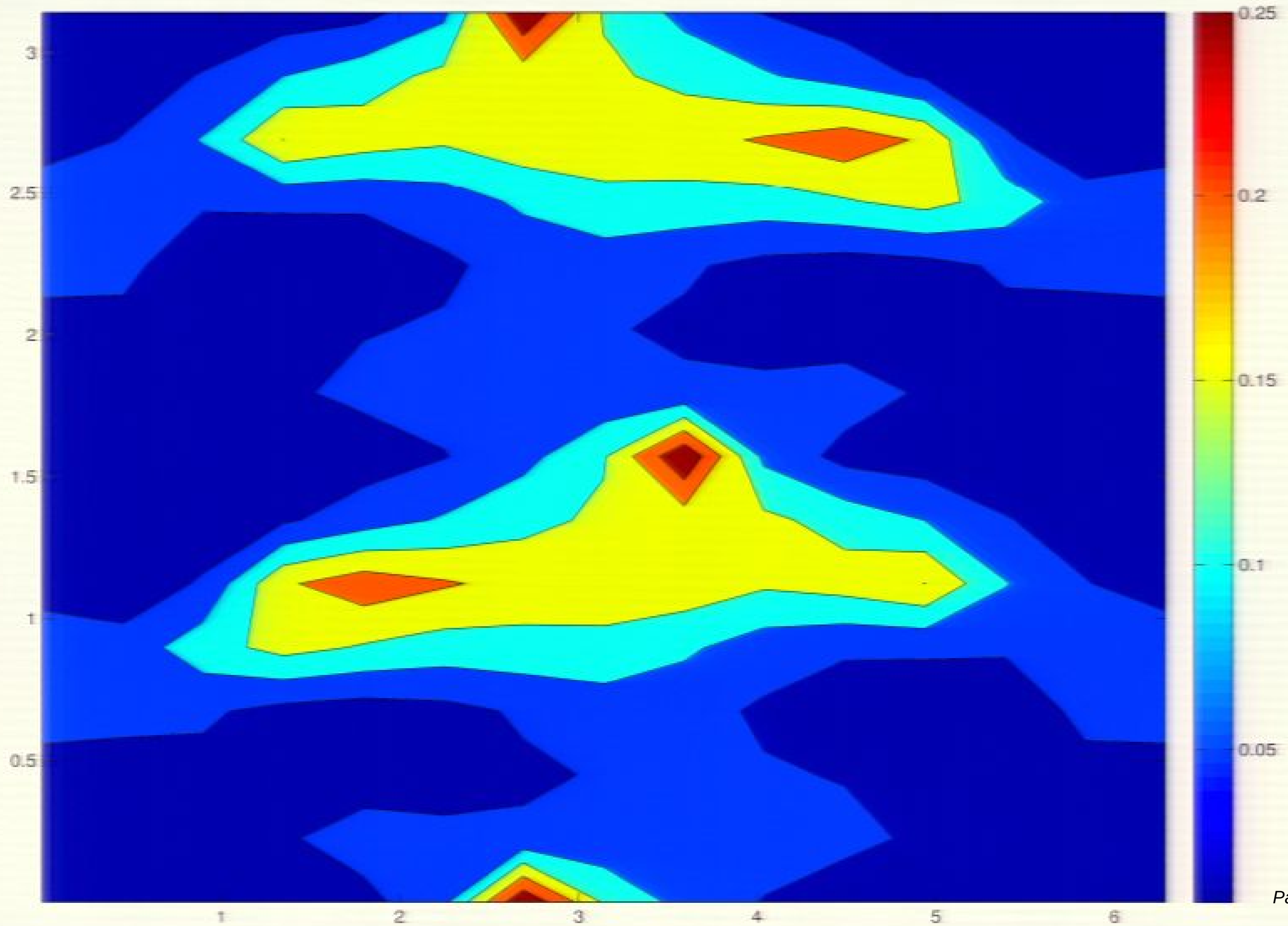
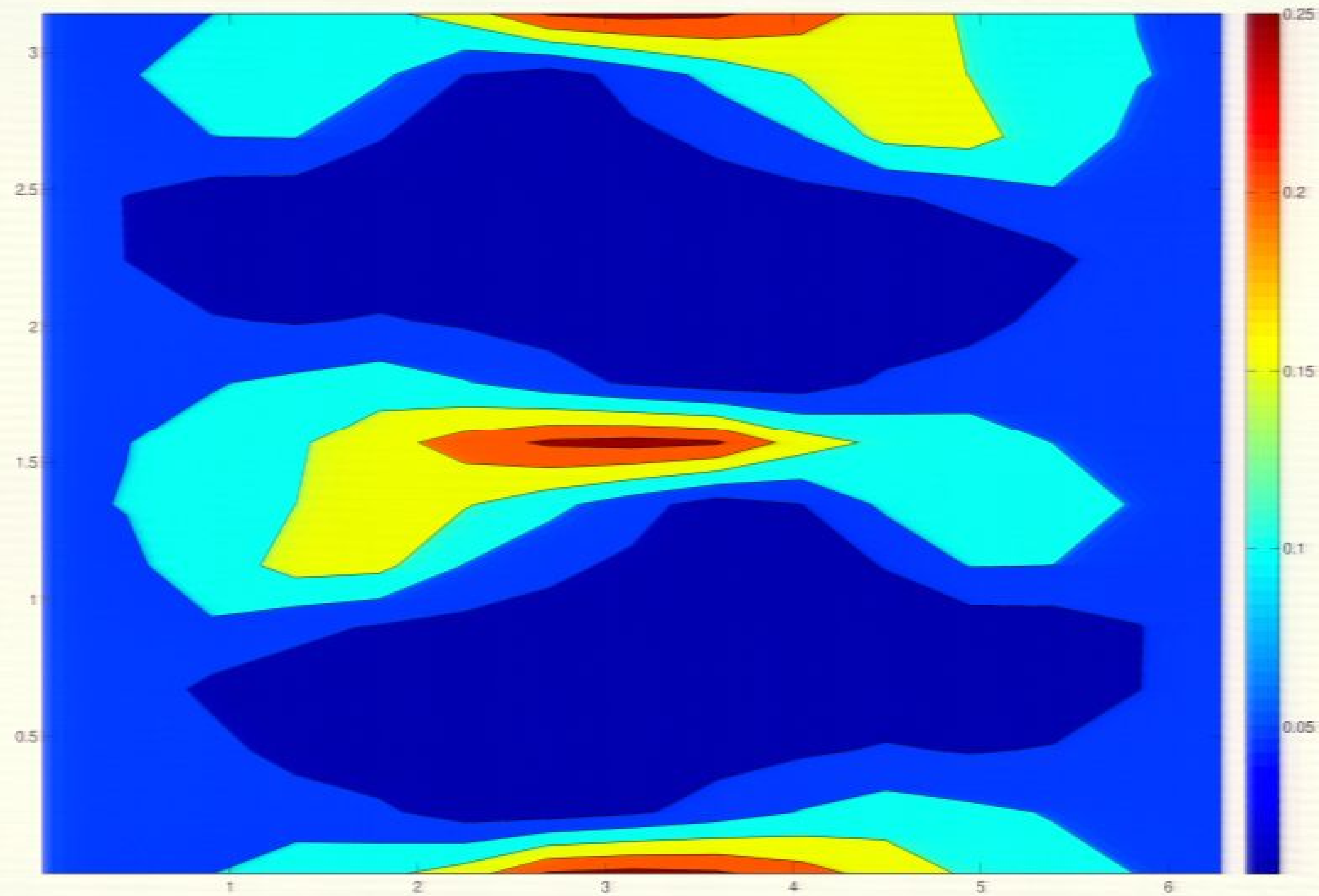


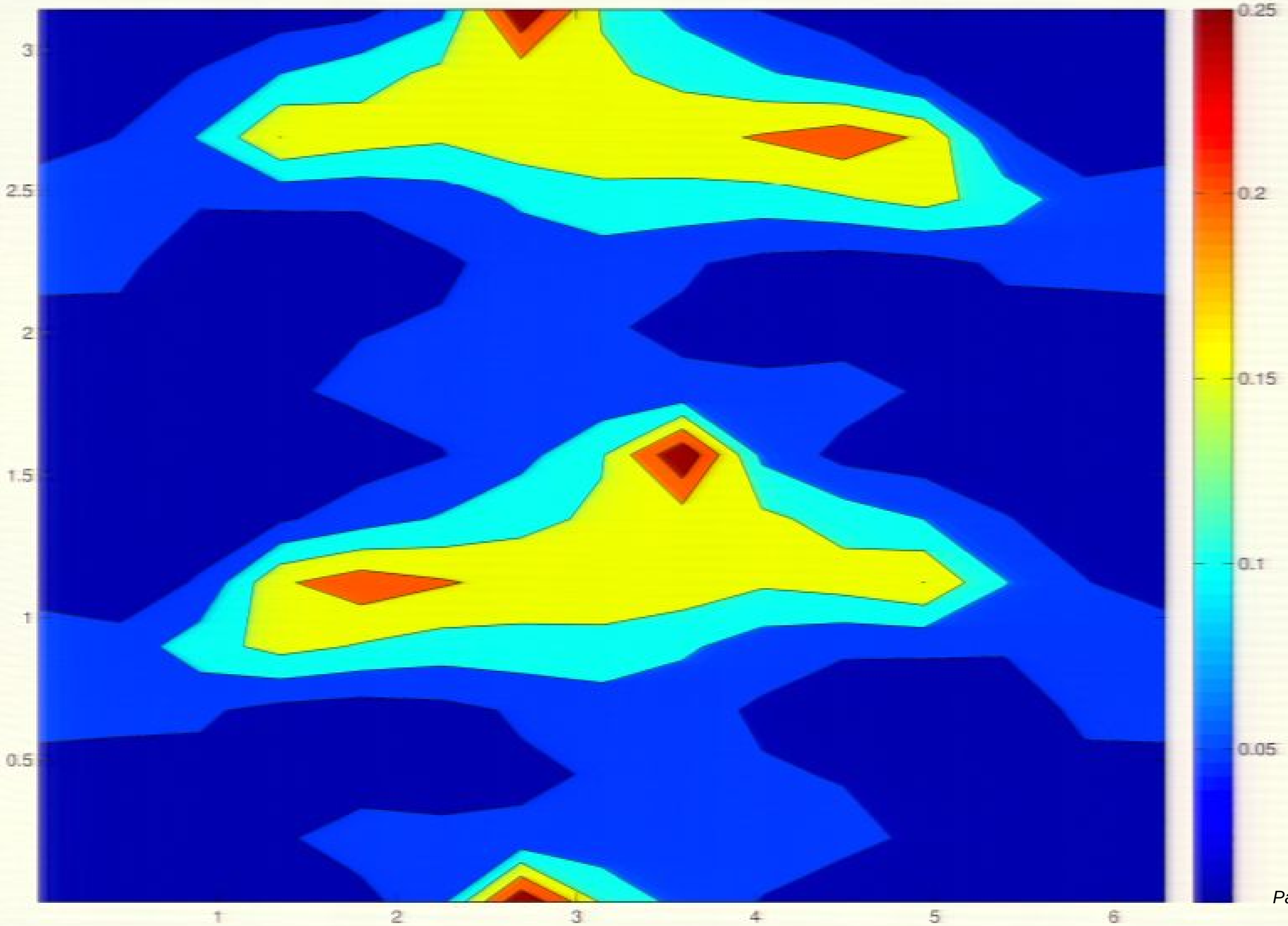
theta



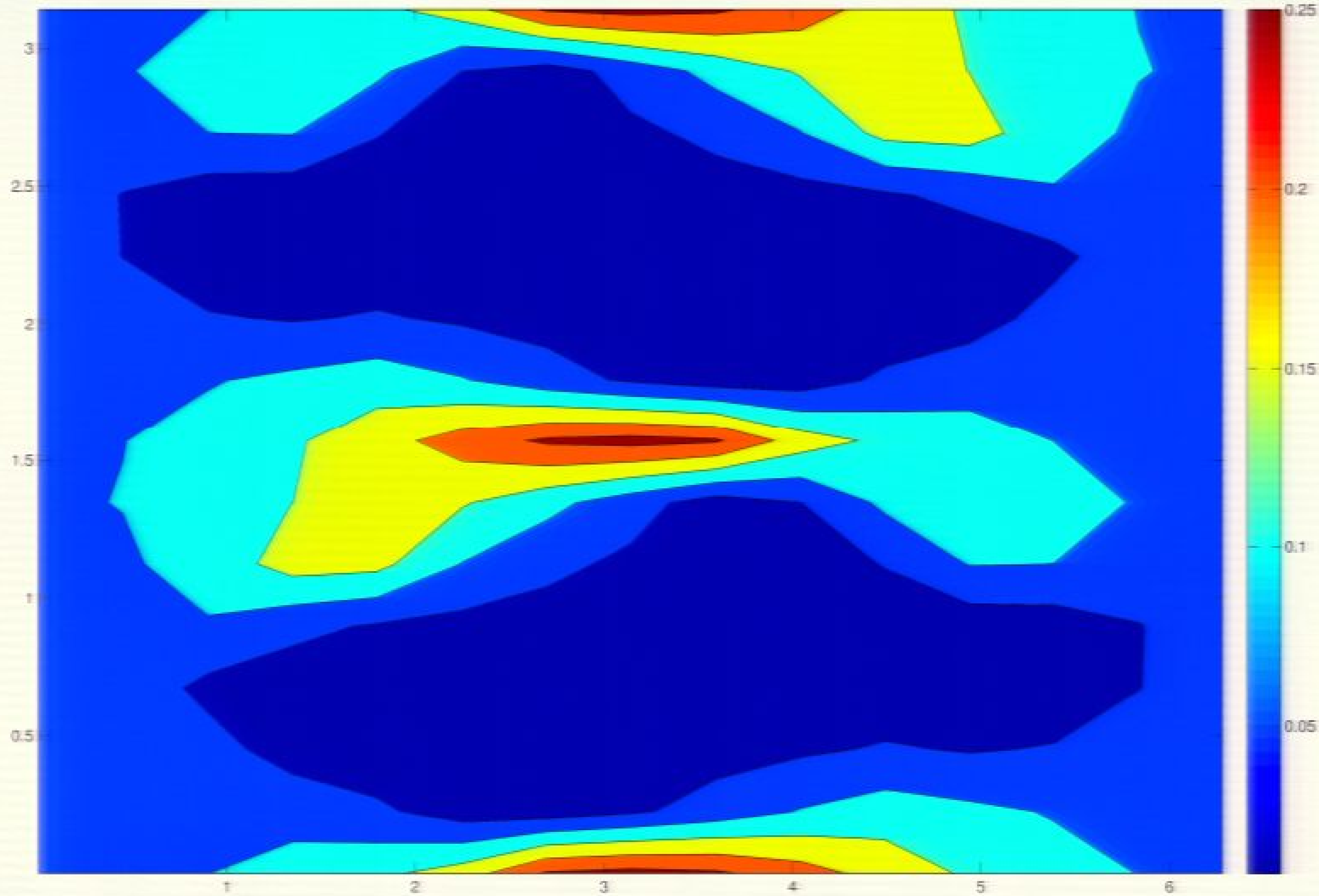
phi



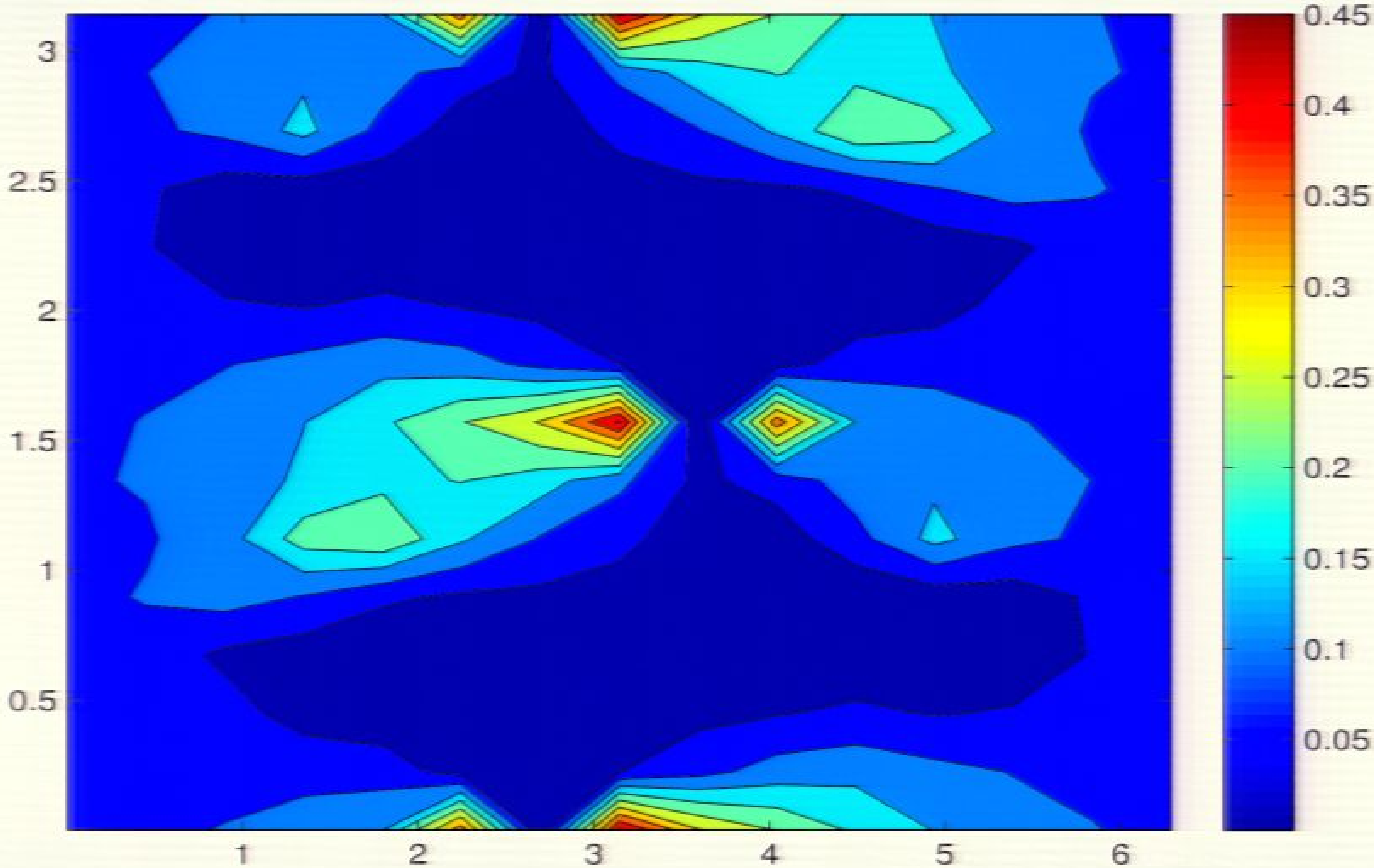
theta



phi



phi



Conclusions

- Complete directional paper
- Extend to Bayesian formalism
- Continue to compare with montecarlo simulations and existing pipelines