Title: Interfacing numerical and analytical relativity for gravitational-wave astronomy: Status and prospects

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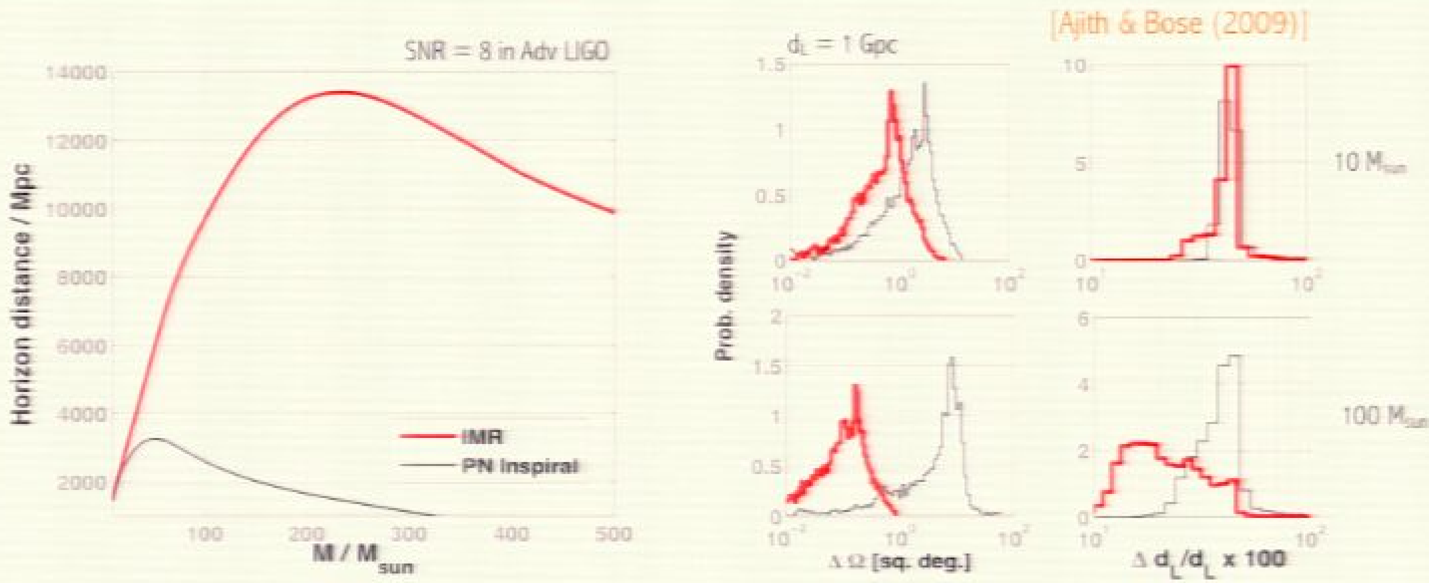
Abstract: Recent progress in numerical- and analytical relativity enables us to construct analytical waveform templates coherently describing the inspiral, merger and ring down of coalescing black-hole binaries. Such waveform templates not only improve the sensitivity of the searches for gravitational waves from high-mass binaries significantly, but also the accuracy of the parameter estimation. This talk summarizes the status and prospects of different approaches of the modeling of gravitational waveform from binary black holes calibrated to numerical-
relativity simulations.

Interfacing numerical- and analytical relativity for GW astronomy: Status and prospects

P. Ajith California Institute of Technology

Theory Meets Data Analysis at Comparable and Extreme Mass Ratios Capra + NRDA Meeting
Perimeter Institute, Waterloo, Canada
24 June 2010

Why NR for GW astronomy?



- Significant improvements in the “distance reach” and parameter-estimation accuracies for “high-mass” binaries.
- NR of sources other than compact binaries, provide useful information for fine tuning the searches.

Constructing analytical IMR waveforms tuned to NR

- Based on the effective-one-body formalism
  - Map the two-body dynamics into one-body dynamics in the presence of an effective ("deformed") metric.
  - Several free parameters can be introduced. Propose some ansatz for the mass-ratio (and spin) dependence of the free parameters, and tune them against NR.
  - Match the calibrated EOB inspiral-plunge waveforms with several QNM modes.

EOB-dynamics adjustable parameters	EOB-waveform adjustable parameters
$a_5(\nu)$	$r^{\ell m}_{\rm match}(\nu)$
$v_{\rm pole}(\nu)$	$\Delta t^{\ell m}_{\rm match}(\nu)$
$a^{\mathcal{F}_1}_{\rm RR}(\nu)$ or $a^{\mathcal{F}_2}_{\rm RR}(\nu)$	$a_i^{h_{\ell m}}(\nu)$; $i = 1, \ldots 4$
A_8	

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- Phenomenological approaches connecting PN with NR
  - Construct hybrid waveforms by matching PN and NR in region where both calculations are assumed to be valid.
  - Parametrize the hybrid waveforms using some appropriate fitting functions. Modelling or inspiral and ring down is motivated by PN and BH perturbation theory.

$$\Psi(f) \equiv 2\pi f t_0 + \varphi_0 + \frac{3}{128\eta v^5}\left(1 + \sum_{k=2}^{7} v^k \psi_k\right)$$

$$\psi_k \text{ and } \mu_k \equiv \{f_1, f_2, \sigma, f_3\}$$

EOB calibrated to NR: nonspin equal-mass waveforms

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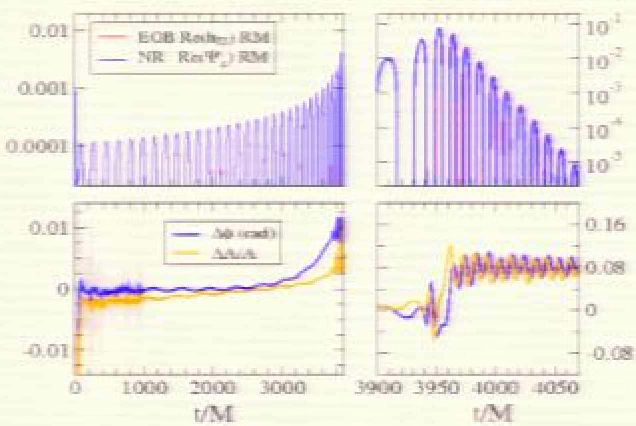
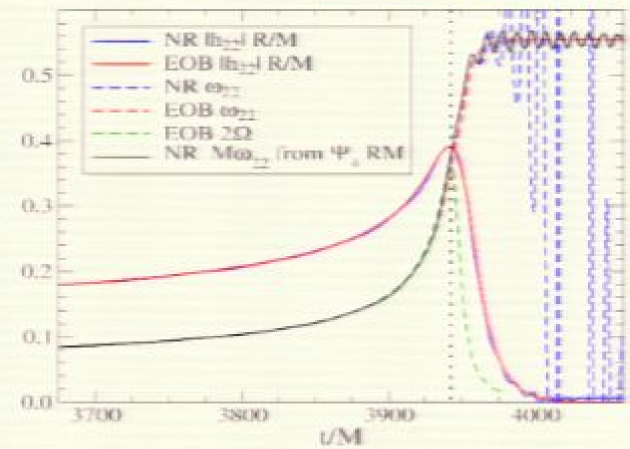
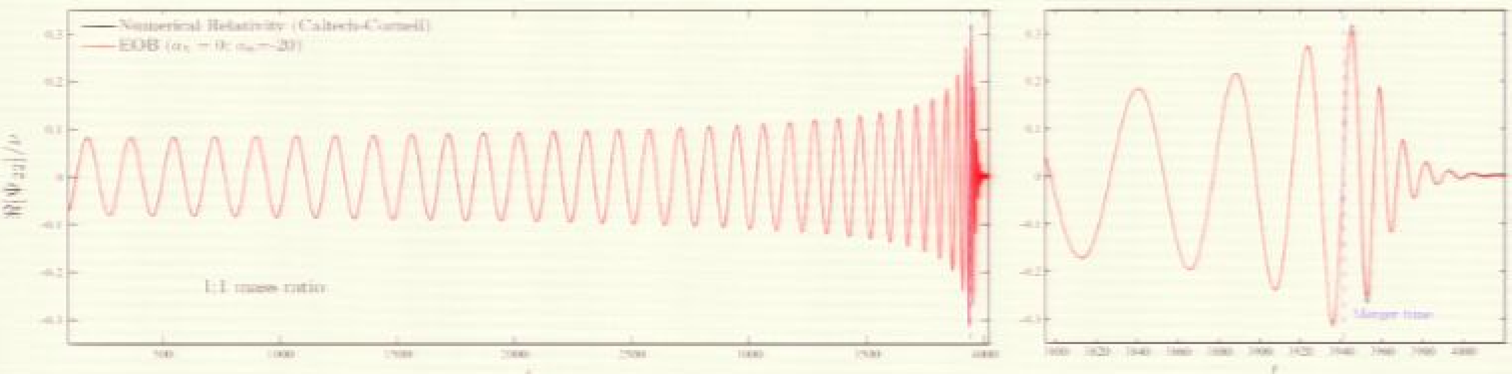


[Buonanno et al. 2009]



- Phase and amplitude difference within numerical errors. Faithfulness > 0.9999 (w.r.t. the non-spinning, equal-mass SpEC NR waveform.
- "Full" (with non-quadrupole modes) non-spin EOBNR being calibrated to NR [Talk by Y. Pan].

Alternative way of calibrating EOB with NR

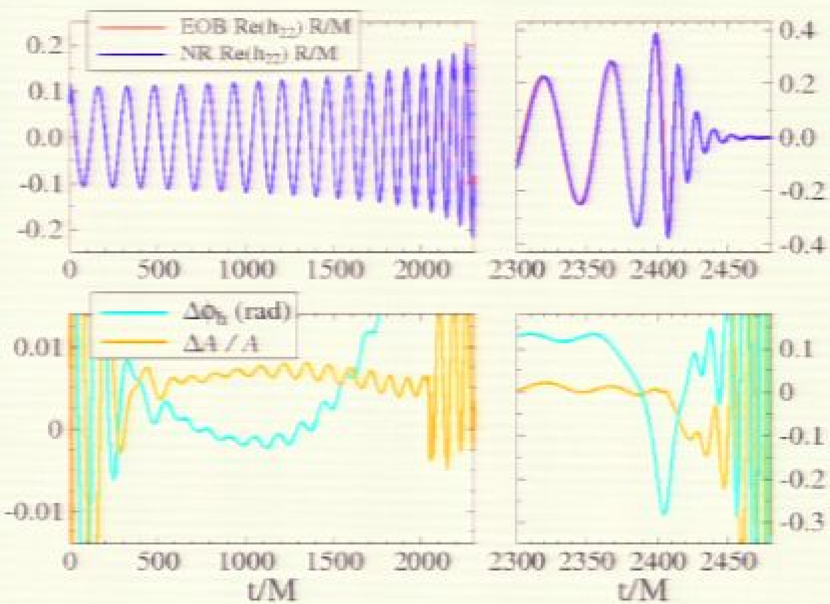


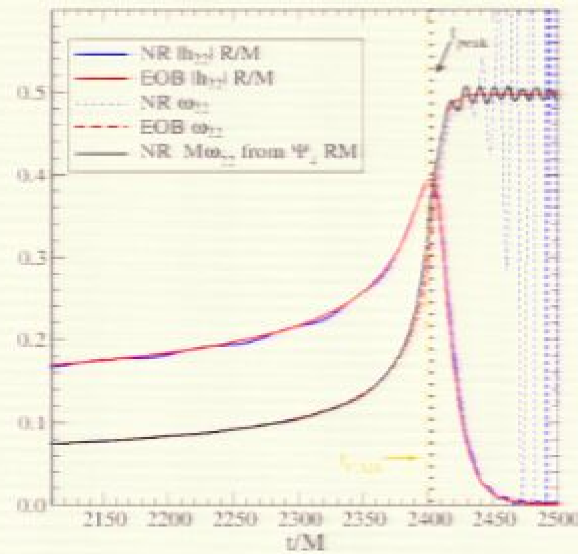
[Damour & Nagar (2009)]

- Two free parameters in the EOB potential: $A(u; a_5, a_6, \nu) \equiv P^1_5[A^{3PN}(u) + \nu a_5 u^5 + \nu a_6 u^6]$
- "Improved resummation" of the RR force: $\mathcal{F}_\varphi = -\frac{1}{8\pi\Omega}\sum_{\ell=2}^{\ell_{max}}\sum_{m=1}^{\ell}(m\Omega)^2|Rh^{(\epsilon)}_{\ell m}|^2$
- "Next to quasi-circular corrections" to the waveforms.
- NR calibration of the maximum GW amplitude.

EOB calibrated to NR: non-precessing equal-mass equal-spin waveforms

[Pan et al. 2009]





- Calibrated to SpEC simulation of an equal-mass binary with equal-amplitude anti aligned spins: $\chi_1 = \chi_2 = 0.43757$
- Improvements made in modeling spin Hamiltonian and radiation reaction [Talks by E. Barausse and Y. Pan]

A data-analysis-based approach to spinning IMR waveforms

- 8 dimensional template bank – large **false alarms**. [e.g., Van Den Broeck et al (2009)]
- Requires systematic and accurate exploration of the **large parameter space** using NR simulations – might not be possible in the near future.
- Several technical difficulties of implementing a search using generic spinning templates.

- A sub-family: **non-precessing spins**: Making use of the degeneracy of the parameter space, try to model the binaries using just **one extra parameter** in the waveform.

Leading term in the spin-orbit coupling

$$\chi \equiv \chi_{\rm s} + \delta\, \chi_{\rm a}$$

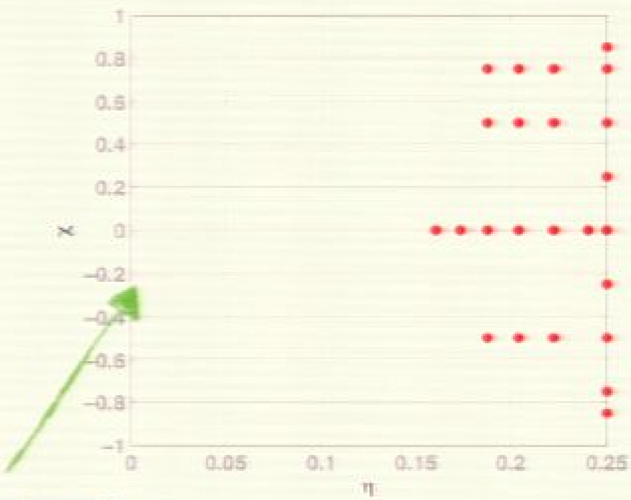
sum of spins

mass asymmetry

difference in spins

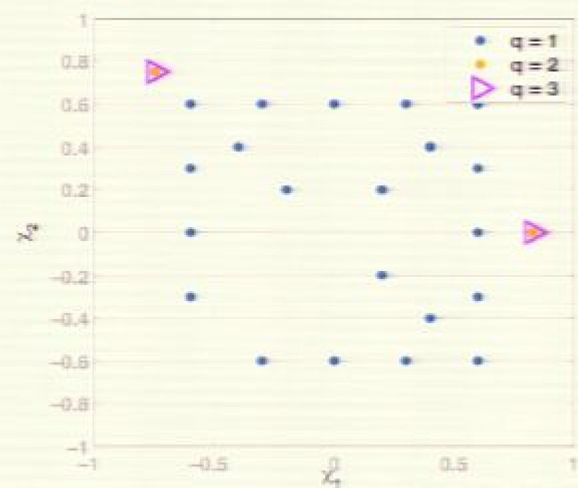
NR Simulations

Used 24 BAM equal-spin simulations ($|\chi| \leq 0.85$, $1 \leq q \leq 4$) to construct the template family.



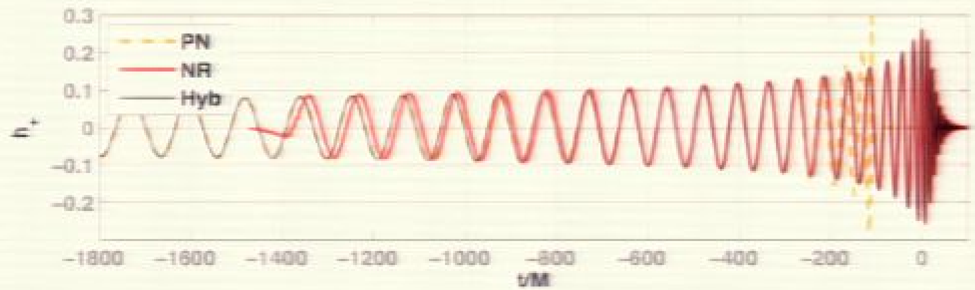
Inputs from BH perturbation

Used more than 30 (BAM, CCATIE, LLAMA, SpeC) different simulations (including unequal spins and precession) to test the template family.



Parametrizing the hybrid waveforms

- Construct NR-PN hybrids:



- Parametrisation:

$$A(f) \equiv C f_1^{-7/6} \begin{cases} f'^{-7/6}\left(1+\sum_{i=2}^{3}\alpha_i \nu^i\right) & \text{if } f < f_1 \\ w_m f'^{-2/3}\left(1+\sum_{i=1}^{2}\varepsilon_i \nu^i\right) & \text{if } f_1 \leq f < f_2 \\ w_r \mathcal{L}(f, f_2, \sigma) & \text{if } f_2 \leq f < f_3, \end{cases}$$

$$\Psi(f) \equiv 2\pi f t_0 + \varphi_0 + \frac{3}{128\eta \nu^5}\left(1+\sum_{k=2}^{7}\nu^k \psi_k\right). \quad (1)$$

Parametrizing the hybrid waveforms

- The phenomenological parameters are written in terms of the physical parameters as:

estimated from the hybrid waveforms

$$\psi_k = \sum_{i=1}^{3} \sum_{j=0}^{N} x_k^{(ij)}\, \eta^i \chi^j + \psi_k^0, \quad \mu_k = \sum_{i=1}^{3} \sum_{j=0}^{N} \frac{y_k^{(ij)}\, \eta^i \chi^j}{\pi M} + \mu_k^0,$$

- such that, in the test-mass ($\eta \to 0$)limit:

LSO frequency of a Kerr BH with appropr. mass & spin

Freq & quality factor of the dominant ring down mode

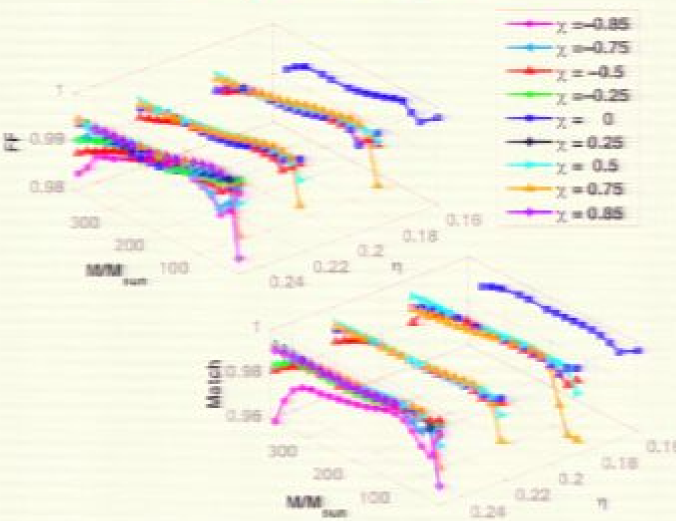
2PN phasing coefficients in the test-mass limit

$$f_1 \to f_{\rm LSO}^0, \quad f_2 \to f_{\rm QNM}^0, \quad \sigma \to f_{\rm QNM}^0/Q^0, \quad \psi_k \to \psi_k^0,$$

Testing the analytical waveforms

Using BAM Simulations

[P. Ajith et al (2009)]

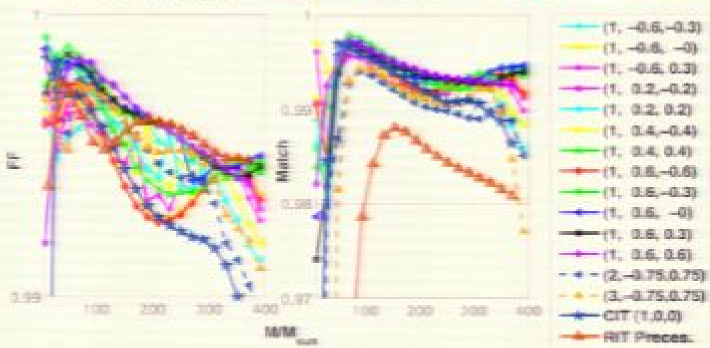


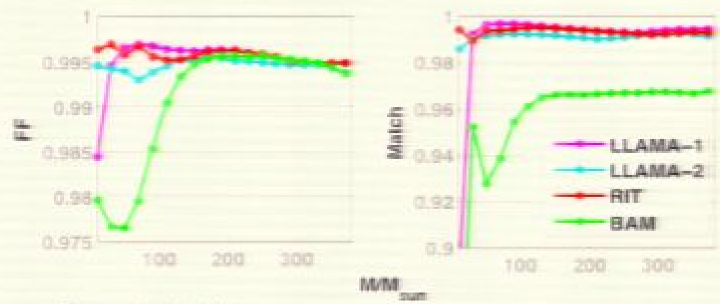
Equal-spin hybrids (used to construct the template family)

FF > 0.98 for $q \leq 4$, $|\chi| \leq 0.85$ in Initial/Enhanced LIGO

Testing the analytical waveforms

Unequal-spin hybrids Using CCATIE, BAM, SpeC Simulations





Precessing hybrids

- **LLAMA-1** $q = 1$, $\chi = [0.42, 0, 0.42], [0, 0, 0]$
- **LLAMA-2** $q = 1$, $\chi = [0.15, 0, 0], [0, 0, 0]$
- **RIT** $q = 1.25$, $\chi = [-0.2\ -0.14\ 0.32], [-0.09\ 0.48\ 0.35]$
- **BAM** $q = 3$, $\chi = [0, 0, 0], [0.75, 0, 0]$

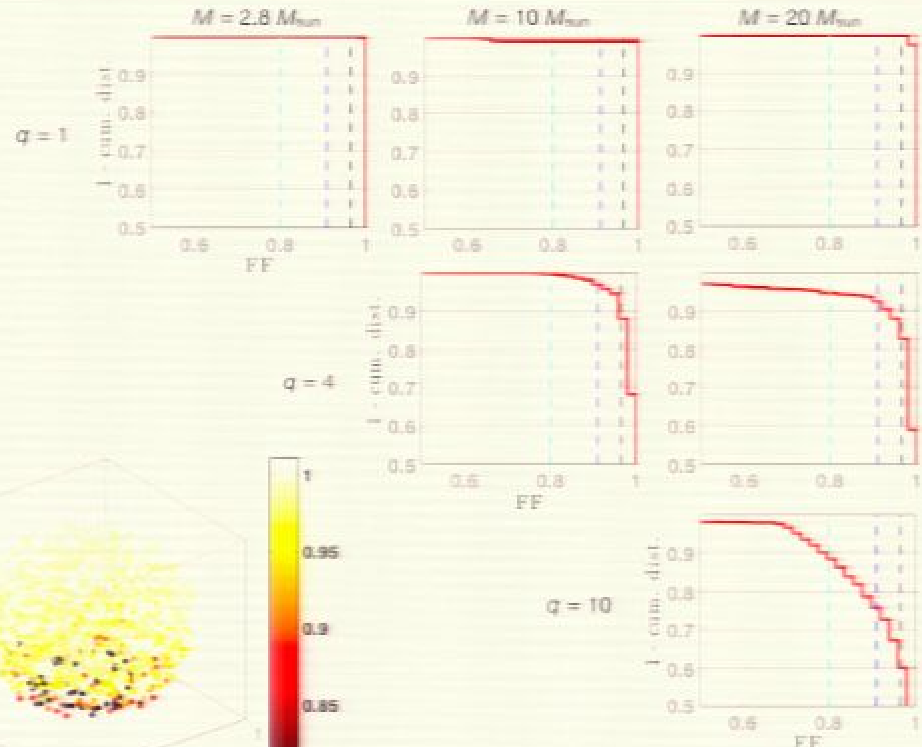
- The template family is able capture the spin-orbit coupling.
- Not enough NR waveforms to make general statements about the efficiency of the template family in detecting generic spinning binaries.
- But, we might be able to get some useful insights from PN.

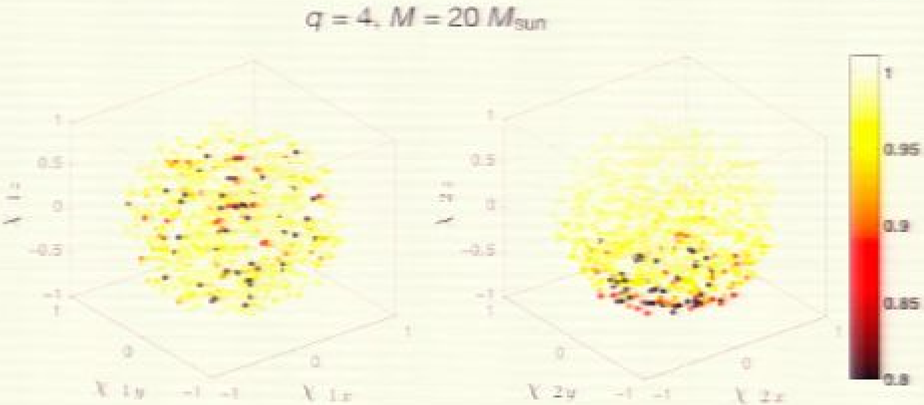
Monte-Carlo simulation of generic spin PN binaries

- Generate generic spinning PN binaries with arbitrary spins and test the efficiency of "aligned-spin" PN templates (where the spins are represented by a single spin parameter) in detecting them.

Monte-Carlo simulation of generic spin PN binaries

- Efficiency of aligned-spin (3-parameter) PN templates in detecting generic spinning PN binaries in Initial/Enhanced LIGO.





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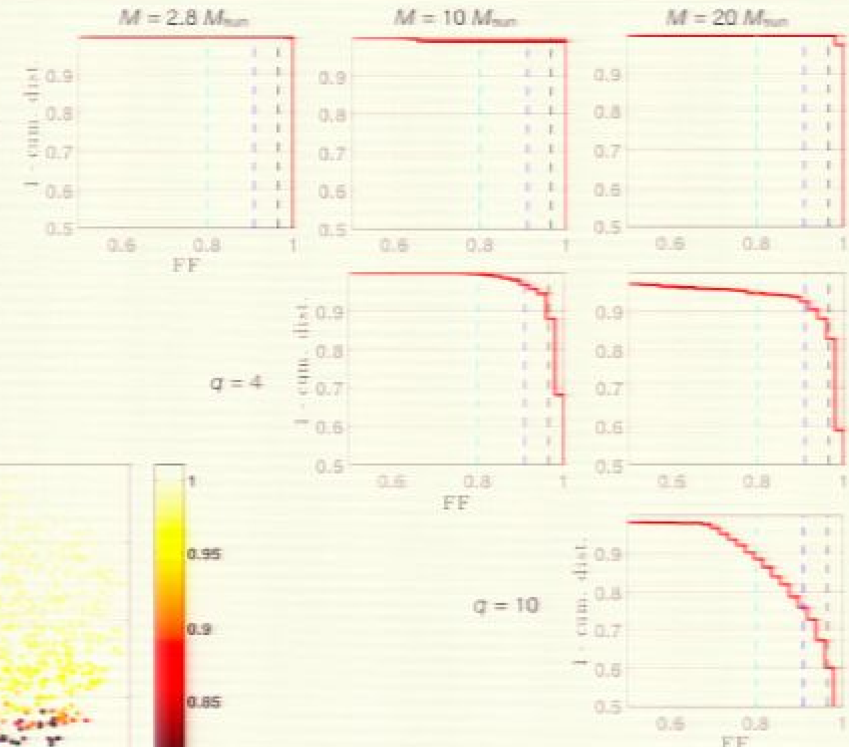
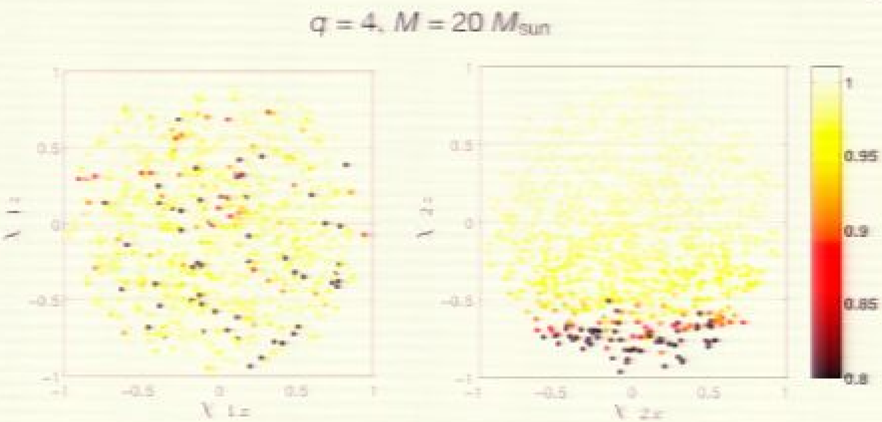
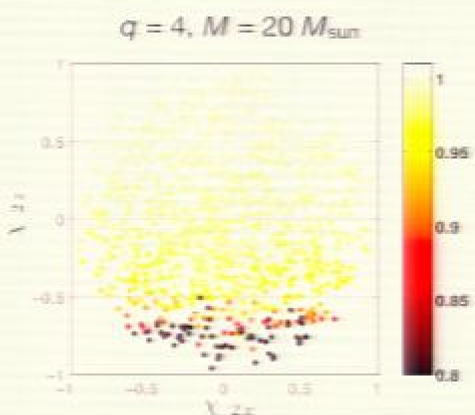


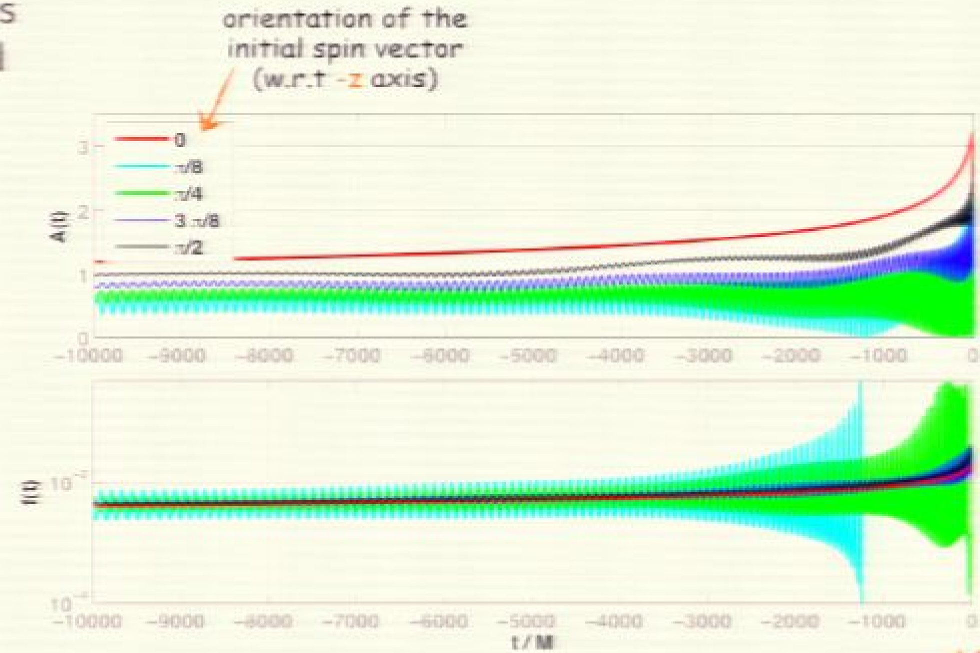


Monte-Carlo simulation of generic spin PN binaries

[P. Ajith et al., in Prep (2010)]

- Aligned-spin templates are less efficient in detecting certain high-precession configurations (involving "flips" of the orbital plane).

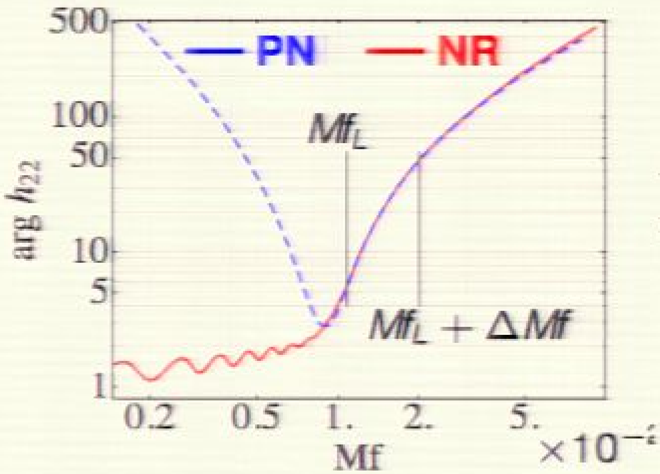


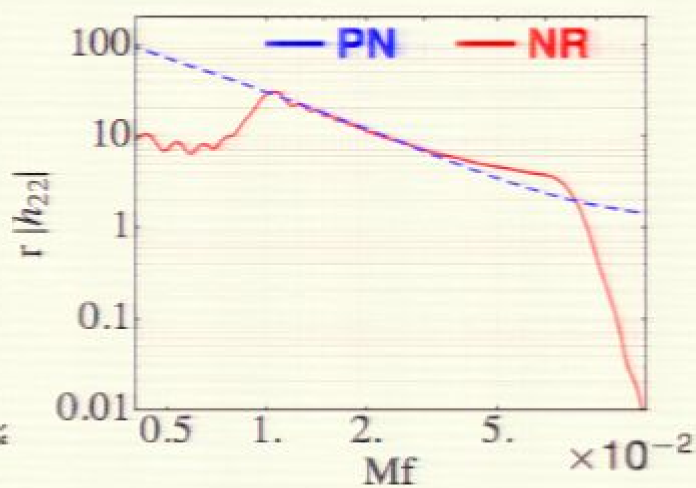


Systematics in the construction of hybrid waveforms

[Santamaria, Ohme et al 2010]

- Alternative ways of constructing hybrid waveforms (e.g., in frequency domain)
  - **PN** 3.5PN TaylorF2 phase, 3PN amplitude
  - **NR** BAM, LLAMA, CCATIE, SpEC waveforms, $q \leq 4$, non-precessing spins





Monte-Carlo simulation of generic spin PN binaries

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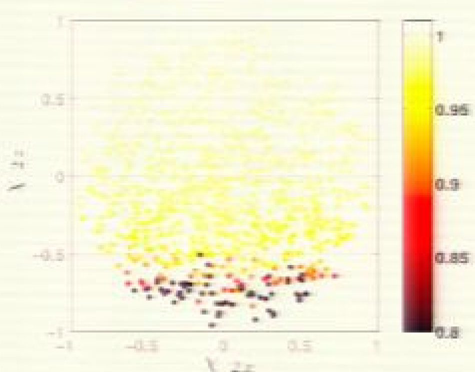
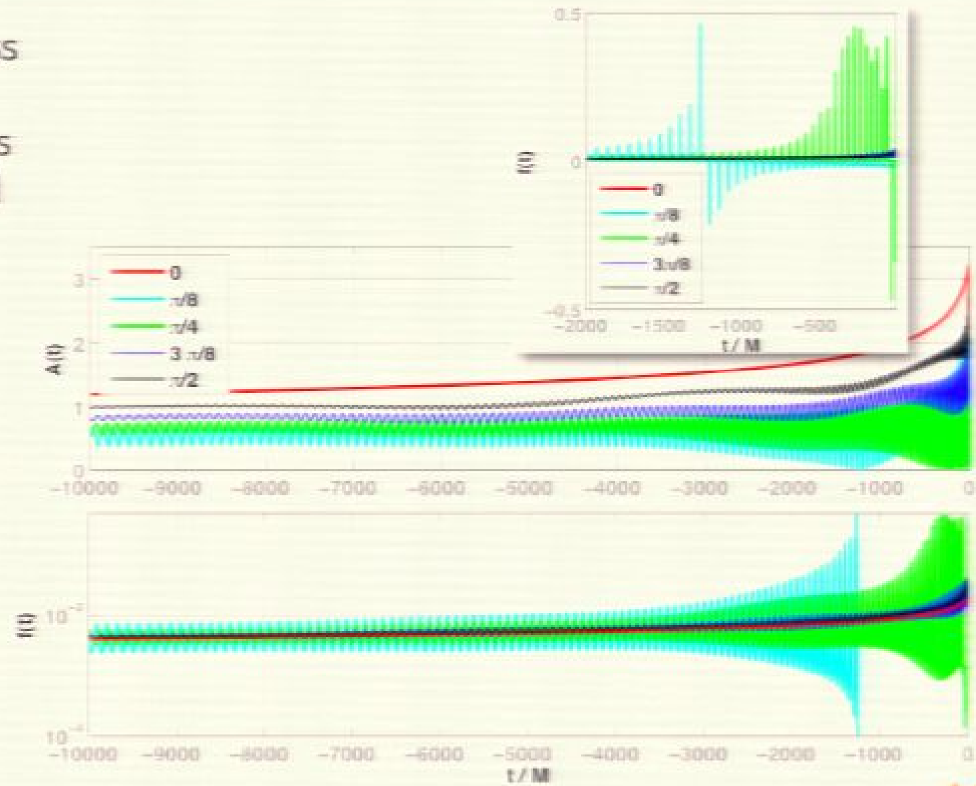
$q = 4,\ M = 20\ M_{\rm sun}$





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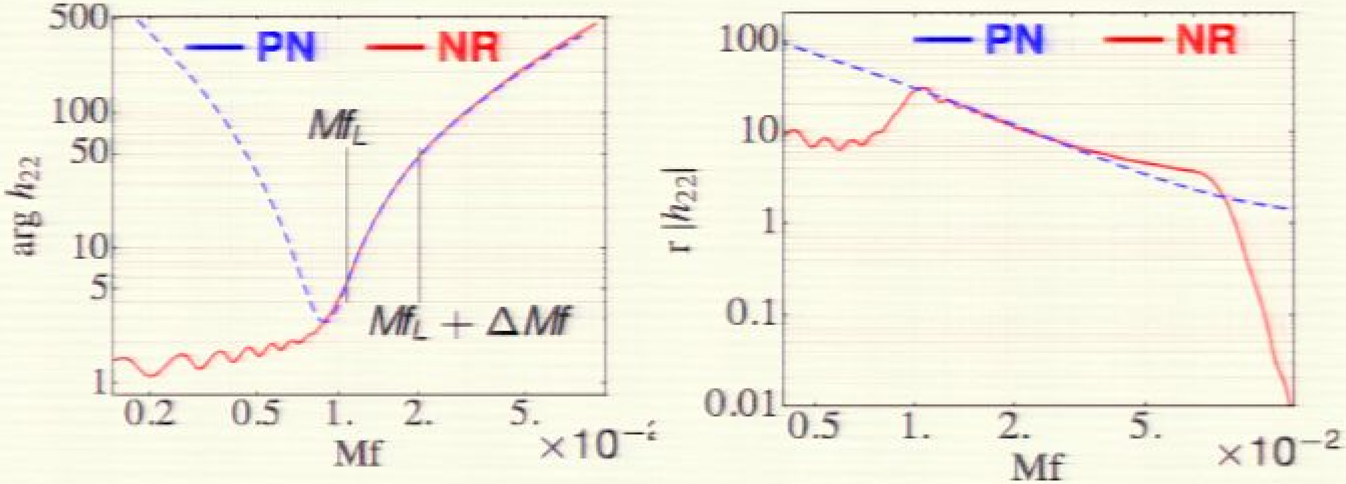
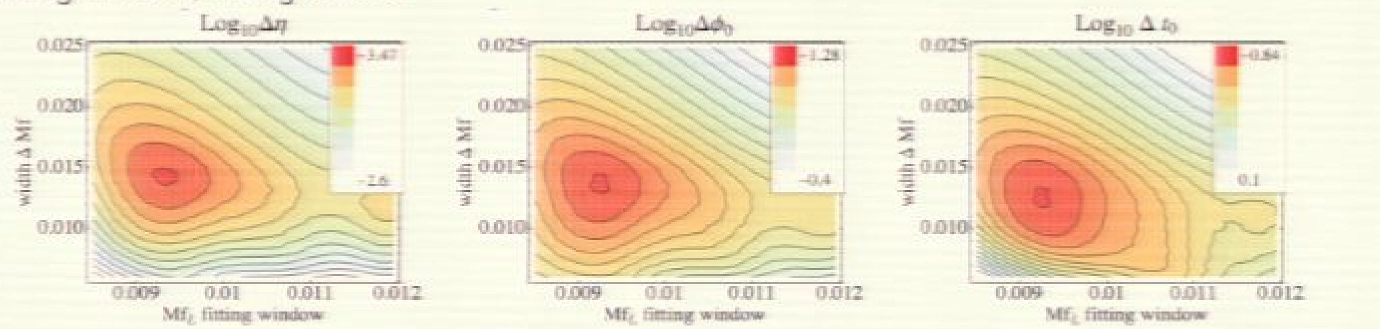
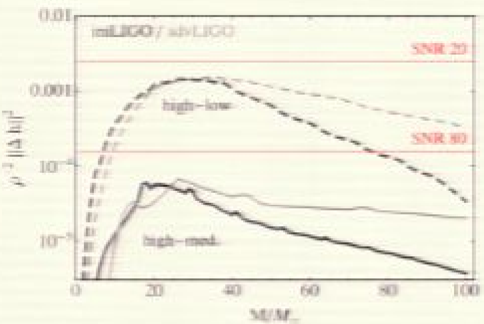


Systematics in the construction of hybrid waveforms

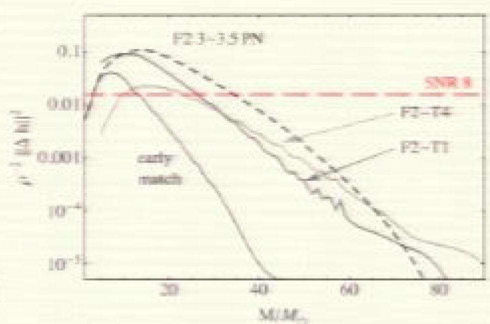
- Matching window, fitting errors



- PN/NR uncertainties



Difference between high and med. resolution indistinguishable for SNR 80



Difference between hybrids constructed using different PN approximants

[F. Ohme's Talk]

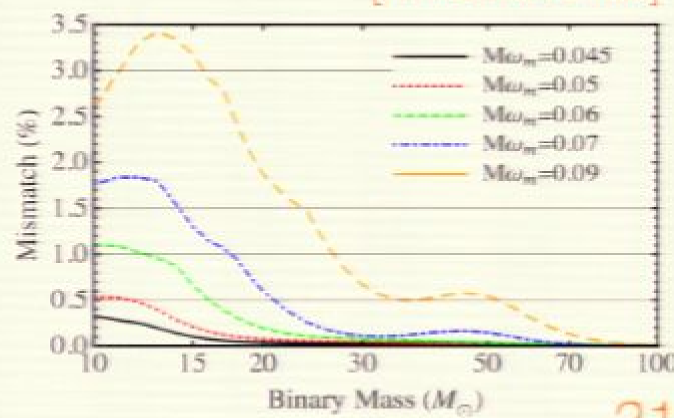
PN/NR comparison, minimum length of NR waveforms ...

- PN-NR Comparison
  - Equal-mass, non-spinning & non-precessing spins.
  - TaylorT4 extremely accurate in the non-spinning case.
  - TaylorT1 is the most robust in the spin case.
  - PN amplitude accuracy < 4%.

[H. Pfeiffer's Poster]

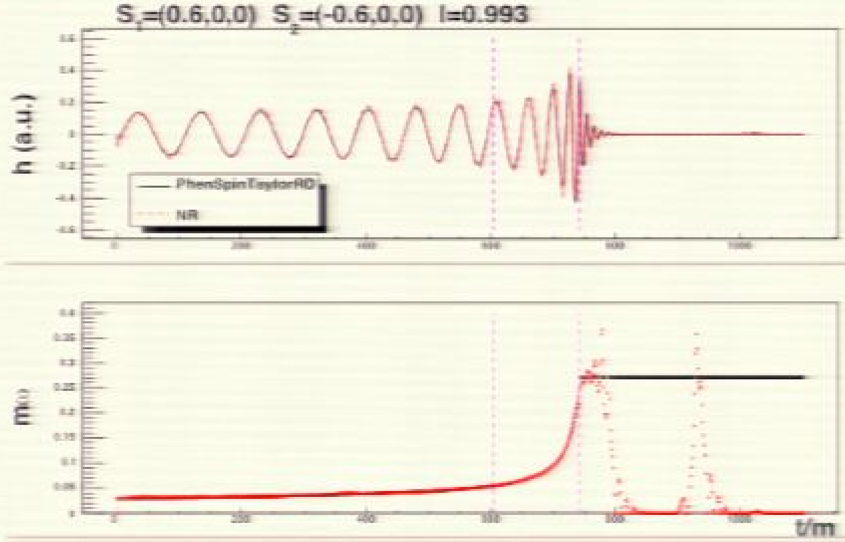
- Minimum length for NR waveforms for detection.
  - Compare NR+PN hybrids by varying the matching frequency ω_m.
  - In the equal-mass, non-spinning case, 4 orbits are sufficient to produce FF > 0.98 between different hybrids.

[M. Hannam's Talk]



Phenomenological IMR waveforms for generic spinning BBHs

- **Inspiral** "Restricted" PN waveforms computed in the TaylorT4 approximation, up to a phenomenological "merger" frequency f_m, which is determined empirically.
- **Merger** Phenomenological model. Free parameters are constrained by imposing continuity on the phase and its derivatives & comparing with NR.
- **Ring down** Attach ring down (with overtones) at f_{RD}. Motivated by the EOBNR approach.
- Phenomenological parameters are currently estimated from equal-mass simulations by GeorgiaTech.
- Overlaps of 0.95-0.99 with NR waveforms.



[R. Sturani's Talk]

NR-AR Collaboration

- **Goal**
  - Produce accurate NR simulations spanning a large region of the binary parameter space.
  - Develop in time for the searches of GWs the best-calibrated template families covering a large parameter space.
- **Resources**
  - NSF has made available 11M CPU-hrs on the Teragrid machine Kraken (April - Summer 2010). NR groups were also encouraged to use their own resources.
- **Current status**
  - 14 NR groups have joined the NR-AR Collaboration.
  - First simulations started at the beginning of April. Several runs are in progress.

Summary

- Steady progress in the construction of analytical IMR waveforms inspired by NR.
- More NR simulations, calculation of higher order PN terms (especially involving spins) are in order.
- The NR-AR effort will provide a "zoo" of NR waveforms. Useful for the construction/calibration of analytical templates as well as for NINJA.
- Lot of efforts to quantify the systematics in the PN-NR matching, hybrid construction, minimum length of NR simulations etc.

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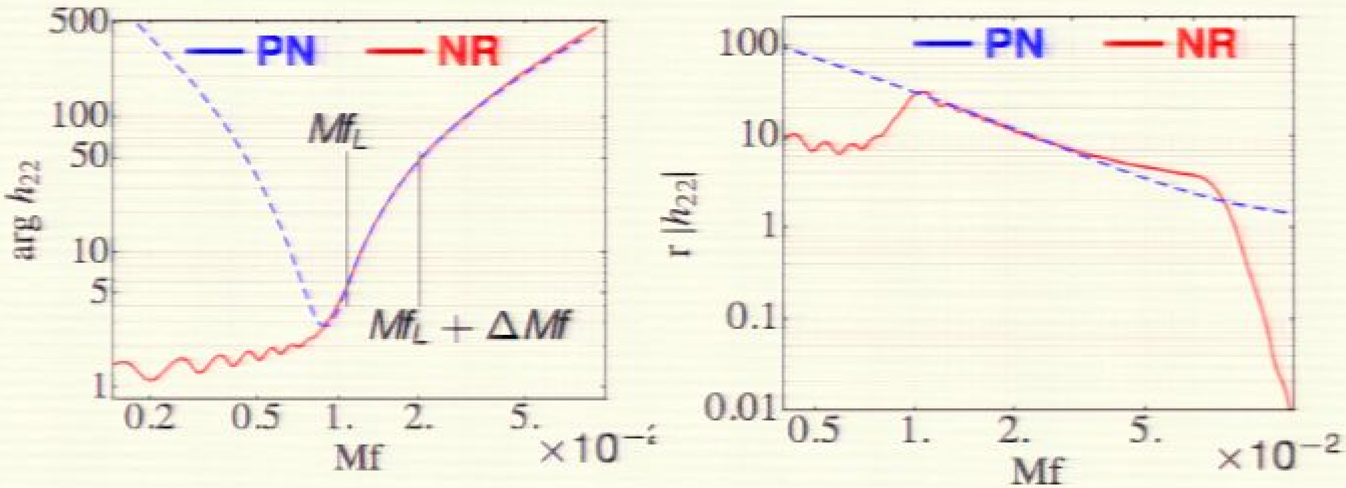
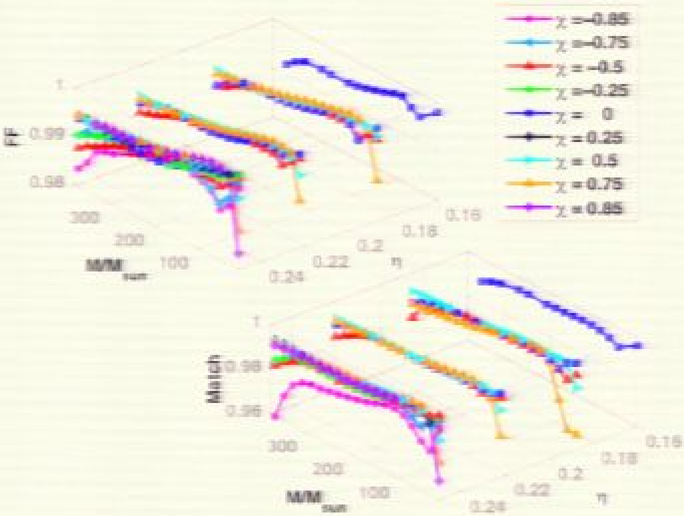


Testing the analytical waveforms

[P. Ajith et al (2009)]

Using BAM Simulations



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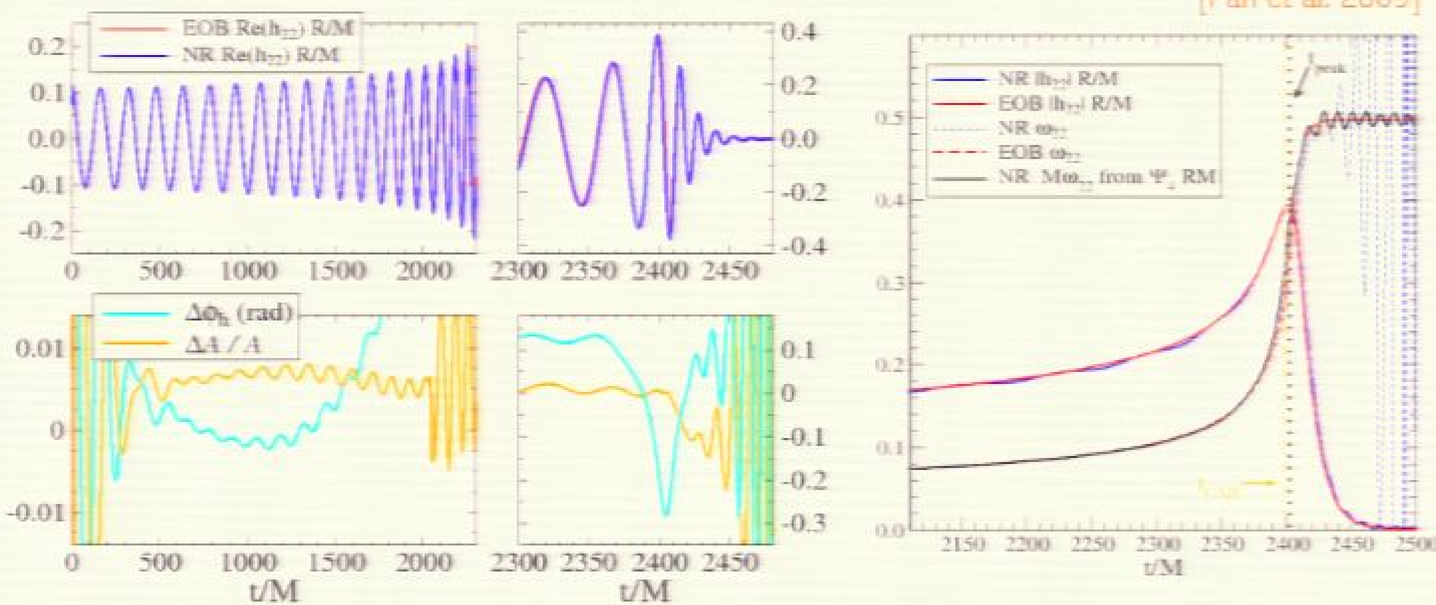

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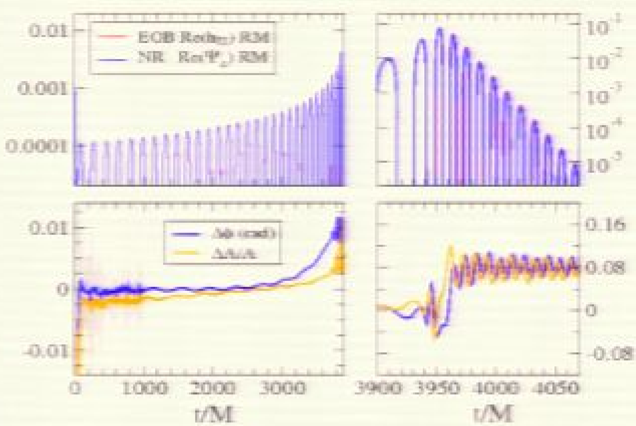

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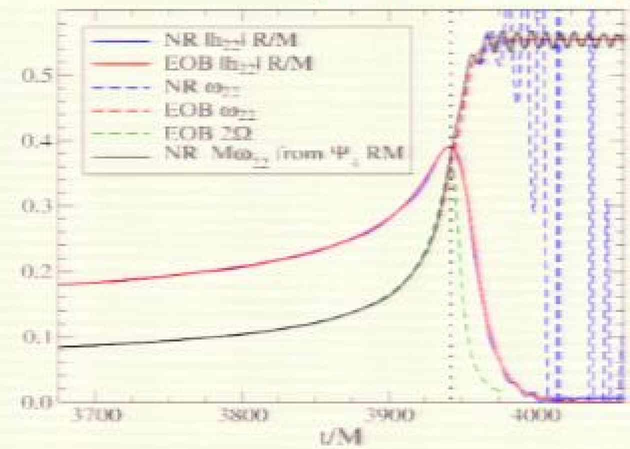

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