Title: Automating the post-Newtonian expansion on a computer

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Abstract: With several ground-based gravitational wave interferometers operating at design sensitivity the need for high-order post-Newtonian (PN) calculations of potentials waveforms etc. especially including spin effects has grown significantly over the last several years. Not only are these calculations necessary for precisely estimating the parameters of detected gravitational wave sources but they are also useful for providing more accurate models of binary evolutions in for example the effective one-body program and for computing the PN contributions to self-force effects in the extreme mass ratio limit. Since these calculations become more demanding to carry out at higher PN orders it is necessary to utilize symbolic computer algebra programs (such as Mathematica). Our aim is to automate PN calculations (of potentials power loss etc.) on the computer using the effective field theory (EFT) approach of Goldberger and Rothstein which is itself a systematic and algorithmic method for computing in the PN approximation. The EFT approach lends itself to automation through definite power counting rules that identify precisely those interactions appearing at a given PN order through Feynman rules and diagrams that provide an elegant way to side-step the need to explicitly solve the wave equation for metric perturbations (unlike in traditional methods) through working at the level of the action (a scalar) instead of equations of motion etc. We discuss our progress in automating these calculations on a computer using the EFT approach.

Automating the post-Newtonian expansion on a computer

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Capra/NRDA -- Perimeter Institute -- 6/23/2010

Collaborators: Manuel Tiglio (UMD)

Introduction

	Equations of motion	Power loss	Restricted waveforms	Amplitude corrections
Nonspinning bodies	3.5 PN (Blanchet et al. 2002)	3.5 PN (Blanchet et al. 2002)	3.5 PN (Blanchet et al. 2002)	3 PN (Blanchet et al. 2008)
Spinning bodies	*Spin-orbit*: 2.5 PN (Tagoshi et al. 2001, Faye et al. 2006) *Spin-spin*: 3 PN (Porto and Rothstein 2008, Steinhoff et al. 2008)	*Spin-orbit*: 2.5 PN (Blanchet et al. 2006) *Spin-spin*: 2 PN (Mikóczi et al. 2005)	*Spin-orbit*: 2.5 PN (Blanchet et al. 2006) *Spin-spin*: 2 PN (Mikóczi et al. 2005)	*Spin-orbit*: 1.5 PN (Kidder 1995) *Spin-spin*: 2 PN (Will and Wiseman 1996)

(Courtesy M. Vallisneri & I. Rothstein)

It's believed that 3.5PN is sufficiently good for the purposes of detection and possibly parameter estimation for binaries with comparable masses.

Results are not complete through 3.5PN for all the relevant GW observables, especially for spinning compact objects.

Still quite a bit of work remaining left to do.

Motivating automation

Higher order post-Newtonian (PN) corrections are useful for:

Building more accurate semi-analytical models (e.g., Effective One Body)

Comparing NR, PN and self-force results (talks by Favata, Le Tiec,...)

Practical computation of self-force effects for e.g., kludge waveforms

Binaries with intermediate mass ratios

More accuracy in parameter estimation of detected GW sources

Asymptotic behavior of PN equations of motion, flux,... (Yunes,Berti,...)

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Higher order PN calculations are not feasible/possible by hand

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For efficient automation, choose a theoretical framework that itself is rule-based and algorithmic -- Effective Field Theory approach (EFT/NRGR)

Work in EFT for classical dynamics

- Potentials through 2PN -- Goldberger, Rothstein, Ross, Gilmore, Chu

  Spin-orbit through 2.5PN -- Porto, Rothstein, Perrodin, Levi

  Spin1-spin2 potential through 3PN -- Porto, Rothstein

  Spin1-spin1 potential through 3PN -- Porto, Rothstein, Perrodin

  Leading order radiation reaction (Burke-Thorne) -- CRG, Tiglio

  Leading order waveform (quadrupole) -- CRG, Tiglio

- EMRIs -- CRG, Hu

- Alternative theories -- Maggiore, Sturani, Sanctuary, Cannella

- Thermodynamics of caged black holes -- Kol, Smolkin

- Higher dimensional black holes, membranes -- ???

- Radiation reaction with finite size corrections in E&M -- CRG, Leibovich, Rothstein

Picosecond overview of EFT

Goldberger & Rothstein, PRD **73**, 104029 (2006)

Post-Newtonian treatment $\quad \dfrac{GR_m}{r} \sim v^2, \qquad \dfrac{r}{\lambda} \sim v$

$R_m \ll r \ll \lambda$ EFT: Muliple scales -- "tower" of EFTs

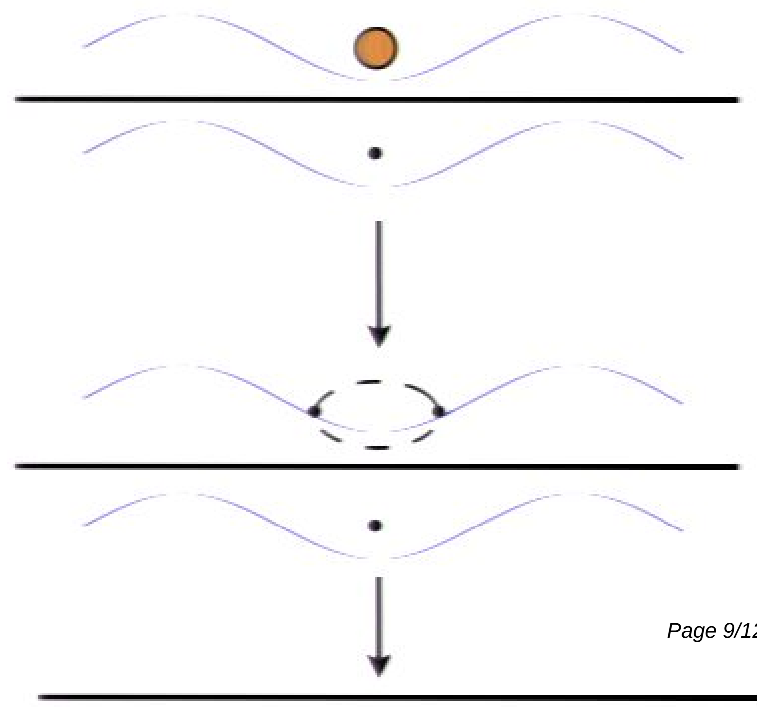
match $\quad \dfrac{\text{GR + compact object}}{\text{GR + point particle}} \quad \ell \sim R_m$

↓

match $\quad \dfrac{\text{Nonrelativistic binary}}{\text{Composite object + radiation}} \quad \ell \sim r$

↓

$\ell \sim \lambda$



How to calculate with EFT

Goal: Calculate a potential that yields PN eom when extremized.

Start with the action: $S[g, x_1, x_2] = S_{EH}[g] + \sum_{n=1}^{2} S_{pp}[x_n, g]$

Expand in metric perturbation *H* and 3-velocities to get many interaction terms.

$$S[g, x_1, x_2] = S_{(0)}[H] + \frac{1}{2}\sum_{n=1}^{2}\int dt\, m_n \mathbf{v}_n^2 + \delta S[H, x_1, x_2]$$

Integrating out *H* from the action:

Extremize S to get wave equation for H

Solve for H (near-zone metric)

Plug solution back into S to get the effective action S_{eff}

A more efficient way is to use Feynman diagrams

Each interaction term has a definite power counting in v and $L = mvr$

$$-\frac{m_n}{2}\int dt\, H_{00}(t,\mathbf{x}_n(t)) \sim v^0 L^{1/2}$$

$$D_{\alpha\beta\gamma\delta}(t,t';\mathbf{k},\mathbf{q}) = \langle H_{\alpha\beta}(t;\mathbf{k}) H_{\gamma\delta}(t';\mathbf{q})\rangle$$

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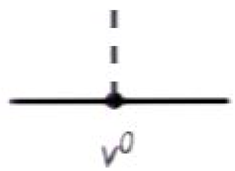

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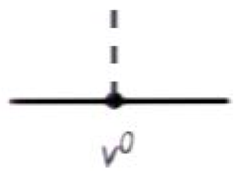

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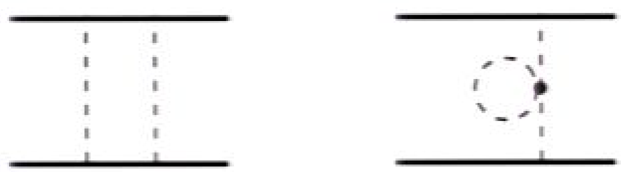
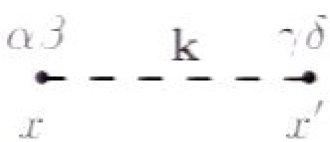
Feynman diagrams: At nPN order form all tree-level (no loops of dashed lines) and connected diagrams that scale as $v^{2n}L$



Lines connecting vertices are Green's functions (or "propagators")



$$= -\frac{1}{\mathbf{k}^2}(2\pi)^3\delta^3(\mathbf{k}+\mathbf{p})\delta(t-t')(32\pi G)P_{\alpha\beta\gamma\delta} \equiv D_{\alpha\beta\gamma\delta}(t,t';\mathbf{k},\mathbf{q})$$

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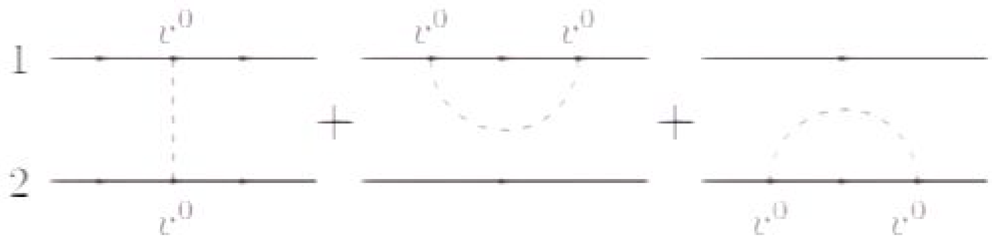
At 0PN the diagrams for the Newtonian potential are



$$\sim v^0 L$$

$$= \left(-\frac{m_1}{2}\right)\left(-\frac{m_2}{2}\right)\int dt \int dt' \int_{\mathbf{k}}\int_{\mathbf{q}} e^{i\mathbf{k}\cdot\mathbf{x}_1(t)}\eta_{0\alpha}\eta_{0\beta}D^{\alpha\beta\gamma\delta}(t,t';\mathbf{k},\mathbf{q})\eta_{0\gamma}\eta_{0\delta}e^{i\mathbf{q}\cdot\mathbf{x}_2(t')}$$

$$= 4\pi G m_1 m_2 \int dt \int_{\mathbf{k}} e^{i\mathbf{k}\cdot(\mathbf{x}_1(t)-\mathbf{x}_2(t))}\frac{1}{\mathbf{k}^2}$$

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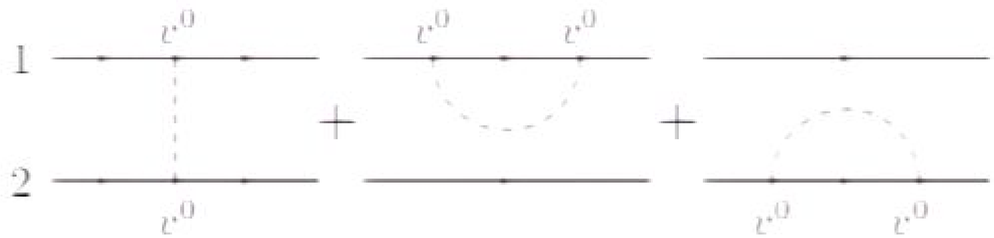

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Divergent integrals: *In dimensional regularization only log divergences are physical (screening effects from gravitational perturbations) but power divergences vanish*

$$= 2\pi G m_1^2 \int dt \int_{\mathbf{k}} \frac{1}{\mathbf{k}^2} \longrightarrow 0$$

Choice of metric variables

Kol & Smolkin (2007) showed that a change of variables can dramatically simplify the calculation of potentials in the harmonic gauge.

$$H_{\mu\nu} \longrightarrow (\phi, A_i, \sigma_{ij})$$

Covariant variables:

$$g_{\mu\nu} = \eta_{\mu\nu} + H_{\mu\nu}$$

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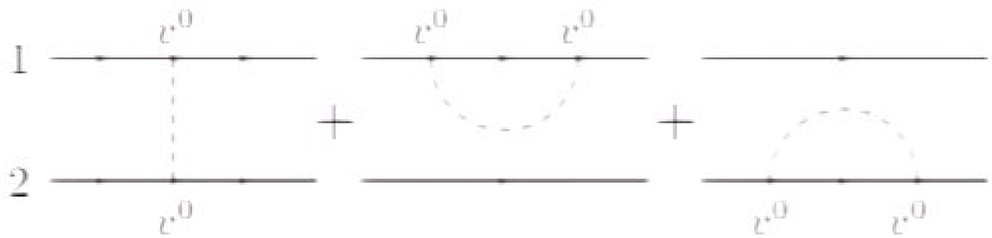

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Corrections to instantaneity

Even though sources move slowly, gravitational perturbations do not propagate infinitely fast.

This implies that the Green's function for the Laplacian is corrected by terms higher order in *v*.

$$D^{rel}_{\alpha\beta\gamma\delta}(t,t',\mathbf{x},\mathbf{x}') = \int_{k_0,\mathbf{k}} \frac{e^{ik_0\cdot(t-t')-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}}{k_0^2-\mathbf{k}^2}$$

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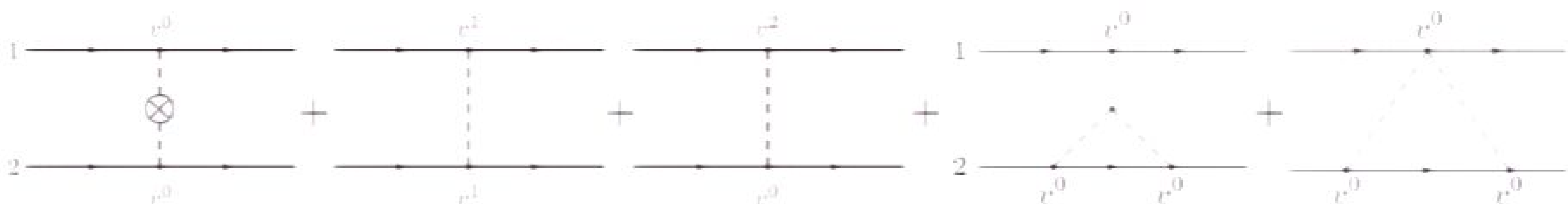

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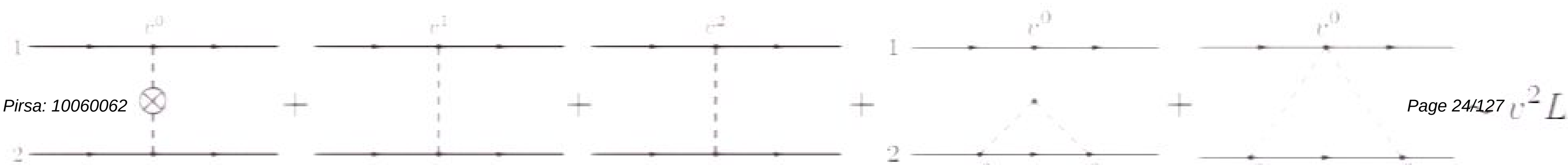

Calculations to be automated

Potentials

Goldberger, Rothstein, Ross, Gilmore, Porto

Radiation reaction "potentials"

CRG

Power loss/Flux

Goldberger, Rothstein, Porto

Restricted waveforms

Amplitude corrections

CRG

Full automation is probably impractical but our goal is to at least provide a set of robust tools for calculating intensive parts of PN computations

Calculating potentials in EFT

Interaction terms (3+1) — Power counting — Feynman rules

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Find combos of interactions ("vertices") at given PN order — Feynman diagrams (Wick contractions)

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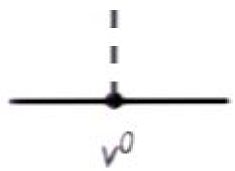

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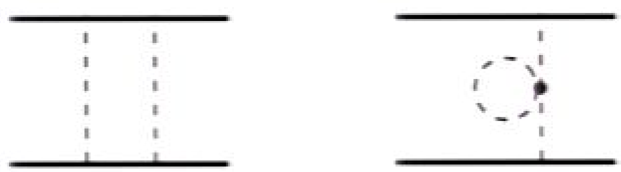

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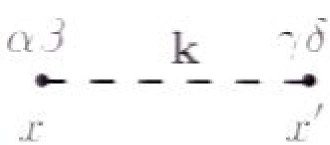

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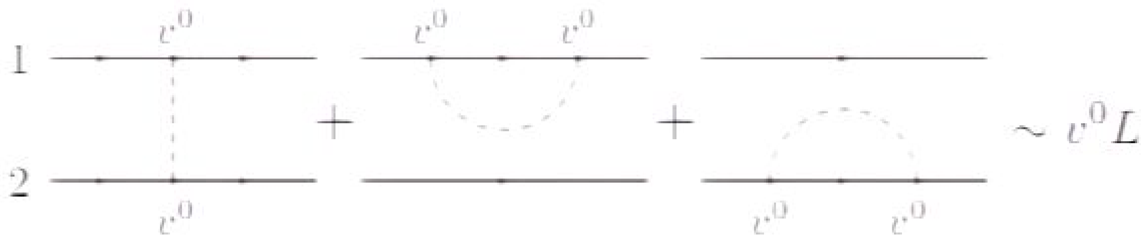

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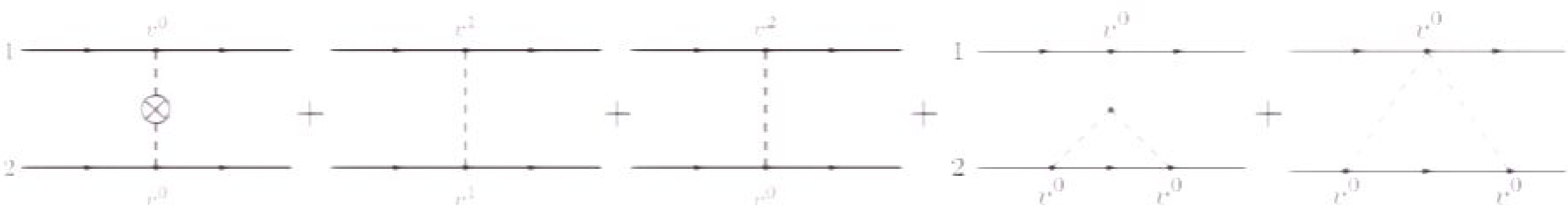
$$g_{\mu\nu} = \begin{pmatrix} e^{2\phi} & -e^{2\phi} A_j \\ -e^{2\phi} A_i & -e^{-2\phi}(\delta_{ij} + \sigma_{ij}) + e^{2\phi} A_i A_j \end{pmatrix}$$
$$= \eta_{\mu\nu} + H^{(1)}_{\mu\nu} + H^{(2)}_{\mu\nu} + \cdots$$



$\sim v^2 L$

Corrections to instantaneity

Even though sources move slowly, gravitational perturbations do not propagate infinitely fast.

This implies that the Green's function for the Laplacian is corrected by terms higher order in *v*.

$$D^{rel}_{\alpha\beta\gamma\delta}(t,t',\mathbf{x},\mathbf{x}') = \int_{k_0,\mathbf{k}} \frac{e^{ik_0\cdot(t-t')-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}}{k_0^2-\mathbf{k}^2}$$

$$\frac{1}{k_0^2-\mathbf{k}^2} = -\frac{1}{\mathbf{k}^2}\left(1+\frac{k_0^2}{\mathbf{k}^2}+\frac{k_0^4}{\mathbf{k}^4}+\cdots\right)$$

$$k_0 = -i\frac{\partial}{\partial t} \qquad k_0^2 = \frac{\partial}{\partial t}\frac{\partial}{\partial t'} \quad \left(= -\frac{\partial^2}{\partial t^2} = -\frac{\partial^2}{\partial t'^2}\right)$$

$$D^{rel}_{\alpha\beta\gamma\delta}(t,t',\mathbf{x},\mathbf{x}') = \delta(t-t')\int_{\mathbf{k}} \frac{e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}}{\mathbf{k}^2} - \frac{\partial}{\partial t}\frac{\partial}{\partial t'}\delta(t-t')\int_{\mathbf{k}} \frac{e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}}{\mathbf{k}^4} + \cdots$$

Corrections to instantaneity

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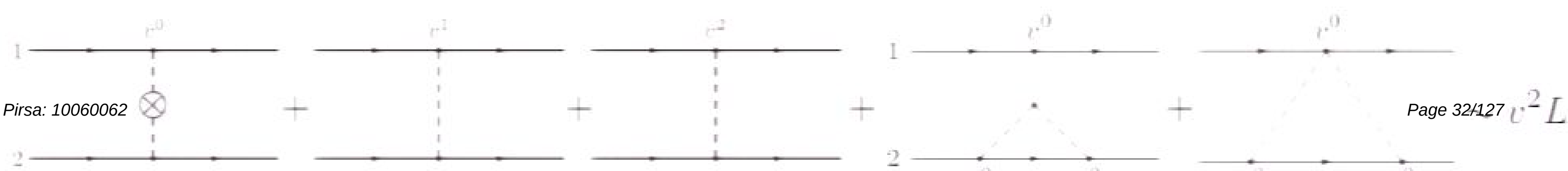
$$D^{rel}_{\alpha\beta\gamma\delta}(t,t',\mathbf{x},\mathbf{x}') = \delta(t-t')\int_{\mathbf{k}} \frac{e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}}{\mathbf{k}^2} + \frac{\partial}{\partial t}\frac{\partial}{\partial t'}\delta(t-t')\int_{\mathbf{k}} \frac{e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}}{\mathbf{k}^4} + \cdots$$



Calculations to be automated

Potentials

Goldberger, Rothstein, Ross, Gilmore, Porto

Radiation reaction "potentials"

CRG

Power loss/Flux

Goldberger, Rothstein, Porto

Restricted waveforms

Amplitude corrections

CRG

Full automation is probably impractical but our goal is to at least provide a set of robust tools for calculating intensive parts of PN computations

Calculating potentials in EFT

Interaction terms (3+1) — Power counting — Feynman rules

Find combos of interactions ("vertices") at given PN order — Feynman diagrams (Wick contractions)

Evaluate integrals — Regularize/ Renormalize

Architecture for automation

Mathematica -- widely used and easily accessible

For tensor calculations we use the xAct suite of packages (Jose Maria Martin-Garcia, David Brizuela,...)

`http://www.xact.es/`

xTensor, xPert, xCoba, Harmonics, Spinors,...

Architecture for automation

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xTensor, xPert, xCoba, Harmonics, Spinors,...

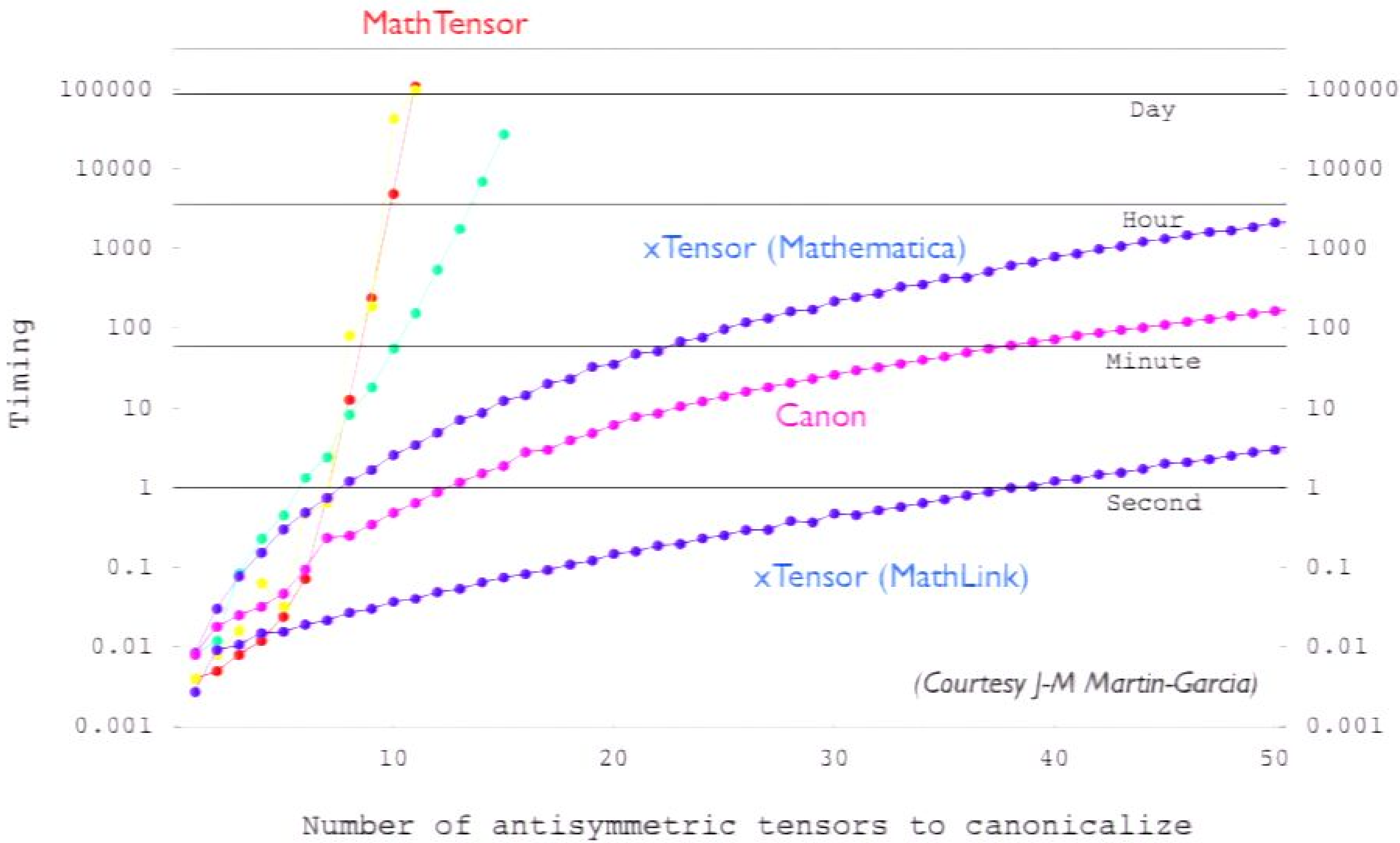
Reasons to choose xAct/xTensor:

1) Free (GNU public license)

2) Fast

3) Native support for manifolds, vector bundles, curvature, etc.

Benchmarking



$$A^a{}_b A^b{}_c \cdots A^{i_n}{}_a$$

Automation -- "xPN"

Full automation is lofty and probably not practical.

Realistic goal -- to provide a set of symbolic computational tools to do many (intensive) parts of the calculations automatically.

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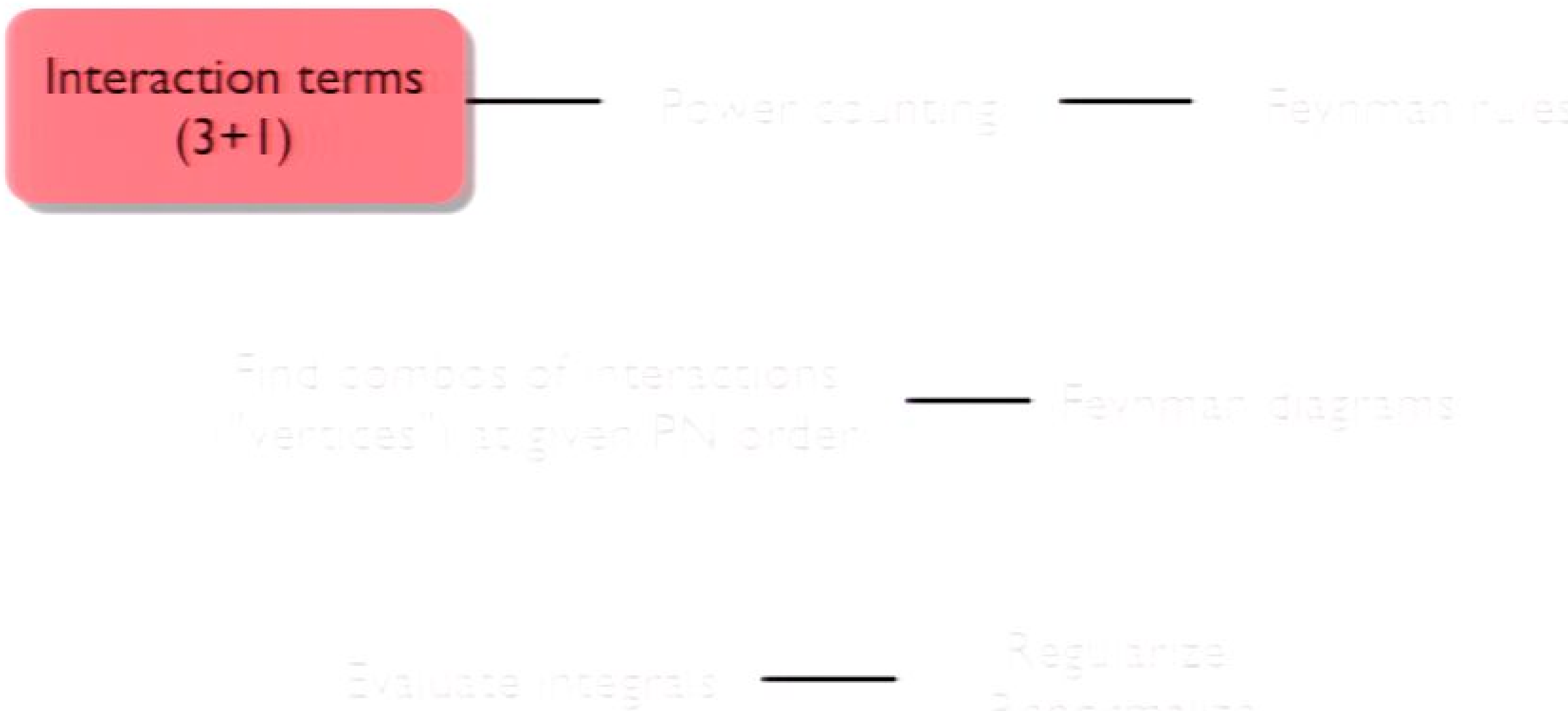
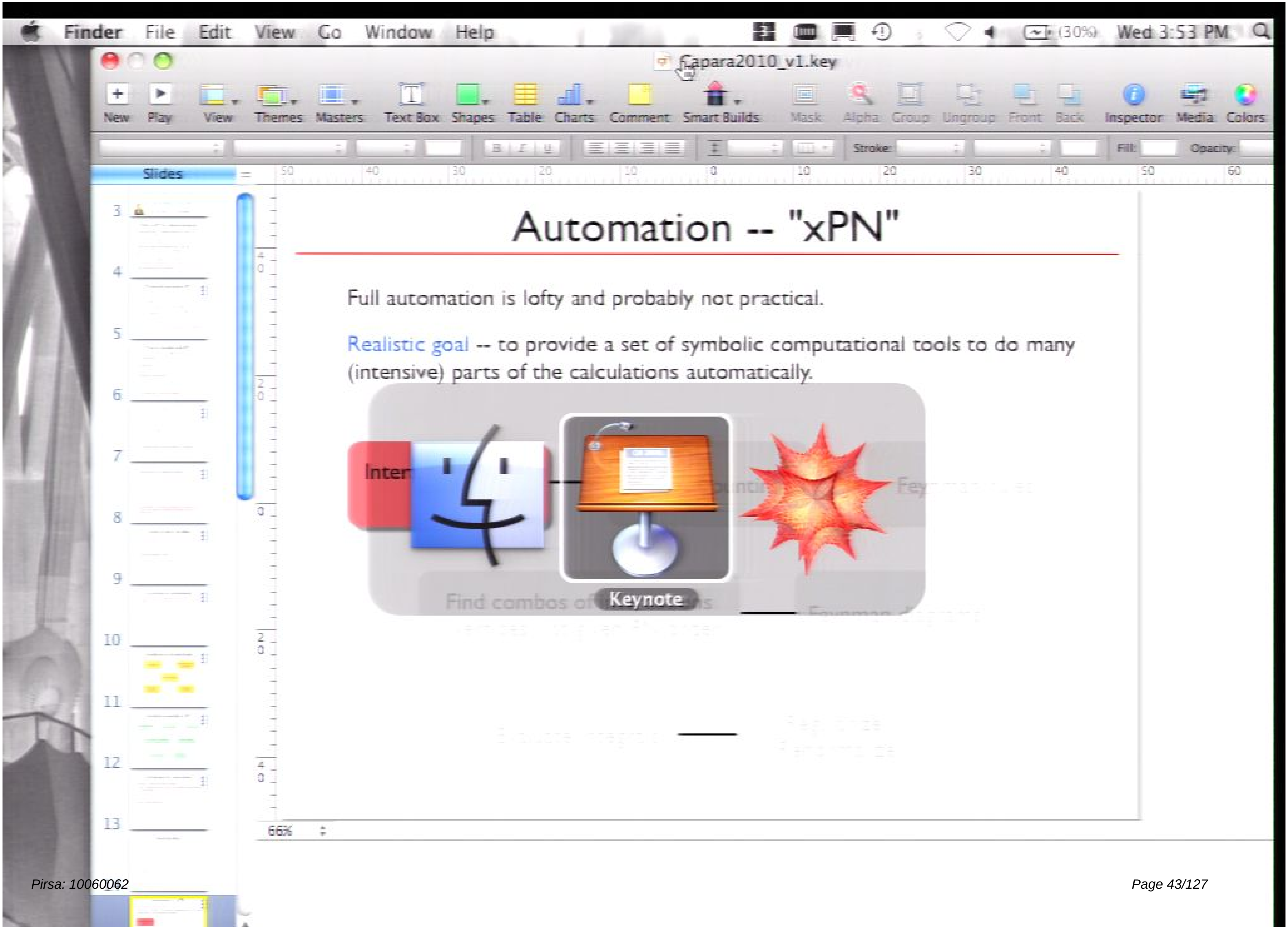


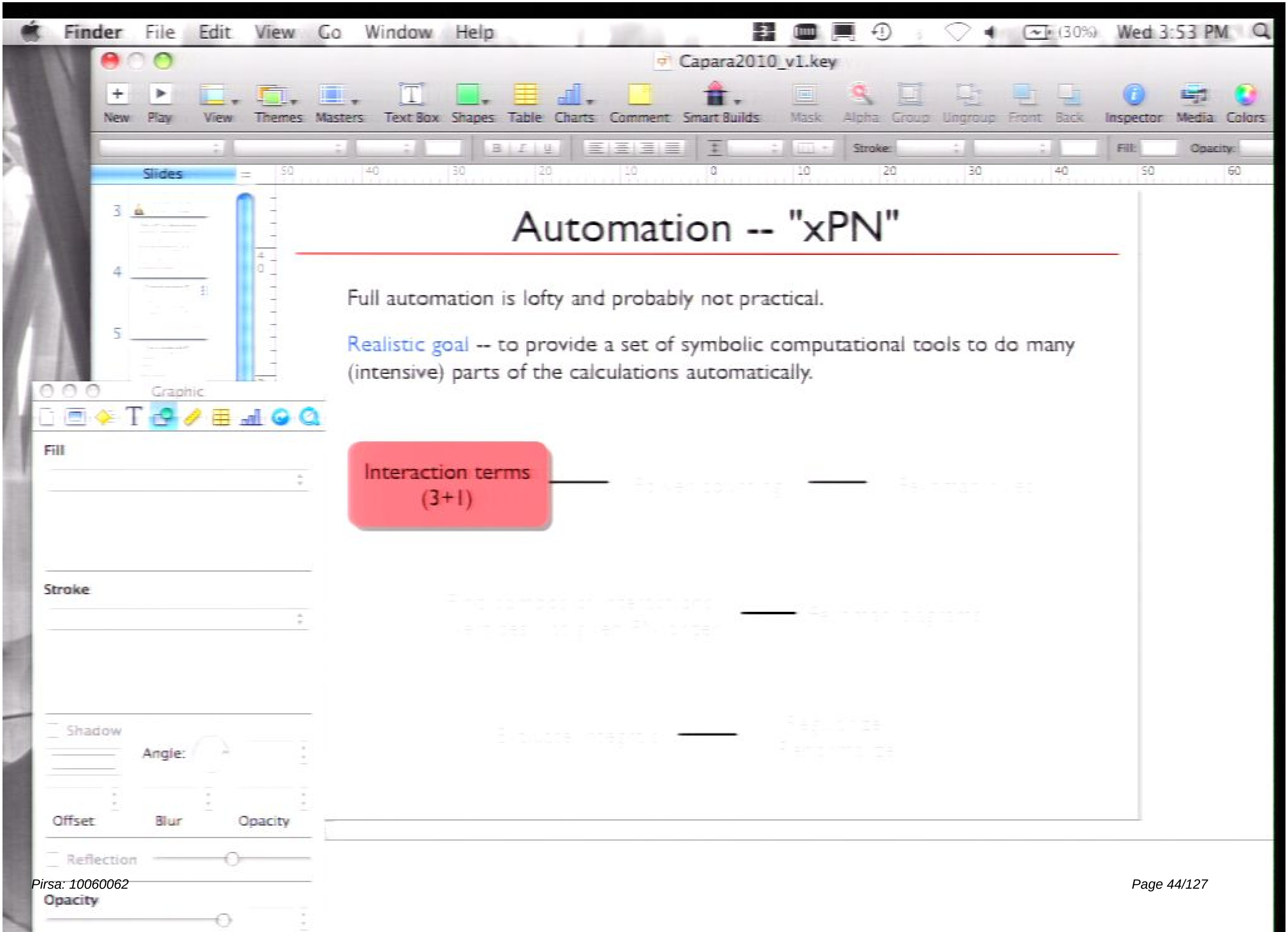
[redacted]


No Signal
VGA-1

No Signal

VGA-1


Finder File Edit View Go Window Help
Wed 3:53 PM
Capara2010_v1.key
New Play View Themes Masters Text Box Shapes Table Charts Comment Smart Builds Mask Alpha Group Ungroup Front Back Inspector Media Colors
Slides
Automation -- "xPN"
Full automation is lofty and probably not practical.
Realistic goal -- to provide a set of symbolic computational tools to do many (intensive) parts of the calculations automatically.
Keynote
66%


Capara2010_v1.key
Automation -- "xPN"
Full automation is lofty and probably not practical.
Realistic goal -- to provide a set of symbolic computational tools to do many (intensive) parts of the calculations automatically.
Interaction terms
(3+1)

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Demo_CapraNRDA_2010.nb

xPN demo for Capra/NRDA 2010 -- Perimeter Institute

Load xTensor, xPert and xPN functions

In[1]:=
```
MemoryInUse[]
```

Out[1]=
```
6 579 832
```

- Load xPert and define geometry
- Initialization functions
- Functions for 3+1 decompositions
- Functions for perturbation theory
- Functions for power counting
- Functions for Feynman rules
- Functions for databasing interaction terms

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Demo_CapraNRDA_2010.nb

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Running...Demo_CapraNRDA_2010.nb

```
SetOptions[CanonicalPerm, MathLink → True]

------------------------------------------------------------

Package xAct`xPerm`  version 1.0.3, {2009, 9, 9}

CopyRight (C) 2003-2008, Jose M. Martin-Garcia, under the General Public License.

Connecting to external mac executable...

Connection established.

------------------------------------------------------------

Package xAct`xTensor`  version 0.9.9, {2009, 9, 14}

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------------------------------------------------------------

Package xAct`xPert`  version 1.0.0, {2008, 6, 30}

CopyRight (C) 2005-2008, David Brizuela, Jose M. Martin-Garcia
  and Guillermo A. Mena Marugan, under the General Public License.

------------------------------------------------------------

These packages come with ABSOLUTELY NO WARRANTY; for details type Disclaimer[].
  This is free software, and you are welcome to redistribute it under
  certain conditions. See the General Public License for details.

------------------------------------------------------------

Out[3]= {MathLink → True, TimeVerbose → False, xPermVerbose → False, OrderedBase → True}
```

Load a package to print the run-times

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Demo_CapraNRDA_2010.nb

- Functions for power counting
- Functions for Feynman rules
- Functions for databasing interaction terms
- Functions to assemble vertices to make diagrams
- Connecting the vertices -- Wick contraction
- Functions for automating the PN expansion

Initialize variables

Lagrangians

Examples with xPN

- 3+1 decomposition & notation
- Perturbation theory
- Kinetic term for gravitational perturbations
- Transforming to Kol-Smolkin variables
- Power counting

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Demo_CapraNRDA_2010.nb

- Connecting the vertices -- Wick contraction
- Functions for automating the PN expansion

Initialize variables

In[114]:=
```
MemoryInUse[]
```

Out[114]=
```
46 593 360
```

In[115]:=
```
DefxPNObjects[g]
```

```
Metric signature is mostly minus.
** DefCovD: Defining covariant derivative $CovD2[-α$1071].
** DefTensor: Defining vanishing torsion tensor Torsion$CovD2[α, -β, -γ].
** DefTensor: Defining
 symmetric Christoffel tensor Christoffel$CovD2[α, -β, -γ].
** DefTensor: Defining Riemann tensor
 Riemann$CovD2[-α, -β, -γ, δ]. Antisymmetric only in the first pair.
** DefTensor: Defining non-symmetric Ricci tensor Ricci$CovD2[-α, -β].
** DefCovD:  Contractions of Riemann automatically replaced by Ricci.
** DefVBundle: Defining vbundle $R3.
```

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Running...Demo_CapraNRDA_2010.nb

```
** DefTensor: Defining tensor $VEL1[LI[1], i].
** DefTensor: Defining tensor $VEL2[LI[1], i].
** DefTensor: Defining tensor $Unit[AnyIndices[TangentM4]].
** DefConstantSymbol: Defining constant symbol Mass1.
** DefConstantSymbol: Defining constant symbol Mass2.
** DefConstantSymbol: Defining constant symbol PlanckMass.
0.116964
```

```
In[118]:= DefxPNRules[MakeKolSmolkinRules → True]
(*
DefxPNRules[ImportKolSmolkinRules→True]
*)
```

```
Rules {1, 2, 3, 4} have been declared as UpValues for FourVel.
Rules {1, 2, 3, 4} have been declared as UpValues for ThreeVel.
Rules {1, 2} have been declared as UpValues for UnitNormal.
Rules {1, 2, 3, 4, 5, 6, 7, 8} have been declared as UpValues for $Flat.
Rules {2, 3} have been declared as DownValues for Delta.
Rules {1} have been declared as UpValues for Delta.
Rules {1} have been declared as UpValues for $Unit.
Generating rules that transform from covariant to Kol-Smolkin variables...
```



```
In[117]:= MemoryInUse[]
```

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Demo_CapraNRDA_2010.nb

```
** DefConstantSymbol: Defining constant symbol PlanckMass.

0.116964

In[118]:= DefxPNRules[MakeKolSmolkinRules → True]
(*
DefxPNRules[ImportKolSmolkinRules→True]
*)

Rules {1, 2, 3, 4} have been declared as UpValues for FourVel.
Rules {1, 2, 3, 4} have been declared as UpValues for ThreeVel.
Rules {1, 2} have been declared as UpValues for UnitNormal.
Rules {1, 2, 3, 4, 5, 6, 7, 8} have been declared as UpValues for $Flat.
Rules {2, 3} have been declared as DownValues for Delta.
Rules {1} have been declared as UpValues for Delta.
Rules {1} have been declared as UpValues for $Unit.
Generating rules that transform from covariant to Kol-Smolkin variables...done.
Exporting rules to /Users/crgalley/Physics/Research/Projects/EFT/NRGR/Automation/I

3.21617

In[119]:= MemoryInUse[]

Out[119]= 47 873 896
```

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Demo_CapraNRDA_2010.nb

Lagrangians

Examples with xPN

- 3+1 decomposition & notation
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- Feynman rules
- Assemble the Feynman diagrams

Fully partial automation at 1PN

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Demo_CapraNRDA_2010.nb

In[119]:=
```
MemoryInUse[]
```
Out[119]=
```
47 873 896
```

Lagrangians

Einstein-Hilbert Lagrangian

In[118]:=
```
LEH = -2 $Sign Sqrt[-Detg[]] RicciScalarCD[]
```
Out[118]=

$-2\sqrt{-\bar{\bar{g}}}\ R[\nabla]$

Gauge-fixing Lagrangian for potential gravitons

In[119]:=
```
DefTensor[GPotHarmonic[α], M4]
```
```
** DefTensor: Defining tensor GPotHarmonic[α].
```

Harmonic gauge condition: (We need to be sure to not contract the metrics so that *Perturb* will expand all of the inverse metrics appearing in the expression.)

In[120]:=
```
IndexSet[GPotHarmonic[α_], ChristoffelCD[α, -β, -γ] g[β, γ]]
LgfPot[Harmonic] ^= $Sign Sqrt[-Detg[]] g[-α, -β] GPotHarmonic[α]
  GPotHarmonic[β]
```

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Demo_CapraNRDA_2010.nb

In[119]:=
```
MemoryInUse[]
```
Out[119]=
```
47 873 896
```

Lagrangians

Einstein-Hilbert Lagrangian

In[120]:=
```
LEH = -2 $Sign Sqrt[-Detg[]] RicciScalarCD[]
```
Out[120]=

$$-2\sqrt{-\tilde{\tilde{g}}}\ R[\nabla]$$

Gauge-fixing Lagrangian for potential gravitons

In[121]:=
```
DefTensor[GPotHarmonic[α], M4]
```
```
** DefTensor: Defining tensor GPotHarmonic[α].
```

Harmonic gauge condition: (We need to be sure to not contract the metrics so that *Perturb* will expand all of the inverse metrics appearing in the expression.)

In[120]:=
```
IndexSet[GPotHarmonic[α_], ChristoffelCD[α, -β, -γ] g[β, γ]]
LgfPot[Harmonic] ^= $Sign Sqrt[-Detg[]] g[-α, -β] GPotHarmonic[α]
  GPotHarmonic[β]
```

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Demo_CapraNRDA_2010.nb

Out[120]= $-2\sqrt{-\bar{\bar{g}}}\ R[\nabla]$

Gauge-fixing Lagrangian for potential gravitons

```
In[121]:= DefTensor[GPotHarmonic[α], M4]
```

** DefTensor: Defining tensor GPotHarmonic[α].

Harmonic gauge condition: (We need to be sure to not contract the metrics so that *Perturb* will expand all of the inverse metrics appearing in the expression.)

```
In[122]:= IndexSet[GPotHarmonic[α_], ChristoffelCD[α, -β, -γ] g[β, γ]]
LgfPot[Harmonic] ^= $Sign Sqrt[-Detg[]] g[-α, -β] GPotHarmonic[α]
  GPotHarmonic[β]
```

Out[122]= $\Gamma[\nabla]^{\alpha}{}_{\beta\gamma}\ g^{\beta\gamma}$

Out[123]= $\Gamma[\nabla]^{\alpha}{}_{\gamma\epsilon}\ \Gamma[\nabla]^{\beta}{}_{\delta\sigma}\ \sqrt{-\bar{\bar{g}}}\ g_{\alpha\beta}\ g^{\gamma\epsilon}\ g^{\delta\sigma}$

Lorenz gauge condition:

```
In[122]:= DefTensor[GPotLorenz[α], $Manifold]

DefTensor[P[α, β, γ, δ], M4]
IndexSet[P[α_, β_, γ_, δ_],
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (31%) Wed 3:54 PM

Demo_CapraNRDA_2010.nb

the inverse metrics appearing in the expression.)

In[122]:=
```
IndexSet[GPotHarmonic[α_], ChristoffelCD[α, -β, -γ] g[β, γ]]
LgfPot[Harmonic] ^= $Sign Sqrt[-Detg[]] g[-α, -β] GPotHarmonic[α]
  GPotHarmonic[β]
```

Out[122]= $\Gamma[\nabla]^{\alpha}{}_{\beta\gamma}\, g^{\beta\gamma}$

Out[123]= $\Gamma[\nabla]^{\alpha}{}_{\gamma\epsilon}\, \Gamma[\nabla]^{\beta}{}_{\delta\sigma} \sqrt{-\bar{\bar{g}}}\, g_{\alpha\beta}\, g^{\gamma\epsilon}\, g^{\delta\sigma}$

Lorenz gauge condition:

In[124]:=
```
DefTensor[GPotLorenz[α], $Manifold]

DefTensor[P[α, β, γ, δ], M4]
IndexSet[P[α_, β_, γ_, δ_],
 1 / 2 (g[α, γ] g[β, δ] + g[α, δ] g[β, γ] - g[α, β] g[γ, δ])]
```

** DefTensor: Defining tensor GPotLorenz[α].

** DefTensor: Defining tensor P[α, β, γ, δ].

Out[126]= $\frac{1}{2}\left(g^{\alpha\delta}\, g^{\beta\gamma} + g^{\alpha\gamma}\, g^{\beta\delta} - g^{\alpha\beta}\, g^{\gamma\delta}\right)$

In[125]:=
```
IndexSet[GPotLorenz[α_], P[α, β, γ, δ] CD[-β][GravitonH[LI[1], -γ, -δ]]]
LgfPot[Lorenz] ^= $Sign Sqrt[-Detg[]] g[-α, -β] GPotLorenz[α] GPotLorenz[β] //
```

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Demo_CapraNRDA_2010.nb

In[127]:=

```
IndexSet[GPotLorenz[α_], P[α, β, γ, δ] CD[-β][GravitonH[LI[1], -γ, -δ]]]
LgfPot[Lorenz] ^= $Sign Sqrt[-Detg[]] g[-α, -β] GPotLorenz[α] GPotLorenz[β] //
  ToCanonical
```

Out[127]= $\frac{1}{2}\left(g^{\alpha\delta} g^{\beta\gamma} + g^{\alpha\gamma} g^{\beta\delta} - g^{\alpha\beta} g^{\gamma\delta}\right)\left(\nabla_\beta H^1{}_{\gamma\delta}\right)$

0.123687

Out[128]= $\frac{1}{4}\sqrt{-\bar{\bar{g}}}\, g^{\beta\alpha} g^{\gamma\delta} g^{\epsilon\theta}\left(\nabla_\gamma H^1{}_{\beta\alpha}\right)\left(\nabla_\delta H^1{}_{\epsilon\theta}\right) + \sqrt{-\bar{\bar{g}}}\, g^{\beta\alpha} g^{\gamma\delta} g^{\epsilon\theta}\left(\nabla_\alpha H^1{}_{\beta\gamma}\right)\left(\nabla_\theta H^1{}_{\delta\epsilon}\right) - \sqrt{-\bar{\bar{g}}}\, g^{\beta\alpha} g^{\gamma\delta} g^{\epsilon\theta}\left(\nabla_\gamma H^1{}_{\beta\alpha}\right)\left(\nabla_\theta H^1{}_{\delta\epsilon}\right)$

Point particle Lagrangians (with no finite size effect terms).

In[129]:=

```
Lpp1 = -Mass1 Scalar[$Sign g[-α, -β] FourVel[α] FourVel[β]]^(1/2)
Lpp2 = -Mass2 Scalar[$Sign g[-α, -β] FourVel[α] FourVel[β]]^(1/2)
```

Out[129]= $-m_1\sqrt{\text{Scalar}\left[u^\alpha u^\beta g_{\alpha\beta}\right]}$

Out[130]= $-m_2\sqrt{\text{Scalar}\left[u^\alpha u^\beta g_{\alpha\beta}\right]}$

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Demo_CapraNRDA_2010.nb

Examples with xPN

- 3+1 decomposition & notation
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Fully partial automation at 1PN

Final Memory

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Demo_CapraNRDA_2010.nb

Examples with xPN

3+1 decomposition & notation

In[129]:=
```
FourVel[α]
UnitNormal[α] + ThreeVel[α]
```

Out[129]= u^{α}

Out[130]= $v^{\alpha} + n^{\alpha}$

In[131]:=
```
GravitonH[LI[1], -α, -β] FourVel[α] FourVel[β]
% // To3plus1
```

Out[131]= $u^{\alpha} u^{\beta} H^{1}{}_{\alpha\beta}$

Out[132]= $H^{1}{}_{ij} v^{i} v^{j} + 2 H^{1}{}_{i\beta} v^{i} n^{\beta} + H^{1}{}_{\alpha\beta} n^{\alpha} n^{\beta}$

In[133]:=
```
CD[-α]@CD[-β]@GravitonH[LI[1], α, β]
% // To3plus1
```

Out[133]= $\nabla_{\alpha} \nabla_{\beta} H^{1\alpha\beta}$

Automation -- "xPN"

Full automation is lofty and probably not practical.

Realistic goal -- to provide a set of symbolic computational tools to do many (intensive) parts of the calculations automatically.

- Perturbative expansion of total action accomplished with `Perturb`

- 3+1 decomposition (implemented in `To3plus1`):

  xTensor can define induced metrics, (in)extrinsic curvature, etc., but is fairly slow in combination with calculating the perturbations

  Make own scheme: Multiply by factors of metric trace and switch indices

- Perturbative expansion of total action accomplished with `Perturb`

- 3+1 decomposition (implemented in `To3plus1`):

  xTensor can define induced metrics, (in)extrinsic curvature, etc., but is fairly slow in combination with calculating the perturbations

  Make own scheme: Multiply by factors of metric trace and switch indices

$$u^\alpha = n^\alpha + v^\alpha \qquad n^\alpha = (1, 0, 0, 0) \qquad v^\alpha = (0, v^i) = (0, -v_i)$$

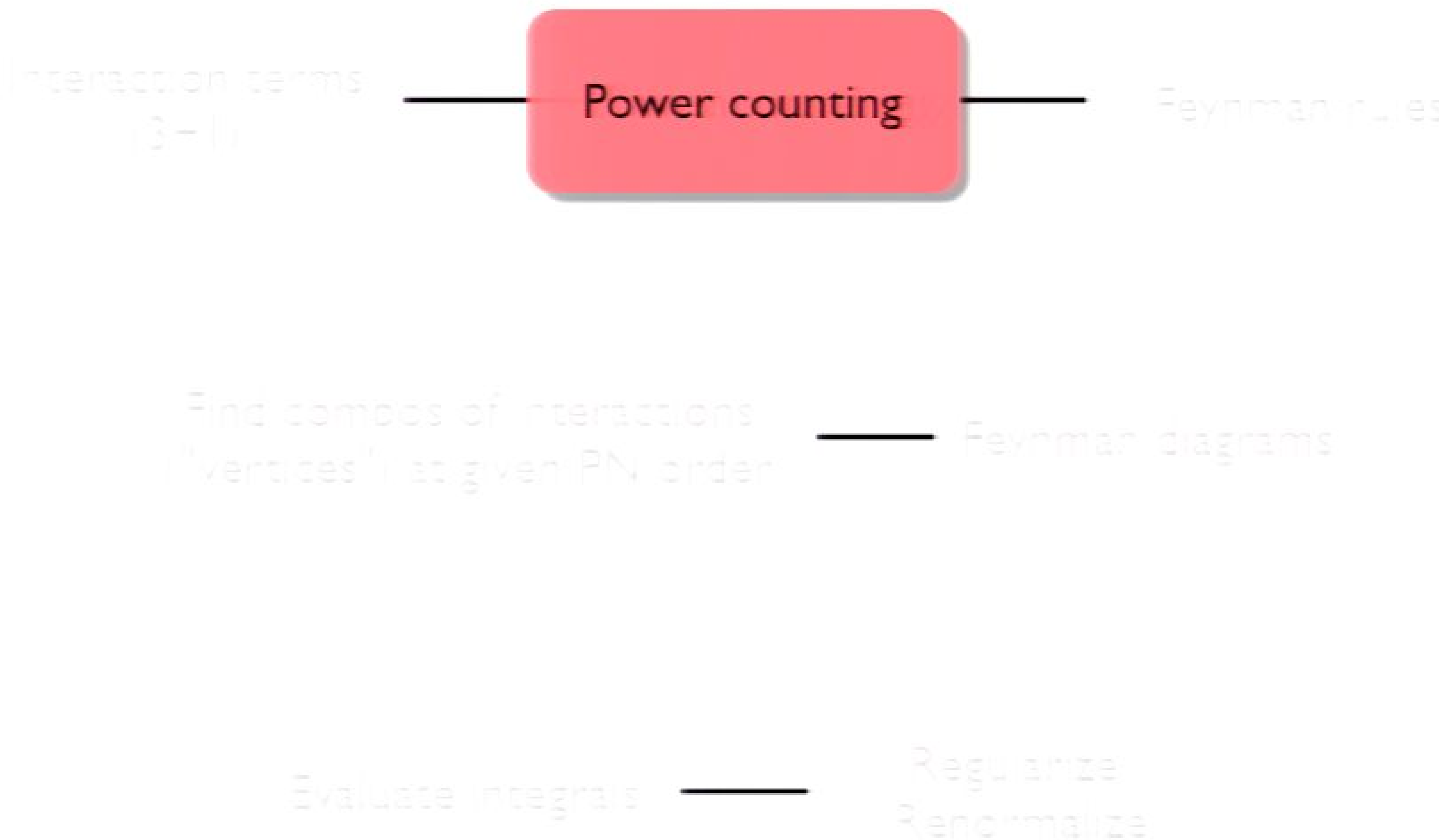
$$n^2 = 1, \quad n \cdot u = 1, \quad v \cdot n = 0$$

Example: $$H_{\alpha\beta} u^\alpha u^\beta \longrightarrow H_{\alpha\beta} u^\alpha u^\beta (n^\mu n_\mu + \delta^\mu{}_\mu)(n^\nu n_\nu + \delta^\nu{}_\nu)$$

$$\delta^\mu{}_\nu = \begin{pmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \delta^i{}_j \end{pmatrix} \qquad \delta^\mu{}_\nu n^\nu = 0$$

$$\longrightarrow H_{\alpha\beta} u^\mu u^\nu (n^\alpha n_\mu + \delta^\alpha{}_\mu)(n^\beta n_\nu + \delta^\beta{}_\nu)$$

$$= n^\alpha n^\beta H_{\alpha\beta} + 2 n^\alpha H_{\alpha i} v^i + H_{ij} v^i v^j$$


Interaction terms
Power counting
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Find combos of interactions ("vertices") at given PN order
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Evaluate integrals
Regularize
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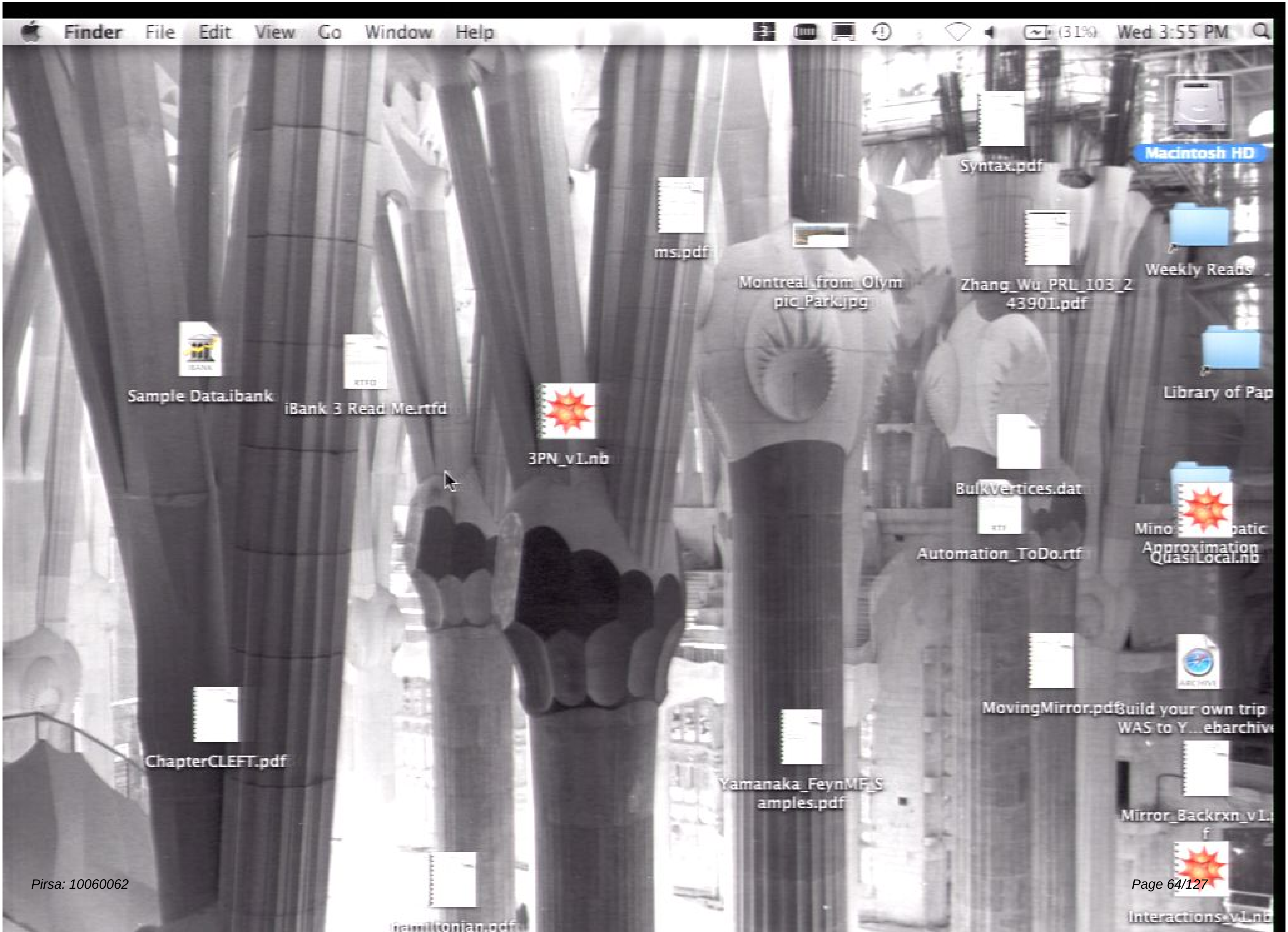
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ms.pdf
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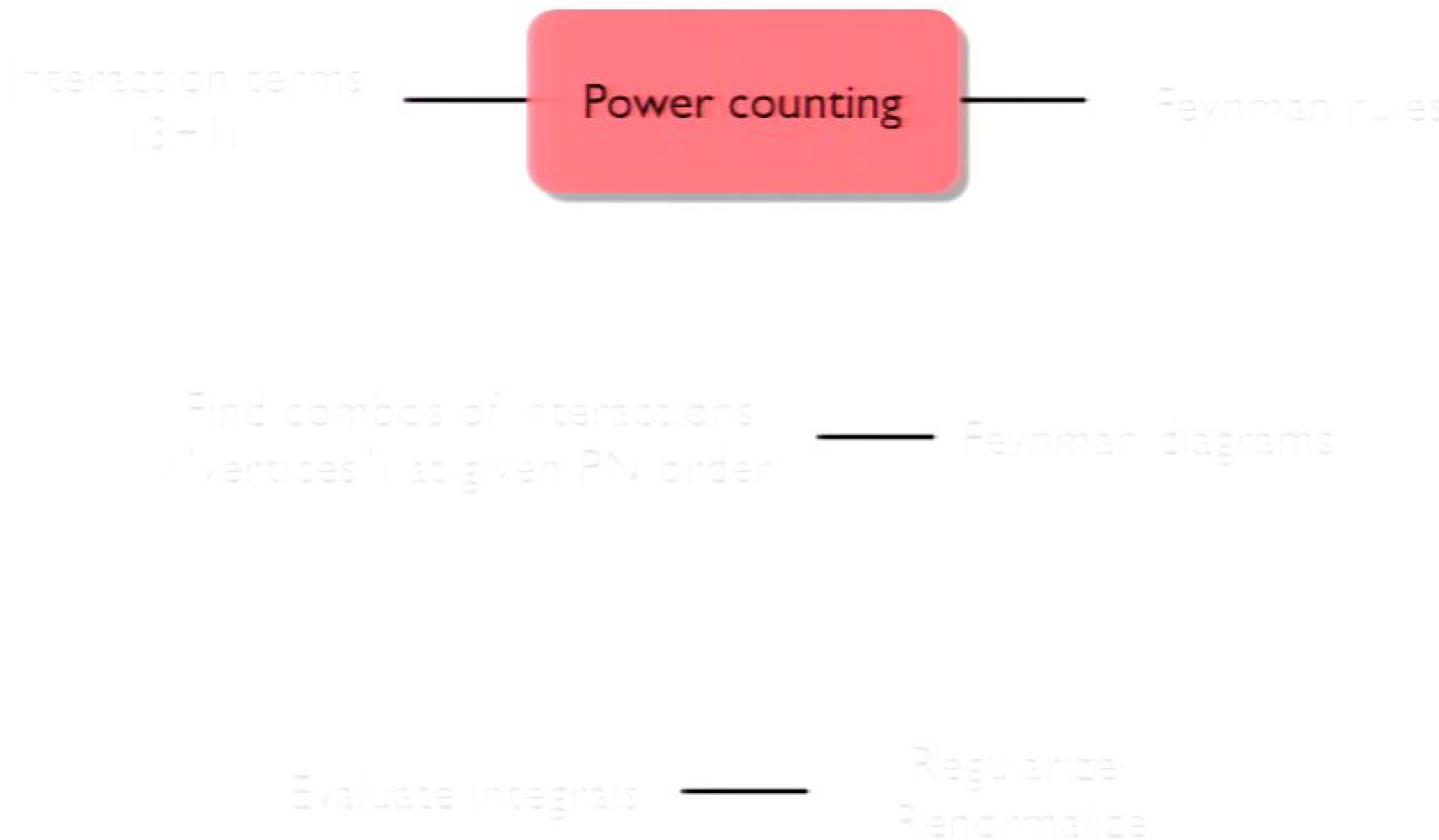
Zhang_Wu_PRL_103_243901.pdf
Weekly Reads
Library of Pap
Sample Data.ibank
iBank 3 Read Me.rtfd
3PN_v1.nb
BulkVertices.dat
Automation_ToDo.rtf
QuasiLocal.nb
MovingMirror.pdf
Build your own trip
WAS to Y...ebarchive
ChapterCLEFT.pdf
Yamanaka_FeynMF_Samples.pdf
Mirror_Backrxn_v1.
Interactions_v1.nb


Interaction terms
(3+1)
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Demo_CapraNRDA_2010.nb

Examples with xPN

3+1 decomposition & notation

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FourVel[α]
UnitNormal[α] + ThreeVel[α]
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% // To3plus1
```

Out[131]= $u^{\alpha} u^{\beta} H^{1}{}_{\alpha\beta}$

Out[132]= $H^{1}{}_{ij} v^{i} v^{j} + 2 H^{1}{}_{i\beta} v^{i} n^{\beta} + H^{1}{}_{\alpha\beta} n^{\alpha} n^{\beta}$

In[133]:=
```
CD[-α]@CD[-β]@GravitonH[LI[1], α, β]
% // To3plus1
```

Out[133]= $\nabla_{\alpha} \nabla_{\beta} H^{1\,\alpha\beta}$

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Examples with xPN

3+1 decomposition & notation

In[129]:=

```
FourVel[α]
UnitNormal[α] + ThreeVel[α]
```

Out[129]= u^{α}

Out[130]= $v^{\alpha} + n^{\alpha}$

In[131]:=

```
GravitonH[LI[1], -α, -β] FourVel[α] FourVel[β]
% // To3plus1
```

Out[131]= $u^{\alpha} u^{\beta} H^{1}{}_{\alpha\beta}$

Out[132]= $H^{1}{}_{ij} v^{i} v^{j} + 2 H^{1}{}_{i\beta} v^{i} n^{\beta} + H^{1}{}_{\alpha\beta} n^{\alpha} n^{\beta}$

In[133]:=

```
CD[-α]@CD[-β]@GravitonH[LI[1], α, β]
% // To3plus1
```

Out[133]= $\nabla_{\alpha} \nabla_{\beta} H^{1\alpha\beta}$

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Examples with xPN

3+1 decomposition & notation

In[129]:=
```
FourVel[α]
UnitNormal[α] + ThreeVel[α]
```

Out[129]= u^{α}

Out[130]= $v^{\alpha} + n^{\alpha}$

In[131]:=
```
GravitonH[LI[1], -α, -β] FourVel[α] FourVel[β]
% // To3plus1
```

Out[131]= $u^{\alpha} u^{\beta} H^{1}{}_{\alpha\beta}$

Out[132]= $H^{1}{}_{ij} v^{i} v^{j} + 2 H^{1}{}_{i\beta} v^{i} n^{\beta} + H^{1}{}_{\alpha\beta} n^{\alpha} n^{\beta}$

In[133]:=
```
CD[-α]@CD[-β]@GravitonH[LI[1], α, β]
% // To3plus1
```

Out[133]= $\nabla_{\alpha} \nabla_{\beta} H^{1\,\alpha\beta}$

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Examples with xPN

3+1 decomposition & notation

In[131]:=
```
FourVel[α]
UnitNormal[α] + ThreeVel[α]
```

Out[131]= u^{α}

Out[132]= $v^{\alpha} + n^{\alpha}$

In[133]:=
```
GravitonH[LI[1], -α, -β] FourVel[α] FourVel[β]
% // To3plus1
```

Out[133]= $u^{\alpha} u^{\beta} H^{1}{}_{\alpha\beta}$

Out[134]= $H^{1}{}_{ij} v^{i} v^{j} + 2 H^{1}{}_{i\beta} v^{i} n^{\beta} + H^{1}{}_{\alpha\beta} n^{\alpha} n^{\beta}$

In[133]:=
```
CD[-α]@CD[-β]@GravitonH[LI[1], α, β]
% // To3plus1
```

Out[133]= $\nabla_{\alpha} \nabla_{\beta} H^{1\,\alpha\beta}$

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```
% // To3plus1
```

Out[133]= $\nabla_\alpha \nabla_\beta H^{1\,\alpha\beta}$

Out[134]= $-n^\alpha n^\beta (\nabla_i \nabla_\beta H^1{}_\alpha{}^i) + \nabla_j \nabla_i H^{1\,ij} - n^\alpha n^\beta (\nabla_\beta \nabla_i H^1{}_\alpha{}^i) + n^\alpha n^\beta n^\gamma n^\delta (\nabla_\delta \nabla_\gamma H^1{}_{\alpha\beta})$

In[135]:=

```
GravitonH[LI[1], α, -α]
% // To3plus1
```

Out[135]= $H^{1\,\alpha}{}_\alpha$

Out[136]= $-H^{1\,i}{}_i + H^1{}_{\alpha\beta} n^\alpha n^\beta$

- Perturbation theory
- Kinetic term for gravitational perturbations
- Transforming to Kol-Smolkin variables
- Power counting
- Storing interactions in data structure
- Generating interactions for computing the 1PN potential
- Automating the perturbation calculations

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Examples with xPN

- 3+1 decomposition & notation
- Perturbation theory
- Kinetic term for gravitational perturbations
- Transforming to Kol-Smolkin variables
- Power counting
- Storing interactions in data structure
- Generating interactions for computing the 1PN potential
- Automating the perturbation calculations
- Feynman rules
- Assemble the Feynman diagrams

Fully partial automation at 1PN

Final Memory

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Examples with xPN

- 3+1 decomposition & notation
- Perturbation theory

In[137]:=
```
Lpp1
```
Out[137]= $-m_1 \sqrt{\text{Scalar}[u^\alpha u^\beta g_{\alpha\beta}]}$

In[138]:=
```
Perturb[Lpp1, 1, 0, 2]
```
Out[138]= $-\frac{1}{2} m_1 H^1{}_{\alpha\beta} v^\alpha v^\beta + \frac{1}{4} m_1 H^1{}_{\beta\gamma} v_\alpha v^\alpha n^\beta n^\gamma$

In[139]:=
```
% // To3plus1
```
Out[139]= $-\frac{1}{2} m_1 H^1{}_{ij} v^i v^j - \frac{1}{4} m_1 H^1{}_{\beta\gamma} v_i v^i n^\beta n^\gamma$

In[140]:=
```
Perturb[Lpp1, 0, 0, 8] / Mass1 // To3plus1
SeriesCoefficient[Series[- Sqrt[1 - ϵ^2 v^2], {ϵ, 0, 8}], 8]
```

24.3228

5 ... i ... j ... k ... l

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Examples with xPN

- 3+1 decomposition & notation
- Perturbation theory

In[135]:=
```
Lpp1
```

Out[135]= $-m_1 \sqrt{\text{Scalar}\left[u^\alpha u^\beta g_{\alpha\beta}\right]}$

In[138]:=
```
Perturb[Lpp1, 1, 0, 2]
```

Out[138]= $-\frac{1}{2} m_1 H^1{}_{\alpha\beta} v^\alpha v^\beta + \frac{1}{4} m_1 H^1{}_{\beta\gamma} v_\alpha v^\alpha n^\beta n^\gamma$

In[139]:=
```
% // To3plus1
```

Out[139]= $-\frac{1}{2} m_1 H^1{}_{ij} v^i v^j - \frac{1}{4} m_1 H^1{}_{\beta\gamma} v_i v^i n^\beta n^\gamma$

In[140]:=
```
Perturb[Lpp1, 0, 0, 8] / Mass1 // To3plus1
SeriesCoefficient[Series[- Sqrt[1 - ε^2 v^2], {ε, 0, 8}], 8]
```

24.3228

5

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In[136]:=
```
Perturb[Lpp1, 1, 0, 2]
```

Out[136]=
$$-\frac{1}{2} m_1 H^1{}_{\alpha\beta} v^\alpha v^\beta + \frac{1}{4} m_1 H^1{}_{\beta\gamma} v_\alpha v^\alpha n^\beta n^\gamma$$

In[137]:=
```
% // To3plus1
```

Out[137]=
$$-\frac{1}{2} m_1 H^1{}_{ij} v^i v^j - \frac{1}{4} m_1 H^1{}_{\beta\gamma} v_i v^i n^\beta n^\gamma$$

In[140]:=
```
Perturb[Lpp1, 0, 0, 8] / Mass1 // To3plus1
SeriesCoefficient[Series[- Sqrt[1 - ϵ^2 v^2], {ϵ, 0, 8}], 8]
```

24.3228

Out[140]=
$$\frac{5}{128} v_i v^i v_j v^j v_k v^k v_l v^l$$

Out[141]=
$$\frac{5 v^8}{128}$$

This interaction term appears as the first contribution to the 1PN potential due to non-linear features of general relativisty.

In[142]:=
```
LHHH = Perturb[LEH, 3, 0, 0]
```

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Demo_CapraNRDA_2010.nb

Out[137]= $-\frac{1}{2} m_1 H^{1}{}_{ij} v^i v^j - \frac{1}{4} m_1 H^{1}{}_{\beta\gamma} v_i v^i n^\beta n^\gamma$

In[140]:=
```
Perturb[Lpp1, 0, 0, 8] / Mass1 // To3plus1
SeriesCoefficient[Series[- Sqrt[1 - ϵ^2 v^2], {ϵ, 0, 8}], 8]
```

24.3228

Out[140]= $\frac{5}{128} v_i v^i v_j v^j v_k v^k v_l v^l$

Out[141]= $\frac{5 v^8}{128}$

This interaction term appears as the first contribution to the 1PN potential due to non-linear features of general relativisty.

In[142]:=
```
LHHH = Perturb[LEH, 3, 0, 0]
```

4.60842

Out[142]= $\frac{3}{2} H^{1\,\alpha\beta} (\nabla_\alpha H^{1\,\gamma\delta}) (\nabla_\beta H^{1}{}_{\gamma\delta}) - \frac{1}{2} H^{1\,\alpha\beta} (\nabla_\alpha H^{1\,\gamma}{}_{\gamma}) (\nabla_\beta H^{1\,\delta}{}_{\delta}) + 2 H^{1\,\alpha\beta} (\nabla_\beta H^{1\,\delta}{}_{\delta}) (\nabla_\gamma H^{1}{}_{\alpha}{}^{\gamma}) + 2 H^{1\,\alpha\beta} (\nabla_\beta H^{1}{}_{\alpha}{}^{\gamma}) (\nabla_\gamma H^{1\,\delta}{}_{\delta}) + 2 H^{1}{}_{\alpha}{}^{\gamma} H^{1\,\alpha\beta} (\nabla_\gamma \nabla_\beta H^{1\,\delta}{}_{\delta}) - H^{1\,\alpha}{}_{\alpha} H^{1\,\beta\gamma} (\nabla_\gamma \nabla_\beta H^{1\,\delta}{}_{\delta}) - 2 H^{1}{}_{\alpha}{}^{\gamma} H^{1\,\alpha\beta} (\nabla_\gamma \nabla_\delta H^{1}{}_{\beta}{}^{\delta}) + H^{1\,\alpha}{}_{\alpha} H^{1\,\beta\gamma} (\nabla_\gamma \nabla_\delta H^{1}{}_{\beta}{}^{\delta}) - H^{1\,\alpha\beta} (\nabla_\gamma H^{1\,\delta}{}_{\delta}) (\nabla^\gamma H^{1}{}_{\alpha\beta}) +$

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Demo_CapraNRDA_2010.nb

This interaction term appears as the first contribution to the 1PN potential due to non-linear features of general relativisty.

In[138]:=
```
LHHH = Perturb[LEH, 3, 0, 0]
```

4.54805

Out[138]=

$$
\begin{aligned}
&\frac{3}{2} H^{1\alpha\beta}(\nabla_\alpha H^{1\gamma\delta})(\nabla_\beta H^{1}{}_{\gamma\delta}) - \frac{1}{2} H^{1\alpha\beta}(\nabla_\alpha H^{1\gamma}{}_{\gamma})(\nabla_\beta H^{1\delta}{}_{\delta}) + 2 H^{1\alpha\beta}(\nabla_\beta H^{1\delta}{}_{\delta})(\nabla_\gamma H^{1}{}_{\alpha}{}^{\gamma}) - \\
&2 H^{1\alpha\beta}(\nabla_\beta H^{1}{}_{\alpha}{}^{\gamma})(\nabla_\gamma H^{1\delta}{}_{\delta}) + 2 H^{1}{}_{\alpha}{}^{\gamma} H^{1\alpha\beta}(\nabla_\gamma \nabla_\beta H^{1\delta}{}_{\delta}) - H^{1\alpha}{}_{\alpha} H^{1\beta\gamma}(\nabla_\gamma \nabla_\beta H^{1\delta}{}_{\delta}) - \\
&2 H^{1}{}_{\alpha}{}^{\gamma} H^{1\alpha\beta}(\nabla_\gamma \nabla_\delta H^{1}{}_{\beta}{}^{\delta}) + H^{1\alpha}{}_{\alpha} H^{1\beta\gamma}(\nabla_\gamma \nabla_\delta H^{1}{}_{\beta}{}^{\delta}) - H^{1\alpha\beta}(\nabla_\gamma H^{1\delta}{}_{\delta})(\nabla^\gamma H^{1}{}_{\alpha\beta}) + \\
&\frac{1}{4} H^{1\alpha}{}_{\alpha}(\nabla_\gamma H^{1\delta}{}_{\delta})(\nabla^\gamma H^{1\beta}{}_{\beta}) - 2 H^{1\alpha\beta}(\nabla_\gamma H^{1}{}_{\alpha}{}^{\gamma})(\nabla_\delta H^{1}{}_{\beta}{}^{\delta}) - 4 H^{1\alpha\beta}(\nabla_\beta H^{1}{}_{\alpha}{}^{\gamma})(\nabla_\delta H^{1}{}_{\gamma}{}^{\delta}) + \\
&H^{1\alpha}{}_{\alpha}(\nabla_\beta H^{1\beta\gamma})(\nabla_\delta H^{1}{}_{\gamma}{}^{\delta}) + 2 H^{1\alpha\beta}(\nabla^\gamma H^{1}{}_{\alpha\beta})(\nabla_\delta H^{1}{}_{\gamma}{}^{\delta}) - H^{1\alpha}{}_{\alpha}(\nabla^\gamma H^{1\beta}{}_{\beta})(\nabla_\delta H^{1}{}_{\gamma}{}^{\delta}) - \\
&2 H^{1\alpha\beta} H^{1\gamma\delta}(\nabla_\delta \nabla_\beta H^{1}{}_{\alpha\gamma}) + 2 H^{1\alpha\beta} H^{1\gamma\delta}(\nabla_\delta \nabla_\gamma H^{1}{}_{\alpha\beta}) - 2 H^{1}{}_{\alpha}{}^{\gamma} H^{1\alpha\beta}(\nabla_\delta \nabla_\gamma H^{1}{}_{\beta}{}^{\delta}) + \\
&H^{1\alpha}{}_{\alpha} H^{1\beta\gamma}(\nabla_\delta \nabla_\gamma H^{1}{}_{\beta}{}^{\delta}) + \frac{1}{2} H^{1}{}_{\alpha\beta} H^{1\alpha\beta}(\nabla_\delta \nabla_\gamma H^{1\gamma\delta}) - \frac{1}{4} H^{1\alpha}{}_{\alpha} H^{1\beta}{}_{\beta}(\nabla_\delta \nabla_\gamma H^{1\gamma\delta}) + \\
&2 H^{1}{}_{\alpha}{}^{\gamma} H^{1\alpha\beta}(\nabla_\delta \nabla^\delta H^{1}{}_{\beta\gamma}) - H^{1\alpha}{}_{\alpha} H^{1\beta\gamma}(\nabla_\delta \nabla^\delta H^{1}{}_{\beta\gamma}) - \frac{1}{2} H^{1}{}_{\alpha\beta} H^{1\alpha\beta}(\nabla_\delta \nabla^\delta H^{1\gamma}{}_{\gamma}) + \\
&\frac{1}{4} H^{1\alpha}{}_{\alpha} H^{1\beta}{}_{\beta}(\nabla_\delta \nabla^\delta H^{1\gamma}{}_{\gamma}) - 2 H^{1\alpha\beta}(\nabla_\beta H^{1}{}_{\gamma\delta})(\nabla^\delta H^{1}{}_{\alpha}{}^{\gamma}) - H^{1\alpha\beta}(\nabla_\gamma H^{1}{}_{\beta\delta})(\nabla^\delta H^{1}{}_{\alpha}{}^{\gamma}) + \\
&3 H^{1\alpha\beta}(\nabla_\delta H^{1}{}_{\beta\gamma})(\nabla^\delta H^{1}{}_{\alpha}{}^{\gamma}) + \frac{1}{2} H^{1\alpha}{}_{\alpha}(\nabla_\gamma H^{1}{}_{\beta\delta})(\nabla^\delta H^{1\beta\gamma}) - \frac{3}{4} H^{1\alpha}{}_{\alpha}(\nabla_\delta H^{1}{}_{\beta\gamma})(\nabla^\delta H^{1\beta\gamma})
\end{aligned}
$$

Integrate by parts on those terms with second derivatives so that the resulting expressions involves products of fields with first derivatives only.

4.54805

Out[138]=

$$
\begin{aligned}
&\frac{3}{2} H^{1\,\alpha\beta} (\nabla_\alpha H^{1\,\gamma\delta}) (\nabla_\beta H^{1}{}_{\gamma\delta}) - \frac{1}{2} H^{1\,\alpha\beta} (\nabla_\alpha H^{1\,\gamma}{}_\gamma) (\nabla_\beta H^{1\,\delta}{}_\delta) + 2 H^{1\,\alpha\beta} (\nabla_\beta H^{1\,\delta}{}_\delta) (\nabla_\gamma H^{1}{}_\alpha{}^\gamma) + \\
&2 H^{1\,\alpha\beta} (\nabla_\beta H^{1}{}_\alpha{}^\gamma) (\nabla_\gamma H^{1\,\delta}{}_\delta) + 2 H^{1}{}_\alpha{}^\gamma H^{1\,\alpha\beta} (\nabla_\gamma \nabla_\beta H^{1\,\delta}{}_\delta) - H^{1\,\alpha}{}_\alpha H^{1\,\beta\gamma} (\nabla_\gamma \nabla_\beta H^{1\,\delta}{}_\delta) - \\
&2 H^{1}{}_\alpha{}^\gamma H^{1\,\alpha\beta} (\nabla_\gamma \nabla_\delta H^{1}{}_\beta{}^\delta) + H^{1\,\alpha}{}_\alpha H^{1\,\beta\gamma} (\nabla_\gamma \nabla_\delta H^{1}{}_\beta{}^\delta) - H^{1\,\alpha\beta} (\nabla_\gamma H^{1\,\delta}{}_\delta) (\nabla^\gamma H^{1}{}_{\alpha\beta}) + \\
&\frac{1}{4} H^{1\,\alpha}{}_\alpha (\nabla_\gamma H^{1\,\delta}{}_\delta) (\nabla^\gamma H^{1\,\beta}{}_\beta) - 2 H^{1\,\alpha\beta} (\nabla_\gamma H^{1}{}_\alpha{}^\gamma) (\nabla_\delta H^{1}{}_\beta{}^\delta) - 4 H^{1\,\alpha\beta} (\nabla_\beta H^{1}{}_\alpha{}^\gamma) (\nabla_\delta H^{1}{}_\gamma{}^\delta) + \\
&H^{1\,\alpha}{}_\alpha (\nabla_\beta H^{1\,\beta\gamma}) (\nabla_\delta H^{1}{}_\gamma{}^\delta) + 2 H^{1\,\alpha\beta} (\nabla^\gamma H^{1}{}_{\alpha\beta}) (\nabla_\delta H^{1}{}_\gamma{}^\delta) - H^{1\,\alpha}{}_\alpha (\nabla^\gamma H^{1\,\beta}{}_\beta) (\nabla_\delta H^{1}{}_\gamma{}^\delta) - \\
&2 H^{1\,\alpha\beta} H^{1\,\gamma\delta} (\nabla_\delta \nabla_\beta H^{1}{}_{\alpha\gamma}) + 2 H^{1\,\alpha\beta} H^{1\,\gamma\delta} (\nabla_\delta \nabla_\gamma H^{1}{}_{\alpha\beta}) - 2 H^{1}{}_\alpha{}^\gamma H^{1\,\alpha\beta} (\nabla_\delta \nabla_\gamma H^{1}{}_\beta{}^\delta) + \\
&H^{1\,\alpha}{}_\alpha H^{1\,\beta\gamma} (\nabla_\delta \nabla_\gamma H^{1}{}_\beta{}^\delta) + \frac{1}{2} H^{1}{}_{\alpha\beta} H^{1\,\alpha\beta} (\nabla_\delta \nabla_\gamma H^{1\,\gamma\delta}) - \frac{1}{4} H^{1\,\alpha}{}_\alpha H^{1\,\beta}{}_\beta (\nabla_\delta \nabla_\gamma H^{1\,\gamma\delta}) + \\
&2 H^{1}{}_\alpha{}^\gamma H^{1\,\alpha\beta} (\nabla_\delta \nabla^\delta H^{1}{}_{\beta\gamma}) - H^{1\,\alpha}{}_\alpha H^{1\,\beta\gamma} (\nabla_\delta \nabla^\delta H^{1}{}_{\beta\gamma}) - \frac{1}{2} H^{1}{}_{\alpha\beta} H^{1\,\alpha\beta} (\nabla_\delta \nabla^\delta H^{1\,\gamma}{}_\gamma) + \\
&\frac{1}{4} H^{1\,\alpha}{}_\alpha H^{1\,\beta}{}_\beta (\nabla_\delta \nabla^\delta H^{1\,\gamma}{}_\gamma) - 2 H^{1\,\alpha\beta} (\nabla_\beta H^{1}{}_{\gamma\delta}) (\nabla^\delta H^{1}{}_\alpha{}^\gamma) - H^{1\,\alpha\beta} (\nabla_\gamma H^{1}{}_{\beta\delta}) (\nabla^\delta H^{1}{}_\alpha{}^\gamma) + \\
&3 H^{1\,\alpha\beta} (\nabla_\delta H^{1}{}_{\beta\gamma}) (\nabla^\delta H^{1}{}_\alpha{}^\gamma) + \frac{1}{2} H^{1\,\alpha}{}_\alpha (\nabla_\gamma H^{1}{}_{\beta\delta}) (\nabla^\delta H^{1\,\beta\gamma}) - \frac{3}{4} H^{1\,\alpha}{}_\alpha (\nabla_\delta H^{1}{}_{\beta\gamma}) (\nabla^\delta H^{1\,\beta\gamma})
\end{aligned}
$$

Integrate by parts on those terms with second derivatives so that the resulting expressions involves products of fields with first derivatives only.

In[143]:=

```
LHHH = MapAt[IntByParts[#, GravitonH] &, LHHH,
  Position[LHHH, expr_ $CovD[a_, b_]@GravitonH[__]]]
```

0.245483

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$$H^{1\alpha}{}_{\alpha} (\nabla_\beta H^{1\beta\gamma})(\nabla_\delta H^{1}{}_{\gamma}{}^{\delta}) + 2 H^{1\alpha\beta}(\nabla^\gamma H^1{}_{\alpha\beta})(\nabla_\delta H^1{}_\gamma{}^\delta) - H^{1\alpha}{}_\alpha (\nabla^\gamma H^{1\beta}{}_\beta)(\nabla_\delta H^1{}_\gamma{}^\delta) -$$
$$2H^{1\alpha\beta}H^{1\gamma\delta}(\nabla_\delta\nabla_\beta H^1{}_{\alpha\gamma}) + 2H^{1\alpha\beta}H^{1\gamma\delta}(\nabla_\delta\nabla_\gamma H^1{}_{\alpha\beta}) - 2H^1{}_\alpha{}^\gamma H^{1\alpha\beta}(\nabla_\delta\nabla_\gamma H^1{}_\beta{}^\delta) +$$
$$H^{1\alpha}{}_\alpha H^{1\beta\gamma}(\nabla_\delta\nabla_\gamma H^1{}_\beta{}^\delta) + \frac{1}{2} H^1{}_{\alpha\beta}H^{1\alpha\beta}(\nabla_\delta\nabla_\gamma H^{1\gamma\delta}) - \frac{1}{4} H^{1\alpha}{}_\alpha H^{1\beta}{}_\beta(\nabla_\delta\nabla_\gamma H^{1\gamma\delta}) +$$
$$2H^1{}_\alpha{}^\gamma H^{1\alpha\beta}(\nabla_\delta\nabla^\delta H^1{}_{\beta\gamma}) - H^{1\alpha}{}_\alpha H^{1\beta\gamma}(\nabla_\delta\nabla^\delta H^1{}_{\beta\gamma}) - \frac{1}{2} H^1{}_{\alpha\beta}H^{1\alpha\beta}(\nabla_\delta\nabla^\delta H^{1\gamma}{}_\gamma) +$$
$$\frac{1}{4} H^{1\alpha}{}_\alpha H^{1\beta}{}_\beta(\nabla_\delta\nabla^\delta H^{1\gamma}{}_\gamma) - 2H^{1\alpha\beta}(\nabla_\beta H^1{}_{\gamma\delta})(\nabla^\delta H^1{}_\alpha{}^\gamma) - H^{1\alpha\beta}(\nabla_\gamma H^1{}_{\beta\delta})(\nabla^\delta H^1{}_\alpha{}^\gamma) +$$
$$3H^{1\alpha\beta}(\nabla_\delta H^1{}_{\beta\gamma})(\nabla^\delta H^1{}_\alpha{}^\gamma) + \frac{1}{2} H^{1\alpha}{}_\alpha(\nabla_\gamma H^1{}_{\beta\delta})(\nabla^\delta H^{1\beta\gamma}) - \frac{3}{4} H^{1\alpha}{}_\alpha(\nabla_\delta H^1{}_{\beta\gamma})(\nabla^\delta H^{1\beta\gamma})$$

Integrate by parts on those terms with second derivatives so that the resulting expressions involves products of fields with first derivatives only.

In[143]:=

```
LHHH = MapAt[IntByParts[#, GravitonH] &, LHHH,
  Position[LHHH, expr_ $CovD[a_, b_]@GravitonH[__]]]
```

0.245483

Out[143]=

$$-\frac{1}{2} H^{1\alpha\beta}(\nabla_\alpha H^{1\gamma\delta})(\nabla_\beta H^1{}_{\gamma\delta}) + \frac{1}{2} H^{1\alpha\beta}(\nabla_\alpha H^{1\gamma}{}_\gamma)(\nabla_\beta H^{1\delta}{}_\delta) -$$
$$H^{1\alpha\beta}(\nabla_\beta H^{1\delta}{}_\delta)(\nabla_\gamma H^1{}_\alpha{}^\gamma) - H^{1\alpha\beta}(\nabla_\beta H^1{}_\alpha{}^\gamma)(\nabla_\gamma H^{1\delta}{}_\delta) + H^{1\alpha\beta}(\nabla_\gamma H^{1\delta}{}_\delta)(\nabla^\gamma H^1{}_{\alpha\beta}) -$$
$$\frac{1}{4} H^{1\alpha}{}_\alpha(\nabla_\gamma H^{1\delta}{}_\delta)(\nabla^\gamma H^{1\beta}{}_\beta) - H^{1\alpha\beta}(\nabla^\gamma H^1{}_{\alpha\beta})(\nabla_\delta H^1{}_\gamma{}^\delta) + \frac{1}{2} H^{1\alpha}{}_\alpha(\nabla^\gamma H^{1\beta}{}_\beta)(\nabla_\delta H^1{}_\gamma{}^\delta) +$$
$$2H^{1\alpha\beta}(\nabla_\beta H^1{}_{\gamma\delta})(\nabla^\delta H^1{}_\alpha{}^\gamma) + H^{1\alpha\beta}(\nabla_\gamma H^1{}_{\beta\delta})(\nabla^\delta H^1{}_\alpha{}^\gamma) - H^{1\alpha\beta}(\nabla_\delta H^1{}_{\beta\gamma})(\nabla^\delta H^1{}_\alpha{}^\gamma) -$$

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$$2\,H^{1}{}_{\alpha}{}^{\gamma}\,H^{1\,\alpha\beta}\left(\nabla_{\delta}\nabla^{\delta}H^{1}{}_{\beta\gamma}\right)-H^{1\,\alpha}{}_{\alpha}\,H^{1\,\beta\gamma}\left(\nabla_{\delta}\nabla^{\delta}H^{1}{}_{\beta\gamma}\right)-\frac{1}{2}H^{1}{}_{\alpha\beta}\,H^{1\,\alpha\beta}\left(\nabla_{\delta}\nabla^{\delta}H^{1\,\gamma}{}_{\gamma}\right)+$$
$$\frac{1}{4}H^{1\,\alpha}{}_{\alpha}\,H^{1\,\beta}{}_{\beta}\left(\nabla_{\delta}\nabla^{\delta}H^{1\,\gamma}{}_{\gamma}\right)-2\,H^{1\,\alpha\beta}\left(\nabla_{\beta}H^{1}{}_{\gamma\delta}\right)\left(\nabla^{\delta}H^{1}{}_{\alpha}{}^{\gamma}\right)-H^{1\,\alpha\beta}\left(\nabla_{\gamma}H^{1}{}_{\beta\delta}\right)\left(\nabla^{\delta}H^{1}{}_{\alpha}{}^{\gamma}\right)+$$
$$3\,H^{1\,\alpha\beta}\left(\nabla_{\delta}H^{1}{}_{\beta\gamma}\right)\left(\nabla^{\delta}H^{1}{}_{\alpha}{}^{\gamma}\right)+\frac{1}{2}H^{1\,\alpha}{}_{\alpha}\left(\nabla_{\gamma}H^{1}{}_{\beta\delta}\right)\left(\nabla^{\delta}H^{1\,\beta\gamma}\right)-\frac{3}{4}H^{1\,\alpha}{}_{\alpha}\left(\nabla_{\delta}H^{1}{}_{\beta\gamma}\right)\left(\nabla^{\delta}H^{1\,\beta\gamma}\right)$$

Integrate by parts on those terms with second derivatives so that the resulting expressions involves products of fields with first derivatives only.

In[139]:=

```
LHHH = MapAt[IntByParts[#, GravitonH] &, LHHH,
  Position[LHHH, expr_ $CovD[a_, b_]@GravitonH[__]]]
```

0.24304

Out[139]=

$$-\frac{1}{2}H^{1\,\alpha\beta}\left(\nabla_{\alpha}H^{1\,\gamma\delta}\right)\left(\nabla_{\beta}H^{1}{}_{\gamma\delta}\right)+\frac{1}{2}H^{1\,\alpha\beta}\left(\nabla_{\alpha}H^{1\,\gamma}{}_{\gamma}\right)\left(\nabla_{\beta}H^{1\,\delta}{}_{\delta}\right)-$$
$$H^{1\,\alpha\beta}\left(\nabla_{\beta}H^{1\,\delta}{}_{\delta}\right)\left(\nabla_{\gamma}H^{1}{}_{\alpha}{}^{\gamma}\right)-H^{1\,\alpha\beta}\left(\nabla_{\beta}H^{1}{}_{\alpha}{}^{\gamma}\right)\left(\nabla_{\gamma}H^{1\,\delta}{}_{\delta}\right)+H^{1\,\alpha\beta}\left(\nabla_{\gamma}H^{1\,\delta}{}_{\delta}\right)\left(\nabla^{\gamma}H^{1}{}_{\alpha\beta}\right)-$$
$$\frac{1}{4}H^{1\,\alpha}{}_{\alpha}\left(\nabla_{\gamma}H^{1\,\delta}{}_{\delta}\right)\left(\nabla^{\gamma}H^{1\,\beta}{}_{\beta}\right)-H^{1\,\alpha\beta}\left(\nabla^{\gamma}H^{1}{}_{\alpha\beta}\right)\left(\nabla_{\delta}H^{1}{}_{\gamma}{}^{\delta}\right)+\frac{1}{2}H^{1\,\alpha}{}_{\alpha}\left(\nabla^{\gamma}H^{1\,\beta}{}_{\beta}\right)\left(\nabla_{\delta}H^{1}{}_{\gamma}{}^{\delta}\right)+$$
$$2\,H^{1\,\alpha\beta}\left(\nabla_{\beta}H^{1}{}_{\gamma\delta}\right)\left(\nabla^{\delta}H^{1}{}_{\alpha}{}^{\gamma}\right)+H^{1\,\alpha\beta}\left(\nabla_{\gamma}H^{1}{}_{\beta\delta}\right)\left(\nabla^{\delta}H^{1}{}_{\alpha}{}^{\gamma}\right)-H^{1\,\alpha\beta}\left(\nabla_{\delta}H^{1}{}_{\beta\gamma}\right)\left(\nabla^{\delta}H^{1}{}_{\alpha}{}^{\gamma}\right)-$$
$$\frac{1}{2}H^{1\,\alpha}{}_{\alpha}\left(\nabla_{\gamma}H^{1}{}_{\beta\delta}\right)\left(\nabla^{\delta}H^{1\,\beta\gamma}\right)+\frac{1}{4}H^{1\,\alpha}{}_{\alpha}\left(\nabla_{\delta}H^{1}{}_{\beta\gamma}\right)\left(\nabla^{\delta}H^{1\,\beta\gamma}\right)$$

Do a 3+1 decomposition of the derivatives on the potential fields to make more explicit the dependence on the spatial and time derivatives.

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Out[139]=

$$-\frac{1}{2} H^{1\alpha\beta} (\nabla_\alpha H^{1\gamma\delta}) (\nabla_\beta H^{1}{}_{\gamma\delta}) + \frac{1}{2} H^{1\alpha\beta} (\nabla_\alpha H^{1\gamma}{}_{\gamma}) (\nabla_\beta H^{1\delta}{}_{\delta}) -$$
$$H^{1\alpha\beta} (\nabla_\beta H^{1\delta}{}_{\delta}) (\nabla_\gamma H^{1}{}_{\alpha}{}^{\gamma}) - H^{1\alpha\beta} (\nabla_\beta H^{1}{}_{\alpha}{}^{\gamma}) (\nabla_\gamma H^{1\delta}{}_{\delta}) + H^{1\alpha\beta} (\nabla_\gamma H^{1\delta}{}_{\delta}) (\nabla^\gamma H^{1}{}_{\alpha\beta}) -$$
$$\frac{1}{4} H^{1\alpha}{}_{\alpha} (\nabla_\gamma H^{1\delta}{}_{\delta}) (\nabla^\gamma H^{1\beta}{}_{\beta}) - H^{1\alpha\beta} (\nabla^\gamma H^{1}{}_{\alpha\beta}) (\nabla_\delta H^{1}{}_{\gamma}{}^{\delta}) + \frac{1}{2} H^{1\alpha}{}_{\alpha} (\nabla^\gamma H^{1\beta}{}_{\beta}) (\nabla_\delta H^{1}{}_{\gamma}{}^{\delta}) +$$
$$2 H^{1\alpha\beta} (\nabla_\beta H^{1}{}_{\gamma\delta}) (\nabla^\delta H^{1}{}_{\alpha}{}^{\gamma}) + H^{1\alpha\beta} (\nabla_\gamma H^{1}{}_{\beta\delta}) (\nabla^\delta H^{1}{}_{\alpha}{}^{\gamma}) - H^{1\alpha\beta} (\nabla_\delta H^{1}{}_{\beta\gamma}) (\nabla^\delta H^{1}{}_{\alpha}{}^{\gamma}) -$$
$$\frac{1}{2} H^{1\alpha}{}_{\alpha} (\nabla_\gamma H^{1}{}_{\beta\delta}) (\nabla^\delta H^{1\beta\gamma}) + \frac{1}{4} H^{1\alpha}{}_{\alpha} (\nabla_\delta H^{1}{}_{\beta\gamma}) (\nabla^\delta H^{1\beta\gamma})$$

Do a 3+1 decomposition of the derivatives on the potential fields to make more explicit the dependence on the spatial and time derivatives.

In[144]:=

```
LHHHto3plus1 = LHHH // To3plus1
```

4.59463

Out[144]=

$$\frac{1}{2} H^{1}{}_{\alpha\beta} n^\alpha n^\beta n^\gamma n^\delta (\nabla_i H^{1j}{}_{j}) (\nabla^i H^{1}{}_{\gamma\delta}) - \frac{1}{2} H^{1}{}_{\alpha\beta} n^\alpha n^\beta n^\gamma n^\delta (\nabla^i H^{1}{}_{\gamma\delta}) (\nabla_j H^{1}{}_{i}{}^{j}) -$$
$$\frac{1}{2} H^{1ij} (\nabla_i H^{1kl}) (\nabla_j H^{1}{}_{kl}) - H^{1ij} n^\alpha n^\beta (\nabla_i H^{1}{}_{\alpha\beta}) (\nabla_j H^{1k}{}_{k}) +$$
$$H^{1}{}_{\alpha}{}^{i} n^\alpha n^\beta (\nabla_i H^{1}{}_{\beta}{}^{j}) (\nabla_j H^{1k}{}_{k}) + \frac{1}{2} H^{1ij} (\nabla_i H^{1k}{}_{k}) (\nabla_j H^{1l}{}_{l}) +$$
$$H^{1}{}_{\alpha}{}^{i} n^\alpha n^\beta (\nabla_i H^{1k}{}_{k}) (\nabla_j H^{1}{}_{\beta}{}^{j}) + H^{1ij} n^\alpha n^\beta (\nabla_i H^{1}{}_{\alpha}{}^{k}) (\nabla_j H^{1}{}_{\beta k}) -$$
$$H^{1}{}_{\alpha}{}^{i} n^\alpha n^\beta n^\gamma n^\delta (\nabla_i H^{1}{}_{\beta\gamma}) (\nabla_j H^{1}{}_{\delta}{}^{j}) + \frac{1}{4} H^{1}{}_{\alpha\beta} n^\alpha n^\beta (\nabla_j H^{1k}{}_{k}) (\nabla^j H^{1i}{}_{i}) +$$
$$\frac{1}{2} H^{1i}{}_{i} n^\alpha n^\beta (\nabla_j H^{1k}{}_{k}) (\nabla^j H^{1}{}_{\alpha\beta}) - 2 H^{1}{}_{\alpha}{}^{i} n^\alpha n^\beta (\nabla_j H^{1k}{}_{k}) (\nabla^j H^{1}{}_{\beta i}) +$$

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$$\frac{}{4} H^{1\,\alpha}{}_{\alpha}\, H^{1\,\beta}{}_{\beta} \left(\nabla_\delta \nabla^\delta H^{1\,\gamma}{}_{\gamma}\right) - 2\, H^{1\,\alpha\beta} \left(\nabla_\beta H^{1}{}_{\gamma\delta}\right) \left(\nabla^\delta H^{1}{}_{\alpha}{}^{\gamma}\right) - H^{1\,\alpha\beta} \left(\nabla_\gamma H^{1}{}_{\beta\delta}\right) \left(\nabla^\delta H^{1}{}_{\alpha}{}^{\gamma}\right) +$$
$$3\, H^{1\,\alpha\beta} \left(\nabla_\delta H^{1}{}_{\beta\gamma}\right) \left(\nabla^\delta H^{1}{}_{\alpha}{}^{\gamma}\right) + \frac{1}{2} H^{1\,\alpha}{}_{\alpha} \left(\nabla_\gamma H^{1}{}_{\beta\delta}\right) \left(\nabla^\delta H^{1\,\beta\gamma}\right) - \frac{3}{4} H^{1\,\alpha}{}_{\alpha} \left(\nabla_\delta H^{1}{}_{\beta\gamma}\right) \left(\nabla^\delta H^{1\,\beta\gamma}\right)$$

Integrate by parts on those terms with second derivatives so that the resulting expressions involves products of fields with first derivatives only.

In[139]:=
```
LHHH = MapAt[IntByParts[#, GravitonH] &, LHHH,
  Position[LHHH, expr_ $CovD[a_, b_]@GravitonH[__]]]
```

0.24304

Out[139]=

$$-\frac{1}{2} H^{1\,\alpha\beta} \left(\nabla_\alpha H^{1\,\gamma\delta}\right) \left(\nabla_\beta H^{1}{}_{\gamma\delta}\right) + \frac{1}{2} H^{1\,\alpha\beta} \left(\nabla_\alpha H^{1\,\gamma}{}_{\gamma}\right) \left(\nabla_\beta H^{1\,\delta}{}_{\delta}\right) -$$
$$H^{1\,\alpha\beta} \left(\nabla_\beta H^{1\,\delta}{}_{\delta}\right) \left(\nabla_\gamma H^{1}{}_{\alpha}{}^{\gamma}\right) - H^{1\,\alpha\beta} \left(\nabla_\beta H^{1}{}_{\alpha}{}^{\gamma}\right) \left(\nabla_\gamma H^{1\,\delta}{}_{\delta}\right) + H^{1\,\alpha\beta} \left(\nabla_\gamma H^{1\,\delta}{}_{\delta}\right) \left(\nabla^\gamma H^{1}{}_{\alpha\beta}\right) -$$
$$\frac{1}{4} H^{1\,\alpha}{}_{\alpha} \left(\nabla_\gamma H^{1\,\delta}{}_{\delta}\right) \left(\nabla^\gamma H^{1\,\beta}{}_{\beta}\right) - H^{1\,\alpha\beta} \left(\nabla^\gamma H^{1}{}_{\alpha\beta}\right) \left(\nabla_\delta H^{1}{}_{\gamma}{}^{\delta}\right) + \frac{1}{2} H^{1\,\alpha}{}_{\alpha} \left(\nabla^\gamma H^{1\,\beta}{}_{\beta}\right) \left(\nabla_\delta H^{1}{}_{\gamma}{}^{\delta}\right) +$$
$$2\, H^{1\,\alpha\beta} \left(\nabla_\beta H^{1}{}_{\gamma\delta}\right) \left(\nabla^\delta H^{1}{}_{\alpha}{}^{\gamma}\right) + H^{1\,\alpha\beta} \left(\nabla_\gamma H^{1}{}_{\beta\delta}\right) \left(\nabla^\delta H^{1}{}_{\alpha}{}^{\gamma}\right) - H^{1\,\alpha\beta} \left(\nabla_\delta H^{1}{}_{\beta\gamma}\right) \left(\nabla^\delta H^{1}{}_{\alpha}{}^{\gamma}\right) -$$
$$\frac{1}{2} H^{1\,\alpha}{}_{\alpha} \left(\nabla_\gamma H^{1}{}_{\beta\delta}\right) \left(\nabla^\delta H^{1\,\beta\gamma}\right) + \frac{1}{4} H^{1\,\alpha}{}_{\alpha} \left(\nabla_\delta H^{1}{}_{\beta\gamma}\right) \left(\nabla^\delta H^{1\,\beta\gamma}\right)$$

Do a 3+1 decomposition of the derivatives on the potential fields to make more explicit the dependence on the spatial and time derivatives.

In[144]:=
```
LHHHto3plus1 = LHHH // To3plus1
```

4.59463

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Out[139]=

$$-\frac{1}{2} H^{1\,\alpha\beta} \left(\nabla_\alpha H^{1\,\gamma\delta}\right) \left(\nabla_\beta H^{1}{}_{\gamma\delta}\right) + \frac{1}{2} H^{1\,\alpha\beta} \left(\nabla_\alpha H^{1\,\gamma}{}_{\gamma}\right) \left(\nabla_\beta H^{1\,\delta}{}_{\delta}\right) -$$

$$H^{1\,\alpha\beta} \left(\nabla_\beta H^{1\,\delta}{}_{\delta}\right) \left(\nabla_\gamma H^{1}{}_{\alpha}{}^{\gamma}\right) - H^{1\,\alpha\beta} \left(\nabla_\beta H^{1}{}_{\alpha}{}^{\gamma}\right) \left(\nabla_\gamma H^{1\,\delta}{}_{\delta}\right) + H^{1\,\alpha\beta} \left(\nabla_\gamma H^{1\,\delta}{}_{\delta}\right) \left(\nabla^\gamma H^{1}{}_{\alpha\beta}\right) -$$

$$\frac{1}{4} H^{1\,\alpha}{}_{\alpha} \left(\nabla_\gamma H^{1\,\delta}{}_{\delta}\right) \left(\nabla^\gamma H^{1\,\beta}{}_{\beta}\right) - H^{1\,\alpha\beta} \left(\nabla^\gamma H^{1}{}_{\alpha\beta}\right) \left(\nabla_\delta H^{1}{}_{\gamma}{}^{\delta}\right) + \frac{1}{2} H^{1\,\alpha}{}_{\alpha} \left(\nabla^\gamma H^{1\,\beta}{}_{\beta}\right) \left(\nabla_\delta H^{1}{}_{\gamma}{}^{\delta}\right) +$$

$$2 H^{1\,\alpha\beta} \left(\nabla_\beta H^{1}{}_{\gamma\delta}\right) \left(\nabla^\delta H^{1}{}_{\alpha}{}^{\gamma}\right) + H^{1\,\alpha\beta} \left(\nabla_\gamma H^{1}{}_{\beta\delta}\right) \left(\nabla^\delta H^{1}{}_{\alpha}{}^{\gamma}\right) - H^{1\,\alpha\beta} \left(\nabla_\delta H^{1}{}_{\beta\gamma}\right) \left(\nabla^\delta H^{1}{}_{\alpha}{}^{\gamma}\right) -$$

$$\frac{1}{2} H^{1\,\alpha}{}_{\alpha} \left(\nabla_\gamma H^{1}{}_{\beta\delta}\right) \left(\nabla^\delta H^{1\,\beta\gamma}\right) + \frac{1}{4} H^{1\,\alpha}{}_{\alpha} \left(\nabla_\delta H^{1}{}_{\beta\gamma}\right) \left(\nabla^\delta H^{1\,\beta\gamma}\right)$$

Do a 3+1 decomposition of the derivatives on the potential fields to make more explicit the dependence on the spatial and time derivatives.

In[144]:=

```
LHHHto3plus1 = LHHH // To3plus1
```

4.59463

Out[144]=

$$\frac{1}{2} H^{1}{}_{\alpha\beta}\, n^\alpha n^\beta n^\gamma n^\delta \left(\nabla_i H^{1\,j}{}_{j}\right) \left(\nabla^i H^{1}{}_{\gamma\delta}\right) - \frac{1}{2} H^{1}{}_{\alpha\beta}\, n^\alpha n^\beta n^\gamma n^\delta \left(\nabla^i H^{1}{}_{\gamma\delta}\right) \left(\nabla_j H^{1}{}_{i}{}^{j}\right) -$$

$$\frac{1}{2} H^{1\,ij} \left(\nabla_i H^{1\,kl}\right) \left(\nabla_j H^{1}{}_{kl}\right) - H^{1\,ij}\, n^\alpha n^\beta \left(\nabla_i H^{1}{}_{\alpha\beta}\right) \left(\nabla_j H^{1\,k}{}_{k}\right) +$$

$$H^{1}{}_{\alpha}{}^{i}\, n^\alpha n^\beta \left(\nabla_i H^{1}{}_{\beta}{}^{j}\right) \left(\nabla_j H^{1\,k}{}_{k}\right) + \frac{1}{2} H^{1\,ij} \left(\nabla_i H^{1\,k}{}_{k}\right) \left(\nabla_j H^{1\,l}{}_{l}\right) +$$

$$H^{1}{}_{\alpha}{}^{i}\, n^\alpha n^\beta \left(\nabla_i H^{1\,k}{}_{k}\right) \left(\nabla_j H^{1}{}_{\beta}{}^{j}\right) + H^{1\,ij}\, n^\alpha n^\beta \left(\nabla_i H^{1}{}_{\alpha}{}^{k}\right) \left(\nabla_j H^{1}{}_{\beta k}\right) -$$

$$H^{1}{}_{\alpha}{}^{i}\, n^\alpha n^\beta n^\gamma n^\delta \left(\nabla_i H^{1}{}_{\beta\gamma}\right) \left(\nabla_j H^{1}{}_{\delta}{}^{j}\right) + \frac{1}{4} H^{1}{}_{\alpha\beta}\, n^\alpha n^\beta \left(\nabla_j H^{1\,k}{}_{k}\right) \left(\nabla^j H^{1\,i}{}_{i}\right) +$$

$$\frac{1}{2} H^{1\,i}{}_{i}\, n^\alpha n^\beta \left(\nabla_j H^{1\,k}{}_{k}\right) \left(\nabla^j H^{1}{}_{\alpha\beta}\right) - 2 H^{1}{}_{\alpha}{}^{i}\, n^\alpha n^\beta \left(\nabla_j H^{1\,k}{}_{k}\right) \left(\nabla^j H^{1}{}_{\beta i}\right) +$$

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$$\frac{1}{4} H^{1\,i}{}_{i}\, n^{\alpha} n^{\beta} \left(\nabla_{\alpha} H^{1\,jk}\right) \left(\nabla_{\beta} H^{1}{}_{jk}\right) - H^{1}{}_{\alpha}{}^{i}\, n^{\alpha} n^{\beta} \left(\nabla_{i} H^{1\,k}{}_{k}\right) \left(\nabla_{\beta} H^{1\,j}{}_{j}\right) +$$

$$H^{1}{}_{\alpha}{}^{i}\, n^{\alpha} n^{\beta} \left(\nabla_{i} H^{1}{}_{jk}\right) \left(\nabla_{\beta} H^{1\,jk}\right) + H^{1}{}_{\alpha}{}^{i}\, n^{\alpha} n^{\beta} \left(\nabla_{j} H^{1}{}_{i}{}^{j}\right) \left(\nabla_{\beta} H^{1\,k}{}_{k}\right) +$$

$$H^{1\,ij}\, n^{\alpha} n^{\beta} \left(\nabla_{j} H^{1}{}_{\alpha i}\right) \left(\nabla_{\beta} H^{1\,k}{}_{k}\right) - \frac{1}{2} H^{1\,i}{}_{i}\, n^{\alpha} n^{\beta} \left(\nabla_{j} H^{1}{}_{\alpha}{}^{j}\right) \left(\nabla_{\beta} H^{1\,k}{}_{k}\right) -$$

$$H^{1\,ij}\, n^{\alpha} n^{\beta} \left(\nabla_{\alpha} H^{1}{}_{ij}\right) \left(\nabla_{\beta} H^{1\,k}{}_{k}\right) + \frac{1}{4} H^{1\,i}{}_{i}\, n^{\alpha} n^{\beta} \left(\nabla_{\alpha} H^{1\,j}{}_{j}\right) \left(\nabla_{\beta} H^{1\,k}{}_{k}\right) +$$

$$H^{1\,ij}\, n^{\alpha} n^{\beta} \left(\nabla_{j} H^{1\,k}{}_{k}\right) \left(\nabla_{\beta} H^{1}{}_{\alpha i}\right) - \frac{1}{2} H^{1\,i}{}_{i}\, n^{\alpha} n^{\beta} \left(\nabla_{j} H^{1\,k}{}_{k}\right) \left(\nabla_{\beta} H^{1}{}_{\alpha}{}^{j}\right) +$$

$$H^{1\,ij}\, n^{\alpha} n^{\beta} \left(\nabla_{k} H^{1}{}_{ij}\right) \left(\nabla_{\beta} H^{1}{}_{\alpha}{}^{k}\right) - H^{1\,ij}\, n^{\alpha} n^{\beta} n^{\gamma} n^{\delta} \left(\nabla_{j} H^{1}{}_{\delta i}\right) \left(\nabla_{\gamma} H^{1}{}_{\alpha\beta}\right) +$$

$$\frac{1}{2} H^{1\,i}{}_{i}\, n^{\alpha} n^{\beta} n^{\gamma} n^{\delta} \left(\nabla_{j} H^{1}{}_{\delta}{}^{j}\right) \left(\nabla_{\gamma} H^{1}{}_{\alpha\beta}\right) - 2 H^{1}{}_{\alpha}{}^{i}\, n^{\alpha} n^{\beta} n^{\gamma} n^{\delta} \left(\nabla_{j} H^{1}{}_{\delta}{}^{j}\right) \left(\nabla_{\gamma} H^{1}{}_{\beta i}\right) +$$

$$H^{1}{}_{\alpha}{}^{i}\, n^{\alpha} n^{\beta} n^{\gamma} n^{\delta} \left(\nabla^{j} H^{1}{}_{\beta\gamma}\right) \left(\nabla_{\delta} H^{1}{}_{ij}\right) + H^{1}{}_{\alpha\beta}\, n^{\alpha} n^{\beta} n^{\gamma} n^{\delta} \left(\nabla^{j} H^{1}{}_{\gamma}{}^{i}\right) \left(\nabla_{\delta} H^{1}{}_{ij}\right) -$$

$$\frac{1}{4} H^{1}{}_{\alpha\beta}\, n^{\alpha} n^{\beta} n^{\gamma} n^{\delta} \left(\nabla_{\gamma} H^{1\,ij}\right) \left(\nabla_{\delta} H^{1}{}_{ij}\right) - \frac{1}{2} H^{1}{}_{\alpha\beta}\, n^{\alpha} n^{\beta} n^{\gamma} n^{\delta} \left(\nabla_{i} H^{1}{}_{\gamma}{}^{i}\right) \left(\nabla_{\delta} H^{1\,j}{}_{j}\right) +$$

$$\frac{1}{4} H^{1}{}_{\alpha\beta}\, n^{\alpha} n^{\beta} n^{\gamma} n^{\delta} \left(\nabla_{\gamma} H^{1\,i}{}_{i}\right) \left(\nabla_{\delta} H^{1\,j}{}_{j}\right) - H^{1}{}_{\alpha}{}^{i}\, n^{\alpha} n^{\beta} n^{\gamma} n^{\delta} \left(\nabla_{j} H^{1}{}_{i}{}^{j}\right) \left(\nabla_{\delta} H^{1}{}_{\beta\gamma}\right) -$$

$$\frac{1}{2} H^{1}{}_{\alpha\beta}\, n^{\alpha} n^{\beta} n^{\gamma} n^{\delta} \left(\nabla_{i} H^{1\,j}{}_{j}\right) \left(\nabla_{\delta} H^{1}{}_{\gamma}{}^{i}\right) + H^{1\,ij}\, n^{\alpha} n^{\beta} n^{\gamma} n^{\delta} \left(\nabla_{i} H^{1}{}_{\alpha\beta}\right) \left(\nabla_{\delta} H^{1}{}_{\gamma j}\right) -$$

$$\frac{1}{2} H^{1\,i}{}_{i}\, n^{\alpha} n^{\beta} n^{\gamma} n^{\delta} \left(\nabla^{j} H^{1}{}_{\alpha\beta}\right) \left(\nabla_{\delta} H^{1}{}_{\gamma j}\right) + 2 H^{1}{}_{\alpha}{}^{i}\, n^{\alpha} n^{\beta} n^{\gamma} n^{\delta} \left(\nabla^{j} H^{1}{}_{\beta i}\right) \left(\nabla_{\delta} H^{1}{}_{\gamma j}\right) +$$

$$\frac{3}{2} H^{1}{}_{\alpha\beta}\, n^{\alpha} n^{\beta} n^{\gamma} n^{\delta} n^{\epsilon} n^{\zeta} \left(\nabla_{i} H^{1}{}_{\zeta}{}^{i}\right) \left(\nabla_{\epsilon} H^{1}{}_{\gamma\delta}\right) - \frac{3}{2} H^{1}{}_{\alpha\beta}\, n^{\alpha} n^{\beta} n^{\gamma} n^{\delta} n^{\epsilon} n^{\zeta} \left(\nabla^{i} H^{1}{}_{\gamma\delta}\right) \left(\nabla_{\zeta} H^{1}{}_{\epsilon i}\right)$$

- Kinetic term for gravitational perturbations
- Transforming to Kol-Smolkin variables

$$2\,H^{1}{}_{\alpha}{}^{i}\,n^{\alpha}\,n^{\beta}\left(\nabla^{k}H^{1}{}_{i}{}^{j}\right)\left(\nabla_{\beta}H^{1}{}_{jk}\right)-2\,H^{1\,ij}\,n^{\alpha}\,n^{\beta}\left(\nabla^{k}H^{1}{}_{\alpha i}\right)\left(\nabla_{\beta}H^{1}{}_{jk}\right)+$$

$$H^{1\,i}{}_{i}\,n^{\alpha}\,n^{\beta}\left(\nabla^{k}H^{1}{}_{\alpha}{}^{j}\right)\left(\nabla_{\beta}H^{1}{}_{jk}\right)+H^{1\,ij}\,n^{\alpha}\,n^{\beta}\left(\nabla_{\alpha}H^{1}{}_{i}{}^{k}\right)\left(\nabla_{\beta}H^{1}{}_{jk}\right)-$$

$$\frac{1}{4}H^{1\,i}{}_{i}\,n^{\alpha}\,n^{\beta}\left(\nabla_{\alpha}H^{1\,jk}\right)\left(\nabla_{\beta}H^{1}{}_{jk}\right)-H^{1}{}_{\alpha}{}^{i}\,n^{\alpha}\,n^{\beta}\left(\nabla_{i}H^{1\,k}{}_{k}\right)\left(\nabla_{\beta}H^{1\,j}{}_{j}\right)+$$

$$H^{1}{}_{\alpha}{}^{i}\,n^{\alpha}\,n^{\beta}\left(\nabla_{i}H^{1}{}_{jk}\right)\left(\nabla_{\beta}H^{1\,jk}\right)+H^{1}{}_{\alpha}{}^{i}\,n^{\alpha}\,n^{\beta}\left(\nabla_{j}H^{1}{}_{i}{}^{j}\right)\left(\nabla_{\beta}H^{1\,k}{}_{k}\right)+$$

$$H^{1\,ij}\,n^{\alpha}\,n^{\beta}\left(\nabla_{j}H^{1}{}_{\alpha i}\right)\left(\nabla_{\beta}H^{1\,k}{}_{k}\right)-\frac{1}{2}H^{1\,i}{}_{i}\,n^{\alpha}\,n^{\beta}\left(\nabla_{j}H^{1}{}_{\alpha}{}^{j}\right)\left(\nabla_{\beta}H^{1\,k}{}_{k}\right)-$$

$$H^{1\,ij}\,n^{\alpha}\,n^{\beta}\left(\nabla_{\alpha}H^{1}{}_{ij}\right)\left(\nabla_{\beta}H^{1\,k}{}_{k}\right)+\frac{1}{4}H^{1\,i}{}_{i}\,n^{\alpha}\,n^{\beta}\left(\nabla_{\alpha}H^{1\,j}{}_{j}\right)\left(\nabla_{\beta}H^{1\,k}{}_{k}\right)+$$

$$H^{1\,ij}\,n^{\alpha}\,n^{\beta}\left(\nabla_{j}H^{1\,k}{}_{k}\right)\left(\nabla_{\beta}H^{1}{}_{\alpha i}\right)-\frac{1}{2}H^{1\,i}{}_{i}\,n^{\alpha}\,n^{\beta}\left(\nabla_{j}H^{1\,k}{}_{k}\right)\left(\nabla_{\beta}H^{1}{}_{\alpha}{}^{j}\right)+$$

$$H^{1\,ij}\,n^{\alpha}\,n^{\beta}\left(\nabla_{k}H^{1}{}_{ij}\right)\left(\nabla_{\beta}H^{1}{}_{\alpha}{}^{k}\right)-H^{1\,ij}\,n^{\alpha}\,n^{\beta}\,n^{\gamma}\,n^{\delta}\left(\nabla_{j}H^{1}{}_{\delta i}\right)\left(\nabla_{\gamma}H^{1}{}_{\alpha\beta}\right)+$$

$$\frac{1}{2}H^{1\,i}{}_{i}\,n^{\alpha}\,n^{\beta}\,n^{\gamma}\,n^{\delta}\left(\nabla_{j}H^{1}{}_{\delta}{}^{j}\right)\left(\nabla_{\gamma}H^{1}{}_{\alpha\beta}\right)-2\,H^{1}{}_{\alpha}{}^{i}\,n^{\alpha}\,n^{\beta}\,n^{\gamma}\,n^{\delta}\left(\nabla_{j}H^{1}{}_{\delta}{}^{j}\right)\left(\nabla_{\gamma}H^{1}{}_{\beta i}\right)+$$

$$H^{1}{}_{\alpha}{}^{i}\,n^{\alpha}\,n^{\beta}\,n^{\gamma}\,n^{\delta}\left(\nabla^{j}H^{1}{}_{\beta\gamma}\right)\left(\nabla_{\delta}H^{1}{}_{ij}\right)+H^{1}{}_{\alpha\beta}\,n^{\alpha}\,n^{\beta}\,n^{\gamma}\,n^{\delta}\left(\nabla^{j}H^{1}{}_{\gamma}{}^{i}\right)\left(\nabla_{\delta}H^{1}{}_{ij}\right)-$$

$$\frac{1}{4}H^{1}{}_{\alpha\beta}\,n^{\alpha}\,n^{\beta}\,n^{\gamma}\,n^{\delta}\left(\nabla_{\gamma}H^{1\,ij}\right)\left(\nabla_{\delta}H^{1}{}_{ij}\right)-\frac{1}{2}H^{1}{}_{\alpha\beta}\,n^{\alpha}\,n^{\beta}\,n^{\gamma}\,n^{\delta}\left(\nabla_{i}H^{1}{}_{\gamma}{}^{i}\right)\left(\nabla_{\delta}H^{1\,j}{}_{j}\right)+$$

$$\frac{1}{4}H^{1}{}_{\alpha\beta}\,n^{\alpha}\,n^{\beta}\,n^{\gamma}\,n^{\delta}\left(\nabla_{\gamma}H^{1\,i}{}_{i}\right)\left(\nabla_{\delta}H^{1\,j}{}_{j}\right)-H^{1}{}_{\alpha}{}^{i}\,n^{\alpha}\,n^{\beta}\,n^{\gamma}\,n^{\delta}\left(\nabla_{j}H^{1}{}_{i}{}^{j}\right)\left(\nabla_{\delta}H^{1}{}_{\beta\gamma}\right)-$$

$$\frac{1}{2}H^{1}{}_{\alpha\beta}\,n^{\alpha}\,n^{\beta}\,n^{\gamma}\,n^{\delta}\left(\nabla_{i}H^{1\,j}{}_{j}\right)\left(\nabla_{\delta}H^{1}{}_{\gamma}{}^{i}\right)+H^{1\,ij}\,n^{\alpha}\,n^{\beta}\,n^{\gamma}\,n^{\delta}\left(\nabla_{i}H^{1}{}_{\alpha\beta}\right)\left(\nabla_{\delta}H^{1}{}_{\gamma j}\right)-$$

$$\frac{1}{2}H^{1\,i}{}_{i}\,n^{\alpha}\,n^{\beta}\,n^{\gamma}\,n^{\delta}\left(\nabla^{j}H^{1}{}_{\alpha\beta}\right)\left(\nabla_{\delta}H^{1}{}_{\gamma j}\right)+2\,H^{1}{}_{\alpha}{}^{i}\,n^{\alpha}\,n^{\beta}\,n^{\gamma}\,n^{\delta}\left(\nabla^{j}H^{1}{}_{\beta i}\right)\left(\nabla_{\delta}H^{1}{}_{\gamma j}\right)+$$

$$\frac{3}{2}H^{1}{}_{\alpha\beta}\,n^{\alpha}\,n^{\beta}\,n^{\gamma}\,n^{\delta}\,n^{\epsilon}\,n^{\zeta}\left(\nabla_{i}H^{1}{}_{\zeta}{}^{i}\right)\left(\nabla_{\epsilon}H^{1}{}_{\gamma\delta}\right)-\frac{3}{2}H^{1}{}_{\alpha\beta}\,n^{\alpha}\,n^{\beta}\,n^{\gamma}\,n^{\delta}\,n^{\epsilon}\,n^{\zeta}\left(\nabla^{i}H^{1}{}_{\gamma\delta}\right)\left(\nabla_{\zeta}H^{1}{}_{\epsilon i}\right)$$

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$$\frac{3}{2} H^1{}_{\alpha\beta}\, n^\alpha n^\beta n^\gamma n^\delta n^\epsilon n^\varsigma \left(\nabla_i H^1{}_\varsigma{}^i\right)\left(\nabla_\epsilon H^1{}_{\gamma\delta}\right) - \frac{3}{2} H^1{}_{\alpha\beta}\, n^\alpha n^\beta n^\gamma n^\delta n^\epsilon n^\varsigma \left(\nabla^i H^1{}_{\gamma\delta}\right)\left(\nabla_\varsigma H^1{}_{\epsilon i}\right)$$

- Kinetic term for gravitational perturbations
- Transforming to Kol-Smolkin variables
- Power counting
- Storing interactions in data structure
- Generating interactions for computing the 1PN potential
- Automating the perturbation calculations
- Feynman rules
- Assemble the Feynman diagrams

Fully partial automation at 1PN

Final Memory

In[204]:=
```
MemoryInUse[]
```

Out[204]= 53 969 776

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$$\frac{3}{2} H^1{}_{\alpha\beta} n^\alpha n^\beta n^\gamma n^\delta n^\epsilon n^\zeta (\nabla_i H^1{}_\zeta{}^i)(\nabla_\epsilon H^1{}_{\gamma\delta}) - \frac{3}{2} H^1{}_{\alpha\beta} n^\alpha n^\beta n^\gamma n^\delta n^\epsilon n^\zeta (\nabla^i H^1{}_{\gamma\delta})(\nabla_\zeta H^1{}_{\epsilon i})$$

Kinetic term for gravitational perturbations

The gauge-fixed kinetic term of the Lagrangian for the potential field is

In[145]:=
```
LHH = Perturb[LEH, 2, 0, 0] + Perturb[LgfPot[Lorenz], 0, 0, 0] // ToCanonical
```

1.10444

Out[145]=

$$-2 H^{1\alpha\beta}(\nabla_\beta \nabla_\alpha H^{1\gamma}{}_\gamma) + 2 H^{1\alpha\beta}(\nabla_\beta \nabla_\gamma H^1{}_\alpha{}^\gamma) + \frac{3}{4}(\nabla_\beta H^{1\gamma}{}_\gamma)(\nabla^\beta H^{1\alpha}{}_\alpha) +$$
$$3(\nabla_\alpha H^{1\alpha\beta})(\nabla_\gamma H^1{}_\beta{}^\gamma) - 3(\nabla^\beta H^{1\alpha}{}_\alpha)(\nabla_\gamma H^1{}_\beta{}^\gamma) + 2 H^{1\alpha\beta}(\nabla_\gamma \nabla_\beta H^1{}_\alpha{}^\gamma) - H^{1\alpha}{}_\alpha(\nabla_\gamma \nabla_\beta H^{1\beta\gamma}) -$$
$$2 H^{1\alpha\beta}(\nabla_\gamma \nabla^\gamma H^1{}_{\alpha\beta}) + H^{1\alpha}{}_\alpha(\nabla_\gamma \nabla^\gamma H^{1\beta}{}_\beta) + (\nabla_\beta H^1{}_{\alpha\gamma})(\nabla^\gamma H^{1\alpha\beta}) - \frac{3}{2}(\nabla_\gamma H^1{}_{\alpha\beta})(\nabla^\gamma H^{1\alpha\beta})$$

Integrate L2H by parts on only those terms that have two derivatives on the potential field.

In[146]:=
```
LHH =
 MapAt[IntByParts[#, GravitonH] &, LHH,
  Position[LHH, expr_ $CovD[a_, b_]@GravitonH[__]]] // ToCanonical
```

Out[146]=

$$-\frac{1}{4}(\nabla_\beta H^{1\gamma}{}_\gamma)(\nabla^\beta H^{1\alpha}{}_\alpha) + (\nabla_\alpha H^{1\alpha\beta})(\nabla_\gamma H^1{}_\beta{}^\gamma) - (\nabla_\beta H^1{}_{\alpha\gamma})(\nabla^\gamma H^{1\alpha\beta}) + \frac{1}{2}(\nabla_\gamma H^1{}_{\alpha\beta})(\nabla^\gamma H^{1\alpha\beta})$$

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$$\frac{3}{2} H^1{}_{\alpha\beta} n^\alpha n^\beta n^\gamma n^\delta n^\epsilon n^\zeta (\nabla_i H^1{}_\zeta{}^i)(\nabla_\epsilon H^1{}_{\gamma\delta}) - \frac{3}{2} H^1{}_{\alpha\beta} n^\alpha n^\beta n^\gamma n^\delta n^\epsilon n^\zeta (\nabla^i H^1{}_{\gamma\delta})(\nabla_\zeta H^1{}_{\epsilon i})$$

Kinetic term for gravitational perturbations

The gauge-fixed kinetic term of the Lagrangian for the potential field is

In[141]:=
```
LHH = Perturb[LEH, 2, 0, 0] + Perturb[LgfPot[Lorenz], 0, 0, 0] // ToCanonical
```

1.10444

Out[145]=

$$-2 H^{1\alpha\beta}(\nabla_\beta\nabla_\alpha H^{1\gamma}{}_\gamma) + 2H^{1\alpha\beta}(\nabla_\beta\nabla_\gamma H^1{}_\alpha{}^\gamma) + \frac{3}{4}(\nabla_\beta H^{1\gamma}{}_\gamma)(\nabla^\beta H^{1\alpha}{}_\alpha) + 3(\nabla_\alpha H^{1\alpha\beta})(\nabla_\gamma H^1{}_\beta{}^\gamma) - 3(\nabla^\beta H^{1\alpha}{}_\alpha)(\nabla_\gamma H^1{}_\beta{}^\gamma) + 2H^{1\alpha\beta}(\nabla_\gamma\nabla_\beta H^1{}_\alpha{}^\gamma) - H^{1\alpha}{}_\alpha(\nabla_\gamma\nabla_\beta H^{1\beta\gamma}) - 2H^{1\alpha\beta}(\nabla_\gamma\nabla^\gamma H^1{}_{\alpha\beta}) + H^{1\alpha}{}_\alpha(\nabla_\gamma\nabla^\gamma H^{1\beta}{}_\beta) + (\nabla_\beta H^1{}_{\alpha\gamma})(\nabla^\gamma H^{1\alpha\beta}) - \frac{3}{2}(\nabla_\gamma H^1{}_{\alpha\beta})(\nabla^\gamma H^{1\alpha\beta})$$

Integrate L2H by parts on only those terms that have two derivatives on the potential field.

In[146]:=
```
LHH =
 MapAt[IntByParts[#, GravitonH] &, LHH,
   Position[LHH, expr_ $CovD[a_, b_]@GravitonH[__]]] // ToCanonical
```

Out[146]=

$$-\frac{1}{4}(\nabla_\beta H^{1\gamma}{}_\gamma)(\nabla^\beta H^{1\alpha}{}_\alpha) + (\nabla_\alpha H^{1\alpha\beta})(\nabla_\gamma H^1{}_\beta{}^\gamma) - (\nabla_\beta H^1{}_{\alpha\gamma})(\nabla^\gamma H^{1\alpha\beta}) + \frac{1}{2}(\nabla_\gamma H^1{}_{\alpha\beta})(\nabla^\gamma H^{1\alpha\beta})$$

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$$\frac{3}{2} H^1{}_{\alpha\beta} n^\alpha n^\beta n^\gamma n^\delta n^\epsilon n^\zeta \left(\nabla_i H^1{}_\zeta{}^i\right) \left(\nabla_\epsilon H^1{}_{\gamma\delta}\right) - \frac{3}{2} H^1{}_{\alpha\beta} n^\alpha n^\beta n^\gamma n^\delta n^\epsilon n^\zeta \left(\nabla^i H^1{}_{\gamma\delta}\right) \left(\nabla_\zeta H^1{}_{\epsilon i}\right)$$

- Kinetic term for gravitational perturbations
- Transforming to Kol-Smolkin variables
- Power counting
- Storing interactions in data structure
- Generating interactions for computing the 1PN potential
- Automating the perturbation calculations
- Feynman rules
- Assemble the Feynman diagrams

Fully partial automation at 1PN

Final Memory

In[204]:=

```
MemoryInUse[]
```

Out[204]=

```
53 969 776
```

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- Kinetic term for gravitational perturbations
- Transforming to Kol-Smolkin variables

In[151]:=
```
UnitNormal[α] UnitNormal[β] GravitonH[LI[2], -α, -β] // ToKolSmolkin
ThreeVel[i] UnitNormal[α] GravitonH[LI[3], -α, -i] // ToKolSmolkin
ThreeVel[i] ThreeVel[j] GravitonH[LI[2], -i, -j] // ToKolSmolkin
```

Out[151]= $4\,\phi^{1\,2}$

Out[152]= $-12\,A^{1\,i}\,\phi^{1\,2}\,v_i$

Out[153]= $-4\,\phi^{1\,2}\,v_i\,v^i + 2\,A^{1\,i}\,A^{1\,j}\,v_i\,v_j + 4\,\phi^1\,\sigma^1{}_{ij}\,v^i\,v^j$

In[154]:=
```
LHHinKS = LHHto3plus1 // ToKolSmolkin
```

0.358574

Out[154]= $-4\left(\nabla_i\phi^1\right)\left(\nabla^i\phi^1\right) + \left(\nabla_j A^1{}_i\right)\left(\nabla^j A^{1\,i}\right) + \frac{1}{4}\left(\nabla_j\sigma^1{}^k{}_k\right)\left(\nabla^j\sigma^1{}^i{}_i\right) - \frac{1}{2}\left(\nabla_k\sigma^1{}_{ij}\right)\left(\nabla^k\sigma^{1\,ij}\right) - n^\alpha n^\beta\left(\nabla_\alpha A^{1\,i}\right)\left(\nabla_\beta A^1{}_i\right) + 4\,n^\alpha n^\beta\left(\nabla_\alpha\phi^1\right)\left(\nabla_\beta\phi^1\right) + \frac{1}{2}\,n^\alpha n^\beta\left(\nabla_\alpha\sigma^{1\,ij}\right)\left(\nabla_\beta\sigma^1{}_{ij}\right) - \frac{1}{4}\,n^\alpha n^\beta\left(\nabla_\alpha\sigma^1{}^i{}_i\right)\left(\nabla_\beta\sigma^1{}^j{}_j\right)$

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- Kinetic term for gravitational perturbations
- Transforming to Kol-Smolkin variables

In[147]:=
```
UnitNormal[α] UnitNormal[β] GravitonH[LI[2], -α, -β] // ToKolSmolkin
ThreeVel[i] UnitNormal[α] GravitonH[LI[3], -α, -i] // ToKolSmolkin
ThreeVel[i] ThreeVel[j] GravitonH[LI[2], -i, -j] // ToKolSmolkin
```

Out[147]= $4\,\phi^{1\,2}$

Out[148]= $-12\,A^{1\,i}\,\phi^{1\,2}\,v_i$

Out[149]= $-4\,\phi^{1\,2}\,v_i\,v^i + 2\,A^{1\,i}\,A^{1\,j}\,v_i\,v_j + 4\,\phi^1\,\sigma^1{}_{ij}\,v^i\,v^j$

In[154]:=
```
LHHinKS = LHHto3plus1 // ToKolSmolkin
```

0.358574

Out[154]= $-4\left(\nabla_i\phi^1\right)\left(\nabla^i\phi^1\right) + \left(\nabla_j A^1{}_i\right)\left(\nabla^j A^{1\,i}\right) + \frac{1}{4}\left(\nabla_j\sigma^{1\,k}{}_k\right)\left(\nabla^j\sigma^{1\,i}{}_i\right) -$
$\frac{1}{2}\left(\nabla_k\sigma^1{}_{ij}\right)\left(\nabla^k\sigma^{1\,ij}\right) - n^\alpha n^\beta\left(\nabla_\alpha A^{1\,i}\right)\left(\nabla_\beta A^1{}_i\right) + 4\,n^\alpha n^\beta\left(\nabla_\alpha\phi^1\right)\left(\nabla_\beta\phi^1\right) +$
$\frac{1}{2}\,n^\alpha n^\beta\left(\nabla_\alpha\sigma^{1\,ij}\right)\left(\nabla_\beta\sigma^1{}_{ij}\right) - \frac{1}{4}\,n^\alpha n^\beta\left(\nabla_\alpha\sigma^{1\,i}{}_i\right)\left(\nabla_\beta\sigma^{1\,j}{}_j\right)$

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Transforming to Kol-Smolkin variables

In[147]:=
```
UnitNormal[α] UnitNormal[β] GravitonH[LI[2], -α, -β] // ToKolSmolkin
ThreeVel[i] UnitNormal[α] GravitonH[LI[3], -α, -i] // ToKolSmolkin
ThreeVel[i] ThreeVel[j] GravitonH[LI[2], -i, -j] // ToKolSmolkin
```

Out[147]= $4\,\phi^{1\,2}$

Out[148]= $-12\,A^{1\,i}\,\phi^{1\,2}\,v_i$

Out[149]= $-4\,\phi^{1\,2}\,v_i\,v^i + 2\,A^{1\,i}\,A^{1\,j}\,v_i\,v_j + 4\,\phi^1\,\sigma^1{}_{ij}\,v^i\,v^j$

In[150]:=
```
LHHinKS = LHHto3plus1 // ToKolSmolkin
```

0.361854

Out[150]= $-4\left(\nabla_i\,\phi^1\right)\left(\nabla^i\,\phi^1\right) + \left(\nabla_j\,A^1{}_i\right)\left(\nabla^j\,A^{1\,i}\right) + \frac{1}{4}\left(\nabla_j\,\sigma^{1\,k}{}_k\right)\left(\nabla^j\,\sigma^{1\,i}{}_i\right) - \frac{1}{2}\left(\nabla_k\,\sigma^1{}_{ij}\right)\left(\nabla^k\,\sigma^{1\,ij}\right) - n^\alpha\,n^\beta\left(\nabla_\alpha\,A^{1\,i}\right)\left(\nabla_\beta\,A^1{}_i\right) + 4\,n^\alpha\,n^\beta\left(\nabla_\alpha\,\phi^1\right)\left(\nabla_\beta\,\phi^1\right) + \frac{1}{2}\,n^\alpha\,n^\beta\left(\nabla_\alpha\,\sigma^{1\,ij}\right)\left(\nabla_\beta\,\sigma^1{}_{ij}\right) - \frac{1}{4}\,n^\alpha\,n^\beta\left(\nabla_\alpha\,\sigma^{1\,i}{}_i\right)\left(\nabla_\beta\,\sigma^{1\,j}{}_j\right)$

In[155]:=
```
LHHHinKS = LHHHto3plus1 // ToKolSmolkin
```

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Out[149]=

$$-4\,\phi^{1\,2}\,v_i\,v^i + 2\,A^{1\,i}\,A^{1\,j}\,v_i\,v_j + 4\,\phi^1\,\sigma^1{}_{ij}\,v^i\,v^j$$

In[150]:=

```
LHHinKS = LHHto3plus1 // ToKolSmolkin
```

0.361854

Out[150]=

$$-4\,(\nabla_i\phi^1)\,(\nabla^i\phi^1) + (\nabla_j A^1{}_i)\,(\nabla^j A^{1\,i}) + \frac{1}{4}\,(\nabla_j\sigma^{1\,k}{}_k)\,(\nabla^j\sigma^{1\,i}{}_i) - \frac{1}{2}\,(\nabla_k\sigma^1{}_{ij})\,(\nabla^k\sigma^{1\,ij}) - n^\alpha n^\beta\,(\nabla_\alpha A^{1\,i})\,(\nabla_\beta A^1{}_i) + 4\,n^\alpha n^\beta\,(\nabla_\alpha\phi^1)\,(\nabla_\beta\phi^1) + \frac{1}{2}\,n^\alpha n^\beta\,(\nabla_\alpha\sigma^{1\,ij})\,(\nabla_\beta\sigma^1{}_{ij}) - \frac{1}{4}\,n^\alpha n^\beta\,(\nabla_\alpha\sigma^{1\,i}{}_i)\,(\nabla_\beta\sigma^{1\,j}{}_j)$$

In[155]:=

```
LHHHinKS = LHHHto3plus1 // ToKolSmolkin
```

4.54379

Out[155]=

$$16\,\phi^1\,(\nabla_i\phi^1)\,(\nabla^i\phi^1) - 2\,\sigma^{1\,j}{}_j\,(\nabla_i\phi^1)\,(\nabla^i\phi^1) - 2\,\sigma^{1\,jk}\,(\nabla_i\sigma^1{}_{jk})\,(\nabla^i\phi^1) - 8\,\phi^1\,(\nabla_i\sigma^{1\,j}{}_j)\,(\nabla^i\phi^1) + 2\,\sigma^{1\,j}{}_j\,(\nabla_i\sigma^{1\,k}{}_k)\,(\nabla^i\phi^1) - A^{1\,i}\,(\nabla_i\sigma^{1\,k}{}_k)\,(\nabla_j A^{1\,j}) + 4\,A^{1\,i}\,(\nabla_i A^{1\,j})\,(\nabla_j\phi^1) + 8\,\phi^1\,(\nabla^i\phi^1)\,(\nabla_j\sigma^{1\,j}{}_i) + \frac{1}{2}\,\sigma^{1\,ij}\,(\nabla_i\sigma^{1\,kl})\,(\nabla_j\sigma^1{}_{kl}) - A^{1\,i}\,(\nabla_i A^{1\,j})\,(\nabla_j\sigma^{1\,k}{}_k) - 2\,\sigma^{1\,j}{}_i\,(\nabla^i\phi^1)\,(\nabla_j\sigma^{1\,k}{}_k) - \frac{1}{2}\,\sigma^{1\,ij}\,(\nabla_i\sigma^{1\,k}{}_k)\,(\nabla_j\sigma^{1\,l}{}_l) - 4\,A^{1\,i}\,(\nabla_j\phi^1)\,(\nabla^j A^1{}_i) + 2\,A^{1\,i}\,(\nabla_j\sigma^{1\,k}{}_k)\,(\nabla^j A^1{}_i) - \frac{1}{2}\,\sigma^{1\,k}{}_k\,(\nabla_i A^1{}_j)\,(\nabla^j A^{1\,i}) + \ldots$$

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In[150]:= `LHHinKS = LHHto3plus1 // ToKolSmolkin`

0.361854

Out[150]=

$$-4\left(\nabla_i \phi^1\right)\left(\nabla^i \phi^1\right)+\left(\nabla_j A^1{}_i\right)\left(\nabla^j A^{1i}\right)+\frac{1}{4}\left(\nabla_j \sigma^{1k}{}_k\right)\left(\nabla^j \sigma^{1i}{}_i\right)-\frac{1}{2}\left(\nabla_k \sigma^1{}_{ij}\right)\left(\nabla^k \sigma^{1ij}\right)-n^\alpha n^\beta\left(\nabla_\alpha A^{1i}\right)\left(\nabla_\beta A^1{}_i\right)+4 n^\alpha n^\beta\left(\nabla_\alpha \phi^1\right)\left(\nabla_\beta \phi^1\right)+\frac{1}{2} n^\alpha n^\beta\left(\nabla_\alpha \sigma^{1ij}\right)\left(\nabla_\beta \sigma^1{}_{ij}\right)-\frac{1}{4} n^\alpha n^\beta\left(\nabla_\alpha \sigma^{1i}{}_i\right)\left(\nabla_\beta \sigma^{1j}{}_j\right)$$

In[151]:= `LHHHinKS = LHHHto3plus1 // ToKolSmolkin`

4.54379

Out[155]=

$$16\phi^1\left(\nabla_i \phi^1\right)\left(\nabla^i \phi^1\right)-2\sigma^{1j}{}_j\left(\nabla_i \phi^1\right)\left(\nabla^i \phi^1\right)-2\sigma^{1jk}\left(\nabla_i \sigma^1{}_{jk}\right)\left(\nabla^i \phi^1\right)-$$
$$8\phi^1\left(\nabla_i \sigma^{1j}{}_j\right)\left(\nabla^i \phi^1\right)+2\sigma^{1j}{}_j\left(\nabla_i \sigma^{1k}{}_k\right)\left(\nabla^i \phi^1\right)-A^{1i}\left(\nabla_i \sigma^{1k}{}_k\right)\left(\nabla_j A^{1j}\right)+$$
$$4A^{1i}\left(\nabla_i A^{1j}\right)\left(\nabla_j \phi^1\right)+8\phi^1\left(\nabla^i \phi^1\right)\left(\nabla_j \sigma^1{}_i{}^j\right)+\frac{1}{2}\sigma^{1ij}\left(\nabla_i \sigma^{1kl}\right)\left(\nabla_j \sigma^1{}_{kl}\right)-$$
$$A^{1i}\left(\nabla_i A^{1j}\right)\left(\nabla_j \sigma^{1k}{}_k\right)-2\sigma^1{}_i{}^j\left(\nabla^i \phi^1\right)\left(\nabla_j \sigma^{1k}{}_k\right)-\frac{1}{2}\sigma^{1ij}\left(\nabla_i \sigma^{1k}{}_k\right)\left(\nabla_j \sigma^{1l}{}_l\right)-$$
$$4A^{1i}\left(\nabla_j \phi^1\right)\left(\nabla^j A^1{}_i\right)+2A^{1i}\left(\nabla_j \sigma^{1k}{}_k\right)\left(\nabla^j A^1{}_i\right)-\frac{1}{2}\sigma^{1k}{}_k\left(\nabla_i A^1{}_j\right)\left(\nabla^j A^{1i}\right)+$$
$$\frac{1}{2}\sigma^{1k}{}_k\left(\nabla_j A^1{}_i\right)\left(\nabla^j A^{1i}\right)-\sigma^1{}_{ik}\left(\nabla_j A^{1k}\right)\left(\nabla^j A^{1i}\right)+4\sigma^1{}_{ij}\left(\nabla^i \phi^1\right)\left(\nabla^j \phi^1\right)+$$

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$$
\begin{aligned}
&\frac{1}{4}\sigma^{1\,i}{}_{i}\left(\nabla_k \sigma^{1\,l}{}_{l}\right)\left(\nabla^k \sigma^{1\,j}{}_{j}\right)+\sigma^{1\,ij}\left(\nabla^k \sigma^{1}{}_{ij}\right)\left(\nabla_l \sigma^{1}{}_{k}{}^{l}\right)-\frac{1}{2}\sigma^{1\,i}{}_{i}\left(\nabla^k \sigma^{1\,j}{}_{j}\right)\left(\nabla_l \sigma^{1}{}_{k}{}^{l}\right)-\\
&2\sigma^{1\,ij}\left(\nabla_j \sigma^{1}{}_{kl}\right)\left(\nabla^l \sigma^{1}{}_{i}{}^{k}\right)-\sigma^{1\,ij}\left(\nabla_k \sigma^{1}{}_{jl}\right)\left(\nabla^l \sigma^{1}{}_{i}{}^{k}\right)+\sigma^{1\,ij}\left(\nabla_l \sigma^{1}{}_{jk}\right)\left(\nabla^l \sigma^{1}{}_{i}{}^{k}\right)+\\
&\frac{1}{2}\sigma^{1\,i}{}_{i}\left(\nabla_k \sigma^{1}{}_{jl}\right)\left(\nabla^l \sigma^{1\,jk}\right)-\frac{1}{4}\sigma^{1\,i}{}_{i}\left(\nabla_l \sigma^{1}{}_{jk}\right)\left(\nabla^l \sigma^{1\,jk}\right)+2A^{1\,i}n^{\alpha}\left(\nabla_j A^{1\,j}\right)\left(\nabla_\alpha A^{1}{}_{i}\right)+\\
&8\phi^1 n^{\alpha}\left(\nabla_i \phi^1\right)\left(\nabla_\alpha A^{1\,i}\right)-2\sigma^{1\,j}{}_{j}n^{\alpha}\left(\nabla_i \phi^1\right)\left(\nabla_\alpha A^{1\,i}\right)-\sigma^{1\,jk}n^{\alpha}\left(\nabla_i \sigma^{1}{}_{jk}\right)\left(\nabla_\alpha A^{1\,i}\right)+\\
&\frac{1}{2}\sigma^{1\,j}{}_{j}n^{\alpha}\left(\nabla_i \sigma^{1\,k}{}_{k}\right)\left(\nabla_\alpha A^{1\,i}\right)-\sigma^{1}{}_{i}{}^{j}n^{\alpha}\left(\nabla_j \sigma^{1\,k}{}_{k}\right)\left(\nabla_\alpha A^{1\,i}\right)+8\sigma^{1}{}_{ij}n^{\alpha}\left(\nabla^j \phi^1\right)\left(\nabla_\alpha A^{1\,i}\right)-\\
&2A^{1\,i}n^{\alpha}\left(\nabla_j A^{1}{}_{i}\right)\left(\nabla_\alpha A^{1\,j}\right)-8\phi^1 n^{\alpha}\left(\nabla_i A^{1\,i}\right)\left(\nabla_\alpha \phi^1\right)+2\sigma^{1\,j}{}_{j}n^{\alpha}\left(\nabla_i A^{1\,i}\right)\left(\nabla_\alpha \phi^1\right)+\\
&8A^{1\,i}n^{\alpha}\left(\nabla_i \phi^1\right)\left(\nabla_\alpha \phi^1\right)-2A^{1\,i}n^{\alpha}\left(\nabla_i \sigma^{1\,j}{}_{j}\right)\left(\nabla_\alpha \phi^1\right)-4\sigma^{1}{}_{ij}n^{\alpha}\left(\nabla^j A^{1\,i}\right)\left(\nabla_\alpha \phi^1\right)-\\
&\sigma^{1\,k}{}_{k}n^{\alpha}\left(\nabla^j A^{1\,i}\right)\left(\nabla_\alpha \sigma^{1}{}_{ij}\right)+4A^{1\,i}n^{\alpha}\left(\nabla^j \phi^1\right)\left(\nabla_\alpha \sigma^{1}{}_{ij}\right)-A^{1\,i}n^{\alpha}\left(\nabla_j \sigma^{1\,k}{}_{k}\right)\left(\nabla_\alpha \sigma^{1}{}_{i}{}^{j}\right)+\\
&2\sigma^{1}{}_{j}{}^{k}n^{\alpha}\left(\nabla^j A^{1\,i}\right)\left(\nabla_\alpha \sigma^{1}{}_{ik}\right)-\sigma^{1\,jk}n^{\alpha}\left(\nabla_i A^{1\,i}\right)\left(\nabla_\alpha \sigma^{1}{}_{jk}\right)+2\sigma^{1}{}_{i}{}^{k}n^{\alpha}\left(\nabla^j A^{1\,i}\right)\left(\nabla_\alpha \sigma^{1}{}_{jk}\right)-\\
&2A^{1\,i}n^{\alpha}\left(\nabla_i \phi^1\right)\left(\nabla_\alpha \sigma^{1\,j}{}_{j}\right)+A^{1\,i}n^{\alpha}\left(\nabla_i \sigma^{1\,k}{}_{k}\right)\left(\nabla_\alpha \sigma^{1\,j}{}_{j}\right)-A^{1\,i}n^{\alpha}\left(\nabla_k \sigma^{1}{}_{i}{}^{k}\right)\left(\nabla_\alpha \sigma^{1\,j}{}_{j}\right)-\\
&A^{1\,i}n^{\alpha}\left(\nabla_i \sigma^{1}{}_{jk}\right)\left(\nabla_\alpha \sigma^{1\,jk}\right)+2A^{1\,i}n^{\alpha}\left(\nabla_k \sigma^{1}{}_{ij}\right)\left(\nabla_\alpha \sigma^{1\,jk}\right)+\frac{1}{2}\sigma^{1\,j}{}_{j}n^{\alpha}\left(\nabla_i A^{1\,i}\right)\left(\nabla_\alpha \sigma^{1\,k}{}_{k}\right)-\\
&\sigma^{1}{}_{ij}n^{\alpha}\left(\nabla^j A^{1\,i}\right)\left(\nabla_\alpha \sigma^{1\,k}{}_{k}\right)+2\sigma^{1\,i}{}_{i}n^{\alpha}n^{\beta}\left(\nabla_\alpha \phi^1\right)\left(\nabla_\beta \phi^1\right)-2\sigma^{1\,ij}n^{\alpha}n^{\beta}\left(\nabla_\alpha \phi^1\right)\left(\nabla_\beta \sigma^{1}{}_{ij}\right)-\\
&\sigma^{1\,ij}n^{\alpha}n^{\beta}\left(\nabla_\alpha \sigma^{1}{}_{i}{}^{k}\right)\left(\nabla_\beta \sigma^{1}{}_{jk}\right)+\frac{1}{4}\sigma^{1\,i}{}_{i}n^{\alpha}n^{\beta}\left(\nabla_\alpha \sigma^{1\,jk}\right)\left(\nabla_\beta \sigma^{1}{}_{jk}\right)+\\
&\sigma^{1\,ij}n^{\alpha}n^{\beta}\left(\nabla_\alpha \sigma^{1}{}_{ij}\right)\left(\nabla_\beta \sigma^{1\,k}{}_{k}\right)-\frac{1}{4}\sigma^{1\,i}{}_{i}n^{\alpha}n^{\beta}\left(\nabla_\alpha \sigma^{1\,j}{}_{j}\right)\left(\nabla_\beta \sigma^{1\,k}{}_{k}\right)
\end{aligned}
$$

Power counting

- Storing interactions in data structure

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$$2\,\sigma^{1}{}_{j}{}^{k}\,n^{\alpha}\,(\nabla^{j} A^{1i})\,(\nabla_{\alpha}\sigma^{1}{}_{ik}) - \sigma^{1jk}\,n^{\alpha}\,(\nabla_{i} A^{1i})\,(\nabla_{\alpha}\sigma^{1}{}_{jk}) + 2\,\sigma^{1}{}_{i}{}^{k}\,n^{\alpha}\,(\nabla^{j} A^{1i})\,(\nabla_{\alpha}\sigma^{1}{}_{jk}) -$$
$$2\,A^{1i}\,n^{\alpha}\,(\nabla_{i}\phi^{1})\,(\nabla_{\alpha}\sigma^{1j}{}_{j}) + A^{1i}\,n^{\alpha}\,(\nabla_{i}\sigma^{1k}{}_{k})\,(\nabla_{\alpha}\sigma^{1j}{}_{j}) - A^{1i}\,n^{\alpha}\,(\nabla_{k}\sigma^{1}{}_{i}{}^{k})\,(\nabla_{\alpha}\sigma^{1j}{}_{j}) -$$
$$A^{1i}\,n^{\alpha}\,(\nabla_{i}\sigma^{1}{}_{jk})\,(\nabla_{\alpha}\sigma^{1jk}) + 2\,A^{1i}\,n^{\alpha}\,(\nabla_{k}\sigma^{1}{}_{ij})\,(\nabla_{\alpha}\sigma^{1jk}) + \frac{1}{2}\,\sigma^{1j}{}_{j}\,n^{\alpha}\,(\nabla_{i} A^{1i})\,(\nabla_{\alpha}\sigma^{1k}{}_{k}) -$$
$$\sigma^{1}{}_{ij}\,n^{\alpha}\,(\nabla^{j} A^{1i})\,(\nabla_{\alpha}\sigma^{1k}{}_{k}) + 2\,\sigma^{1i}{}_{i}\,n^{\alpha}\,n^{\beta}\,(\nabla_{\alpha}\phi^{1})\,(\nabla_{\beta}\phi^{1}) - 2\,\sigma^{1ij}\,n^{\alpha}\,n^{\beta}\,(\nabla_{\alpha}\phi^{1})\,(\nabla_{\beta}\sigma^{1}{}_{ij}) -$$
$$\sigma^{1ij}\,n^{\alpha}\,n^{\beta}\,(\nabla_{\alpha}\sigma^{1}{}_{i}{}^{k})\,(\nabla_{\beta}\sigma^{1}{}_{jk}) + \frac{1}{4}\,\sigma^{1i}{}_{i}\,n^{\alpha}\,n^{\beta}\,(\nabla_{\alpha}\sigma^{1jk})\,(\nabla_{\beta}\sigma^{1}{}_{jk}) +$$
$$\sigma^{1ij}\,n^{\alpha}\,n^{\beta}\,(\nabla_{\alpha}\sigma^{1}{}_{ij})\,(\nabla_{\beta}\sigma^{1k}{}_{k}) - \frac{1}{4}\,\sigma^{1i}{}_{i}\,n^{\alpha}\,n^{\beta}\,(\nabla_{\alpha}\sigma^{1j}{}_{j})\,(\nabla_{\beta}\sigma^{1k}{}_{k})$$

- Power counting
- Storing interactions in data structure
- Generating interactions for computing the 1PN potential
- Automating the perturbation calculations
- Feynman rules
- Assemble the Feynman diagrams

Fully partial automation at 1PN

Final Memory

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- **Automating the perturbation calculations**
- **Feynman rules**
- **Assemble the Feynman diagrams**

In[191]:=

```
data1PN2 // MatrixForm
```

Out[191]//MatrixForm=

G	{0, 0, 3, 0, 0}	$\{2, -\frac{1}{2}\}$	$\frac{1}{2}\sigma^{1\,ij}(\nabla_i\sigma^{1\,kl})(\nabla_j\sigma^1{}_{kl}) - \frac{1}{4}\sigma^{1\,ij}(\nabla_i\sigma^{1\,k}{}_k)(\nabla_j\sigma^{1\,l}{}_l) - \frac{1}{2}\sigma$
G	{0, 2, 1, 0, 0}	$\{2, -\frac{1}{2}\}$	$\frac{1}{2}\sigma^{1\,k}{}_k(\nabla_i A^{1\,i})(\nabla_j A^{1\,j}) - \frac{1}{2}\sigma^{1\,k}{}_k(\nabla_i A^1{}_j)(\nabla^j A^{1\,i}) + \frac{1}{2}\sigma^{1\,k}{}_k$
G	{1, 2, 0, 0, 0}	$\{2, -\frac{1}{2}\}$	$4\phi^1(\nabla_i A^{1\,i})(\nabla_j A^{1\,j}) - 4\phi^1(\nabla_i A^1{}_j)(\nabla^j A^{1\,i}) + 4\phi^1(\nabla_j A^1{}_i)$
G	{2, 0, 1, 0, 0}	$\{2, -\frac{1}{2}\}$	$-2\sigma^{1\,j}{}_j(\nabla_i\phi^1)(\nabla^i\phi^1) + 4\sigma^1{}_{ij}(\nabla^i\phi^1)(\nabla^j\phi^1)$
P1	{0, 0, 0, 0, 4}	{2, 1}	$\frac{1}{8}m_1 v_i v^i v_j v^j$
P1	{0, 0, 1, 0, 2}	$\{2, \frac{1}{2}\}$	$\frac{1}{2}m_1\sigma^1{}_{ij}v^i v^j$
P1	{0, 1, 0, 0, 1}	$\{1, \frac{1}{2}\}$	$m_1 A^{1\,i} v_i$
P1	{1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$	$-m_1\phi^1$
P1	{1, 0, 0, 0, 2}	$\{2, \frac{1}{2}\}$	$-\frac{3}{2}m_1\phi^1 v_i v^i$
P1	{2, 0, 0, 0, 0}	{2, 0}	$-\frac{1}{2}m_1\phi^{1\,2}$
P2	{0, 0, 0, 0, 4}	{2, 1}	$\frac{1}{8}m_2 v_i v^i v_j v^j$
P2	{0, 0, 1, 0, 2}	$\{2, \frac{1}{2}\}$	$\frac{1}{2}m_2\sigma^1{}_{ij}v^i v^j$
P2	{0, 1, 0, 0, 1}	$\{1, \frac{1}{2}\}$	$m_2 A^{1\,i} v_i$
P2	{1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$	$-m_2\phi^1$

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Demo_CapraNRDA_2010.nb

$$
\begin{aligned}
&8\,\phi^{1}\,n^{\alpha}\,(\nabla_{i}\phi^{1})\,(\nabla_{\alpha}A^{1\,i}) - 2\,\sigma^{1\,j}{}_{j}\,n^{\alpha}\,(\nabla_{i}\phi^{1})\,(\nabla_{\alpha}A^{1\,i}) - \sigma^{1\,jk}\,n^{\alpha}\,(\nabla_{i}\sigma^{1}{}_{jk})\,(\nabla_{\alpha}A^{1\,i}) + \\
&\frac{1}{2}\,\sigma^{1\,j}{}_{j}\,n^{\alpha}\,(\nabla_{i}\sigma^{1\,k}{}_{k})\,(\nabla_{\alpha}A^{1\,i}) - \sigma^{1}{}_{i}{}^{j}\,n^{\alpha}\,(\nabla_{j}\sigma^{1\,k}{}_{k})\,(\nabla_{\alpha}A^{1\,i}) + 8\,\sigma^{1}{}_{ij}\,n^{\alpha}\,(\nabla^{j}\phi^{1})\,(\nabla_{\alpha}A^{1\,i}) - \\
&2\,A^{1\,i}\,n^{\alpha}\,(\nabla_{j}A^{1}{}_{i})\,(\nabla_{\alpha}A^{1\,j}) - 8\,\phi^{1}\,n^{\alpha}\,(\nabla_{i}A^{1\,i})\,(\nabla_{\alpha}\phi^{1}) + 2\,\sigma^{1\,j}{}_{j}\,n^{\alpha}\,(\nabla_{i}A^{1\,i})\,(\nabla_{\alpha}\phi^{1}) + \\
&8\,A^{1\,i}\,n^{\alpha}\,(\nabla_{i}\phi^{1})\,(\nabla_{\alpha}\phi^{1}) - 2\,A^{1\,i}\,n^{\alpha}\,(\nabla_{i}\sigma^{1\,j}{}_{j})\,(\nabla_{\alpha}\phi^{1}) - 4\,\sigma^{1}{}_{ij}\,n^{\alpha}\,(\nabla^{j}A^{1\,i})\,(\nabla_{\alpha}\phi^{1}) - \\
&\sigma^{1\,k}{}_{k}\,n^{\alpha}\,(\nabla^{j}A^{1\,i})\,(\nabla_{\alpha}\sigma^{1}{}_{ij}) + 4\,A^{1\,i}\,n^{\alpha}\,(\nabla^{j}\phi^{1})\,(\nabla_{\alpha}\sigma^{1}{}_{ij}) - A^{1\,i}\,n^{\alpha}\,(\nabla_{j}\sigma^{1\,k}{}_{k})\,(\nabla_{\alpha}\sigma^{1}{}_{i}{}^{j}) + \\
&2\,\sigma^{1}{}_{j}{}^{k}\,n^{\alpha}\,(\nabla^{j}A^{1\,i})\,(\nabla_{\alpha}\sigma^{1}{}_{ik}) - \sigma^{1\,jk}\,n^{\alpha}\,(\nabla_{i}A^{1\,i})\,(\nabla_{\alpha}\sigma^{1}{}_{jk}) + 2\,\sigma^{1}{}_{i}{}^{k}\,n^{\alpha}\,(\nabla^{j}A^{1\,i})\,(\nabla_{\alpha}\sigma^{1}{}_{jk}) - \\
&2\,A^{1\,i}\,n^{\alpha}\,(\nabla_{i}\phi^{1})\,(\nabla_{\alpha}\sigma^{1\,j}{}_{j}) + A^{1\,i}\,n^{\alpha}\,(\nabla_{i}\sigma^{1\,k}{}_{k})\,(\nabla_{\alpha}\sigma^{1\,j}{}_{j}) - A^{1\,i}\,n^{\alpha}\,(\nabla_{k}\sigma^{1}{}_{i}{}^{k})\,(\nabla_{\alpha}\sigma^{1\,j}{}_{j}) - \\
&A^{1\,i}\,n^{\alpha}\,(\nabla_{i}\sigma^{1}{}_{jk})\,(\nabla_{\alpha}\sigma^{1\,jk}) + 2\,A^{1\,i}\,n^{\alpha}\,(\nabla_{k}\sigma^{1}{}_{ij})\,(\nabla_{\alpha}\sigma^{1\,jk}) + \frac{1}{2}\,\sigma^{1\,j}{}_{j}\,n^{\alpha}\,(\nabla_{i}A^{1\,i})\,(\nabla_{\alpha}\sigma^{1\,k}{}_{k}) - \\
&\sigma^{1}{}_{ij}\,n^{\alpha}\,(\nabla^{j}A^{1\,i})\,(\nabla_{\alpha}\sigma^{1\,k}{}_{k}) + 2\,\sigma^{1\,i}{}_{i}\,n^{\alpha}\,n^{\beta}\,(\nabla_{\alpha}\phi^{1})\,(\nabla_{\beta}\phi^{1}) - 2\,\sigma^{1\,ij}\,n^{\alpha}\,n^{\beta}\,(\nabla_{\alpha}\phi^{1})\,(\nabla_{\beta}\sigma^{1}{}_{ij}) - \\
&\sigma^{1\,ij}\,n^{\alpha}\,n^{\beta}\,(\nabla_{\alpha}\sigma^{1}{}_{i}{}^{k})\,(\nabla_{\beta}\sigma^{1}{}_{jk}) + \frac{1}{4}\,\sigma^{1\,i}{}_{i}\,n^{\alpha}\,n^{\beta}\,(\nabla_{\alpha}\sigma^{1\,jk})\,(\nabla_{\beta}\sigma^{1}{}_{jk}) + \\
&\sigma^{1\,ij}\,n^{\alpha}\,n^{\beta}\,(\nabla_{\alpha}\sigma^{1}{}_{ij})\,(\nabla_{\beta}\sigma^{1\,k}{}_{k}) - \frac{1}{4}\,\sigma^{1\,i}{}_{i}\,n^{\alpha}\,n^{\beta}\,(\nabla_{\alpha}\sigma^{1\,j}{}_{j})\,(\nabla_{\beta}\sigma^{1\,k}{}_{k})
\end{aligned}
$$

- Power counting
- Storing interactions in data structure
- Generating interactions for computing the 1PN potential
- Automating the perturbation calculations
- Feynman rules

$$8 A\; n\; (\nabla_i \phi)\; (\nabla_\alpha \phi) - 2 A\; n\; (\nabla_i \sigma_j)\; (\nabla_\alpha \phi) - 4 \sigma_{ij}\, n\; (\nabla A)\; (\nabla_\alpha \phi) -$$

$$\sigma^{1\,k}{}_{k}\, n^\alpha \left(\nabla^j A^{1\,i}\right) \left(\nabla_\alpha \sigma^1{}_{ij}\right) + 4 A^{1\,i} n^\alpha \left(\nabla^j \phi^1\right) \left(\nabla_\alpha \sigma^1{}_{ij}\right) - A^{1\,i} n^\alpha \left(\nabla_j \sigma^{1\,k}{}_{k}\right) \left(\nabla_\alpha \sigma^{1}{}_{i}{}^{j}\right) +$$

$$2\, \sigma^1{}_j{}^k\, n^\alpha \left(\nabla^j A^{1\,i}\right) \left(\nabla_\alpha \sigma^1{}_{ik}\right) - \sigma^{1\,jk} n^\alpha \left(\nabla_i A^{1\,i}\right) \left(\nabla_\alpha \sigma^1{}_{jk}\right) + 2\, \sigma^1{}_i{}^k\, n^\alpha \left(\nabla^j A^{1\,i}\right) \left(\nabla_\alpha \sigma^1{}_{jk}\right) -$$

$$2 A^{1\,i} n^\alpha \left(\nabla_i \phi^1\right) \left(\nabla_\alpha \sigma^{1\,j}{}_{j}\right) + A^{1\,i} n^\alpha \left(\nabla_i \sigma^{1\,k}{}_{k}\right) \left(\nabla_\alpha \sigma^{1\,j}{}_{j}\right) - A^{1\,i} n^\alpha \left(\nabla_k \sigma^1{}_i{}^k\right) \left(\nabla_\alpha \sigma^{1\,j}{}_{j}\right) -$$

$$A^{1\,i} n^\alpha \left(\nabla_i \sigma^1{}_{jk}\right) \left(\nabla_\alpha \sigma^{1\,jk}\right) + 2 A^{1\,i} n^\alpha \left(\nabla_k \sigma^1{}_{ij}\right) \left(\nabla_\alpha \sigma^{1\,jk}\right) + \frac{1}{2} \sigma^{1\,j}{}_{j}\, n^\alpha \left(\nabla_i A^{1\,i}\right) \left(\nabla_\alpha \sigma^{1\,k}{}_{k}\right) -$$

$$\sigma^1{}_{ij}\, n^\alpha \left(\nabla^j A^{1\,i}\right) \left(\nabla_\alpha \sigma^{1\,k}{}_{k}\right) + 2\, \sigma^{1\,i}{}_{i}\, n^\alpha n^\beta \left(\nabla_\alpha \phi^1\right) \left(\nabla_\beta \phi^1\right) - 2\, \sigma^{1\,ij} n^\alpha n^\beta \left(\nabla_\alpha \phi^1\right) \left(\nabla_\beta \sigma^1{}_{ij}\right) -$$

$$\sigma^{1\,ij} n^\alpha n^\beta \left(\nabla_\alpha \sigma^1{}_i{}^k\right) \left(\nabla_\beta \sigma^1{}_{jk}\right) + \frac{1}{4} \sigma^{1\,i}{}_{i}\, n^\alpha n^\beta \left(\nabla_\alpha \sigma^{1\,jk}\right) \left(\nabla_\beta \sigma^1{}_{jk}\right) +$$

$$\sigma^{1\,ij} n^\alpha n^\beta \left(\nabla_\alpha \sigma^1{}_{ij}\right) \left(\nabla_\beta \sigma^{1\,k}{}_{k}\right) - \frac{1}{4} \sigma^{1\,i}{}_{i}\, n^\alpha n^\beta \left(\nabla_\alpha \sigma^{1\,j}{}_{j}\right) \left(\nabla_\beta \sigma^{1\,k}{}_{k}\right)$$

- **Power counting**
- **Storing interactions in data structure**
- **Generating interactions for computing the 1PN potential**
- **Automating the perturbation calculations**
- **Feynman rules**
- **Assemble the Feynman diagrams**

In[191]:=

```
data1PN2 // MatrixForm
```

Out[191]//MatrixForm=

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Demo_CapraNRDA_2010.nb

- Power counting
- Storing interactions in data structure
- Generating interactions for computing the 1PN potential

In[168]:=

```
dataP1 = Flatten[{
    ExprToStruct[Perturb[Lpp1, 0, 0, 4] // To3plus1 // ToKolSmolkin,
     "P1", Variables → KolSmolkin],
    ExprToStruct[Perturb[Lpp1, 1, 0, 0] // To3plus1 // ToKolSmolkin,
     "P1", Variables → KolSmolkin],
    ExprToStruct[Perturb[Lpp1, 1, 0, 1] // To3plus1 // ToKolSmolkin,
     "P1", Variables → KolSmolkin],
    ExprToStruct[Perturb[Lpp1, 1, 0, 2] // To3plus1 // ToKolSmolkin,
     "P1", Variables → KolSmolkin],
    ExprToStruct[Perturb[Lpp1, 2, 0, 0] // To3plus1 // ToKolSmolkin,
     "P1", Variables → KolSmolkin]
   }, 1];
% // MatrixForm
```

0.605805

Out[169]//MatrixForm=

P1	{0, 0, 0, 0, 4}	{2, 1}	$\frac{1}{8} m_1 v_i v^i v_j v^j$
P1	{1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$	$-m_1 \phi^1$
P1	{0, 1, 0, 0, 1}	$\{1, \frac{1}{2}\}$	$m_1 A^{1i} v_i$
P1	{1, 0, 0, 0, 2}	$\{2, \frac{1}{2}\}$	$-\frac{3}{2} m_1 \phi^1 v_i v^i$

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```
P1 {1, 0, 0, 0, 2} {2, 1/2} -3/2 m1 φ^1 v_i v^i
P1 {0, 0, 1, 0, 2} {2, 1/2} 1/2 m1 σ^1_ij v^i v^j
P1 {2, 0, 0, 0, 0} {2, 0} 1/2 m1 φ^1^2
```

In[170]:=

```
dataP2 = dataP1 /. {Mass1 → Mass2, "P1" → "P2"};
% // MatrixForm
```

Out[171]//MatrixForm=

```
P2 {0, 0, 0, 0, 4} {2, 1} 1/8 m2 v_i v^i v_j v^j
P2 {1, 0, 0, 0, 0} {0, 1/2} -m2 φ^1
P2 {0, 1, 0, 0, 1} {1, 1/2} m2 A^1i v_i
P2 {1, 0, 0, 0, 2} {2, 1/2} -3/2 m2 φ^1 v_i v^i
P2 {0, 0, 1, 0, 2} {2, 1/2} 1/2 m2 σ^1_ij v^i v^j
P2 {2, 0, 0, 0, 0} {2, 0} 1/2 m2 φ^1^2
```

In[172]:=

```
dataG = Flatten[{
    ExprToStruct[LHHHinKS, "G", Variables → KolSmolkin]
   }, 1];
% // MatrixForm
```

0.560987

Out[173]//MatrixForm=

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0.560987

Out[173]//MatrixForm=

$$
\begin{array}{lll}
G\{3,0,0,0,0\} & \{2,-\tfrac{1}{2}\} & 16\,\phi^1\,(\nabla_i\phi^1)\,(\nabla^i\phi^1) \\
G\{2,0,1,0,0\} & \{2,-\tfrac{1}{2}\} & -2\,\sigma^{1j}{}_j\,(\nabla_i\phi^1)\,(\nabla^i\phi^1) \\
G\{1,0,2,0,0\} & \{2,-\tfrac{1}{2}\} & -2\,\sigma^{1jk}\,(\nabla_i\sigma^1{}_{jk})\,(\nabla^i\phi^1) \\
G\{2,0,1,0,0\} & \{2,-\tfrac{1}{2}\} & -8\,\phi^1\,(\nabla_i\sigma^{1j}{}_j)\,(\nabla^i\phi^1) \\
G\{1,0,2,0,0\} & \{2,-\tfrac{1}{2}\} & 2\,\sigma^{1j}{}_j\,(\nabla_i\sigma^{1k}{}_k)\,(\nabla^i\phi^1) \\
G\{0,2,1,0,0\} & \{2,-\tfrac{1}{2}\} & -\mathbf{A}^{1i}\,(\nabla_i\sigma^{1k}{}_k)\,(\nabla_j\mathbf{A}^{1j}) \\
G\{1,2,0,0,0\} & \{2,-\tfrac{1}{2}\} & 4\,\mathbf{A}^{1i}\,(\nabla_i\mathbf{A}^{1j})\,(\nabla_j\phi^1) \\
G\{2,0,1,0,0\} & \{2,-\tfrac{1}{2}\} & 8\,\phi^1\,(\nabla^i\phi^1)\,(\nabla_j\sigma^1{}_i{}^j) \\
G\{0,0,3,0,0\} & \{2,-\tfrac{1}{2}\} & \tfrac{1}{2}\,\sigma^{1ij}\,(\nabla_i\sigma^{1kl})\,(\nabla_j\sigma^1{}_{kl}) \\
G\{0,2,1,0,0\} & \{2,-\tfrac{1}{2}\} & -\mathbf{A}^{1i}\,(\nabla_i\mathbf{A}^{1j})\,(\nabla_j\sigma^{1k}{}_k) \\
G\{1,0,2,0,0\} & \{2,-\tfrac{1}{2}\} & -2\,\sigma^1{}_i{}^j\,(\nabla^i\phi^1)\,(\nabla_j\sigma^{1k}{}_k) \\
G\{0,0,3,0,0\} & \{2,-\tfrac{1}{2}\} & -\tfrac{1}{2}\,\sigma^{1ij}\,(\nabla_i\sigma^{1k}{}_k)\,(\nabla_j\sigma^{1l}{}_l) \\
G\{1,2,0,0,0\} & \{2,-\tfrac{1}{2}\} & -4\,\mathbf{A}^{1i}\,(\nabla_j\phi^1)\,(\nabla^j\mathbf{A}^1{}_i) \\
G\{0,2,1,0,0\} & \{2,-\tfrac{1}{2}\} & 2\,\mathbf{A}^{1i}\,(\nabla_j\sigma^{1k}{}_k)\,(\nabla^j\mathbf{A}^1{}_i) \\
G\{0,2,1,0,0\} & \{2,-\tfrac{1}{2}\} & -\tfrac{1}{2}\,\sigma^{1k}{}_k\,(\nabla_i\mathbf{A}^1{}_j)\,(\nabla^j\mathbf{A}^{1i}) \\
G\{0,2,1,0,0\} & \{2,-\tfrac{1}{2}\} & \tfrac{1}{2}\,\sigma^{1k}{}_k\,(\nabla_j\mathbf{A}^1{}_i)\,(\nabla^j\mathbf{A}^{1i}) \\
G\{0,2,1,0,0\} & \{2,-\tfrac{1}{2}\} & -\sigma^1{}_{ik}\,(\nabla_j\mathbf{A}^{1k})\,(\nabla^j\mathbf{A}^{1i}) \\
G\{2,0,1,0,0\} & \{2,-\tfrac{1}{2}\} & 4\,\sigma^1{}_{ij}\,(\nabla^i\phi^1)\,(\nabla^j\phi^1)
\end{array}
$$

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G {0, 0, 3, 0, 0} $\{4, -\frac{1}{2}\}$ $-\sigma^{1\,ij}\, n^{\alpha}\, n^{\beta}\, (\nabla_{\alpha}\sigma^{1}{}_{i}{}^{k})\, (\nabla_{\beta}\sigma^{1}{}_{jk})$

G {0, 0, 3, 0, 0} $\{4, -\frac{1}{2}\}$ $\frac{1}{4}\sigma^{1}{}_{i}^{i}\, n^{\alpha}\, n^{\beta}\, (\nabla_{\alpha}\sigma^{1\,jk})\, (\nabla_{\beta}\sigma^{1}{}_{jk})$

G {0, 0, 3, 0, 0} $\{4, -\frac{1}{2}\}$ $\sigma^{1\,ij}\, n^{\alpha}\, n^{\beta}\, (\nabla_{\alpha}\sigma^{1}{}_{ij})\, (\nabla_{\beta}\sigma^{1}{}_{k}^{k})$

G {0, 0, 3, 0, 0} $\{4, -\frac{1}{2}\}$ $-\frac{1}{4}\sigma^{1}{}_{i}^{i}\, n^{\alpha}\, n^{\beta}\, (\nabla_{\alpha}\sigma^{1}{}_{j}^{j})\, (\nabla_{\beta}\sigma^{1}{}_{k}^{k})$

In[174]:=
```
dataG = dataG // CombineSamePowerCounting;
% // MatrixForm
```

Out[175]//MatrixForm=

G {0, 0, 3, 0, 0} $\{2, -\frac{1}{2}\}$ $\frac{1}{2}\sigma^{1\,ij}\, (\nabla_{i}\sigma^{1\,kl})\, (\nabla_{j}\sigma^{1}{}_{kl}) - \frac{1}{2}\sigma^{1\,ij}\, (\nabla_{i}\sigma^{1}{}_{k}^{k})\, (\nabla_{j}\sigma^{1}{}_{l}^{l}) + \sigma^{1\,ij}$

G {0, 0, 3, 0, 0} $\{4, -\frac{1}{2}\}$ $-\sigma^{1\,ij}\, n^{\alpha}\, n^{\beta}\, (\nabla_{\alpha}\sigma^{1}{}_{i}{}^{k})\, (\nabla_{\beta}\sigma^{1}{}_{jk}) + \frac{1}{4}\sigma^{1}{}_{i}^{i}\, n^{\alpha}\, n^{\beta}\, (\nabla_{\alpha}\sigma^{1\,jk})\, (\nabla_{\beta}\sigma^{1}{}_{j}$

G {0, 1, 2, 0, 0} $\{3, -\frac{1}{2}\}$ $-\sigma^{1\,jk}\, n^{\alpha}\, (\nabla_{i}\sigma^{1}{}_{jk})\, (\nabla_{\alpha}A^{1\,i}) + \frac{1}{2}\sigma^{1}{}_{j}^{j}\, n^{\alpha}\, (\nabla_{i}\sigma^{1}{}_{k}^{k})\, (\nabla_{\alpha}A^{1\,i}) - \sigma^{1}$

G {0, 2, 1, 0, 0} $\{2, -\frac{1}{2}\}$ $-A^{1\,i}\, (\nabla_{i}\sigma^{1}{}_{k}^{k})\, (\nabla_{j}A^{1\,j}) - A^{1\,i}\, (\nabla_{i}A^{1\,j})\, (\nabla_{j}\sigma^{1}{}_{k}^{k}) + 2A^{1\,i}\, (\nabla_{j}\sigma$

G {0, 3, 0, 0, 0} $\{3, -\frac{1}{2}\}$ $2A^{1\,i}\, n^{\alpha}\, (\nabla_{j}A^{1\,j})\, (\nabla_{\alpha}A^{1}{}_{i}) - 2A^{1\,i}\, n^{\alpha}\, (\nabla_{j}A^{1}{}_{i})\, (\nabla_{\alpha}A^{1\,j})$

G {1, 0, 2, 0, 0} $\{2, -\frac{1}{2}\}$ $-2\sigma^{1\,jk}\, (\nabla_{i}\sigma^{1}{}_{jk})\, (\nabla^{i}\phi^{1}) + 2\sigma^{1}{}_{j}^{j}\, (\nabla_{i}\sigma^{1}{}_{k}^{k})\, (\nabla^{i}\phi^{1}) - 2\sigma^{1}{}_{i}^{j}\, (\nabla$

G {1, 0, 2, 0, 0} $\{4, -\frac{1}{2}\}$ $-2\sigma^{1\,ij}\, n^{\alpha}\, n^{\beta}\, (\nabla_{\alpha}\phi^{1})\, (\nabla_{\beta}\sigma^{1}{}_{ij})$

G {1, 1, 1, 0, 0} $\{3, -\frac{1}{2}\}$ $-2\sigma^{1}{}_{j}^{j}\, n^{\alpha}\, (\nabla_{i}\phi^{1})\, (\nabla_{\alpha}A^{1\,i}) + 8\sigma^{1}{}_{ij}\, n^{\alpha}\, (\nabla^{j}\phi^{1})\, (\nabla_{\alpha}A^{1\,i}) + 2\sigma^{1\,j}$

G {1, 2, 0, 0, 0} $\{2, -\frac{1}{2}\}$ $4A^{1\,i}\, (\nabla_{i}A^{1\,j})\, (\nabla_{j}\phi^{1}) - 4A^{1\,i}\, (\nabla_{j}\phi^{1})\, (\nabla^{j}A^{1}{}_{i})$

G {2, 0, 1, 0, 0} $\{2, -\frac{1}{2}\}$ $-2\sigma^{1}{}_{j}^{j}\, (\nabla_{i}\phi^{1})\, (\nabla^{i}\phi^{1}) - 8\phi^{1}\, (\nabla_{i}\sigma^{1}{}_{j}^{j})\, (\nabla^{i}\phi^{1}) + 8\phi^{1}\, (\nabla^{i}\phi^{1})$

G {2, 0, 1, 0, 0} $\{4, -\frac{1}{2}\}$ $2\sigma^{1}{}_{i}^{i}\, n^{\alpha}\, n^{\beta}\, (\nabla_{\alpha}\phi^{1})\, (\nabla_{\beta}\phi^{1})$

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G {2, 0, 1, 0, 0} $\{2, -\frac{1}{2}\}$ $-2\,\sigma^{1\,j}{}_{j}\,(\nabla_i \phi^1)\,(\nabla^i \phi^1) - 8\,\phi^1\,(\nabla_i \sigma^{1\,j}{}_{j})\,(\nabla^i \phi^1) + 8\,\phi^1\,(\nabla^i \phi^1)$

G {2, 0, 1, 0, 0} $\{4, -\frac{1}{2}\}$ $2\,\sigma^{1\,i}{}_{i}\,n^\alpha\,n^\beta\,(\nabla_\alpha \phi^1)\,(\nabla_\beta \phi^1)$

G {2, 1, 0, 0, 0} $\{3, -\frac{1}{2}\}$ $8\,\phi^1\,n^\alpha\,(\nabla_i \phi^1)\,(\nabla_\alpha A^{1\,i}) - 8\,\phi^1\,n^\alpha\,(\nabla_i A^{1\,i})\,(\nabla_\alpha \phi^1) + 8\,A^{1\,i}\,n^\alpha\,(\nabla$

G {3, 0, 0, 0, 0} $\{2, -\frac{1}{2}\}$ $16\,\phi^1\,(\nabla_i \phi^1)\,(\nabla^i \phi^1)$

In[176]:=

```
dataX = Flatten[{
    ExprToStruct[LHHinKS, "X", Variables → KolSmolkin]
   }, 1];
% // MatrixForm
```

Out[177]//MatrixForm=

X {2, 0, 0, 0, 0} {0, 0} $-4\,(\nabla_i \phi^1)\,(\nabla^i \phi^1)$

X {0, 2, 0, 0, 0} {0, 0} $(\nabla_j A^1{}_i)\,(\nabla^j A^{1\,i})$

X {0, 0, 2, 0, 0} {0, 0} $\frac{1}{4}\,(\nabla_j \sigma^{1\,k}{}_{k})\,(\nabla^j \sigma^{1\,i}{}_{i})$

X {0, 0, 2, 0, 0} {0, 0} $-\frac{1}{2}\,(\nabla_k \sigma^1{}_{ij})\,(\nabla^k \sigma^{1\,ij})$

X {0, 2, 0, 0, 0} {2, 0} $-n^\alpha\,n^\beta\,(\nabla_\alpha A^{1\,i})\,(\nabla_\beta A^1{}_i)$

X {2, 0, 0, 0, 0} {2, 0} $4\,n^\alpha\,n^\beta\,(\nabla_\alpha \phi^1)\,(\nabla_\beta \phi^1)$

X {0, 0, 2, 0, 0} {2, 0} $\frac{1}{2}\,n^\alpha\,n^\beta\,(\nabla_\alpha \sigma^{1\,ij})\,(\nabla_\beta \sigma^1{}_{ij})$

X {0, 0, 2, 0, 0} {2, 0} $-\frac{1}{4}\,n^\alpha\,n^\beta\,(\nabla_\alpha \sigma^{1\,i}{}_{i})\,(\nabla_\beta \sigma^{1\,j}{}_{j})$

In[178]:=

```
data1PN = Flatten[{dataP1, dataP2, dataG, dataX}, 1];
% // MatrixForm
```

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In[178]:=

```
data1PN = Flatten[{dataP1, dataP2, dataG, dataX}, 1];
% // MatrixForm
```

Out[179]//MatrixForm=

P1	{0, 0, 0, 0, 4}	{2, 1}	$\frac{1}{8} m_1 v_i v^i v_j v^j$
P1	{1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$	$-m_1 \phi^1$
P1	{0, 1, 0, 0, 1}	$\{1, \frac{1}{2}\}$	$m_1 A^{1i} v_i$
P1	{1, 0, 0, 0, 2}	$\{2, \frac{1}{2}\}$	$-\frac{3}{2} m_1 \phi^1 v_i v^i$
P1	{0, 0, 1, 0, 2}	$\{2, \frac{1}{2}\}$	$\frac{1}{2} m_1 \sigma^1{}_{ij} v^i v^j$
P1	{2, 0, 0, 0, 0}	{2, 0}	$\frac{1}{2} m_1 \phi^{12}$
P2	{0, 0, 0, 0, 4}	{2, 1}	$\frac{1}{8} m_2 v_i v^i v_j v^j$
P2	{1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$	$-m_2 \phi^1$
P2	{0, 1, 0, 0, 1}	$\{1, \frac{1}{2}\}$	$m_2 A^{1i} v_i$
P2	{1, 0, 0, 0, 2}	$\{2, \frac{1}{2}\}$	$-\frac{3}{2} m_2 \phi^1 v_i v^i$
P2	{0, 0, 1, 0, 2}	$\{2, \frac{1}{2}\}$	$\frac{1}{2} m_2 \sigma^1{}_{ij} v^i v^j$
P2	{2, 0, 0, 0, 0}	{2, 0}	$\frac{1}{2} m_2 \phi^{12}$
G	{0, 0, 3, 0, 0}	$\{2, -\frac{1}{2}\}$	$\frac{1}{2} \sigma^{1ij} (\nabla_i \sigma^{1kl}) (\nabla_j \sigma^1{}_{kl}) - \frac{1}{2} \sigma^{1ij} (\nabla_i \sigma^{1k}{}_k) (\nabla_j \sigma^{1l}{}_l) + \sigma^{1i}$
G	{0, 0, 3, 0, 0}	$\{4, -\frac{1}{2}\}$	$-\sigma^{1ij} n^\alpha n^\beta (\nabla_\alpha \sigma^1{}_i{}^k) (\nabla_\beta \sigma^1{}_{jk}) + \frac{1}{4} \sigma^{1i}{}_i n^\alpha n^\beta (\nabla_\alpha \sigma^{1jk}) (\nabla_\beta \sigma$
G	{0, 1, 2, 0, 0}	$\{3, -\frac{1}{2}\}$	$-\sigma^{1jk} n^\alpha (\nabla_i \sigma^1{}_{jk}) (\nabla_\alpha A^{1i}) + \frac{1}{2} \sigma^{1j}{}_j n^\alpha (\nabla_i \sigma^{1k}{}_k) (\nabla_\alpha A^{1i}) -$
G	{0, 2, 1, 0, 0}	$\{2, -\frac{1}{2}\}$	$-A^{1i} (\nabla_i \sigma^{1k}{}_k) (\nabla_j A^{1j}) - A^{1i} (\nabla_i A^{1j}) (\nabla_j \sigma^{1k}{}_k) + 2 A^{1i}$
G	{0, 3, 0, 0, 0}	$\{3, -\frac{1}{2}\}$	$2 A^{1i} n^\alpha (\nabla_i A^{1j}) (\nabla_\alpha A^1{}_j) - 2 A^{1i} n^\alpha (\nabla_j A^1{}_i) (\nabla_\alpha A^{1j})$

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G	{0, 2, 1, 0, 0}	$\{2, -\frac{1}{2}\}$	$-A^{1i}(\nabla_i \sigma^{1k}{}_k)(\nabla_j A^{1j}) - A^{1i}(\nabla_i A^{1j})(\nabla_j \sigma^{1k}{}_k) + 2A^{1i}(\nabla_j$
G	{0, 3, 0, 0, 0}	$\{3, -\frac{1}{2}\}$	$2A^{1i} n^\alpha (\nabla_j A^{1j})(\nabla_\alpha A^1{}_i) - 2A^{1i} n^\alpha (\nabla_j A^1{}_i)(\nabla_\alpha A^{1j})$
G	{1, 0, 2, 0, 0}	$\{2, -\frac{1}{2}\}$	$-2\sigma^{1jk}(\nabla_i \sigma^1{}_{jk})(\nabla^i \phi^1) + 2\sigma^{1j}{}_j (\nabla_i \sigma^{1k}{}_k)(\nabla^i \phi^1) - 2\sigma^1{}_i{}^j ($
G	{1, 0, 2, 0, 0}	$\{4, -\frac{1}{2}\}$	$-2\sigma^{1ij} n^\alpha n^\beta (\nabla_\alpha \phi^1)(\nabla_\beta \sigma^1{}_{ij})$
G	{1, 1, 1, 0, 0}	$\{3, -\frac{1}{2}\}$	$-2\sigma^{1j}{}_j n^\alpha (\nabla_i \phi^1)(\nabla_\alpha A^{1i}) + 8\sigma^1{}_{ij} n^\alpha (\nabla^j \phi^1)(\nabla_\alpha A^{1i}) + 2\sigma^1$
G	{1, 2, 0, 0, 0}	$\{2, -\frac{1}{2}\}$	$4A^{1i}(\nabla_i A^{1j})(\nabla_j \phi^1) - 4A^{1i}(\nabla_j \phi^1)(\nabla^j A^1{}_i)$
G	{2, 0, 1, 0, 0}	$\{2, -\frac{1}{2}\}$	$-2\sigma^{1j}{}_j (\nabla_i \phi^1)(\nabla^i \phi^1) - 8\phi^1 (\nabla_i \sigma^{1j}{}_j)(\nabla^i \phi^1) + 8\phi^1 (\nabla^i \phi^1)$
G	{2, 0, 1, 0, 0}	$\{4, -\frac{1}{2}\}$	$2\sigma^{1i}{}_i n^\alpha n^\beta (\nabla_\alpha \phi^1)(\nabla_\beta \phi^1)$
G	{2, 1, 0, 0, 0}	$\{3, -\frac{1}{2}\}$	$8\phi^1 n^\alpha (\nabla_i \phi^1)(\nabla_\alpha A^{1i}) - 8\phi^1 n^\alpha (\nabla_i A^{1i})(\nabla_\alpha \phi^1) + 8A^{1i} n^\alpha ($
G	{3, 0, 0, 0, 0}	$\{2, -\frac{1}{2}\}$	$16\phi^1 (\nabla_i \phi^1)(\nabla^i \phi^1)$
X	{2, 0, 0, 0, 0}	{0, 0}	$-4(\nabla_i \phi^1)(\nabla^i \phi^1)$
X	{0, 2, 0, 0, 0}	{0, 0}	$(\nabla_j A^1{}_i)(\nabla^j A^{1i})$
X	{0, 0, 2, 0, 0}	{0, 0}	$\frac{1}{4}(\nabla_j \sigma^{1k}{}_k)(\nabla^j \sigma^{1i}{}_i)$
X	{0, 0, 2, 0, 0}	{0, 0}	$-\frac{1}{2}(\nabla_k \sigma^1{}_{ij})(\nabla^k \sigma^{1ij})$
X	{0, 2, 0, 0, 0}	{2, 0}	$-n^\alpha n^\beta (\nabla_\alpha A^{1i})(\nabla_\beta A^1{}_i)$
X	{2, 0, 0, 0, 0}	{2, 0}	$4n^\alpha n^\beta (\nabla_\alpha \phi^1)(\nabla_\beta \phi^1)$
X	{0, 0, 2, 0, 0}	{2, 0}	$\frac{1}{2} n^\alpha n^\beta (\nabla_\alpha \sigma^{1ij})(\nabla_\beta \sigma^1{}_{ij})$
X	{0, 0, 2, 0, 0}	{2, 0}	$-\frac{1}{4} n^\alpha n^\beta (\nabla_\alpha \sigma^{1i}{}_i)(\nabla_\beta \sigma^{1j}{}_j)$

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- **Automating the perturbation calculations**
- **Feynman rules**
- **Assemble the Feynman diagrams**

In[191]:=

```
data1PN2 // MatrixForm
```

Out[191]//MatrixForm=

G	{0, 0, 3, 0, 0}	$\{2, -\frac{1}{2}\}$	$\frac{1}{2}\sigma^{1\,ij}(\nabla_i\sigma^{1\,kl})(\nabla_j\sigma^1{}_{kl}) - \frac{1}{4}\sigma^{1\,ij}(\nabla_i\sigma^{1\,k}{}_k)(\nabla_j\sigma^{1\,l}{}_l) - \frac{1}{2}\sigma$
G	{0, 2, 1, 0, 0}	$\{2, -\frac{1}{2}\}$	$\frac{1}{2}\sigma^{1\,k}{}_k(\nabla_i A^{1\,i})(\nabla_j A^{1\,j}) - \frac{1}{2}\sigma^{1\,k}{}_k(\nabla_i A^1{}_j)(\nabla^j A^{1\,i}) + \frac{1}{2}\sigma^{1\,k}{}_k$
G	{1, 2, 0, 0, 0}	$\{2, -\frac{1}{2}\}$	$4\phi^1(\nabla_i A^{1\,i})(\nabla_j A^{1\,j}) - 4\phi^1(\nabla_i A^1{}_j)(\nabla^j A^{1\,i}) + 4\phi^1(\nabla_j A^1{}_i)$
G	{2, 0, 1, 0, 0}	$\{2, -\frac{1}{2}\}$	$-2\sigma^{1\,j}{}_j(\nabla_i\phi^1)(\nabla^i\phi^1) + 4\sigma^1{}_{ij}(\nabla^i\phi^1)(\nabla^j\phi^1)$
P1	{0, 0, 0, 0, 4}	{2, 1}	$\frac{1}{8}m_1 v_i v^i v_j v^j$
P1	{0, 0, 1, 0, 2}	$\{2, \frac{1}{2}\}$	$\frac{1}{2}m_1\sigma^1{}_{ij}v^i v^j$
P1	{0, 1, 0, 0, 1}	$\{1, \frac{1}{2}\}$	$m_1 A^{1\,i} v_i$
P1	{1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$	$-m_1\phi^1$
P1	{1, 0, 0, 0, 2}	$\{2, \frac{1}{2}\}$	$-\frac{3}{2}m_1\phi^1 v_i v^i$
P1	{2, 0, 0, 0, 0}	{2, 0}	$-\frac{1}{2}m_1\phi^{1\,2}$
P2	{0, 0, 0, 0, 4}	{2, 1}	$\frac{1}{8}m_2 v_i v^i v_j v^j$
P2	{0, 0, 1, 0, 2}	$\{2, \frac{1}{2}\}$	$\frac{1}{2}m_2\sigma^1{}_{ij}v^i v^j$

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X {0, 0, 2, 0, 0} {2, 0} $\frac{1}{2} n^{\alpha} n^{\beta} (\nabla_{\alpha} \sigma^{1\,ij}) (\nabla_{\beta} \sigma^{1}{}_{ij})$

X {0, 0, 2, 0, 0} {2, 0} $-\frac{1}{4} n^{\alpha} n^{\beta} (\nabla_{\alpha} \sigma^{1\,i}{}_{i}) (\nabla_{\beta} \sigma^{1\,j}{}_{j})$

Automating the perturbation calculations

In[180]:= `FindAllOrders[2, 0] // MatrixForm`

Out[180]//MatrixForm=

```
P 0 0 4
P 1 0 0
P 1 0 1
P 1 0 2
P 2 0 0
G 2 0 0
G 3 0 0
```

In[181]:= `FindAllOrders[4, 0] // MatrixForm`

Out[181]//MatrixForm=

```
P 0 0 4
P 0 0 6
P 1 0 0
P 1 0 1
P 1 0 2
P 1 0 3
P 1 0 4
P 2 0 0
P 2 0 1
```

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```
G 3 0 0
```

In[181]:=

```
FindAllOrders[4, 0] // MatrixForm
```

Out[181]//MatrixForm=

```
P 0 0 4
P 0 0 6
P 1 0 0
P 1 0 1
P 1 0 2
P 1 0 3
P 1 0 4
P 2 0 0
P 2 0 1
P 2 0 2
P 3 0 0
G 2 0 0
G 3 0 0
G 4 0 0
```

In[182]:=

```
FindAllOrders[2, 0]
data1PN2 = MakeAllInteractions[2, %, GaugeChoice → Harmonic,
   Variables → KolSmolkin];
% // MatrixForm
```

Out[182]=

```
{{P, 0, 0, 4}, {P, 1, 0, 0}, {P, 1, 0, 1},
 {P, 1, 0, 2}, {P, 2, 0, 0}, {G, 2, 0, 0}, {G, 3, 0, 0}}
```

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```
G 3 0 0
```

In[181]:=

```
FindAllOrders[4, 0] // MatrixForm
```

Out[181]//MatrixForm=

```
P 0 0 4
P 0 0 6
P 1 0 0
P 1 0 1
P 1 0 2
P 1 0 3
P 1 0 4
P 2 0 0
P 2 0 1
P 2 0 2
P 3 0 0
G 2 0 0
G 3 0 0
G 4 0 0
```

In[182]:=

```
FindAllOrders[2, 0]
data1PN2 = MakeAllInteractions[2, %, GaugeChoice → Harmonic,
   Variables → KolSmolkin];
% // MatrixForm
```

Out[182]=

```
{{P, 0, 0, 4}, {P, 1, 0, 0}, {P, 1, 0, 1},
 {P, 1, 0, 2}, {P, 2, 0, 0}, {G, 2, 0, 0}, {G, 3, 0, 0}}
```

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Out[182]= {{P, 0, 0, 4}, {P, 1, 0, 0}, {P, 1, 0, 1},
{P, 1, 0, 2}, {P, 2, 0, 0}, {G, 2, 0, 0}, {G, 3, 0, 0}}

```
Calculating interactions for particle 1...done.
Calculating interactions for particle 2...done.
Calculating propagator insertions in Harmonic gauge...done.
Calculating gravitational self-interactions for 3 potential field(s) and 0 radiat
```

47.7271

Out[184]//MatrixForm=

G	{0, 0, 3, 0, 0}	$\{2, -\frac{1}{2}\}$	$\frac{1}{2}\sigma^{1\,ij}\left(\nabla_i \sigma^{1\,kl}\right)\left(\nabla_j \sigma^{1}{}_{kl}\right) - \frac{1}{4}\sigma^{1\,ij}\left(\nabla_i \sigma^{1\,k}{}_{k}\right)\left(\nabla_j \sigma^{1\,l}{}_{l}\right) - \frac{1}{2}\sigma$
G	{0, 2, 1, 0, 0}	$\{2, -\frac{1}{2}\}$	$\frac{1}{2}\sigma^{1\,k}{}_{k}\left(\nabla_i A^{1\,i}\right)\left(\nabla_j A^{1\,j}\right) - \frac{1}{2}\sigma^{1\,k}{}_{k}\left(\nabla_i A^{1}{}_{j}\right)\left(\nabla^j A^{1\,i}\right) + \frac{1}{2}\sigma^{1\,k}{}_{k}$
G	{1, 2, 0, 0, 0}	$\{2, -\frac{1}{2}\}$	$4\,\phi^1\left(\nabla_i A^{1\,i}\right)\left(\nabla_j A^{1\,j}\right) - 4\,\phi^1\left(\nabla_i A^{1}{}_{j}\right)\left(\nabla^j A^{1\,i}\right) + 4\,\phi^1\left(\nabla_j A^{1}{}_{i}\right)$
G	{2, 0, 1, 0, 0}	$\{2, -\frac{1}{2}\}$	$-2\,\sigma^{1\,j}{}_{j}\left(\nabla_i \phi^1\right)\left(\nabla^i \phi^1\right) + 4\,\sigma^{1}{}_{ij}\left(\nabla^i \phi^1\right)\left(\nabla^j \phi^1\right)$
P1	{0, 0, 0, 0, 4}	{2, 1}	$\frac{1}{8} m_1 v_i v^i v_j v^j$
P1	{0, 0, 1, 0, 2}	$\{2, \frac{1}{2}\}$	$\frac{1}{2} m_1 \sigma^{1}{}_{ij} v^i v^j$
P1	{0, 1, 0, 0, 1}	$\{1, \frac{1}{2}\}$	$m_1 A^{1\,i} v_i$
P1	{1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$	$-m_1 \phi^1$
P1	{1, 0, 0, 0, 2}	$\{2, \frac{1}{2}\}$	$-\frac{3}{2} m_1 \phi^1 v_i v^i$
P1	{2, 0, 0, 0, 0}	{2, 0}	$-\frac{1}{2} m_1 \phi^{1\,2}$
P2	{0, 0, 0, 0, 4}	{2, 1}	$\frac{1}{8} m_2 v_i v^i v_j v^j$
P2	{0, 0, 1, 0, 2}	$\{2, \frac{1}{2}\}$	$\frac{1}{2} m_2 \sigma^{1}{}_{ij} v^i v^j$
P2	{0, 1, 0, 0, 1}	$\{1, \frac{1}{2}\}$	$m_2 A^{1\,i} v_i$
P2	{1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$	$-m_2 \phi^1$

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P2	{0, 1, 0, 0, 1}	$\{1, \frac{1}{2}\}$	$m_2 A^{1i} v_i$
P2	{1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$	$-m_2 \phi^1$
P2	{1, 0, 0, 0, 2}	$\{2, \frac{1}{2}\}$	$-\frac{3}{2} m_2 \phi^1 v_i v^i$
P2	{2, 0, 0, 0, 0}	{2, 0}	$-\frac{1}{2} m_2 \phi^{1\,2}$
X	{0, 0, 2, 0, 0}	{2, 0}	$\frac{1}{2} n^\alpha n^\beta (\nabla_\alpha \sigma^{1ij})(\nabla_\beta \sigma^1{}_{ij}) - \frac{1}{4} n^\alpha n^\beta (\nabla_\alpha \sigma^{1i}{}_i)(\nabla_\beta \sigma^{1j}{}_j)$
X	{0, 2, 0, 0, 0}	{2, 0}	$-n^\alpha n^\beta (\nabla_\alpha A^{1i})(\nabla_\beta A^1{}_i)$
X	{2, 0, 0, 0, 0}	{2, 0}	$4 n^\alpha n^\beta (\nabla_\alpha \phi^1)(\nabla_\beta \phi^1)$

Feynman rules

Assemble the Feynman diagrams

In[191]:=

```
data1PN2 // MatrixForm
```

Out[191]//MatrixForm=

G	{0, 0, 3, 0, 0}	$\{2, -\frac{1}{2}\}$	$\frac{1}{2} \sigma^{1ij} (\nabla_i \sigma^{1kl})(\nabla_j \sigma^1{}_{kl}) - \frac{1}{4} \sigma^{1ij} (\nabla_i \sigma^{1k}{}_k)(\nabla_j \sigma^{1l}{}_l) - \frac{1}{2} \sigma$
G	{0, 2, 1, 0, 0}	$\{2, -\frac{1}{2}\}$	$\frac{1}{2} \sigma^{1k}{}_k (\nabla_i A^{1i})(\nabla_j A^{1j}) - \frac{1}{2} \sigma^{1k}{}_k (\nabla_i A^1{}_j)(\nabla^j A^{1i}) + \frac{1}{2} \sigma^{1k}{}_k$
G	{1, 2, 0, 0, 0}	$\{2, -\frac{1}{2}\}$	$4 \phi^1 (\nabla_i A^{1i})(\nabla_j A^{1j}) - 4 \phi^1 (\nabla_i A^1{}_j)(\nabla^j A^{1i}) + 4 \phi^1 (\nabla_j A^1{}_i)$
G	{2, 0, 1, 0, 0}	$\{2, -\frac{1}{2}\}$	$-2 \sigma^{1j}{}_j (\nabla_i \phi^1)(\nabla^i \phi^1) + 4 \sigma^1{}_{ij} (\nabla^i \phi^1)(\nabla^j \phi^1)$
P1	{0, 0, 0, 0, 4}	{2, 1}	$\frac{1}{8} m_1 v_i v^i v_j v^j$
P1	{0, 0, 1, 0, 2}	$\{2, \frac{1}{2}\}$	$\frac{1}{2} m_1 \sigma^1{}_{ij} v^i v^j$

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X	{0, 0, 2, 0, 0}	{2, 0}	$\frac{1}{2} n^{\alpha} n^{\beta} (\nabla_{\alpha} \sigma^{1\,ij}) (\nabla_{\beta} \sigma^{1}{}_{ij})$
X	{0, 0, 2, 0, 0}	{2, 0}	$-\frac{1}{4} n^{\alpha} n^{\beta} (\nabla_{\alpha} \sigma^{1\,i}{}_{i}) (\nabla_{\beta} \sigma^{1\,j}{}_{j})$

- **Automating the perturbation calculations**
- **Feynman rules**
- **Assemble the Feynman diagrams**

In[191]:=

```
data1PN2 // MatrixForm
```

Out[191]//MatrixForm=

G	{0, 0, 3, 0, 0}	$\{2, -\frac{1}{2}\}$	$\frac{1}{2} \sigma^{1\,ij} (\nabla_i \sigma^{1\,kl}) (\nabla_j \sigma^{1}{}_{kl}) - \frac{1}{4} \sigma^{1\,ij} (\nabla_i \sigma^{1\,k}{}_{k}) (\nabla_j \sigma^{1\,l}{}_{l}) - \frac{1}{2} \sigma$
G	{0, 2, 1, 0, 0}	$\{2, -\frac{1}{2}\}$	$\frac{1}{2} \sigma^{1\,k}{}_{k} (\nabla_i A^{1\,i}) (\nabla_j A^{1\,j}) - \frac{1}{2} \sigma^{1\,k}{}_{k} (\nabla_i A^{1}{}_{j}) (\nabla^j A^{1\,i}) + \frac{1}{2} \sigma^{1\,k}{}_{k}$
G	{1, 2, 0, 0, 0}	$\{2, -\frac{1}{2}\}$	$4 \phi^1 (\nabla_i A^{1\,i}) (\nabla_j A^{1\,j}) - 4 \phi^1 (\nabla_i A^{1}{}_{j}) (\nabla^j A^{1\,i}) + 4 \phi^1 (\nabla_j A^{1}{}_{i})$
G	{2, 0, 1, 0, 0}	$\{2, -\frac{1}{2}\}$	$-2 \sigma^{1\,j}{}_{j} (\nabla_i \phi^1) (\nabla^i \phi^1) + 4 \sigma^{1}{}_{ij} (\nabla^i \phi^1) (\nabla^j \phi^1)$
P1	{0, 0, 0, 0, 4}	{2, 1}	$\frac{1}{8} m_1 v_i v^i v_j v^j$
P1	{0, 0, 1, 0, 2}	$\{2, \frac{1}{2}\}$	$\frac{1}{2} m_1 \sigma^{1}{}_{ij} v^i v^j$
P1	{0, 1, 0, 0, 1}	$\{1, \frac{1}{2}\}$	$m_1 A^{1\,i} v_i$
P1	{1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$	$-m_1 \phi^1$
P1	{1, 0, 0, 0, 2}	$\{2, \frac{1}{2}\}$	$-\frac{3}{2} m_1 \phi^1 v_i v^i$
P1	{2, 0, 0, 0, 0}	{2, 0}	$-\frac{1}{2} m_1 \phi^{1\,2}$
P2	{0, 0, 0, 0, 4}	{2, 1}	$\frac{1}{8} m_2 v_i v^i v_j v^j$

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```
{P1}          {{0, 0, 0, 0, 4}}
{P2}          {{0, 0, 0, 0, 4}}
{P1, P1}      {{1, 0, 0, 0, 0}, {0, 0, 1, 0, 2}}
{P1, P1}      {{1, 0, 0, 0, 0}, {1, 0, 0, 0, 2}}
{P1, P2}      {{1, 0, 0, 0, 0}, {0, 0, 1, 0, 2}}
{P1, P2}      {{1, 0, 0, 0, 0}, {1, 0, 0, 0, 2}}
{P2, P1}      {{1, 0, 0, 0, 0}, {0, 0, 1, 0, 2}}
{P2, P1}      {{1, 0, 0, 0, 0}, {1, 0, 0, 0, 2}}
{P2, P2}      {{1, 0, 0, 0, 0}, {0, 0, 1, 0, 2}}
{P2, P2}      {{1, 0, 0, 0, 0}, {1, 0, 0, 0, 2}}
{P1, P1}      {{0, 1, 0, 0, 1}, {0, 1, 0, 0, 1}}
{P1, P2}      {{0, 1, 0, 0, 1}, {0, 1, 0, 0, 1}}
{P2, P2}      {{0, 1, 0, 0, 1}, {0, 1, 0, 0, 1}}
{P1, P1, P1}  {{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}
{P1, P1, P2}  {{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}
{P1, P1, X}   {{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 0, 2, 0, 0}}
{P1, P1, X}   {{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 2, 0, 0, 0}}
{P1, P1, X}   {{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}
{P1, P2, P1}  {{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}
{P1, P2, P2}  {{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}
```

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{P1, P2}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 2}}
{P2, P1}	{{1, 0, 0, 0, 0}, {0, 0, 1, 0, 2}}
{P2, P1}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 2}}
{P2, P2}	{{1, 0, 0, 0, 0}, {0, 0, 1, 0, 2}}
{P2, P2}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 2}}
{P1, P1}	{{0, 1, 0, 0, 1}, {0, 1, 0, 0, 1}}
{P1, P2}	{{0, 1, 0, 0, 1}, {0, 1, 0, 0, 1}}
{P2, P2}	{{0, 1, 0, 0, 1}, {0, 1, 0, 0, 1}}
{P1, P1, P1}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}
{P1, P1, P2}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}
{P1, P1, X}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 0, 2, 0, 0}}
{P1, P1, X}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 2, 0, 0, 0}}
{P1, P1, X}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}
{P1, P2, P1}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}
{P1, P2, P2}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}
{P1, P2, X}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 0, 2, 0, 0}}
{P1, P2, X}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 2, 0, 0, 0}}
{P1, P2, X}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}
{P2, P2, P1}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}
{P2, P2, P2}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}

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Demo_CapraNRDA_2010.nb

```
{P1, P1, P2} {{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}} {{0, 1/2}, {0, 1/2}, {
{P1, P1, X}  {{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}} {{0, 1/2}, {0, 1/2}, {
{P1, P2, X}  {{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}} {{0, 1/2}, {0, 1/2}, {
{P2, P2, P1} {{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}} {{0, 1/2}, {0, 1/2}, {
{P2, P2, P2} {{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}} {{0, 1/2}, {0, 1/2}, {
{P2, P2, X}  {{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}} {{0, 1/2}, {0, 1/2}, {
```

In[199]:=

```
MakeDataForDiagrams[1, data1PN2];
% // TransposeDiagrams // MatrixForm
%% // RemoveMixedPropagators
```

Out[200]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} P1 \\ P1 \end{pmatrix} & \begin{pmatrix} \{1,0,0,0,0\} \\ \{0,1,0,0,1\} \end{pmatrix} & \begin{pmatrix} \{0,\frac{1}{2}\} \\ \{1,\frac{1}{2}\} \end{pmatrix} & \begin{pmatrix} -m_1\,\phi^1 \\ m_1\,A^{1\,i}\,v_i \end{pmatrix} \\ \begin{pmatrix} P1 \\ P2 \end{pmatrix} & \begin{pmatrix} \{1,0,0,0,0\} \\ \{0,1,0,0,1\} \end{pmatrix} & \begin{pmatrix} \{0,\frac{1}{2}\} \\ \{1,\frac{1}{2}\} \end{pmatrix} & \begin{pmatrix} -m_1\,\phi^1 \\ m_2\,A^{1\,i}\,v_i \end{pmatrix} \\ \begin{pmatrix} P2 \\ P1 \end{pmatrix} & \begin{pmatrix} \{1,0,0,0,0\} \\ \{0,1,0,0,1\} \end{pmatrix} & \begin{pmatrix} \{0,\frac{1}{2}\} \\ \{1,\frac{1}{2}\} \end{pmatrix} & \begin{pmatrix} -m_2\,\phi^1 \\ m_1\,A^{1\,i}\,v_i \end{pmatrix} \\ \begin{pmatrix} P2 \\ P2 \end{pmatrix} & \begin{pmatrix} \{1,0,0,0,0\} \\ \{0,1,0,0,1\} \end{pmatrix} & \begin{pmatrix} \{0,\frac{1}{2}\} \\ \{1,\frac{1}{2}\} \end{pmatrix} & \begin{pmatrix} -m_2\,\phi^1 \\ m_2\,A^{1\,i}\,v_i \end{pmatrix} \end{pmatrix}$$

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Demo_CapraNRDA_2010.nb

In[204]:=
```
MemoryInUse[]
```
Out[204]=
```
53 969 776
```

In[205]:=
```
FreeMemory[]
```
```
Memory now in use: 48 548 848
Memory freed: 5 414 424
4.74994
```

Benchmarking

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Demo_CapraNRDA_2010.nb

P2 {0, 1, 0, 0, 1} $\{1, \frac{1}{2}\}$ $m_2 A^{i\,i} v_i$

Out[201]= {}

In[202]:=
```
MakeDataForDiagrams[0, data1PN2];
% // TransposeDiagrams // MatrixForm
```

Out[203]//MatrixForm=

P1 P1	{1, 0, 0, 0, 0} {1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$ $\{0, \frac{1}{2}\}$	$-m_1 \phi^1$ $-m_1 \phi^1$
P1 P2	{1, 0, 0, 0, 0} {1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$ $\{0, \frac{1}{2}\}$	$-m_1 \phi^1$ $-m_2 \phi^1$
P2 P2	{1, 0, 0, 0, 0} {1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$ $\{0, \frac{1}{2}\}$	$-m_2 \phi^1$ $-m_2 \phi^1$

Fully partial automation at 1PN

Final Memory

In[204]:=
```
MemoryInUse[]
```

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Demo_CapraNRDA_2010.nb

P2	{1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$	$-m_2\, \phi^1$
P2	{1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$	$-m_2\, \phi^1$

Fully partial automation at 1PN

In[212]:=

```
AutoPotential[2, GaugeChoice → Harmonic, Variables → KolSmolkin] //
  TransposeDiagrams // MatrixForm
```

```
Calculating interactions for particle 1...done.
Calculating interactions for particle 2...done.
Calculating propagator insertions in Harmonic gauge...done.
Calculating gravitational self-interactions for 3 potential field(s) and 0 radiati

48.6877
```

Out[212]//MatrixForm=

{P1}	{{0, 0, 0, 0, 4}}	{{2, 1}}
{P2}	{{0, 0, 0, 0, 4}}	{{2, 1}}
{P1, P1}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 2}}	$\{\{0, \frac{1}{2}\}, \{2, \frac{1}{2}\}\}$
{P1, P2}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 2}}	$\{\{0, \frac{1}{2}\}, \{2, \frac{1}{2}\}\}$
{P2, P1}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 2}}	$\{\{0, \frac{1}{2}\}, \{2, \frac{1}{2}\}\}$
{P2, P2}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 2}}	$\{\{0, \frac{1}{2}\}, \{2, \frac{1}{2}\}\}$
{P1, P1}	{{0, 1, 0, 0, 1}, {0, 1, 0, 0, 1}}	$\{\{1, \frac{1}{2}\}, \{1, \frac{1}{2}\}\}$

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Running...Demo_CapraNRDA_2010.nb

P2	{1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$	$-m_2\,\phi^1$
P2	{1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$	$-m_2\,\phi^1$

Fully partial automation at 1PN

In[152]:=

```
AutoPotential[2, GaugeChoice → Harmonic, Variables → KolSmolkin] //
  TransposeDiagrams // MatrixForm
```

```
Calculating interactions for particle 1...done.
Calculating interactions for particle 2...done.
Calculating propagator insertions in Harmonic gauge...
```

Final Memory

In[204]:=

```
MemoryInUse[]
```

Out[204]= 53 969 776

In[205]:=

```
FreeMemory[]
```

```
Memory now in use: 48 548 848
```

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Running...Demo_CapraNRDA_2010.nb

P2	{1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$	$-m_2\, \phi^1$
P2	{1, 0, 0, 0, 0}	$\{0, \frac{1}{2}\}$	$-m_2\, \phi^1$

Fully partial automation at 1PN

In[152]:=
```
AutoPotential[2, GaugeChoice → Harmonic, Variables → KolSmolkin] //
  TransposeDiagrams // MatrixForm
```

```
Calculating interactions for particle 1...done.
Calculating interactions for particle 2...done.
Calculating propagator insertions in Harmonic gauge...done.
Calculating gravitational self-interactions for 3 potential field(s) and 0 radiati
```

Final Memory

In[204]:=
```
MemoryInUse[]
```

Out[204]= 53 969 776

In[205]:=
```
FreeMemory[]
```

```
Memory now in use: 48 548 848
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (36%) Wed 4:01 PM

Demo_CapraNRDA_2010.nb

47.2939

Out[152]//MatrixForm=

{P1}	{{0, 0, 0, 0, 4}}	{{2, 1}}
{P2}	{{0, 0, 0, 0, 4}}	{{2, 1}}
{P1, P1}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 2}}	{{0, $\frac{1}{2}$}, {2, $\frac{1}{2}$}}
{P1, P2}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 2}}	{{0, $\frac{1}{2}$}, {2, $\frac{1}{2}$}}
{P2, P1}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 2}}	{{0, $\frac{1}{2}$}, {2, $\frac{1}{2}$}}
{P2, P2}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 2}}	{{0, $\frac{1}{2}$}, {2, $\frac{1}{2}$}}
{P1, P1}	{{0, 1, 0, 0, 1}, {0, 1, 0, 0, 1}}	{{1, $\frac{1}{2}$}, {1, $\frac{1}{2}$}}
{P1, P2}	{{0, 1, 0, 0, 1}, {0, 1, 0, 0, 1}}	{{1, $\frac{1}{2}$}, {1, $\frac{1}{2}$}}
{P2, P2}	{{0, 1, 0, 0, 1}, {0, 1, 0, 0, 1}}	{{1, $\frac{1}{2}$}, {1, $\frac{1}{2}$}}
{P1, P1, P1}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}	{{0, $\frac{1}{2}$}, {0, $\frac{1}{2}$}, {2
{P1, P1, P2}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}	{{0, $\frac{1}{2}$}, {0, $\frac{1}{2}$}, {2
{P1, P1, X}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}	{{0, $\frac{1}{2}$}, {0, $\frac{1}{2}$}, {2
{P1, P2, X}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}	{{0, $\frac{1}{2}$}, {0, $\frac{1}{2}$}, {2
{P2, P2, P1}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}	{{0, $\frac{1}{2}$}, {0, $\frac{1}{2}$}, {2
{P2, P2, P2}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}	{{0, $\frac{1}{2}$}, {0, $\frac{1}{2}$}, {2
{P2, P2, X}	{{1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}	{{0, $\frac{1}{2}$}, {0, $\frac{1}{2}$}, {2

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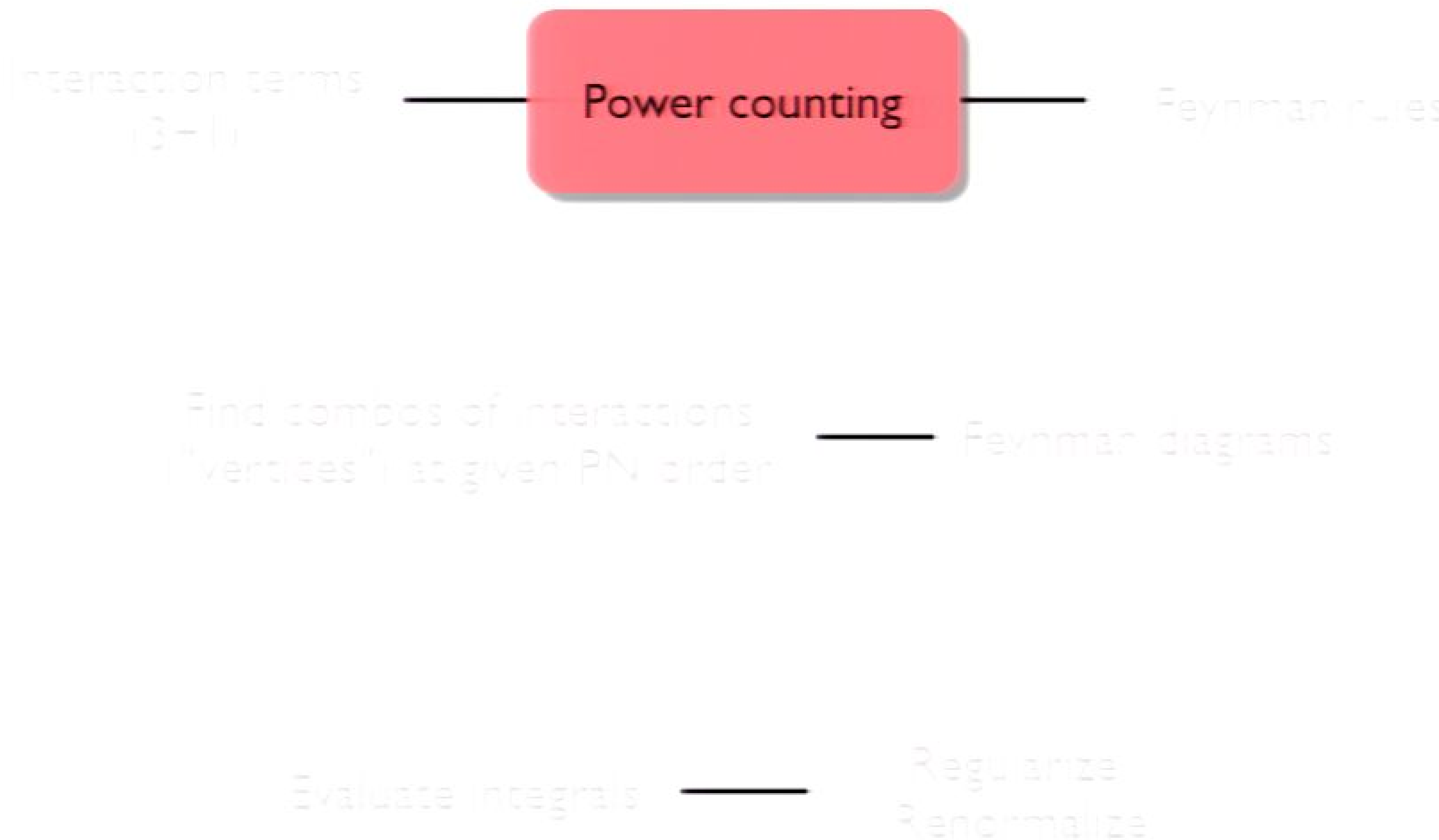
Demo_CapraNRDA_2010.nb

47.2939

	{{2, 1}}	$\{\frac{1}{8} m_1 v_i v^i v_j v^j\}$
	{{2, 1}}	$\{\frac{1}{8} m_2 v_i v^i v_j v^j\}$
, 0, 0, 0, 2}}	$\{\{0, \frac{1}{2}\}, \{2, \frac{1}{2}\}\}$	$\{-m_1 \phi^1, -\frac{3}{2} m_1 \phi^1 v_i v^i\}$
, 0, 0, 0, 2}}	$\{\{0, \frac{1}{2}\}, \{2, \frac{1}{2}\}\}$	$\{-m_1 \phi^1, -\frac{3}{2} m_2 \phi^1 v_i v^i\}$
, 0, 0, 0, 2}}	$\{\{0, \frac{1}{2}\}, \{2, \frac{1}{2}\}\}$	$\{-m_2 \phi^1, -\frac{3}{2} m_1 \phi^1 v_i v^i\}$
, 0, 0, 0, 2}}	$\{\{0, \frac{1}{2}\}, \{2, \frac{1}{2}\}\}$	$\{-m_2 \phi^1, -\frac{3}{2} m_2 \phi^1 v_i v^i\}$
, 1, 0, 0, 1}}	$\{\{1, \frac{1}{2}\}, \{1, \frac{1}{2}\}\}$	$\{m_1 A^{1i} v_i, m_1 A^{1i} v_i\}$
, 1, 0, 0, 1}}	$\{\{1, \frac{1}{2}\}, \{1, \frac{1}{2}\}\}$	$\{m_1 A^{1i} v_i, m_2 A^{1i} v_i\}$
, 1, 0, 0, 1}}	$\{\{1, \frac{1}{2}\}, \{1, \frac{1}{2}\}\}$	$\{m_2 A^{1i} v_i, m_2 A^{1i} v_i\}$
, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}	$\{\{0, \frac{1}{2}\}, \{0, \frac{1}{2}\}, \{2, 0\}\}$	$\{-m_1 \phi^1, -m_1 \phi^1, -\frac{1}{2} m_1 \phi^{12}\}$
, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}	$\{\{0, \frac{1}{2}\}, \{0, \frac{1}{2}\}, \{2, 0\}\}$	$\{-m_1 \phi^1, -m_1 \phi^1, -\frac{1}{2} m_2 \phi^{12}\}$
, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}	$\{\{0, \frac{1}{2}\}, \{0, \frac{1}{2}\}, \{2, 0\}\}$	$\{-m_1 \phi^1, -m_1 \phi^1, 4 n^\alpha n^\beta (\nabla_\alpha \phi^1)(\nabla_\beta \phi^1)\}$
, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}	$\{\{0, \frac{1}{2}\}, \{0, \frac{1}{2}\}, \{2, 0\}\}$	$\{-m_1 \phi^1, -m_2 \phi^1, 4 n^\alpha n^\beta (\nabla_\alpha \phi^1)(\nabla_\beta \phi^1)\}$
, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}	$\{\{0, \frac{1}{2}\}, \{0, \frac{1}{2}\}, \{2, 0\}\}$	$\{-m_2 \phi^1, -m_2 \phi^1, -\frac{1}{2} m_1 \phi^{12}\}$
, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}	$\{\{0, \frac{1}{2}\}, \{0, \frac{1}{2}\}, \{2, 0\}\}$	$\{-m_2 \phi^1, -m_2 \phi^1, -\frac{1}{2} m_2 \phi^{12}\}$
, 0, 0, 0, 0}, {2, 0, 0, 0, 0}}	$\{\{0, \frac{1}{2}\}, \{0, \frac{1}{2}\}, \{2, 0\}\}$	$\{-m_2 \phi^1, -m_2 \phi^1, 4 n^\alpha n^\beta (\nabla_\alpha \phi^1)(\nabla_\beta \phi^1)\}$


Interaction terms
Power counting
Feynman rules
Find combos of interactions ("vertices") at given PN order
Feynman diagrams
Evaluate integrals
Regularize
Renormalize

Choice of metric variables

Kol & Smolkin (2007) showed that a change of variables can dramatically simplify the calculation of potentials in the harmonic gauge.

$$H_{\mu\nu} \longrightarrow (\phi, A_i, \sigma_{ij})$$

Covariant variables:

$$g_{\mu\nu} = \eta_{\mu\nu} + H_{\mu\nu}$$

Kol-Smolkin (Kaluza-Klein) variables:

$$g_{\mu\nu} = \begin{pmatrix} e^{2\phi} & -e^{2\phi} A_j \\ -e^{2\phi} A_i & -e^{-2\phi}(\delta_{ij} + \sigma_{ij}) + e^{2\phi} A_i A_j \end{pmatrix}$$

$$= \eta_{\mu\nu} + H^{(1)}_{\mu\nu} + H^{(2)}_{\mu\nu} + \cdots$$

How to calculate with EFT

Goal: Calculate a potential that yields PN eom when extremized.

Start with the action: $S[g, x_1, x_2] = S_{EH}[g] + \sum_{n=1}^{2} S_{pp}[x_n, g]$

Expand in metric perturbation *H* and 3-velocities to get many interaction terms.

$$S[g, x_1, x_2] = S_{(0)}[H] + \frac{1}{2}\sum_{n=1}^{2}\int dt\, m_n \mathbf{v}_n^2 + \delta S[H, x_1, x_2]$$

Integrating out *H* from the action:

Extremize S to get wave equation for H

Solve for H (near-zone metric)

Plug solution back into S to get the effective action S_{eff}

A more efficient way is to use Feynman diagrams

Each interaction term has a definite power counting in *v* and *L* = *mvr*

$$-\frac{m_n}{2}\int dt\, H_{00}(t, \mathbf{x}_n(t)) \sim v^0 L^{1/2}$$

$$D_{\alpha\beta\gamma\delta}(t, t'; \mathbf{k}, \mathbf{q}) = \langle H_{\alpha\beta}(t; \mathbf{k}) H_{\gamma\delta}(t'; \mathbf{q})\rangle$$

How to calculate with EFT

Goal: Calculate a potential that yields PN eom when extremized.

Start with the action: $S[g, x_1, x_2] = S_{EH}[g] + \sum_{n=1}^{2} S_{pp}[x_n, g]$

Expand in metric perturbation *H* and 3-velocities to get many interaction terms.

$$S[g, x_1, x_2] = S_{(0)}[H] + \frac{1}{2}\sum_{n=1}^{2}\int dt\, m_n \mathbf{v}_n^2 + \delta S[H, x_1, x_2]$$

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