Title: Spin-induced bobbing effects in relativistic systems

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Abstract: Recent numerical simulations of spinning binary black holes have found that the orbital plane tends to bob up and down in phase with the orbit. It will be shown that similar effects occur in nearly all relativistic systems. The reasons for this are essentially kinematic and appear unrelated to those leading to the final "kicks" observed after merger. Simple examples are provided for binary systems bound together by gravitational electromagnetic and mechanical forces.

Spin-induced bobbing effects in relativistic systems

Abraham Harte

with Robert Wald and Samuel Gralla
University of Chicago

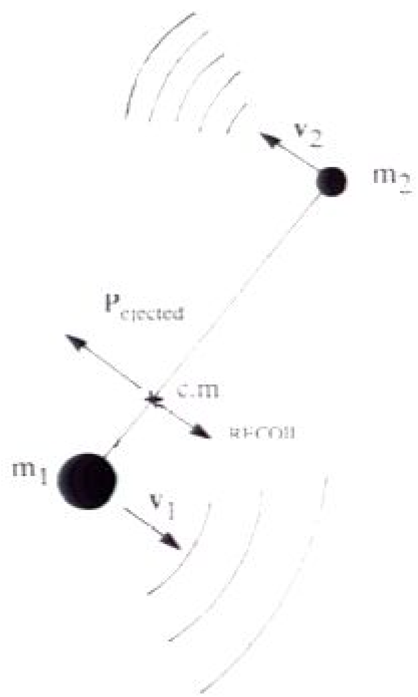
Based on: Phys. Rev. D **81**, 104012 (2010)

Outline

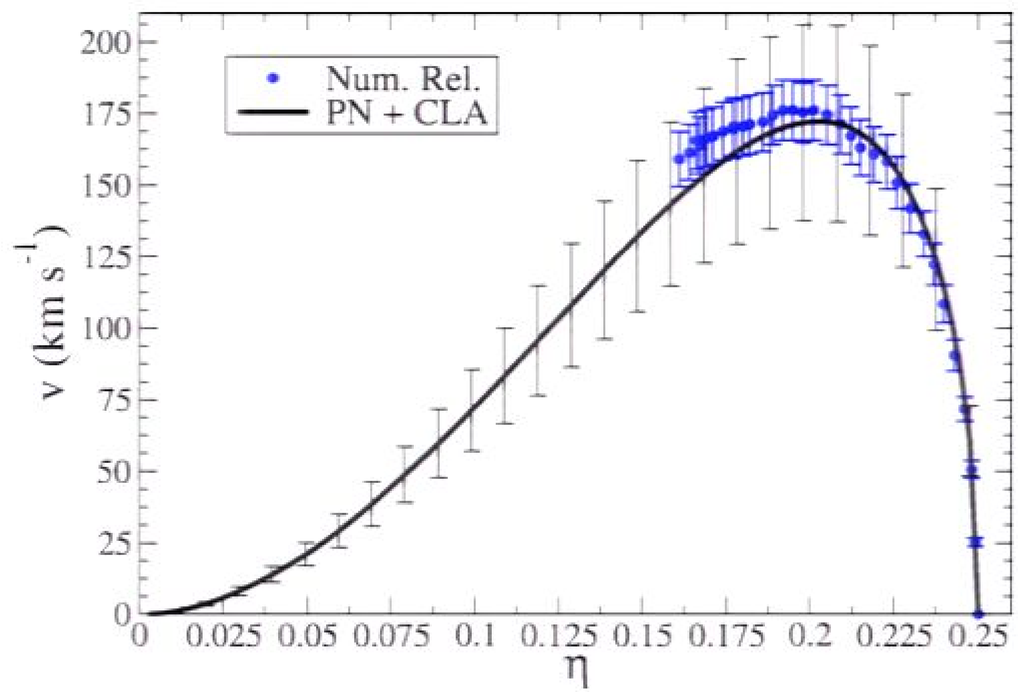
- Simulations of spinning black hole binaries
  - Post-merger kicks and superkicks
  - Pre-merger bobbing
- Mechanics of spinning particles
  - Centroids and mass centers
  - Hidden momentum and bobbing
- Some examples
- Implications for the relation between kicks and bobbing

Binary black hole kicks

Non-spinning black holes can fly off at up to ~150 km/s after merger:



from Wiseman (1992)

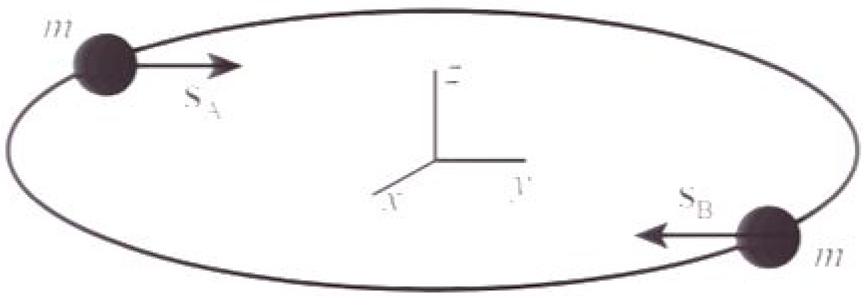


from Sopuerta, Yunes, and Laguna (2007)

Superkicks

Adding spin in the orbital plane gives recoils up to ~3,000 km/s.

Kicks are along the z-axis.



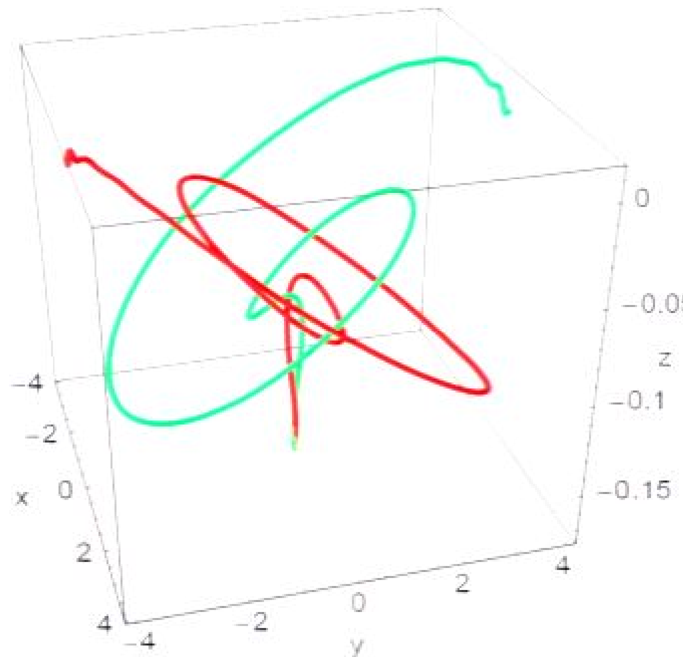
from Keppel et al. (2009)

- Magnitude varies sinusoidally with initial phase
- It's linear in spin
- Maximized for opposite spins and equal masses

Bobbing and superkicks

Spinning black holes bob up and down before merger.

Is the kick an inertial continuation of this? Is it due to frame-dragging?



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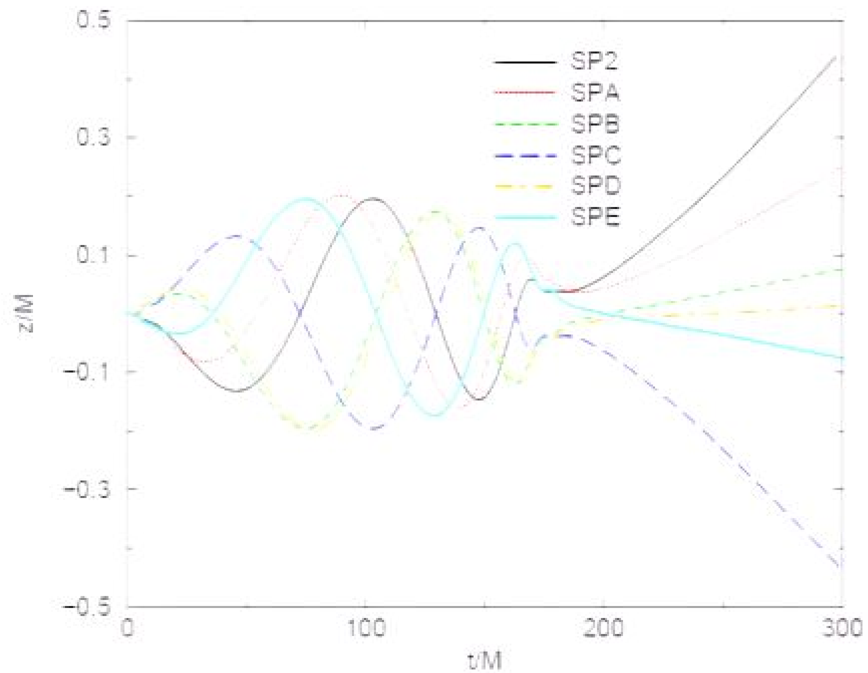

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Mechanics of test bodies

Look at some model systems without strong gravity.

Define momentum, e.g.,

$$p^{\mu}(z,\Sigma) = \int_{\Sigma} T^{\mu\nu} \mathrm{d}S_{\nu}, \qquad S^{\mu\nu}(z,\Sigma) = 2\int_{\Sigma} (x-z)^{[\mu} T^{\nu]\lambda} \mathrm{d}S_{\lambda}$$

More generally, use Dixon's definitions [1974].

Define a centroid such that the mass dipole vanishes in an appropriate frame:

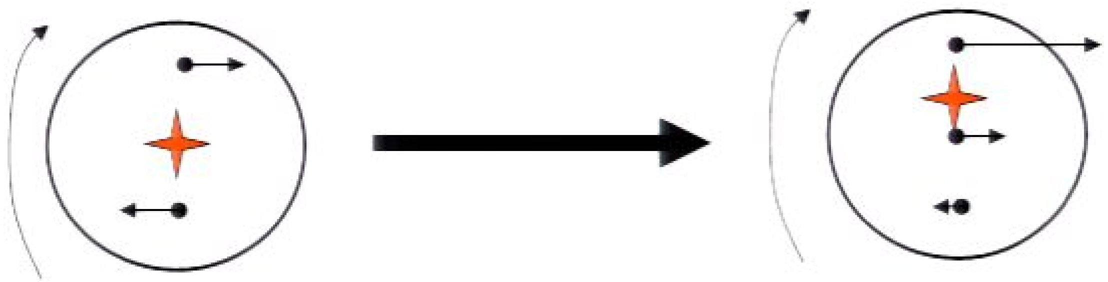
$$S_{ab}(\xi,\Sigma)u^{b} = 0$$

$$S^{i0}(\xi, t = \text{const.}) = \int (x-\xi)^{i} T^{00} \mathrm{d}^{3}x = \int x^{i} T^{00} \mathrm{d}^{3}x - p^{0}\xi^{i} = 0$$

Different centroids

Which u^a? This matters!

$$m\Delta \mathbf{z} \simeq \Delta \mathbf{u} \times \mathbf{S}$$



"Lab frame" centroid $u = \partial/\partial t$?

- Simple
- Useful for global conservation laws (collisions, etc.)
- Not unique
- Need something intrinsic

Center of mass

Use only intrinsic properties of the object to define a unique center of mass:

$$S_{ab}(z_{CM}, \Sigma_{CM}) p^b(z_{CM}, \Sigma_{CM}) = 0; \qquad p^a(z_{CM}, \Sigma_{CM}) \perp \Sigma_{CM}$$

In general, there is a "hidden momentum:"

$$p^a_{CM} \neq m \dot{z}^a_{CM} \Rightarrow \frac{\mathrm{d}p^a_{CM}}{\mathrm{d}s} \neq m \ddot{z}^a_{CM}$$

Be careful with notions of "force!"

Hidden momentum

$$0 = \frac{\mathrm{d}}{\mathrm{d}s}(S_{ab}p^b) = (2p_{[a}\dot{z}_{b]} + N_{ab})p^b + S_{ab}(F^a - \frac{1}{2}R^b{}_{cdf}\dot{z}^c S^{df})$$

Solved exactly for $\dot{z}^a$ by Ehlers and Rudolph [1977]. Approximate solution is easy:

$$m\dot{\bar{z}} \simeq \bar{p} + \overbrace{\bar{S} \times (\bar{F}/m)}^{\text{From setting dipole moment to zero in a non-inertial frame}} - \bar{n}$$

$$n^a \equiv N^a{}_b(p^b/m)$$

e.g. $\bar{\mu} \times \bar{E}$

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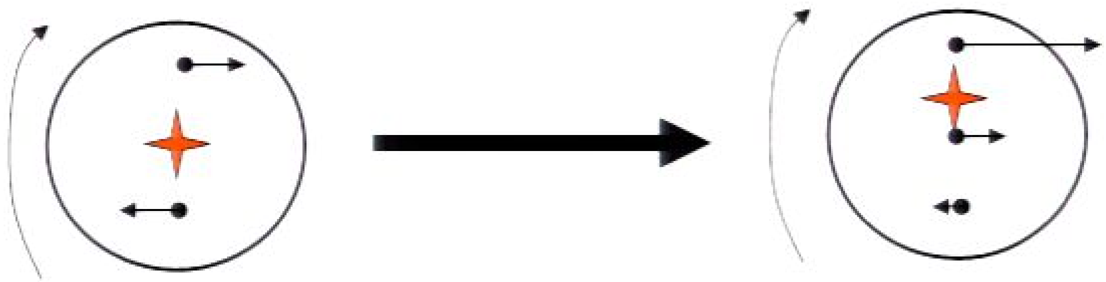

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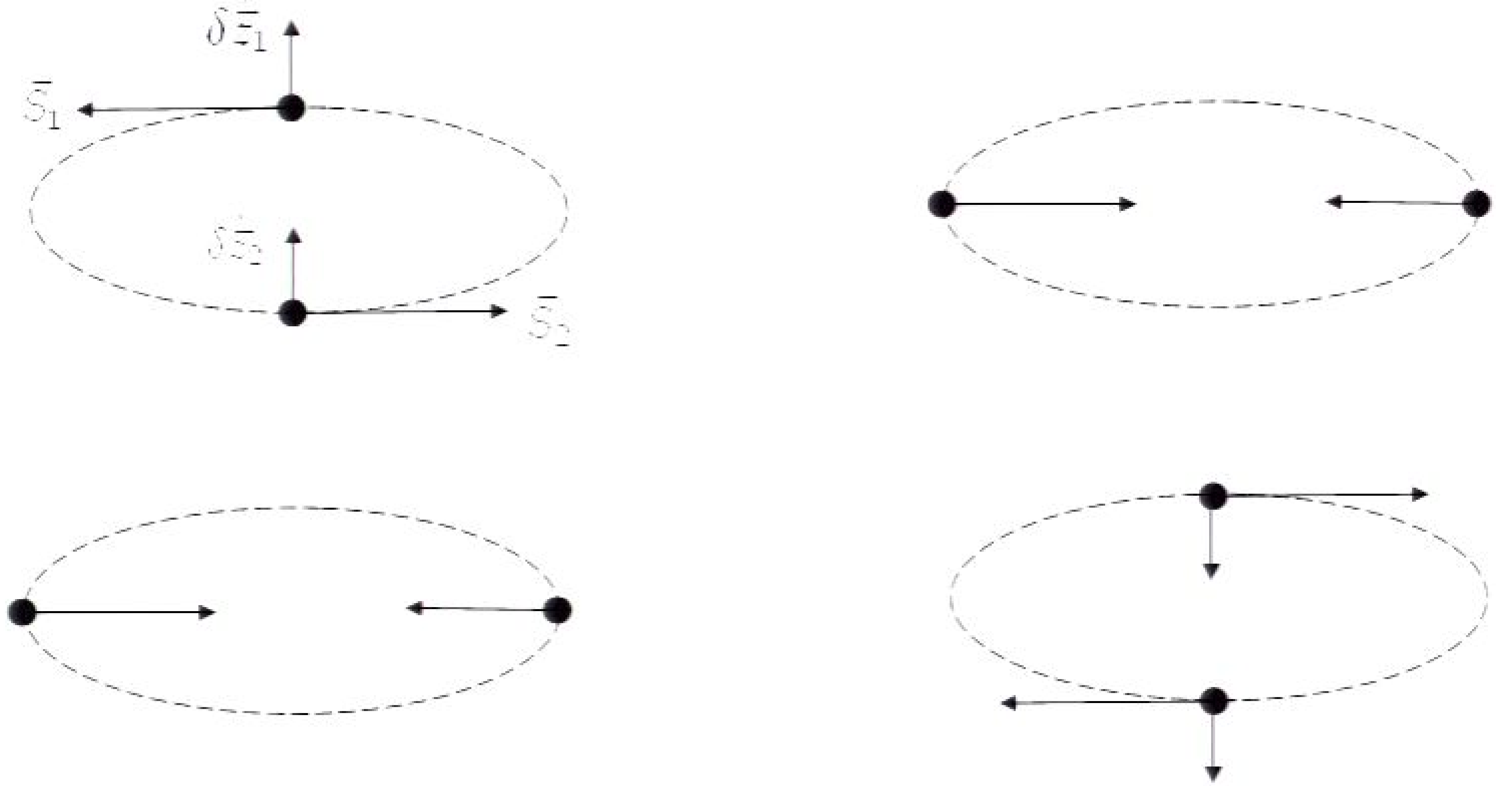
Spin-induced bobbing

$$m\delta\dot{\vec{z}} \simeq \vec{S} \times (\vec{F}/m)$$



Tetherballs

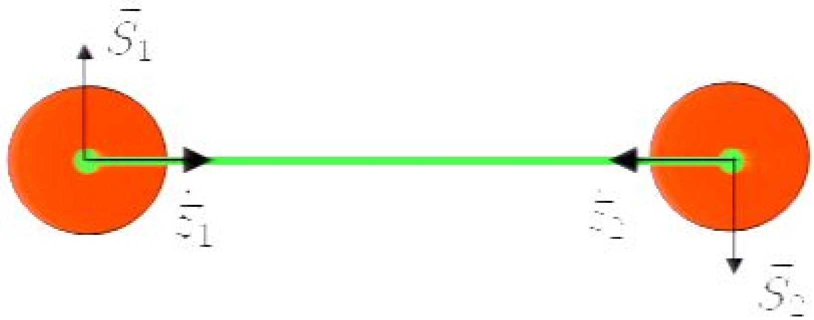
Net bobbing:

$$(m_1 \dot{z}_1 + m_2 \dot{z}_2) \cdot L = \left[\left(\frac{\bar{S}_1}{m_1} - \frac{\bar{S}_2}{m_2} \right) \times \bar{F}_1 \right] \cdot L$$

Anti-parallel spins in the orbital plane maximize bobbing.

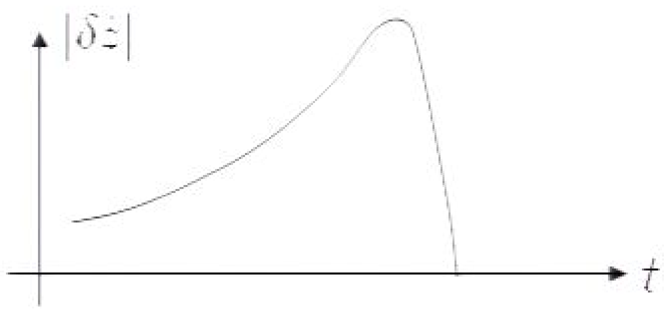
Nothing analogous to frame-dragging here. Just standard SR.

Radial infall

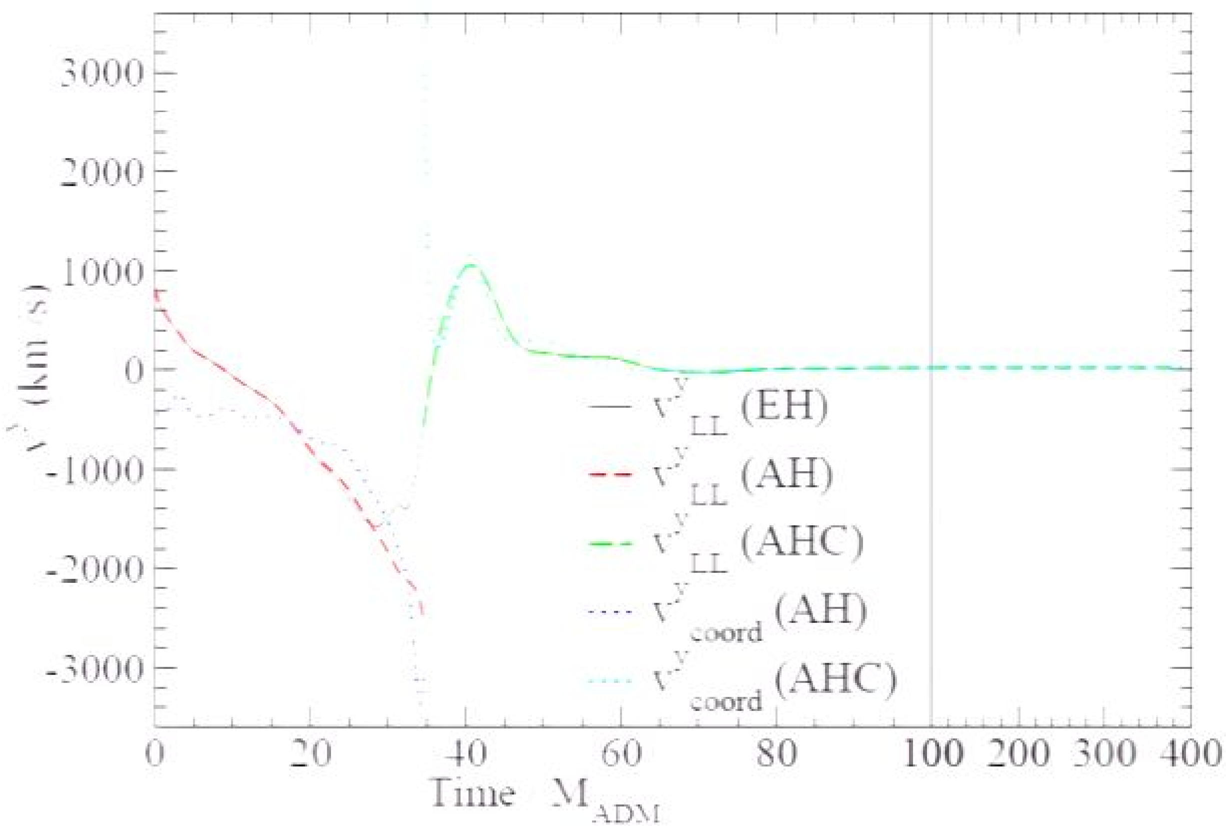


Increasing velocity into the screen as the bodies fall together.

After collision,
- hidden momentum vanishes
- overall momentum is conserved
- motion into the screen abruptly stops



Qualitatively similar behavior observed for the radial infall of two spinning black holes (Lovelace et al. [2009]).



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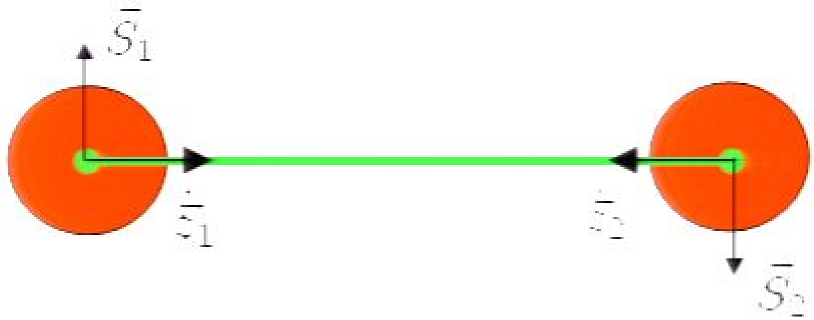

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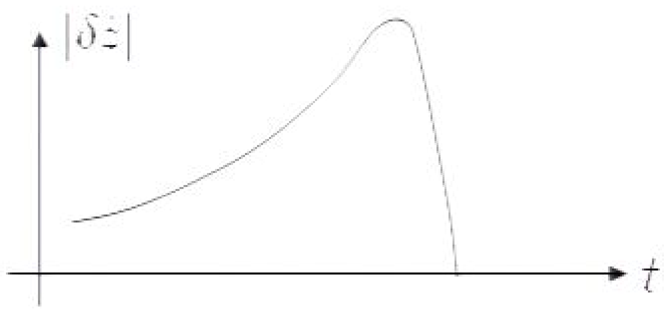

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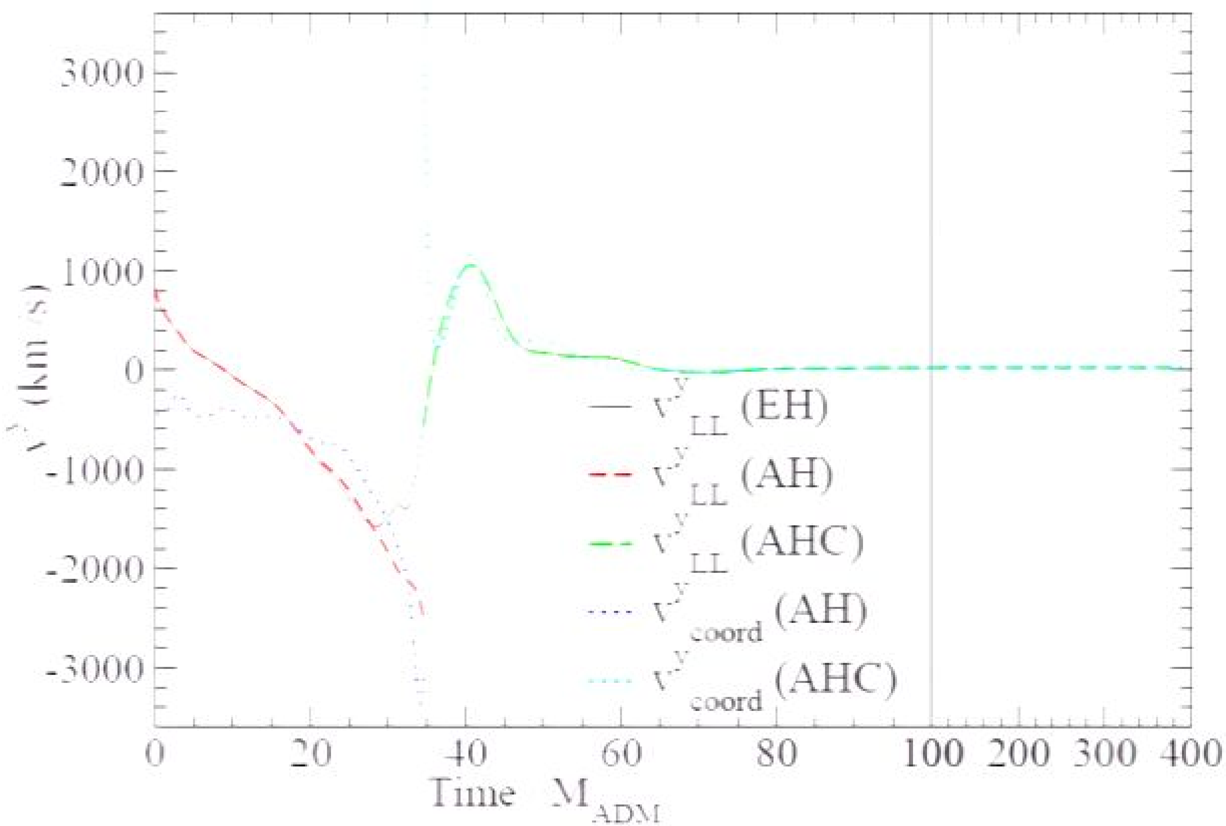

Universality

Bobbing comes from the $\bar{S} \times (\bar{F}/m)$ part of the hidden momentum. What about the other portion?

$$m\dot{\bar{z}} \simeq \bar{p} + \bar{S} \times (\bar{F}/m) + \bar{n}$$

It doesn't matter for the "net" bobbing. Canceled out by magnetic effects. Tetherball result with $\bar{n} = 0$ is preserved in EM (with $\bar{n} \simeq \bar{\mu} \times \bar{E}$) and post-Newtonian GR (with some terminology/interpretation caveats).

$$(m_1 \dot{\bar{z}}_1 + m_2 \dot{\bar{z}}_2) \cdot L = \left[\left(\frac{\bar{S}_1}{m_1} - \frac{\bar{S}_2}{m_2} \right) \times \bar{F}_1 \right] \cdot L$$

Maximum amplitude for quasicircular orbits: $\delta z_{\max} \sim \frac{|\bar{S}|}{\sqrt{mr}}$

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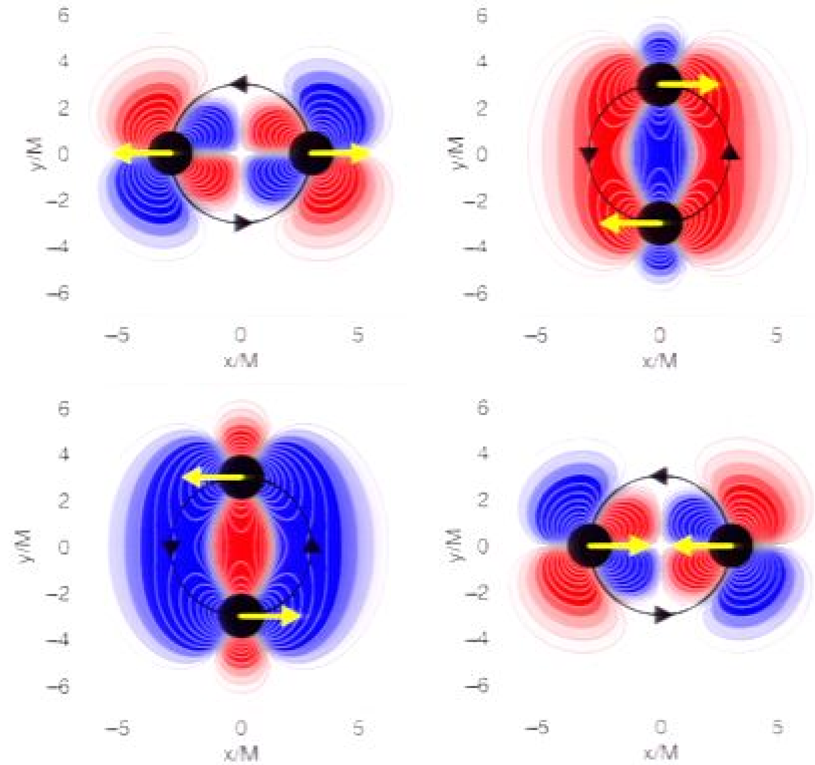
Why: Field momentum?

"Self-external" cross term in the field momentum oscillates, so bodies oscillate in response?

No! Field momentum $\bar{\mu} \times \bar{E}$ exchanges with the "dynamical" portion of the hidden mechanical momentum $-\bar{\mu} \times \bar{E}$.

Net "kinetic mechanical momentum" $m\dot{\bar{z}}$ is not directly affected.



From Keppel et al. [2009] (for PN field momentum, but it's the same in EM)

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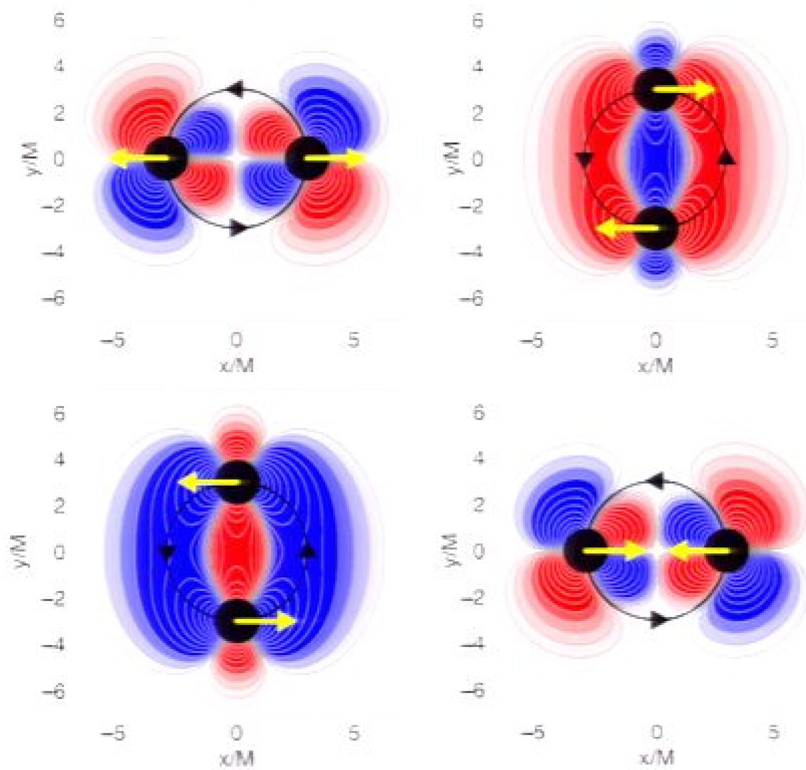

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Kicks

Does this effect have anything to do with kicks?

- Clearly no in the tetherball case
- In EM, probably not:
  - Kicks are related to radiated momentum
  - Quasi-static field momentum right before merger is controlled by magnetic dipole moments.
  - Spin probably has only a weak effect on radiated momentum, yet it almost entirely controls the bobbing.
- In gravity, everything is controlled by the spin. Bobbing and kicks will probably be correlated, so attributing a clear cause might be a matter of taste…

Conclusions

- There are spin-induced bobbing effects in almost all relativistic systems. They have nothing in particular to do with frame dragging or curved spacetime.
- Like $(\bar{E}, \bar{B}) - F_{ab}$, spin and CM position/mass dipole moment are two components of the same mathematical object. They must interact when decomposed wrt to an accelerating frame. This is just kinematics.
- Qualitatively similar effects seem to be observed in numerical simulations of spinning black holes.
- Kicks appear mostly unrelated (but there's room for argument in the gravitational case).

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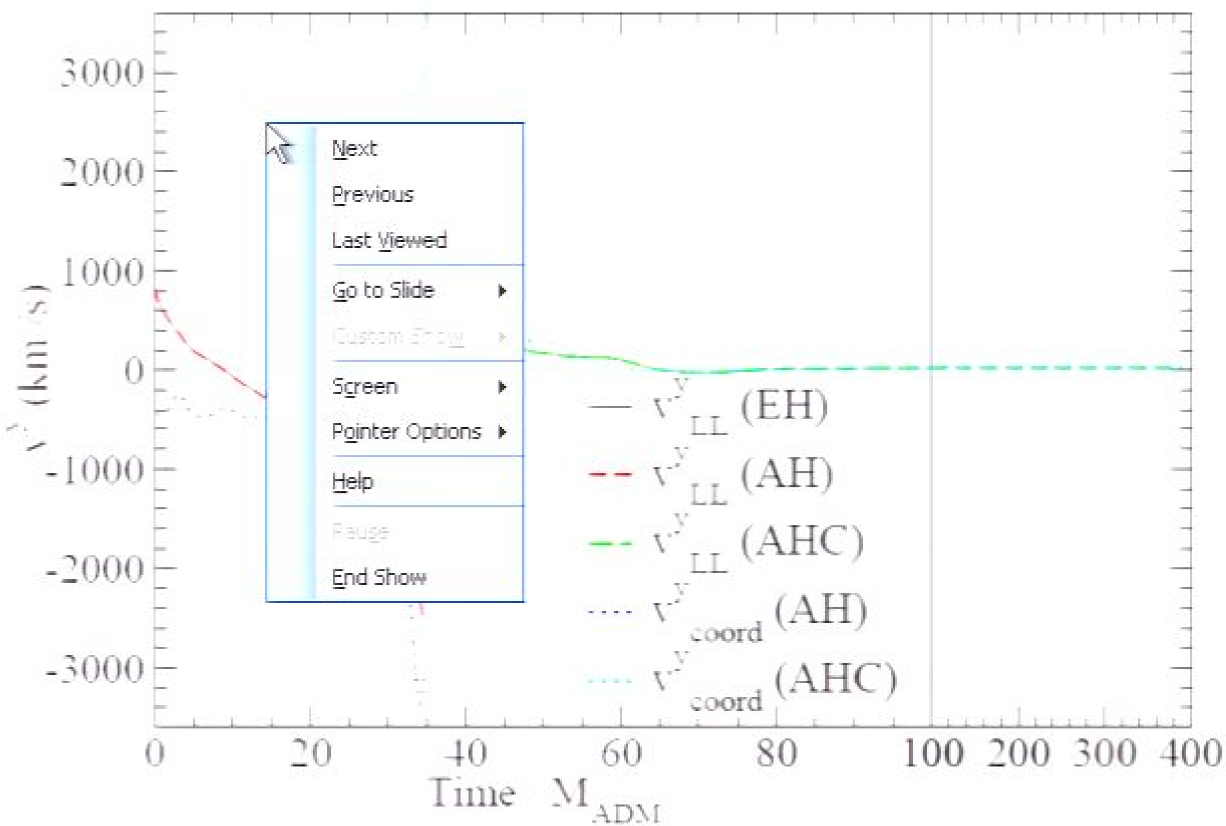

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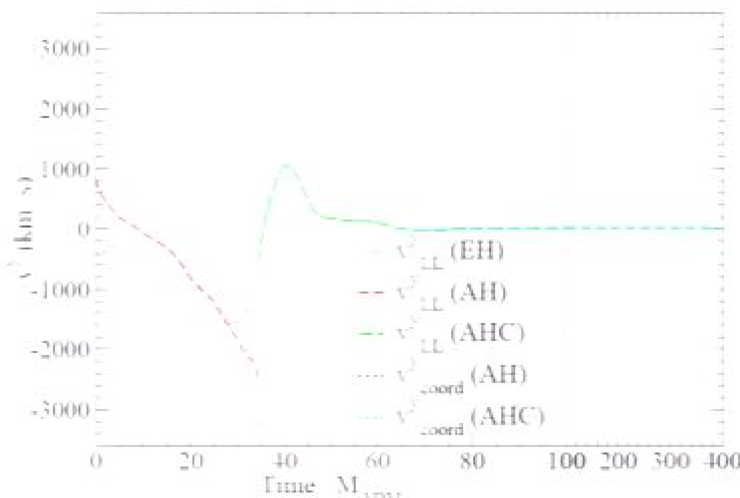

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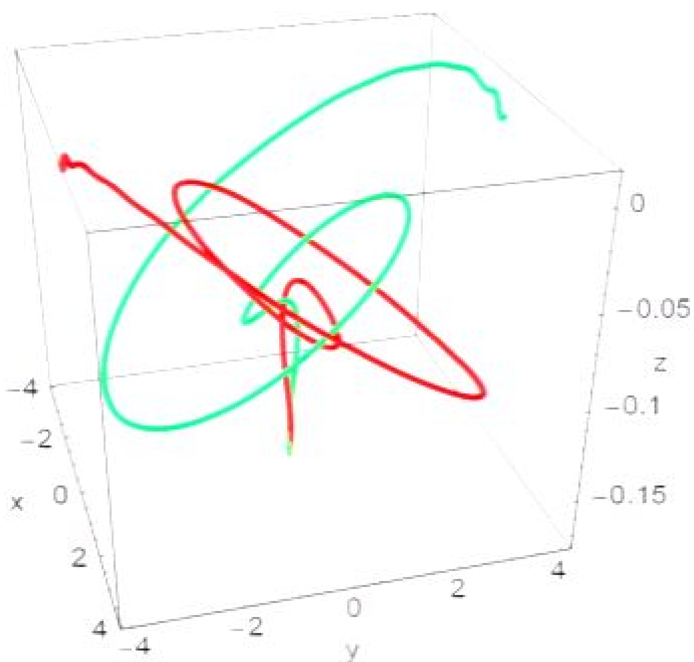

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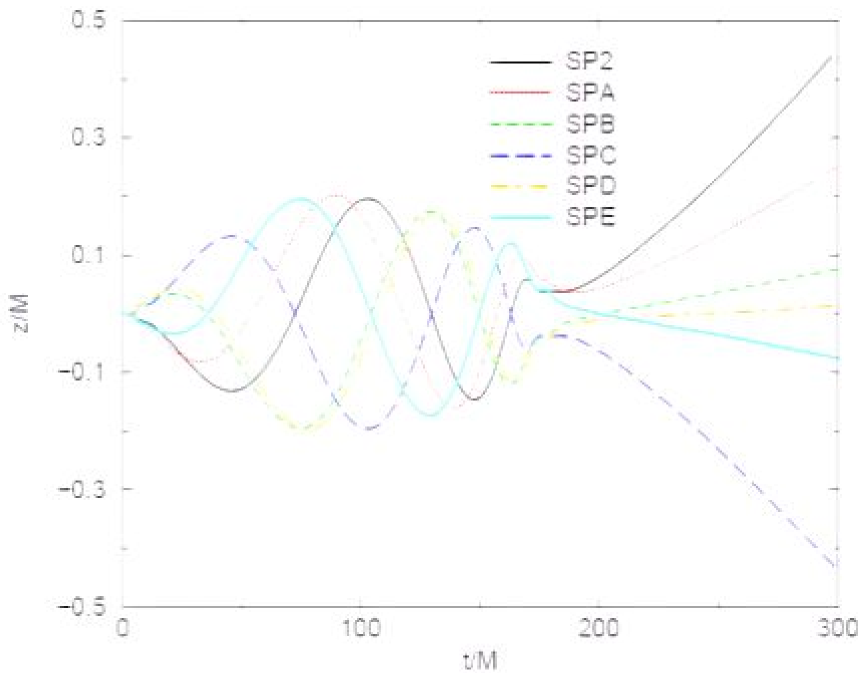

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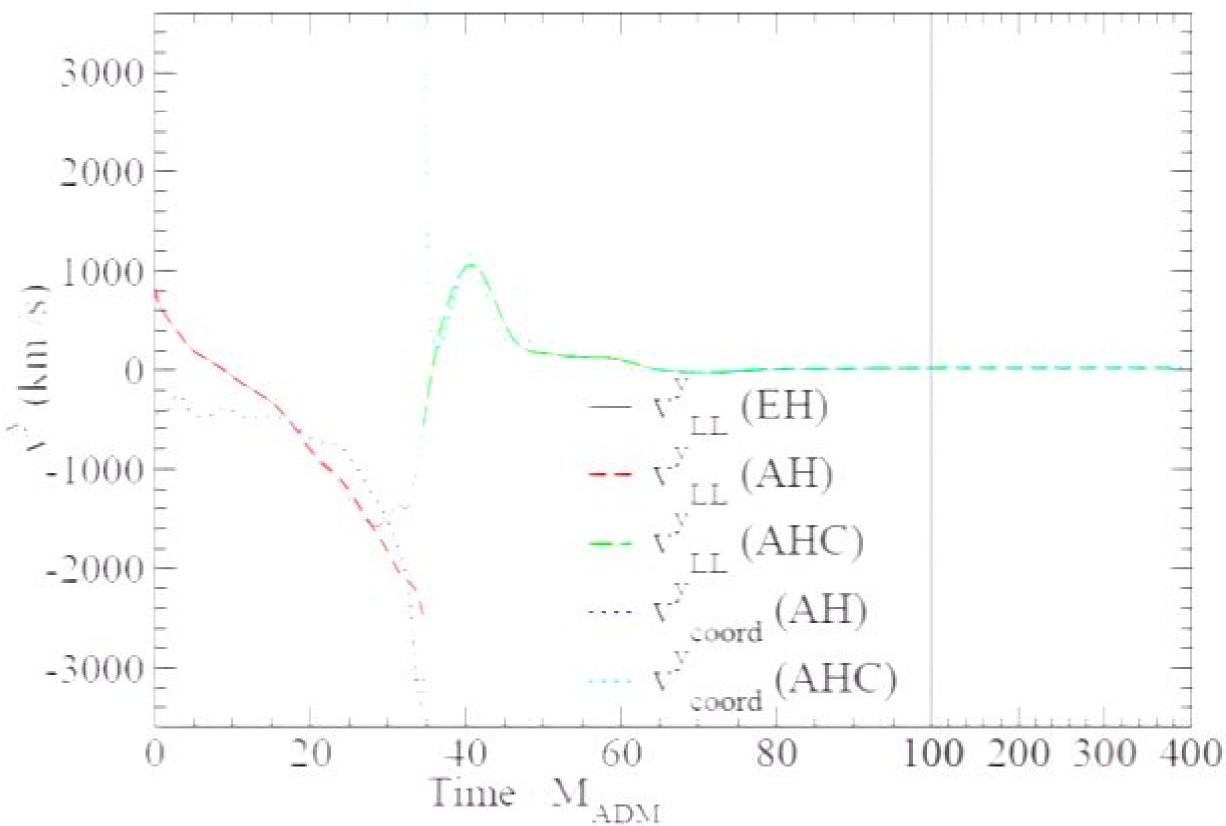

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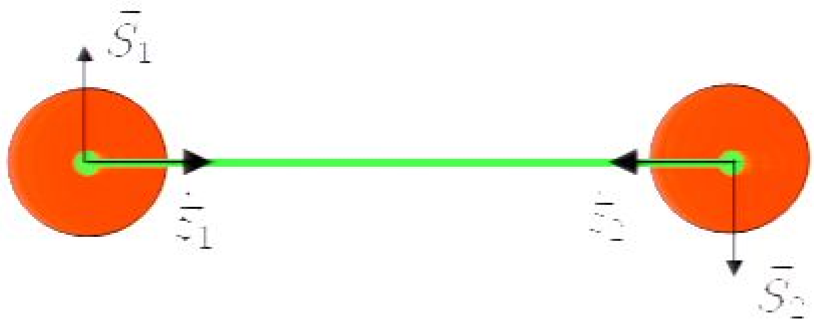

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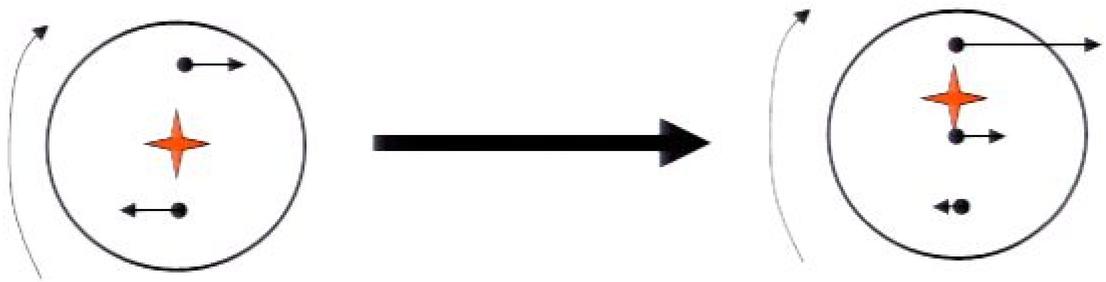

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