Title: Perturbative effects of spinning black holes with applications to full numerical relativity results

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Abstract: We present a second order perturbative formalism that includeperturbative spin effects and apply it to the computation of recoil velocites of merging binary black holes and to the computation of waveforms from small mass ratio binaries.

Perturbative effects of spinning black holes with applications to full numerical relativity results

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1. Introduction

Spin effect on

A. Gravitational recoil

B. Gravitational wave

for black hole binaries
in the black hole perturbation approach.

A. Recoil: Linear momentum flux for binaries (analytic expression)

[Kidder (1995), Racine, Buonanno and Kidder (2008)]
PN approach, inspiral

[Mino and Brink (2008)]
BHP approach, near-horizon (but low frequency)

We (can) use

[Sago et al. (2005, 2007)]
BHP approach (dE/dt, dL/dt, dC/dt for periodic orbits)

* **<u>Spin as a perturbation in the BHP approach</u>**
(Schwarzschild background)

B. Gravitational waveforms (related to NR)

[Lousto et al. (2010)]
To compute waveforms for large mass ratio cases:
use of nonlinear numerical (NR) trajectories
and efficient perturbative (BHP) evolutions

Generally, perturbations using the Teukolsky equations
(Kerr background, 2+1 dimension)

<Non-spinning BH binaries with intermediate mass ratio>

1. Inspiral: Schwarzschild + test particle

2. After merger, one BH with a spin

* **BHP approach with the Schwarzschild background**

2. Regge-Wheeler-Zerilli formalism

Metric perturbation in the Schwarzschild background

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + h^{(1)}_{\mu\nu} \,,$$

Regge-Wheeler-Zerilli formalism

$$\begin{aligned} \boldsymbol{h}^{(1)} &= \sum_{\ell m} \Bigg[f(r) H_{0\ell m}(t,r) \boldsymbol{a}^{(0)}_{\ell m} - i\sqrt{2} H_{1\ell m}(t,r) \boldsymbol{a}^{(1)}_{\ell m} + \frac{1}{f(r)} H_{2\ell m}(t,r) \boldsymbol{a}_{\ell m} \\ &\quad - \frac{i}{r}\sqrt{2\ell(\ell+1)} h^{(e)}_{0\ell m}(t,r) \boldsymbol{b}^{(0)}_{\ell m} + \frac{1}{r}\sqrt{2\ell(\ell+1)} h^{(e)}_{1\ell m}(t,r) \boldsymbol{b}_{\ell m} \\ &\quad + \sqrt{\frac{1}{2}\ell(\ell+1)(\ell-1)(\ell+2)} G_{\ell m}(t,r) \boldsymbol{f}_{\ell m} + \left(\sqrt{2} K_{\ell m}(t,r) - \frac{\ell(\ell+1)}{\sqrt{2}} G_{\ell m}(t,r) \right) \boldsymbol{g}_{\ell m} \\ &\quad - \frac{\sqrt{2\ell(\ell+1)}}{r} h_{0\ell m}(t,r) \boldsymbol{c}^{(0)}_{\ell m} + \frac{i\sqrt{2\ell(\ell+1)}}{r} h_{1\ell m}(t,r) \boldsymbol{c}_{\ell m} \\ &\quad + \frac{\sqrt{2\ell(\ell+1)(\ell-1)(\ell+2)}}{2r^2} h_{2\ell m}(t,r) \boldsymbol{d}_{\ell m} \Bigg] . \end{aligned}$$

a_lm, **b**_lm: tensor harmonics (angular function)

Regge-Wheeler-Zerilli formalism

* Gravitational waves in the asymptotic flat gauge:

$$\mathbf{h}^{(i)} = \sum_{\ell m} \left[\left[\frac{1}{2} \ell(\ell+1)(\ell-1)(\ell+2) \right]^{1/2} G^{(i)AF}_{\ell m}(t,r) \mathbf{f}_{\ell m} + \frac{[2\ell(\ell+1)(\ell-1)(\ell+2)]^{1/2}}{2r^2} h^{(i)AF}_{2\,\ell m}(t,r) \mathbf{d}_{\ell m} \right]$$

$$G^{(i)AF}_{\ell m}(t,r) = \frac{1}{r} \psi^{(i)(even)}_{\ell m}(t,r)\,. \quad \text{Zerilli function}$$

$$h^{(i)AF}_{2\,\ell m}(t,r) = i\, r\, \psi^{(i)(odd)}_{\ell m}(t,r)\,. \quad \text{Regge-Wheeler function}$$

$$\left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^{*2}} - V^{\rm even}_{\ell}(r) \right] \psi^{\rm even}_{\ell m}(t,r) = S^{\rm even}_{\ell m}(t,r)\,.$$

$$\left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^{*2}} - V^{\rm odd}_{\ell}(r) \right] \psi^{\rm odd}_{\ell m}(t,r) = S^{\rm odd}_{\ell m}(t,r)\,.$$

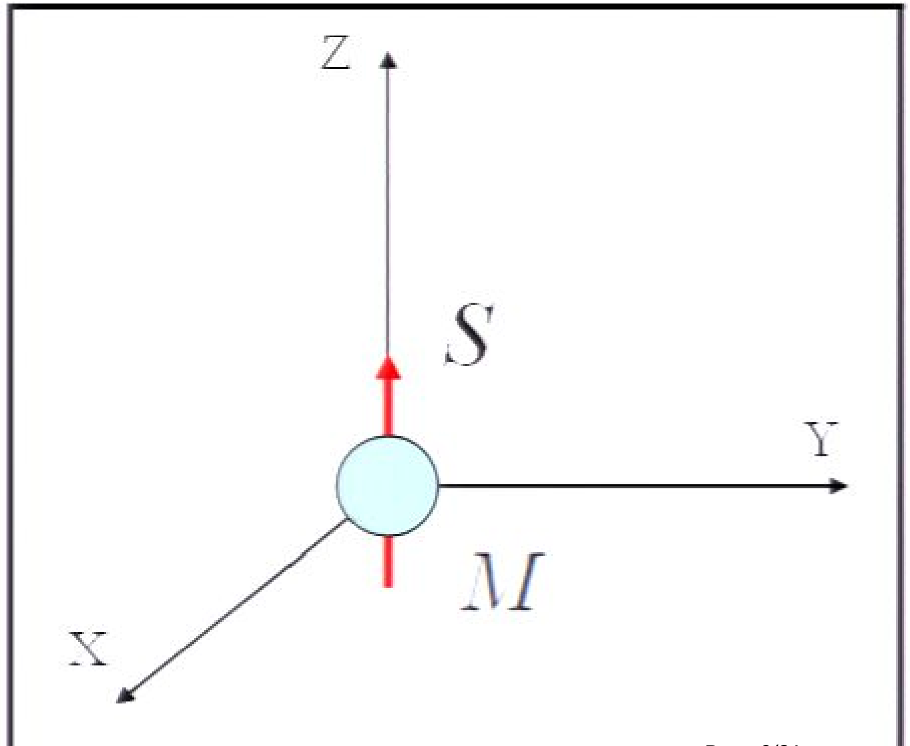
$$r^* = r + 2M \log(r/2M - 1)$$

3. Spin as a perturbation

Kerr metric in the Boyer-Lindquist coordinates,

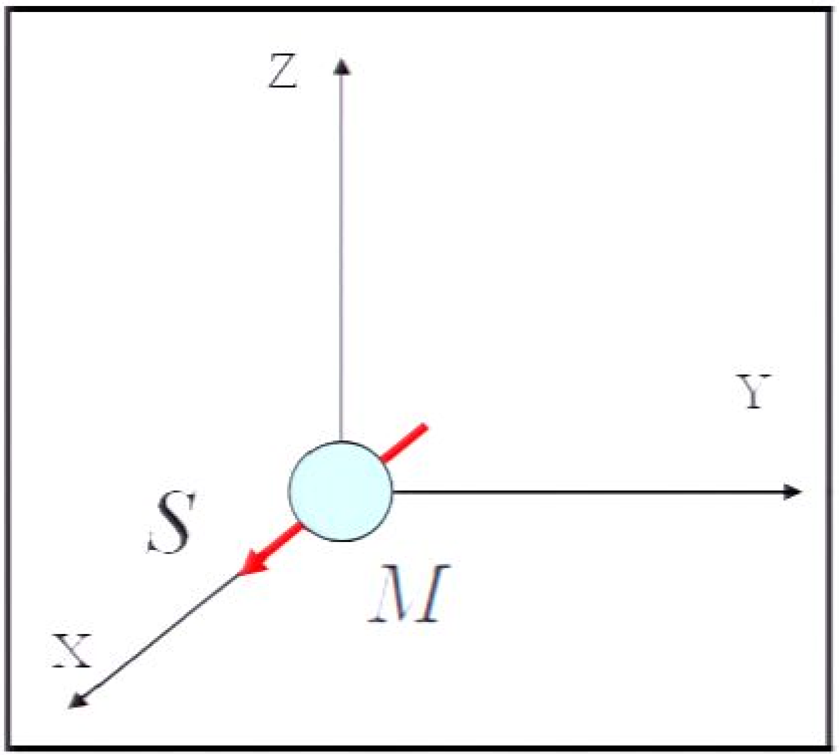
$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2+\left(1-\frac{2M}{r}\right)^{-1}dr^2+r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)$$
$$-\frac{4Ma}{r}\sin\theta^2 dt d\phi+O(a^2)\,.$$

in the Taylor expansion with respect to $a=S/M$.



$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$
$$+ \frac{4Ma}{r} dt \left(\sin\phi d\theta + \sin\theta\cos\theta\cos\phi d\phi\right) + O(a^2)\,.$$

Background Schwarzschild
+ perturbation



$$g_{\mu\nu} = g^{\rm Sch}_{\mu\nu} + h^{(1)S}_{\mu\nu}:$$
$$h^{(1)S}_{t\theta} = h^{(1)S}_{\theta t} = \frac{2S_x}{r}\sin\phi\,.$$
$$h^{(1)S}_{t\phi} = h^{(1)S}_{\phi t} = \frac{2S_x}{r}\sin\theta\cos\theta\cos\phi\,.$$

$$(Sx = M\,a)$$

$$h^{(1)S}_{t\theta} = h^{(1)S}_{\theta t} = \frac{2S_x}{r}\sin\phi\,.$$

$$h^{(1)S}_{t\phi} = h^{(1)S}_{\phi t} = \frac{2S_x}{r}\sin\theta\cos\theta\cos\phi\,.$$

Tensor harmonics expansion for the perturbation:

$$\mathbf{h}^{(1)S} = \sum_{\ell m}\left[-\frac{\sqrt{2\ell(\ell+1)}}{r}h^{(1)S}_{0\,\ell m}(t,r)\mathbf{c}_{0\,\ell m}\right]\,.$$

$$\mathbf{c}_{0\,\ell m} = r[2\ell(\ell+1)]^{-1/2}\begin{pmatrix} 0 & 0 & (1/\sin\theta)(\partial/\partial\phi)Y_{\ell m} & -\sin\theta(\partial/\partial\theta)Y_{\ell m} \\ 0 & 0 & 0 & 0 \\ \mathrm{Sym} & 0 & 0 & 0 \\ \mathrm{Sym} & 0 & 0 & 0 \end{pmatrix},$$

L = 1, m = +1/-1 odd parity modes

$$h^{(1)S}_{0\,1\,1}(t,r) = -\sqrt{\frac{8\pi}{3}}\frac{S_x}{r}\,.$$

$$h^{(1)S}_{0\,1\,-1}(t,r) = \sqrt{\frac{8\pi}{3}}\frac{S_x}{r}\,.$$

4. Gravitational Recoil

Linear momentum loss:

$$\dot{P}_x = -\frac{1}{32\pi}\int d\Omega\, r^2 \sin\theta\cos\phi\, \left\langle h^{\alpha\beta}{}_{:t} h_{\alpha\beta:t}\right\rangle_{\rm TT}\,,$$

$$\dot{P}_y = -\frac{1}{32\pi}\int d\Omega\, r^2 \sin\theta\sin\phi\, \left\langle h^{\alpha\beta}{}_{:t} h_{\alpha\beta:t}\right\rangle_{\rm TT}\,,$$

$$\dot{P}_z = -\frac{1}{32\pi}\int d\Omega\, r^2 \cos\theta\, \left\langle h^{\alpha\beta}{}_{:t} h_{\alpha\beta:t}\right\rangle_{\rm TT}\,.$$

After the angular integration,

$$\dot{P}_i = -\frac{1}{64\pi}\sum_{\ell m}\sum_{\ell' m'}\Bigg[\left(r^2\,\dot{G}^{(i)AF}_{\ell m}(t,r)\dot{G}^{(i')AF}_{\ell' m'}(t,r) - \frac{1}{r^2}\dot{h}^{(i)AF}_{2\,\ell m}(t,r)\dot{h}^{(i')AF}_{2\,\ell' m'}(t,r)\right)P^S_i(\ell m\ell' m')$$
$$+i\left(\dot{G}^{(i)AF}_{\ell m}(t,r)\dot{h}^{(i')AF}_{2\,\ell' m'}(t,r) - \dot{h}^{(i)AF}_{2\,\ell m}(t,r)\dot{G}^{(i')AF}_{\ell' m'}(t,r)\right)P^C_i(\ell m\ell' m')\,.$$

$$G^{(i)AF}_{\ell m}(t,r) = \frac{1}{r}\,\psi^{(i)(even)}_{\ell m}(t,r)\,,$$
$$h^{(i)AF}_{2\,\ell m}(t,r) = i\,r\,\psi^{(i)(odd)}_{\ell m}(t,r)\,.$$

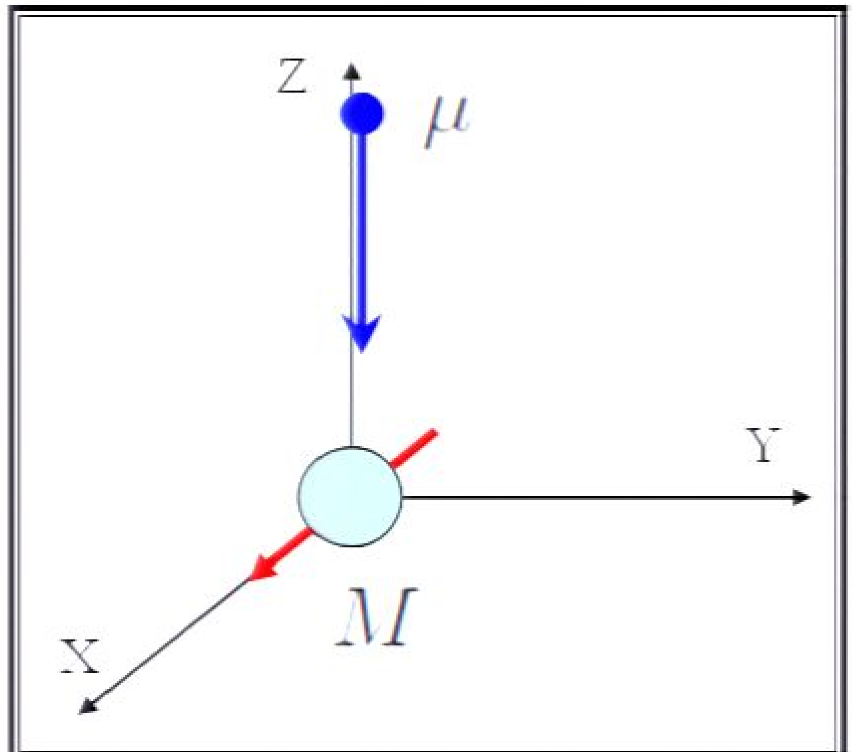
* We calculate the Regge-Wheeler and Zerilli functions.

Particle falling radially into a Schwarzschild black hole

$$\left(\frac{dR}{dt}\right)^2 = -\left(1-\frac{2M}{R}\right)^3\frac{1}{E^2}+\left(1-\frac{2M}{R}\right)^2 .$$

E: particle's energy

$$T^{\alpha\beta}_{mass} = \mu\int d\tau u^\alpha u^\beta \frac{\delta^{(4)}(x-z(\tau))}{\sqrt{-g}} .$$



Slow motion approximation

$dR/dt \sim v$, $M/R \sim v^2$, $v << 1$

Tensor harmonics expansion of the energy-momentum tensor:

$$\mathcal{A}^{(1)}_{\ell m}(t,r) = \mu \frac{E\,R(t)}{R(t)-2M}\left(\frac{dR}{dt}\right)^2 \frac{1}{(r-2M)^2}\delta(r-R(t))Y^*_{\ell m}(0,0)\ .$$

$$\mathcal{A}^{(1)}_{0\,\ell m}(t,r) = \mu \frac{E\,R(t)}{R(t)-2M}\frac{(r-2M)^2}{r^4}\delta(r-R(t))Y^*_{\ell m}(0,0)\ .$$

$$\mathcal{A}^{(1)}_{1\,\ell m}(t,r) = \sqrt{2}i\mu \frac{E\,R(t)}{R(t)-2M}\frac{dR}{dt}\frac{1}{r^2}\delta(r-R(t))Y^*_{\ell m}(0,0)\ .$$

1st order L = 2, m = 0, even parity mode (GW)

$$\psi^{(1)(even)}_{20}(t,r) = \frac{16\pi}{15}\mu\left(\dot{R}^2-\frac{M}{R}\right)Y^*_{20}(0,0)\ .$$

1st order L = 3, m = 0, even parity mode (GW)

$$\psi^{(1)(even)}_{30}(t,r) = \frac{16\pi}{105}\mu\left(\dot{R}^3-\frac{2M}{R}\dot{R}\right)Y^*_{30}(0,0)\ .$$

1st order L = 1, m = 0, even parity mode (not GW mode)

$$H_{0\,10}^{(1)D}(t,r) = \frac{8\pi\mu E}{3M}\frac{R(t)^3}{(R(t)-2M)^2}\left(\frac{dR(t)}{dt}\right)^2 Y_{10}^*(0,0)\,\delta(r-R(t))\,.$$

$$H_{1\,10}^{(1)D}(t,r) = -\frac{8\pi\mu E}{3M}\frac{R(t)^2}{R(t)-2M}\frac{dR(t)}{dt}Y_{10}^*(0,0)\,\delta(r-R(t))\,.$$

$$H_{2\,10}^{(1)D}(t,r) = \frac{8\pi\mu E}{3M}R(t)Y_{10}^*(0,0)\,\delta(r-R(t))\,.$$

$$h_{0\,10}^{(1)(e)D}(t,r) = -\frac{4\pi\mu E}{3M}R(t)^2\frac{dR(t)}{dt}Y_{10}^*(0,0)\,\delta(r-R(t))\,.$$

$$h_{1\,10}^{(1)(e)D}(t,r) = \frac{4\pi\mu E}{3M}R(t)^2 Y_{10}^*(0,0)\,\delta(r-R(t))\,.$$

$$K_{10}^{(1)D}(t,r) = 0\,.$$

Zero except for the particle

"Low multipole contributions" [Detweiler and Poisson (2004)]

2nd order L = 2, m = +1/-1, odd parity mode

$$G^{(1)}_{\mu\nu}[h^{(2)}] = 8\pi T^{(2)}_{\mu\nu} - G^{(2)}_{\mu\nu}[h^{(1)}, h^{(1)}].$$

Leading order

Black hole's Spin [L = 1, m = +1/-1 odd parity mode]
and Particle [L = 1, m = 0 even parity mode]

$$\left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^{*2}} - 6\,\frac{(r-2M)(r-M)}{r^4}\right] \psi^{(2)(odd)}_{2\pm1}(t,r) = \mathcal{S}^{\rm RW}_{2\pm1}(t,r)\,.$$

$$\mathcal{S}^{\rm RW}_{2\pm1} = \pm\frac{\sqrt{10}\pi\mu S_x}{5M} Y^*_{10}(0,0)\left(\frac{2}{R(t)^2}\delta(r-R(t)) - \frac{4}{R(t)}\frac{d}{dr}\delta(r-R(t)) + \frac{d^2}{dr^2}\delta(r-R(t))\right).$$

$$\psi^{(2)(odd)}_{2\pm1}(t,r) = \pm\frac{4\sqrt{10}\pi}{15}\mu\frac{S_2}{R^2}Y^*_{10}(0,0)\,.$$

$$S_2 = S_x\,.$$

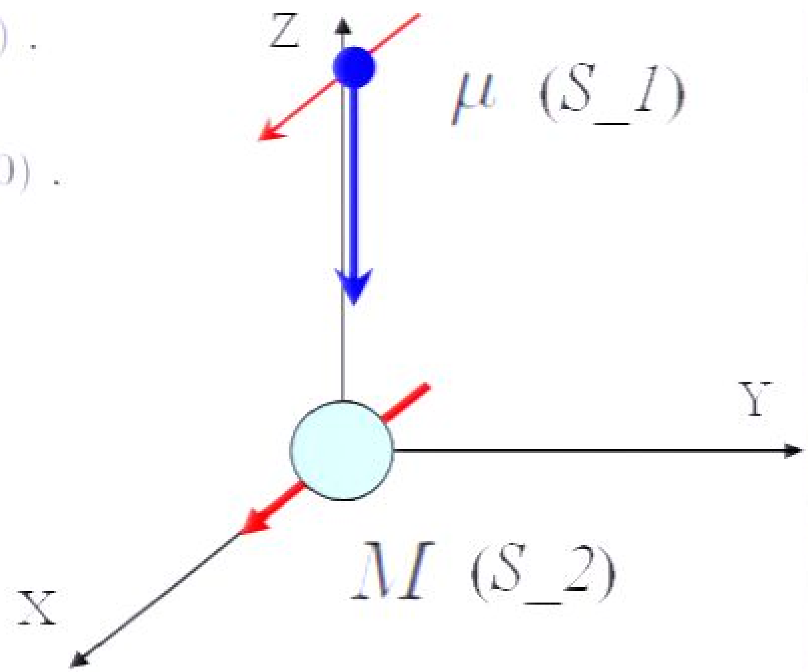
Gravitational wave modes:

A. $\psi_{20}^{(1)(even)}(t,r) = \frac{16\pi}{15}\mu\left(\dot{R}^2 - \frac{M}{R}\right) Y_{20}^{*}(0,0)$.

B. $\psi_{30}^{(1)(even)}(t,r) = \frac{16\pi}{105}\mu\left(\dot{R}^3 - \frac{2M}{R}\dot{R}\right) Y_{30}^{*}(0,0)$.

C. $\psi_{2\pm1}^{(2)(odd)}(t,r) = \pm\frac{4\sqrt{10}\pi}{15}\mu\frac{S_2}{R^2} Y_{10}^{*}(0,0)$.

D. $\psi_{2\pm1}^{(1)(odd)}(t,r) = -\frac{S_1}{S_2}\frac{M}{\mu}\psi_{2\pm1}^{(2)(odd)}(t,r)$.



Linear momentum loss:

$$\dot{P}_x = 0 .$$

$$\dot{P}_y = -\frac{16}{15}\mu^2 M^2 \frac{\dot{R}^2}{R^5}\left(\frac{S_2}{M} - \frac{S_1}{\mu}\right) . \quad \text{(A and C + A and D)}$$

$$\dot{P}_z = -\frac{16}{105}\frac{\mu^2 M^2}{R^4}\left(\dot{R}^3 - \frac{2M}{R}\dot{R}\right) . \quad \text{(A and B)}$$

* Consistent with Kidder's results in the PN approach.

5. Gravitational Waveforms

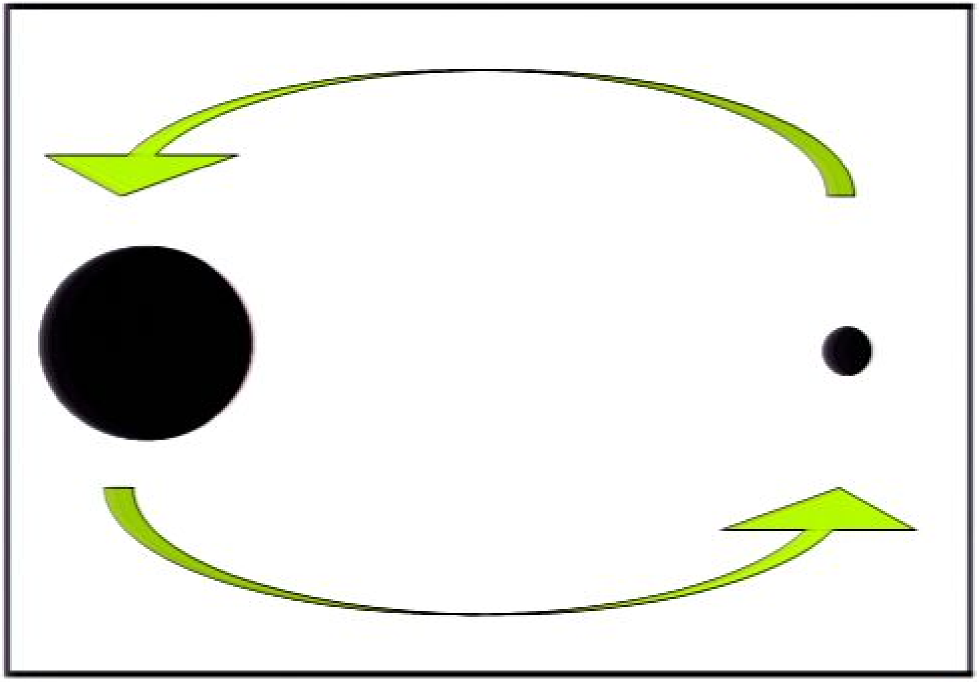
Efficiently solving the intermediate (and also extreme) mass ratio regime by purely nonlinear numerical methods

Difficult!

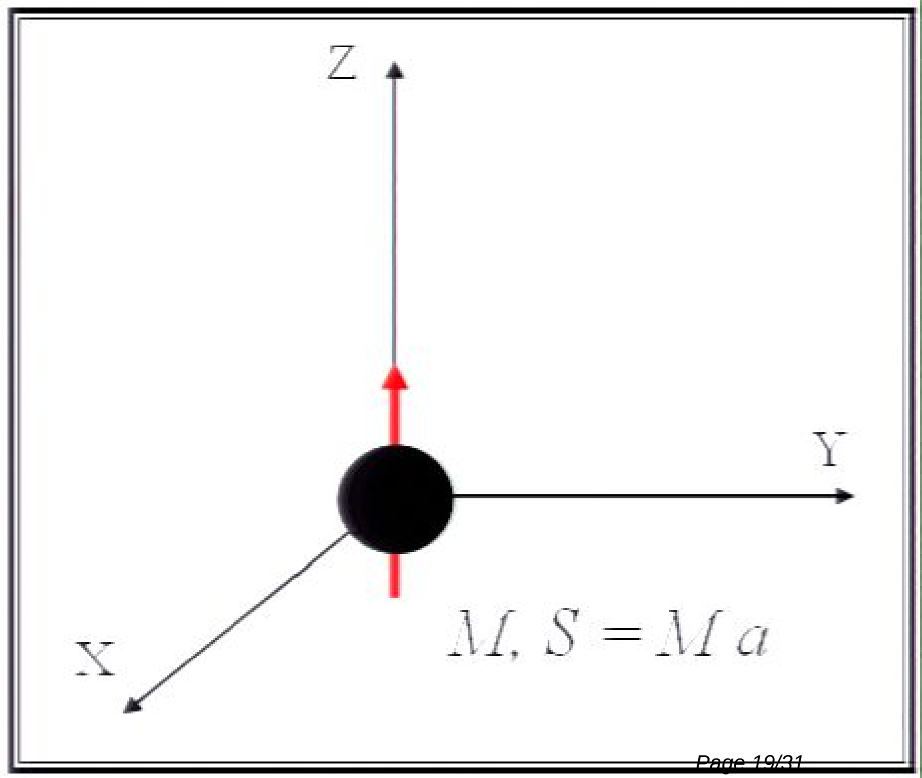
* Use perturbation theory to propagate gravitational waves on a black hole background

* Use trajectory information from a combination of nonlinear numerical and semi-analytic (Capra's self-force) methods

Tomorrow
Yosef Zlochower, *Extreme Black-Hole Binaries*



After merger



Spin as a perturbation

$$h^{(1)}_{0\,10}(t,r) \;=\; \sqrt{\frac{4\pi}{3}}\frac{2Ma}{r}$$

1. Translate the numerical tracks into the Schwarzschild coordinates (from isotropic 'trumpet' stationary 1+log slices) [Brugmann (2009)]

2. Calculate the time component of the particle's four velocity at each time step (instantaneous Schwarzschild geodesic)

3. Introduce the final remnant black hole **spin** [Lousto et al. (2010)] Empirical formula

Second order formulation in the BHP approach

Coupling between First order metric perturbation and Spin

(Ignore square terms of the first order perturbation)

Thanks to Maple!

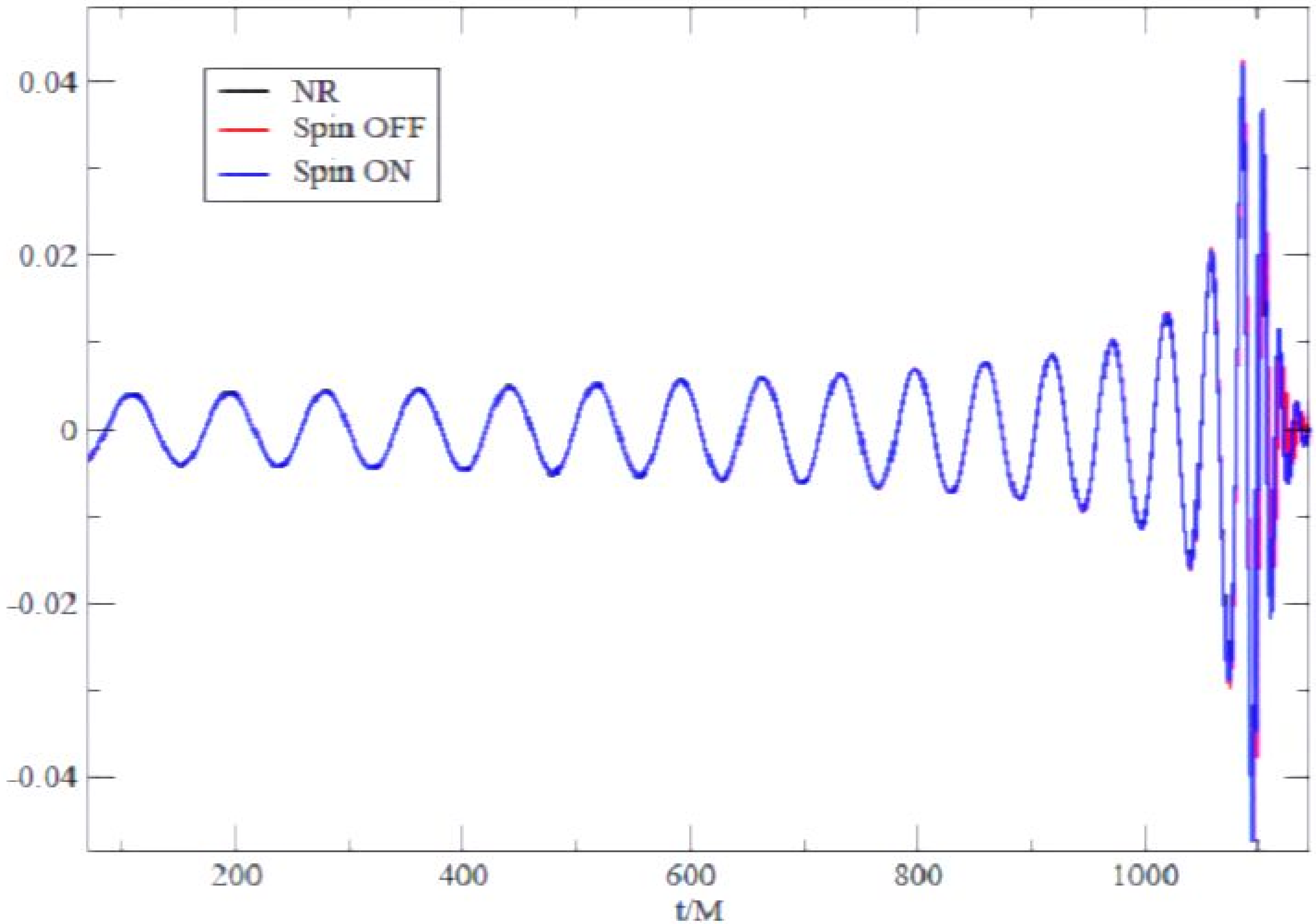
$$\Psi_{\ell m}(t,r) = \Psi^{(1)}_{\ell m}(t,r) + \Psi^{(2)}_{\ell m}(t,r)$$

$$-\frac{\partial^2}{\partial t^2}\Psi_{\ell m}(t,r) + \frac{(r-2M)^2}{r^2}\frac{\partial^2}{\partial r^2}\Psi_{\ell m}(t,r) + 2\,\frac{(r-2M)M}{r^3}\frac{\partial}{\partial r}\Psi_{\ell m}(t,r)$$
$$-(r-2M)\left(4r^3\ell - \ell^4 r^3 + 3\ell^5 r^3 - 7\ell^3 r^3 + \ell^6 r^3 + 12\ell^3 r^2 M - 24 r^2 M\ell - 18 r^2 M\ell^2 + 24 r^2 M\right.$$
$$\left.+6\ell^4 r^2 M - 72 r M^2 + 36\ell^2 r M^2 + 36\ell r M^2 + 72 M^3\right)\Psi_{\ell m}(t,r) \Big/ \left[\left(r\ell^2 + \ell r - 2r + 6M\right)^2 r^4\right]$$
$$-4\,iS\,m\,\left(4r^3\ell^7 + 144 M^3\ell^2 + 16 r^3\ell - 24 r^3 + 18 M\ell^6 r^2 + 144 M^3\ell + r^3\ell^8 - 216 r M^2 - 66\ell^3 r^2 M\right.$$
$$-48 r^2 M\ell + 144 M^3 + 36\ell^2 r M^2 + 22 r^3\ell^2 + 120 r^2 M + 6\ell^3 r^3 - 11\ell^4 r^3 + 54 M r^2\ell^5$$
$$\left.+72 M^2\ell^4 r + 12\ell^4 r^2 M + 144 M^2 r\ell^3 - 90 r^2 M\ell^2 - 36\ell r M^2 - 14\ell^5 r^3\right)\frac{\partial}{\partial t}\Psi_{\ell m}(t,r)$$
$$\Big/\left[r^3(\ell+1)\ell\left(r\ell^2 + \ell r - 2r + 6M\right)^3\right] + \frac{24\,iS\,(\ell+2)(\ell-1)\,m\,(r-2M)^2}{r^2\left(r\ell^2+\ell r-2r+6M\right)^2\ell(\ell+1)}\frac{\partial^2}{\partial t\partial r}\Psi_{\ell m}(t,r)$$
$$= -12\,S\sqrt{\frac{(\ell-m)(\ell+m)}{(2\ell-1)(2\ell+1)}}\,(r-2M)(\ell-2)\left(\ell^5 r^2 + 2r^2\ell^4 + 4r\ell^3 M - 4r^2\ell^2 + 12 r M\ell^2\right.$$
$$\left.-5r^2\ell + 12 r M\ell + 6r^2 - 28 r M + 36 M^2\right)\Psi^{(o)}_{\ell-1\,m}(t,r) \Big/ \left[r^5\ell\left(r\ell^2 + \ell r - 2r + 6M\right)^2\right]$$
$$+12\,S\sqrt{\frac{(\ell+m+1)(\ell-m+1)}{(2\ell+1)(2\ell+3)}}\,(r-2M)(\ell+3)\left(\ell^5 r^2 + 3r^2\ell^4 + 4r\ell^3 M + 2r^2\ell^3 + 2r^2\ell^2\right.$$
$$\left.+32 r M - 8r^2 - 36 M^2\right)\Psi^{(o)}_{\ell+1\,m}(t,r) \Big/ \left[(\ell+1)\,r^5\left(r\ell^2 + \ell r - 2r + 6M\right)^2\right].$$

+ (local source terms)

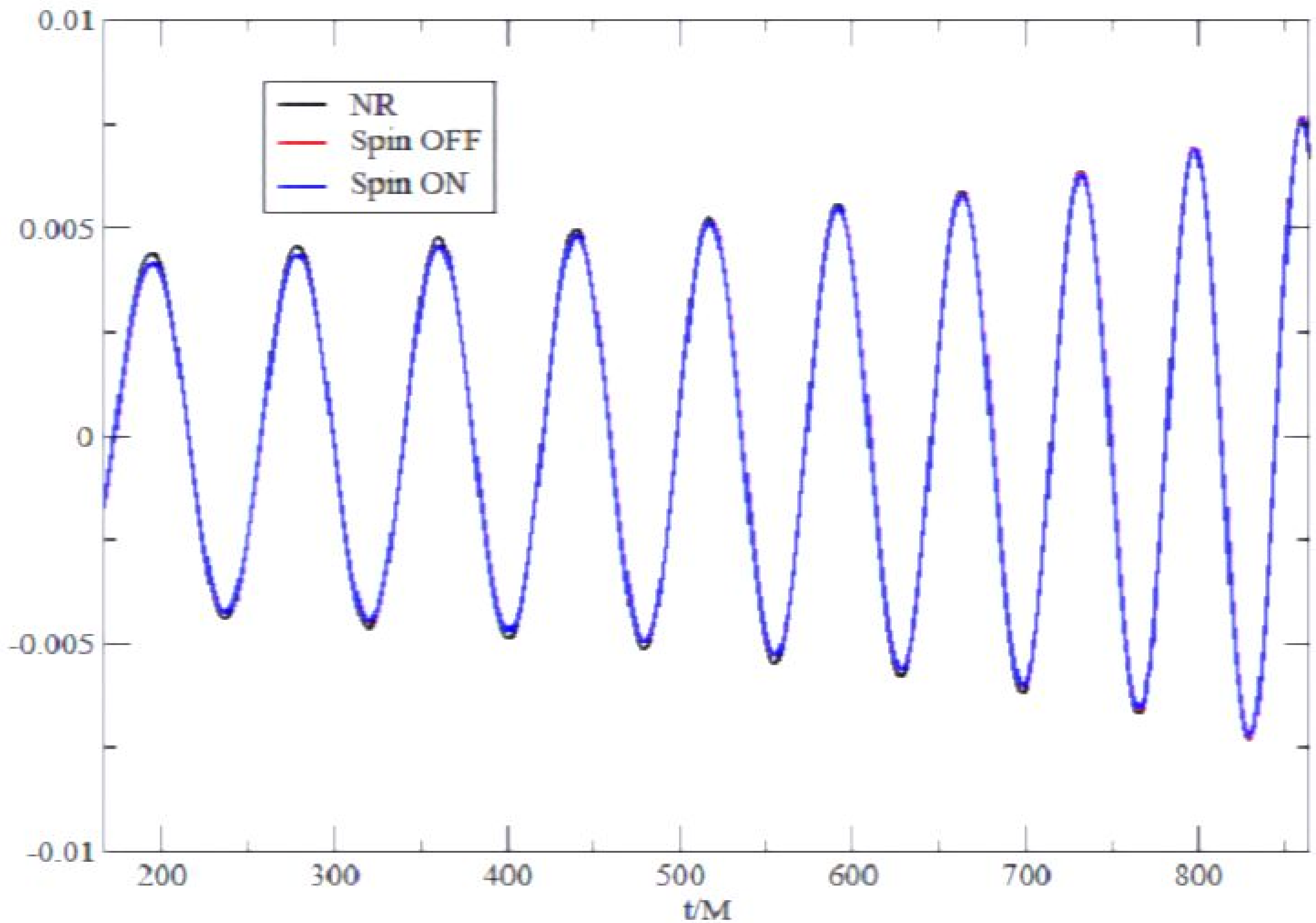
L=2, m=2 real part of dh/dt

10 to 1 case



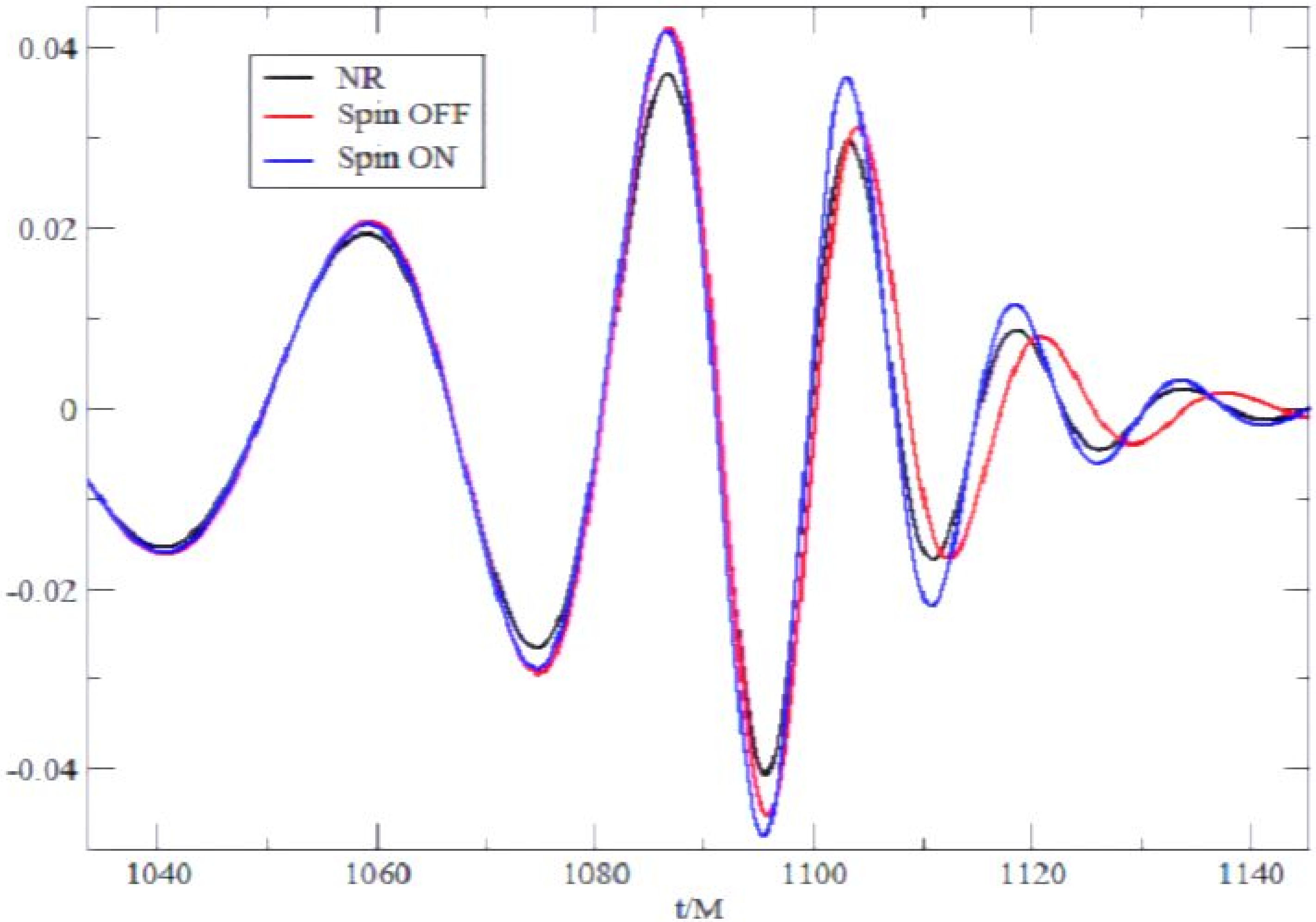
L=2, m=2 real part of dh/dt (Inspiral part)

10 to 1 case



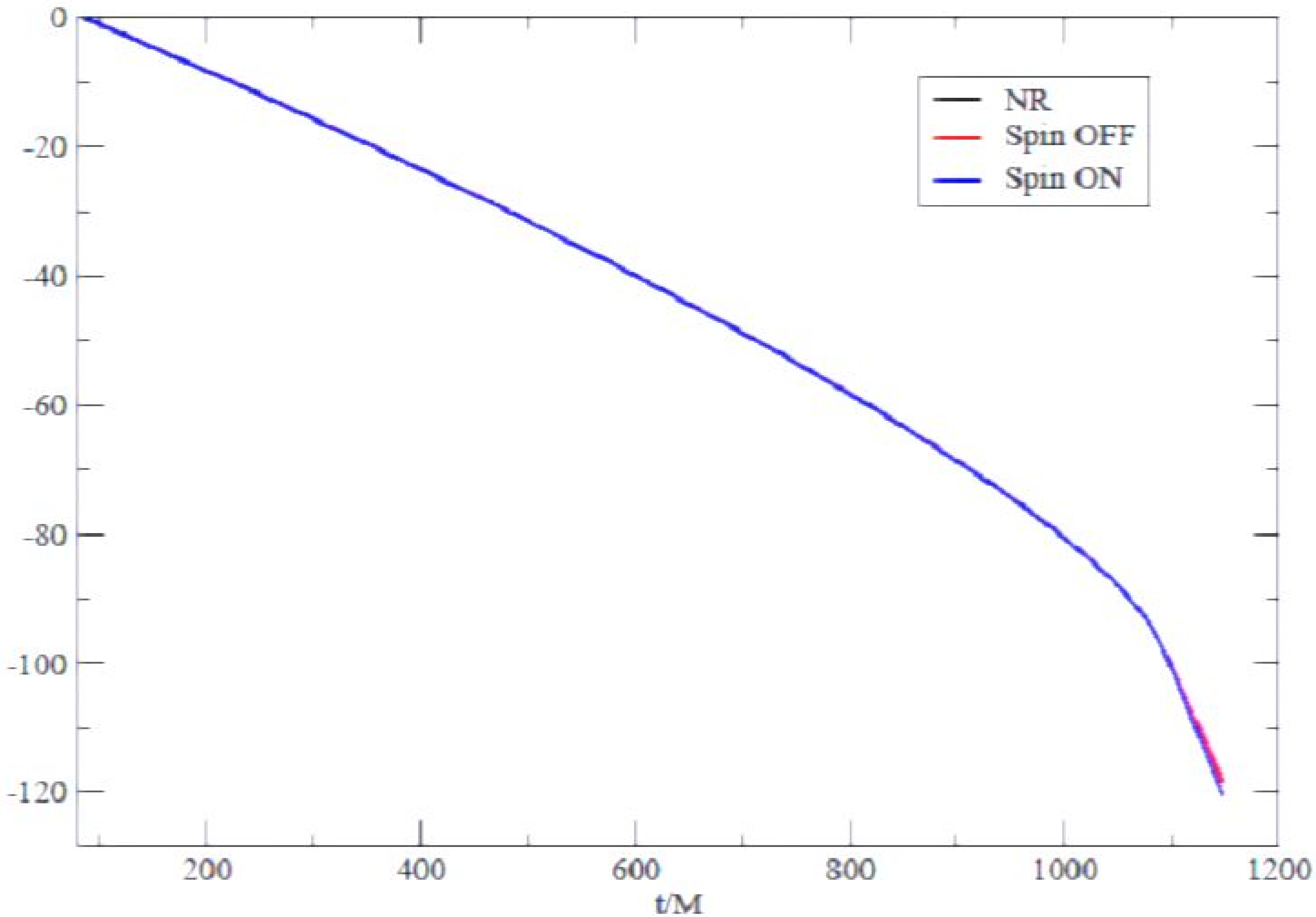
L=2, m=2 real part of dh/dt (Merger part)

10 to 1 case



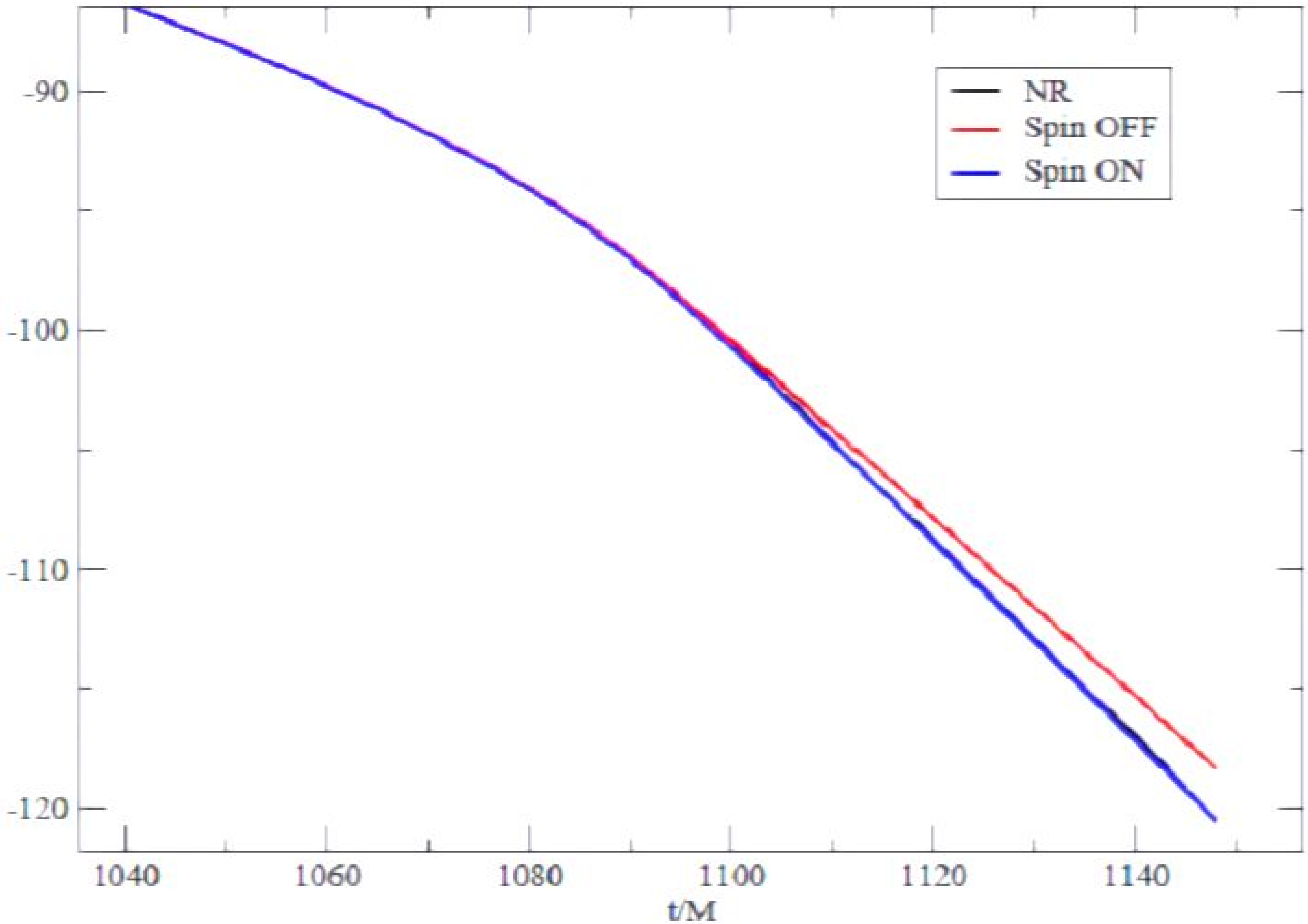
Phase of the L=2, m=2 waveform

10 to 1 case



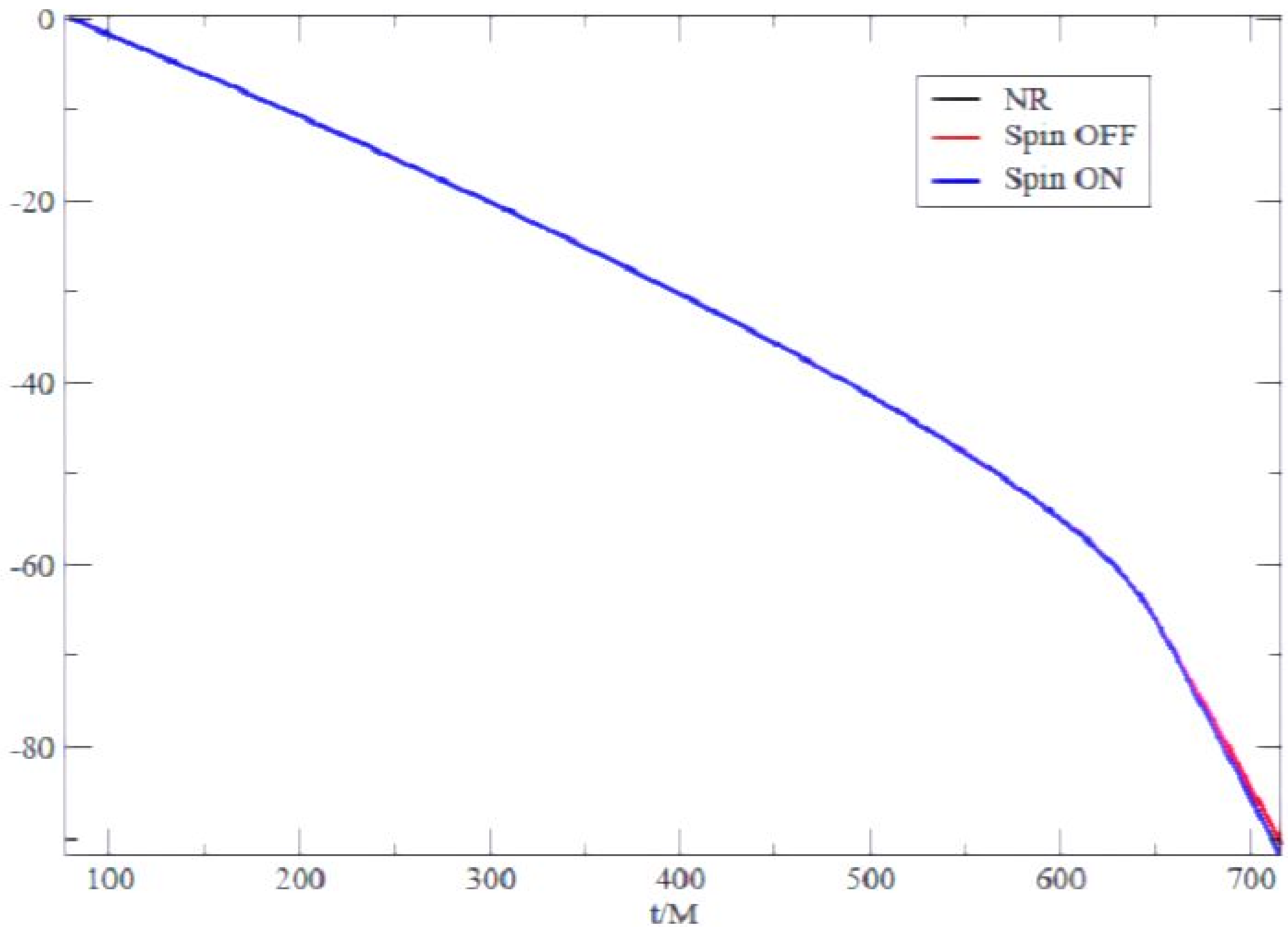
Phase of the L=2, m=2 waveform

10 to 1 case



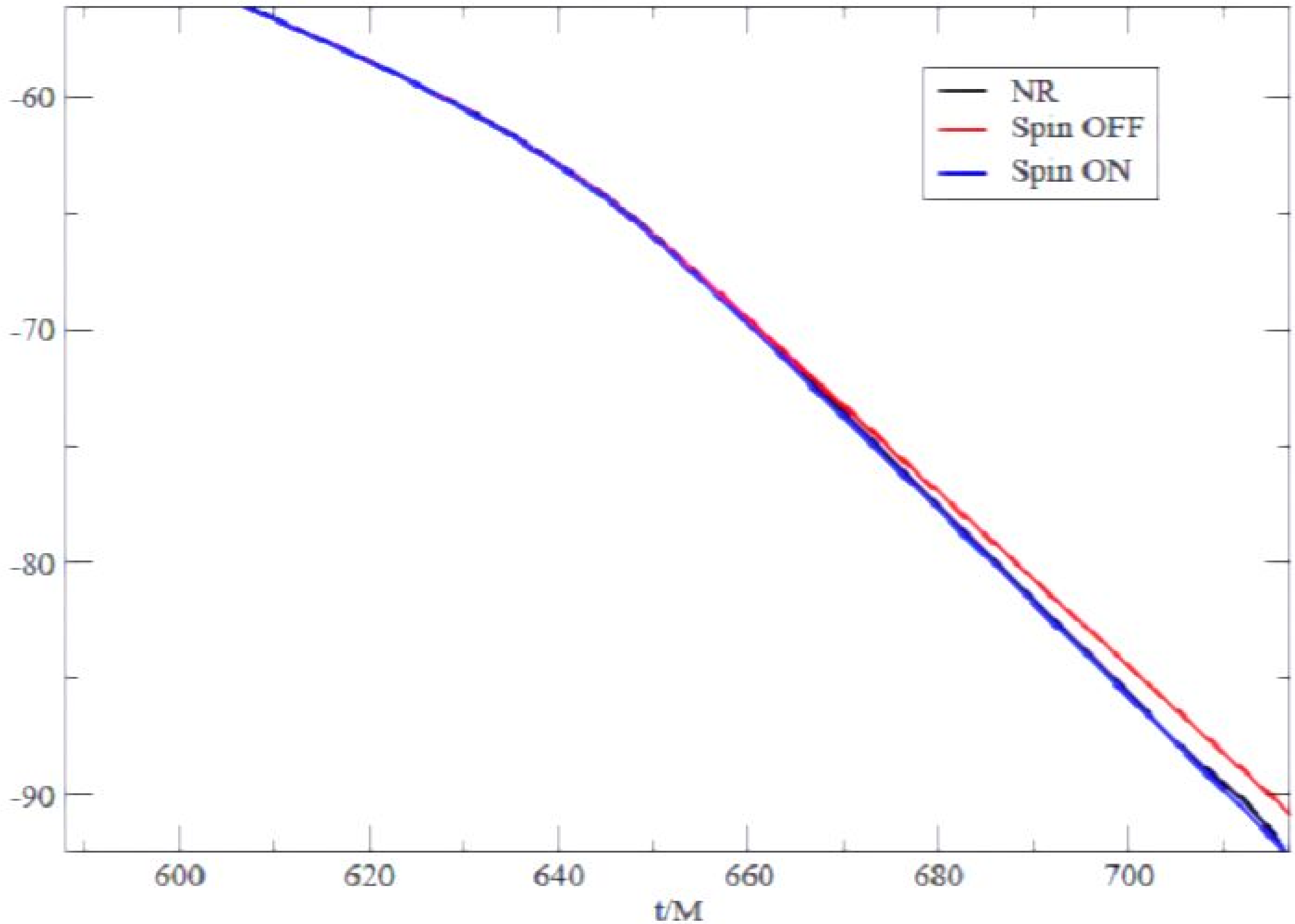
Phase of the L=2, m=2 waveform

15 to 1 case



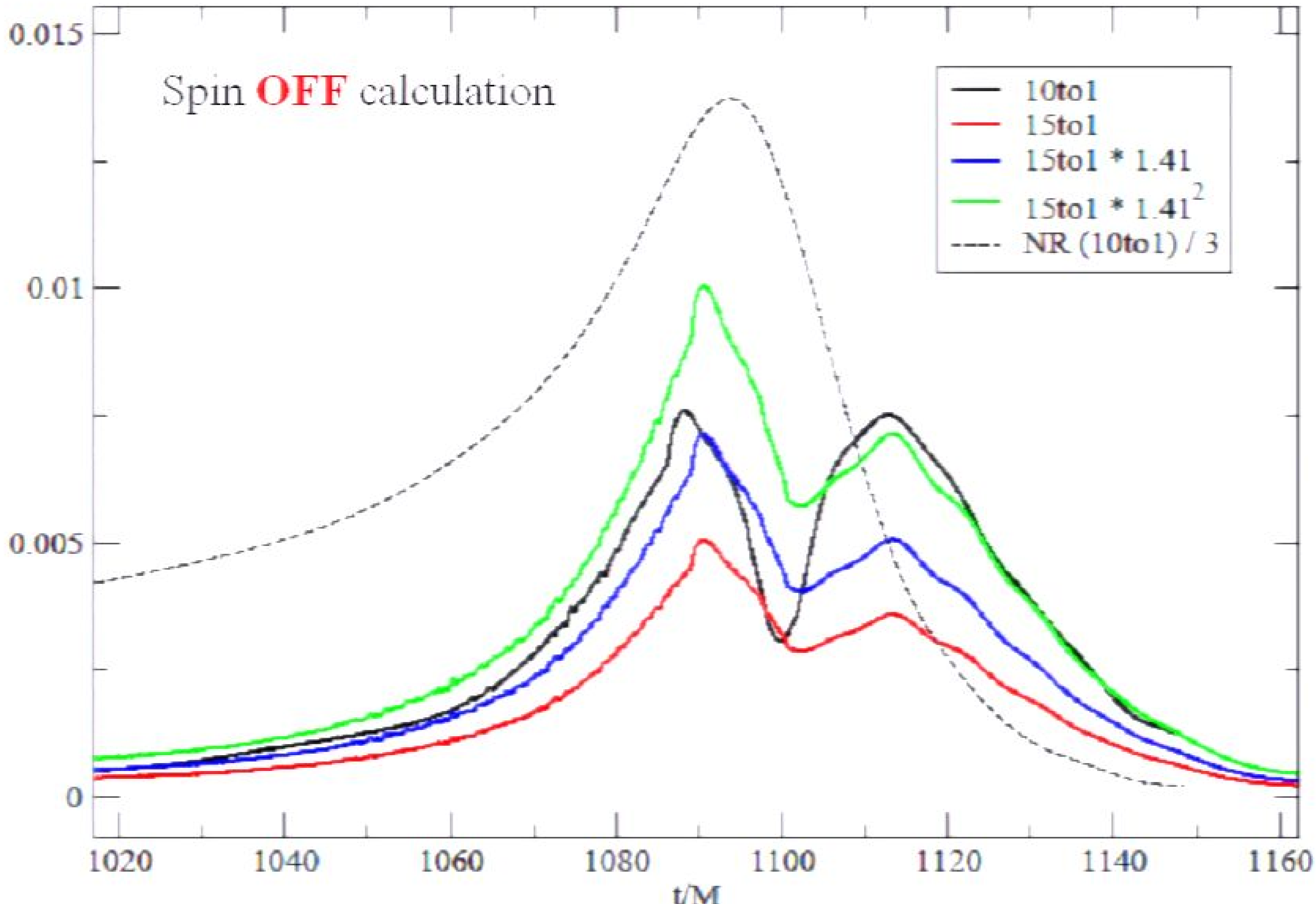
Phase of the L=2, m=2 waveform

15 to 1 case



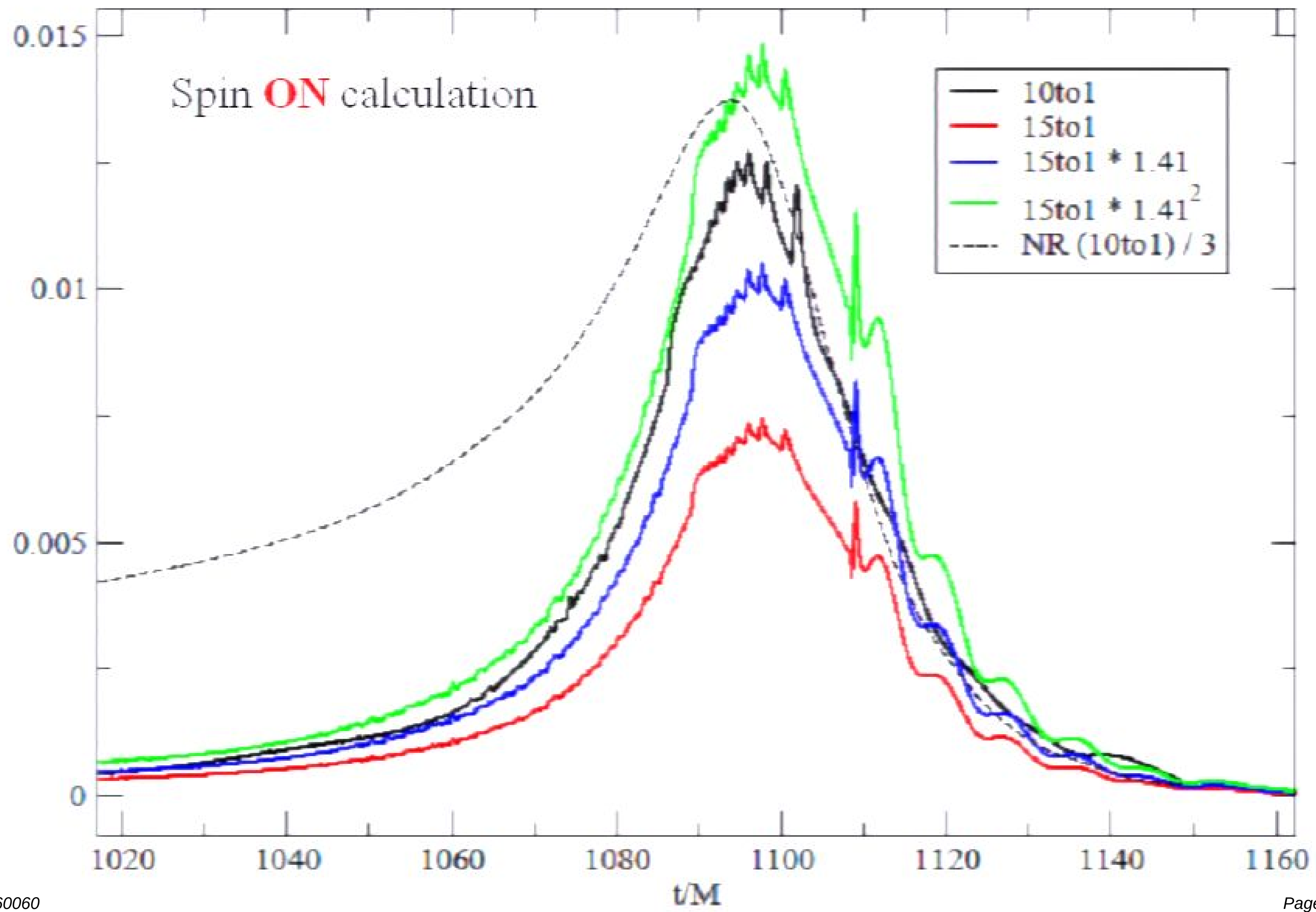
Theory Meets Data Analysis at Comparable

Difference between the NR and BHP (10 to 1 and 15 to 1)



1.41 = mu(10to1) / mu(15to1), 1.41^2 → Particle's second order

Difference between the NR and BHP (10 to 1 and 15 to 1)



Remaining parts: (15to1) * 1.41 < (10to1) < (15to1) * 1.41^2

5. Discussion

A. Gravitational Recoil

* Analytically possible in the BHP approach?

1st order perturbations from local source terms
(delta function) ⇨ O.K. in a finite slow motion order.

2nd order perturbations from extended source terms
(not local) ⇨ ? ? ?

B. Gravitational waveform

non-spinning binary systems → Remnant BH`s spin

Solve Teukolsky equation?
[Sundararajan, Khanna and Hughes (2010)]